energy potentialized at the pinhole.   Each new increment is added to the stored energy or may also be withdrawn from it.

The results for cases in which the primary or secondary, or both primary and secondary are removed, the pipe being tuned for each case, separately, were unsatisfactory.   With the primary only in circuit (secondary removed) a distinctly higher pitch was heard, though the pipe depth is about the same.   In this case and when both *I* and *II* are cut out, the graphs have definite crests and are, as a whole, more curvilinear than the preceding.   With *II* only in place, the largest fringe displacements, *s*, obtainable are too small to be of service.   In fact, taken together, these graphs are, throughout, small in their *s* values, as compared with figures 2 and 3 with both *I* and *II* in place; and the latter, in turn of 4–5 times less sensitivity than the earlier graphs (loc. cit.) for a completely separated primary and secondary.   The marked tendency to preserve linear progress in the present cases has, however, been put in evidence.

* Advance note from a Report to the Carnegie Institution of Washington, D. C.
[1] These PROCEEDINGS, 13, 1927, p. 87.

THERMIONIC EMISSION AND THE "UNIVERSAL CONSTANT" A

BY EDWIN H. HALL

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY

Communicated March 30, 1927

This paper continues my endeavor to show that certain ideas embodied in the dual theory of electric conduction within metals are capable of illuminating the field of electronic emission.

It is probable that nearly every one who has undertaken to discuss the behavior and functions of "free" electrons within metals has begun by assuming these electrons to share the energy of thermal agitation.   This assumption is suggested by the general kinetic theory of the solid state, and is supported by the behavior of electrons that have been emitted by hot metals.   It is embodied in the ordinary formulas, differing from each other only by a constant factor, for electric conduction by means of free electrons within metals.   Only very powerful evidence or argument should be accepted as upsetting this general pre-supposition.

Nevertheless, workers in thermionic emission have seemed disposed, too hastily I think, to admit this disturbing change of conception.   Accordingly, I shall here present and comment upon the various considerations that, so far as I know, have been most influential in forming opinion on the question at issue:

(a)　Richardson, on p. 34 of his "Emission of Electricity" (1916), says that "the optical properties of metals lead us to conclude that the number of free electrons present in them is quite large, and if this large number of electrons possessed the kinetic energy which the classical dynamics endows them with, the specific heats of metals would be very much larger than those actually observed." This is, I believe, substantially equivalent to saying that we cannot explain metallic electric conduction by means of free electrons only without getting into difficulties over specific heats.

But the dual theory of conduction attributes a relatively small share of this operation to the free electrons. Moreover, the general conception as to the effectiveness of a single free electron in conduction has, I think, changed since the passage quoted from Richardson was written. I believe, and I am not alone in believing, that the conclusions to be drawn from the optical properties of metals are by no means unmistakable.

(b)　Experiments like those of Davisson and Germer[1] indicate that the amount of energy needed to maintain a given thermionic emission current is greater than it would be if all the emitted electrons entered the emitting filament as free electrons sharing the energy of thermal agitation.

But I have shown[2] that, according to the dual theory of conduction, most of the emitted electrons *enter* the emitting filament in the "associated" conductive state, without thermal energy. If this view of the matter is taken, there is no need of assuming that the free electrons also within the metal lack energy of thermal agitation. I believe that the same consideration will explain the evidence[3] which has inclined Millikan to the view that most of the conductive electrons do not have thermal energy.

(c)　Finally we have the measure of success, whatever it may prove to be, attained by Dushman and others in calculating the value of the constant A, in Richardson's emission equation,

$$I = AT^2\epsilon^{-\frac{b_0}{T}},\qquad(1)$$

by means of Nernst's "third law" and the quantum theory.

But Bridgman[4] has subjected Dushman's argument to a thorough criticism, and, though he is disposed to admit that the calculated value of A may prove to be approximately correct in the case of all pure metals, he maintains that not all of Dushman's conclusions are justified.

In particular, Bridgman argues that Dushman's success in the case of tungsten, where he treats the "surface heat" as zero, is no proof that this heat is really zero. His conclusion is given as follows: "It is therefore not necessary that the surface heat vanish in order that the emission formula hold [with Dushman's value of the 'universal constant' A]; numerically it may be large or small provided only that it be proportional to temperature."

Bridgman defines "surface heat" as "the heat which must be absorbed by the system to maintain it isothermal when one unit of electricity is added to the surface." Now, according to the dual theory of conduction, as I showed[5] some years ago, this surface heat is due to the ionization which occurs at the surface, in accordance with the mass law of chemical equilibrium, when a positive charge (taking away of free electrons) is added there. Accordingly the surface heat at any temperature should be proportional to what I have called the heat of ionization within the metal and have indicated, per electron freed, by the symbol λ' in the equation

$$\lambda' = \lambda'_c + s\,k\,T, \tag{2}$$

given as equation (3′) in my "Summary"[6] printed six years ago. I have thus far taken λ'_c and s in this equation as constants, though it is possible, as will be shown later, that s should be regarded as a slowly changing function of temperature. The value of λ'_c given for tungsten in the paper referred to is $40k$, while that of s is 11.4. Accordingly, if we take T as 2000°, a reasonable temperature for emission experiments on tungsten, we find that the second term in the value of λ'_c, which term is proportional to T, is 570 times the value of the constant λ'_c. I attach no great importance to the particular values of λ'_c and s here given for tungsten, as they were derived from somewhat conjectural data and should be regarded as tentative; but apparently equation (2), which I have used as one of the fundamental formulas of the dual-conduction theory, may, for approximate purposes at least, be regarded as indicating a "surface heat" proportional to the temperature and so, according to Bridgman's argument, held consistent with the practical correctness of equation (1), with Dushman's value of A, in the case of tungsten.

There is one other particular in which recent discussion seems favorable to one of my cherished notions. Equation (1) of the "Summary" already referred to is

$$n = zT^q, \tag{3}$$

where n is the number of free electrons per unit volume within a metal, while z and q are constants characteristic of the metal. Values of q as given in the "Summary" for eighteen metals, including two alloys, range from 1.2 for iron and bismuth to 1.6 for nickel, palladium and platinum, the average value being 1.47. Now the tendency in current treatment of emission is to establish Richardson's equation (1) of this paper, in preference to his older formula containing $T^{0.5}$ instead of T^2. But Richardson, on p. 34 of his "Emission of Electricity" (1916), writes that, "if n_1 [meaning the same as my n] is proportional to $T^{1.5}$ and ϕ is independent of T,

$$I = A_2 T^2 \epsilon^{-\frac{\phi_0}{kT}}," \tag{4}$$

provided we keep to the "classical kinetic theory" concerning conditions within the metal.   That is, Richardson in the passage referred to recognized that, if the classical theory held, as he had apparently assumed at first, the acceptance of equation (4) implied the relation

$$n = zT^{1.5}, \tag{5}$$

for the number of free electrons per unit volume within a metal, z being a constant.   Comparing this with my equation (3) above, and remembering that my mean value of q is very near 1.5, we find a degree of coincidence that is rather impressive, especially in view of the fact that thermionic literature in general gives little prominence to the idea that n increases with rise of temperature.

What precedes is enough, I trust, to show that the dual-conduction theory, including the classical kinetic conception as to the state of free electrons within a metal, should be reckoned with in future discussions of electronic emission.   Furthermore, it should not be forgotten that this theory gives very definite and simple explanations of the Volta effect and of the Peltier effect, including that phase of it which Bridgman was surprised to find within a single metal crystal.   In addition it makes possible a clear definition of "electron affinity," otherwise a rather vague term.

But I can well go farther than this. Equation (1) of this paper is recognized by Richardson and also by Bridgman as probably only an approximation to the true formula of thermionic emission. I shall, therefore, keeping to the classical kinetic conception of the free electrons within a metal, and following in general that particular method of Richardson which begins on p. 33 of his "Emission of Electricity" (1916), undertake to derive and discuss a form of emission equation that will embody those concepts of the dual-conduction theory which find application here.

If we assume a condition of equilibrium between the free electrons within a metal and the free-electron atmosphere outside the metal, we have a perfectly simple case for the application of Boltzmann's distribution law. That is, letting n be the number of electrons per unit volume in the surface layer of the metal and n' the number in unit volume of space just outside the surface, and defining "electron affinity" in this case as the amount of work, w, done against resisting forces in the passage of one electron from the free state within to the free state just outside the metal, we have

$$\frac{n'}{n} = \epsilon^{-\frac{w}{kT}}. \tag{6}$$

For n I shall substitute zT^q, according to equation (3) above. As to w, I have in these Proceedings for August, 1926, given my reasons for regarding it as a constant, which I will here write as b_0k, minus $(s-2.5)kT$,

where s is the same as in equation (2) of the present paper. That is, putting a for $(s-2.5)$, we have

$$w = k(b_0 - aT). \qquad (7)$$

Thus from (6) we get, by means of (3) and (7),

$$n' = zT^q\epsilon^{-\frac{(b_0-aT)}{T}} = \epsilon^a zT^q\epsilon^{-\frac{b_0}{T}}. \qquad (8)$$

According to Richardson's admittedly venturesome but generally accepted assumption, there is no reflection of electrons from the outer surface of the metal, so that N', the total number of electrons entering unit area of the surface per second from without, is the same as the number reaching this area per second from without. This number is, according to the kinetic theory of gases, proportional to n' and to the square root of the absolute temperature; that is,

$$N' = Fn'T^{0.5}, \qquad (9)$$

where F is a constant, $(k \div 2\pi m)^{0.5}$, m being the mass of an electron. The numerical value of F is 1.56×10^5.

Now in the case of equilibrium this inflow must be offset by an equal out-flow. It is, furthermore, assumed by Richardson and accepted by others that, when equilibrium is upset by applying an external potential-gradient sufficient to prevent any outside electrons from returning to the metal, the outflow of electrons from the surface is the same as in the case of equilibrium conditions. Argument in favor of this assumption is found in the fact that moderate increase of the potential-gradient beyond a certain value does not increase the observed outflow. Accordingly, the saturation current outflow, I, measured for unit surface area, is equal to e times the N' of equation (9). That is,

$$I = Fen'T^{0.5}. \qquad (10)$$

From (10) and (8) we get

$$I = Fe\epsilon^a zT^{(q+0.5)}\epsilon^{-\frac{b_0}{T}}. \qquad (11)$$

Putting A' for the product $Fe\epsilon^a z$, we get from (11)

$$I = A'T^{(q+0.5)}\epsilon^{-\frac{b_0}{T}}. \qquad (12)$$

Equation (12) differs from equation (1) in having $(q + 0.5)$ instead of 2 as the exponent of the T factor and in having instead of the constant factor A the factor A', which may prove to be not altogether constant. I shall presently discuss these differences at some length, but for the moment I wish to comment on the constant b_0. This may be numerically the same as Richardson's b_0, each being defined as $(1 \div k)$ times the value

of w for $T = 0$. But Richardson (loc. cit.) in arriving at equation (1) assumes w, his ϕ, to be independent of temperature, whereas I take its value as expressed by equation (7).

It has become the fashion to discuss the claim of A in equation (1) to be regarded as a "universal constant," the same for all pure metals. My contribution to this discussion must involve a somewhat roundabout argument. Going back to equation (11), which immediately preceded the introduction of my A', I can, using equation (3), write

$$I = Fe\epsilon^a z T^q \cdot T^{0.5} \epsilon^{-\frac{b_0}{T}} = Fe\epsilon^a n T^{0.5} \epsilon^{-\frac{b_0}{T}}, \qquad (13)$$

where Fe is a constant.

I shall deal with the question whether the factor $Fe\epsilon^a z T^q$, or its equivalent $Fe\epsilon^a n$, is likely to be the same in different metals at the temperature T. If I took q to have the same value, 1.5, for all metals, I should in this inquiry be asking whether $Fe\epsilon^a z$, which is the A' of equation (12), is a universal constant.

The definition which I have given to a is such that I can rewrite equation (2) as

$$\lambda' = \lambda'_c + (a + 2.5)kT. \qquad (14)$$

In this equation, $2.5kT$ is the heat energy which one electron, after the process of ionization, possesses as a gas particle within the metal. If we denote by γ the amount of kinetic energy which this electron possessed before the ionization and by i the amount of work done against resisting attractions or repulsions in freeing the electron from the atom of which it formed a part, we shall have

$$i = \gamma + \lambda'_c + akT. \qquad (15)$$

The quantity i, if it needs a name, may be called the work-function of ionization within the metal. It is unnecessary for our present purpose to attempt any evaluation of γ. That part of it which represents energy of thermal agitation is negligibly small, and accordingly γ as a whole may be treated as a constant. It is to be noted in passing that γ may prove to be the larger part, at ordinary temperatures, of i in equation (15).

It is one of the tenets of the dual-conduction theory that, when any two metals α and β are joined at the same temperature, the total energy, including the energy γ, of an "associated" electron, an electron capable of taking part in what I call associated-electron conduction, is the same in an atom of α as in an atom of β. This tenet[7] was not first suggested by but is strongly supported by the observed fact that, when α and β are joined together, the photo-electric work function is the same for both.[8]

Accordingly, the total amount of energy required from without to take an associated electron from an atom of β to the free state in β, thence to

the free state in α and thence to the associated state in α, is zero. Let the work required for the second step of this process be called $w_{\beta\alpha}$. Then we have

$$(\lambda_c' + (a + 2.5)kT)_\beta + w_{\beta\alpha} - (\lambda_c' + (a + 2.5)kT)_\alpha = 0, \quad (16)$$

or

$$w_{\beta\alpha} = (\lambda_c')_\alpha - (\lambda_c')_\beta + (a_\alpha - a_\beta)kT. \quad (17)$$

Now, as a rule, according to the values given in my "Summary," already referred to more than once, λ_c' is decidedly smaller than akT for any metal, even at room temperature, though not to be ignored. At thermionic emission temperatures, as I have already shown in connection with equation (2) in this paper, the value of λ_c' may in the case of some metals be negligibly small compared with skT and even compared with akT. If we can in the case of α and β at high temperatures neglect the λ_c' terms in the value of $w_{\beta\alpha}$, we have from (17)

$$w_{\beta\alpha} = (a_\alpha - a_\beta)kT. \quad (18)$$

But evidently, n_α being the number of free electrons per unit volume in α and n_β the corresponding number in β, we have by the Boltzmann law

$$\frac{n_\alpha}{n_\beta} = \epsilon^{-\frac{w_{\beta\alpha}}{kT}}, \quad (19)$$

and so, from (18)

$$\frac{n_\alpha}{n_\beta} = \epsilon^{-(a_\alpha - a_\beta)}, \quad (20)$$

or

$$(\epsilon^a n)_\alpha = (\epsilon^a n)_\beta. \quad (21)$$

Accordingly, since the Fe of equation (13) is a universal constant, the quantity $Fe\epsilon^a n$, or $Fe\epsilon^a zT^q$, of the same equation is, within the limits of accuracy of the preceding argument, the same for metal α as for metal β.

It is to be noted that, if a is zero and if q is 1.5, equation (13) reduces to the form

$$I = FezT^2\epsilon^{-\frac{b_0}{T}}, \quad (22)$$

which is equivalent to the Richardson equation that I have written as (1). Richardson does, in fact, in that part of his "Emission" to which I have referred, take a as always zero and q as always 1.5 in getting this equation. Accordingly, his A is my Fez of equation (22), and if A is to be taken as a universal constant, Fe being undoubtedly such, z must also be a universal constant, and so $zT^{0.5}$, the number of free electrons per unit volume,

according to Richardson's assumptions here, must be the same for all metals at the same temperatures. This seems an improbable conclusion. We can avoid it in any one of three ways. We can give up the simple classical kinetic conception as to the state of the free electrons within the metal, in spite of the great advantages it offers in several respects, or we can assume either that a is not always zero or that q is not always 1.5. I have made both of these saving assumptions.

In neither case was I influenced originally by any consideration of electronic emission. My assumption regarding q, as introduced in my adoption of equation (3), was made in an endeavor to account for the known facts of thermo-electric action. I brought in a, by making the value of s in equation (2) greater than 2.5, for the purpose of enabling my theory of dual electric conduction to account for the small temperature coefficient of thermal conductivity as compared with the temperature coefficient of electric conductivity. If I should be compelled to give up the attempt to explain the experimentally known relations of these two conductivities and to adopt the conception that thermal conduction in metals is carried on wholly or mainly by means of elastic vibrations, I might drop the assumption that a is not zero. But I feel very sure that in any case the conception of q as not always 1.5 would remain, so long as I might try to understand thermo-electric action.

With given values of I and b_0 the effect of introducing a, which is always positive, is to reduce in equation (13) the factor zT^q, which represents the number of free electrons per unit volume within the metal. This is as it should be; for the greater a is, with a fixed value of b_0, the less is the work, see equation (7), required to remove a free electron from inside to outside the metal, and evidently the less this work the smaller the number of interior free electrons required to maintain a given outflow I.

The particular values of a and q which I have published in my "Summary" are entitled to no great confidence, even if my dual-conduction theory is accepted as sound, for they were derived, as I have explained,[9] from imperfect data taken for the temperature range from 0°C. to 100°C. and by a method lacking in strict accuracy. Nevertheless, I shall undertake to apply them in one or two cases, largely for the purpose of finding what values of n are likely to be indicated by the logic of this paper.

The constant Fe, which must appear in every case, has the numerical value

$$1.56 \times 10^5 \times 1.59 \times 10^{-20} = 2.48 \times 10^{-15}.$$

Tungsten.—I have $q = 1.48$ and $a = 11.4 - 2.5 = 8.9$. From data given by Dushman[10] and his co-workers it appears that, in the case of tungsten, if I is measured in absolute electromagnetic units, the b_0 of equation (1) is about 52,640 and the A is about 6.02. In order, with the same value of

b_0, to make my equation (13) conform to the experimental data, I must have

$$A'T^q = AT^{1.5} = 6.02T^{1.5},$$

whence, if we, for example, take $T = 2000$,

$$A' = 6.02 \times 2000^{0.02} = 7.01.$$

Then, from the definition of A',

$$z = 7.01 \div Fe\epsilon^a = 7.01 \div (2.48 \times 10^{-15} \times 2.72^{8.9}) = 3.86 \times 10^{11},$$

and, for $T = 2000$,

$$n = zT^{1.48} = 2.97 \times 10^{16}.$$

Molybdenum.—I have here $q = 1.3, a = 8-2.5=5.5$. From Dushman's paper I find that the universal constant A is not quite so constant as one might expect from its name. Its $\log_{10}$ is given as 1.776 in the case of tungsten and 2.268 for the case of molybdenum, when I is measured in amperes. When I is measured in absolute units, we have for A in the case of molybdenum the value of 18.5, about three times as great as for tungsten.

Accordingly, for molybdenum we get, taking T as 2000,

$$A' = 18.5 \times 2000^{(1.5-1.3)} = 84.8.$$

Then

$$z = 84.8 \div (2.48 \times 10^{-15} \times 2.72^{5.5}) = 1.40 \times 10^{14}.$$

And, for $T = 2000$,

$$n = zT^{1.3} = 1.40 \times 10^{14} \times 2000^{1.3} = 2.74 \times 10^{18}.$$

I have no estimates of q and a for tantalum, the third metal dealt with by Dushman.

The value of n at 2000° which I have arrived at by the method above shown is about ninety times as great in the case of molybdenum as in the case of tungsten. This ratio seems improbably large, in view of the fact that the electric conductivities of the two metals are not very different; but when due consideration is given to the possibilities of error in the valuations of a, q and A, the ratio is not large enough to discredit seriously the argument that led to it. I shall have something more to say on this matter at the end of this paper.

On the other hand, even the value of n found for molybdenum at 2000° is surprisingly small, about 1 in 20,000 as compared with the number of the atoms. From one point of view this outcome is gratifying, as, if accepted, it banishes all concern about the influence of the free electrons on the specific heat of metals. But it raises at once the question whether so small a number of electrons, and the corresponding number of ions

can reasonably be supposed to provide the observed value of electric conductivity.

This question may well be regarded as an open one at present. The answer will depend upon the admissible length of that very dubious quantity the "mean free path" of the free electrons within the metal and on the conception which may ultimately prevail as to the exact manner in which the ions function in facilitating the passage of electrons which are not free in the ordinary sense. I do not insist, nor have I ever insisted, on any particular mode as the one in which this influence must be exercised, though I have more than once made suggestions which seemed to me at the time to have some degree of plausibility regarding this matter. What I have emphasized and would still emphasize is the great importance of the ions as furnishing leeway, clearance space, terminal facilities, whatever the freedom indicated may best be called, for the movement of "associated" electrons, singly or in files, to progress in the direction of the imposed electromotive force, without maintaining a lock-step relation to each other all around the circuit.

I have once or twice suggested in the preceding pages that the a of my equations may not be strictly a constant. In fact, if equation (21) be written in the equivalent form

$$(\epsilon^a z T^q)_\alpha = (\epsilon^a z T^q)_\beta, \tag{23}$$

we see at once that, z being taken as a constant for each metal, a_α and a_β cannot both be taken as constant unless $q_\alpha = q_\beta$. It is quite probable that both a and q are variables in the case of every metal; for it seems unlikely that equations so simple as (14), with λ'_c and a constant, and (3), with z and q constant, should hold all the way from room temperatures up to $T = 2000°$.

I shall, however, for the present assume z_α, q_α, z_β and q_β to be constants through any range of temperature here dealt with, and accordingly I shall put upon a the responsibility of changing with temperature in such a way as to maintain equation (23). This requires the differential relation

$$d(a_\beta - a_\alpha) = (q_\alpha - q_\beta)d \log T, \tag{24}$$

whence, if we integrate from $T = 325°$, approximately the temperature for which the conclusions of my "Summary" apply, to $T = 2000°$, we have

$$\int_{325}^{2000} d(a_\beta - a_\alpha) = (q_\alpha - q_\beta) \int_{325}^{2000} d \log T = 1.8 \, (q_\alpha - q_\beta). \tag{25}$$

Of course, this change might come partly in a_α and partly in a_β, α indicating tungsten, let us say, and β indicating molybdenum, in which case $(q_\alpha - q_\beta) = 0.18$. I shall, for simplicity, assume that it occurs wholly

in a_β, which accordingly increases from the value 5.5, used above in the case of molybdenum, to 5.8 at 2000°. If with this new value of a I repeat my calculations relating to molybdenum, I have

and
$$z = 84.8 \div (2.48 \times 10^{-15} \times 2.72^{5.8}) = 1.03 \times 10^{14},$$

$$n = zT^{1.3} = 1.03 \times 10^{14} \times 2000^{1.3} = 2.16 \times 10^{18}.$$

Thus the ratio of n for molybdenum to n for tungsten, at 2000°, is reduced from a value about 90 as found above, to a value about 65.

If, furthermore, I were to take for Dushman's A in the case of molybdenum the same value, 6.02, which I, on Dushman's authority, used in the case of tungsten, the ratio of the two n's in question would come out about 22, a value that still seems large.

I have sometimes criticized, as others have, the argument by which Richardson, beginning on page 28 of his "Emission of Electricity" (1916) and ending on page 33, has deduced his emission equation. I have believed, and I still believe, this argument to contain a fallacy, failure to distinguish properly between the work done against resisting forces in removing a free electron from a metal and the absorption of heat which attends this action; but I have not found it easy to decide how much harm had resulted from this fallacy in this particular connection. In the present paper, while differing from Richardson as to certain explicit assumptions, I have found no fault with his logic; for that argument of his which I have here been following is different from the one referred to above.

The fact is that Richardson has in the book mentioned given three derivations of his emission equation, the (1) of this paper. The first is the one criticized, the third, running from page 35 to page 39, follows the quantum theory and reaches a result not very different from that found by Dushman in a similar way. The second is contained in a single page, beginning on 33 and ending on 34. Unfortunately, I think, the author felt obliged to give it up, because of a misgiving as to assigning thermal energy to the free electrons within a metal. The purpose of the present paper has been to show that such a misgiving lacks justification, and to modify, without rejecting, Richardson's "classical kinetic theory" of thermionic emission.

I predict that the A of Richardson's emission equation, equation (1) of this paper, will not prove to be the same for different metals. I propose as a substitute for this equation (1) the equation numbered (12) in this paper, the A' therein having the meaning given in the text. I hope that, when sufficient data concerning the electric conductivity, the heat conductivity and the thermo-electric behavior of metals are available, the factor $A'T^q$, or $Fe\epsilon^a zT^q$, will be found approximately the same for metals in general.

[1] *Physic. Rev.*, **20** (1922), p. 300.

[2] *Proc. Nat. Acad. Sci.*, **13** (1927), pp. 43–46.

[3] *Physic. Rev.*, **27** (1926), pp. 51–67.

[4] *Ibid.*, **27** (1926), pp. 173–180.

[5] *Proc. Nat. Acad. Sci.*, **9** (1923), pp. 207–211.

[6] *Ibid.*, **7** (1921), pp. 98–107.

[7] Hall, "Conditions of Electric Equilibrium, Etc.," *Ibid.*, **11** (1925), pp. 111–116.

[8] Richardson and Compton, *Phil. Mag., London*, **24** (1912), p. 575; Hennings, *Physic. Rev., Ithaca*, **4** (1914), p. 228; Millikan, *Physic. Rev.*, **18** (1921), p. 236.

[9] Illustrations of the "Dual Theory of Metallic Conduction," *Physic. Rev.*, **28** (1926), pp. 392–417.

[10] *Ibid.*, **25** (1925), pp. 356 and 357.

LAGRANGIAN FUNCTIONS AND SCHRÖDINGER'S RULE

By H. Bateman

California Institute of Technology

Communicated March 23, 1927

In a recent paper Schrödinger[1] has extended a rule, used by writers in the theory of gravitation, for deriving a stress energy tensor from a Lagrangian function and has illustrated its application in the case of the tensor which he has associated with the system of equations proposed by Gordon.[2]

We shall now apply the rule to various Lagrangian functions to see if it is generally applicable. Let (a_1, a_2, a_3, a_4) be the components of a typical 4-vector on which the Lagrangian function depends and let a_{mn} denote the derivative of a_m with respect to the coördinate x_n. The rule then states that the component T_{mn} of an associated stress-energy tensor is given by

$$T_{mn} = \sum \left[\sum_{l=1}^{4} a_{lm} \frac{\partial L}{\partial a_{ln}} + \sum_{l=1}^{4} a_{ml} \frac{\partial L}{\partial a_{nl}} + a_m \frac{\partial L}{\partial a_n} \right] - \delta_{mn}L \quad \ldots \ldots \quad (1)$$

where

$$\delta_{mn} = 0 \quad m \neq n$$
$$= 1 \quad m = n$$

and the summation $\sum$ extends over all the four vectors of type a.

Let us now apply this rule to the Lagrangian function

$$L = \frac{1}{2} (E^2 - H^2) \quad (2)$$