

THE ELASTIC PROPERTIES OF CERTAIN BASIC ROCKS AND  
OF THEIR CONSTITUENT MINERALS

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The rapid growth of the science of seismology, providing as it does more and more reliable data on the velocity of elastic waves at various depths within the Earth, calls for a closer study of the elastic constants of rocks and minerals because, given an accurate knowledge of the velocity of earthquake propagation at any depth and the elastic constants of all rocks under conditions prevailing at that depth, it is possible to speculate intelligently upon the nature of the rocks within the interior of the Earth.

Several years ago it was shown<sup>1</sup> that measurements of compressibility taken with the value of Poisson's ratio, which is approximately the same for all rocks, provided one of the best means of determining these elastic constants at high pressures. At that time the acidic rocks were investigated more thoroughly than the basic ones, and yielded more consistent results. The importance which is now being attached to the properties of basic rocks in inquiries concerning the nature of the deeper parts of the Earth's crust has made it seem advisable to carry out additional measurements of the compressibility of these rocks and certain of their constituent minerals.

In this work we have measured the compressibility of several diabases† with an improved apparatus, and where necessary redetermined the compressibility of the minerals in order to formulate more clearly the relation between the compressibility of the rocks and of their minerals. Furthermore, we have examined the compressibility of the garnets, grossularite and almandite, and of the pyroxene, jadeite, with a view to estimating the compressibility of eclogites, or rocks resembling basalt in gross composition but crystallized as high pressure minerals.

*Experimental Method.*—The apparatus, although entirely new, is very similar in construction to that already described in former publications<sup>1</sup> from this Laboratory. A leak-proof piston<sup>2</sup> is forced into a heavy-walled steel cylinder, which contains the specimen completely immersed in a suitable liquid (*n*-butyl ether). The hydrostatic pressure so generated is measured to 1 megabarye (about 1 atmosphere) by an electrical resistance gauge, and the corresponding decrease in volume is determined by measuring the travel of the piston with a dial-micrometer on which one division represents 0.01 mm. The apparatus is calibrated frequently by making a series of measurements on soft steel, the compressibility of which is accurately known.<sup>3</sup> The formula for calculating the decrease in volume for each pressure has been given fully in previous papers.<sup>4</sup>

**Materials.**—The sample of **Whin Sill diabase** was collected by Professor A. Holmes at Scordale Beck, near the Hilton lead mines, Westmorland, England. The rock is a fine-grained quartz-dolerite of density 2.937 at 31.4°. It has been described by Holmes and Harwood,<sup>5</sup> who give on page 512 of their paper a complete analysis. The calculated mode is approximately as follows: quartz, 7; plagioclase ( $Ab_1An_1$ ), 47; augite, 19; hypersthene, 18; magnetite and ilmenite, 9.

TABLE 1  
RESULTS FOR MINERALS

PRES- SURE (MEGA- BARYES)	ALMANDITE (243)			ALMANDITE (241)			GROSSULARITE (239)			GROSSULARITE (245)		
	$10^4a = -0.4$ $10^6b = 0.576$			$10^4a = 0.4$ $10^6b = 0.557$			$10^4a = 1.2$ $10^6b = 0.600$			$10^4a = 2.1$ $10^6b = 0.595$		
	$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.	
(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)
12000	0.565	0.572	-0.7	0.550	0.561	-1.1	0.601	0.613	-1.2	0.606	0.616	-1.0
11000	0.517	0.515	0.2	0.506	0.505	0.1	0.550	0.553	-0.3	0.556	0.556	0.0
10000	0.455	0.457	-0.2	0.456	0.450	0.6	0.489	0.493	-0.4	0.494	0.497	-0.3
9030	0.404	0.399	0.5	0.397	0.394	0.3	0.435	0.433	0.2	0.437	0.437	0.0
8000	0.347	0.342	0.5	0.349	0.338	1.1	0.382	0.373	0.9	0.372	0.378	-0.6
7000	0.287	0.284	0.3	0.288	0.283	0.5	0.315	0.313	0.2	0.332	0.318	1.4
6000	0.231	0.227	0.4	0.228	0.227	0.1	0.263	0.253	1.0	0.263	0.259	0.4
5000	0.166	0.169	-0.3	0.165	0.171	-0.6	0.207	0.193	1.4	0.208	0.199	0.9
4000	0.098	0.111	-1.3	0.117	0.116	0.1	0.141	0.132	0.9	0.161	0.140	2.1
3000	0.055	0.054	0.1	0.054	0.060	-0.6	0.055	0.072	-1.7	(0.109)*	(0.080)*	...
2000	0.000	-0.004	0.4	0.000	0.004	-0.4	0.000	0.012	-1.2	0.000	0.021	-2.1

GROSSULARITE (261) +	JADRITE (251)			JADRITE (249)			LABRADORITE (235)			LABRADORITE (237)				
	$10^4a = 3.8$ $10^6b = 0.611$			$10^4a = 1.6$ $10^6b = 0.735$			$10^4a = 3.1$ $10^6b = 0.775$			$10^4a = 0.2$ $10^6b = 1.457$ $10^{10}c = -0.115$				
	$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.			
(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)		
0.642	0.649	-0.7	0.748	0.751	-0.3	0.784	0.806	-2.2	1.342	1.344	-0.2	1.328	1.333	-0.5
0.591	0.588	0.3	0.669	0.678	-0.9	0.715	0.729	-1.4	1.225	1.220	0.5	1.211	1.208	0.3
0.524	0.527	-0.3	0.603	0.604	-0.1	0.659	0.651	0.8	1.101	1.094	0.7	1.092	1.081	1.1
0.470	0.466	0.4	0.531	0.531	0.0	0.579	0.574	0.5	0.959	0.966	-0.7	0.946	0.952	-0.6
0.409	0.405	0.4	0.460	0.457	0.3	0.508	0.496	1.2	0.830	0.835	-0.5	0.818	0.821	-0.3
0.351	0.344	0.7	0.391	0.384	0.7	0.439	0.419	2.0	0.693	0.702	-0.9	0.683	0.689	-0.6
0.276	0.282	-0.6	0.312	0.310	0.2	0.352	0.341	1.1	0.569	0.566	0.3	0.556	0.555	0.1
0.217	0.221	-0.4	0.243	0.237	0.6	0.276	0.264	1.2	0.444	0.429	1.5	0.426	0.419	0.7
...	...	...	0.178	0.163	1.5	0.192	0.186	0.6	0.286	0.289	-0.3	0.278	0.282	-0.4
...	...	...	0.083	0.090	-0.7	0.104	0.109	-0.5	0.144	0.146	-0.2	0.146	0.143	0.3
...	...	...	0.000	0.016	-1.6	0.000	0.031	-3.1	0.000	0.002	-0.2	0.000	0.001	-0.1

\* Omitted from the least square calculation.

The **Maryland diabase** was collected by Dr. E. S. Shepherd on the Baltimore Pike near Frederick, Md. It is a fine-grained diabase, density 3.033 at 31.0°. An unpublished analysis, by E. S. Shepherd, of the specimen used in this work is as follows:  $SiO_2$ , 51.28;  $Al_2O_3$ , 15.07;  $Fe_2O_3$ , 1.12;  $FeO$ , 9.31;  $MgO$ , 7.97;  $CaO$ , 11.42;  $Na_2O$ , 2.03;  $K_2O$ , 0.27;  $H_2O$  +, 0.39;  $H_2O$ -, 0.09;  $TiO_2$ , 0.78;  $P_2O_5$ , 0.13;  $Cl$ , 0.00;  $S$ , 0.06;  $Cr_2O_3$ , 0.05;  $MnO$ , 0.16. Calculated mode; plagioclase ( $Ab_1An_2$ ), 48; augite, 24; hypersthene, 25; magnetite and ilmenite, 3.

The specimen of **Sudbury diabase** is the same as that used by Adams and Williamson.<sup>6</sup> Density 3.002 at 31.1°. It is a moderately coarse-

grained olivine diabase. Calculated mode: plagioclase ( $\text{Ab}_3\text{An}_2$ ), 60; augite, 15; olivine, 17; magnetite and ilmenite, 8.

The **jadeite** was from a large piece of massive material from Burma and was kindly furnished by Dr. W. F. Foshag of the U. S. National Museum. Density 3.328 at 30.9°. Microscopic analysis showed it to be practically pure jadeite.

The fine specimen of translucent massive **grossularite** was also furnished by Dr. Foshag. Density 3.544 at 30.9°. An analysis of material from the locality has been made by Clarke and Steiger.<sup>7</sup>

TABLE 2  
RESULTS FOR DIABASE ROCKS

PRESSURE (MEGA- BARYES)	MARYLAND DIABASE (227)			MARYLAND DIABASE (229)			MARYLAND DIABASE (233)		
	$10^4a = -0.23$ $10^4b = 1.280$ $10^{10}c = -0.115$			$10^4a = 0.35$ $10^4b = 1.239$ $10^{10}c = -0.135$			$10^4a = 0.21$ $10^4b = 1.228$ $10^{10}c = -0.130$		
	$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.	
	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$
12000	1.164	1.162	0.2	1.111	1.107	0.4	1.103	1.100	0.3
11000	1.048	1.056	-0.8	1.007	1.009	-0.2	1.005	1.002	0.3
10000	0.958	0.948	1.0	0.911	0.908	0.3	0.903	0.901	0.2
9000	0.835	0.837	-0.2	0.797	0.804	-0.7	0.787	0.798	-1.1
8000	0.722	0.724	-0.2	0.694	0.698	-0.4	0.683	0.692	-0.9
7000	0.604	0.609	-0.5	0.582	0.589	-0.7	0.585	0.584	0.1
6000	0.493	0.491	0.2	0.483	0.477	0.6	0.477	0.473	0.4
5000	0.374	0.371	0.3	0.371	0.363	0.8	0.375	0.359	1.4
4000	0.252	0.249	0.3	0.254	0.246	0.8	0.240	0.243	-0.3
3000	0.118	0.125	-0.7	0.120	0.126	-0.6	0.119	0.124	-0.5
2000	0.000	-0.002	0.2	0.000	0.004	-0.4	0.000	0.002	-0.2

PRESSURE (MEGA- BARYES)	SUDBURY DIABASE (217)			SUDBURY DIABASE (219)			WHIN SILL DIABASE (221)			WHIN SILL DIABASE (223).		
	$10^4a = 0.13$ $10^4b = 1.440$ $10^{10}c = -0.130$			$10^4a = 0.05$ $10^4b = 1.351$ $10^{10}c = -0.061$			$10^4a = 0.15$ $10^4b = 1.712$ $10^{10}c = -0.284$			$10^4a = 0.58$ $10^4b = 1.688$ $10^{10}c = -0.280$		
	$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.		$-\frac{\Delta V}{V_0} \times 10^2$	DIFF.	
	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$	(OBS.)	(CALC.)	$\times 10^4$
1.317	1.312	0.5		1.295	1.291	0.4	1.434	1.430	0.4	1.418	1.414	0.4
1.182	1.192	-1.0		1.163	1.168	-0.5	1.309	1.312	-0.3	1.299	1.298	0.1
1.080	1.070	1.0		1.048	1.043	0.5	1.192	1.189	0.3	1.181	1.177	0.4
0.941	0.946	-0.5		0.911	0.917	-0.6	1.054	1.061	-0.7	1.040	1.050	-1.0
0.816	0.819	-0.3		0.784	0.790	-0.6	0.929	0.927	0.2	0.906	0.918	-1.2
0.683	0.689	-0.6		0.659	0.661	-0.2	0.773	0.787	-1.4	0.774	0.780	-0.6
0.563	0.557	0.6		0.537	0.531	0.6	0.648	0.638	1.0	0.644	0.636	0.8
0.428	0.422	0.6		0.407	0.400	0.7	0.498	0.490	0.8	0.502	0.487	1.5
0.282	0.284	-0.2		0.270	0.268	0.2	0.341	0.333	0.8	0.339	0.332	0.7
0.143	0.144	-0.1		0.130	0.135	-0.5	0.161	0.170	-0.9	0.166	0.172	-0.6
0.000	0.001	-0.1		0.000	0.001	-0.1	0.000	0.001	-0.1	0.000	0.006	-0.6

The **almandite**, from Salida, Col., was also furnished by Dr. Foshag. Density 4.160 at 30.9°. An analysis of almandite from the same locality has been made by Penfield and Sperry.<sup>8</sup>

The specimen of **labradorite**, from Nain, Labrador, is the same as that described in a previous paper.<sup>6</sup> The density is 2.696 at 29.5°, and the composition is approximately  $\text{Ab}_{48}\text{An}_{52}$ .

*Experimental Results.*—The results for the three rocks and for the four minerals are shown in tables 1 and 2, which give the fractional decrease

in volume,  $-\Delta V/V_o$ , caused by an increase of pressure from 2000 megabaryes to the pressure indicated. Within the experimental error, the observations for jadeite and for the two garnets, fall on a straight line, when  $\Delta V/V_o$  is plotted against the pressure,  $P$ . Accordingly, they have been represented by the equation:

$$-\Delta V/V_o = a + b(P - 2000) \quad (1)$$

where  $a$  and  $b$  are constants. These constants were calculated from the data by the method of least squares, and are shown in table 1 at the head of each series of results. The agreement between the observed values of  $\Delta V/V_o$  and those calculated by equation (1) is quite satisfactory and indicates that the readings of the dial-micrometer were consistent to better than 0.01 mm.

For labradorite the graph of  $\Delta V/V_o$  against  $P$  is distinctly curved; the results, therefore, are represented by a quadratic equation:

$$-\Delta V/V_o = a + b(P - 2000) + c(P - 2000)^2 \quad (2)$$

the constants,  $a$ ,  $b$ , and  $c$  being obtained from a least-square solution.

The compressibility,  $\beta$  (equal to  $-dV/V_o \cdot dP$ ) may be obtained by differentiating equation (1) or (2). The compressibility of **labradorite** varies considerably with pressure, and is found to be  $1.48 \times 10^{-6}$  at  $P = 0$ , and  $1.28 \times 10^{-6}$  at  $P = 10,000$  megabaryes. These values are somewhat lower than those previously obtained (1.55 and 1.34, respectively), but the difference is partly accounted for by the lower value now taken for the compressibility of the reference substance, soft steel.

The compressibilities of the minerals are summarized in table 3, in which have also been inserted the results previously obtained for other minerals commonly found in basic rocks and the observations on magnetite and ilmenite made by Madelung and Fuchs.<sup>9</sup> For those substances, whose change of compressibility has not been measured, the compressibilities at the various pressures have been determined by the empirical method already described.<sup>10</sup> This method takes advantage of the fact that, approximately, the change of compressibility with pressure is a function of the compressibility alone. If the compressibility at only one pressure, or the mean compressibility over a certain range of pressure, is known, the compressibility at various pressures can be estimated, probably with considerable accuracy, and certainly with less error than if no allowance were made for the change of compressibility.

In the plagioclase series only two members have been investigated, but on the assumption that the compressibility bears a linear relation to the volume-percentage of albite or anorthite the compressibilities of two other members of the series,  $\text{Ab}_{60}\text{An}_{40}$  and  $\text{Ab}_{33}\text{An}_{67}$ , have been calculated. In table 3 those values which have been obtained by interpolation,

extrapolation, or correction for the "statistical" change of compressibility are enclosed in parentheses. For pressures from 0 to 15,000 megabaryes, this table gives a comprehensive view of the compressibilities of the more important minerals found in basic rocks.

TABLE 3  
COMPRESSIBILITIES OF THE IMPORTANT MINERALS IN BASIC ROCKS

MINERAL	COEFFS. OF EQUATIONS (1) AND (2)		0	COMPRESSIBILITY, $10^6\beta$ , AT THE FOLLOWING PRESSURES (MEGABARYES)			
	$10^6b$	$-10^{10}c$		2000	7000	10,000	15,000
Quartz	2.61	0.216	2.70	2.61	2.40	2.27	..
Oligoclase $Ab_{78}An_{22}$	1.70	0.105	1.74	1.70	1.60	1.54	(1.44)
Andesine $Ab_{80}An_{40}$	..	...	(1.59)	(1.55)	(1.45)	(1.39)	(1.29)
Labradorite $Ab_{48}An_{52}$ *	1.44	0.100	1.48	1.44	1.34	1.28	(1.18)
Labradorite $Ab_{33}An_{67}$	..	...	(1.32)	(1.30)	(1.24)	(1.20)	(1.14)
Actinolite	1.29	...	(1.37)	(1.35)	1.29	(1.25)	(1.19)
Diopside	1.06	...	(1.13)	(1.11)	1.06	(1.03)	(0.98)
Augite	1.01	...	(1.07)	(1.05)	1.01	(0.98)	(0.94)
Enstatite	1.00	...	(1.06)	(1.04)	1.00	(0.97)	(0.93)
Hypersthene	0.98	...	(1.04)	(1.02)	0.98	(0.95)	(0.91)
Olivine	0.81	...	(0.85)	(0.84)	0.81	(0.79)	(0.76)
Jadeite*	0.75	...	(0.78)	(0.77)	0.75	(0.73)	(0.71)
Garnet (Grossularite)*	0.60	...	(0.63)	(0.62)	0.60	(0.59)	(0.57)
Garnet (Almandite)*	0.57	...	(0.60)	(0.59)	0.57	(0.56)	(0.54)
Ilmenite	..	...	0.56	(0.55)	(0.53)	(0.52)	(0.50)
Magnetite	..	...	0.55	(0.54)	(0.52)	(0.51)	(0.49)

\* New determinations described in this paper.

With each of the three rocks investigated the volume-decrease was not a linear function of the pressure, a fact which was at once evident from the graphs. Two series of measurements were made with the Whin Sill diabase, two with the Sudbury, and three with the Maryland. There seemed to be a tendency for the second or third series with a given specimen to yield a somewhat lower compressibility than the first series. This was probably due to the liquid more completely filling the fine pores of the rock by the continued application of pressure. In figure 1 is shown graphically the volume-change under pressure for the Whin Sill and for the Maryland diabase (two series for each).

*Discussion.*—In a previous investigation<sup>6</sup> the compressibility of rocks was measured under two different conditions, (1) the rock was placed directly in contact with the pressure-transmitting liquid, and (2) the rock was completely enclosed in a thin jacket of tin. The first method, leading to the "inner" compressibility, gave the volume changes due to the compression of the actual material particles of the rock because the pressure liquid had ample opportunity to fill up the interstices; but in the second method, from which the "outer" compressibility was calculated, the observed volume change included both the change due to the compression of the particles and that due to the closing of the pores.

If we are interested in the propagation of elastic waves through a rock of a certain composition and structure on the surface of the Earth the "outer" compressibility is the modulus which is of most importance to us, but in this paper we are more concerned with rocks formed under pressures at which a very compact structure would be most probable. For this reason and also on account of the greater ease of experimentation we have made all our measurements on unenclosed specimens. At lower pressures the differences between these two types of compressibility are

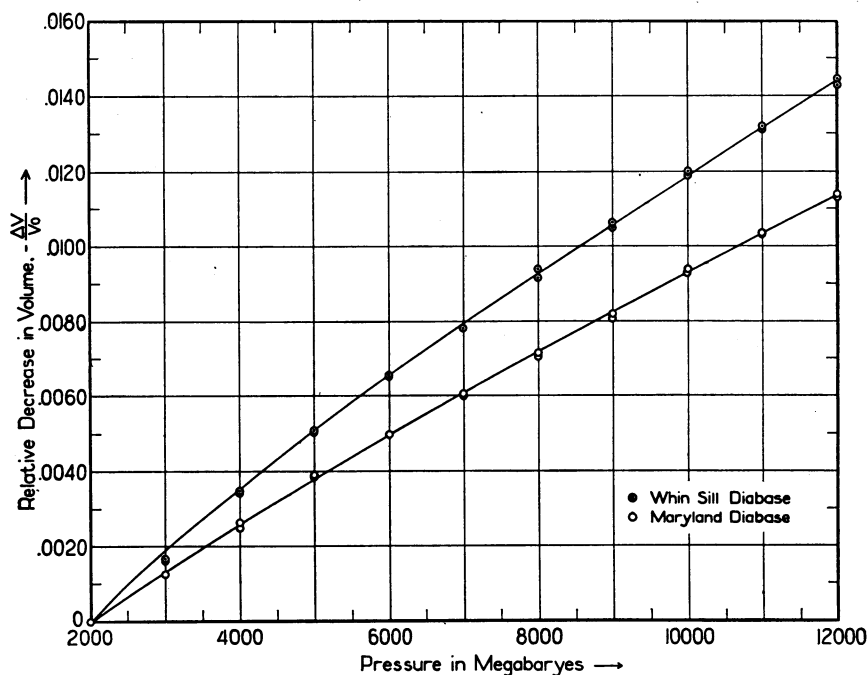


FIGURE 1

Final results for the volume change under pressure of Whin Sill and Maryland diabases.

considerable for many, especially the more porous rocks, but beyond 10,000 megabaryes the differences approach zero asymptotically and so our problem is simplified.

In the paper by Adams and Williamson<sup>6</sup> it is shown that at high pressures the compressibility of rocks tends toward the value obtained by averaging (according to volume-percentage) the compressibilities of the minerals present in the rock. At lower pressures, however, compressibility of the rock may be much greater than that calculated from the minerals and depends as much on the physical condition of the particular rock as on its mineral composition. Figure 2, which is taken (with minor changes) from

the earlier paper, illustrates the way in which the compressibilities of granitic and gabbroic rocks vary with pressure or depth. The large variation at low pressures depends on the more or less accidental structure of the rock, while at high pressures the variation is mainly that due to differences in mineral content of the rock of the type in question.

In table 4 are summarized, first the compositions of the various rocks in terms of the minerals present, and second the measured and calculated compressibilities at certain pressures. The results for the previously investigated Palisade diabase, recalculated to show the change of com-

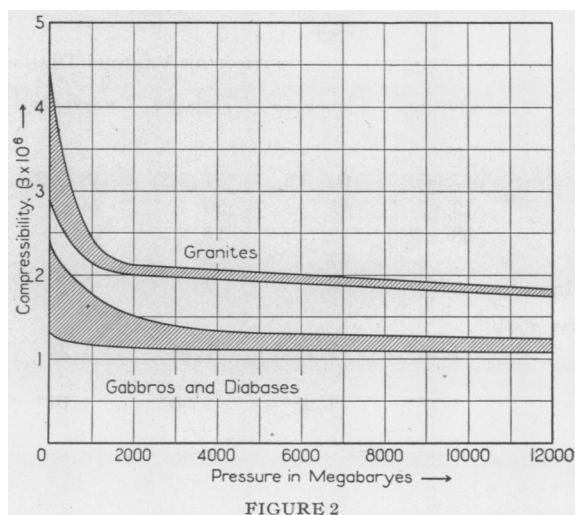


FIGURE 2  
The compressibility of granites and diabases as a function of pressure. The width of the shaded areas indicates the variation which may be expected for ordinary rocks within a given class. At high pressures the variation is governed by the mineral composition of the rock, at low pressures the variation in compactness is the dominating factor.

pressibility with pressure, have been included in the table. There is also included the presumable mineral content and the calculated compressibility of the average plateau basalt as computed by R. A. Daly<sup>11</sup> from 50 analyses of Deccan, Oregonian and Thulean flows.

The measured compressibility, which at 2000 mb. exhibits a considerable variation from one rock to another, shows much less variation at 10,000 mb. Moreover, at 10,000 mb. the compressibilities have approached fairly closely to those calculated from the minerals, being on the average only  $0.10 \times 10^{-6}$  higher. In one case—the Maryland diabase—the figures are the same for the measured and calculated compressibilities. The

greatest divergence is found for the Palisade diabase, the measured and calculated values being  $1.30 \times 10^{-6}$  and  $1.16 \times 10^{-6}$ , respectively. It is significant in this connection that by one criterion of the degree of weathering the Maryland diabase is a much fresher rock than the Palisade diabase. According to unpublished measurements by E. S. Shepherd the latter yields 90 cm.<sup>3</sup> of gas (mostly water vapor) per gram when heated to a high temperature in a vacuum, while the former evolves only 30 cm.<sup>3</sup> per gram, which is close to the minimum observed for ordinary rocks. Only lavas from recent flows show less. In the former paper the presence of saussuritized feldspar in the Palisade diabase was noted as an indication

TABLE 4  
COMPRESSIBILITY AND MINERAL COMPOSITION OF VARIOUS DIABASE ROCKS

MINERAL COMPOSITION	WHIN SILL	SUDBURY	PALISADE	MARYLAND	AVERAGE PLATEAU BASALT (DALY)
Quartz	7	....	....	....	....
Plagioclase	47 (1,1)	60 (3,2)	54 (3,2)	48 (1,2)	50 (3,2)
Augite	19	15	20	24	} 37
Hypersthene	18	....	14	25	
Olivine	....	17	5	....	4
Magnetite and Ilmenite	9	8	7	3	9
COMPRESSIBILITY, $\beta \times 10^6$					
Measured	1.70	1.37	1.54	1.23	....
$P = 2000$ mb.					
Measured	1.26*	1.25*	1.30*	1.07*	....
$P = 10,000$ mb.					
Calcd. from minerals	1.17	1.16	1.16	1.07	1.13
$P = 10,000$ mb.					
Calcd. from minerals	1.09	1.08	1.09	1.02	1.06
$P = 15,000$ mb.					

\* These figures were calculated by passing a least-square straight line through the points from 8000 to 12,000 megabaryes. It is noteworthy that the same result was obtained from the slope of the quadratic equation at 10,000 megabaryes.

that the rock was not perfectly fresh. The Maryland rock, on the other hand, is probably a very perfect specimen of a sound and fresh diabase. Even at 2000 megabaryes the measured compressibility is nearly as low as that calculated from the minerals (1.23 against 1.16). We may conclude from the table that at high pressures the compressibilities of the various rocks approach and finally become equal to those calculated from the mineral content. Indeed, it appears that for any rock a *minimum* value of its compressibility at any pressure may be obtained by averaging the compressibilities of its constituent minerals. The actual compressibility may be considerably higher, especially at low pressures. It is interesting that granites lose their abnormality almost entirely at 2000 mb. or less, but that the basic rocks in most instances still retain some of their abnormality even at 10,000 mb.



In a former paper a slightly higher value was assigned to the compressibility of diabase than will be found here. This is due to two factors: (1) better values now available for the compressibility of the standard of reference, steel, which are lower than those used in the previous work, and which exhibit an appreciable decrease with increasing pressure; and (2) the difference between the old and the new values for the compressibility of labradorite.

From figure 2 it may be seen that the characteristic course of the compressibility of rocks as the pressure is increased is first a rapid and then a much slower decrease of the compressibility. This course should properly be represented by an equation with an exponential term. Even though the volume change of a rock in the pressure-range from 2000 to 12,000 megabaryes may, within the error of experiment, follow a parabolic equation such as eqn. (2), it is very evident that an extrapolation by such an equation to either higher or lower pressures would be very unsafe. Indeed, an extrapolation in either direction would yield compressibilities lower than the true ones. At very low pressures the compressibility of a given sample of rock can be determined only by direct measurement on the particular sample, and at pressures above the experimental range a calculation based on the extrapolated compressibilities of the constituent minerals will give the most satisfactory value, although it is to be remembered that the value so obtained is merely the *minimum* compressibility. The bottom line of table 4 shows the calculated compressibilities at 15,000 megabaryes of the four diabases and also of Daly's average basalt (if crystalline). It does not seem possible that the compressibilities at the pressure stated can be significantly lower than the values given, and it does not seem probable that they can be very much higher.

In a recent paper on "The Outer Shells of the Earth," R. A. Daly<sup>11</sup> has made a painstaking criticism of all previous determinations of the compressibility of rocks, and has concluded that the values which have been given for the compressibilities are too high, mainly because of two factors: first, the change of compressibility with pressure; and second, the tendency of elastic hysteresis to increase the effective elastic moduli of rocks. The first effect has already been discussed in a paper by the authors, and as for the second, we have not been able to observe any hysteresis in the volume change of igneous rocks when subjected to *hydrostatic* pressure.

From the compressibilities given in table 4 the velocity of longitudinal waves through the various diabase rocks has been calculated by the equation:<sup>6</sup>

$$\dot{U}_p = 13.13/\sqrt{10^6\beta\rho}, \quad (3)$$

where  $U_p$  is the velocity in kilometers per second and  $\rho$  is the density. The results are shown in the following table.

TABLE 5  
VELOCITY OF LONGITUDINAL WAVES IN VARIOUS DIABASE ROCKS

ROCK	VELOCITY IN KM./SEC. AT VARIOUS PRESSURES			
	2000 $\beta$ MEASURED	10,000 $\beta$ MEASURED	10,000 $\beta$ CALC.	15,000 $\beta$ CALC.
Whin Sill diabase	5.9	6.8	7.0	7.3
Sudbury diabase	6.5	6.7	7.0	7.2
Palisade diabase	6.1	6.6	7.0	7.2
Maryland diabase	6.8	7.2	7.2	7.4
Av. plateau basalt	...	...	7.1	7.3

The velocities at 15,000 megabaryes equivalent to a depth of slightly more than 50 kilometers are appreciably less than 7.9 km. per sec., which is the velocity of longitudinal waves at this depth as deduced from seismologic data. It should be noted that the velocities given in table 5 apply only if the rock is crystalline. The compressibilities of glass and glassy rocks<sup>4</sup> will be discussed in a subsequent paper.

For plateau basalt the velocity at 15,000 megabaryes is 7.3 km./sec. Although the gap between 7.3 and 7.9 is not large, it seems difficult to close it and thus to allow crystalline basalt—with glassy basalt the gap is wider—at depths below the 50–60 km. discontinuity. We believe that the compressibility of diabase as here given cannot be in error by as much as 4 per cent, which would correspond to 2 per cent for the wave velocity. In order to have material of the composition of diabase below the 50–60 km. discontinuity it seems necessary to assume: (1) that that material is crystalline; (2) that the effect of temperature is negligible; and (3) that the seismologic value for the velocity at this depth is in error by at least 0.6 km./sec.

*The Compressibility of Eclogites.*—The compressibilities of grossularite and almandite, the only two garnets we could obtain as large cylinders, are given in table 1, and may be summarized by the statement that the average compressibility between 2000 and 12,000 megabaryes is  $0.60 \times 10^{-6}$  for grossularite and  $0.57 \times 10^{-6}$  for almandite. Although the compositions of these two garnets differ widely, their compressibilities are nearly the same and we are emboldened to place the average compressibility of pyrope between that of grossularite and almandite.

We also determined the compressibility of jadeite. The results are in table 1 and indicate that the average compressibility of this mineral is  $0.75 \times 10^{-6}$  reciprocal megabaryes.

With the help of these results it is possible to calculate the approximate compressibility of an eclogite. In table 6 we have taken hypothetical eclogites whose mineral compositions are based on three of the Norwegian eclogites so carefully studied by Eskola.<sup>12</sup> The compressibility of each has been calculated at 7000 megabaryes from our data and thence the earthquake wave velocity through such rocks has been obtained in the usual way. It must be emphasized that although we have tried to keep

the compositions of the hypothetical rocks within the bounds of plausibility these calculations must be regarded as approximate and are intended merely to assign probable elastic constants to rocks composed of high-pressure minerals. The wave velocities in these eclogite rocks even at 7000 megabaryes are about the same as those given below the 50–60 km. discontinuity by the seismic data, and hence our results do not conflict with the hypothesis<sup>13</sup> of an eclogite shell at that depth. Further discussion of this is postponed to some future occasion.

TABLE 6  
CALCULATED COMPRESSIBILITIES AT 7000 MEGABARYES OF CERTAIN ECLOGITES AND  
THE VELOCITIES OF LONGITUDINAL WAVES THROUGH THEM

PERCENTAGE BY VOLUME			AVERAGE	AVERAGE	AVERAGE	REFERENCE
GARNET	JADEITE	DIOPSIDE	$\rho$	$\beta$	$U_p$	
50	30	20	3.7	0.73	8.0	1
42	6	52	3.5	0.84	7.7	2
32	24	44	3.5	0.83	7.7	3

1 Analogous to chloromelanite eclogite from Vanelvsdalén, Søndmøre, cf. *Eskola*, p. 31.

2 Analogous to eclogite from Rødhaugen, Almklovdaalen, Søndmøre, cf. *Eskola*, p. 27.

3 Analogous to eclogite from Silden, Nordfjord, cf. *Eskola*, p. 37.

*Summary and Conclusions.*—We have measured directly, at pressures between 2000 and 12,000 megabaryes, the cubic compressibility of the minerals, labradorite, jadeite, grossularite and almandite, and of diabase rocks from Sudbury, Canada; Frederick, Maryland; and the Whin Sill in the North of England. By combining these mineral results with those already known, we are able to give a table of the compressibilities, at various pressures, of all the important constituents of basic rocks, to compare the compressibilities of the rocks with those of their constituent minerals, and to conclude that the compressibility calculated from the mineral content gives a limit to which that of the rocks approaches at high pressure. Our results demonstrate, furthermore, that at 15,000 megabaryes and 30°C. the *maximum* velocity of longitudinal elastic waves through rocks of basaltic composition and mode is 7.4 km. per sec. On the other hand, the compressibilities of garnet and jadeite, being found to be surprisingly low, lead to the conclusion that magmas of this composition, crystallized primarily as, or subsequently metamorphosed to eclogites, may transmit longitudinal waves with velocities exceeding 8 km. per sec. Bearing in mind that the pressures existing at 20, 40 and 60 km. below the surface of the earth are somewhat greater than 5, 10 and 15 thousand megabaryes, respectively, we may readily see the connection between these observations and deductions concerning the composition of the interior of the earth. Indeed, the existing data, in our opinion, place the possible components of the earth, below 60 km. and above the core,

in the following ascending order of probability: holocrystalline basalt, eclogite, peridotite.

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† Throughout this paper the word *diabase* is used in the American sense to indicate a holocrystalline basalt. It should be understood by British readers to mean *dolerite*.

<sup>1</sup> Adams, Williamson and Johnston, *J. Am. Chem. Soc.*, **41**, 1919 (12). Adams and Williamson, *J. Franklin Inst.*, **195**, 1923 (475).

<sup>2</sup> P. W. Bridgman, *Proc. Am. Acad. Arts & Sci.*, **49**, 1914 (634).

<sup>3</sup> P. W. Bridgman, *Proc. Am. Acad. Arts & Sci.*, **58**, 1923 (166); see also Adams and Gibson, *These PROCEEDINGS*, **12**, 1926 (275); *Gerlands Beitr. z. Geophys.*, **15**, 1926 (17).

<sup>4</sup> Adams and Gibson, *Op. cit.*; Adams and Williamson, *Op. cit.*

<sup>5</sup> Holmes and Harwood, *Min. Mag.*, **21**, 1928 (493).

<sup>6</sup> Adams and Williamson, *Op. cit.*

<sup>7</sup> Clarke and Steiger, *U. S. Geol. Surv., Bull. No. 262*, 1905 (72).

<sup>8</sup> Penfield and Sperry, *Am. J. Sci.*, **32**, 1886 (311).

<sup>9</sup> Madelung and Fuchs, *Ann. Physik*, **65**, 1921 (289-309).

<sup>10</sup> L. H. Adams, *J. Wash. Acad. Sci.*, **17**, 1927, 529.

<sup>11</sup> R. A. Daly, *Am. J. Sci.*, **15**, 1928 (108).

<sup>12</sup> Pentti Eskola. *Vidensk. Skrifter I. Mat. Nature*. Klasse 1921, No. 8.

<sup>13</sup> For references to this hypothesis see Bowen, *The Evolution of Igneous Rocks*. Princeton Univ. Press, 1928.

## THE MINIMA OF INDEFINITE QUATERNARY QUADRATIC FORMS

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Let  $f = \sum a_{ik}x_i x_k$  ( $a_{ik} = a_{ki}$ ) be a quadratic form of rank  $n$  with real coefficients in  $n$  variables  $x_1, \dots, x_n$ . The variables assume integer values only, the set  $0, 0, \dots$  excepted. One of the many problems associated with quadratic forms is that of finding the upper bound of  $L(f)$ , the absolute lower bound of  $f$ , when  $f$  runs over the set of forms which have a given hessian  $a = ||a_{ik}||$ . We shall consider only real indefinite quaternary forms. For indefinite binary and ternary forms, Markoff\* has given two theorems, which are needed in the discussion of quaternary forms. By means of these theorems of Markoff, I have proved the following: