NATIONAL BUREAU OF ECONOMIC RESEARCH

Studies in Business Cycles

No. 2

MEASURING BUSINESS CYCLES

MEASURING BUSINESS CYCLES

Arthur F. Burns
and
Wesley C. Mitchell

NATIONAL BUREAU OF ECONOMIC RESEARCH
NEW YORK 1946

REPRINTED 1947, 1964

MANUFACTURED IN THE U.S.A. BY
H. WOLFF, NEW YORK

Relation of the Directors to the Work and Publications

of the

National Bureau of Economic Research

1. The object of the National Bureau of Economic Research is to ascertain and to present to the public important economic facts and their interpretation in a scientific and impartial manner. The Board of Directors is charged with the responsibility of ensuring that the work of the Bureau is carried on in strict conformity with this object.

2. To this end the Board of Directors shall appoint one or more Directors of Research.

3. The Director or Directors of Research shall submit to the members of the Board, or to its Executive Committee, for their formal adoption, all specific proposals concerning researches to be instituted.

4. No report shall be published until the Director or Directors of Research shall have submitted to the Board a summary drawing attention to the character of the data and their utilization in the report, the nature and treatment of the problems involved, the main conclusions and such other information as in their opinion would serve to determine the suitability of the report for publication in accordance with the principles of the Bureau.

5. A copy of any manuscript proposed for publication shall also be submitted to each member of the Board. For each manuscript to be so submitted a special committee shall be appointed by the President, or at his designation by the Executive Director, consisting of three Directors selected as nearly as may be one from each general division of the Board. The names of the special manuscript committee shall be stated to each Director when the summary and report described in paragraph (4) are sent to him. It shall be the duty of each member of the committee to read the manuscript. If each member of the special committee signifies his approval within thirty days, the manuscript may be published. If each member of the special committee has not signified his approval within thirty days of the transmittal of the report and manuscript, the Director of Research shall then notify each member of the Board, requesting approval or disapproval of publication, and thirty additional days shall be granted for this purpose. The manuscript shall then not be published unless at least a majority of the entire Board and a two-thirds majority of those members of the Board who shall have voted on the proposal within the time fixed for the receipt of votes on the publication proposed shall have approved.

6. No manuscript may be published, though approved by each member of the special committee, until forty-five days have elapsed from the transmittal of the summary and report. The interval is allowed for the receipt of any memorandum of dissent or reservation, together with a brief statement of his reasons, that any member may wish to express; and such memorandum of dissent or reservation shall be published with the manuscript if he so desires. Publication does not, however, imply that each member of the Board has read the manuscript, or that either members of the Board in general, or of the special committee, have passed upon its validity in every detail.

7. A copy of this resolution shall, unless otherwise determined by the Board, be printed in each copy of every National Bureau book.

(Resolution adopted October 25, 1926 and revised February 6, 1933 and February 24, 1941)

Studies in Business Cycles

1 *Business Cycles: The Problem and Its Setting*
By Wesley C. Mitchell

2 *Measuring Business Cycles*
By Arthur F. Burns and Wesley C. Mitchell

3 *American Transportation in Prosperity and Depression*
By Thor Hultgren

4 *Inventories and Business Cycles, with Special Reference to Manufacturers' Inventories*
By Moses Abramovitz

5 *What Happens during Business Cycles: A Progress Report*
By Wesley C. Mitchell

6 *Personal Income during Business Cycles*
By Daniel Creamer with the assistance of Martin Bernstein

7 *Consumption and Business Fluctuations: A Case Study of the Shoe, Leather, Hide Sequence*
By Ruth P. Mack

8 *International Financial Transactions and Business Cycles*
By Oskar Morgenstern

9 *Federal Receipts and Expenditures during Business Cycles, 1879–1958*
By John M. Firestone

10 *Business Cycle Indicators:* Volume I, *Contributions to the Analysis of Current Business Conditions;* Volume II, *Basic Data on Cyclical Indicators*
Edited by Geoffrey H. Moore

11 *Postwar Cycles in Manufacturers' Inventories*
By Thomas M. Stanback, Jr.

12 *A Monetary History of the United States, 1867–1960*
By Milton Friedman and Anna Jacobson Schwartz

Preface

THE BASIC features of the plan for measuring business cycles presented in this book were outlined in the last six pages of *Business Cycles: The Problem and Its Setting,* published by the National Bureau in 1927. My 'tentative working plans' of that date were embryonic. They have developed slowly under the solicitous attention of numerous coworkers, and besides secular growth, have undergone some structural changes.

I had thought of analyzing the movements of "all the time series for a given country on the basis of a standard pattern derived from the business annals of that country, not on the basis of the various patterns which might be derived from study of the several series themselves." This plan was promptly amended to include analysis on both bases. We found that the words 'prosperity' and 'depression', to which I clung in 1927, misrepresent some business-cycle phases, and replaced them by 'expansion' and 'contraction'. As our statistical findings accumulated, we refined upon the rough chronologies provided by the collection of *Business Annals* that Willard L. Thorp had compiled for us. To picture the cyclical behavior of a series, I had proposed to draw a separate chart for each of the four phases of a business cycle. We found it better to plot the four phases on a single chart, and to add a 'specific-cycle pattern' based upon the series' own troughs and peaks.

To determine what aspects of cyclical behavior should be measured, and just how each measure should be made, required much experimenting with the wide variety of data we wished to use. For six years, the brunt of this developmental work was borne by Simon Kuznets and several assistants, of whom Cicely Applebaum was chief. When Dr. Kuznets became absorbed in estimating national income and its components, Arthur F. Burns took over. He instituted a searching critique of our methods and rigorous tests of our findings, out of which came many improvements in our conceptions and procedures.

While I shared in building up our technique, my chief function was to study and interpret the results it yielded. In that capacity, I wrote two experimental reports at different stages of our progress, explaining our methods and summarizing what they seemed to show about the cyclical behavior of the activities we had studied. These efforts were useful mainly in a negative way. They demonstrated first that we needed to enlarge our sample of time series in various directions; second that much more intimate knowledge of economic activities than I possessed was necessary to understand their reactions to business cycles; third that our findings could not be adequately presented in a single volume as I had naively expected. The upshot was that we enlarged our staff. The detailed interpretation of our findings was undertaken by a group of collaborators who were or became specialists in such fields as agriculture, construction work, transportation, merchandising, inventories, prices, labor problems, foreign trade, international finance, and banking. I stuck to the task of trying to see how the results in all the fields fitted together, depending on others to supply knowledge I lacked. Meanwhile, Dr. Burns familiarized himself with the uses of our measures by preparing a preliminary analysis of our findings about construction work, and then devoted himself to a final critique and revision of our statistical methods.

The present volume is mainly his work. Though the basic features of my original design have been retained, Dr. Burns has made our technique of measuring cyclical behavior a much better kit of tools than it was when put into his hands. All of the tests of our measures were planned and executed by him. So also were the chapters on the time unit and our treatment of secular and random components. With minor exceptions, the drafts I contributed have been so much improved by him that they have become virtually his products. I have taken advantage of my seniority to insist against his wish that the relative shares we have had in preparing this book be represented by putting Dr. Burns' name first on the title page.

The dozen chapters form three broad groups. Chapters 1-5 describe our methods, first in general terms, then in full detail. Chapters 6-8 elaborate upon three themes treated briefly in the third chapter: our insistence upon using monthly data whenever they can be had, and our peculiar ways of dealing with secular and random movements. Chapters 9-12 examine critically the justification for using averages to express the typical characteristics of cyclical behavior.

We have sought to make the book useful to several groups of readers. While the discussion is focussed throughout upon the National Bureau's technique, certain alternative methods of time-series analysis come in for examination. Also, the abundant illustrations of our results possess substantive as well as methodological interest. Students who desire earnestly to understand business cycles must feel a professional interest in the design and efficiency of the tools used in observing cyclical behavior. On the one hand, these observations show what should be explained; on the other hand, they afford means for testing explanatory hypotheses. For such specialists, there is no short cut. To facilitate use of the book by others, we offer a tabular guide.

A Reader's Guide

	Chapters recommended
Laymen and economists with a general interest in business cycles	1-2
Students of business cycles	
Who have little or no concern with the empirical foundations of the subject, but would like to see what 'economic statisticians' may contribute to 'theory'	1-4[a], 9[b], 12[c]
Who have wider interests but are only incidentally concerned with statistical techniques	1-4[a], 9-12
Statisticians whose primary interest is	
Time-series analysis	2, 3, 5-8
Measurement of economic magnitudes	4
Testing hypotheses regarding time series	2, 9-12[d]

[a] Sec. IV of Ch. 3 may be omitted.
[b] Perhaps add Sec. VII-VIII of Ch. 10, and Sec. VII of Ch. 11.
[c] Omit Sec. III.
[d] Omit Sec. III-IV.

Dr. Burns and I have received generous and varied help from many friends. Acknowledgments of specific suggestions on technical points are made at the appropriate points in the text, and need not be recapitulated here. But this is the place to mention obligations of a more general sort.

Chester I. Barnard, W. Leonard Crum, and Oswald W. Knauth formed the Committee of National Bureau Directors who examined our manuscript. All three raised questions of which we have taken account as best we could. Mr. Barnard's critique incited us to add a section to Chapter I on 'The Symbols Used in Observing Business Cycles'.

Our present and former colleagues in the National Bureau have read parts or all of the successive versions through which the manuscript has passed, and aided in various ways to better it. We are especially indebted to Milton Friedman, Simon Kuznets, Frederick C. Mills, Geoffrey H. Moore, Julius Shiskin, and W. Allen Wallis. We have benefited also from criticisms or suggestions by Moses Abramowitz, James W. Angell, G. Heberton Evans, Gottfried Haberler, Edward E. Lewis, Oskar Morgenstern, George Stigler, Albert Wohlstetter, and Leo Wolman. Harold Hotelling of Columbia University kindly advised us on some points of statistical analysis. Martha Anderson put her editorial skill at our disposal, and Hanna Stern prepared the Index.

We owe an especially heavy debt of gratitude to our statistical assistants. Karl Laubenstein was mainly responsible for compiling data

and verifying sources. H. Irving Forman, Sophie Sakowitz and Denis Volkenau, aided at times by several computers, carried through the extensive calculations. Sophie Sakowitz helped also in the preparation of the printers' copy, and assumed the main burden of reading the proofs. H. Irving Forman drew all the charts in the volume, and assisted with the proofs. Sally Edwards typed our bulky and difficult manuscripts with a diviner's art. Without the care, patience, and skill of all five, our task would have been far harder and less pleasant.

WESLEY C. MITCHELL

Contents

CHAPTER 4

CHAPTER 5

CHAPTER 6

CHAPTER 7

List of Tables

List of Charts

MEASURING BUSINESS CYCLES

THIS AN
ness Cycl
earlier vo
the defini

Business cy
of nations
of expansio
lowed by si
the expansi
periodic; i
years; they
approxima

This c
pending c
attempts t
economic
economy,
a tool of
sciences,
not borne

Whet
cycles int
upon the
ing that r

1 Wesley C.
Research, 19

CHAPTER 1

Working Plans

I The Point of Departure

THIS AND succeeding volumes of the National Bureau's *Studies in Business Cycles* take as their point of departure a definition reached in an earlier volume.[1] With modifications suggested by experience in using it, the definition runs as follows:

> Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

This definition lists the observable characteristics of what we assume, pending closer study, to be a distinct species of economic phenomena. It attempts to differentiate business cycles from the fluctuations in aggregate economic activity that occurred prior to the emergence of our business economy, and from other types of fluctuations in modern times. It is thus a tool of research, similar to many definitions used by observational sciences, and like its analogues is subject to revision or abandonment if not borne out by observation.

Whether an investigator needs to condense his concept of business cycles into a definition, and what kind of definition he needs, depends upon the researches he has in view. Many theorists feel justified in assuming that readers know what business cycles are; the use of that term or one

1 Wesley C. Mitchell, *Business Cycles: The Problem and Its Setting* (National Bureau of Economic Research, 1927), p. 468.

of its equivalents suffices to designate the range of experience they plan to explain. Others pave the way by defining business cycles as recurrent departures from and returns toward 'a normal state of trade', or 'a position of economic equilibrium'. Still others, intent from the start upon some line of explanation, begin by confining the discussion to movements arising from the factors they have in mind; they may say, for example, that business cycles are recurrent alternations of prosperity and depression generated by 'factors originating within the economic system itself', or that business cycles are departures from equilibrium arising from discrepancies between 'the' market rate of interest and 'the natural rate'.

Whatever their working concepts, and we have done no more than illustrate the diversities, all investigators cherish the same ultimate aim—namely, to attain better understanding of the recurrent fluctuations in economic fortune that modern nations experience. This aim may be pursued in many ways. The way we have chosen is to observe the business cycles of history as closely and systematically as we can before making a fresh attempt to explain them.

At the beginning of this investigation our knowledge of business cycles was derived partly from an imperfectly digested mass of factual observations, partly from a great variety of untested hypotheses. Journalists in many lands have been publishing impressions concerning changes in the state of trade week by week and year by year for many decades. Useful as are the business annals that have been compiled from these materials, they give at best vague accounts, in terms that are general and shifting. Within the last generation, statisticians have been making more precise observations by analyzing time series and compiling 'indexes of business conditions'. Their work is instructive; but differences in method make it difficult to compare and combine the results of the many investigations, and the best of the business indexes cover too short a period or are woefully incomplete in coverage. Meanwhile with increasing finesse economic theorists have been tracing causal relationships among the cyclical movements of different activities. Their work is often highly suggestive; yet it rests so much upon simplifying assumptions and is so imperfectly tested for conformity to experience that, for our purposes, the conclusions must serve mainly as hypotheses. Nor can we readily use the existing measures of statisticians to determine which among existing hypotheses account best for what happens. Satisfactory tests cannot be made unless hypotheses have been framed with an eye to testing, and unless observations upon many economic activities have been made in a uniform manner.

If we are to observe the business cycles of actual experience closely and systematically, we need a working definition that tells where to look and for what to look. It must list observable characteristics, particularly such as differentiate business cycles from other movements with which they may be confused. To say that business cycles are departures from and returns toward a normal state of trade or a position of equilibrium, or that they are movements resulting from discrepancies between market and natural rates of interest, will not help, because we cannot observe normal states of trade, equilibrium positions, or natural interest rates. Nor, when we start observing, can we tell whether cyclical movements are due to factors originating within the economic system or outside of it.

Though business annals, time-series analysis, and business-cycle hypotheses in their present forms do not provide the knowledge we need, they put us in the path of learning more. It was by analyzing these three sets of contributions that our definition was developed. Considerable evidence can now be cited to support its every clause. But an intensive study of the best available records is necessary if we are to ascertain conclusively whether many economic activities really fluctuate in unison as the definition states, and how different activities behave with respect to the alleged cycles. Once that is accomplished we can proceed to the next step: to explain how business cycles run their course and their tendency toward variation.

II Questions Raised by the Definition

How much there is to learn about business cycles begins to appear when our working definition is examined critically. Every clause suggests hard questions, some of which raise doubts about the validity of the concept itself.

Thus the definition states that business cycles occur in 'nations' whose economic activities are organized 'mainly in business enterprises'. Does a large nation, such as the United States, have a single set of business cycles, or do the several geographical regions have substantially different cyclical movements? Or is a nation too small, rather than too large, a geographical unit to observe? May it not be best to treat business cycles as international movements? How far back in history can business cycles be traced? If they are associated with a form of economic organization that is itself evolving, are they subject to secular changes that make recent cycles noncomparable with those of earlier times? The term 'business enterprise' connotes a measure of individual initiative and competition. Do business cycles fade out when freedom of enterprise is drastically limited by governmental controls, or when competition is virtually suppressed by private monopoly? Did they disappear in Fascist Italy, Nazi Germany, Soviet Russia? Even in Great Britain and the United States, have not business cycles undergone progressive changes?

The statement that cyclical expansions occur at about the same time in 'many economic activities' is vague. In some contexts 'economic activities' mean specific acts performed by individuals, while in others the same words signify a few broad categories of actions performed continuously by millions of people, as when an economist speaks of the production, exchange and distribution of wealth. Clearly, our definition requires that we go back of broad aggregates or index numbers of economic activity, such as indexes of production and prices, or national aggregates of income, employment, bank clearings, and imports. But how far is it wise to break down the aggregates and examine their constituent parts? If broad aggregates hide differences among their constituents that are significant for understanding business cycles, may not economic activities be atomized to the point where cyclical movements are obscured by the idiosyncrasies of small units?

Our definition presents business cycles as a consensus among expansions in 'many' economic activities, followed by 'similarly general' recessions, contractions, and revivals. How 'general' these movements are, what types of activity share in them and what do not, how the consensus differs from one cyclical phase to another, and from one business cycle to the next, can be learned only by empirical observation. And that is more than a mere matter of counting the series that rise and that fall during a given phase. A rise in bankruptcies when most activities are expanding would be a movement counter to the tide; a fall is what we expect, usually find, and interpret as sharing in the consensus of movements. Copper prices move with the general tide if expressed in cents per pound; they move counter to the tide if expressed as pounds per dollar. Thus not only the character of the activity represented but also the form in which the record is kept determines whether a series is related positively or invertedly to business cycles. Hence, it is necessary to know precisely what each series represents and its bearing on economic activity as a whole.

The statement that similar cyclical movements occur 'at about the same time' in many activities admits the possibility of 'leads' and 'lags'. But the implication is that the cyclical turns of different processes are concentrated around certain points in time; for if there were no bunching of cyclical turns, there would be no business cycles answering to our definition. We can imagine a world in which every economic activity is subject to cyclical fluctuations, but in which the divergences in the timing of these cycles produce an unchanging total. The need for ascertaining leads and lags is thus clear from the start. And the significance of this inquiry grows as the results multiply and demonstrate characteristic differences in the timing of different types of activity.

The sequence of expansion, recession, contraction, and revival is said to be 'recurrent but not periodic'. To determine the respects in which and the regularity with which the sequence recurs, it is necessary to identify and compare successive business cycles. But will our definition enable us to identify business cycles among the other movements—seasonal variations, random changes, and secular trends—with which they are interwoven? And is there only one set of cyclical fluctuations in general business activity? May not the business cycles we identify be the net resultants of several sets of general cycles running concurrently, each set perhaps periodic but combining with cycles of other periods to produce the variability our definition admits?

Another tacit implication of the definition is that business cycles run a continuous round; for the definition says that expansion is followed by recession, recession by contraction, contraction by revival, and revival by a fresh expansion. No intervals are admitted between one phase and its successor, or between the end of one cycle and the beginning of the next. Yet current reports sometimes speak month after month of a confused state of trade, and statistics often indicate that some industries or localities are doing well while others of equal importance are dull or depressed. Granted that our concept of business cycles fits the facts much of the time, can we claim that it fits them all of the time? To answer with assurance, it is again necessary to investigate the behavior of many activities and determine whether the alleged consensus among their cyclical movements is continuous or intermittent.

The definition gives 'more than one year' as the lower limit of duration of business cycles, and 'ten or twelve years' as the upper limit. These figures rest upon an earlier attempt to identify the cycles revealed by a considerable collection of business annals and a smaller collection of business indexes.[2] Neither set of materials is thoroughly satisfactory. We can hardly suppose that current commentators are infallible summarizers, or that the compilers of business annals are infallible digestors. Nor can we be confident that statistical indexes represent correctly the cyclical movements in the general condition of business.[3] Hence we cannot rely exclusively upon annals and business indexes to mark off business cycles and measure their durations. Even for that seemingly simple task, it is necessary to compare individual time series covering many activities, though the data become scantier and we know the results will be less assured the further back we go.

The last clause of the definition brings up a related problem. Our examination of business indexes, and less definitely of business annals, forbade us to think of business cycles "as sweeping smoothly upward from depressions to a single peak of prosperity and then declining steadily to a new trough".[4] On the contrary, the expansion and contraction of many cycles seem to be interrupted by movements in the opposite direction, and some cycles apparently have double or triple peaks or troughs. When the irregularities are slight they do not seriously complicate the task of identifying business cycles; but in some instances, notably in this country since

2 *Ibid.*, pp. 339-43, 391-407.
3 *Ibid.*, pp. 365-75.
4 *Ibid.*, p. 329 ff.

1930, they attain considerable proportions. Hence the need of criteria for deciding what reversals in direction mark the end of a cyclical phase. Most brief movements are excluded by the clause that business cycles cover 'more than one year'. By adding that they 'are not divisible into shorter cycles of similar character with amplitudes approximating their own', a rule is laid down for deciding when to treat movements lasting several years as a single cycle and when to recognize two or more cycles.[5] But this rule cannot be applied without knowing at least approximately what amplitudes are characteristic of business cycles.

III 'Inductive Verification' of Business-cycle Hypotheses

The need to answer the difficult questions suggested by our definition is not peculiar to the plan of research we have chosen. Any investigator who sets out to explain the business cycles of the actual world should know what he is explaining; that is, at some stage of his work, he should identify the cycles of historical record and study their characteristics.

Systematic factual research is often thought of as belonging to the stage of 'inductive verification', and 'inductive verification' as a step to be taken *after* a 'theory' has been excogitated. Of course no writer has ever attempted to devise a hypothesis concerning business cycles entirely apart from the facts to be explained. But theorists have not infrequently been handicapped by a sadly incomplete, sometimes by a badly twisted, knowledge of the facts. Numerous writers have invented plausible explanations of business cycles before they have tried to ascertain what consensus actually prevails among the cyclical movements of different economic activities: which activities have a positive, which an inverted, relation to the supposed consensus, and which follow a path of their own; the timing relations among the movements and the relative amplitudes they attain; how considerable are the checks encountered and overcome by business-cycle expansions and contractions; how long these phases last; what changes in business cycles have accompanied or followed secular and structural changes in economic organization; how the cycles in different countries compare with one another in timing, duration, and amplitude—in short, without knowing definitely the actual behavior for which their explanations should account.

When this order of inquiry is followed—explanation preceding thorough knowledge of what is to be explained—the results are likely to be unhappy. (1) The theorist often stops before his work is finished, leaving 'inductive verification' to others, who may or may not take on the job. (2) When anyone tries to 'verify' a hypothesis about business cycles, he often finds that it rests on assumptions purposely chosen to simplify

[5] This rule is necessary as a 'brake' on an investigator's pattern sense which, while the source of all true knowledge, may lead to mischievous fictions.

the situation that is analyzed. In that case evidence drawn from the actual world has a problematical relation to the simpler world of the theorist's imagination; the hypothesis propounded may be logically impeccable, but it cannot be confirmed or refuted by an appeal to facts. (3) Granted that the hypothesis concerns actual experience, the worker who tries to verify it must examine the processes on which it centers attention; but unless he examines other processes as well, the test will be superficial. As pointed out in another volume:

> Recent writers upon business cycles differ . . . less in principle than in emphasis . . . Each gives chief attention to the one or more factors which he believes to play the chief causal role; but many writers also show how the changes produced by their chosen causes affect other processes, and in so doing they are likely to find use for the work of men whose distribution of emphasis differs from their own.
>
> Among the factors to which the leading role in causing business cycles has been assigned by competent inquirers . . . are the weather, the uncertainty which beclouds all plans that stretch into the future, the emotional aberrations to which business decisions are subject, the innovations characteristic of modern society, the 'progressive' character of our age, the magnitude of savings, the construction of industrial equipment, 'generalized over-production', the operations of banks, the flow of money incomes, and the conduct of business for profits. Each of these explanations merits attention from those who seek to understand business cycles; for each should throw light upon some feature or aspect of these complex phenomena.[6]

Hence, an investigator who seeks earnestly to discover the cause or causes of business cycles should not restrict himself to testing any single hypothesis. If he concludes that the facts of experience are consistent with one hypothesis, he should make sure that they are not equally consistent with other hypotheses. In the measure that he is thorough, his effort will broaden into an attempt to test many hypotheses and determine how they fit together.

Anyone who embarks upon such a venture will presently encounter all the difficulties that confront this investigation. He must satisfy himself whether there really are cyclical fluctuations in general business, and if so what are their characteristics. For that purpose he must study the cyclical behavior of many activities, determine which do and which do not fluctuate in unison, what are the timing relations among their expansions and contractions, what amplitudes these fluctuations attain; in brief, he must identify business cycles, and in the process answer as best he can the hard questions listed in the preceding section.

This work cannot be organized in the most effective way by taking up one hypothesis after another for 'verification'. "The plan of testing theories would indeed lead to work with the facts, but in an artificial order, and one involving much repetition." [7] The investigator can save

[6] *Ibid.*, pp. 11, 12.
[7] *Ibid.*, p. 58.

time, all the more important because the undertaking is so huge, by concentrating upon a systematic examination of the cyclical movements in different economic activities, classified in whatever fashion seems best suited to his purpose. In making this examination, he will not put the 'theories' aside; on the contrary he will use them continuously as hypotheses concerning what activities and what relations among them are worth studying. In that way they will be of inestimable value in his factual inquiries. Also, his detailed studies are likely to suggest new hypotheses from time to time, or modifications of old ones, and they too will direct his researches. But he will not think himself equipped to judge what contribution any hypothesis makes to the understanding of business cycles until he has attained as clear a view as he can of the whole congeries of interrelated movements. If this attitude of suspending judgment taxes patience at times, the investigator can comfort himself with the belief that, so far as he succeeds in showing what cyclical behavior is characteristic of economic activities, he will put others as well as himself in a better position to evaluate hypotheses.

IV The Data Needed for Observing Cyclical Behavior

The longest records of cyclical fluctuations in economic activities are the contemporary opinions of journalists. They show that men whose business it was to report the condition of trade were impressed by the alternations of prosperity and depression long before the concept of business cycles had been formulated. They indicate what years were deemed good and what years bad by contemporaries, and thus are helpful in identifying successive business cycles, and in making rough measures of their duration. Further, they often call attention to the branches of trade that prospered notably or suffered in exceptional degree at particular periods. But the business annals we now have, specifically the compilation made by Willard L. Thorp and published by the National Bureau in 1926, do not provide detailed and continuous observations upon the changing fortunes of many branches of trade.[8]

Better suited to our purposes are time series that record the fluctuations of specific processes or transactions from month to month, quarter to quarter, or year to year, though, as will be demonstrated in Chapter 6, annual data leave much to be desired. These materials must be sufficiently abundant to allow systematic comparisons of the behavior of different activities in the same business cycle, and of the same activity in different

[8] Of course, fuller and probably more representative annals can be compiled by ransacking the sources, and much can be learned about business cycles by work of this sort. But no compilation can transcend the limitations of its data. We believe that at the present stage of research more can be accomplished by analyzing time series than by elaborating annals: though the latter effort promises to contribute heavily to knowledge of those periods and countries for which the statistical record is scanty.

cycles. Obviously, no single time series can reveal business cycles as we have defined them. At best a series reveals only the cyclical changes in one activity or group of activities, such as mining coal, building houses, hauling freight, paying wages, trading in securities, clearing checks. Nor do index numbers—whether of prices, production, employment, or what not—represent more than the average changes of specific factors in business. Even the audacious statistician who constructs what he calls an 'index of business conditions', perhaps basing it upon indexes of production, prices, sales, employment, and financial operations, is not charting the course of business cycles. Apart from the limitations of coverage, his composite shows net resultants, not similar movements in many activities.[9] A business index may establish a presumption that the activities it represents fluctuate in unison, but the presumption must be tested before it is accepted, and an adequate test entails examining the cyclical behavior of many series. To repeat: only by analyzing numerous time series, each of restricted significance, can business cycles be made to reveal themselves definitely enough to permit close observation. If we wish to know what the wholes are like, we must study the parts and then see what sort of wholes they make up.

V Requirements that Technique Must Meet

To determine the cyclical behavior characteristic of different economic activities, we should have a method that yields comparable results when applied to a wide variety of time series. If possible, the results should be in quantitative form, that is, we should *measure* the cyclical behavior of economic activities; otherwise we cannot say definitely what uniformities and what differences appear among the movements.

The questions raised in Section II suggest that we must ascertain, first of all, what economic activities reveal recurrent sequences of expansion, recession, contraction, and revival, lasting more than one year but not more than ten or twelve years. We call such cyclical movements in a time series its 'specific cycles'. According to our methods of observation, specific cycles appear in a preponderant majority of the time series in our collection. But there are some exceptions, and it is as much a part of our task to learn which economic activities are virtually immune to cyclical contagion as to learn which are sensitive to it. Without such knowledge we cannot judge how general is the alleged consensus among cyclical movements.

Next, we need to know how the specific cycles of different activities are related to one another in direction of movement, in the timing of their peaks and troughs, and in the duration of their expansions and contractions. Knowledge of the existence or nonexistence of specific cycles in

9 See *ibid.*, pp. 307-26, for a critique of business indexes.

many economic activities, and of their agreement or nonagreement in direction of movement, timing, and duration are obvious prerequisites for determining empirically whether there are business cycles answering to our definition. Finally, we must have measures of the amplitude of the cyclical movements of individual activities and of their rate of change during cyclical expansions and contractions. These measures together with those showing the sequence in which different activities turn up at business-cycle revivals and turn down at business-cycle recessions are essential in tracing causal relations.

In order to observe these several features of specific cycles closely, the original data of time series must be subjected to several operations. Before the specific cycles of a series are identified, it is desirable to remove the seasonal variations. To compare the amplitudes and patterns of specific cycles in series that run in different physical units, different sums of money, or in the form of ratios, some common denominator is necessary. The simplest plan is to express the original data as percentages of their average value during a specific cycle, and determine in terms of these percentages the rise from trough to peak, the fall from peak to trough, and the change from one stage to another into which the phases of expansion and contraction may be broken. That plan has the further advantage of eliminating in step-wise fashion the secular trend of a series.

More elaborate preparations are required to measure cyclical timing. If we attempted to compare the turning dates of the specific cycles in all the series analyzed, millions of comparisons would be required; for our investigation, though far from complete, already includes over eight hundred monthly and quarterly series for the United States alone, a few of which cover more than twenty specific cycles. Our solution of this difficulty is to draw up a table of 'reference dates' that purport to mark off the troughs and peaks of successive business cycles, and to measure the leads or lags of specific-cycle troughs and peaks from these benchmarks. This step is the crux of the investigation; it involves passing from the specific cycles of individual time series, which readers not embarrassed by experience are likely to think of as objective 'facts', to business cycles, which can be seen through a cloud of witnesses only by the eye of the mind. We prepared for the transition by modeling the definition of specific cycles upon that of business cycles, as the reader may have noticed. Granted that the time series representing many economic activities show recurrent sequences of expansion, recession, contraction, and revival, lasting more than one year but not more than ten or twelve years, we should be able to determine whether there is a consensus among these movements. If there is, the dates of specific-cycle troughs of individual activities must be concentrated around certain points of time, and the like must be true of specific-cycle peaks. We can then proceed to identify business cycles in the country from which the time series come, assign approximate dates to their troughs and peaks, and plunge into a study of the behavior of different economic activities within the periods thus marked off.

The logic of this procedure may seem to imply that the specific cycles in all series we intend to analyze for a country should be identified before attempting to identify its business cycles. That is a counsel of perfection on which we could act only if we knew at an early stage of the investigation many things learned in the course of the work. We did not know at the start just what time series should be analyzed among those readily available, or what gaps should be filled by new compilations. Nor did we know what importance to attach to different time series as indicators of cyclical turns in general business activity. Only as we discovered the relations of many activities to the general consensus among cyclical movements could we form intelligent judgments upon these matters. This process of 'learning on the job' would have been inefficient if we had confined analysis to specific cycles for a long while, and postponed trying to see how they fit together into business cycles.

At an early stage of the investigation we thought it prudent to compare the specific cycles in numerous series.[10] Rough tabulations of specific-cycle turns suggested that they clustered around certain months, which usually came in years when business annals reported a recession or revival. These results were reassuring, but we wished to test them systematically. The best way was to settle on an experimental set of 'reference dates' and see how they met expectations, when applied in the analysis of new series. By a process of trial and error, we were able to work out several years ago a set of dates that fitted fairly well both the annals and what we then knew about specific-cycle turning dates. These reference dates have been utilized in analyzing additional series and thus subjected to further tests. And this process of refinement must continue: the reference dates, like our definition of business cycles, are a tool of research subject to revision as more is learned about the phenomena they help us investigate.

But once a set of fairly well tested reference dates is obtained for a country, we can show in detail how different activities behave during business cycles: how the cyclical turns of different series are related to one another, and how their movements compare in magnitude and direction from stage to stage of business cycles. For the latter purpose, every time series is broken into segments corresponding to the periods occupied by the business cycles of the country to which the series refers, and the seasonally adjusted data for each segment are turned into percentages of their average value. This step enables us to measure in a common unit the rise or fall of different economic activities from stage to stage of business-cycle expansions and contractions.

For an economic historian concerned with what happened in a par-

10 The methods used to identify business cycles and date their troughs and peaks are described in Ch. 4.

ticular period these measures of the behavior of time series during individual specific or business cycles may be sufficient. An economic theorist, however, wishes to know primarily what features have appeared in all or in most cycles, though he should be interested also in the variability of the phenomena, which is one of their most striking characteristics. The thousands of people who nowadays anxiously follow the course of business cycles have practical needs that combine those of the historian and theorist. They are as deeply immersed in what happens at a particular period as any historian; but in trying to foresee what will happen in the near future they require knowledge of the type a theorist strives for. Thus our historical studies of individual business cycles should be supplemented by efforts to learn whatever we can concerning the uniformities and diversities among them.[11] This final requirement laid upon our technique of observation calls for averaging the measures of cyclical behavior during successive specific cycles and during contemporaneous business cycles. Of course, we must examine also the way in which measures of individual cycles are distributed about their means.

VI The Symbols Used in Observing Business Cycles

When we speak of 'observing' business cycles we use figurative language. For, like other concepts, business cycles can be seen only 'in the mind's eye'. What we literally observe is not a congeries of economic activities rising and falling in unison, but changes in readings taken from many recording instruments of varying reliability. These readings have to be decomposed for our purposes; then one set of components must be put together in a new fashion. The whole procedure seems far removed from what actually happens in the world where men strive for their livings. Whether its results will be worth having is not assured in advance; that can be determined only by pragmatic tests after the results have been attained.

This predicament is common to all observational sciences that have passed the stage of infancy. An example familiar to everyone is meteorology. The layman observes the weather directly through his senses. He sees blue sky, clouds, snow, and lightning; he hears thunder; he feels wind, temperature and humidity; at times he tastes a fog and smells a breeze; he sees, hears, and feels storms. The meteorologist can make these direct observations as well as a layman; but instead of relying upon his sense impressions he uses a battery of recording instruments—thermographs, barographs, anemometers, wind vanes, psychrometers, hygrographs, precipitation gauges, sunshine recorders, and so on. That is, he transforms much that he can sense, and some things he cannot sense, into numerous sets of symbols stripped of all the vivid qualities of personal experience.

[11] Cf. *ibid.*, p. 469.

It is with these symbols from his own station and with similar symbols sent to him by other observers dotted over continents and oceans that he works. They show the weather broken down into numerous factors, which he must put together again in his mind. To that end he plots certain of his symbols on a weather map, which he compares with maps drawn a few hours earlier. From these maps, a new set of symbols, he derives conclusions about air masses of different types and about the weather likely to be produced by their movements and internal changes. By these highly artificial operations he arrives at forecasts concerning actual conditions over wide areas, that will soon be judged right or wrong by millions who sense their local weather.

All of us can observe economic activities as easily and directly as we can observe the weather, for we have merely to watch ourselves and our associates work and spend. What we see in this way has a wealth of meanings no symbols can convey. We know more or less intimately the hopes and anxieties, efforts and fatigues, successes and failures of ourselves and a few associates. But we realize also that what happens to us and our narrow circle is determined largely by what is being done by millions of unidentified strangers. What these unknowns are doing is important to us, but we cannot observe it directly.

A man tending an open-hearth furnace has a close-up view of steel production. But what he sees, hears, smells and feels is only a tiny segment of a vast process. He works at one furnace; he cannot see the hundreds of other furnaces in operation over the country. And smelting is only one stage in a process that includes mining and transporting iron ore, limestone, coal, and alloys; the getting of orders for steel, the erection of plants, and the raising of capital; importing and exporting, hiring and training workers, making and selling goods that give rise to a demand for steel, setting prices, and keeping accounts of outgo and income. No man can watch personally all these activities. Yet those engaged in them and in the activities dependent on the steel industry need an over-all view of what is happening. To get it they, like meteorologists, resort to the use of symbols that bear no semblance to actual processes and that are compiled mainly by other men.

For the intermittent process of making steel in a furnace with its heat and noise, its dim shadows and blinding glares, they substitute a column of figures purporting to show how many tons of steel ingots have been turned out by all the furnaces in a given area during successive days or weeks. That colorless record gives no faintest idea of what the operation looks like or feels like; it does not tell whether the work is hard or easy, well or ill paid, profitable or done at a loss. It suggests continuous operation, which is achieved at no furnace. It hides differences of location and types of product. And it separates the one act of turning out tonnage from all the other activities with which it is interwoven. Many, though not all, of these interrelated changes are likewise recorded in columns of figures; but each record is as devoid of reality and as divorced from its matrix as the record of tons produced.

It is with such symbolic records that a 'realistic' investigator who wishes to find out what happens during a business cycle must work. Not all the activities he wishes to study are recorded; some of the most interesting figures are not published; of the published figures many are defective in one way or another. Less obvious are the difficulties of combining what one learns from time series of very limited, very comprehensive, or intermediate coverage. The most detailed series keep closest to individual experience; but they are likely to be so dominated by circumstances peculiar to single enterprises, groups, or localities that common behavior traits are hard to descry. These difficulties can be reduced to a minimum, and much labor saved, by confining study to broad aggregates or indexes. But then the field of vision becomes as dim as it is wide. Highly significant differences in cyclical timing and amplitude may be hidden from sight.[12] Nor can one tell whether the movements in these comprehensive series are net resultants of concomitant or of divergent fluctuations in individual activities. Records of intermediate coverage are less affected by 'disturbing circumstances' than the most detailed and reveal more of individual experience than the most inclusive series; but they have the defects of these qualities, being more remote from individual experience than the first and more affected by disturbing circumstances than the second. If all activities were recorded in equal detail, an investigator might use series belonging to each of these coverage groups, or a larger number than our rough classification recognizes, just because they give him different pictures of the process he wants to understand. In practice the available data differ considerably from one activity to another, so that he cannot maintain uniform standards. On the contrary, he must often compare the cyclical behavior of activities represented by symbols that relate to details in one case and broad aggregates in another. Then he must mix a large element of personal judgment into his comparisons.

In trying to compose a picture of business cycles from these diverse materials, the would-be objective student is forced to devise further artifices. He must take apart every record he uses in order to separate as best he can the cyclical fluctuations from movements of other kinds. In so doing he must solve the technical problems of 'time-series analysis' in some fashion, and the solutions he chooses will shape the new set of symbols he derives from the symbols that constitute his 'raw data'. And a still higher pitch of abstraction is reached if the investigator seeks, as we do,

12 When the differences in timing are considerable, the amplitude of the cyclical fluctuations of an aggregate differs widely from the average cyclical amplitude of its components. See Wesley C. Mitchell and Arthur F. Burns, Production during the American Business Cycle of 1927–1933 (National Bureau of Economic Research, *Bulletin 61*, Nov. 9, 1936), Sec. IV.

to get a picture of the cyclical behavior characteristic of each activity by averaging measures covering as many cycles as his time series include.

Nor is that the end of the story. We conceive business cycles to consist of roughly synchronous movements in many activities. To determine whether this thought symbol represents experience or fantasy, our measures of the cyclical behavior characteristic of many activities must be assembled into the end products of which our definition is the blueprint. In statistical jargon, time-series analysis must be followed by a time-series synthesis. Most of this assembling job is reserved for a future volume. But if the measures described in this volume are later to be fitted together, they must be designed with that use in view. This requirement explains our effort to develop a method of analysis that can be applied in uniform fashion to a wide variety of time series. More than that, it explains one step toward the ultimate synthesis taken in this volume, namely, the fixing of 'reference dates' that are meant to show when successive business cycles reached their peaks and troughs. Having fixed these dates as best we now can, we derive measures of characteristic behavior during business cycles that will serve as the basic symbols for our later synthesis.

Thus the concept of business cycles ties together in our minds, and gives meaning to, a host of experiences undergone by millions of men, few of whom think of themselves as influenced by cyclical pressures and opportunities. The concept, as we develop it, is itself a symbol compounded of less comprehensive symbols representing the cyclical behavior characteristic of many unlike activities. In turn, these symbols are derived by extensive technical operations from symbolic records kept for practical ends, or combinations of such records. We are, in truth, transmuting actual experience in the workaday world into something new and strange, much as a meteorologist transforms our experience of sunshine into new and strange symbols that record solar radiation.

The hazards of our undertaking are many. What makes the venture worth trying is that the symbols constituting our basic data have for the most part been made for practical ends and found useful; also our notions about the ways in which different activities fit into one another are borrowed at first or second hand from experience; our analytic methods when applied to many time series yield results that broadly confirm one another, and these time series cover a goodly part of the activities we should like to include. With such materials, a staff of workers should be able at least to attain a better approximation to knowledge of what happens during a business cycle than has been available hitherto.

VII Range Covered by the Observations

The range of activities whose cyclical behavior can be measured in the manner sketched in Section V, the number of countries that can be covered, and the periods over which the observations can be extended are limited by the time series that have been or might be compiled. But much narrower limits have been set to this investigation by the funds available to us and by our capacity to integrate diverse inquiries.

The subjects we have sought to cover so far include the production of commodities, construction work, transportation and communication, prices at wholesale and retail, sales by merchants and manufacturers, the stocks of goods held by various hands, foreign trade, hours of work, wage rates and employment, the disbursement of money incomes to individuals, the profits or losses of business enterprises, the formation of new businesses, savings and investments, dealings in securities, interest rates, currency, banking, and the aggregate volume of business transactions. We hope to expand this list by analyzing also changes in the balance-sheet items of business enterprises, public finance, and the social concomitants of business cycles.[13] All these processes are stressed in the theoretical literature of business cycles; but our actual selection of individual series has been determined as much by the puzzles that turned up in the course of work with the data as by the suggestions gleaned from theoretical writings.

The adequacy of the statistical records, and therefore of our observations upon cyclical behavior, differs widely from subject to subject. For example, statistics of inventories, sales by manufacturers, savings, and retail trade, are meager. On the other hand, price quotations at wholesale, and data upon exports and imports of individual commodities are so abundant that we cannot afford to analyze all the available series—which does not mean that we can obtain just the data we should like best to have on these subjects.

To investigate adequately how the business cycles of different nations are related to one another,[14] it would be necessary to observe perhaps twenty nations. That we cannot do. But neither can we confine analysis to a single nation. A man interested solely in the business cycles of the United States could not understand them by studying American data alone; for they would not show the changes in foreign business conditions that stimulated or retarded American expansions, and mitigated or aggravated the contractions. Our compromise between what we should like and what we are able to do is to observe business cycles in the four large nations that led in commercial, industrial, and financial developments during the nineteenth century—Great Britain, France, Germany, and the United States. These nations have had close business relations with one another, possess good statistical records as such things go, and present a

[13] Matters such as birth and death rates, marriage and divorce rates, poor relief, unemployment benefits, school attendance, and crime.

[14] Willard L. Thorp's *Business Annals* (National Bureau of Economic Research, 1926) suggests that contemporaneous cycles in nations trading freely with one another have much in common. For a summary of the evidence, see Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 424-50.

variety of economic conditions. In dealing with some activities we may be forced to extend the geographical range of observation.[15] But for the most part we shall have to rest content with a four-country sample, knowing full well that it does not represent adequately the cyclical tides that sweep round the world.

Meanwhile it is important to observe carefully the business cycles of single nations. The domestic idiosyncrasies of these cycles are not less significant than their international similarities. Most time series upon which we are dependent refer to conditions in a single nation or one of its subdivisions: these are the ultimate sources from which we must build up knowledge even of international business cycles. The latter bear much the same relation to the cycles of different nations as the business cycles of a single nation bear to the specific cycles of its different economic activities. As we must get knowledge of domestic business cycles by studying a complex of interrelated movements in many industries and markets, so we must get knowledge of world cycles by studying the interrelated expansions, recessions, contractions, and revivals in many nations.

Regrettably, our analysis cannot be pushed back far enough to answer the questions raised by our definition about the beginnings of business cycles. The periods we can cover in practice are determined mainly by a factor beyond our control—the availability of continuous statistical data. Relatively few time series in monthly form begin before the 1870's. More than half of our collection dates from 1900 or later, and a considerable fraction is confined to years since World War I. Our general rule is to cover as long a period as the data allow. In one series our results may therefore sum up two or three generations of experience, in another they may represent barely a decade. The problem of combining materials with such varied time reference may become acute at a later stage.[16] At present, in trying to ascertain what is characteristic of the cyclical behavior of economic processes taken singly or in small groups, it would be profligate to discard half or more of the evidence concerning one factor merely because we cannot get evidence for the same years about other factors.

Tables 1-3 show the range of time series we had analyzed by July 1, 1942. Of the 1,277 series,[17] 76 per cent relate to the United States, 11 per cent to Great Britain, 7 per cent to Germany, and 6 per cent to France. Over 40 per cent of the series cover production, construction work, and transportation. These and the holding of commodity stocks are the activities we have studied most thoroughly so far, and upon which monographs

15 One colleague who is studying the cyclical behavior of foreign trade has been driven to construct series representing the imports and exports of 'Outlandia', that is, a combination of countries outside the four covered systematically by our compilations.

16 See, however, the tests in Ch. 10-12, which are reassuring as far as they go.

17 This count is confined to series subjected to the full analysis described in subsequent chapters. It omits a large number of series that have been only partially processed or used for special purposes without processing.

TABLE 1

Series Classified According to the Process Represented, Country, and the Time Unit

(Series analyzed by July 1, 1942)

Group	United States	Great Britain	Germany	France	Four countries
PROCESS					
Production	214	23	25	22	284
Construction	133	21	15	15	184
Transportation	53	11	5	8	77
Commodity prices	144	20	10	7	181
Inventories	67	1	..	..	68
Merchandising	41	..	..	..	41
Foreign trade	19	13	4	8	44
Personal incomes	115	12	5	2	134
Profits and losses	40	1	1	..	42
Savings and investments	17	6	3	2	28
Security markets	11	9	1	3	24
Interest rates	26	6	5	3	40
Money and banking	75	14	7	6	102
Aggregate transactions	17	4	3	4	28
Total	972	141	84	80	1,277
TIME UNIT					
Monthly	727	92	52	53	924
Quarterly	84[a]	19	3	1	107
Annual	161	30	29	26	246
Total	972	141	84	80	1,277

[a] Includes 57 series relating to the status of national banks through 1914, at 5 irregularly spaced 'call dates' within the year.

TABLE 2

Periods Covered by American and Foreign Series

(Series analyzed by July 1, 1942)

No. of years covered[a]	United States			Three European countries			Four countries		
	Monthly & quarterly	Annual	All series	Monthly & quarterly	Annual	All series	Monthly & quarterly	Annual	All series
Under 10	13	..	13	1	..	1	14	..	14
10 – 19	338	24	362	40	1	41	378	25	403
20 – 29	204	21	225	15	3	18	219	24	243
30 – 39	69	16	85	55	7	62	124	23	147
40 – 49	98	33	131	38	14	52	136	47	183
50 – 59	43	20	63	39	25	64	82	45	127
60 – 69	16	10	26	8	8	16	24	18	42
70 – 79	19	24	43	10	8	18	29	32	61
80 – 89	10	4	14	13	8	21	23	12	35
90 – 99	1	4	5	..	6	6	1	10	11
100 – 109	..	4	4	..	2	2	..	6	6
110 – 119	..	..	..	1	..	1	1	..	1
120 – 129	..	..	..	..	..	..	..	..	..
130 – 139	..	..	..	..	..	..	..	..	..
140 – 149	..	1	1	..	3	3	..	4	4
Total	811	161	972	220	85	305	1,031	246	1,277

This table shows roughly the periods covered by our analyses, many of which stopped in 1932 or 1933 at the time the table was prepared. Since then another cycle has been added in a majority of the series.

[a] Determined from the years in which our cyclical analysis starts and ends, irrespective of the month or quarter. Cf. Table 17.

TABLE 3
Number of Business Cycles Covered by American and Foreign Series
(Series analyzed by July 1, 1942)

No. of business cycles covered	United States			Three European countries			Four countries		
	Monthly & quarterly	Annual	All series	Monthly & quarterly	Annual	All series	Monthly & quarterly	Annual	All series
1	7	..	7	2	..	2	9	..	9
2	15	..	15	3	..	3	18	..	18
3	100	1	101	22	2	24	122	3	125
4	235	30	265	18	2	20	253	32	285
5	121	6	127	10	3	13	131	9	140
Under 5	357	31	388	45	4	49	402	35	437
5 – 9	262	22	284	111	35	146	373	57	430
10 – 14	134	48	182	46	31	77	180	79	259
15 – 19	49	48	97	17	12	29	66	60	126
20 – 24	9	7	16	..	..	..	9	7	16
25 – 29	..	5	5	1	3	4	1	8	9
Total...	811	161	972	220	85	305	1,031	246	1,277

See note to Table 2.

are nearest completion. As any part of the field is worked over intensively, we find that the series selected in advance should and can be supplemented by additional analyses. Doubtless many more series will eventually be used in the section on prices and in the sections from merchandising to money and banking than is indicated by Table 1; some will be added even to the first three sections.

The analysis rests as far as possible on monthly records which constitute 72 per cent of the series analyzed; quarterly records contribute 8 per cent, and annual series 19 per cent. Annual data as a rule run back much farther than monthly: over half of our annual series cover 50 years or more, a bare 16 per cent of the monthly and quarterly series are so long. Of the monthly and quarterly series 39 per cent include fewer than 5 business cycles, 36 per cent include 5-9 cycles, 25 per cent include 10 or more cycles. The corresponding figures for the annual series are 14, 23, and 63 per cent. The series including 15 or more cycles make 7 per cent of the monthly or quarterly and 30 per cent of the annual group.

VIII The Program as a Whole

This volume, the second of the National Bureau's *Studies in Business Cycles*,[18] sets forth in detail the methods of measuring cyclical behavior that we are using; presents various tests of the methods; discusses their limitations for the purpose they are meant to serve; and analyzes four types of changes that may occur in cyclical behavior and compromise our averages.

[18] The first volume is *Business Cycles: The Problem and Its Setting*, by Wesley C. Mitchell.

The monographs to follow will give the results obtained by applying our technique to time series representing various groups of economic activities. Their aim is to summarize the statistical results, and to explain, so far as possible, the differences found to be characteristic of the cyclical behavior of different activities, the role these activities play in the domestic economy of a country, and their bearing upon international business relations. Since these tasks require much technical knowledge of industries and markets, each monograph is entrusted to a specialist. The dozen monographs thus far initiated include the cyclical behavior of agriculture, mining and manufacturing production, construction work, transportation and communication, inventories, prices at wholesale and retail, wages and employment, consumer income and expenditures, formation of new business firms, money and banking, foreign commerce, and international financial relations. We hope to add, as promptly as conditions permit, monographs on public finances, incomes and expenditures of business firms, security markets, and savings and investment, and to embark on a series of historical studies of business cycles. None of the monographs will attempt to present a general theory of business cycles. If our conception of the problem is sound, that task cannot be performed satisfactorily until the cyclical behavior of all the activities we are attempting to study has been measured and the salient differences in behavior have been examined with care.

Our original plan called for a final volume that would weave the results established by the monographs together with existing knowledge into a theoretical account of how business cycles run their course. The outbreak of war and the need to anticipate post-war adjustments have led us to modify this leisurely program. Several of our collaborators are giving all or part of their time to the government, and it will not do to postpone the theoretical analysis of business cycles until all their investigations can be completed. In view of the pressure of the times, we think it advisable to make available as soon as possible the best summary that can now be framed of what typically occurs in the course of a business cycle. This 'preview' will be published shortly under the title *What Happens during Business Cycles: A Progress Report,* by Wesley C. Mitchell.

When better days return, we hope to expand and to revise the 'preview'. The final volume will attempt to fulfill the many promises made in different places in this monograph. It will draw heavily upon the studies of collaborating specialists, and carry through some fresh investigations. We hope that other students will find the *Studies in Business Cycles* as useful in their theoretical constructions as the National Bureau expects to find them.

CHAPTER 2

Preliminary Sketch of the Statistical Analysis

OUR PLAN of measuring the numerous features of cyclical behavior is so much of one piece that some understanding of the whole is needed for full comprehension of the parts. Hence we preface the detailed exposition of the method by a preliminary sketch designed to bring out its general characteristics. Even the most careful reader will save time by viewing the plan as a whole before plunging into the detailed explanations of its parts.

I Basic Features of the Analysis

The scheme of analysis has two parts. Primarily we seek to determine how each important economic process of which we possess a statistical record behaves during the periods occupied by business cycles. For that purpose we break every time series into segments that coincide with the cycles in general business activity in the country to which the series refers. By analyzing on this basis over one thousand time series representing a wide variety of economic processes in four countries, we are able to get a fairly comprehensive picture of what happens during a business cycle.

While our interest centers in this picture, it cannot stop there. Almost all our series exhibit cyclical fluctuations, but these fluctuations bear widely different relationships in time to business cycles. The dates on which a series makes its cyclical turns may differ little from the turning dates in general business, may usually lead or lag behind them by brief or by considerable intervals, or may have no regular relationship to them. Now the course of business cycles is influenced by cyclical fluctuations in every economic process, whether or not they synchronize with the general tides of activity. Hence, to lay a satisfactory foundation for a theory of business cycles, our analysis based upon business-cycle periods must be supplemented by studies of the cycles peculiar to each series.

The following sections explain and illustrate most of the measures we make, define the technical terms we use, and give a sample of the standard charts upon which we rely heavily in presenting results.

II Reference Dates, Reference Cycles and Specific Cycles

To learn how different economic processes behave in respect of business cycles, their movements must be observed during the revivals, expansions, recessions, and contractions in general business activity. Before we can begin observing we must mark off these periods. To that end we have made for each of the four countries a table of 'reference dates', showing the months and years when business cycles reached troughs and peaks. These tables were based first upon the business annals compiled for the National Bureau by Willard L. Thorp; then we refined, tested, and at need amended the dates by studying statistical series. The turning points of the cyclical movements in general business activity can be made more precise as the field covered by economic statistics expands. Hence we have more confidence in the reference dates for the United States than in those for the three European countries, and more confidence in the later than in the earlier dates.

After eliminating the seasonal variations of a series, we break it into segments marked off by the reference troughs of the country to which the series relates. Since each segment spans an interval between successive reference troughs, we call it a 'reference-cycle segment', or 'reference cycle' for brevity.[1] Next we compute the average of the monthly values during each 'reference cycle' and convert the data into percentages of this base; these percentages are called 'reference-cycle relatives'. The application of a uniform set of dates to all series for a given country, and the reduction of the original data expressed in diverse units to relatives of their average values during the periods thus marked off, put all the materials into comparable form and enable us to see how different processes behave during successive business cycles.

Next, we look in every series for wave-like movements, the duration of which is of the same order as that of business cycles. We call the cyclical movements peculiar to a series its 'specific cycles'. In most series the dates of the troughs and peaks of the specific cycles are fairly clear, but in some series they are obscured by erratic fluctuations. We mark off the specific cycles by the dates of their turning points as well as we can; compute the average value of the monthly data during each cycle, and convert the monthly data into 'specific-cycle relatives' which correspond in every

1 We find it convenient to use the term 'reference cycle' in two senses: first, to denote the section of a time series between the dates of successive reference troughs (or peaks), second, to denote the interval between successive troughs (or peaks). The meaning intended should be obvious from the context.

TABLE 4

Coke Production, United States, 1914–1933

(Thousands of short tons)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1914	2973	3147	3476	3364	2940	2897	2991	2927	2797	2531	2193	2348
1915	2281	2555	2675	2897	2990	3410	3613	3873	3959	4320	4475	4553
1916	4381	4564	4554	4425	4581	4581	4392	4667	4684	4655	4593	4499
1917	4664	4523	4672	4720	4693	4778	4731	4611	4693	4542	4577	4452
1918	3855	3957	4415	4639	4801	4941	5228	5067	5033	5017	4844	4730
1919	4763	4126	3773	3335	2977	3173	3777	3987	3943	3157	3600	3624
1920	4329	4261	4360	3885	4031	4299	4412	4536	4520	4496	4284	3971
1921	3314	2886	2203	1855	1860	1679	1497	1637	1719	2076	2231	2338
1922	2391	2512	2658	2798	2979	3180	3038	2413	2927	3638	4145	4342
1923	4650	4695	4853	5174	5250	5216	5076	4901	4641	4362	4132	4107
1924	4278	4493	4386	4199	3581	3108	2923	2936	3132	3466	3596	4182
1925	4599	4458	4259	4204	3950	3900	3804	3838	4102	4333	4836	5087
1926	5244	5280	4746	4719	4643	4635	4721	4606	4578	4604	4665	4495
1927	4471	4426	4521	4553	4389	4320	4219	4219	4112	4027	3887	3991
1928	4249	4348	4276	4365	4450	4413	4286	4344	4332	4524	4569	4688
1929	4822	4798	4889	5005	5250	5311	5361	5295	5000	4961	4761	4502
1930	4441	4480	4387	4562	4460	4316	4041	3817	3579	3480	3280	3193
1931	3195	3193	3187	3266	3167	2870	2682	2522	2396	2403	2356	2277
1932	2150	2174	2037	1948	1761	1619	1586	1522	1598	1741	1817	1846
1933	1853	1819	1664	1720	1948	2363	2928	3029	2803	2553	2443	2523

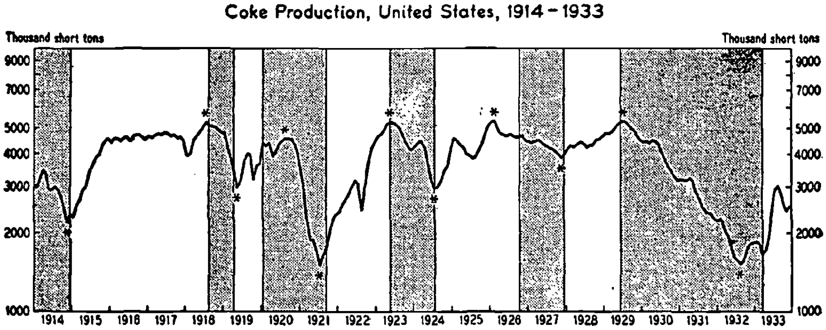
Adjusted for seasonal variations. The original data come from the Bureau of Mines, *Mineral Resources of the United States*, 1925, Part II, p. 545, and later annual numbers (now called *Minerals Yearbook*).

respect to the reference-cycle relatives, except that they show movements during specific cycles.

To exemplify these steps: Table 4 shows by months the seasonally adjusted figures of coke production in the United States from 1914 through 1933, a series chosen because it is relatively short and presents few of the complications we ordinarily encounter. These figures are plotted on Chart 1, which shows also the turning points of business cycles and of the specific cycles in coke production. The average monthly production of coke during the first complete specific cycle (November 1914

CHART 1

Coke Production, United States, 1914 – 1933



Adjusted for seasonal variations. Shaded areas represent reference contractions, white areas represent reference expansions. Asterisks identify peaks and troughs of specific cycles. See Table 4.

Logarithmic vertical scale

to May 1919) was 4,246,000 short tons. With that figure as a base, the monthly values in Table 4 for the months covered by this cycle are converted into specific-cycle relatives. The first reference cycle covered by this series runs from December 1914 to April 1919. Average monthly output was 4,305,000 short tons, which is the base upon which the first set of reference-cycle relatives is computed. During the second specific cycle (May 1919 to July 1921) average monthly output was 3,565,000 short tons; during the second reference cycle (April 1919 to September 1921) it was 3,417,000 short tons. These figures are the bases upon which relatives are computed for the second specific and the second reference cycle. The turning points shown on Chart 1 mark off three more specific and three more reference cycles for each of which we compute cycle relatives.

From the specific-cycle relatives we make what we call 'S' tables, of which there are five. Samples of these tables interspersed with explanatory text appear as Tables 5-9. Similarly, from the reference-cycle relatives we make three or four 'R' tables, samples of which appear as Tables 10-12. Important details of our procedure are omitted in the sketch that follows. A full description of the methods underlying the tables is presented in Chapter 5.

III Timing, Duration and Amplitude of Specific Cycles

The reference dates yield measures of the duration of business cycles and of their expansions and contractions. The dates of the cyclical turns in a given series yield similar measures of the duration of its specific cycles. By comparing the turning dates of specific cycles with the reference dates,

TABLE 5

Sample of Table S1: Timing and Duration of Specific Cycles
Coke Production, United States, 1914–1932

Dates of specific cycles	Timing at reference peak		Timing at reference trough		Duration of cyclical movements (mos.)						Per cent of duration of specific cycles	
					Specific cycles			Excess over reference cycles				
Trough – Peak – Trough	No. of months lead (−) or lag (+)	Date of reference peak	No. of months lead (−) or lag (+)	Date of reference trough	Expansion	Contraction	Full cycle	Expansion	Contraction	Full cycle	Expansion	Contraction
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Nov. 14			−1	12/14								
Nov. 14 – July 18 – May 19	−1	8/18	+1	4/19	44	10	54	0	+2	+2	81	19
May 19 – Aug. 20 – July 21	+7	1/20	−2	9/21	15	11	26	+6	−9	−3	58	42
July 21 – May 23 – July 24	0	5/23	0	7/24	22	14	36	+2	0	+2	61	39
July 24 – Feb. 26 – Nov. 27	−8	10/26	−1	12/27	19	21	40	−8	+7	−1	48	52
Nov. 27 – July 29 – Aug. 32	+1	6/29	−7	3/33	20	37	57	+2	−8	−6	35	65
Average	−0.2		−1.7		24.0	18.6	42.6	+0.4	−1.6	−1.2	57	43
Average deviation	3.4		1.9		8.0	8.3	10.3	3.5	5.5	2.6	12	12

we determine the number of months by which the troughs and peaks in the series precede or follow the reference troughs and peaks.

These procedures are illustrated by the sample of Table S1. After the specific cycles have been marked off, the dates of the turning points are entered in column (1). The reference dates with which the specific-cycle turns are compared are entered in columns (3) and (5). The differences in months between the turning dates of the specific cycles and the reference dates are then shown in columns (2) and (4). The durations of the specific cycles and their phases are shown in columns (6) to (8). The differences between the durations of specific cycles and corresponding business cycles are shown in columns (9) to (11). Finally, in columns (12) and (13) the lengths of the specific-cycle phases are expressed as percentages of the durations of full cycles.

Next, the amplitude of the cyclical swings is ascertained by measuring the rise of the specific-cycle relatives from the initial trough of a cycle to the peak, and the fall from the peak to the terminal trough. In order to diminish the influence of random factors upon the amplitudes, we use three-month averages centered on the troughs and peaks. Of course, the results show the rise and fall in percentages of the average value of the series during each specific cycle.

Table S2 gives these amplitude measures in three forms. Columns (2) to (4) show the three-month averages of the specific-cycle relatives centered on the initial trough, peak, and terminal trough. The second set of measures, columns (5) to (7), shows the rise from trough to peak, the fall from peak to trough, and the total rise and fall; these figures are obtained from the entries in columns (2) to (4). The third set of measures, columns (8) to (10), shows the amplitudes per month; they are obtained

TABLE 6

Sample of Table S2: Amplitude of Specific Cycles
Coke Production, United States, 1914–1932

Dates of specific cycles	3-month average in specific-cycle relatives centered on			Amplitude of			Per month amplitude of		
Trough – Peak – Trough (1)	Initial trough (2)	Peak (3)	Terminal trough (4)	Rise (5)	Fall (6)	Rise & fall (7)	Rise (8)	Fall (9)	Rise & fall (10)
Nov.14–July 18–May19	55.5	119.6	74.5	64.1	45.1	109.2	1.5	4.5	2.0
May19–Aug.20–July 21	88.7	125.9	45.0	37.2	80.9	118.1	2.5	7.4	4.5
July 21–May23–July 24	44.3	144.0	82.6	99.7	61.4	161.1	4.5	4.4	4.5
July 24–Feb. 26–Nov.27	69.4	118.2	92.1	48.8	26.1	74.9	2.6	1.2	1.9
Nov.27–July 29–Aug.32	105.5	141.6	41.7	36.1	99.9	136.0	1.8	2.7	2.4
Average	72.7	129.9	67.2	57.2	62.7	119.9	2.6	4.0	3.1
Average deviation	19.5	10.4	19.1	19.8	22.2	23.0	0.8	1.7	1.2

by dividing the figures in columns (5) to (7) of Table S2 by the corresponding duration figures in columns (6) to (8) of Table S1.

IV Measures of Secular Movements

Our method of computing cycle relatives as percentages of the average value during a specific or a reference cycle eliminates from the original data what we call the 'inter-cycle' portion of the secular trend. The 'intra-cycle' portion of the trend we make no effort to eliminate, because we wish to reproduce as faithfully as may be the 'cyclical units' of actual economic experience.

Table S3 throws into relief the secular component of the specific cycles. Columns (2) and (3) show the average value of the seasonally

TABLE 7
Sample of Table S3: Secular Movements
Coke Production, United States, 1914–1932

Dates of specific cycles	Average monthly standing (thous. short tons)			Per cent change from preceding phase		Per cent change from preceding cycle on base of			
						Preceding cycle		Average of given and preceding cycle	
Trough – Peak – Trough (1)	Expansion (2)	Contraction (3)	Full cycle (4)	Contraction to expansion (5)	Expansion to contraction (6)	Total (7)	Per month (8)	Total (9)	Per month (10)
Nov. 14–July 18–May 19	4193	4479	4246	...	+7	...	...	...	...
May 19–Aug. 20–July 21	3906	3099	3565	−13	−21	−16	−0.40	−17	−0.42
July 21–May 23–July 24	3171	4326	3620	+2	+36	+2	+0.06	+2	+0.06
July 24–Feb. 26–Nov. 27	4107	4488	4307	−5	+9	+19	+0.50	+17	+0.45
Nov. 27–July 29–Aug. 32	4577	3319	3760	+2	−27	−13	−0.27	−14	−0.29
Average	...	...	...	...	...	...	...	−3.0	−0.05
Average deviation	...	...	...	...	...	...	...	12.5	0.30
Weighted average	...	...	...	...	...	...	...	...	−0.08

adjusted data during the phases of specific cycles. Column (4) shows the average value during full specific cycles; these values are the bases on which the specific-cycle relatives are computed. Column (5) shows the percentage change from the average standing during a contraction to the average during the following expansion, and column (6) shows the percentage change from the average standing during an expansion to the average during the following contraction. Column (7) shows the percentage change from the average standing during a full specific cycle to that during the next. And column (8) reduces the measures in column (7) to a per month basis, the divisor being the number of months from the midpoint of one cycle to the midpoint of the next.

Column (9) is the same as column (7) except that the percentages are computed on the base of the average of the two cycles being compared, instead of on the base of the first of the two cycles. This shift of method frees the percentages from secular 'bias' and permits us to strike averages over all cycles. Column (10) bears the same relation to column (9) as column (8) bears to column (7). The weighted average at the bottom of column (10) is obtained by weighting the entries in this column by the intervals between the midpoints of successive cycles.

V Cyclical Patterns

To show the behavior of a series during the course of its specific cycles in greater detail than in Table S2, each specific cycle is broken into nine stages. Stage I covers the three months centered on the initial trough, stage V covers the three months centered on the peak, and stage IX the three months centered on the terminal trough. Stages II to IV cover successive thirds of the length of the expansion, and stages VI to VIII cover successive thirds of the contraction. By averaging the specific-cycle relatives for the months included in each of these stages, we get 'specific-cycle patterns'. These computations are illustrated in the sample of Table S4.

TABLE 8

Sample of Table S4: Specific-cycle Patterns

Coke Production, United States, 1914–1932

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Nov. 14 – July 18 – May 19	55.5	81.3	108.0	107.1	119.6	118.7	112.6	88.2	74.5
May 19 – Aug. 20 – July 21	88.7	101.2	110.9	117.8	125.9	124.4	86.8	50.4	45.0
July 21 – May 23 – July 24	44.3	58.8	78.9	124.3	144.0	137.0	118.1	105.5	82.6
July 24 – Feb. 26 – Nov. 27	69.4	84.8	95.1	106.2	118.2	108.3	105.2	99.0	92.1
Nov. 27 – July 29 – Aug. 32	105.5	113.8	118.4	133.3	141.6	122.4	86.2	55.9	41.7
Average	72.7	88.0	102.3	117.7	129.9	122.2	101.8	79.8	67.2
Average deviation	19.5	15.6	12.2	8.9	10.4	6.9	12.2	21.3	19.1

We make 'reference-cycle patterns' on a similar plan; but here the nine stages are marked off on the basis of the cyclical turning dates of general business. By breaking each reference cycle into nine segments, we show the behavior of different economic processes from stage to stage of business cycles and put both our concept of business cycles and the schedule of reference dates to a critical test. Table R1 presents the reference-cycle patterns. It differs from Table S4 in only two respects: the troughs and peaks are taken from the standard list of reference dates instead of from the turning points of specific cycles, and the entries are expressed in units of reference-cycle relatives instead of specific-cycle relatives.

Additional information concerning cyclical patterns is supplied by Tables S5 and R2. Table S5 shows the rate of change from one stage of specific cycles to the next, and Table R2 shows the rate of change from one stage of reference cycles to the next. The entries in Table S5 are obtained by dividing the differences between successive figures on each

TABLE 9

Sample of Table S5: Rate of Change from Stage to Stage of Specific Cycles
Coke Production, United States, 1914–1932

Dates of specific cycles	Average change per month in specific-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough – Peak – Trough	Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Nov. 14 – July 18 – May 19	+3.4	+1.8	–0.1	+1.7	–0.4	–2.0	–8.1	–6.8
May 19 – Aug. 20 – July 21	+4.2	+2.2	+1.5	+2.7	–0.8	–10.7	–10.4	–2.7
July 21 – May 23 – July 24	+3.6	+2.9	+6.5	+4.9	–2.8	–4.2	–2.8	–9.2
July 24 – Feb. 26 – Nov. 27	+4.4	+1.7	+1.8	+3.4	–2.5	–0.5	–1.0	–1.7
Nov. 27 – July 29 – Aug. 32	+2.4	+0.7	+2.3	+2.4	–3.0	–3.0	–2.5	–2.2
Average	+3.6	+1.9	+2.4	+3.0	–1.9	–4.1	–5.0	–4.5
Average deviation	0.6	0.6	1.6	0.9	1.0	2.7	3.4	2.8
Average interval (mos.)	*4.3*	*7.7*	*7.7*	*4.3*	*3.4*	*5.9*	*5.9*	*3.4*

TABLE 10

Sample of Table R1: Reference-cycle Patterns
Coke Production, United States, 1914–1933

Dates of reference cycles	Average in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Dec. 14 – Aug. 18 – Apr. 19	52.8	83.9	106.7	106.6	118.7	116.7	111.0	91.7	78.1
Apr. 19 – Jan. 20 – Sep. 21	98.4	96.9	116.1	101.3	119.2	123.2	117.1	52.3	53.0
Sep. 21 – May 23 – July 24	48.4	63.3	80.2	124.2	139.5	132.7	114.4	102.2	80.0
July 24 – Oct. 26 – Dec. 27	69.5	90.0	98.4	111.6	107.4	105.0	102.4	94.5	94.0
Dec. 27 – June 29 – Mar. 33	114.4	123.1	124.8	138.9	150.2	128.2	83.3	52.3	49.1
Average	76.7	91.4	105.2	116.5	127.0	121.2	105.6	78.6	70.8
Average deviation	23.8	14.8	12.8	12.0	14.3	8.2	10.2	21.0	15.8

TABLE 11

Sample of Table R2: Rate of Change from Stage to Stage of Reference Cycles
Coke Production, United States, 1914–1933

Dates of reference cycles	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough – Peak – Trough	Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec. 14 – Aug. 18 – Apr. 19	+4.1	+1.6	0.0	+1.6	−1.3	−2.3	−7.7	−9.1
Apr. 19 – Jan. 20 – Sep. 21	−0.8	+7.7	−5.9	+9.0	+1.1	−0.9	−10.0	+0.2
Sep. 21 – May 23 – July 24	+4.3	+2.6	+6.8	+4.4	−2.7	−4.1	−2.7	−8.9
July 24 – Oct. 26 – Dec. 27	+4.1	+1.0	+1.6	−0.8	−1.0	−0.6	−1.8	−0.2
Dec. 27 – June 29 – Mar. 33	+2.5	+0.3	+2.6	+3.2	−2.8	−3.1	−2.1	−0.4
Average	+2.8	+2.6	+1.0	+3.5	−1.3	−2.2	−4.9	−3.7
Average deviation	1.6	2.0	3.2	2.6	1.1	1.2	3.2	4.3
Average interval (mos.)	*4.3*	*7.5*	*7.5*	*4.3*	*3.6*	*6.5*	*6.5*	*3.6*

line in Table S4 by the number of months from the middle of one specific-cycle stage to the middle of the next stage. Table R2 is made from Table R1 just as Table S5 is made from Table S4.

VI Measures of Conformity to Business Cycles

The comparisons in Table S1 between specific and reference cycles show roughly how the wave-like movements in a given series conform to the waves in general business activity. Further light is shed upon this matter by the similarity or the difference between the average specific-cycle and the average reference-cycle patterns of Tables S4 and R1. But it is desirable to measure explicitly the varying degrees of conformity.

Table R3 gives the measures we seek. Column (4), which is derived from columns (2) and (3), supplies essential information on the conformity of the series to business-cycle expansions. That is, the entries in column (4) show the average rise or fall per month during successive reference expansions, while the average near the bottom of this column shows the average rate of change during all the reference expansions covered by the series. Column (7) supplies similar information concerning the behavior of the series during reference contractions. Finally, column (8) expresses the difference between the rates of change during reference expansion and contraction; this measure is needed because some series with rapidly rising trends continue to advance even during reference contractions, and we wish to know how much, if at all, the rate of rise is intensified during expansions in general business and diminished during contractions.

TABLE 12

Sample of Table R3: Conformity to Business Cycles
Coke Production, United States, 1914–1933

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough–Peak–Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[a]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec.14–Aug.18–Apr. 19	+65.9	44.0	+1.50	−40.6	8.0	−5.08	−6.58	−
Apr.19–Jan. 20–Sep. 21	+20.8	9.0	+2.31	−66.2	20.0	−3.31	−5.62	−
Sep.21–May23–July 24	+91.1	20.0	+4.56	−59.5	14.0	−4.25	−8.81	−
July 24–Oct. 26–Dec. 27	+37.9	27.0	+1.40	−13.4	14.0	−0.96	−2.36	−
Dec.27–June29–Mar.33	+35.8	18.0	+1.99	−101.1	45.0	−2.25	−4.24	...
Average	+50.3	...	+2.35	−56.2	...	−3.17	−5.52	...
Average deviation	...	...	0.88	...	...	1.25	1.78	...

Index of conformity to reference
- Expansions .... +100
- Contractions .... +100
- Cycles, trough to trough .... +100
- Cycles, peak to peak .... +100
- Cycles, both ways .... +100

[a]Only the sign of the difference is entered.

While the averages near the bottom of columns (4), (7) and (8) are useful measures of conformity, they do not indicate the *regularity* with which a series 'responds' to the stimuli of general business expansion and contraction. To bring out this feature of cyclical behavior we make a second set of conformity measures, that is, 'indexes of conformity' which take account of the direction of movements but not their magnitude. When a series rises during a reference expansion we mark it +100; when the series remains unchanged we mark it 0; when it falls we mark it −100. By casting up the algebraic sum of these entries for all cycles and dividing by their number, we get an index of conformity to reference expansions. This result, entered at the bottom of column (4), may vary between +100 (positive conformity to all the reference expansions covered) and −100 (inverse conformity to all the expansions). An equal number of positive and inverse movements produces an index of 0.[2] To measure conformity to reference contractions we proceed in a similar way, but a decline in column (7) is now marked +100, and a rise −100; for a decline means positive conformity to reference contractions and a rise means inverse conformity.

Finally, we make indexes of conformity to business cycles as wholes.

2 Again, an index of +50 means positive conformity in 3 and inverse conformity in 1 case out of 4; an index of +33 means positive conformity in 2 and inverse conformity in 1 case out of 3.

Here we wish to take account of the fact that some series rise or decline throughout reference cycles, but at a different rate during reference expansions and contractions. A preliminary index is obtained by crediting each difference in column (8) with +100 when the difference is minus, with −100 when it is plus, and then striking an arithmetic mean of these entries. This index shows merely the conformity to business cycles marked off by troughs; hence it is supplemented in column (9) by a similar index showing conformity to business cycles marked off by peaks. A weighted average of the two preliminary indexes gives our final index of conformity to business cycles taken as wholes. A value of +100 for this index means that the rate of change per month during a reference contraction is without exception algebraically lower than the rate of change during the next preceding and following reference expansions.

The sample Table R3 for coke production illustrates these computations. The 'expansion index' comes out +100 because all entries in column (4) are plus. The 'contraction index' comes out +100 because all signs in column (7) are negative. The preliminary 'full-cycle index', taken on a trough-to-trough basis, comes out +100, and so too does the index of conformity to full cycles on a peak-to-peak basis, since all signs in columns (8) and (9) are negative. The final full-cycle index is obviously +100, since it is an average of two preliminary indexes each of which is +100.

Coke production typically rises from stage I to stage V and declines from stage V to stage IX of the reference cycles. Although many series follow this simple pattern, some show pronounced leads or lags at the reference turns. When a series conforms to business cycles with a characteristic lead or lag, we take this fact into account in Table R4. This table is similar to Table R3 in every respect, except that it recognizes differences in timing between the cyclical movements of a given activity and those of business at large. A sample of Table R4 is given in Chapter 5, where our measures of cyclical behavior are described more fully.

VII Averages and Average Deviations

Most of the measures described above are made for every reference and for every specific cycle covered by a series, and are then averaged for each set of cycles. When averages are struck for all the cycles covered by a series, features peculiar to single cycles tend to fade away, while features common to all or most of the cycles tend to stand out prominently.

In general, the more cycles a series covers, the greater is our confidence that the average discloses faithfully what cyclical behavior is typical of the process represented. But in analyzing price and value series, we usually exclude cycles affected by grave monetary disturbances from the averages. We also make exclusions when some exceptionally powerful random factor, such as a great strike, has warped an individual cycle out of resemblance to other cycles in the array. When the cyclical behavior of a long series gives definite indications of having undergone a secular or structural change, we break the series into relatively homogeneous segments, and strike separate averages for each segment.

Our attempt to find what cyclical behavior is characteristic of different economic processes does not end in the contemplation of average measures; for one of the leading features of specific and of business cycles is that they vary in duration, intensity, and other respects. To keep this feature prominently before our minds and to provide materials for studying it at a later stage of the investigation, we compute average deviations from the averages. These deviations are simple measures of the degree to which the figures for individual cycles in a series are clustered about the arithmetic means that we use to represent 'central tendencies'.

VIII Charts of Cyclical Patterns

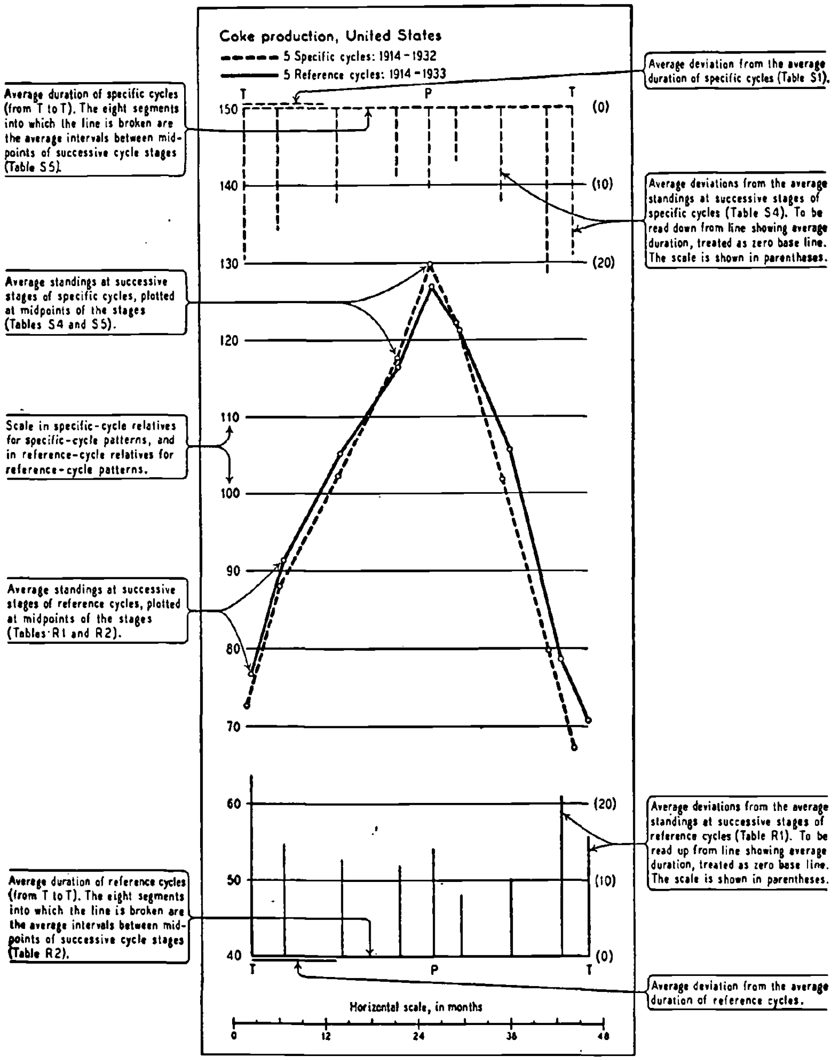
Several results of our statistical analysis that lend themselves readily to graphic presentation are embodied in charts of cyclical patterns. The sample for coke production, shown in Chart 2, pictures the averages and average deviations in Tables S4 and R1, and certain additional measures from Tables S1, S5, and R2.

The curves in the chart trace out the specific-cycle and reference-cycle patterns made by averaging the standings of the individual cycles at each of the nine stages used in Tables S4 and R1. Since coke production corresponds closely in time to business cycles, its two patterns are nearly identical. The more irregular the timing of a series in relation to business cycles, the smaller will be the amplitude of the reference-cycle pattern relative to that of the specific-cycle pattern. The representative value of the two patterns is indicated by the lengths of the vertical lines, which show the average deviations of the individual cycles from their average standings at the nine stages.

The long horizontal lines above and below the cyclical patterns represent the average durations of the specific and reference cycles. We refer to them as 'duration lines'. The vertical lines representing the average deviations from the average standings are dropped from or erected at the midpoints of the cycle stages. The short horizontal lines above and below the duration lines represent the average deviations from the average durations. The ruler at the bottom of the chart defines the time scale; with its aid all durations can be approximated.

When, as in coke production, the specific and reference cycles correspond to one another, the two duration lines are placed so that they show average leads or lags. When the specific and reference cycles of a series do not correspond throughout, the duration lines are so placed that the peak standings of the two patterns are aligned vertically.

CHART 2
Sample Chart of Cyclical Patterns
(Drawn to twice the standard scales)



T represents the trough stage (I or IX), P the peak stage (V). For explanation of how the line representing the average duration of specific cycles is placed in relation to the line representing the average duration of reference cycles, see Ch. 5, Sec. VIII.

The charts of cyclical patterns presented later in this volume have been drawn to a strictly uniform set of scales. However, in order to help the reader learn our method, the scales in Chart 2 are twice the standard scales. The explanatory comments on this chart are not repeated in later charts; nor are the scale numbers for average deviations of the standings in successive cycle stages.

IX Comparison with Customary Techniques

Despite its relative simplicity, our statistical technique may strike the reader as formidable. This impression may be due to the numerous measures we require to reveal the cyclical behavior of a series. But the impression may be due also to the novel features of the technique. It may help the reader to become oriented in our analysis, if our method of finding what cyclical behavior is characteristic of a series is contrasted with the method that has become customary since the publication of Warren M. Persons' basic memoirs on time series analysis.[3]

The customary way of showing the cyclical fluctuations of a time series is to begin by measuring the secular trend and the seasonal variations. These two elements are then combined, and the original data expressed as percentage deviations from corresponding ordinates of the 'secular-seasonal' composite. The percentages are supposed to show cyclical and erratic movements in combination. Sometimes the percentages are expressed in standard deviation units, at other times this step is omitted. In either case the results are presented in a chart where the 'secular-seasonal' composite is reduced to a horizontal line about which oscillate the 'cyclical-erratic' values, smoothed or not as the case may be. What we should call the 'specific cycles' of the series in trend-adjusted form can then be distinguished, though the turning dates may be obscured by erratic movements. The behavior of the specific cycles in relation to business cycles can be made out after a fashion by marking the time scale with the cyclical turning dates in general business activity or by plotting a curve that purports to represent business cycles. Cyclical analysis of economic time series frequently does not go further; although average amplitudes of specific cycles, average leads or lags at the turns of selected 'indicators of business', and coefficients of correlation between the series and some 'indicator of business cycles' are sometimes computed.

Our method follows precedent in eliminating seasonal variations at the outset. It then diverges from the usual technique: first, in breaking the data adjusted for seasonal variations into reference-cycle segments and into specific-cycle segments, which are treated as units of economic experience subject to comparison and averaging; second, in eliminating the inter-cycle portions of the secular trend; third, in preserving the intra-cycle portions of the secular trend in the measures of cyclical behavior, but arranging the measures so as to bring out the secular movements; fourth, in striking averages for a group of cycles to show what cyclical behavior is characteristic of the series; fifth, in charting, not successive specific cycles as a continuum, but average behavior during a group of specific cycles and during the group of contemporaneous business cycles.

[3] Indices of Business Conditions, *Review of Economic Statistics*, Jan. 1919; An Index of General Business Conditions, *ibid.*, April 1919.

CHAPTER 3

Plan of Treating Secular, Seasonal and Random Movements

I The 'Cycle of Experience' as the Unit of Analysis

IN *Business Cycles: The Problem and Its Setting* various methods of 'eliminating' seasonal variations and secular trends from time series were described. Since the publication of that book in 1927, the technical devices available to the investigator of business cycles have multiplied abundantly. In the present studies we might utilize such of these tools, old and new, as approve themselves to our judgment, and begin the analysis of cyclical behavior by striving for the completest attainable isolation of cyclical fluctuations.

We do not follow that plan. In the first place, the isolation of cyclical fluctuations is a highly uncertain operation. Edwin Frickey once diligently assembled 23 trend lines fitted by various investigators to pig iron production in the United States, and found that some of the trend lines yield cycles averaging 3 or 4 years in duration while others yield cycles more than ten times as long.[1] This range of results illustrates vividly the uncertainty that attaches to separations of trends and cycles, though it perhaps exaggerates the difficulties. If an investigator fits a trend line in a mechanical manner, without specifying in advance his conception of the secular trend or of cyclical fluctuations, he may get 'cycles' of almost any duration. But an informed investigator who is seriously studying cycles of a given order of duration will use whatever guidance he can get from history and statistics; he will scrutinize the movements of the original data, seek to mark off in advance the cycles or traces of cycles that

[1] The Problem of Secular Trend, *Review of Economic Statistics*, Oct. 15, 1934; see also Ch. 7-8 below.

correspond to his basic conception, then choose a trend line that cuts through and exposes the cycles in which his interest centers. Yet this procedure also illustrates the difficulties of segregating trends and cycles. For it leaves room for choice of the trend line, the method of fit, and the method of trend elimination. Further, it makes the trend depend upon the cycles, and may not lead to the discovery of cycles that are obscured by the trend. To judge what features of the final result are merely technical and what features are a significant characteristic of the series is likely to require considerable testing and experimenting even on the part of a skilled technician.[2]

It is fairly common for statisticians to assume that the elimination of the secular trend from a time series indicates what the course of the series would have been in the absence of secular movements, and that the graduation of a time series, whether in original or trend-adjusted form, indicates what the course of the series would have been in the absence of random movements. There is no warrant for such simple interpretations. A 'least squares' trend line fitted, for example, to grocery chain store sales in the United States may move majestically on a chart, but the analytic significance of the trend line is obscure. At least some of the 'growth factors' impinging on this branch of business—the addition of meats and vegetables to the grocery line, the rise of supermarkets, special taxes on chain stores—have made their influence felt spasmodically. When a continuous 'trend factor' is eliminated from the data, it is therefore difficult to say what influences impinging on the activity have been removed and what influences have been left in the series. Cyclical graduations are no easier to interpret than trend adjustments. Systematic smoothing of a time series will, indeed, eliminate short-run oscillations produced by random factors; but can it eliminate the influence of powerful random factors—such as a protracted strike, or a succession of bad harvests, or a great war?

There is always danger that the statistical operations performed on the original data may lead an investigator to bury real problems and worry about false ones. When new commodities, new techniques of production, new methods of organizing business, new methods of financing are first introduced on a substantial scale, they affect the general business situation more profoundly than at a later time when they have fully penetrated the economic system and become a part of routine experience. For example, railroad investment in the United States shows long leads at cyclical revivals during the eighteen seventies and eighties; as the decades roll on the leads tend to become shorter, disappear, and finally are replaced by lags. A fact of this sort is of considerable importance

[2] See the interesting study of this problem by Edwin Frickey, *Economic Fluctuations in the United States* (Harvard University Press, 1942). Cf. Arthur F. Burns, Frickey on the Decomposition of Time Series, *Review of Economic Statistics*, Aug. 1944.

historically, for it suggests that railroad investment in this country gradually shifted from an 'active' to a 'passive' role in the process of recovery from business depressions.[3] It is also of theoretical significance, so far as it suggests a point that may be true generally of 'new' versus 'old' industries. But this point is likely to be lost or blurred when the secular trend is completely removed from the data, since that operation tends to standardize the features of successive cycles.[4]

If these observations are well founded, it follows that in attempting to explain business cycles, we should work with cyclical measures that take account of secular trends, and also of substantial random movements. The historical records of processes that represent volume of business, as distinct from prices, usually appear as a series of expansions followed by sharper and briefer but less considerable contractions. Also the few dwindling processes in business do not show a steady movement, but brief and mild expansions followed by larger contractions. Our aim in developing a technique for analyzing cyclical behavior has been to reproduce the *form of development* common to the industrial activities of nations whose economic life has been organized on a business basis. Hence we make no attempt to adjust for that portion of the secular trend which falls within the limits of a single cycle; but we do adjust the original data of time series for seasonal variations, and make allowance in the course of the analysis for the minor oscillations that diversify the cyclical expansions and contractions of economic processes.

As explained in the preceding chapter, most of our cyclical measures are based upon entries for three or more months instead of single months. This practice tends to moderate the influence of erratic movements on the results, and so too does the device of averaging the cyclical measures for all the cycles covered by a series. Our practice of first breaking the seasonally adjusted data of a series into specific cycles and then turning the data included within each of these segments into percentages of their average value during the segment, eliminates the portion of the secular trend that represents shifts in the level of the series from cycle to cycle, but retains the portion that lies within the limits of a specific cycle.[5] A similar result is obtained by breaking a series into reference cycles and then converting the data within each of these segments into reference-cycle relatives.[6] On the whole these statistical procedures yield fairly realistic pictures of the 'cyclical units' of economic experience in modern

[3] See Ch. 10, Sec. VIII.

[4] See Ch. 7, especially Sec. VI. Also, Joseph A. Schumpeter, *Business Cycles* (McGraw-Hill, 1939), Vol. I, Ch. V.

[5] This procedure implies that if the secular trend were represented instead by a continuous line, that line would be a flexible curve cutting through successive specific cycles. Cf. Arthur F. Burns, *Production Trends in the United States since 1870* (National Bureau of Economic Research, 1934), Ch. II.

[6] See Charts 14 and 18.

nations, and afford clues to the actual behavior of men from stage to stage of business cycles.

We treat these units of experience in seasonally adjusted form. Seasonal fluctuations vary endlessly from one business activity to another, but they are a comparatively regular factor within each activity. Since the seasonal pattern, by and large, is much the same in years of 'good' business and years of 'bad' business,[7] our analysis of cyclical movements can be facilitated by putting the seasonal fluctuations provisionally out of sight. The effects on business enterprises of an increase in activity that is expected to last at most a few months are very different from an increase that is expected to continue for years. For one thing, seasonal increases do not lead men to expand their investments as does rapid secular growth. To understand changes in contracting for the construction of new plants, ordering of industrial equipment, issuing of corporate securities, and the like, it is not necessary to differentiate activities that reach their seasonal peaks in January, April, and November; but it is necessary to differentiate activities that, seasonal variations aside, are growing, remaining constant, and shrinking. The business community 'allows for' seasonal fluctuations deliberately or tacitly, and we merely follow suit by erasing as well as may be their influence on time series.[8]

The larger the seasonal fluctuations the more essential is this step, both practically and theoretically. For example, the prices and inventories of many farm products are highly unsteady; but the wide fluctuations are rooted principally in seasonal factors, not in business cycles. If we ignored this knowledge and marked off their specific cycles on the basis of the original figures, we should conclude that their cyclical amplitudes are among the largest in economic records. That conclusion would rest on measures that reflect mainly the amplitude of the seasonal movements, and would be seriously misleading.

II Limitations of the Technique

Our technical procedures, designed as they are to aid in the explanation of the business cycles of experience, have certain disadvantages for the more modest task of describing the business-cycle behavior of individual time series. If secular trends were eliminated at the outset as fully as are

[7] Since 1940, seasonal fluctuations have practically disappeared in numerous branches of industrial activity in this country. Such a result is to be expected when an industry is operating over a protracted period at 'full capacity'. But protracted plateaus of industrial output are not typical of cyclical fluctuations: as a rule, once output stops expanding, a decline sets in rather promptly.

[8] Of course, no exact correspondence is implied. Our seasonal correction for a given year is usually based on the data of that year and of several preceding and following years. But when a businessman 'corrects' for the seasonal movement in recent months, he has only past experience to guide him. Again, the net result of seasonal adjustments applied to the sales of individual firms in an industry may differ materially from the seasonal adjustment applied to a single series representing aggregate sales of the industry.

seasonal variations, we would show that business cycles are a more pervasive and a more potent factor in economic life than our results indicate. For when the secular trend of a series rises rapidly it may offset the influence of cyclical contractions in general business, or make the detection of this influence difficult. In such instances our method may indicate lapses from conformity to contractions in general business, which would not appear if the secular trend were removed. Even when secular trends do not obscure the specific cycles, our cyclical patterns include an element of trend that confuses somewhat the 'cyclical component' proper. Another troublesome factor is the persistence of what seem to be random movements in the reference-cycle patterns of some series. If we could eliminate random movements from the original data we would probably find that our series on the whole conform more closely to business cycles than the present results indicate.

But we are not without safeguards against being misled by these shortcomings of method. Thus, when the secular trend rises with exceptional swiftness, we may put aside the early portions of a series, or recognize diminutive declines as contractions of specific cycles. Our index of business-cycle conformity is designed to show whether the rate of increase in a series is less rapid, or the rate of decrease more rapid, during a contraction in general business than during the next preceding and following expansions. Also we make measures that throw into relief the element of trend retained in the cyclical measurements. Those specific cycles which we know are dominated by random forces we exclude from the averages. We believe that these safeguards and certain others, combined with the checks and counterchecks that our many measures for each series and for related series make possible, overcome to a considerable extent the deficiencies of our technique for describing cyclical behavior.

III Need to Economize Effort

Doubtless the ideal procedure would be to make two sets of measures for each series: one set based on the original data adjusted only for seasonal variations, as is our present practice, the other based on the best attainable isolation of the 'cyclical component' of the data. But the resources at our disposal place grave obstacles to the realization of this ideal.

The isolation of cyclical components is a very costly operation. To assure sensible results, painstaking study of each series and considerable experimentation are likely to be necessary. If we attempted a full analysis, first, of the original data adjusted merely for seasonal variations, second, of data adjusted for secular trend and random movements as well as seasonal variations, we would be able to analyze comparatively few series, perhaps less than a tenth of the number we can analyze by our simple method. This restriction of the coverage would doom our efforts to lay a thorough factual foundation for the explanation of business cycles. As explained in Chapter 1, our working definition of business cycles makes necessary extensive observation on the cyclical behavior of economic activities. These activities embrace at least the production of commodities, construction work, transportation, pricing, carrying stocks of commodities, marketing, foreign commerce, getting and spending personal incomes, making business profits, saving, investing, borrowing, trading in securities, the circulation of money, and banking. Each of these broad processes must be divided into several or many parts, and for each subdivision enough time series are needed to show what cyclical behavior, or what varieties of cyclical behavior, are characteristic.

We might have limited the sample to perhaps a hundred series by working with broad aggregates or index numbers. But experience early convinced us that this labor-saving shift would not do. Though comprehensive series reveal certain facts that might otherwise escape notice, and we analyze many series of this type, they hide differences among their constituents in respect of cyclical timing, duration, amplitude, pattern, and conformity—differences many of which seem highly significant for the understanding of business cycles. To get a clear view of the cyclical behavior of economic activity, it is necessary to go back of broad aggregates or index numbers to the series from which they are made.

In studying an economic process in detail, for example, the production of textiles, the construction of buildings, the prices of foodstuffs at retail, interest rates, we often find marked divergences in cyclical behavior and so must analyze a considerable number of series. If another group of activities behaves so similarly that any one series might be accepted as representative of the group, that important fact can be established best by making several sets of measurements and comparing them. Doubts concerning the trustworthiness of the original data add weight to these considerations. To dispel or confirm doubts we look for series compiled by two or more authorities, or series that record different aspects of the same activity. In so doing we are guided by the belief that comparison of the results yielded by different series relating to a given group of activities is the surest way of judging whether the measurements represent typical characteristics of cyclical behavior.[9] For example, we cannot be certain that an analysis of wheat harvests in the United States will give a representative picture of the cyclical behavior of agricultural production. To make sure, we examine also wheat crops in other countries, add studies of other crops, and investigate the output of animal husbandry. Our simple analysis of some forty series showing production by farmers, supplemented by numerous series on the acreage planted and the acre-yields of leading crops, also by the prices, sales, stocks, exports and imports

9 Cf. Ch. 12, Sec. V.

of farm products and their processed derivatives, gives more trustworthy knowledge of the relation of agriculture to business cycles than could be achieved by expending equal effort on two sets of cyclical measures—one based on the 'original' data and the other on the best attainable isolation of the cyclical component—for perhaps only ten or twelve series.

Our desire to cover as many cycles as possible in each of several countries makes still more formidable the task of a double analysis. That desire has grown stronger as we have become increasingly familiar with the behavior of time series. "Strictly speaking, every business cycle is a unique historical episode, differing in significant ways from all its predecessors, and never to be repeated in the future."[10] The like is true of every specific cycle in every economic activity. It is part of our task to learn the respects in which and the degree to which business cycles and specific cycles vary: in particular, whether there are secular and perhaps cyclical changes in their duration and intensity, as well as the irregular variations that everyone recognizes.[11] Obviously, long series are necessary for such studies. And we can no more discover what are the uniform than what are the variable characteristics of our phenomena unless numerous instances are observed.

The bulk of materials required by our concept of business cycles not only rules out two analyses of each series; it also puts a premium upon simplicity in analysis. Fortunately the measures made from data adjusted only for seasonal variations promise to be more useful in explaining business cycles than would measures made from highly fabricated data. The force of this statement will become clearer after the reader has studied Chapters 7-8, where the influence of trend adjustments and smoothing operations upon our measures of cyclical behavior is analyzed;[12] but its full justification must await the theoretical analysis of the final volume.

IV Treatment of Seasonal Variations

While we consider it desirable to economize effort in handling secular trends and random movements, experience has taught us not to economize effort by working with annual data. As Chapter 6 shows in detail, annual data are exceedingly crude materials for comparing the cyclical behavior of different activities in the same period or of the same activity in different periods. They obscure timing relations, they make it impossible to trace cyclical patterns with confidence, often they obscure and sometimes they obliterate cyclical fluctuations. For these reasons our

10 See the section on The Distinctive Character of Each Business Cycle, in Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 354-7.

11 This problem is tentatively considered in Ch. 9-12.

12 Cf. also Ch. 10, Sec. VIII.

analysis, excepting crop harvests, is based chiefly on monthly and quarterly data; annual data are used only as a last resort.

Monthly and quarterly records are more difficult to compile and to analyze by our standard technique than annual data; they also impose the burden of removing seasonal fluctuations. A preponderant number of monthly and quarterly series show an unmistakable seasonal swing, that is, a repetitive intra-annual fluctuation. Although techniques of seasonal measurement have been greatly improved in recent years, the problem of adjusting time series for seasonal variations remains very troublesome in practice. Hard decisions must be made at every turn: Is the series characterized by a genuine seasonal movement? If definite evidence of seasonality does exist, how should it be measured? What period should the computation cover? Should a constant or a shifting seasonal index be constructed? If a shifting seasonal index seems preferable, may it remain constant over short segments of the data or should it change from year to year? Should the seasonal index be of the 'absolute' type, of the 'relative' type, or some cross between the two? If a relative seasonal index is used, by what method should it be constructed? Should the index be adjusted, and in what manner, for variations in seasonal amplitude? After these questions have been settled, it is still necessary to decide how to remove the seasonal variations and, most important of all, to judge the success of the seasonal adjustment.

A statistician who has struggled with seasonal adjustments of numerous time series is not likely to underestimate the part played by 'hunch' and 'judgment' in his operations. If pressed hard, he may admit that what he does, and not too satisfactorily in many instances at that, is merely to erase the repetitive intra-annual tendency that he observes in the original figures. Seasonally adjusted figures no more show what the behavior of time series would have been in the absence of seasonal forces than indexes of physical production show what production would have been in the absence of changes in relative prices; or than figures on per capita income show what income would have been in the absence of income inequalities. But although it is difficult to give a satisfactory theoretical interpretation of seasonal adjustment, this operation like any other can be subjected to a pragmatic test. In the last resort, its significance must be judged by the results to which it leads. That the removal of seasonal variations can facilitate analysis of the interrelations of cyclical movements in different activities will be plain to any reader who takes the trouble to study Chart 3, which depicts the movement in recent years of industrial production, railroad freight traffic, and department store sales in the United States, before and after adjustment for seasonality. The seasonal variations follow an individual course in each series, and obscure the protracted fluctuations in freight traffic and department store sales. When the variations associated with the seasons of the year are removed, the underlying fluc-

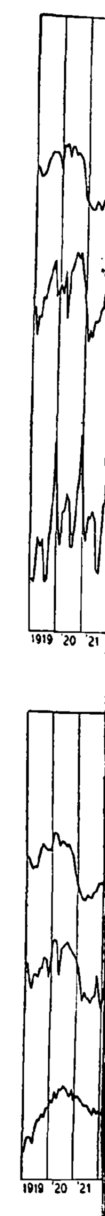
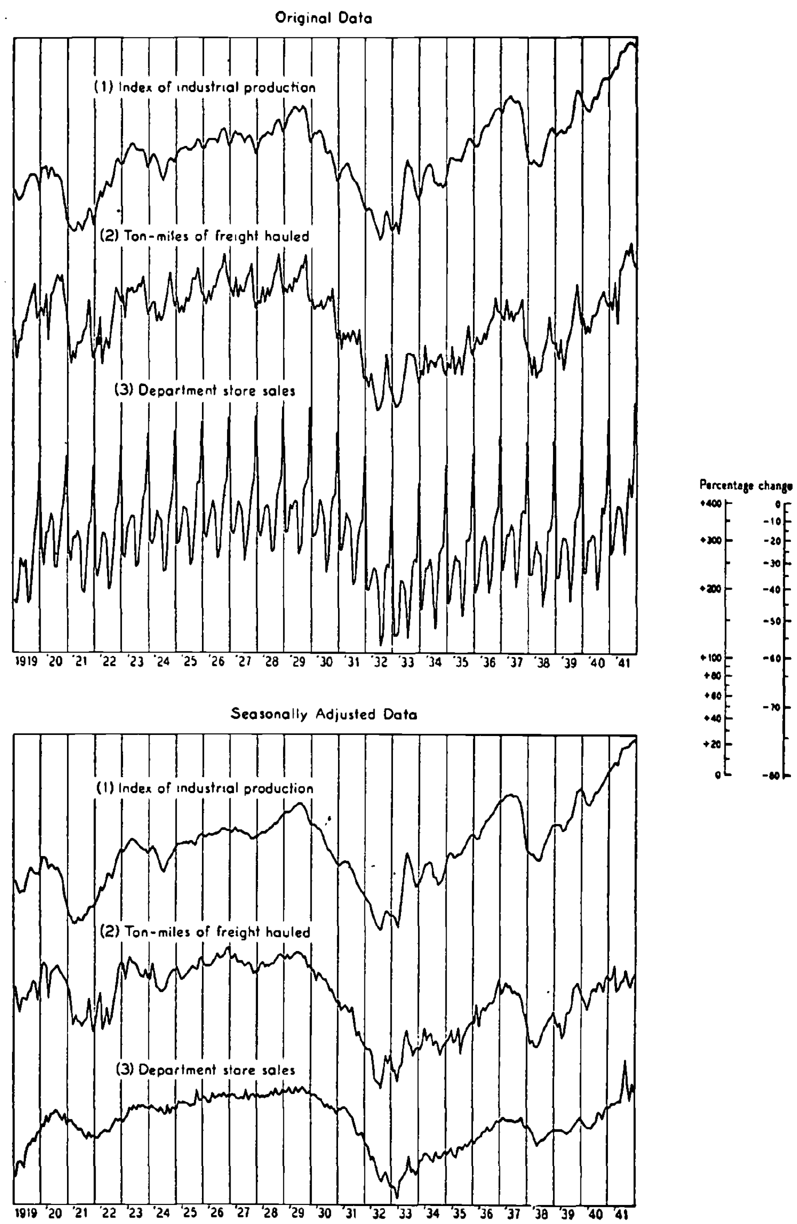


CHART 3
Original and Seasonally Adjusted Data
Three American Series, 1919 – 1941



For sources of data, see Appendix C.

tuations in economic fortune come clearly to the surface and can be readily traced in the several series.

The seasonal variations of some series are removed by their compilers, but in most instances we have had to carry through this operation ourselves. This step in the analysis precedes all others and often takes as much time as the subsequent operations put together. We have measured seasonal variations by a variety of methods, partly because different series require different types of adjustment, partly because our methods changed as we gained experience in using them. A brief description of the methods we currently use will have to suffice; they differ little from the techniques explained in treatises on statistics.[13]

In ascertaining the seasonal variations of monthly data we use two principal methods. Method (1) consists in taking averages of the original figures for successive Januaries, for successive Februaries, and for each of the other months, then adjusting these monthly averages for secular trend. Method (2) entails taking a twelve-month moving average of the original figures, placing each average in the seventh month of shifting twelve-month intervals, expressing the original figures as ratios to the 'centered' moving averages, and striking averages of these ratios, first for the Januaries, then the Februaries, and so on.[14] In applying each method we attempt to protect the twelve monthly averages from distortion by extreme items. Sometimes we compute two or three sets of positional means and select that set which includes the largest number of items without including any extreme item; at other times we drop extreme items or years at the start—that is, before striking averages. In either case the twelve monthly averages are adjusted so that their sum equals 12.[15] When the data come by quarters, the general procedure is the same, but the sum of the values of the seasonal index equals 4. When it is obvious that the amplitude of seasonal fluctuations is more nearly constant in absolute than in percentage units we vary the method; that is, the final seasonal index is expressed in units of the original data, the algebraic sum of the twelve monthly terms being zero.

Our two principal methods of constructing seasonal indexes rest on the same logic: to ascertain the seasonal variations of a time series, all nonseasonal movements must be eliminated. By averaging the arrays for successive Januaries, Februaries, and so on, we allow the random move-

[13] See also Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 233-49.

[14] In practice we use ratios to twelve-month moving totals instead of to moving averages. The two yield the same final results, but the former is a more economical method of calculation. A thirteen-month moving average centered on the seventh month, the first and last months receiving half weight, is preferable in principle to a simple twelve-month moving average; but experiments by one of our colleagues, Julius Shiskin, have demonstrated that the gain is negligible and definitely is not worth the extra cost. Even a simple four-quarter moving average centered on the third quarter is not appreciably inferior to a five-quarter moving average in which the two end quarters receive half weight each.

[15] See Note 1 of the Appendix to this chapter.

ments of a series to cancel one another. The larger the number of observations the better are the chances that the random movements will cancel out in fact; hence extreme values are dropped only when there is fairly clear indication that they distort the averages. Each method rests on the assumption that the process of averaging will tend also to make the cyclical components of a series sum to zero. This desideratum is likely to be met better by method (2) than by method (1); for in (2) we average values each of which contains, as a rule, merely a small portion of the cyclical component, while in (1) we average values that contain the full cyclical component. In applying each method we analyze periods covering whole years instead of periods covering whole cycles. The former practice reduces the chances of eliminating the cyclical component when method (1) is used, but it helps to prevent distortion of the seasonal index by the secular component of the series. In method (1) the secular component is eliminated by adjusting the month-by-month averages of the original data for the average monthly increment of a linear trend; while in method (2) the adjustment for trend is automatically accomplished by expressing the original data as ratios to twelve-month moving averages. The elimination of the secular movement is likely to be more successful in method (2), which takes account of a trend of any degree of flexibility.

Method (1) is obviously less laborious, but method (2) is superior on most other counts. Hence we confine method (1) to series having seasonal variations that seem both clear-cut and fairly constant over many years. When seasonal variations are obscured by other types of fluctuation, or when they seem to change materially over time, we begin the seasonal analysis by expressing the original figures as ratios to twelve-month moving averages. The ratios are then studied with a view to finding periods, if any, during which the seasonal variations were fairly uniform. The periods selected rarely cover less than eight or more than fifteen years. The operations called for by method (2) are then performed separately upon each segment. The final seasonal measurements sometimes seem excellent, more often they are merely tolerable. For some series the best seasonal indexes we can make after much labor are poor approximations. Especially troublesome are the cases in which the timing of what seem to be seasonal peaks and troughs varies irregularly from year to year by a month or two.

In some series the pattern of seasonal variations shifts from one year to the next in regular fashion; in some others the pattern of what seems to be the seasonal movement remains constant but the amplitude varies sharply from year to year. When series of these types are encountered, we do not construct 'constant' seasonal indexes. To take care of the first group we compute 'moving' seasonal indexes; to take care of the second we compute seasonal indexes that are constant in pattern but vary in amplitude from year to year.

Our method of constructing 'moving' seasonal indexes consists of the following steps: (1) a twelve-month moving average of the original data is taken and placed in the seventh month; (2) the original monthly figures are expressed as ratios to the centered moving averages; (3) the ratios for successive Januaries are arranged in chronological order, so also are the ratios for the Februaries and the other months; (4) these figures are plotted on twelve diagrams, one for each month; (5) a moving average, usually covering five points, is taken of the ratios for successive Januaries, and likewise for each of the other months; (6) the moving averages for the Januaries are plotted on the diagram showing the ratios for successive Januaries, and the moving averages for other months are plotted on the other monthly diagrams; (7) the lines of the moving averages are smoothed freehand and are extended to cover the initial and the terminal years for which there are no moving averages; (8) the values for the months of each year are read from the smoothed curves; (9) if the sum of the monthly values for each calendar year is not equal to 12, the smoothed curves are experimentally adjusted to produce this result.

The series in which the amplitude of the seasonal swings varies sharply and irregularly from year to year are treated on a different plan. The first step is to make a 'constant' seasonal index by one of the methods described above. The seasonal observations [16] for a given year are then adjusted so that their sum is 12. Next, the constants of a 'least squares' straight line are computed for the year, the seasonal index being treated as the independent variable and the adjusted seasonal observations for the year as the dependent variable. Finally, monthly values for the year that correspond to the monthly values of the seasonal index are computed from the equation of the straight line; this step yields a seasonal index having the same pattern as the constant seasonal index but a different amplitude. These operations are repeated for each year covered by the series, or each year of that portion of the series subjected to a seasonal correction of shifting amplitude.

This refinement upon current practice has been employed extensively by Simon Kuznets,[17] who refers to the slope of the straight line just described as the 'amplitude ratio'. As Kuznets points out, the amplitude ratio rests on the hypothesis that the nonseasonal movements within each year are uncorrelated with the seasonal movements. The amplitude ratio cannot be used properly except when the seasonal observations for a given year are positively correlated with the constant seasonal index; we do not employ it when the coefficient of correlation between the two is less

[16] That is, the values averaged to make the seasonal index: the original data in method (1), ratios to moving averages in method (2).

[17] See his Seasonal Pattern and Seasonal Amplitude: Measurement of Their Short-time Variations, *Journal of the American Statistical Association*, March 1932; and *Seasonal Variations in Industry and Trade* (National Bureau of Economic Research, 1933), pp. 322-42.

than +.70. The amplitude ratio may break down completely, as when the calculated seasonal index for some month is negative.[18] Even over the range for which the amplitude ratio is serviceable, it makes merely approximate adjustments possible. We believe, nevertheless, that if used judiciously, this device improves the results.[19]

Having constructed by one or another method measures of seasonal behavior, we proceed to free the series from their seasonal variations. As a rule we follow the standard procedure; that is, the original figure for each month is divided by the seasonal index for the month.[20] This operation implicitly assumes that the seasonal correction, regarded as an additive term, depends not only on the trend and cyclical components but also on the random component—an assumption not easy to justify and one that sometimes leads to very poor results. To meet this difficulty, we occasionally substitute for the standard method of eliminating seasonal variations another method based on the assumption that the seasonal and random components are independent. This method involves (1) multiplying a centered twelve-month moving average of a series by its seasonal index, month by month, (2) subtracting these products from the original figures, (3) adding the resulting differences to the twelve-month averages. The first step estimates the nonrandom component of the series, the second estimates the random component, the third estimates the nonseasonal component by combining the trend-cycle and random components.[21] The results yielded by this method will not diverge appreciably from those obtained by using the standard method of eliminating seasonal variations, except when the seasonal and random movements are both very large.

After the laborious process of eliminating the seasonal variations from a time series has been carried out, two tests are applied before we are content to leave the matter of seasonality. First, the mean annual 'level' of the seasonally adjusted data is compared with that of the original data. If a discrepancy of more than 10 per cent turns up in any twelve-month interval, we consider the adjustment unsatisfactory and resort to new

18 This difficulty, however, may be avoided by making the computation in terms of logarithms.

19 The case for applying an amplitude ratio is clearest when the activity represented by a series has a 'natural' business year, with a definite beginning and end, as in movements of products from farms. The amplitude ratio should then apply to the natural year. In the absence of a natural year, there is no basis other than convention for selecting the boundaries of the year; the amplitude ratio and the final seasonal adjustment will then vary with the boundaries selected.

In some series, what look like enormous differences in seasonal amplitude are confined to the values in the immediate vicinity of the seasonal troughs or the seasonal peaks and are not characteristic of the month-by-month differences. If the troughs vary considerably in intensity from year to year, but not the peaks, the simple procedure described in Note 2 of the Appendix to this chapter (method B) is likely to yield better seasonal adjustments than do 'amplitude ratios'.

20 This is the standard procedure for a 'relative' seasonal index. In the case of an 'absolute' seasonal index (one expressed in units of the original data, the algebraic sum of the twelve monthly terms being zero), the index for each month is subtracted from the original value for the month.

21 For a fuller description, see Note 2 of the Appendix to this chapter.

devices, such as computing the seasonal index on the basis of a slightly different period, replacing a constant seasonal index by a changing seasonal index, substituting our modified method of removing seasonal variations for the 'standard' method, or discarding a seasonal index expressed in percentage units in favor of a seasonal index expressed in units of original data. Second, the adjusted figures are scrutinized to see whether they still show similar monthly movements in successive years, or movements correlated with the seasonal index.[22] If not, we deem the operation successful and pass on to the measurement of cyclical behavior. If clear traces of seasonal variations do remain, we must find what is wrong or deficient in the calculation and try again. Perhaps the correction for secular trend was too mechanical; perhaps method (1) was applied to data that call for the more elaborate treatment of method (2); perhaps the need of working with shorter periods was overlooked, or the need of moving seasonal indexes or seasonal indexes of constant pattern but varying amplitude. A second or third trial usually yields results we are willing to accept. But there remain instances in which we feel far from satisfied with the seasonal indexes as we have left them.

To free time series from seasonal variations is not the sole use to which we put indexes of seasonal variations. At a later stage we plan to compare seasonal movements with cyclical behavior. Meanwhile our worksheets show the average amplitude of the seasonal movements during the period covered by a series. The average amplitude is measured in two ways: by the range [23] and by the average deviation of the seasonal index. When more than one seasonal index is computed for a series, we take a weighted mean of the average amplitudes of whatever seasonal indexes are used, the weights being proportionate to the number of years to which each seasonal index is applied. In the few instances where the seasonal index is expressed in units of the original data, it must be converted into percentage units before calculating the average amplitude.

22 See Note 3 of the Appendix to this chapter.

23 Strictly speaking, we take the rise of the seasonal index from trough to peak, plus the fall from peak to trough. This equals twice the range. Of course, the erratic component that remains in the seasonal index tends to increase the range.

APPENDIX

Notes on the Elimination of Seasonal Variations

Note 1

The reasons for adjusting a monthly seasonal index so that the sum of its twelve terms equal 12 should be made explicit. This note attempts to show why the sum is 12, or 1200 in percentage terms, rather than some other number.

Let o_{ij} and n_{ij} stand respectively for the original and seasonally adjusted figures in the i^{th} month of the j^{th} year, and let s_i stand for the seasonal index in the i^{th} month.

In the standard method of removing seasonal fluctuations, the original figure for a month is divided by the seasonal index for the month; that is, $n_{ij}=\frac{o_{ij}}{s_i}$, or $o_{ij}=n_{ij}s_i$. The mean value of the original figures during any twelve-month interval is $\frac{\Sigma o}{12}$ or $\frac{\Sigma ns}{12}$. The mean value of the seasonally adjusted figures during the year is $\frac{\Sigma n}{12}$. The relation between the two means is as follows:

$$\frac{\Sigma ns}{12}\cdot\frac{12}{\Sigma s}=\frac{\Sigma n}{12}+\frac{r_{ns}\sigma_n\sigma_s}{\bar{s}} \tag{1}$$

Here σ_n is the standard deviation of n during the year, σ_s the standard deviation of the seasonal index, r_{ns} the coefficient of correlation between n and s during the year, and $\bar{s}$ the mean value of the seasonal index.[24]

As may be seen from the above expression, $\frac{\Sigma ns}{12}=\frac{\Sigma n}{12}$ if two conditions are fulfilled: (i) $r_{ns}=0$, (ii) $\Sigma s=12$. Now, if seasonal variations have been eliminated properly, we should expect n and s to be uncorrelated. In any given year their numerical values may be correlated positively or inversely. But it seems clear, *a priori,* that in any given series the average value of r_{ns} (over a moving twelve-month interval) will closely approximate zero. The average value of r_{ns} for many series will approximate zero more closely still. It follows that the sum of the seasonal index must equal 12 in order that the *expected value* of the sum of the seasonally adjusted figures for a year equal the sum of the original figures for the year.[25]

[24] G. U. Yule, *An Introduction to the Theory of Statistics* (Charles Griffin, London, 1927), p. 221.

[25] If an 'absolute' seasonal index is used, $n=o-s$. For any year, $\frac{\Sigma n}{12}=\frac{\Sigma o}{12}-\frac{\Sigma s}{12}$. Hence if Σs is made 0, $\frac{\Sigma n}{12}$ will equal $\frac{\Sigma o}{12}$.

Note 2

Call the standard procedure of eliminating seasonal variations 'method A', and the alternative procedure sketched in the text 'method B'. Let the seasonally adjusted figures be represented by n_{ij} when method A is used and by n'_{ij} when method B is used.

Method A assumes that $o_{ij} = n_{ij}s_i$.

Method B assumes that $o_{ij} = m_{ij}s_i + R'_{ij}$, where m represents a composite of the trend and cyclical components of the series, and R' the random component. A rough estimate of m can be obtained by taking a centered twelve-month moving average of the original figures and that is the method we use in practice; but the argument that follows does not depend on this or any other specific method of estimating m. We assume R' to be independent of s. Whether or not R' should also be treated as independent of m is critical for the problem of obtaining an optimum estimate of s. But we are not concerned with that problem at present; we take s as given, and merely consider the consequences of alternative methods of removing s.

From the above definitions it is evident that $n_{ij} = \frac{o_{ij}}{s_i}$, and that

$$n'_{ij} = m_{ij} + R'_{ij}.$$

There is an advantage in writing these expressions in another form:

$$n_{ij} = \left[\frac{o_{ij}}{s_i}(1 - s_i)\right] + o_{ij} \tag{2}$$

$$n'_{ij} = \left[m_{ij}(1 - s_i)\right] + o_{ij} \tag{3}$$

The bracketed expressions in (2) and (3) show the seasonal *correction*, treated as an additive term, in methods A and B respectively. The implicit seasonal *component* is obtained by reversing the sign of the bracketed expressions. Clearly, the seasonal correction in method B is proportional to the trend-cycle component alone, while in method A it is proportional to the seasonally adjusted figure which includes the random component besides the trend-cycle component.

These relations can be put another way. The random component is the seasonally adjusted figure minus the trend-cycle component. Hence in method B the random component $(R'_{ij}) = n'_{ij} - m_{ij} = o_{ij} - m_{ij}s_i$. In method A the (implicit) random component $(R_{ij}) = n_{ij} - m_{ij} = \frac{o_{ij} - m_{ij}s_i}{s_i}$. Algebraically, R_{ij} is the seasonally adjusted equivalent of R'_{ij}; but it should be noted that the seasonal (s) applies to plus and minus values.

We may, finally, write n and n' as follows:

$$n_{ij} = R_{ij} + m_{ij} = \left(\frac{o_{ij} - m_{ij}s_i}{s_i}\right) + m_{ij} \tag{4}$$

$$n'_{ij} = R'_{ij} + m_{ij} = (o_{ij} - m_{ij}s_i) + m_{ij} \tag{5}$$

From these expressions it is plain that the results yielded by methods A and B cannot differ much when the seasonal variations of a series are relatively small. Even when the seasonal variations are large, the results will tend to be similar so long as the random variations are small. But when the seasonal and random variations are both large, n and n' may diverge sharply. If $s_i < 1$, the variance of R_i will exceed the variance of R'_i, and R_{ij} will be greater or less than R'_{ij} according as R'_{ij} is plus or minus; the larger the gap between s and 1, the larger will be these differences. On the other hand, if $s_i > 1$, the variance of R_i must be smaller than the variance of R'_i.

The basic assumption underlying method B is that the random component is independent of s. If this condition is fulfilled, the random component will tend to be a larger part, in percentage terms, of the original values at months of seasonal trough (and immediately adjacent months) than of the original values at months of seasonal peak (and immediately adjacent months). The original values at months of seasonal trough will therefore tend to vary more, logarithmically, than the original values at months of seasonal peak. Hence, if the seasonal fluctuations of a particular series are large, and the logarithms of its original figures at months of seasonal trough vary much and irregularly from year to year while the logarithms of the figures at peaks are relatively stable, method B is preferable to method A despite the added burden of arithmetic calculations. In such a series the application of method A is bound to introduce a seasonal factor into the seasonally adjusted figures, the imprint of s on n being positive in some years and inverted in others. We have found that in series of this type the seasonal adjustment obtained through method B is ordinarily a considerable improvement upon the results got by method A; for the application of method B yields, in effect, an amplitude correction.[26]

Whether method A or B is applied to a given series, the 'level' of the seasonally adjusted figures will normally differ from the 'level' of the original figures; that is, the mean of seasonally adjusted figures for any twelve-month period will normally differ from the mean of the original figures. But method B has a stronger tendency to satisfy the 'equality condition' than method A. If method A is used, $\frac{\Sigma o}{12} - \frac{\Sigma n}{12} = r_{ns}\sigma_n\sigma_s$. This follows from equation (1), since $\bar{s} = 1$. If method B is used, $\frac{\Sigma o}{12} - \frac{\Sigma n'}{12} = \frac{\Sigma ms}{12} - \frac{\Sigma m}{12}$. But since $\bar{s} = 1$,

$$\frac{\Sigma ms}{12} = \frac{\Sigma m}{12} + r_{ms}\sigma_m\sigma_s.$$

where σ_m is the standard deviation of m for a twelve-month interval and r_{ms} the coefficient of correlation between m and s during this interval. Hence $\frac{\Sigma o}{12} - \frac{\Sigma n'}{12} = r_{ms}\sigma_m\sigma_s$. But $|r_{ns}\sigma_n\sigma_s|$ will tend to exceed $|r_{ms}\sigma_m\sigma_s|$ for two reasons: (i) σ_n is likely to be larger than σ_m since n includes the random component while m is a smoothed variable; (ii) inasmuch as m is smooth, low values of $|r_{ms}|$ are likely to rule, while the erratic values of n will now and then produce high values of $|r_{ns}|$.

[26] One technical disadvantage of method B (shared by an absolute seasonal) is that it *may* yield negative figures.

An empirical test demonstrates that method B approximates the 'equality condition' far better than method A. We applied the test to eight series, five of which were chosen because very large inequalities were known to result occasionally from the application of method A. In each series a (moving) twelve-month interval was taken as the unit of comparison. Method A yielded inequalities of 10 per cent or more in 159 instances, of 25 per cent or more in 52 instances, of 50 per cent or more in 19 instances; the largest inequality that turned up was 67 per cent. Method B yielded only eight inequalities of 10 per cent or more, the maximum discrepancy being 11.8 per cent.

A word may be added about the relevance of the 'equality condition'. The problem has been largely ignored by statisticians, and as yet has not entered the stage of open debate. But one way or another, the equality condition is involved in much of the work done on seasonal measurements. As explained in Note 1, this condition underlies the practice of making $\Sigma s = 12$. Again, the equality condition is the only logical defense for not recognizing a seasonal problem in work on annual figures. The statistician who disavows this condition has a seasonal problem on his hands even when he is concerned exclusively with the behavior of annual figures. For from his point of view, it may be desirable to remove seasonal variations from monthly or quarterly figures, and operate with annual sums or averages of the seasonally adjusted figures instead of annual sums or averages of the original figures.

If the 'equality condition' were made a crucial test of the goodness of seasonal adjustment, the standard method of removing seasonal variations (method A) would be ruled out from the start. That would be embarrassing in practice, since 'relative' seasonals usually give satisfactory results when judged by other criteria. The method we have followed represents a compromise, which seems wise in the present rough state of our knowledge of seasonal variations. Other things equal, we regard a seasonal adjustment as satisfactory in the degree to which the equality condition is fulfilled. But there is a point beyond which we are not willing to tolerate differences in the 'level' of a series produced by seasonal adjustments. Our limit of 10 per cent for a twelve-month interval is one of those practical rules for which no more can be said than that it is convenient, leads to recomputation infrequently, and yet yields results that usually look sensible.

Note 3

A common method of judging the goodness of a seasonal adjustment is to see whether the adjusted figures show similar movements in successive years. If a positive (or inverse) correlation exists between n and s, year by year, over the entire period covered, or if the correlation is positive at one end of the period and inverse at the other, it is plain that the seasonal adjustment is defective.[27] But it is more difficult to judge the seasonal adjustment if n and s do not behave in this fashion, yet are correlated in individual years. Some correlation, positive or inverse, is practically bound to exist; that is why the 'equality condition' is not met by method A (see the preceding note). The correlation

[27] Here n stands for seasonally adjusted data, irrespective of the method used in their derivation.

may result from a poor seasonal adjustment but that need not be the case: there is no good reason for assuming that n and s are invariably uncorrelated in single years.

It may be well, therefore, to put the trend-cycle component out of sight, and to judge the goodness of a seasonal adjustment by comparing s with an estimate of the random component of the series. In any single year the patterns of s and of the random component may be correlated; but that is not very likely to happen. Of course, the 'isolation' of the 'random component' is beset with grave difficulties. But if several methods of seasonal adjustment are tried, there will be as many (explicit or implicit) estimates of the random variations. The behavior of these estimates may then be compared with s, year by year, and any correlation between the two noted. The emergence of such correlations must count against the method that yields them.

Other things equal, that method of seasonal adjustment which involves the 'best' estimate of the random component may be regarded as yielding also the 'best' estimate of the nonseasonal movement of the series. For example, methods A and B, described in the preceding note, involve different estimates of the random component. Now, if R were truly a random series, we should expect the variance of R_i to equal the variance of R_{i+1}, of R_{i+2}, etc. The like applies to R'. (The variances should be measured from zero.) In any actual case the twelve variances of R will almost certainly not be the same, nor will the twelve variances of R'. But a preference may be expressed for R or R' according as the one or the other shows substantially smaller variability, when judged, say, by the coefficient of variation of the twelve variances.

This note is intended merely to be suggestive. The problem raised requires careful exploration.

CHAPTER 4

Dating Specific and Business Cycles

Once a time series has been adjusted for seasonal variations, it is ready for analysis on the plan sketched in Chapter 2. That plan, it will be recalled, consists of two parts. Each series is broken into segments corresponding in time to successive business cycles in the country to which the series relates, and the principal characteristics of these segments are measured. Next, the series is broken into segments corresponding to its own specific cycles, and their characteristics are measured. But before the two sets of measurements can be made, the specific and business cycles must be identified and their turning points dated. This chapter is concerned with the technical problem of dating cyclical fluctuations.

I Dating Specific Cycles

To determine whether a time series has specific cycles, and if so, to fix the dates when each cycle began, culminated, and ended, we plot the data, both in their original form and after adjustment for seasonal variations, upon a semi-logarithmic chart and study the whole record in this graphic form. A typical chart of monthly data covering sixty years is about 7 feet long; its width ranges from about 1 to 3 feet, depending mainly on the size of the secular movement. As far as possible the scales are kept uniform. They are varied only in handling annual data, where we compress the horizontal scale; in monthly or quarterly series having exceedingly violent cyclical amplitudes, where we compress the vertical scale; and in the few series with plus or minus values, where the vertical scale eludes standardization.[1]

When charted, almost all monthly and quarterly series show cyclical

[1] The rigidity of the printed page has compelled us to vary rather freely the horizontal and vertical scales, both absolutely and relatively to one another, in the historical charts of this book.

fluctuations, but they are more or less obscured by secular and erratic movements. Our basic criterion for distinguishing these three types of movement is their duration. We conceive of secular trends as drifts toward higher or lower levels that persist in a given direction for periods long in relation to business cycles.[2] Erratic movements appear as alternations of rise and fall that cover usually a few months; the 'saw-tooth contour' characteristic of indexes of business conditions is still more characteristic of series that represent a narrower range of activities.[3] Intermediate between the persistent drifts that often cover decades and the oscillations that occur every few months, there appear in most series well-defined movements of rise and fall, the duration of which from trough to trough and from peak to peak is rarely less than two or more than seven years. These fluctuations varying in duration 'from more than one year to ten or twelve years' are our specific cycles; that is, they are fluctuations of the same order of duration as business cycles.

Occasionally the specific cycles of a series are superimposed upon cycles lasting from fifteen to twenty-five years. Such long waves are typical of building construction; they occur also in certain other activities, the full list of which we do not yet know. When we find specific cycles superimposed upon long cycles, both sets are marked off and analyzed on the same principles.[4] If we found only long cycles in a series we would analyze them, but not call them specific cycles.

The task of identifying specific cycles in a time series is easy or difficult according as their amplitudes are large or small in comparison with those of erratic and secular movements. The relative magnitude of the three types, and hence the difficulty of distinguishing among them, differs widely in the processes we analyze. For example, the specific cycles in employment and disbursements of wages can almost always be recognized with ease, while the specific cycles in security flotations are often obscured. In some series the specific cycles are hard to distinguish because of the difficulty of eliminating large and shifting seasonal fluctuations; for example, inventories of farm products. But in a large majority of our series the cyclical fluctuations stand out clearly on the charts.

When specific cycles are made doubtful by random movements, we smooth the data by moving averages and base judgments upon the curve of moving averages. When the secular trend rises sharply, we allow brief and mild declines to count as contractions of specific cycles. Similarly, when the secular trend falls sharply, brief and mild rises are counted as specific-cycle expansions. We do not recognize a rise and fall as a specific

2 See Arthur F. Burns, *Production Trends in the United States since 1870,* Ch. II.

3 Mitchell, *Business Cycles: The Problem and Its Setting,* pp. 329-30. See also Mitchell and Burns, Statistical Indicators of Cyclical Revivals (National Bureau of Economic Research, *Bulletin 69,* May 28, 1938), pp. 4-8.

4 See Ch. 11, Sec. I and VII.

cycle unless its duration is at least fifteen months, whether measured from peak to peak or from trough to trough. Fluctuations lasting less than two years are scrutinized with especial care; they are not treated as specific cycles unless they are clearly defined and in no sensible part a result of faulty adjustment for seasonal variations. The lower limit of the range of amplitudes of all fluctuations that we class confidently as specific cycles is our rough guide in deciding whether any doubtful fluctuation, of long or short duration, is well enough defined to be accepted as a specific cycle. Our general practice is to identify specific cycles without consulting the 'reference dates of business cycles', which are explained later in this chapter. We depart from this rule only in treating series that conform with great regularity to business cycles. In such series we first determine whether there are any lapses from one-to-one correspondence between specific and business cycles. If there are and they are due to our failure to class movements close to the borderline of our rules as specific cycles, the rules are relaxed and these movements accepted as specific cycles.[5]

Once the specific cycles have been distinguished we proceed to date their turning points. When the cycles are clear in outline, our practice is to take the lowest and highest points of the plotted curves as the dates of the cyclical turns. When the crests or troughs are 'flat', the latest month in the horizontal zone is chosen as the turning date.[6] The chief difficulties arise when erratic movements are prominent in the vicinity of a cyclical turn. Then we examine the several competing peaks or troughs to determine whether any are due to inadequate seasonal correction. That question settled, let us say for the peaks, we compare the average levels of several months centered on each potential peak and select as the actual peak the highest point in the cluster having the highest average level. If the averages of several clusters are approximately the same, we give preference to the highest point in the latest cluster, provided the movement in the period spanning the multiple peaks is not clearly downward.[7] Finally, if the series is especially choppy in the turning zone, moving averages are used to help determine the month of the peak or trough. We rarely deviate from these rules. The main exceptions come when an isolated high point occurs many months before or after the general contour of a curve indicates a cyclical peak, or when an isolated low point occurs many months before or after the general contour indicates a cyclical trough. We aim especially to disregard such extreme isolated

[5] See the illustrations on pp. 314-5.

[6] Horizontal movements may be an intrinsic feature of the data (as in central bank discount rates) or merely a technical effect of rounding. In the latter case the rule in the text imparts a bias, in the sense that turns dated from, say, three-digit figures must coincide with or come later than turns dated from four-digit figures of the same series. This point is usually of slight practical importance, but bears watching when refined comparisons are attempted.

[7] See also pp. 148, 346.

values as we know are associated with strikes, tariff changes, or other random events.

An illustration or two may make the procedure clearer. In a series such as coke production in the United States (see Chart 1 and Table 4) the specific cycles stand out clearly and their dating is easy. One possible doubt is with respect to the movement from June 1917 to January 1918. We do not treat this contraction as a cyclical movement. In the first place, the dip is slight except for the low values in January and February 1918, which reflect the disturbance in railroad operations and in the iron industry caused by extreme winter weather. In any event June 1917 is only thirteen months from the cyclical peak recognized in July 1918, too

TABLE 13

Bituminous Coal Production, United States, 1905–1939

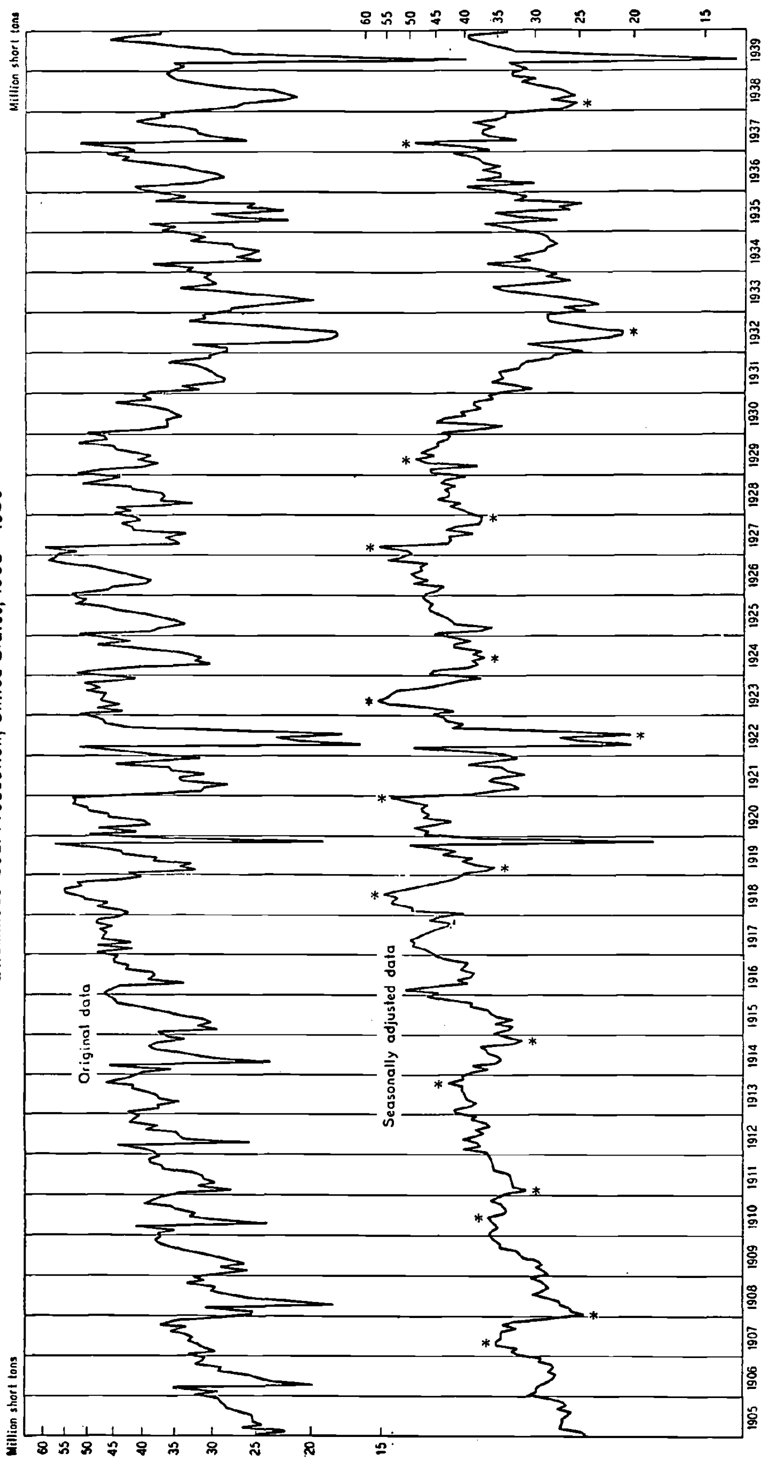
(Millions of short tons)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1905	24.2	24.7	26.9	26.7	26.6	26.6	25.9	25.6	27.1	26.1	26.5	28.1
1906	30.6	29.5	29.5	29.1	28.0	27.5	27.2	28.1	27.5	27.6	29.2	28.9
1907	31.9	32.8	31.8	34.8	34.8	34.1	34.2	34.1	32.1	34.0	32.2	28.2
1908	24.3	25.6	25.8	26.9	26.9	28.2	30.0	28.8	28.1	28.5	29.0	30.2
1909	28.1	28.7	29.5	28.8	29.4	31.2	31.7	32.1	34.3	34.2	34.9	35.6
1910	35.8	35.4	34.5	35.1	35.7	36.0	33.7	33.3	33.7	34.1	35.6	34.6
1911	33.3	30.8	32.4	32.5	32.7	32.8	33.1	35.2	35.3	35.4	35.6	35.9
1912	36.2	39.8	37.1	37.6	40.0	37.7	36.5	37.8	35.6	36.2	38.5	37.7
1913	41.1	41.2	38.3	37.6	39.2	39.4	39.7	40.0	39.8	42.4	39.6	39.9
1914	38.3	35.9	38.2	34.7	34.0	34.5	36.1	36.3	37.1	32.5	31.2	33.6
1915	35.1	32.9	32.4	34.9	32.5	34.7	35.6	35.9	38.7	38.4	43.0	46.3
1916	44.0	50.8	44.7	39.1	40.8	38.5	38.1	40.3	39.7	39.0	43.2	44.5
1917	45.3	46.5	48.9	48.7	49.6	47.8	46.3	44.7	42.5	42.0	45.9	44.4
1918	39.8	49.2	49.1	53.5	53.1	52.1	55.0	52.0	48.3	45.5	42.2	40.6
1919	39.8	36.1	35.0	38.0	40.2	38.5	43.4	41.1	45.5	49.7	18.3	37.6
1920	46.9	46.2	48.9	45.1	41.9	47.0	46.0	47.2	47.4	46.3	50.6	53.8
1921	38.8	35.4	31.7	32.8	35.9	35.3	31.0	33.3	33.9	38.9	35.4	31.9
1922	33.8	40.4	49.0	20.1	25.0	26.9	20.2	27.6	41.7	39.9	43.0	43.3
1923	45.1	41.5	45.7	54.4	56.8	54.8	53.7	52.2	46.9	43.5	40.7	37.1
1924	45.7	45.0	39.0	37.5	38.5	36.6	38.3	37.0	41.6	41.4	38.6	41.7
1925	44.9	37.0	35.3	41.4	42.0	43.0	45.3	46.0	45.7	45.2	46.3	47.3
1926	46.3	44.0	43.1	49.0	46.1	48.4	49.5	47.4	47.6	46.2	54.3	51.5
1927	49.3	50.2	56.5	42.6	42.0	42.4	38.5	42.8	41.0	37.4	37.2	37.0
1928	39.0	40.0	42.2	40.4	44.3	42.4	42.4	43.1	41.2	43.8	42.9	39.7
1929	45.6	45.8	37.8	46.4	48.7	45.1	47.6	46.1	44.4	44.6	42.7	42.3
1930	43.8	38.2	34.2	44.8	43.3	39.7	40.5	37.2	38.3	38.2	35.4	36.2
1931	33.8	30.2	32.3	35.6	34.0	34.3	34.6	31.9	31.7	30.9	27.9	27.6
1932	24.6	27.0	30.8	25.4	22.1	20.9	20.8	23.5	26.2	28.3	28.4	28.4
1933	24.3	26.6	23.0	24.4	26.8	29.7	34.1	35.5	29.1	25.9	28.6	27.3
1934	29.1	31.1	36.3	30.5	32.6	30.1	28.6	28.4	27.4	28.2	28.4	29.3
1935	32.3	33.4	36.8	27.3	32.1	35.2	25.9	27.1	24.8	32.6	30.9	32.3
1936	35.0	39.5	30.0	38.0	34.3	34.4	37.1	34.5	37.0	37.5	39.0	41.8
1937	36.0	40.7	49.0	32.3	36.1	37.2	37.0	35.3	38.6	35.1	33.7	33.7
1938	27.0	26.1	25.2	28.0	25.4	26.2	26.9	29.6	31.7	29.9	32.9	32.9
1939	30.9	32.3	33.3	13.2	21.3	32.4	33.4	35.8	37.5	39.3	39.3	33.6

Adjusted for seasonal variations. The original data for 1905–22 come from the Geological Survey, *Mineral Resources of the United States*, 1922, Part II, pp. 464-5; for 1923–37, Bureau of Mines, annual numbers of *Mineral Resources* (since 1932 *Minerals Yearbook*) through 1939; for 1938–39, *Survey of Current Business*. (Slight revisions in the figures for 1938 and 1939 made later by the Bureau of Mines are ignored.)

CHART 4

Bituminous Coal Production, United States, 1905 – 1939



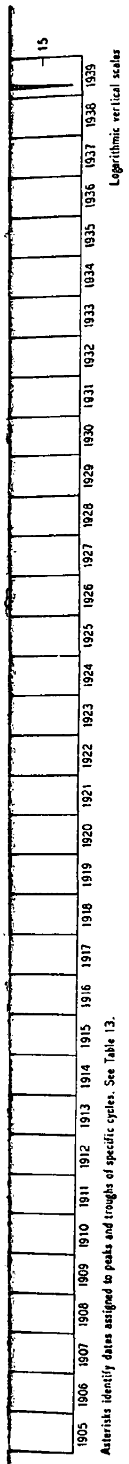
Asterisks identify dates assigned to peaks and troughs of specific cycles. See Table 13.

short an i
from Janu
tion. This
tions: Jan
February
recognized

A bett
cific cycles
(see Table
tions and
variations
impossible
apparent t
and this fa
operated u
'even' year
renewal, th
tions, and
In many m
new agreem
curve of pr
by July or
differences
are sufficie
(Chart 5).
which speci
(Chart 4).

Some o
three succe
specific cycl
the second
March 1919
nitude to b
declines als
May 1917 t
ment. The
1917; the us
notable dro
War I came
traffic and

8 'Peace' and 'st
of strike statisti
state Commerce
chronicle of the
late called *Mine*



short an interval to pass as a specific cycle under our rules. The decline from January to July 1925 also fails to qualify as a specific-cycle contraction. This movement is mild and does not meet our rule concerning durations: January 1925 is only thirteen months from the peak recognized in February 1926, while July 1925 is only twelve months from the trough recognized in July 1924.

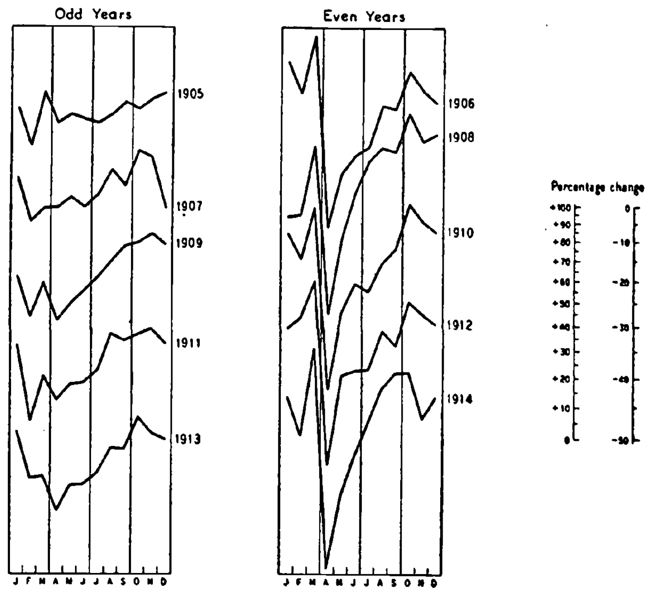
A better illustration of the problems encountered in identifying specific cycles is afforded by bituminous coal production in the United States (see Table 13 and Chart 4), an industry notorious for its seasonal fluctuations and labor disputes. A preliminary attempt to eliminate seasonal variations by our standard devices yielded such poor results that it was impossible to date the specific cycles reliably. After a little study it became apparent that at least from 1904 to 1914 strikes had a clear periodicity, and this fact provided a clue to analysis. A large section of the miners operated under two-year agreements, which expired April 1 of every 'even' year. For several months before the agreements were due for renewal, there was general uncertainty as to the outcome of the negotiations, and consumers sought protection by adding to their stocks of coal. In many mines operations were actually suspended in April. But soon a new agreement was reached, gradually the miners returned to work, the curve of production rose again, at first sharply, then more gently, and by July or August the usual seasonal pattern was being repeated. The differences between the odd (peace) and even (strike) years [8] before 1914 are sufficiently strong and regular to justify separate seasonal indexes (Chart 5). By adopting this expedient we finally attained a curve from which specific cycles up to 1914 could be dated with ease and confidence (Chart 4).

Some of the later years, however, are troublesome. During 1914–19 three successive waves may be distinguished. We could not accept all as specific cycles. The first decline extends from February 1916 to July 1916, the second from May 1917 to January 1918, the third from July 1918 to March 1919. There can be little doubt that the third is of sufficient magnitude to be considered a cyclical movement. But are the two preceding declines also to be treated as cyclical contractions? The decline from May 1917 to January 1918 is suspect on account of the seasonal adjustment. The original data testify to a sustained demand for coal throughout 1917; the usual lull in the spring and summer is absent. Indeed, the only notable drop during the entire period of our participation in World War I came in the winter of 1917–18, when severe storms tied up railroad traffic and caused car shortages. Even if the propriety of our seasonal

[8] 'Peace' and 'strike' years in the coal industry are a matter of degree. For a convenient tabulation of strike statistics, see The Effect of Labor Relations in the Bituminous Coal Industry upon Interstate Commerce (National Labor Relations Board, *Bulletin 2*, June 30, 1938), Table 13. For a chronicle of the industry in all its aspects, see the annual reviews of coal in *Mineral Resources* (of late called *Minerals Yearbook*) by the Bureau of Mines.

CHART 5

Ratios of Original Data to Twelve-month Moving Averages
Bituminous Coal Production, United States, 1905 – 1914



See Chart 4.

adjustments for 1917 and 1918 were not in question, we would have to disregard the decline from May 1917 to January 1918, since only 14 months separate its trough from the trough recognized in March 1919. On the other hand, the wave starting in November 1914 lasted 20 months, and thus manages to meet the duration test. But it lacks impressive amplitude. Moreover, it is due, in considerable part, to a spurt in anticipation of a strike in April 1916, followed by a drop when the strike failed to develop. Our decision, therefore, was to treat the declines of February–July 1916 and May 1917–January 1918 as interruptions of a cyclical expansion running from November 1914 to July 1918.

In addition to these uncertainties in identifying the specific cycles in coal production, several questions arise in dating their turns. The general movement of the data seems a sufficient reason for placing a trough in June 1924, instead of March 1925 when output was somewhat lower. Again, a trough is placed in March 1919, not in November 1919 when output was very much lower. The abrupt drop in November 1919 was due to a general coal strike. It caused merely a brief halt in the expansion that clearly got under way March 1919; for that reason we disregard it. For a similar reason the low value in April 1939 is disregarded, and the cyclical trough is dated March 1938. But we have not hesitated to date a cyclical trough in a month when a strike occurred if that month seemed to coincide roughly with the end of a cyclical decline, or to date a cyclical peak in a month preceding the outbreak of a strike if it seemed roughly coincident with the end of a cyclical expansion. Our decisions to treat March 1927 as a specific-cycle peak and July 1922 as a specific-cycle trough are examples.[9]

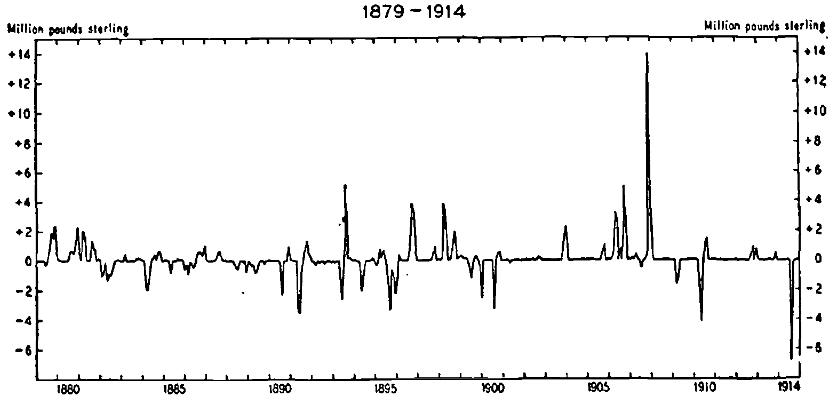
The trough in 1922 exemplifies a 'double bottom'. There is a deep trough in April 1922, when a strike—probably the greatest in the history of this afflicted industry—broke out. A slight revival occurred during the next two months, and a relapse in July, when the railroad shopmen's strike produced an acute car shortage in the non-union field. The seasonally adjusted figure is fractionally higher in July than in April (20.2 against 20.1 million tons). But the difference is negligible, and in line with our rules, the trough is dated in the later month.

Other illustrations of our method of dating specific cycles are provided in Table 19, which accompanies Chart 8, and in Charts 11 and 53. Some of the turning points are problematical, especially in series like share trading and structural steel orders, where erratic movements flourish. But no series in these charts is as extreme as net gold imports by the United States from the United Kingdom, pictured in Chart 6. No sus-

CHART 6

Net Gold Imports by the United States from the United Kingdom

1879 – 1914



Source: Great Britain, Board of Trade, *Accounts Relating to Trade and Navigation.*

tained movements of rise and fall can be detected in this series by the naked eye. By applying moving averages, the semblance of a cyclical movement could be created. But we cannot trust 'cyclical movements'

[9] The trough might have been set in July 1921 instead of July 1922. But output was only a little higher in Dec. 1921 than in July 1921, and it was considerably lower in April–July 1922. It is doubtful if a cyclical rise occurred between July 1921 and March 1922; for the apparent rise would vanish if the peak in a single month, March 1922, were removed. In setting the trough in July 1922, we assume that the sharp peak in March 1922 reflects entirely or largely the preparations for a strike.

that arise from a mathematical spreading of isolated peaks and troughs over surrounding months. Fortunately, series of this erratic type are very rare in our collection.

Our methods of determining specific cycles make no pretensions to elegance. Since no fast line separates erratic or episodic movements from specific cycles, or erratic turns from cyclical turns, there is ample opportunity for vagaries of judgment. At times our rules fail to yield a clear-cut decision. At times the members of our statistical staff disagree in their efforts to apply the rules to a given series. Our experience indicates that this difficulty cannot be removed by multiplying rules. The most effective way of coping with it is to organize the work on a plan that disciplines judgment. Hence when a new series comes up for analysis, three persons mark off its specific cycles, each working independently. If a conflict arises, the reasons for the divergent choices are discussed in conference. Sometimes, agreement is reached quickly. More often, new work must be undertaken, such as the application of moving averages to a troublesome period, or the recomputation of the seasonal adjustment, or detailed comparisons with related series, or research to clarify the background of some puzzling fluctuation. Once the conflict is settled, and the computations involved in the analysis of a series are completed and checked, the whole operation—including the dating of the specific-cycle turns—is 'audited' by an experienced member of our staff, who often has not participated in the work to this point. This process of checking may be repeated two or three times. For when a series previously analyzed is brought up to date, the occasion is made an opportunity for a thorough reexamination of all specific-cycle decisions. A check of this sort may come years after the original work has been done, and thus tests the consistency with which judgment has been applied. When earlier decisions seem faulty, we do not hesitate to reanalyze the series. Still another check on both the uniformity and reasonableness of the procedure comes when the data charts and cyclical measures are used by our collaborators in preparing their monographs on cyclical behavior.

These safeguards are adequate, we believe, to ensure trustworthy pictures of average specific-cycle behavior in the great majority of our series. A whimsical factor nevertheless remains in the measures of many individual cycles, and sometimes plagues even the average measures. These matters are discussed critically later in the book, especially in Chapter 8. Here we need only mention that in order to help readers judge the reliability of our results we plan to indicate in later monographs the amplitudes of erratic movements relatively to the specific cycles in each series analyzed, by a scale that runs from 'mild' to 'moderate', 'pronounced', and 'very pronounced'—terms that indicate roughly what degree of confidence we attach to our analysis of the specific cycles.[10]

10 See pp. 153-4.

It might seem to some readers that the uncertainties connected with dating specific cycles could be overcome by eliminating secular trends and smoothing out erratic movements; but this is an illusion. Unless specific cycles are marked off prior to fitting a line of secular trend, and the line of trend is so fitted as to equilibrate exactly or approximately the plus and minus deviations within each cycle distinguished, widely differing results are possible with respect to the number and duration of specific cycles.[11] Of course, if secular trends are eliminated, some specific cycles not shown by our methods may be revealed. But once more, unless criteria are laid down, on the basis of study of the raw data, for the behavior of the trend line, many sorts of specific cycles may be found. The most refined procedure in computing trends offers no escape from the necessity of distinguishing cycles in the raw data or in their first differences.

Nor are the turns of specific cycles any easier to date in trend-adjusted than in raw data, for it is erratic movements that make most trouble at the turns. Smoothing by a sensitive graduation formula that produces continuous curvatures solves formally the dating of the turns; but it does not meet all the difficulties encountered in identifying specific cycles by our methods and it introduces some new difficulties. For example, it is still necessary to decide by arbitrary rule what movements are too brief or too mild to be considered specific cycles. More important, the process of smoothing may convert a steep random peak with a base of two or three months into a 'cycle' lasting two years or longer, or it may erase entirely a mild and brief cyclical movement in the raw data.[12]

We believe, therefore, that specific cycles dated from seasonally adjusted data are not less trustworthy than they would be if dated from data adjusted for secular and erratic as well as seasonal movements. At the same time we think that the dating of specific cycles could be improved by supplementing study of the charts of the original and seasonally adjusted data by study of several additional curves, one showing the seasonally adjusted data freed from erratic flutterings, another showing them adjusted for secular trend, and a third showing them adjusted for both secular trend and erratic movements. But to adjust many time series satisfactorily on this plan is extremely laborious and costly, and we have had to be satisfied with passing simple moving averages through doubtful portions of time series.

It is easier to mark off specific cycles in annual series than in monthly or quarterly.[13] But secular trends tend to obscure specific cycles far more potently in annual than in monthly data. While annual reporting does not affect the size of secular movements, it chops off the cyclical peaks

[11] See pp. 37-8. But note the comments on pp. 273-6.
[12] For illustrations, see Ch. 8, Sec. II.
[13] See pp. 215-6.

and fills in the cyclical troughs that would appear in data reported at briefer intervals.[14] For this reason, as well as because annual data often misrepresent the timing of peaks and troughs in what specific cycles do appear, we use such series only when dealing with crops that are harvested once a year, or when monthly or quarterly data are unavailable, or when monthly or quarterly data cover such short periods that we doubt their representative value.

II Diffusion of Specific Cycles

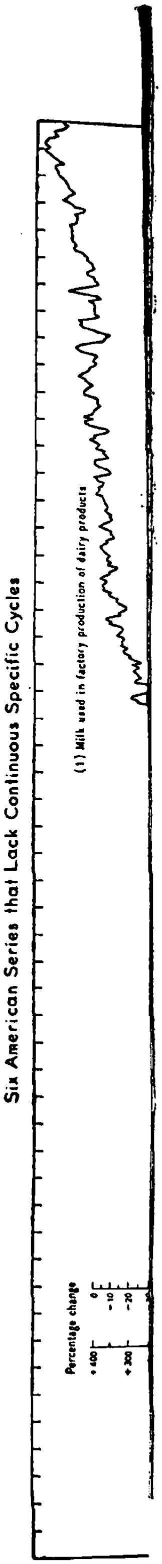
Not all time series exhibit specific cycles. For example, net gold imports (Chart 6) undergo large movements, but they are so brief and irregular that we cannot regard them as cyclical. Chart 7 presents several other series that seemingly are either free from specific cycles or undergo such movements only intermittently. Commutation traffic on railroads is a steady series, except for the shift to a lower level during the 1930's and minor erratic fluctuations throughout. Local transit in New York is also a steady series. The amount of milk used in the production of dairy products is more volatile; but what waves can be distinguished in this series are rendered uncertain by the nature of the seasonal adjustment. The remaining three series do not change at all for months or years, then rise or fall in a vertical step to a new level. We do not treat such movements as specific cycles unless the shifts in level are frequent enough to trace out, after their own fashion, a wave-like pattern with recognizable peaks and troughs.

These series are instructive because they illustrate the diversity of behavior found in time series. They represent, however, exceptional types. A survey of our American series in monthly or quarterly form, made when the number analyzed was 828, showed that there were only 27, or slightly over 3 per cent, in which we found no specific cycles or only intermittent specific cycles. Of course, an investigator who insisted on greater regularity in durations than we do would surely find fewer series moving in specific cycles. The like would happen if the concept of specific cycles were restricted to fluctuations of closely similar amplitude. On the other hand, by adopting refined mathematical techniques, specific cycles might be found in so steady a series as transit rides in New York or in so jerky a series as net gold imports from the United Kingdom. But we do not wish to labor a defense of our detailed results. It may well be that now and then, despite the precautions taken to guard against 'cyclical bias', we have 'seen' cycles in series that have none in fact. After making liberal allowance for this possibility, we can still say that the great majority of series in our collection exhibit continuous fluctuations whose period corresponds to that specified by our working definition of business cycles.

These specific cycles are the basic 'raw materials' of this investigation.

14 The shortcomings of annual data in studying business cycles are treated at length in Ch. 6.

CHART 7
Six American Series that Lack Continuous Specific Cycles

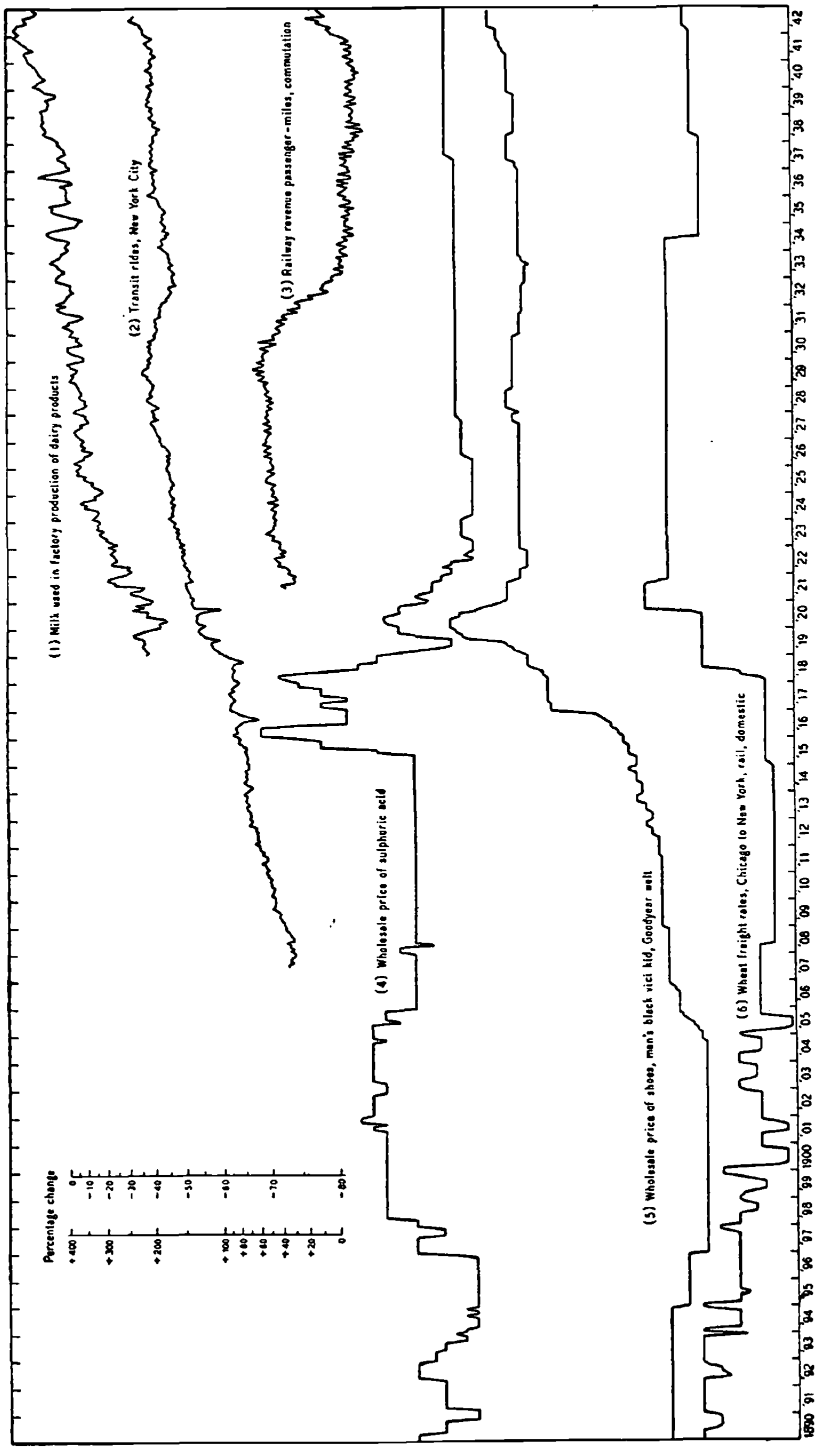


reported at
l data often
ic cycles do
e harvested
le, or when
doubt their

old imports
d irregular
veral other
dergo such
ilroads is a
1930's and
York is also
dairy prod-
is series are
he remain-
e or fall in
as specific
after their
oughs.
iversity of
xceptional
erly form,
were only
les or only
nsisted on
ewer series
of specific
le. On the
cific cycles
k or in so
But we do
that now
ical bias',
ng liberal
ajority of
se period
ess cycles.
stigation.
in Ch. 6.

CHART 7

Six American Series that Lack Continuous Specific Cycles



(1), (2) and (3) adjusted for seasonal variations. For sources of data, see Appendix C

TABLE 14

Chronology of Specific Cycles in Employment
Ten Manufacturing Industries, United States, 1919–1938

Date	Industries reaching a peak or _trough_[a]	Number of industries whose specific cycles: Reach a peak	Reach a trough	Undergo expansion[b]	Undergo contraction[b]
Feb. 1919	_Textiles_*, _Stone_*, _Lumber_*	..	3	..	10
Mar. 1919	_Leather_*	..	1	3	7
Apr. 1919		..	..	4	6
May 1919	_Food_*, _Paper_*, _Iron_, _Transportation equipment_	..	4	4	6
June 1919	_Machinery_	..	1	8	2
July 1919		..	..	9	1
Aug. 1919	_Tobacco_	..	1	9	1
Sep. '19–Nov. '19		..	..	10	..
Dec. 1919	Food, Lumber, Leather	3	..	10	..
Jan. 1920	Stone, Tobacco	2	..	7	3
Feb. 1920	Transportation equipment	1	..	5	5
Mar. 1920	Iron, Machinery	2	..	4	6
Apr. 1920	Textiles	1	..	2	8
May '20–June '20		..	..	1	9
July 1920	Paper	1	..	1	9
Aug. '20–Nov. '20		..	..	..	10
Dec. 1920	_Lumber_	..	1	..	10
Jan. 1921	_Textiles, Leather, Transportation equipment_	..	3	1	9
Feb. 1921	_Stone_	..	1	4	6
Mar. '21–Apr. '21		..	..	5	5
May 1921	_Paper_	..	1	5	5
June 1921		..	..	6	4
July 1921	_Iron_	..	1	6	4
Aug. 1921	_Food_	..	1	7	3
Sep. 1921	Textiles	1	..	8	2
Oct. 1921	_Machinery_	..	1	7	3
Nov. '21–Dec. '21		..	..	8	2
Jan. 1922	_Tobacco_	..	1	8	2
Feb. '22–Apr. '22		..	..	9	1
May 1922	_Textiles_	..	1	9	1
June 1922		..	..	10	..
July 1922	Tobacco	1	..	10	..
Aug. '22–Mar. '23		..	..	9	1
Apr. 1923	Leather	1	..	9	1
May 1923	Textiles	1	..	8	2
June 1923	Iron	1	..	7	3
July 1923	Stone, Lumber, Machinery	3	..	6	4
Aug. '23–Sep. '23		..	..	3	7
Oct. 1923	Food	1	..	3	7
Nov. 1923	Transportation equipment	1	..	2	8
Dec. '23–June '24		..	..	1	9
July 1924	_Textiles, Stone, Lumber, Leather_	..	4	1	9
Aug. 1924	_Iron, Transportation equipment_	..	2	5	5
Sep. 1924	_Machinery_	..	1	7	3
Oct. 1924	_Food_	..	1	8	2
Nov. '24–Feb. '25		..	..	9	1
Mar. 1925	Leather	1	..	9	1
Apr. '25–Oct. '25		..	..	8	2
Nov. 1925	Textiles, Transportation equipment	2	..	8	2
Dec. '25–Mar. '26		..	..	6	4
Apr. 1926	Lumber, _Leather_	1	1	6	4
May '26–June '26		..	..	6	4
July 1926	_Textiles_	..	1	6	4
Aug. 1926	_Tobacco_	..	1	7	3
Sep. 1926	Iron, Stone	2	..	8	2
Oct. 1926	Machinery	1	..	6	4
Nov. 1926	Paper	1	..	5	5
Dec. 1926	Leather	1	..	4	6
Jan. '27–July '27		..	..	3	7
Aug. 1927	Textiles	1	..	3	7
Sep. 1927	Tobacco	1	..	2	8
Oct. 1927		..	..	1	9
Nov. 1927	_Transportation equipment_	..	1	1	9
Dec. 1927		..	..	2	8

TABLE 14—*Continued*

Chronology of Specific Cycles in Employment

Ten Manufacturing Industries, United States, 1919–1938

Date	Industries reaching a peak or *trough*[a]	Number of industries whose specific cycles: Reach a peak	Reach a trough	Undergo expansion[b]	Undergo contraction[b]
Jan. 1928	*Iron, Lumber, Machinery*	..	3	2	8
Feb. '28–Mar.'28	....	..	..	5	5
Apr. 1928	*Paper*	..	1	5	5
May '28–Aug.'28	....	..	..	6	4
Sep. 1928	*Textiles*	..	1	6	4
Oct. 1928	....	..	..	7	3
Nov. 1928	*Leather*	..	1	7	3
Dec. '28–Jan. '29	....	..	..	8	2
Feb. 1929	Transportation equipment	1	..	8	2
Mar.'29–June'29	....	..	..	7	3
July 1929	Textiles	1	..	7	3
Aug. 1929	Iron, Lumber, Machinery	3	..	6	4
Sep. 1929	Paper	1	..	3	7
Oct. 1929	....	..	..	2	8
Nov. 1929	Food, Leather	2	..	2	8
Dec. '29–Dec. '30	....	..	..	..	10
Jan. 1931	*Textiles*	..	1	..	10
Feb. '31–June'31	....	..	..	1	9
July 1931	Textiles	1	..	1	9
Aug.'31–June'32	....	..	..	..	10
July 1932	*Food, Textiles, Leather*	..	3	..	10
Aug. 1932	*Lumber*	..	1	3	7
Sep. '32–Feb. '33	....	..	..	4	6
Mar. 1933	*Iron, Machinery*	..	2	4	6
Apr. 1933	*Paper, Stone, Transportation equipment, Tobacco*	..	4	6	4
May '33–Mar.'34	....	..	..	10	..
Apr. 1934	Tobacco	1	..	10	..
May '34–Sep. '34	....	..	..	9	1
Oct. 1934	Food	1	..	9	1
Nov.'34–Sep. '35	....	..	..	8	2
Oct. 1935	*Food*	..	1	8	2
Nov.'35–Jan. '36	....	..	..	9	1
Feb. 1936	*Tobacco*	..	1	9	1
Mar.'36–Oct. '36	....	..	..	10	..
Nov. 1936	Tobacco	1	..	10	..
Dec. '36–Mar.'37	....	..	..	9	1
Apr. 1937	Stone	1	..	9	1
May 1937	Textiles	1	..	8	2
June 1937	Paper, Leather	2	..	7	3
July 1937	Food, Lumber	2	..	5	5
Aug. 1937	Iron, Machinery, Transportation equipment	3	..	3	7
Sep. '37–May '38	....	..	..	..	10
June 1938	*Textiles, Paper, Stone, Lumber, Leather*	..	5	..	10
July 1938	*Iron, Machinery*	..	2	5	5
Aug. 1938	*Transportation equipment*	..	1	7	3
Sep. 1938	....	..	..	8	2
Oct. 1938	*Food*	..	1	8	2
Nov.'38–Dec. '38	....	..	..	9	1

Based on the Bureau of Labor Statistics indexes of factory employment, adjusted for seasonal variations by the Federal Reserve Board. Source: *Federal Reserve Bulletin*, Oct. 1938, pp. 842-5, and Oct. 1939, p. 880, except that revised figures for textiles in 1933–38, automobiles in 1919–22, and transportation equipment in July–Sept. 1938 were furnished directly.

The full titles of the ten industrial groups covered are iron, steel, and their products; machinery; transportation equipment (limited to automobiles prior to 1923); lumber and allied products; stone, clay, and glass products; textiles and their products; leather and its manufactures; food and kindred products; tobacco manufactures; paper and printing. Three industrial groups (chemicals and allied products, and petroleum refining; rubber products; nonferrous metals and their products) in the Bureau of Labor Statistics index are omitted here because their record does not go back to 1919.

*So far as possible, data on employment before 1919 and related records were consulted in dating the cyclical turns in 1919. The troughs marked with an asterisk are more uncertain than the others.

[a]Industries reaching a trough are italicized.

[b]For simplicity in tabulation, a rise or fall between two months is credited to the second month.

Our central problem is to determine how they fit together. The nature of this undertaking may be glimpsed from a highly simplified illustration. Table 14 lists, month by month, the specific-cycle turns of ten constituents of the Bureau of Labor Statistics index of factory employment during 1919–38. This information is developed in later columns, which show the number of industries in which employment rose or fell, reached a peak or trough, each month. A tendency toward a common rhythm can be detected in the table, but the different industries do not always keep in close step. Six industries—food products, tobacco, leather, textiles, paper and printing, and stone, clay and glass—occasionally 'skip' a cycle common to the other four, or trace out an 'extra' cycle. Even when corresponding cyclical turns are found in all ten series, they are dispersed over several months. For example, every series turned up in 1932–33, but the troughs span 10 months. The 'turning zone' is still longer at the upturn in 1921. On account of the divergencies in the timing of cyclical turns, all industries experience specific-cycle expansion or all experience specific-cycle contraction in only 69 of the 239 months covered, less than a third of the full period.

Let the reader imagine a thousand time series arranged on the plan of Table 14, and he will begin to face in their full complexity the timing relations among the cyclical movements of actual life. Such a table would show every month, or practically every month, some activities in an expanding phase, some beginning to recede from their peaks, some contracting, and some beginning to revive from their troughs. But although the four phases of expansion, recession, contraction, and revival will be found running side by side at all times, the basic conception that emerges from our studies is that at any one time one phase is dominant. Recession is evident in some activities in the first stage of the phase when expansion becomes dominant, and spreads gradually to other activities until recession replaces expansion as the dominant phase. Similarly, revival begins early in the phase when contraction becomes dominant, and spreads gradually until revival becomes dominant. Our hypothesis, in other words, is that a period in which expansions are concentrated is succeeded by another in which cyclical peaks are concentrated, by another in which contractions are concentrated, by another in which cyclical troughs are concentrated; and this round of events is repeated again and again. To expose this process of continual change, it is necessary to observe systematically the direction and amplitude of the cyclical movements of many activities.

If our analysis were restricted to a few time series, it would be simple to compare their specific cycles directly. But when the analysis covers hundreds of series, it is clumsy and wasteful to compare the timing of each series with every other; indeed, as clumsy and wasteful as it would be to express the exchange value of each commodity in terms of every other commodity. Much clearer results can be attained by adopting some common denominator, that is, by setting up a reference scale on which every series can be laid out in strictly uniform fashion. Any scale is bound to be artificial in the sense that it will break apart movements that are continuous. But the scale should be as little artificial as possible. The cyclical changes going on all the time are highly complex: expansions and contractions of economic activities weave into one another. Nevertheless, if our concept of business cycles is valid, there must be certain years and months in which 'general business activity' reaches successive peaks and troughs. Approximations to these 'turning points' will give the reference scale we seek.

This reference scale will rest on two assumptions. First, we shall assume that business cycles run a continuous round, as explained in Chapter 1. Second, we shall assume that it is sufficient to mark off the dates when expansions and contractions in general business activity culminate. This implies that once business activity as a whole has reached a peak or trough, it does not linger there, but commences rather promptly to decline in the one case, to rise in the other. We believe that these assumptions are tolerably close to the run of experience. However, the validity of the reference scale built on this framework of assumptions must be fully tested. Whether the periods marked off by our reference scale actually correspond to business cycles in the sense of our definition, and if so how different activities are related in direction and amplitude from stage to stage of the business cycles, are precisely the questions this investigation must answer.

III Different Methods of Deriving a Reference Scale

The derivation of a reference scale of business cycles raises numerous problems. How should we proceed to ascertain the dates when the expansions and contractions in 'general business activity' culminate? One possible plan is to identify the central tendency in each cluster of turning points in individual economic activities. But how are different activities to be marked off from one another; that is, how are they to be classified and individualized? And how is the central tendency of their cyclical turns to be determined? Another plan would be to identify the turning points of a series measuring aggregate economic activity. But where are the boundaries of aggregate activity to be set? Can this 'quantity' be given a meaning that is at once significant for the purpose at hand and precise enough to be measurable? Suppose that much is done. Then it will still be necessary to check the cycles in the chosen measure of aggregate activity against the evidence of related series. But what series will serve this purpose and how are conflicts to be reconciled? It will also be necessary to determine whether the cycles in aggregate activity are sufficiently diffused through the economy to rate as business cycles. What series or processes should be examined to determine this issue? Should the list be kept the same or varied from country to country and from period to period within a country? Must the fluctuation be reflected in all the items on the list or will some specified set or number or proportion suffice? Should not a fluctuation that satisfies the criterion of diffusion meet also certain criteria of duration and amplitude, phase by phase as well as for the entire fluctuation, before it is admitted to the family of business cycles? Finally, how should we distinguish between cyclical and erratic turns in aggregate activity; in other words, how are the uncertainties connected with dating the turning points of the specific cycles in the series chosen as a measure of aggregate activity, and in other series that may be used in conjunction with it, to be surmounted?

These tantalizing questions cannot be resolved by excogitation alone. The only practicable plan is to work with the facts in the light of tentative judgments and hypotheses, then revise the hypotheses in the light of the findings. Work with the facts can be organized on many different plans. An investigator whose main concern was to establish a chronology of business cycles might wish to spend years in studying specific cycles before venturing to date the peaks and troughs of business cycles. But our interest in a reference scale is largely incidental to theoretical ends. Early in the investigation it seemed wise to determine whether our working definition of business cycles was a promising guide to further inquiry, and if that seemed to be the case, to organize the work in such fashion that we could learn quickly, even if only approximately, how the cyclical fluctuations of leading economic activities are related to one another. For this purpose it was necessary to settle rather promptly on a reference scale of business cycles; conceived of as a tool of analysis that was to be tested, amended or, if need be, rejected in the course of further observation.

The simplest method of deriving such a scale would be to mark off the months in which the specific cycles of an acceptable measure of aggregate economic activity reached successive peaks and troughs. Aggregate activity can be given a definite meaning and made conceptually measurable by identifying it with gross national product at current prices. This total includes the values of all finished commodities and services reaching the final users in a nation during a given period, plus or minus any change in inventories held at points short of their final destination, plus or minus any change in claims on international account.[15] However, for the purpose of analyzing business cycles, it is better to restrict the total to the portion of the national product that passes through the 'market'. In ex-

[15] For a fuller explanation and supplementary definitions of this total, see Simon Kuznets, *National Income and Capital Formation, 1919–1935* (National Bureau of Economic Research, 1937), pp. 3-5, and especially pp. 34-5.

tending measurements into the past, an increasing part of agricultural production should be excluded; also production of all sorts within local, self-sufficient communities. Similar restrictions apply to other measures of aggregate activity that might be used, such as the physical volume of production or the volume of employment. They apply also, though with less force, to total income payments, and to total monetary transactions excluding, of course, trading in assets.

Unfortunately, no satisfactory series of any of these types is available by months or quarters for periods approximating those we seek to cover. Estimates of the value of the gross or net national product on a monthly or quarterly basis are still in an experimental stage. The Department of Commerce estimates of total income payments by months go back only to 1929. Recently, Harold Barger has prepared quarterly estimates of net and gross national product in the United States back to 1921. For Great Britain, Colin Clark has devised quarterly figures on national income since 1929. These statistical efforts represent an important step forward in the measurement of 'national income' by short time units, and bear considerable promise for the future. But as yet they rest heavily on estimates eked out from small samples or purely mathematical interpolations, which leave considerable margins of uncertainty in the final result.[16]

Index numbers of the physical volume of production by short time units are available for longer periods than national income estimates, but they too fail to meet our needs. The 'present' version of the Federal Reserve Board index of industrial production goes back only to 1923; an older version goes back to 1919, and a still older version to 1913. There the record stops. The one monthly index of production with reasonably adequate coverage before World War I is Babson's index of the physical volume of business, which starts in 1904. In our foreign countries tolerable indexes of production by months or quarters are available only since World War I. Even for recent years, indexes of production leave much to be desired. Thus the Federal Reserve Board index is restricted to manufacturing and mining, both covered on a sampling basis. Since the fluctuations of different branches of production synchronize imperfectly, a change in the sample of series included in a production index or in the weights assigned to the series may easily shift the date when the index reaches a cyclical peak or trough. Table 15 shows the turning points of the Federal Reserve Board index at different stages of its statistical history. The cyclical turns of the index developed by the Standard Statistics Company are presented on a similar plan, as are also the turns of the index

16 See Frederick M. Cone, Monthly Income Payments in the United States, 1929–40 (Bureau of Foreign and Domestic Commerce, *Economic Series No. 6*); Harold Barger, *Outlay and Income in the United States: 1921–1938* (National Bureau of Economic Research, 1942); Colin Clark, *National Income and Outlay* (Macmillan, London, 1937), Ch. IX. See also *Review of Economic Statistics*, May 1943, p. 159, and *The Economist*, April 12, 1941, p. 489, and April 18, 1942, pp. 531-2.

of the Federal Reserve Bank of New York.[17] Sometimes the shifts in the cyclical turns from one index to another are negligible, but not always. The instability of cyclical turns disclosed by the table is a warning that the turning points of even the latest production indexes are approximations, not precise measurements.[18]

Employment series by months or quarters are no better suited to our needs than production indexes. Estimates of the number of civil nonagricultural employees in the United States go back to 1929 only. The index of the number of factory employees by the Bureau of Labor Statistics goes back to 1914, but is based on a slender sample before 1923. These series can be supplemented by data on the average number of hours worked per man-week, collected since 1920 (except for January–June 1922) by the National Industrial Conference Board and on a much wider scale by the Bureau of Labor Statistics since 1932. But there are practically no statistics of employment or unemployment on a national basis prior to World War I. Foreign countries, especially Great Britain, are more fortunate in this respect. In Germany trade-union unemployment percentages are available by months back to 1906 and by quarters back to 1903; in France by months since 1894, and in Great Britain by months since 1854. But the coverage of these foreign materials narrows as the records recede into the past: the British data before 1887 are limited to the Friendly Society of Iron Founders.

There are indeed some comparable monthly or quarterly series that extend over several decades in each or all except one of our four coun-

17 Some trend-adjusted indexes are placed deliberately in the table. If a rising trend were removed from a production index and nothing else changed, cyclical troughs of the trend-adjusted index would coincide with or come later than corresponding turns of the unadjusted index, while cyclical peaks would coincide or come earlier. (See Ch. 7 for an analysis of the influence of trend adjustments on cyclical timing and other measures of cyclical behavior.) The reader will note how frequently the actual relations among the several samples of production indexes are not in keeping with this simple model. The reason, of course, is that the trend-adjusted and unadjusted indexes differ in other respects—composition, weighting, etc.

18 This should be plain enough from the composition of any production index. The statistician who sets out to construct a production index must choose, in principle, between an index based on inadequate industrial coverage and an index based in part on crude estimates. In practice, he is sure to compromise. Thus the Federal Reserve Board index of 'industrial production' actually covers only the manufacturing and mining sectors. Of the 81 series included in the index, as revised in the *Federal Reserve Bulletin* of Aug. 1940, only 38 with a weight of 40.4 per cent (that is, the 38 series account for this percentage of the index figure) in 1935–39 are production series in their original state. Another 19 with a weight of 11.3 per cent in 1935–39 also report production, but are partly estimated; in most instances, so as to conform to Census of Manufactures data. The remaining series do not report production as such, but other activities supposed to reflect production. Among these are 8 man-hour series, with a weight of 27.6 per cent in 1935–39, adjusted by smooth interpolated values that allegedly reflect changes in man-hour productivity.

The representation of man-hour series was increased in revisions of the index published in the *Federal Reserve Bulletin*, Sept. 1941 and Oct. 1943. Of late the effective weight of the man-hour series has increased by leaps and bounds, partly because of their increased number, partly because these series predominantly represent war industries. In June 1943 the man-hour series accounted for 58 per cent of the total index of industrial production, and for a still larger percentage of its manufactures component (*ibid.*, Oct. 1943, p. 949).

TABLE 15

Dates of Cyclical Peaks and Troughs
Successive Versions of Three Indexes of 'Industrial Production'
United States, 1919–1938

Cyclical turn[a]	Federal Reserve Board index[b]			Standard Statistics Co. index[c]			Federal Reserve Bank of New York index[d]		
	A	B	C	A[e]	B	C	A[e]	B[e]	C[e]
T	June 1919	Mar. 1919	....	June 1919	May 1919	May 1919	Mar. 1919	Mar. 1919	Mar. 1919
P	Jan. 1920	Feb. 1920	....	Jan. 1920	Mar. 1920	Feb. 1920	Jan. 1920	Jan. 1920	Jan. 1920
T	July 1921	Apr. 1921	....	July 1921	Apr. 1921	Mar. 1921	Jan. 1921	Mar. 1921	Mar. 1921
P	May 1923	June 1923	May 1923	May 1923	May 1923	May 1923	Feb. 1924	Apr. 1923	May 1923
T	Aug. 1924	July 1924	July 1924	June 1924	June 1924	July 1924	July 1924	July 1924	July 1924
P	....	Oct. 1926	Mar. 1927	....	Sep. 1926	Sep. 1926	....	Dec. 1925	Dec. 1925
T	....	Nov. 1927	Nov. 1927	....	Dec. 1927	Nov. 1927	....	Dec. 1927	Dec. 1927
P	....	June 1929	Aug. 1929	....	May 1929	June 1929	....	June 1929	Aug. 1929
T	....	July 1932	July 1932	....	....	Mar. 1933	....	Mar. 1933	Mar. 1933
P	....	Dec. 1936	May 1937	....	....	Mar. 1937	....	Dec. 1936	Dec. 1936
T	....	May 1938	May 1938	....	....	Apr. 1938	....	....	May 1938

All indexes are seasonally adjusted.

[a] T stands for trough, P for peak. When a cylical movement culminates in two or more identical values, the latest month was taken as the date of turn.

[b] For index A, see *Federal Reserve Bulletin*, May 1924–Jan. 1927; index B, Board of Governors of the Federal Reserve System, *Annual Report* for 1937, pp. 173-9, and *Federal Reserve Bulletin*, Oct. 1938, p. 910; index C, *ibid.*, Aug. 1940, p. 825. Slight revisions in the latter index (*ibid.*, Sept. 1941, p. 934) shifted the trough from May to June 1938, but left the other cyclical turns unchanged.

[c] See the following publications of the Standard Statistics Company: for index A, *Standard Daily Trade Service: Annual Statistical Bulletin*, 1926, pp. 32-6; index B, *Standard Trade and Securities: Base Book, Standard Statistical Bulletin*, Jan. 1932, p. 159; index C, *Standard Trade and Securities: Basic Statistics*, April 29, 1938, Sec. D, p. 67, and later issues (title changes).

[d] For index A, see the series on 'productive activity' in Carl Snyder, The Revised Index of the Volume of Trade, *Journal of the American Statistical Association*, Sept. 1925. Indexes B and C were obtained directly from the Federal Reserve Bank of New York. For a description of B, see *ibid.*, Dec. 1931, p. 436 ff; for a description of C, *ibid.*, June 1938, p. 341 ff. Cf. *ibid.*, Dec. 1923, p. 949 ff; June 1928, p. 154 ff; Sept. 1941, p. 423 ff.

[e] Adjusted for secular trend as well as seasonal variations.

tries. But the only series of this type that warrant consideration for setting a reference scale of business cycles are bank clearings, indexes of wholesale prices, open-market interest rates, and indexes of business conditions. Our experience with American data indicates that none of these records, by itself, is a satisfactory gauge of business cycles. Bank clearings, even if limited to cities outside New York, give excessive weight at times to financial transactions. The addition of new cities complicates the interpretation of the figures in the early decades. The increasing reliance upon checks[19] as an instrument of payment is a more serious factor, since checks have come to be used more and more in relatively stable transactions, such as paying salaries, rents, and trading at retail. Indexes of wholesale prices have served more faithfully as 'barometers' of business cycles than many students now believe. It is true, nevertheless, that wholesale prices and the physical volume of business activity have sometimes followed divergent paths for months at a time, as during 1926–29 and 1938–39. Nor can open-market commercial paper rates be trusted implicitly as a criterion for dating business cycles. The substantial business cycle from

[19] That has been the broad secular tendency, at least to 1930.

1933 to 1938 made hardly a ripple in short-term interest rates. At other times short-term interest rates have traced out specific cycles which reflected disturbances confined largely to the financial markets, as during the depression of the 1870's. Indexes of business conditions should be free from the vagaries that mark the cyclical course of individual series; but before the 1890's these composites are made from exceedingly inadequate data. We believe that with the exception of open-market interest rates, foreign records of these various types are, if anything, even less satisfactory than the American.

The conclusion to be drawn from this condensed review of statistical data bearing on aggregate economic activity is obvious. If there is no monthly or quarterly series in any of our countries that can serve by itself as a criterion for setting a reference scale of business cycles, whether because the series is not long enough, or not accurate enough, or not broad enough in its coverage, or not stable enough in its relation to business cycles, or for all these reasons, then it is necessary to use a more laborious method; that is, a reference scale of business cycles must be extracted from the fallible indications provided by time series for varied economic activities. In using this method there is, of course, considerable leeway in the range of data, as well as in the choice of specific techniques. In view of our aims at the start of this investigation, it was not worth while to use time-consuming statistical methods. Hence we followed a simple procedure, which cannot lay claim to elegance or to a high degree of precision, but which at least promised more trustworthy results than did any single statistical series.

IV A Tentative Schedule of Reference Dates

Here, as elsewhere, our working definition of business cycles served as a guidepost. According to this definition, no fluctuation in activity qualifies as a business cycle unless it spreads over many of the economic processes of a country. Descriptive evidence concerning the generality of past fluctuations is provided by *Business Annals,* compiled by Willard Thorp and published by the National Bureau in 1926.[20] Our first step toward identifying business cycles was to identify the turns of general business activity indicated by these annals. Next, the evidence of the annals was checked against indexes of business conditions and other series of broad coverage. In most cases these varied records pointed clearly to some one year as the time when a cyclical turn occurred. When there was conflict of evidence, additional statistical series were examined and historical accounts of business conditions consulted, until we felt it safe to write down an interval

[20] For a continuation of these annals through 1931, see W. L. Thorp, The Depression as Depicted by Business Annals (National Bureau of Economic Research, *News-Bulletin No. 43,* Sept. 19, 1932).

within which a cyclical turn in general business probably occurred. We then proceeded to refine the approximate dates by arraying the cyclical turns in the more important monthly or quarterly series we had for the time and country.

But the outstanding fact about economic time series is wide variation in their behavior traits. It would not do merely to mark off the zone within which a succession of series reached (say) cyclical peaks, then choose the month of their central tendency as the reference peak. (1) Some series 'indicate' a decline in business activity when they rise and an increase when they fall; for example, bankruptcies, unemployment, idle equipment. Their peaks and troughs must be inverted before casting up the evidence. (2) Some series regularly reach their peaks and troughs within the intervals marked by concentrations of turning dates. Others behave erratically. In setting reference dates, the evidence of a series that always or usually keeps in step with others is more significant than that of a series that usually 'walks by itself'. (3) Of the series that fluctuate in unison, some are early to rise and early to fall; others are laggards; a few lead at one turn and lag at the other; many exhibit no consistent timing. These timing characteristics must be taken into account in fixing reference turns. (4) So also must the secular trends of the series. That electric power production in the United States rose from 1924 to 1929 is not good evidence that there was no business-cycle contraction in 1926–27; for during these years the use of electricity was growing so fast that a mild contraction in general business might merely retard its rise. (5) Erratic movements must be taken into account because, when large in proportion to cyclical fluctuations, they cast doubt on the dates chosen to represent specific-cycle turns. (6) These dates may be dubious also because the amplitudes of specific-cycle expansions and contractions are slight. (7) Finally, it is essential to recognize that the relative importance of the economic activities represented may vary considerably from period to period for the same series, as well as from series to series for the same period.

Thus, to ascertain a business-cycle turn from an array of specific-cycle turns, one needs to know a great deal about the individual series. When we first made a reference scale of business cycles, we had to rely on vague knowledge concerning the cyclical behavior and the economic significance of different series. Under the circumstances, it seemed pointless to assign formal weights to the turning dates of individual series. A judgment as to a reference turn was based on a study of whatever evidence had been marshaled, without the aid of any 'objective' statistical controls. In many cases the turning points of different series were bunched so closely that we could not go far astray. But there were cases in which the turning points were widely scattered, and others in which they were concentrated around two separate dates. If there was little else to guide

TABLE 16

Reference Dates and Durations of Business Cycles in Four Countries

Line	Monthly reference dates: Peak	Monthly reference dates: Trough	Duration in months: Expansion[a]	Duration in months: Contraction[b]	Duration in months: Full cycle	Quarterly reference dates: Peak	Quarterly reference dates: Trough	Calendar-year reference dates: Peak	Calendar-year reference dates: Trough	Fiscal-year[c] reference dates: Peak	Fiscal-year[c] reference dates: Trough
					United States						
1	...	...	..	..	..	...	...	...	1834	...	...
2	...	...	..	..	..	...	...	1836	1838	...	...
3	...	...	..	..	..	...	...	1839	1843	...	...
4	...	...	..	..	..	...	...	1845	1846	...	...
5	...	...	..	..	..	...	...	1847	1848	...	...
6	...	Dec. 1854	..	..	..	...	4Q 1854	1853	1855	...	...
7	June 1857	Dec. 1858	30	18	48	2Q 1857	4Q 1858	1856	1858	...	...
8	Oct. 1860	June 1861	22	8	30	3Q 1860	3Q 1861	1860	1861	...	...
9	Apr. 1865	Dec. 1867	46	32	78	1Q 1865	1Q 1868	1864	1867	...	1868
10	June 1869	Dec. 1870	18	18	36	2Q 1869	4Q 1870	1869	1870	1869	1871
11	Oct. 1873	Mar. 1879	34	65	99	3Q 1873	1Q 1879	1873	1878	1873	1878
12	Mar. 1882	May 1885	36	38	74	1Q 1882	2Q 1885	1882	1885	1882	1885
13	Mar. 1887	Apr. 1888	22	13	35	2Q 1887	1Q 1888	1887	1888	1887	1888
14	July 1890	May 1891	27	10	37	3Q 1890	2Q 1891	1890	1891	1890	1891
15	Jan. 1893	June 1894	20	17	37	1Q 1893	2Q 1894	1892	1894	1893	1894
16	Dec. 1895	June 1897	18	18	36	4Q 1895	2Q 1897	1895	1896	1896	1897
17	June 1899	Dec. 1900	24	18	42	3Q 1899	4Q 1900	1899	1900	1900	1901
18	Sep. 1902	Aug. 1904	21	23	44	4Q 1902	3Q 1904	1903	1904	1903	1904
19	May 1907	June 1908	33	13	46	2Q 1907	2Q 1908	1907	1908	1907	1908
20	Jan. 1910	Jan. 1912	19	24	43	1Q 1910	4Q 1911	1910	1911	1910	1911
21	Jan. 1913	Dec. 1914	12	23	35	1Q 1913	4Q 1914	1913	1914	1913	1915
22	Aug. 1918	Apr. 1919	44	8	52	3Q 1918	2Q 1919	1918	1919	1918	1919
23	Jan. 1920	Sep. 1921	9	20	29	1Q 1920	3Q 1921	1920	1921	1920	1922
24	May 1923	July 1924	20	14	34	2Q 1923	3Q 1924	1923	1924	1923	1924
25	Oct. 1926	Dec. 1927	27	14	41	3Q 1926	4Q 1927	1926	1927	1927	1928
26	June 1929	Mar. 1933	18	45	63	2Q 1929	1Q 1933	1929	1932	1929	1933
27	May 1937	May 1938	50	12	62	2Q 1937	2Q 1938	1937	1938	1937	1939
					France						
1	...	...	..	..	..	...	...	...	1840	...	...
2	...	...	..	..	..	...	...	1847	1849	...	...
3	...	...	..	..	..	..	...	1853	1854	...	...
4	...	...	..	..	..	...	...	1857	1858	...	...
5	...	Dec. 1865	..	..	..	...	...	1864	1865	...	...
6	Nov. 1867	Oct. 1868	23	11	34	...	...	1866	1868	...	...
7	Aug. 1870	Feb. 1872	22	18	40	...	...	1869	1871	...	...
8	Sep. 1873	Aug. 1876	19	35	54	...	...	1873	1876	...	...
9	Apr. 1878	Sep. 1879	20	17	37	...	...	1878	1879	...	...
10	Dec. 1881	Aug. 1887	27	68	95	...	...	1882	1887	...	...
11	Jan. 1891	Jan. 1895	41	48	89	...	1Q 1895	1890	1894	...	...
12	Mar. 1900	Sep. 1902	62	30	92	1Q 1900	3Q 1902	1900	1902	...	...
13	May 1903	Oct. 1904	8	17	25	2Q 1903	3Q 1904	1903	1904	...	...
14	July 1907	Feb. 1909	33	19	52	3Q 1907	1Q 1909	1907	1908	...	...
15	June 1913	Aug. 1914	52	14	66	3Q 1913	3Q 1914	1913	1914	...	...
16	June 1918	Apr. 1919	46	10	56	2Q 1918	2Q 1919	1917	1918	...	...
17	Sep. 1920	July 1921	17	10	27	3Q 1920	3Q 1921	1920	1921	...	...
18	Oct. 1924	June 1925	39	8	47	3Q 1924	3Q 1925	1924	1925	...	...
19	Oct. 1926	June 1927	16	8	24	3Q 1926	3Q 1927	1926	1927	...	...
20	Mar. 1930	July 1932	33	28	61	1Q 1930	3Q 1932	1930	1932	...	...
21	July 1933	Apr. 1935	12	21	33	3Q 1933	1Q 1935	1933	1935	...	...
22	June 1937	Aug. 1938	26	14	40	2Q 1937	3Q 1938	1937	1938	...	...

TABLE 16—*Continued*

Reference Dates and Durations of Business Cycles in Four Countries

Line	Monthly reference dates		Duration in months			Quarterly reference dates		Calendar-year reference dates		Fiscal-year[c] reference dates	
	Peak	Trough	Expansion[a]	Contraction[b]	Full cycle	Peak	Trough	Peak	Trough	Peak	Trough
					Great Britain						
1	...	...	..	..	..	...	...	1792	1793	...	...
2	...	...	..	..	..	...	...	1796	1797	...	...
3	...	...	..	..	..	...	...	1802	1803	...	...
4	...	...	..	..	..	...	...	1806	1808	...	...
5	...	...	..	..	..	...	...	1810	1811	...	...
6	...	...	..	..	..	...	...	1815	1816	...	...
7	...	...	..	..	..	...	...	1818	1819	...	...
8	...	...	..	..	..	...	...	1825	1826	...	...
9	...	...	..	..	..	...	...	1828	1829	...	...
10	...	...	..	..	..	...	...	1831	1832	...	...
11	...	...	..	..	..	...	...	1836	1837	...	...
12	...	...	..	..	..	...	...	1839	1842	...	...
13	...	...	..	..	..	...	...	1845	1848	...	...
14	...	Dec. 1854	..	..	..	...	1Q 1855	1854	1855	...	...
15	Sep. 1857	Mar. 1858	33	6	39	4Q 1857	1Q 1858	1857	1858	...	...
16	Sep. 1860	Dec. 1862	30	27	57	4Q 1860	4Q 1862	1860	1862	...	...
17	Mar. 1866	Mar. 1868	39	24	63	2Q 1866	2Q 1868	1866	1868	...	...
18	Sep. 1872	June 1879	54	81	135	4Q 1872	2Q 1879	1873	1879	...	...
19	Dec. 1882	June 1886	42	42	84	1Q 1883	2Q 1886	1883	1886	...	...
20	Sep. 1890	Feb. 1895	51	53	104	3Q 1890	1Q 1895	1890	1894	...	...
21	June 1900	Sep. 1901	64	15	79	2Q 1900	4Q 1901	1900	1901	...	...
22	June 1903	Nov. 1904	21	17	38	2Q 1903	4Q 1904	1903	1904	...	...
23	June 1907	Nov. 1908	31	17	48	2Q 1907	4Q 1908	1907	1908	...	...
24	Dec. 1912	Sep. 1914	49	21	70	1Q 1913	3Q 1914	1913	1914	...	...
25	Oct. 1918	Apr. 1919	49	6	55	3Q 1918	2Q 1919	1917	1919	...	...
26	Mar. 1920	June 1921	11	15	26	2Q 1920	2Q 1921	1920	1921	...	...
27	Nov. 1924	July 1926	41	20	61	4Q 1924	3Q 1926	1924	1926	...	...
28	Mar. 1927	Sep. 1928	8	18	26	2Q 1927	3Q 1928	1927	1928	...	...
29	July 1929	Aug. 1932	10	37	47	3Q 1929	3Q 1932	1929	1932	...	...
30	Sep. 1937	Sep. 1938	61	12	73	3Q 1937	3Q 1938	1937	1938	...	...
					Germany						
1	...	...	..	..	..	...	...	...	1866	...	...
2	...	...	..	..	..	...	...	1869	1870	...	...
3	...	Feb. 1879	..	..	..	...	1Q 1879	1872	1878	...	...
4	Jan. 1882	Aug. 1886	35	55	90	1Q 1882	3Q 1886	1882	1886	...	...
5	Jan. 1890	Feb. 1895	41	61	102	1Q 1890	1Q 1895	1890	1894	...	...
6	Mar. 1900	Mar. 1902	61	24	85	2Q 1900	1Q 1902	1900	1902	...	...
7	Aug. 1903	Feb. 1905	17	18	35	3Q 1903	1Q 1905	1903	1904	...	...
8	July 1907	Dec. 1908	29	17	46	2Q 1907	4Q 1908	1907	1908	...	...
9	Apr. 1913	Aug. 1914	52	16	68	1Q 1913	3Q 1914	1913	1914	...	...
10	June 1918	June 1919	46	12	58	2Q 1918	2Q 1919	1917	1919	...	...
11	May 1922	Nov. 1923	35	18	53	2Q 1922	4Q 1923	1922	1923	...	...
12	Mar. 1925	Mar. 1926	16	12	28	2Q 1925	2Q 1926	1925	1926	...	...
13	Apr. 1929	Aug. 1932	37	40	77	2Q 1929	3Q 1932	1929	1932	...	...

[a] From trough on preceding line to peak.
[b] From peak to trough on same line.
[c] That is, years ending June 30.

us, we placed the reference turn toward the close of the transition period.

The reference dates derived in this rough fashion have been subjected to continual tests, as new series have been studied and old ones restudied. If only a few specific-cycle turns occurred near one of our experimental reference dates, we questioned the existence of a business-cycle turn at that time, reexamined what evidence we had, and sought more. If most of the specific-cycle turns in the vicinity of a reference date came months before that date, we concluded that it had been put too late. After extensive testing on this plan, which necessitated various revisions, we obtained several years ago a reference scale that seemed to fit the specific-cycle movements tolerably well in most instances. This reference scale is still tentative, and will be reviewed thoroughly in the course of future work.

Table 16 presents the reference scale in its present stage of development for the United States, Great Britain, Germany and France.[21] In analyzing monthly series we need the reference dates by months, as is explained fully in Chapter 5. The quarterly reference dates are needed in handling quarterly series, and the annual reference dates in handling annual series. The fiscal-year reference dates are designed to facilitate analysis of some important American records available only by years ending June 30. In each country the annual reference scale covers more cycles than the monthly or quarterly, because annual series extend furthest into the past. But the reference scale by fiscal years is carried back no further than is necessary to analyze the series in our collection that come in this form. A like reason accounts for the relatively brief quarterly reference scale for France.

The monthly reference dates are basic. They alone enable us to observe cyclical behavior in the detail we consider essential.[22] They therefore control the quarterly and annual reference dates. The quarterly dates are virtually derived from the monthly. When the monthly reference date (peak or trough) occurs in the middle of a quarter (February, May, August, or November), we took that quarter as the date of the reference turn. When it occurs in the first month of a quarter, we placed the quarterly turn either in that quarter or in the one just preceding, according to the indications of a sample of important statistical series by quarters. Similarly, when the monthly reference date occurs in the last month of a quarter, we placed the quarterly turn in that quarter or in the one just following.

The annual reference dates have also been set to correspond with the monthly dates, although some mild and short business cycles are obscured

21 Simon Kuznets took a leading part in the preparation of the original set of reference dates. For aid in extending, revising or criticizing the dates, we are indebted to Isaiah Frank, George Garvy, and Walt Rostow; and especially Moses Abramovitz, Cicely Applebaum, Geoffrey H. Moore, Julius Shiskin, and Albert Wohlstetter.

22 Quarterly records, however, are often a satisfactory substitute. For an analysis of the influence of the time unit on cyclical measures, see Ch. 6.

in annual records.[23] The annual reference turn often comes in a year other than that of the monthly turn. The reason is that the annual reference dates purport to state the years in which general business reached a high or low point when comparisons are made by full years; these need not be the same as the years in which business activity on a monthly basis made cyclical turns.

Since our reference dates delimit only expansions and contractions, we may, and often do, speak of business cycles as if they consisted of only two phases. But our working definition of business cycles presents each of them as passing through four phases: expansion, recession, contraction, and revival. The most important practical and the most difficult theoretical problems presented by business cycles do not lie in the processes by which expansions and contractions develop after they have started; they lie in processes by which an expansion is succeeded by a recession and a contraction by a revival. Since a definition designed to guide an investigation should focus attention upon matters that require attention, we include in our definition the transitional phases of which we must later give an account.

V Difficulties in Setting Reference Dates Illustrated

The critical difficulty in setting reference dates is the uneven range and quality of statistical records for different periods and countries. In the United States monthly data are more abundant than in foreign countries. In each country they are more abundant since 1900, and especially since 1919, than in the nineteenth century. For example, in setting the reference dates for France in the 1860's, we were able to utilize a fairly large number of annual series. Our monthly records for this period, however, were limited to open-market interest rates and several series representing activities of the Bank of France. Even in the United States the number of monthly or quarterly series available for the early decades of the chronology is relatively small, as Table 17 indicates.[24] It is obvious, therefore, that the difficulty of setting reference dates, especially if they are expressed by months, must increase, and their reliability diminish, as the chronology is pushed back into the past.

Given the range of available records and the fineness of the time unit in which the reference scale is to be expressed, the ease or difficulty of

[23] Independent dating of annual reference scales would probably yield fewer cycles. Our reasons for forcing the annual chronology to correspond with the monthly are explained on p. 262. In the periods not covered by the monthly scales, the annual reference dates are of necessity independent.

[24] When the present reference scale was developed, the number of series in our collection was much smaller than in the table. Even as it stands, the list in 1860 is dominated by financial and price series. The production group in that year includes only three monthly series: hog receipts in Chicago, cattle receipts in Chicago, boot and shoe shipments from Boston. These activities are of slight value in fixing a reference scale of business cycles.

TABLE 17

Number of Monthly or Quarterly American Series Available at Decennial Dates since 1860

(Series analyzed by July 1, 1942)

Process	Number of series in our collection for							
	1860	*1870*	*1880*	*1890*	*1900*	*1910*	*1920*	*1930*
Production	3	7	13	15	23	29	133	154
Construction	..	1	2	1	1	8	78	81
Transportation	..	4	4	9	13	17	28	29
Commodity prices	8	11	13	66	76	86	141	143
Inventories	..	1	2	3	6	7	50	61
Merchandising	..	4	4	4	4	6	38	32
Foreign trade	..	7	7	7	8	19	19	19
Personal incomes	..	..	1	2	2	6	86	91
Profits and losses	..	..	2	5	9	11	16	22
Savings and investments	..	1	1	1	1	4	15	16
Security markets	2	2	4	7	8	10	9	6
Interest rates	4	4	5	6	11	11	20	24
Money and banking[a]	..	6	35	59	59	59	16	16
Aggregate transactions	2	2	11	12	13	13	16	16
Total	19	50	104	197	234	286	665	710

See note to Table 2.

[a] The drop in 1920 and 1930 is explained by the fact that statistics on the condition of the national banks have been analyzed thus far only through 1914.

setting reference dates depends on the nature of the fluctuations found in different activities and their interrelations. The range and quality of the statistical data for the United States since 1929 surpass anything we have for other countries or periods. Some uncertainty nevertheless surrounds the setting of monthly reference dates in this period. Readers unfamiliar with the complex behavior of economic time series are invited to study Chart 8, both for its own sake and for the illustrations it affords of the practical problems encountered in marking off the turning points of business cycles. The chart exhibits the movements of 40 significant monthly series, each of rather broad coverage,[25] during 1932–39.

The first feature to notice is that business activity fell to a deep trough in the summer of 1932, revived in the autumn, slumped again at the end of the year, and reached a new trough in the spring of 1933. The 'double bottom' appears in most series. Of the two troughs, that in the summer of 1932 is lower in the Federal Reserve Board index of production and in several other series. But a great majority of the series show a lower trough in the spring of 1933. This fact, reinforced by our general rule to accept a later date in cases of doubt, led us to date the trough in March 1933. We believe that the evidence favors this date rather than June or July 1932, whether a business-cycle trough is considered as a turn in aggregate activity, defined in some plausible fashion, or as the central tendency in the turning dates of leading branches of activity.[26] True, the

25 Except commercial paper rates.

26 See Tables 14-15 above; also our *Bulletin 61*, pp. 2-4.

Federal Reserve index of production reached a minimum in July 1932. But an enterprising investigator who sought to date business cycles according to the cyclical turns in physical production would begin, not stop, with that index. Noting the slight difference between its values in July 1932 and March 1933, the roughness of many of the underlying series and of their adjustment for seasonal variations, the limited industrial coverage of the index, and the different showing of other production indexes; he would want to examine many series, of the sort depicted in Chart 8, for the light they throw indirectly on the physical volume of production. In the end he would probably, though by no means certainly, pick March 1933 as the trough date.

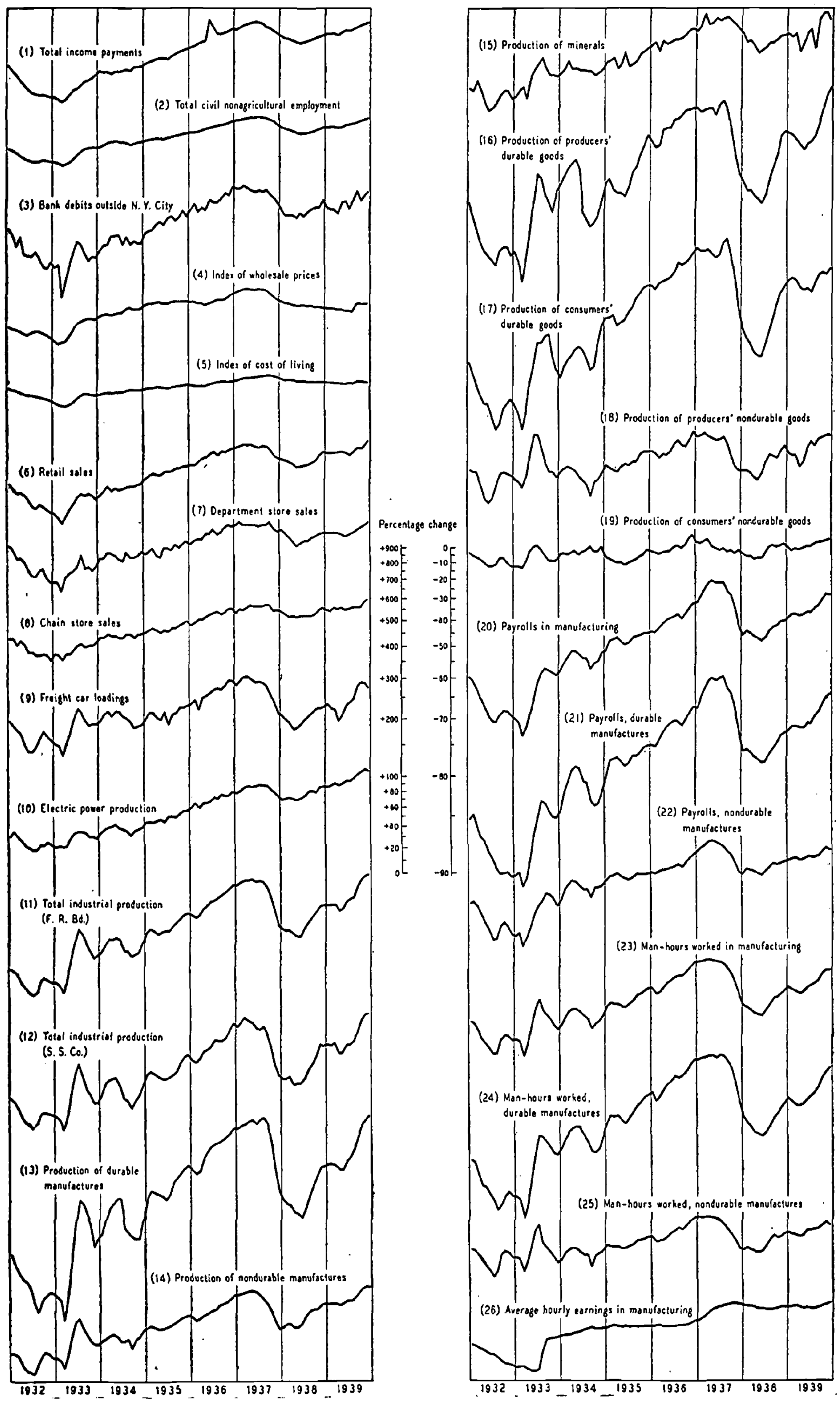
When the cyclical turns of leading branches of economic activity are sharply angular and closely bunched, as in the second quarter of 1938 in the United States, a cyclical turn in general business activity can be dated with considerable assurance. In such a case minor errors in the original data, in the adjustments for seasonal variations, or in the dates assigned to the turns of the specific cycles, are likely to have slight influence on the final judgment. But when the turns in leading activities are comparatively 'flat', crisscrossed by erratic movements, and dispersed over many months, the turn in general business activity becomes elusive. The American business-cycle peak of 1937 approximates this type. To fix the month of this peak, a wide range of evidence, or at least a variety of independent measures or indicators of aggregate activity, is essential, no matter what theoretical criterion or criteria of a business-cycle turn an investigator may set in advance. Tables 18 and 19 summarize a few of the contrasts revealed by Chart 8 between the business-cycle peak in 1937 and the trough in 1938. Table 18 shows that erratic movements in a sample of 23 series on the volume of business activity are more prominent in the vicinity of the 1937 turn than of the 1938 turn; also that the figures reached at the turns are nearly duplicated in more months in the vicinity of the former turn than in the vicinity of the latter. Table 19 shows that the peaks reached in 1937 or thereabouts by our full sample of 40 series are widely dispersed, whereas the troughs are sharply concentrated: 27 series reach a trough in the interval from April to June 1938.[27] In view of unavoidable errors in the data and in the dates assigned to the specific-cycle

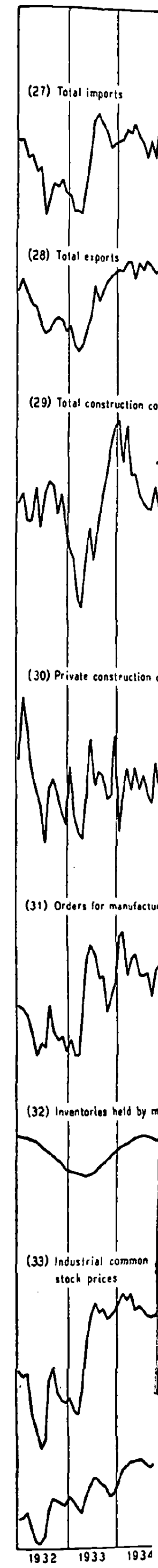
[27] Tables 14-15 supply further contrasts between the business-cycle turns in 1937 and 1938; also between the trough in 1924, which resembles that of 1938, and the peak in 1926, which bears some resemblance to that of 1937. Table 14 indicates that the cyclical turns of employment series are bunched more closely in 1924 than in 1926, and in 1938 than in 1937. Table 15 shows that the divergence among the cyclical turns of different measures of industrial production is at a maximum in 1926, considerable in 1937, but slight in 1924 and 1938.

It may be noted parenthetically that this evidence, so far as it goes, gives no support to J. M. Keynes' thesis that "the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning-point when an upward is substituted for a downward tendency." (*The General Theory of Employment, Interest and Money*, Harcourt, Brace and Co., 1936, p. 314.)

CHART 8

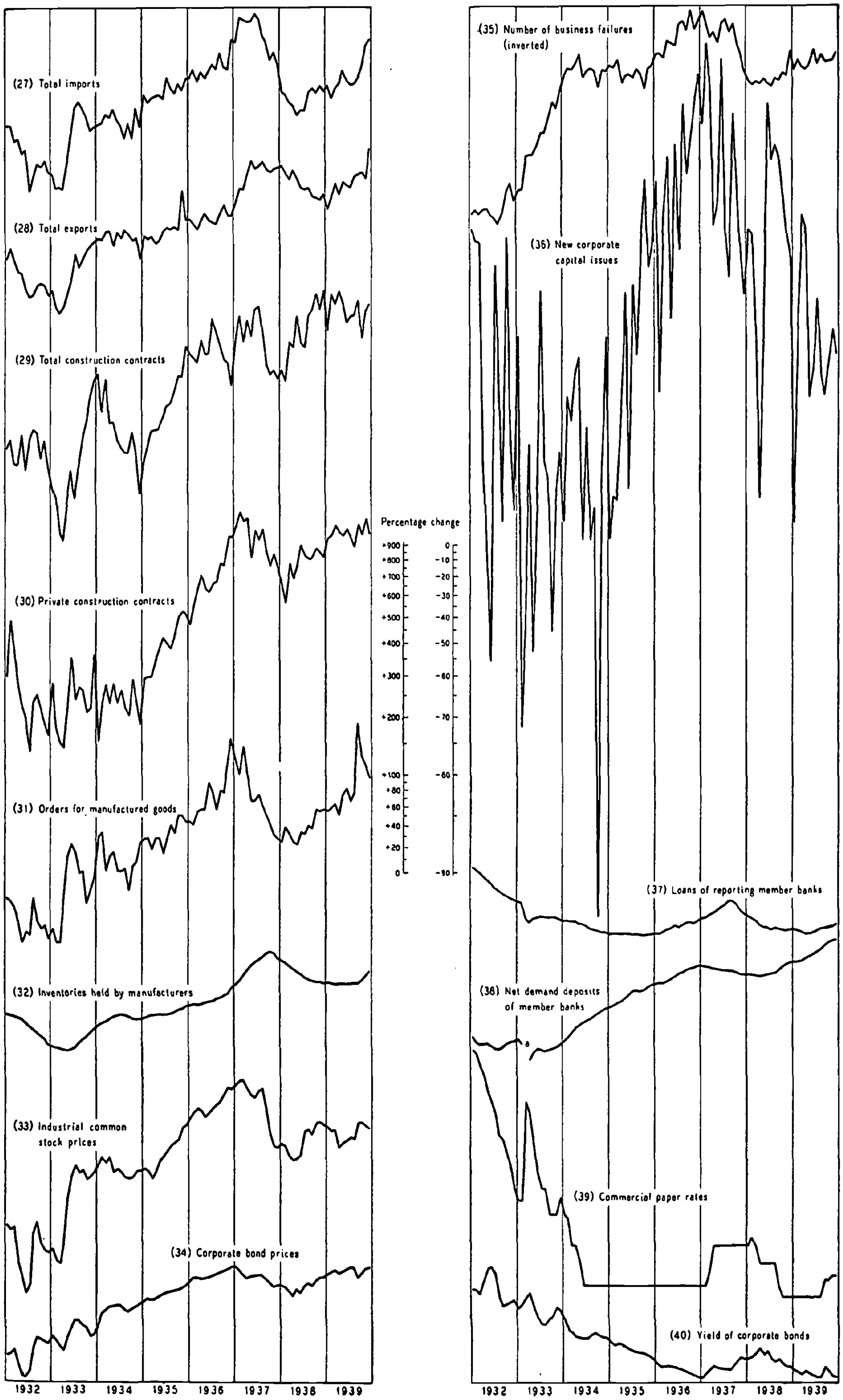
Behavior of Forty American Series, 1932 – 1939





For sources of data and oth

CHART 8 (CONTINUED)



For sources of data and other notes, see Appendix C.

a No data, March 1933.

TABLE 18

Short-term Fluctuations around the Cyclical Turns of 1937 and 1938
23 American Series

No.[a]	Series	In an interval of eleven months centered on the cyclical turn: No. of months that are within 1% of the value at the turn[b]: Specific-cycle Peak[d]	Specific-cycle Trough[d]	Reference Peak (May 1937)	Reference Trough (May 1938)	No. of reversals in direction[c]: Specific-cycle Peak[d]	Specific-cycle Trough[d]	Reference Peak (May 1937)	Reference Trough (May 1938)
	PRODUCTION								
11	Total industrial (F.R.Bd.)	3	1	6	3	3	1	3	1
12	Total industrial (S.S.Co.)	..	2	3	3	4	3	4	3
13	Durable manufactures	1	..	1	1	3	1	3	1
14	Nondurable manufactures	3	..	4	3	1	2	1	2
16	Producers' durable goods	..	..	3	1	3	1	4	1
17	Consumers' durable goods	..	1	4	2	2	1	5	1
18	Producers' nondurable goods	1	..	6	1	5	2	4	2
19	Consumers' nondurable goods	..	..	1	3	5	2	5	2
15	Minerals	2	..	3	1	3	3	6	3
10	Electric power	4	3	7	5	3	4	4	4
9	Freight car loadings	1	..	3	1	4	1	4	1
	DOMESTIC TRADE								
3	Bank debits outside N.Y.City	1	1	7	3	6	3	6	3
6	Retail sales	3	2	7	4	5	1	5	1
7	Department store sales	..	..	7	1	4	2	2	2
8	Chain store sales	5	6	8	7	4	3	5	3
	EMPLOYMENT								
2	Nonagricultural employment	4	3	8	5	1	1	1	1
23	Man-hours, manufacturing	2	2	6	3	1	1	1	1
24	Man-hours, durable manufactures	3	..	6	2	3	1	4	1
25	Man-hours, nondurable manufactures	4	2	7	3	4	4	3	4
	FLOW OF INCOMES								
1	Total income payments	3	2	5	5	1	1	2	1
20	Payrolls, manufacturing	..	..	5	2	1	2	1	3
21	Payrolls, durable manufactures	..	1	2	1	3	2	4	3
22	Payrolls, nondurable manufactures	2	1	3	3	1	4	1	4
	Total	42	27	112	63	70	46	78	48

Comparison between peaks and troughs

	Months within 1%: Specific-cycle	Months within 1%: Reference	Reversals: Specific-cycle	Reversals: Reference
No. of series in which entry at peak				
Exceeds entry at trough	13	18	14	15
Is exceeded by entry at trough	4	1	4	4
Equals entry at trough	6	4	5	4

[a] The series are numbered as in Chart 8. For sources of data and other notes, see Appendix C.

[b] In the case of reference turns, the average value in the three months centered on the turn was treated as the value at the turn. In the case of specific-cycle turns, the actual value in the month of turn was used. Hence the count is based on 10 months for specific-cycle turns, and 11 months for reference turns.

[c] Within an interval of 11 months, there are 10 observations on direction of movement and 9 on reversal of direction; hence the theoretical range is from 0 to 9. Horizontal movements are ignored in the tabulation. Another calculation made on the assumption that a first difference of zero is equally likely to be plus or minus in fact (excepting, of course, a series like commercial paper rates in recent years) did not change the results significantly; it is omitted for reasons of simplicity.

[d] The dates are listed in Table 19.

turns, there is a fair chance that the peak we have dated in May 1937 is misplaced by several months. But it seems rather unlikely from the present evidence that the trough dated in May 1938 can be in error by more than one month.

The series in Chart 8 are, of course, a sample drawn from a large 'parent population'. The sample rests on judgments that inevitably condition the reference dates. The present list is confined to series of rather broad significance. When the reference dates of 1937 and 1938 were originally set, another list was used, which included series on individual industries as well as broad composites. The change from one list to the other represents a shift in judgment on our part.[28] Even the series common to the two lists are in several instances common in name only. These differences arise from shifts in judgment on the part of the original compilers, who—contrary to widespread opinion—are not engaged simply in 'straight reporting'. To make progress, they experiment in handling their problems as we do in handling ours. When our work was first done, we chose May instead of June 1938 as the reference trough, largely because three of the most comprehensive aggregates at our disposal—the Federal Reserve index of industrial production, the Department of Commerce series on total income payments, and an unpublished index of consumer expenditures by the Federal Reserve Board—all showed a trough in May. Later revisions by the compilers shifted the trough in the production index to May and June, in income payments to June, and in the consumption index to July.[29] If we took up the problem of dating anew, we would set the reference trough in June instead of May 1938, whether relying on the series originally used or those in Chart 8.[30] But changes in the underlying sample of time series will not always be so neutral in their effect on a reference date; nor will revisions of the figures of the same series always have so slight an effect.

Chart 8 illustrates another difficulty in developing a reference scale of business cycles; namely, how to distinguish between fluctuations that are and those that are not 'business cycles'. In many branches of business the expansion from March 1933 to May 1937 was seriously interrupted from about July to November 1933 and again from about May to September 1934. So widespread were these reversals that they raise the question whether one or more 'extra' business cycles should not be recognized in this period. Several factors argue against such treatment. (1) Unlike past recoveries in this country, the recovery from the slump of 1929–32 was

28 Compare, for example, the series in Chart 8 with the list in our *Bulletin 61*, p. 3.

29 The index for consumption is not included in Chart 8, because of its uncertain dependability. We use instead a major component of the index: the total dollar value of retail sales (undeflated).

30 Hence this paradoxical result: despite the simplicity of dating the reference turn of 1938 compared with that of 1937, we seem to have erred by one month in dating the turn in 1938, while there is no clear evidence at present that the date of the peak set in May 1937 can be improved.

TABLE 19

Sequence of Cyclical Turns in the 1937 Recession and the 1938 Revival

40 American Series

Year & month	Specific-cycle peak	Year & month	Specific-cycle trough
1936 Dec.	Production of producers' nondurable goods (18) Production of consumers' nondurable goods (19) Orders for manufactured goods (31)		
1937 Jan.	Man-hours worked, nondurable manufactures (25) Corporate bond prices[a] (34) Number of business failures, *inverted*[b] (35) Net demand deposits of member banks (38)		
Feb.	Private construction contracts (30) New corporate capital issues (36)		
Mar.	Bank debits outside N.Y.City (3) Total industrial production, S.S.Co. (12) Industrial common stock prices (33)		
Apr.	Index of wholesale prices (4) Freight car loadings[a] (9) Man-hours worked in manufacturing (23)		
May	Retail sales[c] (6) Total industrial production, F.R.Bd. (11) Production of nondurable manufactures[a] (14) Payrolls in manufacturing (20) Payrolls, nondurable manufactures (22) Total exports (28)		
June	Total income payments[d] (1) Total imports (27)		
July	Total civil nonagricultural employment (2) Electric power production (10) Man-hours worked, durable manufactures (24) Total construction contracts (29)		
Aug.	Production of durable manufactures (13) Production of producers' durable goods (16) Payrolls, durable manufactures (21) Loans of reporting member banks (37)		
Sep.	Production of minerals[e] (15) Production of consumers' durable goods (17)		
Oct.	Index of cost of living[a] (5) Department store sales[f] (7) Chain store sales[g] (8) Average hourly earnings in manufacturing (26) Inventories held by manufacturers (32)		
		1937 Dec.	Production of nondurable manufactures (14)
1938 Feb.	Commercial paper rates (39)	*1938* Feb.	Total construction contracts (29) Private construction contracts (30)
Apr.	Yield of corporate bonds (40)	Apr.	Freight car loadings (9) Total industrial production, S.S.Co. (12) Production of consumers' nondurable goods (19) Industrial common stock prices(33) Corporate bond prices (34) Net demand deposits of member banks (38)
		May	Bank debits outside N.Y.City (3) Retail sales (6) Department store sales (7) Chain store sales[c] (8) Electric power production[h] (10) Production of minerals (15) Production of producers' nondurable goods (18) Man-hours worked, nondurable manufactures (25) Total imports (27) Orders for manufactured goods (31)

TABLE 19—*Continued*

Sequence of Cyclical Turns in the 1937 Recession and the 1938 Revival
40 American Series

Year & month	Specific-cycle peak	Year & month	Specific-cycle trough
		June	Total income payments (1) Total civil nonagricultural employment (2) Total industrial production, F.R.Bd.[a] (11) Production of durable manufactures (13) Production of producers' durable goods (16) Production of consumers' durable goods[a] (17) Payrolls in manufacturing (20) Payrolls, durable manufactures (21) Payrolls, nondurable manufactures (22) Man-hours worked in manufacturing (23) Man-hours worked, durable manufactures (24)
		July	Number of business failures, *inverted* (35)
		Aug.	Average hourly earnings in manufacturing (26)
		1939 Jan.	Total exports (28)
		Apr.	Loans of reporting member banks (37)
		June	Index of cost of living (5) Inventories held by manufacturers (32)
		Aug.	Index of wholesale prices (4)
		Sep.	New corporate capital issues[i] (36)

In this table, as in Chart 8, the series on corporate bond prices is the same as corporate bond yields, except that it is inverted. There is no entry for commercial paper rates and bond yields at the trough; the lower turning points cannot be determined even now (spring 1943). See Appendix C for sources and other notes about the series, which are numbered as in Chart 8.

[a] The same value in the preceding month.
[b] A slightly higher value in Oct. 1936.
[c] The same value two months earlier.
[d] A higher value in June 1936 (payments to veterans).
[e] A higher value in March 1937.
[f] The same value in Feb. 1937.
[g] The same value in May and Sept. 1937.
[h] A slightly lower value in Jan. 1938.
[i] A lower value in April 1938, Jan. and May 1939, and June 1940.

subject to repeated political shocks. (2) Three of the most comprehensive aggregates of economic activity—total income payments, nonagricultural employment, retail sales—show little or no trace of the two sharp fluctuations evident in most production and employment series, the first fluctuation consisting of a rise and fall from about March to November 1933, the second from about November 1933 to September 1934. (3) These fluctuations are similar to that from about July 1932 to March 1933 and from about September 1934 to May 1935. It seems reasonable to treat all four on the same basis. (4) The duration of these fluctuations is well below the lower limit set by our working concept of business cycles. However interesting the short fluctuations may be, we cannot recognize them as business cycles without changing the object of our investigation.[31] (5) The mechanism of fluctuations lasting only a few months cannot be the same as that of fluctuations lasting several years. The transition from one phase of a business cycle to the next comes about gradually through a

[31] But it is important to keep in mind that general business activity is subject to interruptions in its march from a trough to a peak and from a peak to a trough. See Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 329-30; and our *Bulletin 69*, pp. 6-7.

complicated set of cumulative changes in the relations among different factors in the economy. This process takes time.[32] We know, for example, that months elapse before a downturn in contracts for new factories is followed by a downturn in the construction work actually done, and that another few months must elapse before a downturn occurs in the new industrial facilities completed. (6) We may follow our rule governing durations and yet recognize a business cycle with a trough in March 1933, a peak approximately in May 1934, and a terminal trough approximately in September 1934. But this 'cycle' would violate another part of our definition: namely, that no cycle be divisible into shorter cycles with amplitudes approximating its own. Further, the contraction phase of this cycle would be only 4 months, distinctly shorter than any we have recognized.

These considerations seem to us to constitute a reasonably decisive argument against recognizing more than one business cycle in the United States from 1933 to 1938. The case in point is significant, however, because it draws attention to the vagueness of our definition of business cycles; which fails to set a lower limit on the amplitude of business cycles, or to limit the duration of their phases of expansion and contraction, or to specify how extensively a fluctuation must be diffused through the economy in order to rate as a business cycle. The lack of precision in our criteria of business cycles has proved troublesome in a few border-line cases. But so great is the variation in the quantity, quality and economic range of statistical records for different countries and periods, that our definition would not be so useful as a working tool if every element of vagueness in it were removed. Moreover, border-line cases are intrinsic in the historical process itself. Puzzling cases are likely to arise in practice, no matter how precisely the boundary line that sets off business cycles is drawn, or how reverently it is observed. The important thing, therefore, is to organize the statistical analysis on a plan that will force the border-line cases to the surface and thus permit revisions, if that should prove desirable. In the end our statistical analysis will accomplish this aim.

It may perhaps be helpful at this time to describe one puzzling case, and our reasons for treating it as we have. Readers familiar with the disturbed course of business during World War I may wonder why this period counts as an expansion in our reference chronology. Table 20 is not likely to dispel the doubts. In Great Britain and Germany, production of basic commodities dropped, as did employment. At the same time, the price level soared and unemployment practically disappeared. In the United States, production of basic commodities, viewed in the aggregate, changed little from our entry into the war until the Armistice; the same

[32] Note the long average leads or lags of some of the series tabulated in the source last cited.

TABLE 20

Production, Employment and Prices in Three Countries, 1914–1918

Series and country	Unit	*1914*	*1915*	*1916*	*1917*	*1918*
UNITED STATES						
(1) Industrial production...	Av. 1914 = 100	100	110	127	132	127
(2) Employment..........	Av. 1914 = 100	100	102	118	122	125
(3) Unemployment.......	Per cent	8.9	8.4	2.6	3.2	2.4
(4) Wholesale prices.......	Av. 1914 = 100	100	102	126	173	193
GREAT BRITAIN						
(5) Industrial production...	Av. 1914 = 100	100	101	93	90	85
(6) Employment..........	July 1914 = 100	100	92	93	92	91
(7) Unemployment.......	Per cent	4.2	1.3	0.6	0.7	0.8
(8) Wholesale prices.......	Av. 1914 = 100	100	127	160	206	226
GERMANY						
(9) Industrial production...	Av. 1914 = 100	100	[illegible]	77	75	69
(10) Employment..........	Av. 1914 = 100	100	81	81	84	84
(11) Unemployment.......	Per cent	7.1	3.4	2.3	1.0	1.0
(12) Wholesale prices.......	Av. 1914 = 100	100	135	145	170	207

The series are not closely comparable, but they probably suffice to indicate broad tendencies.

SOURCES:

(1) Geoffrey H. Moore, Production of Industrial Materials in World Wars I and II (National Bureau of Economic Research, *Occasional Paper 18*, March 1944), p. 5. (2) Factory employment in three states—New York, New Jersey and Massachusetts; estimated by H. Jerome, *Migration and Business Cycles* (National Bureau of Economic Research, 1926), p. 248. (3) Reported percentage of unemployed trade union members in Massachusetts. Department of Labor and Industries of Massachusetts, *Annual Report on the Statistics of Labor*, 1923, Part III, p. 20. Annual averages derived from 2-point moving averages of end-of-quarter figures. (4) Index of the Bureau of Labor Statistics; see its *Bulletin 493*, p. 9.

(5) W. Hoffmann, Ein Index der industriellen Produktion für Grossbritannien seit dem 18. Jahrhundert, *Weltwirtschaftliches Archiv*, Sept. 1934, p. 398. (6) Estimates of industrial employment (private concerns), for July. Ministry of Munitions, *History of the Ministry of Munitions*, Vol. VI, Part 4, pp. 23-9. (7) Percentage of insured work-people unemployed. Ministry of Labour, *Eighteenth Abstract of Labour Statistics of the United Kingdom*, 1926, p. 50. Annual averages derived from 2-point moving averages of end-of-month figures. Average for 1918 excludes December. (8) *The Statist*, Jubilee Section, June 1928, p. 134. (Series 5-7 for Great Britain include Ireland.)

(9) Rolf Wagenführ, Die Industriewirtschaft: Entwicklungstendenzen der deutschen und internationalen Industrieproduktion 1860 bis 1932 (Institut für Konjunkturforschung, *Vierteljahrshefte zur Konjunkturforschung*, Sonderheft 31), p. 23. (10) Estimates of industrial employment on the basis of accident insurance membership, by Albert Wohlstetter and Fred Lynn of the National Bureau staff. (11) Reported percentage of unemployed trade union members. *Reichsarbeitsblatt*, 1920, p. 25. Annual averages derived from 2-point moving averages of end-of-month figures. (12) Zahlen zur Geldentwertung in Deutschland 1914 bis 1923, *Wirtschaft und Statistik*, Sonderheft 1, 1925, p. 16.

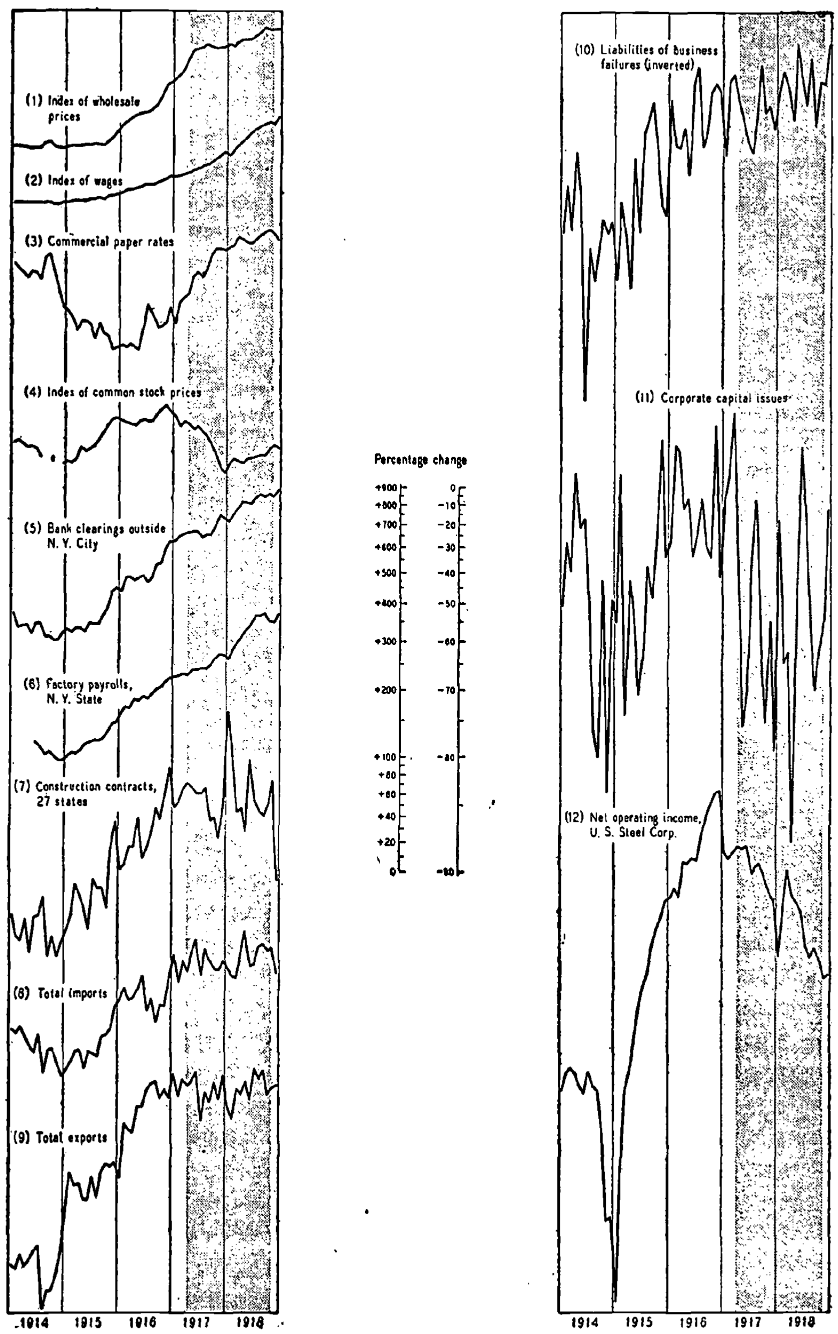
seems to have been true of employment.[33] In each country the output of war industries rose; the output of consumer goods, especially of the durable type, slumped. A few of the sharp contrasts that developed in American industry and finance during our active participation in the War are depicted in Chart 9. The production of trucks rose rapidly; the production of pleasure cars and residential construction fell drastically. Wholesale prices rose vigorously, stock prices declined. Domestic trade rose in dollar value, exports fell a trifle. Security issues declined, also trading in outstanding shares; but the bond market was exceptionally active. On the face of the statistical record, there is little reason for regarding 1917 and 1918 in the United States, or 1914–18 in Great Britain and Germany, as characterized by 'cyclical expansion'.

[33] See the study by Geoffrey H. Moore cited in the note to Table 20; also an expected publication by Albert Wohlstetter on German and British experience during World War I.

CHART 9

Behavior of Twenty-three American Series, 1914–1918

(A) Price or Value Series



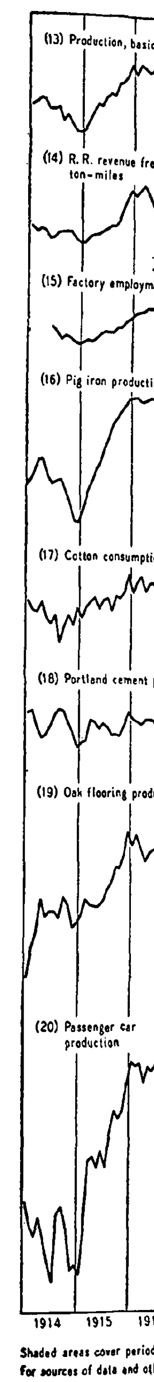
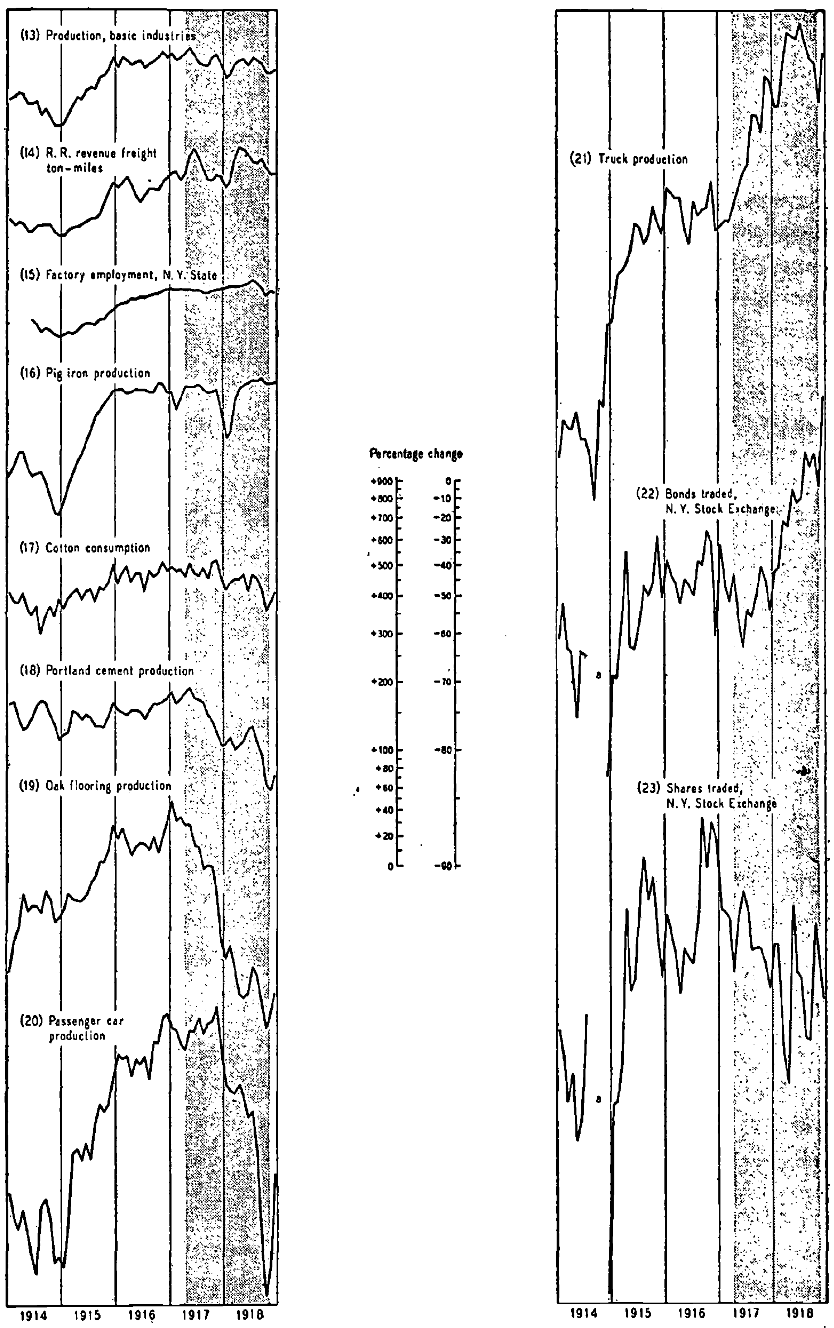


CHART 9 (CONTINUED)

(B) Physical Volume Series



Shaded areas cover period of American participation in World War I.
For sources of data and other notes, see Appendix C.

a N. Y Stock Exchange closed Aug.–Nov. 1914.

However, the bare statistical record is somewhat misleading. If one country ceded part of its area to another, we would want to allow for the change in geographic coverage before pronouncing a judgment on the 'trend' of economic activity in either country. So too must we allow for the shrinkage in the economic sector of a nation's life during a major war, regardless of changes in its geographic area. In Great Britain and Germany a great part of the industrial manpower was diverted to the military forces. This loss was only partly made up by absorption of the unemployed, youths coming of industrial age, and accessions of men and women not normally in the labor force. A reduction in the efficiency of industrial labor probably accompanied the reduction in numbers. Similar factors were at work in the United States; though in view of our relatively brief participation in the war, their influence was much smaller. The virtual disappearance of 'unemployment' and the violent rise of commodity prices in 1916–18 testify to the terrific strain on such resources as were available to industry in each of our countries.[34] Rising money incomes, a relative decline in the output of consumer goods, and increasing inelasticity of supply of commodities are typical of cyclical expansions; these factors were merely magnified by the war.

The sharp divergence in the movements of different branches of production is admittedly troublesome from the viewpoint of our concept of business cycles. But there is only one feasible alternative to treating 1914–18 as a cyclical expansion, and that is to ignore it—a practice frequently followed by writers on business cycles. That solution would be proper if our aim were merely to explain the tendencies toward cyclical fluctuations generated by processes internal to the economy. But as said in Chapter 1, the task before this investigation is to explore the business cycles of history, however 'disturbed' or 'distorted' they may be by random factors. The inclusion of the period 1914–18 as well as other war 'expansions' in the analysis, if it accomplishes nothing else, at least makes it possible to compare peace and war expansions,[35] and thus to gain some insight into differences among business cycles.[36]

VI Dependability of the Reference Dates

As already explained, the reference dates purport to mark the culminations of the cyclical expansions and contractions in 'general business

[34] In France the index of wholesale prices (Statistique Générale) rose from 118 in 1914 to 392 in 1918. Regrettably, no other statistics, similar to those used in Table 20 for other countries, seem to be available for France.

[35] For a preliminary study, see Wesley C. Mitchell, Wartime 'Prosperity' and the Future (National Bureau of Economic Research, *Occasional Paper 9,* March 1943).

[36] Of course, so far as individual activities, such as governmental expenditures or commodity prices or shipbuilding, undergo fluctuations well outside the usual range, they cannot be included in average measures of cyclical behavior without prejudice to their representative value. In such cases, our practice is to omit the war and immediate post-war cycles from the averages. See pp. 381, 492.

activity'. If this concept is somewhat fuzzy, so must be our dating. Neater results could be attained by estimating the cyclical turns of a quantity corresponding to some precise concept of aggregate economic activity. But as we have shown, the existing records virtually rule out this course except for the most recent business cycles. Even in dating recent cycles the plan has limitations. No measure of aggregate activity now available can be trusted implicitly on matters of fine detail. An investigator bent on precision will therefore wish to examine other measures of the same quantity, whether made by the same compiler at different times or by different compilers;[37] also related series that reflect indirectly the behavior of the given quantity. In the end he might present estimates of the cyclical turning dates of, say, the volume of 'employment'; but the operations he will have performed may not be very different from ours. As long as statistical data remain in something like their present state, theoretically distinct methods of dating business cycles—each used in a thoughtful and discriminating fashion—are reasonably certain to merge in practice.

That is not to say that the reference dates must remain in their present stage of rough approximation. Most of them were originally fixed in something of a hurry; revisions have been confined mainly to large and conspicuous errors, and no revision has been made for several years.[38] Surely, the time is ripe for a thorough review that would take account of extensive new statistical materials, and of the knowledge gained about business cycles and the mechanics of setting reference dates since the present chronology was worked out. In the summer of 1941 we projected such a study. The plan was to explore further the methodological problem of dating business cycles,[39] to set reference dates by different methods, reconcile conflicts so far as possible,[40] and with the experience thus gained embark on a more ambitious undertaking—a reasoned history of business cycles. But this project had barely started when the investigators placed in charge were drawn into war work.

For the time being, therefore, we must put up with a reference scale that requires extensive reworking. For example, the American reference trough in 1938 seems predated one month,[41] the trough in 1927 seems

[37] Progress in statistical records, as in other branches of life, is uneven. Despite the labor expended, revisions of statistical series are not always improvements.

[38] Indeed, the monthly (but not the quarterly or annual) American reference dates through 1927 have been allowed to stand as published in 1929 in the National Bureau's *Recent Economic Changes in the United States*, Vol. II, p. 892.

[39] Concerning techniques of ascertaining the consensus of cyclical turning points from a collection of time series, see Arthur F. Burns, *Production Trends in the United States since 1870*, pp. 182-96, and Edwin Frickey, *Economic Fluctuations in the United States*, Part II.

[40] This may, of course, involve replacing some reference turning *points* by turning *zones*; which would force a modification of some features of our technique. Cf. p. 148.

[41] See above, p. 87.

postdated one month,[42] the trough in 1921 is probably postdated several months, and the peak in 1899 predated several months. Revision is certain to shift many reference dates by a month or two or three; some may be shifted six months or a year. Even after the revisions have been carried through, the reference dates will still vary in dependability. Estimates of turning points of business cycles cannot transcend the raw materials on which they are based; hence, annual reference dates are and will continue to be more dependable than monthly,[43] American reference dates than foreign, and reference dates for recent than for early decades.

The matter of primary importance, however, is whether the reference cycles that we have recognized correspond to business cycles in the sense of our definition. That is the essential thing; the precision of the dates assigned to the culminating points of the expansions and contractions in business activity is a matter of detail by comparison. This investigation has reached a stage where we can be reasonably confident that the list of reference cycles, as a whole, identifies with substantial fidelity the cyclical tides that have swept the business world. The full evidence will be presented in subsequent publications. All we can attempt now is to put before the reader a few fragments of the evidence on which this confidence is based.

If business cycles really consist of roughly concurrent fluctuations in many economic activities, and if our reference dates mark approximately the turning points of business cycles, we should find expansion predominating in every period marked off as a reference expansion, and contraction predominating in every period marked off as a reference contraction. Table 21 tests this expectation by recording the movements of 46 monthly or quarterly series in successive phases of the American reference cycles. The sample leaves much to be desired. No series on employment, or the flow of incomes, or retail trade is included. Production, construction work, and banking are inadequately represented. Physical quantity series are relatively few. There are many partial duplications among the series. But these deficiencies must not be permitted to distort judgment. The sample covers fairly well the behavior of commodity prices, short- and long-term interest rates, security prices, trading in securities, foreign trade, payments by check, business failures, the activity of the iron industry, railroad traffic, and railroad investment. Several indexes of business conditions, which combine on different plans a variety of activities, are also included. The sample is thus of sufficient scope to lend serious interest to the results.

42 See our *Bulletin 61*, pp. 2-3.

43 But when an error does occur in an annual date, it is likely to be more serious than an error in a monthly date: the former cannot be less than twelve months.

The fiscal-year reference scale for the United States was derived from smaller samples of time series than the calendar-year scale, and therefore is not so trustworthy as the latter.

The table shows the direction of movements in successive reference phases, without any regard to the size of the movements. Each series covers at least 25 reference phases, ending in 1933. Hence the table includes 46 series in every phase since 1890, and a diminishing number in earlier phases.[44] A plus entry for a reference phase indicates a rise, a minus indicates a decline, zero indicates no change.[45] If a series systematically tends to move early or late at the reference turns, that fact is taken into account in ascertaining its direction of movement; the size of the lead or lag, if any, is entered in the two columns following the title of the series. For example, orders for locomotives tend to lead by one cycle stage at both reference peaks and troughs. Hence the sign for each reference

[44] This sample was selected from our basic collection on the following plan:

Every monthly or quarterly series that covers continuously 12 or more full reference cycles ending in 1933 was listed, the minimum period being therefore from 1891 to 1933. In the interests of simplicity it seemed desirable to limit the sample over the greater part of the period surveyed to a fixed group of series.

But in 1890 begin the price series published by the Bureau of Labor Statistics. Since many are included in our basic collection, strict adherence to the above criterion would have yielded a sample dominated by price series. This difficulty was met by including only those single-commodity price series that covered more than 12 full reference cycles. The criterion thus modified was met by 74 series.

Of the 74 series, we dropped the following 28:

Cattle receipts, Chicago	Price of sheep, Chicago
Sheep and lamb receipts, Chicago	Price of hogs, Chicago
Hog receipts, Chicago	Index of wholesale prices, farm products
Hog slaughter, commercial	Index of wholesale prices, foods
Wool receipts, Boston	Crude rubber imports
Flour shipments, Minneapolis	Raw silk imports
Raw sugar meltings, 4 to 8 ports	Tea imports
Stocks of cotton, visible supply	Coffee imports
Stocks of wheat, visible supply	Tin imports
Stocks of raw sugar, 4 ports	Anthracite coal shipments (or production)
Price of corn, Chicago	Crude petroleum production, Appalachian field
Price of wheat, Chicago	Petroleum wells completed, Appalachian field
Price of cotton, N. Y. City	Price of steel rails, Pennsylvania
Price of cattle, Chicago	Snyder's index of wages

Most (22) of these series represent marketing, crude processing, prices, imports, or stocks of agricultural commodities. Nearly all of the agricultural series are of narrow coverage—geographic, industrial, or both. The behavior of few, if any, can be regarded as important evidence of the existence or nonexistence of business cycles. In view of the dominating role nature plays in the agricultural sphere, it seemed well to exclude the purely agricultural series entirely.

Of the six remaining series, the wage index was excluded because it consists from 1875 to 1913 of a mathematical interpolation of monthly values from annual data. The other five were excluded partly because of narrow coverage, partly for other reasons: the frequency of strikes in the case of anthracite coal; a rigidly maintained price over protracted periods, in rails; limited business control over supply (a condition similar to that found in agriculture), in the two petroleum series; erratic behavior of monthly data, in tin imports.

Some of the decisions are more or less arbitrary. For example, perhaps as good a case can be made for including rubber and tin imports as for including the price series (at least tin) in Group VI of Table 21; though the fact that monthly data on imports of individual commodities are highly erratic must not be overlooked. Apart from a few doubtful cases, the series in the table are, though to an uneven degree, of broad economic significance, in contrast to the specialized nature of the items that dominate the excluded list.

[45] The signs in the columns for successive reference phases, which show direction of movement, should not be confused with the signs in the two 'lead or lag' columns, which show the characteristic timing of each series at the reference turns.

TABLE 21

Directions of Movement in Successive Reference Phases, 1854–1933

46 American Series

Series no.	Series	Lead (−) or lag (+) in cycle stages at reference: Peak	Trough	E 57	C 58	E 60	C 61	E 65	C 67	E 69	C 70	E 73	C 79	E 82	C 85
	I GENERAL BUSINESS ACTIVITY														
1	Ayres' index of business activity	0	0	+	−	+	−	+	−	+	−	+	−	+	−
2	Persons' index of production & trade	0	0	..	..	..	..	..	..	..	..	..	..	+	−
3	Axe-Houghton index of trade & industrial activity	0	0	..	..	..	..	..	..	..	..	..	..	+	−
4	A.T.&T. index of business activity	0	0	..	..	..	..	..	..	..	..	..	..	+	−
5	Pittsburgh index of business	0	0	..	..	..	..	..	..	..	..	..	..	..	..
6	Bank clearings, total	0	−1	..	..	..	..	..	..	..	..	..	..	+	−
7	Bank clearings outside N.Y.City	0	−1	..	..	..	..	..	..	..	..	..	..	+	−
8	Bank clearings outside N.Y.City, deflated	0	−1	..	..	..	..	..	..	..	..	..	..	+	−
9	Snyder's clearings index of business	0	0	..	..	..	..	..	..	..	..	..	..	+	−
10	Snyder's index of deposits activity	−1	−1	..	..	..	..	..	..	..	..	..	..	+	−
	II ORDERS FOR INVESTMENT GOODS														
11	Orders for locomotives	−1	−1	..	..	..	..	..	..	..	..	+	−	+	−
12	Orders for freight cars	−1	−1	..	..	..	..	..	..	..	..	+	+	+	−
13	Orders for passenger cars	−1	−1	..	..	..	..	..	..	..	..	+	+	+	−
14	Plans filed for new buildings, Manhattan	−2	−2	..	..	..	..	..	..	+	−	−	−	+	+
	III PRODUCTION														
15	Railroad freight ton-miles	0	−1	..	..	..	..	..	..	+	+	+	+	+	+
16	Pig iron production	0	0	..	..	..	..	..	..	..	..	..	..	+	−
	IV FOREIGN TRADE														
17	Total imports	0	−1	..	..	..	..	..	..	+	+	+	−	+	−
18	Total exports	0	0	..	..	..	..	..	..	+	+	+	+	+	−
	V INDEXES OF PRICES														
19	Snyder's index of general prices	0	0	..	..	..	−	+	−	−	−	−	−	+	−
20	Wholesale prices, total	0	0	+	−	−	−	+	−	−	−	−	−	+	−
21	Wholesale prices, metals & metal products	0	0	..	..	..	..	..	..	..	..	..	..	..	..
22	Wholesale prices, building materials	+1	0	..	..	..	..	..	..	..	..	..	..	..	..
23	Wholesale prices, fuel & lighting	0	0	..	..	..	..	..	..	..	..	..	..	..	..
24	Wholesale prices, chemicals & drugs	0	0	..	..	..	..	..	..	..	..	..	..	..	..
25	Wholesale prices, textiles	0	0	..	..	..	..	..	..	..	..	..	..	..	..
26	Wholesale prices, housefurnishing goods	+1	+1	..	..	..	..	..	..	..	..	..	..	..	..
27	Wholesale prices, hides & leather products	0	−1	..	..	..	..	..	..	..	..	..	..	..	..
	VI WHOLESALE PRICES OF INDIVIDUAL COMMODITIES														
28	Pig iron, Philadelphia	0	0	−	−	+	−	+	−	−	−	+	−	+	−
29	Steel billets, Pittsburgh	0	+1	..	..	..	..	..	..	..	..	..	..	..	..
30	Slab zinc, N.Y.City	0	0	..	..	..	..	..	..	..	..	..	..	+	−
31	Copper, N.Y.City	0	0	..	..	..	−	+	−	+	−	+	−	+	−
32	Pig lead, N.Y.City	0	0	..	..	..	..	..	..	..	..	+	−	+	−
33	Pig tin, N.Y.City	0	0	..	..	..	..	..	..	..	..	..	..	..	−
	VII MONEY AND SECURITY MARKETS														
34	Bank clearings, N.Y.City	−1	−1	+	−	+	−	+	−	+	−	+	−	+	−
35	Shares traded, N.Y.Stock Exchange	−1	−1	..	..	..	..	..	..	..	..	..	..	+	−
36	Bonds traded, N.Y.Stock Exchange, *inverted*	+2	+1	..	..	..	..	..	..	..	..	..	..	..	..
37	Index of 'all' common stock prices	−1	−1	..	..	..	..	..	..	..	..	..	−	+	−
38	Index of railroad stock prices	−1	−1	..	−	+	−	+	−	+	+	+	−	+	−
39	Call money rates, N.Y.Stock Exchange	0	0	..	−	+	−	−	−	+	−	+	−	+	−
40	90-day money rates, stock exchange loans	+1	0	..	..	..	..	..	..	..	..	..	..	..	..
41	Commercial paper rates, N.Y.City	+1	+1	..	−	+	−	+	+	+	−	−	−	+	−
42	Railroad bond yields	+1	+2	..	−	+	−	+	0	+	−	−	−	−	−
	VIII BUSINESS FAILURES														
43	Number (Dun's), *inverted*	0	0	..	..	..	..	..	..	..	..	..	..	+	−
44	Number (Bradstreet's), *inverted*	0	0	..	..	..	..	..	..	..	..	..	..	..	−
45	Liabilities (Dun's), *inverted*	−1	−2	..	..	..	..	..	..	..	..	..	..	+	−
46	Liabilities (Bradstreet's), *inverted*	−1	−2	..	..	..	..	..	..	..	..	..	..	..	−

For explanation of the signs, see text. 'E 57' stands for the reference expansion ending June 1857, 'C 58' for the reference contraction ending Dec. 1858, etc. The monthly reference dates are listed in Table 16.

See Appendix C for sources and other notes about the series.

Direc

Series no.	E 87	C 88	E 90	C 91	E 93	C 94
1	+	−	+	−	+	−
2	+	−	+	−	+	−
3	+	−	+	−	+	−
4	+	−	+	−	+	−
5	+	−	+	−	+	−
6	+	−	+	−	+	−
7	+	−	+	−	+	−
8	+	−	+	−	+	−
9	+	−	+	−	+	−
10	+	−	+	−	+	−
11	+	−	+	−	+	−
12	+	−	+	−	+	−
13	+	−	+	−	+	−
14	+	−	+	+	−	−
15	+	+	+	+	+	−
16	+	−	+	−	+	−
17	+	+	+	−	+	−
18	−	−	+	+	−	+
19	+	+	−	−	+	−
20	0	+	−	+	−	−
21	..	..	..	−	−	−
22	..	..	..	−	−	−
23	..	..	..	−	−	−
24	..	..	..	+	+	−
25	..	..	..	−	+	−
26	..	..	..	+	−	−
27	..	..	..	−	−	−
28	+	−	−	−	−	−
29	+	−	+	−	−	−
30	+	+	+	−	−	−
31	−	+	−	−	−	−
32	+	+	−	−	−	−
33	+	+	−	−	−	−
34	..	−	+	−	+	−
35	+	−	+	−	+	−
36	..	..	−	−	+	−
37	+	−	+	−	+	−
38	+	−	+	−	+	−
39	+	−	+	−	−	−
40	..	..	..	+	+	−
41	+	−	+	+	+	−
42	+	−	+	+	+	−
43	+	−	−	−	+	−
44	+	−	−	−	+	−
45	+	−	+	−	+	−
46	+	−	+	−	+	−

TABLE 21—*Continued*

Directions of Movement in Successive Reference Phases, 1854–1933

46 American Series

E 87	C 88	E 90	C 91	E 93	C 94	E 95	C 97	E 99	C 00	E 02	C 04	E 07	C 08	E 10	C 12	E 13	C 14	E 18	C 19	E 20	C 21	E 23	C 24	E 26	C 27	E 29	C 33	Series no.
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	1
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	2
+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	3
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	4
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	5
+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	+	+	−	+	+	+	+	+	−	6
+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	+	+	−	+	+	+	−	+	−	+	+	+	−	7
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	+	+	+	−	8
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	9
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	+	+	−	10
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	11
+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	12
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	+	−	+	−	−	−	13
+	−	+	+	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	+	−	14
+	+	+	+	+	−	+	−	+	+	+	+	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	15
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	16
+	+	+	−	+	−	+	+	−	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	17
−	−	+	+	−	+	+	+	+	+	+	−	+	−	−	+	+	+	+	+	+	−	+	−	+	−	+	−	18
+	+	−	−	+	−	+	−	+	+	+	+	+	−	+	+	+	−	+	+	+	−	+	−	+	+	+	−	19
0	+	−	+	−	−	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	20
..	..	..	−	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	21
..	..	..	−	−	−	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	22
..	..	..	−	−	−	+	−	+	+	+	−	+	−	−	−	+	−	+	−	+	−	+	−	+	−	−	−	23
..	..	..	+	+	−	+	+	+	+	+	−	−	−	+	−	−	+	+	−	+	−	−	−	+	−	−	−	24
..	..	..	−	+	−	+	−	+	+	−	+	+	−	+	−	+	−	+	−	+	−	+	−	−	−	−	−	25
..	..	..	+	−	−	−	−	+	+	+	−	+	−	+	−	+	+	+	+	+	−	+	−	−	−	−	−	26
..	..	..	−	−	−	+	−	+	−	+	−	+	−	+	−	+	+	+	+	+	−	−	−	−	+	−	−	27
+	−	−	−	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	28
+	−	+	−	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	29
+	+	+	−	−	−	+	+	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	30
−	+	−	−	−	−	+	+	+	−	−	+	+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	31
+	+	−	−	−	−	−	+	+	−	−	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	32
+	+	−	−	−	−	−	−	+	+	−	+	+	−	+	+	+	−	+	−	−	−	+	+	+	−	−	−	33
.	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	+	+	+	+	+	−	34
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	+	+	−	+	−	+	+	+	−	35
..	..	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	−	−	+	−	+	+	+	−	36
+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	−	+	+	−	+	−	+	+	+	−	37
+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	−	+	−	−	+	−	+	+	+	−	38
+	−	+	−	−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	+	−	39
..	..	..	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	−	+	−	40
+	−	+	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	+	+	−	41
+	−	+	+	+	−	−	−	−	+	+	+	+	−	+	+	+	0	+	+	+	−	+	−	−	+	+	−	42
+	−	−	−	+	−	−	+	+	−	−	−	+	−	+	−	+	−	+	+	+	−	+	−	−	−	+	−	43
+	−	−	−	+	−	−	+	+	−	+	−	+	−	+	−	+	−	+	+	+	−	+	−	+	−	+	−	44
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	+	+	−	−	−	+	−	+	−	45
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	+	+	+	−	+	−	+	−	+	−	46

expansion indicates whether the average volume of orders in the stage just before the reference peak exceeded or fell short of the average in the stage just before the preceding reference trough. Likewise the sign for each reference contraction compares the average volume of orders in the stage just before the reference trough with the average in the stage just before the preceding reference peak. Of the 46 series, 22 have no tendency to lead or lag; in this group of series the signs are determined simply by comparing the standings at successive reference peaks and troughs. In bankruptcies and bond sales, which bear an inverted relation to business cycles, all signs are reversed; but it is simpler to think of these series as treated on the standard plan, after having been inverted at the start, and they are designated thus in the stub.[46]

Table 22 summarizes these entries, series by series. A small group of highly significant items—four indexes of business activity, pig iron production, and locomotive orders—invariably rise during reference expansions and decline during reference contractions. Another two indexes of business activity conform in direction to every reference phase but one. Six more series conform in direction to all but two reference phases; another seven conform to all but three phases, and another twelve to all but four or five phases. This count includes 33 series, among which are the most comprehensive items in the full sample of 46. The number of series whose fluctuations match closely our reference cycles would appear still larger if the entries in Table 21 were refined. For example, although railway freight traffic rose in eight reference contractions, the rise in seven of these instances was at a lower rate than during the adjacent phases of expansion.[47] Taken as a whole, the table clearly supports the hypothesis that many business activities in the United States have shared a common rhythm and that our reference-cycle chronology exposes and expresses this rhythm.

It is a matter of some consequence whether the reference cycles fit the cyclical fluctuations in business activities phase by phase, as well as on the average. The summary in Table 23 is directed to this question. Without exception we find that the proportion of rising series drops abruptly in passing from a reference expansion to a contraction; with equal abruptness it goes up in passing from a reference contraction to an expansion. Further, in every reference expansion the proportion of rises exceeds one-half by a good margin, and the like is true of declines during refer-

[46] This explanation covers the essentials, but is incomplete. It is not possible to go further without assuming knowledge of Ch. 5, or repeating much of that chapter. The entries in Table 21 are taken directly from our standard Table R4, explained in Ch. 5, Sec. X.

[47] Furthermore, declines that we recognize as specific-cycle contractions occurred within four of the eight reference contractions. They fail to register in Table 21, because their timing differs materially from the fixed schedule assumed in the table. See Thor Hultgren, Railway Freight Traffic in Prosperity and Depression (National Bureau of Economic Research, *Occasional Paper 5*, Feb. 1942). Table 3 of that paper is not strictly comparable with our Table 21, since the former does not take account of the tendency to lead at reference troughs.

TABLE 22

Summary of Movements in All Reference Expansions and Contractions
46 American Series, 1854–1933

Series	Number of reference expansions			Number of reference contractions			Number of reference phases		
	Covered	In which series		Covered	In which series		Covered	In which series	
		Rises	Declines		Rises	Declines		Conforms[a]	Fails to conform[b]
I GENERAL BUSINESS ACTIVITY									
Ayres' index of business activity	20	20	..	20	..	20	40	40	..
Persons' index of production & trade	15	15	..	15	..	15	30	30	..
Axe-Houghton index, trade & ind. activity	15	15	..	15	1	14	30	29	1
A.T.&T. index of business activity	15	15	..	15	..	15	30	30	..
Pittsburgh index of business	14	14	..	14	..	14	28	28	..
Bank clearings, total	15	15	..	15	4	11	30	26	4
Bank clearings outside N.Y.C.	15	15	..	15	4	11	30	26	4
Bank clearings outside N.Y.C., deflated	15	15	..	15	2	13	30	28	2
Snyder's clearings index of business	15	15	..	15	1	14	30	29	1
Snyder's index of deposits activity	15	15	..	15	2	13	30	28	2
II ORDERS FOR INVESTMENT GOODS									
Orders for locomotives	16	16	..	16	..	16	32	32	..
Orders for freight cars	16	16	..	16	3	13	32	29	3
Orders for passenger cars	16	14	2	16	1	15	32	29	3
Plans filed for new buildings, Manhattan	17	15	2	17	3	14	34	29	5
III PRODUCTION									
Railroad freight ton-miles	17	17	..	17	8	9	34	26	8
Pig iron production	15	15	..	15	..	15	30	30	..
IV FOREIGN TRADE									
Total imports	17	16	1	17	4	13	34	29	5
Total exports	17	14	3	17	9	8	34	22	12
V INDEXES OF PRICES									
Snyder's index of general prices	18	15	3	19	6	13	37	28	9
Wh. prices, total	20[c]	13	6	20	3	17	40	30	10
Wh. prices, metals & metal products	12	10	2	13	..	13	25	23	2
Wh. prices, building materials	12	9	3	13	..	13	25	22	3
Wh. prices, fuel & lighting	12	9	3	13	1	12	25	21	4
Wh. prices, chemicals & drugs	12	8	4	13	4	9	25	17	8
Wh. prices, textiles	12	9	3	13	2	11	25	20	5
Wh. prices, housefurnishing goods	12	8	4	13	4	9	25	17	8
Wh. prices, hides & leather products	12	8	4	13	3	10	25	18	7
VI WHOLESALE PRICES OF INDIVIDUAL COMMODITIES									
Pig iron, Philadelphia	20	16	4	20	..	20	40	36	4
Steel billets, Pittsburgh	14	12	2	14	..	14	28	26	2
Slab zinc, N.Y.City	15	14	1	15	3	12	30	26	4
Copper, N.Y.City	18	14	4	19	4	15	37	29	8
Pig lead, N.Y.City	16	12	4	16	3	13	32	25	7
Pig tin, N.Y.City	14	8	6	15	5	10	29	18	11
VII MONEY AND SECURITY MARKETS									
Bank clearings, N.Y.City	20	20	..	20	3	17	40	37	3
Shares traded, N.Y.Stock Exchange	15	14	1	15	2	13	30	27	3
Bonds traded, N.Y.Stock Exchange, *inv.*	13	10	3	13	1	12	26	22	4
Index of 'all' common stock prices	15	14	1	16	3	13	31	27	4
Index of railroad stock prices	19	17	2	20	4	16	39	33	6
Call money rates, N.Y.Stock Exchange	19	16	3	20	..	20	39	36	3
90-day money rates, stock exchange loans	12	11	1	13	1	12	25	23	2
Commercial paper rates, N.Y.City	19	18	1	20	3	17	39	35	4
Railroad bond yields	19	14	5	20[d]	6	12	39	26	13
VIII BUSINESS FAILURES									
Number (Dun's), *inverted*	15	11	4	15	2	13	30	24	6
Number (Bradstreet's), *inverted*	14	12	2	15	2	13	29	25	4
Liabilities (Dun's), *inverted*	15	13	2	15	1	14	30	27	3
Liabilities (Bradstreet's), *inverted*	14	13	1	15	1	14	29	27	2

Derived from Table 21.

[a] Rises during reference expansion or declines during reference contraction.

[b] Difference between the two preceding columns.

[c] Includes one reference expansion during which the series shows no change.

[d] Includes two reference contractions during which the series shows no change.

ence contractions. The smallest proportion of rises to declines in a reference expansion, or of declines to rises in a reference contraction, is 2 to 1. Though not conclusive, these results strongly indicate that the successive reference dates under test mark off units of pervasive fluctuations in business life, and thus serve the purpose for which they were designed.

TABLE 23

Summary of Movements in Successive Reference Phases, 1854–1933
Based on 46 American Series

Nature of phase	Period	Number of series covered	Number of series that			Per cent of series that	
			Rise	Decline	Show no change[a]	Rise	Decline
Exp.	1854–57	4	3	1	...	75	25
Con.	1857–58	8	...	8	...	0	100
Exp.	1858–60	8	7	1	...	88	12
Con.	1860–61	10	...	10	...	0	100
Exp.	1861–65	10	9	1	...	90	10
Con.	1865–67	10	1	8	1	15	85
Exp.	1867–69	14	11	3	...	79	21
Con.	1869–70	14	4	10	...	29	71
Exp.	1870–73	18	13	5	...	72	28
Con.	1873–79	19	4	15	...	21	79
Exp.	1879–82	32	31	1	...	97	3
Con.	1882–85	35	2	33	...	6	94
Exp.	1885–87	37	34	2	1	93	7
Con.	1887–88	37	8	29	...	22	78
Exp.	1888–90	38	29	9	...	76	24
Con.	1890–91	46	9	37	...	20	80
Exp.	1891–93	46	31	15	...	67	33
Con.	1893–94	46	1	45	...	2	98
Exp.	1894–95	46	39	7	...	85	15
Con.	1895–97	46	8	38	...	17	83
Exp.	1897–99	46	44	2	...	96	4
Con.	1899–00	46	16	30	...	35	65
Exp.	1900–02	46	41	5	...	89	11
Con.	1902–04	46	7	39	...	15	85
Exp.	1904–07	46	45	1	...	98	2
Con.	1907–08	46	1	45	...	2	98
Exp.	1908–10	46	44	2	...	96	4
Con.	1910–12	46	9	37	...	20	80
Exp.	1912–13	46	42	4	...	91	9
Con.	1913–14	46	4	41	1	10	90
Exp.	1914–18	46	42	4	...	91	9
Con.	1918–19	46	15	31	...	33	67
Exp.	1919–20	46	43	3	...	93	7
Con.	1920–21	46	...	46	...	0	100
Exp.	1921–23	46	41	5	...	89	11
Con.	1923–24	46	5	41	...	11	89
Exp.	1924–26	46	38	8	...	83	17
Con.	1926–27	46	15	31	...	33	67
Exp.	1927–29	46	38	8	...	83	17
Con.	1929–33	46	...	46	...	0	100

Derived from Table 21.

[a] These instances were split equally between the rises and declines, in computing the percentages of the following columns.

The preceding results are tested in Table 24 on the basis of three subsamples. One group includes 18 series on 'commercial and industrial activity'; a second on 'prices and financial activity' includes the remaining 28 series. The second group bears out the full sample, phase by phase.

TABLE 24
Summary of Movements in Successive Reference Phases, 1854–1933
Based on Three Subsamples Drawn from 46 American Series

Nature of phase	Period	Commercial and industrial activity[a]			Prices and financial activity[b]			Non-duplicating sample[c]		
		No. of series	Per cent of series that		No. of series	Per cent of series that		No. of series	Per cent of series that	
			Rise	Decline		Rise	Decline		Rise	Decline
Exp.	1854–57	1	100	0	3	67	33	3	67	33
Con.	1857–58	1	0	100	7	0	100	6	0	100
Exp.	1858–60	1	100	0	7	86	14	6	83	17
Con.	1860–61	1	0	100	9	0	100	7	0	100
Exp.	1861–65	1	100	0	9	89	11	7	86	14
Con.	1865–67	1	0	100	9	17	83	7	21	79
Exp.	1867–69	5	100	0	9	67	33	11	82	18
Con.	1869–70	5	60	40	9	11	89	11	27	73
Exp.	1870–73	8	88	12	10	60	40	15	73	27
Con.	1873–79	8	50	50	11	0	100	16	25	75
Exp.	1879–82	17	100	0	15	93	7	20	95	5
Con.	1882–85	17	12	88	18	0	100	22	9	91
Exp.	1885–87	18	94	6	19	92	8	23	89	11
Con.	1887–88	18	11	89	19	32	68	23	30	70
Exp.	1888–90	18	100	0	20	55	45	24	75	25
Con.	1890–91	18	17	83	28	21	79	25	32	68
Exp.	1891–93	18	89	11	28	54	46	25	68	32
Con.	1893–94	18	6	94	28	0	100	25	4	96
Exp.	1894–95	18	100	0	28	75	25	25	88	12
Con.	1895–97	18	11	89	28	21	79	25	12	88
Exp.	1897–99	18	94	6	28	96	4	25	92	8
Con.	1899–00	18	33	67	28	36	64	25	40	60
Exp.	1900–02	18	100	0	28	82	18	25	96	4
Con.	1902–04	18	6	94	28	21	79	25	12	88
Exp.	1904–07	18	100	0	28	96	4	25	96	4
Con.	1907–08	18	6	94	28	0	100	25	0	100
Exp.	1908–10	18	94	6	28	96	4	25	92	8
Con.	1910–12	18	22	78	28	18	82	25	16	84
Exp.	1912–13	18	100	0	28	86	14	25	88	12
Con.	1913–14	18	6	94	28	12	88	25	18	82
Exp.	1914–18	18	94	6	28	89	11	25	88	12
Con.	1918–19	18	17	83	28	43	57	25	32	68
Exp.	1919–20	18	100	0	28	89	11	25	96	4
Con.	1920–21	18	0	100	28	0	100	25	0	100
Exp.	1921–23	18	100	0	28	82	18	25	84	16
Con.	1923–24	18	17	83	28	7	93	25	8	92
Exp.	1924–26	18	100	0	28	71	29	25	76	24
Con.	1926–27	18	33	67	28	32	68	25	36	64
Exp.	1927–29	18	94	6	28	75	25	25	76	24
Con.	1929–33	18	0	100	28	0	100	25	0	100

See notes to Table 23.

[a] Includes groups I-IV in Table 21.

[b] Includes groups V-VIII in Table 21.

[c] Includes series 8, 11-18, 21-27, 34-37, 39-42, and 46, as numbered in Table 21. Since series 21-27 do not go back of 1890, series 20 and 28-33 are used instead through 1890. Series 46 was preferred to series 45, because its dollar coverage is on the average larger.

In the first group, rises slightly outnumber declines in one reference contraction and equal the number of declines in another; but these anomalies occur before our sample reaches its full size. Finally, the table shows the movements of 25 series that are practically independent, in the sense that their coverage is practically free from duplication. The summary for this subsample is easier to interpret than the summary for the full sample, for the subsample has some claim to statistical purity while the full sample has none. At the same time, the duplications in the latter tend on the whole to give additional weight to the more important activities in the sample. But we need not dwell on the respective merits of the two samples, for the nonduplicating series confirm closely the results for the full sample.

Further confirmation is afforded by Table 25, which records the number of rises and declines determined on three principles. The entries under 'A' repeat the results in Table 23. Method B is the same as method A, except that it makes no allowance for the characteristic lead or lag of some series at reference turns.[48] Method C is a hybrid of A and B. If the movement of a series appears as countercyclical (that is, as a rise in a reference contraction or a decline in a reference expansion) when method A is used but as conforming when method B is used, it is counted as conforming in method C. In all other instances the count in method C is the same as in A.[49] Thus method B ignores leads or lags entirely, A recognizes fixed leads or lags, while C admits some flexibility in timing. Of the three methods, B supplies the severest test of our hypothesis that the reference cycles mark off units of roughly concurrent fluctuations in many economic activities. Method C, on the other hand, is most closely geared to our hypothesis, which admits the possibility of irregular leads or lags.[50] But the merits of the several methods need not detain us; in view of the roughness of each, it is better to regard them as supplements than as alternatives. In any event, each method supports the list of reference cycles.[51]

48 The count was made from a table similar to Table 21. Of course, the entries in the two tables were identical for the 22 series for which no allowance for leads or lags was necessary in Table 21.

49 There is nothing to recommend this method of admitting flexibility, except that it is easy to compute, allows no room for subjective judgments, and is conservative—in the sense that it allows only slight flexibility.

50 Cf. p. 6.

51 So do several other methods, not reproduced for reasons of economy. The one discordant item in these experiments is the result of method B for the reference contraction of 1899–1900. But the peak from which this contraction starts is predated, perhaps by a half year. A rough adjustment for the error in dating may be obtained by measuring the change from the first third of the reference contraction to the trough (that is, from stage VI to IX in our standard Table R1, instead of from V to IX). According to this computation, likely to be an understatement, the percentage of declines is 61. It may also be noted that 72 per cent of the series 'conform' to the reference contraction even as dated; that is to say, the rate of change during this fixed interval is algebraically smaller than in the next preceding and following expansions in 72 per cent of the series in our sample.

Besides the general results already indicated, Tables 23-25 bring out the uneven diffusion of business cycles, which is one of their outstanding characteristics. On the average the proportion of rises during reference expansions exceeds slightly the proportion of declines during reference

TABLE 25

Summary of Movements in Successive Reference Phases, 1854–1933 Based on Different Methods Applied to 46 American Series

Nature of phase	Period	No. of series covered	Per cent of series that rise when: Allowance is made for fixed leads or lags[a] (A)	Per cent of series that rise when: No allowance is made for leads or lags[b] (B)	Per cent of series that rise when: Allowance is made for flexible leads or lags[b] (C)	Per cent of series that decline when: Allowance is made for fixed leads or lags[a] (A)	Per cent of series that decline when: No allowance is made for leads or lags[b] (B)	Per cent of series that decline when: Allowance is made for flexible leads or lags[b] (C)
Exp.	1854–57	4	75	75	75	25	25	25
Con.	1857–58	8	0	0	0	100	100	100
Exp.	1858–60	8	88	75	88	12	25	12
Con.	1860–61	10	0	10	0	100	90	100
Exp.	1861–65	10	90	80	90	10	20	10
Con.	1865–67	10	15	30	5	85	70	95
Exp.	1867–69	14	79	79	79	21	21	21
Con.	1869–70	14	29	25	21	71	75	79
Exp.	1870–73	18	72	56	78	28	44	22
Con.	1873–79	19	21	37	21	79	63	79
Exp.	1879–82	32	97	97	97	3	3	3
Con.	1882–85	35	6	6	6	94	94	94
Exp.	1885–87	37	93	88	93	7	12	7
Con.	1887–88	37	22	32	19	78	68	81
Exp.	1888–90	38	76	58	76	24	42	24
Con.	1890–91	46	20	26	17	80	74	83
Exp.	1891–93	46	67	59	70	33	41	30
Con.	1893–94	46	2	4	2	98	96	98
Exp.	1894–95	46	85	78	85	15	22	15
Con.	1895–97	46	17	34	17	83	66	83
Exp.	1897–99	46	96	93	96	4	7	4
Con.	1899–00	46	35	52	33	65	48	67
Exp.	1900–02	46	89	80	89	11	20	11
Con.	1902–04	46	15	26	15	85	74	85
Exp.	1904–07	46	98	87	98	2	13	2
Con.	1907–08	46	2	7	0	98	93	100
Exp.	1908–10	46	96	91	96	4	9	4
Con.	1910–12	46	20	26	20	80	74	80
Exp.	1912–13	46	91	90	93	9	10	7
Con.	1913–14	46	10	13	10	90	87	90
Exp.	1914–18	46	91	87	93	9	13	7
Con.	1918–19	46	33	37	28	67	63	72
Exp.	1919–20	46	93	91	93	7	9	7
Con.	1920–21	46	0	5	0	100	95	100
Exp.	1921–23	46	89	89	93	11	11	7
Con.	1923–24	46	11	20	9	89	80	91
Exp.	1924–26	46	83	76	85	17	24	15
Con.	1926–27	46	33	29	25	67	71	75
Exp.	1927–29	46	83	77	85	17	23	15
Con.	1929–33	46	0	2	0	100	98	100

In an insignificant number of instances, a series showed zero change during a reference phase. Their treatment is explained in Table 23, note 'a'.

[a] Taken directly from Table 23.

[b] Explained in text.

TABLE 26

Relation between the Amplitude and Diffusion of Business Cycles
United States, 1879–1933

Method of ascertaining direction of movement[a]	Average per cent of series that					
	Rise in expansions whose amplitude is[b]			Decline in contractions whose amplitude is[c]		
	Mild	Moderate	Vigorous	Mild	Moderate	Severe
B........	73	84	91	67	74	95
A........	81	90	94	75	82	98
C........	83	91	95	78	84	98

Derived from Table 25. The averages are unweighted arithmetic means. Weighting by number of series would change the averages by one point at most. It cannot be of much consequence because the sample is fixed, starting with the reference contraction of 1890–91.

[a] See the headings in Table 25 and the explanations in the text.

[b] To classify business-cycle expansions according to intensity of amplitude (see Table 156), we (a) matched the specific-cycle expansions of three indexes of business activity (A.T.&T., Ayres, and Persons) with the reference expansions, (b) ranked the fifteen expansions in each index from 1879 to 1929, (c) averaged the ranks of the three indexes for each reference expansion, (d) sorted the reference expansions into three equal groups on the basis of the average ranks. The results are as follows—
Mild (smallest 5): 1888–90, 1891–93, 1900–02, 1912–13, 1927–29
Moderate (next 5): 1885–87, 1894–95, 1904–07, 1919–20, 1924–26
Vigorous (largest 5): 1879–82, 1897–99, 1908–10, 1914–18, 1921–23

[c] The cyclical contractions were classified on the same plan as the expansions, with the following results—
Mild (smallest 5): 1887–88, 1899–1900, 1902–04, 1910–12, 1926–27
Moderate (next 5): 1890–91, 1895–97, 1913–14, 1918–19, 1923–24
Severe (largest 5): 1882–85, 1893–94, 1907–08, 1920–21, 1929–33

contractions. This is the result to be expected in a progressive economy.[52] However, there are more business-cycle contractions in which every or almost every series of our sample declines than there are expansions in which every or almost every series rises. Consequently, the proportion of falls during reference contractions varies more from case to case than does the proportion of rises during reference expansions. Table 26 demonstrates that the degree of cyclical diffusion is correlated with the amplitude of cyclical fluctuations. Since 1879[53] practically all series in our sample declined during severe business-cycle contractions, while a substantial proportion rose during mild contractions.[54] The diffusion of business-cycle expansions is likewise correlated with their amplitude, though the correlation is not so close as in contractions (see Tables 25 and 156).[55]

[52] The average per cent of rises during reference expansions is 82, 88, 89, according to methods B, A and C, respectively, in the sample of 46 series. The corresponding averages of the per cent of declines during reference contractions are 78, 85, and 87. The discrepancy between the averages for expansion and contraction would be more prominent if the sample excluded series adjusted for secular trend and included more volume series relatively to price series. The averages just cited are weighted arithmetic means, the weight being the number of series covering a reference phase.

[53] That is as far back as it is safe to go at present, in view of the diminishing size of the sample and the uncertainties in grading the severity of cyclical movements. Cf. pp. 455, 462, 464.

[54] It may be of interest to note (Table 24) that, according to method A, every series on 'prices and finance' declined during the five severe contractions between 1879 and 1933. This happened as well in the substantial contractions of 1857–58 and 1873–79. However, back of 1879 the sample shrinks rapidly.

[55] If we may judge from our sample, there is no significant correlation between the duration of reference phases and cyclical diffusion.

From the uneven diffusion of business fluctuations and the correlation of this feature with their amplitude springs discord in chronologies of business cycles. There is a growing tendency among economists to regard mild and severe business contractions as belonging to one species of phenomena, but disagreements on this issue persist. Every competent judge who admits the existence of business cycles will readily agree that violent contractions, such as occurred in 1907–08, 1920–21 and 1929–33, are business-cycle movements. But there is disagreement whether a mild and uneven contraction such as that of 1887–88 or 1926–27 may be justly considered a phase of business cycles.[56] It is still a fairly common practice among theorists, when venturing observations on the history of business cycles, to recite with little ado chronologies restricted to 'booms' and 'severe depressions'. Economic historians still have a special predilection for 'crises' dates. These traditional procedures have been dropped by economic statisticians; not from a love of novelty, but because their method of working trains and disciplines the eye. Observing in their charts a continuous gradation from 'mild' to 'vigorous' fluctuations, they have been impelled to recognize as business cycles many movements that are overlooked or slighted by literary investigators. Their chronologies are very similar to ours,[57] though their language is not always the same.

Table 27 shows compactly how our list of business cycles in the United States between 1854 and 1938 compares with the lists of other investigators, each of whom has chronicled all or a substantial part of this period on some independent basis. Kitchin, Persons, Ayres, and Axe and Houghton speak explicitly of dating 'business cycles'. Eckler observes that some contractions on his list should be regarded as 'recessions' rather than 'depressions'. Hubbard writes in a similar vein: "minor movements, when well defined, have been included, whether such movements are properly to be classified as depressions or not".[58] Both Hubbard and Eckler seem to imply that only a 'depression'—by which they mean a severe contraction—can mark the close of a business cycle. Similar views are held by Gilbert, who argues, for example, that a 'recession', not a 'depression', occurred late in 1887 and early in 1888: "to consider the dip and recovery of 1887 and 1888 as more than a slight adjustment or breathing space in the forward march is to deny the significance of those fundamental changes which have been distinguished as business cycles".[59] It does not seem that more is meant here by 'fundamental changes' than vigorous expansions and severe contractions. Terminological prefer-

[56] These matters are rarely analyzed scientifically. Cf. Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 464-8, and our *Bulletin 61*, pp. 2, 18-20.

[57] Economic statisticians have also called attention to fluctuations of a higher order than business cycles. It is instructive to note in this connection that their chronologies of 'long cycles' differ widely. See Ch. 11.

[58] *Review of Economic Statistics*, Feb. 1936, p. 17.

[59] *Ibid.*, Aug. 1933, pp. 142-3.

TABLE 27

Business Cycles Recognized by the National Bureau and Other Investigators, United States, 1854–1938

Investigator	Period covered by comparison[a]	Number of cyclical phases[b] during this period in N.B.E.R. chronology	Cyclical phases recognized by N.B.E.R. that investigator omits	Cyclical phases not recognized by N.B.E.R. that investigator lists	Criterion for dating business cycles	Source of publication
J. Kitchin	1890–1920	18	None	None	Bank clearings, wholesale prices, and commercial paper rates	Cycles and Trends in Economic Factors, *Review of Economic Statistics*, Jan. 1923, p. 10.
E. W. Axe and R. Houghton	1885–1929	27	Con. 1926–27	None	Axe-Houghton index of trade and industrial activity	Financial and Business Cycles, Manufacturing Growth, and Analysis of Individual Industries, 1883–1930, *The Annalist*, Jan. 16, 1931, pp. 150-1. Numerous misprints; verified from pp. 95, 162-3 of this source, and *ibid.*, Jan. 15, 1926, pp. 115-6, and July 18, 1930, pp. 102-3.
W. M. Persons	1873–1938	33	None	Exp. 1875–76 Con. 1898	Persons' index of production and trade (Barron's index after 1929)	*Forecasting Business Cycles* (John Wiley, 1931), pp. 84-5, 198. Continued after 1929 according to Persons' method, by Elmer C. Bratt, *Business Cycles and Forecasting* (Business Publications, 1940), p. 401.
A. R. Eckler	1873–1933	31	Con. 1918–19	None	Six annual series: railway operating revenues, imports, pig iron production, cotton consumption, coal production (bituminous and anthracite), bank clearings or debits for selected cities.	A Measure of the Severity of Depressions, 1873–1932, *Review of Economic Statistics*, May 1933, p. 79.
D. W. Gilbert	1854–1929	39	Con. 1918–19	Exp. 1866	Ayres' index of business activity, Persons' index of production and trade, Snyder's clearings index of business. Also the following annual records: Thorp's annals, Bullock's analyses of imports and postal revenues, and Cole's index of domestic trade (1843–62).	Business Cycles and Municipal Expenditures, *Review of Economic Statistics*, Aug. 1933, pp. 140-1.
J. B. Hubbard	1873–1912, 1920–33	26	Exp. 1894–95 Con. 1926–27	None	Frickey's clearings index for seven cities outside New York before World War I. Bank debits for 241 cities (excluding Boston, Chicago, Los Angeles, San Francisco, Detroit, Philadelphia and Cleveland, as well as New York) after the war.	Business Declines and Recoveries, *Review of Economic Statistics*, Feb. 1936.
L. P. Ayres	1854–1938	42	Con. 1926–27	None	Ayres' index of business activity	*Turning Points in Business Cycles* (Macmillan, 1939), pp. 19, 35, 51.

[a] The years cited relate to the corresponding monthly reference peaks or troughs in our chronology.

[b] That is, expansions or contractions of business cycles.

ences aside, it is plain that our chronology and the chronologies of the other investigators are designed to describe the same class of facts. Hence we feel free to speak of all of them as chronologies of business cycles.

It appears that only three cyclical movements in our list lack counterparts in one or more of the other chronologies. In fact, the area of real disagreement is even smaller than Table 27 may suggest.[60] Hubbard treats the period from 1893 to 1897 as a single depression. But he observes that "for certain purposes, it is probably desirable to consider the fluctuations of the period as involving two depressions: that of 1893, followed by substantial and persistent recovery, and then relapse into depression again in 1896".[61] Since that is precisely the course we have chosen, there is no genuine difference between us. The contraction of 1918–19 is not recognized by Eckler or Gilbert, whose chronologies run by years,[62] whereas the other chronologies are monthly.[63] In view of the exceptional brevity and moderate amplitude of this contraction, its failure to register in annual summaries is not surprising. Finally, the period from 1924 to 1929 is treated as a single expansion in three of the chronologies. The reasons for this treatment by Ayres and by Axe and Houghton are obscure.[64] In Hubbard's case, the omission of the 1926–27 contraction merely means that it failed to register in bank debits of selected centers outside New

60 However, the table is silent on one point. Although our list of business cycles agrees with Kitchin's in the period for which he supplies actual dates, he intimates a chronology of 'minor cycles' (Kitchin prefers this term but uses it interchangeably with business cycles) that diverges somewhat from ours prior to 1890. See Ch. 11, Sec. V.

Table 27 is confined to the period covered by our monthly reference dates. In the period covered exclusively by the annual reference dates (1834–54), comparisons can be made between our chronology and those of Ayres and Gilbert. Every cyclical movement we list in this period is recognized also by Ayres and Gilbert, except the contraction of 1845–46. The genuineness of this movement had previously been challenged by C. J. Bullock and H. L. Micoleau in their paper on Foreign Trade and the Business Cycle, *Review of Economic Statistics*, Nov. 1931, pp. 153-4. Arthur H. Cole has commented on the difficulty of distinguishing cyclical movements in the vicinity of the alleged contraction. See W. B. Smith and A. H. Cole, *Fluctuations in American Business, 1790–1860* (Harvard University Press, 1935), p. 136. In an able study by David Schwartz the conclusion is reached that 1846 was a year of "mild depression", but that "of all the recessions" during 1843–59 "that of 1846 is certainly by far the mildest and the one which affected the least number of spheres of activity" (*A Reinterpretation of American Business Cycle History, 1843–1859*. Unpublished master's thesis, University of California, May 1941).

61 *Review of Economic Statistics*, Feb. 1936, p. 20. See also Hubbard's earlier study, *ibid.*, Nov. 1930, pp. 183-4.

62 Eckler derived his chronology from annual data and expressed it by calendar years. With minor exceptions Gilbert made qualitative judgments for full calendar years on the basis of both monthly and annual records; in our judgment these records, on the whole, indicate a contraction in 1918–19.

63 Indeed, Kitchin dated peaks and troughs to the hundredth of a year.

64 Ayres writes: "There was a minor business downturn in 1927 which was not sufficiently important to be considered as marking the end of a cycle" (*Turning Points in Business Cycles*, p. 45). But if we may judge from the index he used as a criterion for marking off business cycles, this decline is a trifle longer and at least as large as the 1887–88 decline, and definitely larger though a little shorter than the 1869–70 decline, both of which Ayres considers as marking the end of a business cycle. A similar remark applies to Axe and Houghton: according to their trend-adjusted index of trade and industrial activity (the series they used to mark off business cycles), the contraction of 1926–27 is a trifle milder but appreciably longer than the contraction of 1887–88.

York City, the indicator on which he relied to identify business cycles.[65]

Table 27 calls attention also to several movements that have no place in our chronology. These 'extra' movements raise the question whether we may not have recognized too few, rather than too many, business cycles. Some uncertainty is bound to surround this matter, at least until precise distinctions are drawn between business cycles and shorter fluctuations. We should, however, be able to determine whether the 'line' separating business cycles from other fluctuations has been drawn with tolerable consistency in our chronology; that is, whether our list of cycles excludes any movements that are clearly 'larger' than certain other movements that are included. The preceding tabulations for the sample of 46 series throw no light on this question; for close conformity to a given reference phase does not rule out the possibility that this phase might be subdivided into segments, each characterized also by good conformity, and yet no shorter in duration or smaller in amplitude than certain phases recognized by our chronology.

We have tested this possibility on sundry occasions in the past, when movements not covered by our reference scale seemed to recur in a variety of activities. The matter has been tested again by examining the specific-cycle turns of the sample of 46 series analyzed in this chapter. These studies have yielded negative or inconclusive results. Although it is impossible to predict what more thorough investigation will disclose, it seems highly improbable that our chronology omits any movements that are clearly 'larger' than any now included. It is unlikely that it even omits any movements that rival closely any now recognized. We have been unable to find strong evidence in favor of the two 'extra' cyclical movements suggested by Persons.[66] The evidence seems a shade more favorable

[65] But as he well observes, "a single measure of business activity cannot tell the whole story" (*Review of Economic Statistics*, Feb. 1936, p. 16). 'Outside debits' in 1927 were not uninfluenced by the intense speculative activity of the time. Cf. Edwin Frickey, Outside Bank Debits Corrected for Seasonal Variation: Monthly and Weekly, 1919–31, *ibid.*, May 1931.

[66] Persons' basic chronology of 'phases of business cycles' is derived from his index of industrial production and trade, which starts in Jan. 1875. In 1875–76 the index consists solely of Frickey's series on bank clearings in seven cities, adjusted for trend (p. 91). Persons characterizes Jan. 1875–Jan. 1876 as 'prosperity' (p. 198), defined as "that interval of supra-normal business . . . ending with the month preceding a persistent recession to sub-normal business" (p. 197). Since 'prosperity' is preceded by 'recovery', it would appear that expansion started months before Jan. 1875 and continued through the year. But in other places Persons suggests that the crisis of 1873 was followed by depression in 1874 (pp. 85-7); describes 1875–78 as characterized by "deflation and the struggle for resumption of specie payments; great increase in agricultural production of the world accompanied by declining commodity prices" (pp. 89, 93); and claims that 'business expansion' occurred in the last quarter of 1875 (p. 93). It is impossible to reconcile these statements with an expansion that is supposed to have started before Jan. 1875. (All page references are to W. M. Persons, *Forecasting Business Cycles*.)

Again, the contraction of 1898 is a dubious entry in Persons' list from the standpoint of his criteria, though not necessarily of ours. Persons dates a trough in Oct. 1896, a recovery from Nov. 1896 to Feb. 1898, a recession from March to June 1898, and a trough from July to Oct. 1898. Since the 'recovery' never developed into 'prosperity' (statistically, the index failed to reach 100 in Feb. 1898), this cycle fails to meet Persons' specifications of a business cycle. Cf. Elmer C. Bratt, *Business Cycles and Forecasting*, p. 401.

to the mild 'extra' movement in Gilbert's chronology. But monthly statistical records for 1865–67 are very scanty, and extensive study of contemporary journals will be necessary before a firm decision can be reached.[67]

Another check on our chronology is made possible by the 'pattern' of fluctuations in American business activity, which Edwin Frickey has derived from thirteen important time series,[68] by quarters, from 1866 to 1914. Three variants of this pattern are exhibited in Chart 10. The curve marked 'supplementary standard pattern' is the most useful for our purposes, since it involves no adjustment for the secular trends in the original data. The 'standard pattern' makes approximate adjustments for trend, and the 'revised standard pattern' makes refined adjustments.[69] Except for the secular drift in the 'supplementary standard pattern', the form of the three curves is much the same. Frickey examined in detail the movements of the thirteen series from which the pattern was derived, and tested the results by analyzing an extensive body of materials that played no part in the original derivation of the pattern. At the end of his labors he felt able to conclude that in "the United States over the half-century from the close of the Civil War to the outbreak of the World War in 1914, there is a clearly-defined pattern of short-run fluctuation which permeates the whole structure of the nation's industrial and commercial life".[70]

In view of the importance of this conclusion and the scientific care of its architect, it is especially desirable to see how well our list of reference cycles agrees with the cyclical waves in Frickey's pattern. On Chart 10 we have drawn vertical lines at the months when business activity reaches peaks and troughs according to our chronology, shaded the reference contractions so that they stand out prominently, and marked by asterisks the 'specific cycles' that we recognize in each of Frickey's curves. The turning points of these specific cycles vary somewhat from curve to curve, and vary still more from our reference dates; such differences are natural in

67 On the basis of preliminary explorations, Isaiah Frank, formerly on our staff, has suggested that the reference peak be dated in Oct. 1864 instead of April 1865, and that a trough be recognized in Aug. 1865. He was uncertain, however, whether Aug. 1865–June 1869 should be treated as a single expansion, or broken into three phases: expansion from Aug. 1865 to some later date, contraction from this date to about Dec. 1867, expansion thereafter to about June 1869.

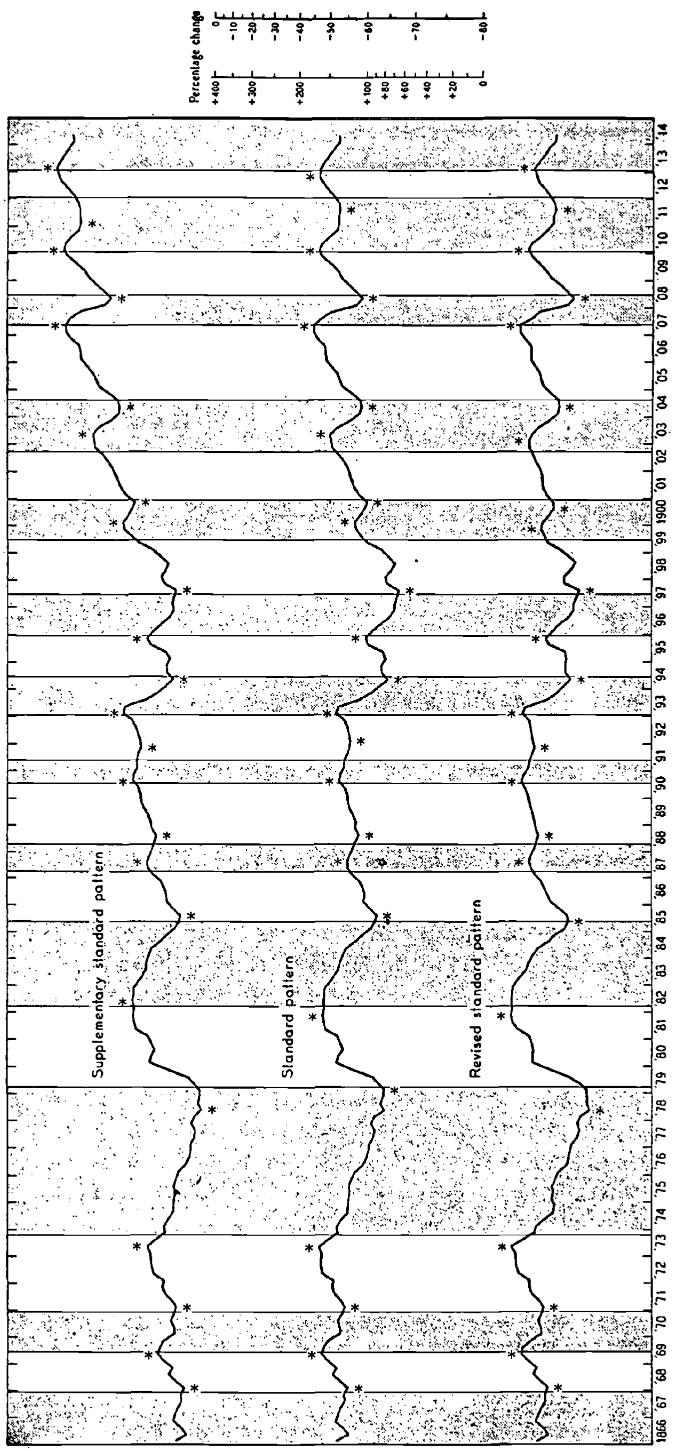
68 Clearings in seven cities outside New York (only Philadelphia before 1875), New York clearings, loans of New York banks, railroad earnings, imports, exports, immigration, sensitive commodity prices, wholesale commodity prices, railroad stock prices, industrial stock prices, bond prices, commercial paper rates.

69 Also, three series (loans of New York banks, exports, and bond prices) were dropped, and the form of two others (outside clearings, railroad earnings) changed. For the derivation of the pattern curves, see E. Frickey, *Economic Fluctuations in the United States,* Ch. III, IV, XII, XIV, and the appendices to these chapters. The figures for the 'revised standard pattern' are taken from p. 328. We are indebted to Frickey for the figures of the 'standard' and 'supplementary standard' patterns, and to the Harvard University Committee for Research in the Social Sciences for permission to reproduce them.

70 *Ibid.,* p. 230.

CHART 10

Frickey's Standard Pattern of Short-term Fluctuations in American Business Activity, 1866-1914



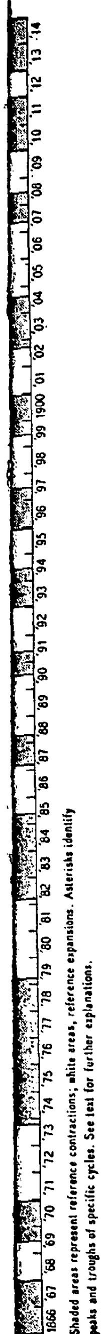
Shaded areas represent reference contractions; white areas, reference expansions. Asterisks identify peaks and troughs of specific cycles. See text for further explanations.

view of
matter o
and the
ence pha
three qu
ment ke
Frickey's
nology;
sponden
three qu
have ma
cyclical
could no
would at
The p
business
for forei
mations
contracti
is dubiou
careful re
marked b
in busine
a differen
made dras
tion conti
enterprise
mental co
freedom i
tion for w
Fragment
started, b
movemen
now in ut
already p
that the f
definition.
rule about

71 The follow
between the s
1870, (c) contr
and third qua
expansion, an
same is true o
'revised stand
expansion (18

view of the differences in the time unit, data, and methods used. The matter of chief interest is the correspondence between our reference cycles and the cycles in Frickey's pattern. Allowing for leads or lags, every reference phase is reflected in a movement of corresponding direction, lasting three quarters or longer in each of Frickey's curves. Again, every movement keeping the same direction for three quarters or longer in any of Frickey's curves is matched by a corresponding movement in our chronology; so that the two sets of cyclical waves are in one-to-one correspondence throughout. Except for the contraction in 1900, which lasted three quarters, the duration of each expansion and contraction that we have marked off in Frickey's curves is at least one year. If any 'extra' cyclical phase were recognized in one or another of Frickey's curves, it could not be of more than two quarters' duration, and its amplitude would at best be marginal.[71]

The preceding analysis is confined to the reference dates of American business cycles. Our experience indicates that the lists of reference cycles for foreign countries are at least tolerable, if not equally good, approximations to the historical course of their business cycles. However, the contraction we list for Germany between August 1903 and February 1905 is dubious. The chronology for France in the 1860's and 1870's requires careful reconsideration. After 1932 the economic situation in France is marked by many confusions and conflicts, which render any description in business-cycle terms uncertain. German developments since 1932 raise a different problem. When the National Socialists came into power, they made drastic changes in economic organization. Production and distribution continued to be carried on mainly by business enterprises, but these enterprises were subjected to increasingly strict and pervasive governmental controls. The Nazi State repudiated the concept of individual freedom in business enterprise as in other matters. Large-scale preparation for war produced a great expansion in employment and output. Fragmentary records indicate that the expansion continued after the war started, but we do not have the data to determine when the peak of this movement was reached. Nor do we know when the German economy, now in utter collapse, will begin to revive. Close to thirteen years have already passed since the cyclical trough we have set in August 1932, so that the full 'cycle' will last longer than the extreme limit set by our definition. But this German episode is not an exception to our working rule about the duration of business cycles where free enterprise prevails.



71 The following 'extra' movements of two quarters' duration can be distinguished: (a) expansion between the second and last quarters in 1866, (b) expansion between the first and third quarters in 1870, (c) contraction between the first and third quarters in 1880, (d) contraction between the first and third quarters in 1898. In the 'standard' pattern, (a) and (b) are of smaller amplitude than any expansion, and (c) and (d) are of smaller amplitude than any contraction, marked on the chart. The same is true of the 'supplementary standard' pattern. The same is true again of (b) and (c) in the 'revised standard' pattern; but in this pattern (a) exceeds slightly the amplitude of one recognized expansion (1891–93), and (d) exceeds slightly the amplitude of one recognized contraction (1887–88).

It is not profitable to push critical evaluation of our reference chronology further at this juncture. A fully dependable list of business cycles in several countries can be attained only as the end product of a thorough study of business cycles. An investigator who strives for such a result must proceed by successive approximations, using what he learns from one approximation to improve the next. For all its faults, we are confident that our present chronology of business cycles in four countries is sufficiently close to the mark to yield trustworthy results of a general character. Future revisions of the reference dates will doubtless change materially the reference-cycle measures for some single cycles or single series, but the broad results portrayed by our average measures of cyclical behavior are practically certain to stand.[72] The force of this observation will be clarified in the course of the tests of our technique carried through in subsequent chapters.

[72] This, on the whole, has been our experience as the reference dates have been revised or brought forward, and the average reference-cycle measures recomputed.

CHAPTER 5

The Basic Measures of Cyclical Behavior

IN THIS chapter we explain in greater detail than in Chapter 2 our methods of measuring specific-cycle and reference-cycle behavior. The calculations are described in full for one series, the monthly records of bituminous coal production in the United States (Table 13). This activity conforms closely to business cycles but exhibits a few irregularities not encountered in the coke illustration given in Chapter 2. We also draw on other series to illustrate different steps in the procedure and the results obtained in practice. The bulk of this chapter is devoted to methods of handling monthly data, but Section XI shows how the methods can be adapted to quarterly and annual records.

I Positive and Inverted Specific Cycles

A large majority of the series we have analyzed show a strong tendency to rise during periods of general business expansion and to fall during periods of general business contraction. But series such as bankruptcies and unemployment have as strong a tendency to move in the opposite fashion. In our jargon the latter series have 'inverted' specific cycles. We can often, but by no means always, anticipate whether the specific cycles of a series will be positive or inverted. For example, the specific cycles of oleomargarine production bear a positive relation to business cycles, while the specific cycles of butter production bear an inverted relation. Stocks of raw materials held by manufacturers tend to be related positively to business cycles, stocks of finished products held by manufacturers tend to be related invertedly, and so on.

In 'positive' series we treat specific cycles as units running from trough to trough; in 'inverted' series we treat them as units from peak to peak. Hence it is necessary to decide whether a series is to be classed as 'positive' or 'inverted' before any measures of specific-cycle behavior are made.

Ordinarily, study of the reference-cycle patterns of a series yields a clear verdict: a series rises during most of the reference expansions and falls during most of the reference contractions, or it falls during most of the former and rises during most of the latter phases. More difficult to classify are series that (1) ordinarily rise during the early stages of reference expansion, fall during the later stages, and turn upward again while general business is still shrinking; (2) ordinarily rise from the later stages of reference expansion to the early stages of reference contraction, and fall during the later stages of reference contraction and the early stages of expansion; (3) show marked independence of business cycles.

As explained in Section X, if a series typically moves in the same direction as general business over more reference-cycle stages than it moves in the opposite direction, we class the series as 'positive' and analyze the specific cycles on a trough-to-trough basis. In the opposite case, we class the series as 'inverted' and analyze the specific cycles on a peak-to-peak basis. If the expansions of a series typically run from the middle of reference expansion to the middle of reference contraction, or from the middle of reference contraction to the middle of reference expansion, we arbitrarily analyze the specific cycles on a trough-to-trough basis. Series that move in virtual independence of business cycles are handled in the same way. Thus 'positive', 'neutral', and 'irregular' series are treated on a *positive plan*—that is, their specific cycles are treated as units running from trough to trough; while 'inverted' series are treated on an *inverted plan,* their specific cycles being marked off by peaks.

These rules are instruments of convenience. As will be explained presently, we at times find it desirable to analyze 'neutral' series on both a positive and inverted basis, and sometimes treat a series as positive when strict application of our rules calls for inverted analysis and vice versa. Positive or inverted analysis is a poor guide to the conformity of a series to business cycles. A series may be related to business cycles in many ways, and several measures are necessary to bring out clearly the actual relations.

II Timing of Specific Cycles

One of the principal aims of this investigation is to measure the time sequence in which various economic activities have followed one another in the revivals and recessions of general business activity. Hence we seek to compare the dates of the peaks and troughs of the specific cycles of each series with the reference dates of business cycles in the country to which the series relates. By noting the number of months that the turns of the specific cycles of a series lead or lag behind the reference turns, striking averages of the leads or lags for peaks and also for troughs, and supplementing the averages by average deviations, we can tell what timing has been characteristic of the series in regard to business cycles. By observing the leads or lags of numerous series at successive reference dates, we can determine also in what respects the sequence of upturns has been similar or different in past business-cycle revivals, and in what respects the sequence of downturns has been similar or different in past recessions.

In some series, as in the production of coke or bituminous coal in the United States, the specific cycles correspond closely in time to business cycles. It is fairly easy to fit the turns of the specific cycles of such series into the chronology of business cycles. More often the correspondence between the two sets of cycles is imperfect; sometimes it is negligible. Thus if two specific cycles occupy approximately the same period as a business cycle, the trough between the two specific cycles and at least one peak will not correspond to any reference turn. Again, if one specific cycle spans two business cycles, its initial and terminal troughs will match reference turns and the peak is likely to do so, but at least two reference turns will not match any specific-cycle turn. In series where the relationship between the turning dates of specific and business cycles is still looser, matching of the two sets of dates becomes very hazardous unless the entire operation can be reduced to a tolerably objective basis.

The first step in the measurement of the cyclical timing of a series is to decide whether to match the specific-cycle turns with like or opposite reference turns. The rules recited in the preceding section [1] must be used discriminatingly. For example, series on new orders usually lead similar turns in general business; most of them are clearly 'positive' according to our criterion, but a few with longer leads are on the margin, while one or two with still longer leads are 'inverted'. To avoid confusion we analyze all series in this group on a positive basis. This seems the more logical treatment, although it may prove desirable later to add inverted measures for the entire group. In analyzing activities such as bond yields or bond sales, which usually have cyclical turns near the middle of reference expansions and contractions, we may make a double analysis from the start. From certain points of view it is desirable to think of upturns and downturns in long-term interest rates as lagging behind the revivals and recessions of general business; from other points of view it is important to emphasize the tendency of upturns in long-term interest rates to lead general recessions and of downturns to lead general revivals. A similar problem arises in handling short-term interest rates in the principal money markets, despite the fact that series of this type are clearly 'positive' according to our rules and have been uniformly handled on this basis. In tracing the developments that generate revivals and recessions in business activity, we are sure to add inverted measures; for a decline in interest rates is a favorable development and a rise an unfavorable development

1 When the relationship in time between specific and business cycles is rather loose, it may be difficult to decide whether the relation is positive, inverted, neutral, or irregular. However, errors in classifying such series cannot be serious: irregular timing is their most prominent feature and that will be demonstrated whether the analysis is carried through on a positive or on an inverted basis.

with respect to the business situation at large. The subtle and shifting demands of analytic work can be anticipated only in part at the stage where series are taken up for examination, one by one.

Once a decision has been reached on the issue of positive versus inverted treatment, the next step is to match individual turns of the specific cycles of a series with individual turns of business cycles. This operation calls for considerable self-restraint, lest the data be forced to suit preconceived notions. We follow fairly rigid rules in making timing comparisons. The method is not 'purely objective', for we have found no mechanical device that insures sensible results; but it is sufficiently objective to minimize if not eliminate bias.

Our basic rule is that a specific-cycle turn (S) must meet two criteria in order to be considered as corresponding to a *like* reference turn (R):

(1) there is no other reference turn in the interval between S and R (including the month of S)
(2) there is no other specific-cycle turn in the interval between S and R (including the month of R).

These criteria help to weed out 'extra' specific cycles, that is, specific-cycle movements unrelated to business cycles. It may happen, however, that two specific-cycle peaks, coming on opposite sides of a given reference peak, deviate by approximately the same number of months from the reference peak, but that the earlier specific-cycle peak meets both criteria while the later peak meets only the first criterion (since the intervening specific-cycle trough happens to follow the reference peak). To say in that event that the earlier peak 'corresponds' to a reference turn and that the later one does not is to settle a problem of causation by mechanical rule. Difficulties of this type are reduced by our third criterion: if S meets criteria (1) and (2) and another *like* specific-cycle turn on the opposite side of R meets just criterion (1), S will be treated as a corresponding turn only in the event that it deviates no more than three months from R. In applying criterion (3) we avoid guessing about the behavior of the series prior to the first or after the last specific-cycle turn; that is, a turn is treated as noncorresponding so long as it might fail to meet this criterion if the series were extended.[2]

If all three criteria are met, S is treated as corresponding to R. If not, S is treated as noncorresponding; that is, no attempt is made to compare its timing with that of business cycles. The three criteria have been phrased for series that are treated on a positive plan. To apply the criteria

[2] Criteria (1) and (2) would yield identical comparisons if the turning dates of the series were the standard against which the reference dates were compared—instead of the other way around, which of course is our practice. Criterion (3) does not satisfy this principle of reversibility. If that criterion were dropped we might gain a formal advantage, but at the cost of a less stringent weeding of 'extra' specific cycles in relation to business cycles.

Criteria (1) and (2) may be combined in the following rule: a specific-cycle turn (S) corresponds to a like reference turn (R) if no other specific or reference turn falls in the interval between S and R. Criterion (3) places a restriction on this rule.

to series treated on an inverted plan, it is necessary merely to substitute *opposite* for *like* in the wording of the rules. A simpler expedient is to think of inverted relations as being transformed into positive ones by plotting figures invertedly, and we therefore continue the exposition as if all series were treated on a positive basis.

Chart 11 illustrates the application of the timing rules. The angular line at the top identifies the American reference cycles from 1905 to 1933 and serves no other purpose.[3] All series are seasonally adjusted. The turns of the specific cycles are marked on the chart, and the dates are listed in Table 28. To facilitate comparisons between the turning dates of the

TABLE 28

Turning Dates of Specific Cycles in Six American Series

COKE PRODUCTION		BITUMINOUS COAL PRODUCTION		CALVES SLAUGHTERED UNDER FEDERAL INSPECTION	
Peak	Trough	Peak	Trough	Peak	Trough
Jan. 1913	Nov. 1914	May 1907	Jan. 1908		Feb. 1908
July 1918	May 1919	June 1910	Feb. 1911	Dec. 1909	Dec. 1911
Aug. 1920	July 1921	Oct. 1913	Nov. 1914	Aug. 1912	Nov. 1914
May 1923	July 1924	July 1918	Mar. 1919	June 1920	Dec. 1920
Feb. 1926	Nov. 1927	Dec. 1920	July 1922	July 1925	June 1929
July 1929	Aug. 1932	May 1923	June 1924	June 1931	Dec. 1932
		Mar. 1927	Dec. 1927		
		May 1929	July 1932		

PETROLEUM OUTPUT APPALACHIAN FIELD		COTTON STOCKS AT MILLS		STRUCTURAL STEEL ORDERS	
Peak	Trough	Peak	Trough	Peak	Trough
	Aug. 1908		Oct. 1914	May 1909	Nov. 1910
Nov. 1909	Jan. 1912	Nov. 1916	Oct. 1917	Oct. 1912	Nov. 1914
May 1912	Oct. 1915	Oct. 1918	Apr. 1919	Dec. 1915	Sep. 1917
July 1919	Feb. 1920	Jan. 1920	Nov. 1920	July 1918	Jan. 1919
Feb. 1921	Nov. 1924	Nov. 1922	Oct. 1924	Feb. 1920	Feb. 1921
Feb. 1930	Aug. 1931	June 1927	Sep. 1928	Mar. 1923	June 1923
Oct. 1931	May 1933	Mar. 1930	Sep. 1931	Oct. 1925	Sep. 1926
		May 1932	Mar. 1933	Mar. 1929	Jan. 1932

See Chart 11.

specific and business cycles, vertical lines are erected at the reference turns and the reference contractions are shaded. In each case where a specific-cycle turn satisfies our three criteria, an arrow is drawn from it to the reference turn with which it is compared. The absence of an arrow at a specific-cycle turn means that one or more of the criteria are not met, and accordingly that no timing comparison is made.

In the series on calves slaughtered, the specific-cycle trough of June 1929 fails to satisfy criterion (1); hence no arrow appears at this point. There is also no arrow at June 1931, for this peak fails to meet criterion (2). In cotton stocks held by mills, every specific-cycle turn can be matched with a reference turn so as to satisfy criterion (1), but four turns

[3] The straight lines joining peaks and troughs are arbitrary; so also are the uniform amplitudes.

(November 1916, October 1917, September 1931 and May 1932) fail to satisfy criterion (2). A failure to meet criterion (3) is illustrated by the peak of July 1919 in Appalachian petroleum output. Both this peak and that of February 1921 are linked by criterion (1) to the reference peak of January 1920; the first specific-cycle peak meets criterion (2) while the second specific-cycle peak does not; but the first specific-cycle peak succumbs to criterion (3) since it deviates more than three months from the reference peak in the face of a competing specific-cycle peak on the opposite side of the same reference peak. There is no arrow at the last turning points of coke production, bituminous coal production, and structural steel orders. These turns satisfy criteria (1) and (2); but since there is no assurance that they would satisfy criterion (3) if the data were carried forward, they are not classed as corresponding turns. These cases illustrate how we handle series when the records cannot be carried forward or backward. Of course, it is now possible to carry all series sufficiently beyond 1933 to decide whether or not criterion (3) is actually met.

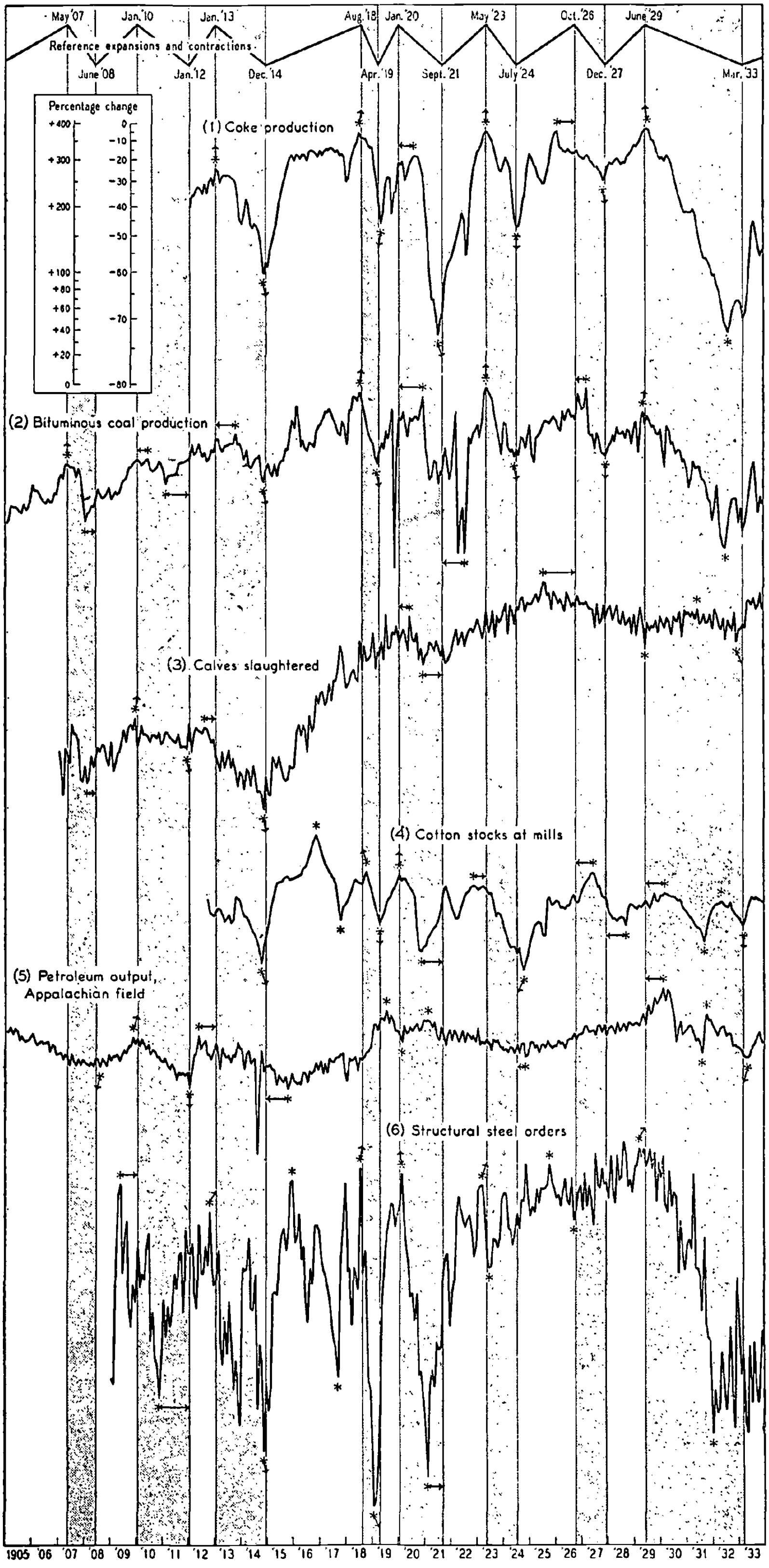
The mechanical rules for timing comparisons separate out the cyclical movements in a series that are closely related to business cycles from others dominated by factors peculiar to the series. They thus serve to restrict our averages to cyclical turns that have some presumptive claim to being connected with the revivals and recessions of general business. But the separation is very rough and we do not consider it final. Obviously, the rules treat with excessive liberality series that move in virtual independence of business cycles. That is not especially disturbing, since timing averages are in any event of slight value in series that conform badly to business cycles. On the other hand, the rules may reduce unduly the number of timing comparisons in series that conform well to business cycles. This difficulty is serious: first, because even our longest series cover relatively few cycles, second, because the rules may weed out turns in a way that biases the averages. Thus criterion (1) weeds out leads or lags equal to, or longer than, a full reference phase. Leads or lags of this length may be of slight value in describing the sequence of cyclical turns in a particular revival or recession of general business, but they should not therefore be ignored in estimating the typical timing of an activity at business-cycle revivals or recessions. When the turns of the specific cycles of a series usually precede reference turns by half a phase, random factors are as likely to produce a lead that is slightly longer than a full reference phase as they are to produce a short lag. To give random movements an opportunity to cancel out in the averages, it seems no less necessary to include the one than to include the other.[4]

We therefore relax the mechanical rules in series that *conform well* to business cycles. If each of two specific-cycle turns deviates less than a full

[4] See the analysis in Ch. 8, Sec. III of the influence of erratic movements on our timing measures.

CHART 11

Illustrations of Mechanical Rules for Comparing the Timing of Specific Cycles with the Reference Dates
Six American Series



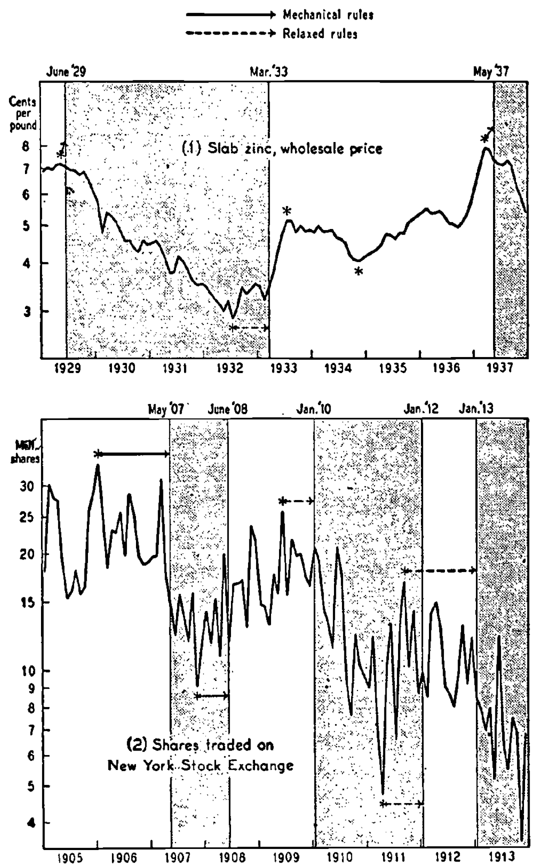
Shaded areas represent reference contractions; white areas, reference expansions. Asterisks identify peaks and troughs of specific cycles. See text for explanation of arrows and other features. Sources are listed in Appendix C.

phase from the same reference turn, we may treat one of them as corresponding regardless of the mechanical rules. When cases of this sort are encountered, we scrutinize the amplitudes and patterns of the competing movements, and draw on whatever knowledge we may have concerning the particular fluctuations. In the absence of special knowledge, if any cyclical movement in the neighborhood of the reference turn is so mild that it might have been disregarded in the first place, we ignore it in making timing comparisons; while if none of the cyclical movements is dubious, we give preference to the turn that represents the culmination of a larger specific-cycle movement. Chart 12 gives an illustration of the procedure. If we followed the mechanical rules, we would disregard the troughs in zinc prices in 1932 and 1934. But no one could reasonably

CHART 12

Illustrations of Relaxed Rules for Comparing the Timing of Specific Cycles with the Reference Dates

Two American Series



See note to Chart 11.

Logarithmic vertical scales

doubt that zinc prices were at a trough in 1932, while the contraction from 1933 to 1934 might be considered a pause in the sharp recovery that started in 1932 and culminated in 1937. We therefore disregard the trough in 1934 and match the trough in 1932 with the reference trough.

We relax the mechanical rules in two additional respects when handling well conforming series. First, in applying criterion (3) we proceed on the assumption that there is no competing turn just before the first or just after the last turn of the specific cycles that can be dated from existing records.[5] Second, we sometimes admit timing measures as long as a full reference phase or even longer. For example, the trough of June 1923, peak of October 1925, and trough of September 1926 in structural steel orders are noncorresponding according to the mechanical rules (Chart 11). But since the series on the whole conforms well to business cycles, we might compare the trough of September 1926 with the reference trough in December 1927, in spite of the fact that this comparison yields a lead longer than a full phase. Once that decision is made, the trough in June 1923 is readily matched with the reference trough in July 1924 and the peak in October 1925 with the reference peak in October 1926. Chart 12 illustrates a similar situation in share trading on the New York Stock Exchange. Here we recognize a lead longer than a phase at the reference peak of January 1913; two additional timing comparisons naturally follow from this decision.

These relaxed procedures make possible a fuller use of the statistical information than could be attained by relying exclusively on the mechanical rules. In practice we begin the analysis of every series by applying the mechanical rules. If a series does not conform well to business cycles, nothing further is done regardless of the number of specific-cycle turns that are classed as noncorresponding. But if a series conforms closely to business cycles, we try to reduce the number of noncorresponding turns by relaxing the mechanical rules. The main difficulty is to draw a reasonable dividing line between series that conform 'closely' to business cycles and those that do not. In general, we accept an index of full-cycle conformity of 50 or higher as indicative of close conformity. But this index is not an infallible indicator; and we therefore take account as well as may be of other evidence before making a decision, such as the patterns of successive reference cycles, the number of reference cycles covered, the number of specific cycles relatively to the number of reference cycles, and the behavior of intimately related series.[6]

Once decisions have been reached, by studying charts like those produced here, concerning what turns of the specific cycles of a series correspond to what reference turns, the timing measures are entered in our

[5] However, to avoid absurdities, no timing comparison longer than 12 months is made unless such timing is typical of the series.

[6] See below, Sec. X.

standard Table S1. Table 29 presents a few samples of the first five columns in Table S1, the part devoted to timing measures. Column (1) shows the turning points of each full specific cycle: the month of the initial trough, of the cyclical peak, and of the low point that marks the end of the con-

TABLE 29
Samples of a Section of Table S1: Timing of Specific Cycles
Three American Series

Dates of specific cycles Trough–Peak–Trough (1)	Timing at reference peak: Number of months lead (−) or lag (+) (2)	Timing at reference peak: Date of reference peak (3)	Timing at reference trough: Number of months lead (−) or lag (+) (4)	Timing at reference trough: Date of reference trough (5)
Bituminous coal production				
May 07–Jan. 08	0	5/07	−5	6/08
Jan. 08–June 10–Feb. 11	+5	1/10	−11	1/12
Feb. 11–Oct. 13–Nov. 14	+9	1/13	−1	12/14
Nov. 14–July 18–Mar. 19	−1	8/18	−1	4/19
Mar. 19–Dec. 20–July 22	+11*	1/20	+10	9/21
July 22–May 23–June 24	0	5/23	−1	7/24
June 24–Mar. 27–Dec. 27	+5	10/26	0	12/27
Dec. 27–May 29–July 32	−1	6/29	(−8)	3/33
Average..............	+3.5		−2.1	
Average deviation......	4.0		4.4	
Structural steel orders				
May 09–Nov. 10	−8	1/10	−14*	1/12
Nov. 10–Oct. 12–Nov. 14	−3	1/13	−1	12/14
Nov. 14–Dec. 15–Sep. 17	...	...	...	...
Sep. 17–July 18–Jan. 19	−1	8/18	−3	4/19
Jan. 19–Feb. 20–Feb. 21	+1	1/20	−7	9/21
Feb. 21–Mar. 23–June 23	−2	5/23	(−13)*	7/24
June 23–Oct. 25–Sep. 26	(−12)	10/26	(−15)**	12/27
Sep. 26–Mar. 29–Jan. 32	−3	6/29	(−14)	3/33
Average..............	−4.0		−9.6	
Average deviation......	3.4		5.1	
Petroleum output, Appalachian field				
Aug. 08			+2	6/08
Aug. 08–Nov. 09–Jan. 12	−2	1/10	0	1/12
Jan. 12–May 12–Oct. 15	−8*	1/13	+10	12/14
Oct. 15–July 19–Feb. 20	...	...	...	...
Feb. 20–Feb. 21–Nov. 24	...	...	+4	7/24
Nov. 24–Feb. 30–Aug. 31	+8	6/29	...	...
Aug. 31–Oct. 31–May 33	...	...	+2	3/33
Average..............	−0.7		+3.6	
Average deviation......	5.8		2.7	

Average is the arithmetic mean; the average deviation is measured from the mean. The entries in parentheses identify the timing measures made under the 'relaxed rules'.

*Indicates that the specific-cycle turn deviates more than a half but less than a full reference phase from the corresponding reference turn.

**Indicates that the soecific-cvcle turn deviates a full reference phase or longer from the corresponding reference turn.

traction. If a specific-cycle peak comes before the first or after the last complete specific cycle, its date is also entered since we wish to utilize fully what information we have on timing. Columns (3) and (5) indicate the reference dates with which the specific-cycle turns are compared. Column (2) states the number of months by which the peak of a specific cycle precedes or follows the corresponding reference peak. Similarly, column (4) states the number of months by which the trough of a specific cycle, as shown by the last set of entries in column (1), precedes or follows the corresponding reference trough. A minus sign indicates a lead at a reference turn, a plus a lag, zero a coincidence; a blank indicates noncorrespondence. Thus the entries for coal production of '—5' in column (4) and '6/08' in column (5) mean that the specific-cycle trough (January 1908) precedes the reference trough (June 1908) by 5 months. To facilitate use of the timing measures, several symbols are used. Parenthetic entries represent timing comparisons in addition to those permitted by the mechanical rules. Thus Appalachian petroleum output is treated throughout according to the mechanical rules; but these rules are relaxed in bituminous coal production and structural steel orders, which conform well to business cycles.[7]

When specific cycles are treated on an inverted plan, the general procedure is the same. But now we match the specific-cycle troughs with the reference peaks and the specific-cycle peaks with the reference troughs. Column (1) gives the month of the initial specific-cycle peak, of the trough, and of the terminal peak. Column (2) indicates the number of months by which the cyclical troughs in the series precede or follow the reference peaks. Similarly, column (4) indicates the number of months by which its cyclical peaks precede or follow the reference troughs.

As explained in Chapter 4, some business-cycle turns are more difficult to date than others. Hence we do not have implicit confidence in the accuracy of the leads and lags, even when the turning points of specific cycles are perfectly clear. Moreover, our practice of setting reference dates toward the close of transition periods whenever cyclical waves seemed to culminate in a flat or double peak or trough produces a bias—leads are more numerous than lags.[8] But since the leads and lags of all series for a given country are measured from a standard set of benchmarks, they are dependable with respect to one another, within the variable margins of

[7] There is no timing entry in Chart 11 for the specific-cycle trough of July 1932 in bituminous coal production. There is an entry in Tables 29 and 30, parenthetic in the former table but not in the latter. The reasons for these differences are as follows. The chart shows what timing comparisons would be made if bituminous coal output were treated according to the mechanical rules; the tables show the comparisons we actually make in this well conforming series. But whereas Table 29 is made on the assumption that the available data stop in 1933 (the period covered by the chart), Table 30 draws upon later data.

[8] Dating specific cycles in a similar manner does not prevent a bias. For example, an expansion in general business might culminate in a plateau maintained for several months, while the peaks in the specific cycles of most individual activities are sharply angular.

error involved in dating the turning points of specific cycles. Any error we may make in fixing the reference dates affects the absolute timing measures, but does not misrepresent the sequence of the cyclical turns in individual series during a business-cycle revival or recession. Nor do errors in reference dates affect the average sequence of individual series so long as they are compared for identical periods; for the average error of the timing measures, if any, is then necessarily the same for all series. If the periods are not identical, the average timing measures may contain an error of varying size, since the average error of one set of reference dates may differ from the average error of another set of reference dates.

Further difficulties arise if a series shows 'extra' cycles or if its timing differs widely in other respects from that of business cycles—complications which we ignore in the preceding paragraph. In such instances slight shifts in reference dates may alter decisions as to what turns of the specific cycles are corresponding, and may even result in a different matching of specific and business cycles. These difficulties are inherent in any set of rules involving rigid boundary lines. There are always some cases near the margin, so that if a reference date is changed, however slightly, a turn that is barely corresponding according to the rules may become noncorresponding or vice versa. Another aspect of the same difficulty appears when the cyclical turns of a group of closely related series are nearly coincident, yet differ materially from the reference dates in some portion of the period covered. In a case of this sort a difference of one or two months in the turning points of individual series may result in a different matching of their cyclical turns with the reference dates. Moreover, the timing of 'extra' cycles will go unrecorded; in other words, part of the information concerning the timing of the specific cycles common to the group will not be used.[9] These difficulties flow from the fact that the measures in Table S1 are designed chiefly to determine what timing is characteristic of a series in regard to general business activity. If another problem is faced, such as comparing the turns of a closely related group of series, our standard measures cannot be trusted implicitly. The safest method is to check timing measures made on the standard plan by direct comparisons among the series.[10]

[9] It is proper to ignore this information if the problem is simply to determine the timing relations among the series in the vicinity of business-cycle revivals and recessions. The text refers to an effort to determine the timing relations among the specific cycles of a closely related group of series, regardless of their conformity to business cycles.

[10] It is always desirable to check timing comparisons made from the dates assigned to the turning points of specific cycles by going back to the data charts. If all series were sine curves of equal period and amplitude, the sequence of their cyclical movements could be determined completely by dating peaks and troughs alone; indeed, the dates of a single cluster of turns including all series would suffice. But in actual life the shapes of cyclical movements are so varied that a mere record of the months in which different series turned up or down may be misleading. Data charts must be studied closely before drawing conclusions about sequences even of series with clearly defined cyclical movements. The charts should show the original as well as seasonally adjusted figures so that faulty seasonal adjustments may be detected. See also below, Sec. VII.

At a later stage of the investigation we shall make tables showing the order in which different series turned up at successive reference troughs and turned down at successive reference peaks, thus establishing a record of the sequence in which various economic activities have followed one another during cyclical reversals of general business activity. In drawing up these tables we shall need to supplement the measures in Table S1. The distinction between 'corresponding' and 'noncorresponding' turns, although vital for the purpose of determining what timing is characteristic of a given activity in regard to business cycles, is of little use when interest shifts to particular business-cycle revivals or recessions. Specific-cycle movements now classed as 'noncorresponding' may be no less important for an understanding of individual business cycles than those classed as 'corresponding'. On the other hand, 'leads' or 'lags' lasting longer than a reference phase may be misleading for this purpose.

As these remarks indicate, the method of measuring sequences must be adapted to the aims pursued in different parts of this investigation. And in this connection, it is well to note briefly our reasons for choosing the particular method that we use to determine what cyclical timing is characteristic of a series instead of the traditional method, which takes as a measure of the typical lead or lag the pairing of months that yields the highest coefficient of correlation between the series and some index of business activity. Our method involves judgment at every step of the calculation: in dating business cycles, dating specific cycles, deciding whether to analyze a series on a positive or inverted basis, and deciding how to match the specific and business cycles. The traditional method, while by no means free from personal factors, is more objective and more elegant. But it has three grave shortcomings: (1) it fails to distinguish between the months of cyclical turn and other months and therefore also between troughs and peaks, (2) it tells nothing about the variation in timing from cycle to cycle, and (3) it reports at best the sequence of trend-adjusted data. For our purposes, it is highly important to distinguish between the timing at upturns and downturns of business cycles, to note the variations in timing from one business cycle to the next, and to follow the sequence of cyclical movements as they occur rather than as they appear after trends are removed. Whatever its defects, our method at least gets directly at matters that, from our viewpoint, are significant for the understanding of business cycles.

Moreover, in trying to determine the timing of cyclical movements characterizing different economic processes, we do not rely exclusively upon the leads and lags of Table S1. The reference-cycle patterns, described in Section VII, afford a check on these measures. Although the check is insensitive to minor variations in timing, it has several valuable features. First, the reference-cycle patterns indicate roughly the timing of the rate of change of a series during the expansions and contractions of business cycles, as well as the timing of its peaks and troughs. Second, though these measures have uncertainties of their own, they are free from practically every uncertainty that surrounds the calculation of leads and lags in Table S1, except the dating of business cycles. Third, the reference-cycle patterns automatically show timing in fractions of business-cycle expansions and contractions.

The last feature raises an important question about the timing measures in Table S1, which are expressed in months. Since a business cycle in a given country is the fundamental unit in our scheme of analysis, timing measures expressed as fractions of the duration of a business cycle or one of its phases may seem more defensible logically than timing measures expressed in months. Yet the issue is full of complexities. For some practical purposes the simple measures in months are best: for example, when one is trying to anticipate business-cycle recessions or revivals from the average leads of related series in the past and their current movements.[11] The critical question from an analytic viewpoint is whether timing measures are more stable in months or in fractions of cyclical units, but that question we are as yet in no position to answer. And if the answer should turn out to be that most economic groups show greater stability on one plan but that a few show greater stability on the other, the question will remain whether the method that is best for most series is best for all.

At a later stage we may want to convert the leads and lags given by months in Table S1 to other forms, but it should be noted that the conversion into cyclical units is not a simple matter. Perhaps the best plan is that implicit in our reference-cycle measures of timing, where the lead or lag of a series at a reference turn takes the form of a fraction of the reference phase within which the turn of the series falls.[12] But there are other methods that cannot be dismissed lightly. To mention just one, the lead or lag at a reference peak might be expressed as a percentage of the cycle running from the preceding to the following reference trough, and the lead or lag at a reference trough as a percentage of the cycle running from peak to peak. Preference for one base or another may shift with the purpose the investigator has at the moment. In any case the measures in months in Table S1 are of direct value as they stand, and they may be converted into other forms more readily than highly fabricated measures.

III Duration of Specific Cycles

Table 30 presents a sample of Table S1 in full. The measures of cyclical duration in columns (6) to (8) are obvious and raise no difficulties. The

11 For a preliminary study of this problem, see our paper, Statistical Indicators of Cyclical Revivals (*Bulletin 69*).

12 See below, note 70.

expansion is the interval from the midpoint of the date of the initial trough to the midpoint of the date of the peak; the contraction is the interval from the midpoint of the peak to the midpoint of the terminal trough. The duration of a full specific cycle is obtained by summing the lengths of the expansion and contraction. If the duration of a contraction or expansion of an incomplete specific cycle at the beginning or end of a series can be ascertained, it is recorded but excluded from the averages; e.g., the contraction from May 1907 to January 1908 in coal production.

TABLE 30

Sample of Table S1: Timing and Duration of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Timing at reference peak		Timing at reference trough		Duration of cyclical movements (mos.)						Per cent of duration of specific cycles	
					Specific cycles			Excess over reference cycles				
Trough - Peak - Trough (1)	No. of mos. lead (−) or lag (+) (2)	Date of reference peak (3)	No. of mos. lead (−) or lag (+) (4)	Date of reference trough (5)	Expansion (6)	Contraction (7)	Full cycle (8)	Expansion (9)	Contraction (10)	Full cycle (11)	Expansion (12)	Contraction (13)
May 07–Jan. 08	0	5/07	−5	6/08	..	8[a]	..	...	−5[a]	...	..	..
Jan. 08–June 10–Feb. 11	+5	1/10	−11	1/12	29	8	37	+10	−16	−6	78	22
Feb. 11–Oct. 13–Nov. 14	+9	1/13	−1	12/14	32	13	45	+20	−10	+10	71	29
Nov. 14–July 18–Mar. 19	−1	8/18	−1	4/19	44	8	52	0	0	0	85	15
Mar. 19–Dec. 20–July 22	+11*	1/20	+10	9/21	21	19	40	+12	−1	+11	52	48
July 22–May 23–June 24	0	5/23	−1	7/24	10	13	23	−10	−1	−11	43	57
June 24–Mar. 27–Dec. 27	+5	10/26	0	12/27	33	9	42	+6	−5	+1	79	21
Dec. 27–May 29–July 32	−1	6/29	−8	3/33	17	38	55	−1	−7	−8	31	69
July 32–Mar. 37–Mar. 38	−2	5/37	−2	5/38	56	12	68	+6	0	+6	82	18
Average[b]	+2.9		−2.1		30.2	15.0	45.2	+5.4	−5.0	+0.4	65	35
Average deviation[c]	4.1		3.9		11.0	6.8	9.8	6.8	4.5	6.6	17	17

[a] Excluded from the average and the average deviation.

[b] Arithmetic mean determined separately for each column. Hence (6)+(7) *may* differ from (8) in the last place; likewise (9)+(10) from (11).

[c] Measured from the mean.

* Indicates that the specific-cycle turn deviates more than a half but less than a full reference phase from the corresponding reference turn.

In columns (9) to (11) the durations of specific cycles are compared with the durations of corresponding business cycles derived from the monthly reference dates. A plus sign in column (9) means that the expansion of the specific cycles is longer than the corresponding reference expansion; a minus sign means that it is shorter. Similar rules govern columns (10) and (11). Columns (9) to (11) are restricted as a rule to well conforming series, in the sense defined in the preceding section on timing measures. In series of this type we compare the durations of all corresponding phases, but base the averages on specific cycles that correspond in full to business cycles. In other words, if an expansion in the series corresponds to a reference phase while the succeeding contraction does not, the former is compared with the corresponding reference expansion but is excluded from the average at the bottom of the table. Similarly, if there is a full phase of an incomplete specific cycle at the beginning or end of a series, its duration is compared with the corresponding duration of business cycles but is excluded from the averages; the first entry in column (10) is a case in point.[13]

Finally, in columns (12) and (13) we transform the entries in columns (6) and (7) into relatives of the entries in column (8). This step facilitates comparisons of the expansion and contraction phases of specific cycles of different durations. In the averages of columns (12) and (13), each cycle receives the same weight regardless of its duration. Weighted averages, if desired, may be obtained by expressing the averages in columns (6) and (7) as percentages of the average in column (8); these percentages would show the proportion of the period covered by the full cycles that consisted of expansion and the proportion that consisted of contraction. In coal production the unweighted average for expansions is 65 per cent and the weighted average is 67 per cent; the difference is negligible because the correlation between the absolute duration of specific cycles and the percentage consisting of expansions is slight.

If a series is analyzed on an inverted plan the procedure is essentially the same. But now the contraction is entered in columns (6) and (12), the expansion is entered in columns (7) and (13), the entries in column (9) compare the contraction of specific cycles with the corresponding reference expansion, and the entries in column (10) compare the expansion of specific cycles with the corresponding reference contraction.

The measures of cyclical duration are peculiarly sensitive to uncertainties in identifying specific cycles and must be used with discrimination. If a contraction or expansion is just on the margin of being counted as a specific-cycle movement, its inclusion or omission may have considerable effect on the average duration of specific cycles and their phases, especially if the series is short. The average durations must be interpreted with care even when the specific cycles are perfectly clear. For example, the specific cycles in mill consumption of cotton in the United States averaged 36 months during 1914–32, or 8 months less than business cycles during a comparable period. Cattle slaughter, on the other hand, shows specific cycles averaging 100 months from 1908 to 1932, in contrast to an average of 42 months for business cycles. The first difference arises entirely from an 'extra' cycle during the great contraction of 1929–33, while the second arises from individual observations that are characteristically longer than those for business cycles.

[13] The entries in col. (2) to (11) check one another. The figure in col. (4) minus that in col. (2) should equal the figure on the same line in col. (10). The figure in col. (2) on a given line minus the figure in col. (4) on the preceding line should equal the figure in col. (9) on the given line, while the figure in col. (4) on a given line minus the figure in the same column on the preceding line should equal the figure in col. (11) on the given line. If these columns check, the entries in col. (6) to (8) are in all likelihood also correct.

IV Amplitude of Specific Cycles

The original data of economic time series are expressed in diverse units: tons, dollars, persons, bushels, miles, square feet, percentages, and so on. Variations of the unit do not affect the comparability of duration measures, but bar direct comparisons of the amplitude of cyclical swings. Our solution of this difficulty is to express the rise from the trough to the peak and the fall from the peak to the trough of each specific cycle as a percentage of the average monthly value of the series during the cycle. This method puts amplitudes of different types of series in a similar unit and thereby facilitates comparisons among them. Also, it eliminates the secular trend in step-wise fashion and thus facilitates comparisons of the cyclical amplitudes at different stages of the development of a given activity.

The first step in measuring the amplitude of a specific cycle is, therefore, to compute the 'cycle base', that is, the average value of the series during the cycle. Since specific cycles run into one another, a trough is as much a part of a given cycle as of the one adjacent to it. Hence we include both the initial and terminal troughs in computing the average value during a specific cycle; but to avoid a downward bias the trough values receive a weight of one-half each.[14] For example, the initial trough of the first complete specific cycle in bituminous coal production comes in January 1908, the terminal trough in February 1911, and the duration of the full cycle is 37 months. To obtain the average monthly standing during the cycle, we sum the values of the seasonally adjusted figures from February 1908 to January 1911 inclusive, add to this sum one-half the value in January 1908 and one-half the value in February 1911, and divide the grand total by 37, the number of months in the cycle. When a series is analyzed on an inverted instead of a positive basis, the procedure is the same except that the peak values are weighted one-half each. The 'cycle bases' yielded by these calculations are recorded in column (4) of Table S3, of which Table 33 gives a sample.

The second step is to compute specific-cycle relatives, which involves expressing the seasonally adjusted figure for each month during a specific cycle as a percentage of the average monthly value for the cycle. In practice it is not necessary to compute specific-cycle relatives for every month. As explained below, nearly all measures based upon these relatives cover two or more months. It is quicker to average the seasonally adjusted data for the months covered by a measure and to reduce this average to a relative than to compute relatives for the individual months and average them.[15] But our explanation can be kept simpler by assuming that the roundabout calculation is made; that is to say, that the monthly values

[14] Here we follow a suggestion by Milton Friedman.

[15] Also, errors from rounding numbers are reduced.

in each specific cycle are converted to specific-cycle relatives, and that subsequent computations are made from these relatives.

The next step is to determine the standing of the series at the successive troughs and peaks of the specific cycles. Since the cycle relatives at the troughs are likely to be lower and at the peaks higher than they would be if the data were adjusted for erratic movements, we represent the limits of the cyclical swings by three-month averages centered on the peaks and troughs.[16] This is the general rule, but now and then exceptions are necessary. If there is an extremely low value in the month following or preceding a cyclical peak, or an extremely high value in the month following or preceding a cyclical trough, we omit it in computing the peak or trough standing. In these instances a two-month average represents better the limit of a cyclical swing than does a three-month average. On the other hand, some series are free from erratic movements; for example, the specific cycles in series on wholesale prices and bank discount rates at times have horizontal tops and bottoms. In such cases the cyclical peaks and troughs are best represented by the highest and lowest values.

Short phases also require special treatment. When the phase of a specific cycle is extremely short, three-month averages centered on the peak and trough may use up all or most of the observations on the phase, and therefore dampen the cyclical movement as well as eliminate erratic flutterings. To reduce this difficulty we use two-month averages in handling cyclical phases of less than four months. Thus, if the short phase is an expansion, the standing at the trough is represented by an average of the values at the trough and in the preceding month, while the standing at the peak is represented by an average of the values at the peak and in the following month. Similarly, if the short phase is a contraction, the standing at the peak is represented by the peak month and the one preceding it, and the standing at the trough by the trough month and the one following. But in some short phases a two-month average fails to reach as high up into the peak or as far down into the trough as would a three-month average, and therefore is even less satisfactory than a three-month average. To provide for such cases we follow the rule that if the average value in the two months specified to represent the peak is equal to or lower than the value in the additional month that would be included in a three-month average centered on the peak, the standing at the peak should be determined from the value in the peak month alone. With obvious changes in wording, this rule applies also to troughs of short phases.

The amplitude measures are shown in three forms in Table S2, of which Table 31 gives a sample. Columns (2), (3) and (4) show the standings at the initial low point of a specific cycle, at the high point, and at the terminal low point. As stated, these standings are usually three-month

16 In this connection, see pp. 326, 334-5.

TABLE 31
Sample of Table S2: Amplitude of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	3-month average in specific-cycle relatives centered on			Amplitude of			Per month amplitude of		
Trough–Peak–Trough (1)	Initial trough (2)	Peak (3)	Terminal trough (4)	Rise (5)	Fall (6)	Rise & fall (7)	Rise (8)	Fall (9)	Rise & fall (10)
May 07–Jan. 08	...	111.1[a]	83.6[a]	...	27.5[a]	...	...	3.4[a]	...
Jan. 08–June 10–Feb. 11	82.7	111.6	102.2	28.9	9.4	38.3	1.0	1.2	1.0
Feb. 11–Oct. 13–Nov. 14	87.3	110.2	88.0	22.9	22.2	45.1	0.7	1.7	1.0
Nov. 14–July 18–Mar. 19	76.0	124.3	85.2	48.3	39.1	87.4	1.1	4.9	1.7
Mar. 19–Dec. 20–July 22	93.4	122.7	64.0	29.3	58.7	88.0	1.4	3.1	2.2
July 22–May 23–June 24	57.2	127.0	86.8	69.8	40.2	110.0	7.0	3.1	4.8
June 24–Mar. 27–Dec. 27	85.9	113.1	85.8	27.2	27.3	54.5	0.8	3.0	1.3
Dec. 27–May 29–July 32	100.1	124.0	57.7	23.9	66.3	90.2	1.4	1.7	1.6
July 32–Mar. 37–Mar. 38	68.4	128.0	83.2	59.6	44.8	104.4	1.1	3.7	1.5
Average[b]	81.4	120.1	81.6	38.7	38.5	77.2	1.8	2.8	1.9
Average deviation[c]	10.6	6.4	10.4	15.4	14.2	23.5	1.3	1.0	0.8
Weighted average	...	...	...	...	...	...	1.3	2.6	1.7

Average rise & fall of seasonal: 68.4 per cent

[a] Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average and the average deviation.
[b] Arithmetic mean determined separately for each column. Hence (3)−(2) *may* differ from (5) in the last place; likewise (3)−(4) from (6), and (5)+(6) from (7).
[c] Measured from the mean.

averages of the specific-cycle relatives centered on the turning points of the specific cycles. If a standing is based on fewer than three months, the exceptional treatment is noted in the table. The 'terminal trough' of each cycle interlocks with the 'initial trough' of the next cycle; that is, the same item is presented as the terminal trough of one cycle and as the initial trough of the next. But the three-month averages of cycle relatives made from the same items differ almost always, sometimes widely, because the average value used as the base of the cycle relatives changes from one cycle to the next. Thus, in bituminous coal production the average of the three values centered on the trough of November 1914 is 88 per cent of the average monthly value during the cycle of February 1911–November 1914, but 76 per cent of the average monthly value during the cycle of November 1914–March 1919.

By scanning the entries on successive lines in columns (2) to (4) one gets a mental image of the specific cycles in the series. This image is severely simple, for it presents only the trough, peak, and trough of each cyclical wave measured from its own mean level. The varying shapes of the cycles from one turning point to the next and from series to series are brought out in the measures of cyclical patterns, described in Section VI. In the meantime columns (5) to (7) show the rise from trough to peak, the fall from peak to trough, and the total rise and fall. These measures are obtained from the entries in the preceding columns. Column (5) shows the absolute differences between columns (2) and (3); column (6) shows the absolute differences between columns (3) and (4); column (7) is the sum of columns (5) and (6). These figures add no information, but they make it easier to compare the amplitudes of different specific cycles in one process and of corresponding specific cycles in different processes. For convenience, the average rise and fall of the seasonal fluctuations is noted at the bottom of the table. Coal production is a highly seasonal industry, but the range of the seasonal fluctuations has been narrower on the average than the range of the cyclical fluctuations.

The third set of amplitude measurements, given in columns (8) to (10), is obtained by dividing the rise and fall figures in columns (5) to (7) of Table S2 by the corresponding duration figures in columns (6) to (8) of Table S1. This step brings out characteristics of the cyclical waves that are hidden by the preceding measurements of rise and fall. In Table S2 for bituminous coal production, the largest cyclical decline occurred in the cycle of 1927–32. But this contraction was uncommonly long, and when we divide the amplitudes of fall by their respective durations, we find that the average rate of fall was smaller in this cycle than in any other cycle except the first and second. Again, the rise exceeds the decline in five cycles out of the eight, but the *rate* of decline exceeds the *rate* of rise in every cycle except one. That contraction is a more violent change than expansion is a common finding, though it is not characteristic of all series. The per month figures are useful not only in comparing relatively long with relatively short cycles in the same country, but also cycles of unlike duration in different countries. For example, the specific cycles in British and American exports have almost the same average amplitudes. But the American cycles are on the average decidedly shorter than the British, and their per month amplitudes are therefore much greater.

The averages of columns (8) to (10) are given in two forms. In the simple averages each entry receives the same weight regardless of the length of the period to which it applies. In the weighted averages successive entries are assigned weights proportionate to the durations. Our plan of treating each cycle as a unit of analysis implies that the average should be unweighted. But we also wish to protect the averages against the distorting influence of extreme items. One way of approximating this objective is to compute weighted average rates of change.[17] Weighting reduces the influence of violently large values, for they are much more likely to come during brief than during long phases. In practice the weighted averages are obtained as a by-product of other calculations; for example, the weighted average of column (8) in Table S2 is derived by dividing the sum of column (5) in Table S2 by the sum of column (6) in Table S1. The differences between the simple and weighted averages are instructive. Thus the unweighted average rate of rise in coal production

[17] See pp. 364-5 and Tables 186-187.

is 1.8 per cent per month, the weighted average 1.3 per cent. The unweighted average rate of fall is 2.8 per cent, the weighted average 2.6 per cent. The weighted figure is commonly lower than the unweighted one, since there is a tendency for the rates of change during cyclical phases to be correlated inversely with their duration.

When a series is analyzed on an inverted basis, columns (2) to (4) show successively the three-month averages of cycle relatives centered on the initial peak of the specific cycle, the trough, and the terminal peak; columns (5) and (8) show the fall instead of the rise, and columns (6) and (9) show the rise instead of the fall. Although the method of calculation is essentially the same, the measure of average amplitude obtained by analyzing a series on an inverted basis is likely to differ from the measure that would be obtained by positive analysis. One reason for the difference is that the period covered by a group of positive cycles is slightly different from the period covered by inverted cycles: the former start and end with a trough, the latter start and end with a peak. But there is also another reason of a more technical character.

This technical reason arises from our practice of eliminating secular trends by steps covering a full cycle, not continuously as is the usual practice of statisticians. When the trend is upward a fall of specific cycles will usually appear larger in units of cycle relatives if the fall is made a part of a positive cycle than if it is made a part of an inverted cycle; for the base of the positive cycle is apt to be smaller than the base of the inverted cycle. But an upward trend will also make a rise of specific cycles smaller when the rise is included in a positive cycle than when it is included in an inverted cycle; for the base of the former is now larger than the base of the latter. If we take positive analysis as a standard, inverted analysis tends to reduce the amplitude of fall and to increase the amplitude of rise when the trend is upward; it tends to produce opposite effects when the trend is downward; but the effects on the rise are set against the effects on the fall when the amplitudes of rise and fall are combined. When the trend is horizontal, the positive and inverted measures should be identical.

These effects and their order of magnitude are illustrated by Table 32, which includes two samples of a rising trend, one of a falling trend, and three of a horizontal trend.[18] The three samples of a horizontal trend are the trend-adjusted forms of the actual data; the latter are designated in the table as 'unadjusted', being corrected merely for seasonal variations. To isolate the difference between the positive and inverted measures that is attributable to differences in the periods covered, the averages for the positive cycles are shown in two forms. One set covers a period beginning and ending with a trough, as is our usual practice. The second set omits the expansion of the first and the contraction of the last

[18] For the rate of secular advance or decline in these series, see Table 84. The series themselves are identified more fully in Ch. 7, Sec. I.

TABLE 32

Average Amplitude of Specific Cycles on Positive and Inverted Plans
Three American Series, Unadjusted and Trend-adjusted

Series and type of analysis	Period covered	No. of cycles	Direction of trend[a]	Average amplitude in specific-cycle relatives		
				Rise	Fall	Rise & fall
			Unadjusted data			
A.T.&T. INDEX						
Positive	1900–1933	9	U	30.5	26.1	56.5
Positive	1903–1929	8	U	30.7	20.2	51.0
Inverted	1903–1929	8	U	31.4	19.1	50.6
PIG IRON PRODUCTION						
Positive	1879–1933	15	U	62.1	54.8	116.8
Positive	1883–1929	14	U	61.9	48.0	110.0
Inverted	1883–1929	14	U	64.5	43.9	108.4
RAILROAD BOND YIELDS						
Positive	1868–1899	8	D	6.3	14.6	20.9
Positive	1869–1896	7	D	6.0	14.4	20.4
Inverted	1869–1896	7	D	5.8	15.0	20.8
			Trend-adjusted data			
A.T.&T. INDEX						
Positive	1900–1933	9	H	23.9	29.6	53.6
Positive	1903–1929	8	H	25.1	23.5	48.6
Inverted	1903–1929	8	H	24.8	23.8	48.6
PIG IRON PRODUCTION						
Positive	1879–1933	15	H	52.2	57.7	109.9
Positive	1882–1929	14	H	53.1	51.2	104.3
Inverted	1882–1929	14	H	52.1	51.9	104.1
RAILROAD BOND YIELDS						
Positive	1868–1899	8	H	8.6	8.4	17.0
Positive	1869–1896	7	H	8.6	8.4	17.0
Inverted	1869–1896	7	H	8.6	8.4	17.0

[a] U stands for upward, D for downward, H for horizontal.

specific cycle, and thus includes exactly the same cycles as do the inverted averages.[19] We find a systematic difference between the positive and inverted measures in the samples in which the secular trend has a definite direction; but the differences are small in relation to the size of the figures, and are practically confined to the rise and fall taken separately. In general, as the table suggests, the inclusion or omission of an additional cycle may well exercise a greater effect on the averages than a shift from positive to inverted analysis or vice versa for the same group of cycles.[20]

It follows, if the analysis is positive, that the amplitude of a declining

[19] That applies to the rise and fall in specific-cycle relatives; the bases, on which the relatives are computed, unavoidably cover the maximum numbers of cycles shown for the positive analysis in the table.

[20] The trend-adjusted forms of pig iron production and the A.T.&T. index behave a little like series with downward trends (Table 32). This or the opposite result can happen, since the cycle bases of trend-adjusted data are almost certain to vary somewhat. See pp. 287-9 for further analysis of the effect of positive versus inverted treatment on measures of amplitude.

phase of an incomplete specific cycle at the beginning of a series, or of a rising phase at the end, can be approximated by treating it as part of an inverted cycle. Similarly, if a series is analyzed on an inverted basis, the amplitude of an extra phase at the beginning or end of the series can be approximated by treating it as part of a positive cycle. We follow this practice in making Table S2, since we wish to record as fully as we can the amplitudes of the cyclical movements recognized in the series. For example, the relatives on the first line in columns (3) and (4) of Table 31 are computed on the base of the inverted cycle from May 1907 to June 1910; the remaining entries on this line are derived in the usual fashion. But we do not include the entries for the incomplete cycle in the averages, mainly because comparisons of the averages for the phases of rise and fall are as a rule facilitated if based on the same number of observations.

A more serious difficulty is that our measure of amplitude is not entirely independent of the unit in which the original data are expressed.[21] For example, the measure is the same whether a series is expressed in dollars or cents, in long tons or short tons, in miles or yards. But we shall get one measure of amplitude if the price of eggs is expressed as so many cents per dozen, and another measure if the price is expressed as so many dozens per dollar; or if London exchange is expressed in dollars per pound, instead of shillings per dollar. Again, a series showing the percentage of workers unemployed is apt to yield very different measures of amplitude than the same data converted into employment percentages. For example, the percentage of trade union members unemployed was exceptionally low in Great Britain during 1914–17, the average for this cycle being 1.091 per cent. At the peak of the cycle in August 1914 unemployment was 5.20 per cent, then fell to a trough of .33 per cent in April 1917, and rose to another peak of 1.17 per cent in December 1917; the total swing is thus 5.71 per cent.[22] When 5.71 is expressed as a relative of 1.091, the small unemployment average for the cycle, we get an impression of an enormous fluctuation—a total swing of 523 per cent. But we get the opposite impression when the unemployment percentages are converted into employment percentages; for now 5.71, expressed as a percentage of 98.909, comes out 6 per cent.

A shortcoming of the amplitude measure is that it cannot be applied to series of figures that are sometimes plus, sometimes minus. Familiar

21 Assume that the original figures of a time series are represented by a_1, a_2, a_3, a_4, etc. Then our measure of amplitude will be unchanged if each term is multiplied by a constant; that is, the measure of amplitude for the series a_1k, a_2k, a_3k, etc. is the same as for the original series. But the measure will be different if the original data are converted to the form $\frac{k}{a_1}, \frac{k}{a_2}, \frac{k}{a_3}$, etc., or if they are converted to the form $k - a_1$, $k - a_2$, $k - a_3$, etc.

22 The figures cited are three-month averages centered on the turns. The figure for 1914 is seasonally adjusted; the others are not, no seasonal calculation being made for the few disturbed years that followed. The original figures come from the Ministry of Labour, *Twenty-first Abstract of Labour Statistics of the United Kingdom (1919–33)*, p. 69.

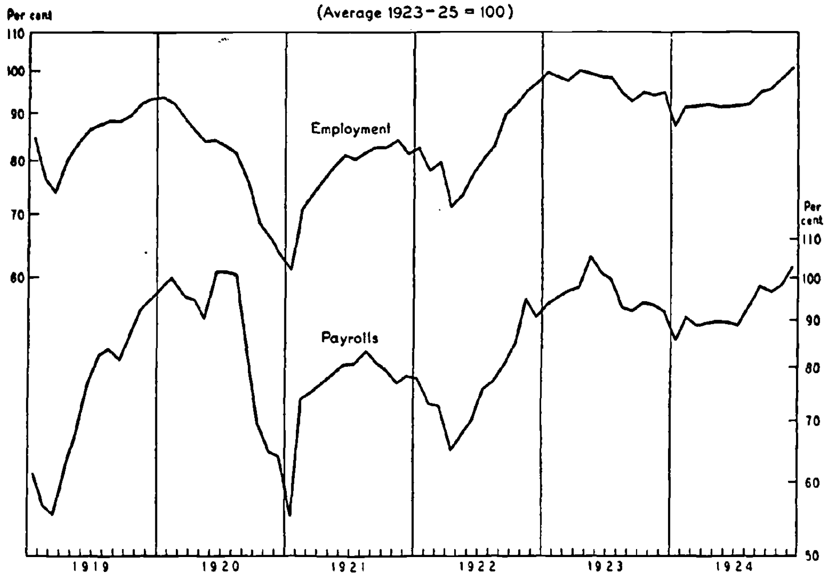
examples are series on profits or losses, net movements of gold from one country to others, differentials between interest rates in the same or different markets, net changes in inventories or plant equipment. In such series the cycle base may approach zero or even be negative, and thus produce preposterous amplitudes. This difficulty is not peculiar to the amplitude measures; it is common to all measures in our scheme of analysis that involve percentages, which means all the measures we make except those in Table S1 and the conformity indexes in Tables R3 and R4. The alternative we have adopted for handling series with plus or minus figures is to work with absolute deviations from the average for each cycle, instead of percentage deviations. Amplitudes measured in this way can be compared for series expressed in the same unit, but not for series in diverse units.

Another troublesome point is that our rules for identifying specific cycles sometimes fail to yield clear-cut results. This difficulty too is not peculiar to the amplitude measures, but applies to all measures of specific cycles. To minimize the difficulty we check carefully the specific-cycle decisions for related series, and thus attempt to prevent inconsistent treatment of marginal cases. Sometimes we leave a period beclouded by violent erratic movements out of the averages, or else use a range instead of a single figure to indicate the central tendency, one average including and the other excluding the doubtful movements.[23] But even when there are no marginal cases in the sense of our rules, the results may be unsatisfactory, for the rules themselves suffer from discontinuity. An extreme instance is illustrated by Chart 13, showing the employment and payrolls in dyeing and finishing textile plants from 1919 to 1924. The cyclical movements of the two series are similar except that payrolls have wider fluctuations than employment, which is to be expected. Yet, so long as we adhere rigidly to the rules, we must recognize one cycle less in payrolls than in employment; for the interval between the peaks of payrolls in 1920 and 1921 is only 13 months, while the minimum interval that can qualify as a specific cycle under our rules is 15 months. Since we must choose between the peaks in 1920 and 1921, it seems better to select the former. If we now recognize a trough in January 1921, which seems unavoidable, we are bound to ignore the contraction of 1921 to 1922, although it is both longer and larger than the corresponding contraction in employment. Decisions of this character are obviously absurd and can only lead to confused results and interpretations. They can be avoided by any of three devices: by omitting the period causing trouble from the averages of both series, by recognizing peaks in both 1920 and 1921 in payrolls despite the rules, or by ignoring the contraction from 1921 to 1922 in employment as well as in payrolls. It is wiser to set the rules aside

[23] Cf. Ch. 8, Sec. X.

CHART 13

Employment and Payrolls in Dyeing and Finishing Textile Plants
United States, 1919 – 1924
(Average 1923 – 25 = 100)



Adjusted for seasonal variations. Source: Bureau of Labor Statistics, Revised Indexes of Factory Employment and Payrolls, 1919 to 1933 *(Bulletin 610)*, Feb. 1935, pp. 60, 102.

Logarithmic vertical scales

momentarily when they lead to absurd results than to stand by and permit them make a mockery of common sense.

Some modification of the standard procedure of measuring amplitudes may be desirable in other situations. Suppose, for example, that we wish to compare the amplitude of the cyclical declines in different branches of production during the contraction of 1929–33 in the United States. It would not do to measure the change during the same interval for each industry, since the timing of the peaks and troughs varies considerably from activity to activity. If the amplitude of the full cyclical declines is to be determined, the measures must be made from the specific cycles of the individual series. But work with specific cycles in our usual fashion is awkward for two reasons. In the first place, some industries skipped the contraction of 1926–27, while a few skipped both this and the 1923–24 contraction, so that the bases on which specific-cycle relatives are computed cover widely varying periods in different series. To meet this difficulty the decline of each series may be expressed as a percentage of its standing at the peak from which the decline started; or better, as a percentage of its average value during the reference cycle 1927–33. The second difficulty is that although the contraction of 1929–33 engulfed virtually the entire industrial system, the long decline was interrupted by several abortive revivals, one of which lasted long enough to produce an 'extra' specific-cycle expansion in the production of many consumer goods industries.[24] The peak of the extra expansion was as a rule lower, sometimes much lower, than the peak in the vicinity of the reference turn in 1929. For the special purpose of comparing the amplitude of the cyclical declines in different branches of industry during the business-cycle contraction of 1929–33, it is probably best therefore to ignore entirely the extra cyclical movements. But the extra cycles cannot be ignored in other connections: they are a conspicuous feature of the Great Depression, and they raise the question whether similar developments occurred in other protracted depressions of general business.

It must always be remembered that the large amplitude of one activity in units of cycle relatives and the small amplitude of another activity tell us little about the relative importance of the two activities in business cycles. For example, industrial building contracts in the United States have larger cyclical amplitudes than residential building contracts; this relation is reversed if the amplitudes are measured in dollars instead of cycle relatives. Again, since repair work has smaller amplitudes in cycle relatives than new building, it may be said to 'moderate' the cyclical fluctuations of total building construction inclusive of repair work. But this observation means merely that the cyclical swing of total building is a smaller percentage of its average level than the swing of new building is of its average. So far as the cyclical timing of repair work is similar to that of new building, repair work is obviously an intensifying factor in building fluctuations; that is, it tends to increase the fluctuations of the industry whether measured in man-hours of employment or in dollars expended. One more example may be cited. In recent business cycles the construction of new railroad lines in this country appears to be nearly as volatile, on a percentage basis, as in the eighteen-seventies or eighties, when thousands of miles of road were added each year to the railroad system. But the enormous percentage fluctuations of recent times excite little interest, since the construction of new railroads has shrunk to insignificance both absolutely and in relation to total industrial activity.

These illustrations point a moral, and one that applies to all the measures we use to describe cyclical behavior, not only to the amplitudes. These measures cannot be interpreted properly unless the relation of each process to the economy at large is brought out. For example, we must know the relative importance of agriculture, of exports and imports, of railroads and other transport agencies, of commercial banks, of construction work, of the steel industry, and so on, in each of the four countries represented in our statistical collection. In these instances, useful criteria of importance are the contribution of a factor to the national income, the number who get their living from the activity in question, and the capital invested. Such indicators of the importance of an activity

[24] See our paper, Production during the American Business Cycle of 1927–1933 (*Bulletin 61*), especially pp. 18-19.

must be supplemented by knowledge of its industrial and financial relation to other activities. It is well to know that railroads rate much higher as employers of labor than banks; yet it is no less essential to recognize that railroads and banks are alike in that all modern business depends upon their continuous functioning, that business cycles existed before railroading though not before commercial banking, that when banks provide more credit the volume of a nation's circulating medium is increased, but that when railroads move more traffic the direct monetary effect is merely an increase in the exchange velocity of 'money'.

V Measures of Secular Movements

The 'intra-cycle' trend of a series is that portion of its secular movement which occurs within the period of a single cycle. The 'inter-cycle' trend is that portion of the secular movement which cumulates from cycle to cycle. The former we retain in our measures of cyclical behavior; the latter we eliminate in computing specific-cycle and reference-cycle relatives.

The element of trend retained in our measures can be judged from the difference between the average rise and the average fall of the specific cycles, or from the tilt of the curves on the charts of cyclical patterns. But these approximations are rough and may be misleading, especially when the series is short. Hence we set out in Table S3 some of the chief facts concerning the secular movements of each series we study. A sample for bituminous coal production appears in Table 33.

Columns (2) and (3) of Table S3 show the average monthly standing of the seasonally adjusted data during the expansions and contractions of specific cycles. If there is a full phase of an incomplete specific cycle at the beginning or end of a series, its average value is also recorded in these columns. In computing the average standing during an expansion or contraction, the values at the peak and trough dates receive a weight of one-half each and the intervening monthly values a weight of one each. Column (4) shows the average standing of the seasonally adjusted data during each specific cycle. This figure may be obtained by striking a mean of the average standings in the expansion and contraction phases, each weighted by its duration; which is equivalent to assigning a weight of one-half each to the values at the initial and terminal troughs and a weight of one to each intervening value. As previously explained, the entries in column (4) are the bases on which the specific-cycle relatives are computed.

The entries in later columns are elaborations of the figures in columns (2) to (4). Column (5) states by what percentage the average standing during an expansion is higher or lower than the average during the preceding contraction. Similarly, column (6) states by what percentage the level of

TABLE 33

Sample of Table S3: Secular Movements
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average monthly standing (million short tons)			Per cent change from preceding phase		Per cent change from preceding cycle on base of			
						Preceding cycle		Average of given and preceding cycle	
Trough – Peak – Trough (1)	Expansion (2)	Contraction (3)	Full cycle (4)	Contraction to expansion (5)	Expansion to contraction (6)	Total (7)	Per month (8)	Total (9)	Per month (10)
May 07–Jan. 08	...	32.3	...	...	...	...	...	...	...
Jan. 08–June 10–Feb. 11	30.8	34.0	31.5	−5	+10	...	...	...	...
Feb. 11–Oct. 13–Nov. 14	37.0	36.5	36.9	+9	−1	+17	+0.41	+16	+0.39
Nov. 14–July 18–Mar. 19	42.5	43.7	42.7	+16	+3	+16	+0.33	+15	+0.31
Mar. 19–Dec. 20–July 22	43.3	34.0	38.9	−1	−21	−9	−0.20	−9	−0.20
July 22–May 23–June 24	42.1	44.7	43.6	+24	+6	+12	+0.38	+11	+0.35
June 24–Mar. 27–Dec. 27	44.8	41.2	44.0	0	−8	+1	+0.03	+1	+0.03
Dec. 27–May 29–July 32	42.3	35.6	37.7	+3	−16	−14	−0.29	−15	−0.31
July 32–Mar. 37–Mar. 38	31.3	34.1	31.8	−12	+9	−16	−0.26	−17	−0.28
Average	...	...	...	...	...	...	...	+0.3	+0.04
Average deviation	...	...	...	...	...	...	...	12.0	0.26
Weighted average	...	...	...	...	...	...	...	...	+0.01

Average is the arithmetic mean; the average deviation is measured from the mean.

a given contraction is above or below that of the preceding expansion. In both columns the percentages are computed on the base of the preceding phase. Together they show the height of the steps from one phase of specific cycles to the next, the width of the steps being already entered in columns (6) and (7) of Table S1. When the secular trend rises, as in the first few cycles of coal output, the average standing during contraction is often higher than during the preceding expansion. The mean levels of successive phases thus bring out important features of cyclical movements in an economic system characterized by secular changes that differ in pace and direction within given processes during different periods and among different processes during the same period.

For our purposes the magnitude of the 'intra-cycle' trend can usually be judged sufficiently well from the line of 'inter-cycle' trend, that is, the step-line formed by the mean levels of successive specific cycles. In column (7) we therefore give the percentage change from the mean level of one specific cycle to the mean level of the next. We take account of the width of the steps between successive cycles in column (8), which shows the percentage change per month between the mean levels of successive cycles. This column is obtained by dividing the number of months from the midpoint of one cycle to the midpoint of the next into the entries in column (7). Thus the duration of the first specific cycle in coal output is 37 months and of the second cycle 45 months; the interval between their midpoints is 41 months; the mean level of the second cycle is 17 per cent higher than the mean level of the first; therefore the percentage change per month between the two cycles is +0.41.

While the measures in columns (5) to (8) show in a simple manner the percentage changes between successive phases or cycles, we would get biased results if we averaged these measures for a number of cycles. Assume, for example, that the average monthly standings of successive cycles are 80, 40, 80, 40, 80. There is, obviously, no secular trend in this series. Yet the percentage changes between successive cycles are −50, +100, −50, +100; hence the average percentage change from cycle to cycle is +25. This result exemplifies the familiar statement that percentage changes have an 'upward bias'. But the bias disappears if the average of the two values being compared is taken as the base of the percentage, instead of the first value. On this plan the bases are uniformly 60 in our example; the percentage changes are alternately −67 and +67, and their average is 0. We enter in column (9) percentages computed in this manner; that is, instead of using the average standing in the preceding cycle as the base, as we do in column (7), we use a simple average of the average standings in the given and preceding cycles as the base.[25] Column (10) differs from (8) in the same way that column (9) differs from (7). Averages for all cycles are restricted in Table S3 to columns (9) and (10), because these alone are free from 'secular bias'.

At the foot of column (10) both simple and weighted averages are entered. The latter weights the individual entries by the intervals to which they relate, that is, the intervals between midpoints of successive cycles. In practice the weighted average is most readily obtained by dividing the sum of the entries in column (9) by the number of months from the midpoint of the first to the midpoint of the last full cycle in the series. Whereas the unweighted average is likely to vary with specific-cycle decisions, the weighted average is virtually independent of those decisions. We use this weighted figure more commonly than any other in the table. We have found empirically that it is a good approximation to the average percentage change per month determined from a 'least squares' exponential.

When a series is analyzed on an inverted basis, the procedure is the same except that the average standing during contraction is entered in column (2), the average standing during expansion in column (3), the

[25] Let A be the average standing of a series during a given specific cycle, KA the average during the next cycle, F the customary measure of percentage change, and J the measure described in the text as free from secular bias. Then

$$F = 100\left(\frac{KA-A}{A}\right) = 100\,(K-1), \text{ and } J = 100\left(\frac{KA-A}{\frac{KA+A}{2}}\right) = 200\,\frac{K-1}{K+1}.$$

The theoretical limits of F are -100 and ∞, while the limits of J are -200 and $+200$. Since $J = \left(\frac{2}{K+1}\right)F = \frac{200F}{200+F}$, in practice we first compute F, then find the equivalent J from a table relating F and J.

percentage change from expansion to contraction in column (5), and the percentage change from contraction to expansion in column (6). In handling series such as net changes in inventories, the successive values of which are sometimes plus and sometimes minus, we show absolute instead of percentage changes in the table.

Table S3 supplies what we wish to know: the shifts in the average level of a series from one phase of specific cycles to the next and from one full cycle to the next. We should use different procedures were we concerned primarily with secular trends instead of cyclical fluctuations.[26] Of course, if a series is free from specific cycles, Table S3 cannot be made. In such series we use measures like those in Table S3, but made from reference cycles instead of specific cycles. Since reference cycles cover uniform periods in all series, this plan facilitates comparisons of different series for some purposes, and it may prove profitable at a later stage to extend this computation to all series. However, the weighted average in column (10) is virtually bound to be the same when computed from reference cycles as when computed from specific cycles, unless a series is short and the specific and reference cycles match very badly at the ends.

VI Specific-cycle Patterns

While Table S1 shows the timing and duration of the specific cycles of economic activities, Table S2 their amplitudes, and Table S3 the changes in their average levels, these tables do not give a clear idea of the form of the mounting wave from trough to crest or of the subsiding wave from crest to trough. Supplemental measures are needed to show in some detail the progress of the cyclical fluctuations. These measures are supplied by Table S4, which records the average standing of a series during nine segments of each specific cycle. A sample for bituminous coal production is shown in Table 34.

The nine-point 'pattern' of a specific cycle is made on the following plan. First, the average of the cycle relatives in the three months centered on the initial trough of the specific cycle is entered in column (2). Next, the expansion phase—strictly speaking, the interval beginning with the month after the trough and ending with the month before the peak—is subdivided into three parts as nearly equal as may be without using fractions of a month. The average standing of the cycle relatives in each third of expansion is computed, and the results entered in columns (3) to (5). The standing at the specific-cycle peak is represented by a three-month average centered on that date; this average is entered in column (6). The contraction phase, that is, the interval beginning with the month after the peak and ending with the month before the terminal trough, is

26 Indeed, as our work on business cycles deepens, we find at times a need for mathematical representations of secular movements or for measures other than those recorded in Table S3.

divided into thirds as is the expansion phase, and the average standings are entered in columns (7) to (9). Finally, the average standing in the three months centered on the trough that links the given cycle to the next is presented in column (10). For convenience, the successive stages into which each specific cycle is broken are designated by Roman numerals from I to IX: the stage of the initial trough is represented by I, of the peak by V, and of the terminal trough by IX.

TABLE 34
Sample of Table S4: Specific-cycle Patterns
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
May 07–Jan. 08	...	...	...	...	111.1[a]	109.7[a]	107.3[a]	97.0[a]	83.6[a]
Jan. 08–June 10–Feb. 11	82.7	87.8	94.9	111.4	111.6	106.4	109.5	107.8	102.2
Feb. 11–Oct. 13–Nov. 14	87.3	92.5	101.9	106.9	110.2	104.3	95.9	96.3	88.0
Nov. 14–July 18–Mar. 19	76.0	86.7	101.3	110.5	124.3	117.5	100.2	88.9	85.2
Mar. 19–Dec. 20–July 22	93.4	108.8	104.1	119.8	122.7	89.9	87.5	83.6	64.0
July 22–May 23–June 24	57.2	83.6	100.5	108.3	127.0	119.1	95.8	91.8	86.8
June 24–Mar. 27–Dec. 27	85.9	90.7	102.8	111.5	113.1	96.2	92.4	87.6	85.8
Dec. 27–May 29–July 32	100.1	109.3	113.1	114.3	124.0	114.3	95.1	74.2	57.7
July 32–Mar. 37–Mar. 38	68.4	87.3	96.3	111.1	128.0	112.2	114.4	94.8	83.2
Average	81.4	93.3	101.9	111.7	120.1	107.5	98.8	90.6	81.6
Average deviation	10.6	7.8	3.6	2.6	6.4	8.3	6.9	7.0	10.4

Average is the arithmetic mean; the average deviation is measured from the mean.
[a] Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average and the average deviation.

Thus the nine-point pattern is an elaboration upon the skeleton framework in columns (2) to (4) of Table S2. The standings in stages I, V and IX are taken directly from that table. When one or two months instead of the usual three are used in Table S2 to represent the standing at a peak or trough, that is done also in Table S4. Stage V overlaps stages IV and VI; that is, of the three months usually included in stage V, the first is included also in IV and the last in VI. In a similar manner stage I (or IX) overlaps stages VIII and II. Since stages IV and VI omit the peak month, and stages VIII and II omit the trough month, the intervals designated as 'expansion' and 'contraction' in Table S4 cover one month less than the full phase. If the duration of the expansion or contraction, thus defined, is exactly divisible by three, the distribution of months into thirds is straightforward. If the division yields a remainder of one, the extra month is placed in the middle third. If the remainder is two, an extra month is assigned to the first and last thirds.[27] Thus the successive stages of expansion or contraction may cover 6, 6, 6 months, or 6, 7, 6, or 7, 6, 7. This plan ensures that the midpoint of stage III is also the midpoint of the expansion, and that the midpoint of stage VII is also the midpoint of the contraction. Further, it tends to make the average number of months in each third of a run of expansions or contractions equal, since the probability that the duration of a phase when divided by three will leave a remainder of zero is presumably the same as the probability that the remainder will be one or two.

TABLE 35

The Computation of Specific-cycle Patterns Illustrated
Bituminous Coal Production, United States, 1908–1919

Cycle (1)	Stage (2)	Period covered (3)	No. of months (4)	Average monthly standing in: Millions of short tons (5)	Average monthly standing in: Cycle relatives (6)	Interval from stage to stage (mos.) (7)
	I	Dec. 1907 – Feb. 1908	3	26.033	82.7	...
	II	Feb. 1908 – Oct. 1908	9	27.644	87.8	5.0
Jan. 1908–Feb. 1911	III	Nov. 1908 – Aug. 1909	10	29.870	94.9	9.5
(Av. monthly	IV	Sep. 1909 – May 1910	9	35.056	111.4	9.5
value = 31.482	V	May 1910 – July 1910	3	35.133	111.6	5.0
million short	VI	July 1910 – Aug. 1910	2	33.500	106.4	1.5
tons)	VII	Sep. 1910 – Nov. 1910	3	34.467	109.5	2.5
	VIII	Dec. 1910 – Jan. 1911	2	33.950	107.8	2.5
	IX	Jan. 1911 – Mar. 1911	3	32.167	102.2	1.5
	I	Jan. 1911 – Mar. 1911	3	32.167	87.3	...
	II	Mar. 1911 – Dec. 1911	10	34.090	92.5	5.5
Feb. 1911–Nov. 1914	III	Jan. 1912 – Nov. 1912	11	37.545	101.9	10.5
(Av. monthly	IV	Dec. 1912 – Sep. 1913	10	39.400	106.9	10.5
value = 36.853	V	Sep. 1913 – Nov. 1913	3	40.600	110.2	5.5
million short	VI	Nov. 1913 – Feb. 1914	4	38.425	104.3	2.5
tons)	VII	Mar. 1914 – June 1914	4	35.350	95.9	4.0
	VIII	July 1914 – Oct. 1914	4	35.500	96.3	4.0
	IX	Oct. 1914 – Dec. 1914	3	32.433	88.0	2.5
	I	Oct. 1914 – Dec. 1914	3	32.433	76.0	...
	II	Dec. 1914 – Jan. 1916	14	37.000	86.7	7.5
Nov. 1914–Mar. 1919	III	Feb. 1916 – Apr. 1917	15	43.207	101.3	14.5
(Av. monthly	IV	May 1917 – June 1918	14	47.143	110.5	14.5
value = 42.667	V	June 1918 – Aug. 1918	3	53.033	124.3	7.5
million short	VI	Aug. 1918 – Sep. 1918	2	50.150	117.5	1.5
tons)	VII	Oct. 1918 – Dec. 1918	3	42.767	100.2	2.5
	VIII	Jan. 1919 – Feb. 1919	2	37.950	88.9	2.5
	IX	Feb. 1919 – Apr. 1919	3	36.367	85.2	1.5

The illustration covers the first three full specific cycles in coal production; see Table 34.

The illustration worked out in Table 35 may clarify the details. The initial trough of the first complete specific cycle recorded for bituminous coal production comes in January 1908. Hence the standing in stage I is an average of the specific-cycle relatives for December 1907, January

[27] We are indebted to W. Allen Wallis for suggesting this arrangement.

1908, and February 1908, each computed on the base January 1908–February 1911. As stated before, this result is obtained more easily by averaging the original values in the three months and then converting the average into a relative, and the calculations in Table 35 are shown in this form. The 'expansion' phase covers the period from February 1908 through May 1910, or 28 months in all. Hence stages II, III and IV cover successively 9, 10 and 9 months. Stage V includes the three months May–July 1910, but May is included also in stage IV and July in stage VI. Stage IX includes the three months January–March 1911, and thus laps over stage VIII of the given cycle and stage II of the next cycle. The three months entering into stage IX of the cycle from January 1908 to February 1911 constitute also stage I of the cycle from February 1911 to November 1914. But the standing in stage IX of the first cycle is 102 and the standing in stage I of the next cycle is only 87; the difference means that the monthly average of the data for January–March 1911 is 102 per cent of the monthly average for January 1908–February 1911, and 87 per cent of the average for February 1911–November 1914.

When the specific cycles of a series are treated as inverted, the procedure is the same but now the sequence of stages runs from the initial peak (stage I) to the terminal peak (stage IX). A sample of an inverted analysis appears in Table 36. If there is a full phase of an incomplete specific cycle at the beginning or end of a positive analysis, as in coal production, this phase is subdivided in the usual fashion, but the standings are expressed as relatives on the base of an inverted cycle and excluded from the averages of all cycles. If a series is treated on an inverted plan, the standings of a full phase of an incomplete specific cycle are

TABLE 36

Sample of Table S4: Specific-cycle Patterns on Inverted Plan

Slab Zinc Stocks at Refineries, United States, 1921–1938

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial peak	Contraction			3 mos. centered on trough	Expansion			3 mos. centered on terminal peak
Peak – Trough – Peak		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
July 21–Apr. 23–July 24	237.8	190.2	102.7	42.5	28.3	53.5	88.9	103.1	135.6
July 24–Nov. 25–Sep. 28	174.4	125.1	63.1	54.6	27.0	67.9	115.0	143.5	157.4
Sep. 28–May 29–Jan. 31	58.4	58.6	55.4	45.9	44.4	64.7	120.7	171.1	182.7
Jan. 31–Oct. 33–Dec. 34	117.7	110.7	106.3	100.3	80.1	87.4	85.1	88.7	95.6
Dec. 34–Aug. 37–June 38	153.8	145.0	107.5	39.8	16.8	35.8	114.8	175.5	194.2
Average	148.4	125.9	87.0	56.6	39.3	61.9	104.9	136.4	153.1
Average deviation	48.3	33.3	22.2	17.5	18.3	13.8	14.3	32.4	30.0

Average is the arithmetic mean; the average deviation is measured from the mean.

expressed as relatives on the base of a positive cycle. In the few series consisting of plus or minus values, whether analyzed on a positive or inverted basis, the standings in successive stages of the cycles are shown as absolute deviations from the cycle base, instead of in percentages.[28]

Our method of tracing cyclical patterns makes implicit assumptions about the general character of cyclical movements. The principal assumption is that the turns of specific cycles come in single months; that is to say, the tops and bottoms of specific cycles are rounded or angular, not flat. On the whole this assumption is amply justified by experience. Specific cycles consisting of a horizontal low level followed by a stationary high level, which in turn is followed by a stationary low level and so on, appear very rarely in practice. But intermediate types are troublesome, as when a specific cycle consists of an expansion, plateau, and contraction; or of an expansion, contraction, and flat bottom; or of an expansion, flat top, contraction, and flat bottom. Our standard practice has been to date the peak towards the end of a flat top and the trough towards the end of a flat bottom—a method that obviously biases the cyclical patterns. But such cases are few in proportion to the number of cycles we have analyzed; they can be readily spotted on the data charts, and qualitative amendments made where needed. A simple quantitative adjustment could be made by including all months within the flat top in the standing at stage V, including all months within the flat bottom in stages I and IX, assigning the months between the end of the flat bottom and the beginning of the flat top to stages II, III and IV, and the months between the end of the flat top and the beginning of the flat bottom to stages VI, VII and VIII. But this method also will not work perfectly in practice, for so-called 'flat' tops or bottoms may be jagged and therefore not easy to delimit. And when the tops and bottoms do happen to be perfectly flat, the expansions and contractions are likely to be staircase movements, and this feature will be concealed by the adjustment just as it is concealed by the standard plan of making patterns.

Another assumption that underlies our method of tracing cyclical patterns is that the phases of specific cycles cover fairly large numbers of months, so that there is reasonable opportunity for erratic flutterings to disappear. The practice of breaking expansions and contractions into thirds irrespective of their duration becomes a thin formality when the phases are very short. When an expansion lasts four months, three months are covered by the interval exclusive of the peak and trough and hence only one month each is left for stages II, III, and IV. When a phase lasts less than four months, as happens occasionally, we must resort to interpolation to get the standings needed for the successive thirds of the phase.

28 To keep the exposition simple, the remainder of this section, except for the closing paragraphs, is written from the viewpoint of positive analysis. With obvious changes in phrasing, the description applies also to inverted analysis.

Interpolated figures, if any, are enclosed in parentheses in Table S4.[29]

Where needed, the interpolations are made on the following plan. If the short phase is an expansion, the standing at stage III is determined from the cycle relatives for the one or two months in the expansion exclusive of the trough and peak months. The standing at stage II is then determined by interpolating along a straight line between the standings at stages I and III, and the standing at stage IV by interpolating between the standings at stages III and V. In these computations each standing is conceived as placed in the middle of the stage. The midpoint of stage V is treated as coming in the middle of the peak month and the midpoint of stage I as coming in the middle of the trough month, although these stages are commonly represented by two months in very brief phases. This procedure is necessary to ensure that the sum of the intervals from midpoint to midpoint of the stages equals the duration of the cycle; and may be rationalized by saying that stages I, V and IX include in principle only the trough or peak months, but that the cyclical standing at these stages is estimated in practice from one, two or three months.[30]

The disregard of differences in cycle durations in Table S4, and in its congener Table R1, is the most considerable distortion we practise upon the original data at any point in the analysis. At a later stage, we plan to make a special study of long, medium, and short cycles to see whether differences in duration are regularly associated with differences in other aspects of cyclical behavior. Meanwhile we express the relations between the amplitude and duration of cyclical fluctuations in the per month figures of Table S5,[31] which show the average change per month, in units of cycle relatives, from stage to stage of the specific cycles. As the sample in Table 37 indicates, the rate of change of coal production has been far from steady during successive intervals of expansion and contraction. As usual, within its briefer span contraction is somewhat more violent than expansion. But in coal production at least, the most violent declines tend to occur at the beginning and close of contraction.

29 For some examples, see Appendix Table B1.

30 In a three-month expansion, stages II to IV cover two months in all, or two-thirds of a month each; the middle of stage II is five-sixths and the middle of stage III is nine-sixths of a month from the middle of stage I. Hence if we interpolate along a straight line between stages I and III, the standing in stage II is equal to the standing in stage I plus five-ninths of the excess of the standing in stage III over the standing in stage I. Again, the middle of stage IV is four-sixths and the middle of stage V nine-sixths of a month from the middle of stage III; hence the standing in stage IV is equal to the standing in stage III plus four-ninths of the excess of the standing in stage V over that in stage III.

By similar reasoning, if the expansion lasts two months, the standing in stage II equals that in stage I plus two-thirds of the excess of stage III over stage I, and the standing in stage IV equals that in stage III plus one-third of the excess of stage V over stage III.

With obvious changes in stage numerals, the procedure is the same in handling short contractions.

31 Also in Tables S2, R2, R3, and R4 the amplitudes are reduced to a per month basis. In Table S3 per month figures are used to express the relation between changes in the average level of successive cycles and the intervals between their midpoints.

TABLE 37

Sample of Table S5: Rate of Change from Stage to Stage of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average change per month in specific-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough-Peak-Trough (1)	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
May 07 – Jan. 08	...	...	...	...	−0.9*	−1.0*	−4.1*	−8.9*
Jan. 08 – June 10 – Feb. 11	+1.0	+0.7	+1.7	0.0	−3.5	+1.2	−0.7	−3.7
Feb. 11 – Oct. 13 – Nov. 14	+0.9	+0.9	+0.5	+0.6	−2.4	−2.1	+0.1	−3.3
Nov. 14 – July 18 – Mar. 19	+1.4	+1.0	+0.6	+1.8	−4.5	−6.9	−4.5	−2.5
Mar. 19 – Dec. 20 – July 22	+3.8	−0.7	+2.4	+0.7	−9.4	−0.4	−0.6	−5.6
July 22 – May 23 – June 24	+13.2	+5.6	+2.6	+9.4	−3.2	−5.8	−1.0	−2.0
June 24 – Mar. 27 – Dec. 27	+0.8	+1.2	+0.8	+0.3	−8.4	−1.5	−1.9	−0.9
Dec. 27 – May 29 – July 32	+3.1	+0.7	+0.2	+3.2	−1.5	−1.5	−1.7	−2.5
July 32 – Mar. 37 – Mar. 38	+2.0	+0.5	+0.8	+1.8	−6.3	+0.6	−5.6	−4.6
Average	+3.3	+1.2	+1.2	+2.2	−4.9	−2.0	−2.0	−3.1
Average deviation	2.6	1.1	0.8	2.0	2.4	2.2	1.5	1.2
Weighted average	+2.3	+0.9	+1.0	+1.6	−4.5	−1.8	−1.8	−3.2
Average interval (mos.)	*5.3*	*9.8*	*9.8*	*5.3*	*2.8*	*4.7*	*4.7*	*2.8*

$$\frac{\text{Duration of irregular movements}}{\text{Duration of specific cycles}} = \frac{16.5}{370} = 4.5 \text{ per cent}$$

Average is the arithmetic mean; the average deviation is measured from the mean.

*Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average, the average deviation, and the average interval.

The figures in Table S5 are obtained by dividing the differences between successive entries in Table S4 by the number of months from the middle of one stage to the middle of the next stage. The calculation of these intervals is illustrated in the last column of Table 35. Whether one, two or three months are used to represent the standing in stage V, the intervals between stages IV and V and between stages V and VI are computed as if the standing in stage V were in the center of the peak month. A like remark applies to stage I (or IX) in relation to adjacent stages. Where the standings of an expansion are interpolated, the average rate of change is computed from stage I to III and from stage III to V; the first represents the change from stage I to II and from stage II to III; the second represents the change from stage III to IV and from IV to V. Contractions are handled on the same principle. The rates of change computed from interpolated values are placed in parentheses, so that they may be easily 'spotted'. There are no instances of this type in coal production.

The average rates of change at the bottom of Table S5 are first shown in unweighted form; that is, the rate of change from stage to stage of every cycle is allowed to count the same regardless of the interval to which it applies. Since the rates of change computed from interpolated values are apt to be of extreme size relatively to corresponding figures in longer cycles, we exclude them as a rule from the averages. But the averages may be distorted also by rates of change during phases that are sufficiently long not to require interpolation, yet too short to reduce erratic movements effectively. One method of handling this difficulty is to omit the 'extreme' items from the averages. Another and perhaps less arbitrary method is to weight the rates of change between successive standings by the intervals to which they apply. Weighted averages are therefore included in Table S5. They are derived, in effect, from the average standings at the bottom of Table S4 and the average intervals at the bottom of Table S5. Thus the excess of the average standing in stage II over the average in stage I, divided by the average interval between these stages, is equivalent to an arithmetic mean of the rates of change in successive cycles weighted by the intervals to which the rates apply; and so on from stage to stage.[32] These weighted averages correspond precisely to the slopes of our graphic pictures of average specific-cycle patterns, described later in this section.

Since stages I, V and IX overlap adjacent stages, the differences between the standings at the cyclical turns and adjacent standings involve a peculiar weighting of individual months. This difficulty could be avoided by not assigning to the 'expansion' any month included in stages I and V, or to the 'contraction' any month included in stages V and IX. However, the use of overlaps adds two months to each phase; and consequently, the chance of wiping out erratic flutterings is improved, the occasions on which interpolating is needed to get cyclical standings are fewer, and the inaccuracy of describing stages II to IV as successive thirds of expansion and stages VI to VIII as successive thirds of contraction is reduced. The advantages of the overlaps seem to outweigh the disadvantages.[33] In practice the two plans yield closely similar cyclical patterns.

The manner in which our method of replacing the full set of monthly values for a cycle by nine average standings irons out erratic movements and discloses the form of the successive cycles is brought out vividly by Chart 14. Curve A shows the original monthly figures of coal production in the United States from 1905 to 1939, and curve B shows these

[32] Let the standings of successive cycles in a given stage be $a_1, a_2, \ldots a_N$, the standings in the next stage $b_1, b_2, \ldots b_N$, and the intervals between the midpoints of the adjacent stages $d_1, d_2, \ldots d_N$. Then the rates of change from one stage to the next in successive cycles are $\frac{b_1 - a_1}{d_1}, \frac{b_2 - a_2}{d_2}$, and so on. If these rates are weighted by the corresponding durations, the weighted average is $\frac{\Sigma b - \Sigma a}{\Sigma d}$, or $\frac{\frac{\Sigma b}{N} - \frac{\Sigma a}{N}}{\frac{\Sigma d}{N}}$. In practice, we derive the weighted averages from sums instead of means.

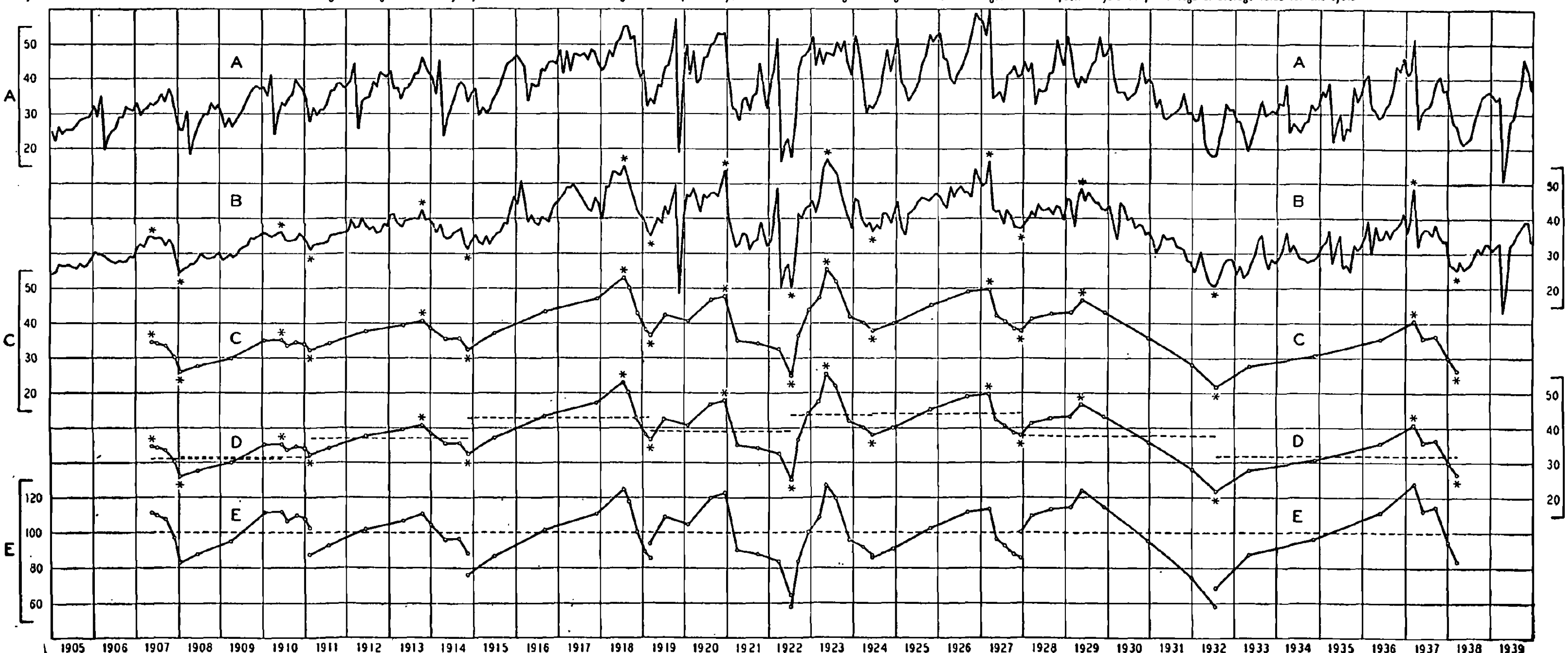
[33] If the overlaps were eliminated, cyclical patterns made from monthly data would be more nearly comparable with patterns made from quarterly data than at present. But our statistical analysis is designed primarily for monthly data, and the analysis of quarterly and anual data adapted as well as may be to the monthly model. See below, pp. 199, 249-50.

CHART 14

Derivation of Specific-cycle Patterns

Bituminous Coal Production, United States, 1905 – 1939

A Original monthly data
B Seasonally adjusted monthly data
C Average standing of seasonally adjusted data in successive stages of the specific cycles
D Step-line of trend and average standing in successive stages of specific cycles
E Average standing in successive stages of each specific cycle as percentage of average value for the cycle



Scale of A, B, C and D in millions of short tons; scale of E in specific-cycle relatives. Asterisks identify peaks and troughs of specific cycles. The contraction May 1907 – Jan. 1908 is treated as a phase of the inverted cycle from May 1907 to June 1910.

figures adjusted for seasonal variations. Curve C, derived from the seasonally adjusted figures, shows the patterns of successive specific cycles expressed in units of the original data, that is, before adjustment for the changing level of the specific cycles. Curve D is the same as C but is cut across by horizontal lines representing the mean levels of successive specific cycles, and curve E shows the cyclical patterns after elimination of the inter-cycle trend—the standard form in which we put the results for practically all series.

When the amplitudes of erratic movements are modest compared with those of specific cycles, they tend to disappear from view when the full set of monthly values within a cycle is replaced by nine average standings. But when the erratic movements are relatively large, they are likely to leave their stamp visibly on the patterns of individual cycles, sometimes even on the average patterns. Chart 14 indicates that the erratic movements in bituminous coal production are removed in large part, but not entirely. In four specific cycles the rise is continuous from stage I to V and the decline is continuous from stage V to IX; each of the other four cycles shows one irregularity. Any reversal of direction within a phase—that is, a decline between stage I and V, or rise between stage V and IX—is an irregular movement. Such reversals of direction may be due to the failure of erratic movements to cancel out, or to recognition of one specific cycle when two should have been taken, which can happen when erratic movements obscure the specific cycles. Therefore a simple though indirect method of judging the amplitude of the erratic movements in a series relatively to the amplitude of its specific cycles is to compare the number of irregularities in its stage-to-stage movements with the total number of stage-to-stage intervals; or better, the total duration of the intervals having irregular signs with the total duration of all the specific cycles. We do the latter and enter the result for each series at the bottom of Table S5.[34]

The qualitative descriptions of erratic movements noted in Section I of Chapter 4 are derived largely from these percentages. Four classes of percentages are distinguished: 0 to 2.0 is taken to indicate 'mild' erratic movements, 2.1 to 6.0 'moderate', 6.1 to 10.0 'pronounced', and over 10.0

[34] The result for coal production is obtained as follows:

List of irregular movements	*Interval in months*
Stage VI to VII, cycle 1908–11	2.5
Stage VII to VIII, cycle 1911–14	4.0
Stage II to III, cycle 1919–22	6.5
Stage VI to VII, cycle 1932–38	3.5

The total duration of intervals showing irregular signs is 16.5 months. The total duration of specific cycles from May 1907 to March 1938 is 370 months. The first figure is 4.5 per cent of the second.

A slight change between consecutive stages in Table S4 may be lost by rounding in Table S5. Hence, if a zero appears in Table S5, it is necessary to refer to Table S4 to make sure whether a sign cannot be determined.

'very pronounced'. Descriptions carried through on this plan are then checked against independent judgments made from the data charts, and frequently revised in the light of the latter. Although the method is very rough,[35] it helps us appraise the analysis of specific cycles. More confidence can be placed in the measures for specific cycles when the erratic movements are set down as mild than when they are set down as pronounced or very pronounced. These descriptions are included in our worksheets for each series, which list in detail any peculiarities of the data or doubtful features of the analysis.

The patterns developed in Chart 14 are rearranged in Chart 15 to bring out the significance of the average pattern of all the cycles in a series. The vertical scale runs in units of cycle relatives, the horizontal scale in units of time, as in Chart 14. The patterns of the single cycles are plotted so that each peak (standing in stage V) is directly under the preceding one. Since the durations of the cycles vary, the standings at other stages are out of alignment; but they can be readily identified from the dots representing the successive standings. The average specific-cycle pattern at the bottom of the chart is drawn from the average standings in Table S4 and the average intervals between successive stages in Table S5. This pattern gives a composite photograph of the durations, amplitudes and stage-to-stage changes of the individual specific cycles. As is to be expected, it is considerably smoother than the patterns of most individual cycles. Its slope represents the average rate of change from stage to stage of the cycles weighted by the intervals between the stages, and thus corresponds to the weighted averages in Table S5.[36] But the average pattern gives no heed to the sequence in which the cycles actually occurred. This disregard of the historical succession of the cycles is implicit in all the averages we strike of cyclical measures: it is a crucial feature of our technique and raises questions of great importance to which Chapters 9-12 are devoted.

Chart 16 shows our standard method of summarizing graphically some leading features of the specific cycles of a series. The diagram for coal production is the same as that at the bottom of Chart 15 except for additional detail. Several other series are included in the chart to give a glimpse of the variety of cyclical patterns found in economic activities.[37]

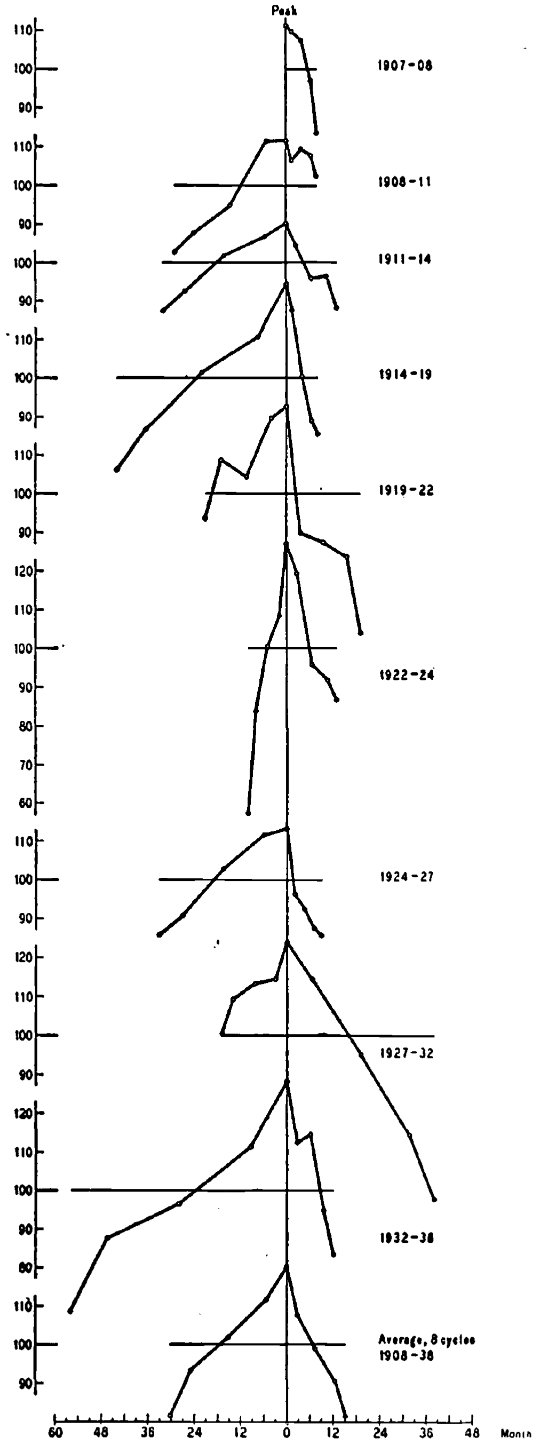
35 Not all irregularities of sign are due to erratic movements. If a series flattens out some months before it reaches a specific-cycle peak and falls sharply in the month following the turn, the standing in stage V may be lower than in stage IV; for similar reasons the standing in stage IX may be higher than in stage VIII. On the other hand, absence of irregular signs need not mean that erratic movements are slight. For example, if a declining segment of a highly choppy series is mistakenly treated as a specific-cycle contraction, there may be a continuous decline from stage to stage of this false contraction. In general, the longer the phase of specific cycles the more effectively will the cyclical patterns wipe out erratic movements; our method takes this principle into account, though very imperfectly.

36 See note 32.

37 For several of these series, the patterns of individual cycles are shown in Appendix Table B1.

CHART 13

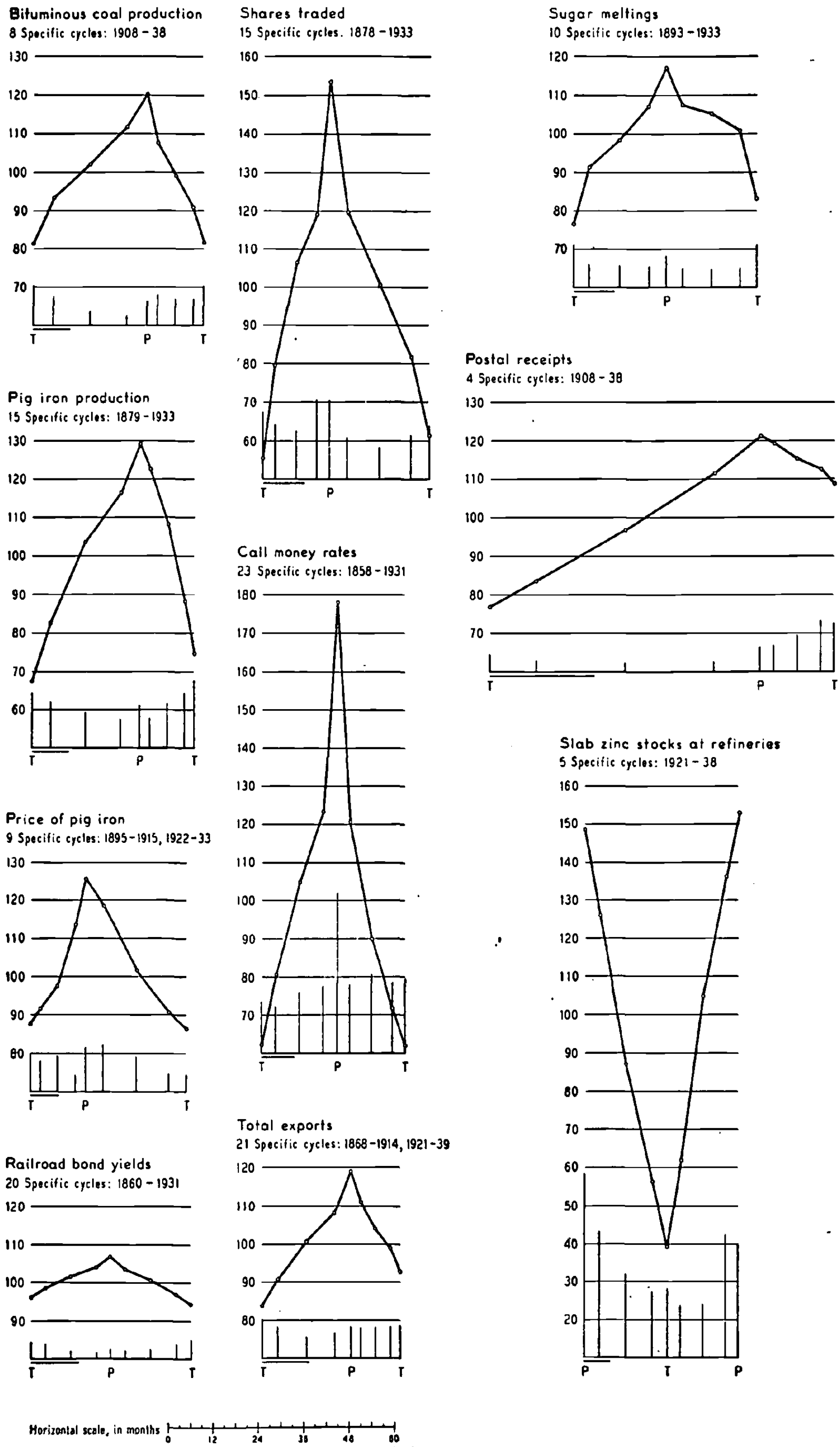
Patterns of Successive Specific Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1907 – 1938



See Chart 14 and explanations in text.

CHART 16

Average Specific-cycle Patterns of Ten American Series



See explanations in text. For sources of data, and other notes, see Appendix C.

The long horizontal line at the bottom of each pattern represents the average duration of the specific cycles; the short horizontal line below it represents the average deviation of the cycle durations. The distances we take to represent time are shown by a ruler, which furnishes a scale for approximating all measurements of duration. The vertical scale at the left runs in units of cycle relatives; with its aid the successive standings of a cyclical pattern and its amplitude can be read. The horizontal line indicating the average duration of the specific cycles is broken by vertical lines erected at the midpoints of the successive cycle stages. The midpoint of the trough stage is marked by 'T' and of the peak stage by 'P' in order that the eye may quickly find these stages. The lengths of the nine vertical lines represent average deviations from the average standings at successive stages of the specific cycles.[38]

When numerous patterns have been worked out, they give lively impressions of the variety of cyclical behavior characteristic of economic processes. Among these patterns we can distinguish various types, and classify the series according to the duration and amplitude of their cyclical movements, the relative magnitude of the average deviations, the rate of change during successive stages of their cycles, the peakedness of the patterns, the presence or absence of 'saw teeth' in the patterns, as well as their positive or inverted shapes.[39] Chart 16 suggests, and our later monographs will demonstrate in detail, that there is little justification for the common notion that cyclical patterns of different activities vary merely in amplitude, or that a sine curve is a satisfactory 'approximation' or 'model' of the specific cycles found in experience.

The patterns in Chart 16 show besides the changing pace of the average rise and fall of the specific cycles, the durations of the expansions and contractions, their amplitude, and the intra-cycle trend. The latter features are standardized in Chart 17, which isolates the information that the cyclical patterns add to the average measures described in preceding sections. The new chart shows merely the rates of change from stage to stage of an average expansion relatively to one another, as well as the rates of change from stage to stage of an average contraction relatively to one another. The computations necessary to make the chart are illustrated in Table 38. The adjusted durations in column (8) are plotted along the horizontal axis, and the adjusted standings in column (4) are plotted along the vertical axis. A similar procedure is used when the analysis is inverted. Here also the expansion is treated separately from the contraction; but in making the adjustments the average standing in stage V is subtracted from the standings in other stages, since the trough now comes in stage V and the peak in stages I and IX.[40]

38 See in this connection the description of specific-cycle patterns in Chart 2.

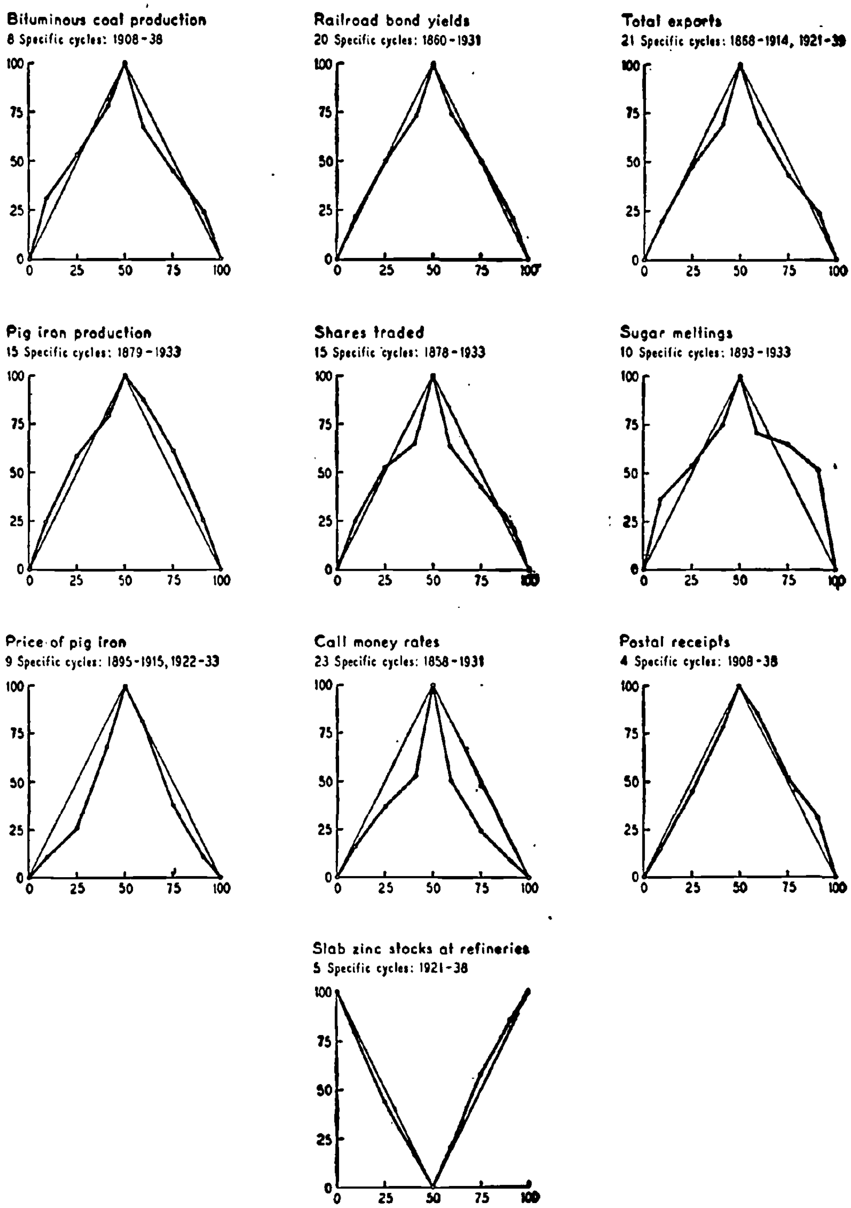
39 Note carefully, however, the last two paragraphs in Sec. I.

40 The principal features of this chart were suggested by Geoffrey H. Moore.

CHART 17

Variation of Average Rates of Change from Stage to Stage of Expansions and Contractions of Specific Cycles

Ten American Series



To facilitate study, the peaks and troughs are joined by straight lines representing uniform rates of expansion and contraction. See explanations in text and Table 38.
For sources of data and other notes, see Appendix C.

TABLE 38

Adjustment of Average Specific-cycle Pattern to Show Relative Variation of Rates of Change from Stage to Stage of Expansion and Contraction Bituminous Coal Production, United States, 1908–1938

Expansion of specific cycles							
Stage	Average standing (see Table 34)	Excess over stage I		Average interval in months from		Average interval from stage I as origin, as % of twice the duration of expansion	Adjusted interval from stage I as origin (same as col. 7)
		Absolute amount	As % of excess for stage V	Preceding stage (see Table 37)	Stage I as origin (cumulatives of col. 5)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	81.4	0.0	0.0	...	0.0	0.0	0.0
II	93.3	11.9	30.7	5.3	5.3	8.8	8.8
III	101.9	20.5	53.0	9.8	15.1	25.0	25.0
IV	111.7	30.3	78.3	9.8	24.9	41.2	41.2
V	120.1	38.7	100.0	5.3	30.2	50.0	50.0

Contraction of specific cycles							
Stage	Average standing (see Table 34)	Excess over stage IX		Average interval in months from		Average interval from stage V as origin, as % of twice the duration of contraction	Adjusted interval from stage I as origin (add 50 to col. 7)
		Absolute amount	As % of excess for stage V	Preceding stage (see Table 37)	Stage V as origin (cumulatives of col. 5)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
V	120.1	38.5	100.0	...	0.0	0.0	50.0
VI	107.5	25.9	67.3	2.8	2.8	9.3	59.3
VII	98.8	17.2	44.7	4.7	7.5	25.0	75.0
VIII	90.6	9.0	23.4	4.7	12.2	40.7	90.7
IX	81.6	0.0	0.0	2.8	15.0	50.0	100.0

The adjusted patterns are of uniform amplitude and have no tilt. In series treated on a positive plan the pattern starts at 0 in stage I, rises to 100 in stage V, and falls to 0 in stage IX; in inverted analyses the movement is from 100 to 0 to 100. The durations are also uniform: the expansion runs from 0 to 50 and the contraction from 50 to 100. The adjusted patterns can therefore differ only in their slopes. These correspond precisely to the slopes in Chart 16, in the sense that whatever set of numbers is formed by the slopes of the latter during an average expansion or contraction, taken separately, precisely the same set of numbers multiplied by some constant is formed by the slopes of Chart 17; so that the *relative* variation of the slopes during expansion or contraction is preserved. A glance at Chart 17 reveals whether the average expansion of a series has been characterized by a steady, increasing or decreasing rate of rise, how the rate of decline has behaved on the average during contractions, and the manner and degree in which the pace of expansions or contractions has varied in different series. It is clear, for example, that neither the production nor price of pig iron has moved at a steady average pace during the expansions and contractions of specific cycles; but whereas three-fifths of the rise in production came in the first half of the expansion and three-fifths of the decline in the last half of contraction, in prices only a fourth of the rise was accomplished from the trough to mid-expansion and less than two-fifths of the decline was accomplished from mid-contraction to the trough.

Our methods of analyzing cyclical patterns are far from perfect. We hope that they may excite the reader sufficiently to undertake fresh experiments. One troublesome question is whether the patterns give sufficient detail, particularly in the neighborhood of the turning points. This question is briefly explored in a later chapter. Two related issues are also considered there: whether three-month averages are satisfactory representatives of cyclical peaks and troughs, and whether the results could be improved by preliminary smoothing of the data.[41]

VII Reference-cycle Patterns

The 'S' tables do not show how different economic activities behave at one and the same time. Were our analysis confined to these measures, we would be left with vague notions concerning the relations in time of the fluctuations of different series. Study of the leads and lags in Table S1 would demonstrate that certain series agree more or less closely in timing with the reference turns, and differ by more or less regular intervals from certain other series. But if business cycles really are units of concurrent fluctuations in many activities, the critical point to establish is how different activities fluctuate during fixed periods. To that end, we transfer measurements of cyclical behavior from specific cycles, which vary in timing from series to series, to reference cycles, which occupy uniform periods in all series for a given country.

The behavior of each series during the periods occupied by successive business cycles is shown in a set of 'R' tables. These are a slighter affair than the 'S' tables, because some of the essential comparisons between specific and business cycles have already been made, and because certain measurements of specific cycles need no counterparts in reference cycles. Thus the leads and lags and the duration measurements of Table S1 have compared the timing of specific with that of business cycles. To measure the full amplitudes of cyclical waves, we must have the cyclical peaks and troughs touched by each series at whatever times these turning points are reached—the procedure followed in Table S2. Nor can we measure secular changes for our purpose any better than in Table S3, so long as we center attention on the cyclical behavior of different activities, one by one. Hence we make the 'R' tables in forms corresponding only to Tables S4 and S5, and add two sections measuring the conformity of the series to business cycles.

41 See Ch. 8, Sec. IV and VI.

TABLE 39

Sample of Table R1: Reference-cycle Patterns
Bituminous Coal Production, United States, 1905–1938

Dates of reference cycles	Average in reference-cycle relatives at stage									Average monthly standing during cycle (million short tons)
	I	II	III	IV	V	VI	VII	VIII	IX	
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough	
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Aug. 04–May 07–June 08	...	...	97.2[a]	103.2[a]	119.9[a]	116.6[a]	102.9[a]	91.2[a]	98.4[a]	28.8[b]
June 08–Jan. 10–Jan. 12	86.6	88.9	89.4	103.2	108.7	105.9	101.8	105.4	113.9	32.7
Jan. 12–Jan. 13–Dec. 14	99.2	102.7	99.3	98.4	106.4	104.6	102.6	91.8	88.5	37.6
Dec. 14–Aug. 18–Apr. 19	77.9	89.4	100.9	111.2	121.1	109.7	95.6	83.2	88.3	42.7
Apr. 19–Jan. 20–Sep. 21	91.8	99.0	105.4	85.7	106.0	111.6	111.1	81.1	86.1	41.1
Sep. 21–May 23–July 24	87.9	95.0	71.5	113.1	137.5	129.0	105.4	94.2	92.7	40.2
July 24–Oct. 26–Dec. 27	84.5	90.3	102.1	106.0	111.8	116.2	100.5	89.7	85.4	44.2
Dec. 27–June 29–Mar. 33	104.2	114.3	117.9	121.6	130.2	115.6	92.2	70.9	68.1	36.2
Mar. 33–May 37–May 38	76.3	92.7	93.2	115.2	108.8	114.5	105.6	82.2	82.0	32.3
Average	88.6	96.5	97.5	106.8	116.3	113.4	101.8	87.3	88.1	...
Average deviation	7.4	6.6	9.6	8.5	10.0	5.4	4.3	8.0	7.7	...

$$\frac{\text{Average rise \& fall of reference cycles}}{\text{Average rise \& fall of specific cycles}} = \frac{56.0}{77.2} = 73 \text{ per cent}$$

Average is the arithmetic mean; average deviation is measured from the mean.

[a] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average and the average deviation.

[b] Incomplete cycle; data start in Jan. 1905.

Table 39 presents a sample of Table R1, which parallels Table S4 in every respect except for the insertion of cycle bases.[42] But now the dates of troughs and peaks are taken from the standard list of reference dates instead of from the turning points of specific cycles, and the standings during successive stages are expressed in reference-cycle relatives instead of specific-cycle relatives. The reference cycles are divided into nine stages in exactly the same way as are the specific cycles.[43] As in the specific-cycle patterns, the standing at the terminal trough in one reference cycle is rarely the same as the standing at the initial trough in the next cycle; for although the standings are made from the same items, the base of the former is the monthly average during that reference cycle, while the base of the latter is the monthly average during the following reference cycle. The successive reference-cycle stages in Table R1 also are designated by Roman numerals from I to IX; V refers to the reference peak. But the standings in stages I, V and IX invariably cover three months, whereas they are sometimes based upon one or two months in the specific cycles. The nine standings within a reference cycle are called the 'reference-cycle pattern'.

[42] The specific-cycle bases are given in Table S3.

[43] Appendix Table A1 shows in full the division of successive reference cycles in the United States, Great Britain, Germany and France, when the analysis is based upon monthly data.

Table 40 illustrates the calculation in detail, and Charts 18-19 show the process graphically. The table and charts are precise analogues of Table 35 and Charts 14-15 for specific cycles. As said before, it is easier to compute the average standing of relatives in a cycle-stage by taking an average of the original figures and converting it to a relative than by computing relatives for individual months and then averaging them; hence the computations in Table 40 are shown on the former plan. Indeed, there is a slightly better method of calculation, though its meaning may be less obvious at first sight. Let T be the total of the monthly values included in a stage, M the number of months in the stage, S the total of the monthly values in the cycle,[44] and N the number of months in the cycle. Then the standing in a reference-cycle stage is given by $\frac{TN}{MS}$. Of course, $\frac{N}{S}$, the reciprocal of the cycle base, is a constant for a given cycle; hence the most convenient formula is $\frac{TV}{M}$, where $V = \frac{N}{S}$.

Since most of our time series do not start at the very trough of a reference cycle, they cover only part of a reference cycle at the beginning. It is desirable to utilize information on even a fraction of a cycle, especially in brief series. We handle fractional cycles as follows. If only one or two stages are missing, as in the reference cycle from 1904 to 1908 in coal production, the standings in the stages covered by the data are expressed as relatives on the base of the incomplete cycle. If more than two stages are missing, the standings in the stages covered by the data are computed on the base of the nearest complete reference cycle, whether marked off by peaks or by troughs. These standings are recorded in Table R1, but are not included in the averages.

Since the aim of Table R1 is simply to show how different series behave during the same periods, it is not necessary to distinguish between series that trace out specific cycles and those that do not. A series that appears to have no specific cycles, and for which therefore no 'S' Tables are made, can be chopped into reference-cycle segments as readily as any other. These segments can be presented in Table R1 and scrutinized to see what 'response' if any the series makes, or what relation it bears, to business cycles. Nor is it necessary to distinguish between positive and inverted analyses in making Table R1; although the decision to analyze *specific cycles* on a positive or inverted basis is actually based on a study of Table R1, which comes first in the order of computation.

Table R1 is therefore computed for every series and in every series the computation is made on a positive plan; that is, every series is broken into segments running from one reference trough to the next. Business-cycle units could be delimited in other ways: for example, from one

[44] As in the specific cycles, this includes the values from the initial to the terminal trough, but only half the value at the troughs.

TABLE 40

The Computation of Reference-cycle Patterns Illustrated
Bituminous Coal Production, United States, 1908–1919

Cycle (1)	Stage (2)	Period covered (3)	No. of months (4)	Average monthly standing in: Millions of short tons (5)	Average monthly standing in: Cycle relatives (6)	Interval from stage to stage (mos.) (7)
June 1908–Jan. 1912 (Av. monthly value = 32.740 million short tons)	I	May 1908 – July 1908	3	28.367	86.6	...
	II	July 1908 – Dec. 1908	6	29.100	88.9	3.5
	III	Jan. 1909 – June 1909	6	29.283	89.4	6.0
	IV	July 1909 – Dec. 1909	6	33.800	103.2	6.0
	V	Dec. 1909 – Feb. 1910	3	35.600	108.7	3.5
	VI	Feb. 1910 – Sep. 1910	8	34.675	105.9	4.5
	VII	Oct. 1910 – Apr. 1911	7	33.329	101.8	7.5
	VIII	May 1911 – Dec. 1911	8	34.500	105.4	7.5
	IX	Dec. 1911 – Feb. 1912	3	37.300	113.9	4.5
Jan. 1912–Dec. 1914 (Av. monthly value = 37.611 million short tons)	I	Dec. 1911 – Feb. 1912	3	37.300	99.2	...
	II	Feb. 1912 – May 1912	4	38.625	102.7	2.5
	III	June 1912 – Aug. 1912	3	37.333	99.3	3.5
	IV	Sep. 1912 – Dec. 1912	4	37.000	98.4	3.5
	V	Dec. 1912 – Feb. 1913	3	40.000	106.4	2.5
	VI	Feb. 1913 – Aug. 1913	7	39.343	104.6	4.0
	VII	Sep. 1913 – Apr. 1914	8	38.600	102.6	7.5
	VIII	May 1914 – Nov. 1914	7	34.529	91.8	7.5
	IX	Nov. 1914 – Jan. 1915	3	33.300	88.5	4.0
Dec. 1914–Apr. 1919 (Av. monthly value = 42.746 million short tons)	I	Nov. 1914 – Jan. 1915	3	33.300	77.9	...
	II	Jan. 1915 – Feb. 1916	14	38.229	89.4	7.5
	III	Mar. 1916 – May 1917	15	43.127	100.9	14.5
	IV	June 1917 – July 1918	14	47.529	111.2	14.5
	V	July 1918 – Sep. 1918	3	51.767	121.1	7.5
	VI	Sep. 1918 – Oct. 1918	2	46.900	109.7	1.5
	VII	Nov. 1918 – Jan. 1919	3	40.867	95.6	2.5
	VIII	Feb. 1919 – Mar. 1919	2	35.550	83.2	2.5
	IX	Mar. 1919 – May 1919	3	37.733	88.3	1.5

The illustration covers the first three full reference cycles in coal production; see Table 39.

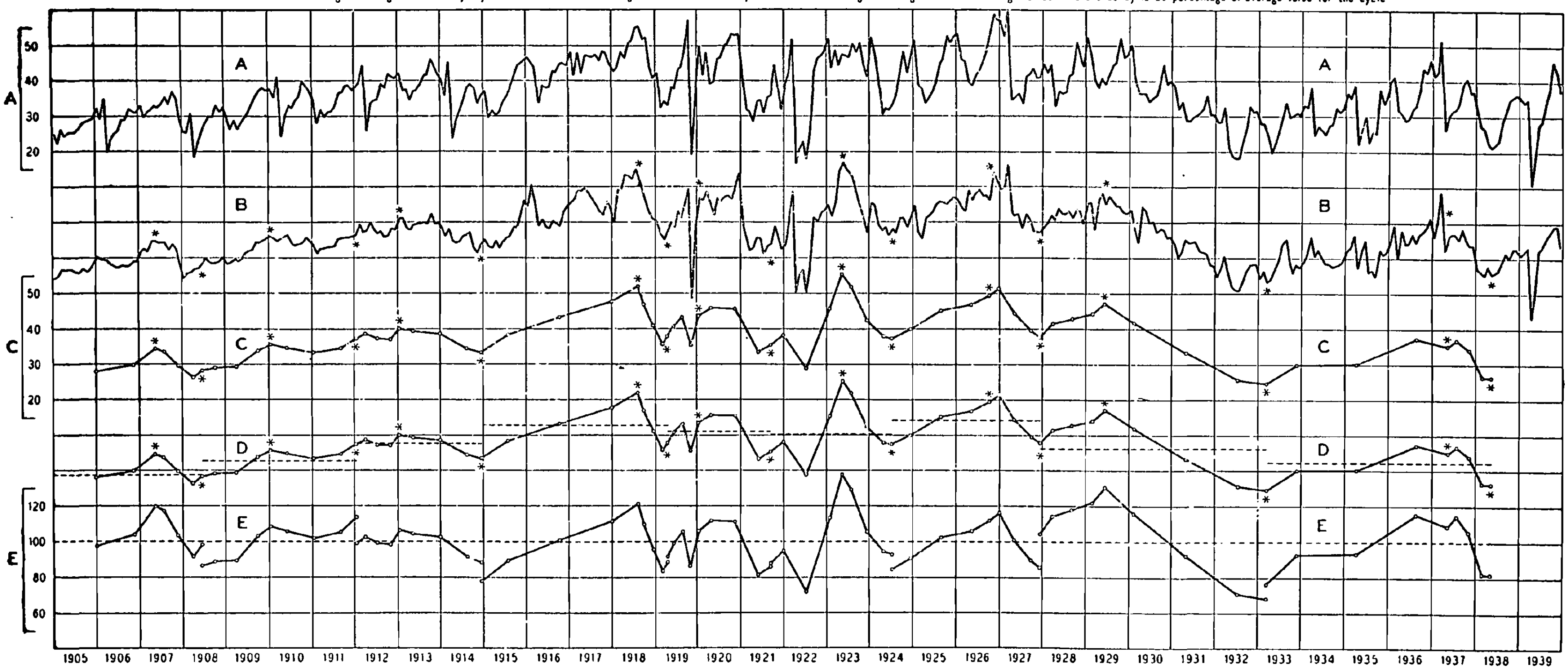
reference peak to the next, or from the midpoint of one reference expansion to the midpoint of the next. Our reason for making Table R1 on a positive plan is merely that this method is more familiar than any other; it involves no theoretical assumptions such as may have influenced other investigators who have also considered business cycles as units running from trough to trough. True, the developments during the transition from expansion to contraction of business cycles can be traced more easily from patterns made on a positive basis than from patterns made on an inverted basis. But just as clearly patterns on an inverted basis facilitate analysis of business-cycle revivals, and that is one reason among others why it is sometimes desirable to put the patterns in this form.[45]

[45] See below, Ch. 11, Sec. V and Ch. 12, Sec. I. In Appendix B we show individual reference-cycle patterns on both a positive and inverted basis for several series; see Tables B3 and B4.

CHART 18

Derivation of Reference-cycle Patterns
Bituminous Coal Production, United States, 1905 - 1939

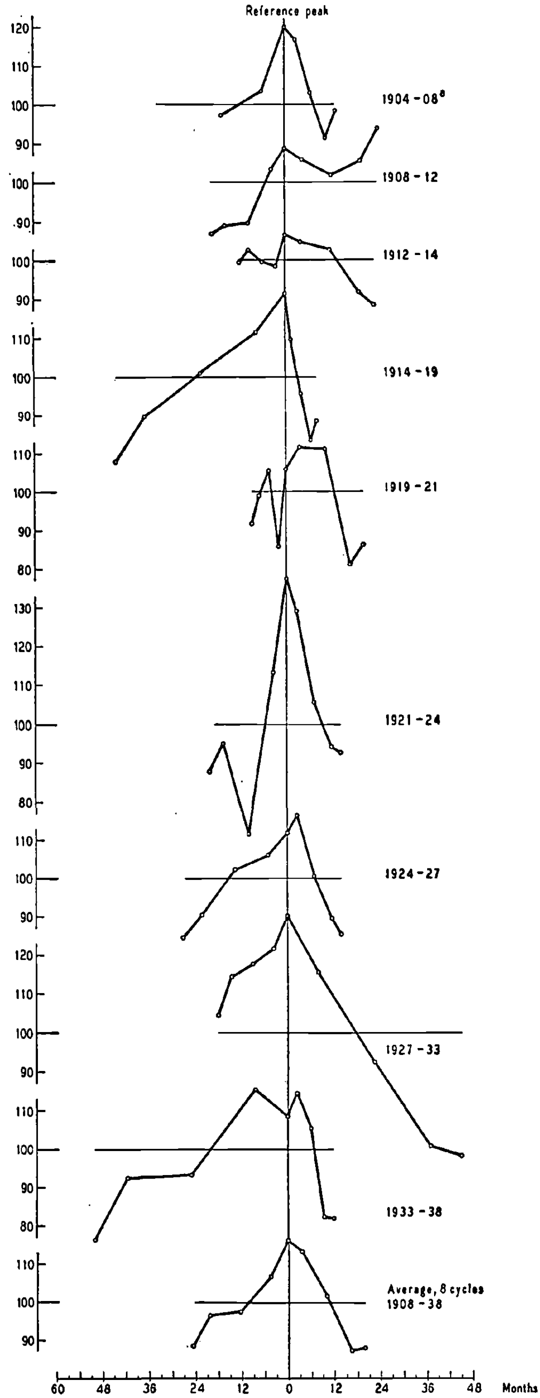
A Original monthly data
B Seasonally adjusted monthly data
C Average standing of seasonally adjusted data in successive stages of the reference cycles
D Step-line of trend and average standing in successive stages of reference cycles
E Average standing in successive stages of each reference cycle as percentage of average value for the cycle



Scale of A, B, C and D in millions of short tons; scale of E in reference-cycle relatives. Asterisks identify peaks and troughs of reference cycles. The pattern of the 1904-08 cycle is incomplete, data not being available.

CHART 19

Patterns of Successive Reference Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1905 – 1938



See Chart 18.

[a] The pattern is incomplete, data not being available.

Inverted average patterns may be derived by a fresh computation along the lines described for positive patterns, or approximated by a simple transformation of the positive patterns.[46] If the average standings in stages I and IX are the same, the inverted pattern can be estimated by accepting the values of the positive pattern, beginning in stage V and going the full round to stage V again. If the average standings in stages I and IX differ, as they usually do, a tolerable approximation can be achieved by using the average of the standings in stages I and IX to represent the trough, then adjusting the standings in stages V-IX by the ratio of this average to the standing in stage IX, and adjusting the standings in stages I-V by the ratio of this average to the standing in stage I.[47]

In the few series having plus and minus values, it is not feasible to work with reference-cycle relatives. But series of this type can still be broken into reference-cycle segments and presented in Table R1. The standings in successive stages of each cycle may then be expressed as simple averages of the original data, or as plus or minus deviations from the average value during the cycle.[48] The latter plan resembles our standard method, since it involves an adjustment for the changing level of successive reference cycles. But the absolute level of a series is frequently of considerable interest.[49] For example, in analyzing the relations among different types of interest rates during a business cycle, it is desirable to know not only the percentage variations of each about its average during the cycle, but also whether and how much one type of interest rate is above or below another. Hence we sometimes find it profitable to express the standings of a series in successive stages of business

46 The term 'inverted pattern', when applied to reference cycles, is slippery; it may refer to the shape of the pattern or to the plan on which Table R1 is computed. Thus the pattern of zinc stocks (Chart 20) is inverted in shape, though made on a positive plan. The statement in the text refers to the plan of computation solely.

47 This transformation will rarely give exactly the same results as direct calculation, for the following reasons. (1) The period covered by inverted cycles must differ somewhat from that covered by positive cycles, since the former begin and end with a reference peak while the latter begin and end with a trough. (2) The cycle bases and therefore also the cycle relatives vary according as the cycles are marked off by troughs or peaks; see Ch. 12, note 3.

It might seem that the estimated pattern should be adjusted so that the average of its nine average standings equals 100. But this average may differ from 100 even when a pattern is computed directly. To explain: In a single cycle the average of the nine average standings would necessarily equal 100 if (a) the standings at reference peaks and troughs were represented by single months, (b) the standings at the troughs (assuming positive analysis) were weighted one-half each, (c) the other standings were weighted by the number of months they cover. For a group of cycles an additional condition would be necessary in order that the average of the nine average standings equal 100: the average for each stage must be a weighted average, where the weights are the periods covered by the stage in successive cycles.

48 The difficulty of plus and minus values may be circumvented by ranking the nine standings within each reference cycle, then averaging the ranks for each stage to obtain the average pattern. This plan of computation is unsatisfactory for most purposes, because it brushes aside differences in amplitude.

49 That is our reason for showing the reference-cycle bases in Table R1.

cycles in the form of simple averages of the original data, not only when the series contains plus and minus values, but also when it consists of positive values throughout.

Table R2 elaborates upon Table R1: that is, it records the figures obtained by dividing the differences between successive stages in Table R1 by the number of months between the middle of one stage and the middle of the next. All that has been said about the relations between Tables R1 and S4 might be rephrased to apply to Tables R2 and S5. Also, Table R2 supplements R1 in the same way that Table S5 supplements S4. The rates of change per month in Table R2 are a highly valuable device for examining the influence of business cycles and random factors upon different business processes. For example, coal production rose in every business cycle from the trough to the first third of expansion; in the middle stages of expansion the rise tended to slacken; there was a fresh burst of activity from the last third of expansion to the peak, and sometimes the rise extended into the first third of contraction; a fairly continuous and vigorous decline followed; but the rate of decline tapered off from the last third of contraction in general business to the trough and in a few of the earlier cycles coal production actually rose during this interval (Table 41).

TABLE 41

Sample of Table R2: Rate of Change from Stage to Stage of Reference Cycles
Bituminous Coal Production, United States, 1905–1938

Dates of reference cycles	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough – Peak – Trough (1)	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
Aug. 04 – May 07 – June 08	...	...	+0.6[a]	+2.8[a]	−1.3[a]	−3.4[a]	−2.9[a]	+2.9[a]
June 08 – Jan. 10 – Jan. 12	+0.7	+0.1	+2.3	+1.6	−0.6	−0.5	+0.5	+1.9
Jan. 12 – Jan. 13 – Dec. 14	+1.4	−1.0	−0.3	+3.2	−0.4	−0.3	−1.4	−0.8
Dec. 14 – Aug. 18 – Apr. 19	+1.5	+0.8	+0.7	+1.3	−7.6	−5.6	−5.0	+3.4
Apr. 19 – Jan. 20 – Sep. 21	+3.6	+2.6	−7.9	+10.2	+1.6	−0.1	−4.6	+1.4
Sep. 21 – May 23 – July 24	+2.0	−3.6	+6.4	+7.0	−3.4	−5.2	−2.5	−0.6
July 24 – Oct. 26 – Dec. 27	+1.2	+1.4	+0.5	+1.2	+1.8	−3.5	−2.4	−1.7
Dec. 27 – June 29 – Mar. 33	+2.9	+0.7	+0.7	+2.5	−1.8	−1.6	−1.5	−0.4
Mar. 33 – May 37 – May 38	+1.9	0.0	+1.3	−0.8	+2.3	−2.5	−6.7	−0.1
Average	+1.9	+0.1	+0.5	+3.3	−1.0	−2.4	−3.0	+0.4
Average deviation	0.7	1.2	2.3	2.7	2.4	1.8	1.9	1.4
Weighted average	+1.8	+0.1	+1.2	+2.1	−0.8	−1.8	−2.3	+0.2
Average interval (mos.)	*4.5*	*7.9*	*7.9*	*4.5*	*3.6*	*6.4*	*6.4*	*3.6*

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average, the average deviation, and the average interval.

Chart 20 shows the average reference-cycle pattern of coal production and of several other American series.[50] The chart is constructed on the same principles as Chart 16 showing specific-cycle patterns,[51] but now the graphs are drawn from the average standings in Table R1 and the average intervals in Table R2. The long horizontal line below each pattern represents the average duration of the business cycles covered by the series, and the shorter horizontal line represents the average deviation of the business-cycle durations. The other features correspond to similar features of Chart 16; for example, the slopes of the average reference-cycle pattern represent weighted averages of the rates of change from one stage of the reference cycles to the next, just as in the specific cycles. The average patterns of most series we have analyzed rise during reference expansions and decline during reference contractions, attesting the existence of business cycles in the sense of our definition. But the patterns of some series rise during contractions though at a retarded rate (e.g., postal receipts), while others move invertedly (zinc stocks) or quasi-invertedly (bond yields), and still others move in haphazard fashion (sugar meltings).

By studying Tables R1 and R2 we can learn in some detail what the behavior of a series has been during successive reference expansions and contractions, and thus supplement the knowledge derived from the timing measures of Table S1, which show merely the timing of the series at reference turns. For example, Table S1 tells us that if we count all turns in the nine specific cycles from 1893 in pig iron production and from 1895 in pig iron prices to World War I and from 1921 or 1922 to 1933, production led the reference troughs by 4.3 months on the average while prices lagged 4.4 months. At reference peaks, production lagged 1.6 months while prices led by 1.7 months. The following summary of Tables R1 and R2, based on the nine reference cycles in 1894–1914 and

	Average in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
Production	73.1	90.5	102.9	116.9	126.1	119.0	96.4	84.5	81.5
Prices	89.5	91.2	94.2	108.9	117.4	114.7	103.2	91.6	86.9

	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
Production	+4.4	+1.9	+2.3	+2.6	−1.6	−3.4	−1.8	−1.0
Prices	+0.2	+0.4	+2.4	+2.0	−0.9	−1.9	−2.0	−1.5

1921–1933, adds significant information about the timing of the advances and declines.[52] Production increased at an extremely fast rate between the trough and the first third of reference expansion, much faster than during

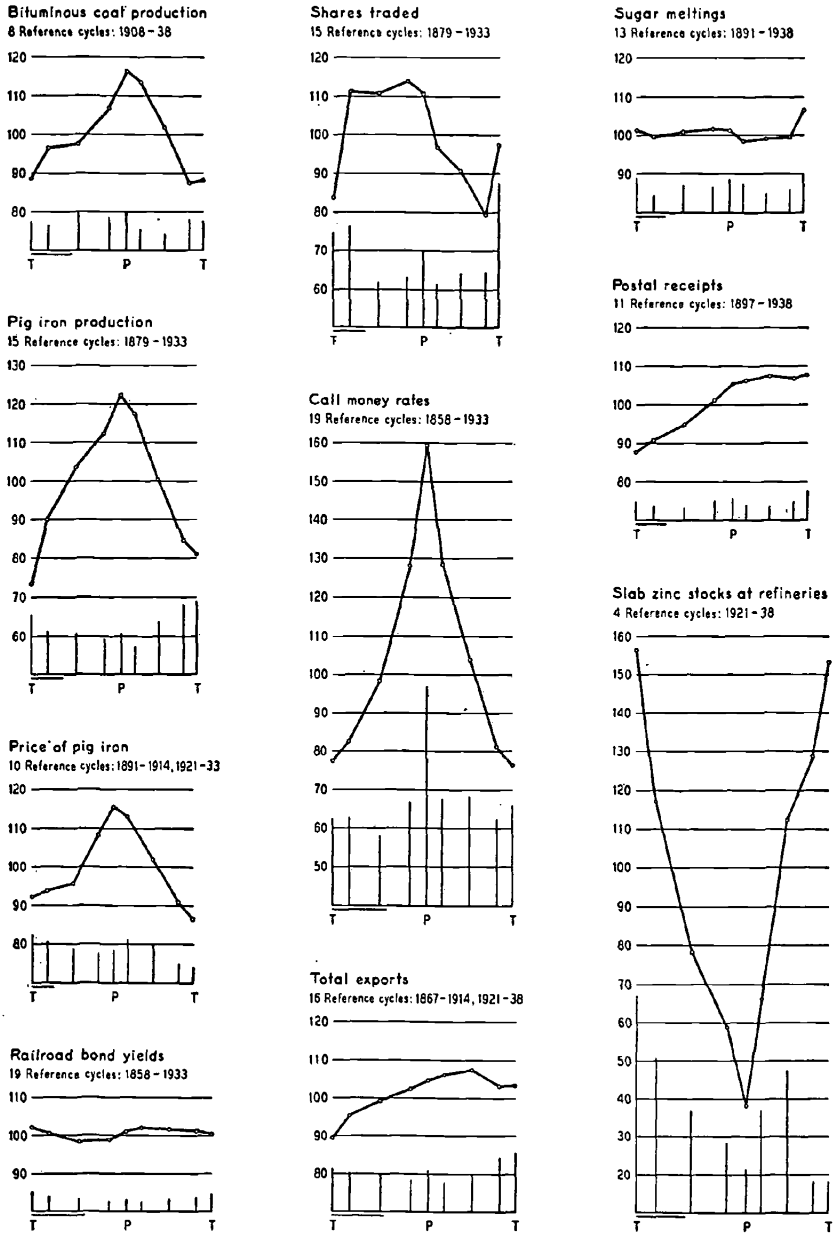
50 For several of these series the patterns of individual cycles are shown in Appendix Table B3.

51 In this connection, see the description of reference-cycle patterns in Chart 2.

52 The rates of change shown are unweighted averages. Weighted averages would demonstrate similar results.

CHART 20

Average Reference-cycle Patterns of Ten American Series



Horizontal scale, in months 0 12 24 36 48 60

See explanations in text. For sources of data and other notes, see Appendix C.

the rest of the expansion. Prices, on the other hand, did not begin to rise appreciably until the middle of the expansion. Both series fell sharply between the first and middle thirds of reference contraction, production much more than prices. After the middle of the reference contraction, the rate of decline abated considerably in production but not in prices. Comparisons of this character for numerous series should give fairly clear pictures of the timing relations among the movements of different processes during the periods when general business activity is experiencing cyclical expansions and contractions.

Tables R1 and R2 provide also a partial check on the timing measures of Table S1. For example, the average reference-cycle pattern of railroad bond yields reaches a trough in stage III and a peak in stage VI. This result is broadly confirmed by Table S1, which shows that bond yields lagged on the average 8 months at reference peaks and 12 months at reference troughs. The average reference-cycle pattern of share trading reveals a pronounced tendency to lead, and this result too is confirmed by Table S1. On the other hand, the patterns of production and prices of pig iron obscure the much smaller departures of these series from the reference dates. Both reach a peak in stage V and a trough in stage IX, although it seems clear from Table S1 that their timing differed significantly: from 1893 to 1933 the cyclical peaks in prices led the peaks in production in eight instances, coincided in three, but lagged in none, while the cyclical troughs in prices lagged behind the troughs in production in ten instances, and coincided in the two remaining instances.[53]

To extract reliable information on cyclical timing, Tables S1, R1 and R2 should be used together. As said before, the measures in Table S1 show the timing of cyclical turns in months, but are surrounded by uncertainty because of the difficulties of dating specific cycles and matching them with business cycles. The measures in Tables R1 and R2 are free from these difficulties, show timing of the rates of change of cyclical movements as well as of their reversals in direction, but indicate timing in coarse fractions of a cyclical phase. The limitations of the two sets of measures to some extent offset one another. The knowledge that can be wrung from time series by checking one set of measures carefully against the other is both more detailed and more reliable than what can be learned by relying on either set exclusively.

VIII Relation between Reference- and Specific-cycle Patterns

To facilitate study of the cyclical measures described so far, we display in a single chart the averages and average deviations of those features of the

[53] For fuller details, see Table 176. That table stops in 1933. It may be well to add here that at the downturn of business activity in 1937, the peak in pig iron prices came 9 months after the downturn in pig iron production. This is the only lag in the record, and may well excite some speculation.

cyclical behavior of a series that lend themselves readily to graphic presentation. Several samples appear in Chart 21. The purpose of this chart is to picture average behavior during the cycles peculiar to a series and during the group of contemporaneous business cycles. Hence we need merely combine the graphs of specific-cycle and reference-cycle patterns already illustrated. But we must ensure that the averages of the specific and reference cycles cover comparable periods, a matter that was of no concern so long as specific and reference cycles were observed separately. And we must also decide how to place the two patterns in relation to each other.

Our method is as follows: If each turning point of the specific cycles is treated in Table S1 as corresponding to a reference turn and every reference turn within the period covered is thus accounted for, the 'duration lines' of the two patterns—that is, the long horizontal lines representing average cyclical duration—are so placed that they show the average lead or lag of the turning dates of the specific cycles compared with the turning dates of business cycles.[54] Arrows are drawn to call attention to the average leads or lags. If the specific cycles have been analyzed on a positive basis, an arrow is drawn from the trough of the average specific-cycle pattern to the trough of the average reference-cycle pattern, and from the peak of the former to the peak of the latter. If the average lead or lag is one month or less, we draw a vertical arrow, which signifies that the timing of specific and business cycles is roughly coincident on the average at that turn. Similarly, if the specific cycles have been analyzed on an inverted basis, an arrow is drawn from the peak of the specific-cycle pattern to the reference trough and from the trough of the specific-cycle pattern to the reference peak. When the specific cycles do not bear a one-to-one correspondence to business cycles, we do not attempt to represent leads or lags on the chart. In these instances, the 'duration lines' of the two patterns are placed so that their average standings in stage V are aligned vertically. In other words, if the specific cycles have been analyzed positively, as in call money rates, the average standings at the peak stage of specific and business cycles are plotted on the same vertical; but if the specific cycles have been analyzed invertedly, as in zinc stocks, the standings at the trough stage of the specific cycles and at the peak stage of business cycles are plotted on the same vertical.

The two patterns are plotted to a common set of vertical scales; hence the average standings of the patterns can be read directly from the scale on the left. The average deviations of the patterns are drawn as vertical lines above and below the curves representing the average patterns. The

[54] The average lead or lag at the initial reference trough need not be the same as the average at the terminal reference trough. For the former is based on the leads or lags at all reference troughs except the last, while the latter includes the leads or lags at all reference troughs except the first. The average lead or lag shown on the chart at the reference peak may also differ from that in Table S1, since the former includes timing comparisons for full specific cycles only.

average deviations of the specific-cycle standings must be read down from the 'duration line' of the specific cycles, which serves as a zero line for this purpose. The average deviations of the reference-cycle standings must be read up from the 'duration line' representing the average duration of the comparable set of business cycles. The short horizontal lines above and below the 'duration lines' represent average deviations of the cyclical durations. The ruler on the chart affords a scale from which all duration measurements can be approximated.

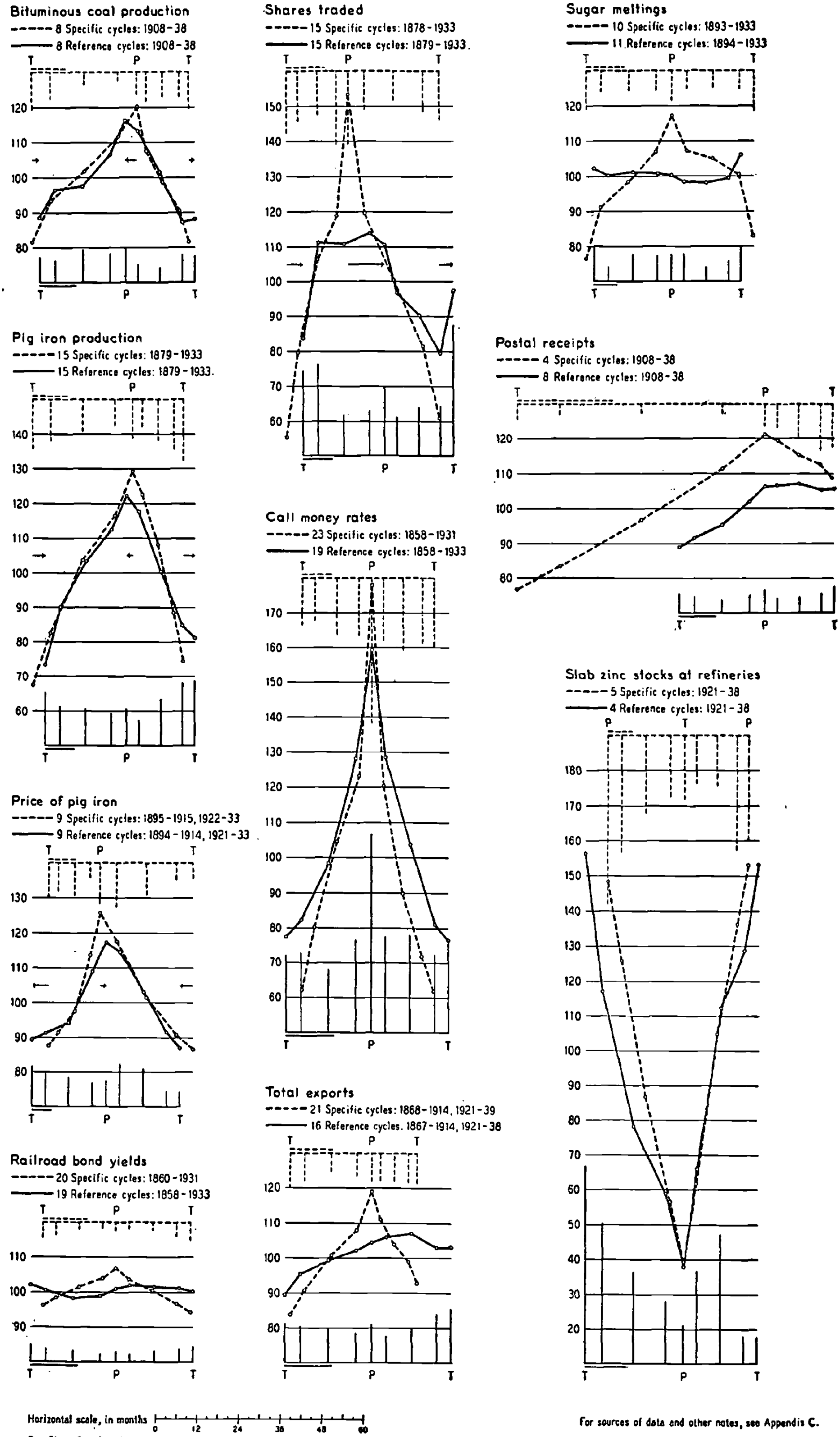
The figures in Chart 21 may at first seem puzzling even to a person who has studied with care the manner in which they are constructed.[55] But the reader will soon learn how to use them if he bears in mind the following points: (1) The patterns indicate average leads or lags at reference turns only when there is a one-to-one correspondence of specific and business cycles. (2) The specific- and reference-cycle patterns cover periods that are as nearly the same as the turning dates of specific and business cycles allow. (3) The representative value of the patterns is indicated by the lines showing average deviations and by the captions stating the number of cycles covered. (4) Even when the number of cycles is 'large' and the average deviations are 'small', the full significance of the patterns cannot be determined without close study of the original data, the arrays from which the patterns are made, and the many measures we compute but do not record on the charts. (5) Differences among cyclical patterns of individual series may reflect persistent differences, or merely special features of the varying periods covered by the series. (6) The amplitude of the reference-cycle pattern relative to that of the specific-cycle pattern provides a quick clue to the relationship in time between a given series and business cycles. If the turning dates of the specific cycles are close to the reference dates, the two patterns will be nearly the same. But if the timing of the specific cycles is fairly independent of that of business cycles, then what may have been a large amplitude in the specific-cycle pattern will be obscured or disappear in the reference-cycle pattern.

The last remark is vital and must be understood by anyone wishing to follow closely the results yielded by our technical apparatus. When we break a series into specific-cycle segments, divide each segment into stages, and average the standings at each stage for all cycles, the cyclical movement of the series is exhibited in full. When this procedure is applied to reference-cycle segments, some portion of the cyclical movement is virtually bound to be erased, for the trough-peak-trough reference dates rarely, if ever, match precisely the trough-peak-trough points in the specific cycles. To be sure, if the specific cycles conform approximately in

[55] See also above, pp. 154-7, 168; and especially the description in Chart 2. The scales in the charts of cyclical patterns in this volume have been kept strictly uniform, so that the features of one chart may be readily compared with the features of any other. For obvious reasons Charts 2 and 50 are exceptions.

CHART 21

Average Cyclical Patterns of Ten American Series



See Chart 2 and explanations in text.

→ Indicates lead at reference peaks, or troughs.

← Indicates lag at reference peaks, or troughs.

For sources of data and other notes, see Appendix C.

timing to business cycles, as in the production of bituminous coal or pig iron, the average patterns of Tables S4 and R1 resemble each other closely. Further, if the turning dates of the specific cycles differ from the reference dates by considerable but tolerably regular intervals, the two patterns will be much alike when the difference in timing is allowed for. But if the specific cycles have no systematic relationship in time to business cycles, then what may have been a clearcut cyclical pattern in Table S4 will disappear or be obscured in the averages of Table R1. In other words, when the movements of a series are independent of the direction of business cycles, and the series covers a fairly large number of business cycles, the average reference-cycle pattern is apt to reveal merely the secular trend. Comparison of the averages in Tables R1 and S4 thus gives a snapshot of the relationship in time between the specific and business cycles.[56]

We have no satisfactory method of measuring the degree of resemblance between the specific- and reference-cycle patterns. But the feature of the patterns that concerns us most is the degree to which the amplitudes of the specific cycles are preserved in the reference-cycle patterns, and this can be readily ascertained. We could get reference-cycle amplitudes by taking the maximum rise and fall in the averages of Table R1. But the figures will be less exposed to random perturbations and more influenced by business cycles if we use the rise during those stages of reference cycles that are the characteristic period of expansion in the series and the fall during the stages that are characteristic of contraction; that is to say, if we determine the rise during the stages assigned to 'expansion' and the fall during the stages assigned to 'contraction' in making Table R4, as explained in Section X. In coal production, for example, the typical period of expansion is from stage I to V, and the typical period of contraction from stage V to IX. The average change from stage I to V of the reference cycles is +27.8, the average change from stage V to IX is −28.2.[57] The average rise and fall of the reference cycles is therefore 56.0 points, that is, +27.8 − (−28.2). According to Table S2 the average rise and fall of the specific cycles is 77.2 points. Hence the average amplitude of the reference-cycle pattern is 73 per cent of the average amplitude of the specific cycles; this figure is placed at the bottom of Table R1.

The percentage ratio of the average reference-cycle amplitude to the average specific-cycle amplitude can be used not only to indicate the relation in time between specific and business cycles, but also the relation in time of any pair of economic series. For example, a price series may be broken into segments on the basis of the turning points in the specific cycles of a corresponding production series instead of on the basis of the

[56] Cf. Ch. 12, Sec. IV.

[57] These figures are taken from col. (2) and (5) of our sample Table R3 (see Table 42). Since Tables R3 and R4 are the same for coal production, no Table R4 is explicitly presented for this series.

turning points of business cycles. In that case the percentage ratio of the amplitude computed on the basis of the special reference dates to the specific-cycle amplitude would indicate the degree of correspondence between the specific cycles in prices and output, since the output series yields reference cycles, so to speak, for the price series. Or if a series on wage rates is broken into segments on the basis of the turning points in the specific cycles of employment, the ratio of the amplitude computed on the basis of these 'reference dates' to the specific-cycle amplitude would indicate the degree of correspondence between the specific cycles in wage rates and employment.

The percentage ratio of the reference-cycle amplitude to the specific-cycle amplitude is a rough indicator of similarity, and must be used cautiously. In the first place, leads or lags do not stand on the same footing as coincidences. If the timing of a series coincides with that of business cycles, the reference-cycle amplitude must be the same as the specific-cycle amplitude; but the latter is nearly certain to exceed the former if the series leads or lags behind the reference dates by a substantial interval, even though this interval be perfectly uniform, whether in months or in fractions of a cycle phase. For example, if a series led the reference turns uniformly by half a reference phase, the peaks would be in the center of stage III and the troughs in the center of stage VII of the reference cycles. But since the values at the peaks and troughs would be averaged with all the months falling in these stages, usually more than three, the reference-cycle amplitude would be smaller for this purely technical reason than the specific-cycle amplitude. Another shortcoming of the measure is that it may be influenced unduly by extreme observations. For example, if the turning points of some specific cycle coincide with the turning points of a business cycle, and this specific cycle happens to be of enormous amplitude relatively to other cycles in the series, the average reference-cycle amplitude may approach in size the average specific-cycle amplitude regardless of the relations in time between the other cycles in the series and business cycles. Finally, the measure may defy logic by falling below 0 or rising above 100. Thus the typical period of expansion may run from stage I to V of the reference cycles, and yet the average reference-cycle pattern may not only rise from stage V to IX, but this rise may exceed that between stages I and V; in that case the reference-cycle amplitude, as we measure it, would be minus. Again, if the specific cycles in a series correspond closely to all business cycles but one, and two specific cycles occur during this cycle, the first marked by a slight decline and the second by a slight rise, the average amplitude of the reference-cycle pattern may exceed that of the specific-cycle pattern. Such absurd results are rare, though the series on zinc stocks (Chart 21) illustrates the difficulty: the average amplitude of its four reference cycles in 1921–38 is a little larger than the average of its five specific cycles within this period.

In presenting the results of our analysis of time series, we rely heavily on charts of cyclical patterns, of the type illustrated in Chart 21. The charts are drawn to a uniform set of scales, so that the features of one series can be compared directly with the corresponding features of any other series. Once a person has become expert in interpreting the charts, he will find that a glance gives a fairly comprehensive notion of the behavior of a series with respect to business cycles. But he will also find that the knowledge so quickly won must be tested by close and patient study of the full evidence. This evidence includes, besides the measures already described, Tables R3 and R4, to which we now turn.

IX Conformity to Business Cycles: Behavior during Fixed Periods

Several of our measures throw light upon the manner and degree in which various economic processes conform to business cycles. Table S1 shows the differences between the timing and the duration of specific and business cycles; Tables R1 and R2 show the movements of a series from stage to stage of business cycles. Whether these movements match business cycles closely or not is indicated by a comparison of the average amplitudes in Tables S4 and R1. But we need a numerical expression of these relationships that takes account of the type of conformity, its regularity, and that shows separately the conformity to the expansion phase of business cycles, to the contraction phase, and to full business cycles.[58]

A simple method of measuring conformity is to observe the direction and rate of movement of a series during successive reference expansions and contractions. Conformity measures on this plan are set out in Table R3, of which Table 42 provides several samples. The first step is to subtract the standing of the series in stage I of each reference cycle from its standing in stage V; this difference is recorded in column (2). Next the durations of the reference expansions covered by the series are entered in column (3). Each of these entries is then divided into the total change during a reference expansion, and the resulting average change per month entered in column (4). The average change per month during reference contractions is determined similarly. The interval from the midpoint of stage V to the midpoint of stage IX is entered in column (6). The standing in stage V of each reference cycle is subtracted from the standing in stage IX, and the difference entered in column (5). This figure is then divided by the duration of the contraction; and the quotient recorded in column (7). In column (8) we subtract the entry in column (4) from the entry in column (7), in order to show whether there is any 'response' to business cycles in the numerous instances where a series rises

58 The methods that follow may be used to measure the conformity of one series to the specific cycles of another, just as conformity to business cycles. In that case, the specific cycles of either series will yield 'reference dates' for the analysis of the other. See pp. 174-5.

TABLE 42

Samples of Table R3: Conformity to Business Cycles
Three American Series

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[a]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bituminous coal production								
Aug. 04–May 07–June 08	...	...	...	−21.5[b]	13.0	−1.65[b]	...	–
June 08–Jan. 10–Jan. 12	+22.1	19.0	+1.16	+5.2	24.0	+0.22	−0.94	–
Jan. 12–Jan. 13–Dec. 14	+7.2	12.0	+0.60	−17.9	23.0	−0.78	−1.38	–
Dec. 14–Aug. 18–Apr. 19	+43.2	44.0	+0.98	−32.8	8.0	−4.10	−5.08	–
Apr. 19–Jan. 20–Sep. 21	+14.2	9.0	+1.58	−19.9	20.0	−1.00	−2.58	–
Sep. 21–May 23–July 24	+49.6	20.0	+2.48	−44.8	14.0	−3.20	−5.68	–
July 24–Oct. 26–Dec. 27	+27.3	27.0	+1.01	−26.4	14.0	−1.89	−2.90	–
Dec. 27–June 29–Mar. 33	+26.0	18.0	+1.44	−62.1	45.0	−1.38	−2.82	–
Mar. 33–May 37–May 38	+32.5	50.0	+0.65	−26.8	12.0	−2.23	−2.88	...
Average[c]	+27.8	...	+1.24	−28.2	...	−1.80	−3.03	...
Average deviation[d]	...	...	0.45	...	...	1.06	1.17	...

Index of conformity to reference
- Expansions .... +100
- Contractions .... +78
- Cycles, trough to trough .... +100
- Cycles, peak to peak .... +100
- Cycles, both ways .... +100

Dates of reference cycles (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Postal receipts in 50 large cities								
June 97–June 99–Dec. 00	+12.9	24.0	+0.54	+12.6	18.0	+0.70	+0.16	–
Dec. 00–Sep. 02–Aug. 04	+18.8	21.0	+0.90	+17.1	23.0	+0.74	−0.16	–
Aug. 04–May 07–June 08	+23.1	33.0	+0.70	+0.5	13.0	+0.04	−0.66	–
June 08–Jan. 10–Jan. 12	+15.7	19.0	+0.83	+14.0	24.0	+0.58	−0.25	–
Jan. 12–Jan. 13–Dec. 14	+10.7	12.0	+0.89	+0.8	23.0	+0.03	−0.86	–
Dec. 14–Aug. 18–Apr. 19	+43.3	44.0	+0.98	+4.9	8.0	+0.61	−0.37	+
Apr. 19–Jan. 20–Sep. 21	+3.7	9.0	+0.41	+8.5	20.0	+0.42	+0.01	–
Sep. 21–May 23–July 24	+16.9	20.0	+0.84	+1.8	14.0	+0.13	−0.71	–
July 24–Oct. 26–Dec. 27	+19.1	27.0	+0.71	+1.5	14.0	+0.11	−0.60	–
Dec. 27–June 29–Mar. 33	+3.2	18.0	+0.18	−30.2	45.0	−0.67	−0.85	–
Mar. 33–May 37–May 38	+26.6	50.0	+0.53	−5.9	12.0	−0.49	−1.02	...
Average[c]	+17.6	...	+0.68	+2.3	...	+0.20	−0.48	...
Average deviation[d]	...	...	0.20	...	...	0.37	0.33	...

Index of conformity to reference
- Expansions .... +100
- Contractions .... −64
- Cycles, trough to trough .... +64
- Cycles, peak to peak .... +80
- Cycles, both ways .... +71

(*Table continued on next page*)

TABLE 42—*Continued*

Samples of Table R3: Conformity to Business Cycles
Three American Series

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[a]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Slab zinc stocks at refineries								
Apr. 19–Jan. 20–Sep. 21	...	...	...	+86.4[e]	20.0	+4.32[e]	...	+
Sep. 21–May 23–July 24	−190.8	20.0	−9.54	+109.3	14.0	+7.81	+17.35	+
July 24–Oct. 26–Dec. 27	−136.4	27.0	−5.05	+96.9	14.0	+6.92	+11.97	+
Dec. 27–June 29–Mar. 33	−2.6	18.0	−0.14	+103.9	45.0	+2.31	+2.45	+
Mar. 33–May 37–May 38	−143.3	50.0	−2.87	+150.6	12.0	+12.55	+15.42	...
Average[c]	−118.3	...	−4.40	+115.2	...	+7.40	+11.80	...
Average deviation[d]	...	...	2.90	...	...	2.78	4.67	...

Index of conformity to reference
- Expansions .... −100
- Contractions .... −100
- Cycles, trough to trough .... −100
- Cycles, peak to peak .... −100
- Cycles, both ways .... −100

[a] Only the sign of the difference is entered.
[b] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average and the average deviation.
[c] Arithmetic mean determined separately for each column. Hence (7)−(4) *may* differ from (8) in the last place.
[d] Measured from the mean.
[e] Computed on base of inverted cycle, 1920–23. Excluded from the average and the average deviation.

or falls throughout a reference cycle. The explanation of column (9) is best postponed to a later point.

Once entries have been made for each reference cycle in Table R3, the results are summarized in two sets of conformity measures. The first set shows arithmetic means, for all business cycles covered by the series, of the rates of change during reference expansions and contractions, and of the differences between the two rates. The averages are given below the entries for individual cycles in columns (4), (7), and (8). For the eight business cycles from 1908 to 1938 covered by bituminous coal, the averages come out +1.2, −1.8, and −3.0 per cent. The first figure indicates that coal output conformed positively to cyclical expansions in business, for output rose on the average during reference expansions. The second figure indicates that coal output also conformed positively to contractions in general business, for it declined on the average during reference contractions. The last figure indicates that coal output conformed positively to full business cycles, for the average rate of change was lower during reference contractions than during reference expansions.

A different type of behavior is illustrated by zinc stocks. For the four business cycles from 1921 to 1938, the averages are —4.4, +7.4, and +11.8; in other words, zinc stocks conformed inversely to reference expansions, contractions, and full cycles. In the series on postal receipts in 50 large American cities, the averages are +0.7, +0.2, —0.5 for eleven business cycles. These results indicate that on the average postal receipts 'responded' to contractions in general business by slackening their rate of increase; the conformity may therefore be said to be positive to expansions, inverted to contractions, but positive to business cycles taken as wholes.

While the average rates of change are significant for many purposes, they do not reveal the degree of consistency in the movements that occurred in successive business cycles. For example, the illustrations just given tell us that the reaction of coal production to business cycles differed, on the average, in direction and degree from the reaction of zinc stocks. They do not tell us whether the reaction of either or both series was uniform or variable from cycle to cycle. Average deviations from the average rates of change during the cycles covered by a series are an important supplement to the averages; so also is the ratio of the average amplitude of the reference-cycle pattern to the average amplitude of the specific-cycle pattern. But these measures still do not show how regularly a series has conformed positively or inversely to business cycles. In the period covered by our illustration, coal production rose during every reference expansion, and declined during every reference contraction but one. Zinc stocks declined invariably during reference expansions, and rose invariably during reference contractions. Postal receipts rose in every expansion, also in nine out of eleven contractions. To measure such differences in the type and regularity of reactions to cyclical changes in general business conditions, we make a second set of conformity measures, namely, 'indexes of conformity'.

We obtain an index of the conformity of a series to reference expansions—more briefly, an 'expansion index'—from column (4) of Table R3 by crediting a series with 100 for every rise, debiting it with 100 for every fall, writing 0 when there is no change, and taking an arithmetic mean of all the entries. In similar fashion column (7) yields a 'contraction index'. But now we weight every decline with +100 and every rise with —100; for a decline means positive conformity to reference contractions and a rise means inverse conformity. Indexes made on this plan range from +100, which means invariable positive conformity, to —100, which means invariable inverse conformity. An index of zero means an equal number of positive and inverse movements, combined perhaps with one or more instances of no change. The magnitude of an expansion index indicates the degree of consistency in the direction of movement of a series from cycle to cycle during reference expansions; its sign indicates whether the process is related in the main positively or inversely to reference expansions. Similarly, the contraction index shows the regularity of movements during reference contractions, and whether the series is characterized by a positive or an inverted relation to reference contractions.[59]

Since these indexes are based simply on the sign of the net difference between three-month averages centered on reference troughs and peaks, they could be computed directly from the original data instead of from reference-cycle relatives. The former method would be easier if the conformity indexes were the sole measure of conformity. But we need the relatives in any case to determine the average rates of change during business cycles—the measure of conformity previously described. Once the relatives are computed and the entries for successive cycles set out in this form in Table R3, it is simpler to work from the relatives. If an entry for a reference cycle appears in column (7) but not in column (4), or vice versa, it is excluded from the average rates of change for the group of cycles covered by the series but not from the conformity indexes. The main reason for the difference in treatment is that the average rates of change bear a simple additive relation to each other, but not the conformity indexes.

A need remains for an index of the conformity of a series to full business cycles, that is, for a 'full-cycle index'. Since the intra-cycle trend is retained in the reference-cycle relatives, it is desirable that the full-cycle index count a decline in the *rate of increase* during reference contractions as equivalent to an actual decline. Hence we consider a series as conforming positively to a business cycle when the sign for that cycle in column (8) of Table R3 is minus, as conforming inversely when the sign is plus, as showing zero conformity when the entry is 0. By counting every minus in column (8) as +100, every plus as −100, and every zero as 0, casting an algebraic sum of these entries, and dividing by the number of business cycles covered, we obtain a preliminary index of conformity to business cycles. The index is entered in the table on the line reading 'Index of conformity to reference cycles, trough to trough'.

This index is based on comparisons of rates of change during reference contractions with the rates of change during the preceding reference expansions. But it is equally important to see how the rate of change during a contraction compares with that during the following expansion, and this requires that business cycles be marked off by successive peaks.

[59] It is sometimes convenient to transform conformity indexes so as to show the number of instances of positive conformity as a percentage of the total number of cycles. This may be done by dividing the conformity index by 2 and adding 50 to the quotient. That is, let P be the number of instances of positive conformity and I the number of instances of inverted conformity; then

$$\frac{100\left(\frac{P-I}{P+I}\right)}{2}+50=100\left(\frac{P}{P+I}\right).$$

The transformation assumes that there are no instances of zero conformity, or that one-half of such instances are allotted to the positive group and one-half to the inverted group.

If the expansion and contraction indexes of a series are both +100, its conformity to full business cycles will be the same whether the cycles are considered as running from peak to peak or from trough to trough. In other instances a difference may arise between indexes made on the two plans, and the difference may be substantial if a series shows no specific cycles. Suppose, for example, that the successive terms of a series fall exactly on a parabola whose second derivative with respect to time is negative; then the full-cycle index will be +100 if the reference cycles are taken as units from trough to trough, but −100 if they are taken as units from peak to peak. If we are not to be misled by the first figure, we should have the second.[60] Hence we supplement our first preliminary full-cycle index by a second, which is entered in Table R3 on the line reading 'Index of conformity to reference cycles, peak to peak'.

The second index is computed from the entries in column (9) in exactly the same manner as the first index is computed from the entries in column (8). Column (9) contains only signs; that is, it shows merely whether the average rate of change during each contraction specified in column (1) is algebraically larger or smaller than the average rate of change during the next reference expansion. In most instances the sign in column (9) can be inferred from the signs in columns (4) and (7). If the entry is minus on a given line in column (7) and plus on the next line in column (4), the entry on the given line in column (9) must be minus. Or if the entry is plus on a given line in column (7) and minus on the next line in column (4), the entry on the given line in column (9) must be plus. A difficulty arises, however, when an entry on one line in column (7) has the same sign as the entry on the next line in column (4), for the two rates are computed on different bases and therefore are incomparable. They can be made comparable by measuring the average rates of change from the original data or from cycle relatives expressed on the same base, which may be any convenient figure. The method we find simplest in practice is to shift the standing in stage V of a given cycle to the base of the next cycle, and then compare the change per month from stage V to IX of the given cycle with the change per month from stage I to V of the next cycle.

We obtain the final full-cycle index by striking an arithmetic mean of the two preliminary indexes, each weighted by the number of cycles it covers, and record the result on the line reading 'Index of conformity to reference cycles, both ways'.[61] The final index must be +100 if a series

[60] Cf. pp. 298-9.

[61] The description in the text follows the method of calculation we have found most convenient in practice. But the computations can be organized on a plan that dispenses with the preliminary indexes.

The final full-cycle index involves a comparison of each phase with the preceding and following phase. Let the reference phase therefore be the unit of observation. If the rate of change during a contraction is algebraically smaller than during the preceding expansion, credit the contraction always rises during reference expansions and declines during reference contractions, or if the average rise per month during contractions is always less than the rise during the next preceding and following expansions, or if the decline during contractions is always more rapid than the decline during the next preceding and following expansions; or, in general, if the average change per month during reference contractions is in every instance algebraically smaller than the average change per month during the next preceding and following reference expansions. The full-cycle index will be -100 if the average change per month during reference contractions is in every instance algebraically larger than the average change per month during the next preceding and following reference expansions.

To illustrate: the conformity indexes of coke production (Table 12) are all $+100$; that is, coke output conformed positively to all the cyclical tides in general business covered by the data, rising and falling in harmony with them. The conformity indexes of zinc stocks are all -100; in other words, this process consistently moved inversely to the few cycles in general business covered by the series. Coal production has an expansion index of $+100$ and a full-cycle index of $+100$; but the contraction index is $+78$ because the series moved counter to the business tide in one of the nine contractions covered $\left(\frac{+800-100}{9}=+78\right)$. Still another result is illustrated by postal receipts, which have conformity indexes of $+100$, -64, $+71$. This series conformed positively to every reference expansion $\left(\frac{+1100}{11}=+100\right)$; it conformed inversely to nine reference contractions and positively to only two $\left(\frac{+200-900}{11}=-64\right)$; yet its conformity was preponderantly positive to full reference cycles, in the sense that its rate of rise was usually lower during contractions than during the next preceding and following expansions $\left(\frac{+1800-300}{21}=+71\right)$. The last result may be clearer if stated another way: the 'index of conformity to reference cycles, trough to trough' is $\frac{+900-200}{11}=+63.6$; the 'index of conformity to reference cycles, peak to peak' is

with $+50$; if it is also smaller than the rate of change during the following expansion, credit the contraction with another $+50$, which makes $+100$ in all; but if the rate of change is larger than during the following expansion, the additional credit is -50 and the total credit is 0; while if the rate of change is larger than during both the preceding and following expansions, the contraction gets a total credit of -100. The accounting is similar for expansions, except that the expansion is credited with $+50$ or -50 according as the rate of change is algebraically larger or smaller than that in a contiguous contraction. Obviously, the first and last phases cannot receive a credit of $+100$ or -100, since they can be compared with only one adjacent phase. Once a credit has been assigned to each reference phase covered by the analysis, the total is cast up and divided by $(N-1)$, where N is the number of reference phases. The quotient is the final index of full-cycle conformity.

$\frac{+900-100}{10}=+80$; hence the 'index of conformity to reference cycles, both ways' is $\frac{11(+63.6)+10(+80)}{21}=+71$.

The conformity indexes resemble coefficients of correlation. Except for the use of percentages, the limits of both are -1 and $+1$. The coefficient of correlation (squared) states the degree to which estimates of a given variable may be improved by taking account of its correlation with other variables. The index of conformity may be interpreted in a similar manner: it states the degree to which errors can be reduced in estimating the direction of movement of a series by taking account of its conformity to business cycles instead of guessing.[62] If in eight reference expansions or contractions there are seven movements in the same direction, that is 75 per cent better than chance, and the conformity index is precisely this figure, plus or minus. The greater the number of observations and the firmer the rational analysis, the greater is our confidence in the significance of a coefficient of correlation, and so it is also with indexes of conformity. An index of conformity as low as +33 indicates that instances of positive conformity preponderate over instances of inverted conformity in the ratio of 2 to 1; but the index commands more serious attention when the cycles number ten than when they number three, and when they number thirty than when they number ten.

Conformity indexes are highly unstable when few cycles are covered (Table 43). If there are only four cycles and no zero movements, the index of conformity to reference expansions or contractions may be -100, -50, 0, $+50$, or $+100$. The gaps between the possible values of the index diminish as the cycles become more numerous, but the gaps are appreciable even for 15 cycles. If the conformity index for four cycles is $+100$, the index will drop to $+60$ if a fifth observation happens to be inverted, and to $+33$ if a sixth observation also is inverted. If the index is $+100$ for thirteen cycles, it will drop to $+86$ if the fourteenth cycle happens to be inverted and to $+73$ if the fifteenth also is inverted. A conformity index of $+100$ or -100 for four expansions (or contractions) may seem significant at first glance; but is unimpressive when we remember that a random series would yield such a result one time in eight. Probability tests are therefore helpful in judging the statistical significance of conformity indexes, just as they help to judge coefficients of correlation. Such tests, however, must be used with fine discrimination, because as a rule the values of economic time series are serially correlated and different series are interdependent. In the case of the full-cycle index there is the additional difficulty that the same phase is used twice.[63]

[62] We are indebted to W. Allen Wallis for suggesting these remarks.

[63] See in this connection Geoffrey H. Moore and W. Allen Wallis, Time Series Significance Tests Based on Signs of Differences, *Journal of the American Statistical Association*, June 1943. Methods of testing the statistical significance of reference-cycle patterns are developed in this paper.

TABLE 43

Conformity Indexes for P Instances of Positive Conformity in N Cycles

No. of observations on cycles (N)	Number of instances of positive conformity (P)																									
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	−100	+100																								
2	−100	0	+100																							
3	−100	−33	+33	+100																						
4	−100	−50	0	+50	+100																					
5	−100	−60	−20	+20	+60	+100																				
6	−100	−67	−33	0	+33	+67	+100																			
7	−100	−71	−43	−14	+14	+43	+71	+100																		
8	−100	−75	−50	−25	0	+25	+50	+75	+100																	
9	−100	−78	−56	−33	−11	+11	+33	+56	+78	+100																
10	−100	−80	−60	−40	−20	0	+20	+40	+60	+80	+100															
11	−100	−82	−64	−45	−27	−9	+9	+27	+45	+64	+82	+100														
12	−100	−83	−67	−50	−33	−17	0	+17	+33	+50	+67	+83	+100													
13	−100	−85	−69	−54	−38	−23	−8	+8	+23	+38	+54	+69	+85	+100												
14	−100	−86	−71	−57	−43	−29	−14	0	+14	+29	+43	+57	+71	+86	+100											
15	−100	−87	−73	−60	−47	−33	−20	−7	+7	+20	+33	+47	+60	+73	+87	+100										
16	−100	−88	−75	−62	−50	−38	−25	−12	0	+12	+25	+38	+50	+62	+75	+88	+100									
17	−100	−88	−76	−65	−53	−41	−29	−18	−6	+6	+18	+29	+41	+53	+65	+76	+88	+100								
18	−100	−89	−78	−67	−56	−44	−33	−22	−11	0	+11	+22	+33	+44	+56	+67	+78	+89	+100							
19	−100	−89	−79	−68	−58	−47	−37	−26	−16	−5	+5	+16	+26	+37	+47	+58	+68	+79	+89	+100						
20	−100	−90	−80	−70	−60	−50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	+60	+70	+80	+90	+100					
21	−100	−90	−81	−71	−62	−52	−43	−33	−24	−14	−5	+5	+14	+24	+33	+43	+52	+62	+71	+81	+90	+100				
22	−100	−91	−82	−73	−64	−55	−45	−36	−27	−18	−9	0	+9	+18	+27	+36	+45	+55	+64	+73	+82	+91	+100			
23	−100	−91	−83	−74	−65	−57	−48	−39	−30	−22	−13	−4	+4	+13	+22	+30	+39	+48	+57	+65	+74	+83	+91	+100		
24	−100	−92	−83	−75	−67	−58	−50	−42	−33	−25	−17	−8	0	+8	+17	+25	+33	+42	+50	+58	+67	+75	+83	+92	+100	
25	−100	−92	−84	−76	−68	−60	−52	−44	−36	−28	−20	−12	−4	+4	+12	+20	+28	+36	+44	+52	+60	+68	+76	+84	+92	+100

This table is made on the assumption that there are no instances of zero conformity. To take account of the zeroes, it would be necessary to insert columns for 0.5, 1.5, 2.5, etc. instances of positive conformity. In computing conformity indexes a series is credited with 0 for every zero rate of change; which is equivalent to counting the zero change as a half instance of positive conformity and a half instance of inverted conformity. Thus if a series rises during four expansions (or falls during four contractions) and shows no change in a fifth, the conformity index is $\frac{+400+0}{5}$ or $\frac{+450-50}{5}$.

In using the table, it must be remembered that, in the case of the index of full-cycle conformity, the count includes business cycles marked off both by peaks and by troughs.

Strictly speaking, an expansion index of +100 does not mean that a series rose consistently *during* the reference expansions covered; nor does a contraction index of +100 mean that a series declined consistently *during* reference contractions. For these indexes as well as the full-cycle index are based simply on the standings at the initial trough, peak, and terminal trough of each reference cycle, and take no account of what happens *within* the phases of expansion and contraction. If the standing in stage V of a reference cycle is higher than the standing in stage I, the series is considered to conform positively to the given expansion, although the movement from stage I to IV may be sharply downward. On the other hand, if the series rises steadily from stage I to IV, but falls to a lower level in stage V than in stage I, the movement as a whole is counted as inverted.[64] Nor does the size of the difference between the standings at peaks and troughs have any influence; a tiny rise and a huge rise count the same in the expansion and contraction indexes. These considerations reinforce what has already been said, namely, that variations in conformity indexes may have little meaning, especially when cycles are few. They suggest also the wisdom of using the average rates of change in conjunction with the indexes.

X Conformity to Business Cycles: Timing Differences Recognized

The measures of conformity just described are valuable because they reveal the consistency with which a given process and general business activity have moved concurrently in the same or opposite directions, but they may obscure regular responses to the cyclical rhythm in business when a process leads or lags by considerable intervals. Imagine a series with no definite secular trend, lagging consistently a quarter of a cycle (strictly, by half a phase) behind the turns in general business. The conformity of such a series to business cycles is in a sense perfect; yet the measures described in the preceding section would probably show zero conformity, for within each reference cycle the direction of movement will be opposite to that of general business over as many stages as it will be similar, and the standing of the series at reference peaks will be about the same as its standing at reference troughs. To avoid such pitfalls, we supplement the measures of conformity based on rigid periods with another set that takes differences in timing into account. The additional measures

[64] During the reference contraction from 1910 to 1912, coal production declined from stage V to VII, then rose to stage IX. This rise carried the figure in stage IX above that in stage V, and thus accounts for the lapse of the contraction index from perfect conformity. Indexes of conformity that take account of every movement from stage to stage of the reference cycles are considered in Ch. 8, Sec. VIII.

are entered in Table R4, a sample of which is shown in Table 47.[65]

The preparation of this supplement follows the lines of Table R3, but requires several preliminary steps. The successive reference-cycle patterns in Table R1 must first be charted, these nine-point patterns studied cycle by cycle, and a decision reached how they can best be divided into two uniform segments—one representing the stages during which the series most regularly advances and the other the stages during which it most regularly declines. For this purpose we frequently draw charts on the plan illustrated by Chart 22. The midpoint of stage I of each reference cycle is taken as the origin. The midpoint of stage II is then placed one-half of a standard unit (which may be any convenient figure) to the right of the origin; in other words, the abscissa of the midpoint of stage II is .5. The abscissas of the midpoints of the later stages are as follows: 1.5 for stage III, 2.5 for stage IV, 3 for stage V, 3.5 for stage VI, 4.5 for stage VII, 5.5 for stage VIII, and 6 for stage IX. This method of plotting reference-cycle patterns blinks the varying durations of reference phases; also the fact that the successive thirds of a reference phase, as we identify them, do not add up to the full duration of the phase, but to this figure minus one month. The advantage of standardizing the durations of all reference phases is that corresponding standings of successive reference cycles, when plotted on a chart, are aligned vertically. The movement of successive reference cycles from one stage to the next can therefore be seen at a glance—in sharp contrast to Chart 19, which represents the varying intervals between successive stages with fidelity. Of course, disregard of the varying durations of reference expansions and contractions may give a very erroneous impression of the behavior of a series that usually or frequently rises or falls throughout a reference cycle; in such cases there is no satisfactory substitute for a chart of the type exemplified by Chart 19.

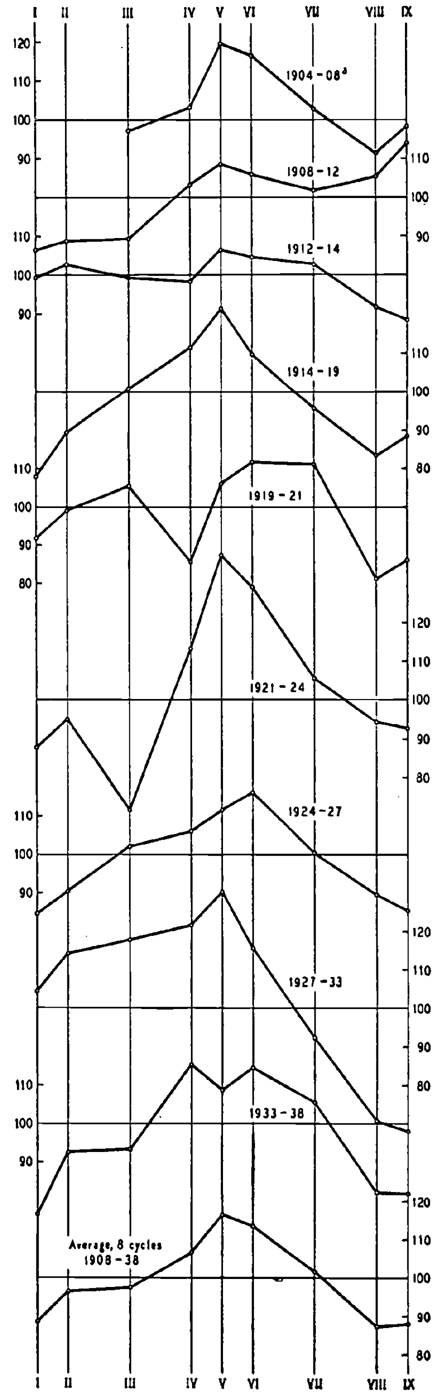
In general, the task of determining what stages are characteristic of expansion and contraction in a series is easy or difficult according as its timing with respect to business cycles is regular or irregular. In a series like coke production (Table 10) a decision is quickly reached. The series typically rises from stage I to V and declines from stage V to IX; no Table R4 is therefore necessary, or more precisely, Table R3 is the equivalent of Table R4. In bituminous coal production (Table 39 and Chart 22) the timing is less regular, especially at reference troughs. In railroad bond yields (Table 45) the timing varies so much from cycle to cycle that it is not easy to decide upon the typical stages of expansion and contraction, although it is plain that the trough was typically reached later than stage I and that the peak was typically reached later than stage V.

To expose and test what parallelism there may be in the successive reference-cycle patterns of a series, we use a variety of devices. For

[65] This step is analogous to that traditionally followed in correlation analysis of time series when one series is 'lagged' after the other, except that our procedure permits differentiation between the timing at peaks and troughs.

CHART 22

Patterns of Successive Reference Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1905 – 1938
(Drawn to a Schematized Time Scale)



See explanations in text; cf. Chart 19.

[a] The pattern is incomplete, data not being available.

example, we may first examine the timing of the average reference-cycle pattern, then see whether this timing is confirmed by averages of subgroups of cycles, by a count of the rises and falls between successive stages of the reference cycles, by patterns formed by rankings of the reference-cycle standings, by averages of the leads or lags expressed in fractions of the duration of a cycle phase (1/6, 1/2, 5/6), and so on. When a decision is finally reached, we usually feel reasonably certain that we have made that division into 'expansion' and 'contraction' which best represents the behavior of the series during successive reference cycles. If the behavior of a series is so irregular from one business cycle to the next that there is serious doubt whether it is correlated at all with business cycles, we dispense with Table R4.

We rarely make divisions more unequal than five stages for expansion and three for contraction, or three for the former and five for the latter. The terms 'expansion' and 'contraction' are used in a relative sense. Thus, if a series shows no specific cycles, we mean by 'expansion' the stages during which the advance is more rapid or the decline less rapid, according as the secular trend slopes upward or downward. Table 44 indicates that twenty-four divisions of reference cycles into expansion and contraction are possible when no division more unequal than five and three stages is allowed. Of these, six are coterminous with the reference cycles (lines 1-3, 12, 14, and 16), two are coterminous with the reference phases (lines 2 and 14), and one is coincident with both reference troughs and peaks (line 2).

Once the division of reference cycles in a series has been decided upon, the next step is to class it as 'positive', 'inverted' or 'neutral'. This classification determines whether the *specific* cycles are to be marked off from trough to trough or from peak to peak,[66] and also plays a part in the computation of Table R4. We class a division of reference cycles as positive when the selected expansion segment contains more stages in reference expansion than in reference contraction; or, what comes to the same thing, when the selected contraction contains more stages in reference contraction than in reference expansion. We class the division of reference cycles as inverted when the selected expansion covers more stages in reference contraction than in reference expansion. When the selected expansion overlaps equally reference expansions and contractions, the division is classed as neutral.

Whatever the division, it is applied uniformly in subsequent operations to all the reference cycles covered by the series.[67] When the division is positive, the expansion segments of the series are matched with reference expansions and the contraction segments with reference contractions. When the division is inverted, the contraction segments are

66 See Sec. I of this chapter.

67 See, however, Ch. 10, Sec. VIII.

TABLE 44

Possible Divisions of Reference Cycles when Three to Five Stages Are Assigned to Expansion

Line	Division of the reference cycles of a series		Classification of the division	In computing Table R4 the following stages are matched with	
	Expansion covers stages	Contraction covers stages		Reference expansion	Reference contraction
1	I-IV	IV-IX	Positive	I-IV	IV-IX
2	I-V	V-IX	Positive	I-V	V-IX
3	I-VI	VI-IX	Positive	I-VI	VI-IX
4	II-V	V-II	Positive	II-V	V-II
5	II-VI	VI-II	Positive	II-VI	VI-II
6	II-VII	VII-II	Positive	II-VII	VII-II
7	III-VI	VI-III	Positive	III-VI	VI-III
8	III-VII	VII-III	Neutral	III-VII	VII-III
9	III-VIII	VIII-III	Inverted	VIII-III	III-VIII
10	IV-VII	VII-IV	Inverted	VII-IV	IV-VII
11	IV-VIII	VIII-IV	Inverted	VIII-IV	IV-VIII
12	IV-IX	I-IV	Inverted	I-IV	IV-IX
13	V-VIII	VIII-V	Inverted	VIII-V	V-VIII
14	V-IX	I-V	Inverted	I-V	V-IX
15	V-II	II-V	Inverted	II-V	V-II
16	VI-IX	I-VI	Inverted	I-VI	VI-IX
17	VI-II	II-VI	Inverted	II-VI	VI-II
18	VI-III	III-VI	Inverted	III-VI	VI-III
19	VII-II	II-VII	Inverted	II-VII	VII-II
20	VII-III	III-VII	Neutral	VII-III	III-VII
21	VII-IV	IV-VII	Positive	VII-IV	IV-VII
22	VIII-III	III-VIII	Positive	VIII-III	III-VIII
23	VIII-IV	IV-VIII	Positive	VIII-IV	V-VIII
24	VIII-V	V-VIII	Positive	VIII-V	V-VIII

matched with reference expansions and the expansion segments with reference contractions. When the division is neutral, the division is treated as if it were positive; that is, the expansion segments are matched arbitrarily with reference expansions and the contraction segments with reference contractions. For each series we indicate the procedure by writing at the top of Table R4 what stages are considered characteristic of expansions, and whether expansions are matched with reference expansions or reference contractions, implying that contractions are matched with the other reference phase (see Table 47).

We then compute the average rate of change per month during the stages matched with reference expansions and during the stages matched with reference contractions, and enter the results in Table R4. The computations correspond in every respect to those in Table R3. For example, if the selected expansion runs from stage I to IV of the reference cycles, the duration of this segment, that is, the number of months between the midpoints of stages I and IV, is entered in column (3), for the segment is matched with reference expansion. The excess of the standing in stage IV over that in stage I is entered in column (2), and this difference divided by the interval between the midpoints of the two stages is entered in column (4). Similarly, column (6) shows the number of months in the segment matched with reference contraction, column (5) the total (net) change during this interval, and column (7) the rate of change during this interval. Column (8) is determined from columns (4) and (7) just as in Table R3; so also is column (9), though here, as previously explained, special calculations may be necessary.

A special calculation is also necessary when the segment matched with reference expansion does not begin in stage I; for in that case the standings necessary to determine the average rates of change are expressed on different bases in Table R1. The series on railroad bond yields (see Tables 45 and 47), in which stages III-VI are matched with reference expansion, may serve as an example. There is no difficulty in determining from Table R1 the average change per month from stage III to VI, since the stand-

TABLE 45

Sample of Table R1: Reference-cycle Patterns
Railroad Bond Yields, United States, 1857–1933

Dates of reference cycles	Average in reference-cycle relatives at stage									Av. monthly standing during cycle (per cent)
	I	II	III	IV	V	VI	VII	VIII	IX	
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough	
Trough – Peak – Trough		First third	Mid. third	Last third		First third	Mid. third	Last third		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Dec. 54–June 57–Dec. 58	...	...	...	...	105.8[a]	115.9[a]	104.6[a]	98.4[a]	95.0[a]	6.438[b]
Dec. 58–Oct. 60–June 61	99.4	100.7	101.6	96.5	96.8	102.5	100.1	102.9	104.2	6.150
June 61–Apr. 65–Dec. 67	112.5	104.8	84.4	87.6	103.9	109.8	111.2	111.0	111.9	5.696
Dec. 67–June 69–Dec. 70	99.6	97.4	97.8	101.0	100.7	103.0	100.8	99.6	100.7	6.396
Dec. 70–Oct. 73–Mar. 79	113.4	111.7	109.0	108.6	112.4	102.6	92.0	89.8	85.3	5.677
Mar. 79–Mar. 82–May 85	112.8	109.5	102.0	96.6	98.2	97.8	97.5	96.7	93.3	4.292
May 85–Mar. 87–Apr. 88	106.3	103.6	98.1	98.2	98.8	98.9	101.4	99.5	98.9	3.767
Apr. 88–July 90–May 91	102.3	100.4	96.7	98.7	100.3	101.8	104.0	104.0	105.8	3.642
May 91–Jan. 93–June 94	102.3	102.8	98.9	98.6	98.8	99.5	103.2	96.9	95.9	3.770
June 94–Dec. 95–June 97	103.6	102.1	101.6	98.0	99.4	99.6	102.7	96.9	94.8	3.488
June 97–June 99–Dec. 00	103.1	102.1	102.9	98.1	96.0	98.3	98.9	99.6	98.7	3.207
Dec. 00–Sep. 02–Aug. 04	94.6	94.5	95.9	96.4	98.5	101.0	105.6	105.7	104.2	3.346
Aug. 04–May 07–June 08	96.6	95.9	96.8	99.8	102.7	104.7	109.0	105.5	104.4	3.612
June 08–Jan. 10–Jan. 12	100.3	98.2	96.4	98.1	99.3	101.4	101.8	102.4	102.2	3.761
Jan. 12–Jan. 13–Dec. 14	96.4	96.6	97.5	98.3	98.4	101.7	101.8	100.8[c]	105.2[d]	3.988[e]
Dec. 14–Aug. 18–Apr. 19	97.3[d]	96.3	93.9	106.1	111.1	110.5	103.5	106.6	107.6	4.312
Apr. 19–Jan. 20–Sep. 21	92.4	92.4	96.7	96.6	99.0	105.4	100.5	101.7	99.0	5.020
Sep. 21–May 23–July 24	109.8	102.2	95.6	99.4	101.3	101.3	101.9	100.6	98.4	4.525
July 24–Oct. 26–Dec. 27	102.7	102.8	102.5	99.9	99.5	98.4	96.7	95.0	93.4	4.336
Dec. 27–June 29–Mar. 33	93.1	94.7	99.5	101.9	103.8	100.5	95.0	105.8	99.8	4.349
Average	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2	...
Average deviation	5.1	4.0	3.3	2.6	3.1	2.5	3.2	3.8	4.7	...

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.
[b] Base of inverted cycle, June 1857–Oct. 1860.
[c] Includes 3 months, instead of 7; see note 'e'.
[d] Includes 2 months, instead of 3; see note 'e'.
[e] Omits Aug.–Nov. 1914. No data (N. Y. Stock Exchange closed).

ings in both stages are expressed as relatives on the base of the same reference cycle. Thus the standing of bond yields in stage III of the reference cycle of 1858–61 is 101.6; the standing in stage VI is 102.5; the total change is +0.9 over an interval of 12.5 months, or an average of +0.07 per cent per month. But we cannot determine directly the average change per month during the segment matched with the contraction of the given reference cycle—that is, from stage VI of the given cycle to stage III of the next cycle, because the required standing in stage III is expressed in Table R1 as a relative on the base of the next reference cycle. We meet the difficulty by shifting the standing in stage III to the base of the given reference cycle. Thus the standing of bond yields in stage VI of the reference cycle of 1858–61 is 102.5; the standing in stage III of the reference cycle of 1861–67 is 84.4; this standing becomes 78.1 when shifted to the base of the reference cycle of 1858–61; [68] the change from stages VI to III is therefore −24.4 (in relatives of the base of the 1858–61 cycle); and since the interval between these stages is 29.5 months, the average change per month is −0.83 per cent.

The procedure is similar whenever shifts of base are required. For example, if the stages matched with reference expansion were VII-IV, we would need to shift the standing in stage VII of the preceding reference cycle to the base of the given reference cycle in order to compute the average rate of change during the segment matched with the reference expansion of the given cycle. On the other hand, if the stages matched with reference expansion were II-V, we would need to shift the standing in stage II of the following reference cycle to the base of the given reference cycle to compute the average rate of change during the segment matched with the reference contraction of the given cycle.

Once the entries in Table R4 have been made, they are summarized by two sets of conformity measures, in exactly the same way as in Table R3. How useful the new measures can be appears from a comparison of Tables 46 and 47. From Table 46 we learn that the synchronous movements of bond yields and general business have been almost as often in the opposite as in the same direction. We cannot infer, however, that there is no relation between the movements of bond yields and general business, since the table ignores not only haphazard but also systematic differences in timing. Table 47 indicates that when the lagging tendency of bond yields is allowed for, they move preponderantly in the same direction as business cycles.

The conformity measures in Tables R3 and R4 thus serve different ends and we compute both for each series.[69] The measures in the former

[68] The base of the 1861–67 cycle is lower than the base of the 1858–61 cycle in the ratio of 5.696 to 6.150, as is indicated by the entries in col. (10) of Table 45.

[69] Except when no typical expansion or contraction segments can be marked off; in such instances Table R4 is omitted. Of course, when the typical expansion of a series runs from stage I to V or from stage V to IX, Table R4 is the same as Table R3.

tell the behavior of different series during fixed periods, and thereby help to characterize these periods. In particular, they impose a severe test of whether the fixed periods are or are not business cycles in the sense of our definition. But when the problem is to ferret out the 'response' or 'relation' of a series to business cycles, it is essential to take account of the adjustment in time between the given activity and business as a whole.

TABLE 46

Sample of Table R3: Conformity to Business Cycles
Timing Differences Ignored
Railroad Bond Yields, United States, 1857–1933

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[g]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec. 54–June 57–Dec. 58	...	...	...	−10.8[a]	18.0	−0.60[a]	...	−
Dec. 58–Oct. 60–June 61	−2.6	22.0	−0.12	+7.4	8.0	+0.92	+1.04	+
June 61–Apr. 65–Dec. 67	−8.6	46.0	−0.19	+8.0	32.0	+0.25	+0.44	+
Dec. 67–June 69–Dec. 70	+1.1	18.0	+0.06	0.0	18.0	0.00	−0.06	+
Dec. 70–Oct. 73–Mar. 79	−1.0	34.0	−0.03	−27.1	65.0	−0.42	−0.39	−
Mar. 79–Mar. 82–May 85	−14.6	36.0	−0.41	−4.9	38.0	−0.13	+0.28	+
May 85–Mar. 87–Apr. 88	−7.5	22.0	−0.34	+0.1	13.0	+0.01	+0.35	+
Apr. 88–July 90–May 91	−2.0	27.0	−0.07	+5.5	10.0	+0.55	+0.62	+
May 91–Jan. 93–June 94	−3.5	20.0	−0.18	−2.9	17.0	−0.17	+0.01	+
June 94–Dec. 95–June 97	−4.2	18.0	−0.23	−4.6	18.0	−0.26	−0.03	+
June 97–June 99–Dec. 00	−7.1	24.0	−0.30	+2.7	18.0	+0.15	+0.45	−
Dec. 00–Sep. 02–Aug. 04	+3.9	21.0	+0.19	+5.7	23.0	+0.25	+0.06	+
Aug. 04–May 07–June 08	+6.1	33.0	+0.18	+1.7	13.0	+0.13	−0.05	+
June 08–Jan. 10–Jan. 12	−1.0	19.0	−0.05	+2.9	24.0	+0.12	+0.17	−
Jan. 12–Jan. 13–Dec. 14	+2.0[b]	12.0	+0.17[b]	+6.8[c]	23.0	+0.30[c]	+0.13[c]	−
Dec. 14–Aug. 18–Apr. 19	+13.8[d]	44.0	+0.31[d]	−3.5	8.0	−0.44	−0.75[d]	−
Apr. 19–Jan. 20–Sep. 21	+6.6	9.0	+0.73	0.0	20.0	0.00	−0.73	+
Sep. 21–May 23–July 24	−8.5	20.0	−0.42	−2.9	14.0	−0.21	+0.21	−
July 24–Oct. 26–Dec. 27	−3.2	27.0	−0.12	−6.1	14.0	−0.44	−0.32	−
Dec. 27–June 29–Mar. 33	+10.7	18.0	+0.59	−4.0	45.0	−0.09	−0.68	...
Average[e]	−1.0	...	−0.01	−0.8	...	+0.03	+0.04	...
Average deviation[f]	...	...	0.24	...	...	0.26	0.35	...

Index of conformity to reference
- Expansions .... −26
- Contractions 0
- Cycles, trough to trough .... −16
- Cycles, peak to peak .... −16
- Cycles, both ways .... −16

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.
[b] Computed on base omitting Aug.–Nov. 1914.
[c] Computed on base omitting Aug.–Nov. 1914; standing in stage IX includes 2 months.
[d] Standing in stage I includes 2 months.
[e] Arithmetic mean determined separately for each column. Hence (7)−(4) *may* differ from (8) in the last place.
[f] Measured from the mean.
[g] Only the sign of the difference is entered.

The measures in Table R4 are therefore more useful for analytical purposes than the measures in Table R3, and we use the former more extensively than the latter. Whenever measures of 'conformity' are referred to without qualification, it is to be understood that Table R4 is meant.

Table R4 shares the defects of Table R3 noted in the preceding section, and suffers besides from the uncertainty inherent in any effort to

TABLE 47

Sample of Table R4: Conformity to Business Cycles
Timing Differences Recognized
Railroad Bond Yields, United States, 1857–1933

(Expansion covers stages III-VI. Expansions are matched with reference expansions.)

Dates of reference cycles	Change in reference-cycle relatives during stages matched with						Av. change per month during stages matched with reference contraction minus that during stages matched with	
	Reference expansion			Reference contraction				
Trough–Peak–Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[e]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec. 54–June 57–Dec. 58	...	...	...	−18.8[a]	25.5	−0.74[a]	...	−
Dec. 58–Oct. 60–June 61	+0.9	12.5	+0.07	−24.4	29.5	−0.83	−0.90	−
June 61–Apr. 65–Dec. 67	+25.4	28.5	+0.89	0.0	35.5	0.00	−0.89	−
Dec. 67–June 69–Dec. 70	+5.2	12.5	+0.42	−6.3	31.5	−0.20	−0.62	+
Dec. 70–Oct. 73–Mar. 79	−6.4	28.0	−0.23	−25.5	72.0	−0.35	−0.12	−
Mar. 79–Mar. 82–May 85	−4.2	24.5	−0.17	−11.7	42.5	−0.28	−0.11	−
May 85–Mar. 87–Apr. 88	+0.8	13.5	+0.06	−5.4	24.0	−0.22	−0.28	−
Apr. 88–July 90–May 91	+5.1	15.5	+0.33	+0.6	18.0	+0.03	−0.30	−
May 91–Jan. 93–June 94	+0.6	13.0	+0.05	−5.5	23.0	−0.24	−0.29	−
June 94–Dec. 95–June 97	−2.0	12.5	−0.16	−5.0	26.5	−0.19	−0.03	+
June 97–June 99–Dec. 00	−4.6	15.5	−0.30	+1.7	25.0	+0.07	+0.37	−
Dec. 00–Sep. 02–Aug. 04	+5.1	14.5	+0.35	+3.5	35.5	+0.10	−0.25	−
Aug. 04–May 07–June 08	+7.9	19.0	+0.42	−4.3	20.0	−0.22	−0.64	−
June 08–Jan. 10–Jan. 12	+5.0	14.0	+0.36	+2.0	25.5	+0.08	−0.28	−
Jan. 12–Jan. 13–Dec. 14	+4.2[b]	10.0	+0.42[b]	−0.2[b]	41.0	0.00[b]	−0.42[b]	−
Dec. 14–Aug. 18–Apr. 19	+16.6	23.5	+0.71	+2.1	11.0	+0.19	−0.52	−
Apr. 19–Jan. 20–Sep. 21	+8.7	8.0	+1.09	−19.2	26.5	−0.72	−1.81	−
Sep. 21–May 23–July 24	+5.7	12.5	+0.46	−3.0	25.0	−0.12	−0.58	+
July 24–Oct. 26–Dec. 27	−4.1	16.0	−0.26	+1.4	20.5	+0.07	+0.33	+
Dec. 27–June 29–Mar. 33	+1.0	17.0	+0.06	−19.4	62.0	−0.31	−0.37	...
Average[c]	+3.7	...	+0.24	−6.2	...	−0.17	−0.41	...
Average deviation[d]	...	...	0.32	...	...	0.20	0.33	...

Index of conformity to reference
- Expansions .... +47
- Contractions .... +30
- Cycles, trough to trough .... +79
- Cycles, peak to peak .... +58
- Cycles, both ways .... +68

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.
[b] Computed on base omitting Aug.–Nov. 1914.
[c] Arithmetic mean determined separately for each column. Hence (7)−(4) *may* differ from (8) in the last place.
[d] Measured from the mean.
[e] Only the sign of the difference is entered.

determine leads or lags. Our leads or lags are expressed in cycle-stage units, since they are derived from Table R1. This method confines the leads or lags to coarse fractions of a cycle. Cycle stages sometimes cover many months, and summation over these intervals may conceal or obscure the true timing. To meet this difficulty we might follow another plan, namely: (1) convert the leads or lags in Table S1 into percentages of the duration of the reference phase within which the turn occurs,[70] (2) determine the typical lead or lag from these percentages, (3) divide the reference cycles into expansion and contraction segments on the basis of the typical percentages, (4) represent the beginnings of the expansion and contraction segments by centered three-month averages of cycle relatives, (5) compute the average rates of change during the expansion and contraction segments from these standings, (6) then proceed as in Table R4. Measures made on this plan might be more sensitive to cyclical influences than our present measures,[71] but they would be more laborious to compute, and it is unlikely that they would yield results materially different from those we get by our simpler method.

Another plan we have considered is to divide reference cycles into segments of expansion and contraction on the basis of typical leads or lags measured in months, instead of in percentages of a cycle phase.[72] If the timing of economic series were more uniform in months than in cyclical percentages,[73] a matter of which we as yet have little knowledge, this plan would be preferable in principle to the method we are using. But this variant upon our current practice may break down when the typical lead or lag is long relatively to a particular reference phase. Thus, if a series typically lagged 5 months at reference peaks but led by 5 months at reference troughs, its conformity to a reference contraction lasting just 10 months could not be determined. If the typical lead were 12 months at both reference peaks and troughs, conformity to a reference phase lasting less than 12 months could be determined arithmetically, but it might be difficult to interpret or justify the computation. On the other hand, a typical lag of 12 months under the assumed circumstances would seem plausible, if it reflected the period required by technical or administrative adjustments. These observations are inconclusive; they mean merely that the division of reference cycles on the basis of monthly leads or lags

70 This statement applies only when the lead or lag is shorter than a reference phase. The following statement covers also longer leads or lags: a lead would be expressed as a percentage of the duration of the reference phase just preceding the reference turn with which the specific-cycle turn is compared, and a lag would be expressed as a percentage of the reference phase just following the reference turn with which the specific-cycle turn is compared.

71 The uncertainties of measuring leads or lags should be recalled in this connection; see Sec. II. Also, the 'typical' lead or lag is not necessarily the arithmetic mean.

72 See above, p. 128.

73 In the course of an intensive study of inventory cycles, Moses Abramovitz found this to be true of the behavior of inventories of finished goods held by manufacturers, and certain other classes of inventories.

would work badly under conditions we are sure to encounter occasionally. For the time being at least, our method has the advantage that it avoids these embarrassments.

Our method of making conformity indexes has been criticized on the ground that the cyclical responses of a series may vary in timing from cycle to cycle, and that a uniform division of reference cycles, whether on the basis of leads or lags expressed in months or in percentages, fails to take this variation into account.[74] This criticism may well be justified, but it raises a host of questions in regard to long economic waves, also cyclical versus random movements, with which we cannot cope adequately at present. At this stage of our work there seems to be no satisfactory alternative to uniform divisions; for unless the same division is applied to each reference cycle of a series there is danger that the results will be indistinguishable from subjective judgments of conformity.[75] The method has been criticized also on the ground that it maximizes the conformity indexes, but this criticism involves a misinterpretation. The conformity measures are based on the division of reference cycles that seems most representative, not on the division that yields the highest conformity indexes. For example, our series on sales per store of 'variety chains' in the United States rises typically from stage I to VI, but in one of the five cycles covered there is an abrupt drop between stage V and VI. The standing in stage V of this cycle is above while that in stage VI is below the standing in stage I. In Table R3 the expansion index is +100, and the contraction index +20. But in Table R4 we take stages I-VI to represent the expansion and VI-IX the contraction, so that the expansion index drops to +60, while the contraction index remains +20. The conformity indexes in Table R4 not only need not be maximum indexes, but—as this example illustrates—may actually be lower than those in Table R3, though as a rule they are of the same size or higher.

There is, however, a difficulty in the fact that the margin of preference for the division we make is sometimes slight. In bituminous coal production there is little to choose between stages VIII-V and I-V as the typical period of expansion. In railroad bond yields we take stages III-VI to represent expansion, and on this basis the indexes of conformity to reference expansions, contractions, and full cycles come out +47, +30, +68. But there is some uncertainty whether the expansion culminates typically in stage VI or stage VII. If we took stages III-VII to represent expansion, we would get another set of conformity indexes: +37, +60, +58. The dependence of conformity indexes on the particular division that is made of the reference cycles is frequently much greater than in bond yields, especially in short series. The difficulty in question must be met by exercising care and caution. If there is little to choose between two

[74] See Ch. 8, Sec. VIII.

[75] See, however, method C in Table 25, and the accompanying text.

divisions of reference cycles, the sensible procedure is to compute Table R4 in a form corresponding to each division and to express each result by a range instead of a single figure.

There is no logical justification of our practice of treating neutral divisions, which overlap equally upon reference expansions and contractions, in the same way as positive divisions. Being marginal (neutral), such divisions could just as well be treated as inverted. However, the consequences of our arbitrary practice are less disturbing than may seem at first glance. If a series consistently rises from stage III to VII and declines from stage VII to III, positive treatment must yield conformity indexes of +100, +100, +100; for stages III-VII, the expansion segments, are matched with reference expansions and stages VII-III, the contraction segments, are matched with reference contractions. Inverted treatment, on the other hand, would yield indexes of —100, —100, —100; since the expansion segments would be matched with reference contractions, and the contraction segments with reference expansions.[76] The signs of the conformity indexes are uncertain, since they reflect the arbitrariness of our procedure. No one who notes that the timing is neutral should be misled. In any case, the absolute magnitudes of the indexes are unequivocal: they tell with what consistency the series rose or fell during the two segments into which the reference cycles are broken. Whether these movements are related positively or inversely to business cycles must be determined on independent grounds.

The marginal divisions of reference cycles merely focus a difficulty that applies also to other, if not to all, divisions. Suppose that stages III-VIII are selected to represent expansions; then we are bound by our rules to match the expansion segments in the series with reference contractions and the contraction segments with reference expansions, and so will get negative conformity indexes.[77] The procedure in this instance is more defensible than in the marginal case, but it rests upon rules that have slight relevance to a theoretical understanding of economic processes. If we can attach no importance to signs of conformity indexes when the expansion runs from stage III to VII, or from stage VII to III, we cannot attach much importance to the signs *per se* when the selected expansion covers stages III-VIII or stages VI-III. If an occasional series on business orders rises typically between stages VI and III, we should probably regard it as conforming positively, although the signs of the conformity indexes were all negative; at any rate we would do so if our

[76] Note that if (1) the selected expansion runs from stage III to VII, (2) the conformity analysis is positive, and (3) the expansion, contraction, and full-cycle indexes come out, respectively, +80, +20, +90: then, a shift to inverted treatment would produce conformity indexes of −20, −80, −90, respectively, not −80, −20, −90.

[77] They will not be negative necessarily. In the first place, the division may be improper. Even if proper, the segment from stage VIII to III may represent a retardation of growth rather than actual decline, so that the expansion index at least will be positive.

immediate task was to trace the transition of general business from contraction to expansion. Of course, the formal inconsistency can be removed by matching stages VI-III with reference expansions, in defiance of the rules, and we have done that in several instances to facilitate comparisons of closely related series. But the thing to note is that even rigid observance of the rules will never lead an investigator astray if he bears in mind that the signs of conformity measures cannot be taken at face value and must submit to the test of reason.[78]

The list of limitations of the conformity measures might be extended, but enough has been said to demonstrate that judgment of the way a process conforms to business cycles cannot be based solely upon the conformity indexes. In addition one should note carefully how the reference cycles are divided into segments of expansion and contraction, remember that the conformity indexes report merely the consistency of rise or fall between the initial and terminal stages of these segments, study the average rates of change, the average deviations about these averages, the full array from which both averages are made, the number of business cycles and the periods they cover, the relation of the average reference-cycle pattern to the average specific-cycle pattern, the behavior in these several respects of related series in the same country and in different countries, and whatever historical and theoretical analysis can teach concerning the causal connections between the given process and business at large. To determine just how much the available time series can be made to reveal about the business-cycle conformity of different economic processes is an elaborate undertaking that requires skillful use of most of the tools in our kit.

XI Analysis of Quarterly and Annual Data

The methods described in this chapter were originally designed to handle monthly records, but they can be adapted to series that run by quarters and even by years. Table 48 shows, measure by measure, how we analyze quarterly and annual data. The description of some of the operations is limited to positive analysis, but with obvious changes in phrasing the descriptions apply also to series treated on an inverted plan.

The analysis of quarterly series is modeled closely upon the analysis of monthly series. But nothing can overcome the greater coarseness of the time unit in measuring leads and lags. Interpolations are more often necessary in making cyclical patterns than when monthly data are used. Nor does our practice of representing peaks and troughs of specific cycles by single quarterly values ensure that the amplitude measurements will be the same as those derived from three-month averages centered on the peaks and troughs of monthly series; for in quarterly series the bound-

[78] See above, p. 116.

TABLE 48

Methods Used in Analyzing Quarterly and Annual Series

Measure	*Quarterly series*	*Annual series*
Table S1, col. (2)-(5)	If the series expresses totals or averages for the full quarter, the middle month of the quarter that marks the turn of the specific cycles is compared with the corresponding monthly reference turn. When the series relates to the first or third month of the quarter instead of the full quarter, the leads or lags are computed from that month. In other respects, the method is the same as in monthly series. See Ch. 6, Sec. VI for contrast with another method of measuring leads or lags; namely, comparison of quarterly specific-cycle turns with quarterly reference turns.	Leads or lags are entered only if the series conforms very well to business cycles, and permits at least a dozen comparisons of specific-cycle and reference turns. In such series the lead or lag is obtained by comparing the midpoint of the year of the specific-cycle turn with the midpoint of the corresponding monthly reference turn. If the series relates to a single month in the year, that month is compared with the monthly reference turn. If the series starts before the monthly reference dates, the specific-cycle turns are compared with the annual reference turns. In other respects, the method is the same as in monthly series. The timing measures for individual annual turns have little value of themselves; their main use is in estimating the average timing. See Ch. 6, Sec. VI for further discussion of the method and its limitations.
Table S1, col. (6)-(13)	Same as monthly.	Col. (6)-(8) and (12)-(13) same as monthly. Entries in col. (9)-(11) are omitted.
Table S2, col. (2)-(4)	The standings are based on single quarters marking the cyclical turns.	The standings are based on years marking the cyclical turns.
Table S2, col. (5)-(10)	Same as monthly.	Same as monthly.
Table S3, col. (2)-(3)	The average includes all quarters from one turn to the next; the values at the turns are weighted one-half each and the intervening quarters one each.	The average includes all years from one turn to the next. The values at the turns are weighted one-half each; the intervening values, if any, are weighted one each.

<table>
<tr><th>Measure</th><th>Quarterly series</th><th>Annual series</th></tr>
<tr><td>Table S3, col. (4)</td><td>The average includes all quarters from trough to trough; the troughs are weighted one-half each and the intervening values one each.</td><td>The average includes all years from trough to trough; the troughs are weighted one-half each and the intervening values one each.</td></tr>
<tr><td>Table S3, col. (5)-(10)</td><td>Same as monthly.</td><td>Same as monthly.</td></tr>
<tr><td>Table S4, col. (2), (6), (10)</td><td>Same as Table S2, col. (2)-(4).</td><td>Same as Table S2, col. (2)-(4).</td></tr>
<tr><td>Table S4, col. (3)-(5)</td><td>The period from the quarter following the initial trough through the quarter preceding the peak is divided into three parts as nearly equal as possible. The period called 'expansion' in this table is therefore 3 months shorter than the true expansion. If exact division of the 'expansion' into thirds is impossible, the remainder, plus or minus one, is placed in the middle third. The operations thus far are, in principle, the same as in monthly series. But interpolations of standings in short phases are made on a different principle: along a straight line connecting the quarterly relatives. For example, if two quarters in all are available for stages II, III and IV, the standing at (the midpoint of) stage II equals the relative at the trough (standing in stage I) plus five-sixths of the excess of the relative in the quarter following the trough over the relative at the trough. Interpolations from quarterly relatives are sometimes identical with interpolations from cycle-stage standings (the method applied in monthly series); that is true of stages II and IV when only one quarter is available for the 'expansion'. If no quarter is available for stages II, III and IV (in other words, if the full length of the expansion is just one quarter) the standings in these stages are interpolated from the standings at stages I and V, being placed three-quarters of a month apart. The slight difference in the method of interpolating for monthly and quarterly series is explained by the fact that single monthly values are more erratic than single quarterly values.</td><td>Only one standing is computed instead of three; namely, at mid-expansion. It is entered in col. (4), which refers to stage III. Col. (3) and (5) are left blank; in other words, there is no stage II or IV. The standing in stage III of each cycle is an arithmetic mean computed according to the following schedule of weights:
<table>
<tr><th>No. of years from turn to turn, counting both</th><th>Weights of successive values in obtaining mid-phase standing</th></tr>
<tr><td>2</td><td>1, 1</td></tr>
<tr><td>3</td><td>0, 1, 0</td></tr>
<tr><td>4</td><td>0, 1, 1, 0</td></tr>
<tr><td>5</td><td>0, 1, 2, 1, 0</td></tr>
<tr><td>6</td><td>0, 0, 1, 1, 0, 0</td></tr>
<tr><td>7</td><td>0, 0, 1, 2, 1, 0, 0</td></tr>
<tr><td>8</td><td>0, 0, 1, 2, 2, 1, 0, 0</td></tr>
<tr><td>9</td><td>0, 0, 1, 2, 2, 2, 1, 0, 0</td></tr>
<tr><td>10</td><td>0, 0, 1, 1, 2, 2, 1, 1, 0, 0</td></tr>
<tr><td>11</td><td>0, 0, 1, 1, 2, 2, 2, 1, 1, 0, 0</td></tr>
</table>
For longer periods the weight scheme is the same as for ten years if the number of years is even, and the same as for eleven years if the number is odd. The long intervals in this schedule occur rarely.</td></tr>
</table>

TABLE 48 — *Continued*

Methods Used in Analyzing Quarterly and Annual Series

Measure	*Quarterly series*	*Annual series*
Table S4, col. (7)-(9)	Same as Table S4, col. (3)-(5), except for obvious changes in wording.	Only one standing is computed instead of three; namely, at mid-contraction. It is entered in col. (8), which refers to stage VII. For the rest, see comments on col. (3)-(5) of this table.
Table S5, col. (2)-(9)	Same as monthly. Note, however, that when the standings in Table S4 are interpolated from quarterly relatives instead of cycle-stage standings, the calculation of the rate of change is made independently for each stage-to-stage interval.	In principle, same as monthly. But now the entries are confined to the rate of change from the trough to the mid-expansion (I-III), the mid-expansion to the peak (III-V), the peak to mid-contraction (V-VII), and mid-contraction to the trough (VII-IX). When the standing in stage III (or VII) is computed by interpolating between the standings in stages I and V (or V and IX), the rate of change from stage I to V represents the rate of change both from stage I to III and from stage III to V (or V to VII and VII to IX). The measure (at the bottom of the table) of the relative amplitude of erratic movements is omitted.
Table R1, col. (2)-(11)	The reference-cycle pattern is made on the basis of quarterly reference dates. The base of the reference cycles is computed on the same principle as the base of the specific cycles. For the rest, see comments on Table S4.	The reference-cycle pattern is made on the basis of annual reference dates. Note that two sets of annual reference dates have been prepared for the United States: by calendar years, and years ending June 30. The base of the reference cycles is computed on the same principle as the base of the specific cycles. For the rest, see comments on Table S4. The comparison of the amplitudes of reference and specific cycles is omitted.
Table R2, col. (2)-(9)	See comments on Table S5.	See comments on Table S5.
Table R3, col. (2)-(9)	Same as monthly.	Same as monthly.

Measure	*Quarterly series*	*Annual series*
Table R4, col. (2)-(9)	Same as monthly.	Stages matched with reference expansion may be I-III, I-V, I-VII, III-V, III-VII, III-IX, V-VII, V-IX, V-III, VII-IX, VII-III, VII-V. In practice, the choice is more restricted, since the duration of reference phases (notably American contractions) is frequently one year. In such cases the standings in stages III and VII are entirely artificial. In view of the difficulty of recognizing leads or lags in cycle-stage units when handling annual data, we often follow another plan. Thus, if an annual series shows a strong tendency to lead the annual reference dates by one year, we postdate the series one year and then compute the conformity on the basis of a typical expansion running from stage I to V, as in Table R3. If the series lags with considerable regularity by one year, we predate the figures one year and then proceed as in Table R3. In handling American series, two sets of annual reference dates are at our disposal. Hence if a calendar-year series shows cyclical turns that lag regularly 6 months behind the fiscal-year reference dates (years ending June 30), we may predate the figures 6 months (the figures for the calendar years 1929, 1930, etc. are treated, respectively, like figures for the fiscal years 1929, 1930, etc.) and then proceed as in Table R3. But we never postdate or predate a series by more than one year.
Chart of cyclical patterns	Same as monthly. Note, however, that leads or lags shown on the chart are obtained by comparing the quarter in which the series makes a cyclical turn with the corresponding quarterly reference date, not by comparing the midmonth of the quarter (or the first or third month if the series refers to the beginning or end of the quarter) with the monthly reference date, as we do in Table S1. The former procedure is necessary because the duration lines of the reference-cycle patterns, as well as the intervals into which they are broken, are derived from the quarterly reference dates.	Same as monthly, except that leads or lags of specific cycles are in no case represented.

aries of the quarters are fixed by conventions of the calendar while in monthly series the boundaries of the quarters are flexible. Nevertheless, as the following chapter demonstrates, the results obtained by applying our analysis to quarterly series are very similar to those we get from monthly series.

When forced to use annual data we simplify our procedure. The observations are not numerous enough to allow the cyclical patterns to be made on the same plan as when the data run by months or quarters. Instead, we take the standing in the year of the initial trough, of the peak, and of the terminal trough as a framework, and complete the pattern by computing the average standing in mid-expansion and in mid-contraction. This procedure gives five-point patterns; but to avoid confusion the numerals I, III, V, VII, and IX are used to indicate the five stages. In Table S1 we usually omit timing comparisons; that is, measures of leads or lags are restricted to fairly long series that conform exceptionally well to business cycles. In Table R4 we modify the procedure in another respect. Since the small number of observations per reference phase makes it difficult to take account of leads or lags in cycle units, we mark off expansion and contraction segments in some series by years instead of by cycle-stages.[79] The brevity of reference phases limits the extent to which we can prudently go in this direction: in making conformity measures we never assume a systematic lead or lag greater than one year.[80]

The results obtained from annual series are less trustworthy than those from monthly or quarterly. Cycle durations become crude approximations, even when the series undergoes cyclical fluctuations that are large in relation to its secular movements. Summation by years tends to reduce or obliterate cyclical swings. Hence cycle durations obtained by our methods from annual series tend to exceed those obtained from monthly or quarterly series. Amplitude measurements, on the other hand, tend to be reduced; so also are the indexes of conformity. Even our simplified patterns involve an over-elaboration of the data when a cycle lasts less than four years, which is fairly common, particularly in American records.[81] These and other shortcomings of annual data for the study of cyclical behavior are analyzed in detail in the next chapter.

79 We are indebted to Moses Abramovitz for this procedure, which is described in Table 48.

80 See above, p. 194.

81 A cycle with a duration of four years gives us five observations; a shorter cycle therefore requires interpolating.

CHAPTER 6

Effects of the Time Unit on Cyclical Measures

In Chapter 1 and more explicitly in Chapter 3 we have explained why we deem it necessary to analyze many time series, why we wish each to cover many cycles, and why we should like to include more countries than we do. By using annual data we could have approximated this far-reaching program better than we have. But we could not meet it nearly so well as we have, if we had spent our resources in elaborating treatment of a relatively small number of monthly or quarterly series; for example, by making one analysis of the raw data adjusted merely for seasonal variations, another of data adjusted for secular trend as well, a third of data adjusted for both erratic and seasonal movements, and perhaps still a fourth of data adjusted in all three respects. The aim of this and the two following chapters is to explain why we have chosen to lavish effort in certain directions and to economize in others. These explanations will not carry conviction to a discerning reader, unless supported by empirical evidence. Hence the chapters abound in detailed demonstrations of the manner in which alternative decisions affect the cyclical measures we make.

I The Problem of This Chapter

Annual data are more abundant than monthly or quarterly. Many important series are available only by years, and the monthly or quarterly series we analyze can often be had for much longer periods in annual form. Also, annual data are simpler to handle; there are no seasonal variations to deal with, and the other computations we make are much abbreviated. By confining ourselves strictly to annual data from the outset, it is not improbable that we could have doubled the number of series analyzed and performed the work in half the time we are taking.

The reason we have not adopted this labor-saving course is that we do not trust pictures of cyclical movements drawn from annual series, except when the process represented consists of operations such as the production of staple crops harvested once a year. Though we think this distrust is justified by experience, there is still sufficient difference of opinion and practice among time-series analysts to call for a thorough examination of the influence of the time unit upon measures of cyclical behavior.[1] This examination will not only make clear why we prefer monthly data despite their relative scarcity and the heavy costs of analyzing them, but will also provide a basis for criticizing the results we get ourselves when, in default of better materials, we resort to series that run by years. We shall take this occasion to deal also with the less acute issue of quarterly versus monthly data.

II Why the Time Unit Matters

The largest number of observations we can make upon a cycle in annual data is one more than the number of years covered. If the cycle lasts two years we can observe only the initial trough, peak, and terminal trough; we treat the last as an observation also upon the initial trough of the following cycle. Quarterly data enable us to make roughly four times and monthly data roughly twelve times as many observations. One-fourth of the business cycles marked off by our monthly reference dates lasted three years or less; more than half of the reference expansions and contractions lasted two years or less, and nearly one-sixth lasted one year or less. Specific cycles have similar durations. The few observations on these short movements yielded by annual data are obviously crude materials for comparing the behavior of different processes in the same cycle, or for comparing the behavior of the same process in different cycles.

Summation by years hides many of the cycles revealed by monthly data, sometimes introduces spurious cycles, and influences the amplitude, pattern, and other features of all cyclical movements. These effects, which we shall presently examine in detail, depend upon the duration of cyclical expansions and contractions in monthly data, their amplitude, and the steepness of the trend. But a less obvious factor also counts heavily: the time of the year when cyclical turns occur.

Suppose that a monthly series has regular two-year cycles that start from 0 at the initial trough, rise by the monthly increment b to $11b$, maintain this peak value for two months, then decline by b each month

[1] We have been influenced most by the experience of one of the authors who in an earlier investigation relied largely upon annual data in trying to get a comprehensive view of 'the rhythm of business activity'. See Wesley C. Mitchell, *Business Cycles* (University of California Press, 1913). For a recent example of reliance upon annual data, see J. Tinbergen, "A Method and Its Application to Investment Activity" and "Business Cycles in the United States of America, 1919–1932" (League of Nations, *Statistical Testing of Business-cycle Theories*, I and II, Geneva, 1939).

to 0 at the terminal trough. So long as we deal with specific cycles of monthly data, it does not matter in what months the troughs and peaks occur. But if the data are summarized by years, the results depend upon the months of the cyclical turns.[2] Calendar-year summation or averaging will wipe out the cycles completely if they make turns in January: [3] for the series will rise one year from 0 to $11b$, it will fall the next year from $11b$ to 0, and the successive annual totals will therefore remain constant. If the turns come in any other month, the cycles in monthly data will be reflected in the calendar-year summations, and their amplitude will be largest when the turning points are at the middle of the year. If the turning month is April, the trough values of monthly averages by years will be $3.25b$ and the peak values $7.75b$. If the turning month is July, the trough values will be $2.5b$ and the peak values $8.5b$. Summation or averaging by years ending June, on the other hand, will wipe out the cycles whose turns come in July, and give maximum amplitude to the cycles with turns in January.

Suppose next that the series has regular three-year cycles and a rising trend, the expansions lasting two years and the contractions one year. A cycle 'starts' from 0 at the initial trough, rises by the monthly increment b to $23b$, maintains this peak value for two months, declines by b each month to $12b$ at the terminal trough, maintains this value for two months, rises again by the monthly increment b to a peak of $35b$, and so on. In this as in the preceding example, summation or averaging by calendar years will wipe out the cycles completely if the turns come in January, and yield maximum amplitudes if the turns come in July. But the results are very

2 The way in which the monthly dates of cyclical turn affect the cyclical movements of annual data can be instructively shown by a formula, for which we are indebted to Edward E. Lewis.

Let Q_1 be the sum of a time series in one calendar year and Q_2 the sum in the next year. Let q_1 represent the January value in the first year, q_2 the February value, etc.; q_{24} represents the December value in the second year. Finally, let $d_1 = q_2 - q_1$, $d_2 = q_3 - q_2$, etc. Then

$$q_{13} - q_1 = d_1 + d_2 + d_3 + \ . \ . \ . + d_{12}$$
$$q_{14} - q_2 = d_2 + d_3 + d_4 + \ . \ . \ . + d_{13}$$
$$\cdots$$
$$q_{24} - q_{12} = d_{12} + d_{13} + d_{14} + \ . \ . + d_{23}$$

If these twelve equations are summed, we get $Q_2 - Q_1$ on the left of the equal sign. On the right we have a weighted sum of the month-to-month differences: d_1 enters this sum just once, d_2 enters twice, d_3 enters three times, and so on through d_{12} which enters twelve times. After that the weights decrease successively by unity.

It is plain that in the hypothetical series discussed in the text, annual data will show a much more substantial movement if the cyclical turns come in the middle of the year than if they come near the boundaries. In the former case the month-to-month movements during the rising phase (which extends from the middle of one year to the middle of the next) will receive a substantially larger weight than the month-to-month movements during the declining halves of the first and second years. This excess will progressively diminish as the cyclical turns come closer to the yearly boundaries.

3 We describe the second of the two months reaching identical peak or trough values as the peak or trough month; or in general, as the month of turn. Similarly, the second of two years having identical maxima or minima is treated as the year of turn. These conventions are adhered to throughout.

TABLE 49

Illustrations of the Dependence of Specific Cycles in Annual Data on the Months of Cyclical Turn

Case A. Cycle period 3 years: expansion 27 months, contraction 9 months, peak and trough values maintained 2 months. Rise per month is *b*; rise per month equals fall per month. Value at trough month in Year I is 0.

Case B. Cycle period 3 years: expansion 30 months, contraction 6 months, peak and trough values maintained 2 months. Rise per month is *b*; rise per month equals fall per month. Value at trough month in Year I is 0.

Case C. Cycle period 3 years: expansion 34 months, contraction 2 months, peak and trough values maintained 2 months. Rise per month is *b*; total rise equals total fall. Value at trough months in Years I, IV, etc. is 0.

Dates of possible specific cycles in monthly data			Total during calendar year (in units of *b*)				Amplitude of specific cycle in annual totals (in units of *b*)		
Trough	Peak	Trough	Year I	Year II	Year III	Year IV	Rise	Fall	Rise & fall
				Case A					
Jan. I	Apr. III	Jan. IV	66	210	273	282	...	...	...
Feb. I	May III	Feb. IV	55	198	278	271	223	7	230
Mar. I	June III	Mar. IV	46	186	281	262	235	19	254
Apr. I	July III	Apr. IV	39	174	282	255	243	27	270
May I	Aug. III	May IV	34	162	281	250	247	31	278
June I	Sep. III	June IV	31	150	278	247	247	31	278
July I	Oct. III	July IV	30	138	273	246	243	27	270
Aug. I	Nov. III	Aug. IV	31	126	266	247	235	19	254
Sep. I	Dec. III	Sep. IV	34	114	257	250	223	7	230
Oct. I	Jan. IV	Oct. IV	39	102	246	255	...	...	...
Nov. I	Feb. IV	Nov. IV	45	90	234	261	...	...	...
Dec. I	Mar. IV	Dec. IV	51	78	222	267	...	...	...
				Case B					
Jan. I	July III	Jan. IV	66	210	318	354	...	...	...
Feb. I	Aug. III	Feb. IV	55	198	317	343	...	...	...
Mar. I	Sep. III	Mar. IV	46	186	314	334	...	...	...
Apr. I	Oct. III	Apr. IV	39	174	309	327	...	...	...
May I	Nov. III	May IV	34	162	302	322	...	...	...
June I	Dec. III	June IV	31	150	293	319	...	...	...
July I	Jan. IV	July IV	30	138	282	318	...	...	...
Aug. I	Feb. IV	Aug. IV	30	126	270	318	...	...	...
Sep. I	Mar. IV	Sep. IV	30	114	258	318	...	...	...
Oct. I	Apr. IV	Oct. IV	30	102	246	318	...	...	...
Nov. I	May IV	Nov. IV	30	90	234	318	...	...	...
Dec. I	June IV	Dec. IV	30	78	222	318	...	...	...
				Case C					
Jan. I	Nov. III	Jan. IV	66	210	318	66	252	252	504
Feb. I	Dec. III	Feb. IV	55	198	341	55	286	286	572
Mar. I	Jan. IV	Mar. IV	78	186	330	78	252	252	504
Apr. I	Feb. IV	Apr. IV	102	174	318	102	216	216	432
May I	Mar. IV	May IV	126	162	306	126	180	180	360
June I	Apr. IV	June IV	150	150	294	150	144	144	288
July I	May IV	July IV	174	138	282	174	144	144	288
Aug. I	June IV	Aug. IV	198	126	270	198	144	144	288
Sep. I	July IV	Sep. IV	222	114	258	222	144	144	288
Oct. I	Aug. IV	Oct. IV	246	102	246	246	144	144	288
Nov. I	Sep. IV	Nov. IV	270	90	234	270	180	180	360
Dec. I	Oct. IV	Dec. IV	294	78	222	294	216	216	432

The second of two months (years) reaching identical peak or trough values is treated as the month (year) of turn. Blank spaces in the last three columns signify that the cycle is skipped.

different if we change one assumption—make the expansions last 27 months and the contractions 9 months. In this case summation by calendar years will wipe out the cycles that have troughs in October, November, December or January; the maximum amplitude will be reached when the troughs occur in May or June, the minimum when they occur in February or September.[4] If we make the contractions last 6 months and the expansions 26 months, the other assumptions being left unchanged, the cycles will disappear regardless of the months of turn. If we change the assumptions by supposing that the trend is horizontal, the fall will equal the rise, and even a two-month contraction will be preserved in annual data.

Table 49 shows the last case, as well as the two preceding ones, worked out in detail. The two-month contraction is reflected in the calendar-year summations no matter in what month the turns come; the maximum amplitude is yielded if the trough occurs in February, and the minimum amplitude if the trough occurs any month from June to October. Another result, not so explicit in the table, is that the two-month contraction is transformed into a one-year contraction if the trough comes between January and May or between October and December, and into a two-year contraction if the trough comes between June and September.[5]

Table 50 shows what combinations of monthly turning points favor the preservation of cyclical contractions (or expansions) lasting 12 months or less, when calendar-year summations are struck. The table is made on the assumption that both the cyclical rise and fall are continuous and linear; also that the rate of rise equals the rate of fall. We see that a contraction starting or ending in January is wiped out in annual data, whether the movement lasts one month or twelve. On the other hand, a contraction starting or ending in July is preserved in annual data, provided it lasts 8 months or longer. July 1 is more favorable to the preservation of cyclical contractions than any other date. This result is reasonable, since a mid-year peak favors a high average for the year and a mid-year trough favors a low average. A contraction lasting 7 months or less is skipped by our imaginary series whatever the months of turn. A contraction lasting 8 to 12 months is skipped if it starts in certain months, but not if it starts in others. As shown in Section XI, a shift from calendar to fiscal years would merely change the months of cyclical turns that are favorable or unfavorable to the preservation of cycles in annual summations.

[4] We consider amplitudes here in absolute units, not in units of cycle relatives; see Table 49, and notes 2 and 3.

[5] This example makes no assumption in regard to the absolute amplitude of the cycles. The successive observations are expressed in units of b, which may be any positive number. The example is also free from special assumptions with respect to the relative amplitude (that is, the total rise and fall expressed as a percentage of the average value during a cycle); for if we use x instead of 0 at the trough month in Year I, where x stands for any positive number, the general results are unchanged.

TABLE 50

How Months of Cyclical Turn Determine Whether Brief Cyclical Phases Remain or Disappear in Calendar-year Summations

ASSUMPTIONS: Cycle period, 3 years. Contraction (or expansion) lasts 12 months or less. The rise is continuous and linear from the trough to the peak; the fall is continuous and linear from the peak to the trough; and the *rate* of rise equals the *rate* of fall. The months are treated as if they were equal in length.

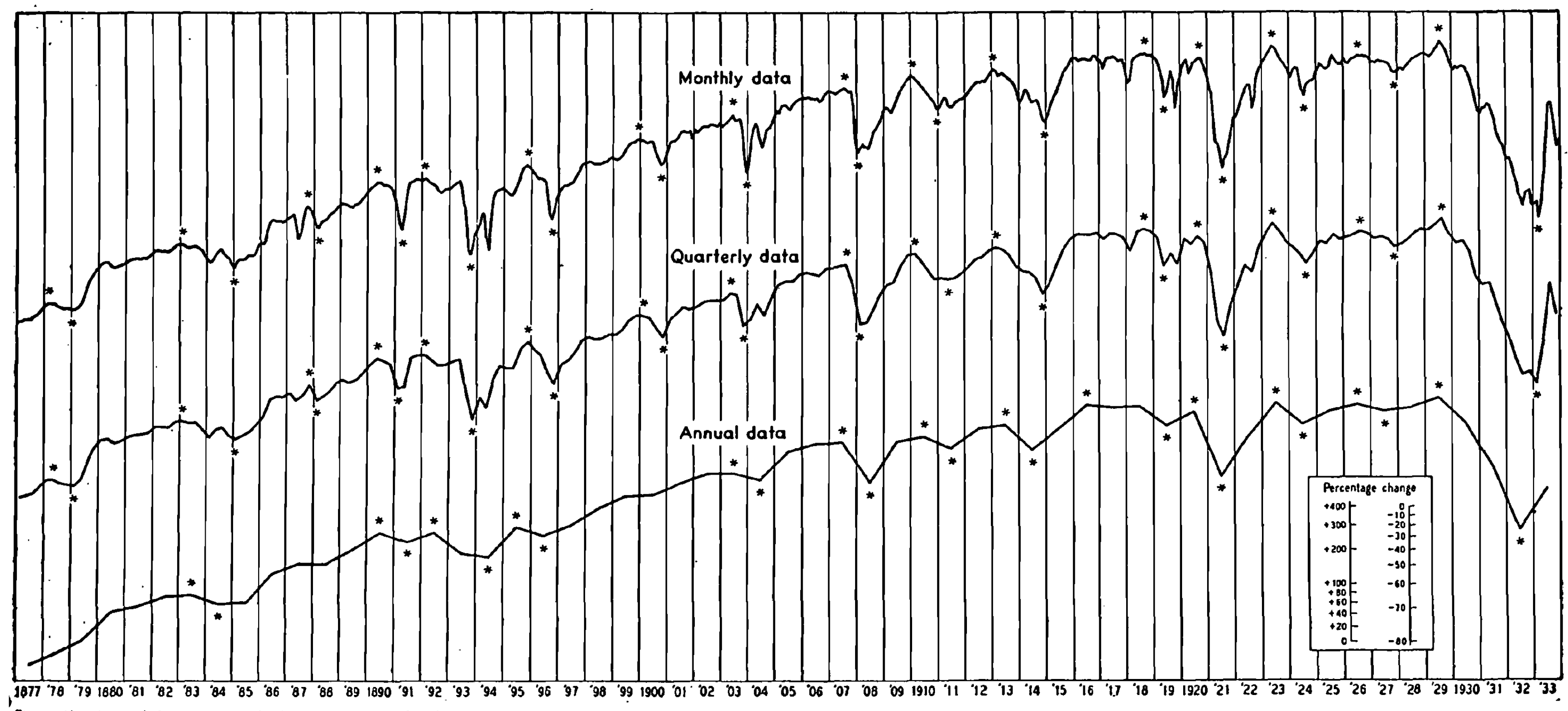
In view of the assumption of continuity, calendar-year totals were obtained by summing areas under the curve. Movements preserved in annual data are marked X. A blank space indicates that the phase is wiped out. If the table is read for contractions, a rising trend is implied; if read for expansions, a declining trend is implied.

Contraction (or expansion) *begins* on the first of	Contraction (or expansion) *ends* on the first of											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.												
Feb.		X										
Mar.		X	X									
Apr.		X	X	X								
May		X	X	X	X							
June			X	X	X	X						
July			X	X	X	X	X					
Aug.				X	X	X	X	X				
Sep.					X	X	X	X	X			
Oct.						X	X	X	X	X		
Nov.							X	X	X	X	X	
Dec.									X	X	X	X

An investigator working with annual data does not know in what months the cyclical turns come, the duration in months of the expansions and contractions, or other features of the basic cyclical movements. He therefore cannot allow for the influence of these factors upon the number of specific cycles that appear in annual data, or upon their duration and amplitude. The most he can do is to make a vague allowance for the influence of the secular trend relatively to the amplitude of the cyclical movements. Whether or not these limitations of annual data are serious in practice cannot be settled by imaginary series. The proper method is to take records of experience and compare cyclical measures made from one observation a year with measures made from four or twelve.

Towards this end we have made quarterly and annual averages or totals for six long monthly series for the United States, and compared in detail cyclical measures derived from these records. The series include pig iron production, 'deflated' bank clearings outside New York City, railroad stock prices, number of shares traded on the New York Stock Exchange, call money rates, and railroad bond yields. Chart 23 illustrates

CHART 23
Pig Iron Production, United States, 1877–1933
Monthly, Quarterly and Annual



The monthly and quarterly figures are seasonally adjusted. Asterisks identify peaks and troughs of specific cycles. For sources of data, see the note at the end of Sec. II.

the movements of the three forms of pig iron production.[6] Whatever differences appear among the measures made from the three forms of the same data must be due solely to differences in the time unit, provided our methods of analysis are kept strictly uniform. That proviso we observe as far as possible.[7]

III The Direction of Movements in Time Series

Annual data cannot reveal changes within a year; they reveal only changes between years. A like remark applies to monthly and quarterly data. But data that reveal changes between months enable us to learn what happens within quarters, and data that reveal changes between quarters enable us to learn what happens within years.

It is necessary to insist upon these distinctions because they are frequently obscured by the charts we draw and the words we use in describing the movements of time series. For example, we are apt to say on the basis of annual data that the output of iron 'rose in 1910', though the data merely tell that output in the calendar year 1910 was larger than in the calendar year 1909. We do not know whether this difference is due to a continuous rise in both years, or to a fall within 1909 and a greater rise within 1910, or to a rise within 1909 and a slower fall within 1910, or to some combination of irregular movements. Charts may prove equally misleading. To represent outputs of successive years, the statistician commonly plots points in the middle of the years at heights indicated by the vertical scale and connects the points by straight lines. The chart therefore pictures a continuous rise during the fiscal year 1910, that is, from the end of June 1909 to the end of June 1910. This graphic presentation not only is more brazen than the verbal statement but also conflicts with it, since the latter suggests that the rise occurred during the calendar year

[6] See also Charts 29-31. The monthly data of all six series are shown in Chart 53. Cycle-by-cycle measures of the monthly data are presented in Appendix B.

[7] All monthly series are adjusted for seasonal variations, except railroad bond yields and stock prices; the seasonal correction of call money rates stops in May 1931. The annual figures were derived from monthly data unadjusted for seasonal variations, which is the form in which they would come to an investigator concerned with annual figures. However, the quarterly figures were derived from seasonally adjusted monthly data. That step saved time and ensured that comparisons between quarterly and monthly data were not confused by differences in seasonal adjustments. (The latter objective could have been achieved another way; viz., computing quarterly seasonal indexes for the same period and by the same method as in the monthly data, and removing the seasonal variations directly from the quarterly data.)

The series on pig iron production and deflated clearings are on an average daily basis.

For the series on call money rates, railroad bond yields and railroad stock prices, see Frederick R. Macaulay, *The Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856* (National Bureau of Economic Research, 1938), Appendix A, Table 10. col. 1, 5, and 6, pp. A142-61. For the series on pig iron production and deflated clearings (continued after 1918 with bank debits in 140 centers outside N.Y.City, adjusted to the level of 'outside' clearings in Jan. 1919, and deflated throughout by Snyder's index of the general price level), see *ibid.*, Appendix A, Table 27, col. 4 and 2, pp. A255-69. The series on number of shares of stock traded through 1897 comes from *Commercial and Financial Chronicle;* since 1898, *New York Stock Exchange Bulletin,* Aug. 1934, pp. 10A-B.

1910.[8] Of course if we read our tables and charts by saying, as cautious students often will, that 'yearly output rose between 1909 and 1910' or 'from 1909 to 1910', instead of saying that output (the time unit unspecified) 'rose in 1910', our words are less misleading, in that they do not conceal our ignorance concerning what happened within either year.

Perhaps the investigators who rely upon annual data in studying cyclical movements remember most of the time that they are dealing solely with changes between years and know nothing of what happens within years. But investigators like ourselves, who use monthly data when available and fall back upon annual data only when nothing better can be had, may be betrayed by their words into assuming that both forms of data show the direction of cyclical movements 'in' successive years. As a sample of the errors that arise in practice from this assumption, we have made the comparisons summarized in Table 51 between the directions in which the monthly and annual data of our six test series move.

The direction of movement *within* years can be judged from the specific cycles marked off in the monthly data. In other words, we consider the direction as upward throughout a specific-cycle expansion and as downward throughout a contraction; the month-to-month flutterings within a specific-cycle phase are disregarded. When a specific-cycle expansion continues for a major fraction of a year, or when there are at least seven months of expansion interrupted by a brief contraction, we say that the 'prevailing' direction within the year is upward. When cyclical contraction covers seven or more months, we say that the prevailing direction within the year is downward. Under this rule, some 8 or 10 per cent of the years covered by our sample lack a prevailing direction; we may speak of them as 'neutral' years. In this way we have classified both the calendar and the fiscal years (that is, years ending in June) covered by the monthly data of each of the six test series.

The direction of movement *between* years has been judged similarly from the specific-cycle expansions and contractions we recognize in the annual data. Two interpretations may be placed on these year-to-year changes. First, it may be assumed that the change from one calendar year to the next indicates the prevailing direction of movement *within* the second year. Let us call this assumption I. Its graphic equivalent is that the annual entries are plotted at the ends of calendar years. Comparisons of annual movements between calendar years with the monthly movements prevailing within calendar years appear in columns (4) to (6) of Table 51. Second, it may be assumed that the change from one calendar year to the next indicates the prevailing direction of movement *within* the 'fiscal year' that begins in July of the first and ends in June of the second year. Call this assumption II. Its graphic equivalent is that the annual data are

8 But the customary graphic presentation leads to errors less frequently than the verbal statement, as is demonstrated later.

TABLE 51

Comparison of the Directions of Movement of Monthly and Annual Data
Six American Series

Series	Period covered	No. of years covered	No. of years in which prevailing direction of monthly data					
			During a given calendar year			During a given fiscal year		
			Agrees with change of annual data from preceding cal. year	Disagrees with change of annual data from preceding cal. year	Is lacking	Agrees with change of annual data between overlapping cal. years[a]	Disagrees with change of annual data between overlapping cal. years[a]	Is lacking
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Deflated clearings...	1878–1933	55	41	9	5	47	4	4
Pig iron production..	1879–1933	54	33	15	6	41	4	9
Railroad stock prices.	1857–1932	75	52	14	9	69	3	3
Shares traded.......	1878–1933	55	35	16	4	42	9	4
Call money rates....	1858–1931	73	54	13	6	64	3	6
Railroad bond yields.	1860–1931	71	53	9	9	61	4	6
Six series								
Absolute number..		383	268	76	39	324	27	32
Percentage.......		100.0	70.0	19.8	10.2	84.6	7.0	8.4

The first entry in col. (2) is the year of the initial trough of the first full cycle covered by the monthly analysis; the second entry is the year in which the terminal trough of the last full cycle occurs. The heading of col. (4)-(6) relates to the period in col. (2) except the first year. The heading of col. (7)-(9) relates to a period starting and ending six months earlier than is covered by col. (4)-(6). In making this table we took account of several peaks at the ends of the series.

As explained in the text, the 'prevailing' direction of monthly data during a year is that which dominates seven months or more of the year. The rise or fall during the year need not be continuous; for example, if the monthly data reach a specific-cycle peak in April and a trough three months later, the prevailing direction during the calendar year is considered as upward. A rise from a trough in January to a peak in July is counted as six months, although seven monthly entries fall within the segment including the trough and peak dates. Similarly, if the rise begins before the given calendar year, culminates in a July peak, and is followed by a decline during the remainder of the year, six months are allotted to expansion and five months to contraction. We consider such a year as not having a prevailing direction of movement. Like rules govern the movements of monthly data in fiscal years, that is, years ending in June.

[a]That is, between the calendar years overlapping the fiscal year.

plotted at the centers of the calendar years. Comparisons of annual movements between calendar years with the monthly movements prevailing within the overlapping fiscal years appear in columns (7) to (9). Of course the results of both sets of comparisons are subject to the uncertainties that beset efforts to identify the specific cycles of time series. Revisions might remove some of the differences between the directions in which the annual and monthly data are said to move; but they might equally well introduce some differences where we now find agreements.

The table demonstrates that the prevailing direction of the monthly data differs from the direction of the annual data in an appreciable fraction of the years covered by our sample, but that the differences are more numerous if we make assumption I than if we make assumption II. If we interpret the changes between calendar-year figures as representing the prevailing direction of movement within calendar years, our test series indicate that we shall be right in 70 per cent of the years. But if we interpret the changes between calendar-year figures as representing the prevailing direction of movements within fiscal years, we shall be right 85 per cent of the time. For future guidance we add an obvious inference:

when forced to work with data that run by fiscal years ending in June, we shall err frequently if we interpret their year-to-year changes as showing the prevailing direction of movement within calendar years, but less frequently than if we interpret them as showing the prevailing direction of movement within fiscal years.

The reason assumption I leads to more serious errors than assumption II is easily explained. Suppose (a) that a series reaches a peak in some month within two calendar years, but makes no other turn in this period; (b) that the series rises to a peak by uniform increments, then falls away by uniform decrements; (c) that the rates of rise and fall need not be the same; and (d) that the average value of the series is higher in the second calendar year than in the first. If the peak comes in May of the second year or some earlier month, the prevailing direction of the monthly data during the second calendar year will be downward under our seven-month rule, and thus contradict the rise assumed in the annual data. A contradiction can occur whether the monthly rate of decline exceeds, is less than, or equals the rate of rise. But to produce a seven-month decline in the monthly data during the fiscal year overlapping the two calendar years, the peak must come not later than November of the first calendar year; under our assumptions this can happen only if the rate of rise to the peak exceeds by some margin the rate of decline after the peak. It follows that comparisons of monthly and annual data based on assumption II are less likely to show differences of direction than comparisons based on assumption I. A similar argument may be formulated regarding declines in annual data associated with troughs in monthly data. Of course, the path of the cyclical expansions and contractions of our test series is usually not linear, erratic movements diversify the expansions and contractions, and two cyclical turns occur in some years; but these differences from our imaginary series are not more favorable to the comparisons based on assumption II than to the comparisons based on assumption I.

Granted the representativeness of our sample, it appears from Table 51 that assumption I leads to wrong conclusions concerning the direction of the cyclical tide prevailing within years three times out of ten, while assumption II leads to wrong conclusions three times out of twenty. Wrong conclusions are likely to be drawn in a still greater proportion of years if annual data are used to compare the direction of cyclical movements of two or more series *within* successive years. In line with the results yielded by assumption I, let us suppose that series A and B both cover 100 years, that the prevailing movement within 20 calendar years of each series runs counter to the change of annual data from the preceding year, and that in another ten years of each series there is no prevailing movement. Call the changes in the annual data in these 30 years 'errors', though the real error lies in treating year-to-year movements as showing the direction of movements within years. If the errors in the two series all occur in different years, which is unlikely, there will be 60 errors in the comparison. If the years of error in the two series coincide, which is still less likely, only 15 errors in the comparison are to be expected.[9] If the errors in the two series are not correlated, which seems the most reasonable assumption, the expectation is that invalid comparisons will number about 46 in 100, or nearly half of the total.[10] Table 52 shows the actual number of valid and invalid comparisons for every possible pair of our six series. The number of invalid comparisons ranges from 36 to 56 per cent for the individual pairs, and is 45 per cent on the average.[11]

[9] Of the 30 years in which errors occur in *A*, 10 have no prevailing direction. The like applies to *B*. The probability that both *A* and *B* will have no prevailing direction in any year of the 30 is therefore 1/9; the probability that both will have a prevailing direction in any year is 4/9; the probability that *A* or *B* but not both will have a prevailing direction in any year is also 4/9.

In the years of error in which the monthly data of both *A* and *B* have a prevailing direction, the comparison of annual data must be valid despite the errors in each series taken separately; that is, the comparisons will show correctly the presence or absence of agreement in prevailing directions. The number of such years is 4/9 of 30. On the other hand, the comparison of annual data must be invalid in years in which one of the two series has no prevailing direction; for annual data invariably move up or down, our criteria of movement being the expansions and contractions of the specific cycles distinguished in the annual series. The number of such invalid comparisons is also 4/9 of the total. The remaining 1/9 are years in which both *A* and *B* have no prevailing direction. We may expect these years to be divided equally between valid and invalid comparisons: the annual comparison will be valid if the changes in *A* and *B* are in the same direction, it will be invalid if the changes are in opposite directions. The expected number of wrong annual comparisons is therefore $30 (4/9 + 1/18)$ or 15, which is also the expected number of valid comparisons in the 30 years considered.

[10] The expected number of years in which an error occurs in *A* but not in *B* is $100 (3/10 \times 7/10) = 21$. The expected number of years in which an error occurs in *B* but not in *A* is likewise 21. In these 42 years the annual comparison is invalid. The expected number of years in which an error occurs in both is $100 (3/10 \times 3/10) = 9$. As explained in the preceding note, the comparison will be valid in one-half of the 9 years, and invalid in the remaining half. Hence the expected number of invalid comparisons is 46.5 per 100. This result is based on what we have called assumption I. If, instead, we adopt assumption II, the expected number of invalid comparisons is 26.6 per 100. See also the next note.

[11] The comparison of year-to-year changes in direction of two annual series is considered valid in Table 52 if (a) the direction of movement in each annual series of the pair is correct according to the monthly data; or (b) if the direction of movement of each annual series is opposite to that of the monthly data (for in that case the annual comparison will still show correctly whether the two monthly series moved in the same or opposite directions); or (c) if both annual series show the same direction while both monthly series lack a 'prevailing' direction (for, once again, the annual series indicate agreement when agreement, in a technical sense, exists in fact according to monthly data). The comparison of year-to-year changes in direction of two annual series is considered invalid if (d) the direction of one is opposite to that of the monthly data while the direction of the other is the same as that of the monthly data; or (e) if one monthly series of the pair lacks a prevailing direction (for in that case annual data will show that both series moved in similar or opposite directions, whereas in fact one series lacks a prevailing direction); or (f) if the two annual series move in opposite directions when both monthly series lack a prevailing direction.

Of course, the calculations could be made on a different principle. Thus the number of valid comparisons might be restricted to type (a), in which case the annual showing would be still worse. On the other hand, the inevitable failure of two annual series to match a 'quasi-inverted' relation of the corresponding monthly series (that is, one having and the other lacking a prevailing direction) might be given a smaller weight than the failure of annual series to report correctly similar or opposite movements of monthly series; if this were done the showing of annual data would be improved.

Note, finally, that the calculations in Table 52 are restricted to what we have called assumption I, that the results are roughly consistent with the expectations developed in the preceding note, and that the expectation based on assumption II is also stated in that note.

TABLE 52

Comparison of the Directions of Movement of Two Series in Annual Form with the Directions of the Same Series in Monthly Form

Every Pair of Six American Series

Paired series	Period covered	No. of years covered	Agreement or disagreement in direction of year-to-year changes of two annual series is: Confirmed by monthly data		Contradicted by monthly data	
			No. of years	Per cent of years	No. of years	Per cent of years
Clearings & iron output....	1880–1933	54	33	61.1	21	38.9
Clearings & stock prices....	1879–1932	54	31	57.4	23	42.6
Clearings & shares traded..	1879–1933	55	30	54.5	25	45.5
Clearings & call rates......	1879–1931	53	34	64.2	19	35.8
Clearings & bond yields....	1879–1931	53	32	60.4	21	39.6
Iron output & stock prices..	1880–1932	53	32	60.4	21	39.6
Iron output & shares traded	1880–1933	54	24	44.4	30	55.6
Iron output & call rates...	1880–1931	52	24	46.2	28	53.8
Iron output & bond yields..	1880–1931	52	23	44.2	29	55.8
Stock prices & shares traded	1879–1932	54	31	57.4	23	42.6
Stock prices & call rates...	1859–1931	73	39	53.4	34	46.6
Stock prices & bond yields..	1861–1931	71	39	54.9	32	45.1
Shares traded & call rates..	1879–1931	53	28	52.8	25	47.2
Shares traded & bond yields	1879–1931	53	29	54.7	24	45.3
Call rates & bond yields....	1861–1931	71	42	59.2	29	40.8
Total..................		855	471	55.1	384	44.9

See text, especially note 11; also note to Table 51.

IV The Number of Specific Cycles

An investigator who relies upon annual data must mark off specific cycles on the basis of what he can see in these materials. To find what effect the time unit has upon cyclical measurements, we should subject ourselves to the conditions under which such an investigator works; that is, when analyzing annual data we should not use the fuller knowledge supplied by monthly figures. We have attempted to follow this rule in marking off specific cycles in annual data; and our decisions have already been applied in the preceding analysis of the direction of cyclical movements.

In marking off specific cycles in annual data our general rule is to treat every rise or fall as cyclical, except when it is well below the typical range of movements characteristic of the series. We ignored only five movements in our test series, in each instance on the ground that the movement seemed to be a trifling pause within a cyclical expansion or contraction. After the work had been done, we found that two of these exceptions were valid according to our analysis of the monthly series; that is, there seems to be only a single cycle between 1914 and 1919 in pig iron production, and between 1899 and 1905 in railroad bond yields. The remaining three exceptions come in railroad bond yields, and our decisions in these instances are wrong if the decisions based on the monthly data are right.[12] If we had treated every rise or fall in annual data, however slight, as a cyclical movement, we should have a slightly different set of specific cycles. A shift in the method of marking off cycles would therefore blur some of our detailed results; but it could not affect the substance of the argument in this chapter.[13]

When annual data fail to reflect two consecutive cyclical turns in monthly data, they skip a contraction or an expansion and therefore have one cycle less than the monthly data. Of the 218 cyclical turns [14] in our six series, 39 are skipped and there is a resulting loss of 20 specific cycles in the annual records.[15] But annual data sometimes show 'extra' cycles, and they may do so frequently if erratic movements are very large in relation to the cyclical movements. When war broke out in 1914, the interest rate on call loans shot up from 2.88 in July to 6.87 the following month, but

12 The magnitude of the movements disregarded in bond yields is shown by the following figures:

Year	Average yield (per cent)	Per cent change
1872	6.185	
1873	6.214	+0.5
1876	5.168	
1877	5.178	+0.2
1900	3.182	
1901	3.181	−0.03
1913	4.057	
1914	4.046	−0.3

The rise between 1917 and 1918 in iron production, which we have also disregarded, is 0.8 per cent. Its magnitude in relation to that of other fluctuations in the series may be judged from Chart 23.

13 To test the consequences of a shift in method, we took another sample of sixteen long series. We recognized 228 specific cycles in this group, and disregarded 17 trifling movements. Upon checking the latter against the lists of specific cycles in the monthly series, we found that nine were cyclical and eight were not. Thus the larger sample confirms the one analyzed in this chapter: by treating every movement in annual data, however small, as cyclical, we would increase the number of specific cycles, but about half of the increase would consist of spurious movements. (For the analysis of this additional sample, we are indebted to Julius Shiskin.)

The recognition of every rise or fall in annual data as a cyclical movement dispenses with judgment. Further, in series with slight or no erratic movements, the 'mechanical' method is superior to ours in that it tends to make the average duration of specific cycles approach more closely the average yielded by monthly data. Our method, on the other hand, tends to give closer approximations to the average amplitudes of monthly data. When a series is characterized by erratic movements that are violent relatively to the cyclical movements, numerous changes in annual data may not be cyclical; so that the mechanical method may yield poorer estimates of both the average duration and the average amplitude of specific cycles.

14 This count is based on complete positive cycles. If the first full specific-cycle phase of the monthly data is a contraction or if the last full phase is an expansion, it is ignored in the analysis of this chapter. The exclusions may be identified from Chart 53. For the periods covered in our analysis of the six series, see Table 55.

15 In pig iron production the annual data skip the initial trough but reflect the peak of the 'first' cycle (1879–85) in the monthly data; that means, of course, that they skip also the peak preceding the first cycle, and explains why an odd number of turns in our monthly analysis is skipped. See Chart 23.

by December it was lower than in July.[16] Annual data convert this violent random movement into a bulge that looks like a genuine cycle: the annual average is raised from 3.19 in 1913 to 3.46 in 1914, after which it declines to 1.91 in 1915. Thus the specific cycle with a trough in November 1911, peak in December 1912, and trough in November 1915 gets subdivided in the annual series into two cycles: 1911 (trough), 1912 (peak), 1913 (trough), 1914 (peak), 1915 (trough). The extra cycle plus 20 skipped cycles make 21 discrepancies between the specific cycles in the monthly and annual data; the net difference is 19.

In general, when annual summations are struck the fate of a cyclical movement depends on (1) its duration, (2) its amplitude, (3) the timing of the turns, (4) the pattern of the movement, (5) the steepness of the underlying trend, (6) the nature of the erratic fluctuations. The basic factor is the duration of the cyclical phase. Other things equal, a phase lasting twelve months has a better chance of turning up in annual data than one lasting nine months, and a phase lasting nine months than one lasting six. A phase lasting well over a year is reasonably certain to appear in annual data whatever its other characteristics may be. Factors (2)-(5) are important in short phases, and the shorter the phase the greater is their importance. Given the duration, a brief phase has a better chance of turning up in annual figures if it begins or ends around the middle of the year than if it begins or ends at the boundaries of the year.[17] A cyclical phase that laps over two years has a better chance of turning up in the annual data if the peak and trough zones are gently rounded than if they are sharply angular. The chances are also better if its amplitude is large than if it is small relatively to a given trend; or if the trend is moderate than if it is steep relatively to a given amplitude.[18] Since no one can be sure what annual figures will do to a cyclical movement unless he knows exactly how the six factors combine to shape the movement, no one can tell with any certainty by examining annual data alone whether they skip many or few cycles; or which cyclical movements are larger and which are smaller than the hidden movements, or which movements, if any, spring from erratic fluctuations.

In our sample the expansions or contractions skipped by annual data are usually brief and mild (Tables 53-54). No movement lasting over 16 months or having an amplitude of more than 66 points is erased by annual data. On the other hand, 9 of the 15 phases lasting less than 9 months and

[16] The figures cited are monthly averages, seasonally adjusted.

[17] See above, Sec. II.

[18] This is the only way in which the secular trend influences directly the number of cycles in annual data unadjusted for trend, the form in which we usually analyze time series. But secular trends influence the number of cycles also indirectly, since the retention of trends tends to reduce durations and amplitudes—of contractions in monthly series if the trend is rising, of expansions if the trend is declining. These effects are already allowed for in factors (1) and (2). See Ch. 7, especially Sec. VII, where cycles of monthly and annual data are compared in trend-adjusted form.

TABLE 53
Characteristics of Cyclical Phases Skipped by Annual Data
Six American Series

Series and skipped phase	Characteristics of skipped phase: Nature[a]	Duration (mos.)	Amplitude in specific-cycle relatives	Number of expansions or contractions that are not skipped although they are of: Equal or shorter duration, In same series	Equal or shorter duration, In six series	Equal or smaller amplitude, In same series	Equal or smaller amplitude, In six series	Shorter duration & smaller amplitude, In six series	Smaller size[d], In six series
DEFLATED CLEARINGS									
June 1887 – Mar. 1888	C	9	6	4	20	2	11	1	4
July 1903 – May 1904	C	10	6	5	23	2	11	1	4
Mar. 1910 – Oct. 1910	C	7	3	4	14	..	1	..	..
Aug. 1918 – Dec. 1918	C	4	5	1	3	..	4	..	..
May 1923 – Sep. 1923	C	4	6	1	3	2	11	..	1
PIG IRON PRODUCTION[b]									
Oct. 1887 – Mar. 1888	C	5	26	..	4	2	66	1	19
Dec. 1899 – Oct. 1900	C	10	30	5	23	2	74	9	43
RAILROAD STOCK PRICES									
Oct. 1857 – Mar. 1858	E	5	18	..	4	12	50	..	11
Apr. 1900 – Sep. 1900	C	5	5	..	4	..	4	..	1
CALL MONEY RATES									
Jan. 1876 – Sep. 1876	C	8	52	1	18	7	110	8	62
Jan. 1878 – Sep. 1878	C	8	66	1	18	10	129	12	72
May 1882 – Sep. 1882	E	4	60	1	3	7	116	1	35
Aug. 1918 – Dec. 1918	C	4	31	1	3	1	75	1	19
RAILROAD BOND YIELDS									
Jan. 1864 – July 1864	C	6	13	..	10	22	36	..	10
Nov. 1866 – Dec. 1867	E	13	3	7	48	1	1	..	1
July 1872 – Nov. 1873[c]	E	16	5	13	73	4	4	1	11
Aug. 1876 – Apr. 1877[c]	E	8	2	2	18	..	..	..	..
Dec. 1913 – June 1914[c]	C	6	2	..	10	..	..	..	..
Sep. 1918 – Nov. 1918	C	2	8	..	1	12	17	..	..

For periods covered, see Table 55. There are no entries for shares traded, since annual data reflect every specific cycle in this series.
[a] C stands for specific-cycle contraction, E for expansion.
[b] See note 15.
[c] See note 12.
[d] See note 19.

having an amplitude under 20 points, and all 3 phases lasting less than 5 months and with an amplitude under 10 points, are wiped out. But we also find that for every movement skipped by a series, there are usually several, sometimes many, phases of equal or shorter duration, or of equal or smaller amplitude, that are not skipped. Some skipped phases are both longer and larger movements than other phases that are not skipped. To cite an example, annual data on pig iron production wipe out the contraction of 1899–1900, which lasted 10 months and attained an amplitude of 30 points. Fully 9 movements in our six series are reflected in annual records although they have both shorter durations and smaller amplitudes than this contraction in the iron series. And the number swells to 43 if the 'size' of cyclical movements is judged from an index that assigns equal

TABLE 54

Joint Distribution of Durations and Amplitudes of All Cyclical Phases and Those Skipped by Annual Data

Amplitude of phase in specific-cycle relatives	Duration of phase in months					
	Under 5	5–8	9–12	13–16	Over 16	Total
Under 10	3 (3)	7 (4)	5 (2)	8 (2)	7 (0)	30 (11)
10 - 19	0 (0)	5 (2)	4 (0)	8 (0)	19 (0)	36 (2)
20 - 29	1 (0)	3 (1)	0 (0)	2 (0)	13 (0)	19 (1)
30 - 39	1 (1)	0 (0)	2 (1)	5 (0)	10 (0)	18 (2)
40 - 49	0 (0)	2 (0)	4 (0)	1 (0)	11 (0)	18 (0)
50 - 59	0 (0)	3 (1)	0 (0)	2 (0)	6 (0)	11 (1)
60 - 69	2 (1)	3 (1)	2 (0)	1 (0)	9 (0)	17 (2)
Over 69	1 (0)	1 (0)	6 (0)	10 (0)	45 (0)	63 (0)
Total..........	8 (5)	24 (9)	23 (3)	37 (2)	120 (0)	212 (19)

Based on six American series. The figures in parentheses show the number of expansions or contractions skipped by annual data; those not in parentheses show the full number, whether skipped or not, in complete positive cycles. For the periods covered, see Table 55; see also note 15.

weight to their duration and amplitude.[19] The use of annual data is sometimes defended on the ground that they wipe out movements that are too small 'to matter'. Of course, everyone is free to decide what movements matter for his purposes; but the investigator who takes annual data as his guide will be frequently misled—he will ignore the contraction of 1899–1900 in iron production and lavish attention on 43 smaller movements in our test series.

Table 55 shows the number of specific cycles in the monthly, quarterly, and annual forms of each of our series. The fraction of the net number of cycles lost in annual data ranges in different series from zero to one-third; the fraction lost in the six series is nearly one-fifth.[20] The shift from monthly to quarterly data affects the number of specific cycles in the

[19] The 'size' of a cyclical movement depends on its duration, amplitude, and pattern. A rough index of size may be obtained by multiplying the duration of a phase by its amplitude. This index is 300 for the contraction in iron production from 1899 to 1900. In all, 43 cyclical movements that are not skipped by the annual data of our six test series have smaller indexes. The entries in the last column of Table 53 are made on this plan. See also below, Sec. XI.

[20] In the sample referred to in note 13, there are 249 specific cycles in the monthly and 226 cycles in the annual data. The annual data 'skip' 37 cycles in the monthly data, but show 14 'extra' specific cycles; hence the net loss of specific cycles is only about one-tenth. This sample is dominated by series of narrow coverage characterized by large erratic fluctuations. In series of this type spurious cycles may easily appear in annual data. (The number of annual cycles given in this note differs slightly from that in note 13. Here the count starts and ends with specific cycles having corresponding turns in the annual and monthly series.) A sample of six series analyzed in the next chapter (see Table 103) shows 8 per cent fewer cycles in annual than in monthly series when both are adjusted for secular trend. The corresponding loss in annual data not adjusted for trend is 26 per cent. (Three series in the latter group are included in the sample analyzed in this chapter.)

same direction as the shift from monthly to annual data but in much slighter degree.[21] Only two specific-cycle movements are lost, the contractions in clearings and call money rates in 1918. Both contractions are exceptionally brief and mild: we might have disregarded them even in the monthly series, were it not for our practice of relaxing rules in the case of dubious but conforming movements in series that on the whole conform well to business cycles.

TABLE 55
Number of Specific Cycles in Monthly, Quarterly and Annual Data
Six American Series

Series	Period covered by monthly cycles	Number of specific cycles: Actual numbers			Number of specific cycles: Relatives of monthly		
		M	Q	A	M	Q	A
Deflated clearings..........	1878–1933	15	14	10	100	93	67
Pig iron production........	1879–1933	15	15	12	100	100	80
Railroad stock prices........	1857–1932	18	18	16	100	100	89
Shares traded..............	1878–1933	15	15	15	100	100	100
Call money rates...........	1858–1931	23	22	20	100	96	87
Railroad bond yields........	1860–1931	20	20	14	100	100	70
Total.....................		106	104	87	100	98	82

M stands for monthly data, Q for quarterly, A for annual. The number of cycles in the quarterly and annual data is the number of complete cycles, taken positively, within the periods covered by the monthly cycles. See Table 57.

V Duration of Specific Cycles

Just as lengths of objects are known less precisely if measured to the nearest foot than to the nearest inch, so measures of cyclical duration are less precise if made to the nearest year than to the nearest month. But there is this difference between the two: telegraph poles are the same poles whether measured in feet or in inches, but cycles are not the same cycles when measured in years as when measured in months, except, of course, when an investigator merely 'rounds' observations of monthly data. The measures of duration made from annual data are thus less precise in two senses: first, they are expressed in a coarser unit; second, this unit changes the form of the cycles—a substantial number of specific cycles are obliterated and those that are left are modified. We may call the first the 'precision effect', and the second the 'twisting effect'.

The two effects are illustrated in Table 56. The table presents three types of duration measures for successive specific cycles in pig iron production: the monthly measures as they come, the monthly measures read

21 In marking off cycles in quarterly data we took full notice of our monthly decisions. That step is not inconsistent with our treatment of annual data. Borderline cases in monthly data may or may not match borderline cases in annual data. But a borderline case in a monthly series is usually a borderline case also in a quarterly series, and vice versa. To isolate the influence of the time unit upon cyclical measures, it seemed best to treat the borderline cases similarly.

TABLE 56
Duration of Specific Cycles Measured by Different Methods
Pig Iron Production, United States, 1879–1933

Specific cycle[a]	Duration in months ascertained from								
	Monthly data			Monthly durations read to the nearest year			Annual data		
	Expansion	Contraction	Full cycle	Expansion	Contraction	Full cycle	Expansion	Contraction	Full cycle
1879–1885.......	49	23	72	48	24	72	([c])	12[d]	([c])
1885–1888.......	33	5	38	36	0	36	{72	12	84}
1888–1891.......	26	11	37	24	12	36			
1891–1893.......	10	20	30	12	24	24 or 36	12	24	36
1893–1896.......	25	11	36	24	12	36	12	12	24
1896–1900.......	38	10	48	36	12	48	{84	12	96}
1900–1903.......	32	6	38	36	0 or 12	36			
1903–1908.......	43	6	49	48	0 or 12	48	36	12	48
1908–1910.......	24	11	35	24	12	36	24	12	36
1910–1914.......	25	23	48	24	24	48	24	12	36
1914–1919.......	45	8	53	48	12	48	24	36	60
1919–1921.......	16	10	26	12	12	24	12	12	24
1921–1924.......	22	14	36	24	12	36	24	12	36
1924–1927.......	24	16	40	24	12	36	24	12	36
1927–1933.......	20	44	64	24	48	60	24	36	60
Average[b]									
All cycles......	28.8	14.5	43.3	29.6	15.2	42.0	31.0	17.0	48.0
10 corresponding cycles........	25.4	16.3	41.7	26.4	17.4	40.2	21.6	18.0	39.6

[a] Years of the initial and terminal troughs of the specific cycles in monthly data.
[b] Where double entries occur, we used their mean in computing the mean of the column.
[c] See note 15.
[d] Excluded from average.

to the nearest year—which show the 'precision effect', and the annual measures taken as they come—which show the 'precision' and 'twisting' effects in combination. If cyclical durations of monthly data are read to the nearest year, we cannot distinguish between a duration of 7 months and one of 17 months or between a duration of 19 months and one of 29; the loss of information may be serious but at least the margins of error are clearly defined. When cyclical durations are measured directly from annual data, the margins of error are indefinite and may be enormous. For example, the 45-month expansion of the 1914–19 cycle in iron production is replaced by a 2-year expansion, the 8-month contraction by a 3-year contraction, and the 53-month cycle by a 5-year cycle.

The mean 'precision effect' may be gauged by averaging, without regard to sign, the differences between cyclical durations of monthly data and the corresponding durations read to the nearest year.[22] This average is 2.4 months for the 45 observations covering all expansions, contractions, and full cycles in iron production, and 2.5 months for the 10 cycles

22 Where double entries occur in Table 56, we use both, each weighted one-half. Thus a duration of 6 months is '0 or 12' when read to the nearest year; hence the difference between '0 or 12' and 6 is 6 months. The expected 'precision effect' is 3 months in general, or somewhat higher than in our sample.

that match the cycles in annual data. Similarly, the 'twisting effect' may be gauged from the differences between the cyclical durations of annual data and the corresponding monthly measures read to the nearest year; this average is 4.8 months for the 10 cycles common to the monthly and annual lists. In other words, the twisting effect is nearly twice as large as the precision effect even if the most flagrant form of twisting—the obliteration of cycles—is disregarded. Averages made on the above plan do not allow opposite errors in single cycles to cancel out; the averages for all cycles at the bottom of Table 56 show that when such cancellation is allowed, the twisting effect remains larger than the precision effect.

Table 57 shows the effects of the annual time unit on average cyclical durations in each of our six series. One effect is obvious: the average dura-

TABLE 57

Average Duration of Specific Cycles in Monthly, Quarterly and Annual Data
Six American Series

Series and form of data	Period covered	No. of specific cycles	Duration of specific cycles in months					
			Average			Average deviation		
			Expansion	Contraction	Full cycle	Expansion	Contraction	Full cycle
DEFLATED CLEARINGS								
Monthly	1878–1933	15	32.6	11.4	44.0	9.9	6.5	11.6
Quarterly	1878–1932	14	34.7	12.0	46.7	10.8	6.4	14.3
Annual	1878–1933	10	49.2	16.8	66.0	27.6	7.7	30.0
PIG IRON PRODUCTION								
Monthly	1879–1933	15	28.8	14.5	43.3	9.0	7.1	9.9
Quarterly	1879–1933	15	28.6	14.6	43.2	9.1	6.8	9.6
Annual	1884–1932	12	31.0	17.0	48.0	16.5	7.5	18.0
RAILROAD STOCK PRICES								
Monthly	1857–1932	18	28.8	21.0	49.8	16.0	12.1	22.6
Quarterly	1857–1932	18	28.8	20.8	49.7	16.4	11.8	22.1
Annual	1859–1932	16	30.0	24.8	54.8	20.2	10.2	28.2
SHARES TRADED								
Monthly	1878–1933	15	17.9	26.2	44.1	8.0	9.1	11.0
Quarterly	1878–1933	15	16.6	27.4	44.0	9.3	10.1	11.6
Annual	1878–1932	15	20.8	22.4	43.2	10.6	5.5	12.8
CALL MONEY RATES								
Monthly	1858–1931	23	19.9	18.0	37.9	7.6	6.4	8.6
Quarterly	1858–1931	22	21.7	18.0	39.7	8.0	6.8	10.2
Annual	1858–1931	20	25.2	18.6	43.8	7.9	7.3	11.8
RAILROAD BOND YIELDS								
Monthly	1860–1931	20	21.0	21.4	42.4	9.0	11.3	12.6
Quarterly	1860–1931	20	20.6	21.9	42.4	8.7	11.0	12.4
Annual	1860–1931	14	27.4	33.4	60.9	15.2	18.6	20.1
			Percentage excess of annual over monthly					
Deflated clearings			+51	+47	+50	+179	+18	+159
Pig iron production			+8	+17	+11	+83	+6	+82
Railroad stock prices			+4	+18	+10	+26	−16	+25
Shares traded			+16	−15	−2	+32	−40	+16
Call money rates			+27	+3	+16	+4	+14	+37
Railroad bond yields			+30	+56	+44	+69	+65	+60

tion of full cycles is increased, the ratio of the average derived from annual data to the average derived from monthly data being approximately equal to the reciprocal of the fraction of specific cycles preserved by the annual data. Another and less obvious result is that annual data make the durations more uneven than they are in the monthly data. The 'precision' and 'twisting' effects combine to produce this result. An opposite result is conceivable; but it cannot be very frequent since the tendency of annual data to combine two and sometimes three cycles in monthly data into a single unit is unrelated to the length of the monthly cycles. In every series covered by our sample, annual data increase the average deviations of full cycles both absolutely and in proportion to the means. The effects of annual data on the phases of expansion and contraction are less predictable than the effects on full cycles. For they depend on the way in which specific cycles are lost—whether by skipping expansions or contractions—as well as on the number lost. Both the precision and twisting effects of annual data tend to be proportionately larger in short than in long durations, hence in cycle phases than in full cycles.

Just as annual measures of duration are less precise than monthly measures, so also are quarterly measures, though in much smaller degree. Sometimes the twisting effect stands out clearly; for example, the two-month contraction of clearings in 1907 becomes a six-month contraction in the quarterly data. At other times monthly and quarterly durations differ still more, but it is problematical whether the quarterly summation or the erratic movements in the vicinity of the turns are principally at fault. In any event, the differences between the monthly and quarterly durations of single cycles practically disappear when averages are struck for all cycles in a series. The only noticeable differences are in clearings and call money rates, which lose one cycle by the shift from monthly to quarterly data.

VI Timing of Specific Cycles

We distrust leads or lags determined from annual data and use them sparingly. The reason is that leads or lags of specific-cycle turns at reference dates are usually minor fractions of a year. More than half of the leads and lags in our test series are six months or shorter; only about a sixth exceed a year (Table 58). Since the shortest lead or lag that an annual series can show is 12 months, most cyclical turns of well conforming annual series coincide with the annual reference dates and the rest usually lead or lag 12 or 24 months. These measures can no more help the economist trace the relations in time of different activities during business cycles than lengths in feet can help the physician trace the growth of infants.

TABLE 58

Frequency Distribution of Leads or Lags of Specific Cycles in Monthly Data
Six American Series

Lead or lag at reference turn (mos.)	Number of leads or lags in monthly data						
	Deflated clearings	Pig iron production	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields	Six series
Under 4....	16	19	11	9	19	4	78
4 – 6....	6	3	3	4	10	7	33
7 – 9....	..	6	8	6	3	5	28
10 – 12....	5	2	6	4	3	5	25
Over 12....	2	1	7	8	3	11	32
Total......	29	31	35	31	38	32	196

In this table leads are not distinguished from lags; in other words, we consider merely the number of months that a specific-cycle turn deviates from a corresponding reference turn.

See Table 55 for the periods covered. The grand total (196) is smaller than the full number of cyclical turns (218), because noncorresponding turns are omitted. For the measures of timing, cycle by cycle, see Appendix Table B3.

In practice, the 'twisting effect' of annual data accentuates the difficulties caused by the 'precision effect'. A few examples may help the reader realize how serious these difficulties can be. (1) In 1929 call money rates reached a peak in March, iron production in July, railroad stock prices and bond yields in September, share trading in October, deflated clearings in November. Annual data tell nothing about the sequence of these changes, since all six series reach peaks in 1929. (2) In nine business cycles clearings lag at downturns but lead at upturns; annual data show similar behavior just once. (3) In eleven business cycles share trading leads at both upturns and downturns; annual data show only four such instances. (4) Clearings lead iron production at upturns in twelve instances, lag in one; in another two instances both series turn up the same month. Annual data show coincident upturns in every instance, except one in which clearings lag in the face of a coincidence in the monthly data. (5) Share trading reaches a peak in May and railroad stock prices in June 1881. Annual data convert the one-month lead into a one-year lag, stock prices showing a peak in 1881 and share trading in 1882. (6) At the reference trough in 1904 stock prices lead the procession of upturns, followed in two months by iron production, in seven months by clearings and share trading, in nine months by call money rates, and in sixteen months by bond yields. Annual data, on the other hand, put share trading in the lead, obliterate the turn in clearings, make iron production, stock prices, and call money rates follow in one year, and bond yields in two years. (7) Monthly data on iron production lag one month at the reference peak of 1918; annual data lead by two years. (8) Monthly data on railroad stock prices show two troughs in the vicinity of the reference trough in 1858, a deep trough in October 1857 and a moderate one in May 1859; annual data manage to skip the deep but not the moderate trough.

Table 59 shows compactly how wide are the discrepancies between the individual timing measures of monthly and annual data. The table is restricted to corresponding turns, that is, turns for which timing observations in monthly and annual data match. If the annual timing measures were merely 'less precise' than the monthly measures, the entries in the table would be restricted to the diagonal cells: monthly measures of −6 to +6 months would be matched invariably by annual measures of 0, monthly measures of −6 to −18 months by annual measures of −12, and so on. In fact, the diagonal cells include only about two-thirds of the entries, even if skipped turns are left out of account. The remaining third are due to the twisting of cycles by annual data.

TABLE 59
Joint Distribution of Corresponding Leads or Lags of Monthly and Annual Data

Lead (−) or lag (+) of monthly data at reference turn (mos.)	Corresponding lead (−) or lag (+) of annual data at reference turn (mos.)							
	−36	−24	−12	0	+12	+24	+36	Total
−42 to −30....	...	1	...	...	...	...	...	1
−30 to −18....	...	2	2	...	...	...	...	4
−18 to −6....	1	1	19	22	1	...	...	44
−6 to +6....	...	1	9	74	6	1	...	91
+6 to +18....	...	...	...	6	21	2	...	29
+18 to +30....	...	...	...	...	...	2	2	4
+30 to +42....	...	...	...	...	...	...	...	...
Total.........	1	5	30	102	28	5	2	173

Based on specific-cycle turns of the monthly and annual data of six American series within the periods shown in Table 55. The monthly data are compared with the monthly reference dates, the annual data with the annual reference dates.

Items falling at the boundaries of the monthly classes (for example, −6) were distributed equally between adjacent classes in a column. There were eleven such items. The fractions that resulted in certain cases were rounded up or down in a manner most advantageous to the showing of annual data.

Quarterly data once again produce effects similar but smaller than those produced by annual data. The number of coincidences is much larger in quarterly than in monthly data, as Table 60 shows in detail. The result is that the quarterly data frequently cover up differences in timing that appear in monthly data. According to the quarterly data all series except call money rates reached a peak in the third quarter of 1929; but the turns in monthly data are scattered from March to November. Further, the monthly data show the peak in call money rates in the first quarter of 1929, not in the second quarter as do the quarterly data. Again, the monthly data show peaks in clearings and share trading in the last quarter, not in the third quarter. These detailed differences are exhibited in Chart 24. With the possible exception of the monthly data on share trading all cyclical turns on this chart are clearly defined. But when erratic movements are very pronounced the advantage of monthly data

TABLE 60

Frequency of Leads or Lags and Average Timing of Specific Cycles
Six American Series, Monthly, Quarterly and Annual

Series and form of data	Timing at reference peaks: Number of Leads	Lags	Coinci-dences	Total	Average lead (−) or lag (+) in months	Timing at reference troughs: Number of Leads	Lags	Coinci-dences	Total	Average lead (−) or lag (+) in months
DEFLATED CLEARINGS										
Monthly (I)	1	10	3	14	+3.2	14	...	1	15	−5.8
Quarterly	...	4	9	13	+2.5	12	...	2	14	−5.1
Annual	...	1	8	9	+1.3	1	1	8	10	0.0
Monthly (II)	1	7	1	9	+3.3	9	...	1	10	−5.4
PIG IRON PRODUCTION										
Monthly (I)	4	8	3	15	+1.9	12	1	3	16	−3.4
Quarterly	2	7	6	15	+1.8	7	...	9	16	−2.2
Annual	1	1	11	13	−0.9	1	...	12	13	−0.9
Monthly (II)	4	6	3	13	+1.2	9	1	3	13	−3.8
RAILROAD STOCK PRICES										
Monthly (I)	13	3	1	17	−5.6	14	4	...	18	−7.4
Quarterly	13	2	2	17	−5.8	14	2	2	18	−7.3
Annual	9	...	7	16	−7.5	5	3	9	17	−2.1
Monthly (II)	13	2	1	16	−6.6	13	4	...	17	−7.7
SHARES TRADED										
Monthly (I)	14	1	...	15	−10.4	12	2	2	16	−4.6
Quarterly	14	1	...	15	−11.8	10	1	5	16	−3.8
Annual	10	...	5	15	−9.6	4	...	12	16	−3.0
Monthly (II)	14	1	...	15	−10.4	12	2	2	16	−4.6
CALL MONEY RATES										
Monthly (I)	8	8	3	19	−0.1	9	10	...	19	+1.5
Quarterly	6	7	5	18	0.0	4	8	6	18	+2.0
Annual	3	2	13	18	−0.7	2	6	10	18	+1.3
Monthly (II)	8	8	2	18	−0.1	8	10	...	18	+1.8
RAILROAD BOND YIELDS										
Monthly (I)	...	16	...	16	+7.8	2	13	1	16	+11.8
Quarterly	...	15	1	16	+7.7	2	13	1	16	+12.4
Annual	...	8	6	14	+8.6	...	13	1	14	+17.1
Monthly (II)	...	14	...	14	+8.8	1	12	1	14	+12.4

See Table 55 for the periods covered. The entries on line (I) include all timing measures for the monthly series within the periods shown in Table 55; the entries on line (II) are restricted to monthly timing measures that correspond to those in the annual series. The timing of the quarterly data is obtained by comparing their specific-cycle turns with the quarterly reference dates. Similarly, the timing of the annual data is obtained by comparing their specific-cycle turns with the annual reference dates. See the next table, and Table 48 in the preceding chapter.

is problematical; in such cases cyclical turns can be dated with greater assurance in quarterly data than in monthly.[23]

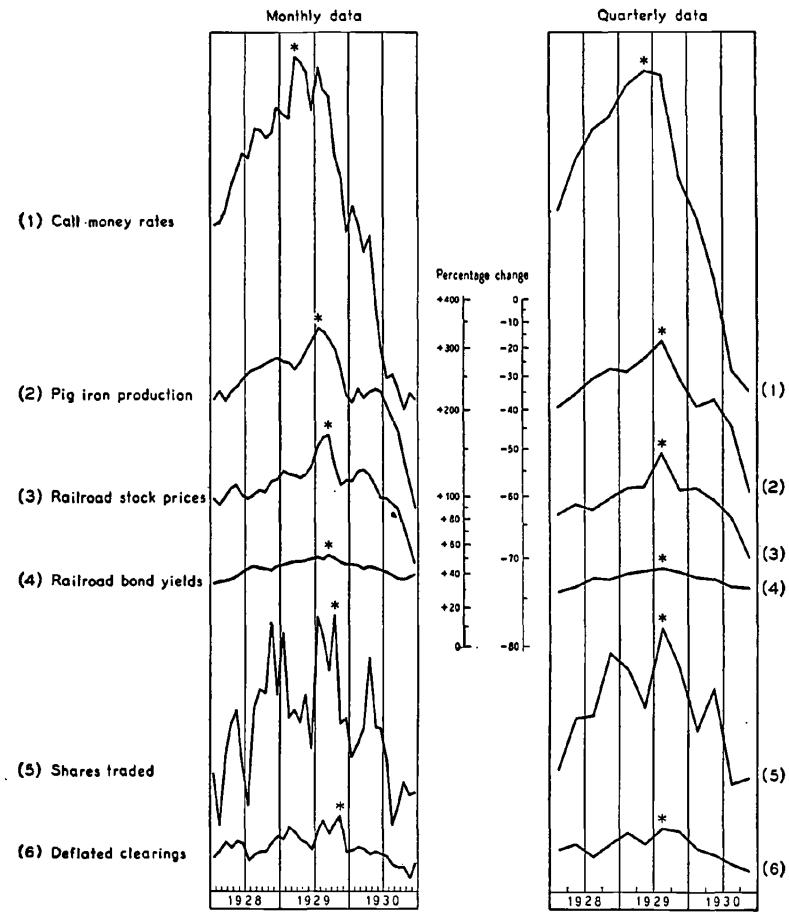
When averages are struck for all cycles the monthly and quarterly results agree remarkably well. The largest difference between their averages is only 1.4 months (Table 60). The small differences between the averages leave the rankings of the six series in quarterly form exactly the same as the rankings in monthly form at upturns and at downturns. The

[23] In handling highly erratic series a three-month moving total (or average) may be preferable to monthly or quarterly figures. Of course, quarterly data represent every third term of a three-month moving total (or average) of monthly figures.

CHART 24

Sequence of Cyclical Downturns in 1929

Six American Series, Monthly and Quarterly



Asterisks identify peaks of specific cycles. See the note at the end of Sec. II, for sources of data, etc.

annual averages, on the other hand, differ widely from the monthly averages in several instances. For example, monthly data on clearings show an average lead of six months at reference troughs; annual data show coincidence on the average. Monthly data on iron production show an average lag of two months at reference peaks; annual data show a lead of one month. Monthly data put share trading in third place at the turn from contraction to expansion; annual data put it first. These differences cannot be ascribed to the skipping of cycles by the annual data: for the monthly-annual discrepancies are about as large when judged from averages confined to corresponding turns as when judged from averages based on all turns.

So far we have compared annual specific-cycle turns with annual reference dates and quarterly specific-cycle turns with quarterly reference dates. This is the only method that can be used by an investigator working along our lines but relying exclusively upon annual or quarterly data. But in our own work, not being subject to this limitation, we can use the monthly reference dates as benchmarks for measuring the cyclical timing of all series regardless of the time unit in which they are expressed. Thus, instead of comparing the midpoints of annual specific-cycle turns with the midpoints of annual reference turns, we can relate the former to the midpoints of monthly reference turns.[24] Table 61 presents average timing

TABLE 61
Average Timing of Specific Cycles Computed in Different Ways
Six American Series, Monthly, Quarterly and Annual

Series	Corresponding turns in monthly and quarterly data				Corresponding turns in monthly and annual data			
		Av. lead (−) or lag (+) in months				Av. lead (−) or lag (+) in months		
	No. of turns	Monthly spec.-cycle turn compared with monthly ref. turn	Quarterly spec.-cycle turn compared with *monthly* ref. turn	Quarterly spec.-cycle turn compared with quarterly ref. turn	No. of turns	Monthly spec.-cycle turn compared with monthly ref. turn	Annual spec.-cycle turn compared with *monthly* ref. turn	Annual spec.-cycle turn compared with annual ref. turn
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				Timing at reference peaks				
Deflated clearings	13	+3.5	+3.2	+2.5	9	+3.3	+1.8	+1.3
Pig iron production	15	+1.9	+2.2	+1.8	13	+1.2	+0.3	−0.9
Railroad stock prices	17	−5.6	−5.7	−5.8	16	−6.6	−7.1	−7.5
Shares traded	15	−10.4	−11.4	−11.8	15	−10.4	−8.3	−9.6
Call money rates	18	−0.1	−0.1	0.0	18	−0.1	−0.4	−0.7
Railroad bond yields	16	+7.8	+7.6	+7.7	14	+8.8	+9.0	+8.6
				Timing at reference troughs				
Deflated clearings	14	−5.9	−5.9	−5.1	10	−5.4	−3.8	0.0
Pig iron production	16	−3.4	−2.9	−2.2	13	−3.8	−3.7	−0.9
Railroad stock prices	18	−7.4	−7.7	−7.3	17	−7.7	−4.9	−2.1
Shares traded	16	−4.6	−4.4	−3.8	16	−4.6	−5.9	−3.0
Call money rates	18	+1.8	+1.5	+2.0	18	+1.8	−1.8	+1.3
Railroad bond yields	16	+11.8	+12.0	+12.4	14	+12.4	+14.4	+17.1

See Table 55 for the periods covered.

measures of annual data computed both ways, and similar measures for quarterly data. When the timing of annual or quarterly data is measured from the monthly reference dates, the resulting average usually approximates better the average determined from monthly data than when annual or quarterly chronologies are used exclusively. This result is not a peculiarity of our sample; it may be expected as a rule, since the use of monthly reference dates limits the error of the timing measures to the distorting effect of quarterly or annual data on the specific-cycle turns.

[24] See Table 48. Of course, if a series starts before the monthly reference dates, we must determine leads or lags of the early cycles from the annual reference dates.

Hence our standard practice when analyzing quarterly or annual series is to measure their timing from the monthly reference dates.[25]

It is important to recognize that monthly benchmarks merely *tend* to improve estimates of average cyclical timing. As Table 61 demonstrates, the estimates are sometimes worsened; also, some of the estimates made from annual series remain poor, though they are not quite so poor as those derived from the annual reference dates. Table 62 shows that when the period covered by an annual series is brief and no adjustment is made for the failure of its cycles to correspond to the monthly, measures of average timing derived from annual data frequently diverge sharply from the monthly measures. For this reason we rarely measure leads or lags of short annual series. We also insist that the series conform well to business cycles; for unless this condition is met it is extremely difficult to determine what cyclical turns correspond to what reference dates. Finally, we attach little value to individual leads or lags of an annual series: we use them chiefly as materials for estimating average timing.

VII Amplitude of Specific Cycles in Annual Data

In annual series that represent continuous processes the standings at the peaks and troughs of specific cycles cover twelve months; in monthly and quarterly series they cover three months. When annual and quarterly data are expressed as monthly averages, and the cyclical peaks in all three forms come in the same year, the standing at the peak must be lower in the annual data than in the other forms; for in monthly series this standing covers the three months centered on the highest value attained during the cycle, and in quarterly series the standing is that of the highest quarter. The relations must be similar when the cyclical turns of the three forms of the data occur in different years, provided the cycles still correspond approximately in time. For the standing at the peak of a monthly series covers the three months centered on the apex of expansion, whether that expansion attained its highest annual average in the year preceding or the year following the monthly peak. Likewise the standing at the

[25] Let S be the average distortion of the timing of specific cycles produced by annual data, and R the average distortion of the timing of reference cycles produced by annual data. Then S is measured by col. (8) minus col. (7) of Table 61; R is measured by col. (8) minus col. (9); while col. (9) minus col. (7) measures $(S-R)$.

Whether col. (8) will give a better approximation to col. (7) than will col. (9) turns on whether $|S| < |S-R|$. For any group of reference cycles R may be plus, minus, or zero. If many series are analyzed for the period covered by these reference cycles, the expectation is that S will be plus in as many series as it will be minus. Now if R is zero, col. (8) must agree with col. (9). If S is zero, the above inequality obviously holds, except, of course, when R too is zero, in which case col. (8) agrees with col. (9). If S and R are of opposite sign, the inequality must again hold. And it will also hold when S and R are of the same sign, whenever $|R| > |2S|$. Hence there is a greater probability that $|S|<|S-R|$ than that $|S| > |S-R|$; in other words, the expectation is that col. (8) will more often approximate col. (7) than will col. (9).

With obvious changes of phrase, this argument applies to quarterly data. Also, as our colleague Geoffrey H. Moore points out, the above argument could be put more rigorously in terms of variances and covariances.

TABLE 62
Average Timing of Specific Cycles during Brief Periods
Six American Series, Monthly and Annual

Series and period covered	No. of reference cycles	No. of observations on timing at reference				Average lead (−) or lag (+) in months at reference			
		Peaks		Troughs		Peaks		Troughs	
		M	A	M	A	M	A	M	A
Deflated clearings									
1879–1897	5	5	4	5	4	+3.8	+0.8	−6.2	−4.2
1897–1914	5	5	3	5	3	+4.2	+2.5	−7.4	−5.5
1914–1933	5	4	2	5	3	+1.2	+3.0	−3.8	−1.5
Pig iron production									
1879–1897	5	5	4	5	3	+0.8	+0.8	−3.2	−2.8
1897–1914	5	5	4	5	4	+3.4	+5.5	−7.2	−4.8
1914–1933	5	5	5	6	6	+1.4	−4.3	−0.3	−3.3
Railroad stock prices									
1858–1888	6	5	5	5	5	−7.6	−6.7	−13.4	−8.3
1888–1908	6	6	5	6	5	−2.2	−5.1	−1.2	+0.7
1908–1933	7	6	6	7	7	−7.3	−9.2	−8.6	−6.4
Shares traded									
1879–1897	5	5	5	5	5	−11.4	−5.9	−2.2	−0.5
1897–1914	5	5	5	5	5	−12.0	−7.5	−4.8	−7.3
1914–1933	5	5	5	6	6	−7.8	−11.5	−6.5	−9.3
Call money rates									
1858–1888	6	6	6	6	6	−3.3	−3.5	−1.2	−7.8
1888–1908	6	6	6	6	6	+4.2	+1.8	+3.0	+1.7
1908–1933	7	7	6	7	6	−1.0	+0.5	+2.6	+0.7
Railroad bond yields									
1858–1888	6	6	5	6	5	+9.2	+8.5	+17.8	+18.1
1888–1908	6	5	5	5	5	+9.2	+9.3	+14.4	+15.1
1908–1933	7	5	4	5	4	+4.8	+9.2	+1.8	+8.8

M stands for monthly data, A for annual. The periods mark off successive thirds of the reference cycles covered by a series. Where the full number is not exactly divisible by 3, the odd item is placed in the last period.

The averages are made on our standard plan; that is, both monthly and annual specific-cycle turns are compared with the monthly reference dates. To avoid duplication, the timing at the terminal trough of the first period is excluded from the average for that period but included in the average for the second, and the timing at the terminal trough of the second period is excluded from the average for that period but included in the average for the third.

apex quarter should be higher than the average standing in the corresponding apex year, whether this year comes before or after the year including the apex quarter. On similar grounds, we expect the annual troughs to be higher than the monthly or quarterly troughs, whether they come in the same, the preceding, or the following year.[26]

Exceptions to these rules are rare. They may arise from disregarding a random peak or trough in marking off specific cycles in monthly or quarterly data.[27] They can scarcely arise from a violently low value in the

[26] Cf. note 44.

[27] For example, monthly data on the value of building plans filed in Manhattan show a rise from July 1927 to Feb. 1928, a decline to Dec. 1928, followed by an extraordinary rise to April 1929, and a sharp and protracted decline to July 1933. The data suggest that the rise from Dec. 1928 to April 1929 was a random interruption of a cyclical decline that began Feb. 1928; this suggestion is confirmed by other statistics and by the fact that the new Multiple Dwellings Act, with its more stringent provisions governing construction of apartment houses and hotels, was debated by the

month following or preceding a cyclical peak or from a violently high value in the month following or preceding a cyclical trough; for when such movements appear we omit them in computing the peak or trough standing. In the 70 specific cycles that correspond in the monthly, quarterly, and annual forms of our test series, the peak standings of the annual data are invariably lower and the trough standings invariably higher than the corresponding standings of the quarterly and monthly data. But the differences between the standings vary erratically, as may be seen from the detailed measures for pig iron production on lines 7-13 of Table 63.

Since the standings at the peaks of specific cycles are lower and the standings at troughs higher in the annual than in the quarterly or monthly data when these standings are expressed in the original units, the absolute rise and fall of specific cycles must be smaller in the annual than in the other data. There are no exceptions in our sample. Lines 19-25 in Table 63 show the discrepancies between the three forms of the data on iron production, and columns (1) to (4) in Table 64 summarize the monthly-annual discrepancies for all six series. In the first cycle of iron production the rise in the annual data is only 24 per cent of the rise in the monthly data; in the eighth cycle this ratio is 72 per cent. The range of the discrepancies is still larger in the cyclical falls. Other series show similar variations in the degree to which the monthly amplitudes are cut by a shift to annual data.

The reduction in amplitude produced by annual summarizing varies inversely with the duration of cyclical expansions and contractions. Chart 25 demonstrates this tendency; but also shows that it is crossed by other factors, and disappears when a phase lasts about four years or longer. As indicated in Section IV, the fate of a cyclical expansion or contraction in annual summations depends partly upon its duration, partly upon other things—the months in which the cyclical turns occur, the amplitude of the movement relatively to the underlying trend, its pattern and that of the erratic fluctuations diversifying it. These factors determine whether a given expansion or contraction will be preserved or wiped out in annual data; also, whether the fraction preserved is large or small. A student working solely with annual data must be ignorant of some of these factors and can have only rough knowledge of others; hence he can no more tell which cycles have lost a large and which a small portion of their amplitude by annual summarizing than he can tell which annual cycles correspond to single cycles in the monthly figures and which combine two or three cycles.

Legislature in the early months of 1929 and became law April 19, 1929. In analyzing the specific cycle from 1927 to 1933 we therefore take Feb. 1928 as the peak, and disregard the much higher value in April 1929. The annual figures are too rigid to be handled in a similar manner; they rise from 1927 to 1928 and again from 1928 to 1929, so that we are forced to recognize 1929 as the peak. The amplitude measures of the 1927-33 cycle come out +141, −252, 393 in the annual and +72, −184, 256 in the monthly data.

TABLE 63

Standings at Peaks and Troughs, Cycle Bases, and Amplitudes of Corresponding Specific Cycles in Monthly, Quarterly and Annual Data
Pig Iron Production, United States

Line	Measure	Cycle 1			Cycle 2			Cycle 3			Cycle 4			Cycle 5		
		M	Q	A	M	Q	A	M	Q	A	M	Q	A	M	Q	A
1	DATE OF TURNS															
2	Initial trough	Apr. 91	1Q 91	1891	Oct. 93	4Q 93	1894	Dec. 03	4Q 03	1904	Jan. 08	1Q 08	1908	Dec. 10	2Q 11	1911
3	Peak	Feb. 92	1Q 92	1892	Nov. 95	4Q 95	1895	July 07	3Q 07	1907	Jan. 10	1Q 10	1910	Jan. 13	1Q 13	1913
4	Terminal trough	Oct. 93	4Q 93	1894	Oct. 96	4Q 96	1896	Jan. 08	1Q 08	1908	Dec. 10	2Q 11	1911	Dec. 14	4Q 14	1914
5	STANDING AT TURNS															
6	*In thousand long tons*															
7	Initial trough	15.12	17.17	21.08	11.03	11.85	17.62	30.77	36.14	44.25	37.32	36.74	42.72	60.37	62.45	63.94
8	Peak	25.31	25.31	23.56	29.72	29.72	25.24	75.05	74.20	69.39	86.38	84.88	73.66	93.19	91.34	84.24
9	Terminal trough	11.03	11.85	17.62	16.69	17.87	22.72	37.32	36.74	42.72	60.37	62.45	63.94	51.75	52.30	63.19
10	*As relatives of M*															
11	Initial trough	100	114	139	100	107	160	100	117	144	100	98	114	100	103	106
12	Peak	100	100	93	100	100	85	100	99	92	100	98	85	100	98	90
13	Terminal trough	100	107	160	100	107	136	100	98	114	100	103	106	100	101	122
14	CYCLE BASE															
15	*In thousand long tons*	22.41	21.74	20.47	21.42	21.60	22.71	60.20	59.38	60.69	62.40	63.10	65.52	72.96	74.63	76.02
16	*As relatives of M*	100	97	91	100	101	106	100	99	101	100	101	105	100	102	104
17	ABSOLUTE AMPLITUDE[a]															
18	*In thousand long tons*															
19	Rise	10.19	8.14	2.48	18.69	17.87	7.62	44.28	38.06	25.14	49.06	48.14	30.94	32.82	28.89	20.30
20	Fall	14.28	13.46	5.94	13.03	11.85	2.52	37.73	37.46	26.67	26.01	22.43	9.72	41.44	39.04	21.05
21	Rise & fall	24.47	21.60	8.42	31.72	29.72	10.14	82.01	75.52	51.81	75.07	70.57	40.66	74.26	67.93	41.35
22	*As relatives of M*															
23	Rise	100	80	24	100	96	41	100	86	57	100	98	63	100	88	62
24	Fall	100	94	42	100	91	19	100	99	71	100	86	37	100	94	51
25	Rise & fall	100	88	34	100	94	32	100	92	63	100	94	54	100	91	56
26	RELATIVE AMPLITUDE[b]															
27	*In specific-cycle relatives*															
28	Rise	45.4	37.4	12.1	87.2	82.7	33.6	73.6	64.1	41.4	78.6	76.3	47.2	45.0	38.7	26.7
29	Fall	63.7	61.9	29.0	60.8	54.9	11.1	62.7	63.1	43.9	41.7	35.5	14.8	56.8	52.3	27.7
30	Rise & fall	109.1	99.3	41.1	148.0	137.6	44.7	136.3	127.2	85.3	120.3	111.8	62.0	101.8	91.0	54.4
31	*As relatives of M*															
32	Rise	100	82	27	100	95	39	100	87	56	100	97	60	100	86	59
33	Fall	100	97	46	100	90	18	100	101	70	100	85	35	100	92	49
34	Rise & fall	100	91	38	100	93	30	100	93	63	100	93	52	100	89	53

TABLE 63 — *Continued*

Line	Measure	Cycle 6			Cycle 7			Cycle 8			Cycle 9			Cycle 10		
		M	Q	A	M	Q	A	M	Q	A	M	Q	A	M	Q	A
1	DATE OF TURNS															

29	Fall	63.7	61.9	29.0	60.8	54.9	11.1	62.7	63.1	43.9	41.7	35.5	14.8	56.8	52.3	27.7
30	Rise & fall	109.1	99.3	41.1	148.0	137.6	44.7	136.3	127.2	85.3	120.3	111.8	62.0	101.8	91.0	54.4
31	*As relatives of M*															
32	Rise	100	82	27	100	95	39	100	87	56	100	97	60	100	86	59
33	Fall	100	97	46	100	90	18	100	101	70	100	85	35	100	92	49
34	Rise & fall	100	91	38	100	93	30	100	93	63	100	93	52	100	89	53

TABLE 63 – *Continued*

Line	Measure	Cycle 6			Cycle 7			Cycle 8			Cycle 9			Cycle 10		
		M	Q	A	M	Q	A	M	Q	A	M	Q	A	M	Q	A
1	DATE OF TURNS															
2	Initial trough	Dec. 14	4Q 14	1914	May19	2Q 19	1919	July 21	3Q 21	1921	July 24	3Q 24	1924	Nov. 27	4Q 27	1927
3	Peak	Sep. 18	3Q 18	1916	Sep. 20	3Q 20	1920	May23	2Q 23	1923	July 26	3Q 26	1926	July 29	3Q 29	1929
4	Terminal trough	May19	2Q 19	1919	July 21	3Q 21	1921	July 24	3Q 24	1924	Nov.27	4Q 27	1927	Mar. 33	1Q 33	1932
5	STANDING AT TURNS															
6	*In thousand long tons*															
7	Initial trough	51.75	52.30	63.19	72.96	72.96	83.91	32.33	31.56	45.45	73.35	75.70	85.08	91.10	91.10	99.32
8	Peak	112.73	113.42	106.68	105.85	104.81	99.50	121.88	121.88	109.74	109.80	110.93	107.04	128.03	128.07	115.86
9	Terminal trough	72.96	72.96	83.91	32.33	31.56	45.45	73.35	75.70	85.08	91.10	91.10	99.32	18.03	17.83	23.77
10	*As relatives of M*															
11	Initial trough	100	101	122	100	100	115	100	98	141	100	103	116	100	100	109
12	Peak	100	101	95	100	99	94	100	100	90	100	101	97	100	100	90
13	Terminal trough	100	100	115	100	98	141	100	103	116	100	100	109	100	99	132
14	CYCLE BASE															
15	*In thousand long tons*	98.52	97.58	94.27	80.87	79.28	82.09	82.80	84.09	82.85	99.80	100.33	99.69	73.86	74.74	83.42
16	*As relatives of M*	100	99	96	100	98	102	100	102	100	100	101	100	100	101	113
17	ABSOLUTE AMPLITUDE[a]															
18	*In thousand long tons*															
19	Rise	60.98	61.12	43.49	32.89	31.85	15.59	89.55	90.32	64.29	36.45	35.23	21.96	36.93	36.97	16.54
20	Fall	39.77	40.46	22.77	73.52	73.25	54.05	48.53	46.18	24.66	18.70	19.83	7.72	110.00	110.24	92.09
21	Rise & fall	100.75	101.58	66.26	106.41	105.10	69.64	138.08	136.50	88.95	55.15	55.06	29.68	146.93	147.21	108.63
22	*As relatives of M*															
23	Rise	100	100	71	100	97	47	100	101	72	100	97	60	100	100	45
24	Fall	100	102	57	100	100	74	100	95	51	100	106	41	100	100	84
25	Rise & fall	100	101	66	100	99	65	100	99	64	100	100	54	100	100	74
26	RELATIVE AMPLITUDE[b]															
27	*In specific-cycle relatives*															
28	Rise	61.9	62.6	46.2	40.7	40.2	19.0	108.3	107.6	77.5	36.5	35.2	22.1	50.0	49.5	19.8
29	Fall	40.3	41.4	24.2	91.1	92.6	65.8	58.6	55.0	29.7	18.7	19.8	7.8	148.9	147.5	110.4
30	Rise & fall	102.2	104.0	70.4	131.8	132.8	84.8	166.9	162.6	107.2	55.2	55.0	29.9	198.9	197.0	130.2
31	*As relatives of M*															
32	Rise	100	101	75	100	99	47	100	99	72	100	96	61	100	99	40
33	Fall	100	103	60	100	102	72	100	94	51	100	106	42	100	99	74
34	Rise & fall	100	102	69	100	101	64	100	97	64	100	100	54	100	99	65

M stands for monthly data, Q for quarterly, A for annual. The output figures refer to average daily production in all three forms of the data.

[a] That is, expressed in the unit of the original data.

[b] That is, our standard measure of amplitude as derived in Table S2 (see Table 31).

TABLE 64

Frequency Distribution of the Differences between Absolute Amplitudes, Cycle Bases, and Relative Amplitudes of Corresponding Specific Cycles in Monthly and Annual Data

Deviation of annual figure from monthly (per cent)	Number of differences between						
	Amplitudes expressed in original units			Cycle bases	Amplitudes expressed in specific-cycle relatives		
	Rise	Fall	Rise & fall		Rise	Fall	Rise & fall
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Under −90	2	1	..	..	2	1	..
−90 to −81	4	3	2	..	3	4	2
−80 to −71	5	6	1	..	7	4	1
−70 to −61	6	6	11	..	7	7	9
−60 to −51	12	19	14	..	8	19	17
−50 to −41	16	13	15	..	18	12	13
−40 to −31	10	7	14	..	9	9	16
−30 to −21	8	7	7	..	10	7	7
−20 to −11	5	8	6	1	5	7	5
−10 to −1	2	..	..	26	1	..	..
0 to +9	..	..	..	41	..	..	..
+10 to +19	..	..	..	2	..	..	..
Total	70	70	70	70	70	70	70

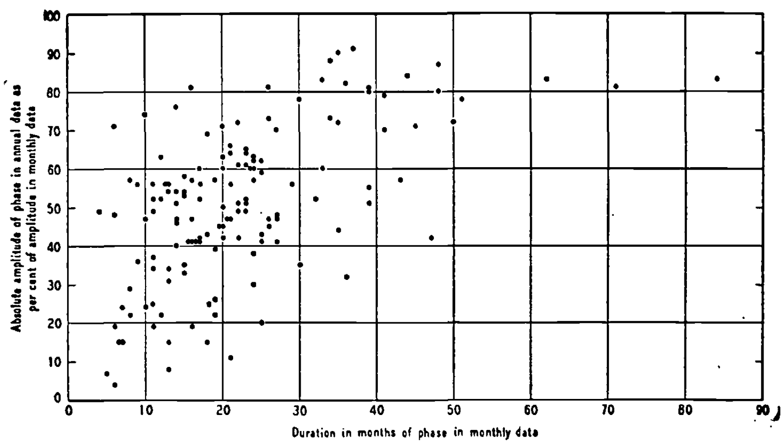
Based on six American series for periods shown in Chart 27.

The preceding remarks apply to cyclical amplitudes expressed in units of the original data, that is, to one stage in our computation. Our standard measures of amplitude convert the 'absolute amplitudes' into percentages of their respective cycle bases. The differences between 'relative amplitudes' made from annual and from monthly or quarterly data thus depend upon the differences between the cycle bases as well as upon the differences between the absolute amplitudes.[28] When the annual cycle base is higher than the monthly, the difference between bases works in the same direction as the difference between the absolute amplitudes; that is, both tend to make the annual measure of amplitude smaller than the monthly. When the annual cycle base is lower than the monthly, the difference between the bases works against the difference between the absolute amplitudes; the relative amplitude *may* therefore be larger in the annual data. Not a single instance of this sort appears in the comparisons afforded by our sample (Table 64). The reason is that the differences between cycle bases are smaller than the differences between absolute rises or falls. In railroad bond yields the largest difference between the

28 Let a and b be the standings, in units of the original data, of a monthly cycle at the initial trough and peak, respectively; and let k be the average value of the data during the cycle. Then the relative amplitude of the rise is $100\left(\frac{b-a}{k}\right)$. Suppose that the absolute rise of the annual data is $x(b-a)$ and that the cycle base of the annual data is yk, where x and y are any positive numbers. Then the relative amplitude of rise of the annual data is $100\left(\frac{b-a}{k}\right)\left(\frac{x}{y}\right)$; that is, the annual amplitude differs from the monthly by the multiplier $\frac{x}{y}$. Similar statements apply to the cyclical fall and to the combined rise and fall.

CHART 25

Relation between the Cycle-dampening Effect of Annual Data and the Duration of Specific-cycle Phases



Based on 140 corresponding phases in monthly and annual data of six American series.
See Chart 26 for the period covered by shares traded, Chart 27 for periods of other series.

monthly and annual cycle bases is 2 per cent, while the smallest difference between the absolute amplitudes is 12 per cent. Even in share trading and call money rates, which have very large cyclical and erratic movements, the difference between monthly and annual cycle bases is 5 per cent or less in 23 out of 30 comparisons, and in no case exceeds 12 per cent. In contrast, the differences between the absolute rises range from 19 to 93 per cent in share trading and from 30 to 69 per cent in call money rates, while the differences between falls range from 30 to 96 per cent in the first series and from 19 to 75 per cent in the second.[29]

The importance of differences between the cycle bases of monthly and annual data relatively to the differences between their absolute amplitudes is still smaller in *average* measures of cyclical amplitude. The second set of differences practically always work in the same direction and therefore register with full force in the averages. But the first set raise the annual amplitudes of some cycles, lower the amplitudes of others, and thus tend to be self-effacing in averages. Table 65 shows these different effects in practice. Columns (3) to (5) present the average amplitudes in the monthly forms of our six series; columns (6) to (8) give corresponding averages for annual data on our usual plan, and columns (9) to (11) give annual averages made by substituting the monthly bases for the annual. The differences between columns (6) to (8) and (9) to (11) gauge the effects of differences between the cycle bases of monthly and annual data; these effects are practically nil. The differences between columns (3) to

29 The difference between the absolute amplitudes of monthly and annual data exceeds the difference between their cycle bases in each of the 210 comparisons afforded by our tests.

TABLE 65

Average Amplitude of Corresponding Specific Cycles in Monthly and Annual Data, Six American Series

Series	No. of corresponding specific cycles	Average amplitude in specific-cycle relatives of								
		Monthly data			Annual data					
					Computed on base of annual cycle			Computed on base of corresponding monthly cycle		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Deflated clearings...	5	28.1	12.8	40.9	20.0	5.4	25.4	20.3	5.3	25.7
Pig iron production.	10	62.7	64.3	127.0	34.6	36.4	71.0	35.1	37.9	73.0
Railroad stock prices	15	35.4	34.9	70.3	24.7	23.9	48.6	24.8	23.8	48.6
Shares traded......	15	98.1	92.4	190.5	49.9	39.3	89.2	48.9	39.2	88.1
Call money rates....	15	134.8	136.1	270.9	67.6	69.1	136.7	67.9	69.6	137.5
Railroad bond yields	10	9.4	13.2	22.6	5.2	9.1	14.3	5.2	9.1	14.3
Six series[a].........	70	69.8	68.4	138.2	37.6	35.2	72.8	37.6	35.5	73.1

See Chart 27 for the periods covered.

[a] The averages on this line are weighted averages of the above; that is, the unit observation is the amplitude of a cycle in a series, not the average amplitude in a series.

(5) and (9) to (11) gauge the effects of differences in absolute amplitudes; these effects are considerable, varying from about 30 to over 50 per cent of the monthly figures in different series.

If the monthly measures of amplitude are accurate, the annual measures not only understate the amplitude of cyclical fluctuations, but they do that unevenly. For example, Table 63 shows that in pig iron production the rise of monthly specific-cycle relatives during the second cycle is exceeded only by the rise in the eighth cycle; but in annual data the rise in the second cycle is exceeded in three additional cycles. The monthly rise in the ninth cycle is the smallest in the table; the annual rise in this cycle exceeds three others. The monthly fall in the second cycle is exceeded in four other cycles, the annual fall is exceeded in eight cycles. Such distortions of relations among the amplitudes of different cycles may be a serious matter to a student concerned with the characteristics of individual cycles. However, although annual data distort relations of magnitude, they by no means obliterate them so long as the annual cycles correspond roughly to the monthly. This fact is demonstrated by the coefficients of rank correlation in Table 66. Moreover, the dampening effect of annual data is more nearly uniform between than within series; hence the distortion of relative magnitudes may be of slight consequence to a student concerned with average cyclical behavior provided, once again, that the specific cycles in annual data match the cycles in monthly data.

This proviso is important. The differences we have so far discussed between annual and monthly amplitudes are restricted to corresponding

TABLE 66

Coefficients of Rank Correlation between Amplitudes of Corresponding Specific Cycles in Monthly and Other Data

Six American Series

Series	No. of corresponding specific cycles in		Coefficient of rank correlation between					
	Monthly and annual data[a]	Monthly and quarterly data[b]	Monthly and annual amplitudes[c] of			Monthly and quarterly amplitudes[c] of		
			Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Deflated clearings	5	13	1.00	.70	.90	.97	.96	.97
Pig iron production	10	15	.76	.82	.73	.97	.97	.96
Railroad stock prices	15	18	.99	.90	.91	.99	.99	.99
Shares traded	15	15	.79	.88	.90	.91	.88	.95
Call money rates	15	21	.90	.92	.87	.97	.96	.97
Railroad bond yields	10	20	.93	.95	.46[d]	.99	1.00	.99

[a] See Chart 27 for the periods covered.

[b] The periods covered are shown in Table 57, except for the omission of the specific cycles during 1914–21 in deflated clearings and 1915–22 in call money rates.

[c] In specific-cycle relatives.

[d] The coefficient of correlation comes out +.95, when based on the actual values instead of the ranks.

cycles, and therefore take no account of the most serious distortions produced by annual summarizing—the elimination of a genuine cycle or the introduction of a spurious one. When annual data skip a cyclical expansion or contraction, two cycles in the monthly data occupy approximately the same period as one cycle in the annual data. When annual data skip two successive contractions, three cycles in the monthly data occupy the same period as one cycle in the annual data. Such 'noncorresponding' cycles account for 15 of the 87 specific cycles in the annual data and for 35 of the 106 cycles in the monthly data of our six series. In addition, two specific cycles in the annual data of one series (call money rates) occupy the same period as one cycle in the monthly data. If our analysis is sound, the noncorresponding cycles of monthly data are no less genuine than the corresponding. In annual data the two sets of cycles stand on a different footing: the 70 cycles that correspond to cycles in monthly data are genuine, the remaining 17 cycles are not. Since a student working solely with annual series cannot be sure which cycles in his data are genuine and which spurious, he may be gravely misled in his judgments of business history. For example, annual data suggest that within the periods covered by our analysis of iron production and clearings the largest cyclical rise occurred from 1884 to 1890; but this rise telescopes two cyclical expansions, from 1884 or 1885 to 1887 and from 1888 to 1890, both of moderate amplitude.

The mixed character of the cyclical units in annual data makes it necessary to distinguish three cases. (a) An annual cycle that corresponds to a monthly cycle is practically certain to be of smaller amplitude. (b) An annual cycle that combines two or three monthly cycles is likely to show an extraordinary rise if it skips contractions, or an extraordinary fall if it skips expansions. (c) On the other hand, if two annual cycles occupy the same period as one monthly cycle, they are likely to have especially small amplitudes; though the opposite may happen if the extra 'cycle' is generated by violent erratic movements in the monthly data. Since annual series skip cycles far more often than they insert cycles, it follows that the tendency of annual and monthly cycles to fall out of correspondence opposes the dampening effect of annual data on the amplitudes of corresponding cycles. Table 67 shows these opposite effects on the averages and their net resultants. In four out of five series the average rise of the noncorresponding cycles is larger in the annual than in the monthly data; in the remaining series the average rise of annual data is closer to

TABLE 67
Average Amplitude of Corresponding, Noncorresponding and All Specific Cycles in Monthly and Annual Data
Six American Series

Series and group of cycles	Number of specific cycles		Average amplitude in specific-cycle relatives						Annual average as per cent of monthly		
	M	A	Rise		Fall		Rise & fall		Rise	Fall	Rise & fall
			M	A	M	A	M	A			
DEFLATED CLEARINGS											
Corresponding	5	5	28.1	20.0	12.8	5.4	40.9	25.4	71	42	62
Noncorresponding	10	5	26.3	39.4	13.7	14.2	39.9	53.6	150	104	134
All	15	10	26.9	29.7	13.4	9.8	40.2	39.5	110	73	98
PIG IRON PRODUCTION											
Corresponding	10	10	62.7	34.6	64.3	36.4	127.0	71.0	55	57	56
Noncorresponding	5	2	60.8	73.9	35.6	12.0	96.4	85.8	122	34	89
All	15	12	62.1	41.1	54.8	32.4	116.8	73.5	66	59	63
RAILROAD STOCK PRICES											
Corresponding	15	15	35.4	24.7	34.9	23.9	70.3	48.6	70	68	69
Noncorresponding	3	1	36.6	78.8	16.1	18.6	52.7	97.4	215	116	185
All	18	16	35.6	28.1	31.8	23.5	67.3	51.6	79	74	77
SHARES TRADED[a]											
All	15	15	98.1	49.9	92.4	39.3	190.5	89.2	51	43	47
CALL MONEY RATES											
Corresponding	15	15	134.8	67.6	136.1	69.1	270.9	136.7	50	51	50
Noncorresponding	8	5	80.6	47.9	78.6	50.2	159.2	98.1	59	64	62
All	23	20	115.9	62.7	116.1	64.4	232.0	127.1	54	55	55
RAILROAD BOND YIELDS											
Corresponding	10	10	9.4	5.2	13.2	9.1	22.6	14.3	55	69	63
Noncorresponding	10	4	12.2	17.4	11.8	16.1	24.1	33.5	143	136	139
All	20	14	10.8	8.7	12.5	11.1	23.3	19.8	81	89	85
SIX SERIES[b]											
Corresponding	70	70	69 8	37.6	68.4	35.2	138.2	72.8	54	51	53
Noncorresponding	36	17	40.1	43.1	30.8	25.2	70.9	68.3	107	82	96
All	106	87	59.7	38.7	55.7	33.3	115.4	72.0	65	60	62

M stands for monthly data, A for annual. See Charts 26-27 for the periods covered.
[a]All cycles correspond in this series.
[b]See Table 65, note 'a'.

the monthly average in the noncorresponding than in the corresponding cycles. The results are similar for cyclical declines, though less striking because the annual forms of our test series skip few expansions. Bond yields have the distinction of skipping three expansions and three contractions, and the average fall of their annual noncorresponding cycles is no less conspicuous than the average rise.

The effects of cycle-skipping by annual data seem to be even more haphazard than the dampening effects. In iron production the inclusion of noncorresponding cycles in the averages reduces only slightly the discrepancy between the monthly and annual averages restricted to corresponding cycles. In clearings, on the other hand, the inclusion of noncorresponding cycles produces a larger average rise in the annual data than in the monthly and practically the same full-cycle average as in the monthly—an odd result, but one that may occur whenever annual data skip many cycles. Despite these uneven effects, the ranks of the averages of joint rise and fall, covering all cycles, are the same for the annual and

TABLE 68

Average Amplitude of Specific Cycles during Brief Periods

Six American Series, Monthly and Annual

Series and period covered	Number of specific cycles		Average amplitude in specific-cycle relatives						Annual average as per cent of monthly		
	M	A	Rise		Fall		Rise & fall		Rise	Fall	Rise & fall
			M	A	M	A	M	A			
Deflated clearings											
1878–1893.......	5	4	29.6	27.0	12.5	5.0	42.1	32.0	91	40	76
1893–1910.......	5	3	27.3	26.7	10.0	5.4	37.3	32.0	98	54	86
1910–1933.......	5	3	23.7	36.2	17.6	20.7	41.3	56.9	153	118	138
Pig iron production											
1879–1896.......	5	3	62.3	42.4	44.4	18.2	106.7	60.6	68	41	57
1896–1914.......	5	4	64.4	45.4	48.3	23.9	112.8	69.4	70	49	62
1914–1933.......	5	5	59.5	36.9	71.5	47.6	131.0	84.5	62	67	65
Railroad stock prices											
1857–1889.......	6	5	45.8	41.0	30.6	23.1	76.4	64.1	90	75	84
1889–1907.......	6	5	29.2	24.4	22.7	14.5	51.9	38.9	84	64	75
1907–1932.......	6	6	31.7	20.4	42.0	31.4	73.7	51.8	64	75	70
Shares traded											
1878–1897.......	5	5	74.5	30.4	73.1	27.8	147.6	58.2	41	38	39
1897–1914.......	5	5	108.0	47.9	111.5	46.3	219.5	94.1	44	42	43
1914–1933.......	5	5	111.7	71.4	92.6	44.0	204.3	115.4	64	48	56
Call money rates											
1858–1880.......	7	5	109.9	59.6	108.2	52.8	218.1	112.4	54	49	52
1880–1904.......	8	7	141.6	75.3	147.9	85.0	289.5	160.3	53	57	55
1904–1931.......	8	8	95.6	53.6	91.2	53.6	186.8	107.2	56	59	57
Railroad bond yields											
1860–1876.......	6	3	12.3	12.5	14.4	23.9	26.6	36.4	102	166	137
1876–1905.......	7	6	7.4	4.5	12.8	7.4	20.2	11.8	61	58	58
1905–1931.......	7	5	13.0	11.5	10.8	7.9	23.7	19.4	88	73	82

M stands for monthly data, A for annual. The periods cover (approximately) successive thirds of the specific cycles in monthly data.

monthly data. This credit would be dimmed if our test series were not so dissimilar in the vigor of their cyclical fluctuations; it is dimmed even in our sample when the averages for all cycles are considered separately for expansions and contractions, or when any of the three amplitude measures are restricted to brief periods, as in Table 68. In the 54 comparisons provided by this table the annual average ranges from 38 to 166 per cent of the monthly average, being less than half of the monthly in 11 instances and exceeding the monthly in 6. Table 69 adds the information that in every series covered by our tests the variability of amplitudes relatively to their mean is larger in the annual than in the monthly measures.

TABLE 69

Variability of Amplitudes of Specific Cycles in Monthly and Annual Data

Six American Series

Series and form of data	No. of specific cycles	Average deviation in specific-cycle relatives			Coefficient of variation[a]		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
DEFLATED CLEARINGS							
Monthly	15	10.6	8.7	13.8	39	65	34
Annual	10	14.5	8.7	17.7	49	89	45
PIG IRON PRODUCTION							
Monthly	15	15.7	21.2	26.8	25	39	23
Annual	12	18.9	20.5	23.1	46	63	31
RAILROAD STOCK PRICES							
Monthly	18	20.5	17.0	35.9	58	53	53
Annual	16	24.1	16.2	38.9	86	69	75
SHARES TRADED							
Monthly	15	31.8	30.8	56.4	32	33	30
Annual	15	30.0	18.9	39.4	60	48	44
CALL MONEY RATES							
Monthly	23	52.0	56.7	105.9	45	49	46
Annual	20	32.2	33.0	59.4	51	51	47
RAILROAD BOND YIELDS							
Monthly	20	5.3	6.2	7.2	49	50	31
Annual	14	6.0	7.9	10.1	69	71	51

See Table 57 for periods covered.

[a] Here taken as the percentage ratio of the average deviation to the mean.

Finally, Table 70 demonstrates that amplitude measures on a per month basis reduce materially the difficulties caused by noncorresponding cycles and therefore have a special claim on the attention of students forced to work with annual data. The reason is simply that if, say, two cyclical expansions are telescoped by annual figures, we are likely to get an unusually large cyclical rise; but since the rise is also likely to be very long, the one tendency will counteract the other when the *rate* of rise is computed. In each series the average per month figure is considerably lower in annual than in monthly data, whether corresponding or noncorresponding cycles are compared. Not only that, but the discrepancies

TABLE 70

Average Per Month Amplitude of Corresponding, Noncorresponding and All Specific Cycles in Monthly and Annual Data

Six American Series

Series and group of cycles	Number of specific cycles		Average per month amplitude in specific-cycle relatives					
			Rise		Fall		Rise & fall	
	M	A	M	A	M	A	M	A
DEFLATED CLEARINGS								
Corresponding	5	5	0.9	0.8	1.6	0.3	1.1	0.7
Noncorresponding	10	5	0.8	0.5	2.1	0.5	0.9	0.5
All	15	10	0.8	0.7	1.9	0.4	0.9	0.6
PIG IRON PRODUCTION								
Corresponding	10	10	2.8	1.6	4.8	2.2	3.2	1.9
Noncorresponding	5	2	1.7	1.0	4.4	1.0	2.2	1.0
All	15	12	2.4	1.5	4.7	2.0	2.9	1.7
RAILROAD STOCK PRICES								
Corresponding	15	15	1.3	0.8	1.7	0.9	1.4	0.9
Noncorresponding	3	1	2.2	1.1	1.4	0.8	1.6	1.0
All	18	16	1.4	0.8	1.7	0.9	1.4	0.9
SHARES TRADED[a]								
All	15	15	7.6	2.5	4.1	1.7	4.6	2.1
CALL MONEY RATES								
Corresponding	15	15	5.7	2.8	8.7	4.3	6.6	3.4
Noncorresponding	8	5	9.0	1.7	5.6	2.4	5.7	2.0
All	23	20	6.9	2.5	7.6	3.8	6.3	3.0
RAILROAD BOND YIELDS								
Corresponding	10	10	0.5	0.2	0.6	0.3	0.5	0.3
Noncorresponding	10	4	0.7	0.5	1.1	0.4	0.7	0.4
All	20	14	0.6	0.3	0.9	0.3	0.6	0.3

The averages in this table are unweighted. M stands for monthly data, A for annual. See Charts 26-27 for the periods covered.

[a] All cycles correspond in this series.

between the monthly and annual averages for noncorresponding cycles are not very different as a rule from the discrepancies between the averages of corresponding cycles. As a result the relations between the all-cycle averages of the monthly and annual data of our series are roughly similar to the relations between averages restricted to corresponding cycles.[30]

VIII Amplitude of Specific Cycles in Quarterly Data

Since the standings at the peaks and troughs of specific cycles cover three months in both quarterly and monthly series, they are not likely to differ so widely as the annual and monthly standings. Indeed, the monthly and quarterly standings are identical whenever the monthly peak or trough occurs in the middle month of the quarter that has the highest or lowest

30 We omit comparisons of per month amplitudes on the detailed basis of Table 68. In every one of the 54 comparisons, the average per month amplitude is lower in annual than in monthly data.

value.[31] When they cover different months the monthly peaks are nearly certain to be higher than the quarterly peaks and the monthly troughs lower than the quarterly, if the cyclical rise is symmetrical with the cyclical fall. When the two phases are not symmetrical, as is the rule in experience, the quarterly peak may easily reach higher than the monthly. This result is most probable if the peak comes in the last month of the quarter and is preceded by a moderate rate of rise and followed by a sharp rate of decline; or if it comes in the first month of the quarter and is preceded by a sharp rise and followed by a moderate decline. Similarly, the three months centered on the trough month may under certain conditions have a higher value than one of the conventional quarters. Again that result is most probable if the trough comes in the last month of the quarter and is preceded by a moderate decline and followed by a sharp rise; or comes in the first month of the quarter and is preceded by a sharp decline and followed by a moderate rise.

Table 63 illustrates in detail the effect of the quarterly time unit upon the standings at cyclical turns and the resulting amplitude measures of iron production, and shows how these effects differ from those produced by annual data. Table 71 summarizes the effects of the quarterly time unit

TABLE 71

Standings at Cyclical Turns and Amplitudes of Monthly Data Compared with Similar Measures of Corresponding Specific Cycles in Quarterly Data

Measure	No. of instances in which monthly measure is		
	Smaller than quarterly	Larger than quarterly	Same as quarterly
STANDING IN ORIGINAL UNITS			
At peak	26	47	29
At trough	44	32	34
AMPLITUDE IN ORIGINAL UNITS			
Rise	39	55	8
Fall	34	55	13
Rise & fall	34	62	6
AMPLITUDE IN SPECIFIC-CYCLE RELATIVES			
Rise	35	61	6
Fall	35	62	5
Rise & fall	37	61	4

Based on specific cycles of six American series. The periods covered are shown in Table 57, except that we omit here the noncorresponding specific cycles during 1914–21 in deflated clearings and during 1915–22 in call money rates.

on all six series. The 102 specific cycles that correspond in the monthly and quarterly data make possible as many comparisons of standings at peaks and a slightly larger number at troughs. The monthly peaks are higher than the quarterly in more instances than they are lower; also the monthly troughs are lower than the quarterly in more instances than they are higher. In neither set of comparisons is there a clear majority, for

31 Except, of course, when the monthly standing is based on one or two months instead of three. See p. 132.

in about 30 per cent of the instances the monthly and quarterly standings are equal. But when differences are struck between the standings at peaks and troughs, the monthly and quarterly results diverge more sharply. Thus the absolute full-cycle amplitude of monthly data exceeds the quarterly in 62 instances, is lower in 34, and the same in 6. These results are not changed materially when the absolute amplitudes are expressed as relatives of their respective cycle bases, since the monthly-quarterly differences between cycle bases are slight.[32]

TABLE 72

Frequency Distribution of the Differences between Amplitudes of Corresponding Specific Cycles in Monthly, Quarterly and Annual Data

Deviation of annual or quarterly amplitude from monthly (per cent)	No. of differences between annual and monthly amplitudes[a]			No. of differences between quarterly and monthly amplitudes[b]		
	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Under −40	45	47	42	1	...	...
−40 to −31	9	9	16	...	1	...
−30 to −21	10	7	7	4	7	3
−20 to −11	5	7	5	12	13	11
−10 to −1	1	...	...	41	37	44
0 to +9	...	...	...	33	32	35
+10 to +19	...	...	...	9	7	7
+20 to +29	...	...	...	1	3	1
+30 to +39	...	...	...	1	...	1
Over +39	...	...	...	...	2	...
Total	70	70	70	102	102	102

Based on amplitudes in specific-cycle relatives of six American series.

[a] See Chart 27 for the periods covered, and Table 64 for a more detailed breakdown.

[b] See note to Table 71 concerning the periods covered. Rounding figures produces some minor discrepancies between the results in this table and the preceding one. In this table the quarterly amplitude is expressed as a percentage deviation, to the nearest one per cent, from the monthly amplitude.

It appears, therefore, that the amplitudes of quarterly data tend to be biased in the same direction as the annual. But it is important to note carefully the differences between the quarterly and annual effects. (1) The annual amplitudes of our test series are invariably smaller than the corresponding monthly amplitudes; the quarterly amplitudes are frequently larger. (2) The differences between the quarterly and monthly amplitudes are usually much smaller than between the annual and monthly. The deviation of the quarterly amplitude from the monthly rarely exceeds 20 per cent (Table 72),[33] while the deviation of the annual

[32] The quarterly base is within 1 per cent of the monthly in 77 of the 102 corresponding cycles. The difference between bases exceeds 3 per cent in share trading only; even in this volatile series only two differences exceed 5 per cent and the largest is 12 per cent.

[33] Further, large percentage differences between the monthly and quarterly amplitudes as a rule arise only when the amplitudes are small. If the monthly rise is 2 points and the quarterly 3 points, the difference between them is 50 per cent; large percentages arising in this manner have slight significance.

from the monthly is rarely below this figure. (3) The relations among the amplitudes of individual cycles are highly similar in the quarterly and monthly forms of each series, but sometimes diverge considerably in the annual and monthly forms. Coefficients of correlation between monthly and quarterly amplitudes (Table 66) are invariably above .88 and usually well above .90, while the monthly-annual coefficients fall below .80 in five instances and below .50 in one. (4) In every series the average amplitudes of corresponding cycles are considerably lower in annual than in monthly data. The quarterly averages are somewhat lower than the monthly in five series, but higher in one (Table 73). Moreover, the differences between the quarterly and monthly measures are frequently dimmed or erased when the averages are read to the nearest one per cent. (5) Since non-corresponding cycles are few in quarterly and numerous in annual data, their effect on the relations among cycles and on the averages is slight in the quarterly but very marked in the annual measures. (6) The average joint rise and fall of all cycles in monthly data exceeds the annual average by margins of 1 to 105 points in different series; in percentages of the monthly averages the discrepancies range from −2 to −53. Thus the annual effects are usually in the same direction and of considerable size; they cannot be slighted. In contrast, the quarterly average rise and fall is within 1 point of the monthly in 4 series and the largest difference is 6 points; in percentages of the monthly averages the discrepancies range

TABLE 73

Average Amplitude of Specific Cycles in Monthly and Quarterly Data
Six American Series

Series and group	Number of specific cycles		Average amplitude in specific-cycle relatives					
			Rise		Fall		Rise & fall	
	M	Q	M	Q	M	Q	M	Q
ALL CYCLES								
Deflated clearings	15	14	26.9	27.7	13.4	13.6	40.2	41.3
Pig iron production	15	15	62.1	59.1	54.8	51.4	116.8	110.5
Railroad stock prices	18	18	35.6	35.2	31.8	31.2	67.3	66.4
Shares traded	15	15	98.1	100.3	92.4	95.8	190.5	196.1
Call money rates	23	22	115.9	115.6	116.1	117.3	232.0	232.9
Railroad bond yields	20	20	10.8	10.5	12.5	12.2	23.3	22.7
Six series[a]	106	104	59.7	59.3	55.7	55.6	115.4	114.9
CORRESPONDING CYCLES								
Deflated clearings	13	13	27.0	26.3	14.2	13.8	41.3	40.1
Call money rates	21	21	118.4	114.4	122.0	118.4	240.3	232.8
Six series[a]	102	102	59.8	58.6	56.6	55.7	116.4	114.3
NONCORRESPONDING CYCLES								
Deflated clearings	2	1	25.8	45.5	7.9	12.0	33.7	57.5
Call money rates	2	1	90.6	139.8	54.2	94.7	144.8	234 5

M stands for monthly data, Q for quarterly. The periods covered by all cycles are shown in Table 57; the noncorresponding cycles cover 1914–21 in deflated clearings and 1915–22 in call money rates. In the other four series the specific cycles in monthly and quarterly data correspond throughout.

[a] See Table 65, note 'a'.

from −5 to +3. In view of the errors to which the monthly measures of amplitude are subject, these differences are so small that they may usually be disregarded. (7) When the amplitudes of all cycles in all six series are thrown together, the averages must be carried to a decimal to show a difference between quarterly and monthly data; so that the downward bias of the quarterly amplitudes, which is tenuous and slight even for corresponding cycles, almost disappears when noncorresponding cycles are included.

IX The Secular Component of Specific Cycles

Measures of percentage change between the levels of successive cycles of monthly, quarterly, and annual data differ for two reasons: the specific cycles in different forms of the same series do not correspond invariably, and the boundaries even of corresponding cycles vary with the time unit. Both factors sometimes produce important differences in individual series; but while the effects of the second factor tend to cancel out in averages, those of the first do not.

Since annual data frequently run two or more monthly cycles together, they tend to show a larger average rise between successive cycles than do monthly data when the trend is upward, and a larger average fall when the trend is downward. When the trend changes direction the result depends in good part on the proportion of cycles skipped along the rising and the declining segments. When there is no clearly defined trend the annual and monthly averages should confirm each other. Quarterly data may be expected to differ from monthly much less but in the same direction as annual data. Table 74 illustrates these effects. The monthly-annual difference is largest in deflated clearings, a series whose annual summations skip one-third of the specific cycles. The difference is negligible in share trading and call money rates; in the first the annual cycles correspond throughout to the monthly, while the second has no clearly defined secular trend.

When the inter-cycle changes are put on a per month basis, an exceptionally large rise or fall that results from cycle-skipping is offset by a lengthened interval between the midpoints of the cycles; hence the systematic biases of annual data tend to disappear. Further, when the rates of change between the levels of successive cycles are weighted by the intervals between their midpoints, the monthly, quarterly, and annual averages become practically identical. In comparing secular movements of different series we rely usually on these weighted averages. That they are practically independent of the time unit attests the soundness of this measure.

The measures of secular movements demonstrate how difficult it is to judge merely from the steepness of the trend what proportion of the

TABLE 74
Average Secular Movement of Monthly, Quarterly and Annual Data
Six American Series

Series and form of data	Number of specific cycles	Average per cent change from cycle to cycle		
		Total	Per month	
			Unweighted	Weighted
DEFLATED CLEARINGS				
Monthly	15	+15.2	+0.36	+0.37
Quarterly	14	+16.4	+0.37	+0.37
Annual	10	+22.6	+0.36	+0.36
PIG IRON PRODUCTION				
Monthly	14	+12.9	+0.32	+0.32
Quarterly	14	+12.8	+0.33	+0.32
Annual	12	+14.5	+0.28	+0.32
RAILROAD STOCK PRICES				
Monthly	18	+10.4	+0.20	+0.21
Quarterly	18	+10.4	+0.20	+0.21
Annual	16	+11.4	+0.16	+0.21
SHARES TRADED				
Monthly	15	+14.1	+0.37	+0.34
Quarterly	15	+14.1	+0.37	+0.34
Annual	15	+14.3	+0.33	+0.35
CALL MONEY RATES				
Monthly	23	−0.1	−0.02	0.00
Quarterly	22	−0.3	−0.05	−0.01
Annual	20	−0.5	−0.03	−0.01
RAILROAD BOND YIELDS				
Monthly	20	−1.6	−0.01	−0.04
Quarterly	20	−1.6	−0.01	−0.04
Annual	14	−2.1	−0.03	−0.03

See Table 57 for the periods covered. Note, however, that the cycle from 1879 to 1885 in pig iron production is excluded from the monthly and quarterly data, since the cycles in the annual series start in 1885. When this cycle is included, the monthly averages in successive columns are +14.5, +0.34, +0.35.

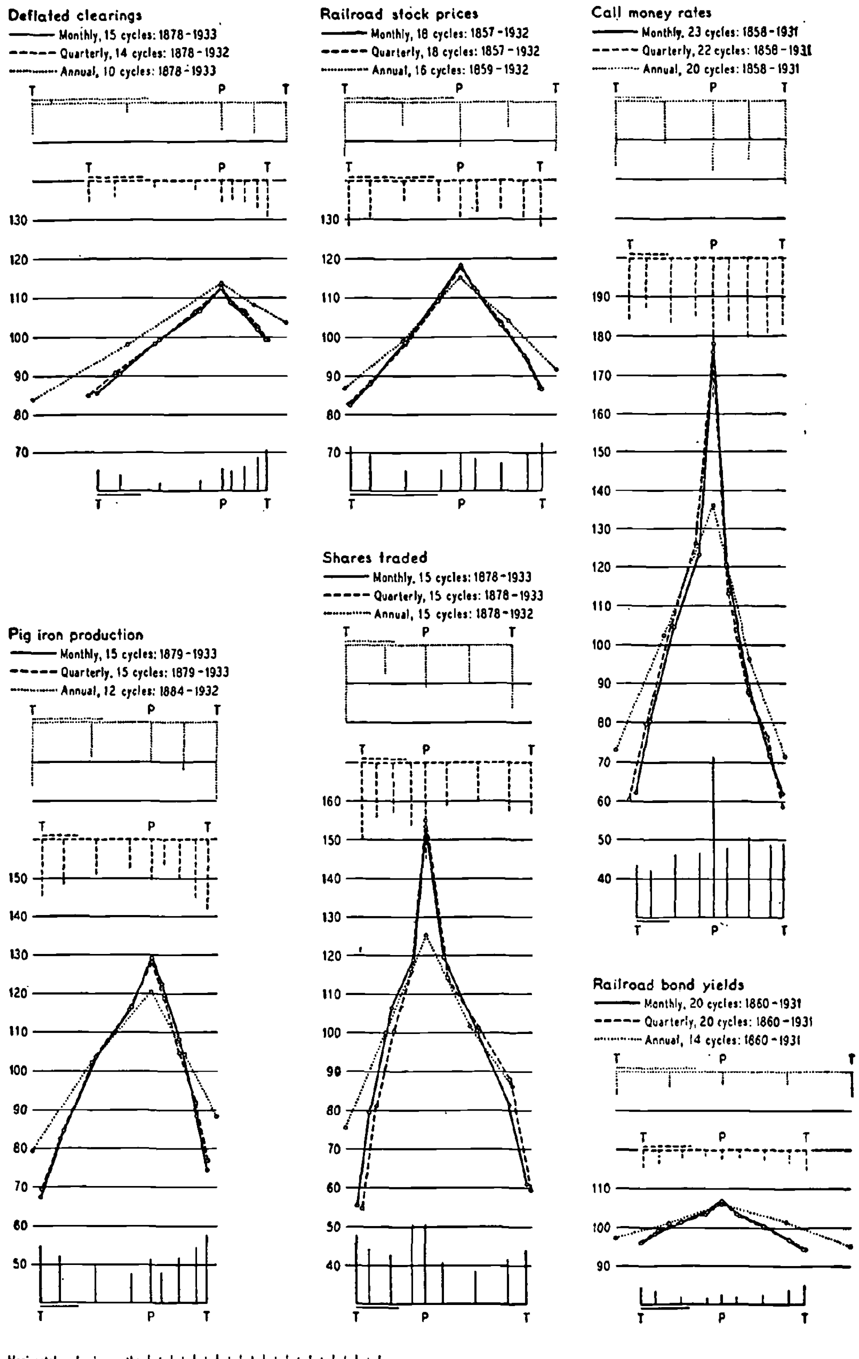
cycles in a series is skipped by annual data. The average rate of secular change is about the same in clearings and share trading, but annual data hide one-third of the monthly cycles in the former and none in the latter. Nor can the effects of the time unit be gauged very well from annual measures of both secular change and cyclical amplitude. For example, since the rate of secular change is larger relatively to the cyclical amplitude in clearings than in share trading, an investigator might expect the first series to skip a greater proportion of cycles than the second. That turns out to be correct; but by the same reasoning, share trading should skip more cycles than call money rates, and that is contrary to fact.

X Specific-cycle Patterns

Charts 26 and 27 recapitulate many of the results established in preceding sections, and add information about the *form* of the mounting wave from trough to peak and of the receding wave from peak to trough. This in-

CHART 26

Average Specific-cycle Patterns of Monthly, Quarterly and Annual Data
Six American Series

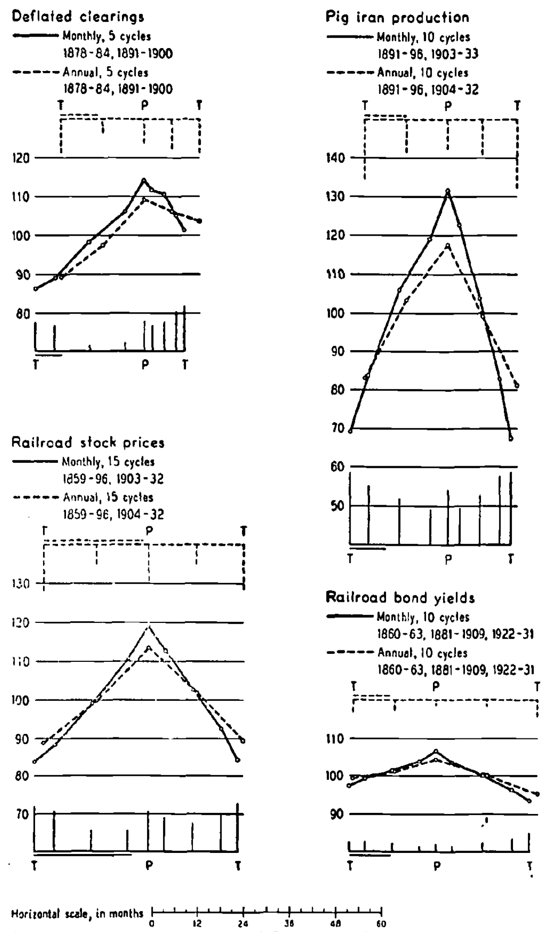


In each figure the three horizontal lines representing the average duration of specific cycles are zero base lines from which the average deviations of the three patterns are to be read. For other explanations of chart, see Ch. 5, Sec. VI and XI. See also Table 75.

CHART 27

Average Patterns of Corresponding Specific Cycles in Monthly and Annual Data

Five American Series



For explanation of chart, see Ch. 5, Sec. VI and XI. For the patterns of shares traded, see Chart 26.

formation is both more detailed and reliable when derived from monthly or quarterly than when derived from annual data.

Two factors limit the detail in which the patterns of specific cycles can be measured: erratic movements and the number of observations available for each cycle. Our experience indicates that in monthly and quarterly data fairly satisfactory results are generally obtained by making nine-point patterns.[34] If a series is free from substantial erratic movements, the monthly patterns articulate the timing of the cycles better; for their framework is fixed by quarters centered on the lowest, highest, and

34 See Ch. 8, Sec. VI.

TABLE 75
Average Specific-cycle Patterns of Monthly, Quarterly and Annual Data
Six American Series

Series and form of data	No. of specific cycles	Average in specific-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
		Initial trough	Expansion			Peak	Contraction			Terminal trough
			First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS										
Monthly	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Quarterly	14	85.0	90.9	98.4	106.4	112.6	108.7	106.5	102.7	99.0
Annual	10	84.0	...	98.1	...	113.6	...	108.2	...	103.8
PIG IRON PRODUCTION										
Monthly	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Quarterly	15	69.3	84.6	103.8	115.9	128.4	118.9	104.7	91.9	77.0
Annual	12	79.4	...	102.2	...	120.5	...	104.2	...	88.2
RAILROAD STOCK PRICES										
Monthly	18	82.8	88.0	98.8	110.8	118.3	112.4	103.2	94.0	86.6
Quarterly	18	82.9	88.3	98.4	109.2	118.1	111.5	103.7	95.2	86.8
Annual	16	87.0	...	99.0	...	115.1	...	103.0	...	91.6
SHARES TRADED										
Monthly	15	55.4	79.7	106.5	119.0	153.4	119.7	100.4	81.6	61.0
Quarterly	15	54.9	81.3	100.5	116.1	155.2	114.2	101.3	88.1	59.4
Annual	15	75.6	...	100.1	...	125.5	...	101.8	...	86.2
CALL MONEY RATES										
Monthly	23	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Quarterly	22	60.3	79.7	105.1	126.1	175.9	113.2	87.8	76.5	58.6
Annual	20	73.3	...	102.6	...	136.0	...	96.4	...	71.6
RAILROAD BOND YIELDS										
Monthly	20	96.1	98.4	101.5	104.0	106.9	103.7	100.6	96.9	94.3
Quarterly	20	96.3	99.3	101.7	103.3	106.7	103.1	100.4	97.0	94.5
Annual	14	97.5	...	101.1	...	106.2	...	101.4	...	95.1

See Chart 26 for the periods covered.

lowest monthly dates of each cycle, while the framework of the quarterly patterns is fixed by the lowest, highest, and lowest calendar-year quarters. Since the durations of expansions or contractions [35] are not always exactly divisible by three, the successive 'thirds' tend to be more uneven in quarterly than in monthly data. Further, since many cyclical phases are shorter than a year, it is frequently necessary to interpolate standings at one or more cyclical stages in the quarterly data.[36] But the advantage is not always on the side of monthly data: if erratic movements are pronounced and cyclical phases short, quarterly data may trace out the cyclical pattern better than monthly data.

On the whole, the quarterly and monthly patterns agree remarkably well, although many differences of detail turn up in Table 75, some con-

35 See pp. 145-6 and Table 48.

36 We interpolated one or more standings during 40 of the 208 phases of expansion and contraction covered by the quarterly data of the six series, but during only 2 of the 212 phases covered by the monthly data.

sistently in the same direction. We pass by the differences between the standings in stages I, V and IX, since these standings are taken from the amplitude tables and we have already commented on them. The point that calls for notice is that the average monthly standing as a rule exceeds the quarterly in stages IV and VI and falls short of the quarterly in stages II and VIII; while the differences in stages III and VII seem random. These results arise from a slight difference in our method of making monthly and quarterly patterns, not from the time unit as such.[37] Imagine a monthly series that reaches cyclical turns in the middle month of the quarter in which the quarterly data make their turn. Then the period covered by stages II-IV will be two months longer in monthly data than in quarterly, since this period includes every month except the months of cyclical turn, but every quarter except the quarterly turns. One of the additional months is adjacent to the trough; it is included in stage II and tends to pull the standing for this stage below the quarterly. The second additional month is adjacent to the peak and therefore tends to pull the standing for stage IV above the quarterly. For similar reasons, the standing of monthly data in stage VI tends to be higher and in stage VIII lower than in quarterly data.[38] If we eliminated the one-month overlap of stages IV and VI upon stage V, and of stages VIII and II upon stage I (or IX), we would find that the quarterly standings agree better with the monthly than they do at present.[39]

The patterns of annual cycles cannot be measured in the same detail as the monthly or quarterly patterns, because annual data provide too few observations on individual cycles. To make nine-point patterns without recourse to interpolations, the annual cycles must cover at least 8 years, and their phases of expansion and contraction at least 4 years. Comparatively few specific cycles meet these specifications: in our six series there are only three, and one is spurious since it telescopes two cycles in monthly data. Hence, when working with annual data we simplify the method, and record merely the standing at the initial trough, mid-expansion, peak, mid-contraction, and terminal trough. Even these simplified patterns do violence to the raw data in many instances. For when an expansion or contraction lasts just one year in the annual data, the stand-

[37] That is to say, the monthly patterns could have been so made as to avoid the systematic differences from the quarterly. See p. 151. But monthly data are basic in our scheme; we therefore used that method of analysis which seems best suited to them, and adapted the quarterly analysis to it.

[38] The text presupposes positive analysis. If the specific cycles are treated on an inverted basis, the relations between the quarterly and monthly patterns are similar; but stages II, IV, VI and VIII of positive cycles become VI, VIII, II and IV, respectively, in inverted cycles.

[39] The very feature of our method that tends to produce a lower standing of monthly than quarterly data in stage II tends to make the interval from stage I to II shorter in monthly than in quarterly data; the one change tends to offset the other so far as the rate of change between stage I and II is concerned. Similar statements apply to the rates of change between other stages: thus the increment from stage II to III tends to be larger in monthly than quarterly data, but the interval between stage II and III also tends to be longer in monthly data, and so on.

ing in the middle of the phase must be determined by interpolation. These poor substitutes for actual observations account for 75 out of 174, or over two-fifths of the mid-phase standings in our six series.

Close observation of the form of specific cycles is a matter of great importance. A sine curve in the strict sense has been a common device of mathematical economists in their work on business cycles. In a looser sense, the sine curve assumption is a favorite of 'literary economists' as well. Numerous hypotheses about cyclical fluctuations, particularly those that make much of the 'acceleration principle', proceed on the tacit assumption that the cyclical movements of time series behave like sine curves, in the sense that the rate of rise reaches a maximum around mid-expansion and the rate of fall attains a maximum around mid-contraction. Sometimes the specific cycles of economic activities conform to this model, probably more often they do not; the crucial point is that monthly, or at least quarterly, data are necessary to grapple with the acceleration principle or any other hypothesis that calls for fairly detailed knowledge of cyclical patterns.[40] For example, monthly data on iron production show that the rise is fastest typically in the early or late stages of expansion, while the decline is sharpest toward the close of contraction; the annual patterns cover up these variations in the progress of the cyclical movements. Again, the annual pattern of call money rates suggests that the rise is faster in the second than in the first half of expansions, and that the decline is faster in the first than in the second half of contractions. In fact the large movements of call money rates are characteristic of brief intervals in the vicinity of cyclical peaks, not of the full period from mid-expansion to mid-contraction.[41]

Chart 28 completes the preceding analysis. The chart shows cyclical patterns of monthly and annual data adjusted for variation in the average duration and amplitude of their separate phases.[42] If annual and monthly data conveyed the same information concerning relative variations in the rate of cyclical movements, their adjusted patterns would be the same. If neither conveyed any information on the subject—which would happen if both represented the standing of a series merely at the initial trough, peak, and terminal trough—the adjusted patterns would be the same regardless of the series or the time unit, as are the triangles formed by connecting peaks and troughs in Chart 28. Actually, the annual patterns, as a rule, hug closely the triangular guidelines, while the monthly patterns move freely; in other words, annual data tell little about the form of specific cycles, and what little they tell is not always trustworthy. The real culprit, of course, is the small number of observations from which the annual patterns are made. When a cyclical phase lasts only

[40] Cf. pp. 340-7.

[41] See Charts 26-27 and Table 76.

[42] See pp. 157-9 concerning this adjustment.

TABLE 76
Position of Fastest and Slowest Rates of Change in Specific-cycle Patterns
Six Monthly American Series

Series	No. of specific cycles	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Distribution of rates of change between stages							
		No. of expansions in which rise is fastest				No. of contractions in which fall is fastest			
Deflated clearings...	15	5	1	2	7	6.5	4.5	3	1
Pig iron production.	15	7	1	2	5	1	2	4	8
Railroad stock prices	18	2	4	4	8	5	2	4	7
Shares traded......	15	4	4	1	6	9	1	1	4
Call money rates...	23	2	1	3	17	16	2	3	2
Railroad bond yields	20	6.5	4	1	8.5	10.5	3	2.5	4
		No. of expansions in which rise is slowest				No. of contractions in which fall is slowest			
Deflated clearings...	15	6	4	3	2	...	7	5.5	2.5
Pig iron production.	15	3	1	11	...	5.5	4.5	2	3
Railroad stock prices	18	6	2	6	4	5	4	8	1
Shares traded......	15	2	2	9	2	2	4	8	1
Call money rates...	23	3	7	12	1	1	8	4	10
Railroad bond yields	20	2	7.5	7.5	3	2.5	8	3.5	6

To minimize the number of ties the rates of change were computed to extra decimals. But some ties remained; hence the fractions. See Table 55 for the periods covered.

one year, as happens frequently, we know nothing about variations in the rate of movement within it. When a phase lasts two or three years we have some knowledge; but since we cannot even be sure that annual figures tell correctly the direction of the cyclical movements, we must accept with serious reservations what they tell about rates of change in cyclical movements.[43]

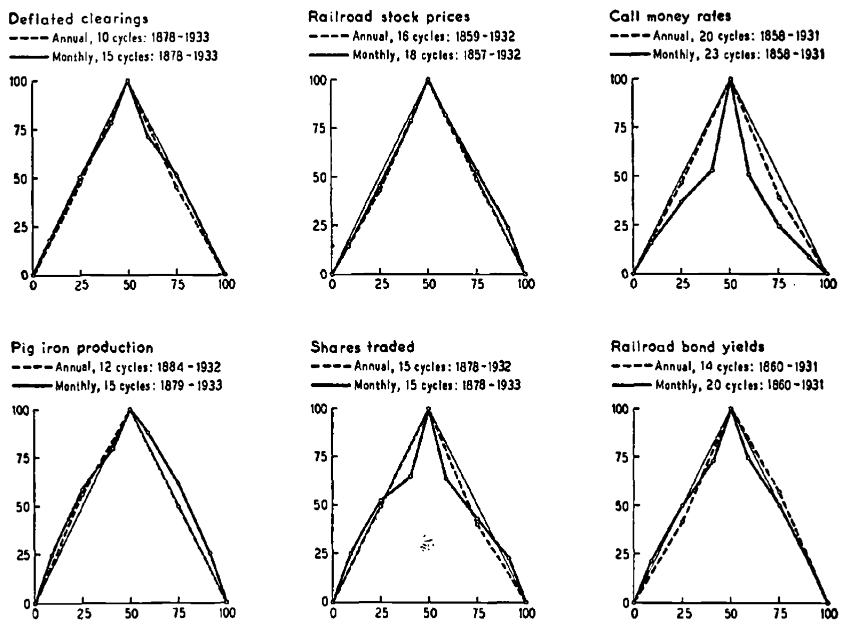
XI Different Forms of Annual Data

The problem of the time unit consists of three distinct parts, only one of which we have considered thus far. Cyclical measures depend, first, upon the number of entries per year, second, upon the boundaries of the time unit to which the entries refer, third, upon the presence or absence of a gap between the entries. If the number of observations per year is large the three factors merge; as the number dwindles the second and third factors become increasingly important. Specific cycles determined from monthly data are practically independent of the limits of the monthly figures; that is, they would not be changed appreciably if calendar-month figures were replaced by figures running from the fifteenth of one month

[43] To observe how the monthly and annual patterns compare when both show equivalent detail, it is necessary merely to connect the standings at the initial trough, mid-expansion, peak, mid-contraction, and terminal trough in the monthly curves on Chart 28. Some enormous differences appear, particularly in iron production, call money rates, and bond yields. Clearly, annual data provide less reliable as well as scantier information about cyclical patterns than do monthly data. See the comparison between the directions of annual and monthly data in Sec. III.

CHART 28

Variation of Average Rates of Change from Stage to Stage of Expansions and Contractions of Specific Cycles in Monthly and Annual Data
Six American Series



To facilitate study, the peaks and troughs are joined by straight lines representing uniform rates of expansion and contraction. For further explanations, see Ch. 5, Sec. VI.

to the fifteenth of the next, or from the tenth to the tenth. Nor would they be changed materially, except in highly erratic series, if full-month averages or aggregates were replaced by daily figures reported once a month. Annual data lack this stability. What annual data do to the specific cycles depends in considerable measure on the boundaries of the figures and on the size of the gap, if any, between the successive figures.

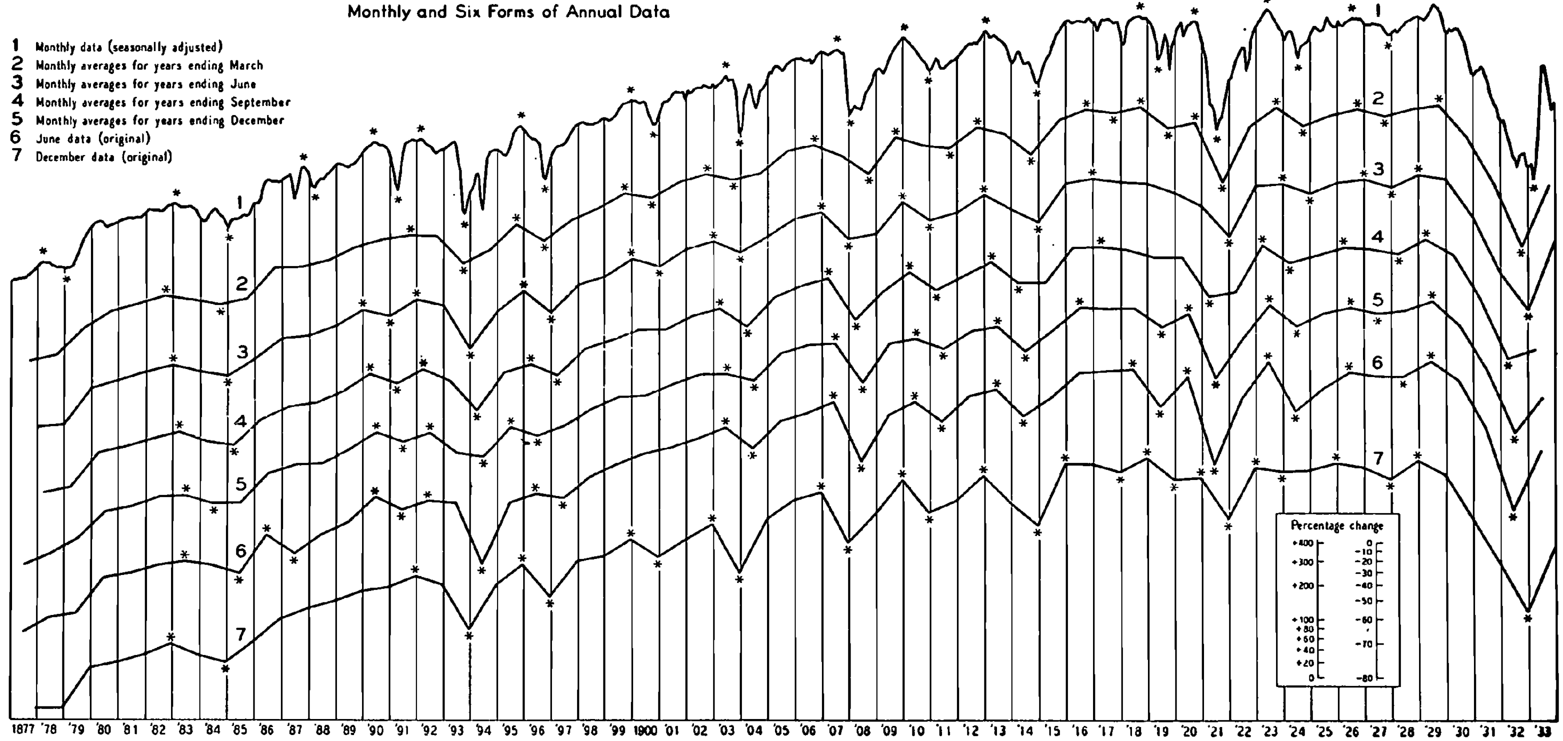
Imagine a twelve-month moving total (or average) passed through a time series, properly centered, and plotted on a chart. Next suppose that the moving totals plotted at the end of June in successive years are singled out and all others ignored. The figures chosen are the calendar-year summations analyzed in preceding sections,[44] but they are merely one out

[44] This interpretation of annual figures is helpful in analyzing various problems raised by the time unit. For example, we know that a twelve-month moving average lops off cyclical peaks and fills in cyclical troughs. But the cyclical amplitude is cut further by selecting every twelfth item of the series of moving averages, for these selected values rarely coincide with the peaks and troughs of the moving averages. It follows that the annual amplitudes must be lower than the monthly at least by the amount of dampening produced by a twelve-month moving average, and are likely to be lower by a larger amount. (See, however, note 27.)

CHART 29

Pig Iron Production, United States, 1877–1933

Monthly and Six Forms of Annual Data

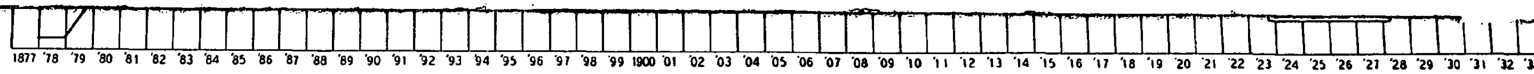


Asterisks identify peaks and troughs of specific cycles, determined on our standard plan. The annual series are derived from monthly figures unadjusted for seasonal variations. See the note at the end of Sec. II.

CHART 30

Bank Clearings outside New York City, Deflated, 1875–1933

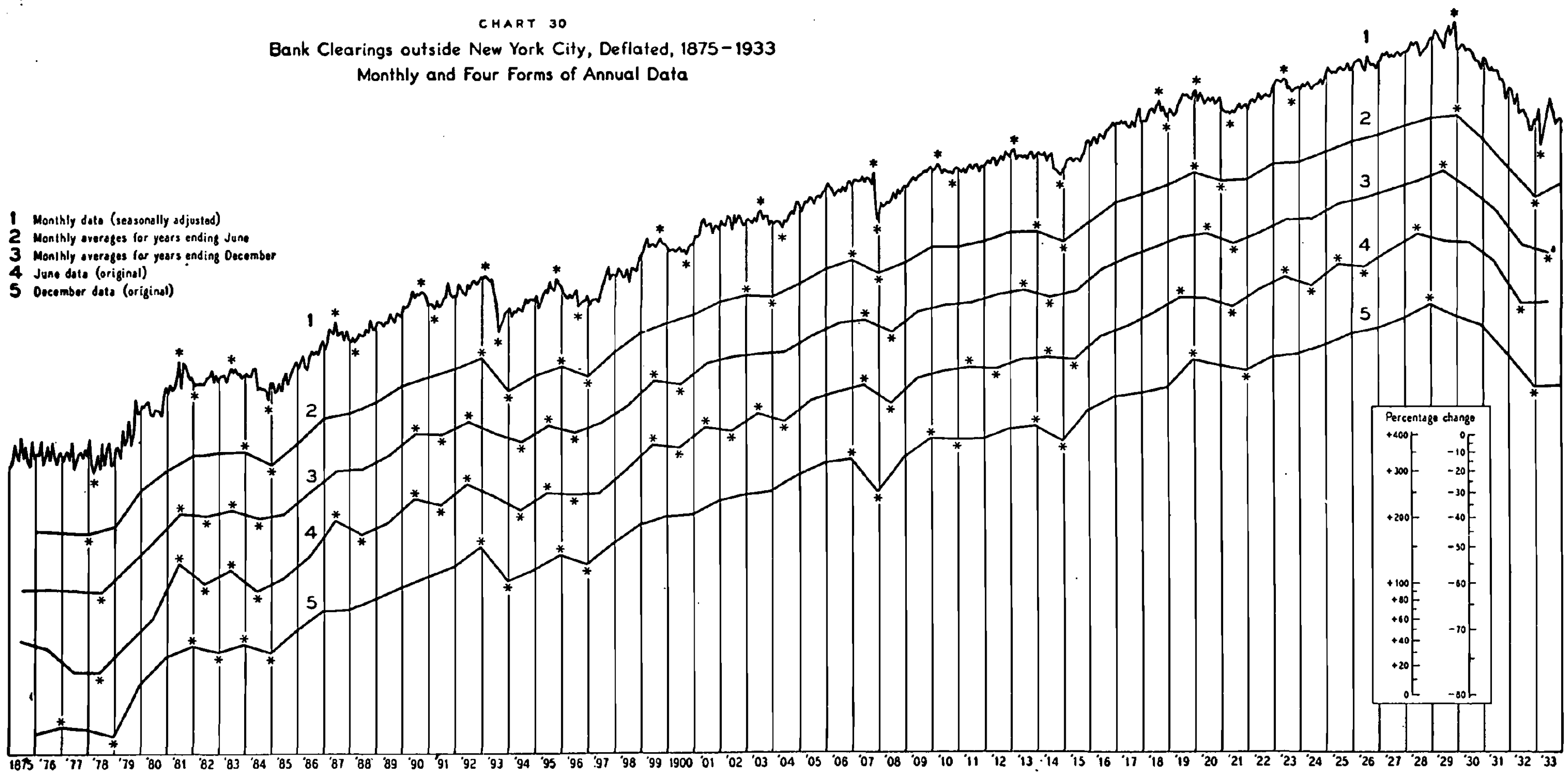
Monthly and Four Forms of Annual Data



Asterisks identify peaks and troughs of specific cycles, determined on our standard plan. The annual series are derived from monthly figures unadjusted for seasonal variations. See the note at the end of Sec. II.

CHART 30

Bank Clearings outside New York City, Deflated, 1875–1933
Monthly and Four Forms of Annual Data



The vertical scale in this chart is double that in Charts 29 and 31.

See note to Chart 29. From Jan. 1919 on, bank debits in 140 cities outside New York are used in place of bank clearings; the level of the former is adjusted to that of the latter in Jan. 1919.

* Data rise after 1933.

TABLE 77

Duration and Amplitude of Successive Specific-cycle Contractions in Monthly and Twelve Forms of Annual Data
Pig Iron Production, United States, 1883–1933

Contraction in monthly data		Duration and amplitude of corresponding contraction in annual summations from											
Peak – trough dates	Duration and amplitude	Aug. to July	Sep. to Aug.	Oct. to Sep.	Nov. to Oct.	Dec. to Nov.	Jan. to Dec.	Feb. to Jan.	Mar. to Feb.	Apr. to Mar.	May to Apr.	June to May	July to June
Feb. 1883 – Jan. 1885	23,21	24,14	24,14	24,15	24,14	24,13	12,10	24, 9	24,10	24,10	24,11	24,11	24,12
Oct. 1887 – Mar. 1888	5,19	S	S	S	S	S	S	S	S	S	12,0.3	S	S
May 1890 – Apr. 1891	11,38	12, 8	12,10	12,11	12,11	12,11	12,10	12, 7	12, 2	S	S	12, 3	12, 7
Feb. 1892 – Oct. 1893	20,56	24,46	24,43	24,39	24,34	24,29	24,25	12,24	12,27	24,29	24,34	24,40	24,45
Nov. 1895 – Oct. 1896	11,44	12,22	12,19	12,12	12, 3	12, 5	12,10	12,14	12,16	12,18	12,20	12,22	12,23
Dec. 1899 – Oct. 1900	10,24	12, 6	12, 3	S	S	S	S	12, 2	12, 4	12, 6	12, 7	12, 8	12, 8
June 1903 – Dec. 1903	6,42	12,15	12,18	12,20	12,19	12,15	12, 7	12, 3	12, 5	12, 7	12, 8	12, 9	12,13
July 1907 – Jan. 1908	6,50	12,32	12,36	12,39	12,42	12,43	12,38	24,35	24,32	24,30	24,28	24,26	12,27
Jan. 1910 – Dec. 1910	11,30	12,21	12,21	12,20	12,19	12,17	12,13	24,10	24,11	24,13	24,14	12,17	12,20
Jan. 1913 – Dec. 1914	23,44	24,27	24,25	12,22	12,24	12,25	12,25	24,24	24,26	24,28	24,29	24,30	24,29
July 1917 – Jan. 1918[a]	6,21	S	S	S	S	S	12, 2	12, 4	12, 4	12, 4	12, 4	12, 4	S
Sep. 1918 – May 1919	8,35	S	S	24,12	36,16	36,19	12,20	12,23	12,24	12,23	12,21	S	S
Sep. 1920 – July 1921	10,70	60,47	60,44	12,38	12,46	12,51	12,54	12,56	12,55	12,52	12,46	36,53	60,50
May 1923 – July 1924	14,40	24, 9	12,14	12,19	12,22	12,23	12,22	12,21	12,20	12,20	12,19	12,16	12,10
July 1926 – Nov. 1927	16,17	12, 8	24, 7	24, 7	24, 7	12, 7	12. 7	12, 7	12, 8	12, 9	12, 9	12, 9	12, 9
July 1929 – Mar. 1933	44,86	48,78	48,75	36,76	36.78	36,79	36,79	36,80	36,81	36,82	36,82	48,82	48,81

S means that the contraction is skipped; see Chart 29. The figure to the left of the comma shows the duration of the contraction (in months), the figure to the right shows its amplitude. To economize effort, the annual amplitudes are expressed as percentages of the peak value from which the contraction starts. The monthly declines are shown on a similar plan, but they are computed from three-month averages centered on the peaks and troughs.

[a] See text below.

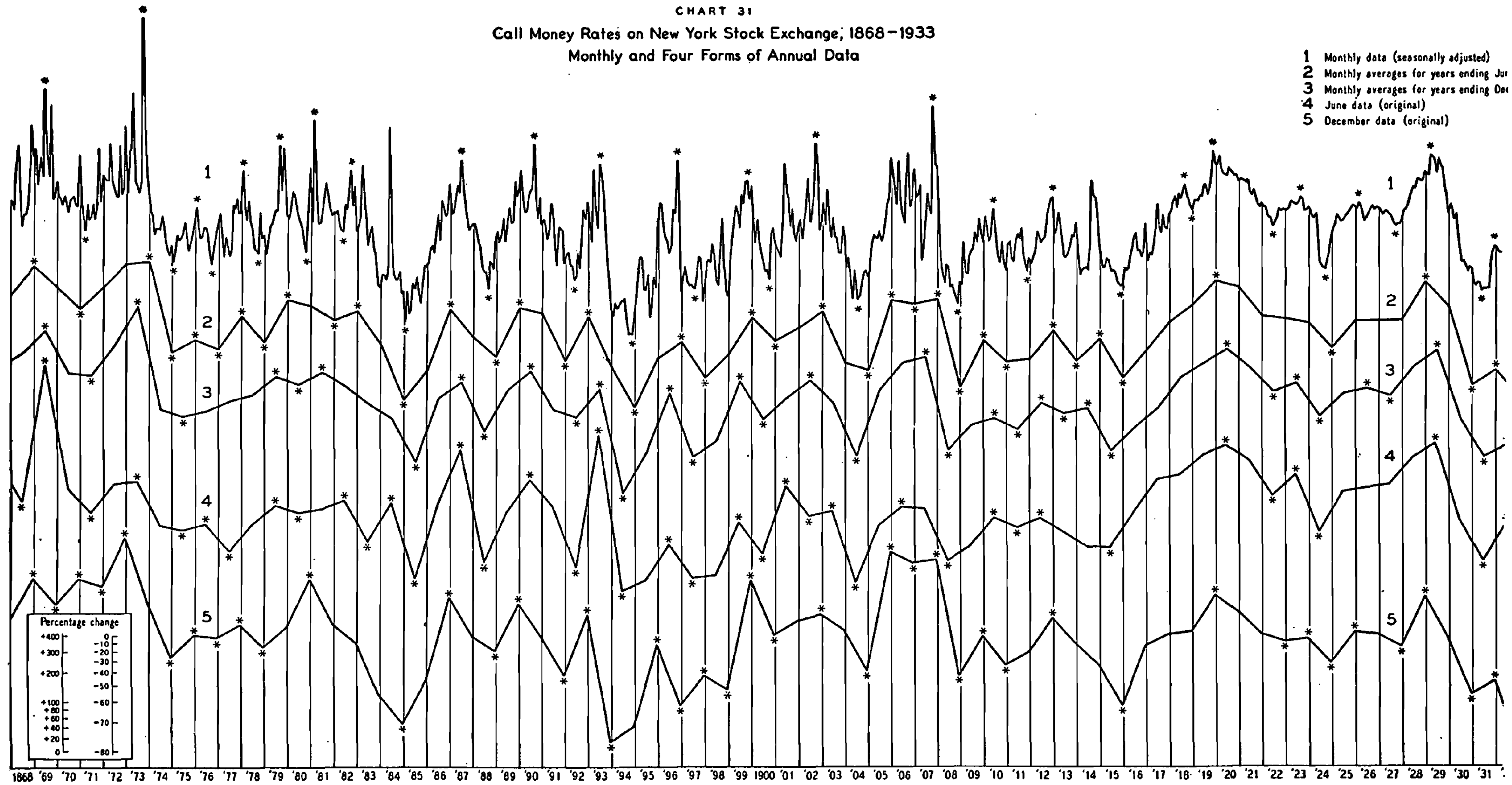
of twelve annual summaries that might have been selected.[45] This point is important; and to give it emphasis, we have carried through an experiment with twelve sets of annual totals for pig iron output. The first two columns of Table 77 list the specific-cycle contractions, their duration and their amplitude. The contraction from July 1917 to January 1918, which we consider a random movement, is also listed because several annual summations reflect it. The succeeding columns give the durations and amplitudes of corresponding declines in each of the twelve annual summations, every decline—however slight—being listed. There is, of course, considerable similarity among the twelve arrangements, but the thing to notice is how whimsically the cyclical movements change now and then with shifts in the annual boundaries (see Chart 29). The contraction of 1892–93 appears in every form of the annual data; but in one summation its duration is one year and its amplitude 24 per cent, in another the duration is two years and the amplitude 46 per cent. The contraction of

[45] On the whole there is an increasing tendency to present annual figures on a calendar-year basis, but this practice is far from universal. For example, the receipts and expenditures of the United States government are reported for the fiscal year ending June 30, similar data for Great Britain and Germany cover years ending March 31, agricultural data are frequently reported by crop years, and so on.

CHART 31

Call Money Rates on New York Stock Exchange, 1868–1933

Monthly and Four Forms of Annual Data



See note to Chart 29. For monthly data, 1857–67, see Chart 53.

TABLE 78

Characteristics of Cyclical Phases Skipped by Calendar- and Fiscal-year Data
Three American Series

Series and phase	Phase (marked S) skipped by		Characteristics of skipped phase[c]		
	Calendar-year data	Fiscal-year data[a]	Nature[b]	Duration in months	Amplitude in specific-cycle relatives
DEFLATED CLEARINGS					
June 1881 – Jan. 1882	..	S	C	7	10.2
June 1887 – Mar. 1888	S	S	C	9	6.5
Sep. 1890 – Mar. 1891	..	S	C	6	8.0
Sep. 1899 – Sep. 1900	..	S	C	12	6.2
July 1903 – May 1904	S	..	C	10	5.9
Mar. 1910 – Oct. 1910	S	S	C	7	3.3
Aug. 1918 – Dec. 1918	S	S	C	4	4.9
May 1923 – Sep. 1923	S	S	C	4	5.7
PIG IRON PRODUCTION					
Oct. 1887 – Mar. 1888	S	S	C	5	25.5
Dec. 1899 – Oct. 1900	S	..	C	10	30.5
May 1919 – Sep. 1920	..	S	E	16	40.7
CALL MONEY RATES					
Jan. 1876 – Sep. 1876	S	..	C	8	51.9
Jan. 1878 – Sep. 1878	S	..	C	8	66.5
Oct. 1880 – Feb. 1881	..	S	E	4	98.0
May 1882 – Sep. 1882	S	..	E	4	60.1
Aug. 1918 – Dec. 1918	S	S	C	4	31.0
June 1922 – Aug. 1923	..	S	E	14	33.0
Feb. 1926 – Sep. 1927	..	S	C	19	27.0

See Table 55 for the periods covered.
[a] Years ending June 30.
[b] C stands for specific-cycle contraction, E for expansion.
[c] See also Table 53 and Charts 29-31.

1907–08 lasted six months according to the monthly data, but one or two years according to the annual data; its amplitude was 50 per cent according to the monthly data, 26 to 43 per cent according to the annual. The contraction of 1910 is reflected in every form of the annual data, while the contraction of 1890–91, which was just as long and more pronounced, is skipped in two instances. The contraction of 1929–33, by far the longest and deepest in the record, is reflected fairly accurately in the annual series, its duration varying from three to four years and its amplitude from 75 to 82 per cent. But even here some annual series are deceptive: in the summations ending June, July, and August, there is a four-year contraction from 1929 to 1933 but a five-year contraction from 1917 to 1922.[46]

These results for iron production are not unusual. Charts 30 and 31 show the monthly, calendar-year, and fiscal-year figures (that is, for years ending June 30) of two additional series, clearings and call money rates.

[46] Compare the present analysis, where we take the monthly turning dates of the cycles as they come and observe shifts in the annual cycles produced by changing the annual boundaries, with the analysis of Sec. II where we treat the years as fixed but shift hypothetical monthly series along the time scale. The principle is the same: the shapes of annual cycles depend upon the months of cyclical turn in relation to the annual boundaries.

TABLE 79

Average Measures of Specific Cycles in Monthly and Four Forms of Annual Data
Three American Series

Series and form of data	Period covered[a]	No. of specific cycles	Av. lead (−) or lag (+) in months at reference[b]		Av. duration in months			Av. amplitude in specific-cycle relatives			Av. per cent change from cycle to cycle	
			Peaks	Troughs	Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Total	Per month (wtd.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
DEFLATED CLEARINGS												
Monthly	1878–1933	15	+3.2	−5.8	32.6	11.4	44.0	26.9	13.4	40.2	+15.2	+0.37
Calendar-year	1878–1933	10	+1.8	−3.8	49.2	16.8	66.0	29.7	9.8	39.5	+22.6	+0.36
Fiscal-year	1878–1933	8	+4.8	−5.9	67.5	15.0	82.5	36.8	13.6	50.4	+28.4	+0.36
June	1878–1932	16	+0.5	−2.8	24.8	15.8	40.5	22.0	8.5	30.5	+14.7	+0.38
December	1878–1932	9	−0.8	−4.3	54.7	17.3	72.0	35.6	14.6	50.2	+25.4	+0.36
PIG IRON PRODUCTION												
Monthly	1879–1933	15	+1.9	−3.4	28.8	14.5	43.3	62.1	54.8	116.8	+14.5[c]	+0.35[d]
Calendar-year	1884–1932	12	+0.3	−3.7	31.0	17.0	48.0	41.1	32.4	73.5	+14.5	+0.32
Fiscal-year	1885–1933	12	−1.5	−2.9	27.0	21.0	48.0	41.8	33.6	75.4	+13.8	+0.30
June	1885–1932	13	+1.7	−2.0	27.7	15.7	43.4	51.9	44.6	96.5	+15.3	+0.35
December	1884–1932	13	−0.6	−3.3	25.8	18.5	44.3	48.8	45.0	93.8	+10.2	+0.25
CALL MONEY RATES												
Monthly	1858–1931	23	−0.1	+1.5	19.9	18.0	37.9	115.9	116.1	232.0	−0.1	0.00
Calendar-year	1858–1931	20	−0.4	−1.8	25.2	18.6	43.8	62.7	64.4	127.1	−0.5	−0.01
Fiscal-year	1859–1931	21	+0.4	+2.9	20.6	20.6	41.1	56.2	58.0	114.2	−0.4	−0.01
June	1858–1931	23	−2.1	+0.7	21.4	16.7	38.1	68.6	70.9	139.5	+0.1	0.00
December	1858–1930	24	−4.2	+1.0	17.0	19.0	36.0	63.7	63.7	127.4	−0.3	−0.01

[a] Applies to col. (6)-(13). Col. (4)-(5) include all timing measures within the period covered by the monthly data.

[b] The timing of each series is measured from the monthly reference dates. The number of turns included in the averages depends on the number of comparisons made with the reference dates. Usually this number is not the same as the number of full specific cycles (see Charts 29-31).

[c] If the monthly average is restricted to the period of the annual, +12.9.

[d] If the monthly average is restricted to the period of the annual, +0.32.

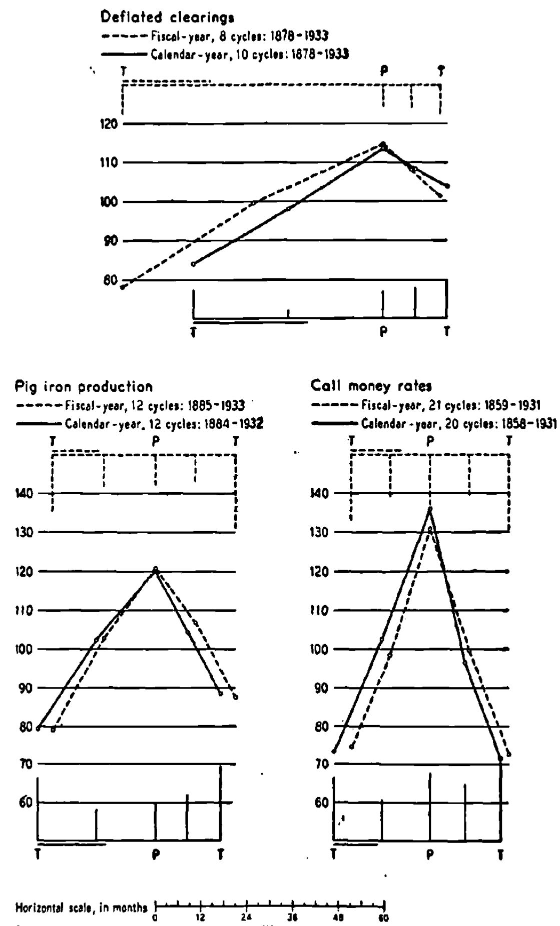
Tables 78-79 and Chart 32 compare the cyclical measures derived from the calendar- and fiscal-year figures of all three series. The discrepancies between the two annual forms are considerable. In clearings the calendar-year figures skip five specific-cycle contractions, the fiscal-year figures skip seven. In call money rates both annual forms skip four cyclical phases, but they skip only one phase in common.[47] Notable discrepancies appear in the timing averages and in the average rates of change within expansions and contractions. The measures of cyclical duration and amplitude differ conspicuously, though they tell a reasonably consistent story. The one measure not affected appreciably by the character of the

[47] Table 78 records merely the skipped cycles. But as stated on p. 217 the calendar-year figures of call money rates show an 'extra' specific-cycle expansion from 1913 to 1914. The fiscal-year figures reflect this movement, and besides seem to show another 'extra' cycle. The decline from Sept. 1906 to April 1907, which we have treated as a random movement in analyzing monthly data, is fairly prominent in the fiscal-year data; we have treated it, after some hesitation, as cyclical.

Only two changes in the direction of the fiscal-year figures were disregarded in marking off specific cycles: a decline of 0.1 per cent in deflated clearings from 1910 to 1911 and in call money rates from 1926 to 1927. The decline from 1906 to 1907 in call money, which we have not disregarded, is 6 per cent of the value in 1906.

CHART 32

Average Specific-cycle Patterns of Calendar- and Fiscal-year Data
Three American Series



For explanation of chart, see Ch. 5, Sec. VI and XI.

annual summation is the average secular change per month over the period covered by all cycles.

Charts 29-31 and Table 79 give us a glimpse also of what happens when an annual series fails to represent full years, but refers instead—as do many financial series—to a single date within the year. For the present purpose we have merely taken the original (that is, unadjusted for seasonal) June and December figures in every year covered by three series, and analyzed each set of June and each set of December figures by the technique we apply to full-year data. Our sample is small and the evidence not clear-cut, but the following conclusions seem justified. (1) Specific cycles tend to be more numerous in single-date than in full annual series.

The reason is that in single-date series erratic values may be joined together into movements that look like specific cycles. (2) The specific cycles that can be distinguished in the June figures are not very good approximations to the specific cycles in the calendar-year data, nor are the December figures very good approximations to the fiscal-year data. (3) The June and December entries are independent observations. But the calendar-year figure for any given year has six months in common with the fiscal-year figure. Hence the full-year forms of a series are more nearly similar in their movements than are the single-date forms. (4) Estimates of average timing and the average monthly rate of secular change tend to be slightly better when made from full-year than from one-date-per-year series. (5) On the average, specific cycles tend to be shorter but of larger amplitude in single-date than in full-year series. Consequently, it may be expected that these average measures will approximate more closely the averages of the underlying monthly data in single-date than in full annual series. This expectation is stronger for durations than for amplitudes.[48] (6) But it is always necessary to make qualitative amendments of average durations and amplitudes derived from annual data; that is to say, some judgment must be made of the biases to which the averages are subject. However difficult this may be in the case of full-year data, it is still more difficult in single-date series, for a change in the boundaries of the data is apt to make a larger difference in single-date than in full-year figures. In comparing annual series of different processes, an investigator is therefore even more likely to err when the series represent only one date a year than when they represent whole years.

XII Reference-cycle Measures

As explained in Section IV of Chapter 4, our procedure in dating business cycles is to set monthly reference dates, and then make the quarterly and annual reference dates match the monthly as well as may be. Once a monthly reference cycle has been recognized, we recognize also a quar-

[48] In the absence of erratic movements, the peak of (say) calendar-year data (conceived of as every twelfth term of a twelve-month moving average of average daily figures, centered at the end of June) is likely to be lower than the peak of actual (last day of) June figures, while the trough of the former is likely to be higher than the trough of the latter. Erratic movements do not affect the expectation. Regardless of erratic movements, so far as the specific cycles in a single-date annual series and in a full-year series correspond, the amplitudes of the single-date series are likely to be larger on the average. However, this tendency is offset by the tendency of single-date series to show more cycles than do full-year series.

The average amplitudes of specific cycles that correspond in the calendar-year and June series are as follows:

	No. of cycles	Calendar-year data: Rise	Fall	Rise & fall	June data: Rise	Fall	Rise & fall
Deflated clearings	6	22.9	5.8	28.7	27.4	9.0	36.4
Pig iron production	11	37.5	34.0	71.4	51.5	48.8	100.3
Call money rates	11	78.3	73.3	151.6	84.4	76.3	160.7

terly and annual cycle in approximately the same period. It is reasonably certain that if we attempted to date quarterly reference cycles independently of the monthly, we would get practically the same results as are yielded by our method. Independent dating of annual reference cycles, on the other hand, would probably yield fewer reference cycles. Such a chronology would be less significant than the one we obtain by forcing the annual reference cycles into correspondence with the monthly. For the new annual cycles would not be homogeneous, some corresponding to single cycles in the monthly chronology and others combining two or even three cycles. We might get one set of reference cycles on a calendar-year basis and a different set on a fiscal-year basis. There is no assurance that the new list, even if restricted to calendar years, would consistently include the largest and exclude the mildest cycles in the monthly list; for as the experiments of this chapter demonstrate, annual data sort out mild and substantial cyclical movements imperfectly. We can even imagine a case in which every annual series reflected a cyclical movement disclosed by monthly data but which we could not recognize as a reference cycle unless we took account of the monthly data: that would happen if some annual series declined one year, others equally important declined the following year, and a dependable composite of all series merely registered a retarded rise over the two years. By forcing quarterly and annual reference cycles into correspondence with monthly cycles, we avoid these difficulties and ensure that the analysis is always focused on the same units of business experience, though its exactness varies with the form of the data.

The small number of annual observations on individual cycles re-

TABLE 80

Number of One-year Phases in Annual Reference Cycles

Four Countries

Country and period covered	Total no. of reference phases	Number of one-year phases		
		Expansion	Contraction	Both
United States				
1834–1938	52	6	17	23
1878–1938	32	3	13	16
Great Britain				
1793–1938	58	3	18	21
1879–1938	24	3	7	10
Germany				
1866–1932	24	1	6	7
1878–1932	20	1	5	6
France				
1840–1938	42	5	12	17
1879–1938	26	3	8	11
Four countries[a]	176	15	53	68

Derived from the calendar-year reference dates in Table 16.

[a]The summary is based on the maximum period shown above for each country.

stricts the detail in which reference-cycle patterns can be measured, just as it restricts the detail in which specific-cycle patterns can be measured. If we attempted to make nine-point reference-cycle patterns of annual data without interpolating, we would succeed in only 4 of the 88 reference cycles covered by our annual chronology in four countries. Table 80 shows how numerous one-year phases have been in these countries, particularly contractions in the United States after 1878. Since interpolations are necessary to obtain mid-phase standings for phases lasting one year, even our five-point patterns elaborate excessively upon the raw data of many annual reference cycles, just as they elaborate excessively upon the raw data of many specific cycles.

That the loss of detail is a serious disadvantage is apparent from Table 81 and Chart 33. The annual patterns are like the monthly in that both show that bank clearings, iron production, share trading, stock prices, and call money rates on the average tend to rise and fall with the general tides in business activity, while bond yields bear a quasi-inverted relation to business cycles. But the annual patterns sacrifice precious information:

TABLE 81

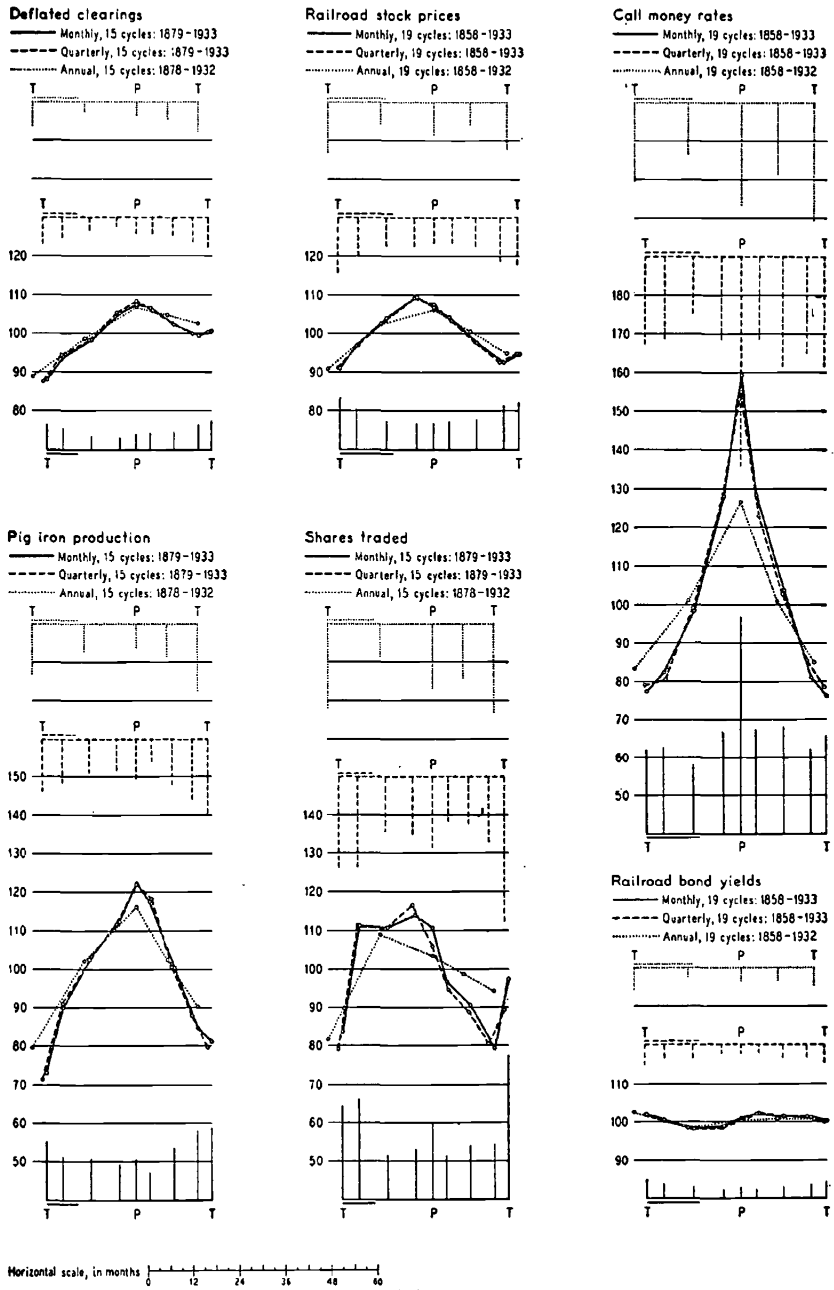
Average Reference-cycle Patterns of Monthly, Quarterly and Annual Data
Six American Series

Series and form of data	No. of reference cycles	Average in reference-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
			Expansion				Contraction			
		Initial trough	First third	Middle third	Last third	Peak	First third	Middle third	Last third	Terminal trough
DEFLATED CLEARINGS										
Monthly	15	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Quarterly	15	87.6	94.3	98.2	105.3	108.2	106.0	102.7	100.0	100.3
Annual	15	88.8	...	98.6	...	106.9	...	104.9	...	102.6
PIG IRON PRODUCTION										
Monthly	15	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Quarterly	15	71.6	91.0	102.5	111.8	122.0	118.3	100.6	87.9	79.8
Annual	15	79.7	...	102.1	...	116.2	...	102.5	...	90.2
RAILROAD STOCK PRICES										
Monthly	19	91.0	96.9	104.0	109.4	106.9	104.3	97.7	92.5	94.7
Quarterly	19	91.1	97.0	103.6	109.2	107.5	103.2	97.8	92.5	94.7
Annual	19	90.8	...	102.5	...	106.1	...	100.5	...	94.9
SHARES TRADED										
Monthly	15	83.8	111.2	110.9	114.0	110.6	96.8	90.5	79.5	97.5
Quarterly	15	79.3	111.2	110.3	116.7	105.9	94.7	89.0	80.4	89.5
Annual	15	81.7	...	109.0	...	103.4	...	98.7	...	94.2
CALL MONEY RATES										
Monthly	19	77.5	82.4	98.4	128.2	159.5	128.5	103.9	81.1	76.2
Quarterly	19	79.1	80.4	99.3	125.9	155.1	123.0	102.9	85.6	78.6
Annual	19	83.5	...	101.2	...	126.7	...	100.9	...	85.1
RAILROAD BOND YIELDS										
Monthly	19	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2
Quarterly	19	102.0	100.3	98.3	98.7	100.4	102.1	101.5	101.4	100.0
Annual	19	102.6	...	98.8	...	100.6	...	100.9	...	100.8

See Chart 33 for the periods covered.

CHART 33

Average Reference-cycle Patterns of Monthly, Quarterly and Annual Data
Six American Series



In each figure the three horizontal lines representing the average duration of reference cycles are zero base lines from which the average deviations of the three patterns are to be read. For other explanations, see Ch. 5, Sec. VII and XI. See also Table 81.

they give no hint of the tendency of clearings, stock prices, and share trading to turn up early in business-cycle revivals; they indicate that share trading tends to turn down early in recessions but fail to disclose a similar tendency in stock prices; they conceal the retardation of the rise in pig iron production in mid-expansion and its accelerated decline in mid-contraction. Quarterly data, on the other hand, trace out the reference-cycle movements with practically the same fidelity as the monthly data.[49]

Also measures of conformity to business cycles are rougher approximations in annual than in monthly or quarterly data. If a reference peak or trough is misdated by one year, it is apt to leave a mark on conformity indexes of annual data, but minor shifts of monthly or quarterly reference dates usually have slight influence. Similarly, if there is a substantial error in a series one month or quarter, the effect on conformity measures is likely to be slight, while even a small error that affects the direction of an annual figure may easily prove serious. If a series conforms badly to business cycles, conformity indexes on an annual basis may be higher or lower than indexes on a monthly or quarterly basis. But if a series conforms well, the chances are that annual indexes will understate conformity. For annual data tend to skip conforming contractions when the secular trend is rising and conforming expansions when the secular trend is declining; they therefore understate the conformity to reference contractions in series of the first type and the conformity to reference expansions in series of the second type. These effects are likely to be reinforced by the greater flexibility of monthly or quarterly data when one is dividing reference cycles into stages of expansion and contraction.

The differences between the conformity indexes of the monthly and quarterly data of our six series are generally slight and seem erratic (Table 82). The annual indexes, on the other hand, betray their coarse origins. The contraction index of annual data is conspicuously lower than the monthly in five series and slightly higher in one; it is lower than the quarterly in all six series. Again, the full-cycle index of annual data is lower than the monthly in four series and the same in two; it is likewise lower than the quarterly in four series and the same in two. The downward bias of annual expansion indexes is slight, but that merely reflects the dominance of series with rising secular trends in our sample. We must, however, chalk up two credits for the annual measures. First, while the indexes of conformity to full business cycles tend to run lower in the annual than in the monthly or quarterly data, the reduction does not seem large in view of the inherent instability of conformity indexes.[50] Second,

[49] Compare the analysis in Sec. X of monthly versus quarterly specific-cycle patterns. In making reference-cycle patterns of American series by quarters, interpolations are needed in the expansion of 1919–20 and the contractions of 1887–88, 1890–91, and 1918–19; in monthly data the need to interpolate never arises.

[50] See pp. 183-5, 195.

TABLE 82

Conformity to Business Cycles of Monthly, Quarterly and Annual Data

Six American Series

Series and form of data	No. of reference cycles[a]	Stages matched with reference expansion	Average change per month in reference-cycle relatives during stages matched with reference: Expansions	Contractions	Index of conformity to reference: Expansions	Contractions	Cycles
DEFLATED CLEARINGS							
Monthly	15	VIII-V	+0.78	−0.50	+100	+73	+86
Quarterly	15	VIII-V	+0.74	−0.63	+100	+73	+93
Annual	15	I-V	+0.65	−0.17	+100	+7	+86
PIG IRON PRODUCTION							
Monthly	15	I-V	+2.26	−2.27	+100	+100	+100
Quarterly	15	I-V	+2.29	−2.47	+100	+87	+100
Annual	15	I-V	+1.46	−1.65	+100	+73	+100
RAILROAD STOCK PRICES							
Monthly	19*	VIII-IV	+0.77	−0.61	+79	+60	+74
Quarterly	19*	VIII-IV	+0.74	−0.62	+68	+70	+68
Annual	19*	I-V	+0.50	−0.32	+68	+40	+63
SHARES TRADED							
Monthly	15	VIII-IV	+1.96	−1.66	+87	+73	+93
Quarterly	15	VIII-IV	+2.06	−1.89	+100	+73	+86
Annual	15	I-III	+1.73	−0.59	+87	+47	+86
CALL MONEY RATES							
Monthly	19*	I-V	+3.62	−3.57	+68	+100	+100
Quarterly	19*	I-V	+3.27	−3.67	+89	+90	+100
Annual	19	I-V	+1.60	−2.12	+79	+68	+84
RAILROAD BOND YIELDS							
Monthly	19*	III-VI	+0.24	−0.17	+47	+30	+68
Quarterly	19*	III-VI	+0.24	−0.17	+37	+40	+74
Annual	19*	III-VII	+0.11	−0.16	+26	+35	+63

[a] See Chart 33 for the periods covered; but note that where an asterisk appears, the contraction and full-cycle indexes cover an additional reference contraction at the beginning of the series.

the average rates of change during reference expansions and contractions have invariably the same sign in the annual as in the monthly and quarterly data. Of course, the fact that the annual averages are smaller than the others merely reflects the smaller amplitudes of annual data.[51]

[51] Our device of recognizing the timing of a series in units of cycle-stages when measuring conformity breaks down partly in annual series, since only one or two observations are commonly available for a reference phase. Hence we often follow another plan when the cyclical turns of a series tend to lead or lag behind the annual reference dates—whether for calendar or fiscal years—by some uniform interval. See the explanation in Table 48.

Three of our six annual series show some tendency to lead or lag by a uniform interval, and yield the following conformity indexes on this basis:

	No. of months lead (−) or lag (+) assumed	Index of conformity to reference: Expansions	Contractions	Cycles
R. R. stock prices	−6	+53	+41	+70
Shares traded	−6	+60	+87	+79
R. R. bond yields	+12	+37	+45	+55

On the whole, these measures do not differ greatly from those obtained by our standard method; but the higher contraction indexes are worth noting.

TABLE 83

Conformity to Business Cycles of Monthly and Four Forms of Annual Data
Three American Series

Series and form of data	No. of reference cycles	Stages matched with reference expansion[a]	Average change per month in reference-cycle relatives during stages matched with reference		Index of conformity to reference		
			Expansions	Contractions	Expansions	Contractions	Cycles
DEFLATED CLEARINGS							
Monthly	15	VIII-V	+0.78	−0.50	+100	+73	+86
Calendar-year	15	I-V	+0.65	−0.17	+100	+7	+86
Fiscal-year	15	I-V	+0.66	−0.14	+100	+20	+86
June	15	I-V	+0.68	−0.21	+87	+33	+79
December	15	I-V	+0.76	−0.25	+100	+7	+86
PIG IRON PRODUCTION							
Monthly	15	I-V	+2.26	−2.27	+100	+100	+100
Calendar-year	15	I-V	+1.46	−1.65	+100	+73	+100
Fiscal-year	15	I-V	+1.38	−1.31	+87	+73	+86
June	15	I-V	+1.90	−2.13	+100	+60	+86
December	15	I-V	+1.57	−1.84	+87	+60	+66
CALL MONEY RATES[a]							
Monthly	17	I-V	+3.92	−3.94	+76	+100	+100
Calendar-year	17	I-V	+1.68	−2.34	+76	+65	+82
Fiscal-year	17	III-V	+3.42	−1.18	+76	+76	+88
June	17	I-V	+1.73	−3.19	+65	+65	+64
December	17	III-V	+4.82	−1.72	+76	+65	+82

See Chart 34 for the periods covered. The calendar-year periods apply also to June data, and the fiscal-year periods to December data. The calendar-year and June figures are analyzed on the basis of the calendar-year reference dates, the fiscal-year and December figures on the basis of the fiscal-year reference dates.

[a] Since our fiscal-year reference dates start in 1868, the analysis of the fiscal-year and December data cannot start earlier. To ensure comparability, the analysis of other forms of the data covers approximately the same period. Cf. Table 82.

To illustrate the influence of the form of annual data on reference-cycle patterns and conformity measures, we present Chart 34 and Table 83. The patterns of calendar- and fiscal-year data differ conspicuously in call money rates. They seem similar in iron production and clearings, although in the former the variations in the slopes of the patterns within reference expansions and contractions are sufficiently different to discredit their value. On the other hand, the conformity measures of the calendar- and fiscal-year data agree fairly well, as do also the measures of single-date and full-year data.[52] But the rate of change during reference phases is somewhat greater on the average in single-date figures, while the indexes of conformity tend to run higher in full-year figures. Discrepancies of this character are to be expected whenever the conformity of the underlying monthly data is high.[53]

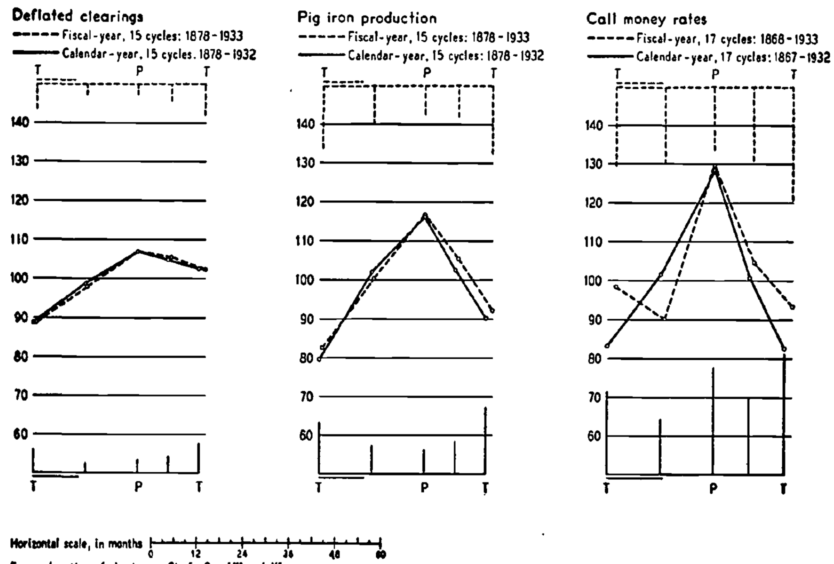
[52] But in call money rates the division of reference cycles is not the same in the fiscal-year and December data as in the other forms. If stages I-V are taken to represent expansions, the indexes of conformity of the fiscal-year data are: expansion +53, contraction +53, full cycles +70. The corresponding indexes of the December data are +71, +53, +76.

[53] For an adaptation of our method of measuring conformity, especially designed to handle single-date annual series, see a forthcoming monograph on inventory cycles by Moses Abramovitz.

CHART 34

Average Reference-cycle Patterns of Calendar- and Fiscal-year Data

Three American Series



For explanation of chart, see Ch. 5, Sec. VII and XI.

XIII Conclusions

At the risk of elaborating the obvious we have attempted in this long chapter, first, to explain why we eschew the easy path of working with annual data, second, to provide an experimental basis for judging cyclical measures made from annual data. Yearly observations are excessively crude materials for studying business cycles, not because they suffer from any inherent flaw, but simply because a year is such a substantial fraction of the usual length of specific and reference cycles. In studies of 'long waves', such as are found in building construction and certain other activities, the annual time unit is not likely to prove a disadvantage, while in studies of broad secular changes data by decades may suffice. On the other hand, in analyzing the financial adjustments that take place during a brief and violent crisis, such as occurred in 1907 in the United States, even monthly data are excessively coarse, and weekly or perhaps daily figures need to be used.[54]

There is no single optimum time unit for economic investigations. A time unit that serves well the needs of one investigation, or some part of one investigation, may serve poorly the needs of another. Nor can an in-

[54] See Wesley C. Mitchell, *Business Cycles* (1913), Ch. XII; or the new edition of Part III of that volume, published under the title *Business Cycles and Their Causes* (University of California Press, 1941), Ch. 3.

vestigator usually do more than approximate the time unit that serves his purposes best. For example, the tests of this chapter demonstrate conclusively that annual data falsify essential features of cyclical fluctuations, but they speak haltingly on the merits of quarterly versus monthly data for the type of analysis we are now making. If a series is characterized by mild erratic fluctuations, subtle responses to cyclical movements of general business may be spotted better in monthly than in quarterly data; monthly data make it possible to trace more faithfully the sequence of changes in revivals and recessions, and to show more accurately the form of the cyclical movements, particularly in short phases. But if erratic movements are pronounced these advantages of monthly data are likely to be lost, and the scale may tip in favor of the quarterly. It is possible that we have carried our preference for monthly data too far; it may be better to handle monthly series with substantial erratic movements on a quarterly basis, which is less expensive than monthly analysis.

The use of annual data, on the other hand, is a poor economy if an investigator seeks earnestly to understand the business cycles of actual life. True, by making careful qualitative allowances, some useful knowledge may be wrung from annual data; but the knowledge thus gained is extremely rough, and may be utterly misleading, as in the case of cyclical patterns. Some shortcomings of annual data may be mitigated by removing secular trends before subjecting time series to analysis, but the elimination of trends raises difficult questions which call for a separate chapter.

CHAPTER 7

Effects of Trend Adjustments on Cyclical Measures

As SHOWN in an earlier volume, business cycles are a development of late modern times. They emerged with the intensification of technical changes, the vast expansion of commercial and industrial activity, and the widening organization of economic life on the basis of making and spending money incomes.[1] Cyclical fluctuations are so closely interwoven with these secular changes in economic life that important clues to the understanding of the former may be lost by mechanically eliminating the latter. It is primarily for this reason that we take as our basic unit of analysis a business cycle that includes the portion of secular trend falling within its boundaries. In this way we prepare materials that we consider more useful for the explanation of business cycles than similar materials based upon trend-adjusted data.

It is desirable, however, to ascertain as definitely as possible how our various measures of cyclical behavior would be affected by the elimination of intra-cycle trends. The method is obvious. We must compare in detail the results obtained by applying our technique to sample series before and after their secular trends are eliminated. That step will aid in interpreting our results at large. In particular it will aid in comparing the results we get from series that come to us with trends eliminated by their compilers with those we get from the far more numerous series not so adjusted.[2] And it will clarify our reasons for retaining the portion of secular trend that falls within the limits of single cycles.

1 See Mitchell, *Business Cycles: The Problem and Its Setting*, Ch. II, Sec. I.

2 The adjustment for secular trend is sometimes explicit, sometimes implicit. An example of the latter is a long series showing the percentage of trade-union workers or insured workers out of employment.

I Materials Used in the Tests

Our tests cover the unadjusted and trend-adjusted forms of six monthly American series. To economize effort we have utilized series whose secular trend has already been calculated by other investigators. Three of the series analyzed in the preceding chapter—pig iron production, 'deflated' bank clearings outside New York, and railroad bond yields—are taken up also in this chapter. For these series we have used materials worked up by F. R. Macaulay for the National Bureau.[3] Our fourth series consists of the index of business conditions prepared by the American Telephone and Telegraph Company, both with and without trend adjustment, for the period since 1899.[4] The fifth consists of Frickey's composite of bank clearings in seven important cities exclusive of New York, covering 1875–1914, before and after trend adjustment.[5] The sixth series is the production of electric power since 1919, from which we have ourselves eliminated the secular trend.[6]

In each series the trend-adjusted data are relatives of the 'unadjusted' data to the corresponding ordinates of secular trend; that is, each monthly figure is expressed as a percentage of the corresponding trend value. But we also investigate how the results would be affected if the 'adjusted' data were taken in the form of absolute deviations from trend. Both the 'unadjusted' and 'adjusted' data are corrected for seasonal variations, except railroad bond yields, where we find no seasonal movement. The adjusted data correspond to the unadjusted data in every respect except that they are freed from their secular trends. As an illustration, both forms of the data for pig iron production are presented in Chart 35.

Our sample of series represents a fair variety of secular movements. Five series have rising trends (Table 84). The sixth, railroad bond yields, has an oscillatory trend; in this series we make separate comparisons for

3 For the sources of the unadjusted forms of these series, see Ch. 6, note 7. The trend-adjusted figures are not given in Macaulay's *Interest Rates, Bond Yields and Stock Prices*. But they are shown graphically (with seasonal unremoved in pig iron production) on pp. 223-6 of this source; see also Macaulay's Appendix, Table 11.

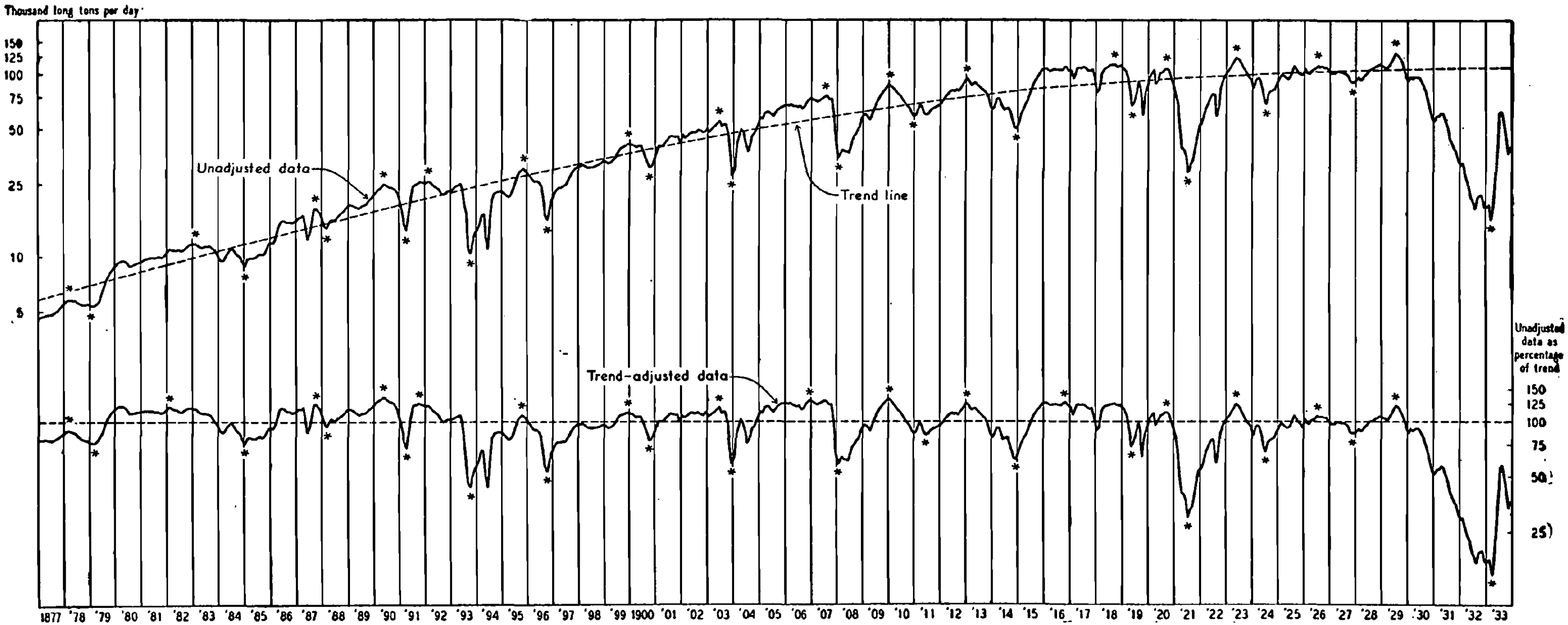
4 Through July 1932, both adjusted and unadjusted figures come from *Index of Industrial Activity in the United States* (a confidential report of the American Telephone and Telegraph Company, Oct. 20, 1932). Since Aug. 1932, the adjusted figures come from the company's *Summary of Business Conditions in the United States;* the unadjusted figures and the equation of trend applied to them. from the Chief Statistician's Division of the company. These series have recently been revised extensively, according to later confidential releases by the Chief Statistician's Division. Since the revisions do not affect the methodological issues treated in this chapter, we have not recomputed our results.

5 Edwin Frickey, Bank Clearings outside New York City, 1875–1914, *Review of Economic Statistics*, Oct. 1925, pp. 260-1. We are indebted to Frickey for sending us the seasonal indexes applied to his composite over the period 1875–1902.

6 The figures for 1919 come from a paper on The Nature of Cyclical Fluctuations in Electric Power Production Data (University of Illinois, Bureau of Business Research, 1927, *Bulletin 16*). After 1919, *Survey of Current Business*, Nov. 1927, p. 26; *ibid.*, 1932 Supplement, pp. 142-3, and 1936 Supplement. p. 85. Slight revisions of these figures back to 1920 have been made by the Federal Power Commission; our computations are based on the unrevised figures. The trend is a straight line fitted by 'least squares' to annual averages of seasonally corrected monthly data in 1919–32.

CHART 35

Pig Iron Production, United States, 1877–1933
Unadjusted and Trend-adjusted



Corrected for seasonal variations. Asterisks identify peaks and troughs of specific cycles. For sources of data, see Sec. I.

Logarithmic vertical scales.

TABLE 84

Change per Decade of Monthly Ordinates of Secular Trend

Six American Series

Series and measure	Unit	1860–1870	1870–1880	1880–1890	1890–1900	1900–1910	1910–1920	1920–1930
RAILROAD BOND YIELDS								
Absolute change	Per cent	0.2	−1.9	−0.8	−0.5	0.6	1.2	−0.6
Percentage change	....	*3*	*−30*	*−18*	*−14*	*20*	*31*	*−13*
DEFLATED CLEARINGS								
Absolute change	Mill. 1913 \$[a]	...	...	29.5	44.3	66.1	99.4	148.1
Percentage change	....	...	...	*66*	*60*	*56*	*54*	*52*
FRICKEY'S CLEARINGS								
Absolute change	Mill. \$	...	...	303	668	1,055	...	...
Percentage change	....	...	...	*61*	*83*	*72*	...	...
A.T.&T. INDEX								
Absolute change	Av. 1899 = 100	...	...	...	...	60.4	60.4	60.4
Percentage change	....	...	...	...	...	*59*	*37*	*27*
PIG IRON PRODUCTION								
Absolute change	Th. long tons[a]	...	...	10.4	19.8	29.0	27.0	13.0
Percentage change	....	...	...	*132*	*108*	*76*	*40*	*14*
ELECTRICITY OUTPUT								
Absolute change	Bill. kil.-hrs.	...	...	...	...	...	...	4.0
Percentage change	....	...	...	...	...	...	...	*113*

This table shows the decennial absolute and percentage increments of *monthly* ordinates of secular trend centered at June 30 of the decennial dates. The increments of deflated clearings from 1910 to 1930 are computed from 'deflated bank debits outside New York' adjusted to the level of 'deflated clearings outside New York' in Jan. 1919.

[a] Per day.

segments of rising and declining trends as well as for the full period covered by the data. The trends vary considerably in steepness and curvature. The percentage rate of growth in pig iron production during the 1880's is somewhat larger than that of electric power production during the 1920's, and is several times its own advance during the 1920's.[7] What is a declining rate of growth in percentage units is a uniform rate of growth in the units of the A.T.&T. business index, an increasing rate of growth in 'deflated' dollars of bank clearings, and first a rising and then a falling rate of growth in tons of iron produced. Regrettably, our sample is less satisfactory in representing the cyclical movements encountered in experience than in representing the secular movements. All the series we treat bear a positive and rather close relation to business cycles. However, in the course of analysis we shall attempt to take account of these deficiencies of the sample.

II The Number of Specific Cycles

It seems reasonable to expect that the elimination of secular trends will tend to increase the number of specific cycles as we count them; for any sharp retardation of growth in the unadjusted data will tend to be con-

[7] Table 84 records the decennial rates of change shown by the lines of secular trend. These may differ appreciably from rates of change computed from the original data for separate decades.

verted into an actual decline once the secular trend is removed. So also will any sharp retardation of decline tend to be converted into a rise. But the removal of trends will not add to the number of specific cycles if the amplitude of the 'cyclical component' of a time series is large compared with the amplitude of the 'secular component'. Indeed, under certain circumstances, a specific cycle will be lost when the secular trend is removed. Thus, if the trend is upward, a cycle will disappear in the adjusted data whenever the rate of rise during a cyclical expansion of the unadjusted data is no greater than the rate of rise of the fitted trend line during the corresponding period.[8] Similarly, if the trend is downward, a specific cycle will disappear whenever the rate of decline of a cyclical contraction in the unadjusted data is no greater than the rate of decline of the trend.

In our present sample the elimination of trends has slight influence on the number of specific cycles (Table 85). In pig iron production the

TABLE 85

List of Specific Cycles in Unadjusted and Trend-adjusted Data
Six American Series

Series	Direction of trend	All cycles			Corresponding cycles		Noncorresponding cycles		
		Period covered	Number in Unadj. data	Number in Adj. data	Period covered	No. in unadj. & adj. data	Period covered	Number in Unadj. data	Number in Adj. data
Deflated clearings	Upward	1878–1933	15	15	1884–1933	13	1878–1884	2	2
Frickey's clearings	Upward	1878–1914	11	11	1884–1914	9	1878–1884	2	2
A.T.&T. index	Upward	1900–1933	9	9	1900–1933	9	....	..	..
Pig iron production	Upward	1879–1933	15	15	1879–1933	15	....	..	..
Electricity output	Upward	1921–1933	2	2	1921–1933	2	....	..	..
Railroad bond yields	Oscillatory	1860–1931	20	21	[a]	16	[b]	4	5
Railroad bond yields	Downward	1868–1899	8	8	1868–1899	8	....	..	..
Railroad bond yields	Upward	1899–1918	5	7	[c]	3	[d]	2	4

[a] 1860–64, 1868–99, 1905–09, 1914–31.
[b] 1864–68, 1899–1905, 1909–14.
[c] 1905–09, 1914–18.
[d] 1899–1905, 1909–14.

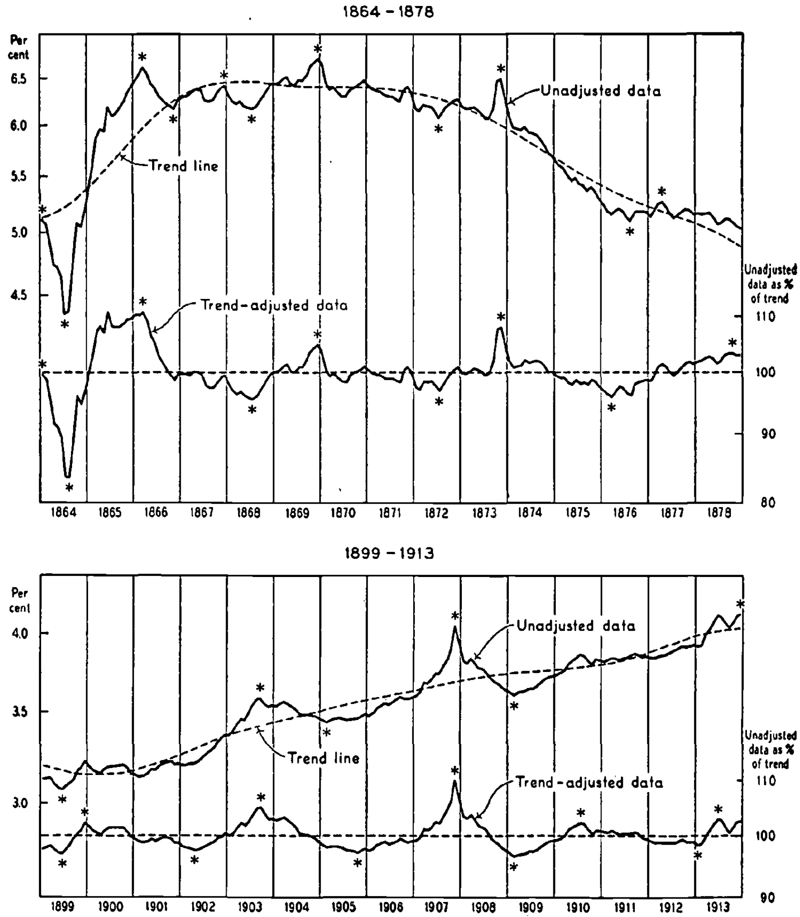
list of specific cycles is the same in the adjusted as in the unadjusted data. The like is true of the A.T.&T. index and the short series on electric power production. In both clearings series the specific cycles in the adjusted and unadjusted data agree in number, yet fail to correspond throughout. On the one hand, the expansion in the unadjusted data during 1878 to 1881 is matched by three phases in the adjusted data; on the other, the cyclical rise of 1882–83 in the unadjusted data is wiped out in the adjusted data. In bond yields, our longest series, the differences between the two lists of specific cycles are more numerous. Here we find two extra cyclical movements in the adjusted data—one during the expansion

[8] The two rates of rise may be expressed in units of the original data or their logarithms. It is mathematically possible for a cycle to disappear when the rates of rise are expressed in units of the original data but not when they are expressed in logarithms, and vice versa.

of 1899–1903 and another during the expansion of 1909–13; but these additions are partly offset by the virtual disappearance of the rise in 1866–67 which we treat as a specific-cycle expansion in the unadjusted data. The manner in which specific cycles are gained or lost in this series is displayed by Chart 36. In summary, we recognize 72 specific cycles in the unadjusted data of our six series and 73 cycles in the adjusted data; the net difference between the two totals is accounted for by bond yields.

Of course, a judgment factor enters into the present comparisons. The representation of the secular trend of a series by a mathematical curve or

CHART 36
Railroad Bond Yields, United States
Unadjusted and Trend-adjusted



Corrected for seasonal variations. Asterisks identify peaks and troughs of specific cycles. For sources of data, see Sec. I.

Logarithmic vertical scales

moving average involves judgment. So too does the decision to eliminate trends by division or by subtraction. Finally, the boundary line between a movement that is cyclical under our rules and one that is not becomes uncomfortably vague at times. But plausible variations in the form of trend lines are not likely to influence appreciably our count of specific cycles, since we do not limit the count to full swings about the trend line. If we consider a fluctuation large enough to qualify as a specific cycle, we recognize it as such irrespective whether it falls on one or both sides of the trend line. The removal of trend by division instead of subtraction rarely changes the cyclical turns, and is hardly likely to change the count of cycles of the particular duration in which our interest centers. True, our method of marking off specific cycles leaves margins of uncertainty. But the cyclical movements in our test series are for the most part very clearly defined, and our decisions have been checked with care. We therefore trust the general character of the results for the series covered by the present tests. And since these series seem fairly representative of the bulk of economic time series,[9] we feel justified in concluding that the removal of secular trends will rarely increase much the number of specific cycles as we count them.

This judgment is limited to monthly, or at most to quarterly data; it definitely excludes annual data. Trend adjustments have a substantial influence on the number of specific cycles in annual data, but we postpone analysis of this effect until Section VII.

III Timing and Duration of Specific Cycles

The effect of removing trends upon the turning points of specific cycles depends on the direction of the 'trend', its slope, and the slope of the 'cyclical component'. Consider a theoretical time series built up by adding a 'cyclical component' to a linear 'trend', the former having sufficient amplitude to stamp its movements on the composite series. If the cyclical component is triangular, both its rising and declining phases being linear, the cyclical turns of the composite should coincide with the turns of the cyclical component; in other words, removal of the 'true' trend will leave the cyclical timing of trend-adjusted data the same as that of unadjusted data. On the other hand, if the cyclical component is gently rounded at tops and bottoms, as for example in a sine curve, the turns of the composite will tend to deviate from those of the cyclical component. If the trend is rising the peaks of the composite will tend to come later and the troughs earlier than corresponding turns of the cyclical component; if the trend is declining, peaks will tend to come earlier and troughs later.

[9] Except in length. The addition of a cycle or two is more disturbing in short than in long series; see bond yields in Table 85.

These relations are based on special assumptions, but they may be generalized and put in a form suited to empirical time series. Assume that a rising trend is removed from a time series. This operation will either leave the dating of cyclical peaks unchanged or make the peaks come earlier than in unadjusted data. It will do the former if the rate of rise towards the close of a cyclical expansion in the unadjusted data is greater than the rate of rise of the secular trend during the same period; it will do the latter if the first rate is smaller than the second. In other words, whether the cyclical peak is pushed back or left unmolested by eliminating the trend depends on how steep is the rise in the closing stages of the cyclical expansion compared with the rise of the secular trend. Similarly, if the rate of rise in the early stages of a cyclical expansion in the unadjusted data is smaller than the rise of the secular trend, the removal of trend will make the cyclical trough come later; while if the first rate is larger than the second the dating will be unaffected. The effects of a declining trend on cyclical timing are opposite to those of a rising trend; that is, the removal of a declining trend tends to make cyclical peaks come later and troughs earlier than in unadjusted data, but in order that this tendency become effective the rate of fall towards the beginning and close of a cyclical contraction in the unadjusted data must be smaller than the rate of decline of the secular trend during the corresponding periods.

Table 86 shows the effects of removing trends in practice. The most interesting result is that slightly more than two-thirds of the cyclical turns

TABLE 86

Size and Frequency of Leads or Lags of Specific-cycle Turns in Trend-adjusted Data at Corresponding Turns of Unadjusted Data Five American Series with Upward Trends

Lead (−) or lag (+) of trend-adj. at turn of unadj. data (mos.)	Number of leads or lags in											
	Deflated clearings		Frickey's clearings		A.T.&T. index		Pig iron production		Electricity output		Five series	
	P	T	P	T	P	T	P	T	P	T	P	T
Below −12	1	..	..	..	..	..	1	..	..	..	2	..
−7 to −12	..	..	..	..	1	..	2	..	1	..	4	..
−1 to −6	5	..	1	..	2	..	2	..	1	..	11	..
0	8	10	9[a]	6	7	6	11	14	..	3	35	39
+1 to +6	..	2	..	2	..	4	..	2	..	..	..	10
+7 to +12	..	3	..	2	..	..	..	..	..	..	..	5
Over +12	..	..	..	1	..	..	..	..	..	..	..	1
Total	14	15	10	11	10	10	16	16	2	3	52	55
Leads	6	..	1	..	3	..	5	..	2	..	17	..
Coincidences	8	10	9[a]	6	7	6	11	14	..	3	35	39
Lags	..	5	..	5	..	4	..	2	..	..	..	16
Total	14	15	10	11	10	10	16	16	2	3	52	55

P stands for peak, T for trough. See Table 87 for the periods covered.

[a] A one-month computed lag included here, because it arises from a slight error in the published figures from which we dated the specific cycles of the adjusted data; that is, we dated a peak in Sept. 1881, whereas the correct month is August.

in our five series with upward trends are unaffected by the trend adjustment.[10] The reason must be that the march of secular trends is usually less vigorous than that of the unadjusted data during early and late stages of cyclical expansions.[11] In those instances in which the elimination of trends alters the dating of cyclical turns, the direction of the shifts conforms to expectations. Most shifts are small, but some exceed six months and a few exceed a whole year. It is plain, therefore, that if cyclical turns were dated from trend-adjusted data, the apparent succession of revivals or recessions of different economic activities would at times be seriously modified.[12] Of course, Table 86 merely indicates the general character and size of the shifts produced by trend adjustments. The specific results are bound to vary with the circumstances surrounding each series, the type of trend used, the manner in which the trend is fitted to the data, and the manner in which the trend is removed. In the present experiments the trends have been removed by division; that is, the unadjusted figures have been expressed as percentages of the corresponding ordinates of secular trend. If our rising trends had been removed by subtraction, the tendency of adjusted data to lag at the troughs of unadjusted data would be slightly stronger, while the tendency to lead at the peaks would be slightly weaker.[13] On the other hand, if our rising trend lines were replaced by others rising still faster, both tendencies would be more prominent than in the present measurements.

The average effect of trend adjustments is larger in our sample at peaks than at troughs. The effect varies appreciably from series to series, and without close regard to the steepness of the secular trend. The average shift is less than two months in 9 out of 16 comparisons (Table 87); it exceeds four months at the cyclical peaks in electric power production, a short series rising with exceptional swiftness. Our one sample of a declining trend, railroad bond yields during 1868–99, shows a shift in average timing at peaks and troughs opposite to that of the several series with

10 Bond yields show a similar result. Of the 41 corresponding turns in this series, 28 coincide in the unadjusted and adjusted data. In all six series, 102 out of 148 corresponding turns coincide (49 out of 73 peaks and 53 out of 75 troughs).

11 Or what comes to the same thing, the rate of fall in the late and early stages of cyclical contractions in the adjusted data is, usually, absolutely larger than the rate of rise of the secular trend during corresponding periods.

12 If trends of different series are not only converted into horizontal lines, but the fluctuations about the trends are expressed in standard deviation units, as is still common practice, about all that can be learned is the relation in time of the cyclical movements. And even this limited knowledge may be false since the timing relations of experience may be distorted.

13 The absolute deviations are, of course, equivalent to percentage deviations multiplied by corresponding ordinates of trend. Upon comparing the adjusted data in the form of absolute deviations from trend with the percentage deviations from trend, we found their troughs coinciding in every instance, but three peaks in the percentage deviations preceded corresponding peaks in the absolute deviations. In two of these three instances the peaks in the absolute deviations coincided with the peaks in the unadjusted data.

rising trends. The rough coincidence of average timing in the two forms of railroad bond yields during the full period 1857–1932 is an instructive example of how the effects of a rising and declining trend neutralize each other.

TABLE 87
Average Timing of Specific Cycles
in Trend-adjusted Data at Corresponding Turns of Unadjusted Data
Six American Series

Series and trend	Period covered	Number of corresponding turns		Average lead (−) or lag (+) of adjusted at turns of unadjusted data (mos.)	
		Peak	Trough	Peak	Trough
UPWARD					
Deflated clearings....	1878–1933	14	15	−3.2	+2.4
Frickey's clearings....	1878–1914	10	11	−0.2	+3.7
A.T.&T. index......	1899–1933	10	10	−1.4	+0.5
Pig iron production...	1878–1933	16	16	−2.9	+0.5
Electricity output....	1921–1933	2	3	−5.5	0.0
Railroad bond yields..	1899–1918	5	6	−3.0	+1.5
DOWNWARD					
Railroad bond yields..	1868–1899	8	9	+3.2	−0.6
OSCILLATORY					
Railroad bond yields..	1857–1932	21	20	+0.9	+0.2

The periods run from the year of the first to the year of the last corresponding turn, whether peak or trough, covered by our analysis of monthly data. They are longer in a few instances than the all-cycle periods listed in Table 85, since the latter start and end with a trough.

The turning points in iron production (both the unadjusted and trend-adjusted data) are shown in Chart 35. The turning points of deflated clearings and railroad bond yields (unadjusted data) are shown in Chart 53 and Appendix Table B3; but see the note introducing Appendix B.

The shifts in cyclical turns produced by trend adjustment must modify the durations of the phases of specific cycles. Every time the removal of a rising trend makes a cyclical peak come earlier, expansions are shortened and contractions lengthened. The effect is similar when a cyclical trough is pushed forward; and is likely to be similar, as far as the relation between the average durations of expansions and contractions is concerned, when the number of specific cycles is increased. The removal of a declining trend has opposite effects. The changes in the cyclical durations of our test series produced by trend adjustments are shown in Table 88. The largest effect appears in the rising segment of railroad bond yields —a result that follows from the gain of two specific cycles. It is interesting to observe that even after the trend is removed, cyclical expansions run longer than contractions in every series except bond yields.

Since secular trends have slight influence on the number of specific cycles in our test series, the average duration of full specific cycles is virtually the same in the adjusted and unadjusted data. The one appreciable discrepancy comes in the rising segment of bond yields.

TABLE 88

Average Duration of Specific Cycles in Unadjusted and Trend-adjusted Data

Six American Series

Series and group	Direction of trend	No. of specific cycles		Average duration in months						Average per cent of duration of specific cycles			
				Expansion		Contraction		Full cycle		Expansion		Contraction	
		Un	Ad	Un	Ad	Un	Ad	Un	Ad	Un	Ad	Un	Ad
CORRESPONDING CYCLES													
Deflated clearings	Up	13	13	33.3	27.7	11.3	16.9	44.6	44.6	76	61	24	39
Frickey's clearings	Up	9	9	30.6	26.4	9.4	13.6	40.0	40.0	76	66	24	34
A.T.&T. index[a]	Up	9	9	25.9	23.8	17.3	19.4	43.2	43.2	60	56	40	44
Pig iron production[a]	Up	15	15	28.8	25.1	14.5	18.1	43.3	43.2	67	59	33	41
Electricity output[a]	Up	2	2	48.5	43.0	22.5	28.0	71.0	71.0	75	62	25	38
Railroad bond yields	Up	3	3	22.7	17.0	11.0	14.3	33.7	31.3	69	53	31	47
Railroad bond yields[a]	Down	8	8	16.8	20.6	29.6	25.8	46.4	46.4	38	44	62	56
Railroad bond yields	Oscil.	16	16	17.4	18.8	24.4	22.6	41.8	41.4	46	46	54	54
ALL CYCLES													
Deflated clearings	Up	15	15	32.6	25.6	11.4	17.9	44.0	43.5	75	59	25	41
Frickey's clearings	Up	11	11	29.8	24.1	9.9	15.1	39.7	39.2	74	63	26	37
Railroad bond yields	Up	5	7	35.4	13.7	11.2	19.7	46.6	33.4	75	40	25	60
Railroad bond yields	Oscil.	20	21	21.0	17.4	21.4	23.1	42.4	40.5	52	43	48	57

'Un' stands for unadjusted, 'Ad' for trend-adjusted data. See Table 85 for the periods covered.

[a]All cycles correspond in the unadjusted and adjusted data.

IV Amplitude of Specific Cycles

When an upward trend is removed from a time series the cyclical rise in the original units must be reduced and the cyclical fall increased. When a downward trend is removed the cyclical rise must be increased and the fall reduced. Table 89 indicates the magnitude of the changes in absolute cyclical amplitudes that result from the removal of the secular trend from pig iron production.[14] The changes vary considerably from phase to phase and from cycle to cycle, as may be expected from the differences in the durations, amplitudes, patterns, and intra-cycle trends of the specific cycles. We also find that the amplitudes of full specific cycles are usually changed much less than their expansions and contractions. This result reflects the opposite effects of the removal of trend on the rise and on the fall. But these opposite effects are not evenly balanced: the amplitude of full specific cycles is reduced in 12 and increased in only 3 instances. Similar reactions appear in other series, whatever the direction of their trends (Table 90). Of the 64 corresponding cycles in the adjusted and unadjusted data of our six series, the absolute amplitude of full specific cycles is smaller in the adjusted data in 50 and larger in only 14. Apparently, if we removed secular trends before taking cyclical measures, the amplitudes of full specific cycles, expressed in the original units, would as a rule be reduced.

[14] Of course, the trends are removed by subtraction. The relative amplitudes in Table 89 are discussed later.

TABLE 89

Absolute Amplitude of Specific Cycles in Unadjusted and Trend-adjusted Data
Pig Iron Production, United States, 1879–1933

Specific cycle[a]	Amplitude in thousand long tons per day						Excess of adjusted over unadjusted data					
	Rise		Fall		Rise & fall		In thousand long tons per day			In percentages[b]		
	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
1879–1885	6.18	3.35	2.37	4.19	8.55	7.54	−2.83	+1.82	−1.01	−59.4	+55.5	−12.6
1885–1888	8.88	5.87	3.50	4.01	12.38	9.88	−3.01	+0.51	−2.50	−40.8	+13.6	−22.5
1888–1891	9.89	6.98	9.34	10.70	19.23	17.68	−2.91	+1.36	−1.55	−34.5	+13.6	−8.4
1891–1893	10.19	9.36	14.28	17.60	24.47	26.96	−0.83	+3.32	+2.49	−8.5	+20.8	+9.7
1893–1896	18.69	14.72	13.03	14.92	31.72	29.64	−3.97	+1.89	−2.08	−23.8	+13.5	−6.8
1896–1900	24.27	17.03	9.74	11.82	34.01	28.85	−7.24	+2.08	−5.16	−35.1	+19.3	−16.4
1900–1903	22.14	15.03	22.59	23.99	44.73	39.02	−7.11	+1.40	−5.71	−38.3	+6.0	−13.6
1903–1908	44.28	33.69	37.73	39.27	82.01	72.96	−10.59	+1.54	−9.05	−27.2	+4.0	−11.7
1908–1910	49.06	42.82	26.01	30.63	75.07	73.45	−6.24	+4.62	−1.62	−13.6	+16.3	−2.2
1910–1914	32.82	27.99	41.44	47.40	74.26	75.39	−4.83	+5.96	+1.13	−15.9	+13.4	+1.5
1914–1919	60.98	52.12	39.77	41.64	100.75	93.76	−8.86	+1.87	−6.99	−15.7	+4.6	−7.2
1919–1921	32.89	29.35	73.52	74.28	106.41	103.63	−3.54	+0.76	−2.78	−11.4	+1.0	−2.6
1921–1924	89.55	86.54	48.53	50.31	138.08	136.85	−3.01	+1.78	−1.23	−3.4	+3.6	−0.9
1924–1927	36.45	33.89	18.70	20.17	55.15	54.06	−2.56	+1.47	−1.09	−7.3	+7.6	−2.0
1927–1933	36.93	35.50	110.00	112.23	146.93	147.73	−1.43	+2.23	+0.80	−3.9	+2.0	+0.5

The trend was removed by subtraction.

[a] Years of the initial and terminal troughs of the specific cycles in monthly data, unadjusted for trend.

[b] The base of the percentages is the mean of each pair of amplitudes being compared; e.g., the rise for the 1879–85 cycle is $-59.4 = 100\left(\frac{-2.83}{\frac{6.18 + 3.35}{2}}\right)$. This method equalizes the theoretical scale of plus and minus percentages.

TABLE 90

Frequency Distribution of the Differences between Amplitudes of Corresponding Specific Cycles in Unadjusted and Trend-adjusted Data

Deviation of amplitude of adj. from amplitude of unadj. data[a] (per cent)	Number of differences between adjusted and unadjusted data							
	Five series with upward trends						Six series	
	Absolute amplitude			Relative amplitude[b]			Absolute rise & fall	Relative rise & fall[b]
	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall		
Below −70.0	6	..	1	7	..	1	1	1
−70.0 to −50.0	11	..	..	11	..	1	1	2
−50.0 to −30.0	15	..	5	14	..	4	5	4
−30.0 to −20.0	4	..	6	4	..	8	10	12
−20.0 to −10.0	8	..	13	8	..	12	16	16
−10.0 to 0.0	4	..	13	4	..	13	17	16
0.0 to +10.0	..	13	10	..	13	9	11	10
+10.0 to +20.0	..	11	..	..	11	..	3	3
+20.0 to +30.0	..	5	..	..	5	..	..	..
+30.0 to +50.0	..	9	..	..	11	..	..	..
+50.0 to +70.0	..	7	..	..	5	..	..	..
Over +70.0	..	3	..	..	3	..	..	..
Total.........	48	48	48	48	48	48	64	64

See Table 85 for the series included and the periods covered by their corresponding cycles.

[a] See note to preceding table for the method used in computing the percentage deviations. Two items at the stated class limits were distributed after carrying the percentages to an extra decimal.

[b] See pp. 284-5 concerning the measures of relative amplitude of the adjusted data (same as method C in Table 92).

This expectation may be readily tested with the aid of a few symbols. Assume that a_t, b_p, c_t, d_p, etc. are ordinates of secular trend at dates of successive troughs and peaks of the specific cycles in unadjusted data, the subscript t indicating a trough date and the subscript p a peak date. Assume further that the unadjusted data at corresponding turns are $(a_t - m)$, $(b_p + n)$, $(c_t - o)$, $(d_p + p)$, etc., and that the removal of the trend leaves the dating of the cyclical turns unchanged. Then the absolute amplitude of a full cycle running from trough to trough in the unadjusted data is $(b_p + n) - (a_t - m) + (b_p + n) - (c_t - o)$, or $2b_p - (a_t + c_t) + (m + 2n + o)$. But $(m + 2n + o)$ is the amplitude of a full cycle running from trough to trough in the adjusted data. Hence the absolute amplitude of the unadjusted data will be larger or smaller than that of the adjusted data according as $2b_p - (a_t + c_t)$ is plus or minus.[15] It may be shown in a similar manner that when the cycles are taken on an inverted basis, the amplitude of a full cycle in the unadjusted data is larger or smaller than that of the adjusted data according as $(b_p + d_p) - 2c_t$ is plus or minus. Table 91 makes explicit some of the relations implicit in this and the preceding expression.[16] It appears that the removal of secular trends may increase the amplitudes of full cycles, decrease them, or leave them unchanged: the result depends upon the direction of the trend, its curvature, the relative duration of expansions and contractions, and the treatment of the cycles as positive or inverted.[17] But the critical point is that when the trend is rising and expansions are longer than contractions, or when the trend is declining and contractions are longer than expansions, it appears that the amplitude of full cycles of adjusted data is more likely to fall short of than to exceed the amplitude of unadjusted data. These are the conditions we usually encounter in time series. They characterize also our present sample, and thus explain the tendency of the

[15] To simplify the argument, the use of three-month averages to represent standings at peaks and troughs is disregarded.

[16] The relations are obvious to common sense. If, for example, the trend is upward and linear, and the cycle phases are of equal duration, the amplitude of rise is increased by the trend component to the same degree that the amplitude of fall is decreased; for the cumulation of the trend component in one phase is the same as in the other, the two being of the same length. But if the expansion is longer than the contraction, the amplitude of rise is increased more by a linear trend than the amplitude of fall is reduced; for the trend component is now greater for the longer of the two phases. And so on from line to line of the table.

[17] The result depends also on the extent to which cyclical turns are shifted by removing the trend. Table 91 is made on the assumption that the trend changes none of the cyclical turns—a valid assumption only two-thirds of the time according to our sample. If this assumption is dropped, the relations between the absolute amplitudes of unadjusted and adjusted data are modified as follows: (1) If removal of the trend affects the dating of peaks but not of troughs, and the cycles are taken positively, $A = m + 2z + o$, where $z > n$. (See the text and Table 91 for the meaning of the symbols.) Hence $U - A$ is algebraically smaller than it would be if the dating of the peaks were unaffected. Similar relations obtain between U and A when the cycles are inverted, if the trend affects the dating of the troughs but not of the peaks. (2) Other things equal, A is increased more than in the preceding case if the dating of both troughs and peaks is affected bv removing the trend.

TABLE 91

Relation between Amplitudes of Full Specific Cycles in Unadjusted and Trend-adjusted Data under Different Conditions

Curvature of trend	Upward trend		Downward trend	
	Cycles taken from trough to trough	Cycles taken from peak to peak	Cycles taken from trough to trough	Cycles taken from peak to peak
	Duration of expansion equals that of contraction			
Linear	$U = A$	$U = A$	$U = A$	$U = A$
Concave	$U > A$	$U < A$	$U > A$	$U < A$
Convex	$U < A$	$U > A$	$U < A$	$U > A$
	Duration of expansion longer than that of contraction			
Linear	$U > A$	$U > A$	$U < A$	$U < A$
Concave	$U > A$	$U \gtreqless A$	$U \gtreqless A$	$U < A$
Convex	$U \gtreqless A$	$U > A$	$U < A$	$U \gtreqless A$
	Duration of expansion shorter than that of contraction			
Linear	$U < A$	$U < A$	$U > A$	$U > A$
Concave	$U \gtreqless A$	$U < A$	$U > A$	$U \gtreqless A$
Convex	$U < A$	$U \gtreqless A$	$U \gtreqless A$	$U > A$

U stands for the amplitude of a full specific cycle (rise and fall) in unadjusted data, A for the amplitude of a full cycle in adjusted data. The comparisons are made on the assumption that the turning points of the specific cycles coincide in the adjusted and unadjusted data. The table applies to relative as well as to absolute amplitudes; provided the relative amplitude of adjusted data is envisaged as in 'method C', described on p. 285.

The troublesome terms 'convex' and 'concave' are used as follows: the trend is said to be convex if its second derivative with respect to time is positive, concave if its second derivative is negative.

amplitudes of full specific cycles to run lower in adjusted than in unadjusted data.

The preceding analysis is based on amplitudes expressed in the original units, whereas our standard practice is to work with amplitudes expressed in cycle relatives. In the unadjusted data the trend affects our amplitude measures in two ways. First, it influences the amplitude expressed in the original units. Second, it influences the final result *via* the cycle bases; for example, if the trend is upward, a set of cyclical rises divided by a set of cycle bases yields smaller quotients if the bases relate to positive than if they relate to inverted cycles. Since the removal of trend lines frees the data from 'intra-cycle' as well as 'inter-cycle' trends, cyclical rises and declines must be roughly equal on the average in relative as in absolute measures of amplitude, providing our general plan of measuring amplitudes is followed. In the unadjusted data, on the other hand, a trend factor must remain in the amplitude measures, whether expressed in units of the original data or in cycle relatives. True, our standard method, which converts the absolute amplitude of a specific cycle into a percentage of the average value of the series during the cycle, involves in effect an adjustment for secular trend. But since the 'trend' is a step-line of cyclical averages, a line that is horizontal within each specific cycle, a trend component remains within each cycle and each cyclical phase, and its impact on any given phase must vary according as the cycles have been marked off by peaks or by troughs. We may expect therefore the removal of secular trends not only to yield measures of the relative amplitude of specific cycles that differ from our standard measures, but to modify these measures in different ways according as the cycles have been analyzed on a positive or on an inverted basis.

For the moment, we concentrate on positive analysis, which is our typical method of handling specific cycles. Table 90 shows the direction and degree in which the relative amplitude of single specific cycles is changed by eliminating trends. In this table we have followed our standard method in computing the relative amplitude of the specific cycles in the unadjusted data, while in the adjusted data we converted the absolute amplitude of each specific cycle into a percentage of the average value of the *unadjusted* data during the period occupied by that cycle. It appears from Table 90 that the effects of eliminating secular trends on the relative amplitudes of expansions, contractions and full cycles are distributed in virtually the same way as are the effects on the absolute amplitudes. The results could hardly be otherwise. For the base used in computing the relative amplitude of a specific cycle is the same in the unadjusted and adjusted data whenever their cyclical troughs coincide, and the difference between the bases is usually slight even in the absence of coincidence. It may be recalled that the effects of trend adjustments analyzed in Table 91 assume coincidence of cyclical turns. Since on this assumption the cycle bases of adjusted and unadjusted data are identical, the removal of trend will have exactly the same effect on the relative as on the absolute amplitudes.[18] Hence the reasons previously advanced for the tendency of amplitude measures of full specific cycles to run lower in adjusted than in unadjusted data apply to the relative amplitudes no less than to the absolute amplitudes.

Of course, this argument is based on a particular method of measuring the relative amplitude of specific cycles in trend-adjusted data—a difficulty that should be faced explicitly. At least four plausible methods may be distinguished. (A) If the adjusted data are expressed as trend relatives, we can simply apply our standard technique to these data. (B) The relative amplitude may also be measured directly from the trend relatives, that is, without adjusting the relatives for differences in their average

[18] See above, note 17, for the effects of trends on absolute amplitudes of full cycles when the trend shifts cyclical turns. Point (1) of that note applies also to relative amplitudes, for under the assumed conditions the cycle bases are the same in adjusted and unadjusted data. So too does point (2) with this additional comment: the increase in the absolute amplitude tends to be counteracted or reinforced in the relative amplitude according as the cycles are positive or inverted, because (regardless of the direction of the trend) the cycle base tends to be raised in the former case and lowered in the latter.

TABLE 92

Average Amplitude of Corresponding Specific Cycles
in Unadjusted and Trend-adjusted Data Computed by Different Methods
Six American Series

Series and trend	No. of corresponding specific cycles	Average relative amplitude of											
		Unadjusted data treated according to our standard technique			Adjusted data treated in the form of								
					Trend relatives						Absolute deviations from trend, but 'correcting' for the level of cycles (*Method C*)		
					According to our standard technique (*Method A*)			As in Method A, but not 'correcting' for the level of cycles (*Method B*)					
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
UPWARD													
Deflated clearings. .	13	25.9	13.7	39.7	14.8	17.6	32.4	14.8	17.5	32.3	14.4	19.0	33.4
Frickey's clearings. .	9	30.2	13.6	43.8	17.4	17.3	34.7	17.2	17.3	34.5	16.5	18.4	34.8
A.T.&T. index. . . .	9	30.5	26.1	56.5	23.9	29.6	53.6	23.5	28.3	51.8	23.4	30.8	54.2
Pig iron production.	15	62.1	54.8	116.8	52.2	57.7	109.9	49.9	54.0	103.9	49.8	60.6	110.5
Electricity output. .	2	44.2	13.8	58.0	18.0	22.2	40.1	17.6	22.2	39.8	16.4	26.0	42.3
Railroad bond yields	3	12.9	8.4	21.3	8.7	10.5	19.2	8.7	10.5	19.2	8.6	10.7	19.3
DOWNWARD													
Railroad bond yields	8	6.3	14.6	20.9	8.6	8.4	17.0	8.6	8.4	16.9	8.8	8.3	17.1
OSCILLATORY													
Railroad bond yields	16	9.4	14.7	24.1	10.2	11.1	21.2	10.1	11.0	21.2	10.3	10.9	21.3

Methods A, B and C are identified more fully in the text. See Table 85 for the periods covered.

level during successive specific cycles. (C) When trend-adjusted data are expressed as absolute deviations from trend—and that is the form in which they must be taken to show the effect of trend adjustments on amplitudes in the original units—a different procedure is necessary. Here the absolute amplitude of each cycle may be expressed as a percentage of the average value of the *unadjusted* data during the period occupied by the cycle, or (D) as a percentage of the average value of the ordinates of secular trend during the period occupied by the cycle. It seems that method A should be better than method B whenever the trend is so fitted that some full cycles are sunk below or raised above the line of trend, for in such cases method A supplies what is in effect a supplementary trend adjustment. For similar reasons method C seems better than method D. But it is more difficult to choose between methods A and C either on theoretical or practical grounds.[19]

Table 92 compares the average amplitudes of unadjusted data with the average amplitudes of adjusted data measured by methods A, B and

[19] If the amplitudes of rise and fall in the adjusted data are equal in the original units, the rise of a specific cycle will still equal the fall if computed by method C but not by method A or B: while if the amplitudes of specific cycles of trend relatives are constant, the rise will equal the fall if ascertained by method A or B but not by method C. Since the amplitude of percentage deviations from trend is apt to vary less than the amplitude of absolute deviations from trend, these considerations seem to argue in favor of methods A and B. So too does the analysis of positive vs. inverted measures, later in this section.

C. The three methods of computing the relative amplitude of trend-adjusted data yield closely similar results, although there are numerous differences of detail. The most prominent is that the average cyclical rise is smaller and the average fall larger in method C than in method A or B in every sample of an upward trend, while the average rise is larger and the average fall smaller in method C than in method A or B in the one sample of a declining trend.[20] But whatever the method, the removal of an upward trend reduces the average cyclical rise and increases the average cyclical fall, the removal of a downward trend produces opposite effects, and the removal of an upward or downward trend reduces the joint rise and fall.

TABLE 93

Comparison of Relative Amplitudes of Corresponding Specific Cycles in Unadjusted and Trend-adjusted Data

Amplitude of adjusted data determined by method[a]	Number of instances in which the relative amplitude of adjusted data					
	Is larger than that of unadjusted data			Is smaller than that of unadjusted data		
	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
A..........	..	44	8	48	4	40
B..........	2	43	8	46	5	40
C..........	..	48	9	48	..	39

Based on 48 corresponding cycles in the unadjusted and trend-adjusted data of five American series with upward trends. See Table 85 for the series and periods covered, and Table 90 for more detailed comparisons involving method C.

[a] The several methods are identified in Table 92, and more fully on pp. 284-5.

Theoretically, these effects are not strictly necessary in any of the methods. The removal of an upward trend must reduce the cyclical rise and increase the cyclical fall when the amplitudes are measured in the original units. But strange as it may seem, the removal of an upward trend

20 Assume that the ordinates of secular trend at the dates of initial trough, peak, and terminal trough of a specific cycle in *adjusted* data are, successively, a, b, and c; that the values of the cyclical component in the original units are $-m$, $+n$, and $-o$ at the corresponding dates; and that these dates are the same no matter how the secular trend is eliminated. Then, according to method B, the cyclical rise is $100\left(\frac{n}{b}+\frac{m}{a}\right)$ and the cyclical fall is $100\left(\frac{n}{b}+\frac{o}{c}\right)$; while according to method C, the rise is $100\left(\frac{n}{k}+\frac{m}{k}\right)$ and the fall is $100\left(\frac{n}{k}+\frac{o}{k}\right)$, where k is the average value of the unadjusted data during the cycle. The cyclical rise will be smaller in method C than in method B, provided k is larger than the weighted harmonic mean of a and b; their respective weights being m and n. And the cyclical fall will be larger in method C than in method B, provided k is smaller than the weighted harmonic of b and c, their respective weights being n and o. It seems reasonable to expect that k will rarely be below the first harmonic or above the second, so long as the secular trend is upward and the specific cycles are taken positively.

If specific cycles are taken invertedly, it may be expected that the average value of the unadjusted data during the inverted cycle running from the date of b to the next peak will usually be above the weighted harmonic of b and c, while the average value of the unadjusted data during the inverted cycle ending with the date of b will usually be below the weighted harmonic of a and b. Hence inverted analysis should tend to produce differentials between methods B and C that are opposite in sign to those produced by positive analysis. See Table 94, where positive and inverted measures are contrasted.

The preceding remarks may be readily rephrased for the case of a declining trend.

may increase the relative amplitude of the cyclical rise or reduce the relative amplitude of the cyclical fall. For example, if a cyclical contraction is exceptionally violent, the average value of the original data during the cycle including this phase may lie considerably below the trend; hence the amplitude of the fall in the adjusted data, if ascertained by method B, may be considerably smaller than the amplitude of the fall in the unadjusted data. The summary of cycle-by-cycle comparisons in Table 93 demonstrates that such curious results are rare in practice. Nevertheless, Table 92 discloses an instance in which the *average* fall of adjusted data covering fifteen specific cycles is smaller than the average fall of unadjusted data, despite the upward trend of the series.[21]

Table 94 compares amplitude measures of positive and inverted cycles on the plan of Table 92. We have already shown in Chapter 5 how measures of amplitude of unadjusted data depend on the decision to analyze specific cycles on a positive or on an inverted basis. In brief, if the secular trend of a series is upward, the amplitude of cyclical rises is likely to be smaller and of cyclical falls larger when the specific cycles are taken positively than when they are taken invertedly; but if the secular trend is downward, rises are likely to be larger and falls smaller in positive than in inverted cycles. The influence of positive versus inverted treatment is similar in the adjusted data if the amplitudes are measured by method C, because cycle bases affect these measures in much the same way as they affect the measures of unadjusted data. Method B, on the other hand, avoids the use of cycle bases; hence there can be no difference between amplitude measures for positive and inverted cycles, provided, of course, they cover the same period. Method A may show differences, but

21 If the cyclical turns in unadjusted and trend-adjusted data are coincident, the cyclical fall in the adjusted data, according to method B, is $100\left(\frac{n}{b} + \frac{o}{c}\right)$, and the cyclical fall in the unadjusted data is $100\left(\frac{b-c}{k} + \frac{n+o}{k}\right)$. (The symbols are defined in the preceding note.) If a specific cycle in the unadjusted data has a mild expansion followed by an exceptionally sharp contraction, and the trend line during this cycle moves upward at a gentle pace, k may be much lower than b or c. If that happens $\frac{n+o}{k}$ will be considerably larger than $\frac{n}{b} + \frac{o}{c}$, and this excess will be only slightly offset by $\frac{b-c}{k}$; in other words, the amplitude of fall in the unadjusted data will be larger than in the adjusted data. An example that approximates this hypothetical case is the decline of iron production from 1929 to 1933; the amplitude of which is 148.9 in the unadjusted data, but only 103.8 in the adjusted data treated by method B. This extreme discrepancy is the main reason for the paradoxical result in Table 92, to which we refer in the text. (See Chart 35.)

Method A also may produce curious results. For example, if the cyclical fall in the adjusted data analyzed by method B is only slightly larger than in the unadjusted data, while the average of the trend relatives during the cycle is well above 100, the fall in the adjusted data treated by method A will be smaller than in the unadjusted data.

Nor is method C devoid of this difficulty. The absolute cyclical fall after an upward trend is removed must exceed the absolute fall of unadjusted data. But the base on which the former is expressed may be higher than the base on which the latter is expressed. Hence the relative amplitude of the fall may be smaller in the adjusted than in the unadjusted data, though the likelihood of such a result is slight.

These remarks may be readily extended to cyclical rises.

TABLE 94

Average Amplitude of Corresponding Specific Cycles
in Unadjusted and Trend-adjusted Data on Positive and Inverted Plans
Five American Series

Series, trend, and measure	Average relative amplitude								Excess of average of adjusted data over average of unadjusted data					
	Unad-justed data		Adjusted data											
			Method A		Method B		Method C		Method A		Method B		Method C	
	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.
UPWARD TREND														
Deflated clearings														
Rise	25.0	25.7	14.0	14.0	14.0	14.0	13.6	14.5	−11.0	−11.7	−11.0	−11.7	−11.4	−11.2
Fall	10.3	9.6	14.1	14.4	14.4	14.4	14.7	13.6	+3.8	+4.8	+4.1	+4.8	+4.4	+4.0
Rise & fall	35.4	35.3	28.1	28.5	28.5	28.5	28.3	28.1	−7.3	−6.8	−6.9	−6.8	−7.1	−7.2
Frickey's clearings														
Rise	29.4	30.5	16.4	16.7	16.2	16.2	15.5	16.9	−13.0	−13.8	−13.2	−14.3	−13.9	−13.6
Fall	13.8	12.7	17.0	17.7	17.1	17.1	18.2	16.7	+3.2	+5.0	+3.3	+4.4	+4.4	+4.0
Rise & fall	43.2	43.2	33.4	34.4	33.3	33.3	33.7	33.6	−9.8	−8.8	−9.9	−9.9	−9.5	−9.6
A.T.&T. index														
Rise	30.7	31.4	25.1	24.8	24.6	24.6	24.5	25.2	−5.6	−6.6	−6.1	−6.8	−6.2	−6.2
Fall	20.2	19.1	23.5	23.8	23.6	23.6	24.4	23.1	+3.3	+4.7	+3.4	+4.5	+4.2	+4.0
Rise & fall	51.0	50.6	48.6	48.6	48.1	48.1	48.9	48.3	−2.4	−2.0	−2.9	−2.5	−2.1	−2.3
Pig iron production														
Rise	61.9	64.5	53.1	52.1	50.5	50.5	50.9	53.1	−8.8	−12.4	−11.4	−14.0	−11.0	−11.4
Fall	48.0	43.9	51.2	51.9	50.4	50.4	54.1	49.4	+3.2	+8.0	+2.4	+6.5	+6.1	+5.5
Rise & fall	110.0	108.4	104.3	104.1	100.9	100.9	105.1	102.6	−5.7	−4.3	−9.1	−7.5	−4.9	−5.8
DOWNWARD TREND														
Railroad bond yields														
Rise	6.0	5.8	8.6	8.6	8.6	8.6	8.8	8.4	+2.6	+2.8	+2.6	+2.8	+2.8	+2.6
Fall	14.4	15.0	8.4	8.4	8.4	8.4	8.3	8.6	−6.0	−6.6	−6.0	−6.6	−6.1	−6.4
Rise & fall	20.4	20.8	17.0	17.0	16.9	16.9	17.1	17.0	−3.4	−3.8	−3.5	−3.9	−3.3	−3.8
OSCILLATORY TREND														
Railroad bond yields														
Rise	9.3	9.1	10.1	10.1	10.0	10.0	10.3	10.0	+0.8	+1.0	+0.7	+0.9	+1.0	+0.9
Fall	15.4	16.0	10.8	11.0	10.9	10.9	10.7	11.2	−4.6	−5.0	−4.5	−5.1	−4.7	−4.8
Rise & fall	24.7	25.1	21.0	21.1	20.9	20.9	21.0	21.1	−3.7	−4.0	−3.8	−4.2	−3.7	−4.0

Methods A, B and C are identified in Table 92, and more fully on pp. 284-5. With one exception, the number of corresponding cycles is one less for each sample than in Table 92, the expansion of the first and the contraction of the last cycle being dropped. In railroad bond yields (oscillatory trend) the number of cycles is 12 in this table, but 16 in Table 92; the expansion of the first and the contraction of the last cycle in each of the four clusters of corresponding cycles shown in Table 85 (note 'a') was dropped. The brief series on electricity and the rising segment of bond yields are omitted because of fewness of cycles.

they are bound to be slight and erratic, since the cycle bases are usually close to 100 and in any case deviate more or less erratically from this value. We can therefore say that in methods A and B the amplitude measures do not depend upon whether the specific cycles are treated positively or invertedly. We also know that the average rise will tend to equal the average fall in methods A and B, while the presence of a trend makes the rise and fall unequal in the case of unadjusted data. It follows that if method A or B is applied to adjusted data, the average rise and fall, taken separately, will be closer to corresponding averages of unadjusted data when their specific cycles are treated positively than when they are treated invertedly—a conclusion that is equally valid for series with upward or downward trends. Or to put the same thing in different words, if method A or B is used, the removal of secular trends will alter our standard measures of amplitude of rise and fall, taken separately, more when the specific cycles have been handled invertedly than when they have been handled positively. No such systematic difference will appear if method C is used. However, as Table 94 shows, the differences on account of positive versus inverted treatment are as a rule very small in relation to the size of the amplitudes of expansions and contractions, taken separately, and they are practically of no consequence whatsoever in the amplitudes of full cycles.[22]

TABLE 95

Average Amplitude of Corresponding and All Specific Cycles in Unadjusted and Trend-adjusted Data

Six American Series

Series and group	Direction of trend	No. of specific cycles		Average amplitude in specific-cycle relatives						Average of adj. data as relative of average of unadj. data		
				Rise		Fall		Rise & fall		Rise	Fall	Rise & fall
		Un	Ad	Un	Ad	Un	Ad	Un	Ad			
CORRESPONDING CYCLES												
Deflated clearings	Upward	13	13	25.9	14.8	13.7	17.6	39.7	32.4	57	128	82
Frickey's clearings	Upward	9	9	30.2	17.4	13.6	17.3	43.8	34.7	58	127	79
A.T.&T. index[a]	Upward	9	9	30.5	23.9	26.1	29.6	56.5	53.6	78	113	95
Pig iron production[a]	Upward	15	15	62.1	52.2	54.8	57.7	116.8	109.9	84	105	94
Electricity output[a]	Upward	2	2	44.2	18.0	13.8	22.2	58.0	40.1	41	161	69
Railroad bond yields	Upward	3	3	12.9	8.7	8.4	10.5	21.3	19.2	67	125	90
Railroad bond yields[a]	Downward	8	8	6.3	8.6	14.6	8.4	20.9	17.0	137	58	81
Railroad bond yields	Oscillatory	16	16	9.4	10.2	14.7	11.1	24.1	21.2	109	76	88
ALL CYCLES												
Deflated clearings	Upward	15	15	26.9	15.6	13.4	17.4	40.2	33.0	58	130	82
Frickey's clearings	Upward	11	11	31.2	19.7	13.7	18.3	44.8	38.0	63	134	85
Railroad bond yields	Upward	5	7	13.3	6.7	6.3	7.1	19.6	13.8	50	113	70
Railroad bond yields	Oscillatory	20	21	10.8	9.9	12.5	10.0	23.3	19.9	92	80	85

'Un' stands for unadjusted, 'Ad' for trend-adjusted data. The amplitudes of the adjusted data were computed by our standard technique from trend relatives. See Table 85 for the periods covered.

[a] All cycles correspond in the adjusted and unadjusted data.

It may be well to pause at this point and sum up the main findings, which are simple enough in essence. Four conclusions stand out. (1) The removal of an upward trend tends to reduce the amplitude of cyclical rise and increase the amplitude of cyclical fall; the removal of a declining trend has opposite effects. These effects must register in amplitudes expressed in the original units, and they are very likely to do so in amplitudes expressed in cycle relatives. (2) The removal of any definite trend tends to reduce the amplitude of full specific cycles, whether expressed in the original units or in cycle relatives. This effect is not necessary mathematically, but it is likely to dominate in the time series with which we deal. (3) The average effect produced by trend adjustments (Table 95) is considerable in the case of amplitudes of expansions and contractions

[22] Cf. pp. 135-6.

taken separately, and is appreciable even in the amplitudes of full cycles.[23] (4) The effects of trend adjustments on measures of cyclical amplitude depend partly upon 'real' factors, that is, the duration, amplitude, pattern, and intra-cycle trend of different specific cycles; and partly upon 'technical' factors, such as the particular trend line used, the method used to eliminate the trend, the method used to measure the amplitude of trend-adjusted data, and whether the specific cycles are analyzed positively or invertedly. The technical factors are more important in the amplitudes of cyclical phases than in the amplitudes of full cycles; but, in general, if the trend line is at all plausible, the 'technical' effects are reasonably sure to be swamped by the 'real' effects.

To complete the present analysis, we show in Tables 96 and 97 how the removal of secular trends affects the per month amplitudes. It appears that the per month amplitude of full specific cycles is affected by trend adjustments in about the same ratio as is the total amplitude proper. This result reflects the slight influence of trend adjustments upon the duration of full cycles.[24] The effects of trends on the *rates* of rise and fall, taken separately, are relatively smaller and less uniform than the effects on the amounts of rise and fall. For the removal of an upward trend tends not only to reduce cyclical rises and increase cyclical falls, but also to shorten expansions and lengthen contractions. Likewise the removal of a downward trend tends to reduce both the amplitude and the duration of contractions, and to increase the amplitude and duration of expansions. Whether the removal of trend increases or diminishes the per month amplitude depends therefore upon three factors: the direction of the trend, its influence on the amplitude of a phase, and its influence on the corresponding duration. The main line of cleavage is between cyclical movements that are in the same direction as the trend and the cyclical movements that oppose the trend. Assume that the removal of trend affects the amplitude of cyclical phases in greater proportion than their duration. Then the rate of cyclical rise will be reduced when an upward trend is eliminated and the rate of cyclical fall will be reduced when a downward trend is eliminated. The dominance of these tendencies in our sample appears clearly in the distribution of single cycles in Table 96, as well as in the averages of Table 97. But in order that the removal of an upward trend reduce the rate of cyclical fall or the removal of a downward trend reduce the rate of cyclical rise, the trend adjustment must increase the amplitudes in smaller proportion than the durations. These tendencies are not prominent in the distribution of single cycles, though they

23 So far as the removal of an upward (downward) trend tends to increase the number of specific cycles, it will tend to intensify the reduction of the average cyclical rise (fall) and, though to a lesser extent, offset the increase of the average cyclical fall (rise); hence it will tend to intensify the reduction of the average amplitude of full cycles. But the noncorresponding cycles in our sample are too few to cut an appreciable figure. See Table 95, and Sec. II of this chapter.

24 But see Ch. 8, note 18.

TABLE 96

Rates of Rise and Fall of Specific Cycles in Unadjusted Data Compared with Corresponding Measures of Trend-adjusted Data
Six American Series

Series	Number of corresponding specific cycles	Number of instances in which: Rise per month in adjusted data is Larger than in unadjusted data	Rise per month in adjusted data is Smaller than in unadjusted data	Fall per month in adjusted data is Larger than in unadjusted data	Fall per month in adjusted data is Smaller than in unadjusted data
			Upward trend		
Deflated clearings	13	2	11	7	6
Frickey's clearings	9	..	9	4	5
A.T.&T. index	9	1	8	6	3
Pig iron production	15	3	12	8	7
Electricity output	2	..	2	1	1
Railroad bond yields	3	1	2	2	1
Total	51	7	44	28	23
			Downward trend		
Railroad bond yields	8	6	2	..	8

The rates of rise or fall in specific-cycle relatives were computed by our standard technique, to as many places as was necessary to establish a difference between the adjusted and unadjusted data. See Table 85 for the periods covered.

TABLE 97

Average Per Month Amplitude of Corresponding and All Specific Cycles in Unadjusted and Trend-adjusted Data
Six American Series

Series and group	Direction of trend	No. of specific cycles: Un	No. of specific cycles: Ad	Average per month amplitude in specific-cycle relatives: Rise Un	Rise Ad	Fall Un	Fall Ad	Rise & fall Un	Rise & fall Ad	Average of adj. data as relative of average of unadj. data: Rise	Fall	Rise & fall
CORRESPONDING CYCLES												
Deflated clearings	Upward	13	13	0.81	0.64	2.05	1.93	0.92	0.74	79	94	80
Frickey's clearings	Upward	9	9	1.00	0.69	1.78	1.65	1.13	0.88	69	93	78
A.T.&T. index[a]	Upward	9	9	1.30	1.12	1.87	1.79	1.35	1.28	86	96	95
Pig iron production[a]	Upward	15	15	2.43	2.49	4.68	4.02	2.89	2.72	102	86	94
Electricity output[a]	Upward	2	2	0.97	0.48	0.80	0.69	0.92	0.56	49	86	61
Railroad bond yields	Upward	3	3	0.58	0.58	1.73	0.74	0.71	0.65	100	43	92
Railroad bond yields[a]	Downward	8	8	0.39	0.49	0.50	0.35	0.46	0.38	126	70	83
Railroad bond yields	Oscillatory	16	16	0.56	0.62	0.94	0.65	0.66	0.60	111	69	91
ALL CYCLES												
Deflated clearings	Upward	15	15	0.83	0.80	1.92	1.77	0.94	0.79	96	92	84
Frickey's clearings	Upward	11	11	1.02	0.98	1.65	1.59	1.15	1.01	96	96	88
Railroad bond yields	Upward	5	7	0.45	0.57	1.16	0.44	0.53	0.45	127	38	85
Railroad bond yields	Oscillatory	20	21	0.57	0.64	0.84	0.56	0.64	0.55	112	67	86

The per month averages are unweighted. 'Un' stands for unadjusted, 'Ad' for trend-adjusted data. The amplitudes of the adjusted data were computed by our standard technique from trend relatives. See Table 85 for the periods covered.

[a] All cycles correspond in the adjusted and unadjusted data.

leave their impress on the averages. In our sample the effect of trend adjustments is generally greater on the amplitudes and smaller on the durations, relatively to the size of the figures, when the cyclical movement is in the same direction as the trend than when it opposes the trend.[25] The tendency of trend adjustments to reduce the per month amplitudes in the different circumstances represented in Table 96 reflects this fact, although it does not follow inevitably from it.

TABLE 98

Average Specific-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of specific cycles	Average in specific-cycle relatives at stage								
			I	II	III	IV	V	VI	VII	VIII	IX
			Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
				First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS											
Unadjusted	Up	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Adjusted	....	15	92.7	96.6	100.8	104.7	108.3	103.7	100.0	94.1	90.9
FRICKEY'S CLEARINGS											
Unadjusted	Up	11	84.2	88.2	98.3	106.6	115.4	113.5	111.1	105.8	101.7
Adjusted	....	11	89.4	95.1	99.2	104.9	109.1	105.0	101.5	96.3	90.7
A.T.&T. INDEX											
Unadjusted	Up	9	84.7	90.4	101.4	109.6	115.1	111.0	104.5	94.0	89.1
Adjusted	....	9	89.7	94.6	103.1	109.4	113.7	108.8	100.9	88.7	84.0
PIG IRON PRODUCTION											
Unadjusted	Up	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Adjusted	....	15	73.5	86.6	106.9	115.3	125.7	118.0	101.5	84.2	67.9
ELECTRICITY OUTPUT											
Unadjusted	Up	2	74.0	81.0	96.6	111.0	118.2	116.2	113.3	107.1	104.3
Adjusted	....	2	92.0	94.8	99.9	106.6	110.0	106.2	100.2	93.2	87.8
RAILROAD BOND YIELDS											
Unadjusted	Up	5	93.4	95.7	99.3	102.8	106.7	104.3	103.4	102.0	100.5
Adjusted	....	7	97.0	97.6	99.9	101.9	103.8	101.7	100.3	98.3	96.7
RAILROAD BOND YIELDS											
Unadjusted	Down	8	100.4	101.8	102.8	104.2	106.7	102.8	99.0	94.9	92.1
Adjusted	....	8	96.3	98.1	100.0	101.8	104.8	101.8	100.1	98.1	96.5
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	20	96.1	98.4	101.5	104.0	106.9	103.7	100.6	96.9	94.3
Adjusted	....	21	95.2	97.4	100.5	102.7	105.1	102.1	100.0	97.1	95.1

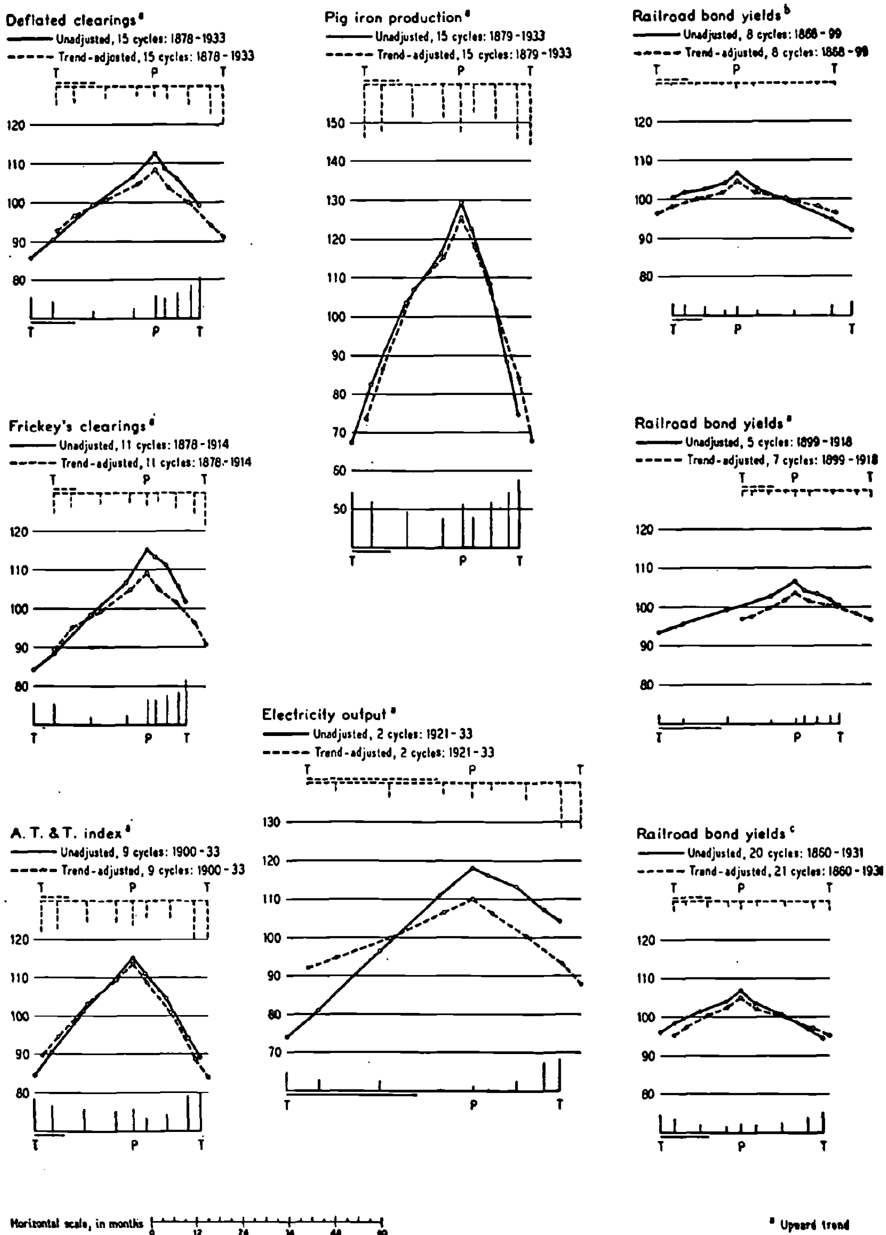
The patterns of the adjusted data were computed by our standard technique from trend relatives. See Chart 37 for the periods covered.

Chart 37 and Table 98 compare the patterns of the specific cycles of the adjusted and unadjusted data, traced out by the average standings in successive stages of the cycles. In the main the chart recapitulates the differences in specific-cycle behavior shown by preceding tables. The largest difference between any two patterns is in electric power production, as is to be expected. The two patterns of bond yields differ notably in periods

[25] See Tables 88 and 95.

CHART 37

Average Specific-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series



See Table 98. For explanation of chart, see Ch. 5, Sec. VI.

[a] Upward trend
[b] Downward trend
[c] Oscillatory trend

over which the trend moves in a single direction, but are fairly similar when the period includes opposing trends. It is worth observing that the patterns of the adjusted data of our sample look no more like sine curves than do the patterns of the unadjusted data.

V Reference-cycle Measures

When we break series on the basis of the turning points in general business activity instead of the turning points peculiar to each series, the shift has scarcely any effect on the trend component. On the other hand, the amplitudes of the cycles are reduced in varying degree, and leads or lags emerge. As a result the trend component of the unadjusted data appears more prominent in the reference-cycle patterns of Chart 38 than in the specific-cycle patterns of Chart 37. The trend obscures the response of bond yields to business cycles if we take periods of rising and falling trends separately, and we must look closely at the figures in Table 99 to detect it.

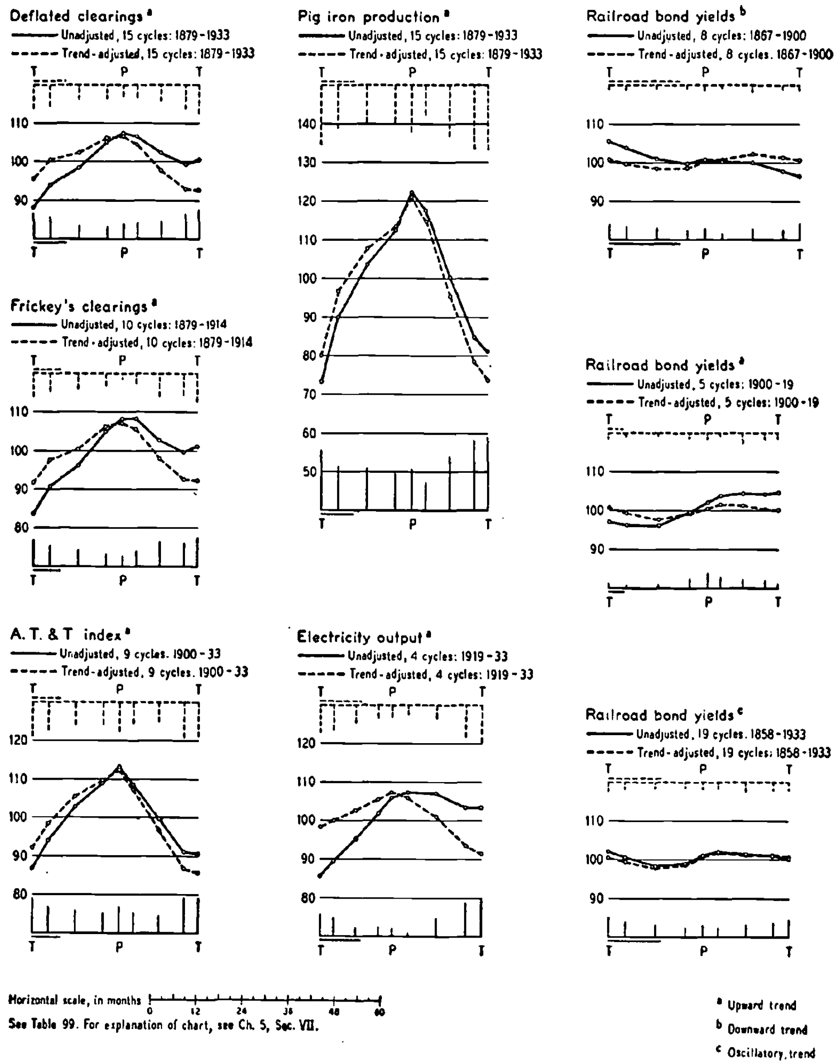
TABLE 99
Average Reference-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of reference cycles	Average in reference-cycle relatives at stage								
			I	II	III	IV	V	VI	VII	VIII	IX
			Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
				First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS											
Unadjusted	Up	15	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Adjusted	....	15	95.6	100.4	102.2	106.2	106.7	104.5	97.9	93.0	92.9
FRICKEY'S CLEARINGS											
Unadjusted	Up	10	83.5	90.7	96.1	104.9	108.1	108.2	102.7	99.7	101.1
Adjusted	....	10	91.6	97.6	100.2	106.0	107.0	105.6	97.9	92.5	92.2
A.T.&T. INDEX											
Unadjusted	Up	9	86.8	94.1	102.7	109.1	113.2	108.8	99.8	91.1	90.8
Adjusted	....	9	92.1	98.5	105.5	109.8	112.7	107.3	96.7	86.8	85.7
PIG IRON PRODUCTION											
Unadjusted	Up	15	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Adjusted	....	15	80.0	96.7	107.6	113.3	121.0	114.8	95.4	78.4	73.7
ELECTRICITY OUTPUT											
Unadjusted	Up	4	85.7	89.5	95.3	101.8	105.9	107.3	107.0	103.4	103.4
Adjusted	....	4	98.2	100.0	102.5	105.3	107.2	105.9	101.0	93.6	91.5
RAILROAD BOND YIELDS											
Unadjusted	Up	5	97.0	96.3	96.1	99.7	102.0	103.9	104.3	104.2	104.7
Adjusted	....	5	100.8	99.4	97.8	99.5	100.4	101.7	101.3	100.2	100.1
RAILROAD BOND YIELDS											
Unadjusted	Down	8	105.4	103.7	100.9	99.7	100.6	100.2	100.1	97.9	96.7
Adjusted	....	8	100.6	99.8	98.4	98.7	100.2	100.7	102.1	101.3	100.8
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	19	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2
Adjusted	....	19	100.3	99.3	97.9	98.8	100.8	101.9	101.4	101.4	100.8

The patterns of the adjusted data were computed by our standard technique from trend relatives. See Chart 38 for the periods covered.

CHART 38

Average Reference-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series



However, when the periods of rising and falling trends in bond yields are combined, the response is as clear in the unadjusted as in the adjusted data.[26]

26 In several series the standing of the cyclical patterns of the adjusted data is lower in stage IX than in stage I. This can be explained partly by the tendency of the trend lines to exaggerate the contraction of 1929–33. But the drift of the cyclical patterns is only a rough guide to the average intra-cycle trend. If, for example, the average value of a series is the same in successive cycles, the average standings in stages I and IX will still differ, except when the standing at the terminal trough of the last cycle is the same as the standing at the initial trough of the first cycle.

TABLE 100

Conformity to Business Cycles of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of reference cycles[a]	Stages matched with reference expansion[b]	Av. change per month in reference-cycle relatives during stages matched with reference: Expansions	Contractions	Cycles[c]	Index of conformity to reference: Expansions	Contractions	Cycles taken: From trough to trough	From peak to peak	Both ways[d]
DEFLATED CLEARINGS											
Unadjusted	Up	15	VIII-V	+0.78	−0.50	−1.28	+100	+73	+87	+86	+86
Adjusted	....	15	VIII-V	+0.43	−0.85	−1.29	+100	+87	+87	+86	+86
FRICKEY'S CLEARINGS											
Unadjusted	Up	10	VIII-V	+0.94	−0.63	−1.57	+100	+60	+100	+100	+100
Adjusted	....	10	VIII-V	+0.55	−1.01	−1.56	+100	+100	+100	+100	+100
A.T.&T. INDEX											
Unadjusted	Up	9*	I-V	+1.33	−1.17	−2.50	+100	+80	+100	+100	+100
Adjusted	....	9*	I-V	+1.08	−1.39	−2.47	+100	+100	+100	+100	+100
PIG IRON PRODUCTION											
Unadjusted	Up	15	I-V	+2.26	−2.27	−4.53	+100	+100	+100	+100	+100
Adjusted	....	15	I-V	+1.93	−2.57	−4.50	+100	+100	+100	+100	+100
ELECTRICITY OUTPUT											
Unadjusted	Up	4	I-VI	+0.98	0.00	−0.98	+100	0	+100	+100	+100
Adjusted	....	4	I-VI	+0.31	−0.62	−0.93	+100	+50	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Up	5	III-VI	+0.45	+0.03	−0.42	+100	−40	+100	+100	+100
Adjusted	....	5	III-VI	+0.24	−0.17	−0.41	+100	+100	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Down	6	III-VI	0.00	−0.25	−0.24	0	+100	+100	+50	+80
Adjusted	....	6	III-VI	+0.19	−0.07	−0.26	+100	+67	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	19*	III-VI	+0.24	−0.17	−0.41	+47	+30	+79	+58	+68
Adjusted	....	19*	III-VI	+0.26	−0.14	−0.40	+74	+70	+79	+68	+74

[a] An asterisk means that an additional reference contraction at the beginning of the series is covered by the contraction and full-cycle indexes. For reasons stated in note 29, only 6 cycles are covered in the period of secular decline of bond yields, whereas 8 cycles are covered in Chart 38. Subject to these exceptions, the periods covered by this table are shown in Chart 38.

[b] See note 31.

[c] Difference between reference contraction and reference expansion (see Table 47, col. 8).

[d] Described in the text as the 'final full-cycle index'.

As explained in Chapter 5, we measure the conformity of each time series to reference expansions, to reference contractions, and to full cycles. The 'expansion' measures report merely the rate at which, or the regularity with which, a series rose or fell during reference expansions, or during the reference-cycle stages matched with reference expansions. The 'contraction' measures provide similar information for reference contractions. It is obvious from their nature that the removal of trend must modify the rates of change during reference expansions and contractions, and that it is likely to change also the conformity indexes for these phases. But the measures of full-cycle conformity contain an automatic adjustment for secular trends, since a decline in the rate of increase from reference expansion to reference contraction is treated the same way as a rise during reference expansion followed by an actual decline during reference contraction. We should find, therefore, that formal removal of secular trends from the original data has slight influence on the measures of conformity to full business cycles.

Imagine a series built up by adding ordinates of a linear trend to a series of monthly cyclical values. This trend must increase the *rate* of change, in the original units, both during the stages matched with reference expansions and during the stages matched with reference contractions. Further, the rate for 'expansion' segments must be increased by exactly the same amount as the rate for 'contraction' segments, since the slope of the trend is assumed to be constant. If the trend is downward instead of upward, the two rates must be reduced the same amount. It follows that the influence of the trend can be wiped out by striking a difference between the two rates. We record this difference, expressed in reference-cycle relatives, in column (8) of our standard Tables R3 and R4, and treat it as an indicator of conformity to full reference cycles.[27] Of course, if the trend is nonlinear this indicator is not completely free from the influence of trend. But since the average slope of the secular trend of the original data is unlikely to differ appreciably during the two segments into which reference cycles are broken, the differential between the two rates of change should be substantially, if not entirely, free from trend.

Table 100 shows the influence of secular trends on average rates of change during the reference cycles covered by our test series. The adjusted data are analyzed in the form of trend relatives, and therefore do not correspond precisely to the theoretical model in the preceding paragraph. Nevertheless, in every comparison an upward trend increases the average rise per month during the stages matched with reference expansions to about the same degree as it reduces the fall per month during the stages matched with reference contractions.[28] A declining trend has similar effects in the opposite direction.[29] Hence the influence of the secular trend tends to cancel out in the average change per month referring

27 See Ch. 5, Sec. IX-X.

28 If a series bears an inverted relation to business cycles, an upward trend will reduce the average fall per month during the stages matched with reference expansions to about the same degree as it increases the rise per month during the stages matched with reference contractions.

29 Only 6 cycles are included in the period of secular decline of bond yields, whereas 8 cycles are covered in Table 99 and Chart 38. The reason for the difference is that Macaulay's trend line is somewhat undulatory even within this period. The rise of the trend line from 1888 to 1892 and the resumption of the rise early in 1900 have a negligible influence on the cyclical measures previously analyzed, but they confuse the conformity analysis. Hence we limit the conformity measures to cycles during which the trend declined unequivocally, that is, the four cycles from 1867 to 1888 and the two from 1891 to 1897. Since stages III-VI are matched with reference expansions, the periods actually analyzed are shifted half a phase forward from the standard reference cycles.

to full cycles, whether the trend rises, falls, or changes direction within the period covered by a series.

In making the index of conformity to full reference cycles, we take severer precautions to control the influence of secular trends. The method is fully described in Sections IX and X of Chapter 5, but a few additional remarks may help to clarify the exact mathematical nature of this index. Assume that absolute deviations from trend show zero conformity to one or more business cycles; that is, that the average change per month is some constant c during both reference expansion and contraction. Then the addition of a linear trend must leave the conformity zero, whether the reference cycles are taken from trough to trough or from peak to peak; for if the slope of the trend is m, the change per month becomes $c + m$ during both expansion and contraction. A concave trend in relation to the axis of time will make the conformity of the trend-cycle composite positive if reference cycles are marked off by troughs, and inverted if the cycles are marked off by peaks; for the change per month during successive reference phases is now $c + d_1$, $c + d_2$, $c + d_3$, etc., where the d's are successively smaller algebraically. Similarly, a convex trend will make the conformity inverted for reference cycles marked off by troughs, and positive for cycles marked off by peaks. But these opposite 'biases' tend to offset one another in the final full-cycle index, since we take the cycles both from trough to trough and from peak to peak in making this index.

The result is similar when the cyclical and trend components are combined by multiplication, instead of addition. If the standings at the three stages from which conformity is measured fall on a straight line, with the equation $a_1 + b_1x$, the insertion of a linear trend, with the equation $a_2 + b_2x$, will make the conformity of the trend-cycle composite positive or inverted for cycles marked off by troughs, and inverted or positive for cycles marked off by peaks, according as b_1b_2 is minus or plus. The insertion of convex or concave trends will now produce one result, now another. But in these instances as when the trend is linear, excepting occasional shifts from concavity to convexity or vice versa in the trend-cycle composite,[30] the 'bias' for reference cycles marked off by troughs must oppose the 'bias' for reference cycles marked off by peaks. Hence the final full-cycle index of unadjusted data should be practically independent of the trend.

30 That is one reason why the index for cycles taken by troughs and the index for cycles by peaks may differ in practice. But a difference might arise merely from rounding numbers, when the indexes are computed from reference-cycle relatives instead of the original data; or from the fact that the index for reference cycles taken by troughs cannot cover exactly the same period as the index for cycles by peaks.

Still another factor may be illustrated by a hypothetical example. Assume that a series has a horizontal 'secular trend'; that each of its specific cycles starts at a reference trough, rises throughout one reference cycle, and falls throughout the next reference cycle; and that the second differences of the monthly values of each specific cycle, considered as a discrete unit, are uniformly minus. In this case the conformity to reference cycles taken by troughs will be invariably positive, while the

In our test series the final full-cycle index is actually the same or almost the same in the unadjusted as in the adjusted data (Table 100). On the other hand, the indexes of conformity to reference expansions and contractions bear clearly the stamp of secular trends. In the one sample of a declining trend the expansion index is lower and the contraction index higher for unadjusted than for adjusted data. In the six samples of a rising trend the contraction index made from unadjusted data is lower than the index made from adjusted data in five series and the same in one. Since the expansion indexes of our several samples of a rising trend are all +100 in the adjusted data, they cannot be higher in the unadjusted data. Barring such limiting cases, secular trends must impart opposite biases to the expansion and contraction indexes. A rising trend tends to increase positive conformity to reference expansions and to diminish positive conformity to reference contractions. A declining trend tends to have opposite effects.[31]

VI Variability of Cyclical Measures

A striking feature of Charts 37-38 is that the cyclical patterns of different series are more alike when made from adjusted data than when made from unadjusted data. The same is true of the separate segments of bond yields, our one series subject to different trends. Thus, in the adjusted data, the reference-cycle pattern of deflated clearings does not differ much from that of electric power production. But the unadjusted data indicate that, in the periods represented, power production rose more vigorously on the average during reference expansions than the volume of clearings;

conformity to cycles taken by peaks will be alternately inverted and positive. The difference between the two conformity indexes cannot be ascribed to the secular trend, since the trend is assumed to be horizontal. Of course, a change of assumptions will change the conclusion. Thus if we regard the long specific cycles as the 'secular trend' and assume that the 'short-run' cyclical component is zero, we must say that the 'trend' is solely responsible for the difference between the two indexes; also, that the 'bias' of the index on a peak basis opposes only in part the 'bias' of the index on a trough basis, the reason being that the timing of the oscillatory 'trend' is correlated with the timing of business cycles.

31 The comparisons between conformity measures of adjusted and unadjusted data in Table 100 are based on the division of reference cycles that seemed most appropriate for the unadjusted data of each series. Consequently, the comparisons reflect the influence of the trend factor alone. But they do not necessarily reflect the *full* influence of the trend, since the trend may modify the stages characteristic of expansion and contraction. If the conformity measures of the adjusted data were made on that division of reference cycles which seemed best for these data, the comparison would reflect the full influence of the trend; but it would reflect also nonsecular factors whenever the division of the reference cycles of adjusted data differed from that of unadjusted data.

As we have seen in Sec. III, the shifts in cyclical timing produced by secular trends are, usually, not very large in our sample. We should expect therefore the division of reference cycles in our standard Table R4 to be similar for the adjusted and unadjusted data. In fact, a difference arises only in Frickey's clearings and electricity output. In Frickey's clearings the expansion stages of the adjusted data are I-V; the average rates of change on this basis are +0.65, −0.83, and −1.48, and the conformity indexes are all +100. In electricity the expansion stages of the adjusted data are also I-V; the average rates of change are +0.44, −0.56, −1.00; the conformity indexes are +100 except the contraction index, which is +50.

also, that its declines during reference contractions were milder. Again, the reference-cycle pattern of the adjusted data on bond yields when the trend is upward is a rough duplicate of the pattern when the trend is downward; but in the unadjusted data the former pattern shows virtually no decline during the stages matched with reference contractions, while the latter shows virtually no rise during the stages matched with reference expansions.

The vertical lines on the charts representing average deviations of the cyclical patterns are in some instances longer in the unadjusted data, in other instances shorter. But when averages are struck, it appears that the elimination of secular trends usually reduces the differences among the successive cycles. In eight out of ten comparisons the average deviations of the patterns of the adjusted data are smaller than those of the unadjusted data (Table 101). Also, the removal of trends reduces the differences among the durations of specific cycles in every series covered by our tests, and among the amplitudes in every series except one.

It seems, therefore, that if we removed secular trends completely from the original data at the start of the analysis we would find that the variability of cyclical measures within a series is usually reduced. And this is likely to mean that the scope and frequency of secular changes in cyclical measures would also be reduced. Table 102 illustrates the point for bond yields. For the present purpose we may consider a shift in average cyclical

TABLE 101

Average Deviations from Average Measures of Cyclical Behavior
Five American Series, Unadjusted and Trend-adjusted

Series and form of data	Number of		Average deviation		Average of average deviations from the nine average standings of	
	Specific cycles	Reference cycles	Duration of specific cycles	Rise & fall of specific cycles	Specific-cycle patterns	Reference-cycle patterns
DEFLATED CLEARINGS						
Unadjusted	15	15	11.6	13.8	5.7	5.2
Adjusted	15	15	10.2	7.9	4.7	4.9
FRICKEY'S CLEARINGS						
Unadjusted	11	10	6.1	10.0	6.3	5.2
Adjusted	11	10	5.9	8.6	4.1	4.6
A.T.&T. INDEX						
Unadjusted	9	9	7.6	15.4	6.6	7.1
Adjusted	9	9	7.4	14.9	6.8	7.2
PIG IRON PRODUCTION						
Unadjusted	15	15	9.9	26.8	11.9	12.9
Adjusted	15	15	9.1	29.5	11.3	12.4
RAILROAD BOND YIELDS						
Unadjusted	20	19	12.6	7.2	3.2	3.6
Adjusted	21	19	9.7	6.7	1.6	2.0

The average deviations of the durations are expressed in months; the others in cycle relatives. See Table 85 for the periods covered by the specific cycles, Chart 38 by the reference cycles. Electricity output is omitted because of its brief statistical record.

TABLE 102

Average Measures of Specific Cycles in Railroad Bond Yields, United States
Unadjusted and Trend-adjusted Data, 1868–1899 and 1899–1918

Measure and form of data	Downward trend 1868–1899		Upward trend 1899–1918		Excess of (5) over (3)
	Number of specific cycles	Average	Number of specific cycles	Average	
(1)	(2)	(3)	(4)	(5)	(6)
DURATION OF EXPANSIONS					
Unadjusted	8	16.8	5	35.4	+18.6
Adjusted	8	20.6	7	13.7	−6.9
DURATION OF CONTRACTIONS					
Unadjusted	8	29.6	5	11.2	−18.4
Adjusted	8	25.8	7	19.7	−6.1
AMPLITUDE OF RISE					
Unadjusted	8	6.3	5	13.3	+7.0
Adjusted	8	8.6	7	6.7	−1.9
AMPLITUDE OF FALL					
Unadjusted	8	14.6	5	6.3	−8.3
Adjusted	8	8.4	7	7.1	−1.3
LAG AT REFERENCE PEAKS					
Unadjusted	7	8.6	4	7.5	−1.1
Adjusted	7	9.7	6	4.5	−5.2
LAG AT REFERENCE TROUGHS					
Unadjusted	7	15.6	4	0.8	−14.8
Adjusted	7	15.6	6	6.7	−8.9

The lags and durations are expressed in months, the amplitudes in specific-cycle relatives. The entries for timing in col. (2) and (4) show the number of observations included in the averages. These are not consistent with the timing averages in Table 87. The averages in Table 87 are based on turns that correspond in the unadjusted and adjusted data, whether or not they correspond to the reference dates; the averages in this table include turns that correspond to the reference dates, whether or not they correspond in the unadjusted and adjusted data; also, the timing of the specific-cycle trough of June 1899 (both unadjusted and adjusted data) is included in the averages of both periods in Table 87, but excluded from both in this table.

behavior from 1868–99, when the trend of bond yields was downward, to 1899–1918, when the trend was upward, as indicating a secular change in cyclical behavior. The table shows that the differences between average cyclical measures of the two periods run smaller in the adjusted than in the unadjusted data; in other words, secular changes in the cyclical measures are less pronounced in the trend-adjusted data.[32] But the measures of timing at reference peaks convey a warning that the elimination of secular trends from the original data will not always make the cyclical measures for different periods more alike.

This warning is important. A fitted trend line may segregate the secular from the cyclical component in such fashion that cyclical measures made from the trend-adjusted data are virtually free from secular change. When this happens, as in the amplitudes of Table 102, we may say that the secular changes which appear in the cyclical measures made from the unadjusted data are due to the trend. But the trend line that effects this

32 Of course, the secular changes may or may not be 'statistically significant'; cf. Ch. 10. Further, they may be interpreted as indicating what happens upon passing from a contraction phase to an expansion phase of a long cycle; on this, see Ch. 11, especially Sec. III.

segregation with respect to some features of cyclical-secular change may be less successful with respect to other features. That is, a secular change may still appear in certain cyclical measures of the trend-adjusted data, and it may be smaller or larger than in the corresponding measures of the unadjusted data. For example, there is a decline from the first to the second period in the duration of contractions in the adjusted data; the declining trend of the unadjusted data during the first period tends to lengthen, and the rising trend during the second period tends to shorten, the contractions; hence the decline in the duration of contractions is larger in the unadjusted than in the adjusted data. A rather different result appears in the measures of timing at reference peaks. According to the adjusted data, the average lag at peaks declines from 9.7 months in the first period to 4.5 in the second; this change is counteracted by the tendency of the declining trend in the unadjusted data during the first period to reduce lags (or increase leads) and of the rising trend during the second period to increase lags (or reduce leads); the net result is that the timing of the unadjusted data is nearly the same in the second period as in the first.

VII The Time Unit and Trend Adjustments

In the preceding chapter we have shown how cyclical measures depend upon the time unit in which the observations are given. In this chapter we have shown how cyclical measures depend upon the retention or elimination of secular trends. These influences cross one another. Other things being equal, the steeper the trend the greater are the changes in cyclical measures induced by a shift from monthly to annual data. And the coarser the time unit the greater are the changes in cyclical measures induced by the elimination of a given trend. If we had carried through the analysis of the preceding chapter on the basis of trend-adjusted data, we would have found less startling differences between cyclical measures made from monthly and annual data. On the other hand, if the analysis of this chapter had been based on annual data, we would have established larger effects of secular trends.

Assume that the secular trend is removed from a monthly time series characterized by a rising trend. This operation is not likely to alter the number of specific cycles appreciably, since the amplitude of the 'cyclical component' is usually large compared with the amplitude of the 'trend component'. True, a retardation of increase in the original data is now and then converted into an actual decline that we must count as a cyclical movement, but this tendency is offset by the conversion under certain circumstances of a cyclical rise in the original data into a retarded decline in the adjusted data.[33] If, now, the original and trend-adjusted monthly series are put into annual form, some specific cycles are likely to be lost

[33] See above, pp. 273-5.

by each. But since the process of trend adjustment tends to lengthen and intensify cyclical contractions, fewer cycles are likely to be lost in the annual summations of the trend-adjusted data than in the annual summations of the original data. The brief and mild contractions that cannot survive in annual summations of the original data stand a good chance of survival in annual summations of the trend-adjusted data.

The dependence of the number of specific cycles upon the form of a time series may be explained in another way. A shift from monthly to annual data has no effect on the 'secular component' of a time series, but it dampens the 'cyclical component', so that a considerable fraction of the cyclical movements are converted into mere retardations of growth if the trend is upward or into mere retardations of decline if the trend is downward. These hidden cycles stand an excellent chance of coming to the surface again when secular trends are removed. For example, three cyclical contractions in pig iron production disappear when monthly figures unadjusted for trend are converted into calendar-year sums, but two of the three contractions are recovered when the trend is removed

TABLE 103
Number of Specific Cycles in Unadjusted and Trend-adjusted Data
Six American Series, Monthly and Annual

Series	Number of specific cycles			
	Monthly data		Annual data	
	Unadjusted	Adjusted	Unadjusted	Adjusted
Deflated clearings	15	15	10	13
Frickey's clearings	11	11	7	10
A.T.&T. index	9	9	9	9
Pig iron production	15	15	12	14
Electricity output	2	2	1	2
Railroad bond yields	20	21	14	19
Total	72	73	53	67

The numbers represent the full cycles within the periods covered by the monthly data, as shown in Table 85. In pig iron production there is no trough in either annual series corresponding to the monthly trough in 1879; hence the count of annual cycles starts in 1884 and ends in 1932.

from the annual sums. Similar results are obtained in other series (Table 103). The number of specific cycles in the unadjusted forms of the test series used in this chapter is 72 in monthly but only 53 in annual data, while the corresponding numbers in the trend-adjusted data are 73 and 67.[34]

[34] The trends removed from the annual (calendar-year) and from the monthly data are, of course, the same.

The proper way to obtain annual trend-adjusted figures is to remove the trend from annual data; or to convert monthly figures adjusted for trend, but not for seasonal, into annual sums or averages. (The last statement implies a 'relative' seasonal, the type we have used.) We followed the latter practice in iron production, deflated clearings, and bond yields. In the other series we took annual sums of monthly figures adjusted for both trend and seasonal; but it is practically certain that this change in method has no influence on the count of their specific cycles.

TABLE 104

Cyclical Measures of Unadjusted and Trend-adjusted Data

Two American Series, Monthly and Annual

Measure	Deflated clearings				Pig iron production			
	Monthly		Annual		Monthly		Annual	
	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.
NO. OF CYCLES								
Specific	15	15	10	13	15	15	12	14
Reference	15	15	15	15	15	15	15	15
AV. DURATION OF SPECIFIC CYCLES[a]								
Expansion	32.6	25.6	49.2	25.8	28.8	25.1	31.0	22.3
Contraction	11.4	17.9	16.8	24.9	14.5	18.1	17.0	18.0
Full cycle	44.0	43.5	66.0	50.8	43.3	43.2	48.0	40.3
AV. AMPLITUDE OF SPECIFIC CYCLES[b]								
Rise	26.9	15.6	29.7	11.4	62.1	52.2	41.1	27.2
Fall	13.4	17.4	9.8	13.8	54.8	57.7	32.4	32.8
Rise & fall	40.2	33.0	39.5	25.1	116.8	109.9	73.5	60.0
Rise per month[c]	0.8	0.8	0.7	0.5	2.4	2.5	1.5	1.2
Fall per month[c]	1.9	1.8	0.4	0.5	4.7	4.0	2.0	1.8
Rise & fall per month[c]	0.9	0.8	0.6	0.5	2.9	2.7	1.7	1.6
AV. LEAD (−) OR LAG (+)[a]								
At reference peaks	+3.2	−1.7	+1.8	−5.1	+1.9	−1.3	+0.3	−2.4
At reference troughs	−5.8	−3.4	−3.8	−1.9	−3.4	−2.9	−3.7	−3.1
CONFORMITY TO BUSINESS CYCLES								
Expansion stages[d]	VIII-V	VIII-V	I-V	I-V	I-V	I-V	I-V	I-V
Av. change per month[e] during stages matched with reference								
Expansions	+0.78	+0.43	+0.65	+0.29	+2.26	+1.93	+1.46	+1.08
Contractions	−0.50	−0.85	−0.17	−0.53	−2.27	−2.57	−1.65	−1.99
Index of conformity to reference								
Expansions	+100	+100	+100	+73	+100	+100	+100	+100
Contractions	+73	+87	+7	+73	+100	+100	+73	+100
Cycles	+86	+86	+86	+93	+100	+100	+100	+100

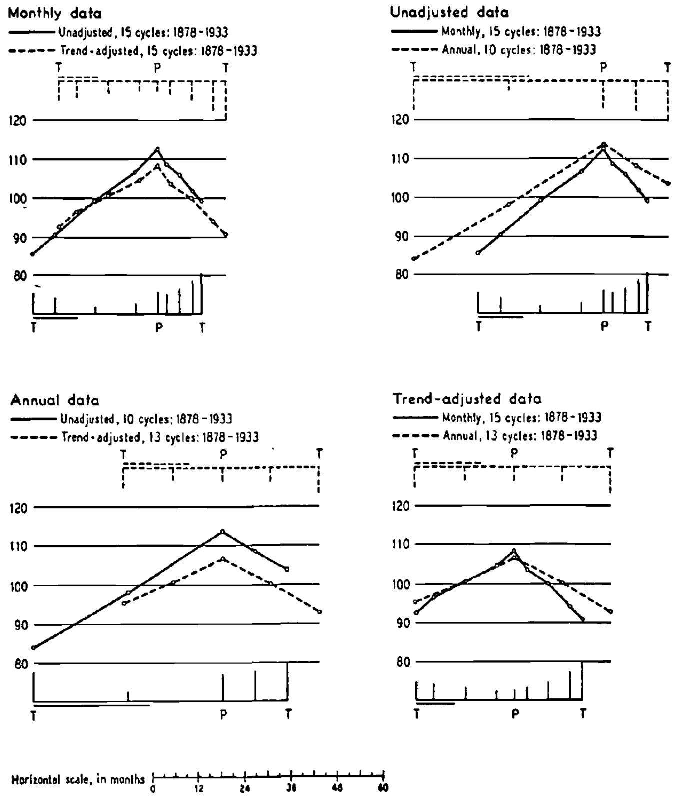
For the periods covered, see Charts 39-42. The average timing measures in this table include turns corresponding to reference dates; hence the apparent inconsistency with the averages in Table 87.

[a] In months.
[b] In specific-cycle relatives.
[c] Unweighted average.
[d] Matched in every instance with reference expansion.
[e] In reference-cycle relatives.

In Table 104 and Charts 39-42 we compare the cyclical measures of different forms of the data on deflated clearings and iron production. Although these series are subject to peculiarities that blur some theoretically interesting effects, they help to round out the preceding argument. Whether a series is monthly or annual, the removal of trends has similar effects on the timing of specific cycles, the duration of expansions and contractions, their amplitudes, and the conformity to reference expansions and contractions. But since trend adjustments influence the number of specific cycles in annual data much more than the number in monthly data, the effects of trend adjustments are likely to be greater on annual than on monthly cyclical measures, especially on the average duration of specific cycles and the indexes of conformity to reference expansions and contractions. In general, the removal of trends will tend

CHART 39

Average Specific-cycle Patterns
Bank Clearings outside New York City, Deflated
Unadjusted and Trend-adjusted Data, Monthly and Annual



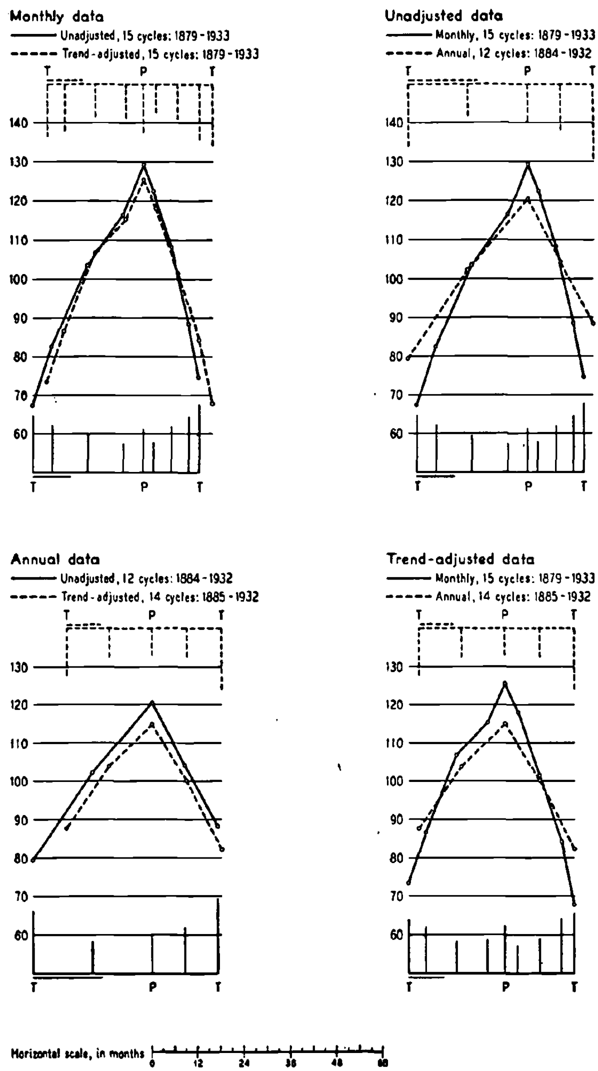
For explanation of chart, see Ch. 5, Sec. VI.

to alter these measures of annual data in the direction of the measures of monthly data unadjusted for trend. Consequently, an investigator restricted to annual data is likely to make better estimates of the number of specific cycles in the underlying monthly data, and of their conformity to reference expansions or contractions,[35] by adjusting the annual figures for trend than by using them as they come. Further, an investigator is likely to have somewhat better success in estimating cyclical measures of monthly trend-adjusted data from annual trend-adjusted data than in estimating cyclical measures of monthly unadjusted data from annual unadjusted data. But no device on this plane can counteract the coarseness of annual figures in measuring cyclical timing, their dampening effect on cyclical amplitudes, or their obfuscation of cyclical patterns.

[35] To reference expansions if the trend is downward, to reference contractions if the trend is upward.

CHART 40

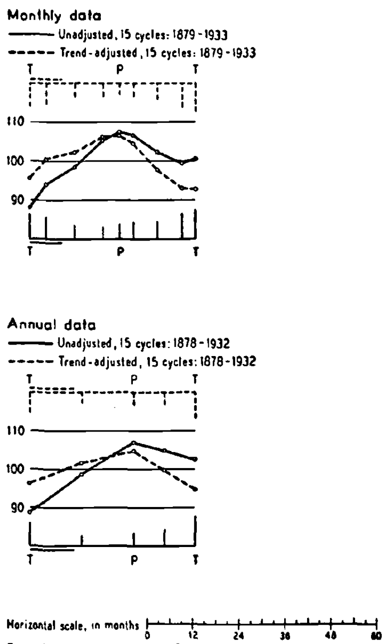
Average Specific-cycle Patterns
Pig Iron Production, United States
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VI.

CHART 41

Average Reference-cycle Patterns
Bank Clearings outside New York City, Deflated
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VII.

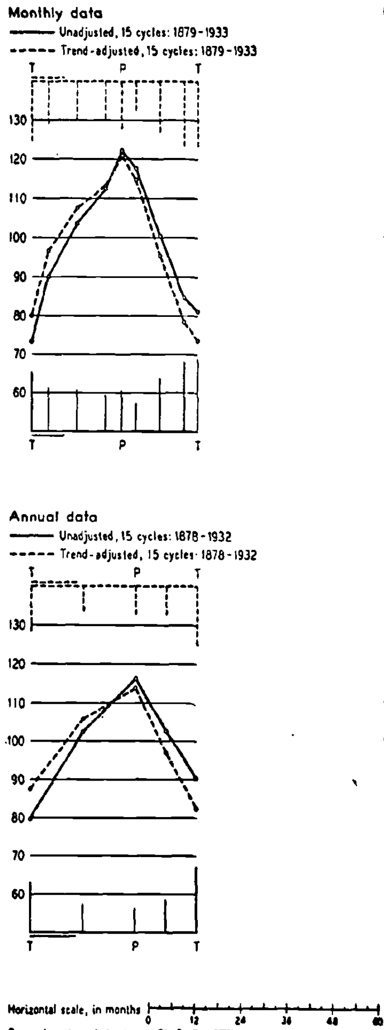
VIII Conclusions

We have seen that cyclical measures of different series, as well as cyclical measures of the same series, tend to be more alike when made from trend-adjusted data than when made from unadjusted data. To us that is a disadvantage of trend adjustments. The variations of cyclical behavior among and within series count in the interplay of forces that produce the business cycles of experience, and we therefore wish to preserve them. An investigator who seeks to gauge the role played by railroad construction, government spending, installment credit, or agricultural production in past business cycles and their probable role in future business cycles cannot remove secular trends without sacrificing the main part of his problem. It may be legitimate for students concerned with secular trends to put cyclical fluctuations out of sight, but students of cyclical behavior cannot take similar liberty with secular trends. If the trends characteristic of different business activities are set aside, inquiry is apt to be limited to the tendency of economic processes to fluctuate in unison. Our aims, as indicated in Chapter 1, are more ambitious.[36]

[36] See also Ch. 3, Sec. I-III, and Ch. 10, Sec. VIII.

CHART 42

Average Reference-cycle Patterns
Pig Iron Production, United States
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VII.

At the same time, as we have argued in Chapter 3, the retention of intra-cycle trends is a disadvantage when the task is chiefly to describe how business cycles manifest themselves in different activities. But this chapter has demonstrated that the disadvantage is less serious than might be supposed. Since secular trends exercise a systematic influence on nearly all of our cyclical measures, we can take rough account of their influence whenever desirable. Sometimes, as in indexes of conformity to full business cycles, no qualification is necessary. In most measures, the nature of the allowance depends on the direction of the trend; while the magnitude of the allowance depends partly upon the steepness of the trend, partly on other factors which may be of equal or greater importance. For instance, in judging the influence of trends on average measures of timing or duration of expansions and contractions, it is necessary to note the shapes of the specific cycles in the neighborhood of turning points. Again, in judging the influence of trends on amplitude measures, note must be taken of the curvature of the trend, as well as its direction and steepness; also the duration of expansions relatively to contractions, and several other factors. The main considerations that are relevant to each cyclical measure have been set out in the body of this chapter. Of course, judgments of the influence of trends are bound to be rough. But they would not be highly precise even if trend lines were fitted and removed by formal methods. As every statistician knows, secular trends of time series are rarely, if ever, susceptible of precise and objective determination. There is an arbitrary element not only in the choice of the trend line, but in every other step of trend adjustment: the period used in fitting the trend, the time unit in which the data are expressed for this purpose, the method used to fit the selected trend, and the method used to remove the trend.

Our standard Table S3 supplies the essential facts concerning the secular movements of each series. By studying this table in conjunction with other measures, we can usually judge roughly what contribution secular trends make to our measures, and thus allow for the deficiencies of our method in describing the scope of cyclical fluctuations. In annual series this check is less effective than in monthly. But annual data at best are very crude approximations for our purposes. Hence the reasons for trend adjustment, although stronger in annual than in monthly data, do not seem to us sufficient to justify the additional cost. We repeat, however, that if the resources at our disposal permitted it, we would analyze all series presented in the following monographs, or at least the more important ones, in both unadjusted and trend-adjusted forms. And we would feel still better equipped for the work ahead if we could supplement analyses of data adjusted and unadjusted for trend by analyses of data freed from erratic flutterings. That we cannot do. As a substitute we present in the following chapter sample measurements of the effects that 'erratic' movements exercise on our averages.

CHAPTER 8

Effects of Smoothing on Cyclical Measures

As explained in Chapter 3, we make cyclical measures directly from the raw data adjusted only for seasonal variations. But since most of our measures cover three or more months, an element of smoothing is inherent in our technique. We use, moreover, simple moving averages to help identify and date the specific cycles of very choppy series, though the cyclical measures proper are invariably computed from the raw data. To some students it might seem that our results would be more trustworthy if the data were first smoothed methodically, and the cyclical measures made from the smoothed figures. It may be argued, for example, that by eliminating 'erratic movements', smoothing exposes unequivocally the underlying form of the cyclical fluctuations; hence that more precise measures of cyclical behavior could be derived from smoothed than from raw data. Or a bolder claim may be staked out: namely, that a cyclical graduation, if carried through properly, will show results of 'systematic' forces alone, all effects of 'random perturbations' being wiped out. To come to grips with the questions thus raised, we have studied the manner in which smoothing—that is, the substitution of smoothed monthly figures for the raw—affects cyclical measures. This chapter presents the main results of our experiments.

I Range of the Tests

Some effects of smoothing time series have already been traced in Chapter 6. Converting monthly data into quarterly form is equivalent to taking every third term of a three-month moving average or total. A quarterly series may therefore be regarded as a smoothed variant of monthly data, but one that contains gaps. Likewise an annual series may be regarded as a smoothed variant of monthly data in which the gaps are wider.

We might begin our analysis by showing what happens to cyclical measures when the gaps are filled. But little can be learned from such calculations. We have seen that the cyclical measures derived from quarterly data are usually very close to those yielded by monthly data, and it is obvious that the use of three-month moving averages would make the margins still narrower. There is more to be learned from experimenting with twelve-month moving averages, but such experiments are bound, in the main, to prove unfavorable to smoothing. Everyone experienced in time series analysis knows that a twelve-month moving average distorts cyclical movements when the phases of expansion and contraction are asymmetrical in the vicinity of cyclical turns, and that even under conditions of symmetry a twelve-month average does not reach far enough up into cyclical peaks or far enough down into cyclical troughs; the result being that cyclical amplitudes are damped and the cyclical patterns distorted.

To evaluate fairly the power of smoothing to improve cyclical measures, we must choose a smoothing 'formula' that meets at least three conditions. First, the formula should cover a period long enough to iron out effectively the seemingly erratic movements which, by changing direction every few months, blur the course of cyclical expansions and contractions. Second, the formula should weight successive months in accordance with a 'smooth' weight diagram; this condition is desirable to insure a smooth result. Third, some of the weights should be negative; this is necessary if the graduation is to reach well up into cyclical peaks and well down into cyclical troughs. Since an indefinite number of formulas will meet these minimum conditions, our experiments of necessity have to be restricted. We have based the experiments largely on one formula, the "43-term summation approximately fifth-degree parabolic graduation" devised by Frederick R. Macaulay. The practical reason for this choice is that Macaulay applied his smoothing formula to several series we have analyzed by our standard technique, and we were able to take over his results. This consideration was reinforced by the fact that Macaulay's method of graduating cyclical data gives results as good for our purpose as any we have yet encountered.

Macaulay's formula is a complicated moving average covering 43 terms of the data. It is alleged to reach well up into cyclical peaks and well down into cyclical troughs. When applied to monthly data, it eliminates seasonal and erratic movements, leaving a curve that is supposed to describe adequately the remaining cyclical and trend components. We selected four American series from the group that Macaulay had smoothed by applying his graduation to logarithms of the data—deflated bank clearings, railroad stock prices, pig iron production, and call money rates. We then compared the measures obtained by applying our technique to each of the four series, first in what we shall call the 'raw' form and second in the 'smoothed' form.[1] The 'raw' data are, of course, adjusted for seasonal variations.

These series form a fair sample for judging the influence of erratic fluctuations on measures of cyclical behavior. One series reports payments aggregating billions of dollars, the second prices, the third production in physical units, and the fourth shows percentages. The erratic movements are pronounced relatively to the cyclical movements in call money rates, but only mild or moderate in the other series. The erratic movements seem to differ in type as well as in size. Bank clearings show a 'saw-tooth' movement with a reversal of direction every month or two. In railroad stock prices such reversals occur at longer intervals. Iron production often rises or falls without interruption for months at a time, but occasionally has sharp 'saw-tooth' movements. In call money rates there are numerous high, narrow peaks, often standing in impressive isolation, but there are few deep, narrow troughs.

Macaulay's formula completely eliminates erratic fluctuations, in the sense that it replaces a 'saw-tooth' movement by a smooth curve, and turns angles, however acute, into gradual curvatures. But it does not necessarily eliminate protracted random movements, such as may last several years. Moreover, smoothing—no matter how skillfully done—may at times distort cyclical movements, besides eliminating erratic fluctuations. We must therefore pay careful attention in this chapter to possible limitations of smoothing, no less than to the presence of erratic components in our cyclical measures. As an aid to the reader, two samples of Macaulay's graduation are shown in Chart 43.

II The Number of Specific Cycles

In Chapter 4 we dwelt at some length, though probably not sufficiently, on the uncertainties connected with the identification of specific cycles. These uncertainties cannot be eliminated by smoothing. For there will always be movements, whether time series are smoothed or not, that seem too small or too brief to qualify as specific cycles. Some uncertainty is therefore bound to surround the results, so long as the attempt is made to distinguish specific cycles, and to fix in time the limits of their expansions and contractions. For example, Table 105 lists every movement of rise and fall in the smoothed figures of pig iron production, irrespective of its length or intensity. If all these movements were treated as specific cycles,

[1] The sources of the raw data are cited in Ch. 6, note 7. The smoothed series come from Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices*, Appendix A, Tables 17, 21, 30, 31, col. 2. The figures are expressed in logarithms, which we converted to natural numbers before analyzing. Macaulay describes his 'cyclical graduation' briefly in the above source, Appendix D, p. A 331, and more fully in *The Smoothing of Time Series* (National Bureau of Economic Research, 1931), pp. 24-6 and 73-5.

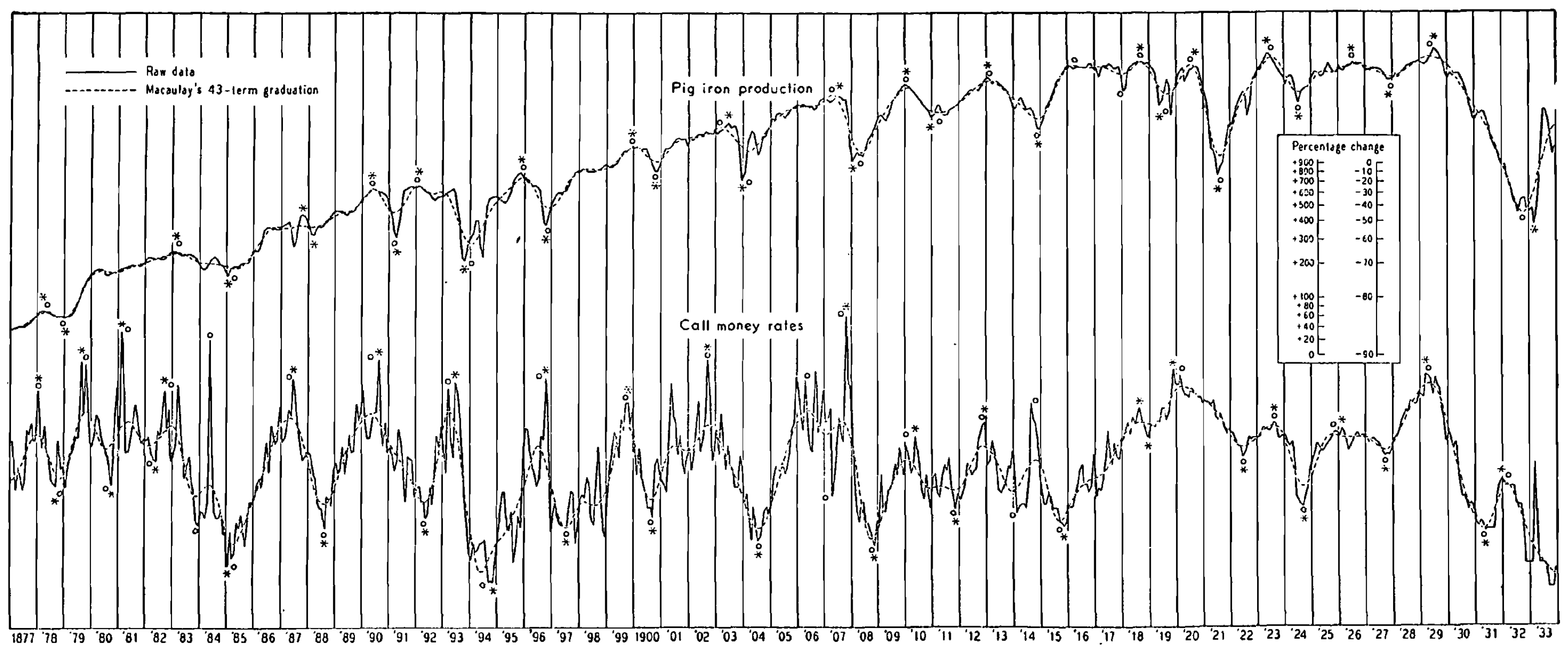
There is no difference between data called 'raw' in this chapter and 'unadjusted' in the preceding chapter.

CHART 43
Pig Iron Production and Call Money Rates, United States, 1877–1933
Raw Data and Macaulay's Graduation

CHART 43

Pig Iron Production and Call Money Rates, United States, 1877–1933

Raw Data and Macaulay's Graduation



* Identifies dates of peaks and troughs of specific cycles in raw data.
o Identifies dates of peaks and troughs of specific cycles in Macaulay's graduation.

Adjusted for seasonal variations. For sources of data, see text, note 1. The graduation of pig iron production in the first 21 months is based on estimated monthly figures.

TABLE 105
Full List of Rises and Declines in Monthly Smoothed Data
Pig Iron Production, United States, 1878–1932

Date of		Average daily production (thous. long tons) during month of		Duration in months from	
Peak	Trough	Peak	Trough	Trough on preceding line to peak	Peak to trough
*May 1878	*Dec. 1878	5.7	5.3	..	7
May 1880	Oct. 1880	9.3	9.2	17	5
*Mar. 1883	*Apr. 1885	11.6	9.5	29	25
Oct. 1886	Mar. 1887	15.7	15.5	18	5
Sep. 1887	Feb. 1888	15.9	15.8	6	5
*May 1890	*Mar. 1891	24.6	18.6	27	10
*Jan. 1892	July 1892	25.6	23.3	10	6
Nov. 1892	*Jan. 1894	24.0	12.7	4	14
*Dec. 1895	*Nov. 1896	28.7	19.6	23	11
May 1898	Sep. 1898	31.5	31.3	18	4
*Dec. 1899	*Oct. 1900	41.3	34.5	15	10
*Feb. 1903	*Mar. 1904	52.0	38.9	28	13
*Apr. 1907	*May 1908	77.1	38.4	37	13
*Jan. 1910	*Apr. 1911	84.0	61.3	20	15
*Feb. 1913	*Nov. 1914	90.7	57.0	22	21
*Apr. 1916	Oct. 1916	108.2	105.5	17	6
Mar. 1917	*Dec. 1917	106.8	98.0	5	9
*Sep. 1918	*Aug. 1919	112.8	76.9	9	11
*July 1920	*Aug. 1921	109.8	36.3	11	13
*July 1923	*July 1924	117.7	78.1	23	12
July 1925	Oct. 1925	100.0	99.1	12	3
*July 1926	*Dec. 1927	108.7	94.3	9	17
*May 1929	*Oct. 1932	119.2	18.1	17	41

*Indicates specific-cycle turns recognized in the smoothed data. Concerning the discrepancies between the specific cycles in the raw and smoothed data, see text.

we should be putting into the same class minor ripples whose very existence is unknown to history, with substantial waves for which there is clear historical evidence. To keep to the aims of this investigation, it is necessary to apply the criteria for identifying specific cycles, described in Chapter 4, to the smoothed data just as to the raw data.

The lists of cycles drawn up on this plan for the two sets of data show occasional discrepancies. In railroad stock prices the two lists agree perfectly. In pig iron production the decline in the raw data from October 1887 to March 1888, which we consider a cyclical movement, is virtually ironed out in the smoothed data. The like is true of the decline from August to December 1918 in both bank clearings and call money rates. On the other hand, the sharp drop in pig iron production in the winter of 1917–18, resulting from the extreme cold of that season and best considered as a random movement, is converted by Macaulay's formula into a wave-like fluctuation which we must treat as a specific cycle. In call money rates three 'extra' cycles turn up in the smoothed data, and at least two of them are spurious (Chart 43). The sharp rise in call money rates from July to August 1914, induced by financial panic upon the outbreak of war, and the prompt decline to a level below that of July, are converted by Macaulay's formula into a substantial wave lasting twenty-one months. Again, the sharp isolated peak in 1884, the base of which extends merely from April to July, becomes a seventeen-month wave in the smoothed data. The interval between the peaks in 1906 and 1907 is exactly fifteen months in the smoothed data, and we have marked it as a specific cycle; the corresponding movement in the raw data is too short to qualify as a cycle.[2]

From this summary it is plain that smoothing by no means offers a solution of the difficulties encountered in identifying specific cycles. Sometimes 'marginal' movements appear in smoothed data just as in the raw data, and one is forced to ponder whether to recognize them as cyclical. Sometimes smoothing spreads a high random peak or deep random trough, with a narrow time base, into a wave broad enough to qualify as a specific cycle under our rules. Sometimes, besides eliminating erratic movements, smoothing obliterates a genuine cyclical movement. It should be noted, however, that the three cyclical movements virtually ironed out by smoothing in our sample series are mild even in the raw data, and might not have been recognized as cyclical, were it not our practice to relax rules in treating dubious movements of closely conforming series.

Furthermore, the comparison of cycle lists in the raw and smoothed data is biased in one important respect against smoothing. The specific cycles in the raw data were originally selected with the aid of simple moving averages passed through doubtful portions of the series. The specific cycles in the smoothed data, on the other hand, were selected without reference to the movements of the raw data. As a result, we are able to demonstrate some significant shortcomings of smoothed data when relied on exclusively, but we are not in a position to show the power of smoothing to expose cyclical movements that are hidden from view by erratic fluctuations. To avoid embarrassment arising from this bias, we disregard in the following sections the effects of smoothing on the lists of specific cycles; that is, the effects of smoothing on cyclical measures are analyzed apart from their effects on the determination of specific cycles. This procedure requires that the lists of cycles in the raw and smoothed data be identical.[3] The simplest plan is to treat as 'cyclical' the three faint movements in iron production, bank clearings, and call money rates that correspond in time to cyclical declines in the raw data, and to drop the 'cycles' in the smoothed data, three in call money rates and one in iron production, that have no counterpart in the raw data. Thus put, the materials are in shape for isolating one set of effects of smoothing on

[2] The interval between the peaks in 1901 and 1902 is less than fifteen months in the smoothed data on call money rates; likewise the interval between the troughs in 1897 and 1898.

[3] For a full list of the specific cycles in the raw data and cycle-by-cycle measures, see Appendix Table B1.

cyclical measures. In Section X we shall return briefly to the effects of smoothing *via* changes in the lists of specific cycles.

III Timing and Duration of Specific Cycles

In the preceding chapter we found that the removal of trends shifted less than one-third of the turning points of specific cycles. Table 106 shows that smoothing shifts over four-fifths of the turning points. Except in call money rates, the smoothed data show some tendency to lead at the peaks and to lag at the troughs of the raw data.[4] But most of the shifts are small. Nearly half the turns in the smoothed data come within one month of the turns in the raw data. The discrepancy exceeds four months in only 9, and seven months in only 2, out of 149 instances.

TABLE 106
Shifts in the Timing of Specific Cycles Produced by Smoothing
Four American Series

Lead (−) or lag (+) of smoothed data at turn of raw (mos.)	Number of leads or lags in									
	Deflated clearings (1878–1933)		Pig iron production (1878–1933)		Railroad stock prices (1857–1932)		Call money rates (1857–1932)		Four series	
	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
Below −7	..	..	..	..	..	..	1[a]	..	1[a]	..
−7 to −5	1	..	..	1	1	..	..	1	2	2
−4 to −2	3	4	4	..	4	3	8	6	19	13
−1 to +1	8	6	10	9	8	5	10	14	36	34
+2 to +4	2	5	2	6	3	11	6	3	13	25
+5 to +7	..	1	..	..	2	..	..	..	2	1
Over +7	1[b]	..	..	..	..	..	..	..	1[b]	..
Total	15	16	16	16	18	19	25	24	74	75
Leads	8	5	6	5	7	4	14	13	35	27
Coincidences	4	..	5	2	5	1	4	6	18	9
Lags	3	11	5	9	6	14	7	5	21	39
Total	15	16	16	16	18	19	25	24	74	75
Av. lead (−) or lag (+) in months	*−0.2*	*+1.1*	*−0.4*	*+0.9*	*+0.1*	*+1.2*	*−1.2*	*−0.7*	*−0.5*	*+0.5*

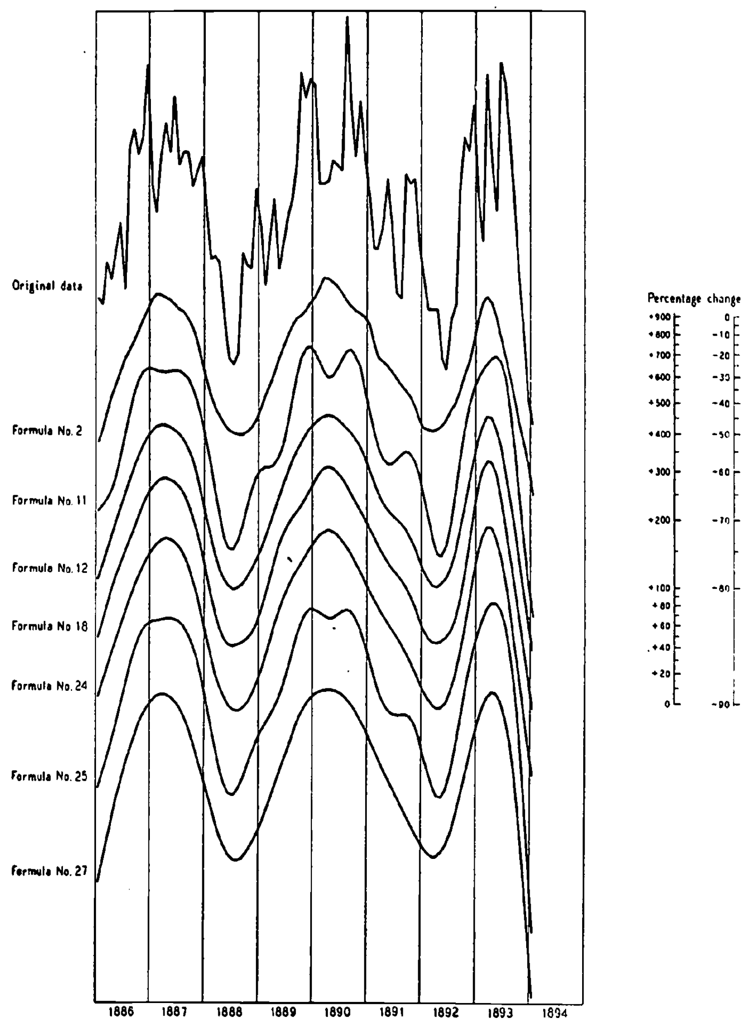
The entries in this table run from the first to the last corresponding turn in the raw and smoothed data, whether peak or trough. In a few instances they cover a longer period than the measures in Table 112.
[a] This value is −17 months.
[b] This value is +9 months.

How are the differences in cyclical timing between the raw and smoothed data to be explained? The simplest explanation is that the smoothed curve represents fairly the cyclical movements, and that the discrepancies between the cyclical turns of the raw and smoothed data are therefore due mainly, if not entirely, to erratic movements in the raw data. This explanation, doubtless, is valid in some instances. We know

[4] This feature of the results is explained later: see Tables 110-112, and pp. 324-6.

that the turning points of specific cycles in the raw data are often blurred by erratic movements and that the process of dating cyclical turns sometimes follows a vacillating course. We therefore suspect an appreciable erratic factor in the turning points that we finally select. It is possible, however, that smoothing frequently distorts the underlying cyclical fluctuation, and that turns in the smoothed data miss the 'true' cyclical turns by margins as wide or wider than do the turns in the raw data. For,

CHART 44
Seven Smoothing Formulas
Applied to Call Money Rates on New York Stock Exchange
1886 – 1894



See Table 107, especially the notes; also Table 118.

strictly speaking, smoothing does not eliminate erratic movements; it merely redistributes the values of the original data in a manner predetermined by the particular formula used. If this formula is poorly suited to some or all of the cycles in a series, smoothing may misrepresent the cyclical movements. Just as there is uncertainty in dating cyclical turns from raw data, so there is uncertainty in choosing the method of smoothing. Chart 44 and Table 107 illustrate, for a small range of smoothing formulas, the dependence of turning points on the particular formula selected. Formula No. 24 is Macaulay's 43-term graduation.[5]

The pitfalls of smoothing can be seen most clearly in artificially designed series. The results of an experiment with such data are presented in Chart 45 and Table 108. To gauge the effects of smoothing exactly, we assumed independent knowledge of the true cyclical curve. The second column of the table shows the dates assigned to the peaks and troughs of the pure cycles. The succeeding columns set against this standard the timing of artificial series consisting of an assumed random component besides the assumed cycles. Series S′ is a sine curve with a period of 44 months, to which a series of random numbers (drawn from a distribution with a mean of zero) has been added. Series S″ is the same as S′, except that the random numbers were doubled before being added. Series C′ and C″ are like S′ and S″, respectively, except that they are based on a cusp-shaped cyclical curve instead of a sine curve. The like applies to series T′ and T″, whose underlying cyclical component is triangular. The cusp-shaped curve (series C), the triangular curve (series T), and the sine curve (series S) reach peaks and troughs on exactly the same dates; so that the dates in the second column of Table 108 apply to all three sets of pure cycles.[6]

Several results stand out in the table. As is to be expected, the larger the amplitude of the random component relatively to the pure cycles, the more does the timing of the series tend to deviate from that of the pure cycles. This result is characteristic not only of the raw data, but also of the data smoothed by Macaulay's formula. Smoothing tends to improve the timing of the two series built up from a sine curve, and of one series built up from triangular cycles. But it fails to improve, indeed it worsens, the dating of the cyclical turns in the three remaining series—one built up from a triangular and two from a cusp-shaped cyclical curve.

[5] The period covered by this experiment seems to be one in which the different methods are especially likely to harmonize. See Chart 43.

The shape of a graduation depends not only on the formula used, but also on the form of the data to which the formula is applied. If the original figures, instead of the logarithms, were smoothed, the results would be very different in a series like call money rates.

[6] Series S, T and C have different cyclical shapes, but their period and amplitude are exactly the same. The period is 44 months. The range is from a maximum of 1200 to a minimum of 800. For the derivation of the series, see the notes on Chart 45 in Appendix C.

TABLE 107

Effects of Fourteen Smoothing Formulas on the Timing of Specific Cycles
Call Money Rates on New York Stock Exchange, 1887–1893

Form of data	Peak	Trough	Peak	Trough	Peak
Raw[a]	June 1887	Aug. 1888	Aug. 1890	May 1892	June 1893
Smoothed by formula[b]					
No. 2	Feb. 1887	Sep. 1888	Mar. 1890	Mar. 1892	Mar. 1893
No. 11	Jan. 1887	July 1888	Dec. 1889	May 1892	May 1893
No. 12	Mar. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 13	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 14	Apr. 1887	Aug. 1888	Apr. 1890	Mar. 1892	Mar. 1893
No. 18	Apr. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 19	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 20	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 21	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 22	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 23	Apr. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 24	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 25	May 1887	July 1888	Dec. 1889	May 1892	May 1893
No. 27	Mar. 1887	Aug. 1888	Apr. 1890	Mar. 1892	Apr. 1893

[a] As usual, adjusted for seasonal variations. Chart 44 shows the data before seasonal adjustment.

[b] The smoothing formulas were applied to the logarithms of the original data, unadjusted for seasonal. The graduations are all taken from Appendix VIII of Macaulay's *Smoothing of Time Series*, where the different formulas are identified. The number of terms covered by the formulas ranges from 13 (No. 2) to the entire body of data (No. 25, 27). But seven of the fourteen (No. 18 through No. 24) are fifth-degree parabolics, covering from 35 to 45 terms.

TABLE 108

Timing of Raw and Smoothed Data of Six Artificial Series
Compared with Timing of the Underlying Pure Cycles

Line	Turning dates of pure cyclical curve	Nature of turn[a]	Lead (−) or lag (+) at turn of pure cyclical curve (mos.)											
			Series S′		Series S″		Series T′		Series T″		Series C′		Series C″	
			Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
1	Dec. 1892	T	0	..	0	..	0	..	0	..	0	..	0	..
2	Oct. 1894	P	+3	+1	+3	+2	+3	+1	+3	+1	+3	+1	+3	+2
3	Aug. 1896	T	−1	0	−1	−1	−1	0	−1	−1	0	0	0	−1
4	June 1898	P	+1	0	+5	+1	+1	0	+1	+1	0	0	0	+1
5	Apr. 1900	T	0	0	+2	0	0	0	0	0	0	0	0	0
6	Feb. 1902	P	0	0	0	−1	0	0	0	−1	0	0	0	−1
7	Dec. 1903	T	+1	−1	+1	−3	+1	−1	+1	−3	+1	−2	+1	−3
8	Oct. 1905	P	−1	+1	−1	+2	−1	+1	−1	+2	0	+1	−1	+2
9	Aug. 1907	T	+3	−1	+3	−2	−1	−1	+3	−2	−1	−2	+3	−3
10	June 1909	P	−1	−2	−3	−4	−1	−2	−1	−4	−1	−2	−1	−5
11	Apr. 1911	T	+3	+1	+3	+1	+3	+1	+3	+1	0	+1	+3	+1
12	Feb. 1913	P	0	−1	+4	−3	0	0	0	−3	0	0	0	−5
13	Dec. 1914	T	+1	..	+1	..	+1	..	+1	..	+1	..	+1	..
Average, signs ignored														
Lines 1–13			1.2	..	2.1	..	1.0	..	1.2	..	0.5	..	1.0	..
Lines 2–12			1.3	0.7	2.4	1.8	1.1	0.6	1.3	1.7	0.5	0.8	1.1	2.2
Average, signs respected														
Lines 1–13			+0.7	..	+1.3	..	+0.4	..	+0.7	..	+0.2	..	+0.7	..
Lines 2–12			+0.7	−0.2	+1.5	−0.7	+0.4	−0.1	+0.7	−0.8	+0.2	−0.3	+0.7	−1.1

Smoothing by Macaulay's 43-term graduation involves the loss of 21 terms at each end; hence there are no smoothed entries in lines 1 and 13, and their average is confined to lines 2-12. See Chart 45.

[a] T stands for the trough of specific cycles, P for the peak.

It is plain from Chart 45 that besides wiping out the erratic movements, smoothing twists in some measure the cyclical movements. The degree of twist depends partly on the size of the erratic movements, partly on the shape of the cyclical movements. Since Macaulay's graduation is designed to follow the outlines of a sine curve or any other with more or less rounded tops and bottoms, it is most satisfactory in the series derived from a sine curve.[7] But it does not cope adequately with sharp cusps[8] such as characterize series C′ and C″. In both, the graduation is much too low in the immediate vicinity of the true peaks and, usually, much too high in adjacent months. It is likewise too high in the immediate vicinity of the true troughs and too low in adjacent months. Since the tops and bottoms become gently rounded in the graduation, the turning dates can easily deviate from the turns of the underlying cusp cycles. In the raw data, on the other hand, the random component is likely to be especially small relatively to the cusped cyclical component precisely on the dates when the latter reaches a maximum or minimum; hence the timing of the raw data tends to match closely the timing of the pure cycles. Of course, this result will tend to be blurred as the amplitude of the random component is increased relatively to the cusped component. But there can be no escape from the conclusion that a cyclical graduation may seem excellent to the eye, perhaps even satisfy formal tests applied to deviations of the raw data from it, and yet seriously misrepresent the true cyclical movements.

While these experiments demonstrate that the results of smoothing may be surrounded by wide margins of error, they do not argue against smoothing as such. Given the assumptions underlying the artificial series, smoothing is valid in principle, though the right formula may be hard to discover and may need to be varied from series to series. And so it would be also in the case of historical series, if we could assume that they correspond to our experimental model; that is, that they record cyclical effects, or combined secular and cyclical effects, on which random effects are superposed. There is rarely any warrant for such an assumption. In the experimental model cyclical and random forces are distinct; their individual effects can be distinguished precisely, month by month, phase by phase, cycle by cycle. In historical series the effects of cyclical and random forces cannot be separated even over the course of a full cycle. Random factors constantly play on business at large and on each of its many branches, and their effects register in different ways under different circumstances. Some series on prices and central-bank discount rates show step movements of extended duration, free of short-term oscillations. Broad series on employment frequently move in the same direction for many months. Call money rates, on the other hand, show frequent re-

[7] See below, p. 330, and note 24.

[8] This difficulty is by no means peculiar to Macaulay's graduation. See below, especially Sec. IV and VI.

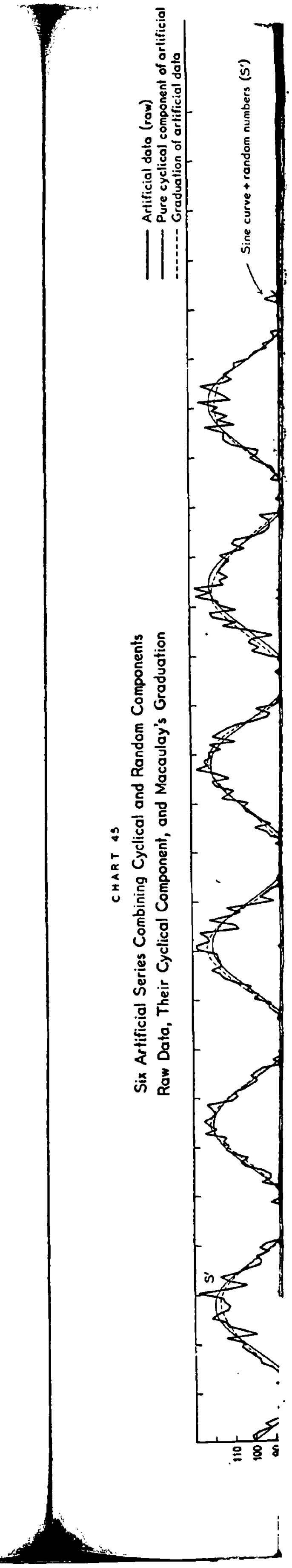
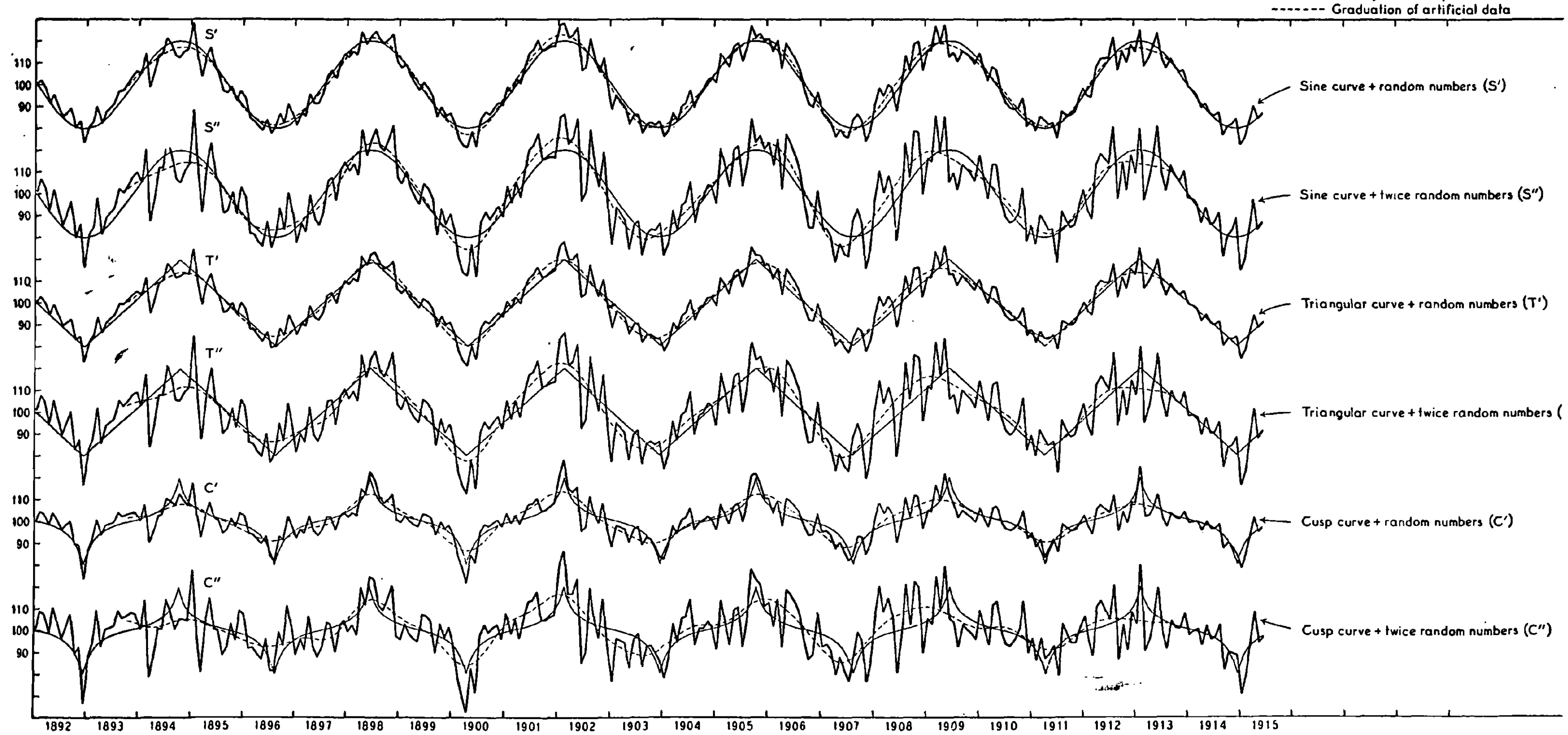


CHART 45

Six Artificial Series Combining Cyclical and Random Components
Raw Data, Their Cyclical Component, and Macaulay's Graduation



For derivation of the series and other explanations, see text and Appendix C.

versals of direction which may be mainly the effects of random happenings; though we know, historically, that most of the high narrow peaks in this series mark cyclical, not random, effects. Series like bank clearings and iron production may seem to approximate our experimental model more closely. Yet we may be sure that random factors have played an important part in shaping their specific cycles—now prolonging an expansion, now intensifying as well as prolonging it, now abbreviating a contraction, and so on, in endless variety. 'Erratic movements' therefore cannot be identified with 'effects of random forces' as in our experimental model. The 'erratic movements' that stand out clearly in time series and are obliterated by smoothing are simply the short-term oscillations, other than seasonal variations, that play, so to speak, on the back of specific cycles.[9] Not clearly revealed by the data and not smoothed out by the usual formulas are other effects of random forces. Indeed, the general contours of cyclical movements themselves may have been shaped, in part, by the same forces that produced the short-term oscillations.

These observations instill caution in the use of smoothing devices. If the turning zone of the specific cycles in a series is crisscrossed by erratic movements, it is likely that the historical limits of the expansions and contractions of the specific cycles will be approximated better from smoothed than from raw data. On the other hand, if the cyclical turns in the raw data are naturally 'smooth'—that is, if erratic movements seem absent in the vicinity of cyclical turns[10]—smoothing is an indulgence, the price of which may be distortion of historical cycles. Table 109 presents several illustrations.[11] Even if erratic movements make their influence felt in the vicinity of cyclical turns, smoothing is of doubtful advantage so long as expansion culminates in a clearly defined peak and contraction in a clearly defined trough. For when a cyclical phase happens to be brief but large, smoothing is likely to spread it out, predating the peak and postdating the trough in the case of contractions, predating the trough and postdating the peak in the case of expansions.[12] The strength

[9] See pp. 57-8, 87-90, concerning difficulties sometimes encountered in distinguishing these short-term variations, here lumped together as 'erratic movements', from specific or business cycles. The interrelations of the short-term fluctuations, other than seasonal variations or specific cycles, in different economic activities are now being investigated by one of our colleagues, Ruth P. Mack.

[10] Here we include curves having cusps, which some readers may not consider as being 'smooth'.

[11] To obtain the list of 'smooth turns' in Table 109, we first recorded every trough (peak) that is preceded by at least a five-month decline (rise) and followed by at least a five-month rise (decline). Of course, this criterion of a 'smooth' turn sometimes proves inadequate. For example, if a specific-cycle peak is preceded by a six-month rise and followed by a six-month decline, which in turn is followed by a two-month rise culminating in a value not much lower than the peak, we can hardly say that the cyclical turn is 'smooth'. For reasons of this nature, five instances conforming to the five-month rule are omitted in Table 109. In view of the precautions taken to omit 'double' tops or bottoms, the five-month rule is severe; that is, the table understates the number of 'smooth turns'.

[12] If a cyclical phase is brief and mild, smoothing may spread it so much that it disappears—virtually if not entirely. But a movement that is both very brief and very small is not likely to be recognized as cyclical. As a rule, therefore, brief cyclical phases have longer counterparts in smoothed data; although brief phases of small amplitude may have shorter counterparts.

TABLE 109

Effect of Smoothing on Cyclical Turns That Are 'Smooth' in Raw Data

Three American Series

Series and 'smooth' specific-cycle turn in raw data	Nature of turn[a]	No. of months raw data move continuously in same direction		Date of corresponding turn in smoothed data	Lead (−) or lag (+) of smoothed data at turn of raw (mos.)
		Before turn	After turn		
PIG IRON PRODUCTION					
Apr. 1891	T	7	6	Mar. 1891	−1
Nov. 1895	P	7	5	Dec. 1895	+1
Oct. 1896	T	5	5	Nov. 1896	+1
Jan. 1910	P	9	11	Jan. 1910	0
July 1921	T	10	11	Aug. 1921	+1
May 1923	P	9	8	July 1923	+2
RAILROAD STOCK PRICES					
Oct. 1857	T	10	5	Jan. 1858	+3
CALL MONEY RATES					
Sep. 1924	T	5	6	Sep. 1924	0

Based on our four series for the periods shown in Table 112. There are no entries for deflated clearings, since no 'smooth' turns occur in this series in the sense of our rules in note 11.

[a] T stands for the trough of specific cycles, P for the peak.

of these tendencies when Macaulay's formula is used is displayed in Table 110.

An historical illustration may help the reader see more vividly the difficulties that are sometimes caused by smoothing. The contraction of 1907–08 was short but steep, like the contraction of 1937–38 of more recent memory. Signs of recession appeared late in 1906, when stock prices began to sag, trading in securities fell off, and bank clearings in New York turned down. General business continued to expand, however, until May 1907 according to our reference chronology. Whatever decline occurred during the summer was small. But in October a succession of bank closures in New York precipitated a financial panic that spread rapidly to the interior.[13] For several months following, acute depression in industry and trade was general. Table 111 shows the timing of the contraction, as manifested in ten series—the four on which the bulk of the tests of this chapter are based, and six additional series to which Macaulay applied his smoothing formula. The course of the crisis can be followed easily in the raw data, but not in the smoothed data. Macaulay's graduation makes the peak in call money rates come in August 1907,[14] instead of October when panic broke out in the New York financial district. The

[13] See Wesley C. Mitchell, *Business Cycles* (1913), pp. 514-38; or the reprint *Business Cycles and Their Causes*, pp. 74-107.

[14] A still higher peak occurs in May 1906. In Table 111 this peak is ignored, whereas in other comparisons of this chapter the 'minor' peak of Aug. 1907 is ignored. We might have recognized both peaks in our standard analysis; but that course was ruled out by the convention of one-to-one correspondence laid down in Sec. II. The high point in the raw data in Sept. 1906, which corresponds to the high of May 1906 in the smoothed data, could not be recognized as a cyclical peak in the raw series, because it is only thirteen months removed from the higher peak in Oct. 1907. See Chart 43.

TABLE 110
Timing and Duration of 'Brief' Specific-cycle Phases
Four American Series, Raw and Smoothed

Series and specific-cycle phase in raw data[a]	Nature of phase[b]	Number of months lead (−) or lag (+) of smoothed data at		Duration of phase in months	
		Initial turn of raw data	Terminal turn of raw data	Raw data	Smoothed data
DEFLATED CLEARINGS					
June 1881 – Jan. 1882	C	+2	+4	7	9
June 1887 – Mar. 1888	C	0	−1	9	8
Sep. 1890 – Mar. 1891	C	−1	+1	6	8
Feb. 1893 – Aug. 1893	C	−3	+4	6	13
Oct. 1907 – Dec. 1907	C	−6	+4	2	12
Mar. 1910 – Oct. 1910	C	−1	+1	7	9
Aug. 1918 – Dec. 1918	C	−1	−2	4	3
May 1923 – Sep. 1923	C	0	+5	4	9
PIG IRON PRODUCTION					
Oct. 1887 – Mar. 1888	C	−1	−1	5	5
June 1903 – Dec. 1903	C	−4	+3	6	13
July 1907 – Jan. 1908	C	−3	+4	6	13
Sep. 1918 – May 1919	C	0	+3	8	11
RAILROAD STOCK PRICES					
Oct. 1857 – Mar. 1858	E	+3	+5	5	7
Sep. 1860 – May 1861	C	0	+3	8	11
May 1890 – Dec. 1890	C	−2	+3	7	12
Mar. 1895 – Sep. 1895	E	−4	0	6	10
Apr. 1900 – Sep. 1900	C	−7	−4	5	8
CALL MONEY RATES					
Jan. 1876 – Sep. 1876	C	−1	+3	8	12
Jan. 1878 – Sep. 1878	C	0	+2	8	10
Oct. 1880 – Feb. 1881	E	−3	+3	4	10
May 1882 – Sep. 1882	E	−3	+3	4	10
Aug. 1918 – Dec. 1918	C	0	+1	4	5

[a] Includes every expansion or contraction lasting 8 months or less in either the raw or smoothed data during the periods listed in Table 112.
[b] C stands for contraction, E for expansion.

raw data on clearings outside New York show a peak in October and a trough in December 1907; Macaulay's graduation shifts the peak back six months and the trough forward four months. But the raw data tell the tale that is cyclically significant: outside clearings slumped sharply as banks generally suspended or limited cash payments, and rose promptly at the end of the year when normal banking operations were resumed.

When we leave single cycles, where smoothing sometimes improves and sometimes worsens the dating of cyclical turns, and examine averages, it appears that the net effect of smoothing is negligible. Table 106 shows that in four instances the smoothed data lag on the average behind the raw data; in another four instances the smoothed data lead. The maximum average difference at peaks or troughs is only 1.2 months. It thus appears that smoothing influences the average timing of specific cycles less than the removal of secular trends, despite the fact that the former affects single cycles more frequently than the latter. Of course the reason is that the effects of removing trends are cumulative (except when the trend changes direction), while the effects of smoothing are largely self-canceling.

But there is a slight twist to the cancellation. As Table 106 indicates, the smoothed data have some tendency to lead at peaks and lag at troughs. Table 112 shows that in each series smoothing reduces somewhat the average duration of expansions and increases the average duration of con-

TABLE 111

Chronology of Specific-cycle Turns in the Contraction of 1907–1908
Ten American Series, Raw and Smoothed

Series	Specific-cycle turns in raw data		Specific-cycle turns in smoothed data		Lead (−) or lag (+) of smoothed data at turn of raw (mos.)	
	Peak	Trough	Peak	Trough	Peak	Trough
Bank clearings, N.Y.C.	Aug. 1906	Dec. 1907	July 1906	Feb. 1908	−1	+2
Railroad stock prices	Sep. 1906	Nov. 1907	Aug. 1906	Dec. 1907	−1	+1
Pig iron production	July 1907	Jan. 1908	Apr. 1907	May 1908	−3	+4
Call money rates, N.Y. Stock Ex.	Oct. 1907	Nov. 1908	Aug. 1907[a]	Oct. 1908	−2	−1
Bank clearings, outside N.Y.C.	Oct. 1907	Dec. 1907	Apr. 1907	Apr. 1908	−6	+4
Deflated clearings, outside N.Y.C.	Oct. 1907	Dec. 1907	Apr. 1907	Apr. 1908	−6	+4
Wholesale prices	Oct. 1907	Feb. 1908	June 1907	June 1908	−4	+4
90-day money rates, stock ex. loans	Nov. 1907	Sep. 1908	Aug. 1907	Nov. 1908	−3	+2
Railroad bond yields	Nov. 1907	Feb. 1909	Dec. 1907	Mar. 1909	+1	+1
Commercial paper rates, N.Y.C.	Dec. 1907	July 1909	Aug. 1907	Jan. 1909	−4	−6

The smoothed data are all given in Appendix A of Macaulay's *Interest Rates, Bond Yields, and Stock Prices*. For the sources of the original data, see our Appendix C (Table 21). The specific cycles of the 'raw data' are dated from seasonally adjusted figures, except when there seemed to be no seasonal movement (railroad bond yields, railroad stock prices, wholesale prices).

Macaulay's smoothed series on bank clearings outside N.Y.City does not correspond to ours. However, the cyclical turns in 1907 in the 'raw' series underlying Macaulay's smoothed series agree with the turns in the 'raw' series that we have used.

[a] See note 14.

TABLE 112

Average Duration of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	Period covered[a]	No. of specific cycles[b]	Average duration in months			Average per cent of duration of specific cycles	
			Expansion	Contraction	Full cycle	Expansion	Contraction
DEFLATED CLEARINGS							
Raw	1878–1933	15	32.6	11.4	44.0	75	25
Smoothed	1878–1932	15	31.0	12.5	43.5	71	29
PIG IRON PRODUCTION							
Raw	1879–1933	15	28.8	14.5	43.3	67	33
Smoothed	1878–1932	15	27.0	16.1	43.1	62	38
RAILROAD STOCK PRICES							
Raw	1857–1932	18	28.8	21.0	49.8	56	44
Smoothed	1858–1932	18	27.7	22.0	49.7	54	46
CALL MONEY RATES							
Raw	1858–1931	23	19.9	18.0	37.9	52	48
Smoothed	1858–1931	23	19.2	18.7	37.9	51	49

[a] Unlike Table 106, the periods covered by this table start and end with a trough.
[b] See Sec. II.

tractions. The differences between the average durations are small, as they must be in view of the small effects of smoothing on average timing. Nevertheless, they are of theoretical interest. As we have seen, when a cyclical expansion or contraction is brief, smoothing tends to spread it over a longer period. In three of our series brief contractions are more numerous than brief expansions.[15] On the average, therefore, smoothing tends to lengthen the contractions and to shorten the expansions. In call money rates, where the number of brief expansions and contractions is practically the same, it also happens that smoothing has the smallest influence on the average duration of cyclical phases.

The average duration of full specific cycles is virtually the same in the raw and smoothed data we are using. The result could not be otherwise; for in railroad stock prices the number of specific cycles is the same in the two records, while in the other three series we have forced the two sets of cycles into correspondence.[16]

IV Amplitude of Specific Cycles

If we had a method of dating accurately the crest of a cyclical upswing, we could claim that whatever erratic thrust occurs on that date is as likely to lower the crest as to raise it. But when we select the highest point of data that are composites of both cyclical and erratic movements, that point is more likely to be above than below the cyclical crest. *Mutatis mutandis,* these remarks apply also to the lowest points of cyclical downswings. Since we seek to mark off specific cycles by selecting the highest and lowest months, there is danger that the amplitudes of the cyclical fluctuations would be exaggerated if we measured them by the differences between specific-cycle relatives at the peaks and troughs. To guard against this danger we measure amplitudes, not from the cycle relatives of the highest and lowest months, but from averages covering three months centered on the peaks and troughs.

This protective device is obviously rough. The relations in size among the raw figures in the vicinity of a cyclical turn differ from case to case, and we therefore cannot expect that the use of three-month averages to represent cyclical peaks and troughs will yield a good measure of the cyclical amplitude in every instance. The most that we can expect is that it will do so on the average, and then only if used with circumspection. The method breaks down (1) if a cyclical expansion or contraction is very short, (2) if the top or bottom of a cycle is horizontal, (3) if the month selected as the peak is preceded or followed by an extremely low value, or if the month of trough is preceded or followed by an extremely high value. In these

15 For partial evidence, see Tables 110 and 117.

16 See Sec. X.

TABLE 113
Average Amplitude of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	No. of months in standing at turns	Average standing in specific-cycle relatives at: Initial trough	Peak	Terminal trough	Average amplitude in specific-cycle relatives: Rise	Fall	Rise & fall
DEFLATED CLEARINGS								
Raw	15	1	83.4	116.0	96.2	32.6	19.8	52.4
Raw	15	3[b]	85.7	112.6	99.2	26.9	13.4	40.2
Smoothed	15	1	86.8	110.8	100.2	24.0	10.6	34.6
PIG IRON PRODUCTION								
Raw	15	1	64.0	131.4	70.7	67.3	60.6	128.0
Raw	15	3	67.3	129.3	74.6	62.1	54.8	116.8
Smoothed	15	1	73.0	126.5	81.6	53.4	44.8	98.3
RAILROAD STOCK PRICES								
Raw	18	1	80.7	120.1	84.8	39.4	35.3	74.7
Raw	18	3	82.8	118.3	86.6	35.6	31.8	67.3
Smoothed	18	1	84.5	116.0	88.0	31.6	28.0	59.6
CALL MONEY RATES								
Raw	23	1	54.6	254.5	54.5	199.8	200.0	399.8
Raw	23	3	62.1	178.0	61.9	115.9	116.1	232.0
Smoothed	23	1	68.2	146.8	68.1	78.6	78.7	157.2
			Relatives of averages based on 3-month standings of raw data					
DEFLATED CLEARINGS								
Raw	15	1	97	103	97	121	148	130
Smoothed	15	1	101	98	101	89	79	86
PIG IRON PRODUCTION								
Raw	15	1	95	102	95	108	111	110
Smoothed	15	1	108	98	109	86	82	84
RAILROAD STOCK PRICES								
Raw	18	1	97	102	98	111	111	111
Smoothed	18	1	102	98	102	89	88	89
CALL MONEY RATES								
Raw	23	1	88	143	88	172	172	172
Smoothed	23	1	110	82	110	68	68	68

See Table 112 for the periods covered.
[a] See Sec. II.
[b] In this series there are several exceptions to the 3-month rule; see Appendix Table B1.

three types of cases we make exceptions from the three-month rule.[17] Yet the exceptions cover only the most glaring defects of the method, and are themselves handled in ways that may raise as many doubts as does the three-month rule.

All these difficulties are swept away by smoothing. Since erratic fluctuations have already been removed in the process of smoothing, amplitudes can be measured directly from the highest and lowest points of the cycles. Measures of cyclical amplitude derived in this way have a simplicity and elegance to which our standard measures can lay no claim.

The use of three-month averages to represent peaks and troughs involves, of course, a form of smoothing. The smoothing effect is substantial, as Table 113 demonstrates. Yet it is smaller on the average than

[17] See p. 132.

the effect produced by Macaulay's formula. The differences between average amplitudes based on three-month standings of the raw data and one-month standings of the smoothed data are sometimes uncomfortably large, especially in call money rates. And since smoothing has a marked effect on average results, it is bound at times to produce still larger effects in single cycles. Table 114 shows the cycle-by-cycle measures for pig iron production. It is plain that smoothing not only tends to reduce the measures of amplitude, but that it does this rather unevenly. Table 115 demonstrates that the uneven effects of smoothing do not disappear completely even when the amplitudes are ranked.

TABLE 114

Amplitude of Specific Cycles in Raw and Smoothed Data
Pig Iron Production, United States, 1878–1933

Form of data	Dates of specific cycles: Trough – Peak – Trough	Standing in specific-cycle relatives at: Initial trough	Peak	Terminal trough	Amplitude in specific-cycle relatives: Rise	Fall	Rise & fall	Amplitude of smoothed data as relative of raw amplitude: Rise	Fall	Rise & fall
Raw	Jan. 79 – Feb. 83 – Jan. 85	55.7	119.7	95.1	64.0	24.6	88.6			
Sm.	Dec. 78 – Mar. 83 – Apr. 85	55.3	120.5	99.5	65.2	21.0	86.2	102	85	97
Raw	Jan. 85 – Oct. 87 – Mar. 88	67.0	131.8	106.3	64.8	25.5	90.3			
Sm.	Apr. 85 – Sep. 87 – Feb. 88	68.6	114.1	113.2	45.5	0.9	46.4	70	4	51
Raw	Mar. 88 – May 90 – Apr. 91	74.0	124.2	76.8	50.2	47.4	97.6			
Sm.	Feb. 88 – May 90 – Mar. 91	80.4	125.2	94.7	44.8	30.5	75.3	89	64	77
Raw	Apr. 91 – Feb. 92 – Oct. 93	67.5	112.9	49.2	45.4	63.7	109.1			
Sm.	Mar. 91 – Jan. 92 – Jan. 94	87.9	121.0	60.0	33.1	61.0	94.1	73	96	86
Raw	Oct. 93 – Nov. 95 – Oct. 96	51.5	138.7	77.9	87.2	60.8	148.0			
Sm.	Jan. 94 – Dec. 95 – Nov. 96	57.9	130.8	89.4	72.9	41.4	114.3	84	68	77
Raw	Oct. 96 – Dec. 99 – Oct. 00	52.3	128.3	97.8	76.0	30.5	106.5			
Sm.	Nov. 96 – Dec. 99 – Oct. 00	61.1	128.7	107.5	67.6	21.2	88.8	89	70	83
Raw	Oct. 00 – June 03 – Dec. 03	69.0	118.0	68.0	49.0	50.0	99.0			
Sm.	Oct. 00 – Feb. 03 – Mar. 04	77.2	116.4	87.1	39.2	29.3	68.5	80	59	69
Raw	Dec. 03 – July 07 – Jan. 08	51.1	124.7	62.0	73.6	62.7	136.3			
Sm.	Mar. 04 – Apr. 07 – May 08	65.5	129.8	64.6	64.3	65.2	129.5	87	104	95
Raw	Jan. 08 – Jan. 10 – Dec. 10	59.8	138.4	96.7	78.6	41.7	120.3			
Sm.	May 08 – Jan. 10 – Apr. 11	59.0	129.1	94.2	70.1	34.9	105.0	89	84	87
Raw	Dec. 10 – Jan. 13 – Dec. 14	82.7	127.7	70.9	45.0	56.8	101.8			
Sm.	Apr. 11 – Feb. 13 – Nov. 14	82.6	122.3	76.8	39.7	45.5	85.2	88	80	84
Raw	Dec. 14 – Sep. 18 – May 19	52.5	114.4	74.1	61.9	40.3	102.2			
Sm.	Nov. 14 – Sep. 18 – Aug. 19	59.1	117.0	79.8	57.9	37.2	95.1	94	92	93
Raw	May 19 – Sep. 20 – July 21	90.2	130.9	39.8	40.7	91.1	131.8			
Sm.	Aug. 19 – July 20 – Aug. 21	97.6	139.3	46.1	41.7	93.2	134.9	102	102	102
Raw	July 21 – May 23 – July 24	38.9	147.2	88.6	108.3	58.6	166.9			
Sm.	Aug. 21 – July 23 – July 24	43.1	139.8	92.8	96.7	47.0	143.7	89	80	86
Raw	July 24 – July 26 – Nov. 27	73.5	110.0	91.3	36.5	18.7	55.2			
Sm.	July 24 – July 26 – Dec. 27	79.0	110.0	95.4	31.0	14.6	45.6	85	78	83
Raw	Nov. 27 – July 29 – Mar. 33	123.3	173.3	24.4	50.0	148.9	198.9			
Sm.	Dec. 27 – May 29 – Oct. 32	121.0	152.9	23.2	31.9	129.7	161.6	64	87	81

The amplitudes of the raw data are derived from 3-month averages of specific-cycle relatives centered on the trough and peak dates. The amplitudes of the smoothed data are derived from the specific-cycle relatives at the trough and peak dates.

On a per month basis also the amplitudes are larger on the average in the raw than in the smoothed data (Table 116). But whereas, in proportion to the size of the figures, smoothing affects the fall only slightly more than the rise, it affects the *rate* of fall considerably more than the *rate* of rise. Further, as Table 115 shows, smoothing affects the ranking of the per month amplitudes within a series more than the ranking of the amplitudes proper. These differences are connected with the tendency of smoothing to stretch out and dampen brief cyclical phases; also with the fact that brief contractions are more numerous in our sample than brief expansions. By reducing more often than it prolongs the duration of ex-

TABLE 115

Coefficients of Rank Correlation between Amplitudes of Corresponding Specific Cycles in Raw and Smoothed Data
Four American Series

Series	No. of specific cycles[a]	Coefficient of rank correlation between measures of raw and smoothed data					
		Total amplitude[b]			Per month amplitude[b]		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Deflated clearings	15	+.95	+.95	+.95	+.94	+.44	+.85
Pig iron production	15	+.91	+.94	+.94	+.92	+.64	+.93
Railroad stock prices	18	+.97	+.97	+.98	+.61	+.79	+.78
Call money rates	23	+.90	+.91	+.88	+.54	+.69	+.82

The periods covered are shown in Table 112.
[a] See Sec. II.
[b] Expressed in specific-cycle relatives. See note to Table 114.

TABLE 116

Average Per Month Amplitude of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	Per month amplitude in specific-cycle relatives					
		Unweighted average			Weighted average		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
DEFLATED CLEARINGS							
Raw	15	0.8	1.9[b]	0.9	0.8	1.2	0.9
Smoothed	15	0.8	0.7	0.8	0.8	0.8	0.8
PIG IRON PRODUCTION							
Raw	15	2.4	4.7	2.9	2.2	3.8	2.7
Smoothed	15	2.3	2.9	2.5	2.0	2.8	2.3
RAILROAD STOCK PRICES							
Raw	18	1.4	1.7	1.4	1.2	1.5	1.4
Smoothed	18	1.1	1.2	1.2	1.1	1.3	1.2
CALL MONEY RATES							
Raw	23	6.9	7.6	6.3	5.8	6.4	6.1
Smoothed	23	4.3	4.6	4.3	4.1	4.2	4.2

The periods covered are shown in Table 112. See note to Table 114; also Table 113, note 'b'.
[a] See Sec. II.
[b] Becomes 1.2 when the contraction of 1907 is excluded; this contraction was severe but lasted only two months.

pansions, smoothing tends to raise the average rate of rise, offsetting part of the decline it effects in the amount of rise. By prolonging more often than it reduces the duration of contractions, smoothing tends to reduce the average rate of decline, thus reinforcing its effect on the amount of decline. Since the mean duration of full specific cycles is practically the same in the raw and smoothed data, smoothing affects the average per month amplitude of full cycles in approximately the same ratio as the average amplitude proper.[18]

The gap between the raw and smoothed figures in Table 116 is narrower in the weighted than in the unweighted average rates of change for expansions and contractions. Weighting has slight influence on the average per month amplitudes of the smoothed data, but reduces materially the averages of the raw data. The reason for the marked effects of weighting on the raw averages is that short specific-cycle phases tend to have larger per month amplitudes than long phases. The inverse correlation between the per month amplitude and the duration of cyclical phases tends to disappear when the data are smoothed, mainly because smoothing tends to increase the duration and decrease the amplitude of brief cyclical phases.

So much for the differences between the raw and smoothed results. The outstanding fact is that our standard method yields higher measures of amplitude than are derived from smoothed data. We cannot say with certainty which method commonly gives the more valid cyclical measures. It is clear, however, that smoothing by Macaulay's formula often cuts off a part of the 'true' cyclical fluctuations in the process of erasing erratic movements. That result is likely whenever the specific cycles of raw data have high and narrow peaks, or deep and narrow troughs; also when the phases of specific cycles are very brief. Macaulay's formula will preserve almost perfectly the amplitude of a sine curve with a period of 30 months or longer.[19] When the period drops below two years, or when the underlying cyclical curve, whatever its period, is not of sine shape, but has cusped peaks or troughs, Macaulay's formula is sure to understate the cyclical amplitude. Since brief phases are fairly numerous in our test series, and since the cyclical patterns of these series are, as a rule, decidedly more angular than sine curves, we should expect smoothing to yield measures of amplitude that, in the first place, are usually below our standard measures and that, in the second place, are sometimes almost as large

18 This result is mathematically necessary only in the weighted average per month amplitude. But in our sample, weighting (by duration) makes little difference in the average per month amplitude of the joint rise and fall; hence the relation governing the weighted per month average applies also to the unweighted average.

19 The percentage of the amplitude preserved is 99.39 for a sine-curve period of 30 months, 100.36 for a period of 36 months, 100.44 for 40 months, 100.33 for 48 months, 100.18 for 60 months; but the percentage is only 94.17 for 24 months, 82.05 for 20 months, 69.76 for 18 months (*The Smoothing of Time Series*, p. 159).

and at other times only a tenth or a fifth as large as our standard measures —precisely the results we have found.

In the preceding section we saw how smoothing tends to draw out brief cyclical phases. When that happens the amplitude of the cyclical phase tends to be reduced. In a cyclical phase that lasts more than a year the amplitude of smoothed data is, typically, only 5 to 15 per cent below our standard measure (Table 117). But when a phase lasts a half year or less, smoothing on the average reduces our standard measure about 50 per cent.[20] The tendency of the margin between the two sets of measures to widen as the duration of cyclical phases becomes shorter is not due to any flaw in our standard method; for, so far as our technique may yield biased measures of amplitude for short phases, that bias is likely[21] to work in the same direction as the bias produced by smoothing.[22]

TABLE 117

Influence of the Duration of Cyclical Phases on the Gap between Amplitudes of Corresponding Phases in Raw and Smoothed Data

Duration of phase in raw data (mos.)	Number of instances			Average ratio of amplitude of smoothed data to corresponding amplitude of raw data		
	Expansion	Contraction	Both phases	Expansion	Contraction	Both phases
Under 7	2	9	11	.43	.57	.55
7 - 12	1	17	18	.73	.77	.76
13 - 18	8	9	17	.84	.85	.85
19 - 24	10	6	16	.89	.84	.87
25 - 36	14	3	17	.85	.95	.87
37 - 48	9	3	12	.92	.93	.92
Over 48	4	1	5	.94	.91	.94

Based on specific cycles in deflated clearings, pig iron production, and railroad stock prices during the periods shown in Table 112. See Sec. II and note to Table 114.

The relations exhibited by Table 117 appear in call money rates, as well as in the three series covered by the table.[23] But in call money rates the chief reason for the wide gap between the amplitude measures derived from raw and smoothed data is the large number of high and narrow

20 Of course, the gap between the two measures for a phase depends (among other factors) on the duration of adjacent phases as well as on its own duration. This consideration is important in long phases. The average ratio of the smoothed to the raw amplitude is .90 for the 34 phases that lasted more than two years. Of these 34 phases, 12 were preceded or/and followed by a phase lasting 6 months or less; the average ratio is .86 for these cases and .92 for the rest.

21 Except in phases lasting less than 4 months (see p. 132); but such extremely short phases are very rare.

22 Compare the effects of the shift from one-month to three-month standings on the average amplitude of expansions and contractions in the raw data on clearings and iron production (Table 113) with the average duration of these phases (Table 112).

23 The averages for call money rates corresponding to the first six lines in the last column in the table are as follows: .36(3), .71(7), .71(15), .71(9), .76(11), .57(1). The figures in parentheses show the number of observations included in the averages. No phase lasting over 48 months occurs in call money rates.

cyclical peaks. Given the organization of the New York money market, particularly as it stood before the adoption of the Federal Reserve system, and the character of the demand for call loans, cyclical forces themselves have tended under certain circumstances to produce high and narrow peaks, and at other times broad and low troughs. Macaulay's formula turns the high, narrow peaks into broad, low hills, and thus misrepresents the cyclical behavior that is characteristic of the call loan market.[24] Nor is Macaulay's formula by any means peculiar in this respect, as Table 118 demonstrates.[25] It seems not unlikely that even our three-month averages understate the 'true' cyclical amplitudes of this series.

TABLE 118

Effects of Fourteen Smoothing Formulas on Amplitude of Specific Cycles
Call Money Rates on New York Stock Exchange, 1887–1893

Form of data	Per cent change from: Peak in 1887 to trough in 1888	Trough in 1888 to peak in 1890	Peak in 1890 to trough in 1892	Trough in 1892 to peak in 1893
Raw	−73	+304	−76	+256
Smoothed by formula				
No. 2	−57	+154	−60	+121
No. 11	−66	+234	−71	+229
No. 12	−62	+183	−64	+177
No. 13	−62	+182	−63	+160
No. 14	−63	+192	−65	+180
No. 18	−63	+193	−65	+199
No. 19	−64	+191	−65	+186
No. 20	−64	+190	−65	+182
No. 21	−64	+190	−65	+180
No. 22	−64	+188	−64	+170
No. 23	−64	+196	−66	+204
No. 24	−64	+196	−66	+198
No. 25	−65	+203	−67	+219
No. 27	−63	+177	−63	+168

In contrast to our standard method, the measures in this table express the change between dates of cyclical turn as a percentage of the standing at the earlier turn. One-month standings are used in the smoothed data, 3-month standings in the raw data. The months of cyclical turn in the raw and smoothed data are given in Table 107. For supplementary information, see Chart 44 and the notes to Table 107.

If the differences between the peaks of the raw data and of Macaulay's graduation were due mainly to erratic movements, we should not expect these differences to differ significantly from one period of call money rates to another. In fact, as Table 119 shows, the average difference between one-month standings at peaks of raw and smoothed data drops from 133

24 One of the criteria for satisfactory smoothing set up by Macaulay is that "if applied to successive points on a sine curve . . . , the graduation should fall as close as possible to the points on the sine curve". His 43-term graduation meets this test satisfactorily for sine curves with a period of 30 months or longer (see note 19). Macaulay recognizes that "if the underlying curve have cusps or be discontinuous", any continuous smoothing, whether by his 43-term or some other formula, will "somewhat obscure such characteristics"; and he cites the behavior of call money rates "during and after a financial panic" as an example. See *The Smoothing of Time Series,* pp. 21-2, 104-5.

25 Macaulay's formula is No. 24. In view of the wide use of a twelve-month moving average to represent the combined trend and cyclical fluctuations of a series, it is worth noting that formula No. 2 is a two-month moving average of a twelve-month moving average.

points in 1858–1915 to 18 points in 1915–31; the average difference between one-month and three-month standings of raw data drops from 94 to 13; the average difference between three-month standings of raw data and one-month standings of smoothed data drops from 38 to 5. The reason for the drop is, of course, that after the introduction of the Federal Reserve system, the high and narrow peaks in call money rates practically disappeared. There can be little doubt that this represents a change in the cyclical behavior of the New York money market, and that we would not be justified in treating the smoothed figures of call money rates before 1915 as even approximate measures of cyclical fluctuations.[26]

TABLE 119
Average Amplitude of Specific Cycles
Call Money Rates on New York Stock Exchange
Before and after Inauguration of Federal Reserve System

Period and form of data	No. of specific cycles[a]	No. of months in standing at turns	Average standing in specific-cycle relatives at			Difference from average on preceding line		
			Initial trough	Peak	Terminal trough	Initial trough	Peak	Terminal trough
1858–1915								
Raw.......	18	1	52.1	281.8	50.6	....	....	....
Raw.......	18	3	60.8	187.6	59.1	+8.7	−94.2	+8.5
Smoothed...	18	1	67.0	149.1	65.0	+6.2	−38.5	+5.9
1915–1931								
Raw.......	5	1	63.7	156.0	68.6	....	....	....
Raw.......	5	3	66.8	143.3	72.2	+3.1	−12.7	+3.6
Smoothed...	5	1	72.6	138.2	79.1	+5.8	−5.1	+6.9

[a] See Sec. II.

In iron production there are numerous instances of deep and narrow troughs, though they are not nearly so deep as the peaks of call money rates are high. In 1891, 1896, 1900, 1914 and 1921, when the troughs are fairly narrow but seemingly free from erratic movements, smoothing raises the troughs by larger amounts than in 1910 and 1927, when the troughs are relatively broad but marked by conspicuous erratic movements (Chart 43). The troughs in 1900 and 1914 are not included in the list of 'smooth' turns in Table 109, but that list is unduly conservative.

From this evidence we judge that the differences between our standard measures of amplitude and those derived from Macaulay's graduations result in considerable part from a tendency of smoothing to eliminate a part of the cyclical movements. But can we be sure that our standard method is free from bias? As in the problem of timing, one seemingly objective way of judging the relative merits of the two methods is to see what happens when each is applied to series whose underlying cyclical shapes are known *a priori*. The results for several artificial series are recorded in Chart 45 and Table 120. It is evident, in the first place, that smoothing gives only rough approximations, sometimes very poor

[26] See below, Chart 50 and pp. 352, 356.

approximations. The relative merits of the two methods are more difficult to judge, partly because the sample covered by the experiments is small. Taking the figures in Table 120 as they come, it seems that when the underlying cyclical movement is a sine curve, smoothing yields somewhat better approximations of the true amplitude than does our standard method. Results of this character may be generally expected, inasmuch as Macaulay's 43-term graduation is designed to fit sine curves closely. On the other hand, when the underlying cycles are cusp-shaped, smoothing yields decidedly poorer approximations than our method. This result, too may be generally expected, except perhaps when the amplitude of the random component is especially violent. When the underlying cycles are triangular, the merits of the two methods depend more closely on the size of the random movements; our method is likely to give better results when the random movements are 'mild', but poorer results when the random movements are 'pronounced'. On the whole, our method surely fares no worse than smoothing in the circumstances envisaged by the experiment.[27]

We must recall, moreover, that the theoretical model underlying the artificial series is not strictly applicable to actual time series. So far as the specific cycles of actual series are affected by cumulations of random movements, we do not want to ignore the random movements. Our aim in measuring the amplitude of a specific cycle is to allow for the random movements that blur its contours, not for the part played by random forces in the observed fluctuation—as is the case with the artificial series.[28] The practice of representing peaks and troughs by three-month standings promotes this objective unevenly, but so too would smoothing. In brief phases, three-month standings may well tend to understate cyclical amplitudes; the use of one-month or two-month standings might be better.[29] In very choppy series, five or seven months may not suffice to get rid of the erratic component at the turns.[30] On the other hand, in a series like call money rates, which is choppy but has sharply cusped cyclical peaks,

[27] It may be of interest to note that the use of one-month standings at cyclical turns of the raw data, instead of three-month standings, would exaggerate the 'true' amplitude of full cycles in every series. The averages based on one-month standings are as follows:

Series S'	104.7	Series T"	124.8
" S"	132.6	" C'	86.9
" T'	98.2	" C"	109.6

The total rise and fall of the pure cyclical component is 80, in units of specific-cycle relatives.

[28] See above, pp. 320-2.

[29] Note the discontinuity in our rules (p. 132) for handling phases less than four months, and those four months or longer.

[30] Imagine a strong case: a random series in which specific cycles are somehow chosen by a cycle-conscious investigator. Then the three-month standing at the peak is sure to be above the three-month standings at the adjacent troughs; if it were otherwise, these turns would not have been recognized in the first place. The case is not very different if we imagine, next, a series having a cyclical component of very small amplitude but large erratic movements; here too the use of three-month standings is likely to overstate the cyclical amplitude and produce cusped patterns.

TABLE 120

Amplitude of Specific Cycles in Raw and Smoothed Data of Six Artificial Series Compared with Amplitude of the Underlying Pure Cycles

Phase	Nature of phase[a]	Excess over amplitude of pure cyclical curve (in specific-cycle relatives)											
		Series S′		Series S″		Series T′		Series T″		Series C′		Series C″	
		Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
1	E	+0.3	...	+3.0	...	−4.3	...	−1.6	...	−14.1	...	−11.6	...
2	C	−1.5	−4.1	−0.4	−8.2	−6.5	−10.5	−5.4	−14.7	−16.9	−23.1	−17.1	−27.3
3	E	+1.8	0.0	+0.7	+0.3	−1.2	−6.3	+1.0	−6.1	−8.1	−18.8	−7.8	−18.8
4	C	+8.8	+4.5	+6.9	+9.1	+6.2	−1.8	+15.2	+2.8	−0.2	−14.3	+8.0	−9.8
5	E	+11.9	+5.3	+15.3	+10.3	+9.7	−0.9	+21.6	+4.2	+3.6	−13.4	+15.3	−7.9
6	C	+6.4	+2.3	+12.2	+5.6	+3.8	−4.0	+10.4	−0.6	−3.0	−16.3	+3.6	−12.5
7	E	+2.9	+1.0	+6.2	+4.1	−0.6	−5.2	+3.2	−2.6	−5.0	−17.5	−4.3	−14.7
8	C	+3.8	+2.9	+9.5	+7.4	+0.2	−3.3	+4.5	+0.9	−4.1	−15.7	−6.3	−11.3
9	E	+0.9	+0.6	+7.1	+3.4	−1.5	−5.9	−0.8	−3.0	−9.3	−18.2	−11.3	−14.8
10	C	−2.2	−2.0	+0.9	−2.5	−6.7	−8.5	−7.0	−8.9	−13.1	−20.8	−17.5	−20.8
11	E	−6.0	−4.3	−2.6	−7.7	−10.6	−10.7	−14.2	−14.2	−15.7	−23.1	−24.2	−26.0
12	C	−1.8	...	+4.4	...	−4.2	...	−5.3	...	−10.7	...	−12.0	...
Av., signs ignored													
Phases 1-12		4.0	...	5.8	...	4.6	...	7.5	...	8.6	...	11.6	...
Phases 2-11		4.6	2.7	6.2	5.9	4.7	5.7	8.3	5.8	7.9	18.1	11.5	16.4
Av., signs respected													
Phases 1-12		+2.1	...	+5.3	...	−1.3	...	+1.8	...	−8.0	...	−7.1	...
Phases 2-11		+2.7	+0.6	+5.6	+2.2	−0.7	−5.7	+2.8	−4.2	−7.2	−18.1	−6.2	−16.4

For explanations of the artificial series, see p. 318 and Appendix C (Chart 45). The specific cycles are treated positively. The amplitudes of the raw data are measured from 3-month standings at cyclical turns; the amplitudes of the smoothed data and of the pure cycles are measured from 1-month standings. The amplitude of the expansion or contraction of the pure cycles is invariably 40. The amplitude of phase 1 in the raw data of Series S′ is 40.3; hence the entry +0.3 in the table, and so on. Since smoothing involves the loss of a phase at each end of the series, there are no entries for phases 1 and 12 in the smoothed data, nor averages based on phases 1-12.

[a] E stands for specific-cycles expansion, C for contraction.

even three-month standings at peaks may understate the cyclical amplitudes. Ideally, the method should be varied from case to case depending on the character of the cyclical and erratic movements; but this counsel of perfection applies with equal force to smoothing. We believe that our rough method yields amplitude measures that are simpler to interpret and more trustworthy, on the whole, than those yielded by Macaulay's graduations or similar devices.

V The Secular Component of Specific Cycles

Since Macaulay's graduation cannot influence much the level about which cyclical fluctuations play, smoothing makes practically no difference in average measures of the percentage change between the levels of successive cycles (Table 121). It affects the percentage changes between successive cycles chiefly by shifting the dates of cyclical troughs or peaks: the former if a series is analyzed on a positive basis, as are our present samples, the latter in inverted series. Though the effects of these shifts are sometimes large in cycle-by-cycle measures, they cancel out in the averages.

TABLE 121

Average Measures of Secular Movements of Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	Average per cent change from						
		Preceding phase		Preceding cycle on base of				
				Preceding cycle		Average of given and preceding cycle		
		Contraction to expansion	Expansion to contraction	Total	Per month	Total	Per month	
							Unweighted	Weighted
DEFLATED CLEARINGS								
Raw	15	+8.9	+7.3	+16.9	+0.40	+15.2	+0.36	+0.37
Smoothed	15	+8.8	+6.9	+16.8	+0.40	+15.1	+0.37	+0.37
PIG IRON PRODUCTION								
Raw	15	+9.5	+7.1	+18.2	+0.43	+14.5	+0.34	+0.35
Smoothed	15	+13.8	+3.3	+18.2	+0.44	+15.0	+0.36	+0.36
RAILROAD STOCK PRICES								
Raw	18	+7.4	+4.7	+13.4	+0.26	+10.4	+0.20	+0.21
Smoothed	18	+9.1	+2.8	+13.3	+0.27	+10.2	+0.20	+0.21
CALL MONEY RATES								
Raw	23	+9.8	−4.9	+4.1	+0.09	−0.1	−0.02	0.00
Smoothed	23	+1.5	+2.7	+4.2	+0.10	−0.1	−0.03	0.00

See Table 112 for the periods covered. Except for the last three columns, the average percentages are subject to an upward bias, explained in Ch. 5, Sec. V. This bias is immaterial in a comparison of raw and smoothed data.
[a] See Sec. II.

The effects of smoothing are more prominent in the phase-to-phase changes than in the cycle-to-cycle changes. The former are affected by shifts of both troughs and peaks. Since the number of months shared by corresponding phases in raw and smoothed data is sometimes a small fraction of the full phase in either body of data, large differences can easily arise. Furthermore, smoothing influences the phase-to-phase changes by altering the form of the cyclical pattern as well as the timing of cyclical turns.

VI Specific-cycle Patterns

Our practice of replacing the full set of monthly values in each specific cycle by nine average standings facilitates comparisons among cycles of different durations, and at the same time is a powerful device for reducing erratic fluctuations. Whereas the monthly figures of time series usually have extremely rough contours, the nine-stage patterns tend to rise continuously to the peak and decline continuously to the trough. A further reduction of erratic movements is effected when the standings in each stage are averaged for all the cycles covered by a series.

Our method, however, is far from perfect. In deflated clearings, for example, only 3 out of 15 cycles rise continuously from stage I to V, and decline continuously from stage V to IX. Corresponding numbers in call

TABLE 122
Average Specific-cycle Patterns of Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	Average in specific-cycle relatives at stage: I Initial trough	II Expansion: First third	III Expansion: Middle third	IV Expansion: Last third	V Peak	VI Contraction: First third	VII Contraction: Middle third	VIII Contraction: Last third	IX Terminal trough
DEFLATED CLEARINGS										
Raw	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Smoothed	15	86.8	90.6	99.0	106.6	110.8	109.4	105.5	101.4	100.2
PIG IRON PRODUCTION										
Raw	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Smoothed	15	73.0	82.9	103.5	118.1	126.5	120.7	102.8	86.2	81.6
RAILROAD STOCK PRICES										
Raw	18	82.8	88.0	98.8	110.8	118.3	112.4	103.2	94.0	86.6
Smoothed	18	84.5	88.4	100.1	111.2	116.0	112.2	102.2	91.7	88.0
CALL MONEY RATES										
Raw	23	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Smoothed	23	68.2	76.0	98.5	130.4	146.8	130.4	99.0	76.4	68.1
		Excess of the raw average over the smoothed								
Deflated clearings	15	−1.1	−0.1	+0.2	+0.1	+1.8	−0.7	+0.5	+0.5	−1.0
Pig iron production	15	−5.7	−0.4	+0.2	−1.6	+2.8	+1.9	+5.4	+2.2	−7.0
Railroad stock prices	18	−1.7	−0.4	−1.3	−0.4	+2.3	+0.2	+1.0	+2.3	−1.4
Call money rates	23	−6.1	+4.3	+6.2	−7.2	+31.2	−10.0	−9.1	−4.6	−6.2

The standings in stages I, V and IX cover 3 months in the raw data, 1 month in the smoothed data; but see Table 113, note 'b'. For the periods covered, see Table 112.

[a] See Sec. II.

money rates are 10 out of 23, railroad stock prices 13 out of 18, pig iron production 11 out of 15.[31] In these four series the *average* pattern rises without interruption from stage I to V and declines without interruption from stage V to IX (Table 122 and Chart 46). Our efforts to measure cyclical patterns are not always attended with such conspicuous success, and even in these instances success is partly an illusion. The fact that the pattern of a single cycle, or the average pattern, is free from reversals of direction within the expansion or contraction may mean that erratic movements have disappeared; but it is much more likely to mean that whatever erratic component remains is merely not large enough to produce reversals of direction.[32] Substantial erratic components may be expected to remain in the patterns of individual cycles during brief phases, and even in the average patterns of short series characterized by sharp erratic fluctuations. The difficulty is most acute at the turning stages (I, V and IX), since the three-month standings may yield a biased framework of the patterns.

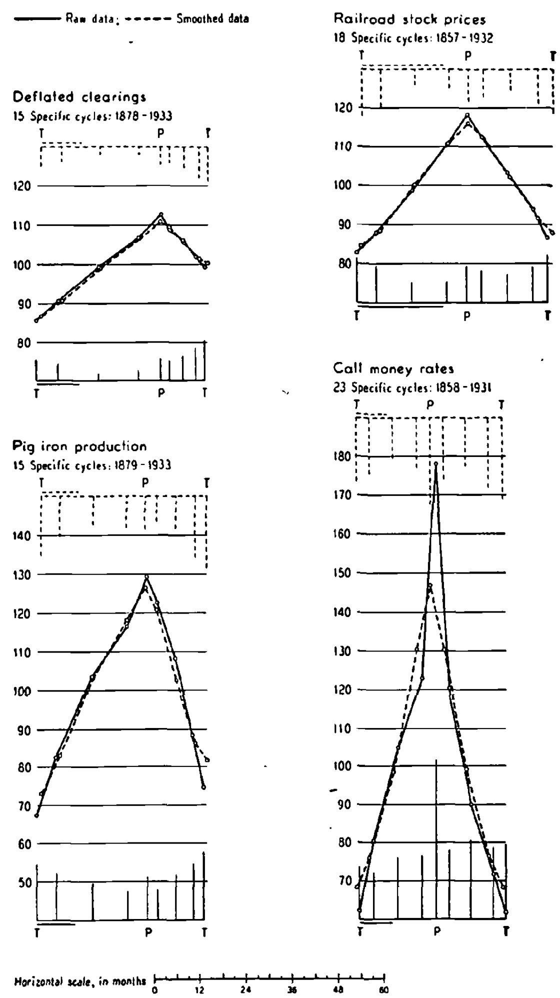
[31] See the cycle-by-cycle measures in Appendix Table B1.

[32] Some reversals of direction may even be cyclical phenomena, as when the rise of a series is regularly retarded, sometimes interrupted by an actual dip, near the center of expansion. However, few reversals of direction in cyclical patterns can be confidently put in this class.

CHART 46

Average Specific-cycle Patterns of Raw and Smoothed Data

Four American Series



See Table 122. The patterns are spaced so as to show average leads or lags between the raw and smoothed data. For other explanations of chart, see Ch. 5, Sec. VI.

So far as an erratic component remains in the standings of the raw data at individual stages of the cycles, it is bound to count more heavily in the rates of change between successive standings than in the standings themselves. Moreover, we should expect the erratic factor to vary inversely with the length of the interval between the cycle stages. Of the eight intervals, those during contraction tend to run shorter, and therefore are more susceptible to erratic movements, than those during expansion. The shortest intervals are usually from the peak to the first third of contraction (V-VI) and from the last third of contraction to the trough (VIII-IX). Also the intervals from the trough to the first third of expansion (I-II) and from the last third of expansion to the peak (IV-V) are short relatively to the other intervals into which we divide expansions.[33] The measures in our standard Table S5 therefore have uneven degrees of reliability as representatives of cyclical behavior, and the question how far measures of cyclical patterns could be improved by preliminary smoothing of the data by Macaulay's formula, or some similar device, assumes considerable importance.

The differences between the raw and smoothed figures showing rates of change from stage to stage of the specific cycles (Table 123) are much larger in proportion to the items compared than the corresponding differences between the standings in successive stages (Table 122). The differences between the rates of change (signs disregarded) in the raw and smoothed data are on the average larger during contractions than during expansions, except in call money rates. They average larger also during the turning intervals of the cycles (that is, during stages I-II, IV-V, V-VI and VIII-IX) than during other intervals. Again, they tend to be larger during the turning intervals within contractions (V-VI and VIII-IX) than during the turning intervals within expansions (I-II and IV-V). In all these respects the figures behave as we should expect them to behave if the smoothed data yielded faithful representations of cyclical patterns, while an appreciable erratic component remained in our standard averages.

But there are serious obstacles to this interpretation. If erratic movements and nothing else stood between the raw and smoothed figures, we should indeed find larger differences between their rates of change in the turning intervals of the cycles than in other intervals, but it would be unlikely that the signs of the differences in the turning intervals would agree from series to series. Table 123 shows a plus difference in every series for the first and last intervals of expansion, and a minus difference for the first and last intervals of contraction. This uniformity of signs reflects, of course, the tendency of smoothing to make cyclical amplitudes smaller than the amplitudes yielded by our standard method. If our judgment that smoothing tends to understate the 'true' cyclical amplitudes in our sample is sound, erratic movements cannot account wholly, perhaps not even largely, for the differences between the rates of change in the raw and smoothed data in the turning intervals of the cycles, or for the larger difference in these intervals than in others. There is a similar difficulty in interpreting the larger differences between the raw and smoothed rates during the contraction than during the expansion stages. As shown in Section IV, smoothing influences the average rates of change more during contractions than during expansions of specific cycles—a difference con-

[33] The intervals for our four series are shown at the bottom of Table 122.

nected with the tendency of smoothing to reduce cyclical amplitudes and to change the relative duration of expansions and contractions. These distortions of the cyclical movements are inevitably reflected in larger differences, generally, between the raw and smoothed rates in the contraction than in the expansion intervals of the cycles.

Table 123 thus confirms what we have said about the distortions produced by smoothing; it also focuses attention upon a feature of smoothing which, although correlated with the tendency to reduce peaks

TABLE 123
Average Rate of Change from Stage to Stage of Specific Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Average change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Expansion				Contraction			
		Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
DEFLATED CLEARINGS									
Raw	15	+0.8	+0.9	+0.7	+1.0	−2.6	−2.1	−1.7	−1.5
Smoothed	15	+0.7	+0.9	+0.8	+0.7	−0.4	−0.9	−0.9	−0.4
PIG IRON PRODUCTION									
Raw	15	+3.2	+2.7	+1.5	+2.7	−2.3	−3.0	−7.2	−6.2
Smoothed	15	+1.9	+2.7	+2.3	+1.9	−1.9	−3.8	−3.4	−1.4
RAILROAD STOCK PRICES									
Raw	18	+1.3	+1.4	+1.6	+1.4	−1.7	−1.6	−1.5	−2.2
Smoothed	18	+0.8	+1.4	+1.3	+0.8	−0.9	−1.5	−1.4	−0.7
CALL MONEY RATES									
Raw	23	+6.6	+5.8	+1.0	+16.9	−20.2	−6.2	−4.5	−2.3
Smoothed	23	+2.1	+4.1	+5.7	+4.4	−4.7	−6.3	−4.4	−2.2
		Excess of the raw average over the smoothed							
Deflated clearings	15	+0.1	0.0	−0.1	+0.3	−2.2	−1.2	−0.8	−1.1
Pig iron production	15	+1.3	0.0	−0.8	+0.8	−0.4	+0.8	−3.8	−4.8
Railroad stock prices	18	+0.5	0.0	+0.3	+0.6	−0.8	−0.1	−0.1	−1.5
Call money rates	23	+4.5	+1.7	−4.7	+12.5	−15.5	+0.1	−0.1	−0.1
		Average interval in months between midpoints of stages							
DEFLATED CLEARINGS									
Raw	15	5.8	10.5	10.5	5.8	2.3	3.4	3.4	2.3
Smoothed	15	5.5	10.0	10.0	5.5	2.4	3.8	3.8	2.4
PIG IRON PRODUCTION									
Raw	15	5.1	9.3	9.3	5.1	2.7	4.6	4.6	2.7
Smoothed	15	4.8	8.7	8.7	4.8	3.0	5.0	5.0	3.0
RAILROAD STOCK PRICES									
Raw	18	5.1	9.2	9.2	5.1	3.8	6.7	6.7	3.8
Smoothed	18	5.0	8.9	8.9	5.0	3.9	7.1	7.1	3.9
CALL MONEY RATES									
Raw	23	3.7	6.3	6.3	3.7	3.3	5.7	5.7	3.3
Smoothed	23	3.5	6.1	6.1	3.5	3.5	5.8	5.8	3.5

The average rates of change are unweighted; weighted averages are given in Table 125. See notes to Table 122.

and raise troughs, becomes evident only in the full patterns. In every series covered by our sample the smoothed data rise fastest on the average from stage II to III or III to IV and fall fastest on the average from VI to VII or VII to VIII; that is, the rise is fastest around mid-expansion and the fall is fastest around mid-contraction. In no series does the average rate of change reach a numerical maximum within the turning intervals of the cycles. Numerical minima, on the contrary, come invariably within a turning interval. That is exactly what we should expect of a curve with gently rounded tops and bottoms. The raw patterns are free from this technical compulsion, and have notably different characteristics. The maximum rates of change are no longer concentrated in the middle range of expansions and contractions; they frequently come in one of the turning intervals. Nor are the phases of expansion and contraction as symmetrical as in the smoothed data. These differences be-

TABLE 124

Position of Fastest and Slowest Rates of Change in Specific-cycle Patterns
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Distribution of rates of change between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		No. of expansions in which rise is fastest				No. of contractions in which fall is fastest			
DEFLATED CLEARINGS									
Raw	15	5	1	2	7	6.5	4.5	3	1
Smoothed	15	1.5	5	5	3.5	0.25	7.25	7.25	0.25
PIG IRON PRODUCTION									
Raw	15	7	1	2	5	1	2	4	8
Smoothed	15	4	4	5	2	1	10.5	3.5	...
RAILROAD STOCK PRICES									
Raw	18	2	4	4	8	5	2	4	7
Smoothed	18	...	9.5	5.5	3	...	11	6	1
CALL MONEY RATES									
Raw	23	2	1	3	17	16	2	3	2
Smoothed	23	1	3	14	5	2	13	5	3
		No. of expansions in which rise is slowest				No. of contractions in which fall is slowest			
DEFLATED CLEARINGS									
Raw	15	6	4	3	2	...	7	5.5	2.5
Smoothed	15	7	1	2	5	5.25	0.25	0.25	9.25
PIG IRON PRODUCTION									
Raw	15	3	1	11	...	5.5	4.5	2	3
Smoothed	15	6	...	5	4	4	1	1	9
RAILROAD STOCK PRICES									
Raw	18	6	2	6	4	5	4	8	1
Smoothed	18	8.5	1	...	8.5	4	2	1	11
CALL MONEY RATES									
Raw	23	3	7	12	1	1	8	4	10
Smoothed	23	15	3	2	3	2	4	4	13

To minimize the number of ties the rates of change were computed to extra decimals. But some ties remained; hence the fractions. Cf. Table 76, and see notes to Table 122.

TABLE 125

Weighted Average Rate of Change from Stage to Stage of Specific Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Weighted average change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
DEFLATED CLEARINGS									
Raw	15	+0.8	+0.8	+0.7	+1.0	−1.7	−0.8	−1.2	−1.2
Smoothed	15	+0.7	+0.8	+0.8	+0.8	−0.6	−1.0	−1.1	−0.5
PIG IRON PRODUCTION									
Raw	15	+3.0	+2.3	+1.4	+2.5	−2.5	−3.1	−4.3	−5.2
Smoothed	15	+2.1	+2.4	+1.7	+1.7	−1.9	−3.6	−3.3	−1.5
RAILROAD STOCK PRICES									
Raw	18	+1.0	+1.2	+1.3	+1.5	−1.6	−1.4	−1.4	−2.0
Smoothed	18	+0.8	+1.3	+1.3	+1.0	−1.0	−1.4	−1.5	−0.9
CALL MONEY RATES									
Raw	23	+5.0	+3.9	+2.9	+15.0	−17.4	−5.4	−3.2	−3.0
Smoothed	23	+2.3	+3.7	+5.2	+4.7	−4.7	−5.4	−3.9	−2.4
		Excess of the raw average over the smoothed							
Deflated clearings	15	+0.1	0.0	−0.1	+0.2	−1.1	+0.2	−0.1	−0.7
Pig iron production	15	+0.9	−0.1	−0.3	+0.8	−0.6	+0.5	−1.0	−3.7
Railroad stock prices	18	+0.2	−0.1	0.0	+0.5	−0.6	0.0	+0.1	−1.1
Call money rates	23	+2.7	+0.2	−2.3	+10.3	−12.7	0.0	+0.7	−0.6

See notes to Table 122.

tween the raw and smoothed averages in Table 123 are broadly confirmed by the summary of individual cycles in Table 124.[34] They are again confirmed by Table 125, in which weighted average rates of change replace the unweighted averages of Table 123.[35]

An independent check on the preceding analysis is provided by Table 126 and Chart 47, which present cyclical patterns of several artificial

[34] Although the slowest rates of change in the smoothed data are concentrated in the turning zones, the concentration is heavier in stages I-II and VIII-IX than in stages IV-V and V-VI. This result is partly due to the fact that Macaulay converted the data to logarithms before smoothing. We converted his smoothed logarithms to natural numbers before making cyclical measures.

[35] The averages in Table 125 correspond precisely to the slopes of the patterns in Chart 46 (see p. 151). They are significant for that reason, and also because they reduce erratic movements more effectively than do the unweighted averages.

The differences (signs ignored) between the raw and smoothed averages in Table 125 are smaller in most instances than the corresponding differences in Table 123. The reduction of the gaps is due chiefly to the effects of weighting on the raw averages. Extreme figures come, as a rule, in very short phases, precisely the instances in which our method handles erratic movements least effectively. For example, in the raw data on call money rates the unweighted average change from stage III to IV is +1.0 points per month; the weighted average is +2.9. The discrepancy is mainly attributable to the expansion from Oct. 1880 to Feb. 1881. Since this expansion lasted only four months, the standing in stage III covers Dec. 1880 and the standing in stage IV Jan. 1881. Between the two months there was a sharp erratic decline in interest rates, which is indicated by a rate of change of −81.5 points from stage III to IV of the cycle. In the unweighted average of Table 123, this figure gets the same weight as any other. But in Table 125 its influence on the average is reduced; for the rates of change in individual cycles are weighted by the interstage intervals. In this instance and several others, the weighted average exceeds numerically the unweighted average. But the opposite relation is the rule, since the rate of change and the length of the interstage interval tend to be correlated inversely.

series, before and after smoothing. These exhibits should be studied in conjunction with Chart 45. The reader should note, especially, how smoothing imposes a 'sine shape' on cyclical fluctuations, regardless of their true characteristics. Many theorists are prone to reason on the assumption that economic cycles are of 'sine shape'; meaning by this phrase not that economic activities fluctuate precisely like sine curves, but that their rates of rise increase in the early stages of a cyclical expansion and taper off towards its close, and that the rates of fall behave similarly in contraction. This belief has deep roots in intellectual history: the extensive use of sine curves in the physical sciences, the ease of fitting a sine curve to observational data, the vogue of the 'principle of continuity', have all contributed to it. But perhaps the most important reason is the image created in the minds of men over several generations by the practice of smoothing. The statistician faced with jerky figures commonly resorts to smoothing. As a rule he relies on simple, unweighted moving averages. Sometimes he uses more sophisticated devices; for example, moving averages having a more or less smooth weight diagram, with some weights negative, such as are described in Macaulay's *Smoothing of Time Series*. In either case he obtains a curve with rounded tops and bottoms.[36] The 'sine shapes' are sometimes faithful representatives of the specific cycles of economic time series. Perhaps more often, they are merely technical creatures of the smoothing process.[37]

The jerky contours of time series present a dilemma. If the data are left as they come, the shapes of cyclical patterns cannot be measured. On the other hand, if smoothing of the ordinary type is employed, it is practically inevitable that the patterns will be 'sine shaped'; for 'sine shapes' emerge the moment the terms included in a moving average cross cyclical phases. One way of escaping from the dilemma is to avoid moving averages altogether, and that is the route we have followed in measuring cyclical patterns. Another way is to use hypothetical values, based on projections of the actual data, in getting moving averages for the months in the vicinity of a cyclical turn. Still another possibility, theoretically, is to vary the smoothing formula from one stretch of the data to the next, the formula being always properly adjusted to the underlying cyclical movement.[38] But this method breaks down when the underlying cyclical curve is triangular or cusp shaped: there is no set of weights which, if applied

[36] See the illustrations of this effect in Charts 44, 45 and 47.

[37] In this connection it is important to bear in mind the distinction between specific and business cycles. For example, the patterns of the specific cycles in most activities might be triangular, yet the pattern of the cycles in aggregate business activity might be 'sine shaped'; though, in fact, we have found little evidence of the latter.

[38] Appropriate devices for parabolic curves of different orders are described by Macaulay in *The Smoothing of Time Series*, and by Max Sasuly, *Trend Analysis of Statistics* (Brookings Institution, 1934). Appendix VII of Macaulay's book shows the closeness with which 19 graduation formulas fit sine series of different periods.

TABLE 126

Average Specific-cycle Patterns of Raw and Smoothed Data of Six Artificial Series Compared with Patterns of the Underlying Pure Cycles

Series and form of data	No. of specific cycles	No. of mos. in stage I, V & IX	Average in specific-cycle relatives at stage								
			I	II	III	IV	V	VI	VII	VIII	IX
(S) Sine curve	..	1	80.0	83.9	100.0	116.1	120.0	116.1	100.0	83.9	80.0
(S′) Sine curve + random											
Raw	6	3	78.7	87.8	102.3	116.0	120.7	114.4	97.5	82.6	78.4
Raw	4	3	78.1	87.6	101.2	116.6	122.5	115.9	97.6	82.2	78.3
Smoothed	4	1	79.1	83.9	100.0	116.4	120.8	116.8	100.1	82.7	78.9
(S″) Sine curve + 2 random											
Raw	6	3	77.9	91.2	104.1	115.7	122.9	111.1	95.3	81.3	77.3
Raw	4	3	77.1	91.4	102.4	117.6	124.4	112.6	95.9	80.2	77.0
Smoothed	4	1	77.6	82.9	99.2	116.8	122.2	117.5	100.9	82.3	77.2
(T) Triangular curve	..	1	80.0	87.3	100.0	112.7	120.0	112.7	100.0	87.3	80.0
(T′) Triang. curve + random											
Raw	6	3	80.4	89.7	101.1	112.7	119.0	111.0	98.9	86.2	80.2
Raw	4	3	79.4	88.7	100.1	113.2	121.0	112.6	99.4	86.0	80.2
Smoothed	4	1	82.2	87.0	100.2	113.3	117.6	113.7	99.9	85.8	82.0
(T″) Triang. curve + 2 random											
Raw	6	3	78.6	93.4	102.1	112.0	120.2	110.0	97.6	85.4	78.1
Raw	4	3	77.0	93.2	101.0	113.3	123.3	112.2	97.3	84.6	77.5
Smoothed	4	1	80.8	86.0	99.8	113.9	119.0	114.4	100.4	85.2	80.4
(C) Cusp curve	..	1	80.0	93.4	100.0	106.6	120.0	106.6	100.0	93.4	80.0
(C′) Cusp curve + random											
Raw	6	3	83.7	94.9	100.8	106.8	115.6	105.4	99.6	92.1	83.6
Raw	4	3	82.6	94.6	100.2	106.8	117.9	106.6	99.3	92.3	83.0
Smoothed	4	1	88.4	92.0	100.4	107.9	111.4	108.2	100.0	91.6	88.2
(C″) Cusp curve + 2 random											
Raw	6	3	83.4	99.4	101.3	105.6	116.1	104.8	98.8	90.4	83.0
Raw	4	3	81.5	99.2	100.5	106.4	119.5	106.9	98.1	90.0	82.5
Smoothed	4	1	86.9	91.2	100.4	108.6	112.8	109.2	99.8	90.4	86.4

Smoothing involves the loss of 2 positive cycles, as explained in Appendix C (Chart 45). The averages for 4 cycles include corresponding cycles in the raw and smoothed data.

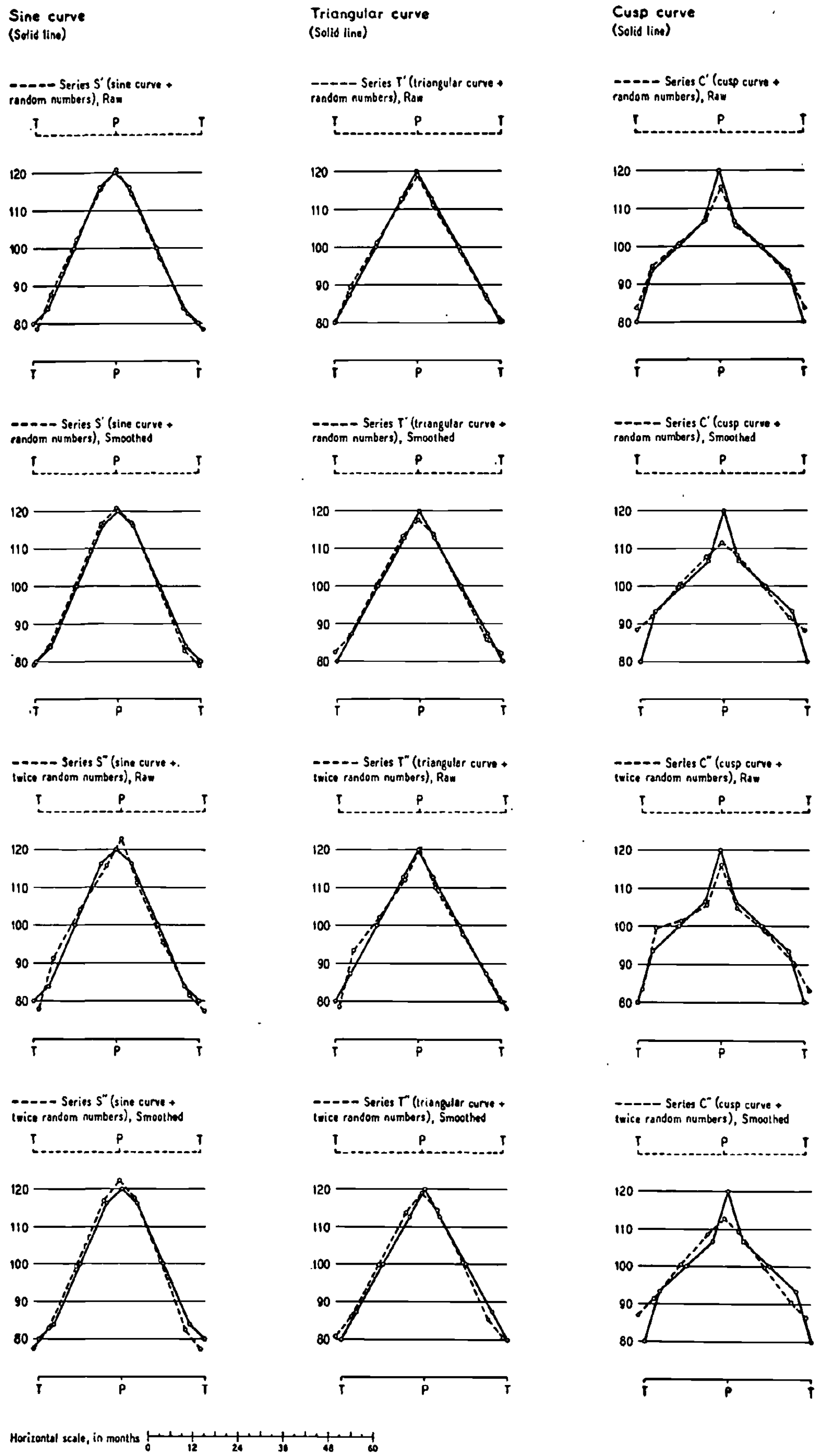
to successive terms in a moving average, will reproduce triangular or cusped cycles.[39]

Granted that our method of measuring cyclical patterns avoids the rounding bias characteristic of smoothing, may it not be subject to other types of bias? Surely, the patterns of the raw artificial series deviate materially from the patterns of the underlying pure cycles. The former tend to have higher standings in expansion and lower standings in contraction; the excess in successive thirds of expansion and the deficiency in successive thirds of contraction tends to shrink; hence the rate of rise during stages II-III and III-IV is moderated, and likewise the rate of fall during VI-VII and VII-VIII. Furthermore, the pattern of the artificial series is asymmetrical in the turning zones: the rate of change from stage V to VI exceeds numerically the rate from IV to V, and the rate from I to

[39] We disregard 'fitting' by drawing on many terms of a parabolic expansion, or by means of harmonic analysis. There is nothing to be said in favor of expending much labor on reproducing, approximately, the original data, unless the process yields a truly helpful classification of different types of fluctuation.

CHART 47

Average Specific-cycle Patterns of Raw and Smoothed Data of Six Artificial Series Compared with Patterns of the Underlying Pure Cycles



Patterns of raw data are based on six cycles, patterns of smoothed data on four cycles. The patterns are spaced so as to show average leads or lags between the raw or smoothed data and the pure cyclical component. See Chart 45 and Table 126.

II exceeds numerically the rate from VIII to IX. All these departures from the true cyclical pattern *could* arise from our practice of dating cyclical turns towards the close of the period of transition from one phase to the next, as when a trough is 'flat' or 'double-bottomed', or a peak is 'flat' or 'double-topped'.

But they could arise also from the peculiarities of the particular set of random numbers that enters into all six artificial series.[40] The latter, in fact, is largely responsible for the asymmetry.[41] Our rule regarding transitions is used sparingly in dating cyclical turns. It serves only as a last resort, when other rules fail to yield a decision. In the great majority of series the rule is of slight importance. If the rule were reversed, one or two turns, at most, would be shifted in our present sample of four series. The rule has serious consequences, however, in series with perfectly flat tops or bottoms, and it may have a telling effect whenever erratic movements are exceptionally pronounced in the vicinity of cyclical turns.[42]

Not a few of the cyclical patterns yielded by our method of analysis seem 'cusp shaped', in the sense that they show maximum rates of change in the turning zones. This feature may lead some readers to inquire whether our method imparts a bias to cyclical patterns opposite to the bias imparted by smoothing: that is, if smoothing tends to create 'sine shapes', may not our method create 'cusp shapes'? As far as we can judge, there is nothing in our method to produce such a bias in series free from pronounced erratic movements. True, three-month standings at cyclical turns may be poorly suited to the underlying cyclical movements. But if this factor introduces a bias towards cusp shapes in some series, it introduces a bias towards sine shapes in others.[43] In very choppy series, on the other hand, our method does promote cusps. In such series there is danger of recognizing spurious cycles. When that happens, a cusp-shaped 'cycle' is likely to emerge.[44] A similar result may arise if a genuine cyclical rise or decline is overlooked, for the skipped movement will then depress the rate of change in one or more intervals outside the turning zones of the cycles that are recognized. Even if specific cycles are marked off properly, the use of three-month standings at peaks and troughs is likely to exaggerate the amplitude of very choppy series, and thereby tend to produce cusp-

40 We have six series, not six independent tests. See p. 318 and Appendix C.

41 The evidence for this statement is partly indirect, partly direct. Table 108 shows that the artificial series, before smoothing, tend to lag at both peaks and troughs of the pure cycles. That is what we should expect, if the rule concerning transitions made an appreciable difference. In that event, however, a reversal of the rule should produce leads. Actually, an experiment with reversing the rule left the turns practically unchanged.

This indirect evidence was checked by constructing 'cyclical' patterns of the random numbers entering into the artificial series, the dates of the pure cycles serving as a framework for the patterns. The patterns clearly indicated that the particular random numbers that we have used produce the asymmetry.

42 See p. 148 for a method of handling such cases.

43 See above, pp. 333-5.

44 See above, note 30.

shaped patterns.[45] But the existence of a bias towards cusp shapes cannot be safely inferred from the mere fact that erratic movements are pronounced, for the combination of cyclical and erratic movements varies from series to series. Call money rates, for example, are subject to pronounced erratic movements. Yet the cusped cyclical peaks of this series are genuine; so far as our method errs, it probably does so in the direction of moderating the cusps.

TABLE 127
'Special' Cyclical Patterns
Pig Iron Production, United States, 1879–1933

Stage	Average specific-cycle pattern		Average reference-cycle pattern	
	Cumulative no. of months to stage	Standing in cycle relatives at stage	Cumulative no. of months to stage	Standing in cycle relatives at stage
1 (Trough)	0.0	65.3	0.0	74.0
2	3.2	77.3	2.6	83.5
3	6.4	85.7	5.2	93.9
4	9.6	94.2	7.8	99.6
5	12.8	101.2	10.4	102.0
6	16.0	106.1	13.0	103.6
7	19.2	109.0	15.5	106.9
8	22.4	113.5	18.1	111.3
9	25.6	120.4	20.7	116.4
10 (Peak)	28.8	130.9	23.3	122.1
11	30.4	125.2	25.5	119.2
12	32.0	121.2	27.7	116.7
13	33.7	114.9	29.9	112.0
14	35.3	108.2	32.1	103.8
15	36.9	103.1	34.4	95.3
16	38.5	98.2	36.6	88.7
17	40.1	90.7	38.8	86.3
18	41.8	81.0	41.0	80.7
19 (Trough)	43.4	72.2	43.2	81.7

Based on 15 specific and reference cycles. See the explanations in the text.

A difficulty of another sort is that our technique yields only nine observations on cyclical patterns. Additional detail would be desirable, especially in the neighborhood of cyclical turns. But to provide much detail is not feasible, in view of the brevity of many cyclical phases and the prevalence of erratic movements. There are no such limitations on cyclical patterns made from data smoothed by Macaulay's formula, but in their case fine detail would often be spurious. One way of steering between these difficulties is to use a smoothing formula of much shorter time span than Macaulay's, and to protect the patterns against the 'rounding effect' of smoothing by special treatment of the data in the vicinity of peaks and troughs. The results of an experiment along these lines on pig iron production are presented in Table 127 and Chart 48. The 'special'

[45] A partial tendency towards cusped patterns is also imparted by the rule on cyclical dating, discussed in the preceding paragraph, which tends to exaggerate the rates of change during stages I-II and V-VI: but this rule imparts an opposite bias to the rates of change during stages VIII-IX and IV-V.

specific-cycle pattern consists of nineteen standings, in contrast to the 'standard' pattern of nine standings.[46]

The 'special' specific-cycle pattern was derived in the following steps. (1) A five-month centered moving average was taken of the raw data. (2) Since a 'rounding effect' emerges when the span of a moving average laps over two phases, hypothetical values were used to get the moving average for each month of cyclical turn and for the month just preceding and following the turn. The hypothetical values were obtained by projecting the actual movements of the data [47] in the vicinity of the turns. (3) The interval from the middle of the trough to the middle of the peak month in each specific cycle was divided into nine equal segments, and the standing at the end of each segment was computed by linear interpolation of the moving averages. (4) The contraction was divided likewise into nine segments, and the standing at the end of each segment computed. (5) The nineteen standings for the cycle were then expressed as relatives of the cycle base, computed from the raw monthly values on our standard plan. (6) Next, the cycle relatives at the end of each segment were averaged for all cycles. (7) Likewise the intervals between successive standings were averaged for all cycles. (8) Finally, the successive average standings of the cycle relatives were plotted on Chart 48 against the cumulatives of the average intervals.

The detailed 'special' pattern derived by this method is surprisingly similar to our standard pattern. The principal difference is the slightly

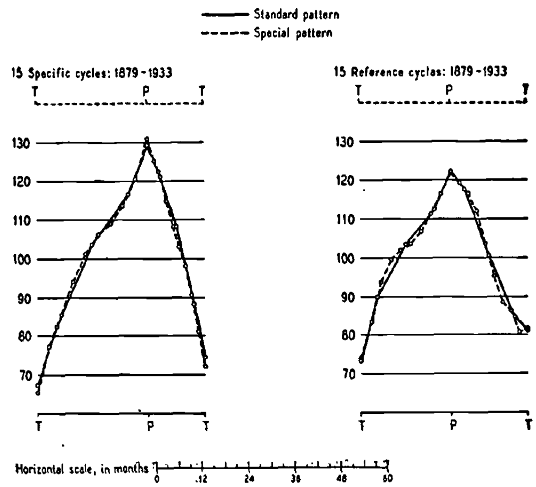
[46] The 'special' reference-cycle pattern is considered at the close of Sec. VII.

[47] Let a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, etc. be the actual values for a run of months, where the subscript identifies the month. Let a_{15} be the value in the peak (or trough) month. Then the hypothetical value (a'_{16}) substituted for a_{16} is $(a_{15}+a_{14}-a_{13})$; and the hypothetical value (a'_{17}) substituted for a_{17} is $(a'_{16}+a_{15}-a_{14})$, or $(2a_{15}-a_{13})$. The five-month moving average is therefore $\frac{a_{12}+2a_{14}+2a_{15}}{5}$ for month 14, and $\frac{-a_{13}+2a_{14}+4a_{15}}{5}$ for month 15. The movement following the peak (or trough) is projected backward in a similar manner, which results in a five-month moving average of $\frac{2a_{15}+2a_{16}+a_{18}}{5}$ for month 16, and $\frac{4a_{15}+2a_{16}-a_{17}}{5}$ for month 15. Since two values for the peak (or trough) month are obtained by this process, we take their average: $\frac{-a_{13}+2a_{14}+8a_{15}+2a_{16}-a_{17}}{10}$. This value is not likely to differ much from the actual value at the peak or trough. It may even exceed the former and fall short of the latter. For example, it will exceed the actual value at the peak if $a_{14}-a_{13}>a_{15}-a_{14}$ and $a_{16}-a_{17}>a_{15}-a_{16}$. In such cases we have used the formula $\frac{2a_{14}+8a_{15}+2a_{16}}{12}$. Obviously a makeshift, it emphasizes the difficulty of making sensible projections.

An alternative method is to project forward a straight line connecting $\frac{a_{10}+a_{11}+a_{12}}{3}$ and $\frac{a_{13}+a_{14}+a_{15}}{3}$ to obtain hypothetical values for months 16 and 17, and to project backward a straight line connecting $\frac{a_{15}+a_{16}+a_{17}}{3}$ and $\frac{a_{18}+a_{19}+a_{20}}{3}$ to obtain hypothetical values for months 13 and 14. This method raises slightly the standing at the peak of the pattern of iron production, and reduces slightly the standing at the terminal trough.

CHART 48

Special and Standard Average Cyclical Patterns
Pig Iron Production, United States



For special patterns, see Table 127 and accompanying text. The standard pattern is our usual nine-point pattern based on the raw data; see Tables 122 and 128.

larger amplitude of the special pattern.[48] Since both patterns are based on turning points dated from the raw data, they share the defect of a chronological framework that is somewhat warped by erratic fluctuations. To reduce this difficulty, the method could be modified so that cyclical turns marred by erratic fluctuations would be dated from, say, a five-month moving average.

The 'special' pattern represents one of many experiments that might well be undertaken to improve the measurements of cyclical patterns we are now making. Further work in this direction is desirable in view of the importance of economic thinking in terms of the rate of change of different variables from stage to stage of business cycles.

VII Reference-cycle Patterns

In the course of the preceding analysis we have seen that our method of measuring specific-cycle patterns eliminates erratic movements imperfectly, and occasionally imparts a systematic twist to the patterns. If the original data were subjected to smoothing by Macaulay's formula before making cyclical patterns, erratic movements would be wiped out, but the cyclical fluctuations would be distorted with intolerable frequency. The defects of both methods count for less in reference-cycle than in specific-

[48] The reason for this difference is apparent from note 47. In the special pattern the standing at the peak (trough) stage is dominated by the actual value at the peak (trough). In the standard pattern the actual value at the peak (trough) gets a weight no larger than that of the two adjacent months.

TABLE 128

Average Reference-cycle Patterns of Raw and Smoothed Data

Four American Series

Series and form of data	No. of reference cycles	I Initial trough	II Expansion: First third	III Expansion: Middle third	IV Expansion: Last third	V Peak	VI Contraction: First third	VII Contraction: Middle third	VIII Contraction: Last third	IX Terminal trough
		Average in reference-cycle relatives at stage								
DEFLATED CLEARINGS										
Raw	15	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Smoothed	15	88.6	93.1	98.9	105.2	107.5	106.2	102.8	100.1	101.5
PIG IRON PRODUCTION										
Raw	15	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Smoothed	15	77.8	88.6	103.8	113.8	119.8	117.1	99.3	85.0	86.7
RAILROAD STOCK PRICES										
Raw	19	91.0	96.9	104.0	109.4	106.9	104.3	97.7	92.5	94.7
Smoothed	19	91.7	96.4	104.7	109.3	107.5	104.6	97.8	93.0	95.7
CALL MONEY RATES										
Raw	19	77.5	82.4	98.4	128.2	159.5	128.5	103.9	81.1	76.2
Smoothed	19	78.4	82.8	101.0	127.2	138.6	128.7	99.7	82.5	76.6
		Excess of the raw average over the smoothed								
Deflated clearings	15	−0.5	+0.9	−0.5	0.0	0.0	+0.5	−0.5	−0.6	−0.9
Pig iron production	15	−4.5	+1.4	−0.3	−1.3	+2.4	+0.5	+1.1	−0.2	−5.6
Railroad stock prices	19	−0.7	+0.5	−0.7	+0.1	−0.6	−0.3	−0.1	−0.5	−1.0
Call money rates	19	−0.9	−0.4	−2.6	+1.0	+20.9	−0.2	+4.2	−1.4	−0.4

See Chart 49 for the periods covered. Three-month standings are used in stages I, V and IX of both the raw and smoothed patterns. The use of 1-month standings in the smoothed specific-cycle patterns was necessary to obtain a valid measure of amplitude, a reason that has no force in the case of reference cycles.

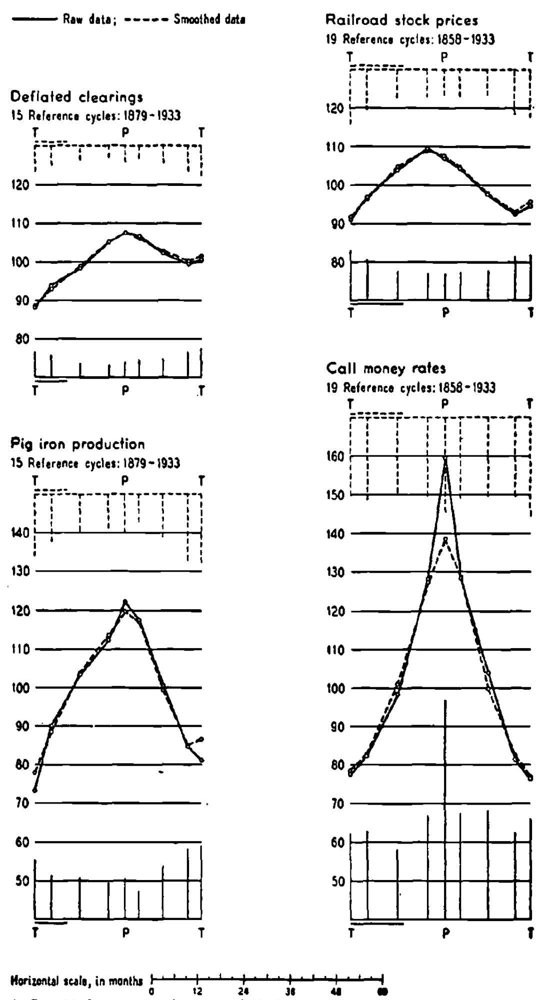
cycle patterns. In the first place, since the months entering into each reference-cycle stage are the same in the raw and smoothed analyses, there can be no differences in duration, whether due to erratic movements or the perversity of smoothing, such as occur in the specific cycles.[49] In the second place, each distortion of the tempo of cyclical movements by smoothing, instead of being concentrated in the same stage of different specific cycles, is dispersed over several stages of the reference cycles—the dispersion being large or small according as the differences in time between the specific and reference cycles are considerable or slight. The like is true of any systematic distortions of cyclical movements that may be produced by our standard method.

The average reference-cycle patterns of the raw and smoothed data of our four series are shown in Tables 128-129 and Chart 49. As expected, the differences between the standings of the two sets of patterns are, on the whole, considerably smaller than the corresponding differences be-

49 Of course, reference dates determined from smoothed data would not match exactly our present reference dates. But it is neither feasible nor (if the analysis of Sec. III is sound) desirable to investigate the influence of smoothing on cyclical measures *via* its influence on reference dates. The practical question is how smoothed data behave relatively to raw data within the periods of reference cycles, as now marked off.

CHART 49

Average Reference-cycle Patterns of Raw and Smoothed Data
Four American Series



See Table 128. For explanation of chart, see Ch. 5, Sec. VII.

tween the specific-cycle patterns. Of course, the rates of change from stage to stage of the reference cycles differ more in proportion to the size of the figures than do the corresponding standings of the patterns, but they differ less than the rates of change from stage to stage of specific cycles. We no longer find a tendency for the differences between the rates of change in the raw and smoothed data to be larger during contractions than during expansions. There is still a tendency for the rates of change to differ more in the turning intervals than in the others, but it is not as

TABLE 129

Average Rate of Change from Stage to Stage of Reference Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Change per month in reference-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Expansion				Contraction			
		Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
		Unweighted averages							
DEFLATED CLEARINGS									
Raw	15	+1.4	+0.5	+0.9	+0.6	−0.2	−0.9	−0.3	+0.5
Smoothed	15	+1.0	+0.8	+0.8	+0.5	−0.3	−0.6	−0.3	+0.5
PIG IRON PRODUCTION									
Raw	15	+3.7	+2.2	+1.1	+2.9	−1.2	−2.7	−3.0	−1.5
Smoothed	15	+2.2	+2.1	+1.6	+1.6	−0.7	−3.0	−2.4	+0.3
RAILROAD STOCK PRICES									
Raw	19	+1.3	+0.7	+0.7	−0.4	−1.0	−0.7	−0.5	+0.6
Smoothed	19	+1.0	+0.9	+0.6	−0.3	−0.8	−0.8	−0.3	+0.6
CALL MONEY RATES									
Raw	19	+1.4	+2.0	+4.8	+6.8	−3.2	−4.2	−5.1	−0.8
Smoothed	19	+1.2	+2.7	+3.8	+3.1	−1.0	−4.5	−3.7	−1.7
		Weighted averages							
DEFLATED CLEARINGS									
Raw	15	+1.4	+0.6	+0.9	+0.5	−0.2	−0.7	−0.4	+0.3
Smoothed	15	+1.0	+0.8	+0.8	+0.5	−0.4	−0.5	−0.4	+0.4
PIG IRON PRODUCTION									
Raw	15	+3.9	+1.8	+1.2	+2.3	−1.3	−2.7	−2.5	−1.0
Smoothed	15	+2.5	+2.1	+1.4	+1.4	−0.7	−2.8	−2.3	+0.5
RAILROAD STOCK PRICES									
Raw	19	+1.3	+0.9	+0.7	−0.6	−0.6	−0.9	−0.7	+0.5
Smoothed	19	+1.0	+1.1	+0.6	−0.4	−0.7	−1.0	−0.7	+0.7
CALL MONEY RATES									
Raw	19	+1.1	+2.0	+3.8	+6.9	−7.8	−3.5	−3.2	−1.2
Smoothed	19	+1.0	+2.3	+3.3	+2.5	−2.5	−4.1	−2.4	−1.5

See note to Table 128.

pronounced as in the specific cycles. Moreover, as Table 130 demonstrates, the ranking of the rates of change from stage to stage of the reference cycles in the smoothed data is very similar to the ranking of the rates of change in the raw data—a condition not found in the specific cycles.

It would not be unreasonable to infer from the similarity between the patterns of the raw and smoothed data that erratic movements play a relatively minor role in reference-cycle patterns made by our standard method. But there is stronger evidence to support this conclusion. If the month-by-month differences between the raw data and Macaulay's graduation were akin to a series of random numbers, we would expect the differences to be uncorrelated with business cycles. This expectation may

TABLE 130

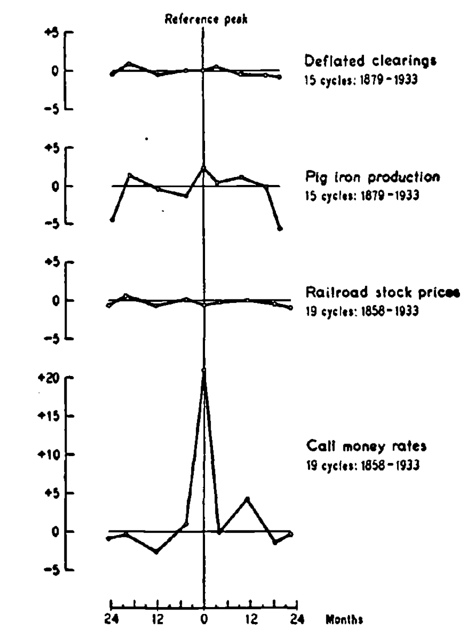
Ranks of Average Rates of Change from Stage to Stage of Reference Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Rank of average change per month between reference-cycle stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
DEFLATED CLEARINGS									
Raw	15	1	3	2	4	6	8	7	5
Smoothed	15	1	3	2	4	6	8	7	5
PIG IRON PRODUCTION									
Raw	15	1	3	4	2	6	8	7	5
Smoothed	15	1	2	4	3	6	8	7	5
RAILROAD STOCK PRICES									
Raw	19	1	2	3	5	6	8	7	4
Smoothed	19	2	1	4	5	7	8	6	3
CALL MONEY RATES									
Raw	19	4	3	2	1	8	7	6	5
Smoothed	19	4	3	1	2	7	8	6	5

Derived from weighted average rates, given in the lower section of Table 129. A rank of 1 is assigned to the highest figure, a rank of 8 to the lowest (taken algebraically). In case of a tie the computations were carried to an additional place before ranking.

CHART 50

Differences between Average Reference-cycle Patterns of Raw and Smoothed Data
Four American Series



The vertical scale shown here is twice the standard scale used in other charts.
See Table 128 and Chart 49.

TABLE 131

Average Patterns of Small Groups of Reference Cycles
Four American Series, Raw and Smoothed

Series, period and form of data	No. of reference cycles	Average in reference-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
Deflated Clearings										
1879–1897										
Raw	5	85.0	91.4	96.6	105.8	108.6	108.6	102.4	100.3	102.0
Smoothed	5	85.4	90.6	97.1	106.1	108.3	106.9	103.2	100.8	103.1
1897–1914										
Raw	5	87.8	93.7	96.9	102.8	105.3	104.6	101.7	100.0	102.2
Smoothed	5	87.9	92.7	97.6	102.7	105.3	104.4	102.0	100.5	102.3
1914–1933										
Raw	5	91.6	97.0	101.8	106.9	108.5	106.9	102.9	98.3	97.7
Smoothed	5	92.6	95.9	102.2	106.7	108.8	107.2	103.1	99.0	99.2
Pig Iron Production										
1879–1897										
Raw	5	67.9	89.1	99.8	110.8	117.3	113.1	97.8	91.9	92.0
Smoothed	5	73.7	86.1	101.9	112.1	115.9	110.9	96.6	93.4	98.8
1897–1914										
Raw	5	72.5	88.3	99.4	109.7	117.5	115.8	100.0	86.7	85.4
Smoothed	5	75.4	86.6	100.7	110.2	117.0	114.9	98.8	87.1	90.1
1914–1933										
Raw	5	79.5	92.7	111.2	117.0	131.9	124.0	103.4	75.9	66.0
Smoothed	5	84.3	93.2	108.6	119.0	126.3	125.5	102.5	74.5	71.2
Railroad Stock Prices										
1858–1888										
Raw	6	82.8	89.0	101.5	112.0	106.2	105.6	102.8	95.3	96.5
Smoothed	6	83.8	88.7	101.9	111.5	107.8	106.7	102.3	95.8	98.0
1888–1908										
Raw	6	89.5	96.8	101.5	107.7	105.8	102.3	92.5	93.7	98.7
Smoothed	6	90.4	96.0	102.3	107.4	105.8	101.3	93.5	94.2	99.3
1908–1933										
Raw	7	99.3	103.7	108.4	108.5	108.4	105.0	97.8	89.1	89.6
Smoothed	7	99.5	103.3	109.1	109.0	108.8	105.6	97.6	89.6	90.8
Call Money Rates										
1858–1888										
Raw	6	80.4	90.6	106.4	139.3	193.7	116.5	80.0	83.8	76.3
Smoothed	6	81.7	92.2	106.1	135.4	133.7	113.3	88.7	81.6	75.8
1888–1908										
Raw	6	67.3	71.1	88.4	107.8	145.1	151.4	137.4	58.8	63.2
Smoothed	6	63.6	70.8	89.4	114.0	142.5	152.8	113.0	69.1	61.2
1908–1933										
Raw	7	83.8	85.1	100.2	136.3	142.5	119.3	95.7	98.0	87.3
Smoothed	7	88.2	85.1	106.6	131.5	139.5	121.2	97.8	94.7	90.4

The periods mark off successive thirds of the reference cycles covered by a series. Where the full number is not exactly divisible by 3, the odd item is placed in the last period. Three-month standings are used in stages I, V and IX of both the raw and smoothed patterns.

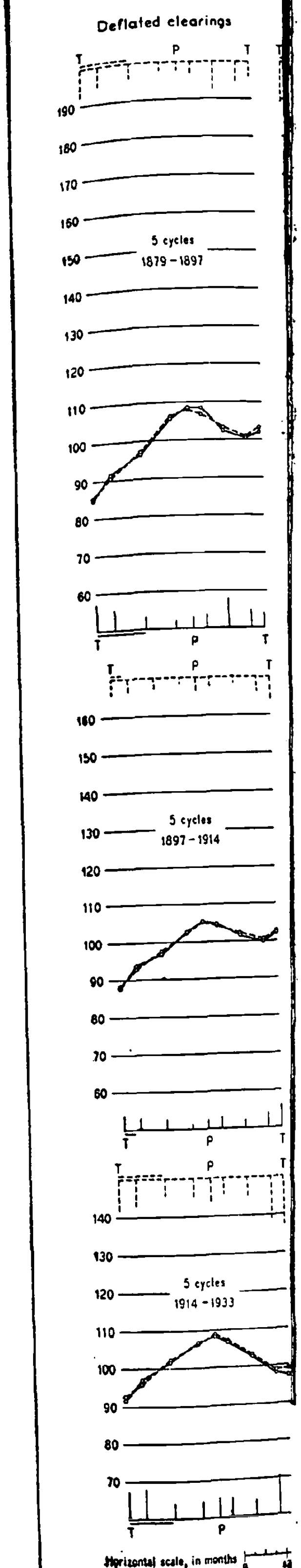
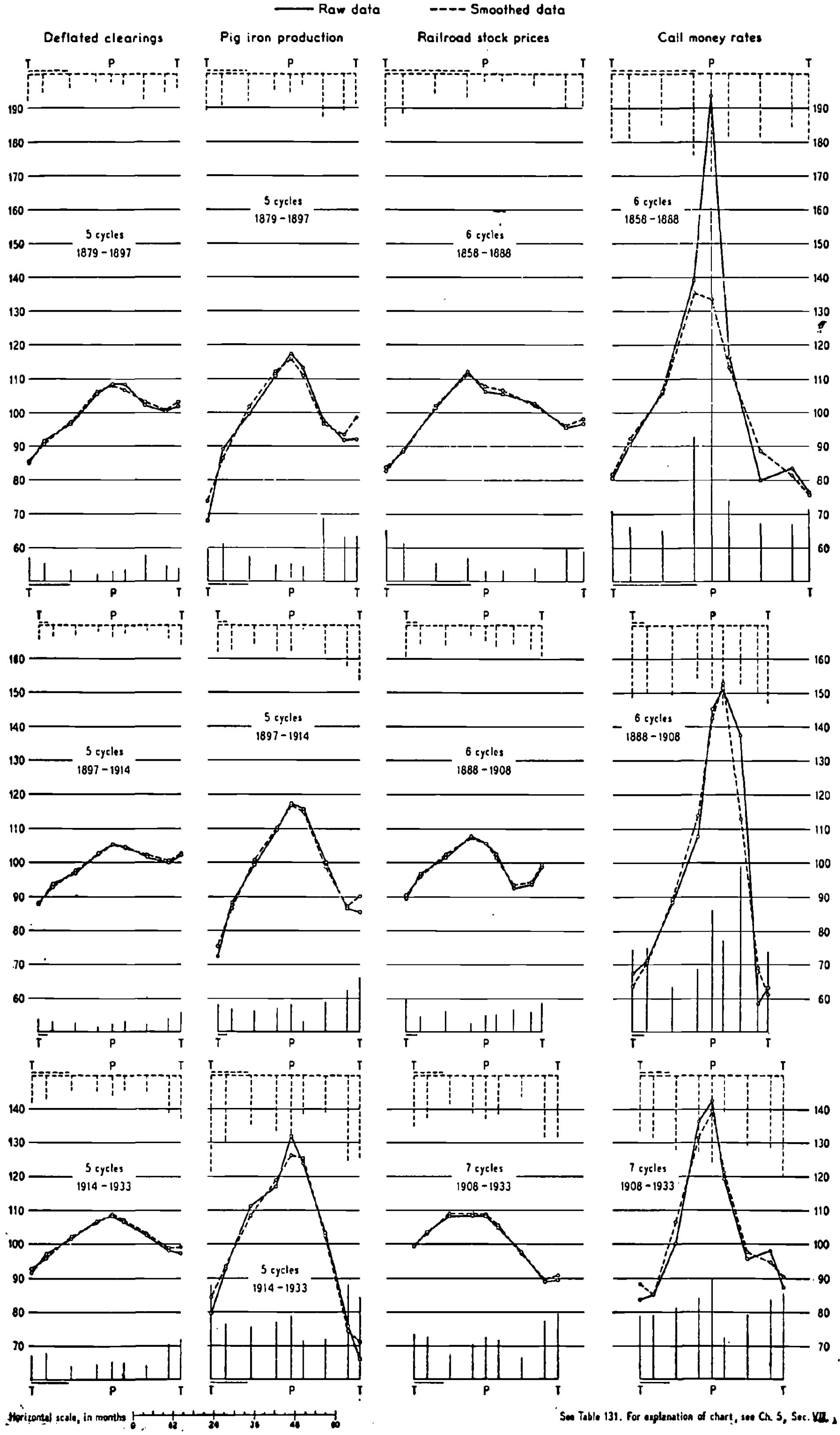


CHART 51

Average Patterns of Small Groups of Reference Cycles
Four American Series, Raw and Smoothed



See Table 131. For explanation of chart, see Ch. 5, Sec. VII.

be tested by constructing reference-cycle patterns of the differences between the raw and smoothed data, as we do in Chart 50.[50] The patterns are irregular in the expansion and contraction stages, as they should be on the hypothesis of randomness; but in each series the standing of the differences is higher at the reference peak than at the reference troughs. The margin is slight in railroad stock prices, considerable in the other series. This striking feature appears in the reference cycles taken singly, not only in the averages, as indexes of full-cycle conformity for the differences demonstrate:[51] deflated clearings +38, iron production +93, railroad stock prices +32, call money rates +53. The marked conformity of the raw-smoothed differences to business cycles confirms a judgment voiced repeatedly in preceding pages: namely, that smoothing by Macaulay's formula or by similar devices, besides eliminating erratic movements, tends to distort cyclical fluctuations. It seems reasonable to infer that whatever erratic components remain in the raw reference-cycle patterns are smaller than the differences between the raw and smoothed patterns.

Of course, the conclusion that erratic movements play a relatively minor role in the reference-cycle patterns must be interpreted with care. It is more likely to be true of our standard Table R1 than of Table R2. The conclusion does not apply to patterns of single reference cycles, and may not apply to average patterns that are based on few cycles, especially if the erratic movements of a series are pronounced. Our comparisons between raw and smoothed patterns have been made from series that cover from 15 to 19 reference cycles. But as Chapter 1 shows, relatively few of our time series are so long; a large fraction cover no more than five cycles and many cover a smaller number. The degree of mutual canceling of erratic movements is likely to be smaller in reference-cycle patterns based on a half-dozen observations than in patterns based on three times that number. To throw some light on this matter, Table 131 and Chart 51 present raw and smoothed patterns for successive thirds of the reference cycles in our four long test series. In a few instances the differences between the raw and smoothed patterns run much larger than any encountered in the patterns for all cycles. There are, moreover, some irregularities in the subgroup patterns of raw data, whereas none occur in the patterns based on all cycles.[52] On the whole, however, the reduction in the

[50] Strictly speaking, the chart shows the differences between the reference-cycle patterns of the raw and smoothed data (see the bottom section of Table 128). This is not a mathematical equivalent of reference-cycle patterns of the differences between the raw and smoothed data. However, the shapes of the patterns yielded by the two methods are very similar. The method used in the chart avoids the need to express the results for different series in the original units.

[51] The indexes of conformity were computed, on the plan of Table R3, from the differences between the raw and smoothed data, with the former as minuend.

[52] A reference-cycle pattern may be considered 'irregular' if it contains more than one peak and trough. A stricter definition would be that the pattern is irregular if any movement runs counter

number of cycles included in the patterns makes less difference than might have been expected.

As in the case of specific cycles, our standard method yields only nine-point patterns for reference cycles. The lack of greater detail would be a source of some embarrassment in tracing the sequence of revivals and recessions in different activities, were it not for the monthly timing measures in Table S1. Even so, greater refinement in the reference-cycle patterns would be an advantage if the additional detail were trustworthy. Table 127 and Chart 48 present the results of an experiment with a nineteen-point pattern. The 'special' pattern for reference cycles was made on exactly the same principle as the special pattern for specific cycles, described towards the end of Section VI. The special reference-cycle pattern is somewhat less regular than the standard pattern. It suggests, however, a slight lead (2.2 months) of pig iron production at the reference trough, which is confirmed by the measure of average timing in our Table S1 (3.4 months).

VIII Measures of Conformity to Business Cycles

In view of the comparatively small differences between the reference-cycle patterns of raw and smoothed data, we should expect their measures of conformity to be similar. Table 132 shows that in each series smoothing by Macaulay's formula tends to reduce (numerically) the average rates of change during the stages matched with reference expansions and with reference contractions. But the differences made by smoothing run smaller in these measures than in the average rates of change during expansions and contractions of specific cycles.

Of the 12 indexes of conformity, smoothing leaves 7 unchanged, raises 2 and lowers 3.[53] None of the changes are of much consequence. Some improve the measure of conformity slightly, others worsen it. In clearings, for example, the smaller contraction index of the smoothed data is due to the virtual disappearance of the decline in 1918, which we consider a genuine cyclical contraction. In call money rates, on the other hand, the expansion index is higher in the smoothed data, in part because erratic movements in the raw data are very prominent in the vicinity of the boundaries of the reference expansion of 1891–93.

---

to the division of stages in our standard Table R4. Note that while irregular movements in specific-cycle patterns are always indicative of erratic movements, subject only to the qualifications in note 35 of Ch. 5 and note 32 of this chapter, a reference-cycle pattern may be irregular for two reasons: (1) erratic movements in the raw data, (2) erratic behavior of specific cycles with respect to business cycles.

53 The count is 6, 2 and 4, if indexes based on stages VIII-V are used for the smoothed data on iron production.

TABLE 132

Conformity to Business Cycles of Raw and Smoothed Data

Four American Series

Series and form of data	No. of reference cycles[a]	Stages matched with reference expansion	Average change per month in reference-cycle relatives during stages matched with reference: Expansions	Contractions	Index of conformity to reference: Expansions	Contractions	Cycles
DEFLATED CLEARINGS							
Raw	15	VIII-V	+0.78	−0.50	+100	+73	+86
Smoothed	15	VIII-V	+0.75	−0.39	+100	+60	+93
PIG IRON PRODUCTION[b]							
Raw	15	I-V	+2.26	−2.27	+100	+100	+100
Smoothed	15	I-V	+1.87	−1.79	+100	+87	+100
Smoothed	15	VIII-V	+1.61	−2.26	+87	+87	+100
RAILROAD STOCK PRICES							
Raw	19*	VIII-IV	+0.77	−0.61	+79	+60	+74
Smoothed	19*	VIII-IV	+0.73	−0.57	+79	+60	+74
CALL MONEY RATES							
Raw	19*	I-V	+3.62	−3.57	+68	+100	+100
Smoothed	19*	I-V	+2.83	−3.07	+89	+80	+100

[a] See Chart 49 for the periods covered; but note that where an asterisk appears, the contraction and full-cycle indexes cover an additional reference contraction (1857–58) at the beginning of the series.

[b] The conformity measures for the smoothed data are shown for two sets of intervals because there is little to choose between them. See pp. 195-6.

In Table 133 we show separate indexes of conformity for each of the eight intervals into which Table R2 divides reference cycles. These indexes are computed in the same way as our standard measures. In the group of stages matched with reference expansions, the conformity index for a given interval (for example, stage I to II in call money rates, or stage VIII to IX in clearings) is the excess of the number of rises over the number of falls, expressed as a percentage of the number of cycles. In the group of stages matched with reference contractions, a fall signifies positive conformity and a rise inverted conformity; hence the excess of the number of falls over the number of rises for a given interval is expressed as a percentage of the number of cycles. Of course this method of computation reduces the indexes; for a movement opposed to the cyclical tide in any one of the three to five stages that are grouped together as 'expansions' or 'contractions' will affect the entries in the present table, whereas in the broader averages of Table 132 that movement is often more than offset by conforming changes in the preceding or following stage. In these stage-by-stage indexes, the smoothed figures stand higher than the raw figures in 17 cases, the same in 7 cases and lower in 8. This drift in the results is to be expected; for smoothing tends to eliminate erratic movements counter to the cyclical tide, which reduce the raw indexes.

TABLE 133

Stage-by-stage Indexes of Conformity to Business Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Stages matched with reference expansion	Index of conformity of changes between reference-cycle stages								Average index of conformity		
			I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX	Turning zones[b]	Other intervals	All intervals
DEFLATED CLEARINGS													
Raw	15	VIII-V	+100	+80	+87	+47 *	+33 *	+33	+13 *	+73 *	+42	+75	+58
Smoothed	15	VIII-V	+100	+100	+100	+73 *	+33 *	+47	+20 *	+87 *	+53	+87	+70
PIG IRON PRODUCTION[a]													
Raw	15	I-V	+87 *	+100	+60	+100 *	+20 *	+73	+73	+47 *	+63	+77	+70
Smoothed	15	I-V	+87 *	+87	+73	+73 *	+7 *	+73	+73	−7 *	+40	+77	+58
Smoothed	15	VIII-V	+87	+87	+73	+73 *	+7 *	+73	+73 *	+7 *	+40	+80	+60
RAILROAD STOCK PRICES													
Raw	19	VIII-IV	+89	+68	+37 *	+16 *	+26	+16	+16 *	+16 *	+21	+50	+36
Smoothed	19	VIII-IV	+58	+79	+47 *	+26 *	+53	+63	+5 *	+5 *	+21	+63	+42
CALL MONEY RATES													
Raw	19	I-V	+16 *	+47	+58	+58 *	+26 *	+79	+16	+26 *	+32	+50	+41
Smoothed	19	I-V	+16 *	+68	+79	+47 *	+26 *	+89	+47	+47 *	+34	+71	+53

See Chart 49 for the periods covered.

[a] See Table 132, note 'b'.

[b] Here treated as two-interval overlaps on the selected expansion and contraction segments of the series. Indexes of conformity for these intervals are marked by an asterisk (*) in the preceding columns.

The addition that Table 133 makes to our knowledge is that the conformity of a series to business cycles is higher outside than within its turning zones. Of course the turning zones vary from series to series. Two of the present four usually turn up in the last third of reference contraction, but call money rates and iron production defer the upturn until the reference trough. The downturn usually occurs in the last third of reference expansion in railroad stock prices, but not until the reference peak in the other series. In Table 133 the indexes for the turning zones are starred, and averages are shown of these four indexes and also of the four indexes outside the turning zones. Without exception, in both the raw and the smoothed data, the average for the turning zones is lower than the average for the other intervals.

We attribute the low conformity in the turning zones to irregularities in the timing of specific-cycle revivals and recessions. If a series is a little prompter or a little slower than usual in responding to a cyclical turn in general business, its turn may not come in the usual reference-cycle stage. That is true whether a series is analyzed in raw or smoothed form. Since our plan of computing indexes of conformity assigns the same stages of reference cycles to expansion in all the cycles covered by a series, and other fixed stages to contraction, any departure from the timing of cyclical turns characteristic of the series as a whole may affect our standard indexes in Table 132, and is much more likely to affect the stage-by-stage indexes of Table 133 that cover briefer periods. Smoothing tends to increase the stage-by-stage indexes outside the turning zones, but has no systematic influence on the indexes within the turning zones.

TABLE 134
Average Deviations from Average Cyclical Measures
Four American Series, Raw and Smoothed

Measure	Deflated clearings		Pig iron production		Railroad stock prices		Call money rates	
	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
(I) DURATION OF SPECIFIC CYCLES								
Expansion	9.9	9.3	9.0	8.8	16.0	15.1	7.6	6.4
Contraction	6.5	4.6	7.1	6.4	12.1	10.4	6.4	6.7
Full cycle	11.6	10.6	9.9	9.7	22.6	21.3	8.6	8.0
Expansion as % of cycle	11	9	13	14	13	11	14	11
(II) AMPLITUDE OF SPECIFIC CYCLES								
Rise	10.6	9.9	15.7	16.1	20.5	20.5	52.0	33.2
Fall	8.7	8.0	21.2	23.0	17.0	15.9	56.7	40.7
Rise & fall	13.8	13.1	26.8	26.6	35.9	34.6	105.9	67.2
Rise per month	0.2	0.2	0.9	0.9	0.6	0.4	3.3	1.5
Fall per month	1.7	0.3	2.1	1.3	0.8	0.5	4.0	2.3
Rise & fall per month	0.2	0.2	0.9	0.9	0.4	0.4	2.4	1.7
(III) SECULAR MOVEMENTS								
Change from								
Contraction to expansion	6.4	5.6	12.6	11.2	14.0	14.6	15.8	16.8
Expansion to contraction	8.2	7.3	15.0	13.4	10.4	9.4	17.3	16.8
Cycle to cycle, total	6.2	5.8	17.4	15.1	16.7	16.7	23.8	23.6
Cycle to cycle, per month	0.15	0.13	0.40	0.33	0.36	0.36	0.68	0.68
(IV) SPECIFIC-CYCLE PATTERN								
Standing								
I	5.5	5.3	14.6	15.3	11.6	12.3	13.7	16.6
II	4.2	4.2	12.1	10.7	9.6	9.9	12.3	14.5
III	2.0	2.1	9.5	7.4	5.3	4.4	16.3	10.7
IV	2.8	2.3	7.5	8.3	5.4	5.1	16.8	13.1
V	5.9	4.9	11.2	8.7	9.5	8.6	41.8	22.7
VI	5.2	4.4	7.9	6.7	8.4	7.3	18.1	16.1
VII	6.6	5.6	11.9	8.5	7.3	5.8	20.7	12.8
VIII	8.8	8.3	14.6	15.9	9.4	9.2	18.9	18.6
IX	10.6	9.1	17.8	18.6	12.3	11.5	19.5	21.2
Rate of change								
I-II	0.5	0.2	1.6	0.7	0.8	0.3	4.1	0.8
II-III	0.4	0.3	1.2	1.1	0.8	0.6	6.2	1.7
III-IV	0.3	0.3	1.1	1.5	1.1	0.5	7.9	2.6
IV-V	0.4	0.4	1.4	0.7	0.8	0.4	11.7	2.6
V-VI	2.6	0.3	1.3	0.9	1.1	0.3	14.6	2.7
VI-VII	3.3	0.4	2.4	2.1	0.9	0.7	5.4	4.2
VII-VIII	2.0	0.4	5.2	1.5	1.2	0.8	3.7	2.7
VIII-IX	1.3	0.2	3.9	0.6	1.4	0.4	2.7	0.8
(V) REFERENCE-CYCLE PATTERN								
Standing								
I	6.6	6.8	15.5	16.1	13.8	14.1	22.3	20.3
II	5.7	5.2	11.4	12.2	10.7	10.8	22.8	21.3
III	3.7	3.8	10.9	9.7	7.7	7.5	18.2	19.4
IV	3.2	3.2	9.4	9.0	7.0	7.4	26.9	20.3
V	4.0	4.2	10.7	10.1	7.0	6.7	56.9	24.6
VI	4.5	3.5	7.3	7.2	7.4	7.2	27.6	21.1
VII	4.8	4.8	13.9	11.3	7.9	6.9	28.3	20.6
VIII	6.6	6.9	18.2	17.2	11.5	11.8	22.4	20.5
IX	7.6	7.8	19.0	17.9	12.2	12.3	26.0	25.8

TABLE 134—*Continued*
Average Deviations from Average Cyclical Measures
Four American Series, Raw and Smoothed

Measure	Deflated clearings		Pig iron production		Railroad stock prices		Call money rates	
	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
(V) REFERENCE-CYCLE PATTERN—*Cont.*								
Rate of change								
I-II	0.5	0.3	2.2	1.3	0.8	0.9	4.8	3.2
II-III	0.3	0.2	1.3	1.1	0.8	0.7	2.7	2.6
III-IV	0.4	0.3	1.9	1.3	0.9	0.8	4.3	2.2
IV-V	0.6	0.4	2.1	1.0	1.4	1.1	9.2	3.8
V-VI	0.7	0.6	2.1	2.4	1.5	0.9	10.5	4.4
VI-VII	1.0	0.6	2.8	1.7	1.4	1.0	6.7	3.1
VII-VIII	0.5	0.5	2.7	1.9	1.1	0.9	7.4	4.2
VIII-IX	0.6	0.5	2.6	1.7	1.2	0.9	4.1	2.7
(VI) CONFORMITY								
Rate of change[a] during reference								
Expansions	0.18	0.16	0.87	0.79[b]	0.57	0.60	2.37	1.38
Contractions	0.44	0.45	1.35	1.22[b]	0.58	0.61	2.08	2.09

The entries in group (I) are expressed in months, except on the last line. The entries in groups (II) and (IV) are expressed in specific-cycle relatives, and in groups (V)-(VI) in reference-cycle relatives. The entries in group (III) correspond to the plan of computation in col. 5, 6, 9 and 10 of our standard Table S3 (see Table 33). The periods and numbers of cycles covered are given in Table 112 for the specific-cycle measures, in Chart 49 for the reference-cycle measures.

[a] That is, during the stages matched with reference expansion and contraction, shown in Table 132.
[b] Based on the assumption that stages I-V are characteristic of expansion. See Table 132, note 'b'.

IX Variability of Cyclical Measures

Table 134 shows in detail how smoothing by Macaulay's formula affects the average deviations about the averages given in the preceding tables, and Table 135 summarizes these effects. Smoothing as a rule reduces the average deviations. The reduction is, typically, some ten or twenty per cent, except in the measures of stage-to-stage changes, where the reduction is usually greater because these measures are especially susceptible to erratic disturbances.

The large average deviations that remain in the smoothed figures are among the more significant results established by this series of tests. One reason for the large average deviations is that while smoothing eliminates erratic movements that change direction every few months, it does not eliminate random movements that spread over years. Strictly speaking, smoothing merely redistributes the original values. In this redistribution the degree to which a movement in the raw data is changed depends upon the extent of its departure from the general level, its duration, and the formula used. As said above, high peaks or low troughs with a time base of two or three months may disappear in Macaulay's graduation or become converted into rather gentle undulations; movements of equal amplitude lasting two or three years are changed much less. Now there are random factors that continue in force for periods reckoned in years; for example, a great war. There are also random events that sometimes

TABLE 135

Effect of Smoothing on Average Deviations from Average Cyclical Measures

Measure	Number of obser-vations	Number of instances in which smoothing			Average ratio of average deviation of smoothed data to average deviation of raw[a]
		Reduces average deviation	Increases average deviation	Leaves average deviation unchanged	
I Duration of specific cycles	16	14	2	..	.91
II Amplitude of specific cycles	24	16	2	6	.82
III Secular movements	16	11	2	3	.94
IV Specific-cycle pattern					
Standing	36	25	10	1	.91
Rate of change	32	29	1	2	.50
V Reference-cycle pattern					
Standing	36	21	13	2	.94
Rate of change	32	29	2	1	.72
VI Conformity	8	4	4	..	.92
Total	200	149	36	15	.81

Derived from Table 134.

[a] The ratio of the average deviation of smoothed data to the average deviation of raw data was computed separately for each measure in each series. The ratios were then averaged for all measures assigned to a group.

recur in two or more successive years; for example, harvest failures. Smoothing by Macaulay's method will not remove the great bulges in American price and value series in 1862–67 and in 1915–21. It will moderate the effects of two bad harvests upon agricultural prices less than it will moderate the effects of speculative maneuvers associated with monthly crop reports, or even the effects of two bad seasons separated by a good season. Random effects of considerable size thus remain in the smoothed forms of the series we have used in our tests, and contribute toward making the average deviations nearly as large in the results[54] obtained from the smoothed as in those obtained from the raw data.

All that Macaulay claims for smoothing is that it eliminates erratic movements. But some students of this problem have made much bolder claims. Anderson, for example, believes that his method of smoothing, which is based upon analysis of the variances of successive differences, makes possible the separation of the 'casual' from the 'essential' components of a time series.[55] But since Anderson's method proceeds on the assumption that 'casual' forces operate independently on each value of a time series, it does not cope with random perturbations that shape the general features of a time series over many months.[56]

[54] Except in the measures of stage-to-stage changes in cyclical patterns.

[55] See Oskar Anderson, Die Korrelationsrechnung in der Konjunkturforschung (*Veröffentlichungen der Frankfurter Gesellschaft für Konjunkturforschung,* Heft 4, 1929), particularly pp. 72-80.

[56] In *Prices in the Trade Cycle* (J. Springer, Vienna, 1935), Gerhard Tintner describes the Anderson method in nontechnical language and applies it to numerous price series. His own work, as he recognizes, is based upon approximations that do not make the fullest use of the Anderson criteria of graduation. Tintner merely uses an unweighted moving average containing from three to eleven monthly items, the period of the moving average being fixed for any one series. For a further development of Anderson's method, see Tintner's recent book, *The Variate Difference Method* (Principia Press, 1940).

X Uncertainties in Identifying Specific Cycles

Our analysis of specific cycles has been restricted by the assumption that Macaulay's graduation leaves the lists of specific cycles unchanged. This assumption is in a sense invalid. As Section II shows, if the specific cycles in smoothed data are marked off independently of the specific cycles in the raw data, the lists of cycles disagree in three of our four series. But it is neither necessary nor wise to follow so artificial a plan in practice. Whether an investigator prefers to make cyclical measures from raw or from smoothed data, he must try to pick 'real' specific cycles. The specific cycles identified in the raw data of the present sample were originally selected with the aid of simple moving averages passed through doubtful portions of the series, as is our usual practice. Study of the raw data can be no less helpful in this regard to other investigators who may prefer to make cyclical measures from smoothed data. Thus viewed, the correspondence forced in Section II between the cycles in raw and smoothed data is more than an analytic convenience; it is a device for making the analysis run, as we best can, in terms of 'real' cycles.

But 'real' cycles can be very elusive. Since some uncertainty surrounds the selection of specific cycles in many series, the question of practical interest is how erratic movements affect cyclical averages, by leading us either to recognize a spurious cycle or to ignore a genuine cycle. The simplest way to illustrate these effects is to compare two sets of averages, one set being based on the cycles we actually recognize, the other on some modification of that list. In line with this plan, we may treat the mild contraction of 1918 in deflated clearings and in call money rates and that of 1887–88 in iron production, which are the least certain of the cyclical movements we recognize in the raw data, as samples of the uncertainties to which erratic movements frequently give rise in the process of identifying specific cycles. Table 136 compares our standard averages of cyclical duration and amplitude both for all cycles and small groups of cycles with averages in which one cycle instead of two is taken during 1885–91 in iron production, during 1914–21 in clearings, and during 1915–22 in call money rates. The standard averages are entered on the lines marked 'Raw I', those omitting a cycle on lines 'Raw II'.

To rationalize the influence of a dubious 'extra' cycle, let us assume, for simplicity, that the 'Raw II' averages are 'true', that the 'Raw I' averages mistakenly include an extra cycle, that the extra movement is mild and brief, and that it comes during a true expansion. It is obvious that the extra cycle must reduce the average duration of expansions; for the sum of the lengths of the expansions is reduced while their number is increased. It must likewise reduce the average duration of full cycles, since their number is larger while the sum of their lengths is unchanged.

TABLE 136

Effect of a 'Dubious' Cycle on Average Duration and Amplitude of Specific Cycles
Three American Series

Series, period, and form of analysis	No. of specific cycles	Average duration in months			Average in specific-cycle relatives					
					Amplitude of			Per month amplitude of		
		Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
DEFLATED CLEARINGS										
1878–1933										
Raw I	15	32.6	11.4	44.0	26.9	13.4	40.2	0.8	1.9	0.9
Raw II	14	35.2	11.9	47.1	28.4	14.1	42.4	0.8	2.0	0.9
1910–1933										
Raw I	5	36.8	17.0	53.8	23.7	17.6	41.3	0.7	1.1	0.8
Raw II	4	47.0	20.2	67.2	28.2	21.1	49.3	0.6	1.0	0.7
PIG IRON PRODUCTION										
1879–1933										
Raw I	15	28.8	14.5	43.3	62.1	54.8	116.8	2.4	4.7	2.9
Raw II	14	31.2	15.2	46.4	64.8	57.5	122.3	2.4	4.7	2.9
1879–1896										
Raw I	5	28.6	14.0	42.6	62.3	44.4	106.7	2.6	3.8	2.8
Raw II	4	37.0	16.2	53.2	72.1	51.3	123.4	2.7	3.7	2.7
CALL MONEY RATES										
1858–1931										
Raw I	23	19.9	18.0	37.9	115.9	116.1	232.0	6.9	7.6	6.3
Raw II	22	21.0	18.6	39.6	119.3	120.8	240.2	6.9	7.6	6.4
1904–1931										
Raw I	8	20.4	19.9	40.2	95.6	91.2	186.8	4.7	5.7	4.5
Raw II	7	23.9	22.1	46.0	103.4	102.5	205.9	4.5	5.5	4.5

'Raw I' stands for the raw data as treated in preceding tables; 'Raw II' for the raw data treated in the manner explained in the text.

The average duration of contractions, on the other hand, may or may not be modified; but it is likely to be lower, since the addition made by the extra cycle to the sum of contractions is likely to be below the 'true' average of this measure. For a similar reason, the average amplitudes of rise, fall, and combined rise and fall are likely to be lowered.[57] The per month amplitudes are affected differently. Consider, first, the fall per month. This average is as likely to be raised as lowered, since there is no more reason for supposing that the fall per month of the 'extra' cycle is below than for supposing that it is above the 'true' average. It may seem that the average rise per month, on the other hand, is more likely to be raised than lowered, for the average rate of rise during the two expansions carved out of one 'true' expansion is practically sure to exceed the rate of rise during the true expansion. But in order that the average rise per month for all cycles actually increase, the sum of the two rates during the true expansion must exceed the rate during this expansion by more than

[57] This and the following remarks concerning amplitudes ignore the complicating influence of specific-cycle bases.

TABLE 137

Effect of a 'Dubious' Cycle on Average Specific-cycle Patterns

Three American Series

Series, period, and form of analysis	No. of specific cycles	Average in specific-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
DEFLATED CLEARINGS										
1878–1933										
Raw I	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Raw II	14	84.8	90.0	98.7	107.0	113.2	109.2	106.1	101.9	99.1
1910–1933										
Raw I	5	88.3	92.1	100.8	107.4	112.0	107.2	104.1	97.6	94.4
Raw II	4	85.7	90.8	99.6	108.5	113.8	108.7	103.9	96.8	92.7
PIG IRON PRODUCTION										
1879–1933										
Raw I	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Raw II	14	65.9	82.3	103.6	117.3	130.8	123.7	108.8	88.3	73.3
1879–1896										
Raw I	5	63.1	78.7	101.5	112.7	125.5	121.4	112.9	96.6	81.1
Raw II	4	57.4	77.1	100.7	114.5	129.5	124.8	116.2	98.4	78.2
CALL MONEY RATES										
1858–1931										
Raw I	23	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Raw II	22	60.9	79.4	105.2	123.5	180.2	120.4	88.6	69.9	59.4
1904–1931										
Raw I	8	60.9	76.2	107.1	123.3	156.5	123.5	97.4	76.2	65.3
Raw II	7	57.0	73.1	108.8	124.1	160.4	123.8	94.2	70.8	57.8

See note to Table 136.

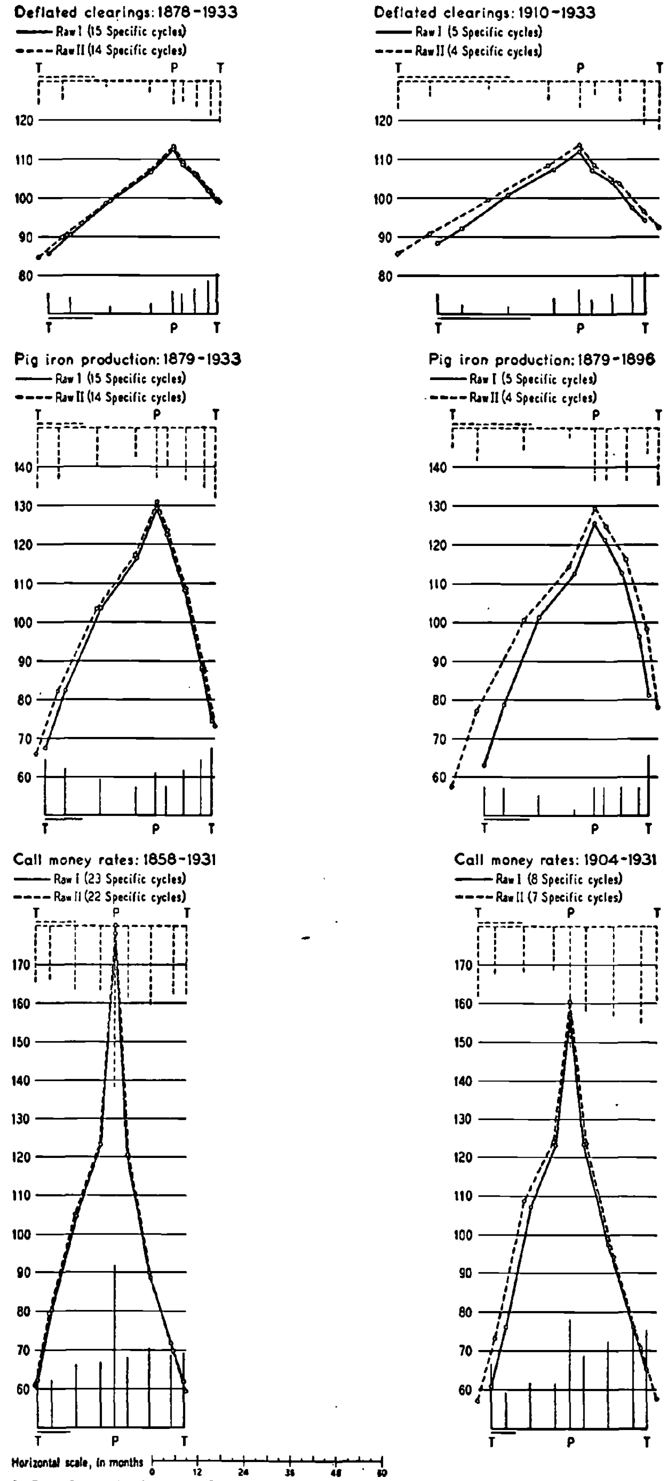
the true average rate. The average joint rise and fall per month is affected in much the same way as the average rise per month.[58]

These expectations could be worked out in greater detail and on altered assumptions, but that is not necessary. The significant results are indicated in Table 136. A single doubtful cycle affects average durations more than average amplitudes, and average total amplitudes much more than average per month amplitudes. When the number of cycles is rather large, the effect on all averages in the table is small. When the number of cycles is only about a half dozen, a single doubtful cycle naturally has larger effects. Even so, really prominent effects are confined to several measures of duration and total amplitude; the per month amplitudes are remarkably stable. Results of this character are fairly typical measures of the influence that doubtful decisions concerning specific cycles exercise on our averages. Of course, when erratic fluctuations are so violent that many cycles are obscured, the effects may be much greater than any in the table. Although such instances are relatively rare, they require cautious handling, and in later monographs we strive to call attention to them as they come up.

[58] Given the assumptions in the text, if the per month amplitudes are weighted by durations, the extra cycle will necessarily increase the average rise per month, and the average joint rise and fall per month, but not the average fall per month.

CHART 52

Effect of a 'Dubious' Cycle on Average Specific-cycle Patterns
Three American Series



See Table 137 and explanations in text. For construction of chart, see Ch. 5, Sec. VI.

TABLE 138
Effect of a 'Dubious' Cycle on
Average Rate of Change from Stage to Stage of Specific Cycles
Three American Series

Series, period, and form of analysis	No. of specific cycles	Change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Unweighted averages							
DEFLATED CLEARINGS 1910–1933									
Raw I	5	+0.5	+0.9	+0.6	+0.7	−1.4	−0.6	−1.3	−1.0
Raw II	4	+0.6	+0.6	+0.6	+0.7	−0.9	−1.6	−0.6	−1.2
PIG IRON PRODUCTION 1879–1896									
Raw I	5	+4.3	+3.2	+1.1	+2.5	−1.8	−2.6	−5.6	−5.4
Raw II	4	+4.9	+3.2	+1.1	+2.2	−1.9	−1.8	−4.8	−7.2
CALL MONEY RATES 1904–1931									
Raw I	8	+4.1	+4.8	+3.1	+8.1	−11.1	−5.5	−4.5	−2.8
Raw II	7	+4.1	+5.1	+2.9	+6.7	−11.2	−5.4	−3.4	−3.3
		Weighted averages							
DEFLATED CLEARINGS 1910–1933									
Raw I	5	+0.6	+0.7	+0.6	+0.7	−1.5	−0.6	−1.2	−1.0
Raw II	4	+0.6	+0.6	+0.6	+0.7	−1.4	−0.8	−1.1	−1.1
PIG IRON PRODUCTION 1879–1896									
Raw I	5	+3.1	+2.5	+1.2	+2.5	−1.6	−1.9	−3.6	−6.2
Raw II	4	+3.0	+2.0	+1.2	+2.3	−1.7	−1.6	−3.4	−7.0
CALL MONEY RATES 1904–1931									
Raw I	8	+4.1	+4.8	+2.5	+8.9	−9.1	−4.1	−3.4	−3.0
Raw II	7	+3.7	+4.7	+2.0	+8.3	−9.2	−4.2	−3.3	−3.2

See note to Table 136.

Tables 137-138 and Chart 52 supply comparisons of specific-cycle patterns on the plan of Table 136. The 'Raw I' pattern lies inside the 'Raw II' pattern, which merely means that its average duration and amplitude are both smaller. An 'extra' cycle sometimes makes a very considerable difference in the unweighted average rates of change from stage to stage of the specific cycles, but the weighted average rates are tolerably stable even when the number of cycles in the sample is only about half a dozen.

XI Conclusions

The reader who has gotten so far in this book must be sufficiently aware of the difficulties that surround our efforts to provide an authentic description of cyclical behavior. The 'facts' of cyclical experience do not tell their own story. They must be wrung from data by tools of observation. If the tools are coarse and the investigator is blind to their defects, his vision of cyclical fluctuations may be distorted. A serious investigator does not save time in the long run by neglecting to understand his tools or to improve upon them.

In this chapter we have examined the influence of erratic movements on our measures of cyclical behavior, and investigated the possibility of refining the measures through preliminary smoothing of the raw data by Macaulay's formula or some similar device. We have found that smoothing does indeed refine cyclical measures, in the sense of freeing them from the influence of erratic, saw-tooth movements of brief duration. But smoothing does not eliminate random movements that extend over years, and it frequently misrepresents cyclical fluctuations.

We do not claim that smoothing by so sensitive a graduation formula as Macaulay's would be valueless in our studies. On the contrary, we believe that it could be a material aid in identifying and, to a lesser extent, in dating specific cycles. Nor do we claim that cyclical measures made from smoothed data are necessarily less trustworthy than those yielded by our rough method. Theoretically, the results may be better. But if that end is to be attained, the investigator will usually need to vary the smoothing formula not only from series to series, but also from part to part of a given series. Furthermore, he will have to devise methods that avoid spurious rounding of cyclical tops and bottoms. Experimentation along these lines is highly desirable. But if we attempted it on any scale, we would be forced to confine our statistical analysis to a small fraction of the time series we can cover by our present methods. Experience has convinced us that knowledge of cyclical fluctuations based upon a small sample of time series is untrustworthy, and that as the range of activities analyzed becomes broader, the need for highly refined measurements is much reduced.[59]

Although our measures for individual cycles are marred by erratic fluctuations, it is important to observe that our technique involves a certain element of smoothing. In most measures made from monthly data, we use, not the entries for single months, but averages covering three or more months. For example, in making Tables S4 and R1, we determine the standing of the series at the initial trough, peak, and terminal trough of each cycle by averaging the cycle relatives for the three months centered on each of these dates. Also the standings during successive thirds of expansion and of contraction are averages for whatever months are included in these stages. Averaging of a similar sort is used in Table S2 which gives the amplitudes of specific cycles, in Table S3 which measures secular movements, in Tables S5 and R2 which show rates of change from stage to stage of specific and reference cycles, in Table R3 which shows rates of change during reference expansions and contractions, and in Table R4 which shows rates of change during the reference-cycle stages characteris-

[59] Cf. Ch. 3, Sec. III, and Ch. 12, Sec. V.

tic of expansions and contractions in each series. Indeed the only measures in which such averaging is not used are the measures of cyclical duration and of leads and lags in Table S1.

When averages are struck for all cycles covered by a series, the erratic factors in the measures for single cycles have an additional opportunity to cancel out. In the course of the tests in this chapter we have found that in some measures the averages made from raw data are practically indistinguishable from the averages made from Macaulay's graduations. In other measures the differences are very large; but these seem to be precisely the cases where Macaulay's formula represents cyclical movements least successfully. We believe, therefore, that so far as erratic movements do not obscure specific cycles, our averages, as a rule, are not gravely affected by erratic movements in the data from which they are made. In our judgment this statement holds for averages based on only half a dozen cycles as well as for averages based on many cycles, except perhaps the averages in Tables S5 and R2, so long as the erratic movements are not especially pronounced.

The tendency of erratic movements to obscure specific cycles is a more serious matter than their influence on the timing and outlines of the cycles that stand out clearly in the data. This factor is very troublesome in some series and its importance should not be minimized. But we also must not exaggerate this difficulty relatively to others—the defects of the original data in most economic time series, the roughness of many seasonal adjustments, the relatively short time span covered by most series, the marked variability of cyclical behavior. The effect upon our averages of uncertainties in identifying specific cycles is certainly no greater—as a rule it is probably smaller—than the effect of 'extreme' cycles that stand out clearly in the records, as the reader may judge by comparing the discrepancies between the averages in Table 136 with the discrepancies between averages based upon different periods or made by different methods, which appear in Tables 180, 186, 187 and 190. The influence of the period covered by a time series on average measures of its cyclical behavior is by far the most important question raised by our technique. The remaining four chapters are devoted to this intricate problem.

CHAPTER 9

Role of Averages in the Analysis

THE SUCCESSIVE volumes in the Bureau's *Studies in Business Cycles* stress average cyclical behavior. In passing from one economic process to another, we note briefly the scope and representative character of the original data, describe any departures from the usual mode of analysis, and call attention to doubtful features of the results. Although the behavior of a series during individual cycles is frequently described, the chief purpose is to point out what features of cyclical behavior are broadly characteristic of the economic activities we treat, to compare and contrast the behavior of the series representing different activities, and to suggest problems that will be encountered in constructing a theory of business cycles. The reader who does not go back to the cycle-by-cycle measures must judge the results mainly from averages, average deviations, and the number of cycles covered. To develop the meaning of the averages and average deviations is the principal task of this and the succeeding chapters.

I Variability of Cyclical Behavior

Like all scientific inquiries, our study of business cycles presupposes the 'uniformity of nature'. Of course, to allow generalization this uniformity need not be perfect. Physiologists are not debarred from making general statements by the differences among human bodies; for certain purposes they can even disregard differences between men and guinea pigs. Statistical methods such as we use are peculiarly adapted to treating phenomena that have a considerable range of variation. Yet we must face the difficulty that results are apt to become less trustworthy as the range of variation becomes larger and the number of observations fewer. This difficulty is especially acute in handling temporal sequences subject to cumulative changes. Relatively few of our time series cover as many as fifteen cycles, most series show great variation from cycle to cycle, and some appear to have experienced secular or cyclical changes in business-cycle behavior.

Table 139 shows how variable the durations of business cycles have been according to our reference dates. At the close of World War I we

TABLE 139
Average Duration of Business Cycles and Their Variability
Four Countries

Country and measure[a]	Reference dates used[b]	Period covered	No. of observations	Duration in months: Average	Average deviation	Standard deviation	Range	Coefficient of variation[c]
UNITED STATES								
Expansion..	M	1854–1929	20	25	8	9	9– 46	38
Contraction	M	1857–1933	20	22	10	14	8– 65	61
Full cycle								
T-T.....	M	1854–1933	20	47	13	18	29– 99	38
P-P......	M	1857–1929	19	45	13	17	17–101	38
T-T.....	A	1834–1888	12	54	21	23	24– 96	43
P-P......	A	1836–1890	12	54	17	22	24–108	40
T-T.....	A	1888–1932	13	41	9	11	24– 60	27
P-P......	A	1890–1929	12	39	8	10	24– 60	25
T-T.....	A	1834–1932	25	47	15	19	24– 96	41
P-P......	A	1836–1929	24	46	14	18	24–108	40
GREAT BRITAIN								
Expansion..	M	1854–1929	15	36	14	17	8– 64	47
Contraction	M	1857–1932	15	27	14	19	6– 81	72
Full cycle								
T-T.....	M	1854–1932	15	62	22	28	26–135	46
P-P......	M	1857–1929	14	62	26	32	17–123	51
T-T.....	A	1793–1858	14	56	15	17	36– 84	31
P-P......	A	1792–1857	14	56	17	21	36–108	38
T-T.....	A	1858–1932	14	63	23	28	24–132	45
P-P......	A	1857–1929	14	62	26	30	24–120	49
T-T.....	A	1793–1932	28	60	18	24	24–132	40
P-P......	A	1792–1929	28	59	22	26	24–120	44
FRANCE								
Expansion..	M	1865–1930	15	31	12	15	8– 62	48
Contraction	M	1867–1932	15	23	13	16	8– 68	72
Full cycle								
T-T.....	M	1865–1932	15	53	19	23	24– 95	43
P-P......	M	1867–1930	14	53	20	26	24–110	49
T-T.....	A	1840–1932	19	58	21	25	24–108	43
P-P......	A	1847–1930	18	55	19	24	24–120	44
GERMANY								
Expansion..	M	1879–1929	10	37	10	13	16– 61	37
Contraction	M	1882–1932	10	27	15	17	12– 61	63
Full cycle								
T-T.....	M	1879–1932	10	64	20	23	28–102	36
P-P......	M	1882–1929	9	63	22	27	34–122	43
T-T.....	A	1866–1932	12	66	22	25	24– 96	37
P-P......	A	1869–1929	11	65	27	31	36–120	47

[a] T-T means that the durations are measured from trough to trough; P-P means that they are measured from peak to peak.

[b] M stands for monthly reference dates, A for calendar-year reference dates. See Table 16.

[c] Standard deviation expressed as a percentage of the mean, each carried to one more place than shown in the table. In computing the standard deviation, we divided the sum of squared deviations from the mean by the number of observations, not the number of 'degrees of freedom'.

find one business cycle in the United States lasting 17 months when measured from peak to peak, and a roughly contemporaneous cycle of the same duration in Great Britain. At the other extreme is a British cycle in 1868–79 which lasted 135 months from trough to trough. Here is an eightfold difference in duration. Among our annual measures the extremes are 2 and 11 years.[1] Even the average duration of business cycles varies appreciably from country to country, and from period to period in the same country. The average ranges from 45 months in the United States to 64 months in Germany according to the monthly reference dates, from 46 to 66 months according to the annual reference dates. If we group the measures made from annual reference troughs into two equal parts, we find in the United States a decline from an average of 54 months in 1834–88 to an average of 41 months in 1888–1932, and in Great Britain an increase from 56 months in 1793–1858 to 63 months in 1858–1932. The coefficients of variation of full-cycle durations range in Table 139 from 25 to 51 per cent. In each country they are about the same for expansions as for full cycles, but they are much higher for contractions than for expansions.

In view of the irregular duration of business cycles, we should expect great variability in the cyclical behavior of most time series. The seven series in Chart 53 illustrate the variability characteristic of single branches of activity in the United States.[2] They cover a considerable stretch of time, from fifty-odd to nearly eighty years. Two represent the production of durable goods, two the money market, two the stock market, and one the volume of payments. More specifically, the series show the number of tons of pig iron produced by months since 1877 and the number of freight cars ordered by railroads of the United States each quarter since 1870; yields of high-grade railroad bonds and call money rates on the New York Stock Exchange by months since 1857; railroad stock prices by months since 1857 and the number of shares traded in round lots on the New York Stock Exchange by months since 1875; bank clearings outside New York City by months from 1875 through 1918, continued by bank debits after 1918, and 'deflated' throughout by Snyder's index of the 'general price level'.

Tables 140-143 and Chart 54 show cyclical measures for these series, from the time they start until about 1933.[3] Only two series, pig iron pro-

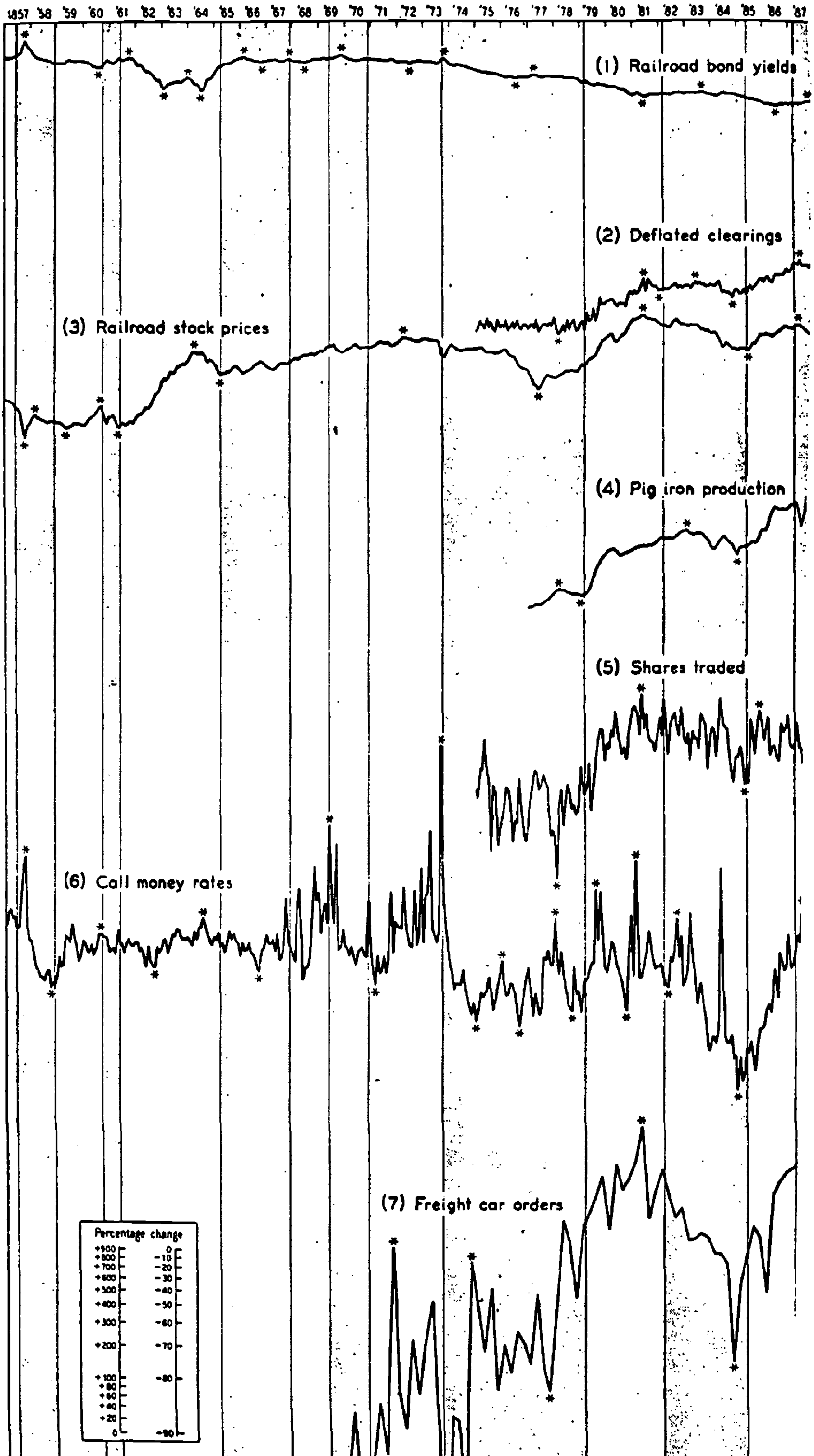
[1] See p. 113.

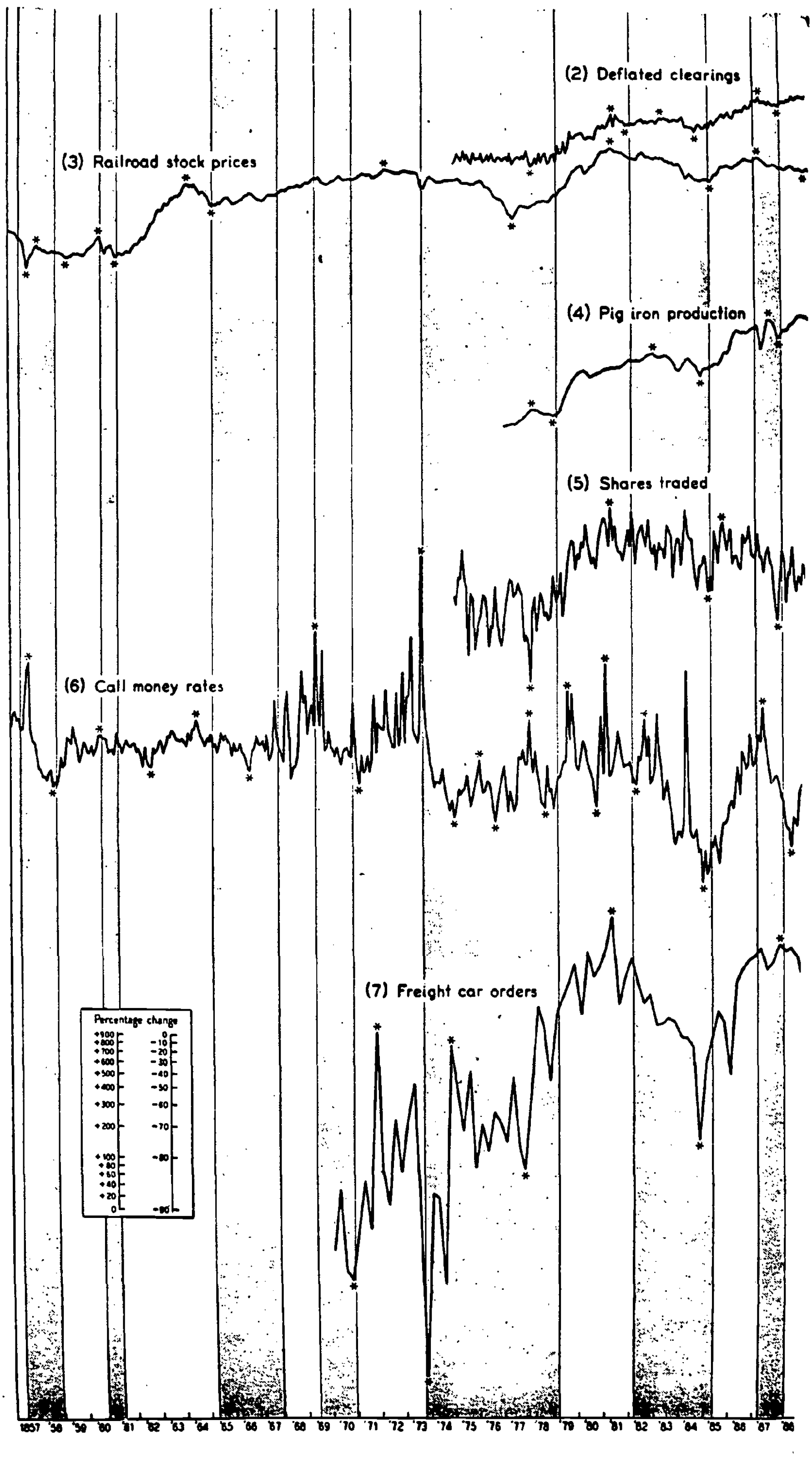
[2] The series on freight car orders comes from John E. Partington, *Railroad Purchasing and the Business Cycle* (Brookings Institution. 1929), pp. 219-26 for 1870–1926; overlapped since 1924 by figures from the *Iron Trade Review*. Partington's series represents orders by domestic railroads; the *Iron Trade Review* data include also orders by foreign railroads and by noncarriers. In Chart 53 Partington's series is plotted through 1924, the *Iron Trade Review* data thereafter. The data are not adjusted for seasonal variations after 1930.

For the sources of the other series in Chart 53, see p. 210, note 7.

[3] The analysis is confined to periods starting and ending with a trough.

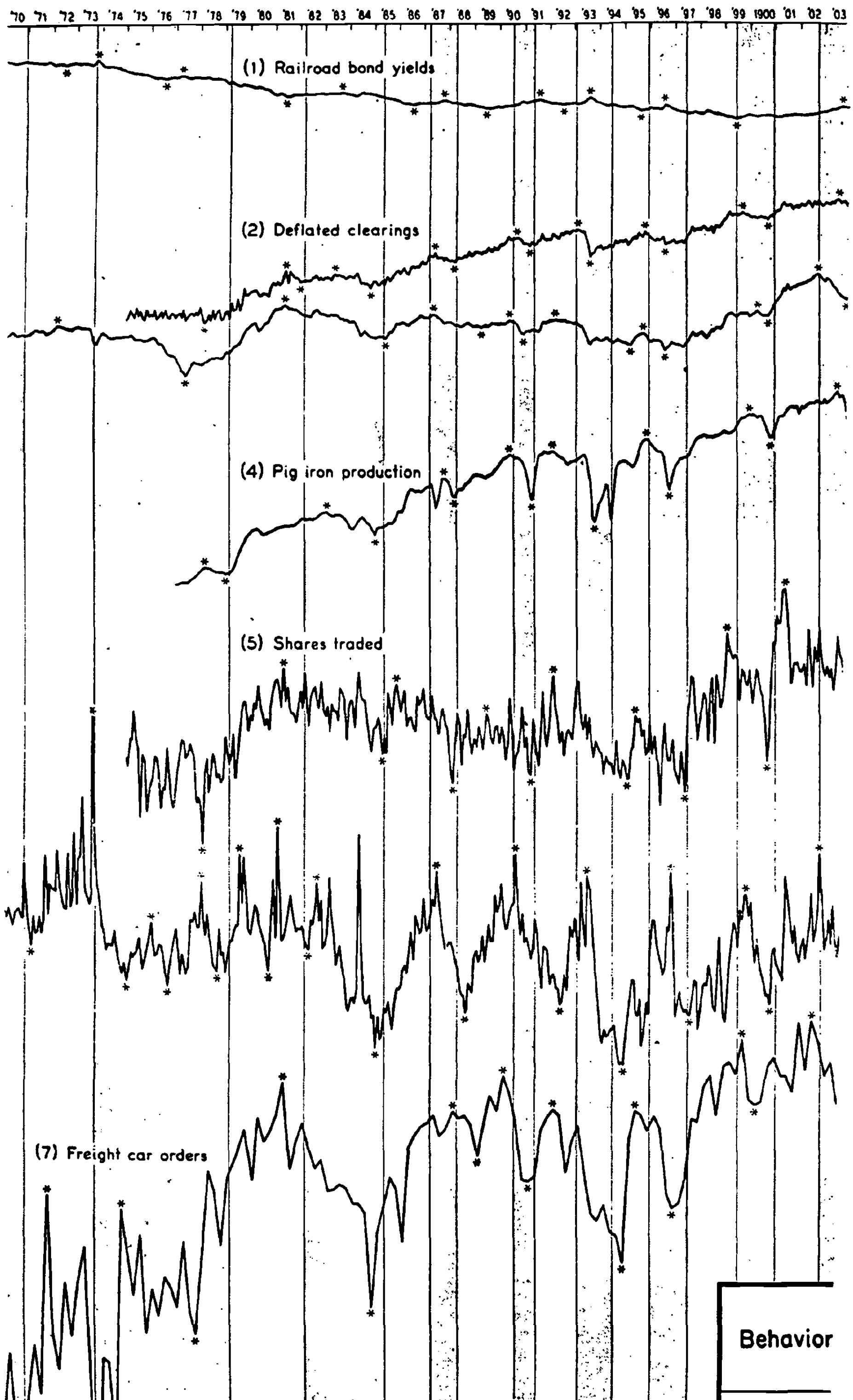
CHART 53


1857 '58 '59 '60 '61 '62 '63 '64 '65 '66 '67 '68 '69 '70 '71 '72 '73 '74 '75 '76 '77 '78 '79 '80 '81 '82 '83 '84 '85 '86 '87
(1) Railroad bond yields
(2) Deflated clearings
(3) Railroad stock prices
(4) Pig iron production
(5) Shares traded
(6) Call money rates
(7) Freight car orders
Percentage change
+900
+800
+700
+600
+500
+400
+300
+200
+100
+80
+60
+40
+20
0
-0
-10
-20
-30
-40
-50
-60
-70
-80
-90



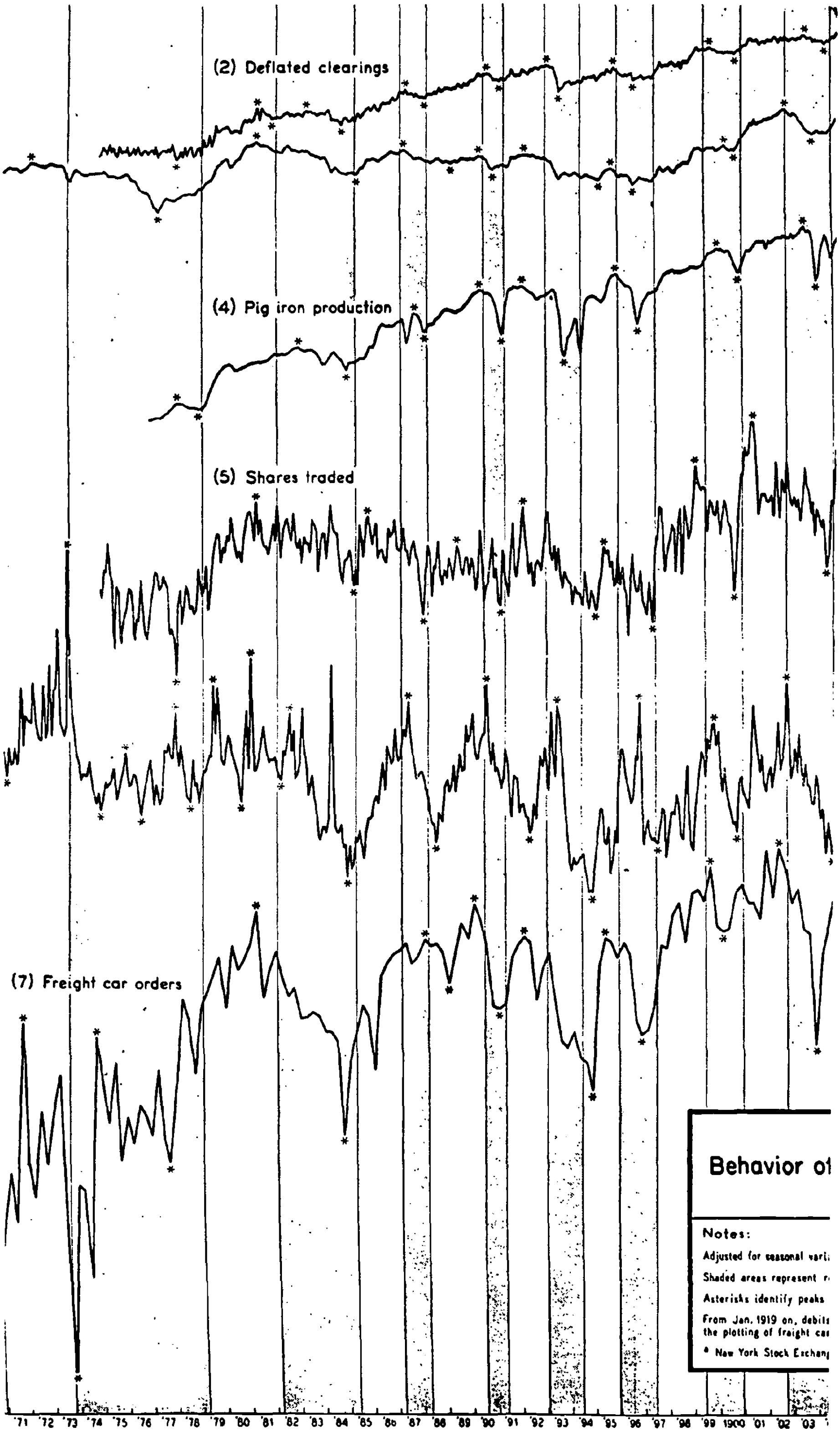
(2) Deflated clearings
(3) Railroad stock prices
(4) Pig iron production
(5) Shares traded
(6) Call money rates
(7) Freight car orders
Percentage change
1857 '58 '59 '60 '61 '62 '63 '64 '65 '66 '67 '68 '69 '70 '71 '72 '73 '74 '75 '76 '77 '78 '79 '80 '81 '82 '83 '84 '85 '86 '87 '88

CHART 53



Behavior

Notes:



(2) Deflated clearings
(4) Pig iron production
(5) Shares traded
(7) Freight car orders
Behavior of
Notes:
Adjusted for seasonal vari
Shaded areas represent r
Asterisks identify peaks
From Jan. 1919 on, debits
the plotting of freight ca
* New York Stock Exchan
'71 '72 '73 '74 '75 '76 '77 '78 '79 '80 '81 '82 '83 '84 '85 '86 '87 '88 '89 '90 '91 '92 '93 '94 '95 '96 '97 '98 '99 1900 '01 '02 '03

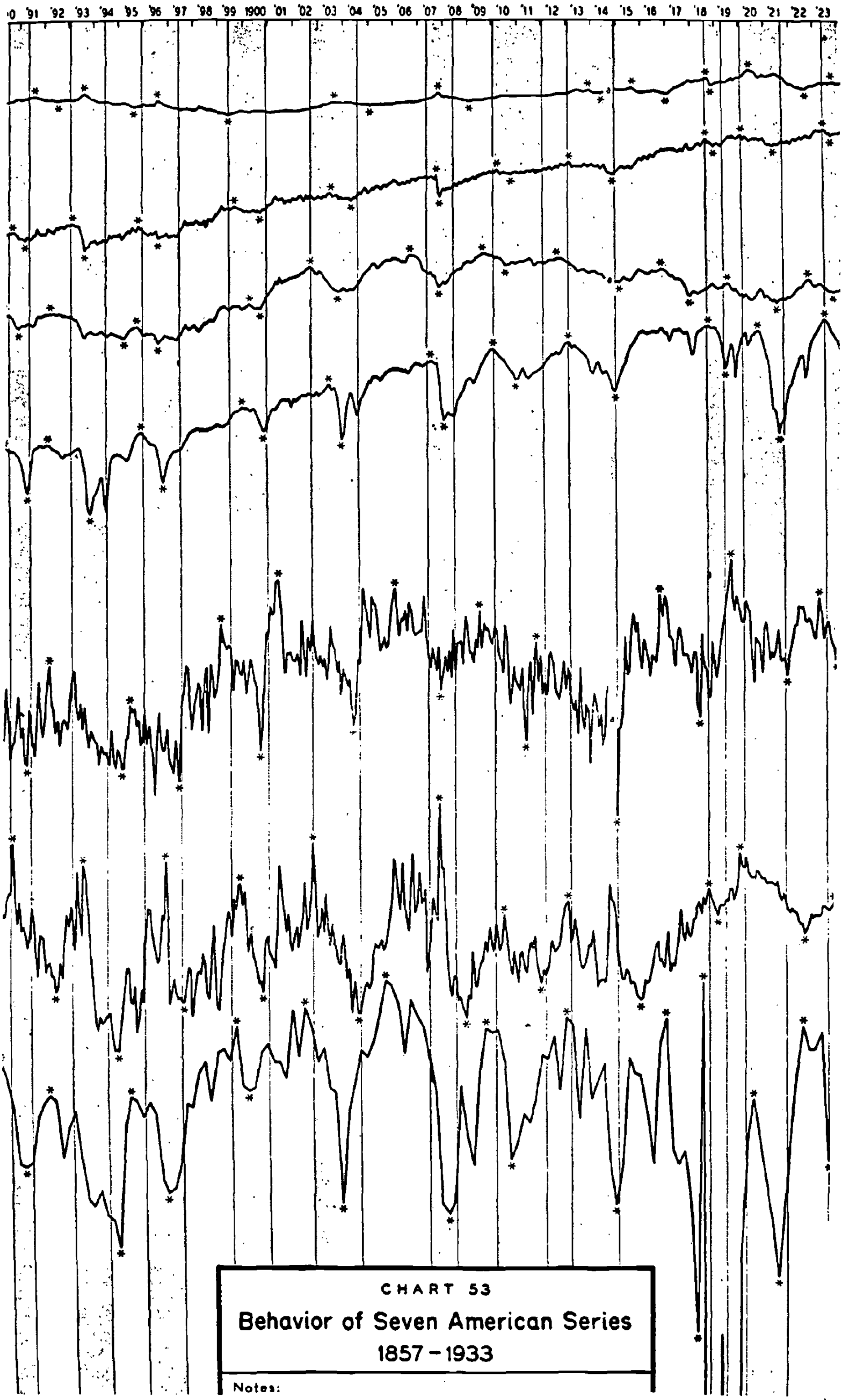


CHART 53

Behavior of Seven American Series

1857 – 1933

Notes:

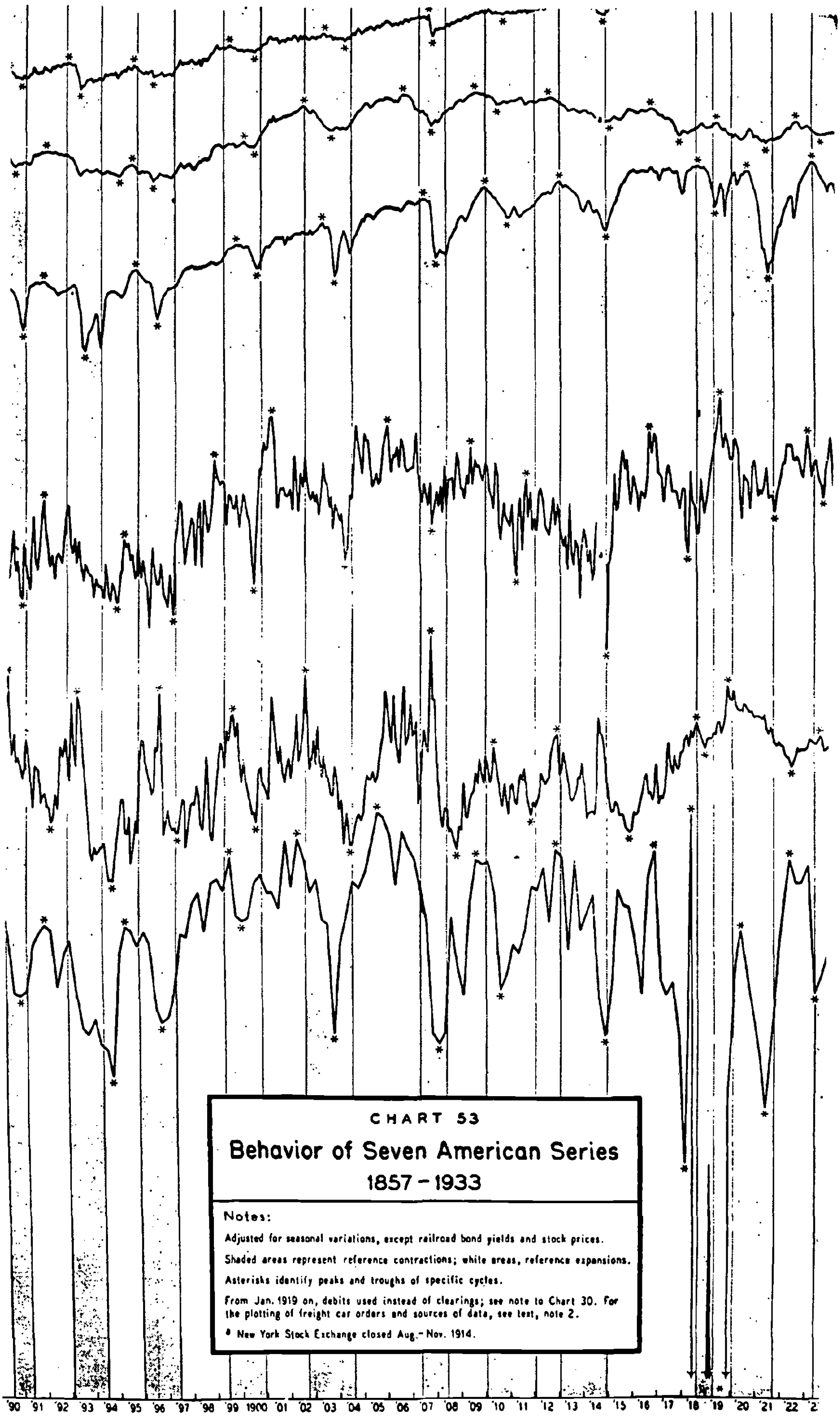


CHART 53

Behavior of Seven American Series

1857 - 1933

Notes:

Adjusted for seasonal variations, except railroad bond yields and stock prices.

Shaded areas represent reference contractions; white areas, reference expansions.

Asterisks identify peaks and troughs of specific cycles.

From Jan. 1919 on, debits used instead of clearings; see note to Chart 30. For the plotting of freight car orders and sources of data, see text, note 2.

* New York Stock Exchange closed Aug.- Nov. 1914.

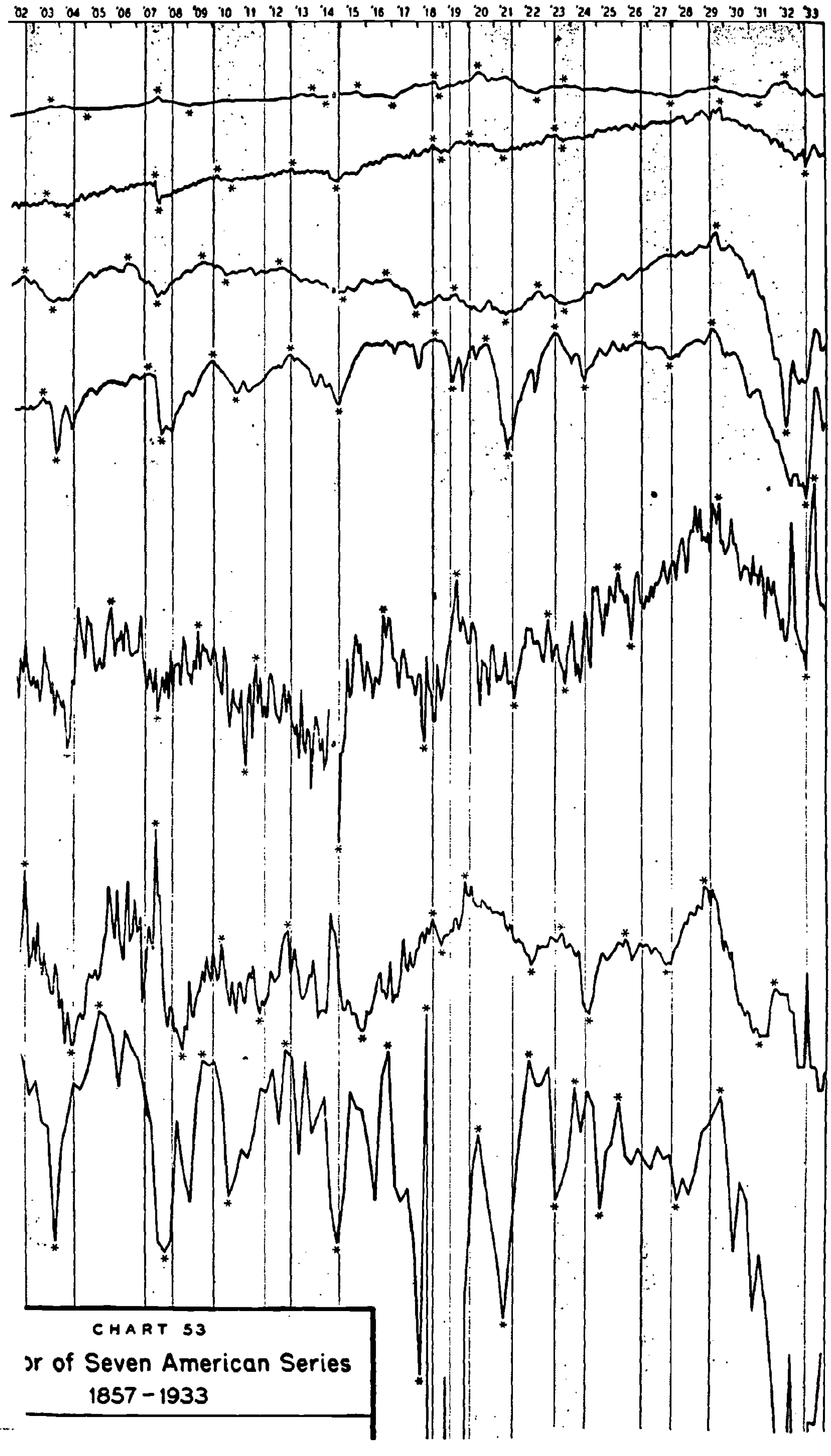


CHART 53

ɔr of Seven American Series

1857 – 1933

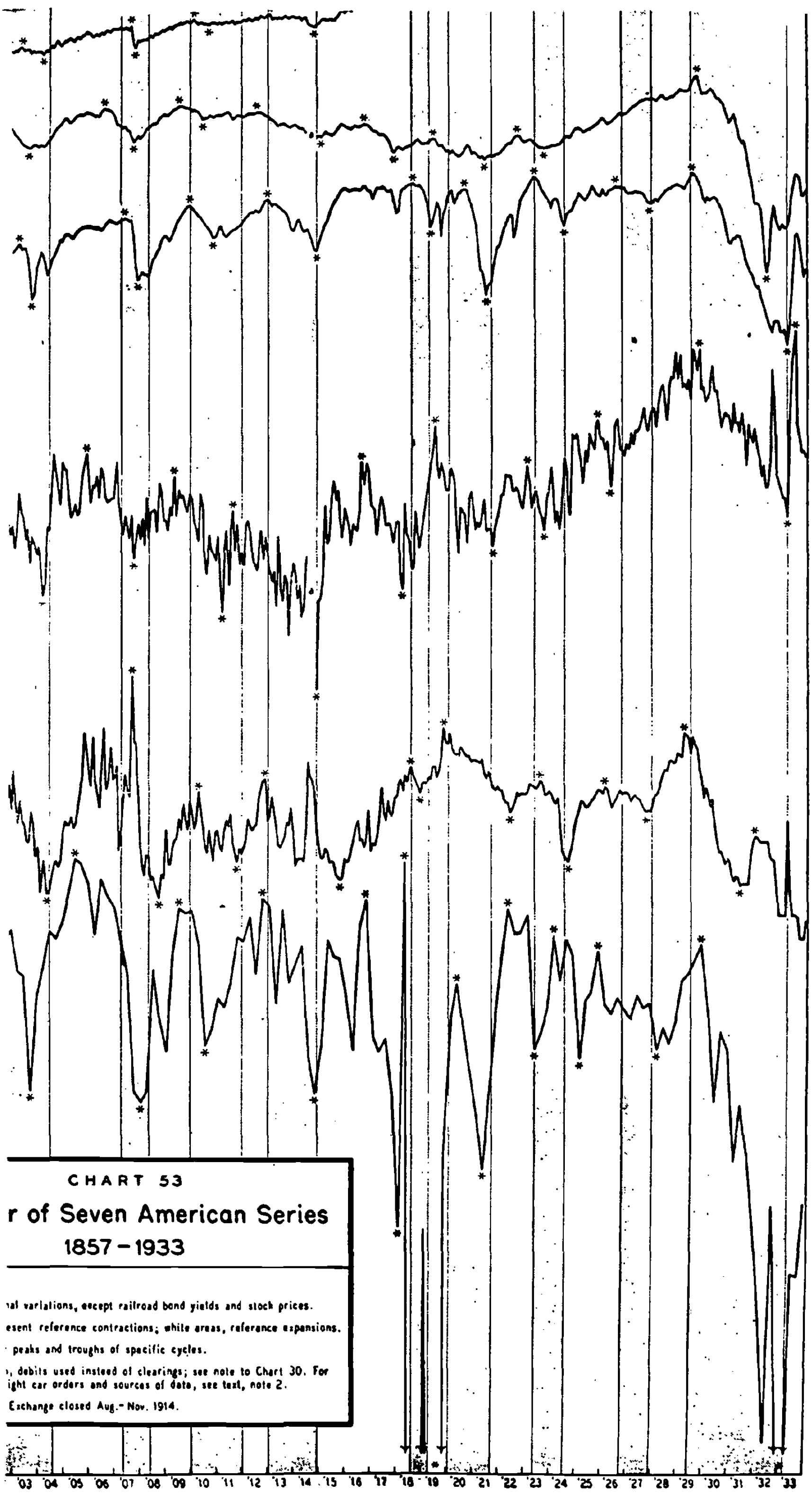


CHART 53
r of Seven American Series
1857 – 1933
al variations, except railroad bond yields and stock prices.
esent reference contractions; white areas, reference expansions.
peaks and troughs of specific cycles.
, debits used instead of clearings; see note to Chart 30. For
ight car orders and sources of data, see text, note 2.
Exchange closed Aug.– Nov. 1914.
'03 '04 '05 '06 '07 '08 '09 '10 '11 '12 '13 '14 '15 '16 '17 '18 '19 '20 '21 '22 '23 '24 '25 '26 '27 '28 '29 '30 '31 '32 '33

TABLE 140

Averages and Ranges of Selected Cyclical Measures of Seven American Series

Measure	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
PERIOD COVERED							
Specific cycles	Mar. 78 – Mar. 33	Jan. 79 – Mar. 33	4Q 70 – 1Q 33	Oct. 57 – June 32	Feb. 78 – Mar. 33	Oct. 58 – May 31	Aug. 60 – May 31
Reference cycles	Mar. 79 – Mar. 33	Mar. 79 – Mar. 33	4Q 70 – 1Q 33	Dec. 58 – Mar. 33[h]	Mar. 79 – Mar. 33	Dec. 58 – Mar. 33[h]	Dec. 58 – Mar. 33[h]
NUMBER OF CYCLES							
Specific	15	15	19	18	15	23	20
Reference	15	15	16	19	15	19	19
DURATION OF SPECIFIC CYCLES[a]							
Expansion	33(13, 74)	29(10,49)	18 (3,42)	29 (5, 84)	18 (4,41)	20 (4,39)	21 (8,58)
Contraction	11 (2, 40)	15 (5,44)	21 (6,42)	21 (5, 62)	26 (6,47)	18 (4,35)	21 (2,50)
Full cycle	44(28,114)	43(26,72)	39(18,84)	50(17,146)	44(24,86)	38(19,53)	42(17,68)
AMPLITUDE OF SPECIFIC CYCLES[b]							
Rise	27 (6,60)	62(36,108)	213(106, 585)	36 (6, 90)	98(45,182)	116(33,316)	11(2,36)
Fall	13 (3,55)	55(19,149)	212 (67, 591)	32 (5,142)	92(38,176)	116(27,337)	13(2,33)
Rise & fall	40(18,91)	117(55,199)	425(209,1176)	67(23,232)	190(89,359)	232(83,653)	23(6,44)
Rise per month[c]	0.8(0.4, 1.5)	2.4(1.3, 4.9)	22.6(3.6,195.0)	1.4(0.3,3.6)	7.6(2.4,20.3)	6.9(2.0,24.5)	0.6(0.2,1.8)
Fall per month[c]	1.9(0.5,12.5)	4.7(1.1,10.4)	11.7(5.0, 39.4)	1.7(0.7,4.3)	4.1(1.3, 9.6)	7.6(1.3,21.1)	0.9(0.2,4.1)
Rise & fall per month[c]	0.9(0.5, 1.5)	2.9(1.2, 5.1)	13.2(4.1, 65.3)	1.4(0.6,2.5)	4.6(1.9, 8.0)	6.3(1.7,14.2)	0.6(0.2,1.5)
SECULAR MOVEMENTS							
% change from cycle to cycle							
Total[d]	+15(−6,+26)	+14(−30,+39)	+9(−59,+136)	+10(−21,+49)	+14(−51,+77)	0(−83,+55)	−2(−20,+21)
Per month[e]	+0.37(−0.18,+0.66)	+0.35(−0.58,+0.96)	+0.23(−3.03,+4.80)	+0.21(−0.55,+1.35)	+0.34(−1.20,+2.30)	0.00(−2.52,+1.39)	−0.04(−0.85,+0.93)
CONFORMITY							
Expansion stages[f]	VIII-V	I-V	VIII-IV	VIII-IV	VIII-IV	I-V	III-VI
Change per month during reference[g]							
Expansions	+0.8(+0.4,+1.1)	+2.3(+1.4,+5.0)	+3.8(+0.4,+9.3)	+0.8(−0.6,+1.9)	+2.0(−0.6,+4.8)	+3.6(−0.6,+11.9)	+0.2(−0.3,+1.1)
Contractions	−0.5(−1.5,+0.7)	−2.3(−4.4,−0.3)	−3.9(−8.0,+1.4)	−0.6(−2.9,+1.6)	−1.7(−3.9,+2.2)	−3.6(−8.7,− 0.2)	−0.2(−0.8,+0.2)
Index of conformity to reference							
Expansions	+100	+100	+100	+79	+87	+68	+47
Contractions	+73	+100	+62	+60	+73	+100	+30
Cycles	+86	+100	+100	+74	+93	+100	+68

The ranges are shown in parentheses; the lowest figure is to the left of the comma, the highest figure to the right. Averages are outside the parentheses. For several measures in this table, cycle-by-cycle figures are given in Appendix Table B1.

[a] In months.

[b] In specific-cycle relatives.

[c] Unweighted average.

[d] Corresponds to col. 9 of our Table S3; see sample in Table 33.

[e] Weighted average. Corresponds to col. 10 of our Table S3; see sample in Table 33.

[f] Matched in every series with reference expansion.

[g] That is, average change per month in reference-cycle relatives during stages matched with reference expansion and contraction.

[h] The index of conformity to reference contractions and full cycles covers an extra phase at the beginning; that is, the period covered is June 1857–March 1933.

TABLE 141
Average Timing of Specific-cycle Turns and Their Variability
Seven American Series

Series	Number of cycles		Lead (−) or lag (+) in months			Number of specific-cycle turns that			
	Specific	Reference	Average	Average deviation	Range	Lead	Lag	Coincide	Lead or lag 3 mos. or less
					Timing at reference peaks				
Deflated clearings	15	15	+3.2	3.2	−2 to +15	1	10	3	10
Pig iron production	15	15	+1.9	4.2	−11 to +11	4	8	3	9
Freight car orders	19	16	−5.8	6.8	−23 to +11	12	4	0	3
Railroad stock prices	18	19	−5.6	5.8	−22 to +10	13	3	1	6
Shares traded	15	15	−10.4	5.2	−23 to +4	14	1	0	1
Call money rates	23	19	−0.1	3.8	−13 to +10	8	8	3	11
Railroad bond yields	20	19	+7.8	3.8	+1 to +18	0	16	0	3
					Timing at reference troughs				
Deflated clearings	15	15	−5.8	3.8	−15 to 0	14	0	1	6
Pig iron production	15	15	−3.4	3.2	−13 to +1	12	1	3	10
Freight car orders	19	16	−3.0	5.8	−17 to +10	12	5	0	5
Railroad stock prices	18	19	−7.4	8.0	−32 to +12	14	4	0	5
Shares traded	15	15	−4.6	5.2	−19 to +7	12	2	2	8
Call money rates	23	19	+1.5	5.6	−14 to +16	9	10	0	8
Railroad bond yields	20	19	+11.8	7.6	−6 to +27	2	13	1	1

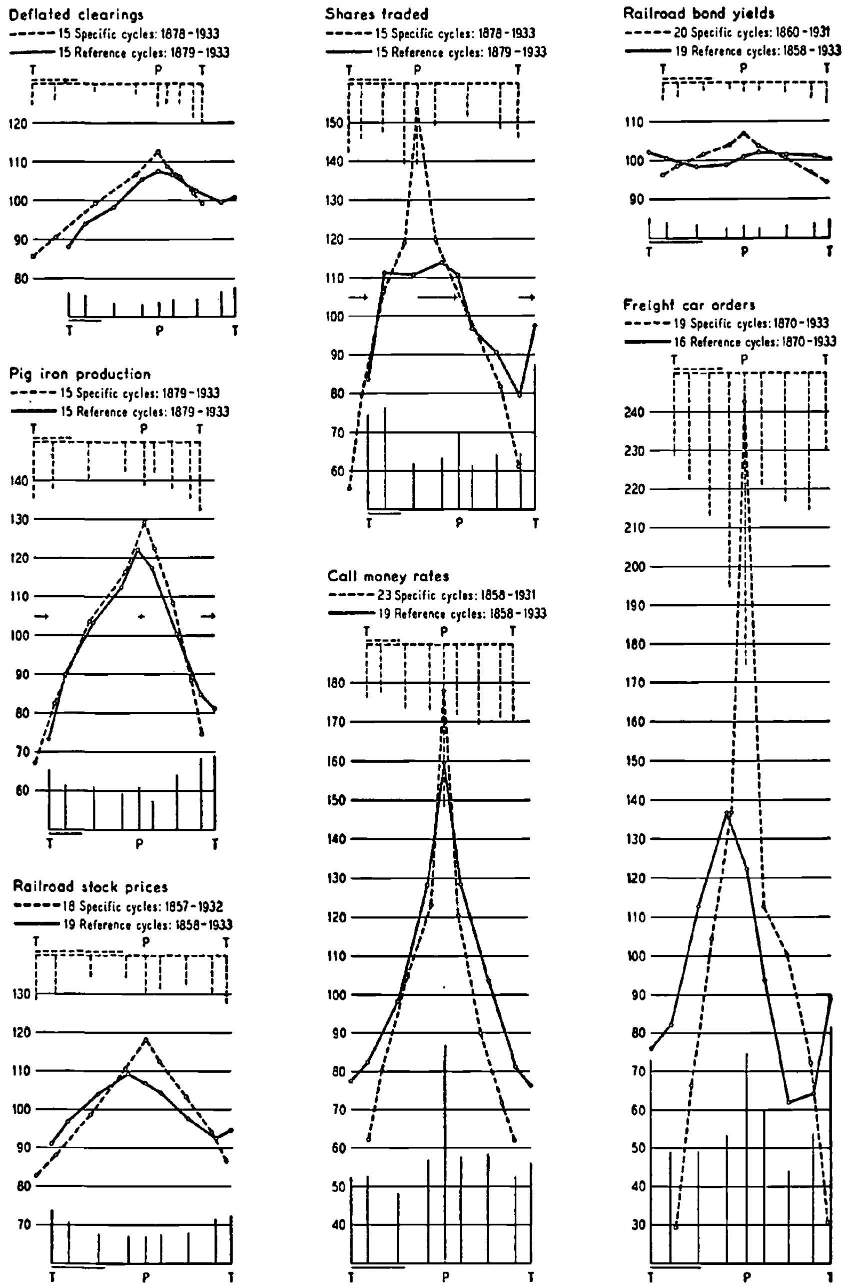
See Table 140 for the periods covered, and Appendix Table B3 for the timing of the individual specific-cycle turns at corresponding reference turns.

duction and share trading, have specific cycles that bear a one-to-one correspondence to business cycles.[4] None of the seven series leads or lags consistently at reference troughs. Railroad bond yields lag at every reference peak to which a specific-cycle peak corresponds, but we lack observations on the timing of this series at several peaks. The timing of the specific-cycle turns at reference troughs ranges from −13 to +1 months in iron production and from −32 to +12 months in railroad stock prices; the range at peaks runs from +1 to +18 months in railroad bond yields, from −2 to +15 in deflated clearings, from −23 to +11 in freight car orders. Iron production, deflated clearings, and freight car orders rose in every cycle during the stages matched with reference expansion; the other series show at least one decline during these stages. Iron production and call money rates declined invariably during the stages matched with reference contraction; every other series exhibits two or more lapses from conformity. In the peak stage of one reference cycle there were no freight car orders, in another they ran 133 per cent above the average for the cycle. At one reference-cycle peak call money rates were 35 per cent below the cycle average, at another 425 per cent above. One expansion in stock prices lasted 84 months, another only 5 months. The shortest contraction in bond yields is 2 months, the longest 50 months. The range of the full

[4] Even iron production is an exception if the record is pushed back of 1879, as is evident from the 'extra' contraction from March 1878 to Jan. 1879.

CHART 34

Average Cyclical Patterns of Seven American Series



Horizontal scale, in months 0 12 24 36 48 60

In series whose specific-cycle turns bear one-to-one correspondence to reference turns, → indicates lead at reference peaks, or troughs; and ← indicates lag at reference peaks.

For sources of data, see text, note 2; for explanation of chart, Ch. 5, Sec. VIII; for average patterns, Tables 142 and 143.

TABLE 142

Average Specific-cycle Patterns and Their Variability
Seven American Series

Series and measure	I Initial trough (3 mos.)	II Expansion First third	III Expansion Middle third	IV Expansion Last third	V Peak (3 mos.)	VI Contraction First third	VII Contraction Middle third	VIII Contraction Last third	IX Terminal trough (3 mos.)
DEFLATED CLEARINGS (15)									
Average	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Lowest value	72.2	78.2	95.5	101.3	104.3	96.8	90.0	74.8	68.4
Highest value	98.0	97.0	103.8	116.2	131.7	126.9	130.2	124.6	121.5
Range	25.8	18.8	8.3	14.9	27.4	30.1	40.2	49.8	53.1
Average deviation	5.5	4.2	2.0	2.8	5.9	5.2	6.6	8.8	10.6
PIG IRON PRODUCTION (15)									
Average	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Lowest value	38.9	54.6	84.9	103.8	110.0	106.0	71.1	31.0	24.4
Highest value	123.3	129.2	147.6	154.7	173.3	137.9	125.1	109.1	106.3
Range	84.4	74.6	62.7	50.9	63.3	31.9	54.0	78.1	81.9
Average deviation	14.6	12.1	9.5	7.5	11.2	7.9	11.9	14.6	17.8
FREIGHT CAR ORDERS (19)									
Average	29.5	66.3	104.5	136.8	242.5	112.6	100.2	72.2	30.9
Lowest value	0.0	19.3	37.4	11.7	141.0	0.0	3.0	0.0	0.0
Highest value	91.3	152.3	298.6	444.8	591.0	180.2	185.6	166.0	86.9
Range	91.3	133.0	261.2	433.1	450.0	180.2	182.6	166.0	86.9
Average deviation	21.5	27.7	37.0	55.3	75.3	28.9	33.4	35.8	20.3
RAILROAD STOCK PRICES (18)									
Average	82.8	88.0	98.8	110.8	118.3	112.4	103.2	94.0	86.6
Lowest value	56.1	60.6	82.8	101.3	104.8	101.0	91.9	41.8	16.0
Highest value	101.9	104.5	106.3	132.2	157.8	140.4	129.4	120.9	115.9
Range	45.8	43.9	23.5	30.9	53.0	39.4	37.5	79.1	99.9
Average deviation	11.6	9.6	5.3	5.4	9.5	8.4	7.3	9.4	12.3
SHARES TRADED (15)									
Average	55.4	79.7	106.5	119.0	153.4	119.7	100.4	81.6	61.0
Lowest value	23.3	51.4	70.8	89.1	120.4	100.3	80.9	57.9	30.2
Highest value	81.5	112.7	145.6	181.9	217.7	142.3	117.2	112.0	95.6
Range	58.2	61.3	74.8	92.8	97.3	42.0	36.3	54.1	65.4
Average deviation	17.7	14.2	12.6	20.8	20.8	10.9	8.3	11.6	14.0
CALL MONEY RATES (23)									
Average	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Lowest value	31.6	43.2	68.5	73.0	116.2	87.4	39.2	33.5	30.1
Highest value	105.5	132.3	154.5	162.1	369.6	163.3	146.3	132.9	132.8
Range	73.9	89.1	86.0	89.1	253.4	75.9	107.1	99.4	102.7
Average deviation	13.7	12.3	16.3	16.8	41.8	18.1	20.7	18.9	19.5
RAILROAD BOND YIELDS (20)									
Average	96.1	98.4	101.5	104.0	106.9	103.7	100.6	96.9	94.3
Lowest value	75.3	83.8	96.1	100.3	101.4	99.2	95.2	84.5	76.9
Highest value	106.1	107.3	106.7	108.4	111.7	109.6	106.7	105.0	104.7
Range	30.8	23.5	10.6	8.1	10.3	10.4	11.5	20.5	27.8
Average deviation	4.6	3.8	2.1	1.7	2.4	2.1	2.6	4.0	5.2

Column heading: Standing in specific-cycle relatives at stage

The number of cycles covered is shown in parentheses after the title of the series. See Table 140 for the periods covered, and Appendix Table B1 for the cycle-by-cycle patterns.

TABLE 143

Average Reference-cycle Patterns and Their Variability
Seven American Series

Series and measure	Standing in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
		First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS (15)									
Average	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Lowest value	71.1	82.5	90.8	100.8	99.5	100.5	87.0	75.5	67.5
Highest value	107.6	112.4	109.6	118.4	115.5	113.7	110.6	110.6	112.2
Range	36.5	29.9	18.8	17.6	16.0	13.2	23.6	35.1	44.7
Average deviation	6.6	5.7	3.7	3.2	4.0	4.5	4.8	6.6	7.6
PIG IRON PRODUCTION (15)									
Average	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Lowest value	41.1	61.5	85.1	96.6	106.1	101.5	66.3	32.1	24.5
Highest value	124.6	132.2	148.4	153.1	169.3	139.8	123.5	108.7	106.1
Range	83.5	70.7	63.3	56.5	63.2	38.3	57.2	76.6	81.6
Average deviation	15.5	11.4	10.9	9.4	10.7	7.3	13.9	18.2	19.0
FREIGHT CAR ORDERS (16)									
Average	76.0	82.1	112.9	136.8	122.3	93.8	62.0	64.1	89.2
Lowest value	0.0	0.0	17.4	59.8	0.0	0.0	2.4	3.9	0.0
Highest value	292.4	142.9	164.2	200.3	232.8	229.5	117.8	173.0	350.5
Range	292.4	142.9	146.8	140.5	232.8	229.5	115.4	169.1	350.5
Average deviation	53.0	29.0	29.1	33.3	54.6	40.2	24.1	33.8	61.6
RAILROAD STOCK PRICES (19)									
Average	91.0	96.9	104.0	109.4	106.9	104.3	97.7	92.5	94.7
Lowest value	52.8	60.1	87.0	89.5	88.6	91.5	80.9	27.8	27.3
Highest value	129.4	131.4	133.4	143.8	153.0	144.2	118.3	125.4	126.1
Range	76.6	71.3	46.4	54.3	64.4	52.7	37.4	97.6	98.8
Average deviation	13.8	10.7	7.7	7.0	7.0	7.4	7.9	11.5	12.2
SHARES TRADED (15)									
Average	83.8	111.2	110.9	114.0	110.6	96.8	90.5	79.5	97.5
Lowest value	24.0	78.0	84.3	89.5	61.9	67.0	63.7	54.9	44.0
Highest value	137.9	180.1	139.6	141.9	161.0	133.3	115.6	133.0	194.6
Range	113.9	102.1	55.3	52.4	99.1	66.3	51.9	78.1	150.6
Average deviation	24.7	26.5	11.9	13.2	20.0	11.4	14.2	14.6	37.7
CALL MONEY RATES (19)									
Average	77.5	82.4	98.4	128.2	159.5	128.5	103.9	81.1	76.2
Lowest value	29.8	40.0	66.1	61.4	65.0	64.8	43.8	36.6	32.6
Highest value	130.7	135.6	167.3	268.5	524.9	195.2	212.3	151.0	157.2
Range	100.9	95.6	101.2	207.1	459.9	130.4	168.5	114.4	124.6
Average deviation	22.3	22.8	18.2	26.9	56.9	27.6	28.3	22.4	26.0
RAILROAD BOND YIELDS (19)									
Average	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2
Lowest value	92.4	92.4	84.4	87.6	96.0	97.8	92.0	89.8	85.3
Highest value	113.4	111.7	109.0	108.6	112.4	110.5	111.2	111.0	111.9
Range	21.0	19.3	24.6	21.0	16.4	12.7	19.2	21.2	26.6
Average deviation	5.1	4.0	3.3	2.6	3.1	2.5	3.2	3.8	4.7

The number of cycles covered is shown in parentheses after the title of the series. See Table 140 for the periods covered, and Appendix Table B3 for the cycle-by-cycle patterns.

amplitudes of the 125 specific cycles in the seven series runs from 6 to 1,176 points. In pig iron production the most violent cycle has three to four times the amplitude of the mildest cycle. In railroad stock prices the most violent cycle has ten times the amplitude of the mildest. In one specific cycle the standing of freight car orders at the peak was about 40 per cent above the cycle average, in another almost 500 per cent above.[5] Even the percentage change from the level of one specific cycle to the next is unsteady within each series. In deflated clearings the percentage change from cycle to cycle varies from −6 to +26, in call money rates from −83 to +55, in freight car orders from −59 to +136.[6]

II Function of Averages and Average Deviations

To extract order from such highly variable observations on the specific and reference cycles of economic activities, we use devices that vary from one group of series to another according to the problems we encounter. But one device is common to our analysis as a whole and distinguishes it from the techniques usually employed by business-cycle statisticians: for virtually every series we strike averages that include all or most of its specific and reference cycles. In this way we attempt to expose the typical characteristics of the cyclical behavior of different activities and of business as a whole, and to establish a base from which the wide variations in cyclical behavior observable in actual life may be readily explored.

The averages of our sample series suggest that stock prices, share trading and freight car orders tend to lead in business-cycle revivals and recessions, that bond yields tend to lag, that call money rates tend to move coincidently, while deflated clearings and iron production tend to lead in revivals and lag in recessions. The amplitudes of the specific cycles of pig iron production are enormous in comparison with bond yields, but seem moderate in comparison with freight car orders. Expansions average three times as long as contractions in deflated clearings but only two-thirds as long in share trading. Contraction is a briefer and more violent phase than expansion in most series; in freight car orders and share trading expansion is briefer and more violent. The specific cycles of call money rates and freight car orders have sharply pointed, narrow tops; the cyclical top of stock prices seems rounded by comparison. Railroad stock prices conform better to business cycles than railroad bond yields, call

[5] The average deviations of the specific-cycle patterns tend to be at a maximum in stages I, V and IX, and at a minimum at or near stages III and VII. In large part this is a technical result of the use of the average standing during a cycle as the base for the relatives. A contributory factor is the brevity of stages I, V and IX, in comparison with the other stages. The latter factor leaves its impress also on the average deviations of the reference-cycle patterns, and it is reinforced by the first factor if the timing of a series matches fairly closely the reference dates.

[6] These percentage changes are computed on the base of the average value of the two cycles being compared. If the preceding cycle is used as the base, the range is still wider; in freight car orders it becomes −46 to +426.

money rates conform better than share trading, and iron production conforms better than freight car orders. Observations of this character for numerous series and several countries enable us to judge what cyclical behavior is characteristic of different activities, and lay the groundwork for a simple and systematic description of business cycles.

When averages are struck for groups of cycles, features peculiar to single cycles tend to fade away, while features common to most cycles tend to stand out clearly. The larger the number of cycles the greater the confidence we usually feel in the representative value of an average. At times, however, the representative quality of an average is improved by reducing the number of cycles. When we find secular changes in cyclical behavior, our usual practice is to break the series into segments that are roughly homogeneous and to compute separate averages for the cycles in each segment. More often we exclude from the averages such cycles as we believe are dominated by random influences. Most exclusions are cycles in price and value series affected by violent changes in monetary conditions. Occasionally, cycles in physical quantity series are excluded; for example, the cycles in vessel construction during World War I, and several cycles in anthracite coal mining distorted by strikes. Sometimes we exclude a cycle in averaging certain measures but include it in averaging others. Thus in price series the final stage of the reference cycle ending in 1914 is often seriously distorted by the outbreak of war; hence this stage is excluded in computing the average reference-cycle pattern but other stages of the cycle are included. All exclusions from our averages, and the reasons for them, will be noted in the succeeding monographs. The averages are as a rule unweighted arithmetic means, though we supplement them by weighted means in some of our measures, and occasionally use medians or other averages.[7]

To keep the varying character of the arrays from which the averages are drawn clearly in mind, we compute also a simple measure of dispersion—the average deviation from the arithmetic mean. Sometimes we find it desirable to use standard deviations, variances, or coefficients of variation; but for most of our work the average deviations seem sufficient. The average deviations are more than safeguards. They have a positive value, for they bring out what we consider one of the most important aspects of cyclical behavior. Some economic processes are fairly uniform in their movements from cycle to cycle, and so have relatively small average deviations; most factors show wide diversity of movement, and so have large average deviations. To know what processes can be counted upon with some assurance to behave in a standard fashion, and what processes vary in an unpredictable fashion from one cycle to the next, is important both theoretically and practically.

[7] To reduce the influence of extreme 'per month' figures, which usually occur during brief phases, we present weighted as well as unweighted averages in our standard Tables S2, S5 and R2. See pp. 134, 330, 342.

III Problems Raised by Averages

But are we justified in assuming that averages of cyclical movements are proper starting points for theoretical inquiries? An average covering a dozen cycles in a series takes no account of the historical sequence in which they occurred. Given a set of measures for individual cycles, the average will be the same whether the cycles show a progressive rise or a progressive decline in amplitude, whether the amplitudes vary haphazardly or depend upon the position of the cycles within 'long waves'. If secular, discontinuous, or cyclical changes of formidable scope occur in specific or business cycles, the repetition that justifies the use of averages becomes a repetition in name only. Under such conditions averages that cover decades hide significant traits of cyclical behavior; their historical value may be slight and their value as bases of future expectations slighter still.

The conclusions reached by some earlier investigators warn us not to dismiss this possibility lightly. Certain economists have held that 'commercial crises' tend to become progressively more severe; others have held the diametrically opposite view. Best known among suggestions of this character is Karl Marx's thesis that the commercial crises characteristic of capitalism tend to become increasingly severe. A generation or two later, Thorstein Veblen argued that the alternation of lively expansions and contractions is giving place to a chronic state of mild depression, from which business revives only when stimulated by favorable random factors.[8] Many American observers in the 1920's were persuaded that business cycles were being 'ironed out'. At the same time many of their foreign contemporaries believed that a structural change had taken place in world economy, and that business cycles shifted as a consequence from the characteristic pattern of pre-War times to a pattern marked by persistent underemployment and intensified fluctuations. Similar remarks have been echoed and re-echoed the world over since the crash of 1929–30.

Besides these contentions that there are secular and discontinuous changes in cyclical behavior, we must note the hypothesis that business cycles are minor subdivisions of 'major' or 'long' cycles. Several investigators have found long cycles by analyzing statistical records. Thus Kondratieff finds 'long waves' lasting about 50 to 60 years; Kuznets finds 'secondary secular movements' averaging 22 years in production and 23 years in price series; Burns finds 'trend cycles' of about 15 to 20 years in production and other business activities; Wardwell finds 'major cycles' that average about 15 years in the United States and 9.5 in Germany; Kitchin finds 'major cycles' lasting usually about 7 years, sometimes 10

[8] For fuller statements of the views of Marx and Veblen and references, see Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 8-9, 42-4, 232, 255.

years.[9] If business cycles really succeed one another in cyclical fashion, then the position that an individual business cycle occupies in a 'long cycle' determines whether it is a mild movement of slight consequence or a convulsive fluctuation, whether the revival with which it begins is vigorous or mild, whether its expansion develops into a 'boom', whether its recession becomes a 'crisis', and whether its contraction turns into a drastic 'depression'.

The hypotheses we have noted are of the greatest importance. If any one of them is valid, we should be missing our opportunities if we did not take full account of it. As stated in Chapter 1, our aim is to determine as thoroughly as we can what business cycles are, which means that we must try to determine whether the variations of successive business and specific cycles are haphazard or follow some regular pattern. This objective is always before us in later monographs, where we prepare materials as well as we can in advance for a systematic attack in the theoretical volume. But we believe that an intelligible notion of what business cycles are can best be reached from available statistical records by a process of successive approximations. The primary objective of our monographs on cyclical behavior is to describe in a preliminary way the typical features of business cycles. Whether our averages yield a useful first approximation to a description of cyclical behavior turns on the question whether such secular, discontinuous, or cyclical changes as may have occurred in business and specific cycles may for a time be slighted, not on the question whether such changes have actually occurred.

All we need to determine at this stage, therefore, is whether secular, discontinuous, or cyclical changes in cyclical behavior have been so pronounced that they discredit the use of averages. For this purpose sample studies should suffice. In Chapter 10 we investigate secular and discontinuous changes in relation to our averages of cyclical behavior; in Chapter 11 we investigate cyclical changes. Finally, in Chapter 12 we consider whether 'random' differences among business cycles are so great as to make averages futile constructions. These studies put us in a fair position to judge to what extent averages of the sort used in our standard tables are open to serious criticism. At the same time they give us an opportunity to explain more fully the flexible features of our technique; in particular, how we modify our procedures when the materials at hand seem to require it.

[9] The different periods found by these investigators are not necessarily contradictory since several cycles may coexist. Schumpeter, for example, works with a three-cycle schema. See the references cited in Ch. 11.

CHAPTER 10

Secular and Discontinuous Changes in Cyclical Behavior

OUR FIRST problem is to determine whether substantial secular changes can be detected in cyclical behavior as we measure it. Towards this end we have tested in detail the seven American series presented in the preceding chapter. These series cover processes that rank high among the activities stressed in theoretical studies of business cycles. Partly for this reason, partly because of the comparatively long stretch of time covered by these records, we regard our small sample as fairly satisfactory for the present purpose. We analyze also durations of business cycles, the one tolerably reliable set of measures that we have of business cycles as wholes.[1]

I Duration and Amplitude of Specific Cycles

To investigate secular changes in our sample, we have fitted straight lines by the method of least squares to the durations and amplitudes of successive specific cycles in each series.[2] Table 144 and Chart 55 present the results. As the chart shows, the trend lines sometimes fit the cyclical observations badly. Hence we supplement them in Table 145 by subgroup means, each set of cyclical measures being divided into three parts as nearly equal as possible.[3]

1 We are under heavy obligations to Milton Friedman for assistance in handling the technical problems encountered in this and the next two chapters.

2 Measures for the individual specific and reference cycles in the seven series are shown in Appendix B.

3 The bulk of the statistical analysis in Ch. 9-12 was completed before measurements for the business cycle of 1933–38 were ready; hence, with minor exceptions, our analysis stops around 1933. The addition of another cycle, while obviously desirable, would not modify the main results.

TABLE 144

Extreme Ordinates of Straight-line Trends
Fitted to Durations and Amplitudes of Successive Specific Cycles
Seven American Series

Series	Period covered	No. of specific cycles	Duration in months		Amplitude in specific-cycle relatives: Rise & fall		Amplitude in specific-cycle relatives: Rise & fall per month	
			First trend value	Last trend value	First trend value	Last trend value	First trend value	Last trend value
Deflated clearings	1878–1933	15	32	56	38	42	1.1	0.8
Pig iron production	1879–1933	15	44	43	95	139	2.4	3.3
Freight car orders	1870–1933	19	48	31	405	444	8.8	17.6
Railroad stock prices	1857–1932	18	51	49	55	80	1.5	1.4
Shares traded	1878–1933	15	46	42	164	217	3.9	5.2
Call money rates	1858–1931	23	37	38	254	210	7.2	5.4
Railroad bond yields	1860–1931	20	34	51	25	21	0.9	0.4

See Chart 55.

TABLE 145

Average Duration and Amplitude of Three Successive Groups of Specific Cycles
Seven American Series

Series and period covered	Number of specific cycles	Average duration in months	Average amplitude in specific-cycle relatives: Rise & fall	Average amplitude in specific-cycle relatives: Rise & fall per month[a]
Deflated clearings				
1878–1893	5	37	42	1.1
1893–1910	5	41	37	0.9
1910–1933	5	54	41	0.8
Pig iron production				
1879–1896	5	43	107	2.8
1896–1914	5	44	113	2.6
1914–1933	5	44	131	3.2
Freight car orders				
1870–1894	6	48	421	9.7
1894–1914	6	40	319	8.3
1914–1933	7	32	518	20.4
Railroad stock prices				
1857–1889	6	63	76	1.5
1889–1907	6	37	52	1.4
1907–1932	6	49	74	1.4
Shares traded				
1878–1897	5	46	148	3.6
1897–1914	5	42	220	5.2
1914–1933	5	44	204	4.9
Call money rates				
1858–1880	7	38	218	5.9
1880–1904	8	36	289	8.4
1904–1931	8	40	187	4.5
Railroad bond yields				
1860–1876	6	32	27	0.9
1876–1905	7	49	20	0.4
1905–1931	7	45	24	0.6

Where the number of cycles is not exactly divisible by 3, an additional cycle is placed in the last group or in both the second and last.

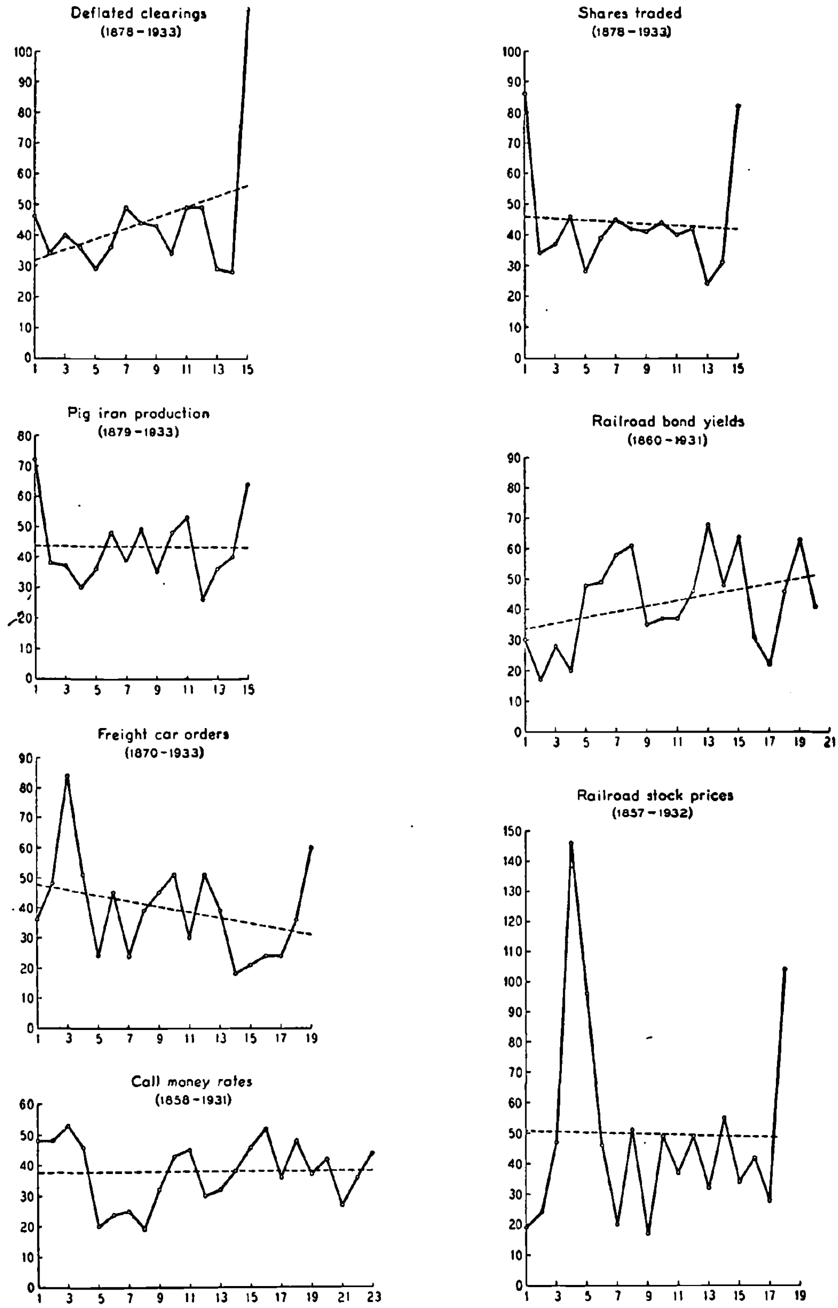
[a] Unweighted average.

CHART 55

Secular Changes in the Duration and Amplitude of Specific Cycles
Seven American Series

Duration of Full Specific Cycles[a]

------ Straight line fitted by 'least squares'



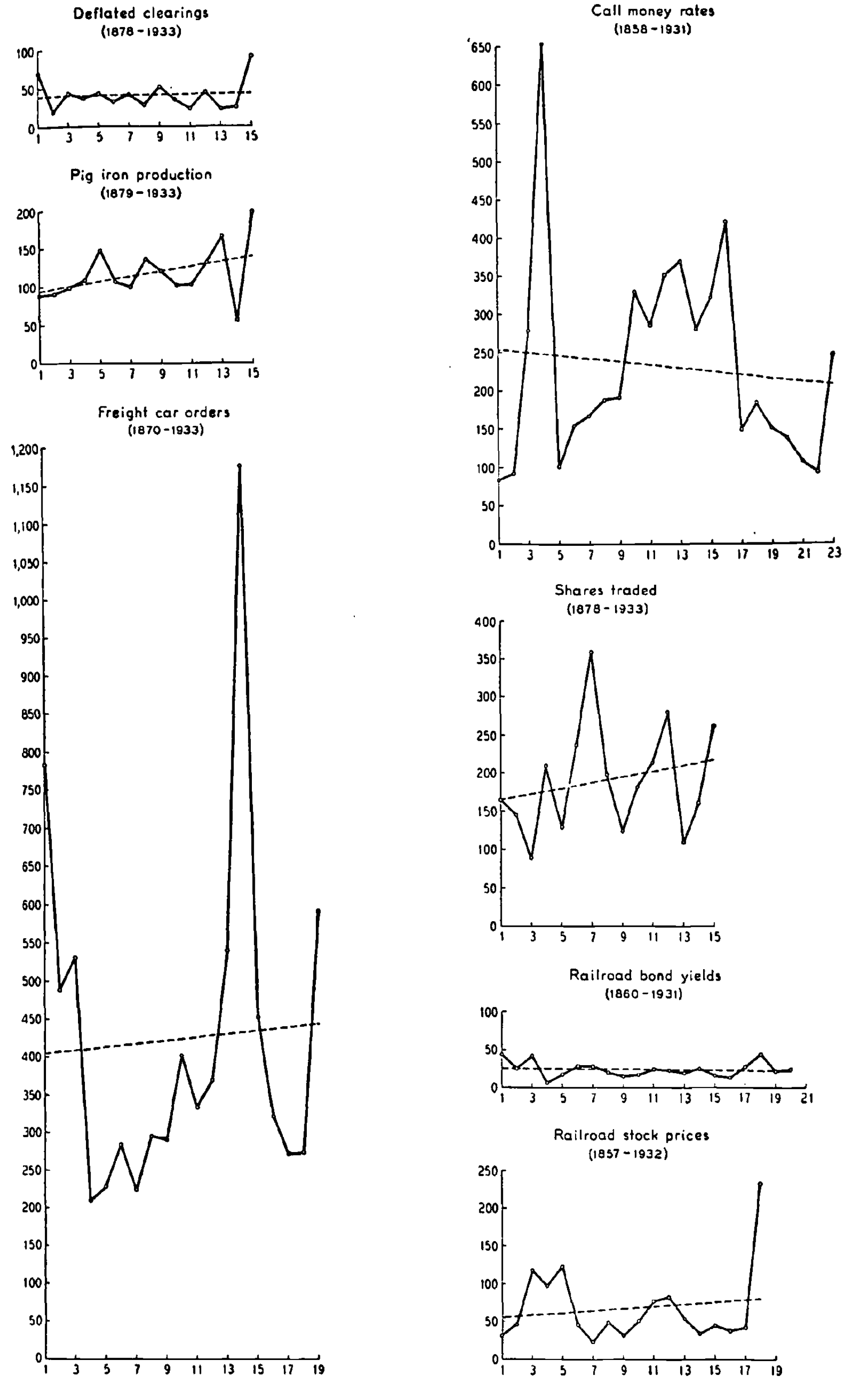
[a] The horizontal scale shows serial numbers of successive specific cycles; the vertical scale represents months

See Table 144 and Appendix Table B1.

CHART 55 (CONT.)

Amplitude of Full Specific Cycles[b]

------ Straight line fitted by 'least squares'



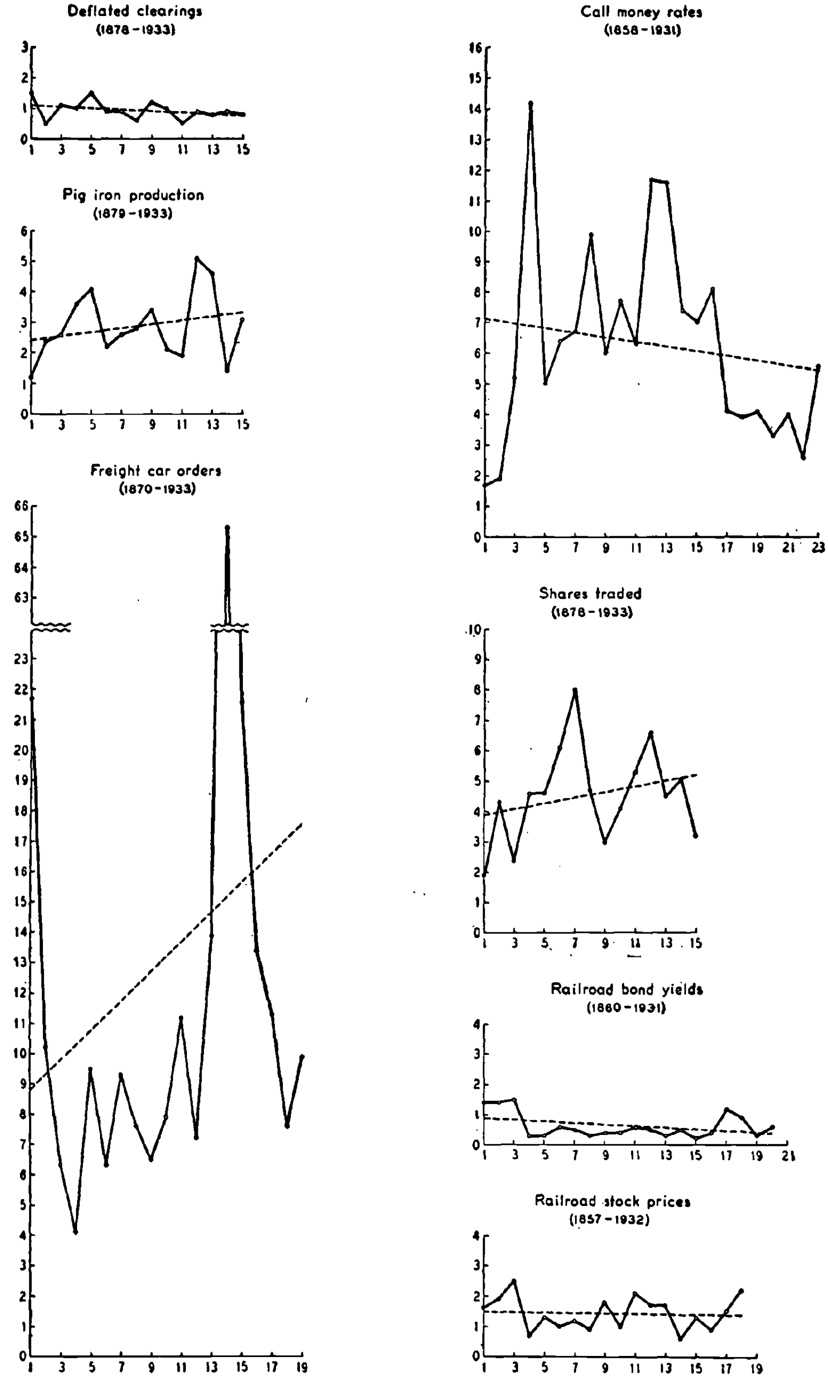
[b] The horizontal scale shows serial numbers of successive specific cycles; the vertical scale represents specific-cycle relatives.

See Table 144 and Appendix Table B1.

CHART 55 (CONT.)

Per Month Amplitude of Full Specific Cycles[c]

------ Straight line fitted by 'least squares'



[c] The horizontal scale shows serial numbers of successive specific cycles; the vertical scale represents specific-cycle relatives.

See Table 144 and Appendix Table B1.

A casual glance at these records might suggest that secular change is a general characteristic of cyclical fluctuations. But the mere fact that the trend lines are not horizontal, or that the subgroup averages differ within each series, does not demonstrate that secular changes have been marked or even that there have been any. Similar results could easily be obtained by fitting trend lines or computing subgroup means for random series; for example, successive values obtained from dice throws or the last places of a table of logarithms. The odds are heavy that a mathematically fitted trend will not be horizontal and that subgroup averages will not be constant. What we have to determine, therefore, is whether the slopes of our trend lines or the variations in the subgroup averages are 'substantial'; and if that does seem to be the case, whether they reflect persistent or haphazard changes. Should we find that secular changes in cyclical measures have as a rule been both 'substantial' and 'statistically significant', we would be forced to regard our notion that long-run averages of cyclical behavior give a useful first approximation of business-cycle behavior as discredited.[4]

A partial answer to our problem is given by measures of correlation between cyclical behavior and time (Table 146). We use two measures of correlation: the square of the coefficient of correlation and the square of the correlation ratio. The former shows the fraction of the total variation of the cyclical measures for each series that is attributable to the linear trend. The latter shows the fraction of the total variation that is attributable to the subgroup averages considered as a step-line of trend.[5] The outstanding feature of the correlation measures is their extremely low level. The square of the coefficient of correlation between cyclical durations and time varies in different series from 0 to .14; the square of the coefficient of correlation between cyclical amplitudes and time varies from 0 to .16 in the total amplitudes and from 0 to .13 in the per month amplitudes. Of course, the correlation ratios squared are higher in most

[4] It is important to distinguish between 'substantial' and 'significant' secular changes. If accounting records show that Mr. X received $1,000.00 during the first year of teaching, $1,000.01 the second year, $1,000.02 the third, and so in regular increments until the fiftieth year when he retired after enjoying a stipend of $1,000.49; then, there has been a definite trend in the teacher's annual earnings. This trend is 'statistically significant', but just as surely it is trifling and of no practical consequence.

Of course, what secular changes are 'substantial' and what 'slight' is a matter of judgment, which is bound to shift from problem to problem.

[5] In each computation the total variation is measured by the sum of the squares of the deviations of individual observations from their mean. Let Y be any observation on (say) durations of successive specific cycles, Y_t successive ordinates of a straight line fitted by least squares, $\overline{Y}$ the mean of all observations, $\overline{Y}_i$ the mean of the i^{th} subgroup, N the total number of observations, and N_i the number of observations in the i^{th} subgroup. Then the square of the coefficient of correlation is $\frac{\Sigma(Y_t - \overline{Y})^2}{\Sigma(Y - \overline{Y})^2}$. The square of the correlation ratio is $\frac{\Sigma N_i(\overline{Y}_i - \overline{Y})^2}{\Sigma(Y - \overline{Y})^2}$. If the number of observations is the same in each subgroup, the latter becomes $\frac{N_i \Sigma(\overline{Y}_i - \overline{Y})^2}{\Sigma(Y - \overline{Y})^2}$.

TABLE 146

Correlation between Specific-cycle Measures and Their Order in Time
Seven American Series

Series	Period covered	Number of		Coefficient of correlation squared			Correlation ratio squared		
		Specific cycles	Subgroups[a]	Duration of full cycles and time	Amplitude of full cycles and time	Per month amplitude of full cycles and time	Duration of full cycles and time	Amplitude of full cycles and time	Per month amplitude of full cycles and time
Deflated clearings...	1878–1933	15	3	.14	.00	.13	.13	.01	.23
Pig iron production.	1879–1933	15	3	.00	.16	.06	.00	.09	.05
Freight car orders...	1870–1933	19	3	.10	.00	.04	.18	.13	.18
Railroad stock prices	1857–1932	18	3	.00	.02	.00	.10	.05	.01
Shares traded......	1878–1933	15	3	.01	.05	.07	.01	.20	.22
Call money rates....	1858–1931	23	3	.00	.01	.03	.04	.11	.29
Railroad bond yields	1860–1931	20	3	.13	.02	.13	.22	.07	.24

The specific-cycle measures (durations in months, amplitudes in specific-cycle relatives) are correlated with serial numbers of the successive specific cycles.

[a] See Table 145 for the number of cycles and periods covered by successive subgroups.

instances than the coefficients of correlation squared, but they are just as unimpressive.[6]

These results are what we should expect from a careful study of Chart 55 and Table 145, apart from formal measures of correlation. Not only do secular changes appear small in comparison with the variability of individual cycles, but there is slight uniformity among the series with regard to the trend of any attribute of cyclical behavior. If we look merely at the trend lines of the full-cycle amplitudes, it seems that cyclical fluctuations have become narrower in the series on interest rates and wider in the series on industrial and speculative activity. But this suggestion is not borne out uniformly by the subgroup averages of the amplitudes or even by the trend lines of the per month amplitudes. The indications of different series concerning secular trends in cyclical duration are likewise contradictory and inconclusive. The dominating impression conveyed by Chart 55 is that the durations and amplitudes of successive cycles have varied in a highly irregular fashion, and that substantial secular changes have not taken place.

On the basis of the evidence presented so far, the most that may be said is that secular changes account for a small part of the variation in the durations and amplitudes of the specific cycles in our sample series. But have we any warrant for believing that secular changes account for *any* part of the variation? This question cannot be answered by measures of correlation alone. Even if measures of correlation were computed from

[6] The cycle measures are grouped by periods in computing correlation ratios, but the correlation coefficients are computed from ungrouped data. In these circumstances the correlation ratio may be smaller than the correlation coefficient. See Jan K. Wiśniewski, Pitfalls in the Computation of the Correlation Ratio, *Journal of the American Statistical Association*, Dec. 1934, pp. 416-7.

purely random series, it is almost certain that they would not be exactly zero. Further, no matter what the nature of the series may be, the size of correlation measures is not independent of the complexity of the regression lines that are fitted, or the number of subgroups into which the data are broken, or the number of observations that are used. To determine whether there is any warrant for believing that the trend lines or subgroup averages truly 'account' for any part of the variability of the cyclical measures in our samples, we should establish whether the correlation measures are larger than might reasonably be expected to arise from chance alone. Such a test is provided by 'variance analysis', a statistical tool devised by R. A. Fisher.

The principles on which the technique of 'variance analysis' rests can be explained most simply in terms of our subgroup analysis. Let us start, for example, with the hypothesis that the durations of successive specific cycles in pig iron production are free from secular change. To determine whether our data are consistent with this hypothesis, we first group the durations of the cycles by periods. Then, as in computing correlation ratios, we divide the total variation in the data—that is, the sum of the squares of the deviations of the individual cyclical measures from their mean—into two parts: the variation among and within the periods. On the hypothesis of no secular change both parts arise from the same source, namely, random variation. Each part can therefore be used to estimate the variability of the universe of cycles from which the original observations are considered to be a sample. The estimates are obtained by dividing each component of the total sum of squares by the number of 'degrees of freedom' on which it is based, that is, by the number of independent comparisons among the relevant observations. For example, if fifteen cycles are grouped into three periods, the estimate of variance derived from the differences among periods is based on two degrees of freedom, while the estimate of variance derived from the differences within periods is based on twelve degrees of freedom. The ratio of the first estimate to the second yields a measure designated F, which furnishes an objective test of the hypothesis that secular changes have not taken place.[7]

We should expect the two estimates of variance, or the two variances for short, to be approximately equal if the hypothesis that no secular

[7] If we use the symbols in note 5, the variance estimated from the variation among subgroups (periods) is $\frac{\Sigma N_i(\overline{Y}_i - \overline{Y})^2}{K-1}$, where K is the number of subgroups; that is, $i = 1, 2, \ldots, K$. The variance estimated from the variation within subgroups is $\frac{\Sigma S(Y - \overline{Y}_i)^2}{N-K}$, where S represents the summation within a subgroup. Hence $F = \frac{(N-K)\,\Sigma N_i(\overline{Y}_i - \overline{Y})^2}{(K-1)\,\Sigma S(Y - \overline{Y}_i)^2}$.

In the straight-line trend analysis, the variance estimated from the variation of the ordinates of trend is $\Sigma(Y_t - \overline{Y})^2$; the variance estimated from the variation about the line of trend is $\frac{\Sigma(Y - Y_t)^2}{N-2}$. Hence $F = \frac{(N-2)\,\Sigma(Y_t - \overline{Y})^2}{\Sigma(Y - Y_t)^2}$.

change has occurred in cyclical durations is valid. On the other hand, if a secular change has taken place, the variance among periods should be larger than the variance within periods, and F should exceed unity. But F may exceed unity, even if calculated from random series. Therefore, to judge any F, we must determine the probability of obtaining a greater value of F from a random sample grouped at random. If the probability is small, the presumption is that the differences among periods are *significantly* greater than the differences within periods. In other words, we may conclude provisionally that a secular change has occurred in cyclical behavior. How 'small' the probability need be is a matter of choice and of the objectives in view; but it has become customary to consider a probability of .05 (one chance in twenty) or smaller as indicating 'significance', and that is the standard to which we shall adhere.[8]

The probability tables used to test variance ratios (F's) are derived from a theoretical population distributed according to the 'normal curve'. Since definite evidence exists that the frequency distributions of our cyclical measures are frequently skewed,[9] the tests we make are in some degree inexact. Further, the probability tables are based on the assumption that the observations entering the sample are independent. It seems reasonably certain that cyclical measures do not fulfill this condition, although they come closer to doing so than the original data of time series. This too means that our tests are inexact, in the sense that the probability tables are not perfectly applicable to our data. But these technical difficulties do not seriously affect the analysis of this chapter. Our main problem is whether secular changes in cyclical behavior have been substantial, and that question we can usually answer sufficiently well from ordinary data charts and tables. It is helpful, however, to check judgments reached in this fashion by determining which of our cyclical measures are and which are not reasonably consistent with the hypothesis of no secular change. Even rough tests of statistical significance used with reserve and discrimination will serve this limited purpose.

Thus Table 147 shows the variance ratios (F's) derived for the durations and amplitudes of full specific cycles in our seven series. The hypothesis of no secular change is tested in two ways: by dividing the data into subgroups, and by computing linear trends. The trend lines and subgroups that we use are indicated in Chart 55 and Table 145. On the whole the variance tests seem to corroborate the hypothesis that the

[8] R. A. Fisher and F. Yates have published a table showing what values of F correspond to the .20, .05, .01, and .001 levels of significance (that is, the values of F that will be exceeded in the stated proportion of cases by chance) for specified degrees of freedom among and within groups. See their *Statistical Tables for Biological, Agricultural and Medical Research* (Oliver and Boyd, London, 1938), pp. 29-35. A more detailed table of the values of F, but limited to the .05 and .01 levels, is given by George W. Snedecor in his *Statistical Methods* (Collegiate Press, Ames, Iowa, 1938 rev. ed.), pp. 184-7. Both tables show directly values of F greater than 1; values between 0 and 1 can be obtained by using reciprocals.

[9] See Ch. 12, Sec. III.

TABLE 147

Tests of Secular Change in Durations and Amplitudes of Specific Cycles
Seven American Series

Series	Number of		Ratio of variance among subgroups to variance within subgroups			Ratio of variance of ordinates of straight-line trend to variance about the trend		
	Specific cycles	Subgroups	Duration of full cycles	Amplitude of full cycles	Per month amplitude of full cycles	Duration of full cycles	Amplitude of full cycles	Per month amplitude of full cycles
Deflated clearings	15	3	0.87	0.07	1.78	2.12	0.05	2.00
Pig iron production	15	3	0.01 †	0.61	0.35	0.01	2.41	0.84
Freight car orders	19	3	1.74	1.18	1.75	1.95	0.05	0.74
Railroad stock prices	18	3	0.87	0.39	0.08	0.01	0.38	0.06
Shares traded	15	3	0.06	1.46	1.64	0.08	0.73	0.91
Call money rates	23	3	0.39	1.23	4.03‡	0.01	0.22	0.63
Railroad bond yields	20	3	2.44	0.67	2.70	2.66	0.33	2.60

See note 7 concerning the variance ratios, Table 145 for the number of cycles and periods covered by the successive subgroups, and Chart 55 for the straight-line trends.

‡ Indicates that the ratio is 'significantly large', i.e., larger than the value that would be exceeded once in twenty times by chance. For the analysis by subgroups this value is 3.88 for deflated clearings, iron production and shares traded, 3.68 for railroad stock prices, 3.63 for freight car orders, 3.59 for railroad bond yields, and 3.49 for call money rates. For the straight-line trend analysis, the corresponding values are 4.67, 4.49, 4.45, 4.41, and 4.32. The differences among these values result from differences in the number of degrees of freedom on which the estimates of variance are based.

† Indicates that the ratio is 'significantly small', i.e., smaller than the value that would be fallen short of once in twenty times by chance. For the subgroup analysis this value is .05 for all series. For the straight-line trend analysis, it is .004 for all series.

durations and amplitudes of the specific cycles in our test series have not been subject to secular changes. Surely, they do not contradict the conclusion already reached: namely, that if secular changes have taken place in these cyclical characteristics, they have in general been slight. In the portion of the table relating to the subgroup analysis, one out of twenty-one variance ratios is greater than the value that would be exceeded by chance once in twenty times, and one is less than the value that would be fallen short of by chance once in twenty times.[10] No value of F in the straight-line trend analysis falls outside either limit. Our data seem to behave the way we should expect data derived from the same universe and grouped at random to behave. Yet we believe that the amplitudes of call money rates have undergone a real change. By facilitating increases of bank reserves in times of pressure and by reducing the dependence of interior banks upon New York, the Federal Reserve system has undoubtedly tended to mitigate the spasmodic fluctuations of call money rates.

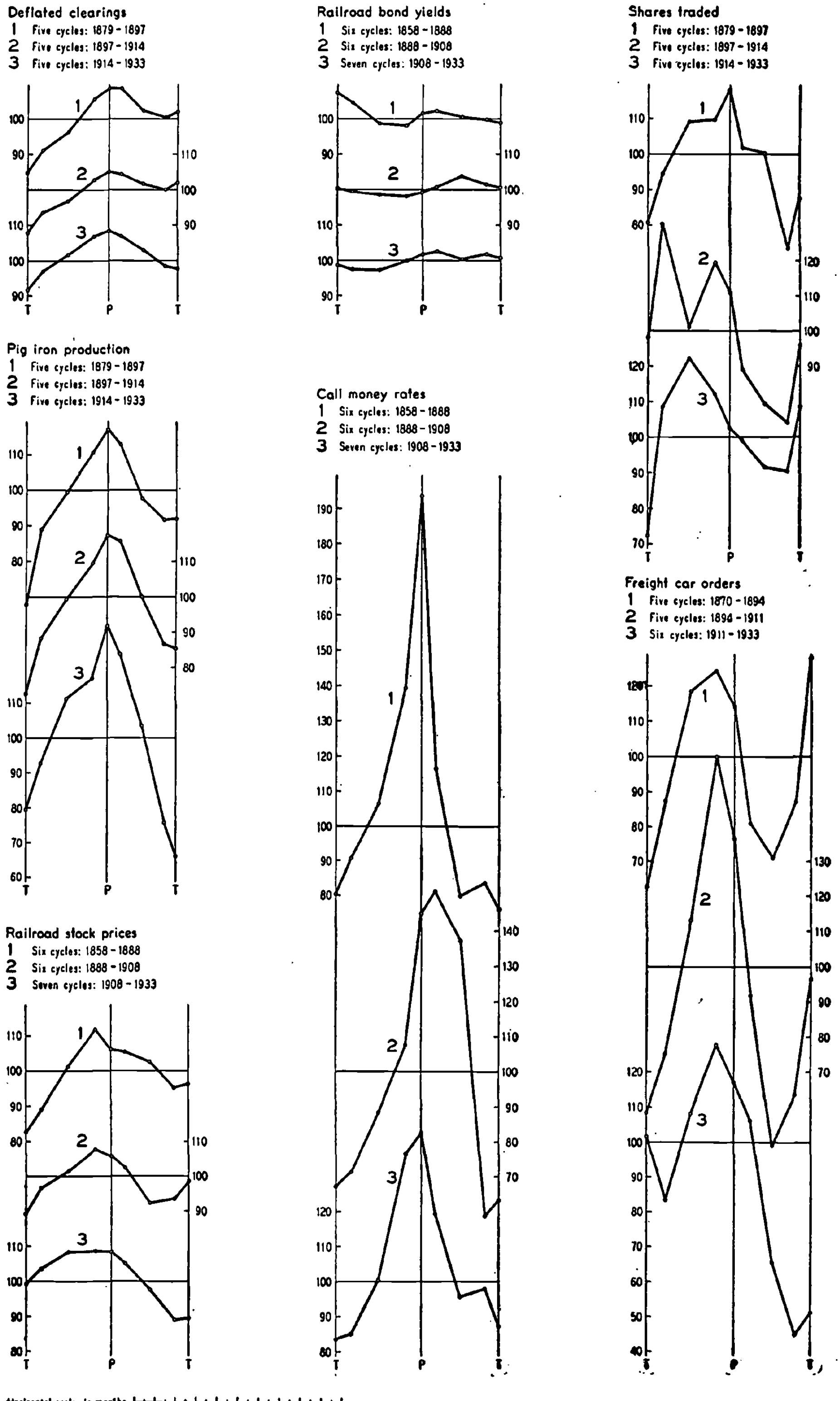
II Reference-cycle Patterns

We pass from these tests of specific cycles to tests of reference cycles. The question now is whether the behavior of our sample series has changed materially during the business cycles marked off by our reference dates.

[10] These values are identified in Table 147. Of course, only the values of F that are greater than the value that would be exceeded once in twenty trials by chance can create a presumption of a secular change in cyclical behavior.

CHART 56

Average Patterns of Three Successive Groups of Reference Cycles
Seven American Series



To simplify comparisons, average deviations are omitted; also, each pattern is drawn to the schedule of intervals appropriate to the full number of cycles covered by the series, instead of the true intervals for each group of cycles (cf. Appendix A). See Table 148.

TABLE 148

Average Patterns of Three Successive Groups of Reference Cycles
Seven American Series

Series and period covered	No. of reference cycles	I Initial trough (3 mos.)	II Expansion First third	III Expansion Middle third	IV Expansion Last third	V Peak (3 mos.)	VI Contraction First third	VII Contraction Middle third	VIII Contraction Last third	IX Terminal trough (3 mos.)
Deflated clearings										
1879–1897.........	5	85.0	91.4	96.6	105.8	108.6	108.6	102.4	100.3	102.0
1897–1914.........	5	87.8	93.7	96.9	102.8	105.3	104.6	101.7	100.0	102.2
1914–1933.........	5	91.6	97.0	101.8	106.9	108.5	106.9	102.9	98.3	97.7
Pig iron production										
1879–1897.........	5	67.9	89.1	99.8	110.8	117.3	113.1	97.8	91.9	92.0
1897–1914.........	5	72.5	88.3	99.4	109.7	117.5	115.8	100.0	86.7	85.4
1914–1933.........	5	79.5	92.7	111.2	117.0	131.9	124.0	103.4	75.9	66.0
Freight car orders										
1870–1894.........	5	62.7	87.4	118.6	124.3	114.1	81.0	70.9	87.2	128.0
1894–1911.........	5	58.3	75.2	113.0	160.0	136.6	91.8	49.1	63.8	96.3
1911–1933.........	6	101.7	83.5	108.1	127.9	117.2	106.1	65.3	45.0	51.0
Railroad stock prices										
1858–1888.........	6	82.8	89.0	101.5	112.0	106.2	105.6	102.8	95.3	96.5
1888–1908.........	6	89.5	96.8	101.5	107.7	105.8	102.3	92.5	93.7	98.7
1908–1933	7	99.3	103.7	108.4	108.5	108.4	105.0	97.8	89.1	89.6
Shares traded										
1879–1897.........	5	80.8	94.7	109.3	109.9	118.2	101.9	100.4	73.7	87.8
1897–1914.........	5	98.2	130.2	101.0	119.7	110.9	89.1	79.5	74.4	96.0
1914–1933.........	5	72.4	108.7	122.4	112.3	102.7	99.3	91.7	90.5	108.8
Call money rates										
1858–1888.........	6	80.4	90.6	106.4	139.3	193.7	116.5	80.0	83.8	76.3
1888–1908.........	6	67.3	71.1	88.4	107.8	145.1	151.4	137.4	58.8	63.2
1908–1933.........	7	83.8	85.1	100.2	136.3	142.5	119.3	95.7	98.0	87.3
Railroad bond yields										
1858–1888.........	6	107.3	104.6	98.8	98.1	101.8	102.4	100.5	99.9	99.0
1888–1908.........	6	100.4	99.6	98.8	98.3	99.3	100.8	103.9	101.4	100.6
1908–1933.........	7	98.9	97.6	97.4	100.0	101.8	102.7	100.2	101.8	100.8

Where the number of cycles is not exactly divisible by 3, an additional cycle is placed in the last group.

TABLE 149

Square of Correlation Ratio between Reference-cycle Standings and Time
Seven American Series

Series	Period covered	Number of Reference cycles	Number of Sub-groups	I	II	III	IV	V	VI	VII	VIII	IX	Pattern as a whole[b]
				Square of correlation ratio between time and standings[a] in: Reference-cycle stage									
Deflated clearings...	1879–1933	15	3	.10	.10	.25	.16	.10	.11	.01	.01	.04	.07
Pig iron production.	1879–1933	15	3	.05	.01	.13	.06	.19	.23	.02	.09	.23	.11
Freight car orders...	1870–1933	16	3	.08	.02	.01	.16	.02	.04	.09	.15	.15	.09
Railroad stock prices	1858–1933	19	3	.16	.17	.11	.03	.01	.02	.18	.02	.04	.08
Shares traded......	1879–1933	15	3	.13	.20	.37	.08	.06	.13	.28	.14	.04	.12
Call money rates....	1858–1933	19	3	.07	.09	.09	.10	.06	.22	.30	.33	.10	.12
Railroad bond yields	1858–1933	19	3	.33	.36	.02	.05	.08	.06	.15	.03	.02	.14

See Table 148 for the number of cycles and periods covered by the successive subgroups.

[a] That is, between standings in reference-cycle relatives and serial numbers of successive reference cycles.

[b] See note 11.

TABLE 150

Tests of Secular Change in Reference-cycle Patterns

Seven American Series

Series	Number of		Ratio of variance among subgroups to variance within subgroups for									
	Reference cycles	Subgroups	Standings in reference-cycle stage									Pattern as a whole
			I	II	III	IV	V	VI	VII	VIII	IX	
Deflated clearings. . .	15	3	0.65	0.69	1.97	1.13	0.70	0.73	0.03†	0.06	0.23	0.46†
Pig iron production. .	15	3	0.33	0.08	0.93	0.37	1.43	1.78	0.12	0.62	1.74	0.75
Freight car orders. . . .	16	3	0.55	0.12	0.09	1.24	0.15	0.24	0.65	1.16	1.11	0.62
Railroad stock prices.	19	3	1.57	1.70	0.96	0.24	0.08	0.13	1.75	0.17	0.32	0.67
Shares traded.	15	3	0.94	1.47	3.53	0.49	0.38	0.86	2.39	0.99	0.23	0.80
Call money rates. . . .	19	3	0.56	0.75	0.84	0.94	0.52	2.23	3.50	3.88J	0.90	1.07
Railroad bond yields.	19	3	4.00J	4.52J	0.16	0.42	0.66	0.51	1.36	0.25	0.14	1.30

See notes 7 and 11 concerning the variance ratios, and Table 148 for the number of cycles and periods covered by the successive subgroups.

J Indicates that the ratio is 'significantly large', i.e., larger than the value that would be exceeded once in twenty times by chance. For the pattern as a whole this value is 1.68 for railroad bond yields, railroad stock prices, and call money rates; 1.70 for deflated clearings, iron production, and shares traded; 1.69 for freight car orders. For the single stages it is 3.63 for railroad bond yields, railroad stock prices, and call money rates; 3.88 for deflated clearings, iron production, and shares traded; 3.80 for freight car orders.

† Indicates that the ratio is 'significantly small', i.e., smaller than the value that would be fallen short of once in twenty times by chance. In all series this value is .51 for the pattern as a whole, and .05 for the single stages.

To answer this question we proceed as before, except that we engage in no curve fitting. The first step is to divide into thirds the full number of cycles covered by each series and compute means of the reference-cycle patterns in each subgroup. Next we compute for each series squares of correlation ratios between (a) time and (b) the standings in individual cycle stages and all stages combined. Finally, we show the statistical significance of the correlation ratios by using variance analysis.[11] The results of these operations are presented in Chart 56 and Tables 148-150.

The new experiments yield results similar to those for specific cycles. The squares of the correlation ratios for whole reference-cycle patterns range from .07 to .14 in different series. Of the sixty-three correlation measures for single stages, only six are above .25; the largest is .37. None of the correlation measures for whole patterns and only three for single stages are 'significantly large'. It might seem that in these instances the hypothesis of absence of secular change is contradicted. That inference is inadmissible without additional evidence; for if sixty-three variance ratios were computed from random samples, approximately three such instances would be expected. Nor is it surprising that two of the *F*'s are

11 In computing the correlation ratio (squared) for the reference-cycle pattern as a whole, the numerator is the sum of the numerators for the nine separate stages and the denominator is the sum of the corresponding denominators. See above, note 5. Consequently, the correlation ratio (squared) for the whole pattern of a series is a weighted arithmetic mean of the correlation ratios (squared) for the nine stages, the weight being the sum of squared deviations about the mean of the stage. This measure takes no account of the actual sequence of the stages.

In making the variance analysis for the whole pattern, the number of degrees of freedom is $9(K-1)$ among subgroups and $9(N-K)$ within subgroups, where N is the number of cycles and K the number of subgroups.

less than the values that would be fallen short of once in twenty times by chance, since approximately one such case is to be expected in twenty random samples.

The preceding statements cannot be accepted literally, as statistical experts will be quick to recognize, for our data fail to satisfy fully the

TABLE 151

Average Duration and Amplitude of Expansions and Contractions of Three Successive Groups of Specific Cycles

Seven American Series

Series and period covered	Number of specific cycles	Average duration in months		Average amplitude in specific-cycle relatives			
		Expansion	Contraction	Rise	Fall	Rise per month[a]	Fall per month[a]
Deflated clearings							
1878–1893	5	28	9	30	13	1.0	1.7
1893–1910	5	33	8	27	10	0.8	3.0
1910–1933	5	37	17	24	18	0.7	1.1
Pig iron production							
1879–1896	5	29	14	62	44	2.6	3.8
1896–1914	5	32	11	64	48	2.1	5.6
1914–1933	5	25	18	59	72	2.6	4.6
Freight car orders							
1870–1894	6	22	26	215	206	13.9	8.3
1894–1914	6	22	18	163	156	9.4	10.4
1914–1933	7	12	19	254	264	41.3	15.9
Railroad stock prices							
1857–1889	6	35	28	46	31	1.8	1.5
1889–1907	6	23	14	29	23	1.4	1.8
1907–1932	6	28	21	32	42	1.1	1.8
Shares traded							
1878–1897	5	15	31	74	73	7.7	2.5
1897–1914	5	15	28	108	112	9.8	4.2
1914–1933	5	24	20	112	93	5.3	5.6
Call money rates							
1858–1880	7	21	17	110	108	5.2	7.3
1880–1904	8	18	17	142	148	10.5	9.8
1904–1931	8	20	20	96	91	4.7	5.7
Railroad bond yields							
1860–1876	6	15	17	12	14	0.8	1.1
1876–1905	7	22	27	7	13	0.4	0.5
1905–1931	7	25	20	13	11	0.6	1.0
	Number of subgroups	Ratio of variance among subgroups to variance within subgroups					
Deflated clearings	3	0.43	1.38	0.21	0.39	0.90	0.49
Pig iron production	3	0.46	0.62	0.07	1.04	0.35	0.49
Freight car orders	3	1.65	1.16	1.03	1.26	1.11	1.61
Railroad stock prices	3	0.47	1.17	0.69	0.59	1.14	0.18
Shares traded	3	1.05	1.11	1.45	1.31	0.86	4.87J
Call money rates	3	0.13	0.32	0.99	1.32	4.35J	1.22
Railroad bond yields	3	1.03	0.97	1.10	0.30	1.88	0.93

See note to Table 145.

[a] Unweighted average.

J Larger than the value that would be exceeded once in twenty times by chance. This value is specified in a note to Table 147.

assumptions underlying variance analysis. In particular, we cannot justifiably regard our several series or the standings within a given series in successive cycle stages as being independent of one another; we know that their movements are intercorrelated. But technical difficulties do not cloud the essential point, which is sufficiently plain without probability calculations. If secular changes have taken place in reference-cycle behavior, they have not left a prominent imprint on our sample series. The outstanding feature of Chart 56 is that the average cyclical patterns are roughly similar from period to period within the same series but differ widely among series. This demonstration suffices for our present purpose.

III Other Cyclical Measures

The results of the preceding tests are confirmed when the range of the tests is extended. Table 151 supplements Tables 145 and 147, by analyzing separately the expansion and contraction phases of the specific cycles. We find that the variance among subgroups is larger than the variance within subgroups in half of the forty-two instances covered by the new table. But in only two of the twenty-one instances does the variance ratio exceed unity to a degree that is significant according to probability calculations: the rate of cyclical rise in call money rates and the rate of cyclical fall in share trading. The latter result is perhaps unexpected. The former is confirmed by independent knowledge, as already stated. No other result in the table is even in the vicinity of the 'level of significance'. As a whole the evidence in Table 151 gives no support to the view that real secular changes of substantial scope have been a common feature of the cyclical behavior of economic activities. This remark applies also to the measures of timing and conformity in Table 152, although the timing of some series seems to have undergone a clear-cut secular change.

In Table 153 we summarize the variance tests thus far presented. We show merely the number of 'significantly large' variance ratios relatively to the number computed, without allowing for the duplication that automatically results when closely related measures are analyzed or when certain of these measures are treated on more than one plan. Of the 189 variance ratios, only nine are 'significantly large'. It is interesting that none of the 'significant' ratios is among the measures of cyclical duration and that seven occur in the two series on interest rates. These results may mean that the duration of specific cycles is one of their most stable features in the long run, and that money markets are more susceptible to secular changes in cyclical behavior than are industrial or security markets. Although the evidence at hand is slender and indefinite, it is noteworthy that the series on deflated clearings outside New York, which comes closer to representing economic activity at large than any other in our sample,

TABLE 152

Average Timing of Specific Cycles and
Rates of Change during Three Successive Groups of Reference Cycles
Seven American Series

Series and period covered	Number of reference cycles	Average lead (−) or lag (+) in months at reference[a]		Average change per month in reference-cycle relatives during stages[b] matched with reference		
		Peaks	Troughs	Expansions	Contractions	Cycles[c]
Deflated clearings						
1879–1897	5	+3.8	−6.2	+0.9	−0.7	−1.5
1897–1914	5	+4.2	−7.4	+0.8	−0.4	−1.2
1914–1933	5	+1.2	−3.8	+0.7	−0.4	−1.1
Pig iron production						
1879–1897	5	+0.8	−3.2	+2.1	−1.7	−3.9
1897–1914	5	+3.4	−7.2	+2.2	−1.8	−4.0
1914–1933	5	+1.4	−0.3	+2.4	−3.3	−5.7
Freight car orders						
1870–1894	5	−7.6	−3.2	+2.4	−2.5	−4.9
1894–1911	5	−7.0	−5.0	+5.2	−5.2	−10.5
1911–1933	6	−3.2	−1.4	+3.8	−3.9	−7.8
Railroad stock prices						
1858–1888	6	−7.6	−13.4	+0.9	−0.6	−1.5
1888–1908	6	−2.2	−1.2	+0.9	−0.8	−1.7
1908–1933	7	−7.3	−8.6	+0.5	−0.5	−1.0
Shares traded						
1879–1897	5	−11.4	−2.2	+1.4	−2.2	−3.5
1897–1914	5	−12.0	−4.8	+1.8	−2.2	−4.1
1914–1933	5	−7.8	−6.5	+2.7	−0.6	−3.3
Call money rates						
1858–1888	6	−3.3	−1.2	+4.2	−3.5	−7.7
1888–1908	6	+4.2	+3.0	+3.4	−5.0	−8.4
1908–1933	7	−1.0	+2.6	+3.3	−2.4	−5.7
Railroad bond yields						
1858–1888	6	+9.2	+17.8	+0.2	−0.3	−0.5
1888–1908	6	+9.2	+14.4	+0.1	−0.1	−0.2
1908–1933	7	+4.8	+1.8	+0.4	−0.1	−0.5
	Number of subgroups	**Ratio of variance among subgroups to variance within subgroups**				
Deflated clearings	3	0.50	0.78	0.57	0.26	0.51
Pig iron production	3	0.26	7.83♪	0.09	1.99	1.55
Freight car orders	3	0.37	0.31	1.29	0.92	2.05
Railroad stock prices	3	0.90	2.11	0.58	0.18	0.56
Shares traded	3	0.54	0.57	1.23	1.61	0.14
Call money rates	3	4.23♩	0.59	0.16	1.86	0.49
Railroad bond yields	3	1.61	7.61♪	1.02	1.44	0.89

See note to Table 148.

[a] The number of timing observations is not defined exactly by the number of reference cycles; see Tables 62 and 141. To avoid duplication, the timing at the last reference trough of the first period is excluded from the average for that period but included in the average for the second, and the timing at the last reference trough of the second period is excluded from the average for that period but included in the average for the third.

[b] These stages are indicated in Table 140.

[c] Difference between contraction and expansion (see Table 47, col. 8).

♩ Larger than the value that would be exceeded once in twenty times by chance.

♪ Larger than the value that would be exceeded once in a hundred times by chance.

TABLE 153

Summary of Variance Tests in Preceding Tables

Measure	Number of variance ratios computed		Number of 'significantly large' variance ratios							
	Per series	For all series	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields	All series
Duration[a]	4	28	...	...	...	...	...	...	...	...
Amplitude[a]	8	56	...	...	...	...	1	2	...	3
Timing[b]	2	14	...	1	...	...	...	1	1	3
Pattern[c]	10	70	...	...	...	...	...	1	2	3
Conformity[b]	3	21	...	...	...	...	...	...	...	...
Total	27	189	...	1	...	...	1	4	3	9

This summary omits 'significantly small' ratios, of which there are two in clearings and one in iron production.
[a] Derived from Tables 147 and 151.
[b] Derived from Table 152.
[c] Derived from Table 150.

has very few ratios exceeding unity, in no instance turns up a 'significantly large' ratio, but does turn up two that are 'significantly small'.

It is no part of our aim to discuss the results for individual series in any detail at present. Nor, to repeat, are 'significant' but 'slight' secular changes of more than incidental interest. It is well to note, nevertheless, that apart from their technical limitations, our probability calculations cannot be taken at face value. Economic analysis and historical knowledge must play a part in whatever judgment is finally reached concerning the presence or absence of secular change in cyclical behavior. For example, our observations suggest that the tendency of pig iron production to lead at general revivals has practically disappeared; the variance test indicates that so large a difference as we find between the timing of recent and earlier cycles could arise from chance causes less frequently than one time out of twenty; which creates some presumption that a secular change in timing has actually occurred. This presumption is materially strengthened by knowledge that the secular trend of iron output has flattened out, for we have independent reasons for believing that a change in cyclical timing tends to be associated with retardation of growth. But retardation in the railroad equipment industry has been even more pronounced than in the iron industry; the mean timing of freight car orders at business-cycle troughs has shifted in the same direction as the timing of pig iron production; the original data on orders leave much to be desired and an erratic factor is bound to enter the placing of orders; hence we are inclined to judge that a secular change may well have occurred also in the timing of freight car orders, in spite of the low variance ratio.[12] Of course,

[12] Freight car orders led the reference trough in 1911 by 17 months. This long lead is placed in the third period in Table 152, by force of the convention adopted in constructing the table. If it were placed instead in the second period, the average timing in the three successive periods would run − 3.2, − 7.0 and + 1.2, and the variance ratio would be 2.02 (the .05 value is 3.74). See below, Sec. VIII.

what we do in such instances is to combine intuitively the knowledge given by probability tests with other knowledge concerning the phenomenon under observation.

IV Duration and Amplitude of Business Cycles

A vital check on some of the preceding results may be obtained by analyzing the business-cycle durations yielded by our reference dates. If there is little or no presumption that the durations of specific cycles have undergone noteworthy secular changes, the same should be true of the durations of business cycles. And since our business-cycle measures cover three countries besides the United States, we are also in a position to determine whether the experience of other countries has been similar to that of the United States.

We therefore group the durations of successive business cycles and their phases of expansion and contraction into three approximately equal classes in each country, and compare the variance among subgroups in each with the variance within subgroups. The subgroup means and the results of the variance analysis are set out in Table 154. There is a rough

TABLE 154
Average Duration of Three Successive Groups of Business Cycles
Four Countries

Country and period covered	Number of business cycles	Average duration in months: Expansion	Contraction	Full cycle
United States				
1854–1885	6	31	30	61
1885–1908	7	24	16	40
1908–1933	7	21	21	42
Great Britain				
1854–1886	5	40	36	76
1886–1914	5	43	25	68
1914–1932	5	24	19	43
France				
1865–1887	5	22	30	52
1887–1914	5	39	26	65
1914–1932	5	30	13	43
Germany				
1879–1902	3	46	47	92
1902–1914	3	33	17	50
1914–1932	4	34	20	54
	Number of subgroups	Ratio of variance among subgroups to variance within subgroups		
United States	3	1.89	1.75	3.11
Great Britain	3	2.09	0.93	1.88
France	3	1.77	1.47	1.07
Germany	3	0.78	4.31	6.18[j]

Derived from the monthly reference dates in Table 16. See note to Table 145.
[j] Larger than the value that would be exceeded once in twenty times by chance.

suggestion that business cycles may have become shorter with the passage of time. But this suggestion is only partly borne out by the annual measures in Table 139, which cover a longer period. Our present data yield a 'significant' variance ratio only in one instance, the duration of full cycles in Germany. True, the variance ratio for durations of full cycles exceeds unity in all four countries. But Table 155 suggests that at least the American result is fortuitous, and we know that the timing of business cycles in each of our countries is correlated with that of business cycles in the others.

TABLE 155
Tests of Secular Change in Durations of Business Cycles
United States, 1854–1933

Test	Average duration of successive groups of cycles (mos.) (1)	(2)	(3)	(4)	(5)	Variance ratio (F)[b]
20 cycles divided into						
Two equal groups	51	43	..	..	..	0.99
Three equal groups[a]	61	40	42	..	..	3.11
Four equal groups	58	44	42	44	..	0.84
Five equal groups	48	61	40	44	42	0.86
Straight line fitted by 'least squares'[c]	54[d]	..	..	..	40[e]	1.19

Derived from the monthly reference troughs in Table 16.
[a] The successive groups include 6, 7 and 7 cycles.
[b] None of these ratios is statistically significant.
[c] The x values are the serial numbers of the successive reference cycles.
[d] First trend value ($x = 1$).
[e] Last trend value ($x = 20$).

Unfortunately, we lack at present reliable measures of the amplitude of successive business cycles. In the absence of anything better, the ratings in Table 156, based on several familiar indexes of American business activity, may perhaps serve as a provisional check on our sample series.[13] These ratings fail to disclose any very clear trend in the intensity of successive business-cycle expansions or contractions, and thus corroborate the broad evidence of our sample.[14] The coefficient of correlation between the graded intensities of expansions and their order in time[15] is −.01, or virtually zero; the coefficient for contractions is +.18. On the basis of these results, we can say at most that secular change accounts for a small part of the variation in the amplitudes of business-cycle declines. But even this statement is problematical, since the coefficient of correlation for contractions is not statistically significant.[16]

[13] Note, however, the difficulty that at least in the early years these indexes lean heavily on two of our series, clearings and iron production.

[14] See also Sec. VI, and Ch. 11, Sec. VI.

[15] That is, between the ranks of the average ranks in Table 156 and the serial numbers of the successive cycles.

[16] The standard error of a coefficient of rank correlation based on 15 observations is .27; which means that the coefficient would have to be (approximately) .54 in order to satisfy the .05 level of significance.

TABLE 156

Ranks of Amplitudes of Cyclical Expansions and Contractions
Three Indexes of American Business Activity, 1879–1933

Reference expansion[a]	Amplitude of expansions ranked					Reference contraction[a]	Amplitude of contractions ranked				
	A.T.&T. index	Persons index	Ayres index	Average of three ranks	Rank of average ranks		A.T.&T. index	Persons index	Ayres index	Average of three ranks	Rank of average ranks
1879–82	11	14	10	11.7	12	1882–85	11	12	9	10.7	11
1885–87	9	10	9	9.3	9	1887–88	1	4	1	2.0	1
1888–90	4	8	2	4.7	5	1890–91	6	6	5	5.7	6
1891–93	3	5	4	4.0	3	1893–94	12	14	13	13.0	13
1894–95	7	12	11	10.0	10	1895–97	7	11	10	9.3	10
1897–99	13	11	14	12.7	13	1899–00	2	5	4	3.7	5
1900–02	1	3	1	1.7	1	1902–04	4	3	3	3.3	3.5
1904–07	10	4	8	7.3	7.5	1907–08	13	10	12	11.7	12
1908–10	12	9	12	11.0	11	1910–12	3	1	6	3.3	3.5
1912–13	2	1	3	2.0	2	1913–14	10	8.5	8	8.8	8
1914–18	14	13	13	13.3	14	1918–19	8	8.5	7	7.8	7
1919–20	6	6	5	5.7	6	1920–21	14	13	14	13.7	14
1921–23	15	15	15	15.0	15	1923–24	9	7	11	9.0	9
1924–26	8	7	7	7.3	7.5	1926–27	5	2	2	3.0	2
1927–29	5	2	6	4.3	4	1929–33	15	15	15	15.0	15

The mildest contraction or expansion is assigned a rank of 1, the next a rank of 2, and so on. The ranks were determined from our standard measures of specific-cycle amplitude, the analysis being positive. Each of the three indexes is trend-adjusted. For this reason, if for no other, the ranks of the successive cycles are very rough approximations.

The A.T.&T. index is available without trend adjustment since 1900. The ranks of the cyclical movements in the unadjusted and trend-adjusted indexes differ considerably at times. The ranks of the two forms of the index (the ranks of the trend-adjusted index are shown in parentheses) for the nine successive expansions since 1900 are 5(1), 8(6), 6(7), 1(2), 7(8), 2(4), 9(9), 4(5), 3(3); and 3(2), 7(7), 1(1), 4.5 (6), 4.5(4), 8(8), 6(5), 2(3), 9(9) for the nine successive contractions since 1902.

For sources, see Appendix C, notes on series (1), (2) and (4) of Table 21. (The notes in the Appendix are phrased for reference cycles; they apply only approximately to specific cycles when overlapping segments of the same series have been used, as in the A.T.&T. and Persons indexes.)

[a] Years of turning points in our monthly reference chronology (Table 16).

V Business Cycles and Economic Stages

It is possible, of course, that so few noteworthy secular or discontinuous changes in cyclical behavior have emerged in our tests because they have been made by mechanical rule. The trend lines and subgroups we have used involve arbitrary arrangements in every instance; they do not follow hints derived from general economic history or the history of business cycles. We have used mechanical schemes not because we prefer them, but because no other method seemed possible in view of the looseness with which hypotheses concerning secular changes in cyclical behavior have usually been formulated. But there are two notable exceptions to this statement, and they warrant careful investigation.

The first is Frederick C. Mills' suggestion that the duration of business cycles is a function of the stage of industrial development.[17] Mills developed his hypothesis to account for the differences in the durations of business cycles shown by Thorp's *Business Annals*. In its author's words:

[17] An Hypothesis Concerning the Duration of Business Cycles, *Journal of the American Statistical Association*, Dec. 1926, pp. 447-57.

When the modern type of economic organization is in the initial stages of development, the average duration of cycles is relatively long. During the stage of rapid growth, when modern types of business enterprise and modern forms of industrial organization are being applied extensively, business cycles are of relatively short average duration. With the decline in the rate of economic change and the attainment of comparative stability, business cycles increase again in length.

Though he had "no objective criterion for distinguishing the stages in a country's industrial development, or for classifying countries according to their present state of development", Mills attempted a "quite experimental and tentative" classification, which applied as follows to the four countries treated in this study:

A. *Early stage of industrialization*
United States: to 1822 (Annals begin in 1796)
Germany: to 1866 (Annals begin in 1848)
B. *Stage of rapid transition*
England: to 1831 (Annals begin in 1793)
France: to 1876 (Annals begin in 1838)
United States: 1822 to date
Germany: 1866 to date
C. *Stage of economic stability*
England: 1831 to date
France: 1876 to date

Making similar tentative judgments about the stage of industrialization prevailing at different periods in all of the seventeen countries covered by Thorp's annals, Mills obtained the following results:

	No. of business cycles	Mean duration in years	Standard deviation in years
A. Early stage of industrialization	51	5.86	2.41
B. Stage of rapid transition	77	4.09	1.88
C. Stage of economic stability	38	6.39	2.42

He then showed by a sampling test that differences in average duration so large as he had found were most unlikely to happen by chance. Hence he concluded that, if his classification of countries by stages of industrial development is valid and if the durations shown by *Business Annals* are correct, "there is proof here of a definite secular change in the factors determining the duration of business cycles."

In Table 157 we present a detailed test of Mills' hypothesis. On applying variance analysis to the data available to Mills, we confirm his results not only in regard to seventeen countries, but also in regard to the four countries we are using at present. The results are again confirmed if we omit Mills' 'early stage of industrialization', which is not represented in the revised annual reference dates shown in Chapter 4 of this volume; also if we omit the early cycles in the second stage that are not covered by our new chronology. These four samples of durations not only yield averages for the several stages that differ in the direction suggested

TABLE 157

Analysis of Durations of Business Cycles Classified According to Mills' Stages of Industrialization

Data and grouping	No. of business cycles covered	Average duration of cycles in months during stage[a]			Ratio of variance among groups to variance within groups (F)	Probability that larger F would be obtained by chance is	
		A	B	C		Less than	More than
(1) Business-cycle durations in 17 countries, determined from annual records in *Business Annals*. Classified into three groups. The data and classification are those used by Mills	166	70.3	49.1	76.7	17.79	.001	
(2) Business-cycle durations in 4 countries: United States, Great Britain, Germany and France. In other respects the same as (1)	84	77.1	49.7	80.4	13.79	.001	
(3) The cycles placed by Mills in stage A are dropped. In other respects the same as (2)	77	...	49.7	80.4	23.72	.001	
(4) A few early cycles not covered by the annual reference dates in Table 16 are dropped. The data and other features of the analysis are the same as in (3)	70	...	49.0	80.4	23.80	.001	
(5) The calendar-year reference dates of Table 16 are substituted for reference dates taken from *Business Annals*. The periods and other features of the analysis are the same as in (4)	71	...	53.4	65.5	3.53	.20	.05
(6) A few cycles completed since *Business Annals* was published are added; that is, all cycles, from peak to peak, covered by annual reference dates through 1928–30 are included. Otherwise the same as (5). Going outside Mills' period, we lack the benefit of his classification. Mills assigned the last cycle he covered in U. S. and Germany to stage B. We arbitrarily assign later cycles (since 1923 in U. S. and since 1925 in Germany) to this stage	80	...	52.6	60.0	1.60		.20
(7) The same as (6) except that, in view of the difficulty just stated, the cycles since 1923 in U. S. and since 1925 in Germany are assigned to stage C	80	...	53.4	58.2	0.69		.20
(8) Differs from (6) only in that the cycles are taken on a trough-to-trough basis. All cycles covered by our annual reference dates through 1932 are included	84	...	54.1	60.4	1.34		.20
(9) The same as (6), except that cycle durations are measured from monthly instead of annual reference dates. As a consequence, the period covered is shorter than in (6)	56	...	50.2	59.4	1.69	.20	.05
(10) The same as (8), except that the durations are measured from monthly instead of annual reference dates and that in consequence the period covered is shorter	60	...	51.8	59.4	1.46		.20

[a] Stages A, B and C are identified in the text.

by Mills' hypothesis, but the differences are 'highly significant' in every instance. However, if we substitute durations derived from the annual reference dates of the present investigation for those derived from Thorp's *Business Annals,* the gap between the averages is reduced sharply and the value of F is no longer statistically significant. If we use the revised annual reference dates and add several business cycles that have run their course since Thorp's work was done, the gap between the averages becomes still narrower and its 'significance' still more doubtful, whether we count durations from peak to peak or from trough to trough. The somewhat smaller sample of durations measured from the monthly reference dates yields similar results.

Since the use of presumably more reliable and certainly more numerous measures reverses the results of variance analysis applied to earlier measurements of business-cycle durations in four countries classified by 'stages of industrialization', we must accept the conclusion that what we take to be the best collection of measures available at present gives slight support to Mills' hypothesis.[18]

VI Business Cycles before and after 1914

Another challenging hypothesis is the idea voiced repeatedly in recent years that World War I marks a 'break' in the history of business cycles. To test this hypothesis of structural change, we divide the leading cyclical measures for our seven American series and the durations of business cycles in our four countries into two groups; the first of which includes all cycles from the time our monthly records start to 1914, and the second the cycles from 1914 to about 1932. The results are presented in Tables 158-160 and Charts 57-58. To facilitate comparisons, the average reference-cycle patterns before the war are restricted to 1879–1914, the period covered by all seven series.

On the whole these measurements indicate a family likeness between the business cycles that come before and after 1914. As we move from pre-War to later cycles, numerous discrepancies appear within each pair of patterns, but they are overshadowed by the basic similarity that the patterns of the several series bear to one another. The order of magnitude of the cyclical durations, leads or lags, even amplitudes is, broadly speaking, not very different in the two periods. Less than half of the variance ratios in Tables 158-159 exceed unity (42 out of 98); in other words, the differences between the two periods exceed the differences within the periods in less than half of the instances tested. Four variance ratios are 'significantly large', but five are 'significantly small'. Doubtless, changes

[18] C. G. W. Schumann finds that the experience of South Africa from 1806 to 1909 does not bear out Mills' hypothesis. See his *Structural Changes and Business Cycles in South Africa, 1806–1936* (P. S. King, London, 1938), pp. 118-21.

TABLE 158

Average Duration and Amplitude of Specific Cycles before and after 1914
Seven American Series

Series and period covered	No. of specific cycles	Average duration in months			Average in specific-cycle relatives					
					Amplitude of			Per month amplitude[a] of		
		Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Deflated clearings										
1878–1914........	11	30	10	40	27	11	38	0.9	2.2	1.0
1914–1933........	4	39	16	55	27	19	46	0.8	1.2	0.8
Pig iron production										
1879–1914........	10	30	13	43	63	46	110	2.4	4.7	2.7
1914–1933........	5	25	18	44	59	72	131	2.6	4.6	3.2
Freight car orders										
1870–1914........	12	22	22	44	189	181	370	11.6	9.3	9.0
1914–1933........	7	12	19	32	254	264	518	41.3	15.9	20.4
Railroad stock prices										
1857–1915........	14	28	21	49	35	26	61	1.5	1.6	1.4
1915–1932........	4	31	21	52	36	52	89	1.2	2.2	1.5
Shares traded										
1878–1914........	10	15	29	44	91	92	184	8.8	3.3	4.4
1914–1933........	5	24	20	44	112	93	204	5.3	5.6	4.9
Call money rates										
1858–1915........	18	20	18	38	127	129	255	7.6	8.5	6.9
1915–1931........	5	19	19	37	77	71	148	4.2	4.6	3.9
Railroad bond yields										
1860–1914........	15	22	21	43	10	13	23	0.5	0.7	0.6
1914–1931........	5	17	23	41	13	13	25	0.7	1.2	0.7
	No. of groups	Ratio of variance between groups to variance within groups								
Deflated clearings....	2	1.09	1.30	1.60	0.00‡	1.00	0.45	0.19	0.31	0.47
Pig iron production..	2	0.70	1.13	0.01	0.12	2.21	1.23	0.11	0.01	0.70
Freight car orders....	2	3.50	0.33	2.74	1.48	2.10	1.84	2.32	3.16	3.67
Railroad stock prices.	2	0.05	0.00§	0.02	0.01	2.67	0.93	0.61	1.14	0.02
Shares traded.......	2	2.25	2.15	0.00†	0.90	0.00‡	0.26	1.42	6.00J	0.41
Call money rates....	2	0.12	0.03	0.03	2.36	2.63	2.66	1.89	2.14	3.89
Railroad bond yields.	2	0.52	0.11	0.09	0.39	0.00†	0.24	0.69	1.18	0.09

[a] Unweighted average.
J Larger than the value that would be exceeded once in twenty times by chance.
† Smaller than the value that would be fallen short of once in twenty times by chance.
‡ Smaller than the value that would be fallen short of once in a hundred times by chance.
§ Smaller than the value that would be fallen short of once in a thousand times by chance.

in cyclical behavior occurred or first became marked around 1914 in some single series. The most prominent change turned up by our sample is in the timing of bond yields. A real change occurred also in the amplitude of call money rates, but it is obscured by lumping together all pre-War cycles. From 1858 to 1885 we find nine specific cycles in call money rates and only five business cycles; from 1885 to 1915 we find another nine cycles in call money rates but each corresponds neatly to a business cycle; thus the pre-War period is apparently not homogeneous. If the cycles in call money rates during 1885–1915 are compared with the cycles during

1915–31, we find a 'significant' decline in amplitude.[19] But as already stated, this change is attributable to the Federal Reserve system, not to the War and its sequelae.

It is worth noting, however, that the average total swing of the specific cycles is larger after 1914 than before in every series except call money rates. True, not one of the differences is statistically significant. But our analysis stops in 1933, and we know that the cycle of 1933–38 was sufficiently violent to make its influence felt on the comparisons we are making. For example, the average amplitudes of pig iron production are + 76, −81, 158 for the six cycles from 1914 to 1938, in contrast to + 59, − 72, 131 for the five cycles from 1914 to 1933. If the 1933–38 cycle is added to the post-War period the variance ratio of the cyclical amplitudes also rises materially, though it still fails to meet the .05 level of significance.[20] Apparently, a tendency toward intensified cyclical fluctuations is impressed on our American samples—though not with any great clarity. It is impressed also on the ratings of business-cycle contractions presented in Table 156, but again not very clearly. If this table were extended in both directions, the severe contraction of 1937–38 would be added at the end, the severer contraction of 1873–79 at the beginning. It may be that no lasting change in the severity of cyclical contractions has taken place. On the other hand, a lasting change may have occurred, but we do not have as yet a sufficient number of cyclical observations to establish this result with confidence.[21]

If we are to judge from Table 160, the duration of business cycles in each of our four countries has been shorter on the average since 1914 than in pre-War times. However, the difference is small for this country,[22]

[19] The measures in full are:

	Average in specific-cycle relatives		Variance ratio
	1885–1915	*1915–1931*	
Rise	151	77	10.62
Fall	149	71	8.82
Rise & fall	299	148	11.80
Rise per month	6.8	4.2	4.99
Fall per month	10.0	4.6	4.36
Rise & fall per month	7.5	3.9	7.62

The .05 and .01 values of the variance ratio are 4.75 and 9.33, respectively.

[20] The variance ratio (F) for the rise is .64, the fall 4.19, the joint rise and fall 3.24. The .05 value of F is 4.60.

[21] It is well to observe specifically that since the sample of series analyzed in this chapter leaves us uninformed about the cyclical amplitude of total industrial production or employment, no inference concerning the presence or absence of secular changes in the amplitude of these fundamental magnitudes is possible. Furthermore, if we suppose for a moment that the amplitude of cyclical fluctuations in industrial employment has been constant in the long run, that would not imply constancy in the amplitude of fluctuations in total employment. On the contrary, it would imply that the amplitude of fluctuations in total employment has actually increased; for the number of persons engaged in agricultural work, which is a rather steady branch of employment (though not of output or income) during business cycles, has been a declining fraction of the gainfully occupied population.

[22] It is scarcely visible in the specific-cycle durations of the series in Table 158.

and it would virtually disappear if the 1933–38 business cycle were included in the analysis. In foreign countries the difference is substantial, though in no instance statistically significant.

TABLE 159

Average Timing of Specific Cycles and
Rates of Change during Reference Cycles before and after 1914
Seven American Series

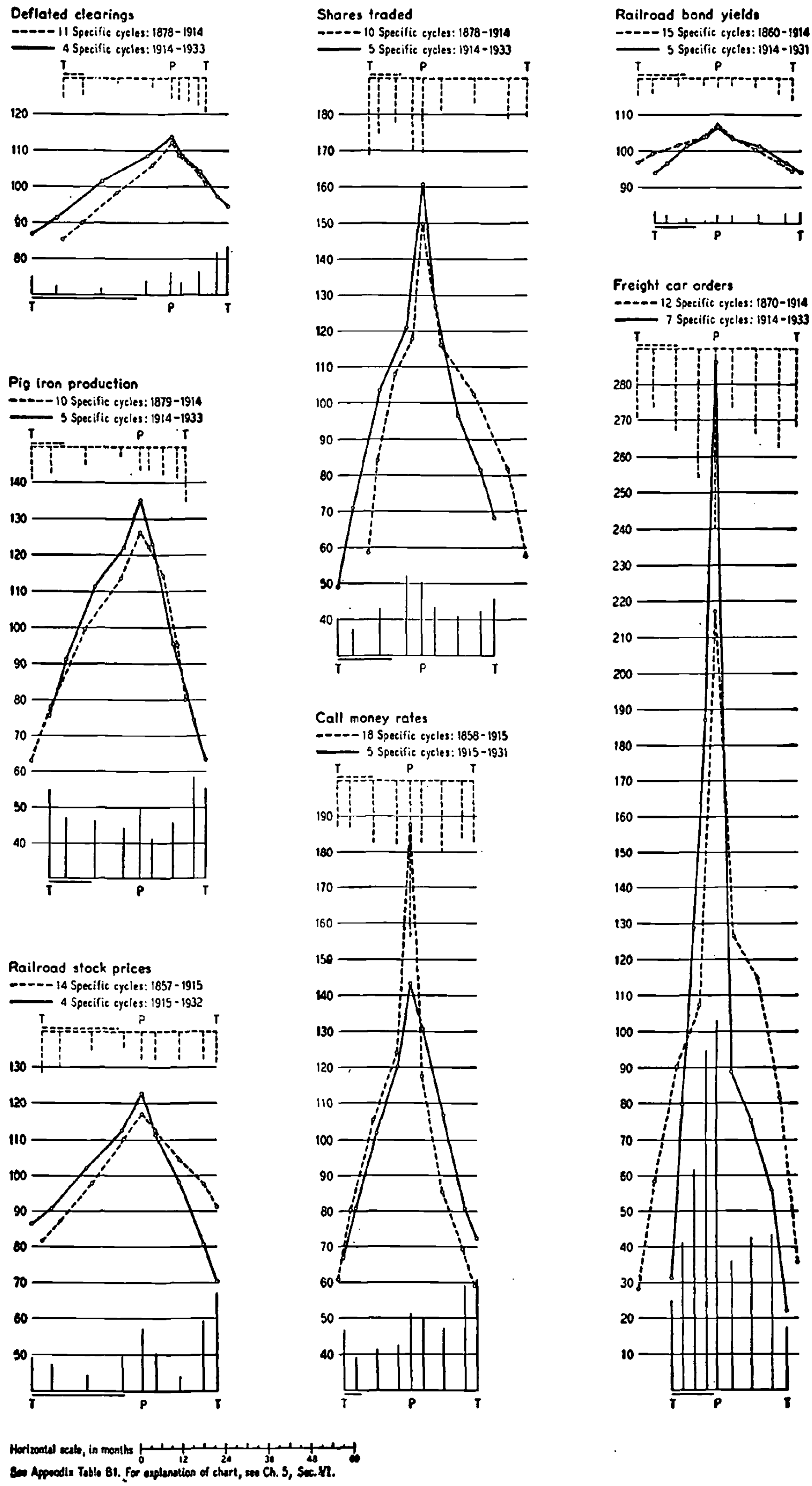
Series and period covered	Number of reference cycles	Average lead (−) or lag (+) in months at reference[a]		Average change per month in reference-cycle relatives during stages[b] matched with reference		
		Peaks	Troughs	Expansions	Contractions	Cycles[c]
Deflated clearings						
1879–1914	10	+4.0	−6.8	+0.8	−0.5	−1.4
1914–1933	5	+1.2	−3.8	+0.7	−0.4	−1.1
Pig iron production						
1879–1914	10	+2.1	−5.2	+2.2	−1.8	−3.9
1914–1933	5	+1.4	−0.3	+2.4	−3.3	−5.7
Freight car orders						
1870–1914	11	−6.8	−5.3	+4.2	−4.1	−8.3
1914–1933	5	−3.4	+1.2	+3.1	−3.5	−6.5
Railroad stock prices						
1858–1914	14	−4.7	−7.6	+0.9	−0.7	−1.6
1914–1933	5	−8.5	−7.0	+0.4	−0.4	−0.8
Shares traded						
1879–1914	10	−11.7	−3.5	+1.6	−2.2	−3.8
1914–1933	5	−7.8	−6.5	+2.7	−0.6	−3.3
Call money rates						
1858–1914	14	+0.6	+1.0	+3.8	−3.8	−7.7
1914–1933	5	−2.0	+3.0	+3.1	−2.8	−5.9
Railroad bond yields						
1858–1914	14	+9.3	+15.6	+0.2	−0.2	−0.3
1914–1933	5	+3.2	+0.2	+0.4	−0.2	−0.6
	Number of groups	Ratio of variance between groups to variance within groups				
Deflated clearings	2	1.08	1.49	1.10	0.15	0.55
Pig iron production	2	0.05	8.46J	0.18	4.30	3.34
Freight car orders	2	0.48	3.29	0.52	0.11	0.45
Railroad stock prices	2	0.73	0.01	2.27	0.29	2.00
Shares traded	2	1.15	0.80	2.27	3.47	0.15
Call money rates	2	0.85	0.27	0.20	0.63	0.47
Railroad bond yields	2	7.02J	14.32♪	1.32	0.01	1.00

[a] The number of timing observations is not defined exactly by the number of reference cycles; see Table 141. To avoid duplication, the timing observation at the reference trough of 1914 is included in the later period only.
[b] These stages are indicated in Table 140.
[c] Difference between contraction and expansion (see Table 47, col. 8).
J Larger than the value that would be exceeded once in twenty times by chance.
♪ Larger than the value that would be exceeded once in a hundred times by chance.

CHART 57

Average Specific-cycle Patterns before and after 1914

Seven American Series

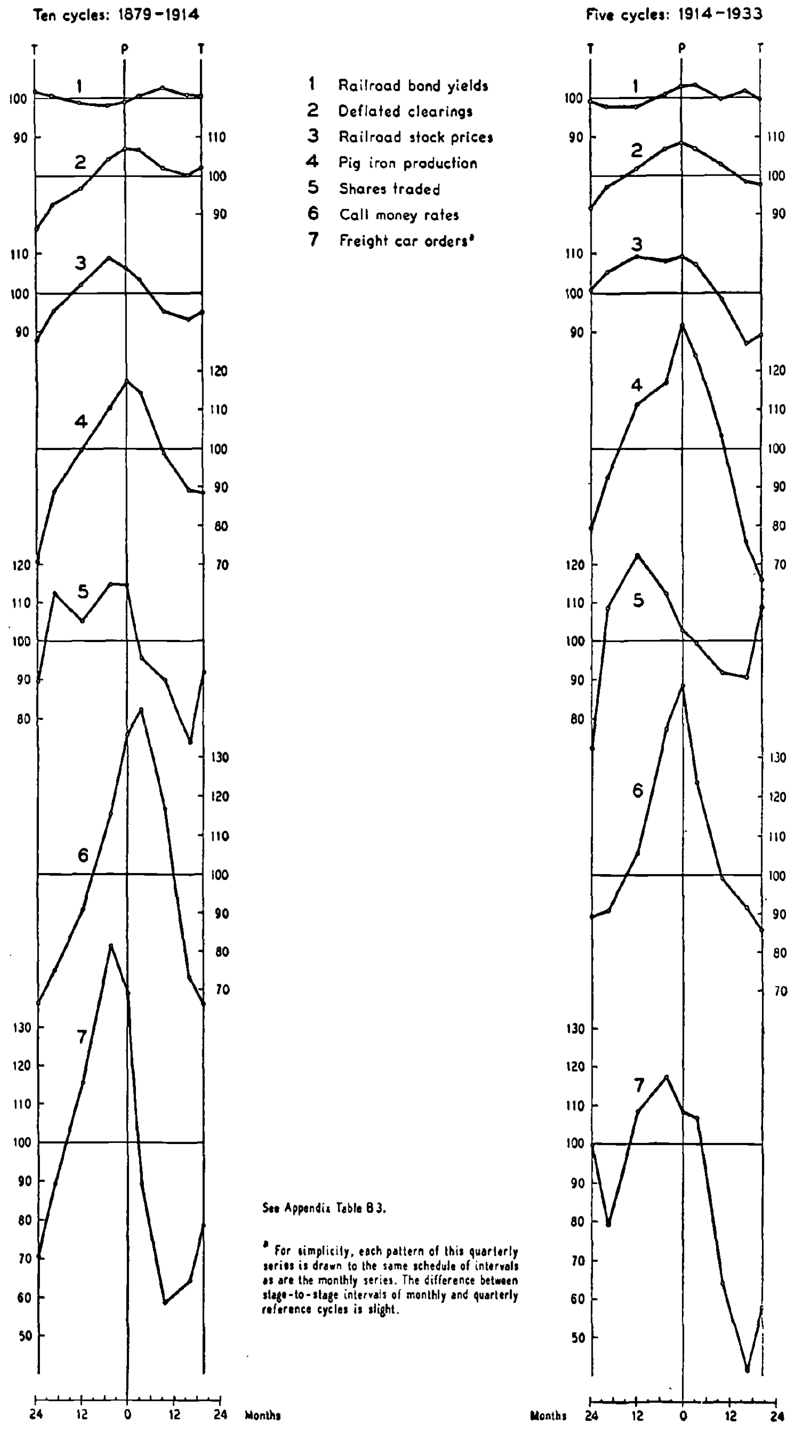


See Appendix Table B1. For explanation of chart, see Ch. 5, Sec. VI.

CHART 58

Average Reference-cycle Patterns before and after 1914

Seven American Series



See Appendix Table B 3.

* For simplicity, each pattern of this quarterly series is drawn to the same schedule of intervals as are the monthly series. The difference between stage-to-stage intervals of monthly and quarterly reference cycles is slight.

TABLE 160
Average Duration of Business Cycles before and after 1914
Four Countries

Country and period covered	Number of business cycles	Average duration in months: Expansion	Contraction	Full cycle
United States				
1854–1914	15	25	23	48
1914–1933	5	24	20	44
Great Britain				
1854–1914	10	41	30	72
1914–1932	5	24	19	43
France				
1865–1914	10	31	28	58
1914–1932	5	30	13	43
Germany				
1879–1914	6	39	32	71
1914–1932	4	34	20	54
	Number of groups	Ratio of variance between groups to variance within groups		
United States	2	0.13	0.10	0.19
Great Britain	2	4.35	1.06	3.81
France	2	0.00†	2.96	1.44
Germany	2	0.36	0.93	1.19

Derived from the monthly reference dates in Table 16.
†Smaller than the value that would be fallen short of once in twenty times by chance.

VII Conclusions from Tests

Before attempting to draw conclusions from the tests in this chapter, it is well to note their limitations. All our tests are based on a small sample of series. They are restricted to this country except for measures of the duration of business cycles. They are arranged for the most part according to mechanical criteria. The probability tests we have used are based on assumptions that are not fully met by cyclical measures. More troublesome still, the number of cycles in our test series is small, only twenty-three at its largest. These series cover a long stretch relatively to most monthly series in our collection, but not relatively to the history of business economy. Hence our experiments may mean that cyclical behavior in fact has usually been free from 'substantial' and 'significant' secular changes; but they also may mean that the influence of secular changes is too small to be detected reliably by means of the data and techniques employed.

These remarks severely limit the conclusions that we may draw concerning secular changes in cyclical behavior. But the aims of this chapter are also modest. We have not been concerned with the question whether secular or structural changes are characteristic of cyclical behavior, but with the narrower question whether such changes have been so prominent as to discredit the use of averages. We can say with confidence that, on the whole, secular or structural changes have not impressed their influence very strongly on the cyclical behavior of our sample of seven American series or on the durations of business cycles in our four countries. From experience in handling numerous time series, we judge that our sample is tolerably representative in this respect of the great bulk of economic series, except those of very narrow coverage. Hence, we see no serious obstacle, so far as secular changes are concerned, to the use of averages to express in a preliminary way what cyclical behavior has been characteristic of different economic activities in recent decades.

If secular changes usually account for only a small portion of the cyclical variability in our time series, two corollaries follow. First, averages covering all cycles in a series can usually generalize the cyclical behavior of the series, if that is to be done at all, about as well as evolving averages or subperiod averages. Second, it is safe, ordinarily, to use in different connections averages based upon different groups of cycles for the same series, and we may even compare averages for different series based upon different periods.

VIII Preparation for Later Work

These conclusions, however, cannot be applied casually or mechanically. Our analysis has disclosed what we take to be several genuine instances of secular or structural change in cyclical behavior. It also suggests, however faintly, that the cyclical behavior of money markets has been more sensitive to secular factors than the cyclical behavior of security or industrial markets, that the duration of specific cycles has been influenced less than their timing or amplitude, and that the amplitude of business fluctuations may have increased since 1914. Quite apart from our specific results, it hardly seems possible that the widespread secular changes that have taken place in economic organization—such as the increasing scale of business enterprise, the spread of absentee ownership, the building up of colonial empires, the disappearance of our frontier, the commercialization of agriculture, the declining rate of population growth, the development of installment selling, the increasing role of government in economic affairs, and many others—have not left their mark on business cycles. Hence, we scrutinize closely the cyclical measures for each series in succeeding monographs, note secular changes in cyclical behavior that appear in the light of the surrounding facts to be significant, and thus prepare materials that should prove suggestive in a later attack on the problem of cumulative changes in business cycles.

These steps would be essential even if we knew in advance that business cycles were uniform in the long run. In an expanding economic system, characterized by continual accessions of new industries and an absolute or relative decline of old industries, secular changes might be found in the cyclical behavior of each industry and yet be absent in industry as a whole; just as retardation in the growth of individual industries is compatible with a uniform rate of growth or even acceleration of total industrial production. To show how changing practices in industry, commerce, and finance penetrate the world of business and what their cyclical repercussions are, is one of the most important problems to be tackled in the theoretical volume of this series.

Studies of secular movements by several investigators have shown that many industries have similar histories. During the experimental years of a new industry, or the period when an old industry gets its first taste of the Industrial Revolution, the rate of growth is usually moderate. When the methods of production have been stabilized and a wide demand for the product at a profitable price has been assured, the rate of growth is rapid. As technical improvements are added, prices fall relatively to the products of older industries, and markets keep expanding. But after a point, every fresh advance encounters increasingly stiff resistance from older industries and even more from new industries endowed with the freshness and vigor of youth. Finally, a stage of decline may set in, when the product is superseded by some other article that gives more satisfaction in proportion to its cost.[23]

Some of our time series are long enough to show industries passing through two or more of these 'stages', and the secular changes ordinarily affect cyclical behavior as we measure it. The intra-cycle trend and the tilt of the cyclical patterns shift with the rate of growth. The duration of the expansions and contractions of specific cycles, the amplitude of rise relative to the fall, the leads or lags at the reference turns, the indexes of conformity, indeed, nearly all of our measures may be influenced.[24] Sometimes the record is long enough to yield two or three sets of averages, one representing the stage of rapid growth, another the stage of moderate growth; or one representing the high tide, another the stage of comparative stability, and perhaps a third the decline of the industry. In other instances, the best we can do is to exclude from the averages two or three early or late cycles that represent a fragment of life history too short to yield significant averages, but too different from most of the record to be lumped with it.

An instructive illustration of secular change in cyclical behavior is provided by the history of American railroads.[25] Chart 59 shows cyclical

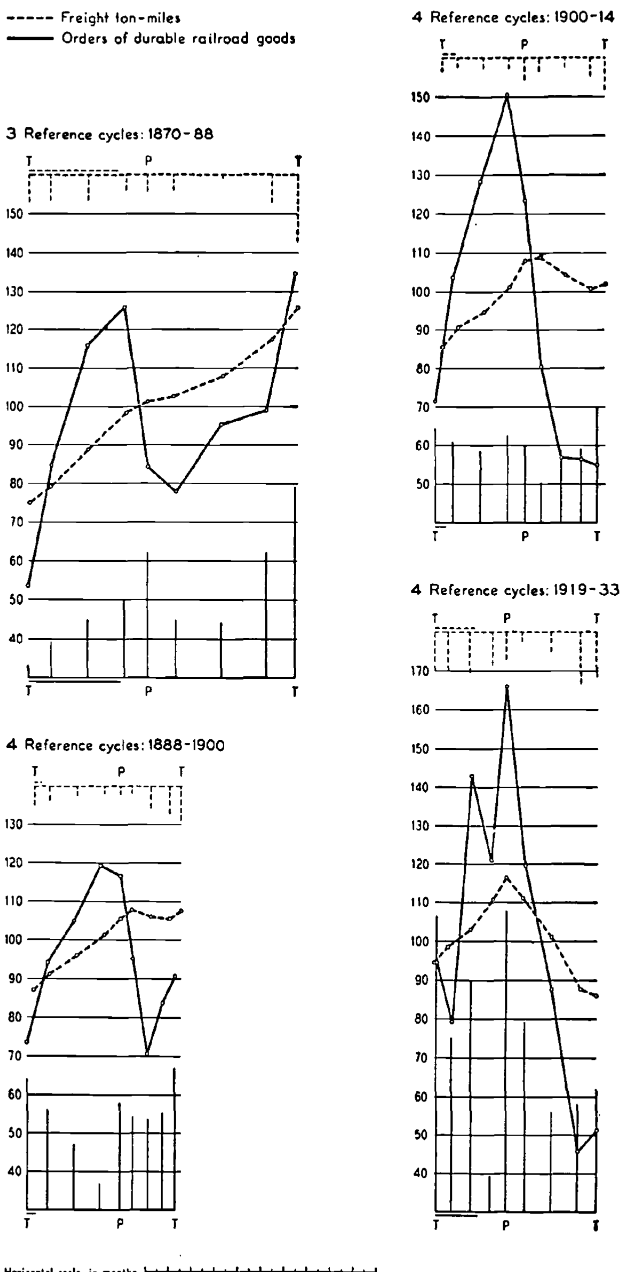
23 See Simon Kuznets, *Secular Movements in Production and Prices* (Houghton Mifflin, 1930), Ch. I, III, V-VI, and Arthur F. Burns, *Production Trends in the United States since 1870*, pp. 79-82, 96-173, and 270-81.

24 See W. C. Mitchell and A. F. Burns, The National Bureau's Measures of Cyclical Behavior (National Bureau of Economic Research, *Bulletin 57*, July 1, 1935), pp. 16-17; also above, Ch. 7, particularly Sec. VI.

25 Secular changes in the cyclical behavior of railroad investment will be discussed at length in our monograph on construction work.

CHART 59

Average Patterns of Four Successive Groups of Reference Cycles
Railroad Traffic and Investment, United States, 1870–1933



The disturbed cycle 1914–19 is omitted. For sources of data, etc., see Appendix C; for explanation of chart, Ch. 5, Sec. VII. The intervals between successive stages for ton-miles and orders differ slightly, since the former series is monthly and the latter quarterly.

patterns of orders for durable railroad goods and of freight ton-miles of traffic during four groups of business cycles since 1870. For a time the opportunities for profits in railroads seemed boundless, and investments in railway plant increased at a vigorous rate. A cyclical depression in general business checked new investment, but the check was comparatively brief; for even during depressions railroads were able to gain new traffic from other transport agencies and to carry that traffic over longer distances. Hence we find railroad construction leading by long intervals and playing an 'active' part in cyclical revivals. But the sharp rate of growth of new railroad lines could not be continued indefinitely. Once the business of rival agencies was fairly well captured and the continent crisscrossed with railroad lines, the addition of new mileage often resulted merely in a duplication of existing facilities. A period of rate wars, maneuvers for control of competing lines, and outright consolidations set in. The incentive to construct new lines diminished, while the need for betterments and additions increased. These changes reinforced one another, so far as their effects on cyclical timing are concerned. The expansion of auxiliary industries to a point where they could meet orders by railroads more quickly than in earlier times worked to the same end. Railroad enterprises tended to become more cautious, to give less weight in making investment decisions to favorable building costs and money market conditions, and to give more weight to the traffic in sight. These tendencies were quickened as new transport agencies—interurban railways, trucks, motor buses, passenger automobiles, pipe lines, airplanes, and revived waterways—emerged and battled the railroads for traffic as mercilessly as the railroads in their youth had battled their rivals. Chart 59 shows the progressive changes in the cyclical timing of fixed railroad investments that resulted from these complex forces. As the trend of railway traffic flattened out and its cyclical fluctuations became intensified, the leads of new investments at business-cycle revivals became irregularly weaker, vanished, and finally were replaced by lags.[26]

In certain activities cyclical behavior is free from secular change, yet is not steady in the long run. For an extended period a series may follow a characteristic pattern, then shift abruptly to another pattern. When we encounter such cases, we strike separate averages for the period preceding and following the discontinuous shift. For example, railroad freight rates in the United States, so far as we may judge from receipts per ton-mile, tended to conform positively to business cycles before about 1890; after that date they usually move invertedly. This shift is due, at least in part, to the regulative activities of the Interstate Commerce Commission established in 1887, which made the adjustment of freight rates to

26 By applying variance analysis to the leads or lags (in cycle-stage units) of the series on orders of durable railroad goods at reference troughs, using the groups shown on the chart, we get a variance ratio of 3.42, which is very close to the .05 point (3.59).

cyclical changes in general business a more deliberate and time-consuming process. Another example is the reduction in the cyclical amplitude of short-term interest rates after the introduction of the Federal Reserve system, on which we commented above. A still neater example is found in the history of silver prices. During the two centuries prior to the 1870's, the price of silver relatively to the price of gold was virtually free from cyclical fluctuations. Since that time, the progressive 'demonetization' of silver has been accompanied by exceptionally violent flucuations in its gold price.[27]

Many of our foreign series seem to support the view that an abrupt change occurred around 1914 in the economic activities of Western Europe. The change is evident in certain series on production, trade and employment. It is reflected most sharply in monetary series, as should be expected from the intermittent existence of the gold standard and the changes in the methods of operating it since 1914. For instance, the metallic reserves of the Bank of France moved invertedly to business cycles without exception from 1872 to 1914, but bear an extremely irregular relation to business cycles since 1914. The ratio of reserves to liabilities in the Bank of England conforms about as well to business cycles after 1914 as before, but the cycles have become more intense. In these and similar instances, as already stated, we strike separate averages for the period before and after the discontinuous shift.

We trust that these illustrations convey a more adequate notion of the manner in which we actually use averages than do the brief statements in preceding chapters. Our primary interest at this stage of our work, to repeat, is in average cyclical behavior. We proceed on the belief that business cycles have been sufficiently stable 'in the long run' to justify the effort to develop a systematic and detailed picture of what happens during an 'average' or 'typical' business cycle. But we conceive of this picture as being merely a first approximation to our ultimate goal, which is to explain the business cycles of actual life. Hence we are also preparing materials as well as we now can on secular and discontinuous changes in cyclical behavior. Once our basic analysis of time series is completed, we plan to collate the results for the longest series, note what regularities they suggest, then scrutinize all our series, those covering only a half-dozen cycles as well as those covering ten or more cycles, in the light of these suggestions. We expect that these studies will help us not only to determine what secular or discontinuous changes have taken place in specific and business cycles, but also to explain how the business cycles of actual life typically have run their course.

[27] See George F. Warren and Frank A. Pearson, *Prices* (John Wiley, 1933), pp. 142-4.

CHAPTER 11

Cyclical Changes in Cyclical Behavior

THE HYPOTHESIS that business cycles are minor subdivisions of 'major' or 'long' cycles raises the same fundamental question concerning the use of averages as the hypothesis of secular change in cyclical behavior. If business cycles differed radically from one another according to their position within major cycles, we would not be justified in striking averages on our standard plan. One of our main objectives would then be to bring out this variation, and for that purpose we would require separate averages of the cycles occupying corresponding positions within major cycles. Even in contrasting the cyclical behavior of different activities in the long run it would be necessary to take account of the major cycles; for the averages would be biased unless they covered periods including an integral number of major cycles.

In this chapter we shall examine several outstanding hypotheses concerning major cycles to see if we can find pronounced and repetitive changes in business cycles within the periods suggested by the hypotheses. Our problem is not whether long cycles exist in general economic activity, but whether they are so strongly impressed on economic time series that they should control working plans at this stage of our study. In making this survey we use the seven time series and the measures of business-cycle duration analyzed in the preceding chapter.

I Long Cycles Marked Off by Long Waves in Building

We begin with a hypothesis suggested by the behavior of the building industry. Our studies indicate that building construction is characterized by long cycles of remarkably regular duration. They run usually from about fifteen to twenty years; they are clear-cut in outline, attain enormous amplitudes, and are paralleled by long cycles in other real estate

TABLE 161

Average Duration and Amplitude of Long Cycles in Building Construction
Twenty-five Annual American Series

Series	Period covered	No. of long cycles	Average duration in months			Average in long-cycle relatives					
						Amplitude of			Per month amplitude[a] of		
			Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
PER CAPITA PERMITS, U.S.											
Unadjusted	1830–1933	6	106	100	206	130	128	259	1.4	1.3	1.4
Unadjusted	1878–1933	3	112	108	220	111	110	221	1.1	1.0	1.1
Adjusted	1830–1933	6	102	104	206	115	120	235	1.2	1.2	1.2
Adjusted	1878–1933	3	112	108	220	107	120	227	1.1	1.2	1.1
ADJUSTED PER CAPITA PERMITS BY REGIONS											
New England	1879–1933	3	124	92	216	117	122	239	1.1	2.3	1.3
Middle Atlantic	1878–1933	3	96	124	220	112	126	238	1.3	1.1	1.1
South Atlantic	1879–1933	3	92	124	216	105	121	226	1.2	1.0	1.1
East North Central	1878–1933	3	128	92	220	118	138	256	1.0	1.5	1.2
West North Central	1877–1932	3	112	108	220	115	122	238	1.1	1.2	1.1
South Central	1878–1933	3	140	80	220	111	130	241	1.0	1.6	1.2
Western	1878–1932	3	100	116	216	157	161	317	1.7	1.5	1.5
NEW BUILDING PERMITS											
Manhattan	1877–1933	3	140	84	224	148	134	282	1.3	1.5	1.3
Brooklyn	1879–1933	3	96	120	216	159	143	302	1.9	1.2	1.4
Chicago	1862–1933	4	126	87	213	213	187	400	1.8	2.2	1.9
Philadelphia	1900–1933	2	96	102	198	140	154	294	1.6	1.6	1.6
Detroit	1878–1933	2	264	66	330	258	232	490	0.9	3.0	1.4
St. Louis	1878–1932	3	108	108	216	163	152	314	1.7	1.5	1.5
Minneapolis	1897–1933	2	144	72	216	193	182	375	1.7	4.8	1.8
Pittsburgh	1918–1934	1	96	96	192	158	186	344	1.6	1.9	1.8
RESIDENTIAL PERMITS											
Manhattan	1877–1932	3	116	104	220	159	158	317	1.5	1.6	1.5
Philadelphia	1900–1933	2	96	102	198	176	191	366	2.0	1.9	2.0
St. Louis	1900–1933	2	90	108	198	202	214	416	2.3	2.0	2.2
COMMERCIAL & INDUSTRIAL PERMITS											
Manhattan	1877–1933	2	252	84	336	250	244	494	1.5	3.0	1.7
Philadelphia	1908–1935	1	204	120	324	238	254	492	1.2	2.1	1.5
SUBDIVISION ACTIVITY											
Chicago	1843–1932	5	122	91	214	323	325	648	2.8	3.9	3.0
Detroit area	1841–1918	4	180	51	231	429	408	837	2.3	11.3	3.6
Pittsburgh area	1831–1932	6	112	90	202	248	248	496	2.5	3.5	2.5

The series on 'per capita permits' are index numbers of building permits issued in a sample of American cities. The 'unadjusted' data are corrected for changes in population only; the 'adjusted' data are corrected for changes in population, construction costs, and secular trend. The index numbers were taken from John R. Riggleman, *Variations in Building Activity in United States Cities* (unpublished Johns Hopkins dissertation, 1934). See John R. Riggleman and Ira N. Frisbee, *Business Statistics* (McGraw-Hill, 1938, 2nd ed.), Appendix VI.

The series on building permits in individual cities are expressed in dollars, except residential permits in St. Louis which refer to number of dwelling units.

Except for St. Louis, the series on 'new' building permits exclude alterations and additions. Full descriptions and references will be supplied in our monograph on construction work, which will discuss in detail long cycles in building construction and related activities in the United States and foreign countries.

All measures in this table are subject to revision. Since the preparation of the table, improved data for a few series have become available. Also, the amplitude measures shown here are expressed as relatives of cycle bases which assign the same weight to every year from the initial through the terminal trough—a plan we formerly followed in handling annual data. At present we compute cycle bases by assigning a weight of one-half each to the initial and terminal troughs.

[a] Unweighted average.

processes. Table 161 shows the average durations and amplitudes of the long cycles found in twenty-five annual series on American building activity. Table 162 shows how the specific cycles in building vary according as they occur during the upswing or the downswing of the long building cycles.[1] The specific cycles that follow a protracted lull in building tend to have long expansions and short contractions, rapid rises and slow declines; therefore the rises are especially large relatively to the declines. Once the tide of building activity turns, these relations between expansions and contractions are reversed. We may say that the structure of the first group of specific cycles constitutes the upswing of a long cycle, and the structure of the second group of specific cycles constitutes the downswing of the long cycle. Or we may look at these relations from the viewpoint of long cycles and say that, as a rule, during long-cycle expansions the specific-cycle expansions are relatively long and pronounced, while during long-cycle contractions the specific-cycle expansions are relatively brief and mild. Opposite relations characterize specific-cycle contractions.

In view of the magnitude of the building industry and the amplitude of its long waves, it is desirable to see whether the cyclical alternations in the specific cycles of building construction are paralleled by similar alternations in business cycles. Presumably the long cycles in building influence industrial activity at large by producing retardations and accelerations in growth, not actual declines and rises. Perhaps in financial and monetary series we may find actual declines and rises as frequently as retardations and accelerations. But whatever may be the intensity of the reaction of other processes to long building cycles, so long as these reactions are regular and recurrent the durations and amplitudes of the expansions of specific cycles in leading activities should be larger relatively to the durations and amplitudes of full specific cycles when the 'trend' of building is upward than when it is downward. According as the 'trend' of building is upward or downward, we may also expect the durations and amplitudes of specific-cycle expansions to be comparatively large or small, and the durations and amplitudes of specific-cycle contractions to be small or large.

To test these expectations we compare averages of the business and specific cycles that come during the upswing of long building cycles with averages of the business and specific cycles that come during the downswing of long building cycles. The first step is to fit business and specific cycles into the periods of long building cycles (Table 163). In distributing business cycles between the upswings and downswings of the successive long cycles, we follow the rule that the period matched with an upswing

[1] By 'long building cycles' or 'long cycles in building construction' we mean long cycles in the construction of buildings, not long cycles in the aggregate volume of construction work. The existence of long cycles in aggregate construction has not been definitely established.

TABLE 162

Average Duration and Amplitude of Specific Cycles during the Upswings and Downswings of Long Cycles in Building Construction Twenty-five Annual American Series

Series	Period covered	Specific cycles during upswings of long cycles						Specific cycles during downswings of long cycles					
		Average duration in months		Average in specific-cycle relatives				Average duration in months		Average in specific-cycle relatives			
				Amplitude of		Per month amplitude* of				Amplitude of		Per month amplitude* of	
		Expansion	Contraction	Rise	Fall	Rise	Fall	Expansion	Contraction	Rise	Fall	Rise	Fall
PER CAPITA PERMITS, U. S.													
Unadjusted	1830 – 1933	41	15	70	17	1.8	1.3	15	34	15	62	1.0	1.8
Adjusted	1830 – 1933	46	22	66	11	2.2	0.5	14	30	11	53	0.8	2.0
ADJUSTED PER CAPITA PERMITS BY REGIONS													
New England	1879 – 1933	28	20	54	21	2.2	1.0	14	21	13	50	0.9	2.2
Middle Atlantic	1878 – 1933	40	16	71	23	3.0	1.5	14	28	22	54	1.7	2.2
South Atlantic	1879 – 1933	32	28	60	15	2.8	0.7	19	24	14	48	0.8	2.4
East North Central	1878 – 1933	34	22	56	23	2.2	1.3	18	48	12	86	0.7	2.4
West North Central	1877 – 1932	23	15	45	17	2.3	1.1	12	28	11	47	0.9	1.7
South Central	1878 – 1933	40	19	58	25	1.4	1.4	12	57	6	94	0.5	1.8
Western	1878 – 1932	63	48	120	53	2.0	1.1	12	36	13	66	1.1	1.7
NEW BUILDING PERMITS													
Manhattan	1877 – 1933	30	17	69	36	2.8	2.4	21	24	77	99	3.3	3.7
Brooklyn	1879 – 1933	34	12	79	18	3.1	1.5	14	31	22	59	1.6	1.7
Chicago	1862 – 1933	30	13	82	23	3.0	1.9	18	34	31	85	1.9	3.2
Philadelphia	1900 – 1933	26	20	86	46	4.3	2.1	16	31	19	69	1.0	2.0
Detroit	1878 – 1933	34	17	69	28	2.5	1.4	12	40	28	91	2.3	2.7
St. Louis	1878 – 1932	23	14	55	9	3.1	0.5	12	28	16	57	1.3	2.0
Minneapolis	1897 – 1933	40	12	76	11	2.6	0.9	18	27	25	90	1.6	3.7
Pittsburgh	1918 – 1934	42	12	80	7	1.8	0.6	12	42	38	119	3.2	2.6
RESIDENTIAL PERMITS													
Manhattan	1877 – 1932	34	14	99	50	3.8	4.1	17	28	77	108	4.5	4.0
Philadelphia	1900 – 1933	22	15	92	44	5.2	3.2	36	56	57	134	1.6	2.4
St. Louis	1900 – 1933	29	12	85	8	3.4	0.7	12	48	32	126	2.7	2.6
COMMERCIAL & INDUSTRIAL PERMITS													
Manhattan	1877 – 1933	41	19	103	60	3.4	3.6	18	33	56	122	2.6	4.0
Philadelphia	1908 – 1935	24	21	93	76	5.7	3.7	18	28	55	97	3.0	3.1
SUBDIVISION ACTIVITY													
Chicago	1843 – 1932	34	15	182	122	7.8	8.3	12	45	99	210	8.3	6.4
Detroit area	1841 – 1918	33	16	124	57	5.1	3.9	16	22	59	142	4.2	7.8
Pittsburgh area	1831 – 1932	34	17	130	70	5.2	4.2	16	37	38	123	2.0	3.9

See note to Table 161. Each series is broken according to the upswings and downswings of the long cycles in that series. The results would not differ materially if all series were broken according to the long cycles in the national index, since the long cycles of different regions and cities synchronize fairly closely. See Ch. 6 concerning biases of cyclical measures made from annual data.

* Unweighted average.

TABLE 163

Relations in Time between Business Cycles and Long Building Cycles

United States, 1853–1933

Long building cycles[a]			Business cycles		
Phase	Nature of phase[b]	Duration of phase (years)	Corresponding period[c]	No. of phases during this period: Expansion	No. of phases during this period: Contraction
1853–1862	D	9	*1853* – June 1861	2	3
1862–1871	U	9	June 1861 – *Oct. 1873*	3	2
1871–1878	D	7	*Oct. 1873* – Mar. 1879	0	1
1878–1890	U	12	Mar. 1879 – *July 1890*	3	2
1890–1900	D	10	*July 1890* – Dec. 1900	3	4
1900–1909	U	9	Dec. 1900 – *Jan. 1910*	3	2
1909–1918	D	9	*Jan. 1910* – Apr. 1919	2	3
1918–1925	U	7	Apr. 1919 – *Oct. 1926*	3	2
1925–1933	D	8	*Oct. 1926* – Mar. 1933	1	2

[a] As shown by Riggleman's annual index of building permits per capita in the United States (unadjusted). See note to Table 161.

[b] D stands for downswing (contraction), U for upswing (expansion).

[c] The italicized dates are cyclical peaks, according to our reference dates in Table 16. The monthly reference dates start in Dec. 1854; hence the peak of 1853 omits the month.

in building must start with a trough and end with a peak and that the period matched with a downswing in building must start with a peak and end with a trough. The specific cycles of our sample series in turn are fitted as closely as possible into the distribution of business cycles.[2] In distributing the expansions and contractions of specific and business cycles we include virtually every cycle covered by our analysis.[3] But in distributing full cycles we omit the cycles that overlap periods of rising and falling building 'trends'.[4]

Chart 60 shows average patterns of the specific cycles of each series that occur during the upswings and during the downswings of long building cycles. Chart 61 shows average reference cycles on the same basis, except that it is restricted to the period since 1879, which is covered by all seven series. The outstanding feature of the charts is the similarity between the two specific-cycle and between the two reference-cycle patterns of each series. There are indeed numerous divergencies; sometimes they are considerable, as in the specific-cycle patterns of railroad stock prices and in the reference-cycle patterns of share trading. But the dominating impression is that the differences among the cyclical patterns of our several series are far greater than the differences between the patterns for

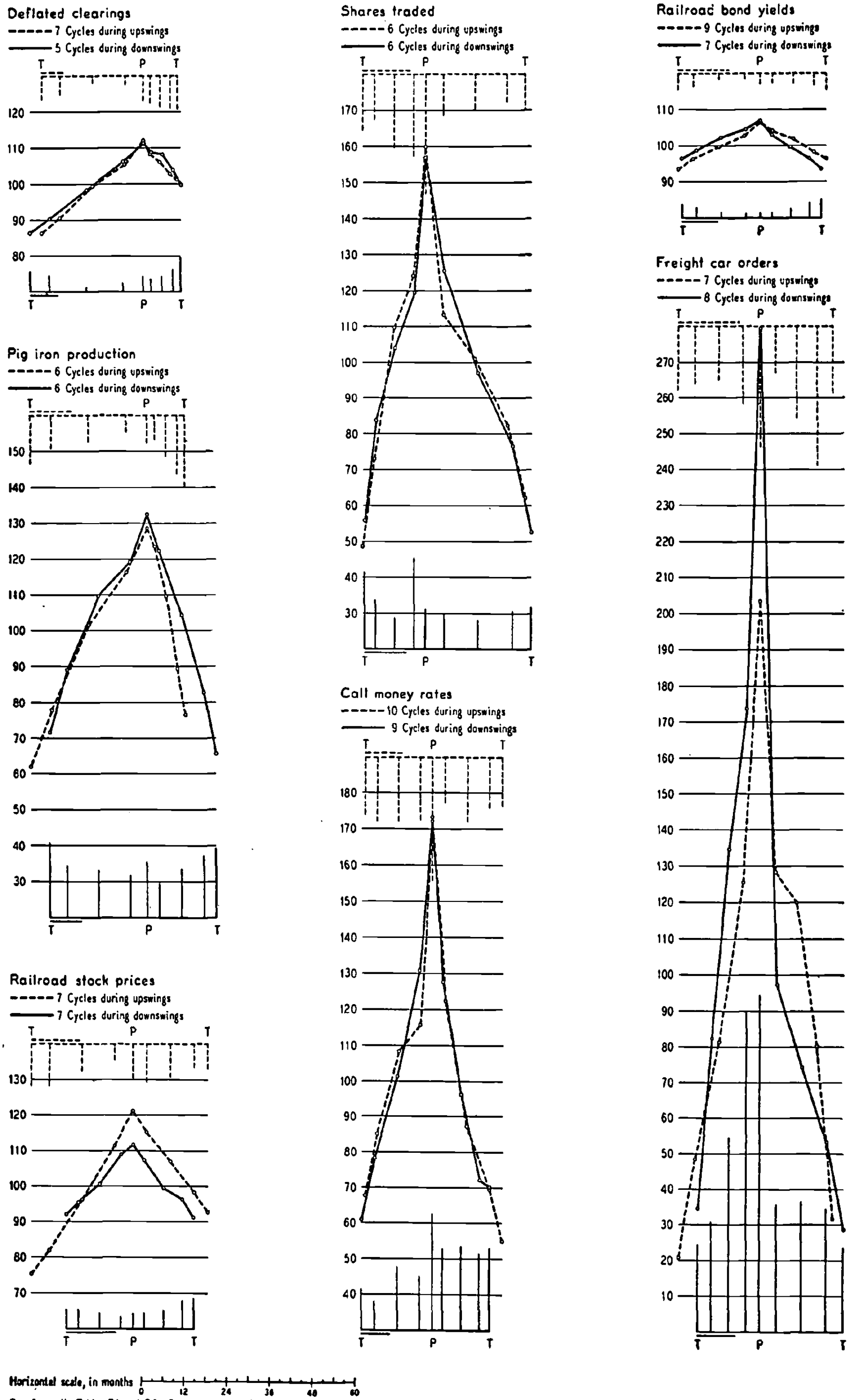
2 In a few instances, especially in railroad bond yields, arbitrary decisions are unavoidable. Appendix Table B5 shows what cycles are placed in each group for the characteristics that we measure. Appendix Tables B1-B4 supply measures for individual cycles, both specific and reference.

3 Four phases are omitted; in these instances it was impossible to fit the specific-cycle phases into the business-cycle phases without infringing the principle of the classification. See Appendix Table B5.

4 If specific cycles were marked off by peaks instead of by troughs, the cycles now excluded would have to be included, while the cycles overlapping periods of falling and rising building 'trends' would be excluded. Inverted analysis can thus serve as a check upon positive analysis.

CHART 60

Average Specific-cycle Patterns during the Upswings and Downswings of Long Cycles in Building Construction
Seven American Series

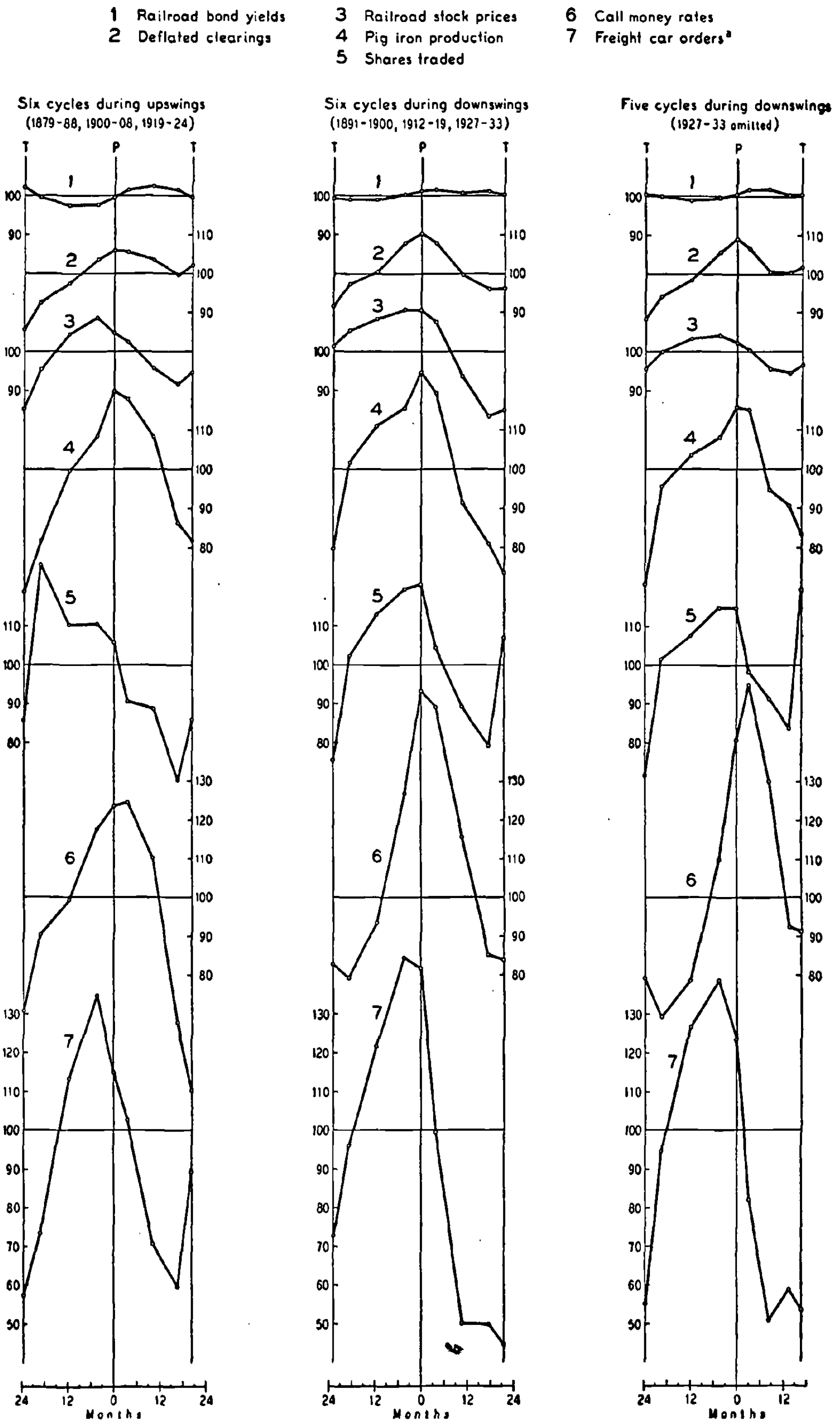


See Appendix Tables B1 and B5. For explanation of chart, see Ch. 5, Sec. VI.

CHART 61

Average Reference-cycle Patterns during the Upswings and Downswings of Long Cycles in Building Construction, 1879–1933

Seven American Series



See Appendix Table B3.

[a] See Chart 58, note 'a'.

each series. Chart 60 suggests that any description we might frame of specific cycles would be fundamentally similar whether we looked at the specific cycles that come during the upswings of long building cycles, at those that come during downswings, or ignored this principle of classification and drew our account from the full list of cycles. Chart 61 suggests that there is also little difference between the business cycles that come during the upswings of long building cycles and those that come during the downswings; though we should bear in mind that no small sample of business activities can depict adequately the characteristics of business cycles. On the whole the reference-cycle patterns during periods of rising building 'trends' appear to be much like the reference-cycle patterns during periods of declining building 'trends'; the similarity is especially close if we ignore share trading and drop the exceptional cycle of 1927–33 from the averages.

We pass to the question whether the differences in cyclical behavior between periods of rising and falling building 'trends' are statistically significant. The long lines representing average deviations in Chart 60 suggest that many of the differences within each pair of patterns arise from 'random' factors. To judge this point objectively we analyze the cyclical durations and amplitudes in Table 164. Columns (2), (3), (5)-(7), and (9) are designed to show whether there are cyclical alternations in specific and business cycles that parallel the cyclical alternations in the specific cycles of building construction. Of the 45 comparisons in these columns the 30 marked by an asterisk show differences in the direction one expects long building cycles to produce, 11 show differences in the opposite direction and 4 are neutral. But the variance ratios indicate that none of the differences in these columns are 'significantly large'; also, that when the averages differ in the expected direction, the variance between periods of rising and falling building 'trends' is frequently smaller than the variance within these periods. The preponderance of differences in the direction suggested by the hypothesis under test would carry weight if our sample series were independent. But we know that they are more or less closely correlated. Chart 61 shows that if we include observations on all business cycles from 1879 to 1933 business-cycle contractions seem to be longer and more intense during downswings than during upswings of long building cycles; but this showing is nearly reversed when the 1927–33 cycle is excluded from the averages.

The specific cycles in building construction do not lead us to expect any regular differences in the duration or amplitude of full cycles between periods of rising and falling building 'trends'. It is conceivable, nevertheless, that business cycles might be shorter during upswings than during downswings of long building cycles, while the secular trends of individual economic activities rose more swiftly during the upswings than during the downswings. Under these circumstances the rise of the

TABLE 164

Average Duration and Amplitude of Specific Cycles in Seven Series and Average Duration of Business Cycles during the Upswings and Downswings of Long Cycles in American Building Construction

Series and phase of long cycles in building	Av. duration of specific or business cycles in months			Av. ratio of expansion to full cycle	Av. amplitude in specific-cycle relatives			Av. ratio of rise to total rise & fall
	Expansion	Contraction	Full cycle		Rise	Fall	Rise & fall	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
DEFLATED CLEARINGS								
Upswings	28	9	38	.74	27	11	37	.66
Downswings	32	13	42	.75	25	16	37	.66
Difference	−3	−4*	−5	−.02	+2*	−5*	0	0
PIG IRON PRODUCTION								
Upswings	30	11	43	.75	63	52	119	.57
Downswings	27	17	46	.58	61	57	128	.50
Difference	+3*	−6*	−3	+.17*	+2*	−4*	−9	+.07*
FREIGHT CAR ORDERS								
Upswings	19	20	43	.51	190	172	354	.52
Downswings	17	22	40	.41	245	235	496	.50
Difference	+2*	−2*	+2	+.10*	−55	−63*	−141	+.02*
RAILROAD STOCK PRICES								
Upswings	32	21	49	.58	41	29	75	.59
Downswings	19	21	36	.50	20	34	41	.47
Difference	+13*	0	+14	+.08*	+21*	−5*	+34	+.12*
SHARES TRADED								
Upswings	18	28	46	.40	96	98	209	.54
Downswings	17	25	46	.34	101	89	205	.49
Difference	+1*	+3	−1	+.06*	−5	+9	+3	+.05*
CALL MONEY RATES								
Upswings	20	20	39	.46	119	118	223	.46
Downswings	20	17	36	.58	111	115	214	.54
Difference	0	+3	+3	−.12	+8*	+4	+10	−.08
RAILROAD BOND YIELDS								
Upswings	22	18	41	.56	12	10	23	.52
Downswings	20	18	39	.56	9	14	24	.47
Difference	+2*	0	+2	+.01*	+2*	−3*	−1	+.05*
BUSINESS CYCLES								
Upswings	26	21	47	.54	..	..	..	..
Downswings	24	22	43	.56	..	..	..	..
Difference	+2*	−1*	+4	−.02	..	..	..	..
Ratio of variance between groups to variance within groups								
Deflated clearings	0.42	0.63	0.95	0.03	0.05	0.46	0.00§	0.00†
Pig iron production	0.21	1.55	0.17	2.44	0.03	0.06	0.19	0.53
Freight car orders	0.12	0.10	0.06	0.99	1.08	1.16	1.34	0.41
Railroad stock prices	2.00	0.00§	1.79	0.66	4.32	0.12	6.14‡	1.65
Shares traded	0.06	0.19	0.01	0.29	0.05	0.17	0.01	1.36
Call money rates	0.00†	0.80	0.35	1.74	0.07	0.01	0.04	3.33
Railroad bond yields	0.06	0.01	0.10	0.00†	0.40	0.81	0.02	0.21
Business cycles	0.31	0.02	0.29	0.04	..	..	..	..

All measures are made from specific or business cycles marked off by troughs. The measures for business cycles are derived from the monthly reference dates in Table 16. The measures for cyclical phases include some cycles not covered by the measures for full cycles; hence the discrepancies between the averages. For a full list of the cycles included in each group, see Appendix Table B5. The entries designated 'difference' are computed from averages carried to one more place than is shown in the table.

*Indicates that the difference accords with expectations stated in the text; no definite expectations are advanced for col. (4) and (8).

‡Larger than the value that would be exceeded once in twenty times by chance.

†Smaller than the value that would be fallen short of once in twenty times by chance.

§Smaller than the value that would be fallen short of once in a thousand times by chance.

specific cycles in individual activities could be the same during upswings and downswings; likewise the fall, the ratio of the rise to the total rise and fall, and the ratio of the expansion to the duration of full cycles. But column (4) in Table 164 does not suggest any cyclical alternation in the durations of specific or business cycles within the periods of long building cycles. Nor does column (8) suggest a cyclical alternation in the amplitudes of full specific cycles. The average durations and amplitudes of full cycles differ from upswing to downswing in almost every instance; but the differences lack consistency, few are substantial, and only one is 'significantly large'.

It does not follow from the evidence we have presented that long cycles in building construction fail to leave their stamp on economic activity at large. This important industry must make its influence felt in many branches of economic life.[5] The fact that most of the differences between the phases of the specific cycles in our series are in the direction that we should expect long building cycles to produce, that this is true without exception of pig iron production, and that several variance ratios for railroad stock prices are moderately high, may well be more than a statistical accident. It is even possible to argue from the evidence as it stands that long building cycles *tend* to give rise to a cyclical rhythm in business cycles, though as already stated we doubt that this argument can carry much conviction. Letting that be as it may, two features of the evidence seem incontestable and they suffice for our present purpose. First, the behavior of specific and business cycles does not differ greatly during periods of rising and falling building 'trends'. Second, the differences between these periods are by no means clear or uniform. We therefore see no compelling reason at the present time, nor even any real justification, for organizing cyclical measures of our time series on the assumption that business cycles undergo cyclical swings within periods of long building cycles.

II Long Cycles as Deviations from Trends

Hypotheses of long economic cycles are usually more sophisticated than the building hypothesis we have just investigated. Most students writing on this subject in recent years have worked out their hypotheses with the aid of collections of time series representing broad ranges of activities. In handling these records they use mathematical techniques to isolate long cycles, since the cycles they have in mind appear usually as accelerations and retardations in the original data, not as actual rises and declines. Thus

[5] Note again, however, that the construction of buildings is only a part of total construction work. During 1925–33 it accounted for approximately 60 per cent of the total value of construction in the United States.

by one device or another they fit lines of 'intermediate trend' which are supposed to free the original data from what we call specific cycles. Next they fit lines of 'primary trend' which are supposed to free the data from intermediate trends as well as specific cycles. Finally, they express the ordinates of intermediate trend as deviations from ordinates of primary trend, and in this way expose what they consider major cycles. The studies of 'major cycles' by Wardwell, 'secondary secular variations' by Kuznets, 'long waves' by Kondratieff, and 'trend cycles' by Burns follow this general plan.

The studies by Kuznets and Burns are the most extensive statistical investigations in this field, but unfortunately they are not suited to our present needs. Burns' investigation of 'trend cycles' covers the period from 1870 to 1930 in the United States. It concludes with a chronology of decades during which production and other economic activities increased at a particularly rapid or moderate pace. This chronology is much too coarse for our needs.[6] We are also forced to pass by Kuznets' investigation which covers a still longer period and several countries besides the United States, because Kuznets did not draw up a list of dates showing the peaks and troughs of his 'secondary secular variations'. In attempting to determine such a chronology from his American series, we found their turning points so widely dispersed that we could have little confidence in any list we ourselves might extract.[7]

Wardwell's investigation supplies what the studies by Kuznets and Burns lack—an annual chronology of major economic cycles. This investigation is based upon ten American, three British and four German quarterly series, covering from 37 to 71 years. Wardwell finds 'major cycles' in all his series. Their average duration varies from seven to nineteen years. He believes that the clusters of the peaks and troughs of the American series are so well defined that "it seems safe to conclude that the year 1890 marked the peak of a major cycle common to those series, the year 1895 the trough of this common major cycle, 1906 the next peak, 1914 the succeeding trough, and 1918 the most recent peak." The British materials seem inconclusive, but the German series "indicate, with considerable probability, the existence of a major cycle common to all of them, reaching a peak in 1890, a trough in 1892, a peak in 1898, and a trough in 1903." Wardwell speaks of his major cycles as comprising typically three or four business cycles; the initial trough of the major cycle is supposed to coincide with the trough of a severe business-cycle depression, and the peak of the major cycle is supposed to come at the time of the peak of the second business cycle in the group of three or four.[8]

[6] See A. F. Burns, *Production Trends in the United States since 1870*, Ch. V.

[7] See Simon Kuznets, *Secular Movements in Production and Prices*, Ch. IV.

[8] Charles A. R. Wardwell, *An Investigation of Economic Data for Major Cycles* (Philadelphia, 1927), especially pp. 83-5, 125-32.

Our reference dates for business cycles do not support Wardwell's chronology of major cycles in Germany. Wardwell finds a major cycle between 1890 and 1898; we find merely one business cycle instead of three or four during a slightly longer period, 1890 to 1900. Our list of American business cycles agrees better with Wardwell's scheme, but again discrepancies turn up. We find three business cycles between 1907 and 1918, but as many as five from 1890 to 1907 and again from 1896 to 1914. The peaks of Wardwell's major cycles come in the third business cycle of the major cycle starting in 1895 and in the first business cycle of the major cycle starting in 1914, instead of in the second business cycle as expected. In view of these discrepancies we must reject Wardwell's suggestion concerning the manner in which business cycles group themselves into major cycles.

Wardwell's findings on major cycles can, of course, be divorced from his suggested grouping of business cycles. But the alleged major cycles cover much too short a period to justify detailed testing. As things stand we can learn merely what happened during a period of less than thirty years, including only two major cycles, one of which is dominated by war. During the upswing from 1895 to 1906 we find two business-cycle contractions, in 1899–1900 and 1903–04. Since they are the only contractions during the upswings dated by Wardwell, we should not regard any difference between them and the contractions during downswings as important evidence for or against the hypothesis that business cycles vary cyclically. The second upswing consists entirely of the war prosperity from 1914 to 1918, which is more properly regarded as a cyclical expansion dominated by 'random forces' than as a member of a sequence of business cycles that regularly generate major cycles. This expansion was exceptionally long; [9] if we include it we are bound to get distorted results, while if we exclude it our observations on expansions during upswings are confined to a single upswing. In view of these facts we do not attempt a detailed analysis of Wardwell's scheme, but merely compare reference-cycle patterns.

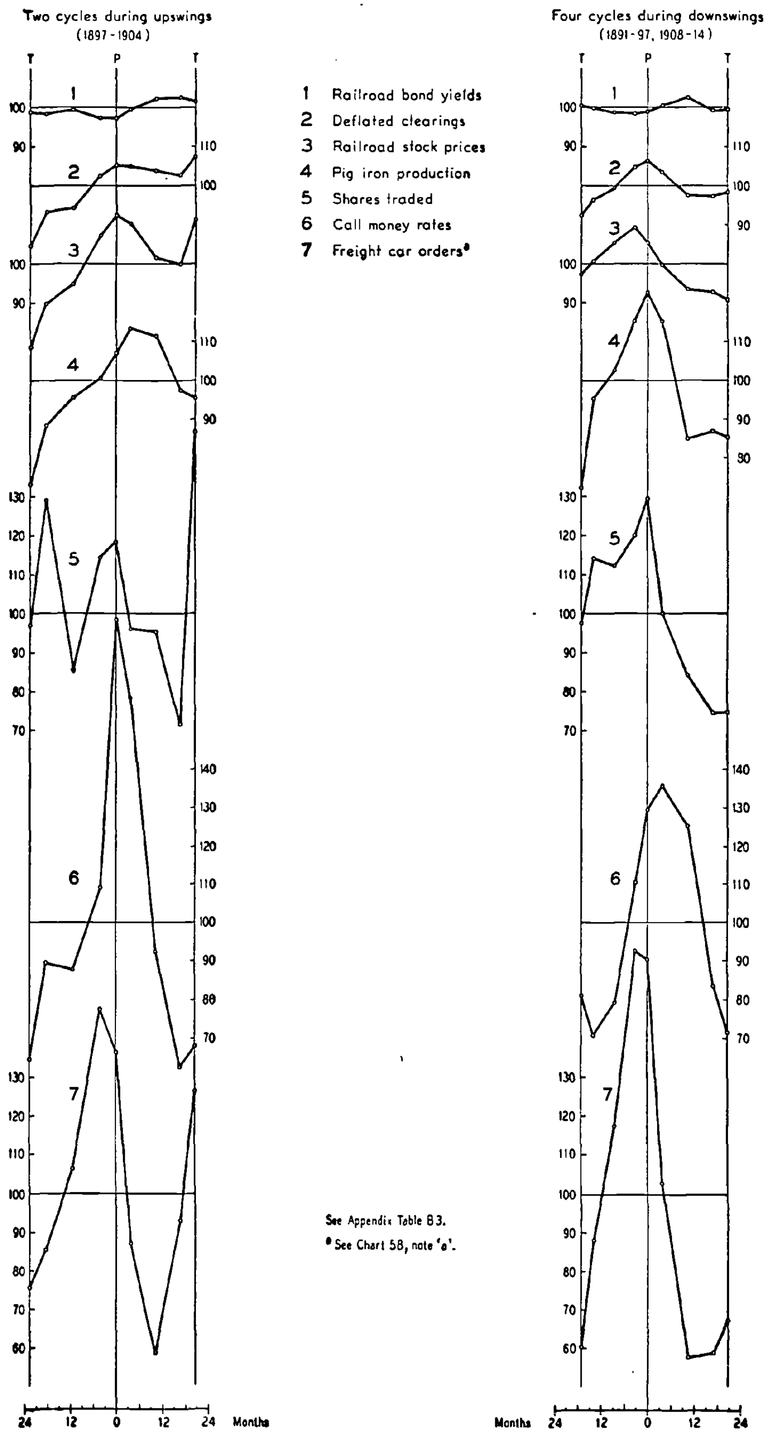
Chart 62 presents the average patterns of our series for the four business cycles during Wardwell's two downswings and for the two business cycles during one of his upswings.[10] We see that the average patterns differ

[9] Our monthly chronology discloses only three longer expansions: one during the Civil War, another following the great contraction of 1929–33, and the expansion in process since May 1938. Thus three of the four longest expansions since 1854 have been war expansions. See pp. 90-4.

[10] The chart omits the business cycles of 1904–08 and 1914–19 which overlap the upswing and downswing of the major cycles; since our rule is that the period matched with an upswing start and end with an expansion and the period matched with a downswing start and end with a contraction. This rule does not cover adequately the business cycle from the trough of June 1894 to the trough of June 1897. Since the annual troughs of this cycle are 1894 and 1896, and Wardwell's major-cycle trough comes in 1895, we have as much warrant for matching the cycle with the upswing as the downswing. By placing it in the downswing we adopt the arrangement that, on the whole, is most favorable to Wardwell's scheme.

CHART 62

Average Reference-cycle Patterns during the Upswings and Downswings of Wardwell's Major Cycles, 1891–1914
Seven American Series



as Wardwell's scheme leads us to expect. The contraction is longer relatively to the expansion during the downswing than during the upswing; it is also deeper in most instances. Consequently the patterns of the reference cycles during the downswing tilt upward slightly or not at all, while most patterns during the upswing have a sharp upward tilt. These differences seem instructive, but we have no good reason for believing that Wardwell's chronology, if extended by his methods, would confirm the traces of a cycle of cycles. We have found it impossible to date from Wardwell's records a representative trough before the peak of 1890, although eight of his ten American series go back to the 1870's or earlier. In any event, apart from the difference in tilt, Chart 62 shows a basic similarity in the relations among our test series—a similarity all the more remarkable because so few cycles are included in the averages.[11]

III Long Cycles Marked Off by Long Waves in Prices

We pass to Kondratieff's hypothesis, the most celebrated of the long-cycle 'theories'. For many years monetary writers have affirmed the existence of long waves in wholesale prices. The waves are supposed to last fifty to sixty years—a much longer period than is found in building construction, or in general economic activity according to Wardwell, Kuznets or Burns. The long waves in prices have usually been explained by accidental discoveries of gold deposits, improvements in gold refining, wars, and changes in the world's monetary systems. Kondratieff presents the more daring hypothesis that the long waves in wholesale prices are an organic part of a long cycle characteristic of capitalism.[12] Kondratieff's statistical investigations cover the leading industrial countries of the world; they are based mainly on value series, but include also small samples of physical volume series.

Table 165 shows the peaks and troughs of the long waves in wholesale prices in the United States, Great Britain, Germany and France.[13] Most of these dates fall within the turning zones of the long waves in the economic life of capitalist countries, as fixed by Kondratieff. Since 1790 there are only two complete long waves and an undetermined fraction of a third. Few series in our entire collection and none of the present sample go back so far. The longest series in our sample starts in 1857, while the monthly reference dates of business cycles in different countries start between

11 See below, Sec. VI.

12 N. D. Kondratieff, Die Langen Wellen der Konjunktur, *Archiv für Sozialwissenschaft und Sozialpolitik*, Dec. 1926. An abridged English translation of this article was published in the *Review of Economic Statistics*, Nov. 1935. See also Joseph A. Schumpeter, *Business Cycles*, Ch. IV-VII, and especially George Garvy, Kondratieff's Theory of Long Cycles. *Review of Economic Statistics*, Nov. 1943.

13 We deliberately follow convention as closely as possible in the dating—a matter to which we return later in this section.

TABLE 165

Peak and Trough Dates of Kondratieff's Long Waves and the Long Waves in Wholesale Prices of Four Countries

Nature of turn	Turns in Kondratieff's long waves	Turns in long waves of wholesale prices[a]			
		United States	Great Britain	Germany	France
Trough....	Late 1780's or early 1790's	1789	1789	1793[b]	....
Peak.......	1810–17	1814	1813	1808	1820[c]
Trough....	1844–51	1843	1849	1849	1851
Peak.......	1870–75	1864	1873	1873	1872–73
Trough....	1890–96	1896–97	1896	1895	1896
Peak.......	1914–20	1920	1920	1923	1926
Trough....	......	1932[d]	1933[d]	1933[d]	1935[d]

[a] See Chart 65. Later in this section we comment on the tendency of this table to oversimplify the facts and to exaggerate similarities among the countries. In dating the peaks and troughs for each country we have relied on annual index numbers based on currency prices.

UNITED STATES: Based on the Warren-Pearson index from 1797 to 1889, and the Bureau of Labor Statistics index since 1890. George F. Warren and Frank A. Pearson, Wholesale Prices in the United States for 135 Years, 1797 to 1932 (Cornell University Agricultural Experiment Station, *Memoir 142*, Part I, Nov. 1932); also U. S. Bureau of Labor Statistics, *Bulletin 572* and later publications. The trough in 1843 is not confirmed by the familiar Aldrich index which shows a trough in 1849, but it is reasonably certain that the trough came in the earlier year. We date a trough in 1789; the trough in prices came that year in Boston, Philadelphia and Charleston, but apparently in 1791 in New York. See the evidence in *Memoir 142*, just cited, and in Arthur H. Cole, *Wholesale Commodity Prices in the United States, 1700–1861* (Harvard University Press, 1938).

GREAT BRITAIN: Based mainly (the exceptions are noted below) on Gayer's index of imported and domestic commodity prices before 1850, and on the Sauerbeck-Statist index since then. The former we owe to an unpublished manuscript by Arthur D. Gayer. For the latter, see *Journal of the Royal Statistical Society*, Vol. 49, p. 648; Vol. 84, p. 260; Vol. 103, p. 348. A peak seems to have come in 1813, although Jevons' index shows a peak in 1810 and Silberling's in 1814 (confirmed by Gayer's index of prices of imported commodities). Some uncertainty surrounds the trough we have dated in 1849. Both Gayer's and Silberling's indexes stop in 1850; the former shows a slight fall and the latter a rise from 1849 to 1850. Sauerbeck's index reached a trough in 1849, Jevons' in 1849–50. See *ibid.*, Vol. 28, pp. 314-5 and *Review of Economic Statistics*, Oct. 1923, Supplement 2, pp. 232-3, for the indexes by Jevons and Silberling, respectively. The trough in 1789 is dated from these indexes.

GERMANY: Alfred Jacobs and Hans Richter, Die Grosshandelspreise in Deutschland von 1792 bis 1934, *Sonderhefte des Instituts für Konjunkturforschung*, No. 37, pp. 82-3, and *Statistisches Jahrbuch für das Deutsche Reich*, 1938, pp. 320-1.

FRANCE: Statistique Générale, *Annuaire Statistique*, 1938, pp. 436*-7*.

[b] Uncertain. The index starts in 1792, stands at 79.8 in that year and 79.7 the following year (av. 1913 = 100).

[c] Uncertain. The index starts in 1820.

[d] Uncertain. Lowest points as of the time of writing.

1854 and 1879. Under the circumstances it is impossible at present to come to serious grips with the problem whether business cycles tend to move in cycles within Kondratieff's periods.

We may, however, examine a simpler question: namely, whether there is evidence that the business cycles occurring during the upswings of the long waves in commodity prices differ substantially from the business cycles during the downswings in prices. Charts 63 and 64 seem to settle this question fairly conclusively. The thing that stands out above everything else is that the relations among the cyclical patterns of the activities represented in our sample are broadly similar during the periods of upswing and downswing in commodity prices. If we drop the extreme cycle of 1927–33, the reference-cycle patterns of at least three series—deflated clearings, pig iron production, and railroad stock prices—are nearly indistinguishable during periods of upswing and downswing in prices. The only series in which the direction of the 'trend' of prices clearly makes a substantial difference is railroad bond yields. That result is not surprising, since it has often been alleged that long-term interest rates are characterized by long waves that roughly parallel the long waves in wholesale prices.[14]

These critical facts suffice for our present purpose. It is of interest, however, to inquire how the cyclical measures of our series differ in detail during periods of opposite price trends. As in the case of the building hypothesis, we may expect (1) that expansions of specific cycles in individual economic activities will tend to be longer and more boisterous, and contractions shorter and milder, when the 'trend' of prices is upward than when it is downward; consequently, (2) that expansions relatively to whole cycles will be longer and the amplitude of rises relatively to full-cycle swings will be larger when the 'trend' of prices is upward than when it is downward. There are also some grounds for believing that "the depressed phases of business cycles are susceptible of greater prolongation than the prosperous phases." [15] Hence we may expect (3) that full cycles will be longer and their amplitudes perhaps larger during periods of declining than during periods of rising price 'trends'.

Table 166 shows that the first expectation is fulfilled in 17 out of 28 instances, the second in 21 out of 28, the third in 9 out of 14.[16] Clearly the averages in the table differ in most instances in the expected direction. But there are considerable differences both among the series and the cyclical measures. Railroad bond yields conform to expectations more consistently than any other series. Shares traded run counter to expectations almost as frequently as bond yields meet expectations. The amplitudes of specific cycles meet expectations less well than the durations —iron production, freight car orders and share trading being poor conformers. The ratios of expansions to full cycles, which are the most sensitive duration measures, meet expectations best of all. Sixteen of the 26 conforming comparisons for durations, but only 8 of the 21 conforming comparisons for amplitudes, yield variance ratios above unity. However, only 5 variance ratios in the table are 'significantly large' and all but one refer to bond yields.

Taken as a whole this evidence is slightly more favorable than the evidence supporting the building hypothesis. But it does not create a strong presumption that a causal relation exists between business cycles and the direction of price trends. For although the movements that conform to expectations preponderate over those that do not, this fact is neutralized by two others: the intercorrelation of the cyclical measures for

[14] For a trenchant analysis of this relation, see F. R. Macaulay, *Interest Rates, Bond Yields and Stock Prices*, Ch. VI.

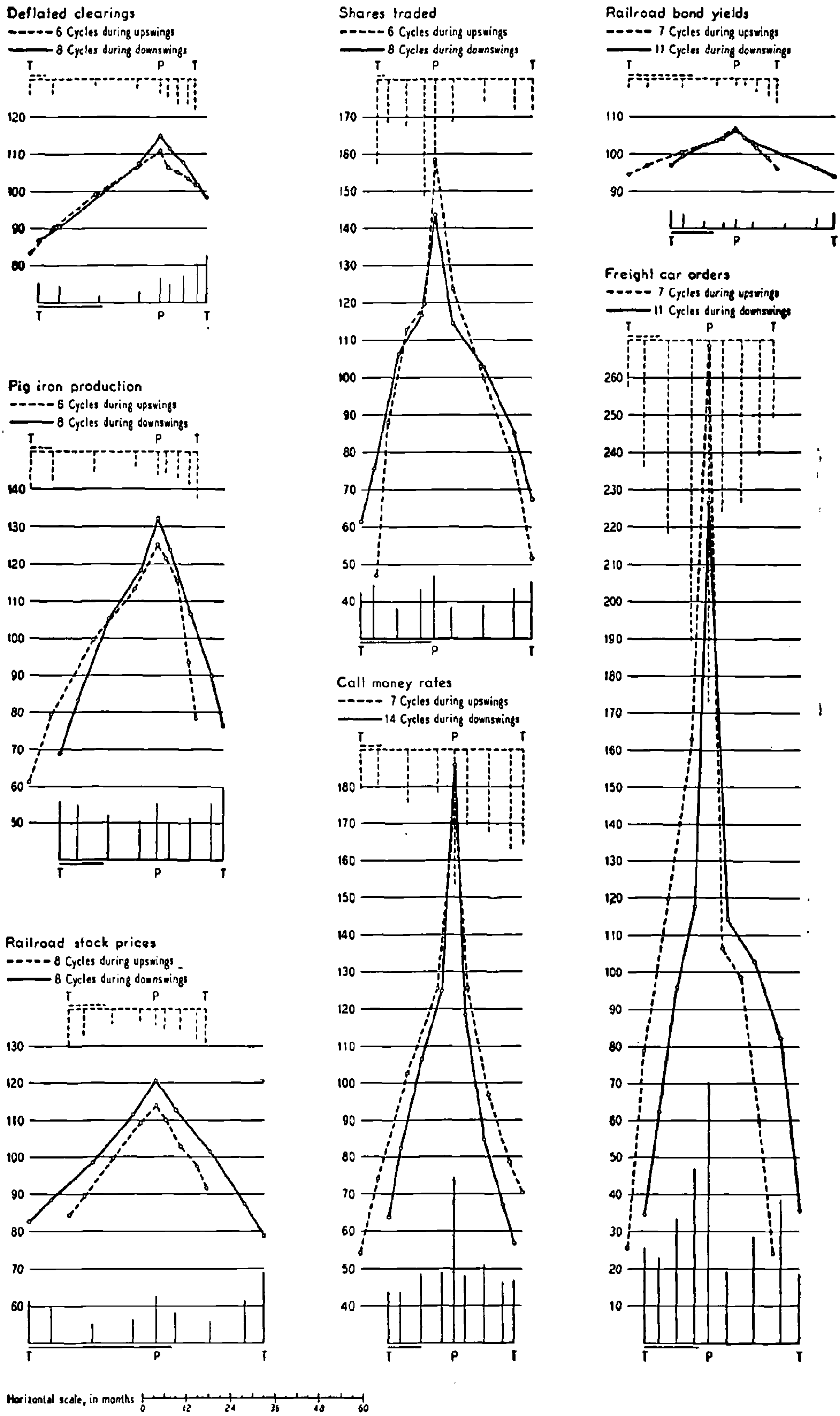
[15] See Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 411-2, 421.

[16] The table shows the ratio of expansions to full cycles marked off by peaks as well as by troughs. The extra computation eliminates the influence of the extreme contraction of 1929–33 and also serves as a check on the analysis for positive cycles; see above, note 4. The distribution of specific cycles between periods of rising and falling price 'trends' is shown in Appendix Table B6.

CHART 63

Average Specific-cycle Patterns during the Upswings and Downswings of Long Waves in Wholesale Prices

Seven American Series



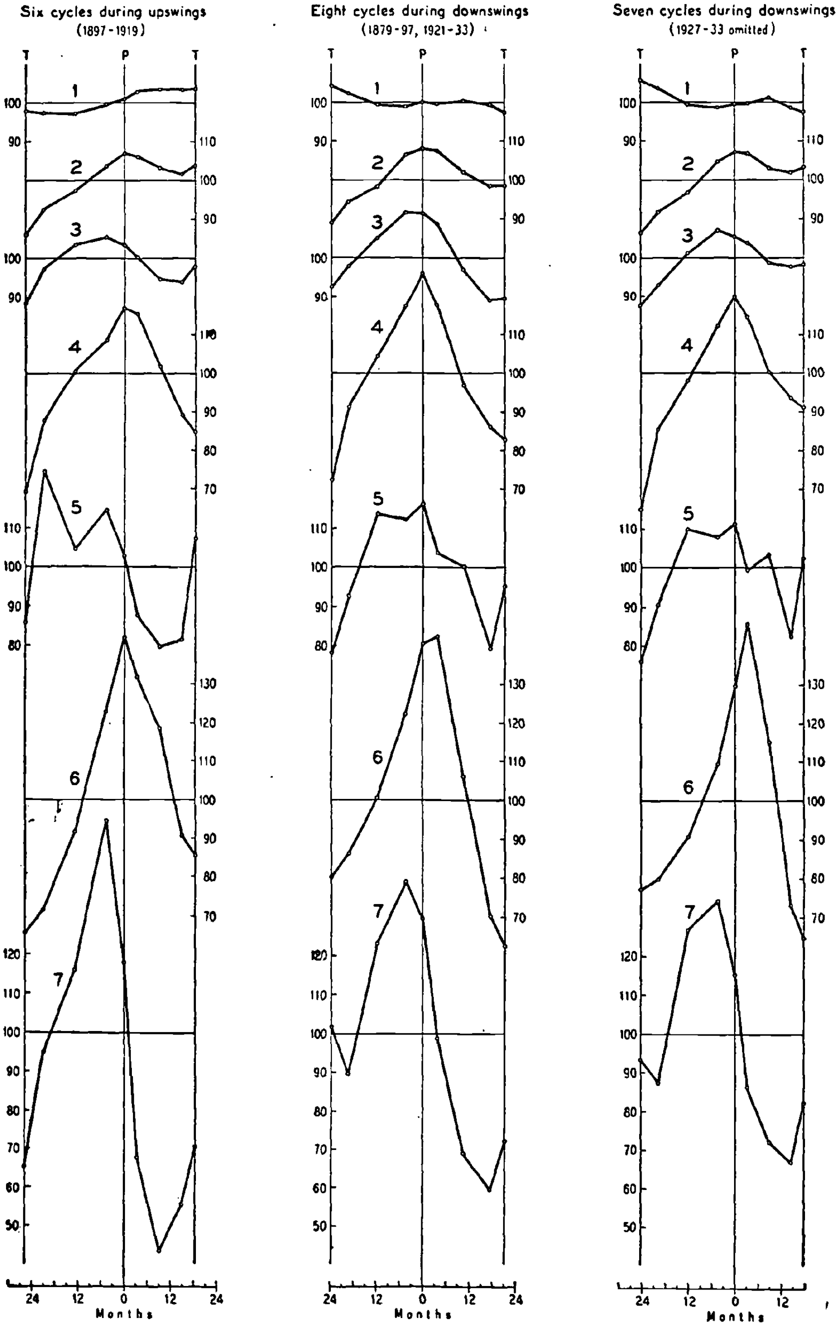
See Appendix Tables B1 and B6. For explanation of chart, see Ch. 5, Sec. VI.

CHART 64

Average Reference-cycle Patterns during the Upswings and Downswings of Long Waves in Wholesale Prices, 1879-1933

Seven American Series

1 Railroad bond yields
2 Deflated clearings
3 Railroad stock prices
4 Pig iron production
5 Shares traded
6 Call money rates
7 Freight car orders[a]



See Appendix Table B3.

[a] See Chart 58, note 'a'.

TABLE 166

Average Duration and Amplitude of Specific Cycles during the Upswings and Downswings of Long Waves in Wholesale Prices Seven American Series

Series and phase of long waves in prices	Av. duration of specific cycles in months			Av. ratio of expansion to full cycles marked off by		Av. amplitude in specific-cycle relatives			Av. ratio of rise to total rise & fall of cycles marked off by	
	Expansion	Contraction	Full cycle	Troughs	Peaks	Rise	Fall	Rise & fall	Troughs	Peaks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
DEFLATED CLEARINGS										
Upswings	32	9	45	.79	.78	25	10	37	.74	.76
Downswings	33	13	45	.75	.75	28	16	45	.64	.68
Difference	−1	−4*	−1*	+.04*	+.03*	−3	−6*	−8*	+.10*	+.08*
PIG IRON PRODUCTION										
Upswings	32	11	45	.76	.74	61	47	111	.57	.59
Downswings	26	17	44	.60	.62	63	60	119	.56	.61
Difference	+6*	−6*	+1	+.16*	+.12*	−3	−13*	−8*	+.01*	−.02
FREIGHT CAR ORDERS										
Upswings	20	17	39	.54	.50	242	244	487	.50	.48
Downswings	17	24	41	.39	.42	192	193	382	.51	.54
Difference	+3*	−6*	−2*	+.15*	+.07*	+50*	+52	+105	−.01	−.06
RAILROAD STOCK PRICES										
Upswings	24	14	38	.61	.64	34	22	52	.55	.56
Downswings	34	27	64	.51	.51	38	39	80	.48	.53
Difference	−10	−13*	−25*	+.10*	+.14*	−4	−17*	−28*	+.07*	+.03*
SHARES TRADED										
Upswings	16	26	42	.38	.35	116	107	218	.50	.52
Downswings	20	26	46	.43	.42	82	83	158	.53	.53
Difference	−4	0	−4*	−.05	−.07	+34*	+24	+60	−.03	−.01
CALL MONEY RATES										
Upswings	23	18	44	.59	.57	107	106	227	.56	.56
Downswings	18	18	34	.50	.48	122	121	251	.49	.52
Difference	+6*	0	+10	+.09*	+.09*	−15	−15*	−24*	+.08*	+.05*
RAILROAD BOND YIELDS										
Upswings	28	14	40	.69	.66	14	12	24	.58	.61
Downswings	17	26	43	.44	.42	9	13	21	.41	.36
Difference	+11*	−12*	−3*	+.26*	+.24*	+5*	−2*	+2	+.17*	+.25*
	Ratio of variance between groups to variance within groups									
Deflated clearings	0.01	0.45	0.00†	0.44	0.30	0.15	0.84	0.51	0.84	1.03
Pig iron production	1.01	1.55	0.02	3.15	4.44	0.06	0.56	0.17	0.02	0.11
Freight car orders	0.40	1.65	0.09	2.81	0.81	0.89	0.76	0.79	0.07	0.51
Railroad stock prices	0.99	3.84	2.33	1.48	1.62	0.12	1.42	1.19	0.53	0.08
Shares traded	0.56	0.00‡	0.17	0.20	0.28	3.24	1.47	2.82	0.40	0.01
Call money rates	2.21	0.01	4.69♩	1.00	2.13	0.28	0.20	0.13	2.43	0.26
Railroad bond yields	3.79	4.43♩	0.18	8.08♩	9.77♪	1.86	0.17	0.22	2.85	6.75♩

The measures in col. (6) and (11) are made from cycles marked off by peaks; all others from cycles marked off by troughs. The analysis by peaks is confined to cycles within the outer boundaries of the cycles marked off by troughs. The measures for cyclical phases include some cycles not covered by the measures for full cycles; hence the discrepancies between the averages. For a full list of the cycles included in each group, see Appendix Table B6. The entries designated 'difference' are computed from averages carried to one more place than is shown in the table.

*Indicates that the difference accords with expectations stated in the text.

♩Larger than the value that would be exceeded once in twenty times by chance.

♪Larger than the value that would be exceeded once in a hundred times by chance.

†Smaller than the value that would be fallen short of once in twenty times by chance.

‡Smaller than the value that would be fallen short of once in a hundred times by chance.

the several series and the unimpressive level of the variance ratios. One point, however, deserves further exploration; namely, there seems to be better evidence of association between price trends and specific cycles in the case of cyclical durations than in the case of amplitudes.

To push analysis of the amplitudes further we would have to enlarge considerably our sample of series—a step we cannot undertake at this time. But in the case of the durations we have at hand a simple and excellent check in the monthly measures of business cycles. In columns (2)-(6) of Table 167 these measures are set out for the United States and also our three foreign countries on the model of Table 166. The average durations of American business cycles meet in every instance the expectations that we set up, but the differences between periods of upswing and downswing

TABLE 167

Average Duration of Business Cycles during the Upswings and Downswings of Long Waves in Wholesale Prices Four Countries

Country and phase of long waves in prices	Based on monthly reference dates					Based on annual reference dates				
	Av. duration of business cycles in months			Av. ratio of expansion to full cycles marked off by		Av. duration of business cycles in months			Av. ratio of expansion to full cycles marked off by	
	Expansion	Contraction	Full cycle	Troughs	Peaks	Expansion	Contraction	Full cycle	Troughs	Peaks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
UNITED STATES										
Upswings...	26	17	42	.59	.58	30	14	45	.66	.64
Downswings.	24	25	49	.53	.53	24	25	49	.52	.53
Difference....	+2*	−8*	−7*	+.07*	+.05*	+6*	−11*	−4*	+.14*	+.11*
GREAT BRITAIN										
Upswings...	38	17	56	.70	.68	42	16	58	.71	.69
Downswings.	30	38	64	.44	.45	34	26	58	.58	.62
Difference....	+8*	−21*	−8*	+.26*	+.22*	+8*	−10*	0	+.13*	+.07*
GERMANY										
Upswings...	40	18	58	.68	.65	39	15	57	.70	.66
Downswings.	32	42	74	.46	.60	39	43	75	.54	.55
Difference....	+8*	−24*	−17*	+.22*	+.06*	0	−28*	−18*	+.16*	+.10*
FRANCE										
Upswings...	31	15	49	.66	.63	35	15	53	.67	.63
Downswings.	30	34	70	.46	.54	43	31	77	.57	.57
Difference....	0	−19*	−22*	+.20*	+.09*	−8	−16*	−24*	+.10*	+.06*
	Ratio of variance between groups to variance within groups									
United States.	0.20	1.86	0.67	0.86	0.53	1.21	4.55♩	0.28	5.15♩	2.68
Great Britain.	0.66	5.91♩	0.45	9.09♩	9.48♩	1.32	3.57	0.00†	5.21♩	1.42
Germany.....	0.69	7.63♩	1.15	11.10♩	0.17	0.00§	10.70♪	1.42	6.11♩	0.69
France.......	0.00†	6.06♩	2.51	5.08♩	0.64	0.50	8.21♩	3.59	1.11	0.26

Derived from Table 16; the annual reference dates are for calendar years. For a full list of the cycles included in each country and group, see Appendix Table B7; for other explanations, see note to Table 166.

* Indicates that the difference accords with expectations stated in the text.

♩ Larger than the value that would be exceeded once in twenty times by chance.

♪ Larger than the value that would be exceeded once in a hundred times by chance.

† Smaller than the value that would be fallen short of once in twenty times by chance.

§ Smaller than the value that would be fallen short of once in a thousand times by chance.

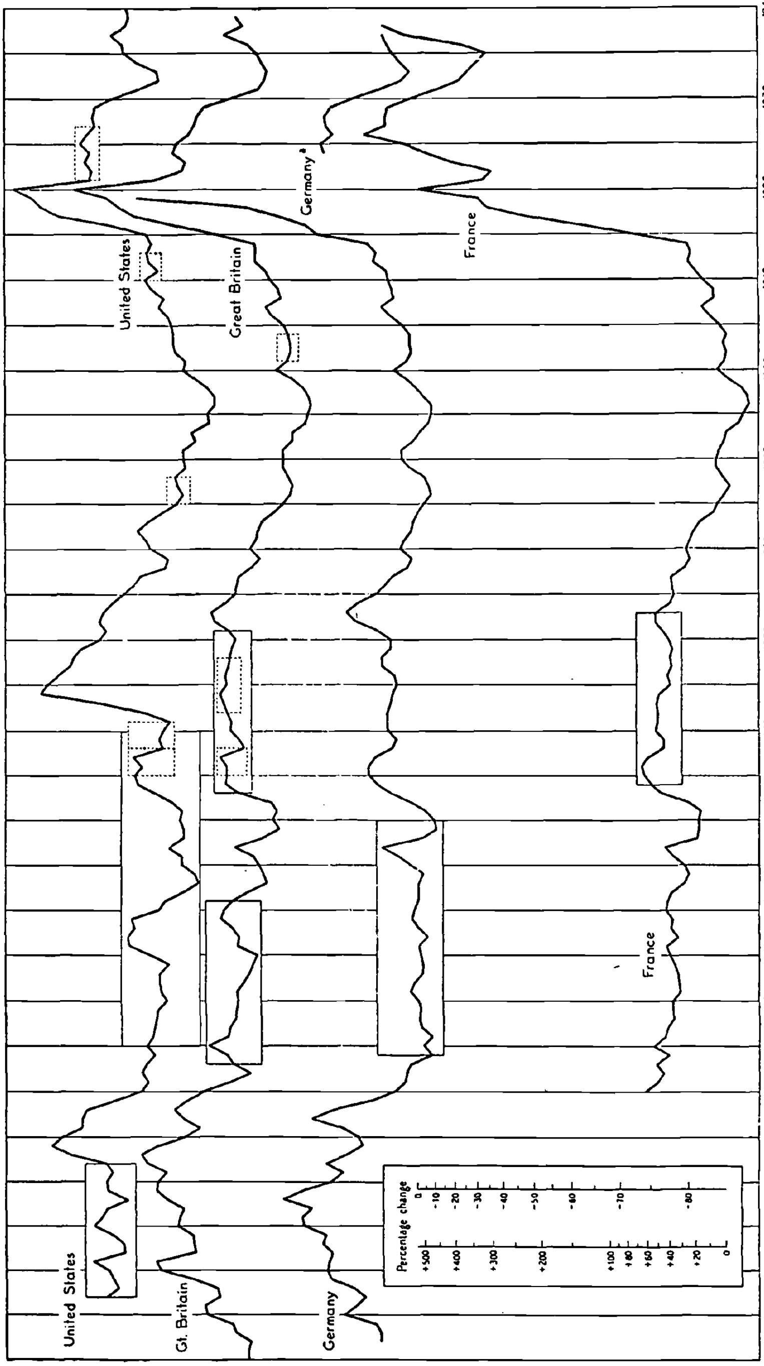
in prices are not statistically significant. Although these results confirm the specific-cycle measures, they appear in a new light when put side by side with other evidence. In practically every instance the foreign monthly measures differ between periods of rising and falling price 'trends' in the expected direction. In each foreign country the durations of business-cycle contractions during periods of rising price trends differ 'significantly' from the durations during periods of falling price trends. Even in the United States the variance ratio for business-cycle contractions, while not 'significantly large', exceeds unity. Further, the analysis based upon annual reference dates, which give coarser measures but cover more business cycles, yields a 'significantly large' variance ratio for business-cycle contractions in the United States, though not in Great Britain. Allowing as best we can for the interrelation of business cycles in our four countries, we judge that the evidence in hand supports the common opinion that there is a real relation between the direction of the trend in wholesale prices and business-cycle contractions: the contractions tend to be long or short according as the trend of prices is falling or rising, and this relation seems to hold for the duration of contractions relatively to full cycles as well as for their absolute duration.[17] At the same time the differences between the variance ratios for business cycles marked off by troughs and by peaks shout warnings that the relation between the direction of price trends and the relative duration of business-cycle phases is heavily overlaid by other factors.

We are unable at this time to undertake a causal analysis of this relation, but it is important to point out that the statistical devices we have used may not be well suited to such an undertaking. Our statistical analysis is built around the abstraction of a 'long-term' price trend, a procedure imposed by the hypothesis we are testing. We say that 'the' trend of prices was upward in the United States from 1897 to 1920, downward from 1920 to 1932, and so on; in other words, the 'trends' that concern us are the movements from trough to peak and from peak to trough of the alleged long waves in wholesale prices. But whether the chain of causation runs from the trend of prices to business cycles, the other way around, or is more complex than either statement implies, it seems plain that the trend of prices that is chiefly relevant to a given business cycle is the 'short-term' trend during that cycle, not the 'long-term' trend. There would be no difficulty if the 'short-term' trend invariably agreed in direction with the 'long-term' trend; but there is no such invariable rule (see Chart 65). For example, we treat 1843–64 and 1897–1920 as periods of rising prices in the United States, but the 'short-term' trend of prices was declining during the business cycles of 1855–58 and 1858–61, and was virtually horizontal during the 1910–13 cycle. We treat 1864–97 and 1920–32 as periods of falling prices, but the 'short-term' trend of prices

17 Cf. Mitchell, *ibid.*, pp. 410-11.

CHART 65

Indexes of Wholesale Prices, Four Countries, 1790–1940



Some periods are boxed in to aid the reader in following the text.
For sources of data, see Table 165, note 'a'.

[a] Indexes for 1920–23, years of great inflation, are much too high to fit on the chart.

was virtually horizontal during the cycle of 1885–88 and the two cycles from 1921 to 1927. In Great Britain we consider the 'trend' of prices as rising from 1849 to 1873 and from 1896 to 1920, but the 'short-term' trend was not rising in the cycle of 1855–58, or in the cycles of 1862–68 and 1901–04.[18]

One more point may be noted. The chronology of 'long waves' in wholesale prices in Table 165 is an abstraction not only in the sense just indicated, but in the additional sense that back of the 1870's the long waves are partly an optical illusion. Our table shows a trough in Germany in 1849, which confirms the entries for the other countries. But Chart 65 shows that the index of wholesale prices of the *Institut für Konjunkturforschung* was almost as low in 1824 and 1826 as in 1849–50; the 'trend' during the intervening period, if any, sloped gently upward. In France we list a peak in 1872–73 that matches the peaks in other European countries. But the index of the *Statistique Générale* was higher in 1854–57 than in 1872–73; from 1854 to 1873 prices were high or falling, definitely not rising. In the United States the long waves in wholesale prices are also to some extent illusory. High rather than rising prices prevailed from 1797 to 1812. Again, if we concentrate on the period from about 1825 to 1860 and disregard the preceding and following years, we might describe prices as moving along a horizontal trend. The long waves are clearest in Great Britain; yet one who did not already know these waves in advance might conclude that the trend of prices was not falling from 1823 to 1841, or rising during 1853–71.

IV Long Cycles as Triplets of Business Cycles

Schumpeter claims that "industrial history" establishes Kondratieff long waves, that each 'Kondratieff cycle' contains six 'Juglar' cycles "of from nine to ten years' duration" and that "every Juglar so far observed . . . is readily . . . divisible into three cycles of a period of roughly forty months". But he adds that Juglar cycles are "less clearly marked" in time series than Kondratieff cycles; while the forty-month cycle, "as well as the others, is more clearly marked in this country than in any other and notably more marked than in England".[19] For our purposes the new

[18] The periods cited are based on our annual reference dates.

[19] Joseph A. Schumpeter, The Analysis of Economic Change, *Review of Economic Statistics,* May 1935, p. 8. Similar statements appear in his *Business Cycles,* though the fuller exposition is less rigid. For example, Schumpeter warns the reader that "there is no rational justification . . . for assuming that the integral number of Kitchins in a Juglar or of Juglars in a Kondratieff should always be the same". He states that "it is possible to count off, historically as well as statistically, six Juglars to a Kondratieff and three Kitchins to a Juglar—not as an average but in every individual case"; but qualifies this claim with the proviso "barring very few cases in which difficulties arise". The Kitchin cycles, Schumpeter explains, are mostly "somewhat less than 40 months": they are "fluctuations, shorter than those of the Juglar group, but which we nevertheless believe to be of similar nature and which we think to be tolerably represented by a typical duration somewhat exceeding three years". See Vol. I, pp. 161-74, especially pp. 173-4.

TABLE 168

Frequency Distribution of Durations of Business Cycles
Four Countries

Duration in months	United States 1854–1933		Great Britain 1854–1932		France 1865–1932		Germany 1879–1932	
	No. of observations	Per cent	No. of observations	Per cent	No. of observations	Per cent	No. of observations	Per cent
16– 22	1	2.6	1	3.4	..	....	..	....
23– 29	1	2.6	4	13.8	5	17.2	1	5.3
30– 36	11	28.2	2	6.9	2	6.9	2	10.5
37– 43	11	28.2	2	6.9	5	17.2	1	5.3
44– 50	4	10.3	3	10.3	4	13.8	4	21.1
51– 57	4	10.3	3	10.3	4	13.8	1	5.3
58– 64	2	5.1	2	6.9	2	6.9	2	10.5
65– 71	1	2.6	4	13.8	2	6.9	2	10.5
72– 78	2	5.1	1	3.4	..	....	1	5.3
79– 85	..	....	2	6.9	..	....	1	5.3
86– 92	..	....	..	....	2	6.9	1	5.3
93– 99	1	2.6	1	3.4	1	3.4	1	5.3
100–106	1	2.6	1	3.4	..	....	1	5.3
107–113	..	....	..	....	2	6.9	..	....
114–120	..	....	1	3.4	..	....	..	....
121–127	..	....	1	3.4	..	....	1	5.3
128–134	..	....	..	....	..	....	..	....
135–141	..	....	1	3.4	..	....	..	....
Total[a]....	39	100.0	29	100.0	29	100.0	19	100.0

Derived from the monthly reference dates in Table 16. Observations include business cycles marked off by both troughs and peaks.

[a]The percentages in this line are rounded to 100.0.

TABLE 169

Relations in Time between Business Cycles and Schumpeter's 'Juglar Cycles'
United States, 1848–1932

JUGLAR CYCLES MARKED OFF BY TROUGHS			JUGLAR CYCLES MARKED OFF BY PEAKS		
Dates assigned by Schumpeter[a]	Corresponding period in our reference chronology[b]	No. of business cycles during this period	Dates assigned by Schumpeter[a]	Corresponding period in our reference chronology[c]	No. of business cycles during this period
1848–1858	1848–1858	2			
1858–1866	1858–1867	2			
1866–1876	1867–1878	2	1872–1881	1873–1882	1
1876–1885	1878–1885	1	1881–1891	1882–1890	2
1885–1895	1885–1894	3	1891–1900	1890–1899	3
1895–1904	1894–1904	3	1900–1909	1899–1910	3
1904–1914	1904–1914	3			
1914–1922	1914–1921	2			
1922–1932	1921–1932	3			

[a]Derived from his *Business Cycles*, Vol. I, pp. 396-7, 426-7, and Vol. II, pp. 786-9, 907. But see note 22. The peaks of several Juglar cycles were not dated by Schumpeter because of the dominance of 'external factors'; hence the smaller number of Juglar cycles marked off by peaks.

We might have matched the period 1885–96 with the Juglar of 1885–95 and 1896–1904 with the Juglar of 1895–1904. But this arrangement is less favorable to Schumpeter's scheme of 3 Kitchins per Juglar, since it assigns 4 business cycles to the Juglar of 1885–95 and 2 to the Juglar of 1895–1904.

[b]Begins and ends with a trough; see the calendar-year dates in Table 16.

[c]Begins and ends with a peak; see the calendar-year dates in Table 16.

question raised by this three-cycle scheme is whether business cycles tend to move cyclically during the periods of 'Juglar cycles'.

Our reference dates confirm Schumpeter's statement that the forty-month cycle is more clearly marked in the United States than in other countries. But even in the United States only about 28 per cent of our measures of business cycles since 1854 fall between 37 and 43 months (Table 168).[20] No arrangement of our monthly measures in groups of three consecutive cycles will produce an approximation to 'Juglar cycles' of from nine to ten years.[21] For example, if we start in 1854 and mark the cycles off by troughs, the durations of the groups of three run, successively, 156, 209, 109, 122, 124, 115 and 166 months; while if we start in 1857 and mark the cycles off by peaks, the durations run 144, 213, 105, 137, 135 and 98 months. Table 169 shows that if we insist upon getting rough groupings approximating nine to ten years, we must disregard a part of the historical record, or take at times three, at times two, and at times only one business cycle as corresponding to a 'Juglar cycle'.[22]

We may, of course, interpret 'Juglar cycles' as triplets of business cycles irrespective of duration. On this interpretation we can examine our cyclical measures to see if triplets of business cycles constitute higher units. Chart 66 shows average patterns of the reference cycles of our seven series grouped according as they come first, second or third in successive triplets of American business cycles from 1879 to 1933. Since fifteen business cycles span this period, the third cycle of the last triplet reaches a terminal trough in March 1933. We find no substantial differences in cyclical behavior on the present classification. The second contraction of the triplets seems on the average to be milder and briefer than either the first or the third, but the measures for single cycles fail to bear out the averages. A grouping of business cycles according to their position within triplets seems to be equivalent to a random grouping.

20 Cf. Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 339-43, 386-407, 416-24. Note, however, that the duration of the nine American business cycles from 1885 to 1914 ranged from 35 to 46 months (Table 16). It seems that this, in the main, is the core of experience on which the widely held notion of a forty-month cycle rests.

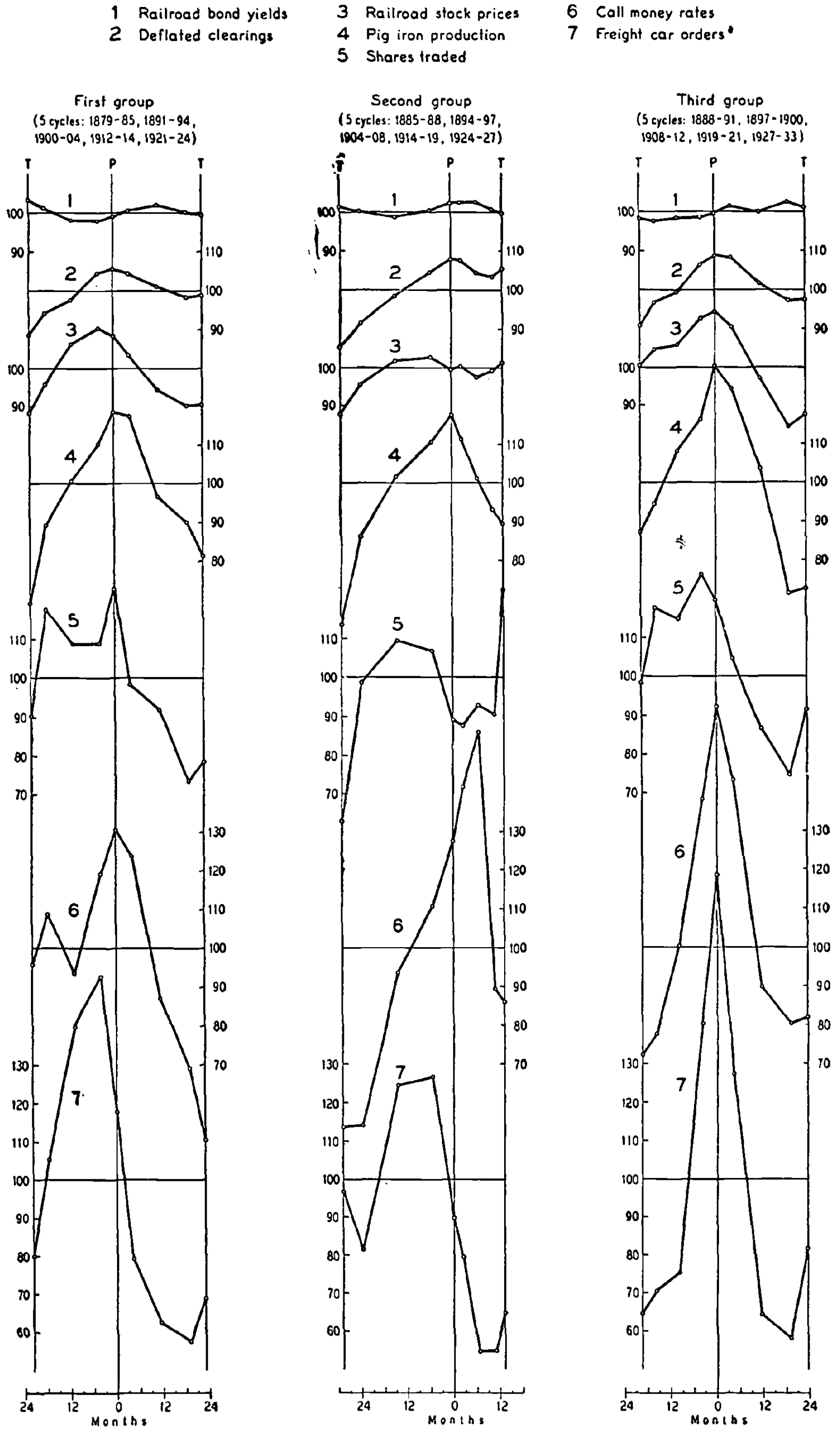
21 Although Juglar called his great work *Des Crises Commerciales et de Leur Retour Périodique*, he did not claim a high degree of regularity for the duration of his cycles. His list of crisis dates during the nineteenth century in France, England and the United States shows crises at widely varying intervals. In England, for example, Juglar listed 14 crises from 1803 to 1882; their successive intervals are 7, 5, 3, 8, 4, 7, 2, 8, 10, 7, 2, 7 and 9 years, and the average interval is about 6 years. See his second edition (Librairie Guillaumin, Paris, 1889), p. 256; also, pp. 162-8, and Part II, History of Crises, *passim*.

22 It should be noted that the dates of successive Juglars in Table 169, which we attribute to Schumpeter, are not true cyclical units in his sense. In Schumpeter's words: a cycle starts "with the neighborhood of equilibrium preceding prosperity" and ends "with the neighborhood of equilibrium following revival. The count from trough to trough or from peak to peak is . . . never theoretically correct. . . . Revival is the last and not the first phase of a cycle. If we count from troughs we cut off this phase from the cycle to which it belongs and add it on to a cycle to which it does not belong." (See his *Business Cycles*, Vol. I, p. 156 and Ch. V.)

CHART 66

Average Patterns of Reference Cycles Occupying First, Second or Third Place within Successive Triplets of Cycles, 1879–1933

Seven American Series



See Appendix Table B3.

* See Chart 58, note 'a'.

This experiment, however, does not bear crucially on Schumpeter's theory that 'Juglar cycles' are higher units of cyclical fluctuations. For our business and specific cycles are not strict equivalents of what Schumpeter calls Kitchin cycles. We do not recognize a cycle unless a fluctuation appears in the data, while Schumpeter regards Kitchin and even Juglar cycles as rhythmic tendencies that may or may not be directly observable in the data. There is therefore no contradiction when Schumpeter testifies to three Kitchins within a Juglar and we find only one or two business cycles. But, if 'Juglar cycles' really are higher cyclical units, we should find that the first specific or business cycle within a Juglar differs significantly from the last specific or business cycle; that is, if both the Juglar and the shorter cycles are marked off by troughs, the rise of the first specific or business cycle should be larger and the fall smaller than the corresponding movement of the last cycle, or at least the rise relatively to the fall should be larger in the first cycle than in the last. Similar expectations may be entertained, though with less confidence, in regard to the durations of the cyclical phases. To test these expectations,[23] it is necessary to disregard the few instances where a single specific or business cycle is roughly coterminous with a Juglar cycle, and to group the others according to their place within the alleged Juglars. We have put the cycles at the beginning of the Juglars in one class, the cycles at the end in another class, and then compared the two classes.[24] The results of this experiment are shown in Table 170 and Charts 67-68.

These charts resemble earlier exhibits in demonstrating considerable similarity between the cyclical patterns of different series in the first group and the cyclical patterns in the second group. But now striking differences also appear in the tilts of the two groups of patterns, the duration of their phases of expansion and contraction, and their amplitudes of rise and fall —differences that, on the whole, seem very favorable to the hypothesis that business cycles vary rhythmically within the periods separating Juglar troughs. Table 170 strengthens this impression. Most differences between the cycles with which the Juglars begin and the cycles with which they end are in the direction to be expected on the present hypothesis, many are substantial, and a goodly number are statistically significant. Clearly, the evidence is better that business cycles vary substantially within periods of Juglar cycles than that they do so within the long-cycle periods suggested by the other hypotheses that we have so far reviewed.

[23] There are several difficulties in making a statistical test. (1) Schumpeter regards cycles as rhythmic tendencies not necessarily observable in the data. (2) He considers the Juglars, as well as the other cycles, as units bounded by 'neighborhoods of equilibrium', not by their turning points. (3) He assumes that when a Juglar reaches a peak a Kitchin reaches a trough, and that at the time of a Juglar trough the Kitchin is at its peak. Our tests blink (as they virtually must if they are to be made at all) these difficulties. They are also confined to intervals separated by troughs; though a full test would require analysis also by peaks, since theoretical expectations might not be met in the former and yet fully met in the latter.

[24] Intervening cycles, if any, were disregarded: see Appendix Table B8.

TABLE 170

Average Duration and Amplitude of Specific Cycles in Seven Series and Average Duration of Business Cycles Occupying First or Last Place within Schumpeter's 'Juglar Cycles' in the United States

Series and place of cycles within Juglars	Av. duration of specific or business cycles in months		Av. ratio of expansion to full cycle	Av. amplitude in specific-cycle relatives		Av. ratio of rise to total rise & fall
	Expansion	Contraction		Rise	Fall	
DEFLATED CLEARINGS						
First	34	6	.85	34	10	.76
Last	32	18	.62	17	20	.49
Difference	+3*	−12*	+.22*	+17*	−10*	+.28*
PIG IRON PRODUCTION						
First	34	9	.78	79	50	.62
Last	21	21	.52	46	82	.38
Difference	+13*	−12*	+.26*	+33*	−32*	+.24*
FREIGHT CAR ORDERS						
First	20	17	.51	179	160	.55
Last	18	27	.40	196	217	.47
Difference	+2*	−10*	+.10*	−16	−57*	+.08*
RAILROAD STOCK PRICES						
First	17	15	.49	24	22	.52
Last	31	25	.54	43	48	.44
Difference	−14	−10*	−.05	−19	−26*	+.08*
SHARES TRADED						
First	14	20	.40	86	73	.53
Last	16	36	.28	122	136	.47
Difference	−3	−16*	+.12*	−37	−62*	+.07*
CALL MONEY RATES						
First	26	13	.66	126	110	.54
Last	15	24	.38	89	115	.44
Difference	+12*	−11*	+.28*	+38*	−5*	+.11*
RAILROAD BOND YIELDS						
First	16	27	.41	6	12	.37
Last	28	21	.55	14	12	.53
Difference	−12	+6	−.14	−8	−1*	−.16
BUSINESS CYCLES						
First	25	13	.65	...	...	...
Last	24	30	.44	...	...	...
Difference	+2*	−17*	+.21*	...	...	...
	Ratio of variance between groups to variance within groups					
Deflated clearings	0.09	6.04♩	11.25♪	4.94	1.52	10.41♪
Pig iron production	4.76	2.98	5.19	14.61♪	2.78	20.00♪
Freight car orders	0.15	2.83	1.04	0.19	1.61	3.79
Railroad stock prices	2.13	4.75	0.24	1.49	1.79	0.70
Shares traded	0.13	14.78♪	1.03	2.30	20.23♪	2.76
Call money rates	9.07♪	7.77♩	14.87♪	1.79	0.02	4.46
Railroad bond yields	3.12	0.71	1.59	3.67	0.03	1.81
Business cycles	0.07	6.98♩	9.19♪	...	...	...

All measures are made from specific or business cycles marked off by troughs. The measures for business cycles are derived from the monthly reference dates in Table 16. For a full list of the cycles included in each group, see Appendix Table B8. The entries designated 'difference' are computed from averages carried to one more place than is shown in the table.

* Indicates that the difference accords with expectations stated in the text.

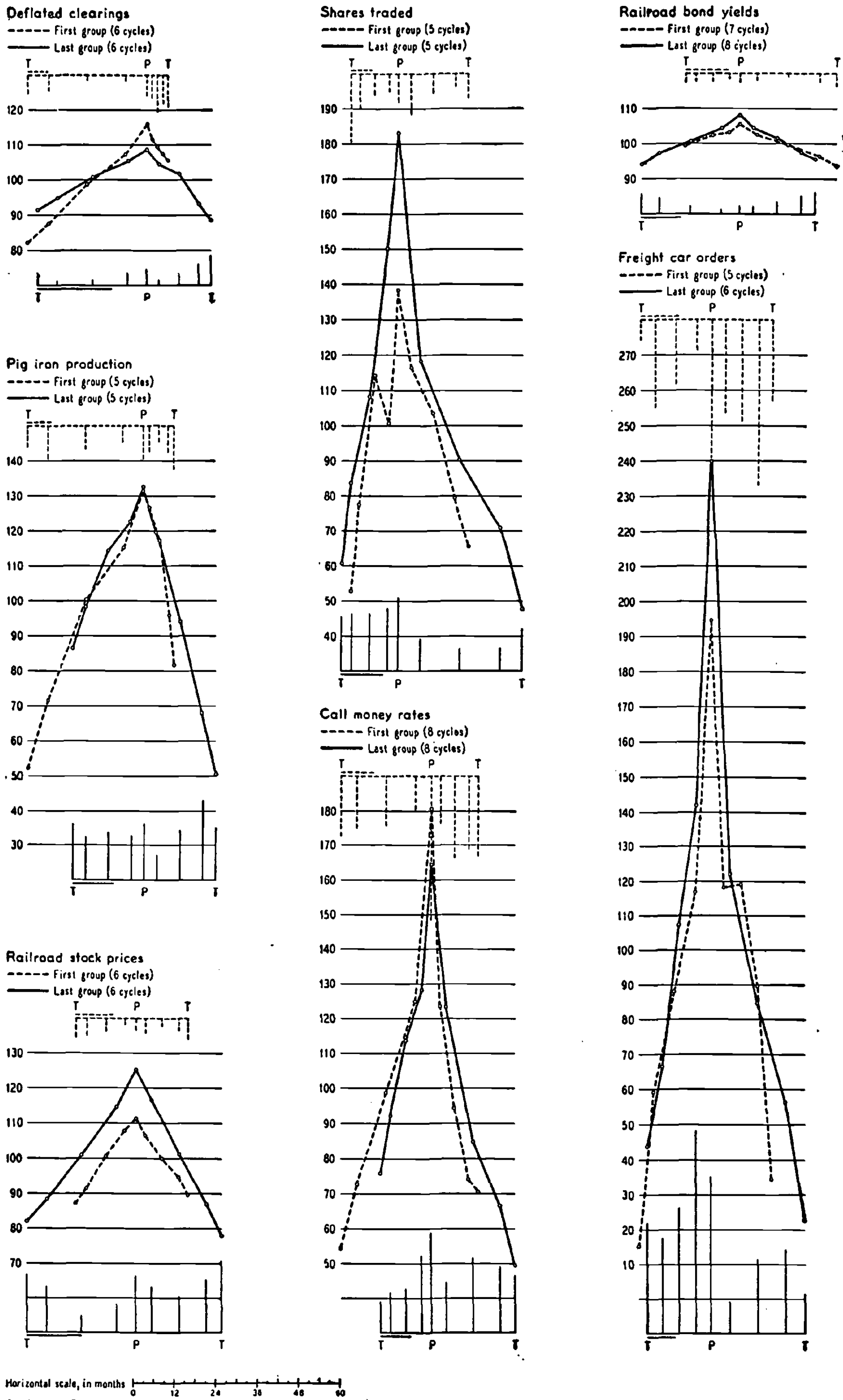
♩ Larger than the value that would be exceeded once in twenty times by chance.

♪ Larger than the value that would be exceeded once in a hundred times by chance.

CHART 67

Average Patterns of Specific Cycles Occupying First or Last Place within Schumpeter's 'Juglar Cycles'

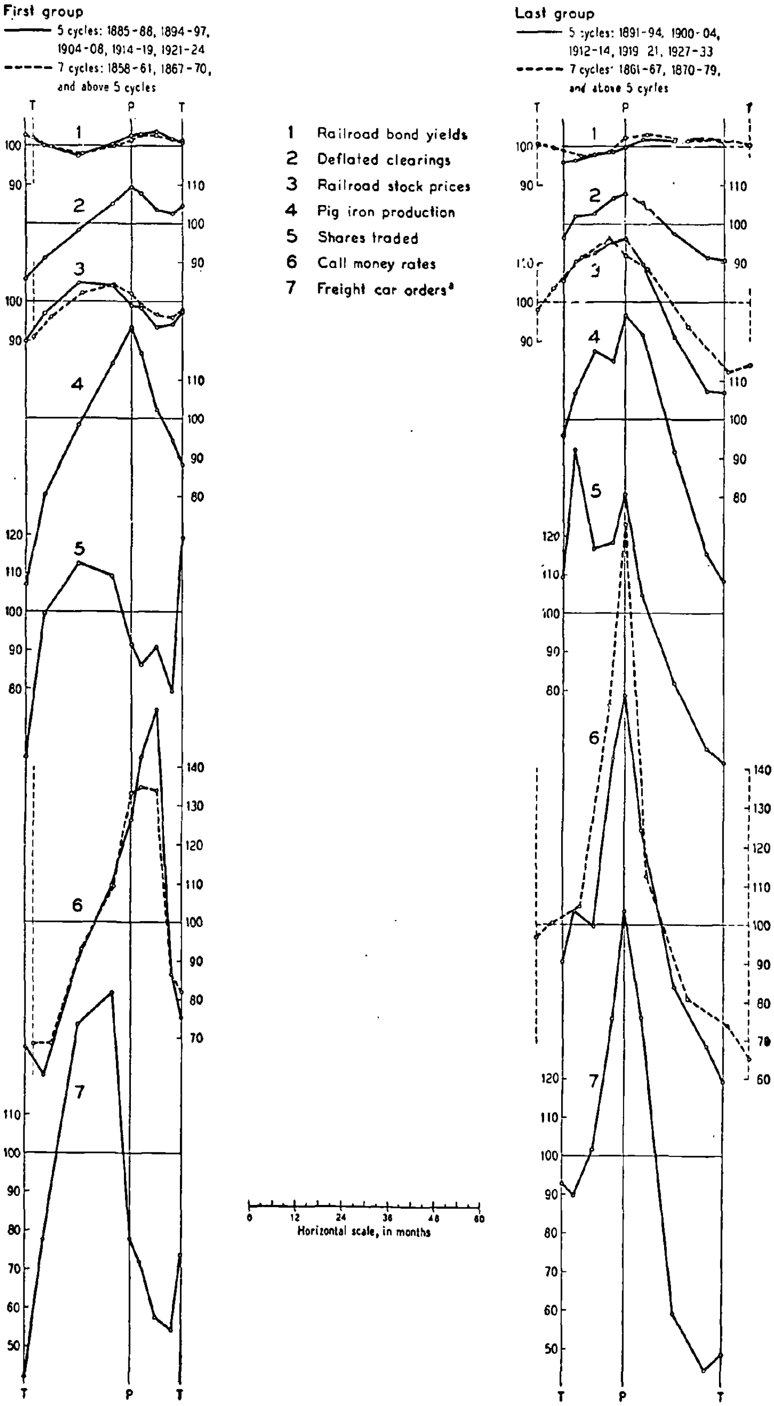
Seven American Series



See Appendix Tables B1 and B8. For explanation of chart, see Ch. 5, Sec. VI.

CHART 68

Average Patterns of Reference Cycles Occupying First or Last Place within Schumpeter's 'Juglar Cycles'
Seven American Series



See Appendix Table B3.

[a] See Chart 58, note 'a'.

May we conclude, therefore, that the 'Juglar cycles' listed by Schumpeter are higher cyclical units, that is, genuine cycles of cycles? It is important to notice, first of all, that the trough dates of the Juglar cycles correspond roughly to the trough dates of severe business depressions. The only clear exception, at least since the 1870's, is the Juglar trough in 1904.[25] So far as Juglar troughs mark off intervals between severe business depressions, it is only natural that substantial differences should appear between the business or specific cycles occupying opposite ends of the Juglars. If the business cycles characterized by long and violent depressions were singled out explicitly, they would of necessity differ materially from the other and milder cycles. Such a classification would demonstrate that some business cycles are in fact milder than others, not that the periods between the troughs of severe depressions are long cycles. Of course, if some internal regularity characterized the business cycles coming within periods separated by troughs of severe depressions, the hypothesis of a cycle of cycles would have surer footing. But the most striking of the internal regularities suggested by Schumpeter—the presence of three Kitchins within a Juglar and the nine to ten year duration of the Juglars—we are unable to confirm at this time. Table 169 shows one American business cycle that is roughly coterminous with a Juglar cycle. There are several instances of this sort in our foreign countries.[26] And the effort to fit our reference dates of business cycles into Schumpeter's chronology of Juglars yields periods that vary from 6.2 to 11.5 years,[27] although most Juglar cycles on this artificial basis do come out close to nine or ten years.

In view of these doubts, we cannot accept Schumpeter's chronological scheme as a solid basis for differentiating among specific or business cycles at this stage of our investigation. But we regard Schumpeter's scheme as a valuable suggestion for future research, and in Section VI say a little about the behavior of business cycles classified explicitly according to their position within periods separating severe business depressions.

V Long Cycles Marked Off by Booms

Kitchin's scheme of economic fluctuations is similar to Schumpeter's. What he calls fundamental movements, major cycles and minor cycles correspond fairly closely to what Schumpeter calls Kondratieff, Juglar

25 The business-cycle contraction of 1902–04, at least on the physical side of economic activity, is one of the mildest on record, definitely not a severe contraction. See at this point Schumpeter's remarks on the 1907–08 contraction (*Business Cycles*, Vol. I, pp. 424-8); also Sec. VI, below.

26 True, the long business cycles may be divisible into so-called Kitchin cycles, but that is a hypothesis of rather uncertain standing. See in this connection the comments on Table 171, below; also Ch. 4, Sec. VI.

27 March 1879–May 1885 is at one extreme, Sept. 1921–March 1933 at the other.

and Kitchin cycles. However, Kitchin's 'major cycles' differ sufficiently from Schumpeter's 'Juglar cycles' to warrant separate notice. According to Schumpeter three 'forty-month' cycles constitute a Juglar cycle. According to Kitchin, major cycles "are merely aggregates, usually of two" and sometimes of three 'minor cycles' lasting forty months on the average. The 'limit' of each major cycle, he explains, is "distinguished by a maximum of exceptional height, by a high bank rate, and sometimes by a panic". He presents his findings for bank clearings, prices of commodities at wholesale, and short-term interest rates by months in Great Britain and the United States; and states that they are supported by a wide range of data, chiefly annual, for other economic factors.[28]

TABLE 171

Number of Minor or Business Cycles within Kitchin's Major Cycles
United States, 1873–1920 and Great Britain, 1848–1921

UNITED STATES			GREAT BRITAIN		
Dates of Kitchin's major cycles[a]	No. of minor cycles found by Kitchin[b]	No. of business cycles shown by our reference dates[c]	Dates of Kitchin's major cycles[a]	No. of minor cycles found by Kitchin[b]	No. of business cycles shown by our reference dates[c]
...	..	..	1848–1858	(3)	2
...	..	..	1858–1866	(3)	2
...	..	..	1866–1874	(2)	1
1873–1883	(3)	1	1874–1882	(3)	1
1883–1893	(3)	3	1882–1891	(2)	1
1893–1900	2	2	1891–1900	3	1
1900–1908	2	2	1900–1907	2	2
1908–1913	2	2	1907–1913	2	1
1913–1920	2	2	1913–1921	2	2

[a] From peak to peak. Kitchin expressed his 'maxima' in hundredths of a year; we round them to the nearest year.
[b] Kitchin presents a chronology of minor cycles since 1890 only. The numbers in parentheses are inferred from his general scheme; see his paper, *loc. cit.*, especially p. 14.
[c] Derived from Table 16.

Table 171 presents Kitchin's chronology of major cycles; it runs solely by peaks since Kitchin found the troughs "often rather indefinite" and therefore did not attempt to date them. We compare in the table the number of business cycles that we find during the period of each major cycle with the number of minor cycles—which term Kitchin uses interchangeably with business cycles—assigned by him to the same periods. There are numerous disagreements: in five instances in Great Britain and one in the United States, the business cycles shown by our reference dates correspond to Kitchin's major cycles, not to his minor cycles. We believe that the wide discrepancies in the two chronologies are due not so much to different senses in which Kitchin and we speak of business cycles, but principally to differences in the materials used to identify business cycles.

28 Joseph Kitchin, Cycles and Trends in Economic Factors, *Review of Economic Statistics*, Jan. 1923, pp. 10-16.

Our chronology of business cycles is based on an extensive range of economic activities; we believe that Kitchin's is dominated by financial processes. Series such as bank clearings, interest rates, and security prices sometimes experience contraction when most business activities are not shrinking, or expansion when most activities are declining.[29] That happens more often in foreign countries than in the United States, but even in the United States we not infrequently find 'extra' cycles in financial series during long business cycles. If this explanation of the disagreements is valid, we must reject Kitchin's hypothesis concerning the relation of business to major cycles.

We might reformulate Kitchin's hypothesis and say that major cycles are coterminous with business cycles at certain times, and include two, three or perhaps more business cycles at other times. So modified the hypothesis may warrant exploration—but not for our present purpose, since it no longer provides a framework for investigating whether business cycles tend to vary cyclically. Another way out is to restrict Kitchin's hypothesis to the United States, and this seems reasonable in view of the evidence. Although Kitchin's and our lists of business cycles are irreconcilable for Great Britain, they differ for the United States only in the period 1873–83 and it is not impossible that we have overlooked some cyclical movements during these years.[30] On this interpretation it is of interest to see whether American business cycles have varied systematically within Kitchin's major-cycle periods since 1883. Kitchin's major cycle from 1883 to 1893 includes three business cycles; the next four major cycles include two business cycles each. If we disregard the middle business cycle within the major cycle of 1883–93, we have two business cycles for each major cycle, and so can determine whether a substantial difference exists between the business cycles that come first and those that come last within the major cycles. Our sample series can be readily treated on a similar basis, since each except railroad bond yields shows specific cycles that correspond closely to business cycles over the entire period from 1883 to 1920.[31]

We follow this plan in Charts 69-70 and Table 172. These exhibits show cyclical movements on an inverted basis, a procedure forced upon us by the fact that Kitchin dates his major cycles by booms. Apart from this technical shift, the charts are similar to those previously presented and repeat the story told in preceding sections. The thing that stands out is that the relations among the patterns of the several series are basically

29 This statement does not apply to wholesale prices which weigh heavily in Kitchin's analysis; but in this instance, if not also in others, Kitchin seems to have been betrayed by his pattern sense. See the chart on pp. 12-13 of the article just cited.

30 Cf. Ch. 4, Sec. VI.

31 We omit bond yields in analyzing specific cycles, but not reference cycles. Appendix Table B8 shows how the cycles in each series are classified. Note that the series on freight car orders is troublesome at one or two points.

TABLE 172

Average Duration and Amplitude of Specific Cycles in Six Series and Average Duration of Business Cycles Occupying First or Last Place within Kitchin's Major Cycles, United States, 1883–1920

Series and place of cycles within Kitchin's major cycles	Av. duration of specific or business cycles in months			Av. ratio of expansion to full cycle	Av. amplitude in specific-cycle relatives			Av. ratio of rise to total fall & rise
	Contraction	Expansion	Full cycle		Fall	Rise	Fall & rise	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
DEFLATED CLEARINGS								
First	12	33	44	.76	16	32	47	.68
Last	7	28	36	.79	5	21	27	.79
Difference	+4*	+4	+8	−.03*	+10*	+10	+20	−.10*
PIG IRON PRODUCTION								
First	16	32	48	.67	44	73	117	.64
Last	9	26	36	.70	41	57	98	.57
Difference	+7*	+5	+12	−.03*	+3*	+16	+19	+.08
FREIGHT CAR ORDERS								
First	27	23	50	.47	193	158	350	.47
Last[a]	15	20	35	.55	308	184	493	.48
Difference	+12*	+2	+14	−.09*	−116	−26*	−142	−.02*
RAILROAD STOCK PRICES								
First	27	19	45	.46	28	29	57	.49
Last	11	27	39	.69	21	26	47	.49
Difference	+15*	−9*	+7	−.23*	+7*	+3	+9	0
SHARES TRADED								
First	32	12	44	.27	88	104	193	.49
Last	24	14	39	.36	82	127	210	.60
Difference	+8*	−2*	+6	−.08*	+6*	−23*	−17	−.11*
CALL MONEY RATES								
First	21	25	46	.57	174	148	322	.48
Last	15	21	35	.58	109	133	242	.58
Difference	+6*	+5	+11	−.01*	+64*	+15	+79	−.10*
BUSINESS CYCLES								
First	22	25	47	.53	...	...	...	...
Last	17	20	36	.54	...	...	...	...
Difference	+5*	+5	+10	0	...	...	...	...
	Ratio of variance between groups to variance within groups							
Deflated clearings	1.31	0.48	0.87	0.27	5.29	2.45	6.95J	1.71
Pig iron production	3.62	0.54	2.00	0.10	0.11	1.87	1.04	1.69
Freight car orders	3.81	0.12	2.17	0.50	0.41	0.64	0.54	0.02
Railroad stock prices	3.91	1.86	0.53	3.50	0.83	0.06	0.64	0.00§
Shares traded	1.75	0.25	0.57	1.02	0.06	0.28	0.12	0.75
Call money rates	1.07	0.64	1.12	0.03	1.71	0.18	1.37	1.29
Business cycles	0.91	0.64	1.18	0.00†	...	...	...	...

All measures are made from specific or business cycles marked off by peaks. The measures for business cycles are determined from the monthly reference dates in Table 16. For a full list of the cycles included in each group, see Appendix Table B8. The entries designated 'difference' are computed from averages carried to one more place than is shown in the table.

[a] If the highly exceptional cycle from 1918 to 1920 (see Ch. 12, Sec. III) is dropped, the averages of the successive amplitude measures are 131, 175, 306, .56.

* Indicates that the difference accords with expectations stated in the text; no definite expectations are advanced for col. (4) and (8).

J Larger than the value that would be exceeded once in twenty times by chance.

† Smaller than the value that would be fallen short of once in twenty times by chance.

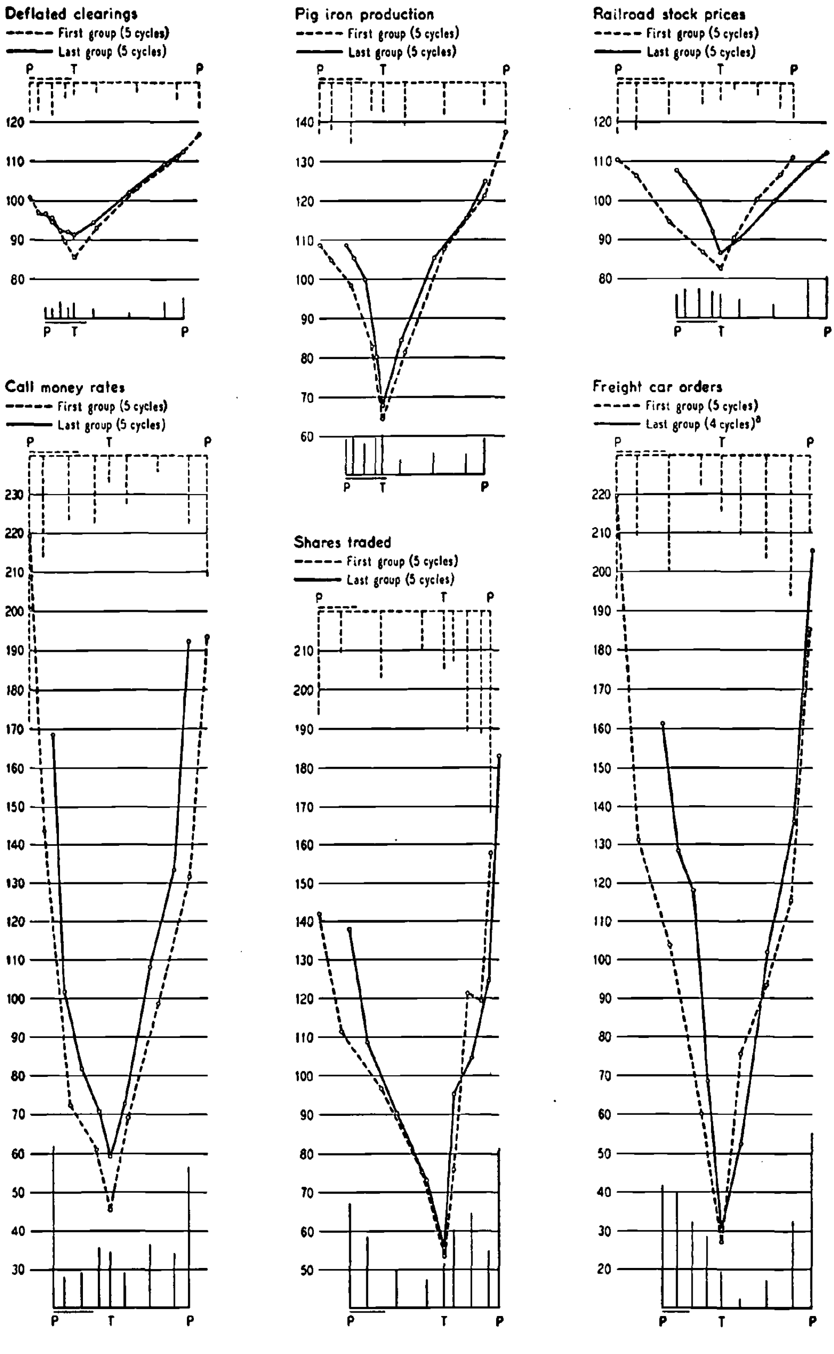
§ Smaller than the value that would be fallen short of once in a thousand times by chance.

CHART 69

Average Patterns of Specific Cycles Occupying First or Last Place within Kitchin's Major Cycles from 1883 to 1920

Six American Series

(Patterns made on inverted basis)

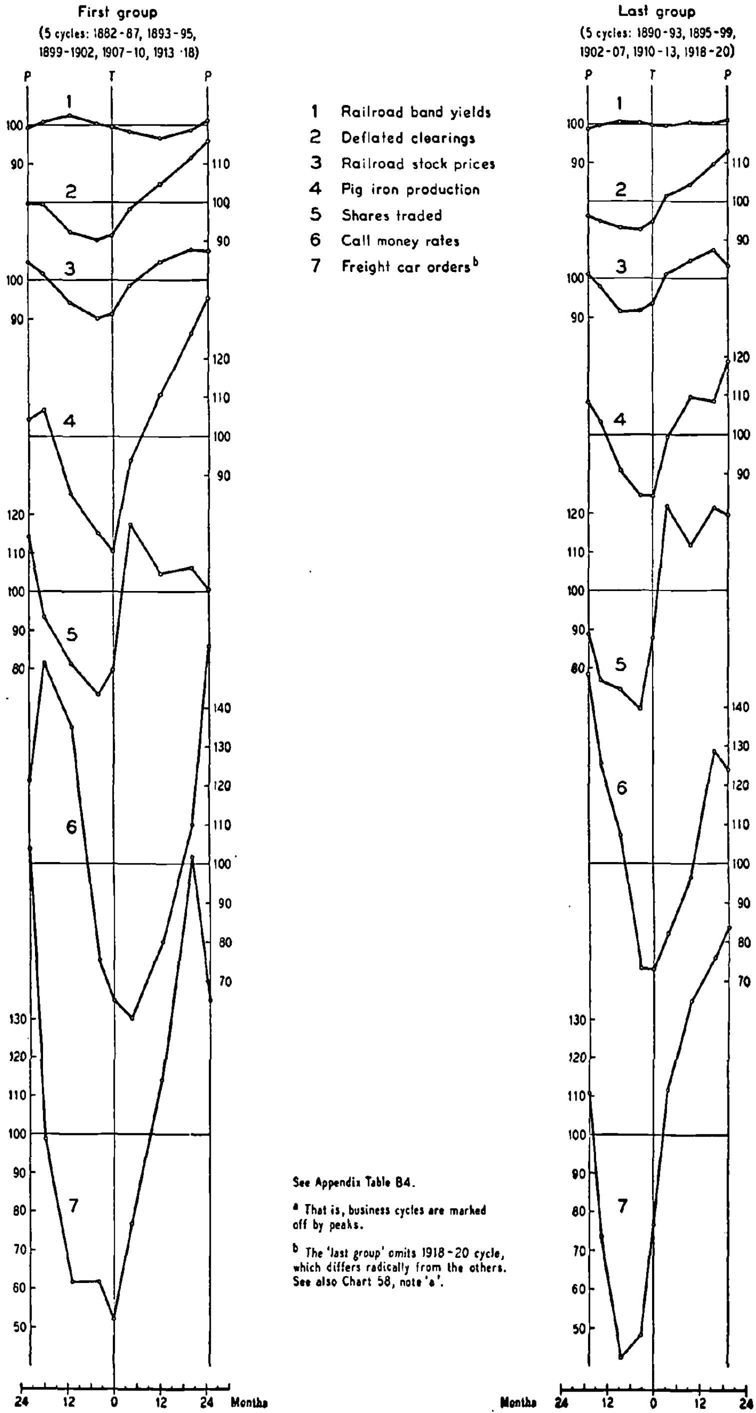


See Appendix Tables B2 and B8. For explanation of chart, see Ch. 5, Sec. VI.

[a] 1918-20 cycle omitted because it differs radically from the others.

CHART 70

Average Patterns of Reference Cycles Occupying First or Last Place within Kitchin's Major Cycles from 1883 to 1920
Seven American Series
(Patterns made on inverted basis[a])



See Appendix Table B4.

[a] That is, business cycles are marked off by peaks.

[b] The 'last group' omits 1918-20 cycle, which differs radically from the others. See also Chart 58, note 'a'.

the same in each group of cycles.[32] One point that may arrest attention is that the business cycles occupying the first place within the major-cycle periods are longer on the average than the business cycles occupying the last place. But the averages represent their arrays badly: the first business cycle is longer than the last in two and shorter in three of the major-cycle periods.

Table 172 analyzes in detail the present classification. If the 'limit' of each major cycle is "distinguished by a maximum of exceptional height", we should find that the ratio of the rise to the full-cycle amplitude is smaller in the first than in the last specific cycle within the major cycles; also, perhaps, that the rise of the first specific cycle is smaller and the fall larger than the corresponding movement of the last specific cycle. Similar expectations may be advanced for the durations of expansions and contractions of the specific cycles.[33] We find that the amplitudes meet expectations in eleven out of eighteen instances; however, in only four of these eleven instances is the variance ratio above unity and in no instance is it statistically significant. Our expectations concerning durations of cyclical phases meet a better fate when we examine specific cycles, but collapse when we turn to business cycles. Nor does the table bring out anything concerning full cycles that warrants particular notice.[34] On the present evidence it seems that there are no substantial differences between business cycles according to their position within Kitchin's major cycles, and that such differences as do exist may well result from chance variations.

As stated before, our principal concern in this chapter is whether business cycles undergo substantial cyclical variations within the periods suggested by certain outstanding hypotheses of long cycles, not whether long cycles are genuine phenomena. We may, however, observe in passing that short-term interest rates, which play a critically important part in Kitchin's scheme, do not fit very closely his chronology of 'major crises' in the United States. In call money rates the supposedly 'minor' peaks of 1887 and 1890 are higher than the 'major' peak of 1893, the 'minor' peak of 1902 is above the 'major' peak of 1899, and the 'minor' peak of 1918 is above the 'major' peak of 1912. In commercial paper rates (a series not included in our sample) the 'minor' peak of 1887 is higher than the 'major' peak of 1883, and the 'major' peak of 1900 is below the 'minor' peaks of 1896, 1898 and 1903. These facts seem to discredit what little support Kitchin's hypothesis derives from the amplitude comparisons of interest rates in the preceding charts and table.

32 For the cycle-by-cycle measures, see Appendix Tables B2 and B4.

33 Kitchin asserts merely that there is a "tendency for minima to be nearer to the lower of two succeeding maxima". His 'ideal curve' of business cycles is drawn on this plan. Note also that according to this 'ideal curve' the rise of the first cycle *is* smaller and the fall larger than the corresponding movement of the last cycle. See Kitchin's article, *op. cit.*, pp. 12-13, 15.

34 In every series specific cycles average longer in the first group than in the last. But the differences between the averages are no more dependable for specific cycles than for business cycles.

VI Long Cycles Marked Off by Severe Depressions

Kitchin's adventure suggests another way of marking off major cycles. The economic development of modern nations has been characterized by a rising secular trend, the peak of each cycle in industrial activity being almost invariably higher than the preceding peak. Cyclical troughs show less uniformity in this respect. In deflated clearings, for example, the trough is lower in November 1884 than January 1882, in August 1893 than March 1891, in November 1914 than October 1910, and in March 1933 than September 1923. But every peak in clearings is higher than its predecessor during the period we analyze, except the peaks of June 1883 and October 1895. Prior to 1920 every cyclical peak in pig iron production tops the preceding peak, but three troughs reach lower than preceding troughs. In view of these facts it should be easier to mark off provisional 'major cycles' by severe depressions [35] in industrial activity than by 'booms'. It is also a more promising method if there is any substance in Schumpeter's scheme of Juglars or in the common opinion that violent depressions are due to the partial character of the liquidations effected during preceding contractions.[36]

American businessmen of today will readily agree that the depressions of 1920–21 and 1929–33 were exceptionally severe; their fathers held the same opinion about the depressions of 1907–08, 1893–94 and 1873–79. On the whole, the leading indexes of business conditions in the United States confirm these ratings.[37] If we judge by Dorothy Thomas' index of business conditions [38] in 1855–1914 and other statistical indicators in later years, the American list of very severe depressions seems to fit also British experience since the 1870's. We dare not carry the list of severe depressions further at present, because we lack confidence in indexes of business for these two countries before 1870 or for other countries before 1914. We regard even the present list as highly tentative.[39]

35 In general, contraction is a less ambiguous term than depression. But for our present purpose, depression is the better term, since we wish to emphasize the 'low level' of industrial activity reached at the end of contraction, rather than the degree of contraction. In an economy in which each cyclical peak is invariably, or almost invariably, higher than the preceding peak, a severe contraction is almost sure to end in severe depression and a mild contraction in a mild depression. Of course, if a cyclical peak is lower than its predecessor, a mild contraction might easily end in what we should have to describe as a severe depression of industry.

36 It may be recalled that the troughs of Wardwell's 'major cycles' are also supposed to be coincident with troughs of severe depressions. See above, Sec. II.

37 We ranked the amplitudes of the cyclical contractions from 1879 to 1933 in the indexes of business conditions compiled by Leonard P. Ayres, Warren M. Persons, and the American Telephone and Telegraph Company, then averaged the ranks, and adjusted the rank of the cyclical contraction in 1873–78 shown by Ayres' index (the only one available for this contraction) to the average ranks after 1878. The highest five average ranks correspond to the severe depressions just named. See Table 156, which however is restricted to 1879–1933; see also note 35.

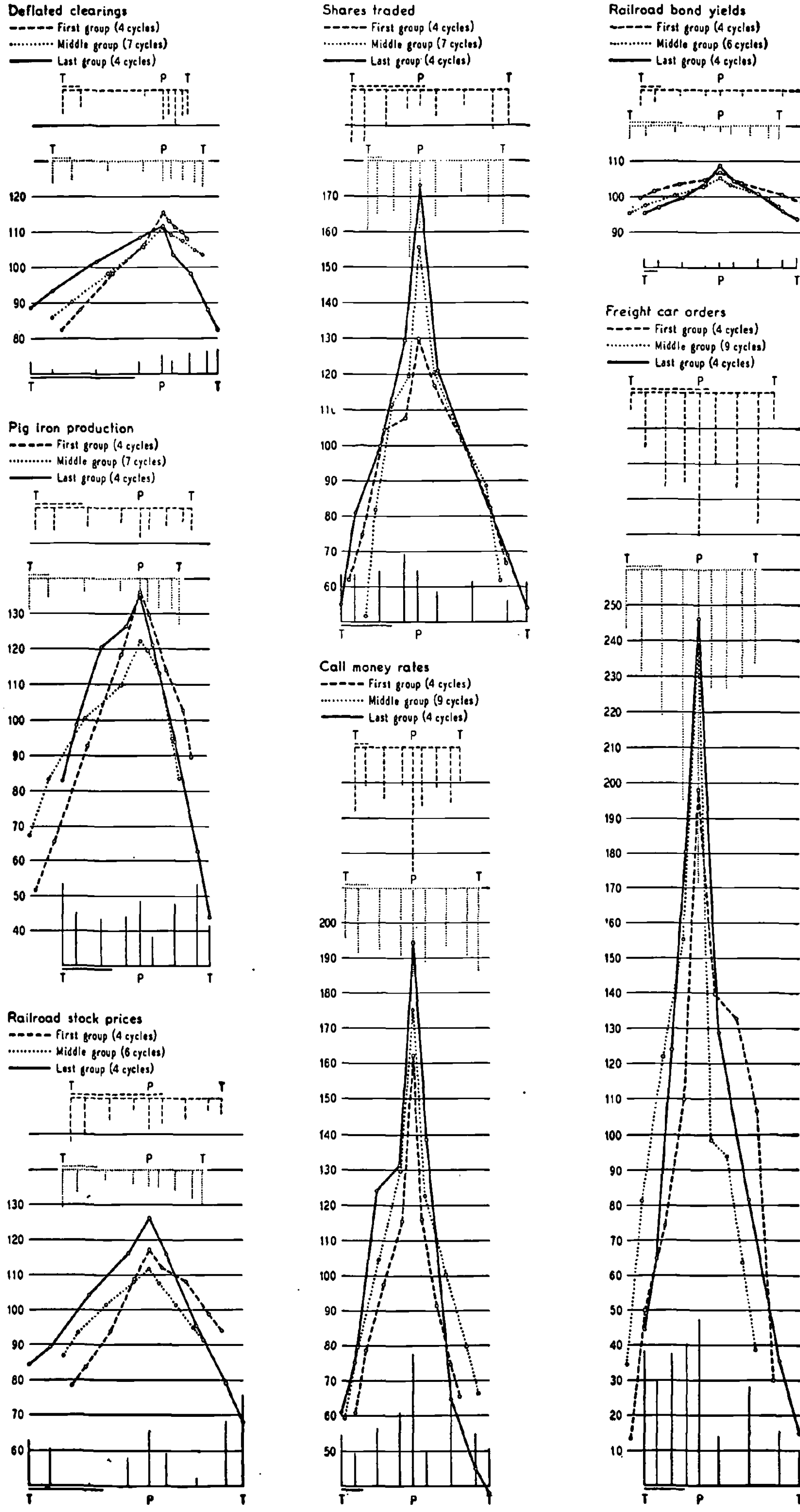
38 An Index of British Business Cycles, *Journal of the American Statistical Association,* March 1925.

39 See below, pp. 462, 464.

CHART 71

Average Patterns of Specific Cycles Occupying First, Middle or Last Place within Periods Marked off by Troughs of Severe Depressions, 1879–1933

Seven American Series



Horizontal scale, in months 0 12 24 36 48 60

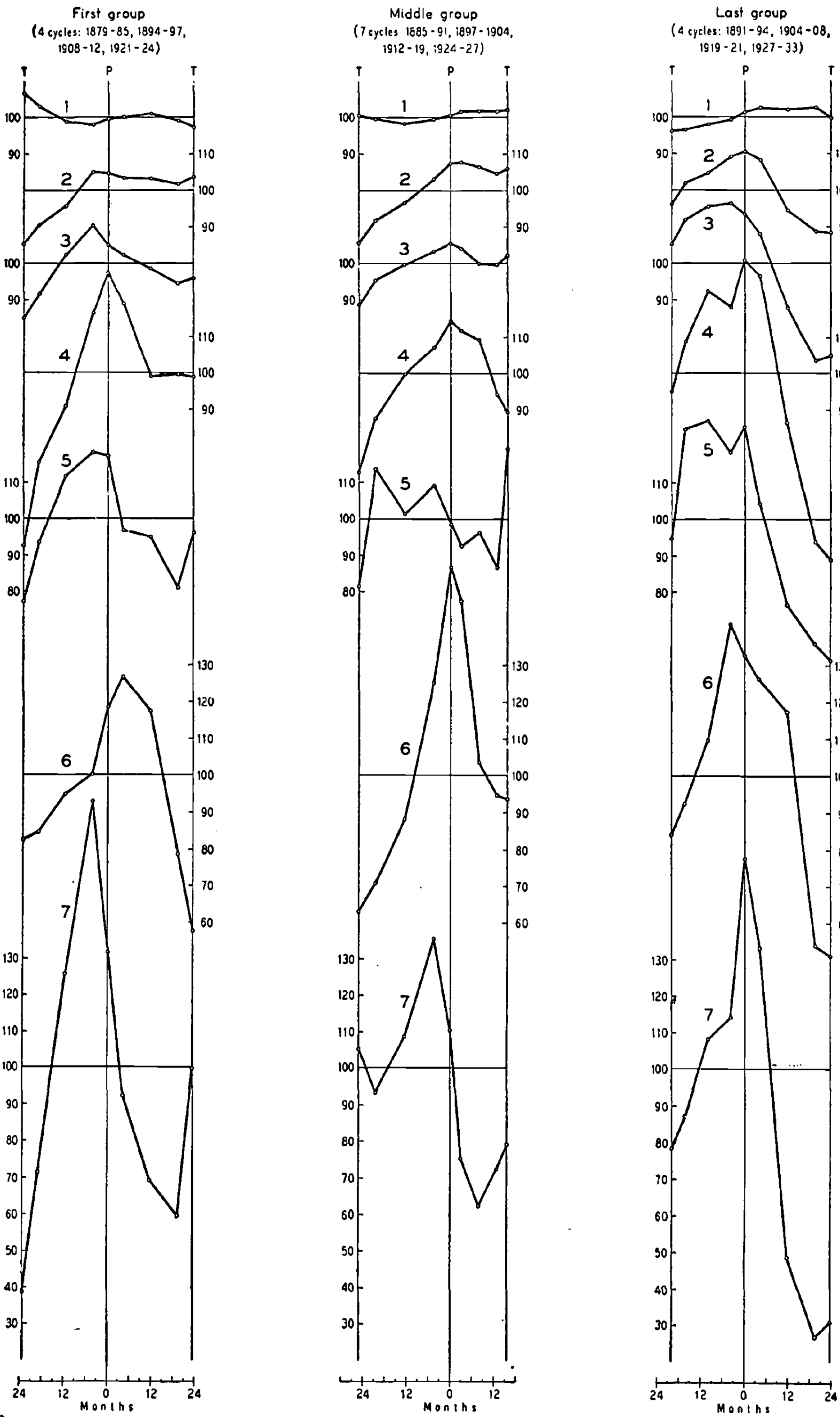
See comment on three horizontal lines in note to Chart 26; for other explanations, Ch. 5, Sec. VI. See also Appendix Tables B1 and B8.

CHART 72

Average Patterns of Reference Cycles Occupying First, Middle or Last Place within Periods Marked off by Troughs of Severe Depressions, 1879–1933

Seven American Series

1 Railroad bond yields
2 Deflated clearings
3 Railroad stock prices
4 Pig iron production
5 Shares traded
6 Call money rates
7 Freight car orders [a]



See Appendix Table B3.

[a] See Chart 58, note 'a'.

The above depression dates mark off four 'major cycles' between 1879 and 1933. According to our American reference dates, their minima came in March 1879, June 1894, June 1908, September 1921 and March 1933; according to the British reference dates, in June 1879, February 1895, November 1908, June 1921 and August 1932. The durations of the successive periods are approximately 15, 14, 13 and 12 years in the United States and 16, 14, 13 and 11 years in Great Britain. If we mark off the 'major cycles' by peaks preceding the severe depressions, the durations are about 19, 14, 13 and 9 years in the United States and 18, 17, 13 and 9 years in Great Britain. These periods include successively 4, 4, 4 and 3 business cycles in the United States and 2, 3, 3 and 3 business cycles in Great Britain. All this suggests a fair degree of uniformity, and gives point to an inquiry whether business cycles have varied cyclically during periods marked off by severe depressions.

Our first task is to distribute business cycles in the United States according to their position within these periods. Since the number of business cycles in these periods is sometimes three and sometimes four, we are forced to make somewhat arbitrary groupings. The 'first' group includes the business cycles, marked off by troughs, just following each severe depression; the 'last' group includes the business cycles within which the severe depressions fall; the remaining business cycles are thrown together in a 'middle' group. We have fitted the specific cycles into the periods of the major cycles, not into the distribution of business cycles within these periods. Thus we put in the 'first' group the specific cycles just following each severe depression, in the 'last' group the specific cycles that cover the severe depression, and in the 'middle' group all intervening cycles—of which there are usually two, occasionally none, and in one instance as many as four. Full details for each series are presented in Appendix Table B8.

Charts 71 and 72 show average patterns of the specific and reference cycles of our seven series distributed on the above plan. The charts suggest that this classification represents a fundamental line of cleavage. The vigor of the expansions in the 'first' group of cycles and the severity of the contractions in the 'last' group are arresting features. But there are no striking differences in the cyclical durations. Moreover, the relations among the cyclical patterns of our series are so similar from one group of cycles to another that it appears that the differences among the business cycles are merely differences of intensity, not differences of kind.[40]

Table 173 presents measures of the average amplitude of the three groups of specific cycles. In every series the average fall of the specific cycles is largest in the last group, as is also the joint rise and fall. Almost invariably the average ratio of rise to the joint rise and fall, in column (5), is lower in the last group than in the first or middle group of cycles. Well

[40] Cf. our *Bulletin 61*, pp. 19-20.

TABLE 173*

Average Amplitude of Specific Cycles Occupying First, Middle or Last Place within Periods Marked Off by Severe Depressions from 1873 to 1933
Seven American Series

Series and order of cycles between severe depressions	Average amplitude in specific-cycle relatives			Average ratio of rise to total rise & fall of cycles marked off by	
	Rise	Fall	Rise & fall	Troughs	Peaks
(1)	(2)	(3)	(4)	(5)	(6)
DEFLATED CLEARINGS					
First	33	7	40	.80	.55
Middle	26	8	33	.71	.76
Last	23	29	52	.46	.80
PIG IRON PRODUCTION					
First	85	46	131	.65	.56
Middle	55	38	93	.59	.64
Last	52	92	144	.38	.57
FREIGHT CAR ORDERS					
First	185	168	353	.53	.61
Middle	205	201	406	.52	.55
Last	202	231	433	.46	.41
RAILROAD STOCK PRICES					
First	39	23	62	.60	.49
Middle	25	21	46	.50	.50
Last	42	58	100	.42	.58
SHARES TRADED					
First	68	63	132	.51	.30
Middle	104	94	198	.53	.59
Last	118	119	237	.50	.66
CALL MONEY RATES					
First	101	97	198	.49	.24
Middle	116	109	225	.53	.55
Last	134	156	290	.45	.66
RAILROAD BOND YIELDS					
First	7	14	21	.39	.29
Middle	10	8	18	.53	.48
Last	14	15	29	.47	.65
	Ratio of variance among groups to variance within groups				
Deflated clearings	0.54	7.43♪	1.23	5.17♩	3.96
Pig iron production	6.01♩	6.56♩	4.70♩	7.64♪	0.75
Freight car orders	0.04†	0.24	0.11	1.11	1.56
Railroad stock prices	0.59	2.01	1.27	0.86	0.18
Shares traded	2.00	2.59	2.73	0.12	4.31♩
Call money rates	0.35	1.49	0.91	0.69	11.53♬
Railroad bond yields	1.47	1.91	3.22	0.44	2.88

The measures in col. (2)-(5) are made from specific cycles marked off by troughs and distributed according to their position within periods separated by the trough dates of severe depressions. The measures in col. (6) are made from cycles marked off by peaks and distributed according to their position within periods separated by the beginning dates of severe depressions. The former cover the period from about 1879 to 1933, the latter from about 1873 (or somewhat later) to 1929. The classification of cycles is shown in full in Appendix Table B8.

♩Larger than the value that would be exceeded once in twenty times by chance.
♪Larger than the value that would be exceeded once in a hundred times by chance.
♬Larger than the value that would be exceeded once in a thousand times by chance.
†Smaller than the value that would be fallen short of once in twenty times by chance.

over half of the variance ratios in columns (2)-(5) exceed unity, and as many as six are statistically significant. Of course, these systematic differences among the amplitudes in our several groups are natural concomitants of the scheme of classification, especially in iron production and clearings, which count heavily in the index numbers we have used to rate business depressions.[41] Column (6), however, stands on a different footing. If specific cycles are taken on an inverted basis and arranged according to their position within periods marked off by the *beginning* dates of severe depressions, there is no longer an inherent tendency for the ratio of the rise to the full-cycle amplitude to vary according to the position of the cycles within these periods. The high variance ratios in this column cannot be regarded as inevitable concomitants of our scheme of classification, and they direct attention to one provocative feature of the evidence.

This feature is that the series representing industrial activity seem to behave in a different way within the provisional long-cycle periods than do the series representing interest rates and speculation. While the average rise is largest in the first and smallest in the last group of specific cycles in both iron production and deflated clearings, it is smallest in the first and largest in the last group in bond yields, call money rates, and shares traded. In the last group of cycles the average rise is nearly the same as the average fall in shares traded and bond yields, not much smaller in call money rates, but considerably smaller in iron production. These differences suggest a hypothesis along the following lines. After a severe depression industrial activity rebounds sharply, but speculation does not. The following contraction in business is mild, which leads people to be less cautious. Consequently, in the next two or three cycles, while the cyclical advances become progressively smaller in industrial activity, they become progressively larger in speculative activity. Finally, the speculative boom collapses and a drastic liquidation follows, which ends this cycle of cycles and brings us back to the starting point. This hypothesis will repay exploration and may turn out to have substance; but what the cycle-by-cycle measures in Table 174 show is that the rises and falls of different activities do not follow invariably the above or any other simple pattern.[42] The picture appears to be one of partial cumulation of successive cycles; but the relations are not sufficiently regular in the records before us to justify us in regarding the business cycles separated by severe depressions as subdivisions of long cycles.

41 The 'significant' variance ratios do not necessarily mean that all three groups of cycles are strongly differentiated: in pig iron production the variance ratio for the rise reflects mainly the difference between the first group of cycles and the others, while the variance ratio for the fall reflects mainly the difference between the last group of cycles and the others.

42 The table shows, besides our standard measures of amplitude, the rise expressed as a percentage of the standing at the trough and the decline expressed as a percentage of the standing at the peak. These measures are perhaps simpler to interpret when successive cycles are being compared. They can be derived readily from columns (2)-(4) of our standard Table S2 (see p. 133).

TABLE 174

Amplitude of Specific Cycles Occupying Successive Places within Periods Bounded by Troughs of Severe Depressions, 1879–1933

Four American Series

Series and period[a] between severe depressions	First specific cycle[b]		Second specific cycle		Third specific cycle		Last specific cycle[c]	
	Rise	Fall	Rise	Fall	Rise	Fall	Rise	Fall
Amplitude expressed as percentage of monthly average during the cycle								
Pig iron production								
1879–1894	64	25	65	26	50	47	45	64
1894–1908	87	61	76	30	49	50	74	63
1908–1921	79	42	45	57	62	40	41	91
1921–1933	108	59	36	19	...	...	50	149
Deflated clearings								
1879–1894	52	13	37	6	28	8	17	26
1894–1908	22	10	36	6	21	6	27	25
1908–1921	31	3	11	12	40	5	12	11
1921–1933	19	6	...	...	...	...	...	55
Call money rates								
1879–1894	124	166	174	156	142	143	160	192
1894–1908	199	171	151	129	149	173	211	211
1908–1921	87	61	84	102	120	31	61	77
1921–1933	33	75	66	27	...	...	102	146
Shares traded								
1879–1894	103	62	62	83	51	38	94	115
1894–1908	62	68	129	108	182	176	113	85
1908–1921	45	79	71	110	129	84	144	135
1921–1933	63	46	102	58	...	...	121	141
Amplitude expressed as percentage of standing at beginning of the phase[d]								
Pig iron production								
1879–1894	115	21	97	19	68	38	67	56
1894–1908	169	44	145	24	71	42	144	50
1908–1921	131	30	54	44	118	35	45	70
1921–1933	278	40	50	17	...	...	41	86
Deflated clearings								
1879–1894	79	11	46	6	33	7	19	24
1894–1908	25	9	44	5	25	6	32	23
1908–1921	39	3	12	11	52	4	13	10
1921–1933	21	5	...	...	...	...	...	44
Call money rates								
1879–1894	174	85	449	73	304	76	256	86
1894–1908	580	73	286	63	248	83	669	87
1908–1921	171	44	118	66	274	19	80	56
1921–1933	36	60	132	23	...	...	139	83
Shares traded								
1879–1894	324	46	84	61	74	31	118	66
1894–1908	85	51	440	68	518	81	348	59
1908–1921	55	62	89	73	554	55	251	67
1921–1933	102	36	196	38	...	...	238	82

[a] The periods refer to groups of business cycles and hence are identical for each series; the corresponding specific-cycle periods differ somewhat.

[b] Two specific cycles in deflated clearings and three in call money rates occupy approximately the same period as the business cycle from 1879 to 1885. To simplify comparisons we ignore here (but not in Tables 173 and 175) the 'extra' cyclical movements; that is, the trough-peak-trough dates in clearings are taken as March 1878, June 1883 and Nov. 1884, and in call money rates Sept. 1878, Feb. 1881 and Jan. 1885.

[c] That is, the fourth cycle in each of the first three periods, but the third cycle in the fourth and last period. During 1921–33 clearings show only two specific cycles; the expansion of the second cycle is omitted, since it runs from 1923 to 1929.

[d] The standings include as a rule the 3 months centered on the month of turn, but in a few instances they include 1 or 2 months only. See Appendix Table B1.

Table 175 seems to support this judgment. Cyclical durations played no part in dating the periods we are investigating and therefore provide an independent test of the hypothesis that business and specific cycles vary cyclically within these periods. We find that the differences between the first and last groups are, on the whole, in the same direction as those between the first and last groups within 'Juglar cycles', but they are less impressive. Few variance ratios exceed unity, and not one in the table is 'significantly large'. There is a rough suggestion that the durations of British business cycles may have varied according to their position within periods separated by severe depressions, but little or no evidence that the durations of American business cycles have done so.

It is important to bear in mind that our ratings of business depressions are surrounded by uncertainty, and that for this reason, if for no other, it is desirable to suspend judgment concerning the nature of the periods marked off by severe depressions. The indexes of business conditions on which we have relied are made from slender samples in the early years. Another and more vital difficulty is that the cyclical depressions of experience do not group themselves readily into mild and severe types.[43] The contraction of 1882–85 has no place in our list of the severest five depressions in the United States between 1873 and 1933. It would have a place if we included the severest six depressions. It would have a place even in a list of five, replacing the contraction of 1907–08, if we made our selection according to Persons' index of 'production and trade' or according to Eckler's ratings,[44] instead of according to the combined indications of the three indexes of business that we have used. But if the contraction of 1882–85 is placed on the list of severe depressions, we get one severe depression succeeding another, and the neat distribution of business cycles that we have been elaborating is disrupted at the start. This would happen automatically if business depressions were rated according to their duration and amplitude, instead of their amplitude alone—as we have done. For when the cyclical contractions from 1879 to 1933 in our three indexes are ranked on the joint criterion,[45] all three agree that the contraction from 1882 to 1885 was only less severe than the contraction of 1929–33. And if we look beyond 1933, even the simple amplitude criterion proves embarrassing since the contraction of 1937–38 in the United States was apparently even severer than that of 1920–21; so that

43 Cf. Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 350-2.

44 See his paper, A Measure of the Severity of Depressions, 1873-1932, *Review of Economic Statistics*, May 15, 1933.

45 The troughs of Schumpeter's Juglars (since the 1870's and barring the trough in 1904) seem to be troughs of severe depressions on the joint criterion. Note the application of this criterion in Table 53.

But as note 19 in Ch. 6 implies, the joint criterion is inadequate so far as it ignores cyclical patterns. The contraction of 1882 to 1885 was very mild in the early stages; if we allowed for its peculiar pattern, it would not rank as high in a list of severe depressions as it does when the ranks are determined on the basis merely of duration and amplitude.

TABLE 175

Average Duration of Specific Cycles in Seven American Series and of American and British Business Cycles Occupying First, Middle or Last Place within Periods Marked Off by Severe Depressions from 1873 to 1933

Series and order of cycles between severe depressions	Average duration of specific or business cycles in months			Average ratio of expansion to full cycles marked off by	
	Expansion	Contraction	Full cycle	Troughs	Peaks
(1)	(2)	(3)	(4)	(5)	(6)
DEFLATED CLEARINGS					
First	29	7	36	.80	.78
Middle	32	11	43	.73	.73
Last	38	16	54	.71	.82
PIG IRON PRODUCTION					
First	30	15	45	.67	.68
Middle	32	11	43	.74	.71
Last	22	20	42	.54	.65
FREIGHT CAR ORDERS					
First	20	21	40	.46	.39
Middle	20	16	36	.52	.51
Last	16	28	44	.36	.46
RAILROAD STOCK PRICES					
First	22	21	43	.51	.39
Middle	25	15	40	.62	.57
Last	35	27	62	.53	.70
SHARES TRADED					
First	20	25	45	.43	.31
Middle	15	24	39	.40	.37
Last	22	31	53	.40	.50
CALL MONEY RATES					
First	16	14	30	.54	.48
Middle	19	19	38	.49	.52
Last	20	22	42	.46	.56
RAILROAD BOND YIELDS					
First	23	35	58	.38	.41
Middle	26	17	42	.62	.52
Last	21	22	43	.48	.60
BUSINESS CYCLES, U. S.					
First	23	24	47	.50	.49
Middle	25	16	41	.61	.54
Last	20	24	44	.46	.59
BUSINESS CYCLES, G. B.					
First	49	24	74	.67	.59
Middle	26	14	40	.58	.52
Last	26	30	56	.44	.55
	Ratio of variance between groups to variance within groups				
Deflated clearings	0.36	0.88	0.72	0.44	1.20
Pig iron production	0.99	0.96	0.04†	1.89	0.30
Freight car orders	0.20	2.28	0.28	0.94	0.72
Railroad stock prices	0.55	0.99	0.92	0.69	2.60
Shares traded	0.55	0.49	0.86	0.04†	0.79
Call money rates	0.16	1.07	2.25	0.15	0.26
Railroad bond yields	0.10	2.32	1.89	1.19	0.79
Business cycles, U. S.	0.40	1.24	0.29	1.19	0.72
Business cycles, G. B.	2.32	1.32	1.94	1.32	0.12

The measures for business cycles are derived from the monthly reference dates in Table 16. For further explanations, see note to Table 173.

†Smaller than the value that would be fallen short of once in twenty times by chance.

once again we get one severe depression following upon another. These difficulties are not necessarily fatal to the chronological scheme we have tested, for it may be better to select severe depressions from small clusters of cycles than from an array covering all cycles. But the difficulties we have recited at least invite caution. It is wiser to wait until the researches now in process yield authentic ratings of business depressions before expending much effort in elaborating the hypothesis that the periods separating severe depressions are genuine cyclical units.

VII Conclusions and Plans for Later Work

From the tests in this chapter we cannot draw any far-reaching conclusions. In the first place, we have analyzed only a small sample of time series and tested only a few hypotheses. In the second place, our technical methods are rough. We have made no allowance for substantial leads or lags that might possibly characterize cyclical variations in the specific cycles of different activities; our probability tests are approximate at best; and our techniques are insensitive to the possible coexistence of several cyclical swings, each with a different period, in business cycles. In view of these limitations we are in no position to say whether business cycles have or have not varied cyclically.

But that basic problem is also beyond the scope of this chapter. Our analysis was designed to determine whether there is any clear presumption for the belief that business and specific cycles have varied substantially according to their position within the long cycles dated by different investigators, rather than whether business cycles are grouped into genuine higher units. We have not found substantial variations within the alleged long cycles, except in the so-called 'Juglar cycles' and the periods explicitly marked off by dates of severe depressions. Even in these periods the systematic variations between business cycles in the different groups that we distinguished seem less impressive than their common features, the systematic differences found in the averages do not invariably mark each of the alleged long cycles, and their dates are surrounded by uncertainties independent of our results. We therefore conclude that strong reasons do not now exist for organizing cyclical measures on the assumption that business cycles have varied cyclically within the periods suggested by any of the hypotheses of long cycles that we have noted. Both the studies in this chapter and our general knowledge of economic time series suggest that if cyclical changes have occurred in business cycles, they cannot have been so pronounced and dominating as to jeopardize the value of the approximate descriptions of cyclical behavior afforded by our over-all averages.

There is a sharp contrast between chronologies of business cycles and chronologies of long cycles. In Chapter 4, Section VI we found that statistical investigators, even when they have used widely different samples of time series and different techniques of measurement, have drawn up similar chronologies of business cycles. That result would be unlikely if business cycles failed to leave a clear imprint on statistical records of important economic activities. Since statistical investigators of long cycles, as we have seen in this chapter, reach widely divergent results, it seems reasonable to infer that cycles in general business activity of a higher order than business cycles are probably far from being a clear or pronounced feature of economic development.

These preliminary conclusions must suffice for the present. At a later time they will be tested thoroughly. As already stated, one of the main problems to be explored in the concluding volume of this series is whether business cycles have been subject to secular, structural or cyclical changes as well as to the haphazard changes that everyone recognizes. When we undertake that task we shall probably find that we need new cyclical measures as well as additional time series. Such things are learned best in working on a problem intensively, but it is also desirable in the course of current work to assemble materials that seem likely to facilitate later studies. In the preceding chapter we explained how we modify our standard technique to take account of secular changes in cyclical behavior that are prominent in some series. We shall now add a few illustrations of how we provide materials concerning cyclical changes in cyclical behavior.

In passing from one economic factor to another, we make an effort to note long cycles whenever they appear clearly in the raw data of individual series. We mark off these cycles, and measure their duration and amplitude, in the same manner as the specific cycles. We also set out their relation in time to the shorter cycles in which our interest centers, and contrast the behavior of specific cycles during the rise and fall of the long cycles. Tables 161-163 illustrate some of these steps. We have analyzed on this general plan railroad stock prices and number of shares traded, two series in our present sample; also the American series on real estate trading, immigration, and numerous series on building construction. All these activities show specific cycles bunched in long cycles averaging fifteen to twenty years, though the timing of the long cycles differs from series to series. In prices of commodities at wholesale and long-term interest rates we find much longer and seemingly more doubtful major cycles. We have not attempted to measure the major cycles in these series, but have found it instructive to take separate averages of their specific and reference cycles for periods of rising and falling 'trends', as well as averages in which the rising and falling 'trends' are allowed to neutralize each other. Whether or not these materials will put us on the path of true cyclical changes in business cycles, they are sure to help us understand better than we do now why no two business cycles in actual life have ever run exactly the same course.

CHAPTER 12

Stable and Irregular Features of Cyclical Behavior

THE TWO preceding chapters suggest that irregular changes in cyclical behavior are far larger in scope than secular or cyclical changes. This tentative finding will not be questioned by students who believe that it is vain to strive after a general theory of crises, or depressions, or business cycles. Their argument is that each of these episodes must be explained by the peculiar combination of conditions prevailing at the time, and that these combinations differ endlessly from one another. Students starting from this theoretical position may see little value in statistical averages of the sort we use to describe cyclical behavior. If the episodic features of historical business cycles are the thing of real importance, averages that conceal the episodic movements are futile if not mischievous constructions—as futile and mischievous as general theories of business cycles. In this chapter we attempt to indicate to what extent averages are justifiably subject to such criticisms.

I Individual Features of Successive Cycles

Every realistic investigator recognizes that business activity at any time is influenced by countless 'random' factors. Some arise from the processes of nature, some from agreements or quarrels among men, some from changes in governmental policy, some from discoveries or inventions—the list of sources is indefinitely long. Some random factors influence directly only a single enterprise; e.g., a new method of keeping accounts, a fire, or the loss of a key employee. Others impinge directly on a whole industry or locality; e.g., unionization of employees, a change in fashion, a new building code, or tariff, or tax on real estate. Still others influence all or most business enterprises of a country, such as a change in the currency system, a war, a general strike. Some 'disturbances' pass quickly, others linger for several years; some exercise a slight and some a potent influence; some affect agents that 'respond' quickly, others affect agents that 'respond' slowly; some tend to stimulate, others to depress business activity, while many are stimulating in certain directions and depressing in others. A cyclical expansion in general business may be curtailed by 'unfavorable' random factors potent enough to overcome the cyclical forces making for further expansion—such as the respending of incomes that are themselves increasing, the accumulation of stocks necessitated by a growing volume of business, new investments in plant fostered by expanding markets and general optimism. But a cyclical expansion may be curtailed also by 'favorable' random factors that intensify business activity and so accelerate the development of whatever stresses bring on a recession.

Each specific and business cycle is therefore an individual differing in countless ways from every other. But what features of a business or specific cycle are 'peculiar', and the ways in which they are 'peculiar', cannot be determined without reference to some 'norm'. Those who accept an episodic theory of business cycles cannot escape having notions of what is 'usual' and what is 'unusual' about any given cycle. By striking averages

TABLE 176

Cyclical Measures of Pig Iron Production and Prices
United States, 1879–1933

Dates of reference cycles	Number of months lead (−) or lag (+) of specific-cycle turns in				Corresponding specific cycles in iron production								
	Iron production at reference		Iron prices[a] at reference		Duration in months			Amplitude[b] of			Per month amplitude[b] of		
Upturn–Peak–Trough	P[c]	T[c]	P[c]	T[c]	Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Mar.79		−2											
Apr. 79–Mar. 82–May 85	+11	−4	...	...	49	23	72	64.0	24.6	88.6	1.3	1.1	1.2
June 85–Mar. 87–Apr. 88	+7	−1	...	...	33	5	38	64.8	25.5	90.3	2.0	5.1	2.4
May 88–July 90–May 91	−2	−1	...	...	26	11	37	50.2	47.4	97.6	1.9	4.3	2.6
June 91–Jan. 93–June 94	−11	−8	...	+9	10	20	30	45.4	63.7	109.1	4.5	3.2	3.6
July 94–Dec. 95–June 97	−1	−8	−2	0	25	11	36	87.2	60.8	148.0	3.5	5.5	4.1
July 97–June 99–Dec. 00	+6	−2	+6	−2	38	10	48	76.0	30.5	106.5	2.0	3.0	2.2
Jan. 01–Sep. 02–Aug. 04	+9	−8	+2	−1	32	6	38	49.0	50.0	99.0	1.5	8.3	2.6
Sep. 04–May 07–June 08	+2	−5	−4	+11	43	6	49	73.6	62.7	136.3	1.7	10.4	2.8
July 08–Jan. 10–Jan. 12	0	−13	−2	+1	24	11	35	78.6	41.7	120.3	3.3	3.8	3.4
Feb. 12–Jan. 13–Dec. 14	0	0	0	+4	25	23	48	45.0	56.8	101.8	1.8	2.5	2.1
Jan. 15–Aug. 18–Apr. 19	+1	+1	−13	+3	45	8	53	61.9	40.3	102.2	1.4	5.0	1.9
May 19–Jan. 20–Sep. 21	+8	−2	+8	+6	16	10	26	40.7	91.1	131.8	2.5	9.1	5.1
Oct. 21–May 23–July 24	0	0	−8	+13	22	14	36	108.3	58.6	166.9	4.9	4.2	4.6
Aug. 24–Oct. 26–Dec. 27	−3	−1	−7	+7	24	16	40	36.5	18.7	55.2	1.5	1.2	1.4
Jan. 28–June 29–Mar. 33	+1	0	0	0	20	44	64	50.0	148.9	198.9	2.5	3.4	3.1
Average	+2	−3	−2	+4	29	15	43	62.1	54.8	116.8	2.4[d]	4.7[d]	2.9[d]

[a] Weighted average of the prices of four leading grades of pig iron, derived from publications of the Bureau of Labor Statistics (*Bulletin* through 1931, *Wholesale Prices* thereafter).

[b] In specific-cycle relatives.

[c] P stands for peaks, T for troughs.

[d] Unweighted average.

of cyclical behavior we make such notions more definite. Averages provide standards for judging individual cycles and thus can be no less useful to the episodic theorist than to students who believe that business cycles have enough common features to admit of a general explanation.

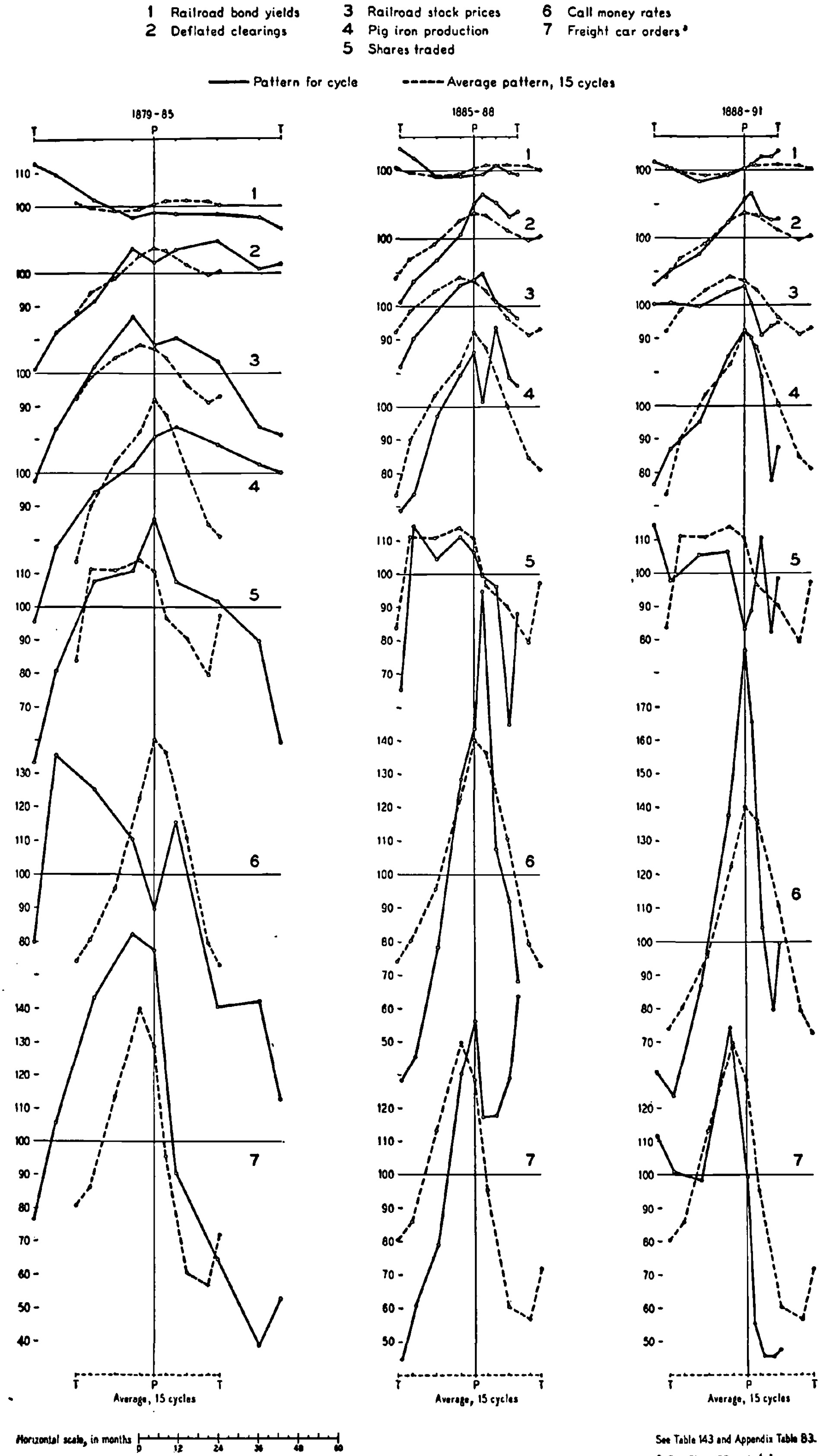
How averages help to identify the peculiarities of successive cycles is simply illustrated by the cyclical measures for the American iron industry in Table 176. The cyclical contraction in iron output during 1907–08 was short but violent, the rate of decline exceeding any other contraction in the period covered. The expansion during the War cycle of 1914–19 was exceptionally long but of average amplitude; hence the rate of rise was well below average. The rise in 1924–26 was unusually small for this volatile activity; it was accompanied by one of the briefest rises in pig iron prices on record, and was followed by a cyclical decline in output much milder than usual but about average in duration. The contraction of 1929–33 was the longest and largest on record, but the rate of decline was below average. The upturn in 1933 coincided with upturns in iron prices and general business, in contrast to past cycles when it usually led the former by substantial and the latter by somewhat shorter intervals.

Chart 73 presents a more elaborate illustration. It shows the reference cycles of the seven series discussed in preceding chapters, cycle by cycle, against a background of averages for the fifteen cycles from 1879 to 1933. The average patterns are so plotted in relation to the patterns for single cycles that the standings of the two at the reference peak are vertically aligned. Of course, the chart contains too few series to convey an adequate notion of the special characteristics of successive business cycles. Nevertheless, it illustrates vividly how averages help to spot idiosyncrasies. For example, we see that the rise of the business cycle of 1879–85 was exceptionally long; apparently it was also of larger amplitude than usual and was marked by irregularly declining interest rates; it culminated in a peak level maintained for some months, was followed by a very long contraction, and experienced a money market panic towards the close of the contraction.[1] By contrast the cycle of 1900–04 seems 'normal', but it too has numerous distinctive features. The stock exchange panic of May 1901, associated with the 'corner' in Northern Pacific stock, is reflected in the patterns of both call money rates and share trading. The amplitude of this cycle was milder than average in series relating to the physical volume of business, but larger than average in call money and share trading. Further, contraction seems to have lasted slightly longer than expansion, whereas the opposite relation is the rule. Dozens of peculiarities of other cycles stand out in the chart: the long expansion of 1914–18,

1 The panic came in May 1884. On the chart it appears merely as a faint rise between stages VII and VIII in call money rates. The standings at these stages are averages based on numerous months, and nearly iron out this violent financial episode. This limitation of the cyclical patterns must be borne in mind; they are an efficient tool for extracting meaning from the raw data, but neither they nor anything else can serve as a substitute for study of the raw data (see Chart 31).

CHART 73

Patterns of Successive Reference Cycles Compared with Their Average Pattern, 1879–1933
Seven American Series



See Table 143 and Appendix Table B3.

* See Chart 58, note 'a'

CHART 73 (CONT.)

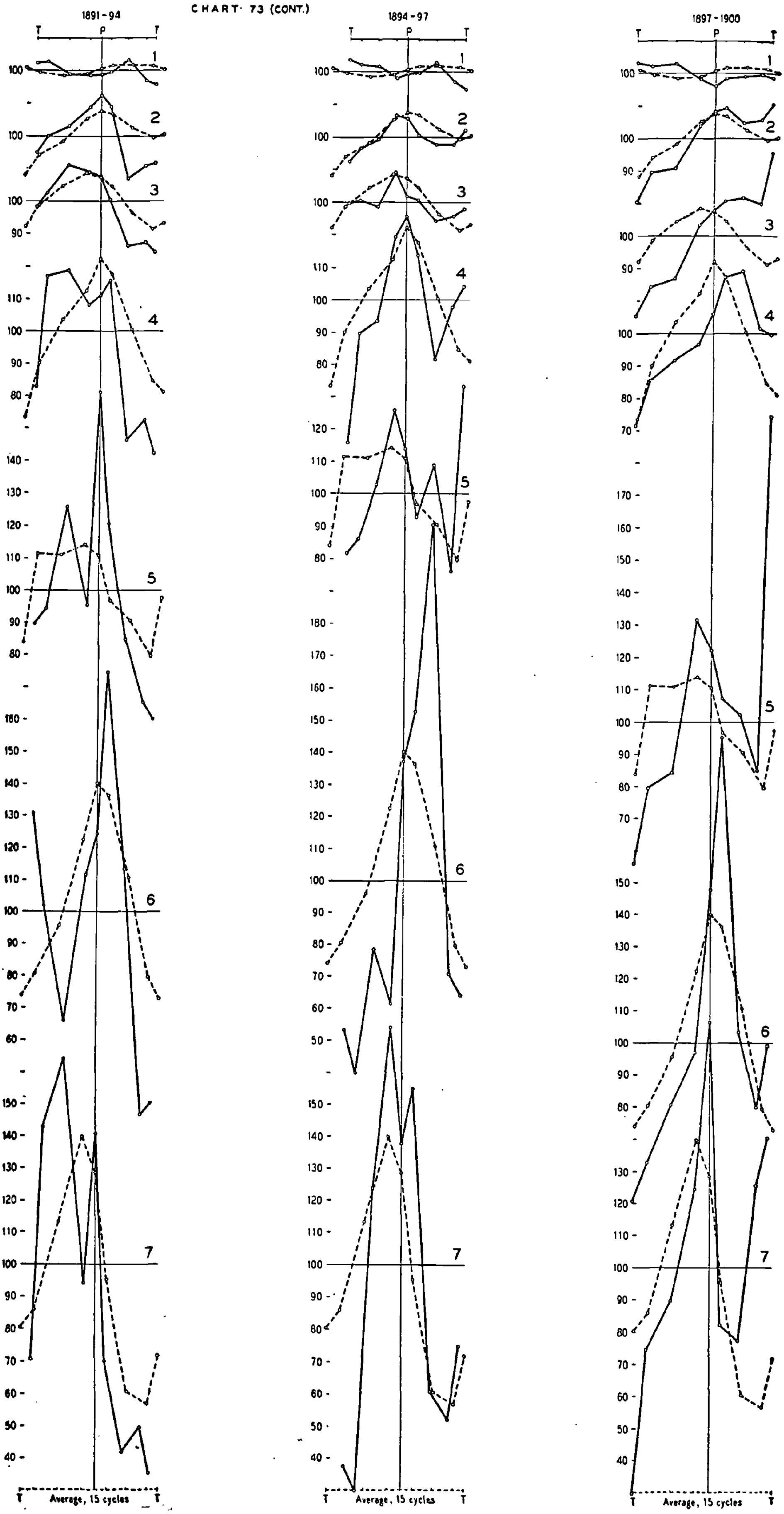


CHART 73 (CONT.)

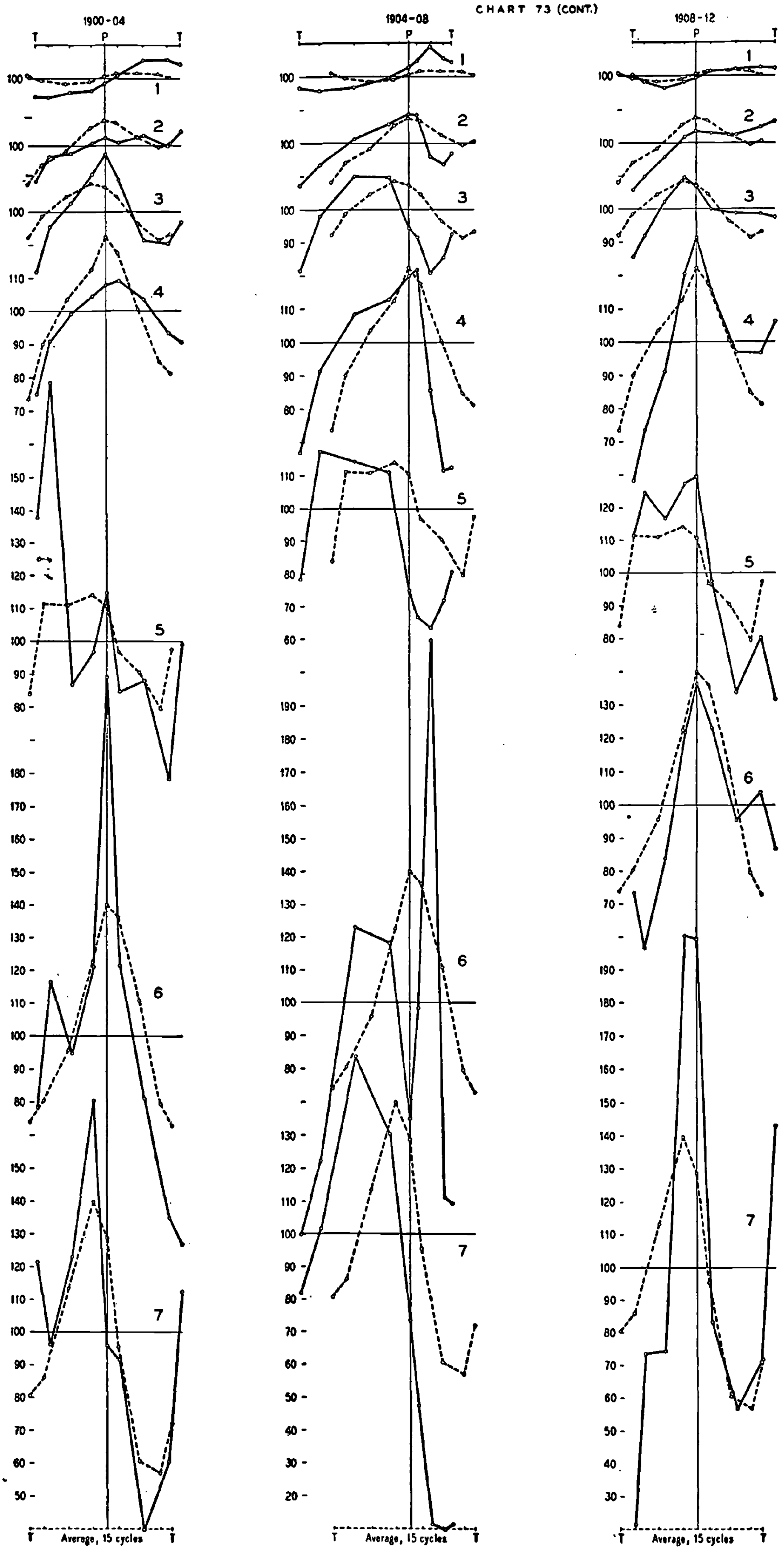
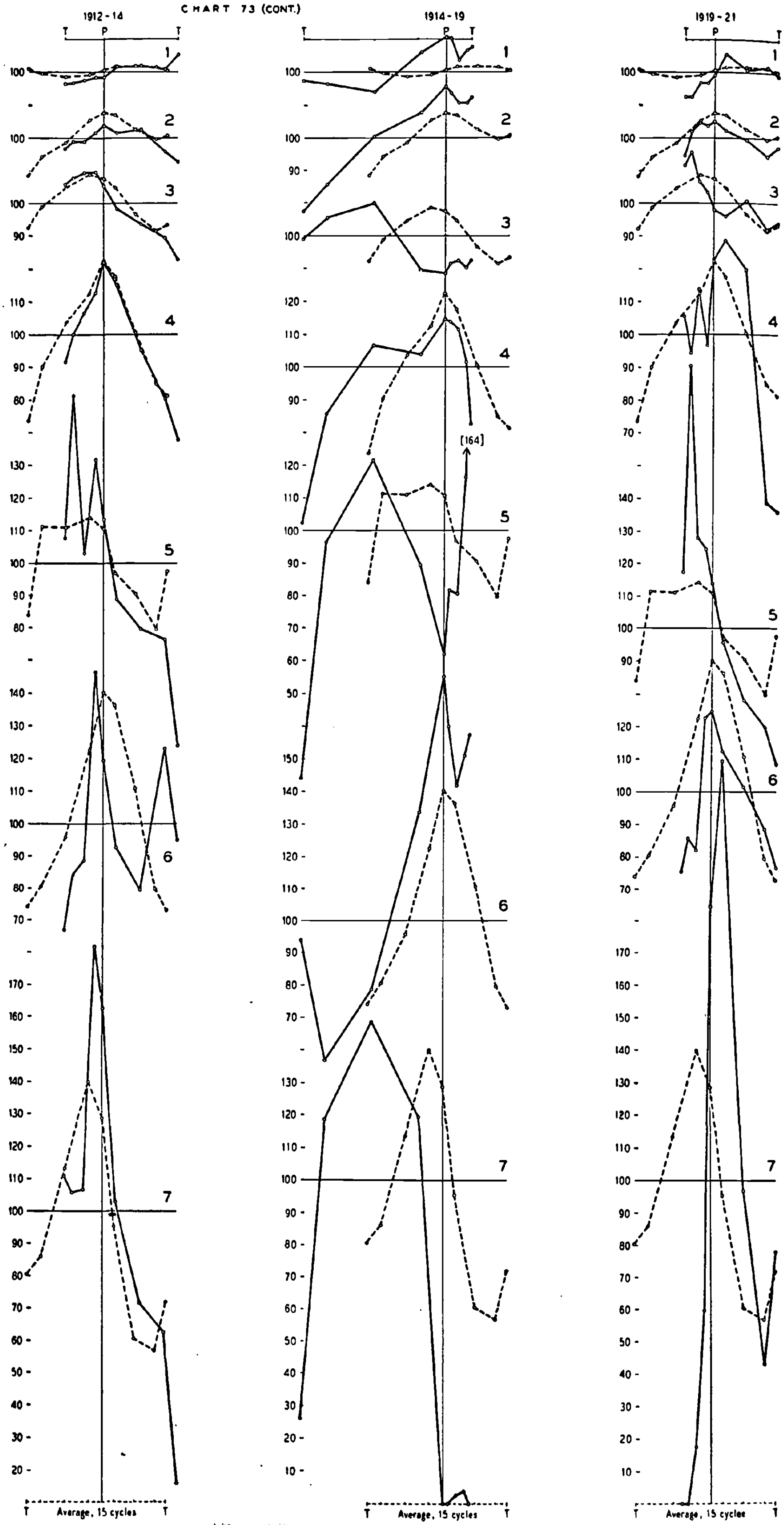
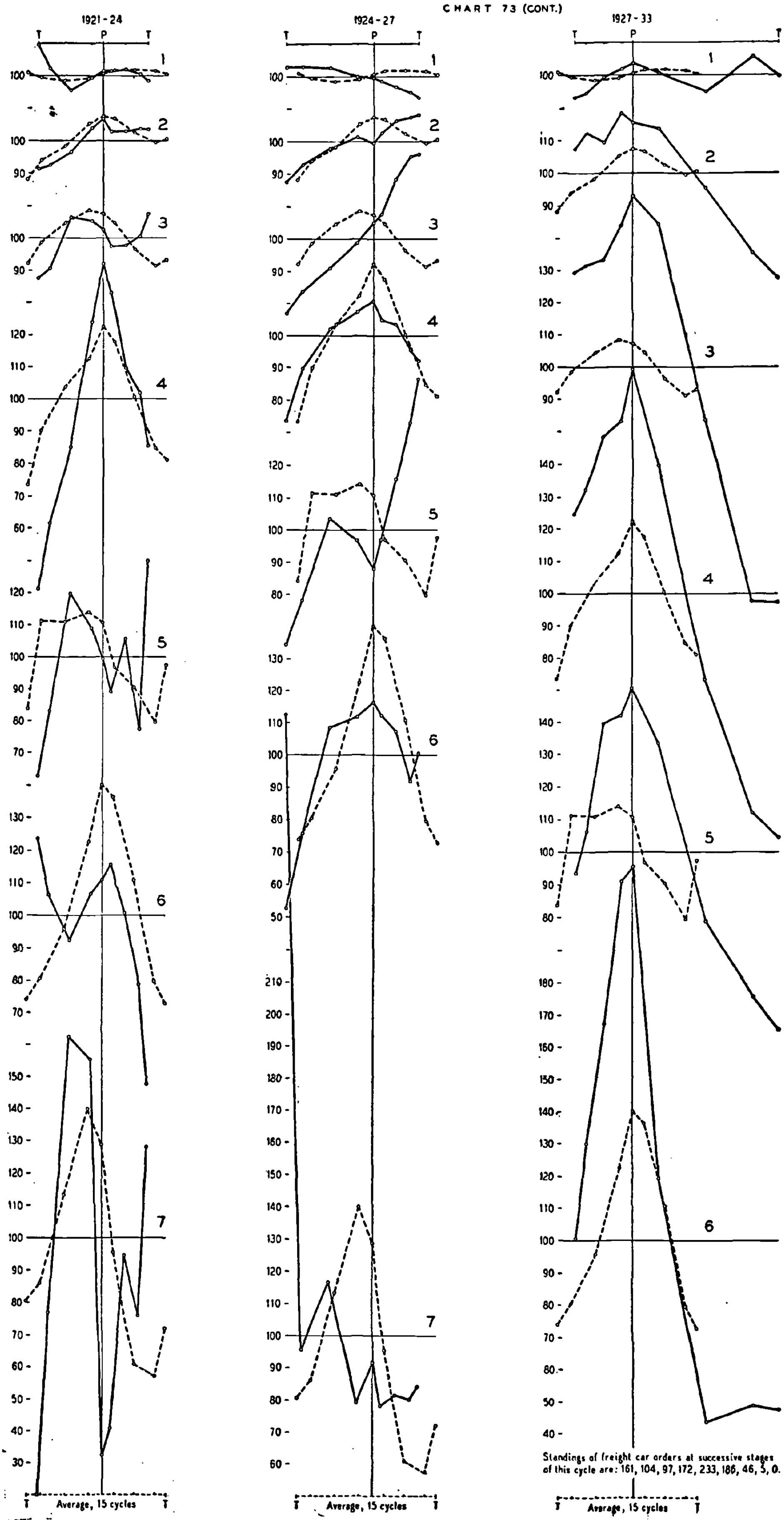


CHART 73 (CONT.)




1921-24
1924-27
1927-33
T
P
T
1
2
3
4
5
6
7
Average, 15 cycles
Standings of freight car orders at successive stages of this cycle are: 161, 104, 97, 172, 233, 186, 46, 5, 0.

the brief and moderate decline of 1918–19, the long and violent decline of 1929–33, the concerted downturn of different activities in 1929, the widely scattered downturns at the reference peak in 1918, the accentuated decline in the last stage of the 1913–14 contraction, the money market panic in 1914, the insensitivity of security markets to the contraction of 1926–27, and so on.

We come closer to the continuum of business activity by analyzing reference cycles marked off by peaks as well as by troughs.[2] Chart 74 illustrates the point: it is made like Chart 73 except that the average patterns, which are now based on fourteen cycles instead of fifteen, are plotted so that their standings and those of individual cycle patterns at the reference trough are vertically aligned.[3] By coupling contractions with the following instead of the preceding expansions, the recovery phase of business cycles becomes clearer. Several characteristics of the cyclical movements are also made more prominent. In particular the cycle of 1918–20 appears below average in amplitude, and certainly as the shortest we have recognized. Because the contraction and expansion are so short,[4] share trading and railroad stock prices, which usually and also in this cycle lead in revivals and recessions by substantial intervals, appear as 'positive' movements during this peak-to-peak cycle.[5]

II Stable Features of Successive Cycles

Averages not only provide benchmarks for judging individual cycles; they also indicate roughly what cyclical behavior is characteristic of different activities, and that is their chief value. Of course, a blind use of averages may lead to worthless conclusions. For example, if two related

[2] As yet our standard measurements include only the latter (see pp. 162-3). But the patterns of the present sample of seven series are shown on both a positive and inverted basis, cycle by cycle, in Appendix Tables B3-B4.

[3] It is well to note the dependence of the reference-cycle standings on the base. The reference-cycle standings of single cycles in Chart 74 differ from corresponding standings in Chart 73 because they are computed on different bases; the average patterns differ for this reason and also because Chart 73 includes one more cycle than Chart 74. Of course, a rise between adjacent standings computed on one base will appear as a rise on any other base, and so will a fall. But a rise between adjacent *average* standings computed on one set of bases may appear as a fall when the average standings are computed on another set of bases. Thus the average pattern of call money rates reaches a peak at the same time as the reference peak in Chart 73 but during the first third of contraction in Chart 74. This difference could arise either from the additional cycle covered by Chart 73 or from the different bases employed in the two charts. The former happens to be responsible; when the additional cycle is eliminated from the averages in Chart 73, the fall from the peak to the first third of contraction is replaced by a rise.

Still another factor affects one discrepancy between the patterns of freight car orders in the two charts. The reference expansion in 1924–26 is based in Chart 73 on figures from the *Iron Trade Review* and in Chart 74 on Partington's compilations. See Ch. 9, note 2.

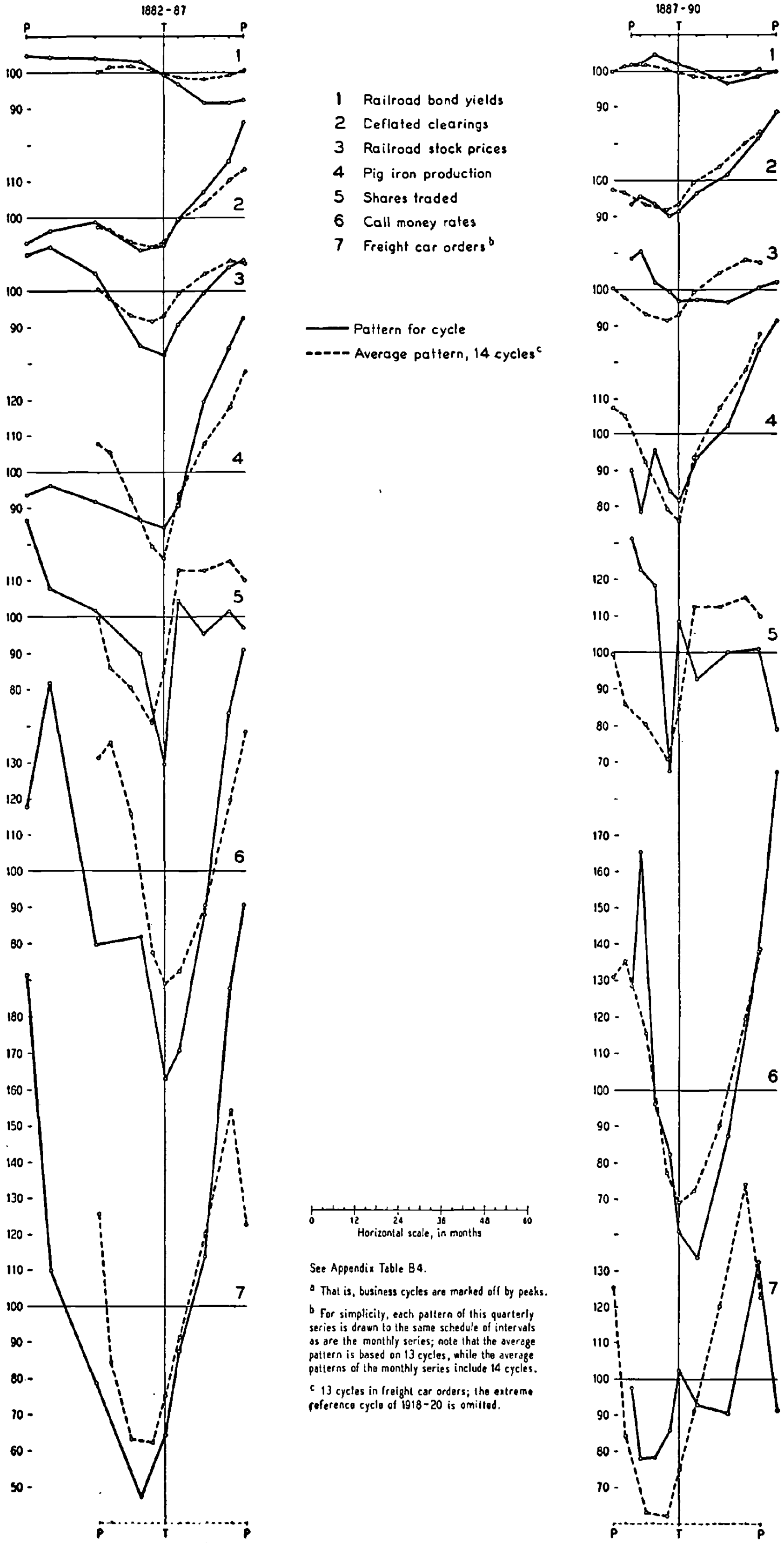
[4] See, however, p. 194.

[5] For a case worked out in detail by the methods illustrated in this section, see our *Bulletin 61*, in which fluctuations in the production of numerous industries during the American business cycle of 1927–33 are compared with averages of preceding cycles.

CHART 74

Patterns of Successive Reference Cycles Compared with Their Average Pattern, 1882–1929
Seven American Series

(Patterns made on inverted basis[a])

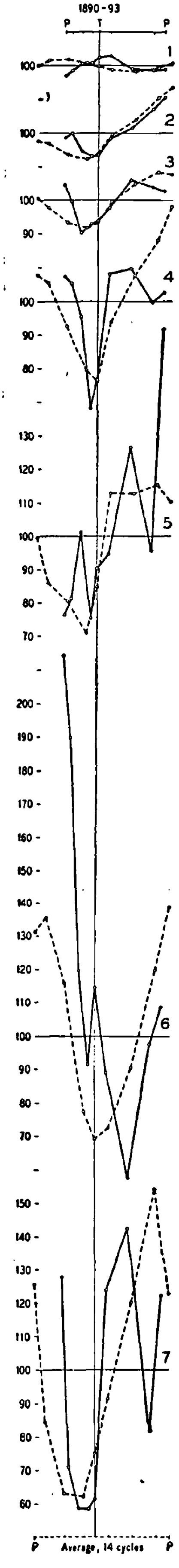


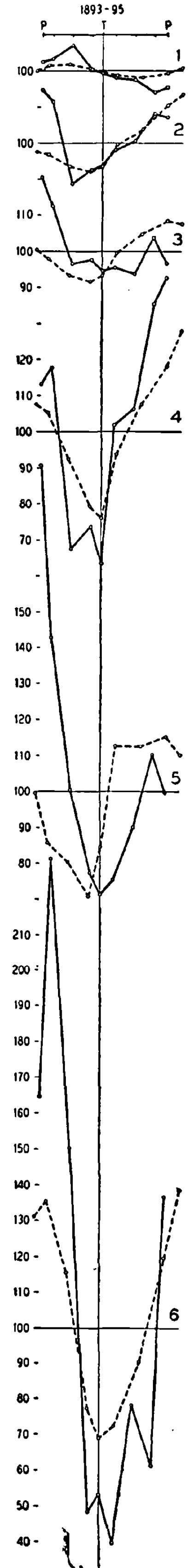
See Appendix Table B4.

[a] That is, business cycles are marked off by peaks.

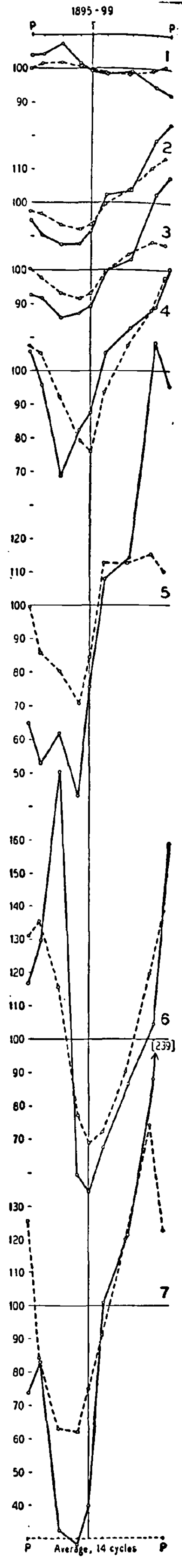
[b] For simplicity, each pattern of this quarterly series is drawn to the same schedule of intervals as are the monthly series; note that the average pattern is based on 13 cycles, while the average patterns of the monthly series include 14 cycles.

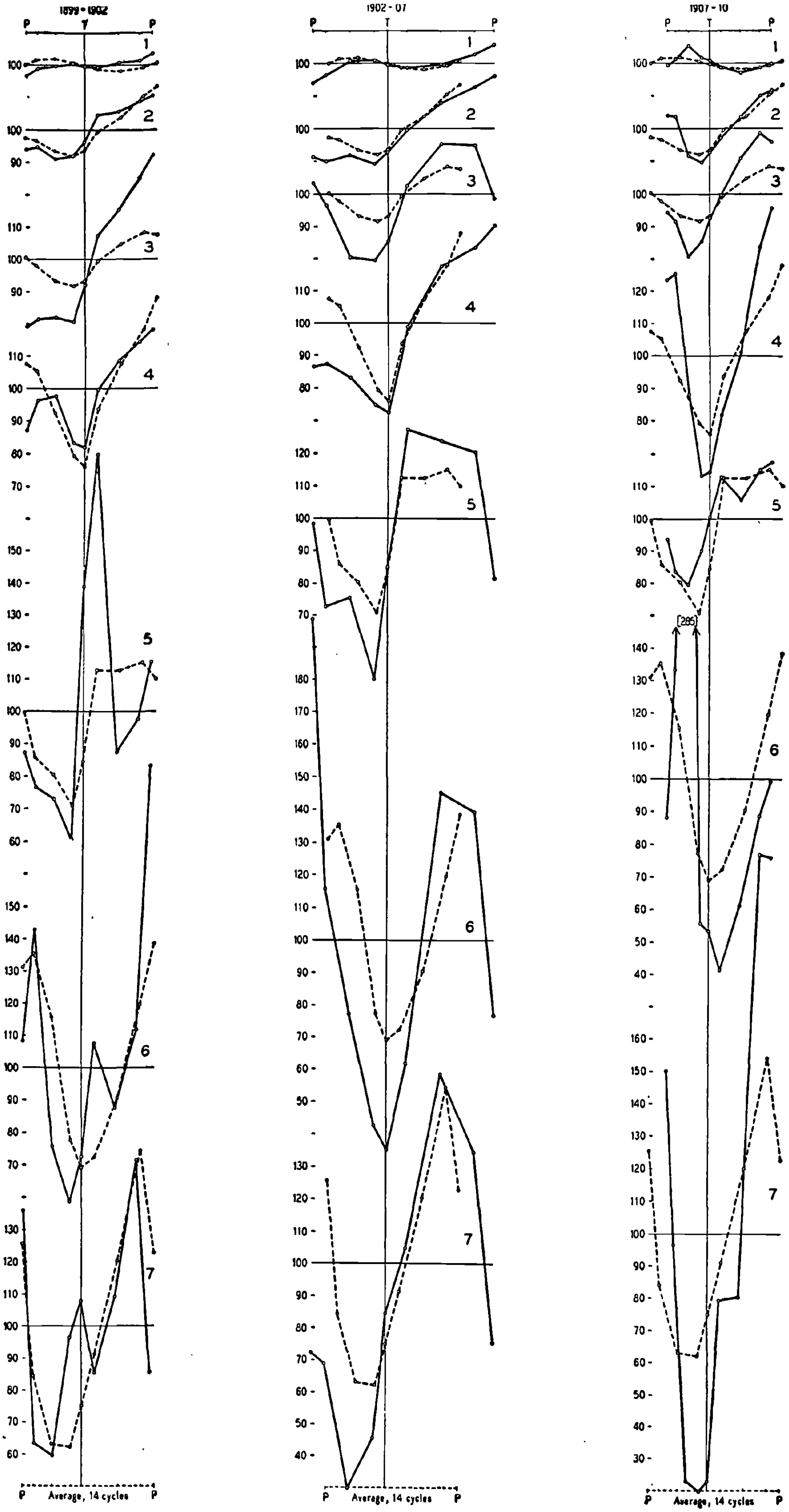
[c] 13 cycles in freight car orders; the extreme reference cycle of 1918–20 is omitted.

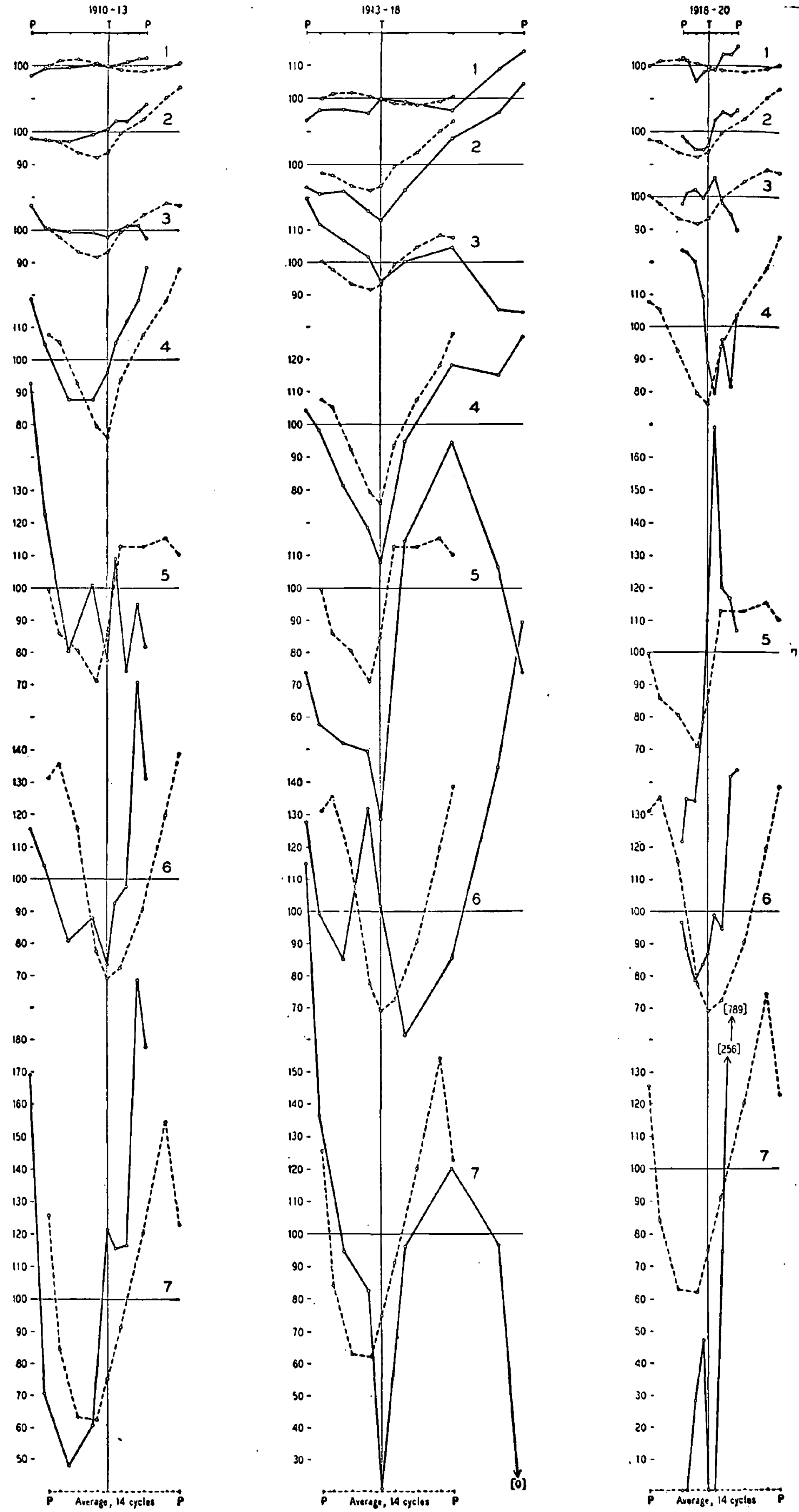


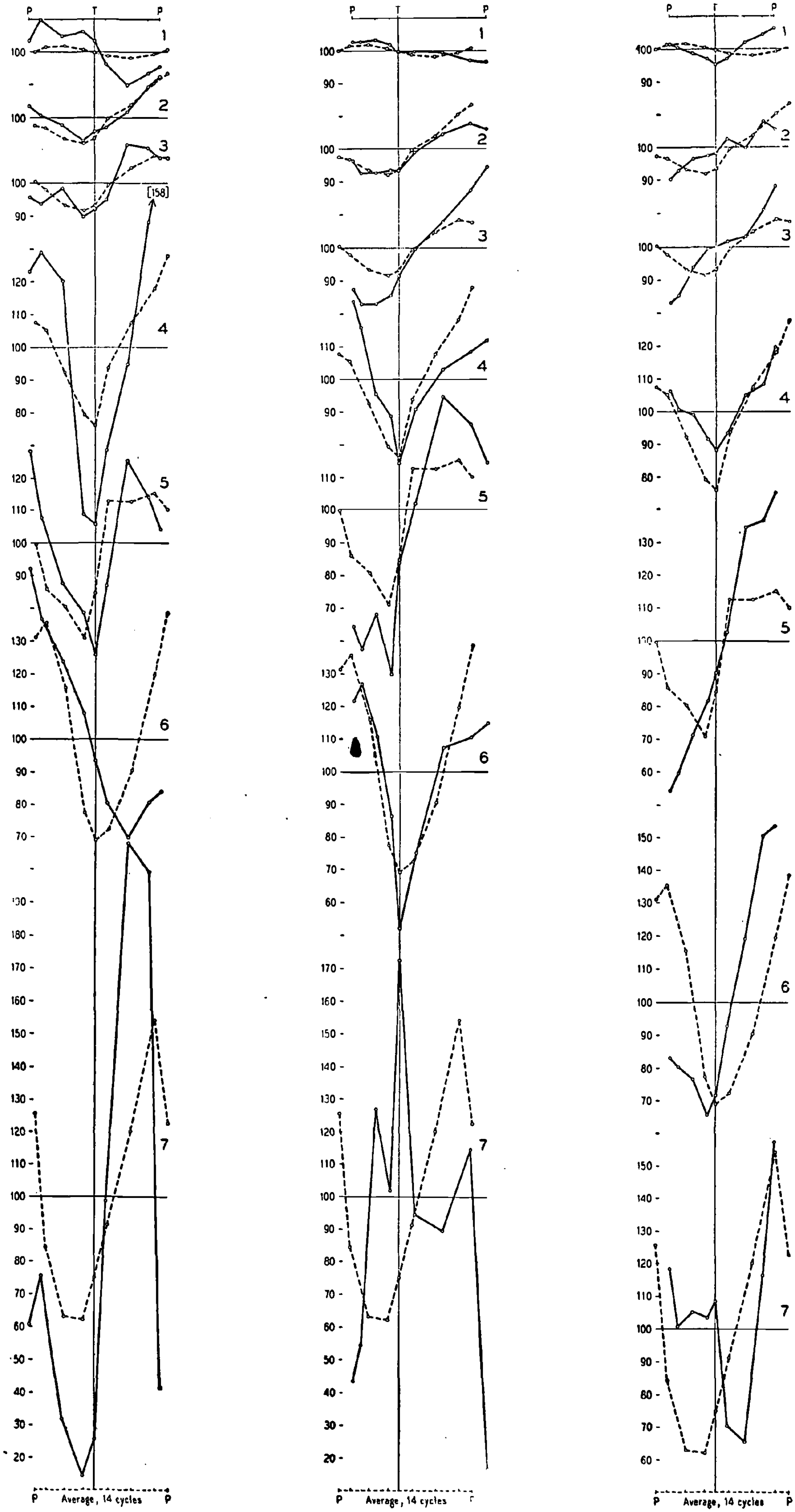


Standings of freight car orders at successive stages of this cycle are: 177, 88, 53, 63, 44, 35, 147, 206, 163.




1899-1902
1902-07
1907-10
P
T
P
1
2
3
4
5
6
7
[285]
P Average, 14 cycles P
P Average, 14 cycles P
P Average, 14 cycles P


1910 - 13
1913 - 18
1918 - 20
P
T
1
2
3
4
5
6
7
[789]
[256]
[0]
Average, 14 cycles



P T P
Average, 14 cycles
[158]

activities have identical amplitudes in nineteen business cycles and widely different amplitudes in one business cycle, averages covering all twenty cycles would be misleading, as would also deviations of individual cycles from the averages. Again, an investigator who finds that the average timing of two series is approximately the same might infer that their characteristic cyclical timing is really the same, in disregard of a persistent lag of one series during all but one or two freakish cycles. Clearly, averages alone may cause an investigator to declare at times that a persistent difference exists in cyclical behavior when none exists in fact, and to declare at other times that no difference exists when ones does exist in fact.

These dangers are real and it is important to stress them. But it is even more important to recognize that averages, if used judiciously, enable us to describe what cyclical behavior has been characteristic 'in the long run' of different factors in our business economy. Our studies in Chapters 10 and 11 have yielded little evidence that secular, structural, or cyclical changes have impressed their influence strongly on the cyclical behavior of single activities or business as a whole. On the other hand, a great deal of evidence exists that random factors constantly influence business activities. Now and then we find secular, or discontinuous, or cyclical changes in the cyclical behavior of single series; but when that happens we restrict averages to periods or groups of cycles that seem homogeneous. Hence we provisionally regard our averages as rough representatives of the cyclical movements characteristic of individual activities and the average deviations as rough representatives of the variability of cyclical measures around fairly stationary means. So far as certain features are peculiar to the individual cycles of a series, they tend to fade out in averages; that is, the averages tend to emphasize the effects common to most cycles in the group.

Charts 73 and 74 teach more than that the behavior of each series differs considerably from cycle to cycle. Another and no less important lesson is that despite these variations the reference cycles of the same series bear a family resemblance, while the reference cycles of different series vary in characteristic ways. What happens upon striking averages for groups of specific or reference cycles is indicated simply by the charts in Chapters 10 and 11. The story is almost always the same: the idiosyncrasies of individual cycles tend to vanish, the average patterns of the same series look much alike in different samples of cycles, the patterns of different series become sharply differentiated, and the relations among the series persist with great regularity from one sample of cycles to the next. This tendency of individual series to behave similarly in regard to one another in successive business cycles would not be found if the forces that produce business cycles had slight regularity.

Table 177 shows how stable are the relations among the amplitudes of

TABLE 177
Amplitude of Successive Reference Cycles
Seven American Series, 1879–1933

Business cycle[a]	Railroad bond yields (III-VI)	Deflated clearings (VIII-V)	Railroad stock prices (VIII-IV)	Shares traded (VIII-IV)	Pig iron production (I-V)	Call money rates (I-V)	Freight car orders (VIII-IV)
			Rise & fall in reference-cycle relatives				
1879–1885	+7.5	+39.1	+90.6	+86.4	+66.3	+66.8	+247.6
1885–1888	+6.2	+34.4	+29.1	+68.9	+57.4	+180.6	+98.7
1888–1891	+4.5	+33.3	+11.1	+59.3	+81.2	+213.3	+149.7
1891–1894	+6.1	+39.4	+33.5	+50.6	+76.8	+78.0	+71.5
1894–1897	+3.0	+21.9	+19.7	+87.1	+91.5	+157.2	+243.1
1897–1900	−6.3	+33.7	+23.1	+146.3	+41.1	+146.0	+102.6
1900–1904	+1.6	+19.6	+60.5	+75.3	+50.4	+302.7	+172.1
1904–1908	+12.2	+40.4	+58.4	+103.9	+110.4	+60.8	+207.1
1908–1912	+3.0	+19.3	+41.7	+75.1	+98.3	+112.1	+310.8
1912–1914	+4.4	+16.9	+22.5	+47.4	+85.0	+76.6	+245.4
1914–1919	+14.5	+40.6	−18.1	+20.9	+94.5	+98.7	+132.5
1919–1921	+27.9	+22.6	+5.9	+95.4	+94.1	+97.1	+75.5
1921–1924	+8.7	+21.7	+24.4	+65.5	+157.4	+50.8	+223.9
1924–1927	−5.5	+3.9	+0.2	+22.3	+55.8	+79.4	+7.3
1927–1933	+20.4	+48.5	+131.1	+143.0	+189.5	+282.8	+186.5
			Rank of rise & fall				
1879–1885	1	2	6	5	3	4	7
1885–1888	1	3	2	5	4	7	6
1888–1891	1	3	2	4	5	7	6
1891–1894	1	3	2	4	6	7	5
1894–1897	1	3	2	4	5	6	7
1897–1900	1	3	2	7	4	6	5
1900–1904	1	2	4	5	3	7	6
1904–1908	1	2	3	5	6	4	7
1908–1912	1	2	3	4	5	6	7
1912–1914	1	2	3	4	6	5	7
1914–1919	2	4	1	3	5	6	7
1919–1921	3	2	1	6	5	7	4
1921–1924	1	2	3	5	6	4	7
1924–1927	1	3	2	5	6	7	4
1927–1933	1	2	3	4	6	7	5
Average for							
1879–1933	1.2	2.5	2.6	4.7	5.0	6.0	6.0
1885–1914	1.0	2.6	2.6	4.7	4.9	6.1	6.2

Roman numerals after the titles indicate what stages of the reference cycles are matched with reference expansion, the remaining stages being matched with reference contraction. Concerning the meaning of the signs, see note 7.
[a] Years of the initial and terminal troughs of the monthly reference cycles (Table 16).

our seven series during the business cycles from 1879 to 1933. The amplitudes are computed from reference-cycle standings in the terminal stages assigned to expansion and contraction, as given in our standard Table R4.[6] For example, in call money rates we match stages I-V with reference expansions and stages V-IX with reference contractions; hence the amplitude of the joint rise and fall of a given reference cycle is the excess of the standing in stage V over the standing in I plus the excess of the standing in

[6] See p. 174 and Table 47.

stage V over the standing in IX.[7] Once the reference-cycle amplitudes of the several series have been computed, we rank them in each cycle by assigning a value of 1 to the smallest and 7 to the largest. Now, if the rankings were purely fortuitous, the mean ranks of the individual series would tend towards equality; but it is plain that the rankings of the series tend to persist from cycle to cycle. By applying a mathematical test devised by Friedman for ranked two-way distributions, we find that the probability that the mean ranks of our series would differ on account of chance factors by more than the observed amount is less than one in a million.[8]

These results are corroborated by specific-cycle measures. The reference-cycle measures in Table 177 have the advantage that they can be computed for every business cycle covered; but they fail to take account of departures from that timing which we consider usual in the series, and thus understate by varying margins the full amplitudes of the specific cycles. The amplitude measures supplied by our standard Table S2 have opposite advantages and disadvantages: they show the full amplitudes of the specific cycles but these cycles do not correspond invariably to business cycles. The latter disadvantage is minimized in Table 178 which is restricted to 1885–1914, a period when the specific cycles of all series except railroad bond yields bear one-to-one correspondence to business cycles. The rankings of the specific-cycle amplitudes of the several series turn out to be even more stable from one business cycle to another than do the rankings of the reference-cycle amplitudes. It is also notable that the ranks of the two sets of amplitudes match fairly well. The principal discrepancies are the lower rank of iron production and the higher rank of call money rates in the specific-cycle than in the reference-cycle measures.

In Table 179 we contrast cyclical amplitudes and cyclical durations. The table arrays averages based on four samples of specific cycles in each of our seven series. The arrays make it possible to perform an experiment. Our concern is, let us say, with the amplitudes of full specific cycles. We select at random any one of the four averages for bank clearings, then do the same for iron production and the other series. It is plain that no matter what seven averages are drawn, bond yields will occupy the first

7 It will be noticed that a few entries are negative. A negative entry usually means inverse conformity and a plus entry positive conformity to full business cycles, as defined in Ch. 5, Sec. IX-X. But this inference may prove false when the reference-cycle 'rise' and 'fall', as recorded in col. (2) and (5) respectively of our standard Table R4, are both plus or both minus; for while the 'rise & fall' shown in Table 177 is computed directly from the 'rise' and 'fall', our conformity measure is derived from the 'per month' figures.

8 See Milton Friedman, The Use of Ranks to Avoid the Assumption of Normality Implicit in the Analysis of Variance, *Journal of the American Statistical Association*, Dec. 1937. Friedman's method involves the computation of a statistic that tends to be distributed as 'chi-square' with $(p-1)$ degrees of freedom, where p is the number of ranks. This statistic comes out 68.8 for the ranks in Table 177. For six degrees of freedom the probability of a value of 'chi-square' greater than 68.8 is .000000.

TABLE 178

Amplitude of Specific Cycles Corresponding to Successive Business Cycles
Seven American Series, 1885–1914

Business cycle[a]	Railroad bond yields	Deflated clearings	Railroad stock prices	Pig iron production	Shares traded	Freight car orders	Call money rates
			Rise & fall in specific-cycle relatives				
1885–1888	14.4	43.1	45.0	90.3	145.0	209.0	329.2
1888–1891	16.3	36.5	23.2	97.6	88.9	228.6	284.9
1891–1894	23.8	42.9	48.0	109.1	209.9	283.7	351.6
1894–1897	21.9	31.7	31.3	148.0	129.3	223.0	369.8
1897–1900	18.7[b]	41.8	50.5	106.5	237.2	295.0	280.1
1900–1904		27.1	76.6	99.0	358.6	290.8	322.0
1904–1908	24.4	51.8	82.0	136.3	197.4	402.6	421.9
1908–1912	15.2[c]	34.3	53.0	120.3	123.3	334.5	148.7
1912–1914		23.0	33.4	101.8	181.2	369.2	185.3
			Rank of rise & fall				
1885–1888	1	2	3	4	5	6	7
1888–1891	1	3	2	5	4	6	7
1891–1894	1	2	3	4	5	6	7
1894–1897	1	3	2	5	4	6	7
1897–1900	1[d]	2	3	4	5	7	6
1900–1904	1[d]	2	3	4	7	5	6
1904–1908	1	2	3	4	5	6	7
1908–1912	1[d]	2	3	4	5	7	6
1912–1914	1[d]	2	3	4	5	7	6
Average....	1.0	2.2	2.8	4.2	5.0	6.2	6.6

The turning points of the specific cycles are given in Appendix Table B1.
[a]Years of the initial and terminal troughs of the monthly reference cycles (Table 16).
[b]Amplitude of the cycle from June 1899 to Feb. 1905.
[c]Amplitude of the cycle from Feb. 1909 to June 1914.
[d]The amplitude of a specific cycle that corresponds approximately to two business cycles is repeated in deriving ranks.

place, clearings the second place, stock prices the third, iron production the fourth, share trading and call money rates the fifth or sixth, and freight car orders the seventh. Of course, an increase in the number of samples would blur the picture. For example, the average amplitude is larger in call money rates than in freight car orders in the sample of cycles covered by Table 178. Again, the rankings of the per month amplitudes of Table 179 are less stable than the total amplitudes. This tendency of the rankings of different series to shade into one another would become more pronounced if we compared seventy instead of seven series. Hence in dealing with numerous series we must think more often of differences among groups of activities than of differences among single series, and we must think more in terms of orders of magnitude than in terms of exact differentials. Subject to this reservation, average amplitudes are indicators of real differences in the cyclical behavior of economic activities.

Let us now perform a similar experiment on the cyclical durations. Again we select any one of the four averages for bond yields, and do the

TABLE 179

Arrays of Average Durations and Amplitudes of Specific Cycles
Based on Four Samples from Seven American Series

Average duration of full cycles in months		AVERAGE AMPLITUDE IN SPECIFIC-CYCLE RELATIVES: Rise & fall		Per month rise & fall[a]	
Car orders	32	Bond yields	20	Bond yields	0.4
Bond yields	32	Bond yields	23	Bond yields	0.6
Call rates	36	Bond yields	24	Bond yields	0.6
Clearings	37	Bond yields	27	Clearings	0.8
Stock prices	37	Clearings	37	Bond yields	0.9
Call rates	38	Clearings	40	Clearings	0.9
Call rates	38	Clearings	41	Clearings	0.9
Car orders	39	Clearings	42	Clearings	1.1
Car orders	40	Stock prices	52	Stock prices	1.4
Call rates	40	Stock prices	67	Stock prices	1.4
Clearings	41	Stock prices	74	Stock prices	1.4
Shares traded	42	Stock prices	76	Stock prices	1.5
Bond yields	42	Iron output	107	Iron output	2.6
Iron output	43	Iron output	113	Iron output	2.8
Iron output	43	Iron output	117	Iron output	2.9
Iron output	44	Iron output	131	Iron output	3.2
Iron output	44	Shares traded	148	Shares traded	3.6
Shares traded	44	Call rates	187	Call rates	4.5
Clearings	44	Shares traded	190	Shares traded	4.6
Shares traded	44	Shares traded	204	Shares traded	4.9
Bond yields	45	Call rates	218	Shares traded	5.2
Shares traded	46	Shares traded	220	Call rates	5.9
Car orders	48	Call rates	232	Call rates	6.3
Bond yields	49	Call rates	289	Car orders	8.3
Stock prices	49	Car orders	319	Call rates	8.4
Stock prices	50	Car orders	421	Car orders	9.7
Clearings	54	Car orders	425	Car orders	13.2
Stock prices	63	Car orders	518	Car orders	20.4

Derived from Table 140, which gives averages for all cycles in each series, and from Table 145, which gives averages for three subgroups containing an equal or approximately equal number of cycles. The order of the items is based on averages carried to more places than shown here.

[a] Unweighted average.

same for bank clearings and the other series. Suppose that the following averages are drawn: freight car orders 32, railroad bond yields 32, call money rates 36, deflated clearings 37, shares traded 42, pig iron production 43, railroad stock prices 63. From this sample we might conceivably jump to the conclusion that the specific cycles of bond yields and car orders are shorter and the cycles of stock prices longer than those of the other series. But we might have made another drawing of averages: railroad stock prices 37, call money rates 40, pig iron production 43, shares traded 44, deflated clearings 44, railroad bond yields 45, and freight car orders 48. These averages practically reverse the showing of the first set. Careful inspection of Table 179 will demonstrate that by changing the samples we may get almost any result for cyclical durations.

In view of this remarkable contrast between the durations and amplitudes of specific cycles, we make in Tables 180-182 a more comprehensive test of the business-cycle behavior of our seven series. From 1879 to 1933 fifteen business cycles occurred in the United States according to our reference dates. We divide these cycles into groups of five: the first from 1879 to 1897, the second from 1897 to 1914, the third from 1914 to 1933. Table 180 shows averages of the duration, amplitude, and several other characteristics of cyclical behavior for each of these periods, series by series. In no instance are the averages for the three periods identical. But whereas the average durations of full specific cycles of the seven series in one period seem uncorrelated with those in another period, the averages of other cyclical measures tend to repeat the same order. This tendency toward repetition is measured by coefficients of rank correlation in Table 181. The correlation between the ranks of the average durations of specific cycles in the first and second periods is negative; the correlation coefficients between the first and third periods and between the second and third, while positive, are of negligible size and much the smallest in the table. For every other cyclical measure the coefficients are invariably positive. They are remarkably high for specific-cycle amplitudes, also for rates of change during reference cycles and for timing measures at reference peaks. The tendency toward repetition is smaller in other cyclical measures, but it is present also in the timing at reference troughs and even in the durations of expansions and contractions of specific cycles. Variance analysis confirms these results. Whether we include all the cycles in each series or restrict analysis to 1879–1933, which is covered by all seven series, Table 182 shows that the differences among series are 'significantly' larger than the differences within series for every measure except the duration of full specific cycles.[9]

These statistical findings are in harmony with theoretical expectations. The sequence of changes is less uniform in revivals than in recessions, since revivals lack the compulsion that attends business recessions. The durations of expansions and contractions differ significantly among individual activities, since some tend to lead at revivals and lag at recessions, others show opposite relations,[10] while still others usually lead or usually lag at both turns. In general, we should expect that average measures of cyclical timing, amplitude, and rate of movement during reference cycles will differ significantly among major economic processes, while average durations of full specific cycles will not. For the periods of business cycles come to be impressed on the components of the economic

[9] The results are similar when the analysis is confined to briefer periods, that is, 1879–97, 1897–1914, 1914–33; though the picture for the last period is somewhat blurred. This detailed evidence is omitted.

The variance ratios in Table 182 cannot be interpreted unambiguously, not only for the reasons adduced earlier (pp. 392, 397-8, 400-1), but also because the variances of most cyclical measures differ from series to series. It does not seem, however, that the latter factor can seriously limit interpretation; first, because most of the variance ratios are very high, second, because the ratios meet in detail expectations based both on theoretical reasoning and on analysis of the cycle-by-cycle measures.

[10] See pig iron prices (Table 176) for an example; none of our seven series behave in this fashion.

TABLE 180

Average Cyclical Measures Covering Successive Periods of Five Business Cycles and All Fifteen Cycles from 1879 to 1933

Seven American Series

Measure and period[a]	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
AVERAGE DURATION OF SPECIFIC CYCLES[b]							
Expansion							
1879–1897	28	29	22	21	15	15	19
1897–1914	33	32	25	30	15	24	47
1914–1933	39	25	12	31	24	19	17
1879–1933	33	29	19	27	18	19	25
Contraction							
1879–1897	9	14	23	25	31	17	25
1897–1914	10	11	18	15	28	20	13
1914–1933	16	18	19	21	20	19	23
1879–1933	11	15	20	20	26	18	21
Full cycle							
1879–1897	37	43	46	46	46	32	43
1897–1914	44	44	43	44	42	44	60
1914–1933	55	44	32	52	44	37	41
1879–1933	44	43	39	47	44	37	46
AVERAGE AMPLITUDE OF SPECIFIC CYCLES[c]							
Rise							
1879–1897	28	62	151	29	74	131	7
1897–1914	25	64	171	36	108	136	14
1914–1933	27	59	254	36	112	77	13
1879–1933	27	62	199	34	98	117	11
Fall							
1879–1897	12	44	144	25	73	137	12
1897–1914	10	48	167	23	112	135	5
1914–1933	19	72	264	52	93	71	13
1879–1933	13	55	200	32	92	117	11
Rise & fall							
1879–1897	40	107	295	54	148	269	19
1897–1914	36	113	338	59	220	272	19
1914–1933	46	131	518	89	204	148	25
1879–1933	40	117	400	66	190	234	22
Rise per month[d]							
1879–1897	0.9	2.6	9.8	1.4	7.7	11.5	0.4
1897–1914	0.8	2.1	7.2	1.3	9.8	5.7	0.3
1914–1933	0.8	2.6	41.3	1.2	5.3	4.2	0.7
1879–1933	0.8	2.4	22.0	1.3	7.6	7.6	0.5
Fall per month[d]							
1879–1897	1.6	3.8	6.5	1.2	2.5	9.2	0.5
1897–1914	2.9	5.6	11.3	1.7	4.2	8.4	0.4
1914–1933	1.2	4.6	15.9	2.2	5.6	4.6	1.2
1879–1933	1.9	4.7	11.8	1.6	4.1	7.6	0.8
Rise & fall per month[d]							
1879–1897	1.1	2.8	7.1	1.2	3.6	8.6	0.4
1897–1914	0.8	2.6	8.1	1.4	5.2	6.1	0.3
1914–1933	0.8	3.2	20.4	1.5	4.9	3.9	0.7
1879–1933	0.9	2.9	12.9	1.4	4.6	6.5	0.5

TABLE 180—*Continued*

Average Cyclical Measures Covering Successive Periods of Five Business Cycles and All Fifteen Cycles from 1879 to 1933

Seven American Series

Measure and period[a]	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
AVERAGE LEAD (−) OR LAG (+)[b]							
At reference peaks							
1879–1897	+3.8	+0.8	−4.4	−4.4	−11.4	+1.2	+10.6
1897–1914	+4.2	+3.4	−6.0	−1.6	−12.0	+2.4	+9.7
1914–1933	+1.2	+1.4	−3.4	−8.5	−7.8	−2.0	+3.2
1879–1933	+3.2	+1.9	−4.6	−4.6	−10.4	+0.5	+7.9
At reference troughs							
1879–1897[e]	−6.2	−3.2	−2.0	−0.8	−2.2	+2.2	+16.6
1897–1914[f]	−7.4	−7.2	−9.4	−9.6	−4.8	0.0	+12.7
1914–1933	−3.8	−0.3	+1.2	−7.0	−6.5	+3.0	+0.2
1879–1933	−5.8	−3.4	−3.1	−5.8	−4.6	+1.7	+10.2
AVERAGE CHANGE PER MONTH DURING[g] REFERENCE CYCLES							
Expansion							
1879–1897	+0.9	+2.1	+3.5	+0.7	+1.4	+2.8	0.0
1897–1914	+0.8	+2.2	+5.3	+1.2	+1.8	+3.8	+0.2
1914–1933	+0.7	+2.4	+3.1	+0.4	+2.7	+3.1	+0.4
1879–1933	+0.8	+2.3	+4.0	+0.8	+2.0	+3.2	+0.2
Contraction							
1879–1897	−0.7	−1.7	−4.1	−0.8	−2.2	−5.0	−0.2
1897–1914	−0.4	−1.8	−5.1	−0.7	−2.2	−3.1	0.0
1914–1933	−0.4	−3.3	−3.5	−0.4	−0.6	−2.8	−0.2
1879–1933	−0.5	−2.3	−4.2	−0.6	−1.7	−3.6	−0.1
Difference[h]							
1879–1897	−1.5	−3.9	−7.6	−1.5	−3.5	−7.8	−0.2
1897–1914	−1.2	−4.0	−10.4	−1.9	−4.1	−6.9	−0.2
1914–1933	−1.1	−5.7	−6.5	−0.8	−3.3	−5.9	−0.6
1879–1933	−1.3	−4.5	−8.2	−1.4	−3.6	−6.9	−0.3

[a] The periods refer to business cycles, as marked off by the monthly reference dates in Table 16. The specific cycles are fitted as closely as possible into these periods; for full details, see Appendix Table B9.

[b] In months.

[c] In specific-cycle relatives.

[d] Unweighted average.

[e] Excludes timing at reference trough of June 1897.

[f] Excludes timing at reference trough of Dec. 1914.

[g] That is, during stages matched with reference expansions and contractions (see Table 177), in units of reference-cycle relatives.

[h] That is, average difference between reference contractions and expansions (see Table 47, col. 8).

TABLE 181

Coefficients of Rank Correlation between Average Cyclical Measures of Seven American Series in Different Periods

Measure[a]	Rank correlation between averages of seven series in		
	1879–1897 and 1897–1914	1897–1914 and 1914–1933	1879–1897 and 1914–1933
DURATION OF SPECIFIC CYCLES			
Expansion	+.56	+.21	+.38
Contraction	+.68	+.39	+.86
Full cycle	−.32	+.11	+.07
AMPLITUDE OF SPECIFIC CYCLES			
Rise	+1.00	+.96	+.96
Fall	+.96	+.89	+.86
Rise & fall	+1.00	+.96	+.96
Rise per month	+.86	+.96	+.89
Fall per month	+.96	+.79	+.71
Rise & fall per month	+.96	+.96	+.89
LEAD OR LAG			
At reference peaks	+.95	+.86	+.83
At reference troughs	+.43	+.46	+.39
CHANGE DURING REFERENCE CYCLES			
Expansion	+.96	+.82	+.89
Contraction	+.96	+.89	+.82
Difference	+.89	+.93	+.96

Based on the averages in Table 180, carried to an additional place.
[a] See the fuller stubs in Table 180, and the appended notes.

system, while the very propagation of business cycles seems causally connected with differences in the timing and amplitude of cyclical movements in different parts of the economy. Tables 183-184 present frequency distributions of the timing and amplitude measures of our seven series. It will be noticed that the distributions of the individual series tend to occupy different portions of the scale. In contrast, the durations of specific cycles, as may be seen from Table 185, cluster in about the same intervals as do the durations of business cycles. Of course, if the lower limit of the duration of movements that we recognize as specific cycles were reduced, we might find greater differences among the cyclical durations of our series. But we define specific cycles, as indeed we must, in a manner consistent with our working definition of business cycles, and we deem it a significant fact that our series tend to show cyclical movements that correspond to that definition.

Two conclusions emerge from this analysis. In the first place our tests, so far as they go, bear out the concept of business cycles as units of roughly concurrent fluctuations in many activities.[11] In the second place, they demonstrate that although cyclical measures of individual series usually vary greatly from one cycle to the next, there is a pronounced tendency towards repetition in the relations among the movements of different

[11] For a demonstration based on a larger sample, see Ch. 4, Sec. VI.

TABLE 182

Tests of the Statistical Significance of Differences among Average Cyclical Measures of Seven American Series

Measure[a]	Ratio of variance among seven series to variance within series based on	
	All cycles in each series	Cycles from 1879 to 1933
DURATION OF SPECIFIC CYCLES		
Expansion	3.30♪	3.03♪
Contraction	2.79♩	2.96♩
Full cycle	0.77	0.69
AMPLITUDE OF SPECIFIC CYCLES		
Rise	28.49𝅘𝅥𝅯	23.83𝅘𝅥𝅯
Fall	25.17𝅘𝅥𝅯	21.52𝅘𝅥𝅯
Rise & fall	28.45𝅘𝅥𝅯	24.39𝅘𝅥𝅯
Rise per month	4.01♪	2.83♩
Fall per month	15.49𝅘𝅥𝅯	12.84𝅘𝅥𝅯
Rise & fall per month	12.22𝅘𝅥𝅯	9.00𝅘𝅥𝅯
LEAD OR LAG		
At reference peaks	14.73𝅘𝅥𝅯	12.97𝅘𝅥𝅯
At reference troughs	12.06𝅘𝅥𝅯	8.66𝅘𝅥𝅯
CHANGE DURING REFERENCE CYCLES		
Expansion	11.44𝅘𝅥𝅯	11.35𝅘𝅥𝅯
Contraction	12.34𝅘𝅥𝅯	13.14𝅘𝅥𝅯
Difference	18.38𝅘𝅥𝅯	21.03𝅘𝅥𝅯

For periods covered by each series and averages based on all cycles, see Table 140. Averages for 1879–1933 are given in Table 180.

[a] See the fuller stubs in Table 180, and the appended notes.

♩ Larger than the value that would be exceeded once in twenty times by chance. This value ranges from 2.18 to 2.20 for the different measures.

♪ Larger than the value that would be exceeded once in a hundred times by chance. This value ranges from 2.96 to 3.01 for the different measures.

𝅘𝅥𝅯 Larger than the value that would be exceeded once in a thousand times by chance. This value ranges from 4.05 to 4.19 for the different measures.

TABLE 183

Frequency Distribution of Amplitudes of Specific Cycles Seven American Series

Rise & fall in specific-cycle relatives[a]	Number of cycles in						
	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
Under 20.0	1	..	..	..	..	..	8
20.0– 29.9	4	..	..	1	..	..	9
30.0– 44.9	7	..	..	6	..	..	3
45.0– 67.4	1	1	..	5	..	..	..
67.5–101.1	2	4	..	3	1	4	..
101.2–151.8	..	8	..	2	4	4	..
151.9–227.7	..	2	2	..	6	5	..
227.8–341.6	..	..	8	1	3	6	..
341.7–512.5	..	..	4	..	1	3	..
512.6–768.8	..	..	3	..	..	1	..
Over 768.8	..	..	2	..	..	..	..
Total	15	15	19	18	15	23	20

For the cycle-by-cycle measures and periods covered, see Appendix Table B1.

[a] Except for the open-end class at the start, the lower limits of successive classes are in geometric progression.

TABLE 184

Frequency Distribution of Leads or Lags of Specific Cycles
Seven American Series

Lead (−) or lag (+) at reference turn (mos.)	Number of specific-cycle turns in						
	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
	Timing at reference peaks						
−24 to −18	..	..	2	2	1	..	..
−17 to −11	..	1	3	1	7	1	..
−10 to −4	..	..	5	7	5	2	..
−3 to +3	10	9	3	6	1	11	3
+4 to +10	3	4	2	1	1	5	7
+11 to +17	1	1	1	..	..	..	5
+18 to +24	..	..	..	..	..	..	1
Total.......	14	15	16	17	15	19	16
	Timing at reference troughs						
−38 to −32	..	..	..	1	..	..	..
−31 to −25	..	..	..	..	..	..	..
−24 to −18	..	..	..	2	1	..	..
−17 to −11	2	1	2	2	2	1	..
−10 to −4	7	5	6	6	4	3	2
−3 to +3	6	10	5	5	8	8	1
+4 to +10	..	..	4	1	1	4	3
+11 to +17	..	..	..	1	..	3	5
+18 to +24	..	..	..	..	..	..	4
+25 to +31	..	..	..	..	..	..	1
Total.......	15	16	17	18	16	19	16

For the cycle-by-cycle measures and periods covered, see Appendix Table B3.

TABLE 185

Frequency Distribution of Durations of American Business Cycles
and Specific Cycles in Seven Series

Duration of cycles (mos.)	Number of cycles in							
	General business activity	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
11 – 20	..	..	..	1	3	..	2	2
21 – 30	2	3	2	6	2	2	4	3
31 – 40	7	5	7	4	3	5	6	4
41 – 50	6	6	3	3	5	6	9	6
51 – 60	1	..	1	4	2	..	2	1
61 – 70	1	..	1	..	..	..	..	4
71 – 80	2	..	1	..	..	..	..	..
81 – 90	..	..	..	1	..	2	..	..
91 – 100	1	..	..	..	1	..	..	..
101 – 110	..	..	..	..	1	..	..	..
Over 110	..	1	..	..	1	..	..	..
Total.....	20	15	15	19	18	15	23	20

The durations of cycles in general business activity are derived from the monthly reference troughs of business cycles, 1854–1933, in Table 16. See Appendix Table B1 for the periods covered by the specific cycles and their successive durations.

activities in successive business cycles. Our analysis of hundreds of time series is sufficiently advanced to give us full confidence in these conclusions. Later monographs will demonstrate in detail that the processes involved in business cycles behave far less regularly than theorists have been prone to assume; but they will demonstrate also that business-cycle phenomena are far more regular than many historically-minded students believe.

These facts have a vital bearing on the value of averages as a tool of theoretical analysis. If business cycles had few if any repetitive features other than the cyclical movement in 'total output' or 'total employment' itself, averages of cyclical measures would have little value. Whether or not a general theory of business cycles would be possible in such a case, average measures of cyclical behavior could be of no aid in the development of a general theory. But so far as business cycles have repetitive features, averages help to expose these features. By showing what cyclical behavior is characteristic of different activities, they put observational records in a form that both reveals concretely what is to be explained and helps in finding explanations. When used competently, average measures of cyclical movements in different economic processes go far toward accounting for one another.

Our theoretical work leans heavily on cyclical averages, such as are described in this book, and we hope that others will find them helpful. Instead of setting out from the dreamland of equilibrium, or from a few simple assumptions that common sense suggests about the condition of business in late 'prosperity' or 'depression', as is the usual procedure of business-cycle theorists, we start our theoretical analysis with cyclical averages; in other words, our 'assumptions' are derived from concrete, systematic observations of economic life. Not only that, but we can test our reasonings as we journey through successive stages of the cycle by referring ever and again to the arrays of averages. This program is essayed in a volume now in preparation, which seeks to explain how business cycles come about and why they differ from one another.[12]

III Influence of Extreme Items on Averages

It follows from the preceding analysis that the tendency of averages to conceal the episodic features of successive cycles is from our viewpoint a virtue rather than a defect. We consider averages faulty not so much because they conceal episodic movements as because they do not do so sufficiently. The larger the variability of cyclical measures, the fewer the cycles covered, the narrower the geographic area or the economic scope represented by a series, the rougher must averages be as gauges of cyclical

12 For the program as a whole, see Ch. 1, Sec. VIII.

behavior. Some of the differences between averages for the same series in Table 180 are considerable. A few reflect structural or secular changes in cyclical behavior: for example, the recent decline in the amplitude of specific cycles in call money rates, and the shortening of the lead of iron production at reference troughs. But we believe that random factors are chiefly responsible for the wide variations sometimes found in averages based on different groups of cycles.

One outstanding example may be cited. After the entry of the United States into World War I, the railroads curtailed purchases of equipment drastically, anticipating that the government would take over their business. On December 28, 1917 the government actually took control and set up the Railroad Administration. No freight cars were purchased for several months. When designs for standard equipment were finally worked out, the Administration entered the market on May 1, 1918 with orders for 99,500 cars. On a quarterly basis the figures of car orders during 1918 and 1919, in thousands of cars, run as follows: 1, 100, 0, 0, 1, 0, 0, 3.[13] We recognize a specific cycle with a trough in the first quarter of 1918, a peak in the second quarter of 1918, and a terminal trough in the third quarter of 1919. The legitimacy of treating the rise between the first and second quarters in 1918 as a specific-cycle expansion instead of a random movement may be questioned. This 'cycle' is certainly one of the most peculiar on record: zero values predominate, the duration is only six quarters, the total amplitude is nearly 1,200 points, the rise per month is 195 points. But we believe that in view of the attendant circumstances this movement is best regarded as a specific cycle dominated by random disturbances. It illustrates how cyclical fluctuations may sometimes be twisted out of their ordinary course by government, which can concentrate its purchases on a single day or month, while there is bound to be some dispersion over time when numerous units act independently. The extraordinary specific cycle of 1918–19 in freight car orders accounts for some of the most striking differences between the averages of subgroups in Table 180.

As explained in Chapter 9, Section II, we attempt to minimize such distortions by excluding from the averages cycles dominated by random factors. We do this freely when we are reasonably sure that certain cycles of a series are dominated by random influences, as in the case of most price and value series during serious monetary disturbances.[14] We sometimes follow this practice even when we lack definite knowledge of random perturbations associated with a given 'extreme' cycle. But our exclusions of cycles on account of size alone are few in comparison with the number

[13] John E. Partington, *op. cit.*, pp. 156-7, 225.

[14] We make no exclusions in many series on interest rates and security prices, since they often escape the extreme fluctuations characteristic of price series during periods of monetary disturbance. Three of the seven series we use for illustrative purposes—railroad bond yields, call money rates, and railroad stock prices—are cases in point.

TABLE 186

Influence of the Highest and Lowest Values on
the Average Duration, Timing and Amplitude of Specific Cycles
Seven American Series

Measure and basis of average[a]	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
AVERAGE DURATION OF SPECIFIC CYCLES							
Expansion							
N	32.6	28.8	18.3	28.8	17.9	19.9	21.0
N−L	34.0	30.1	19.2	30.2	18.9	20.6	21.7
N−H	29.6	27.4	17.0	25.5	16.2	19.0	19.1
Contraction							
N	11.4	14.5	21.2	21.0	26.2	18.0	21.4
N−L	12.1	15.2	22.0	21.9	27.6	18.6	22.4
N−H	9.4	12.4	20.0	18.6	24.7	17.2	19.9
Full cycle							
N	44.0	43.3	39.5	49.8	44.1	37.9	42.4
N−L	45.1	44.6	40.7	51.7	45.5	38.7	43.8
N−H	39.0	41.3	37.0	44.1	41.1	37.2	41.1
AV. LEAD (−) OR LAG (+)							
At reference peaks							
N	+3.2	+1.9	−5.8	−5.6	−10.4	−0.1	+7.8
N−L	+3.6	+2.8	−4.6	−4.6	−9.5	+0.6	+8.3
N−H	+2.3	+1.2	−6.9	−6.6	−11.4	−0.7	+7.1
At reference troughs							
N	−5.8	−3.4	−3.0	−7.4	−4.6	+1.5	+11.8
N−L	−5.1	−2.7	−2.1	−6.0	−3.7	+2.4	+12.9
N−H	−6.2	−3.7	−3.8	−8.6	−5.4	+0.7	+10.7
AVERAGE AMPLITUDE OF SPECIFIC CYCLES							
Rise							
N	26.9	62.1	213.0	35.6	98.1	115.9	10.8
N−L	28.3	63.9	218.9	37.3	101.9	119.7	11.2
N−H	24.5	58.8	192.3	32.4	92.0	106.9	9.5
Fall							
N	13.4	54.8	211.6	31.8	92.4	116.1	12.5
N−L	14.1	57.3	219.6	33.3	96.3	120.1	13.1
N−H	10.4	48.0	190.5	25.3	86.4	106.0	11.5
Rise & fall							
N	40.2	116.8	424.5	67.3	190.5	232.0	23.3
N−L	41.8	121.2	436.5	69.9	197.7	238.8	24.3
N−H	36.6	111.0	382.8	57.6	178.4	212.9	22.3
Rise per month[b]							
N	0.83	2.42	22.56	1.43	7.63	6.88	0.57
N−L	0.86	2.50	23.61	1.49	8.00	7.10	0.59
N−H	0.79	2.24	12.98	1.30	6.72	6.08	0.51
Fall per month[b]							
N	1.93	4.67	11.74	1.69	4.09	7.63	0.86
N−L	2.03	4.93	12.12	1.75	4.29	7.91	0.89
N−H	1.17	4.26	10.21	1.54	3.69	7.01	0.68
Rise & fall per month[b]							
N	0.94	2.87	13.20	1.44	4.56	6.28	0.63
N−L	0.97	2.99	13.71	1.49	4.75	6.49	0.65
N−H	0.90	2.71	10.31	1.38	4.31	5.92	0.58

In order to bring out variations in detail, we use decimals freely in this and the following table. All durations are expressed in months, amplitudes in specific-cycle relatives.

[a] N stands for the full number of cyclical observations on the series, as shown in Tables 140 and 141. 'N−L' excludes the single lowest value in the array, 'N−H' excludes the single highest value. For cycle-by-cycle measures other than the per month amplitudes, see Appendix Tables B1 and B3.

[b] Unweighted average.

TABLE 187

Several Positional Arithmetic Means
Duration, Timing and Amplitude of Specific Cycles
Seven American Series

Measure and no. of items averaged[a]	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
AVERAGE DURATION OF SPECIFIC CYCLES							
Expansion							
N	32.6	28.8	18.3	28.8	17.9	19.9	21.0
N−2	30.9	28.7	17.8	26.8	17.2	19.7	19.7
N−4	30.9	28.4	17.2	25.1	16.3	19.8	18.3
3 or 4	29.7	25.3	14.0	22.0	17.0	19.3	16.5
1 or 2	30.0	25.0	12.0	22.0	16.0	18.0	16.5
Contraction							
N	11.4	14.5	21.2	21.0	26.2	18.0	21.4
N−2	9.9	13.0	20.8	19.4	26.2	17.9	20.9
N−4	9.5	12.7	20.2	18.3	26.5	17.7	20.0
3 or 4	8.7	11.0	17.0	13.8	24.3	17.0	18.2
1 or 2	9.0	11.0	18.0	14.0	24.0	17.0	18.5
Full cycle							
N	44.0	43.3	39.5	49.8	44.1	37.9	42.4
N−2	39.8	42.5	38.1	45.8	42.4	38.0	42.4
N−4	40.0	41.6	37.8	43.6	40.1	38.3	42.5
3 or 4	39.7	38.7	38.0	43.0	41.0	39.0	42.5
1 or 2	40.0	38.0	39.0	44.0	41.0	38.0	43.5
AV. LEAD (−) OR LAG (+)							
At reference peaks							
N	+3.2	+1.9	−5.8	−5.6	−10.4	−0.1	+7.8
N−2	+2.7	+2.2	−5.7	−5.5	−10.5	+0.1	+7.6
N−4	+2.2	+2.0	−5.3	−5.2	−10.7	+0.3	+7.7
3 or 4	+2.0	+0.7	−5.2	−5.7	−10.7	0.0	+7.0
1 or 2	+2.0	+1.0	−5.0	−5.0	−11.0	0.0	+7.0
At reference troughs							
N	−5.8	−3.4	−3.0	−7.4	−4.6	+1.5	+11.8
N−2	−5.5	−3.0	−2.9	−7.1	−4.4	+1.6	+11.9
N−4	−5.4	−2.8	−2.7	−7.3	−4.2	+1.4	+12.3
3 or 4	−4.7	−1.8	−2.7	−7.5	−2.8	+0.7	+13.5
1 or 2	−4.0	−2.0	−3.0	−8.0	−2.5	+1.0	+14.0
AVERAGE AMPLITUDE OF SPECIFIC CYCLES							
Rise							
N	26.9	62.1	213.0	35.6	98.1	115.9	10.8
N−2	25.9	60.5	197.4	34.0	95.7	110.4	9.9
N−4	26.0	59.9	190.1	32.2	95.3	108.6	9.7
3 or 4	25.8	58.7	177.5	29.1	99.8	95.9	9.3
1 or 2	26.8	61.9	177.3	27.4	101.6	98.0	9.4
Fall							
N	13.4	54.8	211.6	31.8	92.4	116.1	12.5
N−2	11.0	50.3	197.7	26.6	90.2	109.8	12.0
N−4	10.2	48.9	190.9	25.5	89.6	108.6	11.7
3 or 4	9.3	51.4	175.9	25.4	83.9	106.7	10.9
1 or 2	9.7	50.0	169.3	25.0	83.8	101.6	10.8
Rise & fall							
N	40.2	116.8	424.5	67.3	190.5	232.0	23.3
N−2	38.0	115.3	393.0	59.8	185.3	219.1	23.2
N−4	36.5	113.0	378.4	57.4	183.8	215.1	22.6
3 or 4	37.5	105.9	341.9	47.3	181.1	187.8	22.4
1 or 2	36.5	106.5	334.5	46.8	181.2	187.4	22.6

TABLE 187—*Continued*
Several Positional Arithmetic Means
Duration, Timing and Amplitude of Specific Cycles
Seven American Series

Measure and no. of items averaged[a]	Deflated clearings	Pig iron production	Freight car orders	Railroad stock prices	Shares traded	Call money rates	Railroad bond yields
AVERAGE AMPLITUDE OF SPECIFIC CYCLES—*Continued*							
Rise per month[b]							
N	0.83	2.42	22.56	1.43	7.63	6.88	0.57
N−2	0.82	2.32	13.53	1.36	7.05	6.28	0.52
N−4	0.82	2.20	12.89	1.35	6.70	6.03	0.51
3 or 4	0.83	1.97	10.83	1.32	6.10	5.63	0.45
1 or 2	0.80	2.00	10.80	1.30	6.20	5.60	0.45
Fall per month[b]							
N	1.93	4.67	11.74	1.69	4.09	7.63	0.86
N−2	1.22	4.51	10.51	1.59	3.88	7.29	0.71
N−4	1.01	4.39	10.30	1.56	3.90	6.98	0.65
3 or 4	0.97	4.10	9.10	1.30	3.63	6.40	0.52
1 or 2	1.00	4.20	9.10	1.30	3.60	6.50	0.50
Rise & fall per month[b]							
N	0.94	2.87	13.20	1.44	4.56	6.28	0.63
N−2	0.93	2.83	10.67	1.42	4.50	6.12	0.61
N−4	0.92	2.80	10.23	1.42	4.50	6.05	0.58
3 or 4	0.90	2.67	9.57	1.42	4.57	5.97	0.48
1 or 2	0.90	2.60	9.50	1.40	4.60	6.00	0.50

All durations are expressed in months, amplitudes in specific-cycle relatives.

[a] That is, number of central items in array that are averaged. N stands for the full number of cyclical observations on the series, as shown in Tables 140 and 141. See text for full explanation of symbols. For cycle-by-cycle measures other than the per month amplitudes, see Appendix Tables B1 and B3.

[b] Unweighted average.

of averages we take. We deem it a better general rule to use fully the comparatively few cyclical observations available for a series than to conjecture that an item that looks extreme would continue to appear in the same light if the series covered two or three times as many cycles.

Table 186 measures the influence exercised on some of our cyclical averages by the highest and lowest items found in the different arrays. This influence is disconcerting at times. For example, the average rise per month during expansions of freight car orders is 22.6 points for the nineteen specific cycles from 1870 to 1933; the average falls to 13.0 when the peculiar expansion lasting one quarter in 1918 is left out. Again, the average fall per month in deflated clearings is 1.9 points for the fifteen specific cycles from 1878 to 1933, but is only 1.2 points when the brief and violent contraction of 1907 is excluded. It is plain that 'extreme' values sometimes dominate the averages, and that they frequently make their influence felt. Nevertheless, the relations among the averages of the seven series in our sample are much the same whether the averages include all items or exclude on a uniform plan the highest or lowest values. Of course, these results would be blurred if the experiment were based on shorter series, if seventy series were compared instead of seven, or if the highest value or two were excluded in some instances and the lowest value or two in others—which we might well do if we knew enough about the play of 'random forces' on the cyclical behavior of different series.

Sometimes statisticians employ positional means as a device for eliminating or reducing the influences of extreme values, and we ourselves have used this device in making seasonal indexes.[15] But it seems sounder practice to proceed cautiously and discriminatingly in each instance than to exclude throughout some fixed number of values at both ends of arrays.[16] Not infrequently these values are extreme only in the sense that they occupy the inescapable first and last places of a variable series, or they are truly extreme at one end but not at the other. To lop off values at both ends of an array by mechanical rule is to discard information indiscriminately.

It is of some interest, however, to compare our standard results with those yielded by positional means. The averages in Table 187 are made from arrays, that is, from cyclical measures arranged in order of size. They show the average duration, timing and amplitude of the specific cycles in the seven series we have been using for experimental purposes. One average is based on N items; that is, it includes all observations on the series. A second includes the middle $(N-2)$ items; that is, the lowest and highest values are excluded. A third includes the middle $(N-4)$; that is, the lowest and highest two values are excluded. The fourth includes the middle three or four items according as the number of cyclical observations is odd or even. The fifth is the median; it includes solely the middle item when the number of observations is odd and the middle two items when the number is even.[17]

If we may judge from Table 187, the exclusion of the single highest and lowest values usually has a slight effect on cyclical averages. The effect is somewhat larger when the highest and lowest two values are omitted, but even these exclusions do not affect materially the relations among the averages. Our samples indicate that if we omitted the extremes at both ends of arrays, average measures of cyclical duration and amplitude, though not of timing, would be far more often below than above the averages we actually make, but the relations among the averages for different series would be substantially the same. Even average durations of full specific cycles, which do not differ significantly among our series, repeat much the same order in the positional as in the full means. Of

[15] In this operation a judgment factor still enters; that is, in deciding what number of central values are to be averaged. See p. 46.

[16] To be sure, mechanical rules have some advantage when work must move speedily, or when expert assistance is not available.

[17] These several averages may be regarded as members of a family of positional arithmetic means, of which the arithmetic mean of ordinary usage includes the maximum number and the median the minimum number of central values in an array. It may be noted parenthetically that this statement defines the median unambiguously.

course, when only one or two central items are included in positional means, greater differences appear. But medians are not formidable rivals of arithmetic means when arrays are short and the gaps between successive values erratic, which is the typical occurrence in our tables of cyclical measures. Despite their greater roughness, the medians rarely differ in order of magnitude from the arithmetic means. Further, the rankings of the medians and the arithmetic means are very similar.

Tables 188-189 and Charts 75-76 illustrate the effects of different methods of averaging on our cyclical patterns. Once again, the shapes of the patterns are substantially the same whether we use all items in making averages or drop one to two values at the ends of the arrays.[18] Even the median patterns of the specific cycles rarely differ radically from the mean patterns.[19] They move in the same directions throughout. Now and then, one pattern shows acceleration and the other retardation; in such instances the cycle-by-cycle measures usually confirm the mean pattern. The median reference-cycle pattern of some series diverges sharply from the mean pattern, especially at the turns. But once again, when the two move in different directions or when one pattern shows acceleration and the other retardation, the cycle-by-cycle measures usually confirm the mean pattern.[20] We conclude that, on the whole, the mean patterns are more faithful representatives of cyclical behavior than the median patterns.

The preceding tests are based upon samples that include a larger number of cycles than most series in our collection. For this reason we make similar tests for smaller groups of cycles in Table 190. Of course, the omission of one or two values at the extremes of an array can make a greater difference when there are six cycles than when there are three times as many. To exclude the highest and lowest two values in a series of five or six items is to convert means into medians. Few of the results for small groups are modified drastically by changing the form of the average; yet the table demonstrates vividly the instability of small samples.

We have already noted that the means of cyclical durations and amplitudes tend to decline as the number of central items included diminishes.

18 Such results may, of course, be expected even for reference cycles of random series.

19 As noted on the chart we use the mean (arithmetic average) intervals between cycle stages in plotting the median as well as the mean patterns. The reason medians of the intervals are not used is that they bear no determinate relation to the median duration of full cycles. This difficulty is shared by positional means generally and is a serious inconvenience in handling measures that bear additive relations to one another, such as durations of cyclical phases, durations of intervals between cycle stages, and amplitudes of cycle phases.

The charts also show average deviations of the patterns about their means. When the average deviation is small, the mean and median must be close together. When the average deviation is large, we cannot tell whether exclusion of items at the ends of arrays will affect the results much or little; but large average deviations are always a warning that the full mean may misrepresent the central tendency of the observations.

20 We omit the detailed evidence supplied by our standard Tables S5 and R2 and first differences of the entries in those tables.

TABLE 188

Several Positional Arithmetic Means of Specific-cycle Patterns
Seven American Series

Series and no. of items averaged[a]	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
		First third	Middle third	Last third		First third	Middle third	Last third	
Deflated clearings									
15......	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
13......	85.8	90.9	99.1	106.4	111.8	108.2	105.4	102.2	99.9
11......	85.9	91.2	99.0	106.4	111.3	108.2	105.4	102.5	100.3
3......	85.7	91.9	98.6	106.6	110.1	107.8	104.5	103.7	101.6
1......	85.5	91.7	98.5	106.8	110.0	108.0	104.4	103.8	100.5
Pig iron production									
15......	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
13......	65.1	81.1	101.8	114.5	127.5	122.8	109.8	91.2	76.0
11......	64.1	81.3	101.6	114.5	127.0	122.7	111.3	93.9	77.3
3......	64.8	82.6	100.4	113.4	126.9	121.7	113.3	91.4	76.3
1......	67.0	83.5	101.2	113.1	127.7	122.2	113.5	90.2	76.8
Freight car orders									
19......	29.5	66.3	104.5	136.8	242.5	112.6	100.2	72.2	30.9
17......	27.6	64.1	97.0	126.1	227.9	115.3	100.9	70.9	29.4
15......	26.1	63.7	95.2	123.4	221.4	117.6	102.4	71.3	27.6
3......	19.8	53.7	95.2	122.2	201.9	123.5	98.7	64.5	26.8
1......	19.4	54.5	97.2	123.0	202.4	123.1	94.6	62.4	26.1
Railroad stock prices									
18......	82.8	88.0	98.8	110.8	118.3	112.4	103.2	94.0	86.6
16......	83.2	88.6	99.3	110.1	116.7	111.4	102.3	95.6	89.1
14......	84.0	89.5	99.9	109.5	115.6	110.5	101.9	94.8	89.4
4......	84.3	90.1	100.7	108.2	114.4	109.6	100.7	94.4	89.5
2......	84.8	90.5	100.9	108.0	114.7	109.4	100.8	94.8	89.6
Shares traded									
15......	55.4	79.7	106.5	119.0	153.4	119.7	100.4	81.6	61.0
13......	55.8	79.4	106.3	116.4	151.0	119.5	100.6	81.1	60.7
11......	56.0	78.9	107.2	114.6	148.8	119.4	100.8	80.6	60.6
3......	57.0	76.3	108.0	111.7	149.5	119.6	100.5	79.1	61.7
1......	57.2	76.3	109.3	110.2	151.0	120.0	100.3	78.7	60.2
Call money rates									
23......	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
21......	61.5	79.5	104.1	123.8	171.8	120.0	89.7	70.7	60.1
19......	61.4	79.7	103.6	123.8	170.9	119.5	90.8	70.8	60.1
3......	63.4	79.1	105.5	122.3	164.1	116.4	99.3	71.0	60.7
1......	63.2	79.7	106.4	121.5	163.8	115.5	99.2	69.4	59.7
Railroad bond yields									
20......	96.1	98.4	101.5	104.0	106.9	103.7	100.6	96.9	94.3
18......	96.6	98.8	101.5	103.9	106.9	103.6	100.5	97.1	94.7
16......	96.6	98.6	101.4	103.8	106.8	103.6	100.4	97.0	94.8
4......	96.7	98.6	101.2	103.6	106.7	103.4	99.8	97.0	94.4
2......	97.0	98.8	101.3	103.6	106.5	103.5	99.8	97.1	94.4

[a]That is, number of central items in array that are averaged. The average on the first line for each series includes all cycles in the series. For the periods covered, see Chart 75; for the cycle-by-cycle patterns, Appendix Table B1.

TABLE 189

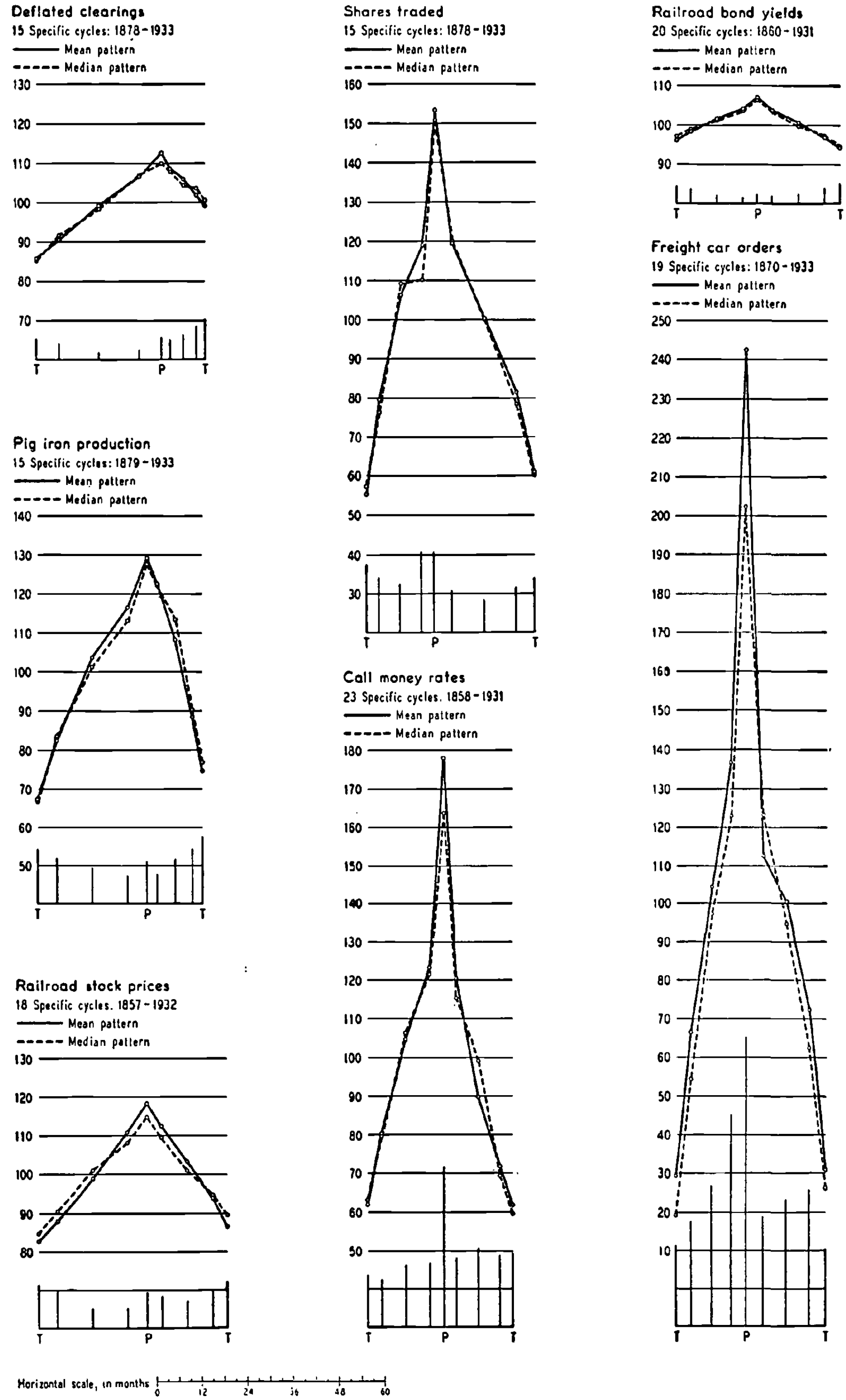
Several Positional Arithmetic Means of Reference-cycle Patterns
Seven American Series

Series and no. of items averaged[a]	Average in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
		First third	Middle third	Last third		First third	Middle third	Last third	
Deflated clearings									
15......	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
13......	88.0	93.5	98.1	104.5	107.5	106.6	102.8	100.5	102.3
11......	88.2	93.5	98.1	104.4	107.2	106.5	102.8	100.7	102.5
3......	88.0	92.9	98.2	104.1	106.9	106.2	102.8	101.6	103.5
1......	87.6	92.9	97.9	103.9	106.9	107.1	102.9	101.2	103.6
Pig iron production									
15......	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
13......	71.8	89.0	101.4	110.6	119.8	117.2	101.3	87.0	83.6
11......	70.5	87.8	100.8	110.6	118.9	116.9	102.1	89.1	85.0
3......	71.3	88.8	99.5	110.0	119.4	115.5	105.3	95.3	87.8
1......	71.3	89.7	99.2	109.4	120.0	115.7	103.7	95.7	87.4
Freight car orders									
16......	76.0	82.1	112.9	136.8	122.3	93.8	62.0	64.1	89.2
14......	65.9	83.6	116.1	137.8	123.2	90.8	62.3	60.6	76.9
12......	62.7	85.5	115.7	139.0	124.8	87.1	63.7	59.5	76.9
4......	57.4	92.4	113.7	140.0	131.0	83.5	63.4	56.2	72.4
2......	57.8	96.0	112.6	137.4	139.3	82.8	62.7	56.3	76.3
Railroad stock prices									
19......	91.0	96.9	104.0	109.4	106.9	104.3	97.7	92.5	94.7
17......	91.0	97.0	103.3	108.5	105.2	102.7	97.5	94.4	96.8
15......	91.2	96.7	103.1	108.2	105.2	102.9	97.5	94.4	95.9
3......	91.1	96.0	102.3	108.7	105.8	101.2	96.9	93.8	95.0
1......	87.6	95.4	102.2	109.0	105.6	100.7	97.7	93.9	94.8
Shares traded									
15......	83.8	111.2	110.9	114.0	110.6	96.8	90.5	79.5	97.5
13......	84.2	108.5	110.7	113.7	110.5	96.2	90.6	77.3	94.2
11......	84.0	104.7	111.3	113.7	110.1	95.3	91.3	75.7	92.3
3......	83.1	100.1	109.1	111.0	113.7	95.1	89.7	76.5	89.1
1......	81.2	97.7	107.8	111.0	113.7	95.6	88.1	76.3	88.1
Call money rates									
19......	77.5	82.4	98.4	128.2	159.5	128.5	103.9	81.1	76.2
17......	77.2	81.8	96.3	123.9	143.6	128.4	101.1	79.6	74.0
15......	76.7	81.0	95.0	119.9	142.4	127.0	96.7	79.3	74.2
3......	74.5	85.6	92.0	118.3	132.7	118.4	99.3	79.5	77.2
1......	74.6	85.7	92.4	118.2	136.2	119.3	100.9	79.8	76.7
Railroad bond yields									
19......	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2
17......	101.9	100.3	98.5	99.0	100.6	101.8	101.4	101.2	100.4
15......	101.8	100.0	98.5	98.6	100.2	101.5	101.4	101.2	100.4
3......	101.6	101.1	97.8	98.4	99.4	101.5	101.7	101.0	99.8
1......	102.3	100.7	97.8	98.3	99.4	101.4	101.8	100.8	99.8

[a] That is, number of central items in array that are averaged. The average on the first line for each series includes all cycles in the series. For the periods covered, see Chart 76; for the cycle-by-cycle patterns, Appendix Table B3.

CHART 75

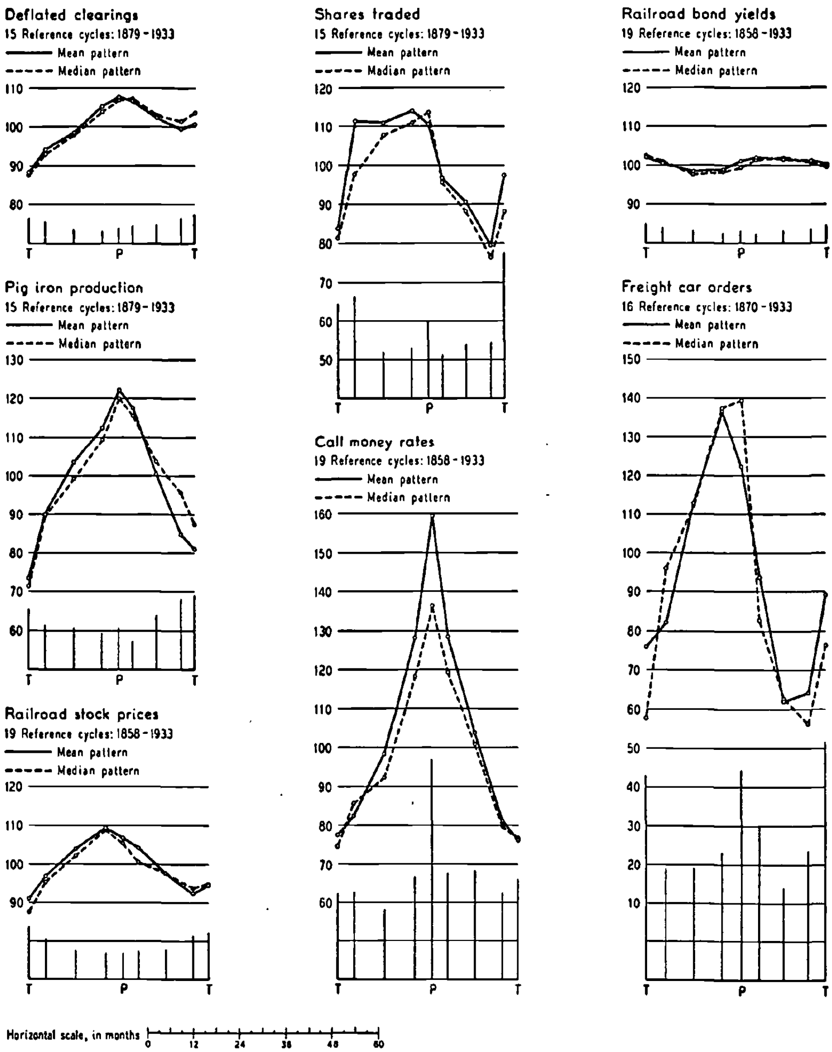
Mean and Median Specific-cycle Patterns
Seven American Series



See *Table 188. In each series the mean (arithmetic average) intervals between cycle stages are used for the median pattern as well as for the mean pattern.* For other explanations, see Ch. 5, Sec. VI.

CHART 76

Mean and Median Reference-cycle Patterns
Seven American Series



See Table 189. In each series the mean (arithmetic average) intervals between cycle stages are used for the median pattern as well as for the mean pattern. For other explanations, see Ch. 5, Sec. VII.

TABLE 190

Several Positional Arithmetic Means
Duration and Amplitude of Small Groups of Specific Cycles
Seven American Series

Series and period covered	No. of specific cycles (N)	Average duration of full cycles in months			Average amplitude in specific-cycle relatives					
					Rise & fall			Per month rise & fall[a]		
		N items	Middle (N−2) items	Middle (N−4) items	N items	Middle (N−2) items	Middle (N−4) items	N items	Middle (N−2) items	Middle (N−4) items
Deflated clearings										
1878–1893.......	5	37	37	36	42	41	43	1.1	1.2	1.1
1893–1910.......	5	41	41	43	37	36	34	0.9	0.9	0.9
1910–1933.......	5	54	42	49	41	31	25	0.8	0.8	0.8
Pig iron production										
1879–1896.......	5	43	37	37	107	99	98	2.8	2.9	2.6
1896–1914.......	5	44	45	48	113	110	106	2.6	2.5	2.6
1914–1933.......	5	44	43	40	131	134	132	3.2	3.2	3.1
Freight car orders										
1870–1894.......	6	48	45	46	421	383	386	9.7	8.1	7.9
1894–1914.......	6	40	41	42	319	322	315	8.3	8.0	7.8
1914–1933.......	7	32	29	28	518	436	438	20.4	14.0	12.9
Railroad stock prices										
1857–1889.......	6	63	53	46	76	76	71	1.5	1.4	1.4
1889–1907.......	6	37	39	43	52	52	49	1.4	1.4	1.4
1907–1932.......	6	49	41	38	74	44	43	1.4	1.4	1.4
Shares traded										
1878–1897.......	5	46	39	37	148	146	145	3.6	3.8	4.3
1897–1914.......	5	42	42	42	220	205	197	5.2	5.0	4.7
1914–1933.......	5	44	38	40	204	211	213	4.9	5.0	5.1
Call money rates										
1858–1880.......	7	38	38	40	218	158	140	5.9	5.0	5.5
1880–1904.......	8	36	37	36	289	293	304	8.4	8.3	8.0
1904–1931.......	8	40	40	40	187	163	156	4.5	4.2	4.0
Railroad bond yields										
1860–1876.......	6	32	32	29	27	28	26	0.9	0.9	1.0
1876–1905.......	7	49	48	47	20	20	20	0.4	0.4	0.4
1905–1931.......	7	45	46	45	24	22	23	0.6	0.5	0.5

Averages are derived from arrays: the average based on the middle (N−2) items excludes the single highest and lowest items, and so on.

[a] Unweighted average.

This tendency indicates a 'skew' towards the higher values.[21] The distributions of durations of business cycles resemble those of specific cycles, in that both are skewed positively.[22] In part these results flow from our method of marking off business and specific cycles; that is, we have a rigid

21 When the highest single value of an array deviates from the mean by more than the lowest single value, the mean with these extremes omitted must be smaller than the full mean; the decline in the average is indicative of a skew towards the higher values. Similarly, when the mean of the values above the median deviates from the mean of the full array by more than the mean of the values below the median, the median must be lower than the full mean; the fact that it is lower is again indicative, though more definitely since the base is broader, of a skew towards the upper ranges.

22 See above, Tables 168 and 185; also Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 416-24.

lower limit but no rigid upper limit to the duration of movements that we recognize as cycles. In the case of cyclical amplitudes we fix no lower limit formally, but our technique is inherently more sensitive to extremely large than to extremely small amplitudes.[23] When there are no limitations of this character, as happens in the timing measures, we find positive skewness no more frequently than negative skewness. But these statements merely scratch the surface of the problem raised by the asymmetries, and we must postpone full analysis. The one point we wish to stress now is that when we find skewness in distributions of cyclical measures, it is rarely so marked as to destroy the usefulness of arithmetic means as rough measures of central tendency.

At the same time we recognize that the samples of cycles covered by our collection of time series are often unstable to a disconcerting degree. In dealing with the amplitudes of specific cycles and with cyclical patterns, we should often count by tens and in extreme instances by hundreds, rather than by integers or decimals. The significant matters—at least at this stage of our work—are, usually, orders of magnitude, not precise figures; the latter often have little value except as guides to the former. Sometimes we regard only the average deviations showing the variability from cycle to cycle as significant, while the averages serve merely as a base from which to measure deviations. To show that the movements of a given factor with respect to business cycles vary within wide limits is an important result, and one that can often be established from a relatively slender sample.

IV Causal Interpretation of Averages

If the analysis of this and the two preceding chapters is valid, our averages may, with the exercise of due caution, be treated as representatives of the cyclical movements that characterize different activities, and the average deviations as representatives of the variability of cyclical measures around fairly stationary means. It is tempting to pass from this statement to another, namely, that averages represent roughly the effects of cyclical forces and that average deviations represent the effects of random forces. Such statements do no harm if carefully interpreted, but they are easily misconstrued and therefore best avoided.[24]

For example, Chart 77 shows clear-cut specific-cycle patterns in crop production in the United States and Great Britain. But the reference-cycle patterns in both countries are nearly straight lines, their slopes reflecting mainly the upward trend of the American series and the down-

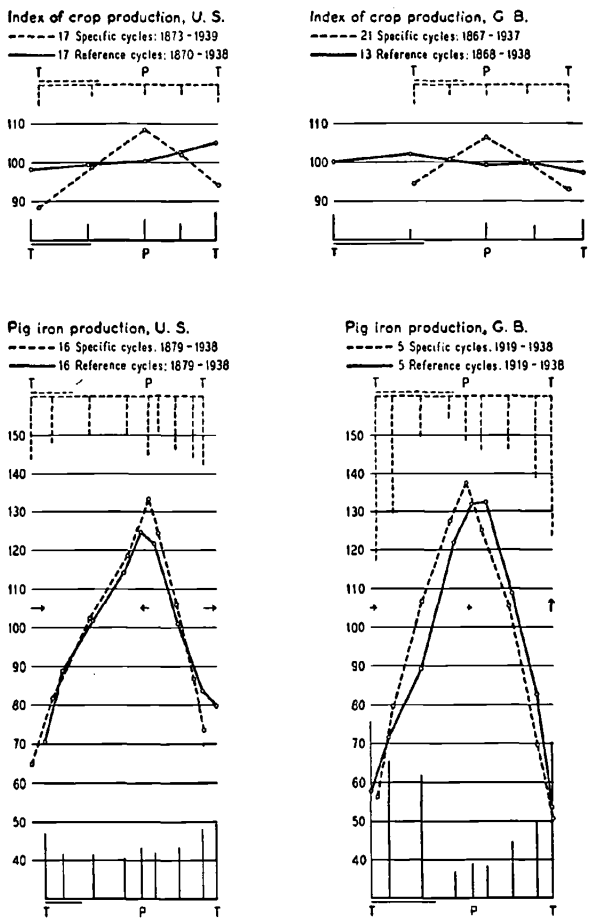
23 There is a slight tendency for the standings of specific cycles to be skewed negatively at troughs and a strong tendency for the standings to be skewed positively at peaks; while the rise, fall, and joint rise and fall of the specific cycles tend to be skewed positively (Tables 187-188).

24 Cf. Ch. 5, Sec. VII.

ward trend of the British. The relation between the specific- and reference-cycle patterns indicates that in each country the specific cycles of crop output have been virtually independent in time of business cycles. It is clear therefore that we cannot regard the specific-cycle averages as measuring the effects of business-cycle forces. We might say that they measure the effects of specific-cycle forces. In that case we should have to include variations in growing conditions (the vagaries of the weather,

CHART 77

Average Cyclical Patterns of Crop and Pig Iron Production
United States and Great Britain



Horizontal scale, in months 0 12 24 36 48 60

→ Indicates lead at reference peaks, or troughs; ← lag at reference peaks; ↑ rough coincidence at reference troughs.

The two crop series are annual; hence the five-point patterns. For sources of data, see Appendix C; for explanation of chart, Ch. 5, Sec. VIII and XI.

plant diseases, insects, and so on) as well as variations in the activities of farmers among specific-cycle forces. But variations in growing conditions are usually classed as random perturbations; if we treat them as such we should have to say that random factors enter substantially into the specific-cycle averages, whether or not rhythmic tendencies characterize the activities of farmers.

As another example, let us take the cyclical behavior of pig iron production in the United States and Great Britain (Chart 77). Now we find that the reference-cycle pattern resembles the specific-cycle pattern very closely in each country. But if we interpret the average reference-cycle pattern as representing business-cycle influences, that precludes an identical interpretation of the specific-cycle pattern. For if the first interpretation is valid, the specific-cycle pattern must contain a random component measured roughly by the difference between the two patterns. We might say that the specific-cycle averages represent specific-cycle forces, but that statement of itself adds nothing to knowledge. We might say instead that business-cycle forces dominate while random forces have little influence on the specific cycles of pig iron production. Even this statement involves many assumptions; among others, (1) that random factors have slight influence upon the timing of business cycles, (2) that variations in the timing of iron production at business-cycle turns do not arise from variations in business-cycle forces, (3) that the timing of random perturbations affecting iron production is uncorrelated with the timing of business cycles, (4) that random perturbations influence the amplitude and pattern of specific cycles as little as they influence their timing in regard to business cycles. Although assumptions of this type may be an intellectual convenience at certain stages of an argument, it is well to recognize that they bury important problems.

In general, when business-cycle influences dominate the movements of a series and random influences count for little, the average specific- and reference-cycle patterns should be closely similar. But we must not lose sight of the fact that similarity might also be produced by factors peculiar to a series, if their influence happens to correspond closely in timing with the cyclical tides of general business. Similarity may also be produced by random influences that affect the same way both the cyclical tides of general business and the specific cycles of the series. On the other hand, wide differences between the two cyclical patterns need not mean that random influences are dominant. For such differences might have been produced by rhythmic specific-cycle influences that are independent of business cycles rather than by random influences. Sometimes the two patterns may appear rather different at first sight though a series is dominated by business cycles, as when it participates in business cycles by rising in the late stages of reference contractions and the early stages of reference expansions, and falling in the late stages of reference expansions and the early stages of reference contractions. However, when the relation is of this type the amplitude of the reference-cycle pattern should approach that of the specific-cycle pattern.

These remarks indicate a few of the difficulties in interpreting averages and average deviations in causal terms. Averages and average deviations provide materials for studying the effects of business-cycle and random forces; they do not solve this problem. Nor can we work with a simple dichotomy of causal factors, unless we assume that secular factors can never express themselves independently of cyclical factors, and treat as random all factors, other than business cycles, that 'produce' specific cycles. From the point of view of a business-cycle theory that puts the subject in a box by itself, these assumptions may be legitimate; but if one wishes to understand the business cycles of actual experience, it is desirable to get as much insight as possible into the miscellaneous and dim category of factors that seem independent of business cycles, whether their influence is cumulative, haphazard or rhythmic. We shall learn more by discriminating among different influences than by lumping as many as possible under one heading.

If, therefore, we sometimes speak of the differences among successive business cycles as 'random', we merely use a vague shorthand expression. It is conceivable that even wars and variations in rainfall, which we regard as two of the most powerful 'random' disturbances of economic life, have sufficient regularity to be predictable. As we press analysis of the variations among business cycles, we may find many significant differences between cycles that come in times of war and those that come in times of peace, between cycles occurring in times of agricultural prosperity and in times of agricultural depression, between cycles characterized by vigorous revivals in investment and lags in consumption and those characterized by vigorous revivals in consumption and lags in investment, between short and long business cycles, mild and violent cycles, and so on.[25] These alluring investigations must stand over until we have attained a tolerably accurate working knowledge of what cyclical fluctuations are typical in different business processes and their broad interrelations.

V Test of Consilience among the Results

As stated many times in this book, such knowledge is not easy to attain. The more we have studied business cycles the more we have become convinced of both the importance and the difficulty of determining reliably what cyclical behavior has been characteristic of different economic activities. Theorists sometimes wrangle about questions of fact as if they were problems in metaphysics. Whether and how wage rates conform to busi-

25 Unfortunately, the small number of cyclical observations places severe limits upon analysis.

ness cycles is a question of fact; so also with building construction, savings, interest rates, and other economic factors. No speculative solution can have any meaning, except as a hypothesis to be tested. To settle these questions of fact, statistics must be marshaled with scientific care, they must be analyzed with the aid of expert knowledge of business processes, and they must be tested for consistency with other leading facts.

Not a few of our time series are rough compilations. Most of them we feel are much too brief for our needs. Our statistical analysis is imperfect: at times the seasonal movements are refractory, now and then the specific cycles are elusive, and there are the many other difficulties on which we have expatiated in preceding chapters. To break through the mist that envelops the facts of cyclical experience, whenever possible we work with several series representing the same process or closely related processes. When results for related series are set side by side, the insufficiencies of our data and methods can to a large extent be overcome. By checking varied bits of evidence against one another, it is usually possible to establish with confidence what cyclical movements are characteristic of different business factors and the relations in time between these movements and business cycles. Where we would be reluctant to generalize on the basis of a single set of fallible observations, the consilience of several sets of observations gives us courage to push forward toward our goal of determining what business cycles are, how they typically run their course, and what tendencies they show toward variation.

Ordinarily we can find several series that represent the behavior of the same economic process in different areas or periods, or of similar economic processes in the same area and period. For example, we have analyzed so far nine series on residential building in the United States. Three show contracts for total residential building in different units—dollar value, square feet of floor space, and number of structures. Another three show separately the value of contracts for one and two family dwellings, apartment houses, and hotels. These six records of contracts by months are supplemented by much longer series of permits for residential building issued annually in Manhattan, Philadelphia and St. Louis. And the American materials are compared with numerous foreign series, some of which measure the volume of residential building completed and thus enable us to observe the stage when construction products are installed. Our analysis of many other business factors is equally detailed. So far we have analyzed eleven series showing purchases or production of railroad equipment, sixteen series on bank clearings or debits, twenty-four on the output of iron and steel, thirteen on stock prices, numerous wholesale price indexes, employment and wage disbursements in upwards of a score of industries, the volume of business estimated in various ways, and so on. Never do two or more samples yield identical results. But when there is marked agreement among the results yielded by several samples, we feel justified in regarding those features of cyclical behavior that characterize the entire group as reliably established. This presumption of reliability is strengthened when the results accord well with *a priori* expectations based upon broader but less exact knowledge, or when they accord well with considerations that come to mind after the statistical analysis is finished, especially when the latter type of confirmation is itself tested by a fresh appeal to facts.

Of course, the test of consistency among different sets of statistical results, or between statistical results and other knowledge, is not limited to cases of agreement. For example, we do not expect exports to have the same relation to domestic cycles in the United States as in Great Britain; we expect new construction and repair work to bear different relations to business cycles; we expect interest rates on 4-6 month loans to behave differently from bond yields; we expect bank clearings in New York City to differ from 'outside' clearings in ways we can define in advance. So too, we expect a combination of similarities and differences to appear when we have series showing physical output, prices, employment, and wage disbursements for a given industry. Our idea of what similarities and differences to expect may be vague when we first make such comparisons; but if we have data of these four types for several industries we can form more definite expectations to test. In a still broader fashion we can tell whether our inclusive measures of production, transportation, employment, prices, inventories, sales, and monetary circulation are consistent with one another.

In the last resort, judgment concerning the significance of our measurements must depend upon the fashion in which they fit together. Whether the conclusions about the cyclical behavior of the processes treated in any one monograph are sound or not can seldom be settled by looking merely at the evidence there presented; almost always the evidence concerning processes treated in other monographs must be considered. A more rigorous test will be supplied by the final volume, where we shall try to weave the many lines of evidence presented in the monographic studies into a systematic account of how business cycles run their course.

APPENDIX A

Division of Reference Cycles into Stages

TABLE A1

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)	Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
			United States				
I	Nov. 1854 – Jan. 1855	3	...	I	Apr. 1885 – June 1885	3	...
II	Jan. 1855 – Oct. 1855	10	5.5	II	June 1885 – Dec. 1885	7	4.0
III	Nov. 1855 – July 1856	9	9.5	III	Jan. 1886 – July 1886	7	7.0
IV	Aug. 1856 – May 1857	10	9.5	IV	Aug. 1886 – Feb. 1887	7	7.0
V	May 1857 – July 1857	3	5.5	V	Feb. 1887 – Apr. 1887	3	4.0
VI	July 1857 – Dec. 1857	6	3.5	VI	Apr. 1887 – July 1887	4	2.5
VII	Jan. 1858 – May 1858	5	5.5	VII	Aug. 1887 – Nov. 1887	4	4.0
VIII	June 1858 – Nov. 1858	6	5.5	VIII	Dec. 1887 – Mar. 1888	4	4.0
IX	Nov. 1858 – Jan. 1859	3	3.5	IX	Mar. 1888 – May 1888	3	2.5
I	Nov. 1858 – Jan. 1859	3	...	I	Mar. 1888 – May 1888	3	...
II	Jan. 1859 – July 1859	7	4.0	II	May 1888 – Jan. 1889	9	5.0
III	Aug. 1859 – Feb. 1860	7	7.0	III	Feb. 1889 – Sep. 1889	8	8.5
IV	Mar. 1860 – Sep. 1860	7	7.0	IV	Oct. 1889 – June 1890	9	8.5
V	Sep. 1860 – Nov. 1860	3	4.0	V	June 1890 – Aug. 1890	3	5.0
VI	Nov. 1860 – Dec. 1860	2	1.5	VI	Aug. 1890 – Oct. 1890	3	2.0
VII	Jan. 1861 – Mar. 1861	3	2.5	VII	Nov. 1890 – Jan. 1891	3	3.0
VIII	Apr. 1861 – May 1861	2	2.5	VIII	Feb. 1891 – Apr. 1891	3	3.0
IX	May 1861 – July 1861	3	1.5	IX	Apr. 1891 – June 1891	3	2.0
I	May 1861 – July 1861	3	...	I	Apr. 1891 – June 1891	3	...
II	July 1861 – Sep. 1862	15	8.0	II	June 1891 – Nov. 1891	6	3.5
III	Oct. 1862 – Dec. 1863	15	15.0	III	Dec. 1891 – June 1892	7	6.5
IV	Jan. 1864 – Mar. 1865	15	15.0	IV	July 1892 – Dec. 1892	6	6.5
V	Mar. 1865 – May 1865	3	8.0	V	Dec. 1892 – Feb. 1893	3	3.5
VI	May 1865 – Feb. 1866	10	5.5	VI	Feb. 1893 – June 1893	5	3.0
VII	Mar. 1866 – Jan. 1867	11	10.5	VII	July 1893 – Dec. 1893	6	5.5
VIII	Feb. 1867 – Nov. 1867	10	10.5	VIII	Jan. 1894 – May 1894	5	5.5
IX	Nov. 1867 – Jan. 1868	3	5.5	IX	May 1894 – July 1894	3	3.0
I	Nov. 1867 – Jan. 1868	3	...	I	May 1894 – July 1894	3	...
II	Jan. 1868 – June 1868	6	3.5	II	July 1894 – Dec. 1894	6	3.5
III	July 1868 – Nov. 1868	5	5.5	III	Jan. 1895 – May 1895	5	5.5
IV	Dec. 1868 – May 1869	6	5.5	IV	June 1895 – Nov. 1895	6	5.5
V	May 1869 – July 1869	3	3.5	V	Nov. 1895 – Jan. 1896	3	3.5
VI	July 1869 – Dec. 1869	6	3.5	VI	Jan. 1896 – June 1896	6	3.5
VII	Jan. 1870 – May 1870	5	5.5	VII	July 1896 – Nov. 1896	5	5.5
VIII	June 1870 – Nov. 1870	6	5.5	VIII	Dec. 1896 – May 1897	6	5.5
IX	Nov. 1870 – Jan. 1871	3	3.5	IX	May 1897 – July 1897	3	3.5
I	Nov. 1870 – Jan. 1871	3	...	I	May 1897 – July 1897	3	...
II	Jan. 1871 – Nov. 1871	11	6.0	II	July 1897 – Feb. 1898	8	4.5
III	Dec. 1871 – Oct. 1872	11	11.0	III	Mar. 1898 – Sep. 1898	7	7.5
IV	Nov. 1872 – Sep. 1873	11	11.0	IV	Oct. 1898 – May 1899	8	7.5
V	Sep. 1873 – Nov. 1873	3	6.0	V	May 1899 – July 1899	3	4.5
VI	Nov. 1873 – July 1875	21	11.0	VI	July 1899 – Dec. 1899	6	3.5
VII	Aug. 1875 – May 1877	22	21.5	VII	Jan. 1900 – May 1900	5	5.5
VIII	June 1877 – Feb. 1879	21	21.5	VIII	June 1900 – Nov. 1900	6	5.5
IX	Feb. 1879 – Apr. 1879	3	11.0	IX	Nov. 1900 – Jan. 1901	3	3.5
I	Feb. 1879 – Apr. 1879	3	...	I	Nov. 1900 – Jan. 1901	3	...
II	Apr. 1879 – Mar. 1880	12	6.5	II	Jan. 1901 – July 1901	7	4.0
III	Apr. 1880 – Feb. 1881	11	11.5	III	Aug. 1901 – Jan. 1902	6	6.5
IV	Mar. 1881 – Feb. 1882	12	11.5	IV	Feb. 1902 – Aug. 1902	7	6.5
V	Feb. 1882 – Apr. 1882	3	6.5	V	Aug. 1902 – Oct. 1902	3	4.0
VI	Apr. 1882 – Mar. 1883	12	6.5	VI	Oct. 1902 – Apr. 1903	7	4.0
VII	Apr. 1883 – Apr. 1884	13	12.5	VII	May 1903 – Dec. 1903	8	7.5
VIII	May 1884 – Apr. 1885	12	12.5	VIII	Jan. 1904 – July 1904	7	7.5
IX	Apr. 1885 – June 1885	3	6.5	IX	July 1904 – Sep. 1904	3	4.0

TABLE A1 — *Continued*

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)	Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
			United States — *Continued*				
I	July 1904 – Sep. 1904	3	...	I	June 1924 – Aug. 1924	3	...
II	Sep. 1904 – July 1905	11	6.0	II	Aug. 1924 – Apr. 1925	9	5.0
III	Aug. 1905 – May 1906	10	10.5	III	May 1925 – Dec. 1925	8	8.5
IV	June 1906 – Apr. 1907	11	10.5	IV	Jan. 1926 – Sep. 1926	9	8.5
V	Apr. 1907 – June 1907	3	6.0	V	Sep. 1926 – Nov. 1926	3	5.0
VI	June 1907 – Sep. 1907	4	2.5	VI	Nov. 1926 – Feb. 1927	4	2.5
VII	Oct. 1907 – Jan. 1908	4	4.0	VII	Mar. 1927 – July 1927	5	4.5
VIII	Feb. 1908 – May 1908	4	4.0	VIII	Aug. 1927 – Nov. 1927	4	4.5
IX	May 1908 – July 1908	3	2.5	IX	Nov. 1927 – Jan. 1928	3	2.5
I	May 1908 – July 1908	3	...	I	Nov. 1927 – Jan. 1928	3	...
II	July 1908 – Dec. 1908	6	3.5	II	Jan. 1928 – June 1928	6	3.5
III	Jan. 1909 – June 1909	6	6.0	III	July 1928 – Nov. 1928	5	5.5
IV	July 1909 – Dec. 1909	6	6.0	IV	Dec. 1928 – May 1929	6	5.5
V	Dec. 1909 – Feb. 1910	3	3.5	V	May 1929 – July 1929	3	3.5
VI	Feb. 1910 – Sep. 1910	8	4.5	VI	July 1929 – Sep. 1930	15	8.0
VII	Oct. 1910 – Apr. 1911	7	7.5	VII	Oct. 1930 – Nov. 1931	14	14.5
VIII	May 1911 – Dec. 1911	8	7.5	VIII	Dec. 1931 – Feb. 1933	15	14.5
IX	Dec. 1911 – Feb. 1912	3	4.5	IX	Feb. 1933 – Apr. 1933	3	8.0
I	Dec. 1911 – Feb. 1912	3	...	I	Feb. 1933 – Apr. 1933	3	...
II	Feb. 1912 – May 1912	4	2.5	II	Apr. 1933 – July 1934	16	8.5
III	June 1912 – Aug. 1912	3	3.5	III	Aug. 1934 – Dec. 1935	17	16.5
IV	Sep. 1912 – Dec. 1912	4	3.5	IV	Jan. 1936 – Apr. 1937	16	16.5
V	Dec. 1912 – Feb. 1913	3	2.5	V	Apr. 1937 – June 1937	3	8.5
VI	Feb. 1913 – Aug. 1913	7	4.0	VI	June 1937 – Sep. 1937	4	2.5
VII	Sep. 1913 – Apr. 1914	8	7.5	VII	Oct. 1937 – Dec. 1937	3	3.5
VIII	May 1914 – Nov. 1914	7	7.5	VIII	Jan. 1938 – Apr. 1938	4	3.5
IX	Nov. 1914 – Jan. 1915	3	4.0	IX	Apr. 1938 – June 1938	3	2.5
I	Nov. 1914 – Jan. 1915	3	...				
II	Jan. 1915 – Feb. 1916	14	7.5				
III	Mar. 1916 – May 1917	15	14.5				
IV	June 1917 – July 1918	14	14.5				
V	July 1918 – Sep. 1918	3	7.5				
VI	Sep. 1918 – Oct. 1918	2	1.5				
VII	Nov. 1918 – Jan. 1919	3	2.5				
VIII	Feb. 1919 – Mar. 1919	2	2.5				
IX	Mar. 1919 – May 1919	3	1.5				
I	Mar. 1919 – May 1919	3	...				
II	May 1919 – July 1919	3	2.0				
III	Aug. 1919 – Sep. 1919	2	2.5				
IV	Oct. 1919 – Dec. 1919	3	2.5				
V	Dec. 1919 – Feb. 1920	3	2.0				
VI	Feb. 1920 – July 1920	6	3.5				
VII	Aug. 1920 – Feb. 1921	7	6.5				
VIII	Mar. 1921 – Aug. 1921	6	6.5				
IX	Aug. 1921 – Oct. 1921	3	3.5				
I	Aug. 1921 – Oct. 1921	3	...				
II	Oct. 1921 – Mar. 1922	6	3.5				
III	Apr. 1922 – Oct. 1922	7	6.5				
IV	Nov. 1922 – Apr. 1923	6	6.5				
V	Apr. 1923 – June 1923	3	3.5				
VI	June 1923 – Sep. 1923	4	2.5				
VII	Oct. 1923 – Feb. 1924	5	4.5				
VIII	Mar. 1924 – June 1924	4	4.5				
IX	June 1924 – Aug. 1924	3	2.5				

TABLE A1 — *Continued*

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)	Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
			Great Britain				
I	Nov. 1854 – Jan. 1855	3	...	I	Jan. 1895 – Mar. 1895	3	...
II	Jan. 1855 – Nov. 1855	11	6.0	II	Mar. 1895 – Nov. 1896	21	11.0
III	Dec. 1855 – Sep. 1856	10	10.5	III	Dec. 1896 – Aug. 1898	21	21.0
IV	Oct. 1856 – Aug. 1857	11	10.5	IV	Sep. 1898 – May 1900	21	21.0
V	Aug. 1857 – Oct. 1857	3	6.0	V	May 1900 – July 1900	3	11.0
VI	Oct. 1857 – Nov. 1857	2	1.5	VI	July 1900 – Nov. 1900	5	3.0
VII	Dec. 1857	1	1.5	VII	Dec. 1900 – Mar. 1901	4	4.5
VIII	Jan. 1858 – Feb. 1858	2	1.5	VIII	Apr. 1901 – Aug. 1901	5	4.5
IX	Feb. 1858 – Apr. 1858	3	1.5	IX	Aug. 1901 – Oct. 1901	3	3.0
I	Feb. 1858 – Apr. 1858	3	...	I	Aug. 1901 – Oct. 1901	3	...
II	Apr. 1858 – Jan. 1859	10	5.5	II	Oct. 1901 – Apr. 1902	7	4.0
III	Feb. 1859 – Oct. 1859	9	9.5	III	May 1902 – Oct. 1902	6	6.5
IV	Nov. 1859 – Aug. 1860	10	9.5	IV	Nov. 1902 – May 1903	7	6.5
V	Aug. 1860 – Oct. 1860	3	5.5	V	May 1903 – July 1903	3	4.0
VI	Oct. 1860 – June 1861	9	5.0	VI	July 1903 – Nov. 1903	5	3.0
VII	July 1861 – Feb. 1862	8	8.5	VII	Dec. 1903 – May 1904	6	5.5
VIII	Mar. 1862 – Nov. 1862	9	8.5	VIII	June 1904 – Oct. 1904	5	5.5
IX	Nov. 1862 – Jan. 1863	3	5.0	IX	Oct. 1904 – Dec. 1904	3	3.0
I	Nov. 1862 – Jan. 1863	3	...	I	Oct. 1904 – Dec. 1904	3	...
II	Jan. 1863 – Jan. 1864	13	7.0	II	Dec. 1904 – Sep. 1905	10	5.5
III	Feb. 1864 – Jan. 1865	12	12.5	III	Oct. 1905 – July 1906	10	10.0
IV	Feb. 1865 – Feb. 1866	13	12.5	IV	Aug. 1906 – May 1907	10	10.0
V	Feb. 1866 – Apr. 1866	3	7.0	V	May 1907 – July 1907	3	5.5
VI	Apr. 1866 – Nov. 1866	8	4.5	VI	July 1907 – Nov. 1907	5	3.0
VII	Dec. 1866 – June 1867	7	7.5	VII	Dec. 1907 – May 1908	6	5.5
VIII	July 1867 – Feb. 1868	8	7.5	VIII	June 1908 – Oct. 1908	5	5.5
IX	Feb. 1868 – Apr. 1868	3	4.5	IX	Oct. 1908 – Dec. 1908	3	3.0
I	Feb. 1868 – Apr. 1868	3	...	I	Oct. 1908 – Dec. 1908	3	...
II	Apr. 1868 – Sep. 1869	18	9.5	II	Dec. 1908 – Mar. 1910	16	8.5
III	Oct. 1869 – Feb. 1871	17	17.5	III	Apr. 1910 – July 1911	16	16.0
IV	Mar. 1871 – Aug. 1872	18	17.5	IV	Aug. 1911 – Nov. 1912	16	16.0
V	Aug. 1872 – Oct. 1872	3	9.5	V	Nov. 1912 – Jan. 1913	3	8.5
VI	Oct. 1872 – Dec. 1874	27	14.0	VI	Jan. 1913 – July 1913	7	4.0
VII	Jan. 1875 – Feb. 1877	26	26.5	VII	Aug. 1913 – Jan. 1914	6	6.5
VIII	Mar. 1877 – May 1879	27	26.5	VIII	Feb. 1914 – Aug. 1914	7	6.5
IX	May 1879 – July 1879	3	14.0	IX	Aug. 1914 – Oct. 1914	3	4.0
I	May 1879 – July 1879	3	...	I	Aug. 1914 – Oct. 1914	3	...
II	July 1879 – Aug. 1880	14	7.5	II	Oct. 1914 – Jan. 1916	16	8.5
III	Sep. 1880 – Sep. 1881	13	13.5	III	Feb. 1916 – May 1917	16	16.0
IV	Oct. 1881 – Nov. 1882	14	13.5	IV	June 1917 – Sep. 1918	16	16.0
V	Nov. 1882 – Jan. 1883	3	7.5	V	Sep. 1918 – Nov. 1918	3	8.5
VI	Jan. 1883 – Feb. 1884	14	7.5	VI	Nov. 1918 – Dec. 1918	2	1.5
VII	Mar. 1884 – Mar. 1885	13	13.5	VII	Jan. 1919	1	1.5
VIII	Apr. 1885 – May 1886	14	13.5	VIII	Feb. 1919 – Mar. 1919	2	1.5
IX	May 1886 – July 1886	3	7.5	IX	Mar. 1919 – May 1919	3	1.5
I	May 1886 – July 1886	3	...	I	Mar. 1919 – May 1919	3	...
II	July 1886 – Nov. 1887	17	9.0	II	May 1919 – July 1919	3	2.0
III	Dec. 1887 – Mar. 1889	16	16.5	III	Aug. 1919 – Nov. 1919	4	3.5
IV	Apr. 1889 – Aug. 1890	17	16.5	IV	Dec. 1919 – Feb. 1920	3	3.5
V	Aug. 1890 – Oct. 1890	3	9.0	V	Feb. 1920 – Apr. 1920	3	2.0
VI	Oct. 1890 – Feb. 1892	17	9.0	VI	Apr. 1920 – Aug. 1920	5	3.0
VII	Mar. 1892 – Aug. 1893	18	17.5	VII	Sep. 1920 – Dec. 1920	4	4.5
VIII	Sep. 1893 – Jan. 1895	17	17.5	VIII	Jan. 1921 – May 1921	5	4.5
IX	Jan. 1895 – Mar. 1895	3	9.0	IX	May 1921 – July 1921	3	3.0

TABLE A1 — *Continued*

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)	Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
			Great Britain — *Continued*				
I	May 1921 – July 1921	3	...	I	Aug. 1928 – Oct. 1928	3	...
II	July 1921 – July 1922	13	7.0	II	Oct. 1928 – Dec. 1928	3	2.0
III	Aug. 1922 – Sep. 1923	14	13.5	III	Jan. 1929 – Mar. 1929	3	3.0
IV	Oct. 1923 – Oct. 1924	13	13.5	IV	Apr. 1929 – June 1929	3	3.0
V	Oct. 1924 – Dec. 1924	3	7.0	V	June 1929 – Aug. 1929	3	2.0
VI	Dec. 1924 – May 1925	6	3.5	VI	Aug. 1929 – July 1930	12	6.5
VII	June 1925 – Dec. 1925	7	6.5	VII	Aug. 1930 – July 1931	12	12.0
VIII	Jan. 1926 – June 1926	6	6.5	VIII	Aug. 1931 – July 1932	12	12.0
IX	June 1926 – Aug. 1926	3	3.5	IX	July 1932 – Sep. 1932	3	6.5
I	June 1926 – Aug. 1926	3	...	I	July 1932 – Sep. 1932	3	...
II	Aug. 1926 – Sep. 1926	2	1.5	II	Sep. 1932 – Apr. 1934	20	10.5
III	Oct. 1926 – Dec. 1926	3	2.5	III	May 1934 – Dec. 1935	20	20.0
IV	Jan. 1927 – Feb. 1927	2	2.5	IV	Jan. 1936 – Aug. 1937	20	20.0
V	Feb. 1927 – Apr. 1927	3	1.5	V	Aug. 1937 – Oct. 1937	3	10.5
VI	Apr. 1927 – Sep. 1927	6	3.5	VI	Oct. 1937 – Jan. 1938	4	2.5
VII	Oct. 1927 – Feb. 1928	5	5.5	VII	Feb. 1938 – Apr. 1938	3	3.5
VIII	Mar. 1928 – Aug. 1928	6	5.5	VIII	May 1938 – Aug. 1938	4	3.5
IX	Aug. 1928 – Oct. 1928	3	3.5	IX	Aug. 1938 – Oct. 1938	3	2.5
			France				
I	Nov. 1865 – Jan. 1866	3	...	I	Aug. 1879 – Oct. 1879	3	...
II	Jan. 1866 – July 1866	7	4.0	II	Oct. 1879 – June 1880	9	5.0
III	Aug. 1866 – Mar. 1867	8	7.5	III	July 1880 – Feb. 1881	8	8.5
IV	Apr. 1867 – Oct. 1867	7	7.5	IV	Mar. 1881 – Nov. 1881	9	8.5
V	Oct. 1867 – Dec. 1867	3	4.0	V	Nov. 1881 – Jan. 1882	3	5.0
VI	Dec. 1867 – Feb. 1868	3	2.0	VI	Jan. 1882 – Oct. 1883	22	11.5
VII	Mar. 1868 – June 1868	4	3.5	VII	Nov. 1883 – Sep. 1885	23	22.5
VIII	July 1868 – Sep. 1868	3	3.5	VIII	Oct. 1885 – July 1887	22	22.5
IX	Sep. 1868 – Nov. 1868	3	2.0	IX	July 1887 – Sep. 1887	3	11.5
I	Sep. 1868 – Nov. 1868	3	...	I	July 1887 – Sep. 1887	3	...
II	Nov. 1868 – May 1869	7	4.0	II	Sep. 1887 – Sep. 1888	13	7.0
III	June 1869 – Dec. 1869	7	7.0	III	Oct. 1888 – Nov. 1889	14	13.5
IV	Jan. 1870 – July 1870	7	7.0	IV	Dec. 1889 – Dec. 1890	13	13.5
V	July 1870 – Sep. 1870	3	4.0	V	Dec. 1890 – Feb. 1891	3	7.0
VI	Sep. 1870 – Feb. 1871	6	3.5	VI	Feb. 1891 – May 1892	16	8.5
VII	Mar. 1871 – July 1871	5	5.5	VII	June 1892 – Aug. 1893	15	15.5
VIII	Aug. 1871 – Jan. 1872	6	5.5	VIII	Sep. 1893 – Dec. 1894	16	15.5
IX	Jan. 1872 – Mar. 1872	3	3.5	IX	Dec. 1894 – Feb. 1895	3	8.5
I	Jan. 1872 – Mar. 1872	3	...	I	Dec. 1894 – Feb. 1895	3	...
II	Mar. 1872 – Aug. 1872	6	3.5	II	Feb. 1895 – Sep. 1896	20	10.5
III	Sep. 1872 – Feb. 1873	6	6.0	III	Oct. 1896 – June 1898	21	20.5
IV	Mar. 1873 – Aug. 1873	6	6.0	IV	July 1898 – Feb. 1900	20	20.5
V	Aug. 1873 – Oct. 1873	3	3.5	V	Feb. 1900 – Apr. 1900	3	10.5
VI	Oct. 1873 – Aug. 1874	11	6.0	VI	Apr. 1900 – Jan. 1901	10	5.5
VII	Sep. 1874 – Aug. 1875	12	11.5	VII	Feb. 1901 – Oct. 1901	9	9.5
VIII	Sep. 1875 – July 1876	11	11.5	VIII	Nov. 1901 – Aug. 1902	10	9.5
IX	July 1876 – Sep. 1876	3	6.0	IX	Aug. 1902 – Oct. 1902	3	5.5
I	July 1876 – Sep. 1876	3	...	I	Aug. 1902 – Oct. 1902	3	...
II	Sep. 1876 – Feb. 1877	6	3.5	II	Oct. 1902 – Nov. 1902	2	1.5
III	Mar. 1877 – Sep. 1877	7	6.5	III	Dec. 1902 – Feb. 1903	3	2.5
IV	Oct. 1877 – Mar. 1878	6	6.5	IV	Mar. 1903 – Apr. 1903	2	2.5
V	Mar. 1878 – May 1878	3	3.5	V	Apr. 1903 – June 1903	3	1.5
VI	May 1878 – Sep. 1878	5	3.0	VI	June 1903 – Oct. 1903	5	3.0
VII	Oct. 1878 – Mar. 1879	6	5.5	VII	Nov. 1903 – Apr. 1904	6	5.5
VIII	Apr. 1879 – Aug. 1879	5	5.5	VIII	May 1904 – Sep. 1904	5	5.5
IX	Aug. 1879 – Oct. 1879	3	3.0	IX	Sep. 1904 – Nov. 1904	3	3.0

TABLE A1 — *Continued*

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
	France — *Continued*		
I	Sep. 1904 – Nov. 1904	3	. . .
II	Nov. 1904 – Sep. 1905	11	6.0
III	Oct. 1905 – July 1906	10	10.5
IV	Aug. 1906 – June 1907	11	10.5
V	June 1907 – Aug. 1907	3	6.0
VI	Aug. 1907 – Jan. 1908	6	3.5
VII	Feb. 1908 – July 1908	6	6.0
VIII	Aug. 1908 – Jan. 1909	6	6.0
IX	Jan. 1909 – Mar. 1909	3	3.5
I	Jan. 1909 – Mar. 1909	3	. . .
II	Mar. 1909 – July 1910	17	9.0
III	Aug. 1910 – Dec. 1911	17	17.0
IV	Jan. 1912 – May 1913	17	17.0
V	May 1913 – July 1913	3	9.0
VI	July 1913 – Oct. 1913	4	2.5
VII	Nov. 1913 – Mar. 1914	5	4.5
VIII	Apr. 1914 – July 1914	4	4.5
IX	July 1914 – Sep. 1914	3	2.5
I	July 1914 – Sep. 1914	3	. . .
II	Sep. 1914 – Nov. 1915	15	8.0
III	Dec. 1915 – Feb. 1917	15	15.0
IV	Mar. 1917 – May 1918	15	15.0
V	May 1918 – July 1918	3	8.0
VI	July 1918 – Sep. 1918	3	2.0
VII	Oct. 1918 – Dec. 1918	3	3.0
VIII	Jan. 1919 – Mar. 1919	3	3.0
IX	Mar. 1919 – May 1919	3	2.0
I	Mar. 1919 – May 1919	3	. . .
II	May 1919 – Sep. 1919	5	3.0
III	Oct. 1919 – Mar. 1920	6	5.5
IV	Apr. 1920 – Aug. 1920	5	5.5
V	Aug. 1920 – Oct. 1920	3	3.0
VI	Oct. 1920 – Dec. 1920	3	2.0
VII	Jan. 1921 – Mar. 1921	3	3.0
VIII	Apr. 1921 – June 1921	3	3.0
IX	June 1921 – Aug. 1921	3	2.0
I	June 1921 – Aug. 1921	3	. . .
II	Aug. 1921 – Aug. 1922	13	7.0
III	Sep. 1922 – Aug. 1923	12	12.5
IV	Sep. 1923 – Sep. 1924	13	12.5
V	Sep. 1924 – Nov. 1924	3	7.0
VI	Nov. 1924 – Dec. 1924	2	1.5
VII	Jan. 1925 – Mar. 1925	3	2.5
VIII	Apr. 1925 – May 1925	2	2.5
IX	May 1925 – July 1925	3	1.5
I	May 1925 – July 1925	3	. . .
II	July 1925 – Nov. 1925	5	3.0
III	Dec. 1925 – Apr. 1926	5	5.0
IV	May 1926 – Sep. 1926	5	5.0
V	Sep. 1926 – Nov. 1926	3	3.0
VI	Nov. 1926 – Dec. 1926	2	1.5
VII	Jan. 1927 – Mar. 1927	3	2.5
VIII	Apr. 1927 – May 1927	2	2.5
IX	May 1927 – July 1927	3	1.5
I	May 1927 – July 1927	3	. . .
II	July 1927 – May 1928	11	6.0
III	June 1928 – Mar. 1929	10	10.5
IV	Apr. 1929 – Feb. 1930	11	10.5
V	Feb. 1930 – Apr. 1930	3	6.0
VI	Apr. 1930 – Dec. 1930	9	5.0
VII	Jan. 1931 – Sep. 1931	9	9.0
VIII	Oct. 1931 – June 1932	9	9.0
IX	June 1932 – Aug. 1932	3	5.0
I	June 1932 – Aug. 1932	3	. . .
II	Aug. 1932 – Nov. 1932	4	2.5
III	Dec. 1932 – Feb. 1933	3	3.5
IV	Mar. 1933 – June 1933	4	3.5
V	June 1933 – Aug. 1933	3	2.5
VI	Aug. 1933 – Feb. 1934	7	4.0
VII	Mar. 1934 – Aug. 1934	6	6.5
VIII	Sep. 1934 – Mar. 1935	7	6.5
IX	Mar. 1935 – May 1935	3	4.0
I	Mar. 1935 – May 1935	3	. . .
II	May 1935 – Dec. 1935	8	4.5
III	Jan. 1936 – Sep. 1936	9	8.5
IV	Oct. 1936 – May 1937	8	8.5
V	May 1937 – July 1937	3	4.5
VI	July 1937 – Oct. 1937	4	2.5
VII	Nov. 1937 – Mar. 1938	5	4.5
VIII	Apr. 1938 – July 1938	4	4.5
IX	July 1938 – Sep. 1938	3	2.5

TABLE A1 – *Continued*

Division of Monthly Reference Cycles into Nine Stages, Four Countries

Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)	Stage	Period covered[a]	No. of months in stage	Interval from stage to stage[b] (mos.)
			Germany				
I	Jan. 1879 – Mar. 1879	3	...	I	July 1914 – Sep. 1914	3	...
II	Mar. 1879 – Jan. 1880	11	6.0	II	Sep. 1914 – Nov. 1915	15	8.0
III	Feb. 1880 – Jan. 1881	12	11.5	III	Dec. 1915 – Feb. 1917	15	15.0
IV	Feb. 1881 – Dec. 1881	11	11.5	IV	Mar. 1917 – May 1918	15	15.0
V	Dec. 1881 – Feb. 1882	3	6.0	V	May 1918 – July 1918	3	8.0
VI	Feb. 1882 – July 1883	18	9.5	VI	July 1918 – Oct. 1918	4	2.5
VII	Aug. 1883 – Jan. 1885	18	18.0	VII	Nov. 1918 – Jan. 1919	3	3.5
VIII	Feb. 1885 – July 1886	18	18.0	VIII	Feb. 1919 – May 1919	4	3.5
IX	July 1886 – Sep. 1886	3	9.5	IX	May 1919 – July 1919	3	2.5
I	July 1886 – Sep. 1886	3	...	I	May 1919 – July 1919	3	...
II	Sep. 1886 – Sep. 1887	13	7.0	II	July 1919 – May 1920	11	6.0
III	Oct. 1887 – Nov. 1888	14	13.5	III	June 1920 – May 1921	12	11.5
IV	Dec. 1888 – Dec. 1889	13	13.5	IV	June 1921 – Apr. 1922	11	11.5
V	Dec. 1889 – Feb. 1890	3	7.0	V	Apr. 1922 – June 1922	3	6.0
VI	Feb. 1890 – Sep. 1891	20	10.5	VI	June 1922 – Nov. 1922	6	3.5
VII	Oct. 1891 – May 1893	20	20.0	VII	Dec. 1922 – Apr. 1923	5	5.5
VIII	June 1893 – Jan. 1895	20	20.0	VIII	May 1923 – Oct. 1923	6	5.5
IX	Jan. 1895 – Mar. 1895	3	10.5	IX	Oct. 1923 – Dec. 1923	3	3.5
I	Jan. 1895 – Mar. 1895	3	...	I	Oct. 1923 – Dec. 1923	3	...
II	Mar. 1895 – Oct. 1896	20	10.5	II	Dec. 1923 – Apr. 1924	5	3.0
III	Nov. 1896 – June 1898	20	20.0	III	May 1924 – Sep. 1924	5	5.0
IV	July 1898 – Feb. 1900	20	20.0	IV	Oct. 1924 – Feb. 1925	5	5.0
V	Feb. 1900 – Apr. 1900	3	10.5	V	Feb. 1925 – Apr. 1925	3	3.0
VI	Apr. 1900 – Nov. 1900	8	4.5	VI	Apr. 1925 – July 1925	4	2.5
VII	Dec. 1900 – June 1901	7	7.5	VII	Aug. 1925 – Oct. 1925	3	3.5
VIII	July 1901 – Feb. 1902	8	7.5	VIII	Nov. 1925 – Feb. 1926	4	3.5
IX	Feb. 1902 – Apr. 1902	3	4.5	IX	Feb. 1926 – Apr. 1926	3	2.5
I	Feb. 1902 – Apr. 1902	3	...	I	Feb. 1926 – Apr. 1926	3	...
II	Apr. 1902 – Aug. 1902	5	3.0	II	Apr. 1926 – Mar. 1927	12	6.5
III	Sep. 1902 – Feb. 1903	6	5.5	III	Apr. 1927 – Mar. 1928	12	12.0
IV	Mar. 1903 – July 1903	5	5.5	IV	Apr. 1928 – Mar. 1929	12	12.0
V	July 1903 – Sep. 1903	3	3.0	V	Mar. 1929 – May 1929	3	6.5
VI	Sep. 1903 – Feb. 1904	6	3.5	VI	May 1929 – May 1930	13	7.0
VII	Mar. 1904 – July 1904	5	5.5	VII	June 1930 – June 1931	13	13.0
VIII	Aug. 1904 – Jan. 1905	6	5.5	VIII	July 1931 – July 1932	13	13.0
IX	Jan. 1905 – Mar. 1905	3	3.5	IX	July 1932 – Sep. 1932	3	7.0
I	Jan. 1905 – Mar. 1905	3	...				
II	Mar. 1905 – Nov. 1905	9	5.0				
III	Dec. 1905 – Sep. 1906	10	9.5				
IV	Oct. 1906 – June 1907	9	9.5				
V	June 1907 – Aug. 1907	3	5.0				
VI	Aug. 1907 – Dec. 1907	5	3.0				
VII	Jan. 1908 – June 1908	6	5.5				
VIII	July 1908 – Nov. 1908	5	5.5				
IX	Nov. 1908 – Jan. 1909	3	3.0				
I	Nov. 1908 – Jan. 1909	3	...				
II	Jan. 1909 – May 1910	17	9.0				
III	June 1910 – Oct. 1911	17	17.0				
IV	Nov. 1911 – Mar. 1913	17	17.0				
V	Mar. 1913 – May 1913	3	9.0				
VI	May 1913 – Sep. 1913	5	3.0				
VII	Oct. 1913 – Feb. 1914	5	5.0				
VIII	Mar. 1914 – July 1914	5	5.0				
IX	July 1914 – Sep. 1914	3	3.0				

[a] Inclusive of the dates given.

[b] That is, from the midpoint of the preceding stage to the midpoint of the given stage.

TABLE A2

Average Interval between Reference-cycle Stages during Selected Periods
United States

Period covered	No. of reference cycles	Average interval in months between stages								Average duration of cycles in months (sum of av. intervals)
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX	
		Expansion				Contraction				
		Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough	
1854–1938	21	4.8	8.3	8.3	4.8	3.9	6.9	6.9	3.9	47.7
1858–1938	20	4.7	8.3	8.3	4.7	3.9	6.9	6.9	3.9	47.6
1861–1938	19	4.8	8.3	8.3	4.8	4.0	7.2	7.2	4.0	48.6
1867–1938	18	4.6	8.0	8.0	4.6	3.9	7.0	7.0	3.9	46.9
1870–1938	17	4.6	8.1	8.1	4.6	4.0	7.1	7.1	4.0	47.6
1879–1938	16	4.6	7.9	7.9	4.6	3.5	6.2	6.2	3.5	44.4
1885–1938	15	4.4	7.7	7.7	4.4	3.3	5.7	5.7	3.3	42.4
1888–1938	14	4.5	7.8	7.8	4.5	3.4	5.9	5.9	3.4	42.9
1891–1938	13	4.4	7.7	7.7	4.4	3.5	6.1	6.1	3.5	43.4
1894–1938	12	4.5	7.8	7.8	4.5	3.5	6.1	6.1	3.5	43.9
1897–1938	11	4.6	8.0	8.0	4.6	3.5	6.2	6.2	3.5	44.6
1900–1938	10	4.6	8.0	8.0	4.6	3.6	6.2	6.2	3.6	44.9
1904–1938	9	4.7	8.2	8.2	4.7	3.5	6.1	6.1	3.5	45.0
1908–1938	8	4.5	7.9	7.9	4.5	3.6	6.4	6.4	3.6	44.9
1912–1938	7	4.6	8.2	8.2	4.6	3.5	6.2	6.2	3.5	45.1
1914–1938	6	5.0	9.0	9.0	5.0	3.4	6.0	6.0	3.4	46.8
1919–1938	5	4.5	7.9	7.9	4.5	3.8	6.7	6.7	3.8	45.8
1921–1938	4	5.1	9.2	9.2	5.1	3.9	6.8	6.8	3.9	50.0
1854–1933	20	4.6	7.9	7.9	4.6	4.0	7.0	7.0	4.0	47.0
1858–1933	19	4.5	7.8	7.8	4.5	4.0	7.1	7.1	4.0	46.9
1861–1933	18	4.6	7.9	7.9	4.6	4.1	7.4	7.4	4.1	47.8
1867–1933	17	4.4	7.5	7.5	4.4	4.0	7.2	7.2	4.0	46.1
1870–1933	16	4.4	7.6	7.6	4.4	4.1	7.3	7.3	4.1	46.7
1879–1933	15	4.3	7.4	7.4	4.3	3.6	6.3	6.3	3.6	43.2
1885–1933	14	4.1	7.1	7.1	4.1	3.4	5.9	5.9	3.4	41.0
1888–1933	13	4.2	7.1	7.1	4.2	3.5	6.0	6.0	3.5	41.5
1891–1933	12	4.1	7.0	7.0	4.1	3.6	6.3	6.3	3.6	41.8
1894–1933	11	4.1	7.0	7.0	4.1	3.6	6.4	6.4	3.6	42.3
1897–1933	10	4.2	7.2	7.2	4.2	3.6	6.4	6.4	3.6	42.9
1900–1933	9	4.2	7.1	7.1	4.2	3.7	6.6	6.6	3.7	43.0
1904–1933	8	4.2	7.2	7.2	4.2	3.6	6.4	6.4	3.6	42.9
1908–1933	7	3.9	6.7	6.7	3.9	3.8	6.8	6.8	3.8	42.4
1912–1933	6	4.0	6.8	6.8	4.0	3.7	6.7	6.7	3.7	42.3
1914–1933	5	4.3	7.5	7.5	4.3	3.6	6.5	6.5	3.6	43.8
1919–1933	4	3.5	5.8	5.8	3.5	4.1	7.5	7.5	4.1	41.8
1854–1914	15	4.7	8.1	8.1	4.7	4.1	7.2	7.2	4.1	48.0
1858–1914	14	4.6	8.0	8.0	4.6	4.1	7.3	7.3	4.1	48.0
1861–1914	13	4.7	8.0	8.0	4.7	4.3	7.7	7.7	4.3	49.4
1867–1914	12	4.4	7.5	7.5	4.4	4.2	7.5	7.5	4.2	47.0
1870–1914	11	4.5	7.6	7.6	4.5	4.3	7.6	7.6	4.3	48.0
1879–1914	10	4.3	7.3	7.3	4.3	3.6	6.2	6.2	3.6	42.9
1885–1914	9	4.1	6.8	6.8	4.1	3.3	5.6	5.6	3.3	39.4
1888–1914	8	4.1	6.8	6.8	4.1	3.4	5.8	5.8	3.4	40.0
1891–1914	7	3.9	6.6	6.6	3.9	3.6	6.1	6.1	3.6	40.4
1894–1914	6	4.0	6.6	6.6	4.0	3.7	6.2	6.2	3.7	41.0
1897–1914	5	4.1	6.8	6.8	4.1	3.7	6.4	6.4	3.7	42.0
1900–1914	4	4.0	6.6	6.6	4.0	3.8	6.6	6.6	3.8	42.0

Derived from Appendix Table A1, which shows intervals between successive stages of the monthly reference cycles.

APPENDIX B

Some Supporting Data

THIS APPENDIX is restricted to the barest essentials. If we attempted to present in full detail the cyclical measures of all series considered in the text, this volume would grow to unmanageable proportions.

Tables B1–B4 present cycle-by-cycle measures of the seven series analyzed in Chapters 9–12. Several of these series are analyzed also in Chapters 5–8. Tables B5–B9 show in detail the classifications of cycles carried through in Chapters 11 and 12.

The arrangement and content of Tables B1–B4 are governed, in the main, by the analysis of Chapters 11 and 12. In Tables B1 and B3 the cycles are treated on a positive basis, in Tables B2 and B4 on an inverted basis. Since the inverted analysis is restricted to the outer boundaries of the positive analysis, some inverted cycles are sacrificed. Our results, however, would not be altered appreciably by lifting this restriction.

TABLE B1

Measures of Successive Specific Cycles Treated on Positive Plan

Seven American Series

Cycle no.	Dates of specific cycles	Duration of specific cycles in months			Ratio of expansion to full cycle	Amplitude in specific-cycle relatives			Ratio of rise to total rise & fall	Average in specific-cycle relatives at stage*								
										I	II	III	IV	V	VI	VII	VIII	IX
										Initial trough	Expansion			Peak	Contraction			Terminal trough
	Trough – Peak – Trough	Expansion	Contraction	Full cycle		Rise	Fall	Rise & fall			First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	Deflated clearings																	
1	Mar.78–June 81–Jan. 82	39	7	46	.85	59.5	10.2	69.7	.85	72.2	78.2	97.3	109.2	131.7[a]	126.9	130.2	124.6	121.5
2	Jan. 82–June 83–Nov. 84	17	17	34	.50	6.3	11.9	18.2	.35	98.0	97.0	100.9	101.3	104.3	103.2	103.5	93.5	92.4
3	Nov. 84–June 87–Mar. 88	31	9	40	.78	36.6	6.5	43.1	.85	80.4	84.2	96.0	108.1	117.0	114.6	113.1	110.4	110.5
4	Mar. 88–Sep. 90–Mar. 91	30	6	36	.83	28.5	8.0	36.5	.78	85.5	90.6	96.4	107.4	114.0	112.7	105.2	106.4	106.0
5	Mar. 91–Feb. 93–Aug. 93	23	6	29	.79	16.9	26.0	42.9	.39	91.5	95.5	99.1	105.6	108.4	106.9	107.4	93.0	82.4
6	Aug. 93–Oct. 95–Aug. 96	26	10	36	.72	22.0	9.7	31.7	.69	87.9	91.6	98.5	104.5	109.9	108.0	102.3	103.8	100.2
7	Aug. 96–Sep. 99–Sep. 00	37	12	49	.76	35.6	6.2	41.8	.85	81.2	82.8	95.5	109.9	116.8	113.3	110.4	111.4	110.6
8	Sep. 00–July 03–May 04	34	10	44	.77	21.2	5.9	27.1	.78	85.2	95.3	100.4	102.7	106.4	104.3	100.3	101.7	100.5
9	May 04–Oct. 07–Dec. 07	41	2	43	.95	26.8	25.0	51.8	.52	83.5	91.7	101.2	108.0	110.3[a]	(96.8)	90.0	(88.4)	85.3[b]
10	Dec. 07–Mar. 10–Oct. 10	27	7	34	.79	31.0	3.3	34.3	.90	79.0[b]	89.4	98.3	106.8	110.0	107.4	107.9	106.5	106.7
11	Oct. 10–Feb. 13–Nov. 14	28	21	49	.57	11.1	11.9	23.0	.48	93.8	95.3	97.9	102.6	104.9	102.8	103.9	99.3	93.0
12	Nov. 14–Aug. 18–Dec. 18	45	4	49	.92	39.7	4.9	44.6	.89	77.3	86.2	101.2	108.9	117.0	113.4	116.8	112.3	112.1
13	Dec. 18–Jan. 20–May 21	13	16	29	.45	11.9	10.9	22.8	.52	93.3	93.9	102.7	104.1	105.2	102.0	101.4	96.1	94.3
14	May 21–May 23–Sep. 23	24	4	28	.86	19.2	5.7	24.9	.77	90.6	92.9	98.3	105.2	109.8	110.0	104.4	105.6	104.1
15	Sep. 23–Nov. 29–Mar. 33	74	40	114	.65	36.6	54.7	91.3	.40	86.5	92.4	103.8	116.2	123.1	108.0	93.9	74.8	68.4
	Pig iron production																	
1	Jan. 79–Feb. 83–Jan. 85	49	23	72	.68	64.0	24.6	88.6	.72	55.7	78.3	97.6	111.9	119.7	117.4	104.3	106.4	95.1
2	Jan. 85–Oct. 87–Mar. 88	33	5	38	.87	64.8	25.5	90.3	.72	67.0	74.5	104.2	113.1	131.8	130.0	120.8	105.2	106.3
3	Mar. 88–May 90–Apr. 91	26	11	37	.70	50.2	47.4	97.6	.51	74.0	83.5	95.2	109.9	124.2	122.6	117.3	87.2	76.8
4	Apr. 91–Feb. 92–Oct. 93	10	20	30	.33	45.4	63.7	109.1	.42	67.5	92.3	111.9	112.1	112.9	106.0	101.8	89.1	49.2
5	Oct. 93–Nov. 95–Oct. 96	25	11	36	.69	87.2	60.8	148.0	.59	51.5	65.0	98.6	116.4	138.7	131.1	120.1	94.9	77.9

Pig iron production—*Continued*

6	Oct. 96–Dec. 99–Oct. 00	38	10	48	.79	76.0	30.5	106.5	.71	52.3	74.5	97.7	112.5	128.3	127.4	125.1	109.1	97.8
7	Oct. 00–June 03–Dec. 03	32	6	38	.84	49.0	50.0	99.0	.49	69.0	87.8	101.2	109.6	118.0	114.8	116.8	89.2	68.0
8	Dec. 03–July 07–Jan. 08	43	6	49	.88	73.6	62.7	136.3	.54	51.1	76.4	106.5	116.3	124.7	122.2	122.0	83.7	62.0
9	Jan. 08–Jan. 10–Dec. 10	24	11	35	.69	78.6	41.7	120.3	.65	59.8	64.0	89.8	120.4	138.4	132.2	118.0	104.3	96.7
10	Dec. 10–Jan. 13–Dec. 14	25	23	48	.52	45.0	56.8	101.8	.44	82.7	86.0	95.7	114.5	127.7	120.3	99.5	83.9	70.9
11	Dec. 14–Sep. 18–May 19	45	8	53	.85	61.9	40.3	102.2	.61	52.5	87.4	107.3	105.1	114.4	112.6	109.6	90.2	74.1
12	May 19–Sep. 20–July 21	16	10	26	.62	40.7	91.1	131.8	.31	90.2	95.6	115.9	122.4	130.9	119.2	80.6	46.9	39.8
13	July 21–May 23–July 24	22	14	36	.61	108.3	58.6	166.9	.65	38.9	54.6	84.9	124.7	147.2	137.9	113.5	105.5	88.6
14	July 24–July 26–Nov. 27	24	16	40	.60	36.5	18.7	55.2	.66	73.5	88.8	101.5	103.8	110.0	109.4	102.6	99.5	91.3
15	Nov. 27–July 29–Mar. 33	20	44	64	.31	50.0	148.9	198.9	.25	123.3	129.2	147.6	154.7	173.3	136.6	71.1	31.0	24.4

Freight car orders

1	4Q 70 – 4Q 71 – 4Q 73	12	24	36	.33	385.8	396.8	782.6	.49	14.8	30.2	57.1	30.2	400.6	54.1	107.1	121.4	3.8
2	4Q 73 – 4Q 74 – 4Q 77	12	36	48	.25	269.7	218.3	488.0	.55	3.2	40.2	37.4	11.7	272.9	123.1	94.6	108.1	54.6
3	4Q 77 – 2Q 81 – 4Q 84	42	42	84	.50	268.0	263.3	531.3	.50	10.4	54.5	104.2	158.0	278.4	125.0	80.5	58.7	15.1
4	4Q 84 – 1Q 88 – 1Q 89	39	12	51	.76	141.6	67.4	209.0	.68	12.2	49.7	91.1	128.8	153.8	141.0	146.9	130.8	86.4
5	1Q 89 – 1Q 90 – 1Q 91	12	12	24	.50	106.2	122.4	228.6	.46	57.8	86.5	128.4	104.8	164.0	131.8	91.9	42.9	41.6
6	1Q 91 – 1Q 92 – 4Q 94	12	33	45	.27	117.3	166.4	283.7	.41	75.2	79.9	149.1	172.9	192.5	130.1	83.0	44.3	26.1
7	4Q 94 – 2Q 95 – 4Q 96	6	18	24	.25	121.6	101.4	223.0	.55	19.4	(71.4)	97.4	(111.9)	141.0	120.5	131.4	79.9	39.6
8	4Q 96 – 3Q 99 – 1Q 00	33	6	39	.85	179.5	115.5	295.0	.61	22.9	44.0	97.2	142.5	202.4	(130.4)	94.4	(91.9)	86.9
9	1Q 00 – 2Q 02 – 4Q 03	27	18	45	.60	121.5	169.3	290.8	.42	62.2	94.2	91.0	116.4	183.7	115.1	109.3	62.4	14.4
10	4Q 03 – 3Q 05 – 1Q 08	21	30	51	.41	200.6	202.0	402.6	.50	11.6	46.2	81.9	120.6	212.2	143.9	136.6	46.0	10.2
11	1Q 08 – 3Q 09 – 3Q 10	18	12	30	.60	175.8	158.7	334.5	.53	17.1	54.8	47.0	69.1	192.9	180.2	185.6	120.8	34.2
12	3Q 10 – 4Q 12 – 4Q 14	27	24	51	.53	177.3	191.9	369.2	.48	33.0	51.9	101.4	123.0	210.3	122.4	115.7	72.3	18.4
13	4Q 14 – 4Q 16 – 1Q 18	24	15	39	.62	259.8	280.6	540.4	.48	25.5	109.0	121.5	123.0	285.3	51.8	46.8	25.7	4.7
14	1Q 18 – 2Q 18 – 3Q 19	3	15	18	.17	584.9	591.0	1175.9	.50	6.1	(152.3)	(298.6)	(444.8)	591.0	0.0	3.0	0.0	0.0
15	3Q 19 – 2Q 20 – 2Q 21	9	12	21	.43	238.0	214.8	452.8	.53	0.0	(24.0)	90.6	(166.7)	238.0	141.2	80.1	47.8	23.2
16	2Q 21 – 2Q 22 – 2Q 23	12	12	24	.50	173.0	148.9	321.9	.54	7.0	19.3	48.5	100.0	180.0	133.3	133.9	166.0	31.1
17	2Q 23 – 1Q 24 – 2Q 25[c]	9	15	24	.38	136.1	136.2	272.3	.50	42.5	(51.6)	62.0	(88.7)	178.6	100.0	152.4	50.0	42.4
18	1Q 25[d] – 4Q 25 – 1Q 28	9	27	36	.25	139.4	133.9	273.3	.51	48.4	(91.8)	106.4	(124.9)	187.8	95.2	84.4	98.0	53.9
19	1Q 28 – 4Q 29 – 1Q 33	21	39	60	.55	250.1	341.3	591.4	.42	91.3	109.0	174.6	261.6	341.4	100.5	25.9	4.2	0.1

TABLE B1—*Continued*

Measures of Successive Specific Cycles Treated on Positive Plan

Seven American Series

Cycle no.	Dates of specific cycles	Duration of specific cycles in months			Ratio of expansion to full cycle	Amplitude in specific-cycle relatives			Ratio of rise to total rise & fall	Average in specific-cycle relatives at stage*								
										I	II	III	IV	V	VI	VII	VIII	IX
										Initial trough	Expansion			Peak	Contraction			Terminal trough
	Trough – Peak – Trough	Expansion	Contraction	Full cycle		Rise	Fall	Rise & fall			First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	Railroad stock prices																	
1	Oct. 57–Mar. 58–May 59	5	14	19	.26	17.9	13.0	30.9	.58	89.3	92.1	97.3	107.8	107.2	103.1	100.6	98.4	94.2
2	May 59–Sep. 60–May 61	16	8	24	.67	23.8	21.9	45.7	.52	91.6	92.9	94.2	106.7	115.4	107.4	101.0	102.3	93.5
3	May 61–Apr. 64–Apr. 65	35	12	47	.74	84.7	33.0	117.7	.72	57.5	60.6	85.9	122.0	142.2	140.4	129.4	120.9	109.2
4	Apr. 65–Apr. 72–June 77	84	62	146	.58	42.0	54.9	96.9	.43	78.4	84.2	99.5	109.8	120.4	114.4	105.1	89.7	65.5
5	June 77–June 81–June 85	48	48	96	.50	78.0	44.2	122.2	.64	56.1	65.1	82.8	115.2	134.1	124.4	116.9	95.1	89.9
6	June 85–May 87–Apr. 89	23	23	46	.50	28.4	16.6	45.0	.63	83.0	93.4	100.5	108.3	111.4	103.5	97.4	97.0	94.8
7	Apr. 89–May 90–Dec. 90	13	7	20	.65	8.1	15.1	23.2	.35	96.7	98.3	101.6	101.3	104.8	105.1	101.0	92.4	89.7
8	Dec. 90–Mar. 92–Mar. 95	15	36	51	.29	17.8	30.2	48.0	.37	97.3	100.6	103.2	112.7	115.1	112.3	91.9	88.1	84.9
9	Mar. 95–Sep. 95–Aug. 96	6	11	17	.35	13.2	18.1	31.3	.42	94.4	99.2	104.1	106.7	107.6	102.5	98.0	95.5	89.5
10	Aug. 96–Apr. 00–Sep. 00	44	5	49	.90	45.4	5.1	50.5	.90	75.6	81.9	95.1	117.6	121.0	118.5	115.2	116.0	115.9
11	Sep. 00–Sep. 02–Oct. 03	24	13	37	.65	46.5	30.1	76.6	.61	70.3	86.4	101.3	109.7	116.8	111.2	104.5	90.8	86.7
12	Oct. 03–Sep. 06–Nov. 07	35	14	49	.71	44.3	37.7	82.0	.54	75.5	79.1	103.7	113.4	119.8	118.3	100.1	91.6	82.1
13	Nov. 07–Aug. 09–July 10	21	11	32	.66	37.8	15.2	53.0	.71	76.5	81.9	95.6	107.2	114.3	113.7	111.0	105.8	99.1
14	July 10–Aug. 12–Feb. 15	25	30	55	.45	6.5	26.9	33.4	.19	101.9	104.5	105.4	105.7	108.4	102.4	92.9	86.0c	81.5
15	Feb. 15–Oct. 16–Dec. 17	20	14	34	.59	14.7	29.6	44.3	.33	93.9	96.3	105.1	105.8	108.6	105.0	97.5	87.5	79.0
16	Dec. 17–June 19–June 21	18	24	42	.43	14.5	23.1	37.6	.39	97.5	99.8	106.3	106.8	112.0	101.0	94.4	94.6	88.9
17	June 21–Sep. 22–Oct. 23	15	13	28	.54	26.4	15.2	41.6	.63	86.6	88.9	93.0	105.6	113.0	107.5	106.6	98.8	97.8
18	Oct. 23–Sep. 29–June 32	71	33	104	.68	90.3	141.8	232.1	.39	67.5	78.2	103.9	132.2	157.8	132.4	94.7	41.8	16.0

Shares traded

1	Feb. 78–May 81–Apr. 85	39	47	86	.45	103.2	61.6	164.8	.63	31.8	51.4	87.7	118.3	135.0	120.4	110.4	103.6	73.4
2	Apr. 85–Nov. 85–Feb. 88	7	27	34	.21	62.0	83.0	145.0	.43	73.8	67.1	120.5	110.2	135.8	103.5	114.5	84.1	52.8
3	Feb. 88–June 89–Mar. 91	16	21	37	.43	51.1	37.8	88.9	.57	69.3	103.0	101.7	98.0	120.4	105.1	99.6	94.6	82.6
4	Mar. 91–Feb. 92–Jan. 95	11	35	46	.24	94.5	115.4	209.9	.45	79.9	95.6	110.6	121.3	174.4	112.4	105.0	67.4	59.0
5	Jan. 95–May 95–May 97	4	24	28	.14	61.5	67.8	129.3	.48	72.3	68.6	99.9	103.9	133.8	120.0	97.1	83.6	66.0[a]
6	May 97–Jan. 99–Aug. 00	20	19	39	.51	129.4	107.8	237.2	.55	29.4[a]	80.6	82.0	89.1	158.8	138.4	109.0	98.9	51.0
7	Aug. 00–May 01–May 04	9	36	45	.20	182.5	176.1	358.6	.51	35.2	76.3	145.6	181.9	217.7	100.3	91.9	78.7	41.6
8	May 04–Jan. 06–Nov. 07	20	22	42	.48	112.6	84.8	197.4	.57	32.4	93.6	113.0	98.6	145.0	114.7	106.9	71.6	60.2
9	Nov. 07–June 09–Apr. 11	19	22	41	.46	44.7	78.6	123.3	.36	81.5	92.5	109.3	107.9	126.2	124.8	100.3	64.2	47.6
10	Apr. 11–Sep. 11–Dec. 14	5	39	44	.11	71.0	110.2	181.2	.39	80.0	112.7	109.8	150.5	151.0	121.5	88.9	71.7°	40.8°
11	Dec. 14–Sep. 16–Apr. 18	21	19	40	.52	129.2	83.8	213.0	.61	23.3°	71.9	117.1	89.4	152.5	142.3	101.7	80.0	68.7
12	Apr. 18–July 19–Oct. 21	15	27	42	.36	143.9	134.7	278.6	.52	57.2	62.6	70.8	134.0	201.1	138.7	84.3	78.6	66.4
13	Oct. 21–Feb. 23–Oct. 23	16	8	24	.67	63.1	45.5	108.6	.58	62.0	86.8	120.4	101.2	125.1	101.8	97.9	77.8	79.6
14	Oct. 23–Nov. 25–May 26	25	6	31	.81	101.6	57.8	159.4	.64	51.8	61.3	104.5	117.1	153.4	133.7	117.2	112.0	95.6
15	May 26–Oct. 29–Mar. 33	41	41	82	.50	120.6	141.1	261.7	.46	50.7	71.9	104.9	163.3	171.3	118.4	80.9	57.9	30.2[a]

Call money rates

1	Oct. 58–Sep. 60–Oct. 62	23'	25	48	.48	51.3	31.9	83.2	.62	68.1	81.6	108.0	102.6	119.4	107.9	105.6	93.3	87.5
2	Oct. 62–Aug. 64–Oct. 66	22	26	48	.46	44.2	47.6	91.8	.48	79.9	94.0	107.5	107.8	124.1	104.6	99.2	87.6	76.5
3	Oct. 66–June 69–Mar. 71	32	21	53	.60	136.6	141.6	278.2	.49	64.5	79.7	92.6	121.5	201.1	122.8	72.2	81.2	59.5
4	Mar. 71–Sep. 73–Jan. 75	30	16	46	.65	315.8	337.3	653.1	.48	53.8	74.0	95.1	124.7	369.6	90.3	46.1	36.9	32.3
5	Jan. 75–Jan. 76–Sep. 76	12	8	20	.60	48.3	51.9	100.2	.48	78.6	92.3	106.4	97.7	126.9	104.9	107.0	87.4	75.0
6	Sep. 76–Jan. 78–Sep. 78	16	8	24	.67	86.8	66.5	153.3	.57	63.2	90.1	70.4	124.9	150.0	115.5	111.6	69.4	83.5
7	Sep. 78–Aug. 79–Oct. 80	11	14	25	.44	86.4	80.4	166.8	.52	67.0	75.4	68.5	94.1	153.4	148.9	104.8	76.6	73.0
8	Oct. 80–Feb. 81–May 82	4	15	19	.21	98.0	89.4	187.4	.52	64.8	91.8	154.5	73.0	162.8	87.4	100.3	77.2	73.4
9	May 82–Sep. 82–Jan. 85	4	28	32	.12	60.1	130.7	190.8	.31	105.5	132.3	123.8	153.4	165.6	128.1	61.4	90.4	34.9
10	Jan. 85–June 87–Aug. 88	29	14	43	.67	173.5	155.7	329.2	.53	38.6	43.2	89.2	152.4	212.1	140.7	103.4	64.3	56.4
11	Aug. 88–Aug. 90–May 92	24	21	45	.53	142.1	142.8	284.9	.50	46.8	71.0	111.0	141.8	188.9	103.4	87.3	64.3	46.1
12	May 92–June 93–Nov. 94	13	17	30	.43	159.6	192.0	351.6	.45	62.5	80.4	147.9	162.1	222.1	138.9	39.2	34.9	30.1
13	Nov. 94–Oct. 96–July 97	23	9	32	.72	199.1	170.7	369.8	.54	34.4	65.8	99.6	130.7	233.5	103.6	71.7	60.2	62.8
14	July 97–Oct. 99–Sep. 00	27	11	38	.71	151.0	129.1	280.1	.54	52.8	67.6	69.3	142.4	203.8	163.3	98.4	66.2	74.7
15	Sep. 00–Sep. 02–July 04	24	22	46	.52	149.0	173.0	322.0	.46	60.0	97.0	108.0	118.7	209.0	121.6	81.1	51.7	36.0

TABLE B1—*Continued*

Measures of Successive Specific Cycles Treated on Positive Plan

Seven American Series

Cycle no.	Dates of specific cycles	Duration of specific cycles in months			Ratio of expansion to full cycle	Amplitude in specific-cycle relatives			Ratio of rise to total rise & fall	Average in specific-cycle relatives at stage*								
										I	II	III	IV	V	VI	VII	VIII	IX
										Initial trough	Expansion			Peak	Contraction			Terminal trough
	Trough – Peak – Trough	Expansion	Contraction	Full cycle		Rise	Fall	Rise & fall			First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	Call money rates – *Continued*																	
16	July 04–Oct. 07–Nov. 08	39	13	52	.75	211.0	210.9	421.9	.50	31.6	55.9	133.7	120.7	242.6	131.9	43.2	33.5	31.7
17	Nov. 08–May 10–Nov. 11	18	18	36	.50	87.3	61.4	148.7	.59	51.0	73.5	110.9	119.9	138.3	99.4	87.8	104.4	76.9
18	Nov. 11–Dec. 12–Nov. 15	13	35	48	.27	83.7	101.6	185.3	.45	70.8	77.3	102.7	144.6	154.5	102.9	114.8	68.6	52.9
19	Nov. 15–Aug. 18–Dec. 18	33	4	37	.89	120.1	31.0	151.1	.79	43.7	65.7	85.7	129.9	163.8	153.1	146.3	132.9	132.8
20	Dec. 18–Nov. 19–June 22	11	31	42	.26	61.1	77.4	138.5	.44	76.0	81.0	93.5	99.6	137.1	126.0	105.1	74.9	59.7
21	June 22–Aug. 23–Sep. 24	14	13	27	.52	33.0	74.6	107.6	.31	90.6	100.4	110.7	117.1	123.6	112.0	101.3	57.4	49.0
22	Sep. 24–Feb. 26–Sep. 27	17	19	36	.47	66.2	27.0	93.2	.71	50.0	72.6	98.4	112.3	116.2	104.6	110.0	101.5	89.2
23	Sep. 27–Mar. 29–May 31	18	26	44	.41	102.4	145.5	247.9	.41	73.6	83.4	121.3	142.4	176.0	158.2	71.0	36.2	30.5
	Railroad bond yields																	
1	Aug. 60–Oct. 61–Feb. 63	14	16	30	.47	10.6	32.9	43.5	.24	99.2	103.5	106.0	108.4	109.8	105.2	95.2	84.5	76.9
2	Feb. 63–Jan. 64–July 64	11	6	17	.65	11.6	12.7	24.3	.48	93.9	96.8	100.6	102.6	105.5	104.0	98.5	97.4	92.8
3	July 64–Mar. 66–Nov. 66	20	8	28	.71	35.5	6.1	41.6	.85	75.3	83.8	100.5	107.4	110.8	109.6	106.7	104.8	104.7
4	Nov. 66–Dec. 67–July 68	13	7	20	.65	2.7	3.2	5.9	.46	98.7	100.1	100.9	100.3	101.4	99.8	99.1	98.4	98.2
5	July 68–Dec. 69–July 72	17	31	48	.35	7.9	8.8	16.7	.47	96.9	99.0	101.3	102.9	104.8	100.4	99.8	98.0	96.0
6	July 72–Nov. 73–Aug. 76	16	33	49	.33	5.3	22.5	27.8	.19	105.7	107.3	106.6	106.7	111.0	103.1	95.8	90.5	88.5
7	Aug. 76–Apr. 77–June 81	8	50	58	.14	2.4	24.5	26.9	.09	106.1	107.0	106.7	107.8	108.5	106.5	99.9	90.3	84.0
8	June 81–Sep. 83–July 86	27	34	61	.44	4.6	14.5	19.1	.24	99.2	101.7	102.4	103.1	103.8	102.0	99.9	92.6	89.3
9	July 86–Oct. 87–June 89	15	20	35	.43	5.1	9.3	14.4	.35	98.9	100.1	100.7	101.7	104.0	101.8	99.4	96.8	94.7
10	June 89–Aug. 91–July 92	26	11	37	.70	10.7	5.6	16.3	.66	94.3	95.8	98.9	103.2	105.0	103.9	101.1	99.9	99.4

Railroad bond yields—*Continued*

11	July 92–Aug. 93–Aug. 95	13	24	37	.35	8.1	15.7	23.8	.34	100.4	101.2	101.4	104.2	108.5	102.1	97.6	95.8	92.8
12	Aug. 95–Aug. 96–June 99	12	34	46	.26	6.0	15.9	21.9	.27	102.0	102.4	104.4	104.0	108.0	102.2	98.6	95.0	92.1
13	June 99–Sep. 03–Feb. 05	51	17	68	.75	14.9	3.8	18.7	.80	93.0	95.8	96.1	101.6	107.9	107.0	106.5	105.0	104.1
14	Feb. 05–Nov. 07–Feb. 09	33	15	48	.69	14.5	9.9	24.4	.59	94.7	95.2	97.7	102.1	109.2	105.7	103.2	100.5	99.3
15	Feb. 09–Dec. 13–June 14	58	6	64	.91	12.8	2.4	15.2	.84	93.4	96.4	99.3	103.0	106.2	104.1	103.8	103.7	103.8
16	June 14–Sep. 15–Jan. 17	15	16	31	.48	5.7	7.0	12.7	.45	97.9	98.5[e]	102.0	102.3	103.6	99.2	98.8	98.2	96.6
17	Jan. 17–Sep. 18–Nov. 18	20	2	22	.91	18.6	8.2	26.8	.69	88.2	92.6	101.3	105.1	106.8[a]	(105.4)	104.7	(102.7)	98.6[a]
18	Nov. 18–May 20–Sep. 22	18	28	46	.39	20.0	23.1	43.1	.46	91.7[a]	94.7	98.3	103.9	111.7	106.2	104.0	91.4	88.6
19	Sep. 22–Oct. 23–Dec. 27	13	50	63	.21	7.6	12.9	20.5	.37	97.2	100.2	103.5	104.1	104.8	102.5	99.9	95.7	91.9
20	Dec. 27–Sep. 29–May 31	21	20	41	.51	11.5	11.8	23.3	.49	94.4	96.7	101.3	104.7	105.9	102.8	98.6	95.9	94.1

* Entries in parentheses are interpolated.

[a] Average of two months.

[b] One month.

[c] Based on the first segment of the series (see Ch. 9, note 2), which we use through cycle 17.

[d] Based on the second segment (see preceding note).

[e] Omits Aug.–Nov. 1914 No data (N. Y. Stock Exchange closed).

TABLE B2

Measures of Successive Specific Cycles Treated on Inverted Plan

Seven American Series

Cycle no. (1)	Dates of specific cycles: Peak – Trough – Peak (2)	Duration of specific cycles: Full cycle (mos.) (3)	Duration of specific cycles: Ratio of expansion to full cycle (4)	Amplitude in specific-cycle relatives: Fall (5)	Amplitude: Rise (6)	Amplitude: Fall & rise (7)	Ratio of rise to total fall & rise (8)	Average in specific-cycle relatives at stage[*]: I Initial peak (9)	II Contraction First third (10)	III Contraction Middle third (11)	IV Contraction Last third (12)	V Trough (13)	VI Expansion First third (14)	VII Expansion Middle third (15)	VIII Expansion Last third (16)	IX Terminal peak (17)
	Deflated clearings															
1	June 81 – Jan. 82 – June 83	24	.71	8.1	6.3	14.4	.44	105.4[a]	101.5	104.2	99.7	97.3	96.3	100.2	100.6	103.6
2	June 83 – Nov. 84 – June 87	48	.65	11.1	39.3	50.4	.78	97.6	96.5	96.8	87.5	86.5	90.6	103.2	116.2	125.8
3	June 87 – Mar. 88 – Sep. 90	39	.77	5.3	29.7	35.0	.85	94.6	92.7	91.5	89.2	89.3	94.6	100.7	112.2	119.0
4	Sep. 90 – Mar. 91 – Feb. 93	29	.79	6.9	17.1	24.0	.71	99.6	98.5	91.9	93.0	92.7	96.8	100.4	107.0	109.8
5	Feb. 93 – Aug. 93 – Oct. 95	32	.81	27.8	22.0	49.8	.44	115.9	114.3	114.8	99.5	88.1	91.8	98.7	104.7	110.1
6	Oct. 95 – Aug. 96 – Sep. 99	47	.79	8.3	37.9	46.2	.82	95.0	93.4	88.5	89.8	86.7	88.3	101.9	117.2	124.6
7	Sep. 99 – Sep. 00 – July 03	46	.74	5.0	22.1	27.1	.82	93.8	91.0	88.7	89.4	88.8	99.3	104.6	107.1	110.9
8	July 03 – May 04 – Oct. 07	51	.80	5.0	27.5	32.5	.85	90.9	89.1	85.7	86.8	85.9	94.3	104.0	111.1	113.4[a]
9	Oct. 07 – Dec. 07 – Mar. 10	29	.93	23.7	31.8	55.5	.57	104.9[a]	(92.0)	85.6	(84.1)	81.2[b]	91.8	100.9	109.7	113.0
10	Mar. 10 – Oct. 10 – Feb. 13	35	.80	2.9	11.3	14.2	.80	98.9	96.5	97.0	95.8	96.0	97.4	100.1	104.9	107.3
11	Feb. 13 – Nov. 14 – Aug. 18	66	.68	10.5	42.3	52.8	.80	92.5	90.6	91.6	87.6	82.0	91.5	107.4	115.6	124.3
12	Aug. 18 – Dec. 18 – Jan. 20	17	.76	4.1	12.0	16.1	.75	98.4	95.4	98.2	94.4	94.3	94.9	103.8	105.2	106.3
13	Jan. 20 – May 21 – May 23	40	.60	10.7	19.6	30.3	.65	103.4	100.3	99.8	94.5	92.7	95.1	100.6	107.7	112.3
14	May 23 – Sep. 23 – Nov. 29	78	.95	4.5	35.4	39.9	.89	88.2	88.4	83.9	84.8	83.7	89.4	100.4	112.4	119.1
	Pig iron production															
1	Feb. 83 – Jan. 85 – Oct. 87	56	.59	19.5	73.0	92.5	.79	95.0	93.2	82.8	84.5	75.5	83.9	117.3	127.5	148.5
2	Oct. 87 – Mar. 88 – May 90	31	.84	18.9	53.4	72.3	.74	97.5	96.2	89.3	77.8	78.6	88.7	101.1	116.8	132.0
3	May 90 – Apr. 91 – Feb. 92	21	.48	41.9	45.8	87.7	.52	109.7	108.3	103.6	77.1	67.8	92.9	112.5	112.7	113.6
4	Feb. 92 – Oct. 93 – Nov. 95	45	.56	68.4	89.5	157.9	.57	121.2	113.7	109.2	95.5	52.8	66.7	101.1	119.4	142.3
5	Nov. 95 – Oct. 96 – Dec. 99	49	.78	45.0	83.7	128.7	.65	102.5	96.9	88.7	70.1	57.5	82.0	107.5	123.8	141.2

	Pig iron production—*Continued*																
6	Dec. 99 – Oct. 00 – June 03	42	.76	22.4	51.0	73.4	.69	94.4	93.8	92.1	80.3	72.0	91.6	105.6	114.3	123.0
7	June 03 – Dec. 03 – July 07	49	.88	38.8	76.0	114.8	.66	91.7	89.2	90.8	69.3	52.9	79.0	110.2	120.2	128.9
8	July 07 – Jan. 08 – Jan. 10	30	.80	64.6	84.1	148.7	.57	128.6	126.1	125.9	86.4	64.0	68.4	96.1	128.8	148.1
9	Jan. 10 – Dec. 10 – Jan. 13	36	.69	35.7	45.1	80.8	.56	118.8	113.5	101.3	89.5	83.1	86.3	96.1	115.0	128.2
10	Jan. 13 – Dec. 14 – Sep. 18	68	.66	46.1	67.9	114.0	.60	103.8	97.8	80.9	68.2	57.7	96.0	117.8	115.3	125.6
11	Sep. 18 – May 19 – Sep. 20	24	.67	42.4	35.0	77.4	.45	120.2	118.3	115.1	94.7	77.8	82.5	99.9	105.5	112.8
12	Sep. 20 – July 21 – May 23	32	.69	103.6	126.1	229.7	.55	149.0	135.6	91.7	53.3	45.4	63.6	98.9	145.3	171.5
13	May 23 – July 24 – July 26	38	.63	49.8	37.4	87.2	.43	124.9	117.0	96.3	89.5	75.1	90.8	103.7	106.1	112.5
14	July 26 – Nov. 27 – July 29	36	.56	17.8	35.1	52.9	.66	104.4	103.8	97.4	94.4	86.6	90.7	103.6	108.6	121.7
	Freight car orders																
1	4Q 71 – 4Q 73 – 4Q 74	36	.33	413.4	342.4	755.8	.45	417.4	56.4	111.6	126.5	4.0	51.0	47.5	14.9	346.4
2	4Q 74 – 4Q 77 – 2Q 81	78	.54	60.7	392.3	453.0	.87	75.9	34.2	26.3	30.1	15.2	79.8	152.5	231.3	407.5
3	2Q 81 – 4Q 84 – 1Q 88	81	.48	261.0	174.0	435.0	.40	276.0	123.9	79.8	58.2	15.0	61.0	112.0	158.2	189.0
4	1Q 88 – 1Q 89 – 1Q 90	24	.50	45.6	107.3	152.9	.70	104.1	95.4	99.4	88.5	58.5	87.5	129.8	105.9	165.8
5	1Q 90 – 1Q 91 – 1Q 92	24	.50	147.0	78.0	225.0	.35	197.0	158.3	110.4	51.6	50.0	53.1	99.1	115.0	128.0
6	1Q 92 – 4Q 94 – 2Q 95	39	.15	179.7	176.9	356.6	.50	207.9	140.5	89.6	47.8	28.2	(103.9)	141.7	(162.8)	205.1
7	2Q 95 – 4Q 96 – 3Q 99	51	.65	70.0	214.9	284.9	.75	97.4	83.3	90.8	55.2	27.4	52.7	116.3	170.6	242.3
8	3Q 99 – 1Q 00 – 2Q 02	33	.82	82.0	120.8	202.8	.60	143.7	(92.6)	67.0	(65.2)	61.7	93.5	90.4	115.6	182.5
9	2Q 02 – 4Q 03 – 3Q 05	39	.54	166.9	245.1	412.0	.59	181.1	113.5	107.8	61.5	14.2	56.5	100.1	147.4	259.3
10	3Q 05 – 1Q 08 – 3Q 09	48	.38	243.2	126.1	369.3	.34	255.5	173.2	164.5	55.4	12.3	39.3	33.7	49.6	138.4
11	3Q 09 – 3Q 10 – 4Q 12	39	.69	139.3	161.5	300.8	.54	169.3	158.1	163.0	106.0	30.0	47.2	92.4	112.0	191.5
12	4Q 12 – 4Q 14 – 4Q 16	48	.50	196.7	192.1	388.8	.49	215.6	125.5	118.6	74.1	18.9	80.6	89.9	91.0	211.0
13	4Q 16 – 1Q 18 – 2Q 18	18	.17	306.5	498.9	805.4	.62	311.7	56.6	51.1	28.1	5.2	(129.9)	(254.6)	(379.4)	504.1
14	2Q 18 – 3Q 19 – 2Q 20	24	.38	1017.2	223.0	1240.2	.18	1017.2	0.0	5.1	0.0	0.0	(22.5)	84.8	(156.1)	223.0
15	2Q 20 – 2Q 21 – 2Q 22	24	.50	135.2	363.3	498.5	.73	149.8	88.9	50.4	30.1	14.6	40.5	101.8	209.8	377.9
16	2Q 22 – 2Q 23 – 1Q 24	21	.43	146.9	98.0	244.9	.40	177.5	131.4	132.0	163.6	30.6	(37.2)	44.7	(63.8)	128.6
17	1Q 24 – 2Q 25 – 1Q 26[c]	24	.38	122.4	110.5	232.9	.47	160.5	89.9	136.9	44.9	38.1	(69.7)	99.3	(126.9)	148.6
18	4Q 25[d] – 1Q 28 – 4Q 29	48	.44	130.5	143.9	274.4	.52	183.1	92.8	82.4	95.6	52.6	62.8	100.5	150.6	196.5

TABLE B2—*Continued*

Measures of Successive Specific Cycles Treated on Inverted Plan
Seven American Series

Cycle no.	Dates of specific cycles	Duration of specific cycles		Amplitude in specific-cycle relatives			Ratio of rise to total fall & rise	Average in specific-cycle relatives at stage*								
								I	II	III	IV	V	VI	VII	VIII	IX
									Contraction				Expansion			
	Peak – Trough – Peak	Full cycle (mos.)	Ratio of expansion to full cycle	Fall	Rise	Fall & rise		Initial peak	First third	Middle third	Last third	Trough	First third	Middle third	Last third	Terminal peak
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	Railroad stock prices															
1	Mar. 58 - May 59 - Sep. 60	30	.53	12.8	24.3	37.1	.65	106.1	102.1	99.6	97.4	93.3	94.7	96.0	108.8	117.6
2	Sep. 60 - May 61 - Apr. 64	43	.81	15.8	99.8	115.6	.86	83.6	77.8	73.1	74.1	67.8	71.4	101.3	143.8	167.6
3	Apr. 64 - Apr. 65 - Apr. 72	96	.88	24.3	43.2	67.5	.64	104.9	103.6	95.5	89.2	80.6	86.6	102.3	112.9	123.8
4	Apr. 72 - June 77 - June 81	110	.44	53.4	88.5	141.9	.62	117.1	111.3	102.2	87.2	63.7	73.9	94.0	130.8	152.2
5	June 81 - June 85 - May 87	71	.32	39.7	27.7	67.4	.41	120.7	112.0	105.2	85.7	81.0	91.1	98.1	105.7	108.7
6	May 87 - Apr. 89 - May 90	36	.36	16.7	8.0	24.7	.32	112.4	104.5	98.3	97.9	95.7	97.3	100.5	100.3	103.7
7	May 90 - Dec. 90 - Mar. 92	22	.68	15.4	16.7	32.1	.52	107.0	107.3	103.1	94.3	91.6	94.7	97.1	106.1	108.3
8	Mar. 92 - Mar. 95 - Sep. 95	42	.14	31.1	12.3	43.4	.28	118.8	115.8	94.8	90.9	87.7	92.2	96.7	99.1	100.0
9	Sep. 95 - Aug. 96 - Apr. 00	55	.80	16.1	47.8	63.9	.75	95.5	91.0	87.0	84.8	79.4	86.1	99.9	123.6	127.2
10	Apr. 00 - Sep. 00 - Sep. 02	29	.83	3.3	49.4	52.7	.94	78.1	76.5	74.4	74.9	74.8	92.0	107.7	116.7	124.2
11	Sep. 02 - Oct. 03 - Sep. 06	48	.73	27.2	46.1	73.3	.63	105.7	100.7	94.6	82.2	78.5	82.3	107.8	117.9	124.6
12	Sep. 06 - Nov. 07 - Aug. 09	35	.60	36.9	39.7	76.6	.52	117.2	115.7	97.9	89.6	80.3	86.0	100.3	112.5	120.0
13	Aug. 09 - July 10 - Aug. 12	36	.69	14.4	6.0	20.4	.29	109.2	108.7	106.2	101.1	94.8	97.2	98.0	98.3	100.8
14	Aug. 12 - Feb. 15 - Oct. 16	50	.40	29.1	13.9	43.0	.32	117.3	110.9	100.5	93.1[e]	88.2	90.4	98.7	99.4	102.1
15	Oct. 16 - Dec. 17 - June 19	32	.56	33.0	13.1	46.1	.28	121.1	117.0	108.7	97.5	88.1	90.2	96.1	96.5	101.2
16	June 19 - June 21 - Sep. 22	39	.38	23.7	27.8	51.5	.54	114.8	103.5	96.7	96.9	91.1	93.5	97.8	111.1	118.9
17	Sep. 22 - Oct. 23 - Sep. 29	84	.85	10.4	90.5	100.9	.90	78.1	74.4	73.7	68.3	67.7	78.4	104.1	132.4	158.2
	Shares traded															
1	May 81 - Apr. 85 - Nov. 85	54	.13	56.0	56.2	112.2	.50	122.7	109.4	100.3	94.1	66.7	60.7	109.0	99.7	122.9
2	Nov. 85 - Feb. 88 - June 89	43	.37	90.9	42.7	133.6	.32	148.8	113.4	125.4	92.1	57.9	86.0	85.0	81.9	100.6
3	June 89 - Mar. 91 - Feb. 92	32	.34	36.0	93.1	129.1	.72	114.6	100.0	94.7	90.0	78.6	94.1	108.8	119.4	171.7
4	Feb. 92 - Jan. 95 - May 95	39	.10	122.2	53.1	175.3	.30	184.6	118.9	111.1	71.4	62.4	59.3	86.3	89.8	115.5
5	May 95 - May 97 - Jan. 99	44	.45	47.8	204.8	252.6	.81	94.3	84.6	68.5	59.0	46.5[a]	127.5	129.8	140.9	251.3

Shares traded—*Continued*

6	Jan. 99 – Aug. 00 – May 01	28	.32	76.8	187.9	264.7	.71	113.1	98.6	77.6	70.4	36.3	78.6	149.9	187.3	224.2
7	May 01 – May 04 – Jan. 06	56	.36	166.6	136.9	303.5	.45	205.9	94.8	86.9	74.4	39.3	113.7	137.3	119.8	176.2
8	Jan. 06 – Nov. 07 – June 09	41	.46	96.2	37.5	133.7	.28	164.4	130.0	121.3	81.2	68.2	77.5	91.5	90.3	105.7
9	June 09 – Apr. 11 – Sep. 11	27	.19	85.4	45.9	131.3	.35	137.1	135.6	109.0	69.7	51.7	72.8	71.0	97.3	97.6
10	Sep. 11 – Dec. 14 – Sep. 16	60	.35	90.8	186.8	277.6	.67	124.5	100.2	73.3	59.1[a]	33.7[a]	103.9	169.3	129.2	220.5
11	Sep. 16 – Apr. 18 – July 19	34	.44	75.8	156.2	232.0	.67	138.0	128.7	92.0	72.4	62.2	68.0	76.9	145.4	218.4
12	July 19 – Oct. 21 – Feb. 23	43	.37	127.4	64.0	191.4	.33	190.2	131.1	79.7	74.3	62.8	88.0	122.0	102.6	126.8
13	Feb. 23 – Oct. 23 – Nov. 25	33	.76	34.1	117.1	151.2	.77	93.8	76.4	73.4	58.4	59.7	70.7	120.5	135.0	176.8
14	Nov. 25 – May 26 – Oct. 29	47	.87	28.6	112.5	141.1	.80	75.8	66.1	57.9	55.3	47.2	67.1	97.8	152.3	159.7

Call money rates

1	Sep. 60 – Oct. 62 – Aug. 64	47	.47	29.7	45.2	74.9	.60	111.3	100.6	98.5	87.0	81.6	96.1	109.8	110.2	126.8
2	Aug. 64 – Oct. 66 – June 69	58	.55	43.4	147.5	190.9	.77	113.0	95.3	90.3	79.8	69.6	86.1	100.0	131.1	217.1
3	June 69 – Mar. 71 – Sep. 73	51	.59	126.5	312.5	439.0	.71	179.7	109.7	64.6	72.6	53.2	73.2	94.1	123.4	365.7
4	Sep. 73 – Jan. 75 – Jan. 76	28	.43	526.0	31.0	557.0	.06	576.3	140.8	71.9	57.5	50.3	59.1	68.2	62.6	81.3
5	Jan. 76 – Sep. 76 – Jan. 78	24	.67	46.8	93.0	139.8	.67	114.4	94.5	96.4	78.8	67.6	96.4	75.3	133.6	160.6
6	Jan. 78 – Sep. 78 – Aug. 79	19	.58	63.3	102.3	165.6	.62	142.7	109.8	106.1	66.0	79.4	89.3	81.1	111.4	181.7
7	Aug. 79 – Oct. 80 – Feb. 81	18	.22	68.1	93.4	161.5	.58	129.8	125.9	88.6	64.8	61.7	87.4	147.3	69.6	155.1
8	Feb. 81 – May 82 – Sep. 82	19	.21	94.5	44.2	138.7	.32	172.1	92.4	106.1	81.7	77.6	97.2	91.0	112.8	121.8
9	Sep. 82 – Jan. 85 – June 87	57	.51	144.3	172.6	316.9	.54	182.7	141.3	67.7	99.7	38.4	43.0	88.8	151.6	211.0
10	June 87 – Aug. 88 – Aug. 90	38	.63	126.5	139.1	265.6	.52	172.3	114.3	84.0	52.2	45.8	69.6	108.6	138.9	184.9
11	Aug. 90 – May 92 – June 93	34	.38	154.6	127.4	282.0	.45	204.5	111.9	94.6	69.7	49.9	64.2	118.0	129.3	177.3
12	June 93 – Nov. 94 – Oct. 96	40	.58	230.6	209.4	440.0	.48	266.7	166.7	47.1	41.8	36.1	69.2	104.7	137.4	245.5
13	Oct. 96 – July 97 – Oct. 99	36	.75	157.3	165.5	322.8	.51	215.2	95.5	66.1	55.5	57.9	74.1	75.9	156.2	223.4
14	Oct. 99 – Sep. 00 – Sep. 02	35	.69	99.8	143.5	243.3	.59	157.6	126.3	76.1	51.2	57.8	93.4	104.1	114.4	201.3
15	Sep. 02 – July 04 – Oct. 07	61	.64	158.5	220.4	378.9	.58	191.5	111.4	74.3	47.4	33.0	58.3	139.6	126.1	253.4
16	Oct. 07 – Nov. 08 – May 10	31	.58	297.3	76.6	373.9	.20	342.0	185.9	60.8	47.2	44.7	64.5	97.3	105.1	121.3
17	May 10 – Nov. 11 – Dec. 12	31	.42	57.3	84.8	142.1	.60	129.1	92.8	82.0	97.4	71.8	78.3	104.0	146.5	156.6
18	Dec. 12 – Nov. 15 – Aug. 18	68	.49	96.7	138.0	234.7	.59	147.0	97.9	109.1	65.3	50.3	75.6	98.6	149.3	188.3
19	Aug. 18 – Dec. 18 – Nov. 19	15	.73	19.4	67.2	86.6	.78	102.9	96.2	91.9	83.5	83.5	89.0	102.8	109.5	150.7
20	Nov. 19 – June 22 – Aug. 23	45	.31	83.3	23.4	106.7	.22	147.7	135.7	113.2	80.7	64.4	71.3	78.6	83.2	87.8
21	Aug. 23 – Sep. 24 – Feb. 26	30	.57	81.8	71.0	152.8	.46	135.5	122.8	111.1	63.0	53.7	77.9	105.6	120.5	124.7
22	Feb. 26 – Sep. 27 – Mar. 29	37	.49	22.1	101.2	123.3	.82	94.8	85.3	89.7	82.8	72.7	82.4	119.8	140.6	173.9

TABLE B2—*Continued*

Measures of Successive Specific Cycles Treated on Inverted Plan

Seven American Series

Cycle no.	Dates of specific cycles	Duration of specific cycles		Amplitude in specific-cycle relatives			Ratio of rise to total fall & rise	Average in specific-cycle relatives at stage*								
								I	II	III	IV	V	VI	VII	VIII	IX
									Contraction				Expansion			
	Peak – Trough – Peak	Full cycle (mos.)	Ratio of expansion to full cycle	Fall	Rise	Fall & rise		Initial peak	First third	Middle third	Last third	Trough	First third	Middle third	Last third	Terminal peak
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	Railroad bond yields															
1	Oct. 61 – Feb. 63 – Jan. 64	27	.41	36.8	10.7	47.5	.23	122.6	117.5	106.3	94.3	85.8	88.5	92.0	93.8	96.5
2	Jan. 64 – July 64 – Mar. 66	26	.77	11.0	38.0	49.0	.78	91.5	90.2	85.4	84.5	80.5	89.7	107.6	114.9	118.5
3	Mar. 66 – Nov. 66 – Dec. 67	21	.62	5.7	2.7	8.4	.32	103.8	102.7	100.0	98.2	98.1	99.5	100.3	99.7	100.8
4	Dec. 67 – July 68 – Dec. 69	24	.71	3.1	7.9	11.0	.72	99.8	98.3	97.6	96.9	96.7	98.8	101.1	102.8	104.6
5	Dec. 69 – July 72 – Nov. 73	47	.34	8.9	4.9	13.8	.36	106.2	101.8	101.1	99.3	97.3	98.8	98.2	98.2	102.2
6	Nov. 73 – Aug. 76 – Apr. 77	41	.20	23.7	2.2	25.9	.08	116.6	108.3	100.6	95.1	92.9	93.8	93.5	94.5	95.1
7	Apr. 77 – June 81 – Sep. 83	77	.35	25.9	4.1	30.0	.14	114.7	112.6	105.6	95.5	88.8	91.0	91.7	92.3	92.9
8	Sep. 83 – July 86 – Oct. 87	49	.31	15.2	4.8	20.0	.24	108.2	106.3	104.1	96.5	93.0	94.2	94.7	95.7	97.8
9	Oct. 87 – June 89 – Aug. 91	46	.57	9.4	10.9	20.3	.54	104.5	102.3	99.8	97.2	95.1	96.7	99.8	104.1	106.0
10	Aug. 91 – July 92 – Aug. 93	24	.54	5.6	8.0	13.6	.59	103.4	102.3	99.6	98.3	97.8	98.7	98.9	101.6	105.8
11	Aug. 93 – Aug. 95 – Aug. 96	36	.33	16.1	5.6	21.7	.26	111.6	105.0	100.4	98.5	95.5	95.8	97.8	97.3	101.1
12	Aug. 96 – June 99 – Sep. 03	85	.60	16.3	15.1	31.4	.48	110.6	104.7	100.9	97.2	94.3	97.2	97.5	103.0	109.4
13	Sep. 03 – Feb. 05 – Nov. 07	50	.66	3.5	14.8	18.3	.81	100.3	99.4	98.9	97.6	96.8	97.3	99.9	104.4	111.6
14	Nov. 07 – Feb. 09 – Dec. 13	73	.79	9.4	12.9	22.3	.58	103.7	100.4	98.0	95.5	94.3	97.3	100.2	104.0	107.2
15	Dec. 13 – June 14 – Sep. 15	21	.71	2.3	5.7	8.0	.71	99.7	97.8	97.4	97.3	97.4	98.0[e]	101.5	101.9	103.1
16	Sep. 15 – Jan. 17 – Sep. 18	36	.56	6.7	19.5	26.2	.74	99.1	94.9	94.5	93.9	92.4	97.0	106.1	110.1	111.9[a]
17	Sep. 18 – Nov. 18 – May 20	20	.90	7.7	20.2	27.9	.72	100.4[a]	(99.1)	98.4	(96.5)	92.7[b]	95.7	99.4	105.0	112.9
18	May 20 – Sep. 22 – Oct. 23	41	.32	23.5	7.0	30.5	.23	113.7	108.0	105.8	93.0	90.2	92.9	96.0	96.6	97.2
19	Oct. 23 – Dec. 27 – Sep. 29	71	.30	13.0	11.3	24.3	.47	105.8	103.6	100.9	96.7	92.8	95.0	99.6	102.9	104.1

* Entries in parentheses are interpolated.

[a] Average of two months.

[b] One month.

[c] Based on the first segment of the series (see Ch. 9, note 2), which we use through cycle 17.

[d] Based on the second segment (see preceding note).

[e] Omits Aug.-Nov. 1914. No data (N.Y. Stock Exchange closed).

TABLE B3

Measures of Successive Reference Cycles Treated on Positive Plan
Seven American Series

Cycle no. (1)	Dates of reference cycles: Trough – Peak – Trough (2)	No. of months lead (−) or lag (+) of corresponding specific-cycle turn at reference: Peak (3)	Trough[a] (4)	I Initial trough (5)	II Expansion: First third (6)	III Expansion: Middle third (7)	IV Expansion: Last third (8)	V Peak (9)	VI Contraction: First third (10)	VII Contraction: Middle third (11)	VIII Contraction: Last third (12)	IX Terminal trough (13)	Average change per month in reference-cycle relatives during stages[b] matched with reference: Expansion (14)	Contraction (15)
	Deflated clearings													
	Mar. 79		−12											
1	Mar. 79 – Mar. 82 – May 85	+15	−6	71.1	82.5	91.6	107.4	103.3	107.1	109.8	101.2	102.8	+0.79	−0.07
2	May 85 – Mar. 87 – Apr. 88	+3	−1	81.0	87.3	93.6	101.2	110.2	113.0	110.5	106.3	107.9	+1.07	−0.37
3	Apr. 88 – July 90 – May 91	+2	−2	85.8	90.5	95.2	104.6	111.5	113.2	106.8	105.2	105.5	+0.92	−0.79
4	May 91 – Jan. 93 – June 94	+1	−10	94.9	100.0	103.0	108.9	112.4	109.0	87.0	90.8	91.6	+0.81	−1.54
5	June 94 – Dec. 95 – June 97	−2	−10	92.4	96.7	99.4	106.7	105.6	100.5	97.7	97.8	102.1	+0.67	−0.54
6	June 97 – June 99 – Dec. 00	+3	−3	80.3	89.5	90.8	103.9	108.2	109.2	104.8	105.8	110.7	+1.14	−0.17
7	Dec. 00 – Sep. 02 – Aug. 04	+10	−3	89.1	96.8	97.7	100.8	102.3	100.9	103.0	99.9	104.1	+0.70	−0.13
8	Aug. 04 – May 07 – June 08	+5	−6	87.2	93.4	101.2	105.8	108.9	108.6	96.0	93.7	97.1	+0.68	−1.45
9	June 08 – Jan. 10 – Jan. 12	+2	−15	85.9	89.9	95.8	101.9	103.4	102.9	102.3	104.6	106.5	+0.95	+0.06
10	Jan. 12 – Jan. 13 – Dec. 14	+1	−1	96.7	98.9	98.9	101.6	103.8	101.5	102.4	95.8	92.6	+0.54	−0.42
11	Dec. 14 – Aug. 18 – Apr. 19	0	−4	77.1	85.6	100.2	107.6	115.5	113.6	110.6	110.6	112.2	+0.74	−0.75
12	Apr. 19 – Jan. 20 – Sep. 21	0	−4	94.3	101.9	104.5	103.4	104.9	102.0	99.2	94.2	97.2	+1.13	−0.65
13	Sep. 21 – May 23 – July 24	0	−10	91.2	92.3	96.6	103.9	106.9	102.8	102.9	103.7	103.6	+0.79	−0.28
14	July 24 – Oct. 26 – Dec. 27	...	...	87.6	92.9	97.9	101.2	99.5	102.5	106.3	107.4	108.1	+0.40	+0.69
15	Dec. 27 – June 29 – Mar. 33	+5	0	107.6	112.4	109.6	118.4	115.5	113.7	95.3	75.5	67.5	+0.41	−1.08
	Pig iron production													
	Mar. 79		−2											
1	Mar. 79 – Mar. 82 – May 85	+11	−4	55.5	77.9	94.3	102.5	111.0	114.0	108.7	102.7	100.2	+1.54	−0.28
2	May 85 – Mar. 87 – Apr. 88	+7	−1	68.9	73.9	97.3	109.4	116.2	101.5	123.5	108.7	106.1	+2.15	−0.78
3	Apr. 88 – July 90 – May 91	−2	−1	76.4	87.0	95.1	114.8	122.5	120.2	108.8	77.7	87.4	+1.71	−3.51
4	May 91 – Jan. 93 – June 94	−11	−8	82.9	117.1	118.9	108.1	111.0	115.8	66.3	72.5	62.3	+1.40	−2.86
5	June 94 – Dec. 95 – June 97	−1	−8	55.8	89.7	93.5	119.2	125.7	113.8	81.6	97.8	104.1	+3.88	−1.20

TABLE B3—*Continued*

Measures of Successive Reference Cycles Treated on Positive Plan

Seven American Series

Cycle no.	Dates of reference cycles	No. of months lead (−) or lag (+) of corresponding specific-cycle turn at reference		Average in reference-cycle relatives at stage*									Average change per month in reference-cycle relatives during stages[b] matched with reference	
				I	II	III	IV	V	VI	VII	VIII	IX		
				Initial trough	Expansion			Peak	Contraction			Terminal trough		
	Trough – Peak – Trough	Peak	Trough[a]		First third	Middle third	Last third		First third	Middle third	Last third		Expansion	Contraction
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Pig iron production—*Continued*													
6	June 97 – June 99 – Dec. 00	+6	−2	71.3	85.5	91.7	96.7	106.1	117.5	119.1	101.5	99.8	+1.45	−0.35
7	Dec. 00 – Sep. 02 – Aug. 04	+9	−8	74.9	91.0	99.2	104.6	107.9	109.1	103.7	93.5	90.5	+1.57	−0.76
8	Aug. 04 – May 07 – June 08	+2	−5	66.9	91.3	108.4	114.0	120.0	121.9	85.6	61.5	62.7	+1.61	−4.41
9	June 08 – Jan. 10 – Jan. 12	0	−13	58.2	73.7	91.1	120.4	131.3	115.7	96.7	96.6	106.1	+3.85	−1.05
10	Jan. 12 – Jan. 13 – Dec. 14	0	0	91.4	100.0	106.6	112.6	122.1	115.0	95.1	80.2	67.8	+2.56	−2.36
11	Dec. 14 – Aug. 18 – Apr. 19	+1	+1	52.3	85.7	106.7	103.9	114.6	113.9	111.4	101.3	82.4	+1.42	−4.02
12	Apr. 19 – Jan. 20 – Sep. 21	+8	−2	105.6	94.2	113.9	96.6	122.7	128.5	119.7	48.4	45.7	+1.90	−3.85
13	Sep. 21 – May 23 – July 24	0	0	41.1	61.5	85.1	123.9	142.0	133.0	109.5	101.8	85.5	+5.04	−4.04
14	July 24 – Oct. 26 – Dec. 27	−3	−1	73.7	89.8	102.0	107.3	110.8	104.9	103.4	95.7	92.1	+1.37	−1.34
15	Dec. 27 – June 29 – Mar. 33	+1	0	124.6	132.2	148.4	153.1	169.3	139.8	73.2	32.1	24.5	+2.48	−3.22
	Freight car orders[c]													
	4Q 70		−1											
1	4Q 70 – 3Q 73 – 1Q 79	−23	−16	10.1	26.7	108.5	90.5	26.6	71.9	84.7	173.0	350.5	+2.14	+1.38
2	1Q 79 – 1Q 82 – 2Q 85	−10	−6	76.7	105.5	143.3	162.2	157.6	90.5	64.8	38.9	53.0	+3.07	−3.16
3	2Q 85 – 2Q 87 – 1Q 88	+11	+10	44.7	60.8	79.0	130.2	146.3	(117.2)	117.8	(129.0)	153.8	+3.61	−0.11
4	1Q 88 – 3Q 90 – 2Q 91	−5	−3	111.2	100.9	98.2	144.2	99.3	(55.2)	45.6	(45.4)	47.7	+1.92	−7.90
5	2Q 91 – 1Q 93 – 2Q 94	−11	+5	70.9	142.9	164.2	94.3	140.8	70.1	41.8	49.7	35.2	+1.42	−2.70
6	2Q 94 – 4Q 95 – 2Q 97	−7	−7	37.4	29.9	123.9	174.0	137.8	154.9	60.6	52.0	74.8	+7.34	−6.78
7	2Q 97 – 3Q 99 – 4Q 00	+2	−10	29.6	74.6	89.8	124.3	176.5	82.3	77.4	125.4	140.2	+4.07	+0.06
8	4Q 00 – 4Q 02 – 3Q 04	−4	−9	121.5	96.2	123.1	170.7	96.1	91.6	39.7	60.6	112.4	+2.76	−5.24
9	3Q 04 – 2Q 07 – 2Q 08	−21	−4	81.9	101.7	153.8	130.5	73.4	47.2	11.2	9.7	11.2	+2.74	−8.05
10	2Q 08 – 1Q 10 – 4Q 11	−5	−17	21.2	73.5	74.2	200.3	199.3	83.2	56.7	71.4	143.1	+9.33	−6.14

	Freight car orders—*Continued*												
11	4Q 11 – 1Q 13 – 4Q 14	−2	−1	111.0	105.7	106.6	181.6	162.4	102.9	71.5	62.4	15.9	+7.65	−6.11
12	4Q 14 – 3Q 18 – 2Q 19	−3	+4	26.0	118.6	148.4	119.2	0.0	(0.0)	2.4	(3.9)	0.0	+0.42	−7.44
13	2Q 19 – 1Q 20 – 3Q 21	+4	−4	0.0	(0.0)	17.4	(59.8)	184.4	229.5	96.9	43.0	77.8	+6.52	−1.05
14[d]	3Q 21 – 2Q 23 – 3Q 24	−12	+7[e]	20.0	76.9	162.3	155.3	32.2	40.4	94.3	75.7	128.0	+6.87	−4.82
15	3Q 24 – 3Q 26 – 4Q 27	−11	+2	292.4	95.7	116.7	79.1	91.4	78.0	81.3	80.0	84.0	+0.36	+0.05
16	4Q 27 – 2Q 29 – 1Q 33	+5	−1	160.8	104.2	97.0	172.4	232.8	185.6	45.5	5.3	0.1	+1.18	−4.13
	Railroad stock prices													
	Dec. 58		−14											
1	Dec. 58 – Oct. 60 – June 61	−1	−1	98.7	94.6	94.8	108.0	112.1	99.0	106.0	95.6	93.6	+0.41	−1.18
2	June 61 – Apr. 65 – Dec. 67	−12	−32	52.8	60.1	99.3	122.4	101.2	105.8	110.8	111.2	116.6	+1.74	−0.32
3	Dec. 67 – June 69 – Dec. 70	...	...	87.6	92.6	95.6	100.7	106.6	102.4	103.1	105.1	103.9	+0.86	+0.24
4	Dec. 70 – Oct. 73 – Mar. 79	−18	−21	108.2	112.7	118.4	117.9	100.9	105.9	91.6	77.5	87.6	+0.27	−0.67
5	Mar. 79 – Mar. 82 – May 85	−9	+1	67.7	83.5	102.1	117.2	108.7	110.8	103.8	83.9	81.4	+1.41	−0.88
6	May 85 – Mar. 87 – Apr. 88	+2	+12	81.9	90.3	98.7	106.1	107.7	109.7	101.2	98.6	96.1	+0.88	−0.52
7	Apr. 88 – July 90 – May 91	−2	−5	100.2	100.6	99.8	103.9	105.6	100.7	91.0	93.9	94.8	+0.04	−0.77
8	May 91 – Jan. 93 – June 94	−10	+9	98.2	102.4	111.0	109.0	107.5	100.3	86.1	87.2	84.4	+0.63	−1.25
9	June 94 – Dec. 95 – June 97	−3	−10	99.6	100.5	98.7	109.1	101.7	100.6	94.2	95.7	98.0	+0.36	−0.74
10	June 97 – June 99 – Dec. 00	+10	−3	75.4	84.2	87.0	103.3	107.8	110.8	111.8	109.9	125.6	+1.29	+0.35
11	Dec. 00 – Sep. 02 – Aug. 04	0	−10	81.9	95.4	102.6	111.2	117.5	109.8	91.2	90.2	96.9	+1.93	−0.91
12	Aug. 04 – May 07 – June 08	−8	−7	81.5	97.9	110.1	109.9	94.4	91.6	80.9	85.5	92.7	+1.10	−1.48
13	June 08 – Jan. 10 – Jan. 12	−5	−18	85.5	92.0	102.2	109.5	107.0	100.0	98.7	98.5	97.4	+1.71	−0.48
14	Jan. 12 – Jan. 13 – Dec. 14	−5	+2	105.7	107.6	109.2	109.4	105.2	98.2	93.6	89.4[f]	82.6[f]	+0.18	−0.93
15	Dec. 14 – Aug. 18 – Apr. 19	−22	−16	98.9[f]	105.3	110.0	89.5	88.6	91.5	92.4	90.1	92.4	−0.43	+0.04
16	Apr. 19 – Jan. 20 – Sep. 21	−7	−3	111.3	115.4	106.7	103.1	97.9	95.8	100.5	91.8	94.0	−0.64	−0.61
17	Sep. 21 – May 23 – July 24	−8	−9	87.6	90.6	106.4	105.3	102.6	97.5	97.7	100.5	107.3	+0.98	−0.32
18	July 24 – Oct. 26 – Dec. 27	...	...	76.9	83.8	90.9	98.8	104.8	107.6	118.3	125.4	126.1	+1.09	+1.61
19	Dec. 27 – June 29 – Mar. 33	+3	−9	129.4	131.4	133.4	143.8	153.0	144.2	83.7	27.8	27.3	+0.89	−2.86
	Shares traded													
	Mar. 79		−13											
1	Mar. 79 – Mar. 82 – May 85	−10	−1	53.3	80.9	107.8	110.9	126.4	107.8	101.8	89.9	59.5	+1.61	−0.55
2	May 85 – Mar. 87 – Apr. 88	−16	−2	65.3	114.6	104.5	111.2	106.4	99.5	96.2	54.9	88.1	+0.51	−3.88
3	Apr. 88 – July 90 – May 91	−13	−2	114.2	97.7	105.3	106.4	83.3	88.7	110.4	82.3	98.5	+1.44	−1.85
4	May 91 – Jan. 93 – June 94	−11	+7	89.8	94.4	126.0	95.4	161.0	120.6	84.8	65.2	60.1	+1.10	−1.73
5	June 94 – Dec. 95 – June 97	−7	−1	81.2	85.9	102.7	125.6	113.7	92.7	108.7	76.0	133.0	+2.14	−2.76

TABLE B3—*Continued*

Measures of Successive Reference Cycles Treated on Positive Plan

Seven American Series

Cycle no.	Dates of reference cycles	No. of months lead (−) or lag (+) of corresponding specific-cycle turn at reference		Average in reference-cycle relatives at stage[a]									Average change per month in reference-cycle relatives during stages[b] matched with reference	
				I	II	III	IV	V	VI	VII	VIII	IX		
				Initial trough	Expansion			Peak	Contraction			Terminal trough		
	Trough – Peak – Trough	Peak	Trough[a]		First third	Middle third	Last third		First third	Middle third	Last third		Expansion	Contraction
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Shares traded—*Continued*													
6	June 97 – June 99 – Dec. 00	−5	−4	56.0	79.6	84.3	131.6	122.2	107.3	102.2	84.9	194.6	+4.33	−2.46
7	Dec. 00 – Sep. 02 – Aug. 04	−16	−3	137.9	178.5	86.9	96.9	114.6	84.7	88.1	58.4	99.1	+1.80	−1.67
8	Aug. 04 – May 07 – June 08	−16	−7	78.3	117.3	114.2	111.0	75.0	67.0	63.7	72.0	80.6	+2.09	−2.36
9	June 08 – Jan. 10 – Jan. 12	−7	−9	111.2	124.7	116.7	127.4	129.4	97.7	63.8	80.3	61.8	+1.56	−2.05
10	Jan. 12 – Jan. 13 – Dec. 14	−16	0	107.8	151.1	103.0	131.8	113.5	88.9	79.7	76.3[f]	44.0[f]	−0.58	−2.58
11	Dec. 14 – Aug. 18 – Apr. 19	−23	−12	24.0[f]	96.2	121.5	89.5	61.9	81.6	80.4	116.4	163.6	+1.18	+1.92
12	Apr. 19 – Jan. 20 – Sep. 21	−6	+1	117.2	180.1	127.9	124.3	113.8	95.6	77.9	69.8	58.5	+4.81	−2.95
13	Sep. 21 – May 23 – July 24	−3	−9	62.9	83.0	119.6	108.9	99.4	89.0	105.5	77.3	130.0	+1.70	−2.11
14	July 24 – Oct. 26 – Dec. 27	−11	−19	64.2	78.0	103.2	96.7	87.9	96.9	115.6	133.0	146.5	+2.39	+2.20
15	Dec. 27 – June 29 – Mar. 33	+4	0	93.6	106.3	139.6	141.9	150.6	133.3	78.9	55.9	45.3	+3.35	−2.12
	Call money rates													
	Dec. 58		−2											
1	Dec. 58 – Oct. 60 – June 61	−1	+16	64.9	92.2	103.4	101.6	114.6	112.0	95.6	106.2	109.3	+2.26	−0.66
2	June 61 – Apr. 65 – Dec. 67	−8	−14	104.9	92.2	100.9	111.1	103.6	101.3	89.1	105.0	97.5	−0.03	−0.19
3	Dec. 67 – June 69 – Dec. 70	0	+3	74.6	86.8	97.5	115.6	186.1	120.3	69.4	66.7	87.3	+6.19	−5.49
4	Dec. 70 – Oct. 73 – Mar. 79	−1	−6	119.7	91.5	133.3	268.5	524.9	64.8	57.8	70.6	63.1	+11.92	−7.10
5	Mar. 79 – Mar. 82 – May 85	−13	−4	80.0	135.6	125.2	110.5	89.7	115.7	60.6	62.3	32.6	+0.27	−1.50
6	May 85 – Mar. 87 – Apr. 88	+3	+4	38.2	45.3	78.3	128.3	143.5	184.9	107.5	91.9	68.2	+4.79	−5.79
7	Apr. 88 – July 90 – May 91	+1	+12	60.8	53.4	87.0	137.5	186.9	165.6	104.2	79.8	99.7	+4.67	−8.72
8	May 91 – Jan. 93 – June 94	+5	+5	130.7	101.8	66.1	111.7	124.4	174.7	113.4	36.6	40.1	−0.32	−4.96
9	June 94 – Dec. 95 – June 97	+10	+1	53.3	40.0	78.4	61.4	137.3	152.7	212.3	70.4	64.1	+4.67	−4.07
10	June 97 – June 99 – Dec. 00	+4	−3	50.6	62.9	80.7	97.1	147.9	195.2	103.3	80.0	99.2	+4.05	−2.71

	Call money rates—*Continued*												
11	Dec. 00 – Sep. 02 – Aug. 04	0	−1	78.6	116.4	94.9	121.1	209.1	121.6	81.1	45.0	36.9	+6.21	−7.49
12	Aug. 04 – May 07 – June 08	+5	+5	29.8	52.3	123.1	118.2	65.0	98.3	209.9	41.1	39.4	+1.07	−1.97
13	June 08 – Jan. 10 – Jan. 12	+4	−2	73.5	56.8	84.0	121.8	136.2	123.1	95.4	103.9	86.8	+3.30	−2.06
14	Jan. 12 – Jan. 13 – Dec. 14	−1	+11	66.9	84.2	88.7	146.3	119.2	92.6	79.3	123.0	94.9	+4.36	−1.06
15	Dec. 14 – Aug. 18 – Apr. 19	0	−4	93.9	56.6	78.7	133.5	174.9	159.9	141.9	151.0	157.2	+1.84	−2.21
16	Apr. 19 – Jan. 20 – Sep. 21	−2	+9	75.4	85.7	82.0	122.8	124.6	112.2	101.5	88.5	76.7	+5.47	−2.40
17	Sep. 21 – May 23 – July 24	+3	+2	123.5	106.3	92.4	106.6	110.9	115.5	100.9	78.6	47.5	−0.63	−4.53
18	July 24 – Oct. 26 – Dec. 27	−8	−3	52.8	75.9	108.5	111.9	116.3	112.3	107.3	92.0	100.4	+2.35	−1.14
19	Dec. 27 – June 29 – Mar. 33	−3	...	100.6	130.0	167.3	211.1	215.5	119.3	43.8	48.9	47.6	+6.38	−3.73
	Railroad bond yields													
	Dec. 58		+20											
1	Dec. 58 – Oct. 60 – June 61	+12	+20	99.4	100.7	101.6	96.5	96.8	102.5	100.1	102.9	104.2	+0.07	−0.83
2	June 61 – Apr. 65 – Dec. 67	+11	+7	112.5	104.8	84.4	87.6	103.9	109.8	111.2	111.0	111.9	+0.89	0.00
3	Dec. 67 – June 69 – Dec. 70	+6	+19	99.6	97.4	97.8	101.0	100.7	103.0	100.8	99.6	100.7	+0.42	−0.20
4	Dec. 70 – Oct. 73 – Mar. 79	+1	+27	113.4	111.7	109.0	108.6	112.4	102.6	92.0	89.8	85.3	−0.23	−0.35
5	Mar. 79 – Mar. 82 – May 85	+18	+14	112.8	109.5	102.0	96.6	98.2	97.8	97.5	96.7	93.3	−0.17	−0.28
6	May 85 – Mar. 87 – Apr. 88	+7	+14	106.3	103.6	98.1	98.2	98.8	98.9	101.4	99.5	98.9	+0.06	−0.22
7	Apr. 88 – July 90 – May 91	+13	+14	102.3	100.4	96.7	98.7	100.3	101.8	104.0	104.0	105.8	+0.33	+0.03
8	May 91 – Jan. 93 – June 94	+7	+14	102.3	102.8	98.9	98.6	98.8	99.5	103.2	96.9	95.9	+0.05	−0.24
9	June 94 – Dec. 95 – June 97	+8	+24	103.6	102.1	101.6	98.0	99.4	99.6	102.7	96.9	94.8	−0.16	−0.19
10	June 97 – June 99 – Dec. 00	...	...	103.1	102.1	102.9	98.1	96.0	98.3	98.9	99.6	98.7	−0.30	+0.07
11	Dec. 00 – Sep. 02 – Aug. 04	+12	+6	94.6	94.5	95.9	96.4	98.5	101.0	105.6	105.7	104.2	+0.35	+0.10
12	Aug. 04 – May 07 – June 08	+6	+8	96.6	95.9	96.8	99.8	102.7	104.7	109.0	105.5	104.4	+0.42	−0.22
13	June 08 – Jan. 10 – Jan. 12	...	...	100.3	98.2	96.4	98.1	99.3	101.4	101.8	102.4	102.2	+0.36	+0.08
14	Jan. 12 – Jan. 13 – Dec. 14	+11	−6	96.4	96.6	97.5	98.3	98.4	101.7	101.8	100.8[f]	105.2[f]	+0.42	0.00
15	Dec. 14 – Aug. 18 – Apr. 19	+1	−5	97.3[f]	96.3	93.9	106.1	111.1	110.5	103.5	106.6	107.6	+0.71	+0.19
16	Apr. 19 – Jan. 20 – Sep. 21	+4	+12	92.4	92.4	96.7	96.6	99.0	105.4	100.5	101.7	99.0	+1.09	−0.72
17	Sep. 21 – May 23 – July 24	+5	...	109.8	102.2	95.6	99.4	101.3	101.3	101.9	100.6	98.4	+0.46	−0.12
18	July 24 – Oct. 26 – Dec. 27	...	0	102.7	102.8	102.5	99.9	99.5	98.4	96.7	95.0	93.4	−0.26	+0.07
19	Dec. 27 – June 29 – Mar. 33	+3	...	93.1	94.7	99.5	101.9	103.8	100.5	95.0	105.8	99.8	+0.06	−0.31

[*] Entries in parentheses are interpolated.

[a] The entries relate to the terminal trough shown on the same line in col. (2).

[b] The stages matched with reference expansion are as follows:

Deflated clearings (VIII-V) — Railroad stock prices (VIII-IV)
Pig iron production (I-V) — Shares traded (VIII-IV)
Freight car orders (VIII-IV) — Call money rates (I-V)
Railroad bond yields (III-VI)

[c] Col. (3)-(4) for this series compare the timing of specific cycles with the monthly reference dates, not with the quarterly reference dates given here.

[d] Based on the first segment of the series (see Ch. 9, note 2), which we use through cycle 14.

[e] Based on the second segment; the corresponding entry for the first segment is +10. See the preceding note.

[f] Omits Aug.–Nov. 1914. No data (N.Y. Stock Exchange closed).

TABLE B4

Patterns of Successive Reference Cycles Treated on Inverted Plan
Seven American Series

Cycle no.	Dates of reference cycles	Average in reference-cycle relatives at stage[a]								
		I	II	III	IV	V	VI	VII	VIII	IX
			Contraction				Expansion			
	Peak – Trough – Peak	Initial peak	First third	Middle third	Last third	Trough	First third	Middle third	Last third	Terminal peak
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Deflated clearings									
1	Mar. 82–May 85–Mar. 87	93.1	96.6	99.0	91.2	92.7	99.9	107.1	115.8	126.2
2	Mar. 87–Apr. 88–July 90	93.4	95.7	93.6	90.1	91.4	96.4	101.4	111.5	118.9
3	July 90–May 91–Jan. 93	98.6	100.0	94.5	93.0	93.3	98.3	101.3	107.0	110.4
4	Jan. 93–June 94–Dec. 95	114.9	111.4	88.9	92.8	93.7	98.1	100.8	108.1	107.1
5	Dec. 95–June 97–June 99	94.8	90.2	87.7	87.8	91.6	102.2	103.6	118.6	123.5
6	June 99–Dec. 00–Sep. 02	94.0	94.8	91.0	91.9	96.2	104.5	105.5	108.8	110.5
7	Sep. 02–Aug. 04–May 07	91.2	90.0	91.9	89.1	92.8	99.5	107.8	112.7	116.0
8	May 07–June 08–Jan. 10	103.9	103.6	91.5	89.4	92.6	97.0	103.4	110.0	111.6
9	Jan. 10–Jan. 12–Jan. 13	97.9	97.5	96.9	99.0	100.8	103.2	103.1	106.0	108.2
10	Jan. 13–Dec. 14–Aug. 18	93.1	91.0	91.9	85.9	83.0	92.2	107.9	115.9	124.4
11	Aug. 18–Apr. 19–Jan. 20	98.4	96.8	94.3	94.2	95.6	103.2	105.9	104.8	106.3
12	Jan. 20–Sep. 21–May 23	103.4	100.6	97.8	92.9	95.8	96.9	101.5	109.2	112.3
13	May 23–July 24–Oct. 26	96.3	92.6	92.7	93.4	93.3	99.0	104.3	107.8	106.0
14	Oct. 26–Dec. 27–June 29	90.3	93.0	96.5	97.5	98.1	102.4	99.9	107.9	105.3
	Pig iron production									
1	Mar. 82–May 85–Mar. 87	93.7	96.3	91.8	86.7	84.6	90.7	119.4	134.3	142.6
2	Mar. 87–Apr. 88–July 90	90.1	78.7	95.8	84.3	81.9	93.6	102.3	123.5	131.8
3	July 90–May 91–Jan. 93	107.3	105.3	95.3	68.1	76.5	108.1	109.8	99.8	102.5
4	Jan. 93–June 94–Dec. 95	113.1	117.9	67.5	73.9	63.4	101.9	106.2	135.4	142.8
5	Dec. 95–June 97–June 99	105.8	95.8	68.7	82.4	87.6	105.2	112.8	119.0	130.5
6	June 99–Dec. 00–Sep. 02	87.1	96.4	97.8	83.3	81.9	99.6	108.5	114.4	118.1
7	Sep. 02–Aug. 04–May 07	86.5	87.4	83.0	74.9	72.5	98.9	117.5	123.5	130.1
8	May 07–June 08–Jan. 10	123.4	125.3	88.0	63.2	64.5	81.7	101.0	133.5	145.5
9	Jan. 10–Jan. 12–Jan. 13	118.8	104.7	87.5	87.4	96.0	105.0	111.9	118.3	128.2
10	Jan. 13–Dec. 14–Aug. 18	104.2	98.2	81.2	68.4	57.9	94.9	118.2	115.0	126.9
11	Aug. 18–Apr. 19–Jan. 20	123.5	122.8	120.0	109.2	88.8	79.2	95.8	81.2	103.2
12	Jan. 20–Sep. 21–May 23	123.2	129.0	120.2	48.6	45.8	68.6	94.9	138.2	158.4
13	May 23–July 24–Oct. 26	123.6	115.8	95.4	88.7	74.4	90.8	103.0	108.4	112.0
14	Oct. 26–Dec. 27–June 29	106.1	100.5	99.0	91.6	88.2	93.5	105.0	108.4	119.8
	Freight car orders									
1	3Q 73 – 1Q 79 – 1Q 82	9.2	24.7	29.1	59.5	120.5	165.8	225.1	254.8	247.6
2	1Q 82 – 2Q 85 – 2Q 87	191.3	109.9	78.7	47.2	64.3	87.5	113.8	187.6	210.7
3	2Q 87 – 1Q 88 – 3Q 90	97.2	(77.9)	78.2	(85.8)	102.2	92.8	90.3	132.6	91.3
4	3Q 90 – 2Q 91 – 1Q 93	127.8	(71.0)	58.8	(58.4)	61.4	123.9	142.3	81.8	122.1
5	1Q 93 – 2Q 94 – 4Q 95	177.4	88.3	52.6	62.6	44.3	35.4	146.6	206.0	163.0
6	4Q 95 – 2Q 97 – 3Q 99	73.8	82.9	32.4	27.9	40.0	100.9	121.4	168.1	238.8
7	3Q 99 – 4Q 00 – 4Q 02	135.9	63.4	59.6	96.6	108.0	85.4	109.3	151.7	85.4
8	4Q 02 – 3Q 04 – 2Q 07	72.3	68.9	29.8	45.6	84.5	104.8	158.6	134.6	75.7
9	2Q 07 – 2Q 08 – 1Q 10	150.1	96.6	22.9	19.9	23.0	79.6	80.5	217.1	216.1
10	1Q 10 – 4Q 11 – 1Q 13	169.0	70.6	48.0	60.6	121.3	115.5	116.5	198.6	177.6
11	1Q 13 – 4Q 14 – 3Q 18	215.0	136.2	94.7	82.7	21.0	96.1	120.2	96.6	0.0
12	3Q 18 – 2Q 19 – 1Q 20	0.0	(0.0)	28.2	(47.1)	0.0	(0.0)	74.6	(255.8)	788.7
13	1Q 20 – 3Q 21 – 2Q 23	60.7	75.6	31.9	14.2	25.6	98.6	208.2	199.2	41.3
14[a]	2Q 23 – 3Q 24 – 3Q 26	43.3	54.4	127.0	101.9	172.3	94.6	89.5	114.6	16.9
15	3Q 26 – 4Q 27 – 2Q 29	118.2	100.8	105.1	103.4	108.6	70.4	65.6	116.4	157.3

TABLE B4 — *Continued*

Patterns of Successive Reference Cycles Treated on Inverted Plan
Seven American Series

Cycle no.	Dates of reference cycles	Average in reference-cycle relatives at stage[a]								
		I	II	III	IV	V	VI	VII	VIII	IX
			Contraction				Expansion			Terminal peak
	Peak – Trough – Peak	Initial peak	First third	Middle third	Last third	Trough	First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Railroad stock prices									
1	Oct. 60–June 61–Apr. 65	71.6	63.2	67.8	61.1	59.8	68.2	112.7	138.9	114.8
2	Apr. 65–Dec. 67–June 69	87.2	91.1	95.4	95.7	100.4	106.2	109.7	115.5	122.2
3	June 69–Dec. 70–Oct. 73	98.1	94.2	94.8	96.7	95.6	99.6	104.6	104.2	89.2
4	Oct. 73–Mar. 79–Mar. 82	95.8	100.5	87.0	73.5	83.2	102.6	125.4	143.9	133.5
5	Mar. 82–May 85–Mar. 87	110.0	112.1	105.0	85.0	82.4	90.8	99.3	106.7	108.4
6	Mar. 87–Apr. 88–July 90	108.5	110.5	101.9	99.4	96.9	97.2	96.5	100.4	102.1
7	July 90–May 91–Jan. 93	104.5	99.6	90.1	92.9	93.8	97.8	106.0	104.0	102.7
8	Jan. 93–June 94–Dec. 95	120.6	112.5	96.5	97.8	94.7	95.6	93.9	103.8	96.7
9	Dec. 95–June 97–June 99	92.7	91.7	85.9	87.2	89.3	99.8	103.1	122.4	127.7
10	June 99–Dec. 00–Sep. 02	79.1	81.3	82.0	80.6	92.1	107.4	115.4	125.1	132.2
11	Sep. 02–Aug. 04–May 07	103.3	96.5	80.2	79.4	85.2	102.3	115.1	114.9	98.6
12	May 07–June 08–Jan. 10	94.2	91.4	80.7	85.3	92.6	99.6	110.7	118.6	115.8
13	Jan. 10–Jan. 12–Jan. 13	107.6	100.5	99.2	99.1	97.9	99.7	101.2	101.3	97.5
14	Jan. 13–Dec. 14–Aug. 18	119.8	111.9	106.6	101.7[b]	94.1[b]	100.2	104.6	85.1	84.3
15	Aug. 18–Apr. 19–Jan. 20	97.9	101.1	102.1	99.5	102.1	105.8	97.9	94.6	89.8
16	Jan. 20–Sep. 21–May 23	95.8	93.8	98.4	89.9	92.0	95.0	111.6	110.5	107.6
17	May 23–July 24–Oct. 26	87.2	82.9	83.0	85.5	91.3	99.5	107.9	117.3	124.5
18	Oct. 26–Dec. 27–June 29	83.1	85.3	93.9	99.4	100.0	101.5	103.1	111.1	118.2
	Shares traded									
1	Mar. 82–May 85–Mar. 87	126.5	107.9	101.9	90.0	59.6	104.6	95.4	101.5	97.1
2	Mar. 87–Apr. 88–July 90	131.2	122.7	118.6	67.7	108.6	92.9	100.1	101.1	79.2
3	July 90–May 91–Jan. 93	76.3	81.2	101.1	75.3	90.2	94.7	126.5	95.7	161.6
4	Jan. 93–June 94–Dec. 95	190.6	142.8	100.4	77.2	71.2	75.3	90.0	110.1	99.7
5	Dec. 95–June 97–June 99	64.9	52.9	62.0	43.4	75.9	107.9	114.2	178.3	165.5
6	June 99–Dec. 00–Sep. 02	87.2	76.6	73.0	60.6	139.0	179.9	87.6	97.7	115.5
7	Sep. 02–Aug. 04–May 07	98.3	72.7	75.6	50.1	85.0	127.3	123.9	120.5	81.4
8	May 07–June 08–Jan. 10	93.7	83.6	79.6	90.0	100.7	112.9	105.6	115.3	117.2
9	Jan. 10–Jan. 12–Jan. 13	162.7	122.8	80.2	100.9	77.7	109.0	74.2	95.0	81.8
10	Jan. 13–Dec. 14–Aug. 18	73.7	57.7	51.8	49.5[b]	28.6[b]	114.3	144.4	106.3	73.6
11	Aug. 18–Apr. 19–Jan. 20	41.6	54.8	54.0	78.2	109.9	168.9	119.9	116.6	106.7
12	Jan. 20–Sep. 21–May 23	128.2	107.7	87.8	78.7	65.9	87.0	125.4	114.1	104.2
13	May 23–July 24–Oct. 26	64.1	57.4	68.0	49.8	83.8	101.9	134.8	126.3	114.8
14	Oct. 26–Dec. 27–June 29	54.1	59.6	71.1	81.8	90.1	102.4	134.4	136.7	145.0
	Call money rates									
1	Oct. 60–June 61–Apr. 65	108.8	106.4	90.8	100.9	103.8	91.2	99.8	109.9	102.5
2	Apr. 65–Dec. 67–June 69	92.0	90.0	79.2	93.3	86.7	100.9	113.3	134.3	216.2
3	June 69–Dec. 70–Oct. 73	166.6	107.8	62.1	59.8	78.2	59.8	87.1	175.4	342.9
4	Oct. 73–Mar. 79–Mar. 82	683.9	84.4	75.3	92.0	82.2	139.5	128.7	113.6	92.3
5	Mar. 82–May 85–Mar. 87	117.8	151.8	79.5	81.8	42.9	50.8	87.9	143.9	161.0
6	Mar. 87–Apr. 88–July 90	128.4	165.5	96.2	82.3	61.0	53.6	87.3	138.0	187.5
7	July 90–May 91–Jan. 93	214.2	189.9	119.4	91.5	114.3	89.0	57.8	97.7	108.8
8	Jan. 93–June 94–Dec. 95	164.7	231.3	150.1	48.4	53.1	39.8	78.1	61.2	136.8
9	Dec. 95–June 97–June 99	116.6	129.7	180.3	59.8	54.4	67.7	86.8	104.5	159.3
10	June 99–Dec. 00–Sep. 02	108.3	142.9	75.7	58.6	72.6	107.6	87.7	111.9	193.3

TABLE B4—*Continued*

Patterns of Successive Reference Cycles Treated on Inverted Plan
Seven American Series

Cycle no.	Dates of reference cycles	Average in reference-cycle relatives at stage[a]								
		I	II	III	IV	V	VI	VII	VIII	IX
		Initial peak	Contraction			Trough	Expansion			Terminal peak
	Peak – Trough – Peak		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Call money rates — *Continued*									
11	Sep. 02–Aug. 04–May 07	198.9	115.7	77.1	42.8	35.1	61.7	145.2	139.3	76.6
12	May 07–June 08–Jan. 10	88.1	133.3	284.7	55.8	53.5	41.4	61.2	88.7	99.2
13	Jan. 10–Jan. 12–Jan. 13	115.5	104.3	80.8	88.0	73.6	92.5	97.5	160.8	131.0
14	Jan. 13–Dec. 14–Aug. 18	127.7	99.2	85.0	131.8	101.7	61.3	85.3	144.5	189.4
15	Aug. 18–Apr. 19–Jan. 20	96.7	88.4	78.4	83.5	86.9	98.9	94.5	141.6	143.7
16	Jan. 20–Sep. 21–May 23	152.0	136.9	123.8	107.9	93.5	80.5	69.9	80.7	84.0
17	May 23–July 24–Oct. 26	121.6	126.6	110.7	86.2	52.1	75.0	107.2	110.5	114.9
18	Oct. 26–Dec. 27–June 29	83.1	80.3	76.6	65.8	71.7	92.6	119.2	150.5	153.6
	Railroad bond yields									
1	Oct. 60–June 61–Apr. 65	109.9	116.3	113.6	116.8	118.3	110.2	88.7	92.0	109.2
2	Apr. 65–Dec. 67–June 69	93.8	99.1	100.4	100.2	101.0	98.8	99.2	102.4	102.1
3	June 69–Dec. 70–Oct. 73	102.0	104.3	102.0	100.8	101.9	100.4	97.9	97.6	101.0
4	Oct. 73–Mar. 79–Mar. 82	126.7	115.6	103.7	101.2	96.1	93.3	86.9	82.3	83.7
5	Mar. 82–May 85–Mar. 87	104.6	104.2	103.9	103.1	99.4	96.9	91.7	91.8	92.4
6	Mar. 87–Apr. 88–July 90	101.9	102.0	104.6	102.7	102.0	100.2	96.5	98.5	100.0
7	July 90–May 91–Jan. 93	96.9	98.4	100.5	100.6	102.3	102.9	99.0	98.6	98.8
8	Jan. 93–June 94–Dec. 95	102.4	103.3	107.1	100.5	99.4	98.0	97.5	94.0	95.4
9	Dec. 95–June 97–June 99	103.9	104.2	107.4	101.4	99.2	98.1	99.0	94.3	92.3
10	June 99–Dec. 00–Sep. 02	96.7	99.0	99.6	100.3	99.4	99.2	100.7	101.2	103.5
11	Sep. 02–Aug. 04–May 07	94.0	96.4	100.7	100.8	99.5	98.8	99.7	102.7	105.8
12	May 07–June 08–Jan. 10	99.2	101.1	105.3	101.9	100.8	98.8	97.0	98.7	99.8
13	Jan. 10–Jan. 12–Jan. 13	97.0	99.1	99.4	100.1	99.9	100.1	101.0	101.9	102.0
14	Jan. 13–Dec. 14–Aug. 18	93.6	96.6	96.8	95.7[b]	100.0[b]	99.0	96.4	109.0	114.2
15	Aug. 18–Apr. 19–Jan. 20	102.1	101.5	95.1	98.0	98.8	98.8	103.5	103.3	105.9
16	Jan. 20–Sep. 21–May 23	103.3	110.0	104.9	106.1	103.3	96.1	89.9	93.5	95.3
17	May 23–July 24–Oct. 26	102.6	102.7	103.2	101.9	99.7	99.8	99.5	97.0	96.6
18	Oct. 26–Dec. 27–June 29	101.6	100.5	98.7	97.0	95.3	97.0	101.9	104.3	106.3

[*] Entries in parentheses are interpolated.
[a] Based on the first segment of the series (see Ch. 9, note 2), which we use through cycle 14.
[b] Omits Aug.–Nov. 1914. No data (N. Y. Stock Exchange closed).

TABLE B5

List of Cycles Included in Table 164 and Chart 60

The entries for the seven series refer to the cycle numbers in Table B1. The entries for business cycles refer to the line numbers in Table 16 for the United States; that is, to cycles whose peaks are shown on these lines.

Series and phase of long cycles in building	Expansion (Col. 2 & 6 of Table 164)	Contraction (Col. 3 & 7 of Table 164)	Full cycle (Col. 4-5 & 8-9 of Table 164, and Chart 60)
DEFLATED CLEARINGS			
Upswings	1-4,8-10,13-14	1-3,8-9,13-14	1-3,8-9,13-14
Downswings	5-7,11-12	4-7,10-12,15	5-7,11-12
PIG IRON PRODUCTION			
Upswings	1-3,7-9,12-14	1-2,7-8,12-13	1-2,7-8,12-13
Downswings	4-6,10-11,15	3-6,9-11,14-15	4-6,10-11,15
FREIGHT CAR ORDERS			
Upswings	1,3-5,9-11,15-18	3-4,9-10,15-17	3-4,9-10,15-17
Downswings	2,6-8,12-14,19	1-2,5-8,11-14,18-19	2, 6-8, 12-14, 19
RAILROAD STOCK PRICES			
Upswings	3-7,11-13,16-17	3,5-6,11-12,16-17	3,5-6,11-12,16-17
Downswings	1-2,8-10,14-15	1-2,4,7-10,13-15,18	1-2,8-10,14-15
SHARES TRADED			
Upswings	1-3,7-9,12-14	1-2,7-8,12-13	1-2,7-8,12-13
Downswings	4-6,10-11,15	3-6,9-11,14-15	4-6,10-11,15
CALL MONEY RATES			
Upswings	2-4,7-11,15-17,20-22	2-3,7-10,15-16,20-21	2-3,7-10,15-16,20-21
Downswings	1,5-6,12-14,18-19,23	1,4-6,11-14,17-19,22-23	1,5-6,12-14,18-19,23
RAILROAD BOND YIELDS			
Upswings	2-6,8-10,13-14,18-19	2-5,8-9,13-14,18	2-5,8-9,13-14,18
Downswings	1,7,11-12,15-17,20	1,6,10-12,15-17,20	1,11-12,15-17,20
BUSINESS CYCLES			
Upswings	9-14,18-20,23-25	9-10,12-13,18-19,23-24	9-10,12-13,18-19,23-24
Downswings	7-8,15-17,21-22,26	7-8,11,14-17,20-22,25-26	7-8,15-17,21-22,26

TABLE B6

List of Cycles Included in Table 166 and Chart 63

The entries in the last column refer to the cycle numbers in Table B2; the entries in the preceding columns refer to the cycle numbers in Table B1.

Series and phase of long waves in prices	Expansion (Col. 2 & 7 of Table 166)	Contraction (Col. 3 & 8 of Table 166)	Full cycle, positive (Col. 4-5 & 9-10 of Table 166, and Chart 63)	Full cycle, inverted (Col. 6 & 11 of Table 166)
DEFLATED CLEARINGS				
Upswings	7-13	7-12	7-12	7-12
Downswings	1-6, 14-15	1-6, 13-15	1-6, 14-15	1-5, 13-14
PIG IRON PRODUCTION				
Upswings	6-12	6-11	6-11	6-11
Downswings	1-5, 13-15	1-5, 12-15	1-5, 13-15	1-4, 12-14
FREIGHT CAR ORDERS				
Upswings	8-15	8-14	8-14	8-14
Downswings	1-7, 16-19	1-7, 15-19	1-7, 16-19	1-6, 15-18
RAILROAD STOCK PRICES				
Upswings	1-3, 10-16	1-2, 10-15	1-2, 10-15	1-2, 10-15
Downswings	4-9, 17-18	3-9, 16-18	4-9, 17-18	3-8, 16-17
SHARES TRADED				
Upswings	6-12	6-11	6-11	6-11
Downswings	1-5, 13-15	1-5, 12-15	1-5, 13-15	1-4, 12-14
CALL MONEY RATES				
Upswings	1-2, 14-20	1, 14-19	1, 14-19	1, 14-19
Downswings	3-13, 21-23	2-13, 20-23	3-13, 21-23	2-12, 20-22
RAILROAD BOND YIELDS				
Upswings	1-2, 13-18	1-2, 12-17	1-2, 13-17	1, 12-17
Downswings	3-12, 19-20	3-11, 18-20	3-11, 19-20	3-11, 18-19

TABLE B7

List of Cycles Included in Table 167

The entries refer to the line numbers in Table 16 in the sense explained at the head of Table B5; except that in the last column, number x is the cycle running from the peak on line x to the peak on line $x+1$.

Country and phase of long waves in prices	Expansion (Col. 2 & 7 of Table 167)	Contraction (Col. 3 & 8 of Table 167)	Full cycle, positive (Col. 4-5 & 9-10 of Table 167)	Full cycle, inverted (Col. 6 & 11 of Table 167)
UNITED STATES				
Upswings	4*-6*, 7-9, 17-23	4*-6*, 7-8, 17-22	4*-6*, 7-8, 17-22	4*-6*, 7-8, 17-22
Downswings	2*-3*, 10-16, 24-26	2*-3*, 9-16, 23-26	2*-3*, 10-16, 24-26	2*, 9-15, 23-25
GREAT BRITAIN				
Upswings	2*-6*, 14*, 15-18, 21-26	2*-5*, 14*, 15-17, 21-25	2*-5*, 14*, 15-17, 21-25	2*-5*, 14*, 15-17, 21-25
Downswings	7*-13*, 19-20, 27-29	6*-13*, 18-20, 26-29	7*-13*, 19-20, 27-29	6*-12*, 18-19, 26-28
GERMANY				
Upswings	2*-3*, 6-11	2*, 6-11	2*, 6-11	2*, 6-10
Downswings	4-5, 12-13	3*, 4-5, 12-13	4-5, 12-13	3*, 4, 12
FRANCE				
Upswings	3*-5*, 6-8, 12-19	3*-5*, 6-7, 12-18	3*-5*, 6-7, 12-18	3*-5*, 6-7, 12-18
Downswings	2*, 9-11, 20	2*, 8-11, 19-20	2*, 9-11, 20	8-10, 19

* Included in the analysis based on annual reference dates, but not in the analysis based on monthly reference dates.

TABLE B8

List of Cycles Included in Tables 170, 172, 173 and 175, and Charts 67, 69 and 71

The entries for the seven series in columns (a) and (c) refer to the cycle numbers in Table B1; the entries in columns (b) and (d) to the cycle numbers in Table B2. The entries for business cycles in the United States and Great Britain refer to the line numbers in Table 16 in the sense explained at the head of Table B5; except that in columns (b) and (d), number x is the cycle running from the peak on line x to the peak on line $x+1$.

Series and place of cycles	Table 170 and Chart 67 (a)	Table 172 and Chart 69 (b)	Col. 2-5 of Tables 173 and 175, and Chart 71 (c)	Col. 6 of Tables 173 and 175 (d)
DEFLATED CLEARINGS				
First	1,3,6,9,12,14	2,5,7,9,11	1,6,10,14	5,9,13
Middle	....	....	2-4,7-8,11-12	1-3,6-7,10-11
Last	2,5,8,11,13,15	4,6,8,10,12	5,9,13,15	4,8,12,14
PIG IRON PRODUCTION				
First	2,5,8,11,13	1,4,6,8,10	1,5,9,13	4,8,12
Middle	....	....	2-3,6-7,10-11,14	1-2,5-6,9-10,13
Last	4,7,10,12,15	3,5,7,9,11	4,8,12,15	3,7,11,14
FREIGHT CAR ORDERS				
First	4,7,10,13,16	3,6,8,10,12	3,7,11,16	2,6,10,15
Middle	....	....	4-5,8-9,12-14,17-18	3-4,7-8,11-13,16-17
Last	2,6,9,12,15,19	5,7,9,11,14[a]	6,10,15,19	5,9,14,18
RAILROAD STOCK PRICES				
First	1,6,9,12,15,17	5,8,10,12,14	5,9,13,17	4,8,12,16
Middle	....	....	6-7,10-11,14-15	5-6,9-10,13-14
Last	3,8,11,14,16,18	7,9,11,13,15	8,12,16,18	7,11,15,17
SHARES TRADED				
First	2,5,8,11,13	1,4,6,8,10	1,5,9,13	4,8,12
Middle	....	....	2-3,6-7,10-11,14	1-2,5-6,9-10,13
Last	4,7,10,12,15	3,5,7,9,11	4,8,12,15	3,7,11,14
CALL MONEY RATES				
First	1,3,6,10,13,16,19,21	9,12,14,16,18	7,13,17,21	4,12,16,20
Middle	....	....	8-11,14-15,18-19,22	5-10,13-14,17-18,21
Last	2,5,9,12,15,18,20,23	11,13,15,17,19	12,16,20,23	11,15,19,22
RAILROAD BOND YIELDS				
First	4,7,9,12,14,16,19	....	7,12,15,19	6,11,14,18
Middle	....	....	8-10,13,16-17	7-9,12,15-16
Last	3,6,8,11,13,15,18,20	....	11,14,18,20	10,13,17,19
BUSINESS CYCLES, U.S.				
First	8,10,13,16,19,22,24	12,15,17,19,21	12,16,20,24	11,15,19,23
Middle	....	....	13-14,17-18,21-22,25	12-13,16-17,20-21,24
Last	7,9,11,15,18,21,23,26	14,16,18,20,22	15,19,23,26	14,18,22,25
BUSINESS CYCLES, G.B.				
First	....	....	19,21,24,27	18,20,23,26
Middle	....	....	22,25,28	21,24,27
Last	....	....	20,23,26,29	19,22,25,28

[a] Omitted in Chart 69.

TABLE B9

List of Specific Cycles Included in Table 180

(The entries refer to the cycle numbers in Table B1)

Series	1879–1897	1897–1914	1914–1933
Deflated clearings	1–6	7–11	12–15
Pig iron production	1–5	6–10	11–15
Freight car orders	3–7	8–12	13–19
Railroad stock prices	5–9	10–14	15–18
Shares traded	1–5	6–10	11–15
Call money rates	7–13	14–18	19–23
Railroad bond yields	8–12	13–15	16–20

APPENDIX C

Sources of Data

THIS APPENDIX lists sources of data, so far as they are not already given in the text. Explanatory notes are occasionally added. Their main purpose is to help the interested reader identify the particular series we have used.

Chapter 3

Chart 3

(1) INDEX OF INDUSTRIAL PRODUCTION
Through 1939, Board of Governors of the Federal Reserve System, *New Federal Reserve Index of Industrial Production* (published apparently at the end of 1941), pp. 23, 29. Since 1940, *Federal Reserve Bulletin*, Aug. 1943, p. 773. (For revised data starting 1939, see *ibid.*, Oct. 1943, p. 964.)

(2) TON-MILES OF FREIGHT HAULED
Nonrevenue ton-miles included. Derived from publications of the Interstate Commerce Commission. Through 1922, *Statistics of Railways*, 1922, p. XCV. For 1923–34, *Freight and Passenger Service Operating Statistics of Class I Steam Railways*. Since 1935, *Freight Train Performance of Class I Steam Railways*. Seasonal removed by the National Bureau.

(3) DEPARTMENT STORE SALES
Federal Reserve Bulletin, Aug. 1936, p. 631; Feb. 1938, p. 160; Jan. 1941, p. 65; April 1942, p. 372. (For revised data, see *ibid.*, June 1944.)

Chapter 4

Chart 7

Some of the titles on the chart are abbreviated here.

(1) MILK USED IN FACTORY PRODUCTION
Through 1941, furnished by Bureau of Foreign and Domestic Commerce. For 1942, *Survey of Current Business*, March 1943, p. S–25. (Slightly revised figures, starting in 1920, may be obtained from the Bureau of Foreign and Domestic Commerce.)

(2) TRANSIT RIDES, NEW YORK CITY
Bus rides, which constituted 3 per cent of total transit rides in 1927, are excluded before 1927. Traffic of Hudson and Manhattan Railroad Company is excluded throughout. In general, the data come from the *Annual Report* of the Transit Commission (prior to 1920, Public Service Commission, First District), State of New York. Figures on subway and streetcar rides in 1941–42, and bus rides in 1927–42, furnished by Transit Commission.

(3) RAILWAY PASSENGER-MILES, COMMUTATION
Interstate Commerce Commission, *Revenue Traffic Statistics of Class I Steam Railways.*

(4) WHOLESALE PRICE OF SULPHURIC ACID
From publications of the Bureau of Labor Statistics: through 1931, various issues of the *Bulletin;* since 1932, *Wholesale Prices.*

(5) WHOLESALE PRICE OF SHOES
Through Sept. 1931, *Bulletin* of the Bureau of Labor Statistics, various issues. For 1931–42, furnished by Bureau of Labor Statistics. The figures of the first segment run higher than the figures of the second. Hence the figures of the second segment, starting Oct. 1931, were raised by 1.026, which is the average ratio of the old to the new series during Jan.–Sept. 1931.

(6) WHEAT FREIGHT RATES, CHICAGO TO NEW YORK
Average daily rate during month. Basic data through 1934, furnished by Chicago Board of Trade. Since 1935, Chicago Board of Trade, *Annual Report.*

Chart 8 and Tables 18–19

All series are adjusted for seasonal variations, except (4)–(5), (33)–(34), (38)–(40). The following series have been adjusted by the National Bureau: (3), (8), (10), (20)–(22), (26)–(30), (35)–(37), also component (b) of (23)–(25).

(1) TOTAL INCOME PAYMENTS
For 1932–38, furnished by Bureau of Foreign and Domestic Commerce. For 1939, *Survey of Current Business,* March 1943, p. 27. For revised data in 1939, see *ibid.,* April 1944.

(2) TOTAL CIVIL NONAGRICULTURAL EMPLOYMENT
Self-employed persons, casual workers, and domestic servants excluded. *Federal Reserve Bulletin,* June 1941, pp. 534–5.

(3) BANK DEBITS OUTSIDE N.Y. CITY
Survey of Current Business, 1936 Supplement, p. 44; June 1933, p. 30; 1938 Supplement, p. 53; 1940 Supplement, p. 48.

(4) INDEX OF WHOLESALE PRICES
Bureau of Labor Statistics, *Wholesale Prices,* Dec. issues.

(5) INDEX OF COST OF LIVING
National Industrial Conference Board index. *Survey of Current Business,* Dec. 1936, p. 19; Jan. 1941, p. 18.

(6) RETAIL SALES
Sales by department stores, chain and independent grocery stores, automobile dealers, restaurants, service stations, and other retail outlets. Estimates furnished by V. Lewis Bassie, War Production Board.

(7) DEPARTMENT STORE SALES
See this appendix, note on series (3) of Chart 3.

(8) CHAIN STORE SALES
Survey of Current Business, 1936 Supplement, p. 25; 1940 Supplement, p. 27.

(9) FREIGHT CAR LOADINGS
Federal Reserve Bulletin, June 1941, pp. 532–3. See p. 529 for description.

(10) ELECTRIC POWER PRODUCTION
Survey of Current Business, Dec. 1940, p. 17.

(11) TOTAL INDUSTRIAL PRODUCTION (F.R.BOARD)
See this appendix, note on series (1) of Chart 3.

(12) Total Industrial Production (S.S.Co.)
From publications of Standard Statistics Company. Through 1936, *Standard Trade and Securities: Basic Statistics,* April 29, 1938, Sec. D, p. 67. Since 1937, *Standard Trade and Securities: Current Statistics,* Dec. 1939, p. 13; Dec. 1940, p. 13.

(13) Production of Durable Manufactures
Board of Governors of the Federal Reserve System, *New Federal Reserve Index of Industrial Production* (published apparently at the end of 1941), pp. 30, 45, 82. (For revised data, starting 1939, see *Federal Reserve Bulletin,* Oct. 1943.)

(14) Production of Nondurable Manufactures
Same as (13).

(15) Production of Minerals
Same as (13).

(16) Production of Producers' Durable Goods
Federal Reserve Bank of New York index; furnished by that agency. Adjusted for secular trend. Data reflect revisions as of Jan. 1942. See description in *Journal of the American Statistical Association,* June 1938, pp. 341–8; Sept. 1941, pp. 423–5.

(17) Production of Consumers' Durable Goods
Same as (16).

(18) Production of Producers' Nondurable Goods
Same as (16).

(19) Production of Consumers' Nondurable Goods
Same as (16).

(20) Payrolls in Manufacturing
Bureau of Labor Statistics index. *Survey of Current Business,* Dec. 1938, p. 16; Oct. 1939, p. 17; 1942 Supplement, pp. 48, 50.

(21) Payrolls, Durable Manufactures
Same as (20).

(22) Payrolls, Nondurable Manufactures
Same as (20).

(23) Man-hours Worked in Manufacturing
Derived by multiplying seasonally adjusted figures of two series: (a) index of factory employment (*Survey of Current Business,* Dec. 1938, p. 15; Oct. 1939, p. 17; March 1941, p. 18) and (b) average hours worked per week (Bureau of Labor Statistics, "Hours and Earnings in the United States, 1932–40", *Bulletin 697,* pp. 48–9, 156).

(24) Man-hours Worked, Durable Manufactures
Same as (23).

(25) Man-hours Worked, Nondurable Manufactures
Same as (23).

(26) Average Hourly Earnings in Manufacturing
Same as (23), series (b).

(27) Total Imports
General imports. Bureau of Foreign and Domestic Commerce, *Monthly Summary of Foreign Commerce.*

(28) Total Exports
Re-exports of foreign merchandise included. Same as (27).

(29) Total Construction Contracts
Value of contracts in 37 states, reported by F. W. Dodge Corporation. *Survey of Current Business.* 1936 Supplement, p. 16; 1940 Supplement, p. 16.

(30) Private Construction Contracts
Value of contracts in 37 states, on account of private owners, reported by F. W. Dodge Corporation. *Ibid.,* Aug. 1937, p. 18; 1940 Supplement, p. 16.

(31) Orders for Manufactured Goods
Index of value of new orders, by National Industrial Conference Board. C. L. Rogers, "Inventories, Shipments, Orders, 1929–1940: Revised Indexes", *The Conference Board Economic Record,* Vol. II, Supplement, Dec. 26, 1940, pp. 3, 7.

(32) Inventories Held by Manufacturers
Index of value of inventories. Same as (31).

(33) Industrial Common Stock Prices
Index of prices of 354 stocks. Standard and Poor's Corporation, *Trade and Securities Statistics: Long Term Security Price Index Record,* p. 7; see description on pp. 3–4. (The issue cited is Vol. 96, No. 9, Sec. 2 of the Corporation's publications.)

(34) Corporate Bond Prices
Same as (40), but inverted.

(35) Number of Business Failures
Data represent 'all commercial' failures. Through 1933, *Dun's Review,* Jan. 1935, p. 18. Since 1934, *Dun's Statistical Review,* Feb. 1940, p. 14; these figures are not strictly comparable with earlier data.

(36) New Corporate Capital Issues
Domestic issues, dollar value, compiled by *Commercial and Financial Chronicle.* Reported in *Survey of Current Business,* Feb. 1938, pp. 18, 20; 1940 Supplement, p. 68; 1942 Supplement, p. 80.

(37) Loans of Reporting Member Banks
From reporting member banks in 101 cities. Derived from *Federal Reserve Bulletin,* Feb. 1933, p. 75; Dec. 1935, pp. 806, 876; and later issues.

(38) Net Demand Deposits of Member Banks
All member banks. Through 1937, Board of Governors of the Federal Reserve System, *Annual Report.* Since 1938, *Federal Reserve Bulletin.*

(39) Commercial Paper Rates
Through Jan. 1937, Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices,* Appendix A, Table 10, col. 3, pp. A160–1. Since Feb. 1937, *Bank and Quotation Record.* See Macaulay, pp. A335–51, for description.

(40) Yield of Corporate Bonds
Moody's 120 domestic bonds. Reported in *Survey of Current Business,* Nov. 1937, p. 19; 1940 Supplement, p. 73.

Chart 9

All series have been adjusted for seasonal variations by the National Bureau, except (1), (2), and (4), which are left in their original form; and (13)–(14), which were adjusted by the compilers. Series (11) is adjusted through 1916 only.

(1) Index of Wholesale Prices
Bureau of Labor Statistics, *Bulletin 543,* pp. 7–8.

(2) Index of Wages
Snyder's index of composite wages. Furnished by Federal Reserve Bank of New York.

(3) Commercial Paper Rates
Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices*, Appendix A, Table 10, col. 3, pp. A156–7.

(4) Index of Common Stock Prices
Alfred Cowles 3rd and Associates, *Common-Stock Indexes, 1871–1937* (Principia Press, 1938), p. 67.

(5) Bank Clearings outside N.Y. City
Daily averages. *Commercial and Financial Chronicle*, Vol. 102, p. 191; Vol. 104, p. 106; Vol. 106, p. 16; Vol. 108, p. 208; Vol. 110, p. 302.

(6) Factory Payrolls, N.Y. State
New York State, Department of Labor, *The Industrial Bulletin*, Jan. 1942, p. 9.

(7) Construction Contracts, 27 States
Furnished by F. W. Dodge Corporation.

(8) Total Imports
See this appendix, note on series (27) of Chart 8.

(9) Total Exports
See this appendix, note on series (28) of Chart 8.

(10) Liabilities of Business Failures
Bradstreet's.

(11) Corporate Capital Issues
Bonds, notes and stocks. Refunding issues included; also foreign issues. *Journal of Commerce*, Jan. 2d or 3d numbers, 1915–19.

(12) Net Operating Income, U.S. Steel Corp.
Commercial and Financial Chronicle, Vol. 106, p. 494; Vol. 110, p. 463. See description in Vol. 106, p. 493.

(13) Production, Basic Industries
Federal Reserve Bulletin, May 1924, p. 422.

(14) R.R. Revenue Freight Ton-miles
Estimates furnished by Babson Statistical Organization, Inc.

(15) Factory Employment, N.Y. State
Same as (6).

(16) Pig Iron Production
Daily averages. Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices*, Appendix A, Table 27, col. 4, pp. A264–5.

(17) Cotton Consumption
In running bales. Bureau of the Census (bulletins on Cotton Production and Distribution), *Bulletin 135*, pp. 57–8; *Bulletin 145*, p. 57.

(18) Portland Cement Production
W. M. Persons, W. T. Foster, and A. J. Hettinger, Jr. (editors), *The Problem of Business Forecasting* (Houghton Mifflin, 1924), pp. 160–1.

(19) Oak Flooring Production
Survey of Current Business, May 1924, pp. 36–7.

(20) Passenger Car Production
Ibid., June 1927, p. 22.

(21) TRUCK PRODUCTION
Same as (20).

(22) BONDS TRADED, N.Y. STOCK EXCHANGE
In dollars, par value of bonds sold. *New York Stock Exchange Bulletin*, Aug. 1934, p. 10A.

(23) SHARES TRADED, N.Y. STOCK EXCHANGE
Number sold. Same as (22).

Table 21

All series are adjusted for seasonal variations, except (19)–(23), (25)–(26), (28)–(29), (31)–(33), (37)–(38), and (42). No seasonal adjustment of (12)–(13) after 1930, (24) before 1921, (27) after 1929, (39) after May 1931. Except for (1)–(5), (9)–(10), and the Babson segment of (15), all seasonal adjustments are by the National Bureau.

All series have been analyzed on a monthly basis, except the following which are quarterly: (11)–(13), (43) and (45) through 1897, and (44) and (46) through 1894.

(1) AYRES' INDEX OF BUSINESS ACTIVITY
Adjusted for trend. From publications by Cleveland Trust Co. Through 1928, chart on *American Business Activity since 1790* (7th ed., Jan. 1933). Since 1929, *Business Bulletin*, Jan. 15, 1935. (For revised figures since 1923, see Leonard P. Ayres, *Turning Points in Business Cycles*, pp. 197–9.)

(2) PERSONS' INDEX OF PRODUCTION AND TRADE
Adjusted for trend. Through 1902, Warren M. Persons, *Forecasting Business Cycles*, pp. 93–125. From 1903 to trough in 1919, *Review of Economic Statistics*, April 1923, pp. 75–6. From trough in 1919 to 1933, *ibid.*, Aug. 1933, pp. 157–60. (For revised data starting in 1919, see Edwin Frickey, *Barron's Index of Business since 1899*, Barron's Publishing Co., 1943.)

(3) AXE-HOUGHTON INDEX OF TRADE AND INDUSTRIAL ACTIVITY
Based on pig iron production, imports, bank clearings outside N.Y. City, and revenue per mile of selected railroads. (Not adjusted for trend.) Furnished by E. W. Axe and Company, Inc., N.Y. City.

(4) A.T.&T. INDEX OF BUSINESS ACTIVITY
Adjusted for secular trend. Through trough in 1900, *Harvard Business Review*, Jan. 1923, p. 159. From trough in 1900 to July 1932, *Index of Industrial Activity in the United States* (a confidential report of the American Telephone and Telegraph Company, Oct. 20, 1932). Since Aug. 1932, the company's *Summary of Business Conditions in the United States*. (The series has been revised according to later confidential releases of the Chief Statistician's Division.)

(5) PITTSBURGH INDEX OF BUSINESS
Adjusted for secular trend. Furnished by Bureau of Business Research, University of Pittsburgh. Described in *Pittsburgh Business Review*, Oct. 1933.

(6) BANK CLEARINGS, TOTAL
Daily averages for a varying (generally increasing) number of cities. Through 1883, *The Public* (formerly *The Financier*). Since 1884, *Commercial and Financial Chronicle*.

(7) BANK CLEARINGS OUTSIDE N.Y. CITY
Same as (6).

(8) BANK CLEARINGS OUTSIDE N.Y. CITY, DEFLATED
The series on clearings (before deflation) is not the same as (7). Continued after 1918 with bank debits in 140 centers outside N.Y. City, adjusted to the level of 'outside' bank clearings in Jan. 1919. Deflated by Snyder's index of the general price level. See Ch. 6, note 7.

(9) SNYDER'S CLEARINGS INDEX OF BUSINESS
Adjusted for secular trend and changes in the general level of prices; smoothed by a 3-month moving average. Based on bank clearings outside N.Y. City through 1918, on bank debits in 140 centers outside N.Y. City since 1919. Furnished by Federal Reserve Bank of New York.

(10) SNYDER'S INDEX OF DEPOSITS ACTIVITY
Adjusted for secular trend through 1918; smoothed throughout by a 3-month moving average. Through 1918, *Review of Economic Statistics,* Oct. 1924, p. 258. Since 1919, furnished by Federal Reserve Bank of New York.

(11) ORDERS FOR LOCOMOTIVES
Number of locomotives. Through trough in 1924, John E. Partington, *Railroad Purchasing and the Business Cycle,* pp. 219–26. Later data from *Survey of Current Business.*

(12) ORDERS FOR FREIGHT CARS
Number of cars. Through trough in 1924, same as (11). Later data from *Iron Trade Review.*

(13) ORDERS FOR PASSENGER CARS
Number of railroad cars. Same as (11).

(14) PLANS FILED FOR NEW BUILDINGS, MANHATTAN
In dollars. Bronx included through trough in 1904; later data confined to Manhattan. Through 1879, *The City Record* (New York), Jan. 19, 1880, p. 104. For 1880–1909 and 1917–19, *Real Estate Record and Builder's Guide.* For 1910–16 and after 1919, Borough of Manhattan, Dept. of Buildings, *Annual Report.*

(15) RAILROAD FREIGHT TON-MILES
Data for 1908–12 and 1919–21 cycles include nonrevenue ton-miles. Through trough in 1908, and from trough in 1912 to trough in 1919, the data are estimates furnished by Babson Statistical Organization, Inc. From trough in 1908 to trough in 1912, American Railway Association, *Proceedings,* May 20, 1914, pp. 525, 527. From trough in 1919 to trough in 1921, Interstate Commerce Commission, *Statistics of Railways,* 1922, p. XCV. Later data from the Commission's *Revenue Traffic Statistics of Class I Steam Railways.*

(16) PIG IRON PRODUCTION
See Ch. 6, note 7.

(17) TOTAL IMPORTS
General imports. Derived from publications of Bureau of Foreign and Domestic Commerce (or its predecessors): through June 1914, *Monthly Summary of Commerce and Finance,* Dec. 1910, pp. 1120–6, and later issues; since July 1914, *Monthly Summary of Foreign Commerce.*

(18) TOTAL EXPORTS
Re-exports of foreign merchandise included. Same as (17).

(19) SNYDER'S INDEX OF GENERAL PRICES
Through 1912, a composite of indexes of wholesale prices, cost of living, wages, security prices, urban realty prices, and rents. For the period since 1913, see Carl Snyder, The Measure of the General Price Level, *Review of Economic Statistics,* Feb. 1928, pp. 40–52. Data furnished by Federal Reserve Bank of New York.

(20) WHOLESALE PRICES, TOTAL
Through trough in 1891, George F. Warren and Frank A. Pearson, *Prices,* pp. 12–13. Later data from publications of the Bureau of Labor Statistics: through 1931, *Bulletin,* various issues; since 1932, *Wholesale Prices.*

(21) METALS AND METAL PRODUCTS, WHOLESALE PRICES
Publications of the Bureau of Labor Statistics, as in (20).

(22) BUILDING MATERIALS, WHOLESALE PRICES
Same as (21).

(23) Fuel and Lighting, Wholesale Prices
Same as (21).

(24) Chemicals and Drugs, Wholesale Prices
Same as (21).

(25) Textiles, Wholesale Prices
Same as (21).

(26) Housefurnishing Goods, Wholesale Prices
Same as (21).

(27) Hides and Leather Products, Wholesale Prices
Same as (21).

(28) Pig Iron, Wholesale Price, Philadelphia
Through trough in 1912, price of 'No. 1 anthracite foundry' pig iron; later, 'basic' pig iron. American Iron and Steel Institute, *Annual Report.*

(29) Steel Billets, Wholesale Price, Pittsburgh
Price of 'Bessemer' steel billets through 1928; 'open hearth, rerolling' steel billets since 1929. Through 1889, *Iron Age,* Jan. 7, 1915, p. 12. Later data from publications of the Bureau of Labor Statistics, as in (20).

(30) Slab Zinc, Wholesale Price, N.Y. City
Through trough in 1900, *The Mineral Industry,* 1892, p. 471, and later issues. Later data from *Iron Age* (Jan. annual review numbers).

(31) Copper, Wholesale Price, N.Y. City
Through trough in 1912, 'lake' copper; later, 'electrolytic' copper. Through 1895, *The Mineral Industry,* 1893, p. 253, and later issues. Since 1896, *Engineering and Mining Journal.*

(32) Pig Lead, Wholesale Price, N.Y. City
Through 1889, *The Mineral Industry,* 1893, p. 423. Since 1890, *Engineering and Mining Journal.*

(33) Pig Tin, Wholesale Price, N.Y. City
Through 1899, *The Mineral Industry,* 1893, p. 612, and later issues. Since 1900, *Metal Statistics* (published by American Metal Market).

(34) Bank Clearings, N.Y. City
Daily averages. Through Sept. 1860 and for 1864–74, New York State Chamber of Commerce, *Annual Report.* From Oct. 1860 through 1861, *The Bankers' Magazine.* For 1862–63, *The Merchants' Magazine and Commercial Review.* For 1875–83, *The Public.* Since 1884, *Commercial and Financial Chronicle.*

(35) Shares Traded, N.Y. Stock Exchange
See Ch. 6, note 7.

(36) Bonds Traded, N.Y. Stock Exchange
In dollars, par value of bonds sold. Through 1897, *New York Times,* 1905. different issues. Since 1898, *New York Stock Exchange Bulletin,* Aug. 1934, pp. 10A–B.

(37) Index of 'All' Common Stock Prices
Alfred Cowles 3rd and Associates, *op. cit.,* pp. 66–7.

(38) Index of Railroad Stock Prices
See Ch. 6, note 7.

(39) Call Money Rates, N.Y. Stock Exchange
See Ch. 6, note 7.

(40) 90-DAY MONEY RATES, STOCK EXCHANGE LOANS, N.Y. CITY
Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices,* Appendix A, Table 10, col. 2, pp. A150–61.

(41) COMMERCIAL PAPER RATES, N.Y. CITY
Ibid., col. 3, pp. A142–61.

(42) RAILROAD BOND YIELDS
See Ch. 6, note 7.

(43) NUMBER OF FAILURES
Dun's Review, successive issues. Includes manufacturing and mining concerns, builders, employers of labor in mechanic arts; also trading concerns, agents and brokers; but not professional men, banks (after 1892), or railroads. See *Dun's Review,* Dec. 30, 1893, pp. 2–3, and Feb. 1936, p. 21.

(44) NUMBER OF FAILURES
Bradstreet's, successive issues. Apparently differs from (43) in including banks, and excluding stock brokers and real estate dealers.

(45) LIABILITIES OF FAILURES
Same as (43).

(46) LIABILITIES OF FAILURES
Same as (44).

Chapter 5

Chart 11

Every series except that for petroleum is plotted from the beginning of its statistical record by months. The output of petroleum from the Appalachian field is available by months from 1868 on. All series are adjusted for seasonal variations.

(1) COKE PRODUCTION
See text, Table 4.

(2) BITUMINOUS COAL PRODUCTION
See text, Table 13.

(3) CALVES SLAUGHTERED
Federally inspected slaughter. Bureau of Agricultural Economics, *Livestock, Meats, and Wool: Market Statistics and Related Data,* 1933, p. 59.

(4) COTTON STOCKS AT MILLS
Derived from Bureau of the Census bulletins on *Cotton Production and Distribution.*

(5) PETROLEUM OUTPUT, APPALACHIAN FIELD
Daily averages. Through 1931, *Mineral Resources* (by Geological Survey through 1923, later by Bureau of Mines). Since 1932, Bureau of Mines, *Minerals Yearbook.*

(6) STRUCTURAL STEEL ORDERS
Through 1922, Bureau of the Census, *Record Book of Business Statistics,* Part II, p. 32. For 1923–32, *Survey of Current Business,* 1932 Supplement, pp. 216–7, and later monthly issues. For 1933, American Institute of Steel Construction, Inc., *Annual Report,* 1941, p. 89.

Chart 12

The series are adjusted for seasonal variations.

(1) SLAB ZINC, WHOLESALE PRICE
Iron Age, Jan. 5, 1939, p. 209.

(2) SHARES TRADED, N.Y. STOCK EXCHANGE
See Ch. 6, note 7.

Charts 16, 17, 20, 21

All series are adjusted for seasonal variations except (3), (4) and (10). The seasonal correction of (6) stops in May 1931.

(1) BITUMINOUS COAL PRODUCTION
See text, Table 13.

(2) PIG IRON PRODUCTION
See Ch. 6, note 7.

(3) PRICE OF PIG IRON
Weighted average of prices of four leading grades. Derived from publications of the Bureau of Labor Statistics: through 1931, *Bulletin*, various issues; since 1932, *Wholesale Prices*.

(4) RAILROAD BOND YIELDS
See Ch. 6, note 7.

(5) SHARES TRADED
See Ch. 6, note 7.

(6) CALL MONEY RATES
See Ch. 6, note 7.

(7) TOTAL EXPORTS
See this appendix, note on series (18) of Table 21.

(8) SUGAR MELTINGS
Data relating to four ports overlapped in 1921 with data for eight ports. For four ports, *Weekly Statistical Sugar Trade Journal*. For eight ports, *Survey of Current Business*, Oct. 1937, p. 17; 1940 Supplement, p. 113.

(9) POSTAL RECEIPTS
Receipts at 50 selected (largest) cities. Through 1919, furnished by Post Office Department. Since 1920, *Survey of Current Business*.

(10) SLAB ZINC STOCKS AT REFINERIES
Data refer to end of month. Furnished by American Zinc Institute.

Chapter 8

Chart 45

Series S consists of monthly readings from a simple sine curve, ranging from a trough of 800 to a peak of 1200, with a period of 44 months. Its equation is $y = 1000 + 200 \sin x$, where x is the number of months multiplied by 8°11′.

Series C also ranges from a trough of 800 to a peak of 1200, and has a period of 44 months. It is made from a segment of a tangent function, $y = 1000 + 38\,[(\tan x) - 1]$. The selected segment falls between $x = 45°$ and $x = 80°\,56'$ inclusive. By taking a monthly reading every 3°16′, the second half of the cyclical rise (starting with the mid-expansion month and ending with the peak month) was obtained. By using the deviations of these figures from 1000, in reverse order and changing signs, the first half of the rise (from the trough month through the mid-expansion month) was obtained. The contraction is the same as the expansion, with the order of the figures reversed.

Series T has 'triangular' cycles, with a period of 44 months. It starts from a trough of 800, rises by a monthly increment of 18.18 to a peak of 1200, and then falls away at the rate of 18.18 per month to a trough of 800.

The *basic random series* was obtained as follows. First, we took the two digit random numbers given in Table XXXIII of *Statistical Tables*, by R. A. Fisher and F. Yates. This is a series of random numbers from a rectangular frequency distribution, with a range from 0 to 99 inclusive. In order to obtain a random series with a normal distribution, the numbers from 1 to 99 (zeroes were skipped) were assumed to represent percentiles of a normal distribution, and the corresponding values of the normal deviate were taken from Table IX in the above source. For example, the normal deviate corresponding to the 1st percentile is −2.3263 (which means that one per cent of the observations in a normal population, with mean of zero and variance of unity, have deviates algebraically less than −2.3263). After transforming the numbers 1 to 99 to normal deviates, the latter were multiplied by 50; so that the range of the *basic random series* is 232 (from −116 to +116), or somewhat more than one-half the range of the cyclical series.

Series S, C and T are aligned: their troughs come in the same month, as do their peaks. The basic random series is fixed in time for all series. Series S' is the sum of the sine function and the basic random series. Series S'' is the sum of the sine function and twice the basic random series. And so on for the other series, as explained in the text.

Each series covers six full cycles, counting from troughs, with a few additional values placed before the first and after the last trough. The number of monthly values used in a series is 282. Smoothing by Macaulay's formula involves a loss of 21 items at each end. Hence the smoothed data cover only four cycles from trough to trough, and five cycles from peak to peak.

Series S', S'', C' and C'' are adaptations of artificial series originally worked out by Geoffrey H. Moore and W. Allen Wallis, to whom we owe thanks.

Chapter 10

Chart 59

The index of orders is based upon quarterly figures for locomotives, freight cars, passenger cars, and rails. These series come from John E. Partington, *Railroad Purchasing and the Business Cycle*, pp. 219–26, through the trough in 1924; continued with data from the *Railway Age* for locomotives and passenger cars, and from the *Iron Trade Review* for freight cars. The index was obtained by averaging the reference-cycle patterns of the four series for each business cycle (except that rail orders are not used after the trough in 1924). This operation combines two steps: the construction of a rough index of orders and a reference-cycle analysis of the index. The implicit index number is an unweighted arithmetic mean of relatives, the average standing of each series during a reference cycle being the base.

The data on freight ton-miles are compiled from various sources. See this appendix, note on series (15) of Table 21.

Chapter 12

Chart 77

The index of American crop production includes 12 basic crops and comes from the Bureau of Agricultural Economics, *Agricultural Situation*, Jan. 1935, Oct. 1937; figures for recent years were furnished by the Bureau of Agricultural Economics. For the British index from 1866 to 1931, see Leo Drescher, Die Entwicklung der Agrarproduktion Grossbritanniens und Irlands seit Beginn des 19. Jahrhunderts, *Weltwirtschaftliches Archiv*, Vol. 41, March 1935, pp. 293–4. To cover recent years, we extended this index by methods similar to Drescher's.

For the source of the American pig iron series, see Ch. 6, note 7. The British iron series comes from *The Iron and Coal Trades Review*.

Index