SUMMARY

1. Attempts to prepare pinacones of the type C_6H_4 $(CR_2OH)_2$ by the action of Grignard reagents upon terephthalic or aminoterephthalic esters, failed in the case of the alkyl derivatives, but succeeded with the aryl magnesium halides.

2. The tetralkyl pinacone types lost water easily with formation of diolefins which then generally polymerized to gums.

3. The tetramethyl pinacone has been investigated pharmacologically and found to possess little, if any, physiological activity. In the case of the other alkylated pinacones, only the di-olefin or gums were obtained.

4. The tetraryl pinacone types were too difficultly soluble to be of pharmacological interest.

5. Various new compounds have been synthesized and investigated, and some old ones have been prepared by new methods.

6. The apparatus of Bogert and Harris, for carrying out the Grignard reaction with difficultly soluble compounds, has been simplified.

[1] See these PROCEEDINGS.

[2] We are indebted to the Hollingsworth and Whitney Company, of Waterville, Maine, for the "spruce turpentine condensate" which served as the initial material for this investigation.—M. T. B. and P. S. N.

[3] Bogert and Harris, *J. Am. Chem. Soc.*, **41,** 1676 (1919).

[4] Berend and Herms, *J. prakt. Chem.*, [2], **74,** 123 (1906).

[5] Perkin and Weizmann, *J. Chem. Soc.*, **89,** 1655 (1906).

[6] Ingle, *Ber.*, **27,** 2526 (1894).

[7] Noelting and Kohn, *Ibid.*, **19,** 147 (1886).

[8] Yocich, *J. Russ. Phys.-Chem. Soc.*, **34,** 971 (1902); **36,** 8 (1904).

[9] Pink, *J. Chem. Soc.*, **123,** 3418 (1923).

[10] Kauffmann and Weissel, *Ann.*, **393,** 10 (1911).

A STATISTICAL DISCUSSION OF SETS OF PRECISE ASTRONOMICAL MEASUREMENTS, IV; THE MASS-RATIO IN BINARIES

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Communicated, July 16, 1924

From lack of data, the assumption has often been made that the masses of the components of a binary system tend toward equality.[1] Although there is little good material for statistical discussion, its implications are important. We have fairly accurate total masses and mass-ratios for a dozen visual binaries and accurate mass-ratios for about 60 spectroscopic binaries. If we put $x = \log_{10}(m'/m)$ and take the mass-ratio both ways (m'/m and m/m') in each pair we obtain a necessarily symmetrical distribution of x about its mean, 0. We found 69 systems that seem worthy of inclusion in the discussion and they give 138 values of x. The value of the standard deviation of x is $\sigma_x = 0.26 \pm .02$ and that of Pearson's constant β_2 (the ratio of the fourth moment about the mean to the square of the second moment about the mean) is $\beta_2 = 4.3 \pm 0.4$. The distribution therefore, is statistically speaking, not significantly different from the normal error curve ($\beta_2 = 3.0$).[2] In our previous note (these PROCEEDINGS, **10,** 394, 1924) we showed that for the sun and 14 nearby binaries the standard deviation of the logarithms of the masses from their mean was $\sigma = 0.36 \pm 0.03$. The value of σ for the logarithmic mass ratios of these 14 systems is 0.30 ± 04. It follows that the inclusion of the additional data from the spectroscopic pairs makes no significant change in the dispersion of x about its mean.

We saw before that the value of σ was almost the same (0.36) whether we treated the stars as individuals or as systems. Now if r be the coefficient of correlation between the logarithmic masses of the components in binaries we may write

$$\sigma_x^2 = 2\sigma^2 - 2r\sigma^2 \quad \text{or} \quad r = 1 - \sigma_x^2/2\sigma^2.$$

If we use $\sigma_x = 0.26$ and $\sigma = 0.36$ we find $r = 0.74$ and there appears to be a high degree of correlation between the masses in the pairs, or, otherwise expressed, a great tendency for the masses to be equal, which is in accord with the assumption generally made. It should not be overlooked, however, that there is probably a strong observational selection at work. Very dissimilar masses would mean statistically very different magnitudes and diminish the probability of detection and measurement of the binary. Another way to put the matter is to ask: What would be the standard deviation of the logarithmic mass-ratio of the 406 hypothetical binaries that could be constructed by pairing in all possible ways the 29 stars of our previous note with $\sigma = 0.36$? The result would be $\sigma_x' = 0.50$ which is much larger than the value 0.26 found above. But in our present list of 69 binaries there is none with a mass ratio greater than 5; whereas in the 406 hypothetical binaries there are 79. Now if it be assumed that these high mass-ratios if existent would not be observed and measured, we find that the value of σ for the remaining 327 pairs would be only 0.31 which is a great reduction from 0.50 and is not far from the value 0.26 actually found.

To discuss the observational selection we may proceed as follows: Let the total number of binaries be N and the number of those discovered and measured $N'(= 69)$. If the ratio of the masses were in the manner that would arise from random mating of stars with $\sigma = 0.36$, the frequency distribution would be

$$f(x) = \frac{2N}{0.50\sqrt{2\pi}}\, e^{-x^2/2(0.50)^2} = \frac{2Ne^{-2x^2}}{0.5\sqrt{2\pi}}.$$

The frequency distribution calculated for the 69 measured binaries is

$$\varphi(x) = \frac{2N'}{0.26\sqrt{2\pi}} e^{-x^2/2(0.26)^2} = \frac{2N' e^{-7.4x^2}}{0.26\sqrt{2\pi}}.$$

The average chance of finding a binary in our set would be N'/N; but the chance of finding one of specified logarithmic mass ratio x would be

$$\varphi(x)/f(x) = 1.9\, e^{-5.4x^2} .N'/N,$$

and there would thus be about double the average chance (N'/N) for finding on our list a binary with equal masses.

Let it be assumed that the relation between the logarithm of the mass and the luminosity M is linear and for definiteness that $x_1 - x_2 = -0.07\ (M_1 - M_2)$, which is close to the value obtained by several investigators. Then the following table is readily computed.

MASS RATIO	DISCOVERY CHANCE*	LUMINOSITY RATIO
20 : 1	1 : 9000	10^8: 1
10 : 1	1 : 220	4×10^5: 1
5 : 1	1 : 14	25000 : 1
3 : 1	1 : 3.7	600 : 1
2 : 1	1 : 1.6	70 : 1
1.5 : 1	1 : 1.2	11 : 1
1 : 1	1 : 1	1 : 1

* Here the chance of discovery is estimated relative to the chance of discovering stars of equal mass, i.e., it is ϕ/f divided by $1.9N'/N$.

In view of the very great luminosity ratios it might appear reasonable to believe that the discovery chances have been over-estimated and that, when the discovery chance is considered, the observed mass-ratios are more scattered than would be the case if the two components were selected at random. And this confirms our remark in the previous note that whenever the masses of enough additional binaries shall have been measured to re-establish the statistical homogeneity, it will not unlikely be found that the dispersion of logarithmic masses is greater than 0.36.

Thus the general conclusion from the meager data of reasonable precision that we possess is that there is a considerable probability that the apparent clustering of the mass-ratios about unity is due to observational selection and that the actual association may be not materially different from random mating.

[1] Cf. Eddington, *Stellar Movements*, p. 23, and Boss, *Preliminary General Catalogue*, p. xxiii.

[2] The series of 138 is too small to make a departure of three times the probable error significant; the numerical constants of the distribution would be altered somewhat by discarding one binary for which the mass ratio is nearly double (or half) that of the next highest (smallest).