# *A REDETERMINATION OF THE NEWTONIAN CONSTANT OF GRAVITATION*[*]

By P. R. Heyl

Bureau of Standards, Washington, D. C.

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The accepted value of the constant of gravitation for the past 30 years has rested upon the independent work of Boys[1] and of Braun.[2] These two experimenters obtained mean results agreeing to 4 significant figures ($6.658 \times 10^{-8}$ cm.$^3$ g.$^{-1}$ sec.$^{-2}$) but because their separate values varied in the third figure (by as much as 4 units in Braun's work, and by 2 units in that of Boys), Poynting,[3] in his discussion of the subject, assigns the value $6.66 \times 10^{-8}$, with a possible uncertainty of one unit in the third figure.

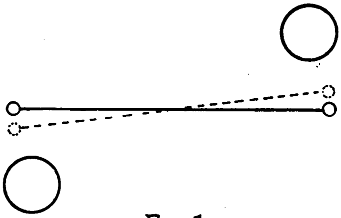

Fig. 1.

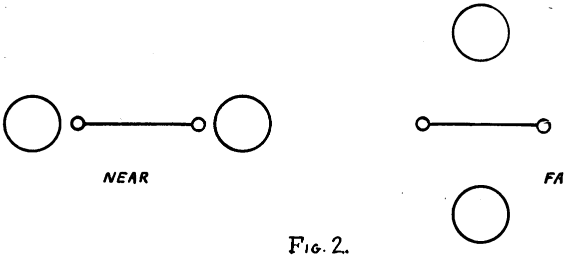

Fig. 2.

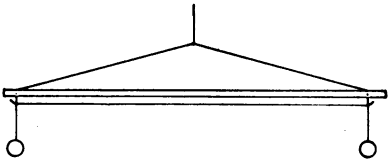

Fig. 4.

Thirty years having elapsed since the work of Boys and Braun, it was felt that it might now be possible to push the precision of this determination one decimal place farther.

The apparatus employed was the torsion balance, used in one form or another by half a dozen experimenters in this subject since the days of Cavendish. The general dimensions of Braun's apparatus were followed,

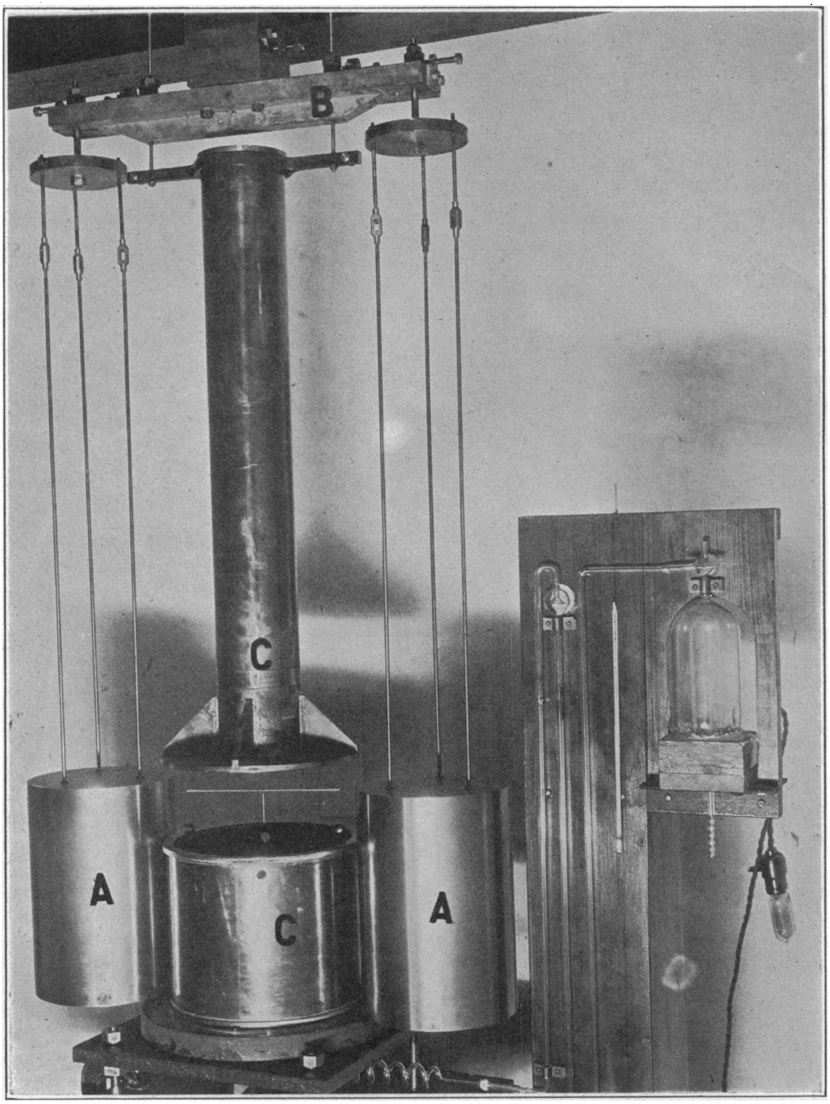

FIGURE 3

except that the large masses were greatly increased in size. The torsion pendulum was operated in a vacuum.

Two methods have been employed in measuring the constant of gravitation by means of the torsion balance. By one method, the large attracting masses are set in the position of maximum attraction (Fig. 1), and the deviation of the pendulum from its neutral position is measured. This method was used by Boys, who pushed it so far that it seemed doubtful whether his work could be much improved on.

The second method, that of the time of swing, was used by Braun, who remarks in his paper that he had by no means exhausted its possibilities. In this method the time of swing of the pendulum is measured in two positions (Fig. 2), which may be called "near" and "far." Braun, working with large masses of about 9 kg. each and small masses of 54 grams each, with a radius of revolution of about 10 cm. obtained a difference in the two times of swing of about 46 seconds. In the present work, the large masses weighed about 66 kg. each, the other quantities being nearly the same as those employed by Braun. In consequence of the greater mass, the times of swing differed by about 330 seconds.

The present paper gives a brief description of the experimental arrangements and states the results obtained during the year 1926. An additional set of observations is being made during 1927. Publication of the complete results will be made eventually as a Bureau of Standards Scientific Paper.

The work was done in the constant temperature room of the Bureau of Standards located about 35 feet below ground. The photograph (Fig. 3) shows the general arrangement of the apparatus.

The large masses *A*,*A*, were of steel, cylindrical in shape. This form was adopted because of practical difficulties in shaping spheres of the weight employed (66 kg. each). The cylinders were turned from ingots which had been forged down from 12″ to 9″ in diameter to close up blow holes, and as finished were about 28.5 cm. long and 19.5 cm. in diameter.

As a check on uniformity of density, samples were taken from the bosses cut off from the ends of each cylinder. The density of each sample agreed with the calculated density of the corresponding cylinder to 1 part in 4000. Calculation shows that the error introduced in the result by neglecting a non-uniformity of density of this order is less than 1 part in 50,000.

The cylinders were hung by steel rods from a beam *B*, movable about a vertical axis, and supported ultimately by two *I*-beams let into the walls of the room. This axis carried at its upper end a divided circle and a vernier, reading to 0.1°.

The bell jar, *C*, was of metal, resting on a plate glass base. The lower wide portion was made of a piece of wrought-iron tubing for magnetic shielding; the upper narrow portion of brass. Windows for observation were provided at a number of strategical points, two of which are seen in the photograph.

The moving system may be seen hanging below the elevated upper portion of the bell jar. It consisted of a light aluminum rod (Fig. 4) provided with a copper truss wire which passed through holes near the end of the rod, and was bent into hooks at its ends. The small masses were provided with hooks and tied together by a piece of tungsten lamp filament 0.025 mm. in diameter. This filament was laid over the hooks as shown in the figure. By this construction over 99% of the moment of inertia of the system was located in the balls.

The moving system was supported by a filament of tungsten of the same diameter and about a meter long. Tungsten has the advantages over silica of being readily tied to its connections and of being not readily broken. Boys used suspensions of silica, and says that he lost much time and previous observational labor by the frequent breaking of his suspensions for no apparent reason. It was found that with a tungsten suspension the moving system came back accurately to its resting point after large displacements.

The small masses used in 1926 were of platinum, weighing about 50 grams each, fused in a vacuum furnace to insure absence of cavities. They were spherical in form. When the upper part of the bell jar was lowered in place the balls hung nearly opposite the centers of the cylinders. The centers of the balls were about 20 cm. apart.

The usual pressure maintained in the bell jar was from 1 to 2 mm.

To start the pendulum swinging gravitational attraction was employed. Two bottles holding about 2 kg. of mercury in each were placed in the maximum attraction positions and changed to the opposite sides of the pendulum in time with its time of swing. By repeating this process for two hours it was possible to produce an amplitude of $3^1/_2$ degrees.

The time of a complete cycle swing in the near position was about 29 minutes, and in the far position $34^1/_2$ minutes. The time of swing was observed by introducing a beam of light through the glass base of the bell jar to a mirror on the moving system, and out by the same path. The motion of the reflected beam was observed by a telescope at a distance of $3^1/_2$ meters.

Successive transits of the beam (usually 24 in number) were recorded on a chronograph simultaneously with second signals from a Riefler clock.

Length measurements requiring precision, such as the distances between centers of balls or of cylinders, were measured by an optical compass similar to that used by Boys, consisting of two micrometer microscopes mounted on a rigid bar. The microscopes were sighted upon the two points whose distance was to be determined, and then upon a specially graduated standard scale. Such measurements could be repeated with a precision of 0.01 mm. Less important measurements, such as vertical differences of elevation, were made by a cathetometer reading to 0.1 mm.

The calculations involved were laborious, but not prohibitively so.

The attraction of a finite cylinder on any external point can be expressed to a precision of 1 part in 100,000 by seven terms of a zonal harmonic series.

The observations during 1926 gave five values of the constant of gravitation as follows:

| | |
|---|---|
| | 6.661 |
| | 6.661 |
| | 6.667 |
| | 6.667 |
| | 6.664 |
| mean | $6.664 \times 10^{-8}$ |
| Average departure from mean | 0.002 |

No previous investigator has obtained results agreeing to more than 2 significant figures.

It is planned to carry out an additional series of observations during 1927, by which it is hoped to obtain six or seven more values of this important constant.

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[1] C. V. Boys, *Phil. Trans. Roy. Soc. A.*, **1895**, Part 1, p. 1.

[2] Carl Braun, *Denkschriften k. Akad. Wissens. (math. und naturwiss. Classe)*, **64**, 1897, p. 187.

[3] Article "Gravitation," *Encyclopedia Britannica*, XI Edition.

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# *CONTACT TRANSFORMATIONS OF THREE-SPACE WHICH CONVERT A SYSTEM OF PATHS INTO A SYSTEM OF PATHS*[1]

BY JESSE DOUGLAS[2]

PRINCETON UNIVERSITY

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*Introduction.*—A contact transformation of the surface elements of three-dimensional space[3] converts every point, curve or surface into a point, curve or surface, but not necessarily a point into a point, a curve into a curve or a surface into a surface.

An important special contact transformation is the duality. This is characterized among proper contact transformations, i.e., those which do not reduce to point transformations by the property of converting each of the straight lines of space again into a straight line.

The $\infty^4$ straight lines of space are an example of a system of paths. By this we mean a system of analytic curves analytically distributed[4] over a region of space so that there is one and only one curve joining any two