THE GENERAL DISTRIBUTION OF COSMICAL VELOCITIES

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Our knowledge of the stellar motions has for a long time been limited to the brighter stars. During the last decade, however, radial velocity determinations have been extended to faint stars and nebulae, and it seems now possible to determine the general form of the distribution law of cosmical velocities. The writer has recently finished an analysis of all the known radial velocities of stars and nebulae, the results of which indicate strongly that the general velocity distribution can be expressed in a very simple form.

The radial velocities are in this case superior to the proper motions because they are given in linear measure, and further because we do not have to wait hundreds of years to get a result, as we must in determining the proper motions of extremely distant objects.

The objects used in this study have been grouped on the basis of spectral type, nebulous character, period of light variation, etc. For spectral types B to M the stars have been subdivided on the basis of proper motion (μ) and apparent magnitude (m), and a certain quantity H has been computed, which is intimately connected with the absolute magnitude (M) and the linear tangential motion (T). We have, for T in km. /sec.,

$$H = m + 5 \log \mu = M + 5 \log T - 8.378. \tag{1}$$

By using H as the basis of grouping it was possible to combine in the same groups stars of a certain spectral type and nearly the same intrinsic brightness and velocity dispersion.

Fifty groups in all were studied, covering all known classes of objects. For each group the velocity distribution was described in terms of the constants of the ellipsoidal distribution law.

$$F\,(xyz)\,dx\,dy\,dz = \frac{N\,dx\,dy\,dz}{(2\pi)^{3/2}\,abc} \exp\left[-\frac{(x-\bar{x})^2}{2a^2} - \frac{(y-\bar{y})^2}{2b^2} - \frac{(z-\bar{z})^2}{2c^2}\right]. \tag{2}$$

The velocity components are here represented by x, y and z and are projected on the principal axes of the distribution; N is the number of objects in the group; a, b and c denote the dispersions along the principal axes and $\bar{x}$, $\bar{y}$ and $\bar{z}$ represent the velocity components of the centroid of the group. The principal axes of the velocity distribution were assumed to have the same directions for the different groups, an assumption known to be nearly correct for the stars in general.

The principal result of this analysis is that the group-motion varies greatly, from about 12 km./sec. for the Cepheids of long period to about 300 km./sec. for stars of maximum velocity dispersion, globular clusters, and spiral nebulae. The solar motion, which is the motion of the sun relative to the stars, is thus not a constant vector, but is dependent upon the class of object to which its motion is referred.

The general features of this variability in group-motion are illustrated

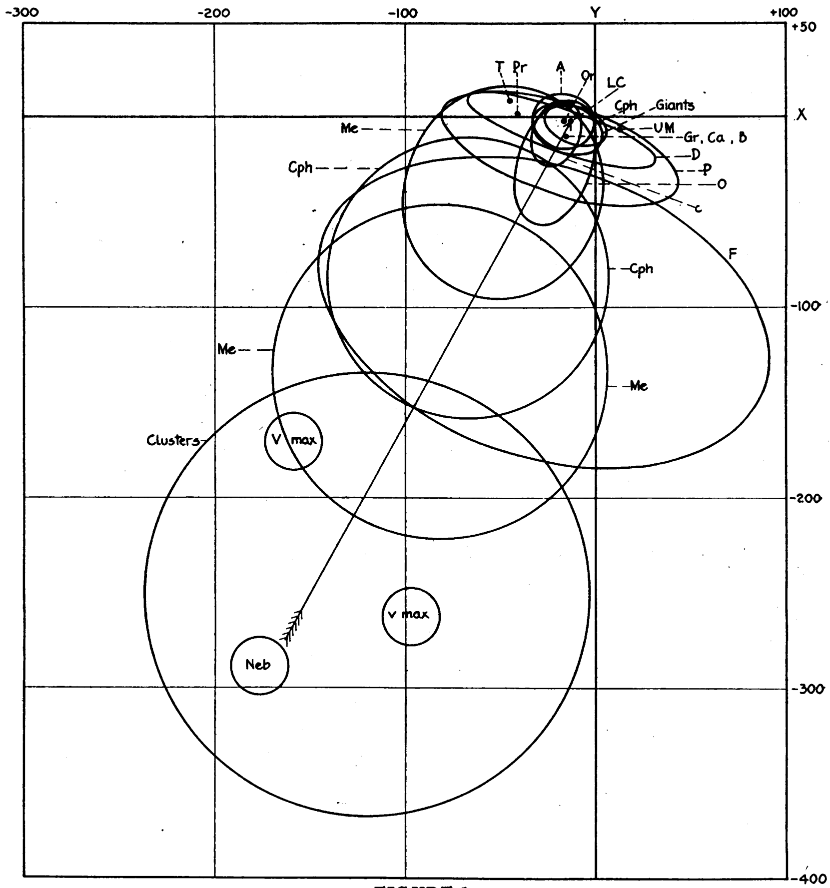

FIGURE 1

in figure 1, which shows the projection of the different velocity ellipsoids in the plane of the galaxy, with the x-axis towards the intersection of the galactic plane with the equator (in Aquila). The sun is at the origin and a vector from the origin to the center of an ellipse or circle indicates the group-motion of a particular class of objects relative to the sun and projected in the galactic plane. The axes of the ellipses indicate the maximum and minimum values of the internal velocity dispersion. The points represent groups of stars whose motions are nearly parallel and equal to one another (Moving Groups). The small circles marked "V max," "v max" and "Neb" indicate group-motion, but the radii of the circles

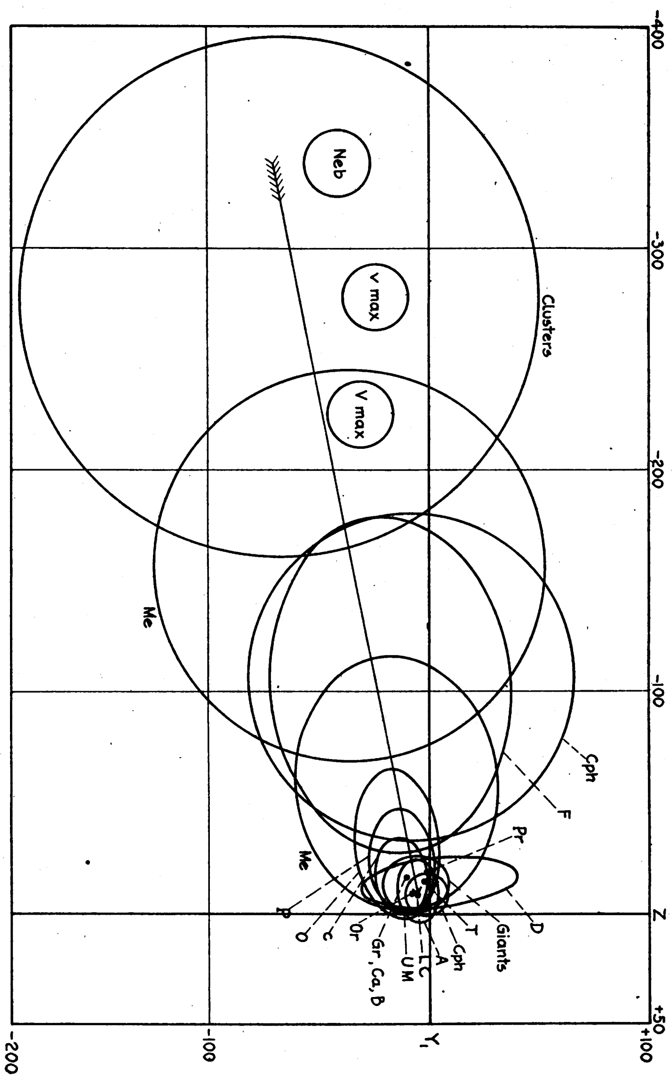

FIGURE 2

have nothing to do with the velocity-dispersion. The first two of these groups are selected directly on the basis of motion and the dispersion is thus affected by the selection; the third group consists of the non-galactic nebulae, the majority of which have spiral structure.

The diagram shows a steady increase in group-motion along a definite axis with increase in dispersion, in harmony with previous results based mainly upon space-velocities.[1] This systematic change in group-motion causes an asymmetry in the distribution of velocities which is of a surprisingly general nature. The direction of this change is in galactic longitude 241.5°. Figure 2 shows the projection of the velocity ellipsoids in a plane perpendicular to the galactic plane and through an axis parallel to the systematic shift. This new projection shows that the displacement of the centers of the velocity ellipsoids is nearly but not quite parallel to the galactic plane. The displacements of the centers define a fundamental axis the direction of which is $\alpha = 310°$, $\delta = +57°$, which is the direction of motion of objects with small velocity dispersion relative to objects of high velocity dispersion.

The group motion does not seem to depend primarily upon spectral type, intrinsic brightness, mass or density, but can be shown to vary with the velocity dispersion of the objects studied. This dependence is expressed by the quadratic relation

$$y = -pb_2 + \beta, \qquad p = 0.0192 \text{ sec./km.}, \ \beta = -10.0 \text{ km./sec.} \quad (3)$$

The y-axis is here in the direction of the shift, and b is the dispersion along an axis parallel to the y-axis. The relation is represented by a parabola in figure 3, the values of $\bar{y}$ and b being plotted for all the groups studied.

The B stars and the highly luminous A stars do not lie on the parabola, which indicates that so far as motions are concerned these stars form a separate system. This is not surprising, as it has been found by Shapley[2] that the bright B stars form a subordinate system within a larger system, and it is probable that the brighter A stars belong to this system, which thus moves as a unit and has a small internal velocity dispersion.

If we accept the quadratic relation, equation (3), the velocity distribution for all the objects studied, except the B star system, can be written in a very simple form. We find

$$F\,(xyz)dx\,dy\,dz = dx\,dy\,dz \exp\,[-p(y-\beta)] \times \sum_v A_v \exp\left[-\frac{(x-\alpha)^2}{2a_v^2} - \frac{(y-\beta)^2}{2b_v^2} - \frac{(z-\gamma)^2}{2c_v^2}\right]. \quad (4)$$

The function expressed by the summation sign is a symmetrical distribution whose center has the velocity components α, β and γ. In polar coördinates this velocity is 14.8 km./sec. in the direction $\alpha = 85.7°$, $\delta = -22.0°$.

The symmetry of the velocity distribution is now restored, but we have instead introduced the functional factor $\exp\,[-p(y-\beta)]$, which seems to be constant for all the groups. If we assume that there is a limit to the

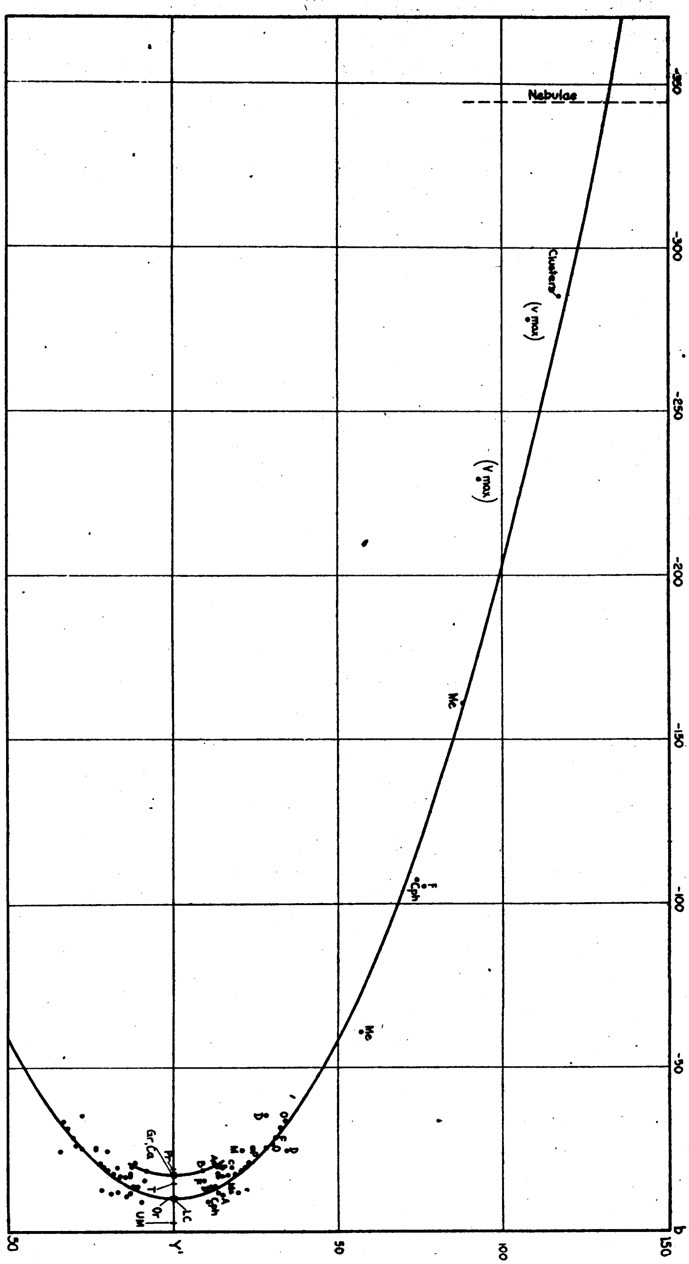
Nebulae
Clusters
(v max)
(v max)
Me
Cph
F
Me
Gr, Ca
UM
LC
Y'
-350
-300
-250
-200
-150
-100
-50
b
50
100
150

FIGURE 3

series of velocity ellipsoids, and that we have reached this limit when we study the fastest moving stars, the globular clusters, and the spiral nebulae, all of which seem to have the same group-motion, the general velocity distribution can be expressed in the form of a product of two symmetrical distributions, thus

$$F(xyz)dx\,dy\,dz = S_1\,S_2\,dx\,dy\,dz$$

$$S_1 = \sum_v B_v \exp\left[-\frac{(x-\alpha)^2}{2}\left(\frac{1}{a_v^2}-\frac{1}{l^2}\right)-\frac{(y-\beta)^2}{2}\left(\frac{1}{b_v^2}-\frac{1}{l^2}\right)-\frac{(z-\gamma)^2}{2}\left(\frac{1}{c_v^2}-\frac{1}{l^2}\right)\right]$$

$$S_2 = \exp\left[-\frac{(x-\alpha)^2}{2l^2}-\frac{(y-\beta+pl^2)^2}{2l^2}-\frac{(z-\gamma)^2}{2l^2}\right]. \qquad (5)$$

The sun's velocity relative to the center of the distributions S_1 is 14.8 km./sec. in the direction $\alpha = 265.7°$, $\delta = +22.0°$. Its velocity relative to the center of the distribution S_2, which is identical with its velocity relative to fast moving stars, clusters and spirals, is found to be about 300 km./sec. in the direction $\alpha = 307°$, $\delta = +56°$. With this velocity known, we can determine the value of l, which is the dispersion in the distribution S_2. We thus find $l = 123$ km./sec., which is about the same as the dispersion for the globular clusters (117 km./sec.).

The physical meaning of the general distribution function is that the motions of the stars in our neighborhood are determined not only by the gravitational field due to a local system of stars, but also by another field connected with the system of globular clusters and spiral nebulae. The function S_2 may be due to the general gravitational field of this vast system. There are many difficulties in the way of this explanation, and if this second field is found incapable of producing such an asymmetrical velocity distribution, the only alternative seems to be the assumption of a fundamental reference frame or medium referred to which high cosmic velocities are non-existant. We must in this connection remember that a star is not a simple object, but is itself a complicated system of atomic nuclei and electrons, which already have extremely high relative velocities. Further, the observed motion of a star is due to its initial velocity and the accumulated effect of all the impulses during its whole life time. We are hardly justified in saying that the nature of all these impulses, even the most feeble ones, are known to us.

[1] *Astrophys. J., Chicago*, **59**, 228 (1924). These PROCEEDINGS, **9**, 312 (1923).

[2] *Astr. Nachr., Kiel*, Jubiläumsnummer, p. 25 (1921).