the Riemann tensor has the same expression in s as in the simpler case (comp. form. (11) of the paper cited under[4]); it is easy to see that in the case we are interested in $s(x)$, like $F(x)$, has two pairs of equal roots of the secular equation and the same planes of invariant directions as $F(x)$, and the Codazzi equation has for its consequence the *vanishing of both vectors p and q;* since the vanishing is a particular case of being a gradient, a Maxwellian field can be extended over the skeleton but it is a very particular case of the general Maxwellian field since both invariants of the electromagnetic tensor are constant throughout the field.

The problem to find all the spaces of the type considered in this section is analogous to the problem to find all the Weingarten surfaces of a given type; using the representation with the aid of the generalized stereographical projection[5] it can be reduced to the finding of one scalar function of four variables which satisfies three partial differential equations of the second order (not linear).

It seems to be natural to look for further solutions in the form of four-dimensional space imbedded in a six-dimensional flat space.[6]

[1] These PROCEEDINGS, **9,** No. 12, 1923, p. 405.

[2] Comp. Whittaker, E. T., *Edinburgh, Proc. R. Soc.*, **42** (1–23).

[3] Baltimore, *Amer. J. Math.*, **43,** 1921 (126–129).

[4] These PROCEEDINGS, **9,** No. 6, 1923 (179–183); a more detailed presentation to appear in the *Amer. J. Math.*, **46.**

[5] These PROCEEDINGS, **9,** No. 12, 1923 (401–403).

[6] See papers by Kasner, *Amer. J. Math.*, **43,** 1921 (130–133), and (217–221); *Science*, **54,** 1921 (304); *Math. Ann. Leipzig*, **85,** 1922 (227–236).

PRINCIPAL DIRECTIONS IN AN AFFINE-CONNECTED MANIFOLD OF TWO DIMENSIONS

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Communicated, February 12, 1924

1. None of the principal directions defined in a previous paper (these PROCEEDINGS, **8,** No. 7, p. 198) exist in a manifold possessing components of affine connection (Γ^i_{jk}) without a fundamental tensor (g_{ij}). In the present paper two types of principal directions are defined in an affine-connected two-space; in the case where the space is Riemannian, by which we understand the existence of a fundamental tensor g_{ij} such that

$$\left\{ {jk \atop i} \right\} = \Gamma^i_{jk}, \qquad (1.1)$$

it will be shown that these principal directions reduce to the null-directions.

By "principal directions" are understood directions defined by invariant relations.

2. In an affine-connected two-space the equations

$$F^i_{jkl} = \frac{\partial \Gamma^i_{jl}}{\partial x^k} - \frac{\partial \Gamma^i_{jk}}{\partial x^l} + \Gamma^i_{rk}\,\Gamma^r_{jl} - \Gamma^i_{rl}\,\Gamma^r_{jk}, \tag{2.1}$$

$$F_{jl} = F^i_{jil}, \tag{2.2}$$

define the curvature tensor and the contracted curvature tensor, respectively. The equation

$$F_{ij}\xi^i\xi^j = 0 \tag{2.3}$$

is invariant: we shall call the directions ξ^i so defined the *principal directions of Type I*.

If a vector ξ^i be propagated parallelly around an infinitesimal circuit of parametric lines of a coördinate system (u, v), and the limit of the increments in its components, divided, by the product of the increments in u and v which define the circuit, be denoted by η^i, then

$$\eta^i = -F^i_{jkl}\xi^j \frac{\partial x^k}{\partial u}\frac{\partial x^l}{\partial v}, \tag{2.4}$$

and the direction defined by η^i is independent of the coördinate system (u, v) (*Annals of Mathematics*, Dec., 1923). We may, therefore define as *principal directions of Type II* the directions of those vectors which regain their original directions after propagation around an infinitesimal circuit. The equations for such directions are

$$\eta^i = \theta\,\xi^i, \tag{2.5}$$

where θ is indeterminate. Choosing $u = x^1$, $v = x^2$, we obtain

$$\theta\xi^i + F^i_{j12}\xi^j = 0, \tag{2.6}$$

from which the equation for θ is

$$\begin{vmatrix} \theta + F^1_{112} & F^1_{212} \\ F^2_{112} & \theta + F^2_{212} \end{vmatrix} = 0. \tag{2.7}$$

3. If the two-space is Riemannian, we have

$$F^i_{jkl} = g^{ri}\,F_{rjkl}, \tag{3.1}$$

where

$$F_{ijkl} = -F_{jikl} = F_{klij}. \tag{3.2}$$

Thus (cf. Laue: *Relativitätstheorie* Bd., 2, p. 98)

$$F_{jl} = g^{ri}\,F_{rjil} = \tfrac{1}{2}g_{jl}\,F, \tag{3.3}$$

where

$$F = g^{ij}\,F_{ij}. \tag{3.4}$$

Thus the directions of Type I are given by

$$F.g_{ij}\xi^i\xi^j = 0, \tag{3.5}$$

or, if F is not zero, by the null-directions.

If equations (2.6) are multiplied by $g_{ik}\xi^k$ and summed, the directions of Type II are given by

$$\theta g_{ik}\xi^i\xi^k + F_{kj12}\,\xi^j\xi^k = 0. \qquad (3.6)$$

But from (3.2),

$$F_{kj12}\,\xi^j\xi^k = 0, \qquad (3.7)$$

and the directions are given by

$$\theta g_{ik}\xi^i\xi^k = 0. \qquad (3.8)$$

We find

$$\begin{cases} F^1_{112} = \tfrac{1}{2}g_{12}\,F, & F^1_{212} = \tfrac{1}{2}g_{22}\,F, \\ F^2_{112} = -\tfrac{1}{2}g_{11}\,F, & F^2_{212} = -\tfrac{1}{2}g_{12}\,F, \end{cases} \qquad (3.9)$$

and the equation (2.7) for θ becomes

$$\theta^2 + \tfrac{1}{4}gF^2 = 0. \qquad (3.91)$$

Thus, if neither F nor g is zero (3.8) gives the null-directions.

A STATISTICAL DISCUSSION OF SETS OF PRECISE ASTRONOMICAL MEASUREMENTS: PARALLAXES

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Communicated February 29, 1924

In discussing the precision of the determination of the velocity of light Simon Newcomb remarked,[1] "So far as could be determined from the discordance of the separate measures the mean errors of Newcomb's result would be less than 10 km., but making allowance for the various sources of systematic error the actual probable error was estimated at ±30 km." Thus did Newcomb give expression to a belief that the probable error as calculated from the data was likely to represent only a fraction of the real probable error connected with the determination of a mean, and although he would have liked to claim the maximum precision for his value of the velocity of light, and probably did claim all he thought safe, his judgment led him to give as the actual probable error the triple of that calculated by his formula. If Newcomb had had faith in this value ±10 km. he would have asserted that the chance of the departure of the velocity of light from his mean by so much as 30 km. was less than one in twenty-two; instead, he said not more than one in two.

In view of the ever widening application of statistical methods to fields of investigation less precise, less controlled, and less controllable than the