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ELECTRICAL THEORY

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MODERN ELECTRICAL THEORY

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MODERN
ELECTRICAL THEORY

by

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PREFACE.

THE scope and intention of this volume need some explanation. It was begun with the idea of providing a text-book from which students, well grounded in the elementary branches of physics, might obtain some knowledge of the later developments; there is no doubt that such a book is needed urgently. But, in the course of writing, departures from the original scheme have been made. There are lengthy descriptions of theories which may be assumed to be familiar to such students, and there are discussions of controversial questions which may be thought undesirable in the early stages of study: it is improbable that a reader, who could gain much new information from the first chapter, could gain much benefit from the last. On the other hand, the book does not aspire to the dignity of a treatise designed to bring the results of the latest original research to the notice of those who have made a special study of the newer branches of physics: there is little that is not well-known to all professed students of the subject.

However, I hope that the following pages are not wholly without use or interest. I have attempted to expound the subject in its logical order, to analyse the arguments by which the various phenomena are correlated, to draw special attention to the assumptions that are made, and to show which of these assumptions are fundamental in the modern theory of electricity and which may be expected to be abandoned in the further progress of investigation.

The distrust of the more advanced developments of the electron theory, which is displayed in some quarters and is apt to extend to its simplest and most certain results, arises in part from the failure of many eminent writers on the subject to distinguish clearly between the two classes of hypotheses. Though the distinction is doubtless clear to their minds, their writings are apt to convey the impression that the assumption of the discontinuous distribution of electric charges must stand or fall with those concerning the nature of electromagnetic mass or the structure of the positively electrified portion of the atom. The exposition which is offered here is marred by traces of the original purpose of the book, but it may be of some service to those who have felt such difficulties.

It is with this object in view, and not with the idea of making the work accessible to those who are capable only of so-called 'elementary methods,' that all mathematical analysis has been excluded. My principle, which has not been followed quite consistently, has been to resort to calculation only when, and so far as, it is necessary in order to show the nature of the argument. It is easier to point out the stage at which the various hypotheses have been introduced, if only the simplest algebraical processes are employed: a page of symbols covers a multitude of assumptions. To all but a few students the treatment of electrical vibrations is nothing but a juggle with differential equations: in Chapters II. and III. an attempt is made to deduce the most important results directly from the fundamental conceptions of the science. In the last chapter I have tried to disentangle our knowledge of the properties of moving systems from the maze of analysis which has been woven round it by the physicists of the German school.

The same principle has determined for the most part the selection of the material to be treated. Subjects have been selected not for their intrinsic importance or for the interest of the accessory studies with which they are connected, but for the light which they throw upon the central problem, the relation of electricity and matter. The references at the end of each chapter are intended not as a complete bibliography, but as an outline sketch of the history of our present conceptions.

In a book which professes no originality it is unnecessary to give a detailed list of acknowledgments. But I must offer my best thanks to Prof. Thomson, who, although he has had no part in the preparation of the book and is not responsible for any of the errors of fact or opinion which it contains, has inspired this and every other portion of my scientific work. My gratitude is also due to Mr Whetham, who suggested the book, and to Mr Alexander Wood for their revision of the manuscript and proofs, and for the correction of many blunders: and, lastly, to Mr Crowther and many other fellow students, whose views expressed in conversation have modified profoundly the scheme of the work. Indeed, if any reader shall learn from studying the book half as much as its author has learnt from writing it, no further justification of its production will be needed.

N. R. C.

CAMBRIDGE,
November, 1907.

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ERRATUM.

Page 25, line 11 from the bottom, 'mass of $\frac{4\pi\mu N \sin^2 \psi}{v^2}$,' should *read* 'mass of $4\pi\mu N \sin^2 \psi$.'

Page 11, line 24, from top, "ends" should read "tends"

NOTE.

In the references at the end of chapters the abbreviation *A.* and *L.* denotes the volumes entitled "Ions, Electrons, Corpuscles," published by MM. Abraham and Langevin under the auspices of the French Physical Society.

PART I.

THE ELECTROMAGNETIC FIELD.

CHAPTER I.

FARADAY'S THEORY OF THE ELECTRIC FIELD AND THE BASIS OF MODERN ELECTRICAL THEORY.

The Old and the New Physics.

1. It is not mere historical interest which makes it desirable to begin our study of the most modern physical theories with the consideration of the views of an author who died two generations ago. The connection of Faraday with modern physics is closer than that of any of his contemporaries and predecessors, and, indeed, of many of his successors in time. Men of his own and of the preceding era had founded 'natural philosophy': they had made discoveries and had elaborated theories which still form part of the frame-work of the physical sciences. But their work has little interest for us to-day. Their aims, their conceptions, their whole attitude toward the problems which they investigated differ so widely from our own, that, while their results may be the basis of modern research, their methods afford little inspiration for it. But though the development of the views which we shall consider in this volume can be traced continuously back to the *Experimental Researches in Electricity and Magnetism*, it is not only the results of Faraday's work that are of interest at the present time: even if his observations had never led to any conclusions of fundamental importance, the spirit of his work would entitle him to be regarded as the founder of modern electrical theory. Since the object of this book is to present that theory as a connected whole

it will not be out of place to notice briefly what were the qualities characteristic in so marked a degree of the work of Faraday, which distinguish the newer from the older science.

At the end of the eighteenth century, when first experimental physics began to make any considerable advance, the study of mathematics was already in a high state of development. The labours of the great mathematicians from Newton to Laplace had brought the science to such a state of perfection that its further growth, at least in those aspects which concern the physicist, has been merely the continuation of their work. It was natural that the younger of the two closely connected branches of knowledge should be considered as subsidiary to the older, and that the growth of the former should be determined by the actual state of the latter. To Coulomb, Ampère or Gauss, experimental physics was useful mainly because it provided them with material on which to exercise their great analytical skill.

Electrical problems appeared to be suited admirably to their needs. Here were new problems in attractions similar to those which had led Newton to his greatest achievements: indeed when it was known that two small charged bodies exerted on each other forces which varied as the inverse square of the distance between them, the problem of electrostatics differed only from those of gravitation in the choice of the particular cases which it was convenient or desirable to investigate. Later, it was felt that the chief problems of electromagnetism had been solved when the classic work of Ampère reduced the mechanical forces between current circuits to actions between the elements of the circuits, which could be studied by similar mathematical methods. About the time that Faraday began to write it must have seemed to the mathematical physicists that the problems which electricity had to offer were almost exhausted.

This earliest school of physicists regarded physical forces as a series of 'actions at a distance'—to use a phrase which had been in vogue since the time of Newton; the total force between two reacting systems was viewed as the sum of forces between particles of the systems which depended only on the relative position and condition of those particles. But it had always been objected to this view by a minority, who were not for the most part professed students of science, that a direct action of one body upon another distant from it was impossible and unthinkable: that some medium occupying the space between the distant systems must be involved in the process. To the theory of 'action at a distance' was opposed the theory of 'action through a medium.'

We need not consider the long controversy that raged between the supporters of the rival theories: it is sufficient for our purpose to note that the two views are not always, if ever, incompatible, and may be entertained simultaneously as affording different interpretations of the same facts. All that is assumed by the 'action at a distance' theory is that the forces between two bodies can be described in terms of the states (including relative position) of the bodies at the moment of observation. Even if the bodies are known to be connected by a material medium, the properties of which exert an influence on the forces between the bodies, it will still be possible to use the conception of 'action at a distance,' if the forces can be defined in terms of the states of the bodies. Long after Faraday had rediscovered the difference in the dielectric constants[1] of different materials, a theory of electrostatics was employed and developed, which, though based upon 'action at a distance,' took into account the variable effect of the material medium separating the charges. The choice between the rival theories cannot be determined by experiment, but mainly by the view that we take of the object of physical science.

In cases where both of the two theories which we have contrasted are applicable each has its peculiar advantages. 'Action at a distance' is especially suited for mathematical treatment, chiefly, perhaps, because of the large part which it has actually played in the development of our mathematics: it permits accurate predictions of the results of certain distributions of the attracting bodies to be made. For such purposes it is indispensable: though Ampère's ideas on the nature of the electric current have been long superseded, no one would think of employing for the practical calculation of the constants of electrical instuments any methods which were not similar in principle to those which he brought into prominence. But the conception of action through a

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[1] I shall use the term 'dielectric constant' in place of 'specific inductive capacity,' which is commonly employed in English.

medium also has advantages of its own; it enables us to visualise the changes in the electrical system. In everyday experience the action of one body upon another at a distance always takes place through the agency of some material medium, which may be continuous (like fluid) or discrete (like elastic strings). Accordingly, if we find that the medium which it is necessary to introduce, in order to account for the forces in the electric field, has properties similar to those of some medium with which we are familiar, we shall be able to predict roughly from 'common sense' and without detailed calculation the action consequent on any given distribution of the electric charges. In addition, it might be urged that the mere fact that the introduction of a medium allows us to form a mechanical picture of electrical phenomena is of overwhelming importance: but to discuss this view would lead us too far. For our present purpose it is only important to notice that the two views which have been held as to the nature of electrical forces are not mutually incompatible, but that one is especially suited for the needs of the mathematician, the other for the needs of the non-mathematician.

Now Faraday was no mathematician: his knowledge of analysis scarcely extended beyond the processes of elementary algebra. But he had unrivalled mechanical insight—a power of devising mechanisms which would simulate the behaviour of the observed systems and of seeing how they would work in any particular case: in his hands the 'medium' interpretation of physical forces became as fruitful as the older theory. The researches which he based upon it are the foundation of the science which is the subject of this book.

The new method of physics has had other and much more important uses than that of merely allowing persons devoid of mathematical instinct to study the subject. The new view has proved, as we shall see, much more suggestive than the old: it has led to investigations which would never have been undertaken by the adherents of the older conceptions: and it has also led to the development of the experimental side of the subject. To the mathematician the difference between various material substances, such as the difference in the dielectric constant, is not a source of great interest: it merely introduces into his work a quantity, which, if it is constant, is unimportant, and, if it is variable, leads to complications which cannot be resolved by exact methods. But to the mechanist, if I may so term the holder of the contrasted view, this difference is essential: it is the clue to the nature of the mechanism. The mathematician is concerned with the resemblances between different materials, the mechanist with their differences: it is to the latter that we owe the great development of experimental physics, which has made such enormous advances in recent years[1].

It has been my object in the foregoing paragraphs to sketch the fundamental distinction between the older and the newer physics. We must now descend from vague generalisations and examine in greater detail the views to the publication of which such fundamental importance has been attributed.

Faraday's lines of force.

2. The outlines of Faraday's theory of the electric field and its application to the simpler problems of electrostatics are described so fully and completely in many well-known text-books that a mere sketch will be sufficient.

If iron filings be scattered on a card resting on the poles of a horse-shoe magnet, they will arrange themselves in chains stretching from one pole of the magnet to the other. The direction of the chain at any point represents the direction of the magnetic intensity (sometimes called the 'magnetic force') of the field of the magnet at that point; so that a small single pole would tend to move along the chain to one or other of the poles. Faraday imagined that the distribution of the iron filings reproduces a real structure in the space covered by the field of the magnet: that there exist in that space lines joining the two poles, which had properties different from those of the surrounding region. A line along which the filings accumulate coincides with one of these characteristic lines of the magnetic field, and a magnetic pole is

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[1] It should be mentioned that much of Faraday's work was anticipated by Cavendish, but since Cavendish took little interest in the publication of his results, they did not become known generally till after the lapse of 70 years, and exerted no influence on the history of the science. It would be extravagant to class among the moderns one who was ready to perform research without caring to obtain 'priority' for it.

that portion of a body towards which the ends of the lines of force converge.

Further he pointed out that the peculiar properties of these 'lines of force,' which distinguish them from the surrounding medium, could be deduced, at least qualitatively, from a consideration of the mechanical stresses on bodies placed in the field. Every line must tend to contract in the direction of its length, like an elastic string, and must also exert a lateral pressure on its neighbours. Since the ends of the lines are attached to the poles of the magnet, a tendency of the lines to contract is equivalent to a tendency of the poles to approach, or, in other words, to an attraction between the poles.

Though it is less easy to produce in the case of the electric field an experiment analogous to that of the distribution of the filings, the close resemblance between the laws of magnetism and of electrostatics led Faraday to extend to the latter his conception of lines of force. He supposed that in the neighbourhood of charged bodies lines in the medium exist following at each point the direction of the electric intensity, tending to contract in the direction of their length and to repel their neighbours.

What is usually termed a charged body is a body on which the ends of lines of force are situated. The arrangement of the lines of force around a few simple distributions of electric charges can be drawn by calculating the direction of the electric intensity at each point in the field: a glance at the resulting diagrams will show that the properties which have been attributed to the tubes account for such simple actions as the attraction of opposite charges, Fig. 1 A, and their repulsion of like charges, Fig. 1 B.

The discovery that the force between two charged bodies varies with the nature of the material surrounding them shows that the properties of the lines of force depend on the substance through which they pass. But since it is well known that electric and magnetic actions can take place across vacuous spaces, from which all known substances have been extracted, it is clear that the lines of force cannot be regarded as part of such substances. Accordingly there has been introduced the conception, already familiar in the theory of light, of an 'aether,' or substance present universally, in which the lines of force could exist in the absence of matter. I shall have more to say of this conception in the last chapter.

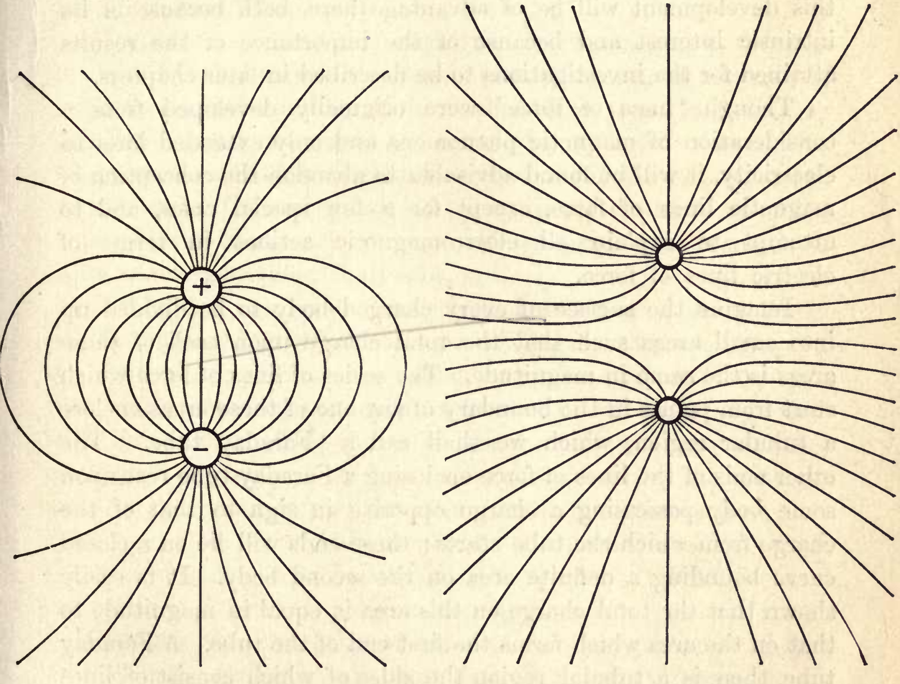

Fig. 1.

Development of Faraday's theory. Tubes of Force.

3. Faraday's theory as it was left by its author is adapted admirably for visualising the processes in the electrostatic field and predicting their general tendency: but it affords no basis for quantitative calculation. The quantitative development of Faraday's fundamental notion of the importance of the medium in electric or magnetic action was carried out by Maxwell: but that author abandoned completely Faraday's terminology and made little attempt to retain a visualisation of electrical processes. His arguments could not possibly be rendered comprehensible to readers untrained in analysis. However J. J. Thomson has

shown that a slight extension of Faraday's original scheme will render it suitable for quantitative treatment and for application to phenomena other than those considered by its author, without the use of any but the simplest mathematics. A brief sketch of this development will be of advantage here, both because of its intrinsic interest and because of the importance of the results attained for the investigations to be described in later chapters.

Though 'lines of force' were originally developed from a consideration of magnetic phenomena and only extended later to electricity, it will be found advisable to abandon the conception of magnetic lines of force, except for a few special cases, and to attempt to explain all electromagnetic actions in terms of electric lines of force.

Imagine the surface of every charged body to be divided up into small areas such that the total charge upon each of these areas is the same in magnitude. The series of lines of force which start from points in the boundary of any one of these areas enclose a tubular region, which we shall call a 'Faraday tube.' The other ends of the lines of force enclosing a Faraday tube rest upon some body possessing a charge opposite in sign to that of the charge from which the tube starts: these ends will lie on a closed curve bounding a definite area on the second body. It is easily shown that the total charge on this area is equal in magnitude to that on the area which forms the first end of the tube. A Faraday tube, then, is a tubular region the sides of which consist of lines of force following at each point the direction of the electric intensity, and the ends of which are small areas on oppositely charged bodies such that the magnitude of the total charge on each of these areas is the same.

If the distribution of lines of force is that which obtains in the space between two very large, parallel plates bearing equal and opposite charges, the tubes are right cylinders the axes of which are perpendicular to the plates. In the case of a conducting sphere insulated at a great distance from all other bodies, the lines of force are the radii of the sphere and the Faraday tubes are cones of which the apical angles are all equal and the apices lie at the centre of the sphere.

One definition and four fundamental propositions define the relations between Faraday tubes and the quantities which appear in the usual mathematical theory of electrostatics. They will be quoted here without formal proof, for such proof may be found in many well-known text-books, e.g. Thomson's *Elements of Electricity and Magnetism*, Chapter II.

DEFINITION. Since the magnitude of the charge on the end of every Faraday tube is the same, the total charge on any area is proportional to the number of tubes which have their (like) ends on it. Nothing has been said yet as to the absolute value of the charge on the end of a single tube: if we make this charge equal to the unit of quantity of electricity in the system of units employed, the charge on any area will be *equal* to the number of tubes which have their (like) ends on it.

Throughout this volume the electrostatic C.G.S. system of units will be employed, in which the units of length, mass and time are the centimetre, gramme and second, and the unit of electricity that which placed at unit distance from a similar charge repels it with unit force[1]. The choice of these units will lead to the occurrence of fractions in the numbers representing the charges upon various areas, so that the definition that all tubes have the same charge on their ends cannot be maintained strictly. But the relations about to be stated will not be invalidated: the equations are true even if N is not integral.

Of course the conception of tubes of force suggests that if the charge on the end of a tube were defined rightly, the number of tubes would always be integral: that is, that there is an atomic charge of electricity of which there can be no submultiples. We shall see that there are cogent reasons for believing that there is such a natural unit and, if it were not for the objection that its value is not known with sufficient accuracy, it would be desirable to make that charge the basis of electrical units.

The direction of a tube is usually taken as from its positive to its negative end.

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[1] I would draw the attention of the reader to this statement, for with two 'scientific' and one 'practical' system of units in common use there is great danger of confusion. The choice that has been made seems the most logical in developing electrical theory from the electrostatic standpoint, but it has the disadvantage that almost every formula, when put in a form suitable for treating experimental results, is cumbered with 'V's.'

PROPOSITION I. If a closed surface be drawn in the electric field, the excess of the number of tubes passing out through the surface over that of the tubes passing in through the surface is equal to the total charge contained within the closed surface. In particular, if the number of tubes passing through the surface in each direction is the same, there is no total charge within the surface.

PROP. II. The electric intensity (called by some writers the 'electric force') coincides in direction with the tubes passing through the point where it is measured. In magnitude it is proportional to the number of tubes passing through a surface of unit area drawn about the point at right angles to the direction of the tubes at that point. This number of tubes is called the *electric polarisation* at the point and will be denoted by N: it is equivalent mathematically to the quantity which Maxwell called the 'electric displacement.' If R is the electric intensity, K the dielectric constant of the medium

$$R = \frac{4\pi}{K} N \qquad \text{.............................(1).}$$

In the present chapter the only medium considered is a vacuum, so that $K = 1$. In a later chapter the interpretation in terms of Faraday's theory of a value of K greater than 1 will be considered.

PROP. III. The tension along the length of a Faraday tube can be shown to be of an amount $\frac{1}{2}R$, where R is the electric intensity at the point considered. If N is the electric polarisation

$$R = 4\pi N,$$

and the pull on unit area drawn at right angles to the tubes is

$$F = \frac{1}{2} \times 4\pi N \times N = 2\pi N^2 \qquad \text{..................(2).}$$

In a similar manner it may be proved that the lateral pressure which a tube exerts on its neighbours perpendicular to the surface of the tube is of an amount $2\pi N^2$ per unit area of the surface of the tube. A knowledge of these stresses is important for some essential calculations in Chapter II.

PROP. IV. The energy in the electrostatic field is usually represented as $\Sigma \frac{1}{2} Ve$, or the sum of half the product of the charge on each body by its potential. Since we are now viewing electric processes as determined by the properties of the medium and not by the properties of the charged bodies immersed in it, we must find an equivalent expression for the electrostatic energy in terms of the medium. It may be proved that the electrostatic energy of any system may be represented by attributing to each tube energy amounting to $\frac{2\pi N^2}{K}$ per unit volume of the tube.

From these propositions all the results of electrostatics might be deduced by appropriate mathematical reasoning. But it is not in the treatment of electrostatics that the highest importance of Faraday's theory for our present purpose consists. We must now turn to the logical extension of Faraday's views beyond the point to which they were carried by their author.

Faraday Tubes in Motion.

4. For the purposes of electrostatics all materials may be roughly divided into two great classes, conductors and insulators. A conductor, according to the language of the older theory, is a body along which a charge of electricity can move from one place to another: an insulator is a body along which electricity cannot move. According to Faraday's view a charge on a body means that Faraday tubes end on that body. Accordingly a conductor is a body along the surface of which the ends of Faraday tubes can move.

Now a tube is subject to tension in the direction of its length and always tends to become shorter: sometimes it is unable to contract because its ends are fixed to points from which they cannot escape, *i.e.* the ends lie on an insulated body. But, if both ends of a tube lie on the same conductor, there is nothing to prevent them from moving to any part of that conductor, and the tube, under the influence of its tension, will contract until the opposite ends meet and the tube ceases to exist for the purposes of electrostatics.

Suppose that the two plates A and B (Fig. 2) carry charges of equal magnitude but opposite sign. The arrangement of the tubes is indicated in the figure where the lines drawn between the plates represent 'lines of force' or the cross sections of the tubes by the plane of the figure. Now let the points M, N be joined by a conducting wire *MPN*. Since both the ends of the tube *MON* lie on this conductor, the tube will contract and collapse. The removal of this tube will relieve the neighbouring tubes *M′O′N′*, *M″O″N″* of the lateral pressure which it has exerted previously. Accordingly they will move in towards the wire: in course of time, their ends will reach the wire and they also will collapse. The process will be continued until all the tubes joining *A* and *B* have disappeared and the system is 'discharged.'

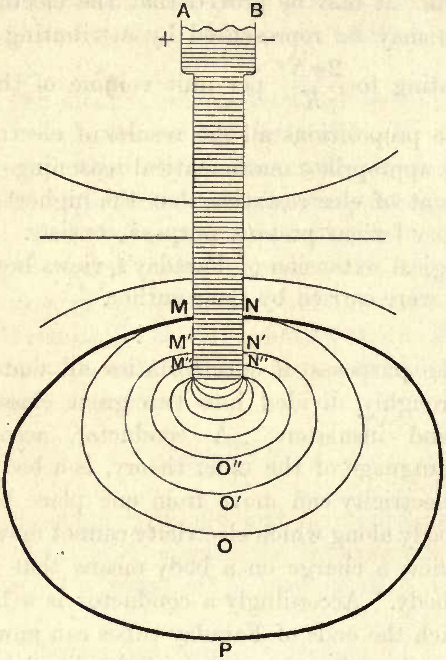

Fig. 2.

If by some means or other, such as a Voltaic cell, fresh tubes are produced continually stretching from one plate to the other, there will be a continuous movement of the tubes from their place of production to their place of disappearance, in which the velocity of the tubes will be such that the number disappearing by collapse is equal to the number produced by the cell in the same time.

Now the discharge of such a system through a wire is accompanied by all the phenomena which are described as the 'passage of a current of electricity.' Heat will be developed in the connecting wire and a magnetic field will be established in the neighbourhood of the circuit. *The passage of an electric current is the motion of Faraday tubes,* and the conductor 'through which the current flows' is the surface along which the ends of the tubes move. Since the end of each Faraday tube is attached to unit charge, the quantity of electricity which has flowed round the circuit is the number of tubes which have crossed any closed curve drawn round the conductor and the strength of the current is the number of tubes which cross such a curve in unit time.

The chief results of the passage of the current or the motion of the tubes are

(1) the production of heat in the conductor, and
(2) the establishment of a magnetic field.

The former is outside the scope of this chapter, for it depends greatly on the nature of the material of the conductor, and for the present we are leaving out of consideration the electrical properties of material bodies. But the latter does not depend on the nature of the conductor but only on its form and on the strength of the current: it is of the utmost importance for our present purpose.

Magnetic Intensity.

5. Let us first attempt to establish a quantitative relation between the motion of the tubes and the magnetic intensity produced by them.

Let *A* (Fig. 3) represent the cross section of the conductor *MPN* of Fig. 2 through which a steady current is flowing perpendicularly to the plane of the paper in such a direction that positive electricity is passing through the paper away from the reader. Let *PQR* be any closed curve in the plane of the paper,

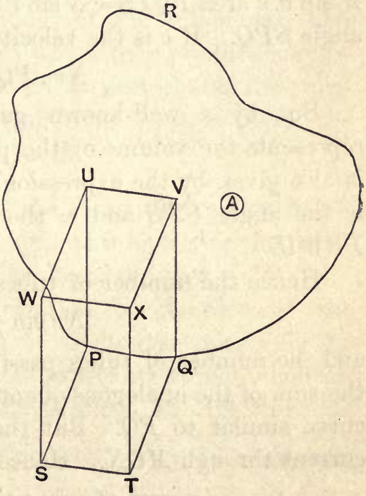

Fig. 3.

enclosing A but not any other portion of the circuit of which MON is part. Then all the tubes which finally collapse into MON must pass across the curve PQR: and since the current is measured by the number of tubes collapsing into MON per second, it is also equal to the total number of tubes passing across PQR per second; for it is assumed that the current is steady. The direction of the tubes passing across PQR is such that their + ends are on the same side of the paper as the reader.

Let PQ be a portion of the curve so small that it may be considered a straight line. Draw PS, QT outwards from A parallel to the direction in which the tubes are travelling across PQ and of a length equal to the velocity of the tubes, or the distance travelled in one second. Draw PU, QV, SW, TX, parallel to the tubes themselves (not in the plane of the paper) and of a length equal to the electric polarisation or the number of tubes passing through unit area perpendicular to PU. Complete the parallelopiped $PQTSWUVX$.

Then the number of tubes included within the curve $PQTS = N \times \text{area } PQTS \times \sin\theta$, where θ is the angle between PU and the plane $PQTS$. In one second all these tubes will pass across PQ, since they travel a distance PS in one second. Accordingly the number of tubes passing across PQ per second is $N \sin\theta \times \text{area } PQTS = N \sin\theta \times PS \times PQ \times \sin\phi$, where ϕ is the angle SPQ. If v is the velocity of the tubes this quantity is

$$Nv \,.\, PQ \,.\, \sin\theta \sin\phi.$$

But by a well-known geometrical theorem this quantity represents the volume of the parallelopiped $PQTSWUVX$, which is also given by the expression $PS \,.\, PU \sin\chi \times PQ \sin\omega$, where χ is the angle UPS and ω the angle between PQ and the plane $PSWU$.

Hence the number of tubes crossing PQ per second is

$$Nv \sin\chi \times PQ \sin\omega,$$

and the number of tubes passing across the whole curve PQR is the sum of the analogous quantities for each of the elements of the curve similar to PQ. But the whole number is equal to i, the current through MON. Hence

$$i = \text{sum of quantities } Nv \sin\chi \times PQ \sin\omega \ldots\ldots\ldots (3).$$

Now the fundamental proposition of electromagnetism states that the work done by taking a single magnetic pole round the circuit PQR is $4\pi i$. If H is the magnetic intensity (also called 'magnetic force') at any element PQ, this work is the sum of the quantities $H \sin \psi \times PQ$, where ψ is the angle between the direction of the magnetic intensity and the element PQ. Hence we may write

$$\text{sum of } H \sin \psi \times PQ = 4\pi \times \text{sum of } Nv \sin \chi . PQ \sin \omega \ldots (4).$$

Now if we suppose that the moving tubes produce a magnetic intensity perpendicular to the plane $PSWU$, $\omega = \psi$ and if we suppose further that H, the magnitude of this intensity, is $4\pi Nv \sin \chi$, the two sides of (4) become identical. Hence we can represent the magnetic effects due to the current by the motion of the tubes if we suppose that the tubes produce a magnetic intensity in their neighbourhood of an amount $4\pi Nv \sin \chi$, the intensity being perpendicular both to the direction of the tubes and to the direction of their motion[1].

When the tubes are moving perpendicularly to themselves so that $\sin \chi = 1$, the expression becomes very simple. Each tube moving at right angles to itself produces a magnetic intensity $4\pi v$, perpendicular both to itself and to its direction of motion. On the other hand, if the tubes move along their own axes $\sin \chi = 0$, and no magnetic field is produced.

The relation between the directions of the tubes and their motion and that of the magnetic field which they produce may be worked out from this simple case. It will be seen that if the tubes are directed upwards and are moving from West to East, the magnetic intensity is from North to South.

Conduction currents and convection currents.

6. We must notice now some of the consequences that follow from the view that a magnetic intensity is the result of the presence of moving Faraday tubes.

Between the enunciation by Ampère of the fundamental principles of electrodynamics and the publication of Maxwell's treatise which expressed in mathematical language the view that we are considering, an interminable

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[1] This proof was given by Prof. J. J. Thomson in his lectures and is reproduced with his kind permission.

controversy raged as to the magnetic action of 'open circuits.' An 'open circuit' is a system of conductors carrying a current, which do not form a closed curve. A condenser in the process of being charged by a battery is an instance of such a circuit. The battery, the plates of the condenser and the wires connecting them are conductors through which a current flows: but there is no conductor joining the plates of the condenser, and accordingly, it seemed natural to suppose, there is no current flowing between them. In this instance, the open current is only transient, but continuous open currents can be produced. Thus, if a continuous succession of insulating bodies is shot across from one plate of the condenser to the other, each will carry across a certain charge which will be given up on reaching the further plate. According to the older view there is a current circulating through the plates of the condenser but, since there is no conductor between the plates of the condenser, there can be no current there.

Many views were held as to the magnetic action of such 'open currents,' and it was difficult to distinguish between them by actual experiment, since the currents were either so weak or so transient that it was impossible to find sufficiently delicate means of detection.

But on Faraday's theory the distinction between an open and a closed current in respect of magnetic action is altogether meaningless. The magnetic intensity is due to the motion of Faraday tubes and will be the same for the same motion of those tubes, whether the ends of the tubes rest during the whole of their passage on conducting bodies (closed circuits) or for part of their passage on insulators (open circuits). For consider the disposition of the tubes in the two cases mentioned. The process of charging a condenser consists in the production of tubes by the battery, which move down with their ends on the connecting wires and the plates of the condenser until they have accumulated in the condenser to such an extent that the lateral pressure due to their mutual action is sufficient to prevent any further motion. The motion of the tubes is precisely the same as that which we noted when the plates of the condenser were joined by a wire, except that the final stage of that motion, the collapse into the wire, is omitted.

The second case is even more instructive. When one of the insulated bodies which are shot across is in contact with one of the plates (A) some of the tubes which proceed from *A* will end on it. As it moves across it will carry these tubes with it; when it touches the other plate (B), the ends of the tube will be joined and the tube will collapse. Hence the effect of the passage of the insulated body from *A* to *B* is exactly the same as that of joining *A* and *B* by a wire. In both cases the tubes running from *A* to *B* are continually collapsing and making room for other tubes to come down and fill their places. If the stream of insulated bodies and the nature of the wire are adjusted so that the same number of tubes collapse in a given time with either arrangement, the motion of the tubes will be precisely similar in the two cases, and, if our view is correct, there will be no difference between the magnetic effects.

The conclusion appears so obvious that it is well to insist that it is not certain without further experimental proof. In our calculation of the magnetic effect of a moving tube it was assumed that the strength of the current—the quantity to which the magnetic intensity is proportional—depends only on the number of tubes crossing a closed curve round the circuit and not on what happens to them inside the curve. But it is conceivable, though it is not likely, that the magnetic effect of the tubes depends, not on their motion, but upon their collapse into the conductor: if this were so, there would be a difference between the magnetic effects of the 'open' and 'closed' circuits. This question can only be settled by observation. The adoption of a new point of view cannot prove anything apart from experiment—the importance of the point of view lies in the experiments which it suggests. It was a direct consequence of Faraday's view to imagine that moving charges are equivalent in their magnetic effects to currents in conducting circuits, whereas the Amperean theory not only failed to suggest such a proposition but actually indicated that it was untrue by connecting too closely the idea of a current and of a circuit.

The actual experimental proof of the magnetic action of charges in motion was first carried out by Rowland of Baltimore in 1876 shortly after the publication of Maxwell's treatise. The observations require extraordinary care and delicacy, and doubt has been thrown on them from time to time. But they have now been repeated by several observers, among whom Roentgen, Pender and Cremieu are prominent, and there is now no doubt of the correctness of Rowland's conclusions. The principle of the experiment is extremely simple. Two discs of insulating material bearing metallic sectors, which are kept at a large difference of potential by periodical contacts with brushes, are spun very rapidly in the neighbourhood of a sensitive magnetic needle. The rotating charged sectors are equivalent in magnetic effect to a current flowing in a circular circuit, and produce a corresponding deflection of the needle. For a full description of the experimental arrangements the reader should refer to Rowland's own account.

No magnetic effect of an open current of the first kind—a 'displacement' current—has been detected directly: but on the assumption of its existence Maxwell based his prediction of the velocity of electromagnetic waves, which was confirmed by the experiments of Hertz. The importance of the proofs of the existence of these magnetic effects of 'convection' and 'displacement' currents cannot be overestimated: for on them rests the whole structure of the modern science of electricity.

Moving charged sphere.

7. As an instance of the actual calculation of the magnetic field of a moving charged body the case of a moving sphere may be considered in further detail, since it will be of especial importance in later chapters.

Let O be the centre of the sphere which is moving with a velocity v in the direction OX. Then the magnetic intensity at any point P outside the sphere is $4\pi Nv \sin\chi$, where N is the electric polarisation, and χ is the angle between the direction of the tubes and OX. If $OP = r$, $N = \frac{e}{4\pi r^2}$, where e is the charge on the sphere, or the number of tubes issuing from it. Hence the magnetic intensity at P is $\frac{ev \sin\chi}{r^2}$. The direction of the magnetic intensity is perpendicular both to the tubes and to their direction of motion, that is, perpendicular to the plane POX. But it is well known that the magnetic intensity due to a current element of strength i and length ds at a point distant from it r in a direction making an angle θ with the direction of the current, is $\frac{i \sin \theta}{r^2} ds$. Hence the magnetic effect of the moving charged sphere is the same as that of an Amperean current element of strength i, coinciding in position at each instant with the sphere and in the direction in which the sphere is moving, where $ids = ev$. All the conclusions applicable to the current element may be applied to the moving sphere: thus we find that if the sphere move in a magnetic field of intensity H, it is subject to a force $\mu Hev \sin \phi$, where ϕ is the angle between H and v, μ the permeability, tending to deflect it in a direction perpendicular both to the field and to the direction of its motion. This result will be quoted frequently in later chapters.

If a charged body is moving in a closed orbit it is often desirable to find the Amperean current flowing in a circuit coincident with that orbit, the magnetic effect of which is the same as that of the moving sphere. (It must be noted that the magnetic intensity due to the sphere at any point varies with the position of the sphere in its orbit, while that of the current is constant. We have to find a current such that its magnetic effect at any point is the same as the *average* effect of the moving charge at that point.) For this purpose the principle may be used that the magnetic intensity produced by a current is proportional to the rate at which electricity is carried round its circuit. If the charge on the sphere is e and the time in which it travels round its orbit T, a charge $\frac{e}{T}$ is carried across any point in its orbit every second. But, if i is the equivalent current, i units of electricity must be carried across any cross section of the current circuit in one second. Hence

$$i = \frac{e}{T} \quad \text{.........................(5).}$$

8. A difficulty in this view of magnetic action may have occurred already to the reader.

Electrostatic and electromagnetic action.

We have attributed both electrostatic and electromagnetic action to the properties of the same tubes: the former has been attributed to the presence of tubes at rest, the latter to the presence of tubes in motion. It would seem, then, that, if any region is by any means cut off from the electrostatic influence of any system, it must also be cut off from its electromagnetic influence; for the absence of electrostatic influence means that the tubes from the system do not reach that region, and if there are no tubes in it there can be no effects which are due to the motion of tubes. But it is well known that any region surrounded by a closed conductor is isolated completely from the electrostatic influence of any system outside that conductor, while the presence of the closed conductor has no effect whatever on the magnetic action of steady currents outside it.

This apparent contradiction can be overcome by altering slightly our notion of the relation of a tube to its charge. Hitherto we have regarded each tube as joining two opposite charges, to both of which it belongs equally. The difficulty which has just been pointed out forces us to the conclusion that each charge carries its own tubes with it independently of the presence of the other charges, and that tubes attached to charges of opposite sign have oppositely directed properties.

The use of an analogy will enable me to make my meaning clearer: we will take a hydrodynamical analogy which we shall find very useful in dealing with the magnetic properties of Faraday tubes[1]. Let us suppose then that the tubes are in some way like 'vortex filaments' in a fluid. Such filaments are tubular portions of the fluid which have a rotary motion different from that of the remainder. We will imagine that the electrostatic properties of the tubes, their tension and their pressure at right angles to their length, is determined by the velocity of the rotation. Then, by the tubes attached to the positive charges possessing 'oppositely directed properties' to those attached to the negative charges, I mean that if the rotation in the former is related to the axis of the tube, drawn outward from the charge, as rotation to translation in a right-handed screw, the rotation in the latter is related to the axis as rotation to translation in a left-handed screw (Fig. 4).

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[1] I must point out that it is not pretended that the analogy is complete even in the aspects which are mentioned here: and that we are not committing ourselves to the extravagant statement that the 'aether is a perfect fluid' or anything of the kind.

Consider then the case of parallel plates carrying equal and opposite charges. The same number of tubes radiate from each of the plates and these tubes will stretch out from *both* sides of the plates at right angles to them, so that the whole space, both between the plates and outside them, is occupied by coincident positive and negative tubes. (In Fig. 5 the positive tubes are represented by continuous, the negative by dotted lines.) A reference to the figure will show that in the space between the plates the direction of absolute rotation in both classes of tubes is the same, whereas in the space outside the plates the direction of absolute rotation is opposite in the opposite classes. Hence between the plates the tubes will reinforce each other's action in respect of any property which is determined by the magnitude of the rotation, and in the space outside they will neutralise each other with respect to such properties. Since we have supposed

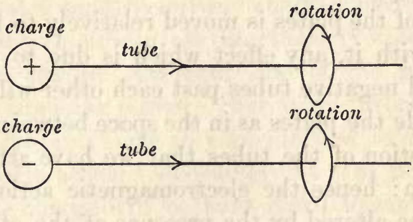

Fig. 4.

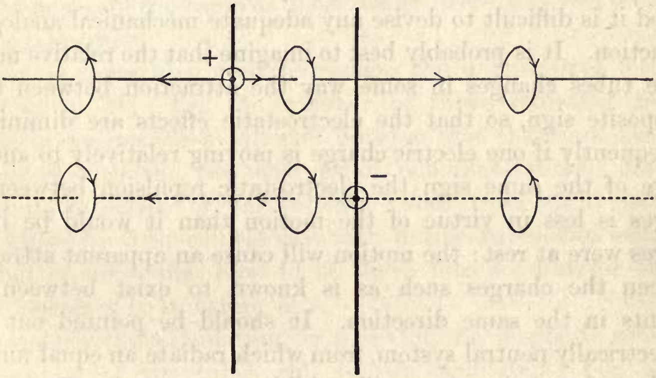

Fig. 5.

that the electrostatic effects of the tubes are determined by this rotation, there will be electrostatic effects due to the tubes in the space between the plates but none in the space outside it, precisely as is found in experiment.

But, if one of the plates is moved relatively to the other, carrying its tubes with it, any effect which is due to the shearing of the positive and negative tubes past each other will be the same in the space outside the plates as in the space between them. It is to this relative motion of the tubes that we have attributed electromagnetic action: hence the electromagnetic action of one set of tubes will not be altered by the presence of the other set, and the magnetic effect of moving charges will be observed in places where there is no electrostatic effect.

We have not attempted hitherto to visualise the magnetic action of the tubes, but have been content to obtain the mathematical relation between the velocity and the magnetic intensity. Indeed it is difficult to devise any adequate mechanical analogy to this action. It is probably best to imagine that the relative motion of the tubes changes in some way the attraction between tubes of opposite sign, so that the electrostatic effects are diminished. Consequently if one electric charge is moving relatively to another charge of the same sign the electrostatic repulsion between the charges is less in virtue of the motion than it would be if the charges were at rest: the motion will cause an apparent attraction between the charges such as is known to exist between two currents in the same direction. It should be pointed out that an electrically neutral system, from which radiate an equal number of tubes of opposite sign, will exhibit no magnetic effects when it is set in motion, for these effects depend on the shearing of tubes of one sign past those of the other sign, and no such relative motion exists if both sets of tubes are attached to the same body and move with it.

It is necessary to alter slightly our ideas of the electrostatic action of the tubes in order to bring them into harmony with the view that we have taken of their electromagnetic action. We must suppose that all tubes, both those which are attached to positive and those which are attached to negative charges, have the same tension in the direction of their length, but that, while like tubes (*i.e.* tubes in which, according to our analogy, the direction of absolute rotation is the same) repel each other as before, unlike tubes (those in which the direction of absolute rotation is opposite) attract each other and tend to coincide. Imagine then that a small positively charged particle from which tubes are radiating equally in all directions is brought close to a similar negative charge. On that side of the negative charge which is remote from the positive charge the tubes from the two charges are unlike and tend to coincide and to neutralise each other's electrostatic action: accordingly in this region the tubes from the positive charge will be bent down towards the negative charge and, when equilibrium is reached, they will all pass through the negative charge. But in the space between the two charges all the tubes are like and will distribute themselves in the manner that is shown in the ordinary diagrams of the lines of force between opposite charges, and the electrostatic effects of the tubes will be precisely the same as those which were deduced from our earlier conception.

Nor is there any need to change the quantitative formulae given on p. 10. For the number of tubes radiating from a charge of magnitude e will still be e, so that there are $2e$ tubes connected with the two charges of our illustration. But the electrostatic action of half of the tubes from each charge is neutralised by their coincidence with unlike tubes, and hence the number of tubes that have to be taken into account for electrostatic purposes is, as before, e.

Similarly the calculation on p. 14 of the magnetic effects of the motion of the tubes will remain unaltered. For, though there are twice as many tubes in the field as there were according to our earlier conception, half of these are to be considered stationary, while the others move past them: it is to this shear of unlike tubes past each other that the magnetic effect is attributed. Since, therefore, our calculations will be precisely the same whether we consider the tubes attached at both or only at one end to charges, the first conception will be used in all cases where the difference is not essential: a little consideration will enable the reader to translate the arguments into terms of the more accurate conceptions.

One difference between the two views should be pointed out. If positive and negative charges are considered as merely manifestations of the two ends of the same tube it is clear that the production of a positive charge without an equal negative charge is impossible. But, if the tubes attached to the two charges of opposite sign are quite independent, the experimental fact that opposite charges are always produced in equal quantities must be regarded as requiring further explanation.

Energy in the magnetic field.

9. The hydrodynamic analogy is useful in representing the distribution of the energy in the magnetic field. It is proved in any treatise on the theory of electromagnetism that the energy contained in unit volume of a magnetic field is $\frac{\mu H^2}{8\pi}$, where μ is the permeability of the medium and H the magnetic intensity at the point round which the unit volume is taken. If the magnetic intensity is due to the presence of moving Faraday tubes, $H = 4\pi N v \sin\psi$, and the energy is

$$E = 2\pi\mu N^2 v^2 \sin^2\psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$$

The occurrence of the quantity v^2 suggests that the energy may be represented as the kinetic energy of some moving system, for the kinetic energy of a body of mass m moving with velocity v is $\frac{1}{2}mv^2$. If the unit of volume taken be a cube of unit side with its edges parallel and perpendicular to the tubes, there will be N tubes, each of unit length, within this volume.

If each unit length of a tube be supposed to possess a mass $4\pi\mu N \sin^2\psi$, the kinetic energy of all the tubes will be

$$\tfrac{1}{2}N \times (4\pi\mu N \sin^2\psi) \times v^2 = 2\pi\mu N^2 v^2 \sin^2\psi,$$

which is the magnetic energy in the field.

At first sight the supposed distribution of mass among the tube appears so arbitrary and so unlike that which occurs in any mechanical system that it might be thought that the analogy was misleading rather than useful. But it will be seen that the analogy used in the previous section puts the matter in a new light. It is a well-known result of hydrodynamics that a body moving in a fluid behaves as if it possessed, in addition to its own mass, the mass of a certain volume of the fluid. The magnitude of this volume depends on the shape of the moving body: if it be a sphere, the additional mass is that of a portion of the fluid of the same volume as the sphere: if it be a right cylinder moving perpendicularly to its axis the volume is that of the cylinder: if the cylinder be moving along its axis the volume will diminish with the area of the cross section of the cylinder and will be nil if the cylinder is sufficiently thin.

Now the vortex filaments to which we have likened Faraday tubes will resemble in some respects cylinders moving in a fluid; if they move at right angles to their axes the additional mass which they derive from the disturbance which their passage causes in the fluid will be different to the mass which they gain when they are moving along their axes. If they are regarded as very thin cylinders the additional mass which they gain when moving along their axes will be very small. Accordingly the apparent mass of the tubes will vary with the direction of their motion in much the same manner as that given by the occurrence of the factor $\sin^2\psi$ in the expression for the mass of the tubes in the preceding section.

Again, if several bodies be moving together through a fluid the additional mass which must be attributed to each will be different from that which would be attributed if each were moving singly remote from all other bodies. The apparent mass of each body will depend on the distribution of the surrounding bodies, just as, according to (6), the mass of each tube depends upon N, the concentration of the tubes around it.

There is nothing extravagant in attributing to each tube a mass of $\frac{4\pi\mu N\sin^2\psi}{\text{[illegible]}}$ per unit length, and in regarding the magnetic energy in the neighbourhood of moving Faraday tubes as the kinetic energy of those tubes. The conception will be found of the greatest value in the next chapter, and when we come to consider the present views which are held as to the relations between electricity and matter.

10. Hitherto we have only considered the production of a magnetic intensity by an electric current: there is another branch of electromagnetism which deals with the currents produced by a variation of the magnetic field—or the 'induction of currents.'

Induction of currents.

It was discovered by Faraday that when a current in one circuit was started, stopped or varied in any way, a transient current was produced in any neighbouring closed circuit. The direction of the induced current was such that it tended to counteract the change in the magnetic field of the first circuit due to the variation of the current in it. Let us consider how this induction of currents is to be explained in terms of the mechanical analogies which have been adopted.

Let *PQRS* and *P′Q′R′S′* (Fig. 6) be two circuits lying one inside the other in the same plane. *PQRS* contains a battery or some other source of the production of Faraday tubes so that a current flows in it accompanied by the motion of tubes. These tubes may be imagined to stretch across from *PQ* (+) to *SR* (−) and to move down the circuit from top to bottom with their axes parallel to the portion *QR* into which they collapse at the end of their passage. Under the influence of these tubes there will be 'induced electrification' in *P′Q′R′S′*, or, in the terms of Faraday's theory, the tubes of the metal of the circuit (see Chap. III.) will be orientated in such a way that there is no electric intensity in the substance of the conductor. Now suppose that the conductor *QR* is removed and the circuit broken. The tube which was just about to collapse into *QR* will now be unable to do so: it will remain stretching across the gap and exerting a lateral pressure on the tubes above it tending to prevent them from moving down the circuit. But it must be remembered that the tubes possess mass and that a finite force will require a finite time in order to reduce them to rest. For a short interval the tubes will continue to move down the wires *PQ* and *RS*, but, being unable to escape at the ends, they will remain piled up about the region *QR*. The distribution of the tubes over the area of the circuit is changed: there are, relatively to the former condition, more tubes in the lower part of the circuit than in the upper.

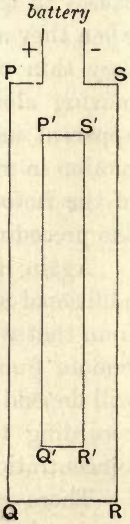

Fig. 6.

This change in the distribution of the tubes will cause a readjustment of the 'induced electrification' in the circuit $P'Q'R'S'$. It was in equilibrium previously, but now the number of tubes in the region $Q'R'$ is greater relatively to that in the region $P'S'$. Accordingly there must be a movement of electricity in the sense determined by the action of the tubes in the lower part of the circuit: that is to say, positive electricity will move in the direction $Q'R'S'P'$ and tubes having their positive ends on the part $P'Q'$ must move down the circuit. The motion of these tubes will give rise to a magnetic intensity, and since the motion of the tubes is in the same direction as that of the tubes in $PQRS$ before the circuit was broken the magnetic intensity will be in the same direction as that due to the current in the inducing circuit. But this latter intensity has disappeared by the breaking of the circuit: hence the intensity due to the induced current will be such as to counteract the change in the field due to the stoppage of the inducing current.

The reader may work out for himself the processes which occur when the current is started in $PQRS$, and he will find that they are such as to give rise to the effects which are observed experimentally. It will be noted that no attempt has been made to prove that the quantitative relations obtained are the same as those given by the usual theories: the calculations would be exceedingly complicated, and since they are not required for future reference and would not throw further light on Faraday's conceptions of the electromagnetic field, they may be omitted without detriment. For a consistent mathematical development of Faraday's theory the reader is referred to Prof. Thomson's *Recent Researches in Electricity and Magnetism*, Chap. I.

REFERENCES FOR CHAPTER I.

FARADAY, *Experimental Researches in Electricity and Magnetism*: especially I. 1231, 1613–1616, III. 3070–3299, and pp. 438–443.

J. J. THOMSON, *Recent Researches in Electricity and Magnetism*, Chap. I. and *Electricity and Matter*, Chap. I.

For § 6 ROWLAND, *Collected Papers*, p. 251, or *Phil. Mag.* (5) XXVII. 445–460, 1889.

CHAPTER II.

THE ELECTROMAGNETIC THEORY OF LIGHT.

Finite velocity of propagation of electric action.

1. We have already noticed one important discovery which was suggested by Faraday's theory of electric action but concealed by the older theories—the magnetic effect of moving charges. We must now turn to another such discovery of equal importance—the finite velocity of propagation of electromagnetic action. It has often been said that this discovery was necessarily the death blow of the 'action at a distance' theories and the final vindication of 'action through a medium.' So it has proved historically, but logically the result could not have been predicted with certainty. Weber tried to reconcile 'action at a distance' with such effects as the induction of currents by supposing that the effect of one charge on another depended not only upon the position of the charges but upon their velocity, acceleration and other characteristics of their motion. The attempt might have been successful; but we know now that the analytical expression necessary for the accurate definition of such action must be so complex, that, even if it could be found successfully from the experimental data, our mathematical powers would be insufficient to enable any results to be deduced from it. 'Action at a distance' has been shown to be totally useless as a description of electromagnetic phenomena.

It was stated in the previous chapter that a Faraday tube is subject to stresses both in the direction of its length and at right angles to it, and further that it must be regarded as possessing a definite mass when it moves in a direction perpendicular to its length. Suppose then that a Faraday tube stretches between two points A and B and that the end A is moved in a direction perpendicular to that of the tube. The tube is distorted and the stresses on it are no longer in equilibrium: there is a

resultant force on it. But, since the tube possesses a finite mass, it requires a finite time for a finite force to set up a finite displacement in it: the parts of the tube remote from the end A will not take up the positions corresponding to the new conditions immediately, but require a finite time after the motion has been given to the end A. That is to say, the action will be transmitted from A to B with a finite velocity.

An analogy may make the process clearer and will help us in our subsequent investigations. If one end of a rope is tied to a heavy body and the other end shaken at right angles to its length, the disturbance caused by the shaking may be seen to travel out along the rope and, after a definite time, reach the body and cause it to move. A Faraday tube is very like a rope in that it has a tension along its length and possesses mass: the action of one charged body on another will be transmitted along the Faraday tubes joining them in just the same way as the action is transmitted from the hand to the heavy body along the rope. But there is an important difference between the rope and the tube. If the rope be stretched taut and jerked in the direction of its length, a compressional disturbance will travel out along the rope, the velocity of which is, in general, different from that of the transverse disturbance. Such longitudinal disturbances cannot be propagated along a Faraday tube, for the tension of the tube is independent of its length. If one of two infinite parallel charged plates be moved in a direction perpendicular to its plane and parallel to the tubes attached to it, the force acting on the other plate is not changed and there is no disturbance of equilibrium. *Only transverse disturbances can be propagated along Faraday tubes.*

Calculation of the velocity of propagation.

2. The use of this illustration will enable us to calculate from the known relation between the tension and mass of the rope and the velocity of the disturbances along it, the velocity with which the transverse disturbances are propagated along the Faraday tubes.

Consider a tube $ABCDEFGH$ slightly displaced into the position $AB'C'DEF'G'H$ and let us calculate the forces tending to restore the portion $B'C'F'G'$ to its normal position.

These forces will be due (1) to the tension along the tube and (2) to the hydrostatic pressure on its surface. Take (1) first and consider a portion of the tube of small cross section ω represented by the line $AB'C'D$. The tension along this line is $T\omega$ and the component of it acting at B' along $B'B$ is $T\omega \cos LB'B$, where $B'L$ is the tangent at B'. But

$$\cos LB'B = \sin B'LB$$
$$= \sin BB'O$$

where $B'O$ is the normal at B': therefore the force along $B'B$ is $T\omega \sin BB'O$. But the angle $BB'O$ is small, since the displacement is small, and $\sin BB'O$ may be put equal to $BB'O$, so that the force is $T\omega . BB'O$. Similarly the force at C' acting in the direction CC' is $T\omega . CC'O$, hence the total force tending to restore $B'C'$ to its normal position is $T\omega(BB'O + CC'O) = T\omega . B'OC'$ and the force on the whole portion of the tube $B'C'F'G'$ is $T . B'OC' \times$ area of the cross section of the tube or

$$T . B'OC' . B'G' . t,$$

where t is the thickness of the tube perpendicular to the plane of the paper.

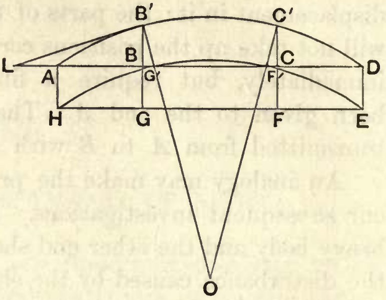

Fig. 7.

Now consider (2). There is a hydrostatic pressure R per unit of surface all over the tube. Hence the pressure on the upper side of the portion of the tube $B'C'F'G'$ forcing it down is $R . B'C' . t$, since $B'C' . t$ is the area of the upper portion of $B'C'F'G'$. Similarly the force on the lower side of the tube forcing it upwards is $R . F'G' . t$, and the resultant force tending to restore the tube to its normal position is

$$R . t . (B'C' - F'G') = R . t . B'OC' \, (B'O - G'O) = R . t . B'G' . B'OC'.$$

The whole force due to (1) and (2) is $(T + R) . t . B'G' . B'OC'$. But T, the tension along the tube per unit area of cross section, is equal to R, the pressure on the tube per unit area of surface, and both are equal to $\frac{2\pi N}{K}$, where K is the dielectric constant of the medium surrounding the tube. Hence the whole force tending to restore $B'C'F'G'$ is $2T.t.B'G'.B'OC'$ and is equivalent to the force that would act if there were only a tension $2T.t.B'G'$ acting along the tube.

But it is well known that the velocity with which a disturbance is propagated along a rope of mass m per unit of length and under a tension P is $\sqrt{\frac{P}{m}}$. Hence the velocity of the disturbance along our tube is $\sqrt{\frac{2T.t.B'G'}{m}}$, where m is the mass of the tube per unit length. But it was shown on p. 24 that the mass of the tube per unit volume is $4\pi\mu N$ (since each point of the tube is moving at right angles to its length and $\sin\psi = 1$): therefore the mass of the part $B'C'F'G'$ is $4\pi\mu N.t.B'G'.F'G'$, and the mass per unit length is $4\pi\mu N.t.B'G'$. Therefore the velocity V with which electrical effects will be propagated along the tube is given by

$$V = \sqrt{\frac{2.T.t.B'G'}{m}} = \sqrt{\frac{\frac{4\pi N}{K}.t.B'G'}{4\pi\mu N.t.B'G'}}$$

or

$$V = \frac{1}{\sqrt{\mu K}} \quad \text{.........................(1).}$$

3. The next step is to measure the value of $\frac{1}{\sqrt{\mu K}}$ for some medium, say air. Of course the value of either μ or K will vary with the system of electrical units that we adopt, but the value of the product μK will depend only on the units of time and length that are adopted. This statement can be proved readily by considering the dimensions of the quantities concerned, but all arguments from dimensions, though perfectly conclusive, seem unconvincing to so many, that it is worth while to sketch the principle of the entire method by which the measurement may be made and to show that the value obtained is independent of the electrical units used.

Determination of the velocity of propagation.

The value of the dielectric constant for a medium is most easily ascertained by finding the capacity of a condenser of known dimensions of which that medium is the dielectric. The value of the permeability μ is most easily ascertained by measuring the magnetic induction through a circuit of known dimensions, and this induction is ascertained by observing the electromotive force which a known variation of the induction produces in a closed circuit. We have then to compare the capacity of a condenser with the E.M.F. in a circuit: but, when the condenser is discharged or the E.M.F. applied to a conducting circuit, a current is obtained, and the comparison which has to be effected will be made most conveniently in terms of this current.

Suppose then that we take a parallel plate guard-ring condenser, in which the plates have the area A and are a distance d apart. The capacity of that condenser is $\frac{AK}{4\pi d}$, where K is the dielectric constant of the dielectric (air): the formula is true whatever system of units is employed so long as the same system is used for all the quantities concerned. If a potential difference V is established between the two plates, the charge in the condenser will be $\frac{AKV}{4\pi d}$. If the condenser is charged n times in t seconds and discharged each time through a conducting circuit, a quantity of electricity $\frac{AKV}{4\pi d}$ will flow through the circuit at each discharge, equivalent to an average current $\frac{n}{t} \cdot \frac{AKV}{4\pi d}$. We have thus got an expression for K in terms of a current: let us turn to μ.

Take a long solenoid (A) with N turns in L units of length, filled with the same medium (air) as the condenser. Inside this place a second solenoid (B) with M turns on it and let the area of its cross section be s. If the axes of the solenoids are parallel there is a total magnetic induction $4\pi\mu \frac{N}{L} . MsC$ through (B) when a current C flows through A. If B be rotated round an axis perpendicular to the axis of (A) through n' complete revolutions in t' seconds, then the rate of change of induction through (B) will be $\frac{16\pi\mu NMsC}{L} \cdot \frac{n'}{t'}$, and this quantity again, by Faraday's law, will be equal to the E.M.F. between the terminals of the coils of (B). (The E.M.F. is supposed to be commutated, as in the direct current dynamo, so as to be always in the same direction.)

If the same current C flows through a resistance R, the potential difference between the ends of the resistance is RC. This P.D. can be compared with that due to the change of magnetic induction in (B) by a potentiometer method: let the comparison be made with the result that

$$RC = \alpha\,\frac{16\pi\mu NMsCn'}{Lt'}.$$

Then
$$R = \frac{16\alpha\pi\mu NMsn'}{Lt'}.$$

Now apply to the terminals of R the P.D. V which was used in charging the condenser: a current $\frac{V}{R}$ will flow and this current may be compared with that due to the discharge of the condenser: let the ratio of the second current to the first be β.

Then
$$\frac{n}{t}\cdot\frac{AKV}{4\pi d} = \beta\frac{V}{R} = \frac{\beta VLt'}{16\pi\alpha\mu NMsn'},$$

or
$$\mu K = \frac{\beta}{\alpha}\cdot\frac{dLtt'}{4NMsnn'A}.$$

All the quantities on the right-hand side of this equation can be measured in terms of the units of time and length without any reference to the electrical units which have been employed in the experiments. n, n', α, β, N and M are numbers or ratios and have no dimensions, t and t' are times, A and s are areas, d and L are lengths. Thus the dimensions of μK are found to be

$$\frac{(\text{length})^2\times(\text{time})^2}{(\text{area})^2} = \frac{(\text{time})^2}{(\text{length})^2} = \frac{1}{(\text{velocity})^2}.$$

Therefore $\frac{1}{\sqrt{\mu K}}$ is a velocity.

The measurement has been actually carried out by methods that differ only in detail from that just described, and it has been found that $\frac{1}{\sqrt{\mu K}}$, the velocity calculated for the propagation of electrical disturbances in air, is very nearly $3{\cdot}0\times10^{10}$ centimetres per second and is equal within the limits of experimental error to the velocity of light in air as deduced from the best results of Fizeau, Newcomb, Michelson and the rest. *This velocity will henceforth be always denoted by the symbol V.*

Some twenty years after Maxwell had published his deduction of the velocity of propagation of electrical disturbances his results were confirmed by the brilliant experiment of Hertz. His classical research is described in all recent text-books of electrical theory[1] and a sketch of the principle of his method will suffice.

Suppose that one end of a pair of charged condenser plates are connected momentarily by a conductor, as when a spark passes through the dielectric medium between the plates. The Faraday tubes in the region of the spark are annihilated and the tubes from the other end of the condenser move down to fill their place. Since the tubes possess mass, their motion will not cease immediately the distribution corresponding to the new position of equilibrium has been reached: their inertia will carry them a little further. Too many tubes will accumulate at the end of the condenser at which the spark had passed, and, when the tubes have come to rest, there will be a force urging them to move back towards the other end of the conductor. The tubes will be set into oscillations, which will gradually die away owing to the damping effect of the resistance of the condenser circuit. According to the theory which has been put forward in the foregoing paragraphs these transverse vibrations of the ends of the tubes attached to the condenser plates should be propagated out along the tubes with the velocity V. Hertz succeeded in measuring the wave length (λ) of the vibrations and in calculating their frequency T. Since these quantities are connected with V by the relation

$$\lambda = VT \qquad \text{.............................(2)},$$

his experiments led to the determination of V. He found that its value agreed with that predicted by Maxwell within the limits of error of observation. The correctness of Maxwell's theory could be doubted no longer and was admitted by the entire scientific world.

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[1] See for example Whetham's *Experimental Electricity*, § 70.

The electromagnetic theory of light.

4. This coincidence of the values of the velocity of propagation of electrical and optical effects led Maxwell to suggest that the vibrations which constitute light are nothing but vibrations of the Faraday tubes which stretch throughout all space.

It has been said that this conclusion was in some measure anticipated by Faraday, and certainly the writer of the letter to Richard Phillips (*Experimental Researches*, vol. III. p. 477) came very near to anticipating Maxwell's theory. But I think that his search after an effect of electrical conditions on optical phenomena show that his ideas were not quite clear and that in this instance his intuition, the most marvellous in the history of physics, must be judged to have been at fault. His famous sentence at the beginning of the third volume gives us his reasons for undertaking that search. "I have long held an opinion," he says, "almost amounting to a conviction, in common I believe with many other lovers of natural knowledge, that the various forms under which the forces of matter are made manifest have one common origin." Now that expression of opinion is certainly an anticipation of the Principle of the Conservation of Energy, but it is not an anticipation of the electromagnetic theory of light. The most convincing evidence of the intimate connection of electrical and optical phenomena is that in both cases the effects are propagated through a vacuum in which there is no matter and that they are the only known effects (except gravitation) which are so propagated. If, then, there is not a close connection between electricity and light we shall have to attribute to vacuous spaces two sets of entirely distinct and mutually independent properties. In other words, both electrical theory and optical theory require an aether, and if there is no connection between the electrical aether and the optical aether, space will have to be filled twice over with distinct aethers—an assumption so complicated that every effort ought to be made to render it unnecessary.

If we are going to look for a relation between electricity and light, we should search first among the effects of these agents in a vacuum (or in those media, such as gases, which differ but slightly from vacua). Faraday, on the other hand, tried the effect of passing light through dense media in a magnetic field. He was rewarded by finding an effect, but it is of so complex a nature that his theory was quite incapable of explaining it without the introduction of subsidiary hypotheses. It is difficult to imagine what experiment he could have tried in which vacuous spaces were concerned, unless it were some modification of Hertz' research; but the fact that he does not appear to have attempted to devise such an experiment would seem to show that his insight into the connection was not so deep as has been imagined frequently.

Optical laws from the electromagnetic standpoint.

5. Let us now investigate whether the known properties of Faraday tubes will enable us to deduce the optical from the electrical laws.

In the first place it may be noted that the explanation of a great many optical phenomena, in fact of all those which are concerned with interference or diffraction and among them the rectilinear propagation of light, follows directly from the theory that light is a form of wave disturbance and makes no assumption whatever as to the physical nature of the waves. All interference and diffraction laws are consequences of our theory as they are of the older elastic solid theory of light or of any other wave theory, such as that which deals with sound or water waves. The only reason why some laws are regarded as optical laws and not as acoustic laws is that the smallness of the wave length of light makes some phenomena important in optical theory which are not important for the longer waves of sound. But of course the wave length on the electromagnetic theory is identical with that of the elastic solid theory or any other theory: the hypothesis which distinguishes the different theories concerns only the physical nature of the vibration that is taking place.

The laws of refraction and reflection are also common to all forms of vibration. It can be shown that a periodic disturbance falling on the bounding surface between two media, in which the velocity of the disturbance is different, is broken up into a reflected portion travelling back through the first medium and a refracted part travelling on through the second medium. Further Huyghens' well-known construction shows that since the velocities of the incident and reflected disturbances (travelling in the same medium) are the same, the angle of reflection is equal to the angle of incidence: while the angle of refraction is connected with that of incidence by the familiar 'sine law,' in which the index of refraction is the ratio of the velocity of the disturbance in the second to that in the first medium.

Now according to our theory the velocity of light in a medium of which the dielectric constant is K and the permeability μ is $\frac{1}{\sqrt{\mu K}}$: hence the refractive index of any medium (A) relative to another (B) is $\sqrt{\frac{\mu_A K_A}{\mu_B K_B}} = n_{AB}$. Experiment shows that the permeability of every medium, including even substances which are classed as magnetic materials, is the same, when the medium is subjected to very rapid alternations of magnetic intensity such as occur in light waves. Accordingly we may write $\mu_A = \mu_B$ and

$$n_{AB} = \sqrt{\frac{K_A}{K_B}}.$$

If the medium A be a vacuum, for which on the electrostatic system of units $K = 1$, the quantity n_{AB} becomes that which is usually called the refractive index (n) of the medium A: and we have

$$n = \sqrt{K} \text{ or } K = n^2 \quad\text{.........................(3).}$$

Here we have an opportunity of subjecting our theory to experimental verification. Both K and n may be measured for the same material, and the relation between the values obtained compared with that predicted by (3). Table I gives the results of a few such experiments.

It will be seen that in some cases the agreement is excellent, but in other cases there is a large discrepancy. It is clear that the relation cannot be universally true, for the value of n is not constant but varies with the frequency of the light vibrations, and the value of K also is different for steady fields and for very rapid oscillations. In order to make the comparison quite fair, K and n should be measured for oscillations of the same frequency, a condition that is impossible in practice; but it is found that the values of K and n^2 become more nearly equal when n is measured for the slowest possible light vibrations and K for the most rapid electrical oscillations. The rough agreement between physical constants apparently so diverse as the dielectric constant and the refractive index must be judged to afford some support for our theory, but we cannot rest content until the reason for the variation of K and n with the frequency has been elucidated completely.

TABLE I.

Substance	K	n^2
Air	1·000590	1·000588
Hydrogen	1·000264	1·000276
Benzol	2·21	2·20
Carbon bisulphide	2·65	2·69
Paraffin wax	2·01	2·28
Quartz	4·6	2·39 (ordinary ray)
Alcohol	25·0	1·85
Water	76·0	1·78

The values of n given above are measured for yellow light.

Polarised light.

6. It is in dealing with the polarisation[1] of light that the necessity first arises of introducing an hypothesis as to the nature of light vibrations. It is found that there are certain light waves, known as plane polarised waves, which have not the same relation to all planes drawn through the direction of propagation. If light reflected from one glass surface be reflected subsequently at a similar surface, the intensity of the doubly reflected light varies not only with the angle of incidence, but also with the angle between the planes of the reflecting surfaces; for a fixed position of the first surface, the plane containing the direction of the ray and the normal to the second reflecting surface, when the intensity of the light reflected from it is a maximum, is called the plane of polarisation of the light. These phenomena are explicable, if the vibrations which constitute

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[1] I hope it is unnecessary to point out that the 'polarisation' of light has nothing to do with 'electric polarisation,' which has been mentioned so often. It is unfortunate that the same word has to be used in two different senses, but I do not think that any ambiguity need arise.

light do not take place (like those of sound) in the line of propagation, but in a direction at right angles to that line. Light must consist of 'transverse' vibrations such as we concluded could be propagated along Faraday tubes.

But further information is necessary before the direction of the vibrations with regard to the plane of polarisation can be fixed. Two hypotheses concerning this direction have been put forward: one, connected with the name of Fresnel, that the vibrations are perpendicular to the plane of polarisation; the other, due to MacCullagh, that the vibrations are in that plane. In order to consider these hypotheses in the light of the electromagnetic theory, we shall have to examine more closely the processes associated with the vibration of a Faraday tube.

Let a train of regular transverse vibrations such as are indicated in Fig. 8 be travelling out along a series of parallel tubes. In any plane at right angles to the direction of propagation the vibrations of all the tubes are in the same phase: such a plane is a wave front and the disturbance is a plane wave.

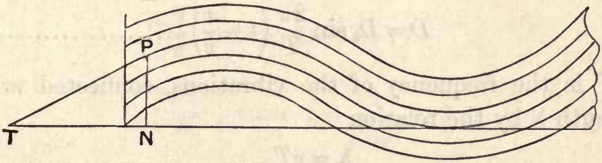

Fig. 8.

The electric polarisation at any point P is in the direction PT and may be regarded as compounded of the undisturbed polarisation N in the direction NT and a polarisation D due to the vibration, in the direction PN, where $\frac{D}{N}=\frac{PN}{PT}$. Since the undisturbed polarisation is supposed to be the same all along the path of the wave, if a curve (Fig. 8 A) be drawn such that the abscissae represent distances along the direction of propagation and the ordinates represent the values of the ratio $\frac{PN}{PT}$, any ordinate pn will be proportional to the magnitude of the transverse polarisation D at the point n in the path of the wave. If the wave is a periodic disturbance such as is shown in the figure, the disturbance may be regarded as a distribution of polarisation given by the portion of the curve *aa′* travelling out with the velocity of light in the medium considered (v). If the wave is a

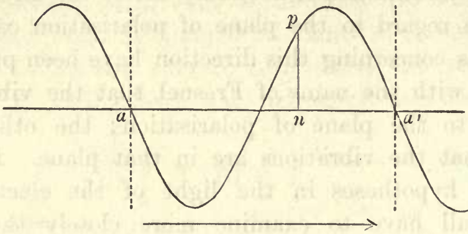

Fig. 8 A.

simple harmonic vibration, the value of D at a point at a distance x from the origin of the vibrations and at a time t is given by[1]

$$D = D_0 \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4),$$

where T is the frequency of the vibrations, connected with the wave length λ by the relation

$$\lambda = vT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

The vibrating polarisation will give rise to a vibrating electric intensity Y, where $Y = \frac{4\pi D}{K}$, or

$$Y = \frac{4\pi D_0}{K} \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \ldots\ldots\ldots\ldots\ldots\ldots (6).$$

The direction of Y coincides with that of D.

Again, at any point the tubes which give rise to the polarisation D are travelling at right angles to their length with velocity v. Hence they will give rise to a magnetic intensity H where

$$H = 4\pi D v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7),$$

or since
$$v = \frac{1}{\sqrt{\mu K}},$$

$$H = \frac{4\pi D_0}{\sqrt{\mu K}} \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

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[1] See R. W. Wood, *Physical Optics*, p. 7.

The direction of H is at right angles both to the tubes and to their motion, that is, in the wave front perpendicular to the plane of Fig. 8, which contains the direction of the electric polarisation and the electric intensity.

Hence we see that there are at least three important quantities which are vibrating transversely in the wave front, of which two, the electric polarisation and the electric intensity, are in the same direction, while the magnetic intensity is in a perpendicular direction.

The energy of the light disturbance is made up of two parts: (1) electrostatic energy due to the displacement of the tubes, and (2) magnetic energy due to their motion. The magnitude of the former is given by the usual formula as

$$\frac{2\pi D^2}{K} \text{ (see p. 11):}$$

that of the latter is

$$\frac{\mu H^2}{8\pi} = 2\pi\mu \,.\, D^2 v^2, \text{ by (7),}$$

or

$$\frac{2\pi D^2}{K}, \text{ since } v = \frac{1}{\sqrt{\mu K}}.$$

The energy of the light wave at every point is equally divided into electrostatic and magnetic energy: the two forms of energy are propagated together, and if we determine the distribution of one form, the distribution of the other is known.

In order to determine the relation of the direction of the vibrating quantities to the plane of polarisation, let us calculate the intensity of the reflected and refracted rays consequent on the incidence of plane polarised light (that is light the vibrations of which, like those of the wave under consideration, take place along straight lines in the wave front). The relative intensity of these rays varies according as the plane of polarisation coincides with, or is perpendicular to, that of incidence, and by comparing the formulae which we deduce with experiment the solution of the problem may be found.

Let the reflecting surface be perpendicular to the plane of Fig. 9: and let the electric polarisation of the incident light ray AO be in the plane of incidence, *i.e.* in the direction PQ. Let OA', OB

be the reflected and refracted rays, i and r the angles of incidence and refraction, $P'Q'$, P_1Q_1 the direction of the electric polarisation in the reflected and refracted rays. Let D_1, D_1', D_2 be the amplitudes of the vibrating electric polarisation in the incident, reflected and refracted rays respectively, K_1 and K_2 the dielectric constants of the first and the second medium. The electrostatic conditions which must be satisfied at the bounding surface are[1]

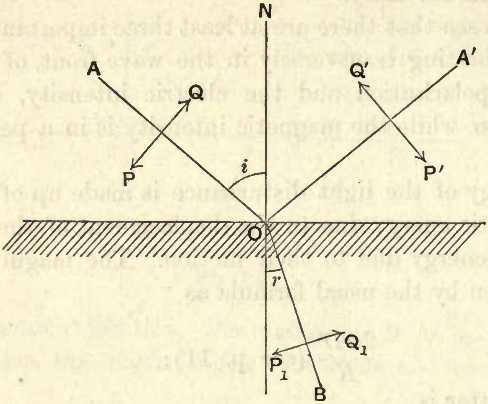

Fig. 9.

(1) The electric polarisation normal to the surface must be the same in magnitude and direction on both sides of the surface and

(2) The tangential electric intensity must be the same on both sides of the surface.

Since the electric intensities in the three rays are

$$\frac{4\pi D_1}{K_1},\ \frac{4\pi D_1'}{K_1}\ \text{and}\ \frac{4\pi D_2}{K_2},$$

respectively, these two conditions give the equations

$$(D_1 + D_1')\sin i = D_2 \sin r \quad \ldots\ldots\ldots\ldots\ldots\ldots(9).$$

$$\frac{4\pi}{K_1}\cos i\,(D_1 - D_1') = \frac{4\pi}{K_2} D_2 \cos r \quad \ldots\ldots\ldots\ldots(10).$$

Solving these equations and remembering that

$$\frac{\sin i}{\sin r} = n = \sqrt{\frac{K_2}{K_1}},$$

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[1] See for example J. J. Thomson's *Electricity and Magnetism*, 3rd ed., § 74.

we find

$$D_1 : D_1' : D_2 = 1 : \frac{\tan(i-r)}{\tan(i+r)} : \frac{2\sin^2 i \cos i}{\sin r \sin(i+r)\cos(i-r)}.$$

But if V_1 and V_2 are the velocities of light in the first and second medium respectively, so that $n = \frac{V_1}{V_2}$, the intensities of the three rays are in the ratio[1]

$$\frac{2\pi D_1^2 V_1}{K_1}\cos i : \frac{2\pi D_1'^2 V_1}{K_1}\cos i : \frac{2\pi D_2^2 V_2^2}{K_2}\cos r$$

or

$$1 : \frac{\tan^2(i-r)}{\tan^2(i+r)} : \frac{\sin 2i \sin 2r}{\sin^2(i+r)\cos^2(i-r)}.$$

Now experiment shows that this ratio holds between the intensities of three rays when the plane of polarisation is perpendicular to the plane of incidence: but the electric polarisation is in the plane of incidence in the case which we have investigated. We may conclude—and a rigorous investigation would confirm the conclusion—that the electric polarisation is perpendicular to the plane of polarisation of plane polarised light and agrees in direction with the 'vibration' which Fresnel used in developing his theory of light. Since the magnetic intensity is perpendicular to the electric polarisation, it must lie in the plane of polarisation and is identical with MacCullagh's vibration. The settlement of the long controversy between these distinguished physicists is found, like that of most of the great disputes of science, in the conclusion that both of the disputants were right. The 'vibration of the light' is both in and perpendicular to the plane of polarisation, for there is more than one vibrating quantity concerned in the propagation of a light disturbance.

Optical properties of conductors.

7. The conditions described in (9) and (10) which must hold at the boundary between two media become especially interesting when one of the media is an insulator and the other a conductor. There can be no electric intensity in a conductor, and hence, from (10),

$$D_1 = D_1',$$

and the intensity of the reflected is equal to the intensity of the incident ray: that is to say the conductor acts as a perfect reflector.

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[1] See Preston, *Theory of Light*, 2nd ed., § 68.

This conclusion agrees with experiment to the extent that the best conductors of electricity (metals) are also the best reflectors of light: but different metals differ in reflecting power and none of them are perfect reflectors. Indeed, if the matter be considered more closely, such variations are to be expected. When a conductor is subjected to an electric intensity a current flows in such a direction as to neutralise that intensity, but, unless the conductivity be infinite, the neutralisation will not be instantaneous. Accordingly, when a light wave falls on a conductor, vibrations of electric intensity will be set up in it; some portion of the energy will travel into the conductor and will not be reflected. But the refracted wave will be able to travel only a very short distance before the electric intensity is neutralised by currents in the metals and the energy of the wave is 'absorbed.'

It is possible to calculate the relation that should hold between the conductivity of the metal and its optical properties. The calculation will not be given here because we shall have to consider the matter in greater detail in another chapter. It is found that if σ be the conductivity, T the period of vibration of the light, n the refractive index of the metal, α its coefficient of absorption, the relation that should hold between these four quantities is

$$n^2\alpha = \sigma T \qquad \text{(11)}.$$

Table II gives the values of the two sides of this equation determined experimentally for several metals. The quantities n and α can be measured by making measurements on extremely

TABLE II.

Substance	$n^2\alpha$	σT
Silver	0·66	3·6
Gold	1·04	2·4
Platinum	8·8	5·25
Copper	1·68	3·5
Iron	8·2	4·4
Sodium	0·013	1·5
Mercury	8·6	6·0

The measurements are made with yellow light for which $T = 2 \times 10^{-15}$ secs. It should be remembered that σ is expressed in electrostatic units: the electrostatic unit of resistance is 9×10^{11} ohms.

thin films of metals and also indirectly from observation on the elliptic polarisation of light reflected from metals.

It will be seen that the agreement and the discrepancy of the experimental values compared with the calculated formula are similar to those which were noticed when we considered the refractive index of transparent bodies: there is a general concordance but no numerical identity. Even the general concordance vanishes when conductors other than metals, such as electrolytic solutions, are taken into account. The conductivity of a solution of sulphuric acid is much nearer to that of silver than to that of the most highly resistant liquids, but, while copper is almost perfectly reflecting, the reflecting power of the aqueous solution is no greater than that of the many insulating liquids.

Extension of the theory necessary.

8. There are other optical phenomena of which the simple electromagnetic theory of light can offer no explanation whatever. Of these the 'Faraday effect'—the rotation of the plane of polarisation of light travelling through a material medium in a magnetic field—is perhaps the most striking: Maxwell considered it at length but failed to bring it into harmony with his hypothesis. It was certainly unfortunate that Faraday, when searching for an experiment which should prove conclusively his cherished conviction that light and electricity were intimately connected, should chance on one of the few phenomena on which the view of the nature of electrical action associated with his name could throw no light. But when all its failures are set in array, the fundamental truth of the electromagnetic theory becomes yet more certain. So long as the behaviour of light in a vacuum is considered the harmony of theory and experiment is complete: it is only when material media are taken into account that difficulties arise.

Like all the truly useful theories of science, the Maxwellian theory of light introduced more problems than it removed. It answered the question as to the physical nature of the vibrations in a light wave, but it raised the vast question of the relation of electricity to matter. Instead of being content with representing the properties of the extremely complex systems which we call material bodies by the introduction into our equations of three constants, we ask ourselves what are the processes which give rise to the necessity of those constants. From that question start all the investigations with which we shall be concerned in the rest of this volume. Logically and historically the development of the 'electron theory' is based on the discrepancy between the values of K and n^2; but it was aided by researches of a totally different nature which at first had no connection with this problem. At the present time the lines of inquiry have converged, but as far as possible we shall keep them separate. In the next part we shall sketch the optical and allied investigations: we shall then turn to the second investigation, which concerns the electrical properties of gases: and finally we shall consider the results, still awaiting completion, which are due to the combination of evidence from all sources.

REFERENCES FOR CHAPTER II.

Since the only subject treated in this chapter is the simple electromagnetic theory of light, the development of which by more advanced analytical methods is given in every modern text-book of optics, it is unnecessary to give detailed references.

PART II.

THE ELECTRON THEORY. DEDUCTIVE METHODS.

CHAPTER III.

THE DIELECTRIC CONSTANT AND THE REFRACTIVE INDEX.

Electric induction.

1. IF there be brought near to a charged body *A* a rod composed of material of which the dielectric constant is different from that of the medium surrounding *A*, the phenomena of electric induction are observed. The ends of the rod near to and remote from the charged body appear to carry respectively charges of the opposite and of the same sign as that upon *A*. If the rod is made of conducting material it can be charged permanently 'by induction': on cutting the rod at any point between its two ends and removing it from the neighbourhood of *A* the separated fragments are found to retain the charges which they appeared to carry under the influence of the charge on *A*. But, if the rod is made of insulating material, the separated fragments will be without charge at whatever point the rod be cut.

The earliest theories of the nature of electricity—the 'fluid' theories for example—offered an obvious explanation of these observations. The neutral rod is supposed to contain equal quantities of electricity of opposite sign which normally counterbalance each other exactly. But when the body is brought into the neighbourhood of a charged body, the electricity on that body attracts that one of the two fluids contained in the neutral body which is of opposite sign, and repels that which is of the same sign. But no clear account of the difference in physical constitution which distinguishes a conducting from an insulating material could be offered. A well-known hypothesis connected with the name of Mosotti suggested that a substance having a dielectric constant greater than that of a vacuum consists of conducting molecules immersed in a perfectly insulating medium: but such a view emphasises rather than removes the distinction between conductor and insulator. Moreover it is perfectly incapable of offering any solution of the immediate problem before us—the explanation of the observed relation between the dielectric constant and the refractive index.

Lorentz' hypothesis. Electrons.

2. The hypothesis that was to lead to a complete explanation of that relation was first made—or at any rate first utilized—by H. A. Lorentz, the great Dutch physicist, in a memoir published in 1878. Lorentz assumed that the electricity of opposite signs, whether in a charged or in a neutral body, is not distributed continuously through the body as supposed by the two fluid theory, but is concentrated on a large number of discrete particles scattered through the body; further he assumed that all these particles bear charges of the same magnitude but of either sign. A body is charged or uncharged according as it contains unequal or equal numbers of particles of opposite sign.

The charge on one of these particles is an 'atom of electricity,' for, since the particles cannot be subdivided, no submultiple of that charge can take part in any change. The idea of such an atom of electricity was not new to physics: it had been employed by several earlier writers, including Maxwell. We noted in Chap. I. that Faraday's theory leads to the conception of such an atom, the charge on the end of a single tube, but the strongest support for the hypothesis comes from observations made by the same great philosopher upon electrolytic conduction.

The proportionality of the quantity of electricity which has passed through an electrolyte and the mass of any element liberated at an electrode indicates that all atoms of the same element must carry the same charge: the examination of the masses of different elements liberated by the same quantity of electricity shows that the magnitudes of the charges upon the atoms of different elements bear a simple relation to each other—they are all either equal to, or a small integral multiple of, the charge carried by an atom of hydrogen. This charge carried by an atom of hydrogen, when in the form of an electrolytic ion, is the smallest charge which can take part in electrolytic conduction: it is the electrolytic 'atom of electricity.' The magnitude of this charge has been estimated from several sources of evidence that will concern us later. The most probable value is $3{\cdot}4 \times 10^{-10}$ electrostatic C.G.S. units: this quantity of electricity will be represented by the symbol ϵ.

In his earliest memoir Lorentz did not state explicitly that he regarded the charge on one of his hypothetical particles as identical with the electrolytic atom of electricity: he made no supposition concerning the magnitude of that charge for his theory was not applied immediately to numerical calculations. But as soon as the question was raised, the answer was obvious: the charge on each of these particles is the quantity ϵ. Johnstone Stoney had suggested the use of the word 'electron' to denote the 'atom of electricity': when the importance of Lorentz' theory was recognized, the word was transferred to denote one of his charged particles carrying an 'atom of electricity.' In a later chapter we shall have to add a further connotation to the term electron, but for the present we will use it to indicate one of the hypothetical charged particles of Lorentz.

Distinction between a conductor and an insulator.

3. On this view a neutral body is one which contains equal numbers of oppositely charged electrons: a charged body is one which contains an excess of electrons of one sign. The forces acting on the electrons in a neutral body, due to their mutual attractions and repulsions, depend on many factors which cannot be taken into account at present. We will consider only two simple cases:—(1) when the mutual actions neutralise each other and the electrons are free to move without restraining forces, and (2) when each electron is attached to one of opposite sign by a force which varies directly as the distance between the electrons.

In the first case, if the body be placed in an electric field, the electrons of opposite sign move to opposite ends of the body, until the number of Faraday tubes connecting them is equal at each point to the number of 'unlike' tubes of the external field. The electric intensity of the field is then neutralised at all points inside the body and further action ceases. By cutting the body at any cross section and separating the fragments they will be found to be charged. Such *a body in which the electrons are free to move behaves as a conductor* in electrostatic experiments.

In the second case, the separation of the electrons will be opposed by the restraining force between opposite pairs, and it will cease, not when the electric intensity in the body is zero, but when the force on any electron due to the external field is equal and opposite to that due to the attraction of the attached electron: the body behaves as a dielectric with a constant greater than 1. It might seem at first sight that, since the electrons of opposite sign are separated to a finite distance, it must be possible by choosing suitably the plane of section to divide the body into two oppositely charged portions. But it must be remembered that there is a limit to our powers of mechanical division: we cannot divide a molecule. If the pair of electrons are contained within the same molecule and are not dragged out of that molecule by the external field, no process of mechanical division can separate them and, wherever the section of the body be made, the two fragments will be neutral. *In a dielectric*[1]*, then, we must imagine that the electrons are contained within the molecules and cannot be dragged out of them by an external electric field.*

Calculation of the dielectric constant.

4. A simple calculation enables us to establish a relation between the dielectric constant and the circumstances of the electrons contained in it.

Let a slab of the dielectric be placed in a uniform field of electric polarisation F in a direction from left to right perpendicular to the face of the slab. Let the electric intensity in the slab, when equilibrium is reached, be X: then the action

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[1] Of course the distinction between a conductor and an insulator is quite arbitrary, for bodies can be found having any conductivity intermediate between those of silver and sulphur. But it is important for electrostatics. In the next chapter we shall consider more closely the nature of conductivity.

of the external field will cause the electrons of opposite sign in the slab to be displaced relatively to each other along the direction of the external field. Let the force of attraction between connected pairs of electrons be fx, where x is the distance between the electrons. Equilibrium will be reached when the relative displacement of the electrons is such that this restoring force is equal and opposite to the force exerted on the electrons by the electric intensity in the slab, *i.e.* when

$$\epsilon X = fx \quad \text{..............................(1).}$$

Let N be the number of electrons of either sign in unit volume of the slab; this quantity will be termed the concentration of the electrons; let A be the area of the faces of the slab. Before the external field acted, the electrons of opposite sign were distributed uniformly through the slab: by its action all the negative electrons have been displaced a distance x to the left relatively to the positive electrons. Consequently there will be a layer of thickness x, covering the left face of the slab, which contains only negative electrons, and there will be a similar layer on the right side of the slab containing only positive electrons. The charges on these layers will be $Ax \times N \times (-\epsilon)$ and $Ax \times N \times (+\epsilon)$ respectively, and they are equivalent to charges of surface density $\mp Nx\epsilon$.

The electric intensity in the slab is made up of two parts: (1) that due to the external Faraday tubes and (2) that due to these induced surface charges. The medium surrounding the electrons is, of course, supposed to have a dielectric constant 1, for it is the deviations from this value due to the presence of material bodies that we are investigating. Since the electric polarisation of the external field is F, the part of the electric intensity (1) will be $4\pi F$ in a direction from left to right. A well-known electrostatic proposition states that the electric intensity in the space between parallel opposite surface charges of density σ is $4\pi\sigma$: consequently the part of the electric intensity (2) is $4\pi Nx\epsilon$ and is directed from right to left. The whole electric intensity in the slab is given by

$$X = 4\pi F - 4\pi Nx\epsilon = 4\pi F - 4\pi N\epsilon \cdot \frac{\epsilon X}{f} \quad \text{.........(2),}$$

hence

$$4\pi F = X\left(1 + \frac{4\pi N\epsilon^2}{f}\right) \quad \text{......................(3).}$$

But the dielectric constant of a body is defined by the statement that the electric intensity in that body due to a polarisation D is $\frac{4\pi D}{K}$. Consequently if K is the dielectric constant of the slab containing the electrons

$$K = 1 + \frac{4\pi N\epsilon^2}{f} \text{ ....(4).}$$

We have then interpreted the dielectric constant in terms of the number and properties of the electrons in the medium. Since we do not know anything yet about these electrons from other sources, it is impossible to test the result directly. But an indirect test is available. If the dielectric is a gas, in which the molecules are so far apart that they exert no appreciable action on one another except in collision, the only quantity which will vary with the density of the gas is N. N will be simply proportional to the density; for an increase in the density of the gas means a proportional increase of the number of atoms in unit volume. The quantities e and f do not depend on the density, and hence we may write $K = 1 + \alpha d$, where α is a constant given by

ϵ

$$\frac{K-1}{d} = \alpha \text{ ....(5).}$$

The experiments of Boltzmann have shown that this relation holds for gases with great accuracy.

Formula (5) cannot be applied to substances of which the density is so great that the molecules are sufficiently near to exert appreciable forces upon one another. For the restraining force fx on an electron is made up of two parts:

(1) the part due to the action of electrons in the same molecule: we will call this $f'x$.

(2) the part due to the action of the electrons in other molecules. If the simplest assumptions are made as to the distribution of the molecules and the electrons in them, it appears that this part amounts to $-\frac{4\pi}{3}Nex$. Since N is proportional to the density, part (2) depends on the density while (1) is independent of it.

Since $$fx = f'x - \frac{4\pi}{3} N\epsilon x \quad\text{......................(6)},$$

we obtain from (4) $$K = \frac{f' + \frac{8\pi}{3} N\epsilon^2}{f' - \frac{4\pi}{3} N\epsilon^2} \quad\text{......................(7)},$$

therefore $$\frac{K-1}{K+2} = \frac{4\pi N\epsilon^2}{3f'} = \beta d \quad\text{......................(8)},$$

where β is a constant, since N is proportional to d and ϵ, f' are constants.

Table III. shows how far this relation agrees with that found experimentally for the change of the dielectric constant with the change of density due to rise of temperature.

TABLE III.

	d	*K* (observed)	*K* (calculated)
Water			
	0·9971	75·7	
	0·9954	73·7	72·6
	0·9988	78·7	79·2
	0·9997	80·2	81·2
	1·0000	82·3	81·8
Benzol			
	0·883	2·336	
	0·841	2·243	2·246
Alcohol			
	0·8009	25·7	
	0·8129	27·5	29·6

It should be noted that both formulae (5) and (8) can be deduced equally well from other hypotheses concerning the physical nature of the dielectric constant; for instance, from that of Mosotti. As yet we have found no evidence which supports Lorentz' theory rather than any other: but, as soon as we turn our attention to the refractive index, the superiority of the electronic hypothesis is apparent.

Calculation of the refractive index.

5. When the light wave falls on an insulator and the periodic electric intensity in its wave front acts upon the electrons contained in it, these electrons will be set into motion. The characteristics of that motion will be determined by two factors—the restraining force fx acting on the electrons and the mass of the electrons. Hitherto nothing has been said or implied about the mass of the electrons: we will assume henceforward that the electrons possess mass, but we will leave open for the present the question whether they all possess the same mass. This assumption is the basis of our whole theory of the relation of light to material media.

The calculation of the refractive index on this assumption requires merely the application of well-known results in the dynamics of vibrations. These results will be stated without proof: any reader to whom they are not familiar will find an excellent elementary treatment of the subject in Poynting and Thomson's *Sound*.

It is known that, if a particle of mass m, subject to an attraction μx towards a centre of force which varies directly as its distance x from that centre, be disturbed from its position of equilibrium, it will execute a simple harmonic vibration of which the period p is given by

$$p = 2\pi\sqrt{\frac{m}{\mu}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9);$$

p is termed the 'natural' or 'free' period of vibration of the particle. But if the particle, in place of being merely disturbed, is acted upon by a periodic force with a period T it will execute 'forced vibrations,' not in its own period, but in the period of the force. It can be shown that if the periodic force P varies harmonically so that it can be represented by

$$P = P_0 \sin\frac{2\pi}{T}(t+\alpha),$$

the displacement of the particle in the forced vibration will be represented by[1]

$$x = \frac{P_0 \sin\frac{2\pi}{T}(t+\alpha)}{4\pi^2 m\left(\frac{1}{p^2} - \frac{1}{T^2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10).$$

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[1] **The expression given in Poynting and Thomson's *Sound*, Chap. IV., is incorrect in sign.**

Now the electrons in the molecule possess a mass m, and they are subject to a force fx varying directly as the distance from a centre: accordingly they have a free period of vibration p where

$$p = 2\pi\sqrt{\frac{m}{f}} \quad\text{.......................(11).}$$

When a light wave of period T falls on such electrons they will be subject to a periodic force ϵX, or

$$\epsilon X_0 \sin\frac{2\pi}{T}(t+\beta),$$

where X is the electric intensity at a point inside the medium containing the electrons. They will be set into forced vibrations of period T, and their displacement x at any time will be given by

$$x = \frac{\epsilon X_0 \sin\frac{2\pi}{T}(t+\beta)}{4\pi^2 m\left(\frac{1}{p^2}-\frac{1}{T^2}\right)} \quad\text{..................(12).}$$

In the last paragraph we saw that, if the electrons in the body were displaced a distance x when the medium is under the action of an external field of electric polarisation F, the electric intensity X in the interior of the slab is related to the external polarisation by the relation

$$X = 4\pi F - 4\pi N\epsilon x \quad\text{....................(13).}$$

Accordingly, if the polarisation D in the incident light wave is given by

$$D = D_0 \sin\frac{2\pi}{T}(t+\beta),$$

we must have

$$X_0 \sin\frac{2\pi}{T}(t+\beta) = 4\pi D_0 \sin\frac{2\pi}{T}(t+\beta) - \frac{4\pi N\epsilon^2 X_0}{4\pi^2 m\left(\frac{1}{p^2}-\frac{1}{T^2}\right)}\sin\frac{2\pi}{T}(t+\beta) \quad\text{......(14)}$$

and

$$4\pi D_0 = X_0\left\{1 + \frac{N\epsilon^2 p^2}{\pi m\left(1-\frac{p^2}{T^2}\right)}\right\}.$$

But the polarisation and the electric intensity in the wave front of the light wave must be connected by the equation

$$X_0 = \frac{4\pi D_0}{K}.$$

Thus $$K = 1 + \frac{N\epsilon^2 p^2}{\pi m\left(1 - \frac{p^2}{T^2}\right)} \quad \text{...............(15)}.$$

Since the wave is propagated with a velocity $\frac{1}{\sqrt{\mu K}}$, the refractive index n of the medium is given by $n = \sqrt{K}$,

or $$n^2 = 1 + \frac{N\epsilon^2 p^2}{\pi m\left(1 - \frac{p^2}{T^2}\right)} \quad \text{..................(16)}.$$

Physical interpretation.

6. Perhaps a little further examination is required to make the foregoing calculation quite convincing, for it might be doubted whether the quantity which has been introduced as the dielectric constant is identical with that which was used in calculating the velocity of propagation, and further, whether the value of μ may not be changed by the presence of the electrons in the material. If we turn back to p. 30 where the velocity of propagation was deduced, any such difficulties will vanish. The dielectric constant was introduced in estimating the tension of the Faraday tubes, which depends on the electric intensity in the field: if the arguments advanced be analysed, it will be seen that it has precisely the same physical significance as the quantity K which is calculated above; it is defined by the relation

$$\text{electric intensity} = \frac{4\pi}{K} \times \text{electric polarisation}.$$

On the other hand μ occurs in the expression for the mass of the tubes. We have already concluded (p. 23) that while the electrostatic properties of Faraday tubes—including their tension—may be influenced by the presence in their neighbourhood of other tubes belonging to independent systems, the magnetic properties of the tubes, which are determined by their motion, will be uninfluenced by the presence of other tubes which are not in motion relative to them. The induced charges in the dielectric will change the electric intensity, but it will be without influence on the magnetic intensity.

We see then that the difference in the value of the dielectric constant measured for steady fields and its value for the rapidly varying field in light waves is due to the influence of the free periods of vibration of the electrons. The direction of the electric intensity changes so rapidly that the electrons cannot follow its variations and fail to take up the same positions as they would occupy if a steady electric intensity of the same magnitude were applied. If we make the period of the incident light vibrations very long compared to the free period of the electrons, formula (16) becomes

$$K = 1 + \frac{N\epsilon^2 p^2}{\pi m} \quad \text{.......................(17)}$$

$$= 1 + \frac{4\pi N\epsilon^2}{f}, \quad \text{since } p = 2\pi\sqrt{\frac{m}{f}}:$$

as might be expected the expression for K for such long period vibrations is the same as that which was deduced previously for steady fields.

It is interesting to consider the mechanism of reflection and refraction of light at the inter-surface between two media in the light of our hypothesis. The vibrations of the tubes along which light is travelling set up vibrations in the electrons of the medium through which they pass. Disturbances are set up in the Faraday tubes attached to these electrons and new light vibrations will be propagated along them. That part of the disturbance which travels forward in the direction of the original light combines with that light giving rise to the refracted ray: while that portion which travels backwards emerges from the medium as the reflected ray.

Resonance. Selective absorption.

7. It will be observed that if T, the period of the incident light, is equal to p, that of the free vibration of the electrons, n^2 becomes infinite. The occurrence of an infinite quantity in a physical equation always means that some factor of importance has been neglected, and that we must investigate the matter further.

When any system whatever is acted upon by a force, the period of which coincides with the period of free vibration of the system, a forced vibration of very great amplitude is set up. 'Resonance'—as the occurrence of this large amplitude is called—is of especial importance in the theory of sound and the construction of musical instruments, but it is well illustrated by many experiences of common life: there is no better example than a swing. If a small impulse be given to the swing at the same instant in successive vibrations, so that the period of the impulse coincides with that of the swing, a large vibration can be set up in the swing by a very small impulse. But the amplitude of the resonant vibrations never becomes infinite, for the motion is opposed by a frictional force which increases with the velocity of the swing: the maximum amplitude, that can be attained is such that the energy communicated by the impulse in each vibration is equal to that lost in overcoming the frictional forces.

Now we have reasons for believing (Chap. VI.) that the motion of electrons is opposed by forces which resemble frictional forces, in so far that their magnitude increases with the speed of the electron. When the period of the incident light is far removed from that of the free vibrations of the electrons, the amplitude and the maximum speed of their motion is comparatively small and this frictional force is of little importance: but when the amplitude of vibration becomes large, owing to resonance, the effects of these forces must be taken into account. The amplitude of the vibration will not be fixed by the considerations which led to the deduction of (10), but will continue to increase until the energy communicated to the electron in each vibration by the electric intensity in the light is equal to the amount of energy spent in overcoming the frictional forces. The electrons behave as if the restraining force which we have supposed to act on them were removed: they move like the electrons in a conductor (Chap. VI.). They will follow the vibrations of the tubes and neutralise the electric intensity due to them as the electrons in a conductor neutralise the electric intensity in a steady field. But we saw (p. 44) that a body in which there can be no electric intensity does not transmit light incident upon it but reflects it; *the substance will act as an almost perfect reflector for light, the* *period of which is the same as that of the free vibration of its electrons.* The reflecting power will not be quite perfect, because the frictional forces on the electrons prevent them from following the vibrations of the tubes perfectly: a small fraction of the light will travel into the substance, but it will be absorbed rapidly and its energy used to maintain the vibration of the electrons against the frictional forces.

Instances of such 'selective absorption' for visible light are very common: every transparent coloured body shows such absorption. A solution of potassium permanganate is transparent for most visible light but shows strong absorption for green light. In a colourless body the region of absorption lies outside the visible spectrum: glass and water are transparent for all visible light but the former absorbs strongly light of very short wave length (ultra-violet light), the former absorbs strongly light of very long wave length (ultra-red). We conclude, therefore, that there are electrons in the permanganate which have free periods identical with that of green light, that glass contains electrons with free periods in the ultra-violet, that water contains electrons which have periods in the ultra-red. The study of the *selective absorption of a substance gives us information as to the period of free vibration of the electrons contained in it.*

Formula (16) for the refractive index must only be applied to light, for which the medium considered is transparent: it becomes inaccurate when applied to light which falls within an 'absorption band' of the medium. It would be possible, by making suitable assumptions about the nature of the frictional forces which impede the motion of the electrons, to deduce a corrected formula which would be correct for light of all wave lengths: but it would serve no useful purpose. We shall see that the formula which has been given already represents with qualitative accuracy all the phenomena of dispersion: to attain to quantitative accuracy we should require to know the exact relation between the speed of the electron and the frictional forces acting on it. Such information we do not possess: indeed it seems probable that the relation is so complex that if it were introduced into our equations they would no longer be capable of exact solution.

Sellmeyer's dispersion formula.

8. The hypothesis that the dispersion of a substance, or the variation of its refractive index with the period of the incident light, is due to the existence of natural periods of vibration in the substance is not peculiar to the electronic hypothesis of Lorentz: it had been introduced previously by Maxwell and developed by Sellmeyer, who showed that the connection of the refractive index with the wave length should be given by a formula of the form

$$n^2 = 1 + \frac{B\lambda^2}{\lambda^2 - \lambda_k^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18),$$

where B is a constant, λ the wave length of the incident light and λ_k the wave length (*in vacuo*) of light, the period of which is the same as that of the natural period of the substance.

An extension of this formula, associated with the names of Helmholtz and Ketteler, takes into account the possibility of more than one natural period:

$$n^2 = B_0 + \frac{B_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{B_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{B_3\lambda^2}{\lambda^2 - \lambda_3^2} \quad \ldots\ldots\ldots(19),$$

which we may write

$$n^2 = B_0 + \Sigma \frac{B_k\lambda^2}{\lambda^2 - \lambda_k^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20).$$

It would be no test of the accuracy of this formula to determine experimentally the values of n for various values of λ over a range for which the substance had no absorption band: for if we have enough undetermined constants λ_1, λ_2 etc. we can always find values for them which make any reasonable formula agree with the observed values within the limit of experimental error. The only way to test the formula is to determine first the values of λ_1, λ_2 etc., *i.e.* the wave lengths of light which has the same period as the free vibrations of the substance; the other constants B_k must then be determined from the observed variation of the refractive index with the wave length, but unless the values of the periods of free vibration have been chosen correctly, it will be impossible to fit the formula to the observations.

Such a test has been applied to the formula by Rubens working in conjunction with Aschkinass and Nichols. In order to determine the wave length of the light which has the same period as the free vibrations of the electrons, he made use of the metallic reflection described in § 7. If a beam of light containing rays of all wave lengths falls upon a medium such as we considered, which is transparent for most vibrations but exhibits selective absorption, all the rays will be transmitted with very slight reflection except those the period of which coincides with the free periods of the electrons: these rays will be reflected with little loss. If the beam is subjected to several such reflections at the surface of the medium, the resulting beam will consist almost entirely of vibrations for which the medium exhibits selective absorption. Measurements of the wave length of the vibrations in this reflected beam—'reststrahlen' as they were called by Rubens and Nichols—give directly the wave length in air of the vibrations which have the same period as the free vibrations of the electrons contained in the medium: that is, they will give us the quantities λ_k.

In this manner Rubens and Nichols showed that quartz (the 'ordinary ray') has selective absorption for vibrations of wave length $88{\cdot}5 \times 10^{-5}$ cm. and $207{\cdot}5 \times 10^{-5}$ cm., far away in the ultra-red.

Hence $\lambda_1 = 88{\cdot}5 \times 10^{-5}$ and $\lambda_2 = 207{\cdot}5 \times 10^{-5}$.

It is well known that quartz absorbs also ultra-violet light of very short wave length and that, consequently, there must be a quantity λ_k corresponding to this part of the spectrum. The exact position of this absorption band could not be measured by the same methods as were employed for the infra-red bands by reason of experimental difficulties: it was deduced from an application to measurements of the dispersion for short wave lengths of a simple Sellmeyer formula (18) suitable for a single free period. The natural periods in the ultra-red were so far removed from the period of this light that they had no influence on the dispersion in this region, and the observations were found to agree well with the presence of an absorption band at

$$1{\cdot}0309 \times 10^{-5} \text{ cm.}$$

Thus $$\lambda_3 = 1{\cdot}0309 \times 10^{-5}.$$

The constants B_k were calculated from the observed dispersion and the following values found:

$$B_0 = 1{\cdot}35428$$
$$B_1 = 1{\cdot}00254$$
$$B_2 = 0{\cdot}56549$$
$$B_3 = 1{\cdot}65646.$$

On comparing the value for the refractive index given by formula (19) with the use of these constants the results given in Table IV. were obtained. It will be seen that the agreement is remarkably good and justifies entirely the use of the Ketteler-Helmholtz formula.

TABLE IV.

$\lambda \times 10^5$ (cm.)	n (obs.)	n (calc.)	
1·98	1·65070	1·65077	
3·58	1·56400	1·56395	visible spectrum
5·34	1·54663	1·5466	
16·17	1·5271	1·5271	
26·0	1·5099	1·5111	
36·7	1·4790	1·4861	
64·5	1·274	1·234	
560·0	2·18	2·14	

Comparison of Theory with Experiment.

9. At the same time it confirms the accuracy of the formula (16) which we deduced from our electronic hypothesis. In deducing it we only considered the possibility of the presence of one kind of electron: if there are several kinds having different masses, different values of f and therefore different free periods p, it is easily seen that

$$n^2 = 1 + \frac{N\epsilon^2 p_1^2}{\pi m_1\left(1 - \frac{p_1^2}{T^2}\right)} + \frac{N\epsilon^2 p_2^2}{\pi m_2\left(1 - \frac{p_2^2}{T^2}\right)} + \dots,$$

or

$$n^2 = 1 + \Sigma \frac{N\epsilon^2 p_k^2}{\pi m_k\left(1 - \frac{p_k^2}{T^2}\right)} \qquad \dots\dots\dots\dots\dots(21).$$

Now $p_k = 2\pi\sqrt{\frac{m_k}{f_k}}$, and if λ_k is the wave length *in vacuo* of light with the period p_k

$$\lambda_k = Vp_k \qquad \text{.........................(22)},$$

also

$$\lambda = VT \qquad \text{.........................(23)}.$$

Hence

$$n^2 = 1 + \Sigma \frac{\frac{N\epsilon^2 p_k^2}{\pi m_k} \cdot \lambda^2}{\lambda^2 - \lambda_k^2} \qquad \text{.................(24)},$$

or

$$n^2 = 1 + \Sigma \frac{N\epsilon^2 p_k^2}{\pi m_k} + \Sigma \frac{\frac{N\epsilon^2 p_k^2}{\pi m_k} \cdot \lambda_k^2}{\lambda^2 - \lambda_k^2} \qquad \text{...........(25)}.$$

Now by a simple transformation we can write the Ketteler-Helmholtz formula (19)

$$n^2 = b^2 + \Sigma \frac{M_k}{\lambda^2 - \lambda_k^2} \qquad \text{....................(26)},$$

where

$$b^2 = B_0 + B_1 + B_2 \ldots\ldots$$

and

$$M_k = B_k \cdot \lambda_k^2.$$

Comparing (25) and (26) we see that they are identical if we put

$$1 + \Sigma \frac{N\epsilon^2 p_k^2}{\pi m_k} = b^2 \qquad \text{.....................(27)},$$

and

$$\frac{N\epsilon^2 p_k^2}{\pi m_k} \cdot \lambda_k^2 = M_k \qquad \text{.................(28)}.$$

From Rubens' measurements we can obtain the values of b^2 and M_k which are expressed by (27) and (28) in terms of the properties of the electrons. We find for quartz

$$b^2 = 4{\cdot}58 \quad M_1 = {\cdot}0106 \quad M_2 = 44{\cdot}224 \quad M_3 = 713{\cdot}55.$$

It will be observed that if we put $\lambda = \infty$ in (26) we obtain $n^2 = b^2$, hence b^2 is the refractive index for infinitely long waves, that is those of which the period is infinite. Hence we must have $b^2 = K$, where K is the dielectric constant for steady fields. The same conclusion arises directly out of (27), for putting

$$p_k = 2\pi\sqrt{\frac{m_k}{f_k}},$$

we have

$$b^2 = 1 + \Sigma \frac{4\pi N\epsilon^2}{f_k}.$$

This is the value that would have been obtained for K in (4) if more than one kind of electron had been taken into account.

The value of b^2 is not known for many substances, but in such cases as it has been ascertained, it agrees well with K. Thus we find

	b^2	K
Flint-glass	6·77	6·7 – 9·1
Fluor spar	6·09	6·7 – 6·9
Quartz	4·58	4·55 – 4·73
Rock salt	5·18	5·81 – 6·29
Sylvine	4·55	4·94

Again we see from (27) and (28) that

$$b^2 - 1 - \Sigma \frac{M_k}{\lambda_k^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(29).$$

Testing this relation for quartz we find $b^2 - 1 = 3{\cdot}58$, $\Sigma \frac{M_k}{\lambda_k^2} = 3{\cdot}22$. Hence the expression on the left of (29) becomes $+0{\cdot}36$. It is found that for all substances this quantity has a small positive value—a result the meaning and importance of which will be discussed in a later chapter (Part IV.).

Another dispersion formula which has been used for expressing empirically the relation between the refractive index and the period T of the incident light is the following:—

$$n^2 = -A'T^2 + A + \frac{B}{T^2} + \frac{C}{T^4} \ldots\ldots\ldots\ldots\ldots(30).$$

Let us put (21) in a form suitable for comparison with this formula.

The electrons in the refracting medium may be divided into two classes, those which have free periods longer, and those which have free periods shorter than that of the incident light: the former will be denoted by the suffix r, the latter by the suffix v. We shall suppose that the period of the refracted light is so far removed from any free period in the medium that all the quantities $\frac{p_v}{T}$ and $\frac{T}{p_r}$ may be considered small compared to 1. Then we have from (21)

$$n^2 = 1 + \Sigma \frac{4\pi N_v \epsilon^2}{f_v\left(1 - \frac{p_v^2}{T^2}\right)} - \Sigma \frac{4\pi N_r \epsilon^2}{f_r\left(1 - \frac{T^2}{p_r^2}\right)} \cdot \frac{T^2}{p_2^2} \quad \ldots\ldots(31).$$

p_r^2

Expanding in powers of $\frac{p_v}{T}$ and $\frac{T}{p_r}$ and neglecting powers higher than the third we have

$$n^2 = 1 + \Sigma \frac{4\pi N_v \epsilon^2}{f_v} + \Sigma \frac{4\pi N_v \epsilon^2}{f_v} \cdot \frac{p_v^2}{T^2} - \Sigma \frac{4\pi N_r \epsilon^2}{f_r} \cdot \frac{T^2}{p_r^2} \text{...(32).}$$

Comparing (30) and (32)

$$A' = \Sigma \frac{4\pi N_r \epsilon^2}{f_r} \cdot \frac{1}{p_r^2}, \qquad A = 1 + \Sigma \frac{4\pi N_v \epsilon^2}{f_v}.$$

Now by (4) the dielectric constant K is given by

$$K = 1 + \Sigma \frac{4\pi N_v \epsilon^2}{f_v} + \Sigma \frac{4\pi N_r \epsilon^2}{f_r}.$$

Hence $$K - A = \Sigma \frac{4\pi N_r \epsilon^2}{f_r} \text{ ....(33),}$$

and $$p_r^2 = \frac{K - A}{A'} \text{ ....(34).}$$

A is the part of n^2 which is independent of the period of the light: and since, for a substance which shows 'normal dispersion' (see next paragraph), the variation of n with T over the range of the visible spectrum is not a large fraction of the values of n within this range, A will be the most important term of the sum (30). Hence A will be nearly equal to the values of the refractive index given in Chap. II. Table I., and we see from (33) that the *discrepancies there noted between the values of* K *and* n^2 *are due to the presence in the medium of electrons having free periods in the ultra-red part of the spectrum.* The conclusion might have been anticipated on general grounds, for since these periods are long compared with that of the visible light vibrations, but are short compared with that of the electrostatic fields used in the measurement of K, they will influence the values of n^2 and K in opposite directions: while free periods in the ultra-violet, which are short compared with the periods both of visible light and of the electrostatic fields, influence both quantities in the same direction.

Further, we can use (33) and (34) to determine from the discrepancy between the values of K and n^2 the free period in the ultra-red. For water, which shows the greatest discrepancy of any substance measured,

$$K = 76 \quad A = 1{\cdot}77 \quad A' = 0{\cdot}1152 \times 10^{28};$$

$p_r^2 = \frac{\lambda_r^2}{V^2}$, where λ_r is the wave length *in vacuo* of light having the same period as the ultra-red absorption band.

Hence
$$\lambda_r^2 = 9 \times 10^{20} \times \frac{74{\cdot}23}{0{\cdot}1152} \times 10^{-28}$$
$$= 58 \times 10^{-6} \text{ cms.},$$
and
$$\lambda_r = 7{\cdot}61 \times 10^{-3} \text{ cms.}$$

[The wave length of yellow light is about 5×10^{-5} cms.]

As a matter of fact water has been shown to possess several absorption bands in the ultra-red, so that the value which has just been calculated cannot be expected to be more than an average, agreeing in order of magnitude with the wave lengths determined by experiment. Paschen has found that the wave length corresponding to the most marked absorption band is 4×10^{-3}, which is not very far from the value found above.

Normal and Anomalous Dispersion.

10. The general nature of the relation between the refractive index and the wave length of the incident light denoted by formula 21 is worthy of a brief notice. It will be observed that n increases as T decreases, provided that T does not become equal to one of the quantities p_k. As the value of T decreases past one of these values, the quantity $\left(1 - \frac{p_k^2}{T^2}\right)^{-1}$ changes from a very large positive value to a very large negative value and the refractive index undergoes a large decrease. Of course, we have noted that, when T is very near to p_k, the formula ceases to hold accurately, but it still represents roughly the nature of the variation of n: that quantity will not change from an infinite positive value to an infinite negative value, but we may be certain that it will change from a large to a small value. The variation of the refractive index with the wave length of the incident light must resemble that represented in fig. 10, where the abscissae ON_1, ON_2 represent wave lengths corresponding to the free periods of the electrons.

Such variations are actually observed. For colourless substances, that is those which have no absorption band within the visible spectrum, the refractive index increases with decreasing wave length throughout that range. This so-called 'normal dispersion' is characteristic of a substance over a range in which it has no absorption band. Substances, such as fluorescein, which have an absorption band in the visible spectrum exhibit 'anomalous dispersion': the refractive index increases regularly with decreasing wave length on either side of the band, but undergoes a large decrease as the wave length of the incident light passes through the band. Thus if a spectrum is formed on a screen by means of a prism of fluorescein, which has an absorption band in the green, the colours from the red to the green will be arranged in their normal order. But the rays which are of a wave length just shorter than those corresponding to the absorption band will be refracted less than the red rays, and will appear on the side of the red remote from the rest of the green rays. As the wave length is decreased still further, the refractive index increases rapidly: some of the blue-green rays will overlap the red rays and the violet rays will appear in their normal position on the more refrangible side of the red.

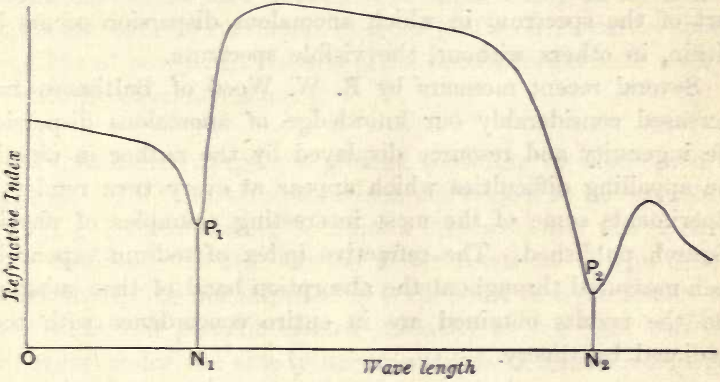

Fig. 10.

It is to be noted that normal and anomalous dispersion are not entirely different phenomena each of which is characteristic of certain bodies: they are different aspects of the same phenomenon.

All substances show 'normal' dispersion in some parts of the spectrum and 'anomalous' dispersion in others: the difference between various substances, which has given rise to the use of the distinguishing terms, lies in the fact that for some substances the part of the spectrum in which anomalous dispersion occurs lies within, in others without, the visible spectrum.

Several recent memoirs by R. W. Wood of Baltimore have increased considerably our knowledge of anomalous dispersion: the ingenuity and resource displayed by the author in meeting the appalling difficulties which appear at every turn render his experiments some of the most interesting examples of physical research published. The refractive index of sodium vapour has been measured throughout the absorption band of that substance and the results obtained are in entire concordance with those predicted by theory.

The Optical Properties of Crystals.

11. In the present state of our knowledge a detailed discussion of the optical properties of crystals cannot lead to any important results: only the barest outline of an explanation of those properties can be given.

In a doubly-refracting crystal light of the same wave length travelling in the same direction through the crystal is propagated with a different velocity according as it is polarised in or perpendicular to a certain plane which is determined by the form of the crystal: that is, according as the electric intensity in the wave front is perpendicular to or in that plane. Now the motion of the electrons in the refracting medium coincides in direction with the electric intensity in the light wave, and the velocity of the light in the medium is determined by the free periods of vibration of the electrons. Accordingly we must conclude that in a doubly refracting medium the electrons have different periods of free vibration for different directions of motion relative to the crystallographic axes of the crystal. But of what determines this difference in period we have at present no certain knowledge: it must be related in some way to the structure of the molecule in which the electrons are contained, but our information concerning that structure is too fragmentary for any speculations on the subject to be useful.

The rotation of the plane of polarisation which occurs when light is propagated along the axis of quartz and other crystals is still more mysterious. Drude has attempted to calculate the relation that must hold between the forces acting on an electron and the direction of its motion in order that such a rotation should occur, but at present the results of his calculations remain without any definite physical meaning.

Dispersion in Metals.

12. In metals there are present, according to the hypothesis of p. 50, electrons which are subject to no restraining force tending to resist a displacement: the electrons when disturbed will not vibrate about positions of equilibrium. If their motion be not impeded by a frictional resistance, they will move under the periodic electric intensity in the light wave, as they move under the steady electrostatic field considered in § 4: the electric intensity in the metal is neutralised completely, the light cannot travel forward through the conductor but is reflected at its surface. But, since the metals of our experience do not behave as perfect reflectors, we must suppose that the motion of the electrons is impeded by a frictional resistance: the neutralisation of the electric intensity is not quite complete: a small fraction of the incident light travels into the medium, but its energy is absorbed speedily in maintaining the vibrations of the electrons.

The frictional resistance increases with the velocity of the electrons and, therefore, with the rapidity of their vibrations. We find accordingly that metals reflect light of long wave length better than light of short wave length. Silver reflects very nearly all of the long heat waves investigated by Rubens and Nichols, absorbing only about ½ %, but it absorbs about 25 % of the shortest ultra-violet rays on which measurements can be made conveniently.

But in some metals the reflecting power does not simply decrease with the wave length of the incident light: there is a distinct selective absorption: such metals are those which show a surface colour—copper and gold for example. A leaf of gold so thin as to be transparent appears green by transmitted light—the colour complementary to its red surface colour: the substance has a strong absorption band in the red. In transparent bodies we attributed the presence of these absorption bands to the action of electrons having definite free periods, and there is no reason why the explanation should not be extended to metals. These 'bound' electrons, as they may be called in distinction to the 'free' electrons which are subject to no restraining force, take no part in electrostatic actions, or, as we shall see in the next chapter, in the conduction of currents: the number of free electrons is always sufficient to neutralise the intensity of an external field within the metal, and there is no residual force acting on the bound electrons.

It is only in optical phenomena that the action of the bound electrons appear, and it must be taken into account in calculating the optical constants of the metal in terms of the properties of its electrons. This calculation has been performed by Drude, but we shall not attempt to reproduce it here: for without further knowledge concerning the nature of the frictional forces which impede the motion of the free electrons, such as we shall attempt to obtain in the next chapter, the necessary hypotheses must appear somewhat arbitrary. Drude assumes that an electron travelling with a velocity v is subject to a retarding force rv, and he finds the following relation between the refractive index n of the metal, its absorption coefficient α and the period T of the incident light:

$$n^2(1-\alpha^2) = 1 + \Sigma \frac{N_k \epsilon^2 p_k^2}{\pi m\left(1 - \frac{p_k^2}{T^2}\right)} - 4\pi\Sigma \frac{N_k' \frac{\epsilon^2}{m_k'}}{\left(\frac{r_k'}{\epsilon^2}\right)^2 + \left(\frac{2\pi\epsilon^2 m_k'}{T}\right)^2} \quad (35),$$

$$n^2\alpha = T\Sigma \frac{\frac{r_k'}{\epsilon^2} N_k'}{\left(\frac{r_k'}{\epsilon^2}\right)^2 + \left(\frac{2\pi\epsilon^2 m_k'}{T}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots(36),$$

where the suffix k denotes the various groups of bound electrons and the suffix k' the groups of free electrons.

Measurements on the optical properties of metals are so difficult that the necessary facts are not available for subjecting this formula to tests of the same nature as those applied in § 9: such comparison of experiment with theory as is possible will be postponed until we have investigated further the nature of the conductivity of metals and other conductors of electricity.

13. We are not yet in a position to consider fully the mechanism of the emission of light, but our investigations have led to one conclusion so obvious that it is desirable to notice it.

The emission of light.

The light emitted by a body constitutes, in general, either a continuous spectrum, containing rays of all wave lengths, or a bright line spectrum, consisting of rays of a few definite wave lengths. The former is emitted by the body under the action of heat, the latter is never excited by heat alone. It is well known that in many cases the light for which a body shows selective absorption is identical with that for which it shows selective emission, *i.e.* with the light that is found in the bright line spectrum emitted by it. (This generalisation is not, as is sometimes thought, universal and deducible from Kirchhoff's law, for that principle applies only to temperature radiation emitted under the action of heat.) Now we have found that the selective absorption of a body is conditioned by the free periods of its electrons, and that the light which it absorbs has a period identical with that of its free vibrations. It is unnecessary therefore to insist further on the obvious conclusion that the bright line spectrum of a body consists of light emitted by the free vibrations of its electrons. The manner in which those electrons are excited into vibration must be considered later, but it will be assumed henceforward as indubitable that the spectrum of a body represents the radiation set up by the vibrations of the electrons contained in it along lines at right angles to the direction of propagation of the light.

REFERENCES FOR CHAPTER III.

The original paper of Lorentz containing the fundamental assumption of the electronic theory is to be found in the *Proceedings of the Amsterdam Academy*, 1878 (*A.* and *L.* vol. I. pp. 430—435).

Measurements of the dielectric constant from which Table III. is taken were made by LEBEDEV, *Wied. Ann.* 44, pp. 309, 1891.

The chief original memoirs developing the theory of the dispersion of light according to this theory are those of

LORENTZ, *Arch. Néerland.* 1892, p. 363 (*A.* and *L.* pp. 435—450).

LARMOR, *Phil. Trans.* 1897, pp. 232—236.

See also DRUDE, *Physik des Aethers*, Chap. X. §§ 9, 10, 11.

The treatment of the subject given above is a paraphrase of that in DRUDE'S *Theory of Optics*, which is reproduced in WOOD'S *Physical Optics.* To these text-books the reader may refer for a more analytical and a stricter deduction of the formulae : the chapter on the Theory of Dispersion in the latter is admirable throughout.

For § 8 see RUBENS and NICHOLS, *Wied. Ann.* 60, p. 418, 1897.

§ 10, see R. W. WOOD on the Anomalous Dispersion of Sodium Vapour, *Phil. Mag.* Sept. 1904.

§ 13, see WOOD, *Physical Optics*, Chap. XIX.

CHAPTER IV.

MAGNETISM.

Para-magnetism and Dia-magnetism.

1. WITH respect to their magnetic properties bodies may be divided into two classes: paramagnetic and diamagnetic. When a paramagnetic body is placed in a magnetic field, magnetism is induced in it such that it appears to be a magnet with its positive pole facing the negative pole of the inducing magnet. When a diamagnetic body is placed in a magnetic field the induced magnetism is such that the positive pole faces the positive pole of the inducing magnet. Accordingly in a paramagnetic body the magnetic induction is greater than the magnetic intensity, the permeability (μ)—or ratio of the induction to the intensity—is greater than unity, the susceptibility (k)—or ratio of the induced magnetisation to the intensity of the inducing field—is positive. In a diamagnetic body the induction is less than the intensity, the permeability is less than unity and the susceptibility is negative.

In addition to this qualitative difference there is also an important quantitative difference in the magnetic properties of material bodies. The susceptibility of the vast majority of substances which have been investigated is so small that it requires the use of powerful magnetic fields and sensitive detecting instruments to perceive that they are not absolutely non-magnetic: but the three paramagnetic elements, iron, cobalt and nickel, together with most of their salts, exert a magnetic action which can be detected by the roughest experiments[1]. The susceptibility of the most diamagnetic element (Bismuth) is about -14×10^{-6}: that of iron is about 280, of cobalt 14 and of nickel 13. Moreover

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[1] To these must now be added certain alloys, of which that known as 'Heusler's alloy' composed of manganese, aluminium and copper is the most prominent.

these three elements exhibit the phenomenon of magnetic hysteresis which has not been observed in the less strongly magnetic substances: their magnetic properties are so different from those of other bodies that a special term is usually employed to designate them: the 'magnetic metals' are said to show 'ferromagnetism.' The only substance possessing a susceptibility of the same order as the magnetic metals and their salts is liquid oxygen, which is also paramagnetic.

Weber's theory.

2. In the mathematical theory of magnetism the permeability plays exactly the same part as that played by the dielectric constant in the theory of electrostatics. It was shown in Chap. III. that the properties of a body with a dielectric constant greater than unity could be explained by the presence in the body of opposite electric charges which were separated under the influence of an electric field. A theory due to Weber gives an analogous explanation of the properties of a paramagnetic body. Weber supposed that the molecules of a paramagnetic body were little magnets, which, in the absence of an external magnetic field, lay with their axes equally distributed in all directions, so that the body as a whole showed no resultant magnetisation; under the influence of an external magnetic field these small magnets tend to place themselves so that their poles face the poles of opposite kind belonging to the external field.

It must also be supposed that there is a force of restitution opposing this orientation of the magnets, so that the resultant magnetisation varies with the intensity of the field. A small intensity will only orientate them partially, but a sufficiently strong field will bring about complete orientation.

The theories of Ampère and Maxwell.

3. A development of Weber's theory, which is of great importance for our present purpose, was suggested by Ampère. It is well known that a circuit in which an electric current is flowing behaves in a magnetic field like a magnet. Ampère suggested that the molecules of paramagnetic substances owe their magnetic properties to currents flowing round perfectly conducting circuits in the molecule. Since the circuits have no resistance, there will be no expenditure of energy due to the passage of the current, which, when once started, will flow indefinitely without diminution. This theory was generally accepted, so far as any theory can be said to be accepted, when there is no evidence either for or against it, and formed the basis of Maxwell's explanation of diamagnetism. Maxwell pointed out that if the molecules contained perfectly conducting circuits, induced currents would be generated in those circuits during the excitation of a magnetic field: and that, since the circuits had no resistance, these currents would flow undiminished until they were stopped by the production of equal and opposite currents by the annihilation of the magnetic field. Now, by Lenz' law, the lines of induction due to the induced currents will be in the opposite direction to those of the inducing field: hence, when a body containing these perfectly conducting molecules is placed in a magnetic field, lines of magnetic induction will be produced opposite in direction to those of the field: that is to say, the body will appear diamagnetic[1]. According to Maxwell, a body was paramagnetic or diamagnetic according as the effect due to the currents induced in the molecular circuits was less or greater than that due to the original currents in the circuits.

The Electronic Theory.

4. Maxwell's views only need to be translated into terms of the conceptions of the electronic theory to provide a thoroughly satisfactory theory of paramagnetism and diamagnetism.

It was shown on p. 19 that the magnetic effects of a current element of length ds and of strength i flowing in a conducting circuit are identical with those produced by the motion of a charged particle in an orbit coincident with the current circuit, and that if e is the charge on the particle and v its velocity $ev = ids$. We have supposed already that the atoms contain charged particles, the electrons: we have only to suppose further that these electrons are moving in closed orbits within the atoms in order to obtain the atomic circuits postulated by Weber. Moreover these revolving electrons will provide the resistanceless circuits in which the induced currents of Maxwell's theory of diamagnetism may travel. For, if the atoms be placed in a magnetic field, new

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[1] See note at the end of this chapter.

forces due to that field will act upon the moving electrons: a change in the orbit of the electrons will be produced thereby, and we shall see presently that this change will be of such a nature as to produce a magnetic intensity in the same direction as that of the external magnetic field. The induced current of Maxwell is the change in the electronic orbit.

Paramagnetism, then, represents the tendency of the electronic orbits to place themselves in the magnetic field so that the lines of magnetic intensity due to them are opposed in direction to that of the external field: the complete orientation is prevented by elastic forces due to the presence of neighbouring electrons. Diamagnetism represents the change in the nature of the orbits themselves due to the action of the magnetic field on the moving electron. A body will appear paramagnetic or diamagnetic according as the former or the latter change is the greater.

It should be noted that there is a finite limit to the intensity of paramagnetic magnetisation which can be induced in a body, for when all the electronic orbits have been turned round so that the lines of induction through them coincide in direction with those of the external field, a further increase in the intensity of that field can produce no further change in the position of the orbits. Such a finite limit has been observed in the magnetic metals, but not in other bodies: when the intensity of magnetisation is such that an increase in the magnetising field produces no increase in the induced magnetisation, the substance is said to be 'saturated.' On the other hand there is no reason to believe that a diamagnetic body could be saturated: an increase in the external field must always produce a further disturbance in the motion of the electron, and a limit could only be reached if the disturbance were so great as to eject the electron from the atom. Even if nickel, the least magnetic of the magnetic metals, were as diamagnetic as bismuth, the most diamagnetic substance known, a field strength of 9×10^5 C.G.S. electro-magnetic units would be required to excite an induced diamagnetic magnetisation as great as the saturated paramagnetic magnetisation; such intense fields cannot be procured experimentally.

The distinction between the paramagnetic and the ferromagnetic substances cannot be brought within the range of the simple form of the electronic theory with which we have been concerned hitherto: it doubtless depends on the internal constitution of the atom. But it seems probable that in paramagnetic substances the orientation of the orbits takes place within the atom, whereas in ferromagnetic substances the atom moves as a whole under the action of the field. But further speculation in this direction is of no service at present.

The theory of magnetism which has just been sketched has been worked out in some detail by Langevin. Only a brief summary of his results will be in place in this volume.

Calculation of the Diamagnetic Susceptibility.

5. On this theory we can calculate the susceptibility of a diamagnetic body in terms of the properties of the electronic orbits contained in it.

Consider an electron of mass m and charge ϵ revolving with velocity v in an orbit of radius r, the plane of which is perpendicular to a magnetic field of intensity H. For the sake of explicitness we will suppose that the relation between the directions of the field and of the velocity is that shown in Fig. 11: such a direction of revolution will be termed clockwise. It should be observed that, if the revolution is clockwise and the charge on the electron positive, the magnetic intensity due to the electronic orbit at points within the orbit is in the same direction as that of the field.

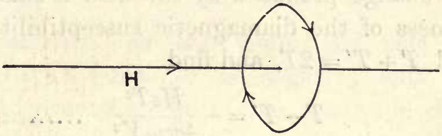

Fig. 11.

In the absence of the magnetic field the centrifugal force on the electron is balanced by elastic forces, which we will suppose to be directed to the centre of the orbit and to be proportional to the radius of the orbit. Then

$$\frac{mv^2}{r} = f \, . \, r \text{ ....(1).}$$

When the magnetic field is acting, the electron is subject to a force at right angles to the field and to the direction of its motion, *i.e.* along the radius of its orbit, of a magnitude $\mu H\epsilon v$. We assume, of course, that the medium around the electron has the same magnetic properties as a vacuum, and therefore that, on the electrostatic system of units, $\mu = \frac{1}{V^2}$. Hence the force on the electron is $\frac{H\epsilon v}{V^2}$. An application of the Amperean rule shows that, if the electron is positively charged, the force is directed outwards along the radius: if the electron is negatively charged, inwards along the radius[1].

Since the forces on the electron are perpendicular to the direction of its motion, the magnitude of its velocity v is unchanged by the action of the field. Hence, denoting the period and radius of the new orbit by T' and r' respectively, we have in place of (1)

$$\frac{mv^2}{r'} = fr' - \frac{H\epsilon v}{V^2} \quad\text{.....................(1 }a\text{)},$$

or from (1)

$$\frac{mv^2}{r'^2} - \frac{mv^2}{r^2} = -\frac{H\epsilon v}{r'V^2} \quad\text{.....................(2)}.$$

But

$$T = \frac{2\pi r}{v}, \quad T' = \frac{2\pi r'}{v} \quad\text{.....................(3)}.$$

Hence

$$\frac{1}{T'^2} - \frac{1}{T^2} = -\frac{H\epsilon}{2\pi m T' V^2} \quad\text{................(4)}.$$

Since the change produced by the field is small, as evidenced by the smallness of the diamagnetic susceptibility, we may put $TT' = T^2$, and $T + T' = 2T$, and find

$$T - T' = -\frac{H\epsilon T^2}{4\pi m V^2} \quad\text{.....................(5)},$$

or

$$\frac{1}{T'} - \frac{1}{T} = -\frac{H\epsilon}{4\pi m V^2} \quad\text{.....................(6)}.$$

In order to calculate the intensity of the induced magnetisation, let us replace the revolving electron by an equivalent current

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[1] If an electron carrying a negative charge moves from West to East in a magnetic field from North to South, the force tends to deflect it upwards.

The following rule may be useful for remembering this important relation:

If the eLectric intensity (*i.e.* the direction in which a positive charge moves) is towards the Left, the magnetic intensity H (*i.e.* the direction in which a north seeking pole moves) towards the Head, the mechanical Force is towards the Front.

flowing in a circuit coincident with its orbit. By Chapter I. (5) the strength (i) of the equivalent current is given by

$$i = \frac{\epsilon}{T} \quad \text{.............................(7).}$$

Now the magnetic moment of a circuit of area A carrying a current i is $\mu i A$: hence the magnetic moment of the undisturbed electronic orbit is $\frac{\epsilon \pi r^2}{T V^2}$ and of the disturbed orbit $\frac{\epsilon \pi r'^2}{T' V^2}$, since $\mu = \frac{1}{V^2}$. The change of the magnetic moment per unit volume of the material is, by definition, the induced magnetisation M. Hence if there are N electronic orbits in unit volume,

$$M = N \frac{\epsilon \pi}{V^2} \left(\frac{r^2}{T} - \frac{r'^2}{T'} \right)$$

$$= N \frac{\epsilon \pi r^2}{V^2} \left(\frac{1}{T} - \frac{T'}{T^2} \right) \text{ by (3),}$$

or by (5)
$$M = - \frac{N H \epsilon^2 r^2}{4 m V^4} \quad \text{.......................(8).}$$

The susceptibility k is defined by the relation

$$M = kH \quad \text{.................................(9),}$$

hence
$$k = - \frac{N \epsilon^2 r^2}{4 m V^4} \quad \text{...........................(10).}$$

All the quantities on the right-hand side of (10) are essentially positive: hence k is negative and the body is diamagnetic whatever the sign of the electronic charge ϵ.

In the calculation it has been assumed that all the electronic orbits are so arranged that their axes are in the direction of the magnetic intensity in the inducing field and their planes perpendicular to that direction. It would probably be more accurate to assume that the axes are distributed in all directions: but the change introduced by that assumption would only consist in multiplying the right-hand side of (10) by a proper fraction not very different from 1. Since, as we shall see when we come to evaluate the formula later, our numerical data are somewhat vague, the alteration would be of little importance.

Paramagnetic Susceptibility.

6. No useful calculation of the susceptibility of paramagnetic bodies can be made in the absence of information concerning the nature of the forces which tend to prevent the orientation of the orbits under the action of the external field: but we can calculate the saturated intensity of magnetisation when all the orbits are orientated completely. Using the notation of the last paragraph, the magnetic moment of each electronic orbit is $\frac{\epsilon \pi r^2}{TV^2}$: since there are N orbits in unit volume the greatest intensity of induced magnetisation which can be attained is

$$M = \frac{N \epsilon \pi r^2}{TV^2} \quad \text{..........................(11).}$$

Later we shall attempt to evaluate this formula and to compare it with experiment.

It should be noted that the observed diamagnetic or paramagnetic effect of a body is the average of all the effects of a very large number of atoms. There is no reason to believe that the actual magnetic field inside the atoms of the body is uniform: its intensity and even its direction may vary from place to place, and it is probable that the intensity acting upon any one of the many electronic orbits is very different from that calculated from observations on large portions of matter. This possibility should be borne in mind in view of some discrepancies which will be noted in future chapters.

We will now proceed to consider the effect of magnetism on the optical properties of matter—a subject which we shall find productive of most interesting and important evidence on fundamental questions connected with our theory.

Note on magnetic induction and magnetic intensity.

In the mathematical theory of electricity and magnetism the relation between electric polarisation and electric intensity is so similar to that between magnetic induction and magnetic intensity that an important physical difference is often overlooked.

Consider a bar carrying a positive charge of electricity or magnetism on the left end and a negative charge on the right.

The lines of electric polarisation run both in the bar and outside it from the positive to the negative electric charge. But the lines of magnetic induction are always taken to be closed curves and, by convention, they are supposed to run from the positive to the negative outside the bar, so that they run from negative to positive inside it. *Inside the magnetic material the lines of induction run in the opposite direction to the lines of magnetic intensity*, which are always drawn in the direction in which a positive charge would move, *i.e.* from positive to negative.

In considering the magnetic induction in an iron bar placed in a magnetic field this difference between the electric intensity and magnetic intensity is of no importance: for the magnetic intensity actually inside the bar cannot be observed: an opening of some kind must be made to insert a measuring instrument. But it becomes important when we consider an electric circuit. A plane circuit is equivalent in its magnetic effect to an infinitesimally thin magnet: the space which corresponds to the region inside the magnetised bar has no volume; it is enclosed between two imaginary planes both of which are coincident with the plane of the circuit. However we must take account of it in considering the direction of the magnetic induction.

Suppose that the direction of the current is such that a positive pole tends to move from left to right through the circuit and back again from right to left outside it: then inside the closed circuit the magnetic intensity is from left to right. But in this region the convention as to magnetic induction determines that the direction of the magnetic induction is in the opposite direction to the magnetic intensity: hence the magnetic induction is from right to left.

If such a circuit, in which no current is flowing, is brought between the poles of a magnet, the positive pole being on the left, Lenz' law states that the direction of the magnetic induction due to the current induced in the circuit must be opposite in direction to that of the inducing field. The latter is from left to right, hence the former is from right to left, and, in the plane of the circuit, the magnetic intensity due to the induced current is from left to right, in the same direction as that of the external field. The circuit will become equivalent to a thin magnet with its positive pole on the left opposite to the positive pole of the external magnet: *i.e.* the circuit will behave like a diamagnetic substance.

It has been thought useful to include this note, though there is nothing in it that is not contained implicitly in all text-books, because students sometimes find it difficult to see how Maxwell's theory of diamagnetism can be correct when the magnetic intensity inside the diamagnetic substance is in the *same* direction as that of the inducing field.

REFERENCES FOR CHAPTER IV.

For WEBER'S and MAXWELL'S view of induced magnetisation see MAXWELL'S Treatise, Vol. II. Chap. XXII.

The electronic theory of magnetism was put forward in the first instance by LARMOR, *Phil. Trans.* 1897, pp. 286–289, and developed in greater detail by LANGEVIN, *Ann. Chim. et Phys.* May 1905.

CHAPTER V.

THE ACTION OF MAGNETISM ON LIGHT[1].

1. **Faraday** discovered that if a plane polarised ray of light passes through certain bodies along the lines of a strong magnetic field, the plane of polarisation of the emergent light is different from that of the incident light. The direction of the rotation of the plane of polarisation depends, in any given substance, only on the direction of the field and is independent of the direction in which the light travels along the field, so that if plane polarised light is reflected back along its course after it has traversed the field, the rotation of the plane of polarisation is doubled. This observation distinguishes at once the magnetic rotation from the rotation of the plane of polarisation observed when plane polarised light travels through certain 'naturally rotating' substances, such as quartz or a solution of cane-sugar: for, when the direction of the propagation of the light in such substances is reversed, the direction of the rotation is also reversed: if the light retraces its course, the plane of polarisation is restored to its original position.

The Faraday effect.

The discovery of this phenomenon was the result of a long search by Faraday for some influence of magnetisation upon light, but, when the influence had been found, Faraday's theory was unable to account for it. Indeed until the advent of the electronic theory no satisfactory explanation was offered: Maxwell attacked the problem but was forced to confess that his theory of the electromagnetic field, identical in all its physical assumptions

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[1] It will be assumed in this chapter, unless the contrary is stated explicitly, that the media considered are diamagnetic, as are the great majority of non-magnetic substances.

with that of Faraday, was unable to cope with it. The explanation offered by the electron theory will become clearer if we consider first a phenomenon which was discovered long after the Faraday effect—the Zeeman effect.

Principle of the Zeeman effect.

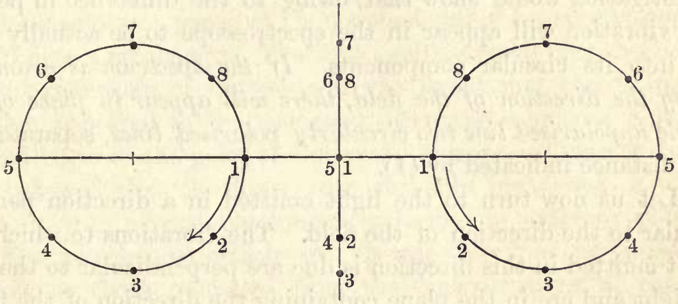
2. In Chapter III. it was argued that the emission spectrum of a body consists of a series of electromagnetic disturbances set up by the vibrations of the bound electrons in the atoms: that each line of the spectrum represents the vibration of one particular electron in each atom and that the period of vibration of the light in that line is the period of vibration of the electron which emits it. Since the vibrations of the electron are transverse to the direction of propagation of the light, an electron which is revolving in a circular orbit in a plane at right angles to the direction of the magnetic field will be emitting circularly polarised light along the direction of the field.

Now we have just seen in Chapter IV. § 5 that the period of the electron revolving in such an orbit will be changed by the action of the field: if the revolution is in the clockwise direction the period is decreased by $-\frac{H\epsilon T^2}{4\pi m V^2}$: if the revolution is in the contrary or 'anti-clockwise' direction, the sign of the term $-\frac{H\epsilon v}{V^2}$ in (2) must be changed and the period is increased by $-\frac{H\epsilon T^2}{4\pi m V^2}$.

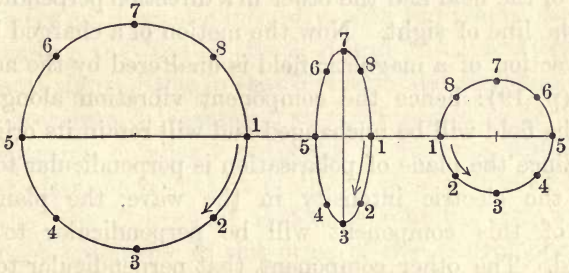
Since the body is not paramagnetic the axes of the orbits will be distributed in all directions. In some orbits the electron will be rotating clockwise, in other orbits which happen to lie in the reversed position the electron will be rotating anti-clockwise. In the absence of a magnetic field the period of the clockwise electrons will be the same as that of the anti-clockwise; but when the field is in action the periods are changed in opposite directions: that of the former will be decreased by $-\frac{H\epsilon T^2}{4\pi m V^2}$ and that of the latter increased by the same amount. If T_0 is the period of the electrons in the absence of a magnetic field, T_c that of the clockwise electron and T_a that of the anti-clockwise in the magnetic field, we have

$$T_c - T_a = +\frac{H\epsilon T_0^2}{2\pi m V^2} \quad \text{......................(1).}$$

Accordingly when we examine the emission spectrum of the body along the direction of the field, we shall see two lines in place of a single line: the periods of the light in these two lines will differ by $\frac{H\epsilon T_0^2}{2\pi m V^2}$ and in one line the light will be circularly polarised in one sense, in the other in the opposite sense.



Rectangular vibration



Elliptic vibration

Fig. 12.

We have only considered the special case of electrons revolving in circular orbits perpendicular to the direction of the field, but there has been no loss of generality in our conclusions. It is well known that any harmonic vibration may be regarded as the resultant of two superimposed circular vibrations of the same period with suitably chosen amplitudes and phases. Thus (Fig. 12) a rectangular vibration may be regarded as the resultant of two circular vibrations in opposite senses and of equal amplitude, while an elliptic vibration can be compounded of two opposite circular vibrations of unequal amplitude (see Fig. 12). All the light emitted along the direction of the field is due to the vibration of electrons in a plane perpendicular to that direction: any vibration in that plane may be regarded as compounded of circular vibrations in the same plane, and we have seen that the period of those circular vibrations under the action of a magnetic field will depend upon the sense of the revolution. A complete dynamical investigation would show that, owing to the difference in period, any vibration will appear in the spectroscope to be actually split up into its circular components. *If the spectrum is examined along the direction of the field, there will appear in place of the single unpolarised line two circularly polarised lines*, separated by the distance indicated by (1).

Let us now turn to the light emitted in a direction perpendicular to the direction of the field. The vibrations to which the light emitted in this direction is due are perpendicular to the line of sight and are in the plane containing the direction of the field: they may all be resolved into two rectangular components, one in the direction of the field and the other in a direction perpendicular to it and to the line of sight. Now the motion of a charged body along the direction of a magnetic field is unaltered by the action of the field (p. 19): hence the component vibration along the direction of the field will be unchanged and will retain its original period T_0. Since the plane of polarisation is perpendicular to the direction of the electric intensity in the wave, the plane of polarisation of this component will be perpendicular to the magnetic field. The other component, that perpendicular to the direction of the field, may be regarded as one of the circular vibrations which have just been considered viewed in profile from a point in its own plane. Its period will be changed by the action of the magnetic field, being increased or decreased by an amount $-\frac{H\epsilon T_0^2}{4\pi m V^2}$, according as the corresponding circular vibration is anti-clockwise or clockwise.

Hence, *if the light be viewed in a direction perpendicular to the* *field, there will appear three lines in place of the single line of unpolarised light*:—(1) a central line of the same period as the undisturbed line, plane-polarised in a plane perpendicular to the field, (2) and (3) two lines, plane-polarised at right angles to the central line, with periods differing from that of the central line by

$$\pm \frac{H\epsilon T_0^2}{4\pi m V^2}.$$

The Zeeman Effect.

3. The first attempt to observe an alteration in the spectrum of a body consequent on the action of a magnetic field appears to have been made by Faraday: owing to the insufficient power of his magnets and of his spectroscope he could detect no change. The first positive result was obtained in 1896 by Zeeman of Amsterdam, who found that the D lines of sodium appeared to be broadened when the source of light was placed between the poles of a powerful magnet: a closer investigation showed that the lines were not only broadened, but were split up into several components polarised in different planes.

The phenomena which are observed are not in general so simple as those predicted from the considerations of the last paragraph. In a few cases, notably that of the red line of cadmium, the simple triplet is found when the spectrum is examined along the lines of magnetic intensity, with the vibrations of the central line along the line of the field and the vibrations of the outer lines at right angles to that direction, and the simple doublet when the spectrum is examined at right angles to the field, with its components circularly polarised in opposite directions. But in most cases the resolution of the lines is more complex. Thus one of the two D lines of sodium is split up into a quadruplet, a doublet appearing in the place of the central line: the other D line is split into a sextet: one of the mercury lines has been resolved into no less than thirteen components. The number of components into which a line can be split seems to depend on the strength of the field and on the resolving power of the spectroscope and to have no definite limit.

But in one respect the observations agree completely with our predictions. The lines when viewed at right angles to the direction of the field are always split into plane-polarised components, and, in all but a very few cases which are probably susceptible of a special explanation, the inner lines are polarised at right angles to the field, while the outer components are polarised in the perpendicular direction: the lines viewed along the direction of the field are always split into opposite circularly polarised components. Nor need the discrepancies throw any doubt on the substantial accuracy of the explanation, which was given by Lorentz immediately after the publication of Zeeman's work and has been sketched above. There are two disturbing factors which have not been taken into account. In the first place the beautiful work of Michelson on the structure of spectral lines has shown that most of these lines do not arise from simple harmonic vibrations: those lines which he has shown to approximate most nearly to that type, of which the best known example is the red line of cadmium, show the Zeeman effect in its simplest form. In the second place the action which we have considered probably does not represent the whole effect of the magnetic field. The changes induced in neighbouring orbits may well affect the forces restraining the electron in its orbit: if they cause an alteration in the restraining forces, which depends on the direction of revolution of the electron, a second resolution of the lines will be effected for the reasons that have been given already. To some such secondary action of the field may be attributed perhaps the splitting of the central component which was noted in the case of the first-mentioned D line.

But the most remarkable feature of the Zeeman effect is connected with the quantitative separation of the components. From (1) we have

$$T_c - T_a = +\frac{H\epsilon T_0^2}{2\pi m V^2},$$

or since $\lambda = VT$,

$$\frac{\lambda_c - \lambda_a}{\lambda_0^2 H} = +\frac{\epsilon}{m}\frac{1}{2\pi V^3} \quad \text{.....................(2)},$$

where λ_c, λ_a, λ_0 are the wave-lengths in air of light having periods T_c, T_a, T_0 respectively. Hence the quantity $\frac{\lambda_c - \lambda_a}{\lambda_0^2 H}$ is proportional to ϵ/m, the ratio of the charge to the mass of the electron. It is found that the value of this quantity, when measured for the most widely separated constituents of a line is very nearly the same for all lines whether in the spectrum of the same or of different elements, and that the value of the same quantity for the less widely separated constituents is some simple sub-multiple of this standard value. This observation is of the greatest importance. We have assumed already for reasons based on the phenomena of electrolysis that the value of the charge upon all electrons is the same: if the value of the ϵ/m is the same for the electrons of all elements we must conclude that the electrons of all elements have the same mass[1]. Since the only physical attributes which we shall have to ascribe to electrons are charge and mass, we may say that the electrons of all elements, the vibrations of which give rise to the spectral lines, are identical in all their characteristics: *the electrons are common constituents of all bodies.*

By suitable optical means the directions of the circularly polarised vibrations can be distinguished. It is found that the 'clockwise' vibration has a shorter period than the anti-clockwise. Hence from (1) the quantity $+\frac{H\epsilon T_0^2}{2\pi m V^2}$ is negative. Since the only term in this quantity which can change sign is ϵ (for the sign of H has been fixed by hypothesis), we must conclude that ϵ is negative and that *the electrons are negatively charged.*

The charge and the mass of an electron.

4. Further from (2) we can calculate the value of the quantity ϵ/m, for λ_c, λ_a, λ_0 and H are all capable of measurement and the value of the other constants is known. Thus in a magnetic field of 10,000 C.G.S. electromagnetic units (gauss) or 3×10^{14} of the electrostatic units that we employ, one D line of sodium, which has a wave-length of $5{\cdot}9\times10^{-5}$ cm., is split into components of which the two extremes differ in wave-length by $3{\cdot}13\times10^{-9}$ cm.

Hence
$$\epsilon/m = \frac{\lambda_c - \lambda_a}{\lambda_0^2 H}\,2\pi V^3$$
$$= \frac{3{\cdot}13\times10^{-9}}{(5{\cdot}9)^2\times10^{-10}}\cdot\frac{2\pi\times(3\times10^{10})^3}{3\times10^{14}},$$
and
$$\epsilon/m = 5{\cdot}1\times10^{17}\,{}^{2}.$$

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[1] It may be remarked that it is impossible to explain the occurrence of constituents with a separation smaller than the standard value by the action of electrons of greater mass: for in the absence of a magnetic field the constituents coalesce into a single line.

[2] By an unfortunate convention, which can only be justified as a means of setting traps for unwary students in the future, when these topics shall have become a subject for examination, the value of ϵ/m is always given with the quantity ϵ measured in electromagnetic units, while the value of ϵ itself is always given in

In Chapter II. reasons were given for believing that the charge on an electron is that carried by a univalent electrolytic ion, namely $3{\cdot}4 \times 10^{-10}$. If this assumption is correct, we can find the value of the mass of an electron from the value for ϵ/m which hast just been obtained: but first we will examine whether that value for the charge is in accordance with such numerical data as we can obtain.

We saw, p. 56 (16), that the dispersion of light by a medium, in which all the electrons which influence the dispersion have the same natural period, can be represented by the equation

$$n^2 = 1 + \frac{N\epsilon^2}{\pi m\left(\frac{1}{p^2} - \frac{1}{T^2}\right)}.$$

It is found that such a simple formula, which takes into account only one natural period, represents with great accuracy the dispersion of light colourless gases. We will take the case of hydrogen. Then since all the electrons in hydrogen have the same free period and all the atoms in an elementary gas are identical, it is a reasonable guess that each atom of the element contains one and only one electron. (It must be remembered in view of certain discussion in a later chapter, that the only electrons with which we are concerned here are those which influence the dispersion.) We will assume, then, that the number N, which gives the number of electrons in unit volume of the gas, is equal to the number of atoms in unit volume of the gas.

If we measure the values n_1 and n_2 of the refractive index for light of two wave-lengths λ_1, λ_2, we can eliminate p and obtain, remembering that

$$\lambda_1 = VT_1 \text{ and } \lambda_2 = VT_2,$$

$$\frac{(n_1^2 - 1)(n_2^2 - 1)}{n_1^2 - n_2^2}\left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right) = \frac{N\epsilon^2}{\pi m V^2} \quad \ldots\ldots\ldots\ldots(3).$$

Measurements by Ketteler on the dispersion of hydrogen give for this gas

$$n = 1{\cdot}00014217 \text{ when } \lambda = 6867 \times 10^{-8},$$
$$n = 1{\cdot}00014294 \text{ when } \lambda = 5893 \times 10^{-8},$$
$$n = 1{\cdot}00014554 \text{ when } \lambda = 4308 \times 10^{-8}.$$

electrostatic units. In the calculation above electrostatic units are used, as always in this volume: but in every other treatise or original memoir which I know, the value of ϵ/m is given as about $1{\cdot}7 \times 10^7$—electromagnetic units being used.

Taking the first two observations we obtain

$$\frac{N\epsilon^2}{V^2\pi m} = 3{\cdot}08 \times 10^6,$$

and from the last two observations

$$\frac{N\epsilon^2}{V^2\pi m} = 3{\cdot}20 \times 10^6;$$

taking the mean we may put

$$\frac{N\epsilon^2}{V^2\pi m} = 3{\cdot}14 \times 10^6.$$

But $$\epsilon/m = 5{\cdot}1 \times 10^{17}.$$

Hence $$N\epsilon = \frac{\pi \times 3{\cdot}14 \times 10^6 \times 9 \times 10^{20}}{5{\cdot}1 \times 10^{17}} = 1{\cdot}74 \times 10^{10}.$$

Now in electrolysis, 1 coulomb or 3×10^{10} electrostatic units of electricity liberates 1·23 c.c. of hydrogen. Hence if E is the charge on the hydrogen ion and N the number of atoms in 1 c.c. of the gas

$$1{\cdot}23\, NE = 3 \times 10^{10}$$

or $$NE = 2{\cdot}45 \times 10^{10},$$

and $$\frac{\epsilon}{E} = 0{\cdot}7.$$

We have assumed that $E = \epsilon$. If we had no further evidence as to the value of ϵ, perhaps the calculation just given could not be held to prove that assumption rather than that $\epsilon = \frac{1}{2}E$. But as we proceed further evidence will accumulate in favour of the equality of the electronic and ionic charges, which we shall regard as established henceforward. At any rate the numbers deduced from the combined evidence of electrolysis and optics is in no way incompatible with that conclusion when the uncertainty of our data is taken into account.

We can now estimate the mass of an electron.

$$\epsilon/m = 5{\cdot}1 \times 10^{17}, \quad \epsilon = 3{\cdot}4 \times 10^{-10}, \quad \text{thus } m = 6{\cdot}7 \times 10^{-28}.$$

The mass of an atom of hydrogen, which can be deduced from the electrolytic data just given, is $1{\cdot}1 \times 10^{-24}$. Hence the electron has a mass only $\frac{1}{1640}$th part of that of an hydrogen atom: we have discovered that *electrons are very much less massive than the lightest atom.* There is no need to insist on the importance of this conclusion taken in conjunction with that reached already, that the electrons are common constituents of all bodies.

Simple method of observing the Zeeman effect.

5. There is a method of observing the Zeeman effect without the use of elaborate apparatus which illustrates so well the principles of optical theory that it should be noted. A flame fed with sodium is placed between the poles of a powerful electromagnet: in front of it and outside the field is placed a less luminous flame also supplied with sodium, and the eye is placed in such a position that the second flame is seen on the background of the first. Now a body absorbs vibrations of the same period as those which it emits: accordingly the second flame absorbs the rays from the sodium in the first flame. Since it emits fewer rays than the first flame, being less luminous, it absorbs more than it emits, and appears dark on the bright background of the first flame. But when the magnet is excited the period of the vibrations emitted by the first flame is changed, and they are no longer absorbed by the second flame: the whole area of the first flame appears bright.

The radiation from the electronic orbits.

6. One question which may have occurred to the reader should be noted here. In order to explain the diamagnetism of bodies, we have assumed that their atoms contain revolving electrons, and in accordance with our optical theory the emission of light from a substance is due to the revolution or other vibration of the electrons. Why then are not all bodies constantly emitting light? But it has not been assumed that the revolving electrons which give rise to magnetic properties are the electrons which cause the emission of light, and we shall see later that, if more than one electron is revolving in the same orbit, the intensity of the radiation emitted by that circuit is reduced immensely. If we suppose then that circuits which give rise to magnetism consist of a considerable number of electrons permanently revolving one after the other in the same orbit, and that the circuits which cause the emission of light represent the vibrations of electrons which have been disturbed temporarily from their position of equilibrium, the difficulty will be removed. Still the permanent circuits must be emitting some radiation, though its intensity may be very small. It is probable that the period of these vibrations is so very much shorter than that of visible light that they cannot be detected by any of the ordinary optical methods and also exert no appreciable effect upon the refractive index of the body. But we must remember the existence of these circuits, for it is likely that they play an important part in phenomena which have not yet been considered.

Principle of the Faraday effect.

7. We are now in a position to consider the Faraday effect, the rotation of the plane of polarisation which occurs when plane polarised light is transmitted through a material medium along the direction of a magnetic field.

We have seen in Chapter III. that the velocity of light in any medium is influenced by the excitation of forced vibrations in the bound electrons of the medium, and depends upon the relation between the periods of these forced vibrations and that of the incident light. Now a plane polarised wave may be regarded as the resultant of two oppositely circularly polarised waves of opposite sense and equal amplitude. When such a wave is incident upon the medium in the magnetic field, each component will excite forced vibrations: those excited by the clockwise component will be clockwise and those excited by the anti-clockwise component will be anti-clockwise. But it has been shown that the natural period of the clockwise component is different from that of the anti-clockwise: hence the influence of the forced vibrations on the velocity of the two circular components of the incident light will be different. If only one kind of electron is taken into consideration so that there is only one free period in the undisturbed medium, (16) p. 56 shows that

$$n_c^2 = 1 + \frac{N\epsilon^2}{\pi m\left(\frac{1}{T_c^2} - \frac{1}{T^2}\right)} \qquad \text{..................(4)},$$

$$n_a^2 = 1 + \frac{N\epsilon^2}{\pi m\left(\frac{1}{T_a^2} - \frac{1}{T^2}\right)} \qquad \text{..................(5)},$$

where n_c is the refractive index of the light vibrating clockwise, n_a that of the light vibrating anti-clockwise, T the period of the incident light, T_a and T_c the free periods of the clockwise and anti-clockwise vibrations of the electron which has the undisturbed free period T_0. The two circularly polarised components will travel through the medium with different velocities v_c and v_a and different wave-lengths λ_c and λ_a, where

$$\lambda_c = v_c T \text{ and } \lambda_a = v_a T.$$

If the light passes through a slab of thickness d, one component will have passed over d/λ_c wave-lengths and the other over d/λ_a wave-lengths. Now a difference of path of one wave-length corresponds to a difference of phase of 2π in the vibrations, hence the phase-difference introduced between the two components during their passage through the medium is $2\pi d\left(\frac{1}{\lambda_c} - \frac{1}{\lambda_a}\right)$. If the two circularly polarised components are recompounded into a rectangular vibration on emergence from the medium, it is easy to show[1] that the plane of polarisation of the emergent vibration will be inclined to that of the incident vibration at an angle

$$\delta = \tfrac{1}{2} \times 2\pi d\left(\frac{1}{\lambda_c} - \frac{1}{\lambda_a}\right) = \pi d\left(\frac{1}{\lambda_c} - \frac{1}{\lambda_a}\right) \quad \text{.........(6).}$$

Since $\lambda_c = v_c T$ and $n_c = \frac{V}{v_c}$, where V is the velocity of the light in a vacuum,

$$\lambda_c = \frac{VT}{n_c} \text{ and } \lambda_a = \frac{VT}{n_a}.$$

---

[1] A plane polarised wave, the vibrations of which are in the plane xy and parallel to the axis of x represented by $x = 2A \sin wt$, can be resolved into the opposite circular components

$$1\begin{cases} x = A \sin wt \\ y = A \cos wt \end{cases} \text{ and } 2\begin{cases} x = A \sin wt \\ y = -A \cos wt \end{cases}.$$

If a phase difference ϕ is introduced between 1 and 2, they become

$$\begin{matrix} x = A \sin(wt + \phi) & x = A \sin wt \\ y = A \cos(wt + \phi) & y = -A \cos wt. \end{matrix}$$

Recombining into a plane polarised wave, we have

$$x = A \sin wt + A \sin(wt + \phi) = 2A \sin\left(wt + \frac{\phi}{2}\right) \cos\frac{\phi}{2},$$

$$y = A \cos wt - A \cos(wt + \phi) = 2A \sin\left(wt + \frac{\phi}{2}\right) \sin\frac{\phi}{2}.$$

The angle through which the plane of polarisation has been turned is

$$\tan^{-1}\frac{y}{x} = -\frac{\phi}{2}.$$

Hence

$$\delta = \frac{\pi d}{VT}(n_a - n_c) = \frac{\pi d}{VT} \cdot \frac{n_a^2 - n_c^2}{n_a + n_c} = \frac{\pi d}{2nVT}(n_a^2 - n_c^2) \text{ ...(7)},$$

where n is the refractive index of the body for light of period T in the absence of a magnetic field.

Hence

$$\delta = \frac{\pi d}{2nVT} \cdot \frac{N\epsilon^2}{\pi m} \left[\frac{1}{\frac{1}{T_c^2} - \frac{1}{T^2}} - \frac{1}{\frac{1}{T_a^2} - \frac{1}{T^2}} \right] \text{.........(8)}$$

$$= \frac{\pi d}{2nVT} \cdot \frac{N\epsilon^2}{\pi m} \frac{T_c^2 - T_a^2}{\left(1 - \frac{T_c^2}{T^2}\right)\left(1 - \frac{T_a^2}{T^2}\right)} \text{ ....(9).}$$

But $T_c + T_a = 2T_0$

$$= \frac{\pi d}{2nVT} \cdot \frac{N\epsilon^2}{\pi m} \cdot 2T_0 \frac{1}{\left(1 - \frac{T_c^2}{T^2}\right)\left(1 - \frac{T_a^2}{T^2}\right)} \cdot (T_c - T_a) \text{ ...(10).}$$

According to (1)

$$T_c - T_a = + \frac{H\epsilon T_0^2}{2\pi m V^2}.$$

But in Chapter III., when we were discussing the influence of the natural periods on the refractive index, it was assumed implicitly that the elastic forces determining the free periods were independent of the time of vibration. From (1 a) p. 78 we see that part of the forces acting on the electrons, represented by the term $\frac{H\epsilon v}{V^2}$, depends on the period of vibration; and the assumption of p. 55 is not justified. A complete investigation would show that in place of (1) we must write

$$T_c - T_a = + \frac{H\epsilon T_0^2}{2\pi m V^2} \cdot \frac{T_0}{T} \text{ ....(11)}$$

in the above equation.

Hence

$$\delta = \frac{d}{2n(VT)^2} \cdot \frac{\frac{N\epsilon^3 T_0^4}{\pi m^2}}{\left(1 - \frac{T_c^2}{T^2}\right)\left(1 - \frac{T_a^2}{T^2}\right)} \cdot \frac{H}{V} \text{ ....(12).}$$

Comparison with Experiment.

8. The agreement between the values given by this formula and those found by experiment is quite satisfactory. It is found that the rotation of the plane of polarisation is proportional to the thickness of the substance traversed and to the strength of the field, as predicted: the variation of the rotation with the incident light also accords well with that given by (12). Further the order of magnitude of the rotation is in accordance with observation. Remembering that by (16), p. 56,

$$n^2 - 1 = \frac{N\epsilon^2 T_0^{\,2}}{\pi m\left(1 - \dfrac{T_0^{\,2}}{T^2}\right)} \quad \text{..................(13)},$$

and that T_c and T_a are both nearly equal to T_0, we may write

$$\delta = d\frac{H}{V}\frac{1}{2n\,(VT)^2}\frac{(n^2-1)^2\,\pi}{N\epsilon} \text{ approximately } \text{......(14)}.$$

Substituting in (14) values determined by experiment, we find that the rotation of the plane of polarisation of sodium yellow light by hydrogen should be

$$\delta = d\frac{H}{V}\frac{1}{2 \times 1{\cdot}0001429 \times (V \times 1{\cdot}96 \times 10^{-15})^2}\cdot\frac{(2{\cdot}9 \times 10^{-4})^2\,\pi}{1{\cdot}74 \times 10^{10}}$$

$$= 2{\cdot}17 \times 10^{-9}\, d\frac{H}{V}.$$

From the experiments of Becquerel and those of Du Bois the observed rotation is found to be $1{\cdot}59 \times 10^{-9}\, d\dfrac{H}{V}$.

Since the only terms on the right-hand side which can change their sign are $\left(1 - \dfrac{T_c^{\,2}}{T^2}\right)$ and $\left(1 - \dfrac{T_a^{\,2}}{T^2}\right)$, the direction of the rotation must be the same for all values of T, the period of the incident light, except when T lies between T_c and T_a, that is to say when the period of the incident light lies between that of the two components of the natural vibration. Experiments on this point are extremely difficult since the separation of the components is so small, but the observations of Schmauss would seem to be in accordance with the theory. It has been shown by Wood that the rotation is in the same direction on each side of the absorption band of Sodium.

As regards the direction of the rotation of polarisation it should be clear that the plane is rotated in the sense of that one of the two circularly polarised components which has the shorter period. For this component will have made a larger number of vibrations in passing through the substance than the other: its phase will be more advanced than that of the other, and, when the two components are reunited, the line along which they vibrate will have been rotated in the direction of the advanced phase. On p. 89 we saw that the clockwise component must have the shorter period, if the charge on the electron is negative, and the anti-clockwise, if the charge is positive. Experiment shows that in all diamagnetic substances[1], with the doubtful exception of the chloride of titanium, the rotation is in the clockwise direction, *i.e.* in the direction in which a current must flow in order to give rise to a magnetic field in the same direction as that which acts upon the medium. We arrive again at the conclusion that *the charge upon the electrons which are chiefly concerned in the emission and absorption of light is of the negative sign.*

Light viewed perpendicular to the magnetic field.

9. The vibrations of light travelling at right angles to the field are in a plane containing the direction of the field and perpendicular to the direction of propagation. The velocity of the light will be influenced only by forced vibrations excited in this plane. On p. 86 it was shown that the natural period of the electrons vibrating in this plane is different according as the electron vibrates parallel to or perpendicular to the direction of the field. Accordingly an application of the principles of the last paragraph will show that the light transmitted at right angles to the field will be split up into two portions travelling with different velocities—one vibrating parallel to the direction of the field and one vibrating at right angles to it: these two portions will be polarised at right angles. But a decomposition of the incident light into two portions polarised at right angles is characteristic of a doubly refracting crystal and, in fact, the medium will behave in a magnetic field

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[1] The theory that has been given does not apply to paramagnetic substances, for any change of the position of the orbit as a whole has been left out of account. In this respect paramagnetic substances must be classed with the magnetic metals considered below.

precisely like a uniaxial crystal the axis of which coincides with the direction of the field.

The separation of the two rays is extremely small unless the period of the light is very near to the free period of the electrons. The experimental investigations are difficult, but Voigt and Wiechert have shown that the phenomena in sodium vapour are in entire concordance with the predictions of theory.

Zeeman effect in absorption bands.

10. The Faraday effect as well as the Zeeman effect can be explained by the direct action of the magnetic field on the free periods of vibrations of the electrons. But we saw in Chapter III. that the selective absorption is determined by the same free periods: it is to be expected that the absorption of light should be influenced by a magnetic field and the absorption bands split up and displaced in the same manner as the lines of the emission spectrum. The absorption bands of most bodies are so broad and diffuse that no accurate measurements can be made of their position and the resolution of the band into several components cannot be perceived. But certain elements and their compounds, notably didymium, show absorption bands so sharp that it might be possible to detect in them changes under the action of a magnetic field similar to those in the emission spectrum which are known as the Zeeman effect. Ingenious experiments in this direction have been made recently by Jean Becquerel on crystals containing didymium, which show sharp absorption bands in the visible spectrum.

In the main his results accord with expectation. When the light is transmitted through a crystal in a direction at right angles to the magnetic field, the single absorption band is split into three or more components, of which some refuse transmission to light polarised in a plane perpendicular to the field and others to light polarised in a plane containing the direction of the field: while, when the light is transmitted along the direction of the field, the single absorption band is split into an even number of components, pairs of which are circularly polarised in opposite directions. But the quantitative relation between the separation of the components shows none of the order which is characteristic of the simple Zeeman effect. The quantities which, if the theory of the Zeeman effect were applicable, should be proportional to ϵ/m are not constant, nor are they all simple sub-multiples of a single constant: there seems no simple connection between the separation of the components of the same absorption band, far less between that of the components of different bands. Moreover examination of the direction of vibration of the circularly polarised components shows that the quantity ϵ/m has not even always the same sign.

Becquerel himself would attribute this complexity to the existence in the crystal of electrons of many different kinds, some positive, some negative; but it is hardly necessary to resort to such an heroic upheaval of the entire electronic theory in order to bring his experiment into line with other optical phenomena. A much simpler explanation is suggested by the fact that this great complexity is not found in gases, nor, if the theory of the Faraday effect just given is correct, in amorphous solids and liquids, but only in crystals.

Now in a doubly refracting crystal, such as Becquerel used, the refractive index is different according to the plane of polarisation of the refracted light: or, on our theory, the period of free vibration of the electrons is different according as they vibrate along or perpendicular to the optic axis. But in Lorentz' explanation of the Zeeman effect, which was sketched above, it is assumed implicitly that the period of vibration of the electron is the same in whatever direction it vibrates: his explanation is not applicable to doubly refracting crystals and there is no reason to believe that in Becquerel's experiments the quantity $\frac{\lambda_c - \lambda_a}{\lambda_0^2 H}$ is proportional to ϵ/m. A complete theory of magneto-optics, which should include crystals, is impossible until further information is obtainable of the nature of the restraining forces on the electrons in such bodies: but a very little consideration will show that irregularities such as were found by Becquerel must be the normal consequence of the application of a magnetic field. Moreover we may note that since the absorption bands of didymium are so very much sharper than those of other bodies, it is probable that the mechanism which gives rise to them is also somewhat different.

The Kerr Effect.

11. If our explanation of the magnetic properties of bodies is correct, all the optical effects which have been described in diamagnetic substances should also appear in paramagnetic substances: for the revolving electrons which formed the basis of our explanation are present in all substances alike. There is nothing in the experimental evidence to contradict such a conclusion, but changes of a nature peculiar to the magnetic metals and of much greater intensity than the diamagnetic effects would prevent the observation of the latter even if they existed.

Of the phenomena peculiar to the magnetic metals the best known is the 'Kerr Effect.' In 1877 Kerr observed that if plane polarised light is reflected from the polished surface of the pole of a magnet the plane of polarisation[1] of the light is rotated. The direction of the rotation is in general that which we have previously called clockwise, but for particular angles of incidence polarisation and magnetisation the rotation is in the reverse direction.

For the explanation of these effects the electron theory is sufficient but is not necessary. With suitable assumptions concerning the nature of the boundary between the metal of the magnet and the surrounding air, Weber's theory of magnetism and the electromagnetic theory of light are all that are required: on this basis an almost complete account of the observations was given by Drude and by Goldhammer in 1892. The few remaining discrepancies are not removed by the adoption of the hypotheses of this chapter. These phenomena are not, therefore, of prime importance for our present purpose, and, since the mathematical methods necessary for their adequate discussion are either too advanced or too cumbrous for the scheme of this work our consideration of them will be confined to noting the general physical aspects of the problem.

It has been shown in Chapter II. that there exists in the wave-front of a light disturbance a periodic magnetic force. It has

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[1] Plane polarised light after reflection from a metallic surface is in general elliptically polarised: but, if the plane of polarisation of the incident light is either in or perpendicular to the plane of incidence, no elliptical polarisation is introduced by reflection. The plane of polarisation of the incident light was in one or other of these positions in all Kerr's experiments.

also been shown that there are present in a magnetic metal under the influence of a magnetic field certain electronic orbits, from which issue lines of induction in a direction opposite to those of the field. When the electric intensity in the wave-front of the light disturbance acts upon these revolving electrons, it will excite in them forced vibrations, and cause the whole orbit to vibrate in the plane of the wave-front with the period of the light. But the orbit will carry its lines of induction with it and the periodic movement of these lines will be equivalent to a periodic magnetic force of the same period as that which exists in the wave-front owing to the motion of the Faraday tubes along which the light is propagated.

Just as the additional electric intensity due to the movement of the electrons produces a change in the velocity of the light (see Chap. III. § 5), so the additional magnetic intensity will produce a change in the velocity of the light: and it can be shown by reasoning similar to that of p. 93 that the velocity of circularly polarised light will depend on the sense of the circular vibration. Accordingly it must be concluded, as in § 7, that the medium under the action of a magnetic field rotates the plane of polarisation of light passing through it. Of course the magnetic metals are extremely opaque, but layers so thin that light can pass through them can be obtained. It is found that the rotation of the plane of polarisation of light passing through fully 'saturated' iron is at the rate of 200000° per centimetre, whereas that caused by the most strongly rotating diamagnetic substance in the strongest field attainable is less than 10° per centimetre.

The Kerr effect is a result of this rotation but it would take us too far from our main purpose to inquire into it further.

The direction of the rotation is clockwise in the magnetic metals themselves but is anti-clockwise in many of their salts. However, the direction of the rotation does not depend, as in diamagnetic metals, only on the sign of the electronic charge, but also on the form and nature of the circuits in which the electrons move. In the present state of our knowledge nothing is gained by making elaborate hypotheses on such subjects.

REFERENCES FOR CHAPTER V.

Faraday Effect. FARADAY'S *Experimental Researches*, Vol. III. §§ 2146–2309.

Kerr Effect. *Phil. Mag.* (3), p. 321, 1877.

Zeeman Effect. The first account of Zeeman's experiments, which were made in 1896, is to be found in the *Proc. Amst. Acad.* 1898. See also *Phil. Mag.* XLIV. p. 55, 1897.

Lorentz' theory of magneto-optics is to be found in its most accessible form in the *Rapports presentés au congrès international* 1900, Vol. III. p. 1. It is given in a slightly different form, based on the work of DRUDE, in WOOD'S *Physical Optics*, Chap. XVII. A similar theory has been worked out with great elaboration by VOIGT.

For an account of modern methods of spectroscopy, including observations on the Zeeman effect, the reader is recommended to study MICHELSON'S *Light Waves and their uses* (Chicago, 1903), which contains an admirable account of the author's own work.

A detailed account of work on the Zeeman effect is given in KAYSER'S *Handbuch der Spectroscopie.*

CHAPTER VI.

CONDUCTIVITY, ELECTRICAL AND THERMAL.

Electrolytic and metallic conduction.

1. Conductors of electricity may be divided into two classes: Electrolytic conductors and metallic conductors. In the former the passage of a current is accompanied by chemical changes in the material of the conductor, in the latter no such change takes place.

For many years past a theory of the mechanism of electrolytic conduction has been generally accepted. It is held that the current is conveyed between the electrodes by the motion under the electric intensity in the circuit of charged particles derived from the electrolyte. When these charged particles or 'ions' arrive at the boundary of the electrolyte, they give up their charge and appear as uncharged substance. Ions of different kinds carrying opposite charges accumulate at opposite boundaries of the electrolyte. No attempt is made in the theory, as usually stated, to explain how the ions become charged or uncharged: this question will concern us in a later chapter: for the present we shall accept the ionic theory as giving a satisfactory explanation of the phenomena of electrolytic conduction.

Many attempts have been made to apply a similar theory to metallic conduction, but until recently none of them were successful. The difficulty arose from the absence of any sign of a transport of matter accompanying the transport of electricity. Every current circuit, except those in which the current is induced by a change of magnetic induction, must consist of two or more portions composed of different materials which may contain no element in common, and we must suppose either that the particles which carry the current can cross from one material to the other, or that they cannot. Either supposition brings grave difficulties. If the particles cannot cross the boundary between different metallic conductors, they must remain piled up at those boundaries: even if they are identical with the atoms of the material in which they move, this piling up must alter the distribution of the mass of the conductor, while, if they are composed of some substance different from that of the rest of the material, some sort of chemical separation should occur. But the most careful experiments on pure metals and on alloys have failed to show the slightest change in any of the properties[1] of a metallic conductor induced by the passage of a current through it.

On the other hand if we suppose that the particles can pass freely from one material to another new difficulties arise. We know that the atoms and all congeries of atoms of different elements differ markedly in their properties: and we could certainly detect the presence in one substance of atoms derived from a foreign element. Since the properties of the materials forming a non-uniform circuit are unchanged by the passage of the current, if the electricity is conveyed at all by the agency of material particles, those particles must be of a nature common to all elements, or at least to all elements which form metallic conductors.

Until the development of the electronic hypothesis this conclusion would have invalidated necessarily any theory which led to it: not because it was thought impossible that there should be any common constituent of all elements, but because nothing was known about such a constituent. Ever since the conception of an element was introduced, physicists have probably entertained privately the belief that these elements would be found to be but many forms of one primordial substance, and the belief derived some support from such facts as those which gave rise to Prout's hypothesis. But whatever arguments there might be for the existence of such a primordial substance, nothing was known about its properties. All research in chemistry and physics had tended to emphasize the difference between different elements: there was no trace of a common constituent. Any theory of conduction based upon the properties of such a substance would

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[1] Except those due to the rise of temperature, which disappear when the current is stopped.

be useless, for it would not connect the conduction of electricity with any other known phenomenon or enable relations to be established between the conductivity and any other characteristics of a body.

Electronic hypothesis.

2. But now this objection is removed. Our study of optics and of magnetism have led us to the belief that the negatively charged electrons, which cause the dispersion of light, are identical in respect of charge and mass, the only characteristics that we shall have to attribute to them, in whatever material they are contained. Here we have charged particles which can be supposed to cross the boundary from one substance to another without giving rise to any of the difficulties that we have just considered.

In order to explain the main features of metallic conduction little need be added to the hypothesis that metallic conductors contain electrons which are held by no restraining force, but are free to move under the action of an electric field. Since the magnetic effect of a current is proportional to the rate of transfer of electricity, the relation between the current (measured by its magnetic effect), the number of the electrons N, and their velocity v along the direction of the current is

$$N\epsilon v = i \qquad (1).$$

It must be remembered that the direction of the current is always taken as the direction in which positive electrification is transferred: since the electrons, to the motion of which is ascribed the production of current phenomena, carry a negative charge, the direction of motion of the electrons is in the opposite sense to that of the current.

Since a metal is electrically neutral as a whole in its normal state, there must be present in it in addition to the negative electrons, positively charged bodies, the total charge on which is equal and opposite to that on the electrons. The investigation of the last chapter showed that these particles play no important part in magneto-optics, and we have been able to obtain no information about them. We shall suppose, for reasons which appear in the next part of this volume, that the positive charge is attached to the atoms of the metal and that the failure of the positively charged particles to show any magneto-optical effect is due to the largeness of the mass of the atoms compared with that of the negative electrons. Since it is known, for reasons given in the preceding paragraph, that the atoms of the metal do not take any part in conveying the current, this supposition involves the view that the positively charged particles play no part in metallic conduction, and their existence may be left out of account in the present chapter. (See also Chapter VII. § 5.)

The heat motion of the electrons.

3. But to arrive at a quantitative estimate of the relation between the conductivity of the metal and the properties of the electrons in it, some further assumption must be made as to the nature of the motion of the electrons in an electric field. The assumption that has been adopted generally is derived from the kinetic theory of gases. According to that theory the molecules of any substance which is not at the absolute zero of temperature are in a state of constant agitation, and the mean kinetic energy of the molecules in virtue of their speed of agitation is proportional to the temperature of the gas on the absolute scale. If a substance contains molecules of two kinds, the theorem known as the equipartition of energy states that the mean kinetic energy of each kind of molecule is the same. Now the electrons which are free to move about among the molecules of the metal are equivalent in this respect to any other kind of molecule, and we shall assume that the mean kinetic energy of an electron in the metal, which it possesses in virtue of a heat-motion agitation, is equal to the mean kinetic energy of a molecule of the metal.

It is not pretended that the truth of this assumption is indubitable or that it is even the most probable assumption that can be made. All that it is really necessary to assume, and all that is proved by the result of the calculations given below, is that the electrons partake in the motion of heat agitation of the molecules of the metal. In the absence of more definite information the best assumption to make is that which is simplest for the purposes of calculation; that which we have chosen will enable us to apply at once all the results of the highly developed kinetic theory of matter.

A theory of metallic conduction on these lines was first proposed by Riecke, stated more definitely in terms of electronic conceptions by Drude and J. J. Thomson and developed recently by many workers among whom Lorentz and Einstein are prominent. In the rest of this chapter and in the following chapter a sketch of their main results will be given.

Calculation of the electrical conductivity.

4. The law of the distribution of velocities among the molecules of a gas is investigated in all treatises on the kinetic theory of gases. But our calculations will be simplified and our main results unaffected if we assume, in place of the complicated 'Maxwell-Boltzman' law, that all the electrons are moving with the same velocity of heat agitation. This velocity is the 'mean velocity' and is connected with the temperature by the relation

$$\tfrac{1}{2}mu^2 = \alpha T \text{ .... (2)},$$

where u is the mean velocity, m the mass of an electron, T the absolute temperature, and α a constant which is the same for all solid, liquid or gaseous bodies.

If there is no external electric field, the velocity of the electrons will be distributed equally in all directions: there will be no tendency for them to move in one direction rather than another. But if the metal is placed in an electric field, the electrons during their free flight between two collisions are subject to a force in virtue of their charge and move along the direction of the electric intensity. By a collision with a molecule the previous history of the electron is wiped out, and it starts afresh, but again it tends to move in the same direction. On the other hand the atoms of the metal are unable to move more than a very small distance from their position of equilibrium: even if they are charged they are unable to move away from that position. Accordingly all the electrons move on the average relatively to the atoms: it is easy to calculate the velocity v of this general translation of the electrons.

If an electron of mass m carrying a charge ϵ is placed in an electric field of intensity X it will be subject to an acceleration f along the direction of X, where

$$f = \frac{X\epsilon}{m} \text{ .... (3)}.$$

In a time t it will move a distance d, where

$$d = \tfrac{1}{2} f t^2 \qquad \text{.........................(4).}$$

In the present case t must be taken to be the time occupied in travelling between two collisions: hence

$$t = \frac{l}{u} \qquad \text{.............................(5),}$$

where u is the mean velocity of the electron and l its free path or the distance travelled between two collisions.

Therefore
$$d = \tfrac{1}{2} \frac{X\epsilon}{m} \cdot \frac{l^2}{u^2} \qquad \text{.........................(6).}$$

But v, the average velocity of translation, is the distance travelled along the direction of X in unit time.

Hence
$$v = \frac{d}{t} = \tfrac{1}{2} \frac{X\epsilon}{m} \cdot \frac{l}{u} \qquad \text{......................(7).}$$

But by (2)
$$\tfrac{1}{2} m u^2 = \alpha T$$

or
$$m = \frac{2\alpha T}{u^2}.$$

Thus
$$v = \frac{X\epsilon l u}{4\alpha T} \qquad \text{........................(8).}$$

If there are N electrons in unit volume, a number vN will cross unit area in unit time: these electrons carry a charge $\epsilon v N$. Hence the current flowing across unit area in unit time, or the current density i, is $\epsilon v N$, or

$$i = \frac{X\epsilon^2 l u N}{4\alpha T} \qquad \text{......................(9).}$$

But ϵ, l, u, N, α, T are all constants for any particular conductor: hence i is proportional to X and Ohm's Law is satisfied. The conductivity σ is given by

$$i = \sigma X \qquad \text{.........................(10),}$$

whence
$$\sigma = \frac{1}{4\alpha T} \cdot \epsilon^2 l u N \qquad \text{......................(11).}$$

It should be pointed out that it has been assumed that the velocity imposed upon the electrons by the electric field does not alter the time between two collisions. Of course if v were greater than u, this assumption would be ridiculous, for some of the electrons would be turned back before they reached the end of their natural flight: but if v is only a small fraction of u, the change of the average time between two collisions will be hardly affected, for while some collisions will be accelerated, others will be retarded to nearly the same extent. At present, while we do not know the values of all the constants involved in (7) the assumption can only be justified by results. If the time of flight were altered appreciably by the field, it is easy to show that Ohm's law would not be satisfied.

It is a well-known experimental law that the resistance of a pure metal is proportional to the absolute temperature. This relation will be represented by (11) if the product Nul is independent of the temperature, for ϵ is always constant.

It is easy to devise a hypothesis concerning the production of the ions which will lead to this relation. Thus we may suppose that the atoms of the metal are throwing off free electrons at a rate which is independent of the temperature and of everything except the nature of the metal, and that, on the other hand, when an electron collides with an atom it ceases to be free until it is released again spontaneously. The number of collisions in unit time is $N\frac{u}{l}$, for each electron travels in unit time over a distance u and makes a collision at the end of each distance l. $N\frac{u}{l}$ is, then, the number of electrons disappearing by collision in unit time. On our view this is equal to q, the number produced from the atoms in the same time, and q is independent of the temperature and of everything but the nature of the metal. If we may take l, the free path, as independent of the temperature, we get $q = N\frac{u}{l}$, or

$$Nul = ql^2 = \text{constant} \quad \text{.....................(12)},$$

in accordance with experiment. The assumption of the constancy of l means that the electrons only collide with the atoms and not with each other: for if there were an appreciable number of collisions between two electrons, l would vary with the number of electrons present. But since we have every reason to believe that the electrons are very much smaller than the atoms, the assumption is by no means improbable.

From (12) it follows that

$$N = \frac{ql}{u} = \frac{\text{constant}}{u} \quad \text{.................(13)},$$

but from (2)

$$u = \left(\frac{2aT}{m}\right)^{\frac{1}{2}} = \text{constant} \times T^{\frac{1}{2}}.$$

Hence N, the number of electrons in unit volume, should be proportional to the inverse of the square root of the absolute temperature, or

$$N = \beta / T^{\frac{1}{2}} \quad \text{.........................(14)}.$$

But it must be admitted that this view of the production of the free electrons is not necessary apart from the experimental relation between the temperature and the conductivity. The observed proportionality between the temperature and the resistance must be held to give us information about that production rather than to confirm our calculations of the conductivity.

In alloys the variation of the resistance with the temperature is more complicated and elaborate hypotheses, undesirable in the present state of our knowledge, would be required to account for it.

Joule's Law and the Mechanism of Resistance.

5. According to Joule's law the heat developed in unit time in a conductor of resistance R carrying a current C is RC^2. If the conductor is a cube of unit side, this quantity is σX^2, where X is the electric intensity and σ the specific conductivity. This relation can be deduced directly from the definition of resistance, Ohm's law and the conservation of energy: any hypothetical mechanism which leads to Ohm's law will lead also to a correct deduction of the Joule development of heat, but it is desirable to note in what manner the electrical energy is converted into heat energy.

Consider an electron moving in a direction making an angle θ with that of the electric field. During its free flight it is subject to an acceleration ft (using the notation of the previous paragraph) in the direction of the field. At the beginning of its flight the components of its velocity, parallel to and perpendicular to the field, are $u \cos \theta$ and $u \sin \theta$ respectively: at the end of its flight they are

$$u \cos \theta + ft \quad \text{and} \quad u \sin \theta.$$

Consequently the electron has gained an amount of kinetic energy

$$\tfrac{1}{2} m \{(u\cos\theta + ft)^2 + u^2\sin^2\theta\} - \tfrac{1}{2} mu^2 = \tfrac{1}{2} m\,(2uft\cos\theta + f^2t^2).$$

Summing this quantity over the N electrons in unit volume, which are moving in all directions, the term containing $\cos\theta$ disappears, since $\cos\theta = -\cos(\pi - \theta)$. Hence the whole gain of kinetic energy in the time t is $\tfrac{1}{2}Nmf^2t^2$, or the electrons gain per second an amount of kinetic energy $\tfrac{1}{2}Nmf^2t$.

But by the fundamental assumption of our theory the kinetic energy of the electrons is equal to that of the molecules of the metal and is proportional to the absolute temperature of the metal. Accordingly the kinetic energy gained by the electrons is shared with the molecules and appears as heat energy of the metal. Hence the amount of heat energy developed in the metal in one second by the passage of the current is

$$\tfrac{1}{2}Nmf^2t = \tfrac{1}{2}Nm\,.\,\frac{X^2\epsilon^2}{m^2}\,.\,\frac{l}{u} = X^2\,.\,\frac{N\epsilon^2 lu}{4\alpha T} = \sigma X^2$$

in accordance with Joule's Law.

According to (11) the difference of the conductivity of different materials can arise either from a difference in the value of N or a difference in the value of l. Part of the variation in the conductivity of different metals may be attributed to the latter cause, but there is no doubt that the enormous difference in conductivity between (say) copper and sulphur arises from the former cause. In a later chapter we shall see reason for believing on chemical grounds that the very badly conducting substances, which are classed as insulators, are those which contain very few free electrons.

Optical Conductivity.

6. Having obtained some idea as to the mechanism of metallic conduction we may return to the optical properties of metals. The hypothesis of a frictional resistance proportional to the velocity impeding the motion of the electrons will be found to be equivalent mathematically to that which has been adopted in the present chapter. If a body moving under a force F is impeded by a resistance rv proportional to its velocity, it will attain ultimately to a steady velocity v in the direction of the force where

$$v = \frac{F}{r} \quad \text{........................} (15).$$

Since the electrons acted on by a force $X\epsilon$ attain a steady velocity $v=\frac{X\epsilon lu}{4\alpha T}$ in the direction of that force, we may consider that they are subject to a frictional resistance rv where

$$r=\frac{4\alpha T}{lu} \qquad \text{.........................}(16).$$

It will now be seen that the equation (36) p. 70 is identical, for very long light waves, with that deduced from the simple Maxwellian theory (p. 44), so long as the bound electrons are left out of account. The 'free' electrons are electrons which have an infinitely long natural period. Hence, putting T infinite, (36) becomes

$$n^2\alpha = T\Sigma\frac{N_k\epsilon^2}{r_k} = T\Sigma N_k\frac{\epsilon^2 l_k u_k}{4\alpha T}$$
$$=\sigma T.$$[1]

Rubens and Hagen have shown that for very long waves, far in the infra red, having a period of about 2×10^{-13} seconds, the Maxwellian relation is fulfilled with great accuracy, so that it is possible to deduce the specific resistance of a metal from observations on its optical properties. But it must be confessed that their result is rather surprising, for it would appear at first sight unlikely that the number of collisions made by an electron during a single vibration of so short a period would be large enough to justify the application of the statistical conceptions of the kinetic theory of matter. It is possible to make some estimate of this number of collisions. The velocity of agitation u of the electrons can be found readily: for it is known that the velocity of a hydrogen molecule at 0° C. is $1{\cdot}7\times 10^5$ cms. per sec.: since the kinetic energy of an electron is equal to that of a molecule at the same temperature, the ratio of the velocity of the electron to that of the hydrogen molecule is the inverse of the ratio of the square roots of their masses. Using the value for the mass of an electron found on p. 91 we find 10^7 cms. per sec. as the velocity of the electron. The mean free path of the electron in a solid is probably comparable with the diameter of a molecule, or 10^{-8} cms.: hence the electron must make about 10^{15} collisions per second.

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[1] The symbol T under the sign Σ signifies the absolute temperature, outside that sign the period of the incident lights. The confusion only occurs here.

Since the period of the wave used by Rubens and Hagen is 2×10^{-13} secs., it appears that the electron only makes about 50 collisions in each vibration. Some considerations advanced in a later chapter make it probable that even this estimate must be reduced.

It appears then from experiment, contrary to expectation, that this number is sufficient to justify the application of the value of r deduced from (16) to the formula. But it is quite clear that it is not permissible to apply it for waves of the period of visible light, *i.e.* about 2×10^{-15}, which can only make one or two collisions in each vibration. It is remarkable therefore, not that Maxwell's formula does not agree with measurements made on visible light, but that there is any sort of agreement even in order of magnitude between the calculated and the observed values: it might almost be said that the agreement, so far from confirming our hypothesis, requires us to modify in some measure our conception of the motion of the electrons in a metal.

On the other hand the difficulty connected with the transparency of electrolytes is removed. The value of r will be the same for an electrolyte and for a metallic conductor of the same conductivity, but the value of m in formula (36), p. 70, will be very different. The vibrating particle in the metallic conductor is an electron, in an electrolyte it is a charged atom, many thousand times more massive: and the right hand of (36), p. 70, which measures the absorption of light, will be reduced correspondingly[1]. The ions in the electrolyte are so massive that the light vibrations are too rapid to set them into motion.

The conduction of heat.

7. It is a well-known generalisation that good conductors of electricity are also good conductors of heat. A list of substances arranged in order of their thermal conductivities is not identical with a list in order of their electrical conductivities, but the general agreement is so close that there would be grave objection to any theory which professed to describe

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[1] From the information which we shall obtain subsequently as to the number of electrons in an atom it appears that the two terms in the denominator of (36) are of the same order of magnitude for a metal. For an electrolyte the second term is much larger than the first.

the mechanism of electrical conduction, if it could not also explain thermal conduction.

On the kinetic theory of matter the conduction of heat is represented as the result of collisions between molecules coming from regions at different temperatures. Those coming from the hotter regions possess the greater kinetic energy: in a collision the kinetic energies of the colliding systems tend to be equalised: the colder regions gain kinetic energy and the hotter regions lose it: heat is conducted from hot to cold.

In a metal this transference of energy can take place through the agency of the electrons as well as of the atoms. In considering the electrical conductivity we could leave the atoms out of account, but, though they are fixed and cannot move along carrying their charge, they can transfer their kinetic energy to their neighbours by colliding with them. However, since we know that good electrical conductors which contain many free electrons are incomparably better conductors of heat than electrical insulators, which contain few free electrons, it is legitimate to assume for the present in dealing with metals, that the electrons are much the most important agency in the transference of heat and to neglect the influence of the atoms.

If the ends of a metal bar are at different temperatures, we have seen that they must contain different numbers of free electrons per unit volume. If the electrons could not move from one end of the bar to the other, there would be no resultant charge on any part of the bar, for at any point the numbers of electrons would be equal to the number of oppositely charged atoms which had lost an electron. But since the electrons can move, they will diffuse from the regions where they are plentiful to those where they are scarce. The atoms cannot diffuse away, and will be left behind, giving rise to a positive charge. Hence there will be a positive charge where the electrons are plentiful and a negative charge where they are scarce—a result that might seem at first sight paradoxical. But the diffusion cannot go on indefinitely: the opposite ends of the bar will be charged with opposite signs, and there will be an electric field between them tending to oppose the diffusion. Equilibrium will be reached when the force of diffusion is balanced by the electrical forces, and there will be no further general drift of electrons.

However the transference of heat will still proceed through collisions. It is shown in treatises on the kinetic theory of gases that the quantity of heat Q transferred in unit time through a slab of unit cross section and small thickness d is given by

$$Q = \frac{ulN}{3} \cdot \frac{E - E'}{d} \qquad \text{.....................(17)},$$

where E and E' are the mean kinetic energies of a molecule on either side of the slab, and the other quantities have the same meaning as before. But from (2),

$$E = \tfrac{1}{2}mu^2 = \alpha T \text{ and } E' = \tfrac{1}{2}mu'^2 = \alpha T',$$

hence

$$Q = \frac{\alpha ulN}{3} \cdot \frac{T - T'}{d} \qquad \text{.....................(18)}.$$

But if k is the thermal conductivity of the medium

$$Q = k \cdot \frac{T - T'}{d} \qquad \text{.........................(19)},$$

so that

$$k = \frac{\alpha ulN}{3} \text{................................(20)}.$$

The Wiedemann-Franz law.

8. On comparing this value for k with that obtained for σ in equation (11), the interesting result appears that

$$k/\sigma = \frac{4}{3}\left(\frac{\alpha}{\epsilon}\right)^2 T \text{ ....(21)},$$

or the ratio of the thermal to the electrical conductivity is independent of the nature of the metal and proportional to the absolute temperature. This law has already been discovered empirically and is known by the name of those who propounded it, as the 'Wiedemann-Franz' law. The extent of the agreement is shown in Table V., in which the first column gives the metal considered, the second the ratio k/σ at 18° C., the third the ratio of k/σ for 100° C. to k/σ at 18° C. This latter ratio should be equal to the ratio of the absolute temperatures, *i.e.*

$$\frac{273 + 100}{273 + 18} = 1{\cdot}28.$$

It should be noted that the electrical conductivity is measured in electrostatic units and is 9×10^{20} times the value usually given, which is in electromagnetic units: also the value of thermal conductivity is the number of ergs (not calories) transferred through a surface of 1 cm.² cross section by a temperature gradient of 1° C. per centimetre.

TABLE V.

Metal	k/σ at 18°	$\frac{k/\sigma \text{ at } 100°}{k/\sigma \text{ at } 18°}$
Aluminium	$\cdot 706 \times 10^{-10}$	1·32
Copper	·738 „	1·30
Zinc	·745 „	1·29
Silver	·760 „	1·28
Nickel	·776 „	1·30
Cadmium	·784 „	1·28
Lead	·794 „	1·31
Gold	·807 „	1·27
Mercury	·815 „	1·30
Tin	·816 „	1·26
Platinum	·836 „	1·35
Palladium	·837 „	1·35
Iron	·890 „	1·33
Bismuth	1·068 „	1·12

All the constants in (21) have been determined by experiments of various kinds, and we can calculate from them the value which our theory predicts for the ratio k/σ at 18° C.

We have $\alpha T = \frac{1}{2}mu^2$: but, by the kinetic theory of gases, if p is the pressure and V the volume of one gramme of any gas, $pV = \frac{1}{3}u^2$. Now 1 c.c. of hydrogen at a pressure of 760 mm. of mercury and a temperature of 0° C. weighs $8{\cdot}96 \times 10^{-5}$ grms.: or one gramme occupies $1{\cdot}189 \times 10^4$ c.c. at the same pressure and at 18° C. A pressure of 760 mm. of mercury is the weight of 76 c.c. of mercury per cm.² or $76 \times \rho \times g$, where ρ is the sp. gr. of mercury (13·596) and g is 981. Hence

$$p = 76 \times 13{\cdot}596 \times 981 = 1{\cdot}014 \times 10^6,$$

$$u^2 = 3 \times 1{\cdot}014 \times 10^6 \times 1{\cdot}189 \times 10^4 = 3{\cdot}617 \times 10^{10},$$

$$\frac{\alpha T}{m} = 1{\cdot}808 \times 10^{10} \text{ or } \frac{\alpha}{m} = 6{\cdot}216 \times 10^7,$$

where m is the mass of the hydrogen molecule. It is also known that 1 coulomb of electricity, or 3×10^9 electrostatic units, liberates

$1{\cdot}036 \times 10^{-5}$ grammes of hydrogen. But each atom of hydrogen carries a charge ϵ and each molecule a charge 2ϵ. Hence $1{\cdot}036 \times 10^{-5}$ grammes of hydrogen contain $\frac{3 \times 10^9}{2\epsilon}$ molecules. Therefore m, the mass of one molecule

$$= \frac{1{\cdot}036 \times 10^{-5} \times 2}{3 \times 10^9} \epsilon = 6{\cdot}907 \times 10^{-15} \epsilon.$$

Hence

$$\frac{\alpha}{\epsilon} = 6{\cdot}216 \times 6{\cdot}907 \times 10^{-8} = 4{\cdot}292 \times 10^{-7}.$$

Therefore $$\frac{k}{\sigma} = \frac{4}{3}\left(\frac{\alpha}{\epsilon}\right)^2 T = {\cdot}715 \times 10^{-10}.$$

It will be seen that the calculated value of the ratio k/σ is very near the value observed for the best metallic conductors. We are able to predict from observations on the electrolysis of sulphuric acid and the density of hydrogen a relation between the thermal and electrical conductivities of copper. No more striking evidence can be offered of the truth of the essential features of our theory, or of its importance: the greatest merit which a scientific theory can have is that it connects diverse phenomena.

(It should be pointed out that the only assumptions, peculiar to the electron theory, which have been introduced in the calculation are:—(1) that the current is carried by particles each of which carries the charge ϵ, and (2) that these particles share in the motion of heat agitation of the molecules of the metal. No additional support has been obtained for the results of Chapter V. concerning the mass of the electrons and the sign of the charge on them.)

No importance must be attributed to the fact that the observed value of k/σ are all slightly larger than those calculated: for the precise value of the numerical coefficient of the expression obtained in (11) depends on the nature of the assumption concerning the collisions between electrons and atoms. Drude, Lorentz and Thomson have all made equally plausible hypotheses on this point and have arrived at slightly different results. But all theories agree in making the quantity k/σ the same for all metals and hence the discrepancies shown in the table are of some importance. Part of them no doubt may be attributed again to the fact that the correct assumption as to the nature of the heat motion of the electrons has not been made, and that the ratio of the kinetic energy of the electrons to that of the atoms differs in different substances: but there is another obvious cause of error in our calculation.

We have left the action of the atoms out of account. It is probable that they cannot carry any part of the electric current, but they must play some part in the conduction of heat by colliding with their neighbours and thereby communicating to them part of their energy. Since the worse conductors contain fewer electrons than the good conductors, the part played by the atoms in the former must be relatively more important than in the latter, and it is to be expected, as is found in experiment, that the divergency from the Wiedemann-Franz law will be greater for the worse conductors.

However it is unlikely that this is the whole explanation of the matter. The thermal conductivity of such a material as sulphur, the electrical conductivity of which is practically nil, must be due wholly to the atoms: some estimate of the part played by the atoms in thermal conductivity is thus obtained. But the thermal conductivity of sulphur is only about one fortieth part of that of bismuth: even if we subtract the thermal conductivity of sulphur from that of bismuth, and suppose that the remainder is the conductivity due to the electrons, the discrepancy between the value of k/σ calculated and observed is not removed. The difficulty must be left unsolved, as indicating one of the points to which further investigation must be directed.

Thermo-electricity.

9. It has been proved that, if the concentration of the electrons in a metal (*i.e.* the number in unit volume) varies with the temperature, there will be an electric field in a metal which is unequally heated: but there will be no current because the force on the electrons due to their unequal concentration will balance that due to the electric field. But if an external field is applied so that a current flows, the effect of the internal field will become apparent.

To fix our ideas, suppose that the distribution of the electrons is such that the end on the right possesses a negative charge in virtue of the unequal heating, and that on the left a positive charge. Let the potential difference between the ends of the bar due to these charges be v. Suppose that two terminals maintained at a difference of potential V, by some external source be applied to the ends of the bar, and let the sign of V be opposite to that of v, so that the external and internal fields are in opposite directions. Then there is a total potential difference $V-v$ tending to send a current through the bar from right to left. If R is the resistance of the bar the current will be

$$C=\frac{V-v}{R} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22).$$

But the heat developed by the current in its passage is equal to the electrical energy lost: this energy is the product of the current and the total fall of potential between the external terminals, *i.e.* CV. But $C=\frac{V-v}{R}$, or $V=CR+v$, hence the total heat developed is C^2R+Cv, and the heat developed by unit current is R^2+v. If the bar had not been heated unequally, the heat developed by unit current would have been R^2, by Joule's law. Hence the unequal heating causes the development of an 'extra heat' per unit of current equal to v, the internal potential difference.

If the external potential difference be applied in the opposite direction, the same calculation can be carried through with the sign of V changed: it will be found that the extra heat per unit of current is $-v$. *Accordingly the rate at which heat is developed in the bar will depend upon the direction in which the current flows.*

If the law of the relation between the concentration of the electrons and the temperature given by (14) be correct, the concentration of the electrons will decrease with the temperature, and the hotter end of the bar will be negative. The current will flow from the positive to the negative terminal of the external field: hence, if the current flows from the hotter end to the colder, the external field will be in the opposite direction to the internal field: the extra heat will be positive in sign and the heat developed in the bar by unit current will be greater than if the bar were equally heated.

There is another cause which tends to make the heat developed in an unequally heated bar vary with the direction of the current. When the current flows, a stream of electrons is flowing from one end of the bar to the other. If the stream is flowing from the hotter end to the colder, the electrons from the hot end, possessing a great kinetic energy, will be distributed over the bar, while those from the cold end, possessing a smaller kinetic energy, will be carried out of the bar. Hence if the stream flows from the hotter end to the colder the heat developed in the bar will be greater than if the stream flows in the opposite direction. The electrons are negatively charged, and the current is said to flow in the direction of the equivalent stream of positive electricity: hence the direction of the stream of electrons is opposite to that of the current. Accordingly if the current flows from the hotter end to the colder, the heat developed will be less than if it flows in the reverse direction.

These two effects in an unequally heated bar are opposed: which of the two is the greater and determines the sign of the total effect can only be determined by a knowledge of the properties of the actual metal investigated. We should expect that in some metals one effect and in others the other would prevail.

The Thomson effect.

10. The phenomena that have just been described are well known to physicists under the name of the Thomson effect: they were first observed by Lord Kelvin (Sir William Thomson), and described by him in terms of his conception of the 'specific heat of electricity.' He imagined a current of electricity as a stream of some substance having a specific heat like ordinary materials: just as greater heat will be developed in an unequally heated tube through which water is flowing if the stream flows from the hotter end to the colder, so a bar through which electricity is flowing will be hotter if the current flows from the hotter end to the colder. This effect has been detected in many metals, among which bismuth is preeminent. But, in other metals such as antimony, the bar is hotter if the current flows from the colder end to the hotter: in such cases the specific heat of electricity is said to be negative—a conception which is inapplicable to material bodies.

If we assume that the relation (14) between the temperature and the concentration of the electrons is correct, it can be shown that the two opposing effects nearly balance one another, the total effect being only one-third of that due to the second cause: and that the magnitude of the effect should be the same in all metals and of the same sign as that observed in bismuth. It is not to be expected that a calculation based on the neglect of so many complications as have been excluded in our work will lead to complete accordance with observation: but it is worth noting that the magnitude of the Thomson effect is of the same order in all metals, and that this order is in rough agreement with that predicted. Calculation shows that the quantity of heat which should be developed when unit current flows down a temperature gradient of one degree centigrade per centimetre is 10^{-4} calories, whereas the largest observed effect (in bismuth) is $\cdot 3 \times 10^{-4}$ calories.

The concordance, though not so complete as that of § 8, must be judged satisfactory. The phenomena are more complicated and the experimental data are less precise. Indeed it has been doubted whether the conduction of heat by the metal under observation has not prevented the true Thomson effect from being measured with any approach to accuracy.

The Peltier effect.

11. Intimately connected with the Thomson effect is the Peltier effect. When a current passes across the junction of two metals, there is, in general, an evolution or absorption of heat: and, conversely, if the two junctions of two metals forming a circuit are maintained at different temperatures, a current will flow round the circuit.

An explanation of these important phenomena is given at once by our theory. We have already noted that the electrons, like the molecules of a gas, tend to diffuse from a place where they are plentiful towards a place where they are scarce. If, then, two metals, in which the concentration of the electrons is different, are placed in contact, the electrons, being able to pass freely across the boundary of the metals, will diffuse from that metal in which the concentration is large towards that in which it is small. This diffusion will continue, as we saw when considering the Thomson effect, until the electric field set up by the motion of the electrons away from their compensating positive charges is sufficient to balance the difference of pressure due to the difference of concentration.

There will accordingly be a 'contact difference of potential' between the metals. If an external potential difference is applied across the junction so that a current flows, there will be a development of heat if the external potential difference is of the opposite sign to the contact potential difference, and an absorption of heat if it is of the same sign.

The relation between the concentration of the electrons, the temperature and the contact difference of potential can be calculated easily. It appears that

$$V = \frac{4}{3}\frac{\alpha T}{\epsilon}\log_e\left(\frac{N_1}{N_2}\right) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23),$$

where V is the contact potential difference, and N_1, N_2 the concentration of the electrons in the two metals. The calculation has been made on the assumption that the whole of each of the two metals in contact is at the same temperature. This condition is never realised in actual measurements of the Peltier effect, for as soon as a current passes round a circuit of two metals, one of the junctions across which it flows is cooled and the other is heated. This unequal heating of the metals causes the Thomson effect to appear: there is a disturbance of the distribution of the electrons in virtue of diffusion along each metal as well as in virtue of diffusion across the junction. The expression (22) would only be accurate if there were no Thomson effect. But the thermodynamical theory shows that if there were no Thomson effect the Peltier effect should be proportional to the absolute temperature in accordance with (23).

There are many considerations which show that the concentration of the electrons in different metals is not very different, and that therefore the term $\log\frac{N_1}{N_2}$ in (22) is not very large. If we assume that the formula is correct, we can calculate from the observed Peltier heat the ratio of the concentration of the electrons in different metals. Thus let us take the case of copper and zinc.

When one coulomb of electricity (or 3×10^9 electrostatic units) passes across the junction from copper to zinc, $3{\cdot}66 \times 10^{-4}$ calories are absorbed. But heat is absorbed when the external field is in the same direction as the internal field: hence the copper is positive to the zinc. Now when one unit of electricity passes through a difference of potential of one unit, one erg of work is performed and the equivalent quantity of heat absorbed. If V is the contact difference of potential in electrostatic units, the number of ergs of work done when 3×10^9 electrostatic units pass across the junction is $V \times 3 \times 10^9$. But the work done is $3{\cdot}66 \times 10^{-4}$ calories, or

$$3{\cdot}66 \times 10^{-4} \times 4{\cdot}19 \times 10^7 = 1{\cdot}53 \times 10^4 \text{ ergs.}$$

Hence $V = \dfrac{1{\cdot}53 \times 10^4}{3 \times 10^9}$ electrostatic units (or $1{\cdot}53 \times 10^{-3}$ volts).

Therefore, if we put in (23)

$$\frac{\alpha}{\epsilon} = 4{\cdot}29 \times 10^{-7} \text{ (see § 8)},$$

$$V = {\cdot}51 \times 10^{-5} \quad \text{and} \quad T = 291 \text{ (18° C.)},$$

we obtain

$$\log_e \left(\frac{N_{Cu}}{N_{Zn}}\right) = \frac{{\cdot}51 \times 10^{-5}}{1{\cdot}33 \times 4{\cdot}29 \times 10^{-7} \times 291}$$

$$= {\cdot}306,$$

and

$$\frac{N_{Cu}}{N_{Zn}} = 1{\cdot}03.$$

The Peltier effect between copper and zinc is rather smaller than that between most pairs of metals. It is greatest between bismuth and antimony: for these metals the ratio is about 3·3. It is probable on other grounds that the ratio of the concentration of the electrons in different metals as calculated from (23) is somewhat too small. One reason for this underestimation is evident. It must be remembered that the number of electrons in the metal is determined by an equilibrium reached between the number produced from the atoms and those disappearing by collision in the same time. If the electrons diffuse away, as we have supposed, from the place where they were produced, the number disappearing by collision at that place will decrease: the concentration of the electrons will rise and the diminution caused by diffusion will partially be compensated. Accordingly the difference of potential observed will be less than that calculated on the assumption that the total number of electrons present in the two metals is uninfluenced by their distribution.

In a later chapter we shall return to the problem of the number of electrons in a metal.

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CHAPTER VII.

CONDUCTION IN A MAGNETIC FIELD.

General Principles.

1. WHEN a current of electricity flows through a metal, the free electrons (supposed to carry a negative charge) move in the opposite direction to the current. But a charged body moving in a magnetic field is subject to a force at right angles both to the direction of its motion and the direction of the field. Consequently if a metal bar, along which a current is flowing, be placed in a magnetic field perpendicular to the direction of the current, the electrons will tend to be deflected from their course and to move towards the one or other of the edges of the bar. Fig. 13 shows the relation which will hold between the directions of the current, the magnetic field and the deflection of the electrons.

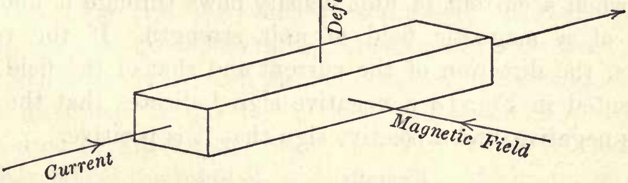

Fig. 13.

It may be observed, in passing, that the relation of the direction of the deflection of the electrons to that of the current will be the same whichever sign they carry: for, if the sign of the electron is reversed, the direction of its motion as well as the direction of the force upon it, when moving in a specified magnetic field, is reversed.

This flow of the electrons across the breadth of the bar will not be permanent: the electrons will accumulate in the upper side of the bar (according to Fig. 13), which will acquire a negative charge, while the lower edge being deprived of electrons will acquire a positive charge: an electric field will be set up tending to oppose the transverse current. At the same time a diffusion of electrons will take place from the parts where they are plentiful to those where they are scarce, and equilibrium will be attained when the number flowing upwards under the deflecting action of the magnetic field and the counteracting transverse electric field is equal to the diffusion current flowing downwards. In this state of equilibrium there will be a difference of potentials between the upper and lower edges of the plate, which may be observed by connecting those edges to the opposite terminals of an electrometer while a current flows through the plate under the influence of a strong magnetic field.

The Hall effect.

2. This difference of potential was first observed in 1879 and has been called after the name of its discoverer the 'Hall effect': it has been studied subsequently by many observers, among whom Ettingshausen and Nernst are prominent, in view of its great importance for electronic theory. The following table gives the value of the Hall effect for several metals: the numbers in the second column represent the potential difference between the opposite edges of a bar one centimetre broad when a current of unit density flows through it under the action of a magnetic field of unit strength. If the relation between the direction of the current and that of the field are as represented in Fig. 13 a negative sign indicates that the upper edge is negative, and a positive sign that it is positive.

Bismuth	−10·1
Nickel	−·024
Sodium	−·0025
Silver	−·0008
Cobalt	+·0046
Iron	+·0113
Antimony	+·192
Tellurium	+530

According to the theory just given, the direction of the difference of potential ought to depend only on the sign of the charge carried by the electrons. Whatever that sign, they will accumulate at the top of the plate in Fig. 13, and hence the upper side ought to have the same sign as the electrons. We have assumed the electrons to carry a negative charge, and hence the sign of the difference of potential predicted is that found in bismuth. The simple considerations that have been advanced will not explain the reverse sign which is found in antimony and tellurium. This discrepancy is important, but the discussion of it will be postponed till some phenomena closely connected with the Hall effect have been considered.

Thermal Changes.

3. When the ends of a metal bar are kept at different temperatures, so that there is a flow of heat along the bar, we have seen (p. 114) that there is no motion of the electrons as a whole from one end of the bar to the other: there are the same number traversing it in one direction as in the other. But the electrons which are travelling from the hot end to the cold end possess a greater kinetic energy and a greater velocity than those which are travelling in the opposite direction. Now the force on an electron travelling in a magnetic field tending to deflect it at right angles to its motion and to the field is proportional to the velocity of the body. Hence the force tending to deflect the electrons travelling from the hot end to the cold will be greater than that tending to deflect (in the opposite direction) those travelling from the cold end to the hot. The electrons will tend to move on the whole in the direction in which those moving from the hot end to the cold end are deflected by the magnetic field. If the directions of the flow of heat and of the field are those represented in Fig. 13, the negative electrons will accumulate on the lower side of the bar. Hence according to the arguments of the last paragraph, there will be a transverse difference of potential, the upper side of the bar being positive to the lower whatever the sign of the electrons. This transverse difference of potential accompanying a flow of heat in a magnetic field has been studied by Ettingshausen and Nernst and shown to be in the direction indicated by theory in all the metals investigated, even in those in which the Hall effect is of the wrong sign.

In addition to these transverse electrical effects there will be transverse thermal effects. For, when an electric current flows through the bar, the negative electrons which are deflected upwards according to the scheme of Fig. 13 have acquired kinetic energy by moving under the force exerted on them by the electric field: they will possess a kinetic energy greater than that due to their heat motion. When they strike the molecules at the top of the bar they will communicate to them some part of their energy, so that the molecules in the top of the bar will possess a greater kinetic energy than the molecules in the lower part of the bar. In other words, the top of the bar will be at a higher temperature than the bottom. Similarly, when the current of heat flows through the bar, the electrons moving in the direction of the current are deflected downwards and carry their excess of energy into the lower part of the bar: the bottom of the bar will be at a higher temperature than the top.

Experiments on these transverse thermal effects are exceedingly difficult, but they have been detected by the workers who have done so much research in this department: it appears that the transverse difference of temperature due to the passage of a current of electricity is in the same direction in all metals, but that due to the passage of a current of heat shows the discrepancies which we noted in dealing with the Hall effect: it is not, as theory would predict, in the same direction for all metals.

To sum up, then, there are four transverse conduction effects in a magnetic field:—

(1) A difference of potential due to the passage of an electric current.

(2) A difference of temperature due to the passage of a heat current.

In some metals both of these show divergence from the direction predicted by our simple theory.

(3) A difference of temperature due to the passage of an electric current.

(4) A difference of potential due to the passage of a heat current.

In all metals these effects are in the direction predicted by our theory.

Longitudinal, Electrical and Thermal effects.

4. The existence of these transversal effects will bring fresh forces into action. We have seen that there is a difference of the velocities not only of those electrons which are moving in a direction parallel to the electric current or the thermal current, but also of those electrons which are moving at right angles both to the current and to the magnetic field, that is from top to bottom or bottom to top in Fig. 13. The velocity of the electrons which are moving upwards is greater than that of those which are moving downwards. These electrons are moving at right angles to the magnetic field, and will be subject to forces tending to deflect them perpendicularly both to the magnetic field and to the direction of motion, that is along the direction of the current. Those moving in opposite directions will be deflected in opposite directions, and, since they possess different velocities, by different forces. There will be a tendency for the electrons to accumulate on the left side of the bar (in Fig. 13), and there will be as before a difference of potential and a difference of temperature between the ends of the bars superimposed on the difference of potential or of temperature which is sending the current through the bar. A little consideration will show that the direction of the difference of potential or of temperature is independent of the sign of the charge on the electrons and will always be such as to oppose the difference which is sending the current through the bar. The difference will be very small, and the difficulty of detecting it very great. But in one case it is of some importance. When a current of electricity is flowing through the bar in a magnetic field there will be a difference of potential tending to oppose the passage of the current. A smaller current will flow when the magnetic field is in action than when it is removed, and the presence of the magnetic field will appear to increase the resistance of the metal. This change of resistance in a magnetic field has been observed and measured: it is considerable in the case of bismuth, which shows a large Hall effect, and is often employed for measuring the intensity of strong magnetic fields. In accordance with the predictions of theory, it is found that in most cases there is an increase of resistance in a magnetic field, but in others, especially those of the magnetic metals, it is found that there is an increase of resistance in small magnetic fields but a decrease in stronger fields.

The difficulty with the 'Hall Effect.'

5. We must now consider briefly the discrepancy noted already, that the sign of the transverse effects (1) and (2) (p. 128) is not in all metals that predicted by our simple theory. The discrepancy has attracted a good deal of attention—in my opinion far more than it deserves.

A little consideration will show that, while the sign of the effects (3) and (4) does not, on our theory, depend on the sign of the charge carried by the electrons, the sign of the effects (1) and (2) does depend on that sign. Accordingly, if we were to suppose that the current is carried by freely moving positive particles as well as by negative, the difference in sign in different metals could be attributed to the preponderance of the action of one or other of the groups of oppositely charged particles. To a reader, who is accustomed from the older theories of electrostatics to regard the two kinds of electricity as identical in everything except their mutual action, no explanation could seem simpler or more desirable: but if there is one thing that recent research in electricity has established, it is the fundamental difference between positive and negative electricity. We have met with one of these differences already in dealing with the phenomena of magneto-optics, but the most important distinctions will be considered in the next part, the conclusions of which we must anticipate.

We have seen (p. 104) that, whatever is the nature of the charged particles carrying the electric current, it is essential that the particles in all metals should be identical. Reasons have been given already for a belief in the existence of negatively charged particles common to all materials and further evidence will accumulate as we proceed: but until quite recently no evidence whatever could be produced of the existence of similarly universal positive particles. At the present time, however, it seems possible that such particles may exist, though it seems unlikely that they could be present in the conditions obtaining in a solid metal. The hypothesis of 'positive electrons' cannot be dismissed summarily as it might have been a few months ago.

But though universal positive particles may exist, it is extremely improbable that they could play any part in metallic conduction. While the negative electrons are very much smaller than atoms, the 'positive electrons,' if there are such things, are as large as the atom of hydrogen. While there is no difficulty in conceiving that the very small negative electrons could move freely in a solid body, it is hard to imagine a body as large as an atom flying about among the closely packed molecules of a solid body. Even in a liquid electrolyte the frictional resistance on the ion (or charged atom) is about four thousand times[1] that on the negative electron in a metal, while the non-existence of any solid electrolyte may be attributed perhaps to the impossibility of ions moving at all in a solid. At any rate it appears that, even if the freely moving 'positive electrons' in a solid were as numerous as the negative, they could certainly not carry more than one four-thousandth part of the current[2].

There are other reasons less direct but none the less conclusive for rejecting any attempt to explain the varying sign of the Hall effect by the action of free positive particles. The conditions which must necessarily be imposed upon the relative motion of the positive and negative particles, in order that the metal as a whole may acquire no charge of electricity, are difficult to reconcile with the observed phenomena of thermoelectricity. Again, though the discrepancy of the Hall effect might be explained by positive free electrons, the variation of the sign of the longitudinal effect cannot be attributed to such a cause, nor can the variation of the Wiedemann-Franz ratio. Some fresh hypothesis must be

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[1] For the *electrons* in a metal r may be calculated thus:

From (16), p. 112, $$r = \frac{4\alpha T}{lu} = \frac{N\epsilon^2}{\sigma}.$$

For an average metal $\sigma = 10^{17}$ about: N is probably about 10^{23} (see Chap. X.),

hence $$r = \frac{10^{23} \times (3{\cdot}4 \times 10^{-10})^2}{10^{17}} = 10^{-13}.$$

For an *electrolyte* the velocity of the hydrogen ion in an electric field of 1 volt ($\frac{1}{300}$ E.S.U.) per cm. is 3×10^{-3}:

$$r = \frac{X\epsilon}{v} = \frac{3{\cdot}3 \times 10^{-3} \times 3{\cdot}4 \times 10^{-10}}{3 \times 10^{-3}} = 4 \times 10^{-10}.$$ See (15), p. 111.

[2] Later we shall have to consider positive particles moving with enormous speeds penetrating solid bodies, but they only penetrate by the expenditure of a large amount of energy in breaking up the obstructing molecules in their path.

introduced to reconcile these phenomena with theory, and, if we cannot now produce one which is perfectly adequate, it is clearly better to wait in the hope of finding a single assumption which will clear up all the difficulties. I do not think that, apart from historical reasons, anyone would have even troubled to argue against the hypothesis of positive electrons: such discussion as there has been arose from the fact that Drude, in his earlier theory, developed before all our present knowledge of positive electricity had accumulated, based his calculations on the assumptions of two kinds of electrons. In his latest memoirs on the subject he adopted the simpler conception which has formed the basis of the present chapter.

The failures of the theory which have been noted in this chapter will doubtless be solved when we have a more complete knowledge of the relation of the electrons to the atoms: and it is not difficult to see how further information on this point may throw light on the Hall effect. It must be remembered that the electrons carrying the electric current are not moving uniformly in the same direction. They are flying about in all directions, but on the whole move in the opposite direction to the current. When they are moving with the current, they are deflected in one direction, when they are moving against it, they are deflected in the other. If it be assumed, as we have assumed, that the electrons, apart from the influence of the electric field, move in equal numbers in all directions, then the direction of the resultant deflection of the whole number of electrons will be that of the electrons moving against the current. But if, as is quite possible, the electrons tend to be ejected from their atoms along the line of the current, then the resultant deflection may be in the direction of the deflection of those which are moving with the current and the Hall effect will appear to have the 'wrong' sign.

It is probable that some such explanation as this will be adopted eventually to explain the discrepancy in the electrical transverse effect. But, in considering the thermal effect, the positively charged atoms, which, though not free to move continuously in one direction, are free to vibrate about a fixed position, must be taken into account. Their vibrations will doubtless be influenced by the forces due to the magnetic field, and may play an important part in determining the direction of the whole transversal thermal effect.

It will be convenient to sum up concisely the matter contained in the last five chapters.

Summary.

Fundamental assumption of the electronic theory. The electrical properties of matter can be attributed to the action of discrete particles bearing an electric charge.

The dielectric constant. In conductors these particles are free to move except in so far as their motion may be hindered by a frictional resistance. In insulators the particles are held to fixed centres by elastic forces.

The refraction of light. These charged particles possess mass. The light which is selectively absorbed or selectively emitted by a substance has a period of vibration identical with that of the vibrations which the charged particles execute under the elastic forces. These forces are determined by the structure of the atoms of the substance.

Magnetism. Magnetic induction in a body represents the change of the motion of the charged particles in it due to the action of the external magnetic field. The phenomena of magneto-optics show that some at least of the particles are common to all elements. These universal particles bear a negative charge of $3{\cdot}4 \times 10^{-10}$ electrostatic units: their mass is about $\frac{1}{1700}$ of that of a hydrogen atom. They are probably constituents of all atoms. To these universal particles the name 'electron' is given. That term will henceforward be used in a narrower sense than that given to it on p. 49; it will not be applied to all particles which bear the charge ϵ, but only to those which are common constituents of all bodies. Accordingly the only electrons, of the existence of which there is evidence derived from magneto-optical phenomena, bear a charge of negative sign.

Conduction of electricity and heat. In a conductor there are free charged particles which share in the heat motion of the molecules of a solid and can move through the metal under the action of an electric field. It is almost certain that these particles are the universal electrons.

These hypotheses provide an explanation qualitatively sufficient of all the phenomena which we have taken into consideration. The calculated numerical constants do not always agree perfectly with those found by experiment; the discrepancy must be explained by further hypotheses concerning the relation of the negative electrons to the atoms, which remain positively charged after the loss of one or more electrons.

We will now turn our attention to an entirely new set of researches, which differ both in their subject-matter and in their method from those which we have considered hitherto, but confirm in the most remarkable manner the conclusions at which we have arrived.

REFERENCES FOR CHAPTER VII.

Hall Effect, E. H. HALL, *Phil. Mag.* (5) x. p. 136.

Thermal Effect, ETTINGSHAUSEN and NERNST, *Wied. Ann.* XXIX. p. 343, 1886. A summary of experimental results is given in WIEDEMANN'S *Handbuch der Physik*, Vol. v. pp. 449–468 : for later work see GRUMNACH, *Phys. Zeit.* VII. p. 729, 1906.

For the theory of the phenomena see the memoirs of DRUDE, THOMSON and LORENTZ given in the references of the last chapter.

A long series of papers by EVERDINGEN dealing with all the phenomena described in this chapter is to be found in the *Proc. Amst. Acad.*, 1896–1899. An important Memoir dealing with the theory by the same author—*Arch. Néerl.* (2) VI. 294–302, 1902—should also be consulted.

PART III.

THE ELECTRON THEORY. INDUCTIVE METHODS.

CHAPTER VIII.

THE CONDUCTION OF ELECTRICITY THROUGH GASES.

Preliminary.

1. THE electrical conductivity of gases in their normal state and at atmospheric pressure is extremely small: it is only within the last few years that it has been established irrefutably that gases conduct at all. If two oppositely charged plates, supported on the most perfect solid or liquid insulators obtainable, are placed side by side in a vessel full of air or other gas at atmospheric pressure, the charges gradually neutralise each other: but it remained doubtful for a long time whether the conduction might not take place through the supports and not through the gas. It might be thought that the point could be settled easily by pumping out the gas from the vessel and noting whether the conduction decreased, but even when the experiment had been performed and a decrease found, it might still be attributed to an improvement in the insulation of the supports, due to the drying of their surface: for it is known that in a moist atmosphere a film of water condenses on the surface of most insulators and increases their conductivity. Even when the question was answered finally, the conductivity of gases in their normal condition was found to be so small that it could not have been investigated satisfactorily unless light had been thrown on the problem from other sources.

It has been known for a long time that the conductivity of a gas can be increased by subjecting it to certain influences: for instance, in the neighbourhood of a red-hot body a gas conducts quite well, as do also the gaseous products of combustion proceeding from a flame. Again, it was known that, though only a very small current could be sent through a gas by an electric field of moderate intensity, when the electric intensity was increased sufficiently a point was reached suddenly at which a very considerable current would pass in the form of a 'spark discharge'; and, further, that the smaller the pressure of the gas the smaller the electric intensity under which the spark would pass. Accordingly the idea arose that a gas prevented in some way the passage of a current, but, when subjected to a force sufficiently great, broke down and permitted the passage. From such an idea it must naturally follow that a current would pass with perfect ease in a space devoid of gases and of all matter; but unfortunately it was found that when the exhaustion of the gas was pushed beyond a certain limit the resistance increased instead of diminishing, and that in the best vacua obtainable no current whatever could be made to pass.

In 1896 a large number of facts had accumulated, but no adequate theory had been suggested to connect them. The whole subject was in a state of confusion, and yet it was felt by many that in the explanation of the electrical properties of gases was to be found the answer to many of the most pressing questions in physics. When at last the clue was found, the subject developed with a rapidity unprecedented in the history of science, and the study of the conduction of electricity through gases has proved, as was expected, the path to a new and undeveloped world of science. Every reader should glance at the chapter on conduction in gases in Prof. J. J. Thomson's *Recent Researches in Electricity and Magnetism*, written in 1893, just before the clue was found, and compare it with the same author's *Conduction of Electricity through Gases*, published exactly ten years later. The attitude of the writer towards electrical problems had changed entirely in that short interval.

In 1895 Prof. W. C. Roentgen, of Warburg, discovered the famous rays which bear his name. The attention of the public has been drawn to the discovery chiefly in its connection with medical problems: but great as is the revolution which the use of Roentgen rays has effected in surgical methods and great as is the alleviation which they have brought to suffering mankind, their importance is almost greater to the physicist than to the surgeon, by reason of the light that they have thrown upon the branch of physics which forms the subject of this chapter. It is true that their importance is historical rather than logical, and that we know now that the fundamental discoveries might have been made as well if the rays had never been known, but as a matter of fact they proceeded from the study of those rays.

In what follows, the historical method will be abandoned entirely and an attempt made to present our knowledge as it now stands in a logical order.

Conductivity under the action of Roentgen rays.

2. When Roentgen rays pass through a gas they render it a conductor of electricity, or, to speak more accurately, they increase its conductivity enormously: if the rays cease to act the conductivity dies away rapidly, and after a few seconds the gas is no more conducting than before exposure to the rays. To show this action of the rays air may be drawn through the apparatus sketched in Fig. 14. The gas first enters the vessel *B*, in which it is exposed to the action of the rays, then passes down the tube *C* into the vessel *A*, in which is placed a gold-leaf electroscope. So long as the rays do not act on the gas in the vessel *B*, the electroscope will lose its charge, as indicated by the divergence of the leaves, extremely slowly: but as soon as the air is rendered conducting in *B* and drawn into *A*, the charge escapes to the walls and the leaves collapse. However, if

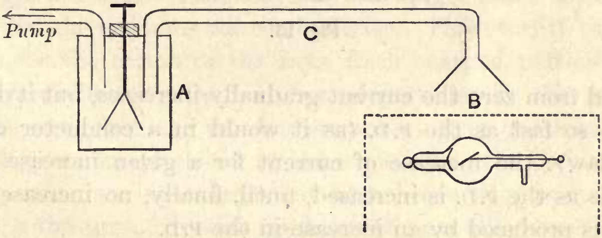

Fig. 14.

the air be drawn through very slowly, the air will have lost the conductivity that it had gained in *B* before it reaches *A* and the rate of discharge of the electroscope (commonly known as the 'leak') will be unaffected by the action of the rays. If a plug of cotton-wool be placed in the tube *C*, however fast a stream of air may be drawn through the apparatus, the leak will be unaffected even when the rays are acting: the air is robbed of its conductivity by passage through the cotton-wool.

The relation between the potential-difference between the boundaries of the gas and the current through it is deserving of attention: it is shown by the diagram in Fig. 15. As the P.D. is increased from zero the current gradually increases, but it does not increase so fast as the P.D. (as it would in a conductor obeying Ohm's law): the increase of current for a given increase of P.D. decreases as the P.D. is increased, until, finally, no increase in the current is produced by an increase in the P.D.

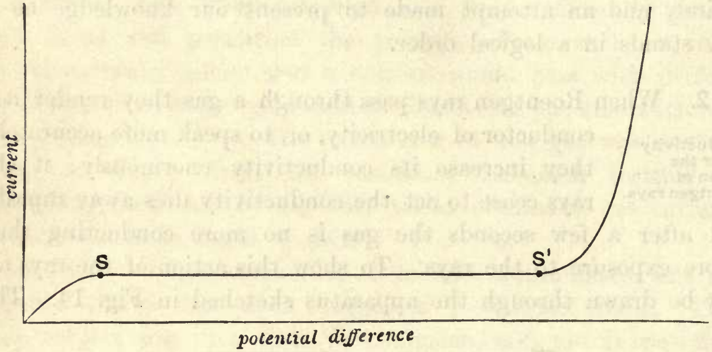

Fig. 15.

This limiting current, the strength of which is independent of the electric intensity and depends only on the volume and nature of the gas and on the rays acting, is termed the 'saturation current.' It is represented by the part of the curve *SS'*. If the electric field be increased still further a stage will be reached at which a spark will pass through the gas and the current will increase once more: in vessels of such size and shape as are ordinarily used in these experiments the intervals between the saturating P.D. and the P.D. required to send a spark through the gas is very large. Unless the conductivity be very great an electric field of 1,000 volts per centimetre will send a saturation current through any gas at atmospheric pressure, whereas a field of the order of 30,000 volts per centimetre is required to produce a spark discharge at the same pressure.

Gaseous ions.

3. These facts were described and an explanation of them offered by J. J. Thomson and Rutherford in 1896. They supposed that when a gas is rendered conducting there are introduced into it, by some means or other, a number of particles charged, some with positive and some with negative electricity. The fundamental hypothesis of the electronic theory is thus introduced; it is assumed that conduction in gases, like all other electrical phenomena, may be attributed to the action of discrete charged particles. When the gas is subjected to an electric field in the vessel *A*, the positive particles move to the negative boundary and the negative to the positive: this motion of the charges constitutes the current through the gas. But, if the gas is left to itself under the action of no external field, the oppositely charged particles attract each other and coalesce in pairs, or 'recombine.' When once the particles have all recombined and their charges are neutralised, they move no longer under the action of a field and the gas loses its conductivity. However, if the gas is kept under the action of the rays, fresh charged particles are produced continuously: the number of particles in the gas at any time is such that the number recombining in one second is equal to the number produced in the same time: the gas is then in a steady state, and the number of particles in the gas does not change with the time. In this condition the gas shows no sign of possessing a charge as a whole, and hence the total charge on all the positive particles must be equal to the total charge on all the negative.

When the gas is passed through a plug of cotton-wool, the charged particles, which differ from the molecules of the gas, are retained, while the molecules pass through. It is not necessary to conclude that the particles are larger than the molecules, for the fact that they are charged, while the molecules are neutral, might account for their retention. The particles can also, and more conveniently, be removed from the gas by passing it through a tube in which it is subjected to a strong electric field: the charged particles, of course, adhere to the oppositely charged boundaries of the field.

Consider now the variation of the current with the P.D. in such a mixture of molecules and charged particles as has been imagined. The greater the strength of the field, the greater is the velocity with which the particles move and the shorter the time that is required for them to get from any part of the field to the boundary. But, while any particle is moving to the boundary, it is liable to encounter a particle of the opposite sign and coalesce with it, losing thereby its conducting power. The shorter the time of passage, the less likely will such an encounter be, and the greater the number of particles which start from any part of the field and arrive, still charged, at the boundary: hence the current, which is measured by the number of charged particles arriving at the boundary, will increase with the P.D. But it is clear that the current cannot increase indefinitely: it must reach a limit when the time occupied by the particles in reaching the boundary is so short that none recombine and the number arriving at the boundary per second is the number produced in the gas in the same time. This number multiplied by the charge on each particle gives the 'saturation current' through the gas. The further increase in the current observed with still stronger fields can only be attributed to an increase in the rate of production of the particles.

According to this explanation, the process of conduction is essentially the same, whether it take place in a metal or in an electrolyte or in a gas. In each case the current consists of a stream of charged particles, and the great difference between the laws governing the conduction in different states of matter is due only to a difference in the nature of the charged particles, in their mode of production, and in their relation to the medium which surrounds them. In view of the similarity between gaseous and electrolytic conduction, the nomenclature of the latter has been applied to the former. The moving charged particles are called 'ions,' and a gas, when it is rendered conducting, is said to be 'ionised': the process of ionising a gas is named 'ionisation': this word is also used quantitatively to express the concentration of the ions in the gas or the number per unit volume. The negative electrode is called the kathode and the positive the anode.

The great difference between the relation of current to potential difference in gases and that in solids or liquids arises from the difference in the mode of production of the ions. In neither solids nor liquids are the carrying particles produced at a constant rate independent of the field strength; consequently in neither solids nor liquids can a saturation current be produced. In solids the rate of production of the electrons probably increases with the field strength: in an electrolyte the whole number of particles available is determined by the quantity of solution present, and so long as any of the solution is left, the current increases with the potential difference. The state of 'saturation' is only attained when all the electrolyte has been removed.

For the present we will leave on one side questions as to the method of production of the ions: with a few exceptions, which are not of any essential importance, the properties of the ions in a gas depend only on the nature and state of the gas and not on the mode of their production. These properties we will now proceed to consider. Two of them, the charge and the velocity, are susceptible of direct measurement, and from our knowledge of them our information concerning the other properties of the ions must depend.

The charge on an ion.

4. At this stage of our investigation it is almost a foregone conclusion that the charge on an ion will turn out to be the electronic charge ϵ, or a simple multiple of it. As a matter of fact, the question as to the charge on an ion was asked before the evidence recounted in the last part was developed. But, even if the historical order had been reversed, the experimental researches described below would have been of fundamental importance. For, though we know roughly the value of the unit charge, the information is by no means as accurate as must be desired. The most definite information is derived from electrolysis: observation enables the product of the charge by the number of molecules in one cubic centimetre of hydrogen to be determined with some accuracy, but for the knowledge of this number of molecules recourse must be had to the uncertain results of the kinetic theory of gases, from which values ranging from 10^{19} to 10^{20} have been deduced.

It may be noted that proof of the identity of the ionic (gaseous) and the electrolytic charges was available before the accurate determination of either. Zeleny showed that from his estimate of the velocity of the ions (see below), combined with certain results of Townsend on the diffusion of ions through the gases in which they are contained, it is possible to calculate the product of the ionic charge by the same number of molecules. For this product he found values ranging from $1{\cdot}00 \times 10^{10}$ to $1{\cdot}65 \times 10^{10}$, which agree well with the value $1{\cdot}22 \times 10^{10}$ deduced from electrolysis.

The direct determination of the charge on an ion was carried out by J. J. Thomson in 1898: his experiments afford one of the most admirable examples of scientific ingenuity and insight in the history of physics, but it is impossible here to do more than sketch their principle.

The charge on each ion in a gas can be ascertained if (1) the total charge on all the ions present in a gas at any given time and (2) the number of such ions present can be measured. The first quantity presents no difficulty. Let the gas be subjected to an ionising influence, such as Roentgen rays, until a state of equilibrium is reached between the number of ions produced per second by the rays and the number disappearing in the same time through recombination. Let the action of the rays be stopped suddenly and the gas exposed immediately to the action of a strong electric field. All the ions of either sign present in the gas will be driven to the boundary of the field, and, if the field be sufficiently strong, the time occupied in reaching the boundary will be so short that no appreciable recombination will have taken place in the meanwhile: the number reaching the boundary is equal to the number present in the gas before the field was put on. If the charge received by the boundary is measured by connecting it to an electrometer, the total charge on all the ions of one sign present in the gas can be ascertained. It may be remarked that, if the ionising agent consist of Roentgen rays, it is found that the charge on all the ions of either sign is the same[1].

The measurement of the number of ions present is a much more difficult matter, but has been effected by a most ingenious method devised by J. J. Thomson and based on a discovery due to C. T. R. Wilson.

It is known that the quantity of water that can exist in the state of vapour in a given volume of gas depends greatly on the temperature of the gas, and that, if a volume of gas saturated with water vapour is cooled, the excess of the water, or the amount representing the difference between that which the gas could hold at the higher temperature and that which it can hold at the lower, is deposited in the form of rain or mist. It was found by Aitken that this deposition of the superfluous water depends on the presence in the gas of solid particles, or dust, which act as nuclei for the formation of the liquid drops: if a gas is rendered perfectly free from dust by filtering it through cotton-wool, it can be cooled to a temperature very much lower than that at which a cloud would form in the presence of dust without any consequent condensation. The necessity for the presence of nuclei in the formation of drops can be shown readily to be a consequence of the existence of a surface tension in liquids, and it appears that, *ceteris paribus*, the larger the particle of dust the more efficient it is as a nucleus and the smaller the supersaturation of water vapour that it is possible to produce in its presence. (By 'supersaturation' is meant the ratio of the mass of water actually present in the gas to that which it could hold without condensation in the presence of very large nuclei.)

C. T. R. Wilson found that ions possess peculiar properties with respect to the formation of clouds in gases supersaturated with moisture. In his investigations he cooled the gases by expanding them adiabatically, thus producing a fall in temperature which could be calculated from the known values of the initial and final volume and the ratio of the specific heats of the gas. He found that it requires less supersaturation to produce a cloud in air, when it is ionised, than when it is not ionised: the ions act as

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[1] The method actually employed was somewhat different from that described, but was based on the same principle.

nuclei for condensation. This does not imply necessarily that the ions are larger than the molecules which are always present and available as nuclei, for it can be shown on theoretical grounds that a charged body should act as a more efficient nucleus than an uncharged body of the same size. In the absence of ionisation eightfold supersaturation is required to produce condensation, whereas fourfold supersaturation is sufficient for the same purpose in the presence of negatively charged ions, and sixfold supersaturation in the presence of positively charged ions. These values are independent of the degree of the ionisation and also, with a few exceptions, of the means by which it is produced. Further, no increase in the amount of condensation is caused by increasing the supersaturation between fourfold and sixfold.

These experiments give us valuable information as to the nature of the ions. (1) They show that there is a difference in the properties of positive and negative ions: it cannot be concluded that the negative ion is larger than the positive because it is a more efficient nucleus, for, even if the charge were the same in the two cases, a difference in the distribution of the charge might bring about the observed result; (2) that the properties of the ions are not determined solely by the method by which they are produced; (3) that the properties of all the ions of the same sign are the same: for if there were more than two kinds of ions present, more than two changes in the supersaturation required to produce condensation would be expected.

Wilson's discovery was used by J. J. Thomson to determine the number of the ions in a gas. The ionised gas, in which the total charge on all the ions had been determined, was expanded adiabatically and cooled to such an extent that a supersaturation between sixfold and eightfold was produced: a cloud formed round all[1] the ions of the gas, but not on the molecules. As soon as the cloud was formed it began to fall, because the drops were heavier than the air surrounding them: owing to the viscosity of the air,

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[1] If the number of ions present had been large, the water set free on cooling would not have been sufficient to go round, and a drop would not have been formed on *every* ion. In that case the observed result would have varied with the degree of the ionisation: but, by using sufficiently weak ionisations, a value for the charge was obtained which was independent of the degree of the ionisation. The independence of the ionisation also proved that few drops included more than one ion.

they soon attained a small and constant velocity which could be measured readily by observing the rate at which the upper boundary of the cloud moved down the vessel. Now Stokes has calculated the steady velocity with which a body of known dimensions moves through a medium of known viscosity under the action of a constant force, such as its own weight. He finds that if F is the force, a the radius of the body (supposed to be spherical), ρ its density, k the coefficient of viscosity of the medium, v, the steady velocity, is given by

$$v = \frac{F}{6\pi a k} \quad \text{.........................(1)},$$

or, if $F = mg$, the weight of the body, where m is its mass,

$$v = \frac{2}{9}\frac{ga^2\rho}{k} \quad \text{.........................(2)},$$

since $$m = \tfrac{4}{3}\pi a^3 \rho.$$

By applying this formula to the case under consideration, where the velocity, the force and the viscosity are known, a, the radius of the drop, can be determined. The total mass of all the drops is the mass of the water set free by the cooling, for the mass of the ion contained in each drop is so small, compared to the whole mass of the drop, that it may be neglected. The calculation of this total mass is somewhat complicated, but it depends on well-known principles, and for further details the reader may be referred to Thomson's account. If, then, the total mass of all the drops is M, the number of the drops n, which is the number of the ions, can be calculated from the relations:

$$n = \frac{M}{m} \quad \text{.........................(3)},$$

$$m = \tfrac{4}{3}\pi \rho a^3 \quad \text{.........................(4)},$$

$$a = \sqrt{\frac{9kv}{2g\rho}} \quad \text{.........................(5)}.$$

The charge on each ion, e, is the total charge on all the ions divided by the number of the ions.

As a first result of this research the result was obtained that the charge on each ion in a gas is $6{\cdot}5 \times 10^{-10}$ E.S.U. Subsequently more accurate observation led to the corrected value:

$$e = 3{\cdot}4 \times 10^{-10} \text{ electrostatic units.}$$

Experiments made in air and hydrogen gave results identical within experimental error.

H. A. Wilson has used a slight modification of the method, which obviates the necessity for the cumbrous calculation of the total quantity of water set free. The total charge on the ions is measured as before. The gas is then expanded and cooled so as to give a supersaturation between four and six, and to cause condensation on the negative and not on the positive ions. The velocity, v, with which the cloud falls is observed. The experiment is then repeated, with the difference that the falling cloud is exposed to the influence of a vertical electric field X, which exerts on the charged drops a force Xe accelerating or retarding the fall. The new velocity v' is measured.

Since, by (1), the velocity of the drop is proportional to the force on it, we have

$$\frac{v'}{v} = \frac{Xe + mg}{mg}, \text{ where } m = \tfrac{4}{3}\pi\rho a^3 \quad \ldots\ldots\ldots\ldots(6),$$

but from (2) $$v = \frac{2}{9}\cdot\frac{ga^2\rho}{k},$$

hence $$e = \frac{9\pi\sqrt{2}}{X}\sqrt{\frac{k^3v^3}{g\rho}}\cdot\frac{v'-v}{v}.$$

By this method Wilson found $e = 3{\cdot}1 \times 10^{-10}$. A few drops were found to bear double and treble this charge, but their numbers were too small to have exerted any influence in Thomson's measurements.

Since Thomson measured the average charge on both positive and negative ions and Wilson that on negative ions only, the coincidence between their results shows that the charge on an ion is independent of its sign.

It will be seen that the charge on an ion is that which has been used in Part II. for the electronic charge ϵ and has been found to accord perfectly with experiments. Wherever we can measure the charge on one of the discrete particles which form the basis of our theory, the same result is attained; whether the measurement is based on electrolytic or gaseous conduction, optics or the conductivity of metals, the charge is a universal constant. Its value is still uncertain within 10%, but the accuracy is sufficient for all the numerical calculations which we shall make. In passing it is well to note that if $\epsilon = 3{\cdot}4 \times 10^{-10}$, N, the number of molecules in 1 c.c. of gas under standard conditions (commonly known as Loschmidt's number), is $3{\cdot}6 \times 10^{19}$.

The velocity of the ions.

5. Measurements have been made of the velocity with which the ions move in an electric field and have led to some interesting results. The methods of measurement have been various and only two of them will be mentioned briefly.

The principle of the first method is to compare the velocity of the ion with that of a stream of air in which it is present. If a stream of air be blowing in one direction and an ion in that stream be moving under an electric intensity with an equal speed in the opposite direction, the resultant effect will be that the ion does not move at all, and will give up its charge to neither of the electrodes. This method was used by Zeleny, who first detected a difference in the velocity of the positive and negative ions.

The second method is less obvious but more convenient experimentally. Suppose that two plates A and B, parallel to each other, are maintained at a constant difference of potential, and that ions are produced by some means close to the plate A. Under the action of the electric field the ions of one sign will give up their charges to A while the others will move towards B. Just as they are going to reach B let the direction of the field be reversed, so that the plate which was formerly positive is now negative: these ions will be driven back towards A, while those of opposite sign move towards B; but as they are about to touch B in their turn, the field is again reversed and with it the direction of their motion. If then the direction of the current is reversed at constant intervals and these intervals are so short that they do not permit the ions to travel from one plate to the other before the field is reversed, the plate B will receive no charge: but if the interval is longer, so that the ions have time to get across before the reversal of the field, B will receive a charge. By measuring the time of reversal for which B just begins to receive a charge, the time required for the ions to pass from one plate to the other is known, and hence the velocity of the ions ascertained.

It should be noted that if the velocities of the ions of opposite sign are equal, the time of reversal for which B just receives a positive charge will be equal to that for which it just receives a negative charge, and, if the number of the ions is the same, it will gain no total charge. But as a matter of fact the velocities of the ions of different sign are not the same, and the plate receives a negative charge before a positive. Many other methods have been used for measuring these velocities, for which the memoirs quoted in the references must be consulted. The two just described are the most direct.

It is found that the velocity of an ion is proportional to the strength of the field in which it moves, *i.e.* to the force acting upon it: its motion, like that of an electrolytic ion, is similar to that of a solid body moving in a viscous medium. The velocity in a given field varies with the sign of the charge on the ion, with the nature of the gas and with the temperature and pressure. The following table gives some of the measurements that have been made. V_+ and V_- are the values of the velocity of the positive and negative ion respectively in a field of 1 volt per centimetre at atmospheric pressure.

TABLE VII.

	V_+	V_-
Air (dry)	1·36	1·87
Air (moist)	1·37	1·51
Oxygen (dry)	1·36	1·80
Carbon Dioxide	0·76	0·81
Hydrogen	6·70	7·95
Sulphur Dioxide	0·5 (mean)	0·5
Helium	1·42	2·03

(These values are for ions produced by Roentgen rays.)

The relation between the velocity of the ions and the pressure of the gas is remarkable and suggestive. From atmospheric pressure (or higher pressures) down to about 50 cms. of mercury the velocity of the ions varies inversely as the pressure. Now if an ion were a particle of visible dimensions, the viscous resistance to its motion through a gas would be independent of the pressure throughout this range and its velocity would be constant: but if an ion were a body of about the same magnitude as a molecule, or smaller than a molecule, so that as a rule it is in collision with only one molecule at a time, the velocity with which it travels through the gas could be connected with its velocity of heat agitation and the mean free path by the considerations of Chapter VI. From (8), p. 108, we see that at constant temperature the velocity would vary directly as the mean free path, *i.e.* inversely as the pressure, as is found by experiment. We may conclude, therefore, that at pressures greater than 50 cms. the ions are particles which are not very much larger than molecules.

Experiments on the diffusion of one gas through another enable the velocity with which the molecules of the first gas move in the second under a given force to be calculated. Since we know the force which acts on an ion in a field of given intensity, by comparing the velocity of the ions with that of a molecule of known dimensions under the same force, information can be obtained as to the size of the molecules. But so many considerations have to be taken into account that the calculation is too complex to be inserted here. It appears that the radius of the ion in air is about three or four times as large as that of the oxygen molecule, and a similar relation holds in most gases with the possible exception of hydrogen, in which the ion is abnormally large.

As the pressure is reduced below 50 cms. the increase in the velocity of the ions becomes much more rapid. For the negative ion the rapid increase begins at about 20 cms., for the positive at 7·5 cms. Since no sudden change in the nature or motion of the molecules occurs at this stage, the change must be attributed to an alteration in the properties of the ions, which we will consider in the next chapter.

Some experiments have been made by Phillips on the change of the velocity with the temperature: a steady increase with rising temperature was found, but the observations do not seem to throw any direct light on the constitution of the ions.

So far the evidence that has been offered seems to show that the carriers of the current in gases are more like those in liquids than the electrons of the metals. But a study of the discharge of electricity through gases at a low pressure throws a new light on the question.

Conduction at low pressures.

6. We will leave for the present the consideration of conduction of electricity through gases at pressures approximately atmospheric and turn to the low pressure phenomena, which, though in appearance more complicated, are found to be more illuminating. Of course the transition from the high pressure to the low pressure state is continuous, and the particular pressure at which the transition occurs will depend on the nature of the gas and of the vessel in which it is contained: but the general course of the change is the same in all cases.

Suppose that a vessel of the size and shape shown in Fig. 16 is taken, and the potential difference between the kathode and the anode is continually increased, while the pressure remains constant at that of the atmosphere. Until the potential difference has reached a certain limit (in the case of the vessel chosen the limit will be several hundred thousand volts), only an extremely small current will pass through the gas, requiring the most delicate means to detect it. But as soon as the limit is reached a 'spark' passes through the air and a very considerable current flows. Now reduce the potential difference to some value (say 10,000 volts) which is insufficient to cause the spark and gradually diminish the pressure of the gas in the vessel by connecting it to a pump. Eventually a pressure will be reached (about 1 cm. of mercury) at which the spark passes once more and a large current flows through the gas: the spark appears much broader than before. As the pressure is reduced further the spark

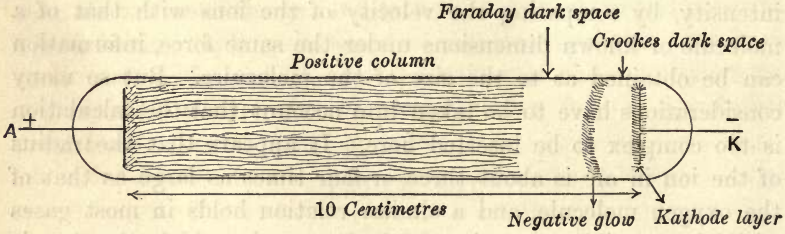

Fig. 16.

becomes still more diffuse, until finally the luminosity of the discharge fills the whole volume of the tube between the electrodes. It will be seen now that the luminosity is not continuous, but is different in different parts of the tube, which will present an appearance similar to that sketched in the figure.

Covering the kathode is a thin luminous layer, the 'kathode-layer': next comes a dark space, the 'Crookes dark space': a luminous layer follows, the 'negative glow,' then another dark space, the 'Faraday dark space,' and, lastly, a region of light, which is sometimes divided into striae, extending up to the anode and known as the 'positive column.' As the pressure is reduced, the Crookes dark space grows at the expense of all the other regions, which diminish in volume, except the kathode layer, which remains practically constant throughout all changes: when this dark space has become so large that its boundaries touch the glass walls of the vessel a curious green phosphorescence is excited in the glass. At first this phosphorescence appears on a few isolated patches near the kathode, but by decreasing the pressure sufficiently the whole of the tube may be filled by the Crookes dark space, with the exception of a thin layer on the kathode and another layer somewhat similar in appearance on the anode: the latter represents the whole of the negative glow and the positive column, which has shrunk to a small fraction of its original size. The whole of the tube, with the exception of the parts behind the kathode and behind the anode, is now glowing with the green phosphorescence: it is in this condition that the tube is emitting Roentgen rays plentifully.

If the pressure be reduced still further, the current diminishes and finally vanishes: at sufficiently high vacua no current can be made to pass through a tube with the greatest electric intensity that can be produced.

The great complexity of the electric discharge is not really peculiar to low pressures. A spark passing in air at atmospheric pressures is, in all probability, made up of a succession of luminous portions separated by dark spaces exactly similar to those which have just been described: but the whole arrangement is crowded into such a small space that the separate parts cannot be distinguished. At high pressures the luminous discharge consists almost entirely of 'positive column,' at very low pressures the existence of the positive column can hardly be detected.

Kathode rays.

7. In 1859 Pluecker discovered that if a magnet is brought near to the tube in which the luminous discharge is passing all the luminous portions are distorted in some measure from their original positions. But the most marked movement in a magnetic field is shown by the green patches of phosphorescence near the kathode: attention was called to the great importance of these patches.

Soon afterwards Hittorf discovered that if a solid body is placed in the tube, near to the kathode, the phosphorescence ceases at all points which were shielded from the kathode by the solid body: a shadow of the body is thrown on the walls of the tube. It appeared that the influence which caused the phosphorescence on the walls must proceed in straight lines from the surface of the kathode to the walls of the tube. According to the ordinary language of physics, any influence which travels in straight lines is called a 'ray': hence the phosphorescence on the walls is due to the action of rays proceeding from the kathode, to which Goldstein subsequently gave the name of kathode rays (kathodenstrahlen). It may be noted also that it was found that the nature of the shadow is such as to prove that the rays do not proceed in all directions from the kathode, like light rays from a source occupying the position of the kathode, but are emitted at right angles to its surface.

A long controversy raged for many years round the nature of these rays. Goldstein and the German physicists maintained that they were aethereal waves essentially of the same nature as light: Crookes and his countrymen held that they were streams of particles electrically charged. The former relied largely on an observation of Hertz, who showed that solid bodies were not absolutely opaque to the rays, which could pass through thin layers of gold or aluminium. The passage of particles even as small as atoms—and nobody had the temerity to suggest that the kathode particles might be smaller than atoms—through a solid body presented great difficulties, while the known properties of light showed that aethereal waves could traverse easily certain solid bodies. However there appeared to be no relation between the transparency of a body for light and that for kathode rays: gold leaf was more transparent to the latter than mica.

On the other hand the deflection of the rays in a magnetic field is an immediate consequence of the view that they consisted of charged particles, while the upholders of the opposite contention could only suggest vaguely that the deflection was in some way analogous to the unexplained rotation of the plane of polarisation of light in a magnetic field. Perhaps the chief reason for the support which the wave theory received is to be found in an unwillingness to accept an alternative resembling so closely the corpuscular theory of light which had proved fallacious.

Crookes sought to maintain his view by direct experiment. He placed in the path of the rays (which can be seen at low pressures as a faint streak of light stretching from kathode to anode) a little mill-wheel with mica vanes. When the current passed the wheel was rotated in the direction in which it would be rotated by the impact of massive particles travelling from kathode to anode. He also attempted to show that if two parallel beams of the rays were sent through the tube each was deflected by the presence of the other, as they might be if the rays consisted of particles carrying electrostatic charges. It is known now that neither of Crookes' experiments can be interpreted in the sense given by their author: the first observation is due to 'radiometer' action and not directly to the impact of the rays[1], while a closer investigation of the apparent repulsion of the rays shows that it cannot be attributed to simple electrostatic action. Indirectly, however, Crookes' work was of immense value, for it drew attention to the wide field of research which the low pressure discharge offered for future investigation.

As regards the nature of the particles, it appears to have been held generally that they were charged atoms, but Crookes, by one of those flights of intuition which are only remembered when they

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[1] See Poynting and Thomson's *Heat*, ch. IX. Even if the motion of the vanes were due to the action of the rays it would not be held nowadays to prove that the rays were massive particles: there is such a thing as the 'pressure of vibrational radiation.'

prove successful, maintained that they consisted of matter in a 'fourth state'—neither solid, liquid, nor gaseous.

The charge and the mass of a kathode ray particle.

8. Many scientific controversies have ended in the comforting conclusion that both sides were right and both wrong. But in this case one of the alternatives has been definitely accepted and the other rejected. An experiment by Perrin was deemed conclusive, in which he demonstrated the electric charge received by a body when the kathode rays fell upon it[1]. This charge was found to be negative. His apparatus, as improved by J. J. Thomson, is shown diagrammatically in Fig. 17. The discharge was produced at a very low pressure between the anode *A* and the kathode *K*: the rays pass through holes in the diaphragms *C*, *D*, and fall on the glass at *P*, causing a green patch. Near *P* is an electrode *E*, surrounded by an earthed metallic case *M*, with a slit *S*: *E* is connected to an electrometer. A magnet is now brought near the tube and the rays deflected so that the patch moves from *P* and falls on *S*. The electrode immediately receives a negative charge, which increases so long as the rays fall on the slit, until the charge communicated in any time by the rays is equal to that lost by conduction to the earthed case. We may conclude, then, that a negative charge is

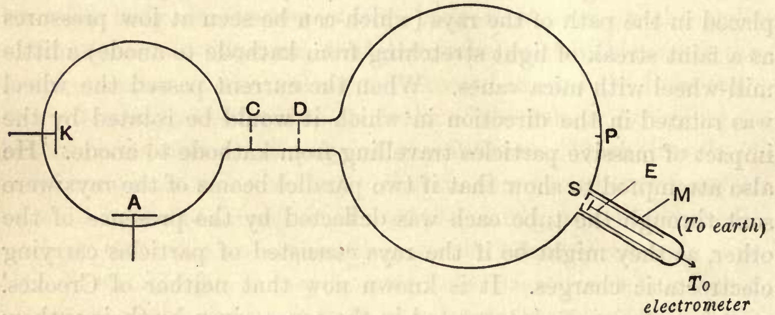

Fig. 17.

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[1] If all our present information as to 'secondary radiation' (see Chap. IX.) had been available, Perrin's experiment might have seemed less convincing. But I fear that few of the "crucial experiments" in the history of physics would stand a searching examination in the light of subsequent knowledge.

carried by the influence which causes the green phosphorescence on the glass of the tube.

But, before it can be affirmed definitely that the rays consist of charged *particles*, some further information is required. The term 'particle' connotes a finite mass and a finite number of carrying agents. Can we measure the number of the particles and the mass of each? The number will be known if we can measure the charge carried by each particle; hence we want to determine the mass of each particle and the charge carried by it. Unfortunately, no direct method has been devised for dealing with individual particles in the kathode rays, and neither of these quantities has been determined directly, but the ratio of the mass to the charge of a kathode particle can be ascertained.

Suppose that a body of mass m carrying a charge e moves along the axis of x with a velocity v under the action of a magnetic field of intensity H parallel to the axis of y. Then according to p. 78 the body will be subject to a force $\frac{Hev}{V^2}$ perpendicular both to H and v, *i.e.* along the axis of z. The acceleration of the body parallel to z will be $f = \frac{Hev}{V^2 m}$, and if it moves for a time t it will have travelled a distance $\frac{1}{2} ft^2$, or $\frac{Hevt^2}{2mV^2}$ in that direction. In the same time t it will have travelled a distance vt parallel to x. Hence the body while travelling a distance l parallel to x is deflected a distance δ parallel to z, where

$$\delta = \frac{Hevt^2}{2mV^2} = \frac{Hev}{2mV^2}\left(\frac{l}{v}\right)^2 \quad \text{......................(7)}$$

$$= \frac{l^2}{2V^2} \cdot H \cdot \frac{e}{m} \cdot \frac{1}{v} \quad \text{........................(8).}$$

By suitable arrangements described below, the quantities l, H, and δ can be measured for the case where the body is a kathode particle. Hence a relation can be obtained between the quantities $\frac{e}{m}$ and v.

In order to measure either of these quantities separately a second relation between them must be ascertained. Suppose, for instance, that the body is moving under the action of an electric field of intensity Z parallel to the axis of z. Then it will have an acceleration $f' = \frac{eZ}{m}$ parallel to z and will undergo a deflection δ' in the same direction where

$$\delta' = \tfrac{1}{2} f' t^2 = \tfrac{1}{2} \frac{eZ}{m} \cdot \left(\frac{l}{v}\right)^2 = \frac{l^2}{2} \cdot Z \cdot \frac{e}{m} \cdot \frac{1}{v^2} \ldots\ldots\ldots\ldots\ldots (9),$$

δ', l, z can be measured and the required second relation obtained.

Another method of obtaining the required second relation consists in measuring the total charge and the total energy in the rays: the first quantity can be ascertained by Perrin's method, the second by allowing the rays to fall on a solid body and measuring the amount of heat which they communicate to it. If N ray particles have passed in a given time, the total charge is Ne and the total energy $N \cdot \tfrac{1}{2} mv^2$: if the ratio of the two quantities is P, we have

$$P = \frac{2e}{m} \cdot \frac{1}{v^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10).$$

A third method is simpler in appearance than either of the foregoing, but liable to considerable experimental difficulty. The kinetic energy of the particles is derived from the electrical energy expended when they fall through the difference of potential between the kathode and the anode. If this P.D. is measured and its value V, we have

$$eV = \tfrac{1}{2} mv^2,$$

$$\therefore V = 2v^2 \cdot \frac{1}{\frac{e}{m}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11).$$

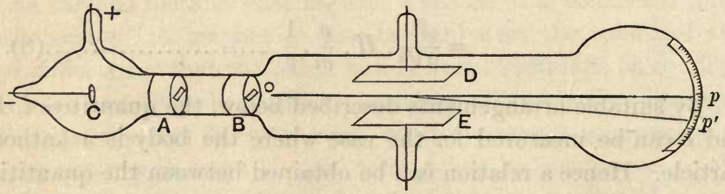

Fig. 18.

By the application of these principles the value of $\frac{e}{m}$ for a kathode ray particle was measured by J. J. Thomson. The apparatus used for the first method is sketched in Fig. 18. The kathode rays emerging from the kathode pass through small holes in the plates A and B: the fine pencil of rays defined by these holes strike the glass at p. The plates D and E can be maintained at any desired difference of potential by means of a suitable battery, and an electric field of any given intensity established in the region between them. The poles of a magnet are placed above and below the tube with reference to the plane of the paper, so that a magnetic field is established, the lines of which are perpendicular to the plane of the paper. If the length of the tube be taken as the axis of x, a line perpendicular to D and E as the axis of z, and the direction of the magnetic intensity as the axis of y, the directions of the various quantities concerned will be those assumed in the calculations of the last paragraph.

When either an electric or a magnetic field is acting, the rays will be deflected from their course, and the spot of green phosphorescence will move from p to some other point p'. Now by the reasoning of the last paragraph it was shown that the deflection due to the electric field was in the same line as that due to the magnetic field. If the sign of these fields be chosen suitably, the deflections due to the fields will be in opposite directions: and further, if the magnitudes be suitably chosen, the magnitudes of the two opposite deflections will be the same and the total deflection will be null. Suppose then that a constant electric field is applied and the deflection δ' measured; and then such a magnetic field imposed by an adjustment of the current in the electromagnet that the spot of phosphorescence returns to its original position at p; the magnetic deflection will be equal and opposite to the electric deflection, or from (8) and (9),

$$\delta = \delta', \text{ or } \frac{l^2}{2V^2}\,.\,H\,.\,\frac{e}{m}\,.\,\frac{1}{v} = \frac{l^2}{2}\,.\,Z\,.\,\frac{e}{m}\,.\,\frac{1}{v^2}.$$

Thus $$v = V^2\,.\,\frac{Z}{H} \text{ and } \frac{e}{m} = \delta'\,.\,\frac{2V^4}{l^2}\,.\,\frac{Z}{H^2} \quad \text{............(12).}$$

As a result of his experiments Thomson found that

$$e/m = 2{\cdot}3 \times 10^{17} \text{ and } v = 3 \times 10^9 \text{ cm. per sec. (about).}$$

He found that the value of v, as might be expected, varied with the conditions of the discharge: but it was usually about the magnitude given, or about one-tenth of the velocity of light. On the other hand, e/m always had the same value whatever the nature of the gas in the tube, and whatever the material of which the kathode was composed. Under no circumstances could any value for this quantity be obtained which differed by more than the possible error of experiment from the mean of all the values.

Thomson's experiments have been repeated later by many experimenters by methods which vary considerably in detail, but all depend on the same principles. The only considerable divergence is in the use of other observations in place of the electrostatic deflection to determine the second relation between e/m and v. The conclusion that e/m is always constant has been confirmed abundantly. The experiments are difficult and complete concordance in the results is not to be expected: but the discrepancies are well within the errors of observation. The most accurate researches are probably those of Simon and Kaufmann, and the best value obtained may be taken as

$$e/m = 5{\cdot}595 \times 10^{17}.$$[1]

On referring back to p. 89 the reader will see that this value of e/m is very close to that deduced from the observations of optical phenomena for the corresponding ratio for an electron. We are not able to measure directly the value of the charge for the kathode ray particle, but since we have found that the value of the charge on all carriers of the current that we have been able to investigate is identical, it is needless to insist on the cogency of the conclusion that the charge on a kathode particle is equal to that on an electron, and that the kathode particle is indeed nothing but an electron moving very rapidly in a straight path. Its velocity is seldom less than one-tenth of that of light, and far greater than that which can be given to any body of sensible size.

The identity of the particle from whatever source it is derived confirms once more our conclusion that the electrons are common constituents of all atoms. It may be well to repeat that, since charge and mass are the only two characteristics which are

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[1] *I.e.* $1{\cdot}865 \times 10^7$ in electromagnetic units.

attributed to electrons in modern theory, identity in respect of these two characteristics is complete identity in any sense which has physical meaning.

Historical note.

9. Though little attention has been paid to the chronological order of development in other parts of this book, the sequence adopted in this chapter is so different from that of discovery that it would be misleading without a brief explanatory note.

The quantity e/m for the kathode ray particle was first measured in 1897. Since that value proved to be a natural constant independent of the material investigated, Thomson was led to suggest that the charge which forms one part of the ratio was identical with the charge which is known to be a natural constant in electrolysis, namely the charge on a univalent ion. On this view, the mass of the kathode particle is smaller than that of the smallest atom, and to these particles he gave the name 'corpuscles.' His measurement of the charge on a gaseous ion, which lent additional support to his speculations, was made in 1898. Meanwhile Lorentz had published in 1898 his calculation of e/m for an electron based on the Zeeman effect, showing that Thomson's 'corpuscles' were identical with his own 'electrons.' Since the latter name was first in the field and has been adopted almost universally, it has been used throughout this volume. But if the reader meets with the term 'corpuscle' elsewhere (as in Thomson's books or papers), he should remember that it is synonymous with negative electron. If at any time a positive electron (see next paragraph) should have to be recognised, it would probably be well to retain the term 'corpuscle' for the negative electron.

'Kanalstrahlen.'

10. It appears then that in the low pressure discharge through gases, as in the optical phenomena considered previously, the most important part is played by negative electricity; the negative electrons seem to be the only active agents, and positive electricity has to be considered only in order to account for the electrical neutrality of the whole body. There is nothing surprising in this great difference between the functions of positive and negative electrification: it is suggested by the most superficial observations. The appearance of the electric discharge even at atmospheric pressure is different at the positive and negative ends, especially in the so-called 'brush discharge.' At low pressures the difference becomes still more noticeable: a practised eye has not the smallest difficulty in telling at a glance at a discharge tube the direction in which the current is passing in it. In electrolytic conduction there is also a difference between the electrodes of opposite sign in the nature of the elements which accumulate there: indeed it is only in metallic conduction that the fundamental distinction is not obvious. If it had not been for the importance of metallic conductors in electrostatic experiments, it is likely that the earliest theories of electricity would have been based on the difference and not on the similarity of positive and negative electrification.

But it is still open to question whether positively charged particles may not play some active part in the mechanism of the electric discharge. The complicated appearance of the discharge and the difference which it shows in different gases suggests that some agent other than the universal negative electrons must have an influence on the phenomena; but for a long time after the discovery of the kathode rays no such agent had been detected directly.

In 1886 Goldstein, working with a discharge tube in which the kathode was a plate perforated by several holes, observed faintly luminous streaks stretching out from the holes in the kathode into the space remote from the anode. Where these streaks meet the walls of the tube, they excite a slight phosphorescence, usually of a mauve colour, but always totally different from the green phosphorescence excited by the kathode rays. He imagined that these streaks represented the path of rays, similar in some manner to the kathode rays, to which he gave the name of Kanalstrahlen; the name has been generally used without translation, and for the present we will employ it.

Since the kanalstrahlen are travelling in a direction from the anode to the kathode, it is natural to suppose that they are positively charged: but at first no experiments could detect any sign of the charge. An attempt to repeat Perrin's experiment gave indefinite results, and no deflection in a magnetic field could be noted. However in 1898 Wien showed that, if fields of sufficient strength be employed, a magnetic deflection can be obtained, and that the direction of this deflection, being opposite to that of the kathode rays, indicates that the rays are positively charged, as had been anticipated.

Great interest attached to the measurement of the quantity e/m for these rays since the results might be expected to throw important light on their nature. The experiments are difficult, but, by an application of the method of electrostatic and magnetic deflections used for the kathode rays, Wien succeeded in measuring the quantities e/m and v for the kanalstrahlen. He found that v was usually about 10^8 cms. per sec., or $\frac{1}{100}$ of the velocity of light, and far in excess of any velocity which we can produce by mechanical means: but his measurements of e/m are of more importance. The smaller deflection of the rays in a given field shows that this quantity must be much larger for the kanalstrahlen than for kathode rays, and in all his experiments Wien never obtained a value greater than 3×10^{14}. Moreover the value showed no approach to constancy: in the same tube there were some particles for some of which e/m was 3×10^{14} and others for which it was so small that they suffered no appreciable deflection: the number of particles having any particular value differs with the gas in the tube. But in all the gases tried the maximum value of e/m is the same.

Now 3×10^{14} is the value of e/m for the electrolytic ion of hydrogen, the lightest atom; and since we have every reason to believe that the charge on a kanalstrahlen particle will be the natural unit of charge which we have met so often—that on a univalent electrolytic ion—it is a natural conclusion that the particles in the kanalstrahlen which have this particular value of e/m are charged atoms of hydrogen. In oxygen we should expect to get rays consisting of charged atoms of oxygen, which would give a value for e/m 16 times smaller: but since in all gases there are particles which have apparent values of e/m ranging continuously from 3×10^{14} to zero, such particles would escape detection, even if they were present. Wien's explanation of his observations was that the particles with the maximum value of e/m were indeed hydrogen atoms derived from impurities in the gas, and that the smaller values represented complex particles consisting of one hydrogen ion loaded with several neutral molecules.

No further progress was made until J. J. Thomson attacked the problem in 1907. The principle of his method was exactly the same as that of Wien, but several discoveries of no great intrinsic importance, which had been made in the interval, enabled him to improve greatly the experimental arrangements. For a detailed account of his observations the reader must consult the original memoir: the chief improvements consisted of making the discharge pass at very low pressures and of being able to observe the deflection of the rays more accurately by means of the phosphorescence which they excite in the mineral Willemite. At higher pressures it was found, in agreement with Wien, that the value for e/m ranged continuously from 3×10^{14} to zero, and that in some cases particles deflected in the opposite direction, indicating a negative charge, could be observed. But at low pressures the phenomena became much simpler. The particles were divided into two, or in one case three groups, each characterised by a definite value of e/m.

In explanation of these observations Thomson suggested that the apparently continuous variation of e/m was due to a variation of the charge on the particles and not to a variation of the mass. Particles start with a positive charge ϵ; as they pass down the tube from anode to kathode they collide with charged ions which are known to be present in the space through which they pass. If a particle strike against a negative ion, its charge may be neutralised: on striking another ion, the charge may be either restored to the positive sign or may become negative. The charge which is measured in the magnetic and electric deflections is the *average* charge which the particle has possessed throughout its course: this average charge can never be greater than that with which it starts, for a carrier at low pressures can never bear a charge greater than ϵ, but it may have any smaller value. When the pressure is so low that there are very few ions in the space through which the particles pass, the latter will retain their charge unchanged throughout their course, and all particles which are similar when they start will give the same apparent value of e/m.

But the greatest interest of Thomson's experiments lies in the values of e/m which he found. (For the sake of brevity we will call the quantity 3×10^{14}, the value of e/m for the hydrogen ion, E.) In hydrogen, air, carbon dioxide, neon and helium, at the lowest pressure at which experiments could be made, there were only two groups of particles present in the kanalstrahlen: for one of these the value of e/m was E, for the other it was $\frac{1}{2} E$. In all these gases, with the exception of helium, no sign whatever of the existence of any other kind of particle could be found at any pressure: the only effect of increasing the pressure was to introduce particles for which e/m varied *continuously* from E downwards. But in helium a stage could be attained in which a third well-defined group appeared for which e/m was $\frac{1}{4} E$. Since the atomic weight of helium is 4, this third group may be regarded with some certainty as consisting of atoms of helium bearing the electronic charge ϵ.

There are two important points in these results which call for consideration. In the first place no gas, except hydrogen and helium, produces kanalstrahlen particles which consist of charged atoms derived from the elements of the gas: there was no trace of particles having values of e/m corresponding to the atoms of oxygen, nitrogen, carbon or neon. Thomson suggests that it might be possible to obtain such particles, if a discharge could be obtained with a lower potential difference between the electrodes without raising the pressure, and introducing thereby the effect of the neutralisation of the particles by their collision with other ions. His suggestion can only be tested by further experiment.

In the second place it is to be noted that the particles for which e/m is E and $\frac{1}{2} E$ occur in all gases at the higher pressures. The nature of the latter particle is not certain, but these experiments suggest strongly that it is a molecule of hydrogen (consisting of two atoms) bearing the charge ϵ, but it is also possible that it might be an atom of helium (atomic weight 4) bearing a charge 2ϵ. The latter supposition would be extremely improbable if it were not for some observations to be described in the next chapter: and the latest work seems to remove a great part of the probability derived from those observations. We shall assume that the first particle is an atom and the second a molecule of hydrogen.

But whence come these omnipresent atoms and molecules of hydrogen? They may be derived from hydrogen contained as impurity in the gas, or we may have to recognise a transmutation of the elements resulting in the production of an atom of hydrogen from the atom of another element. The first hypothesis is not so improbable as it appears at first sight: it is notoriously difficult to free a gas completely from hydrogen: the slightest trace of moisture, which it is extremely difficult to remove from glass walls, may give rise to it. A gas which would be passed by any chemist as 'chemically pure,' contaminated with only one part in a million of hydrogen, would yet contain many thousand of molecules of hydrogen in a cubic centimetre at the lowest pressures reached in these experiments. But Thomson's later experiments show that the appearance of the discharge is the same whether the tube contain nearly pure hydrogen or the most elaborate precautions be taken against the presence of that gas. And, even if it could be supposed that a sufficient quantity of hydrogen were present in all tubes, the question would remain why it is that the hydrogen atoms, which must be quite a small fraction of the whole number present, should alone be capable of becoming charged at these low pressures and high potential differences. However, the electrical properties of hydrogen are different in many respects from those of other gases. The cautious writer in making any general statement about gases always adds the words 'except hydrogen.'

On the other hand, no conclusive reason can be urged against the alternative explanation that the discharge may split up the atoms of other elements with the production of atoms of hydrogen. We know that the atoms in the discharge can lose a negative electron: may they not also be capable of losing a positive portion which is the same for all atoms and identical with the hydrogen atom—a positive electron? The only direct objection that can be suggested is that the atom of hydrogen, even when it has lost an electron, appears to be a very complicated system (see Part IV.), and that if any portion of an atom is to be split off, it is natural to suppose that it would be a perfectly simple system, without internal structure, such as the negative electron appears to be. But these experiments, especially when viewed in connection with those described in Chapter IX., § 11, afford distinct support for the hypothesis that there may be positive electrons common to all atoms—an hypothesis which would have been rejected unhesitatingly as contradictory of all experimental evidence a very short time ago.

(The discussion which has just been given raises an interesting point in connection with the value of scientific evidence. Why do we accept so readily the conclusion that the negative electrons are common constituents of all atoms and yet feel great hesitation in admitting the existence of similar positive electrons? The evidence in the two cases is precisely the same: particles of the same nature are produced from all substances, and yet we are willing to consider an hypothesis of an unavoidable impurity in the latter case, which no rational person would entertain for a moment in the former. The answer is that the first conclusion is in accordance with our preconceived notions, and the latter is not: the former introduces at once a pure simplification of our physical conceptions, the latter introduces at the same time some difficulties. The fallacy that scientific propositions are capable of direct and irrefutable proof like mathematical propositions—or, perhaps it would be more correct to say, like legal propositions—is only entertained by those persons whose acquaintance with science consists in talking about it rather than in studying it at first hand.)

11. **Summary.** Our study of conduction in gases has adduced still further evidence for the fundamental assumption of the electronic theory that electrical phenomena can be interpreted in terms of the action of discrete charged particles. In gases at high pressures these particles are bodies slightly larger than the molecules of the gas, in which they are contained, carrying the charge ϵ: in gases at low pressures the negative carrier is the negative electron, the positive carriers are bodies which are never smaller than the atom of hydrogen: it is possible that the charged atoms of hydrogen are positive electrons derived from other atoms in the gas.

In the next chapter we shall again have reason to consider the great difference between the carriers of positive and negative electrification, which has been insisted on already so often and so strongly. But enough has been said already to justify the attitude which was adopted at the end of Chapter VII., § 5. In many phenomena, such as those of optics, the carriers of the positive electrification play a notable part which has not yet been considered. But, since those carriers are of atomic dimensions, it is impossible that they should play any important part in the conduction of electricity through metals, and, since their mass is at least 1600 times that of the negative electron, it is impossible that they should play any important part in magneto-optical phenomena.

We must now proceed to investigate how these carriers may be produced from the uncharged molecules of the gas in its normal non-conducting condition.

REFERENCES FOR CHAPTER VIII.

A full account of all the researches (up to Sept. 1906) mentioned in the foregoing chapter, and of many more, can be found in J. J. Thomson's *Conduction of Electricity in Gases.* This treatise is the standard authority on the subject and should be studied by all who desire fuller information than that contained in the brief abstract given in the preceding and the following chapter.

The following is a list of the more important original memoirs upon the subjects just treated :—

Mechanism of Conduction in Gases. J. J. THOMSON and RUTHERFORD, *Phil. Mag.* (5), 42, 392, 1896. (*A.* and *L.*, p. 837.) J. J. THOMSON, *Phil. Mag.* (5), 47, 253, 1899. (*A.* and *L.*, 764.)

Charge on an ion. C. T. R. WILSON, *Phil. Trans. A.*, 1899, p. 403. (*A.* and *L.*, 1076.) J. J. THOMSON, *Phil. Mag.* (6), 5, 346, 1903. (*A.* and *L.*, 874.) H. A. WILSON, *Phil. Mag.* (6), 5, 429, 1903. (*A.* and *L.*, 1107.)

Velocity of the ions. ZELENY, *Phil. Trans. A.*, 1901, p. 193. (*A.* and *L.*, 1117.) RUTHERFORD, *Phil. Mag.* (5), 44, 422, 1897. (*A.* and *L.*, 619.) LANGEVIN, several papers collected in *A.* and *L.*, pp. 310–325.

Low pressure discharge. The earlier original work is extremely complex and the reader is advised to turn to the summary of it in Wiedemann's *Elektricität* rather than to the memoirs themselves.

Kathode Rays. Pluecker, *Pogg. Ann.*, 103, 89, 1858. (*A.* and *L.*, 566.) Goldstein, *Berl. Monat.*, 284, 1876. Crookes, *Phil. Trans.*, 1879, I. 135, and II. 641. Perrin, *Compt. Rend.*, cxxi. 1130, 1895. J. J. Thomson, *Phil. Mag.* (5), 44, 293, 1897. (*A.* and *L.*, 779.)

Kanalstrahlen. Goldstein, *An. Ph.*, 64, 38, 1898. (*A.* and *L.*, 243.) Wien, *An. Ph.*, 65, 440, 1898. (*A.* and *L.*, 1061.) J. J. Thomson, *Phil. Mag.* (6), 13, 561, 1907.

CHAPTER IX.

METHODS OF IONISATION.

Ions and Electrons.

1. AT low pressures the carriers of the current in gases are negative electrons and positively charged bodies of the dimensions of atoms: at high pressures they are ions which are all larger than the uncharged molecules of the gas. At intermediate pressures the rapid increase of the velocity of the ions, which was noted in Chapter VIII., § 5, indicates a change from the larger to the smaller carriers.

It is an obvious conclusion, undisputed at the present time, that the large ions, which are present at the higher pressures, consist of the smaller carriers, which exist at lower pressures, loaded up by the adherence of neutral molecules. Just as the water molecules in Wilson's condensation experiments (p. 144) gather round the charged ions in circumstances in which they do not gather round uncharged molecules, so the molecules of the gas itself show a special tendency to gather round the charged carriers: the ions may be regarded as very minute 'drops,' formed of the material of the gas. The charged particles collect neutral molecules just as an electrified glass rod collects fragments of paper.

When the pressure of the gas is reduced, the number of collisions made by an electron or positive carrier diminishes in proportion: it is less likely to meet with suitable molecules, and the number adhering to it decreases. At the lowest pressures it passes its whole career, from the time of production until it reaches the electrode, free from the influence of neutral molecules. It is not a necessary consequence of this view that the velocity of the ion must increase in discontinuous jumps as the molecules are detached from its *entourage* one by one by a reduction in pressure: the quantity which is measured experimentally as the 'velocity of the ions' is doubtless an average quantity taken over a large number of ions: as the pressure is reduced the proportion of the ions which possess a large number of molecules attached to them diminishes continuously.

Accordingly the conductivity of a gas, whether at high or at low pressures, is due to the admixture with the molecules of the gas of free electrons and positively charged carriers. Since the negative electrons are regarded as portions of atoms, it is natural to regard the free electrons in a gas as detached in some manner from atoms and the positive carriers as the portions of the (neutral) atoms which are left when the negative electrons have been extracted from them: we shall speak of the positively charged remnant of an atom left when an electron has been extracted as a 'positive atom.' The remaining problem of the conductivity of gases, which we have to investigate, concerns the method in which the electrons are extracted from the atoms, and it is because it enables us to obtain information on this point of fundamental importance that the study of gaseous conductivity is of such immense interest. Since it is not supposed that the internal constitution of an atom of a solid differs from that of a similar atom in a liquid or gas, our investigation of the methods by which electrons can be extracted from gaseous atoms may be applied (with certain reservations) to all atoms, whatever the state of aggregation of the material in which they are contained.

A gas may be rendered conducting either by the extraction of electrons from its own atoms or from the atoms of the vessel in which it is contained, but for our present purpose the distinction is immaterial. The following list enumerates the agencies by which electrons may be extracted from atoms, *i.e.* by which any gas, to which those atoms have access, may be rendered conducting:—

(1) Kathode rays and kanalstrahlen.
(2) Roentgen rays.
(3) Rays from radio-active substances.
(4) Light, especially light of very short period.
(5) Chemical reactions.
(6) Heat (doubtful).

We will consider these agencies in order. In dealing with some of them, *e.g.* (2) and (3), we shall have to consider phenomena of great intrinsic importance apart from the ionisation produced in gases, but the order of treatment adopted seems to lead to the most logical development of the subject.

(1) Kathode Rays and Kanalstrahlen.

Ionisation by kathode rays.

2. It may seem illogical to start the consideration of the methods by which conductivity may be produced in a gas by treating of an agent which appears only when the gas has already been put in a conducting condition by the action of an intense electric field. But a closer consideration will justify our procedure. A very slight examination will show that it is impossible to suppose that the action of the field in producing conductivity is a direct dragging of electrons out of the atoms by a sufficiently great electric intensity. For we can estimate the order of magnitude of the intensity which would be required for this purpose. According to (9), p. 54, the natural period of vibration of an electron is $2\pi\sqrt{\frac{m}{f}}$, where fx is the force on the electron when it is displaced a distance x. In order to drag the electron out of the atom it must be displaced a distance which is almost certainly greater than 10^{-9} cms. Hence the minimum electric intensity X_0, which can drag the electron out of the atom is given by

$$X_0\epsilon = 10^{-9} f.$$

The period of the electrons in the atoms of gases is not greater than 10^{-16} secs. Hence, substituting the known values of $\epsilon\,(3{\cdot}4 \times 10^{-10})$ and $m\,(6 \times 10^{-28})$, we have

$$X_0 = \frac{10^{-9} \times 4\pi^2 \times 6 \times 10^{-28}}{3{\cdot}4 \times 10^{-10} \times 10^{-32}} = 7 \times 10^6 = 2{\cdot}1 \times 10^9 \text{ volts per cm.}$$

—very much greater than the electric intensity which will produce the discharge.

Again, if the electrons were dragged out by the direct action of the field, the density or pressure of the gas could have no influence on the intensity required to start the discharge, in contradiction to the experimental results. We must look elsewhere for the agent which starts the passage of the electric discharge.

The ionising action of the kathode rays was first observed directly by Lenard. Since the rays can penetrate thin layers of solid bodies, if the wall of the tube at the point where the rays strike it is made of a very thin sheet of metal (*e.g.* aluminium foil), the rays can emerge into the space outside the tube and be examined there. By the use of such tubes Lenard found that the rays were rapidly absorbed in the outside air at atmospheric pressure, producing luminosity in it and rendering it a conductor; subsequent experiment has shown that the penetrating power of the rays increases rapidly with their velocity. Since the nature of the kathode rays was not understood thoroughly at the time of Lenard's experiments, it was supposed that the rays which emerged from the tube were different in nature from the kathode rays, and to them the name of 'Lenard rays' was given. But it is now known that the Lenard rays are nothing but kathode rays which have emerged from the vessel in which they were produced—(in so far as they are not Roentgen rays).

The conductivity conferred by the kathode rays on a gas through which they pass is also illustrated well by Perrin's experiment (Chapter VIII., § 8). If the electrode be charged up by deflecting the rays so as to fall on it, and the rays restored to their normal course, the charge acquired will leak away rapidly owing to the conductivity of the surrounding gas: on stopping the discharge so as to stop the action of the rays the leak of the charge ceases. This conductivity of the residual gas in the tube, even if it is not in the direct line of the discharge, renders it necessary that the pressure should be very low before the electrostatic deflection of the rays can be observed. For so long as the medium between the plates (Fig. 17, p. 154) is a conductor, the electric intensity between the plates will not be that calculated from the potential difference of the plates and the distance between them on the assumption that the medium separating them is an insulator.

Experiments similar to that of Perrin have shown that the kanalstrahlen also render any gas through which they pass a conductor: but, since the rays have a very much smaller penetrating power than the kathode rays, it has not been possible hitherto to make them pass through a window in the tube into another vessel, in which they can be examined as in Lenard's experiments.

Of course the mere presence of the charged particles which constitute the kathode rays and kanalstrahlen will confer some conductivity on the gas in which they are contained: but it is easily shown that the number of ions formed in a gas is very much larger than that of the kathode particles entering it: it is necessary to suppose that the rays cause the ejection of electrons from the atoms of the gas as they pass. The mechanism of this ejection is easily imagined. When the charged electron approaches an atom it will exert an intense repulsion on the electrons contained in the atom; the repulsion may be greater than the attraction of the positive atom, so that an electron is set free. But now the kathode ray particle itself is attracted to the positive atom and it can only proceed on its course by overcoming that attraction, in virtue of its kinetic energy. The ejection of the electron from the atom requires the expenditure of energy, which is derived from the kinetic energy of the rays. The speed of the particles is diminished by successive actions until it is too small to provide the energy required for the ejection of an electron: its ionising power is lost and it remains in the gas, attracted to one of the positive atoms which it has brought into existence: the rays are absorbed. It is clear, as experiment shows, that there must be an intimate connection between the absorption of the rays in a gas and the ionisation which they cause in it. (See further § 7.)

The Electric Discharge.

3. The conductivity in a gas at all pressures is due to the presence in it of electrons and positive atoms. At low pressures these particles remain free, but at high pressures they collect neutral molecules round them and form ions. But even in a gas at high pressure the electrons spend in the free state some part of the period between their production and their absorption in the walls of the vessel or elsewhere: the collection of the neutral molecules cannot be instantaneous. But while the electrons are in the free state they move like the kathode ray particles under the electric intensity to which the gas is exposed:

if before they collide with a neutral molecule they move a distance so great that they acquire sufficient energy to ionise an atom, as a result of the encounter the electron does not adhere to the neutral atom, but ionises it. The new electron which is produced moves through the field and acquires sufficient energy to ionise the atom with which it collides at the end of its free flight. The number of electrons present in the gas is not now merely that due to the action of the external ionising agent, but is very much greater. The current, which had been saturated, increases once more with an increase in the electric intensity: the phenomena of the electric discharge appear.

Accordingly the production of the electric discharge is due to ionisation by collision—to the liberation of new electrons by the electrons existing already through their collisions with neutral molecules. If X is the electric intensity acting on the gas, d the free path of an electron, the difference of potential through which the electron falls is Xd: the loss of electrostatic potential energy $Xd\epsilon$ is equal to the energy gained by the electron. Hence the electric discharge is produced when the quantity $Xd\epsilon$ becomes equal to the work which must be performed in order to drag an electron out of an atom.

The quantity $Xd\epsilon$ can be increased in two ways:—(1) by increasing the field and (2) by increasing d: the free path varies inversely as the pressure of the gas and hence d is increased by decreasing the pressure. We have noted already that the passage of the electric discharge may be attained by either of these two methods.

Further we can make a rough estimate of the energy required to ionise an atom. The mean free path of a very small body like an electron in air at atmospheric pressure is about 6×10^{-5} cm.: the electric intensity required to produce a spark is about 30,000 volts per centimetre, or 1·8 volts over the length of the free path. Hence the energy required to ionise an atom is not less than that acquired by a particle bearing a charge ϵ in falling through a potential difference of 1·8 volts or 6×10^{-3} elec. stat. units. We may be sure that the estimate which has been made is too small for two reasons. Firstly, even when the most powerful discharge is passing through a gas at atmospheric pressure, only a small fraction of the whole number of atoms of the gas are ionised. It is only those electrons which have travelled over a free path which is considerably greater than the mean which acquire the power of ionisation. In taking the mean value of the free path in our calculation we have made the estimated potential difference too small. Secondly, it is not certain that the ionised atom acquires all the energy required to ionise it at one collision. Previous collisions may have set the electrons into vibrations without ejecting them from the atom: a subsequent impulse, if it is in such a direction as to increase the amplitude of the vibration, may secure the liberation of the electron without communicating to the atom all the energy required for that process.

The complete theory of the electric discharge at high and low pressures is beyond the scope of this volume: but it will be well to note some of the conclusions that have been reached by the application of the hypothesis described to measurements of the distribution of the electric intensity along the path of the luminous low pressure discharge. (1) It is found that a large number of negative carriers are emitted from the kathode under the bombardment of the positive carriers, but it is not certain that positive carriers are emitted from the anode: they may all come from the molecules of the gas. The positive carriers require a greater energy before they can act as ionising agents than the negative. (2) The important conclusion is attained that the luminous regions of the discharge are those in which ionisation of the molecules is proceeding, while the non-luminous portions are those in which the carriers are travelling without forming fresh ions. Thus the kathode glow represents the emission of electrons from the kathode: the Crookes dark space the region through which these electrons are travelling under the action of the field with very high speed and producing few ions (see § 13): when they reach the negative glow they begin to ionise more strongly the molecules of the gas: the fresh ions produced have to travel through the Faraday dark space to the positive column before they in their turn begin to act as ionisers[1].

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[1] This description does not pretend to complete accuracy: see further the references at the end of the chapter.

The mechanism of the conduction of the electric discharge is similar in its main features to metallic conduction: in both cases the atoms are ionised and the charged fragments moving under the electric field constitute the current. The main differences are (1) that in metals the carriers of one sign are fixed, (2) the mode of production of the carriers may be somewhat different, (3) the free path of the carriers in gases is so much greater that the discontinuity of the conduction may be seen by the naked eye. This last difference makes the phenomena easier to analyse, and nowadays conduction in gases seems more enlightening and more interesting than metallic conduction, in spite of the complicated relations between current and electric intensity which replace the perfectly simple Ohm's law. We have travelled far from the mathematical conception of physics.

(2) Roentgen Rays.

Roentgen's discovery.

4. In 1895 Roentgen found that if a screen covered with platino-cyanide of barium, a substance which shows marked fluorescence under the action of light, is brought near a tube in which an electric discharge is passing at such a pressure that vivid green phosphorescence is excited on the walls, the screen glows brightly. If thick solid bodies be brought between the tube and the screen, shadows are thrown on the screen: these shadows may be attributed to the interception of rays coming from the tube and exciting the phosphorescence in the screen. But solids are not perfectly opaque to the rays: if layers sufficiently thin are used the rays can pass without appreciable absorption. The absorbing powers of different materials when measured in layers of equal thickness was found to be very different. On the whole less dense bodies are more transparent than more dense bodies: for instance, the flesh of animals is more transparent than their bones: but the absorption of a material bears no relation to its opacity to light.

These rays were called by Roentgen 'X rays,' but they are now generally known by the name of their discoverer. They were found by Roentgen to be able to affect a photographic plate in the same manner as light: J. J. Thomson and others showed that they produce conductivity in any gas through which they pass. All subsequent investigation of the nature and properties of the rays has depended on one or other of these two characteristics rather than on the excitement of fluorescence in a suitable screen.

The nature of the rays. Stokes' theory.

5. It appeared at once that the rays exhibit a remarkable resemblance to light. Their rectilinear propagation, as evidenced by the sharp shadow thrown by bodies which intercept them, their power of affecting a photographic plate and their power of passing through solid bodies, are obvious examples of this resemblance. But there are equally striking differences between Roentgen rays and light. They are not refracted in their passage from one medium to another: they show some sort of reflection, but the laws governing it are totally different from those of the corresponding phenomenon in light: the evidence for their diffraction is not wholly conclusive. In some of these properties the rays resemble light of very short wave-length, but since, in the case of ordinary light, the rays with the shorter wave-length are the more refrangible, the absence of refraction seemed conclusive against the identification of the unknown rays with very short waves of light.

On the other hand, the rays resemble strongly the kathode rays, the properties of which were well known at the time of discovery: but they differ from such rays in carrying no electric charge and in being undeflected in the strongest magnetic field that can be produced. Many theories of the nature of these rays were produced in rapid succession at the time when their discovery was exciting universal interest: that which has ultimately been accepted was put forward by Stokes and elaborated by J. J. Thomson and Sommerfeld.

It had been shown by various observations that the rays proceed from any point of a solid body which was struck by the kathode rays travelling inside the exhausted tube: in the original experiments this solid body had been the tube itself, but in improved arrangements a special 'anti-kathode' is inserted to receive the impact of the kathode rays. *Roentgen rays are emitted from any place where the kathode rays are stopped and absorbed.* Now the kathode rays are electrons moving with a high velocity, and the

Faraday tubes, which are attached to them in virtue of their charge, partake in their motion. When the electron is stopped, the tubes attached to it will not take up immediately the position which they will occupy finally, corresponding to their position of rest: their inertia will carry them on in the direction of the original motion of the electron. The tubes will be distorted and a disturbance will travel out along the tube, reducing to its final position each portion of the tube over which it passes. According to the arguments of Chapter II., this disturbance must be of the nature of a transverse 'pulse' or vibration and will travel with the velocity V.

Let OPQ (Fig. 19) represent one of the tubes of the electron just when the electron O begins to be stopped: O' is the position of the electron when it is finally brought to rest. At this moment the position of the tube will be $O'P'Q'$: the 'kink' at P' represents the disturbance set up in the tube by the first shock on the electron: it has travelled out along the tube a distance MP' with the velocity of light, while the electron has moved from O to O' in a time t (say). Hence the distance $P'M = d = Vt$. (The figure is not drawn to scale even approximately.) At a subsequent moment, after the electron has been finally brought to rest, the position of the tube will be $O'P''Q''R$: the portion of the tube ($Q''R$), which has not yet been reached by the disturbance travelling out with the velocity of light, has moved on with the original velocity of the particle: the portion ($O'P''$), which has been

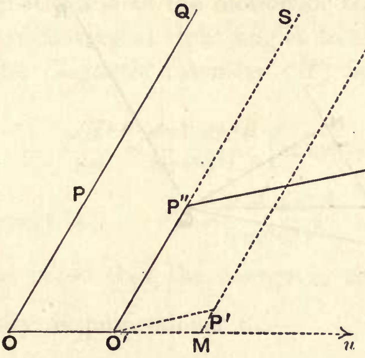

Fig. 19.

reached by the disturbance, has been brought into its final position. After an infinite time when the whole tube has been brought to rest it will occupy the position $O'S$.

We can calculate the energy contained in this pulse which is travelling out along the tube. In Fig. 20 $O'P''Q''R$ represents the position of the tube after a time T since the final stopping of the electron. The electric polarisation in the pulse is in the direction $P''Q''$ and may be regarded as compounded of a polarisation in the direction $P''N$, which will exist when the tube has been brought to rest, together with a polarisation in the direction $Q''L$, which is due to the distortion of the tube. If $OP''=r$, the former polarisation is $\frac{\epsilon}{4\pi r^2}$: let the latter polarisation be D.

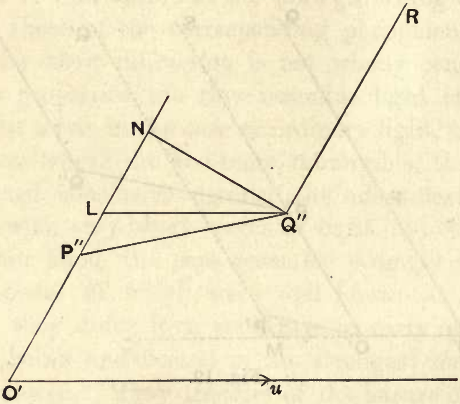

Fig. 20.

Then, from the triangle $P''Q''L$,

$$\frac{D}{\epsilon/4\pi r^2}=\frac{Q''L}{P''L} \quad \text{.........................(1).}$$

But in Chapter II. we saw that the only electric polarisation, which need be taken into account in investigating the propagation of a disturbance along a Faraday tube, is that perpendicular to the tube. Let the electric polarisation along $Q''N$, perpendicular to the tube, be R. Then $R=D\sin\theta$, when θ is the angle between the tube $O'P$ and the direction of the original motion of the electron.

Again, if u is the velocity of that motion, the distance $Q''L = uT$, and $r = VT$: hence $Q''L = \frac{u}{V}r$. Since $Q''L$ is parallel to OO' (Fig. 19), $P''L = P'M = d$.

Hence $R = \frac{Q''L}{P''L} \sin\theta \times \frac{\epsilon}{4\pi r^2} = \frac{ru}{Vd} \times \frac{\epsilon}{4\pi r^2} \sin\theta$,

or
$$R = \frac{\epsilon u \sin\theta}{4\pi r d V} \quad \text{.......................(2).}$$

As in the case of the light vibration treated in Chapter II., the energy of the disturbance will be composed of two parts: (1) electrostatic of an amount

$$\frac{2\pi R^2}{K} = \frac{\epsilon^2 u^2 \sin^2\theta}{8\pi r^2 d^2 V^2} \text{ (since } K = 1\text{)},$$

and (2) magnetic, due to the motion of the tubes. The tubes in number R are moving at right angles to themselves with velocity V, hence the magnetic intensity (H) is $4\pi VR$: the magnetic energy is

$$\frac{\mu H^2}{8\pi} = \frac{\epsilon^2 u^2 \sin^2\theta}{8\pi r^2 d^2 V^2} \left(\text{since } \mu = \frac{1}{V^2}\right).$$

The total energy is
$$E = \frac{\epsilon^2 u^2 \sin^2\theta}{4\pi r^2 d^2 V^2} \quad \text{.......................(3).}$$

It will be noted that the energy in the rays, which measures their intensity, is proportional to $\frac{1}{r^2}$.

We have only considered one tube attached to the electron: to obtain the whole energy of the pulse we must sum up the expression (3) for all the tubes radiating in all directions throughout the volume of the spherical pulse. The result, which is attained by simple integration, is

$$E = \frac{2}{3} \frac{\epsilon^2 u^2}{dV^2}.$$

The quantity d will be called the 'breadth' of the pulse.

Consequences of Stokes' theory.

6. According to Stokes' theory, it is these pulses emitted along the tubes attached to the electron, when it is stopped by collision, which constitute the Roentgen rays. The ionisation produced by the rays represents the action of the electric intensity in the wave-front in dragging electrons out of the atoms. On this view the rays only differ from light in the nature of the disturbance which is propagated along the tubes: in the case of light the disturbance is a series of periodic vibrations, in the case of the Roentgen rays it is an isolated pulse. The direct measurement of the velocity of propagation of the Roentgen rays presents grave experimental difficulties, but the work of Marx and others indicates that, in accordance with this view, it is identical with that of light.

But this difference in the nature of the two kinds of electromagnetic disturbance is sufficient to account for the difference in their properties. The quantity which corresponds to the wave-length of light vibrations is the breadth of the pulse—the distance which light travels between the beginning and the end of the stopping of the electron. Since the electron in the kathode rays is travelling with a velocity comparable with that of light, and since it is probable that the distance in which it is stopped is comparable with the radius of an atom (say 10^{-8} cm.), it is likely that the breadth of the pulse will be about 10^{-8} cm. Thus the wave-length of the disturbance is very much less than that of any known form of light, and the magnitude of the diffraction effects observable will be diminished correspondingly. Haga and Wind have made observations which are generally attributed to a diffraction of the rays, and on that hypothesis have deduced $1{\cdot}3 \times 10^{-8}$ cm. as the breadth of the pulse. On the other hand, Wien has attempted to deduce the breadth of the pulse from a formula similar to (3) by measuring the ratio of the energy of the rays emitted to that of the kathode rays exciting them. If m is the mass of an electron, the ratio of the energy of the pulse (E_r) to that of the electron (E_k) is given by

$$\frac{E_r}{E_k} = \frac{\frac{2}{3}\frac{e^2u^2}{dV^2}}{\frac{1}{2}mu^2} = \frac{4}{3}\frac{e^2}{mdV^2}.$$

From such considerations Wien deduced the value $1{\cdot}15 \times 10^{-10}$ cm. for the breadth of the pulse: the discrepancy between this value and that of Haga and Wind can be explained if it is supposed, as is probable, that the kathode electron is not stopped suddenly, but that it executes several vibrations before it is brought to rest.

It should be noted that if the distance in which the electron is brought to rest is approximately the same whatever its velocity, the breadth of the pulse increases as the velocity of the electron decreases.

The absence of refraction or regular reflection is also connected with the nature of the disturbance. In the case of light we saw that these phenomena were due to the displacement of the electrons by the electric field of the light wave, which gave rise to an opposing electric field. A displacement of the same nature must take place when a Roentgen pulse passes over the atom, but since the time of its passage is extremely short, the pulse has left the atom before the acceleration caused in the electron has produced a sensible displacement, and the opposing electric field set up has no influence on its motion. Nevertheless the setting of the electrons into motion requires the expenditure of energy which must be drawn from that of the pulse[1]: accordingly the energy in the pulse will be diminished and the rays absorbed, even if the electric intensity is not sufficient to cause the ejection of the electrons from the atom and to produce ionisation.

Secondary radiation and polarisation.

7. The reflection of light is connected intimately with its absorption: the reflected waves represent the vibrations excited in the electrons by the incident wave. Since the disturbance in the Roentgen rays is not periodic, the disturbance which it excites in the electrons over which it passes will not be a similar forced vibration. But movements of any nature set up in the electrons must lead to the emission from them of electro-magnetic disturbances, which may be expected to be of two kinds:

(1) Pulses, somewhat similar to the incident pulses, due to the jerk given to the electrons as the rays pass over them, and

(2) Vibrations of the free natural period of the electrons as they return to their positions of equilibrium after displacement.

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[1] In the case of the light wave the energy required for the setting up of the vibrations is drawn from the first part of the train of waves which falls upon the substance and is strongly absorbed. There is little doubt that, if we could make observations during the infinitesimal fraction of a second which elapses between the first incidence of the light rays on a substance and the establishment of the steady vibration, we should find that the behaviour of the light during that interval was more like that of Roentgen rays than that of the light after the interval had elapsed.

Disturbances of the second kind are probably represented by the phosphorescence which the rays cause in many substances: disturbances of the first kind have been detected as 'secondary Roentgen rays' emitted in all directions from bodies struck by the primary beam. A great deal of valuable work on the subject of these secondary rays has been published recently by Barkla: some of his most interesting results concern the nature of the secondary rays emitted from the various elements. He finds that the lighter atoms—all those which have an atomic weight less than that of calcium[1]—produce secondary rays which are exactly similar to the primary rays: while the heavier atoms produce rays of less penetration than the primary rays.

Since the electrons, the impacts of which give rise to the Roentgen rays, are all travelling in the same direction, it is to be expected that the electric polarisation in the pulses emitted from them will lie in planes passing through that line and not in planes perpendicular to it: that is to say, the rays should be polarised in a plane perpendicular to the direction of motion of the electrons. Signs of such polarisation have been detected by Barkla, who finds that the intensity of the secondary radiation excited by the passage of a beam of the rays through a gas depends on the direction of emission: the secondary rays emitted at right angles to the path of the kathode rays in the tube from which the primary rays emerge are more intense than those emitted in any other direction. Since the rays are not completely polarised, it must be supposed that the path of the electron while it is being stopped is not a straight line, but an irregular path, parts of which may lie in all directions.

Absorption and ionisation.

8. Many quantitative experiments have been performed on the absorption of the Roentgen rays in passing through matter. It is found that the ratio of the energy E_x of the rays emergent from a layer of thickness x to the energy E_0 of the rays entering the layer is connected with the thickness of the layer by an expression of the form

$$\frac{E_x}{E_0} = e^{-\lambda x} \quad \text{.........................} (4),$$

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[1] Potassium was not investigated.

where e is the base of Naperian logarithms and λ a constant depending on the nature of the rays and of the absorbing substance. Since we shall meet with a relation of this form in investigating several of the phenomena with which we shall be concerned in this chapter, it will be well to investigate its physical meaning.

Let us suppose that the amount of energy given up to the electrons in an atom through which the rays pass is proportional to the energy contained in the rays. Consider a ray initially containing an amount of energy E_0 travelling through a layer of material of thickness x. Suppose this layer to be subdivided into n layers each of thickness d, n being a very large number. Since the energy lost in each layer is proportional to the energy entering it, the energy emerging from the first layer is

$$E_1 = E_0 - E_0 . \lambda d = E_0 (1 - \lambda d) \quad \text{..............(5)},$$

where λ is the 'absorption coefficient' of the medium and measures the fraction of the energy absorbed in a very thin layer divided by the thickness of that layer. Similarly, the energy emerging from the second layer is

$$E_2 = E_1 (1 - \lambda d) = E_0 (1 - \lambda d)^2 \quad \text{..............(6)},$$

and that emerging from the nth is

$$E_n = E_0 (1 - \lambda d)^n, \quad \text{or since } nd = x, \quad E_n = E_0 \left(1 - \frac{\lambda x}{n}\right)^n \quad \text{.........(7)}.$$

If n is made very large and each layer very thin, the expression on the right-hand of (7) becomes $E_0 e^{-\lambda x}$, where e is the base of natural logarithms. Consequently the energy from the whole slab of thickness x is

$$E_x = E_0 e^{-\lambda x} \quad \text{..........................(8)}.$$

It will be observed that $1/\lambda$ is the distance to which the rays penetrate in the substance before their energy is reduced to $\frac{1}{e}$ of its original value. The greater the value of λ the smaller is the penetration of the rays.

The value of λ depends on the nature of the rays, and on the nature of the absorbing substance. It is found that the lower the pressure of the gas in the tube from which the rays are emitted the greater is the penetration of the rays. But the lower the pressure, the greater is the electric intensity in the tube and the greater the velocity of the kathode rays: and again (§ 6), the greater the velocity of the kathode rays, the less is the breadth of the pulse. Hence penetrating rays represent thin pulses. This conclusion might have been anticipated: for the thinner the pulse, the less is the time it takes to pass over an electron in the absorbing material and the less the time it has to communicate energy to that electron. Since the rays are absorbed by communicating their energy to the electrons in the absorbing substance, the thinner the pulse the smaller will be the absorption. It is usual to speak of the more penetrating rays as 'hard,' and of the more absorbable rays as 'soft.'

Since the penetration of the rays for any given substance varies with the nature of the rays, comparisons of the absorbing power of different materials must be made for the same kind of rays: quantitative experiments are rendered difficult by the impossibility of securing the emission of rays of the same hardness from the same tube for more than a very limited period of time. But the results quoted in the table below show the nature of the measurements that have been made. The third column (q) gives the relative values of the ionisation produced by rays of the same nature in the substances named in the first column.

TABLE VIII.

	λ	q (relative to air)	density (relative to air)
Hydrogen	v. small	0·5	·07
Air	·001	1·0	1·0
Carbon Dioxide	·001	1·2	1·52
Sulphuretted Hydrogen	·0037	6·0	1·18
Chlorine	·0095	13·0	2·44
Methyl Iodide (vapour)	·07	?	4·59
Aluminium	10·0 (about)	?	2020

It will be observed, as has been anticipated, that the ionisation produced in the gas by the rays increases with its absorbing power (λ): but there is no accurate proportionality between absorption and ionisation: the ratio of the energy spent in ejecting electrons from the atoms to that spent in causing merely temporary disturbances in them, depends to some extent on the nature of the atom. It will be observed also that the absorbing power and the ionisation show a tendency to increase with the density of the gas, but here, again, there is no strict proportionality.

Experiments on the absorbing power of chemical compounds show that an atom has the same absorbing power whatever its state of chemical combination. The hypothesis that the forces restraining the electrons are determined by the internal structure of the atom receives further confirmation.

It is interesting and important to observe that, when the most intense Roentgen rays that we can produce are sent through a gas, only an extremely small fraction (less than one-billionth) of the whole number of atoms present are ionised. It might have been anticipated that the action of the rays on all atoms of the same substance would be the same, and that, if a given beam of rays could ionise one atom, they could ionise every atom that they reached, until all their energy was exhausted: but experiment shows that this anticipation is incorrect.

This is another similar difficulty which calls for remark connected with the electric intensity in the Roentgen pulse. Choosing our values so as to make this quantity as large as possible we may put

$$u = 10^{10}\text{ cm./sec.}, \quad d = 10^{-10}, \quad \sin\theta = 1.$$

Then from (2)

$$R = \frac{3\cdot4 \times 10^{-10} \times 10^{10}}{4\pi \times 10 \times 10^{-10} \times 3 \times 10^{10}} = \frac{0\cdot11}{4\pi}$$

at a distance of 10 cms. from the source.

The electric intensity $X = 4\pi R$:

hence $X = 0\cdot11$ electrostatic units

or 33 volts per centimetre: whereas we concluded on p. 170 that an electric intensity of $2\cdot1 \times 10^9$ volts per cm. was required to drag an electron out of an atom. How then do the rays come to produce any ionisation at all? The answer to this question will be discussed in the last paragraph of the chapter, when we have gained more light on the general processes of ionisation.

Secondary rays from metals.

9. It is not only the atoms of gases that can be ionised by the Roentgen rays. When the rays fall upon metals secondary rays are emitted, which consist, at least in part, of electrons expelled with a velocity between 10^9 and 10^{10} cm./sec. The examination of these electrons leads to some interesting and suggestive conclusions. In the first place it appears that the velocity with which the electrons are ejected is independent of the intensity of the rays. A still more interesting result is that the energy of the secondary rays, or at any rate of that part of it which is spent in ionisation, may be greater than that of the incident rays: it has been suggested that the rays can liberate potential energy contained in the atoms of the metal. This conclusion is confirmed by observations by Bumstead, who found that the quantity of heat developed when a given beam of rays is totally absorbed in a metal depends in some degree on the nature of the metal: it is twice as large in lead as in zinc. It is difficult to explain these experiments except by the assumption that part of the heat energy is derived, not from the rays, but from the metal in which they are absorbed. The importance of these conclusions will appear later.

It seems unlikely that similar phenomena can occur in gases, for the velocity with which the secondary rays from metals are emitted is sufficient to enable them to act as ionising agents. If such electrons were emitted from the atoms of gases under the action of the rays, the ionisation produced would not be simply proportional to the amount of energy absorbed from the rays.

(3) The Rays from Radioactive Substances.

Becquerel rays.

10. In 1896, just after the discovery of the Roentgen rays, Becquerel investigated the action of various phosphorescent substances on a photographic plate, with the idea that the emission of Roentgen rays was connected with the phosphorescence of the walls of the tube. None of the substances that he tried had any action on a photographic plate through a screen opaque to light, with the exception of uranium salts: and the action of these salts he showed to be independent of

the phosphorescence of the substance, for it persisted long after all phosphorescence, due to exposure to light, had died away. A further investigation showed that the emission of rays resembling Roentgen rays in several important respects, such as their power of penetrating opaque bodies and exerting photographic and ionising effects, was an intrinsic property of all the compounds of uranium. To these rays the name of 'Becquerel rays' was given at the time of their discovery: but the importance of the results to which the investigation of the rays has led is so great and the nature of the rays so complicated that the original name has been almost dropped in favour of a later and more distinctive nomenclature.

From Becquerel's discovery proceeded the whole of the new science of radioactivity, which first began to attract universal attention both among professed physicists and among laymen when M. and Mme. Curie announced the discovery of a new element, to which they gave the name Radium, possessing the peculiar properties of uranium with far greater intensity.

It would be impossible, within the compass of this volume, to treat in a manner approaching completion the new branch of science which has arisen out of the observations of Becquerel: only those facts will be selected from the immense number that have been recorded within the last few years which seem to have some bearing on the subjects which will be discussed in the next part. But the fresh information is accumulating with such rapidity that it is likely that some at least of the statements made will have been found to be erroneous before the book is in the hands of the reader.

α rays.

11. There are three distinct kinds of Becquerel rays, or ionising rays given out by radioactive substances—α rays, β rays and γ rays. (We will postpone for the present the discussion of the characteristics which entitle a substance to be regarded as 'radioactive.' At the present time there is little, if any, difference of opinion on this point.)

The α rays are the least penetrating of the Becquerel rays: no α rays are known which can penetrate 10 cms. of air at atmospheric pressure, or a couple of sheets of note paper, without losing their ionising properties: but, at the same time, they produce very nearly all the ionisation caused in a gas exposed to radium without the interposition of an absorbing screen. At first no sign could be detected of their deflection in a magnetic field, but Rutherford has shown that they suffer a slight deflection in such a direction as to indicate that they are positively charged, thus confirming an hypothesis put forward previously by Strutt. Lately this positive charge has been detected directly by J. J. Thomson and by Rutherford.

The value of e/m and the velocity of the rays has been determined by several observers by the method of magnetic and electrostatic deflections. The experiments of Rutherford lead to the remarkable result that the value of e/m is the same from whatever element the rays are emitted: this value is $1{\cdot}56 \times 10^{14}$: the velocity of the rays varies with their origin, but it is never very different from 2×10^9 cm./sec. It will be observed that the value of e/m is identical with that of one of the groups of universal kanalstrahlen found by J. J. Thomson. The α rays differ from those kanalstrahlen only in their immensely greater velocity. Henceforward the kanalstrahlen will be included in the general term α rays.

Absorption of α rays.

12. The absorption of the α rays has been the object of much recent research. Bragg and Kleeman showed that the relation between the thickness of the absorbing layer and the proportion of the rays absorbed is entirely different from that which was noted in dealing with Roentgen rays. The ionisation which the rays can produce at a point in their path does not, as in the case of Roentgen rays, diminish gradually with the distance which the rays have travelled through the absorbing substance: over a certain distance, which is termed the 'range' of the rays, the ionisation does not vary greatly, but when the limit of the range is reached the rays can produce no further ionisation. Suppose, then, that the rays of range r (in air) are emitted normally from a thin layer of the radioactive substance: then at all points, the distance of which from the layer is not greater than r, the rays will cause ionisation. A sheet of thickness d of a substance in which the range is r' (less than r) is now placed over the layer: this sheet will deduct from the range of the rays in air a distance $\frac{r}{r'}d$: hence, there will be no ionisation at a point the distance of which from the layer is greater than $r\left(1-\frac{d}{r'}\right)$. The effect of the absorbing sheet is not to reduce the ionisation at all points on the other side of the sheet: indeed, since the ionisation produced by the rays is greater near the end of their range, the ionisation just beyond the screen is greater than before: but the thickness of the layer of air in which there is any ionisation at all is diminished and with it the total amount of ionisation.

But if the rays, instead of being emitted normally from a very thin layer, are emitted in all directions from a thick layer, the interposition of the sheet of absorbing material decreases not only the thickness of the layer of air in which there is ionisation, but also the intensity of ionisation in that layer. For, firstly, the rays which reach any point in the air do not all come from the surface of the radioactive substance: some come from the interior and have performed part of their range before they emerge. The presence of the sheet diminishes the range in air of the rays, and, while those which come from the surface may still be able to reach a given point, others which come from the interior may be able to reach that point no longer. Secondly, the rays, which reach a given point travelling obliquely to the radioactive layer, have performed a greater part of their range than those which are travelling normally; the sheet prevents some of the oblique rays reaching a point at which the normal rays can still arrive. Accordingly if the absorption be studied by observing the decrease in the ionisation due to the rays from a thick layer caused by the interposition of successive sheets of absorbing material, each successive sheet will cause a decrease in the ionisation, and the law of absorption appears somewhat similar to that observed in the case of Roentgen rays. In fact before the publication of Bragg's work it was supposed that the α rays were absorbed in matter according to the same law as the Roentgen rays.

It is found that the range in air of the α rays varies from about 3 cms. to 8·6 cms. according to the radioactive element from which they are emitted, and that the number of ions of either sign produced by an α particle is passing through 1 cm. of air is about 29,000. This last result indicates that the particles must ionise almost every molecule with which they collide. For, since the value of e/m for the particles shows that they are of atomic dimensions, it may be concluded that their radius is about 10^{-8} cm. Hence the number of molecules with which the particle collides in passing through 1 cm. of air is the number contained in a cylinder of 1 cm. in length and of a radius of 2×10^{-8}. Since the number of molecules in 1 c.c. of air is $3{\cdot}6 \times 10^{19}$, the number of molecules with which the particle collides is

$$1 \times \pi \times (2 \times 10^{-8})^2 \times 3{\cdot}6 \times 10^{19} = 43500,$$

which is nearly equal to the number of ions formed.

Experiments have been made by Bragg and Kleeman, Laby and others on the range of the rays in different materials. It appears that the effect of an atom in stopping the particle depends only on its nature and not on its state of chemical combination or on the state of aggregation of the substance. Further it is found that, if a particle passes through layers of different elements containing the same number of atoms, the diminution of range is proportional to the square root of the atomic weight of the substance. In the following table the second column gives the diminution of the range of a particle in passing over a molecule of the substance named in the first column, compared with the diminution in passing over a molecule of air: the third column gives the square root of the molecular weight of the substance. It will be observed that Bragg's law holds accurately over a large number of diverse elements.

TABLE IX.

	stopping power	$\sqrt{\text{molecular weight}}$
Hydrogen	0·246	0·266
Air	1·000	1·000
Aluminium	1·53	1·38
Silver	3·11	2·75
Platinum	4·12	3·7
Methyl Iodide (CH_3I)	2·49	2·35
Carbon Tetrachloride (CCl_4)	3·8	3·61

The connection of the ionisation caused in a gas with its 'stopping power' is not very simple. Bragg and Kleeman deduce the following important results from their observations:—

(1) The energy required to ionise an atom of a gas is independent of the nature of the atom.

(2) The total number of ions formed by an α particle is the same in all gases.

Hence, (3) the number of ions formed in passing over a single atom is proportional to the square root of the atomic weight of the element. This last conclusion is somewhat surprising and difficult to reconcile with deductions from other considerations, but it seems to follow directly from the experiments mentioned.

Rutherford has made some extremely interesting measurements on the velocity of the α particles at various points of their range, and has attained the remarkable conclusion that rays cease to exert any ionising action when their velocity is less than $1{\cdot}12 \times 10^9$ cm./sec. He finds also that other actions of the rays, such as their effect on a photographic plate or in producing phosphorescence, cease at the same velocity as their ionising action. The α particle seems to become inactive when its velocity sinks to this value. On the other hand it is known that the particles in the kanalstrahlen, which never attain to so great a velocity in any practicable conditions, exert a very powerful action in ionisation and in exciting phosphorescence. The most probable explanation of this discrepancy is that the rays lose their ionising power by losing their charge and not by losing their velocity. The α particles in the course of their flight must meet frequently with charged particles of opposite sign, derived from the atoms which have been ionised by previous particles: it is possible that a neutralisation of the charge cannot take place when the velocity of the particle exceeds the minimum ionising velocity found by Rutherford. Thomson's experiments (p. 162) indicated that the kanalstrahlen were liable to lose their charge by combination with negative ions, but, since kanalstrahlen can only be investigated in gases at very low pressures, they are able to travel a considerable distance and produce a notable ionisation before they meet with neutralising particles. It would be interesting to investigate from this point of view the range of the α particles in gases at low pressures: it is to be expected, if the explanation offered is correct, that the 'stopping power' of an atom would be found to decrease rapidly with the pressure of the gas in which it is contained, after a certain limit had been reached.

The energy required to ionise an atom.

13. Rutherford's measurements enable us to calculate the energy required to ionise an atom of a gas. The α particles are ejected from radium with a velocity of $2{\cdot}14 \times 10^9$ cm./sec.: in passing over its range of 3 cms. each particle produces 86,000 ions and its velocity falls to $1{\cdot}12 \times 10^9$ cm./sec.: hence, if m is the mass of the α particle the kinetic energy lost is

$$\tfrac{1}{2} m \left\{(2{\cdot}14)^2 - (1{\cdot}12)^2\right\} \times 10^{18} = m \times 1{\cdot}66 \times 10^{18}.$$

If we assume that the kinetic energy lost is spent in ionisation, the energy required to ionise an atom is

$$\frac{m \times 1{\cdot}66 \times 10^{18}}{86000} = m \times 1{\cdot}94 \times 10^{13}.$$

The value of e/m for the particle is $1{\cdot}56 \times 10^{14}$; assuming that e is equal to the electronic charge ϵ, the mass of the particle is $2{\cdot}17 \times 10^{-24}$ grm., and the energy required to ionise an atom is

$$2{\cdot}17 \times 10^{-24} \times 1{\cdot}94 \times 10^{13} = 4{\cdot}2 \times 10^{-11} \text{ ergs}.$$

It is usual to express this energy in terms of the potential difference, V, through which a particle carrying the charge ϵ must fall in order to acquire that energy. We have then

$$\epsilon V = 4{\cdot}2 \times 10^{-11}, \text{ and } V = {\cdot}122$$

or

$$V = 36{\cdot}6 \text{ volts}.$$

It must be remembered that this value applies only to the atoms of a gas which are so far apart as to be free from mutual influences. The work required to ionise the atom of a solid or liquid may be much less—indeed, no work at all may be required.

β rays.

14. In general the β rays are very much more penetrating than the α rays, but their penetration varies over a much wider range. Some of them appear to be no more penetrating than the α rays[1], while others can produce

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[1] Before the nature of the α and β rays was understood the easily absorbed α rays were confused with β rays.

ionisation after passing through $\frac{1}{2}$ cm. of lead. Usually only 1 % of the total ionisation caused by a radioactive substance is due to the action of the β rays (unless it is covered with an absorbing sheet to cut off the α rays), but most of the photographic action is due to them: the penetration of the α rays is so small that they cannot pass through the gelatine covering the sensitive salts.

The nature of the β rays was elucidated simultaneously by several observers very soon after the publication of Becquerel's discoveries. It was found that they were easily deflected by a magnetic field, and measurements of the quantity e/m by the usual method showed that it was almost identical with that for the kathode rays. It was concluded that the rays were rapidly moving negative electrons, and the conclusion was confirmed when Curie, Strutt and others detected the negative charge directly. The β rays are identical with the kathode rays and the Lenard rays in all essential respects, but in general they possess a far greater velocity. We shall speak henceforward of the kathode rays under the more general name of β rays.

The most accurate measurements of e/m and of v, quantities the value of which is ascertained by the same observations, have been carried out by Simon and Kaufmann. Velocities as high as $2{\cdot}85 \times 10^{10}$ cm./sec., 95 % of the velocity of light, have been noted, but it is not certain whether there is a lower limit to the velocity. The most remarkable result attained is that the value of e/m is not strictly constant but depends on the velocity. For particles the velocity of which is not greater than 10^{10} cm./sec., e/m is constant at the value $5{\cdot}59 \times 10^{17}$: but at higher velocities values notably smaller are found: for rays the velocity of which is $2{\cdot}85 \times 10^{10}$, e/m is only $1{\cdot}89 \times 10^{17}$. The explanation of this variation will concern us in a later paragraph.

Absorption of β rays.

15. The relation between the proportion of the rays which pass through a layer of absorbing material and the thickness of that layer is the same as that for the Roentgen rays. The intensity may be measured by the ionisation which the rays produce in a gas, or by the charge which they carry. If I is the intensity of the rays which enter a layer of thickness x, and I' the intensity of the rays which emerge, then

$$I' = Ie^{-\lambda x},$$

where λ is a constant characteristic of the rays and of the absorbing substance.

λ decreases rapidly with an increase in the velocity of the rays, being approximately proportional to the inverse of the fourth power of the velocity of the rays. Such a result might have been anticipated. The mechanism of absorption and of ionisation is doubtless similar to that which we considered in the case of Roentgen rays. The higher the speed of the particle, the less is the time in which it can give energy to an atom over which it passes, and the larger the number of atoms it can pass before its energy is exhausted. Rapid β rays correspond in this respect to thin Roentgen pulses. Accordingly the ionisation caused by the rays in passing through a small volume of gas increases as the velocity decreases: but the total ionisation in a volume of gas so large as to absorb all the rays is greater for the more rapid rays, since they carry a larger supply of energy. If the velocity of the rays is so small that the particles do not possess enough energy to ionise a single atom, they produce no ionisation at all. Since the electrons composing the β rays are very small compared to the molecules of a gas through which they pass, they only collide with an extremely small fraction of those molecules: but it appears that the slower particles, having a velocity of about 10^9 cm./sec., ionise every molecule with which they collide.

All the β rays emitted by uranium, unlike those emitted by most radioactive substances, appear to have the same velocity, $1\cdot6 \times 10^{10}$ cm./sec.: accordingly these rays are peculiarly suitable for comparing the relative absorbing powers of different materials. The earlier experiments led to the conclusion that the value of λ for any material was proportional to its density: but Crowther has shown that this generalisation is not accurately true. The following table gives his results for a large number of elements; λ is the absorption coefficient and β the density of the element. A most interesting connection can be traced between the value of λ for an element and its position in Mendeleef's periodic table. In passing along a single row from the lighter atom to the heavier, λ at first decreases slightly but increases rapidly at the end of the row: its value for the first element of the next row is nearly the same as that of the last element in the preceding row. It appears also that each atom exerts the same absorbing power whatever its state of chemical combination.

TABLE X.

Substance	at. wt.	λ/ρ	Substance	at. wt.	λ/ρ
Boron	11	4·65	Arsenic	75	8·2
Carbon	12	4·4	Selenium	79	8·65
Sodium	23	4·95	Strontium	87·5	8·5
Magnesium	24·4	5·1	Zirconium	90·7	8·3
Aluminium	27	5·26	Palladium	106	8·0
Silicon	28	5·5	Silver	108	8·3
Phosphorus	31	6·1	Tin	118	9·46
Sulphur	32	6·6	Antimony	120	9·8
Potassium	39	6·53	Tellurium	126	10·8
Calcium	40	6·47	Iodine	127	10·8
Titanium	48	6·2	Barium	137	8·8
Chromium	52	6·25	Platinum	195	9·4
Iron	56	6·4	Gold	197	9·5
Cobalt	59	6·48	Lead	207	10·8
Copper	63·3	6·8	Uranium	240	10·1
Zinc	65·5	6·95			

Secondary β rays.

16. If the theory of the Roentgen rays described in the previous section is correct, the β rays must produce Roentgen rays when they fall upon a solid body and are stopped by it. Such rays do not appear to have been detected, but there is a secondary radiation consisting of rapidly moving electrons excited when the β rays fall on a solid. The nature of these secondary rays depends on the nature of the substance in which they are excited: in some substances (such as lead) the secondary radiation appears to be identical in penetrating power with the primary: in other substances there is, in addition to the secondary rays identical with the primary, secondary rays of considerably less velocity and penetration. It would seem probable that the first class of secondary radiation consists simply of scattered primary radiation, and that the electrons constituting the primary rays are deflected from their course by the attraction of the electrons in the absorbing substance, emerging in a different direction to that in which they entered. There are certain objections to this view into which we need not enter, but it seems preferable to the alternative that the electrons in the secondary rays consist of electrons shot out of the atoms of the absorbing substance with a velocity which happens to be identical with that of the primary rays. It is found, as might be expected, that the intensity of the secondary rays decreases with an increase in the velocity of the primary rays.

The existence of these secondary rays complicates somewhat the interpretation of experiments on absorption. For the rays which emerge from a plate of an absorbing substance have been decreased, not only by the number of electrons which have spent all their energy in the substance, but also by those which have been deflected and sent out in other directions. It is only when such a thickness of the substance has been traversed that the incident rays have been scattered by the substance equally in all directions that there will be no further diminution of the number of emergent rays due to the second cause. Since it is not found that the coefficient of absorption of a substance for homogeneous rays is greater for a small thickness of the substance than for a greater thickness, we may conclude that, in the ordinary method of measurement, the rays emitted from the radioactive substance have undergone already complete scattering. A great deal of work is being done at the present time on the absorption of β rays and the emission of secondary rays by their action. It seems that these phenomena are by no means so simple as has been represented above, and it is likely that many of the statements made will be proved to be untrue.

δ rays.

17. It was noted above that, if the velocity of a β ray particle were less than a definite limit, it would be unable to produce any ionisation because it would not possess the requisite energy. The magnitude of this velocity can be calculated easily. The energy required to ionise an atom is $4{\cdot}2 \times 10^{-11}$ ergs: hence v, the limiting velocity, must be given by

$$\tfrac{1}{2} mv^2 = 4{\cdot}2 \times 10^{-11},$$

where m is the mass on an electron, $6{\cdot}5 \times 10^{-28}$,

hence $$v = 3{\cdot}6 \times 10^8 \text{ cm./sec.}$$

Such very slow β rays have been detected by J. J. Thomson by means of the charge that they carry: it has been proposed to call them δ rays. The velocity of the rays has been measured by Ewers who finds that it is $3\cdot25 \times 10^8$, that is, less than the velocity necessary for producing ionisation.

Electromagnetic Mass.

18. We must return to the problem arising out of the variation of the value of e/m for the β rays with their velocity. To assume that this variation represents a change in the charge on the electrons would be to abandon the whole theory of electricity with which this book is concerned. The charge on a body represents the number of Faraday tubes radiating from it and the assumption of the constancy of the properties of those tubes is the basis of our discussion. We must suppose that the mass of the electron changes with the speed and may proceed to consider whether such a change is explicable in terms of the theory adopted hitherto.

In Chapter I. we saw that, when Faraday tubes move at right angles to their own direction, they behave as if they possessed mass. Now an electron is a charged body, and from it radiate Faraday tubes, which it carries with it in its motion. The velocity of all these tubes, with the exception of those which lie exactly along the direction of motion, will have a component at right angles to the direction of the tubes. Accordingly an electron, or any charged body, possesses a certain mass in virtue of the Faraday tubes attached to it.

The magnitude of this mass can be calculated readily from the considerations of Chap. I. Let the electron be taken as a sphere of radius a with its centre at O. Consider a small volume u drawn about P a point where $OP = r$: let the electric polarisation at P, which will be along OP, be N. Then there will be Nu tubes passing through u and each of these tubes will have a mass of $4\pi\mu N \sin^2\phi$ or $\frac{4\pi N \sin^2\phi}{V^2}$ per unit volume, where ϕ is the angle between OP and the direction of motion OX. Accordingly the total mass of all the tubes in u is $\frac{4\pi N^2 \sin^2\phi \,.\, u}{V^2}$. Since the number of tubes radiating from the electron is ϵ, ϵ is the total number of the tubes passing through the surface of the sphere drawn about O through OP: the area of the surface of this sphere is $4\pi OP^2$ and so the number of tubes passing through unit area perpendicular to P is $\frac{e}{4\pi OP^2}$. Therefore $N = \frac{e}{4\pi OP^2}$ and the mass of the tubes in u is

$$\frac{4\pi e^2 \sin^2\phi \,.\, u}{OP^2 . V^2}.$$

To obtain the total mass of all the tubes in the whole space around the electron, we have to calculate the mass for each of the small volumes u and add them together. Any reader who is acquainted with the elements of the integral calculus can perform the summation for himself, and those who have not that acquaintance would be more likely to be confused than aided by the cumbrous calculations necessary for obtaining the result by so-called elementary methods. It will be found that the total mass (m) of all the tubes is given by

$$m = \frac{2}{3}\frac{e^2}{a V^2},$$

where a is the radius of the electron.

This quantity represents the mass which the electron will possess in virtue of the charge which it carries. It should be noted that the smaller the radius of the electron the greater is its mass, a result seemingly paradoxical to those who have been accustomed to associate mass with bulk. For bodies of visible dimensions, or even of molecular dimensions, the 'electrical mass' due to the largest charge which such bodies can carry is quite inappreciable compared with the total mass. Thus if we take a sphere of 1 cm. radius with a mass of one milligramme and charge it to a potential of 50,000 volts, an operation which is just beyond the possibilities of practice, it will carry a charge of 200 electrostatic units. Accordingly the electrical mass would be

$$\frac{2}{3}\frac{(200)^2}{(3 \times 10^{10})^2} = 3 \times 10^{-15},$$

or about one-billionth part of the total mass.

Or consider an electrolytic ion which carries a charge of

$3{\cdot}4 \times 10^{-10}$ units, and has a radius of about 10^{-8} cms. Its electrical mass is

$$\frac{2}{3} \cdot \frac{(3{\cdot}4 \times 10^{-10})^2}{(3 \times 10^{10})^2 \times 10^{-8}} = 10^{-32} \text{ (about)},$$

whereas, even if it be a hydrogen ion, its total mass will be about 10^{-24}, a hundred million times as great. But an electron carries the same charge as the ion, and since it is part of an atom it must be much smaller than the ion. We have no information as yet as to its size and may attribute to its radius any value we please, subject to a possible revision hereafter if contradictions arise. By taking the radius sufficiently small we can make the electrical mass as great as we please: for instance we may make its electrical mass equal to the total mass found on p. 91. On this supposition the radius will be given by

$$m = 6{\cdot}5 \times 10^{-28} = \frac{2}{3} \cdot \frac{(3{\cdot}4 \times 10^{-10})^2}{(3 \times 10^{10})^2 \times a},$$

whence $$a = 10^{-13} \text{ cm. (about)},$$

i.e. about one hundred thousandth of that of the atom.

The first calculation of the electrical mass of a charged body was given by J. J. Thomson in 1881, at a time when few physicists foresaw the importance which the result would have in the future. The calculation showed that the electrical mass is not strictly constant, but depends on the velocity of the charged body. We can arrive at some suggestion of the relation between the mass and the velocity by the method of Chapter I. In calculating above the total mass of all the tubes attached to the charge, it was assumed that these tubes radiate out from the centre of the sphere equally in all directions. If the charge is at rest, the tubes are equally distributed as we have imagined, for the lateral pressure on any tube due to the repulsion of its neighbours is the same in all directions. But we have seen (p. 22) that a motion of the tubes in a direction perpendicular to their length alters the lateral pressure which they exert on one another. In Fig. 21, where the electron is moving in the direction OX, the tubes OA, OA' are moving approximately perpendicular to their direction: accordingly the lateral pressure which they exert on their neighbours is less than when the charge is at rest. But the tubes OX, OX' are moving parallel to their own direction and their lateral pressure is undiminished. Hence the motion of the sphere will tend to make the tubes crowd up into the equatorial plane AOA', perpendicular to the direction of motion. But the mass of a tube increases when it

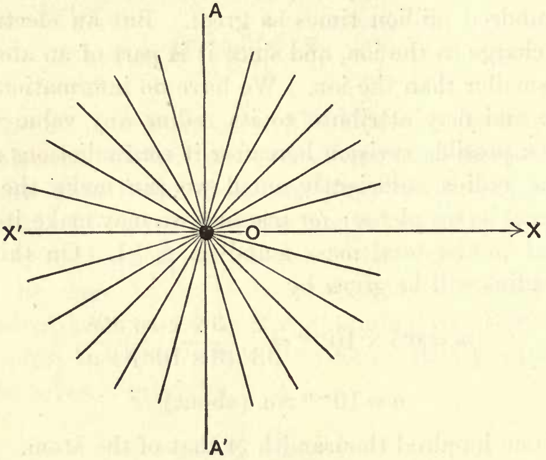

Fig. 21.

moves more nearly perpendicular to its own direction: the crowding of the tubes into the equatorial plane means that they are, on the whole, moving more nearly at right angles to their length and hence the total mass of the tubes will be increased by the motion. The greater the speed of the charge, the greater will be its electrical mass.

Accordingly the change in the magnitude of the electrical mass of the electron is in the direction necessary to explain the observed variation of e/m. But it is still open to question what part of the mass is the electrical mass, due to the charge carried: is it, as in the case of the electrolytic ion, only a small fraction of the whole, or is it, as has been suggested, the whole mass? The latter conclusion, if it could be maintained, would be of the greatest interest and importance. The question can be answered, if the variation of the mass with the velocity can be determined with sufficient accuracy: if it is found that this relation is that predicted for mass of electric origin on theoretical grounds, it cannot be doubted that the whole mass of the electron is the mass of the Faraday tubes connected to it.

The requisite measurements were made by Kaufmann with exquisite skill. The rays passed through superimposed electrostatic and magnetic fields, the intensities of which were measured with great accuracy: the directions of the two fields were parallel so that the deflections were at right angles. At the end of their passage the rays fell on a photographic plate and produced an impress in the form of a curve (Fig. 22). Let *O* be the impress caused by the rays in the absence of electric or magnetic field, *ON* the direction of the magnetic deflection, *OM* the direction of the electric deflection. Any point *P* on the curve represents the action of an electron such that its electrostatic deflection is *PN* and its magnetic deflection *PM*: these deflections can be measured on the plate and the values of e/m and v calculated as on p. 155.

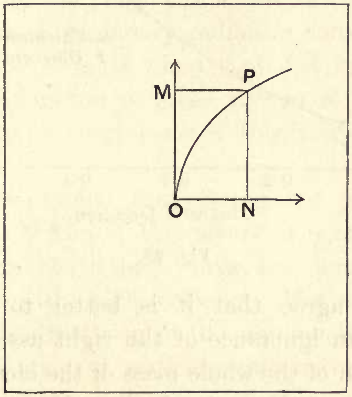

Fig. 22.

Fig. 23 shows the relation calculated by Abraham between the magnetic and electric deflections and compares it with that found experimentally by Kaufmann. The difference between the observed and theoretical curves is perhaps greater than the error of experiment, but this discrepancy need not shake our belief that the mass is purely electrical. The exact form of the theoretical curve depends on certain assumptions made as to fundamental points in electro-dynamics. Different physicists have used different assumptions and the difference between their results is greater than that between the experimental and the best of the theoretical curves. All will agree that it is better to assume that the error arises from an ignorance of the right assumption than that some small fraction of the whole mass of the electron is of a nature different from that of the rest. In Part IV. we shall take up the question again and find that it leads to some of the most interesting problems of modern science.

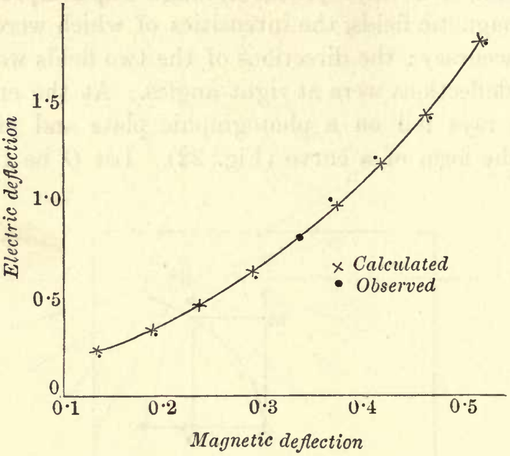

Fig. 23.

19. The third class of Becquerel rays are known as γ rays. They are more penetrating than the β rays and can exert an appreciable ionising action through several inches of solid lead: they appear to be undeflected in the strongest magnetic fields that can be attained experimentally. It has been generally held that they consist of very thin electromagnetic pulses, similar to Roentgen rays except in their very great hardness, and that they are emitted by the β rays at the moment of their expulsion: for the considerations of § 5, which led to the conclusion that pulses are given off when a charged body is stopped, apply with equal force to a charged body which is started.

γ rays.

On the other hand it has been suggested that they are β rays possessing a very high velocity: for equation (8), p. 155, shows that the deflection suffered by a charged body in an electric or magnetic field diminishes indefinitely as the velocity of the body increases. Attempts have been made to settle the question by examining whether the rays carry a charge with them, but the observations are complicated by the occurrence of secondary radiation such as was mentioned on p. 181. At present it can only be said that there is no proof whatever that the γ rays carry a charge: and since it is theoretically probable that rays having the properties of hard Roentgen rays must be given off by radioactive substances, it is well to accept for the present this view of their nature.

The γ rays excite a secondary radiation consisting of rapidly moving electrons (or β rays) when they fall upon solid bodies. But a further consideration of these rays or of the γ rays themselves would lead to no conclusions of theoretical importance and may be omitted.

It should be mentioned that Bragg has proposed recently an entirely different theory of the nature of γ rays and Roentgen rays. He suggests that these rays are uncharged material particles travelling with high velocity. His memoir has been published too late for discussion in this volume, but his arguments, though suggestive, are not entirely conclusive. The proposed theory gives no adequate explanation of the constant association of γ rays with β rays.

Radioactive Energy.

20. A radioactive substance is constantly emitting Becquerel rays which can perform work in ionising the atoms over which they pass: the liberation of a considerable amount of energy is one of the most striking characteristics of these substances. This emission of energy is illustrated in a very striking manner by experiments first performed by Curie and Danne. A small quantity of a radium salt was placed in a cavity bored in a mass of lead, in which was also placed a thermo-couple for measuring the temperature. It was found that the temperature of the radium was always several degrees higher than that of the outside of the mass: since heat is conducted and radiated away by the lead, the maintenance of this constant temperature-difference indicates that heat energy is given off by the radium. By comparing the temperature-difference caused by a known weight of radium with that given by a source from which the rate of emission of heat was known (a wire heated by an electric current), the rate of emission of heat from the radium was determined. The experiments of Curie, Rutherford and Barnes and others show that one gramme of pure radium emits heat at the rate of about 105 gramme calories per hour or

$$\frac{105 \times 4{\cdot}2 \times 10^7}{3{\cdot}6 \times 10^3} = 1{\cdot}22 \times 10^6 \text{ ergs per sec.}$$

It is interesting to compare the amount of this energy with that kinetic energy of the α rays, which carry at least 99 per cent. of the energy spent by radium in ionisation. The experiments of Rutherford have shown that four groups of α rays are emitted by radium in radioactive equilibrium (see § 21): the velocities of the particles in these groups are $2{\cdot}14 \times 10^9$, $2{\cdot}26 \times 10^9$, $2{\cdot}34 \times 10^9$, $2{\cdot}6 \times 10^9$ cm./sec. respectively, and the number of particles of each group emitted per second, assuming that each particle carries a charge ϵ, is $6{\cdot}25 \times 10^{10}$. The mass of the α particle on the same assumption is $2{\cdot}17 \times 10^{-24}$ grm.: hence we find for the energy emitted in the form of kinetic energy of the α rays

$$6{\cdot}25 \times 10^{10} \times \tfrac{1}{2} \times 2{\cdot}17 \times 10^{-24} \times \{(2{\cdot}14)^2 + (2{\cdot}26)^2 + (2{\cdot}34)^2 + (2{\cdot}6)^2\} \times 10^{18} = 1{\cdot}49 \times 10^6 \text{ ergs per sec.}$$

Considering the nature of the data the agreement between the observed and calculated quantities is remarkable. It should be noted that the α particles do not spend all their energy in producing ionisation, for when their velocity sinks to $1{\cdot}12 \times 10^9$ their ionising power is lost, but it seems certain on general grounds that the remainder of their energy must appear as heat in the substance in which they are absorbed. The total energy of the α rays spent per sec. in ionisation is $1{\cdot}16 \times 10^6$, a value which agrees better with that observed than that deduced above, but since the difference between the two values hardly falls without the limit of experimental error it is better to avoid difficulties by accepting the first calculation.

It should be noted that the assumption has been made in our calculation that no part of the heat energy developed by the radium is drawn from the material in which it is absorbed.

The Theory of Radioactivity.

21. The most interesting question raised by the discovery of radioactivity is concerned with the origin of this continuous stream of energy poured forth by radioactive substances. If the principle of the conservation of energy is to be maintained, some system must be discovered the energy of which is diminished, as energy is given to the α particles at their emission. Many theories of the source of this energy, of which a few were worthy of serious consideration, were put forward in the early days of the discovery of radioactivity: but the only suggestion, which is really satisfactory and is maintained by any considerable body of physicists to-day, is that proposed by Rutherford and Soddy on the evidence of a class of observations which started from some experiments made by Crookes. In 1900 Crookes found that a great part of the activity of uranium could be separated from it by suitable chemical treatment. If a solution of an uranium salt is mixed with ammonium carbonate, the whole of the uranium is precipitated but can be redissolved by the addition of more ammonium carbonate. A small residue is left undissolved which consists, as far as chemical tests can show, entirely of known elements which were contained as impurities in the original salt. But it was found that this small residue contained almost all of the activity of the uranium: it was intensely active, whereas the solution which contained all the uranium showed no activity whatever. Further experiments have shown that the uranium retains unaltered the power of emitting α rays, but that the power of emitting β rays has been transferred to the residue. Since Crookes was using the photographic method for the detection of the activity, his experiments could only deal with the presence of β rays: if he had used the ionisation method, he would have observed little change in the activity of the uranium precipitated and redissolved.

It appears then that uranium salts, as ordinarily prepared, contain two distinct radioactive substances. One of these has the chemical properties associated with uranium and gives out α rays, the other has different chemical properties and gives out β rays: this latter substance is present in such small quantities that the ordinary methods of chemical analysis are not sufficiently delicate to detect it: Crookes gave to it the name uranium X.

But the most remarkable feature of these experiments did not appear until Crookes happened to examine his preparations after the lapse of a year. He found that the uranium had recovered completely its power of emitting β rays, and that the uranium X had lost that property completely. On repeating his original experiments he could separate from the uranium a further quantity of uranium X. Experiments similar but much more exhaustive and conclusive were made by Rutherford and Soddy on another radioactive element, thorium. They found that they could separate out from thorium a substance, which they called thorium X, possessing radioactive properties similar to those of the original thorium; at the same time the activity of the thorium decreased by the amount of the activity of the thorium X prepared from it. They also showed that the thorium regained its activity with the lapse of time, while the thorium X lost it: the rate at which it was regained by the one was equal to that at which they were lost by the other, so that the sum of the activity of the two substances remained constant.

It is clear, then, that, if uranium be left untouched, it undergoes some change which results in the production of another substance differing from it in chemical and radioactive properties. As the metal is obtained ordinarily some months after its preparation, the progress of this change is imperceptible, for the rate at which the new substance is produced is equal to that at which it is destroyed, while the concurrent loss of uranium itself is so slow that observations extended over several years fail to detect it. When this condition is reached, the mixture of uranium and uranium X is said to be in radioactive equilibrium. It is only when the second substance, which is destroyed far more rapidly than the first, is separated from the parent uranium, that the progress of the change becomes perceptible.

The change from uranium into uranium X differs entirely from any chemical change with which we were familiar before the discovery of radioactivity—such for instance as the change of one salt of a certain element into another compound of the same element. The differences may be enumerated in order:—

(1) The rate of change of uranium, or of any other radioactive element, is independent of the condition of the element.

The rate of production of uranium X is the same whether the uranium from which it is produced is at a high temperature or at a low temperature: it is independent of the state of chemical combination of the uranium—uranium in the state of oxide produces uranium X at the same rate as uranium in the state of metal: it is independent of the physical state of the uranium compound, or of its state of subdivision: it is independent of the action of light or of any ionising agent. No indisputable evidence has been brought forward that the rate of change of any radioactive element is influenced by any change of circumstances which it is in our power to produce.

No such independence of the rate of change is known in any chemical action: such actions are almost always influenced by change of temperature and always by a change of chemical constitution.

(2) The change of energy accompanying this radioactive change is vastly greater than that accompanying any known chemical change. The energy associated with the production of such a substance as uranium X is represented by the total amount of energy which the substance can emit, in the form of ionising rays or otherwise, during the change from its original condition to that in which it emits those rays no longer. Now radium emits in a day and a half as much energy as can be obtained by the combination of one gramme of hydrogen with oxygen: but the mass of the radium transformed, or the mass of the products arising from it in that period is very much less than one gramme: indeed the mass is so small that it is far beyond the reach of our most sensitive measurements.

(3) The law connecting the rate of change with the time is peculiar. If E_0 is the activity of a radioactive element at time t_0 and E_1 that at time t_1, then

$$E_1 = E_0 e^{-\lambda(t_1 - t_0)} \quad \text{........................(9)}$$

where λ is a constant characteristic of the element.

The investigation by Van 't Hoff and others of the laws governing the rate of chemical action have shown that this relation between the rate of change and the amount of the substance present is characteristic of those cases in which only one system is taking part in the change. Thus, this relation would be found to hold for the change of sulphur from one allotropic modification to another, since the molecule of sulphur is the only system concerned: but in such a change as the combination of hydrogen and chlorine, where at least two systems (the hydrogen atom and the chlorine atom) are concerned, the relation between the rate of change and the amount of either system present is perfectly different.

But the difference in the chemical properties of the original radioactive substance and that into which it is converted, shows that there is some difference in the atoms of the substance in the two cases: for the whole theory of chemistry is built on the assumption that the atom is the unit in all chemical change. The dissimilarity between radioactive change and chemical changes proves conclusively that the atomic change which has taken place is not a change in the relations of the atoms of the substance to each other, such as is characteristic of chemical change of the ordinary type: we are reduced to the view that the atomic change is a change in the nature of the atom concerned—that is, that the change which accompanies radioactivity is a change from one element to another element.

Such was the explanation put forward by Rutherford and Soddy: it will account for all the peculiar features of radioactivity, which have been enumerated above.

(1) It is well known that all the changes of condition which we are able to produce experimentally are incapable of transmuting one element into another, and it is to be expected that all such changes should be incapable of accelerating or retarding such transmutation when it is in progress.

(2) As yet we have no information as to the amount of the energy contained in an atom which is available for radioactive processes, and there is no objection to supposing that it may be large enough to account for the phenomena observed. In the next part, when we come to consider the internal structure of the atom, we shall see that the very large estimate of the internal atomic energy to which we have been led by the study of radioactivity is in complete accordance with evidence derived from other considerations.

(3) Since the radioactive change is one taking place in the interior of the atom, the number of atoms which change in a given time is proportional to the number of atoms present. Suppose, for example, that the life of each radioactive atom, from the time of its production until it breaks up with the emission of Becquerel rays, is T. If this period be divided into m very short periods t, so that $mt = T$, and if N be the total number of atoms present at any given moment, the number of atoms the age of which lies in any period t is $\frac{N}{m}$. In particular the number the age of which lies between $T - t$ and T is $\frac{N}{m}$: these atoms all break up in the ensuing period t and hence the number of atoms breaking up in any period t is $\frac{N}{m}$ and the number surviving is $N - \frac{N}{m}$. The number which break up in the consecutive period t is $\frac{N - \frac{N}{m}}{m}$ and the number surviving is $N\left(1 - \frac{1}{m}\right)^2$; the number N' which survive after the lapse of a period nt is $N\left(1 - \frac{1}{m}\right)^n$.

If m and n are very large numbers this quantity becomes $Ne^{-\frac{nt}{mt}}$. Hence $N' = Ne^{-\lambda . nt}$, where $\lambda = \frac{1}{T}$.

Hence the quantity λ in the empirical formula (p. 207) is the reciprocal of the life of an atom of the radioactive element.

In the five years which have elapsed since the publication of Rutherford and Soddy's work an immense amount of research has been performed which is based on their theory. Many new elements have been discovered by subjecting the known radioactive substances to various chemical operations and examining the rate of change of the activity of the products obtained. (In estimating the amount of a substance present from its ionising power, it is always assumed that the ionisation produced is proportional to the mass of the substance present.) No attempt will be made here even to sketch the results which have been obtained or the methods of experiment: it need only be said that the life of an atom may have any value over a very wide range—

600 million years for uranium to three seconds for actinium emanation. The principle of 'a short life and a merry one' seems general: the more active an element the shorter its life. One further feature of interest may be noted: it appears that there may be 'rayless' radioactive changes—if the words may be used without committing a contradiction. A radioactive atom may emit Becquerel rays and change into another atom: this second atom after a life of definite duration breaks up into a third atom, but this latter change is not accompanied by the emission of ionising rays: it can only be detected by its influence on the rate of decay of the substances descended from it. It appears that many changes which are usually classed as 'rayless' are accompanied by the emission of β rays without α rays, but the liberation of energy in such changes is so small compared to those in which α rays are emitted that it seems desirable to make the distinction implied by the term 'rayless.'

The transmutation of the elements.

22. If radioactive elements are continually disintegrating and turning into other elements, it is highly probable that at some stage of the process an element should be produced which is known to us already and may be detected by the ordinary methods of analysis. It must be remembered, however, that the proportion of the whole number of atoms present which break up in the few years over which we have been able to extend our experiments is so small, even in the case of the substances best suited for observation, that the most refined methods would be required to detect any new element which was produced. Nevertheless, in one instance direct proof had been adduced of the production of an element from a totally different radioactive element.

Two lines of argument pointed to the probability that the elementary gas helium was one of the disintegration products of the radioactive elements. In the first place, it was found that helium was present in all those minerals which contained a radioactive element and in those minerals alone. In the second place it appeared from the measurements of the value of e/m for the α particle that, if e is identified with the electronic charge ϵ, the mass of the particle is twice that of a hydrogen atom: if e is 2ϵ, the mass is four times that of a hydrogen atom, *i.e.* the mass of an helium atom. When the first measurements of this quantity were made, Rutherford suggested that the α particle was indeed a doubly charged atom of helium.

Now helium is a gas which it is easy to detect by spectroscopic methods: accordingly Ramsay and Soddy endeavoured to prove directly the production of helium from radium. Their experiments involved no novel principle, but required only the most elaborate care in the avoidance of sources of error: a description of them would be out of place here. It is sufficient to say that they detected the production of helium in a vacuum tube to which only the disintegration products of radium had access. Their observations have been confirmed by independent workers and at the present time it is undisputed by any physicist of reputation that radium produces helium. The transmutation of the elements has been actually observed.

However, later developments have made the identification of the α particle with helium improbable. For it is difficult to resist the conclusion that the particle observed in the Kanalstrahlen by Thomson (p. 163) is identical with the α particle, which has the same value of e/m: the constant presence of the hydrogen atom in the Kanalstrahlen and the occurrence of the helium atom only when helium is put into the tube, suggest that this particle is a hydrogen molecule and not a helium atom. More direct evidence has been adduced lately by Greinacher and Kernbaum, who endeavoured to collect the α rays given off by one of the descendants of radium and to detect helium in the space into which they passed. They failed however to find any trace of any gas but hydrogen: it seems likely that the α particle is a hydrogen molecule carrying a single charge, and that the helium produced from radium is derived from some other portion of the atom than that which is shot off as the α particle.

In a recent controversy it was suggested that, since helium can be produced from radium, it would be well to call radium a 'compound' of helium and some other element. The proposed change of nomenclature cannot be deemed desirable. If we are going to call anything, from which a certain element can be produced, a compound of that element, we must call air a compound of oxygen and nitrogen. At present the term 'compound' has a definite and important connotation. When water is called a compound of oxygen and hydrogen, or hydrochloric acid a compound of hydrogen and chlorine, it is implied that the relation between the elements named and their 'compound' is the same in the two cases. This relation is utterly unlike the relation between oxygen and nitrogen in air, or that between helium and any other element in radium: air possesses the qualities which are summed up in the world 'mixture' and radium those summed up in the word 'element': and these qualities are quite different from those usually associated with the word 'compound.' A consistent nomenclature is one of the most valuable aids to exact knowledge: when a new relation is discovered it is highly important that it should be denoted by a new term.

Are all elements radioactive?

23. The transmutation into another element may be regarded as a necessary and sufficient criterion that the element transmuted is radioactive: such a definition seems to be used implicitly by all prominent workers in this branch of physics. In order to prove that an element is undergoing such a transmutation, it is sufficient to show that its mass decreases at a rate which is independent of the external conditions and proportional to the mass of the element present. (It will be observed that in order to include the 'rayless' changes in the definition it has been necessary to omit all mention of the emission of rays: an element may be radioactive in the accepted sense without showing any 'radioactivity.') The emission of rays and the liberation of an amount of energy far in excess of that which can be regarded as potential energy of chemical combination is also a sufficient, but not a necessary, criterion that an element is radioactive.

Of the elements which were known before the study of radioactivity began only uranium, thorium, and, I think we may add, potassium have been shown indubitably to be radioactive: but it is interesting to inquire whether many or all of the other ordinary elements may not be radioactive in some slight degree. Elements are known of all degrees of activity from that of actinium emanation to uranium—a range of more than a billion-fold: there is no reason to believe it impossible that there may be elements of which the activity is as much less than that of uranium as the activity of actinium emanation is greater, and is imperceptible to our instruments.

No certain answer can be given to this question. On the one hand, few would doubt on theoretical grounds that every element must possess some radioactivity (see Part IV.): and, on the other hand, it is generally accepted that many 'ordinary elements,' especially lead and tin, as prepared by ordinary processes, show a distinct radioactivity. But it is not certain that this activity may not be due to an impurity contained in those elements and that the true radioactivity is too small to be detected.

In favour of the impurity hypothesis, at first sight improbable, it is urged that small quantities of radium appear to be ubiquitous. The air, the oceans and springs, and all soils can be shown to be contaminated with minute quantities of radium or its products: we cannot be sure that the ordinary metals which show a minute activity do not also contain the same universal impurity.

On the other side it is urged that the presence of radium in commercial lead or in its salts cannot be detected by the ordinary tests: and that the ores from which lead, the most active of the 'ordinary metals,' is derived, contain less and not more radium than many other ores, so that it is improbable that the contamination can come from this source. However, recently Elster and Geitel by chemical treatment have isolated from lead a distinct radioactive substance. They are of the opinion that this substance is one of the descendants of radium derived from an impurity in the lead, but the conclusion cannot be regarded as fully established: it may be a descendant of the lead. Some evidence, the conclusiveness of which is not admitted generally, has been offered to show that the nature of the rays from ordinary substances varies with the nature of the element from which they are emitted. Such a conclusion, if correct, would show that the rays were not emitted by a common impurity and would leave no reasonable alternative to the view that emission of rays is an intrinsic property of the elements concerned.

It appears that all gases under all conditions known to us contain a few ions. This 'spontaneous ionisation,' which exists in the absence of any ionising agent introduced artificially, is generally regarded as due to the slight radioactivity of the walls of the vessel in which the gas is contained and to the traces of radium and other radioactive elements distributed in the neighbourhood of the vessel.

(4) Light as an Ionising Agent.

The photo-electric effect.

24. The first observation showing that light could confer conductivity on a gas was made by Hertz, who found that if ultraviolet light fell on a spark gap a spark could be produced by an electric intensity which was too small to produce the discharge in the absence of the light. Hallwachs, investigating this action, found that when light of short wave-length fell on certain metals and other substances a stream of negative electrification was given off: subsequent research has shown that this stream consists of electrons with a velocity between 10^7 and 10^8 cm./sec. Lenard has made the extremely interesting observation that the velocity of the electrons is independent of the intensity of the light falling on the substance, while the number of the electrons liberated is proportional to the intensity of the light.

Elster and Geitel have made elaborate investigations into the relation of the magnitude of this 'photo-electric' effect, *i.e.* the rate at which electrons are liberated, to the nature of the illuminated surface and the nature of the light. They find that there is an intimate connection between the intensity of the effect and the behaviour in electrolysis of the element from which they are emitted. The more electropositive is an element (or the more readily it forms a positive electrolytic ion), the greater is the stream of electricity given off by it under a given illumination. In general, the shorter the wave-length of the light the more efficient it is in producing the photo-electric effect: zinc and aluminium are only active when the incident light is in the ultraviolet; potassium, rubidium and caesium, the most electropositive metals, give off negative electricity under the action of red rays. It seems probable that the light of short period is the more efficient because its period is nearer to that of the free vibrations of the electrons in the atoms of the illuminated substance, for the light, which is absorbed selectively by a substance, is usually especially efficient in producing the photoelectric effect. It has been shown also that light of which the plane of polarisation is perpendicular to the plane of incidence produces a greater stream that light of the same intensity polarised in the perpendicular direction. It is not certain whether ultraviolet light can ionise the atoms of a gas.

The mechanism of the photo-electric effect.

25. The mechanism of the photo-electric effect should require little explanation after the foregoing chapters. The incident light sets the electrons in the substance into forced vibrations, the amplitude of which may become so large that the electron is shot out of the atom: the nearer the period of the incident light is to that of the free vibrations of the electrons the greater is the amplitude of the forced vibrations and the greater the likelihood of the ejection of an electron. For reasons which we shall consider later it seems likely that an electron can escape more easily from a pure element than from a chemical compound: in accordance with this expectation it is found that elements are in general more active in this respect than their compounds.

The influence of the plane of polarisation of the light is easily comprehensible: for light polarised in a plane at right angles to the plane of incidence produces an electric intensity perpendicular to the surface of the substance, while that polarised in the perpendicular direction only produces an electric intensity parallel to the surface: the former and not the latter tends to drag the electrons set free out of the metal into the surrounding region.

Since the velocity of the electrons is independent of the intensity of the light, it has been suggested that the light does not drag the electrons out directly, but sets up a disturbance in the atom which leads to a 'radioactive' explosion resulting in the spontaneous ejection of the electrons. But such an explanation, which introduces some difficulties which will be considered hereafter, does not seem necessary. It must be remembered that the quantity of energy which an electron can take up from light vibrations during a single vibration is extremely small. The electric intensity in the most powerful light is not more than 0·01 electrostatic unit: if such a field acted steadily on an electron for 10^{-16} sec. it would only produce a velocity of 0·56 cm./sec. While the electron is in the atom and capable of executing forced vibrations, it can continue to take up energy from the light until a velocity is reached, depending only on the nature of the atom and not on the intensity of the light, when its kinetic energy is sufficient to carry it clear of the attraction of the positive atom. It will then escape with a velocity which depends only on the nature of the atom. But once it has escaped it cannot take up further appreciable energy from the light, for the very small additional velocity which it acquires during the first half of the vibration is lost during the reverse half.

Photo-electric fatigue.

26. There is a remarkable phenomenon known as 'photo-electric fatigue.' If a clean surface of a metal be exposed to ultraviolet light, the intensity of the stream of negative electricity from it decreases rapidly with the time. The observations made on this fatigue are numerous and contradictory: in spite of many experiments it is not known definitely whether the fatigue depends on the action of the light or whether it is influenced by the presence of a gas. It is certainly due to some surface alteration of the substance, but of what nature is not ascertained. Ramsay and Spender have suggested that the atoms at the surface of the solid are 'exploded' like radioactive atoms under the action of the light, and that the resulting atoms are less active than their parents. But this explanation can be shown to be untenable, for the quantity of electricity given off by the time that fatigue begins is so small that, if each atom has contributed only one electron, not more than one ten-thousandth part of the whole number of the atoms in the surface layer of the substance could have been affected. It must be remembered also that the substance does not lose negative electricity continuously without compensation: fresh electrons flow in to take the place of those which are lost through the conductor to which the illuminated surface is attached: if the surface were insulated, the flow of electricity would soon stop owing to the restraining attraction of the residual positive charge on the surface.

(5) Chemical Reactions.

Ionisation by chemical action.

27. It has been known for more than a century that gases which have been just prepared by some chemical reaction—for instance, hydrogen prepared by the action of an acid on zinc,—possess an abnormal conductivity. Townsend investigated the subject with great completeness and found that the gas is not only conducting, but also carries a total charge which may be of either sign according to the nature of the action. Many other actions, such as the hydration of quinine sulphate, which do not evolve any considerable quantity of gas, cause the emission of streams of charged particles, which confer both a total charge and a conducting power on the surrounding gas.

An important example of ionisation by chemical reaction occurs in the case of flames: both the gases which actually constitute the flame and those immediately surrounding the flame possess considerable conductivity. The effects observed are somewhat complicated by the action of the high temperature of the flame in causing the emission of charged particles from the electrodes used to measure the conductivity (see § 28).

In a later chapter reasons will be given for believing that the progress of a chemical reaction is connected intimately with the liberation of electrons from the atoms concerned. Some of these electrons and some of the positive atoms, from which they have been derived, escape from the reacting substances still carrying their charge and confer conductivity on the surrounding gas.

A caution must be entered against confusing ionisation by chemical action with radioactivity. The chemical processes which are employed in the study of radioactivity are merely devices for separating radioactive elements or their compounds from a mechanical mixture in which they are contained. A salt of radium, which has been stored for some months after preparation, contains appreciable quantities of at least four radioactive elements, besides radium, which have been produced by the disintegration of the parent substance. One of these new elements, which happens to be a gas at ordinary temperatures, can be separated from the mixture by the simple physical process of heating it: the most convenient methods of separating the others depend on their chemical dissimilarity. The chemical reactions have no effect whatever on the activity of the individual elements: the disintegration proceeds at the same rate whatever their state of chemical combination.

The ionisation produced by a chemical reaction has none of the characteristics of radioactivity; it is not independent of the state, chemical or physical, of the reacting elements. Moreover no ionising rays which can penetrate appreciable thicknesses of matter are emitted, similar to the Becquerel rays which we considered above: the ions produced have no more power of forming other ions than the ions produced by Roentgen rays. The total amount of energy, which appears as electrical energy in the separated electrical charges on the ions, is only a very small fraction of the whole amount of energy, which appears as heat or otherwise, derived from the energy of chemical combination.

(6) Ionisation by Heat.

Electrification by incandescent solids.

28. It has been known ever since 1725 that a gas in the neighbourhood of an incandescent body becomes a conductor of electricity. Guthrie first drew attention to the most important feature of the conductivity so obtained, namely, that it is not symmetrical: at comparatively low temperatures only positive electricity, and not negative electricity, can pass from the hot body to its surroundings: but at higher temperatures either sign can pass with freedom. Elster and Geitel made very elaborate investigations into the complicated phenomena that may be observed: recently our knowledge has been extended considerably by Richardson. A bare outline of all that has been discovered will suffice for our purpose.

When such a metal as platinum is heated in a vacuum at a dull red heat, it begins to give off streams of positive electricity which may be detected by the charge communicated to an insulated conductor placed near the hot metal. As the temperature is raised, this stream appears to grow less, to vanish and then to change sign. At a white heat a stream of negative electricity is given off which is, in general, considerably greater in amount than the greatest stream of positive electricity given off at the lower temperatures. It appears then that at all temperatures above a dull red heat streams of electricity are given off by the metal. At low temperatures the quantity of positive electricity given off is greater than that of the negative: but the latter increases much more rapidly with the temperature than the former, and at high temperatures masks the presence of the positive stream. Measurement of the magnetic deflection of the positive and negative particles given off by the metal indicate that the latter are electrons, whereas the former have masses varying between that corresponding to an atom of the metal bearing an electronic charge and that of an atom of the gas bearing the same charge.

If the incandescent body is not in a vacuum but in a gas at atmospheric pressure, the electrons and atoms from the metal condense molecules of the gas around them to form ions. It might be expected that the negative ions at least would possess exactly the same properties as those of the ions produced by ionising the gas itself, since on the theory that we are adopting all ions consist of electrons surrounded by a cluster of molecules of the gas. But the presence of the hot body seems to influence in some way the formation of the ions, for both the positive and the negative ions are considerably slower than those produced by the action of the Roentgen rays, and the velocity varies with the temperature of the hot body. No adequate explanation of this variation has been offered as yet.

The negative 'leak.'

29. Richardson undertook an investigation of the negative stream or 'negative leak' at high temperatures based on the theory that the emission of the electrons was connected with the presence of free electrons which give the metal its conductivity. In the metal itself such an electron is subject to electric forces which are approximately equal in all directions: but if the electron once emerged through the surface it would tend to be dragged back by the attraction of the positive atoms which were left behind it. Accordingly an electron, in order to escape from the metal, must possess a kinetic energy greater than that which is required to carry it free of the restraining forces. Since the electrons share in the heat energy of the body, the greater the temperature of the body the greater is the average kinetic energy possessed by each electron, and, consequently, the greater the probability of its being able to get free from the metal.

At first sight this hypothesis seems to lead to conclusions admirably in accordance with the facts. The leak increases very rapidly with the temperature, and the relation between the temperature and the leak can be shown to be the same as that predicted by Richardson's hypothesis. From the study of that relation it is possible to calculate the number of electrons present in unit volume of the metal and the minimum kinetic energy which is required by an electron in order that it may escape from the metal. But further investigation has disclosed difficulties. The presence in the metal of a small quantity of hydrogen, quite insufficient to produce any change in the conductivity of the metal, produces an enormous change in the estimated number of electrons, while the considerations of Chapter VI. showed that the conductivity of a metal is proportional to the concentration of the electrons in it. It should not be inferred, however, that the discrepancy is in any way fatal to the general theory which has been propounded: there are many plausible hypotheses which will enable the apparent contradiction to be removed. It might be assumed, for example, that the change of the concentration of the electrons affects only the surface film of the metal from which the electrons are emitted, and that the average concentration is altered only by an inappreciable amount. But this observation and others of a similar nature indicate that though the general outline of the explanation may be correct, our knowledge on the subject is by no means complete.

It should be noted in passing that Wehnelt has shown that certain oxides, such as lime, give very large streams of negative electricity when heated to a high temperature. In such substances it seems that the concentration of the electrons is not especially large, but that the kinetic energy required for an electron to break free is exceptionally small.

The 'positive leak.'

30. In the investigation of the positive leak which is predominant at lower temperatures, phenomena are encountered even more complex than those met with in the study of the negative leak. Our information on the subject is still very incomplete, in spite of the recent work of Richardson. No explanation professing to cover all the facts has been proposed. We must leave this branch of physics as one of those on which more light is required. In this case, as in many others, our knowledge of the properties of positive electricity is much less complete than that of the properties of negative electricity. On the extension of this knowledge depends the further development of our views on the fundamental problems of electricity. The positive leak doubtless consists of positive atoms which have lost their electrons, but we are ignorant as to how or why they become detached from the metal.

The production of conductivity in a gas by the neighbourhood of an incandescent body is thus somewhat different to the cases of ionisation which have been considered hitherto. The heat does not ionise the atoms of the metal, but only causes the free electrons already present to escape into the gas. No evidence has been obtained that the action of heat alone can produce ions from the atoms of a gas or from any other non-conductor in which free electrons are not present already. Such a conclusion could be anticipated from general considerations, for it is clear that a molecule cannot acquire sufficient energy at any attainable temperature to ionise the atom of a gas by collision with it. For, from p. 192, the energy required to ionise the atom of a gas is $4{\cdot}2 \times 10^{-11}$ ergs. Now the kinetic energy of a molecule at temperature T is αT (see p. 107), where α is the constant $1{\cdot}5 \times 10^{-16}$. Therefore the value of T at which a molecule acquires the requisite energy is given by $T = \frac{4{\cdot}2 \times 10^{-11}}{1{\cdot}5 \times 10^{-16}} = 2{\cdot}8 \times 10^5$, whereas it is not in our power to produce a temperature of more than 4×10^3.

The mechanism of ionisation.

31. We must now return to the consideration of the problems left unsettled in § 8. How do the Roentgen rays come to ionise atoms at all, since the electric intensity in their wave front is so small: and why, if they can ionise one atom, do they not ionise every atom they meet until their ionising energy is exhausted?

It is clear that the most satisfactory explanation will be one that will answer both questions. Accordingly we should view with disfavour a supposition that our estimate of the electric intensity required to drag an electron out of an atom was far too great, for, though it would solve the first difficulty, it would leave the second untouched. But there are innumerable direct objections to such an hypothesis which will occur to the reader of the next Part: here it may be noted that the experimental value of the energy required to ionise an atom agrees well with the estimate of the electric intensity required for that purpose. For if the restraining force on the electron displaced a distance x is fx, the work required to drag the electron a distance x_0 to the boundary of the atom is $\frac{1}{2}fx_0^2$. From the numbers of p. 170 we see that $f = 2{\cdot}60 \times 10^6$: the value of x_0 is probably between $10^{-8} \times 10^{-9}$ cm.; the former value gives $1{\cdot}3 \times 10^{-10}$, the latter $1{\cdot}3 \times 10^{-12}$ for the energy of ionisation: the value found experimentally is intermediate ($4{\cdot}2 \times 10^{-11}$).

A much more plausible explanation is of the same nature as that proposed in dealing with the photo-electric effect. It is possible that the energy required to ionise an atom is not derived from a single pulse but from a very large number of pulses: the first pulse sets the electron in vibration, and succeeding pulses increase the amplitude of the vibration, till the kinetic energy of the electron is sufficient to carry it out of the atom. Since there is no phase connection between succeeding pulses, they would, on the average, neutralise each other's action: the first would increase the vibration, the next diminish it: but in a small number of cases there might be a large number of pulses following each other, the effects of which were in the same direction, so that the necessary kinetic energy was given to the electron. But some numerical calculations will show the impossibility of this view.

If the electric intensity in the pulse is X, the acceleration of the electron is $X\frac{\epsilon}{m}$. If the breadth of the pulse is d and the time of passage of the pulse is $\frac{d}{V}$, the velocity conferred on the electron is $X\frac{\epsilon}{m}\frac{d}{V}$ and its kinetic energy U is given by

$$U = \frac{1}{2}\frac{X^2\epsilon^2 d^2}{mV^2}.$$

(A little consideration will show that, since the time of passage of the pulse is small compared to the period of free vibration of the electron, we may leave out of account the restraining force on the electron and the potential energy due to its displacement.)

Now $$X = \frac{\epsilon u}{r d V} \text{ by (2), p. 179,}$$

if we put $$\sin\theta = 1,$$

hence $$U = \frac{1}{2}\frac{\epsilon^4 u^2}{r^2 m V^4} \quad \text{......................(10).}$$

At a distance of 10 cms. from the place of origin of the rays

$$U = \frac{1}{2}\frac{(3{\cdot}4 \times 10^{-10})^4 \times 10^{20}}{10^2 \times 6{\cdot}5 \times 10^{-28} \times 81 \times 10^{40}} = 1{\cdot}2 \times 10^{-35}$$

if $u = 10^{10}$ cm./sec.

Since the whole energy required to ionise the atom is $4{\cdot}2 \times 10^{-11}$, it would require $3{\cdot}5 \times 10^{24}$ pulses to provide the necessary energy. Since each electron in the kathode rays provides one pulse, a quantity of electricity $3{\cdot}5 \times 10^{24} \times 3{\cdot}4 \times 10^{-10} = 1{\cdot}2 \times 10^{15}$ electrostatic units, or 4×10^5 coulombs, must pass through the Roentgen ray tube before such an amount of energy could be given to an atom. With such currents as it is possible to use in practice, several days would elapse after turning on the current in the discharge tube before a single atom could be ionised, even if all the pulses tended to increase the energy of the electron!

In passing it may be noted that we may calculate similarly the coefficient of absorption of the rays. If the area of the surface of the pulse at any distance from the source is A, in moving forward a small distance x the pulse will pass over NAx electrons, where N is the number of electrons per unit volume. To each of these electrons the pulse communicates the energy

$$U = \frac{1}{2}\frac{X^2 \epsilon^2 d^2}{m V^2}:$$

hence the total energy lost by the pulse is

$$\frac{1}{2}\frac{X^2 \epsilon^2 d^2}{m V^2} \cdot NAx.$$

But the whole energy in the pulse is

$$\frac{X^2}{4\pi} \times \text{volume of the pulse} = \frac{X^2}{4\pi} \times Ad.$$

Hence the proportion of energy lost is

$$\frac{2\pi\epsilon^2 d}{mV^2} N . x.$$

But, according to p. 183, the proportion of energy lost is λx, where λ is the coefficient of absorption. Hence

$$\lambda = \frac{2\pi\epsilon^2 d}{mV^2} N \quad \text{......................(11).}$$

Choosing our values so as to make λ as small as possible, we may put $d = 10^{-10}$ and $N = 3 \times 10^{19}$, the number of molecules in unit volume of the gas: hence

$$\lambda = \frac{2\pi \times 1{\cdot}16 \times 10^{-19} \times 10^{-10} \times 3 \times 10^{19}}{6{\cdot}5 \times 10^{-28} \times 9 \times 10^{20}} = 3{\cdot}7 \times 10^{-3},$$

which is not very different for the value found experimentally for air at atmospheric pressure (10^{-3}). It is probable that the values of d and N have been chosen so as to make this quantity too small, but on the other hand our calculations have tended to make it too large. For, if the electron in the kathode rays is not stopped in one motion, but executes an oscillation, the electric intensity is not in the same direction throughout the wave front: the back of the pulse tends to undo the work of the front, and the energy communicated to the electron by the passage of the pulse is considerably smaller than that given by (10).

In any case too much stress must not be laid on the coincidence of the absorption coefficient calculated on these principles with that found experimentally: for nearly the same value for the absorption coefficient will be found whatever view we take of the nature of ionisation by Roentgen rays. The difficulties which have been noted remain undiminished and it is clear that we must seek elsewhere for a satisfactory theory of ionisation by Roentgen rays.

An explanation that has met with some support is based on observations on the secondary kathode rays emitted from metals under the action of the rays. It was noted on p. 186

that the velocity of these rays is very large, that the velocity is independent of the intensity of the Roentgen rays, and that the energy of the secondary rays appears to be greater than that of the primary. Accordingly it has been suggested that these rays are emitted, not by the direct action of the electric intensity in the pulse, but by a radioactive explosion of the atoms of the metal set up by the action of the pulse. All three difficulties could be explained by this suggestion, for we know that a radioactive substance emits high speed electrons, the velocity of which depends only on the nature of the substance, and that radioactive change is accompanied by the liberation of a large amount of energy.

However the magnitude and constancy of the velocity of the secondary rays could be explained by any theory which makes the electric intensity in the pulse sufficiently great and thus answers the most fundamental question as to how the rays cause any ionisation at all. It is to be expected that the velocity of the secondary rays should be independent of the intensity of the rays; for the electric intensity in the pulse, which determines the velocity of the ejected electron, depends only on the nature of the pulse, while the intensity of the rays, as measured by the ionisation which they produce, is proportional to the number of pulses emitted per second. If the discharge tube emits rays of constant hardness, the velocity of the secondary rays should be constant and their number should be proportional to the intensity of the primary rays.

On the other hand, the radioactive theory seems alone capable of explaining the liberation of energy: if that liberation is fully established, there is at present no alternative to the acceptance of that theory. But, though the evidence is very strong, it is not quite conclusive: it is possible that the secondary rays only spend a greater proportion of their energy in ionisation, and that their total energy is not really greater than that of the primary rays. The objections to the radioactive theory seem to me so strong that I am ready to take advantage of any loophole for escape from it. We may proceed to consider those objections.

(1) The liberation of energy appears to take place only in solids and, perhaps, liquids; there is no trace of it in gases. The

energy spent in ionisation, when a pulse passes through a gas, is less and not greater than the whole energy lost by the rays: the remainder of the energy is probably spent in heating the gas. But there is no evidence whatever that the radioactivity of a body is affected by its state of aggregation.

(2) All direct evidence is against the view that the action of the rays can affect the radioactivity of a body: so far as we know the action of Roentgen rays or of any other physical agent has no influence in increasing the rate of radioactive disintegration of elements admitted to be radioactive. It is to be noted that, according to the explosion theory of secondary radiation, there must be an actual increase in the rate of disintegration due to the rays: for, if the rays merely exploded those atoms which were near a state of instability, owing to their age being near that at which disintegration occurs, the emission of secondary radiation would only occur at the first incidence of the rays: when all the old atoms had been exploded, no further radiation could be emitted until processes occurring spontaneously had brought more atoms to the condition of instability. Accordingly the secondary radiation emitted by the substance after the first moment would be comparable in intensity with the radiation due to the spontaneous activity of the metal.

We have now investigated the result of denying three of the assumptions made in the simple treatment of the ionisation by Roentgen rays:—(1) that the same energy is required to ionise an atom by whatever agent the ionisation is effected, (2) that the energy of ionisation is derived from a single pulse, (3) that the energy is derived from the energy of the rays. None of these methods of overcoming the difficulties encountered appear to me satisfactory. There remains one further assumption that may be denied with some show of reason, the assumption that the electric intensity in the pulse can be calculated by the method of § 5. It was assumed that the electron of the kathode rays when stopped sends out pulses carrying energy in all directions: if, on the contrary, we assume that all the energy is concentrated in one direction, our difficulties will vanish. This new assumption is, of course, contrary to the current theory of electromagnetism, but, when we come to revise our fundamental ideas of electrical phenomena in the last chapter, it will appear that it is by no means contrary to the view which has been adopted in this volume. The complete consequences of the assumption cannot be worked out here; it is enough for our purpose to note that, if the entire energy of the pulse is concentrated into a sufficiently small space, the electric intensity in the wave front can be made as great as we please: so long as the total energy in the pulse is equal to the amount of work which must be performed in the ionisation of the atom, an electron may be ejected however great the restraining forces.

Again, if the whole pulse is concentrated into a very small space, it can travel through the gas without bringing within the range of its influence more than an extremely small fraction of the total number of molecules in the gas: both the difficulties from which the discussion of this section started will be removed. There remains only the question of the liberation of energy in the excitation of secondary rays. As has been remarked already, if there is indeed such a liberation of energy, there seems no escape from the radioactive hypothesis; but for the present, in my opinion, we shall do best to hold by the simple theory of ionisation and to consider that the whole energy of the Roentgen pulse, like the whole energy of the β rays, is concentrated in a space of very small volume.

If this view be correct, the Roentgen rays, like the α rays, must ionise every atom they meet, until the energy of the pulse is exhausted, and the law of absorption of the rays must be the same as that for the α rays. There is no objection to this view, for the peculiarity of the absorption of the α rays only appears when it is possible to deal with particles all moving from a thin layer with the same initial velocity in the same direction. The analogous condition is unattainable in the case of Roentgen rays, and an approximation to the law represented by (8), such as was found by the earlier workers for the α rays, is to be expected.

In dealing with ionisation by α rays, no difficulties occurred: the process appears to be perfectly simple and to accord in almost every particular with expectation. This simplicity of ionisation by α rays throws doubt on every theory which would lead to the conclusion that the obscurities connected with ionisation by other agents is due to the complexity of the methods by which the electrons are dragged out of the atoms rather than to our ignorance of the distribution of the electric intensity in the neighbourhood of the atom which is ionised.

There is one further problem connected with the β rays, which demands a brief consideration. Experiments on the diffraction of the Roentgen rays show that the breadth of the pulse must be very small—not greater than 10^{-7} cm.—and that the distance in which the electrons which emit that pulse are brought to rest has about the same value. This conclusion is quite independent of the answer given to the questions which were discussed above. But, on the other hand, it is found that β rays, which have a velocity equal to that of the electrons in the kathode rays, can penetrate through considerable thicknesses of solid bodies without losing their ionising power. How are these inconsistent conclusions to be reconciled? If the electrons are stopped in a distance comparable with the radius of an atom, how can they penetrate several millimetres of a solid?

The question is not difficult to answer. In order that an electron may emit a Roentgen pulse it is not necessary that it should lose its speed but only that a sudden change in its velocity should be produced: if the electron is turned back on its course and pursues its way, a pulse will be emitted of the same nature as if the electron had been stopped.

Now, when an electron encounters the atom of a solid, it is subject to an intense electric field which deflects it from its course, so that it emits a pulse. Proceeding further it meets with another atom, suffers another deflection and emits another pulse: and so it goes on, pursuing a zigzag course, until the direction of its motion has no immediate connection with that of its original motion. A stream of such electrons having traversed a comparatively small distance in the solid will have suffered an immense number of deflections and the direction of their motion will be distributed at random: the rays are completely scattered, but a considerable number are travelling in their original direction. Further progress in the metal will not diminish this number, for the electrons lost by further deflections away from this direction will be replaced by others deflected into this direction. The conception of the electron being stopped by the anti-kathode in the Roentgen ray tube was only introduced to secure simplicity in calculation.

In this process of deflection the electrons lose part of their kinetic energy through emitting pulses and also by doing work on the electrons in the atoms which deflect them: in some cases these forces will be so great as to secure the ejection of the electron from the atom. This loss of energy will continue after the rays have been scattered completely, and probably represents the greater part of the 'absorption' measured by the usual method for rays which have been completely scattered by travelling through the layer of material from which they are emitted.

References for Chapter IX.

As in the last chapter, a complete account of all the work sketched in this chapter may be found in Thomson's *Conduction of Electricity in Gases.* For the science of radioactivity the standard authority is Rutherford's *Radioactivity.* The following original memoirs may be consulted by those who wish to obtain their knowledge first hand:—

Lenard Rays. Lenard, *Wied. Ann.* li. p. 225, 1895 (*A.* and *L.* p. 362).

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PART IV.

ELECTRICITY AND MATTER.

CHAPTER X.

THE CONSTITUTION OF THE ATOM.

The structure of the atom.

1. THE results of our inquiry, though they are of the most far-reaching importance, may be epitomized in a few words. We have learnt all about electrons. We have learnt that they are parts of atoms from which they may emerge in suitable conditions: we know their mass and the sign and magnitude of their charge, and it seems that these facts are all that there is to know. So long as we have dealt with the behaviour of electrons free from the influence of the atoms, from which they are derived, there has been a perfect concordance between our expectations and the results of experiment.

But as soon as that influence is taken into account difficulties arise: it is perfectly clear that the atom, or the positively charged system which is left when an electron is abstracted from it, is not a perfectly simple system, like the electron, but is a structure of very considerable complexity. The determination of that structure is the most important problem of modern physics. For, according to the theory which has survived the discoveries of a century, material bodies are collections of atoms, and the properties of those bodies must depend on the properties of the atoms of which they are composed: a complete elucidation of the structure of the atom would enable us to correlate all the properties, physical and chemical, of every substance and to achieve the aim of inorganic science, as it would have appeared to a philosopher of the last generation.

It is needless to explain that, as yet, we are far from that achievement: only a few dim suggestions can be offered in place of an all-embracing theory, but the insufficiency of our best hypotheses is no reason against advancing them. A false hypothesis is better than none, for it indicates clearly the problems on which our knowledge is deficient. Accordingly, in the next chapters we shall attempt to suggest a structure of the atom, as plausible as can be devised, and shall discuss in what respects it is adequate. The method of procedure is similar to that of Part II., in which a guess was made and its results compared with experiment, but the greater complexity of the problem makes a correct guess more difficult and renders the comparison with experiment less certain.

In making our guess we have but few facts to guide us: we know that the atom contains at least one electron, and that the positively charged portion left when an electron is extracted is different for different atoms, but no definite information has been obtained as to its structure. There are three alternative hypotheses that may be made with regard to that structure. It may be considered either that it is essentially different in the various atoms, or that it is composed of a varying number of constituents common to all atoms, or that it is the same in all atoms, the differences in which arise from variations in the number and distribution of electrons which still remain in the atom. The first hypothesis would remove finally all hope of explaining the intimate connection between different elements in the same chemical group and the transmutation of one element into another. The second hypothesis is that of 'positive electrons,' which has been discussed in some measure on p. 164. Arguments for rejecting it additional to those given in that place will be noted in the course of this chapter: for the present it is sufficient to repeat that positive electricity is always found to be associated with bodies of such complex structure, that they cannot be regarded with satisfaction as ultimate constituents of atoms. If we supposed that the atom is built up of α ray particles, the question would still remain of what are those particles built up: if an hypothesis of the atom is to be framed at all it is advisable to proceed at once to the ultimate constituents. But, if the α particle is not a universal constituent of the atom, then there is no reason to believe that there is any such constituent positively charged: the introduction of a 'positive electron' would afford no aid in correlating the properties of matter and might lead to error by suggesting that there was information obtainable on subjects where there is complete ignorance.

Accordingly, the third hypothesis will be adopted: it has an immense advantage over all others in attempting to explain all physical and chemical phenomena by the properties of one system, the negative electron, which appears in the present state of knowledge to be the true indivisible 'atom.' But, since the chemical atom, as a whole, is electrically neutral, it is necessary to introduce somewhere a positive charge equal in magnitude to the total charge on the electrons. Since there is no evidence as to the distribution of this charge, it is best to adopt an hypothesis which renders most easy the task of deducing the properties of the atom from its structure. It will be supposed, therefore, that the positive electrification is distributed uniformly through a sphere—which will be called the 'positive sphere'—and that the whole atom consists of such a positive sphere, throughout which are distributed a number of electrons. Such is the model atom which was sketched by Kelvin and developed in considerable detail by J. J. Thomson. Of course, it is not pretended that the model is the last word on the constitution of matter, but it will be seen that it is capable of correlating a large number of diverse experimental facts. Even if it were far less successful, it would be necessary to base on it the discussion of this chapter, for there is no rival theory of importance in the field.

The number of electrons in the atom.

2. The first problem to which attention must be directed is the determination of the number of electrons in each atom: unfortunately we shall find that it is not capable of accurate and final solution. Neither the analytical deduction of the consequences of our hypothesis nor the experimental data with which to compare its results are sufficiently complete, but it is likely that, even if these deficiencies were removed, difficulties would still remain. One of the most stringent tests of the correctness of a physical hypothesis is the comparison of the numerical values, determined by different methods, of some quantity introduced by it: thus, no more complete proof of the essential accuracy of the ionic theory of conduction in gases could be adduced than the coincidence of the values of the velocity of the ions obtained by the two distinct methods of Chapter VIII., § 5. So long as the hypotheses involved in the model atom are not perfectly satisfactory, it is useless to expect agreement between the values of the number of electrons in an atom as deduced from different considerations. But, by taking careful account of the assumptions which are introduced by the various estimations, valuable information may be obtained by considering the problem in some detail. Accordingly, we will examine in turn each of the series of phenomena which have been explained on the electronic hypothesis and consider what light they have to throw on the question of the number of electrons in the atom.

Number of electrons, (a) optical evidence.

3. The first source of information is the optical dispersion of transparent bodies, which has been used for this purpose by Drude and J. J. Thomson. In Chapter III. it was shown that the dispersion of any substance might be represented by the formula

$$n^2 = b^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} + \frac{M_2}{\lambda^2 - \lambda_2^2} + \frac{M_3}{\lambda^2 - \lambda_3^2} + \ldots + \frac{M_p}{\lambda^2 - \lambda_p^2} + \ldots \quad (1),$$

and that the constants b^2, M_1, M_2, M_3 ... were connected with the properties of the electrons in the body by the relations:—

$$b^2 = 1 + \frac{4\pi N_1 e_1^2}{f_1} + \frac{4\pi N_2 e_2^2}{f_2} + \ \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

$$M_1 = \frac{4\pi N_1 e_1^2 \lambda_1^2}{f_1}, \quad M_2 = \frac{4\pi N_2 e_2^2 \lambda_2^2}{f_2}, \ \ldots\ M_p = \frac{4\pi N_p e_p^2 \lambda_p^2}{f_p} \ \ldots\ldots (3),$$

where the suffixes 1, 2, 3 ... distinguish the quantities belonging to electrons of different groups. N_p, e_p, λ_p are respectively the number of electrons of the group p in unit volume of the substance, the charge carried by each of them and the wave-length of the radiation emitted by their free vibration. f_p is the elastic force controlling the free vibration and is connected with the period of that vibration p_p by the relation (11), p. 55,

$$p_p^2 = \frac{4\pi^2 m_p}{f_p} \text{ ....(4)},$$

where m_p is the mass of the electron. From these formulae we we obtain, remembering that $\lambda_p = Vp_p$,

$$M_p = \frac{4\pi N_p e_p^2 \lambda_p^4}{4\pi^2 m_p V^2} \text{ or } e_p N_p = \pi \frac{M_p}{\lambda_p^4} \cdot \frac{m_p}{e_p} \cdot V^2 \text{ ......(5)}.$$

In Chapter III. the term 'electron' was used in the sense of any charged portion of an atom: it has been restricted since to designate only the negatively charged portion. Hence, in the language which has been used since Chapter VII., the quantities *e*, *N*, *m*, etc. may refer either to an electron or to a positively charged portion of the atom which has lost an electron. Since the total charge on the absorbing body is zero, the sum of the terms $e_p N_p$ must be zero. Hence

$$\Sigma e_p N_p = V^2 \pi \Sigma \frac{M_p}{\lambda_p^4} \cdot \frac{m_p}{e_p} = 0 \text{ ....(6)}.$$

If the substance be transparent, it will have no absorption bands in the visible spectrum, and these bands may be divided into two series, one in the ultra-red and one in the ultra-violet. For the sake of simplicity, we will suppose for the present that there is only one band in each of these regions. Then, denoting by the suffix *v* the quantities which characterise the system which has its natural period in the ultra-violet and by *r* those which characterise the system which has its natural period in the ultra-red, we have

$$\frac{M_v}{\lambda_v^4} \cdot \frac{m_v}{e_v} + \frac{M_r}{\lambda_r^4} \cdot \frac{m_r}{e_r} = 0 \text{ ....(7)}.$$

All these quantities are necessarily positive, except e_v and e_r: hence e_v and e_r must have different signs: that is to say, the bands in the ultra-red represent the vibrations of a system oppositely charged to those in the ultra-violet. Further, observation shows that $\frac{M_v}{\lambda_v^4}$ is always much greater than $\frac{M_r}{\lambda_r^4}$: hence $\frac{e_v}{m_v}$ is much greater (numerically) than $\frac{e_r}{m_r}$. The conclusion is obvious that the system *v* is a negatively charged electron and the system *r* a positively charged atom. Hence the ultra-violet bands are due to the vibrations of electrons, the ultra-red bands to the vibrations of positively charged atoms.

Now let P_p be the number of electrons of the kind p in each molecule of the substance. Let W be the molecular weight referred to hydrogen as unity, and let H be the mass of an atom of hydrogen. Then the mass of the molecule is WH. Let d be the density of the substance: then the number of molecules in unit vol. is $\frac{d}{WH}$. But this number is also given by $\frac{N_p}{P_p}$: hence

$$N_p = \frac{dP_p}{WH} \text{ ....(8).}$$

But from (5)
$$\frac{e_p}{m_p} = \frac{\pi V^2}{e_p N_p} \cdot \frac{M_p}{\lambda_p^4}:$$

substituting the suffix v for p we have for the ultra-violet bands

$$P_v \cdot \frac{e_v}{m_v} = \pi \cdot \frac{1}{\frac{e_v}{H}} \cdot \frac{W}{d} \cdot \frac{M_v}{\lambda_v^4} \cdot V^2 \text{ ....(9).}$$

If our conclusion is correct that the system v is an electron, $e_v = \epsilon$ and $\frac{e_v}{H}$ is the ratio of the charge to the mass of a hydrogen ion, *i.e.* 3×10^{14}. Hence

$$P_v \left(\frac{e}{m}\right)_v = 1{\cdot}04 \times 10^{-14} \times \frac{W}{d} \cdot \frac{M_v}{\lambda_v^4} \times 9 \times 10^{20}.$$

If we apply this formula to fluor-spar (CaF_2) for which

$$W = 78,\ d = 3{\cdot}18,\ M_v = {\cdot}612 \times 10^{-10},\ \lambda_v^2 = {\cdot}888 \times 10^{-10},$$

we have

$$P_v \left(\frac{e}{m}\right)_v = 1{\cdot}04 \times 10^{-14} \times 24{\cdot}5 \times {\cdot}778 \times 10^{10} \times 9 \times 10^{20}$$
$$= 1{\cdot}78 \times 10^{18}.$$

Measurements on β rays have shown that the value of e/m for an electron is $5{\cdot}6 \times 10^{17}$: but the same quantity as deduced from the behaviour of electrons in optical phenomena, such as the Zeeman effect (Chapter V.), is found to be considerably smaller, about $4{\cdot}5 \times 10^{17}$. The reason for this discrepancy is not fully explained. If, in the above equation, we put $P_v = 4$, we get $\left(\frac{e}{m}\right)_v = 4{\cdot}5 \times 10^{17}$. Accordingly, we conclude that *the number of electrons in the molecule of fluor-spar which give rise to the ultra-violet absorption bands is* 4.

I give two further examples which the reader may work out for himself.

Sylvine (KCl).

$$b^2 = 4{\cdot}55,\ M_r = 10{\cdot}747 \times 10^{-5},\ M_v = 1{\cdot}50 \times 10^{-10},\ \lambda_r^2 = 4{\cdot}57 \times 10^{-5},$$

$$\lambda_v^2 = 2{\cdot}34 \times 10^{-10}.$$

$$W = 74{\cdot}5,\ d = 1{\cdot}95.$$

Thus $$P_v = 2 \quad \text{if } \epsilon/m_v = 5{\cdot}10 \times 10^{17}.$$

Rock Salt (NaCl).

$$b^2 = 5{\cdot}18,\ M_r = 8{\cdot}977 \times 10^{-5},\ M_v = 1{\cdot}85 \times 10^{-10},\ \lambda_r^2 = 3{\cdot}149 \times 10^{-5},$$

$$\lambda_v^2 = 1{\cdot}62 \times 10^{-10}.$$

$$W = 58{\cdot}5,\ d = 2{\cdot}15.$$

Thus $$P_v = 4 \quad \text{if } \epsilon/m_v = 4{\cdot}17 \times 10^{17}.$$

Unfortunately the whole matter is not nearly so simple as has been represented. Our assumption that there is only one ultra-violet natural period is known to be incorrect, and our conclusions based on it will only be correct as far as general 'orders of magnitude' are concerned: they can not be trusted to give us the precise numerical information which we want. If we knew precisely the number of ultra-violet natural periods, the problem, though complicated, might be exactly soluble, but we have not accurate information even on this subject. Ideally we can find from formulae (1), (2), (3) the values of all the constants b, M_p and λ_p: but practically, a complete determination is impossible. If we assume that there are four natural periods in the molecule, we shall have nine constants (b, 4 M's and 4 λ's), the values of which we can adjust to make the theoretical formula correspond with the observed dispersion: values of these nine constants can always be found which make the curve fit within the limits of experimental error, and no better coincidence could be obtained if we assumed that there were five or more natural periods. Accordingly, if there be more than four such vibrating systems, the information that we can attain can never be accurate.

Drude's discussion of this tangled problem is too complicated to be given at length: the indication of its principle, which has been given already, and a quotation of his conclusions must suffice. But there is one further consideration which occurs in the development of the analysis, which is of such importance that it must be mentioned.

From (2) and (3) we have

$$b^2 - 1 + \frac{M_1}{\lambda_1^2} + \frac{M_2}{\lambda_2^2} + \dots \frac{M_p}{\lambda_p^2} + \dots = 0 \quad \text{.........(10).}$$

On substituting the observed values of b, M_p and λ_p in the left-hand side of (10) it is found that the expression always differs considerably from zero. The discrepancy can be explained if we suppose that there are in the atom electrons which have no influence on the dispersion, because their natural periods are very short compared with those of the incident light. A group of charged particles (s) contributes to the expression for n^2 two factors. Firstly, there is the part

$$\frac{4\pi e_s^2 N_s}{f_s} = \frac{N_s e_s^2 p_s^2}{\pi m_s},$$

which is independent of λ, the wave-length of the incident light, and is included in b^2; secondly, there is the term

$$\frac{M_s}{\lambda^2 - \lambda_s^2} = \frac{N_s e_s p_s^4 V^2}{\pi m_s} \cdot \frac{1}{\lambda^2 - \lambda_s^2},$$

which varies with λ and influences the dispersion (*i.e.* the variation of n with λ). Now, if the natural period of the group s is very small, so that p_s and λ_s are very small, the second term will be much less than the first, for it contains p_s^4 in place of p_s^2: the second term may be undetectable by experimental measurements, while the first term is still appreciable. Accordingly a term $\frac{Ne_s^2 p_s^2}{\pi m_s}$ may be included in b^2 on the left-hand side of (2), but, since M_s will have been put equal to zero, there will be no corresponding term on the right-hand side. If our equation is to be correct, we must write in place of (10)

$$b^2 = 1 - \Sigma \frac{M_p}{\lambda_p^2} + \ldots + \Sigma \frac{N_s e_s^2 p_s^2}{\pi m_s},$$

or

$$b^2 - 1 + \Sigma \frac{N_p e_p^2 p_p^2}{\pi m_p} = \Sigma \frac{N_s e_s^2 p_s^2}{\pi m_s} \quad \ldots\ldots\ldots\ldots(11).$$

It is found that the term on the right-hand side of (11) is quite appreciable, compared with the terms $\frac{N_p e_p^2 p_p^2}{\pi m_p}$. But since, by hypothesis, p_s is much smaller than p_p, N_s must be much greater than N_p. From the non-fulfilment of the equation (10) we arrive at the interesting conclusion that the number of electrons in the molecule, which have free periods so short that they cannot influence the dispersion, is very much larger than the number of electrons which can influence the dispersion.

It appears, as might be expected, that the number of electrons in a molecule is an 'additive' quantity, and can be regarded as the sum of those contained in the atoms of which it is composed. In Table XI., which gives some of Drude's results, the numbers *P* are calculated from observations on compounds in which the element is contained. The third column gives the chemical valency of the atom.

TABLE XI.

Substance	*P* (calculated)	valency
H	1·5	1
C	3·7	4
O in form $\begin{matrix}C\\C\end{matrix} > O$	2·7	2 ?
O in form $C = O$	4·4	2 ?
Cl	6·2	7 ?
B_2	5·2	7 ?
I	3·9	7 ?
Ca	1·5	2
Na	2·0	1
Si	3·9	4

Of course, *P* is the number of electrons influencing the dispersion of light: no information can be obtained from optical considerations of the whole number of electrons. The occurrence of fractions in the value of P need cause no surprise, in view of the uncertainty of some of the assumptions which have to be made arising out of ignorance of the number of absorption bands.

The number of electrons, (b) magnetism.

4. The calculations on pp. 77—80 of the diamagnetic susceptibility and the paramagnetic magnetisation of saturation would lead directly to an estimate of the number of electrons in the atom, if there were available more definite information as to the magnitude of some of the other quantities concerned. Thus we found that, if N is the number of atoms per unit volume of the material and P the number of electronic orbits in each atom, r the radius of those orbits, the diamagnetic susceptibility k is given by

$$k = P\frac{N\epsilon^2 r^2}{4mV^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12).$$

If the radius of the atom is 10^{-8} cm. it is reasonable to suppose that the radius of the electronic orbits is between 10^{-8} and 10^{-9}. Hence

$$n > \frac{k\,.\,4mV^4}{N\epsilon^2} \times 10^{16} \quad \text{and} \quad < \frac{K\,.\,4mV^4}{N\epsilon^2} \times 10^{18}.$$

In the case of the elementary gases the value of k seems to be about 3×10^{-29}: N is 6×10^{19} for a diatomic gas, hence P lies between 850 and 85000. On the other hand, for bismuth, the most diamagnetic substance, $k = 1{\cdot}4 \times 10^{-26}$ and N is 5×10^{22}; hence P lies between 50 and 5000; but the numbers derived from measurements of solid bodies are much less reliable than those made on gases, on account of the impossibility of taking into account the mutual actions of the atoms, which are important in the former but not in the latter.

Apart from the uncertainty in the value of r, there are two other causes which render this source of evidence incapable of giving accurate results. Firstly, it is probable that the intensity of the magnetic field inside the atom is very different from the average value throughout the whole substance, which can alone be observed by our instruments. Secondly, it is by no means certain that all the electrons are revolving in orbits: those which are at rest are uninfluenced by the action of the external magnetic field, and contribute nothing to the diamagnetic susceptibility.

According to (11), p. 80, the saturated intensity of magnetisation I is given by

$$I = NP \cdot \frac{\epsilon v r}{2V^2},$$

where v is the velocity of the electron in its orbit. To our uncertainty concerning the value of r is added ignorance of the value of v, but it is not unlikely that the product of vr lies between 1 and 10^{-2}. Hence P may lie between

$$\frac{2IV^2}{N\epsilon} \text{ and } \frac{2IV^2}{N\epsilon} \times 100.$$

For iron $I = 4 \times 10^{-7}$, for nickel $1{\cdot}6 \times 10^{-8}$, for cobalt 4×10^{-8}, and N is about $1{\cdot}4 \times 10^{23}$ for each of these metals: accordingly, for these three metals we find that N lies between 15 and 1500, 0·6 and 60, 1·5 and 150 respectively.

It is to be expected that the value of n derived from considerations of paramagnetism should be less than that derived from diamagnetism, for it is probably only a very small proportion of the electrons in the atom which give rise to the former effect. In an iron atom the axes of the electronic orbits are distributed in all directions, but not equally distributed: it is only the small number which represent an excess in one particular direction which give rise to paramagnetism.

Number of electrons, (c) conductivity and specific heat.

5. By applying to (35) and (36), p. 70, arguments analogous to those used for determining the number of electrons from the optical dispersion of a transparent body, Drude has concluded that the number of free electrons in unit volume of a conducting elementary substance is nearly equal to the number of atoms in the same volume; in fact, the number of free electrons given off by each atom of the element appears to be similar to the number of dispersional electrons which that atom contributes to a transparent substance in which it is contained.

This conclusion has an important bearing upon the specific heat of solid bodies. It is proved in all elementary treatises on the theory of gases that the specific heat at constant volume of a gramme-molecule (*i.e.* a number of grammes equal to its molecular weight) of a gas which possesses no intra-molecular energy is approximately 3, while the specific heat at constant pressure of the same mass is 5. Measurements on monatomic gases lead to values which accord well with this theoretical prediction, and it is concluded that such gases have no intra-atomic thermal energy. The specific heats both at constant volume and at constant pressure of a gramme-molecule of a gas with more complex molecules are greater than these values: the additional energy is regarded as kinetic energy of the atoms moving as separate systems within the molecule. It is readily shown that if there is no intra-atomic (as distinct from intra-molecular) thermal energy, the specific heat of a gramme-atom of a gas can never be greater than 3 and will be less than that value for all gases with complex molecules. Since no value greater than 3 has been observed, it may be concluded that no atom of a gas possesses intra-atomic thermal energy. But, according to our view, intra-atomic energy can be nothing but energy of the intra-atomic electrons, and we may conclude that the electrons within the atoms of a gas do not share in the thermal energy of the gas: the speed of those electrons is not increased by raising the temperature of the gas—a result which is of importance when we consider the mechanism of the emission of spectra.

On the other hand, according to Dulong and Petit's law, the specific heat of a gramme-atom of a solid element is about 6·3 for nearly all elements—a value twice as great as the maximum for a gramme-atom of a gas. (It is clear that the specific heat of a solid must be compared with the specific heat of a gas at constant volume.) But, according to the kinetic theory of matter, the kinetic energy of a molecule of a body depends only on its temperature and is independent of its state of aggregation: hence, if we suppose that the molecules of a solid are monatomic, we must conclude that just half of the heat energy of a solid is intra-atomic energy: if we suppose that the molecules are more complex, a still larger fraction must be attributed to intra-atomic energy.

Now our study of metallic conduction leads us to the belief that a solid conductor contains free electrons, each of which possesses the same kinetic energy as a molecule of the solid. If Drude's result, that the number of free electrons is nearly equal to that of the atoms, is correct, half the heat energy of the solid is kinetic energy of the atoms (molecules) and the other half is kinetic energy of the free electrons. The 'intra-atomic' energy of a solid may be identified satisfactorily with the energy of the free electrons, and the remarkable fact, that an atom should possess intra-atomic energy in the solid but not in the gaseous state, finds an explanation in the presence of free electrons in the former but not in the latter state.

However a difficulty must be noted. It has been assumed hitherto that only conductors contain free electrons which share in the heat energy of agitation: hence we should expect that only conductors should possess 'intra-atomic heat energy,' and that only those bodies should obey Dulong and Petit's law. However, though it is found that all the bodies which disobey that law are non-conductors, it appears that some bodies, such as sulphur and phosphorus, which are non-conductors obey this law. But I do not think that it is necessary on this account to abandon the explanation suggested. It is not difficult to conceive that a body may contain electrons which are 'free' in the sense of being able to share in the heat energy of the atoms, but are not 'free' in the sense of being able to move under an electric field and confer conductivity: such electrons may have escaped outside the parent atoms and yet be unable to break wholly free from their restraining attraction. The divergence of such elements as carbon, which possess a smaller specific heat than that predicted by Dulong and Petit's law, may be attributed to the formation of complex molecules containing more than one atom.

On the other hand, an objection to Drude's estimate of the number of free electrons in a metal arises from the considerations advanced in Chapter VI., § 6. From (11), p. 108,

$$\sigma = \frac{N\epsilon^2 lu}{4\alpha T} = \frac{l}{v} \cdot \frac{N\epsilon^2}{2m}.$$

For silver $\sigma = 5{\cdot}53 \times 10^{17}$; if the number of free electrons is equal to the number of atoms, $N = 8{\cdot}2 \times 10^{22}$. Accordingly

$$\frac{l}{v} = 7{\cdot}12 \times 10^{-14}.$$

But $\frac{l}{v}$ is the time required for an electron to perform its 'free path,' and it seems clear that the optical conductivity of a metal cannot be identical with the conductivity for steady currents, unless the time required for the electron to perform its free path is not greater than half the period of vibration of the light. Rubens has found that for light of period $1{\cdot}1 \times 10^{-14}$ the optical conductivity is identical with the conductivity for steady currents, and yet in half the period of vibration of this light, the electron cannot, on our calculations, perform much more than $\frac{1}{13}$ of its free path.

The discrepancy is doubtless due to an error in the assumptions made concerning the motion of the electrons in the solid: it is likely that the electrons have no true 'free' path, in which they are free from all forces. J. J. Thomson has worked out a theory of conduction in which the difficulty just noted does not arise: it differs considerably from that expounded in Chapter VI., but it retains the fundamental assumptions on which that theory was based (p. 133).

Number of electrons, (d) ionisation of gases.

6. It was noted on p. 191 that Bragg has concluded that the energy required to form an ion is the same from whatever atom the ion is formed and that the number of ions which can be formed from a given atom is proportional to the square root of the weight of the atom. Since an ion is formed by the extraction of an electron from an atom, which subsequently collects round it neutral molecules, the last result would seem to indicate that the number of electrons which can be extracted from an atom by the α rays is proportional to the square root of the atomic weight. But this conclusion does not follow inevitably, for it is not certain that all the molecules in a gas through which the rays pass come within the influence of the rays: the calculation on p. 190 can only be held to show that, in the case of air, the number of molecules ionised is a considerable fraction, not less than a tenth, of the whole number with which the α ray particle comes into collision. It appears more probable that the number of electrons which can be extracted from an atom by ionising agents depends rather on the nature of the atom than upon its weight, and that the variation of the ratio of the number of ions formed to the number of atoms in the gas traversed by the rays is determined by the number of collisions which the particles make in passing through that gas: there is no direct evidence that more than one electron can be extracted from an atom in the process of ionisation of the gas.

But, though the direct examination of the number of ions formed in a gas under the action of ionising agents leads to no definite estimate of the number of electrons in an atom, information may be obtained indirectly by the study of gaseous ionisation. Two such lines of argument have been put forward recently by J. J. Thomson. The first is based on the measurement of the ratio of the intensity of the secondary Roentgen radiation excited in a gas to that of the exciting radiation. We have noted that the emission of this secondary radiation is due to the jerk given to the electrons in the atom when the pulse passes over it, and a simple calculation similar to that of p. 223 enables the intensity of the secondary to be connected with that of the primary pulse. From the value thus found for the intensity of the radiation given out by each electron, combined with measurements of the intensity of the total secondary radiation, the number of electrons present can be deduced immediately: the value obtained by Thomson for the number of electrons in a molecule of air is 25. But it has been noted that difficulties apparently insuperable are encountered in connection with ionisation by Roentgen rays, unless it be assumed either that the number of atoms which come within the action of the rays is a very small proportion of the whole number present in the gas, or that the part of the energy required for ionisation is drawn from some source other than the pulses constituting the rays. Either of these assumptions would invalidate the argument which deduces the number of electrons in the gas from the intensity of the secondary radiation, and accordingly no confidence can be placed in such an argument, until the mechanism of ionisation by Roentgen rays is elucidated more completely.

The second argument is based upon the scattering of β rays in passing through matter. When the electrons constituting the β rays approach the electrons in the atoms of the scattering medium they are deflected from their course: the magnitude of the deflection suffered by any particle is determined by the distance of its nearest approach to a deflecting electron. Some particles are more, and some less deflected, but, after each particle has suffered a considerable number of deflections, the rays, which entered the medium as a parallel beam, will be scattered equally in all directions. By measuring the thickness of the layer of the medium requisite to scatter the rays completely, an estimate of the number of electrons in a given volume of the medium can be formed. The complete calculation of the connection between these two quantities would be of great complexity, but the nature of the argument may be indicated roughly.

If b is the length of the perpendicular from the deflecting electron (supposed fixed) on the line of approach of the deflected

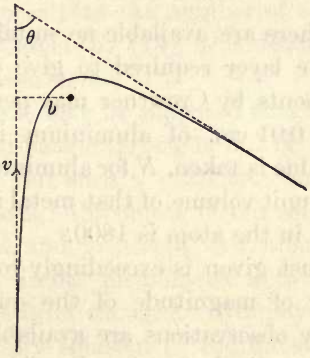

Fig. 24.

electron, m the mass of the electron, v its velocity, the angle of deflection θ is given by

$$\sin^2\frac{\theta}{2} = \frac{1}{1+\frac{b^2v^4}{\epsilon^4}m^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13).$$

Substituting for ϵ and m their known values, for v the value $1{\cdot}6 \times 10^{10}$ corresponding to the β rays of uranium, and for b the values 10^{-11}, 10^{-12}, 10^{-13} cm. successively, we obtain

$b = 10^{-11}$, $\theta = 3°$; $b = 10^{-12}$, $\theta = 73°$; $b = 10^{-13}$, $\theta = 161°$.

Consequently the ray particles suffer no appreciable deflection unless b is not greater than 10^{-12}: making a rough estimate, we may suppose that the incident rays are not completely scattered until each particle has passed five times so near to a deflecting electron that b is not greater than 10^{-12}. Let d be the thickness of the substance required to scatter the rays completely, and let N be the number of electrons per unit volume: then in a layer of unit surface area and of thickness d there are Nd electrons. If a circle of radius 10^{-12} cm. is drawn round each electron perpendicular to the direction of the original motion of the rays, the total area of these circles is $Nd\pi \times 10^{-24}$ cm.^{2}: if each electron in the β rays has passed five times within a distance of 10^{-12} of a deflecting electron, the total area of these circles must be about five times that of the area of the layer: hence

$$Nd\pi \times 10^{-24} = 5.$$

Unfortunately there are available no suitable measurements of the thickness of the layer required to give complete scattering, but some measurements by Crowther may perhaps be interpreted as indicating that 0·01 cm. of aluminium is sufficient for the purpose. If this value is taken, N for aluminium is $1{\cdot}6 \times 10^{26}$: the number of atoms in unit volume of that metal is 9×10^{22}, hence the number of electrons in the atom is 1800.

The argument just given is exceedingly rough but it probably indicates the order of magnitude of the quantity investigated. When the necessary observations are available on which to base the calculation, this method of estimating the number of electrons in the atom will probably give more definite results than any other.

The number of electrons, (e) mass.

7. When the experiments of Kaufmann indicated that the mass of an electron is due to the charge which it carries, and that its kinetic energy is the magnetic energy of the field surrounding that moving charge, it was natural that physicists should proceed at once to the further conclusion that not only the mass of the electron, but the mass of all material bodies is of a similar origin. Until quite recently, it seems to have been assumed implicitly by all those who had followed the development of the electron theory that the mass of an atom was the sum of the masses of the electrons contained in

it and that the number of electrons in an atom (P) was given by the expression

$$P = A \frac{M}{m} \quad \text{......................(14)},$$

where M is the mass of an atom of hydrogen, A the atomic weight, and m the mass of an electron. (We have noted that the mass of an electron is not an invariable quantity but depends on its velocity: but it seems certain that the velocity of the electrons within the atom is so small that their mass differs inappreciably from that corresponding to an infinitely small velocity.)

Lately some doubt has been thrown upon the correctness of this conclusion. The objection does not appear to me to have been established, but since further investigation may show that it is impossible to suppose that the number of electrons in the atom is so great as that given by (14), it is desirable that the alternatives should be considered.

In the first place it may be supposed that the mass of material bodies is not of electromagnetic origin, but is 'mechanical mass' of a different nature. But such a statement only begs the question and hides the real issue: the object of our science is to coordinate different phenomena, and to accept as fundamental a distinction between apparently similar phenomena is to violate the canons of every school of scientific thought. No physicist can regard as a final and satisfactory solution of the question one which does not reduce in some manner electromagnetic and mechanical mass to a common basis.

But, if all the mass of an atom is electromagnetic, there are two further alternatives open: either it may be supposed that the principle of the conservation of mass is not true, and that, while the mass of the atom is due to the presence of the electrons, it is not the sum of the masses associated with those electrons in the free state: or it may be supposed that part of the mass is associated with the positively charged portion of the atom. Since the radius of the positive sphere (which must be taken as equal to that of the atom, or about 10^{-8} cm.) is very large compared with that of the electrons, the considerations of p. 198 show that any mass which that sphere may possess cannot be explained on the simple view of electromagnetic action that has been adopted hitherto. Either alternative, if accepted, must lead to a revision of our ideas on the fundamental laws of electromagnetism, but in the next chapter we shall see that the limits of our ignorance of those laws are very wide, and no decisive objection can be taken to such explanation. But, until a more conclusive disproof is adduced, I think it is wise to retain our confidence in the accepted laws of the electromagnetic field and to suppose that the number of electrons in the atom is that given by (14).

The correctness of this view might be tested if sufficiently accurate determinations of atomic weights were available: for if the mass of the atom is the sum of the masses of the electrons contained in it, its mass must be an integral multiple of that of an electron: but, since the mass of an electron is less than one-thousandth of that of the hydrogen atom and the masses of few, if any, atoms are known with an accuracy of one in a thousand, no reliable argument can be based on such considerations.

It must be admitted to be strange that, if the hydrogen atom contains more than 1000 electrons, no simpler atom should exist containing a smaller number, but the difficulty does not seem to be peculiar to the theory that has been adopted: it is just as hard to see why no positive sphere should exist having a mass smaller than that of the hydrogen atom. It may be noted in passing that, since it has been proved abundantly that the masses of the atoms of the elements are not integral multiples of that of the hydrogen atom, it is impossible to regard the atom as built up of 'positive electrons' having the mass of the hydrogen atom, unless the principle of the conservation of mass is abandoned for structures existing inside an atom.

Light might be thrown on these problems if it were possible to investigate the variation of the mass of a material body with its velocity: but the experimental difficulties seem insuperable, for no appreciable change in the mass is to be expected unless the velocity of the body was greater than 10^{10} cm. per sec. The speed of the hydrogen atom at the highest attainable temperature is only 6×10^5 cm. per sec.—which is much too small for our purpose. Nor does the use of larger portions of matter seem hopeful. In order to give one milligramme of matter a speed of 10^{10} cms. per second, there must be communicated to it an amount of energy 30 times as great as that of the projectile of the latest twelve-inch gun (850 lbs. shell with an initial velocity of 2900 ft. per sec.), which is produced by the explosion of 220 lbs. of cordite.

The number of electrons, (f) radioactivity.

8. A minimum estimate of the number of electrons in a radioactive atom may be obtained by the consideration of the energy liberated in radioactive processes. It has been concluded that this energy is derived from the internal structure of the atom, and, if the model atom which we have adopted is to be satisfactory, the number of electrons in it must be sufficient to provide the requisite amount of energy.

The internal energy of the atom is probably partly kinetic energy, and partly electrostatic potential energy due to the separation of the oppositely charged portions of the atom: general considerations render it improbable that the kinetic energy is greater than the potential energy. The electrostatic energy of a single electron within a sphere of positive electricity of a total charge $P\epsilon$, where P is the number of electrons in the atom, and a the radius of the atom, is $\frac{P\epsilon^2}{2a}$: hence the whole electrostatic energy of the P electrons cannot be greater than $\frac{P^2\epsilon^2}{2a}$ and their whole energy, potential and kinetic, cannot be greater than $\frac{P^2\epsilon^2}{a}$.

One gramme of radium during its life can liberate $1{\cdot}6 \times 10^9$ gramme calories or $6{\cdot}7 \times 10^{16}$ ergs. The weight of a radium atom is 225 times that of a hydrogen atom or $2{\cdot}5 \times 10^{-22}$ grms.: hence the amount of energy liberated by one radium atom is $1{\cdot}7 \times 10^{-5}$ ergs. The amount of energy liberated by other radioactive atoms is probably of the same order of magnitude, though the period during which it is liberated—the life of the atom—may vary over very wide limits.

Hence we have

$$\frac{P^2\epsilon^2}{a} > 1{\cdot}7 \times 10^{-5},\quad \epsilon = 3{\cdot}4 \times 10^{-10} \text{ and } a = 10^{-8} \text{ cm.; or } P > 1220.$$

This estimate of the number of electrons is certainly much too low, for the forces of electrical origin acting on an electron are not those due to the whole positive charge on the atom: the action of that positive charge will be neutralised in part by the action of the other electrons distributed through it. Since the potential energy of an electron increases with the intensity of the electric field in which it is placed, in overestimating that field we have overestimated the energy associated with a single electron and underestimated the number of electrons necessary to provide the energy liberated in radioactivity. A nearer approximation may be obtained if we consider that it is unlikely that any single electron in the atom possesses an amount of energy greater than that required to liberate an electron from the atom. The amount of this energy is known: it is the energy of ionisation and was estimated on p. 192 as $4{\cdot}2 \times 10^{-11}$ ergs. Inserting this value we must have for the radium atom

$$4{\cdot}2 \times 10^{-11} P = 1{\cdot}7 \times 10^{-5}, \text{ and } P = 4 \times 10^{5}.$$

It will be noted that this minimum value is not very far from that deduced from (14) on the assumption that the whole mass of the radium atom is the sum of the masses of the electrons contained in it ($4{\cdot}2 \times 10^{5}$).

Of course the considerations of this paragragh apply only to radioactive atoms, but since radioactive elements differ from other chemical elements only in their radioactivity, it would be unjustifiable to assume that the structure of their atoms is fundamentally different.

Summary.

9. Of the above arguments, those which appear the most reliable, by reason of the accuracy of the experimental data available, and the certainty of the assumptions introduced in the calculations, are based on the consideration of the optical dispersion, the conductivity, the number of ions formed in a gas, and, in a lesser degree, paramagnetism. All these arguments agree in giving for the number of electrons in the atom a small value not greater than ten, while those which are more doubtful give very much larger values amounting to several thousands. But it is not necessary to conclude that the latter estimates are incorrect or that there is any inconsistency in this result. The reliable estimates are all minimum estimates; the electrons which take part in the phenomena on which they

are based may be only a small proportion of the whole number of electrons present. Indeed Drude's result noted on p. 237 points to such a conclusion. The 'dispersional electrons' are those which have natural periods of vibration considerably less than those of the rest of the electrons in the atom: therefore, in accordance with (9), p. 54, they must be those which are held by restraining forces smaller than those which act upon the other electrons: the dispersional electrons are those which are most likely to be extracted from the atoms and to appear as free electrons in metals, or to form the nuclei of gaseous ions. We are thus led to the conclusion that there are a few electrons in every atom which bear a peculiar relation to the rest of the atom from which they are more readily detached than the rest. It is possible that this peculiarity is only apparent, for when one electron has been detached, the extraction of another will require the exertion of a greater force to overcome the attraction of the positively charged remainder of the atom: but the considerations of the next chapter render it likely that it depends on some important feature of the structure of the atom.

In the present state of our knowledge no certain statement can be attained as to the whole number of the electrons in any atom, but the conclusion that it is such that the mass of the atom is the sum of the masses of the electrons contained in it is so attractive that it seems desirable to accept it provisionally in the absence of any conclusive evidence to the contrary.

If the number could be determined with certainty it would be possible ideally to deduce from the structure which we have attributed to the atom, the exact arrangement of the electrons and all the properties of the atom: but as a matter of fact the calculation would not be practicable, even if the electrons were at rest. Since some at least of them appear to be moving in closed orbits with unknown velocities, the determination of the positions of the electrons in the atom is quite impossible. For the present all that we can do is to consider carefully in the light of our theory all the known properties of atoms, to examine how far the theory is capable of explaining those properties, and to endeavour to form new hypotheses to cover any discrepancies that may arise. We shall therefore proceed to investigate the evidence concerning the structure of atoms, and shall begin with the two most important sources of such evidence, chemistry and spectroscopy.

REFERENCES FOR CHAPTER X.

The first suggestion of the model atom sketched in the foregoing chapter is to be found in Thomson's paper on kathode rays mentioned already, *Phil. Mag.* (5) XLIV. pp. 310 etc., 1897 (*A.* and *L.* p. 779): the developments of that model will concern us in the next chapter.

For a discussion of the number of electrons in the atom the reader may refer to

DRUDE, *An. der Phys.* XIV. pp. 677—725 and 936—961, 1904. (There is a full summary of conclusions on pp. 958—961.)

J. J. THOMSON, *Phil. Mag.* (6) XI. pp. 769—781, 1906.

N. R. CAMPBELL, *Proc. Camb. Phil. Soc.* XIV. 287, 1907.

CHAPTER XI.

THE EVIDENCE OF CHEMISTRY.

Chemistry and Electrolysis.

1. Ever since the time of Berzelius, the view has been generally accepted that the combination of two elements to form a chemical compound is determined by electrical attractions between the atoms of those elements. This theory is, of course, based on the phenomena of electrolysis. When an electric current passes through a solution of sodium chloride, the sodium travels to the negative electrode and the chlorine to the positive: that is to say, the sodium behaves as if it were positively charged, the chlorine as if it were negatively charged. The conclusion is irresistible that the combination of sodium and chlorine arises from the attraction of their oppositely charged atoms.

It is found that, on this view, elements may be divided roughly into two great classes, electropositive and electronegative. The first class includes those elements such as sodium, calcium, iron, the electrolytic ions of which carry a positive charge: the second class such elements as chlorine, the ions of which carry a negative charge. Some elements, such as carbon or nitrogen, are not known in the form of simple ions at all; they are placed in one or other of these two classes according as their chemical behaviour is similar to that of the electropositive or electronegative elements.

Electrolytic phenomena lead to the conception of valency. The valency of any element is the number of electronic charges (ϵ) which is carried by its atom when it becomes an electrolytic ion. Since that charge can be of either sign, we shall speak of positive and negative valency: calcium has a positive valency of two (+2), chlorine has a negative valency of one (−1). The definition of the valency of an element which forms no ions is derived from the observation that the number of ions with which any given ion can enter into combination is determined by the principle that the total valency (reckoned algebraically) of the resulting molecule must be zero: thus one atom of calcuim (+2) combines with two atoms of chlorine (−1). Extending this principle to such an element as carbon, we say that it has a valency of +4 when it unites with four atoms of chlorine (−1) to form the compound CCl_4, and that it has a valency of −4 when it unites with four atoms of hydrogen (+1) to form CH_4. There is nothing contrary to ordinary chemical conceptions in regarding the valency of an element as variable with the conditions in which it is placed.

On the theory which has been developed hitherto, an atom becomes charged positively or negatively according as it loses or gains electrons above the number required to neutralise the charge on the positive portion of the atom. Accordingly the difference between the various elements, so far as their chemical relations are concerned, arises from a difference in the number of electrons which they can gain or lose in any given circumstances. A chemical compound is a neutral system composed of two or more atoms, of which some have gained electrons lost by the others, held together by the electrical attraction between the oppositely charged portions. When such a compound is formed out of its constituent elements, the electrons in passing from the electropositive to the electronegative elements generally lose electrostatic energy and gain kinetic energy: this additional kinetic energy appears eventually as heat energy of the reacting substances, just as the additional kinetic energy gained by the free electrons of a conductor in moving through an electric field appears as heat energy of the conductor. The more powerful the forces tending to drag the electron from the electropositive to the electronegative element the greater is the kinetic energy which it acquires during the transference: hence, as in the ordinary theory of 'chemical affinity,' (which merely restates the facts in such a way as to obscure their significance,) the heat of reaction may be taken as some measure of the 'chemical attraction' between two diverse elements: of two reactions possible in any given conditions, that one will tend to take place which has the greater heat of reaction.

In order, then, to bring the facts of chemistry within the range of the electronic theory of matter, two main problems have to be solved. Firstly, we have to explain the difference in the valency of different elements, that is to say, we have to establish a connection between the tendency of an atom to lose or to gain electrons and its other properties: secondly, we have to account for the influence of the physical state of the reacting substances upon the progress of their reaction.

The nature of valency. The size of atoms.

2. There is one very simple difference in the structure of atoms which will account in part for the difference in facility with which atoms lose or gain electrons: it is easily shown that the smaller atom tends to gain electrons at the expense of the larger. Let us take the simplest possible case, that of two atoms in contact containing one electron apiece, and let us calculate the ratio between the radii of the atoms for which the electron will just leave the larger and enter the smaller atom. As usual, the positive spheres are supposed to carry a charge equal in magnitude to that on the electrons contained in them.

Let a be the radius of the larger, b that of the smaller atom, A and B their centres, and P their point of contact. When the electron from A is just on the point of entering B, it will be in equilibrium at the point P. The electron in B will be displaced

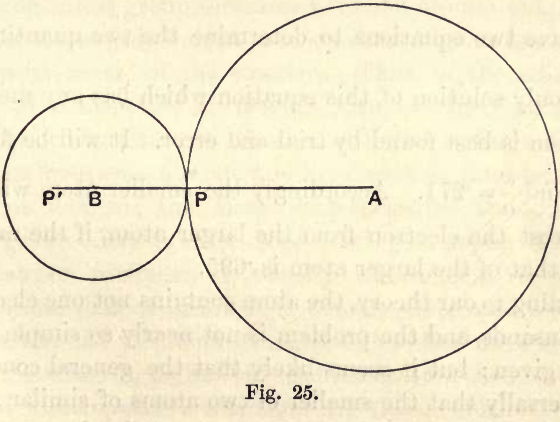

Fig. 25.

from P by the repulsion of the electron which is emerging from A: let it rest at P', where $BP' = x$.

Then the forces acting on the electron at P are:

(1) the attraction towards A due to the larger atom of a magnitude $\frac{\epsilon^2}{a^2}$,

(2) the attraction towards B due to the smaller atom of a magnitude $\frac{\epsilon^2}{b^2}$,

(3) the repulsion towards A due to the electron at P' of a magnitude $\frac{\epsilon^2}{(b+x)^2}$.

When the system is in equilibrium, the relations between a, b, ϵ, x, must be such that

$$\frac{\epsilon^2}{a^2} + \frac{\epsilon^2}{(b+x)^2} = \frac{\epsilon^2}{b^2}.$$

A consideration of the forces acting on the electron at P' will give in a similar manner the relation

$$\frac{e^2}{(a+b+x)^2} + \frac{e^2 x}{b^3} = \frac{e^2}{(b+x)^2},$$

since the force on an electron due to the positive sphere of radius r, when it is at a distance d from the centre of that sphere, is $\frac{\epsilon^2 d}{r^3}$.

We have two equations to determine the two quantities $\frac{b}{a}$ and $\frac{x}{a}$. The only solution of this equation which has any meaning for our problem is best found by trial and error. It will be found that $\frac{b}{a} = \cdot 695$ and $\frac{x}{a} = \cdot 271$. Accordingly the smaller atom will just be able to wrest the electron from the larger atom, if the ratio of its radius to that of the larger atom is ·695.

According to our theory, the atom contains not one electron, but many thousands, and the problem is not nearly so simple as in the case just given: but it seems likely that the general conclusion is true universally that the smaller of two atoms of similar structure tends to gain electrons at the expense of the larger. Perhaps the actual forces acting on the electrons are not so different from those deduced in the simple illustration as might appear at first sight. We have seen that a distinction must be made between the 'dispersional' electrons which are held loosely within the atom and the 'non-dispersional' electrons which are held more firmly, and it is natural to identify with the former the 'valency' electrons, that is the electrons the transference of which from one atom to another produces chemical combination. But spectroscopic evidence appears to indicate that the forces acting on these electrons are very similar to those which would act, if the electrons were alone, within a sphere bearing a total charge of positive electricity equal in magnitude to the sum of the charges carried by the dispersional electrons. The non-valency electrons seem to be distributed almost uniformly through the atom, and to neutralise the effect upon the valency electrons of that part of the positive charge which is equal in magnitude to the total charge which they carry.

It is well known that there is an intimate connection between the valency of the elements and their atomic volume, such as has been suggested by the foregoing argument. (The atomic volume, obtained by dividing the atomic weight by the specific gravity of the element in its solid state, agrees in order of magnitude with the volume of the atoms deduced from considerations based on the kinetic theory of gases.) For the atomic volume of elements of the same chemical group increases with the atomic weight, and an increase in the atomic weight is accompanied by an increase in the 'electropositiveness' of the element. Thus, of the alkali metals, the most electropositive is caesium with the heaviest atom, and the least electropositive is lithium with the lightest atom. Of the halogen elements the most electronegative (and least electropositive) is fluorine, the least electronegative is iodine. The nitrogen group offers an even more striking example. The lightest atom, that of nitrogen, is usually electronegative, while the heaviest atom, that of bismuth, is usually electropositive: in this case, however, the elements cannot be classified directly by their behaviour in electrolysis, but only indirectly from chemical evidence. Since the electropositive elements are those which tend to lose electrons and the electronegative elements are those that tend to gain them, it appears that the smaller element has the greater tendency to acquire electrons and the larger element the greater tendency to lose them. So far then the facts bear out the rule deduced from the structure of our hypothetical atom.

The nature of valency. The structure of atoms.

3. But difference in size will not explain the whole matter: for, while the atomic volumes of the alkali metals are greater than those of any other group, the atomic volumes of the halogen group (the most electronegative) come next in order. It is clear that the arrangement of the electrons in the atom must have an important influence in determining the valency. It is only among elements of similar constitution that the size of the atom determines the valency. But before proceeding to investigate the arrangement of the electrons, one or two further remarks should be made concerning the general nature of the property of 'valency.'

It is important to bear in mind the distinction between positive and negative valency, which is obscured by most chemical theories: an element which has positive valency tends to lose electrons, an element which has negative valency tends to gain them. From the foregoing discussion it is clear that the feature which determines the positive valency of an atom, is the number of 'dispersional' or 'valency' electrons it contains: these electrons, and these only, can be dragged out of the atom, leaving a resulting positive charge equal in magnitude to the sum of those on the lost electrons.

It is not so easy to connect the negative valency of an atom—the number of electrons which it can gain—with any other of its properties: but a relation may be established, if we note that elements which have a negative valency usually, if not always, have at the same time a positive valency which shows itself in suitable circumstances. Thus carbon, which has a valency -4 in the compound CH_4, has a valency $+4$ in CCl_4: phosphorus, which has a valency -3 in PH_3, has a valency $+5$ in PCl_5: sulphur, which has a valency -2 in SH_2, probably has a valency $+6$ in SO_3: iodine, which has a valency -1 in HI, has been held to have a valency $+7$ in I_2O_7: the arithmetical sum of the greatest positive and negative valencies of an element never amounts to more than eight. The sum of the positive and negative valencies is the number of valency electrons which the atom can contain: it is the sum of those which belong to the atom, and those which are collected from outside. It seems that the greatest number of valency electrons which any atom whatever can contain is eight, and that in chemical reactions each atom strives either to reduce the number of valency electrons to zero (positive valency) or to raise that number to eight (negative valency).

Nothing has been said so far about the group of elements composed of the inert gases, helium, argon and so on. Many attempts have been made to discover some chemical reaction in which these gases take part, but no proof whatever has been offered that they can form compounds distinguished by the constancy of composition which is the first characteristic of chemical combination: since they can form no such compounds, they possess a valency of zero. We must suppose accordingly that they possess no valency electrons and do not show the tendency, characteristic of the electronegative elements, to acquire extraneous electrons.

Results of the theory of valency.

4. This view of valency is supported by the calculations of Drude mentioned on pp. 233 etc. It will be observed that the numbers which he deduced for the number of dispersional electrons in an atom agree in some measure with the number of valency electrons estimated on chemical grounds—*i.e.* the positive valency of the element. Perfect agreement is not to be expected by reason of the many simplifying assumptions which have to be introduced into his arguments, but it will be noted that the halogen elements which have the greatest positive valency have also the largest number of dispersional electrons. On the other hand, measurements which have been made of the dispersion of light in helium show that that dispersion is peculiarly small, in agreement with the conclusion reached on chemical grounds that that element contains no valency or dispersional electrons.

Again we can establish a connection between the chemical properties of an element and its conductivity. The elements classified as metals are distinguished from the non-metals by three characteristic properties. The metals have positive valency, form positive electrolytic ions and are conductors of heat and electricity in the solid state. With the exception of hydrogen, all elements which give simple kations conduct in the solid state; and, with the exception of carbon, the converse statement is true. This constant association of properties is explained immediately on our theory; the possession of all the properties is an immediate consequence of the ability on the part of the atom concerned to lose one or more electrons. The metals can lose electrons, the non-metals cannot. It is not necessary that the more electropositive element should be the more conducting, for the conductivity depends on the total number of electrons lost by each atom, as well as on the ease with which each is lost. Caesium loses electrons more easily than bismuth, but it has fewer to lose.

On this view there is no essential difference between electrolytic and metallic conduction: in both cases the presence of conducting particles is due to the loss of electrons by the electropositive atoms present. In a solid conductor the positive remnants of the atoms cannot move under the electric field, and the electrons which are set free do not attach themselves to other atoms because there are no electronegative atoms present. In an electrolyte, on the other hand, the electrons set free from the electropositive atoms attach themselves to electronegative atoms, or groups of atoms, while both the positively and the negatively charged atoms can move through the liquid under the action of an electric field. It is remarkable that, with the exception of the extremely electronegative halogen atoms, no single atom seems capable of acquiring an electron in the liquid state: such atoms as carbon, nitrogen or oxygen are incapable of forming simple electrolytic ions and only appear in electrolysis in complex ions made up of several atoms. Our theory must be developed further before it can cover all these experimental facts.

Many gaps in our theory of valency could be pointed out, especially if organic chemistry were considered, but two well known chemical difficulties may be mentioned which seem to find some explanation. The first of these is concerned with the formation of molecules of an elementary gas, such as oxygen, which consist of two atoms in chemical combination: how can two atoms of identical valency combine? But it is to be noted that these diatomic molecules only occur in the case of elements which can show valency of either sign: the atoms of such elements can either lose or gain electrons, and it is in no way strange that some of the atoms should possess a negative and some a positive charge. The molecules of those elements, such as helium or mercury, which either possess no valency or only valency of one sign are uniformly monatomic[1].

The other difficulty is that of variable valency: some elements, such as iron or copper, can form two kinds of electrolytic ions distinguished by the magnitude of the charge that they carry. On our view such variable valency hardly requires explanation, for it is obvious that the number of electrons lost by an element may vary with the conditions in which its atoms are placed: the extraction of a second electron may require the exertion of a greater force than the extraction of the first, and it is easy to conceive circumstances in which the first, but not the second, electron would be lost.

The Periodic Law.

5. The difference in the chemical properties of the different groups of elements is to be attributed to differences in the arrangement of the valency electrons within their atoms, while the differences between elements of the same group, in which it may be supposed that the arrangement is generally similar, is to be attributed to a difference in the volume of the atoms. In considering the relations of the different chemical groups, we must clearly take into account the most remarkable generalisation in the science of chemistry, Mendeleef's periodic law. The law may be stated in a form suitable for our purpose, somewhat as follows:—If the elements be arranged in a series of increasing atomic mass, the series shows a periodicity in chemical properties. Leaving out of account the more complicated features, it is found that, starting from an element of zero valency, we meet in succession with elements of positive valency 1, 2, 3, 4, 5, 6, 7: the next element is of zero valency and the sequence is repeated.

Now since, on our theory, the order of increasing atomic mass is the order of increasing number of electrons in the atom, the

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[1] Hydrogen might seem to form an exception, but in view of such compounds as ammonium chloride it appears by no means certain that the sign of the valency of hydrogen is always constant.

periodic classification indicates that, as the number of electrons in the atom is increased, any given arrangement of the valency electrons recurs periodically. Any complete explanation of this remarkable conclusion could only arise from an entire knowledge of atomic structure: it is impossible to calculate accurately the positions of the electrons within their sphere of positive electricity, but an interesting analogy due to J. J. Thomson, based on the simpler case of the distribution of attracting centres in a plane, suggests some of the chief features of the arrangement.

If the electrons were confined to one plane within the positive sphere, they would arrange themselves in concentric circles: thus 60 electrons would be arranged in five rings containing 20, 16, 13, 8 and 3 electrons respectively. If one, two, or three electrons are added (while the compensating charge on the positive sphere is increased proportionally) the number of electrons in the inner ring (3) is not changed: the added electrons settle themselves in the outer rings. But, on adding one more electron, the inner ring is increased to four. If the addition of electrons is continued, the number in the inner ring is increased further to five, but it can increase no further: the addition of more electrons causes the appearance of a sixth ring interior to the others, the number in which increases progressively from one to five until a seventh ring is formed.

This analogy suggests that the electrons in the atom which are distributed, not only in one plane, but throughout the positive sphere, are arranged upon concentric spherical shells similar to the concentric circles which have just been described. Suppose then that any property of an element depends on the presence in the atom of an innermost shell containing three electrons. Starting from such an atom and increasing the whole number of electrons progressively, we should expect to find several systems of nearly identical atomic mass containing that property: a further increase in the atomic mass would cause the disappearance of that property and the substitution for it of the property depending on the presence of an innermost shell of 4, 5, ... electrons. But further progress in the same direction would lead to the formation of a new shell, interior to the former shell, the number of electrons in which would increase until it reached three, when the property which we are considering would reappear. In fact the distribution of this property in the series of the elements arranged in the order of their atomic masses would be precisely similar to that of the chemical properties among the actual elements, except that in place of each element of the actual scheme we should find in the hypothetical scheme a group of elements, all possessing the property in question and differing only very slightly in atomic weight.

However this difference is not sufficiently serious to cause the analogy to be rejected as useless: two plausible hypotheses may be suggested to cover the discrepancy. Firstly, we may suppose that of each group of similar elements in the hypothetical system only one is stable and can exist permanently. Some support is lent to this hypothesis by the discovery by means of researches into radioactivity of a large number of elements, existing for a short time only, for which no place can be found in Mendeleef's table. Secondly, we may suppose that the substances which are regarded as chemical elements are in reality mixtures of a large number of very closely similar but not identical atoms. Such a suggestion may appear rather startling, but I do not think that any valid evidence can be urged against it. Even the observed constancy of the periods of the light in the spectrum emitted by the elements is not conclusive, for close examination shows that no spectrum 'line' consists of light of absolutely homogeneous wave-length: the finite breadth of the lines, attributed to various causes, might conceal variations arising from the presence of atoms which are not quite identical.

The occurrence of a definite number of electrons on the innermost shell has only been used as an illustration in the above description: any other feature of the arrangement will recur periodically in the same manner, if the distribution in shells which we have imagined obtains. But it is a rather attractive inference that the electrons on the innermost shell represent the 'valency' or 'dispersional' electrons, and that the number of these electrons can never exceed eight. However, this is sheer speculation. It is not pretended that a complete theory of valency has been given: many features of the actual grouping of the elements, such as the anomalous position of the iron and platinum metals, have been perforce left out of account: but it will be generally admitted that the considerations which have been put forward are worthy of serious notice in the absence of any competing theory.

‘Physical Chemistry.’

6. A few remarks should be made concerning the influence of the physical state of the reacting substances on the progress of a chemical reaction.

If hydrogen and oxygen, two gases which are capable of undergoing chemical combination, are mixed at ordinary temperatures, no change takes place until the reaction is started by some agency such as the passage of an electric spark. It appears that the oxygen atom is not capable of dragging an electron out of the hydrogen atom directly without external aid, and that the difference in the valency of the two elements consists in a superior attraction of the oxygen atoms for electrons which have been introduced in some manner into the gas. This conclusion affects in no way the truth of what was said about the valency of the different elements, but it is extremely suggestive in explaining the influence of physical conditions on chemical reactions, for it appears at once that those agencies must promote a reaction which produce free electrons in the reacting substances. However all ionising agents are not equally efficient, for, while combination is started in a mixture of hydrogen and oxygen by the action of the electric discharge or by contact with an incandescent body, the action of ionising rays has no effect. ‘Physical Chemistry’ is the study of the effects of various methods of ionisation on chemical reaction.

Our previous studies have shown that it requires less work to drag an electron from an atom of a solid or a liquid than from the atom of a gas: when the atoms are so close together that they are subject to mutual influences, the restraining force which any atom exerts upon the electrons within it is neutralised, wholly or in part, by the oppositely directed attractions of neighbouring atoms. Accordingly it is found in accordance with expectation that chemical reactions proceed much more readily between substances in the liquid than in the gaseous state: reactions between solids are hindered by the inability of the reacting atoms to move and come into contact with each other.

The prevalent theory of electrolytic dissociation connects the facility with which reaction takes place in solutions with the acquisition of electric charges by the atoms of the dissolved substances, but it has always assumed that the dissociation, or, as we must term it, ionisation, only takes place in those solutions which possess electrolytic conductivity. However, Kahlenberg and his fellow workers have shown that reactions take place with equal readiness in solutions in organic fluids which possess no appreciable conductivity; and it seems that we must conclude that the ionisation takes place in all solutions, but that different solvents differ in their power of permitting the passage of ions under the action of an electric field: on other grounds Sutherland has recently arrived at the same conclusion. No explanation can be offered at present of the difference of various solvents in their powers of conferring conductivity.

REFERENCES FOR CHAPTER XI.

The original memoirs mentioned in the references of the last chapter also deal with the problems just considered. The development of chemical theory from the results of the electronic hypothesis is also attempted in J. J. THOMSON'S *Electricity and Matter*, and in his more recent *Corpuscular Theory of Matter*; from these two volumes almost all of the ideas expressed in the foregoing chapter have been taken.

CHAPTER XII.

SPECTROSCOPIC EVIDENCE.

General considerations.

1. It is probable that in the future spectroscopy will be found to afford the most valuable evidence concerning the constitution of the atom. By examining the characteristic spectrum of a substance the periods of vibration of the electrons contained within its atom can be deduced, and it might seem that, with a complete knowledge of those periods at our disposal, the determination of the whole structure of the system which emits the vibrations could be deduced directly by suitable mathematical processes. But there are two insuperable difficulties in the way of such a direct deduction which illustrate so well the difference between the method by which scientific theories are often supposed to be established, and that by which discoveries are actually made, that attention may be drawn to them.

In the first place no amount of experiment alone can yield information sufficiently definite to form the foundation of a mathematical argument. Suppose that it is found that a certain quantity A is related to another x by the empirical formula

$$A = 1 + x + \frac{x^2}{2},$$

where x is a quantity so small that a term containing x^3 would be less than the error of experiment. The form of that term cannot be decided by experiment, and we can never tell whether the true relation between A and x is really that given by the empirical formula or whether it may not be given by such an infinite series as

$$A = 1 + x + \frac{x^2}{2\,!} + \frac{x^3}{3\,!} + \ldots + \frac{x^n}{n\,!} + \ldots$$

But the conclusions at which we should arrive would be completely different according to our choice between the alternatives.

In the second place our analytical methods are far from perfect: in fact, the bare mention of the word 'perfect' in connection with the development of those branches of mathematics which are applicable to physics suggests a condition too advanced. It is only in the very simplest cases that the periods of vibration of a given system can be deduced with any completeness, and it is still harder to deduce from the periods of vibrations the structure of a system of which nothing else is known. The immense mass of spectroscopic evidence which has accumulated can only be used by first making a guess at the structure of the atom, deducing the consequences of the guess and then comparing these with the results of observation. But the atom is a very complicated structure, and the difficulty of making a correct guess is enormous.

While, then, spectroscopic evidence is of the greatest value in gaining an insight into the structure of the atom, it cannot be made at present the basis for a complete determination: the results can only be used qualitatively.

Before proceeding to a discussion of such data as are available, it will be well to note that the periods of vibration of the electrons in our model atom are not so different from those actually observed as spectral lines as to prejudice the acceptance of the hypotheses on which that model is based. If m is the mass of an electron, fx the restoring force upon it when displaced a distance x, T, the time of vibration, is given by

$$T = 2\pi\sqrt{\frac{m}{f}}.$$

But in a simple atom consisting of one electron in a positive sphere of radius a, the restoring force is $\frac{\epsilon^2 x}{a^3}$: hence

$$T = 2\pi\sqrt{\frac{m}{\epsilon} \cdot \frac{1}{\epsilon} \cdot a^3}.$$

Putting $\epsilon/m = 5\cdot6 \times 10^7$, $\epsilon = 3\cdot4 \times 10^{-10}$, $a = 10^{-8}$ we have

$$T = 4\cdot6 \times 10^{-16}.$$

The observed periods lie between 3×10^{-15} and 10^{-15}, agreeing in order of magnitude with those calculated for the structure of our model atom.

Spectral 'series.'

2. The work of Balmer, Rydberg, Kayser and Runge and many others has led to the discovery of remarkable relations between the wave lengths (or the frequencies of vibration) of the light corresponding to the various lines in the spectrum of the same and of different elements. The complete description of the results attained would be beyond the scope of this volume: for an excellent account of the whole matter the reader may refer to Baly's *Spectroscopy*. Here we need only note the relations that have been found between the lines in the spectra of the alkali elements, the group for which the most complete results have been attained.

It is found that the lines in these spectra may be divided into certain groups or 'series,' such that the frequencies of the lines in any one series are connected by simple numerical relations. The frequency n of the lines in any one series can be found by substituting for the quantity m in the following formula the positive integers 1, 2, 3,...

$$n = n_0 - \frac{N_0}{(m+\mu)^2},$$

where n_0 and μ are constants characteristic of the series, n_0 being the frequency of the line corresponding to $m = \infty$, and N_0 is a constant which has the same value for all series and all elements. Further, all the lines in any one series have the same character: they are either all weak or all strong, all sharp or all diffuse, all suffer the same type of resolution in a magnetic field and all undergo the same shift with change of pressure.

The spectra of the alkali metals can be divided into eight such series, which are not wholly independent, for numerical relations can be established between the values of the constants of the different series. Thus the eight series can be divided into four pairs, such that the corresponding lines in each pair lie very near together, forming a doublet like the familiar D lines of sodium: for three of these pairs the difference in the frequencies of the members of a doublet is the same throughout the series, while for the fourth the frequency difference between the components of a doublet decreases regularly with the wave length of the doublet.

Other similar relations might be mentioned both in the spectra of the alkali metals and in those of other elements, but the results given are sufficient to indicate the nature of the deductions that may be made from them concerning the structure of the atom from which the light is emitted. It may be concluded that the vibrations of the electrons are not independent, but that each series represents the many modes of vibration of a single system—a statement of which the meaning may be made clear by an acoustical analogy. It is known that an organ pipe emits, besides its fundamental tone, a number of overtones of higher pitch. In the case of a pipe these overtones are all harmonics, that is, their frequencies are multiples of that of the fundamental: but in more complicated systems, such as a bell, the overtones are not necessarily harmonics. It is to be supposed that the lines in a single series are related to the line of lowest frequency, as the overtones of a bell are related to its fundamental tone.

Relations between the spectra of different elements.

3. The importance of this conclusion becomes clear when we consider the relations between the spectra of different elements. The spectra of all elements in the same chemical group are of essentially similar character: all have the same number of series, and the lines in corresponding series have the same character. The only difference between the spectra of such elements concerns the absolute frequencies of the lines, and even in this matter relations may be traced. Thus in the alkali group of elements the spectrum of the heavier atom is displaced towards the violet compared with that of the lighter element: the differences between the wave lengths of corresponding lines in corresponding spectra of two elements of the group has been shown to be proportional to the difference of the squares of the masses of the atoms: and the frequency difference between the components of a doublet, which is constant throughout the spectrum of any one element, is proportional to the square of the atomic mass when different elements are compared.

It seems clear, then, that the systems which give out the vibrations represented by the spectra of similar elements are essentially similar in structure, but are placed in somewhat different environment: we may liken the resemblance between the vibrating systems in the atoms of sodium and potassium to that between bells of the same shape but different sizes placed in media of different density. The arrangement of the electrons which emit the spectrum is the same in all the atoms of elements of the same group: the effect of the additional electrons which give to the heavier elements their greater mass is not to change that arrangement, but only to alter somewhat the external forces to which the system is subjected. Returning to the analogy used in Chapter XI, § 5, it seems permissible to conclude that the vibrating system consists of the innermost shell or shells of electrons, while the electrons which are added in order to form the more massive atoms are situated on the outer shells: the outer electrons do not emit vibrations of periods at all near to those of the inner electrons and only exert on them a perturbing influence, the exact nature of which cannot be deduced in the present state of our knowledge.

Since the arrangement of the valency electrons which determines the chemical properties of the atoms also remains unchanged in passing from one element to another of the same group, it is clear that the valency electrons must form part of the vibrating system. It would be interesting if we could identify the vibrating system with the valency electrons, but it is impossible that such complex series of vibrations should be emitted by such a small number of particles. On the other hand, we have seen that calculations based on the dispersion of transparent substances indicate that the number of electrons which have natural periods of the same order of magnitude as that of visible light is not greater than the number of valency electrons. Of course, it is not to be expected that the periods of vibration of the electrons in the atom when it forms part of a solid should be precisely the same as those when it is in the free condition in which it emits its luminous spectrum (see below), but it is a little difficult to explain the discrepancy satisfactorily. Perhaps the clue may be found in a suggestion due to J. J. Thomson, that some of the electrons which emit the spectrum are not those which form part of the permanent structure of the atom, but are collected temporarily round the atom from the free electrons which are always present when a substance is giving out its characteristic spectrum.

4. Further light is thrown on the nature of spectra by the mode of their origin. If we can determine the circumstances in which a system of electrons may be caused to vibrate, we have made a good step towards determining the nature of the system.

The origin of spectra.

It is probable that an element can only emit its characteristic spectrum when it is in the state of a gas, and its atoms are free from the influence of their neighbours. The spectrum of an element which is solid at ordinary temperatures is observed by placing the element in a non-luminous flame, or by examining the light from an electric spark or arc between terminals made of that element. It was supposed formerly that the emission of the spectrum was caused by the high temperature to which the element was raised by these means, but it seems probable that the only influence of temperature is to vaporise the substance and convert it into the only state in which it can emit its spectrum. The actual excitation of the spectrum must be due to other causes: for there is no evidence that a substance can be made to emit its spectrum by raising it to any attainable temperature in a closed vessel. The electrons in the atom of a gas do not share in the heat motion of the molecules and cannot be set into vibration by merely increasing the temperature.

The spectrum of a gas is usually obtained by passing through it an electric discharge at low pressures (about 1 mm. of mercury). In Chapter VIII the luminous phenomena accompanying the passage of such a discharge was described, and it was stated that the examination of the electric intensity at different points in the gas proved that the luminous regions are those in which ionisation and recombination are proceeding, and the non-luminous regions those in which the charged particles are moving without recombining with particles of the opposite sign. It appears then that a gas can only emit its spectrum when its atom is either being ionised or restored to electrical neutrality. This explanation will cover also the cases of the excitation of spectra in the flame, spark or arc. It is well known (see p. 217) that the gases in a flame are strongly ionised: it is probable that this ionisation is a bye product of the chemical actions which are proceeding in the flame, and are accompanied by the extraction of electrons from some atoms and their absorption by others. Both the spark and the arc are forms of the electric discharge in gases, and, like all forms of that discharge, are dependent on the ionisation of the atoms of the gas.

It is easy to see how the emission of the spectrum is produced by the ionisation of the atom. When the atom contains its full quota of electrons, these electrons will take up positions of equilibrium under the various forces acting upon them; if one of the electrons is extracted, the equilibrium is disturbed, since the force due to the extracted electron vanishes. In passing from the old to the new position of equilibrium the electrons vibrate about their final positions, and these vibrations cause the emission of light. In the same way, when the electron is restored to the atom, the equilibrium is upset once more by the action of the additional force due to that electron, and again vibrations are set up in the electrons as they change their positions. But it should be noted that on this explanation every atom should emit at least two distinct spectra; for the vibrations of the electrons when they are reaching the equilibrium position which obtains when the electron is not present, cannot be the same as the vibrations set up when they are reaching the positions of equilibrium when that electron is present; for the forces acting on the remaining electrons in the two cases differ by the force due to the electron which is removed or restored. If the atom be capable of losing two electrons, it should emit three spectra representing (1) the vibration of the neutral atom set up on the restoration of both electrons, (2) the vibrations of the atom without one electron set up either by the extraction of one electron from the neutral atom or the restoration of one electron to the atom which has lost two, (3) the vibrations set up in the atom without two electrons set up by the removal of the second electron. Now it is well known that most, if not all, elements, can emit two distinct kinds of spectra—line spectra and band spectra—and further that the line spectrum emitted by a substance varies with the conditions of excitation. Thus the spectrum of an element excited by the electric oscillating discharge is usually different according as the discharge circuit contains little or much self-induction, that is, according as the discharge is the passage of a small current for a long time or the passage of a greater current for a less time. There is also an important difference between the flame, spark and arc spectra. Stark has connected these differences with the electronic theory of the atom by a most illuminating suggestion. He has adduced evidence to show that the band spectrum represents the vibrations of the neutral atom, and the line spectrum those of the atom which has lost one or more electrons. It is probable, according to this view, that the difference between the various line spectra represents a difference in the number of electrons which the atom has lost; for it is easy to conceive that an atom which could only lose one electron in a flame could lose a greater number under the more intense action of the arc; but there is little direct evidence for this latter speculation. Stark's most conclusive argument in favour of his view is so ingenious that it deserves some notice here.

When the electric discharge passes through a gas, the atoms which have lost an electron carry a charge and move along the line of the current, but the neutral atoms do not move on the whole in one direction or the other. For reasons which will be explained in the last chapter the period of vibration of light emitted by a moving body along the direction of its motion is slightly different from that which it emits in a direction perpendicular to its motion; this latter period is the same as that of light emitted from the body at rest. Accordingly, if we examine the spectrum of a gas, first along the direction of the current and then in a direction perpendicular to the current, any given line in the spectrum due to the vibrations of a moving body will occupy a slightly different place in the two observations; but the position of a line due to the vibrations of a body at rest will be the same in the two cases. Stark found that the periods of vibration of the lines in a line spectrum of an element differ according as they are observed along the direction of the current or perpendicular to it by just the amount which would be expected if the particles emitting that spectrum were travelling with the velocity which the positive particles in the discharge are known to possess: but that the band spectrum was unaltered whatever the direction in which it was observed. From such observations, of which I have only given the principle and not the actual experimental arrangements, Stark arrived at the conclusion recorded at the end of the preceding paragraph. It should be mentioned that Stark's results have been subjected to some criticism, but it is unlikely that his main conclusions will be affected.

These conclusions are supported by some evidence of a general nature. It is well known that the elements which are peculiarly liable to emit band spectra are carbon and nitrogen: it is these elements which show by their chemical behaviour, as well as by their behaviour in electrolysis, that their atoms can only lose electrons and become charged with great difficulty. Again, we have little evidence on the subject of the emission spectra of chemical compounds, for chemical compounds are usually split up into their component elements by the processes necessary to excite a spectrum: but all the spectra of compounds which have been observed are band spectra, as might be expected on Stark's theory: for a compound is essentially an uncharged system, which would be broken up immediately if it became charged. It is not to be expected that the spectrum of a chemical compound should contain the line spectra of its component elements, for, though the internal structure of the electropositive atom is probably the same as when it is giving its line spectrum, the disturbing influence of the neighbouring atoms is so great that the periods of vibration are changed completely.

Some experiments by Lenard upon flame spectra seem to indicate that not only are the different spectra which an atom can emit due to differences in its state of ionisation, but that different series in the same spectrum proceed from atoms bearing different charges: but the evidence on this point is not always consistent.

Rayleigh has pointed out that the relation between the frequencies of the lines in the same series is not that which would be expected to hold between the frequencies of the various modes of vibration of a system excited by such shocks as we have imagined to be the cause of the emission of spectra: but that it resembles more nearly that of the frequencies of a system in steady motion. But the experimental evidence in favour of the conclusion that the emission of light is excited by the ionisation or recombination of the atom is so strong that it is better to accept it and leave the explanation of the discrepancy to a time when we have a more complete knowledge of the structure of the vibrating system in the atom.

Some recent researches by R. W. Wood seem likely to throw further light upon the origin of spectra. When light of a suitable wave length falls upon a 'fluorescent' substance, it excites the emission of light of a wave length different from that of the incident radiation: the period of the fluorescent light is usually greater than that of the exciting light, as in the well-known case of sulphate of quinine, which fluoresces visibly under the action of ultra-violet light. Wood has shown that as the wave length of the exciting light varies, the nature of the fluorescent light varies also. There seem to be groups of lines in the fluorescent spectrum, presumably all due to the vibrations of a single system, which can only be excited by radiation of a definite wave length: these groups show similar intimate connection in such phenomena as magnetic rotation. His work suggests the possibility that we may soon be able to analyse the atom experimentally: that instead of having to work with the complicated results of the vibrations of several systems, we may be able to sift out the radiation from any one system and study it at our leisure. There is little doubt that in the near future the study of spectra will lead to important information as to the structure of the atom: this brief reference is made merely to indicate one of the many promising lines of research that have been started already.

Continuous spectra. The laws of radiation.

5. The foregoing pages sum up almost all that is known on the subject of the relation of the characteristic spectra of elements to the constitution of their atoms. A few words must be said about continuous spectra, the medley of vibrations of all possible wave lengths which is emitted by solid bodies when they are heated.

It is probable that the continuous spectrum does not represent the vibrations of intra-atomic electrons at all, but is emitted by the 'free electrons' of Chapter VI, which determine the conductivity of metals. The electrons are in rapid motion, and, when at the end of each free flight they suffer a collision with an atom, they will emit pulses along the Faraday tubes attached to them in exactly the same way that the kathode rays in a vacuum tube emit pulses (Roentgen rays) when they strike a solid body. Since the velocity of the electrons is very much less than that of the kathode rays, the pulses emitted by them are very much broader: the velocity of the electrons is about 10^7 cm./sec., and, if they are stopped in a distance comparable with the radius of an atom (10^{-8} cm.), the breadth of the pulses emitted is about 10^{-5} cm. The time which such pulses take to pass over an electron in the atom of any transparent medium is sufficiently long to enable the motion of the electron to follow the variations of the electric intensity in the pulses. It can be shown that, consequently, these broad pulses, unlike the pulses of the Roentgen rays, will be refracted at the surface of such a medium and may be split up by a prism into periodic disturbances of all wave lengths, such as we find in the continuous spectrum. The exact processes by which the resolution of the pulses takes place would lead us too far from our immediate subject: it is sufficient to say that the view that the disturbance, which, when split up by a prism, gives rise to a continuous spectrum, is of the nature of an irregular pulse is in complete harmony with all modern views on radiation.

Lorentz and Thomson have investigated mathematically the nature of the radiation which is emitted by the collisions of the electrons in a conducting substance. The results that they attain are of great complexity, but seem to indicate that the part which is emitted in accordance with the accepted laws of thermal radiation from a hot body is only an extremely small fraction of the whole. An interesting question is raised as to what happens to the larger portion of the radiation, which must be either of immensely greater or immensely smaller wave length than that which is observed in measurements of the radiation from a 'black body.' An upheaval of our ideas concerning the laws of radiation and a proof of the insufficiency of all the thermodynamical arguments which have been advanced concerning them, appears to be one of the results which may possibly follow from the development of the electron theory.

REFERENCES FOR CHAPTER XII.

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The following papers discuss the evidence as to atomic structure which may be obtained from spectroscopic considerations:—

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JEANS, *Phil. Mag.* XI. pp. 604—607, 1906.

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CHAPTER XIII.

OTHER PROPERTIES OF THE ATOMS.

Mechanical properties.

1. Elements of the same group in the periodic classification resemble each other, not only in their chemical behaviour and in their spectra, but also in their mechanical properties: it is well known that such properties as rigidity, hardness, density, boiling and melting points, follow Mendeleef's law closely in their distribution among the elements. Since it has been concluded that the resemblance between the members of each of these groups consists in a similarity in the arrangement of some of the electrons in the atom, an endeavour must be made to explain mechanical properties also in terms of that arrangement.

But little progress has been made as yet in this direction, though it is generally held that cohesion, which is the basis of all these properties, represents the residual attraction between atoms which are on the whole electrically neutral. Unless the density of the electrification within the atom is exactly zero at every point, there will be some electric intensity at points outside the atom and close to it, though there will be no appreciable intensity at points the distance of which from the atom is large compared with the dimensions of that body. Thus, if we take the simple case of two atoms each consisting of a positive sphere containing two electrons, these atoms will attract each other if they are situated in the positions shown in Fig. 26 A while they will repel each other in the positions shown in Fig. 26 B. The arrangement of the electrons in the atom must influence profoundly the residual forces between neighbouring atoms in a solid or liquid, and so determine the mechanical properties of the body. Sutherland has attempted with some success to work out in this manner an electrical theory of rigidity and viscosity: though his results are interesting, the limited powers of mathematical analysis have forced him to confine his consideration to distributions of electrons so simple that they cannot be expected to throw much light on the relationships to be observed between the very complicated systems which make up chemical atoms.

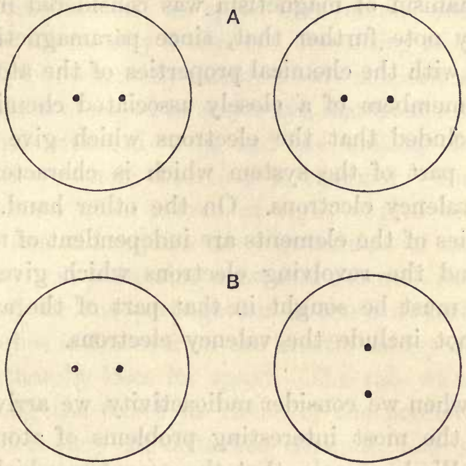

Fig. 26.

Magnetism and conductivity.

2. Of the remaining properties of the atoms of which some explanation must be attained before the electronic theory of matter can be held to be complete, the chief are conductivity, magnetism, radioactivity and gravitation. There is little to add to what has been said already as to conductivity: our knowledge of the circumstances which determine the production of free electrons and of their collisions with the atoms is not complete and must await development by further research. We may note that it has become clear why the chemical compounds of an element which is a conductor in the solid state are not also conductors. The electrons lost by the electropositive element, which act as 'free electrons' in the pure metal, are retained by the electronegative element, and cease to be available for the purposes of conduction.

The mechanism of magnetism was considered in Chapter V: we need only note further that, since paramagnetism seems to be connected with the chemical properties of the atom, appearing only in the members of a closely associated chemical group, it must be concluded that the electrons which give rise to that property are part of the system which is characteristic of that group—the valency electrons. On the other hand, the diamagnetic properties of the elements are independent of their chemical properties, and the revolving electrons which give rise to that phenomenon must be sought in that part of the atomic system which does not include the valency electrons.

Radioactivity.

3. But, when we consider radioactivity, we arrive at some of the most interesting problems of atomic structure. We have seen that the processes which give rise to the phenomena of radioactivity must take place in the interior of the atom and must be determined by the internal structure of the atom: but it appears at once that these processes must be of a very different nature to any intra-atomic processes which we have considered so far. All other changes of this nature are completed in a very short period of time: the period of vibration of the light emitted by the natural vibrations in the atom is only about 10^{-15} second and the total time required for the atom to regain equilibrium after it has been disturbed by ionisation or recombination is probably less than 10^{-9} second. On the other hand, radioactive processes require a time of an altogether different order for their completion: the life of a radioactive atom may vary from three seconds to 10,000 years according to our present knowledge. The time scale of radioactive processes is vastly greater than that of other intra-atomic processes, and to explain them some gradual change in the atom must be found which produces no effect during billions of free vibrations of the electrons, but eventually leads to a convulsion.

The existence of such a change was suggested by Larmor in 1895, several years before Rutherford and Soddy disclosed the true nature of radioactivity: it might almost be said that radioactivity was explained before it was discovered.

If an electron is revolving in a closed orbit, it is constantly subject to an acceleration: its speed may remain unchanged, but the direction of its velocity is altered. According to the principles of Chapter IX, § 5, when any charged body is accelerated (that is, when its motion is changed in any way), disturbances are emitted along the tubes attached to it, and the motion of these tubes at right angles to their direction is changed. But, since the tubes possess mass when moved at right angles to themselves, these disturbances will require the expenditure of energy, for the energy of any system possessing mass changes with its speed. The energy required for the propagation of these disturbances along the Faraday tubes is drawn from the kinetic energy of the moving charge, which thereby loses its speed. The rate at which energy is radiated along the tubes in virtue of the acceleration of the charges to which they are attached depends, not only on that acceleration, but also on the conditions in which the charge is moving. Thus, if the charge is revolving in an orbit of a radius of 10^{-8} cm. with a velocity of 10^8 cms. per sec. (values which probably represent roughly the magnitude of these quantities in an electronic orbit), it will lose half its kinetic energy by radiation in 10^{-12} sec. But, if there are other charges revolving with the same period in the same orbit, the rate at which energy is radiated will be very much less, for the disturbances set up in any tube by the acceleration of one charge will tend to be counteracted by those set up by the others. Thus, if there are two charges revolving in the orbit with a velocity of 3×10^8 cm./sec., the rate of radiation of energy will be only $\frac{1}{500}$ of that due to a single electron with the same velocity, if there are six it will only be 10^{-16} of that due to a single electron.

It is clear then that, if the magnetic properties of an atom are due to the presence in it of electrons revolving in closed orbits, these electrons must be losing energy and losing speed, and that the rate at which energy is lost will be greater or less according as there are fewer or more electrons revolving in each of the orbits. However great the number of electrons in a circuit (so long as it is finite) there will be some loss of energy and, consequently, a gradual retardation of the revolution.

Now among the forces which determine the arrangement of the electrons in the atom are the 'centrifugal forces' arising from the motion of the electrons in closed orbits. As the speed of the electrons decreases, the centrifugal forces diminish and changes in the arrangement follow. But, if we turn back to our original analogy of Chapter XI, § 5, we shall see that it is probable that this arrangement can only change continuously over a very limited range, and that any further change must cause a very important alteration in the distribution of the electrons.

Four electrons in a positive sphere can form a single stable ring, but, if there are five electrons, the arrangement in a single ring is no longer stable, if the electrons are at rest: the system will break up into two rings, an outer with four electrons and an inner with one. However, if the electrons are revolving in closed orbits with sufficient speed, the centrifugal forces make the single ring of five stable. Suppose, then, that we have such a single ring of five revolving electrons and that the speed of revolution is gradually reduced. At first the only effect of the reduction of speed is to make the radius of the single ring a little smaller without impairing its stability. But, if the decrease of speed is continued, a stage will be reached when the single ring is no longer stable, and the smallest disturbance will cause the single ring to break into the two rings containing four and one electron respectively.

Though we cannot carry out the precise calculation, there is no doubt that there must be similar changes in the stability of the more complicated system of revolving electrons in the atom when the speed of revolution is reduced. The speed can be reduced over a limited range without causing any other change than a decrease of the dimensions of the orbit: but, once the limits of that range are passed, a further decrease of speed must involve a fundamental alteration in the distribution of the electrons in the atom: the single ring will be broken up in favour of some more stable system. Now, by a well-known principle of dynamics, the passage from a less stable to a more stable system is the passage from a system with greater to a system with less potential energy: and the difference in the potential energies of the two systems appears in the form of kinetic energy of some part of the system. Hence the breaking up of the arrangement that has become unstable by the reduction of the centrifugal forces will be accompanied by a large increase in the kinetic energy of some part of the system. The speed which this part may attain may be so great as to carry it clear of the attractive forces in the atom: it will break off from the atom and pursue a separate existence. The view is now generally accepted that it is this 'explosion,' due to the conversion of a certain quantity of potential energy into kinetic energy in the passage from a less stable to a more stable configuration, that is detected in our observations as radioactivity: the ejected α and β particles are the portions of the system which have received the large increase in kinetic energy sufficient to carry them clear of the attraction of the rest of the atom.

The explanation of radioactivity that has just been offered alone seems capable of accounting, even qualitatively, for all the features of that remarkable phenomenon. The most striking of those features, and that which is the stumbling-block of all alternative theories proposed hitherto, is the independence of the activity of the radioactive element of the conditions in which it is placed—of the temperature, of the state of chemical combination and of all other changes physical and chemical which influence so profoundly all the other known changes in the atom. The rate of radiation of energy from an electronic orbit would not be permanently changed by the action of any external forces such as are supposed to be associated with magnetic changes or the presence of atoms in chemical combination with the radioactive atom: but it is possible that in the first moment of action they might cause the breaking up of atoms that were already very near the condition of instability. Observations on this point are extremely difficult: the only effect that is to be expected is a slight momentary increase in the emission of rays when the magnetic field is turned on, or when the atoms are entering into chemical combination[1]: the increase would be small, and I do not think that any experiments of sufficient accuracy have been made upon elements known to be radioactive, to determine with certainty whether such a change occurs.

---

[1] Such a temporary increase in the emission of *rays* must not be confused with the formation of *ions*, which is known to accompany many, if not all, chemical actions.

The second remarkable feature of radioactivity, an explanation of which must be provided by any satisfactory theory, is the liberation of an amount of energy vastly in excess of that given off in the most violent chemical reaction. It has been noted already (p. 249) that there is no difficulty in attributing to the atom a sufficiently large store of energy, if the number of electrons in the atom be great enough.

The third characteristic feature of radioactivity, the production of new elements, which was used in Chapter IX for the purpose of defining a radioactive process, needs no further explanation on any theory which holds that the α and β particles emitted are fragments broken off from the radioactive atom. No reason can be offered for the remarkable identity in the nature of the α particles given off by the different radioactive elements, until more detailed knowledge of the internal arrangement of the atom is available. But it seems clear that the positively charged portion of the atom must be considered to have a more complex structure than the simple sphere of uniform density which has formed the basis of our argument.

On this theory then, radioactivity is to be considered as another aspect of diamagnetism: both properties are due to the same structure in the atom, the electrons revolving in nearly closed orbits. It may be noted that, in accordance with this conclusion, radioactivity, like diamagnetism, is not associated with any special chemical properties: members of at least four of the chemical groups have been shown to be radioactive in various degrees, and it is possible, as pointed out on p. 213, that all elements are radioactive. If the bolder speculations advanced be correct, the electronic orbits which become unstable must be those which lie on the outer shells of the structure of the atom; for it is these orbits which give rise to the phenomena of diamagnetism (see § 2).

An interesting question arises as to what happens to the energy which is radiated by the atoms in the process of reaching instability, but its discussion, like that of the similar problem raised in connection with the heat radiation from solid bodies, would take us too far from our immediate purpose and would lead to no definite conclusion.

Gravitation.

4. There remains for consideration the one universal property of all material bodies—gravitation. Here the electron theory fails as completely as every other theory of matter has failed: our knowledge of gravitation is scarcely greater than it was 200 years ago. Several attempts have been made at various times to connect gravitational forces with electrical processes: of these the best known, proposed in slightly different forms by many speculators, is the suggestion that the repulsion between two equal electrical charges of the same sign is not quite equal to, but a little less than the attraction between equal charges of opposite signs: a neutral body being supposed to contain equal charges of opposite signs will thus exert a slight residual attraction on a similar neutral body. For the development of this and other hypotheses the reader may refer to the summary mentioned in the references at the end of this chapter.

But to this theory and to every theory which would connect gravitational with electrical forces there seems a fatal objection: all electrical actions are propagated with a velocity not greater than that of light, while gravitational actions are propagated with a velocity which may be infinite as far as we know, but is at any rate many million times greater than that of light. If the attraction between the sun and a planet depended not on the instantaneous positions of those bodies, but upon their positions at a slightly previous period, the orbit of the planet would not be the closed curve in which the observations of centuries have shown no change, but a spiral along which the planet would travel outward from the sun. If the velocity of the propagation of gravitational actions were only a million times greater than that of light a change in the orbits of the planets would have made itself perceptible to the observations of astronomers.

There is only one way in which I can imagine that the theory which has been developed in this volume can throw any light upon the problem of gravitation: if we could determine whether an electron has any weight, and, if so, whether the ratio of its weight to its mass is the same as that observed for atoms, it cannot be doubted that the experiment, whichever result it gave, would throw much light on this obscure department of physics. But the difficulties of such a determination seem, to me at least, insuperable. If it were not for the earth's magnetic field the observation might be carried out: owing to the action of that field it seems impossible to obtain an electron moving in a straight line over a path of sufficient length to allow the gravitational forces, if they exist, to produce an appreciable effect on its motion.

Summary.

5. Here ends our discussion of the bearing of the electron theory on the properties of material bodies: if the reader has followed the arguments advanced, there will be no need to emphasize the importance of that theory. If it is the purpose of science to correlate diverse phenomena, the development within the last twenty-five years of the original hypothesis of Lorentz constitutes the most important advance that has been made in the history of the inorganic sciences. The barriers which had been erected between the various branches of science by the writers of text-books have been broken down: optics, heat and mechanics seem likely to become in the near future special departments of the all-embracing science of electricity: the gulf which separated the two halves of the study of matter, physics and chemistry, has been bridged. Nothing illustrates the change that has taken place better than the difficulties of classification which meet the editor of a modern encyclopaedia of physics: only a reference to the index can tell an inquirer in which volume of the new *Handbuch* he must seek for an account of recent physical work.

On the other hand it is hardly necessary to point out that the theory is far from complete. Many of the speculations advanced must be abandoned and many new hypotheses framed in the light of further knowledge: but it cannot be doubted that the fundamental results which have been attained, the existence of electrons of constant charge and very small mass, which form part of the atoms and are the chief agents in physical and chemical changes, will remain as part of the fabric of the science. Meanwhile, to exaggerate or to minimise the difficulties that are encountered would be to prejudice further progress; and of the two faults the former is the greater. If every theory had to be abandoned which did not provide immediately a satisfactory explanation of all the experimental data, we should have never got beyond the physics of the middle ages: nothing is more characteristic of the leaders of science than their power of distinguishing intuitively between the discrepancies which are destructive of a hypothesis and those which will be removed by further investigation.

But, when all the difficulties are removed and all the questions that can be asked at present concerning the correlation of phenomena have been answered, the work of the electronic theory will not be ended. It is the function of a theory to raise problems as well as to solve them, and the investigations which have been described suggest doubts concerning the fundamental electromagnetic laws which could not have been formulated a short time ago. To a brief consideration of these new problems the last chapter of the volume will be devoted.

REFERENCES FOR CHAPTER XIII.

The most notable attempt to explain the mechanical properties of matter by the arrangement of electrons is to be found in the papers of SUTHERLAND, e.g. *Phil. Mag.* IV. 625–645, 1902 and VII. pp. 417–144, 1904.

LARMOR'S explanation of radioactive instability is to be found in *Phil. Trans.* A, 1895, pp. 740—742.

An excellent summary of the theory of radioactivity is given by RUTHERFORD in the *Jahrbuch d. Rad. u. Elek.* I. pp. 103–127, 1904.

An account of the theories of gravitation (electrical and non-electrical) which have been put forward is given by ZENNECK in the *Encyc. d. Math. Wiss.* Vol. V. pp. 25 et seq.

The papers by THOMSON cited already and his book, *Electricity and Matter*, contain discussions of most of the problems raised in Chapters XII., XIII. and XIV.

CHAPTER XIV.

THE LAWS OF ELECTROMAGNETISM.

'The aether.'

1. In modern discussions of the fundamental laws of electromagnetism it is customary to employ frequently the conception of the 'aether.' Hitherto we have been able almost completely to avoid reference to that conception and thereby to leave on one side all the difficulties which attend its use. The dictionary may be searched in vain for a word which has given rise to more confusions and misunderstandings: the amazing pronouncements about the 'aether' which have been made by many philosophers are rivalled by the statements which are to be found in the writings of men of science of the highest repute. There seems to be a strange attractiveness about the word: a student never feels so important as when he is able to introduce it into his examination papers: it is to be found in monographs on the geometrical fourth dimension and in treatises on the more obscure issues of theology. But, though it is desirable that the word should be excluded from physics, or at least its use confined to those who display a proper sense of its meaning, it would be pedantic and misleading to omit it from a discussion of the problems of this chapter. But the reader should bear in mind throughout the first paragraphs that all conclusions are to be subjected to a subsequent revision.

I am not acquainted with any authoritative formal definition of the term 'aether,' but we shall not diverge from current usage, if we take it to mean a medium in which electromagnetic actions take place in the absence of substances generally recognized as matter. Of these actions the vibrations which constitute light are the most important: studies of the properties of the aether usually resolve themselves into optical investigations. So long as the bodies of which the optical properties are examined are at rest, the phenomena which are observed can be interpreted according to the principles of Chapters II. and III.: the difficulties which we are about to consider arise in the first instance from the study of light emitted or received by bodies in motion. It will be best to start examination of the limits of our knowledge and of our ignorance concerning the aether from a description of the optical properties of moving bodies. The experimental facts are described admirably in many modern text-books, such as R. W. Wood's *Physical Optics,* but in order to put the problem in the form which will be most convenient for our purpose, it will be advisable to recapitulate these facts briefly together with the explanation that is generally offered of them.

2. There are two well-known effects of the motion of material bodies upon optical phenomena: Aberration and the Doppler Effect.

Aberration.

Aberration is of great importance in astronomy; it consists of an apparent change in the position of a star with the motion of the earth in its orbit round the sun. The nature of this change is best described by an analogy due to Wood. Suppose that a ship is pierced by a shot fired from a gun on the shore. Then, if the ship is at rest, a line drawn through the holes where the shot leaves and enters the ship will, if prolonged, pass through the gun from which the shot is fired: the direction of the gun is given by the line joining the two holes. (It is assumed that the trajectory of the shot is a straight line.) But, if the ship is in motion, it will have moved a certain distance in the interval that elapses between the entrance of the shot through one side and its exit through the opposite side, and the exit hole will lie a corresponding distance behind the position which it would have occupied, if the ship had been at rest and the shot had entered at the place at which it actually enters. A line drawn through the exit and entrance holes will not now point to the gun, but to a point on that side of it towards which the ship is moving. Accordingly, if the direction of the gun be determined by the line drawn through the two shot holes, that direction will appear to be changed by reason of the motion of the ship.

Another excellent example of aberration may be seen by travelling in a train during a heavy shower of rain. When the train is at rest, the direction in which the rain is falling, as shown by the direction of the splashes which it makes on the windows, is almost or quite vertical: but, when the train is moving, the splashes on the window become nearly horizontal, and the rain appears to be coming from the direction of the engine.

Returning to the first analogy, we may argue similarly that, if the earth were at rest, the direction of a star, the image of which lies on the cross wires of a telescope, would be indicated accurately by the line drawn through those cross wires and the centre of the object lens: but, if the earth is in motion carrying the telescope with it, the instrument will move a short distance while the light is passing from the object glass to the cross wires, and the star will appear to be displaced in the direction to which the earth is moving; by turning the telescope through a small angle, known as the 'angle of aberration,' the star image can be made to coincide once more with the cross wires. If the star be observed at opposite seasons of the year when the earth is moving with equal speeds in opposite directions, half the angular difference between the apparent positions of the star will give the angle of aberration.

The value of this angle in terms of the velocity of the earth may be ascertained easily. Let OA (Fig. 27) represent the axis

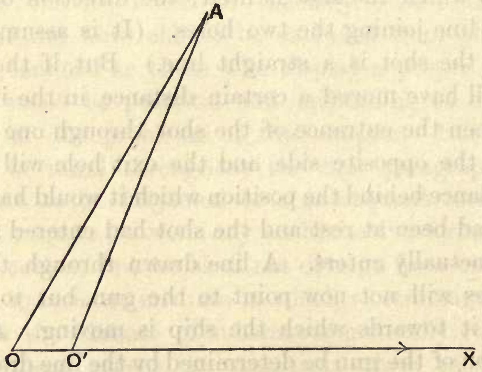

Fig. 27.

of the telescope, A being the centre of the object glass, O the position of the cross wires on which the star image lies when the telescope is at rest: let OX be the direction of the motion of the earth and u its magnitude. Let the angle $AOX = \chi$. Then, when the telescope moves, the image will lie at some point O', such that the time required by the light to traverse the distance AO' is equal to the time required by the earth to move the distance OO'.

Therefore
$$\frac{O'O}{AO'} = \frac{u}{V}.$$

But
$$\frac{O'O}{AO'} = \frac{\sin O'AO}{\sin AOO'} = \frac{\sin O'AO}{\sin \chi}.$$

Now $O'AO$ is the angle of aberration, ϕ.

Hence
$$\sin \phi = \frac{u}{V} \sin \chi \quad \text{.....................(1).}$$

The quantity $\frac{u}{V}$ is called the 'constant of aberration.' The velocity u of the earth in its orbit is about 3×10^6 cms. per sec., so that $\frac{u}{V} = 10^{-4}$, a value which agrees with that found by observation.

It should be noted that ϕ is independent of the length of the telescope.

The Doppler effect.

3. The other effect of the motion of material bodies on optical phenomena is a change in the frequency of the light vibrations.

Suppose that an endless train of waves of frequency n is travelling with a velocity v from a stationary source towards a stationary observer: then a length of the train v containing n waves will pass him every second. If he moves towards the source with a velocity v', a length of the train $v + v'$ will pass him in one second containing $n \cdot \frac{v+v'}{v}$ waves. But the frequency of the waves is the number of waves which pass the observer in unit time: hence in virtue of his motion the frequency of the vibrations changes from n to $n \cdot \frac{v+v'}{v}$. Similarly, if he moves away from the source with the same velocity the frequency will change from n to $n\frac{v-v'}{v}$. This change of frequency is known as the *Doppler effect.*

The most familiar illustration of this effect, that which is given in all text-books, concerns waves of sound; it is pointed out that the note of the whistle of a locomotive appears to change as the locomotive passes the observer and the direction of its velocity relative to the observer is changed. The principle is also applied largely in modern astronomy for determining the velocity of the stars along the line joining them to the earth by observing the displacement of some well-marked line in the spectrum of the light emitted by the star from the position which it would occupy if the light came from a stationary source. On p. 273 an application of exactly the same principle to the case of particles moving in a vacuum tube was mentioned. However, though the Doppler principle is of great use in physical research, for the questions that we are about to consider it is of less importance than aberration.

Airy's experiment.

4. Aberration seems perfectly simple until further inquiries are made about it. The first difficulty was raised by an experiment performed by Airy. Airy argued that, since the angle of aberration depends on the velocity of the light, that angle should be changed if the light travels through a medium in which its velocity is different from that in empty space: if V' is the velocity of light in water the angle of aberration should be given by $\sin\phi=\frac{u}{V'}\sin\theta$. Accordingly he filled a telescope tube with water and observed the angle of aberration of the same star, firstly, when the light travelled within the telescope through air and, secondly, when it travelled in water. To his surprise he found that the angle of aberration was the same in the two cases: it was independent of the medium filling the tube. The only explanation that he could offer was that the water 'carries the aether with it in its motion,' so that the velocity u of § 2 is reduced by the presence of the water in the same proportion as V. It is necessary that the water should not carry

the aether along with its own velocity, for then there would be no aberration at all: it must communicate to the aether some fraction θ of its own velocity, so that the relative velocity of the telescope and the aether is $(1-\theta)u$. Let us investigate what must be the value of θ in order that the angle of aberration should be unchanged by the presence of the water.

The calculations will be simplified and the results unaffected, if we substitute for the water in the telescope tube a slab of water with parallel sides in the plane of which the direction of motion of the earth lies. Let the light be travelling in the direction BA and the earth in the direction AX. When the water is at rest the ray BA is refracted along the path ACQ and emerges parallel to its original direction: but when the water is moving it will travel along a different path $AC'Q'$. We have to find the value of θ such that the angle between CQ and $C'Q'$ is equal to the angle of aberration calculated from (1).

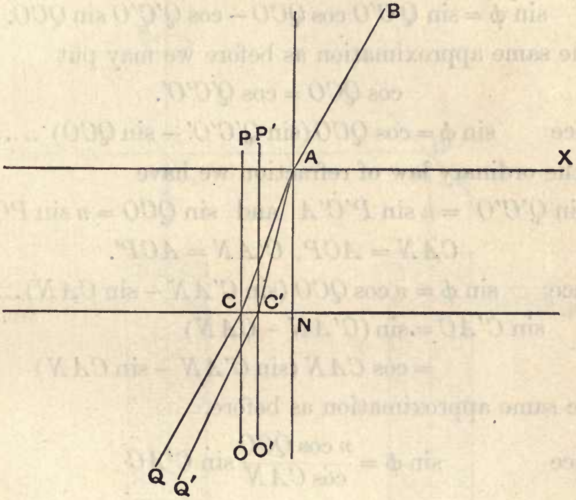

Fig. 28.

Let n be the refractive index of the water: then the velocity of the light in the water is V/n: u is the velocity of the earth and θu the velocity with which the aether is carried along by the water. Inside the water the light is travelling with the velocity V/n in the direction AC relative to the water and with the velocity

$(1-\theta)u$ in the direction CC' relative to the water. Hence, since AC' is the resultant of AC and CC' we must have

$$\frac{CC'}{AC}=\frac{\sin C'AC}{\sin AC'N}=\frac{(1-\theta)\,un}{V}\quad\ldots\ldots\ldots\ldots\ldots(2).$$

The angle of aberration is very small—so small that any quantity which depends upon its square is too small to be detected by experiment and may be neglected. Accordingly in (2) we may substitute ACN for $AC'N$ and write

$$\frac{\sin C'AC}{\sin ACN}=\frac{(1-\theta)\,un}{V}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Since the angle of aberration ϕ is $Q'C'O'-QCO$,

$$\sin\phi=\sin Q'C'O\cos QCO-\cos Q'C'O\sin QCO.$$

With the same approximation as before we may put

$$\cos QCO=\cos Q'C'O'.$$

Hence $$\sin\phi=\cos QCO\,(\sin Q'C'O'-\sin QCO)\quad\ldots\ldots\ldots(4).$$

By the ordinary law of refraction we have

$$\sin Q'C'O'=n\sin P'C'A\quad\text{and}\quad\sin QCO=n\sin PCA,$$

and $$CAN=ACP,\quad C'AN=ACP'.$$

Hence $$\sin\phi=n\cos QCO\,(\sin C'AN-\sin CAN)\ldots\ldots.(5).$$

But $$\sin C'AC=\sin(C'AN-CAN)$$
$$=\cos CAN\,(\sin C'AN-\sin CAN)$$

with the same approximation as before.

Hence $$\sin\phi=\frac{n\cos QCO}{\cos CAN}\sin C'AC$$

$$=\frac{n\sin BAX}{\sin ACN}\sin C'AC$$

by (3) $$=\frac{(1-\theta)\,un^2}{V}\sin BAX\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

But from (1) we have $\sin\phi=\frac{u}{V}\sin\chi$, and here $\chi=BAX$.

Hence, if the angle of aberration is uninfluenced by the pressure of the water,

$$\frac{(1-\theta)\,un^2}{V}=\frac{u}{V}$$

or $$\theta=\left(1-\frac{1}{n^2}\right)\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7).$$

Hence a medium of refractive index n communicates to the aether a fraction of its velocity $\left(1-\frac{1}{n^2}\right)$. If $n=1$, the expression becomes 0.

Fizeau's experiment.

5. Fizeau tried to test the correctness of Airy's hypothesis by direct experiment (Fig. 29). A beam of parallel light was split into two parts by the semi-transparent mirror A: one part which is reflected follows the course $ABCDAPO$, being reflected by mirrors placed at suitable angles, and is brought

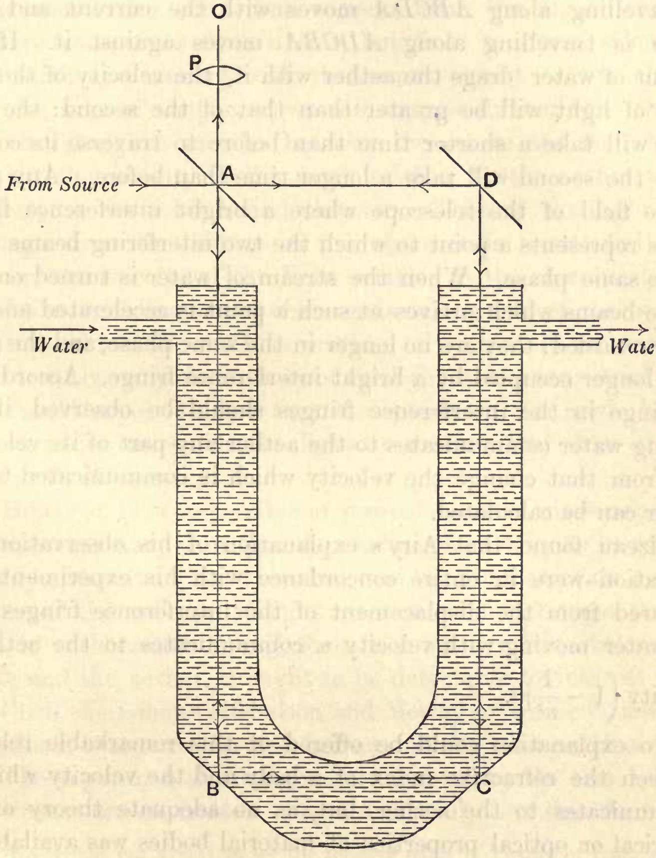

Fig. 29.

to a focus at *O* by the lens *P*. The other part of the light is transmitted in the first instance by *A*, follows the course *ADCBAPO* and is also brought to a focus at *O*. At this point the two rejoined beams will interfere in just the same way as the separated and reunited beams in the well-known experiment with Fresnel's mirrors: if the region round *O* be observed with the eye-piece of the telescope of which *P* is the object glass, a series of interference fringes will be seen crossing the field.

In the path of these two beams was placed a **U**-shaped vessel through which a rapid current of water could be passed from one end of the **U** to the other: when this current flows, the beam which is travelling along *ABCDA* moves with the current and that which is travelling along *ADCBA* moves against it. If the current of water 'drags the aether with it,' the velocity of the first beam of light will be greater than that of the second: the first beam will take a shorter time than before to traverse its course, while the second will take a longer time than before. Any point in the field of the telescope where a bright interference fringe occurs represents a point to which the two interfering beams come in the same phase. When the stream of water is turned on, one of the beams which arrives at such a point is accelerated and the other retarded: they are no longer in the same phase, and the point is no longer occupied by a bright interference fringe. Accordingly a change in the interference fringes should be observed, if the moving water communicates to the aether any part of its velocity: and from that change the velocity which is communicated to the aether can be calculated.

Fizeau found that Airy's explanation of his observations on aberration were in entire concordance with his experiments: it appeared from the displacement of the interference fringes that the water moving with velocity u communicates to the aether a velocity $\left(1 - \frac{1}{n^2}\right)u$.

No explanation could be offered of this remarkable relation between the refractive index of a body and the velocity which it communicates to the aether, but, as no adequate theory of the electrical or optical properties of material bodies was available, it appeared that such an explanation might well be attained by further research. But a difficulty, to the consideration of which we must now proceed, arose to throw doubt upon the whole theory of aberration.

The Michelson-Morley experiment.

6. We have seen that a perfectly consistent explanation of aberration can be attained, if we imagine that the aether, in regions where there is no matter, or only matter at rest, is devoid of motion, and that the change of direction of a star as seen from the earth is due to the motion of the earth relative to the aether; and that, further, the velocity of the earth relative to the aether is its velocity in its orbit round the sun. But suppose that the light which we observe comes, not from a star, but from a source situated on the earth and moving with the same velocity as the observer. For the sake of simplicity, we will suppose further that the light travels through a vacuum between the source and the observer, so that the velocity of the earth relative to the aether is u, its velocity in its orbit round the sun. Then if the light in its passage from the source of the observer is 'contained in the aether,' the velocity of the light relative to the observer is $V-u$ or $V+u$, according as the observer is on the side of the source to or from which the earth is moving. If we could measure with sufficient accuracy the velocity of the light from a terrestrial source, we should expect to find that it depends on the angle between the direction in which the light is moving and that in which the earth is moving[1].

However, it is impossible at present to make measurements of sufficient accuracy directly: the change of velocity to be expected is only one part in ten thousand, and the most accurate measurements of the velocity of light are not reliable to less than one part in a thousand: but, if there is relative motion between the earth and the aether, it ought to be detectable by indirect means, of which the famous Michelson and Morley experiment, now to be described, is at once the best known and the most conclusive.

The experimental arrangements adopted by the two American physicists are indicated diagrammatically in the figure. A

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[1] We should not expect to see the Doppler effect of change of frequency: see Poynting and Thomson's *Sound*, p. 49.

parallel beam of light from a source S is split into two halves by a semi-transparent mirror A: the reflected portion is reflected again at B, transmitted by A and brought to a focus by the telescope D: the transmitted portion is reflected at C, reflected again at A and brought to the same focus by D. The lines AB and AC are equal and are at right angles. Interference fringes will be formed in the field of the observing telescope as in the Fizeau experiment.

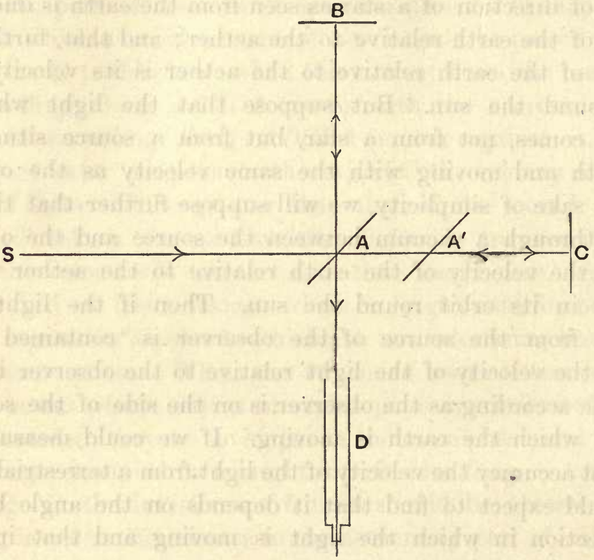

Fig. 30.

Now suppose that the whole apparatus is carried by the orbital motion of the earth in the direction AC with a velocity u. If T is the time required for light to pass from A to C, the mirror at C will have moved a distance uT from the position which it occupied, when the light left A, before the light strikes it: consequently the whole distance travelled by the light is

$$VT = l + uT,$$

where l is the distance AC. Similarly the time T' required for the light to travel back from C to A is given by

$$VT' = l - uT'.$$

Hence the whole distance travelled by that part of the light which proceeds along AC is

$$V(T+T')=2l\frac{V^2}{V^2-u^2}=2l\left(1-\frac{u^2}{V^2}\right)^{-1} \quad\text{.........(8).}$$

Now consider the distance travelled by the light which leaves A for B. If it falls perpendicularly on B and returns along its course, it will not strike A again, for A has moved on during its passage. In order to reach B on its return it must travel along $AB'A'$, where AA' is the distance uT, through which the earth travels during the passage of the light from A back to A. But during this time the light has travelled the distance

$$ABA'=VT_1.$$

Hence $$(\tfrac{1}{2}VT_1)^2=l^2+(\tfrac{1}{2}uT_1)^2,$$

or $$VT_1=2l\left(1-\frac{u^2}{V^2}\right)^{-\frac{1}{2}}.$$

Since $\frac{u}{V}$ is small, we may neglect terms in $\frac{u^4}{V^4}$ as unimportant, and write $$V(T+T')-VT_1=l\frac{u^2}{V^2}.$$

When the apparatus is at rest, the two parts of the beam traverse the same distance before their reunion: but when it is in motion, a difference of path $l\frac{u^2}{V^2}$ is introduced between the two beams, and the interference fringes which they form in the field of the telescope will be displaced. If the apparatus is made to travel in the direction AB, the difference of path will have the opposite sign and the fringes will be displaced in the opposite direction.

In the actual experiment the whole apparatus was mounted on a slab of stone floating in mercury so that it could be rotated without inducing strains. An observer with his eye at the telescope followed the stone in its rotation and watched for a displacement of the fringes: but no such displacement was observed. From calculations of the sensitiveness of their apparatus Michelson and Morley concluded that the velocity of their apparatus, or of the earth, relative to the aether, is certainly less than one sixth of the whole velocity of the earth in its orbit.

We seem to have arrived at mutually inconsistent conclusions from irrefutable experiments. From observations on aberration it must be concluded that the earth moves relatively to the aether with the velocity of its orbital motion: from the Michelson and Morley experiment it must be concluded that the earth is at rest relative to the aether, or at least moves relative to it with a velocity much smaller than that deduced from aberrational effects.

As no flaw can be detected in the arrangement of the experiments or in the logical processes of the argument, there is only one way out of the dilemma: we must deny some assumption that has been made in the process of the argument. Any one of the many assumptions made could be chosen and denied, but of course the choice will be determined by its convenience for the general purposes of the science. At the present time there is only one assumption the denial of which is generally regarded with favour.

The Lorentz-Fitzgerald hypothesis.

7. In the calculations of the last paragraph it was assumed that the dimensions of the apparatus are the same when the structure is at rest and when it is in motion—that the quantity l does not depend on the velocity. Soon after the publication of the research which has just been sketched, Lorentz and Fitzgerald suggested almost simultaneously that this assumption was incorrect: that the dimensions of the apparatus in the direction of motion were diminished relatively to those in the perpendicular direction by just such an amount as to compensate the increase of path caused by the relative motion of the apparatus and the aether. The contraction necessary is extremely small and could not be detected by any other means than that of the research concerned: when AC coincides with the direction of motion, l must be decreased in the ratio

$$1 : 1 - \frac{u^2}{2V^2} \quad \text{or} \quad 1 : 1 - \tfrac{1}{2} \times 10^{-8}:$$

such a contraction would be equivalent to a decrease of 2½ inches in the diameter of the earth (8000 miles). On the other hand the dimensions of the apparatus perpendicular to the direction of motion must remain unchanged.

The hypothesis doubtless sounds somewhat speculative in the form in which it has been stated, but a closer consideration shows that it has a theoretical basis, which was pointed out by Larmor.

On p. 200 it was shown that the Faraday tubes attached to a charged sphere are displaced when the body is set in motion: instead of being equally distributed in all directions round the charge, they tend to congregate in the plane at right angles to the direction of motion. The number of tubes passing through any given small volume, fixed relatively to the charged body, will be diminished by the motion, if the volume lies on the line of motion, and increased if it lies on a line perpendicular to that direction: in order to find a volume which contains the same number of tubes, when the charge is in motion, as the first volume contains, when the charge is at rest, a volume *on the line of motion* must be taken, which is nearer to the charge than the first volume, and a volume on the *line perpendicular to the motion* must be taken which is further from the charge than the first volume. A strict investigation into the relation between the positions of the first and the second volumes is beyond our scope, but one result may be stated simply thus:

Suppose that a charged sphere is moving in the direction of the axis of z with a velocity w. When the sphere is at rest, the polarisation at any point, the coordinates of which, relative to axes of which the centre of the charged sphere is the origin, are x, y, z, have a certain value N: when the sphere is in motion, the polarisation has the same value N at a point A', the coordinates of which, relative to axes, which still have their origin at the centre of the sphere, are x, y, $\dfrac{z}{\left(1+\dfrac{w^2}{V^2}\right)^{\frac{1}{2}}}$. Or, in other words, the electric and magnetic fields (which are proportional at any point to the value of N at that point) around the moving sphere can be deduced from the corresponding fields around the sphere at rest by compressing the whole space along the direction of motion, so as to reduce all lengths in that direction in the ratio

$$1 \text{ to } \frac{1}{\left(1+\dfrac{w^2}{V^2}\right)^{\frac{1}{2}}}.$$

Now we have seen that it is probable that all the properties of material bodies—chemical, mechanical, optical, thermal and the rest—can be reduced to electrical properties, and that therefore the value of any physical constant at any point depends upon the value of N at that point. If, then, the electrons are regarded as spheres, any physical constant having the value P at any point in a material medium at rest, which has the coordinates x, y, z, relative to axes fixed in that medium, will have the same value at a point the coordinates of which are x, y, $\dfrac{z}{\left(1+\dfrac{w^2}{V^2}\right)^{\frac{1}{2}}}$, relative to axes fixed in the medium, when the medium moves with a velocity w parallel to the axis of z. In other words, the effect of moving the medium will be an apparent contraction of the medium along the line of its motion in the ratio 1 to $\dfrac{1}{\left(1+\dfrac{w^2}{V^2}\right)^{\frac{1}{2}}}$: for the length of any given portion of a material body is measured by observing the distance between two points at which a certain physical constant has two assigned values. It may be noted that the contraction could never be observed directly, for, since all bodies contract in the same ratio when moving with the same velocity, the scale on which it might be proposed to measure the length of the moving body would contract in the same ratio as the body itself.

If the value of $\dfrac{w}{V}$ is so small that $\dfrac{w^4}{V^4}$ is inappreciable, we may write the ratio of contraction $1:1-\dfrac{w^2}{2V^2}$. This is the ratio of contraction which is necessary to explain the negative result of the Michelson-Morley experiment. Accordingly, the Lorentz-Fitzgerald hypothesis is competent to explain that result, in which the value of $\dfrac{w}{V}$ was 10^{-4}. But it should be noted that if it were possible to experiment on systems moving with a much greater velocity relative to the aether, there should be found, on that hypothesis, some change due to that velocity.

An alternative hypothesis.

8. From this point of view then, this negative result proves that the optical properties of matter (on which the experiment is based) are electrical in origin—a conclusion that nobody doubts nowadays. Accordingly, the Lorentz-Fitzgerald hypothesis has met with general acceptance, and it has been concluded that the difficulties which were raised by the Michelson-Morley experiment have been solved satisfactorily. But recently the more general and searching inquiry into the electrical properties of moving systems, which has arisen from the development of the conception of electromagnetic mass, has thrown doubt upon the complete sufficiency of this explanation of the difficulties connected with aberration. It is worth while to consider whether the abandonment of some other of the assumptions on which the older theory of aberration is based may not lead to an equally satisfactory reconciliation of theory and experiment.

The negative result of the Michelson-Morley experiment would be explained if it were possible to suppose that the velocity through the aether of the light emitted by a moving source is increased by the velocity of that source. An examination of the arguments of the preceding paragraphs will show that, if such an hypothesis were accepted, in place of the velocity (u) of the observing system relative to the aether there must be substituted the velocity (u') of the system relative to the source: and aberrational effects would only be observed when there was relative motion between the observer and the source of light. For, returning to the analogy of the shot fired from a gun, it is clear that if the gun is carried on a second ship moving with the same velocity as the first ship, the positions of the holes made by the shot are independent of that velocity. In the astronomical observations, where an effect due to motion is observed, there is relative motion between the source (in a star) and the observer (on the earth), while in the physical experiment both source and observer are attached to the earth and there is no relative motion between them. If the aberrational effect depend upon the relative velocity of the reacting systems, the negative result in the latter experiment is to be expected. It is to be noted also that such an explanation would lead to the expectation of a negative result for all velocities, and not only for velocities so small that $\frac{w^4}{V^4}$ is negligible. It does not seem satisfactory that the explanation of such an important observation should be based upon the limitations of our experimental resources.

But, so long as we retain the conception of the aether and make use of the crude mechanical analogy that has been employed hitherto, such a hypothesis will seem difficult to accept. It will be said that, if the light vibrations, when once they have left the source, are contained in the aether, the only motion which can affect the velocity of propagation of those vibrations is velocity of (or relative to) the aether: the motion of the source can have nothing to do with the matter: the velocity of sound waves through air is independent of the velocity of the source. However, if we abandon that conception and return to the Faraday tubes, which served us so well in the earlier part of this volume, the whole problem will appear in a perfectly new light. If the electromagnetic theory of light is true, it must be possible to find the explanation of aberration as of all other optical phenomena in the simple laws of the electromagnetic field which are represented for us by the properties which have been attributed to the Faraday tubes.

Refraction in a moving medium.

9. Consider a light vibration travelling with velocity V along the direction AB (Fig. 31), through a medium moving with velocity u in the direction XY at right angles to AB. Suppose that the light is plane polarised, so that at a given instant the direction of the electric intensity in the wave front is OE, in the same direction as XY, and the magnetic intensity in the wave front is in the direction OM perpendicular both to AB and OE. As the medium moves, it carries its electrons along with it: but we know (p. 18) that a charged body moving in a magnetic field is subject to a force at right angles to the field and to the direction of motion. Accordingly there is a force acting on the electrons, due to the magnetic field in the light vibration, tending to deflect them in the direction OB (see footnote, p. 78). The magnitude of this force will be $\frac{\epsilon u H}{V^2}$, if H is the magnetic intensity in the light at the point O. At the same time, if the electric intensity at O

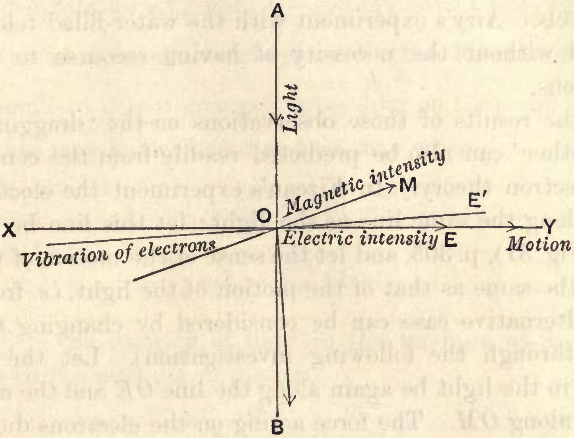

Fig. 31.

is X, there will act on the electrons a force $-\epsilon X$ in the direction EO: the resultant of these forces will be along the line OE', where

$$\sin E'OX = \frac{\epsilon u H}{V^2} \Big/ \epsilon X = \frac{u}{V},$$

since $H = VX$ ((6) and (7) p. 40). The electrons will vibrate along this line and not along OE, as they would if the medium were at rest. But the light emitted by the electrons travels at right angles to their direction of motion: hence the light wave, instead of travelling along OB, travels along OB', where $BOB' = XOE'$. The angle BOB' through which the direction of the light is turned is the angle of aberration: the value found above is the same as that determined from the mechanical analogy which we considered previously[1].

It will be observed that we have made no assumption about the nature of the forces restraining the vibrations of the electrons: the calculation is applicable over the same range as the calculation

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[1] The analysis here and in the succeeding calculation does not pretend to accuracy, but it may be of some service in conveying more clearly than a rigid investigation the physical principles which are involved.

of the refractive index based on the same principles given in Chapter III.: the conclusion is valid for all isotropic bodies, whatever their refractive index, and we may conclude that the angle of aberration is the same whatever the medium through which the light travels. Airy's experiment with the water-filled telescope is explained without the necessity of having recourse to Fizeau's observations.

But the results of those observations on the 'dragging along of the aether' can also be predicted readily from the conceptions of the electron theory. In Fizeau's experiment the electrons are moving along the same line as the light: let this line be denoted by AB (Fig. 31), p. 305, and let the sense of the motion of the electrons be the same as that of the motion of the light, *i.e.* from A to B (the alternative case can be considered by changing the sign of u all through the following investigation). Let the electric intensity in the light be again along the line OE and the magnetic intensity along OM. The force acting on the electrons due to the magnetic field is now in the same line as that due to the electric field, and a reference to the footnote, p. 78, will show further that the directions of the two forces are opposite. Hence the force acting on the electron will be $\left(\epsilon X - \frac{\epsilon u H}{V^2}\right) = \epsilon X\left(1 - \frac{u}{V}\right)$.

In Chapter III., § 5, the refractive index was calculated on the assumption that the force on the electrons due to the light wave was $\epsilon X_0 \sin\frac{\epsilon u t}{T_1}$: a reference to the analysis in that place will show that in order to find n', the refractive index of the medium in motion, we must substitute everywhere $\epsilon\left(1 - \frac{u}{V}\right)X_0$ for ϵX_0. Hence we obtain from equation (17) of that chapter

$$n'^2 = 1 + \left(1 - \frac{u}{V}\right)\frac{4\pi N\epsilon^2}{\epsilon\left(1 - \frac{p^2}{T^2}\right)}$$

$$= n^2 - \frac{u}{V}(n^2 - 1).$$

Hence $$\frac{n'^2}{n^2} = 1 - \frac{u}{V}\left(1 - \frac{1}{n^2}\right).$$

As a matter of fact this conclusion is not accurate: for the change in the motion of the electrons due to the magnetic field will produce a change in that magnetic field, which has not been taken into account. A complete investigation shows that we must substitute for ϵX_0, not $\left(1 - \frac{u}{V}\right) \epsilon X_0$ but $\left(1 - \frac{nu}{V}\right)^2 \epsilon X_0$. The velocity u is so small compared to V that Fizeau's experiment is too small to detect changes depending on terms involving $\frac{u^2}{V^2}$: such terms may be neglected and we may write

$$1 - \frac{2un}{V} \text{ for } \left(1 - \frac{nu}{V}\right)^2.$$

Hence $$\frac{n'^2}{n^2} = 1 - \frac{2un}{V}\left(1 - \frac{1}{n^2}\right).$$

If V_1 is the velocity of light in the medium at rest, V_1' the velocity in the medium in motion,

$$\frac{n'}{n} = \frac{V_1}{V_1'} \text{ and } \frac{V}{n} = V_1.$$

Hence $$\frac{V_1^2}{V_1'^2} = 1 - \frac{2u}{V_1}\left(1 - \frac{1}{n^2}\right)$$

or $$V_1' = V_1\left\{1 - \frac{2u}{V_1}\left(1 - \frac{1}{n^2}\right)\right\}^{-\frac{1}{2}}$$

$$= V_1\left\{1 + \frac{u}{V_1}\left(1 - \frac{1}{n^2}\right)\right\}$$

(making the same approximation as before),

or $$V_1' = V_1 + u\left(1 - \frac{1}{n^2}\right).$$

That is to say, the velocity of the light in the moving medium is increased by a fraction $\left(1 - \frac{1}{n^2}\right)$ of the velocity of the medium. This is Fizeau's result.

10. It should be clear from the preceding paragraphs that the representation of aberration as a 'leaving behind of the light in the aether,' while the earth moves on, is based on an analogy which is far from complete: the change of the direction of the light with the motion of the earth must be

The two hypotheses.

viewed as an effect of the magnetic field in the light vibration on the electrons in the observing system. If the difficulty introduced by the Michelson-Morley experiment be reconsidered from this new point of view, the simple solution suggested in § 8, which seemed so improbable when stated in terms of the mechanical analogy, will appear far more acceptable.

That difficulty arose from the apparent necessity of considering the velocity of the moving observer, which determines the aberration, as the velocity relative to the aether and not the velocity relative to the source of light. In the last paragraph no use of the term 'aether' has been made: the influence of the moving system has been considered in terms of Faraday tubes. It was shown that that influence depends on a reaction between the tubes and the electrons in the moving system, and it is natural to assume provisionally that the velocity which determines that reaction is the velocity of the electrons relative to the tubes along which the light is propagated. But these tubes are attached to the source of light and move with it, for their ends are the electrons the vibrations of which cause the emission of the light. From this point of view it must appear obvious that the velocity which determines aberration is velocity relative to the source of light, and that only a negative result should be expected from the Michelson-Morley experiment.

But no analogy can prove anything. The representation of aberration in terms of tubes can no more decide whether aberration is determined by motion relative to the aether or by motion relative to the source, than can the mechanical representation with which we started. Such a decision can only be made by experiment. If it were shown that the circumstances which determine some optical effect could not be described in terms of the relative velocity of the emitting and observing systems and their velocity relative to other material bodies, then it would be necessary to introduce the conception of an aether, the velocity relative to which has some influence on optical phenomena. But we have seen that both the astronomical observations and the Michelson-Morley result can be stated in a perfectly satisfactory manner in terms of the relative velocity of the source and the observer: and, further, the principles on which the Lorentz-Fitzgerald contraction is predicted theoretically lead also to the conclusion that by no optical experiment, which seems likely to be practicable in the near future, could we detect any effect which did not depend on that relative velocity. Accordingly no experimental decision between the two hypotheses can be expected (unless the phenomena described in § 12 should lead to such a decision), and we are free to adopt whichever of the two seems on general grounds to be the most convenient. Since we have seen that to those who make use of the conception of Faraday tubes the hypothesis of motion relative to the source is far the more convenient, and since it seems undesirable to introduce into our explanations a quantity, 'velocity relative to the aether,' the magnitude of which cannot be detected in any possible experiment, that hypothesis will be adopted throughout the rest of our discussion. I believe that the only reason why anyone has entertained the other hypothesis for a moment is that they have not thought out precisely what they mean by the term 'aether': but since that term is so familiar to physicists that they will be inclined to regret its abandonment, it will be desirable to consider its significance more closely and to show that its use does not make the hypothesis that aberration is determined by 'velocity relative to the aether' any more probable than it appeared in the discussion based on the conception of Faraday tubes. I must ask the reader to pardon a somewhat lengthy digression.

Aether and energy.

11. The fundamental conception of modern physics is energy, and its fundamental principle the conservation of energy. (By a scientific principle I mean a proposition which is assumed to be true universally, so that no explanation can be satisfactory unless it is in harmony with that proposition.) By definition, the total energy of a system is constant throughout all changes. If a change occurs which involves a change in the sum of all the previously recognized forms of energy, a new form of energy is introduced, in order that the total energy should remain unchanged. Thus, if, being only acquainted with mechanical energy, we began to investigate electrical phenomena, the necessity would arise speedily for the invention of such a new form of energy. Mechanical energy is spent in separating two charged bodies and no increase of any previously recognised form of energy can be detected. Accordingly it is necessary, in order to maintain the law of the conservation of energy, to introduce a new quantity, electrical energy, which increases as the mechanical energy is decreased.

Further, energy is regarded as distributed in space; when any physical change occurs in any part of a material system, it is usually found that a simultaneous change occurs in some other part of the reacting system. It is supposed that these changes, detected by our observations, are the manifestation of changes in the energy localised in the parts of the system which undergo the apparent change: energy is transferred from the space occupied by one of the changing parts to that occupied by the other. Accordingly it is imagined that any alteration of the properties of any part of a material body is accompanied by an increase or a decrease of the energy localised in that part of the system, and, conversely, that a loss or gain of energy by any part of a system is accompanied by a change in the physical properties of that part.

Let us view the changes that accompany the propagation of electrical influences—of which light is an important example—from a system A to a distant system B, from the standpoint of this brief sketch of the modern doctrine of energy. Suppose that the system A begins to emit light vibrations all of which fall upon B: then A loses energy in one of its recognised forms (say heat energy) and B gains the same amount of energy as is lost by A. But close examination shows us that the changes in B, which result from its gain of energy (such as rise of temperature), do not begin at the same instant as the changes accompanying loss of energy by A: there is a finite interval between the two events, the magnitude of which increases with the distance between A and B. In this interval A has suffered some change and has a smaller amount of energy localised in it than before, but B has suffered no alteration and its energy content is unchanged: the total energy of the system $A+B$ has decreased. If we are to retain the principle of the conservation of energy, some other system must be found which has gained the energy lost by A. But on investigation it appears that no material body within the range of our knowledge has undergone such a change as might be attributed to the absorption of the energy lost by *A*: that energy has not been gained by any known body, and if we are to imagine that it has not been lost altogether we must invent some body to which the energy has been transferred. This hypothetical body is called 'the aether': it is imagined that *A* gives up its energy to 'the aether' which again gives it up after a suitable interval to *B*.

The aether then, by definition, is a body occupying the space where the energy is localised during the interval between leaving *A* and arriving at *B*. It is a conception which is introduced in order to avoid the inconvenience which would arise if the principle of the conservation of energy were abandoned: nothing is known of it except that it can be the seat of energy. But in Part I. the energy during its transfer from *A* to *B* was regarded as localised in the Faraday tubes, part of it being potential energy and part of it kinetic energy of the tubes. Accordingly, if the aether is the place where the energy is localised, the position of the aether becomes identical with the position of the Faraday tubes, and the two rival hypotheses of 'velocity relative to the aether' and 'velocity relative to the tubes' become the same assumption stated in slightly different words; even if we admit that aberration is determined by 'velocity relative to the aether,' no positive result is to be expected of the Michelson-Morley experiment.

However it may seem that this account of the conception of the aether is not quite adequate. 'It is true,' it may be said, 'that the aether exists where the energy is localised, but it does not exist only there. It is supposed to be some sort of substance that can contain energy in the same way that material bodies contain energy: it is present everywhere, but the amount of energy which it contains is not the same at all points. Electromagnetic changes accompanied by changes in aethereal energy do not involve motion of the aether, but only transfer of energy from one part of the aether to another. The aether itself is always at absolute rest.'

These sentences describe fairly, I think, the view of the aether which is generally accepted, without any very deep inquiry, by most physicists. It is unnecessary to stop and point out all the difficulties into which such a view leads us: it will suffice for our purpose to show that it does not render the 'velocity relative to the aether' hypothesis any more illuminating.

The position of a body can only be determined by means of the spatial variation of some one of its properties: the position of a homogeneous material body is determined by examining the rapid variation of its properties which occur at its boundaries: if our field of research is limited so that we can never arrive at its boundaries, it is impossible to come to any definite conclusion as to the position, or change of position, of the body. There is no difficulty in fixing the position of Faraday tubes, for the energy contained in the tubes varies from point to point; imaginary axes can be drawn through points identified and distinguished from other points by their content of energy, and the rate of change of position of a body relative to these axes is 'velocity relative to the tubes.' But, if the energy can move without the aether moving with it, this means of fixing the position of the aether fails, and there is no other to take its place. The only property which has been attributed to the aether—the power of containing energy—is possessed by all parts, and, by hypothesis, we cannot reach the boundaries of the aether: there is no variation of properties by which the position of the aether can be identified.

Accordingly the velocity of the aether relative to any system cannot be determined, like that of a material body, by noting the rate of change of its position relative to axes fixed in that system. Before any significant statement can be made about the relative velocity of the aether and any other system, the method of determining that velocity must be fixed by definition: the conventional definition, which is implied in every statement about the velocity of material bodies, is applicable no longer. Therefore, whatever grammatical form we may give to our first statement about the velocity of the aether, or the velocity of any body relative to the aether, the statement either must be a definition or must be meaningless.

In the ordinary electromagnetic theory of aberration the velocity of the aether is introduced by the proposition that the magnetic intensity due to a charged body is proportional to the velocity of the charge relative to the aether. This statement is not a profound scientific proposition of intense interest: it is either meaningless, or it is a bad paraphrase of the following definition:—

If that part of the magnetic intensity at a point within a system A due to the motion of a charged body B is proportional to $v + v_0$, where v is the velocity of B relative to A, then it is proposed to call v_0 the velocity of A relative to the aether, and $-v_0$ the velocity of the aether relative to A.

(In many cases the velocity of the aether is introduced by the still more objectionable phrases (1) that the aether is at absolute rest, and (2) that the velocity to which the magnetic intensity of a moving charge is proportional is its absolute velocity. The first statement can have no significant meaning whatever, for neither of the terms 'absolute velocity' or 'velocity of the aether' have been defined. A sentence containing one undefined term is a definition: one that contains two undefined terms is mere nonsense.)

Now, if the Lorentz-Fitzgerald hypothesis were not introduced, optical experiments would show that, if A is the observing system and B the emitting system, the magnetic intensity in A due to the motion of B with velocity v relative to A, is proportional to v: hence the quantity v_0 is zero, and the system A is at rest relative to the aether. If then we have two observing systems in the same part of space, which are moving with finite velocity relative to each other, each of them is at rest relative to the aether: a different value of the velocity of the aether relative to some third body in this region of space is obtained according as it is deduced from one or the other of these systems. It appears that in the space between the two systems the 'aether,' the velocity of which is defined as above, is made up of two portions: one of these moves with A and should be regarded as part of A, the other moves with B and should be regarded as part of B. But, if we insist upon regarding the 'aether' as an indivisible system, we must either suppose that it can have two different velocities at the same point, or we must introduce the elaborate Lorentz-Fitzgerald hypothesis. To me it seems preferable to give up the conception of the 'aether.'

No doubt the orthodox man of science will spurn these arguments as savouring of 'philosophy' and therefore unworthy of the notice of any serious person, but in truth it is he who has been led astray by philosophy. He has fallen into the most glaring errors of the crudest nominalism. Because a name has been given to the conception of the 'aether,' it has been assumed straightway that the aether is a substance, separate from the material bodies of the system and moving independently of them: for no other reason than the existence of the name it has been assumed that this aether has mass, elasticity, even weight: qualities which are only known to be associated with material bodies have been attributed to it, one after the other, until Mendeleef suggested that it is a chemical element with a place in the periodic series! Of course there was no reason why the conception should not have been introduced or why the subsidiary hypothesis should not have been made provisionally, but if scientific investigators in their contempt of 'logical hair-splitting' had not been deceived unawares by a base metaphysical doctrine which would have been rejected with scorn by Thomas Aquinas, they would have re-examined those hypotheses when the aberrational difficulties were met and saved themselves from fruitless discussions about 'velocity relative to the aether.'

We shall therefore adopt the hypothesis, which furnishes a theory of aberration consistent with all the facts known, that the magnetic intensity at any point in an observing system due to the motion of a charged body is proportional to the relative velocity of the charged body and the observing system. It might be thought difficult in all cases to divide the observing from the observed system, but I have not been able to devise any imaginary experimental arrangement in which the distinction is not perfectly clear, if the principles on which our argument has been based are borne in mind. The consequences of this hypothesis, in dealing with such matters as the 'Lorentz-Fitzgerald contraction,' are best considered in connection with another series of electromagnetic phenomena in moving systems, to which we must now proceed.

Electromagnetic mass.

12. In considering the emission of β rays from radioactive substances, the conception of the electromagnetic mass of a charged body was introduced. The acceleration of such a body produces a change in the magnetic energy of the field due to its motion: when the speed of the body does not approach the velocity of light, the energy of this magnetic field is proportional to the square of the speed of the body and may be regarded conveniently as the kinetic energy of a mass, which the body possesses in virtue of its charge. At greater speeds the energy of the field increases more rapidly than the square of the speed: if the conception of electromagnetic mass is to be retained, it must be supposed that that mass increases with the speed. If electromagnetic phenomena are interpreted in terms of Faraday tubes, this increase of mass must be regarded as due to the crowding of the tubes attached to the charge into the plane perpendicular to the direction of motion, in virtue of the forces exerted on them by the tubes in the space through which the charged body passes. This change of the distribution of the tubes was also used in § 7 as a theoretical basis for the hypothetical Lorentz-Fitzgerald contraction: in considering the circumstances which determine the variation of the electromagnetic mass we shall be investigating the effects which may be expected from that contraction in optical or other phenomena.

Several formulae have been deduced on theoretical grounds for the relation between the electromagnetic mass of a charged body and its speed: the differences between them arise mainly from differences in the assumptions made as to the distribution of the tubes round the charged body when at rest. But, with one exception, all investigators appear to have assumed explicitly or implicitly that the velocity upon which the variation depends is 'velocity relative to the aether.' Since we have rejected this conception as either useless or misleading, we must first inquire more nearly into this question.

In the equations (8) and (9) pp. 155 and 156, by the use of which the mass of an electron may be determined from the observations of Kaufmann, or in the more accurate equations based upon a complete theory which take their place in any actual estimation, the velocity of the electron enters in two ways. It is introduced in calculating the distance relative to the photographic plate over which the electron travels in a given time: it is clear that this velocity is velocity relative to the observing apparatus. It is also introduced by the term $\frac{H\epsilon v}{V^2}$, which measures the force on the electron due to the reaction between the tubes attached to it and the tubes which give rise to the imposed magnetic field: since we have assumed that the magnetic action of one system on another is determined by the relative motion of the charges which give rise to that action, this velocity again must be taken to be the velocity of the electron relative to the apparatus which sets up the imposed field. Accordingly a correct estimate of the mass of the electron will be attained, if the quantity v in the equations mentioned is identified with the velocity of the electron relative to the apparatus employed for measurement. This conclusion has been adopted in all discussions of Kaufmann's observations, but its adoption by those who believe that magnetic action is determined by velocity relative to the aether and that the velocity of the apparatus relative to the aether is the velocity of the earth in its orbit can only be justified on the grounds that this velocity is so small compared with that relative to the electron that it may be regarded as zero without introducing an error greater than the error of experiment.

But it is not quite so easy to determine on what velocity depends the change of the distribution of the tubes around the electron which gives rise to the variation of the electromagnetic mass and the Lorentz-Fitzgerald contraction. This change of distribution cannot arise from a reaction between the different tubes attached to the moving electron, for there is no relative motion between these tubes: it must be determined by a reaction between these tubes and those of some system relative to which they are moving with a finite velocity. But in Chapter I. we were forced to the conclusion that the tubes attached to any charge do not end on a compensating charge of opposite sign but extend indefinitely in all directions: consequently the region through which the electron travels must be traversed by tubes issuing from the electrons and positive charges in the atoms of all the surrounding bodies, which may be moving with various velocities relative to the electron. If it is with these tubes that the tubes of the electron react, the variation of the distribution of the tubes with the velocity is indeterminate, until the velocity of the electron relative to all the surrounding atoms is known. However, if the surrounding bodies are neutral, their tubes are arranged at random and their resultant effect on the tubes attached to the electron is nil. But there is one system the tubes of which are not arranged at random with regard to the tubes of the electron, namely, the atom which originally contained the electron and carries the corresponding positive charge: the tubes of this positive charge are arranged in such a way as to give rise to the electric field round the electron, and it is with these tubes that the tubes of the moving electron react. Accordingly these considerations lead to the conclusion that the velocity which determined the variation of the electromagnetic mass is the velocity of the charged system relative to the system which bears the compensating charge of opposite sign; in no neutral system can there be any variation of the electromagnetic mass or any Lorentz-Fitzgerald contraction due to a motion of the system as a whole, whatever be the motion of the observing system.

It should be noted that it is possible for a system which is neutral on the whole, such as an atom, to possess electromagnetic mass which does not vary with its motion. For though there is no magnetic effect due to the motion of the atom at points distant from it, there is an intense field around each electron, the abolition of which, when the atom is stopped by the forces brought into play by its near approach to another atom, gives rise to a redistribution of energy such as is associated with the impact of a massive body.

Too much importance must not be attached to the considerations that have just been offered. Our knowledge of the properties of Faraday tubes is not complete. Indeed the conception may prove as misleading as that of the aether: at least it has the merit of raising questions in connection with unsolved problems which were concealed by the latter. It might seem unlikely that the difference between the two views of the velocity, which determines the magnetic effect of charges, has any bearing upon experimental results; for the relative velocity of the bodies surrounding the electrons of Kaufmann's experiment is so small compared with their velocity relative to the electron, that it is immaterial to which of them the velocity of the electron is referred. But Bucherer, who has developed lately a mathematical theory in which the conception of the aether is abandoned and all phenomena described in terms of the relative motion of charges and magnets, has concluded that his arguments have an important bearing on the interpretation of Kaufmann's observations.

The remaining theories of the variation of the electromagnetic mass with the velocity of the electron differ in the assumptions that they make regarding the distribution of the tubes around the electron. The word 'electron' signifies that there is a fixed relation between certain systems of tubes, namely, that all the tubes of one such system share in a steady velocity of translation: each of such systems has been said to be attached to an electron. But in order to investigate the present problem some assumption must be made about the distribution of the tubes in 'attached to an electron.' The simplest assumption that can be made is that all the tubes radiate equally in all directions from a single point, but this assumption introduces insuperable difficulties. For at the point where all the tubes join the polarisation is infinite, the energy is infinite and therefore the mass of the system is infinite. In order to avoid this conclusion and at the same time retain the assumption that the tubes radiate equally in all directions (*i.e.* that the polarisation follows the inverse square law), it is necessary to suppose that, within a certain region just round the point where all the tubes join, the properties of the tubes are altered in some way so that the polarisation does not become infinite. The different calculations of the electromagnetic mass that have been made differ in the assumptions that are made concerning this region[1].

The earliest calculation, that of Thomson, assumed that within a small sphere drawn round the point from which the tubes radiate the polarisation is zero. The region in which the alteration of the properties of the tubes occurs may be considered as the space occupied by the electron: the radius of Thomson's small sphere

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[1] None of the calculations mentioned below, except that of J. J. Thomson, involve explicitly the conception of Faraday tubes. The assumptions that are made by Abraham, Bucherer, and Lorentz, though perfectly simple when stated in terms of the nomenclature that they adopted, could only be stated in the most cumbrous manner in terms of Faraday tubes. The assumptions mentioned below are not accurately those that were made, but they are those which would have arisen from the same physical ideas in the mind of an investigator who used the conceptions of tubes.

is the quantity which was called the 'radius of the electron,' on p. 199.

The assumption of Abraham (a rigidly electrified sphere) is very similar to that of Thomson. Lorentz assumed that the electron is a sphere when the system of tubes is at rest, but suffers the Lorentz-Fitzgerald change of form when the system is set in motion. Bucherer in his earlier work also assumes that the electron is a sphere when at rest and undergoes when in motion a change of form rather different from that hypothecated by Lorentz. Finally, Righi assumes that the polarisation can never exceed a certain finite value: on the surface of the electron the polarisation has that maximum value, but within it the polarisation is zero. This brief summary is sufficient to indicate the nature of the assumptions that are made.

The only method which is known at present for attempting to distinguish between the various formulae based on these assumptions is to compare the results predicted with the experimental results of Kaufmann. But this test cannot possibly be judged decisive: it can exclude certain theories, but the limits of experimental error are necessarily so large that it must always be possible to invent new assumptions which will give results as satisfactory as the best of these pre-existing[1]. Kaufmann finds that the theories of Abraham and Bucherer agree with his experiments within the limits of error, but that those of Lorentz and Thomson must be rejected. Righi's theory has not yet been put in a form which enables the test to be applied.

Until a more satisfactory means is found of deciding between rival theories of the structure of the electronic systems of tubes, there is little use in inventing new assumptions and deducing their consequences. But it is worth while to point out a new direction which such investigation may take.

Has Faraday's theory physical meaning?

13. All these theories without exception have assumed that the electronic tubes radiate in all directions, *i.e.* that the electric polarisation due to an electron varies as the inverse square of the distance from the electron. But there is not the smallest experimental evidence for that

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[1] Cf. remarks on dispersion formulae, p. 266.

assumption, and it is worthy of consideration whether some other distribution of the tubes may not lead to better results. This statement may surprise a student who has been taught that the law of the inverse square has been established conclusively by the Cavendish experiment: but it must be remembered that that experiment did not deal with the effect of a single electron, but with the effect due to the millions of electrons which go to make up the smallest charge which can be measured directly in such observations. The charge on the sphere in that classical experiment was due to the presence of a very large number of electrons over and above that required to neutralise the positive charge on the atoms of the metal of the sphere. Whatever distribution we assume for the tubes round a single electron, the tubes round an immense number of electrons orientated in all directions will be distributed nearly equally in all directions. For all we know, the tubes of the electronic system may all lie in one plane, or even in one line: in fact there may be only one tube attached to each electron: an electron may be simply the ends of one Faraday tube. The possibility of making such assumptions opens up a wide field for investigation, and leads us to consider the conception of Faraday tubes a little more closely. The conception has been used throughout this volume mainly because it enables the theory of electricity to be expounded in non-mathematical language. But we may ask ourselves whether there is no other justification for their employment: have the Faraday tubes any 'physical meaning'?

That question requires some explanation. If Faraday's theory is used only to translate the results of the modern mathematical theory of electricity—based on the work of Hertz—into non-mathematical language, of course precisely the same results will be attained whether the nomenclature of the first or of the second theory is employed. But, as has been remarked before, the importance of scientific conceptions lies in what they suggest. A great deal of optics can be treated equally well by either the undulatory or the corpuscular theory of light, but the former and not the latter suggests the performance of Young's experiments on interference: since the experimental results were in accordance with the suggestions of the undulatory theory, we say that the conceptions of that theory and not those of the other have a physical meaning.

Now in the same way Faraday's theory suggests experimental results which are not suggested by Hertz' theory. The latter assumes that the distribution of the energy in the electric field is continuous, the former suggests that it is discontinuous: the use of the term 'tube' suggests a difference between the properties of the regions inside and outside the tubes and a discontinuity in those properties in passing through the walls of the tube. (It is just this difference between continuity and discontinuity which makes Hertz' theory so well adapted for mathematical development and Faraday's theory for visualisation.) If we consider the space between two parallel condenser plates Hertz' theory assumes that the energy is distributed uniformly as in Fig. 32 A, while Faraday's theory suggests the distribution shown in B (the shaded portions are those in which energy is localised). We can apply Faraday's conceptions to the distribution A by dividing up the space by imaginary planes (indicated by the dotted lines), and calling each of the cells into which the region is divided by these planes a Faraday tube: but there is no change of properties of the medium in crossing the boundary of a tube. If experiment forces us to the conclusion that the distribution of the energy is indeed continuous, we must admit that the tubes have no physical meaning. If, on the other hand, Faraday's theory has physical meaning, it should be possible to find places in the unshaded

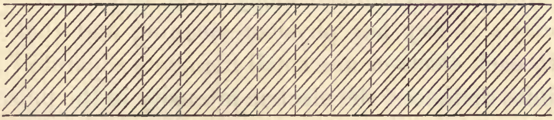

Fig. 32 A.

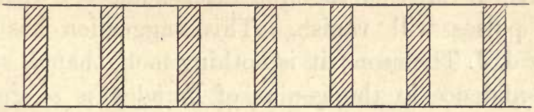

Fig. 32 B.

parts of B where a system is free from the influence of the charges on the plates. Of course the intervals between successive tubes must be very small, and it would be impossible to fit a body so large as a molecule into the spaces. But an electron might be subject to different forces according as it lay within or without the region occupied by a tube.

Now it may be remembered that, in dealing with the difficulties connected with the theory of ionisation by Roentgen rays, the conclusion was reached that the most satisfactory solution of the problem would be attained, if it were possible to suppose that only a small fraction of the whole number of electrons in the gas, through which the rays pass, come under the influence of the rays. The rays are pulses emitted along Faraday tubes: if the distribution of these tubes in the neighbourhood of the kathode rays, which give rise to the Roentgen rays, be indeed similar to that sketched in Fig. 32 B, only those electrons which lie in the shaded portion of the figure will be subject to any forces which may arise from pulses emitted along those tubes. If we imagine that Faraday's theory has physical meaning and that the tubes attached to each electron are small filaments separated from each other by finite distances, the electric intensity in the pulses emitted along them may be supposed to be as great as we please, without making the total energy in the pulse greater than the limit fixed by experiment. Most of the difficulties connected with ionisation by those pulses will vanish. This suggestion has been put forward by J. J. Thomson; it is nothing more than a speculation, but my confidence in the genius of Faraday is so great that I should be inclined to accept any hypothesis, however rash, which was suggested by the ideas that he developed.

On the further question whether the tubes have exactly the properties that have been attributed to them, in particular, whether the relation between the motion of the tubes and the magnetic field which they produce is represented accurately on p. 15, there is little to be said. It is desirable to draw attention once more to our ignorance of the electrostatic and magnetic fields which surround a single electron, which may be supposed to exhibit the properties of the tubes in their simplest form. It seems likely that the forces which exist between the parts of a system of charges of atomic dimensions is somewhat different from those deduced from the analogy of systems of very much larger dimensions. Thus, we have noted already that the value of e/m for an electron, deduced from phenomena which depend upon processes taking place within the atom, is slightly different from that deduced from processes taking place outside the atom. The great difference in the properties of positive and of negative electricity, on the discovery of which the development of the electronic theory of matter is based, is most suggestive: when it is remembered that positive electrification only occurs in structures which, we have every reason to believe, contain a considerable number of electrons, it is impossible to abstain from the speculation that the use of the phrase 'positive electricity' may be a mere cloak for our ignorance, and that the forces which are attributed to it may be found to be manifestations of undiscovered properties of the Faraday tubes attached to the electrons.

Mechanical Theories of Electricity.

14. A few words on mechanical theories of electricity may close this volume appropriately.

At the present time these theories are generally expressed in terms of the conception of the aether, and take the form of attempts to devise a system built up of massive particles reacting according to the accepted laws of mechanics, in which the distribution and transference of energy shall simulate that which is supposed to obtain in the aether. Such a problem is, and, as far as I can see, always must be, indeterminate without further assumptions. The only mechanical function which occurs in the definition of the aether is energy: experiment can only give information concerning the variation of the energy content of the aether with time and space. But the energy of a material system is regarded as a function of at least two mechanical quantities, mass and speed, stress and strain, and so on: observation can only tell us the value at any point of the aether of a single function of two such quantities: it cannot tell us anything about the value of either of those two quantities separately. Accordingly, if part of the energy of the aether is to be regarded as the kinetic energy of a mechanical system, there are an infinite number of systems in which the distribution of kinetic energy is the same as that in the aether: any value of the mass (say) can be assigned to any point so long as the appropriate value is assigned to the velocity at that point, so that the product $\frac{1}{2}mv^2$ shall have the required value. In all actual mechanical theories that have been proposed, an assumption has been made as to the value of the velocity at each point of the aether and the corresponding value of the mass has been deduced: many disputes have arisen out of such theories owing to a failure to observe that different assumptions have been made by the disputants. The two assumptions that have been made most frequently are (1) that the magnetic intensity is proportional to the velocity of the aether, and (2) that the velocity of the aether at any point is the velocity of some set of Faraday tubes about that point. The first of these two assumptions is very similar to that discussed on p. 313 and leads to the same difficulties: the second dispenses with the necessity of introducing the conception of the aether and reduces the theory to a mechanical theory of the tubes. But the problem is not thereby rendered determinate, for the energy of the tubes cannot be described in terms of the motion of the tubes as a whole: the electrostatic energy must either be represented as energy of strain or as kinetic energy of some part of the tubes. Either alternative requires further assumptions to be made before the strain or the velocity of each part of the tubes can be determined uniquely.

J. J. Thomson has sketched the outline of a mechanical theory of the properties of the tubes, which introduces at the same time the conception of the aether, based on the hydrodynamical analogy mentioned in Chapter I. It is suggested that the properties of the aether are similar to those of a fluid possessing mass and that the apparent mass of the tubes represents the mass of this fluid which they drag along with them in their motion. But, for reasons which have been stated already, it seems to be best to avoid the conception of the aether altogether. If the conception of Faraday tubes be employed, the only use of the aether is to afford satisfaction to those who still adhere to the dogma, at least as old as Aristotle, that the ultimate substance, of which material bodies are the various forms, must be continuous in space. It is remarkable that this belief should be retained, while the development of physics from the proposition of Dalton's atomic theory to the present day has been a steady process of rejecting the interpretation in terms of continuity in favour of the interpretation in terms of discontinuity: at least it does not appear to me to justify the use of a term so full of potentialities for error as 'the aether.'

There is a more general objection to all mechanical theories of electricity to which attention should be drawn. The modern theory of electricity attempts to explain the mass of material bodies in terms of the electromagnetic properties of charged systems: is it wise to attempt to explain those electromagnetic properties in their turn in terms of mass? It is true that the new system to which mass is attributed is not a material body, so that there is no circuitousness in the argument, but the mere fact that it differs so widely from material bodies suggests doubts as to the propriety of attributing to it a property associated in our minds so intimately with such bodies as mass. The mention of mass immediately suggests weight, and, as long as we talk about the mass of the aether, there will be a tendency among some persons to fly off into fruitless speculation about the weight of the aether. The modern doctrine of energy must necessarily lead us to attempt to reduce all the forms of energy to one single form, but we have yet to agree upon the choice of that single form. Is it to be kinetic energy or is it to be electromagnetic energy? My own vote is cast, for what it is worth, for the latter: but to discuss the question fully would require a treatise on the principles of science.

REFERENCES FOR CHAPTER XIV.

An excellent account of the mathematical theory of the electromagnetic properties of moving bodies is given in Bucherer's little book, *Mathematische Einführung in die Elektronentheorie.* The classical original memoirs on the subject are Lorentz' *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Koerpern* (Leyden, 1905) and Larmor's *Aether and Matter*, especially Chapter X. The latter book contains much that was published previously in the *Phil. Trans.*

The following memoirs on electromagnetic mass may be quoted from among the large number that have been published on this subject:--

J. J. THOMSON, *Phil. Mag.* XI. p. 229, 1881.

Heaviside, *Phil. Mag.* April, 1889.

Abraham, *Ann. d. Phys.* xi. pp. 105–179, 1907 (*A.* and *L.* p. 1).

Righi, *Phys. Zeit.* viii. pp. 161–169, 1907.

Planck, *Phys. Zeit.* vii. pp. 753–761, 1906 (containing a discussion in which Abraham, Bucherer and others took part).

Bucherer, *Phil. Mag.* xiii. 413–420, 1907.

An account of some mechanical theories of the aether is given by Larmor in *Aether and Matter*, appendices D and E.

APPENDIX.

It may be useful to collect together the values of the most important quantities which have been introduced so frequently in the foregoing chapters.

There are five natural constants independent of the peculiar properties of any material body:—

(1) the charge on an electron $\epsilon = 3{\cdot}4 \times 10^{-10}$ elec. st. units.

(2) the mass of an electron $m = 6{\cdot}1 \times 10^{-28}$ grammes.

(3) the radius of an electron $l = 10^{-13}$ cms. (about).

(4) the velocity of light *in vacuó* $V = 3 \times 10^{10}$ cm./sec.

(5) the 'gas-constant' $\alpha = 1{\cdot}46 \times 10^{-16}$ ergs/degree.

The ratio of (1) to (2) is also important:—

(6) e/m for an electron $\epsilon/m = 5{\cdot}6 \times 10^{17}$ elec. st. units/grammes.

Two other quantities should be included:—

(7) the mass of a hydrogen atom $M = 1{\cdot}1 \times 10^{-24}$ grammes.

(8) the radius of an atom $r = 10^{-8}$ cms. (about).

Those who are interested in units and dimensions may observe that (2), (3) and (4) can be made the foundation of a 'natural' system of units in which the units of mass, length, and time are approximately:—

$$[M] = 6{\cdot}1 \times 10^{-28} \text{ grammes.}$$

$$[L] = 10^{-13} \text{ cms.}$$

$$[T] = 3{\cdot}3 \times 10^{-24} \text{ secs.}$$

Since electric charge on the electrostatic system of units has the dimensions $[K]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{-\frac{3}{2}}[T]^{-1}$, where K is the dielectric constant, we find that

$$[K] = \frac{(3{\cdot}4 \times 10^{-10})^2}{6{\cdot}1 \times 10^{-28} \times 10^{-39} \times (3{\cdot}3 \times 10^{-24})^{-2}}$$

$$= 2{\cdot}1,$$

a value which does not differ from unity by more than the possible error involved by the uncertainty regarding the precise value of l.

Since a is of the dimensions $[M][L]^2[T]^{-2}[\theta]^{-1}$, where θ is the temperature, the 'natural' unit of temperature would be given by

$$[\theta] = \frac{6{\cdot}1 \times 10^{-28} \times 9 \times 10^{20}}{1{\cdot}46 \times 10^{-16}} = 3{\cdot}7 \times 10^9 \text{ degrees centigrade.}$$

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PREFACE. Some words are perhaps necessary to explain the publication of another book dealing with Elementary Electricity. A considerable portion of the present work has been in type for a long time; it was used originally as a part of the practical work in Physics for Medical Students at the Cavendish Laboratory in connexion with my lectures, and was expanded by Mr Wilberforce and Mr Fitzpatrick in one of their Laboratory Note-books of Practical Physics.

When I ceased to deliver the first year course I was asked to print my lectures for the use, primarily, of the Students attending the practical classes; the lectures on Mechanics, Heat and Light have been in type for some years. Other claims on my time have prevented the issue of the present volume until now, when it appears in response to the promise made several years ago.

Meanwhile the subject has changed; but while this is the case the elementary laws and measurements on which the science is based remain unaltered, and I trust the book may be found of service to others besides my successors at the Cavendish Laboratory.

The book is to be used in the same way as its predecessors. The apparatus for most of the Experiments is of a simple character and can be supplied at no great expense in considerable quantities.

Thus the Experiments should all, as far as possible, be carried out by the members of the class, the teacher should base his reasoning on the results actually obtained by his pupils. Ten or twelve years ago this method was far from common; the importance to a School of a Physical Laboratory is now more generally recognized; it is with the hope that the book may be of value to those who are endeavouring to put the method in practice that it is issued now.

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