[5] Cf. abstract, *Bull. Amer. Math. Soc.*, **34**, 1928 (551). Whyburn's paper, "Concerning Points of Continuous Curves Defined by Certain im Kleinen Properties," which contains this example in full, is to appear in an early issue of *Math. Ann.* My ability to reproduce it here is due to Professor Whyburn's kindness in communicating it to me.

[6] Hausdorff, *Grundzüge der Mengenlehre*, Leipzig, 1914, p. 213.

[7] If K is a set of points, by K' we denote the set consisting of K together with its limit points.

[8] These Proceedings, **12**, 1926 (761–767).

[9] Compare the first part of this proof with that given by Whyburn for the case cited above.

[10] Wilder, R. L., *Bull. Amer. Math. Soc.*, **32**, 1926 (338–340). It is clear from its proof that this theorem is true in any topological space.

[11] Moore, R. L., *Math. Zs.*, **15**, 1922 (254–260).

[12] Wilder, R. L., *Bull. Amer. Math. Soc.*, **34**, 1928 (649–655). The theorem quoted here was proved for euclidean spaces, but it is clear that it holds in any locally compact metric space, and, indeed, may be proved directly by use of the imbedding theorem employed in proving Theorem 4 below, and hence so that N is of the same dimension (Menger-Urysohn) as H, etc.

[13] Kline, J. R., abstract, *Bull. Amer. Math. Soc.*, **34**, 1928 (263).

[14] Wilder, R. L., these Proceedings, **11**, 1925 (725–728). Although the proof is given in the terminology of the plane, it is clear that the theorem referred to is true in any locally compact metric space; in fact, the necessity proof holds in any topological space.

COMBINATORY TOPOLOGY OF CONVEX REGIONS

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In a recent paper[1] I have shown that a set of rectilinear simplexes filling a convex region in R_n (Euclidean n-space) form an E_n, i.e., an n-element in the combinatory sense. The first part of the proof there given may be replaced by the considerably shorter argument which follows. For the idea of this simplification I am indebted to Professor J. W. Alexander.

The theorem to be proved is as follows:

If A_n is a finite collection of (closed) rectilinear n-simplexes in R_n satisfying the conditions: (R_1) any two simplexes have either nothing or a k-component in common ($0 \leq k \leq n - 1$); (R_2) the points of A_n form a convex region in R_n; then the set of simplexes form an E_n.

Consider a convex region divided into convex rectilinear n-cells of any type, where by a convex rectilinear n-cell is meant a closed bounded region cut out of R_{n+1} by a finite number of flat $(n - 1)$-folds. A set of simplexes satisfying R_1 and R_2 can be obtained from such a collection of

cells by regular subdivision, i.e., by replacing the 1-, 2-, ..., n-cells successively by 1-, 2-, ..., n-stars of simplexes. An argument used in the paper referred to[2] shows that if it could be proved that regular subdivision of any such convex collection of convex cells (in particular of A_n), leads to an E_n, it would follow that A_n itself is an E_n. We wish then to prove

THEOREM A. *If K_n is a set of convex rectilinear n-cells filling a convex region in R_n, in such a way that the common part of two cells is either nothing or a k-cell of both $(0 \le k \le n - 1)$, then the set of simplexes obtained by regular subdivision of K_n is an E_n.*

The theorem being trivial if $n = 1$, suppose it true of sets of cells in R_{n-1}.

We can derive from K_n another collection of cells K_n^*, of a more special type, by "producing" all the $(n - 1)$-dimensional faces, i.e., introducing as barriers the complete intersection of K_n with all flat $(n - 1)$-folds containing an $(n - 1)$-face of an n-cell. This has the effect of replacing each n-cell of K_n by a collection of n-cells of the type of K_n itself. By the argument already referred to[2] if Theorem A can be proved for K_n^* its truth follows for K_n.

K_n^* has these properties: (P_1) no two boundary $(n - 1)$-cells of the same n-cell lie in the same flat $(n - 1)$-fold, (P_2) if a flat $(n - 1)$-fold contains an $(n - 1)$-cell all its points in K_n^* belong to $(n - 1)$-cells.

(a) Suppose K_n^* consists of a single cell. Let C_{n-1} be any boundary $(n - 1)$-cell, Π_{n-1} the sum of the remaining $(n - 1)$-cells,[3] π_{n-1} the $(n - 1)$-fold containing C_{n-1}. Let V be a point separated from the interior of K_n^* by π_{n-1}, but so near the center of gravity of C_{n-1} that if a straight line through V meets K_n^* it meets C_{n-1}. Since K_n^* is convex it follows that every line through V which contains a point of K_n^* contains one point of C_{n-1} and one point of Π_{n-1}. Thus projection of Π_{n-1} on to π_{n-1} from V gives a (1,1)-continuous representation of Π_{n-1} as a set of convex rectilinear $(n - 1)$-cells filling the convex region C_{n-1}. The conditions of Theorem A are satisfied (with $n - 1$ for n) and Π_{n-1}, on regular subdivision, becomes an E_{n-1}. Hence, by a known theorem,[4] $C_{n-1} + \Pi_{n-1}$, the complete boundary of K_n^*, becomes on regular subdivision a Σ_{n-1}— a combinatory $(n - 1)$-sphere. It now follows[5] that regular subdivision of K_n^*—the join of a central vertex to a Σ_{n-1}—gives an E_n.

(b) The proof for any set of cells K_n^* satisfying (P_1) and (P_2) now proceeds by induction through the number of cells. Let ρ_{n-1} be any $(n - 1)$-fold containing an $(n - 1)$-cell interior to K_n^*. No cell of K_n^* is divided in two by ρ_{n-1}: two cells lying on its two sides form two collections of n-cells satisfying (P_1) and (P_2), which by an inductive hypothesis, may be assumed to become two E_n's on regular subdivision. Their common points form a set of $(n - 1)$-cells, satisfying (P_1) and (P_2), i.e., on regular subdivision they form an E_{n-1}. Hence[6] the sum of the two E_n's is itself an E_n.

This completes the proof of Theorem A.

[1] *Proc. Lond. Math. Soc.*, **30**, 1930 (339–346). For definitions of combinatory entities see two papers, *Proc. Roy. Soc. Amsterdam*, **29**, 1926 (610–641), here cited as *F* I and *F* II.

[2] Loc. cit., p. 343, par. (d).

[3] All simplexes and cells in this paper are closed sets.

[4] *F* II, Theorem 6, Coroll. 1.

[5] *F* I, 12.

[6] *F* II, Theorem 8a, Coroll.

SOME REMARKS ON THE PROBLEM OF PLATEAU

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1. The following remarks are concerned with a recent paper of R. Garnier on the problem of Plateau.[1] This problem, as investigated by Garnier according to the ideas of Riemann, Weierstrass and Schwartz, may be stated in the following analytic form:

Given, in the xyz-space, a simple closed curve Γ^*. Determine three functions $x(u,v)$, $y(u, v)$, $z(u, v)$ with the following properties.

I. $x(u, v)$, $y(u, v)$, $z(u, v)$ are harmonic for $u^2 + v^2 < 1$

II. Satisfy for $u^2 + v^2 < 1$ the relations

$$x_u^2 + y_u^2 + z_u^2 = x_v^2 + y_v^2 + z_v^2,$$
$$x_u x_v + y_u y_v + z_u z_v = 0.$$

III. $x(u, v), y(u, v), z(u, v)$ remain continuous on the unit circle $u^2 + v^2 = 1$ and carry the unit circle in a one-to-one and continuous way into the given curve Γ^*.[2]

We shall see (§11), that at least for a certain class of curves Γ^* the surface $x = x(u, v)$, $y = (u, v)$, $z = z(u, v)$ is uniquely determined by the above conditions; therefore it is not advisable to restrict the problem by any further conditions.

2. The solution, given by Garnier, of the problem of Plateau, can be greatly simplified by the following

Theorem of Approximation.—Let Γ_n^* be a sequence of simple closed curves in the xyz-space with the following properties.

(α) The length $l(\Gamma_n^*)$ of Γ_n^* is uniformly bounded: $l(\Gamma_n^*) \leq L$.

(β) The problem of Plateau (as stated above) is solvable for Γ_n^* ($n = 1, 2, \ldots$).

(γ) The sequence Γ_n^* converges in the sense of Fréchet toward a simple closed curve Γ^*.[3]