$$L_{\sigma 1}^{\sigma+1} = \frac{l_{\sigma+1}}{\rho_\sigma}, \; L_{\sigma 1}^{\sigma-1} = -\frac{l_{\sigma-1}}{\rho_{\sigma-1}}, \qquad (6.2)$$

so that

$$\frac{1}{\rho_\sigma} = \frac{1}{2}\left(\frac{\partial g_{1\sigma+1}}{\partial u^\sigma} - \frac{\partial g_{1\sigma}}{\partial u^{\sigma+1}}\right) \qquad (6.4)$$

or

$$\frac{1}{\rho_\sigma} = \mu_{(\sigma+1\sigma)1} \qquad (6.3)$$

that is, the curvatures of a curve correspond to the μ's of a subspace. The analogues of the Ω's are, of course, zero.

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[1] *Gesammelte Werke*, **1876**, p. 261.

[2] The Latin indices $h, i, j, \ldots$ range through the values $1, 2, \ldots n$, the Greek $\alpha, \beta, \gamma, \ldots$ through $1, 2, \ldots m$, and the Greek $\pi, \sigma, \tau, \ldots$ through $m + 1, m + 2, \ldots n$. Repetition of an index indicates the sum obtained by allowing that index to take on all values of its range.

[3] *Rend. Lincei*, Rome, **31**^{1}, 21, 51 (1922).

[4] These PROCEEDINGS, **8**, 192–197 (1922).

[5] "Non-Riemannian Geometry," *Amer. Math. Soc. Colloq. Publ.* (1927), p. 64.

[6] *Ann. Math.*, **31**^{2}, 151 (1930).

[7] L. P. Eisenhart, *Riemannian Geometry*, Princeton, pp. 159–163.

[8] "Lezioni di Geometria Differenziale," Bologna, **2**, 450–455 (1924).

[9] *Riemannian Geometry*, pp. 106–107.

THE CRYSTAL CLOCK

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Read before the Academy, April 29, 1930

The crystal clock is a relatively new device for keeping accurate time. It consists essentially of a generator of constant frequency controlled by a resonator made of quartz crystal, with suitable means for producing continuous rotation controlled by it to operate time indicating and related mechanisms.

A crystal clock of this sort has been set up in the Bell Telephone Laboratories and has been operating over a considerable period. The apparatus was designed especially as a reference standard of frequency for the Bell System, but it was recognized that it might also serve as a reference standard of time, if developed for that purpose. As a matter of fact, since time interval and frequency are so closely related, it would be very desirable to have the same device serve as both a standard of time and a standard of frequency. Some types of time measurement can be made more readily by means of a time piece of the crystal clock type than by any other clock mechanism. This is the chief reason for describing the crystal clock at this time.

The apparatus for a crystal clock consists of a resonator made from quartz crystal, with means for controlling its temperature and the surrounding atmospheric pressure; a vacuum tube oscillating circuit which is controlled by the crystal; an electrical circuit, known as a submultiple generator, which is used to obtain a low frequency at an exact submultiple of the crystal frequency; and a time-indicating mechanism operated from this low frequency by means of a small synchronous motor.

The crystal used is especially designed to have a low temperature coefficient of frequency. It is made in the form of a ring with the plane of the ring parallel to the optic and electric axes. By using the proper proportions, it is possible to make the temperature coefficient of the ring as near to zero as desired at a given temperature. This is because the temperature coefficient for different modes of vibration is in some cases positive and in others negative. By adjusting the shape of the crystal the vibration can be apportioned between these modes so that their temperature coefficients annul each other. Other shapes[8] can be used to obtain a low temperature coefficient but the ring shape has some additional advantages concerned with the mounting. One of the ring-shaped crystals adjusted for a frequency of 100,000 cycles is shown in figure 1.

The crystal is mounted within a temperature controlled chamber and kept within about 0.01°C. of a constant temperature. The temperature coefficient of the crystal is adjusted to be less than one part in 10^6 per degree Centigrade. The change of frequency due to temperature variations alone is, therefore, less than one part in 10^8. The crystal with its mounting, including the temperature control equipment, is mounted under a hermetically sealed bell jar in order to keep the pressure surrounding the crystal at a constant value. The effect of pressure on the frequency is about one part in 10^7 per cm. of mercury.

The electrical circuit of the oscillator controlled by the crystal, shown in figure 2, is somewhat similar to one first described by Professor G. W. Pierce.[3] The two electrodes of the crystal are connected between the grid and filament of the oscillator tube, a grid leak being connected across the crystal to provide the proper grid bias. A parallel resonant circuit in the plate lead is tuned to a frequency approximately that of the crystal. It has been found possible to choose values for the inductance and capacity such that a small change in either has a negligible effect upon the frequency. Under this condition ordinary changes in filament and plate voltage also have very small effects. The oscillating tube is coupled to the output circuits through an intermediate amplifier stage in order to avoid reaction on the crystal due to variations in the load circuit.

A complete crystal controlled oscillator with temperature and pressure controlled crystal is shown in figure 3. The dial on the front is used to adjust the frequency by means of a small variable condenser connected in parallel with the crystal electrodes. Predicted changes in rate as small as one part in a hundred million can be made readily by this means. The meters indicate the filament and plate currents in the oscillator circuit.

A submultiple generator circuit, controlled by current at the crystal frequency, delivers current at a much lower frequency, an exact submultiple of it. This circuit serves like a reducing gear, in dividing the original high frequency by a definite integral amount. In the apparatus used at present, the reduction is made in two steps of 10 each, so that one cycle in the output corresponds to exactly 100 vibrations of the crystal.

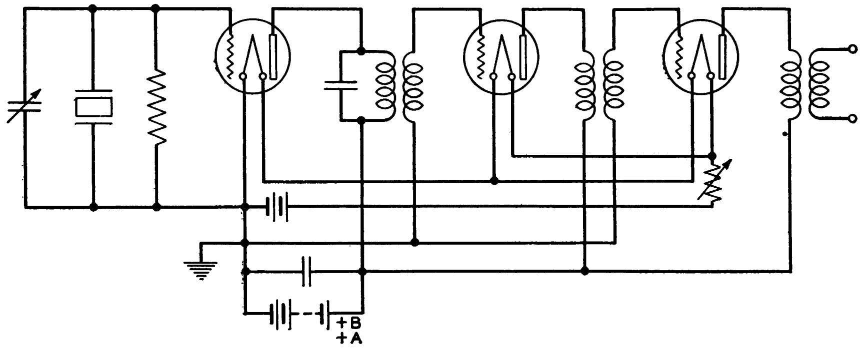

FIGURE 2

Electrical circuit of crystal oscillator.

The frequency thus obtained is 1000 cycles per second. This is used to operate a clock by means of a small synchronous motor.

The 1000-cycle motor and clock are shown in figure 4. The rotor of the motor is shown in figure 5. The motor unit shown, in addition to the 1000-cycle motor, includes a small induction starting motor, two generators for producing current at 100 cycles and 10 cycles, and a mercury damped flywheel used to reduce hunting. As a clock, all that is required is the synchronous motor, some means for starting and a suitably damped flywheel. A cam-operated contact device for giving seconds pulses is also shown. This is not essential in a clock but is convenient for making accurate time comparisons, as, for example, when checking against time signals.

The frequency reduction in the submultiple generator circuit, and the

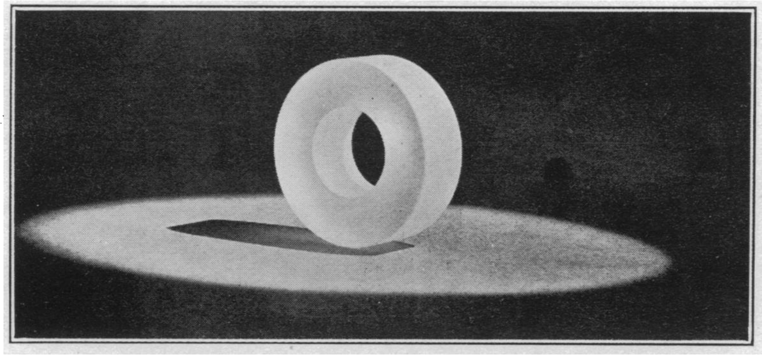

FIGURE 1

100,000-cycle low temperature coefficient crystal.

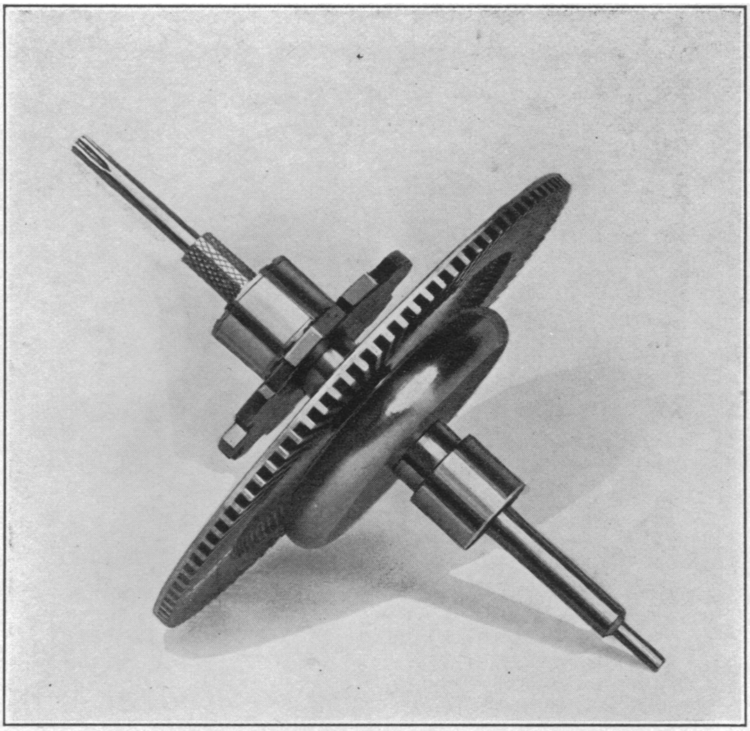

FIGURE 5

Rotor of 1000-cycle motor.

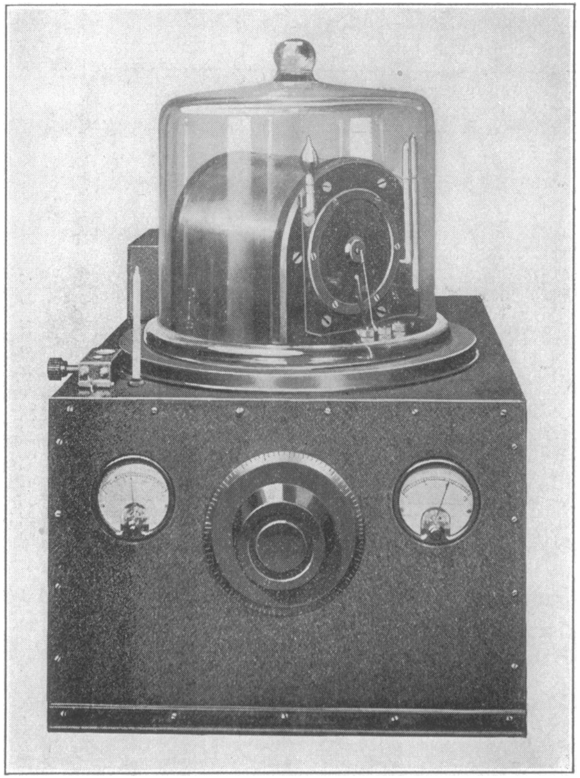

FIGURE 3

Complete crystal oscillator with mounted crystal.

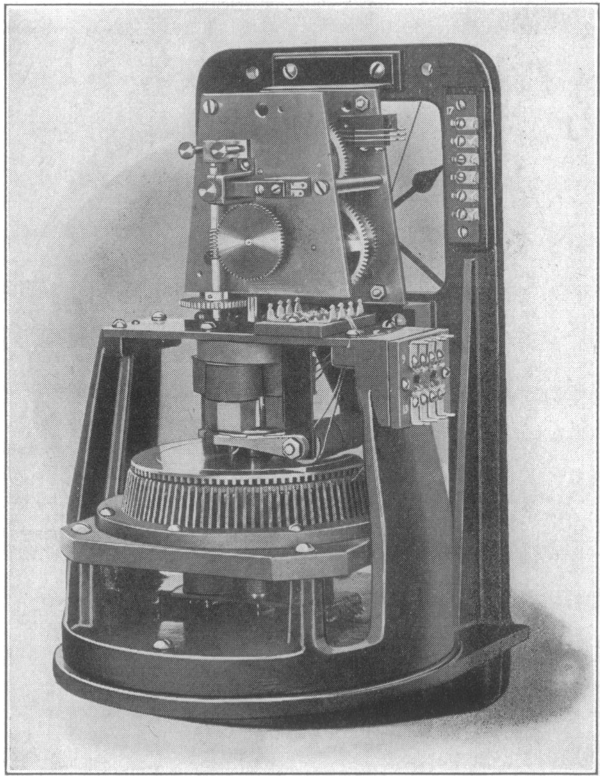

FIGURE 4

1000-cycle synchronous motor geared to clock.

gear reduction in the clock mechanism, are so chosen that, when the crystal has its nominal frequency exactly, the clock keeps accurate time. The time-keeping properties of the crystal clock, therefore, depend entirely upon the performance of the crystal. These, in turn, depend to some extent on certain external conditions such as pressure, temperature and vibration, which in most cases may be controlled accurately.

All of the tests made so far on the crystal clock have been over relatively short periods of time and it has not been possible, so far, to obtain what is perhaps the most interesting data from the standpoint of precision clock performance, that is, the undisturbed long time performance. However, certain results have been obtained which indicate that there may be many uses for a clock of this type and the prospects for further improvement are very good, inasmuch as the total development up to date has covered a period of only a few years.[2,6]

Some performance data indicating the constancy of rate are shown in figures 6, 7 and 8. Figure 6 gives the daily gain of the crystal clock during December, 1929, and part of January, measured in terms of U. S. Naval Observatory time signals. During this period the total change in rate was 0.14 second per day, the average variation being very much less. Toward the end of the record the observed variations were in the order of only one or two-hundredths of a second a day. This record shows a gradual increase in the rate of the crystal which has been going on for several months. It has not been determined as yet whether this change in rate is due to the crystal itself or to some variable in the mounting or in the electrical circuit.

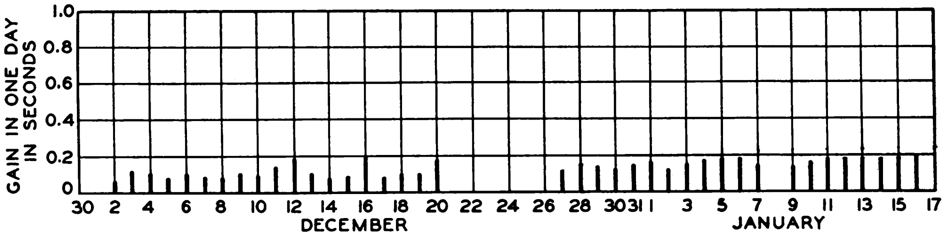

FIGURE 6

Daily rate of crystal clock against Naval Observatory time signals (corrected).

Figure 7 shows the comparison between the crystal used in obtaining the data of figure 7 and two others maintained with the same care at nearly the same frequency. It can be seen that during this period the three crystals remained much more constant relative to each other than relative to time signals.

Figure 8 shows a comparison made between two 100,000-cycle crystals to indicate the constancy over short time intervals. This comparison was made by means of a special spark chronograph[6] which measures accurately the period of each beat between two oscillators and gives a final accuracy of about one part in 10^{10}. In the figure the greatest deviation from the mean was about one part in a hundred million.

The crystal clock has a number of inherent advantages over other timekeeping devices. Some of these are due to properties of the crystal and others are due to properties of the system.

The properties of quartz crystal which makes it especially adaptable for this purpose are, chiefly, the low elastic hysteresis, the low temperature coefficient when suitably made, the chemical stability, and the resistance to deformation. The stability of shape and composition is perhaps the most important factor. Being hard and crystalline, and having been in its present form for a very long time, it could be expected not to change a great deal, except possibly due to relieving of strains after first being cut from the original crystal. A slight aging might be expected also, due to the continual vibration, but this can be reduced to a small amount by limiting the amplitude of vibration, or might be studied so that the effect could be predicted. Thus, from the standpoint of long continued performance, quartz crystal seems to be an ideal material to use as the controlling element in the clock.

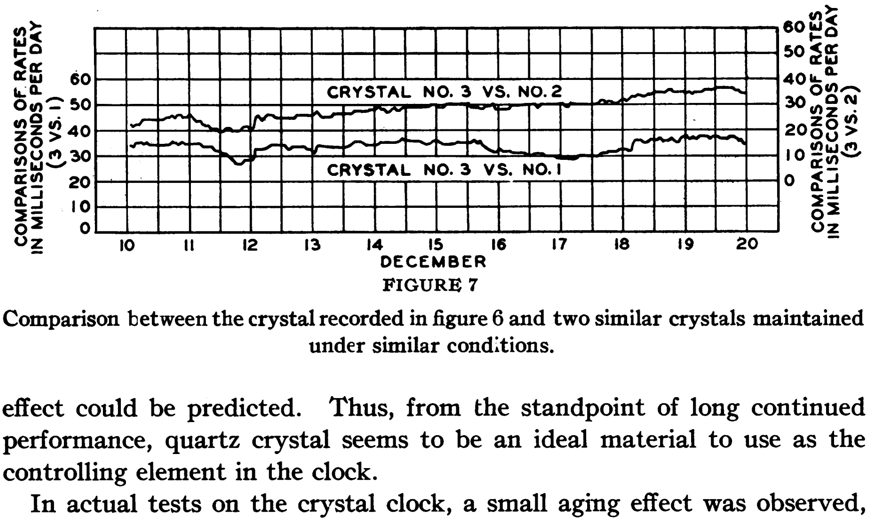

FIGURE 7

Comparison between the crystal recorded in figure 6 and two similar crystals maintained under similar conditions.

In actual tests on the crystal clock, a small aging effect was observed, continuing over several months, but decreasing steadily in amount. From the available data there is as yet no way to determine with certainty whether this aging was due to the crystal itself or due to some uncontrolled variable in the system. This will be investigated in further tests using fresh crystals.

On account of the small size of the controlling element and the nature of the vibration, it is affected to a relatively small extent by ordinary earth vibrations and by slight changes in level. Because of this, it is possible to set up a crystal clock in places that would not be at all suitable for an accurate pendulum clock. Also, on account of the nature of the controlling element, it is possible to operate a number of them near together without any observable mutual interference. The reaction between the crystal resonator and its support can be made very small so that slight changes in the supporting member itself will have a negligible effect upon the frequency of the crystal.

As far as is known, the frequency of the crystal is not affected by gravitation or magnetic fields, and it can be shielded perfectly from electrostatic fields. On account of the independence of the effect of gravity, the rate of the crystal would not be expected to vary due to a direct attraction of the sun or the moon, or due to a change in gravity caused by earth tides. The freedom from effects of magnetic and gravitational fields makes it possible to operate a crystal clock in the neighborhood of electrical machinery or large moving masses.

With the crystal clock, as with no other high precision clock, it is possible to make a continuous adjustment of the mechanism in order that the dial, indicating the time, shall always be correct within very narrow limits. This adjustment can be made either by the use of differential gearing in the clock mechanism, or by a continuous phase-shifting device in the electrical circuit. With a special form of phase-shifter, the alternating current operating the synchronous motor may be advanced or retarded gradually by any fraction or by any whole number of cycles. Since one whole cycle in this part of the circuit corresponds to 0.001 second, this provides a very accurate adjustment.

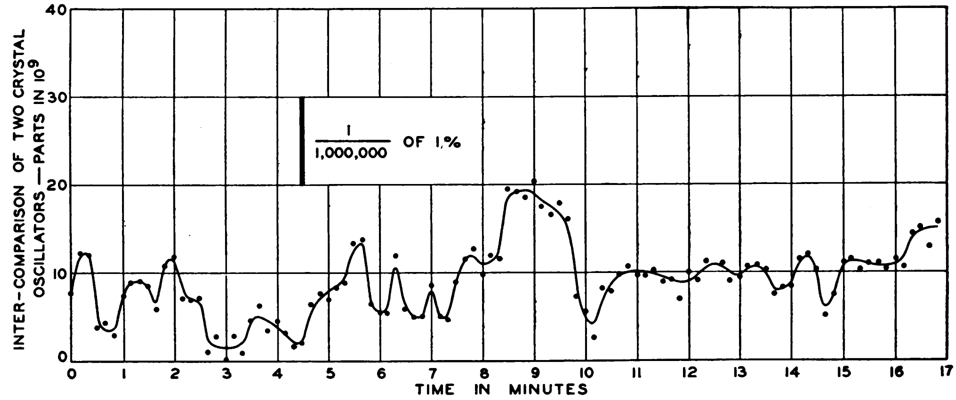

FIGURE 8

Comparison of two similar crystals over a relatively short interval.

The rate of the clock also may be adjusted by predetermined amounts, without, in any other way, disturbing its time-keeping qualities. This can be done by adjusting a small condenser connected in parallel with the crystal electrodes. The effect of varying this condenser is to change very slightly the effective stiffness of the crystal. Other variables in the circuit might be used for this adjustment, but the change of capacity is the simplest and the most easily controlled.

Any time-keeping installation, set up for the purpose of keeping accurate time, should include at least three clocks so that they can be interchecked continuously to expose any erratic behavior that may develop. The clocks used in such an installation should be as nearly independent of each other, and as independent of common disturbing factors, as possible. It should be possible to make continuous records of the intercomparisons entirely automatically and with a precision at least one order better than that required of the clocks as timekeepers.

All of these factors can be taken care of quite simply with crystal clocks. To compare them against each other with high precision, it is only necessary to count the beats between the separate oscillators, by means of an electrical circuit that has been developed for the purpose, and to record the beat numbers at regular intervals. Even the recording of these numbers can be done automatically. If one of the crystals is used to control an actual clock mechanism, the performance of each of the other crystals as clocks can be determined readily, and with extreme accuracy, from these data. As a matter of fact, the accuracy of intercomparison by this method over a few minutes' time is as great as the absolute accuracy with which the rate of a timekeeper can be determined over a year. The crystal previously described makes as many complete vibrations in five minutes as a seconds pendulum makes in two years.

The chief uses for a crystal clock would be those in which it is desired to obtain a continuous indication of time, accurate timing signals of various sorts, or a continuous and very accurately controlled motion. In the event, of course, that the accuracy of the crystal vibration can be increased by another order over that obtained at present, there would be many obvious uses as an accurate timekeeper, entirely apart from the other special properties of the system.

The crystal clock principle could be used in various problems requiring a uniform and very accurately controlled motion. Low frequency currents controlled by the crystal can be used to operate synchronous motors at a wide variety of speeds, depending on the frequency of the crystal, the ratio used in the frequency reducing circuits, and the number of poles on the motor. A motor so controlled could be used to control a chronograph, high speed oscillograph or even the motion of a telescope.

A special application of considerable value for the crystal clock would be in the transmission of time signals by radio or wire, as is done from a number of stations throughout the world. This could be done directly from the crystal clock motor by means of a cam-operated contact or by a light shutter used in conjunction with a photoelectric cell. If the photoelectric cell method were used, signals could be transmitted without the use of physical relay contacts anywhere in the system. By the use of amplifiers, the light signal could be modulated directly on the carrier wave.

Using an auxiliary sectored shutter, or the equivalent, the light going to the photoelectric cell could be modulated at exactly 1000 cycles per second, controlled by the crystal. With the signals thus produced, direct time-signal measurements could be made accurate to a thousandth of a second. The beginning of a signal would be indicated by the beginning of a pulse, the individual milliseconds would be determined by counting the cycles of the 1000 cycles' modulation from the beginning of a pulse. This would involve special indicating or recording equipment but would present no real difficulty. It would not in any way interfere with the ordinary use of the time signals. Special indications every 10 cycles to aid in counting could be provided easily by adjusting the slot widths in the light chopper disc. Instead of a complete interruption of tone every second, a change of amplitude of say 50 per cent could be used. This would indicate the seconds pulses just as effectively as in the present method, and, in addition, would allow measurements accurate to a millisecond to be made in terms of the radio signal without interpolating. Such accuracy in time measurement would be useful in a study of the velocity of wave propagation. One suggested arrangement for such a time-signal transmitting device is shown in figure 9.

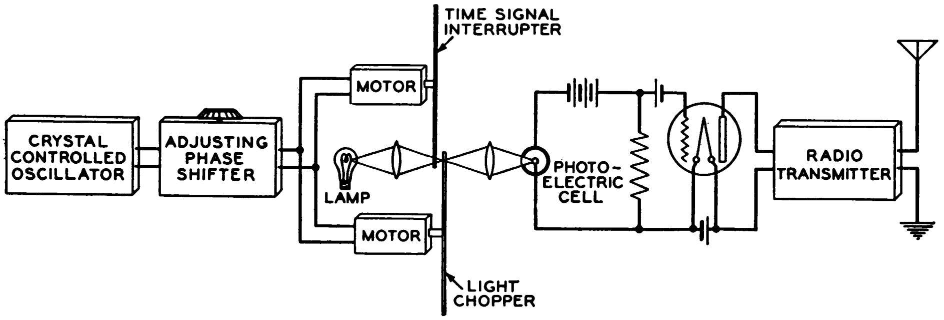

FIGURE 9

Schematic of proposed time signal transmitter.

It would be easy to insert any identifying signal, such as on the half-minute and on the minute. Also it would be possible to divide the minute into any number of equal parts, such as 59 to 61, for the purpose of obtaining "rhythmic" signals.

By the same device that is used to correct the indicated time, it is possible also to adjust the time-signal transmitting mechanism so that signals can always be sent out as accurately as the time is known. By thus eliminating the known clock error and most of the errors of mechanism, such as might be introduced by relay contacts, it should be possible to transmit very accurate signals. It might even be possible, except in work of the very highest precision, to avoid the necessity for publishing a table of time-signal corrections.

As a timekeeper in astronomical observatories, the crystal clock has some interesting possibilities. It would be possible to operate two clock mechanisms from the same crystal, one keeping mean solar time and the other keeping sidereal time. Both mechanisms are similar, but in the simplest mechanical arrangement the gear ratios are different. The electrical circuit of one of the frequency reducing units also is changed by the addition of a device to make a small correction in the frequency controlling the sidereal clock. The ratio of the rates can be adjusted in this way as accurately as the ratio of the sidereal to the mean solar day is known. According to data given in the American Ephemeris this ratio is 1.002,737,803,11 in 1930. The following is a simple design for a combined sidereal and mean solar clock adapted to indicate sidereal and mean solar time and to produce, if desired, electrical signals for sidereal seconds and mean solar seconds. A schematic drawing of such a dual clock arrangement is shown in figure 10.

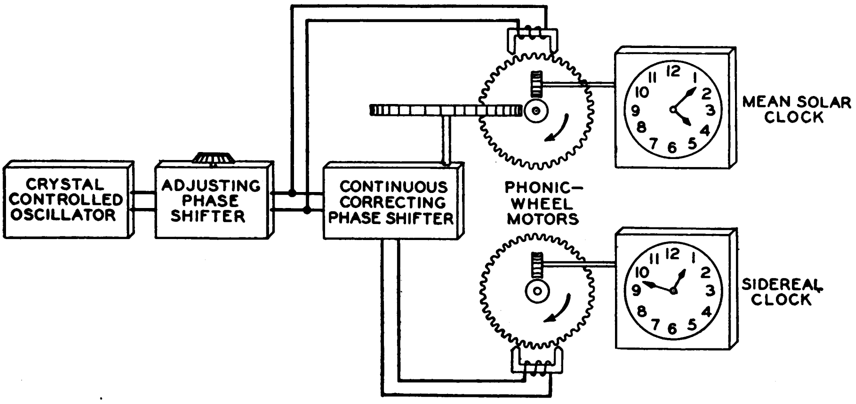

FIGURE 10

Schematic of combined mean solar and sidereal clock controlled by a single crystal.

The best fraction with three figure numbers to approximate the desired ratio is 366:365. These two numbers factor simply into 61 × 6 and 73 × 5. If two synchronous motors have 61 and 73 poles, and are geared down in the ratios of 6 to 1 and 5 to 1, respectively, the two slow speed shafts will rotate at speeds in the ratio $(61 \times 6):(73 \times 5) = 1.002{,}739{,}726{,}03$ when the motors are operated from the same frequency. These motors may then be geared to clocks through the usual clock train. If the operating frequency is such that the clock with the 61-pole motor keeps mean solar time, it will operate the other clock at a fair approximation to true sidereal time, the error being about one second a week. The frequency required for the solar clock is 366 cycles per second exactly. That required for the sidereal clock is 365 × 1.002,737,803,11 = (366 − 0.000,701,865) cycles per second.

The necessary correction in the frequency for controlling the sidereal clock is most easily made by means of a continuously operated phase shifter connected in the input circuit to the motor, which can be used to subtract a small amount from the base frequency. This phase shifter may be driven through suitable gearing from either clock motor or from a separate motor operating from the same source of current. In the following it will be assumed to be driven from the seconds shaft of the mean solar clock motor. The frequency used to operate the mean solar clock motor must, in the present case, be 366 cycles per mean solar second. This is a little too high for the sidereal clock motor, and 0.000,701,865 of a cycle per second must be subtracted from it in order to make the ratio of the rates correct (according to the astronomical tables) to the eleventh significant figure.

A suitable continuous phase shifter can be built in which one complete revolution of the driving shaft adds or subtracts one whole cycle. The problem then reduces to that of selecting a gear train that will obtain 0.000,701,865 revolutions per second from a driving shaft speed of one revolution per second. This ratio factors into 1:1000 and 701865:10^6. The first reduction may be obtained by a double worm gear such as 40 × 25 or 50 × 20. The second, to a sufficiently good approximation, may be obtained by a pair of spur gears having 113 and 161 teeth. The actual reduction obtained with these gears, used in combination, is 0.000,701,863 which gives the desired correction accurate to 0.000,000,002 cycles in 366, or better than one part in 10^{11}. With the phase shifter and the gear arrangement just described, therefore, the actual ratio of rates obtained would differ from the postulated true rate by only one second in thirty centuries.

During thirty centuries the true ratio of the sidereal to the mean solar day will probably change by more than that amount. From time to time a new gear reducing unit would have to be substituted for the one having a ratio of 113 to 161. This would present no difficulty since it is always possible to obtain a set of gears that will give any ratio, nearly unity, with an accuracy of one part in 10^5.

The current at 366 cycles to operate this dual clock can be obtained in a number of ways from a crystal-controlled oscillator as previously described. If successive reductions in the submultiple generator were taken as 10, 6 and 5 the crystal frequency would have to be 109,800 cycles.

Such a value would not be so convenient for use as a standard of frequency as an integral power of 10, such as 10^5. If this were an important consideration, a dual clock system as above described could be designed to operate from a crystal frequency of 100,000 cycles, or any other power of 10.

In one such possible arrangement both clock motors have 100 poles and rotate at one revolution per second. The mean solar clock motor operates from current at exactly 100 cycles obtained from a 100,000-cycle crystal by means of a three-stage submultiple generator. The sidereal clock motor requires current at 100.273,780,311 cycles to give the ratio of rates previously considered. This requires a phase shifter rotating about once in four seconds used in this case to increase the frequency driving the sidereal clock. A good approximation to the desired ratio can be obtained with two pairs of gears having the ratios 13:37 and 60:77 which, used in combination, give a ratio of 0.273,780,274. The ratio of rates of the two clocks with this arrangement will differ from the assumed true ratio by about one second in a century.

With the dual clock arrangement, it would be possible to make accurate time determinations in terms of either sidereal or mean solar time, always using the same time-keeping element. For example, star observations could be referred directly to the sidereal clock and any necessary corrections in the rate could be made in terms of that mechanism by the proper adjustment of the crystal oscillator. Such corrections, however, would also affect the mean solar clock by just the right amount, so that it would be unnecessary to make any computations to determine the corrections for the indicated mean solar time.

It might be possible to make observations on the rate of the crystal sidereal clock directly by means of a moving cross-hair, traveling across the field of vision in the transit instrument, under the control of a crystal controlled motor. The rate of travel would be made the same as that of the star image and would be adjusted in space so that, when the clock keeps accurate time from day to day, the cross-hair falls exactly on the star image. Any error in the clock, therefore, would be indicated by a displacement of the star image to one side or the other of the moving cross-hair. In order to allow a number of time stars to be used in this method, a number of cross-hairs could be used on the same moving element, preferably a disc, one corresponding to each favorite star. In using such a method, it would be necessary, of course, to allow for precession and other known systematic corrections. Real corrections in the rate of the clock, and in the absolute setting of the clock, which should be made from time to time, could be made either at the crystal oscillator or in the electrical circuit between the crystal and the clocks, so that such corrections affect both clocks at the same time and in the proper amounts.

It would thus be possible to combine, in a single system, mean solar and sidereal time-indicating mechanisms, means for rating the clocks in terms of time star observations and means for transmitting time and frequency signals with the absolute accuracy of the time determinations.

The following references cover some of the developments leading up to the crystal clock.

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THE BAND SPECTRUM OF OZONE IN THE VISIBLE AND PHOTOGRAPHIC INFRA-RED

By Oliver R. Wulf

Smithsonian Institution and U. S. Bureau of Chemistry and Soils

Read before the Academy April 29, 1930

In view of the fact that the molecule of ozone has held a prominent place in the studies of photochemical and thermal reaction rates, a knowledge of its spectrum and molecular structure is of interest. It is, furthermore, one of the simplest of triatomic molecules. The gas absorbs in the infra-red, rather generally though weakly across the visible, and, as is better known, very strongly in the ultra-violet. In view of this, and the fact that it exists in the upper atmosphere, ozone is of considerable meteorological interest. In all three regions of the spectrum it reduces the intensity of the incoming solar radiation, and it was owing to its strong ultra-violet absorption, which constitutes the termination of the solar spectrum in the ultra-violet, that the gas was discovered to exist in the upper atmosphere. The existing data on the visible absorption of ozone does not permit one to gain an adequate idea of the structure of the spectrum. In 1880, Chappuis[1] found eleven bands or regions of absorption in the visible and Schoene[2] added two more soon after. Later, Ladenburg and Lehmann[3] studied the visible bands in further detail. Colange[4]