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Cost Curves and Supply Curves

By

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It is the primary purpose of this article to develop a graphical exposition of the manner in which supply curves are dependent upon the different possible types of technological and pecuniary cost situations, under the usual assumptions of atomistic competition and of rational economic behavior on the part of the producers. No attempt is made here at realistic description of the actual types of relationship between costs and supply, and the purpose is the more modest one of presenting the formal types of relationship which can be conceived to exist under certain simplifying assumptions. Analysis of this kind derives obviously from the path-breaking contribution of Alfred Marshall in his *Principles of Economics*. Interest in this type of problem has been largely confined to the Anglo-Saxon countries, and in these countries there has been a tendency until recent years for economists to accept and reproduce the general lines of Marshall's analysis somewhat uncritically and without much further elaboration. I have no very serious fundamental criticism to make of Marshall's analysis of the supply side of the exchange value problem. But Marshall's treatment is highly elliptical. A striking illustration of his tendency to telescope his argument is his common practice in his graphs of labelling cost curves and supply curves alike with the symbols *ss*, conventionally used for supply curves, and thus diverting the attention of his readers, and perhaps also occasionally his own attention, from the necessity of selecting from among the possible types of cost curve that one which in the given circumstances alone has claims to being considered as also a supply curve. Marshall, moreover, although he made valuable additions to the conceptual terminology necessary for analysis of this type, nevertheless worked with vocabulary lacking sufficient terms to distinguish clearly from each other all the significant types of cost phenomena, and here also the terminological poverty tended to lead to inadequate classification not only on the part of his followers but on his own part. Marshall's analysis was excessively simple even on the basis of his own simplifying assumptions, and inadequately precise in formulation, and his followers have standardized an even simpler type of exposition of the relationship of cost to price.

In recent years a number of English economists, notably Pigou, Sraffa, Shove, Harrod and Robertson, have presented in the *Economic Journal* a series of criticisms, elaborations, and refinements of the Marshallian analysis which, in my opinion, go a long way both towards bringing out clearly the contribution contained in its implications as

well as in its explicit formulations, and towards completing and correcting it where that is necessary. The indebtedness of the present paper to their writings is considerable and is freely acknowledged. But I have been presenting charts such as those contained in this article to my students at the University of Chicago for a long period antedating the writings referred to above, and if in the course of years these charts have undergone substantial revision and, as I am convinced, correction, chief credit is due to the penetrating criticisms of my students.

The analysis which follows is based on the usual assumptions and presuppositions of the Marshallian type of economics. As compared to the Lausanne School type of analysis, it contents itself with examination of the conditions of a partial equilibrium of a special sort, and does not inquire into the repercussions of the postulated changes in cost or demand conditions on the general equilibrium situation. Like all partial equilibrium analysis, including the allegedly "general" equilibrium theories of the Lausanne School, it rests on assumptions of the *caeteris paribus* order which posit independence where in fact there is some degree of dependence. For such logically invalid assumptions there is the pragmatic defense that they permit of more detailed analysis of certain phases of economic interdependence than would be possible in their absence, and that to the extent that they are fictions uncompensated by counterbalancing fictions, it is reasonable to believe that the errors in the results obtained will be almost invariably quantitative rather than qualitative in character, and will generally be even quantitatively of minor importance. As compared to the Austrian School, there is, I believe, no need either for reconciliation or for apology. On the somewhat superficial level on which analysis of the present type is conducted the basic issue as between the English and the Austrian Schools does not enter explicitly into the picture, and in so far as it has any bearing on the conclusions, this bearing is again quantitative rather than qualitative in character. The Austrian School starts with the assumption, usually tacit, never emphasized, that the supplies of all the elementary factors of production are given and independent of their rates of remuneration. The English School emphasizes, perhaps overemphasizes, the dependence of the amounts of certain of the elementary factors, notably labor and waiting, on their rates of remuneration. The techniques of analysis of each school are in essentials identical, and each school, if it were to apply its techniques to the situation postulated by the other, would reach identical conclusions. The difference in the assumptions of the two schools has bearing on the quantitative but not on the qualitative behavior of the prices of the elementary factors and therefore also of the money costs of their products, as the demands for these factors and products change. The conflict between the two schools has greater significance for the theory of the value of the elementary factors of production, i. e., for the theory of distribution, than for the theory of particular commodity price determination. For the present analysis, where it is assumed either that the prices of the elementary factors remain unaltered

or that they undergo changes of a kind consistent with the basic assumptions of either school, the differences between the two schools would not affect qualitatively the character of the findings. All of the propositions laid down in this paper should, I believe, be acceptable to, or else should be rejected by, both schools.

The procedure which will be followed, will be to begin in each case with the mode of adjustment of a particular concern to the given market situation when the industry as a whole is supposed to be in stable equilibrium. This particular concern is not to be regarded as having any close relationship to Marshall's "representative firm". It will not be assumed to be necessarily typical of its industry with respect to its size, its efficiency, or the rate of slope of its various cost curves, but it will be assumed to be typical, or at least to represent the prevailing situation, with respect to the general qualitative behavior of its costs as it varies its own output or, in certain situations, as the industry of which it is part varies its output. All long-run differences in efficiency as between concerns will be assumed, however, to be compensated for by differential rates of compensation to the factors responsible for such differences, and these differential rates will be treated as parts of the ordinary long-run money costs of production of the different concerns. In the long-run, therefore, every concern will be assumed to have the same total costs per unit, except where explicit statement to the contrary is made. It will be assumed, further, that for any industry, under long-run equilibrium conditions, the same relationships must exist for every concern between its average costs, its marginal costs, and market price, as for the particular concern under special examination. But the reasoning of this paper would still hold if the realistic concession were made that in every industry there may be a few concerns which are not typical of their industry with respect to the qualitative behavior of their costs as output is varied either by themselves or by the industry as a whole, and which therefore do not wholly conform to these assumptions. It may be conceded, for instance, that in an industry in which for most producers expansion of their output means lower unit costs there should be a few producers for whom the reverse is true.

Short-Run Equilibrium for an Individual Concern

Chart I, which represents the behavior of money costs in the short-run for a single concern with a plant of a given scale, is the fundamental graph, and is incorporated in or underlies all the succeeding ones¹.

¹ The charts were drawn for me by Y. K. Wong of the University of Chicago. Where in any chart one curve is derived from another or a combination of other curves presented in the same chart, it is drawn mathematically to scale. No attempt has been made, however, to maintain the same scales as between different charts. An attempt has been made to use mnemonic symbols for the various curves, *MC* for instance indicating marginal cost, *P* indicating price, and so forth. It is hoped that this will facilitate reading of the charts.

It is assumed that this concern is not of sufficient importance to bring about any change in the prices of the factors as a result of a change in its output. Since unit money costs of production are the sum of the products of the amounts of the factors used in the production of one unit multiplied by the prices of the factors, any change in unit money costs as output varies must in this case be due, therefore, to changes in the amounts of the factors required for the production of one unit, or to use Walras' term, to changes in the "technological coefficients of production". The "short-run" is taken to be a period which is long enough to permit of any desired change of output technologically possible without altering the scale of plant, but which is not long enough to permit of any adjustment of scale of plant. It will be arbitrarily assumed that all of the factors can for the short-run be sharply classified into two groups, those which are necessarily fixed in amount, and those which are freely variable. "Scale of plant" will be used as synonymous with the size of the group of factors which are fixed in amount in the short-run, and each scale will be quantitatively indicated by the amount of output which can be produced at the lowest average cost possible at that scale. The costs associated with the fixed factors will be referred to as the "fixed costs" and those associated with the variable factors will be called the "direct costs". It is to be noted that the "fixed costs" are fixed only in their aggregate amounts and vary with output in their amount per unit, while the "direct costs" are variable in their aggregate amount as output varies, as well as, ordinarily at least, in their amount per unit. Amounts of output are in this as in all the succeeding charts measured along the horizontal axis from O , and money costs and prices along the vertical axis from O .

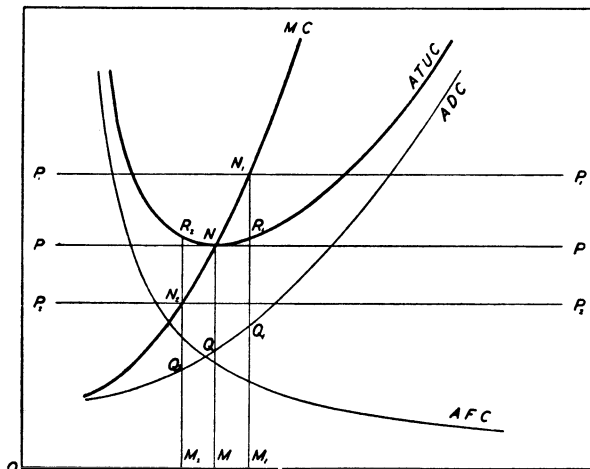


Chart I. Short-Run Cost Curves

The curve AFC represents the trend of the average fixed costs per unit as output is increased. Since these are the costs associated

with the parts of the working combination which, by hypothesis, are absolutely fixed in their aggregate amount, this curve must be a rectangular hyperbola¹). The curve *ADC* represents the trend of average direct costs per unit as output is increased. Since the increase in output is the result of the application, to a constant amount of "fixed" factors, of increased amounts of the variable factors, the law of diminishing returns, if it is operative, should make the output per unit of the variable factor employed diminish, i. e. should make the "direct" technical coefficients of production increase, as total output increases. As the prices of the factors by assumption remain constant, the average direct costs must also increase as output increases, if the law of diminishing returns is operative. It is assumed, not, I believe, without justification, that within the useful range of observation the law of diminishing returns is operative, and the average direct cost curve is therefore drawn positively inclined throughout²). The curve *ATUC* represents the trend of average total (i. e. fixed plus direct) unit costs as output is increased, and is, of course, the sum of the ordinates of the *ADC* and *AFC* curves. It is necessarily *U*-shaped for all industries having any substantial fixed costs, and is in this respect a universal short-run curve qualitatively descriptive of the short-run behavior of average costs of practically all concerns and all industries which cannot quickly and completely adjust the amounts of all the factors they use to variations in their rates of output. But the relative lengths and the relative rates of inclination of the negatively inclined and the positively inclined portions of the curve will differ from concern to concern and from industry to industry, depending upon the relative importance of the fixed to the total costs and upon the degree of sharpness with which the law of diminishing returns is operative for the variable factors. The curve *MC* represents the trend of marginal costs as output is increased. Any point on it represents the increase in aggregate costs as output at that point is increased by one unit³).

The marginal cost curve must cut the average cost curve at the lowest point of the latter. At the point of intersection, average cost and marginal cost are of course equal. But average cost is equal to marginal cost only when average cost is constant, i. e. when the average

¹) I. e., the equation to the curve will be of the form $xy = c$.

²) It is also drawn concave upward, to indicate the progressively sharper operation of the law of diminishing returns as the fixed factors are more intensively exploited.

³) If y_a = average fixed cost per unit, y_b = average direct cost per unit, and x = output, then $ATUC = y_a + y_b$, and $MC = \frac{d[(y_a + y_b)x]}{dx}$. It is important to note that no consideration need be given to the fixed costs, if they really are absolutely fixed, in computing the marginal cost. Since $xy_a = c$, and $\frac{dc}{dx} = 0, \dots \frac{d[(y_a + y_b)x]}{dx} = \frac{d(xy_b)}{dx}$.

cost curve is a horizontal line¹). The point of intersection of the marginal cost curve with the average cost curve when the latter is concave upwards must therefore be at the lowest point of the latter, where its tangent is a horizontal line²).

If this particular producer is an insignificant factor in his industry, i. e., if atomistic competition prevails, he may reasonably assume that no change in his output, and especially no change consistent with the maintenance of the scale of plant at its original level, will have any appreciable effect on the price of his product. Under these conditions, the partial demand curve for his product may be taken as a horizontal line whose ordinate from the base is equal to the prevailing price³). It will be to his interest to carry production to the point where marginal cost equals price, i. e. his short-run *MC* curve will also be his rational short-run supply curve. If price is *MN*, this will mean an output of *OM* and no extra profit or loss on his operations, i. e. the quasi-rent on his fixed investment per unit of output, *NQ*, would be equal to the fixed costs per unit. If price is P_1 , output will be OM_1 , and the quasi-rent per unit of output, $N_1 Q_1$, will be in excess of the fixed costs per unit, $R_1 Q_1$. If P_2 is the price, the output will be OM_2 , and the quasi-rent per unit of output will be $N_2 Q_2$, or less than the fixed costs per unit, $R_2 Q_2$. All of these situations are consistent with short-run equilibrium, which, as far as individual producers are concerned, requires only that marginal cost equal price. The short-run supply curve for the industry as a whole is not shown in this chart, but is simply the sum of the abscissae of the individual short-run marginal cost (= individual supply) curves⁴).

Long-Run Equilibrium

The long-run is taken to be a period long enough to permit each producer to make such technologically possible changes in the scale of his plant as he desires, and thus to vary his output either by a more or less intensive utilization of existing plant, or by varying the scale of his plant, or by some combination of these methods. There will therefore be no costs which are technologically fixed in the long-run⁵), and

¹) If x = output, and y = average cost, marginal cost = $\frac{d(xy)}{dx}$. If $y = c$, then $\frac{d(xy)}{dx} = y$. If y is an increasing function of x , then $\frac{d(xy)}{dx} > y$. If y is a decreasing function of x , then $\frac{d(xy)}{dx} < y$.

²) For a mathematical proof, see Henry Schultz, "Marginal Productivity and the Pricing Process", *Journal of Political Economy*, XXXVIII (1929), p. 537, note 33.

³) This is equivalent to saying that the partial demand for his product has infinite elasticity.

⁴) It is shown in Chart II.

⁵) This is, of course, not inconsistent with the proposition that at any moment within the long-run there will be costs which from the short-run point of view are fixed.

if in fact the scale of plant is not altered as long-run output alters, it will be the result of voluntary choice and not of absolute technological compulsion. For an industry as a whole long-run variations in output can result from more or less intensive use of existing plants, or from changes in the scale of plants, or from changes in the number of plants, or from some combination of these. Under long-run equilibrium conditions changes in output, whether by an individual producer or by the industry as a whole, will be brought about by the economically optimum method from the point of view of the individual producers, so that each producer will have the optimum scale of plant for his long-run output. To simplify the analysis, it will be assumed that in each industry the optimum type of adjustment to a long-run variation in output for that industry as a whole will not only be alike for all producers but will involve only one of the three possible methods of adjustment listed above; namely, change in intensity of use of existing plants, change in scale of plants, and change in number of plants. The theoretical static long-run, it should be noted, is a sort of "timeless" long-run throughout which nothing new happens except the full mutual adjustment to each other of the primary factors existing at the beginning of the long-run period. It is more correct, therefore, to speak of long-run equilibrium in terms of the conditions which will prevail after a long-run, rather than during a long-run. Long-run equilibrium, once established, will continue only for an instant of time if some change in the primary conditions should occur immediately after equilibrium in terms of the pre-existing conditions had been reached. The only significance of the equilibrium concept for realistic price theory is that it offers a basis for prediction of the direction of change when equilibrium is not established. Long before a static equilibrium has actually been established, some dynamic change in the fundamental factors will ordinarily occur which will make quantitative changes in the conditions of equilibrium. The ordinary economic situation is one of disequilibrium moving in the direction of equilibrium rather than of realized equilibrium.

For long-run equilibrium not only must marginal cost of output from existent plant equal price for each individual producer, but it must also equal average cost. If this were not the case, there would be either abnormal profits or losses, which would operate either to attract capital into the industry or to induce withdrawal of capital from the industry, and in either case would tend to bring about a change in output. For long-run equilibrium it is further necessary not only that each producer shall be producing his portion of the total output by what is for him, under existing conditions, the optimum method, but that no other producer, whether already in the industry or not, shall be in a position to provide an equivalent amount of output, in addition to what he may already be contributing, at a lower cost. The relations of costs to supply in the long-run will depend on the technological conditions under which output can be most economically varied, and the succeeding discussion will consist in large part of a classification and analysis of these conceivable types of technological conditions.

"Ricardian" Increasing Costs

Chart II illustrates a special case corresponding to the Ricardian rent theory in its strictest form. Let us suppose that a given industry is already utilizing all of the supply available at any price of a necessary factor of production, so that the output of the industry as a whole can be increased only by the more intensive utilization of the absolutely limited factor. Suppose also that no appreciable economies are to be derived, whatever the output of the industry as a whole, by a combination into larger productive units, or a subdivision into smaller productive

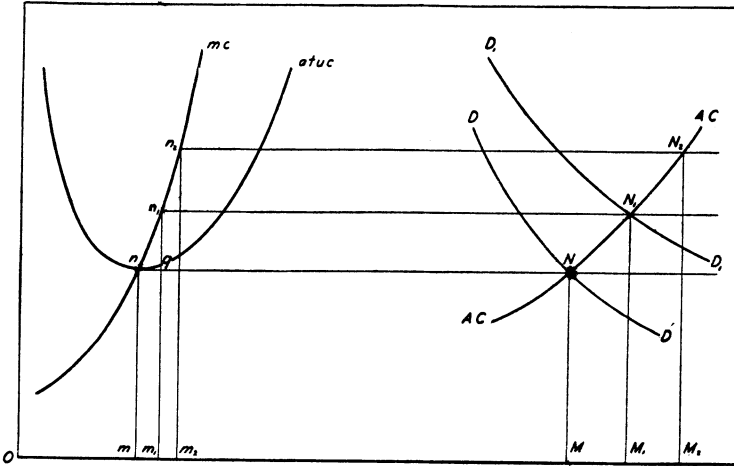


Chart II. "Ricardian" Increasing Costs

units, of the existing concerns. In order further to simplify the analysis, it is assumed that the identical portions of the working-combination which in this case remain technologically fixed in amount whatever may be the short-run variations in output also remain economically fixed in amount whatever long-run variations in output may occur. If the particular concern whose costs are indicated in the left-hand portion of Chart II and the particular concern with which Chart I is concerned were identical, and if the two charts were drawn to the same scale, the MC curve in Chart I and the mc curve in Chart II would be identical, although the former represents the short-run trend and the latter represents the long-run trend of marginal costs as output is varied, i. e., for these assumptions, the short-run and the long-run marginal cost curves would be identical. The $atuc$ curve in Chart II, continuing these assumptions, would simply represent the short-run variations in average cost for this particular concern as output was varied, when long-run price was mn or MN ¹⁾, and would be in all respects identical with the $ATUC$ curve of Chart I. When long-run price was MN , this

¹⁾ The qualifying phrase in italics is important. Its significance is explained in the next paragraph of the text.

concern would be in both short-run and long-run equilibrium when its output was Om , and its average cost, its marginal cost, and price were all equal.

Suppose now, that owing to a long-run increase of market demand from DD to $D_1 D_1$, long-run price rises to $M_1 N_1$. It will pay our producer to increase his output to Om_1 , at which point the new marginal cost, $m_1 n_1$, will be equal to the new price. If the prices of all the factors remain the same, the new price will be higher than the new average cost $m_1 q$. But it is impossible, for a case such as this, to adhere to the assumption that the prices of all the factors remain the same. Given an absolutely limited amount of one of the factors, no change in the prices of the other factors, and a rise in the long-run demand for and in the long-run price of its product, and the long-run price of this absolutely scarce factor must rise. Let us suppose that the fixed factor is land. Its price or rent will rise until there ceases to be any excess of marginal over average cost. The *atuc* curve in Chart II therefore has only short-run significance. A long-run increase in the price of the product will cause an increase in the price of land-use, and therefore a rise in the entire *atuc* curve. The increase in land-rent, however, will have no effect on marginal costs, and therefore on the long-run *mc* curve, for it will be due to the increase in price of the product and not to the increase in output of this particular concern. Even if this producer maintained his output at Om , after long-run price had risen to $M_1 N_1$, the *atuc* curve would rise in the same manner and degree. It would always shift upward in such a way, however, that the *mc* curve would intersect it at its lowest point¹⁾, i. e. rent for land would rise just sufficiently to make the new lowest average cost equal the new equilibrium marginal cost. When the long-run price was $M_1 N_1$, therefore, average cost, marginal cost, and price would be equal for each producer under long-run equilibrium.

The *AC* curve in the right-hand portion of Chart II represents the long-run supply curve for the industry as a whole, and is simply the sum of the abscissas of the individual *mc* curves. It is also a long-run average cost curve for the industry as a whole inclusive of rent, and a long-run marginal cost curve for the industry as a whole exclusive of rent. For the individual producer, the changes in rent payments required as demand changes are due primarily to the changes in demand, secondarily to the changes in output of the industry as a whole, and

¹⁾ Each successive short-run *atuc* curve of a particular producer, as the long-run price of his product rises, consists of the ordinates of his former *atuc* curve plus a new rent charge fixed in total amount regardless of his output, and therefore of the form $xy=c$. As was pointed out in note 3, page 27, the vertical addition of a rectangular hyperbola to an average cost curve does not affect the marginal cost curve derivable from it. The same *mc* curve can, therefore, continue to be the short-run marginal cost curve, even when the short-run average cost curve is undergoing long-run changes consistently with the conditions assumed in this case.

only to an insignificant degree to his own changes in output. The individual producer will therefore not take the effect on his rent payments of increased output on his own part into account, and the supply curve for the industry as a whole will therefore be the marginal cost curve for the industry as a whole exclusive of rent¹).

This appears to be the case usually designated in the textbooks as the case of "increasing costs". I have labelled it as "Ricardian increasing costs" to indicate its close relationship to the Ricardian rent theory. It is to be noted that as output increases the long-run average costs rise even if the increase of rents is disregarded and that there are increasing unit technological costs, therefore, whether the technical coefficients are weighted by the original or by the new prices of the factors. There are increasing marginal costs in every possible sense of the term costs.

If mc were the short-run marginal cost curve for a scale adapted to a long-run equilibrium output of Om , and if not all the factors which were technologically fixed in the short-run remained economically fixed in the long-run as output was increased, then, since there would be less scope for the operation of the law of diminishing returns, the long-run marginal cost curve for the particular concern would be different from and less steeply inclined than the mc curve, and the new short-run $atuc_1$ curve for a long-run equilibrium scale of output of, for example, Om_1 would have no simple relationship to the $atuc$ curve in Chart II. Similarly, the long-run supply curve for the industry as a whole, since it is the sum of the abscissas of the individual long-run marginal cost curves, would then also be less steeply inclined than the AC curve in Chart II, which would then be only a short-run supply curve for the industry as a whole, when the long-run equilibrium output of the industry was OM .

Constant Costs

In the short-run, for industries which have any fixed costs whatsoever, constant marginal costs as output is varied are wholly inconceivable if the law of diminishing returns is operative, and constant average costs are inconceivable if there are increasing marginal costs as required by the law of diminishing returns²).

¹) For the industry as a whole, however, the increase of output as demand increases will affect rent, on the one hand by influencing price and gross receipts, and on the other hand by influencing gross expenses. Depending upon the shift in position and the elasticity of the demand curve and upon the rate of slope of the industry marginal cost curve exclusive of rent, an increase of output when demand increases may make rent either greater or less than if output were kept constant. But under atomistic competition the possible results of keeping output constant when demand rises will play no part in the determination of output, of price, or of rent.

²) Let x = output, y_a = average fixed costs per unit, y_b = average direct costs per unit, and c and k be two different constants. Suppose that short-

In the long-run, however, constant costs are theoretically conceivable under two kinds of circumstances. The first case is when each producer can vary his scale of production without affecting his long-run average costs. The situation in this case for any individual concern will be as represented in Chart III. The curves $atuc_1$ and mc_1 represent,

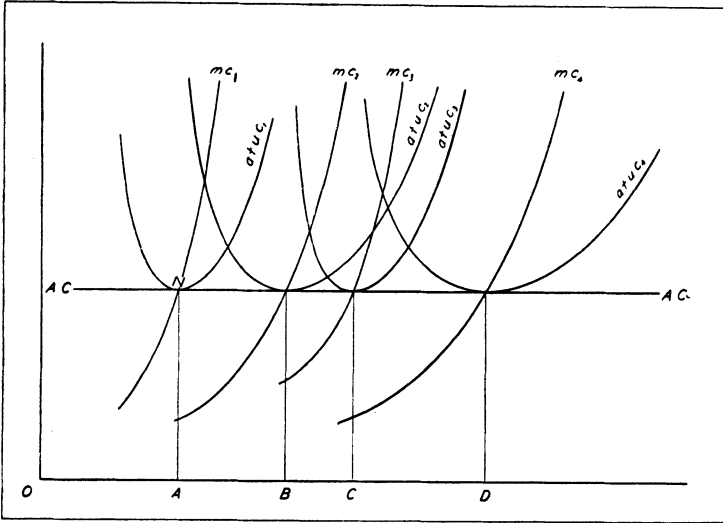


Chart III. Constant Costs

respectively, the short-run trends of average and marginal costs as output is varied from a plant of scale OA . The curves $atuc_2$ and mc_2 , similarly represent, respectively, the short-run trends of average and marginal costs as output is varied from a plant of scale OB ; and similarly, for scales OC and OD . In the long-run any output would be produced from the optimum scale for this output. The long-run average cost curve would therefore be the horizontal line AC , which passes through the lowest points of all the short-run $atuc$ curves. Where average costs are constant as output varies, average cost and marginal cost are always identical¹). This horizontal line would therefore also be the individual producer's long-run supply curve.

This case presents certain difficulties when perfect competition prevails which make it impossible to indicate graphically the relationship between the long-run supply curves of the individual concern and the industry as a whole. Read as an ordinary supply curve, the AC line indicates that in the long-run this concern would be unwilling to operate

run average costs are constant, i. e. that $y_a + y_b = k$. But $xy_a = c$. Then $xy_b = kx - c$, and marginal cost, or $\frac{d(xy_b)}{dx} = \frac{d(kx - c)}{dx} = k$, which is inconsistent with the law of diminishing returns.

¹) See note 1, page 28.

at any price under AN , would be willing to produce any amount at a price AN , and would be anxious to produce unlimited quantities at any price over AN . If the costs of different producers in the industry are not uniform, then the lowest cost concern would tend to monopolize the industry. If the costs of different producers are uniform, the supply curve for the industry would be indefinite, and in the long-run there would be a constant tendency toward overproduction, with consequent losses and a reaction toward underproduction. Actual long-run price and output would be unstable, but would oscillate above and below stable points of equilibrium price and equilibrium output.

The second conceivable case of long-run constant costs, not illustrated graphically here, would be presented by a situation in which all of the concerns within the industry and an indefinite number of potential members of the industry can operate at long-run minimum average costs uniform as between the different concerns, but with average costs increasing for each as its output increases. The long-run output of the industry would then consist of the sum of the outputs of all the member concerns, each operating at that scale at which its costs are at the minimum common to all, and variations of output for the industry as a whole would result wholly from variations in the number of producers, each of whom would maintain a constant output while he remained in the industry. For the industry as a whole, therefore, long-run production would take place under conditions of constant long-run average and marginal cost, uniform for all producers and equal to each other, although each concern would be operating subject to short-run increasing average and marginal costs. Here also actual long-run price and output for the industry as a whole would tend to be unstable, but would oscillate above and below stable points of equilibrium price output.

The situation would in these two cases be somewhat analogous to that of a thermostatic control which aims at maintaining a uniform temperature, which is stimulated into operation only when there is a significant degree of variation from the desired temperature, and which succeeds only in keeping the ever-present variations from the desired temperature from exceeding narrow limits in either direction. Completely stable equilibrium under constant cost conditions is only conceivable on the assumption of some departure from perfect competition, in consequence of which variations in output by individual producers, or entrance into the industry by new producers or withdrawal of old, are subject to some difficulty even in the long-run after the equilibrium price and output have once been momentarily established.

Net Internal Economies of Large-Scale Production

We owe to Marshall the important distinction between the "internal" and the "external" economies resulting from increased output. For present purposes we will use the term "net internal economies of large-scale production" to mean net reductions in costs to a particular

concern resulting from a long-run expansion in its output when each output is produced from a plant of the optimum scale for that output. The word "net" is introduced to make it clear that increase in output may result at the same time in economies and in diseconomies and that it is only the excess of the former over the latter to which reference is made here. Internal economies of large-scale production are primarily a long-run phenomenon, dependent upon appropriate adjustment of scale of plant to each successive output. They should not be confused with the economies resulting from "spreading of overhead", which are a short-run phenomenon, represented by the negative inclination of the average fixed cost curve in Chart I. Internal economies of large-scale production need not be relatively greater for those particular costs which in the short-run are the fixed costs than for those particular costs which in the short-run are the direct costs. In the long-run, in any case, there are no technologically fixed or overhead costs, if the definitions here followed of "long-run" and of "fixed costs" are adhered to. Internal economies of large-scale production are independent of the size of output of the industry as a whole, and may be accruing to a particular concern whose output is increasing at the same time that the output of the industry as a whole is undergoing a decline. It is for this reason that Marshall gave them the name of internal, to distinguish them from the external economies which are dependent on something outside the particular concerns themselves, namely, the size of output of the industry as a whole.

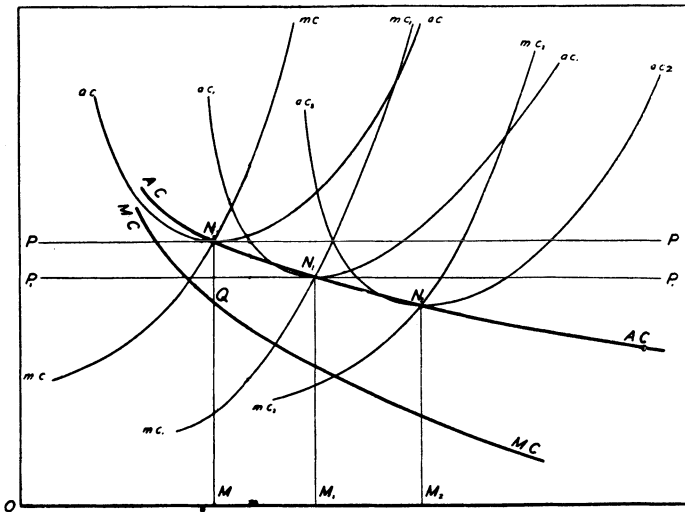


Chart IV. Net Internal Economies of Large-Scale Production

Internal economies may be either technological or pecuniary, that is, they may consist either in reductions of the technological coefficients of production or in reductions in the prices paid for the factors as the result of increases in the amounts thereof purchased. Illustrations of

technological internal economies would be savings in the labor, materials, or equipment requirements per unit of output resulting from improved organisation or methods of production made possible by a larger scale of operations. Pecuniary internal economies, on the other hand, would consist of advantages in buying, such as "quantity discounts" or the ability to hire labor at lower rates, resulting from an increase in the scale of purchases¹⁾.

Chart IV illustrates the behavior of the cost curves for a particular concern which enjoys net internal economies of large-scale production. As in Chart III the *ac* curves and the *mc* curves represent the short-run variations in average and marginal costs respectively, as output is varied from plants of each indicated scale. The *AC* curve represents the long-run trend of average costs, that is, the trend of average costs when each output is produced from a plant of the optimum scale for that output, and is drawn so as to connect the points of lowest average cost for each scale of plant²⁾. The *MC* curve is the long-run marginal curve for this particular concern when the *AC* curve is interpreted as a continuous curve. It represents the increment in aggregate costs resulting from a unit increase in output, when each output is produced from a plant of the optimum scale for that output. It is to be noted that while the short-run marginal cost curves are positively inclined, the long-run marginal cost curve is negatively inclined³⁾.

¹⁾ Pecuniary internal economies are, theoretically, as likely to result from expansion of output from a given plant as from expansion of output brought about by increase of scale of plant. But it is only the latter form of expansion of output which is likely to be great enough to result in significant pecuniary internal economies.

²⁾ The *AC* curve would represent a continuous trend only if it is assumed that scale of plant can be modified by small increments. If the curve is interpreted as a discontinuous one, then only the points N, N_1, N_2, \dots on it are significant, and the significant long-run costs for the intervals between are the lowest short-run average costs available for the indicated outputs. It may be noticed that at certain points the short-run *ac* curves are drawn so as to sink below the long-run *AC* curve. If the *AC* curve is interpreted as having significance only at the N points, this is of no consequence. But if the *AC* curve is interpreted as a continuous curve, this is an error. My instructions to the draftsman were to draw the *AC* curve so as never to be above any portion of any *ac* curve. He is a mathematician, however, not an economist, and he saw some mathematical objection to this procedure which I could not succeed in understanding. I could not persuade him to disregard his scruples as a craftsman and to follow my instructions, absurd though they might be.

³⁾ If y, y_1, y_2 are the short-run average costs for scales of plant, OM, OM_1 , and OM_2 , respectively, as indicated by the *ac* curves; Y = long-run average cost, as indicated by the *AC* curve; x = output; mc, mc_1 , and mc_2 indicate the short-run marginal costs as represented by the *mc* curves; and *MC* indicates the long-run marginal cost, as represented by the *MC* curve, then:

$$mc = \frac{d(xy)}{dx}; \quad mc_1 = \frac{d(xy_1)}{dx}; \quad mc_2 = \frac{d(xy_2)}{dx}; \quad \text{and} \quad MC = \frac{d(xY)}{dx};$$

$$\text{and} \quad \frac{d^2(xy)}{dx^2} > 0; \quad \text{and} \quad \frac{d^2(xY)}{dx^2} < 0.$$

The familiar proposition that net internal economies of large-scale production and long-run stable equilibrium are inconsistent under competitive conditions is clearly illustrated by this chart. When price is MN , this concern, if operating with the scale of plant represented by the short-run curves ac and mc , is in short-run equilibrium when its output is OM , for its short-run marginal cost is then equal to price. It will not be in long-run equilibrium, however, for its long-run marginal cost will then be only MQ , or less than price. Provided that no change in its output will affect market price, it will pay this concern to enlarge its plant whatever the price may be, and whatever its existing scale of plant may be. If thereby it grows so large that its operations exert a significant influence on price, we pass out of the realm of atomistic competition and approach that of partial monopoly. Even then, however, it would still be profitable for this concern to enlarge its plant and increase its output as long as long-run marginal cost was lower than long-run marginal revenue, or the increment in aggregate receipts resulting from a unit increment in output, after allowance for any reduction in price¹).

For any particular concern operating under these conditions, and *a fortiori* for an industry as a whole consisting of such concerns, there is no definite long-run supply curve. At any price MN higher than the asymptote of the AC curve, this producer will be willing to produce any quantity not less than OM .

To negatively-inclined long-run cost curves such as the AC and MC curves in Chart IV, Marshall has denied the characteristic of "reversibility", i. e., of equal validity whether output is increasing or decreasing, on the ground that some of the economies accruing when the output of a concern, or of an industry as a whole, is increased will be retained if the output of the concern or of the industry returns to its original

¹) If Y_p = long-run price, X = long-run output, and Y_c = long-run average cost, long-run marginal cost would be $\frac{d(\bar{X}Y_c)}{d\bar{X}}$, long-run marginal revenue would be $\frac{d(\bar{X}Y_p)}{d\bar{X}}$, and it would pay to carry production to the point where long-run marginal cost equalled long-run marginal revenue, or $\frac{d(\bar{X}Y_c)}{d\bar{X}} = \frac{d(\bar{X}Y_p)}{d\bar{X}}$. Under atomistic competition, $\frac{d(\bar{X}Y_p)}{d\bar{X}} = Y_p$, which is independent of this particular concern's output. Whatever the price, therefore, this concern would always have an incentive to increase its long-run output as long as long-run marginal cost remained less than that price. If partial monopoly resulted, however, marginal revenue, or $\frac{d(\bar{X}Y_p)}{d\bar{X}}$, would become a function of market demand and of competitor's supply and would be smaller than Y_p , and a point of stable long-run equilibrium might exist, depending on how the other producers reacted to variations in output by this one. If complete monopoly resulted, there would probably be a definite point of stable equilibrium. These questions, however, are beyond the range of this paper.

dimensions¹). This reasoning appears to involve a confusion between static and dynamic cost curves. The reductions in costs as output is increased indicated by curves such as the *AC* and *MC* curves in Chart IV are purely functions of size of output when scale is adjusted to output and not of lapse of actual time during which improved processes may happen to be discovered. The economies associated with output *OM* are economies which are not available for any output less than *OM*. The only basis on which the irreversibility of these curves, as static curves, could logically be posited would be the existence of possible economies of a type adapted to any scale of output but discoverable only when output is great, where invention, but not its exploitation, was a function of scale of output.

Net Internal Diseconomies of Large-Scale Production

Cases are clearly conceivable where increase of scale of plant would involve less efficient operation and consequently higher unit costs. The prevailing opinion in the United States that for most types of agriculture the one-family farm is still the optimum mode of agricultural organization would indicate that in this country at least agriculture was subject to net internal diseconomies of large-scale production after an early stage in the size of the farm-unit had been reached. But when increase of output by means of the increase of scale of existing plants involves a substantial increase in unit costs, it will always be possible for the industry as a whole to avoid the net internal diseconomies of large-scale production by increasing its output through increase in number of plants without increase in their scale²). This case has no practical importance, therefore, except as it represents an economic barrier against increase in scale of plants, and it is not worth while to illustrate it graphically.

Net External Economies of Large Production

External economies are those which accrue to particular concerns as the result of the expansion of output by their industry as a whole, and which are independent of their own individual outputs. If an industry which enjoys net external economies of large production increases its output—presumably through increase in number of plants—the average costs of the member concerns of that industry will fall even though each concern maintains a constant scale of plant and a constant output. Like internal economies, external economies may be either technological or pecuniary. Illustrations of technological external economies are

¹) Principles of Economics, eighth ed., 1922, p. 808.

²) Increase of scale should be distinguished from increase in output from the same scale of plant. In the former, all the factors are increased in about the same proportions; in the latter some factors remain fixed in amount. Whenever it is generally possible to increase all the factors in about the same proportion, i. e. to increase scale of plant, it is also possible, alternatively at least, to increase the number of plants.

difficult to find, but a better organization of the labor and raw materials markets with respect to the availability of laborers and materials when needed by any particular plant, and improvement in productive technique resulting from "cross-fertilization", or the exchange of ideas among the different producers, appear to be possible sources of technological external economies resulting from the increase in size of the industry as a whole. Illustrations of pecuniary external economies would be reductions in the prices of services and materials resulting from the increase in the amounts of such services and materials purchased by the industry as a whole. Pecuniary external economies to industry *A* are likely to be internal or external economies to some other industry *B*. If industry *A* purchases materials in greater quantity, their price may fall because industry *B* can then produce them at lower unit cost. But cases are theoretically conceivable where pecuniary external economies to industry *A* may not be economies to any other industry, as, for instance, if laborers should have a preference, rational or irrational, for working in an important rather than in a minor industry, and should therefore be willing to accept lower wages as the industry expands.

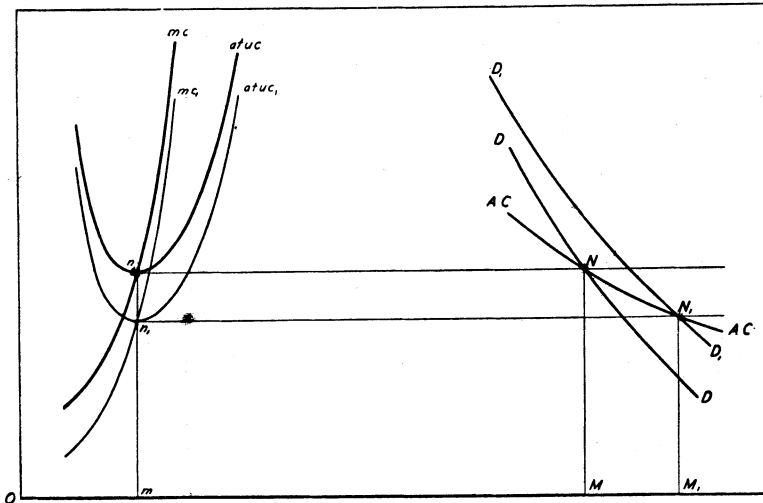


Chart V. Net External Economies of Large Production

Chart V illustrates the case of net external economies of large production, irrespective of whether these economies are technological, or pecuniary, or both. As always, each concern will in the long-run tend to produce its output from the optimum scale for that output, and given that scale, to carry production to the point where its average and marginal costs are both equal to price. If Om represents the optimum scale of plant for the particular producer, i. e., the scale at which he can produce at the lowest average cost, if the long-run price is mn or MN , and if the long-run output for the industry as a whole is OM , this

producer will be in long-run equilibrium when his output is om , and his average and his marginal cost are both mn . Suppose now that long-run demand rises from DD to D_1D_1 , and that long-run output of the industry as a whole increases, as the result of increase in the number of producers, from OM to OM_1 . Since, by assumption, this industry is subject to net external economies of large production, the short-run average and marginal cost curves of each particular concern will fall in the manner indicated in the left-hand portion of Chart V. This particular concern will be in long-run equilibrium with the new situation when its output is om , as before, but its long-run average and marginal costs will have fallen from mn to mn_1 . The AC curve represents the trend of the individual average (and also marginal) costs as output of the industry as a whole changes by the amounts indicated on the horizontal axis. Any point on this curve represents the long-run average cost for every individual producer, and therefore for the industry as a whole, when the output of the industry as a whole is as indicated. It is theoretically the same as the supply curve for the industry as a whole. The long-run marginal cost curve for the industry as a whole is not shown on the chart. It would fall below the AC curve¹). Its only relationship to the short-run marginal cost curves of the individual concerns would be that it was a function of the downward shifting of the lowest points on the individual short-run $atuc$ and mc curves as the output of the entire industry increased. Under atomistic competition this marginal cost curve would have no influence on supply, since individual producers would not take it into account in deciding either upon their continuance in or their entrance into the industry or upon their scale of output when in the industry²).

Net External Diseconomies of Large Production

Although it has not ordinarily been given consideration, the case of net external diseconomies of large production is of indisputable practical importance. Pecuniary diseconomies of this kind will always tend to result from the expansion of output of an industry because the increased

¹) If X = output of the industry as a whole, and Y_a = long-run average cost for the industry as a whole as represented by the AC curve, the MC curve for the industry as a whole would be $\frac{d(XY_a)}{dX}$, $< Y_a$. If average cost for a particular producer = y_a , then $y_a = f(X)$, and at long-run equilibrium, $y_a = \bar{Y}_a$.

²) Employing terminology resembling that used by Pigou in his *The Economics of Welfare*, the marginal private net cost would exceed the marginal industry net cost. If the output of an additional producer be represented by ΔX , and the average cost of his output and of the outputs of the other producers by $y_a = f(X)$, then the marginal private net cost would be y_a , and the marginal industry net cost would be $\frac{\Delta(XY_a)}{\Delta X}$, $< y_a$.

purchases of primary factors and materials which this entails must tend to raise their unit prices. In order that pecuniary diseconomies shall not result from the expansion of an industry's output, it is necessary, for both primary factors of production and materials, that the increase in demand by this industry shall be accompanied by a corresponding and simultaneous decrease in demand by other industries or increase in supply of the factors and materials themselves, or, failing this, that the materials, because of net external or internal economies in the industries producing them, should have negatively inclined supply curves¹). These pecuniary external diseconomies, however, may be more than counterbalanced by technological external economies, and need not necessarily result therefore in net external diseconomies. External technological diseconomies, or increasing technical coefficients of production as output of the industry as a whole is increased, can be theoretically conceived, but it is hard to find convincing illustrations. One possible instance might be higher unit highway transportation costs when an industry which provides its own transportation for materials and products expands its output and thereby brings about traffic congestion on the roads.

Chart VI illustrates the case of net external diseconomies of large production, whether technological or pecuniary. When the long-run equilibrium outputs of the industry as a whole are OM and OM_1 , respectively, the $atuc$ and $atuc_1$ curves represent the respective trends of short-run average costs, the mc and mc_1 curves represent the trends of short-run marginal costs and mn and mn_1 represent the long-run equilibrium average and marginal costs, for one individual producer. The reverse of the conditions when net external economies of large

¹) It is worth pointing out that negative supply curves for the primary factors of production will not prevent an increased demand for them from a particular industry from resulting in an increase in their unit prices and therefore are not a barrier to pecuniary external diseconomies for that industry in so far as their primary factor costs are concerned. The negatively inclined supply curves of primary factors have a different meaning from the negatively inclined supply curves for commodities. If labor has a negatively inclined supply curve that means not that willingness to hire labor in greater quantities will result in a fall in the wage-rate, but, what is very different, that fewer units of labor will be offered for hire when a high rate of wages is offered than when a lower rate is offered. In the case of commodities, any point on a negatively inclined supply curve must be interpreted to mean that at the indicated price, the indicated quantity or more of the commodity can be purchased. In the case of labor, any point on a negatively inclined supply curve must be interpreted to mean that when the indicated wage-rate is obtainable, the indicated quantity of labor, but no more, will be available for hire. If the negatively inclined supply curve for labor has an elasticity of less than unity, as seems probable, it must be assumed that labor will prefer a high wage rate and partial employment to a low wage rate and fuller employment, and therefore will resist any movement toward the lower points on its supply curve.

production are present, in this case the long-run equilibrium average and marginal costs of the individual concern rise as the output of the industry as a whole increases. The AC curve represents the trend of the individual average (and also marginal) long-run costs and therefore also of the industry long-run average cost as the industry as a whole varies its output. This is also the long-run supply curve for the industry as a whole. The long-run marginal cost curve for the industry as a whole

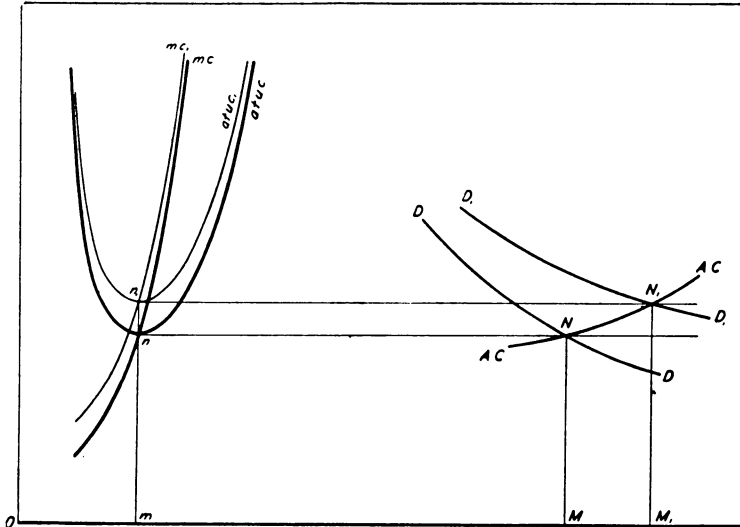


Chart VI. Net External Diseconomies of Large Production

is not shown on the Chart. It would rise above the AC curve¹). Since the individual producers will not concern themselves with the effect on the costs of other producers of their own withdrawal from or entrance into the industry, and since in this case it is assumed that variation in output takes place only through variation in number of producers, the marginal cost curve for the industry as a whole will, under competitive conditions, have no influence on output²).

¹) As for Chart V, if X = output of industry as a whole and Y_a = long-run average cost for industry as a whole, as represented by the AC curve, the marginal cost curve for the industry as a whole would be $\frac{d(XY_a)}{dX}$. If for the individual concern, y_a = average cost, then $y_a = f(X)$, and at long-run equilibrium $y_a = Y_a$.

²) In Pigou's terminology, the marginal industry net cost would exceed the marginal private net cost. If the output of an additional concern be represented by ΔX , and his average cost by $y_a = f(X)$, then the marginal private net cost would be y_a , and the marginal industry net cost would be $\frac{\Delta(XY_a)}{\Delta X} > y_a$.

Particular Expenses Curves

In the foregoing analysis of the relation of cost to supply, it has been throughout maintained, explicitly or implicitly, that under long-run static competitive equilibrium marginal costs and average costs must be uniform for all producers. If there are particular units of the factors which retain permanently advantages in value productivity over other units of similar factors, these units, if hired, will have to be paid for in the long-run at differential rates proportional to their value productivity, and if employed by their owner should be charged for costing purposes with the rates which could be obtained for them in the open market and should be capitalized accordingly. In the short-run, the situation is different. There may be transitory fluctuations in the efficiency of particular entrepreneurs or of particular units of the factors, and it would neither be practicable nor sensible to recapitalize every unit of invested resources with every fluctuation in their rate of yield. Even in the short-run, there must be equality as between the marginal costs of different producers under equilibrium conditions¹), but there may be substantial variations as between the average costs, and therefore as between the net rates of return on original investment, of different producers.

Statistical investigations of individual costs in the United States, based in the main on unrevised cost accounting records, have shown that the variations in average costs as between different producers in the same industry at the same time are very substantial, and that ordinarily a significant proportion of the total output of an industry appears to be produced at an average cost in excess of the prevailing price. To some extent these variations in cost can be explained away as due (1) to different and, from the point of view of economic theory unsatisfactory, methods of measuring costs, and especially the costs associated with the relatively fixed factors of production, (2) to regional differences in f. o. b. factory costs and in prices which, in an area as large as the United States, can be very substantial for bulky commodities without implying the absence of keen competition and (3), to the absence of atomistic competition. But even aside from such considerations, it should be obvious that such findings are in no way inconsistent with the propositions of equilibrium price theory as outlined above. Under short-run equilibrium the average

¹) Since a time-interval is always present between the sale contract and at least some of the stages of hiring of factors and of actual production, there is opportunity under short-run equilibrium for some divergence between price and marginal cost, and therefore, between the marginal costs of different producers. It would be a more precise way of formulating the short-run theory to say that since all producers, if acting rationally, carry production to the point where anticipated marginal cost will equal anticipated price, and since price, in a perfect market, is uniform for all, marginal cost tends to be uniform for all producers, and variations as between different producers result only from errors in anticipation.

costs, including the fixed costs, of any particular producer need bear no necessary relationship to price, except that the average direct costs must not exceed price. These statistical costs, moreover, are not the equilibrium costs of the theoretical short-run, but are the costs as they exist at an actual moment of time when short-run equilibrium with the fundamental conditions as they exist at the moment may not have been attained, and when these fundamental conditions are themselves liable to change at any moment.

It may be worth while, however, to show the relationship of the distribution of particular average costs within an industry at particular actual moments of time to the general supply conditions of the industry under assumptions of long-run equilibrium. To a curve representing the array of actual average costs of the different producers in an industry when the total output of the industry was a given amount, these individual costs being arranged in increasing order of size from left to right, Marshall gave the name of "particular expenses curve"¹), and American economists have called such curves "bulk-line cost curves"²), "accountants' cost curves", and "statistical cost curves". In Chart VII, the curves AN , BN_1 , and CN_2 , are supposed to be the appropriate particular expenses curves for an industry subject to net external economies of large production, when the output of the industry as a whole is OM , OM_1 , and OM_2 , respectively. Because the industry is subject to net external economies, the entire particular expenses curves are made to shift downward as the output of the industry expands. (If the industry were subject to net external diseconomies of production, the particular expenses curves would shift upwards as the output of the industry expands. Corresponding modifications in the chart would have to be made as other assumptions with respect to the conditions under which the industry can expand its output were introduced.) It is to be understood also that no dynamic changes in prices of the factors or in average technological cost conditions for the industry as a whole are occurring except such as are associated with variations in output of the industry as a whole.

The HC curve is a curve connecting the points of highest-cost for each successive output. These highest-costs, though often so designated,

¹) See Principles, eighth ed., Appendix H p. 911. It will be noticed that his particular expenses curve, SS , is drawn so as to project somewhat beyond the point of total output for the industry as a whole A . This is an error, and no significance can be given to the part of the curve projecting beyond the point of total output of the industry as a whole. If the output of the industry were to increase up to the terminal point of this curve, the entire curve would acquire a different locus.

²) "Bulk-line cost curves" because if a perpendicular is dropped to the horizontal axis from the point of intersection of the price-line and the curve, the greater part of the "bulk" of the output would be to the left of this "bulk-line". See F. W. Taussig, "Price-Fixing as seen by a Price-Fixer", Quarterly Journal of Economics, XXXIII.

are not marginal costs in the strict sense of the term, but are in each case simply the average costs of that producer whose average costs are the highest in the industry. If the statistical indications and also certain *a priori* considerations are to be followed, these highest average costs are likely to be, except in "boom years", distinctly higher than

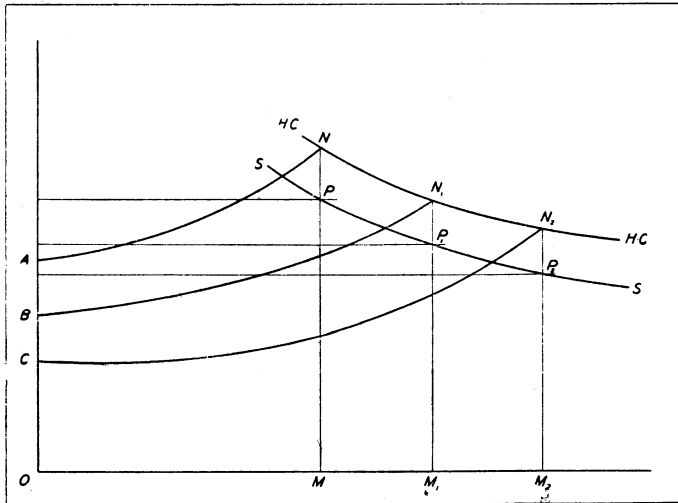


Chart VII. Particular Expenses Curves

the true marginal costs¹⁾, and are so drawn in this graph. The P, P_1, P_2 lines represent price, and are drawn to intersect the particular expenses curves below their highest points, in conformity with the statistical findings. The curve SS , drawn through the P, P_1, P_2 points representing actual prices prevailing when the outputs are OM, OM_1 and OM_2 , respectively, is a sort of actual semi-dynamic²⁾ supply curve.

What is the ordinary relationship between the HC curve and the SS curve under fully dynamic conditions cannot be postulated on *a priori* grounds, and only statistical investigation can throw much light on it. American investigators of particular expenses curves believe that they have already demonstrated stable and predictable relations between them and price, but a reasonable degree of scepticism still seems to

¹⁾ If the AN, BN_1 , and ON_2 curves were the actual particular expenses curves when the actual outputs of the industry as a whole were OM, OM_1 and OM_2 respectively, the actual marginal cost curve for the industry as a whole would be a curve representing the differences per unit increase of output between the aggregate costs represented by the successive areas, $AOMN, BOM_1N_1, COM_2N_2, \dots$ as output was increased from OM to OM_1 , to OM_2 , to \dots It would be negatively inclined, and would be much below the HC curve.

²⁾ "Semi-dynamic" because certain types of dynamic changes have been assumed not to occur.

be justified. One point, however, is clear on *a priori* even more than on inductive grounds. If the *SS* curve in Chart VII were not ordinarily below, and substantially below, the *HC* curve, the familiar and continuously present phenomenon of bankruptcy would be inexplicable.

It is possible, moreover, to devise a theory of even long-run static equilibrium which still leaves room for an excess of the *HC* over the *SS* curves, and therefore for bankruptcy as a phenomenon consistent with long-run equilibrium. For such a theory, however, long-run equilibrium would apply only to the industry as a whole, and would be a sort of statistical equilibrium between rate of output and rate of consumption. None of the individual producers under this theory need be in long-run equilibrium at any time. At any moment, some producers would be enjoying exceptional profits, and others incurring heavy losses. The particular expenses curve could remain positive in its inclination and fixed in its locus, but there would be necessarily a constant process of shifting of their position on that curve on the part of the individual producers, and an equality in rate of withdrawal of producers from the industry through bankruptcy or otherwise, on the one hand, and of entrance of new producers into the industry, on the other hand. A theory of this sort would leave room for pure profits even in a static state.