THE THEORY AND PRACTICE OF BRIDGE CONSTRUCTION

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THE THEORY AND PRACTICE

OF

BRIDGE CONSTRUCTION

IN TIMBER, IRON AND STEEL

BY

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PREFACE

A SAD, an almost tragic note is necessarily imported into this short preface by reason of the conditions under which it was produced. The author provided the outlines when his death in the plenitude of his mental powers was impending, and when the revision of the proofs of a work that had occupied eight or nine years of the limited leisure of a busy life was taxing a physical strength which up to a point had seemed abundant. Mr. Morgan Davies completed the task he had undertaken—that was a source of comfort and solace to him in his last hours—and whilst still comparatively young, died as he had lived, the embodiment of patient courage, devoted to the profession he had enriched by his labours.

This work is based upon notes of lectures delivered from time to time to students of Civil Engineering at the Swansea Technical College. In his capacity as lecturer the author found that in many, if not in most books dealing with Bridge Construction, the formulae for determining the stresses in bridge structures were of too general a character. They involved the use of higher mathematics and consequently could not be intelligently employed by the average student, nor indeed in all cases by engineers who had not applied themselves particularly to the study of Bridge Construction. As a rule, engineers in active practice have neither the time nor the inclination to enter upon laborious theoretical calculations, especially when sufficiently accurate results for all practical purposes can be obtained by more simple means.

The aim of this work is primarily to furnish easily understood rules whereby problems connected with Bridge Construction may be treated analytically and graphically: Examples are given of the various types of existing bridges constructed either of timber or of steel. In every instance the calculations and designs are set out step by step in their development and the illustrations have been reproduced from the actual working drawings.

It was the wish of the author that his indebtedness should be acknowledged to the Engineering Standards Committee for permission to include in the appendix tables of the Standard Sections of steelwork ordinarily used in constructional work.

DAVID DAVIES, F.I.J.

SWANSEA.

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BRIDGE CONSTRUCTION.

CHAPTER I.

Definition of Terms.

THE terms Strain and Stress are frequently used indiscriminately and mistaken to be synonymous.

Strain is the effect of a force or forces acting upon a beam or framed structure tending to distort its form or alter its length, and stress is the producing or causing force, or, as defined by Professor A. J. Du Bois in his well-known work on *Strains in Framed Structures,* the outer forces acting at various portions of a framed structure tending to cause motion of its parts are called stresses, and the inner forces which resist this motion of the parts and hold them in equilibrium are called strains.

The strains which may be produced in a beam or framed structure under various conditions are:

1. Tension or tensile strain.
2. Compression or crushing strain.
3. Transverse or bending strain.
4. Shearing.
5. Torsion.

A tensile strain is the result of forces acting in opposite directions, and tending to pull asunder or elongate a beam or piece of material in the direction of its length.

A compressive strain is the result of forces acting in opposite directions in a manner diametrically the reverse to tension, that is to say, tending to crush or shorten a beam or piece of material in the direction of its length and force its particles together.

Transverse or bending strain is a compound strain composed of tension and compression. Thus in Fig. 1 a weight

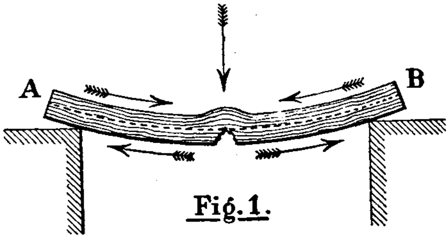


Fig. 1.

The Arrows indicate the direction of the forces

placed at the centre of the beam AB would in the first instance exert a downward vertical force, the effect of which would be to compress or force together the fibres on the upper surface of the beam and pull asunder the fibres on its under surface,

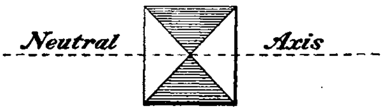
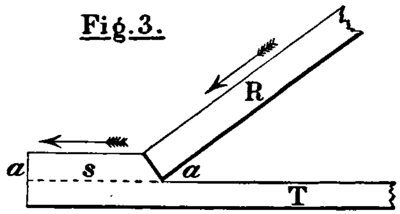


Fig. 2.

and the intensity of these compressive and tensile strains, which is greatest at the top and bottom surfaces, gradually diminishes inwards until at the centre of the beam there is no strain. That point or plane is called the neutral axis or the neutral plane. If we represent a section of the beam AB (Fig. 1) by the rectangle in Fig. 2 and assume that the neutral axis is at the centre of the section, then the shaded areas included by the two triangles will represent the total amount of compression and tension acting on the section the triangle above the neutral axis representing compression, and the one below tension.

Shearing, or detrusion as it is called when applied to woodwork, is a force that may act in a direction parallel to the fibre as in the case of a rafter abutting on a tie-beam, as shown in Fig. 3, wherein the rafter R tends to force out the section S along the plane *a*, *a*, represented by the dotted line. In

Fig. 3.



ironwork or steelwork a shearing strain is generally recognized to be caused by two forces acting parallel to each other but in opposite directions at right angles to the axis or fibre of the particular member acted upon, tending to sever it into two sections, acting in the same manner as the blades of a pair of shears or scissors in the act of cutting.

Torsion or twisting is a strain that occurs oftener in machinery, and more especially in shafts and cranks, than in framed structures, but it may under certain conditions be produced in beams and columns by a transverse force applied at a distance from the axis tending to cause rotation around the axis.

The moment of a force about any point or section is the force multiplied by the leverage or lever arm at which it acts.

Thus in Fig. 4, let a force of 100 lbs. act at the extremity of the lever AB with a leverage of 5 feet, the distance from the fulcrum at C, the moment of that force about the point C is 100 lbs. × 5 feet = 500 foot lbs., and to maintain that force in

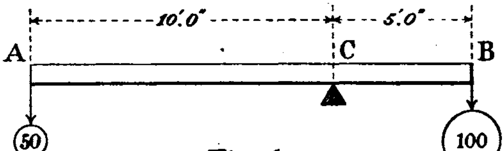
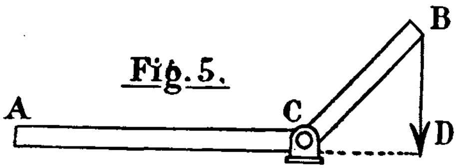


Fig. 4.

equilibrium there must be applied at the point A, distant 10 feet from the fulcrum at C, a force that will have an equal moment, which must obviously be = 500 lbs. ÷ 10 feet = 50 lbs.

In the case of a bent lever, such as that shown in Fig. 5,

Fig. 5.



the lever arm at which the force acts with reference to the point C is the distance CD and not CB, and the moment of the force is its product into its lever arm CD.

The Bending Moment or Moment of Flexure with reference to any section is the resultant moment or the algebraic sum of all the moments due to the external forces acting on either side of that section.

Moment of Inertia is the summation of the area of each particle of a section multiplied by the square of its distance from the axis. The moment of inertia is an important factor in arriving at the moment of resistance, as will be explained.

The Moment of Resistance is an expression for the moment of the internal forces resisting the moment of the external forces, and with reference to any cross section it is the product of the effective area on one side of the neutral axis by the effective leverage at which it acts; or in other words, the intensity of the resistance of any fibre or layer of fibres multiplied into its distance from the neutral axis, and it is derived, as will be seen, from the moment of inertia.

The Elastic Limit is the limit to which a beam or piece of material may be strained by a load or force producing in it tension or compression, so that when that load or force ceases to act and it is relieved of stress it shall return to its original form and dimensions. Up to the limit of elasticity the elongation or compression will be proportional to the load or force producing it; or in other words, the stress and strain are equal and proportional to one another, any increment of the one causing an equal increment of the other. Once the limit of elasticity is exceeded the strain increases more rapidly than the stress, and when the stress is relieved there will be a permanent set in the body acted upon, and it cannot recover its original form and dimensions. The load corresponding to the limit of elasticity is essentially much less than the breaking load. *E.g.* If we suppose a beam loaded at its centre with a weight the beam will slightly deflect under that weight, causing its upper surface to contract and its lower surface to elongate, but once the weight is removed it will spring back to its original level and length.

If that weight be replaced by another of twice its intensity the distortion of the beam will be practically twice as great, and if this process of loading be continued, increasing with each application the intensity of the weight, there would result a correspondingly increased distortion until the elastic limit is attained, after which there would be a permanent set in the beam, which, by still continuing the process of loading, must increase until the ultimate strength of the material is reached, when rupture must result.

Modulus of Elasticity is a purely theoretical force, and affords the means of determining the stress in a body from the strain. It is the force which would extend a bar 1 inch square to an extent equal to its own length. If a known force F will lengthen or shorten to the extent of 1 inch a bar 1 inch square, L inches long, then a force FL would lengthen or shorten the same bar L inches, and this force is called the modulus of elasticity.

CHAPTER II.

Stresses in Beams. Bending Moments and Shearing Forces.

In connection with the Bending Moments and Shearing Forces in beams the following cases will be investigated, viz.:

(A) Cantilevers or beams fixed at one end and unsupported at the other.

1. Loaded with a concentrated load at its unsupported or free end.
2. Loaded with a uniformly distributed load.
3. Loaded with a partially uniformly distributed load.
4. Loaded unsymmetrically at intermediate points.
5. Loaded with a uniformly distributed load and a concentrated load at the unsupported end.

(B) Beams supported at both ends.

1. Loaded with a concentrated load at centre.
2. Loaded with a concentrated load at any point between centre and abutment.
3. Loaded with a uniformly distributed load.
4. Loaded with a partially uniformly distributed load.
5. Loaded unsymmetrically at intermediate points.
6. Loaded with a uniformly distributed load and a concentrated load at one or more intermediate points.

(C) Beam supported at two points with overhanging or cantilever end or ends.

1. Loaded with concentrated weights at centre and ends with both ends overhanging.
2. Loaded with uniformly distributed load with both ends overhanging.
3. Loaded with a uniformly distributed load over the portion of the beam supported, and a concentrated load at the overhanging end.

(D) Beam fixed at one or both ends.

1. Loaded with concentrated load at centre, both ends fixed.
2. Loaded with a uniformly distributed load, both ends fixed.
3. Loaded with concentrated load at centre, one end fixed and one end resting on support.
4. Loaded with a uniformly distributed load, one end fixed and one end resting on support.

Cantilevers.

Case 1. *Loaded with a concentrated weight at free end as in Fig.* 6.

Let $W =$ load.

$l =$ length of cantilever.

$x =$ distance of section from free end where bending moment is required.

$BM =$ bending moment.

$S =$ shearing stress.

BM at $AB = Wx$ and at any other intermediate point distant x from the free end, $BM = W \times x$.

Hence it follows that BM will have a maximum value when $x = l$ and BM will then $= Wl$. The value of S which is uniform throughout $= W$.

Assuming the length of the cantilever to be 10 feet, the weight W to be 8 cwt., and the distance $x = 5$ feet. Then the BM at AB = 8 cwt. × 5 feet = 2 tons, and the bending moment at the point of support C = 8 cwt. × 10 feet = 4 tons.

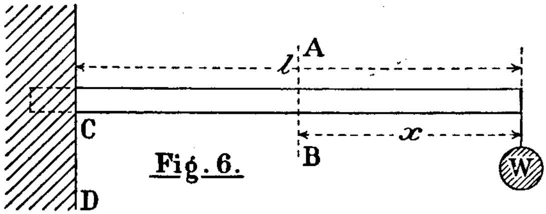


Fig. 6.

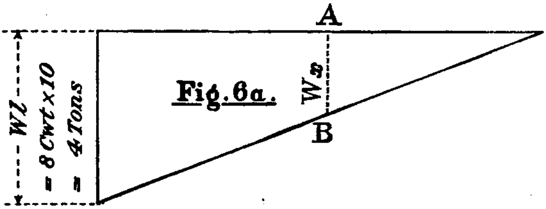


Fig. 6a.

Fig. 6b.

In Figs. 6*a* and 6*b* the bending moment and shearing stress are respectively graphically represented. In Fig. 6*a* DF is set off on any convenient scale = Wl in this case = 4 tons. The ordinate at any section such as at AB will give the bending moment at that section.

In Fig. 6*b* the diagram represents the shearing stress, whose value W is constant throughout in this case = 8 cwt.

Case 2. *Loaded with a uniformly distributed load throughout as in Fig. 7.*

In the case of a cantilever loaded with w units per lineal foot, the total load at any section such as AB distant x from the free end $= wx = \mathrm{W}$, the centre of gravity of which is at the centre of the length x, so that the leverage with respect to that section $= \frac{x}{2}$, hence the BM $= \mathrm{W} \times \frac{x}{2}$.

The BM in this case also becomes a maximum when

$$x = l, \text{ and then BM} = \frac{wx^2}{2} \text{ or } \frac{\mathrm{W}l}{2}.$$

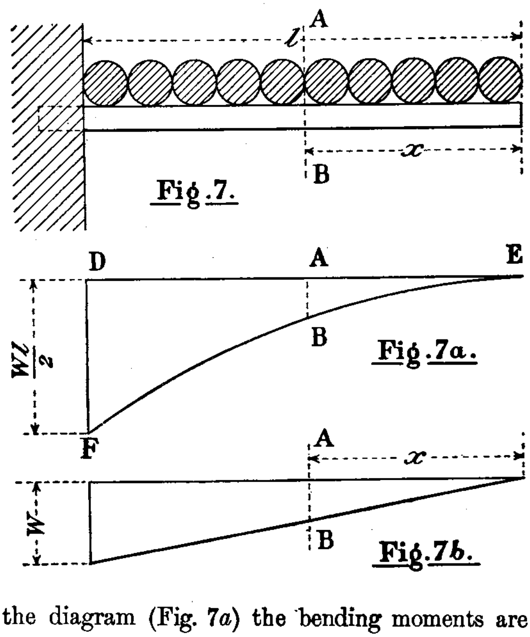


Fig. 7.
Fig. 7a.
Fig. 7b.

In the diagram (Fig. 7a) the bending moments are represented by the parabolic curve EBF, and the ordinate to that curve at any intermediate point in DE, such as at AB, will give the bending moment at section AB.

The curve EBF may be constructed geometrically as in Fig. 8, or by ordinates calculated from the equation of the parabola as in Fig. 9.

In the diagram (Fig. 8) draw AB by any scale to represent the length of cantilever, and draw $AC = \frac{Wl}{2}$.

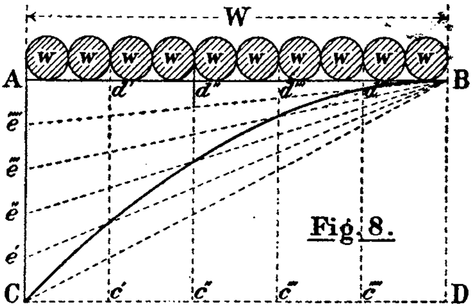


Fig. 8.

Divide CD drawn parallel to AB into a number of parts corresponding with the points at which the ordinates are required. Through those divisions draw the vertical lines c', d', etc. On AC lay off an equal number of divisions $e' \ldots e''''$ and join them by the radiating lines with B. Then the points of intersection of the vertical and radiating lines will be points in the curve.

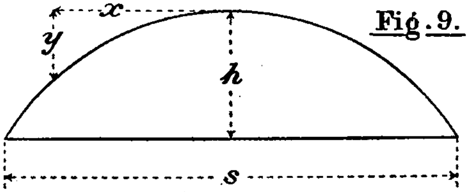


Fig. 9.

The ordinates at d', $d'' \ldots d''''$ may be calculated from the equation of the parabola (Fig. 9), in which y represents the ordinate at a distance x from the vertex. y is calculated from the equation

$$x^2 = 2yp, \text{ hence } y = \frac{x^2}{2p} \text{ and } p = \frac{s^2}{8h}.$$

In Fig. 7, let $l=10$ feet and $w=1$ ton, so that $\mathbf{W}=10$ **tons.** Then the ordinates AC and BD in Fig. 8

$$=\frac{Wl}{2}=\frac{10\times 10}{2}=50 \text{ tons},$$

the bending moment at A, and if moments are taken successively at d'''', d''', d'', d', and A we have

$$\text{BM at } d'''' \quad x=2 \text{ and } W=2, \text{ then } \frac{Wx}{2}=2 \text{ tons.}$$

$$\text{BM at } d''' \quad x=4 \text{ and } W=4, \text{ then } \frac{Wx}{2}=8 \text{ tons.}$$

$$\text{BM at } d'' \quad x=6 \text{ and } W=6, \text{ then } \frac{Wx}{2}=18 \text{ tons.}$$

$$\text{BM at } d' \quad x=8 \text{ and } W=8, \text{ then } \frac{Wx}{2}=32 \text{ tons.}$$

$$\text{BM at A} \quad x=10 \text{ and } W=10, \text{ then } \frac{Wx}{2}=50 \text{ tons.}$$

The ordinates at d', d'' ... d'''' in the diagram (Fig. 8) if drawn to scale will have the same value as the bending moments found by calculation to be equal to $\frac{Wx}{2}$.

In Fig. 7*b* the shearing forces are represented graphically. They are greatest at the point of support, and diminish towards the unsupported end.

At C, or the point of support, $S=W=10$ tons, and at any intermediate point, such as AB distant x or 5 feet from the unsupported end, $S=wx=5$ tons.

Case 3. *Loaded with a partially uniformly distributed load as in Fig.* 10.

In the case of a cantilever loaded with a partially uniformly distributed load the bending moment at any section = the load × the distance from the section to the centre of gravity of the load lying between that section and the unsupported end.

In Fig. 10 the bending moment at the section AB

$$=\frac{w}{2}(y+z-x)^2.$$

Let $l = 10$ feet. Then BM at section AB

$x = 6$ feet. $= \frac{1}{2}(4+4-6)^2.$

$y = 4$ feet. $= 2$ tons.

$z = 4$ feet.

$w = 1$ ton.

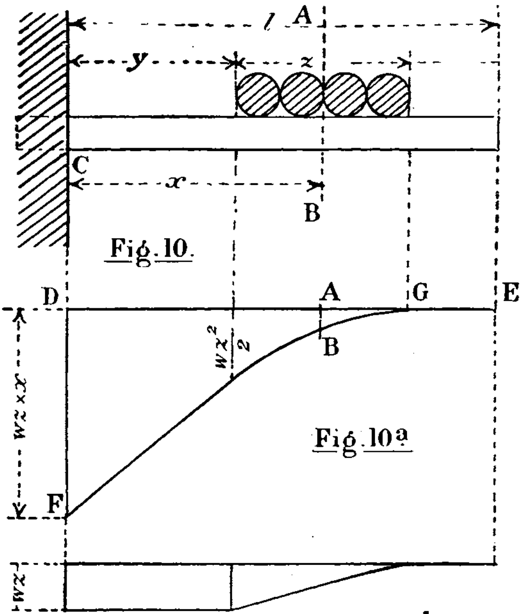


Fig. 10.

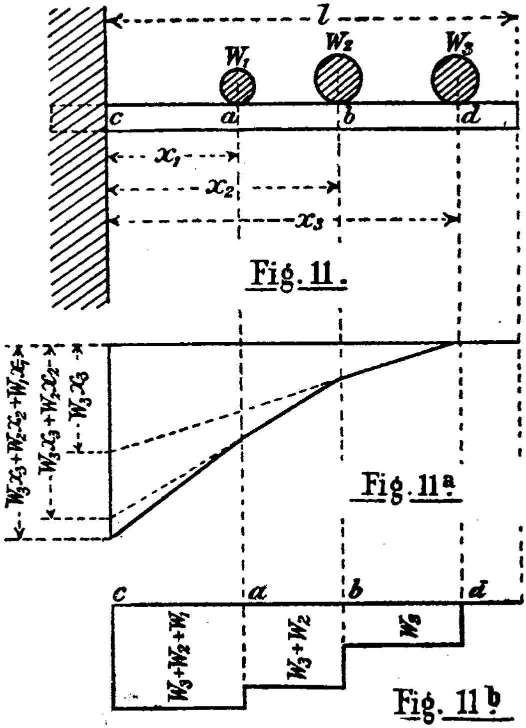
Fig. 10a

Fig. 10b

The maximum bending moment occurs at the point of support C, where it

$$= wz\left(\frac{z}{2}+y\right) = wz \times x = 1 \times 4 \times 6 = 24 \text{ tons.}$$

In the diagram (Fig. 10a) the bending moment is represented by the parabolic curve whose ordinate at the extremity of the load nearest to the point of support $= \frac{wz}{2}$, and thence to the point of support by a straight line, terminating in the point F set off from D $= wz \times x$. This diagram, it will be seen, is a combination of the diagrams in Cases 1 and 2, as also is Fig. 10*b*, in which the shearing forces are graphically represented.



Case 4. *Loaded unsymmetrically at intermediate points as in Fig.* 11.

In this case the bending moment at $b = w_3 \times (x_3 - x_2)$, at A the BM $= W_3(x_3 - x_1) + W_2(x_2 - x_1)$, and at the point of support C, where it becomes a maximum, the BM $= W_1x_1 + W_2x_2 + W_3x_3$.

The shearing stress from the free end to d is nil, in the section db it is $= W_3$, in ba it $= W_3 + W_2$, and in AC it $= W_3 + W_2 + W_1$.

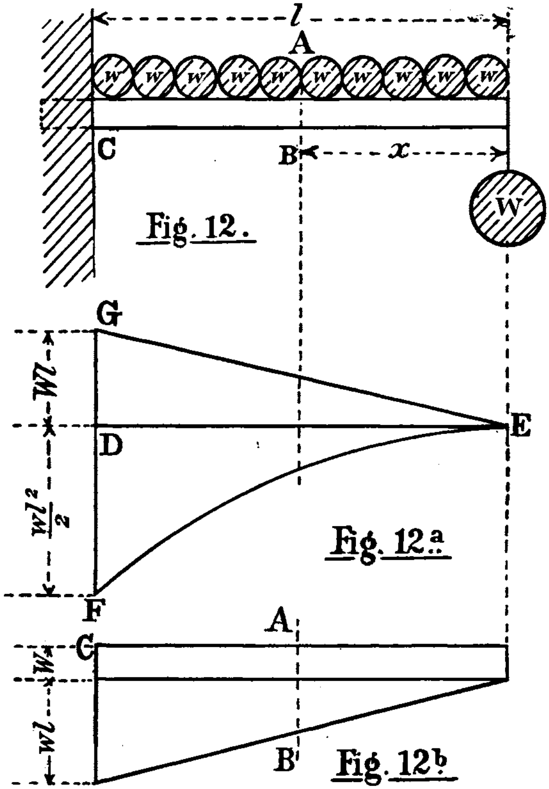


Fig. 12.

Fig. 12a.

Fig. 12b.

Let $\quad x_1 = 3$ feet and $W_1 = 1$ ton.

$x_2 = 5$ feet and $W_2 = 2$ tons.

$x_3 = 8$ feet and $W_3 = 2$ tons.

Then BM at $d = \text{nil}.$
$b = 6$ tons.
$a = 14$ tons.
$c = 29$ tons.

Case 5. *Loaded with a uniformly distributed load and a concentrated load at the unsupported end, as in Fig.* 12.

In this case, which is a combination of cases 1 and 2, the bending moment at any section such as $\text{AB} = \frac{wx^2}{2} + \text{W}x$. The bending moment becomes a maximum at the point of support C, where $x = l$ and the $\text{BM} = \frac{wl^2}{2} + \text{W}l$.

The shearing stress at AB in Fig. 12*b* $= \text{W} + wx$, and at C, where it becomes a maximum, it $= \text{W} + wl$.

Let $w = 1$ ton.
$\text{W} = 3$ tons.
$l = 10$ feet.
$x = 5$ feet.

Then BM at $\text{AB} = 3 \times 5 + \frac{25}{2} = 27\tfrac{1}{2}$ tons, and at C – BM

$$= \frac{10 \times 10}{2} + 3 \times 10 = 80 \text{ tons.}$$

The shearing stress at $\text{AB} = 3 + 5 = 8$ tons, and the point of support $\text{CS} = 3 + 10 = 13$ tons.

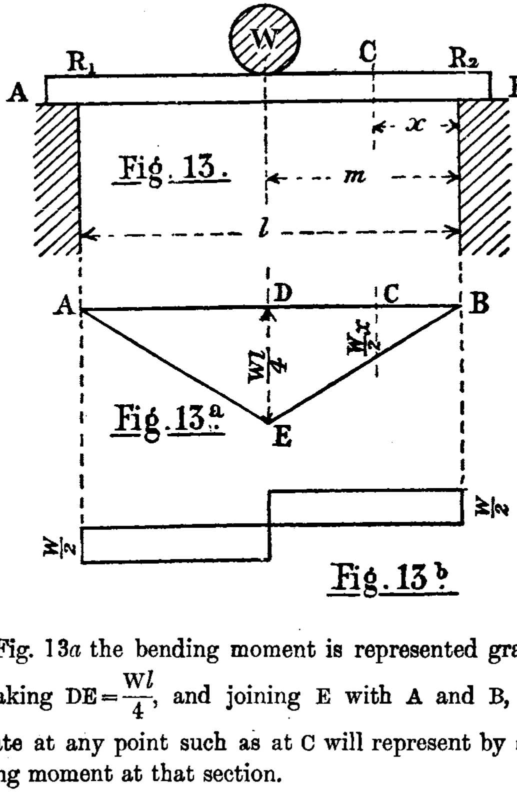
Beams supported at both ends.

Case 1. *Loaded with a concentrated load at centre as in Fig.* 13.

R = Reaction at abutment.

In a beam loaded at the centre with a concentrated load, as W in Fig. 13, the vertical pressure due to the load is transmitted in equal halves to the points of support at A and B, where they are resisted by equal vertical reactions which must therefore be equal to half the load.

The bending moment therefore at the centre where it is a maximum $= \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$. The bending moment at any section distance x from either abutment as at C in Fig. 13 $= \frac{W}{2}x$.



In Fig. 13a the bending moment is represented graphically by making DE $= \frac{Wl}{4}$, and joining E with A and B, then an ordinate at any point such as at C will represent by scale the bending moment at that section.

In Fig. 13b the shearing force is represented by the two rectangles, the depths of which are made equal to the reactions $= \frac{W}{2}$.

In this example let $l=10$ feet, $W=2$ tons, and $x=2$ feet. Then the BM at the centre of the beam $=\frac{2\times 10}{4}=5$ tons, and at the section C it is $=\frac{2}{2}\times 2=2$ tons.

The reactions R_1 and $R_2=1$ ton each, and the shearing stress $=\frac{W}{2}$, which is constant from the points of support to the centre, where there is no shear.

Case 2. *Loaded with a concentrated load at any intermediate point between centre and abutment as in Fig.* 14.

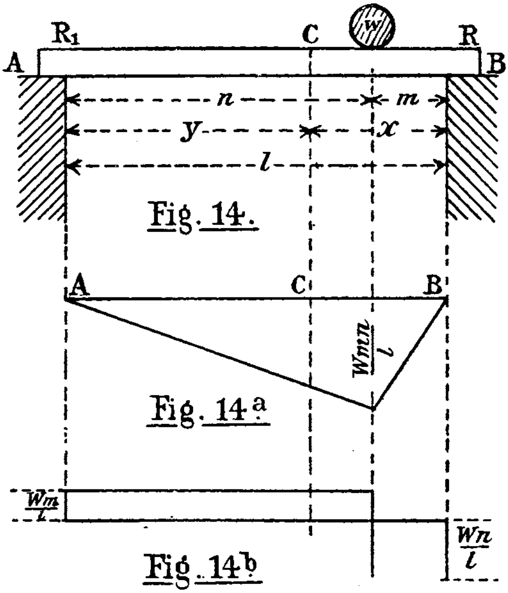


Fig. 14.

Fig. 14a

Fig. 14b

The reactions due to any load are inversely as the distance of its centre of gravity from its supports. In this case the reaction R_1 at $A=W\frac{m}{l}$, and the reaction R_2 at $B=W\frac{n}{l}$. The bending moment which is a maximum at the centre of gravity of the load $= \frac{Wmn}{l}$, at any other section, as at C in Fig. 14, the BM $= \frac{Wmy}{l}$.

The shearing stresses, which are equal to the reactions, are constant from the points of support to the centre of gravity of the load, as represented in the diagram (Fig. 14*b*).

The bending moments are represented in the diagram (Fig. 14*a*).

In this example let $l = 10$ feet, $m = 3$ feet, and $n = 7$ feet.

$$x = 4 \text{ feet and } W = 2 \text{ tons.}$$

Then R_1 at $A = 2 \times \frac{3}{10} = \cdot 6$ tons and R_2 at $B = 2 \times \frac{7}{10} = 1 \cdot 4$ tons.

BM at load $= \frac{2 \times 7 \times 3}{10} = 4 \cdot 2$ tons, and at C the BM

$$= \frac{2 \times 3 \times 6}{10} = 3 \cdot 6 \text{ tons.}$$

The shearing stresses are as shown in diagram 14*b*.

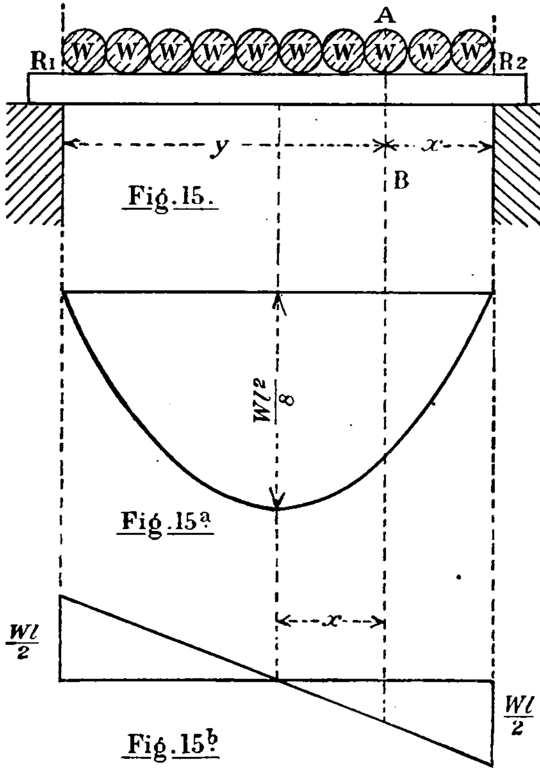
Case 3. *Loaded with a uniformly distributed load as in Fig.* 15.

In this case the reactions at the supports are equal to one another, and equal to half the distributed load carried by the beam $= \frac{wl}{2}$. The maximum bending moment occurs at the centre of the span, and is equal to half the total load acting with a leverage corresponding with the distance of its centre of gravity from its support, or one fourth of the span

$$= \frac{W}{2} \times \frac{l}{4} = \frac{Wl}{8}.$$

At any other section, such as at AB in Fig. 15, the bending moment $= \frac{wx}{2} \times y$ or $\frac{wy}{2} \times x$.

The shearing force at any section is equal to the weight carried by the beam between that section and the centre of the beam, that is to say, in Fig. 15*b* $S = wx$. S becomes a maximum when $x = \frac{l}{2}$, and is then $= \frac{wl}{2}$.



For a beam loaded with a uniformly distributed load the curve of moments is a parabola whose vertex $= \frac{wl^2}{8}$, as shown in Fig. 15*a*. The diagram for shearing forces is given in Fig. 15*b*, and in either diagram an ordinate drawn at any intermediate point will represent the bending moment or shearing force at that section.

Let $l = 10$ feet, $x = 2 \cdot 5$ feet, and $y = 7 \cdot 5$ feet, $w = 1$ ton.

Then $R_1 = R_2 = 5$ tons.

BM at centre of beam $= \frac{1 \times 10^2}{8} = 12 \cdot 5$ tons, and at section AB, BM $= \frac{1 \times 2 \cdot 5}{2} \times 7 \cdot 5 = 9 \cdot 375$ tons.

The shearing stress at either point of support $= 5$ tons, and at section AB $= 2 \cdot 5$ tons.

Case 4. *Loaded with a partially uniformly distributed load as in Fig.* 16.

In this case the reactions at the supports are also equal to one another, and equal to half the distributed load, because the load is symmetrically spaced, and $y = y^1$;

$$\therefore \quad R_1 = R_2 = \frac{wz}{2}.$$

In the case of an unsymmetrical load, as in Fig. 17, the reactions at A and B must be determined

$$R_1 = wz \times \frac{x_1}{l}, \text{ and } R_2 = wz \times \frac{x}{l}.$$

Let y in Fig. 17 $= 5$ feet, $y_1 = 1$ foot, $z = 4$ feet, and $w = 1$ ton.

Then $x = 7$ feet and $x_1 = 3$ feet, and $R_1 = 1 \times 4 \times \frac{3}{10} = 1 \cdot 2$ tons, and $R_2 = 1 \times 4 \times \frac{7}{10} = 2 \cdot 8$ tons.

The bending moment at any section not under the load, as at GH in Fig. 17, $=$ reaction of support on section side of load $= R_1 \times$ distance d of section from that support. For a section under the centre of a partial uniform load the bending moment

$$= wz \frac{l - (\frac{1}{2}z + y)}{l} x + \frac{w(x-y)^2}{2}.$$

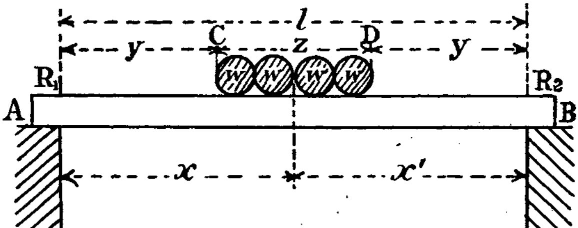


Fig. 16.

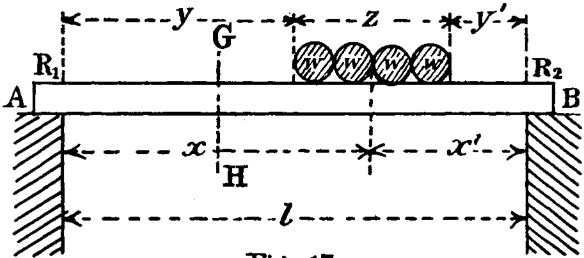


Fig. 17.

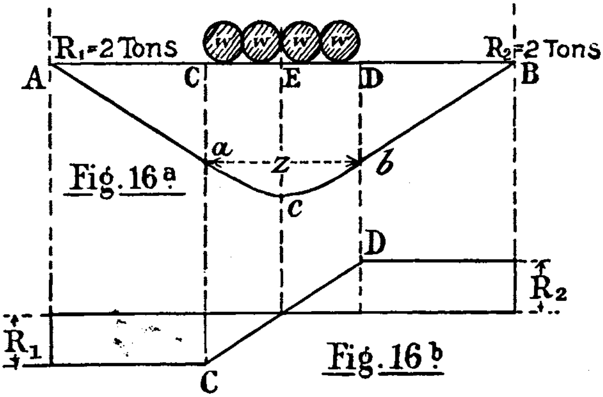


Fig. 16 a.

Fig. 16 b.

Example. In Fig. 16, let $l=10$ feet, $w=1$ ton, $z=4$ feet.

$R_1=R_2=2$ tons, $y=y_1=3$ feet.

Then BM at the section C

$=R_1\times 3=6$ tons, and at the section D

$=R_2\times 3=6$ tons.

The bending moment at the centre of the beam at E, under the load

$$=-1\times 4\frac{10-(\frac{1}{2}4+3)}{10}5+\frac{1(5-3)^2}{2}=-10+2=-8 \text{ tons.}$$

If therefore we construct the parabolic curve a, c, b, making the ordinate $EC=8$ tons and joining its ends at a, b with the lines Aa and Bb, the ordinates to which at C and D have already been determined to be 6 tons, the diagram of bending moments is completed, and any intermediate ordinate will represent by scale the bending moment at that section. The shearing stress from either support to the end of the load is equal to the reaction at that support.

The shearing stress at any section intersecting the load

$$=wz\frac{l-(y+\frac{1}{2}z)}{l}-w(x-y).$$

Thus at the section C at the centre of the beam the shearing stress $=1\times 4\frac{10-(3+2)}{10}-1(5-3)=2-2=0$, which proves the accuracy of the diagram in Fig. 16b, in which the line joining the points C and D cuts the centre of the horizontal line, and the stress there is zero.

Another very useful example of this case is that given in Fig. 18, in which is graphically represented the shearing stress at any section due to a rolling or moving load. The diagram in Fig. 18 represents a railway bridge with a train advancing over it from right to left, as indicated by the arrow, and Fig. 18a represents graphically the distribution of the shearing forces due to the dead and rolling load. the dead load on the bridge $=$ W being the weight of the bridge itself and the rolling load the weight of the train $=W_1$.

The shearing stresses due to the dead load are represented by the two triangles ACD and BDE, the ordinates to which at the points of support AC and BE are made equal to half the total distributed dead load, as in Case 3.

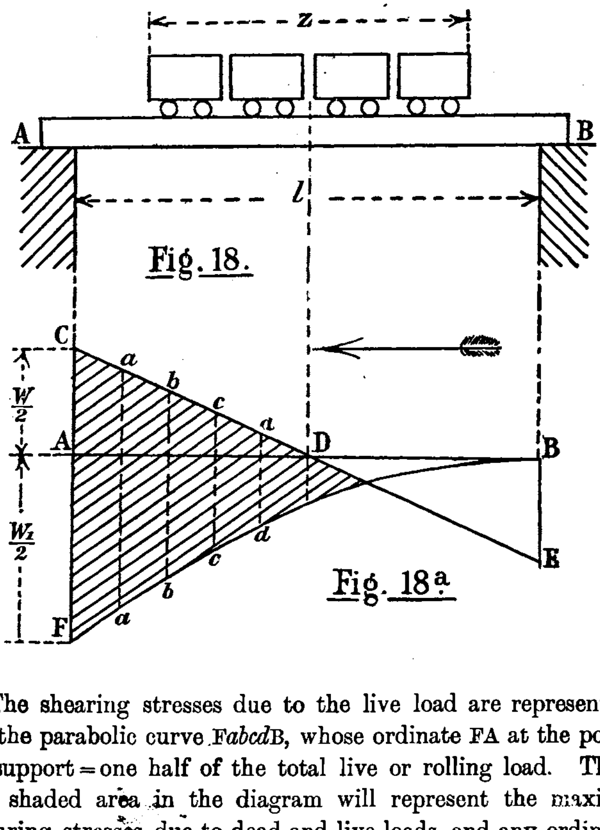


Fig. 18.

Fig. 18a.

The shearing stresses due to the live load are represented by the parabolic curve F*abcd*B, whose ordinate FA at the point of support = one half of the total live or rolling load. Then the shaded area in the diagram will represent the maxima shearing stresses due to dead and live loads, and any ordinate *aa* ... *dd* will represent at that point the maximum shearing force.

Case 5. *Loaded unsymmetrically at intermediate points as in Fig.* 19.

The reactions at the abutments must in this case be first determined. At the abutment A the reaction

$$R_1 = W_1 \times \frac{y_1}{l} + W_2 \times \frac{y_2}{l} + W_3 \times \frac{y_3}{l},$$

and similarly the reaction at the abutment B

$$= R_2 = W_1 \times \frac{x_1}{l} + W_2 \times \frac{x_2}{l} + W_3 \times \frac{x_3}{l}.$$

The bending moment at W_1 in Fig. 19, or at a in Fig. 19a $= R_1 \times x_1$, at W_2, or at b in Fig. 19a, the

$$BM = R_1 \times x_2 - W_1(x_2 - x_1),$$

and at W_3 or C the $BM = R_1 \times x_3 - W_2(x_3 - x_2) - W_1(x_3 - x_1)$.

The bending moment at any section as at MM distant z from left abutment $= R_1 \times z - W_1(z - x_1)$.

The shearing stress at each abutment is equal to the reaction at that abutment, thus at A the shearing stress $= R_1$, and from A to a it is undiminished, from a to b it $= R_1 - W_1$, and from b to c it $= R_1 - W_1 - W_2$.

In Fig. 19a the bending moments are shown graphically. The diagram is constructed as follows: First lay off on the ordinates a, b, c corresponding with the centres of gravity of the loads W_1, W_2, and W_3 the greatest moment due to each load separately as in Case 2, and draw the inclined lines to each point of support as shown in the Fig. by the dotted lines. When the diagram for each load has been so drawn prolong the ordinate a, making it equal to $fk = fg + fa + fh$. Proceed similarly at the ordinates b and c. Then any ordinate from the line AB to the curve AkmoB will represent by scale the bending moment at that section.

In the diagram for shearing forces the reactions R_1 and R_2 are laid off at A and B, as shown in Fig. 19b. The ordinate

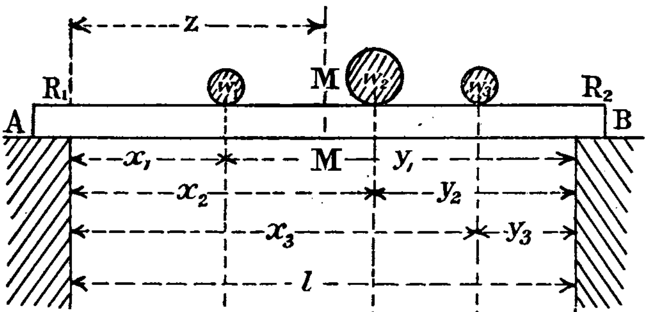


Fig. 19.

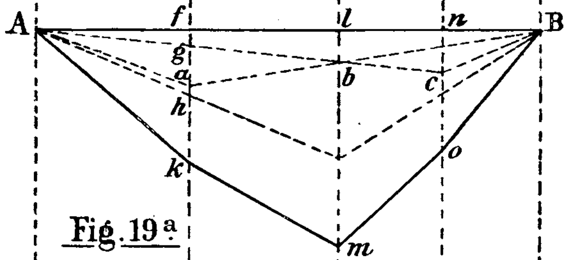


Fig. 19ª.

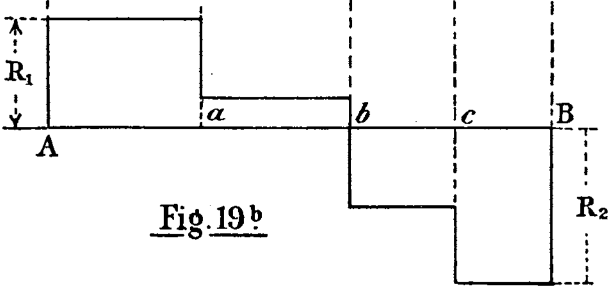


Fig. 19ᵇ.

at $a = R_1 - W_1$, and similarly at C the ordinate $= R_2 - W_3$ in each case representing the shearing forces at these points.

Example. Let l in Fig. 19 = 10 feet.
$W_1 = 5$ tons, $W_2 = 10$ tons, and $W_3 = 5$ tons.
$x_1 = 3$ feet, $x_2 = 6$ feet, and $x_3 = 8$ feet.

Then $R_1 = 5 \times \frac{7}{10} + 10 \times \frac{4}{10} + 5 \times \frac{2}{10} = 8 \cdot 5$ tons.
$R_2 = 5 \times \frac{3}{10} + 10 \times \frac{6}{10} + 5 \times \frac{8}{10} = 11 \cdot 5$ tons.

The bending moments with reference to the abutment $A = R_1$.
$=$ at W_1, $8 \cdot 5 \times 3$ .... $25 \cdot 5$ tons.
$=$ at W_2, $8 \cdot 5 \times 6 - 5 \times 3$ .... $36 \cdot 0$ tons.
$=$ at W_3, $8 \cdot 5 \times 8 - 10 \times 2 - 5 \times 5$ .... $23 \cdot 0$ tons.

Similarly the bending moments with reference to the abutment $B = R_2$.
$=$ at W_3, $11 \cdot 5 \times 2$ .... $23 \cdot 0$ tons.
$=$ at W_2, $11 \cdot 5 \times 4 - 5 \times 2$ .... $36 \cdot 0$ tons.
$=$ at W_1, $11 \cdot 5 \times 7 - 10 \times 3 - 5 \times 5$ .... $25 \cdot 5$ tons.

The shearing stress at each abutment is equal to the reaction, at A it is 8·5 tons and at B 11·5 tons. At a in Fig. 19b the shearing stress $= 8 \cdot 5 - 5 = 3 \cdot 5$ tons and at C it $= 11 \cdot 5 - 5 = 6 \cdot 5$ tons.

Case 6. *Loaded with a uniformly distributed load and a concentrated load at centre.*

This is a combination of cases 1 and 3.

The bending moment at the centre of the beam $= \frac{Wl}{4} + \frac{wl^2}{8}$.

The bending moment at any section, as at AB distant x from the abutment $= \frac{W}{2}x + \frac{wx}{2}y$.

The shearing force at each abutment $= \frac{W}{2} + \frac{wl}{2}$, and at any section, as at AB, Fig. 20b, the shearing force $= \frac{W}{2} + wx$.

Example. Let l in Fig 20 = 10 feet,
and $w = 1$ ton.
and $W = 4$ tons.
and $x = 2$ feet.

Then the bending moment at centre of beam

$$=\frac{4\times 10}{4}+\frac{10\times 10}{8}=22\cdot 5 \text{ tons,}$$

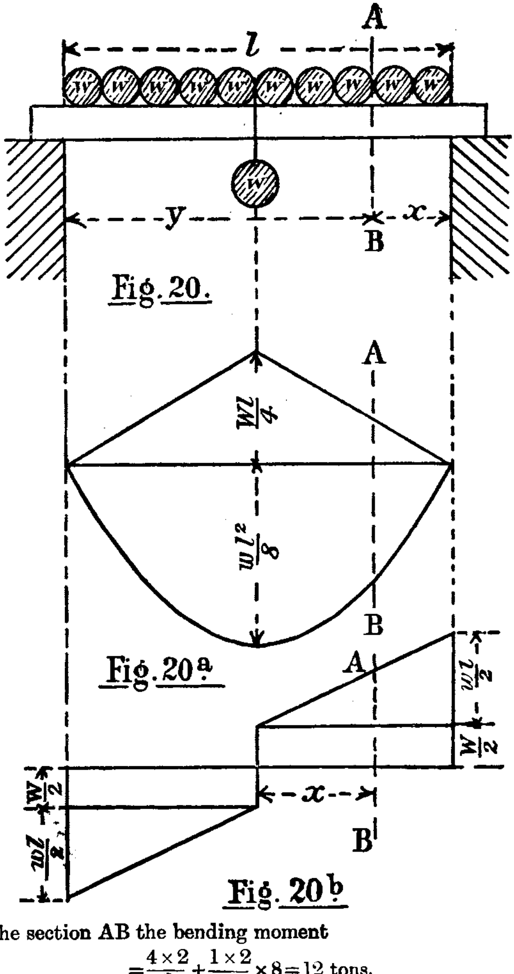


Fig. 20.

Fig. 20a.

Fig. 20b.

and at the section AB the bending moment

$$=\frac{4\times 2}{2}+\frac{1\times 2}{2}\times 8=12 \text{ tons.}$$

The shearing stress at the abutments

$$=\frac{4}{2}+\frac{10}{2}=7 \text{ tons,}$$

and at the section AB the shearing stress

$$=\frac{4}{2}+1\times 3=5 \text{ tons.}$$

Case 7. *To determine the equivalent centre load in the case of a beam loaded symmetrically or unsymmetrically at different points, multiply each load by twice its distance from the nearest point of support, sum the products and divide by the span.*

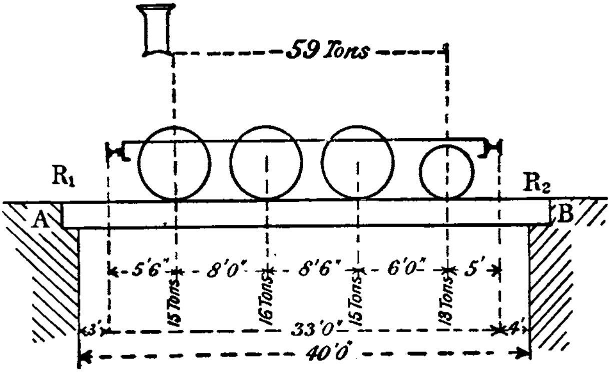


Fig. 21.

Let it be required to find the equivalent centre load of a locomotive engine, whose weight and distribution of same is given in Fig. 21, on a beam of 40 feet span.

Then 13 tons × 9 feet × 2 = 234
15 tons × 15 feet × 2 = 450
15 tons × 8·5 feet × 2 = 255
16 tons × 16·5 feet × 2 = 528

1467 ÷ 40 = 36·675 tons,

the equivalent centre load = 73·350 tons uniformly distributed load.

The bending moment due to a centre load of 36·675 tons on a span of 40 feet $= \frac{36 \cdot 675 \times 40}{4} = 366 \cdot 75$ tons.

This may be verified by taking moments from either abutment, thus the reaction at abutment $B = R_2$

$$= 13 \times \frac{31}{40} + 15 \times \frac{25}{40} + 16 \times \frac{16 \cdot 5}{40} + 15 \times \frac{8 \cdot 5}{40} = 29 \cdot 237 \text{ tons,}$$

and the bending moment at the centre of the beam

$= 29 \cdot 237$ tons $\times$ 20 feet $-$ 15 tons $\times$ 5 feet $-$ 13 tons $\times$ 11 feet

$= 366 \cdot 75$ tons as above.

Beam supported at two points with overhanging or Cantilever ends.

Case 1. *Beam supported at two points with overhanging ends of equal lengths and loaded at centre and at each extremity with concentrated loads as in Fig. 22.*

The bending moments at A and B are

$$M = W_1 x_1 = W_2 x_2 \text{ or} = W_2(l + x_2) + Wy - R_2 l.$$

The bending moment at the centre $C = \frac{Wl}{4} - Wx$.

The bending moment at any section between A and B distant y from $A = M = R_1 y - W_1(x_1 + y)$.

The bending moment at any section of the overhanging part of the beam distant x from the end of the beam, or from W_1 or $W_2 = M = W_1 x$ or $M = W_2 x$.

The bending moment due to the weights W_1 and W_2 produces a reactionary moment between the supports A and B which is uniform and equal to $W_1 x_1 = W_2 x_2$. The load at the centre of the beam C, as in an ordinary beam loaded with a concentrated load, produces a bending moment $= M = \frac{Wl}{4}$, and the bending moment at the centre of the beam due to the whole system of loading $= \mathrm{M} = -\mathrm{W}x + \frac{\mathrm{W}l}{4}$.

M may be positive or negative, varying with the position and intensities of the loads, W_1, W_2, W, etc.

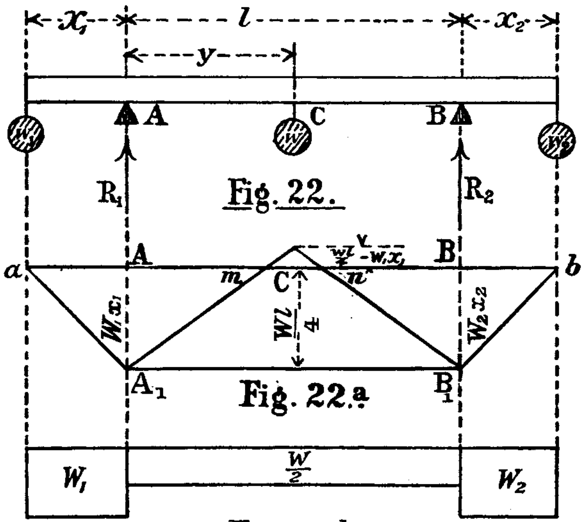


Fig. 22b

The shearing stress throughout the projecting portions is equal to W_1 over x_1, and W_2 over x_2 as in a cantilever, and in the length between the supports A and B the shearing stress $= \frac{\mathrm{W}}{2}$ as in an ordinary beam.

In Fig. 22*a* is shown a diagram of bending moments, the ordinates at A and B being laid out respectively equal to W_1x_1 and W_2x_2, and the ordinate at C the value of which $= \frac{\mathrm{W}l}{4} - \mathrm{W}_1x_1$ is laid down in an opposite direction to those at

A and B. At the point m and n it will be seen that there is no bending moment.

In Fig. 22b is shown a diagram of shearing forces.

Example Let $l=10$ feet, $x_1=x_2=3$ feet, $y=5$ feet, $W=W_1=W_2=2$ tons.

Then $R_1=R_2=W_1+\frac{W}{2}=W_2+\frac{W}{2}=3$ tons.

Then the bending moment at the supports A and B

$$=W_1x_1=W_2x_2=2\times3=6 \text{ tons},$$

and the bending moment at centre of span C

$$M=\frac{Wl}{4}-W_1x_1=\frac{2\times10}{4}-2\times3=5-6=-1 \text{ ton}.$$

The shearing stress in the overhanging portions x_1 and $x^2=2$ tons, and in the length l between supports A and B the shearing stress $=1$ ton.

Case 2. *Beam supported at two points, with overhanging ends of equal lengths and loaded with a uniformly distributed load as in Fig.* 23.

In this case the bending moments at A and B are respectively equal to $\frac{wx_1^2}{2}$ and $\frac{wx_2^2}{2}$ as in an ordinary cantilever, and an equivalent uniform upward bending moment is produced between the supports A and B by this downward bending moment which $=\frac{Wx_1^2}{2}$.

The downward bending moment due to the weights supported between A and B is, as in an ordinary beam $\frac{wl^2}{8}$, so that the bending moment at the centre of the girder due to the whole system of loading $=M=-\frac{wx_1^2}{2}+\frac{wl^2}{8}$.

The shearing stress at the support A due to the weights on the length $x_1=wx_1$ and the stress due to the weights carried on the length $l=\frac{wl}{2}$. The beam being symmetrically loaded the shearing stress at B = the shearing stress at A.

The diagrams, Figs. 23*a* and 23*b*, which need no explanation, show the bending moments and shearing forces graphically.

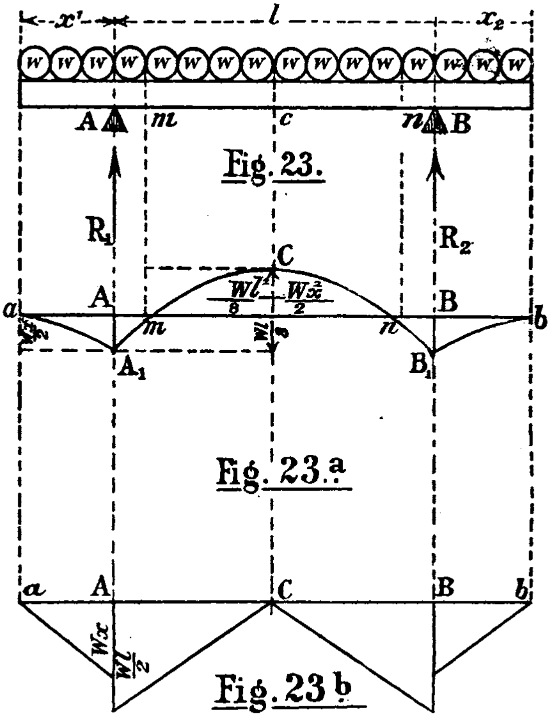


Fig. 23.

Fig. 23.a

Fig. 23 b

Example. Let $l=10$ feet, $x_1=x_2=3$ feet, and let $w=1$ ton per foot run.

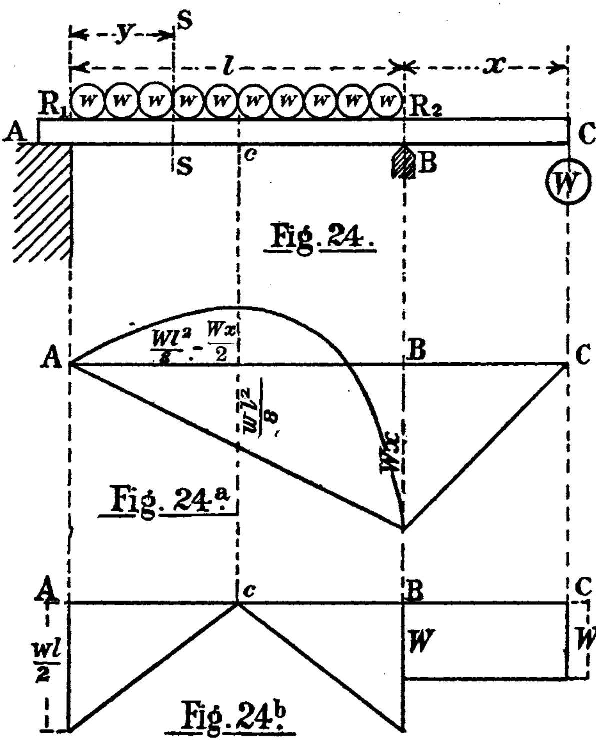
The reactions $R_1=R_2=wx_1+\frac{wl}{2}=3+5=8$ tons.

The bending moment at the centre of the span C

$$=M=-\frac{1\text{ ton}\times 3^2}{2}+\frac{1\times 10^2}{8}=-4\tfrac{1}{2}+12\tfrac{1}{2}=8\text{ tons.}$$

The points *m* and *n* are points of contrary flexure where there is no bending moment.

Case 3. *Beam supported at one end and at an intermediate point with one overhanging end loaded with a uniformly distributed load between the two supports and with a concentrated load at its free end as in Fig.* 24.



The bending moment at the support A is zero and at the support B it is $= Wx$.

The bending moment at the centre of the span AB due to the uniformly distributed load $= \frac{wl^2}{8}$.

The bending moment at the centre of the span due to the whole system of loading $=\frac{wl^2}{8}-\frac{Wx}{2}$.

$$\text{Reaction } R_1 \text{ at } A=\frac{wl\times\frac{l}{2}-Wx}{l},$$

and

$$\text{Reaction } R_2 \text{ at } B=\frac{W(l+x)+wl\times\frac{l}{2}}{l}.$$

Bending moment at any section SS distant y from support A

$$=R_1y-\frac{wy^2}{2}.$$

The shearing stresses at each support $=\frac{wl}{2}$, due to the uniformly distributed load on AB, and the shearing stress at the support B due to the load W at the extremity C = W.

Example. Let $l=10$ feet, $x=5$ feet, $W=3$ tons, $w=1$ ton per foot run, and let $y=3$ feet.

Then $$R_1=\frac{10\times5-3\times5}{10}=\frac{35}{10}=3\tfrac{1}{2}\text{ tons},$$

and $$R_2=\frac{3\times15+10\times5}{10}=\frac{95}{10}=9\tfrac{1}{2}\text{ tons}.$$

Bending moment at A $=0$,

Bending moment at B $=3\times5=15$ tons,

Bending moment at centre C $=\frac{10^2}{8}-\frac{15}{2}=5$ tons,

and Bending moment at SS $=3{\cdot}5\times3-\frac{3^2}{2}=6$ tons.

The shearing force at A and B due to load on AB = 5 tons, and the shearing force to the right of B due to the concentrated load W = 3 tons.

Beam fixed at one end and at both ends.

Case 1. *Beam fixed at both ends, load at centre, and section uniform throughout as in Fig. 25.*

In this case the ends of the beam are firmly held down or fixed, and at the points of support, when the beam is loaded, its upper fibres are in tension and its lower fibres in compression, whereas at the centre these stresses change and become reversed, the upper fibres being in compression and the lower

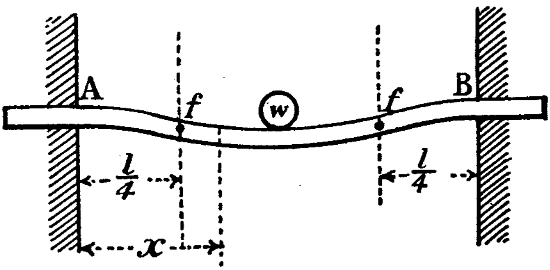


Fig. 25.

fibres in tension, as in an ordinary beam supported at both ends. It is therefore evident that at some intermediate point between the centre of the beam and the point of support there must be a point where there is no direct stress, excepting only the shearing stress, and that point is called the point of contra-flexure or inflexion.

When the load is concentrated at the centre of the beam the position of the point of contra-flexure is $\frac{1}{4}l$ from end, and the maximum bending moment $= \frac{Wl}{8}$.

W in this case = total load,

and l in this case = length of span.

At any point distant x from the support A the bending moment $= \frac{Wl}{2}\left(\frac{x}{l} - \frac{1}{4}\right)$.

The maxima bending moments which occur at the centre and at the points of fixing are equal in amount but opposite in direction.

In Fig. 25a is given a graphical method of arriving at the bending moments.

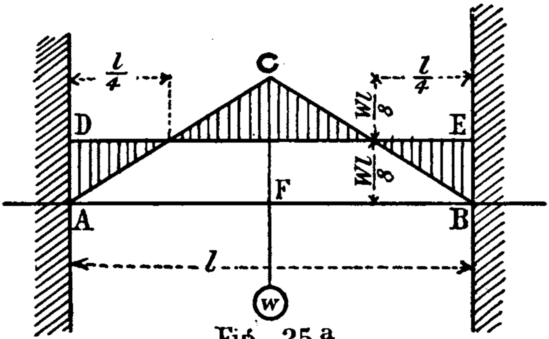


Fig. 25 a.

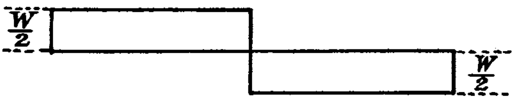


Fig. 25 b.

AB = length of span, and CF $= \frac{Wl}{4}$. Then will the ordinates from the line DE to AC and CB give the bending moments at these places.

Example. Let $l = 10$ feet, $x = 4$ feet, $W = 4$ tons.

Then f the point of contra-flexure $= 2{\cdot}5$ feet, and M the bending moment at centre $=\frac{10\times 4}{8}=5$ tons,

and at a distance x from A the

$$\text{bending moment} = Mx = \frac{4\times 10}{2}\left(\frac{4}{10}-\frac{1}{4}\right)=3 \text{ tons.}$$

The reactions due to a central load in a beam fixed at both ends are the same as in a beam supported at both ends.

The diagram of shearing stress, as shown in Fig. 25*b*, will therefore be the same as in Fig. 13*b*.

Case 2. *Beam of uniform section throughout fixed at both ends and loaded with a uniformly distributed load as in Fig.* 26.

In case 2 the ends of the beams are firmly fixed as in the preceding case, but the beam is loaded with a uniformly distributed load $W=wl$, w being the load per unit or foot of span.

The distance y of the point of inflexion f

$$=\frac{l}{2}\pm\frac{l}{2\sqrt{3}}=0{\cdot}2113l.$$

The maximum bending moment at the points of fixing A and B $=\frac{Wl}{12}$, and at the centre of the beam C it is $=\frac{Wl}{24}$.

At any other point g distant x from A or B the bending

$$\text{moment} = \frac{W}{2}x-\frac{wx^2}{2}-\frac{\left(\frac{W}{2}\right)^2}{3}=\frac{W}{2}x-\frac{wx^2}{2}-\frac{wl^2}{12}.$$

Example. Let $l=10$ feet, $W=10$ tons $=w\times 10$, and $x=4$ feet.

Then the bending moment M at A and B $=\frac{10\times 10}{12}=8\frac{1}{3}$ tons,

and the bending moment M at centre C $=\frac{10\times 10}{24}=4\frac{1}{6}$ tons,

and Mx at distance x from the point of fixing at A

$$=\frac{10}{2}\times 4-\frac{1\times 4^2}{2}-\frac{5^2}{3}=3\frac{2}{3} \text{ tons.}$$

The shearing stress in this case is the same as for a beam uniformly loaded and supported at both ends, and the diagram of shearing stress in Fig. 26*b*, it will be observed, is precisely the same as that given in Fig. 15*b*.

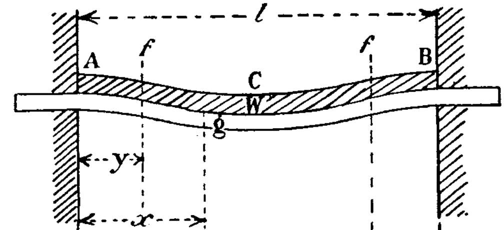


Fig. 26.

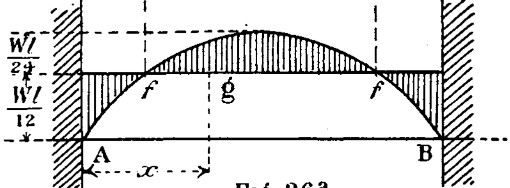


Fig. 26a

Fig. 26b

Case 3. *Beam of uniform section throughout fixed at one end, supported at other end, and loaded at centre as in Fig.* 27.

In this case the reactions at A and B are unequal. At A the reaction $= \frac{11}{16}$W, and at B it $= \frac{5}{16}$W.

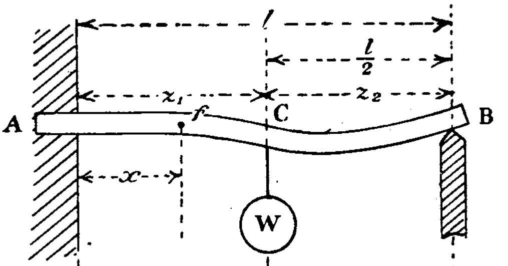


Fig. 27.

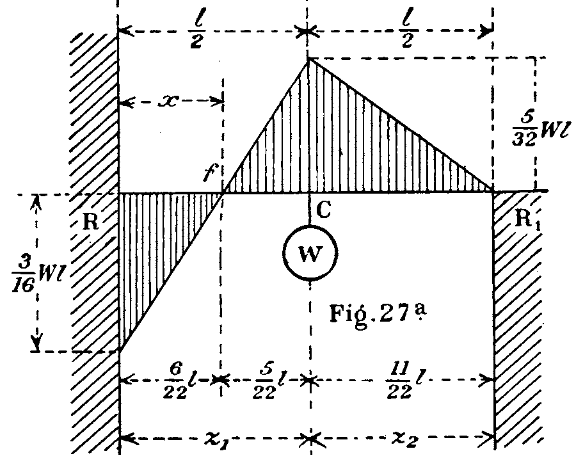


Fig. 27a.

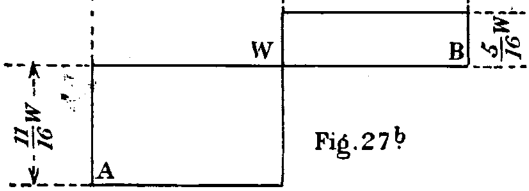


Fig. 27b.

Let reaction at $A = R$, and at $B = R_1$.

The distance of the point of inflexion f

$$= x = \frac{Wz_1 - 2R_1\frac{l}{2}}{W - R_1} = \frac{6}{22}l.$$

The maximum bending moment which occurs at

$$A = \frac{11}{16}W \times \frac{3}{11}l = \frac{3}{16}Wl.$$

The bending moment at the point C under W

$$= \frac{5}{16}W \times \frac{l}{2} = \frac{5}{32}Wl.$$

The shearing stresses are shown in the diagram, Fig. 27*b*, in which it will be observed that between the weight W and B the shearing stress = the reaction at B, and between W and A the shearing stress = the reaction at A.

Example. Let $l = 10$ feet, $W = 4$ tons.

Then reaction at $A = R = 4 \times \frac{11}{16} = 2\frac{3}{4}$ tons,

and at $B = R_1 = 4 \times \frac{5}{16} = 1\frac{1}{4}$ tons.

$$x = \frac{6 \times 10}{22} = 2{\cdot}72 \text{ feet}.$$

Max. bending moment at $A = \frac{3}{16} \times 4 \times 10 = 7\frac{1}{2}$ tons,

and at c under $W = Mc = \frac{5}{16} \times 4 \times 5 = 6\frac{1}{4}$ tons.

Case 4. *Beam of uniform section throughout, fixed at one end, supported at the other end, and loaded with a uniformly distributed load as in Fig.* 28.

In this case the point of inflexion f is distant x from $a = \frac{l}{4}$.

The point of inflexion being at a distance $\frac{l}{4}$ from the fixed end A, fB may be regarded as a beam uniformly loaded throughout and supported at the points f and B.

Then reaction at $B = R_1 = \frac{1}{2}$ load on $fB = \frac{3}{8}W$.

W being the total load $= wl$, w being the unit load. At the fixed end A the reaction $R = W - \frac{3}{8}W = \frac{5}{8}W$.

The maximum bending moment is at the fixed end A where

$$\text{M max.} = -\tfrac{3}{8}\text{W} \times l + \text{W} \times \frac{l}{2} = \frac{\text{W}l}{8}.$$

The bending moment at the point $\text{C} = \dfrac{9\text{W}l}{128} = \dfrac{\text{W}l}{14{\cdot}2}$.

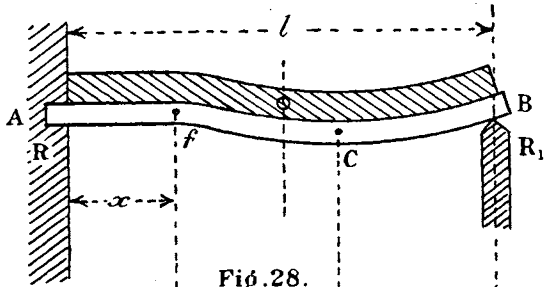


Fig. 28.

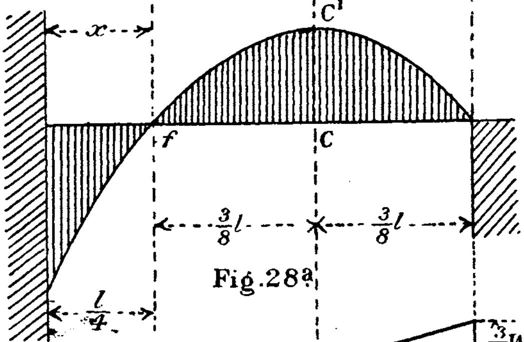


Fig. 28a.

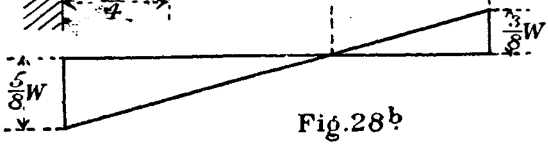


Fig. 28b.

The bending moment at the centre ⊙

$$=\tfrac{3}{8}W\times\frac{l}{2}-\frac{W}{2}\times\frac{l}{4}=\frac{Wl}{16}.$$

Fig. 28 is a diagram of the shearing stresses.

Example. Let $l=10$ feet, $w=1$ ton, and $wl=W=10$ tons.

Then x the distance from A of the point of inflexion $f=\frac{l}{4}=2\cdot5$ feet.

Reaction at $B=R_1=10\times\frac{3}{8}=3\frac{3}{4}$ tons,

and Reaction at $A=10-3\frac{3}{4}=6\frac{1}{4}$ tons.

The max. bending moment at $A=M$ max. $=\frac{10\times10}{8}=12\frac{1}{2}$ tons,

and the bending moment at $C=\frac{10\times10}{14\cdot2}=7\cdot03$ tons,

and the bending moment at the centre $=\frac{10\times10}{16}=6\frac{1}{4}$ tons.

CHAPTER III.

Stresses in framed Structures.

The triangular framework or truss illustrated in Fig. 29 represents a Kingpost truss, which is so largely used in roof work, inverted. In its inverted form it constitutes the simplest type of a trussed girder and has been largely employed in bridge construction.

In Fig. 29 the truss is loaded with a concentrated load W at the centre of the span. Let it be required to determine the stresses due to the load W in the various members.

The weight W is in the first instance transmitted in its entirety through the vertical strut CD to the point D, whence the weight is taken up in equal moieties by the inclined tie-rods AD and DB to the abutments or points of support A and B.

A weight or force acting in a vertical downward direction at the upper extremity of a vertical strut in the form of a column or pillar exerts in that strut a compressive stress, whose magnitude or intensity is equal to that of the weight or force, hence the stress due to the weight W in the strut CD is a compressive stress = W.

Similarly a weight or force applied to the lower extremity in a downward direction, or suspended from a rod or tie, the axis of which is vertical, produces in that rod or tie a tensile stress equal to the weight or force.

The stress in an inclined bar is determined by multiplying the weight or force acting on the bar by the length of the bar and dividing the product by the perpendicular height of the summit of the bar above its base.

Thus the stress in the inclined bars AD or DB

$$= \frac{W}{2} \times \frac{DA}{CD} \text{ or } \frac{W}{2} \times \frac{DB}{CD}.$$

In this instance one half of the weight W is taken up by each bar AD and DB.

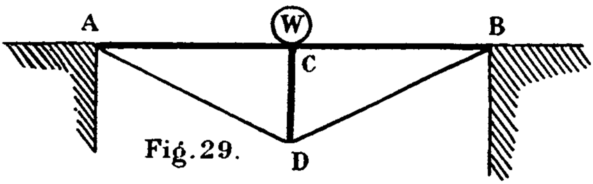


Fig. 29.

The stress in a horizontal bar, or in a bar the direction of which is at right angles to that of the force, is determined by multiplying the weight or force acting upon it by the horizontal distance and dividing the product by the vertical distance.

Thus in AC the stress due to the weight $W = \frac{W}{2} \times \frac{AC}{CD}$.

The stresses in the truss shown in Fig. 29 may now be written down as under:

$$\text{Stress in } AC = BC = \frac{W}{2} \times \frac{AC}{CD}.$$

$$\text{Stress in } CD = W.$$

$$\text{Stress in } AD = BD = \frac{W}{2} \times \frac{AD}{CD}.$$

Example. Let $AC = \frac{AB}{2} = 5$ feet, and $AB = 10$ feet.

Let $CD = 2\frac{1}{2}$ feet.

Let $W = 5$ tons.

Let $AD = \sqrt{AC^2 + CD^2} = 5{\cdot}6$ feet.

Then stress in $AC = BC = \frac{5}{2} \times \frac{5}{2\frac{1}{2}} =$ .... $+5$ tons.

Then stress in $CD = 5$ tons .... $+5$ tons.

Then stress in $AD = BD = \frac{5}{2} \times \frac{5 \cdot 6}{2 \cdot 5} =$ .... $-5 \cdot 6$ tons.

The sign + denotes compression, and the sign − denotes tension.

The stresses in this type of truss can be very readily determined by a graphic process. Thus, in Fig. 30, let ABD be a skeleton diagram of the truss.

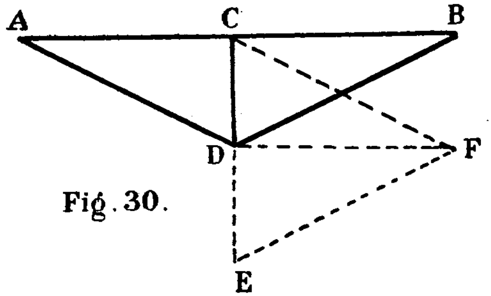


Fig. 30.

Prolong the line CD, and with any convenient scale lay off $CE = W = 5$ tons. From E draw EF parallel to DB, and from C draw CF parallel to AD. Join DF, which should be parallel to AB. Then will CE represent by scale the stress in CD, CF and EF the stresses in AD and DB, and DF the stresses in AC and CB.

In Fig. 31 is represented the same truss, but loaded with a uniformly distributed load. The calculation of the stresses on the various members is effected in precisely the same manner as in the last case, the load being assumed to be divided into three sections

$$w_1 = \frac{W}{4}, \; W_2 = \frac{W}{2}, \text{ and } W_{11} = \frac{W}{4}.$$

W_2, being half of the total load, is transmitted through CD and carried in equal moieties by the tie-rods AD and DB to the abutments A and B, the two sections W_1 and W_{11}, each one quarter of the total weight, rest directly upon the abutments A and B.

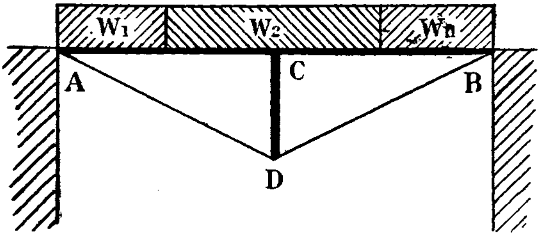


Fig. 31

Let AB = 10 feet, CD = 2·5 feet, W = 10 tons.
Then W_1 and W_{11} will each be = $2\frac{1}{2}$ tons, and $W_2 = 5$ tons.
The stresses in the several members will be as under:

$$AC = BC = \frac{5}{2} \times \frac{5}{2\frac{1}{2}} = \dots\dots \ \dots\dots + 5 \text{ tons.}$$

$$CD = 5 \text{ tons} \dots\dots = \dots\dots\dots\dots + 5 \text{ tons.}$$

$$AD = BD = \frac{5}{2} \times \frac{5{\cdot}6}{2{\cdot}5} = \dots\dots\dots\dots - 5{\cdot}6 \text{ tons.}$$

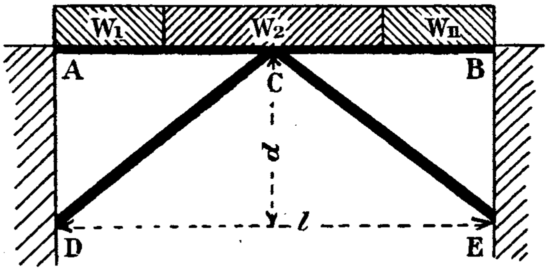


Fig. 32.

The arrangement shown in Fig. 32 is a simple form of trussing a timber beam.

The beam is supported at the three points A, B, and C, and AC and CB are practically two beams.

The sections of the load W_1 and W_{11} rest directly on the abutments at A and B, and the section W_2 is supported by the two inclined struts CD and CE, each of which transmits one half of the load W_2 to the abutments at D and E.

The longitudinal stress in $CD = CE \quad = \frac{W_2}{2} \times \frac{CD}{d}$.

The thrust at the abutments D and $E = \frac{W}{2} \times \frac{AC}{d}$.

Example. Let $AB = DE = 20$ feet, and $AC = CB = 10$ feet, $d = 10$ feet.

Then $CD = CE = \sqrt{AC^2 + AD^2} = \sqrt{10^2 + 10^2} = 14{\cdot}14$.

Let $W_1 = W_{11} = 5$ tons, and $W_2 = 10$ tons.

Then the longitudinal stress in $CD = \frac{10}{2} \times \frac{14{\cdot}14}{10}$ .... $= 7{\cdot}07$ tons.

The thrust at D and E $= \frac{10}{2} \times \frac{10}{10}$ .... $= 5{\cdot}00$ tons.

In bridge structures, beams and trussed girders are subjected to the action of dead and live loads, the dead load. or as it is sometimes called, the "uniform" or "static" load, consisting of the weight of the bridge itself together with its permanent load, comprising the permanent way and ballasting in a railway bridge, and a metalled roadway and paved sidewalks in a highway bridge. The live or variable load consists of the moving or travelling load, which at more or less frequent intervals passes over the bridge.

The moving load in its passage over the bridge produces in the different members varying bending moments for different positions of the load, frequently reversing the stress in certain members from tension to compression and *vice versa*.

In all the cases that have so far been investigated the bending moments and shearing forces due to dead or static loads have only been considered, but in future examples the stresses due to both dead and live loads will be investigated.

In Fig. 33 is represented a section of a truss which is a type of a system of timber bridge construction that has been extensively employed on the Norwegian State Railways, and introduced by the late Mr. Carl Pihl, M.I.C.E., the engineer-in-chief.

The vertical trestles are spaced about 20 feet apart, and the beams are supported at mid-span by two inclined braces in precisely the same manner as illustrated in Fig. 32, with the exception that the thrust which is taken up by the abutments in Fig. 32 is, in this instance, resisted and counteracted by the ties C, C′, C″.

In these trusses each span $A'A = AA = AA''$, and each span for the purposes of determining the stresses may be regarded as consisting of two sections AB and BA resting on their supports A, B, and A.

Let $W = W_1 = W_2 = W_3$ = the dead and live load carried at each point of support A, B, A, etc.

S = maximum stress in any member—then in any vertical post AC or A″C″, $S = W_1 = W_3$, and in any inclined strut as CB whose lower extremities rest against the verticals in the same horizontal plane as in the middle span of Fig. 33,

$$S = \frac{W}{2} \times \frac{BC}{AC} = \frac{W}{2} \; \frac{(\sqrt{AC^2 + AB^2})}{AC}.$$

The maximum tension on the horizontal tie CC

$$= S = \frac{W}{2} \times \frac{\frac{1}{2}CC}{AC}.$$

When the diagonal struts are unsymmetrically inclined, as in the two side spans of Fig. 33, it is necessary to determine the proportion of the load carried by each strut. This can readily be done by the aid of the diagram in Fig. 34, which represents the stresses in the diagonals and tie in span 3,

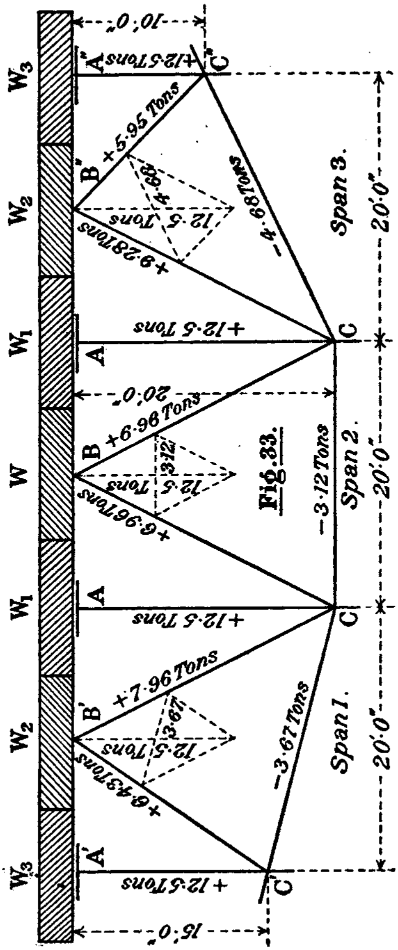

W
W₁
W₂
W₃
A
B
C
A'
B'
C'
A"
B"
C"
+12·5 Tons
+5·95 Tons
+9·28 Tons
12·5 Tons
4·68
−4·68 Tons
+6·96 Tons
3·12
−3·12 Tons
+7·96 Tons
+9·43 Tons
3·67
−3·67 Tons
Span 1.
Span 2.
Span 3.
20'0"
10'0"
15'0"

Fig. 33.

The construction of the diagram is as under:

Draw a vertical line *ab* to any convenient scale $= W = 12\frac{1}{2}$ tons. From *a* draw *ac* parallel to B″C″ and *bc* parallel to B″C, and from *c* draw *cd* parallel to CC″.

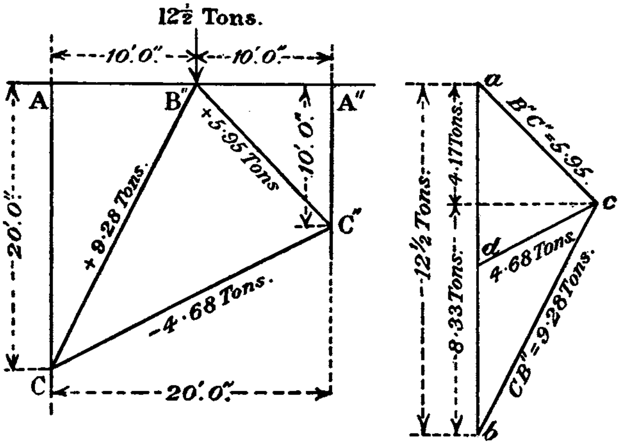


Fig. 34.

Then will *ac*, *cb*, and *cd* give the stresses in B″C″, CB″, and CC″ respectively. The stresses in the other spans determined in a similar manner are written down on each member in the diagram, Fig. 33.

The loads assumed in this case are as follows, viz.:

Live load, 1 ton per foot run, Dead load, 5 cwt. per foot run $=$ Total load of 25 tons on each span, and $12\frac{1}{2}$ tons at each point of support.

The maxima stresses in this truss occur when each span is fully loaded.

Queen Post or Trapezoidal Truss.

This method of strutting beams has been largely employed in timber bridges.

Let w = panel load $= \frac{W}{3}$.

Let W = total uniform load

$$AC = CD = DB = \frac{L}{3}.$$

L = span.

Stress on tie rods CE and DF $= w = \frac{W}{3}$.

Stress on AE = BF $= w\frac{AE}{CE}$.

Stress on EF = stress on AB $= w\frac{AC}{CE}$.

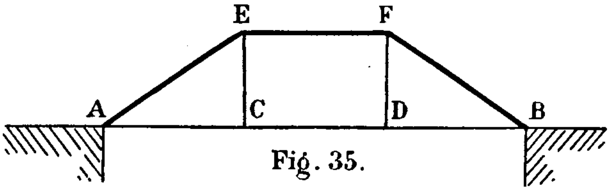


Fig. 35.

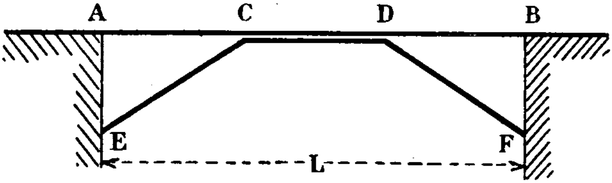


Fig. 36.

These stresses are the stresses due to a full load, that is, when the points C and D are loaded with $w = \frac{W}{3}$, and they have the same value in both trusses, Figs. 35 and 36, but the tension in the straining beam AB in Fig. 35 is converted into thrust against the abutments at E and F in Fig. 36.

Gantry Truss.

Example. w = dead load = 3 cwt. per lin. foot = 3 tons per panel.
w' = live load = 9 cwt. per lin. foot = 9 tons per panel.
$w + w' = W$.

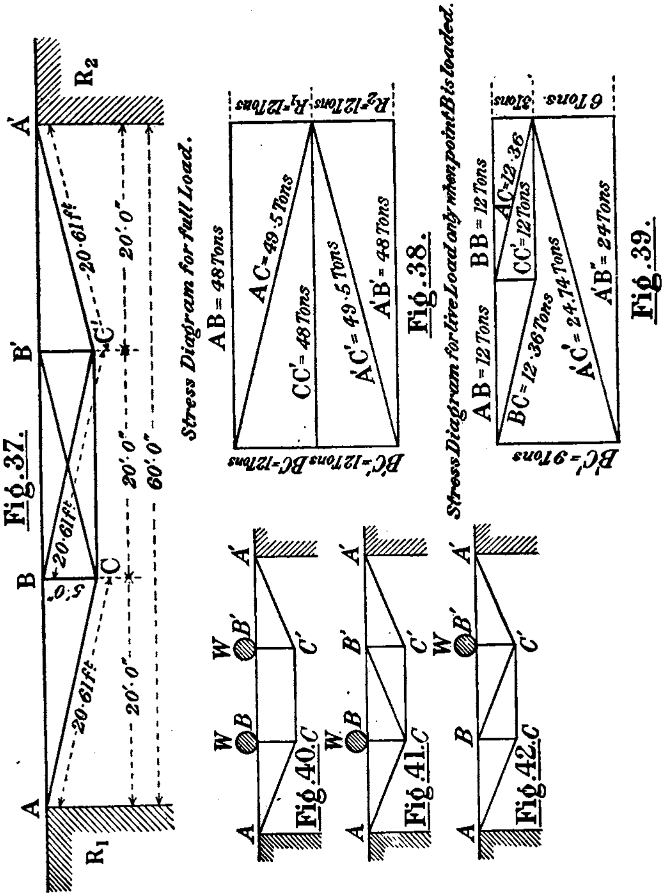


Fig. 37.

Stress Diagram for full Load.

Fig. 38.

Stress Diagram for live Load only when point B is loaded.

Fig. 39.

Fig. 40.

Fig. 41.

Fig. 42.

The maxima stresses in the top and bottom chords occur when the truss is fully loaded as in Fig. 40. Under this condition of loading the diagonals in the centre panel BC′ and B′C bear no strain and may be omitted, but under a moving load, advancing from left to right or right to left, when only one panel point B or B′ is covered as in Figs. 41 and 42, the diagonals become indispensable. Thus in Fig. 41 the stress due to the weight W is transmitted to the point C by the strut BC. From the panel point C two thirds of the weight are transmitted to the abutment A by the tie-rod AC, the remaining one third being carried from C to B′ by the tie rod CB′, thence downward along the strut B′C′, and finally along the rod C′A′ to the abutment A′.

The weight at B is transmitted in the same manner in the proportions of two thirds to the right-hand abutment and one third to the left-hand abutment.

The maximum compression in AB = BB′, and the tension in

$$CC' = (w + w')\frac{AB}{BC} = \frac{12 \times 20}{5} = 48 \text{ tons.}$$

The maximum tension in

$$AC = (w + w')\frac{AC}{BC} = \frac{12 \times 20{\cdot}61}{5} = 49{\cdot}5 \text{ tons.}$$

The maximum compression on

$$BC = B'C'' = w + w' = 12 \text{ tons.}$$

The stress in either diagonal

$$= \frac{w'}{3} \times \frac{BC'}{BC} = \frac{9}{3} \times \frac{20{\cdot}61}{5} = 12{\cdot}36 \text{ tons.}$$

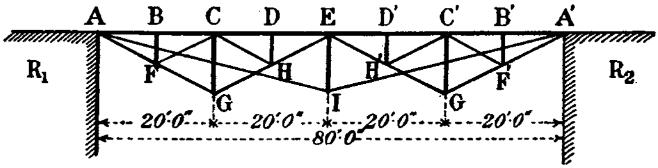
Figs. 41 and 42 indicate the conditions under which CB′ and BC′ are stressed.

Fink Truss.

This truss is so called after the late Mr. Albert Fink, Civil Engineer, by whom it was introduced. The first bridge of importance built by him was the bridge over the Monongahela River at Fairmont, W. Va., in 1852, consisting of three spans of 205 feet each, and a large number of bridges on this plan have since been built in the United States of America.

The construction of the Fink truss is very simple, and the stresses, which can be readily determined, are always either compressive or tensile, with no alternating stresses in any member.

Fig. 43.



The maxima chord stresses occur when all panels are loaded.

The vertical members are all posts, and capable only of taking compression.

The stresses are as under:

Let w = panel dead load.

w' = panel live load.

W = total weight of truss and load.

S = span in feet.

Then stress on centre or half post $EI = (w + w')4 = \frac{W}{2}$.

stress on quarter post $CG = (w + w')2 = \frac{W}{4}$.

stress on eighth post $BF = DH = (w + w') = \frac{W}{8}$.

The tension on suspending rod $AI = \frac{W}{4} \frac{AI}{EI}$.

tension on suspending rod $FG = GH = \frac{W}{8} \frac{AG}{CG}$.

The tension on suspending rod $FC = CH = \frac{W}{16}\frac{FC}{BF}$.

tension on suspending rod $AF = HE = \frac{W}{8}\frac{AG}{CG} \times \frac{W}{16}\frac{FG}{BF}$.

The stress on the chord, which is uniform throughout, is found by resolving the stresses transmitted by the tension rods thus:

$$\frac{W}{4}\frac{\frac{1}{2}S}{EI} + \frac{W}{8}\frac{\frac{1}{4}S}{CG} + \frac{W}{16}\frac{\frac{1}{8}S}{BF}.$$

Example. Span 80 feet, depth of truss FG, EI = 10 feet.
Dead load $\frac{1}{2}$ ton per foot run = 5 tons per panel.
Live load 1 ton per foot run = 10 tons per panel.
Total weight of truss and load = 120 tons.

Then stress on post $EI = (5+10)4 = \frac{120}{2}$ .... = 60 tons.

stress on post $CG = (5+10)2 = \frac{120}{4}$ .... = 30 tons.

stress on post $BF = DH = (5+10) = \frac{120}{8}$ .... = 15 tons.

The tension on suspending rod $AI = \frac{120}{4} \times \frac{41 \cdot 3}{10}$ .... = 123·9 tons.

tension on suspending rod $FG = GH = \frac{120}{8} \times \frac{22 \cdot 3}{10}$ = 33·45 tons.

tension on suspending rod $FC = CH = \frac{120}{16} \times \frac{11 \cdot 18}{5} = 16 \cdot 77$ tons.

tension on suspending rod AF = HE

$$= \frac{120}{8} \times \frac{22 \cdot 3}{10} + \frac{120}{16} \times \frac{11 \cdot 18}{5} = 50 \cdot 22 \text{ tons.}$$

The stress on the top chord which is uniform throughout

$$= \frac{120}{4} \times \frac{40}{10} \times \frac{120}{8} \times \frac{20}{10} + \frac{120}{16} \times \frac{10}{5} = 120 + 30 + 15 = 165 \text{ tons.}$$

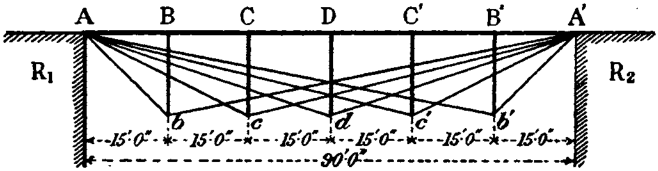
Bollman Truss.

This truss was introduced by Mr. Wendel Bollman of Baltimore, U.S.A., about the year 1840, and is named after him.

The horizontal strain in the chord will be uniform throughout, as in the Fink truss, and will be greatest when all panels are fully loaded.

Let w = panel dead load,
w' = panel live load,
W = total weight of truss and load,
D = depth of truss,
S = span in feet,
N = number of panels.

Fig. 44.



The maximum strain in chord $= \frac{\text{WS}}{\text{D}} \frac{\text{N}^2 - 1}{6\text{N}^2}$.

Strain in any short tie

$$= \frac{(w + w') \times \begin{array}{c}\text{horizontal distance from post}\\ \text{to farthest end of chord}\end{array}}{\text{S}} \times \frac{\text{length of short tie}}{\text{D}}.$$

Thus strain in $\text{A}c = \frac{(w+w')\text{CA}'}{\text{S}} \times \frac{\text{A}c}{\text{D}}$.

Strain in any long tie

$$= \frac{(w + w') \times \begin{array}{c}\text{horizontal distance from post}\\ \text{to nearest end of chord}\end{array}}{\text{S}} \times \frac{\text{length of long tie}}{\text{D}}.$$

Thus strain in $b\text{A}'' = \frac{(w+w')\text{AB}}{\text{S}} \times \frac{b\text{A}'}{\text{D}}$.

Strain on any vertical post Bb, Cc, or D$d = (w + w')$.

Example. Let span $= 90$ feet $= S$,
depth $= 15$ feet $= D$,
dead panel load $= 6$ tons $= w$,
live panel load $= 12$ tons $= w'$,
total weight and load $= 108$ tons $= W$.

Then maximum strain in chord $= \frac{108 \times 90}{15} \times \frac{36-1}{6 \times 36} = 105$ tons.

The tension in $Ab = \frac{18 \times 75}{90} \times \frac{21 \cdot 21}{15} \ldots\ldots\ldots\ldots = 21 \cdot 21$ tons.

tension in $Ac = \frac{18 \times 60}{90} \times \frac{33 \cdot 6}{15} \ldots\ldots\ldots\ldots = 26 \cdot 88$ tons.

tension in $Ad = \frac{18 \times 45}{90} \times \frac{47 \cdot 5}{15} \ldots\ldots\ldots\ldots = 28 \cdot 50$ tons.

tension in $Ac' = \frac{18 \times 30}{90} \times \frac{62}{15} \ldots\ldots\ldots\ldots = 24 \cdot 80$ tons.

tension in $Ab' = \frac{18 \times 15}{90} \times \frac{76 \cdot 5}{15} \ldots\ldots\ldots\ldots = 15 \cdot 30$ tons.

In both the Fink and Bollman trusses the chords must be strong enough not only to bear the horizontal compression to which they are subject, but also to sustain safely as beams the transverse strain from the load carried by them.

Diagram of Stresses.

The stresses in this truss can be readily determined by scale from a diagram, as in Fig. 45, as follows:

From the lower extremity of each vertical post lay off to scale on each post *be*, *ch*, *dk*, *c'n*, and *b'q*, the full panel load $= w + w'$, in this case 18 tons. From *e* draw *ae* parallel to *b*A′ and *ef* parallel to A*b*, then will *ae* represent by scale the stress in *b*A′, *ef* the stress in A*b*, and the horizontal line from *d* will represent the stress in the top chord AB due to the panel load at B. By proceeding similarly at each of the other panel points we have at once the stresses in all the inclined tie rods and vertical members by scaling them off. The stress in the chord may be obtained by adding to-

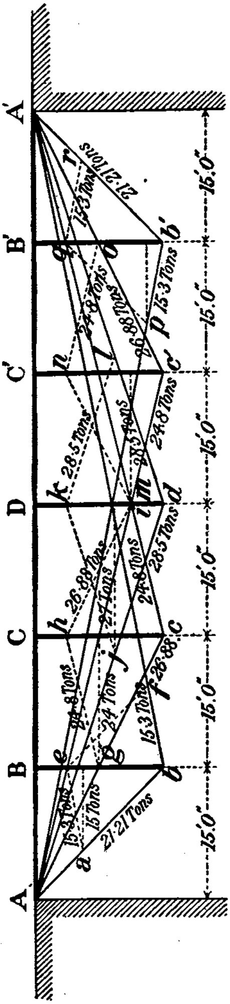

A
B
C
D
C'
B'
A'
15'0"
21·21 Tons
15 Tons
15·3 Tons
24·8 Tons
26·88 Tons
28·5 Tons
24 Tons
27 Tons
15·37 Tons
21·21 Tons

Fig. 45.

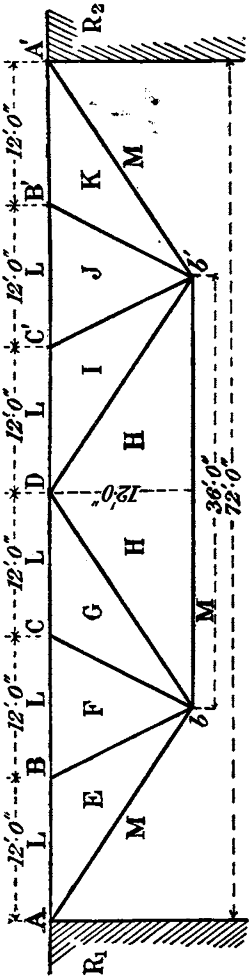

A
B
C
D
C'
B'
A'
R1
R2
L
E
F
G
H
I
J
K
M
b
b'
12'0"
36'0"
72'0"

Fig. 46.

gether the horizontal components represented by the horizontal lines thus

$$15 + 24 + 27 + 24 + 15 \text{ tons} = 105 \text{ tons},$$

as already determined.

The stresses on the various members are written down on each member.

Triangular Truss (Fig. 46).

In this truss the top boom or chord AA′ is supported at five intermediate points B, C, D, C′, and B′. The span AA′ is 72 feet, the depth of the truss is 12 feet, and the distance between each point of support is 12 feet. The static or dead load is 5 cwt. per foot run and the variable or live load 15 cwt. per foot run. The maxima stresses in the various members occur when the truss is fully loaded.

The lengths of the members are as under, viz.:

$\text{A}b = b\text{D} = \sqrt{18^2 + 12^2} = 21{\cdot}63$ ft.
$\text{B}b = b\text{C} = \sqrt{12^2 + 6^2} = 13{\cdot}41$ ft.
$\text{AB} = \text{BC} = \text{CD} = 12$ ft. and $bb' = 36$ ft.

	Tons.
Panel dead load	$= 3 = w$.
Panel live load	$= 9 = w'$.
Total panel load	$= 12 = \text{W}$.

The stresses are determined as follows:

$$\text{Reaction at each abutment due to full load} = \frac{12 \times 5}{2} = 30 \text{ Tons}.$$

Then

$$\text{max. compression in AB} = \frac{30 \times 18}{12} = 45,$$

$$\text{max. compression in BC} = \frac{30 \times 18 - 12 \times 6}{12} = 39,$$

$$\text{max. compression in CD} = \frac{30 \times 18 - 12 \times 6 + 12 \times 6}{12} = 45,$$

$$\text{max. tension in } bb' = \frac{30 \times 36 - 12 \times 12(1 + 2)}{12} = 54,$$

$$\text{compression in B}b = b\text{C} = 12 \times \frac{13{\cdot}41}{12} = 13{\cdot}41,$$

$$\text{compression in } b\text{D} = 10\tfrac{1}{2} \times \frac{21{\cdot}63}{12} = 18{\cdot}83,$$

$$\text{tension in } b\text{D} = \qquad 3 \times \frac{21{\cdot}63}{12} = \overset{\text{Tons.}}{5{\cdot}4},$$

$$\text{tension in A}b = 12 \times 2\tfrac{1}{2} \times \frac{21{\cdot}63}{12} = 54{\cdot}07.$$

Analysing in detail the effect of the various conditions of loading on this truss, we find that the maximum stress, that is to say, the maximum compression in the members Bb and bC, occurs when the full load covers the panel points B and C.

The stress then in Bb or bC $= 12$ tons $\times \dfrac{13{\cdot}41}{12} = 13{\cdot}41$. These members under any condition of loading are only subject to compression due to one panel dead and live load $= 3 + 9$ tons. The maximum tension in bD occurs when the panel points B and C are loaded with the live load when the member bD has to transmit to the right abutment at A one-fourth of the vertical component of the live load at the panel point $b = \left(\dfrac{18}{4} - 1\tfrac{1}{2}\right)\dfrac{21{\cdot}63}{12} = 5{\cdot}4$ tons.

The maximum compression in Db' occurs when the panel points BC and D are covered by the live load, when the stress in D$b' = \tfrac{1}{2}w + \dfrac{1+2+3}{6}w' = \tfrac{1}{2}3 + \dfrac{69}{6} = 13$, and $13 \times \dfrac{21{\cdot}63}{12} = 23{\cdot}43$ tons.

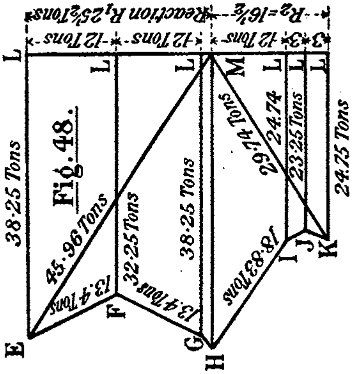
The maximum tension in

$$\text{A}b = 2\tfrac{1}{2}(w + w')\frac{21{\cdot}63}{12} = 2\tfrac{1}{2} \times (3+9) \times \frac{21{\cdot}63}{12} = 54{\cdot}07 \text{ tons.}$$

The maximum compression in AB = the horizontal component of the stress in A$b = 2\tfrac{1}{2}(3+9) \times \dfrac{18}{12} = \dfrac{30 \times 18}{12} = 45$ tons.

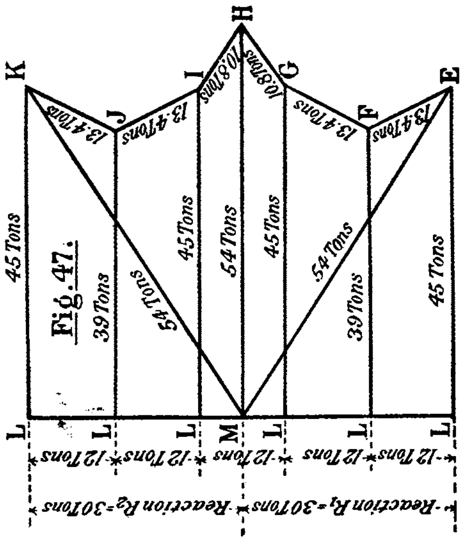
The stresses in the members BC and CD are determined by the method of moments, and their values are as stated.

The stresses in this truss may be ascertained by a graphic process, by drawing a diagram as in Fig. 47.



Stress Diagram when live load covers panel points B, C & D, and the panel points C' & B' are unloaded.

Scale 26·6 Tons to an Inch.



Stress Diagram for Full Load.

In that diagram the reactions at each abutment and the panel loads due to a full load are laid out to any convenient scale on the vertical load line LL. The lines LK, LJ, ... LE are drawn horizontally, delimiting each panel load.

From the centre M the line MK is drawn parallel to the member $b'A'$. From the point K, where it intersects the horizontal line LK, the line KJ is drawn parallel to the member $B'b'$ until it intersects the line LJ at J. From J the line JI is drawn parallel to the member $C'b'$ and IH to the member Db'. The other half of the diagram is constructed in a similar manner.

From this diagram the stresses in the various members due to a full load may be readily scaled off, but the maximum stresses in all the members do not occur when the truss is fully loaded, so it becomes necessary to draw diagrams for other conditions of loading. One such diagram is shown in Fig. 48, in which the panel points BC and D are loaded with the live load and the panel points C' and B' are unloaded. The stresses due to the different conditions of loading are written down on the various members.

The Warren or Triangular Girder.

In the Warren Girder proper the web diagonals are so arranged as to form with the top and bottom chords a series of equilateral triangles, but girders in which the struts and ties are equally inclined, irrespective of the magnitude of the angle subtended by them, are generally known as Warren Girders.

The stresses in this type of girder may be found as follows: In the web members the stresses due in the several members to each position of the live load may be written down as shown in the tabular statement subjoined to Fig. 50.

Thus, when the live load covers the first panel from the left abutment, it is transferred to each abutment in the proportions of $\frac{7}{8}$ to the left abutment a and $\frac{1}{8}$ to the right abutment a''. The live panel load is 6 tons, and $\frac{7}{8}$ of 6 tons $= 5\frac{1}{4}$ tons are carried up in tension by the tie Ab, and in compression by aA to the abutment a. The longitudinal stress in $a\text{A} = \text{A}b$, due to this load $= 5{\cdot}25$ tons $\times 1{\cdot}155 = 6{\cdot}06$ tons, which are accordingly entered in the columns aA and Ab opposite W_1. The remaining $\frac{1}{8}$th of the panel load W_1 is carried to the right abutment at a'' by the web diagonals bB, Bc ... A$''a''$, producing in each member alternate tension, and compression $= \frac{1}{8}$ of 6 tons $= \frac{3}{4}$ ton $\times 1{\cdot}155 = {\cdot}87$ ton, which is accordingly entered in the columns bB, Bc ... De, etc.

Similarly, when the live load covers the second panel point from the left abutment, the proportions carried to the left and right abutments are respectively $\frac{6}{8}$ or $\frac{3}{4}$ and $\frac{2}{8}$ or $\frac{1}{4}$.

The proportion carried to the left abutment $= \frac{3}{4}$ of 6 tons $= 4\frac{1}{2}$ tons produces alternate tension and compression in the diagonals cB, Bb, bA, and Aa amounting to $4{\cdot}5$ tons $\times 1{\cdot}155$ $= 5{\cdot}20$ tons, which is written down in the columns aA, Ab, bB, and Bc opposite W_2. The proportion of the panel load W_2 amounting to $\frac{1}{4}$ of 6 tons $= 1\frac{1}{2}$ tons $\times 1{\cdot}155 = 1{\cdot}73$ is carried to the right abutment by cC, Cd ... A$''a''$, producing in each alternate tension and compression.

By continuing this process and writing down in each column the stresses produced by the live load as it advances over each panel point we have in each diagonal the maximum tension and compression due to the live load.

The stresses in the web members due to the dead load must next be determined, and they are as follows:

Commencing with the centre panel W_4 one half of this weight is transferred to each abutment, the one half supported by the right abutment being carried by eD$''$ and Dd''. At the panel point d'' is suspended a full panel load which must be transferred to a'' so that the diagonals d''C$''$ and Cc'' carry $1\frac{1}{2}$ panel loads. At the panel point c'' another panel load is added, so that the load on c''B$''$ and B$''b''=2\frac{1}{2}$ panel loads, and in a similar manner b''A and Aa carry $3\frac{1}{2}$ panel loads

$$=(w+w')\frac{N-1}{2}.$$

Multiplying each of these values by 1·155 and adding the products, duly entered in their proper columns to the live load stresses already determined, we have the total stress in each diagonal.

The maxima stresses in the top and bottom chords occur when the girder is fully loaded, and are found by adding the horizontal components of the stresses in the web due to a full load, thus:

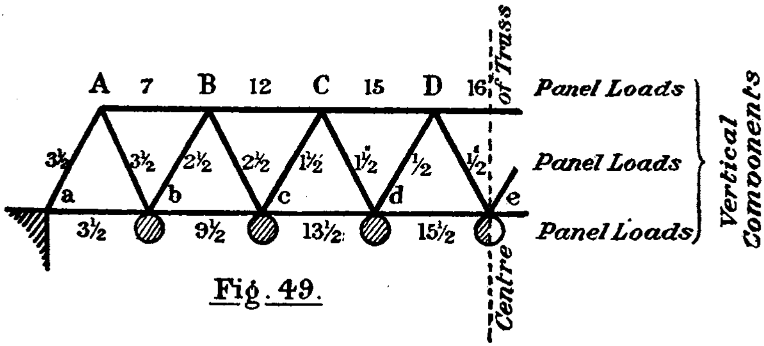


Fig. 49.

The vertical component of the stress in $Aa=3\frac{1}{2}$ panel loads, carried down to a by the end brace aA $=3\frac{1}{2}\times 10{\cdot}5=36{\cdot}75$, and the horizontal component $=36{\cdot}75\times\frac{6}{10{\cdot}4}=36{\cdot}75\times{\cdot}578=21{\cdot}21$ tons.

Similarly the vertical component of the stress in AB = the sum of the stresses in the two diagonals meeting at the apex A = 7 panel loads $=10{\cdot}5\times 7=73{\cdot}5$ tons, and the horizontal component $=73{\cdot}5\times{\cdot}578=42{\cdot}42$ tons.

The stresses in the top and bottom chords may also be found by the method of moments, as shown at the foot of the tabulated statement of the web stresses.

The stresses in this girder may also be determined graphically. The chord stresses may be found by either of the diagrams in Fig. 52 or Fig. 54.

In Fig. 52 it is only necessary to draw the parabolic curve aYa'', making its centre ordinate $XY = \frac{WL}{8d}$ and the ordinates in the manner shown, each of which will represent by scale the stress, in the top or bottom chord member, whose centre it would if prolonged intersect.

In Fig. 54 the various members are drawn parallel to the members bearing the corresponding letters in the diagram of the girder (Fig. 50), starting from the load line on which the panel loads have been laid out by scale.

In Fig. 53, which gives the maxima web stresses, the ordinates ay and az are drawn by scale at right angles to the horizontal line a, a'', the latter representing the reaction at either abutment due to the dead load, and the former the reaction due to the live load when the girder is fully loaded. From y the parabolic curve yxa'' is drawn and from z the line zo. Then the vertical ordinates between the line zo and the parabolic curve yxa'', as shown in the diagram in dotted lines, will give the vertical component of the stresses in any pair of diagonals whose apex they intersect, and the longitudinal stress in those diagonals may be scaled from the diagonal lines drawn from the ordinates as shown.

Warren Girder.

Span 96 feet....depth $= 96$ feet $\times 0{\cdot}1082 = 10{\cdot}38$ feet.

$\left.\begin{array}{l}\text{Dead load 15 cwt. per foot run,}\\ \text{Live load 1 ton per foot run,}\end{array}\right\}$ for bridge.

$\left.\begin{array}{l}\text{Dead load per panel} = 4\tfrac{1}{2}\text{ tons,}\\ \text{Live load per panel} = 6\ \text{ tons,}\end{array}\right\}$ for each girder.

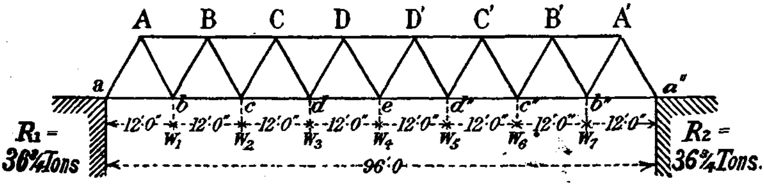
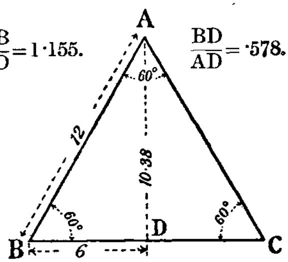


Fig. 50.

The stresses are calculated according to the various positions of the live load advancing from one end of the bridge to the other, and the results are tabulated in the subjoined form:

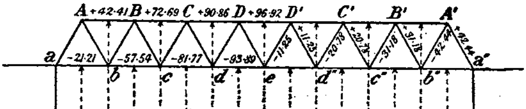
$\frac{AB}{AD} = 1{\cdot}155.$ $\frac{BD}{AD} = {\cdot}578.$



Apices.		aA.	Ab.	bB.	Bc.	cC.	Cd.	dD.	De.
Load at		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
W 1, - - - - -		+6·06	−6·06	−0·87	+0·87	−0·87	+0·87	−0·87	+0·87
W 2, - - - - -		+5·20	−5·20	+5·20	−5·20	−1·73	+1·73	−1·73	+1·73
W 3, - - - - -		+4·33	−4·33	+4·33	−4·33	+4·33	−4·33	−2·60	+2·60
W 4, - - - - -		+3·46	−3·46	+3·46	−3·46	+3·46	−3·46	+3·46	−3·46
W 5, - - - - -		+2·60	−2·60	+2·60	−2·60	+2·60	2·60	+2·60	−2·60
W 6, - - - - -		+1·73	−1·73	+1·73	−1·73	+1·73	1·73	+1·73	−1·73
W 7, - - - - -		+0·87	−0·87	+0·87	−0·87	+0·87	−0·87	+0·87	−0·87
Live Load.	Compression +	+24·25	..	+18·19	+0·87	+12·99	+2·60	+8·66	+5·20
	Tension −	..	−24·25	−0·87	−18·19	−2·60	−12·99	−5·20	−8·66
Dead Load, - -		+18·19	−18·19	+12·99	−12·99	+7·79	−7·79	+2·59	−2·59
Max. Compression, -		+42·44	..	+31·18	..	+20·78	..	+11·25	+2·61
Maximum Tension, -		..	−42·44	..	−31·18	..	+20·78	−2·61	−11·25

Stress Diagrams.

Fig. 51.



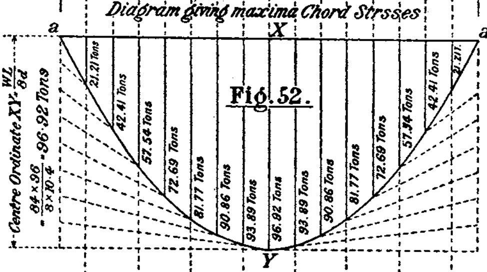


Fig. 52.

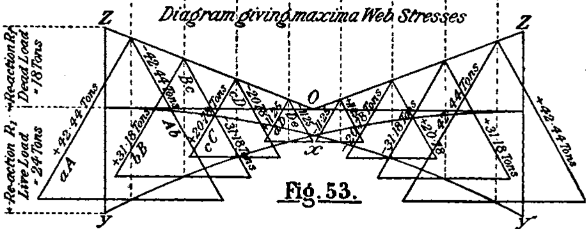


Fig. 53.

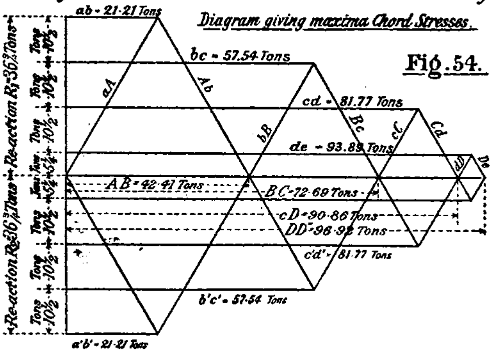


Fig. 54.

Stresses in Top and Bottom Booms.

$$ab = \frac{36{\cdot}75 \times 6}{10{\cdot}4} \ldots\ldots\ldots\ldots = 21{\cdot}21 \text{ tons.}$$

$$AB = \frac{36{\cdot}75 \times 12}{10{\cdot}4} \ldots\ldots\ldots\ldots = 42{\cdot}41 \text{ tons.}$$

$$bc = \frac{36{\cdot}75 \times 18 - 10{\cdot}5 \times 6}{10{\cdot}4} \ldots\ldots\ldots\ldots = 57{\cdot}54 \text{ tons.}$$

$$BC = \frac{36{\cdot}75 \times 24 - 10{\cdot}5 \times 12}{10{\cdot}4} \ldots\ldots\ldots\ldots = 72{\cdot}69 \text{ tons.}$$

$$cd = \frac{36{\cdot}75 \times 30 - 10{\cdot}5(6 + 18)}{10{\cdot}4} \ldots\ldots\ldots\ldots = 81{\cdot}77 \text{ tons.}$$

$$CD = \frac{36{\cdot}75 \times 36 - 10{\cdot}5(12 + 24)}{10{\cdot}4} \ldots\ldots\ldots\ldots = 90{\cdot}86 \text{ tons.}$$

$$de = \frac{36{\cdot}75 \times 42 - 10{\cdot}5(6 + 18 + 30)}{10{\cdot}4} \ldots\ldots\ldots\ldots = 93{\cdot}89 \text{ tons.}$$

$$DD'' = \frac{36{\cdot}75 \times 48 - 10{\cdot}5(12 + 24 + 36)}{10{\cdot}4} \ldots\ldots\ldots\ldots = 96{\cdot}92 \text{ tons.}$$

Inverted Triangular Truss.

This type of girder has been largely used especially in America in deck bridges or bridges in which the load is carried on the top chord.

The stresses in this girder, as in the Warren Girder, may be readily determined on inspection and written down as under.

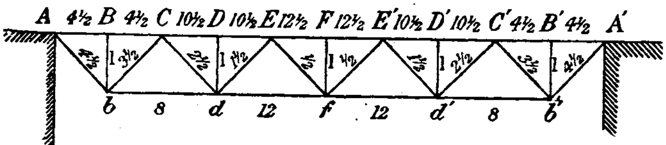


Fig. 55.

Let w = dead or uniform panel load = 5 tons,
w' = live or variable panel load = 10 tons,
S = span = 100 feet and d = depth = 10 feet.

Then maximum tension in

$$\mathrm{A}b\left(4\tfrac{1}{2}w+\frac{1+2+3+4+5+6+7+8+9}{10}w'\right)\frac{\mathrm{A}b}{\mathrm{B}b}$$

$$=(4\tfrac{1}{2}\times 5+4\tfrac{1}{2}\times 10)\times 1\cdot414=67\cdot5\times 1\cdot414=95\cdot44 \text{ tons,}$$

Maximum compression in

$$\mathrm{B}b=\mathrm{D}d=\mathrm{F}f=(w+w')=15\cdot00 \text{ tons,}$$

Maximum compression in

$$b\mathrm{C}=\left(3\tfrac{1}{2}w+\frac{1+2+3+4+5+6+7+8}{10}w'\right)\frac{\mathrm{A}b}{\mathrm{B}b}$$

$$=(3\tfrac{1}{2}\times 5+\tfrac{36}{10}\times 10)\times 1\cdot414=53\cdot5\times 1\cdot414=75\cdot65 \text{ tons,}$$

Maximum tension in

$$\mathrm{C}d=\left(2\tfrac{1}{2}w+\frac{1+2+3+4+5+6+7}{10}w'\right)\frac{\mathrm{A}b}{\mathrm{B}b}$$

$$=(2\tfrac{1}{2}\times 5+\tfrac{28}{10}\times 10)\times 1\cdot414=40\cdot5\times 1\cdot414=57\cdot26 \text{ tons,}$$

Maximum compression in

$$d\mathrm{E}=\left(1\tfrac{1}{2}w+\frac{1+2+3+4+5+6}{10}w'\right)\frac{\mathrm{A}b}{\mathrm{B}b}$$

$$=(1\tfrac{1}{2}\times 5+\tfrac{21}{10}\times 10)\times 1\cdot414=28\cdot5\times 1\cdot414=40\cdot30 \text{ tons,}$$

Maximum tension in

$$\mathrm{E}f=\left(\tfrac{1}{2}w+\frac{1+2+3+4+5}{10}w'\right)\frac{\mathrm{A}b}{\mathrm{B}b}$$

$$=(\tfrac{1}{2}\times 5+\tfrac{15}{10}\times 10)\times 1\cdot414=17\cdot5\times 1\cdot414=24\cdot74 \text{ tons.}$$

Maximum compression in

$$\mathrm{AB}=\mathrm{BC}=4\tfrac{1}{2}(w+w')\frac{\mathrm{AB}}{\mathrm{B}b}=4\tfrac{1}{2}\times 15=67\tfrac{1}{2} \text{ tons.}$$

Maximum compression in

$$\mathrm{CD}=\mathrm{DE}=(4\tfrac{1}{2}+3\tfrac{1}{2}+2\tfrac{1}{2})(w+w')\frac{\mathrm{AB}}{\mathrm{B}b}=10\tfrac{1}{2}\times 15=157\tfrac{1}{2} \text{ tons.}$$

Maximum compression in

$$\mathrm{EF}=\mathrm{FE}'=(4\tfrac{1}{2}+3\tfrac{1}{2}+2\tfrac{1}{2}+1\tfrac{1}{2}+\tfrac{1}{2})(w+w')\frac{\mathrm{AB}}{\mathrm{B}b}$$

$$=12\tfrac{1}{2}\times 15=187\tfrac{1}{2} \text{ tons.}$$

Maximum tension in

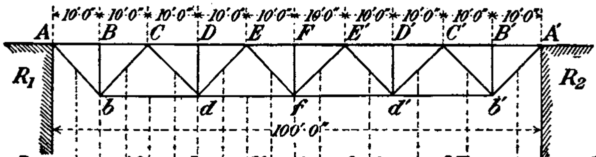
$$bd = (4\tfrac{1}{2} + 3\tfrac{1}{2})(w + w')\frac{\text{AB}}{\text{B}b} = 8 \times 15 = 120 \text{ tons.}$$

Maximum tension in

$$df = (4\tfrac{1}{2} + 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2})(w + w')\frac{\text{AB}}{\text{B}b} = 12 \times 15 = 180 \text{ tons.}$$

Inverted Triangular Truss.

Fig. 56.



Dead or uniform Load-10 cwt per foot run-5 Tons per panel.
Live or variable Load-1 Ton per foot run-10 Tons per panel.

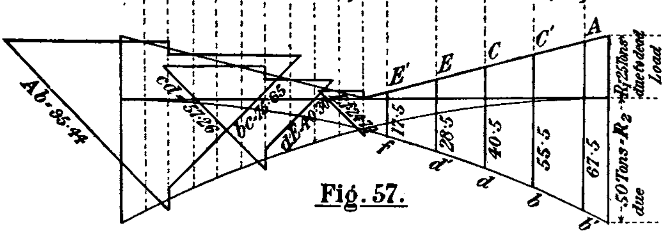


Fig. 57.

Diagram giving maxima in Web Diagonals.
Scale 80 Tons to an Inch.

The stresses in this girder may also be determined graphically from the diagrams in Figs. 57 and 58, which it will be recognized are constructed in precisely the same manner as the diagrams for the Warren girder, with the exception that in Fig. 57 there is a set off in the ordinate coinciding with each vertical which gives a different stress in the two diagonals meeting the vertical due to the load transmitted by the vertical.

In the diagram Fig. 57 the various members are lettered to correspond with the members in Fig. 56, to which they are drawn parallel, and the ordinates in Fig. 57 appertaining to the diagonals in Fig. 56 are indicated by dotted lines.

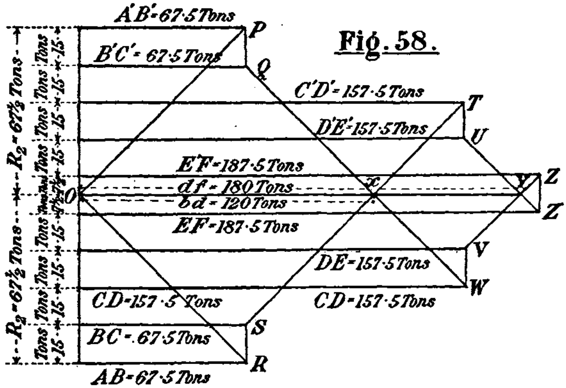


Diagram giving maxima stresses in top & bottom chords.
Scale 80 Tons to an Inch.

In Fig. 58 the abutment re-actions and panel loads are laid off on the vertical load line, and horizontal lines are drawn defining each load limit. From the centre O the diagonal OP is drawn parallel to A'b', PQ parallel to B'b', QX parallel to b'C', XT parallel to C'd', TU parallel to D'd', UY parallel to d'E', and YZ parallel to E'f'. Then by scaling the various lines as shown the stresses in the top and bottom chords may be obtained.

NOTE.—In the diagram Fig. 57 the diagonal members have been omitted in the right hand half of the diagram, but the vertical components of the stresses in the web diagonals are given by the ordinates E'f, E'd' ... A'b', and the longitudinal stress in any diagonal will be in the proportion of the length of the diagonal to the depth of the truss, $=1·414$ in this case, multiplied by the ordinate appertaining to it.

Lattice or Double Intersection Warren Girder.
(Fig. 59.)

This form of girder has been very extensively and generally employed, and for spans of ordinary limits it is probably the simplest and most economical type of girder with parallel chords that has yet been devised.

It will be observed that this girder, shown in Fig. 59, is composed of two distinct and separate systems of web diagonals, each of which is shown in Figs. 60 and 61 respectively. The stresses in each of these systems may be readily determined by the same process as that adopted for arriving at the stresses in the Warren girder, or they may be written down from inspection of the diagrams with the aid of the co-efficients written down on the various members.

The stresses in the top and bottom chords are obtained by adding together the co-efficients in those members due to each system of triangulation; that is to say, the chord co-efficients in Fig. 59 are obtained by adding together the chord co-efficients in Figs. 60 and 61.

Example.

Let $L = \text{span} = 100$ feet, and $d = \text{depth} = 16$ feet.
$w = $ dead or static panel load $= 5$ tons $= \cdot 5$ tons per foot run.
$w' = $ live or rolling load $= 10$ tons $= 1$ ton per foot run.
$l = $ length of panel $= 10$ feet.
$k = $ length of diagonal $= \sqrt{10^2 + 16^2} = 18 \cdot 86$.

Then $\frac{k}{a} = \frac{18 \cdot 86}{16} = 1 \cdot 18$, and $\frac{l}{d} = \frac{10}{16} = \cdot 625$.

The stresses will then be as under:

Maximum compression in Tons.

$$AB = \{(w + w') \times 2\tfrac{1}{2}\}\frac{l}{d} = \{(5 + 10) \times 2\tfrac{1}{2}\} \cdot 625 = 23 \cdot 43.$$

$$BC = \{(w + w') \times 6\tfrac{1}{2}\}\frac{l}{d} = \{(5 + 10) \times 6\tfrac{1}{2}\} \cdot 625 = 60 \cdot 93.$$

Maximum compression in

Tons.

$$CD=\{(w+w')\times\ 9\tfrac{1}{2}\}\frac{l}{d}=\{(5+10)\times\ 9\tfrac{1}{2}\}\cdot625=\ 89\cdot06.$$

$$DE=\{(w+w')\times 11\tfrac{1}{2}\}\frac{l}{d}=\{(5+10)\times 11\tfrac{1}{2}\}\cdot625=107\cdot81.$$

$$EF=\{(w+w')\times 12\tfrac{1}{2}\}\frac{l}{d}=\{(5+10)\times 12\tfrac{1}{2}\}\cdot625=117\cdot18.$$

Maximum tension in

$$ab=\{(w+w')\times\ 2\}\frac{l}{d}\ =\ \{(5+10)\times\ 2\}\cdot625=\ 18\cdot75.$$

$$bc=\{(w+w')\times\ 6\}\frac{l}{d}\ =\ \{(5+10)\times\ 6\}\cdot625=\ 56\cdot25.$$

$$cd=\{(w+w')\times\ 9\}\frac{l}{d}\ =\ \{(5+10)\times\ 9\}\cdot625=\ 84\cdot37.$$

$$de=\{(w+w')\times 11\}\frac{l}{d}\ =\ \{(5+10)\times 11\}\cdot625=103\cdot12.$$

$$ef=\{(w+w')\times 12\}\frac{l}{d}\ =\ \{(5+10)\times 12\}\cdot625=112\cdot50.$$

Maximum stress in

$$Ab=\left(5\times 2\tfrac{1}{2}+10\,\frac{1+3+5+7+9}{10}\right)\times 1\cdot18=\ldots-44\cdot25.$$

$$Bc=\left(5\times 2\ \ +10\,\frac{2+4+6+8}{10}\right)\quad\times 1\cdot18=\ldots-35\cdot40.$$

$$Cd=\left(5\times 1\tfrac{1}{2}+10\,\frac{1+3+5+7}{10}\right)\quad\times 1\cdot18=\ldots-27\cdot73.$$

$$De=\left(5\times 1\ \ +10\,\frac{2+4+6}{10}\right)\quad\times 1\cdot18=\ldots-20\cdot06.$$

$$Ef=\left(5\times \tfrac{1}{2}\ \ +10\,\frac{1+3+5}{10}\right)\quad\times 1\cdot18=\ldots-13\cdot61.$$

Maximum stress in

Tons.

$$aB = \left(5 \times 2 \quad + 10\frac{2+4+6+8}{10}\right) \quad \times 1{\cdot}18 = \ldots + 35{\cdot}40.$$

$$bC = \left(5 \times 1\tfrac{1}{2} + 10\frac{1+3+5+7}{10}\right) \quad \times 1{\cdot}18 = \ldots + 27{\cdot}73.$$

$$cD = \left(5 \times 1 \quad + 10\frac{2+4+6}{10}\right) \quad \times 1{\cdot}18 = \ldots + 20{\cdot}06.$$

$$dE = \left(5 \times \tfrac{1}{2} \quad + 10\frac{1+3+5}{10}\right) \quad \times 1{\cdot}18 = \ldots + 13{\cdot}61.$$

$$eF = \left(10\frac{2+4}{10}\right) \quad \times 1{\cdot}18 = \ldots + \ 7{\cdot}08.$$

The stresses may also be determined graphically from the diagrams Figs. 62 and 63. The construction of the diagram in Fig. 62 is very similar to that given for the Warren girder in Fig. 53, with the exception that the total load is assumed to be divided between the two systems of triangulation, so that the re-actions yz and $y''z''$ may be considered as the re-actions due to the two simple trusses shown in Figs. 60 and 61, the re-action on the right-hand side $y''z''$ being the re-action due to dead and live load on truss No. 1 shown in Fig. 60, and the re-action on the left-hand side yz being the re-action due to truss No. 2 shown in Fig. 61. In all other respects the procedure is the same as in Fig. 53.

Fig. 63, which gives a diagram of the maxima chord stresses, is very similar to Fig. 54. In this case the panel loads transferred by the combined systems of triangulation, as shown in Fig. 59, to the abutment are laid off on the vertical load line $= \frac{N-1}{2} = 4\frac{1}{2}$, two panel loads being transferred by simple truss No. 1, Fig. 60, and two and a half panel loads by simple truss No. 2, Fig. 61. The starting-point in the diagram will therefore be at D, which divides the load line into two sections,

Lattice or Double Intersection Warren Girder.

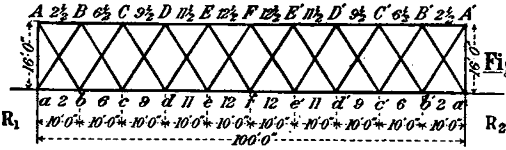


Fig. 59.

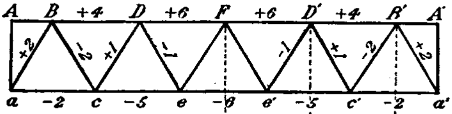


Fig. 60.

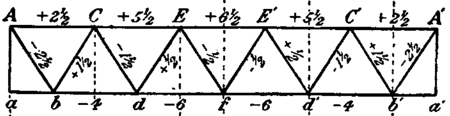


Fig. 61.

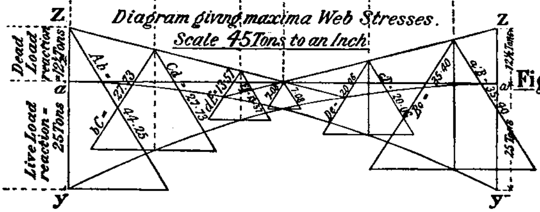


Fig. 62.

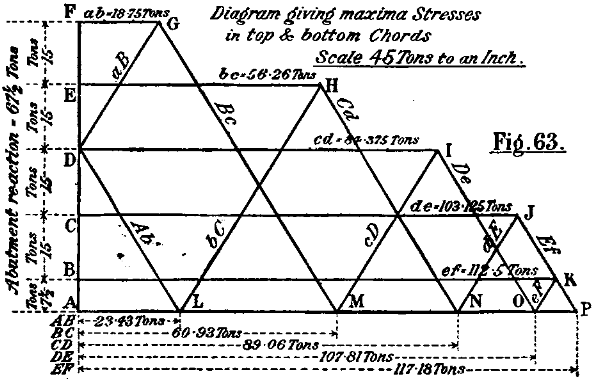


Fig. 63.

the one of two panel loads and the other two and a half panel loads.

From D, DG is drawn parallel to *a*B and DL parallel to A*b*.

From G, GM is drawn parallel to B*c*, and from L, LH is drawn parallel to *b*C, and the other members as shown.

The stresses may then be determined by scale and written down on the various members as shown, and on comparing the stresses so found by scale with those determined analytically it will be seen that they agree.

Lattice Girder with Verticals and Cross Diagonals.

(Figs. 64, 65, 66, and 67.)

This type of girder, of which the Charing Cross Bridge, designed by Sir John Hawkshaw, may be instanced as a prominent example, is, when built of iron or steel, open to the objection on theoretical grounds that either the verticals or one system of diagonals are redundant, and therefore they are useless and so much waste of material, but in practice the verticals enable a more rigid connection to be made between the main girders and the cross girders than would be possible in a simple lattice truss of the type shown in Fig. 59.

On account of the redundant members the accurate investigation of the stresses is attended with certain difficulties, but for practical purposes it will suffice to assume that they are equally distributed over the two systems of diagonals and the verticals. We have only therefore to dissect the two systems of triangulation into two simple trusses (Figs. 65 and 66), determine the stresses in each, and divide the results by 2, and we have the stresses in the dual system as set forth in Fig. 67 and the following analysis.

Example. Let span $=100$ feet $=$ L. and depth $=10$ feet $=d$.
Let dead panel load $=$ 5 tons $=w$ } Let $w+w'=w''=15$ tons.
Let live panel load $=10$ tons $=w'$ } Let W $=(w+w')\times n=150$ tons.
Let $n=$ number of panels $=10$.

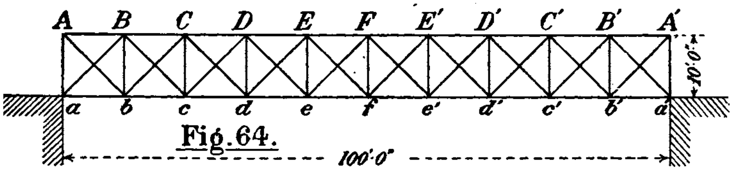


Fig. 64.

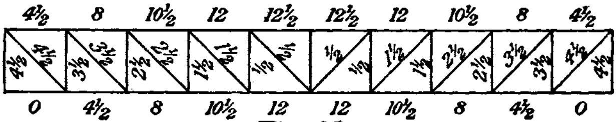


Fig. 65.

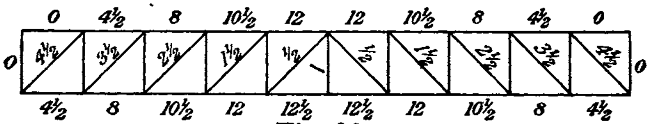


Fig. 66.

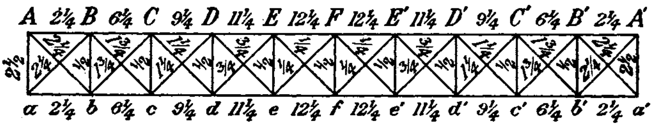


Fig. 67.

Max. stress in $Ab=a$B

$$=\left(w2\tfrac{1}{4}+w'\frac{1+2+3+4+5+6+7+8+9}{10\times 2}\right)\frac{\mathrm{A}b}{\mathrm{B}b}=(5\times 2\tfrac{1}{4}+10\times 2\tfrac{1}{4})1\cdot 414$$

$=47\cdot 70$ tons tension in Ab and compression in aB.

Max. stress in B$c=b$C

$$=\left(w1\tfrac{3}{4}+w'\frac{1+2+3+4+5+6+7+8}{10\times 2}\right)\frac{\mathrm{A}b}{\mathrm{B}b}=(5\times 1\tfrac{3}{4}+10\times 1\cdot 8)1\cdot 414$$

$=37\cdot 75$ tons tension in Bc and compression in bC.

Max. stress in $Cd = cD$

$$= \left(w1\tfrac{1}{4} + w'\frac{1+2+3+4+5+6+7}{10 \times 2}\right)\frac{Ab}{Bb} = (5 \times 1\tfrac{1}{4} + 10 \times 1{\cdot}4)1{\cdot}414$$

$= 28{\cdot}63$ tons tension in Cd and compression in cD.

Max. stress in $De = dE$

$$= \left(w\tfrac{3}{4} + w'\frac{1+2+3+4+5+6}{10 \times 2}\right)\frac{Ab}{Bb} = (5 \times \tfrac{3}{4} + 10 \times 1{\cdot}05)1{\cdot}414$$

$= 19{\cdot}13$ tons tension in De and compression in dE.

Max. stress in $Ef = eF$

$$= \left(w\tfrac{1}{4} + w'\frac{1+2+3+4+5}{10 \times 2}\right)\frac{Ab}{Bb} = (5 \times \tfrac{1}{4} + 10 \times \tfrac{3}{4})1{\cdot}414$$

$= 12{\cdot}37$ tons tension in Ef and compression in eF.

Maximum compression in AB = maximum tension in

$$ab = w'' \times 2\tfrac{1}{4} = 15 \times 2\tfrac{1}{4} = 33{\cdot}75 \text{ tons.}$$

Maximum compression in BC = maximum tension in

$$bc = w'' \times 6\tfrac{1}{4} = 15 \times 6\tfrac{1}{4} = 93{\cdot}75 \text{ tons.}$$

Maximum compression in CD = maximum tension in

$$cd = w'' \times 9\tfrac{1}{4} = 15 \times 9\tfrac{1}{4} = 138{\cdot}75 \text{ tons.}$$

Maximum compression in DE = maximum tension in

$$de = w'' \times 11\tfrac{1}{4} = 15 \times 11\tfrac{1}{4} = 168{\cdot}75 \text{ tons.}$$

Maximum compression in EF = maximum tension in

$$ef = w'' \times 12\tfrac{1}{4} = 15 \times 12\tfrac{1}{4} = 183{\cdot}75 \text{ tons.}$$

Stress on all verticals except $Aa = \tfrac{1}{2}w'' = 7{\cdot}5$ tons in tension. Stress on $Aa = 2\tfrac{1}{2}\ w'' = 37\tfrac{1}{2}$ tons in compression. With load on top flange all stresses are the same, substituting in all verticals compression for tension.

Trellis or Multiple Lattice Girder.

In girders of which the web consists of triangular bracing it frequently happens that the length of a panel or bay for a single system, such as in the Warren girder, is inconveniently large, and it becomes advisable to connect the web with the flanges at closer intervals. This can be done by duplicating or multiplying the systems of bracing. When one additional

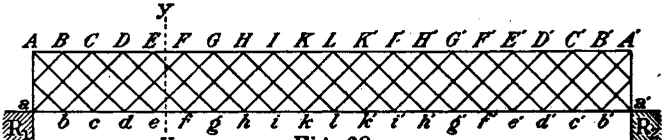


Fig. 68.

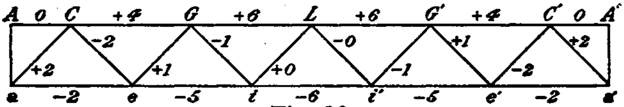


Fig. 69.

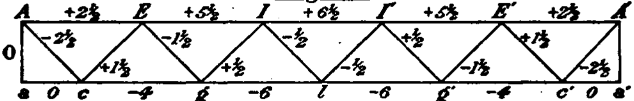


Fig. 70.

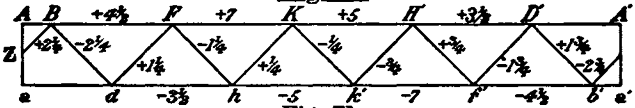


Fig. 71.

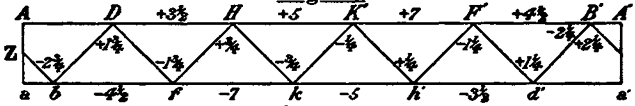


Fig. 72.

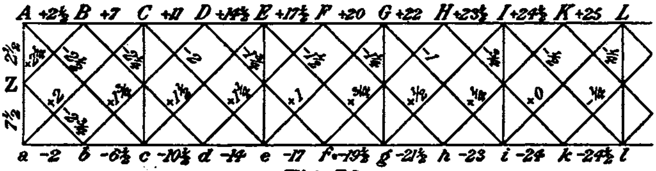


Fig. 73.

system of braces is introduced there results a lattice girder, as illustrated in Fig. 59, but when more than two systems of braces are employed the system is said to be treble or quadruple, according to the number of diagonals cut by a vertical section.

In the girder illustrated in Figs. 68 and 73 there are four distinct triangulations, each of which may be theoretically considered as a separate system, as shown subdivided in Figs. 69, 70, 71, and 72. The stresses in this girder may be determined by combining the stresses in each separate system, which can be arrived at by the same process as that adopted in the case of the Warren girder (Fig. 50).

The simplest method of arriving at the chord or flange stresses is by writing down on each diagonal co-efficients of the panel load, which gives the shear for full load, in each system, as in Figs. 69, 70, 71, and 72. By combining these co-efficients in a diagram, as in Fig. 73, and adding together successively the co-efficients on the two diagonals meeting in an apex at the top and bottom flanges, we have the chord co-efficients, which, multiplied by the panel load and by the length of panel divided by the depth of truss, will give the maxima stresses in the top and bottom flanges.

Example.

Let $L = \text{span} = 100$ feet, and $l = \text{panel length} = \frac{L}{20} = 5$ feet.

$d = \text{depth of truss} = \frac{L}{10} = 10$ feet.

$w = \text{dead load per panel} = 2{\cdot}5$ tons $= {\cdot}5$ ton per foot run.

$w' = \text{live load per panel} = 5{\cdot}0$ tons $= 1{\cdot}0$ ton per foot run.

Let the diagonals be inclined at an angle of 45°, so that the length of the diagonal divided by the depth of the truss $= 1{\cdot}414$. The length of the base of any triangle formed by the diagonals, as compared with the perpendicular, will be as 1 to 1.

Then the maximum compression in

	Tons.
$AB = (2\frac{1}{2} + 5) \times 2\frac{1}{2} =$	$18\frac{3}{4}$.
$BC = (2\frac{1}{2} + 5) \times 7 =$	$52\frac{1}{2}$.
$CD = (2\frac{1}{2} + 5) \times 11 =$	$82\frac{1}{2}$.

The maximum compression in

Tons.

$$DE = (2\tfrac{1}{2} + 5) \times 14\tfrac{1}{2} = 108\tfrac{3}{4}.$$
$$EF = (2\tfrac{1}{2} + 5) \times 17\tfrac{1}{2} = 131\tfrac{1}{4}.$$
$$FG = (2\tfrac{1}{2} + 5) \times 20 = 150.$$
$$GH = (2\tfrac{1}{2} + 5) \times 22 = 165.$$
$$HI = (2\tfrac{1}{2} + 5) \times 23\tfrac{1}{2} = 176\tfrac{1}{4}.$$
$$IK = (2\tfrac{1}{2} + 5) \times 24\tfrac{1}{2} = 183\tfrac{3}{4}.$$
$$KL = (2\tfrac{1}{2} + 5) \times 25 = 187\tfrac{1}{2}.$$
$$AZ = (2\tfrac{1}{2} + 5) \times 2\tfrac{1}{2} = 18\tfrac{3}{4}.$$
$$aZ = (2\tfrac{1}{2} + 5) \times 7\tfrac{1}{2} = 56\tfrac{1}{4}.$$

The maximum tension in

$$ab = (2\tfrac{1}{2} + 5) \times 2 = 15.$$
$$bc = (2\tfrac{1}{2} + 5) \times 6\tfrac{1}{2} = 48\tfrac{3}{4}.$$
$$cd = (2\tfrac{1}{2} + 5) \times 10\tfrac{1}{2} = 78\tfrac{3}{4}.$$
$$de = (2\tfrac{1}{2} + 5) \times 14 = 105.$$
$$ef = (2\tfrac{1}{2} + 5) \times 17 = 127\tfrac{1}{2}.$$
$$fg = (2\tfrac{1}{2} + 5) \times 19\tfrac{1}{2} = 146\tfrac{1}{4}.$$
$$gh = (2\tfrac{1}{2} + 5) \times 21\tfrac{1}{2} = 161\tfrac{1}{4}.$$
$$hi = (2\tfrac{1}{2} + 5) \times 23 = 172\tfrac{1}{2}.$$
$$ik = (2\tfrac{1}{2} + 5) \times 24 = 180.$$
$$kl = (2\tfrac{1}{2} + 5) \times 24\tfrac{1}{2} = 183\tfrac{3}{4}.$$

The maximum compression in

Tons.

$$zB = \left\{(2\tfrac{1}{2} \times 2\tfrac{1}{4}) + \left(5 \times \frac{1+5+9+13+17}{20}\right)\right\} \times 1{\cdot}414 = 23{\cdot}85.$$

$$aC = \left\{(2\tfrac{1}{2} \times 2) + \left(5 \times \frac{4+8+12+16}{20}\right)\right\} \times 1{\cdot}414 = 21{\cdot}22.$$

$$bD = \left\{(2\tfrac{1}{2} \times 1\tfrac{3}{4}) + \left(5 \times \frac{3+7+11+15}{20}\right)\right\} \times 1{\cdot}414 = 18{\cdot}87.$$

$$cE = \left\{(2\tfrac{1}{2} \times 1\tfrac{1}{2}) + \left(5 \times \frac{2+6+10+14}{20}\right)\right\} \times 1{\cdot}414 = 16{\cdot}6.$$

The maximum compression in

Tons.

$$dF = \left\{(2\tfrac{1}{2} \times 1\tfrac{1}{4}) + \left(5 \times \frac{1+5+9+13}{20}\right)\right\} \times 1{\cdot}414 = 14{\cdot}3.$$

$$eG = \left\{(2\tfrac{1}{2} \times 1) + \left(5 \times \frac{4+8+12}{20}\right)\right\} \times 1{\cdot}414 = 11{\cdot}97.$$

$$fH = \left\{(2\tfrac{1}{2} \times \tfrac{3}{4}) + \left(5 \times \frac{3+7+11}{20}\right)\right\} \times 1{\cdot}414 = 10{\cdot}05.$$

$$gI = \left\{(2\tfrac{1}{2} \times \tfrac{1}{2}) + \left(5 \times \frac{2+6+10}{20}\right)\right\} \times 1{\cdot}414 = 8{\cdot}12.$$

$$hK = \left\{(2\tfrac{1}{2} \times \tfrac{1}{4}) + \left(5 \times \frac{1+5+9}{20}\right)\right\} \times 1{\cdot}414 = 6{\cdot}17.$$

$$iL = \left(5 \times \frac{4+8}{20}\right) \times 1{\cdot}414 = 4{\cdot}25.$$

$$kK = \left\{(2\tfrac{1}{2} \times -\tfrac{1}{4}) + \left(5 \times \frac{3+7}{20}\right)\right\} \times 1{\cdot}414 = 2{\cdot}68.$$

The maximum tension in

$$Zb = \left\{(2\tfrac{1}{2} \times 2\tfrac{3}{4}) + \left(5 \times \frac{3+7+11+15+19}{20}\right)\right\} \times 1{\cdot}414 = 29{\cdot}17.$$

$$Ac = \left\{(2\tfrac{1}{2} \times 2\tfrac{1}{2}) + \left(5 \times \frac{2+6+10+14+18}{20}\right)\right\} \times 1{\cdot}414 = 26{\cdot}47.$$

$$Bd = \left\{(2\tfrac{1}{2} \times 2\tfrac{1}{4}) + \left(5 \times \frac{1+5+9+13+17}{20}\right)\right\} \times 1{\cdot}414 = 23{\cdot}85.$$

$$Ce = \left\{(2\tfrac{1}{2} \times 2) + \left(5 \times \frac{4+8+12+16}{20}\right)\right\} \times 1{\cdot}414 = 21{\cdot}15.$$

$$Df = \left\{(2\tfrac{1}{2} \times 1\tfrac{3}{4}) + \left(5 \times \frac{3+7+11+15}{20}\right)\right\} \times 1{\cdot}414 = 18{\cdot}87.$$

$$Eg = \left\{(2\tfrac{1}{2} \times 1\tfrac{1}{2}) + \left(5 \times \frac{2+6+10+14}{20}\right)\right\} \times 1{\cdot}414 = 16{\cdot}60.$$

$$Fh = \left\{(2\tfrac{1}{2} \times 1\tfrac{1}{4}) + \left(5 \times \frac{1+5+9+13}{20}\right)\right\} \times 1{\cdot}414 = 14{\cdot}30.$$

The maximum tension in

$$\text{G}i = \left\{(2\tfrac{1}{2} \times 1) + \left(5 \times \frac{4+8+12}{20}\right)\right\} \times 1{\cdot}414 = 11{\cdot}97 \text{ Tons.}$$

$$\text{H}k = \left\{(2\tfrac{1}{2} \times \tfrac{3}{4}) + \left(5 \times \frac{3+7+11}{20}\right)\right\} \times 1{\cdot}414 = 10{\cdot}05.$$

$$\text{I}l = \left\{(2\tfrac{1}{2} \times \tfrac{1}{2}) + \left(5 \times \frac{2+6+10}{20}\right)\right\} \times 1{\cdot}414 = 8{\cdot}12.$$

$$\text{K}k = \left\{(2\tfrac{1}{2} \times \tfrac{1}{4}) + \left(5 \times \frac{1+5+9}{20}\right)\right\} \times 1{\cdot}414 = 6{\cdot}17.$$

In the foregoing calculations of stresses it is assumed that the web members act independently of one another, and that they are not connected by rivetting at each intersection. In bridges of short span this type of girder is frequently employed, in which the diagonals consist of flat bars rivetted together at intersections, and vertical members are introduced as shown in Fig. 73 to act as stiffeners. Where the diagonals are so connected together there would probably result an equal distribution of the shearing stress amongst the diagonals cut by a vertical section, so that the process adopted of determining the stress in each individual diagonal would not in that case be strictly accurate.

It therefore follows that the greater the number of systems contained in the web the more nearly do we approach the plate girder, in which the bending moments in the top and bottom flanges at any vertical section are equal, and the shearing forces are equally distributed over the section of the web.

Example. The shearing force at the section y, y, Fig. 68, may be determined as under:

The re-action due to the dead or permanent load at $a = \text{R}1 = w' \times 9\tfrac{1}{2}$, and the shear due to the dead load $= (9\tfrac{1}{2} - 4)w = 5\tfrac{1}{2}w$. The re-action due to the live or rolling load is also equal to $w' \times 9\frac{1}{2}$, and the shear at yy due to the live load $= 9\frac{1}{2} - \frac{19+18+17+16}{20} = 9\frac{1}{2} - 3\frac{1}{2} = 6w'$.

Hence the shearing force at yy due to dead and live loads $= (5\frac{1}{2}w + 6w')$, and this must be transmitted through the diagonals, so that by substituting a mean stress for the stresses in the several bars we have $\frac{(5\frac{1}{2}w + 6w') \times 1 \cdot 414}{4}$, the stress in a diagonal cut by the section

$$\frac{5\frac{1}{2}w + 6w' \times 1 \cdot 414}{4} = 15 \cdot 46 \text{ tons.}$$

From a practical point of view an advantage obtains in this type of girder in rivetting together the diagonals at their intersections, inasmuch as it admits of the compression mem bers of the web system being supported at frequent intervals, and thus they can be made of slighter dimensions than would otherwise be possible. Where vertical members are introduced for stiffening the web, they serve to distribute the moving load between the upper and lower flanges, but they may without material error be assumed to be no part of the structure in the calculation of the stresses.

Howe Truss.

This truss is so named after William Howe, an American engineer, who introduced it in 1840, and for spans up to 150 feet it is probably the best combination of iron and wood that can be adopted.

The top and bottom chords and the diagonal web members are made of timber, and the vertical web members of round iron provided at their ends with screws.

The chords are made of uniform section throughout, and both the top and bottom chords of the same number of timbers. The bottom chord is made deeper than the top chord on account of the loss of section from splicing, the lower strains allowed in tension and to resist the bending strains due to the transverse floor beams, which are carried directly by the lower chord.

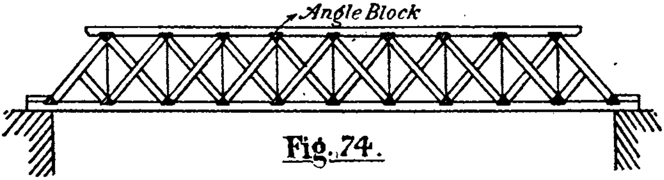


Fig. 74.

The braces and counter braces are square ended and rest upon the faces of the angle-blocks which are let into the chord timbers. The angle-blocks were originally made of oak, but now they are invariably made of cast iron.

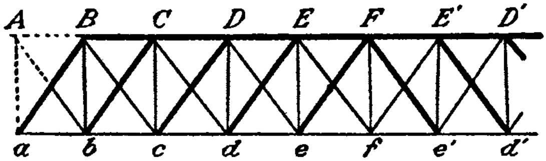


Fig. 75.

The stresses in a Howe truss are determined as under:

Let n = number of panels or bays.
W = static or dead load on one panel.
W″ = variable or live load on one panel
H = length of diagonal braces.
D = depth of truss, from centre to centre of chords.
L = panel length.

Maximum compression on

$$aB = \frac{n-1}{2}(W + W'')\frac{H}{D}.$$

$$bC = \left\{\frac{n-3}{2}W + \frac{(n-2)(n-1)}{2n}W''\right\}\frac{H}{D}.$$

Maximum compression on

$$cD = \left\{\frac{n-5}{2}W + \frac{(n-3)(n-2)}{2n}W''\right\}\frac{H}{D}.$$

$$dE = \left\{\frac{n-7}{2}W + \frac{(n-4)(n-3)}{2n}W''\right\}\frac{H}{D}.$$

$$eF = \left\{\frac{n-9}{2}W + \frac{(n-5)(n-4)}{2n}W''\right\}\frac{H}{D}.$$

By continuing this process beyond the centre of the truss the maxima stresses in the counter braces are obtained, until the result becomes a minus quantity, when the counter braces are no longer subject to stress, and may be omitted.

Maximum stress on

$$ab = BC = \frac{n-1}{2}(W + W'')\frac{L}{D}.$$

$$bc = CD = \left\{\frac{(n-1)+(n-3)}{2}(W + W'')\right\}\frac{L}{D}.$$

$$cd = DE = \left\{\frac{(n-1)+(n-3)+(n-5)}{2}(W + W'')\right\}\frac{L}{D}.$$

$$de = EF = \left\{\frac{(n-1)+(n-3)+(n-5)+(n-7)}{2}(W + W'')\right\}\frac{L}{D}$$

$$ef \quad = \left\{\frac{(n-1)+(n-3)+(n-5)+(n-7)+(n-9)}{2}(W + W'')\right\}\frac{L}{D}.$$

Maximum tension on

$Bb = \frac{n-1}{2}(W + W'')$ for a through bridge, or W'' less for a deck bridge.

$Cc = \frac{n-3}{2}W + \frac{(n-2)(n-1)}{2n}W''$ for a through bridge, or W'' less for a deck bridge.

$Dd = \frac{n-5}{2}W + \frac{(n-3)(n-2)}{2n}W''$ for a through bridge, or W'' less for a deck bridge.

$Ee = \frac{n-7}{2}W + \frac{(n-4)(n-3)}{2n}W''$ for a through bridge, or W'' less for a deck bridge.

etc., etc.

Maximum compression in Tons.

$$aB = (4\tfrac{1}{2}w + \tfrac{45}{10}w')\frac{aB}{Bb} = (10{\cdot}8 + 36)\,1{\cdot}28 \quad = 59{\cdot}90.$$

$$bC = (3\tfrac{1}{2}w + \tfrac{36}{10}w')\frac{aB}{Bb} = \ (8{\cdot}4 + 28{\cdot}8)\,1{\cdot}28 = 47{\cdot}61.$$

$$cD = (2\tfrac{1}{2}w + \tfrac{28}{10}w')\frac{aB}{Bb} = \ (6{\cdot}0 + 22{\cdot}4)\,1{\cdot}28 = 36{\cdot}35.$$

$$dE = (1\tfrac{1}{2}w + \tfrac{21}{10}w')\frac{aB}{Bb} = \ (3{\cdot}6 + 16\ 8)\,1\ 28 = 26{\cdot}11.$$

$$eF = \ (\tfrac{1}{2}w + \tfrac{15}{10}w')\frac{aB}{bB} = \ (1{\cdot}2 + 12{\cdot}0)\,1{\cdot}28 = 16{\cdot}89.$$

$$fE' = (-\tfrac{1}{2}w + \tfrac{10}{10}w')\frac{aB}{Bb} = (-1{\cdot}2 + 8{\cdot}0)\,1{\cdot}28 = \ 8{\cdot}70.$$

Maximum tension in Tons.

$$Bb = 4\tfrac{1}{2}w + \tfrac{45}{10}w' = 10{\cdot}8 + 36 \quad = 46{\cdot}8.$$
$$Cc = 3\tfrac{1}{2}w + \tfrac{36}{10}w' = \ 8{\cdot}4 + 28{\cdot}8 = 37{\cdot}2.$$
$$Dd = 2\tfrac{1}{2}w + \tfrac{28}{10}w' = \ 6{\cdot}0 + 22{\cdot}4 = 28{\cdot}4.$$
$$Ee = 1\tfrac{1}{2}w + \tfrac{21}{10}w' = \ 3{\cdot}6 + 16{\cdot}8 = 20{\cdot}4.$$
$$Ff = \ \tfrac{1}{2}w + \tfrac{15}{10}w' = \ 1{\cdot}2 + 12{\cdot}0 = 13{\cdot}2.$$

Maximum tension on ab = maximum compression on BC Tons.

$$= 4\tfrac{1}{2}(w + w') \times \frac{ab}{Bb} = (4\tfrac{1}{2} \times 10{\cdot}4) \times \tfrac{8}{10} = 37{\cdot}44.$$

Maximum tension on bc = maximum compression on CD

$$= \{4\tfrac{1}{2} + 3\tfrac{1}{2}(w + w')\}\frac{ab}{Bb} = (8 \times 10{\cdot}4) \times \tfrac{8}{10} = 66{\cdot}56$$

Maximum tension on cd = maximum compression on DE

$$= \{4\tfrac{1}{2} + 3\tfrac{1}{2} + 2\tfrac{1}{2}(w + w')\}\frac{ab}{Bb} = (10\tfrac{1}{2} \times 10{\cdot}4) \times \tfrac{8}{10} = 87{\cdot}36.$$

Maximum tension on de = maximum compression on EF

$$= \{4\tfrac{1}{2} + 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2}(w + w')\}\frac{ab}{Bb} = (12 \times 10{\cdot}4) \times \tfrac{8}{10} = 99{\cdot}84.$$

Maximum tension on ef

$$= \{4\tfrac{1}{2} + 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2} + \tfrac{1}{2}(w + w')\}\frac{ab}{Bb} = (12\tfrac{1}{2} \times 10{\cdot}4)\tfrac{8}{10} = 104.$$

The stresses in this type of girder may be readily determined by a graphic process, as shown in Figs. 77 and 78, the former diagram giving the web stresses and the latter the top and bottom chord stresses. Fig. 78 also gives the web stresses

Howe Truss Railway Bridge 80 Feet span 10 Feet deep.

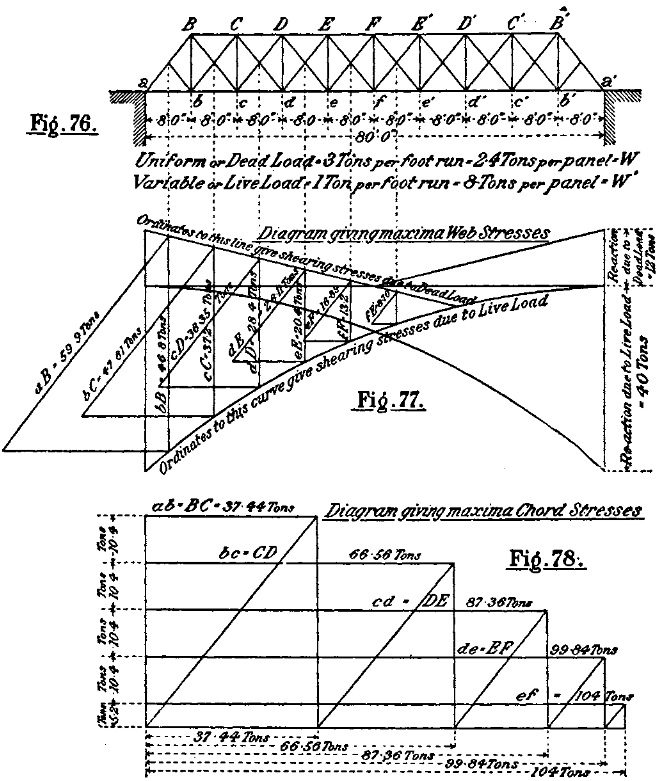


Fig. 76.

Fig. 77.

Fig. 78.

due to a uniformly distributed dead or stationary live load covering the whole truss, but for a rolling or moving load the web stresses must be obtained from the diagram in Fig. 77. The construction of the diagrams will be readily understood, and it is thought that a detailed description is unnecessary. The right half of the diagram in Fig. 77 is not completed so as to avoid complicating the figures.

Murphy Whipple or Pratt Truss.

$L = \text{span} = 96$ feet.	$w = \text{panel dead load} = 6$ tons.
$D = \text{depth} = 12$ feet.	$w_1 = \text{panel live load} = 12$ tons.
$H = $ length of diagonal.	$W = (w + w_1)N$.
$l = $ length of panel.	$N = $ number of panels.

This truss, shown in Fig. 79, is very similar in outline to the Howe truss, but the vertical members take compression and the diagonal braces tension.

The stresses in this truss may be written down in the same manner as in the Howe truss, thus:

The maximum compression in the end vertical Aa

$$= 3\tfrac{1}{2}w + \tfrac{28}{8}w' = 3\tfrac{1}{2} \times 6 + \tfrac{28}{8} \times 12 = 63 \text{ tons.}$$

The maximum compression in

$$Bb = 2\tfrac{1}{2}w + \tfrac{21}{8}w = 2\tfrac{1}{2} \times 6 + \tfrac{21}{8} \times 12 = 46\tfrac{1}{2} \text{ tons.}$$

$$Cc = 1\tfrac{1}{2}w + \tfrac{15}{8}w = 1\tfrac{1}{2} \times 6 + \tfrac{15}{8} \times 12 = 31\tfrac{1}{2} \quad ,,$$

$$Dd = \tfrac{1}{2}w + \tfrac{10}{8}w' = \tfrac{1}{2} \times 6 + \tfrac{10}{8} \times 12 = 18 \quad ,,$$

$$Ee = -\tfrac{1}{2}w + \tfrac{6}{8}w' = -\tfrac{1}{2} \times 6 + \tfrac{6}{8} \times 12 = 6 \quad ,,$$

The maximum tension in

Tons.

$$Ab = (3\tfrac{1}{2}w + \tfrac{28}{8}w') \times \frac{Ab}{Bb} = (3\tfrac{1}{2} \times 6 + \tfrac{28}{8} \times 12) \times 1{\cdot}414 = 89{\cdot}08.$$

$$Bc = (2\tfrac{1}{2}w + \tfrac{21}{8}w') \times \frac{Bc}{Bb} = (2\tfrac{1}{2} \times 6 + \tfrac{21}{8} \times 12) \times 1{\cdot}414 = 65{\cdot}75.$$

$$Cd = (1\tfrac{1}{2}w + \tfrac{15}{8}w') \times \frac{Cd}{Cc} = (1\tfrac{1}{2} \times 6 + \tfrac{15}{8} \times 12 \times 1{\cdot}414 = 44{\cdot}54.$$

$$De = (\tfrac{1}{2}w + \tfrac{10}{8}w') \times \frac{De}{Dd} = (\tfrac{1}{2} \times 6 + \tfrac{10}{8} \times 12) \times 1{\cdot}414 = 25{\cdot}45.$$

$$Ed' = (-\tfrac{1}{2}w + \tfrac{6}{8}w') \times \frac{Ed}{Ee} = (-\tfrac{1}{2} \times 6 + \tfrac{6}{8} \times 12) \times 1{\cdot}414 = 8{\cdot}48.$$

Tension in $ab = $ nil.

Murphy Whipple or Pratt Truss *(single intersection)*

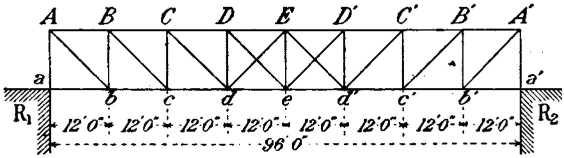


Fig. 79.

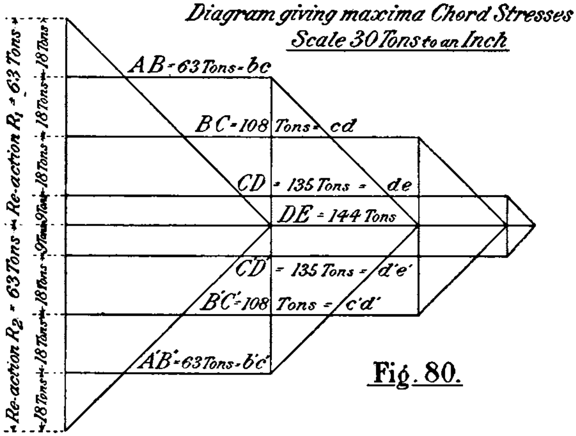


Fig. 80.

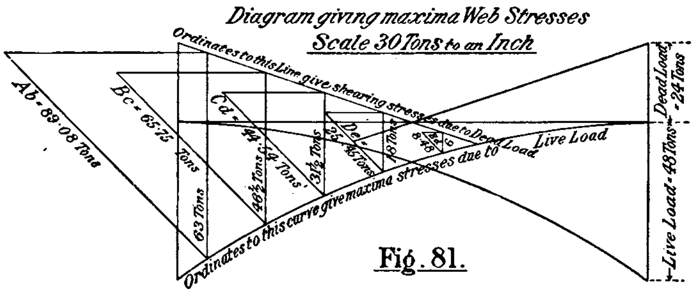


Fig. 81.

Maximum compression in AB = maximum tension in bc

$$= 3\tfrac{1}{2}(w + w') = 3\tfrac{1}{2} \times 18 = 63 \text{ tons.}$$

Maximum compression in BC = maximum tension in cd

$$= 3\tfrac{1}{2} + 2\tfrac{1}{2}(w + w') = 6 \times 18 = 108 \text{ tons.}$$

Maximum compression in CD = maximum tension in de

$$= 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2}(w + w') = 7\tfrac{1}{2} \times 18 = 135 \text{ tons.}$$

Maximum compression in DE

$$= 3\tfrac{1}{2} + 2\tfrac{1}{2} + 1\tfrac{1}{2} + \tfrac{1}{2} = 8 \times 18 = 144 \text{ tons.}$$

The stresses may also be obtained graphically in the manner indicated in the diagrams, Figs. 80 and 81.

L = span = 96 feet.	w = panel dead load = 6 tons.
D = depth = 12 feet.	w' = panel live load = 12 tons.
H = length of diagonal.	$W = (w + w') \times N$.
l = length of panel.	N = number of panels.

In American practice, in which this type of girder is very extensively used, the end vertical in Fig. 79 is invariably omitted, and the end of the truss assumes the form illustrated

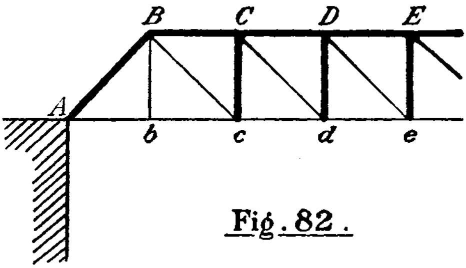


Fig. 82.

in Fig. 82. In this form the stress in the end diagonal AB $= \frac{n-1}{2} \times (w + w') \times \frac{AB}{Bb}$, the stress in $Ab = bc = \frac{n-1}{2}(w + w')\frac{Ab}{Bb}$, and the stress in the hip-vertical $Bb = w + w'$.

In large spans the panel length in a single intersection truss would become too long, and so as to support the top and bottom chords at intermediate points double or triple systems of triangulation are used.

Fig. 83 illustrates a double-intersection truss, which is in some instances known in America as the Linville truss, because Mr. J. H. Linville, an eminent American engineer, built several large bridges in which he employed this type of construction.

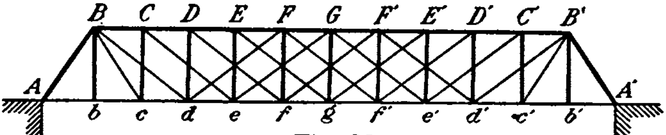


Fig. 83.

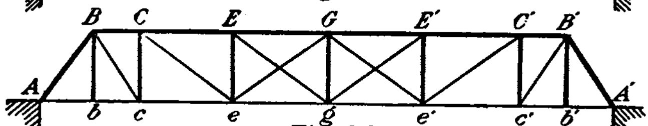


Fig. 84.

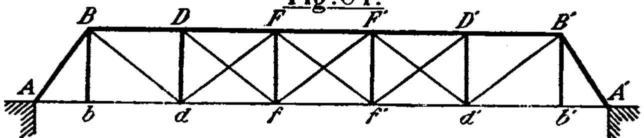


Fig. 85.

The stresses in this type of truss may be determined by decomposing the truss into the two simple systems illustrated in Figs. 84 and 85 and calculating the stresses in each system as if it formed a separate truss. Then will the stresses so obtained represent the actual stresses in the web members of the composite or double intersection truss, but the stresses in the end diagonals AB and in the several panels of the top and bottom chords must be added together to arrive at the stresses in the corresponding members of the composite truss.

The stresses may also be written down by simple inspection as follows:

Let w = panel dead load. Let P = panel length.
w' = panel live load. H = depth of truss.

$$\text{Maximum compression in BC} = 12(w+w')\frac{P}{H}.$$

$$\text{,, ,, CD} = 15(w+w')\frac{P}{H}.$$

$$\text{,, ,, DE} = 17(w+w')\frac{P}{H}.$$

$$\text{,, ,, EF} = 18(w+w')\frac{P}{H}.$$

$$\text{,, ,, FG} = 18(w+w')\frac{P}{H}.$$

$$\text{Maximum tension in } ab = bc = 5\tfrac{1}{2}(w+w')\frac{P}{H}.$$

$$\text{,, ,, } cd = 8(w+w')\frac{P}{H}.$$

$$\text{,, ,, } de = 12(w+w')\frac{P}{H}.$$

$$\text{,, ,, } ef = 15(w+w')\frac{P}{H}.$$

$$\text{,, ,, } fg = 17(w+w')\frac{P}{H}.$$

$$\text{Maximum compression in AB} = 5\tfrac{1}{2}(w+w')\frac{\sqrt{H^2+P^2}}{H}.$$

$$\text{,, ,, C}c = 1\tfrac{1}{2}w + \frac{20\tfrac{1}{2}}{12}w'$$

$$\text{,, ,, D}d = w + \frac{15\tfrac{1}{2}}{12}w'.$$

$$\text{,, ,, E}e = \tfrac{1}{2}w + \frac{12\tfrac{1}{2}}{12}w'.$$

Maximum compression in $\mathrm{F}f=\frac{8\frac{1}{2}}{12}w'$.

” ” $\mathrm{G}g=\frac{6\frac{1}{2}}{12}w'$.

” ” $\mathrm{B}b=w+w$.

Maximum tension in $\mathrm{B}c=\left(2\frac{1}{2}w+\frac{30\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{B}d=\left(2w+\frac{24\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{C}e=\left(1\frac{1}{2}w+\frac{20\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{D}f=\left(w+\frac{15\frac{1}{2}}{12}w\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{E}g=\left(\frac{1}{2}w+\frac{12\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{F}f'=\left(\frac{8\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{G}e'=\left(-\frac{1}{2}w+\frac{6\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

” ” $\mathrm{F}'d'=\left(-1w+\frac{3\frac{1}{2}}{12}w'\right)\frac{\sqrt{\mathrm{H}^2+\mathrm{P}^2}}{\mathrm{H}}$.

Post Truss.

This is a form of double quadrilateral truss that has been extensively used in the United States, and is so called after the name of the engineer who introduced it, Mr. S. S. Post.

The web members are so arranged that the diagonals in tension extend over one and a half panels, and those acting as struts in compression over one half panel, and the apices in one chord are midway between those of the other, thus constituting a sort of compromise between the lattice and Murphy Whipple or Pratt truss.

This type of construction has been principally adopted in America, where a number of bridges of the Post truss have been built, some exclusively of iron and some of a combination of iron and wood, the tension members being of iron and those in compression of wood.

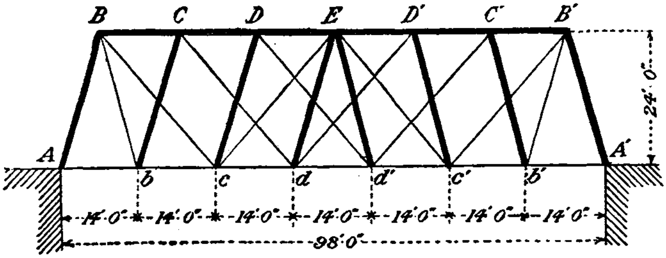


Fig. 86.

An illustration of a Post truss bridge built on the East Glenham, Newburgh, Dutchess, and Connecticut Railway, built of iron and wood, is given in Fig. 86, the stresses in which are worked out.

Dead load, 1000 lbs. per foot run.
Live load, 4000 lbs. per foot run.
Panel dead load per truss, 7000 lbs.
Panel live load per truss, 28,000 lbs.

$$\frac{\sqrt{7^2+24^2}}{24} = 1{\cdot}042 \sec\theta.$$

$$\frac{\sqrt{21^2+24^2}}{24} = 1{\cdot}329 \sec\theta.$$

$$\frac{21}{24} = 0{\cdot}875 \tan\theta.$$

WEB STRESSES.

Maximum compression in

$$\text{AB} = (7000 \times 3 + 28{,}000 \times 3) \times 1{\cdot}042 = 109{,}410 \text{ lbs.}$$

$$b\text{C} = \left(7000 \times 1 + 28{,}000 \times \frac{1+2+4}{7}\right) \times 1{\cdot}042 = 36{,}470 \text{ lbs.}$$

Maximum compression in

$$cD = \left(28{,}000 \times \frac{1+2+3}{7}\right) \times 1{\cdot}042 = 25{,}008 \text{ lbs.}$$

$$dE = \left(28{,}000 \times \frac{1+2}{7}\right) \times 1{\cdot}042 = 12{,}504 \text{ lbs.}$$

Maximum tension in

$$Bb = (7000 \times 2 + 28{,}000 \times 2) \times 1{\cdot}042 = 72{,}940 \text{ lbs.}$$

$$Bc = \left(7000 \times 1 + 28{,}000 \times \frac{5+3}{7}\right) \times 1{\cdot}329 = 51{,}831 \text{ lbs.}$$

$$Cd = \left(7000 \times 1 + 28{,}000 \times \frac{4+2+1}{7}\right) \times 1{\cdot}329 = 46{,}515 \text{ lbs.}$$

$$Dd = \left(28{,}000 \times \frac{3+2+1}{7}\right) \times 1{\cdot}329 = 31{,}896 \text{ lbs.}$$

$$Ec = \left(28{,}000 \times \frac{2+1}{7}\right) \times 1{\cdot}329 = 15{,}948 \text{ lbs.}$$

Chord Stresses.

Maximum compression in

$$BC = (35{,}000 \times 3 + 2) \times \tfrac{7}{24} + (35{,}000 \times 1) \times \tfrac{21}{24} = 81{,}667 \text{ lbs.}$$

$$\left.\begin{array}{l} CD = \\ DE = \end{array}\right\} (35{,}000 \times 3 + 2 + 1) \times \tfrac{7}{24} + (35{,}000 \times 1 + 1) \times \tfrac{21}{24} = 122{,}500 \text{ lbs.}$$

Maximum tension in

$$Ab = (35{,}000 \times 3) \times \tfrac{7}{24} = 30{,}625 \text{ lbs.}$$

$$bc = (35{,}000 \times 6) \times \tfrac{7}{24} = 61{,}250 \text{ lbs.}$$

$$cd = (35{,}000 \times 3 + 2 + 1) \times \tfrac{7}{24} + (35{,}000 \times 1) \times \tfrac{21}{24} = 91{,}875 \text{ lbs.}$$

$$dd_1 = (35{,}000 \times 3 + 2 + 1) \times \tfrac{7}{24} + (35{,}000 \times 1 + 1) \times \tfrac{21}{24} = 122{,}500 \text{ lbs.}$$

Note.—The compression members of the web system shown in thick black lines are assumed to be incapable of taking tensile stress.

Sub-divided Triangular or Baltimore Bridge Co's Truss.

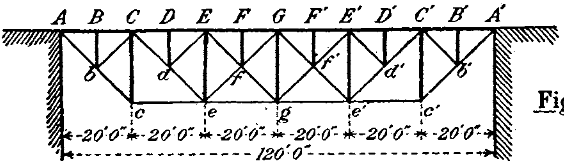


Fig. 87.

Dead Load 10 Cwt per Foot run = 5 Tons per panel = W
Live Load 1 Ton per Foot run = 10 Tons per panel = W'

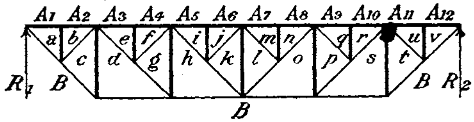


Fig. 88.

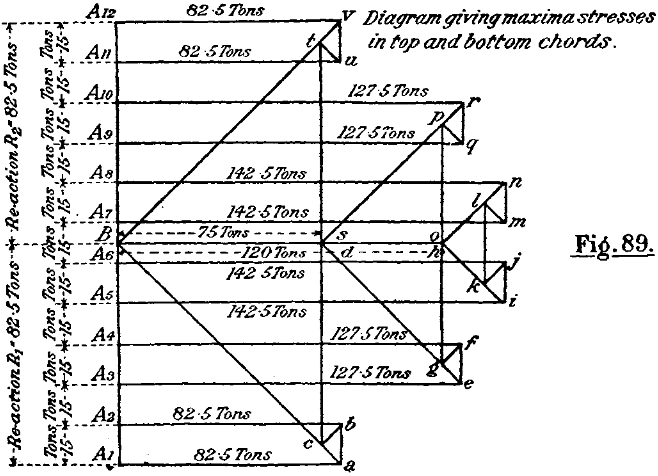


Fig. 89.

The maxima chord stresses occur when the truss is fully loaded, when the re-action at each abutment

$$= 82{\cdot}5 \text{ tons} = R1 = R2.$$

Maxima Web Stresses.

Maximum compression in short verticals Bb, Dd, Ff ... B$'b'$

$= 5 + 10$ tons $= 15$ tons.

Maximum tension in short diagonals bC, dE, fG ... b'C$'$

$= \frac{15}{2} \times 1{\cdot}414 = 10{\cdot}6$ tons.

Maximum tension in Ab = R_1

$= 82{\cdot}5 \times 1{\cdot}414 = 116{\cdot}6$ tons.

Maximum tension in $bc = 5 \times 4\tfrac{1}{2} + 10 \times \dfrac{1+2+3 \ldots 10}{12} + \dfrac{5}{2}$

$= (22{\cdot}5 + 45{\cdot}83 + 2\tfrac{1}{2}) \times 1{\cdot}414 = 100{\cdot}15$ tons.

Maximum compression in Cc

= the vertical component of the stress in bc

$= 70{\cdot}83$ tons.

Maximum tension in Cd

$$= \left(5 \times 3\tfrac{1}{2} + 10 \times \frac{1+2+3 \ldots 9}{12}\right) \times 1{\cdot}414$$

$= 77{\cdot}77$ tons.

Maximum tension in de

$$= \left(5 \times 2\tfrac{1}{2} + 10 \times \frac{1+2+3 \ldots 8}{12} + \frac{5}{2}\right) \times 1{\cdot}414$$

$= 63{\cdot}63$ tons.

Maximum compression in E

= vertical component of stress in de

$= 45{\cdot}00$ tons.

Maximum tension in Ef

$$= \left(5 \times 1\tfrac{1}{2} + 10 \times \frac{1+2+3 \ldots 7}{12}\right) \times 1{\cdot}414$$

$= 43{\cdot}60$ tons.

Maximum tension in fg

$$= \left(5 \times \tfrac{1}{2} + 10 \times \frac{1+2+3 \ldots 6}{12} + \frac{5}{2}\right) \times 1{\cdot}414$$

$= 31{\cdot}81$ tons.

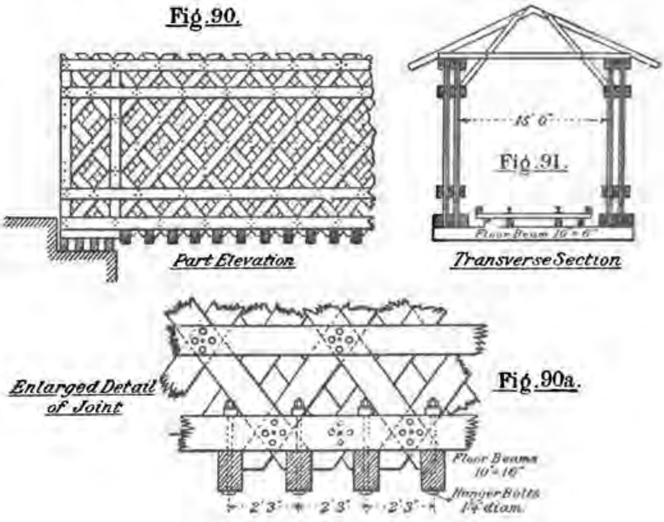
Maximum compression in Gg

= vertical component of stress in fg

$= 22{\cdot}50$ tons.

Town Lattice Truss.

The Town lattice truss is so named after the inventor, Mr. Ithiel Town, an American engineer, by whom it was patented in January, 1820, and it is probably the prototype of the modern lattice girder, and the first marked step in the direction of a braced girder with parallel booms.



The Town truss, which is built almost entirely of timber, consists of top and bottom chords, so arranged that the web members, composed of diagonal lattice bars, pass between them, being fastened at each point of intersection and at their junction with the top and bottom chords with oak treenails.

This type of truss has been extensively used in the United States of America on account of its great simplicity and the ease with which it can be put together by comparatively unskilled hands, but it possessed several defects which gradually brought it into disfavour. The principal of these defects was its tendency to warp through insufficient lateral stability, and the uniform dimensions throughout of the web members, which consequently are no stronger where the strain is greatest than where the strain is least.

It is not, however, obsolete, and the fact that on the Boston and Main Railway there are several bridges built on this principle still in use, a number of which have been renewed on the same lines within the last six years, seems to indicate that where suitable timber is readily available, it makes a firm, strong, durable, and economical structure.

The construction of the Town truss is shown in Figs. 90, 90a, and 91. The web members are composed of spruce planks from 2 to 3 inches thick and from 9 to 12 inches wide, forming lattice bars crossing one another at an angle varying from 60° to 45° with the horizontal, the spaces between the lattice bars measuring about 2′ 6″.

At their intersections the lattice bars are fastened with oak pins 2 in. in diameter, made of well-seasoned timber, carefully turned, so as to drive tightly when the bridge is erected. At the web intersections two pins are used, but at the chord intersections four pins are introduced, and a $\frac{3}{4}$ in. bolt which holds the members firmly together, as shown in Fig. 90a.

The top and bottom chords are generally built of pine planks consisting of two or three layers, each $2\frac{1}{2}$ to 3 inches thick and from 12 to 15 inches wide. A second chord is generally introduced, as shown in the figures, which serves not only to resist the stresses, but also to stiffen the diagonals and to distribute the shear between the tension and compression members of the web.

For short spans under 75 feet single web trusses can be made sufficiently strong to resist lateral flexure, but double trusses built on the box-girder principle, as shown in the illustrations, are much more rigid.

On the Boston and Main Railway, of which Mr. J. Parker Snow was the engineer, to whom the author is indebted for these particulars, the chord strength of these trusses is computed by assuming the distance between the centres of outer chords as the effective strain depth of the truss and reducing the section of the inner chords in the ratio of the squares of their distances from the neutral axis.

The floor beams are invariably hung below the chords, two beams in each panel, by bolts passing through the open spaces in the chords and through washer blocks on top of same.

The value of a pin joint to transmit stress between two members is taken at 1100 lbs. for each pin and 600 lbs. for one bolt, making a total of 5000 lbs. for one joint of four pins and one bolt.

When there are four pins and one bolt at a joint, the net area of the plank is taken as the depth in inches minus five multiplied by the width.

A camber of 1 inch for every 25 feet of span is generally given to trusses of this type.

CHAPTER IV.

Braced Girders with Inclined or Polygonal Chords.

1. The Parabolic Bowstring Girder.

The parabolic bowstring girder is so called because the bow or inclined chord is in the form of a polygon inscribed in a parabola.

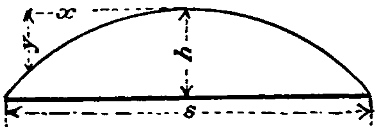


Fig. 92.

The bracing may be arranged as in Fig. 93, in which the diagonals perform the duties of struts and ties, and are alternately subject to tension and compression, or, as in Fig. 101, in which the vertical members are struts and the diagonal members ties.

The apices of the inclined or polygonal chord must lie in the segment of the parabola within which it is inscribed, and the vertical ordinate from the horizontal to the polygonal chord may be determined by the following equation of a parabola (Fig. 92):

$$x^2 = 2yp, \text{ hence } y = \frac{x^2}{2p} \text{ and } p = \frac{S^2}{8h}$$

Thus the ordinate to any point in the curve can be determined by assuming a value for x and solving for y.

In the subjoined parabolic bowstring truss, with isosceles bracing of 96 feet span and the centre ordinate of the parabola 16 feet, the values of y are as given, viz.:

$x = 8$, $y = \cdot 44$, and $H = h - y = 16\cdot 00 - \cdot 44 = 15\cdot 56$.
$x_1 = 24$, $y_1 = 4\cdot 00$, $H_1 = h - y_1 = 16\cdot 00 - 4\cdot 00 = 12\cdot 00$.
$x_2 = 40$, $y_2 = 11\cdot 11$, $H_2 = h - y_2 = 16\cdot 00 - 11\cdot 11 = 4\cdot 89$.

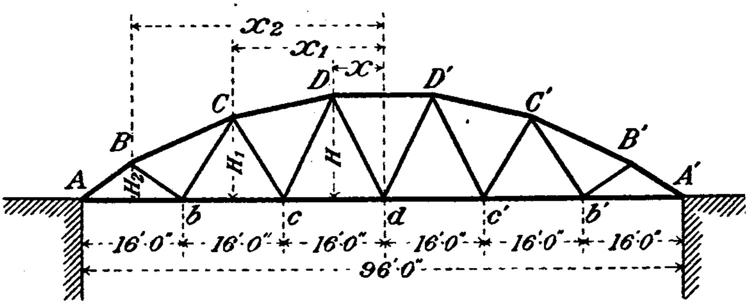


FIG. 93.

$$\text{Here } p = \frac{S^2}{8h} = p = \frac{96^2}{8 \times 16} = 72\,;\ h = 16.$$

The lengths of the several members are as follows, viz.:

AB = 9·38,	Bb = 9·38.
BC = 17·50,	bC = Cc = 14·42.
CD = 16·40,	cD = Dd = 17·50.
DD′ = 16·00,	Ab = bc = cd = 16·00.

Assuming the girder in Fig. 93 to be one of two girders in a railway bridge, the dead or static weight of which is 48 tons and the live or travelling load 2 tons per foot run; let it be required to determine the maxima stresses in the various members.

The static weight carried by each girder will thus be 24 tons and the dead panel load 4 tons. The live or rolling load will be 96 tons and the live panel load 16 tons.

The maxima stresses in top and bottom chords occur when the bridge is fully loaded. Each panel load will then be a maximum, and $=4$ tons $+16$ tons $=20$ tons, and the vertical re-action at each abutment $=\dfrac{20\times(6-1)}{2}=50$ tons.

The maximum tension in

$$\begin{aligned} & & \text{Tons.} \\ Ab &= \frac{50\times 8}{4{\cdot}89} &= 81{\cdot}80, \\ bc &= \frac{50\times 24-20\times 8}{12} &= 86{\cdot}66, \\ cd &= \frac{50\times 40-20\times 8(1+3)}{15{\cdot}56} &= 87{\cdot}40. \end{aligned}$$

The maximum compression in

$$\begin{aligned} AB &= \frac{50\times 9{\cdot}38}{4{\cdot}89} &= 95{\cdot}93, \\ BC &= \frac{50\times 16}{8{\cdot}44}\times\frac{17{\cdot}5}{16} &= 103{\cdot}67, \\ CD &= \frac{50\times 32-20\times 16}{13{\cdot}78}\times\frac{16{\cdot}4}{16} &= 95{\cdot}21, \\ DD' &= \frac{50\times 48-20\times 16(1+2)}{15{\cdot}56} &= 92{\cdot}54. \end{aligned}$$

$$\frac{LW}{8D}=\frac{120\times 96}{8\times 15{\cdot}56}=92{\cdot}54, \text{ as above.}$$

A general equation may be adopted for determining the maxima stresses in the top and bottom chords as under.

Let $L=$ length of truss.

$d=$ versed sine of the curve at the centre.

$l=$ length of panels on horizontal chord.

Let l' = length of an arc member between two apices.

W = maximum load (dead and live).

x = horizontal distance of any panel point from one abutment.

x' = distance of centre of any panel from one abutment.

y = the vertical height of any upper or arc chord panel point from horizontal chord.

Then the maximum horizontal chord strain in any horizontal chord member whose centre or mid-length is distant x' from the abutment

$$=H=\frac{WL}{8d}-\frac{Wl^2L}{32dx'(L-x')},$$

and the maximum longitudinal strain in any arc member whose centre is distant horizontally x' from the abutment, with the exception of the first or end member AB in diagram,

$$=H'=\left\{\frac{WL}{8d}+\frac{Wl^2L}{32d\left(Lx'-x'^2-\frac{l^2}{4}\right)}\right\}\frac{l'}{l}.$$

The strain in the first or end member AB is found from the strain in the horizontal end member Ab by multiplying the strain by the length of the inclined member AB and dividing the product by its horizontal component $=\frac{Ab}{2}$.

The maximum stress in any diagonal web member is found by computing the horizontal strain at its summit and its base, that is to say, at the points where it is connected with the top and bottom chords, due to the conditions of loading that will produce in that member the maximum tension or compression, as the case may be. The difference between these two quantities will be the horizontal component of the strain in the member and its longitudinal equivalent, or the axial strain will be to the horizontal component as the length of the diagonal is to the horizontal distance of its summit from its base.

The maximum tension in Bb occurs when the truss is fully loaded with the live load.

Re-action at A then

$$=\frac{5}{2}20=50 \text{ tons},$$

and the horizontal stress in Bb

$$=\frac{50\times 16}{8{\cdot}44}-\frac{50\times 8}{4{\cdot}89}=12{\cdot}98 \text{ tons},$$

and the longitudinal tension

$$=\frac{12{\cdot}98\times 9{\cdot}38}{8}=15{\cdot}18 \text{ tons}.$$

There can be no compression in Bb under any conditions of loading.

The maximum tension in bC occurs when the panel point b only is loaded with the live load.

Re-action at A then

$$=\frac{5}{2}4+\frac{5}{6}16=23{\cdot}33 \text{ tons},$$

and the horizontal stress in bC

$$=\frac{23{\cdot}33\times 16}{8{\cdot}44}-\frac{23{\cdot}33\times 24-20\times 8}{12}=10{\cdot}90 \text{ tons},$$

and the longitudinal tension

$$=\frac{10{\cdot}90\times 14{\cdot}42}{8}=19{\cdot}64 \text{ tons}.$$

The maximum compression in bC occurs when the live load extends from the right abutment at A′ to the panel point c.

Re-action at A then

$$=\frac{5}{2}4+\frac{1+2+3+4}{6}16=36{\cdot}66 \text{ tons},$$

and the horizontal stress in bC

$$=\frac{36{\cdot}66\times 24-4\times 8}{12}-\frac{36{\cdot}66\times 16}{8{\cdot}44}=1{\cdot}11 \text{ tons},$$

and the longitudinal compression

$$=\frac{1{\cdot}11\times 14{\cdot}42}{8}=2{\cdot}00 \text{ tons}.$$

The maximum tension in Cc occurs when the live load extends from the right abutment at A′ to the panel point c.

Re-action at A then

$$=36{\cdot}66, \text{ as already computed},$$

and horizontal stress in Cc

$$=\frac{36{\cdot}66\times 32-4\times 16}{13{\cdot}78}-\frac{36{\cdot}66\times 24-4\times 8}{12}=9{\cdot}83,$$

and the longitudinal stress

$$=\frac{9{\cdot}83\times 14{\cdot}42}{8}=17{\cdot}72 \text{ tons}.$$

The maximum compression in Cc occurs when the panel point b only is loaded with live load.

Re-action at A then

$$=\frac{5}{2}4+\frac{5}{6}16=23{\cdot}33 \text{ tons},$$

and the horizontal stress in Cc

$$=\frac{23{\cdot}33\times 24-20\times 8}{12}-\frac{23{\cdot}33\times 32-20\times 16}{13{\cdot}78}=2{\cdot}37 \text{ tons},$$

and the longitudinal stress

$$=\frac{2{\cdot}37\times 14{\cdot}42}{8}=4{\cdot}27 \text{ tons}.$$

The maximum tension occurs in cD when the panel points b and c are loaded with live load.

Re-action at A then

$$=\frac{5}{2}4+\frac{5+4}{6}16=34 \text{ tons},$$

and horizontal stress in cD

$$=\frac{34\times32-20\times16}{13\cdot78}-\frac{34\times40-20\times8(1+3)}{15\cdot56}=9\cdot46 \text{ tons},$$

and the longitudinal strain

$$=\frac{9\cdot46\times17\cdot5}{8}=20\cdot69 \text{ tons}.$$

The maximum compression in cD occurs when the live load extends from the right abutment at A′ to the panel point d.

Re-action at A then

$$=\frac{5}{2}4+\frac{1+2+3}{6}16=26 \text{ tons},$$

and horizontal stress in cD

$$=\frac{26\times40-4\times8(1+3)}{15\cdot56}-\frac{26\times32-4\times16}{13\cdot78}=2\cdot88 \text{ tons},$$

and the longitudinal strain

$$=\frac{2\cdot88\times17\cdot5}{8}=6\cdot30 \text{ tons}.$$

The maximum tension in Dd occurs when the live load extends from the right abutment at A′ to the panel point d.

Re-action at A then

$=26$ tons, as in the last case,

and horizontal stress in Dd

$$=\frac{26\times48-4\times16(1+2)}{15\cdot56}-\frac{26\times40-4\times8(1+3)}{15\cdot56}=9\cdot25 \text{ tons},$$

and the longitudinal stress

$$=\frac{9\cdot25\times17\cdot5}{8}=20\cdot23 \text{ tons}.$$

The maximum compression in Dd occurs when the panel points b and c are loaded with the live load.

Re-action at A then

$$=\frac{5}{2}4+\frac{5+4}{6}16=34 \text{ tons},$$

and horizontal stress in Dd

$$=\frac{34\times 40-20\times 8(1+3)}{15\cdot 56}-\frac{34\times 48-20\times 16(1+2)}{15\cdot 56}=3\cdot 09 \text{ tons,}$$

and the longitudinal strain

$$=\frac{3\cdot 09\times 17\cdot 5}{8}=6\cdot 75 \text{ tons.}$$

The stresses in the various members may also be determined by a graphic process, and it may be conveniently adopted if only as a check upon the calculations which, although simple, are lengthy and tedious.

For the stresses in the top and bottom chord members, a ready and simple method of solution is that shown in the diagram, Fig. 95.

The maxima stresses in the chords occur when the truss is fully loaded with the live load, in which case the load at each panel point or lower apex would be 20 tons.

The re-action at each abutment would therefore be

$$\frac{20\times 5}{2}=50 \text{ tons.}$$

In the diagram, Fig. 95, the stresses are only given for the right half of the girder; the loading being uniform and symmetrical, the other half of the diagram would be precisely alike and the stresses in the corresponding members on each side of the centre the same.

To construct the diagram, draw to any convenient scale the truss as shown in Fig. 94, and letter the fields included by each three members as shown. Draw at right angles to the bottom chord the vertical load line GL, and to any convenient scale lay off the re-action at the abutment and the panel loads, in this case $2\frac{1}{2}\times 20$ tons. From L draw LA parallel to the first inclined member of the upper chord LA until it intersects at A the line GA. From A draw AB parallel to the first web diagonal AB until it intersects the line LB

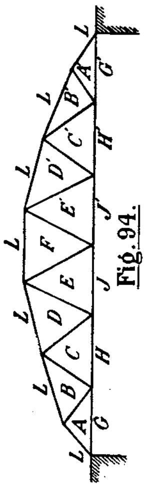


Fig. 94.

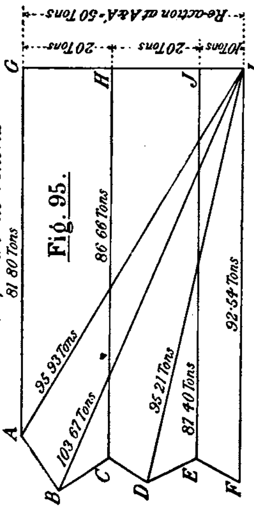


Fig. 95.

Stress Diagram for full Load giving maxima Stresses in top and bottom Chords

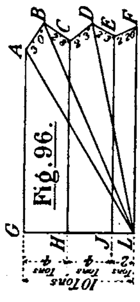


Fig. 96.

Stress Diagram for Dead Load

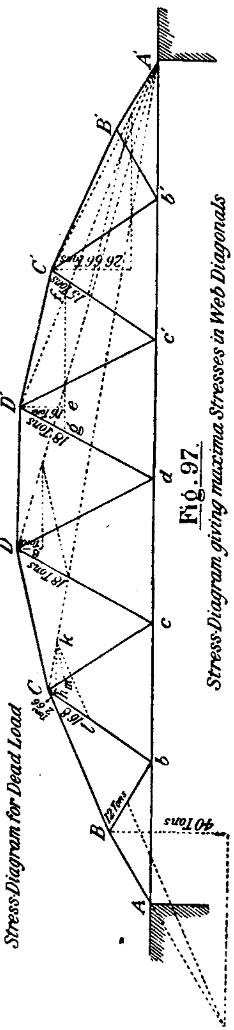


Fig. 97.

Stress-Diagram giving maxima Stresses in Web Diagonals drawn from L parallel with the second inclined member of the top chord LB. In a similar manner the other lines in the diagram are drawn. Then will the lines AL, BL, DL, and FL represent, by the same scale as that used for laying off on the load line the reaction and panel loads, the stresses in the several members of the top chord correspondingly lettered in Fig. 94, and similarly the horizontal lines AG, CH, and EJ will represent by scale the tension in the lower chord members correspondingly lettered.

Fig. 96 is a similar diagram for the dead load only, and the lines AB, BC, CD, DE, and EF represent the stresses in the web diagonals similarly lettered, due to the dead load.

The maximum stress in any diagonal such as *c*D (Fig. 93 and Fig. 97) occurs when the live or travelling load extends from the left abutment at A up to and including the panel point *c*, and the maximum tension in C*c* occurs when the rolling load extends from the abutment A up to and including the panel point *c*′. It is therefore obvious that a different system of loading must be assumed for the maximum stress in each diagonal brace, and if the stress is to be determined graphically by the same process as that adopted in Figs. 95 and 96 a separate diagram must be drawn for each brace based upon the particular system of loading producing the maximum stress, tension, or compression in any particular brace.

This process is very tedious, especially in the case of a truss divided into a large number of panels. A much simpler method of arriving at the web stresses is that shown in Fig. 97, representing a diagram from which all the stresses can be measured at once. The truss is drawn to any convenient scale (the larger the more accurate).

From either end of the truss, let it be A′, draw lines to the panel points or apices of the upper chord, C′, D′, D, and C, which will be the resultants of the pressure transmitted from those points to the abutment.

In the diagram, Fig. 97, the varying live loads producing maxima stresses in the web diagonals will only be considered, the stresses in the diagonals due to the dead or static load having already been determined in the diagram, Fig. 96. It will therefore be necessary to add the stress due to the dead load to the tension resulting from the live load, and to subtract the dead load stress from the compression due to the live load.

Let it be required to determine the maximum tension in the diagonal dD′. The maximum tension in dD′ occurs when the live load extends from A to the panel point d, and dD′ will transmit to the abutment at A′ $\frac{1}{6}$ of the weight at b, $+\frac{2}{6}$ of the weight at c, and $\frac{3}{6}$ of the weight at d. The live panel load is 16 tons, so that the proportion transmitted by dD′ to A′

$$=\frac{1+2+3}{6}\ 16 \text{ tons} = 16 \text{ tons}.$$

From D′ lay off vertically downward to scale D′e = 16 tons, from e draw ef horizontally to intersect the line of resultant, A′D. From f draw fg parallel to DD′ to intersect the diagonal line D′d at g. Then will gD represent the longitudinal stress or tension in D′d due to the live load, which measures by scale 18 tons. The dead load stress in D′d = EF in Figs. 94 and 96 = 2·20 tons, so that the total tension in dD′ = 18 + 2·20 = 20·20 tons, which agrees very closely with the calculated result.

Again, let it be required to determine the maximum tension in bC, which would result from a live load at the panel point b only. The proportion of that load transmitted to the right abutment at A through bC = $\frac{1}{6}$ of 16 tons = 2·66. From the apex C lay off vertically downwards Ch = 2·66 tons, as shown in Fig. 98, which is an enlargement of the panel point C. From h draw hk horizontally to intersect the line of resultant CA′, and from k draw kl parallel to BC and km parallel to CD. Then Cl will represent by scale the tension due to the live load in bC = 16·8 tons. The tension due to the dead load in BC, Figs. 94 and 96, = 2·8 tons, so that the total tension in

$bC = 16{\cdot}8 + 2{\cdot}8$ tons $= 19{\cdot}6$ tons, which also agrees very closely with the calculated result.

The loading for tension in bC is also the loading for compression in Cc, and the distance Cm represents by scale the

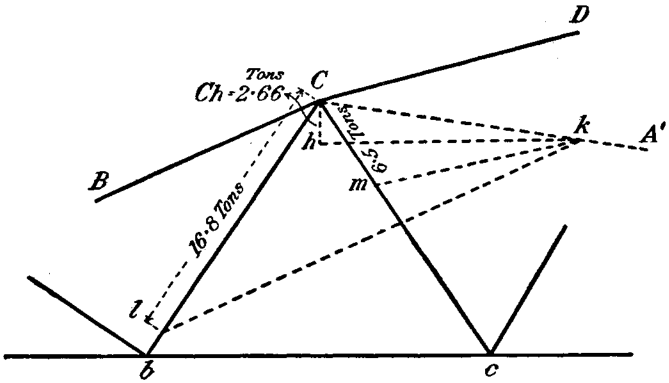


Fig. 98.

compression in $Cc = 6{\cdot}5$ tons, but the effect of the dead load concentrated at c exerts tension in Cc, so that the compression is reduced to that extent. The tension resulting from the dead load in CD, Figs. 94 and 96, $= 2{\cdot}3$ tons and $6{\cdot}5 - 2{\cdot}3 = 4{\cdot}2$ tons, the nett compression in Cc as compared with $4{\cdot}27$, the calculated result.

In a similar manner the stresses in all the other diagonals, both compressive and tensile, can be readily determined.

Bowstring Girder, with two Systems of Diagonal Braces, as in Fig. 99.

In this truss there are two systems of web braces, each acting independently of the other, and it may therefore be considered as divided into two simple trusses, the stresses in which can be determined as in the preceding example so far as the web members are concerned.

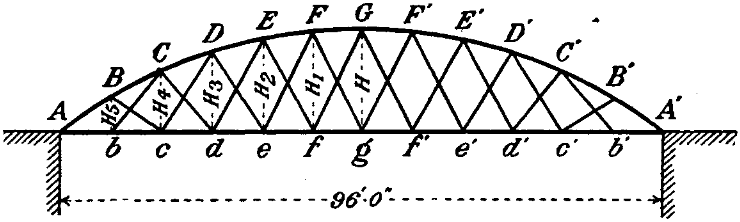


Fig. 99.

$H = 16{\cdot}00.$	$H_2 = 14{\cdot}22.$	$H_4 = 8{\cdot}89.$
$H_1 = 15{\cdot}56.$	$H_3 = 12{\cdot}00.$	$H_5 = 4{\cdot}89.$

The chords are common to both systems, and a general equation may be adopted for the stresses, for which purpose the following notation will be used:

Let

L = length of truss = 96 feet.

d = versed sine of curve = 16 feet.

l = length of panel = 8 feet.

l' = length of an arc member between two apices.

W = maximum load (dead and live).

x = horizontal distance of any panel point from abutment.

$h = \frac{4dx(L-x)}{L^2}$, the vertical distance between the chords at any panel point x.

Then H = strain in the horizontal chord of the two systems, excluding the end members Ab and $A'b'$;

$$H = \frac{WL}{8d} - \frac{WLl^2}{16dx(L-x)},$$

and the strain in the end member Ab

$$= \mathrm{A}'b' = \mathrm{H} = \frac{\mathrm{WL}}{16d} - \frac{\mathrm{WL}l}{16d(\mathrm{L}-l)} + \frac{\mathrm{WL}^2}{16d(\mathrm{L}-2l)}.$$

The longitudinal strain in any arc member, excluding the end members AB and A′B′,

$$= \mathrm{H}' = \left(\frac{\mathrm{WL}}{8d} + \frac{\mathrm{WL}l^2}{16dx(\mathrm{L}-x)}\right)\frac{l'}{l}.$$

In the last equation x is the distance to the lower panel points c, e, and g, and for each value of x there are two values of l', the lengths of the two arc members meeting at that point.

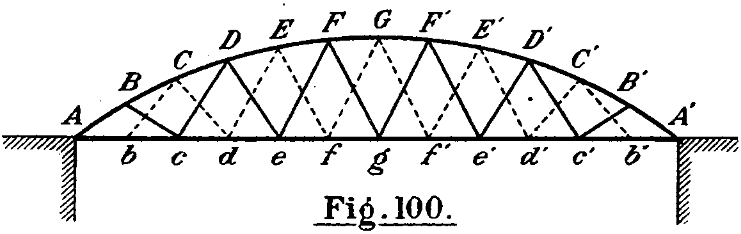


Fig. 100.

The longitudinal or axial compression in the end arc members is equal to the strain in the end members of the lower chord Ab and A′b' $\times \frac{l'}{l}$, or, in other words, the horizontal component of the compression in the end arc member is equal to the longitudinal strain in the end horizontal member, and

$$\left(\frac{\mathrm{WL}}{16d} - \frac{\mathrm{WL}l}{16d(\mathrm{L}-l)} + \frac{\mathrm{WL}^2}{16d(\mathrm{L}-2l)}\right)\frac{l'}{l}$$

will give the longitudinal strain in the end arc members.

The strains in the diagonal web members may be determined in the same manner as that described in the preceding example by assuming the truss to be divided into two simple trusses, as shown in Fig. 100, in which No. 1 truss is represented by bold lines and No. 2 by dotted lines.

The lengths of the members of the top chord are as under:

AB, 9·38.	CD, 8·58.	EF, 8·11.
BC, 8·94.	DE, 8·30.	FG, 8·01.

The stresses in the top and bottom chords are as under:

AB = + 111·56 tons.	Ab = − 94·91 tons.
BC = + 103·08 „	bc = − 85·91 „
CD = + 98·70 „	cd = − 87·75 „
DE = + 95·09 „	de = − 88·33 „
EF = + 92·92 „	ef = − 88·60 „
FG = + 91·89 „	fg = − 88·72 „

The chord stresses given above are based on a dead load of 48 tons and live load of 192 tons, making a total load of 240 tons, or 120 tons per girder, as in the preceding example. In this truss there are 12 panels, so that the panel load is 10 tons.

The stresses in the web members may be determined by either a graphic or analytical process, assuming for each brace the condition of loading that would produce in it the maximum strain in precisely the same manner as for a simple truss, with one system of bracing. The stresses so determined, which should be written down on each brace, will be the maxima stresses.

Bowstring Girder, with vertical Struts and Diagonal Ties.

In this truss, which is the most generally adopted form of the bowstring girder, the leading dimensions and conditions of loading are as follows:

Span, 80 feet; depth of truss at centre, 10 feet = $\frac{1}{8}$ of span.

Length of	AB = 10·90 feet.	Length of	Bc = 10·90 feet.
,,	BC = 10·48 ,,	,,	bC = Cd = 12·50 ,,
,,	CD = 10·17 ,,	,,	cD = De = 13·70 ,,
,,	DE = 10·01 ,,	,,	dE = 14·14 ,,

Dead or static load on one truss = 20 tons = 2·5 tons per panel.
Live or rolling load on one truss = 60 tons = 7·5 tons per panel.

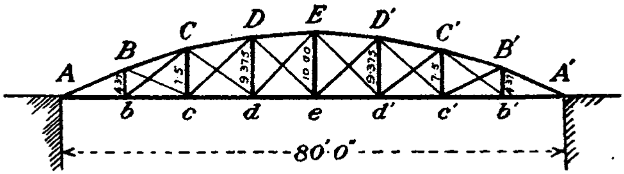


Fig. 101

The stresses in this truss may be obtained as follows:

Let L = span of truss = 80 feet.

l = horizontal length of a panel in lower chord = 10 ft.

l' = length of any arc member in upper chord.

w = dead load per panel = 2·5 tons.

w' = live load per panel = 7·5 tons.

W = total dead and live load = 80 tons.

N = number of panels = 8.

x = horizontal distance of any panel point from abutment.

h = ordinate between chords at x.

d = depth at centre.

D = length of any diagonal.

In a parabolic bowstring girder of this type, with a load uniformly distributed throughout its length, the horizontal components of the stress will be constant in all the chord pieces. In the lower chord the tension will be uniform throughout, and in the upper chord members it will increase from the centre to each abutment in proportion to the length of the inclined member. The maximum stress in the top and bottom chords occurs when the truss is fully loaded, the expression $\frac{WL}{8d}$ will give the maximum stress in the bottom chord, which is uniform throughout, and in any panel of the upper chord, whose length is l', the maximum stress will be $\frac{WL}{8d} \times \frac{l'}{l}$.

Under a uniformly distributed load the diagonal members of the web are unstressed and have no duty to perform, but under a moving load each diagonal in its turn, according to the distribution of the load, is subject to a tensile strain.

The horizontal component of the stress in any diagonal is equal to the difference of the horizontal stresses in the two chords of the panel in which the diagonal is situated, that is to say, at the points where it is connected with the top and bottom chords, and it becomes a maximum in any diagonal inclining from right to left, such as Cd, when the live load extends from the right abutment to the foot of the diagonal at d, Fig. 101.

The vertical members under the effect of a uniformly distributed load are all in tension, but under a variable load they are subject to a compressive strain brought upon them by the diagonals. The maximum compression in any vertical, such as C$'c$, occurs when the live load extends from the right abutment A$'$ to the panel point d, and it is equal to the vertical component of the maximum tension in C$'d'$ less the tension of one panel of dead load. Thus the maximum tension in C$'d'$ occurs when the live load extends from the right abutment at A$'$ to the panel point d, when re-action due to live load at A

$$= \frac{3+4+5+6+7}{8} \times 7 \cdot 5 = 23 \cdot 44 \text{ tons},$$

and horizontal component of stress in $C'd'$

$$=\frac{23{\cdot}44\times 50-7{\cdot}5\times 10(1+2+3+4)}{9{\cdot}375}$$

$$=\frac{23{\cdot}44\times 60-7{\cdot}5\times 10(1+2+3+4+5)}{7{\cdot}5}=7{\cdot}5 \text{ tons,}$$

the vertical component of which

$$=7{\cdot}5\times\frac{7{\cdot}5}{10}=5{\cdot}625 \text{ tons.}$$

This is also equal to the compression brought upon the vertical $C'c'$ by the live load. Deducting from this the panel dead load of $2\frac{1}{2}$ tons at c', we have $5{\cdot}625-2{\cdot}5$, the nett compression in $C'c'$, $=3{\cdot}125$ tons.

The maximum tension in any vertical

$$=w+w'=2{\cdot}5+7{\cdot}5=10 \text{ tons.}$$

In any diagonal the horizontal component of stress

$$=\frac{w'\mathrm{L}}{8d},$$

and the longitudinal stress

$$=\frac{w'\mathrm{L}}{8d}\times\frac{\mathrm{D}}{l}.$$

The stresses in each member of the truss can now be easily and readily determined by these simple formulae.

Thus the maximum tension in the horizontal or lower chord, which is constant throughout,

$$=\frac{\mathrm{WL}}{8d}=\frac{80\times 80}{10\times 8}=-80 \text{ tons.}$$

Top boom

$$\mathrm{AB}=\frac{\mathrm{WL}}{8d}\times\frac{l'}{l}=\frac{80\times 80}{8\times 10}\times\frac{10{\cdot}9}{10{\cdot}0}=+87{\cdot}2 \text{ tons.}$$

$$\mathrm{BC}=\frac{\mathrm{WL}}{8d}\times\frac{l'}{l}=\frac{80\times 80}{8\times 10}\times\frac{10{\cdot}48}{10{\cdot}00}=+83{\cdot}8 \quad ,,$$

$$\mathrm{CD}=\frac{\mathrm{WL}}{8d}\times\frac{l'}{l}=\frac{80\times 80}{8\times 10}\times\frac{10{\cdot}17}{10{\cdot}00}=+81{\cdot}36 \quad ,,$$

$$\mathrm{DE}=\frac{\mathrm{WL}}{8d}\times\frac{l'}{l}=\frac{80\times 80}{8\times 10}\times\frac{10{\cdot}01}{10{\cdot}00}=+80{\cdot}08 \quad ,,$$

The horizontal component of the maxima stresses in the diagonals

$$=\frac{w'L}{8d}=\frac{7\cdot5\times80}{10\times8}=7\cdot5 \text{ tons},$$

and the actual or longitudinal stresses will be as under:

	Tons.		Tons.
$Bc=7\cdot5\times\frac{10\cdot9}{10}=$	$-\ 8\cdot17.$	$bC=7\cdot5\times\frac{12\cdot5}{10}=$	$-\ 9\cdot37.$
$Cd=7\cdot5\times\frac{12\cdot5}{10}=$	$-\ 9\cdot37.$	$cD=7\cdot5\times\frac{13\cdot7}{10}=$	$-10\cdot27.$
$De=7\cdot5\times\frac{13\cdot7}{10}=$	$-10\cdot27.$	$dE=7\cdot5\times\frac{14\cdot14}{10}=$	$-10\cdot6.$

The stresses in this type of truss can be determined graphically by a very simple and expeditious process as follows:

The maxima stresses in the top and bottom chords occur when the truss is fully loaded, when the re-action at each abutment $=(w+w')\frac{N-1}{2}=35$ tons. On a line LP drawn at right angles to the horizontal or bottom chord, Fig. 103, lay off LP to any convenient scale equal to the abutment re-action $=35$ tons, and mark off LM equal to half the panel load at e, Fig. 102, and MN, NO, and OP equal to the panel loads at d, c, and b. Draw the horizontal line LQ and the lines MQ parallel to DE, NQ parallel to CD, OQ parallel to BC, and PQ parallel to AB. Then will LQ give the maximum tension in the bottom chord, PQ the maximum compression in the arc member AB, OQ the compression in BC, NQ the compression in CD, and MQ the compression in DE.

The stresses in the web diagonals can be determined in a similarly simple manner by drawing a diagram of the truss to scale, making AA′ in Fig. 102, or the length of the lower chord, equal to

$$\frac{w'NL}{8d}=\frac{7\cdot5\times8\times80}{8\times10}=60 \text{ tons}.$$

In the diagram, Fig. 104, one half only of the truss is shown, the stresses in the other half being similar and equal. Ae is equal to 30 tons, or one half of $\frac{7 \cdot 5 \times 8 \times 80}{8 \times 10}$, and by the same scale the length of each diagonal will represent the

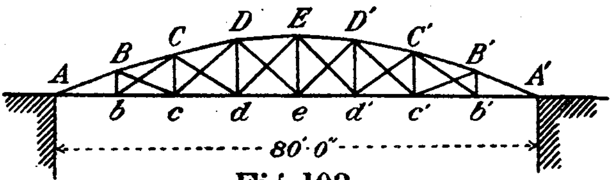


Fig. 102.

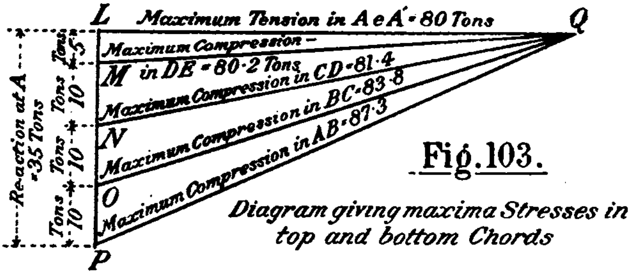


Fig. 103.

Diagram giving maxima Stresses in top and bottom Chords

longitudinal or actual stress in that diagonal, for by construction the horizontal component, which is constant throughout, $= \frac{w'L}{8d}$.

The length of each vertical will also give the compression in that vertical due to the live load, and by deducting the dead panel load at the foot of any vertical which $= 2\frac{1}{2}$ tons, and produces tension, in that vertical we have the nett compression.

The bowstring girder is frequently used in an inverted form, which is attended with many practical advantages, inasmuch as the longer chord or curved member can be more conveniently designed to resist tension than compression, and where vertical braces are introduced they are subject to com-

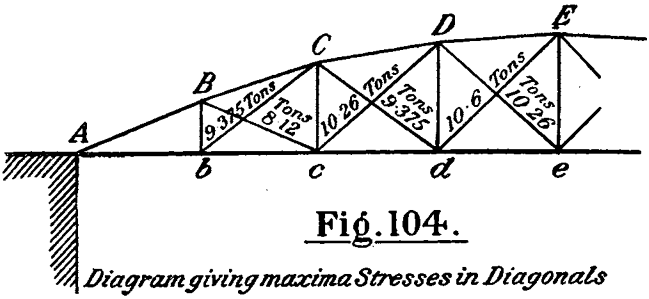


Fig. 104.

Diagram giving maxima Stresses in Diagonals

pression only. The inverted form also admits of the introduction of an efficient system of transverse bracing to resist lateral flexure.

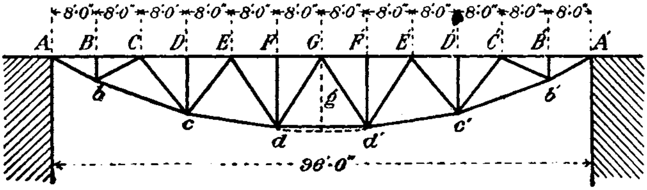


Fig. 105.

In an inverted bowstring the stresses are quantitively precisely the same, but they become reversed in kind; the horizontal or upper chord being subject to compression and the curved or lower member to tension. Where there are web verticals they will only be subject to compression under all conditions of loading, and the dead load at each panel point will exert compression upon them, which must be added to the compression resulting from the live load instead of being subtracted, as in the preceding example.

Probably a better type of an inverted bowstring than either of the trusses that have been described is that illustrated in Fig. 105, inasmuch as it admits of the support of the upper or compression chord at intermediate panel points, and the introduction of the vertical struts materially facilitate the arrangement of the transverse bracing.

Lengths of members :

$Ab = 8{\cdot}80.$	$Bb = 3{\cdot}66.$	$bC = 8{\cdot}80.$
$bc = 16{\cdot}86.$	$Dc = 9{\cdot}00.$	$Cc = cE = 12{\cdot}04.$
$cd = 16{\cdot}21.$	$Fd = 11{\cdot}66.$	$Ed = dG = 14{\cdot}14.$
$dd' = 16{\cdot}00.$	$Gg = 12{\cdot}00.$	

Let the dead load be $\frac{1}{2}$ ton per foot run $= 4$ tons per panel,
and the live load be 1 ton per foot run $= 8$ tons per panel,
then the total panel load $= 4 + 8$ tons $= 12$ tons.

The reaction at each abutment resulting from a full load

$$= \frac{12 - 1}{2} \times 12 = 66 \text{ tons.}$$

The maximum compression in

Tons.

$$AB = BC = \frac{66 \times 8}{3{\cdot}\dot{6}} = 144.$$

$$CD = DE = \frac{66 \times 24 - 12 \times 8(1 + 2)}{9} = 144.$$

$$EF = FG = \frac{66 \times 40 - 12 \times 8(1 + 2 + 3 + 4)}{11{\cdot}\dot{6}} = 144.$$

The maximum tension in

$$Ab = \frac{66 \times 8{\cdot}80}{3{\cdot}\dot{6}} = 158{\cdot}4.$$

$$bc = \frac{66 \times 16 - 12 \times 8}{6{\cdot}\dot{3}} \times \frac{16{\cdot}86}{16} = 159{\cdot}8.$$

The maximum tension in

$$cd = \frac{66 \times 32 - 12 \times 8(1+2+3)}{10\cdot\dot{3}} \times \frac{16\cdot21}{16} = 150\cdot64. \quad \text{Tons.}$$

$$dd' = \frac{66 \times 48 - 12 \times 8(1+2+3+4+5)}{11\cdot\dot{6}} = 148\cdot11.$$

The web stresses in the diagonals may be determined in the manner already described as follows:

The maximum compression in bC occurs when the live load extends from the right abutment at A′ to the panel point C, when the reaction at A

$$= \frac{12-1}{2} \times 4 + \frac{1+2+3+4+5+6+7+8+9+10}{12} \times 8$$

$$= 22 + 36\cdot66 \text{ tons} = 58\cdot66 \text{ tons},$$

and the compression in bC

$$= \frac{58\cdot66 \times 16 - 4 \times 8}{6\cdot\dot{3}} - \frac{58\cdot66 \times 8}{3\cdot\dot{6}}$$

$$= 15, \text{ the horizontal stress,}$$

and $$15 \times \frac{8\cdot8}{8} = 16\cdot5, \text{ the compression in } b\text{C}.$$

The maximum tension in bC occurs when the panel point B only is loaded with the live load, when the reaction at A

$$= \frac{12-1}{2} \times 4 + \frac{11}{12} \times 8 = 29\cdot\dot{3} \text{ tons},$$

and the tension in bC

$$= \left(\frac{29\cdot\dot{3} \times 8}{3\cdot\dot{6}} - \frac{29\cdot\dot{3} \times 16 - 12 \times 8}{6\cdot\dot{3}}\right) \times \frac{8\cdot8}{8} = 5\cdot56 \text{ tons}.$$

The maximum compression in Cc occurs when the panel points B and C are loaded with live load, when reaction at A

$$= \frac{12-1}{2} \times 4 + \frac{11+10}{12} \times 8 = 36 \text{ tons},$$

and the compression in Cc

$$=\left(\frac{36\times 16-8\times 12}{6\cdot\dot{3}}-\frac{36\times 24-12\times 8(1+2)}{9}\right)\times\frac{12\cdot 04}{8}$$

$$=17\cdot 80 \text{ tons.}$$

The maximum tension in Cc occurs when the live load extends from the right abutment at A′ to the panel point D, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{9+8+7+6+5+4+3+2+1}{12}\times 8$$

$$=52 \text{ tons,}$$

and the tension in Cc

$$=\left(\frac{52\times 24-4\times 8(1+2)}{9}-\frac{52\times 16-4\times 8}{6\cdot\dot{3}}\right)\times\frac{12\cdot 04}{8}$$

$$=2\cdot 54 \text{ tons.}$$

The maximum compression in cE occurs when the live load extends from the right abutment at A′ to the panel point E, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{1+2+3+4+5+6+7+8}{12}\times 8=46 \text{ tons,}$$

and the compression in cE

$$=\left(\frac{46\times 32-4\times 8(1+2+3)}{10\cdot 3\dot{3}}-\frac{46\times 24-4\times 8(1+2)}{9}\right)$$

$$\times\frac{12\cdot 04}{8}=(123\cdot 91-112)\frac{12\cdot 04}{8}=17\cdot 92 \text{ tons.}$$

The maximum tension in cE occurs when the live load extends from left abutment at A to the panel point D, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{11+10+9}{12}\times 8=42 \text{ tons,}$$

and the tension in cE

$$=\left(\frac{42\times 24-12\times 8(1+2)}{9}-\frac{42\times 32-12\times 8(1+2+3)}{10\cdot 3\dot{3}}\right)$$

$$\times\frac{12\cdot 04}{8}=8\cdot 54 \text{ tons.}$$

The maximum compression in Ed occurs when the live load extends from left abutment at A to the panel point E, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{11+10+9+8}{12}\times 8=47\cdot 3\dot{3} \text{ tons,}$$

and compression in Ed

$$=\left(\frac{47\cdot 3\dot{3}\times 32-12\times 8(1+2+3)}{10\cdot 3\dot{3}}-\frac{47\cdot 33\times 40-12\times 8(1+2+3+4)}{11\cdot 6\dot{6}}\right)\times\frac{14\cdot 14}{8}$$

$$=19\cdot 14 \text{ tons.}$$

The maximum tension in Ed occurs when the live load extends from the right abutment at A′ to the panel point F, when reaction at A

$$=\frac{12-1}{2}\times 4-\frac{1+2+3+4+5+6+7}{12}\times 8=40\cdot\dot{6} \text{ tons,}$$

and tension in Ed

$$=\left(\frac{40\cdot\dot{6}\times 40-4\times 8(1+2+3+4)}{11\cdot\dot{6}}-\frac{40\cdot\dot{6}\times 32-4\times 8(1+2+3)}{10\cdot 33}\right)\times\frac{14\cdot 14}{8}$$

$$=(112-107\cdot 35)\times\frac{14\cdot 14}{8}=8\cdot 21 \text{ tons.}$$

The maximum compression in dG occurs when the live load extends from the right abutment at A to the panel point G, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{1+2+3+4+5+6}{12}\times 8=36 \text{ tons,}$$

and compression in dG

$$=\left(\frac{36\times 48-4\times 8(1+2+3+4+5)}{11\cdot 6\dot{6}}-\frac{36\times 40-4\times 8(1+2+3+4)}{11\cdot 66}\right)\times\frac{14\cdot 14}{8}$$

$$=(107-96)\times\frac{14\cdot 14}{8}=19\cdot 44 \text{ tons.}$$

The maximum tension in dG occurs when the live load extends from the left abutment at A to the panel point F, when reaction at A

$$=\frac{12-1}{2}\times 4+\frac{11+10+9+8+7}{12}\times 8=52 \text{ tons,}$$

and the tension in dG

$$=\left(\frac{52\times 40-12\times 8(1+2+3+4)}{11\ 6\dot{6}}-\frac{52\times 48-12\times 8(1+2+3+4+5)}{11\ 66}\right)\times\frac{14\cdot 14}{8}$$

$$=9\cdot 7 \text{ tons.}$$

The compression in any vertical is a maximum when that vertical is under a full panel load, and it equals

$$w+w'=4+8=12 \text{ tons.}$$

Bowstring Suspension or Lenticular Truss.

Lengths of members:

Bb = 8·75 ft.	AB = Ab = 15·62 ft.	Bc = bC = 19·13 ft.
Cc = 15·00 „	BC = bc = 15·31 „	Cd = cD = 22·58 „
Dd = 18·75 „	CD = cd = 15·10 „	De = dE = 24·50 „
Ee = 20·00 „	DE = de = 15·01 „	

Dead or static load, ·5 ton per foot run = 7·5 tons per panel.
Live or rolling load, 1 ton „ = 15·0 „

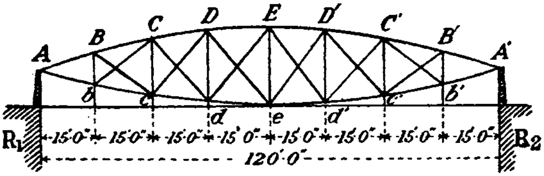


Fig. 106.

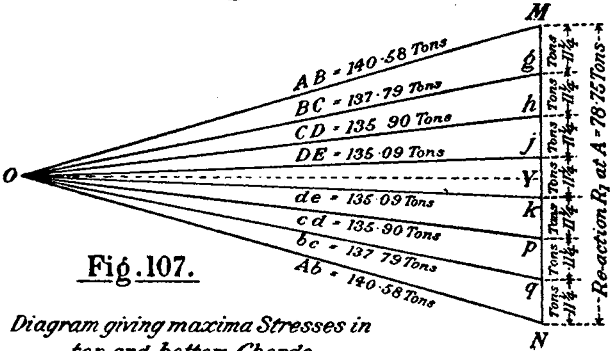


Fig. 107.

Diagram giving maxima Stresses in top and bottom Chords

Fig. 108.

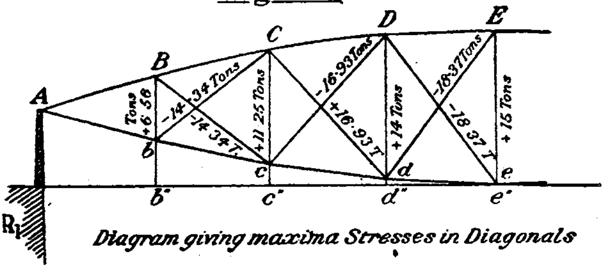


Diagram giving maxima Stresses in Diagonals

Scale for dimensions of truss, - 10 feet to an inch.
Scale for stresses in diagonals, - $7\frac{1}{2}$ tons ,,

The stresses in this truss can be determined graphically by adopting the same process as in Figs. 102, 103, and 104, and proceeding as follows:

On a vertical line MN lay off MN to any convenient scale equal to the reaction due to a full load at either abutment $=(w+w')\frac{N-1}{2}$. From M and N in Fig. 107 draw the ray lines MO and NO respectively parallel to AB and A*b* in Fig. 106. From the point O draw O*g*, O*h*, and O*j* respectively parallel to the members of the upper chord BC, CD, and DE, and again from O draw O*k*, O*p*, and O*q* parallel to *bc*, *cd*, and *de* of the lower chord. Then will these ray lines represent by scale the stresses in the several members of the top and bottom chords to which they are drawn parallel.

The horizontal dotted line OY represents the horizontal component of the stress throughout the top and bottom chords, which $=\frac{WL}{8d}$. The stress in any member of the top and bottom chord $=\frac{WL}{8d}\times\frac{l'}{l}$.

The diagram in Fig. 108 gives the maxima stresses in the diagonals which are subject only to tension due to the live load. In this diagram, as in Fig. 104, AA′, or the length of the horizontal axis, should be laid off by scale $=\frac{w'NL}{8d}$, in this case $=\frac{15\times 8\times 120}{8\times 20}=90$ tons.

Then the length of each diagonal on the scale will be the longitudinal tension in that diagonal, and the length of any vertical on the scale will be the compression due to the live load on that vertical.

Each vertical is subject to a tensile strain of intensity $w+w'$. The stress in any diagonal $=\frac{w'L}{8d}\times\frac{D}{l}$.

The double bowstring or lenticular truss is sometimes known as the Pauli truss, which it resembles in outline, but it differs from the Pauli truss, in which latter the curvature of the chords is so calculated that the strain in them is constant throughout. The double bowstring offers no advantage over a simple parabolic bowstring girder of equal span and depth. Indeed, from a practical point of view, it has several disadvantages.

The principal examples of this type of truss are:

1. The Saltash Viaduct over the Tamar, near Plymouth, with spans of 445 feet, designed by the late Mr. Brunel.
2. The Monongahela Bridge at Pittsburg, in two spans of 360 feet each, designed by Mr. Gustave Lindenthal.
3. The bridge over the Rhine at Mayence, having spans of 326 feet each.

Truncated Bowstring or Hogged-back Girder.

This type of girder, which has been largely used in railway bridges in England and on the Continent, is intermediate between a girder with parallel chords and a bowstring girder. The upper chord is curved similarly to the bowstring girder, but the curved chord is not continued to meet the lower or horizontal chord. It may therefore be described as resembling a bowstring girder with both ends lopped off, hence the term "truncated bowstring"

In this girder as compared with a parallel girder of equal depth the end chord stresses are greater, but the stresses in the web members are less. On the other hand, when it is compared with the bowstring girder the end chord stresses are less, but the stresses in the web members are greater.

The stresses may be most conveniently determined by the same process as that employed in the case of Fig. 93 and Fig. 105.

The dimensions of each member of the girder have been calculated and inscribed thereon as shown in Figs. 110 and 111, and the conditions of loading are given at the foot of Fig. 112.

The maxima stresses in the top and bottom flanges occur when the truss is loaded throughout its length with the live load, and inasmuch as the web consists of two systems of triangulation, the one acting independently of the other, the girder for the purpose of determining the stresses must be assumed to be decomposed into two trusses, shown in the diagram, Figs. 110 and 111, as simple trusses, Nos. 1 and 2 respectively.

Simple Truss, No. 1.

The abutment re-action due to full load in this truss

$$=15 \times \frac{7-1}{2} = 45 \text{ tons.}$$

The maximum compression in

$$AB = \frac{45 \times 10}{13{\cdot}06} \times \frac{10{\cdot}45}{10} = 36{\cdot}1 \text{ tons,}$$

$$BD = \frac{45 \times 30 - 15 \times 20}{17{\cdot}5} \times \frac{20{\cdot}48}{20} = 61{\cdot}44 \text{ tons,}$$

$$DF = \frac{45 \times 50 - 15 \times 20(1+2)}{19{\cdot}72} \times \frac{20{\cdot}12}{20} = 68{\cdot}86 \text{ tons,}$$

$$FF' = \frac{45 \times 70 - 15 \times 20(1+2+3)}{19{\cdot}72} \times \frac{20}{20} = 68{\cdot}45 \text{ tons.}$$

The maximum tension in

$$ab = \text{nil,}$$

$$bd = \frac{45 \times 10}{13{\cdot}06} = 34{\cdot}45 \text{ tons,}$$

$$df = \frac{45 \times 30 - 15 \times 20}{17{\cdot}5} = 60{\cdot}00 \text{ tons,}$$

$$ff' = \frac{45 \times 50 - 15 \times 20(1+2)}{19{\cdot}72} = 68{\cdot}45 \text{ tons.}$$

The maximum tension in Ab occurs when the live load covers the bridge when the abutment re-action $=45$ tons as above.

The tension in A$b=$ the horizontal component of the stress in

$$\mathrm{AB} \times \frac{\mathrm{A}b}{ab} = \frac{45 \times 10}{13{\cdot}06} \times \frac{14{\cdot}14}{10} = 48{\cdot}71 \text{ tons.}$$

There is no compression in Ab under any condition of loading.

The maximum tension in Bd occurs when all the panel points except b are covered with the live load when re-action at

$$a = 5 \times \tfrac{11}{12} + 15 \times \frac{9+7+5+3+1}{12} = 35{\cdot}83 \text{ tons.}$$

Then horizontal stress at $b = \dfrac{35{\cdot}83 \times 10}{13{\cdot}06} = 27{\cdot}43$ tons,

and the horizontal stress at $d = \dfrac{35{\cdot}83 \times 30 - 5 \times 20}{17{\cdot}5} = 55{\cdot}70$ tons.

Then the horizontal component of the tension in

$$\mathrm{B}d = 55{\cdot}7 - 27{\cdot}43 = 28{\cdot}27,$$

and the longitudinal tension $= 28{\cdot}27 \times \dfrac{23{\cdot}83}{20} = 33{\cdot}68$ tons.

The maximum tension in Df occurs when the live load extends from the right abutment to the panel point f when re-action at

$$a = 5 \times \frac{11+9}{12} + 15 \times \frac{7+5+3+1}{12} = 28{\cdot}33 \text{ tons.}$$

Then horizontal stress at

$$d = \frac{28{\cdot}33 \times 30 - 5 \times 20}{17{\cdot}5} = 42{\cdot}85,$$

and horizontal stress at

$$f = \frac{28{\cdot}33 \times 50 - 5 \times 20(1+2)}{19{\cdot}72} = 56{\cdot}61.$$

Then the horizontal component of the tension in

$$Df = 56{\cdot}61 - 42{\cdot}85 = 13{\cdot}76,$$

and the longitudinal stress

$$= 13{\cdot}76 \times \frac{26{\cdot}57}{20} = 18{\cdot}28 \text{ tons.}$$

The maximum tension in Ff occurs when the live load covers the panel points b', d', and f' only, when re-action at

$$a = 5 \times \frac{11+9+7}{12} + 15 \times \frac{5+3+1}{12} = 22\tfrac{1}{2} \text{ tons.}$$

Then horizontal stress at

$$f = \frac{22{\cdot}5 \times 50 - 5 \times 20(1+2)}{19{\cdot}72} = 41{\cdot}83,$$

and horizontal stress at

$$f' = \frac{22{\cdot}5 \times 70 - 5 \times 20(1+2+3)}{19{\cdot}72} = 49{\cdot}44.$$

Then the horizontal component of the tension in

$$Ff = 49{\cdot}44 - 41{\cdot}83 = 7{\cdot}61,$$

and the longitudinal stress

$$= 7{\cdot}61 \times \frac{28{\cdot}12}{20} = 10{\cdot}69 \text{ tons.}$$

The maximum compression in the end vertical Aa = the vertical re-action due to a full load = 45 tons.

The maximum compression in Bb occurs when the live load extends from the right abutment to the panel point d when the re-action at the abutment a

$$= 5 \times \tfrac{11}{12} + 15 \times \frac{9+7+5+3+1}{12} = 35{\cdot}83 \text{ tons.}$$

The maximum compression in Bb = the vertical component of the stress in the diagonal Ab due to this condition of loading less one panel dead load at b which produces tension in Bb.

The horizontal component of the stress in

$$\text{A}b = \frac{35 \cdot 83 \times 10}{13 \cdot 06} = 27 \cdot 43 \text{ tons,}$$

the vertical component of which

$$= 27 \cdot 43 \times \tfrac{10}{10} = 27 \cdot 43 \text{ tons and } 27 \cdot 43 - 5 = 22 \cdot 43 \text{ tons,}$$

the compression in Bb.

The maximum compression in Dd occurs when the live load extends from the right abutment to the panel point f when the re-action at the abutment a

$$= 5 \times \frac{11+9}{12} + 15 \times \frac{7+5+3+1}{12} = 28 \cdot 33 \text{ tons.}$$

The maximum compression in Dd = the vertical component of the stress in the diagonal Bd due to this condition of loading less the panel dead load at d.

The horizontal component of the stress in

$$\text{B}d = \frac{28 \cdot 33 \times 30 - 5 \times 20}{17 \cdot 5} - \frac{28 \cdot 33 \times 10}{13 \cdot 06} = 21 \cdot 16 \text{ tons,}$$

the vertical component of which

$$= 21 \cdot 66 \times \frac{13 \cdot 06}{20} = 13 \cdot 81 \text{ and } 13 \cdot 81 - 5 = 8 \cdot 81 \text{ tons,}$$

the compression in Dd.

The maximum compression in Ff occurs when the panel points b', d', and f' are loaded with the live load when the re-action at the abutment a

$$= 5 \times \frac{11+9+7}{12} + 15 \times \frac{5+3+1}{12} = 22\tfrac{1}{2} \text{ tons.}$$

The maximum compression in Ff = the vertical component of the stress in the diagonal Df due to this condition of loading less the panel dead load at f.

The horizontal component of the stress in

$$\text{D}f = \frac{22 \cdot 5 \times 50 - 5 \times 20(1+2)}{19\ 72} - \frac{22 \cdot 5 \times 30 - 5 \times 20}{17 \cdot 5} = 8 \cdot 97 \text{ tons,}$$

the vertical component of which

$$=8{\cdot}97\times\frac{17{\cdot}5}{20}=7{\cdot}85 \text{ tons and } 7{\cdot}85 \text{ tons}-5 \text{ tons}=2{\cdot}85 \text{ tons,}$$

the compression in F*f*.

Simple Truss, No. 2.

The abutment re-action due to full load in this truss

$$=15\times\frac{6-1}{2}=37{\cdot}5 \text{ tons.}$$

The maximum compression in

$$\mathrm{AC}=\frac{37{\cdot}5\times 20}{15{\cdot}56}\times\frac{20{\cdot}75}{20}\qquad=50{\cdot}00 \text{ tons,}$$

$$\mathrm{CE}=\frac{37{\cdot}5\times 40-15\times 20}{18{\cdot}89}\times\frac{20{\cdot}27}{20}\qquad=64{\cdot}38 \text{ tons,}$$

$$\mathrm{EG}=\frac{37{\cdot}5\times 60-15\times 20(1+2)}{20}\times\frac{20{\cdot}03}{20}=67{\cdot}60 \text{ tons.}$$

The maximum tension in

$$ac=\text{nil}\qquad=00{\cdot}00 \text{ tons,}$$

$$ce=\frac{37{\cdot}5\times 20}{15{\cdot}56}\qquad=48{\cdot}20 \text{ tons,}$$

$$eg=\frac{37{\cdot}5\times 40-15\times 20}{18{\cdot}89}=63{\cdot}52 \text{ tons.}$$

The maximum tension in A*c* occurs when the truss is fully loaded and the abutment re-action at $a=37{\cdot}5$ tons. Then the tension in A*c* = the horizontal component of the stress in

$$\mathrm{AC}\times\frac{\mathrm{A}c}{ac}=\frac{37{\cdot}5\times 20}{15{\cdot}56}\times\frac{22{\cdot}36}{20}=53{\cdot}9 \text{ tons.}$$

The maximum tension in C*e* occurs when all the panel points except *c* are covered with the live load when re-action at

$$a=5\times\tfrac{5}{6}+15\times\frac{4+3+2+1}{6}=29{\cdot}13 \text{ tons.}$$

Then horizontal stress at $c = \frac{29{\cdot}13 \times 20}{15{\cdot}56} = 37{\cdot}44$ tons,

and the horizontal stress at $e = \frac{29{\cdot}13 \times 40 - 5 \times 20}{18{\cdot}89} = 56{\cdot}39$ tons.

Then the horizontal component of the tension in

$$\mathrm{C}e = 56{\cdot}39 - 37{\cdot}44 = 18{\cdot}95,$$

and the longitudinal tension $= 18{\cdot}95 \times \frac{25{\cdot}33}{20} = 24$ tons.

The maximum tension in Eg occurs when the panel points c', e', and g are loaded with the live load when re-action at

$$a = 5 \times \frac{5+4}{6} + 15 \times \frac{3+2+1}{6} = 22\tfrac{1}{2} \text{ tons.}$$

Then horizontal stress at

$$e = \frac{22{\cdot}5 \times 40 - 5 \times 20}{18{\cdot}89} = 42{\cdot}35 \text{ tons,}$$

and horizontal stress at

$$g = \frac{22{\cdot}5 \times 60 - 5 \times 20(1+2)}{20} = 52{\cdot}5 \text{ tons.}$$

Then the horizontal component of the tension in

$$\mathrm{E}g = 52{\cdot}5 - 42{\cdot}35 = 10{\cdot}15,$$

and the longitudinal tension

$$= 10{\cdot}15 \times \frac{27{\cdot}51}{20} = 13{\cdot}9 \text{ tons.}$$

The maximum tension in Ge' occurs when the panel points c' and e' only are loaded with the live load when re-action at

$$a = 5 \times \frac{5+4+3}{6} + 15 \times \frac{2+1}{6} = 17\tfrac{1}{2} \text{ tons.}$$

Then horizontal stress at

$$g = \frac{17{\cdot}5 \times 60 - 5 \times 20(1+2)}{20} = 37{\cdot}5 \text{ tons,}$$

and horizontal stress at

$$e' = \frac{17{\cdot}5 \times 80 - 5 \times 20(1+2+3)}{18{\cdot}89} = 42{\cdot}35 \text{ tons.}$$

Then the horizontal component of the tension in

$$\text{G}e' = 42{\cdot}35 - 37{\cdot}5 = 4{\cdot}85,$$

and the longitudinal tension

$$= 4{\cdot}85 \times \frac{28{\cdot}28}{20} = 6{\cdot}84 \text{ tons.}$$

The maximum compression in the end vertical Aa = the vertical re-action due to a full load $= 37\frac{1}{2}$ tons.

The maximum compression in Cc occurs when the live load extends from the right abutment to the panel point e when the re-action at the abutment a

$$= 5 \times \tfrac{5}{6} + 15 \times \frac{4+3+2+1}{6} = 29{\cdot}13 \text{ tons.}$$

The maximum compression in Cc = the vertical component of the stress in Ac due to this condition of loading less one panel dead load at c which produces tension in Cc.

The horizontal component of the stress in

$$\text{A}c = \frac{29{\cdot}13 \times 20}{15{\cdot}56} = 37{\cdot}44 \text{ tons,}$$

the vertical component of which

$$= 37{\cdot}44 \times \tfrac{10}{20} = 18{\cdot}72 \text{ tons,}$$

and 18·72 tons less 5 tons, the dead panel load,

$$= 13{\cdot}72 \text{ tons, the compression in C}c.$$

The maximum compression in Ee occurs when the live load extends from the right abutment to the panel point g when the re-action at the abutment a

$$= 5 \times \frac{5+4}{6} + 15 \times \frac{3+2+1}{6} = 22\tfrac{1}{2} \text{ tons.}$$

The maximum compression in Ee = the vertical component of the stress in the diagonal Ce due to this condition of loading less the panel dead load at e.

The horizontal component of the stress in

$$Ce = \frac{22{\cdot}5 \times 40 - 5 \times 20}{18{\cdot}89} - \frac{22{\cdot}5 \times 20}{15{\cdot}56} = 12{\cdot}43,$$

the vertical component of which

$$= 12{\cdot}43 \times \frac{15{\cdot}56}{20} = 9{\cdot}67 \text{ tons and } 9{\cdot}67 - 5 = 4{\cdot}67 \text{ tons},$$

the compression in Ee.

The maximum compression in Gg occurs when the live load extends from the right abutment to the panel point e' when the re-action at the abutment a

$$= 5 \times \frac{5+4+3}{6} + 15 \times \frac{2+1}{6} = 17\tfrac{1}{2} \text{ tons.}$$

The maximum compression in Gg = the vertical component of the stress in the diagonal Eg due to this condition of loading less the panel dead load at g.

The horizontal component of the stress in

$$Eg = \frac{17{\cdot}5 \times 60 - 5 \times 20(1+2)}{20} - \frac{17{\cdot}5 \times 40 - 5 \times 20}{18{\cdot}89} = 5{\cdot}74 \text{ tons},$$

the vertical component of which

$$= 5{\cdot}74 \times \frac{18{\cdot}89}{20} = 5{\cdot}4 \text{ tons and } 5{\cdot}4 - 5 \text{ tons} = {\cdot}4 \text{ tons},$$

the compression in Gg.

Having thus determined the stresses in the two component trusses Nos. 1 and 2, it only remains to add together the top and bottom flange stresses and the stresses in the end verticals to arrive at the maxima stresses in the combined truss, Fig. 112.

The maximum compression in the end vertical which has to transmit to the abutment the load on each truss = the vertical re-action due to a full load on each truss

$$= 45 \text{ tons} + 37\tfrac{1}{2} \text{ tons} = 82\tfrac{1}{2} \text{ tons.}$$

The upper chord member AB has to resist a compression of 36·1 tons in simple truss No. 1 + 50·00 tons in simple truss No. 2 = 86·1 tons.

Similarly, BC = 61·5 tons + 50·00 tons = 111·5 tons,
CD = 64·38 tons + 61·5 tons = 125·8 tons,
DE = 64·38 tons + 68·86 tons = 133·24 tons,
EF = 68·86 tons + 67·60 tons = 136·46 tons,
FG = 67·60 tons + 68·45 tons = 136·05 tons.

In the lower chord the stresses are as under:

ab = 00·00,
bc = 34·45 tons,
cd = 34·45 tons + 48·20 tons = 82·65 tons,
de = 48·20 tons + 60·00 tons = 108·20 tons,
ef = 60·00 tons + 63·52 tons = 123·52 tons,
fg = 63·52 tons + 68·45 tons = 131·95 tons.

The stresses in this truss may also be determined by a graphic process. The chord stresses may be obtained by scale from the diagram in Fig. 115 in which the various members are lettered to correspond with those to which they are drawn parallel in Figs. 113 and 114.

The upper half of the diagram gives the stresses in simple truss No 1, Figs. 110 and 113, and the lower half the stresses in simple truss No. 2, Figs. 111 and 114.

Fig. 109.

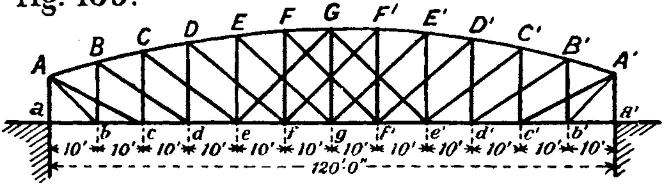
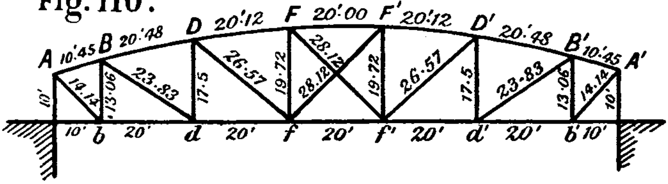
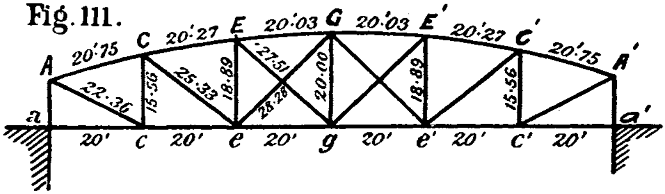


Fig. 110.



SIMPLE TRUSS, NO. 1.

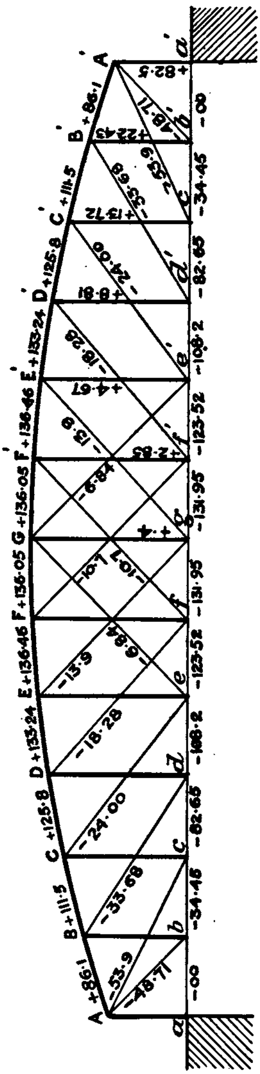
Fig. 111.



SIMPLE TRUSS, NO. 2.

NOTE. The figures in Figs 110 and 111 represent the lengths of the various members.

Fig. 112.



Dead or static load = $\frac{1}{2}$ ton per foot run = 5 tons per panel.
Live or rolling load = 1 ton per foot run = 10 tons per panel.

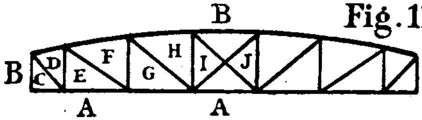


Fig. 113.

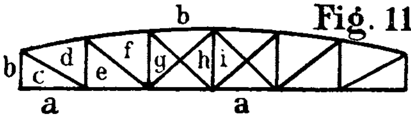


Fig. 114.

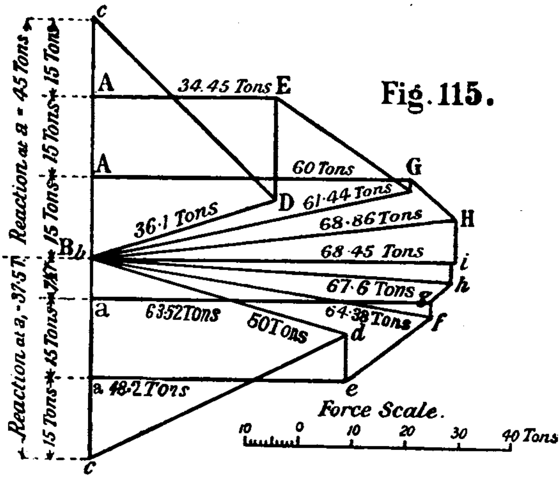


Fig. 115.

Schwedler Truss.

The Schwedler Truss which is so called after Herr Baurath Schwedler, an eminent German engineer who introduced this principle of construction on the Prussian State Railways, is an intermediate form of girder between a parabolic or bowstring girder and a girder with parallel chords. The lower chord, as in a bowstring girder, is horizontal, and the upper chord is curved in such a manner that no web diagonal can under any condition of loading be strained in compression, and also that the number of diagonal members required is reduced to a minimum.

The depth of the truss at the centre is arbitrary and may within usual limits vary from $\frac{1}{8}$th to $\frac{1}{10}$th of the span. The depth at any intermediate point between the centre and the point of support when the centre depth is given may be determined by the following formula,

$$y = \frac{dw}{l}\left[\sqrt{1 + \frac{W_1}{W}} + 1\right]^2 \frac{x(l - x)}{Wl + W_1 x},$$

in which y = depth at any point distant x from end of girder,

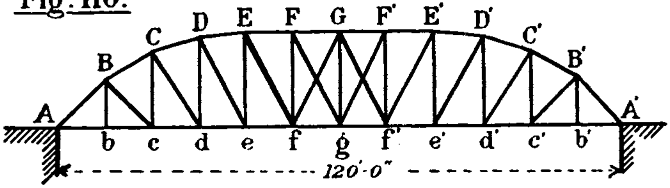
l = span in feet,

d = depth of truss at centre,

W = dead load per foot run,

W_1 = live or moving load per foot run.

Fig. 116.



Applying this formula to the example illustrated in Fig. 116, in which the span is 120 feet, the depth at the centre 20 feet, and the dead or static load is $\frac{1}{2}$ ton per foot run, and the live or moving load 1 ton per foot run, the results are as under, viz.:

$x = 10$ feet, $y = 9{\cdot}75$ feet = Bb.

$x = 20$ feet, $y = 15{\cdot}52$ feet = Cc.

$x = 30$ feet, $y = 18 \cdot 60$ feet $=$ Dd.

$x = 40$ feet, $y = 19 \cdot 87$ feet $=$ Ee, Ff and G$g = 20$ feet.

When the depths at the various panel points have in this way been determined, the stresses in the various members may be ascertained in precisely the same way as for a bow-string girder in accordance with the preceding examples.

Polygonal Truss of 60 feet span.

Fig. 117.

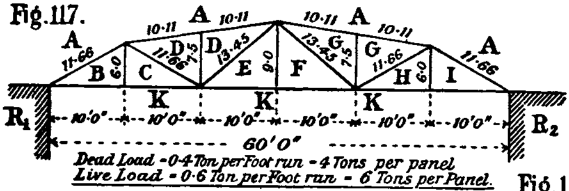


Fig. 118.

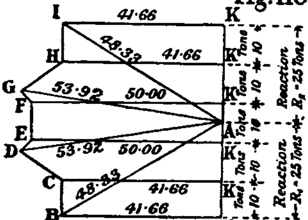


Diagram giving Maxima Stresses in Top and Bottom chords.

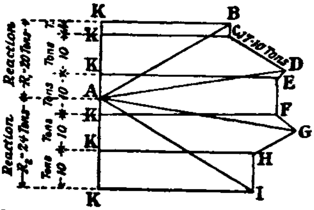


Fig. 119. Diagram giving Maximum Tension in CD.

Fig. 120.

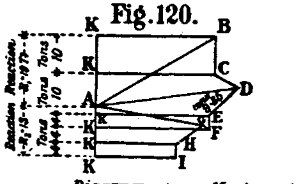


Diagram giving Maximum Tension in DE.

Fig. 121.

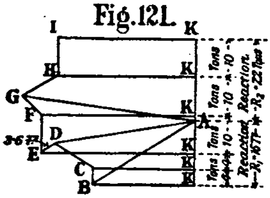


Diagram giving Maximum Compression in DE.

Force Scale of Diagrams = 50 tons to an inch.

Stress Diagram.

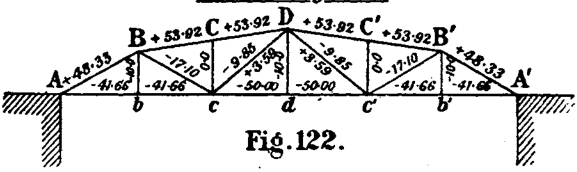


Fig. 122.

This girder has been selected as an example from a bridge built over the Bindjey River in Sumatra by the Harkort Company of Duisburg, an eminent firm of German bridge builders.

The stresses in this truss may be checked by an analytical process, as follows:

The maxima stresses occur in the top and bottom booms when the girder is fully loaded, when the re-action at each abutment

$$=\frac{(4 \text{ tons} + 6 \text{ tons})5}{2} = 25 \text{ tons.}$$

The maximum tension in

$$Ab = bc = \frac{25 \times 10}{6} = 41{\cdot}66 \text{ tons,}$$

and in $$cd = \frac{25 \times 30 - 10 \times 10(1+2)}{9} = 50{\cdot}00 \text{ tons.}$$

The maximum compression in

$$AB = \frac{25 \times 10}{6} \times \frac{11{\cdot}6}{10} = 48{\cdot}33 \text{ tons,}$$

and in $$BC = CD = \frac{25 \times 20 - 10 \times 10}{7{\cdot}5} \times \frac{10{\cdot}11}{10} = 53{\cdot}92 \text{ tons.}$$

The maximum tension in

$$Bb = 4 \text{ tons} + 6 \text{ tons} = 10{\cdot}00 \text{ tons.}$$

The maximum tension in Bc occurs when the live load extends from the right abutment at A_1 to the panel point c, when the re-action at

$$A = 4 \times 2\tfrac{1}{2} + 6 \times \frac{1+2+3+4}{6} = 20 \text{ tons.}$$

Then the horizontal stress at

$$b = \frac{20 \times 10}{6} = 33\tfrac{1}{3} \text{ tons,}$$

and the horizontal stress at

$$c = \frac{20 \times 20 - 4 \times 10}{7 \cdot 5} = 48 \text{ tons.}$$

Then horizontal component of stress in

$$\text{B}c = 48 - 33\tfrac{1}{3} = 14\tfrac{2}{3} \text{ tons,}$$

and the longitudinal tension in

$$\text{B}c = 14\tfrac{2}{3} \times \frac{11 \cdot 66}{10} = 17 \cdot 10 \text{ tons.}$$

There is no compression under any condition of loading in Bc.

There is no stress in Cc.

The maximum tension in D$d = 4 + 6$ tons $= 10 \cdot 00$ tons.

The maximum tension in cD occurs when the panel points b and c are covered by the live load, when the re-action at

$$\text{A} = 4 \times 2\tfrac{1}{2} + 6 \times \frac{5 + 4}{6} = 19 \text{ tons.}$$

Then horizontal stress at

$$c = \frac{19 \times 20 - 10 \times 10}{7 \cdot 5} = 37 \cdot 33 \text{ tons,}$$

and horizontal stress at

$$d = \frac{19 \times 30 - 10 \times 10(1 + 2)}{9} = 30 \cdot 00 \text{ tons.}$$

The horizontal component of the tension in

$$c\text{D} = 37 \cdot 33 - 30 \cdot 00 = 7 \cdot 33 \text{ tons,}$$

and the longitudinal stress in

$$c\text{D} = 7 \cdot 33 \times \frac{13 \cdot 45}{10} = 9 \cdot 85 \text{ tons.}$$

The maximum compression in cD occurs when the live load extending from the abutment on the right-hand side at A_1 covers the panel point d, when re-action at

$$\text{A} = 4 \times 2\tfrac{1}{2} + 6 \times \frac{1 + 2 + 3}{6} = 16 \text{ tons.}$$

Then horizontal stress at

$$c = \frac{16 \times 20 - 4 \times 10}{7 \cdot 5} = 37\tfrac{1}{3} \text{ tons,}$$

and horizontal stress at

$$d = \frac{16 \times 30 - 4 \times 10(1+2)}{9} = 40 \text{ tons.}$$

The horizontal component of the compression in

$$c\text{D} = 40 - 37\tfrac{1}{3} = 2\tfrac{2}{3} \text{ tons,}$$

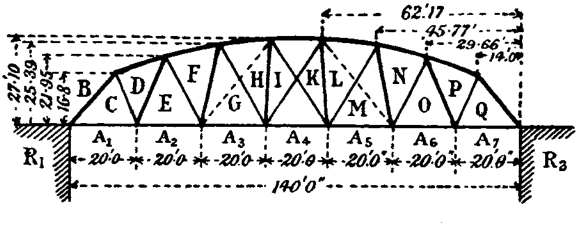
and the axial or longitudinal stress

$$= 2\tfrac{2}{3} \times \frac{13 \cdot 45}{10} = 3 \cdot 59 \text{ tons.}$$

Pegram Truss (Fig. 123).

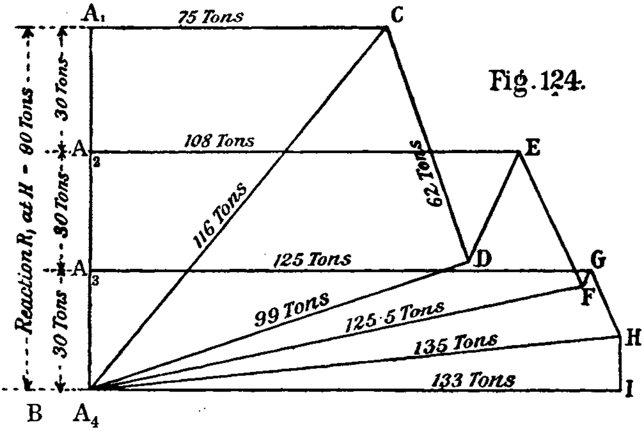
In this truss, which is regarded as an economical type of girder, the panel points of the upper boom lie in the arc of a circle, the length of the chord of which is about one and one half panel lengths shorter than the lower boom. The web members are inclined at varying angles and so arranged that the lengths of the posts or compression members are nearly equal.

Fig. 123.



Dead load = 0·5 ton per foot run = 10 tons per panel.
Live load = 1·0 ton per foot run = 20 tons per panel.

Diagram of Maxima Stresses in Top and Bottom Booms.



The diagram for one half of the truss only is given. The diagram for the other half will be similar.

The stresses may be checked analytically in some of the members; thus the stress in

$$A_3G = \frac{90 \times 45{\cdot}77 - (30 \times 5{\cdot}77 + 30 \times 25{\cdot}77)}{25{\cdot}39} = 124{\cdot}9 \text{ tons.}$$

Again, the stress in

$$A_4I = BK = \frac{90 \times 62{\cdot}17 - (30 \times 2{\cdot}17 + 30 \times 22{\cdot}17 + 30 \times 42{\cdot}17)}{27{\cdot}10}$$
$$= 132{\cdot}4 \text{ tons.}$$

The results obtained graphically are respectively 125 tons and 133 tons, which are in very close agreement with the computations.

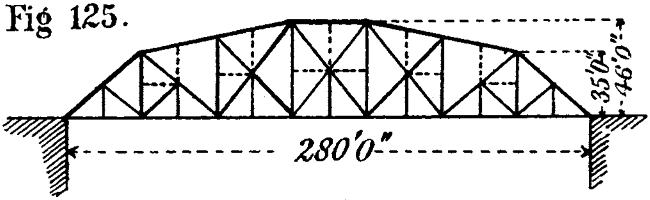
The stresses in the web members may be determined graphically from a series of diagrams drawn for the condition of loading producing the maxima stresses in the several members, or analytically by the same general rules as have been employed for obtaining the stresses in bowstring girders.

The Petit Truss.

The Petit Truss is rapidly growing into favour in the United States of America, and for long spans it has practically become the standard type of girder. In outline it resembles the Baltimore Truss, with the exception that the top chord is inclined. The usual form of Petit Truss is shown in Fig. 125.

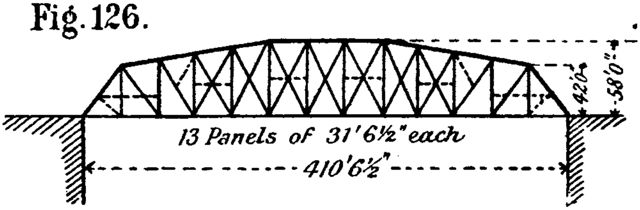
A similar type of truss has been adopted in the Hawkesbury Bridge, New South Wales. It is shown in Fig. 126.

Fig 125.



In both trusses the members shown in dotted lines serve merely as stiffeners, and form no part of the structure for the purpose of calculating the stresses.

Fig. 126.



The Hawkesbury River Bridge consists of seven spans of 416 feet each from centre to centre of piers. The girders are 58 feet deep at centre and 42 feet at end verticals. It was built by the Union Bridge Company of New York.

The stresses in both of these trusses can be determined, either graphically or analytically, by the general methods that have already been given.

CHAPTER V.

Cantilever Bridges.

Bridges built on the cantilever principle appear to have been in use amongst the inhabitants of Kashmir, Tibet and the countries bordering on the Himalayas from time im-

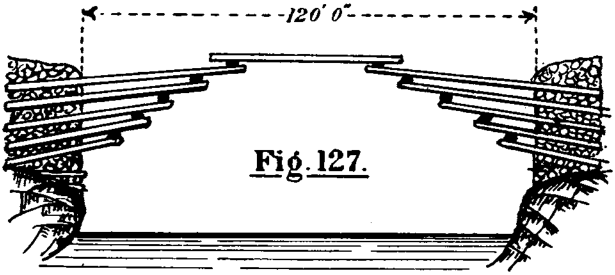


Fig. 127.

memorial. In the professional papers of the Corps of Royal Engineers, an interesting account is given of these bridges in connection with the "Bridging Operations with the Chitral Relief Force." The type of construction, which is practically the same for small and large spans, the limiting dimensions of which are given as 120 feet, is exceedingly simple.

A suitable site is selected where the banks of the river (preferably composed of rock) stand well above the level of the water, and beams are corbelled out over the river or stream, the overhanging or cantilever end of each one projecting beyond the end of the beam upon which it rests, the extent of the projection being regulated by the span. The shore ends are enveloped in stone packing by which they are held in position and counterpoised. The opening between the overhanging ends is spanned by a beam or girder as shown in Fig. 127.

In the light of modern practice a cantilever bridge broadly speaking can only be applied with advantage where the span is too great to be bridged over by a simple girder.

Probably the simplest example of a cantilever is that shown in Fig. 128 which represents a pier for shipping iron ore on the coast of Spain, the particulars of which are taken from the *Annales des Travaux Publics de Belgique*, Deuxième série, Tome V., 4e Fascicule.

The conditions of loading are given at the foot of Fig. 129, and the stresses are determined graphically by means of the diagram in Fig. 130 collected together and tabulated as on p. 153. In the upper part of the diagram, Fig. 130, the stresses are given due to the two conditions of loading indicated in Fig. 129, from which the maximum stress in each member is taken and entered in the tabulated statement.

Under both conditions of loading it becomes necessary to anchor down the shore end, and the pull on the anchor-rod or the weight of the counterpoise W will be as under:

1st case of loading:

$$(W + 4{\cdot}4)\,62 + 8{\cdot}8 \times 13\,(1 + 2 + 3 + 4)$$
$$= 7{\cdot}9 \times 91 + 6{\cdot}1 \times 78 + 3{\cdot}4 \times 65 + 3{\cdot}4 \times 52 + 6{\cdot}1 \times 39 + 8{\cdot}8 \times 26 + 8{\cdot}8 \times 13$$
$$= W \times 62 + 1416{\cdot}8 = 2172{\cdot}7.$$

$$\text{Then } W = 12{\cdot}19 \text{ tons } = \frac{2172{\cdot}7 - 1416{\cdot}8}{62}.$$

2nd case of loading:

$$(W + 1{\cdot}7)62 + 3{\cdot}4 \times 13(1 + 2 + 3 + 4) = 2172{\cdot}7 \text{ as above,}$$

$$W \times 62 + 547{\cdot}4 = 2172{\cdot}7.$$

$$\text{Then } W = 26{\cdot}21 \text{ tons} = \frac{2172{\cdot}7 - 547{\cdot}4}{62}.$$

Tabulated Statement of Maxima Stresses.

Member.	Stress in Tons.	Member.	Stress in Tons.
AB	13·00	LM	24·50
BZ	10·50	AM	89·00
BC	6·50	MN	40·00
AC	10·50	NZ	110·50
CD	19·30	NO	33·00
DZ	24·50	A_1O_1	103·00
DE	11·00	O_1P	18·50
AE	25·00	PZ	99·50
EF	20·00	PQ	30·00
FZ	38·60	A_1Q_1	91·00
FG	12·50	QR	15·00
AG	39·00	R_1Z	87·00
GH	21·30	RS	28·50
HZ	52·50	A_1S_1	74·00
HI	14·30	S_1T_1	12·00
AI	53·00	T_1Z	72·00
IJ	26·00	T_1U_1	31·00
JZ	69·30	A_1U_1	48·00
JK	18·00	U_1V_1	15·30
AK	69·80	V_1Z	47·00
KL	33·30	A_1V_1	54·00
LZ	89·00		

Cantilever Pier for tipping Iron-ore at Los Capuchines, near Pasajes.

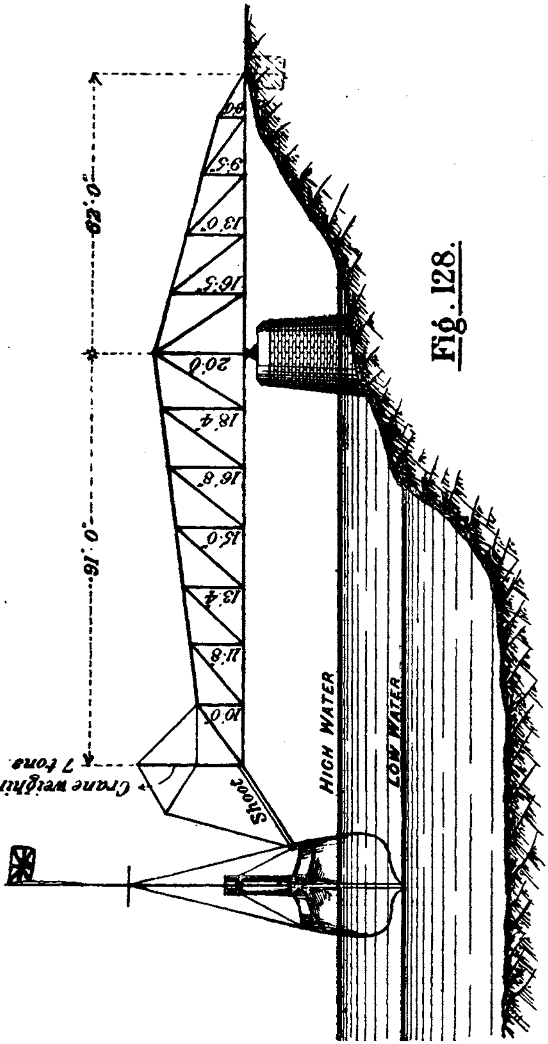


Fig. 128.

Dead Load per panel = 3·4 tons. Live load per panel = 5·4 tons.

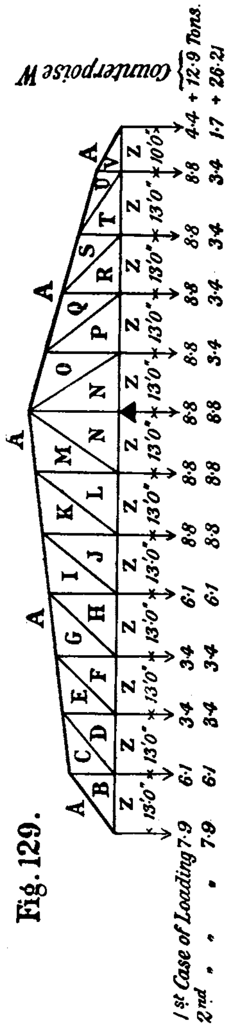
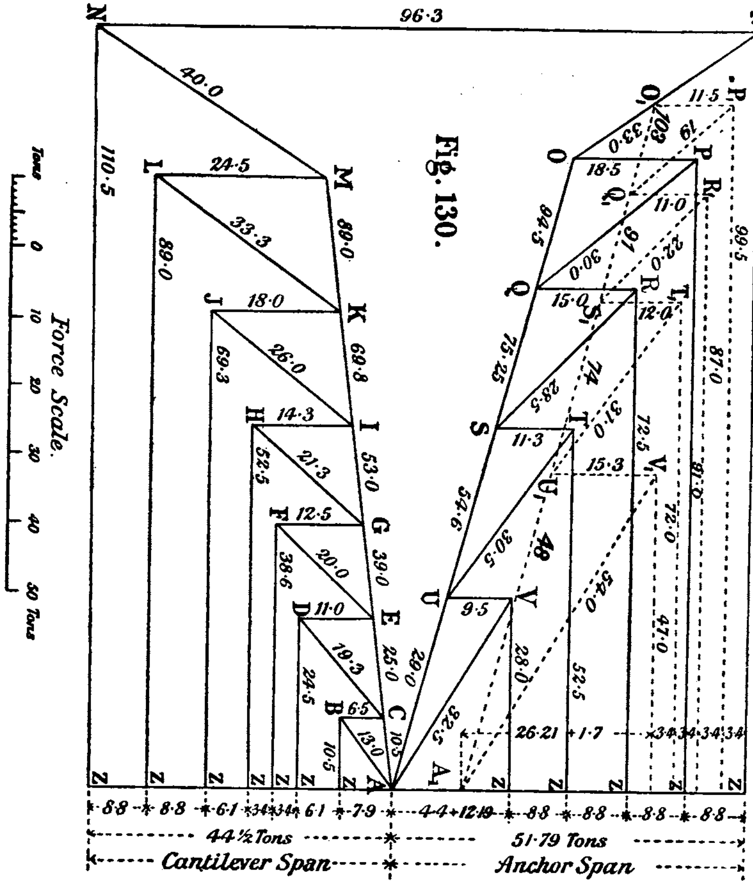

Counterpoise W
1st Case of Loading 7·9 6·1 3·4 3·4 6·1 8·8 8·8 8·8 8·8 8·8 8·8 8·8 8·8 4·4 + 12·9 Tons.
2nd " " 7·9 6·1 3·4 3·4 6·1 8·8 8·8 8·8 3·4 3·4 3·4 3·4 1·7 + 26·21
13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 13′0″ 10′0″

Fig. 129.



NOTE. The upper half of the stress diagram shows the two conditions of loading, the second condition of loading being represented by the dotted lines.

In Figs. 131 and 132 are shown two distinct types of cantilever bridges. In the bridge over the River Warthe there are two simple bowstring trusses at the ends and a centre

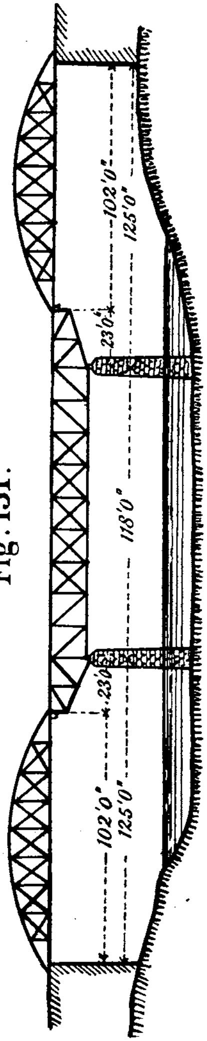


Fig. 131.

Cantilever Bridge at Posen over the River Warthe.

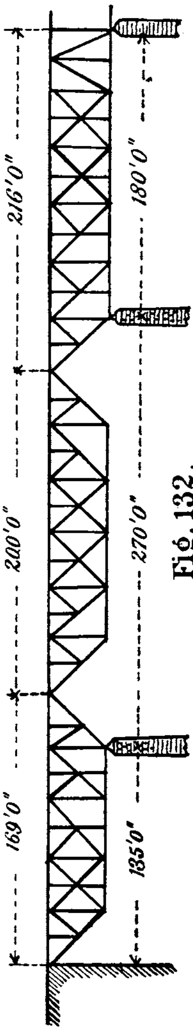


Fig. 132.

Cantilever Bridge at Fort Snelling over the Mississippi River.

truss upon two supports with projecting or cantilever arms. In the bridge over the Mississippi River there are two side trusses with cantilever arms supporting a simple centre truss.

In Figs. 133 to 136 is shown a type of cantilever truss that may be employed as a foot bridge or for purposes of light road traffic. The compression members being straight, may consist either of timber or iron or steel. Figs. 137, 138 and 139 show the stresses due to the several conditions of loadings.

Cantilever Truss.

Static or dead load $= \frac{1}{2}$ ton per foot run $=$ 6 tons per panel.
Live or rolling load $=$ 1 ton per foot run $=$ 12 tons per panel.

Fig. 133.

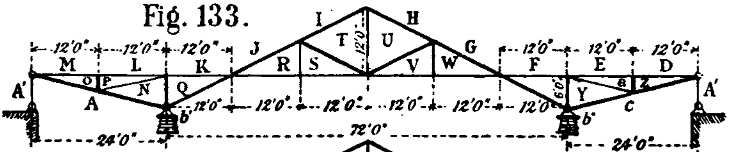


Fig. 134.

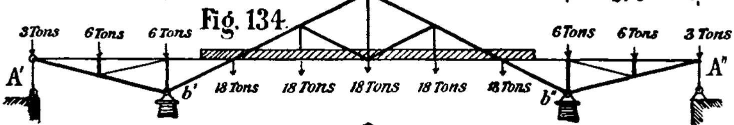


Fig. 135.

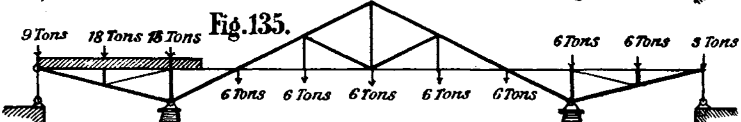
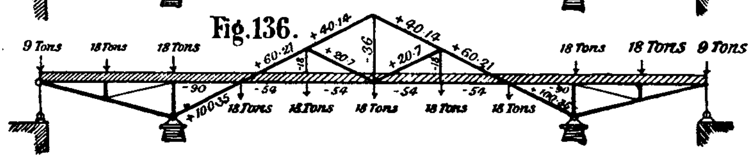


Fig. 136.



Stress Diagrams.

Force Scale, 40 tons to an inch.

Diagram of stresses due to the condition of loading illustrated in Fig. 134.

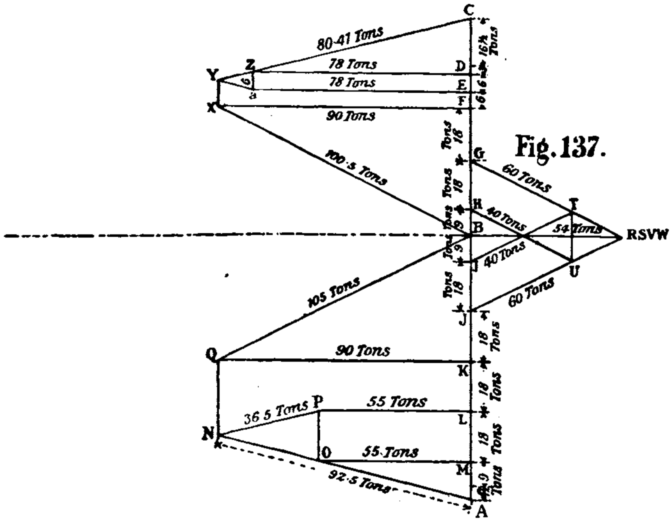


Fig. 137.

Diagram of stresses due to full load as shown in Fig 136.

This type of truss may be considered as composed of two independent systems, the one consisting of the central truss, hinged at the points of support on the piers, and the other of the two side trusses.

The maxima stresses in the various members occur under different conditions of loading, each of which must be considered. In the main or central truss the maxima stresses occur when the truss is fully loaded.

The stress in RS = VW = 1 panel load = 18 tons,

$$ST = UV = \tfrac{1}{2} \text{ panel load} = \tfrac{18}{2} \times \frac{ST}{SB} = 9 \times 2{\cdot}23$$
$$= 20{\cdot}7 \text{ tons,}$$
$$TU = 2 \text{ panel loads} = 18 \times 2 = 36 \text{ tons,}$$
$$HU = IT = \tfrac{36}{2} \times 2{\cdot}23 = 40{\cdot}14 \text{ tons,}$$
$$GW = JR = 18 \times 1\tfrac{1}{2} \times 2{\cdot}23 = 60{\cdot}21 \text{ tons,}$$
$$RB = SB = 18 \times 1\tfrac{1}{2} \times \frac{RB}{RS} = 27 \times \tfrac{12}{6} = 54 \text{ tons,}$$
$$QB = 18 \times 2\tfrac{1}{2} \times \frac{QB}{QN} = 45 \times 2{\cdot}23 = 100{\cdot}35 \text{ tons,}$$
$$KQ = 18 \times 2\tfrac{1}{2} \times \frac{KQ}{NQ} = 45 \times 2 = 90 \text{ tons.}$$

The maximum stress in AN = the horizontal component of the thrust in

$$BQ \times \frac{AN}{LP} = 90 \times \frac{12{\cdot}37}{12} = 92{\cdot}77 \text{ tons.}$$

The stress in MO = LP when the central truss only is loaded = the stress in KQ less the load transmitted by the diagonal PN = the horizontal component of one half panel dead load.

Then stress in $MO = 90 - \dfrac{3 \times 12}{3} = 78$ tons tension.

The stress in

$$AO = \text{the stress in } MO \times \frac{AO}{MO} = 78 \times \frac{12{\cdot}37}{12} = 80{\cdot}41 \text{ tons.}$$

When the centre truss only is loaded, the tension on the horizontal members MO and LP exerts an upward pull on the anchor rods A' and A'' which must be determined by equating the bending moments on either side of the hinge b'.

The re-action due to the loads to the right of

$$b' = 18 \times 2\tfrac{1}{2} = 45 \text{ tons.}$$

Then $$45 \times 12 = 6 \times 12 + 3 \times 24 + x \times 24$$
$$= 540 = 72 + 72 + 24x$$
$$= 540 = 144 + 24x.$$
$$24x = 540 - 144 = 396.$$

Then $x = 396 \div 24 = 16\frac{1}{2}$ tons,
the weight that must be attached to the anchor rod A′ to keep the system in equilibrium.

It is necessary next to consider the stresses when the side truss only is loaded as in Fig. 135. The compression on the anchor rod A is found by equating the bending moments on either side of the hinge b'.

The re-action due to the loads to the right of

$$b' = 6 \times 2\frac{1}{2} = 15 \text{ tons.}$$

Then $$15 \times 12 + x \times 24 = 18 \times 12 + 9 \times 24,$$

$$180 + 24x = 216 + 216 = 432,$$

$$24x = 432 - 180 = 252,$$

and $$x = 252 \div 24 = 10\frac{1}{2} \text{ tons compression.}$$

The stress in

$$\text{MO} = \text{LP} = \text{W}_1 \times \frac{\text{MO}}{\text{OP}} - \text{W}_4 \times \frac{\text{KQ}}{\text{NQ}} = \frac{9 \times 12}{3} - \frac{15 \times 12}{6} = 36 - 30$$

$$= 6 \text{ tons.}$$

The tension in

$$\text{KQ} = \frac{\text{KQ} \times \text{W}_4}{\text{NQ}} = \frac{12 \times 15}{6} = 30 \text{ tons.}$$

The compression in

$$\text{BQ} = 30 \text{ tons} \times \frac{\text{BQ}}{\text{KQ}} = 30 \times \frac{2{\cdot}23}{2} = 33{\cdot}45 \text{ tons.}$$

The tension in

$$\text{NP} = \frac{\text{W}_2}{2} \times \frac{\text{NP}}{\text{OP}} = 9 \times \frac{12{\cdot}32}{3} = 36{\cdot}96 \text{ tons.}$$

The tension in AO = the inclined component of the compressive stress in

$$\text{MO} = 6 \times \frac{\text{AO}}{\text{MO}} = 6 \times \frac{12{\cdot}32}{2} = 6{\cdot}16 \text{ tons.}$$

The compression in AN = the horizontal component of the stress in

$$\text{BQ} \times \frac{\text{AN}}{\text{LP}} = 30 \times \frac{12{\cdot}32}{12} = 30{\cdot}80 \text{ tons.}$$

The compression in OP = 18 tons and in NQ = 27 tons.

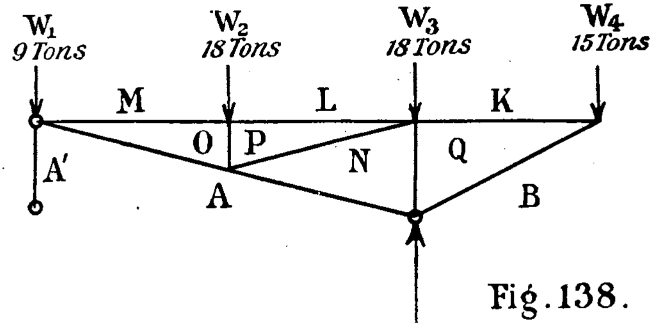


Fig. 138.

Diagram of stresses due to the condition of loading illustrated in Fig. 135 and Fig. 138.

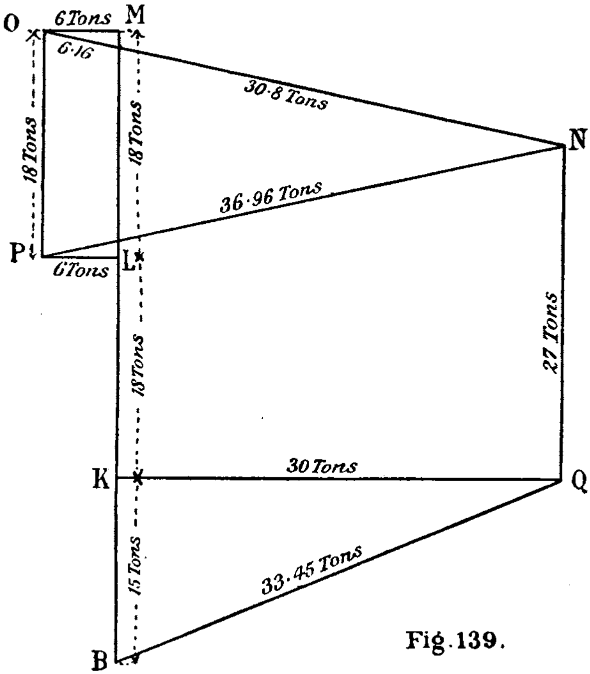


Fig. 139.

Figs. 140 and 141 represent the outline of a truss which is probably the simplest and most effective application of the cantilever principle to bridging over a wide span.

It is the principle that has been adopted in a modified form in the construction of some of the principal cantilever bridges, including the Forth Bridge by Sir Benjamin Baker and the Niagara and Fraser River cantilever bridges in America.

The stresses are worked out analytically and graphically as shown in the following diagrams.

Cantilever Truss of three Spans.

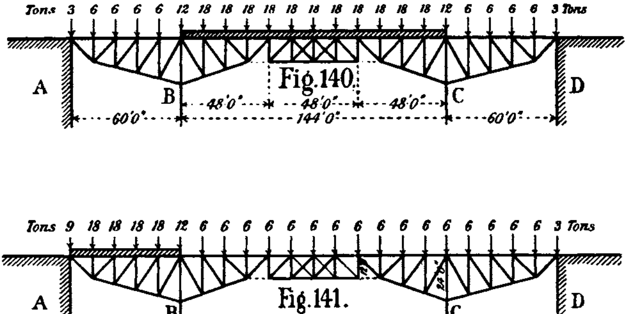


Fig. 140.

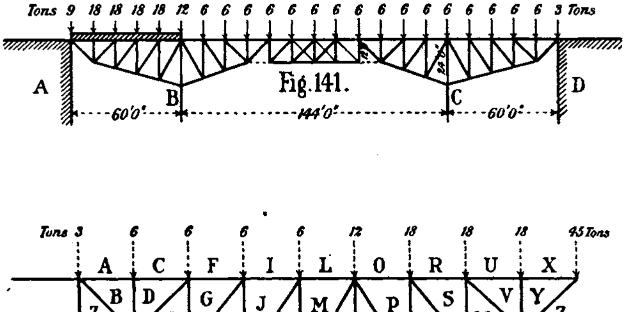


Fig. 141.

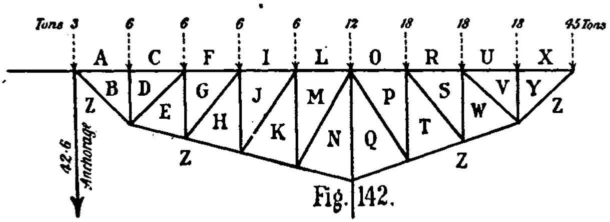


Fig. 142.

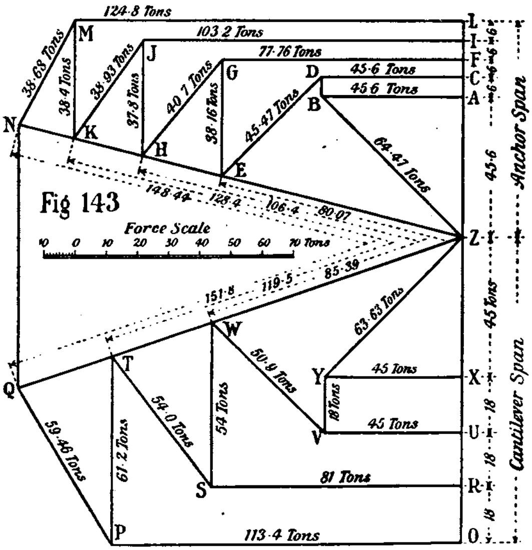
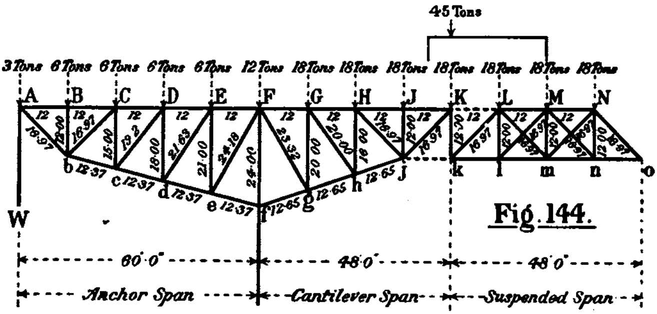


Diagram of stresses due to the condition of loading indicated in Figs. 140 and 142.

Cantilever Span.



Dead or static load is assumed to be $\frac{1}{2}$ ton per foot run
$= 6$ tons per panel.

Live or moving load is assumed to be 1 ton per foot run
$= 12$ tons per panel.

Maximum dead and live load per panel $= 6 + 12$
$= 18$ tons per panel.

Cantilever Span. Full load gives maxima stresses in all members of this span, as under:

Maximum stress in $j\text{K} = 45 \times 1{\cdot}414 = 63{\cdot}63$ tons,

$$\text{J}j = 18 \text{ tons} \quad = 18{\cdot}00 \text{ tons},$$

$$\text{HJ} = \text{JK} = 45 \times \tfrac{12}{12} \quad = 45{\cdot}00 \text{ tons}.$$

Maximum stress in $\text{H}j$ = the difference between the horizontal components of the stresses at h and $j \times 1{\cdot}414$.

$$\left.\begin{array}{l} \text{Stress at } j = 45 \times \tfrac{12}{12} \qquad = 45 \text{ tons} \\ \text{Stress at } h = \dfrac{45 \times 24 + 18 \times 12}{16} = 81 \text{ tons} \end{array}\right\} \left\{\begin{array}{l} \text{Stress in H}j \\ = (81 - 45)\,1{\cdot}414 \end{array}\right\}$$

$$= 50{\cdot}90 \text{ tons}.$$

Maximum stress in $\text{H}h$ = vertical component of stress in $\text{H}j$ + the panel load at $\text{H} = 36 + 18 = 54{\cdot}00$ tons.

Maximum stress in $\text{G}h$ = the difference between the horizontal components of the stresses at g and $h \times \frac{20}{12}$.

$$\left.\begin{array}{l} \text{Stress at } h \text{ as above determined} \\ \qquad\qquad = 81 \text{ tons} \\ \text{Stress at } g \\ = \dfrac{45 \times 36 + 18 \times 12(1+2)}{20} = 113{\cdot}4 \text{ tons} \end{array}\right\} \left\{\begin{array}{l} \text{Stress in G}h \\ = (113{\cdot}4 - 81)\tfrac{20}{12} \end{array}\right\}$$

$$= 54{\cdot}00 \text{ tons}.$$

Maximum stress in $\text{G}g$ = the vertical component of the stress in $\text{G}h$ + the panel load at $\text{G} = 32{\cdot}4 \times \frac{16}{12} + 18 = 43{\cdot}2 + 18$
$= 61{\cdot}20$ tons.

Maximum stress in $\text{F}g$ = the difference between the horizontal components of the stresses at f and $g \times \dfrac{23{\cdot}32}{12}$.

Stress at g as above determined

$$= 113{\cdot}4 \text{ tons}$$

Stress at f

$$= \frac{45 \times 48 + 18 \times 12(1+2+3)}{24} = 144 \text{ tons}$$

Stress in Fg

$$= (144 - 113{\cdot}4)\frac{23{\cdot}32}{12}$$

$$\doteq 59{\cdot}46 \text{ tons.}$$

Maximum stress in

$$\text{GH} = \frac{45 \times 24 + 18 \times 12}{16} = 81{\cdot}00 \text{ tons,}$$

$$\text{FG} = \frac{45 \times 36 + 18 \times 12(1+2)}{20} = 113{\cdot}40 \text{ tons,}$$

$$hj = \frac{45 \times 24 + 18 \times 12}{16} \times \frac{12{\cdot}65}{12} = 85{\cdot}39 \text{ tons,}$$

$$gh = \frac{45 \times 36 + 18 \times 12(1+2)}{20} \times \frac{12{\cdot}65}{12} = 119{\cdot}50 \text{ tons,}$$

$$fg = \frac{45 \times 48 + 18 \times 12(1+2+3)}{24} \times \frac{12{\cdot}65}{12} = 151{\cdot}80 \text{ tons.}$$

Under the conditions of loading indicated in Fig. 5, a counterweight W must be attached to the end of the anchor span at A to balance the negative re-action caused by the live load on the cantilever and suspended spans. The magnitude of this counterweight is obtained by equating the moments on either side about the point of support f, thus: Let x = the total weight acting downwards at the point A including the half-panel dead load = 3 tons.

Then

$$\overset{\text{tons}}{45} \times \overset{\text{ft.}}{48} + \overset{\text{tons}}{18} \times \overset{\text{ft.}}{12}(1+2+3) = \overset{\text{ft.}}{60} \times x + \overset{\text{tons}}{6} \times \overset{\text{ft.}}{12}(1+2+3+4)$$

$$= 2160 + 1296 = \overset{\text{ft.-tons}}{3456} = 60x + \overset{\text{ft.-tons}}{720} \text{ foot-tons}$$

$$= 3456 - 720 = 2736 = 60x, \text{ and}$$

$$2736 \div 60 = x = 45{\cdot}6 \text{ tons; } \therefore \text{W} = 45{\cdot}6 \text{ tons} - 3 \text{ tons} = 42{\cdot}6 \text{ tons.}$$

Anchor Span. The condition of loading indicated in Fig. 144 the maxima stresses in the booms of the anchor span which are as under:

Maximum stress in

$$Ab = 45{\cdot}6 \text{ tons} \times 1{\cdot}414 \qquad = 64{\cdot}47 \text{ tons,}$$

$$AB = BC = 45{\cdot}6 \times \tfrac{12}{12} \qquad = 45{\cdot}6 \text{ tons,}$$

$$CD = \frac{45{\cdot}6 \times 24 + 6 \times 12}{15} \qquad = 77{\cdot}76 \text{ tons,}$$

$$DE = \frac{45{\cdot}6 \times 36 + 6 \times 12(1+2)}{18} \qquad = 103{\cdot}20 \text{ tons,}$$

$$EF = \frac{45{\cdot}6 \times 48 + 6 \times 12(1+2+3)}{21} \qquad = 124{\cdot}80 \text{ tons,}$$

$$bc = \frac{45{\cdot}6 \times 24 + 6 \times 12}{15} \times \frac{12{\cdot}37}{12} \qquad = 80{\cdot}07 \text{ tons,}$$

$$cd = \frac{45{\cdot}6 \times 36 + 6 \times 12(1+2)}{18} \times \frac{12{\cdot}37}{12} \qquad = 106{\cdot}40 \text{ tons,}$$

$$de = \frac{45{\cdot}6 \times 48 + 6 \times 12(1+2+3)}{21} \times \frac{12{\cdot}37}{12} \qquad = 128{\cdot}40 \text{ tons,}$$

$$ef = \frac{45{\cdot}6 \times 60 + 6 \times 12(1+2+3+4)}{24} \times \frac{12{\cdot}37}{12} = 148{\cdot}44 \text{ tons.}$$

The maxima stresses in the braces of the anchor span occur when the cantilever and suspended spans are fully loaded and the panel points, Fig. 144, on the left or abutment side of the brace considered are also loaded with the full load, the remaining panel points up to the support or pier f being unloaded. Thus the maximum stress in the diagonal bc occurs when the panel points A and B are fully loaded and CD and E unloaded.

Then the re-action at

$$A = W = \overset{\text{tons}}{45} \times \overset{\text{ft.}}{48} + \overset{\text{tons}}{18} \times \overset{\text{ft.}}{12}(1+2+3) = \overset{\text{tons}}{60}\overset{\text{ft.}}{x} + 6 \times 12(1+2+3) + 18 \times 48$$

$$= 3456 \text{ tons} = 60x + 1296 \text{ tons,}$$

$$3456 - 1296 \text{ foot-tons} = 60x,$$

and $$2160 \div 60 = x = 36 \text{ tons.}$$

Maximum stress in bC = difference between the horizontal components of the stresses at b and $c \times 1{\cdot}414$.

$$\left.\begin{array}{l}\text{Stress at } b = 36 \times \frac{12}{12} \qquad = 36 \\ \text{Stress at } c = \dfrac{36 \times 24 + 18 \times 12}{15} = 72\end{array}\right\}\left\{\begin{array}{l}\text{Stress in } b\text{C} \\ = (72 - 36)1{\cdot}414\end{array}\right\}$$

$$= 50{\cdot}9 \text{ tons.}$$

Maximum stress in Bb = 18·0 tons.

The maximum stress in the diagonal cD occurs when the panel points A, B, and C are fully loaded and DE unloaded.

Then the re-action at

$$\text{A} = \text{W} = \overset{\text{tons}}{45} \times \overset{\text{ft.}}{48} + \overset{\text{tons}}{18} \times 12(1+2+3) = \overset{\text{tons}}{60x} + 6 \times 12(1+2) + 18 \times 12(4+3)$$

$$= 3456 \text{ tons} = 60x + 1728,$$

$$3456 - 1728 \text{ foot-tons} = 60x,$$

and $$1728 \div 60 = x = 28{\cdot}8 \text{ tons.}$$

Maximum stress in cD = difference between the horizontal components of the stresses at c and $d \times \dfrac{19{\cdot}2}{12}$.

$$\left.\begin{array}{l}\text{Stress at } c = \dfrac{28{\cdot}8 \times 24 + 18 \times 12}{15} = 60{\cdot}5 \\ \text{Stress at } d \\ \quad = \dfrac{28{\cdot}8 \times 36 + 18 \times 12(1+2)}{18} = 93{\cdot}8\end{array}\right\}\left\{\begin{array}{l}\text{Stress in } c\text{D} \\ = (93{\cdot}8 - 60{\cdot}5)\dfrac{19{\cdot}2}{12}\end{array}\right\}$$

$$= 53{\cdot}28 \text{ tons.}$$

Maximum stress in the vertical Cc = the vertical component of the stress in bC + the panel load at C.

Horizontal component of the stress in bC = difference between the stresses at b and c.

$$\left.\begin{array}{l}\text{Stress at } b = 28{\cdot}8 \times \frac{12}{12} \qquad = 28{\cdot}8 \\ \text{Stress at } c \\ \quad = \dfrac{28{\cdot}8 \times 24 + 18 \times 12}{15} = 60{\cdot}5\end{array}\right\}\left\{\begin{array}{l}\text{Horizontal stress in } b\text{C} \\ = 60{\cdot}5 - 28{\cdot}8 = 31{\cdot}7.\end{array}\right\}$$

In this member, because it forms an angle of 45° with the vertical, the vertical component = the horizontal component = 31·7.

The stress in Cc therefore

$$= 31{\cdot}7 + 18 \text{ tons, the load at C} = 49{\cdot}7 \text{ tons.}$$

The maximum stress in the diagonal *d*E occurs when the panel points ABC and D are fully loaded and E unloaded.

Then the re-action at A = W =

$$\overset{\text{tons}}{45} \times \overset{\text{ft.}}{48} + \overset{\text{tons}}{18} \times 12(1+2+3) = \overset{\text{tons}}{60x} + 6 \times 12 + \overset{\text{tons}}{18} \times 12(4+3+2)$$

$$= 3456 \text{ tons} = 60x + 2016,$$

$$3456 - 2016 \text{ tons} = 60x,$$

and $$1440 \div 60 = x = 24 \text{ tons.}$$

Maximum stress in *d*E = difference between the horizontal components of the stresses at *d* and $e \times \dfrac{21{\cdot}63}{12}$.

Stress at *d*

$$= \frac{24 \times 36 + 18 \times 12(1+2)}{18} = 84{\cdot}00$$

Stress at *e*

$$= \frac{24 \times 48 + 18 \times 12(1+2+3)}{21} = 116{\cdot}60$$

Stress in *d*E

$$= (116{\cdot}6 - 84)\frac{21{\cdot}63}{12}$$

$$= 58{\cdot}76 \text{ tons.}$$

Maximum stress in the vertical D*d* = the vertical component of the stress in *c*D + the panel load at D.

Horizontal component of stress in *c*D = difference between the stresses at *c* and *d*.

Stress at *c*

$$= \frac{24 \times 24 + 18 \times 12}{15} = 52{\cdot}8 \text{ tons}$$

Stress at *d*

$$= \frac{24 \times 36 + 18 \times 12(1+2)}{18} = 84{\cdot}0 \text{ tons}$$

Horizontal stress in

$$c\text{D} = 84 - 52{\cdot}8 = 31{\cdot}2$$

and the vertical component $= 31{\cdot}2 \times \frac{15}{12} = 39$ tons.

The maximum stress in D*d* therefore = 39 + 18 = 57 tons.

The maximum stress in the diagonal eF occurs when the anchor span is fully loaded.

Then the re-action at $A = W =$

$$\overset{\text{tons}}{45} \times \overset{\text{ft.}}{48} + \overset{\text{tons}}{18} \times 12(1+2+3) = 60x + 18 \times 12(4+3+2+1)$$

$$= 3456 \text{ tons} = 60x + 2160 \text{ tons},$$

$$3456 - 2160 \text{ tons} = 60x,$$

and $$1296 \div 60 = x = 21{\cdot}6 \text{ tons}.$$

Maximum stress in eF = difference between the horizontal components of the stresses at e and $f \times \frac{24{\cdot}18}{12}$.

$$\text{Stress at } e = \frac{21{\cdot}6 \times 48 + 18 \times 12(1+2+3)}{21} = 111{\cdot}00 \text{ tons.}$$

$$\text{Stress at } f = \frac{21{\cdot}6 \times 60 + 18 \times 12(1+2+3+4)}{24} = 144{\cdot}00 \text{ tons.}$$

Stress in

$$\text{E}f = (144 - 111)\frac{24{\cdot}18}{12} = 33 \times \frac{24{\cdot}18}{12} = 66{\cdot}5 \text{ tons.}$$

Maximum stress in the vertical Ee = the vertical component of the stress in dE + the panel load at E.

Horizontal component of stress in dE = difference between the horizontal stress at d and e.

$$\text{Stress at } d = \frac{21{\cdot}6 \times 36 + 18 \times 12(1+2)}{18} = 79{\cdot}20 \text{ tons.}$$

$$\text{Stress at } e = \frac{21{\cdot}6 \times 48 + 18 \times 12(1+2+3)}{21} = 111{\cdot}00 \text{ tons.}$$

Horizontal stress in

$$d\text{E} = 111 - 79{\cdot}2 = 31{\cdot}8 \text{ tons,}$$

and the vertical component

$$= 31{\cdot}8 \times \tfrac{18}{12} = 47{\cdot}7 \text{ tons.}$$

The maximum stress in Ee therefore

$$= 47{\cdot}7 + 18 \text{ tons} = 65{\cdot}7 \text{ tons.}$$

The maximum compression in Ff = the sum of the panel loads on the anchor and cantilever spans when fully loaded

+the re-actions at A and K less the vertical components of the stresses in ef and fg which exert a downward or tensile stress at f.

Horizontal component of stress in

$$ef = \frac{148{\cdot}44 \times 12}{12{\cdot}37} = 144 \text{ tons},$$

$$fg = \frac{151{\cdot}8 \times 12}{12{\cdot}65} = 144 \text{ tons}.$$

Vertical component of stress in

$$ef = 144 \times \tfrac{3}{12} \quad = \ 36 \text{ tons},$$

$$fg = 144 \times \tfrac{4}{12} \quad = \ 48 \text{ tons}.$$

Then maximum stress in

$$\text{F}f = (18 \text{ tons} \times 8 + 21{\cdot}6 \text{ tons} + 45 \text{ tons}) - (36 + 48 \text{ tons})$$

$$= 126{\cdot}6 \text{ tons}.$$

The stresses in the various members may also be obtained by a graphic process once the re-action at the abutment A due to the particular condition of loading is determined. The lower half of the diagram in Fig. 143 gives the maxima stresses in all the members of the cantilever span which occur when the cantilever and suspended spans are fully loaded and the anchor span unloaded.

Diagram giving maxima stresses in B*b* and *b*C.
Panel point B only fully loaded.

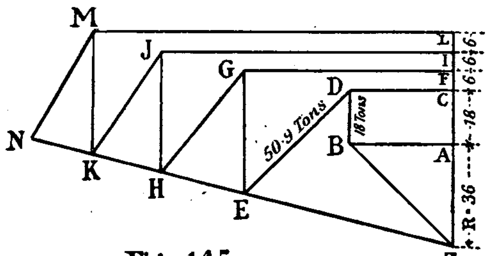


Fig. 145.

Diagram giving maxima stresses in Cc and cD.
Panel points B and C fully loaded.

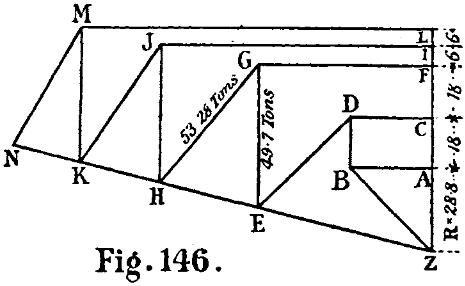


Fig. 146.

Diagram giving maxima stresses in Dd and dE.
Panel points B, C, and D fully loaded.

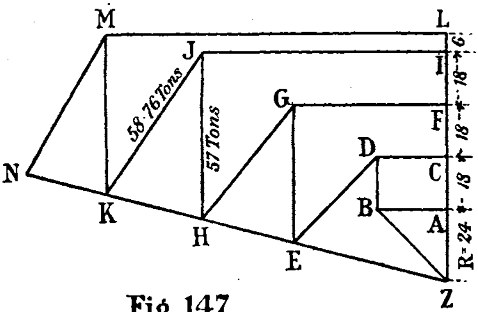


Fig. 147.

Diagram giving maxima stresses in Ee **and** *e*F.
All panel points fully loaded.

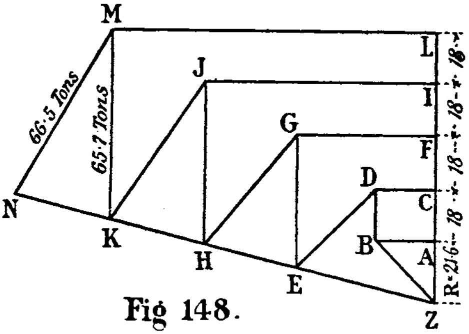


Fig 148.

The same condition of loading produces the maxima stresses in the top and bottom chords of the anchor span, which are set forth in the upper half of the diagram, Fig. 143, in which the re-action at A = 45·6 tons is determined by calculation as explained. The diagrams otherwise, it is thought, will be readily understood.

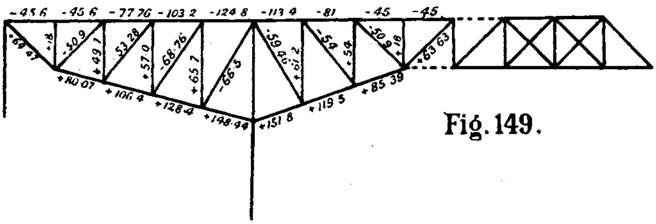


Fig. 149.

The maxima stresses in the web members of the anchor span occur under different conditions of loading, and the re-action at A varying with each condition of loading must be separately determined for each case. The diagrams in Figs. 145 to 148 give the maxima stresses in each member and the conditions of loading producing the same with the corresponding re-actions.

The stresses obtained from these diagrams are collected and written on each member in Fig. 149, which gives the maxima stresses in the cantilever truss. The stresses in the suspended span may be readily determined in the manner already explained.

CHAPTER VI.

Moment of Inertia and Moment of Resistance.

Moment of inertia, as already explained, is the summation of the product of the area of each particle or lamina of a section into the square of its distance from the axis, and it constitutes a geometrical expression in the determination of the moment of resistance.

To illustrate this definition by an example, let it be supposed that a beam of rectangular section, as shown in Fig. 150, is split up into a number of laminae whose areas are respectively represented by a, a_1, a_2, etc., and their distances from the axis running through the centre of gravity of the section by y, y_1, y_2, etc. Then, if we multiply the area of each lamina by the square of its distance from the centre of gravity or the neutral axis and add together the products, multiplying the result by 2, because each half of the section is similar, we have the moment of inertia of the whole section of the beam = I, and $\mathrm{I} = \Sigma ay$, which is the sum of an infinite series, and its value for a beam of rectangular section $= \frac{bd^3}{12}$.

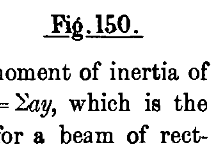
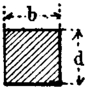


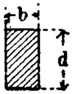
Fig. 150.

The moments of inertia for the several sections of beams and girders about an axis passing through the centre of gravity of the section are as under.

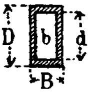
Let I represent the moment of inertia.



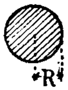
Square section. $I = \frac{d^4}{12}$.



Rectangular section. $I = \frac{bd^3}{12}$.

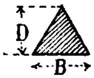


Hollow rectangle. $I = \frac{BD^3 - bd^3}{12}$.

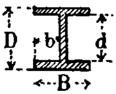


Circular section. $I = \cdot 7854 R^4$.

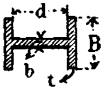
Hollow circle. $I = \cdot 7854 (R^4 - r^4)$.



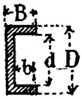
Triangular section. $I = \frac{BD^3}{36}$.



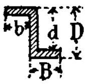
I beam. $I = \frac{BD^3 - 2bd^3}{12}$.



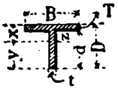
H beam. $I = \frac{2tB^3 + db^3}{12}$.



Channel bar. $I = \frac{BD^3 - bd^3}{12}$.



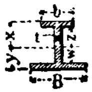
Zed bar. $I = \frac{BD^3 - 8b(d - \frac{1}{2}D)^3}{12}$.



Tee bar.

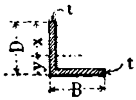
$$y = \frac{d}{2} + \frac{\frac{1}{2}\mathrm{BDT}}{\mathrm{BT} + dt},$$

$$\mathrm{I} = \frac{\mathrm{B}(x^3 - z^3) + t(y^3 + z^3)}{3}$$



Double-flanged beam.

$$\mathrm{I} = \frac{\mathrm{B}(y^3 - w^3) + b(x^3 - z^3) + t(w^3 + z^3)}{3}.$$



Angle bar.

$$y = \tfrac{1}{4}(\mathrm{B} + \tfrac{3}{2}t) \text{ nearly,}$$

$$\mathrm{I} = \frac{tx^3 + \mathrm{B}y^3 - (\mathrm{B} - t)(y - t)^3}{3}.$$

Inertia of built I beam (Figs. 151 and 151*a*).

Fig. 151.

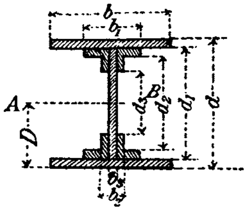
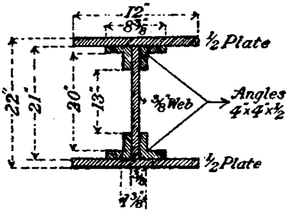


Fig. 151a.



$$\mathrm{I} = \tfrac{1}{12}[b(d^3 - d_1{}^3) + b_1(d_1{}^3 - d_2{}^3) + b_2(d_2{}^3 - d_3{}^3) + b_3 d_3{}^3],$$

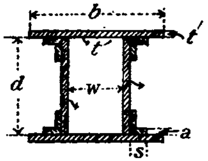
$$\mathrm{I} = \tfrac{1}{12}[12(22^3 - 21^3) + 8\tfrac{3}{8}(21^3 - 20^3) + 1\tfrac{3}{8}(20^3 - 13^3) + \tfrac{3}{8} \times 13^3]$$

$$= \frac{12 \times 1387 + 8\frac{3}{8} \times 1261 + 1\frac{3}{8} \times 5803 + \frac{3}{8} \times 2197}{12}$$

$$= \frac{16644 + 10560{\cdot}875 + 7979{\cdot}125 + 823{\cdot}875}{12} = 3000{\cdot}65.$$

Inertia of Box Girder (Fig. 152).

Fig. 152.



$$I = \frac{bt'^3}{6} + bt'\frac{(d+t')^2}{2} + \frac{(s+t)d^3}{6} - \left[\frac{(s-a)(d-2a)^3 + a(d-2s)^3}{6}\right].$$

If one plate is replaced by latticing,

$$I = \frac{bt'^3}{12} + bt'\frac{(d+t')^2}{4} + \text{etc.}$$

Moment of Resistance.

Having found the moment of inertia of a section, the moment of resistance may be determined as follows:

Let I = moment of inertia,

f = intensity of working stress at a distance,

$y = y$ from the neutral axis,

R = moment of resistance.

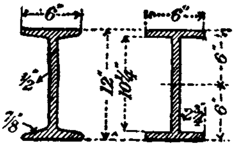
Then $R = \frac{fI}{y}$ and $f = \frac{Ry}{I}$. Here f = the modulus of rupture.

The following example will serve to explain the application of these principles, viz.:

It is required to determine the total distributed load that a rolled steel beam 12 inches deep and 6 inches wide over flanges, as shown in subjoined section, will carry on an effective span of 20 feet from centre to centre of bearings so that the material may not be strained beyond 6·4 tons per square inch.

$$I = \frac{BD^3 - 2bd^3}{12} = \frac{6 \times 12^3 - 2 \times 2\frac{3}{4} \times 10\frac{1}{4}{}^3}{12}$$

$$= \frac{4445}{12} = 370{\cdot}4.$$



$$R = \frac{fI}{y} = \frac{6{\cdot}4 \times 370{\cdot}4}{6} = 395{\cdot}09 \text{ inch-tons, say } 395.$$

The bending moment $= \frac{Wl}{8} = \frac{W \times (12 \times 20)}{8} = 30W.$

Equating the bending moment $= M$ with the moment of resistance $= R$, $M = R$, we have as the value of $M = 30W$; $\therefore$ $30W = 395$ and $W = 395 \div 30 = 13{\cdot}17$ tons $=$ the distributed load.

As another example, let it be required to determine to what extent the metal will be strained in a built girder of the following section, the effective span being 26 feet and the distributed load on the girder being 4 tons per foot run:

Let $a =$ net area of tension flange,
$d =$ effective depth of girder.

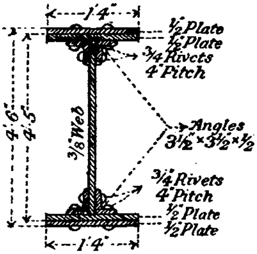
Then $R = f, a, d$.

Area of tension flange in square inches after deducting rivet holes

$= 2$ plates $= 2(16'' - 2 \times \frac{3}{4})\frac{1}{2} = 14{\cdot}50$

2 angles $= 2(6\frac{1}{2} - \frac{3}{4})\frac{1}{2} \quad = \underline{\ 5{\cdot}75\ }$

Then $\qquad a = \underline{\underline{20{\cdot}25}}$



The effective depth $= d = 4'\,5'' = 4{\cdot}4$ feet.

Then $R = f \times 20{\cdot}25'' \times 4{\cdot}4$ feet $= f\,90{\cdot}2$ foot-tons.

The bending moment $M = \frac{Wl}{8} = \frac{4l^2}{8} = \frac{104 \times 26}{8} = 338$ foot-tons.

Equating the bending moment with the moment of resistance $= M = R$, we have

$338 = f\,90{\cdot}2$ foot-tons and $338 \div 90{\cdot}2 = f = 3{\cdot}75$ tons per square inch, the intensity of the working stress.

CHAPTER VII.

Strength and Fatigue of Timber, Iron, and Steel.

The stresses in beams due to various conditions of support and loading have been investigated in Chapter II. It now remains to show how the dimensions of a beam are to be determined, so that it may be of sufficient strength and stiffness to safely and permanently resist the stresses resulting from any given condition of loading.

Timber Beams.

The timbers used for constructive purposes in this country are chiefly pine or fir imported from the Baltic ports and North America. The Baltic timbers are known by the names of the ports from which they are imported, viz.: Stettin, Dantzig, Riga, and Memel.

These timbers are procurable in lengths up to 50 feet and from 15 to 17 inches square. The American timbers are red and yellow pine, which are principally imported from Quebec and other Canadian ports.

They may be had in balks up to 50 feet in length, but not exceeding 14 inches square. Of all these timbers long leaf yellow pine of mature growth is beyond doubt the best timber for bridge building that is imported into this country, but unfortunately it is now very scarce since all the primeval forests have been cut down and there is nothing but young timber left available.

Pitch pine, which is derived from the Southern States of North America and imported from Pensacola and Mobile, is obtainable in abundance to almost any dimensions ranging up to 80 feet in length and to 20 inches square.

Pitch pine when new is a very fine timber, but with age and as the resin dries up it becomes brittle, and it is also subject to dry rot. Notwithstanding these defects it is extensively used in bridge building. Oak is used in small quantities where the strain it has to resist is one of compression, such as in joggles, keys, distance blocks, etc., and sometimes in corbels and cells.

In a simple beam or girder, when loaded either uniformly or with a concentrated load at the centre, its dimensions may be determined by equating its bending moment with its moment of resistance.

Let W = weight or load to be carried,

l = length of span,

b = breadth of beam,

d = depth of beam,

f = modulus of rupture or co-efficient of bending strength,

I = moment of inertia which for a rectangular beam $= \frac{bd^3}{12}$,

y = distance of extreme fibre from the neutral axis which for a rectangular section $= \frac{d}{2}$,

M = bending moment,

Mr = moment of resistance.

The greatest bending moment in a beam due to a concentrated load at the centre of the span $= \text{M} = \frac{\text{W}l}{4}$, and for a uniformly distributed load $\text{M} = \frac{\text{W}l}{8}$.

The moment of resistance of a beam (Rankine) $= \text{M}r = f\frac{\text{I}}{y}$.

But $$f\frac{\text{I}}{y} = f\frac{bd^3}{12} \div \frac{d}{2} = f\frac{bd^2}{6}.$$

Then, by equating the bending moment with the moment of resistance, we have

$$\left(\text{M} = f\frac{\text{I}}{y}\right) = \left(\frac{\text{W}l}{4} = f\frac{bd^2}{6}\right).$$

If b and d are expressed in inches, l in feet, and W and f in lbs., then

$$bd^2 = \frac{\text{W}l}{\frac{f}{18}} \quad \text{and} \quad \text{W} = \frac{bd^2}{l} \times \frac{f}{18}.$$

If F = factor of safety, then

$$bd^2 = \frac{6\text{MF}}{f}, \text{ or } bd^2 = \frac{6\text{M}}{\frac{f}{\text{F}}}, \text{ or } bd^2 = \frac{\text{FW}l}{\frac{f}{18}}, \text{ and } \text{WF} = \frac{bd^2}{l} + \frac{f}{18}.$$

The modulus of rupture is an expression for the intensity in pounds per square inch of the stress upon the extreme fibres of a beam at the point where, and at the time when, rupture begins.

The stress on the extreme fibre (f) is a function of the size of the load, the method of loading, and shape of the piece. For rectangular beams of uniform cross section

$$*f = \frac{3\text{W}l}{4bd^2} \text{ for beams uniformly loaded,}$$

and $$f = \frac{3\text{W}l}{2bd^2} \text{ for concentrated load at centre.}$$

* Bulletin No. 12, U.S. Department of Agriculture, Division of Forestry.

Here W = total load in lbs., and l, b, and d, as above, expressed in inches.

Table giving the values of the modulus of rupture and the modulus of elasticity for different kinds of timber.* No. 1.

Name of Timber.	Modulus of rupture in lbs. per square inch.	Modulus of elasticity in lbs. per square inch.	Authority.
Yellow Pine,	7,486	1,757,900	Professor Lanza.
Spruce,	5,327	1,412,818	Bauschinger.
White Pine,	4,451	1,222,000	Professor Lanza.
Pitch Pine,	7,626	1,539,000	D. Kirkaldy.
Dantzig Fir,	4,581	571,760	D. Kirkaldy.
Swedish,	6,258	1,149,600	D. Kirkaldy.
Red Pine,	5,000	1,200,000	American Association Ry. Supt.*
Norway Pine,	4,000	1,200,000	American Association Ry. Supt.
White Oak,	6,000	1,100,000	U.S. Division of Forestry.
English Oak,	9,762	1,786,000	D. Kirkaldy.

The strength of beams of equal lengths and under equal loads increases directly in proportion to the breadth and as the square of the depth, thus a beam 10 feet long, 12 inches deep and 12 inches wide, would be stronger than a beam of equal length, 12 inches deep and 8 inches wide, in the proportion of 12 to 8 or $1\frac{1}{2}$ to 1.

Similarly a beam 10 feet long, 8 inches wide, and 12 inches deep, would be stronger than a beam of equal length, 8 inches wide and 8 inches deep, in the proportion of 12^2 to 8^2 or 144 to 64 or 9 to 4.

As an example of the principles that have been laid down, let it be required to determine what weight applied at the

* American Association of Railway Superintendents of Bridges and Buildings. Report of fifth Annual Convention, New Orleans, October, 1895.

centre of a beam of pitch pine 10 feet long between supports and 8 inches wide by 12 inches deep would break it.

Let $\quad W = \text{breaking weight.}$

Then $\quad \frac{Wl}{4} = f\frac{bd^2}{6}.$

The value of f as given in the table $= 7626$ lbs per square inch for red pine.

Then $\quad \frac{Wl}{4} = \frac{7626 \times 8 \times 12^2}{6}.$

Reducing l to inches, we then have

$$W \times \frac{10 \times 12}{4} = \frac{7626 \times 8 \times 144}{6}$$

$$= W \times 30 = 1{,}464{,}192;$$

$$\therefore \quad W = 48{,}806 \text{ lbs.}$$

An alternative and probably a simpler method of arriving at practically the same result is given in "Tredgold's Carpentry," from which the following tabulated statement and formulae have been derived:

Table giving the Tensile, Crushing, and Transverse strength of various timbers. No. 2.

Name of Timber.	Specific gravity.	Weight per cubic foot.	Tensile strength per square inch.	Crushing strain per square inch.	Transverse Strength.
			A.	B	C
		lbs.	lbs.	lbs.	lbs
Red Pine,	0·512	32·00	10,500	5,395	483
Yellow Pine,	0·437	27·25	10,500	5,375	474
Pitch Pine,	0·682	42·50	10,500	6,790	495
Larch,	0·622	39·00	7,000	3,201	557
Riga Fir,	0·480	30·00	9,000	5,400	530
Dantzig Fir,	0·708	43·50	9,000	5,400	611
Memel Fir,	0·553	34·50	9,000	5,400	545
Spruce,	0·555	34·50	10,000	6,499	465
English Oak,	0·748	46·50	9000 to 15,000	6,484	710
Canadian Oak,	0·802	50·00	10,000	4,231	572

The above values of A, B, and C are averages deduced from experiments with good specimens of selected woods of small size. In large beams there would be a greater tendency to defects from knots, crooked fibres, etc. The values of the constants in the column C representing the co-efficient for centre breaking loads are arrived at by experiment thus:

If we take a beam of white pine, 12 feet long between supports, 6 inches wide, and 10 inches deep, and gradually load it at its centre until it breaks; and let the intensity of the breaking load be 22,500 lbs.

Let b = breadth in inches,
d = depth in inches,
l = length in feet,
BW = breaking weight in lbs.

Then
$$\frac{b \times d^2}{l} = \frac{6 \times 10^2}{12} = 50.$$

Then the proportion between $\frac{bd^2}{l}$ and BW is as 50 to 22,500 or 1 in 450 which is the co-efficient for the centre breaking load, and the breaking weight of any beam of similar timber may be found by multiplying $\frac{bd^2}{l} \times 450$ or the co-efficient C.

In the same manner, having once determined by experiment the co-efficient C for any quality of timber, the breaking weight of any beam of that particular timber may be readily ascertained.

With the aid of Table No. 2, which gives the various resisting properties of the principal constructive timbers used in this country, the breaking weight of any rectangular beam can be readily determined.

E.g. Let it be required to determine the breaking weight of a beam of pitch pine 10 feet long between supports, 8 inches wide, and 12 inches deep, as in the former example, the weight being applied at the centre.

Then $$BW = \frac{bd^2}{l} \times C.$$

The values of $\frac{bd^2}{l}$ are $\frac{8 \times 12^2}{10}$ and the value of C is 495.

Then $$BW = \frac{8 \times 12^2 \times 495}{10} = 57{,}024 \text{ lbs.}$$

In the preceding example the breaking weight was found to be 48,806 lbs. or about 14 per cent. less than in this case. This may be accounted for by the fact that the experiments, the results of which are set forth in Table No. 2, were made with specimens of selected wood. In view of the varied character of the experiments and of the variety of circumstances influencing the strength of timber, such as climate, age of tree, time of felling, seasoning, etc., the disparity between the two results is not so great as would at first appear, and it only tends to show the importance of allowing a considerable margin of safety in designing timber structures.

In the case of a distributed load the weight that would break a beam would be double that of the load concentrated at the centre, and the rule would be

$$BW = \frac{bd^2 \times C \times 2}{l}.$$

Factor of Safety.—In designing timber structures regard should be had not only to the variable qualities of timber, but to the class of structure and the condition of loading in allocating the working stresses and in the selection of a proper "factor of safety" which must be largely the outcome of personal judgment and experience. In this country a factor of safety of 8 to 10 is generally adopted, which, in such structures as bridges generally exposed to the weather and to alternate compressive and tensile stresses, is not too high a factor.

Much must depend upon the character of the structure whether it is to be temporary or permanent, the nature of the strain which each member has to resist, and its position in the structure with regard to its importance and the possible damage that might result from its failure.

The American Association of Railway Superintendents of Bridges and Buildings in their report, to which reference has already been made, recommend the following factors of safety, viz.:

Tension with and across grain	10
Compression with grain	5
Compression across grain	4
Transverse rupture, extreme fibre stress	6
Transverse rupture, modulus of elasticity	2
Shearing with and across grain	4

Strength and Fatigue of Iron and Steel.

The working stresses or the maximum permissible safe load per square inch which may be imposed on iron or steel in bridge construction must vary very considerably not only in proportion to the quality of the material, but also with the position in the structure of any particular member having regard to the intensity and nature of stress to which it may be subjected.

It has been the rule in this country to adopt certain arbitrary factors of safety; for example, the Board of Trade regulations provide for 5 tons and 6½ tons per square inch as the respective limits of working stresses for iron and steel, and any members of a bridge in which the maxima stresses do not exceed these limits are considered regardless of their particular functions to comply with all the requirements of safety.

Herr Wöhler has laid down the law that rupture may result not only from the application of a steady load producing stress equal to the ultimate resistance of the material, but by the application of much smaller stresses rapidly repeated. In other words, the less the intensity of the load the greater may be the number of repetitions until a limit is reached when rupture will take place only after an infinite number of repetitions.

Following up Wöhler's discovery Professor Launhardt and Weyrauch independently conducted series of experiments, the results of which clearly demonstrate that the adoption of any arbitrary factor of safety as applying generally to all members of a bridge structure is insufficient, and it is now generally recognised that some rational method must be adopted of adjusting the working stresses in the several members, having regard to their special functions.

The principle enunciated by Wöhler's law and further investigated by Launhardt's experiments is that the working stresses should never exceed the elastic limit. It therefore remains to determine how near we can in practice approach this limit. Various rules have been proposed by different authorities for determining the intensities of safe working stresses, but so far the question is one of individual judgment on the part of the engineer.

All engineers are agreed that greater provision should be made for a live than for a dead load, and it has been suggested that the adoption of a higher factor of safety for the live load would meet the circumstances. It has also been suggested that an addition should be made to the calculated stresses in order to provide for the effect of impact and vibration due to variable or dynamically applied loads.

Launhardt's Formula.

A bar of unit sectional area is subjected to a stress of one kind which may be tensile or compressive and which may vary from a minimum B to a maximum B.

Then the breaking weight or working strength is given by the expression

$$a=\eta\left(1+\frac{y-\eta}{\eta}\,\frac{\text{min. B}}{\text{max. B}}\right),$$

in which y = the ultimate strength of the material or the resistance to rupture under a gradually applied load;

η = primitive strength or the resistance to rupture under an infinite number of repetitions of stresses of the same kind;

a = breaking weight or working strength of the material.

Example. If in any member of a bridge, the material being steel, the ratio of dead load to live load is $\frac{1}{3}$, then $\frac{\text{min. B}}{\text{max. B}}=\frac{1}{4}$, and

$$a=16\left(1+\frac{28-16}{16}\times\frac{1}{4}\right)=19 \text{ tons},$$

so that the ultimate breaking strength of the bar = 19 tons, and if we adopt a factor of safety of 4, the working stress will be $\frac{19}{4}=4{\cdot}75$ tons per square inch and not $\frac{28}{4}=7$ tons per square inch.

This goes to prove that although the ultimate resistance of a bar under a quiescent load may be 28 tons per square inch, 19 tons may cause rupture if repeated often enough under the conditions we have assumed.

Launhardt's formula only applies to repeated stresses of one kind. In bridge structure it frequently happens that certain members are subjected to repeated stresses of alternate kinds, viz.: tension and compression. To meet this case a formula has been introduced by Herr Weyrauch giving the value of a as follows:

$$a = \eta\left(1 - \frac{\eta - S}{\eta}\,\frac{\text{max. B}'}{\text{max. B}}\right),$$

in which S = vibration strength or the resistance to fracture under alternating stresses of equal intensity due to a vibratory motion or the greatest stress that can be reversed an unlimited number of times = 7·15 tons for iron and 8·5 tons for steel;

max. B = greatest stress of tension or compression,

B′ = greatest stress of the opposite kind.

Example. If the maximum tension on a lattice bar be 10·12 tons and the maximum compression 3·37 tons, then

$$\frac{\text{max. B}}{\text{max. B}'} = \frac{3{\cdot}37}{10{\cdot}12} = \frac{1}{3}.$$

The material being steel, then

$$a = 16\left(1 - \frac{16 - 8{\cdot}5}{16} \times \frac{1}{3}\right) = 13{\cdot}5.$$

Adopting a factor of safety of 4, there results $\frac{13{\cdot}5}{4} = 3{\cdot}375$ as the safe working stress.

Probably the first practical attempt at any systematic allocation of working stresses to different members in a bridge is due to Mr. Theodore Cooper, whose general specifications for iron and steel highway and railway bridges are well known and widely adopted throughout the United States of America.

In these specifications Mr. Cooper assumes that the stresses produced by a live load are twice as destructive as a dead load, and recommends the proportioning of the parts on this supposition, allowing as the limits of working stresses for railway bridges:

	Live Load.	Dead Load.
Chord segments in tension,	8000 lbs. per sq. in.	16000 lbs. per sq. in.
Chord segments in compression,	$8000 - 30\frac{l}{r}$.	$16000 - 60\frac{l}{r}$.
All posts,	$7000 - 40\frac{l}{r}$.	$14000 - 80\frac{l}{r}$.

And for highway bridges:

Chord segments in tension,	10000 lbs.	20000 lbs.
Chord segments in compression,	$10000 - 40\frac{l}{r}$.	$20000 - 80\frac{l}{r}$.
All posts,	$8750 - 50\frac{l}{r}$.	$15000 - 100\frac{l}{r}$.

l = length of compression member in inches.
r = least radius of gyration of section in inches.

A formula that has been extensively used in American practice and one that appears to have given satisfactory results is the following which is known as the Pennsylvania Railway Company's formula:

$$b = c\left(1 + \frac{\text{min. stress}}{\text{max. stress}}\right),$$

b = permissible stress per square inch.
c = constant depending on the quality of the material used, the value of which for iron in tension is taken at 7500 lbs., and in compression 6500 lbs.

In any member subject to a reversal of stress the formula becomes

$$b = c\left(1 - \frac{\text{max. B}}{2\ \text{max. B}'}\right),$$

B being the stress of one kind, either tension or compression, the lesser in numerical value; B′ the stress of opposite kind the greater in numerical value.

In the department of "Ponts et Chaussées" in France, the following formulae are used:

$$\text{For iron,} \quad b = 3{\cdot}81 + 1{\cdot}9\ \frac{\text{min. stress}}{\text{max. stress}},$$

$$\text{for steel,} \quad b = 5{\cdot}08 + 2{\cdot}54\frac{\text{min. stress}}{\text{max. stress}}.$$

The intention of all these formulae is to provide for the effect of impact or the fatigue of the metal, or of both, in addition to the calculated stresses due to the load, but the relation between impact and fatigue or indeed between statically and dynamically applied loads cannot be satisfactorily determined.

The Wöhler Launhardt theory is based on a series of experiments on prepared test pieces, the load being applied gradually and repeated at regular intervals about four times every minute.

The effect of impact or dynamic action cannot therefore have been considered in this theory and it follows that if provision is to be made for impact the permissible working stress as deduced from Launhardt's formula must be still further reduced so that a limit may be reached that must at once raise doubt as to the safety of most of our present bridge structures.

It seems reasonable that in bridges situated in large towns and busy centres over which hundreds of trains may pass in course of the day, the dynamic stresses produced are probably greater than it has been usual to estimate and that a greater margin of safety should be adopted, but the extent of that margin must be left to the judgment of the engineer.

The specifications usually adopted for wrought iron and steel are as follows:

The plates, bars, angles, tees and other shapes shall be well and cleanly rolled to the full dimensions specified, and shall be free from scales, blisters, laminations, cracked edges, rust and all other defects, and the name of the maker shall be stamped upon every piece, whether of steel or of wrought iron.

Wrought Iron.

In the English iron trade there are several well-known brands of iron, and engineers frequently specify that a certain brand shall be used.

The average values of the working, ultimate and elastic strength of cast iron, wrought iron, and steel are as under:

Material.	Working Strength.			Ultimate Strength.			Elastic Limit.			Modulus of Elasticity.	Specific Gravity	Weight per Cubic Foot.
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.			
	Tons per sq. inch.	Tons per sq. inch.	Tons per sq. inch.	Tons per sq. inch.	Tons per sq. inch.	Tons per sq. inch.	lbs.	lbs.	lbs.	lbs.		lbs.
Cast iron,	1·5	8·0	2·25	8·0	44	6	8,000	21,000	8,000	15,000,000	7·26	450
Wrought iron,	5·0	4·0	4·0	20 to 24	22	18	30,000	33,000	22,000	26,000,000	7·69	480
Steel,	6·5	6·5	5·0	24 to 30	30	22·5	40,000	40,000	28,000	29,000,000	7·80	489·6

Tensile tests. Test pieces cut from any plate intended to be used in the work must have an ultimate tensile strength of not less than 20 tons per square inch of original section, and an elongation of not less than 10 per cent. in eight inches.

Similar test pieces cut from bars, tees, angles or channels must have an ultimate tensile strength of not less than 22 tons per square inch of original section, and an elongation of not less than 10 per cent. in a length of eight inches.

Bending test. Test pieces from plate, angle and other shapes must stand bending cold through 90 degrees to a curve, the radius of which is not over one and a half times the thickness of the piece, without cracking. Iron tested transversely must show 65 per cent. of the strength in the direction of the fibre.

Rivet iron. The iron used for rivets must be of the very best quality, and capable of resisting an ultimate tensile strain of not less than 24 tons per square inch, and of being bent through an angle of 120° when cold, without showing any signs of fracture.

Steel.

The steel used in bridgework is generally mild steel produced by the open-hearth process—if by acid process, containing not more than 0·08 per cent. of phosphorus, and if by basic process, not more than 0·05 per cent. of phosphorus.

Tensile tests. Test pieces from plates, angles, channels, tee bars and other shapes must have an ultimate tensile strength of not less than 26 tons per square inch or more than 30 tons per square inch of original section, with an elongation of not less than 20 per cent. in eight inches.

Bending tests. Strips of steel $1\frac{1}{2}$ inches wide, cut lengthwise or crosswise from any plate, bar, angle, tee or other shape, must be capable of being bent cold to a curve, the radius of which shall not be more than one and a half times the thickness of the piece, without signs of fracture, and must stand the same test if heated uniformly to a low cherry red and cooled in water at a temperature of 82° Fahrenheit.

Rivet steel. The steel used for rivets must have an ultimate tensile strength of not less than 28 tons per square inch of original section, and an elongation of not less than 25 per cent. in a length of eight inches, and it must be capable of being bent double, both hot and cold, without showing cracks or other defects.

Cast-iron Girders.

Cast-iron girders of any form are now practically obsolete in bridge construction, and the occasions, to say the least, will be very rare when an engineer will be called upon to design a cast-iron girder for a bridge, but he may be called upon to investigate the strength of an existing structure, and in that light it is thought that an examination of the stresses in a simple girder of cast iron may be of utility.

It has already been shown that the resistance of cast iron to compression is much greater than when it is exposed to a tensile stress, the limits of the safe working stresses being 8 tons per square inch in compression, and $1\frac{1}{2}$ tons per square inch in tension. It is therefore obvious that the best form in which cast iron can be employed in bridge building is in the form of arched ribs, in which the material is mainly in compression.

There are many cast-iron arched bridges in existence, and doing good duty even to this day, but inasmuch as the strength of arched structures will be fully inquired into in another chapter, we shall for the present confine our attention to the analysis of the strength of a simple cast-iron girder.

In a simple beam or girder of cast iron, the resistance to compression is from 5 to $5\frac{1}{2}$ times the resistance to tension, and if the material is to be effectively distributed, the sectional area of the tension flange must be inversely as the working resistance, or from 5 to $5\frac{1}{2}$ times as great as that of the compression flange; hence it follows that the correct section for the beam is that shown in Fig. 153.

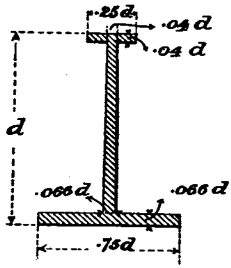


Fig. 153.

The breaking weight at centre in tons of a beam of this section is given approximately by the formula

$$\mathrm{BW} = \mathrm{C}\,\frac{ad}{\mathrm{L}},$$

in which

L = length of girder between supports in feet,

l = length of girder between supports in inches,

d = depth in inches,

a = net sectional area of tension flange in inches,

C = a constant whose value for L = 2·2 and for l = 26.

The bending moment at the centre due to a uniformly distributed load $= \frac{wl^2}{8}$, in which w = the unit load, and the bending moment of any section distant x from the centre of the girder $= \frac{wl^2}{8}\left(1 - \frac{4x^2}{l^2}\right)$.

If we represent the unit stress by f, then

$$afd = \frac{wl^2}{8}\left(1 - \frac{4x^2}{l^2}\right).$$

Then the sectional area at the centre $= a = \frac{wl^2 f}{8d}$, and at any point distant x from the centre the sectional area

$$= a_x = a\left(1 - \frac{4x^2}{l^2}\right).$$

In a cast-iron girder there should be little or no variation in the thickness of the metal in the various members, because of the difficulty in obtaining a sound casting where the thickness is variable, the thinner parts having a tendency to cool sooner than the thicker parts, thereby setting up internal stresses in the metal. For this reason the web is invariably made much thicker than is necessary to resist the shearing force. This additional thickness of metal in the web, however, adds to the strength of the girder, although it is not usually taken into consideration.

The depth generally adopted for a cast-iron girder varies from $\frac{1}{12}$ to $\frac{1}{16}$ of the span.

Rolled Steel Girders.

Girders of I section in steel are rolled up to 24 inches in depth. The steel of which they are made is manufactured by the Siemens-Martin or the Bessemer process, and the makers usually guarantee a tensile strength of not less than 28 tons or more than 32 tons per square inch, with an elongation of 20 per cent. in length of 8 inches.

The following simple rules, given in the Handbook of the Pencoyd Ironworks, Philadelphia (Messrs. A. P. Roberts & Co.), will be found useful in giving, with close approximation, the greatest safe load which a rolled steel girder will carry, and the amount of deflection based on a fibre strain of 16,000 lbs., or 7 tons per square inch.

Let L = length in feet between supports.

A = sectional area of beam in square inches.

D = depth of beam in inches.

w = working load in net tons.

W = greatest safe load in lbs.

Δ = deflection in inches.

Then for load applied at centre of beam

$$W = \frac{1695AD}{L}, \text{ and } \Delta = \frac{wL^3}{58AL^2}.$$

Then for distributed load

$$W = \frac{3390AD}{L}, \text{ and } \Delta = \frac{wL^3}{93AL^2}.$$

To be strictly correct, the greatest safe load is determined by equating the moment of resistance of a girder with the bending moment, and find the value of W, thus:

Let f = intensity of working stress at a distance y = y – from the neutral axis (in a symmetrical girder = ½ the depth).

I = moment of inertia.

Mr = moment of resistance.

Mb = bending moment.

Then

$$\left.\begin{aligned} Mb &= Mr, \\ \frac{Wl}{8} &= \frac{fI}{y}. \end{aligned}\right\}$$

Example. Let it be required to determine the safe load that a rolled girder of the following dimensions will carry on a span of 20 feet, the load to be evenly distributed.

Depth of girder, 15 inches.

Width of flanges, 5 inches.

Mean thickness of flanges, $\frac{5}{8}$ inch.

Thickness of web, $\frac{7}{16}$ inch.

The moment of inertia for this girder is 414·5.

The intensity of working stress is 7 tons per sq. inch.

Let
$$\frac{Wl}{8}=\frac{fI}{y}=\frac{W\times 20\times 12}{8}=\frac{414\cdot 5\times 7}{7\cdot 5}$$
$$=W30=386\cdot 82,$$
$$\text{and}\quad W=\frac{386\cdot 82}{30}=12\cdot 89 \text{ tons.}$$

Applying the simpler formula by way of comparison, we have

$$W=\frac{3390\times 13\times 15}{20}=32{,}052 \text{ lbs.}=14\cdot 3 \text{ tons.}$$

The slight difference between the two results is due to the fact that in the latter formula the value of f has been taken at 16,000 lbs., which is slightly in excess of 7 tons.

The following lists of rolled girders as published by the manufacturers, Messrs. Dorman, Long & Co., Ltd., of Middlesborough, and Messrs. Redpath, Brown & Co., Ltd., of Edinburgh, which give the dimensions, weight per foot, moment of inertia, sectional area and modulus of section, will be found useful for reference.

The modulus of section in these lists $=\frac{I}{y}$, and if multiplied by f or the intensity of working stress it gives the moment of resistance.

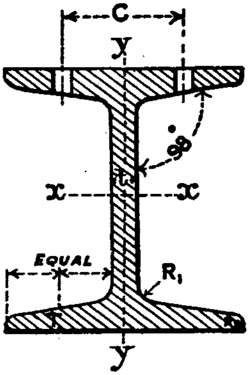


Fig. 153a.

NOTES. In selecting beams the load must not be so great as to cause excessive deflection, which may occur if the span exceeds 20 times the depth.

The deflection (in inches) of girder supported at both ends and uniformly loaded $= \Delta = \dfrac{5Wl^3}{384EI}$.

W = load in tons on whole span uniformly distributed.

l = length of span in inches.

E = modulus of elasticity = 12,000 tons.

I = vertical or greatest moment of inertia.

Messrs. Dorman, Long & Co.'s List.

Normal size. Inches.	Weight per foot in lbs.	Area of Section in sq. inches.	Dimensions in Diagram.				Moment of Inertia		Radius of Gyration in inches		Modulus of Section about $x—x$ in sq. inches.	Cross Centres of Holes C in inches.	Distributed Load in Tons that 1 Foot will carry at a working stress of 6·4 Tons per sq. inch.
			Web t.	Flange T.	Radius R_1.	Radius R_2.	about $x—x$.	about $y—y$.	about $x—x$.	about $y—y$.			
24×7½	100	29·4	·6	1·07	·7	·35	2654	66·92	9·5	1·5	221·1	4·5	943·36
20×7½	89	26·17	·6	1·01	·7	·35	1670	62·63	7·99	1·54	167·0	4·5	712·53
18×7	75	22·06	·55	·928	·65	·325	1149	47·04	7·21	1·46	127·6	4	544·42
16×6	62	18·23	·55	·847	·65	·325	725·7	27·08	6·31	1·21	90·71	3·5	387·02
15×6	59	17·35	·5	·88	·6	·3	628·9	28·22	6·02	1·27	83·85	3·5	357·76
15×5	42	12·35	·42	·647	·52	·26	428·0	11·81	5·88	·978	57·06	2·75	243·45
14×6	57	16·76	·5	·873	·6	·3	532·9	27·96	5·63	1·29	76·12	3·5	324·77
14×6	46	13·53	·4	·698	·5	·25	440·5	21·6	5·7	1·26	62·92	3·5	268·46
12×6	54	15·88	·5	·883	·6	·3	375·5	28·3	4·86	1·33	62·58	3·5	267·00
12×6	44	12·94	·4	·717	·5	·25	315·3	22·27	4·93	1·31	52·55	3·5	224·21
12×5	32	9·41	·35	·55	·45	·225	220·0	9·753	4·83	1·01	36·66	2·75	156·41
10×8	70	20·60	·6	·97	·7	·35	344·9	71·67	4·09	1·86	68·98	4·75	298·58
10×6	42	12·35	·4	·736	·5	·25	211·5	22·95	4·13	1·36	42·3	3·5	180·48
10×5	30	8·82	·36	·552	·46	·23	145·6	9·79	4·06	1·05	29·12	2·75	124·24
9×7	58	17·06	·55	·924	·65	·325	229·5	46·3	3·66	1·64	51·0	4	217·6
9×4	21	6·176	·3	·46	·4	·2	81·1	4·2	3·62	·824	18·02	2·25	76·88
8×6	35	10·29	·44	·597	·54	·27	110·5	17·95	3·27	1·32	27·62	3·5	117·84
8×5	28	8·24	·35	·575	·45	·225	89·32	10·26	3·29	1·11	22·33	2·75	95·27
8×4	18	5·294	·28	·402	·38	·19	55·69	3·578	3·24	·822	13·92	2·25	59·39
7×4	16	4·706	·25	·387	·35	·175	39·21	3·414	2·88	·851	11·2	2·25	47·78
6×5	25	7·35	·41	·52	·51	·255	43·61	9·116	2·43	1·11	14·53	2·75	61·99
6×4½	20	5·88	·37	·431	·47	·235	34·62	5·415	2·42	·959	11·54	2·5	49·28
6×3	12	3·53	·26	·348	·36	·18	20·21	1·339	2·39	·616	6·736	1·5	28·74
5×4½	18	5·29	·29	·448	·39	·195	22·69	5·664	2·07	1·03	9·076	2·5	38·72
5×3	11	3·235	·22	·376	·32	·16	13·61	1·462	2·05	·672	5·444	1·5	23·23
4¾×1¾	6·5	1·912	·18	·325	·28	·14	6·73	·263	1·87	·37	2·833	..	12·08
4×3	9·5	2·794	·22	·336	·32	·16	7·52	1·281	1·64	·677	3·76	1·5	16·04
4×1¾	5·0	1·47	·17	·24	·27	·135	3·668	·186	1·58	·355	1·834	..	7·82
3×3	8·5	2·50	·2	·332	·3	·15	3·787	1·262	1·23	·71	2·524	1·5	10·77
3×1½	4·0	1·176	·16	·248	·26	·13	1·659	·124	1·18	·324	1·106	..	4·72

For safe distributed loads on any girder divide figures in last column by clear span in feet.
NOTE.—The figures in the last column have been calculated by the Author.

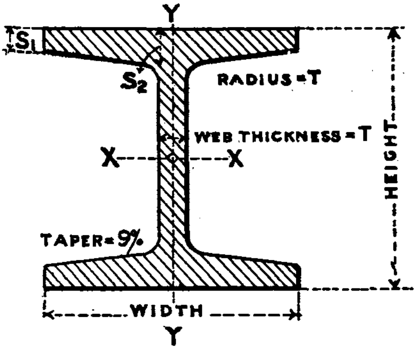


Fig. 153b.

The deflection of a beam supported at both ends and uniformly loaded is

$$\Delta = \frac{5}{24E} \cdot \frac{l^2}{h} \cdot s.$$

l = span in inches.
h = depth in inches.
E = modulus of elasticity in tons.
s = working stress in tons per sq. inch.

This formula may also be written as follows:

$$\frac{\Delta}{l} = \frac{5s}{24E} \cdot \frac{l}{h}.$$

The greatest shearing stress in a beam supported at both ends and loaded uniformly with the load Q occurs at the supports, and is

$$s' = \frac{Q}{2W}.$$

The greatest flange stress is $s = \frac{12QL}{8R}$.

In the last two equations

L = span in feet.
Q = uniform load.
R = moment of resistance of the section.
W = sectional area of web in sq. inches.

Rolled Steel Beams with Broad Flanges. Differdange Beams.

Agents.—H. J. Skelton & Co., 71 Finsbury Pavement, London, E.C.

Weight in lbs. per foot.	Dimensions in Inches.			Flange thickness.		Sectional Area in inches.	Axis X X			Axis Y Y		
	Height.	Width.	Web thickness.	S_1.	S_2.		Moment of Inertia in inches^4	Radius of Gyration in inches.	Moment of Resistance in inches^3	Moment of Inertia in inches^4	Radius of Gyration in inches.	Moment of Resistance in inches^3
51·1	9½	9½	·39	·41	·81	15	246·240	4·052	52·155	73·032	2·206	15·494
55·4	10	10	·41	·43	·86	16·29	289·584	4·216	58·865	85·800	2·295	17·446
61	10¼	10¼	·43	·46	·9	17·92	344·448	4·384	67·344	102·264	2·388	20·008
65	10⅝	10⅝	·44	·47	·93	19·10	396·696	4·557	74·664	118·080	2·486	22·265
69·5	11	11	·45	·49	·96	20·43	457·248	4·731	83·021	136·104	2·581	24·705
74·5	11½	11½	·47	·5	1·0	21·87	524·784	4·898	91·988	154·008	2·778	27·023
80·23	12	12	·5	·52	1·03	23·58	604·824	5·065	102·480	179·856	2·761	30·500
84·8	12½	12	·51	·55	1·06	24·91	722·856	5·386	114·802	188·808	2·753	31·964
88·3	13½	12	·53	·58	1·08	25·94	845·784	5·711	126·543	194·328	2·737	32·940
95·8	14	12	·55	·63	1·14	28·15	1019·496	6·019	143·960	211·032	2·738	35·746
100·87	15	12	·58	·66	1·17	29·64	1187·804	6·331	158·905	220·200	2·725	37·332
107·39	15¾	12	·61	·71	1·22	31·56	1388·016	6·631	176·412	233·304	2·718	39·528
112·83	16¾	12	·62	·75	1·25	33·16	1637·976	7·029	195·932	241·872	2·701	40·992
120·96	17¾	12	·66	·78	1·3	35·55	1941·288	7·389	219·215	256·032	2·683	43·371
127·68	18¾	12	·69	·83	1·34	37·52	2275·464	7·787	243·512	267·408	2·669	45·323
138·1	19¾	12	·76	·88	1·38	40·57	2670·792	8·111	271·511	281·232	2·632	47·641
152	21¾	12	·8	·96	1·45	44·65	3502·056	8·856	323·666	301·968	2·601	51·179
165·9	25½	12	·82	1·0	1·47	48·76	5166·528	10·294	404·003	307·536	2·511	52·094
177	29½	12	·82	1·0	1·5	52·02	6790·968	11·426	460·184	307·752	2·432	52·115

Messrs. Redpath, Brown & Co.'s List.

Dimensions.	Weight per Foot. in lbs.	Area in sq. ins.	Moments of Inertia.		Modulus of Section.	Safe Distributed Load for One-Foot Span.
			Max.	Min.		
24 × 7¼	100	29·41	2380·30	48·56	198·35	925·67
20 × 7½	90	26·47	1557·80	48·98	155·78	726·97
18 × 6¼	70	20·59	921·30	24·62	102·36	477·71
16 × 6	62	18·23	719·92	30·20	89·99	419·95
15 × 5	42	12·35	414·1	12·8	55·2	257·6
14 × 6	52	15·29	499·2	26·7	71·3	332·8
12 × 6	52	15·29	366·8	27·8	61·1	285·3
12 × 6(a)	42	12·35	291·2	23·8	48·5	226·5
12 × 5	36	10·59	237·1	10·9	39·5	184·4
10 × 6	42	12·35	211·4	24·1	42·3	197·3
10 × 5	29	8·53	142·1	11·4	28·4	132·6
9 × 7	54	15·88	212·7	47·5	47·3	220·6
9 × 4	22	6·47	80·4	4·8	17·9	83·3
8 × 6	30	8·82	101·3	19·8	25·3	118·2
8 × 5	27	7·94	86·9	10·9	21·7	101·4
8 × 4	19	5·58	57·7	4·8	14·4	67·3
7 × 3¾	18	5·28	42·6	3·9	12·2	56·8
6 × 5	24	7·04	42·1	10·9	14·0	65·6
6 × 3	13	3·81	21·6	1·8	7·2	33·5
5 × 4½	19	5·57	23·2	7·2	9·3	43·3
5 × 3	11	3·22	13·5	1·6	5·4	25·3
4 × 3	10	2·93	7·6	1·7	3·8	17·6
4 × 2	6½	1·91	5·1	·40	2·5	11·8
3 × 3	9	2·64	4·1	1·6	2·7	12·7
3 × 1½	4½	1·32	1·7	·15	1·1	5·4

The safe distributed loads on these girders are based on a maximum working stress of 7 tons per sq. inch.

CHAPTER VIII.

Strength of Columns.

In a bar, the length of which is small as compared with its least sectional dimension or diameter, subjected to a compressive force in the direction of its length as in the case of a column, failure can only result from the distortion, crushing or disintegration of the material, but in a bar of any considerable length, under the influence of a similar compressive force, a bending stress is produced as well as a compressive stress, and failure will result from flexure before the limit of resistance to direct crushing is reached.

The line of demarcation between a long and short column, or the limit of length as compared with the least sectional dimension at which a strut or column will fail under direct compression only, is assumed to be approximately for

Cast iron......... 5 to 1
Wrought iron...10 to 1
Dry timber......20 to 1
} *See* "Instruction on Construction," Col. Wray.

Professor Hodgkinson proposed a division into three clauses, viz.:

A. Short pillars failing under direct compression, of which the ratio of length to diameter is not more than 5 to 1.

B. Medium pillars failing partly by crushing and partly by bending, ratio of length to diameter exceeding 5 and less than 30 if of cast iron and 60 if of wrought iron.

C. Long pillars failing wholly by bending, ratio of length exceeding 30 if of cast iron and 60 if of wrought iron.

The strength of columns is investigated under four conditions, viz.:

1. One end free and the other fixed (Fig. 154).
2. Both ends pivoted or rounded (Fig. 155).
3. One end pivoted or rounded and the other end fixed (Fig. 156).
4. Both ends fixed (Fig. 157).

Fig. 154.

Fig. 155.

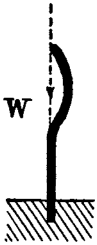


Fig. 156.

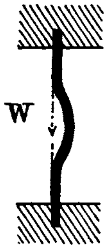


Fig. 157.

The formula in most general use for determining the strength of columns is that deduced by Professor Lewis Gordon from Professor Hodgkinson's experiments and known as "Gordon's formula," which is thus:

Gordon's Formula.

$$p=\frac{f}{1+a\frac{l^2}{h^2}}$$ for columns with both ends flat or fixed,

in which p = crushing stress per square inch of sectional area,

f = ultimate crushing strength of the material in lbs. per square inch, the values being for

Cast iron	80,000	lbs. per sq. inch;
Wrought iron	36,000	,, ,,
Mild steel	67,200	,, ,,
Pure timber	5,000	,, ,,

l = length of column } in same units of
h = diameter or least dimension } measurement;

r = radius of gyration of cross section;

a = constant depending upon the material and upon the form of the column, the values being as under

For cast-iron solid and hollow rectangular and round columns $\frac{1}{500}$.

For wrought iron solid, rectangular, or round $\frac{1}{2500}$.

For cylindrical hollow wrought-iron columns $\frac{1}{5000}$.

For solid, rectangular, and round steel columns $\frac{1}{2000}$.

For hollow cylindrical columns of mild steel $\frac{1}{2500}$.

For wrought iron HL+⊥U, $a=\frac{1}{900}$ and $f=42{,}560$.

If Gordon's rule is applied to columns with rounded or pivoted ends, the formula becomes

$$p=\frac{f}{1+4a\frac{l^2}{h^2}};$$

and if to columns with one end pivoted and one end square or fixed,

$$p=\frac{f}{1+\frac{9}{5}a\frac{l^2}{h^2}}.$$

Rankine's Formulae.

Professor Rankine proposed a modification of Gordon's formula, substituting for h the diameter or least dimension r = the least radius of gyration of the section, which takes into account the variation due to the sectional shape of the column.

The formula thus becomes for columns flat or fixed at both ends:

$$\frac{p}{s}=\frac{f}{1+\frac{l^2}{cr^2}}.$$

For a column with both ends pivoted or rounded,

$$\frac{p}{s}=\frac{f}{1+\frac{4l^2}{cr^2}};$$

In these equations $\frac{p}{s}$ = breaking strain per square inch of sectional area.

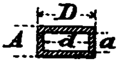
and for a column with one end pivoted and one end fixed,

$$\frac{p}{s}=\frac{f}{1+\frac{16l^2}{9cr^2}}.$$

	Value of f in lbs. per sq. inch.	Value of c.
Malleable iron............	36,000 ...	36,000.
Cast iron	80,000 ...	3,200.
Dry timber................	7,200 ...	3,000.

The radius of gyration $=\sqrt{\text{moment of inertia} \div \text{area}}$.

Square of radius of gyration = moment of inertia ÷ area.

Section.		Least Radius of Gyration.	Square of Least Radius of Gyration.
	Solid square.	$\frac{D}{\sqrt{12}}$.	$\frac{D^2}{12}$.
	Hollow square.	$\sqrt{\frac{D^2+d^2}{12}}$.	$\frac{D^2+d^2}{12}$.
	Solid rectangle.	$\frac{A}{\sqrt{12}}$.	$\frac{A^2}{12}$.
	Hollow rectangle.	$\frac{\sqrt{A^3D-a^3d}}{12(AD-ad)}$.	$\frac{A^3D-a^3a}{12(AD-ad)}$.
	Solid circle.	$\frac{D}{4}$.	$\frac{D^2}{16}$.
	Cylinder.	$\sqrt{\frac{D^2+d^2}{16}}$.	$\frac{D^2+d^2}{16}$.
	Rolled beam.	$\frac{D}{4{\cdot}58}$.	$\frac{D^2}{21}$.
	Channel.	$\frac{D}{3{\cdot}54}$.	$\frac{D^2}{12{\cdot}5}$.
	Angle equal sides	$\frac{D}{5}$.	$\frac{D^2}{25}$.
	Angle unequal sides.	$\frac{Dd}{2{\cdot}6(D+d)}$.	$\frac{D^2+d^2}{13(D^2+d^2)}$.

SECTION.		LEAST RADIUS OF GYRATION.	SQUARE OF LEAST RADIUS OF GYRATION.
	Tee $D=d$	$\frac{D}{4{\cdot}74}$.	$\frac{D^2}{22{\cdot}5}$.
	Cross $D=d$		

This tabulated statement is taken from Trauturne's "Civil Engineer's Pocket Book." The values of r and r^2 for the last six shapes are only approximate.

Gordon's formula as modified by Professor Rankine is probably the best known and most extensively used in the designing of columns, but within comparatively late years several other formulae have been proposed, as being more closely in agreement with the results of experiments and more convenient of application. Of these the formula introduced by Professor Claxton Fidler is accepted as one of the most reliable and comprehensive, which is as follows:

$$P=\frac{f+\rho-\sqrt{(f+\rho)^2-2{\cdot}4f\rho}}{1{\cdot}2}.$$

$\rho=\pi^2 E\left(\frac{r}{l^2}\right)^2$, in which

P = load in lbs. producing stress f;

f = ultimate crushing stress of materials in lbs. per sq. inch

= for	cast iron	80,000 lbs.,
	wrought iron	36,000 „
	hard steel	70,000 „
	mild steel	48,000 „

E = modulus of elasticity of material for

cast iron	14,000,000,
wrought iron	26,000,000,
steel	29,000,000;

ρ = resilient force of an ideal column in lbs. per sq. inch ;

r = radius of gyration ;

l = length of column in inches. For a column fixed at ends $l = \cdot 6l$.

Johnson's Straight line Formula.

This is a formula proposed by Mr. Thos. H. Johnson, M.Am.Soc.C.E., in a paper presented to the American Society of Civil Engineers in 1886, and is the simplest yet introduced. It is as follows :

Wrought iron, hinged ends, $P = 42{,}000 - 157\,\frac{l}{r}$.

„ „ flat ends, $P = 42{,}000 - 128\,\frac{l}{r}$.

Mild steel, hinged ends, $P = 52{,}500 - 220\,\frac{l}{r}$.

„ „ flat ends, $P = 52{,}500 - 179\,\frac{l}{r}$.

In this formula P = elastic limit of material per sq. inch of section,

r = radius of gyration,

l = length in inches.

Mr. Theodore Cooper has adopted this formula in his specifications for proportioning compression members.

The following formula taken from Messrs. Redpath, Brown & Co's Section Book has been found to be reliable in practice, and it is simple of application :

$$BS = \frac{17\cdot 8a}{1 + \frac{l^2}{18000r^2}}$$

BS = breaking strength in tons.
l = length of column in inches.
a = sectional area in inches.
r = radius of gyration².

The factor of safety is determined by the expression

$$= 4 + \cdot 07 \frac{l}{h}.$$

l = length in inches.
h = least diameter.

For the purpose of comparing these several formulae let it be required to determine the breaking weight of a wrought-iron column of the section shown in Fig. 158, and 14 feet in length, the ends being flat or fixed.

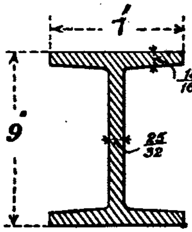


Fig. 158.

The weight of the section is 54 lbs. per foot run.

The area of section = 15·88 square inches.

Greatest or vertical moment of inertia = 216·81.

Least or horizontal moment of inertia = 47·5.

Least radius of gyration squared $= \frac{47 \cdot 5}{15 \cdot 88} = 3.$

Then, by Rankine's formula, $p = \dfrac{36000}{1 + \dfrac{168^2}{36000 \times 3}} \times 15 \cdot 88$

$$= 202 \cdot 6 \text{ tons.}$$

By Fidler's formula,

$$p = \frac{36000 + 75768 - \sqrt{(36000 + 75768)^2 - 2 \cdot 4 \times 36000 \times 75768}}{1 \cdot 2}$$

$$= 204 \cdot 74 \text{ tons.}$$

By Johnson's straight line formula,

$$p = 42000 - 128 \times \frac{168}{1 \cdot 732} = 209 \cdot 7 \text{ tons.}$$

By formula given in Redpath, Brown & Co.'s Section Book,

$$p = \frac{17{\cdot}8 \times 15{\cdot}88}{1 + \frac{168^2}{18000 \times 3}} = 185{\cdot}6 \text{ tons.}$$

For timber pillars the following simple formula is given in Tredgold's "Carpentry":

$$\text{BW} = \frac{\text{B} \times a}{1{\cdot}1 + \frac{l^2}{2{\cdot}9 \times t^2}},$$

in which

BW = breaking weight in lbs.,
a = area of cross section,
B = constant in column B of table on page 184.
l = length in feet,
t = least side or thickness.

CHAPTER IX.

Rivetted Joints and Connections.

In the building of girders the plates, bars, angles, tees or channels of which they are composed are united together by rivets, bolts, gibs, and cotters, pins or screws, and a girder is only properly designed when the connections and joints of the various members are so arranged that the resistance of the joint or connection shall be equal to the stress developed in the members joined or connected.

Rivetted joints. There are two kinds of rivetted joints, viz.: Lap joint in which the plates or bars overlap and are rivetted together by one or more rivets or rows of rivets as shown in Fig. 159; and

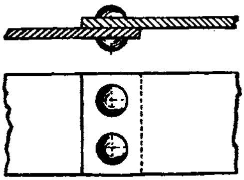
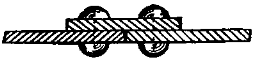
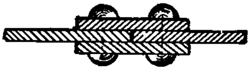


Fig. 159.

Butt joint in which the two plates are brought together in the same plane, their ends butted against one another and the joint covered by a plate on one or on both sides and rivetted together by one or more rivets or rows of rivets as shown in Fig. 160.



Butt joint single cover.



Butt joint double cover.

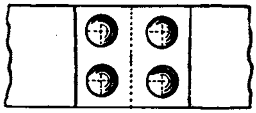


Fig. 160.

Lap joints are principally used in boilerwork, shipbuilding, and in connecting the web members to the chords and principals of bridge and roof trusses.

Butt joints with coverplates are largely used in all kinds of girderwork.

Rivetted joints may fail in three ways:

1st. By the tearing of the plates or the covers or of both through the rivet holes.

2ndly. By the rivets cutting through or crushing the plates, or the plates through the rivets, from an insufficient bearing area, or an excessive pressure on the bearing area.

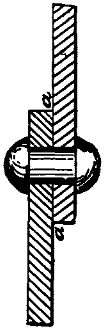
3rdly. By the shearing of the rivets.

Rivets may be shown in one, two, or more planes as shown in Figs. 161, 162, and 163.

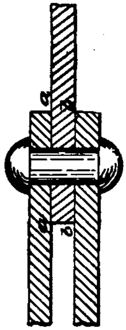
In Fig. 161 if the joint should fail by the shearing of the rivet, the rivet can only be shorn through one section along the plane *aa*.

The joint shown in Fig. 162 can only fail by the shearing of the rivets through two sections simultaneously along the planes *aa* and *bb*.

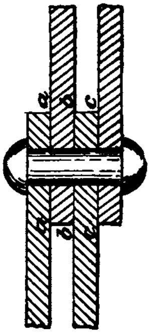
Failure by shearing of the joint shown in Fig. 163 can only result from the simultaneous shearing of the rivet through three sections along the planes *aa*, *bb*, and *cc*.



1 *Shear.*
Fig. 161.



2 *Shears.*
Fig. 162.



3 *Shears.*
Fig. 163.

Rivets in double shear are therefore doubly as strong as if they were in single shear, that is to say, it would be necessary in a joint where the rivets would be subject to single shear to employ twice as many as if they were in double shear.

The bearing area of a rivet or rivet hole is its diameter × the thickness of the plate on which it bears.

Let b = breadth of plate or bar in inches,
t = thickness of plate or bar in inches,
d = diameter of rivet in inches,
n = number of rivets in one line parallel to joint.

Then the shearing area of rivet $= \cdot 7854 d^2 n$,
and tearing area of plate $= (b - dn)t$,
and bearing area $= ndt$.

The total shearing resistance of the rivets on each side of the joint should be at least equal to the tensile resistance of the net section of the plate through the rivet holes or the shearing area of the rivets should be at least equal to the tearing area of the plate.

Let b_1 = breadth of plate after deducting rivet holes,
t = as above,
a = area of rivet,
n = number of rivets required on each side of joint.

Then, neglecting pressure on bearing area, we have for iron rivets

$$1 \times bt = na \text{ in single shear,} \qquad \therefore\ n = \frac{1 \times bt}{a},$$

and $$1\,bt = 1{\cdot}75\,na \text{ in double shear,} \qquad \therefore\ n = \frac{0{\cdot}571\,bt}{a}$$

and for steel rivets

$$1{\cdot}3\,bt = na \text{ in single shear,} \qquad \therefore\ n = \frac{1{\cdot}3\,bt}{a},$$

$$1{\cdot}3\,bt = 1{\cdot}75\,na \text{ in double shear,} \qquad \therefore\ n = \frac{0{\cdot}742\,bt}{a}.$$

NOTE.—Rivets in double shear are theoretically doubly as strong as in single shear, but in practice it is not considered advisable to allow a higher co-efficient than 1·75 times the strength of a rivet in single shear, hence the adoption of a co-efficient of 1·75 for rivets in double shear in the above equations.

The working stresses for rivets may be taken as follows:

Iron.	Safe tensile resistance of plate,			5	tons per	sq inch.
Steel.	do.	do.	do.	7½	do.	do.
Iron.	Safe bearing resistance of plate,			5	do.	do.
Steel.	do.	do.	do.	7½	do.	do.
Iron.	Safe shearing resistance of rivets,			4	do.	do.
Steel.	do.	do.	do.	5	do.	do.

The ordinary modes of finishing the rivet heads are shown in Fig. 164, the most generally adopted in bridge work being the cup or snap head. The hammered head is principally used in boiler work. Rivet heads are only countersunk on the under side of the bearing, flange ends of girders, or on

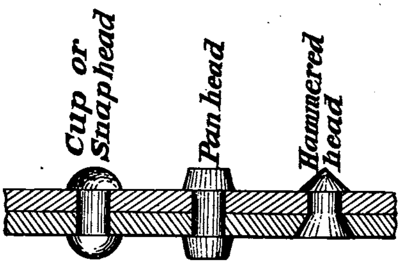


Fig. 164.

the outside or exposed face of plates that are intended to have a smooth surface.

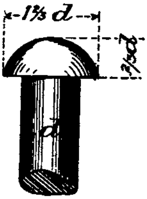


Fig. 165.

Rivets in bridge building vary in diameter from $\frac{3}{4}$ inch to 1 inch and occasionally $1\frac{1}{8}$ inch. The pitch of the rivets or the distance from the centre of one rivet to the centre of another rivet is usually 3 or 4 inches and occasionally 6 inches.

The proportions of the rivet head to the rivet in terms of the diameter of the rivet are given in Fig. 165.

The rivet holes must be spaced at such distances from each other or from the edge of the plate as not to incur any risk of cracking or damaging the plate. The minimum distance from the edge of the hole to the edge of the plate should not be less than the diameter of the rivet.

Rivet holes, especially in steel plates, should be drilled and not punched, and the holes should be of such a size as to admit a rivet $\frac{1}{16}$ of an inch less in diameter than that of the hole being driven in hot without reaming or drawing the plates by drifts, and the rivets when driven must completely fill the holes.

TABLE GIVING DIAMETER OF RIVETS as usually adopted for various thicknesses of plate, sectional area of rivet, the area of its effective lateral bearing surface in the plate = diameter of rivet × thickness of plate and its safe working strength in single shear.

Thickness of Plate.	Diameter of Rivet in fraction of an inch.	Diameter of Rivet in decimals.	Area in square inch.	Bearing Area.	Safe working strength in Tons single shear at 4 tons per sq. inch.
inch.					
$\frac{1}{4}$	$\frac{5}{8}$	·625	·306	·156	1·224
$\frac{5}{16}$	$\frac{5}{8}$	·625	·306	·195	1·224
$\frac{3}{8}$	$\frac{3}{4}$	·750	·441	·281	1·764
$\frac{7}{16}$	$\frac{3}{4}$	·750	·441	·328	1·764
$\frac{1}{2}$	$\frac{3}{4}$	·750	·441	·375	1·764
$\frac{9}{16}$	$\frac{7}{8}$	·875	·601	·492	2·404
$\frac{5}{8}$	$\frac{7}{8}$	·875	·601	·547	2·404
$\frac{11}{16}$	$\frac{7}{8}$	·875	·601	·601	2·404
$\frac{3}{4}$	1	1·000	·785	·750	3·140
$\frac{13}{16}$	1	1·000	·785	·812	3·140
$\frac{7}{8}$	1	1·000	·785	·875	3·140
1	$1\frac{1}{8}$	1·125	·994	1·125	3·976

In well-rivetted joints there is an additional resistance due to the friction of the plates sliding one upon another, and this friction undoubtedly adds to the strength of the joint; but inasmuch as it is impossible to determine to what extent it increases the strength, it should not be taken into account as a factor in the design of the joint.

Example No. 1. Let it be required to join together two lengths of plates 8 inches wide × $\frac{1}{2}$ inch thick by a butt joint. In a joint of this kind wherever it is feasible there should be double covers so that the rivets may be in double shear. The width of the plate 8″ is sufficient to admit of two rows of rivets being employed, and on reference to table giving the dimensions of rivets, etc., it will be seen that for a plate $\frac{1}{2}$ inch thick a $\frac{3}{4}$ inch rivet would be necessary.

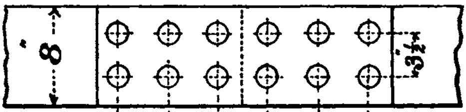
Let the thickness of the cover plates be $\frac{3}{8}$ inch, which is perhaps a little in excess of what is actually required, and let the working strength of the plate and cover plates be 5 tons per sq. inch. The effective strength of the plate through the two rivet holes $=(8-1\frac{1}{2})\times\frac{1}{2}\times 5$ tons $=16{\cdot}25$ tons.

Equating the net section of the plate through the rivet holes $=16{\cdot}25$ tons with the total shearing resistance of the rivets on each side of the joint we have

$$(bt \times 5 \text{ tons} = na \times 4 \text{ tons} \times 1{\cdot}75) = 16{\cdot}25 = (n \times {\cdot}441 \times 4) \times 1{\cdot}75\,;$$

$\therefore\ n = 16{\cdot}25 \div 3{\cdot}087 = 5{\cdot}2$ rivets, say 6 rivets, which would be arranged as shown in Figs. 166 and 166a. The pitch of the rivets transversely is

Fig. 166.



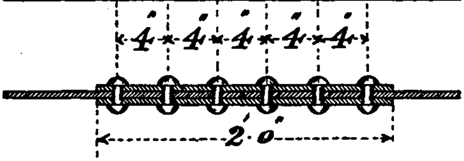


Fig. 166*a*.

$3\frac{1}{2}$ inches, which gives a sufficient margin between the edges of the rivet holes and the edges of the plate. The pitch of the rivets longitudinally is 4 inches, and the length of cover plates necessary to cover the joint is 2 feet 0 inch.

Example No. 2. It is required to join together two lengths of angle bar, and the wrapper or cover must be equal in area to the main angle bar.

The dimensions of the angle bar are $4'' \times 4'' \times \frac{1}{2}$.

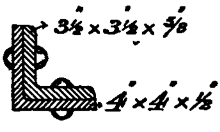


Fig. 167.

The working area of an angle bar $4'' \times 4'' \times \frac{1}{2}$ after making deduction for one $\frac{7}{8}$ rivet hole

$$=(7\tfrac{1}{2}-\tfrac{7}{8})\tfrac{1}{2}=3{\cdot}3.$$

The working area of an angle bar

$$3\tfrac{1}{2}\times 3\tfrac{1}{2}\times\tfrac{5}{8}$$

after making deduction for one $\frac{7}{8}$ rivet hole

$$=(6\tfrac{3}{8}-\tfrac{7}{8})\tfrac{5}{8}=3{\cdot}4$$

so that is in excess of the angle bar which it covers.

The number of rivets required on each side of the joint (in this case all in single shear) $= N = \frac{3 \cdot 3}{0 \cdot 60} = 5 \cdot 6$ say 6. The divisor $0 \cdot 60$ being the area of a $\frac{7}{8}$ in. rivet.

Adopting a pitch of 4 inches the length of the angle wrapper would 2 ft. 4 in. as shown in Fig. 168.

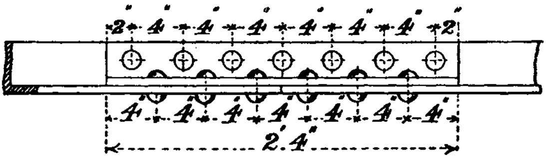


Fig. 168.

Example No. 3. Let it be required to design a joint for the tension flange of a plate girder the dimensions of which are 14 in. $\times \frac{5}{8}$.

A $\frac{5}{8}$ in. plate requires $\frac{7}{8}$ in. rivets, and let there be four rows of rivets, as shown in Figs. 169 and 170.

The net area of the flange plate after deducting two rivet holes $\frac{7}{8}$ in. diameter $= [14 - (\frac{7}{8} \times 2)]\frac{5}{8} = 7 \cdot 66$ inches and $7 \cdot 66 \times 5$ tons $=$ the effective resistance of the plate $= 38 \cdot 30$ tons.

The safe resistance of rivets to shearing is 4 tons per square inch, so that $38 \cdot 30$ tons $\div 4 =$ the total shearing area of the rivets on each side of the joint $= 9 \cdot 57$ inches, and $9 \cdot 57$ in. $\div \cdot 6$, the area of a $\frac{7}{8}$ in. rivet $= 15 \cdot 9$, say 16 rivets in single shear required on each side of the joint *ab*.

By the introduction of the cover strips *cc* which should be provided in every joint of this kind, the outer row of rivets on each side of the plate would be in double shear, so that the number of rivets would be reduced to the extent of the difference in total rivet section between single and double shear.

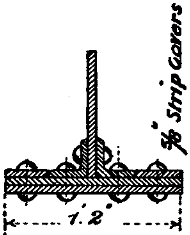
In this case there are on each side of the joint

6 rivets in single shear	$= 6 \times \cdot 6$	$= 3 \cdot 6$
and 6 rivets in double shear	$= 6 \times \cdot 6 \times 1 \cdot 75$	$= 6 \cdot 3$
Total shearing area of rivets		$9 \cdot 9$

which is slightly in excess of the actual area required, which $= 9 \cdot 57$, and is therefore on the safe side.

The net section of the plate through the rivet holes has been shown to be 7·66 inches, so that the total shearing resistance of the rivets is in excess of the tearing area of the plate, and the joint is efficient.

Fig. 169.



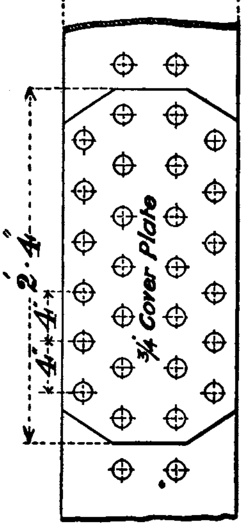


Fig. 170.

Example No. 4. Let it be required to design the joint of a tension bar of a lattice girder 5 in. × $\frac{1}{2}$ in. rivetted to the lower flange as in Fig. 171. The effective breadth of the bar through the section *aa* or *bb* $=(5-\frac{3}{4})\frac{1}{2}=2\cdot125$ inches, and the strength of the bar, allowing a working stress of 5 tons per square inch $=2\cdot125\times5=10\cdot625$ tons.

The number of rivets required to resist shearing, allowing a working stress of 4 tons per square inch $= N \times$ area of rivet $\times 4$ tons $= 10\cdot625$; $\therefore N \times \cdot44 \times 4 = 10\cdot625$.

Then $$N = \frac{10\cdot525}{1\cdot76} = 6 \text{ rivets.}$$

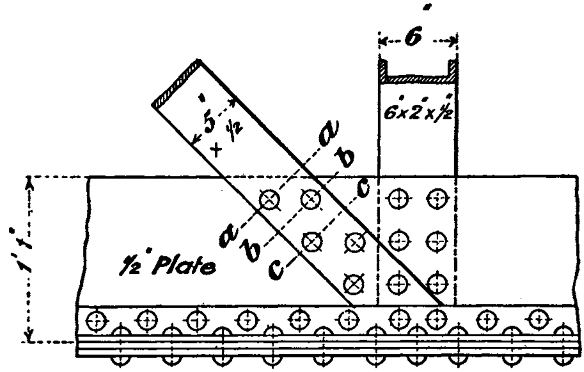


Fig. 171.

The bearing area of each rivet $= \frac{3}{4} \times \frac{1}{2}$ and the bearing resistance $\frac{3}{4} \times \frac{1}{2} \times 5$ tons $= 1\cdot875$ tons. Then by equating as under we have the number of rivets required $= N$. Then $N \times 1\cdot875 = 10\cdot625$, whence $N = 10\cdot625 \div 1\cdot875 = 5\cdot6$, say 6 rivets, which as already determined will be sufficient.

Example No. 5. The joint shown in Fig. 172, known as the lozenge joint, is an efficient and economical mode of joining together two sections

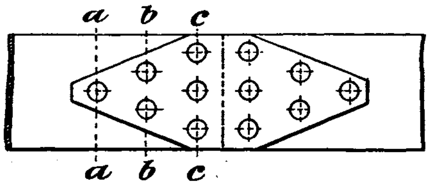


Fig. 172.

of a plate or bar by a butt joint with a cover plate on each side. The rivets are so disposed that the strength of the plate or bar is only weakened by one rivet hole.

Let the dimensions of the bar be 9 in. $\times \frac{5}{8}$ and the rivets be $\frac{7}{8}$ in. in diameter.

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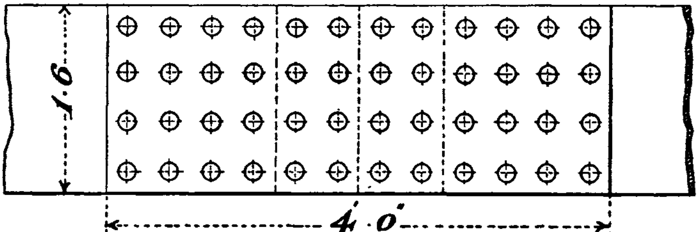
The strength of the bar through the rivet hole at the section *aa* $=(9\text{ in.}-\frac{7}{8})\frac{5}{8}=8\frac{1}{8}\times\frac{5}{8}\text{ in.}=5$ inches, and 5 inches × 5 tons = 25 tons = the net strength of the bar, and 25 tons ÷ 4 = 6·25 tons, the total shearing area of the rivets on either side of the joint if in single shear; but inasmuch as there are two cover plates and the rivets are consequently in double shear, the number of rivets required = 6·25 ÷ (·6 × 1·75) = 6·25 ÷ 1·05 = 6 rivets = the number provided as shown in Fig. 172.

In this joint the strength of the plate is only reduced by one rivet hole in the section *aa*, for the plate could not give way through the section *bb* without shearing, at the same time the rivet through the section *aa*, and similarly the plate could not fail through the section *cc* without shearing the two rivets in the section *bb*.

Fig. 173.



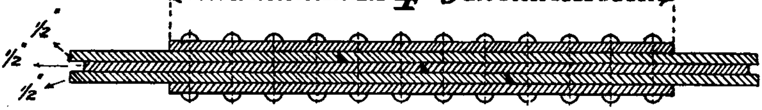


Fig. 174.

Example No. 6. A form of rivetting that has been largely adopted for uniting together several plates in the booms of girders is that known as chain rivetting, consisting of placing the rivets in parallel rows both longitudinally and transversely. Let it be required to design a chain rivetted grouped joint consisting of 3 plates ½ in. thick with double covers and four rows of rivets parallel with the length of the plate as shown in Figs. 173 and 174

Let T = total thickness of plates jointed $=3\times\frac{1}{2}=1\frac{1}{2}$ in.

d = diameter of rivet holes $=\frac{7}{8}$ in.

N = total number of rivets required for shearing strength.

n = number of rivets for bearing strength in each end group.

b = effective breadth of plate $=18\text{ in.}-(4\times\frac{7}{8})=14\frac{1}{2}$ in.

r = safe bearing resistance = 5 tons per square inch.

Let t = safe tensile or tearing resistance = 5 tons per square inch.
s = safe shearing resistance = 4 tons per square inch.

The bearing resistance of the rivet holes in the plates in each end group must at least be equal to the tearing resistance of the plates, hence

$$n = \frac{bt}{dr} = \frac{b}{d} = 14{\cdot}5 \div \tfrac{7}{8} = 16{\cdot}6.$$

The total number of rivets required to resist shearing so as to be equal to the safe tensile resistance of the jointed plates

$$= N = \frac{bTt}{{\cdot}7864d^2s} = \frac{14{\cdot}5 \times 1{\cdot}5 \times 5}{6 \times 4} = 45 \text{ rivets,}$$

and as there are three plates to join together there are three joints and four groups of rivets, and the number in each group $= 45 \div 4 = 11\frac{1}{4}$, but the number in each group must be some multiple of 4, hence the number must be 12.

In grouped joints the rivets may be equally divided between all the groups if all the jointed plates are of the same thickness; but in the end groups the number of rivets may be increased and reduced in the central groups, but they must not be less than $\frac{1}{2}n$ for 2 plates, $\frac{2}{3}n$ for 3 plates, $\frac{3}{4}n$ for four plates, $\frac{4}{5}n$ for 5 plates, and $\frac{5}{6}n$ for 6 plates.

For shearing 48 rivets must be provided, and as there are four groups the minimum number of rivets in the central groups $= (\frac{48}{4})\frac{2}{3} = 12 \times \frac{2}{3} = 8$ rivets, which will be arranged in two rows of 4 rivets each in each of the central groups. There will therefore remain to be distributed between the two end groups, $48 - (8 \times 2) = 32$ rivets, or 16 rivets to each end group, which will be arranged in four rows of four rivets each, as shown in Figs. 15 and 16.

The number of rivets required for bearing area has been shown to be 16·6, so that in practice the 16 rivets required to resist shearing would be also of sufficient bearing area.

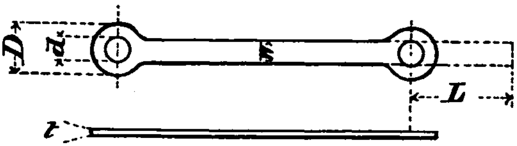
The pitch of the rivets both transversely and longitudinally is 4 inches and the length of the cover plates is 2 feet. The thickness of the cover plates should be equal to the thickness of the plates jointed. In this case the thickness of the cover plates should be at least $\frac{1}{2}$ inch and preferably $\frac{5}{8}$ inch.

Eye Bars.

Eye bars with pin connections are largely employed in bridge and roof structures, especially in American practice, in which all bridges of any magnitude have pin-connected joints.

The bars are generally made of steel and rarely of iron, and the eye is forged or upset in a steel die in such a manner as to develop the full strength of the bar.

In Figs. 175 and 176 and in the table underneath are set forth the dimensions and proportions of standard steel eye bars, made by Messrs. A. P. Roberts & Co., of the Pencoyd Iron and Steel works, Pa.



Figs. 175 and 176.

W Width of Bar.	t Minimum thickness of Bar.	D Diameter of Head.	d Diameter of Largest Pin.	L Additional length of Bar beyond centre of eye required to form one head.	
inch.	inch.	inch.	inch.	ft.	in.
3	$\frac{3}{4}$	7	3	1	3
3	$\frac{3}{4}$	8	$3\frac{7}{8}$	1	$5\frac{1}{2}$
4	$\frac{3}{4}$	$9\frac{1}{2}$	$4\frac{1}{8}$	1	$7\frac{1}{2}$
4	$\frac{3}{4}$	$10\frac{1}{2}$	$5\frac{1}{16}$	1	10
5	$\frac{3}{4}$	$11\frac{1}{2}$	$4\frac{13}{16}$	1	9
5	1	$12\frac{1}{2}$	$5\frac{11}{16}$	2	$0\frac{3}{4}$
6	$\frac{3}{4}$	$13\frac{1}{2}$	$5\frac{1}{2}$	1	11
6	1	$14\frac{1}{2}$	$6\frac{5}{16}$	2	$2\frac{1}{4}$
7	$\frac{7}{8}$	16	$6\frac{11}{16}$	2	$2\frac{3}{4}$
7	$\frac{15}{16}$	17	$7\frac{1}{2}$	2	$7\frac{3}{4}$
8	1	17	6	2	$2\frac{3}{4}$
8	$1\frac{1}{16}$	18	7	2	6
8	$1\frac{1}{8}$	$18\frac{1}{2}$	$7\frac{1}{2}$	2	$9\frac{3}{4}$

Mr. Theodore Cooper, in his general specifications for iron and steel railroad bridges and viaducts, provides that the pins shall be turned straight and smooth and shall fit the pin-holes within $\frac{1}{50}$ of an inch for less than $4\frac{1}{2}$ inches diameter, and for pins of a larger diameter the clearance may be $\frac{1}{32}$ of an inch.

The diameter of the pin shall not be less than two thirds the largest dimension of any tension member attached to it.

Sometimes the eye is thickened out as shown in Figs. 177 and 178.

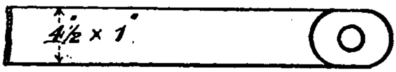


Fig. 177.

Fig. 178.

Let the tensile resistance of the bar = 5 tons per square inch $= 5 \times 4 \cdot 5 = 22 \cdot 5$ tons.

Let the shearing resistance = 4 tons per square inch and the bearing resistance 5 tons.

The pin is assumed to be in double shear. The tension on the bar has been determined to be 22·5 tons, the total working resistance of the pin should therefore be

$$\frac{22 \cdot 5}{4 \times 2 \text{ shears}} = 2 \cdot 81 \text{ inches sectional area,}$$

and the diameter $= \sqrt{\dfrac{2 \cdot 81 \times 4}{3 \cdot 1416}} = \sqrt{3 \cdot 56} = 1 \cdot 89$ inches.

Resistance to bearing $= 22 \cdot 5 \div 5 = 4 \cdot 5 = \dfrac{4 \cdot 5}{1 \cdot 89} = 2 \cdot 38$ inches.

Pin connected joints are theoretically more perfect than rivetted joints, inasmuch as the stresses in the several bars or members hinging upon the pins are uniformly distributed along the axes of those members, but in practice there is a difficulty in securing a tight fit of the pin in the eyes and there is always a hammering action attending the passage of a live load as the result of which the eyes or the holes in the bar have a tendency to grow larger and become elliptical in shape. Notwithstanding these practical objections pin connections with machine-made bearings and pins made a driving fit may be advantageously employed in many structures, especially on account of economy and simplicity of erection.

Least diameter of pin.

Let $\left.\begin{array}{l} t = \text{thickness of eye bar} \\ w = \text{width of eye bar} \end{array}\right\}$ Then tw = area of cross section.

St = working tensile stress of bar.

Then $tw\mathrm{S}$ = stress transmitted from the bar to the pin.

If d = diameter of pin, and if the thickness of the head of the bar is equal to the thickness of the body of the bar $= t$, we have td = bearing area of pin and $td\mathrm{S}c$ for its bearing resistance, $\mathrm{S}c$ being the compressive working stress.

Then the smallest permissible value of d

$$= td\mathrm{S}c = tw\mathrm{S}t \quad \text{or} \quad d = \frac{\mathrm{S}t}{\mathrm{S}c} w.$$

The ratio of the tensile working stress $\mathrm{S}t$ to the compressive working stress $\mathrm{S}c$ or $\frac{\mathrm{S}t}{\mathrm{S}c}$ may be taken at $\frac{3}{4}$. Then the least diameter of pin $= d = \frac{3}{4}w$.

The horizontal stress upon one side of the pin is equal to the area of all the bars on that side multiplied by the working stress of the bars, and the horizontal stress on the other side of the pin must be equal.

Let $\mathrm{M}h$ = the maximum moment of all the horizontal stresses

and $\mathrm{M}v$ = the maximum moment of all the vertical stresses.

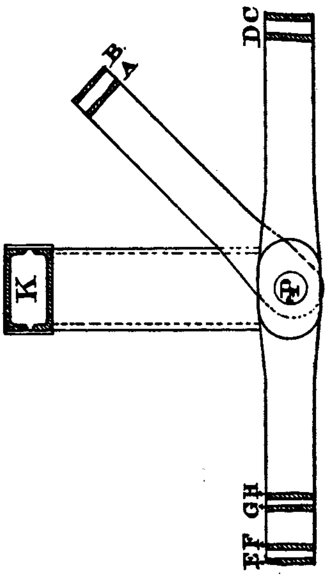


Fig. 179.

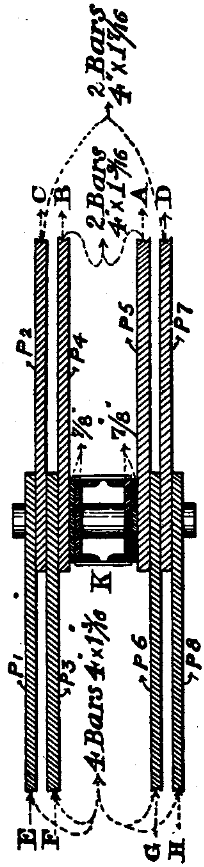


Fig. 180.

Then $\quad M =$ the resultant moment $= \sqrt{Mh^2 + Mv^2}$,

and $\quad d =$ diameter of pin,

$$= d = \frac{\sqrt[3]{32M}}{\pi R},$$

or $\quad d = 0{\cdot}089\sqrt[3]{M}$ (iron).

$d = 0{\cdot}081\sqrt[3]{M}$ (steel).

R being the working stress which for iron may be taken at 15,000 lbs., and for steel 20,000 lbs., according to Professor A. J. Du Bois.

The stress in every member must be resolved into its vertical and horizontal components, and must be considered as acting in each member along the centre line or axis so that the point of application of each vertical and horizontal component is at the centre of the bearing of the corresponding member.

Example. Let it be required to determine the size of pin required in Figs. 179 and 180, the tensile working stress being limited to 10,000 lbs.

On the one side of the pin there are four chord bars EFGH each 4 in. $\times 1\frac{3}{16}$ in., and on the other side 2 chord bars CD each 4 in. $\times 1\frac{7}{16}$. The inclined tie bars AB are $1\frac{9}{16}$ in. thick.

The thickness of the channel forming the one side of the vertical post and the cover plate is $\frac{7}{8}$ in. The vertical compression in the half post for full loading is 40,000 lbs.

$$P_1 = P_3 = 4 \text{ in.} \times 1\tfrac{3}{16} \text{ in.} \times 10{,}000 = 47{,}500.$$
$$P_2 \quad = 4 \quad \times 1\tfrac{7}{16} \quad \times 10{,}000 = 57{,}500.$$

The horizontal component of the stress in the tie P_4

$$= (47{,}500 \times 2) - 57{,}500 = 37{,}500 \text{ lbs.}$$

The distances from centre to centre of the bars $= l_1\ l_2\ l_3$, etc. Thus the distance from centre of E to centre of $C = l_1$, and from centre of C to centre of $F = l_2$, etc., or

,, distance from P_1 to $P_2 = l_1 = \frac{1}{2}(1\frac{3}{16} + 1\frac{7}{16}) = 1\frac{5}{16}$,

,, ,, ,, P_2 to $P_3 = l_2 = \frac{1}{2}(1\frac{3}{16} + 1\frac{7}{16}) = 1\frac{5}{16}$,

,, ,, ,, P_3 to $P_4 = l_3 = \frac{1}{2}(1\frac{3}{16} + 1\frac{9}{16}) + \frac{7}{8} = 2\frac{1}{4}$.

Then at P_2 the moment $P_1 l_1 = 47{,}500 \times 1\frac{5}{16} = 62{,}344$ inch lbs.

,, at P_3 the moment $= 62{,}344 + (P_1 - P_2)l_2 = 49{,}219$ inch lbs.

,, at P_4 the moment $= 49{,}219 + (P_1 - P_2 + P_3)l_3 = 133{,}594$ inch lbs.

The maximum horizontal moment Mh is therefore

133,594 inch lbs. $= 59{\cdot}6$, say 60 inch tons.

The vertical compression in the post is 40,000. Its lever arm

$$= \tfrac{1}{2}(1\tfrac{9}{16} + \tfrac{7}{8}) = 1\tfrac{7}{32}.$$

Then $Mv = 40{,}000 \times 1\frac{7}{32} = 48{,}750$ inch lbs. $= 21{\cdot}76$, say 22 inch tons.

The resultant moment

$$= M = \sqrt{Mh^2 + Mv^2} = \sqrt{60^2 + 22^2} = 63{\cdot}9 \text{ inch tons} = 143{,}136 \text{ inch lbs.}$$
$$d = \sqrt[3]{143{\cdot}136} \times 0{\cdot}089 = 52{\cdot}3 \times 0{\cdot}089 = 4\tfrac{5}{8} \text{ in. diameter.}$$

The bearing resistance of the pin should be equal to the greatest pressure upon it due to any plate through which it passes.

Let d = diameter of pin in inches,

t = thickness of plate through which it passes in inches,

Sc = working compressive stress = 6·25 tons.

Then dt = bearing area in square inches,

and $dtSc$ = bearing resistance of pin,

and $dtSc$ = stress transmitted through plate.

For a stress of 1 ton $dt = \frac{1}{6{\cdot}25}$ and lineal bearing in inches per ton of stress $= \frac{1}{6{\cdot}25d}$.

Let a stress of 25 tons be transmitted through a 12 in. × 3 in. channel whose web is ⅝ in. thick and the diameter of pin is 3 inches. It is required to determine the thickness of re-inforcing plate.

Then $\frac{1}{6{\cdot}25d} = \frac{1}{6{\cdot}25 \times 3} = 0{\cdot}0533$ inches and $0{\cdot}0533 \times 25$ tons $= 1{\cdot}3325$.

The thickness of the web of the channel is

⅝ in. = ·625 and 1·3325 − ·625 = ·707, say ·75

for the thickness of the cover plate, which might be expediently arranged by placing ⅜ in. plates one on each side as shown in Fig. 181.

The cover or re-inforcing plates are shown in black on both sides of the channel bar.

Fig. 181.

In designing a pin joint the bars should be so arranged that no two adjacent bars coupled on the same pin shall pull in the same direction unless by doing so the bending moment is reduced. The diagonal ties should be placed close to the vertical post as shown in Fig. 180.

CHAPTER X.

Deflection of Beams.

When a beam or girder is loaded it has been shown that the upper fibres or members undergo a compressive strain and the lower a tensile strain which produce a bending or deflection of the beam or girder.

So long as the beam is able to recover its original form and dimensions when the load or force producing deformation is removed, its strength is not impaired. All materials are more or less elastic, and the limit to which a beam may be loaded without losing its recovering power or without exceeding the limit of its elasticity depends upon the material of which the beam consists, the form in which it is employed, and the manner in which the load is applied.

The deflection test, as ordinarily applied to a girder under a moving load, may be accepted as a measure of its stiffness but not of its stability; for it is quite possible that a badly designed girder may not exhibit any abnormal degree of deflection under a test load, whereas at the same time it may be subject to a very severe if not indeed a dangerous strain.

In a trussed girder there is frequently a little play at the joints which under the first application of the test load may have the effect of increasing the deflection beyond the theoretical limit and produce a permanent set; but if that set is not increased by subsequent applications of the load, it does not necessarily detract from the strength of the structure.

An investigation of the laws of deflection is a little complex and beyond the aim and scope of this work, but an inquiry into the principles, and the application of the formulae which are the outcome of such investigation becomes indispensable in the designing of continuous girders and long columns. To this end we shall therefore devote our attention to the following rules and formulae.

When a properly designed beam or girder is acted upon by an external force it is uniformly strained throughout, and bent or deflected in the form of an arc of a circle, provided the limit of elasticity is not exceeded and the versed sine of the curve is the measure of the deflection.

Let Δ = maximum deflection in inches.

C and C$'$ = factors for various kinds of beams (see tables).

W = uniformly distributed or concentrated load.

l = length of beam in inches (in cantilevers from point of fixing to extreme point of loading).

I = moment of inertia.

E = modulus of elasticity = cast iron, ... 15,000,000 lbs.
wrought iron, 26,000,000 lbs
steel, ... 29,000,000 lbs.
timber, ... 1,500,000 lbs.

f = limiting intensity of resistance.

y = distance to outside of beam on the weaker side from neutral axis.

General formula for maximum deflection under any given load,

$$\Delta = \frac{\mathrm{CW}l^3}{\mathrm{EI}}.$$

Formula for deflection under any load giving a known limiting intensity of stress,

$$\Delta = \frac{\mathrm{C}'fl^2}{\mathrm{E}y}.$$

Table giving values of C and C′.

Particulars of Section.	Form of Support and Load.	Values of C	Values of C
Beams of uniform Cross Section.	Cantilever loaded at any point,	$\frac{1}{3}$	$\frac{1}{3}$
	Cantilever loaded uniformly,	$\frac{1}{8}$	$\frac{1}{4}$
	Beam supported, both ends loaded at centre,	$\frac{1}{48}$	$\frac{1}{12}$
	Beam supported, both ends loaded uniformly,	$\frac{5}{384}$	$\frac{5}{48}$
	Beam fixed, both ends loaded at centre,	$\frac{1}{192}$	
	Beam fixed, both ends loaded uniformly,	$\frac{1}{384}$	
Beams of uniform strength and depth.	Cantilever loaded at any point,	$\frac{1}{2}$	$\frac{1}{8}$
	Cantilever loaded uniformly,	$\frac{1}{4}$	
	Beam supported, both ends loaded at centre,	$\frac{1}{32}$	
	Beam supported, both ends loaded uniformly,	$\frac{1}{64}$	
Beams of uniform strength and breadth.	Cantilever loaded at any point,	$\frac{2}{3}$	$\frac{2}{3}$
	Cantilever loaded uniformly,	$\frac{1}{2}$	1
	Beam supported, both ends loaded at centre,	$\frac{1}{24}$	$\frac{1}{6}$
	Beam supported, both ends loaded uniformly,	·0178	·1426

As a practical example of the application of these formulae, let it be required to determine what uniformly distributed load W will deflect a timber beam 20 feet span, 8 inches wide, and 12 inches deep, $\frac{3}{4}$ of an inch.

Then $$\Delta = \tfrac{3}{4}\text{ in.} = \frac{CWl^3}{EI},$$

and $$W = \frac{EI\Delta}{Cl^3} = \frac{1{,}500{,}000 \times 1152 \times \cdot 75}{\frac{5}{384} \times (20 \times 12)^3} = 7200 \text{ lbs.}$$

The value of E is taken at 1,500,000.

The value of I for a beam

$$12\text{ in.} \times 8\text{ in.} = \frac{bd^3}{12} = \frac{8 \times 12^3}{12} = 1152.$$

To further illustrate the application of some of the equations employed in connection with the deflection of beams and girders, let AB (Fig. 182) represent a girder resting upon the two supports A and B whose neutral axis under the action of an external force has been bent from its original form, the straight line ACB, into an arc of a circle ADB whose versed sine $CD = \Delta$ the deflection.

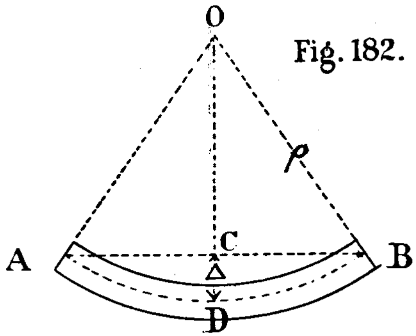


Fig. 182.

The centre of the circle is at the point O and the radius $OB = \rho$.

The dimension CD is so very small as compared with ρ that the length ADB may without appreciable error be assumed to be equal to the horizontal length ACB.

We have therefore a simple expression for the value of Δ when ρ is known, or of ρ when Δ is known.

$$\Delta = \frac{l^2}{8\rho}, \quad \rho = \frac{l^2}{8\Delta}.$$

Let $f =$ unit stress in the beam at a distance y from the neutral axis and $M =$ moment of flexure

Then $$\frac{f}{y} = \frac{M}{I} = \frac{E}{\rho} \text{ and } f = \frac{8\Delta E y}{l^2}.$$

It has been shown (*see* timber beams) that the moment of flexure M = the moment of resistance $R = \frac{fI}{y}$.

$$\therefore \quad M = \frac{EI}{\rho} \text{ and } \frac{1}{\rho} = \frac{M}{EI} \text{ and } E = \frac{M\rho}{I}.$$

Example. A timber beam 12 inches deep by 6 inches wide on a span of 20 feet deflects 1 inch under a load uniformly distributed of 4800 lbs. What is the modulus of elasticity?

Here $E = \frac{M\rho}{I}$.

Then $E = \frac{144,000 \times 7200}{864}$

$= 1,200,000.$

$$M = \frac{4800 \times (20 \times 12)}{8} = 144,000.$$

$$\rho = \frac{240^2}{8 \times 1} = 72,000.$$

$$I = \frac{6 \times 12^3}{12} = 864.$$

Let st, ft, dt and sc, fc, and dc respectively be the length, unit stress, and distance from the neutral axis of the extended and compressed outside fibres of the beam, illustrated in Fig. 182, $dt + dc = d =$ total depth of girder,

$$\frac{st}{l} = \frac{\rho + dt}{\rho} \text{ and } \frac{sc}{l} = \frac{\rho - dc}{l};$$

$$\therefore \quad \frac{st - sc}{l} = \frac{dt + dc}{\rho} = \frac{d}{\rho}.$$

Further

$$\frac{ft}{E} = \frac{st - l}{l} = \frac{dt}{\rho} \text{ and } \frac{fc}{E} = \frac{l - sc}{l} = \frac{dc}{\rho};$$

$$\therefore \quad \frac{ft + fc}{E} = \frac{st - sc}{l} = \frac{dt + dc}{\rho} = \frac{d}{\rho}.$$

Example. In a plate girder bridge of 60 feet effective span and 6 feet deep the stresses are as under. It is required to find the deflection and the difference of length between the extreme fibres.

Dead load $= 0{\cdot}8$ ton per foot run.
Live or test load $= 1{\cdot}75$ tons per foot run.
Ratio of test load to total load $= {\cdot}686$.
Maximum intensity of stress in tension flange $= ft = 6$ tons.
Maximum intensity of stress in compression flange $= fc = 5$ tons.
Mean intensity of stress in flanges $= 5{\cdot}5$ tons.
Mean intensity of stress due to test load $= f = 3{\cdot}77$ tons.

In this case the girder is assumed to be of uniform strength and breadth.

Then $$\frac{ft+fc}{\mathrm{E}}=\frac{st-sc}{l}=\frac{d}{\rho}.$$

But $ft+fc$ in this case may be taken as $f+f=3{\cdot}77$ tons $+\,3{\cdot}77$ tons $=7{\cdot}54$ tons.

Then $$\frac{7{\cdot}54\times2240}{30{,}000{,}000}=\frac{st-sc}{60\times12}=\frac{6}{\rho}=\frac{1}{1778{\cdot}9}=\frac{{\cdot}405}{720};$$

$\therefore\ st-sc={\cdot}405$ in the difference of length,

and $$\rho=\frac{720\times6}{{\cdot}405}=10{,}666 \text{ feet.}$$

Then $$\Delta=\frac{l^2}{8\rho}=\frac{60\times60}{8\times10{,}666}\times12={\cdot}506 \text{ in. the deflection.}$$

The value of Δ may also be obtained by the following simple expression:

$$\Delta=\frac{fl^2}{4\mathrm{E}d}=\frac{3{\cdot}77\times2240\times720^2}{4\times30{,}000{,}000\times72}={\cdot}506 \text{ in. as above.}$$

In the case of a cantilever as in Fig 183, bent under the action of external forces till the neutral axis is deflected from the horizontal line into the form of the circular arc AC, the following general expressions may be written down:

$$\frac{f}{y}=\frac{\mathrm{E}}{\rho}=\frac{2\Delta\mathrm{E}}{x^2} \quad\text{or}\quad f=\frac{2\Delta\mathrm{E}y}{x^2},$$

in which $x=$ the distance from A.

Then the value of Δ at any point distant x from $\mathrm{A}=\dfrac{x^2}{2\rho}$.

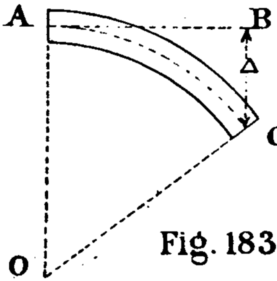


Fig. 183.

Example. Let it be required to determine the maximum deflection BC in a cantilever girder AC of wrought iron whose length is 40 feet, and depth 6 ft. 8 ins. and the unit stresses in the top and bottom flanges are 4 tons per square inch. The section of each flange is assumed to be throughout proportional to the bending moment.

Then
$$\frac{f}{y}=\frac{E}{\rho}$$
$$=\frac{4\times 2240}{40}=\frac{26,000,000}{116,071\cdot 4}; \quad \therefore \rho=116,071\cdot 4,$$

and
$$\Delta=\frac{x^2}{2\rho}=\frac{480\times 480}{2\times 116,071\cdot 4}=\cdot 992 \text{ inch.}$$

By another formula
$$\Delta=\frac{fl^2}{dE}=\frac{4\times 2240\times 480^2}{80\times 26,000,000}=\cdot 992 \text{ inch.}$$

Deflection of Trusses.

The deflection of any point in any framed structure due to any given load is obtained by the formula

$$\Delta=\Sigma\frac{pul}{E}.$$

This formula, known as the "pull over E" formula given in the *Theory and Practice of Modern Framed Structures*, by Professor Johnson, is much used in American practice.

Δ = deflection of point under consideration.

p = stress per square inch in any member for any given load.

P = total stress due to load in any member.

l = length of any member.

E = modulus of elasticity = 26,000,000 lbs. or 11,607 tons.

u = factor of reduction or the stress due to a load of one pound or one ton at centre.

A = area of cross section of member.

This formula may be alternatively written

$$\Delta=\Sigma\frac{Pul}{AE}.$$

Example. Let it be required to calculate the deflection at the centre of the truss shown in Fig. 184. The length of span is 48 feet, the depth 8 feet, and the total panel load $13\frac{1}{3}$ tons.

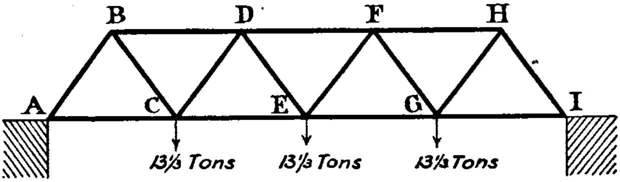


Fig. 184.

The stresses in the various members are tabulated as follows, and also the length, sectional area, and value of u for each member.

Member.	Total Stress due to load in tons=P.	Length of member in feet=l.	Stress due to a centre load of 1 ton=u.	Area of Section in sq. inches =A.	$\frac{Pul}{A}$.
AC	−15	12	− ·375	3	22·50
BD	+30	12	+ ·750	6	45·00
CE	−35	12	−1·125	7	67·50
DF	+40	6*	+1·500	8	45·00
AB	+25	10	+ ·625	5	31·25
BC	−25	10	− ·625	5	31·25
CD	+8·3	10	+ ·625	1·66	31·25
DE	−8·3	10	− ·625	1·66	31·25

Then $\frac{1}{2}\Sigma\frac{Pul}{A} = 305{\cdot}00.$

As a further example, let it be required to calculate the deflection at the centre of the Pratt truss illustrated in

* The truss being symmetrically loaded, the stresses for one half the truss only have been tabulated, and in the case of the centre chord member DF one half of its length has been taken.

If therefore we multiply the result of the last column by 2 and by 12 to reduce the value of l into inches and divide by E whose value in tons=11,607, we then have the value of

$$\Delta = \frac{305 \times 2 \times 12}{11{,}607} = {\cdot}63 \text{ inch.}$$

Fig. 185, the span being 60 feet, the depth 10 feet, and the total load 1 ton per foot run.

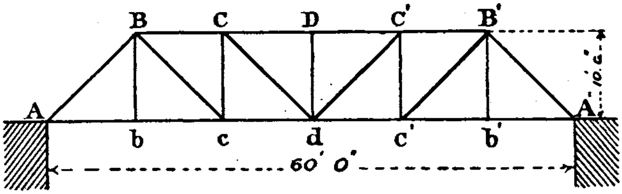


Fig. 185.

Member.	P.	l.	u.	A.	$\frac{Pul}{A}$.
				sq. inches.	
A*b*	−25·00	10·00	·5	5·0	25·0
bc	−25·00	10·00	·5	5·0	25·0
BC	+40·00	10·00	1·0	8·0	50 0
CD	+45·00	10·00	1·5	9·0	75·0
cd	−40·00	10·00	1·0	8·0	50·0
AB	+35·35	14·14	·707	7·07	50·0
B*c*	−21 21	14·14	·707	4·24	50·0
C*c*	+ 5·00	10·00	·500	1·00	25·0
C*d*	− 7·07	14·14	·707	1·41	50·0

Then $\frac{1}{2}\Sigma\frac{Pul}{A} = 400·0.$

The deflection will therefore be $\Delta = \frac{400 \times 2 \times 12}{11.607} = ·827$ inch.

In any Pratt truss the total deflection at the centre due to a full load is given by the formula

$$\Delta = \Sigma\frac{pul}{E} = \frac{pt + pc}{2Eh}\left[(n+2)\frac{nd^2}{4} + (n-2)h^2\right],$$

in which pt = average unit stress of tension members.
pc = average unit stress of compression members.
E = modulus of elasticity of material for all members.
h = height of truss in inches.
d = length of panel in inches.
n = number of panels.

In a girder or truss of proper design and construction the deflection should not exceed $\frac{1}{1200}$ part of its effective span, or

1 inch in every 100 feet after the girder has taken its permanent set, and in a timber beam $\frac{1}{360}$ part of its length, or 1 inch in every 30 feet.

Camber.

To provide for deflection it is usual in building a girder to give the flanges a certain amount of camber or upward convexity, otherwise when the intended maximum load is applied the flanges must sag and curve downwards below a horizontal or straight line. The initial camber, which is introduced more particularly on grounds of appearance, and not in any respect as an addition to the strength of the structure, should be equal to the total deflection due to the load, and also to any play at the joints.

The elastic deflection due to the load may be calculated by the rules we have laid down, and probably an addition of 25% will in most cases suffice to cover any set due to play at the joints.

In trussed girders the camber is given by making the upper chords and all compression members slightly longer, and the lower chord and tension members slightly shorter than the normal or designed length in the proportion of,

for compression members $\left(1+\frac{f}{E}\right)$ to 1,

and for tension members $\left(1-\frac{f}{E}\right)$ to 1.

The increase in length of the top chord over the bottom may also be obtained by the formula

$$i=\frac{8cd}{l} \quad \text{and} \quad c=\frac{il}{8d},$$

in which l = span in feet.
d = depth of girder in feet.
c = camber in inches.
i = length of top chord over bottom in inches.

Example. In a trussed girder built of steel 60 feet span and 6 feet effective depth, the deflection due to a flange stress of a mean intensity of 5 tons per square inch is ·672 inches; it is required to determine how much longer the top chord must be than the bottom chord so that when the maximum load is applied the girder may not deflect below a straight line.

$$i = \frac{8 \times \cdot 672 \times 6}{60} = \cdot 537 \text{ inch},$$

or

$$\left(1 + \frac{f}{E}\right) = 1 + \frac{5 \times 2240}{30,000,000} = 1 + \cdot 27,$$

$$\left(1 - \frac{f}{E}\right) = 1 - \frac{5 \times 2240}{30,000,000} = 1 - \cdot 27,$$

and $(1 + 0 \cdot 27) - (1 - 0 \cdot 27) = \cdot 54$ inch.

CHAPTER XI.

Continuous Girders.

A beam or girder is said to be continuous when it rests upon more than two supports. Thus the girder shown in Fig. 186 is continuous over two spans, and Fig. 187 over three spans.

Fig 186.

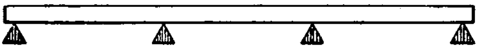


Fig 187

A simple example of a continuous girder is an ordinary railway rail, the spaces between the bearings on each sleeper representing the spans.

Continuous girders have been extensively used in bridge construction, but experience has shown that in many instances there has been no advantage because of the thermal stresses producing a large amount of expansion and contraction in long girders, and the liability to great changes in the stresses due to any slight neglect in the initial adjustment, or to subsequent alteration caused by settlement in the levels of the piers or supports. For that reason, in a bridge of any great length, girders of simple or independent spans are now invariably preferred to continuous girders.

The principles of continuity enter into the construction of swing bridges and of bridge floors, and it will be useful to investigate these principles so far only as is necessary to understand their application to practice.

In the following investigation it is assumed that the points of support are on one uniform plane, or so arranged that the

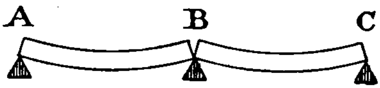


Fig. 188.

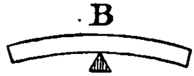


Fig. 189

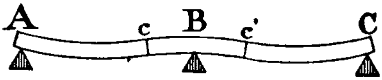


Fig. 190.

girder or beam may rest upon them without being strained. Fig. 188 represents two simple beams resting upon supports A, B and C and deflected under a uniform load, and Fig. 189 represents a beam supported at the centre of its length, or a balanced cantilever deflected at the ends under a uniform load.

If these two diagrams be combined the result will be as shown in Fig. 190, and this is the form that a continuous girder over two spans would assume under a uniform load.

In the segments Ac and cc' it will be observed that the curvature is reversed, and the stress changes in the upper flange or layer of fibres from compression in Ac to tension in cc', and in the lower flange or layer of fibres from tension in Ac to compression in cc'. The points cc', where the curvature changes, are therefore points of inflexion or contra-flexure where there is no bending moment.

When a beam or girder is supported at more than two points the vertical re-actions cannot be determined as for a simple beam on the principle of the lever, because the elasticity of the material must be taken into account.

For the purpose of illustrating this, let it be assumed that the girder AB, Fig. 191, is 96 feet long, composed of two equal

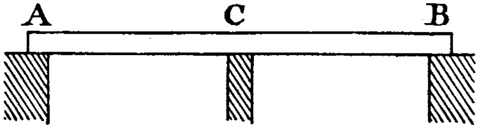


Fig. 191.

spans of 48 feet each, 4 feet effective depth, and each flange having a sectional area of 24 inches uniform throughout, and that it supports a uniformly distributed load of 1 ton per foot run.

Then let it first be assumed that the load is carried entirely by the end supports A and B when the bending moment at the centre $C = \frac{96 \times 96}{8 \times 4} = 288$ tons, and $\frac{288}{24} = 12$ tons, as the intensity of the compressive stress in the upper flange at C.

The deflection at the centre C due to this condition of loading will be $\Delta = \frac{c'fl^2}{Ey} = \frac{5 \times (12 \times 2240) \times (96 \times 12)^2}{48 \times 26,000,000 \times (2 \times 12)} = 5{\cdot}95$ inches.

If again it be assumed that the whole load is supported by the central pier C as in the case of a balanced cantilever, the bending moment at the point of support C will be

$$\frac{48 \times 24}{4} = 288 \text{ tons, and } \frac{288}{24} = 12 \text{ tons,}$$

as the intensity of the tensile stress in the upper flange at C.

The deflection of the ends A and B will therefore be

$$\Delta = \frac{1 \times (12 \times 2240) \times (48 \times 12)^2}{4 \times 26,000,000 \times (2 \times 12)} = 3{\cdot}57 \text{ inches.}$$

It will thus be seen that a small variation in the level of the supports is the cause of very considerable change in the stresses. In the preceding example, it is shown that a displacement of $9\frac{1}{2}$ inches in the level of the central pier, that is to say from 5·95 inches below to 3·57 inches above its normal level, would reverse the stress from + 288 tons to − 288 tons.

If now we suppose the three supports to be effective and to be on the same level, then the girder will, under a uniformly distributed load, assume the outline shown in Fig. 190 in which the girder is divided into three segments Ac, cc' and c'C. In the central segment cc', which acts as a balanced cantilever, the stress in the upper flange is tensile, and in the two segments Ac and c'C the stress is compressive, as in the case of two simple beams supported at their ends Ac and c'C, so that virtually the spans AB and BC are reduced to Ac and c'C. At the points of contra-flexure c and c' there is evidently no stress.

The strains on the flanges must vary with the position of the point of contra-flexure, which changes with the condition of the loading and the mode of its application. The limit of this variation of the point of contra-flexure must therefore be ascertained for each segment under the different conditions of loading.

Theorem of Three Moments.

Let M_1, M_2, M_3, etc. = the bending moments at the consecutive points of support, 1, 2, 3, etc.

l_1, l_2, l_3, = the length of spans between the supports, 1, 2, 3, etc.

w_1, w_2, w_3, = the unit loads on l_1, l_2, l_3, etc.

R_1, R_2, R_3, = the vertical re-actions of the supports.

S_1, S_2, S_3, = the shear immediately to the right of any support.

S'_2, S'_3, S'_4, = the shear immediately to the left of any support.

Then the relation between the bending moments at the points of support is expressed by the equation

$$M_1 l_1 + M_2(l_1 + l_2) + M_3 l_2 = \tfrac{1}{4}(w_1 l_1^3 + w_2 l_2^3),$$

and this is Clapeyron's Theorem of Three Moments.

This theorem enables an equation to be written down for each support of a continuous girder, except at the ends where there is no moment, and from it the following equations are deduced.

1. Continuous girder of two unequal spans.

$$M_2 = \frac{w_1 l_1^3 + w_2 l_2^3}{8(l_1 + l_2)}.$$

$$S_1 = \tfrac{1}{2} w_1 l_1 - \frac{M_2}{l_1}.$$

$$S_2 = \tfrac{1}{2} w_2 l_2 + \frac{M_2}{l_2}.$$

$$S'_2 = S_1 - w_1 l_1.$$

$$S'_3 = S_2 - w_2 l_2.$$

$$R_1 = S_1.$$

$$R_2 = S_2 + S'_2.$$

$$R_3 = S'_3.$$

2. Continuous girder of two equal spans.

In this case the expression for determining the value of M_2 reduces to

$$M_2 = \frac{(w_1 + w_2) l_2^2}{16}.$$

All the other equations are the same as for case 1.

3. Continuous girder of three unequal spans.

$$M_2 = \frac{w_1 l_1^3 (2l_2 + 2l_3) + w_2 l_2^3 + (2l_3 + l_2) - w_3 l_3^3 l_2}{16(l_1 + l_2)(l_2 + l_3) - 4l_2^2}.$$

$$M_3 = \frac{w_1 l_1^3 l_2 + w_2 l_3^3 (2l_1 + l_2) + w_3 l_3^3 (2l_1 + 2l_2)}{16(l_1 + l_2)(l_2 + l_3) - 4l_2^2}.$$

$$S_1 = \tfrac{1}{2} w_1 l_1 - \frac{M_2}{l_1}.$$

$$S_2 = \tfrac{1}{2} w_2 l_2 + \frac{M_2 - M_3}{l_2}.$$

$$S_3 = \tfrac{1}{2} w_3 l_3 + \frac{M_3}{l_3}.$$

$$S'_2 = S_1 - w_1 l_1.$$

$$S'_3 = S_2 - w_2 l_2.$$

$$S'_4 = S_3 - w_3 l_3.$$

$$R_1 = S_1.$$

$$R_2 = S_2 + S'_2.$$

$$R_3 = S_3 + S'_3.$$

$$R_4 = S_4.$$

4. Continuous girder of three spans, of which the two side spans are equal.

$$M_2 = \frac{2w_1(l_1^4 + l_1^3 l_2) + w_2(l_2^4 + 2l_1 l_2^3) - w_3(l_1^3 l_2)}{16l_1^2 + 12l_2^2 + 32l_1 l_2}.$$

$$M_3 = \frac{-w_1 l_1^3 l_2 + w_2(l_2^4 + 2l_1 l_2^3) + 2w_3(l_1^4 + l_1^3 l_2)}{16l_1^2 + 12l_2^2 + 32l_1 l_2}.$$

$$S_1 = \tfrac{1}{2} w_1 l_1 - \frac{M_2}{l_1}.$$

$$S_2 = \tfrac{1}{2} w_2 l_2 + \frac{M_2 - M_3}{l_2}.$$

$$S_3 = \tfrac{1}{2} w_3 l_3 + \frac{M_3}{l_1}.$$

$$S'_2 = S_1 - w_1 l_1.$$

$$S'_3 = S_2 - w_2 l_2.$$

$$S'_4 = S_3 - w_3 l_1.$$

$$R_1 = S_1.$$

$$R_2 = S_2 + S'_2.$$

$$R_3 = S_3 + S'_3.$$

$$R_4 = S_4.$$

5. Continuous girder of three equal spans.

$$M_2 = \frac{(4w_1 + 3w_2 - w_3)l^2}{60}.$$

$$M_3 = \frac{(-w_1 + 3w_2 + 4w_3)l^2}{60}.$$

The other equations are the same as for cases 3 and 4.

6. Continuous girder of four spans, the two central spans being of equal lengths, and the two side spans of equal lengths.

$$M_2 = \frac{w_1(8l_1^4 + 7l_1^3l_2) + w_2(5l_2^4 + 6l_1l_2^3) - w_3(l_1l_2^3 + l_2^4) + w_4l_1^3l_2}{4(16l_1^2 + 12l_2^2 + 28l_1l_2)}.$$

$$M_3 = \frac{-w_1l_1^3 + w_2(2l_1l_2^2 + l_2^3) + w_3(2l_1l_2^2 + l_2^3 - w_4l_1^3)}{2(16l_1 + 12l_2)}.$$

$$M_4 = \frac{w_1l_1^3l_2 - w_2(l_1l_2^3 + l_2^4) + w_3(5l_2^4 + 6l_1l_2^3) + w_4l_1^3l_2}{4(16l_1^2 + 12l_2^2 + 28l_1l_2)}.$$

$$S_1 = \tfrac{1}{2}w_1l_1 - \frac{M_2}{l_1}.$$

$$S_2 = \tfrac{1}{2}w_2l_2 + \frac{M_2 - M_3}{l_2}.$$

$$S_3 = \tfrac{1}{2}w_3l_3 + \frac{M_3 - M_4}{l_2}.$$

$$S_4 = \tfrac{1}{2}w_4l_4 + \frac{M_4}{l_1}.$$

$$S'_2 = S_1 - w_1l_1$$

$$S'_3 = S_2 - w_2l_2.$$

$$S'_4 = S_3 - w_3l_2.$$

$$S'_5 = S_4 - w_4l_1.$$

$$R_1 = S_1.$$

$$R_2 = S_2 + S'_2.$$

$$R_3 = S_3 + S'_3.$$

$$R_4 = S_4 + S'_4.$$

$$R_5 = S_5.$$

7. Continuous girder of four equal spans.

In this case the equations for determining the values of M reduce to

$$M_2 = \frac{(15w_1 + 11w_2 - 3w_3 + w_4)l^2}{224}.$$

$$M_3 = \frac{(-w_1 + 3w_2 + 3w_3 - w_4)l^2}{56}.$$

$$M_4 = \frac{(w_1 - 3w_2 + 11w_3 + 15w_4)l^2}{224}.$$

Similarly equations might be written for any greater number of spans, but it is thought that the equations given are sufficient to deal with any ordinary case of a continuous girder that is likely to arise in practice.

A simple and elegant method of arriving at the stresses in continuous girders, by a graphic process has been introduced by Professor Claxton Fidler, the application of which will be readily understood by the aid of the following example.

In Fig. 192, the girder AB is one of two girders of a railway bridge continuous over the central pier C, and consisting of two equal spans AC and CB each 48 feet in length.

The dead load carried by the girder is one-half ton per foot run, and the live or rolling load 1 ton per foot run.

The girder being continuous, two conditions of loading have to be considered, viz.

1. The two spans fully loaded.
2. The one span only covered by the live load.

The second case will be investigated first.

In Fig. 193 the two spans are laid down to any convenient scale, and the parabolic curves of moments AEC and CGB are drawn for each span as for a simple beam supported at the ends, the centre ordinates DE and FG being made equal to $\frac{wl^2}{8}$ in each case, the values of these ordinates in this case being respectively as follows:

$$\text{DE} = \frac{1{\cdot}5 \times 48^2}{8} = 432 \text{ tons, and FG} = \frac{{\cdot}5 \times 48^2}{8} = 144 \text{ tons.}$$

Then divide each span into three equal parts and from each point of division draw the verticals hj, $h_1 j_1$ etc., and draw the lines AE, EC and CG, GB, marking the points L and O where the lines CE and CG respectively cross the verticals $h_1 j_1$ and $h'' j''$.

Assume any point K in the vertical line CH drawn through the centre of the centre pier, and locate it so that the line AK

Continuous Girder of two equal Spans.

Fig. 193.

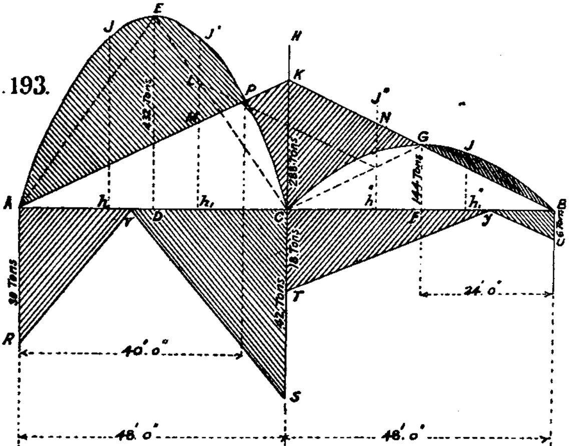


Fig. 192.

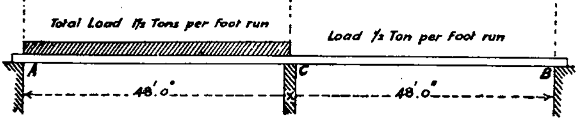
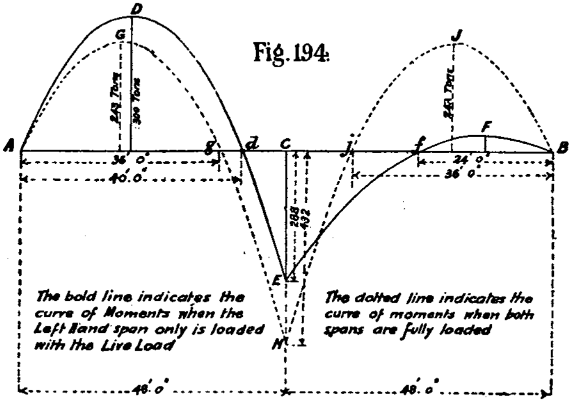


Fig. 194.



shall cut the vertical $h_1 j_1$ so much below the point L as BK shall cut the vertical $h'' j''$ above O, that is to say, the distances LM and NO must be equal to one another.

Then will the points P and G be the points of contra-flexure, and the bending moment over the centre pier is obtained by scaling the distance CK.

Having determined the points of contra-flexure, the shearing forces which are shown by the hatched areas below the horizontal line AB at the re-actions at the abutments and piers can be at once obtained as for a simple beam.

In Fig. 194 the diagram is reduced to a horizontal base, that is to say, the curves of moments are set off by ordinates from the horizontal base line ACB instead of from AKB in Fig. 193, for the purpose of more clearly showing curves of moments.

In Figs. 195 to 200 the bending moments and shearing forces in a continuous girder of three spans are given, the central span being twice the length of the side spans.

In this case the base line A*bc*D is drawn in the same way as in diagram 193, but the adjacent spans being unequal, the ordinates Hh, ... $H_3 h_3$ must be inversely proportional to the span; thus in Fig. 196 the span AB being half the length of the span BC, the ordinate Hh must be double the length of $H_1 h_1$, that is to say, the base line A*bc*D must pass above the point H at twice the vertical distance it passes below the point H_1.

The several diagrams give the bending moments and shearing forces due to the several conditions of loading indicated in Fig. 199. In Fig. 200 the several curves of moments are reduced to a horizontal base in one diagram, so as to exhibit at once the bending moments at any section of the girder due to all four cases of loading.

A similar diagram might be readily constructed to indicate the shearing forces.

Continuous Girder of three Spans.

Fig. 195.

Case 1.

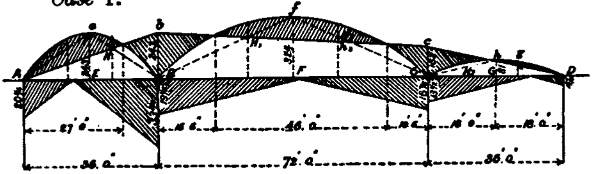


Fig. 196.

Case 2.

The shaded areas above the Horizontal Line indicate the Bending Moments

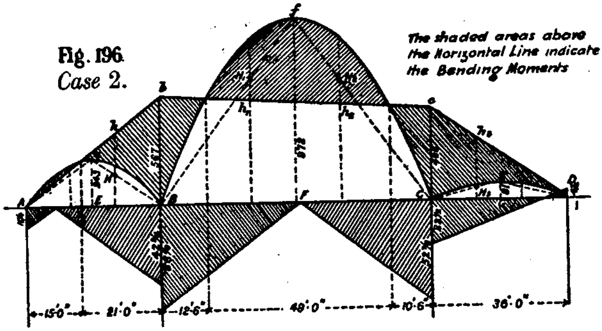
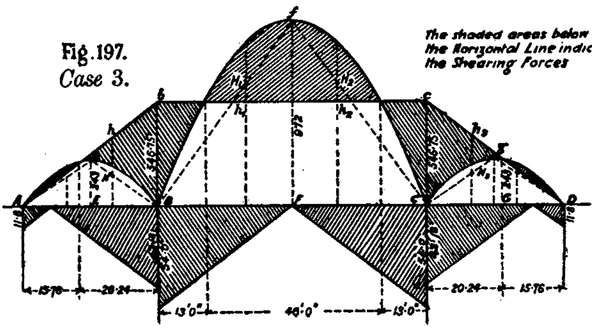


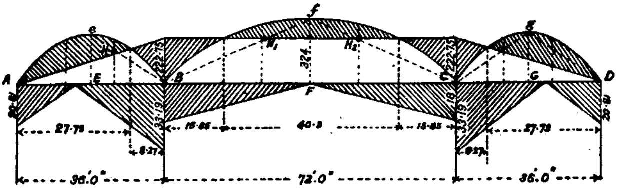
Fig. 197.

Case 3.

The shaded areas below the Horizontal Line indicate the Shearing Forces



Case 4. Fig. 198.



Case 1.
Case 2.
Case 3.
Case 4.

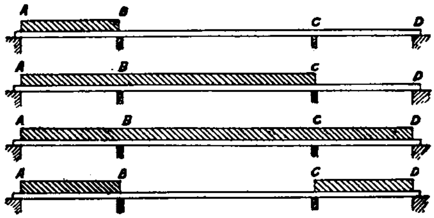
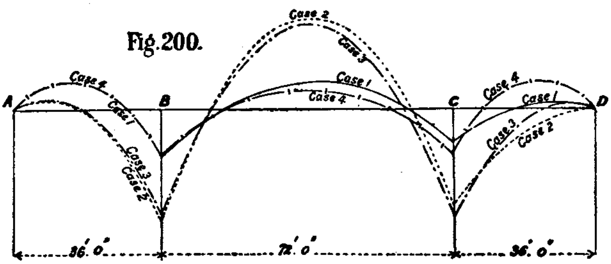


Fig. 199.

Fig. 200.



The following is given as an example of the application of the preceding formulae in the calculations of a plate girder continuous over two spans of 48 feet each, as shown in Fig. 192. The effective depth of the girder is 4 feet. The live load is taken at 1 ton per foot run and the dead load at 10 cwts. per foot run.

It is required to determine the bending moment over the central pier, the shearing forces, and vertical re-actions at each point of support, and from these values the points of contra-flexure and bending moments in the two spans due to the following conditions of loading:

Case 1. The span AC loaded and BC unloaded.
2. Both spans fully loaded.

Case 1. The bending moment over the central pier (see Equation 2 of the Theorem of Three moments)

$$=M_2=\frac{(w_1+w_2)l_2^2}{16}=M=\frac{(1{\cdot}5+0{\cdot}5)48^2}{16}=288 \text{ foot-tons.}$$

The effective depth of the girder is 4 feet and the bending moment $=288 \div 4 \qquad =72$ tons.

$$S_1=\tfrac{1}{2}w_1l_1-\frac{M_2}{l_1}=\tfrac{1}{2}\times 1\tfrac{1}{2}\times 48-\frac{288}{48}=30 \text{ tons.}$$

$$S_2=\tfrac{1}{2}w_2l_2+\frac{M_2}{l_2}=\tfrac{1}{2}\times \ \tfrac{1}{2}\times 48+\frac{288}{48}=18 \text{ tons.}$$

$$S_2'=S_1-w_1l_1=30-(1\tfrac{1}{2}\times 48) \qquad =-42 \text{ tons.}$$

$$S_3'=S_2-w_2l_2=18-\ (\tfrac{1}{2}\times 48) \qquad =-\ 6 \text{ tons.}$$

$$R_1=S_1 \qquad =30 \text{ tons.}$$

$$R_2=S_2+S_2'=18+42 \qquad =60 \text{ tons.}$$

$$R_3=S_3 \qquad =\ 6 \text{ tons.}$$

The re-action R_1 at the support $A=30$ tons and the length of span; in the case of a simple beam supported at both ends, over which a uniformly distributed load of $1\frac{1}{2}$ tons per foot run must extend to produce this re-action, $=\frac{30\times 2}{1{\cdot}5}=40$ feet, which is the distance of the point of contra-flexure from the support A in the span AC.

Similarly the point of contra-flexure in the span BC is distant from the support $B=\frac{6\times 2}{\cdot 5}=24$ feet.

The bending moments on these segments Ad and Bf (Fig. 194), which are the same as for simple beams, are as under:

$$\text{Bending moment in A}d = \frac{40 \times 40 \times 1{\cdot}5}{8 \times 4} = 75 \text{ tons.}$$

$$\text{Bending moment in B}f = \frac{24 \times 24 \times 0{\cdot}5}{8 \times 4} = 9 \text{ tons.}$$

Bending moment over central pier in the segment dCf has already been determined to be 72 tons.

The bending moments in the segments Ad and Bf are positive, that is to say, as in the case of a simple beam, the lower flange of the girder being subject to a tensile stress, and the upper flange to a compressive stress.

In the segment dCf the bending moment is negative, the stress being reversed, the upper flange undergoing tension and the lower flange compression.

Case 2. The bending moment over the central pier

$$= M = \frac{(w_1 + w_2) l_2^2}{16}.$$

$$M = \frac{(1{\cdot}5 + 1{\cdot}5) 48^2}{16} = 432 \text{ foot-tons,}$$

and divided by the depth of girder $= \frac{432}{4} = 108$ tons.

$$S_1 = \tfrac{1}{2} \times 1\tfrac{1}{2} \times 48 - \frac{432}{48} = 27 \text{ tons.}$$

$$S_2 = \tfrac{1}{2} \times 1\tfrac{1}{2} \times 48 + \frac{432}{48} = 45 \text{ tons.}$$

$$S_2' = 27 - 1{\cdot}5 \times 48 \quad = - \ 45 \text{ tons.}$$

$$S_3' = 45 - 1{\cdot}5 \times 48 \quad = - \ 27 \text{ tons.}$$

$$R_1 = 27 \text{ tons.}$$

$$R_2 = 45 + 45 \text{ tons} \quad = 90 \text{ tons.}$$

$$R_3 = 27 \text{ tons.}$$

The two spans in this case being of equal lengths and symmetrically loaded, the point of contra-flexure will be equally distant from the abutments in both spans. The re-action is 27 tons.

Hence $$\frac{27 \times 2}{1 \cdot 5} = 36 \text{ feet.}$$

The bending moments on the segments Ag and Bj will be

$$= M = \frac{36 \times 36 \times 1 \cdot 5}{4 \times 8} = 60\tfrac{3}{4} \text{ tons.}$$

The bending moment over the central pier in the segment gCj has been determined to be 108 tons.

In the diagram in Fig. 194, the maxima positive and negative bending moments may be seen at a glance, and the condition of loading which produces the greatest stress at any part of the girder.

In designing the girder the maximum positive bending moment to be provided for is 75 tons, which, if we allow a working stress of $4\frac{1}{2}$ tons per square inch, would be met by a net sectional area of $75 \div 4\frac{1}{2}$ in the tension flange $= 16\frac{2}{3}$ square inches.

The maximum negative bending moment is 108 tons which at the same unit stress would require in the tension flange a net sectional area of $108 \div 4\frac{1}{2} = 24$ square inches.

The flanges at the centre of the girder, that is to say, in the segment gCj, for a length of 12 feet on either side of the centre support CE would have to be strengthened by an additional plate, the net sectional area of which must not be less than $24 - 16\frac{2}{3} = 7\frac{1}{3}$ square inches.

The greatest shear occurs over the central pier, amounting to 45 tons, which at the same unit stress would require the effective section of the web to be not less than $45 \div 4 \cdot 5 = 10$ square inches.

CHAPTER XII.

Loads in Bridges. Wind Pressure, etc.

In designing a bridge the loads and forces to be considered are:

(1) The weight of the structure itself, which is termed the dead, static or permanent load.

(2) The load moving over the bridge, termed the live, rolling, moving or variable load.

(3) The pressure of the wind acting upon the structure.

(4) The effect of changes of temperature.

Dead Load.

The dead load consists of:

(*a*) In the case of railway bridges—the main girders, cross girders, lateral bracing, deck or floor, handrails, rails, chairs, sleepers or rail-bearers, and in many instances the ballasting laid continuously over the bridge.

(*b*) In the case of highway bridges—the main girders, cross girders, lateral bracing, deck or floor, handrails, kerbs or wheel guards, the roadway generally supported on a bed of concrete, which may consist of either stone or wood sets, or road metal, and occasionally gas and water mains.

In either case, the arrangement of the floor system must largely depend upon the type of bridge, its span and width, the character and intensity of the live load, the assigned working stresses, and other considerations, so that an adequate estimate of the weight can only be obtained by actual calculation for any particular case.

The live load having a known value, and the weight of the floor system, having, it is assumed, been obtained by computation, it only remains to determine the weight of the main girders, but this, until the dimensions of the main girders are settled, is an unknown quantity. It is therefore necessary to assume some approximate estimate of their weight, before the dimensions of the main girders can be calculated, and if, as the result of the calculations, the difference between the resultant and assumed weights is at all appreciable, a correction must be made in the assumed weights, and the calculations revised, until, by a system of trial and error, the result is satisfactory. Engineers frequently, in estimating the weight of main girders, draw upon their experience of the weights of similar structures, but several formulae have been proposed which give sufficiently accurate results.

For plate girders, Professor Cawthorne Unwin has proposed the following formula, which gives very reliable results:

Let W = total external distributed load in tons (exclusive of the weight of girder),

W_1 = weight of girder itself in tons,

l = clear span in feet,

d = effective depth in feet,

s = average stress in tons per square inch on the gross section of the booms at the centre,

r = ratio of span to depth $= \frac{l}{d}$,

C = a constant whose value may be taken at 1500 for bridges of moderate span,

A = gross area of two booms at centre in square inches.

Then
$$W_1 = \frac{Wl^2}{Cds - l^2} = \frac{Wlr}{Cs - lr}.$$

To deduce C from girders of known weight,
$$C = \frac{(W - W_1)l^2}{W_1 ds} = \frac{4Al}{W_1}.$$

An approximate estimate of the weight of a main girder may be obtained by the following expression, which has been proposed by Professor Henry Adams:
$$W = \frac{Wl}{375} \times \sqrt{\frac{l}{d}}.$$

l, length of span in feet.

d, depth in inches.

W, total weight on girder in tons.

w, weight of girder.

In the preceding examples in Chapters II., III., IV., etc., it has been assumed that the dead load is concentrated upon the upper boom, or chord, in deck bridges, and upon the lower boom in through bridges, and for all practical purposes this is sufficiently accurate; but if great exactitude is desired, the weight of the girder itself should be divided between the two booms, and the weight of the floor system added to the boom upon which it rests.

Live Load.

Railway Bridges. The maximum live or rolling load that is liable to pass over a railway bridge, consists of a train of two locomotive engines of the heaviest type, followed by loaded waggons covering the entire bridge. For spans up to 100 feet, it is usual to assume that the bridge is covered by locomotives from end to end.

The weight of such a train is transmitted to the girders in the form of a moving system of concentrated loads at various distances apart, and by following in detail the stresses produced by these loads in the various members, the maximum stress in each member can be arrived at This method of calculation, known as the "concentrated load system," which has been extensively used in American practice, gives accurate results for any particular case, but it involves a considerable amount of labour in tabulating the stresses for the different positions of the live load

An alternative method employed in American practice is to consider the train as a uniformly distributed live load of one ton, and sometimes 3,000 lbs per foot run, headed by a concentrated load equal to the engine excess, and for each member of the bridge, to take the condition of loading that produces the greatest stress in that member

On British and continental railways it is customary to adopt a uniformly distributed live load which would produce stresses equivalent to those due to the heaviest engines and trains passing over the bridge. This method, although not strictly exact, will give results that are sufficiently accurate for all practical purposes, and it saves a considerable amount of labour involved in the computations by the more complicated method of taking the actual wheel loads.

In a paper appearing in the *Proceedings of the Institute of Civil Engineers*, by Mr. W. B. Tarr, Assoc M.I.C.E., the use of uniformly distributed moving loads on railway bridges has been exhaustively investigated, and the effect of impact caused by the sudden application of the load.

In that paper, the author had calculated the bending moment curves for the heaviest locomotives in use on all of the principal English railways, and by enveloping these curves for the various positions of the moving load in a parabola, the following table of equivalent uniformly distributed loads had been arrived at:

Main Girders.

Spans in feet.	Maximum uniformly distributed loads in tons per foot run.	Suggested percentage allowance to be added for impact.	Suggested maximum uniformly distributed live loads per foot run.
5·0	7·60	30·00	9·88
7·5	5·55	27·50	7·07
10·0	4·85	25·00	6·06
15·0	3·74	22·50	4·58
20·0	3·20	20·00	3·84
25·0	2·81	17·50	3·30
30·0	2·63	15·00	3·01
35·0	2·48	14·75	2·84
40·0	2·40	14·50	2·75
45·0	2·32	14·25	2·65
50·0	2·24	14·00	2·55
60·0	2·17	13·50	2·46
70·0	2·11	13·00	2·38
80·0	2·06	12·00	2·30
90·0	2·01	11·00	2·23
100·0	1·97	10·00	2·16

Cross Girders.

Distance apart in feet.	Maximum concentrated load in tons on each single line in tons.	Suggested percentage allowance for impact.	Suggested maximum concentrated loads.
3	19·00	50·0	28·50
4	19·00	47·5	28·03
5	19·00	45·0	27·55
6	19·00	42·5	27·08
7	19·00	40·0	26·60
8	21·50	35·0	29·02
9	23·50	30·0	30·55
10	25·11	25·0	31·39

A similar system of uniformly distributed live loads is being used on the Midland Railway, a tabulated statement of which is given hereunder.

No provision is made in this latter table for impact, but apart from this, it will be seen that the two tables are very closely in accord.

Span in feet.	Uniformly distributed load in tons.	Span in feet	Uniformly distributed load in tons.
8	44	80	165
11	50	85	174
15	56	90	184
20	63	95	193
25	70	100	203
30	77	105	212
35	85	110	222
40	93	115	231
45	101	120	241
50	109	125	250
55	118	130	260
60	127	135	269
65	136	140	279
70	146	145	288
75	155	150	297

Highway Bridges. In the case of highway bridges it is difficult to define what the conditions of loading should be, in view of the frequent and growing use of steam road rollers and traction engines. This has lately become so prominent a question that the establishment of a rational system of loads for various classes of bridges of varying spans would to the engineer greatly simplify his labours, and to the County or District Council be a great advantage, if it only put a definite interpretation upon that expression which is so generally used "The ordinary traffic of the district."

For general purposes highway bridges may be divided into three classes viz.:

(1) For town and suburban traffic.

(2) For main roads in country districts.

(3) For parish roads and country highways where the traffic is very light.

1st. **Town and Suburban Bridges.** In the case of town and suburban bridges there is in addition to the vehicular traffic a constant and extensive pedestrian traffic, and the

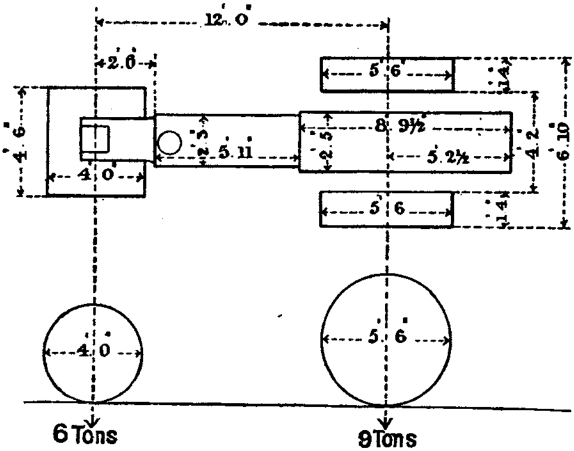


Fig. 201.

greatest live load is generally produced by a crowd of people passing over the bridge, which if densely packed together may be estimated at 140 lbs. per square foot. The vehicular traffic may consist of large boilers or heavy castings weighing from 25 to 30 tons, so that in designing a bridge provision should be made for supporting in addition to a densely packed crowd of people a concentrated load of 25 tons supported on two axles 12 feet centres, wheel centres 5 feet gauge, allowing for the displacement of the crowd that would be occupied in floor area by such concentrated weight.

2nd. For bridges carrying main roads in country districts the pedestrian traffic must essentially be lighter, and a limit of 1 cwt per square foot of floor area would sufficiently meet the requirements. The heaviest moving load that is likely to pass over such a bridge would be a traction engine or road roller, the weight of which may be taken from the diagram shown in Fig. 201.

3rd. For bridges carrying parish and country highways on which traction engines are not employed, the heaviest load consists of a timber carriage or limber the weight of which may be taken at 6 to 7 tons, supported on a wheel base of 15 to 25 or 30 feet measured from centre to centre of axles. In addition to this the bridge should be capable of carrying a crowd of people which may be estimated at one cwt. per superficial foot of floor area.

On a road bridge a vehicle or traction engine does not follow any defined track, and it is liable at times to pass closely to the side of the main girder as shown in Fig. 202, in which case that girder is strained to a greater extent than if the load were confined to the centre of the bridge. The web stresses in consequence are greater, and in designing the girders of a road bridge regard should be had to this contingency, especially in the case of any type of lattice or trellis girder in which the loads are applied at the panel points.

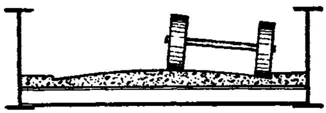


Fig. 202.

Wind Pressure.

In bridges of any considerable height and span, especially if built in exposed situations, the pressure of the wind is often very considerable, and the stresses produced in the structure due to this pressure must be provided for in calculating its strength.

The velocity of the wind is determined by an anemometer, and the pressure may be deduced from the velocity by the following simple formulae, the one introduced by Smeaton based upon Rouse's experiments and the other by Mr. W. H. Dines, as the result of a long and exhaustive series of experiments carried out at Hersham, Surrey.

Smeaton's formula $\quad P = \frac{V^2}{200}.$

Dines' formula $\quad P = V^2 \times 0{\cdot}003.$

P = pressure in lbs. per square foot.

V = velocity in miles per hour.

It will be seen on comparing these formulae that they give widely different values.

According to British Board of Trade regulations, a maximum normal wind pressure of 56 lbs. per square foot of surface exposed should be provided for. The exposed surface to be calculated as including the height of a train multiplied by the length of the girder, and the actual vertical surface of that portion of one girder which may be below rail level or more than the height of a train above rail level.

A wind pressure of 30 to 32 lbs. per square foot would be sufficient to overturn railway carriages and derail trains, but accidents of this nature are fortunately of very rare occurrence; it would therefore appear that to adopt so high a limit of pressure as 56 lbs. per square foot is unnecessary.

In American practice the wind is taken as acting in either direction horizontally, (*a*) when the train is on the bridge at a pressure of 30 lbs. per square foot on the exposed surface of all trusses and the floor as seen in elevation in addition to a train surface of 10 feet average height, the bottom to be taken at 2 ft. 6 in. above base of rail; (*b*) if the train is not on the bridge the pressure is estimated at 50 lbs. per square foot on the exposed surface of all trusses and the floor system.

The greatest result to be taken.

For highway bridges in some instances, the surface exposed to wind pressure is calculated at twice the superficial area of the actual surface of the truss, and multiplied by 35 to 40 lbs. per sup. foot. When this area is not known, Professor Mansfield Merriman has suggested that its approximate value may be found by taking the lineal feet in the skeleton outline of the truss as square feet, or, in other words, by assuming that the average width of each member in the truss that is exposed normally to the pressure of the wind is one foot.

In the American Bridge Company's general specifications, the wind pressure is taken at (*a*) 30 lbs. per square foot of the exposed surface of all trusses, and the floor as seen in elevation in addition to a horizontal live load of 150 lbs. per lineal foot of the span moving across the bridge when it is loaded; (*b*) or at 50 lbs. per sq. foot on the exposed surface of all trusses and the floor system, when there is no load on the bridge.

Maximum result to be taken.

In calculating the stresses in a truss due to wind pressure, the wind is assumed to be blowing horizontally, broadside on, or at right angles to the axis of the bridge. The pressure is taken up at the panel points in the form of concentrated loads, by the lateral or wind bracing, and transmitted to the abutments or points of supports, the cross girders acting as struts and the diagonals as ties.

As an example, let it be required to compute the stresses due to wind pressure in the truss shown in Figs. 203 to 205.

The span is 90 feet, the depth of truss 20 feet, the width of the bridge is 15 feet, and the wind pressure is assumed to be 40 lbs. per square foot.

Adopting Professor Mansfield Merriman's approximate rule, the exposed area

$$= 90 + 60 + 20 \times 5 + 25 \times 8 = 450 \text{ sq. feet,}$$

and 450×40 lbs. $= 18{,}000$ lbs. the total wind pressure.

There are 10 panel points, so that the panel load $= 1800$ lbs.

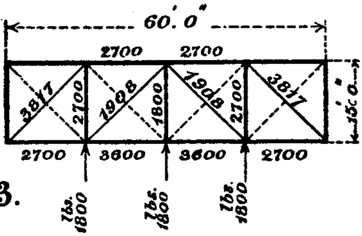


Fig. 203.

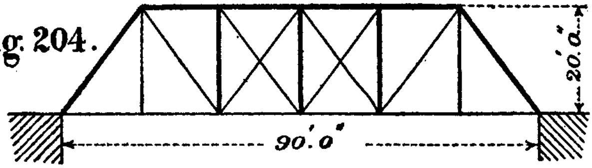


Fig. 204.

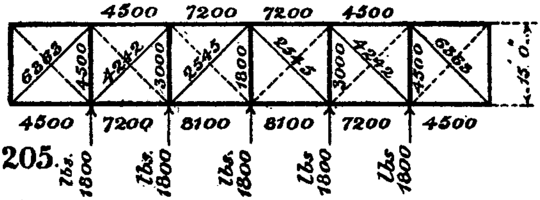


Fig. 205.

The stresses, which are computed as for a simple truss supporting vertical loads, are written down on the several members. When the wind blows from the opposite direction, the stresses in the chords and diagonal ties are reversed, but the stresses in the cross girders, acting as struts, remain the same.

Wind Stresses in Braced Bents or Piers.

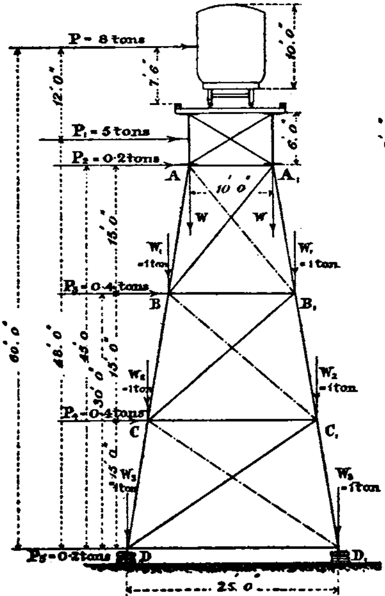


Fig. 207

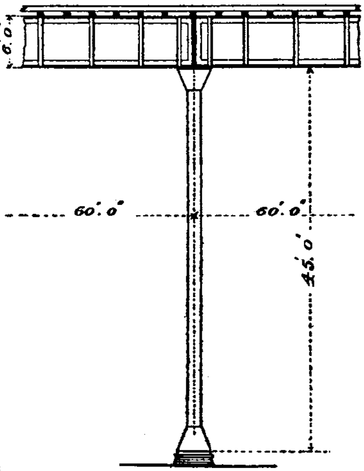


Fig. 206.

The wind pressure is taken at 30 lbs. per square foot of exposed surface of the bridge itself and of a train passing over.

The span between the piers is 60 feet, the wind pressure on the train surface will therefore $= 60$ feet $\times 10$ feet $\times 30$ lbs. $= 8$ tons $=$ P.

The wind pressure on the girder surface $= 60$ feet $\times 6$ feet $\times 30$ lbs $= 4{\cdot}8$, say 5 tons.

The wind pressure on the face of the pier at each panel point, *i.e.* B or C $= 15$ feet $\times 2$ feet $\times 30$ lbs $= {\cdot}4$ ton. At the points A and D the values of P_2 and P_5 will be one half of the pressure at B and C $= {\cdot}4 \div 2 = {\cdot}2$ ton.

The vertical loads acting at the points A and A_1 will be as under, viz.: Dead load $=20$ tons, and the live or rolling load 45 tons. The dead loads acting at the panel points BB′, CC′, and DD′ will be 1 ton at each point.

The moment tending to overturn the pier about the point D_1' will therefore be 8 tons × 60 ft. + ·5 tons × 48 ft. + ·2 tons × 45 ft. + ·4 ton × 30 + ·4 ton × 15 ft. = 747 tons.

The moment of stability = (90 tons + 40 tons + 6 tons)$\frac{25}{2}$ = 1700 tons, or 953 tons in excess of the overturning moment. If the results were reversed, that is to say, the overturning moment = 1700 tons and the moment of stability = 747 tons, it would have been necessary to anchor the windward member at the base D, and there would be exerted a pull on a vertical anchor bolt of $\frac{953}{25} = 38{\cdot}12$ tons.

The forces P and P_1 due to the wind pressure on the surface of the train and the girder have the tendency to lift the superstructure on the windward side and therefore to relieve the pressure at the point A and to increase by an equal amount the pressure at A_1.

Let p represent the amount of this pressure.

Then $$p = \frac{Pa + P_1 b}{AA'} = \frac{8 \times 15 + 5 \times 3}{10} = 13{\cdot}5 \text{ tons.}$$

a and b = respectively the height of the lines of action of P and P_1 above the top of the pier

Then the re-action at $A = \frac{W}{2} - p = 65 - 13{\cdot}5 = 51{\cdot}5$ tons,

and the re-action at $A' = \frac{W}{2} + p = 65 + 13{\cdot}5 = 78{\cdot}5$ tons.

The stresses in the various members may be obtained by a graphic process, the stress diagram being drawn as shown in Fig. 209. The construction of the diagram, it is thought, will be readily understood without a detailed explanation.

On the vertical load line bE the re-actions at A and A_1

= respectively 51·5 and 78·5 tons, are laid down to any convenient scale and EI, IM each = 1 ton, representing the vertical weights at the panel points B and C.

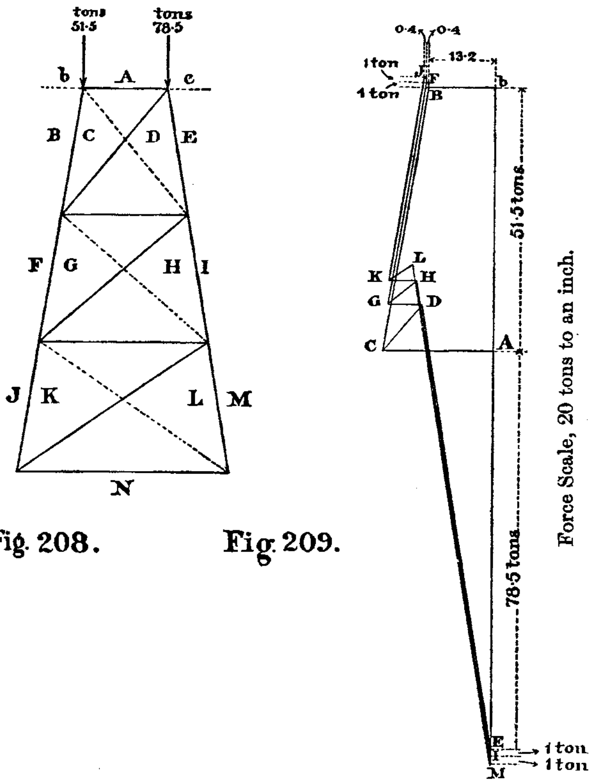


Fig. 208. Fig. 209.

Bb is laid down $= P + P_1 + P_2 = 13·2$ tons. The remaining lines in the diagram are drawn parallel to the lines bearing the same letters in Fig. 208, and they represent by scale the stresses in the corresponding members.

The stresses measured by scale are as under:

DE = 88·37 tons.	AC = 21·78 tons.	CD = 11·35 tons.
HI = 94·08 tons.	DG = 6·20 tons.	GH = 7·15 tons.
LM = 98·17 tons.	HK = 5·05 tons.	KL = 5·45 tons.

The stresses may also be obtained by the method of moments (Fig. 210).

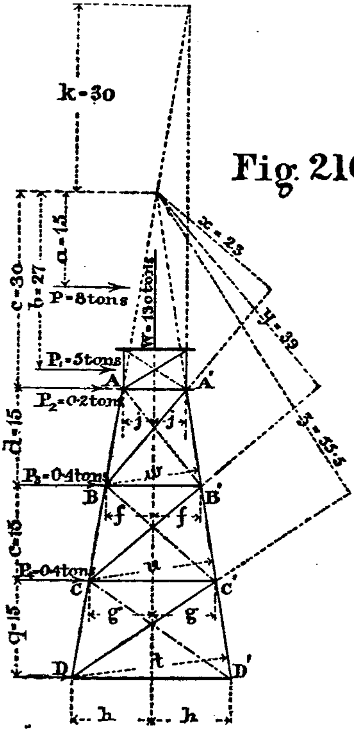


Fig. 210.

Thus the maximum stress in AB or A′B′

$$= [P(c+d-a) + P_1(c+d-b) + P_2 d + Wf] \div w$$

$$= \frac{8 \times 30 + 5 \times 18 + \cdot 2 \times 15 + 130 \times 7\cdot 5}{15\cdot 00} \times \frac{15\cdot 2}{15} = 88\cdot 37 \text{ tons.}$$

The maximum stress in BC = B′C′

$$=[P(c+d+e-a)+P_1(c+d+e-b)+P_2(d+e)+P_3e+Wg]\div u$$

$$=\frac{8\times 45+5\times 33+\cdot 2\times 30+\cdot 4\times 15+132\times 10}{20\cdot 00}\times\frac{15\cdot 2}{15}$$

$$=94\cdot 08 \text{ tons.}$$

The maximum stress in CD = C′D′

$$=\frac{8\times 60+5\times 48+\cdot 2\times 45+\cdot 4\times 30+\cdot 4\times 15+134+12\cdot 5}{25}$$

$$\div\frac{15\cdot 2}{2}=98\cdot 17 \text{ tons.}$$

The maximum stress in AA′

$$=[P(a+k)+P_1(b+k)+P_2(c+k)+Wj]\div c+k$$

$$=\frac{8\times 45+5\times 57+\cdot 2\times 60+130\times 5}{60} \qquad =21\cdot 78 \text{ tons.}$$

The maximum stress in BB′

$$=[Pa+P_1b+P_2c+P_3(c+d)]\div c+d$$

$$=\frac{8\times 15+5\times 27+\cdot 2\times 30+\cdot 4\times 45}{45} \qquad =6\cdot 2 \text{ tons.}$$

The maximum stress in CC′

$$=\frac{8\times 15+5\times 27+\cdot 2\times 30+\cdot 4\times 45+\cdot 4\times 60}{60}=5\cdot 05 \text{ tons.}$$

The maximum stress in A′B

$$=[Pa+P_1b+P_2c]\div x=\frac{8\times 15+5\times 27+\cdot 2\times 30}{23}=11\cdot 35 \text{ tons.}$$

The maximum stress in B′C

$$=[Pa+P_1b+P_2c+P_3(c+d)]\div y$$

$$=\frac{8\times 15+5\times 27+\cdot 2\times 30+\cdot 4\times 45}{39} \qquad =7\cdot 15 \text{ tons.}$$

The maximum stress in C′D

$$=[Pa+P_1b+P_2c+P_3(c+d)+P_4(c+d+e)]\div z$$

$$=\frac{8\times 15+5\times 27+\cdot 2\times 30+\cdot 4\times 45+\cdot 4\times 60}{55\cdot 5}=5\cdot 45 \text{ tons.}$$

The stresses should be determined when the live load is not on the pier, as under that condition of loading the stresses in the diagonals may be greater than under a full load. The various members must be designed so as to be capable of resisting the maxima stresses due to either condition of loading, whichever may produce the greatest stress in any member.

CHAPTER XIII.

Arches.

In an ordinary beam or girder resting on two supports the loads or vertical forces are resisted by the vertical re-actions of the abutments. In an arched structure the loads or vertical forces exert vertical and horizontal forces at the points of support, the latter tending to displace or spread out the extremities of the arched or curved member. In a bowstring girder this horizontal thrust is resisted by a horizontal tie uniting the extremities of the curved member, but in an arch the horizontal thrust must be resisted solely by the abutments.

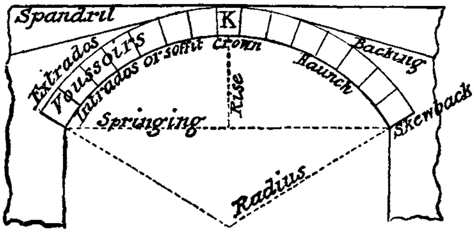
An arch may be built of masonry, that is to say, of stone, brick, or concrete, or of timber or metal, either cast iron, wrought iron, or steel, or it may be built of a combination of concrete and metal.

Masonry Arch. A masonry arch consists of a number of wedge-shaped blocks of stone or of brickwork, laid in courses forming a number of concentric rings, and a timber or metal arch consists of a curved beam or girder.

In masonry arches the stone blocks forming the arch are called voussoirs. The lowest extremities of the arch are called the springing lines, and the radial bed joints from which the arch springs are called the skew backs.

The highest part of the arch at its centre is called the crown, and the central voussoir K in Fig. 211 is called the keystone. The concave or under-surface of the arch is called the intrados, or soffit, the latter term being generally used in connection with small arches. The convex or outside surface of the arch is called the extrados. The spaces contained between the extrados and a horizontal line at the level of the summit of

Fig. 211.



the arch are called the spandrils. Those portions of the arch midway between the springing and the crown are called the haunches.

There are two outer spandril walls to almost every bridge, and there may be intermediate spandril walls in the intervening space, the openings between them being covered by stone flags or arches, to carry the roadway, or the whole of the space between the outside spandril walls, from the level of the backing of the arch to that of the roadway, may be filled in with earth.

A stone or brick arch differs from a timber or metal arch in so far as the elasticity of the material is concerned. In the former case the arch, composed of a number ot separate blocks of stone or bricks pressed together in the form of a series of wedges and relying upon one another for mutual support, is unable to resist a tensile stress of any appreciable magnitude.

In masonry and brick arches it is therefore assumed that no tensile stresses are to be permitted, and to satisfy this condition, according to the theories enunciated by Rankine and Navier, the resultant line of thrust at any section of the arch due to any condition of loading, must be confined within the middle third of that section, and under a uniform system of loading the resultant line of thrust must nearly approach the line of mid-section or the axis of the arch.

This theory may be explained thus: If we imagine two sections of the arch, or two of the voussoirs, A and B in Fig. 212, pressed together by the forces P and P_1, and that the pressure is so distributed over the surface of the section *mn* that it shall be zero at the point *m* and a maximum at the point *n*, then the total pressure on the surface *nm* will be equal to the area of the triangle *mno*, and the centre of pressure will pass through the centre of gravity of the triangle *mno* at a distance of one-third of the depth of the arch ring *mn* from the edge where the pressure is greatest.

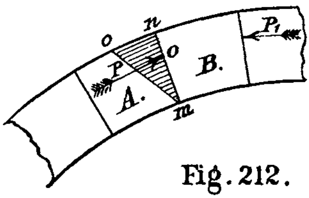


Fig. 212.

Professor Rankine in arriving at this conclusion appears to have considered that the theory of flexure did not apply to masonry, but in a series of exhaustive experiments on the strength of arches, undertaken by the Austrian Society of Engineers and Architects, extending from 1890 to 1895, it was conclusively proved that stone, brick, and concrete arches were subject to the laws of elasticity, and in the discussion on a paper on the "Design of Masonry dams," vide *Proceedings Institution of Civil Engineers*, Vol. CXV., 1894, Sir Benjamin Baker prominently calls attention to the fact.

It would therefore appear that to arbitrarily confine the resultant line of thrust within the middle third of the section is not a satisfactory solution of the problem, but in view of the variable qualities of masonry and the consequent uncertainty, the adoption of the middle third theory is an error on the safe side. Modern writers on the subject, including Doctor Herman Scheffler, have suggested that the limit may be safely extended to the middle half, and for a properly designed arch built in good mortar this appears to be an acceptable solution, at all events there are many arches now standing and doing good duty which if analysed on the basis of the middle third theory would be condemned as no longer being capable of performing their duty.

It is upon this theory, viz., that the line of thrust is assumed to occupy a position well within the arch ring, that is to say, that it shall not pass beyond the middle third or middle half of the section whichever limit may be adopted, that the graphic treatment of the stresses in a masonry arch now in general use is based. The usual mode of procedure is to find the required probable depth of the arch at the crown in terms of the span or the radius of curvature by an empirical formula, and then to investigate its stability, having regard to the special conditions of loading, etc.

The following are some of the formulae most generally known, in which

$D =$ thickness of arch at crown in feet,
$S =$ span in feet,
$r =$ radius at crown in feet.

1. According to Trautwine

$$D = \frac{\sqrt{r + \frac{S}{2}}}{4} + \cdot 2 \text{ feet.}$$

This formula gives the thickness for first-class cut stone work. For second-class work the depth should be increased $\frac{1}{8}$th, and for brick or fair rubble $\frac{1}{3}$rd part.

2. According to Dupuit

$$D = \sqrt{S} \times \cdot 36 \text{ for a full arch.}$$

$$D = \sqrt{S} \times \cdot 27 \text{ for a segmental arch.}$$

3. According to Perronet and de Gauthey

$$D = 0\cdot 32 + \tfrac{1}{24}S - \tfrac{1}{144}S.$$

4. According to Rankine

$$D = \sqrt{\cdot 12r} \text{ for single arch.}$$

$$D = \sqrt{\cdot 17r} \text{ for a series of arches.}$$

Applying these formulae for the sake of comparing one with the other to an arch of 40 feet span and 8 feet rise, we have the following results:

Here $$r = \frac{v^2 + \left(\frac{S}{2}\right)^2}{2v} = \frac{8^2 + 20^2}{2 \times 8} = 29 \text{ feet, } v \text{ being the rise.}$$

Then for an arch of first-class cut stone the thickness at crown should be according to

Trautwine,......................	1·95 feet.
Dupuit,..........................	1·70 feet.
Perronet and de Gauthey,...	1·70 feet.
Rankine,.........................	1·87 feet.

These empirical formulae must only be accepted as giving approximate results inasmuch as they do not take into account the nature of the load and roadway, which may vary very considerably, nor do they sufficiently differentiate between the qualities of the materials and workmanship.

Having by one of the preceding formulae found the thickness required at the crown of an arch of a given span and rise, we can then proceed to determine the curve of equilibrium or line of thrust for a given distribution of the dead and live load and investigate whether the thickness of the arch is sufficient to include that line of thrust within its assigned limits and thus satisfy the condition that there shall be no tension.

The solution of this problem can be best illustrated by the aid of the following example.

Let it be required to design an arched roadway bridge of 40 feet span and 8 feet rise, to be built of dressed stone work, to carry a uniformly distributed live load of 2 cwt. per superficial foot. The arch being symmetrical and the load being uniformly distributed over the arch, it will only be necessary to apply our investigations to a section of the arch one foot in width and extending from the one abutment to the centre of the span, as the other half must obviously be the same, see Fig. 213.

First a diagram is drawn of the half arch, showing the backing and spandril filling up to the level of the roadway, and the arch ring is divided into a convenient number of segments, 1, 2, 3, 4 in Fig. 213, and the vertical lines W, W_1, W_2 and W_3 are drawn through the centres of gravity of the separate voussoirs and their loads.

In Fig. 213 the half arch is divided into four segments only, so as not to complicate the drawing, but in practice it would probably be advisable to adopt a greater number of divisions.

The centre third of the arch ring is then marked off by drawing the two concentric curves AB and CD shown in Fig. 213 by dotted lines.

The line of minimum horizontal thrust is assumed to pass through A at the springing, and D at the crown of the arch.

On the load line *ae*, Fig 214, lay off by scale $ab = W$, $bc = W_1$, $cd = W_2$, and $de = W_3$; select any pole O and draw the radiating dotted lines O*a*, O*b*, O*c*, O*d*, and O*e*.

Commencing at the point D, Fig. 213, draw D*f* horizontally, and from *f* draw f_1 parallel to O*b*, f_1, f_2 parallel to O*c*, f_2, f_3 parallel to O*d*, and from the point f_3, draw f_3g parallel to O*e*, until it meets the horizontal line D*f* prolonged at *g*. Then will a vertical line drawn through the point *g* represent the centre of gravity of the half arch and its load, and the dotted line Df·f_1 f_2 f_3 will represent the trial line of minimum

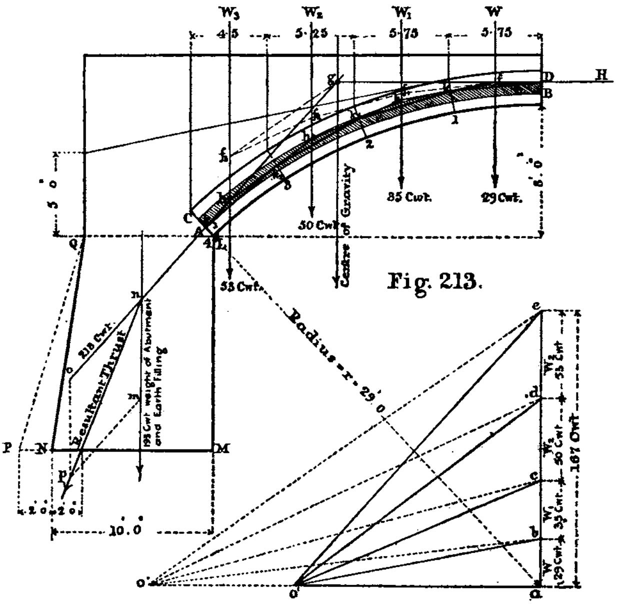


Fig. 213.

Fig. 214.

Force Scale, 60 cwt. to an inch.

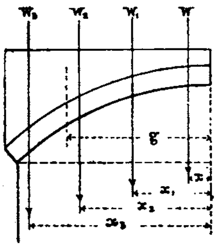


Fig. 215.

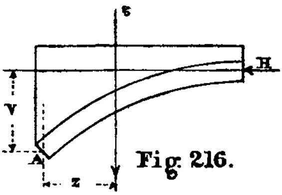


Fig. 216.

horizontal thrust. But it lies wholly beyond the middle third of the section, and it becomes necessary to draw a new force polygon so that the line of thrust shall if possible be made to pass well within the arch ring. For that purpose join gA, A being the lowest point at the springing of the mid section AC. Join Ag, and in Fig. 214 draw from the point e, eO′ parallel to Ag. Then will O′ be the pole of the new force polygon; join O′a, O′b, O′c, O′d, and in Fig. 213 draw the lines fh parallel to O′b, hh_1 parallel to O′c, and h_1h_2 parallel to O′d. Then by connecting the points k, k_1, k_2, k_3, where these lines intersect the joints of the segments forming the arch ring, the line of minimum horizontal thrust for the half arch is shown by the thick black line to pass within the middle third of the section which satisfies the conditions laid down.

The centre of gravity of the half arch and its load may also be determined by an analytical process as follows:

Let w = weight at each point

W = sum of weights between crown and any point P.

m = moment of weight at each point $= wx$.

x = distance of weight from crown.

M = sum of moments between crown and P

$= m + m_1 + m_2 + m_3$, etc.

g = distance of centre of gravity measured from crown.

Then $g = \dfrac{M}{W}$. (See Fig. 215).

$$\left.\begin{aligned} w &= 29 \text{ cwts. and } x = 2{\cdot}875 \\ \text{and } m &= w\,x = 29 \times 2{\cdot}875 \end{aligned}\right\} = 83{\cdot}375$$

$$\left.\begin{aligned} w_1 &= 35 \text{ cwts. and } x_1 = 8{\cdot}625 \\ \text{and } m_1 &= w_1x_1 = 35 \times 8{\cdot}625 \end{aligned}\right\} = 301{\cdot}875$$

$$\left.\begin{aligned} w_2 &= 50 \text{ cwts. and } x_2 = 14{\cdot}125 \\ \text{and } m_2 &= w_2x_2 = 50 \times 14{\cdot}125 \end{aligned}\right\} = 706{\cdot}250$$

$$\left.\begin{aligned} w_3 &= 53 \text{ cwts. and } x_3 = 19{\cdot}000 \\ \text{and } m_3 &= w_3x_3 = 53 \times 19{\cdot}000 \end{aligned}\right\} = 1007{\cdot}000$$

$$M = m + m_1 + m_2 + m_3 \qquad = \underline{\underline{2098{\cdot}500}}$$

$$W = w + w_1 + w_2 + w_3 = 167 \text{ cwts.}$$

and $\frac{M}{W} = \frac{2098.5}{167} = 12.56$ feet, the distance of the centre of gravity of the half arch and its load from the crown, which accords as nearly as possible with the scaled dimensions in the diagram Fig. 213.

The line of resistance in a voussoir arch is really statically indeterminate, but for practical purposes it is assumed that the line drawn on the principles we have laid down may be accepted as a sufficiently close approximation, and on that supposition the pressure can be calculated.

In Fig. 213 the greatest pressure is at the springing where the centre of pressure is 8″ from the inner edge or line of intrados of the arch, and the amount of pressure measured by scale on the diagram Fig. $214 = O'e = 218$ cwts. This pressure is distributed over a width $= 3d$ as shown in Fig. 217 in which ABCD represents a section of the arch at the springing, P the direction of the line of resultant pressure, the intensity of which is represented by the area of the triangle ABE.

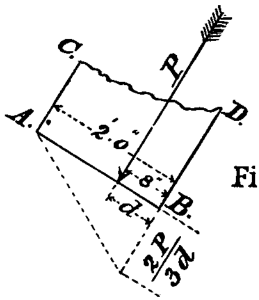


Fig. 217.

Then $P \div 3d =$ the mean pressure and $2P \div 3d =$ the maximum pressure.

Then $\frac{2 \times 218}{3 \times 8 \text{ in.}} = \frac{436}{2 \text{ feet}} = 218$ cwts. or 10·9 tons per square foot, or nearly 170 lbs. per square inch, which is well within the limits of permissible crushing strain in ordinary masonry or even in brickwork, so that the depth of arch ring is sufficient.

In this case the amplitude of the line of resultant thrust being confined to the middle third of the section the value of the expression $3d$ includes the whole depth of the arch ring, but if the amplitude of the line of resultant thrust had been extended to the middle half of the section the value of $3d$ would have been reduced to $\frac{3}{4}$ of the depth of the arch ring, and the resultant pressure being distributed over a lesser surface would be correspondingly increased. Thus, assuming the distance d from the edge of the intrados at B, Fig. 217, to be $\frac{1}{4}$ of the depth of the arch ring AB, the surface acted upon by the pressure P would then $= 3d$ or 1 ft. 6 in., and the total pressure upon that surface would be $= \frac{2 \times 218}{3 \times \cdot 5} = 290 \cdot 6$ cwts. instead of 218 cwts.

In any arch of stone, wood, or iron, the strain at the centre under a uniform load is a horizontal strain, and for any particular condition of loading it is uniform throughout the arch. Each half of the arch may be regarded as a lever loaded with the weights w, w_1, w_2, w_3, etc., which latter can be replaced for purposes of calculation by one single weight W, Fig. 216, representing the centre of gravity of the half arch and its load. This weight W tends to cause the half arch to move about its fulcrum A and fall, having a moment $= W \times z$, which is resisted by the counter pressure of the other half arch acting horizontally against it in the direction indicated by H having a moment $= \frac{W \times z}{V}$. Applying this expression to our example we have $\frac{167 \text{ cwts.} \times 8 \text{ ft.}}{8 \cdot 7 \text{ ft.}} = 153$ cwts.

In a masonry or brickwork arch of any considerable magnitude the live or moving load is so small in proportion to the dead load that for practical purposes the arch may be assumed to be symmetrically loaded, but the line of resultant thrust due to any unsymmetrical condition of loading can be drawn in the same manner as that explained in the preceding example.

Thickness and Stability of Abutment.

In the first instance, the thickness of the abutment, at the springing of the arch, must be tentatively determined by formula, and then it remains to investigate its stability.

The following empirical formula may be used:

$$T = \frac{R}{5} + \frac{V}{10} + 2,$$

R being the radius of curvature in feet, and V the versed sine. In this formula the height LM, Fig. 213, must not exceed $1\frac{1}{2}$ times the base MN.

Applying this formula to our example we have T = thickness of abutment at springing in feet $= \frac{29}{5} + \frac{8}{10} + 2 = 8{\cdot}4$ feet.

In proceeding to investigate the stability of the abutment, it is generally regarded that the masonry abutment should in itself be sufficiently stable to resist the thrust of the arch without the aid of the pressure of earth at its back, although that pressure is of material assistance. In Fig. 213 determine the weight and centre of gravity of the mass of masonry, and the superincumbent load of earth filling, and on a vertical line drawn through the centre of gravity, lay off to scale the distance mn = to the load = 193 cwts. From the point of intersection at n, continue the line gAn and make nO = O$'e$, Fig. 214, = 218 cwts. Then will the two lines mn and no represent respectively the direction and intensity of the resistance of the abutment and the thrust of the arch, and the line np will represent in magnitude and intensity the resultant of these two forces, which should fall within the middle third of the base MN.

In Fig. 213 the resultant passes within 2 feet of the point N, so that the condition of middle third is not satisfied, and it becomes necessary to widen the base as shown by the dotted line PQ.

An alternative method of constructing the curve of equilibrium or resultant line of thrust is illustrated in Fig. 218, which may be proceeded with as follows. First the horizontal thrust at the crown must be determined. Then from the point a at the intersection of the line of horizontal thrust with the vertical, through the centre of gravity of the first segment of the arch $= w_1$ draw aa_1 in continuation of the horizontal line Da, and make $aa_1 = \mathrm{H} = 153$ cwts. From a_1 let fall the

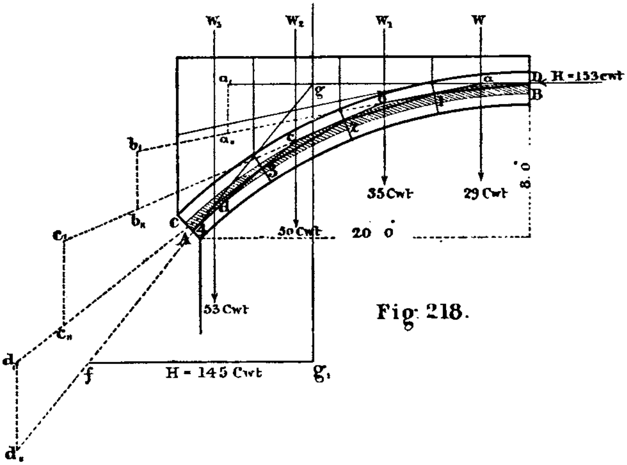


Fig. 218.

perpendicular a_1a_{11} and make it equal to $w = 29$ cwts. and join aa_{11}. From b at the point of intersection with the second vertical $= w_1$ continue the line ba_{11} to b_1 making $bb_1 = \mathrm{H} = 153$ cwts., and from b_1 let fall the perpendicular b_1b_{11} making it $= w_1 = 35$ cwts. Join bb_{11}. From c at the point of intersection with the third vertical $= w_2$ continue the line cb_{11} to c_1 making $c_1c_1 = \mathrm{H} = 153$ cwts. From c_1 let fall c_1c_{11} making it equal to $w_2 = 50$ cwts. Join cc_{11}. From d at the point of intersection with the fourth vertical $= w_3$ continue dc_{11} to d_1

making $dd_1 = \text{H} = 153$ cwts. From d_1 let fall $d_1 d_{11}$ making it $= w_3 = 53$ cwts. Join $d_{11}d$ and prolong the line until it intersects the horizontal line aa_1 at g. Then will a vertical line drawn through g represent the centre of gravity of the half arch and its load, and the thick red line drawn through the points 1, 2, 3, and 4 will be the line of equilibrium or resultant pressure. The horizontal line fg_1 will represent the horizontal thrust at crown, and the diagonal fg will represent the direction and magnitude of the thrust at the springing at A.

Metallic Arches. Three-hinged Arch.

Spandril-Braced Arch hinged at Crown and Springing.

Span, 120 feet. Rise of Arch, 20 feet. Depth of Crown, 3 feet.

Dead Load $\frac{1}{2}$ ton per foot run = 10 tons per panel.
Live Load 1 ton per foot run = 20 tons per panel.

Fig. 219

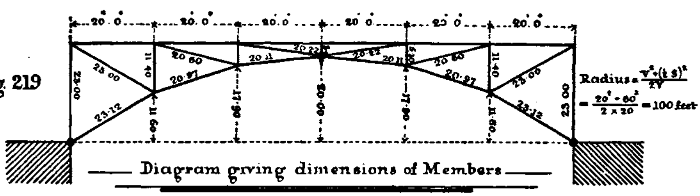


Fig. 220

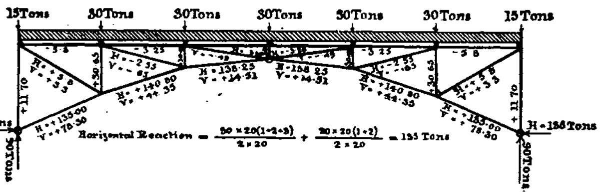


Fig. 221

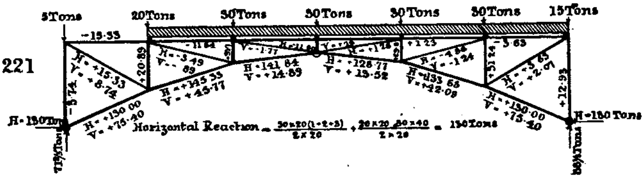


Fig. 222

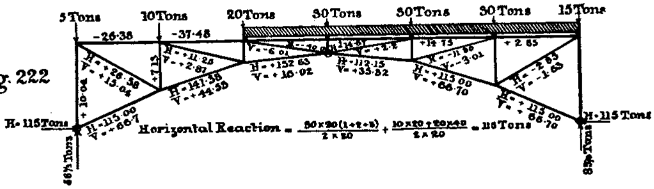


Fig. 223

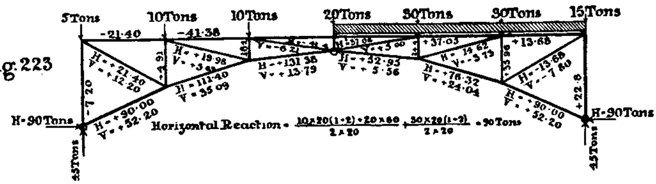


Fig. 224

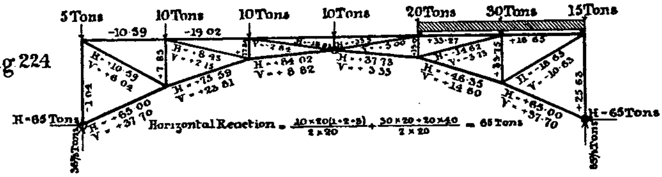


Fig. 225

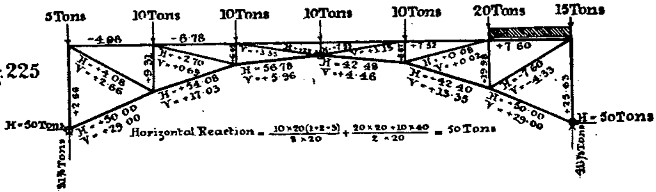


Fig. 226

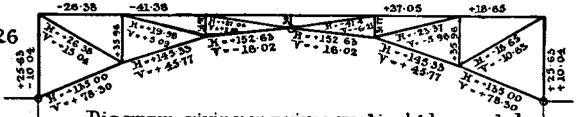


Diagram giving maxima vertical & horizontal components of stresses in the several members

Fig. 227

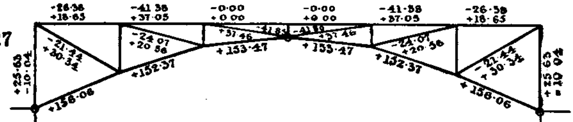


Diagram giving maxima stresses in each member

By reason of the centre hinge at D the members CD and DC cannot take any stress.

On reference to Fig. 228 it will be seen that the horizontal thrust in the member *ab* is 135 tons, the vertical component of which is 78·3 tons. The vertical re-action at the abutment *a* is 90 tons; there must therefore be a vertical downward stress or compression in A*a* in order to produce equilibrium $= 90 - 78{\cdot}3$ tons $= 11{\cdot}7$ tons. The vertical component of the stress in A*b* = the difference between the load at A, and the compression in A*a* $= 15 - 11{\cdot}7 = 3{\cdot}3$ tons, the horizontal component of which $= 5{\cdot}8$. The horizontal component of the stress in AB is also 5·8 tons, and it is a tensile stress because it resists the thrust or compression in AB.

At the panel point *b* there are concentrated two forces of the same kind, viz., a compressive force acting along the member *ab* and also a compressive force acting along the member A*b*, which two forces must be resisted by an equal force acting along the member *bc*. The horizontal components of these two forces are respectively 135·00 tons and 5·8 tons, so that the horizontal component of the stress in *bc* must be

13·500 + 5·80 = 140·80 tons, the vertical component of which = 44·35 tons.

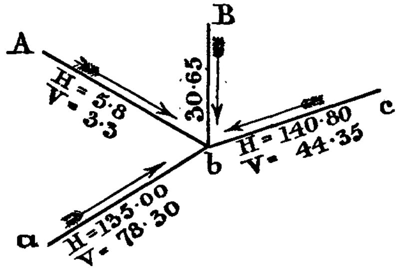


Fig. 228.

We have four members meeting at the point *b* and there are three known forces, viz.,

an upward thrust along the member *ab* = V 78·30 tons,
a downward thrust along A*b* = V 3·3 tons,
a downward thrust along *bc* = V 44·35,
47·65 „
30·65 tons.

Deducting one from another the forces acting in opposite directions we have as the result which must be resisted by a downward or compressive stress in B*b* if equilibrium is to be maintained 30·65 tons compression in B*b*.

The vertical component of the stress in B*c* is equal to the difference between the load at B and the compression in B*b* = 30·65 − 30·00 = 0·65 tons, the horizontal component of which = 2·55, and this must be a tensile stress because it exerts a pull at the panel point B.

The tension in BC is equal to the tension in AB less the horizontal component of the tension in B*c* = 5·8 − 2·55 = 3·25 tons.

At the panel point c the thrust along bc is diminished in its horizontal component by the tension in

$$Bc = 140 \cdot 8 - 2 \cdot 55 = 138 \cdot 25 \text{ tons,}$$

the vertical component of which is 14·51 tons.

At the panel point c we again have three known forces operating in the direction of the arrows, and it is required to determine the force acting along Cc. The tension in Bc reduces the thrust along cd equal in value to the difference

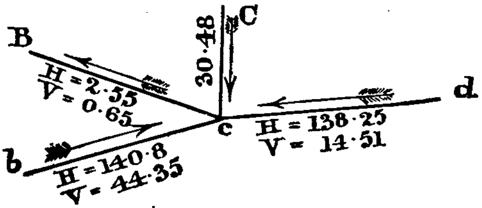


Fig. 229.

of their vertical components = 14·51 − 0·65 = 13·86, so that the difference between the vertical components of the thrusts in bc and cd

$$= 44 \cdot 35 - (14 \cdot 51 - 0 \cdot 65) = 44 \cdot 35 - 13 \cdot 86 = 30 \cdot 49$$

amount of compression in Cc.

The vertical component of the stress in Cd = 30·49 − 30, the panel load at C = ·49, the horizontal component of which = 3·26 tons.

Proceeding in like manner the stresses in the other members due to the various conditions of loading indicated in the diagrams Figs. 220 to 225 may be determined, and finally the horizontal and vertical components of the maxima stresses in the several members may be obtained by inspection of the diagrams, which may expediently be written down in a diagram as shown in Fig. 226. From this diagram the axial or longitudinal stresses in each member may be derived and indicated on each member as shown in Fig. 227.

The stresses in this type of truss may also be determined by a graphic process as shown in Figs. 230 to 236. A diagram must be drawn for each condition of loading by setting off the loads and abutment re-actions on a vertical load line, and drawing from the boundary of the vertical re-actions a horizontal line equal by scale to the horizontal re-action. Then lines drawn parallel to the several members will represent by scale, the axial or longitudinal stresses in those members, and the maxima stresses may be obtained by mere inspection, or they may be written down in tabular form.

The stress diagrams are fully lettered and figured, and it is thought that they will be readily understood without further description.

Calculations of Stresses in a Three-hinged Arched Rib.

Let $S = \text{span} = 120$ feet,

$R = \text{rise of arch} = \frac{S}{6} = 20$ feet,

$$r = \text{radius of arch} = \frac{\left(\frac{S}{2}\right)^2 + R^2}{2R} = \frac{60^2 + 20^2}{2 \times 20} = 100 \text{ feet,}$$

V = vertical re-actions at abutments,

H = horizontal re-action at abutments,

T = stress at crown of arch C,

Tx = stress at any point distant x from crown C,

W = dead load per unit,

w = live load per unit,

W_1 = concentrated load at any point of arch.

The vertical re-actions for an arch hinged at the abutments and at the crown under any conditions of loading are precisely

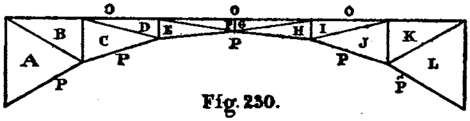


Fig. 230.

Force Scale

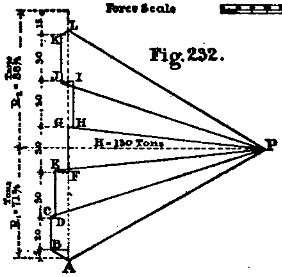


Fig. 232.

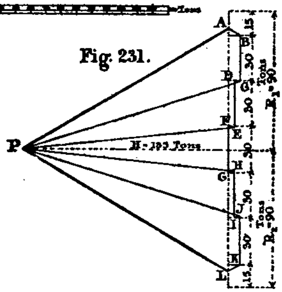


Fig. 231.

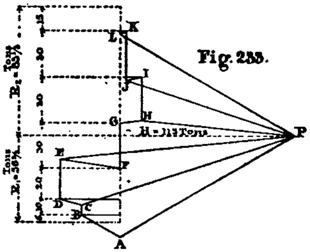


Fig. 233.

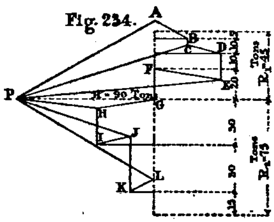


Fig. 234.

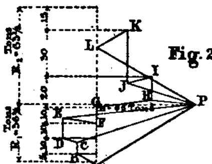


Fig. 235.

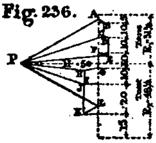


Fig. 236.

the same as for a simple beam. In Fig. 237 the vertical re-action at $A = \frac{W_1(S-y)}{S}$ and at the abutment $B = V_2 = W_1 \frac{y}{S}$. The horizontal re-action at the abutment $A = H_1 = \frac{W_1 y}{2R}$, and at

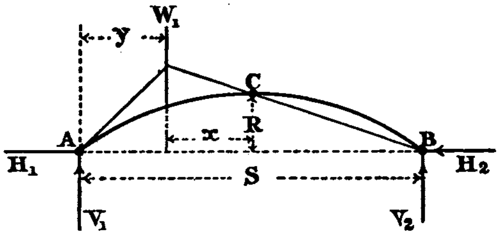


Fig. 237.

the abutment B the horizontal re-action H_2 must be equal and opposite to that at A, that is to say, $H_1 = H_2$, and

$$H_2 = W_1 \frac{1 - \left(\frac{S-y}{S}\right)S}{2R}.$$

Example. Let $x = y = 30$ feet and $W_1 = 12$ tons.

Then
$$V_1 = 12 \times \frac{(120-30)}{120} = 12 \times \frac{90}{120} = 9 \text{ tons},$$

$$V_2 = 12 \times \frac{30}{120} = 12 \times \frac{1}{4} \qquad = 3 \text{ tons},$$

$$H_1 = \frac{12 \times 30}{2 \times 20} = \frac{360}{40} \qquad = 9 \text{ tons},$$

$$H = 12 \times \frac{\left(1 - \frac{120-30}{120}\right)120}{2 \times 20} \qquad = 9 \text{ tons}.$$

The values of V_1, V_2, and H can be determined graphically, as shown in Fig. 238. In a three-hinged arch the thrust produced by a weight W_1, as in Fig. 238, must pass through the hinge C and also through the hinge B. The intersection at d of the line of re-action CB with the vertical line de, representing the direction of the force W_1, must be a point in the line of re-action from A. From the point d let fall the vertical line de, making it by scale $= W_1 = 12$ tons. From e draw ef parallel to CB. Complete the parallelogram by drawing eg parallel to Ad. Draw the horizontal lines fi and hg, which will be equal to one another, and representing by scale the horizontal re-actions at A and B. Similarly, the distance di will give the vertical re-action at A, and dh the vertical re-action at B. In this manner the re-actions for any number of loads, or systems of loading, can be readily determined by adding together the re-action due to each load.

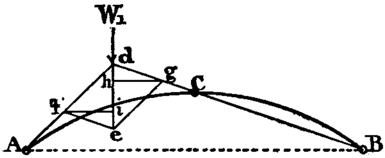


Fig. 238.

Example. Let it be required to determine the vertical and horizontal re-actions due to the system of loading given in Fig. 239, viz., the arch

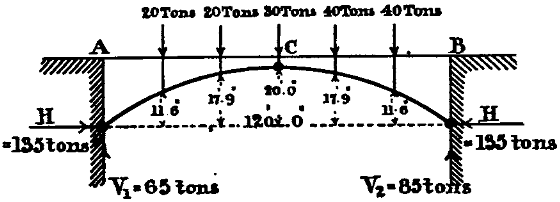


Fig. 239.

supporting from the abutment B to the crown C its maximum load, and from the crown C to the abutment A supporting only the dead or static load due to its own weight with that of the horizontal girder and flooring.

$$V_1 = 40\frac{1+2}{6} + 30\frac{3}{6} + 20\frac{4+5}{6} = 65 \text{ tons},$$

$$V_2 = 20\frac{1+2}{6} + 30\frac{3}{6} + 40\frac{4+5}{6} = 85 \text{ tons}.$$

The horizontal re-action H

$$=\frac{20\times 20(1+2)}{2\times 20}+\frac{30\times 60}{2\times 20}+\frac{40\times 20(1+2)}{2\times 20}=30+45+60=135 \text{ tons.}$$

To determine the extent and position of a moving load to produce maxima bending moments at different points of the arched rib, as at p_1, p_2, etc.,

Let p = the point at which it is required to determine the maximum moment,

x = the co-ordinate having its origin at C or the centre of the arch, or the distance of the point p from the centre of the arch,

a = the distance from the centre of the arch to the extremity of the load, or the load limit exercising a positive influence on the one side and negative on the other,

S = span as before.

Then, for any point p, $$a=\frac{x\times\frac{S}{2}}{S+x}.$$

Let it be required to determine the extent and position of a moving load to produce maximum tension and compression in the arched rib AB, Figs. 240 and 241, at the point p distant $x=40$ feet from the crown C.

Then by the expression $$a=\frac{x\times\frac{S}{2}}{S+x}$$

we have $$a=\frac{40\times\frac{120}{2}}{120+40}=\frac{40\times 60}{160}=15 \text{ feet,}$$

the distance from the crown C to which the live or rolling load should extend from either abutment to produce positively or negatively the maxima stresses at p the point 2. The panel weights due to each system of loading are set forth in the Figs. 240 and 241. When the live or rolling load assumes the position given in Fig. 240, that is to say, extending from the right abutment at B to the load limit represented by the

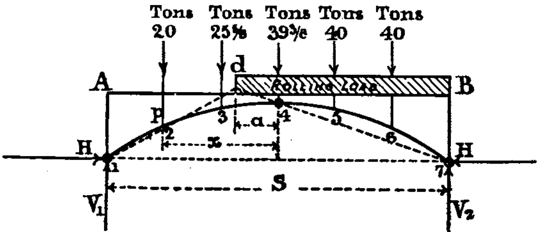


Fig. 240.

vertical line at d, it gives the maximum compression in the intrados or on the inner flange of the rib at p_2, and produces tension in the extrados or at the outer flange Let this be

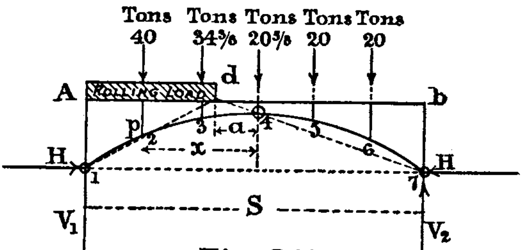


Fig. 241.

called the positive bending moment. The moving load in the position indicated in Fig. 241 gives the maximum compression in the extrados and tension in the intrados. Let this be called the negative bending moment.

Let H and V be respectively the horizontal and vertical components of a force F that would replace the action of AC or BC assuming either segment AC or BC removed (Fig 242).

Then $\quad H = \frac{1}{2R}[\Sigma_C^B 1wa + \Sigma_C^A 1w(S-a)],$

and $\quad V = \frac{1}{S}[\Sigma_C^B 1wa - \Sigma_C^A 1w(S-a)] = \frac{1}{S}\left[\frac{wS^2}{8} - wa\frac{(S-a)}{2}\right].$

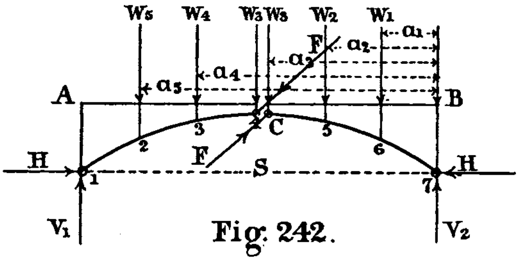


Fig. 242.

Using these expressions to find the values of H and V due to the condition of loading indicated in Fig. 240, we have...........positive,

$$H = \frac{1}{2 \times 20}[w_1a_1 + w_2a_2 + w_3a_3 + w_4(S - a_4) + w_5(S - a_5)]$$

$$= \frac{1}{40}[40 \times 20 + 40 \times 40 + 39\tfrac{3}{8} \times 60 + 25\tfrac{5}{8} \times 40 + 20 \times 20]$$

$$= \frac{800 + 1600 + 2362\tfrac{1}{2} + 1025 + 400}{40} = 154{\cdot}69 \text{ tons.}$$

$$V = \frac{1}{120}[(w_1a_1 + w_2a_2 + w_3a_3)$$
$$- \{(w_3(S - a_3) + w_4(S - a_4) + w_5(S - a_5)\}]$$

$$= \frac{1}{120}[(40 \times 20 + 40 \times 40 + 20 \times 60)$$
$$-(19\tfrac{3}{8} \times 60 + 25\tfrac{5}{8} \times 40 + 20 \times 20)]$$

$$= \frac{(800 + 1600 + 1200) - (1162\tfrac{1}{2} + 1025 + 400)}{120} = 8{\cdot}43 \text{ ton.}$$

Then the maximum positive bending moment at the point p_2, Fig. 240,

$$= 8{\cdot}43 \times 40 - 154{\cdot}69 \times 8{\cdot}4 + 19{\cdot}37 \times 40 + 25{\cdot}625 \times 20 = 325 \text{ tons.}$$

Negative—condition of loading indicated in Fig. 241,

$$H = \frac{1}{2 \times 20}[20 \times 20 + 20 \times 40 + 20\tfrac{5}{8} \times 60 + 34\tfrac{3}{8} \times 40 + 40 \times 20]$$

$$= \frac{400 + 800 + 1237{\cdot}5 + 1375 + 800}{40} = 115{\cdot}3 \text{ tons.}$$

$$V = \frac{1}{120}[(40 \times 20 + 34\tfrac{3}{8} \times 40 + 10\tfrac{5}{8} \times 60) - (20 \times 20 + 20 \times 40 + 10 \times 60)]$$

$$= \frac{(800 + 1375 + 637{\cdot}5) - (400 + 800 + 600)}{120} = 8{\cdot}43 \text{ tons.}$$

Then the maximum negative bending moment at the point p_2, Fig. 241,

$$= -8{\cdot}43 \times 40 - 115{\cdot}3 \times 8{\cdot}4 + 10\tfrac{5}{8} \times 40 + 34\tfrac{3}{8} \times 20 = -197{\cdot}22 \text{ tons.}$$

Proceeding in like manner, it is found that the load boundary for the maximum bending moments at the section p_3 distant 20 feet from the crown of the arch $= \frac{20 \times 60}{120 + 20} = 8{\cdot}57$, say $8{\cdot}5$ feet.

For maximum positive bending moments—condition of loading as in Fig. 243.

$$H = \frac{1}{40}[40 \times 20 + 40 \times 40 + 36{\cdot}7 \times 60 + 21{\cdot}8 \times 40 + 20 \times 20]$$

$$= \frac{800 + 1600 + 2202 + 872 + 400}{40} = 146{\cdot}85 \text{ tons.}$$

$$V = \frac{1}{120}[(40 \times 20 + 40 \times 40 + 20 \times 60) - (16{\cdot}7 \times 60 + 21{\cdot}8 \times 40 + 20 \times 20)]$$

$$= \frac{(800 + 1600 + 1200) - (1002 + 872 + 400)}{40} = 11{\cdot}05 \text{ tons.}$$

Then the maximum positive bending moment at the point p_3, Fig. 243,

$$=11{\cdot}05\times 20-146{\cdot}85\times 2{\cdot}1+16{\cdot}7\times 20=246{\cdot}62 \text{ tons.}$$

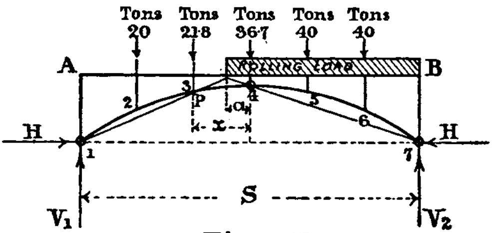


Fig. 243.

For negative bending moment the condition of loading is as indicated in Fig. 244.

$$H=\frac{1}{40}[20\times 20+20\times 40+23{\cdot}3\times 60+38{\cdot}2\times 40+40\times 20]$$

$$=\frac{400+800+1398+1528+800}{40}=123{\cdot}15 \text{ tons.}$$

$$V=\frac{1}{120}[(40\times 20+38{\cdot}2\times 40+13{\cdot}3\times 60)$$

$$-(20\times 20+20\times 40+10\times 60)]$$

$$=\frac{(800+1528+798)-(400+800+600)}{120}=11{\cdot}05 \text{ tons.}$$

Then the maximum negative bending moment at the point p_3, Fig. 244

$$=-11{\cdot}05\times 20-123{\cdot}15\times 2{\cdot}1+13{\cdot}3\times 20=-213{\cdot}61 \text{ tons.}$$

The bending moment may also be found by the expression

$$M=M'-Hr.$$

Here M' = the bending moment for a simple truss.

H = the horizontal re-action at abutments.

V_1 = vertical re-action at abutment A.

r = height of point of moment above horizontal line through hinges AB.

E.g. Let it be required to determine the bending moment at p_2, Fig. 240.

$$H = \frac{1}{2 \times 20}\left[40 \times 20 + 40 \times 40 + 39\tfrac{3}{8} \times 60 + 25\tfrac{5}{8} \times 40 + 20 \times 20\right]$$
$$= 154{\cdot}69 \text{ tons.}$$

$V_1 = 20\tfrac{5}{8} + 25\tfrac{5}{8} \times \tfrac{4}{8} + 38\tfrac{3}{8} \times \tfrac{3}{8} + 40\tfrac{2}{8} + 40\tfrac{1}{8} = 72{\cdot}94$ tons.

Then $M = 72{\cdot}94 \times 20 - 154{\cdot}69 \times 11{\cdot}6 = -335{\cdot}6$ tons, which agrees with the bending moment already determined.

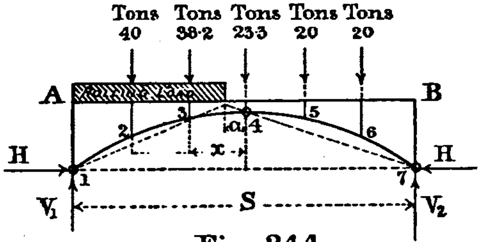


Fig. 244.

Similarly, it may be shown that in taking moments at the point p_3, Fig. 243, $H = 146{\cdot}85$ and $V_1 = 69{\cdot}54$.

Then for a simple truss the bending moment

$= 69{\cdot}54 \times 40 - 20 \times 20 = 2381{\cdot}6$ and $2381{\cdot}6 - 146{\cdot}85 \times 17{\cdot}9$

$= 247$ tons, the bending moment at the point p_3,

which also agrees with the result already determined.

The maxima bending moments are as under:

at the point 2 + 335·00 tons positive and − 197·22 tons negative,

„ 3 + 246·62 tons positive and − 213·61 tons negative.

The effective depth of the web which is generally taken at $\frac{1}{50}$ of the span $= 120 \div 50 = 2{\cdot}4$, say 2·5 feet.

Then the maximum stress due to bending

$$= 335 \div 2{\cdot}5 = 134 \text{ tons.}$$

It is next required to determine the compressive stress in the extrados and the intrados of the rib and the shearing stress in the rib.

The resultant thrust at any point m of a linear arch AO, Fig. 245, $= R = \sqrt{H^2 + V^2}$, in which H is the horizontal thrust and V the vertical shear at that point. The resultant R may be

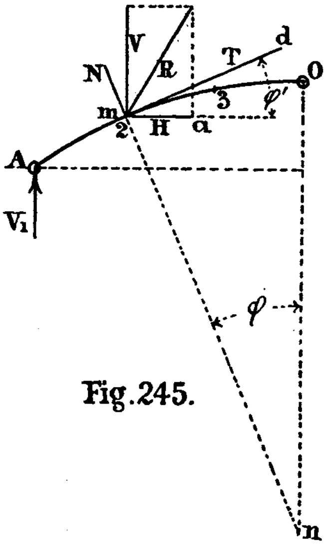


Fig. 245.

considered as a direct stress or thrust T in the direction of the tangent, a shear N at right angles to T, and a bending moment M. If ϕ' is the angle between the line of horizontal thrust and the tangent T.

Then $$T = H \cos \phi' + V \sin \phi',$$
and $$N = V \cos \phi - H \sin \phi.$$

But the angle amd = the angle Onm, therefore the angle $\phi' = \phi$.

The maximum stress due to bending moments in the top and bottom flanges is 130 tons, and if the material used is steel having a safe working stress of $6\frac{1}{2}$ tons per square inch the

V_1 = the vertical reaction at A. Σp the loads on the left of the Section. Then $V = V_1 - \Sigma p$.

Collecting the various results together we have the following tabulated Statement.

Panel Point.	Bending Moments $+ = M$.	Stresses due to Bending Moments $= M \div 2{\cdot}5$ foot.		Sin ϕ.	Cos ϕ.	V = Sum of Vertical Forces = $V_1 - \Sigma p$.	Horizontal Thrust = H.	$\frac{V \sin \phi + H \cos \phi}{2}$.		Resultant Stresses.		Shearing Stresses $= V \cos \phi - H \sin \phi$.
		Extrados Tension.	Intrados Compression.					Extrados Compression.	Intrados Compression.	Extrados Tension.	Intrados Compression.	
				POSITIVE BENDING MOMENTS.								
2	325·00	130·00	130·00	0·400	0·916	73·43	154·69	85·66	85·66	44·34	215·66	5·39
3	246·62	98·65	98·65	0·200	0·979	49·55	146·85	76·83	76·83	21·82	175·48	19·13

Panel Point.	Bending Moments $- = M$.	Stresses due to Bending Moments $= M \div 2{\cdot}5$.		Sin ϕ.	Cos ϕ.	V = Sum of Vertical Forces = $V_1 - \Sigma p$.	Horizontal Thrust = H.	$\frac{V \sin \phi + H \cos \phi}{2}$.		Resultant Stresses.		Shearing Stresses $= V \cos \phi - H \sin \phi$.
		Extrados Compression.	Intrados Tension.					Extrados Compression.	Intrados Compression.	Extrados Compression.	Intrados Tension.	
				NEGATIVE BENDING MOMENTS.								
2	197·22	78·88	78·88	0·400	0·916	76·57	115·3	68·11	68·11	146·99	10·77	24·01
3	213·61	85·44	85·44	0·200	0·979	40·45	123·15	64·32	64·32	149·76	21·12	14·97

sectional area of either flange to resist this strain will be $130 \div 6\frac{1}{2} = 20$ square inches. The rib is also subject to a compressive stress due to the direct or axial thrust, the maximum resultant of which $= 215{\cdot}66$ tons. This stress is assumed to be uniformly distributed over the two flanges, so

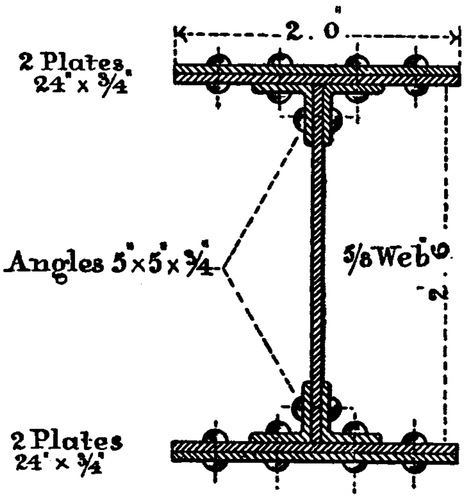


Fig. 246.

that the stress taken by each flange will be $215{\cdot}66 \div 2 = 107{\cdot}83$ tons. This will require in each flange an additional sectional area of $107{\cdot}83 \div 6\frac{1}{2}$ tons $= 16{\cdot}6$ square inches, so that the net section of each flange must not be less than $20 + 16{\cdot}6 = 36{\cdot}6$ square inches. This sectional area would be provided by a rib of the section given in fig. 246.

2 angles 5 in. × 5 × $\frac{3}{4}$ less rivet holes	$= 12{\cdot}375$ sq. in.
2 plates 24 in. × $\frac{3}{4}$ in. less rivet holes	$= 30{\cdot}000$
Net sectional area	$42{\cdot}375$

The maximum shearing stress is 24 tons, which, divided by the depth of the rib, $=\frac{24}{2\cdot5}=9\cdot6$ tons per foot.

The sectional area of the web plate $=30$ in. $\times \frac{5}{8}=18\cdot75$ inches, so that the intensity of the shearing stress on the web is very small.

We have next to determine the dimensions of the pins on which the ribs hinge at the centre and at the springing, the pressure on which becomes a maximum when the whole arch is covered by the maximum load, in which case the value of the horizontal re-action $=\mathrm{H}=180$ tons and V the vertical re-action $=\mathrm{O}$.

At the crown and springing the arched ribs terminate in steel castings forming bearing boxes, their webs being specially strengthened so as to distribute the pressure over the whole length of the pin, which is assumed to be 24 inches.

The pressure on the pin in the centre hinge is 180 tons. Let the diameter of the pin be assumed at 6 inches. Then the pressure per square inch on the pin

$$=\frac{180}{24\times6}=1\tfrac{1}{4} \text{ tons per square inch.}$$

The pressure at the springing

$$=\mathrm{V}\sin\phi+\mathrm{H}\cos\phi \ldots \phi=36^\circ,\ 52'$$
$$=120\times\cdot6+180\times\cdot8=216 \text{ tons.}$$

Then the pressure per square inch on the pin

$$=\frac{216}{24\times6}=1\tfrac{1}{2} \text{ tons per square inch.}$$

The stress in this type of arch may also be determined graphically as indicated in the diagrams Figs. 248 to 251.

Let $\quad \mathrm{V}=$ versed sine of arch.

$x=$ abscissa measured horizontally from crown to any point P.

w = weight of arch and load concentrated at any point P.

W = sum of weights between crown and any point $P = w + w$, etc.

m = moments at each point $P = wx$.

M = sum of moments between crown and $P = m + m$, etc.

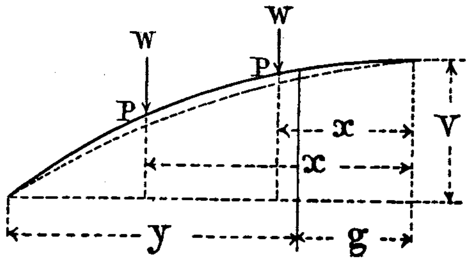


Fig. 247.

g = horizontal distance of centre of gravity of arch measured from crown $= \frac{M}{W}$.

y = horizontal distance of centre of gravity of arch measured from $P = x - g$.

H = horizontal thrust of arch.

k = ordinate from tangent to line of equilibrium

$$k = \frac{Wy}{H}.$$

From these expressions the diagrams in Figs. 248 to 251 are constructed and the stresses are scaled off as under.

Axial Thrusts.

Panel points.	When positive B.M.	When negative B.M.
2	171·06	136·22
3	153·66	128·64

Similarly, the bending moments may be obtained by multiplying the vertical intercept between the force polygon, and the axis of the rib, by the horizontal thrust thus.

Bending Moments.

Panel points.	Positive.	Negative.
No.	H × vertical intercept.	H × vertical intercept.
2	$154·69 \times 2·10 = 324·84$ tons.	$115·3 \times 1·71 = 197·16$ tons.
3	$146·85 \times 1·68 = 246·70$ tons.	$123·15 \times 1·73 = 213·04$ tons.

On comparing these results with the calculated results it will be seen that they are in very close accord.

Probably the most prominent example of this type of construction is the bridge over the Seine in Paris (Pont Alexandre III.) recently built under the direction of Monsieur Resal, Ingénieur en chef des Ponts et Chaussées, having a span of 352·6 feet from centre to centre of hinges and a rise of 20·6 feet.

Parabolic Arched Rib Hinged at Ends and Continuous at Crown.

$H =$ horizontal thrust at abutments due to any load W placed at a distance x from the centre of the span.

$c =$ half span of arch.

$r =$ rise or versed sine of arch.

$l =$ length of span.

V_1 and $V_2 =$ the vertical re-actions at the left and right abutments.

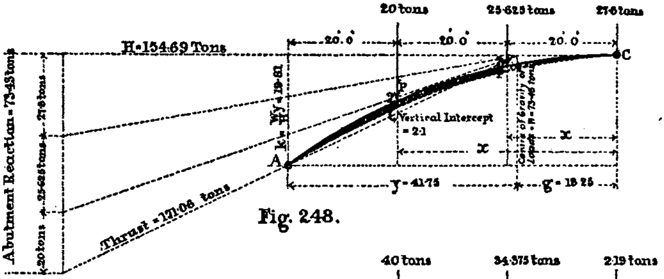


Fig. 248.

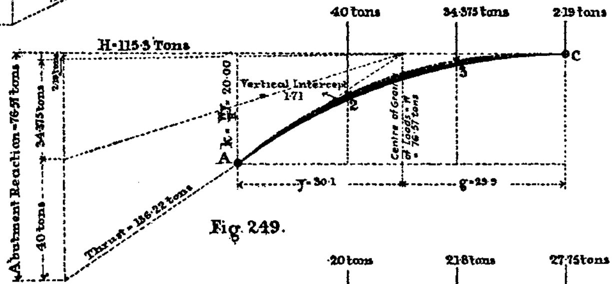


Fig. 249.

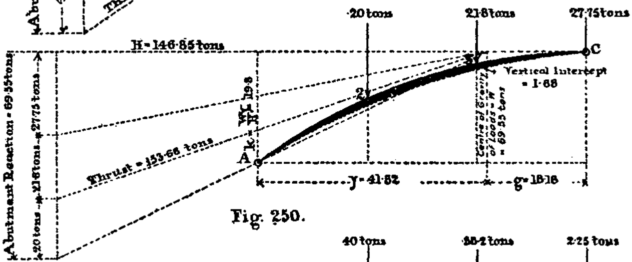


Fig. 250.

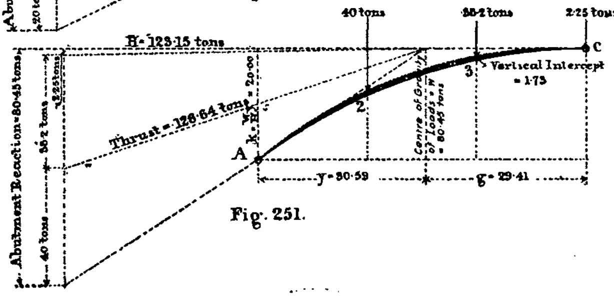


Fig. 251.

y = ordinate to axis of arch at point of bending moment M, this becomes y_0 at application of load W.

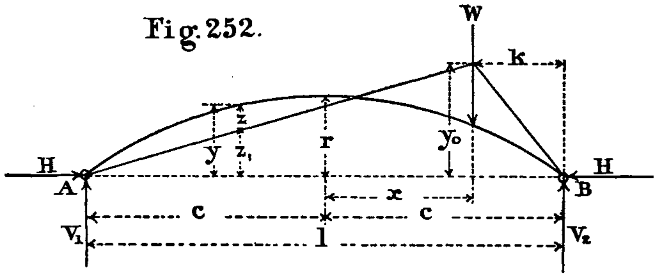
z_1 = ordinate to load polygon at point of bending moment M.

$z = z_1 - y$.

w = dead or static load per foot run.

w_1 = live or moving load per foot run.

Fig. 252.



In this instance, as in the case of an arch with three hinges, the resultant pressures pass through the hinges at the abutment where there is therefore no bending moment. The point of intersection of the lines of re-action of any load $W = y_0$ lies in a curve, the ordinate to which is found from the following equations:

$$y_0 \text{ at centre of span} = \frac{32}{25}r,$$

$$y_0 \text{ at ends of span} = \frac{32}{20}r,$$

$$y_0 \text{ at any intermediate point} = \frac{V_1 kl}{H} = \frac{1{\cdot}6r}{1+k-k^2},$$

or the value of y_0 at any point $= y_0 = \dfrac{32l^2r}{25l^2 - 20x^2}$,

k = fractional length of span = ·1, ·2, ·3, ·4, ·5,

$$H = \frac{5Wl}{8r}(k - 2k^3 + k^4) \text{ or } H = \frac{V_2 \times (c-x)}{y_0}.$$

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The vertical components V_1 and V_2 are the same as for an ordinary beam or girder supported at its ends.

M = the bending moment at any section $= Hz = H(z_1 - y)$.

Example. Let the span of a parabolic arched rib be 120 feet, the rise 20 feet, the dead or static load ½ ton per foot run and the live or rolling load be 1 ton per foot run, the horizontal girder supporting the road or railway as the case may be being supported at 9 intermediate points, as illustrated in Fig. 253.

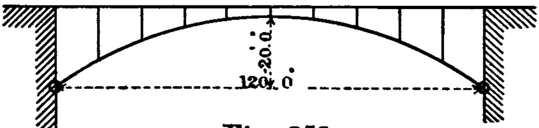


Fig. 253.

It is first required to determine the values of y_0 for each point of application of the load by the formula $\frac{1 \cdot 6r}{1 + k - k^2}$, the results of which are as under:

Panel points 0 and 10	$y_0 = 32{\cdot}00$
" " 1 and 9	$y_0 = 29{\cdot}36$
" " 2 and 8	$y_0 = 27{\cdot}58$
" " 3 and 7	$y_0 = 26{\cdot}44$
" " 4 and 6	$y_0 = 25{\cdot}80$
" " 5	$y_0 = 25{\cdot}60$

These values are set off by scale on vertical lines at the points 1, 2,... 9, which are the ordinates to the re-action locus or curve a, b, c,... k, Fig. 254.

Having once determined the values of y_0, the horizontal thrust = H for each load can be readily obtained by a graphic process from the diagram Fig. 255, the construction of which is as follows:

The vertical line AB is laid out on any convenient scale to represent one ton. From the extremity A the line A9 is

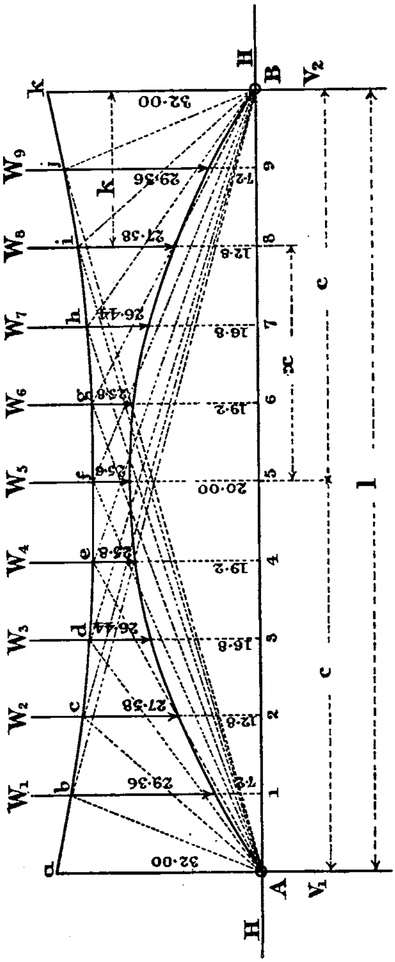
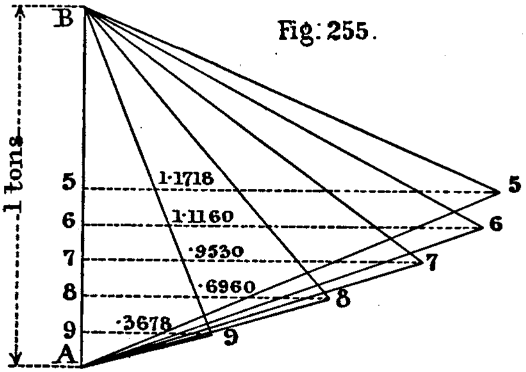

W1
W2
W3
W4
W5
W6
W7
W8
W9
a
b
c
d
e
f
g
h
i
j
k
k
H
H
A
B
V1
V2
32·00
29·36
27·58
26·44
25·8
25·6
7·2
12·8
16·8
19·2
20·00
1
2
3
4
5
6
7
8
9
x
c
c
l

Fig. 254.

drawn parallel to Aj in Fig. 254, and from the point B the line B9 parallel to the line Bj. The horizontal line 9, 9, drawn through the intersection of the two inclined lines A9 and B9,

Fig: 255.



represents on the same scale as AB the horizontal thrust produced by the load W_9. The horizontal thrust due to all the other loads can be determined in a similar manner, and all that is necessary is to multiply the horizontal ordinates 5, 6, 7, 8, 9, by the value of W.

The values of H, V_1 and V_2 are as under:

Position of Load.	H.	V_1.	V_2.
1	·3678	·9	·1
2	·6960	·8	·2
3	·9530	·7	·3
4	1·1160	·6	·4
5	1·1718	·5	·5
6	1·1160	·4	·6
7	·9530	·3	·7
8	·6960	·2	·8
9	·3678	·1	·9
Totals,	7·4374	4·5	4·5

Applying these results to the condition of loading indicated in Fig. 256, that is to say, multiplying each value of H, V_1 and V_2, the horizontal thrust and vertical re-actions will be as follow:

Position of Load.	Load.	H.	V_1.	V_2.
1	6 tons	2·2068 tons	5·4 tons	·6 ton
2	6 ,,	4·1760 ,,	4·8 ,,	1·2 ,,
3	6 ,,	5·7180 ,,	4·2 ,,	1·8 ,,
4	6 ,,	6·6960 ,,	3·6 ,,	2·4 ,,
5	12 ,,	14·0616 ,,	6·0 ,,	6·0 ,,
6	18 ,,	20·0880 ,,	7·2 ,,	10·8 ,,
7	18 ,,	17·1540 ,,	5·4 ,,	12·6 ,,
8	18 ,,	12·5280 ,,	3·6 ,,	14·4 ,,
9	18 ,,	6·6204 ,,	1·8 ,,	16·2 ,,
	108 tons	89·2488 tons	42·0 tons	66·0 tons

The values of H can also be determined readily by the formula $H = \frac{V_2(c-x)}{y_0}$ thus: let it be required to determine the value of H due to the load W_8. The value of V_2 for $W_8 = W \times \cdot 8$ and the value of $x = 36$. Then

$$H = \frac{18 \times \cdot 8 (60 - 36)}{27 \cdot 58} = 12 \cdot 53.$$

The bending moment at any point $= Hz$, and once the value of H for any condition of loading is known, the value of z can be easily determined by scale by means of a diagram as shown in Fig. 256, which represents the equilibrium polygon for the arch when half the span is covered by the live load. Fig. 257 represents the stress diagram the construction of which is as follows.

On the load line AB lay off the panel loads to any convenient scale, making AC = the re-action V_2 and BC = the re-action V_1. From the point C the boundary of the vertical re-actions draw CO horizontally = the horizontal re-action H. Draw the lines AO, 1O, ... BO and commencing at the point A (Fig. 256) draw Ab parallel to BO, bc parallel to 8O, etc. Then will the line Abcd...B be the equilibrium polygon for the condition of loading given, and the vertical intercepts between the force polygon and the axis of the rib $1'b$, $2'c$...$j9$ will give the values of z. Multiplying the values of z by H we have at once the bending moment.

Thus the values of z as scaled from the diagram, Fig. 256, are as under :

Point.	Value of z.	H.		Bending moment = H × z.	
1	$-1{\cdot}56$	$\times 89{\cdot}24$	=	$139{\cdot}21$	tons
2	$-2{\cdot}32$	$\times 89{\cdot}24$	=	$207{\cdot}03$	,,
3	$-2{\cdot}29$	$\times 89{\cdot}24$	=	$204{\cdot}36$	,,
4	$-1{\cdot}47$	$\times 89{\cdot}24$	=	$131{\cdot}18$	,,
5	$+0{\cdot}15$	$\times 89{\cdot}24$	=	$13{\cdot}38$	,,
6	$+1{\cdot}76$	$\times 89{\cdot}24$	=	$157{\cdot}06$	,,
7	$+2{\cdot}55$	$\times 89{\cdot}24$	=	$227{\cdot}56$	,,
8	$+2{\cdot}52$	$\times 89{\cdot}24$	=	$224{\cdot}88$	,,
9	$+1{\cdot}62$	$\times 89{\cdot}24$	=	$149{\cdot}03$	,,

An approximate value, generally within 5 % of the horizontal thrust, is given by the formula $H = \frac{l^2}{8r}\left(w + \frac{w_1}{2}\right)$ when half the span is covered by the live load.

Applying this equation to the condition of loading indicated in Fig. 256 the value of H will be $= \frac{120^2}{8 \times 20}\left(\frac{1}{2} + \frac{1}{2}\right) = 90$ tons, as against 89·24 tons, the actual value.

The values of z and M may in many instances be determined with sufficient precision by measurement from a diagram carefully drawn to a large scale in the manner that has been indicated in Fig. 256, but to be strictly accurate the values of z should be calculated and collected together in tabular form.

As an example, let it be required to determine the value of M at the point 7 in rib.

Weight on	$z_1 =$	$y =$	$z_1 - y = z =$	H =	W =		M =
1	9·79	− 16·8	= − 7·01 ×	·3678 ×	6 tons	=	− 15·47
2	10·34	− 16·8	= − 6·46 ×	·6960 ×	6 „	=	− 26·97
3	11·33	− 16·8	= − 5·47 ×	·9530 ×	6 „	=	− 31·28
4	12·90	− 16·8	= − 3·90 ×	1·1160 ×	6 „	=	− 26·11
5	15·36	− 16·8	= − 1·44 ×	1·1718 ×	12 „	=	− 20·25
6	19·35	− 16·8	= + 2·55 ×	1·1160 ×	18 „	=	+ 51·22
7	26·44	− 16·8	= + 9·54 ×	·9530 ×	18 „	=	+ 161·65
8	24·13	− 16·8	= + 7·33 ×	·6960 ×	18 „	=	+ 91·83
9	22·83	− 16·8	= + 6·03 ×	·3678 ×	18 „	=	+ 39·92
				Bending moment,			+ 226·54

A similar calculation must be made for each point in the arch at which the bending moment is to be determined. In our example, the arch is divided into ten segments so that the values of M must be determined at nine intermediate points, and they are as given in the table on p. 316.

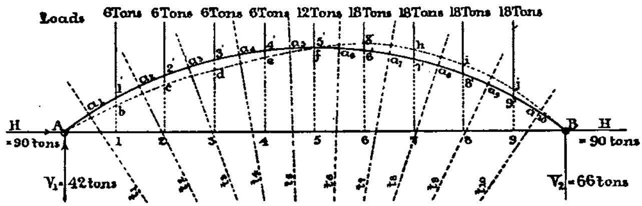


Fig. 256.

The dotted line in Fig. 256 represents the curve of equilibrium for the condition of loading indicated, and the dotted lines t_1, t_2, ... t_{10} represent the lines of action of the shearing stresses.

Load on	Bending Moments at Points 1.	2.	3.	4.	5.	6.	7.	8.	9.
1	+48·90	+29·35	+13·29	+ 0·81	− 8·14	− 13·57	− 15·47	− 13·84	− 8·69
2	+27·51	+61·72	+30·61	+ 6·18	−11·52	− 22·59	− 26·97	− 24·68	− 15·66
3	+ 9·20	+27·62	+55·12	+19·78	− 6·40	− 23·38	− 31·28	− 30·00	− 19·61
4	− 5·02	+00·67	+17·07	+44·19	+10·04	− 13·39	− 26·11	− 28·12	− 19·41
5	−29·24	−36·00	−20·25	+18·00	+78·75	+ 18·00	− 20·25	− 36·00	− 29·24
6	−58·25	−84·36	−78·34	−40·17	+30·13	+132·52	+ 51·22	+ 2·00	− 15·06
7	−58·83	−90·05	−93·83	−70·16	−19·21	+ 59·35	+163·65	+ 82·80	+ 27·81
8	−46·98	−74·04	−80·93	−67·90	−34·58	+ 18·54	+ 91·83	+185·04	+ 82·56
9	−26·07	−41·50	−46·47	−40·71	−24·43	+ 2·50	+ 39·92	+ 88·01	+146·70
Totals,	−138·78	−205·59	−203·73	−129·98	+14·64	+157·98	+226·54	+225·21	+149·40

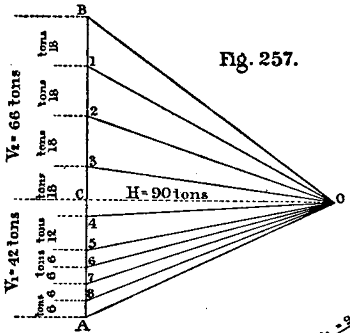


Fig. 257.

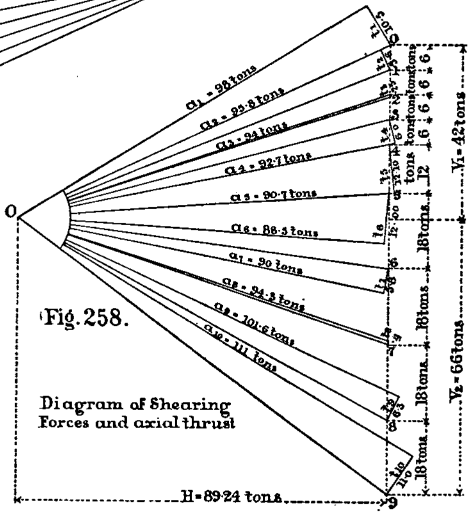


Fig. 258.

For a uniformly distributed load over the whole span the horizontal thrust is obtained by adding together the values of H for each load, that is to say in the example we have selected, assuming the maximum load at each point of division of the rib to be 18 tons, the horizontal thrust would be $= 18 \times 7{\cdot}4374 = 133{\cdot}87$ tons. If the loads were applied at each linear unit of length, thus making the distribution practically continuous as shown in the left-hand half of Fig. 259, instead of being transmitted at intervals as shown in the right-hand half of Fig. 259, the horizontal thrust would be equal to $\frac{Wl}{8d}$, the corresponding value of which in our example would be $\frac{180 \times 120}{8 \times 20} = 135$ tons.

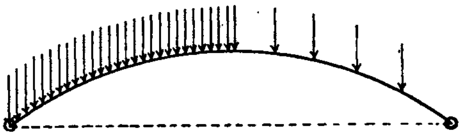


Fig. 259.

Shear.

In arched ribs of I-shaped girders or of open lattice webs connecting the two flanges the vertical force or shear must be determined and provided for, but in ribs of rectangular or circular section the shear will be so small that it need not be considered.

The shearing forces can be readily determined in a simple manner by the aid of a diagram as shown in Fig. 258, thus:

Lay off on the load line 0·9 the vertical re-actions and the panel loads as in Fig. 257. Draw the ray lines O0, O1, ... O9. From the polar point O draw a_1 parallel to the tangent at a_1 (Fig. 256), and from a_1 let fall at right angles t_1, intersecting the point 0. Similarly draw a_2 parallel to the tangent at the point a_2, and from a_2 let fall t_2 at right angles to a_2, intersecting the point 1. Proceeding in like manner, the diagram is completed and the values of $t_1, t_2, \dots t_{10}$ by scale will represent the shear at the corresponding points in the rib.

The directions of the tangents to a parabolic curve are obtained by making the centre ordinate equal to $2r$ and dividing it into a number of equal parts corresponding with the number of panels or segments in the half span. If their ray lines are drawn from the left-hand hinge for the left half of the span to each of these points then will these lines be parallel to the tangents to each segment of the rib.

In the diagram (Fig. 258) the shears as determined by scale are as follow:

Segment of rib.	By diagram. Value of shear.	By calculation. Value of shear.
a_1	10·50 tons	− 11·53
a_2	5·60 "	− 5·65
a_3	0·25 "	+ 0·27
a_4	6·00 "	+ 6·16
a_5	12·10 "	+ 12·04
a_6	12·00 "	+ 11·98
a_7	5·80 "	+ 5·82
a_8	0·40 "	− 0·25
a_9	6·30 "	− 6·35
a_{10}	11·00 "	− 12·47

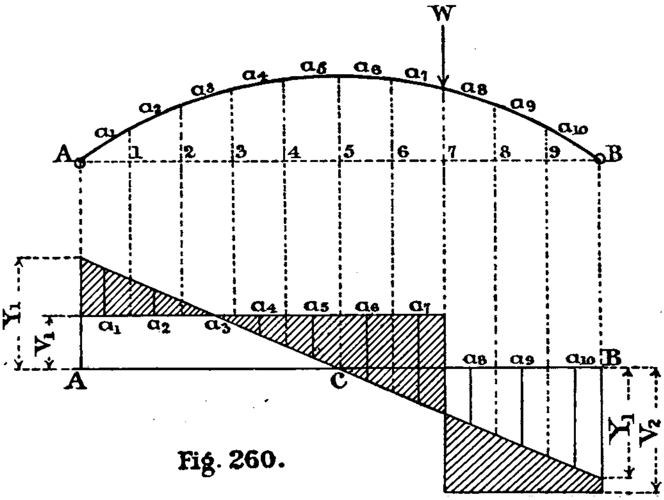


Fig. 260.

The shears may also be obtained by computation, thus:

Let S = shear.

Y_1 = vertical re-action at abutments necessary to combine with the horizontal thrust H in order to bring the resultant re-action at the abutment tangent to the parabolic rib axis.

$$Y_1 = \frac{2r}{c} H \quad \text{and} \quad S = V_1 - Y_1.$$

As an example of the application of this formula, let it be required to determine the shear at the points a_1, a_2, ... a_{10} due to the load $W_7 = 18$ tons.

Then
$$V_1 = 18 \times \cdot 3 = 5 \cdot 4,$$
$$V_2 = 18 \times \cdot 7 = 12 \cdot 6,$$
$$H = 17 \cdot 154,$$
$$Y_1 = \frac{2 \times 20}{6} \times 17 \cdot 154 = 11 \cdot 43.$$

The value of Y_1 at $A = 11 \cdot 43$, at a_1 it $= 11 \cdot 43 \times \cdot 9$, and at a_2 $11 \cdot 43 \times \cdot 7$, a_3 $11 \cdot 43 \times \cdot 5$, etc.

Direct Thrust.

The arched rib is subject to a direct thrust throughout its length uniformly distributed over the area of its cross section if the section be symmetrical, and the value of that thrust increases at all points with any increment of the load, so that it attains its maximum value when all points are fully loaded.

If under a uniformly distributed load the curve of equilibrium due to that condition of loading coincide with the neutral curve or axis of the rib, there will be no bending moments and the stresses will be those of compression simply, acting at right angles to its normal section.

Under this condition of loading the thrust at the crown of the arch

$= T = \dfrac{w_0 l^2}{8r}$ and this also represents the value of H.

Points in rib.	a_1.	a_2.	a_3.	a_4.	a_5.	a_6.	a_7.	a_8.	a_9.	a_{10}.
Values of Y_1, - -	10·29	8·01	5·72	3·44	1·15	−1·15	−3·44	− 5·72	− 8·01	−10·29
Values of V, - -	5·40	5·40	5·40	5·40	5·40	5·40	5·40	−12·60	−12·60	−12·60
Values of $V - Y_1$, -	−4·89	−2·61	−0·33	+1·96	+4·24	+6·55	+8·84	− 6·88	− 4·59	− 2·31

Computing the value of $V - Y_1$ at each point in the rib due to each load, we have the following Table of Shears:

Number and intensity of weight.	V_1.	V_2.	Y_1.	Value of shear $= V - Y_1$ at points in the rib.									
	Tons	Tons	Tons	a_1.	a_2.	a_3.	a_4.	a_5.	a_6.	a_7.	a_8.	a_9.	a_{10}.
Tons 1 = 6	5·4	0·6	1·47	+ 4·07	−1·63	−1·33	−1·04	− 0·75	− 0·45	−0·16	+0·13	+0·43	+ 0·72
2 = 6	4·8	1·2	2·78	+ 2·30	+2·85	−2·59	−2·03	− 1·48	− 0·92	−0·37	+0·19	+0·75	+ 1·30
3 = 6	4·2	1·8	3·81	+ 0·77	+1·53	+2·30	−2·94	− 2·17	− 1·41	−0·68	+0·11	+0·87	+ 1·63
4 = 6	3·6	2·4	4·46	− 0·41	+0·48	+1·37	+2·26	− 2·84	− 1·95	−1·06	−0·17	+0·72	+ 1·61
5 = 12	6·0	6·0	9·37	− 2·44	−0·56	+1·32	+3·19	+ 5·06	− 5·06	−3·19	−1·32	+0·56	+ 2·44
6 = 18	7·2	10·8	13·39	− 4·85	−2·17	+0·51	+3·18	+ 5·86	+ 8·54	−6·79	−4·11	−1·43	+ 1·25
7 = 18	5·4	12·6	11·43	− 4·89	−2·61	−0·33	+1·96	+ 4·24	+ 6·55	+8·84	−6·88	−4·59	− 2·31
8 = 18	3·6	14·4	8·35	− 3·91	−2·25	−0·57	+1·10	+ 2·76	+ 4·44	+6·11	+7·78	−8·55	− 6·88
9 = 18	1·8	16·2	4·41	− 2·17	−1·29	−0·41	+0·48	+ 1·36	+ 2·24	+3·12	+4·00	+4·89	−12·23
				−11·53	−5·65	+0·27	+6·16	+12·04	+11·98	+5·82	−0·25	−6·35	−12·47

At any intermediate point distant x from the centre, when the curve of the arched rib is a parabola,

$$T = \sqrt{\left(\frac{w_0 l^2}{8r}\right)^2 + (w_0 x)^2};$$

T = direct or axial thrust;

w_0 = dead and live loads per unit of length = $w + w_1$.

These expressions are only strictly correct when the load is uniformly distributed over the arched rib, which in practice

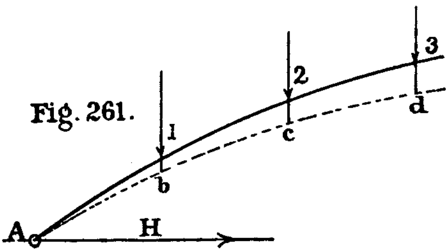


Fig. 261.

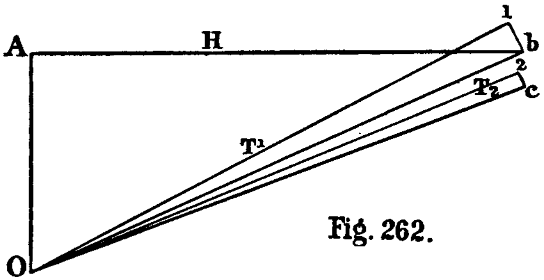


Fig. 262.

could only be accomplished by the introduction of a solid plate web between the horizontal girder supporting the roadway and the arch. By the usual mode of construction the load is transferred to the arch by vertical members equidistantly spaced, and there would be produced in the arched rib between each point of application of the load bending moments.

The value of T under a variable load can be readily determined by a graphic process, thus:

In Fig. 261 the curve A123 represents the axis of the arch, and the dotted curve A*bcd* the curve of pressure due to the loading.

In Fig. 262 draw the horizontal line A*b*, making it equal to H. From the point *b* draw *b*O parallel to the tangent to the curve of pressure at *b*. From A let fall the perpendicular AO intersecting *b*O at O. From O draw O1 parallel to the tangent at 1 to the axis of the arch. From *b* draw *b*1 at right angles to O1 intersecting O1 at 1. Then will $O1 = T_1$ represent by scale the direct or axial thrust at the point 1, and similarly O2 at the point 2 while 1*b* and 2*c* will represent respectively the shears at the points 1 and 2.

In Fig. 258 the axial thrusts and shearing forces have in this manner been graphically determined for each section of the arch.

The thrust in the section A1 = 98 tons and in 1, 2 the thrust = 95·8 tons.

If, therefore, we take the mean of these two values, we obtain with sufficient accuracy for practical purposes the direct thrust at the point 1, where the bending moment has been determined.

The values of T at each point of application of the load will therefore be as follow:

0	98·0	tons
1	96·9	„
2	94·9	„
3	93·3	„
4	91·7	„
5	89·6	„

6...	89·2	tons
7..........	92·1	„
8...........	97·9	„
9....	106·3	„
10..........	111·0	„

Temperature Stresses.

In an arch either hinged or fixed at the abutments the length of span from end to end must always remain invariable, and under the influence of changes in temperature the expansion or contraction which the structure undergoes has the effect of altering its shape, causing the crown of the arch to rise or fall, and producing stresses of compression or tension, increasing or diminishing to that extent the intensity of the horizontal thrust at the abutments. As the abutments are fixed in position, the horizontal force is resisted and transmitted throughout the arch exerting bending moments which must be added to the bending moments due to the loading.

The value of the bending moment is obtained by multiplying the horizontal thrust due to change of temperature $= \mathrm{H}t$ by the ordinate to the axis of the arch at any desired section $= y$. We have therefore to obtain the value of $\mathrm{H}t$, but before we can do so, the moment of inertia of the section of the arch must be determined, because the stiffness of a beam depends upon its form or section.

Let $\mathrm{I} =$ moment of inertia of section,

$\mathrm{E} =$ modulus of elasticity of the material $=$ for steel 29,000,000,

$t =$ number of degrees Fahrenheit range of temperature above or below mean $= \pm 60°$,

$\epsilon =$ coefficient of expansion $= \cdot 0000066$ for steel.

(A bar of iron, if its temperature be raised from freezing point to boiling point, will expand

$\frac{1}{8}$ per cent., or $\frac{1}{800}$ part of its length. Its coefficient of expansion will therefore be for one degree $\frac{1}{800 \times 180} = \frac{1}{144,000} = \cdot 0000067$, and similarly for steel $\frac{1}{920 \times 180} = \frac{1}{165,600} = \cdot 0000066$.

Then $Ht = \pm \frac{15}{8} \times \frac{t\epsilon EI}{r^2}$.

Circular Arch-ribs hinged at springing.

In a circular arch the calculations involved in determining the value of H, the horizontal thrust, are very tedious, but with the aid of tables, such as those prepared by M. Bresse giving the values of certain angular co-efficients employed in the formulae, much labour is saved, the calculations are much simplified. When the rise of the arch does not exceed $\frac{1}{8}$th of the span a circular arch may be submitted to the same treatment as a parabolic arch with sensible error; where the ratio of rise to span exceeds this proportion, the divergence between the axes of a parabolic and circular arch of equal span and rise becomes too pronounced; but if the rise of the parabolic arch be increased so that the two curves shall include an equal area above the chord or springing line, the formulae for parabolic arches can be applied to determine the value of H and then apply that value to the given circular arch. The difference between the value so found and the correct value will be practically unappreciable.

Professor Malverd A. Howe in his work on Arches has proposed the following formula for determining H for a circular arch:

$$H = \frac{\Sigma W\{k(l - k - 2sa) - s(l\phi - la - 2y)\}}{4s^2\phi + 2R^2\phi - 3sl},$$

in which H = horizontal thrust at abutments,

l = length of span,

r = rise or versed sine of arch,

R = radius of arch,

S = R − r,

y = ordinate axis at point of application of any load W,

k = distance from abutment to point of application of load W,

ϕ = angle of support from crown of arch in radians,

α = angle of point of application of load from crown of arch in radians = length of arc divided by radius.

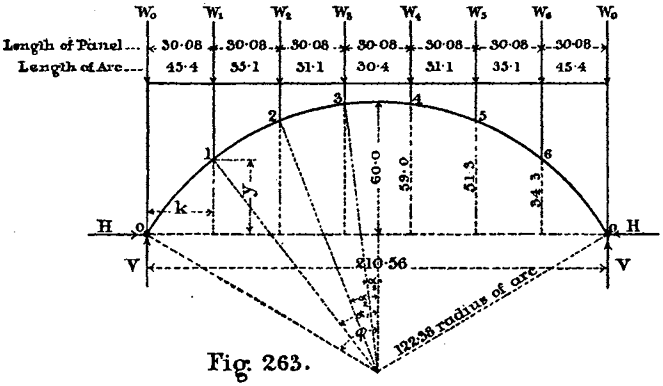


Fig. 263.

$\phi = 59·36° = 1·036.$ Circumference of circle = 768·92 feet.

$\alpha_1 = 38·11° = ·665.$ Length of arc = 253·6 feet.

$\alpha_2 = 21·67° = ·370.$

$\alpha_3 = 7·11° = ·124.$

As an example of this type of arch, Fig. 263 represents a diagram of a two-hinged circular arched bridge built to carry

South Market Street across the Mahoning River at Youngstown, Ohio, from the designs and under the direction of Mr. C. E. Fowler, M.Am.Soc.C.E., to whom the author is indebted for particulars and a description of the structure.

Applying the above formula, the values of H for unity load at the several panel points will be as under:

Panel points

1 and 6.

$$H = \frac{30{\cdot}08(210{\cdot}56 - 30{\cdot}08 - 2 \times 62{\cdot}38 \times {\cdot}665) - 62{\cdot}38(210{\cdot}56 \times 1{\cdot}036 - 210{\cdot}56 \times {\cdot}665 - 2 \times 34{\cdot}3)}{(4 \times 62{\cdot}38^2 + 2 \times 122{\cdot}38^2)1{\cdot}036 - 3 \times 62{\cdot}38 \times 210{\cdot}56}$$

$$= {\cdot}302.$$

2 and 5.

$$H = \frac{60{\cdot}16(210{\cdot}56 - 60{\cdot}16 - 2 \times 62{\cdot}38 \times {\cdot}370) - 62{\cdot}38(210{\cdot}56 \times 1{\cdot}036 - 210{\cdot}56 \times {\cdot}370 - 2 \times 51{\cdot}3)}{(4 \times 62{\cdot}38^2 + 2 \times 122{\cdot}38^2)1{\cdot}036 - 3 \times 62{\cdot}38 \times 210{\cdot}56}$$

$$= {\cdot}506.$$

3 and 4.

$$H = \frac{90{\cdot}24(210{\cdot}56 - 90{\cdot}24 - 2 \times 62{\cdot}38 \times {\cdot}124) - 62{\cdot}38(210{\cdot}56 \times 1{\cdot}036 - 210{\cdot}56 \times {\cdot}124 - 2 \times 59)}{(4 \times 62{\cdot}38^2 + 2 \times 122{\cdot}38^2)1{\cdot}036 - 3 \times 62{\cdot}38 \times 210{\cdot}56}$$

$$= {\cdot}625.$$

Multiplying each value of H by its panel load, and summing the products will give the total horizontal thrust at the abutments due to any given condition of loading.

By way of comparing results, it will be interesting to apply the formulae for parabolic arches to determine the values of H for unit loads on the several panel points.

The area of a parabolic segment is given by the expression $= lr\frac{2}{3}$, and that of a circular segment $= \frac{4r}{3}\sqrt{(0{\cdot}626r)^2 + \left(\frac{l}{2}\right)^2}$.

By equating these expressions and substituting, it is determined that a parabola of equivalent area to the circular arch of 210·56 feet span and 60 feet rise must have a rise of 63·71 feet. Then by the formula

$$H = \frac{5wl}{8r}(k - 2k^3 + k^4),$$

the values of H for unity loads will be as under:

Panel point 1, H = ·2839,
2, H = ·5075,
3, H = ·6297.

Having determined the values of H for unit loads, the two following conditions of loading will be investigated.

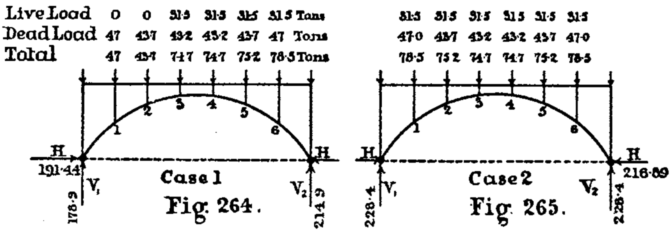


Fig. 264. Fig. 265.

The total horizontal thrust for the condition of loading indicated in Fig. 264 will be

Panel points.	Treated as circular arch.		W = panel loads in tons.	Treated as parabolic arch of equal segmental area.	
	Value of H for unit load = H.	Value of H for actual load = HW.		Value of H for unit load = H.	Value of H for actual load = HW.
1	0·302	14·194	47·0	0·2839	13·343
2	0·506	22·112	43·7	0·5075	22·177
3	0·625	46·687	74·7	0·6297	47·038
4	0·625	46·687	74·7	0·6297	47·038
5	0·506	38·051	75·2	0·5075	38·164
6	0·302	23·707	78·5	0·2839	22·286
	Total,	191·438	393·8		190·046

Similarly, the total horizontal thrust for the condition of loading given in Fig. 265 will be

Panel points.	Treated as circular arch.		W =panel loads in tons.	Treated as parabolic arch of equal segmental area.	
	Value of H for unit load =H.	Value of H for actual load =HW.		Value of H for unit load =H.	Value of H for actual load =HW.
1	0·302	23·707	78·5	0·2839	22·286
2	0·506	38·051	75·2	0·5075	38·164
3	0·625	46·687	74·7	0·6297	47·038
4	0·625	46·687	74·7	0·6297	47·038
5	0·506	38·051	75·2	0·5075	38·164
6	0·302	23 707	78·5	0·2839	22·286
	Total,	216·890	456·8		214·976

These computations show that the difference between the approximate and correct methods does not amount to quite 1 per cent., which for all practical purposes is inappreciable.

It now remains to determine the vertical re-actions for both conditions of loading.

Case 1.

$$V_1 = \frac{47 \times 6 + 43{\cdot}7 \times 5 + 74{\cdot}7 \times (4+3) + 75{\cdot}2 \times 2 + 78{\cdot}5 \times 1}{7}$$

$$= 178{\cdot}9.$$

$$V_2 = \frac{47 \times 1 + 43{\cdot}7 \times 2 + 74{\cdot}7 \times (4+3) + 75{\cdot}2 \times 5 + 78{\cdot}5 \times 6}{7}$$

$$= 214{\cdot}9.$$

Case 2. $V_1 = 456{\cdot}8 \div 2 = 228{\cdot}4.$

$V_2 = 456{\cdot}8 \div 2 = 228{\cdot}4.$

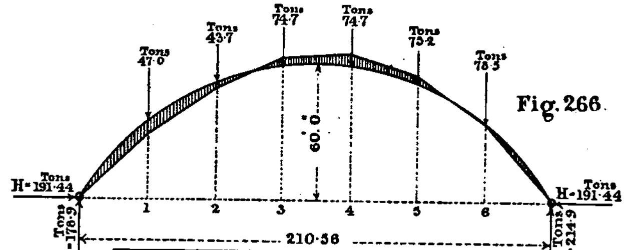


Fig. 266.

Points of Loading	1	2	3	4	5	6
Ordinates to Force Polygon	28.11	48.83	62.69	64.80	55.19	33.76
Ordinates to Axis of Rib	34.30	51.30	59.00	59.00	51.30	34.30
Vertical Intercepts	-6.19	-2.47	+3.69	+5.80	+3.69	-0.54

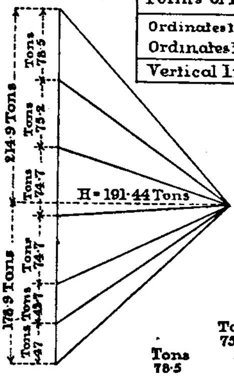


Fig. 267.

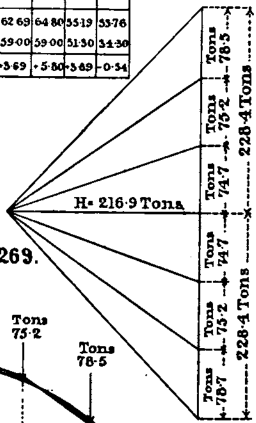


Fig. 269.

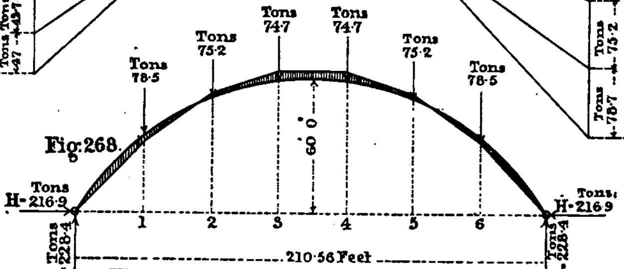


Fig. 268.

Points of Loading	1	2	3	4	5	6
Ordinates to Force Polygon	31.65	52.43	62.78	62.78	52.43	31.65
Ordinates to Axis of Rib	34.30	51.30	59.00	59.00	51.30	34.30
Vertical Intercepts	-2.65	+1.13	+3.78	+3.78	+1.13	-2.65

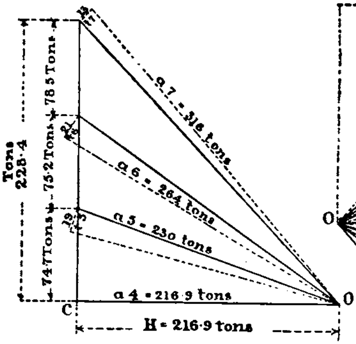


Fig. 272.

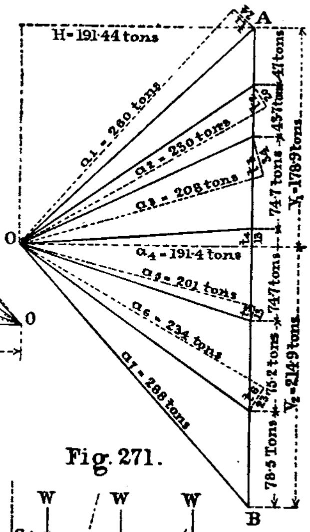


Fig. 271.

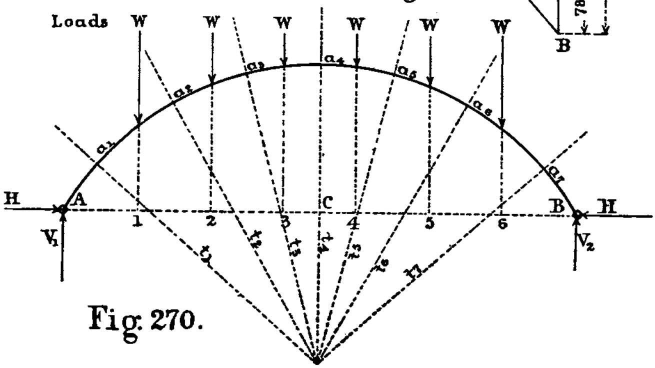


Fig. 270.

The maximum shear occurs at A, and it

$$= N = V \cos \phi - H \sin \phi.$$

The value of V_1 producing maximum shear $= 178 \cdot 9$ tons, and of the corresponding $H = 191 \cdot 4$. The value of $\phi = 59 \cdot 36°$.

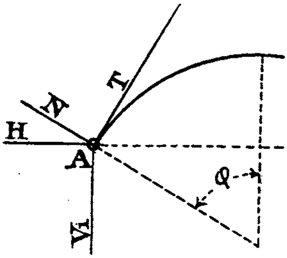


Fig. 273.

Then

$N = V_1 \cos\phi - H \sin\phi = N$

$= 178{\cdot}9 \times {\cdot}5097 - 191{\cdot}44 \times {\cdot}860$

$= -73{\cdot}45$ tons.

Similarly, the value of the direct thrust $= T$ is given by the expression

$$T = H \cos\phi + V_1 \sin\phi.$$

Substituting the value of T at A

$= T$

$= 191{\cdot}44 \times {\cdot}5097 + 178{\cdot}9 \times {\cdot}860$

$= 251{\cdot}43$ tons.

The stresses due to a change of temperature must next be considered. An increase of temperature would have the effect of lengthening the span, and a fall of temperature would shorten it, but the abutments being fixed, any change of length cannot take place; there must, therefore, as the temperature rises or falls above or below the normal temperature, be exerted throughout the rib a bending moment and direct thrust. The range of temperature is usually taken at $\pm 60°$.

The horizontal component of this thrust for a circular arch is given by the expression

$$Ht = \pm \frac{2EIt\epsilon \sin\phi}{R^2(\phi + 2\phi\cos^2\phi - 3\sin\phi\cos\phi)}.$$

For a parabolic arch rib the equivalent expression is

$$Ht = \pm \frac{15}{8} \cdot \frac{t\epsilon EI}{r^2},$$

I = moment of inertia of section,

E = modulus of elasticity of the material = 29,000,000,

t = range of temperature in degrees Fahr. = $\pm 60°$,

ϵ = co-efficient of expansion = ·0000066,

R = radius of circular arch,

r = rise of parabolic arch.

The value of I is so far an unknown quantity, and it cannot be definitely determined until the calculations are completed, which must include the stresses produced by change of temperature, in which the value of I is the most important factor. Some approximate value of I must therefore, in the first instance, be assumed. It will be seen that the effect of a change of temperature in deforming an arched rib in this instance is small as compared with the bending moments, and it may in the first instance be neglected and the section of the rib may be proportioned, without for the moment making any provision for temperature stresses, so as to arrive at some approximate value of I. The calculations may afterwards be re-cast, taking into account the effect of the thrust due to temperature.

In this manner the provisional value of I is taken to be 240,000.

Then

$$\mathrm{H}t = \pm \frac{2\mathrm{EI}t\epsilon \sin\phi}{\mathrm{R}^2(\phi + 2\phi\cos^2\phi - 3\sin\phi\cos\phi)},$$

$$\mathrm{H}t = \pm \frac{2 \times 29{,}000{,}000 \times 240{,}000 \times 60 \times \cdot 0000066 \times \cdot 860}{(122 \cdot 38 \times 12)^2 \times \cdot 25475}$$

$$= 8663 \text{ lbs.}$$

$$= 3 \cdot 86 \text{ tons.}$$

Similarly, by way of comparison, it may be shown that the value of Ht for a parabolic arch is very nearly in accord.

$$\mathrm{H}t = \pm \frac{15 \times 60 \times \cdot 0000066 \times 29{,}000{,}000 \times 240{,}000}{8 \times (63 \cdot 7 \times 12^2)}$$

$$= 8845 \text{ lbs.}$$

$$= 3 \cdot 94 \text{ tons.}$$

Let the value of Ht be taken at 4 tons.

On reference to the diagrams it will be seen that the maximum bending moment is at the point 1 (Fig. 266), when the panel points 3, 4, 5, and 6 are covered by full loads. The vertical intercept from the force polygon to the centre of moments at point 1 is 6·19 feet. The bending moments will then be as under:

Vertical loads, 191·44 tons × 6·19 ft. = 1185 foot-tons.
Temperature, - 4 tons × 34·3 ft. = 137 „

Maximum bending moment = M = 1322 foot-tons.

The tangential or axial thrust is shown in the diagram (Fig. 274) to have a maximum value of 316 tons, due to vertical loads. To this must be added the axial thrust due to change of temperature, which is given by the expression $T = Ht \cos \phi$.

ϕ in this case being the angle of point where thrust is required from crown of arch,

$Ht = 4$ tons and $\cos \phi$ for panel point $1 = \cdot 787$.

Then $T = 4 \times \cdot 787 = 3 \cdot 184$ tons.

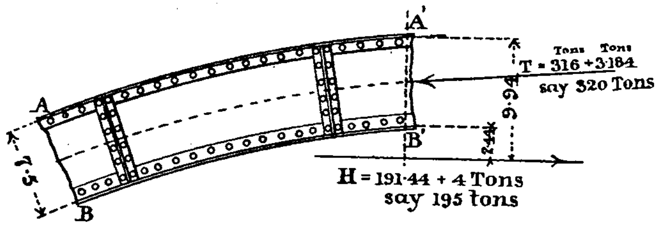


Fig. 274.

The tangential stresses and bending moments are taken entirely by the flanges and the shear by the web. The tangential stress is assumed to be uniformly distributed over the two flanges so that the stress taken by each flange will be $320 \div 2 = 160$ tons.

The stress in the upper flange A . A′ will be

$$= \frac{195 \times 9 \cdot 94}{7 \cdot 5} = 258 \cdot 44 \text{ tons,}$$

and in the lower flange B . B′

$$= \frac{195 \times 2 \cdot 44}{7 \cdot 5} = 63 \cdot 44 \text{ tons.}$$

The vertical intercept in this case being a minus quantity, the stress in the upper flange will be tensile, and in the lower flange compressive.

By combining these stresses under their proper signs the maximum fibre stresses are arrived at.

Stresses at Point 1.	Upper Flange.	Lower Flange.
Axial thrust,	+ 160 tons	+ 160 tons
Bending moments,	− 258·44 tons	+ 63·44 tons
Maximum fibre stress,	− 98 tons	+ 223·44 tons

If the working stresses be limited to 5 tons per square inch, the sectional area of the flanges must not be less than $223 \div 5 = 44{\cdot}6$, say 45 inches.

The sectional area of the built girder shown in Fig. 275 is as under:

		sq. in.
3 flange plates, $16'' \times \frac{11}{16}''$	=	33·00
2 angles, $6'' \times 6'' \times \frac{7}{8}''$	=	19·46
Gross area,	-	52·46

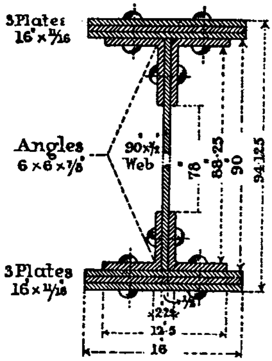


Fig. 275.

The net area, allowing for rivet holes 1″ diameter, will be

3 flange plates $(16-2)\frac{11}{16}$	=	28·875
2 angles, less 2 rivet holes	=	17·720
Net area,	-	46·595

Moment of inertia

$$I = \frac{1}{12}\left[16(94{\cdot}125^3 - 90^3) + 12{\cdot}5(90^3 - 88{\cdot}25^3)\right.$$
$$\left. + 2{\cdot}25(88{\cdot}25^3 - 78^2) + {\cdot}5 \times 78^3\right]$$
$$= \frac{1{,}678{,}400 + 521{,}300 + 478{,}674 + 237{,}276}{12} = 242{,}900.$$

The value of I as a first approximation in the calculation of the horizontal thrust due to a change of temperature was taken at 240,000, which is sufficiently near the actual value.

Two-hinged Parabolic Braced Arch.

As an example of this type of construction, Fig. 276, which represents a diagram of the main span of the Manhattan Valley Viaduct on the New York Rapid Transit Railway recently carried out from the designs and under the direction of Mr. William Barclay Parsons as chief engineer, will serve to illustrate how the stresses are determined.

The horizontal thrust at the abutments due to any vertical load W is found in the same way as in the preceding example for a parabolic arched rib hinged at the ends.

Using the same notation as in that example, the horizontal thrust for vertical loads is given by the expression

$$\mathrm{H} = \frac{5}{64} \times \frac{(c^2 - x^2)(5c^2 - x^2)}{c^3} \times \frac{\mathrm{W}}{r},$$

and for temperature

$$\mathrm{H}t = \pm \frac{15}{8} \times \frac{t\epsilon \mathrm{EI}}{r^2}.$$

The vertical components of the abutment re-actions are found in the same manner as for a simple beam.

The above formulae for H and Ht are based on the assumption that the moment of inertia of the section of the rib varies in the ratio of the secant of inclination of the rib to the horizontal.

Stress in a chord member due to a vertical load.—Let it be required to find the stress in the member GH due to a single concentrated load W applied at joint S. Let $\mathrm{W} = 160{,}000$ lbs. applied at a distance $x = 36{\cdot}5$ feet to the right of the centre of the span. Applying the formula to

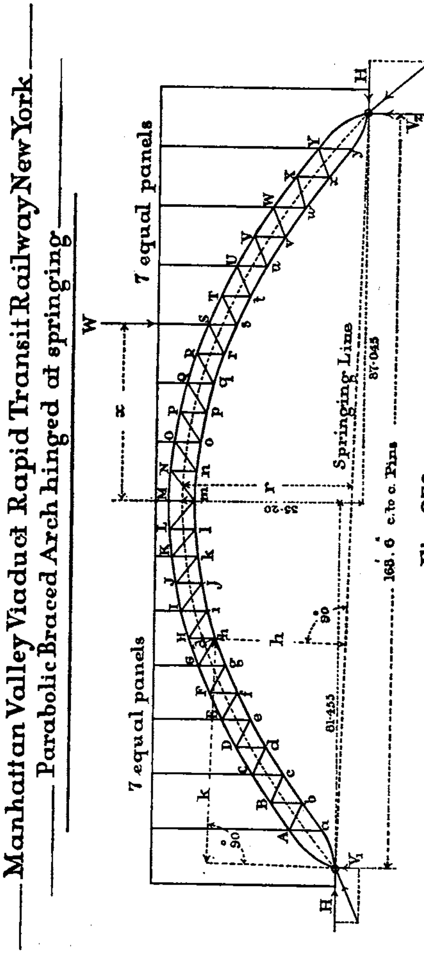

Manhattan Valley Viaduct Rapid Transit Railway New York
Parabolic Braced Arch hinged at springing
7 equal panels
7 equal panels
W
x
r
h
k
Springing Line
87·045
81·455
168.6 c. to c. Pins
H
H
90°
90°

Fig. 276.

determine H and substituting, we derive the value of H to be 109,000 lbs.

By the principle of moments $V_1 = \frac{c-x}{2c} = 45,000$ lbs.

Taking moments about the joint h, $M = V_1 k - Hh$.

Stress in $GH = \frac{V_1 k - Hh}{d}$, in which $k = 50{\cdot}8$ ft., $h = 28{\cdot}7$ ft., and $d = 6{\cdot}0$ ft. are the lever arms of forces V, H and GH respectively.

Substituting, stress in GH $= 141,000$ lbs.

Let it next be required to find the range of stress in the member GH caused by the maximum variation of $\pm 60°$ Fahr. in temperature.

In the formula $Ht = \pm\frac{15}{8} \times \frac{t\epsilon EI}{r^2}$; $\quad I = 97{\cdot}700$, $E = 29,000,000$, $\epsilon = {\cdot}0000066$.

Substituting, $Ht = \pm 10,000$ lbs.

By the principle of moments, stress in $GH = \pm\frac{Ht \times h}{d}$.

Substituting, stress in GH $= \mp 49,000$ lbs., or tension for an increase, and compression for a decrease in temperature.

Let it again be required to find the stress in the member Hh caused by a single concentrated load W applied at the joint S. The resultant on the section perpendicular to the parabola at its intersection with Hh is obtained by algebraically adding the separate products of H and P by their respective cosines of inclination to the section. The product of this shear by the secant of inclination of Hh to the right section is the direct stress.

Shear on right section $= -H \times {\cdot}33 + V_1 \times {\cdot}95 = +6780$ lbs.
Stress in Hh $= \text{shear} \times 1{\cdot}05 = +7122$ lbs.

In this manner the shears and bending moments at all points in the arch can be calculated for any system of loading, and the stresses in any individual member can be readily determined.

The most prominent example of a braced arch is the Niagara Falls and Clifton steel arch, which has been recently built over the Niagara River, on the site of the old suspension bridge, from the designs and under the direction of Mr. L. L. Buck, M.I.C.E. The span of the arch is 840 feet from centre to centre of the end pins and the rise from the level of the end pins to the centre of the rib-trusses at the crown of the arch is 150 feet. Each rib is 26 feet deep, and it is divided into 40 panels of 21 feet each.

Spandril Braced Arch of Two Hinges.

A spandril braced arch is generally understood to mean a structure composed of an arched or curved lower chord and a

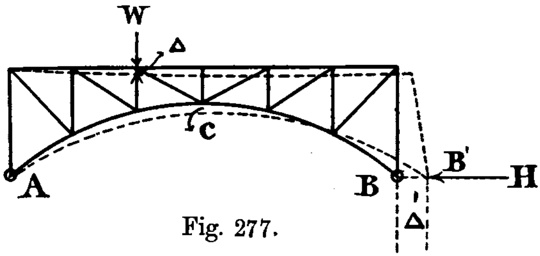


Fig. 277.

horizontal upper chord, tied together by a system of diagonal and vertical bracing, as shown in Fig. 277. A spandril braced arch, as compared with an arched rib, stands in much the same relation as a trussed girder to a simple beam, and the stresses which are statically indeterminate must be obtained from the elastic deformation of the structure, or in other words, from the elastic properties of the material.

The indeterminate quantity involved is the value of the horizontal component of the thrust at the abutments, and once this is obtained, the stresses in the various members can be readily ascertained by simple statical methods, either graphically or analytically.

It is assumed for purposes of calculation that the end A (Fig. 277) is fixed and the end B free to move horizontally to B′ under the influence of stress produced by a load W and that the movement is resisted, or the arched member ACB is compressed back to its original length AB by the force H, or in other words, the internal work of strain equals the external work of resistance.

If W represents the load at any panel point,

H the horizontal thrust produced by W,

Δ the vertical deflection of the point at which the load is applied due to a horizontal force of unity acting at B,

Δ' the horizontal displacement of the end B due to a horizontal force acting at B.

Then $H = W\dfrac{\Delta}{\Delta'}$.

The calculation of the stresses on this principle becomes a tedious operation, inasmuch as the stress in each member and its sectional area must be determined before its deformation under the action of a horizontal force H can be obtained.

To reduce the great amount of labour involved in such calculations for a spandril braced arch, Professor J. B. Johnson, of Washington University, has proposed the following simple method as an approximate solution of the problem, with a resulting error of less than 2 per cent., and probably in the majority of cases, well within 1 per cent. This method consists of constructing an intersection locus curve in terms of the constants, span $= l$, rise of arch $= r$, and depth of crown $= Dc$, so that the lines of re-action drawn to the intersection of the verticals with this curve would give at once the directions and amounts of the re-actions in the same way as for a parallel rib.

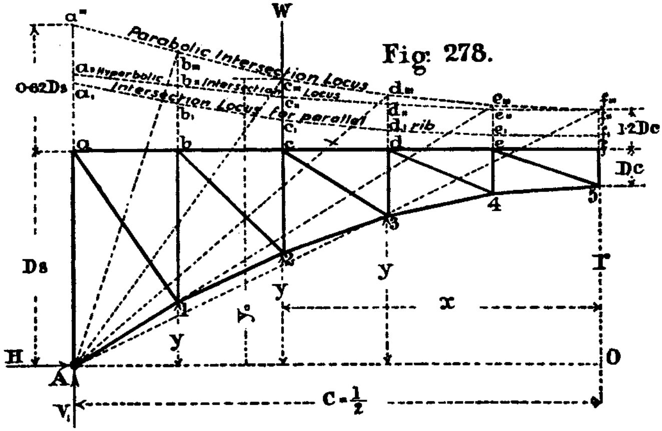


Fig: 278.

Let l = span of arch = 120 feet.

„ c = half span = 60 feet.

„ r = rise of arch = 20 feet.

„ x = distance of any load W from centre of span.

„ Dc = depth of arched truss at centre = 4 feet.

„ Ds = depth of arched truss at ends = 24 feet.

„ y = ordinate to axis of arch.

„ y_0 = ordinate to intersection locus curve.

„ H = horizontal thrust at abutments due to any load W.

„ V_1 = vertical re-action at abutments.

Taking O as the origin, the equations become

$$\text{Intrados,}\quad y = r - \frac{4x^2}{l^2}l,$$

and the equation of the parabolic locus, represented by the upper curve marked $a_{111}b_{111}c_{111} \cdots f_{111}$ is

$$y_0 = \frac{2{\cdot}5(r - \mathrm{D}c)x^2}{l^2} + r + 2{\cdot}2\mathrm{D}c.$$

Professor Johnson suggests that a somewhat closer approximation of the true locus may be found by using a hyperbola having the equation

$$y_0 = \sqrt{\frac{8}{3}\left(\frac{x}{l}\right)^2 (Q^2 - K^2) + K^2},$$

$$Q = 1{\cdot}35r + 1{\cdot}85Dc,$$

$$K = r + 2{\cdot}2Dc.$$

This curve, represented in Fig. 278 by the intermediate dotted line $a_{11}b_{11}c_{11} \ldots f_{11}$, coincides at the centre with the parabolic curve here used, but drops at the end.

The lowest curve represented by the line $a_1b_1c_1 \ldots f_1$ is the intersection locus for a parallel arched rib, the equation of which with reference to an origin at the centre O is

$$y_0 = \frac{32l^2r}{25l^2 - 20x^2}.$$

Adopting the dimensions given at foot of Fig. 278 for the purpose of comparing these three curves, the ordinates are as under:

Ordinates.	Parallel Rib. $y_0 = \frac{32l^2r}{25l^2 - 20x^2}$.	Parabolic Intersection locus. $y_0 = \frac{2{\cdot}5(r - Dc)x^2}{l^2} + r + 2{\cdot}2Dc$.	Hyperbolic Intersection locus. $y_0 = \sqrt{\frac{8}{3}\left(\frac{x}{l}\right)^2 (Q^2 - K^2) + K^2}$.
A	32·00	38·80	32·64
1	29·36	35·20	31·31
2	27·58	32·40	30·23
3	26·44	30·40	29·44
4	25·80	29·20	28·96
5	25·60	28·80	28·80

Having obtained the value of y_0, the value of H in terms of W is given by the expression $H = \frac{V_2(c - x)}{y_0}$.

To further compare the three equations, the values of H for load unity on each panel point have been computed, and are given in the following tabulated statement:

Position of load.	V_1.	V_2.	Values of H.		
			For Parallel Rib.	Spandril braced arch. Parabolic locus.	Spandril braced arch. Hyperbolic locus.
1	·9	·1	·3678	·3068	·3449
2	·8	·2	·6960	·5926	·6351
3	·7	·3	·9530	·8289	·8559
4	·6	·4	1·1160	·9863	·9944
5	·5	·5	1·1718	1·0416	1·0416
6	·4	·6	1·1160	·9863	·9944
7	·3	·7	·9530	·8289	·8559
8	·2	·8	·6960	·5926	·6351
9	·1	·9	·3678	·3068	·3449
Totals,	4·5	4·5	7·4374	6·4708	6·7022

Once the value of H is known for any condition of loading, the stresses in all the members can be readily determined by ordinary static methods, either graphically or analytically, as explained in the case of a three-hinged spandril braced arch.

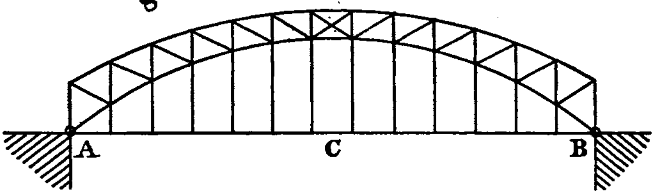
This type of arched truss has been extensively used on the Continent, but the most prominent example is the Niagara Railway arch, having a span of 550 feet, rise of arch 114 feet, and depth of crown 20 feet, designed by Mr. R. S. Buck.

Another type of spandril braced arch to which the same analysis are applicable is that shown in Fig. 279.

In this truss the horizontal steel tension member ACB ties the two hinges together and supplies the horizontal resistance which in other forms of arches is furnished by the abutments.

The road bridge over the Moselle at Trarbach, consisting of four spans 211 ft. 3 in., the road bridge over the Rhine at Düsseldorf, in two spans of 595 feet each, the road bridge over the Rhine at Bonn, one span of 614 feet, and the railway bridge over the Rhine at Worms in three spans, one central span of 383 feet and two side spans of 335 feet each, are

Fig. 279.



prominent examples of this form of construction—the tied arch.

The only advantage of the tied arch over other forms of arches is that the horizontal re-action is taken up by the steel tension member, and in a bridge consisting of several spans the piers need only supply vertical re-actions, as in the case of an ordinary girder.

Parabolic Arch with ends fixed, continuous at crown.

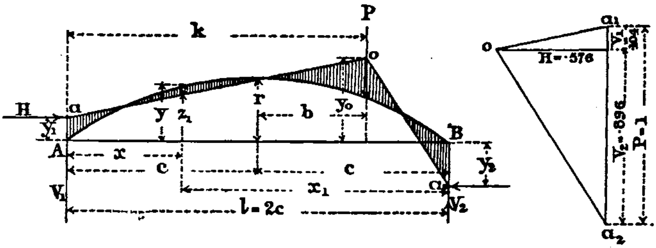


Fig. 280. Fig. 281.

Let H = horizontal thrust at abutments due to any load P placed at a distance b from the centre of the span.

Let l = length of span of arch.

" c = half span of arch.

" r = rise of arch.

" V_1 and V_2 = vertical re-actions at right and left abutments.

" y = ordinate to axis of arch at point of bending moment, M thus becomes y_0 at point of application of load P.

" z_1 = ordinate to load polygon at point of bending moment M.

" z = vertical intercept $= z_1 - y$.

" k = fractional length of span $= \cdot 1, \cdot 2, \cdot 3, \ldots \cdot 9$.

" $n = \frac{r}{e}$ or rise over length of span.

$$y_0 = \tfrac{6}{5} r.$$

$$y_1 = \tfrac{6}{5} \cdot \frac{5k-2}{9k} r.$$

$$y_2 = \tfrac{6}{5} \cdot \frac{3-5k}{9(1-k)} r.$$

If y_1 or y_2 becomes negative it is to be laid off below the line AB but otherwise above.

$$H = \frac{15}{32} P \frac{(c^2-b^2)^2}{c^3 r} \text{ or } H = \frac{15}{4n} P k^2 (1-k)^2.$$

$$V_1 = H \frac{y_0 - y_2}{c-b} \text{ or } V_1 = P(1-k)^2 (1+2k).$$

$$V_2 = H \frac{y_0 - y_1}{c+b} \text{ or } V_2 = P - V_1.$$

M = Bending moment at left support A $= M_1 = Hy_1$.

" = Bending moment at right support B $= M_2 = Hy_2$.

" = Bending moment at any intermediate point $= H(z_1 - y)$.

$$z_1 = y_1 + \frac{y_0 - y_1}{c+b} x \text{ on the left of the weight P.}$$

$$z_1 = y_2 + \frac{y_0 - y_2}{c-b} x_1 \text{ on the right of the weight P.}$$

Once the values of y_0, y_1, and y_2 are determined H, V_1 and V_2 may be obtained by the aid of a diagram as shown in Fig. 281. In Fig. 281 H, V_1 and V_2 are given in terms of the unit value

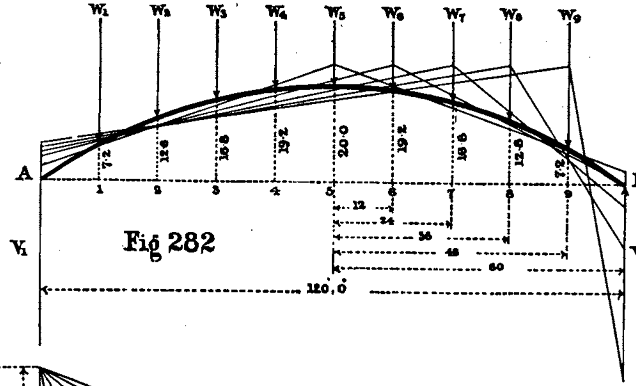


Fig 282

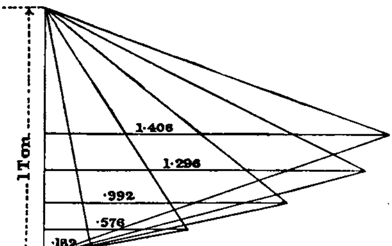


Fig. 283

of P say 1 ton. In Fig. 280 y_0 is laid off $= \frac{6}{5}r$ and from the point O the lines Oa_1 and Oa_2 are drawn respectively parallel to the lines Oa_1 and Oa_2 in Fig. 281.

In the diagram Fig. 280, y_1 and y_2 are laid off on the vertical lines at A and B to the same scale as the diagram, y_2 in this case having a negative or minus value is laid off below the chord line AB. On the vertical line drawn through P y_0 is laid off $= \frac{6}{5}r$ and the lines Oa_1, Oa_2 are drawn. In Fig. 281, the line a_1a_2 is laid off = P which for convenience is assumed to be 1 ton. The lines Oa_1 and Oa_2 are drawn parallel to the corresponding lines in Fig. 281, and at their intersection at O the horizontal line Oq is drawn at right angles to a_1a_2, then will Oq represent by scale the horizontal thrust and a_1q and qa_2 the vertical re-actions V_1 and V_2.

Calculated Values of y_1, y_2, H, V_1 and V_2 for unit loads.

Load at Points.	y_1	y_2	H	V_1	V_2
1	– 40·00	7·40	·182	·972	·028
2	– 13·33	6·66	·576	·896	·104
3	– 4·44	5·71	·992	·784	·216
4	0·00	4·44	1·296	·648	·352
5	2·66	2·66	1·406	·500	·500
6	4·44	0·00	1·296	·352	·648
7	5·71	– 4·44	·992	·216	·784
8	6·66	– 13·33	·576	·104	·896
9	7·40	– 40·00	·182	·028	·972
		Totals,	7·498	4·500	4·500

In Figs. 282 and 283 the equilibrium polygons for one half of the arch are shown the construction of which has already been explained. From the diagram in Fig. 280 the horizontal thrusts and the vertical re-actions at the abutments can be determined by scale, which on being compared with the calculated values will be found to agree.

Panel Points	1.	2.	3.	4.	5.	6.	7.	8.	9.
Value of z_1.	7·65	11·97	15·47	18·21	20·13	20·45	18·37	13·89	7·01
Value of y.	7·20	12·80	16·80	19·20	20·00	19·20	16·80	12·80	7·20
Vertical Intercept = z.	+0·45	– 0·83	– 1·33	– 0·90	+0·13	+1·25	+1·57	+1·09	– 0·19

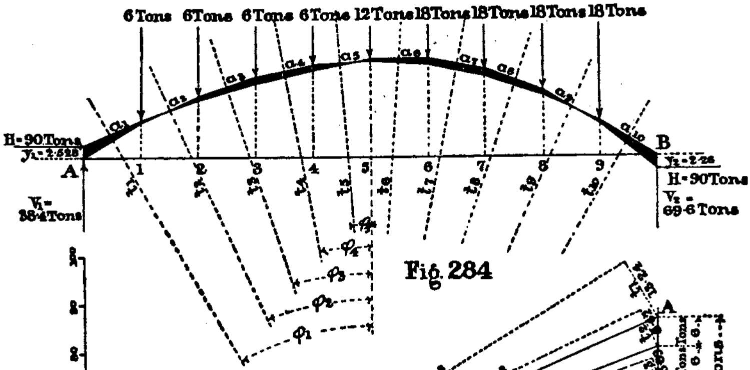


Fig. 284

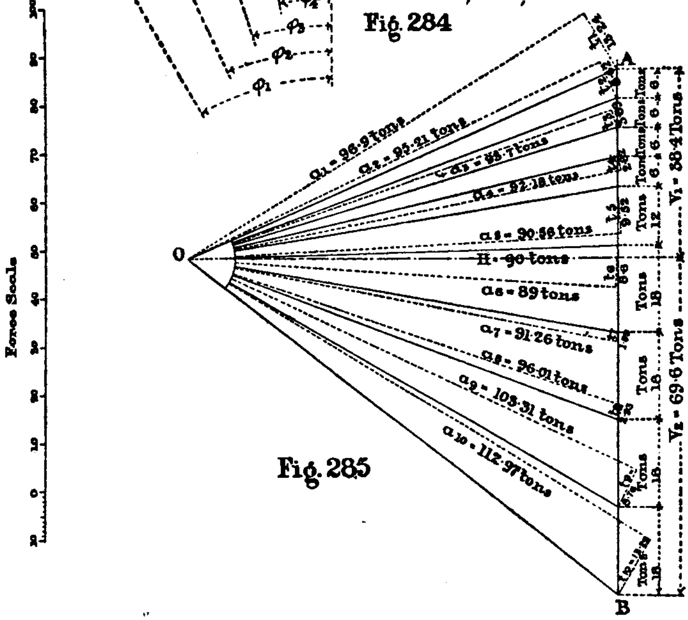


Fig. 285

Applying the foregoing formulae to a practical example let it be required to determine the values of H, V_1 and V_2 for the condition of loading given in the diagram Fig. 284.

Point of Loading.	Load = P.	Horizontal Thrust = H × P.	Vertical re-action at A = V_1 × P.	Vertical re-action at B = V_2 × P.
1	6 Tons.	1·092 Tons.	5·832 Tons.	·168 Tons.
2	6 ,,	3·456 ,,	5·376 ,,	·624 ,,
3	6 ,,	5·952 ,,	4·704 ,,	1·296 ,,
4	6 ,,	7·776 ,,	3·888 ,,	2·112 ,,
5	12 ,,	16·872 ,,	6·000 ,,	6·000 ,,
6	18 ,,	23·328 ,,	6·336 ,,	11·664 ,,
7	18 ,,	17·856 ,,	3·888 ,,	14·112 ,,
8	18 ,,	10·368 ,,	1·872 ,,	16·128 ,,
9	18 ,,	3·276 ,,	·504 ,,	17·496 ,,
Totals,		89·976 Tons.	38·400 Tons.	69·600 Tons.

If we find the line of action of the resultant horizontal thrust or the arm with which it acts at either or both springings $= y_1'$ or y_2', by multiplying the value of H for each load by the corresponding values of y_1 and y_2 and dividing the sum of the products by the sum of the several values of H for any given condition of loading, an equilibrium polygon may be drawn from which the bending moments may be obtained graphically

$$y_1' = \frac{\Sigma H \times y_1}{\Sigma H} \text{ and } y_2' = \frac{\Sigma H \times y_2}{\Sigma H}.$$

These values by way of example have been calculated in the following tables :

Panel Points.	y_1	H.	$y_1 \times$ H.
1	– 40·00	1·092	– 43·680
2	– 13·33	3·456	– 46·068
3	– 4·44	5·952	– 26·426
4	0·00	7·776	0·000
5	+ 2·66	16·872	+ 44·879
6	+ 4·44	23·328	+ 103·576
7	+ 5·71	17·856	+ 101·957
8	+ 6·66	10·368	+ 69·051
9	+ 7·40	3·276	+ 24·242
		89·976	+ 227·531
	say	90·000	

Panel Points.	y_2	H.	$y_2 \times$ H.
1	+ 7·40	1·092	+ 8·080
2	+ 6·66	3·456	+ 23·017
3	+ 5·71	5·952	+ 33·986
4	+ 4·44	7·776	+ 34·525
5	+ 2·66	16·872	+ 44·876
6	0·00	23·328	0·000
7	– 4·44	17·856	– 79·280
8	– 13·33	10·368	– 138·205
9	– 40·00	3·276	– 131·040
		89·976	– 204·038
	say	90·000	

Then $\frac{+227{\cdot}531}{90} = +2{\cdot}528 = y_1'$, and $\frac{-204{\cdot}038}{90} = -2{\cdot}265 = y_2'$,

y_1' having a positive value is laid off to scale in the diagram, Fig. 284, at A above the line and y_2' having a negative value is laid off at B below the line. An equilibrium polygon may then be drawn, commencing at the upper extremity of y_1', and if correctly drawn it should close at the lower extremity of y_2'.

The bending moment at any section $= \mathrm{H}z$, that is to say the horizontal thrust multiplied by the vertical intercept at the point of section, which latter dimension may be scaled from the diagram. If the vertical intercept z lies above the axis of the arch the bending moment $\mathrm{M} = \mathrm{H}z$ is positive, and if below it is negative.

The following general expression will give the bending moment M at any section distant x from the left abutment,

$$\mathrm{M} = \mathrm{M}_1 + \mathrm{V}_1 x - \mathrm{H}y - \Sigma\mathrm{P}(x - k).$$

Thus at panel point 3 distant 36 feet from the left abutment at A the bending moment

$\mathrm{M} = 227 + 38{\cdot}4 \times 36 - 90 \times 16{\cdot}8 - 6 \times 12\,(1+2) = 119$ Foot tons.

The vertical intercept at the point $3 = 1{\cdot}33$.

Then $\mathrm{H}z = 1{\cdot}33 \times 90 = 119{\cdot}7$ Foot tons,

which is practically of equal value.

Axial thrust and Shearing forces.

The axial thrust and shearing forces may be determined graphically from a diagram as shown in Fig. 285, the construction of which is very similar to that already explained in detail in Figs. 256, 257, 258.

A general expression may also be given for determining the axial thrust and shearing forces.

Let T = the axial thrust at any section,
N = the shear „ „ „
ϕ = the angle of section from centre of span.

$$\text{Then } T = H\cos\phi + V\sin\phi,$$
$$N = V\cos\phi - H\sin\phi.$$

Let it be required to determine the axial thrust and shear at $a3$ midway between the panel points 2 and 3.

Here $H = 90$,

$V = 38{\cdot}4 - 6 \times 2 = 26{\cdot}4$,

$\phi = 18^\circ{\cdot}45$. $\sin\phi = {\cdot}321$ and $\cos\phi = {\cdot}947$.

Then $T = 90 \times {\cdot}947 + 26{\cdot}4 \times {\cdot}321 = \ldots 93{\cdot}70$ Tons.

and $N = 26{\cdot}4 \times {\cdot}947 - 90 \times {\cdot}321 = -3{\cdot}89$ Tons.

Temperature Stresses

$$Ht = \pm \frac{45\ \mathrm{EI}\epsilon t}{4 \times r^2}, \quad \begin{cases} \text{minus for a fall in temperature.} \\ \text{plus for a rise in temperature.} \end{cases}$$

The bending moment at the crown will therefore be

$$M = Ht \times \tfrac{1}{3} r = \frac{15\ \mathrm{EI}\epsilon t}{4 \times r},$$

and at the springing $M = Ht \times \frac{2}{3} r = \dfrac{15\ \mathrm{EI}\epsilon t}{2 \times r}$.

$$\left.\begin{array}{l} \mathrm{I} = \text{moment of inertia of section,} \\ \mathrm{E} = \text{modulus of elasticity of material,} \\ t = \text{range of temperature in degrees Fahr.,} \\ \epsilon = \text{co-efficient of expansion.} \end{array}\right\} \text{values already given.}$$

Simple formulae for determining the dimensions of arched ribs.

The following simple formulae given by Sir Benjamin Baker, K.C.M.G., in his work on "Beams, Columns, and Arches" published in 1870, give general results that are in reasonably close accord with the more detailed investigation of modern practice, and they will be found very useful if only as the means of checking calculations by the more precise methods, and especially in determining tentatively the approximate dimensions of an arched rib in cases of calculations involving an assumed value of the moment of inertia of the section.

Let A = sectional area in square inches at the springing,

A_1 = sectional area in square inches at the centre,

A_2 = sectional area in square inches at the haunch.

For round-ended arched ribs,

$$A = \frac{a}{t},$$

$$A_1 = \frac{a(b+1)}{2t - \frac{2rx}{d}},$$

$$A_2 = \frac{ab}{t - \frac{3rx}{4d}}.$$

For square-ended arched ribs,

$$A = \frac{ac}{t - \frac{15rx}{8d}},$$

$$A_1 = A_2 = \frac{a(c+1)}{2t - \frac{3rx}{d}}.$$

In these expressions,

d = ratio of span to depth of rib, usually = 50,

r = ratio of span to rise of arch,

W = total distributed loads in tons,

w_1 = rolling load,

w = ratio of total load to rolling load $= \frac{W}{w_1}$,

t = permissible working stress per square inch on the metal,

x = in England 1·62 for cast iron,
2·75 for wrought iron,
3·44 for steel.

In America and other countries subject to great extremes of temperature the value of x will be $2\frac{1}{2}$ times those amounts.

$$a=\frac{\mathrm{W}r}{8}\sqrt{1+\frac{16}{r^2}},$$

$$b=\frac{\frac{d}{4r}+\frac{3}{4}+w-1}{w},$$

$$c=\frac{\frac{d}{5r}+\frac{3}{4}+w-1}{w}.$$

When the sectional area of the web of the arched rib forms a considerable proportion of the whole, as in cast-iron bridges, the gross sectional area should be somewhat greater than the preceding formulae indicate. The necessary corrections may be effected, when designing the cross section of the rib, by assuming the effective area of the web to be the actual area divided by $1+\frac{3}{w}$.

Relative advantages of the several types of Arches.

In a three-hinged arch the introduction of the hinge at the crown enables the re-actions to be statically determined, and the elastic deformation of the structure does not enter into the calculations of the stresses, no more than in the case of a simple beam or girder, but by reason of the articulations it lacks in stiffness under a heavy live or rolling load, especially where the weight of the structure itself is small in proportion to the live load. It is therefore, except where the dead load is very considerable as compared with the live load, more suitable for highway than railway traffic.

In an arch hinged at the springing and in a hingeless arch the stresses are statically indeterminate, and the calculations are based on the assumption that any deformation of the structure produced by direct axial, bending or thermal stresses must be well within the elastic limit. To conform strictly with this hypothesis, the stresses must not exceed the prescribed limits, and the workmanship in building the various parts of the arched member and in fitting it on its points of support must be carried out with mathematical accuracy, a feat that it is almost impossible to accomplish in practice. The theory therefore at best is only an approximate one, and in case of the yielding of the supports it at once fails. However, the rules that have been laid down, if properly observed, and followed up by careful supervision in construction and erection, will be found to be sufficiently accurate for all practical purposes.

Comparing an arch fixed at the abutments with a two-hinged arch, the former is the more rigid structure, but it is subject to greater stresses due to changes in temperature than the latter.

As a general axiom it may be taken that the abutments of an arch must be absolutely rigid and unyielding.

Combined Arch and Cantilever Trusses or Balanced Arches.

A number of bridges have been recently built on this principle, amongst which the following may be instanced as prominent examples.

1. The White Pass and Yukon Railway Bridge, Alaska.

This bridge was erected under the supervision of Mr. C. E. Fowler, M.Am.Soc.C.E., of Seattle, Washington, from the designs of Mr. H. S. Wood, C.E., of the Atlantic Gulf and

Pacific Co. of New York, to whom the author is indebted for particulars of the structure, published by him in the *American Engineering News* of March 28th, 1901.

The structure consists of three spans, two of 80 feet and one of 240 feet. The side spans were designed to act as shore or anchorage arms during erection, and were anchored down to concrete piers by anchor rods passing through adjusting wedges. The central span was erected as two cantilevers with a central hinge pin. After erection the shore arms were wedged up at the shore ends, so that the side spans acted as simple trusses and the central span as a three hinged arch.

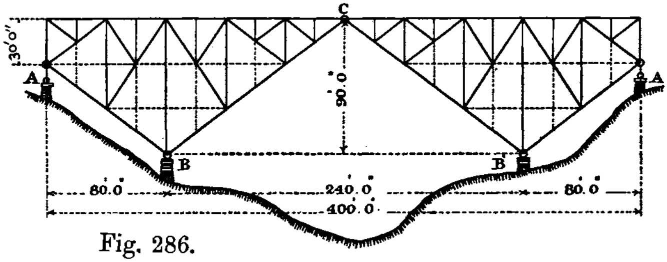


Fig. 286.

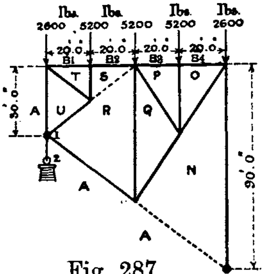


Fig. 287.

Diagram of shore span after erection and after rocker post 1, 2 has been wedged up until strain in AN = 0, that is to say, when cantilever strain is entirely relieved.

Stresses.

		lbs.
TS = OP	=	+ 52000
TU = $26000 \times \frac{25}{15}$	=	− 43333
RS	=	00000
RU = $26000 \times \frac{25}{15}$	=	− 43333
PQ = $26000 \times \frac{36}{30}$	=	− 31200
QR = $26000 + 52000$	=	+ 78000
NQ = $\frac{2}{3} \times 78000 \times \frac{36}{30}$	=	− 62400
NO = $(52000 + 26000) \times \frac{36}{30}$	=	− 93600
AR = $26000 \times \frac{50}{30}$	=	− 43333
AN	=	00000
KN = $52000 + 26000$	=	+ 78000
$B_2S = B_1T = 26000 \times \frac{20}{15}$	=	+ 34666
$B_1O = B_3P = (26000 + 5200) \times \frac{20}{30}$	=	+ 52000
AU = 26000×3	=	− + 78000

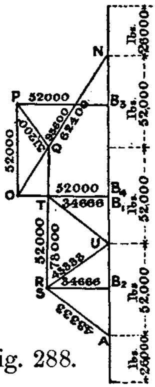


Fig. 288.

Diagram of stresses due to a partial live load covering one-half of the span, the other half being unloaded.

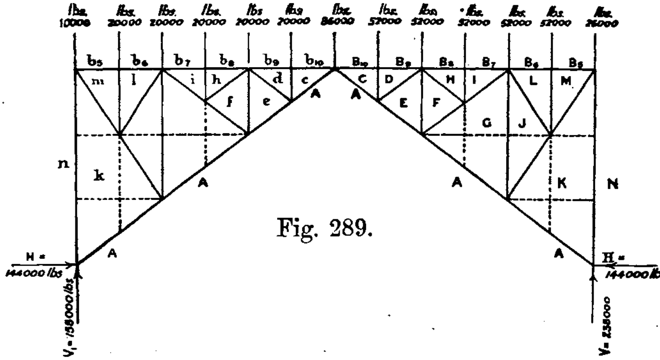


Fig. 289.

$$H = \frac{\overset{\text{lbs.}}{20000} \times \overset{\text{ft.}}{20}(1+2+3+4+5) + \overset{\text{lbs.}}{36000} \times \overset{\text{ft.}}{20} \times 6 + \overset{\text{lbs.}}{52000} \times \overset{\text{ft.}}{20} \times (1+2+3+4+5)}{2 \times 90}$$

$= 144000$ lbs.

$$V_1 = \overset{\text{lbs.}}{52000} \times \frac{1+2+3+4+5}{12} + 36000 \times \frac{6}{12} + 20000$$

$$\times \frac{7+8+9+10+11}{12}$$

$$= 158000 \text{ lbs.}$$

$$V_2 = \overset{\text{lbs.}}{20000} \times \frac{1+2+3+4+5}{12} + 36000 \times \frac{6}{12} + 52000$$

$$\times \frac{7+8+7+10+11}{12}$$

$$= 238000 \text{ lbs.}$$

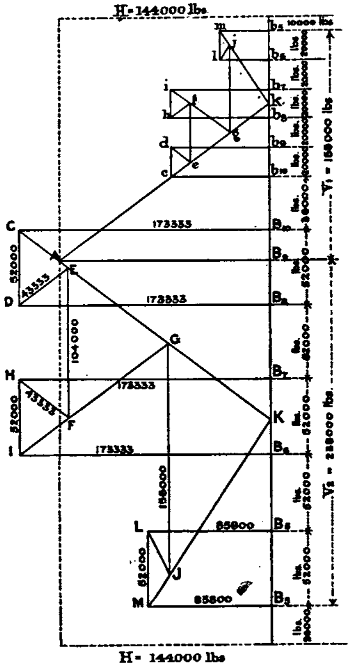


Fig. 290.

The stresses as measured by scale from the diagram are as under. Half truss fully loaded.

AK......	+ 181,440 lbs.
AG......	+ 94,000 lbs.
AE......	+ 6,200 lbs.
AC......	− 37,000 lbs.

The stresses in the other members are the same as when the whole span is fully loaded.

To arrive at the maxima stresses, diagrams of a similar character must be drawn for each condition of loading as the train advances panel by by panel, and thus eliminate the secondary maxima of different algebraic signs.

Diagram of stresses when whole span is fully and uniformly loaded.

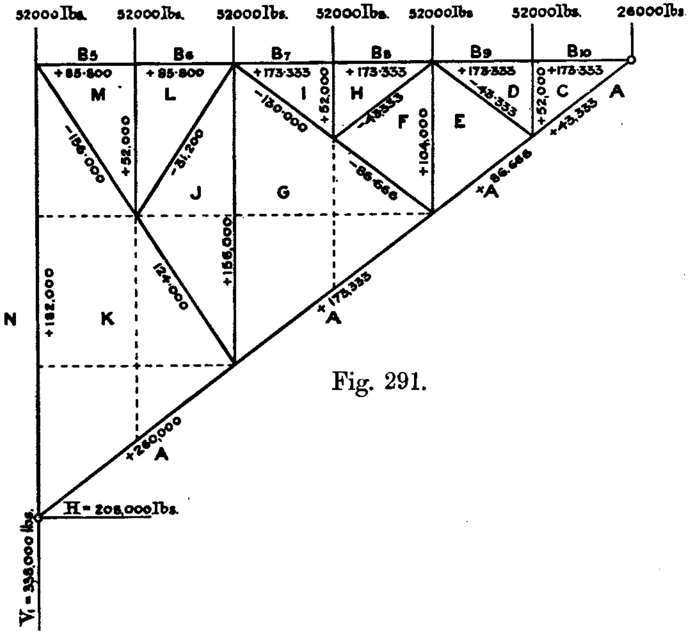


Fig. 291.

$$H = \frac{\overset{\text{lbs.}}{52000} \times \overset{\text{ft.}}{20} \times (1+2+3+4+5+6+5+4+3+2+1)}{2 \times 90}$$

$= 208{,}000$ lbs.

$V_1 = V_2 = 52000 \times 6 + 26000$

$= 338{,}000$ lbs.

The stresses as measured by scale from the diagram are as under.

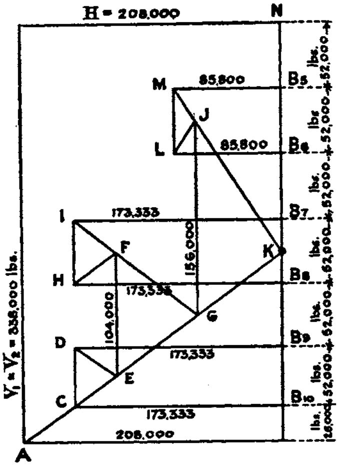


Fig. 292.

		lbs.
$B_5M = B_6L$		$= +\ 85,800$
$B_7I = B_8H = B_9D$ $= B_{10}C$		$= +173,333$
CD	$= HI = LM$	$= +\ 52,000$
FE		$= +104,000$
JG		$= +156,000$
DE	$= FH$	$= -\ 43,333$
AC		$= +\ 43,333$
AE		$= +\ 86,666$
AG		$= +173,333$
AK		$= +260,000$
FG		$= -\ 86,666$
GI		$= -130,000$
JL		$= -\ 31,200$
JK		$= -124,800$
KM		$= -156,000$
KN		$= +182,000$

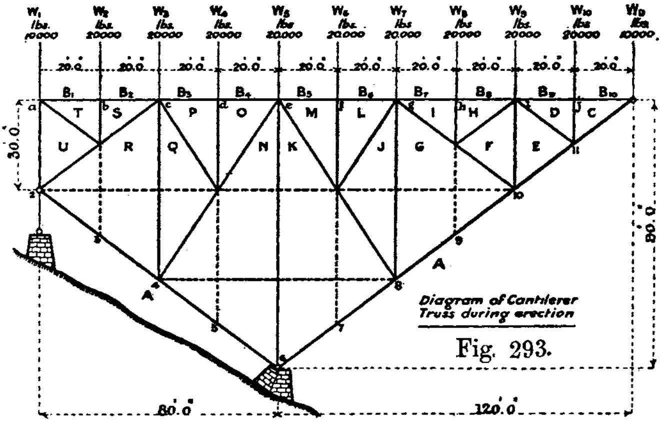


Fig. 293.

Diagram of stresses in one half of the truss acting as a cantilever during erection.

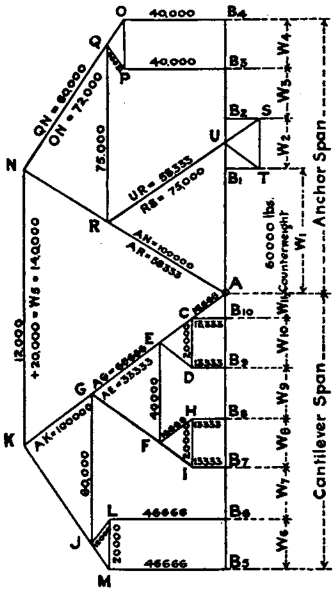


Fig. 294.

Anchor Span.

	lbs.
Stress in TU $=10000 \times \frac{5}{3}$	$= -$ 16,666
Stress in $B_2S = B_1T = 10000 \times \frac{4}{3}$	$= +$ 13,333
Stress in ST	$= +$ 20,000
Stress in PQ $= \frac{20000}{2} \times \frac{6}{5}$	$= -$ 12,000
Stress in OP	$= +$ 20,000

$$\text{Stress in AN} = \frac{60000 \times 80 + 20000 \times 20(1+2+3)}{90} \times \frac{5}{4}$$

$= +100,000$

$$\text{Stress in AR} = \text{UR} = \frac{60000 \times 40 + 20000 \times 20}{60} \times \frac{5}{4}$$

AR $= +$ 58,333

UR $= -$ 58,333

Stress in RS = stress in UR + UT = 74,999

Stress in RQ = vertical components of stresses in SR and PQ $+ W_3 + 45000 + 10000 + 15000$ = 75,000

Stress in QN = difference between the horizontal components of the stresses in AR and AN $\times \frac{36}{20}$

$= \{\frac{4}{5}(100000 - 58333)\}\frac{36}{20}$ = 60,000

Stress in ON = stresses QN + PQ = 72,000

Stress in $B_4O = B_3P$ = horizontal component of stress in NO

$= 72000 \times \frac{20}{36}$ = 40,000

Cantilever Span.

		lbs.
Stress in AC	$= 10000 \times \frac{5}{3}$	$= +$ 16,666
Stress in DE	$=$ FH $= 10000 \times \frac{5}{3}$	$= -$ 16,666
Stress in CD	$=$ HI $=$ LM	$= +$ 20,000
Stress in EF	$= 20000 \times 2$	$= +$ 40,000
Stress in FG	$= 20000 \times \frac{5}{3}$	$= -$ 33,333
Stress in GI	$= (20000 + 10000)\frac{5}{3}$	$= -$ 50,000
Stress in GJ	$= 20000 \times 3$	$= +$ 60,000
Stress in JL	$= 10000 \times \frac{6}{5}$	$= -$ 12,000
Stress in JK	$= \frac{2}{3} \times 60000 \times \frac{6}{5}$	$= -$ 48,000
Stress in KM	$=$ sum of stresses in JL and JK	$= -$ 60,000

Stress in AK $= \dfrac{10000 \times 120 + 20000 \times 20(1+2+3+4+5)}{90} \times \dfrac{5}{4}$

$= +100,000$

Stress in AG $= \dfrac{10000 \times 80 + 20000 \times 20(1+2+3)}{60} \times \dfrac{5}{4}$

$= +$ 66,666

Stress in AE $= \dfrac{10000 \times 40 + 20000 \times 20}{30} \times \dfrac{5}{4}$ $= +$ 33,333

Stress in $B_{10}C = B_9D = B_8H = B_7I = 10000 \times \frac{4}{3}$ $= -$ 13,333

Stress in $B_6L = B_5M$ = stress in B_4O + horizontal component of stress in KM = 46,666

2. Viaduct over the Viaur Valley carrying the line of railway from Carmaux to Rodez.

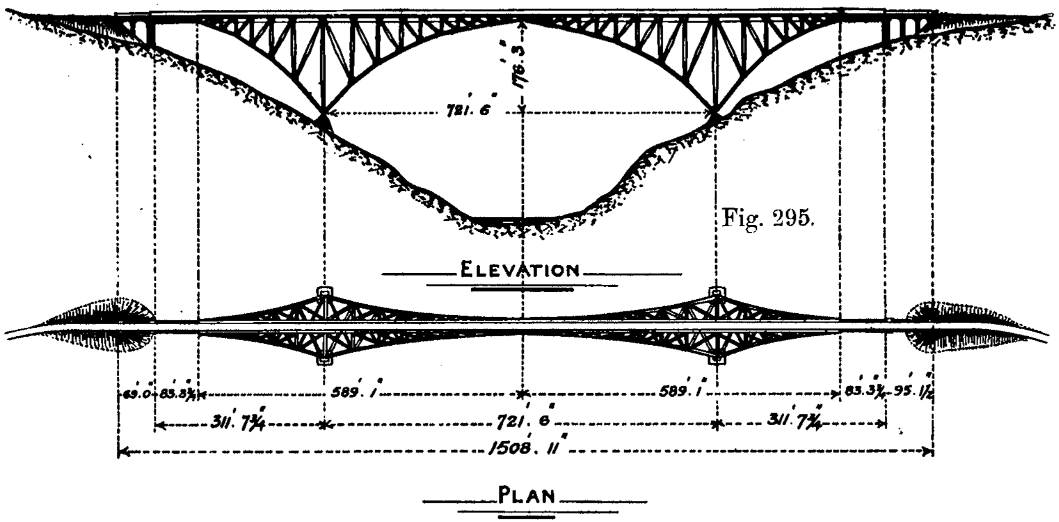


Fig. 295.

The magnitude of the counterweight including the load W_1 at the anchorage end of the truss at a is found thus:

$$10000 \times 120 + 20000 \times 20\,(1+2+3+4+5) = 80x + 20000\,(1+2+3),$$
$$7{,}200{,}000 \text{ foot-tons} = 80x + 2{,}400{,}000 \text{ foot-tons},$$
and
$$7{,}200{,}000 - 2{,}400{,}000 = 80x = 4{,}800{,}000\,;$$
$$\therefore\ 4{,}800{,}000 \div 80 = x = 60{,}000 \text{ lbs.}$$

This bridge, of which the leading dimensions are given in Fig. 295, is of a very similar type of construction to the Yukon Railway Bridge, with the exception that the lower or arched members are curved instead of being straight. The structure is designed on the principle of a three-hinged arch; the stresses therefore are statically determinate, and may be evaluated by either of the methods that have already been laid down.

It will be observed that in this structure the cantilever girder does not extend to the masonry abutment, and that the intervening distance is spanned by a simple girder, the one end of which rests on the abutment, and the other on the extremity of the cantilever (Fig. 296).

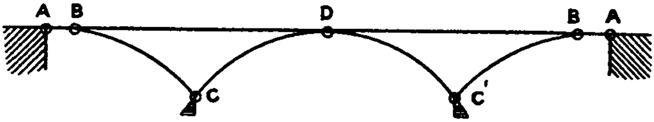


Fig. 296.

By the introduction of this girder AB, the ends of the cantilever B, B′ and the crown of the arch D are free to move under the influence of a passing load or of changes of temperature, and any complication due to displacement of levels produced by this movement is avoided, and the calculations are brought absolutely within the rules of statics.

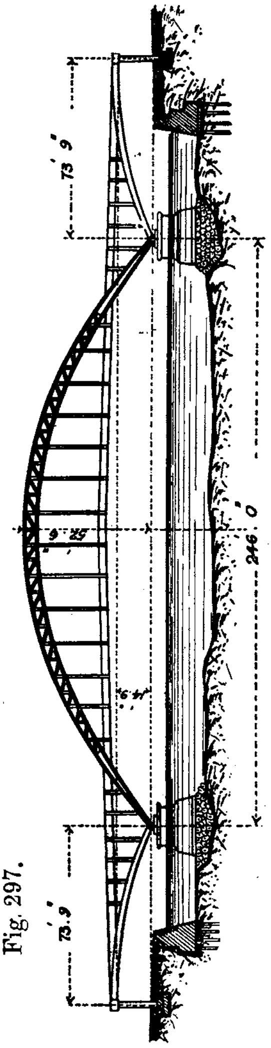
This structure, of which the outline is illustrated in Fig. 297, is a bridge for pedestrian traffic that was erected over the River Seine, in Paris, immediately prior to the Exhibition of 1900. It has a total length of 393·6 feet, consisting of a central arched span of 246 feet and two cantilever side spans of 73·8 feet each. The width of the bridge between the hand-rails is 26·25 feet, and the rise of the arch in the central span is 49·2 feet. The extradosal and intradosal curves are parabolic, and the depth of the arch at the centre is 6·56 feet, reducing gradually to 3·28 feet at the springing.

For the purpose of analysing the stresses, the structure may be considered as composed of two independent systems, viz.:

1. The central span, consisting of an arched rib, hinged at the springings.

2. The two side spans, composed of two cantilever arches, having their extremities tied by the chord forming the floor system.

The stresses in the central span may be determined by the



3. Passerelle sur la Seine entre le Pont de l'Alma et le Pont d'Iéna, Paris.

Fig. 297.

methods that have already been given in the section on arched ribs hinged at their extremities.

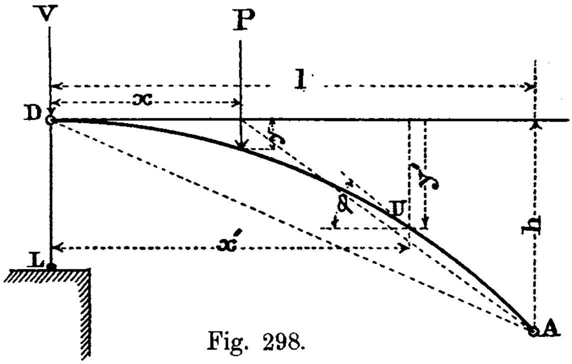


Fig. 298.

In the side span (Fig. 298) a vertical force P will produce in the tie a tension $= q = \dfrac{P(l-x)}{h}$. The moment at a point U due to the force $P = m = P(x'-x) - qy'$ and the tangential compression at the same point $= N = P\sin\phi + q\cos\phi$.

Similarly, a force V will exert in the tie a tension $= q = \dfrac{Vl}{h}$.

A detailed description of this bridge appears in the *Génie Civil* of the 26th May, 1900, from which these particulars have been taken.

This structure (Fig. 299) designed by Mr. Theodore Cooper, the "doyen" of American bridge engineers, is a combination of a two-hinged braced arch with cantilever shore arms. The stresses in the shore spans can be readily determined in the same manner as for an ordinary cantilever, but in the central or arched span the stresses are statically indeterminate; there are, therefore, assuming any system of loading, four unknown quantities, viz. the two components of the re-action at each hinge.

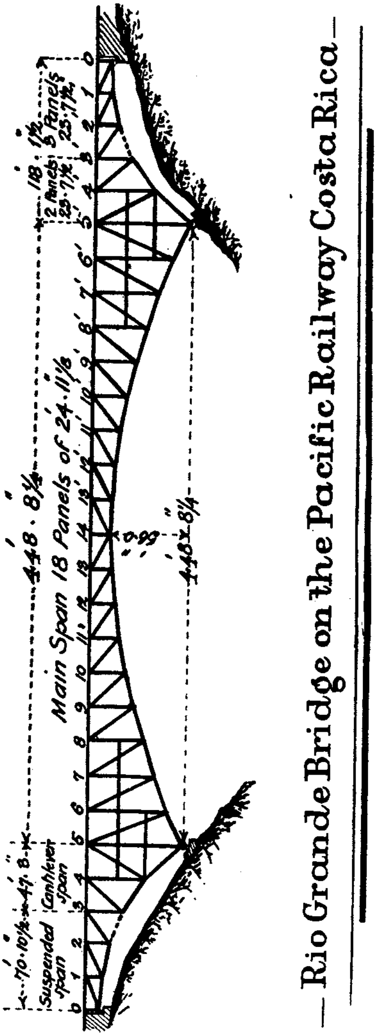

Rio Grande Bridge on the Pacific Railway Costa Rica
Main Span 18 Panels of 24'·11⅛"
448'·8¼"
Suspended Span
Cantilever Span
2 Panels 23'·7½"
3 Panels 23'·7½"

Fig. 299.

Three equations may be written down from the equilibrium of the arch, viz.:

1. The sum of all the horizontal components of the loads and re-actions must equal zero.

2. The sum of all the vertical components of the loads and re-actions must equal zero.

3. The sum of the moments and re-actions about any section must equal zero.

The fourth equation involves the finding of an expression for the horizontal re-actions due to the elasticity of the material.

In these calculations a value of the sectional area of each member must be assumed, but since the object of the calculations is to determine the sectional area of each member, it is difficult, if not impossible, to estimate what the sectional area must be.

Professor A. J. Du Bois has proposed that for a first trial all members shall be assumed to be of equal area, and that after finding the re-actions and corresponding maxima stresses under this assumption of equal areas, the members can be proportioned for these stresses so as to have a constant unit stress. A second calculation based on these areas can then be made, and thus by a system of successive approximation the correct stresses may be arrived at.

Mr. Gunvald Aus, who assisted Mr. Theodore Cooper in the calculations of the stresses in this structure, based his calculations on the formula

$$H = \frac{\Sigma S_1 S_0 \frac{m}{a}}{\Sigma S_0^2 \frac{m}{a}},$$

in which

H = horizontal re-action,

S_1 = total stress in a member from horizontal unit re-action,

S_0 = total stress in a member from vertical unit re-action only,
m = length of a member,
a = area of cross section of a member.

Under any circumstances the method of calculation is necessarily long and tedious and beyond the scope of this work to follow out in detail. Once the value of the horizontal thrust due to the various systems of loading is determined, the maxima stresses, by tabulating the various results, can be readily determined, but since the calculations at best for this type of structure are only an approximation, it is thought that the approximate formula given in the article on two-hinged braced arches may be employed with sufficiently accurate results.

CHAPTER XIV.

Suspension Bridges.

The prototype of the modern suspension bridge probably consisted of a rope or cord stretched across a river or ravine, having its extremities securely attached to the trunks of trees, and it is on record that in the Himalayas and in some of the countries of South America devices of this kind have been in use from prehistoric times for spanning gorges and chasms which could not by any other means then available be crossed.

Suspension bridges are so called because they consist of a platform suspended from chains or cables carried over masonry or metallic piers and having their extremities securely anchored in the ground.

If we imagine a chain or cable perfectly flexible, of uniform section and material, so that the weight of any part of it is proportional to its length, suspended loosely from two points and having only its own weight to support, the curve described by it would be a catenary; but, if from that cable or chain there be suspended a series of weights of equal intensity and at equal intervals horizontally, then the curve assumes a form closely approximating to a parabola.

The case of a chain or cable supporting a load consisting only of its own weight cannot occur in practice in any form of bridge; it will therefore be needless to investigate the properties of a catenary curve.

The simplest form of suspension bridge consists of a chain or cable supporting a platform hung from the chain by a number of equidistant vertical rods.

The weight of the platform or roadway and the load carried by it usually is far in excess of the cable, and for practical purposes it is assumed that the whole weight, that is to say of the cable, roadway, and extraneous load, is uniformly distributed along the horizontal plane of the platform, and the load will be transmitted to the cable by the vertical rods or hangers, the proportion carried by each hanger being the unit load × the length of panel, or the distance between each hanger.

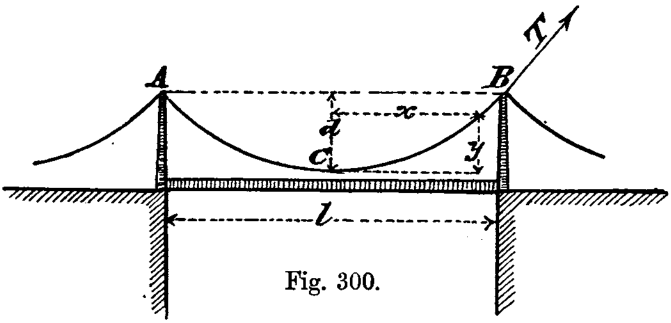


Fig. 300.

Let AB represent the span of a suspension bridge in Fig. 300, l = length of span, d = the dip of the cable below the top of the towers, or the versed sine of the curve.

Let x and y be the co-ordinates of any point P with respect to the centre C.

Let H = horizontal tension acting at C.

w = unit dead load.

w' = unit live load.

W = total distributed dead and live load.

T = tension tangent to the curve at B.

t = tension at any point P.

Then $\text{H}=\frac{\text{W}l}{8d}$, which is the constant horizontal component of the tension in the cable, and this must be so because all the external forces acting on the cable are vertical and have no horizontal component.

The tension in the cable, whose horizontal component throughout is H, increases from the centre C to the point B where it becomes a maximum. The co-ordinate x will then $=\frac{l}{2}$ and $y=d$. At any point P, distant x from C, the tension

$$=t=\text{H}\sqrt{\left(\frac{2y}{x}\right)^2+1},\text{ or } t=\sqrt{\left\{\frac{(w+w_1)l^2}{8d}\right\}^2+\left\{(w+w_1)x\right\}^2}.$$

The maximum tension at the top of the towers A and B

$$\text{T}=\sqrt{\left(\frac{\text{W}l}{8d}\right)^2+\left(\frac{\text{W}}{2}\right)^2},\qquad y=\frac{d\times x^2}{\left(\frac{l}{2}\right)^2}.$$

Length of chain or cable $=2\sqrt{\left(\frac{l}{2}\right)^2+\frac{4}{3}d^2}.$

If from a chain a series of loads be suspended at given intervals, and the dip of the chain, or the horizontal component of the stress, is given, then the curve of equilibrium or the form in which the chain will remain undisturbed under that condition of loading, as well as the stresses in the several links or sections intervening between the points of attachment of each hanger, may be readily determined by a graphic process as follows:

Let the span be 160 feet and the dip or versed sine 20 feet, and let 7 weights of 10 tons each be suspended at equal intervals of 20 feet. Then the horizontal component of the tension will be $\frac{70\times 160}{8\times 20}=70$ tons.

On the vertical line AH (Fig. 301) lay off the whole load to be carried and the subdivisions AB, BC, CD, etc., equal to the load carried by each vertical. In this case the loads are equal and symmetrically distributed so that the reactions at each

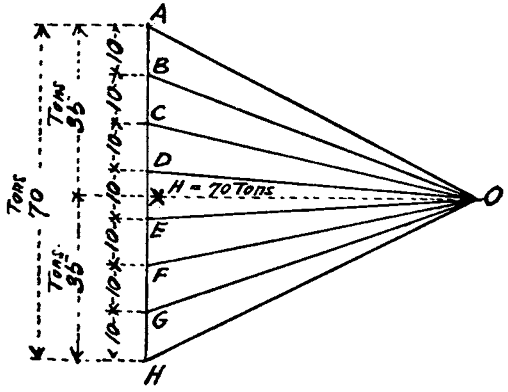


Fig. 301.

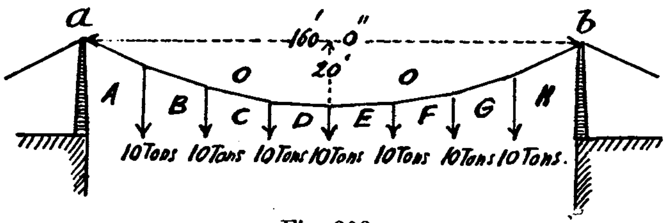


Fig. 302.

point of support will be equal, that is to say $= \frac{N-1}{2} = \frac{8-1}{2} = 3\frac{1}{2}$ panel loads, N being the number of panels.

Lay off $OX =$ to the horizontal component of the stress and join AO, BO, ..., HO.

In Fig. 302 lay off to any convenient scale the horizontal line ab to correspond with the length of the span $=160$ and divided into 8 equal panels of 20 feet each. Commencing at the point a draw the lines AO, BO, CO, etc., parallel to the lines correspondingly lettered in Fig. 301. Then will those lines represent the curve of equilibrium due to those loads, and the longitudinal tension in each member, as AO, BO, etc., will be

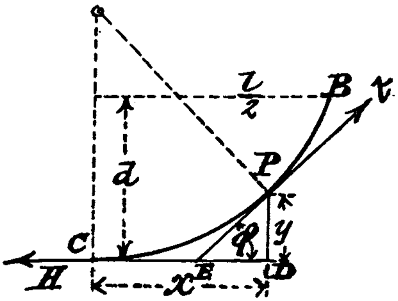


Fig. 303.

represented by scale by the corresponding lines in the diagram, Fig. 301.

The tension in the cable at any point can be determined graphically as shown in Fig. 303. Thus, at the point P, whose co-ordinates, with respect to C, are x and y, the tension

$$=t=\mathrm{H}\times\sec\phi=\mathrm{H}\times\frac{\mathrm{EP}}{\mathrm{ED}}.$$

Stresses on the Piers and Backstays.

If the chains are carried over the piers on rollers in one continuous length the tension on either side will be the same irrespective of the angles of the inclination of the chain with the vertical. If the angles are equal or, in Fig. 304, the distance AE is equal to the distance $\mathrm{AD}=\frac{1}{2}\mathrm{AC}$, the compression on the pier will be vertical; but if AE is less or greater than AD there will be a bending stress in the pier, the magnitude of which can be determined by the parallelogram of forces as shown in Fig. 305.

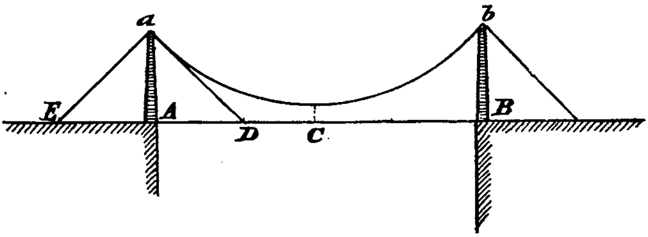


Fig. 304.

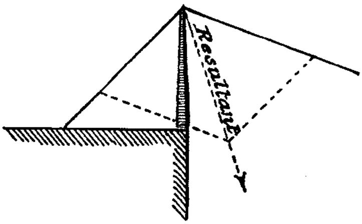
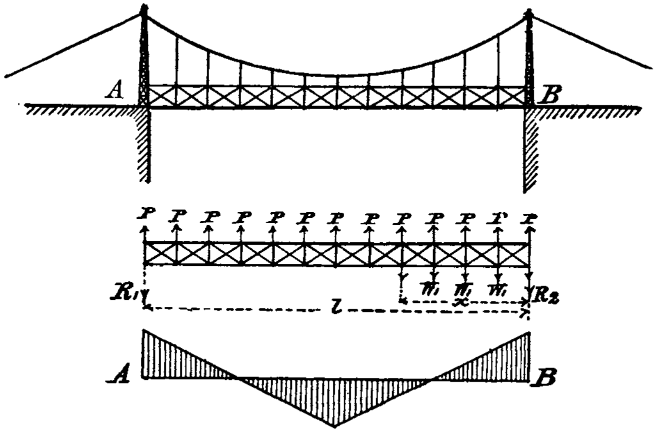


Fig. 305.

It has been shown that when the cable is uniformly loaded throughout its length the curve of equilibrium is a parabola, but, when the loading is not uniform or when a moving load passes over the bridge, the cable will assume a new curve of equilibrium, varying with each altered position of the load, and the more unsymmetrical the condition of loading the greater is the distortion produced in the cables. This is the great disadvantage of a suspension bridge, that is to say, its lack of rigidity, both vertically and horizontally, under the effect of a moving load. To resist this tendency to distortion in the cables a stiffening truss is introduced, the duty of which is to distribute a passing load uniformly over the cable and so prevent, or at all events minimise, the oscillation and deformation to which it would otherwise be subject. The suspenders are generally connected to the lower chords of the truss, as shown in Fig. 306, but the ends A and B are anchored or held



Figs. 306, 307, 308.

down, but in such a manner as to be free to move horizontally. The truss simply serves for distributing the load uniformly along the cable and does not assist in carrying it to the abutments.

Let P = upward pull of the suspender per lineal foot so that the total upward pull of the cable on the truss $= pl$.

Let w = unit dead load.

w' = unit live load.

x = distance from point of support covered by live load.

R_1 and R_2 = re-action at A and B.

Then $R_1 = \frac{w'x}{2l}(l - x) = R_2$. The shear at the head of the live load is equal to the re-action at each end and it becomes a positive and negative maximum when the live load covers half the span when the maximum shear $= \frac{w'l}{8}$. Fig. 308 shows graphically a diagram of the maximum shears.

The maximum bending moment which occurs when x has the respective values of $\frac{2}{3}l$ and $\frac{l}{3} = \frac{w'l^2}{54}$, that is to say, the greatest positive moment occurs when the live load covers two thirds of the span, and the greatest negative moment occurs when the live load covers one third of the span.

In a simple girder with parallel chords the maximum shear $= \frac{w'l}{2}$ and the maximum bending moment $= \frac{w'l^2}{8}$, but in the stiffening truss the corresponding values are $\frac{w'l}{8}$ and $\frac{w'l^2}{54}$. In a simple truss the maximum shears are at the end of a truss and the maximum bending moment is at the centre. In a stiffening truss the maximum shears occur at the ends and at the middle, as shown in Fig. 308, and the maximum bending moment occurs at a point distant one-sixth of the span from the centre.

In a simple truss the boom and web members are designed to resist the stresses due to live and dead loads, but in a stiffening girder the live or moving load only produces stress.

For practical purposes the shearing stresses and the bending moments are supposed to be uniform throughout the girder, and it is customary to design each web member of uniform section so that it may be capable of resisting a shearing force of $\frac{w'l}{8}$, and the top and bottom booms also of uniform section to resist a bending moment of $\frac{w'l^2}{54}$.

The stiffening truss is sometimes provided at the centre of its span with a sliding joint forming so to speak a hinge, so as to counteract temperature stresses. In this, as in the preceding case, the girder is designed so that the web members may be of uniform dimensions throughout and capable of resisting a maximum shearing force $=\frac{w'l}{6}$, and the top and bottom booms also of uniform section so as to resist a maximum bending moment, positive and negative $=\frac{w'l^2 \times 8}{425}$, or approximately $=\frac{w'l^2}{53}$. It will thus be seen that in this case the girder must be made slightly stronger than where it is continuous at the centre.

In both cases the girder is fastened at the ends to the abutments or the towers.

These formulae are based upon the supposition that the suspenders are subject to the same tension, and the curve assumed by the cable is a parabola, both of which conditions may be accepted for all practical purposes.

Various other methods and devices have been suggested and employed for stiffening suspension bridges and making them better able to withstand the oscillations resulting from the passage of a moving load or wind-pressure.

Among these may be mentioned a form of suspension bridge suggested by Mr. Dredge, an English engineer, in which the suspenders, instead of being vertical, are inclined as shown in Fig. 309. By this arrangement there results a stiffer structure than with vertical suspenders.

Another method that has been employed in American practice is the introduction of diagonal stays radiating from the tops of the towers and supporting the platform, as shown in Fig. 310.

It is a general rule that the cables of a suspension bridge are cradled or drawn inwards from the summits of the towers towards the centre of the span, so that the suspension rods or hangers incline inwards instead of hanging in vertical planes, the intention being that the cradling of the cables offers greater resistance to lateral forces than if they were allowed to hang in vertical planes, and to a certain extent

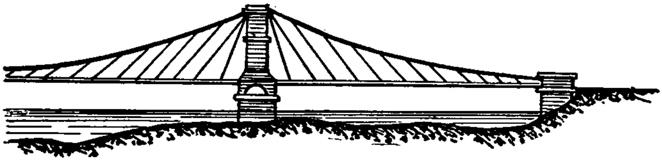


Fig. 309.

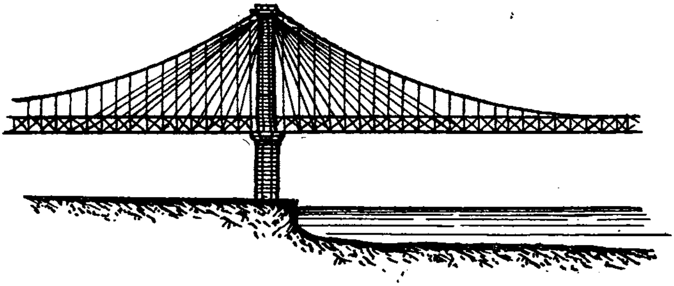


Fig. 310.

the principle is undoubtedly correct. The design, however, is much simplified in many respects if the cables are allowed to hang in vertical planes, and it is a question as to how far it may be advisable to sacrifice simplicity of design for the comparatively little extra lateral resistance obtained by cradling the cables.

In Figs. 311 and 312 are shown, in exaggerated form, the effects of a lateral force acting on the stiffening trusses of a suspension bridge, the cables in Fig. 311 being cradled and in Fig. 312 hanging in vertical planes. From these diagrams it will be seen that the distortion of the truss with cradled cables is greater than when the cables are in a vertical plane, when exposed sideways to wind pressure.

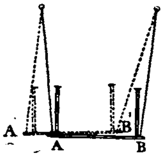


Fig. 311.

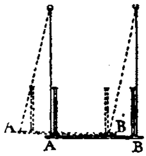


Fig. 312.

Fig. 314 illustrates a form of truss that was suggested by Mr. Thomas M. Cleemann, C.E., of Philadelphia, as an economical form of suspension bridge. It consists of two cables, the one LBCEH ... L′, a horizontal cable, and the other, LGADF ... L′, having its angles in a parabola. Under a uniform load the diagonals experience no stress. If the extremities of the curved cable are anchored to the abutments

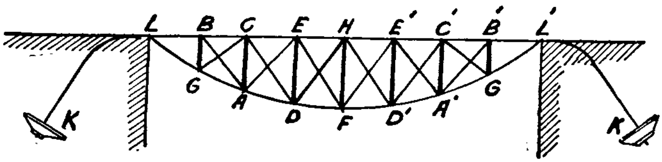


Fig. 313.

at K, K, then the lower chord would take all the stress under a full or uniform load, and the horizontal chord could be removed without affecting the stability of the bridge, but, under a variable or partial load, it has its duty to perform. If there were a load placed only at C it would be transmitted by the vertical C to the point A, where it would be sustained by the ties BA and EA, and, in an ordinary bowstring girder, these stresses would be resolved at B and E into compression on BC and BG, CE and ED. If, however, BC and CE are incapable of taking compression and the members LB and EH are designed to take tension and are so connected to the adjacent members as to transmit it to anchorages at the abutments the compression chord becomes unnecessary and the members in the structure which have to sustain compression are the vertical posts.

The stresses in this type of truss may be written as follow:

Let W = uniform or dead load on one panel.
W' = variable or moving load on one panel.
S = span.
h = depth of truss at centre of span.
n = number of panels supposed to be even.

Then

$$\text{Maximum tension in AB} = \frac{W'S}{8h} \times \frac{AB}{BC}.$$

$$\text{Maximum tension in CD} = \frac{W'S}{8h} \times \frac{CD}{BC}.$$

$$\text{Maximum tension in EF} = \frac{W'S}{8h} \times \frac{EF}{BC}, \text{ etc.}$$

$$\text{Maximum compression in BG} = W + W'.$$

$$\text{Maximum compression in AC} = \frac{W'S}{8h} \times \frac{DE}{BC} + \frac{3}{n}W' + W.$$

$$\text{Maximum compression in DE} = \frac{W'S}{8h} \times \frac{FH}{BC} + \frac{4}{n}W' + W.$$

$$\text{Maximum compression in FH} = \frac{W'S}{8h} \times \frac{IK}{BC} + \frac{5}{n}W' + W.$$

$$\text{Maximum tension in LB, BC, CE, EH, etc.,} \ldots = \frac{W'S}{8h}n.$$

$$\text{Maximum tension in LG} = \frac{(W + W')S}{8h} \times \frac{LG}{BL}n.$$

$$\text{Maximum tension in GA} = \frac{(W + W')S}{8h} \times \frac{GA}{BL}n.$$

A suspension bridge of this type, having a clear span of 92 feet and 100 feet between the points of suspension, carrying a roadway 8 feet wide, was built about twenty years ago in the Roorkee workshops in India and erected in Gurhwal, but in that structure the horizontal top chord was omitted and the vertical and diagonal members of the web only served for the support and bracing of the roadway platform which rested freely at each end on the abutments.

A very economical type of stiffened suspension bridge is that shown in Fig. 314, which has been proposed by Professor Claxton Fidler, M.I.C.E., by whom a very interesting analysis of this type of structure was published in *Engineering*, Vol. XX., 1875.

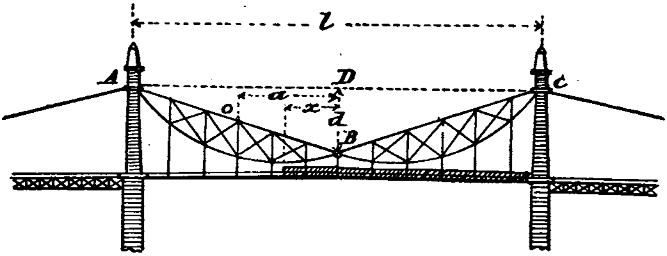


Fig. 314.

Let l = span.
d = depth of central hinge B below the points A and C.
w = unit dead load.
w' = unit live load.

The neutral axis ABC is a parabolic curve and each half girder AB and BC is a parabolic girder, having its upper member AB or BC tangent at the point B to the curve of the other. For a uniformly distributed dead and live load covering the whole span the horizontal component of the stress at any point of the upper or lower member $= \mathrm{H} = \frac{(w+w')l^2}{16d}$ and there is no stress in the diagonals.

For a uniformly distributed live load over the half span AB the horizontal component of the stress at any point in the lower members of the rib AB or in the upper members of the rib $\text{BC} = \text{H} = \frac{w'l^2}{8d}$, and for this condition of loading there is no stress in the diagonal bracing.

The maximum and minimum stress H in any part of the lower member due to any position of the rolling load upon the bridge will be as follows:

$$\text{Max. H} = -(w + w')\frac{l^2}{16d}.$$

$$\text{Min. H} = -\frac{wl^2}{16d}.$$

The greatest stress in the upper member of the semi-girder AB at any point O, distant a from D, produced by the rolling load occurs when the load extends from C to E, the distance $\text{DE} = x = a \times \frac{\frac{1}{2}l}{\frac{1}{2}l + a}$, and the value of H will then be a maximum

$$= \text{Max. H} = \frac{w'l^2}{16d}\left(1 + \frac{a}{\frac{l}{2} + a}\right).$$

If the live load covered the remaining segment only, that is to say from A to E, the segment CE being unloaded

$$\text{Min. H} = \frac{w'l^2}{16d}\left(\frac{a}{\frac{l}{2} + a}\right).$$

The maximum and minimum values of the stresses at the point O will therefore be

$$\text{Max. H} = -\frac{l^2}{16d}\left\{w + w'\left(1 + \frac{a}{\frac{l}{2} + a}\right)\right\}.$$

$$\text{Min. H} = -\frac{l^2}{16d}\left\{w - w'\left(\frac{a}{\frac{l}{2} + a}\right)\right\}.$$

The stresses in the diagonals are the same as in a parabolic bowstring girder, which have been investigated in a preceding chapter.

This truss may be inverted, with all its advantages, into a three hinged arch; the analysis of the stresses being precisely similar.

Various other types of suspension bridges have been introduced in which the tendency to deformation of the cables and the oscillations due to extraneous forces have been overcome by bracing the chords. In some instances two cables are braced together in the same vertical plane by a system of diagonal struts and ties, while in others the cables are braced to the platform in such a manner that the structure partakes more of the principles of an inverted arch or cantilever than a suspension bridge. It is beyond the scope of this elementary work to attempt to describe these various modifications, some of which would involve lengthy and difficult investigations of very advanced theories.

Details of Construction.

In the earlier suspension structures the cables or chains consisted of flat bar iron or steel links, the heads of which were enlarged and joined together by means of pins, as indicated in Figs. 315 and 316. The lengths of the links were made to coincide with the intervals between the vertical suspenders depending from the pins by which the links are joined.

In the more modern type of suspension bridges the cables invariably consist of ropes made of steel wire.

In spinning wire ropes difficulty is experienced in effecting a uniform distribution of the tension amongst the wires. This has been overcome by making the cables of parallel steel wires without twist. The cables of the Brooklyn Suspension Bridge are so constituted, each cable consisting of 5282 parallel wires

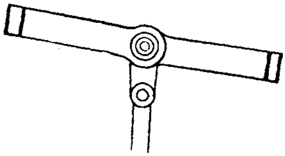


Fig. 315.

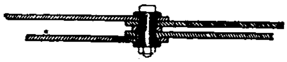


Fig. 316.

of galvanized steel, all closely wrapped together in a cylindrical cable $15\frac{3}{4}$ inches diameter.

Fig. 317.

Fig. 318.

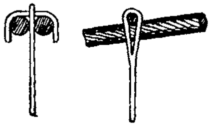


Fig. 319.

Monsieur F. Arnodin, of Chateauneuf-sur-Loire, a French engineer of eminence in the construction of suspension bridges, employs cables built of groups of wires twisted alternately in

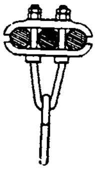


Fig. 320.

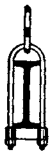


Fig. 321.

Fig. 322.

opposite directions, as shown in Fig. 317, which he calls "cables tordus alternatifs," in contradistinction to the ordinary "cables tordus simples."

The usual methods of connecting the suspenders with the cables are shown in Figs. 318, 319, and 320, and a simple form of attaching the suspender to the cross girder is shown in Figs. 321 and 322. The suspenders should be provided with screws so that any deflection of the floor due to the sagging of the cables may be adjusted.

The cables in passing over the piers are supported on saddles which may be arranged so as to allow the cable to

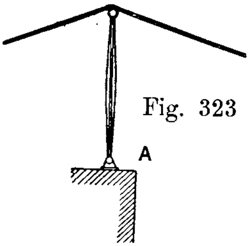


Fig. 323

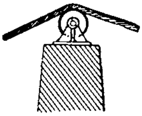


Fig. 324.

move over them with comparatively little friction, or the cables may be attached to the saddles and the saddles resting on rollers may be free to move. An alternative method that

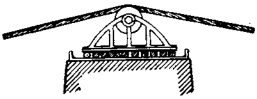


Fig. 325.

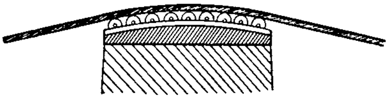


Fig. 326.

may be used for moderately small spans is to pivot the pier at its lower extremity, as shown in Fig. 323, in which case it is free to move around the pivot at A. For small spans, and where one cable only is employed, the arrangement shown in Fig. 324 would probably be found the simplest, consisting of a pedestal supporting a pulley over which the cable is carried. Where the span is of greater proportions and where it may possibly be necessary to employ two or more cables, either of the arrangements indicated in Figs. 325 or 326 would be applicable. In case the angles of inclination on either side of the pier are not equal there would occur in the arrangements, indicated in Figs. 324 and 326, a horizontal component of stress tending to overturn the pier, which would have to be provided for; but in the arrangements given in Figs. 323 and 325 the supports are free to move, and the piers are only subject to a direct vertical compression.

The anchorage, which is an important factor in a suspension bridge, consists of a heavy mass of masonry or concrete, to which the end of the cable is made fast, and it must be of sufficient bulk and solidity to resist the pull on the cable. Where the abutments consist of rock the cables are usually carried down through tunnels or galleries excavated in the rock and secured to heavy anchor plates or bars at their nether ends.

When cables of wire rope are employed, the ends passing into the anchorage invariably consist of iron or steel bars secured at the one end to the cables and at the other end, at the back of the anchor plates, by means of bolts or keys. In any case those sections of the cables passing through the galleries to the anchor plates should always be accessible for inspection because of their liability to rust and corrosion.

As an example of a modern wire rope suspension bridge the following description and illustrations have been selected of a suspension bridge recently built over the Rhone at Vernaison, near Lyons, by Messrs. Teste, Moret & Co., engineers, Lyons, to whom the author is indebted for the drawings and following particulars.

The bridge consists of three spans, viz.:

Right hand shore span, . . .	172·2 feet.
Central span, ...	763·6 feet.
Left hand shore span, . . .	139·0 feet.

The width of the roadway between the parapets is 16 ft.

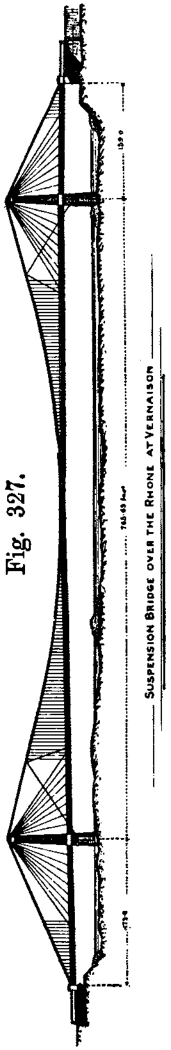


Fig. 327.

10 in. The structure is built throughout of steel with the exception of the flooring, which is of wood.

The cross-girders supporting the floor are spaced 4 ft. $1\frac{1}{4}$ in. apart, centre to centre, and consist of built I beams. The bottom flange is curved, forming, so to speak, a fish-bellied girder, the depth at the ends being

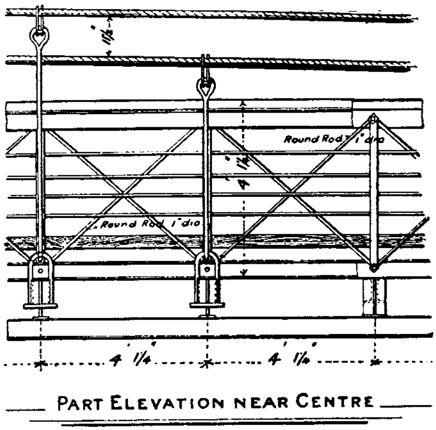


Fig. 328.

8 ins. and at the centre 13 ins. The flanges are composed of two angle bars $2\frac{3}{4}$ in. by $2\frac{3}{4}$ in. by $\frac{3}{8}$ in., and the web of $\frac{5}{16}$ in. plate.

The bottom layer of the floor consists of 16 oak planks, each $10\frac{1}{2}$ in. wide by 4 in. thick, laid longitudinally, over which is nailed the top layer of pine planks $2\frac{3}{8}$ in. thick.

The fenders, or wheel guards, consist of two longitudinal beams of oak 10 in. by $5\frac{1}{2}$ in.

The parapet is 4 ft. $1\frac{1}{4}$ in. in height and is divided into panels of 4 ft. $1\frac{1}{4}$ in. long by verticals of I section, 4 in. by

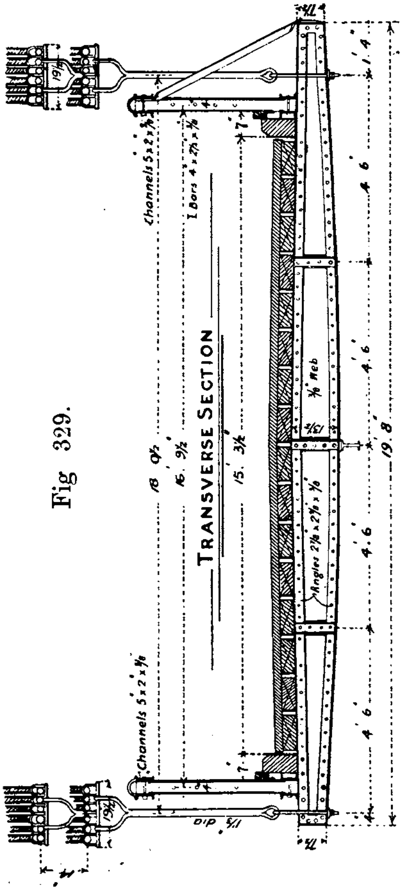
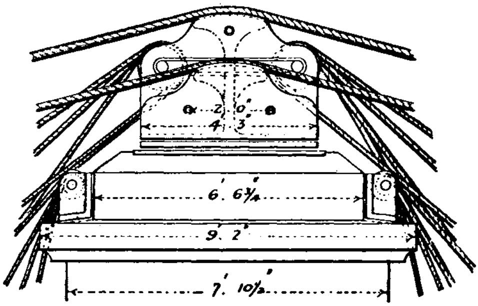


Fig. 329.

$2\frac{1}{2}$ in. by $\frac{3}{8}$ in., the top and bottom members consisting of channel bars, 5 in. by 2 in. by $\frac{3}{8}$ in. The suspension rods depending from the cables and supporting the cross-girders by means of stirrups, as shown on the drawing Figs. 328 and 329, at each panel point are $1\frac{3}{8}$ in. diameter.

Underneath the centre of the cross girders a stiffening girder of I section, 5 in. by 3 in., extending longitudinally throughout the length of the floor, is riveted to the bottom flanges of the cross-girders.



DETAIL OF EXPANSION CARRIAGE

Fig. 330.

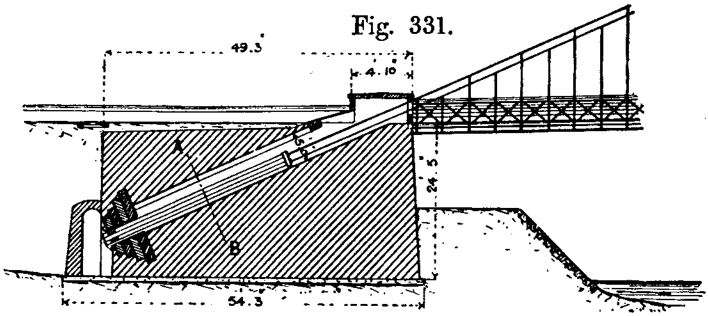
The cables consist of 24 steel wire ropes, arranged as shown in the drawing Figs. 327 and 329, and made up of concentric layers of wires alternately twisted in opposite directions, each rope being composed of 127 wires of 0·15 in. diameter, giving a total sectional area of 2·4 square in., and having an ultimate tensile strength of 86 tons per square inch.

The cables, which are continuous throughout, are supported on the piers on cast-iron carriages on roller bearings, which are free to move with any variation of temperature, and therefore exert only a vertical re-action on the piers.

For a distance of 100 feet on both sides from each pier the vertical suspension rods are omitted and the floor is supported by diagonal cables radiating from the summits of the piers and attached at their lower ends to a longitudinal girder of I section, 12 in. by 6 in. by $\frac{1}{2}$ in., forming cantilevers on each side of the piers.

The main cables were supported during the process of erection from a temporary cable by hangers placed at regular

Fig. 331.



DETAIL AT RIGHT ABUTMENT

intervals. The one end of the temporary cable was anchored down and the other end wound on a drum, so that when the main cables were properly anchored at each end, the temporary cable was paid out until the main cables were freely suspended. Then, by means of screws, the cables were adjusted to the versed sine they were designed to have.

On completion the bridge was subjected to the following tests:

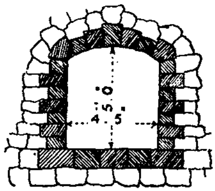
1. Dead Load.

The three spans were each separately loaded with a uniformly distributed load of 41 lbs. per square foot, consisting of a layer of gravel, and subsequently all three spans were similarly loaded simultaneously.

The maximum deflection at the middle of the central span under the most unfavourable condition of loading, viz., when the central span alone was loaded, was $19\frac{1}{2}$ inches. Under this test the carriage on the summit of the right-hand pier moved forward towards the middle of the river $1\frac{1}{4}$ inches and that on the left $1\frac{5}{16}$ inches.

2. Rolling Load.

The tests consisted of the passage over the bridge of vehicles of 8 and 11 tons.



SECTION A.B.

Fig. 332.

In the first test two vehicles, each supported on one axle and weighing 8 tons, drawn by five horses in single file and moving in opposite directions, produced, on passing each other at the middle of the central span, a deflection of $12\frac{3}{4}$ inches.

In the second test a wagon weighing 11 tons, supported on two axles and drawn by nine horses in single file, crossing the bridge from left to right, caused a deflection at the centre of the middle span of 13 inches.

The total cost of the structure was as under, viz.:

Masonry, . .	£7,200
Superstructure, .	13,800
	£21,000

The total weight of the metallic part of the structure is 410 tons.

Figs. 327 to 332 show the details of construction.

Fig. 333 shows the outline in elevation of the design for the new Manhattan Bridge, New York City, by Mr. Gustave Lindenthal, Chief Commissioner of Bridges.

The bridge consists of three spans, viz., one central span of 1,470 feet and two side spans of 725 feet each. The width of floor, which is designed to carry an exceptionally heavy traffic, is 141 feet.

The peculiar feature of this structure is that the cables are built of eye-bars with stiffening frames attached directly to the four chains which form the upper chords.

Mr. Lindenthal estimates that these frames are equal in rigidity to stiffening trusses in the average 62 feet high.

This bridge, when built, will be one of three of the largest suspen-

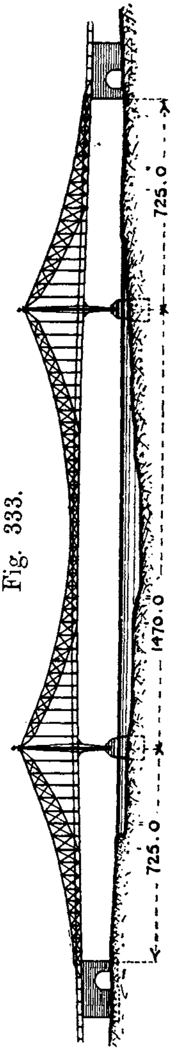


Fig. 333.

Design for the Manhattan Bridge

sion bridges in existence, the comparative dimensions of which are as under:

Name of Bridge.	Length of Central Span.	Width of Floor.	Sectional Areas of Cables or Chains.
Brooklyn Bridge, . . .	1,595·5 feet	78·5 feet	590·4 uniform
Williamsburgh, . . .	1,600 feet	141 feet	891·28 uniform
Manhattan (as designed),	1,470 feet	141 feet	2,156·00 average

CHAPTER XV.

Opening or Draw Bridges.

The origin of the movable or opening bridge is probably to be found in the bascule bridge or draw bridge, which in mediaeval times was used at every castle or fortress to span the moat or ditch by which it was surrounded and protected, thus affording means of access and defence.

An opening or draw bridge in its modern form is used to cross navigable canals, rivers, or waterways, and it may consist of:

A bascule bridge.
A draw bridge or traversing bridge.
A swing bridge.
A vertical lift bridge.

Bascule Bridges.

A simple example of a bascule bridge is a type of bridge that has been, and still is, being extensively used in Holland, for spanning the numerous canals intersecting that country.

The bridge, as will be seen by the diagram, Fig. 334, consists of a platform AB hinged at the end A, and the end B resting on the masonry support hinged, and free to move upwards. At the point D two chains are attached to the platform, one on each side, the upper extremities being secured to the ends C of overhead beams CD, which are braced together laterally, and pivoted at the point E on the vertical support EF. A counterpoise is introduced at the end D, and so arranged that its weight with that of the beams shall balance the

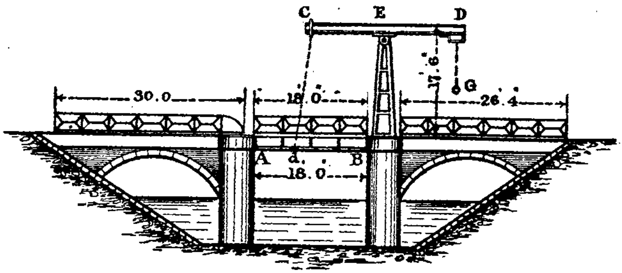


Fig. 334.

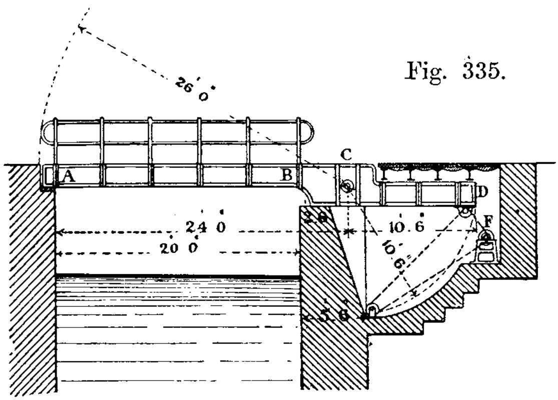
weight of the platform AB, so that the bridge on being lifted shall in every position be in equilibrium. The bridge may, therefore, be lifted by the slightest effort applied at the end of the chain at G.

In our own country, where there are canals, we frequently find bridges of this type in use even to this day where the traffic is not extensive, but they are of limited dimensions, the span rarely exceeding 18 or 20 feet.

Another form of bascule bridge is shown in Figs. 335, 336, and 337. The bridge shown in Fig. 335 represents a single leaf bascule bridge, consisting of the platform AB, the main girders supporting which are pivoted at the point C, and are provided at the point D, or in the space between C and D immediately adjacent to D, with a counterpoise so arranged that it shall in all positions of the bridge in course of being lifted be as nearly as possible in equilibrium.

At F below D a crabwinch or windlass is installed, which, if operated by manual labour, is connected by suitable gearing with a driving wheel placed in a convenient position above the level of the road actuating an endless chain, by means of which the bridge can be easily lifted or lowered at will.

Fig. 335.



The counterpoise it will be seen, as the platform is being lifted, descends into a well at the back of the pier or support on which the bridge is pivoted. The dimensions of the well must depend upon the length of the tail end of the girders carrying the counterpoise, and the longer that length the greater must evidently be the depth of the well.

Where the span is at all considerable and the normal height of the bridge above the water line is small the bottom of the well would probably be below water level, and therefore liable to inundation The inconvenience of deep pits, often necessitating the employment of pumps to keep them free from water, and the resistance due to wind when the bridge is being raised, have been advanced as serious objections to bascule bridges generally.

In Fig. 336 is shown a very similar style of structure, consisting of a double leaf bascule bridge, provided at the tail of the overhanging or cantilever portion of the girders with a counterpoise freely suspended by two hangers, one attached to the extremity of each girder. The counterpoise, in the

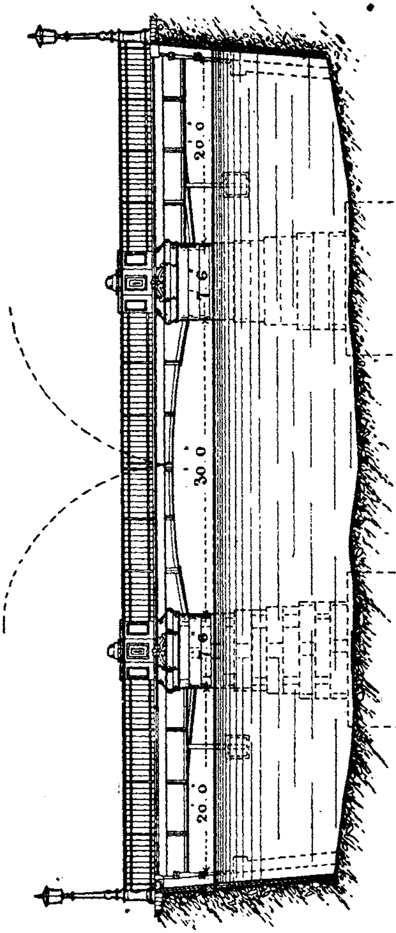
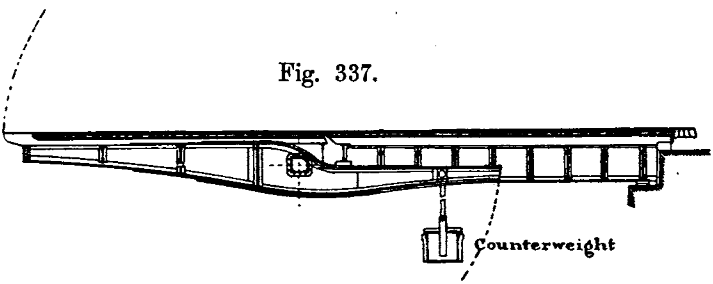


Fig. 336.

form of a rectangular box, extends over the width of the bridge, and is so arranged as to balance freely the weight of the platform on the opposite side of the fulcrum or trunnion. In this illustration, which is a representative type of a number of opening bridges erected at the docks of Copenhagen, under the direction of the late Commodore F. W. W. Luders (for many years the General Superintendent and Engineer of the Harbour of Copenhagen, to whom the author is indebted for these particulars), the mechanical arrangements for opening and closing the bridge, consisting of spur and pinion gearing applied at the trunnion, may be actuated by manual, steam, electrical, or hydraulic power, depending upon the magnitude of the bridge and the frequency of its operation.

In Fig. 337 the girder in the one-half bascule span is shown in detail, as well as the arrangement of the counterweight.

Fig. 337.



The Tower Bridge over the Thames, having a clear span of 200 feet, is built upon the principle illustrated in Fig. 336, and is the most important example of this type of structure.

An improved type of bascule bridge has, within the last ten years, been introduced in America, which is known as the Scherzer Rolling Lift Bridge, and is so called after the inventor, the late Mr. William Scherzer, civil engineer of Chicago.

Several bridges of this type have already been built in America, replacing in several instances swing bridges of modern construction, and erected within recent years.

The Scherzer Bridge, instead of revolving upon hinged pivots or trunnions as in the case of an ordinary bascule bridge, is supported at its tail end on rocker bearings in the form of circular arcs, constituting so to speak the segments of huge wheels, on the faces of which are fixed strong steel rims, provided with indents or pockets, so as to engage the teeth of the horizontal tracks along which the rockers roll when the bridge is being opened or closed. The cantilever projection of the main girders at the tail of the rolling segments carry counterweights calculated to balance the bascule or overhanging leaf in any position, open, shut, or intermediate.

In several bridges the movable spans are counterweighted, so as to be at rest at an angle of about 40 degrees. By this arrangement the operation of opening and closing the spans is greatly facilitated. The bridge when closed is brought down to its bearings, and there securely fastened down by means of suitable locking devices. The opening and closing of the movable span is accomplished by means of an operating strut, provided on its upper or under surface with a steel rack engaging a pinion connected by suitable gearing with, and driven by, a motor actuated by steam, hydraulic or electric power, the position of the counterbalance weights and the radius of the rolling segments being so arranged that the line of travel of the centre of gravity of the whole system is horizontal. The first bridge of the Scherzer type was that built across the Chicago River, at Van Buren Street, Chicago, which was opened for traffic in 1895. The river span is 115 feet from centre to centre of bearings, and the clear waterway 109 feet. It carries a roadway 41 feet wide, and two sidewalks, each 8 feet wide. Since then it has come prominently into existence, and at the present moment there are some thirty-eight bridges built and in course of being built, the length of movable span varying from 35 feet to 275 feet.

In Fig. 338 is shown in elevation a Scherzer Rolling Lift

Bridge, carrying the two lines of the Boston, Revere Beach, and Lynn Railway over Crystal Cove, near Boston, Massachusetts, the clear span of which is 30 feet.

The substructure is built of piles and timber work, so designed as to be capable of being replaced by masonry at any future time if desired. The main girders are about 40 feet long, 8·6 deep at the rocker end, and 3·6 at the nose end.

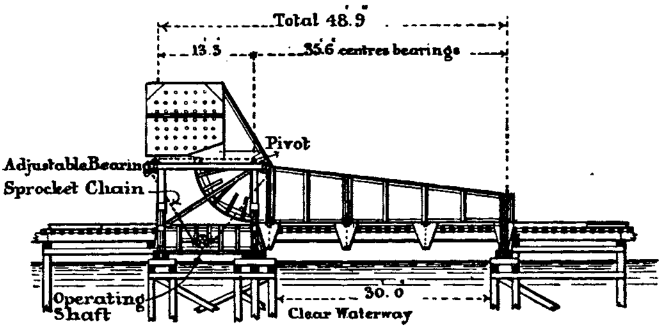


Fig. 338.

The Floor System consists of sleepers laid on longitudinal beams, supported on cross girders attached to the under side of the main girders.

This structure was designed to meet the special circumstances of the limited freeway between the high water line and the under side of the girder, hence it became necessary to fix the counterweight in the position indicated in the diagram. The bridge is opened and closed by means of three sprocket wheels mounted in a braced framework in a vertical plane, and engaging an endless chain, which forms a triangle with approximately equal sides. A special link of the chain is attached to the girder web, and when the chain is moved it causes the bascule girders to revolve. The machinery is so well counterbalanced that it can be operated by one man.

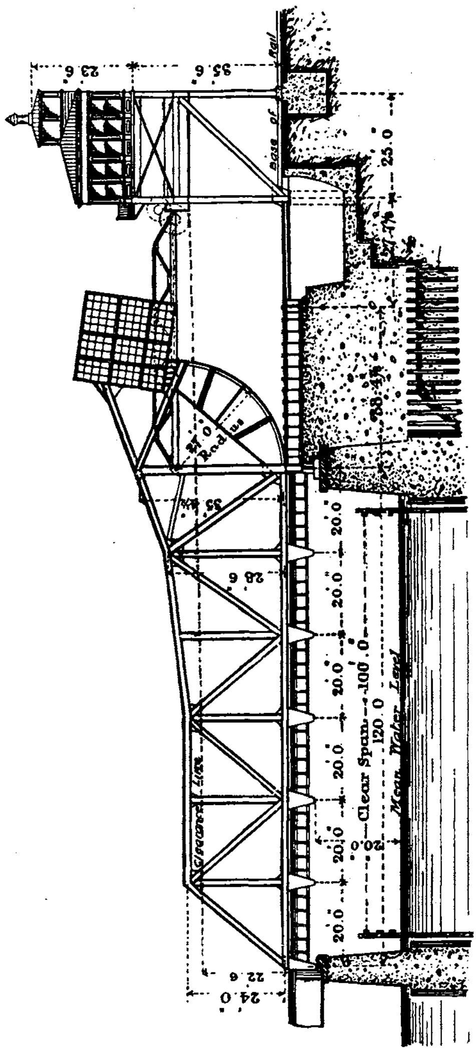

Base of Rail
23.6
35.6
25.0
27.0 Radius
35.4½
28.6
Clearance Line
20.0
20.0
20.0
20.0
20.0
20.0
Clear Span 100.0
120.0
Mean Water Level
22.6
24.0

Fig. 339.

In Fig. 339 is shown in outline elevation a single leaf Scherzer bridge carrying two lines of railway over the Cuyahoga River at Cleveland, Ohio. The total length of movable span from centre to centre of bearings is 120 feet, leaving a clear waterway of 100 feet, and the width is 45 feet.

The main or movable span consists of two girders of the Warren or Triangular type, and the side span is a deck plate girder. The movable span, it is estimated approximately, has to be opened 6000 times annually, and the time occupied in opening or closing is 30 seconds.

This bridge is a more typical illustration of the Scherzer principle than the preceding example, in which the power required to open or close the bridge is applied by means of an endless chain and sprocket wheels. In this case the motion is accomplished by means of an operating strut, consisting of a trussed girder having its one end pivoted to a horizontal shaft, and provided on its under side with a steel rack engaging a pinion geared to and actuated by an electric motor installed in the cabin at the rear of the cantilever end. To open the bridge the pinion is set in motion and the strut is drawn back, thereby causing the girders to revolve on the rockers, the counterweight descending into the pit provided for the purpose. To close the bridge the operations are simply reversed.

The moving span revolves through an angle of about 83 degrees, and the counterweights are so disposed that it moves through an initial angle of 40 degrees without mechanical effort.

Fig. 340 shows details of the operating strut. The pinion P engages a rack R fixed to the underside of the girder, and as the pinion revolves the strut is moved backwards and forwards following the direction in which the pinion revolves at the point of contact.

Fig. 340*a* shows the strut in transverse section on an enlarged scale.

Fig. 341 shows a plan and section of a track along which the rockers roll. The teeth are 12 inches long, 5 inches wide, and 2 inches high, and are spaced at intervals of 2·7″ from centre to centre. These teeth engage corresponding indents or pockets in the track plate of the rolling segment.

Fig. 340.

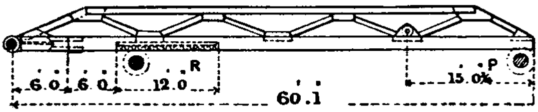
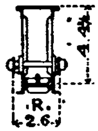


Fig. 340a.



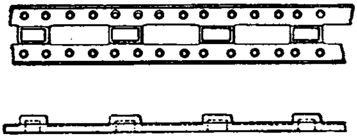


Fig. 341.

Another type of Scherzer Bridge is shown in Fig. 342. It represents an outline elevation of the Taylor Street Bridge carrying the main road and Electric Railway over the Chicago River at Chicago. In this structure the movable span consists of two leaves opening and coming together at the centre of the span, and the rocker segments, with the operating machinery, are placed below the level of the road. The main girders have their lower chords curved, and when the bridge is closed it forms a spandril-braced three-hinged arch.

The span from centre to centre of bearings is 148 feet 7 inches, which leaves a clear waterway of 120 feet, and the width between handrails is 34 feet.

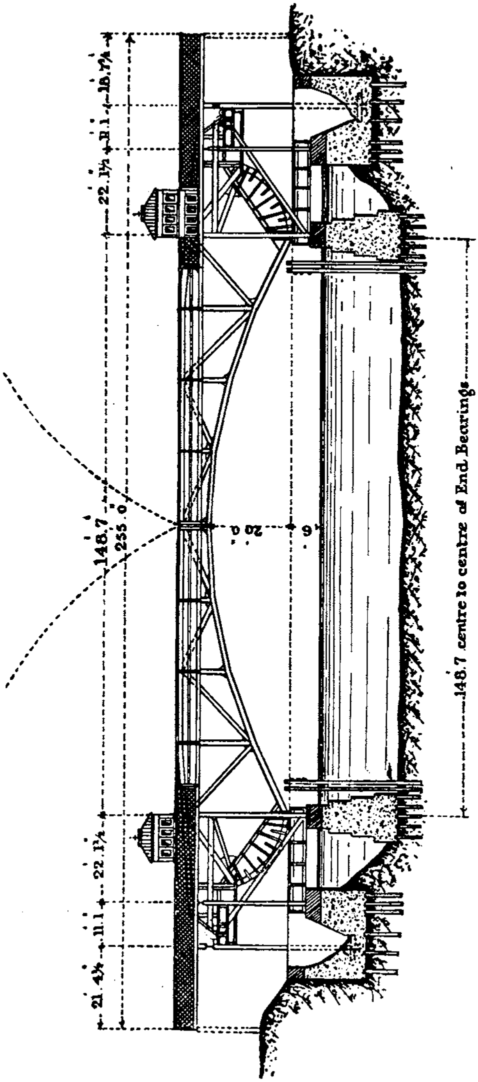


Fig. 342.

The time occupied in fully opening or closing the bridge is 30 seconds. A bridge of this type in which the main girders are given the outline of an arch is capable of very effective treatment from an artistic or aesthetic point of view. The roller segments and operating machinery would be screened by the masonry abutments and wingwalls, which might be made more or less ornamental according to the dictates of architectural taste.

The special advantages claimed for the Scherzer Bridge over the ordinary trunnion bascule bridge are the following, viz.:

First. That it can be easily and quickly opened or closed, and so perfectly counterbalanced as to require the minimum expense of power in its manœuvring.

Secondly. That it requires a minimum of freeway between the underside of the girder and the surface of the water, because all the moving parts can be placed entirely above the road level.

Thirdly. That it can be erected in the open or upright position, and when completed lowered ready to receive the traffic it is designed to carry, thus avoiding any obstruction of the navigable waterway.

Draw or Traversing Bridges.

This type of structure, in which the truss or girder spanning the waterway is moved horizontally on rollers, when the bridge is required to be opened or closed, is only applicable to spans of limited dimensions, and where the circumstances of the situation enable the movable span to be drawn back in the line of the bridge, the required distance corresponding with the dimensions of the opening.

A simple form of draw bridge is shown in Figs. 343 and 344, which represent in outline and diagram form the opening span of the bridge carrying the Cambrian Railway over the estuary of the Mawddach at Barmouth.

The main girders ABD are each pivoted on two wheels, one on each side at the point C, on which the opening span of the bridge AB may be rolled back along the rails BFH until the end A has reached the support B, thus leaving the span AB open for navigation.

The cantilever portions of the tail end of the main girders, viz., BD, are provided on their under side with racks, and at their extremities D with small wheels.

By depressing the ends D the ends A are tilted up, and the racks are brought into contact with and engage pinions F by revolution, of which the span AB is gradually moved backwards, the small wheel D being brought into contact with the underside of the guide rail EG, the end of which ED is turned up at right angles.

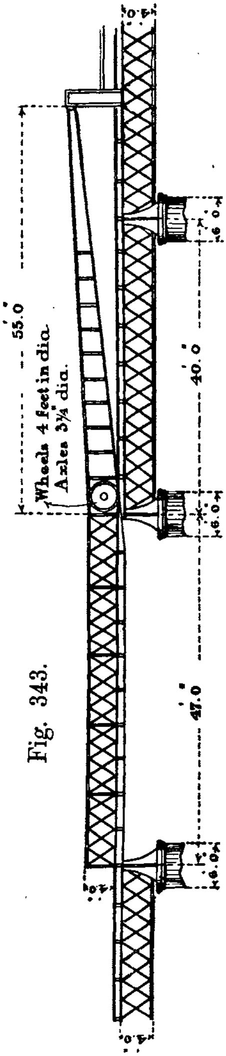


Fig. 343.

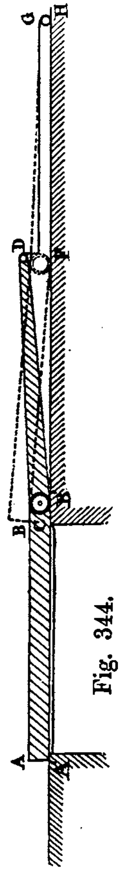


Fig. 344.

The counterpoise on the cantilever arm BD is so disposed that it shall nearly balance the weight of the arm AB, thus enabling the end B to be depressed with comparatively little effort.

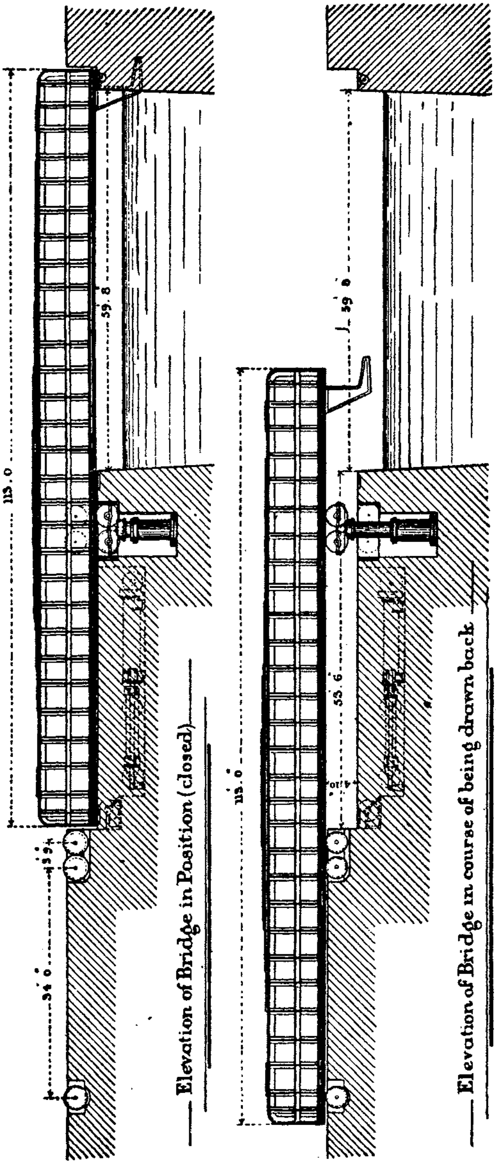
To move the bridge back again to its closed position, the manœuvring has simply to be reversed.

A description of this bridge is given in the minutes of *Proceedings of the Institution of Civil Engineers*, Vol. XXXI., from which some of these particulars are taken.

A more modern and substantial type of this class of structure is the new bridge now in course of construction over the North Dock Lock at Swansea, which is illustrated in Figs. 345 and 346.

This bridge also is of the lift-up and draw-back type, opened and closed by hydraulic power.

Immediately at the back of the west wall of the lock two cast-iron vertical cylinders are fixed, fitted with plungers. Each plunger is provided with two cast-steel travelling rollers, carried on a cast-steel bracket working in a ball and socket joint on the outer end, so that the weight carried by each lifting press is distributed equally on the two rollers. The plunger of each cylinder is fitted with cast-iron guide frame, and guides, and is provided with automatic cut-off gear, which comes into action when the bridge has been raised to the required height. A locking bolt is also provided for securing each lifting plunger at the top of its stroke, worked automatically by the movement of the bridge, the bolt being in position when the bridge is travelling, and withdrawn before it is lowered into place for carrying the road traffic. Immediately below the tail end of the bridge, on the centre line, two cylinders are fixed in a pit side by side, fitted with plungers, carrying at their ends three sheaves in a frame. The back ends of the cylinders also carry similar sheaves. The chain for drawing back the bridge is reeved round the



DRAWBRIDGE OVER NORTH DOCK LOCK SWANSEA

Fig. 345.

Elevation of Bridge in Position (closed)

Fig. 346.

Elevation of Bridge in course of being drawn back sheaves, and the "fall" end is carried out to the nose end of the bridge, to which it is made fast. By this arrangement of multiplying sheaves the bridge travels six times the stroke of each plunger.

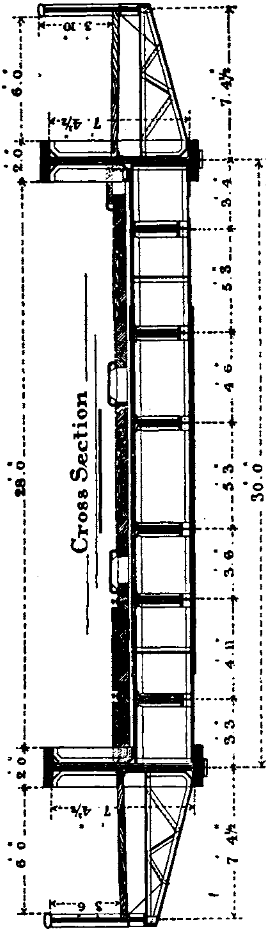


Fig. 347.

The nose end of the bridge is provided with a horn plate on the centre line, which engages with "live" rollers fixed into the masonry abutment at the nose end to guide the bridge while being lifted or lowered into position.

At the tail end of the bridge two sets of cast-steel travelling rollers are fixed in pits in the roadway, the top of each roller projecting about a couple of inches above the surface of the road. These rollers support the tail end of the bridge when it is drawn back over the roadway.

The operating valves are in a pit clear of the bridge altogether, so that the attendant working the bridge can see all that takes place.

To open the bridge when in position across the lock, water under pressure is admitted into the lifting cylinder, the bridge is then lifted bodily until the horn plate at the nose end comes in contact with the friction roller. This end of the bridge then ceases to rise, but the tail end being still free, continues to rise until it is perfectly clear, and ready to be drawn back, the bridge when in this position being tilted at a slight angle. Water under pressure is then admitted into the draw cylinders, and the bridge begins to travel backwards over the roadway, and the moment the horn plate at the nose end is disengaged from the "live" rollers with which it is in contact, the tail being the heavier end drops on to the rollers in the roadway, and the bridge continues to travel backwards until the lock entrance is free and clear for navigation. To close the bridge these operations are merely reversed.

This bridge is designed by Mr. A. O. Schenk, M.I.C.E., the Engineer of the Swansea Harbour, to whom the author is indebted for these particulars.

Swing Bridges.

Swing Bridges may be divided into two classes, viz.:

(1) Those with both ends swinging free.

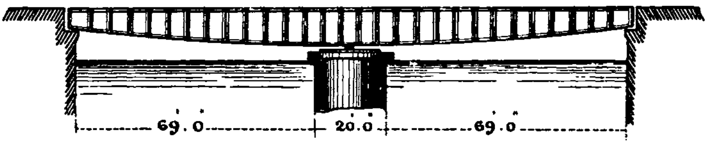
(2) Those with the tail end carried on wheels revolving on a circular path.

CLASS 1.—A bridge with both ends swinging free may be supported on a centre pivot or on a turntable, and may be employed for spanning single or double channels, the full advantage being obtained in a structure symmetrically balanced about its centre support and dividing the waterway into two passages of equal width.

A bridge supported on a centre pivot, as shown in Fig. 348, is, when closed, a continuous girder of two spans, and, when open, a balanced cantilever. In the latter case, the stresses, which are due to the dead load, are readily determined by simple methods. When the bridge is closed the stresses may be obtained by the aid of the "Theorem of Three Moments," as applied to "continuous beam of constant moment of inertia."

This method, involving the assumption of constant moment of inertia, although not strictly accurate, is the method in general use, and it has been found to be sufficiently accurate for all practical purposes.

Fig. 348.

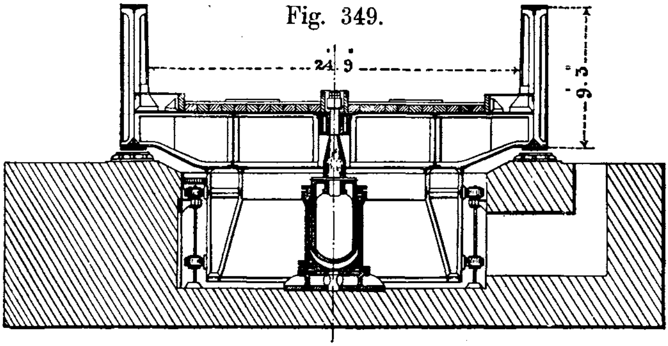


An alternative method of arriving at the stresses is based upon the distortion of each member of the girder, necessitating the application of the principle of least work, for which purpose the length and sectional area of each member must be predetermined, but inasmuch as the objects of the calculations is to ascertain those dimensions, they must be made on a trial and error system, and must, therefore, be very tedious, and any useful description of the process would be quite beyond the scope of this work.

Bridges of this type, supported on long conical centre pivots, have been largely in use on the Continent, especially in Holland, Germany, and, in several instances, in France, where they appear to work satisfactorily, but in most of these instances the structures are deck bridges, that is to say, the roadway is carried on their upper flanges, and, therefore, the pivots have the advantage of a length and full bearing coinciding with the depth of the girder, and the point of support is above the centre of gravity. In some bridges the pivot is made sufficiently strong and firm to guide the bridge in swinging without the aid of rollers, while in others steadying wheels revolving in a horizontal plane in contact with the walls of a cylindrical well are employed, an example of which is shown in Fig 349, illustrating the centre support of a swing bridge at Bordeaux.

The more ordinary method of supporting a swing bridge at the centre is that known as the run-bearing turntable, consisting of a drum or circular girder, carried on conical rollers filled in a ring frame, and resting on a cast iron or steel roller path, securely bolted down to the masonry of the pier.

Fig. 349.



In a rim bearing turntable the weight of the bridge is carried entirely by the rollers, and the function of the centre pivot is merely to maintain the turntable in position, for which purpose the spindles on which the rollers revolve are prolonged radially inwards, and their ends are made fast to a projecting flange or collar fitted round the pivot.

In an alternative arrangement of turntable a portion of the weight is carried by the pivot as well as the rollers. By this arrangement the load on the rollers is slightly relieved, and the weight on the pivot helps to hold it in its proper position. In a rim-bearing turntable the reaction at the centre support is transmitted through the rollers, on which the weight of the superstructure should, therefore, be distributed as uniformly as possible. To this end the weight should be applied to the drum at a number of points, and the greater the number of these points the more even must be the distribution.

Fig. 350.

R - Radial Girder
E - Equalising Girder

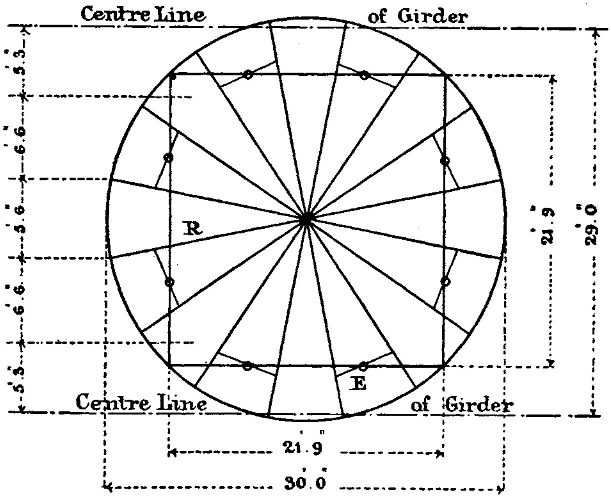
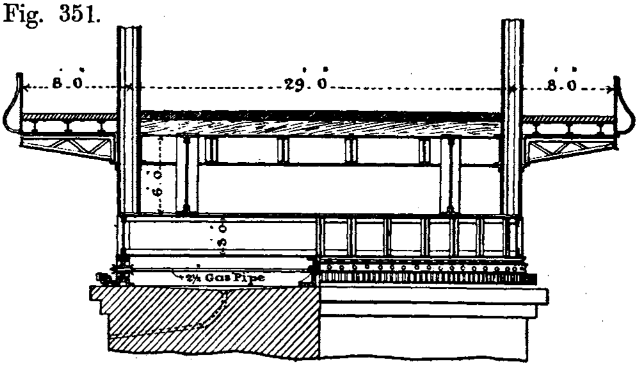


Fig. 351.



Figs 350 and 351 show in plan and elevation the arrangement of a rim-bearing turntable, from which it will be seen in Fig. 350 that the weight is equally distributed over eight points, and transmitted through radial girders to the drum at sixteen points, all symmetrically spaced.

The rollers which carry this weight may be made of cast-iron, with chilled faces, or of cast-steel, and may vary in diameter from twelve to thirty inches.

The safe load which a roller will carry in pounds per lineal inch is given by the expression:

$$600\sqrt{d},$$

in which d = diameter in inches.

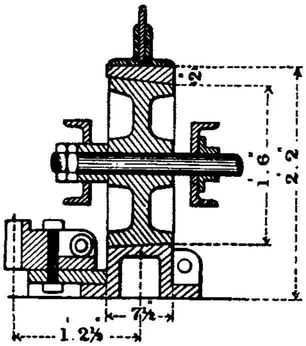


Fig. 352.

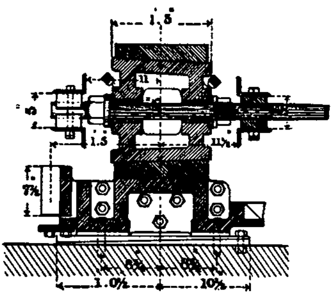


Fig. 353.

Figs 352 and 353 show two types of rollers, the one with a single web and the other with a double web and hollow inside.

The objection to the latter, although the extreme edges are well supported, is that water may find its way into the interior.

A girder carried on a rim-bearing turntable, when closed, may be continuous, partially continuous, or non-continuous.

In Fig. 354 is represented a type of swing bridge that has been extensively adopted in England and France. When the bridge is closed the main girders are continuous over four supports.

Fig. 355 illustrates a type of swing bridge that is much favoured in the United States of America.

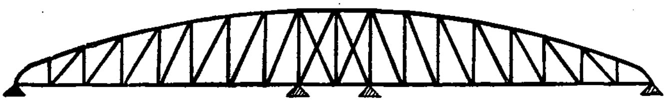


Fig. 354

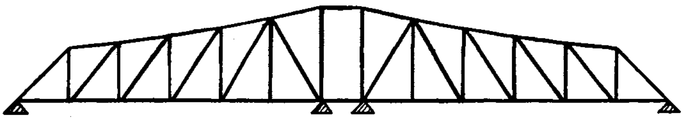


Fig. 355

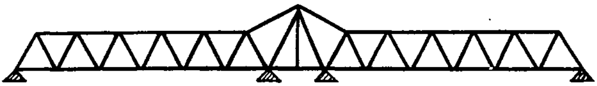


Fig. 356

In this bridge the girders are only partially continuous, because the diagonal bracing is omitted in the central panel over the pier, and, therefore, no shear can be transmitted through this panel.

Fig. 356 illustrates a swing bridge consisting of two simple spans when the bridge is closed. When the bridge is to be swung tension is brought to bear upon the links BC – BC′ by mechanical power, so as to lift the ends AA′ from their seatings. When the ends are thus lifted the bridge is free to move on its turntable. In this case it will be seen that each member is subject to a reversal of stress. When the bridge is closed the stress in the several members may be determined as for two simple trusses, but when in the act of being revolved the stresses must be calculated as for a cantilever girder.

Fig. 357

Elevation

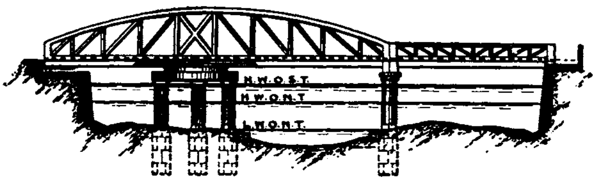
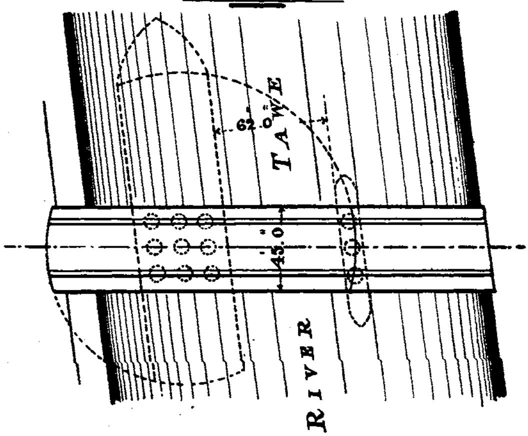


Fig. 358

Plan

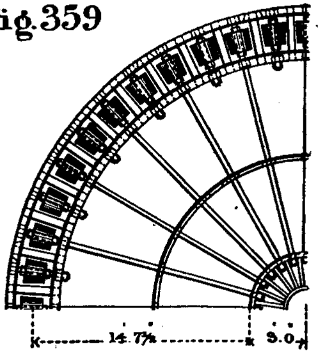


Figs. 357 and 358 illustrates in outline plan and elevation a swing bridge that has recently been built across the river

Tawe at Swansea, of which the following are the leading particulars:

When the bridge is closed and in position across the navigable channel it is supported at the nose and tail ends on four cast-iron blocks, one at each end of each main girder, and at the centre on a turntable of live rollers.

Fig. 359



A sectional plan showing the live rollers is given in Fig. 359, and a vertical section in Fig. 360.

Each of the bearing blocks is in two parts, the lower is bolted solidly to the masonry, while the upper is free to slide on the lower. The nose and tail ends of the bridge also carry an arrangement of knuckle gear. When the knuckles are straightened the weight at each end of the bridge is taken off the bearing blocks, the upper portions of which are then drawn back,

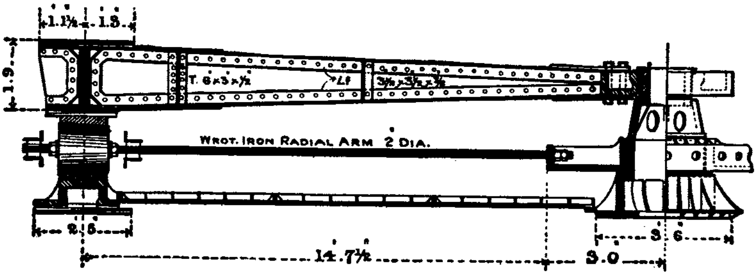
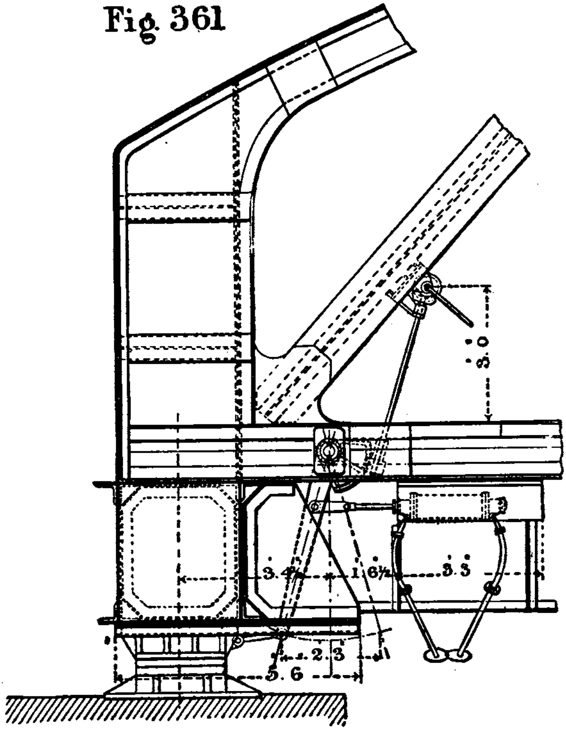


Fig. 360

leaving the whole weight of the bridge upon the rollers of the turntable, the bridge being thus free at each end, is caused to revolve by a vertical shaft extending from the platform of the valve house over the centre of rotation down to a circular rack attached to the fixed part of the turntable foundation.

Fig. 361



The arrangement of the knuckle gear and the bearing blocks is illustrated in Fig. 361.

To the bottom of the shaft is keyed on a pinion which engages the rack, and the top end is connected by means of bevel gearing to the crank shaft of a "three throw" hydraulic engine. An automatic locking gear is provided at the tail end to prevent the bridge over-swinging itself.

The method of working the bridge is as follows :—As soon as the operator in the valve-house receives the signal that the road traffic is stopped, he, after having first released the locking gear, admits pressure into the cylinders of the knuckle gear at each end of the bridge. the rams of which are forced outwards, and straighten out the knuckles just sufficiently to take weight of each end of the bridge off the resting blocks; he then opens a valve and admits pressure into four cylinders, two at the nose end and two at the tail end of the bridge. The piston rods of these cylinders are attached to the short arms of levers, and pull them back. The long and bottom end of the lever travels in the same direction, and each lever pulls out the sliding block on which the ends of the bridge rest. Directly the blocks move and electric communication is established between them and the valve-house, the current deflects a needle on a dial facing the operator as he stands with his hand on the valve levers. If the four sliding blocks are drawn out as they should be, the needle indicates the same. The operator is thus assured before he begins to put the rotating machinery in motion that the bridge is clear at both ends.

An indicating gear is also attached to the crank shaft of the turning engine, so that the operator may ease down the pressure as the bridge approaches the limit of its swing. To bring back the bridge to its original position across the channel the operations described are reversed.

The whole of the machinery and gear is worked by hydraulic power, at a pressure of 700 lbs. per square inch, and every operation is controlled by one man from the valve house, standing on top of the main girders over the centre of rotation.

The water under pressure is conveyed in a 2-inch diameter pipe from the edge of the quay into the turntable, passing up through the centre of rotation, and fitted at that point with an oscillating joint, the upper portion of the pressure pipe revolving round the lower. The machinery in the valve house is in duplicate, so that if the one set should by accident break down, the other is immediately available, and hand gear is provided for emergencies should the hydraulic power for withdrawing the sliding blocks fail.

The bridge can, if required, be opened to its full extent and brought back to its original position within two minutes. The structure was built from the designs and under the supervision of Mr. O. A. Schenk, M.I.C.E., the engineer of the Swansea Harbour, to whom the author is indebted for these particulars and illustrations.

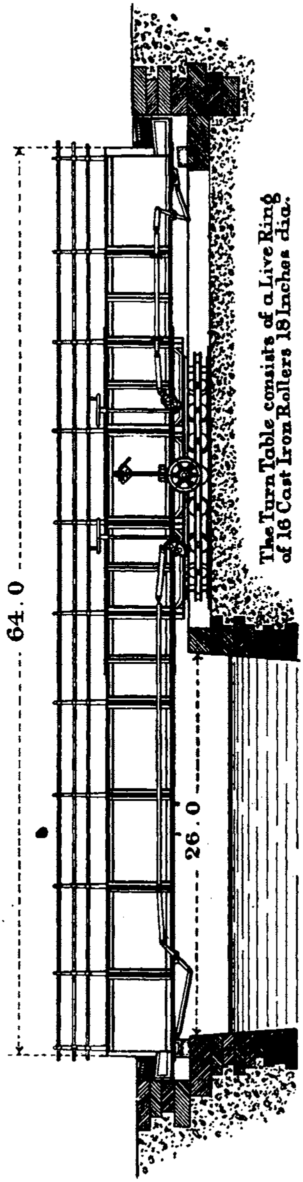
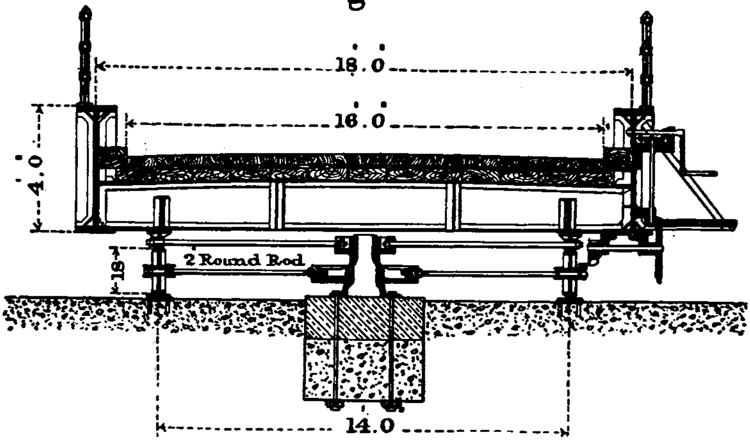


Fig. 362

In Figs. 362 and 363 is shown a swing bridge of very efficient and simple construction, built across the harbour at Castletown, in the Isle of Man, from the designs of Mr. Charles Waun, M.I.C.E. The turning gear, consisting of an arrangement of spur and pinion, is worked by hand, from a small platform carried on the side of the main girder facing the shore when the bridge is open. The bridge can be easily turned by one man in less than a minute and a half.

Fig. 363



In this bridge the rack, instead of being fixed to the masonry, is carried on the live roller ring, and revolves with the bridge, but at half the speed.

The arrangement is very simple, compact, and efficient, and completely self-contained.

The same system of turning gear has been applied to several other bridges designed by Mr. Waun.

Class 2.—In the type of bridge described in this class the girders whilst swinging are supported at their tail ends on wheels, and counterweighted so as give the shore span a preponderance over the channel or opening span.

An example of this type of structure is illustrated in Figs. 362 and 363.

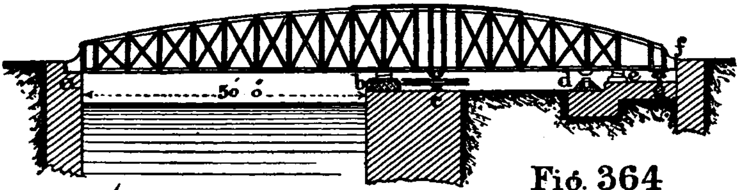


Fig. 364

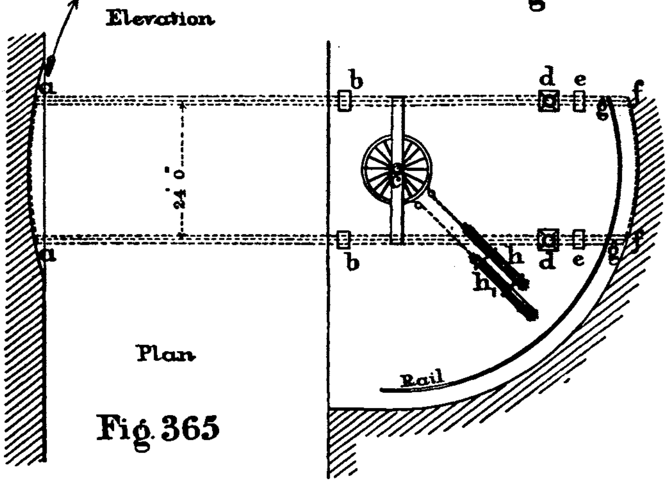


Fig. 365

The bridge when closed rests upon the supports a, b, and the wedge blocks e, e. To open the bridge water under pressure is admitted into the presses d, d, by which the tail end is slightly lifted, so as to allow the wedges e, e to be withdrawn, as well as the locking bolts at the extremities a and f. The pressure is then withdrawn from the presses, and the whole structure oscillating lightly, on the centre pivot c, descends to its former position, but the tail end being heavier than the channel span, is tilted downward by a slight bascule movement about the centre pivot, until the wheels *f*, *f* at the tail end rest on their roller path.

By this operation the nose of the bridge is lifted, and the structure being now entirely supported on the centre pivot and the tail wheels, is free to move.

The movement is effected by two hydraulic cylinders *h*, *h′*, placed horizontally, the one serving to open the bridge and the other to close it, having the ends of the rams connected with chains carried round the circular drum, as shown in Fig. 365. To close the bridge these operations are reversed.

The Pollet Bridge at Dieppe having an opening span of 131′ 3″ is another example of this type of bridge, but it differs from that already described, in so much that when the bridge is being lifted to withdraw the wedge blocks, etc., the centre pivot has a vertical movement by means of hydraulic pressure, as well as the two presses at the tail end corresponding with *d*, *d* in the preceding example.

Power required to Operate a Swing Bridge.

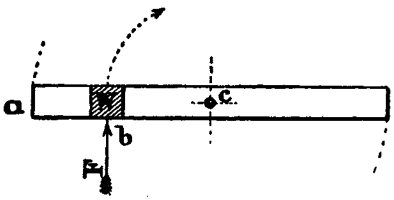
The power required to turn a bridge is absorbed in overcoming the resistances due to friction, inertia, and wind pressure. The frictional resistance is that due to the gearing and the moving parts.

The resistance of inertia is the force required to put the structure into motion, starting from a state of rest until it acquires its maximum velocity, and then retarding its motion until it is again brought to rest.

If in Fig. 366 a force F be applied at the point *b*, of the arm *a*, *c*, to the concentrated weight W, and it causes W to be moved at a velocity $= v$ expressed in feet per second. Then the value of the force $F = W\frac{v}{g}$, g being the velocity of a falling body due to gravity $= 32{\cdot}16$ feet per second.

The resistance of wind pressure can at best be only approximately estimated. It is, however, an important factor in the design of swing bridges, especially in exposed situations, where frequently the wind blows in gusts. In such circumstances the provision to be made for wind pressure must be left to the judgment of the engineer.

Fig 366



An approximate rule, very simple in its application, for determining the power to operate a swing bridge of the type illustrated in Figs. 350 to 360, that is to say, a bridge in which the whole weight of the structure is carried by a drum supported on live rollers, is given in a paper read before the American Society of Civil Engineers, Vol. XXV., by Messrs. Boller and Schumacher, Civil Engineers, which is as follows:—

$$HP = \frac{{\cdot}01 \text{ or } {\cdot}015\ (Wv)}{550}$$

in which :—

W = weight of bridge.

V = maximum velocity per second at circumference of rack.

·01 and ·015 = co-efficients ·01 being for careful work and construction, and 0·15 for second-rate or hasty work.

Lift Bridges.

Bridges of this type, that is to say, bridges capable of being vertically lifted to a sufficient height to enable the channel span to be used for navigation purposes, are of comparatively rare occurrence. There are, however, some instances in which they have been employed to bridge over canals and rivers where the span and headway are of no considerable magnitude, but the only example on a large scale is the Halsted Street bridge of Chicago, of which an outline elevation is given in Figs. 367 and 368. This bridge, designed by Mr. J. A. Waddell, M.I.C.E., of Kansas City, has a span of 130 feet from centre to centre of bearings, and a clear headway when raised to its maximum limit of 155 feet. When the bridge is lowered for traffic the headway is 14′ 6″. The structure is designed to carry a roadway 36 feet wide, including two lines of tramway, and two side walks each 7 feet wide, the distance from centre to centre of trusses being 40 feet. The live load was calculated at 4500 lbs. per foot run, and the dead load at 4000 lbs. per foot run.

For lifting the bridge steam is employed, and two 70 horsepower engines are installed under one of the side spans, which, by means of suitable gearing, actuate the pulleys, giving motion to the operation cables, consisting of steel wire ropes.

The weight of the bridge is balanced as nearly as possible by counterweights, so that little mechanical effort has to be exerted in lifting or lowering it, beyond that due to friction. The wire ropes pass over pulleys 12 feet in diameter, carried on the tops of the towers, one on each side of the river. The towers are tied together at their summits by two lightly-braced girders, which also serve to counteract the pull of the ropes, and to guard against flexure inwards under the weight of the bridge while it is being raised.

The principal advantage claimed for this type of structure

Fig. 367.

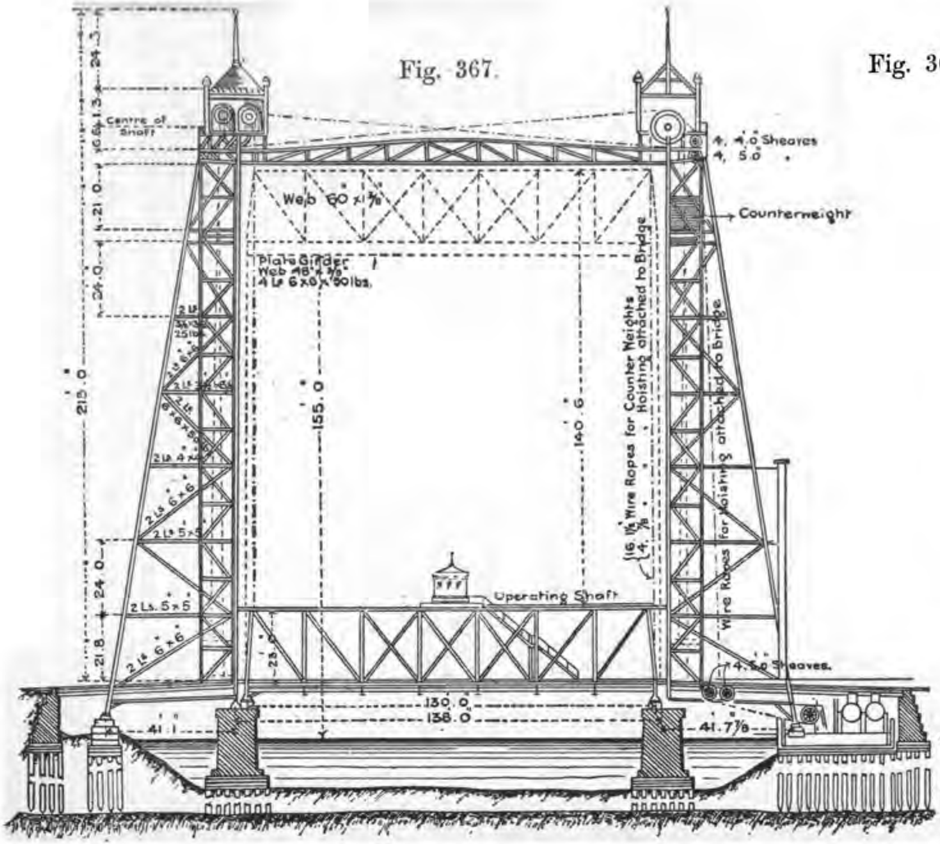
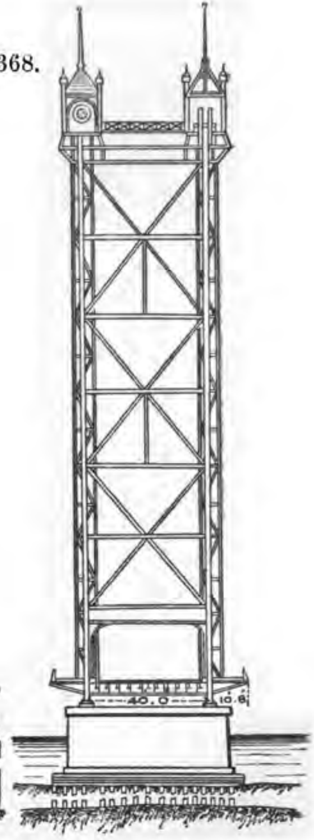


Fig. 368.



is that the dead weight can be exactly balanced by counterpoises, leaving only the resistance to inertia to be overcome by the lifting machinery, but it is a question whether the advantages are not outweighed by the costly character of the superstructure, including the lofty towers and the mechanism for operating the bridge.

Amongst other bridges of this type the next most prominent example is probably the Chitpore Lifting Bridge of the port of Calcutta. This bridge, built some 25 years ago, carries a single line of railway 5′ 6″ gauge, and has a span of 116 feet between supports, with a clear waterway of 110 feet between abutment walls for the passage of boats, and a headway of 20 feet at high water, when the platform is lifted to its maximum limit.

The main girders are 18 feet deep, and are spaced 17 feet apart, centre to centre.

The operation of raising and lowering the bridge is effected by a system of balance.

At the ends of the bridge counterweights are attached to the platform at the four corners by means of chains passing over pulleys. These counterweights exceed the weight of the platform by about two tons, which was found to be just sufficient to overcome the frictional resistance.

Under the platform are placed two tanks containing water ballast, which is admitted when the bridge is about to be lowered, and allowed to remain in the tanks when the platform is down. Each of these tanks contains 75·5 cubic feet of water.

When the bridge is to be lifted the water ballast is let out of the tanks, and the counterweights, by their preponderance over the weight of the platform, lift the bridge. When the bridge has to be lowered water is admitted into the tanks from an elevated cistern or reservoir, which is supplied by a small pump. The weight of water in the tanks, about two tons, produces the same preponderance over the weight of the counterpoise as does the counterpoise over that of the platform.

While the bridge is being lifted or lowered the speed is controlled by suitable brake arrangements. The bridge has been found to work very satisfactorily, and is remarkably steady under the passage of trains.

CHAPTER XVI.

Traversing and Transporter Bridges.

Besides the various types of bridges that have been described in the last chapter other devices have been employed for crossing navigable waterways, amongst which may be mentioned:

(*a*) The traversing bridge at St. Malo in France.

(*b*) A system of bridges introduced by M. F. Arnodin, civil engineer of Chateauneuf-sur-Loire, France, known as Transporter Bridges.

The traversing bridge at St. Malo consists of a movable platform supported by a braced framework on a carriage running on rails and moved by a hauling chain running along the bottom of the channel as shown in Fig 369.

This contrivance in reality has no more claim to be called a bridge than a ferry-boat moved along a rope or a chain.

The only other instance of the employment of this device is at Greenock, where in 1885 a bridge similar in principle was built at the entrance to the west harbour, which has worked daily ever since with every satisfaction.

The span or distance traversed is 103 feet 5 inches, width of platform 20 feet, and height of platform above rail level 32 feet.

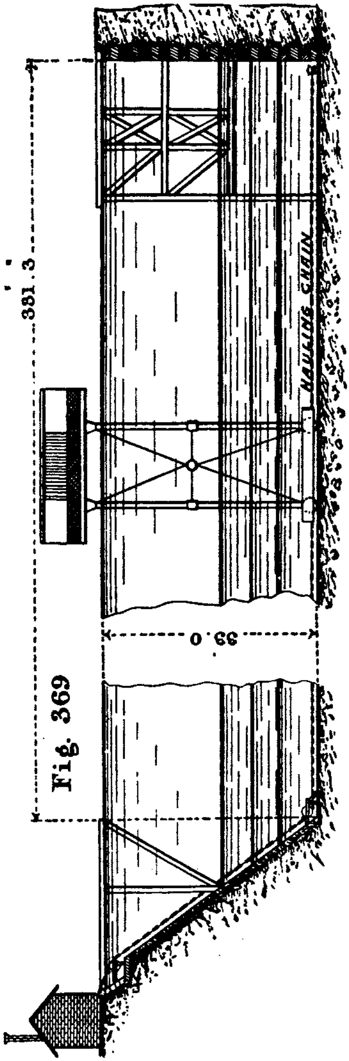


Fig. 369

Longitudinal Section

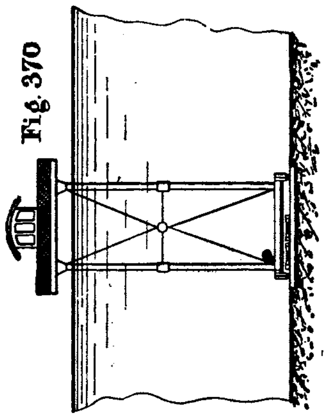


Fig. 370

Transverse Section

The platform travels on rails laid along the bottom of the harbour entrance at a depth of 26 feet below high-water level.

On the one side of the channel there is a chamber recessed in the wall underneath the quay into which the bridge is drawn back when the channel is required to be freed for the passage of a vessel into or out of the harbour, and as the bridge is being drawn back the platform and handrailings automatically fall low enough to enable the bridge to be housed in the chamber under the quay. The bridge is very easily moved as a large portion of its weight being supported on water-tight tanks is water-borne.

The conditions under which a structure of this type can be applied are very rare, involving a clear rocky bottom, exposed at low water, a sheltered situation and a current that is hardly perceptible. In channels given to silting its use at once becomes impracticable.

The system introduced by Monsieur Arnodin and his collaborator, M. de Palacio, and called by them Transporter or Transhipping Bridge (*Pont à transbordeur*) may be described as a bridge carrying a railway spanning the channel at such a height as to enable the highest masted vessels to pass underneath.

On this railway rolls a carriage consisting of a framework on small wheels, the number of wheels varying in proportion with the load. From this carriage is suspended, by means of rods or cables, a platform hanging at a level corresponding with that of the quays or landing stages on each bank. To prevent as far as possible any oscillatory motion caused by wind pressure or otherwise, the suspending rods or cables are arranged in the form of a series of triangles, efficiently bracing and counter-bracing, longitudinally and laterally, the entire suspended system connecting the platform with the overhead carriage by which it is moved to and fro.

The difference between this type of bridge and the traversing bridge at St. Malo is that in the one case the travelling platform is suspended from an overhead railway, at a considerable height above the level of the water, whereas in the other it is supported on a lofty framework carried on wheels rolling on a railway laid along the bottom of the channel.

The illustration in Fig. 371 represents in a crude form the principle of the transporter bridge. In the device outlined in this illustration the cradle or vehicle is directly supported by the cables, and is moved from one side of the channel to the other by manual effort applied to an endless hauling rope attached to the hangers by which the cradle is suspended.

In sparsely populated districts or new countries where the cost of a bridge would be prohibitive some such device as this may, even in these days, be employed with advantage, and erected at a comparatively small cost.

M. Arnodin has built and is now building a number of bridges

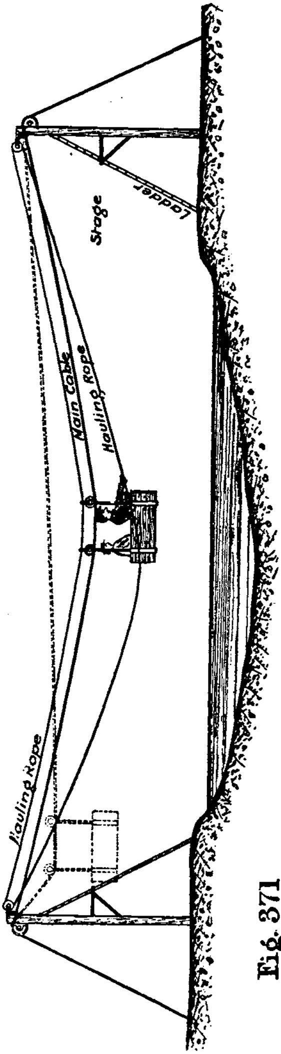


Fig. 371

of this type, amongst which the following may be instanced as prominent examples:

Particulars of Dimensions, Weights, etc.	Names of Bridges.				
	Bilbao, over the Nervion.	Rouen, over the Seine.	Bizerte (Tunis) over the Canal.	Martion, over the Charente.	Newport, over the Usk.
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
Span centre to centre of supports	525 0	469 0	357 6	458 6	644 9
Height from high water to under side of stiffening girder	147 6	167 0	147 6	164 0	177 0
Total height of towers	202 9	217 8	189 9	219 3	240 0
Dimensions of travelling carriage	26′ 3″ × 20′ 6″	42′ 8″ × 33′ 3″	29′ 6″ × 24′ 8″	46′ 0″ × 37′ 9″	39′ × 32′
Total weight of travelling carriage and load for test load	40 tons	100 tons	80 tons	110 tons	116 tons

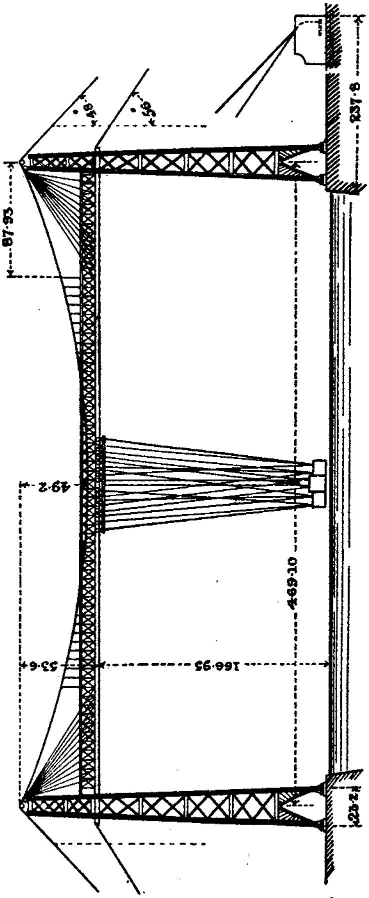


Fig. 372

Elevation

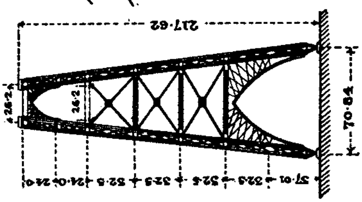
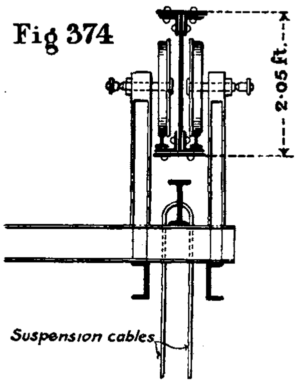


Fig. 373

Section

Figs. 372 and 373, which may be regarded as typical illustrations of this system of bridge construction, represent in elevation an outline of the transporter bridge over the Seine at Rouen. Figs. 374 and 375 illustrate the suspending arrangements.

Fig 374



The structure as shown in the illustration consists of a Stiffened Suspension Bridge, this type of construction being adopted because of the facility it offers for spanning wide channels, and the advantage of erection without scaffolding, which would interfere with navigation.

This bridge is built over the Seine between the Boulevard Cauchoise and rue Jean Rondeaux, and so as to ensure ample headway for navigation at the highest tides, the stiffening girder is suspended from the main cables at a clear height of 164 feet above the level of the quay walls and 167 feet above high-water mark of the highest spring tides.

The span from centre to centre of the towers supporting the cables is 469 feet, reducing the width of the navigable channel between the quay walls to 437·8 feet, and the sag or dip of the cables is 49·25 feet, or nearly $\frac{2}{19}$ of the span.

The main cables from which the stiffening girders and the platform are suspended are built of steel wire ropes, each cable having a sectional area of 4 square inches. The cables are carried over the summits of the towers on saddles, by means of which a certain amount of freedom of movement is given to the whole suspended system. To further stiffen the whole structure, and to provide against the effect of unequal loading, the ends of the stiffening girders are tied down to the same anchorages by 4 cables, each of 3 square inches sectional area. The platform, weighing about 37 tons, is 33 feet 3 inches long and 42 feet 8 inches wide, and it is capable of accommodating 200 passengers and six vehicles. Motion is imparted to the platform by a steel cable passing round a drum mounted on a shaft, to the ends of which two electro motors are coupled. Since its opening in September 1899 it has made as many as 240 trips daily to and fro, and carried 300 vehicles and up to 10,000 passengers. The stresses in this type of structure are determined in the same manner as for a suspension bridge.

Monsieur Arnodin has recently constructed at Nantes a transporter bridge on the principle of a cantilever girder, instead of the usual type of suspension bridge hitherto employed by him. This departure became necessary because of the difficulty and expense of obtaining suitable sites for the advantages of the cables at both ends for a bridge of the suspension type.

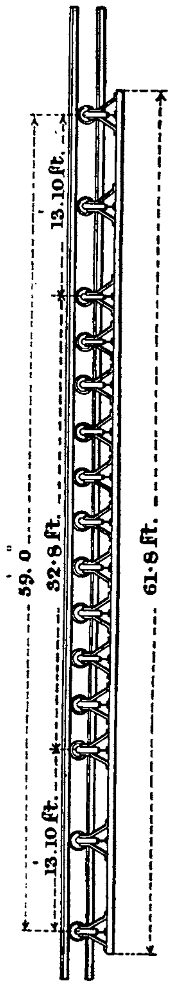


Fig. 375

An outline of this structure is given in Figs. 376 and 377.

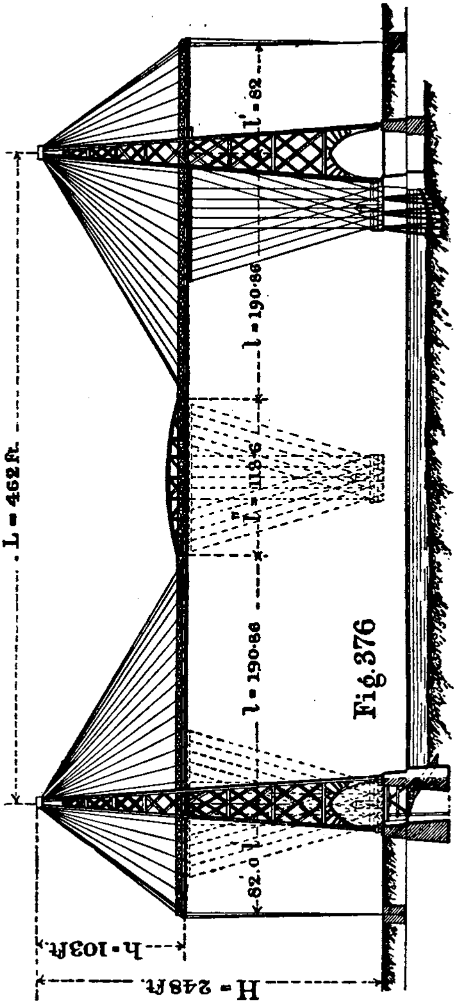
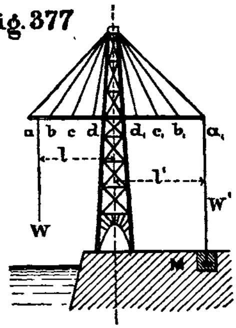


Fig. 376

Let it be supposed that the segment a, b, c, d in Fig. 377 is loaded with a weight W at a distance l from the axis of the pier or support, then there must be attached to the extremity a_1 of the balanced cantilever $a\,a_1$ a weight W′ the value of which so that it may counterbalance W

$$= W' = \frac{Wl}{l'}.$$

Fig. 377



The extremity a' instead of being held down by the counterpoise W′ may be anchored to a block of masonry or concrete M, the mass of which must be so disposed as to exert a sufficient re-action under any given condition of loading. The stresses in the several members of this structure may be readily determined as for a cantilever bridge.

A later and more prominent example of this type of structure is the Widnes and Runcorn transporter bridge now being built across the Mersey and the Manchester Ship Canal, having a clear span of 1000 feet, outline elevations and plan of which are given in Figs. 378, 379, 380.

The towers, constructed entirely of steel, are 190 feet above high-water level, and are supported on cast-iron cylinders filled with concrete sunk into the solid rock foundation.

The main cables, consisting of 19 steel ropes bound together, each rope being built up of 127 wires 0·16 inch in diameter, are 12 inches in diameter and have a sectional area of 50 sq. inches. The steel wire employed in the cables has an ultimate resistance of 95 tons per square inch, or 4750 tons for each cable.

The stiffening girders, which are hinged at the centre, are 18 feet deep and are suspended from the cables at a distance

Fig. 378

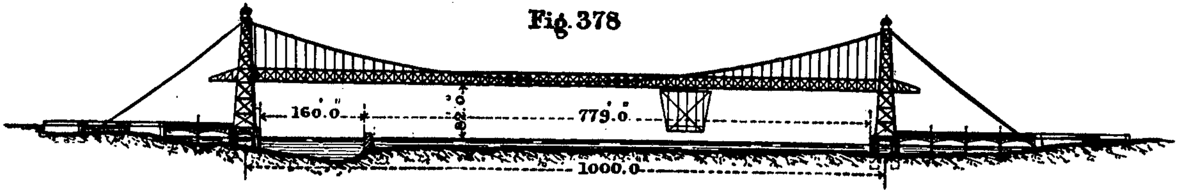


Fig. 379

Fig. 380

of 35 feet apart, at an elevation that gives 82 feet of clear headway above high-water level.

The platform, 25 feet long by 21 feet wide, is suspended from an overhead trolley carried on the stiffening girder by steel wire ropes grouped together so as to constitute an efficient system of bracing against all lateral forces. The trolley is supported on 32 wheels, 16 on each rail, and is propelled by two electric motors each of about 35 B.H.P. The engineer is Mr. J. H. Webster, M.I.C.E., of Westminster, to whom the author is indebted for these particulars.

CHAPTER XVII.

Floors of Bridges.

The floor of a bridge is usually carried on cross girders or floor beams and rail bearers on which the deck or platform is laid.

In Highway Bridges it is the practice invariably, except in new countries and where the traffic is very light, to cover the floor or platform with road-metalling, asphalte or wooden paving. In Railway Bridges the floor is frequently exposed, the rails being laid on longitudinal timbers resting on the cross girders, but wherever practicable it is wise to cover it with ballast and so enable the permanent way or what is called a "free sleeper road" to be laid continuously throughout. The disadvantage of this is that it adds considerably to the dead load to be carried by the bridge, but this is more than counterweighed by the advantage that it distributes the load over the whole width of floor, it allows expansion and contraction of the rails to take place, independently of the bridge floor, it avoids impact at the ends of the spans due to the passage of trains, and in case of repairs the permanent way can be got at easily.

Railway Bridges.

In bridges of which the deck or platform is laid with timber, the spaces between the rails should always be covered with a thin layer of ballast, concrete, or some other non-combustible material, if only as a protection against fire, which is always liable to result from the dropping of hot cinders from the fire-box of a passing locomotive engine. In bridges crossing over main roads or important thoroughfares the Local Authorities invariably require that they shall be watertight, which necessitates the use of some type of steel plate flooring, unless the bridge consists of a masonry arch.

A railway bridge may be a "through" bridge or a "deck" bridge, that is to say the track may be carried between the girders in a through bridge or on top of the girders in a deck bridge. Generally a deck bridge is a more economical type of bridge than a through bridge, because the main girders may be spaced much about the same distance apart as the gauge of the road, so that each rail is continuously supported on the top of each girder.

In a deck or through bridge the maximum limit of distance from centre to centre of cross girders should not exceed 4 feet, unless rail bearers, extending from cross girder to cross girder, are employed, and in the latter case the space should coincide as nearly as possible with the distance between the centres of the driving wheels or between the driving wheel and trailing wheel of a locomotive engine, which may be taken at 7 to 8 feet, on the principle that the greater the interval between the cross girders, the greater must be the load transmitted by them at intermittent points to the main girders, and therefore the greater the hammering action produced thereby in the main girders.

Where way beams or longitudinal timbers under the rails are used, it has been the custom to assume that they serve as continuous girders to distribute the load over the cross girders, but the modern practice is to design each cross girder so that it may be capable of carrying the greatest load that can come upon it, consisting of the maximum concentrated load on the axle of a locomotive engine, which may be estimated at 20 tons or 10 tons on each wheel.

Steel plates rolled in the form of troughs, corrugations, or ridges and furrows are now extensively employed in the construction of bridge floors. The troughing may be laid transversely, that is to say the corrugations at right angles to the rails, resting on the flanges of the main girders, in which case cross girders are dispensed with, or they may be laid longitudinally, the corrugations coinciding with the axis of the bridge, and supported at intervals on cross girders.

For short spans this system of flooring laid longitudinally is a very simple and efficient type of structure, combining the function of girder and flooring.

These trough sections are rolled in various forms and to various dimensions and thickness of plate. Some of the principal sections are shown in the following figures, 381 to 383.

Fig. 381

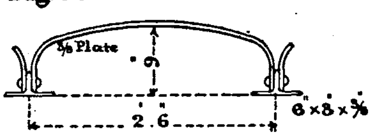


Fig. 382

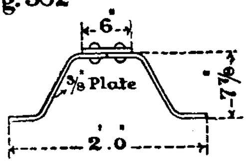
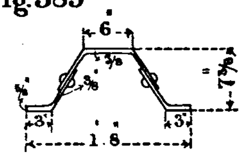


Fig. 383



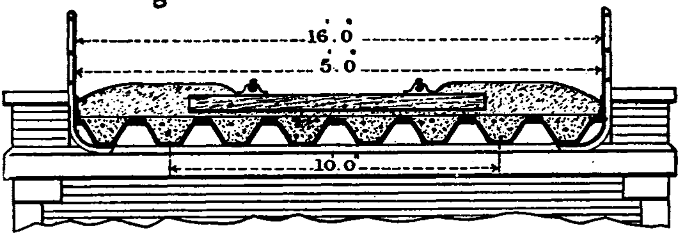
The section in Fig. 381 is that known as Hobson's.
Weight of floor per square foot = 26·5 lbs.
Modulus of section 36·7.
Fig. 382 represents J. Westwood's section.
Weight of floor per square foot = 27·7 lbs.
Modulus of section 34·5.

Fig. 383 represents Messrs. Dorman Long & Co's.
Section C maximum.
Weight of floor per square foot = 32·97 lbs.
Modulus of section 30·6.

The following example is given to illustrate the application of trough flooring to carry a single line of railway over an occupation road 12 feet wide. For this purpose either of the sections shown in above figures is applicable. Let the section shown in Fig. 382 be selected.

The width of the bridge, which should not be less than 15 feet between the handrails, must be some multiple of the pitch of the corrugations, in this case 2 feet. The width should therefore be 8 corrugations or 16 feet, as shown in Fig. 384.

Fig 384



The troughs or furrows are to be filled up with concrete, and the whole surface covered over with ballast.

The dead weight will therefore be as under, for a length of 1 foot of the bridge:

Steel flooring, 27·7 lbs. × 16 ft.	= 443 lbs.
Concrete filling	= 600 „
Ballast	= 1200 „
Permanent way	= 160 „
Total dead load per foot run	= 2403 lbs.
Total dead load per square foot = $\frac{2403}{16}$	= 150 lbs.

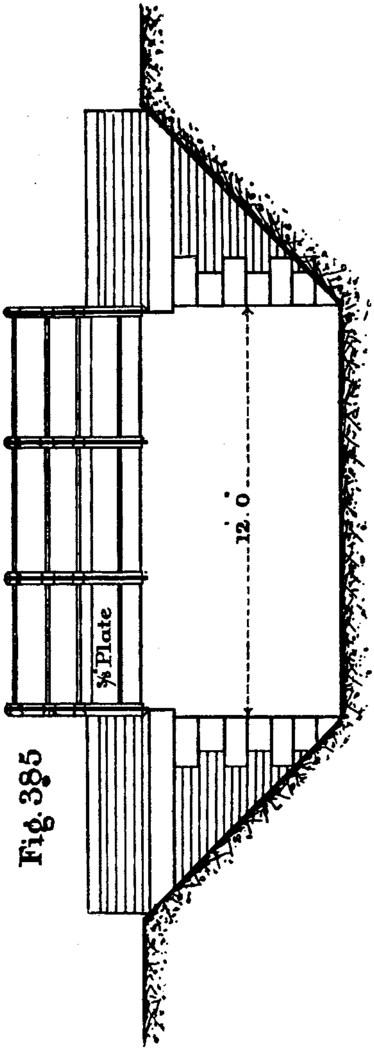


Fig. 385

The equivalent uniformly distributed live load may be taken at 50 tons $= 50$ tons.

The dead load is distributed over a width of five corrugations $= 10$ feet, so that the total intensity of dead load $= 10$ ft. $\times 12$ ft. $\times 150$ lbs. $= 8$ tons.

Total dead and live load, 58 tons.

The bending moment in inch tons

$$= \frac{58 \times 12 \times 12}{8} = 1044 \text{ in. tons.}$$

The resistance of 5 corrugations $= 34{\cdot}5 \times 5 = 172{\cdot}5$.

Then by equating the bending moment with the moment of resistance, we have 1044 inch tons $= 172{\cdot}5f$; whence $f =$ working stress

$$= \frac{1044}{172{\cdot}5} = 6{\cdot}05 \text{ tons per sq. in.}$$

Figs. 384 and 385 illustrate a bridge of this type.

Fig. 386 illustrates the application of steel troughing in the floor of a "Through" plate girder bridge, in which the troughing is laid transversely and takes the place of cross girders.

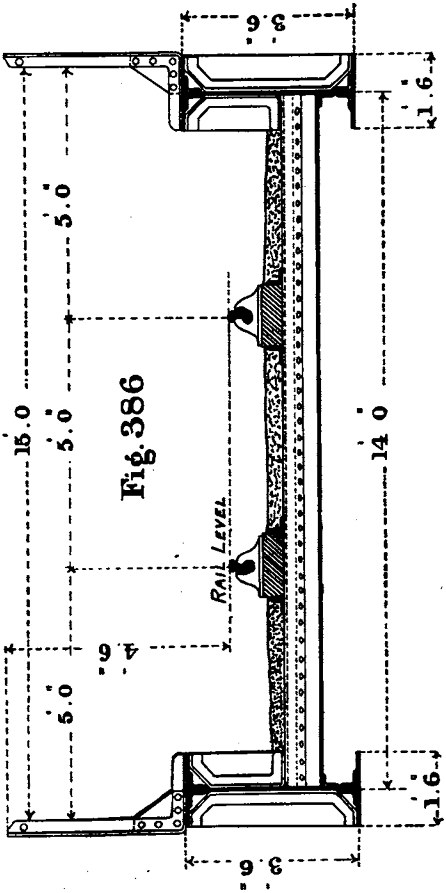


Fig. 386

The section here used is Messrs. Dorman, Long's "C' maximum," the weight of which is 35·02 lbs. per square foot, and the moment of resistance 51·45, as shown in Fig. 387.

The dead load in this as in the preceding example may be taken at 150 lbs. per square foot, because the depth of ballast is slightly less, which would more than cover the additional weight of the flooring.

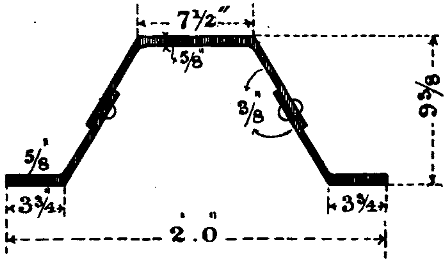


Fig. 387

The dead load per foot run will therefore be

$$150 \times 14 \text{ feet} = 18{\cdot}75 \text{ cwt.}$$

The live load on any cross section of the bridge, allowing for the heaviest type of locomotive engine and for impact, is as shown in Fig. 388, and this is assumed to be distributed over three flutes in cross section = 6 feet.

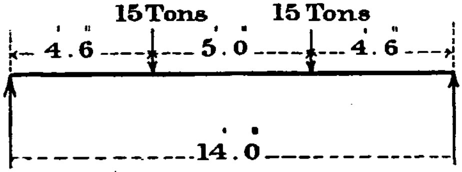


Fig. 388

Bending moments.

$$\text{Dead load} = \frac{18{\cdot}75 \text{ cwt.} \times 6 \text{ ft.} \times 14 \text{ ft.} \times 12 \text{ in.}}{8} = 118 \text{ inch tons.}$$

$$\text{Live load} = 15 \text{ tons} \times 4{\cdot}5 \text{ ft.} \times 12 \text{ in.} \qquad = 810 \quad ,,$$

$$\text{Total bending moment} = 928 \quad ,,$$

Moment of resistance.

Resistance of 3 flutes of flooring $= 51{\cdot}45 \times 3 = 154{\cdot}35$ inch tons.

$$\therefore \frac{BM}{MR} = \frac{928}{154{\cdot}35} = 6 \text{ tons per } \square'' \text{ the working stress.}$$

Fig. 389

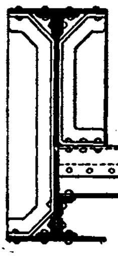
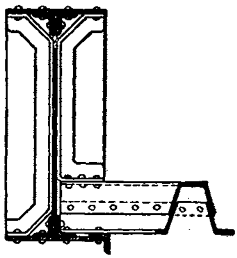


Fig 390



Figs. 389 and 390 show two methods of connecting and supporting the trough flooring on the main girders, the one illustrated in Fig. 390 being the more suitable, where the height from the surface of the road to be bridged over to rail level is limited.

Fig. 391

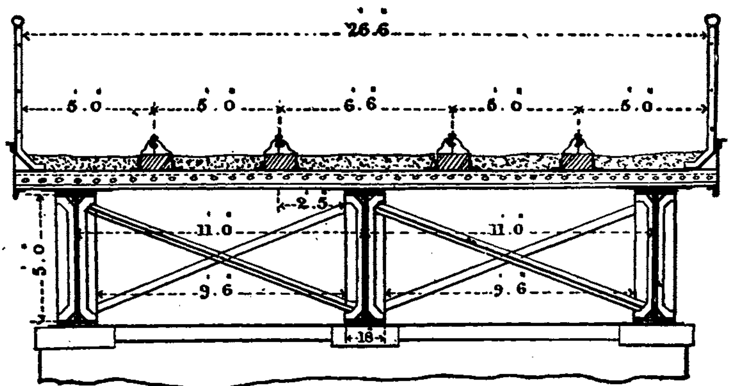


Fig. 391 illustrates the application of steel troughing in the floor of a deck bridge carrying a double line of railway. The section here used is "Maximum Section C" in Messrs. Dorman, Long's list, weighing 32·97 lbs. per square foot, and having a modulus of resistance of 30·6 per flute, 1′ 8″ wide.

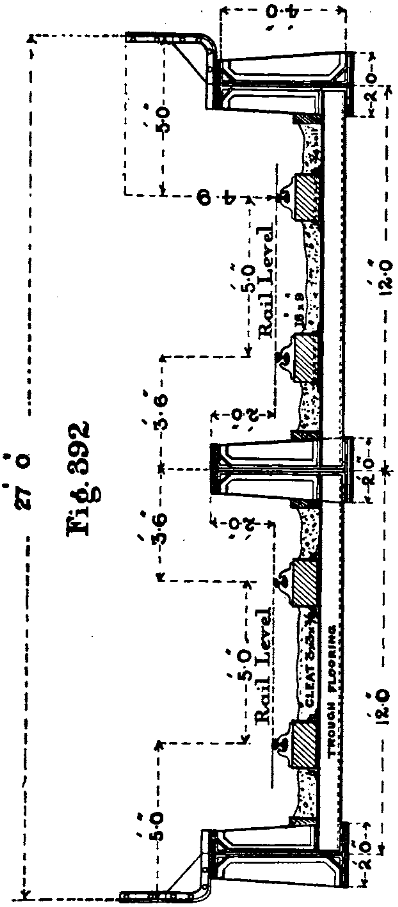


Fig. 392

In Fig. 392 is shown a cross section of a Through Plate girder bridge, the floor of which is laid with Steel Troughing supported on the bottom flanges of the main girders. The section of troughing used is "C′ maximum" in Messrs. Dorman, Long's list, as shown in Fig. 387.

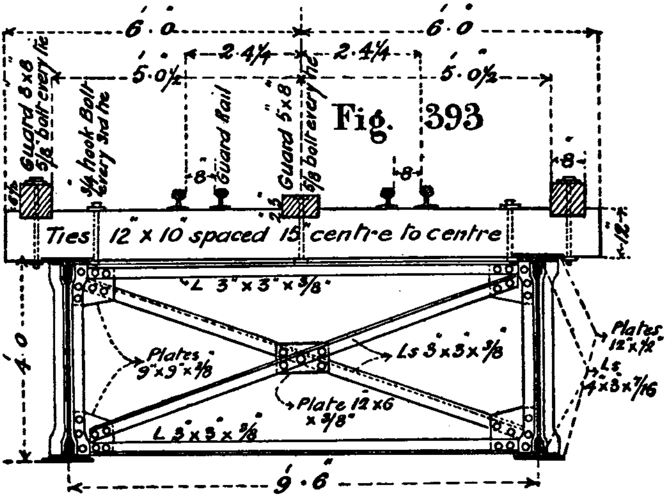


Fig. 393 illustrates the Standard Plan of floor for Deck Plate girder bridges on the Northern Pacific Railway. The rails, as will be seen, are spiked down on the ties or sleepers, spaced 15 inches apart centre to centre, which are secured to the upper flanges of the main girders by hook bolts grasping the under side of the flange. In case of derailment, guard rails are spiked down to the ties on the inner side of each rail at a distance of 8 inches from edge to edge of rail.

Fig. 394 represents the cross section of a deck plate girder bridge, carrying a single line of railway, in which the main girders are placed directly under the rails. This type of construction is very economical, and it has the advantage of affording a continuous support to each rail, but in case of a derailed train the cantilever brackets, which are spaced about 7 feet apart from centre to centre, would not be capable of supporting the load accidentally thrown upon them.

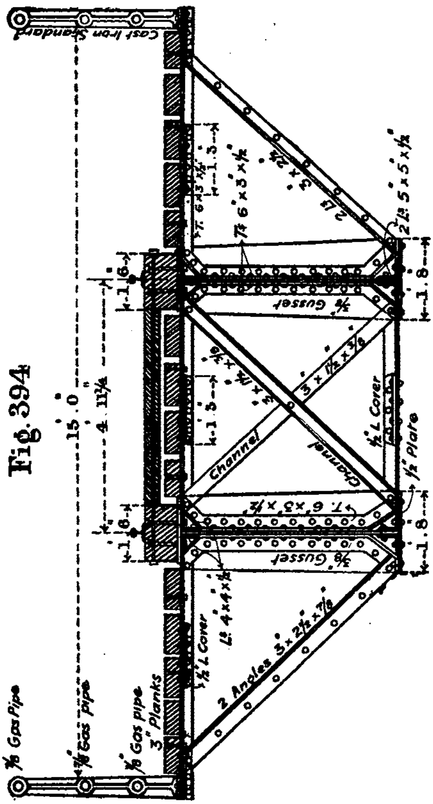


Fig. 394

In Fig. 395 is shown a cross section of a deck girder bridge carrying a double line of railway, in which the main girders are placed directly under each rail. In this case it will be seen

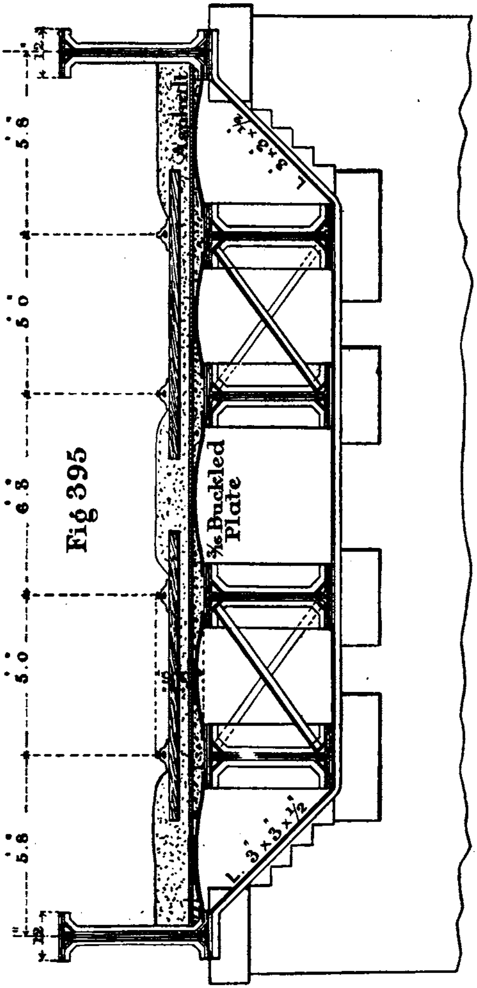


Fig 395

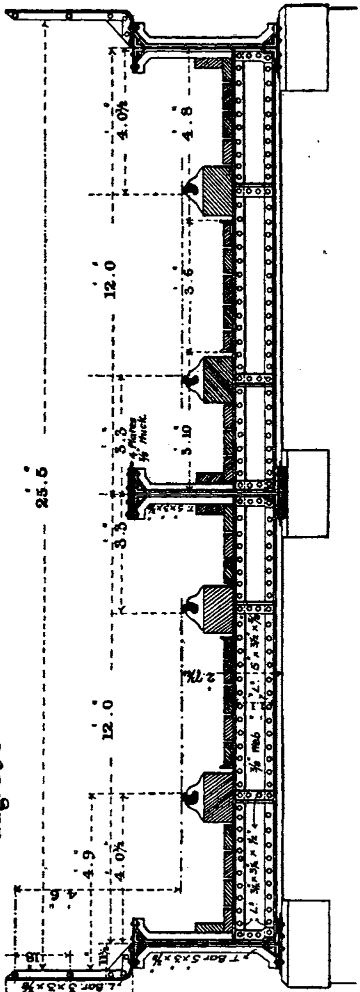


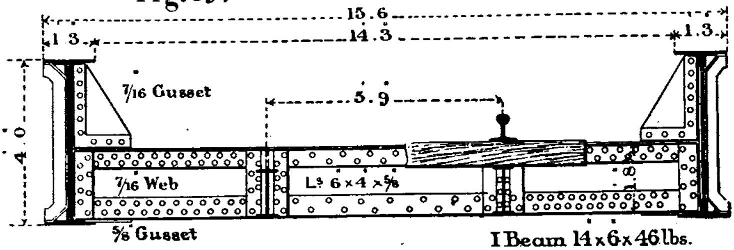
Fig. 396

that the running road is carried on sleepers instead of on way beams, as in the preceding cases. This is what is termed a "free sleeper road." This type of construction, where there is no restriction as to the limit of headway, is very efficient and economical. The side girders, which only form parapets, may consist of light plate or lattice girders.

Fig. 396 illustrates a type of floor in which the rails are carried on way beams resting on the cross girders. The cross girders are supported on the lower flanges of the main girders with their ends rivetted to the webs and spaced 4 feet apart centre to centre. The decking is laid with 3 inch planks.

Fig. 397 represents the floor of a plate girder bridge of 40 feet span on the Indian State Railways, 5 ft. 6 in. gauge, built from the designs of Sir Alexander Rendel and Mr. F. E. Robertson. The cross girders resting on gusset plates, supported on the lower flanges of the main girders and securely rivetted to the webs by angle gussets, are spaced

Fig. 397



8′ 6″ apart from centre to centre. Rail bearers, consisting of 14″ × 6″ × 64 lbs. rolled steel beams, extending from cross girder to cross girder, are laid one under each rail, on which the sleepers are supported and secured by hook bolts.

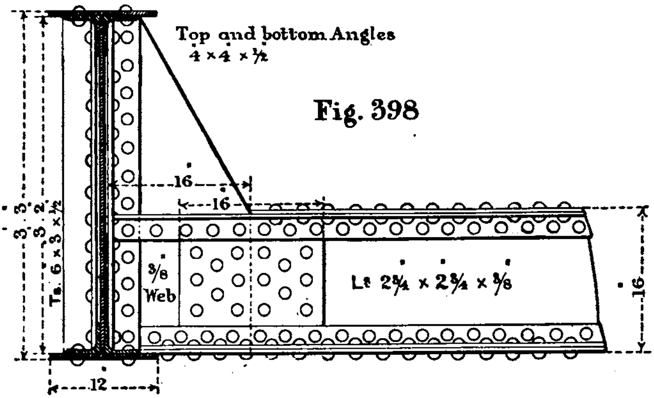


Fig. 398

Fig. 398 shows another method of connecting the cross girders with the main girders by means of gussets. It differs

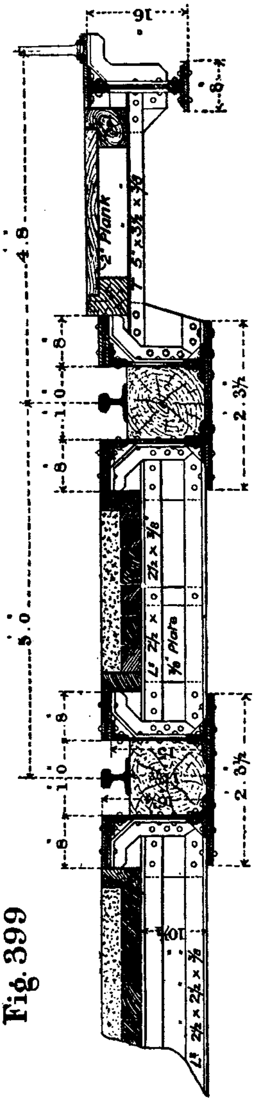


Fig. 399

from the arrangement shown in Fig. 397, inasmuch as the gusset plate, instead of being rivetted through a bent angle bar connection to the upper flange of the cross girder, is continued downwards the depth of the girder and made to form part of the web of the cross girder, the junction of the web being made good by cover plates.

The type of floor illustrated in Fig. 399 is adopted for short spans up to 25 feet in length, where the headway to be bridged over is limited. The rails are carried on way beams laid in trough girders, the depth from the upper surface of the rail to the underside or soffit of the superstructure not exceeding 17 inches for a span of 25 feet. For shorter spans the depth would be less.

If a water-tight floor is desired, steel plates rivetted to light channel bars supported by the cross girders should be substituted for the timber decking, and the floor then covered with a layer of concrete, as shown in Fig. 395.

Fig. 400 illustrates a cross girder and flooring of a triangular girder of the type shown in the diagram. The cross girders are supported on the lower chords of the main girders, which are spaced 8 feet 3 inches apart. Rail bearers extend from cross girder to cross girder, their ends being securely rivetted to the webs of the cross girders.

The main girders are spaced 27 ft. 8 in. apart from centre to centre.

Fig. 400

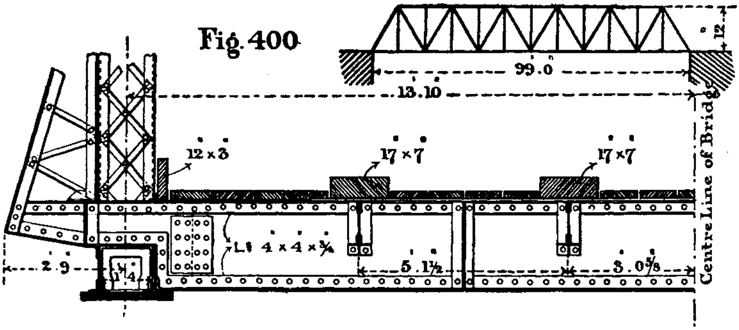
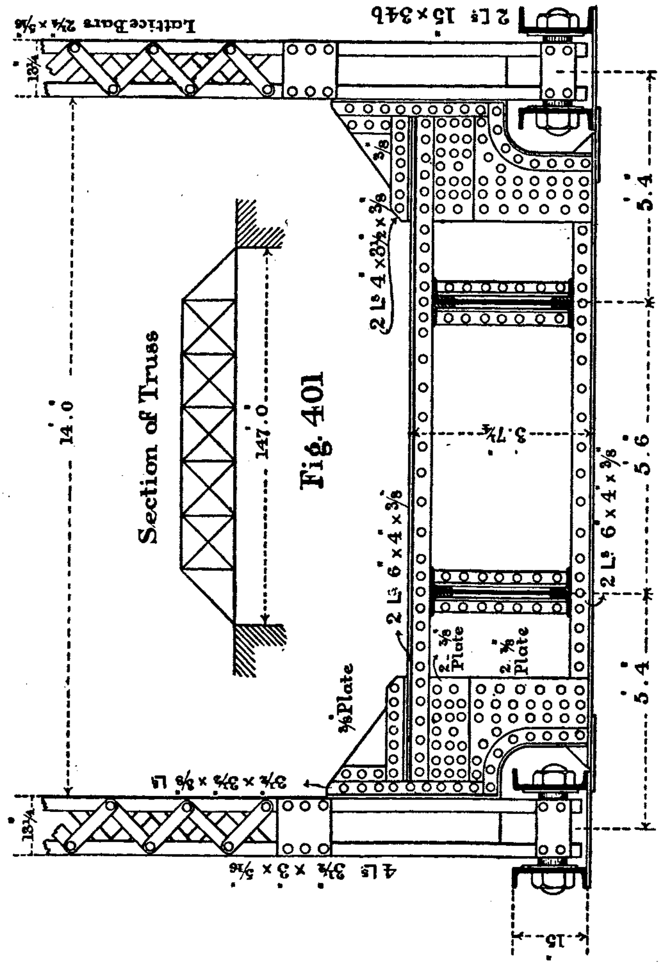


Fig. 401 illustrates in section the floor system of the Atbara Bridge built for the Egyptian Government by the Pencoyd Iron Works. The main girders consist of Prutt trusses of 7 panels of 21 feet each. The cross girders are attached to the vertical members of the trusses at each panel point by strong rivetted connections and offset in the lower part, so as to clear the bottom chord pins and chords to which they arc connected by rivetted horizontal lateral plates. Longitudinal rail girders extend from one cross girder to another, to which they are securely rivetted by angle connections, as shown in the illustration.

Fig. 402 illustrates the cross section of a bridge of 27 feet span carrying a single line of railway and built of Differdange rolled steel beams.

The live load is assumed to be 81 tons, or 3 tons per foot



run, or 20 tons on an axle, and the dead load 10·8 tons or 8 cwt. per foot run.

The main girders are of section No. 75B $29\frac{1}{2}'' \times 12'' \times 177$ lbs.

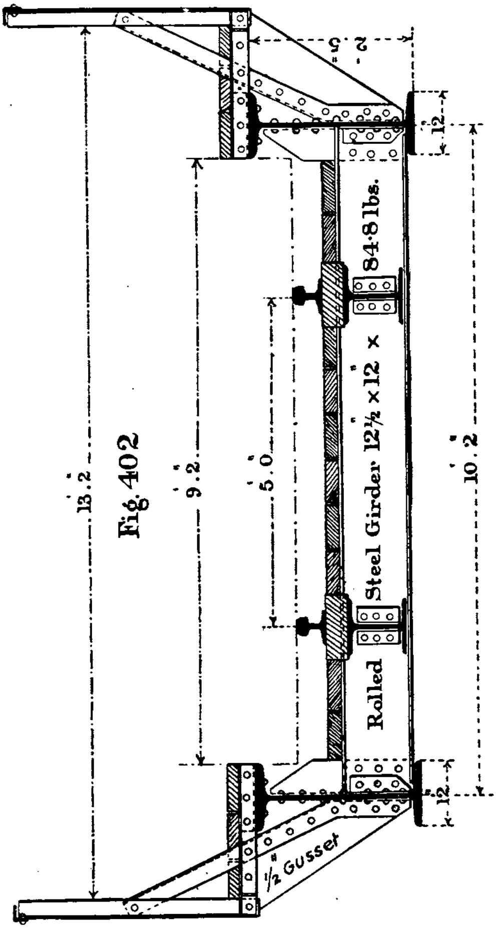


Fig. 402

per foot run; the cross girders, which are spaced 8′ 3″ apart centre to centre, are of section No. 32B 12½″ × 12″ × 84 lbs. per foot run; and the rail beams, which are supported on the lower flanges and firmly rivetted by angle cleats to the webs of the cross girders, are of section No. 26B 10¼″ × 10¼″ × 61 lbs. per foot run. The working stress on the metal due to this condition of loading is about 4 tons per square inch.

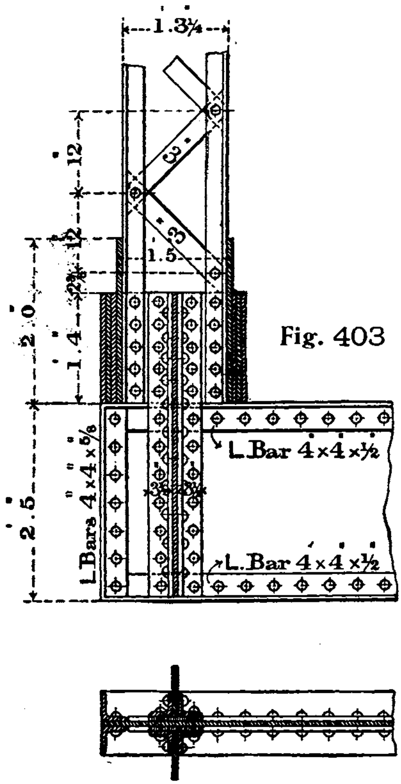


Fig. 403

By rivetting flanges to the main girders the span in this type of bridge may be increased to 50 feet.

Fig. 403 shows a method of connecting the floor system with the main girders that is often adopted in girders of the open web type. The cross girders are suspended at each panel point from the main girders by means of angle bar hangers, which are securely rivetted to a diaphragm plate carried by the web verticals.

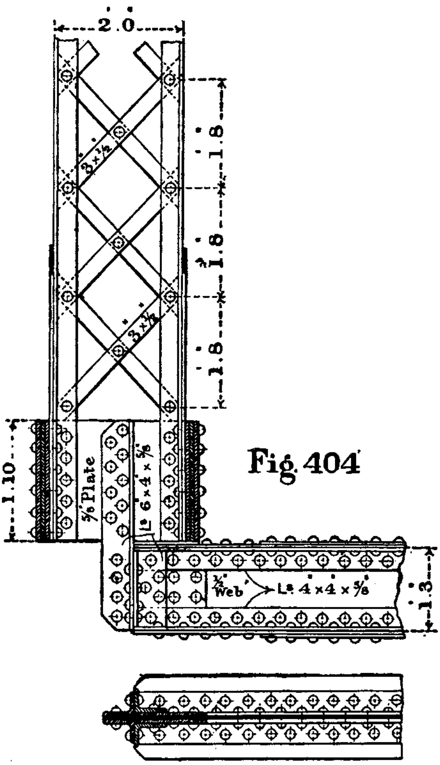


Fig. 404

The web of the cross girder at the point of attachment and the diaphragm plate are re-inforced by packing pieces so as to provide sufficient bearing area for the rivets.

The angle bar hangers are rivetted back to back against a stiffening plate, the axis of which coincides with the centre line of the main girder chords. The load carried by the cross girders is thus transmitted symmetrically about the centre line in vertical section of the main girder.

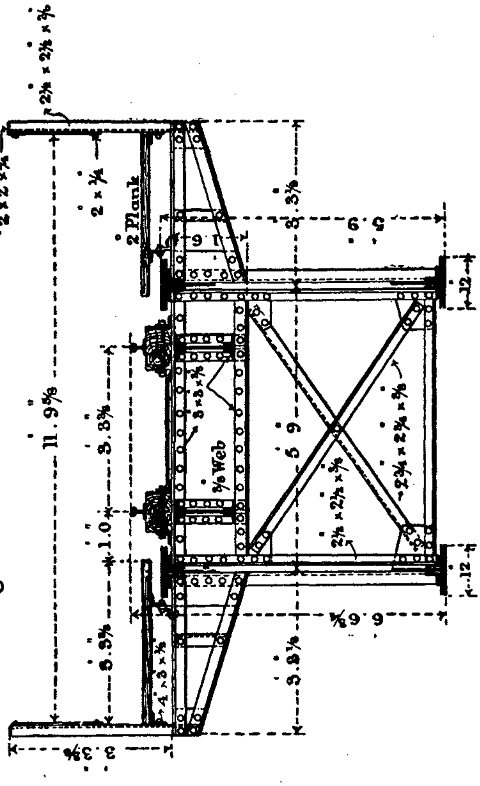


Fig. 405

Fig. 404 shows a somewhat similar form of attaching the cross girder to the main girder. In this case one pair of angle hangers is employed instead of two, as shown in Fig. 403. The web of the cross girder is continued beyond the end angles a sufficient length to give the angle hangers a grip upon it, as shown in section AB, so that the hangers are rivetted to the ends as well as the web of the cross girder. In this form of attachment the load carried by the cross girder is also transmitted to the main girder on its centre line, so as to prevent eccentricity of stress.

Fig. 405 illustrates a cross section of a deck bridge for carrying a light railway, as adopted on the Continent, especially in France and Italy, which may be regarded as a representative type of construction in those countries, where the headway admits of the adoption of a deck bridge.

As shown in the illustration, the rails are carried on way beams, supported on cross girders at intermediate points between the main girders. The footways on each side, consisting of timber planks, are carried on cantilever brackets projecting outwards from the main girders, and the space between the way beams is covered over with perforated steel plates for the reception of the ashes and hot cinders dropping from the fire-box of the locomotive engines passing over, so as to obviate any risk of setting fire to the superstructure, to which there is always a liability where a timber floor is exposed. It will be seen that in this illustration the main girders are exceedingly well braced laterally, and must therefore be very rigid against all extraneous forces and especially wind pressure.

Highway Bridges.

Under this general heading is included all classes of road bridges, ranging from a main road bridge in an Urban District capable of carrying the heaviest traffic, to a light rural road bridge capable of carrying the traffic of the district.

By reason of the very variable conditions of loading, the width of roadway to be provided and the special circumstances involved in each case, it at once becomes an exceedingly difficult if not indeed an impossible problem to propose any general rules for the treatment of the design of a highway bridge. Suffice it then to say that each case must be dealt with on its own merits and requirements, and that the economical

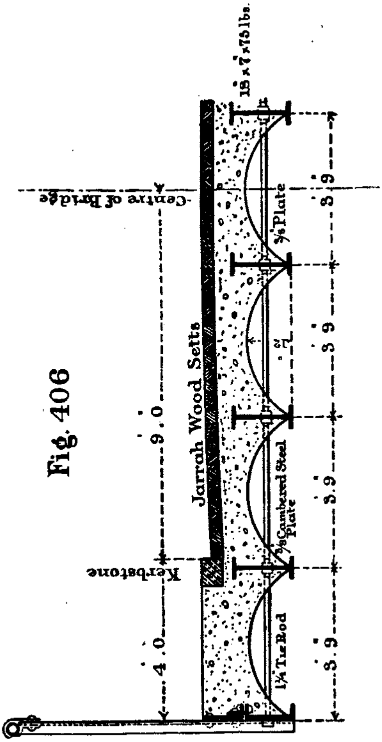


Fig. 406

design of an efficient highway bridge is frequently a more complex problem than that of a railway bridge.

Fig. 406 illustrates a simple and efficient type of road bridge suitable for spans of 20 to 30 feet. It is built of rolled steel girders carrying on their lower flanges cambered steel plates or corrugated steel sheeting and covered over with cement concrete. The cambered steel sheets, although offering substantial support to the concrete floor, act more particularly as centres for the concrete arches. Any thrust due to the arches is taken up by steel tie rods passing through the webs of the girders and tightly screwed up against them. The two outside girders are built up of a channel bar and plate rivetted together by an angle bar running longitudinally on the top flange of the channel.

Fig. 407

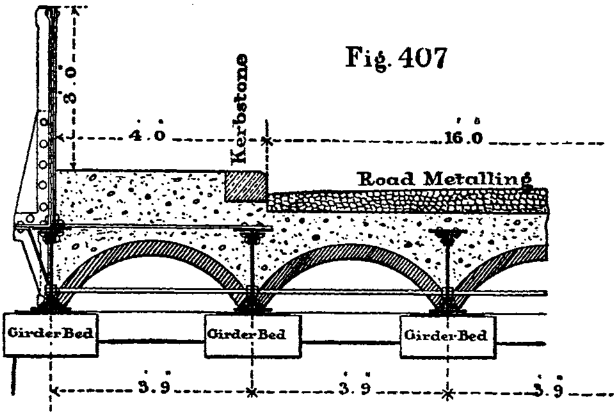


Fig. 407 illustrates a somewhat similar type of flooring, the difference consisting of the introduction of brick arches having their abutments on the lower flanges of the main girders, instead of the cambered steel plates. In other respects the construction is very much alike.

Both types of floor are suitable for carrying the heaviest class of road traffic for spans up to 30 feet, provided the main girders are spaced and proportioned for the load.

The dimensions given in Figs. 406 and 407 are of sufficient strength to carry a uniformly distributed load of 1 cwt. per square foot of floor area, or a traction engine weighing 20 tons.

Figs. 408 and 409 illustrate a type of flooring that is frequently adopted for highway bridges in the United States

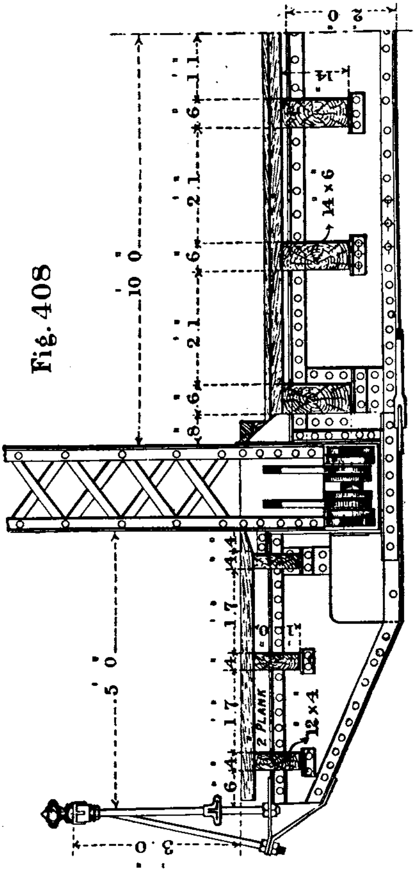


Fig. 408

of America. In both illustrations the floor is laid with planks of pine or other suitable timber. In Fig. 408 the floor beams extending from cross girder to cross girder are of timber, whereas in Fig. 409 they consist of rolled steel girders. In other respects the construction in both illustrations is very similar.

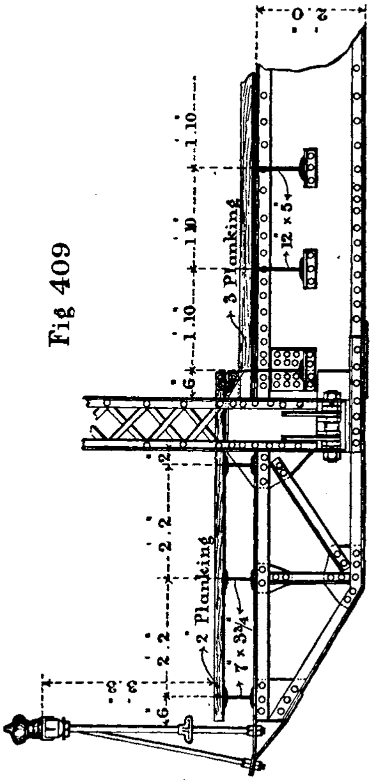
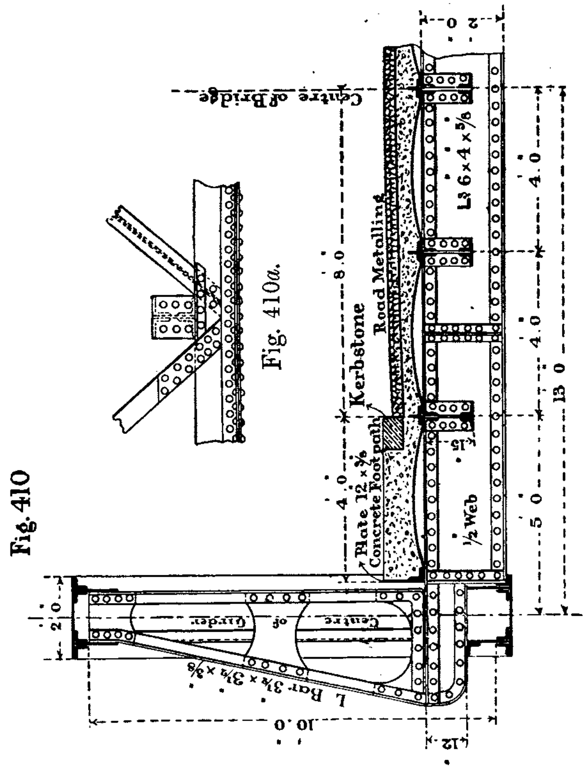


Fig 409

Fig. 410 illustrates the attachment of the cross girder to main girder and the floor system of a highway bridge of 100 feet span. The main girders are of the Lattice or double intersection Warren type, and the cross girders, spaced 10 feet apart centre to centre, are supported at the panel points or intersection of the web diagonals, as shown in Fig. 410*a*. The flooring consists of buckled steel plates, rivetted at their edges to longitudinal beams extending from cross girder to cross



girder, and one intermediate beam laid between each cross girder. The plates are covered with concrete and road metalling. This makes a very efficient and strong floor.

In Fig. 411 is shown half the cross section of a highway bridge of 72 feet span, the main girders of which consist of two parabolic bowstring trusses, 9′ 9″ deep at centre, and divided into 8 panels of 9′ 9″ each. The cross girders are suspended from the vertical web members at each panel point in the manner indicated in the illustration. The floor is laid with steel sheeting of the ridge and furrow type, which is covered over with concrete and a surface layer of road metalling. The cross girders project on each side 5 feet beyond

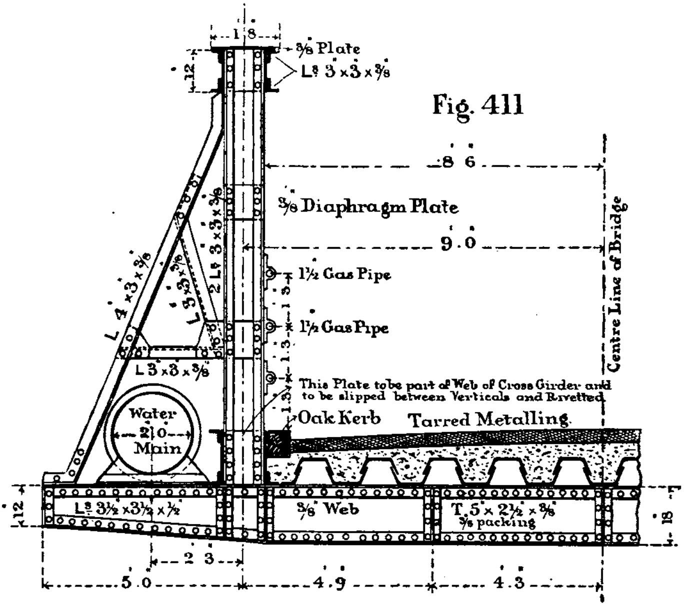


Fig. 411

the centre line of the main trusses, forming cantilevers for carrying water mains.

Fig. 412 illustrates a type of flooring that is largely used in France for highway bridges carrying heavy traffic up to 50 or 60 feet span. The structure is composed of two outside plate girders and two or more intermediate plate girders, carrying on their lower flanges jack arches built of masonry or brickwork and backed with concrete to the formation level of the roadway.

The thrust of the arches on the longitudinal girders is taken up by cross girders, tying the main girders together at intervals

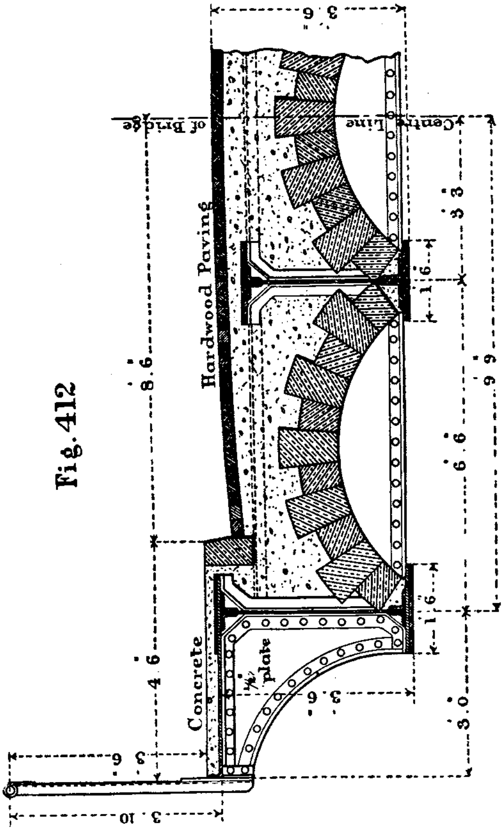
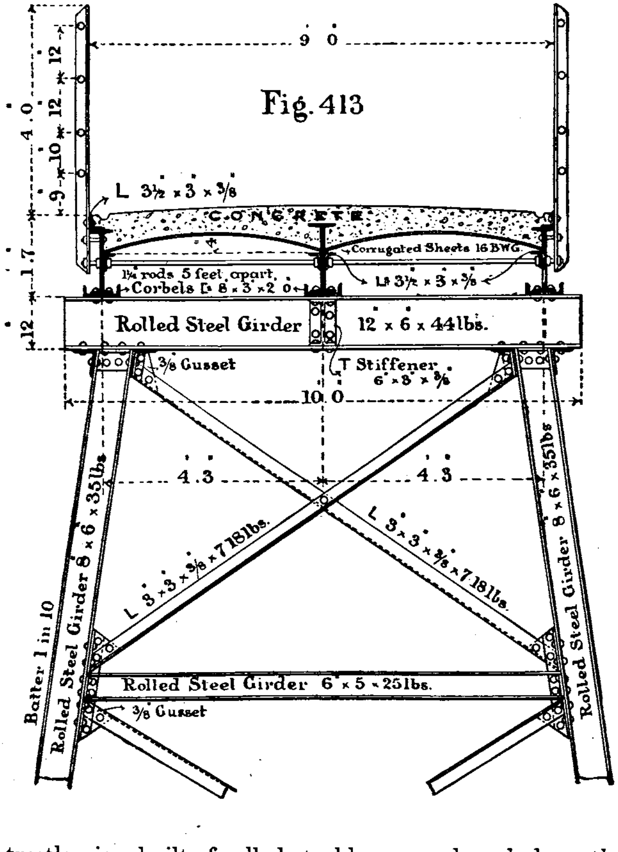


Fig. 412

of 5 feet to 7 feet 6 inches. The footpaths on each side are carried on cantilever brackets, the spaces between which coincide with those between the cross girders. This class of floor is very heavy, but it is exceedingly strong and solid.

Fig. 413 represents a cross section of a highway bridge to carry light traffic suitable to the requirements of a rural district. The bridge consists of 3 spans of 25 feet each carried



on trestle piers built of rolled steel beams and angle bars, their bases being supported on a bed of concrete.

The main girders are rolled steel beams of H section, and the structure is designed to carry a uniformly distributed load of 10 tons, or a concentrated load of 5 tons on each span. The floor is composed of a bed of concrete supported on cambered corrugated galvanized sheeting No. 16 B.W.G., the ends of which abut on angle bars rivetted to the webs of the main girders, as shown in the illustration.

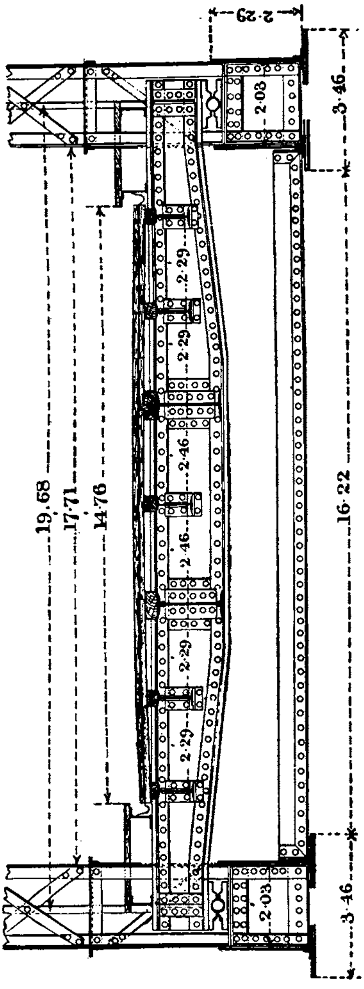


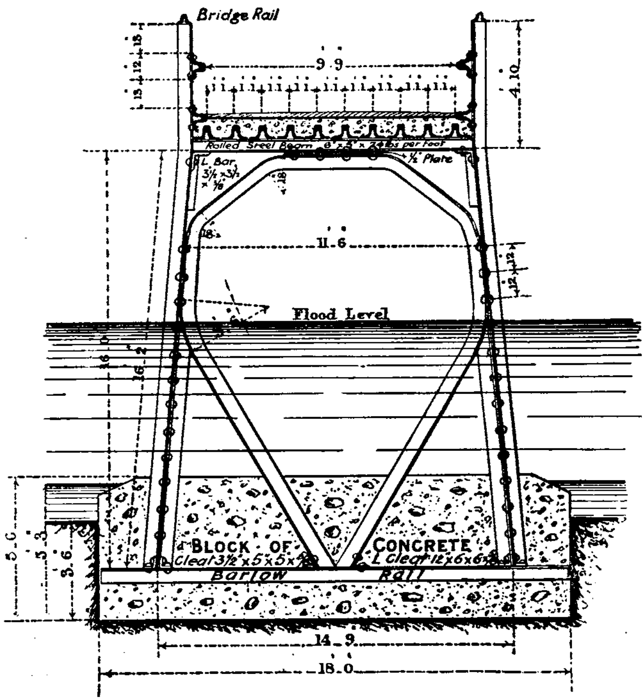
Fig. 414

Fig. 414 represents a cross section of a highway bridge built in Holland under the administration of the Dutch State Railways. The principal feature in this illustration is the method of supporting the cross girder on the main girders, in which it will be seen that the ends of the cross girders rest on saddles, forming hinge joints, thereby bringing the lines of action of the supporting forces into vertical planes coinciding with the centre lines of the main girders, thus avoiding eccentricity of stress.

Fig. 415 illustrates the cross section of a highway bridge and pier built of old Barlow rails, which for the last seven years has admirably served the requirements of the district, where light but constant traffic has been passing over it.

The span from centre to centre of piers is 20 feet, but knee braces extend outwards from each pier, abutting against a

Fig. 415



transverse bearer, thus reducing the central unsupported span to 14 feet.

This illustration is given, not as an example of modern construction, but as what may be done with old materials at hand.

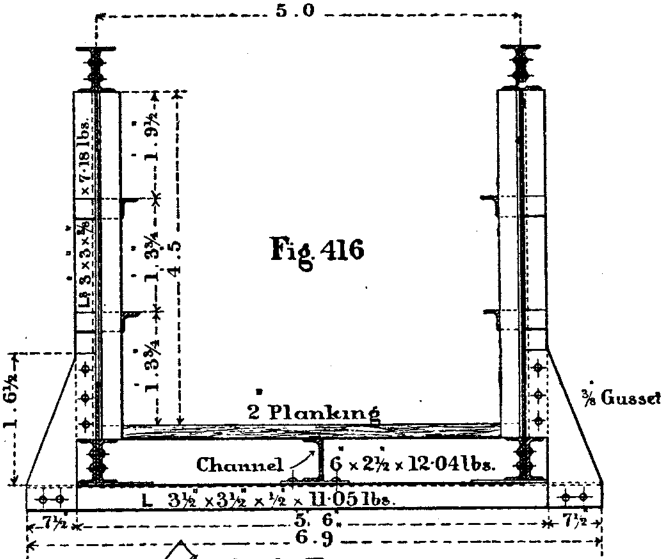


Fig. 416

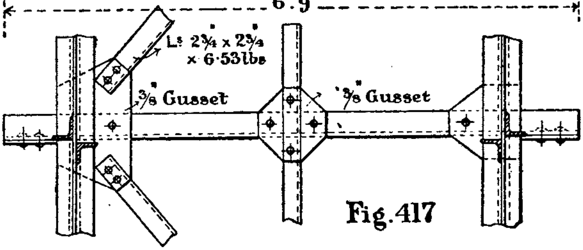


Fig. 417

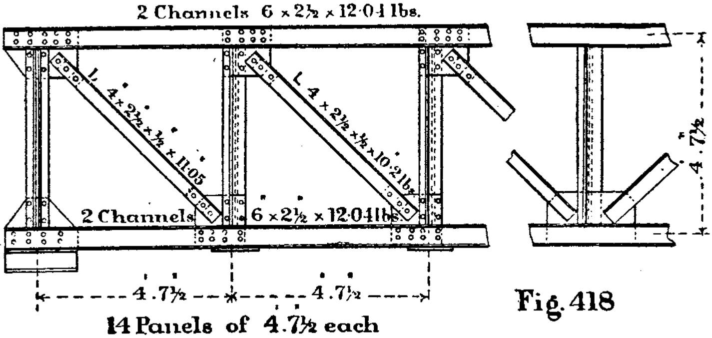


Fig. 418

Figs. 416, 417, and 418 illustrate the details of a simple but efficient type of foot bridge, in which the floor system is rigidly connected with the main girders by means of gusset plates.

Buckled Plates.

Buckled plates are usually made from 3′ 0″ to 4′ 0″ square and from $\frac{1}{4}$″ to $\frac{1}{2}$″ in thickness. The resistance of buckled plates rivetted down on all four edges is double the resistance of the same plates merely supported all around, and if the two opposite sides are only rivetted the resistance is reduced in the ratio of 8 to 5.

Buckled plates if used inverted, that is to say with the buckle in the form of a dish, are much stronger than if used in the form of a flattened dome.

The strength of buckled plates may be approximately deduced from the following formula given by "Winkler."

$$D = \frac{100kht - 0{\cdot}175gl^2}{6h + 15t} \times t.$$

D = total concentrated load in lbs.

g = uniform load in pounds per square foot.

h = depth of buckle in inches.

l = length of buckle in inches.

t = thickness in inches.

k = permissible stress in pounds per square inch.

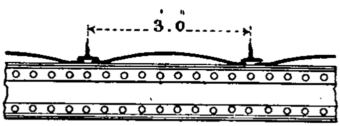


Fig 419

Fig. 419 illustrates a bridge floor formed of buckled plates, the transverse joint being supported by a ⊥ bar.

Corrugated Sheets.

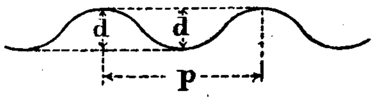


Fig. 420

l = unsupported length of sheet in inches.
t = thickness of sheet in inches.
b = width of sheet in inches.
d = depth of corrugations in inches.
W = Breaking weight distributed in tons.
w = Breaking weight distributed in pounds.

$$W = \frac{49{\cdot}95 \times t \times b \times d}{l} \text{ and } w = \frac{99{,}900 \times t \times b \times d}{l}.$$

Bridge Piers, End Bearings of Girders, Bedplates and Expansion Rollers, Handrails and Parapets.

Bridge Piers or intermediate supports may be divided into three categories, viz. :

1. Masonry Piers.
2. Cylinders or Hollow Columns of steel, wrought or cast iron.
3. Trestle or Braced Piers built of rolled steel or iron beams, angles, channels, etc., or of timber.

Masonry Piers.

Masonry Piers may in many instances be advantageously built in localities where suitable stone is readily available at a reasonable cost. In such circumstances it would probably be found desirable that the superstructure should also consist of arches in masonry, the difference in first cost being more than compensated by the more permanent and durable character of the structure.

In bridges crossing rivers or waterways, if the foundations have to be carried down to any considerable depth, the construction of the piers becomes a difficult and costly undertaking, frequently involving a very appreciable proportion of the total outlay.

Fig. 421

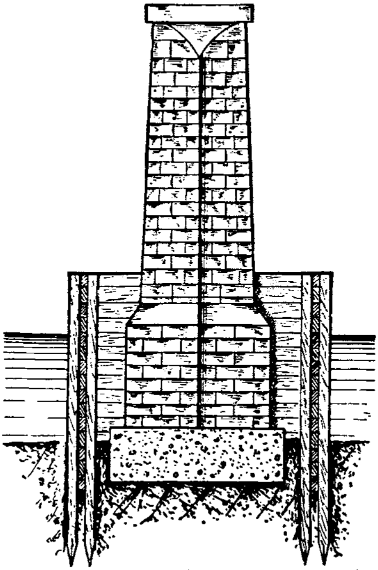
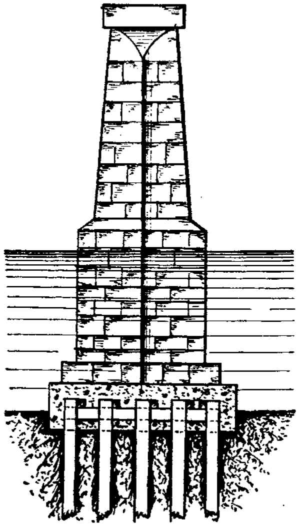


Fig 422



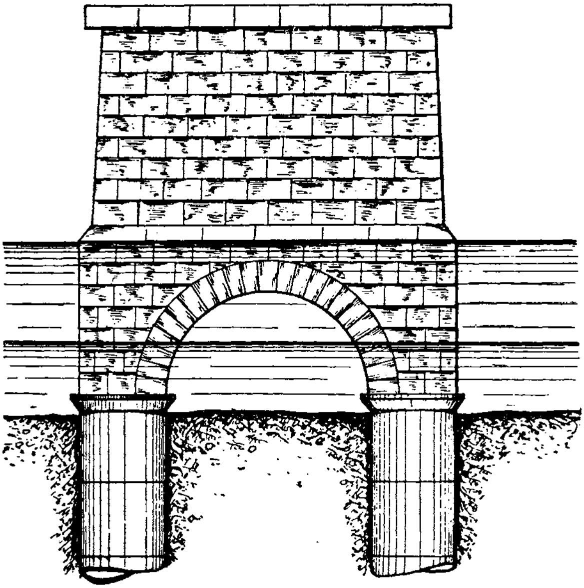
Where in such cases the piers are built of masonry, the foundations and work below water level may either consist of:

1. Masonry or concrete laid down within a coffer dam or caisson, that is to say a water-tight box or compartment

surrounding and including the entire space occupied by the pier, as shown in Fig. 421.

2. A bed of concrete carried on a grillage of timber beams securely bolted to the heads of tiers of piles firmly driven down into the river bed, as shown in Fig. 422.

Fig. 423



3. Cylinders sunk into the river bed by the aid of pneumatic pressure and filled with concrete, as shown in Fig. 423.

In districts where, as the result of working the minerals, there may be a liability to subsidence of the surface or in countries subject to seismic disturbances or in instances where Bridge Piers have to be built to any considerable height, a pier built of masonry stands at a considerable disadvantage as compared with a braced steel structure, and speaking generally under such conditions a masonry construction should not be adopted.

Cylindrical Piers or Hollow Columns of Steel or Cast Iron.

In this category are included, first, screw piles consisting of cast-iron tubes provided at their lower ends with screw blades, by revolving which the pile or tube is made to penetrate into the ground in precisely the same way as a wood screw is forced down by the application of a screwdriver; and, secondly, hollow cylinders open at the bottom and wholly or partially filled with masonry, brickwork, or concrete.

Screw Piles.

Piers built of screw piles have been largely used because of the facility with which they can be driven down, especially in

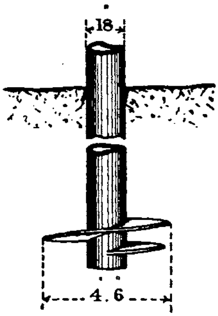


Fig 424

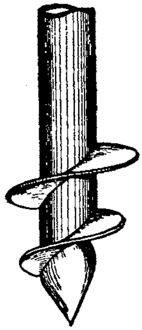


Fig. 425

river beds, where the whole of the work can be carried on above water level; but by reason of the limited dimensions of the columns or tubes ranging from 12 to 30 inches in diameter this kind of column is only adapted for bridges of limited spans.

In Figs. 424 and 425 are illustrated two types of screw piles; that in Fig. 425 represents a tapered screw.

Screw piles, if the screw blades are able to successfully resist the process of being driven down to a stable bed, afford an excellent and firm foundation for a pier by reason of the large base area exposed by the blades for the support of the columns.

Too much reliance should not, however, be placed upon the support afforded by the screw blades, which, being composed of cast iron or cast steel, are liable to be broken without displaying any sign of fracture, in which event the supporting power may be reduced to the base upon which the bottom of the pile rests and the friction of the ground against its sides.

Hollow Columns or Cylinders.

For bridges of any considerable span, where a rock or other stable foundation is not obtainable at a shallow depth, cylindrical piers are the most efficient foundations. The columns may be built of cast iron or wrought iron or steel plate, and may vary in dimensions from 5 feet to 15 feet in diameter, depending upon the nature of the foundation, the height and the total load to be borne by each column.

In piers of large diameter, where wrought iron or steel plates are used, the metal work only serves as a shell or casing to protect the masonry, and the tube need only be of sufficient strength to safely carry the weight with which it is loaded for sinking, the permanent weight of the superstructure being carried by the enveloped column of masonry, as shown in Fig. 426. In tubes of small diameter the interior is generally filled with concrete, but the superstructure is carried by the iron cylinder on suitable bedplates resting on flanged caps bolted to the heads of the columns. Cast-iron rings from 6 to 10 feet in diameter are generally cast in one piece, in lengths of 4 to 6 feet, each length being provided at its ends with internal flanges, which are machined, so that when

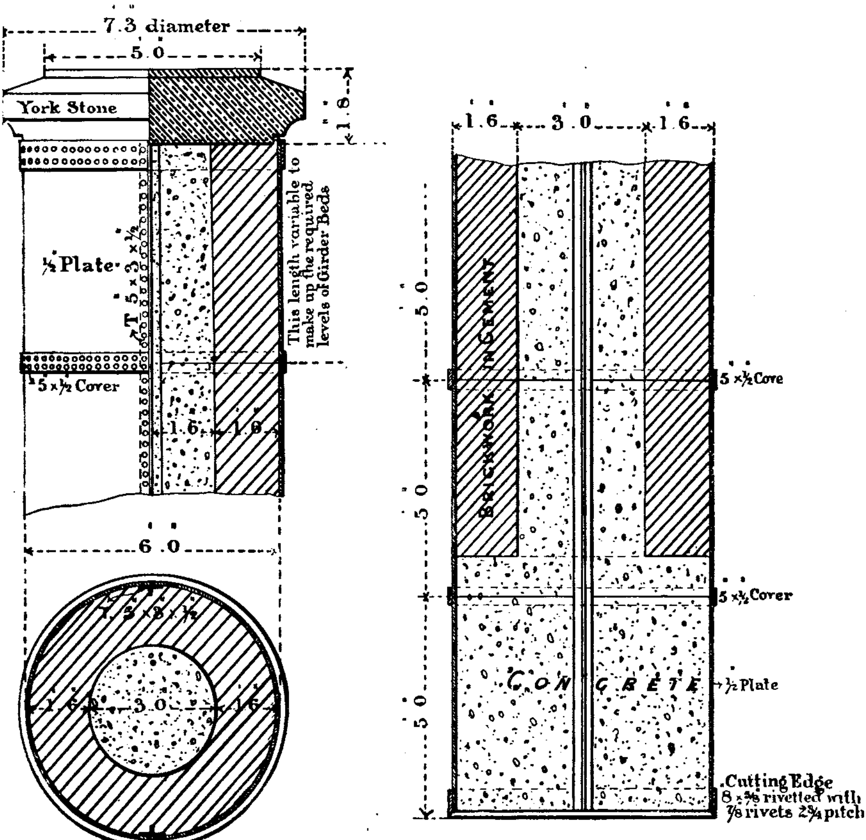


Fig. 426.

bolted together they form a water-tight joint. See Figs. 427 and 428.

The rings in the lower tier of the cylinders up to ground

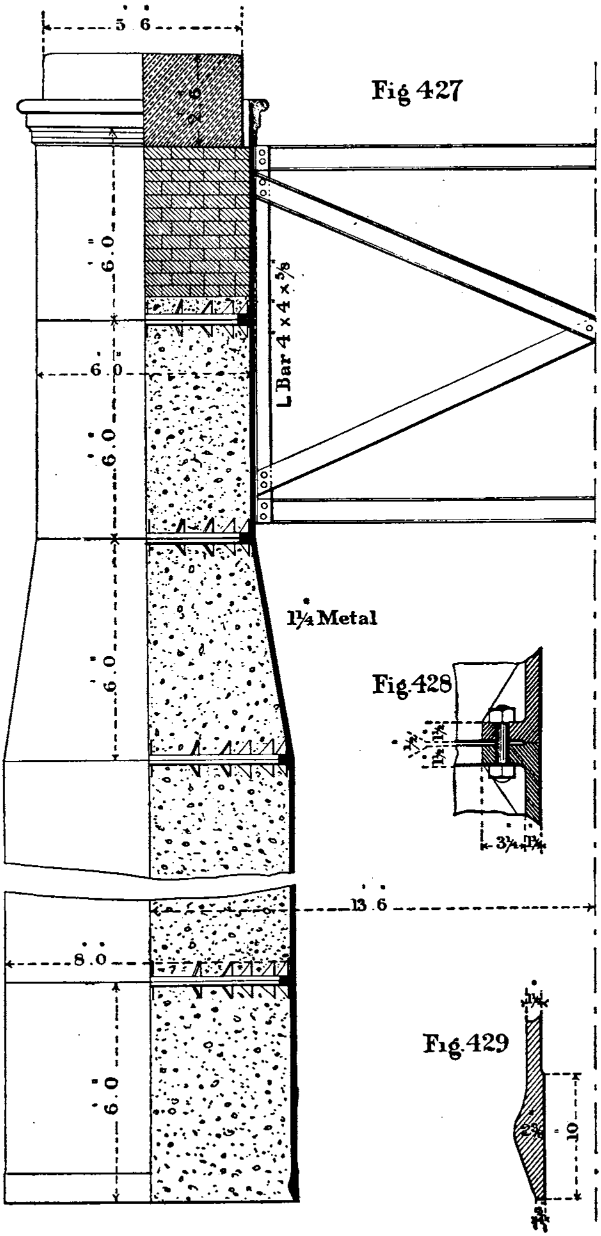


Fig 427

Fig. 428

Fig. 429

level, unless a rock foundation is available, are usually made of larger diameter than in the upper by the introduction of a conical ring, so as to distribute the weight over a greater base area. The bottom ring is provided with a cutting edge, as shown in Fig. 429.

The thickness of the metal varies from 1″ to $1\frac{1}{4}$″.

Sinking of the Cylinders.

The simplest method of sinking cylindrical columns is to load them with a sufficient weight at the top to cause them to descend as the ground is excavated from the inside by grab dredging until a clayey or nearly water-tight bottom is reached. The water given off can then be disposed of by pumps, and the remaining excavation carried out by manual labour under normal conditions until a solid foundation is obtained.

It frequently happens that a cylinder has to be sunk through loose gravelly soil containing boulders or other obstructions which cannot be removed by the grab dredger. In such cases recourse must be had to the pneumatic process. This process enables the sinking to be carried on under water by means of compressed air forced into the cylinder at a sufficiently high pressure to exclude the water from the interior. The cylinder at its upper end terminates in a chamber called the air lock, by means of which the pressure of the air is equalised on entering and in quitting the chamber.

The pressure of the air within the cylinder must necessarily depend upon the depth below the surface of the water at which the operations are carried on. The limit at which the process ceases to be applicable without danger to the workmen corresponds to a vertical head of about 90 to 100 feet, and even at this pressure the workmen must be very cautious in

Fig 430

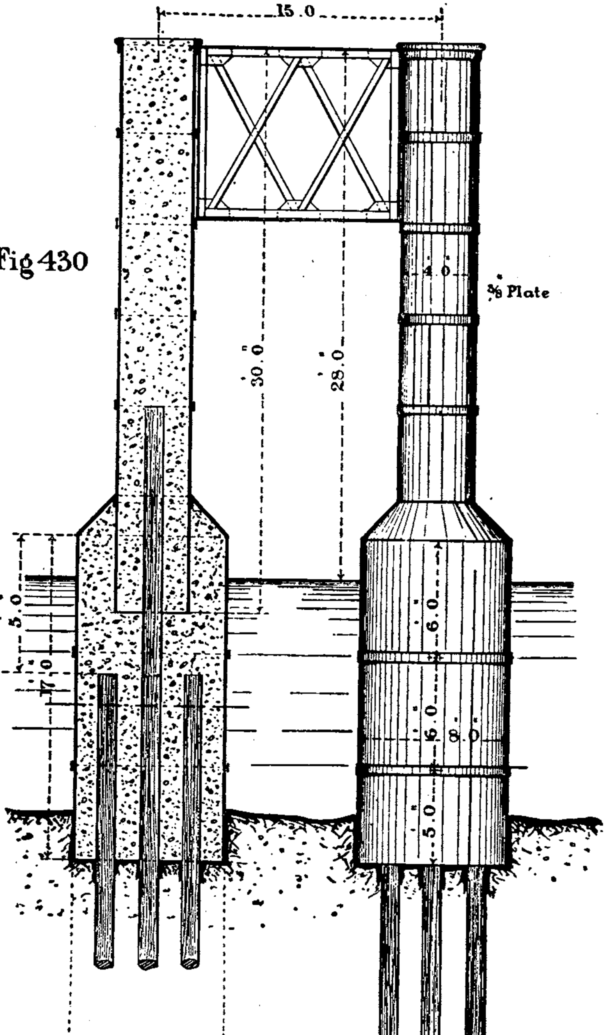
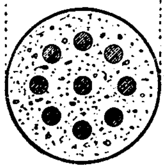


Fig 431



entering and leaving the artificial atmosphere and avoid the effects of too sudden a change, which are liable to produce fatal disorders. Men of sober habits are able to work under such pressure from 3 to 4 hours a day with the observance of the necessary precautions without much inconvenience.

Where foundations have to be carried down to greater depths the excavation inside the cylinder must be exclusively performed by means of grab dredging. A notable example of such method of sinking is that of the No. 6 pier of the Hawkesbury River Bridge in Australia, the foundation of which was carried down to a depth of 170 feet below H.W.M., the greatest depth of water being 77 feet and the range of the tide 7 feet.

A simple and efficient form of pier is to drive down to a firm foundation a group of timber piles and to encase the group with a cylinder built of cast iron, wrought iron, or steel plates, its base firmly resting on the bottom and the spaces between the piles and the walls of the cylinder being filled in with concrete. The piles are thus protected against decay and the attack of the "Teredo navalis" by the concrete packing in which they are enveloped.

An illustration of this type of pier is given in Figs. 430 and 431.

Trestle or Braced Piers.

A Timber Trestle Bridge is the simplest form of spanning an opening, and when properly designed and put together constitutes an efficient and strong structure.

In the United States and other countries where suitable timber is readily and cheaply obtainable trestle bridges are extensively used on account of the facility with which they can be put together and the saving of time and money as compared with the formation of embankments or the erection of more permanent structures.

Fig. 432

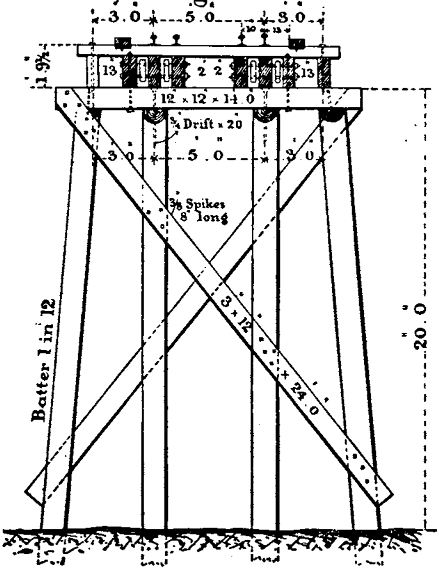


Fig. 433

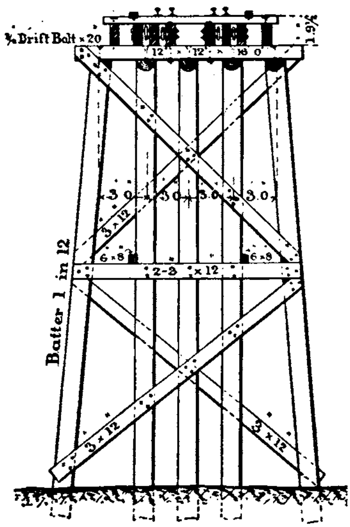


Fig. 434

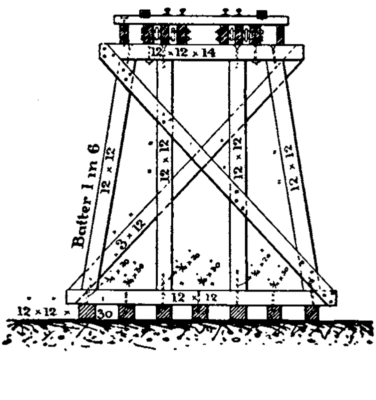
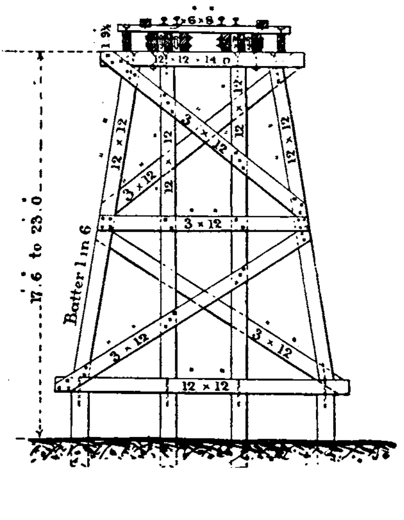
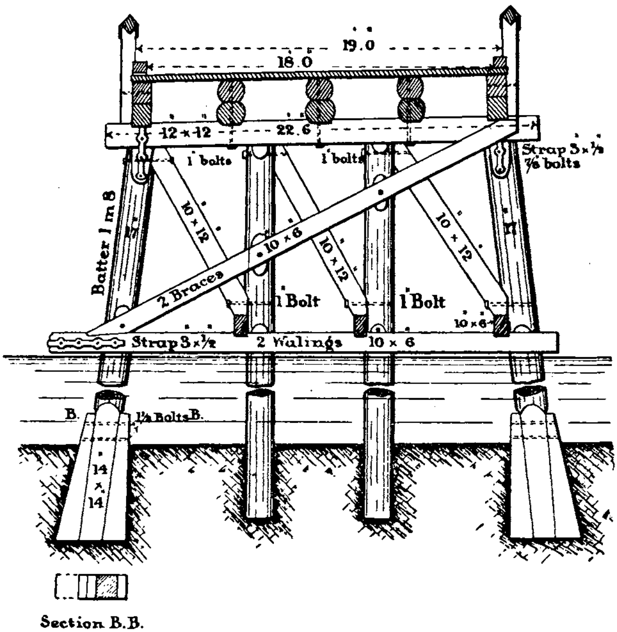


Fig. 435



In such cases skilled labour is frequently not available; the construction must therefore be of the simplest character, and all complicated joints involving mortice and tenon work should be avoided and butt joints adopted as far as possible.

Fig 436



The trestles may consist of Pile Bents or Framed Bents.

Fig. 432 illustrates a Pile Bent, in which the columns consist of four piles driven into the ground and firmly tied together by diagonal or sway bracing.

Fig. 433 illustrates a similar type of bent suitable for greater heights and braced together in two tiers or panels.

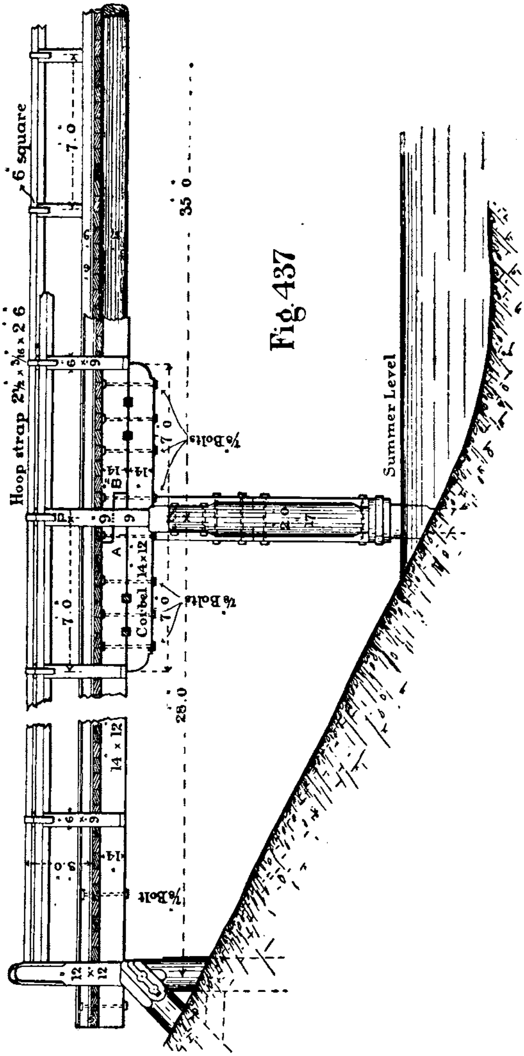

Hoop strap 2½ x 3/16 x 2 6
6 square
7.0
35 0
28.0
7/8 Bolts
Corbel 14 x 12
14 x 12
7/8 Bolt
12 x 12
Summer Level

Fig. 437

Fig. 434 illustrates a Framed Bent resting on cills.

Fig. 435 illustrates a Framed Bent resting on a pile foundation.

In bents up to 8 feet in height the sway bracing can be dispensed with.

Up to 20 feet in height the sway bracing may be in one panel, as indicated in Figs. 432 and 434.

When the height exceeds 20 feet the bent should be divided into panels of about 15 feet each and diagonally braced in each panel, as shown in Figs. 433 and 435.

The outer members are generally given a batter varying from $1\frac{1}{2}$ to 3 inches per foot, but generally 2 inches per foot, or 1 in 6.

Figs 436 and 437 illustrate in transverse and longitudinal views one of the trestle piers of the Balonne River Bridge, St. George, Queensland, for the drawings of which the author desires to acknowledge his indebtedness to Mr. A. B. Brady, M.I.C.E.

This bridge is built of Bloodwood (Eucalyptus corymbosa), and consists of twelve spans of 35 feet each with two end spans of 28 feet each.

Each pier consists of four piles spaced at 6′ 2″ centres at the level of the cap cill, the two outer piles having a batter of $\frac{1}{2}$ inch to the foot.

The outer piles, as will be seen on reference to Fig. 436, are secured to the rock bottom of the river by an arrangement of Lewis wedges of seasoned wood, which are made to bear evenly against the rock. The feet of the piles are squared for a length of 6′ 6″, and are inserted in position between the wedges and firmly driven home, the whole being securely bolted together above the surface of the rock.

Timber Piles and Pile-driving.

Timber piles for bridge construction in this country generally consist of pine, and occasionally, in foundations, of oak, elm, or beech. In foreign countries other woods of a more durable nature where available are employed.

The head of the pile, to protect it from splitting under the blows of the hammer when it is being driven, should be bound

Fig. 438.

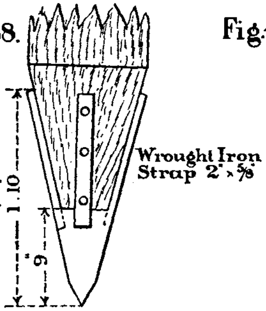
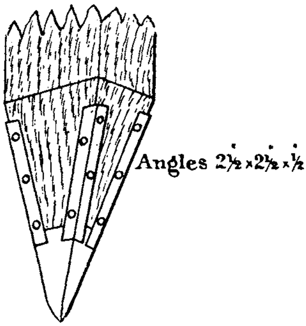


Fig. 439



by a band or ring of wrought iron about 3" in depth and ¾" thick. The foot should be provided with a shoe spiked to the pile by means of wrought-iron straps, as shown in Figs. 438 and 439.

The simplest form of driving piles is by means of a ram or monkey consisting of a block of cast iron, to which a free fall is given through any desired distance, the ram being guided in its fall in a vertical or inclined direction as occasion may require. The weight of the ram, which may range from 5 cwt. to 3 tons, is regulated by the depth and the resistance offered by the ground through which the pile has to be driven. The greater the weight of the ram the less fall required, and experience has shown that a heavy ram having a small fall is more effective than a light ram falling from a greater height, and that a series of blows continuously delivered are more effective than if intermittently delivered.

There are, however, instances on record where the piles could be driven no further under the application of three or four blows in succession, but after a few days' rest their driving could be continued with renewed ease. It is therefore of importance that the pile after being driven home should rest for some little time before the final blow is applied to test its resistance.

Supporting Capacity of Piles.

Let L = safe load on the pile in cwts.
l = do. do. in pounds.
H = height through which ram has fallen in feet.
h = do. do. do. in inches.
D = penetration or set of pile by last blow in feet.
d = do. do. do. in inches.
W = weight of ram in cwts.
w = do. in pounds.
p = weight of pile in pounds.

The energy accumulated in the ram on striking the pile = WH.

Several formulae are in use for determining the safe load, from which the following three have been selected as being the simplest of application:

1. Major Saunders' rule:

$$l=\frac{wH}{8D} \text{ or approximately } L=\frac{Wh}{8d}.$$

2. Wellington's rule:

$$l=\frac{2wH}{d+1}.$$

3. Weisbach's rule:

$$l=\frac{wH}{d(w+p)}.$$

Applying these formulae to two cases of varying height of fall, weight of ram and set of pile, the following tabulated statement exhibits the results :

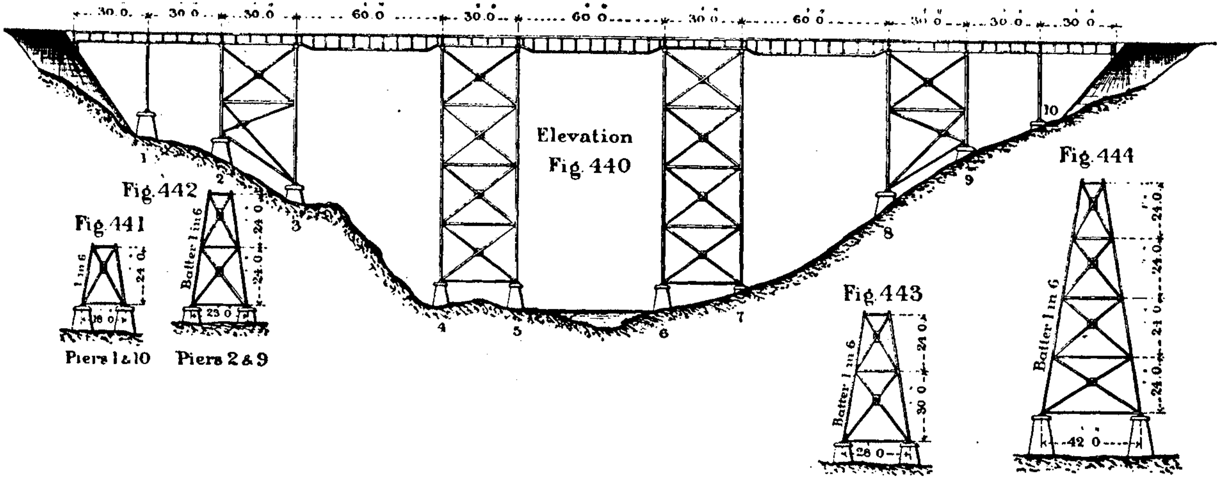
Formula.	Case 1. Height of fall, 20 ft. Weight of ram, 10 cwt. Set of pile, ½ inch.	Case 2. Height of fall, 10 ft. Weight of ram, 25 cwt. Set of pile, ¼ inch.
	Safe load in lbs.	Safe load in lbs.
Saunder's Rule, - -	65,120	166,660
Wellington's Rule, - -	29,867	44,800
Weisbach's Rule, - -	21,628	78,400
Mean, - - -	38,872	96,620

By this statement it will be seen that there is a great disparity between the results, but having regard to all the circumstances Wellington's rule appears to be the most rational. For practical purposes the maximum safe load for piles ranging from 10″ to 15″ square well driven home may be taken at 500 pounds, or say 4½ cwt. per square inch of sectional area.

Trestle Piers of Iron or Steel.

This type of structure has been extensively used in America and other foreign countries for bridging over deep valleys, a notable example of which is afforded by the Pecos Viaduct in the State of Texas on the Southern Pacific Railway, in which the rail level over the central piers is 321 feet above low water.

Figs. 440 to 444 illustrate a typical example of a steel



Elevation
Fig. 440

Fig. 441
Piers 1 & 10

Fig. 442
Piers 2 & 9

Fig. 443
Piers 3 & 8

Fig. 444
Piers 4, 5, 6, & 7

trestle railway bridge designed according to modern American practice. The structure, as will be seen, consists of a number of plate girders of 30 and 60 feet spans alternately, the 30 feet spans being supported at their ends on a pair of bents or trestles firmly braced together longitudinally and transversely, and the 60 feet spans extending over the intermediate spaces between the towers. The length of the intermediate span is limited to 60 feet, because this is the maximum length and weight of girder that can be conveniently dealt with by the overhanging traveller, by means of which the girders during erection are moved into position.

The tower spans are 4 feet deep and the intermediate spans 6 feet, but at the ends by inclining the lower chords the depth is reduced to 4 feet so as to match the depth of the tower spans.

The towers are formed of four columns, two to each bent or trestle, the plane of each bent being vertical. To guard against any force tending to overturn them in a longitudinal direction, which can only consist of impact caused by the sudden application of a brake to a moving train, the bents are firmly braced together longitudinally. Transversely the force tending to overturn the piers is that of wind pressure, to resist which the columns are given a sufficient outward batter (varying from $1\frac{1}{2}$ to 3 inches to the foot) to prevent tension at the base of the windward column under the most unfavourable conditions.

Figs. 445 and 446 illustrate in detail the construction of the piers.

The length of the spans is determined by the height of the piers, and from an economical point of view the best results are obtained when the cost of the piers and their foundations is equal to that of the superstructure, but generally speaking the length of the intermediate spans should be double that of the tower spans.

Fig. 445

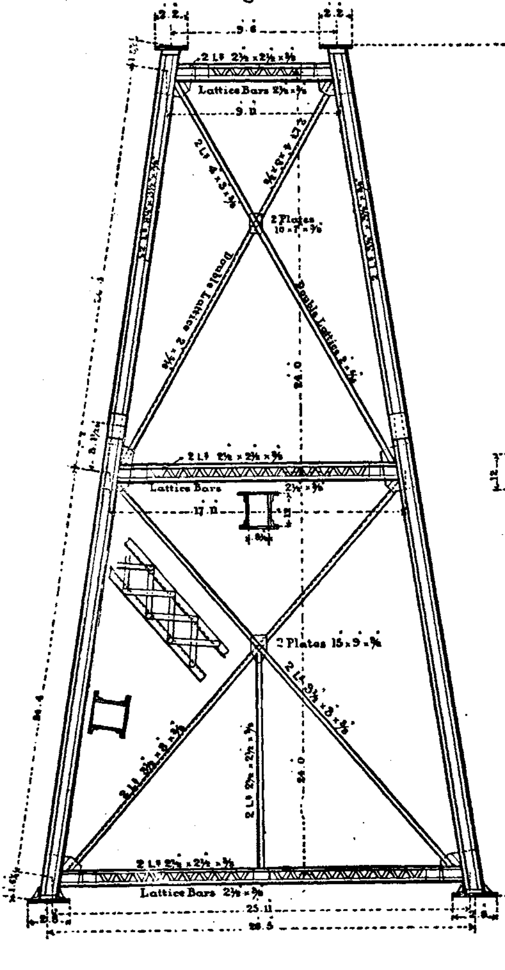
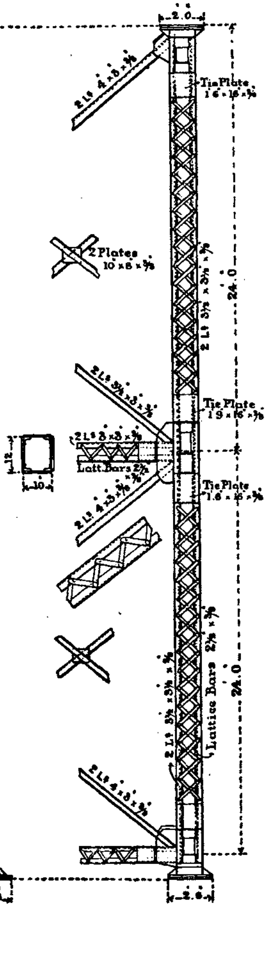


Fig. 446



The columns are built of rolled steel beams of various shapes, some of which are shown in the following illustrations contained in Figs. 447 to 450.

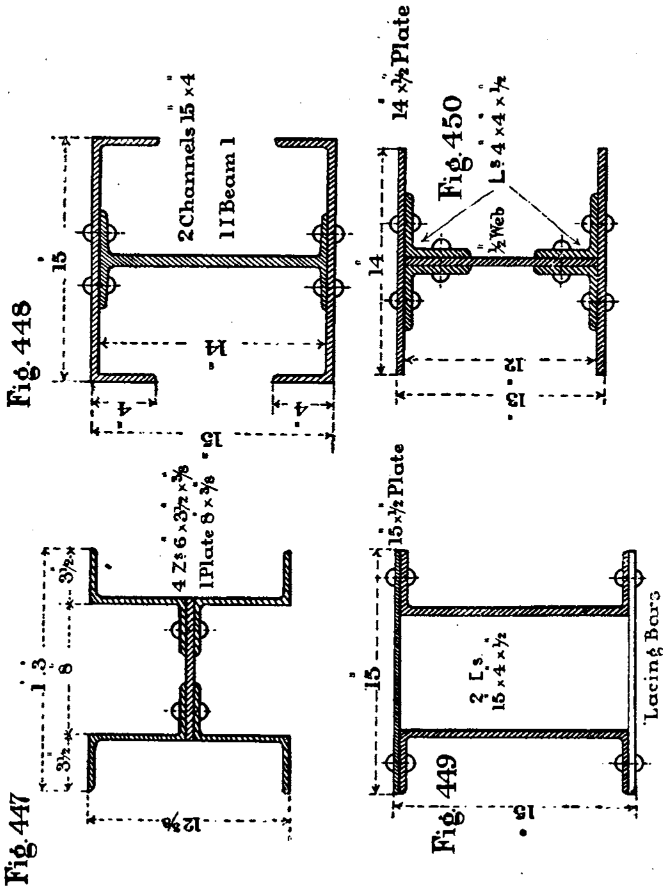


Fig. 447 Fig. 448 Fig. 449 Fig. 450

Figs. 451 to 455 illustrate one of the trestle piers of the Lossnitz Bridge on the Eppendorf-Hertzdorf State Railway in Saxony. The bridge is composed of two continuous plate girders spanning three equal openings of 46 feet each, the girders being supported at the two intermediate points on piers which by a hinge arrangement at the top and bottom are free to move longitudinally, and thus allow for expansion and contraction of the continuous girders, the ends of which are carried on roller bearings in the abutments.

A full and interesting description of this bridge appeared in the *Engineer*, July 22nd, 1898.

Several bridges with rocking or pendulum piers of this type have been built on the Norwegian State Railways.

Fig. 456 illustrates one of the piers of the viaduct across the Manhattan Valley in New York. The bent, which is one of a pair forming the tower, consists of three columns arranged in two rows 33′ 9″ apart centre to centre longitudinally and 24′ 3″ apart from centre to centre transversely. The columns, as will be seen by the illustration, are well braced together in all directions, and the details are in every respect well designed. The author is indebted to Mr. W. Barclay Parsons, the engineer-in-chief of this work, for a number of drawings and descriptions of this important work, from which Fig. 456 has been selected for the purpose of illustrating this article.

End Bearings, Bedplates, and Expansion Rollers.

In the end bearings of bridges provision should be made for the horizontal movement of the girders in a longitudinal direction caused by expansion and contraction produced by variations of temperature usually estimated to correspond to 150 degrees Fahrenheit, and also for vertical motion at the ends of the girders due to deflection.

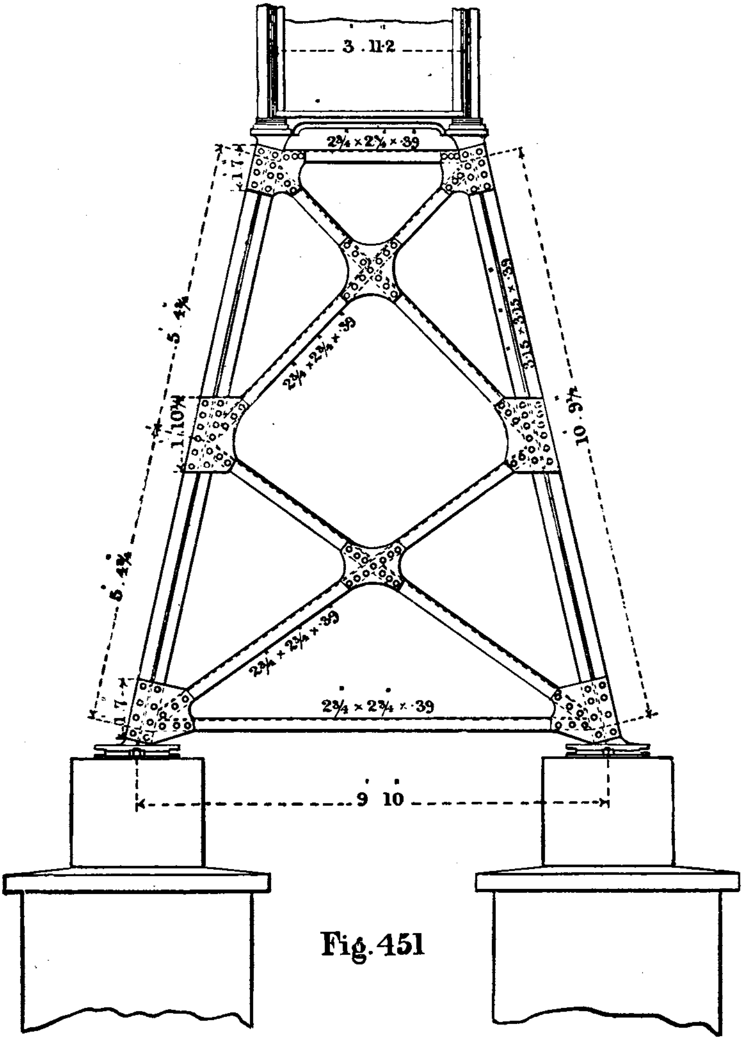

3 . 11·2
2¾ × 2¾ × ·39
1 . 7
5 . 4⅝
1 . 10¾
5 . 4⅝
1 . 7
2¾ × 2¾ × ·39
2¾ × 2¾ × ·39
3·15 × 3·15 × ·39
10 . 9¼
2¾ × 2¾ × ·39
9 10

Fig. 451

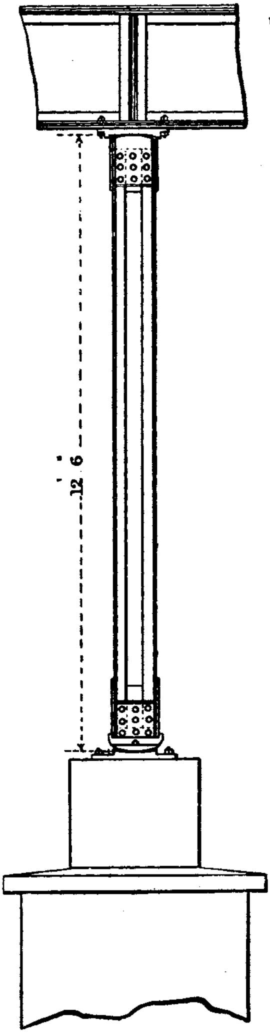


Fig. 452

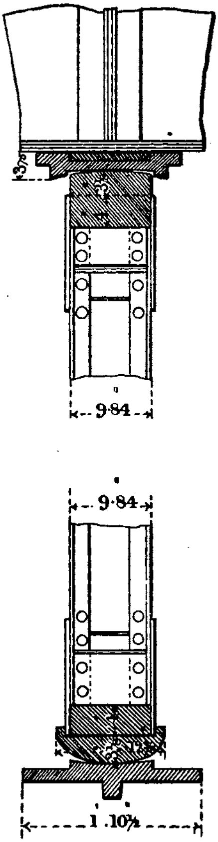


Fig. 454

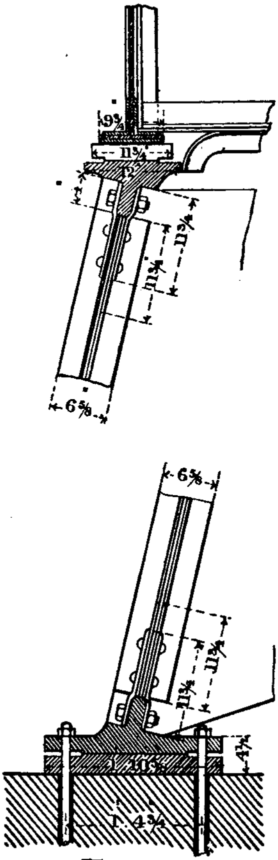


Fig. 455

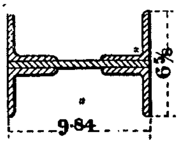


Fig. 453

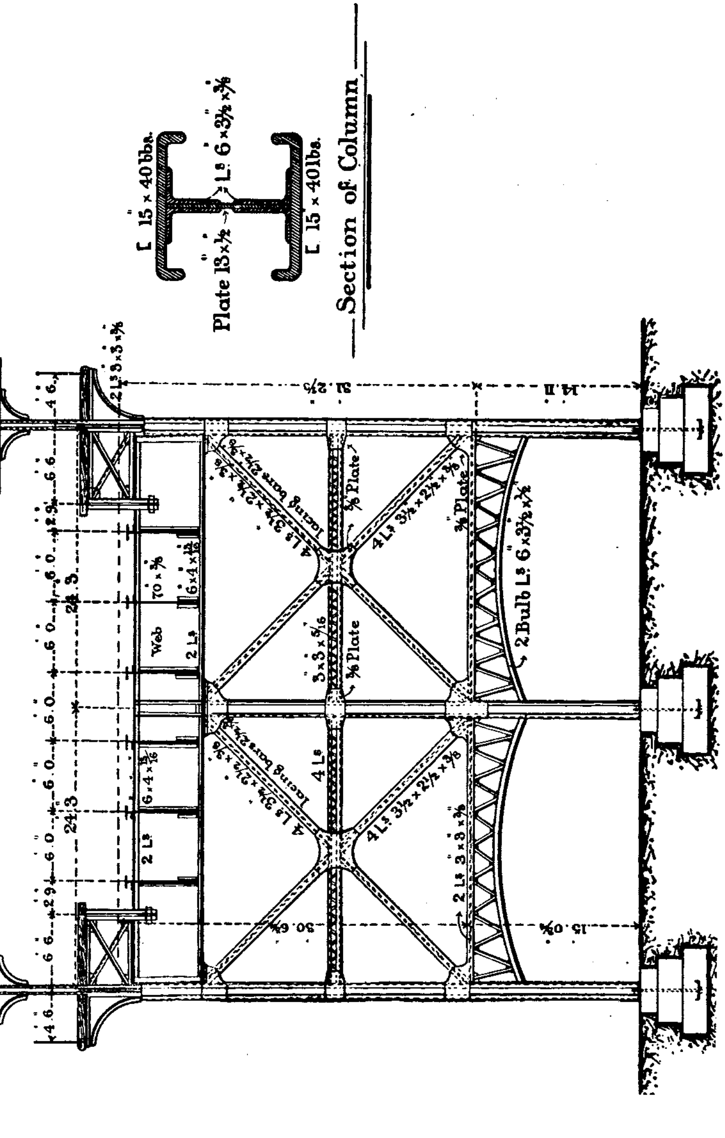


Fig. 456

In all bridges over 100 feet span the ends of the girders should be supported on bolsters or shoes having hinged or

Fig. 457

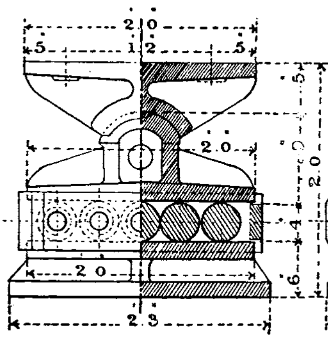


Fig. 458

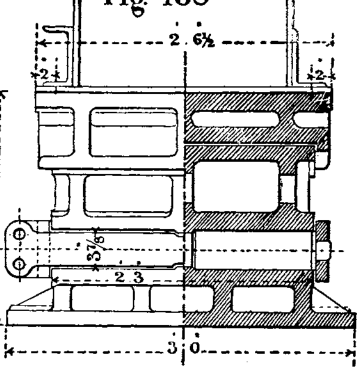


Fig. 459

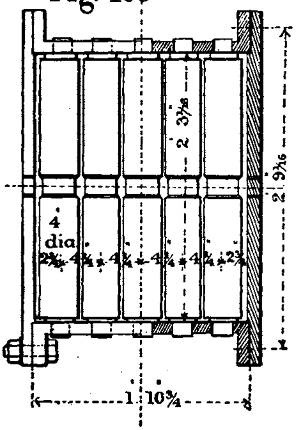
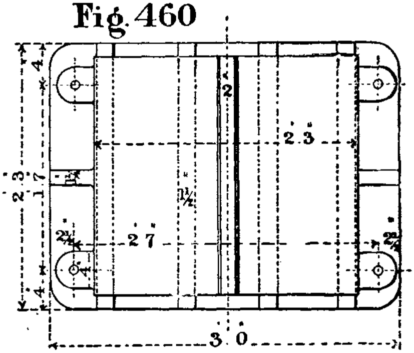


Fig. 460



knuckle bearings on a steel casting, so as to distribute the pressure equally over the bearing plates, which at one end of the girder are firmly bolted down to the supports. At the other end the bolsters are carried on cylindrical or segmental steel rollers revolving between planed surfaces. By means of this arrangement the bridge is free to move longitudinally and vertically.

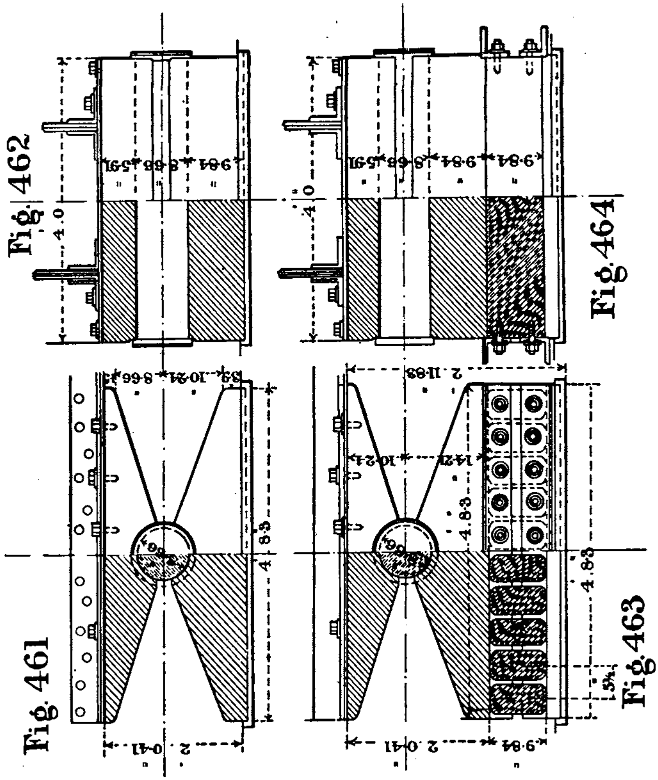


Fig. 461. Fig. 462. Fig. 463. Fig. 464.

For spans of 100 feet and under the girders at one end are bolted down to the supports, and at the other end they are free to move on planed surfaces, the under surface of the bolster attached to the end of the girder and the upper surface of the bedplate being machined so as to admit of a sliding longitudinal movement. To provide for vertical movement

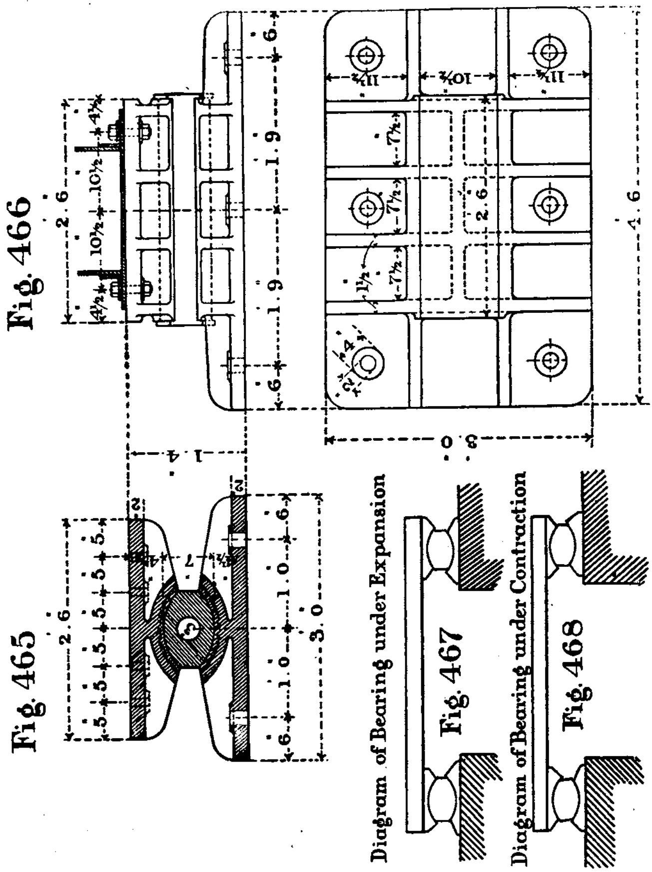


Fig. 465

Fig. 466

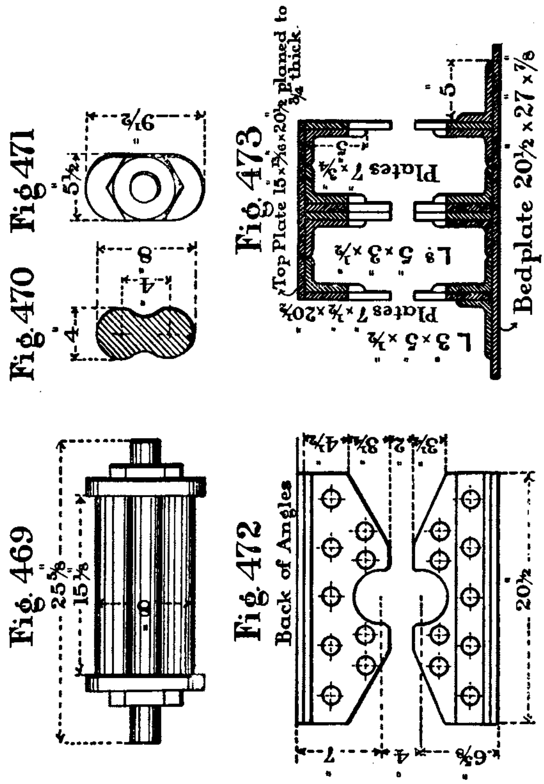
Fig. 467 Diagram of Bearing under Expansion

Fig. 468 Diagram of Bearing under Contraction

due to deflection a sheet of lead is introduced as a cushion between the bedplate and the support.

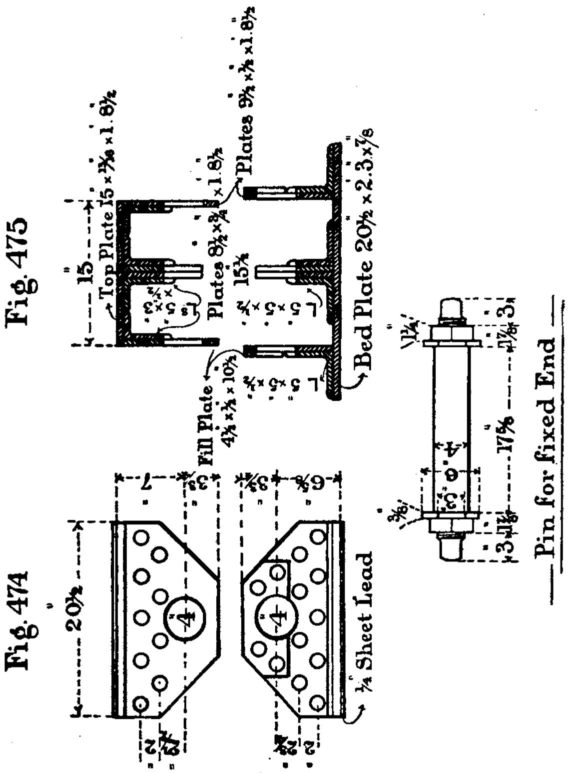
The bedplates should be so proportioned that the greatest pressure on the masonry shall not exceed 250 lbs. per square inch.

The rollers, which should be made of machined steel, shall be of such dimensions that the maximum pressure in pounds



per lineal inch on each roller shall not exceed $300d$, d being the diameter of the roller in inches. No roller should be less than 3 inches in diameter, with an increment of 1 inch for each additional 100 feet span in excess of 100 feet.

Figs. 457 to 460 illustrate the roller bearings under the expansion end of a railway bridge of 100 feet span on the Burma Railways designed by Messrs. Rendel & Robertson, the consulting engineers.



Figs. 461 and 462 show the knuckle bearing under fixed end, and Figs. 463 and 464 the roller bearings under the expansion end of a heavy girder. In this case the rollers are segmental.

Fig. 476

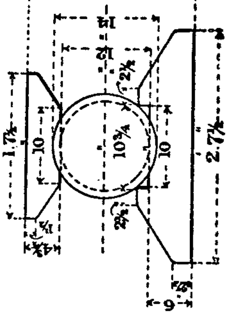


Fig. 477

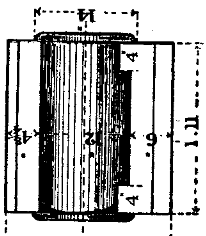


Fig. 478

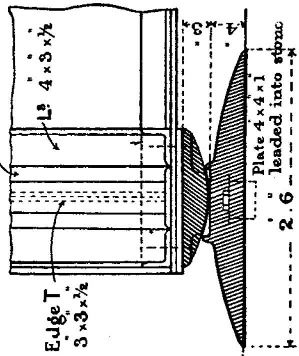


Fig. 479

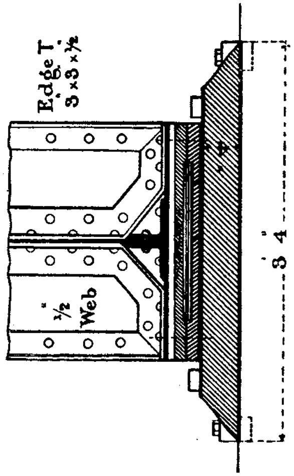
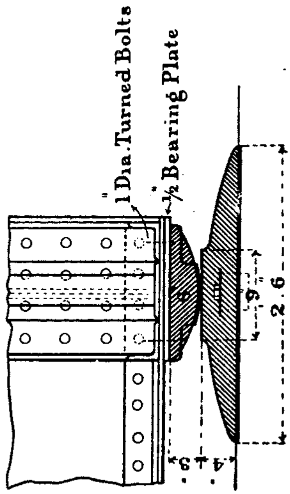


Fig. 480



Figs. 465, 466, 467, and 468 illustrate the "Horseley Bearing," "Westwood's Patent."

The special advantages of this bearing are that the bearing surfaces are very large with a corresponding small pressure per square inch thereon, and the machined surfaces of the bearings being in close contact dust cannot possibly get between. The bearings are made entirely of cast iron, and are therefore much less expensive than appliances used for the same purpose, which are usually composed of steel rollers, wrought iron or steel frames, and castings combined.

Figs. 469 to 475 illustrate an arrangement of bearings under the ends of girders that is used on the Northern Pacific Railroad of the United States of America. Figs. 469 to 473 show the details of a rocker stilt under the expansion end of the girder, and Figs. 474, 475 the knuckle bearing under the fixed end.

Figs. 476 and 477 illustrate a new type of expansion bearing that is used on the Dutch State Railways for which many advantages are claimed, and especially that it combines the action of the knuckle and roller bearing.

Figs. 478 to 480 show an arrangement of rocker bearing under the fixed and expansion ends suitable for a girder up to 100 feet span. The rocker at the expansion end of the girder is free to move longitudinally on a planed bedplate, as shown in Fig. 480.

Parapets.

For railway bridges an open parapet is usually employed composed of two longitudinal rails, which may consist of angle bars or gas tubing supported at suitable intervals by vertical stanchions of steel, wrought iron, or cast iron.

In highway bridges it is customary to make the parapets of lattice work or of vertical bars spaced at close intervals, so as to afford the necessary protection to pedestrian and vehicular traffic passing over the bridge.

Figs. 481 and 482 illustrate a simple and useful type of parapet, consisting of lattice work surmounted by a handrail in the form of a hollow cast-iron trough or of wood as may be

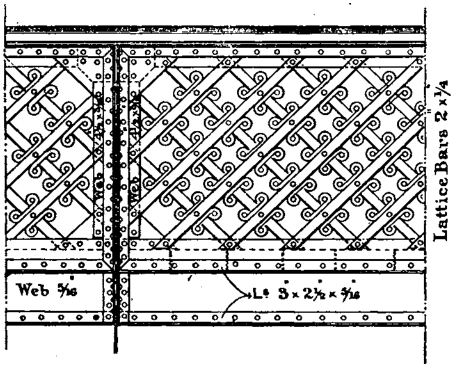


Fig. 481

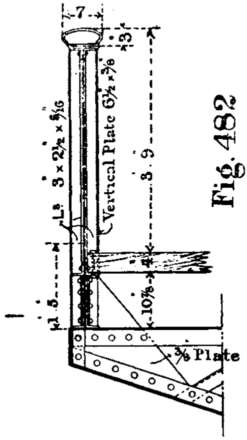


Fig. 482

desired. The lattice bars at their intersections are provided with ornamental fitments of a geometrical pattern made of malleable cast iron.

Fig. 483 illustrates a very simple and efficient type of parapet which is used largely on highway bridges in France. It is simple in construction and has a light and elegant appearance.

Fig. 484 shows another type of parapet, composed of vertical twisted bars spaced 6 inches apart centre to centre secured to two horizontal angle bars and surmounted by a cast-iron handrail.

Figs. 485 and 486 illustrate a type of parapet that may be almost regarded as a standard type for American highway bridges The lower portion is composed of lattice bars

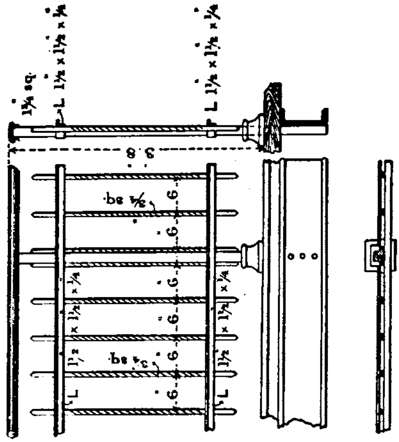


Fig. 484

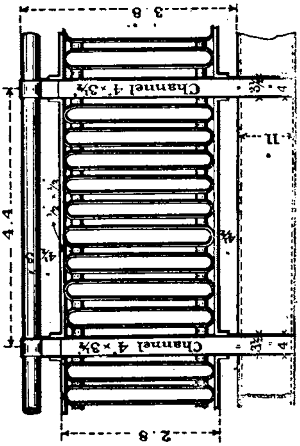


Fig. 483

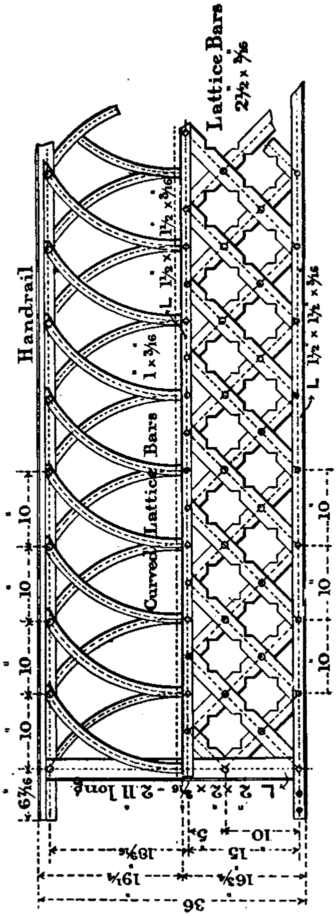
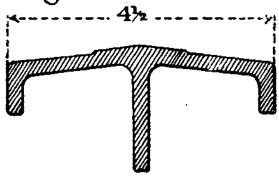


Fig. 485

indented in such a manner as to present a series of openings of a stellular form. The upper portion is built of curved lattice bars intersecting one another and riveted to the web of a flanged T bar, a section of which is given in Fig. 486.

Fig 486.



Section of Handrail

For bridges built in large towns parapets of a more elaborate pattern are frequently employed. These parapets are generally made of cast iron, and the design and colouring may be made as ornate as taste may dictate.

CHAPTER XVIII.

Examples of Bridge Designing.

EXAMPLE No. I.

Timber Trestle Bridge to carry a Single Line of Railway in spans of 15 feet each.

This is a type of bridge that has been extensively used in American railway construction, and where timber is available it is a very economical and efficient kind of structure.

The dead load on one span is assumed to be 6 tons = ·4 ton per foot run.

The live load on one span is assumed to be 42 tons = 2·8 tons per foot run.

These loads are assumed to be equally distributed over two tiers of beams, each tier supporting ·2 ton + 1·4 tons = 1·6 tons per foot run, or a total distributed load of 1·6 tons × 15 = 24 tons.

If W = distributed load to be carried,

l = length of span,

b = breadth of beam,

d = depth of beam,

f = modulus of rupture,

M = bending moment,

M_0 = moment of resistance,

I = moment of inertia, which, for a rectangular beam, $= \frac{bd^3}{12}$,

y = distance of neutral axis from extreme fibre, which for a rectangular section $= \frac{d}{2}$.

Then for a uniformly distributed load

$$M=\frac{Wl}{8}=\frac{24\times 15\times 12}{8}=540 \text{ inch tons.}$$

The values of b and d are so far unknown, so that the values of I and y cannot be determined.

The moment of resistance of a beam $=M_0=\frac{fI}{y}$, and by equating the bending moment with the moment of resistance the dimensions may be obtained.

For a distributed load

$$\frac{Wl}{8}=\frac{fI}{y}=\frac{Wl}{8}=f\frac{bd^2}{6}.$$

Let $b=7$ inches and let each tier consist of four beams, so that $b=7\times 4=28$,

$f=$ for pitch pine $=7626$ lbs. $=3\cdot 4$ tons, and let

$F=$ factor of safety $=6$.

Then by transposing the expression $\frac{Wl}{8}=f\frac{bd^2}{6}$ and substituting, it is found that

$$bd^2=\frac{6MF}{f}=bd^2=\frac{6\times 540\times 6}{3\cdot 4},$$

whence $\qquad 28\times d^2=5728$ and $d^2=\frac{5728}{28}$;

$$\therefore \quad d=\sqrt{204}=14\cdot 28, \text{ say } 14'',$$

so that the dimensions of the beams must be not less than $14''\times 7''$.

Each beam should be the length of two spans, or 30 feet each, with alternating joints, so that over each point of support two beams may always be continuous.

It is well, as a preventive against fire and a protection against the weather, to cover the upper surface of the main girders from end to end with a thin sheet of galvanised iron or tin, thus

or better still, to plank over the cross beams and cover them with a thin layer of concrete.

It is next necessary to calculate the length of grip at end bearing on pier.

Let g = required length of grip in inches,
S = safe crushing strength across the grain,
d = depth of beam in inches,
l = length of beam in inches,
R = safe modulus of rupture $= \frac{f}{F}$.

Then
$$g = \frac{2Rd^2}{3Sl}.$$

The safe modulus of rupture for pitch pine, allowing a factor of safety of 6, $= 7626 \div 6 = 1271$ lbs. per sq. inch, and the safe crushing strength across the grain per square inch, allowing a factor of safety of 4, $= 800 \div 4 = 200$.

Then
$$g = \frac{2 \times 1271 \times 14^2}{3 \times 200 \times 15 \times 12} = 4{\cdot}6 \text{ inches} = 4\tfrac{5}{8}'',$$

so that the width of the cap must not be less than $4\frac{5}{8}'' \times 2 = 9\frac{1}{8}''$.

Columns.—Each bent or pier is to consist of four columns, each of which will support $\frac{1}{4}$ of the total load on the span $= 48 \div 4 = 12$ tons. The width of the cap, it has been shown, must not be less than $9\frac{1}{8}$ inches, let it be 10 inches, so that the side of the column in one direction must not be less than the width of the cap. Let the dimension of the other side be assumed at 7 inches, and let the length of column be 15 feet.

It is therefore required to determine whether a column of pitch pine 15′ 0″ long 10″ × 7″ will safely support a load of 12 tons.

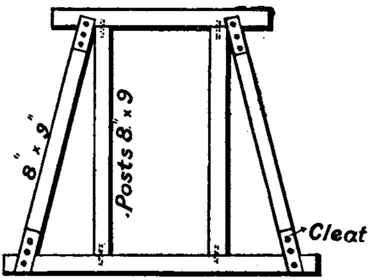
Let BW = breaking or crippling weight in lbs.,
a = area of cross section = 7 × 10 = 70″,
B = a constant representing the crushing strain of pitch pine in lbs. per sq. inch = 6790,
l = length in feet,
t = least side or thickness.

Then

$$\text{BW} = \frac{\text{B} \times a}{1{\cdot}1 + \dfrac{l^2}{2{\cdot}9 \times t^2}}, \text{ and substituting, BW} = \frac{6790 \times 70}{1{\cdot}1 + \dfrac{15^2}{2{\cdot}9 \times 7^2}} = 79{\cdot}2 \text{ Tons.}$$

Allowing a factor of safety of 5, the safe load will be 79 ÷ 5 = 15·8 tons, which is in excess of the actual maximum load, so the dimensions assumed are sufficient.

FIG. 2a



Figs. 1 and 2, Plate 1, show in elevation and section one type of construction in which the ends of the beams rest directly on the caps. In this case the bent or trestle is framed, and its base may be supported on a block of masonry, vertical

Plate 1.

Fig. 1.

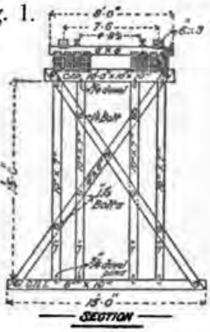


Fig. 2.

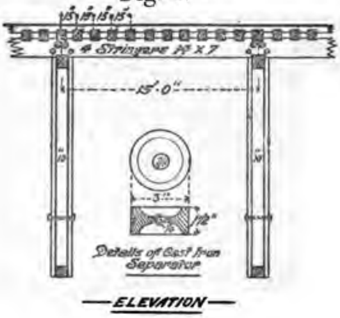


Fig. 3.

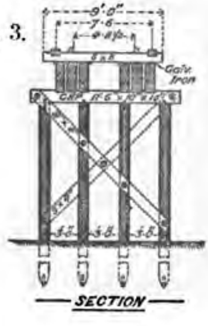
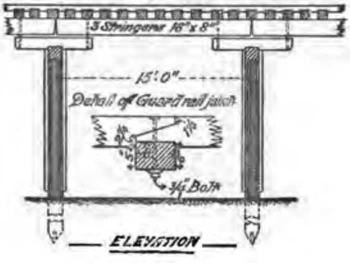


Fig. 4.



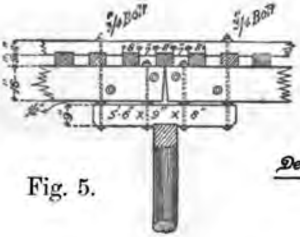


Fig. 5.

Details of Corbel.

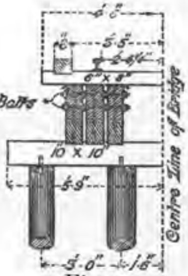
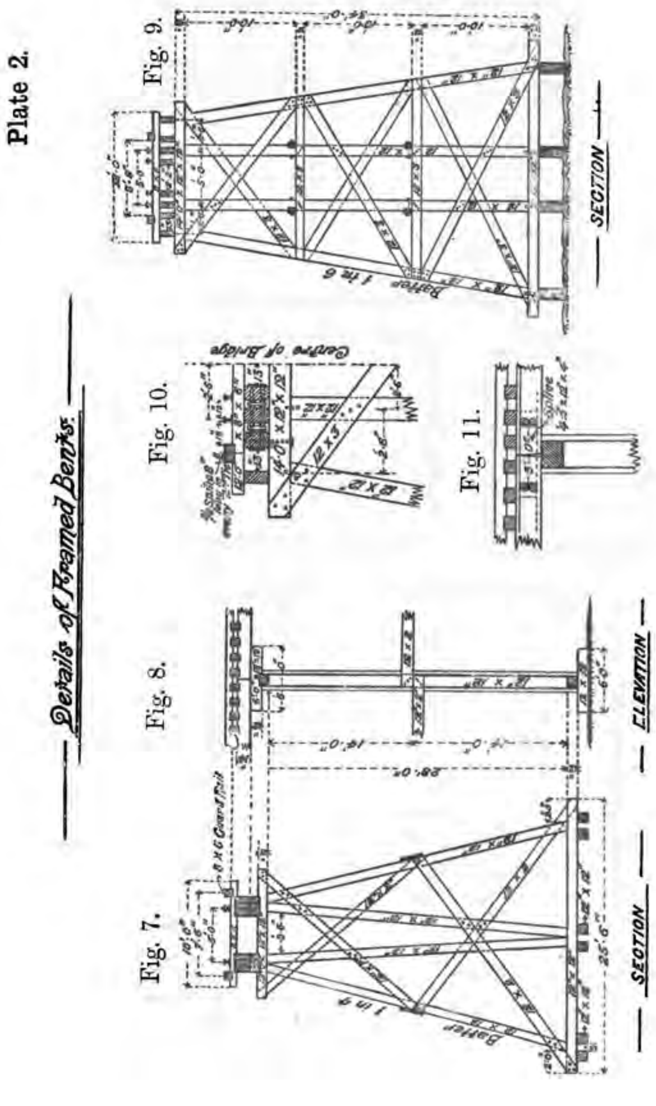


Fig. 6.

Plate 2.

Details of Framed Bents.



piles, or for temporary purposes on longitudinal timbers laid on the ground. An alternative form of trestle that is extensively used is that shown in Fig. 2 (*a*), but the construction shown in Fig. 1 is much simpler, inasmuch as the four vertical legs can at once be cut to the same lengths with ends squared and thus avoid the labour of fitting the outside slanting legs in Fig. 2 (*a*).

In Figs. 3 and 4, Plate 1, is shown a structure supported on piles and the beams resting on corbels. A corbel has several advantages: it unites together the two abutting beams, and to some extent it imparts to them the principle of continuity; it also has the effect of shortening the span, and in many instances it affords the only means of giving the girders a sufficient bearing on the supports.

Amongst the disadvantages advanced against the corbel are the increase in the cost of labour and iron, in bolts, etc., and increase of joints and of surfaces in contact tending to increase the liability to decay.

The structure shown in Figs. 3 and 4 is of equal strength with that of Figs. 1 and 2; the main girders, although only three in number, are of greater depth.

Figs. 5 and 6 represent the details of the corbels and the bearings of the girders on the supports.

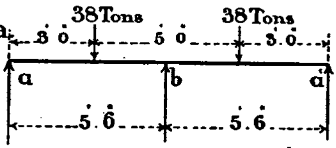
Figs. 7 and 8, Plate 2, show in section and elevation a type of framed trestle bent suitable for heights varying from 20 to 32 feet.

In Fig. 9, Plate 2, another type of construction is shown suitable for a height of 28 to 34 feet. In Figs. 10 and 11 are given details of the joints over the piers.

EXAMPLE No. 2.

The illustrations in Figs. 1, 2, 3 (Plate 3), 1*a*, and 2*a* represent a simple and useful type of timber bridge consisting of a series of longitudinal beams supported at intervals and carrying a single line of railway. The spans are 20 feet each from centre to centre of piers.

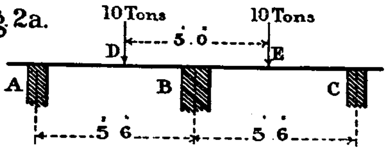
Fig. 1a.



The dead load, including the weight of permanent way, ballasting, and of the main girders, is estimated at 10 cwt. per foot run. The live load, including an addition for impact, is assumed to be 3·8 tons per foot run.

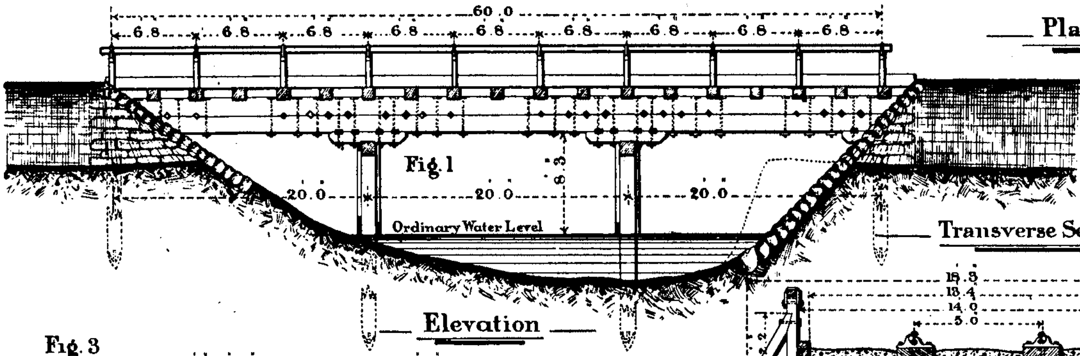
The dead load may be taken to be distributed uniformly over the three girders, so that the total load on each span of 10 tons divided by 3 will give the dead load on each girder $= 3\frac{1}{3}$ tons.

Fig. 2a.



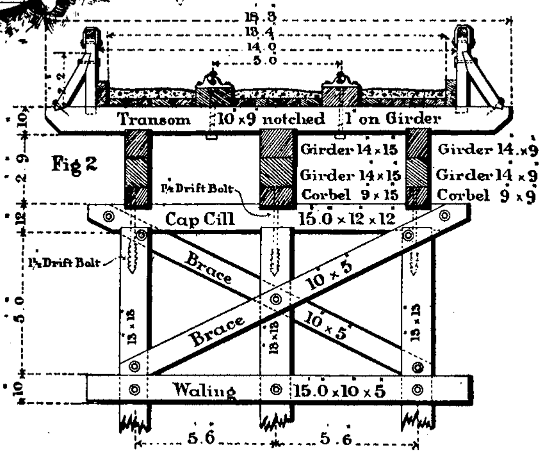
The live load, which is to be carried on two rails spaced 5 feet apart, centre to centre, one half on each rail, will be supported by the three girders in proportion to the respective reactions of the load upon its supports. Fig. 1*a* shows in

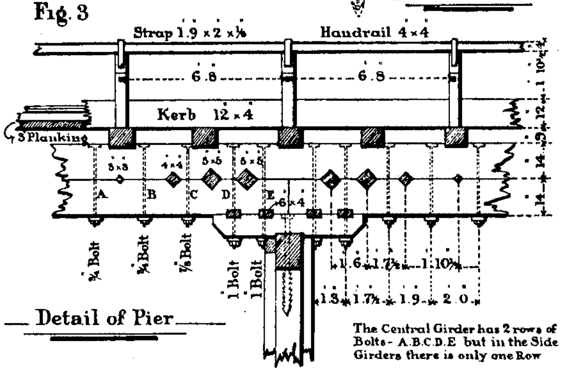
Plate 3.



Elevation

Transverse Section





Detail of Pier

diagram a transverse section of the bridge in which a, b, and a' represent the centres of the girders. The total live load on the span = 3·8 tons × 20 = 76 tons, one half of which = 38 tons, is carried by each line of rail. The proportion supported by each girder will therefore be as follows:

$$\text{Each outside girder,} \quad \frac{38 \text{ tons} \times 2 \cdot 5 \text{ ft.}}{5 \cdot 5} = 17 \cdot 27 \text{ tons.}$$

$$\text{Central girder,} \quad \frac{38 \text{ tons} \times 3 \text{ ft.}}{5 \cdot 5} \times 2 = 41 \cdot 46 \quad ,,$$

The total load to be carried by each girder will therefore be as follows:

	Dead Load.	Live Load.	Total distributed Load.	Equivalent concentrated Load.
Each outside girder,	3·33 tons +	17·27 tons.	20·60 tons.	10·30 tons.
Central girder,	3·33 tons +	41·46 tons	44·79 tons.	22·40 tons.

The girders are to be of pitch pine. It is now required to determine the cross-sectional dimensions of a pitch pine beam capable of carrying in the first instance a concentrated load of 22·40 tons = 50,176 lbs., with a factor of safety of 6 = 50,176 lbs. × 6 = 301,056 lbs. ultimate strength. Assuming the breadth of the girder to be 15 inches, then, by Tredgold's rule,

$$d^2 = \frac{\text{B}\text{W} \times l}{b \times c}.$$

Substituting, we have $d^2 = \dfrac{301 \cdot 056 \times 20}{15 \times 500} = 803,$

and $d = \sqrt{803} = 28 \cdot 33$ inches, say 28 inches,

the remaining $\frac{1}{3}$ of an inch being amply compensated by the shortening of the span by the introduction of corbels over the points of support.

Similarly, the cross-sectional dimensions of each outside girder may be determined, or they may be proportioned according to the load if the same depth be adopted. In this instance the load on each outside girder is less than one half of that on the central girder, so that the breadth need not be more than $7\frac{1}{2}$ inches, but so as to give the girder lateral rigidity let the breadth be 9 inches.

In the foregoing formula

BW = breaking weight in lbs.,
l = span in feet,
b = breadth of girder in inches,
d = depth of girder in inches,
C = ultimate resistance of pitch pine to transverse or bending stress = 495, say 500 lbs. per square inch.

It is extremely difficult, if not impossible, to obtain a beam of pitch pine of any appreciable length, 27 inches deep; a built beam or girder must therefore be adopted.

For the central girder two beams each 14 inches in depth and 15 inches in breadth would answer the requirements.

If one beam be merely laid upon another their strength will be equal to that of two separate beams of equal dimensions, but if the one beam be prevented from sliding upon the other by the insertion of joggles or transverse keys and then firmly bolted together the strength is practically equal to that of one solid beam of equal depth, which strength varies as the square of the depth.

The joint depth of all the keys, according to Tredgold, should not amount to less than $1\frac{1}{3}$ the total depth of the beam. The keys should be of greater depth and spaced at closer intervals at the bearing ends of each span, because the shearing stress is there the greatest. At any section the resistance of the wedges and bolts must be equal to the maximum horizontal and vertical shearing stresses.

If V = the vertical shear at any section,
H = the maximum horizontal shear per unit of neutral surface,
b = breadth of beam in inches,
d = depth of beam in inches,

then $H = \frac{3V}{2bd}$.

It is next required to determine the dimensions of the transoms which are spaced 3′ 4″ apart from centre to centre.

The maximum weight to be carried by a transom is the weight carried by a pair of driving wheels, which for the heaviest class of locomotive engine may be estimated at 20 tons, or 10 tons on each wheel, distributed as shown in Fig. 2*a*.

The span from centre to centre of the main girders is 5′ 6″, but by reason of the width of the main girders the effective span may be taken as 5 feet.

The equivalent centre load on AB or BC will therefore be 9 tons if treated as two simple beams resting on the supports AB and BC, and the dimensions of a pitch pine beam capable of carrying a concentrated load of 9 tons on a span of 5 feet is given by the following formula:

$$d^2 = \frac{BW \times l}{b \times c}.$$

Assuming the width of the beam to be 9 inches, allowing a factor of safety of 6, and substituting, we have

$$d^2 = \frac{120{,}960 \text{ lbs.} \times 5 \text{ ft.}}{9 \times 500} = 134{\cdot}4 \text{ and } d = \sqrt{134{\cdot}4} = \text{say } 11 \text{ in.},$$

but the transom is continuous over three supports, and its strength as compared with a beam merely supported at the ends is increased in the proportion of 4 to 3, so that a depth of 10 inches will amply suffice.

Piers.

The strength of the columns or vertical pillars supporting the bridge may be determined by the following formula:

Let BW = breaking weight in pounds,
a = area of cross section,
B = constant in column B of table, page 184,
L = length in feet,
T = least side, or thickness,

then
$$BW = \frac{B \times a}{1{\cdot}1 + \frac{L^2}{2{\cdot}9 \times T^2}}.$$

The central pile has to support the greatest load, which is 44·79, say 45 tons.

The piles are 12 inches square, and the unsupported length between the braces is taken as 6 feet.

Then
$$BW = \frac{6790 \times 144}{1{\cdot}1 + \frac{36}{2{\cdot}9 \times 144}} = 824{\cdot}418 \text{ lbs.}$$

Allowing a factor of safety of 8, the safe working load = 103,052 lbs., or 46 tons, which is sufficient.

The two outside columns are of the same cross-sectional area, and having only a load 20·60 tons each to support are amply strong.

The details of construction are shown in Figs. 1, 2, and 3, Plate 3.

It is here supposed that the piles have been driven well home and capable of carrying their intended load with safety in accordance with the requirements of the formulae that have been given in a preceding chapter.

EXAMPLE No. 3.

Howe Truss of 40 feet clear span to carry a single line of railway, the maximum load consisting of two locomotive engines each weighing 30 tons, the length over buffers and wheel base being as shown in Fig. 1.

Fig. 1

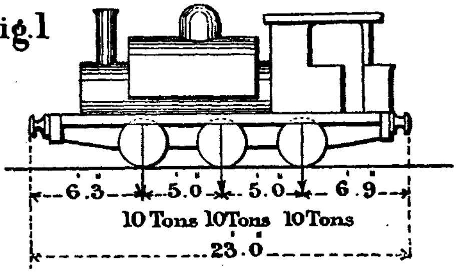
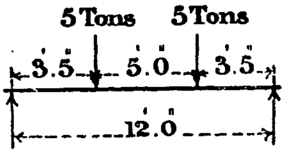


Fig. 2.

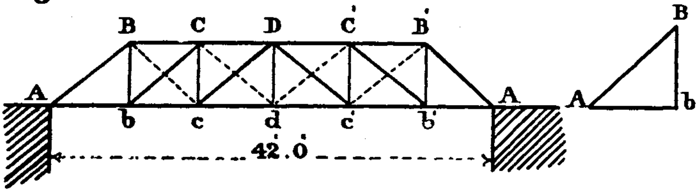


Dead load of bridge = ·3 ton per foot run, and for one girder ·15 ton.

Live load = 60 tons + 20 % for impact = 72 tons on bridge = 1·71 tons, say 1·75, and for one girder = ·875 ton.

The truss is to have an effective span of 42 feet and to consist of 6 panels of 7 feet each in length, the effective depth being 5 feet.

Fig. 3



$A b = 7$ ft., $\quad B b = 5$ ft., $\quad AB = \sqrt{7^2 + 5^2} = 8\ 6,$

$\frac{AB}{Bb} = \frac{8 \cdot 6}{5} = 1 \cdot 72, \qquad \frac{Ab}{Bb} = \frac{7}{5} = 1 \cdot 4.$

Max. dead load on panel $= \cdot 15 \times 7 = 1 \cdot 05$ tons.
Max. live load on panel $= \cdot 875 \times 7 = 6 \cdot 125$ tons.

Max. compression in Tons.

$$AB = \left(1 \cdot 05 \times 2\tfrac{1}{2} + 6 \cdot 125 \times \frac{1+2+3+4+5}{6}\right) 1 \cdot 72 = 30 \cdot 85$$

$$bC = \left(1 \cdot 05 \times 1\tfrac{1}{2} + 6 \cdot 125 \times \frac{1+2+3+4}{6}\right) 1 \cdot 72 \quad = 20 \cdot 27$$

$$CD = \left(1 \cdot 05 \times \tfrac{1}{2} + 6 \cdot 125 \times \frac{1+2+3}{6}\right) 1 \cdot 72 \quad = 11 \cdot 44$$

$$dC' = \left\{(1 \cdot 05 \times -\tfrac{1}{2}) + \left(6 \cdot 125 \times \frac{1+2}{6}\right)\right\} 1 \cdot 72 \quad = 4 \cdot 36$$

Max. tension in

$$Bb = 1 \cdot 05 \times 2\tfrac{1}{2} + 6 \cdot 125 \times \frac{1+2+3+4+5}{6} \quad = 17 \cdot 94$$

$$Cc = 1 \cdot 05 \times 1\tfrac{1}{2} + 6 \cdot 125 \times \frac{1+2+3+4}{6} \quad = 11 \cdot 78$$

$$Dd = 1 \cdot 05 \times 1 + 6 \cdot 125 \times \frac{1+2+3}{6} \quad = 7 \cdot 175$$

Maximum panel load for top and bottom booms

$$= 1 \cdot 05 + 6 \cdot 125 = 7 \cdot 175$$

Then maximum tension in Ab = maximum compression in BC

$$= 7{\cdot}175 \times 2\tfrac{1}{2} \times 1{\cdot}4 = 25{\cdot}12 \text{ tons.}$$

The maximum tension in bc = maximum compression in CD

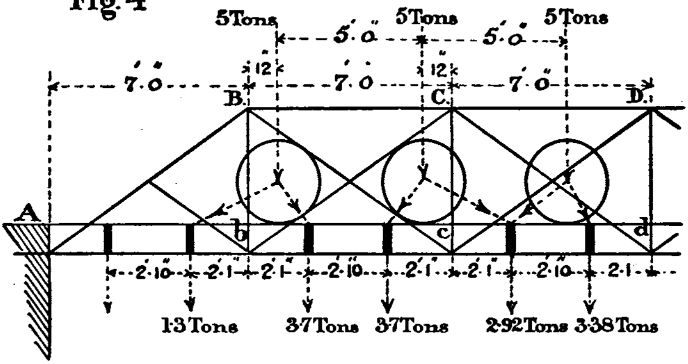
$$= 7{\cdot}175 \times (2\tfrac{1}{2} + 1\tfrac{1}{2}) \times 1{\cdot}4 = 40{\cdot}17 \text{ tons.}$$

The maximum tension in cd

$$= 7{\cdot}175 \times (2\tfrac{1}{2} + 1\tfrac{1}{2} + \tfrac{1}{2}) \times 1{\cdot}4 = 45{\cdot}20 \text{ tons.}$$

In the top chord and all the inclined web members the strain is compression, and in the bottom chords and the vertical web members tension.

Fig. 4



The timber of which the bridge is to be built is pitch pine, the ultimate breaking and safe unit stresses of which are assumed to be as under:

	Compression.	Tension.
Ultimate breaking stress in lbs. per sq. in. = R	= 6630 lbs.	10,000 lbs.
Factor of safety = F	= 8	10
Safe unit stress = $\frac{R}{F}$	= 828 lbs.	1000 lbs.
Safe unit stress in tons per sq. inch	= 0·37 tons	0·45 tons.

The safe working stress for the wrought-iron tension rods is limited to 4 tons per square inch.

The dimensions of the various members of the truss may now be determined as follows:

Particulars of Member.	Total Stress on Member in tons.	Theoretical Cross Sectional Area in sq. inches.	Number of Members Instressed.	Actual Cross Sectional Area in sq. inches.	Dimensions of each Member.
Braces—					
AB	+30·85	80·60	2	81	2 pieces 9″ × 4½″
*b*C	+20·27	54·80	2	72	2 pieces 8″ × 4½
*c*D	+11·44	30·90	2	72	2 pieces 8″ × 4½
Counterbraces—					
*d*C′	+ 4·36	11·80	1	48	1 piece 8″ × 6″
Top Chord—					
BC	+25·12	68·00	2	} 144	2 pieces 12″ × 6″
CD	+40·17	108·56	2		
Bottom Chord—					
A*b*	−25·12	56·00	2	} 156	2 pieces 13″ × 6″
bc	−40·17	89·60	2		
cd	−45·20	100·44	2		
Tension Rods—					
B*b*	−17·94	4·49	1	5·93	1 rod 2¾″ dia.
C*c*	−11·78	2·95	1	3·54	1 rod 2⅛″ dia.
D*d*	− 7·175	1·80	1	2·40	1 rod 1¾″ dia.

The actual cross-sectional area of the bottom chord is 156 square inches, whereas the theoretical sectional area is only 100·44 square inches. This excess is provided because the bottom chord has to perform the two functions of serving as tension member in the truss and as a beam to support the cross girders between the panel points, in which latter capacity it is subject to an additional bending stress.

It is next necessary to determine the additional sectional area required to resist this stress.

The greatest bending stress in any panel of the bottom chords occurs when two of the locomotive driving wheels are contained within one panel. Thus in Fig. 4 the two driving wheels are contained within the panel *b*, *c*, exerting re-actions of 3·7 tons each, which are transmitted to the chord through the cross girders *e* and *f*.

In determining the re-actions at *e* and *f* the continuity of the way beam has been disregarded, so as not to complicate the calculations. To be strictly accurate the way beam should have been treated as a continuous girder, in which case the re-actions on the cross girders would be less than has been assumed. This, however, is an error on the safe side.

On reference to Fig. 4 it will be observed that the maximum load to be supported by the bottom chord is as given in Fig. 5.

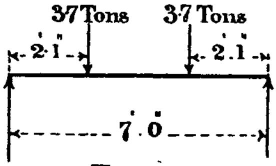


Fig. 5

The equivalent centre load

$$=\frac{(3\cdot 7\times 2+3\cdot 7\times 2)25 \text{ inches}}{84 \text{ inches}}=4\cdot 4 \text{ tons}.$$

The depth of the bottom chord is 13 inches. It is now required to determine the width of beam necessary to support a load of 4·4 tons applied at the centre of a span of 7 feet, allowing a factor of safety of 6. The equivalent breaking load will therefore be $4\cdot 4\times 6=26\cdot 4$ tons for a beam simply supported at both ends, but in this case, the bottom chord being supported at each panel point, it is a continuous girder, and its strength as compared with a beam only supported at the ends may be taken as 4 to 3, so that the breaking load may be estimated at

$$26\cdot 4 \text{ tons}\times \tfrac{3}{4}=19\cdot 8, \text{ say } 20 \text{ tons}=44{,}800 \text{ lbs}.$$

Let l = length of span in feet,
b = breadth of beam in inches,
d = depth of beam in inches,
W = breaking load in lbs.
C = co-efficient of transverse strength = 500 for pitch pine.

The formula for breaking weight of a beam supported at both ends

$$= \mathrm{W} = \frac{bd^2\mathrm{C}}{l}.$$

Transposing and substituting we have

$$b = \frac{\mathrm{W}l}{d^2\mathrm{C}} = \frac{44800 \times 7}{13^2 \times 500} = 3{\cdot}7 \text{ inches},$$

and 3·7 inches × 13 inches = 48·1 sq. inches, which added to the theoretical sectional area of the beam = 100·44 = 148·54 square inches. The actual sectional area is 156 square inches, thus leaving a margin of about 7·5 square inches.

In all the other members the actual area is in excess of that theoretically required. This excess could not be avoided without impairing the strength and stiffness of the structure, because some of the members, if proportioned strictly to meet the theoretical requirements, would be liable to rupture from any slight extraneous cause and no provision would be made for weathering. In this instance the web braces have been made larger than was necessary for the sake of adopting one pattern of angle block.

Cross Girders.

At any transverse section of the bridge immediately under any pair of driving wheels the condition of loading will be that illustrated in Fig. 2, and the equivalent centre load

$$= \frac{5 \text{ tons} \times 7 \times 2}{12} = \text{say } 6 \text{ tons}.$$

This equivalent centre load of 6 tons is assumed to be distributed by the way beam over two cross girders, so that each cross girder has to support 3 tons live load.

The clear span of each cross girder is 11 feet and the effective span is taken as 12 feet. Allowing as before a factor of safety of 6, the total weight to be supported by a cross girder will be as follows:

Dead load, say	·5 tons × 6	= 3 tons.
Live panel load,	3·0 tons × 6	= 18
Total load,	= 3·5 and breaking load	= 21 tons.

Then $bd^2 = \frac{Wl}{C} = bd^2 = \frac{21 \times 2240 \times 12}{500} = 1129$ square inches.

Assuming the breadth of the cross girder to be 8 inches, the depth will be $\sqrt{\frac{1129}{8}} = \sqrt{142}$, say 12 inches.

The dimensions of each cross girder may therefore be 12 inches × 8 inches.

Way Beams.

The way beams are 12 inches wide and 6 inches deep, and the maximum span from centres of cross girders is 4 ft. 2 ins. The beam is continuous, so its strength as compared with a beam merely supported at the ends will be as 4 to 3, hence the span may be regarded as reduced from 4 feet 2 inches to 3 feet $1\frac{1}{2}$ inches. The maximum load at the centre of any span of the way beam = 5 tons. Multiplying this by 6 the breaking load = 30 tons.

The breaking weight of a pitch pine beam $3\frac{1}{8}$ feet span, 12 inches wide, by 6 inches deep,

$$= \frac{bd^2C}{l} = \frac{12 \times 6^2 \times 500}{3\frac{1}{8}} = 31 \text{ tons.}$$

Plate 4.

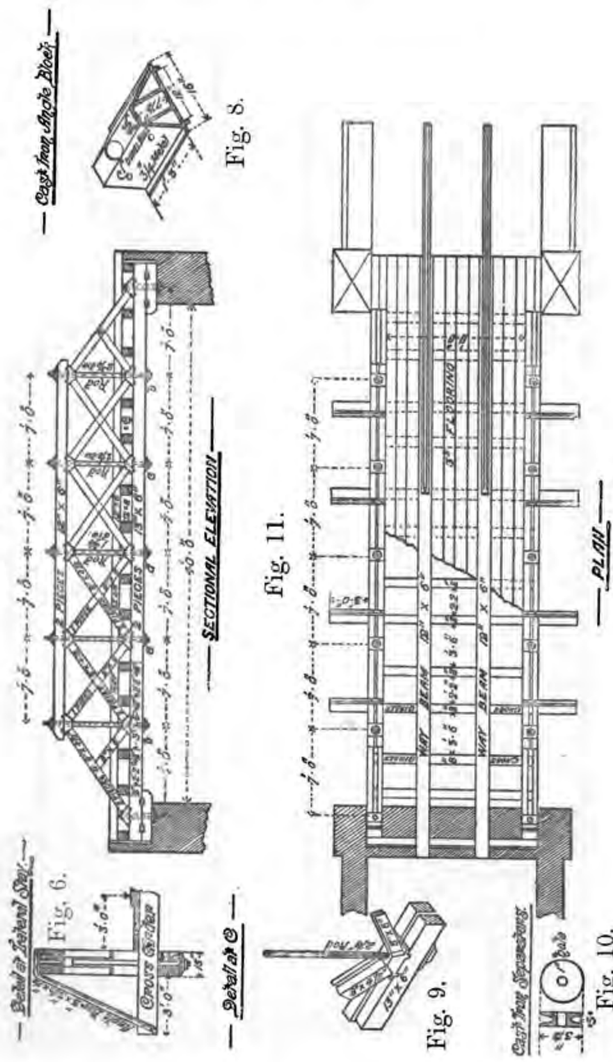


Fig. 6. Fig. 7. Fig. 8. Fig. 9. Fig. 10. Fig. 11.

Plate 5

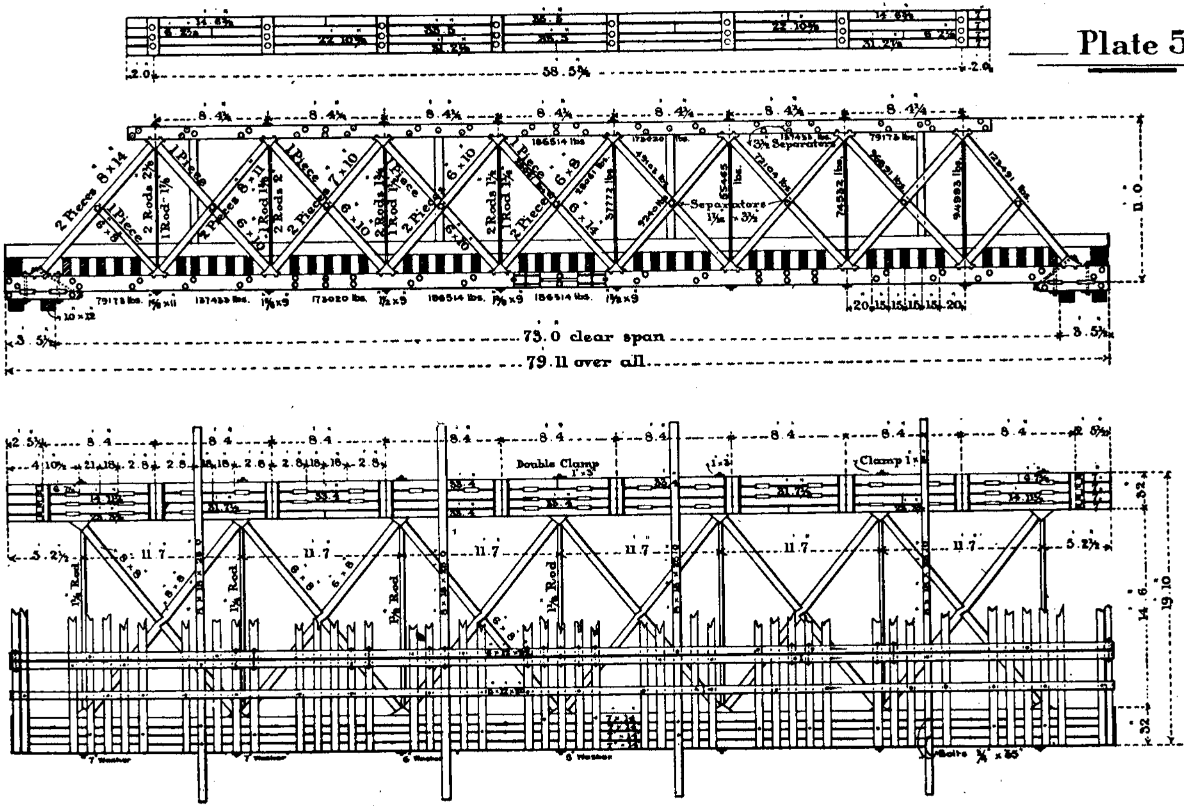
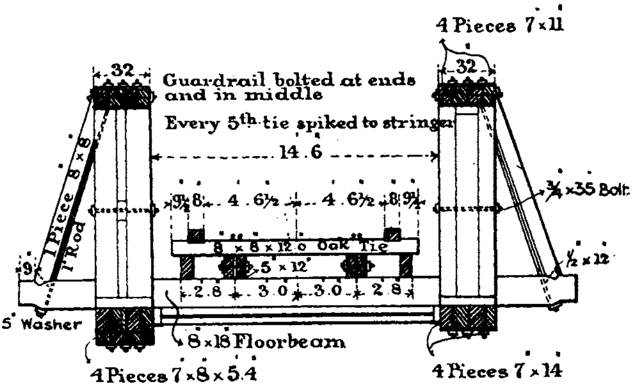

58.5¾
2.0
8.4¼
11.0
2 Pieces 8×14
1 Piece
2 Rods 2⅛
1 Rod 1⅞
Separators
3½ Separators
73.0 clear span
79.11 over all
3.5⅝
8.4
11.7
5.2½
Double Clamp
Clamp
14.6
19.10
32
Bolts

Plate 5^{A}



Top Chord

Packing Bolts ¾ dia

Cast Iron Separators for Packing Bolts 5 dia.

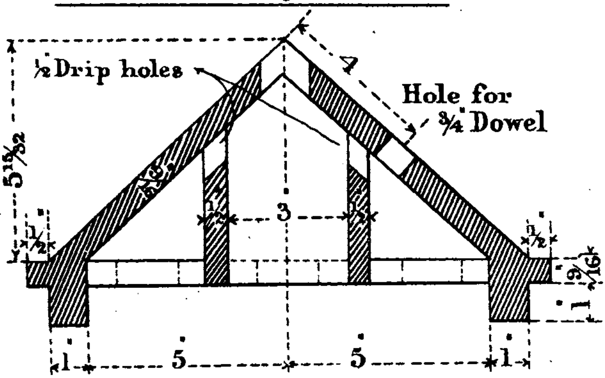
Bottom Chord

iron packing Keys

Packing Bolts ¾ dia.

Separators for Packing Bolts 1⅞×3½

Dead Load: 1200 lbs. per ft.
Live Load: Consol. Engine and Tender, 225000 lbs. followed by train of 3000 lbs. per foot



Detail of Angle Block

Figs. 6-11, Plate 4, illustrate the details of the construction of the bridge.

This bridge was built on a mineral railway to replace an existing structure, and the width between the trusses is less than would be sanctioned by Board of Trade requirements for a main line of railway. The example will, however, serve to illustrate the principles upon which a similar structure may be designed to carry a main line of railway.

A Howe truss is better adapted as a deck bridge than a through bridge for heavy traffic because of the difficulty of arranging cross girders in timber of sufficient strength to carry the traffic of a main line of railway of standard gauge. In a deck bridge the main trusses can be placed immediately under the rails, so that a very light system of flooring would be required.

For temporary work the angle blocks may be made of oak or some other hard wood instead of cast iron.

In Plate 5 is shown the Standard Plan of Howe Truss, 80 feet pony span, adopted on the Northern Pacific Railroad of the United States of America, for which the author is indebted to Mr. K. E. Hilgard, the engineer of bridges on this railroad.

The several figures given in Plate 5 and 5*a* fully illustrate the details of construction. The method of calculating the stresses in a Howe Truss have already been fully explained, and it is unnecessary to repeat the operation in the case of this particular structure.

EXAMPLE No. 4.

Highway Bridge for Light Traffic in a Rural District.

Span 40 ft., width of roadway 10 ft. The bridge to be capable of carrying the following moving or live loads:

(*a*) A uniformly distributed load of 1 cwt. per square foot of floor area due to a crowd of people passing over it;

(*b*) A timber carriage or limber weighing when fully loaded 10 tons carried on four wheels or 5 tons concentrated load on each pair of wheels, the distance from centre to centre of wheels being not less than 10 ft.

Dead Load.

The dead load carried by a cross girder is made up as follows:

	Tons	Cwts.	Qrs.	Lbs.	
Joists, ...	0	6	3	7	
Kerbs, ...	0	0	2	10	
Planking, ...	0	7	3	7	
Cross girder, ...	0	3	3	2	
	0	18	3	26	
Allow for mud and snow	0	11	0	2	= 12·34 lbs. per sq. foot.
	1	10	0	0	

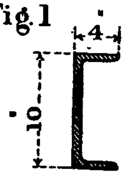
The live load carried by a cross girder is 5 tons, and the span of a cross girder measured from centre to centre of main girders = 10 feet 6 inches.

The bending moment will therefore be as follows:

		Inch Tons.
Dead load,	$\frac{1{\cdot}5 \text{ tons} \times 10{\cdot}5 \text{ feet} \times 12''}{8} =$	23·62
Live load,	$2{\cdot}5 \times 2{\cdot}75 \times 12 =$	82·50
	Total bending moment,	106·12

Let the cross girder be a channel bar of the dimensions indicated in Fig. 1.

Fig. 1



		Lbs.
Weight per foot run,	=	30·16
Sectional area, ...	=	8·87 inches^2
Moment of inertia, ...	=	130·715 inches^4
Moment of resistance,	=	26·143 inches^3

Equating the moment of resistance with the bending moment, and denoting the working stress of the metal by f, we have

$$\text{M}rf = \text{B}m, \text{ and substituting}$$

$$26{\cdot}143f = 106{\cdot}12\,;$$

$$\therefore\ f = 106{\cdot}12 \div 26{\cdot}143 = 4{\cdot}06 \text{ tons per inch}^2,$$

say 4 tons per inch^2.

It is next required to determine the strength of the joists, composed of Pitch-pine beams 10″ × 5″, the span from centre to centre of cross girders being 10 feet.

The joists are 6 in number, and it is assumed that the live load is equally distributed over 5 of the joists.

Let d = depth of joist in inches.

b = breadth of joist in inches.

l = length of span in feet.

C = coefficient of transverse strength, which for Pitch-pine = 500 lbs.

F = factor of safety = 8.

S = safe working load in lbs.

$$\text{Then S} = \frac{d^2 \times b \times \text{C}}{l \times \text{F}} = \frac{10^2 \times 5 \times 500}{10 \times 8} = 1{\cdot}4 \text{ tons.}$$

The strength of 5 joists will therefore = 1·4 × 5 = 7 tons.

The maximum concentrated live load on any section of the floor is 5 tons, so that there is available an ample margin of safety.

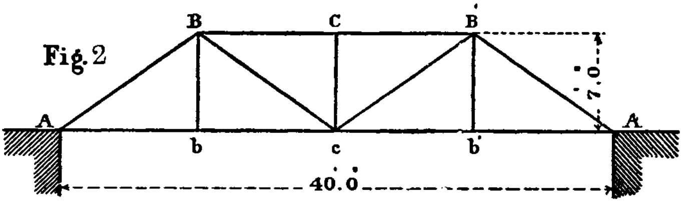
Weight of Main Girders.

The load to be carried by a cross girder which is equivalent to one panel load has been shown to be

	Tons	Cwts.
Dead load, ...	1 .	10
Live load, ...	5 .	0
Total panel load,	6 .	10

The main trusses consist of four panels, so that the total weight to be carried by the bridge $= 6\frac{1}{2}$ tons $\times 4 = 26$ tons, or 13 tons on each truss exclusive of the weight of the trusses.

Fig. 2



The weight of the main trusses may with sufficient accuracy be estimated as a first approximation by the formula

$$W_1 = \frac{Wl}{400}$$ in which $W_1 =$ weight of main girder in tons.

$W =$ total extraneous load on span in tons.

$l =$ length of span in feet.

Substituting, $$W_1 = \frac{13 \times 40}{400} = 1{\cdot}3 \text{ tons, say } 1{\cdot}5 \text{ tons.}$$

The total load to be carried by the girder will therefore be

Floor, ...	3 tons
Girder, ...	$1\frac{1}{2}$
Total dead load,	$4\frac{1}{2}$ tons, or 1·125 tons on each panel.

The live load carried by one girder = 10 tons and on one panel 2·5 tons.

The stresses in the several members can now be determined.

Stresses.

Tons.

AB maximum compression

$$= \left(1{\cdot}125 \times 1\tfrac{1}{2} + 2{\cdot}5 \times \frac{1+2+3}{4}\right)\tfrac{12}{7} = \quad 9{\cdot}320$$

Bb maximum tension

$$= 1{\cdot}125 + 2{\cdot}5 \qquad = \quad 3{\cdot}625$$

Bc maximum tension

$$= \left(1{\cdot}125 \times \tfrac{1}{2} + 2{\cdot}5 \times \frac{1+2}{4}\right)\tfrac{12}{7} \qquad = \quad 4{\cdot}180$$

Bc maximum compression

$$= (2{\cdot}5 \times \tfrac{1}{4} - 1{\cdot}125 \times \tfrac{1}{2})\tfrac{12}{7} \qquad = \quad 0{\cdot}108$$

Ab = bc maximum tension

$$= (1{\cdot}125 + 2{\cdot}5) \times 1\tfrac{1}{2} \times \tfrac{10}{7} \qquad = \quad 7{\cdot}770$$

BC = CB′ maximum compression

$$= (1{\cdot}125 + 2{\cdot}5) \times (1\tfrac{1}{2} + \tfrac{1}{2})\tfrac{10}{7} \qquad = 10{\cdot}360$$

Cc no stress = 0·00

The girders are to be built of steel which is to have a safe working stress of $6\frac{1}{2}$ tons per sq. inch.

Let the working stress for dead load = $6\frac{1}{2}$ tons per sq. inch and for live load 5 tons per sq. inch.

The ratio of total dead load to total live load = $1\frac{1}{8}$ to $2\frac{1}{2}$, or approximately 1 to $2\frac{1}{4}$.

Then the working stress for combined dead and live loads

$$= \frac{6\frac{1}{2} \times 1 + 5 \times 2\frac{1}{4}}{1 + 2\frac{1}{4}} = 5{\cdot}46 \text{ tons per sq. inch,}$$

the permissible working stress.

For compression members this permissible working stress of 5·46 tons per square inch must be reduced in proportion to the ratio of the length to the least radius of gyration of the section by the following formula:

$$p = \frac{15000}{1 + \dfrac{l^2}{13500 r^2}}$$

in which p = permissible working stress.

l = length of member, centre to centre of connection in inches.

r = least radius of gyration of section in inches.

The member that has to resist the greatest compressive stress having reference to its length is AB in Fig. 2, the length of which = 12 feet. The section of this member is as shown in Fig. 3, and the least radius of gyration is 1·51, say 1·5 inches.

Substituting we have

$$p = \frac{15000}{1 + \dfrac{144^2}{13500 \times 1{\cdot}5^2}} = \overset{\text{Tons.}}{3{\cdot}975}, \text{ say 4 tons.}$$

The permissible working stresses will therefore be as follow:

Compression members 4 tons per square inch.

Tension members 5·46, say 5·5 tons per square inch.

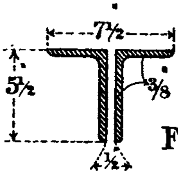


Fig. 3

In proportioning the members to the working stresses they have to resist regard must be had to the effects of corrosion and to the fatigue which the metal must undergo after years of work.

Stresses and Dimensions of Members.

Member.		Maximum Compression.	Maximum Tension.	Total Stress.	$\frac{l}{r}$	Permissible Working Stress per sq. inch.	Section required.	Make up.	Section given.	Actual Working Stress per sq. inch.
		Tons.	Tons.	Tons.		Tons.	sq. in.		sq. in.	Tons.
Top Chord,	AB	9·32	......	9·32	$\frac{144}{1·5}$	4·0	2·33	2 Ls. 5″ × 3½″ × ⅜″ × 10·37 lbs.	6·09	1·53
,,	BC	10·36	......	10·36	$\frac{120}{1·5}$	4·0	2·59	2 Ls. 5″ × 3½″ × ⅜″ × 10·37 lbs.	6·09	1·72
Bottom Chord,	A*b*	......	7·77	7·77		5·5	1·41	1 L 4″ × 3″ × ½″ × 11·05 lbs.	2·81	2·76
,,	*bc*	......	7·77	7·77		5·5	1·41	1 L 4″ × 3″ × ½′ × 11·05 lbs.	2·81	2·76
Web,	B*c*	0·108	4·18	4·29		5·5	0·78	1 L 3″ × 2½″ × ½″ × 8·5 lbs.	2·06	2·08
,,	B*b*	......	3·62	3·62		5·5	0·66	1 L 3″ × 2½″ × ½″ × 8·5 lbs.	2·06	1·80
,,	C*c*	nil.	nil.	nil.		......	......	1 L 3″ × 2½″ × ½″ × 8·5 lbs.	2·06	nil.

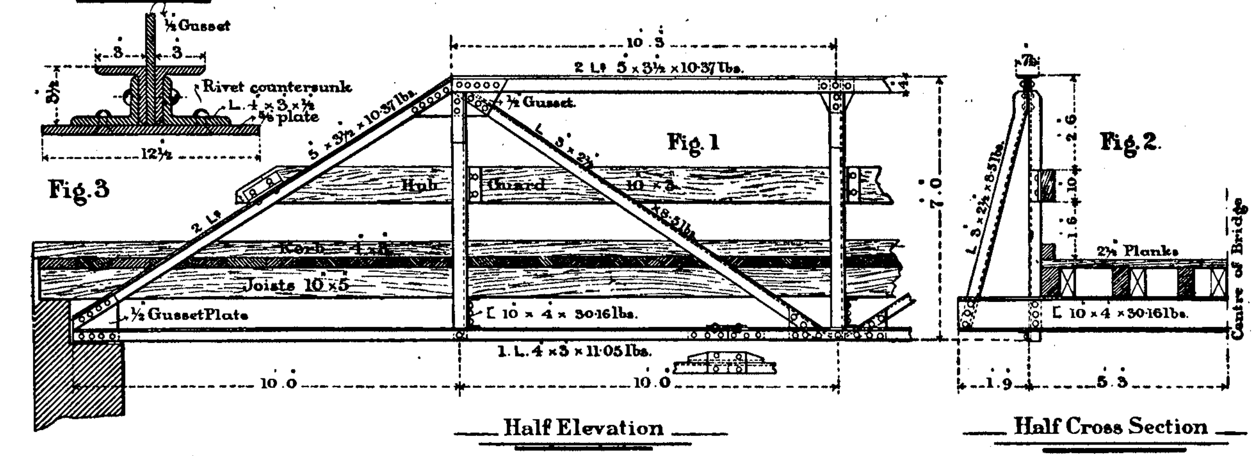
Riveted Connections.

The total stress in the member Bc is 4·29 tons. Assuming the rivets to be $\frac{7}{8}''$ dia., the resistance to shearing of one rivet is 3·006 tons, or alternatively for each ton of stress in the bar, the resistance of a $\frac{7}{8}''$ rivet to shearing is ·333. This multiplied by the stress to be carried by the member = the total number of rivets required in single shear. In this case the number required = $\cdot 333 \times 4 \cdot 29 = 1 \cdot 43$ rivets. The number of rivets provided is 3, so that there is an ample margin of security. The other connections are similarly worked out.

The details of construction are shown in the illustrations on Plate 6. A number of light bridges of this type has been constructed by the Pittsburg Bridge Company, Pennsylvania, for use in and beyond the Andes Mountains. These had to be designed so as to be capable of being transported in sections of limited weight and length on mule back, and the connections arranged so as to be easily put together with unskilled labour.

For remote situations, and especially in mountainous countries, this type of structure will be found to be strong, simple, serviceable and inexpensive, and fully capable of accommodating all the ordinary traffic of the district.

Plate 6



Half Elevation

Half Cross Section

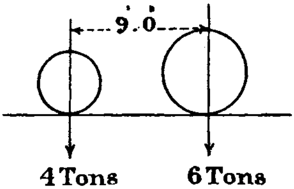
EXAMPLE No. 5.

Highway Bridge over the River Sawdde at Llangadock, Carmarthenshire.

Calculations of Stresses.

The bridge consists of three spans of 45 feet each in the clear between supports, and the width of the roadway is 12 feet. The main girders are of the lattice type divided into 10 bays or panels, each one 4′ 6″ in length and the depth over all is 5 feet.

Fig. 1



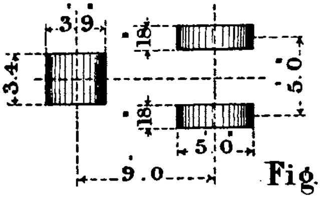
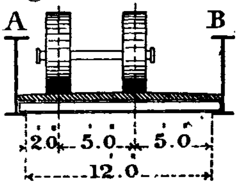


Fig. 2

The bridge is designed to carry a steam road roller of 10 tons weight as shown in subjoined sketch, and upon the remaining portion of the floor a load of 1 cwt. per square ft.

For the purpose of calculating the stresses in the web members the roller is assumed to pass close to the side of the one girder as shown in Fig. 3.

Fig. 3



Reaction on girder

$$A = 3 \times \frac{10}{12} + 3 \times \frac{5}{12} = 3\tfrac{3}{4} \text{ tons}$$

as the maximum live load that can come on any panel point which will produce the maxima stresses at that point in the web members.

Dead Load.

The weight of concrete in flooring was estimated at per foot run.	7 cwts.
The weight of road metalling was estimated at per foot run.	3 ,,
The weight of metal in superstructure at	6 ,,
	16 cwts.

The actual weight of the latter was 5·8 cwts.

Total weight per foot run on both girders	16 cwts.
and on one girder	8 cwts.

The total dead load on one panel of the main girder will therefore be 8 cwts. × 4·5 ft. = 1·8 tons.

The maximum live load on one panel of the main girder, allowing for the eccentricity of loading as shown in Fig. 3 = 3·75 tons.

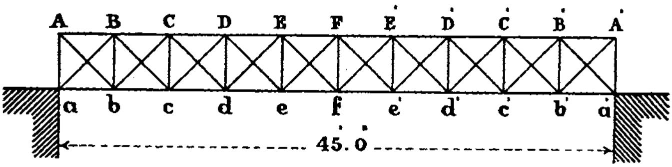


Fig. 4

Max. stress in Ab = aB

$$= \frac{(1\cdot8 \times 2\tfrac{1}{4} + 3\cdot75 \times 1 + 2 \ldots + 9)1\cdot41}{10 \times 2} = 17\cdot60 \text{ tons.}$$

Max. stress in Bc = bC

$$= \frac{(1\cdot8 \times 1\tfrac{3}{4} + 3\cdot75 \times 1 + 2 \ldots + 8)1\cdot41}{10 \times 2} = 13\cdot96 \text{ tons.}$$

Max. stress in Cd = cD

$$= \frac{(1\cdot8 \times 1\tfrac{1}{4} + 3\cdot75 \times 1 + 2 \ldots + 7)1\cdot41}{10 \times 2} = 10\cdot57 \text{ tons.}$$

Max. stress in $De = dE$

$$= \frac{(1{\cdot}8 \times \ \tfrac{3}{4} + 3{\cdot}75 \times 1 + 2 \ldots + 6)1{\cdot}41}{10 \times 2} = \ 7{\cdot}44 \text{ tons.}$$

Max. stress in $Ef = eF$

$$= \frac{(1{\cdot}8 \times \ \tfrac{1}{4} + 3{\cdot}75 \times 1 + 2 \ldots + 5)1{\cdot}41}{10 \times 2} = \ 4{\cdot}60 \text{ tons.}$$

Max. comp. in AB = max. tension in ab

$$= 1{\cdot}8 \times \ 2\tfrac{1}{4} + 3{\cdot}75 \times \ 2\tfrac{1}{4} = 12{\cdot}5 \text{ tons.}$$

Max. comp. in BC = max. tension in bc

$$= 1{\cdot}8 \times \ 6\tfrac{1}{4} + 3{\cdot}75 \times \ 6\tfrac{1}{4} = 34{\cdot}7 \text{ tons.}$$

Max. comp. in CD = max. tension in cd

$$= 1{\cdot}8 \times \ 9\tfrac{1}{4} + 3{\cdot}75 \times \ 9\tfrac{1}{4} = 51{\cdot}33 \text{ tons.}$$

Max. comp. in DE = max. tension in de

$$= 1{\cdot}8 \times 11\tfrac{1}{4} + 3{\cdot}75 \times 11\tfrac{1}{4} = 62{\cdot}43 \text{ tons.}$$

Max. comp. in EF = max. tension in ef

$$= 1{\cdot}8 \times 12\tfrac{1}{4} + 3{\cdot}75 \times 12\tfrac{1}{4} = 68{\cdot}00 \text{ tons.}$$

Max. comp. in $Aa = (1{\cdot}8 + 3{\cdot}75)2\tfrac{1}{2} \qquad = 13{\cdot}75$ tons.

Max. tension on all other verticals

$$= (1{\cdot}8 + 3{\cdot}75)\tfrac{1}{2} = \ 2{\cdot}75 \text{ tons.}$$

In calculating the moment of resistance of the tension flange it is assumed that one half of the sectional area of the vertical web is effective in addition to the nett sectional area of the flange plate and angle bars. The nett sectional area of the tension flange, after deducting rivet holes, will be as under:

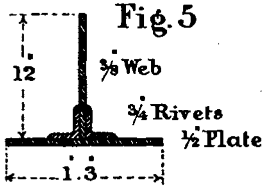


Fig. 5

Bottom flange plate $(15'' - 1\tfrac{1}{2}'')\tfrac{1}{2}$	=	$6{\cdot}75$ sq. ins.
Angle bars $(5\tfrac{1}{8}'' - \tfrac{3}{4}'')\tfrac{3}{8} \times 2$	=	$3{\cdot}28$ sq. ins.
Web plate (half assumed to be effective) $(12 \times \tfrac{3}{8}) \div 2$	=	$2{\cdot}25$ sq. ins.
Nett sectional area of tension flange ...	=	$12{\cdot}28$ sq. ins.

The maximum stress in the tension flange is 68 tons and the intensity of maximum working stress $= 68 \div 12{\cdot}28 = 5{\cdot}53$ tons per square inch. The material throughout is steel assumed to be capable of a safe working stress of $6\frac{1}{2}$ tons per square inch.

In the compression flange the gross sectional area is effective. The sectional area is as under:

Top flange plate $15'' \times \frac{1}{2}''$	= 7·50 sq. ins.
2 angle bars $3'' \times 2\frac{1}{2}'' \times \frac{3}{8}''$	= 3·84 sq. ins.
Web plate (half assumed to be effective)	= 2·25 sq. ins.
Gross sectional area of compression flange	= 13·59 sq. ins.

The working stress $= 68 \div 13{\cdot}59 = 5$ tons per square inch.

The working stresses in the web members are given in the following tabulated statement.

Web members.	Total max. stress in tons.	Nett sectional area in sq. ins.	Intensity of max. working stress in square ins.
*a*B	+17·60	4·76	3·69 tons.
A*b*	−17·60	3·28	5·36 ,,
*b*C	+13·96	3·375	4·13 ,,
B*c*	−13·96	2·65	5·26 ,,
*c*D	+10·57	2·875	3·64 ,,
C*d*	−10·57	2·34	4·51 ,,
*d*E	+ 7·44	2·20	3·38 ,,
D*e*	− 7·44	1·72	4·32 ,,
*e*F	+ 4·60	2·20	2·09 ,,
E*f*	− 4·60	1·125	4·08 ,,
A*a*	+13·75	4·90	2·80 ,,
B*b*, C*c*, D*d*, E*e*, and F*f*	− 2·75	4·60	1·69 ,,

NOTE.—The vertical web members act as stiffeners in addition to tension members, hence the low limit of working stress.

Stress Diagram for one half of the Truss.

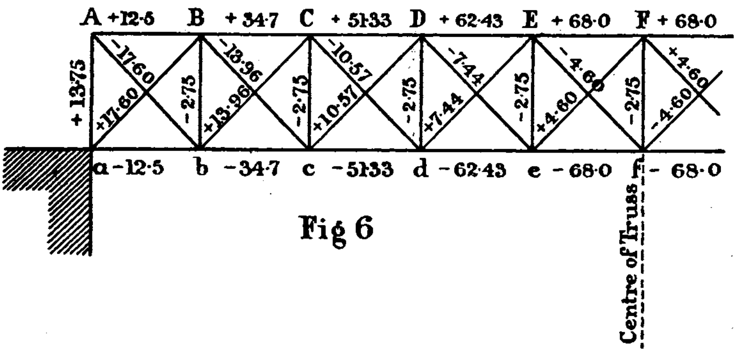


Fig 6

Calculations of the strength of the Cross Girders.

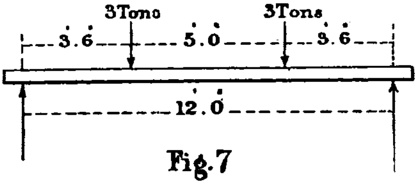


Fig.7

Diagram giving position of load for maximum bending moment in cross girder.

Equivalent centre load due to road roller in the position indicated in diagram as above

$$= \frac{3 \times 3{\cdot}5 \times 2 + 3 \times 3{\cdot}5 \times 2}{12} = 3{\cdot}6 \text{ tons.}$$

Equivalent uniform distributed load $= 3{\cdot}6 \times 2 = 7{\cdot}2$ tons.

The total dead load carried by one cross girder is as follows:

Concrete flooring, ...	31·5 cwts.
Road metalling, ...	13·5 cwts.
Cambered steel flooring, ...	5·3 cwts.
Cross girder 8″ × 6″ × 38 lbs.,	4·5 cwts.
Total dead load on one cross girder, say 55 cwts. or 2·75 tons.	54·8 cwts.

The total distributed load on one cross girder will therefore be 2 tons 15 cwt. + 7 tons 2 cwts. = 9 tons 17 cwts., say 10 tons.

Maximum Bending Moment.

(1) Cross girder taken as supported at both ends

$$=\frac{Wl}{8}=\frac{10\times 12\times 12}{8}=180 \text{ inch tons.}$$

(2) Cross girder taken as fixed at both ends

$$=\frac{Wl}{12}=\frac{10\times 12\times 12}{12}=120 \text{ inch tons.}$$

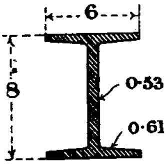


Fig. 8

The moment of inertia of a rolled steel beam of the section given in Fig. 8 is 111·6. Then moment of resistance

$$=M=\frac{fI}{y}=\frac{111\cdot 6}{4}f=27\cdot 9f \text{ in square inches.}$$

Equating the bending moment with the moment of resistance, we have:

Case (1). $27\cdot 9f=180$ inch tons, and

$f=180\div 27\cdot 9=6\cdot 44$ tons per square inch.

Case (2). $27\cdot 9f=120$ inch tons, and $f=120\div 27\ 9=4\cdot 30$ tons per square inch.

It is thought that to take the mean between Case 1 and Case 2, giving as the limit of the unit stress 5·37 tons per square inch, will be sufficiently correct.

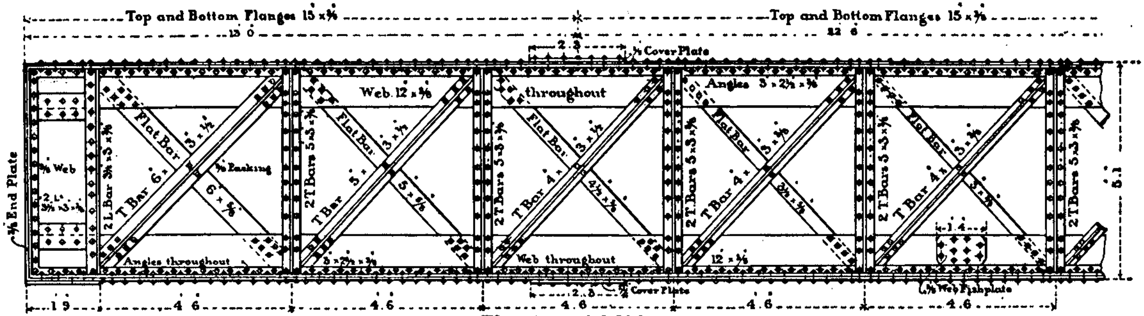

WILSON & HUDSON

The total weight of the steel superstructure in the three spans was 39 tons 10 cwts., including the steel flooring, and the cost of the bridge was as under:

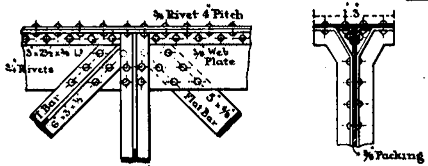
Piers and abutments, ...	£605	17	3
Steelwork, ...	722	12	6
Approaches, ...	902	10	3
Total cost,	£2231	0	0

The details of the superstructure of this bridge are given in Plate 7, and a photograph of the structure as erected is given in Plate 7*a*.

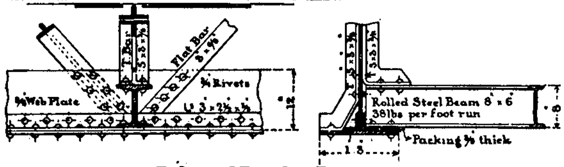
Plate 7.



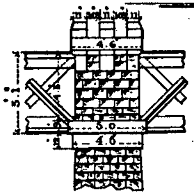
Elevation of Half Girder



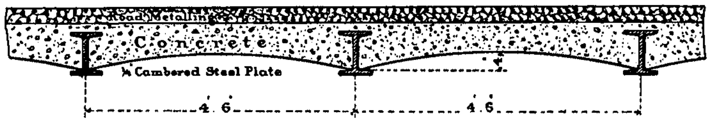
Enlarged Detail at A



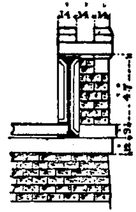
Enlarged Detail at B. inside Girder



Elevation Pier



Section on Centre Line of Bridge



Transverse Section Pier

EXAMPLE No. 6.

Details of Calculation for a Plate Girder Railway Bridge of 48 feet clear span.

Effective span, ...	50 feet.
Depth of girder, ...	4 feet.
Width from centre to centre of main girder	= 14 feet.

The bridge to be capable of carrying a live load of 2·25 tons per foot run, and in calculating the strength of the floor system a maximum load of 20 tons on a pair of driving wheels.

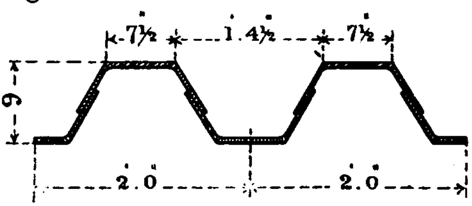
The floor is to be laid with Dorman, Long & Co.'s trough decking section C′ maximum, the weight of which is 35 lbs. per square foot of floor area. Fig. 1.

The dead load to be carried per lineal foot will therefore be as follows:

Trough flooring 35 lbs. × 14 feet	= 490 lbs.
Concrete filling at 135 lbs. per cub. foot	= 705 lbs.
Ballasting at 120 lbs. per cub. foot	= 1260 lbs.
Permanent way	= 136 lbs.
Total dead load per lineal foot, exclusive of weight of main girders,	= 2591 lbs. say $1\frac{1}{4}$ tons.

Section of Flooring.

Fig. 1



Weight per square foot = 35·02 lbs.
Modulus of section ... = 51·45 lbs.

The maximum live load is assumed to be 20 tons on a pair of driving wheels, which load is distributed over three flutes or corrugations, thus extending longitudinally over 6 feet of floor.

Strength of Trough Flooring.
Bending Moments.

	Inch tons.
Live load $= 10$ tons $\times 4' \ 6'' \times 12'' =$ distance from centre of girder to rail	$= 540$
Dead load $= \dfrac{1\frac{1}{4} \text{ tons} \times 6 \text{ ft.} \times 14 \text{ ft.} \times 12 \text{ ins.}}{8}$	$= 157{\cdot}5$
BM $=$ total bending moment in inch tons	$697{\cdot}5$

The modulus of section $= 51{\cdot}45$ for one flute and for three flutes $51{\cdot}45 \times 3 = 154{\cdot}35$ as the moment of resistance $=$ MR. Then $\dfrac{\text{BM}}{\text{MR}} = \dfrac{697{\cdot}5}{154{\cdot}35} = 4{\cdot}52$ tons per square inch as the working stress.

It is next required to estimate the weight of the main girders, which may be found from Professor Unwin's formula as under:

$$W_1 = \frac{Wlr}{cs - lr}, \text{ in which}$$

W $=$ total external distributed load in tons, exclusive of the weight of girder.

$W_1 =$ weight of girder itself in tons.

$l =$ clear span $= 50$ feet.

$d =$ effective depth in feet $= 4$ feet.

$s =$ average working stress in tons $= 6$ tons per sq. inch.

$r =$ ratio of span to depth $= \frac{50}{4} =$ practically 12.

$c =$ constant, value of which in this case $= 1280$.

	Tons.
The total distributed live load $= 50 \times 2{\cdot}25$ tons $=$	$112{\cdot}5$
The total distributed dead load $= 50 \times 1{\cdot}25$ tons $=$	$62{\cdot}5$
Total distributed load on two girders, exclusive of the weight of the girders	$175{\cdot}0$
Total distributed load on one girder	$87{\cdot}5$

Then $$W_1 = \frac{87 \cdot 5 \times 50 \times 12}{1280 \times 6 - 50 \times 12} = 7 \cdot 4, \text{ say } 7\tfrac{1}{2} \text{ tons.}$$

The total weight to be carried by one girder will therefore be:

	Tons.	Tons.
Flooring and permanent way	= 31·25	
Weight of girder...............	= 7·50	
Total dead load	= 38·75	= 0·775 per foot.
Total live load	= 56·25	= 1·125 per foot.
	95·00	1·900 per foot.

The bending moment at the centre of the girder will therefore be: $$\frac{95 \times 50}{4 \times 8} = 148 \cdot 44 \text{ foot tons.}$$

The curve of bending moments for a uniformly distributed load is a parabola whose vertex at the centre of the span $= \frac{Wl}{8d}$ and the ordinates to that curve will give the bending moments at any section between the centre of the girder and the points of support.

Let the curve be drawn as shown in Fig. 2, setting off the

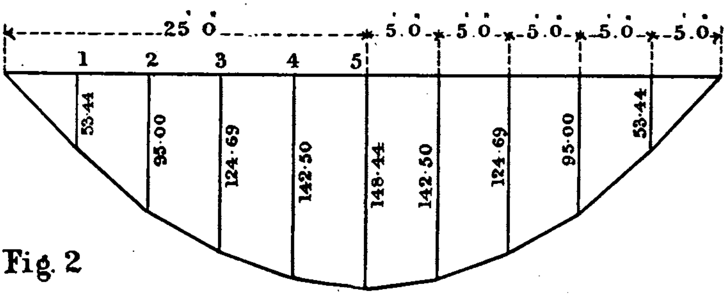


Fig. 2

centre ordinate $= 148 \cdot 44$ units, then the other ordinates will give the bending moments at those points.

The values of the bending moments, obtained by scaling the ordinates in the diagram, Fig. 2, may be checked by calculation, by the following formula introduced by Professor Unwin:

Let x = distance of section from either abutment.
W_1 = unit dead load per foot run,
W_{11} = unit live load per foot run.

Then $$\text{BM at } x = \frac{(W_1 + W_{11})\frac{x}{2}(l - x)}{d}.$$

The girders are to be built of steel which is assumed to have a maximum working stress of 7 tons per square inch for dead load and 5 tons per square inch for live load.

The ratio of total dead load to total live load = 38·75 to 56·25 tons or 1 to 1·45.

Then the working stress for combined dead and live loads

$$= \frac{7 \times 1 + 5 \times 1{\cdot}45}{1 + 1{\cdot}45} = 5{\cdot}81 \text{ tons, say } 5{\cdot}8 \text{ tons per square inch.}$$

The greatest stress in the flanges is produced when the girder is fully loaded and it is a maximum at the centre of the span where it = 148·44 tons.

The nett sectional area at the centre of the span must not therefore be less than 148·44 ÷ 5·8 .... = 25·6 square inches.

The angles in the top and bottom flanges are $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ of which the gross sectional area = 3·25 × 2 = 6·5 square inches.

The nett sectional area after deducting one $\frac{7}{8}$ rivet hole = $(6\frac{1}{2} - \frac{7}{8})\frac{1}{2} \times 2 = 5{\cdot}625$ square inches.

The sectional area of the angles is 5·625 square inches, there must therefore be provided by the flange in the tension member 25·6 − 5·625 = say 20 square inches.

The width of the flange in girders of this type should not be less than from $\frac{1}{36}$ to $\frac{1}{40}$ of the span, let it be in this case 18 inches. The thickness must, therefore, after deducting 4 rivet holes each $\frac{7}{8}''$ dia. $=(18-3\frac{1}{2})t=20$ square inches, whence $t=20\div 14\cdot 5=1\cdot 38$ inches, say $1\cdot 5$ inches.

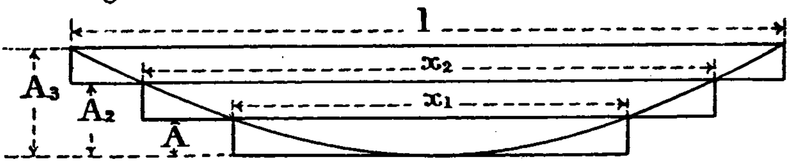
This thickness would be most conveniently made up of three plates each one $\frac{1}{2}$ inch thick.

The total sectional area in the tension flange will therefore be as follows:

Angles	$=$	$5\cdot 625$ sq. inches.
Flange plates $18'' \times 3 \times \frac{1}{2}$ less rivet hole	$=$	$21\cdot 750$ sq. inches.
Effective area of tension flange	$=$	$27\cdot 375$ sq. inches.

The bending moments, it has been shown in the diagram, Fig. 2, vary in proportion to the ordinates of a parabola from a maximum at the centre of the span to zero at the ends. If therefore a moment parabola be constructed as shown in Fig. 3 setting off the centre ordinate = total thickness of the plates at the centre of the span it will at once show to what extent the sectional area of the flange may be reduced towards the abutments and the lengths of the cover or outer plates can be readily determined by scale.

Fig. 3



The lengths of the cover plates may also be ascertained by the following simple formula based on the equation of the parabola:

Let x_1 and x_2 represent the lengths of plates required;
A_1 represent the area of the outer plate;
A_2 represent the area of the two outer plates;
A_3 represent the area of the three plates;
l represent the length of span.

Then

$$x_1 = \text{length of outer plate} = l\sqrt{\frac{A_1}{A_3}} = 50\sqrt{\frac{7{\cdot}25}{21{\cdot}75}} = 28{\cdot}9 \text{ feet.}$$

$$x_2 = \text{length of second plate} = l\sqrt{\frac{A_2}{A_3}} = 50\sqrt{\frac{14{\cdot}50}{21{\cdot}75}} = 40{\cdot}60 \text{ feet.}$$

The actual length should be at least 1 foot more than given by this formula.

Let the length x_1 be 30 feet and $x_2 = 42$ feet.

In the compression flange the gross sectional area is effective inasmuch as the rivet holes are filled up with the driven rivets, and the flange area, to meet the strict requirements of theoretical calculation, should be reduced either by diminution of the width of flange or the thickness of the plates, but in practice it is the custom to make the compression flange of the same dimensions as the tension flange.

The gross sectional area of the compression flange at the centre of the span will therefore be as follows:

Angles, ...	6·5 sq. inches.
3 plates, $18'' \times \frac{1}{2}$, ...	27·0 sq. inches.
Effective area of compression flange,	33·5 sq. inches.

The unit working stress in the compression flange will therefore be lower than in the tension flange

Web Shearing Stresses.

The shearing stress in the web due to a uniform load is nil at the centre of the span, and at the bearing ends of the girder it is equal to half the total load $= 95$ tons $\div 2 = 47\frac{1}{2}$ tons.

The thickness of the web must be sufficient at any section to resist buckling and to give a sufficient bearing area on the rivet.

The shear is a maximum at the points of support where its value is $47\frac{1}{2}$ tons. The effective depth of the girder is 4 feet, the shear per foot run vertically and horizontally at that point will therefore be $47{\cdot}5 \div 4 = 11{\cdot}875$ tons, say 12 tons.

Assuming the resistance to shearing to be 5 tons per square inch there would be required a sectional area of $12 \div 5 = 2 \cdot 4$ square inches per foot run, and $2 \cdot 4 \div 12 = 0 \cdot 2$ of an inch as the theoretical thickness of the web plate, say $\frac{3}{8}$ of an inch, because no plate of less thickness than $\frac{3}{8}''$ should be used in girder construction.

The working stress of a $\frac{7}{8}''$ rivet in single shear $= 3$ tons and in double shear $3 \times 1 \cdot 75 = 5 \cdot 25$ tons. The shear per foot run $= 12$ tons, so that $12 \div 5 \cdot 25$ will give the number of rivets per foot run $= 2 \cdot 4$, but as this must be a whole number let it be $3 = 4$ inch pitch.

It next remains to examine whether the pressure on the bearing area with this pitch will be too great.

The bearing area of a $\frac{7}{8}''$ rivet in a $\frac{3}{8}''$ plate $= \frac{7}{8}'' \times \frac{3}{8}'' = \cdot 3281$. Allowing a working pressure of $7\frac{1}{2}$ tons per square inch the total bearing area required will be $12 \cdot 0 \div 7\frac{1}{2} = 1 \cdot 6$.

Dividing $1 \cdot 6$ by $\cdot 3281$ will give $4 \cdot 8$ as the number of rivets required, but as the rivets are in double shear there will be required $4 \cdot 8 \div 1 \cdot 75 = 2 \cdot 74$, say 3 rivets per foot run, that is to say, the rivets must have a pitch of 4 inches so that the bearing area is sufficient.

It has already been stated that no plate of a less thickness than $\frac{3}{8}$ of an inch should be used in girder construction, and inasmuch as a $\frac{3}{8}''$ plate only meets the requirements in this case it would be advisable to provide some margin for weathering, corrosion, and other contingencies, so the web will be made of $\frac{7}{16}''$ plate.

The shearing stresses being a maximum at the bearing ends of the girder and diminishing to practically nothing at the centre, the thickness of the web should, if theoretical calculations are strictly followed, be proportionately reduced towards the centre of the girder, but within such limits as to be capable of resisting buckling, and of affording a sufficient bearing area upon the rivets.

From a practical point of view it is not desirable to have many variations in the thickness of the web plate, and it frequently happens that by the adoption of a uniform thickness throughout the cost of the little extra metal that is used is more than compensated by the labour avoided in making the web joints. In the case under consideration the web will be throughout the length of the girder of the uniform thickness of $\frac{7}{16}''$.

Web Stiffeners.

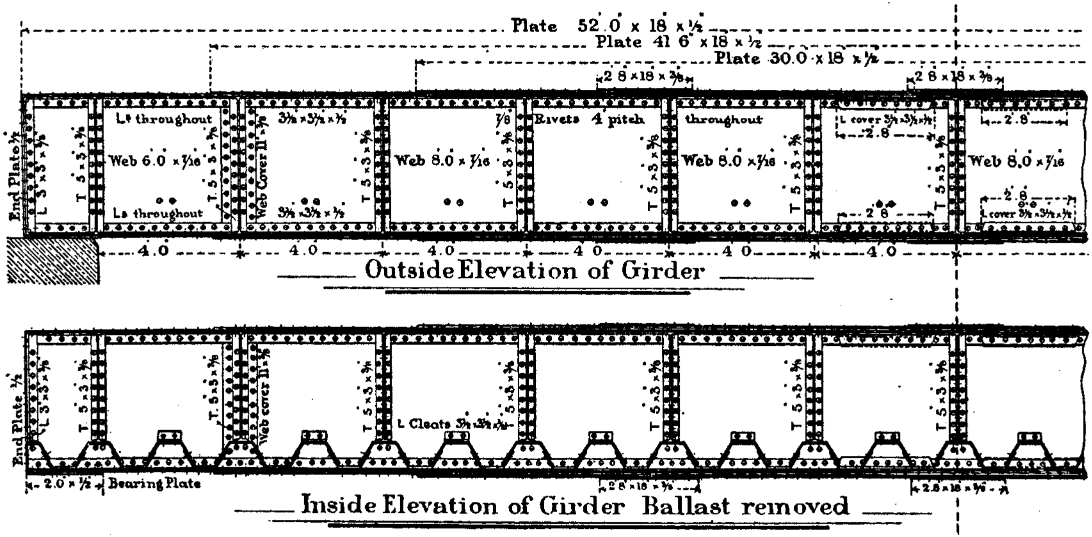
The web of a plate girder should be stiffened at intervals not exceeding the depth of the girder or a maximum of 5 feet by the introduction of angle or tee bars.

The depth of the girder in this case is 4 feet, and the vertical stiffeners will be placed 4 feet apart centre to centre.

No definite rules can be laid down for determining the most convenient lengths of plates and angle bars and the positions of the joints. These are questions that largely depend upon circumstances, and must be left to the judgment and experience of the designing engineering.

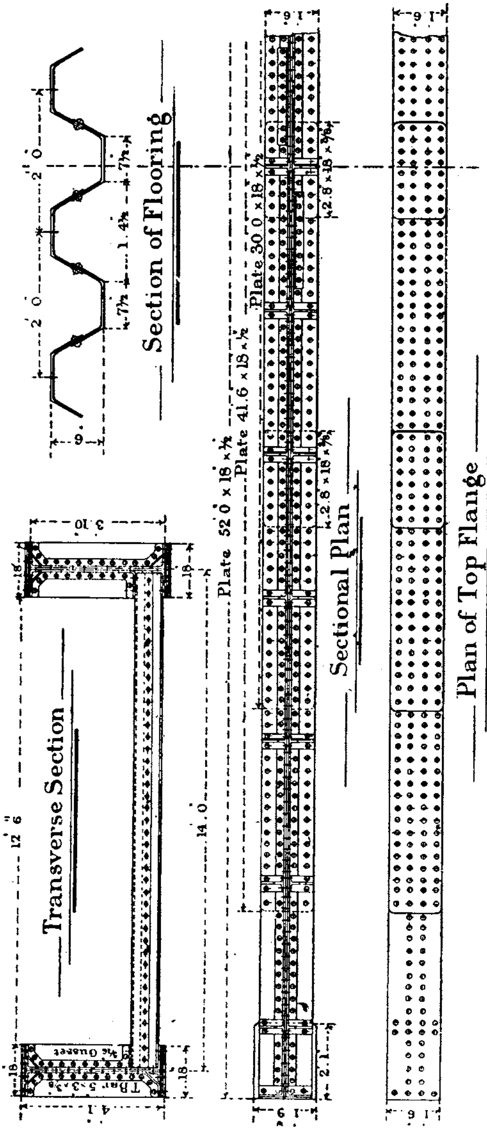
The details of construction of the girder forming the subject of this example, which are fully illustrated in Plate 8, may be accepted as a modern type of a railway plate girder bridge capable of carrying the heaviest traffic on a single line of railway.

Plate 8



Outside Elevation of Girder

Inside Elevation of Girder Ballast removed


Section of Flooring
Transverse Section
Sectional Plan
Plan of Top Flange
Plate 52 0 x 18 x ½
Plate 41.6 x 18 x ½
Plate 30 0 x 18 x ½
T Bar 5 x 3 x ⅜
⅜ Gusset
12 6
14 0
3 10
4 1
1 6
2 0
1 4½
7½

EXAMPLE No. 7.

Highway Bridge 78 feet span from centre to centre of end bearings. Width of roadway 24 feet.

The live load to be carried to be 1 cwt. per square foot of floor area, which is assumed to represent the weight of a densely packed crowd of people, or alternatively, a road roller weighing 15 tons, 6 tons being carried on the leading wheel and 9 tons on the trailing wheels. The distance between wheel centres = 12 feet, and total floor space covered = $19' \times 7'$.

The main girders to be of the outline indicated in Fig. 1, composed of 6 main panels, each 13 feet long, with sub-verticals depending from each apex of the triangulation of the upper chord, to the lower ends of which the cross girders are attached.

Floor System.

The floor is composed of $\frac{3}{8}$ cambered steel plates, their edges carried on and rivetted to longitudinal steel bearers or joists, extending from cross girder to cross girder, and spaced 4 feet apart, centre to centre, as shown in Fig. 2.

Longitudinal Bearers.

It is first required to determine the dimensions of the longitudinal bearers, the span of which is 15 feet.

The dead load to be carried by a longitudinal bearer $= 13' \, 0'' \times 4' \, 0''$ of floor area, the weight of which is made up as follows:

	Lbs.	Cwt.
$\frac{3}{8}$ cambered steel flooring, ...	845	
Concrete filling, ...	1750	
Road metalling, ...	2080	
Estimated weight of floor, ...	4675	
Estimated weight of beam, say 6 cwt.,	672	
Total dead load, ...	5347	= 47·75, say $2\frac{1}{2}$ tons.

Fig. 1.

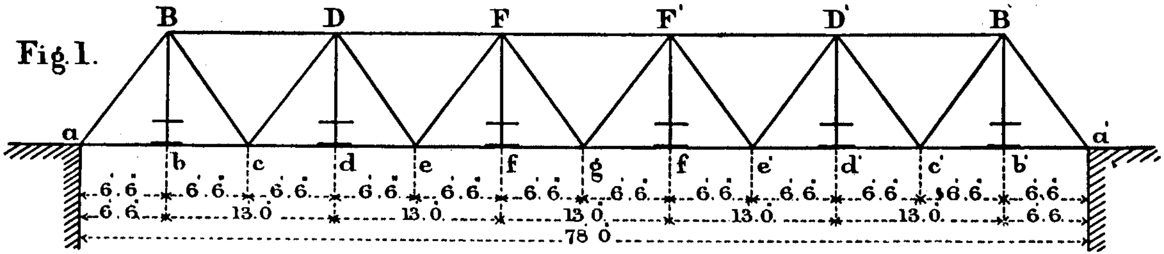
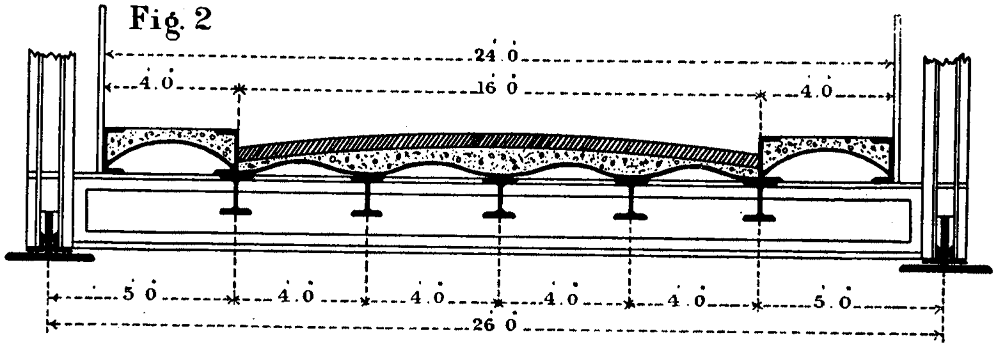

B
D
F
F'
D'
B'
a
b
c
d
e
f
g
f'
e'
d'
c'
b'
a'
6'.6"
13'.0"
78'.0"

Fig. 2


24'.0"
4'.0"
16'.0"
4'.0"
5'.0"
4'.0"
4'.0"
4'.0"
4'.0"
5'.0"
26'.0"

The greatest live load to be carried by any bearer is the leading wheel of a road roller weighing 6 tons. The total load will therefore be a concentrated live load of 6 tons, equivalent to a distributed load of 12 tons and a distributed dead load of $2\frac{1}{2}$ tons.

The bending moment will therefore be

$$\frac{14\cdot5 \text{ tons} \times 13 \text{ ft.} \times 12 \text{ in.}}{8} = 282\cdot75 \text{ inch tons.}$$

A rolled steel beam, $14'' \times 6''$, weighing 46 lbs. per foot run, having a moment of resistance of 62·94 inches^3, would be sufficiently strong for the purpose.

The working stress in the metal would be

$$\frac{\text{BM}}{\text{MR}} = \frac{282\cdot75}{62\cdot94} = 4\cdot5 \text{ tons per square inch.}$$

Cross Girders.

The dead load to be carried by a cross girder, consisting of a floor area of $13'\,0'' \times 24'\,0''$, with the weight of the cross girder itself, is made up as follows:

	Lbs.	Tons.
Steel flooring, ...	5070	
Concrete filling, ...	10500	
Road metalling, ...	8320	
Longitudinal bearers, ...	4347	
	28237 =	12·61
The weight of cross girder is estimated at		1·50
		14·11, say $14\frac{1}{2}$ tons.

The maximum live load to be carried by a cross girder is a uniformly distributed moving load of 1 cwt. per square foot of floor area, or the driving wheels of a road roller, whichever may be the greater.

The distributed load of 1 cwt. per sq. foot on one panel $24'\,0'' \times 13'\,0''$ .... = 15·6 tons.

The equivalent distributed load on the driving wheels of a road roller .... = 14·26 „

The former, being the greater of the two quantities, will be adopted as the live load to be carried, which may in round numbers be taken as 16 tons.

$$\text{Then the bending moment} = \frac{(16 \text{ tons} + 14{\cdot}5) \times 26}{8}$$

$$= 99{\cdot}1, \text{ say } 100 \text{ foot tons.}$$

The depth of the cross girder over all is 2′ 3″, but the effective depth is taken at 2′ 0″, which is a slight error on the safe side, and the working stress of the metal is assumed to be 6 tons per sq. inch.

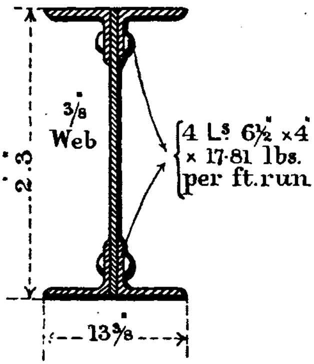
Then $\qquad fad = 6 \times 2' \, 0'' \times a = 12a,$

and $\qquad 12a = 100\,;$

$\qquad \therefore \; a = 100 \div 12 = 8{\cdot}33$ inches.

This sectional area would be provided by a girder of the section given in Fig. 3, the precise area of the tension flange

Fig. 3.



allowing for one $\frac{7}{8}''$ rivet in each angle being 9·55 sq. inches, which would give a margin of nearly 15 per cent. on the side of safety.

Shearing Stresses in Web of Cross Girder.

The shearing stress which is a maximum at the points of support = 15·25 tons. The depth of the girder is 2 feet; the stress per foot run vertically and horizontally will therefore be $15 \cdot 25 \div 2 = 7 \cdot 62$ tons. Assuming resistance to shear to be 6 tons per square inch, there would be required a sectional area of $7 \cdot 62 \div 6 = 1 \cdot 27$ sq. inch per foot run and $1 \cdot 27 \div 12 = \cdot 11$ as the theoretical thickness of the web, so that a $\frac{3}{8}''$ web will suffice.

Main Girders.

It is next required to estimate the weight of the main girders.

The total dead load per panel, exclusive of weight of main girders, ... = $14\frac{1}{2}$ tons.

The total live load per panel .... = 16 ,,

The total extraneous load to be carried by the bridge will therefore be $(14 \cdot 5 + 16) \times 6 = 183$ tons, and by each girder 91·5, say 92 tons.

Then, by Professor Unwin's formula, $W' = \dfrac{Wlr}{Cs - lr}$, in which

W = total external distributed load in tons = 92 tons.
W′ = weight of girder.
d = effective depth in feet = 8·5 ft.
l = clear span in feet = 78.
r = ratio of span to depth = 9.
s = stress on metal in tons per sq. inch = 6.
C = constant value = 1500.

Then, substituting,

$$W' = \frac{92 \times 78 \times 9}{1500 \times 6 - 75 \times 9}$$

$= 7 \cdot 8$, say 8 tons.

Total dead load on one panel of a main girder .... = $7 \cdot 25 + \frac{8}{6} = 8 \cdot 58$, say 9 tons.

Total live load on one panel of a main girder .... = 8 tons.

Stresses in Top and Bottom Chords.

The re-action at either abutment when the truss is fully loaded $=(8+9)\times 2{\cdot}75=46{\cdot}75$ tons.

Then

Max. tension in $ab=bc$

Tons.

$$=\frac{46{\cdot}75\times 6{\cdot}5}{8{\cdot}5} = 35{\cdot}75$$

Max. tension in $cd=eg$

$$=\frac{46{\cdot}75\times 19{\cdot}5-12{\cdot}75\times 13}{8{\cdot}5} = 87{\cdot}74$$

Max. tension in $eg=gh$

$$=\frac{46{\cdot}75\times 32{\cdot}5-(17\times 13+12{\cdot}75\times 26)}{8{\cdot}5} = 113{\cdot}75$$

Max. compression in BD

$$=\frac{46{\cdot}75\times 13-12{\cdot}75\times 6{\cdot}5}{8{\cdot}5} = 61{\cdot}75$$

Max. compression in DF

$$=\frac{46{\cdot}75\times 26-(17\times 6{\cdot}5+12{\cdot}75\times 19{\cdot}5)}{8{\cdot}5} \quad 100{\cdot}75$$

Max. compression in FF′

$$=\frac{46{\cdot}75\times 39-(17\times 6{\cdot}5+17\times 19{\cdot}5+12{\cdot}75\times 32{\cdot}5)}{8{\cdot}5} = 113{\cdot}75$$

Web Stresses.

Apices.		*a*B.	B*c*.	*c*D.	D*e*.	*e*F.	F*g*.
Load at		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
Point 1, ...		+ 9·24	+ 0·80	− 0·80	+ 0·80	− 0·80	+ 0·80
,, 3, ...		+ 7·56	− 7·56	+ 7·56	+ 2·52	− 2·52	+ 2·52
,, 5, ...		+ 5·88	− 5·88	+ 5·88	− 5·88	+ 5·88	+ 4·20
,, 7, ...		+ 4·20	− 4·20	+ 4·20	− 4·20	+ 4·20	− 4·20
,, 9, ...		+ 2·52	− 2·52	+ 2·52	− 2·52	+ 2·52	− 2·52
,, 11, ...		+ 0·80	− 0·80	+ 0·80	− 0·80	+ 0·80	− 0·80
Live Load.	Compression,	+ 30·20	+ 0·80	+ 20·96	+ 3·32	+ 13·40	+ 7·52
	Tension, ...	—	− 20·96	− 0·80	− 13·40	− 3·32	− 7·52
Dead Load, ...		+ 34·02	− 22·68	+ 22·68	− 11·34	+ 11·34	—
Max. compression,		+ 64·22	+ 0·80	+ 43·64	+ 3·32	+ 24·74	+ 7·52
Max. tension, ...		—	− 43·64	− 0·80	− 24·74	− 3·32	− 7·52

The stress on all the sub verticals is unform throughout, except the two end ones, and it is equal to the re-action of the cross girder, and its load = 30·5 tons ÷ 2 = 15·25 tons. In the two end sub-verticals the stress = 11·45 tons.

$\frac{a'B}{Bb} = 1{\cdot}26.$

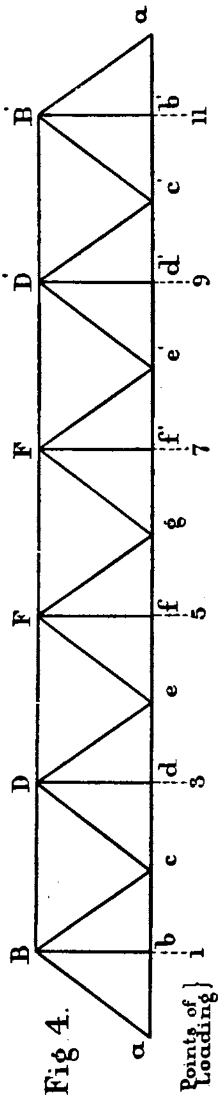


Fig 4.

STRESSES AND DIMENSIONS OF MEMBERS.

Member.		Max. Comp.	Max. Tension.	Total Stress.	Section required.	Make up.	Section given.
		Tons.	Tons.	Tons.	Sq. Ins.		Sq. Ins.
Top Chord	BD	+ 61·75	—	+ 61·75	10·30	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 15\cdot31$ lbs. and 1 web plate $14'' \times \frac{3}{8}''$	14·25
,,	DF	+100·75	—	+100·75	16·80	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 1 web plate $14'' \times \frac{3}{8}''$ and 1 flange plate $24'' \times \frac{3}{8}''$	23·25
,,	FF	+113·75	—	+113·75	18·96	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 1 web plate $14'' \times \frac{3}{8}''$ and 1 flange plate $24'' \times \frac{3}{8}''$	23·25
Bottom Chord	*ab*=*bc*	—	- 35·75	- 35·75	5·96	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 15\cdot31$ lbs. and 1 web plate $14'' \times \frac{3}{8}''$	12·18
,,	*cd*=*de*	—	- 87·74	- 87·74	14·62	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 1 web plate $14'' \times \frac{3}{8}''$ and 1 flange plate $24'' \times \frac{3}{8}''$	20·52
,,	*ef*=*fg*	—	-113·75	-113·75	18·96	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 1 web plate $14'' \times \frac{3}{8}''$ and 1 flange plate $24'' \times \frac{3}{8}''$	20·52
Web	*a*B	+ 64·22	—	+ 64·22	10·70	2 Angles, $6'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' \times 11\cdot64$ lbs. and 1 web plate $14'' \times \frac{3}{8}''$	12·07
,,	B*c*	+ 0·80	- 43·64	± 44·44	7·41	4 Angles, $5'' \times 5'' \times \frac{1}{2}'' \times 16\cdot15$ lbs. per foot run	+19·0 - 8·25
,,	*c*D	+ 43·64	- 0·80	± 44·44	7·41	4 Angles, $5'' \times 5'' \times \frac{1}{2}'' \times 16\cdot15$ lbs. per foot run	+19·0 - 8·25
,,	D*e*	+ 3·32	- 24·74	± 28·06	4·68	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 11\cdot05$ lbs. per foot run	+13·00 - 5·25
,,	*e*F	+ 24·74	- 3·32	± 28·06	4·68	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 11\cdot05$ lbs. per foot run	+13·00 - 5·25
,,	F*g*	+ 7·52	- 7·52	± 15·04	2·50	4 Angles, $3'' \times 3'' \times \frac{1}{2}'' \times 9\cdot36$ lbs. per foot run	+11·0 - 4·5
,,	B*b*	—	- 11·45	- 11·45	2·0	4 Angles, $3'' \times 3'' \times \frac{1}{2}'' \times 9\cdot36$ lbs. per foot run	4·5
,,	D*d*	—	- 15·25	- 15·25	2·64	4 Angles, $3'' \times 3'' \times \frac{1}{2}'' \times 9\cdot36$ lbs. per foot run	4·5
,,	F*f*	—	- 15·25	- 15·25	2·64	4 Angles, $3'' \times 3'' \times \frac{1}{2}'' \times 9\cdot36$ lbs. per foot run	4·5

PLATE 9.

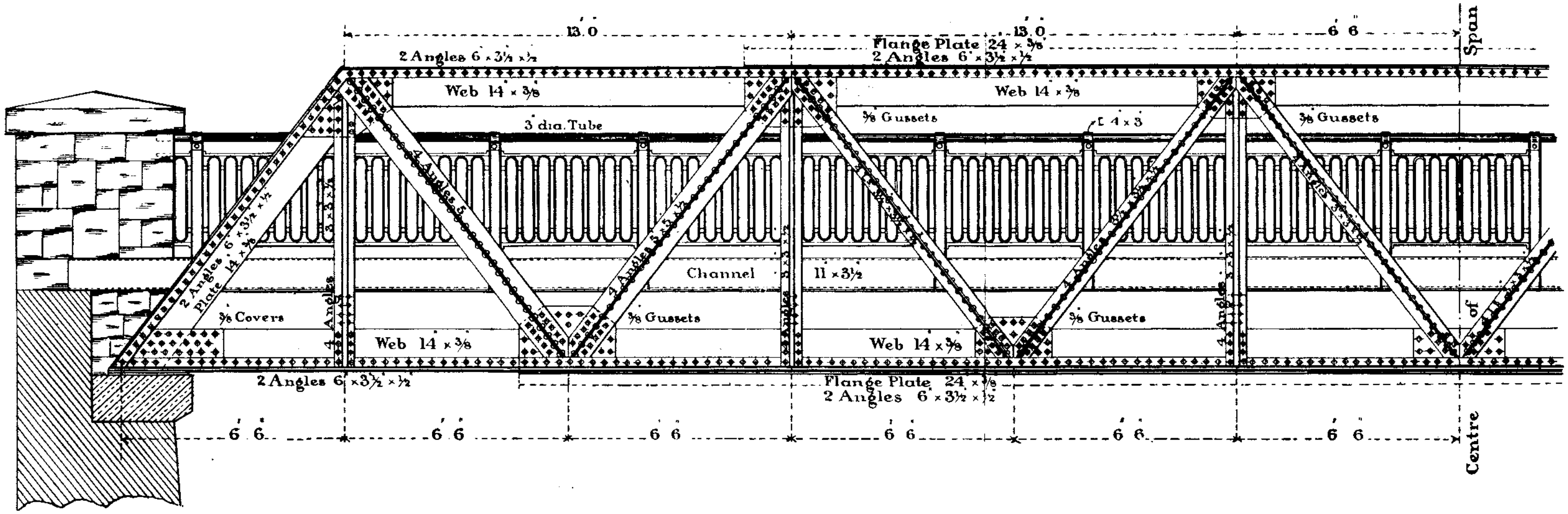


PLATE 9A.

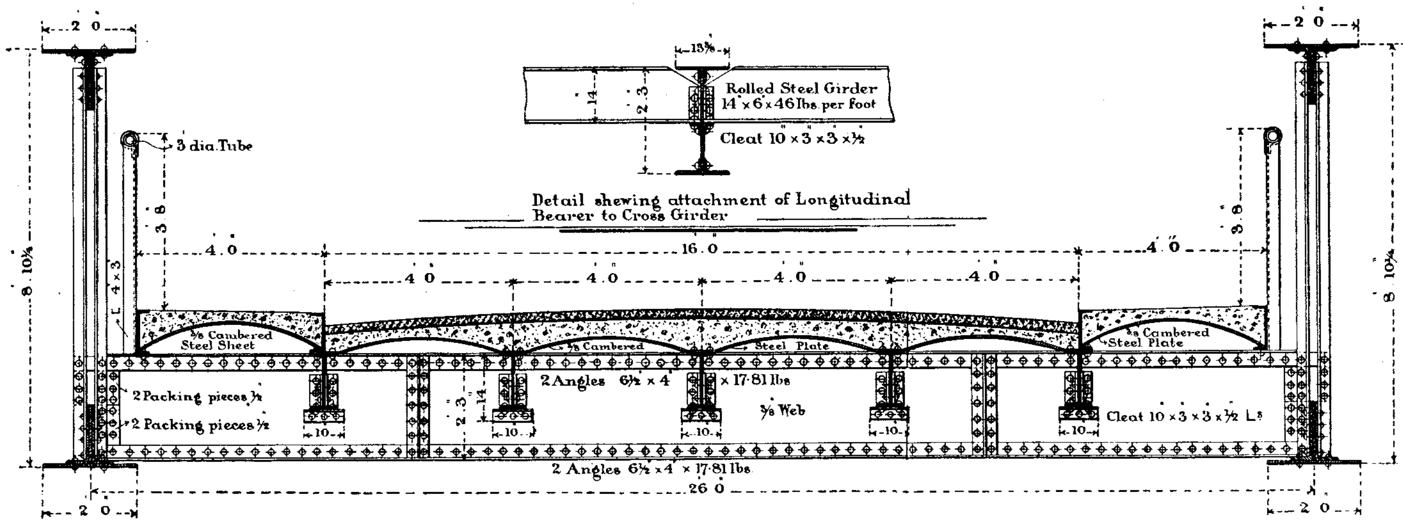
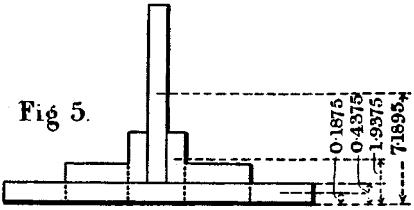


Fig 5.



Eccentricity.

$14\frac{3}{8} \times \frac{3}{8}$	....	$5{\cdot}3906 \times 7{\cdot}1875 =$	$38{\cdot}7449$
$3\frac{7}{8} \times \frac{1}{2} \times 2$	....	$3{\cdot}8750 \times 1{\cdot}9375 =$	$7{\cdot}5078$
$5\frac{1}{2} \times \frac{1}{2} \times 2$	....	$5{\cdot}5000 \times 0{\cdot}4375 =$	$2{\cdot}4062$
$11\frac{5}{16} \times \frac{3}{8} \times 2$	....	$8{\cdot}4840 \times 0{\cdot}1875 =$	$1{\cdot}5907$
		$23{\cdot}2496$	$50{\cdot}2496$

$\frac{50{\cdot}2496}{23{\cdot}2496} = 2{\cdot}16''$ the distance from the bottom of the lower flange to the centre of gravity of the section.

Full details of the construction of this bridge are given in Plates 9 and 9A.

EXAMPLE No. 8.

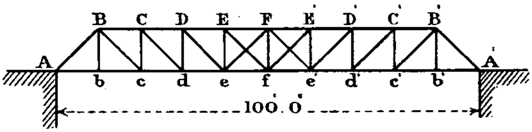
Murphy-Whipple or Pratt Truss Railway Bridge of 100 feet effective span, to carry a single line of railway.

Effective span, ... 100 ft.
Depth of girder, ... 10 ft.
Width from centre to centre of girders, ... 16 ft. 6 in.

The cross girders are fixed to the main girders at each panel point 10 feet apart from centre to centre. The floor to consist of Dorman, Long & Co.'s steel corrugated plates, section C′ maximum, weighing 35 lbs. per sup. foot, the troughs to be filled with concrete and the floor covered with a layer of ballast.

A diagram of the girder is shown in Fig. 1.

Fig. 1.



The maximum live or rolling load to be carried by the bridge is 216 tons, the half of which (= 108 tons) is carried by each girder, so that the panel live load for one girder = 10·8 tons.

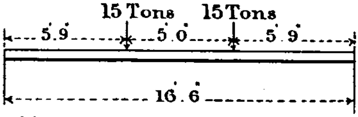


Fig. 2

The load on each cross girder, allowing for the heaviest type of locomotive engine and for impact, is as shown in Fig. 2.

The equivalent uniformly distributed load

$$=\frac{15 \times 5{\cdot}75 \times 2 \times 4}{16{\cdot}5}=41{\cdot}8 \text{ tons.}$$

The dead load to be carried by a cross girder is estimated to be as follows:

Steel trough flooring,	2·2	tons.
Concrete filling,	2·0	„
Ballasting,	5·0	„
Permanent way,	0·75	„
Total dead load on cross girder, excluding weight of girder,	9·95,	say 10 tons.

It is next required to estimate the weight of the cross girder.

The equivalent distributed live load has been shown to be 42 tons, and the weight of floor, etc., 10 tons, so that the total extraneous load $= 42 + 10 = 52$ tons.

Then, by Professor Unwin's formula, the weight of the cross girder $=\frac{\mathrm{W}lr}{\mathrm{C}s - lr}$. Substituting, and taking the value of r, that is to say, the ratio of span to depth as 10, we have

$$\frac{52 \times 16{\cdot}5 \times 10}{1280 \times 6 - 16{\cdot}5 \times 10}=1{\cdot}14 \text{ tons.}$$

The total distributed load (dead and live) carried by a cross girder will therefore be $52 + 1{\cdot}14$ tons $= 53{\cdot}14$, say 53 tons, and the maximum bending moment $=\frac{53 \times 16{\cdot}5}{8 \times 1{\cdot}65}=66{\cdot}25$ tons.

The total dead load carried by a cross girder is				11·14 tons.
The total live load	„	„	„	42·00 „

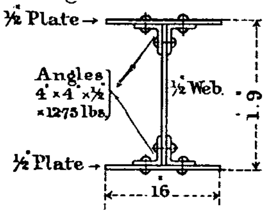
The ratio of total live load to total dead load is therefore in round numbers 4 to 1.

Allowing a working stress of 5 tons for live load and 7 tons for dead load, the working stress for combined dead and live loads $= \frac{7 \times 1 + 5 \times 4}{1 + 4} = 5{\cdot}4$ tons per square inch.

The total nett area required in the flanges of the cross girder at the centre will then be $66{\cdot}25 \div 5{\cdot}4 = 12{\cdot}3$ sq. inches.

This area would be supplied by a girder of the section given in Fig. 3.

Fig. 3



The dimensions of the cross girder having been determined its weight can now be accurately obtained, thus:

	Lbs.
66′ 0″ L bars. 4″ × 4″ × ½″ × 12·75 lbs....... =	841·5
16′ 5″ × 1′ 9″ × ½″ × 20·4 lbs. web .... =	589·0
15′ 0″ × 1′ 4″ × ½″ × 2 × 27·2 lbs. flanges ..... =	816·0
6′ 8″ × 5″ × 4″ × ½″ T stiffeners × 14·5 .... =	86·0
4′ 0″ × 5 × ½″ × 8·5 lbs. per foot packing .. =	34·0
	2366·5
Allow for rivets 5 per cent., ...	118·0
Total weight of cross girder = 1·11 tons, . .	2484·5

The estimated weight was 1·14 tons, which may therefore be accepted as correct.

It now remains to estimate the weight of the main girders.

The live or rolling load is assumed to be 216 tons, the half of which is carried by each girder = 108 tons.

The load carried by each main girder, excluding the weight of the girder itself, will be as follows:

		Tons.
Dead load due to floor system	$= \frac{11 \cdot 14 \times 10}{2} =$	55·7
Live load	$=$	108·0
Total extraneous load = say 164 tons	$=$	163·7

Adopting Professor Unwin's formula, the weight of a main girder $= \frac{164 \times 100 \times 10}{1500 \times 6 - 100 \times 10} = 20 \cdot 5$, say 21 tons.

The total load on each main girder will then be

Dead load $= 55 \cdot 7 + 21$ tons	$=$ say	76	tons.
Live load	$=$	108	„
Dead load on each panel point $= 76 \div 10 =$		7·6	„
Live load „ „ $= 108 \div 10 =$		10·8	„

Stresses in Top and Bottom Chords.

The re-action at either abutment when the truss is fully loaded $= (7 \cdot 6 + 10 \cdot 8) 4 \cdot 5 = 82 \cdot 8$ tons.

Then

$$\text{Max. tension in } \mathrm{A}b = bc = \frac{82 \cdot 8 \times 10}{10} = 82 \cdot 8 \text{ Tons.}$$

$$\left.\begin{array}{l}\text{Max. tension in } cd = \\ \text{Max. comp in BC} =\end{array}\right\} \frac{82 \cdot 8 \times 20 - 18 \cdot 4 \times 10}{10} = 147 \cdot 2$$

$$\left.\begin{array}{l}\text{Max. tension in } de = \\ \text{Max. comp. in CD} =\end{array}\right\} \frac{82 \cdot 8 \times 30 - 18 \cdot 4 \times 10(1+2)}{10} = 193 \cdot 2$$

$$\left.\begin{array}{l}\text{Max. tension in } ef = \\ \text{Max. comp. in DE} =\end{array}\right\} \frac{82 \cdot 8 \times 40 - 18 \cdot 4 \times 10(1+2+3)}{10} = 220 \cdot 8$$

$$\text{Max. comp. in EF} = \frac{82 \cdot 8 \times 50 - 18 \cdot 4 \times 10(1+2+3+4)}{10} = 230 \cdot 0$$

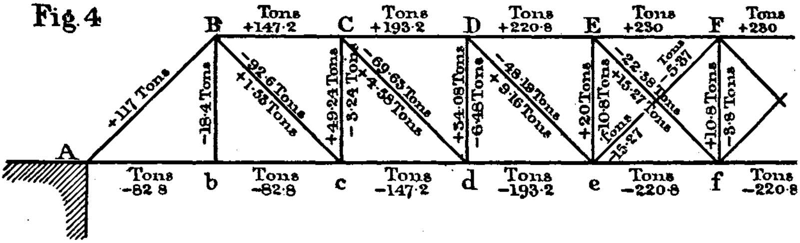
WEB STRESSES.

Apices.		AB.	B*b*.	B*c*.	C*c*.	C*d*.	D*d*.	D*e*.	E*e*.	E*f*.	F*f*.	*e*F.
Load at		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
b, ...		+13·74	−10·8	+ 1·53	− 1·08	+ 1·53	− 1·08	+ 1·53	− 1·08	+ 1·53	+1·08	− 1·53
c, ...		+12·22	—	−12·22	− 2·16	+ 3·05	− 2·16	+ 3·05	− 2·16	+ 3·05	+2·16	− 3·05
d, ...		+10·69	—	−10·69	+ 7·56	−10·69	− 3·24	+ 4·58	− 3·24	+ 4·58	+3·24	− 4·58
e, ...		+ 9·16	—	− 9·16	+ 6·48	− 9·16	+ 6·48	− 9·16	− 4·32	+ 6·11	+4·32	− 6·11
f, ...		+ 7·64	—	− 7·64	+ 5·40	− 7·64	+ 5·40	− 7·64	+ 5·40	− 7·64	—	—
e′, ...		+ 6·11	—	− 6·11	+ 4·32	− 6·11	+ 4·32	− 6·11	+ 4·32	− 6·11	—	—
d′, ...		+ 4·58	—	− 4·58	+ 3·24	− 4·58	+ 3·24	− 4·58	+ 3·24	− 4·58	—	—
c′, ...		+ 3·05	—	− 3·05	+ 2·16	− 3·05	+ 2·16	− 3·05	+ 2·16	− 3·05	—	—
b′, ...		+ 1·53	—	− 1·53	+ 1·08	− 1·53	+ 1·08	− 1·53	+ 1·08	− 1·53	—	—
Live load.	Comp., ...	+67·72	—	+ 1·53	+30·24	+ 4·58	+22·68	+ 9·16	+16·20	+15·27	+10·8	—
	Tension,	—	−10·8	−54·98	− 3·24	−42·76	− 6·48	−32·07	−10·80	−22·91	—	−15·27
Dead load, ...		+ 48·36	− 7·6	−37·61	+19·00	−26·87	+11·40	−16·12	+ 3·80	− 5·37	− 3·8	+ 5·37
Max. Comp., ...		+117·08	—	+ 1·53	+49·24	+ 4·58	+34·08	+ 9·16	+20·00	+15·27	+10·8	+ 5·37
Max. Tension,		—	−18·4	−92·59	− 3·24	−69·63	− 6·48	−48·19	−10·80	−28·28	− 3·8	−15·27

STRESSES AND DIMENSIONS OF MEMBERS.

Member.		Max. Comp.	Max. Tension.	Total Stress.	Section required.	Make up.	Section given.
		Tons.	Tons.	Tons.	Sq. Ins.		Sq. Ins.
Top Chord,	BC	147·2	—	147·2	24·53	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 2 webs $15'' \times \frac{1}{2}''$ and flange plate $2'\ 3'' \times \frac{1}{2}''$	41·5 gross.
,,	CD	193 2	—	193·2	32·20	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 2 webs $15'' \times \frac{1}{2}''$ and flange plate $2'\ 3'' \times \frac{1}{2}''$	41·5 ,,
,,	DE	220·8	—	220·8	36·80	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 2 webs $15'' \times \frac{1}{2}''$ and flange plate $2'\ 3'' \times \frac{1}{2}''$	41·5 ,,
,,	EF	230·0	—	230·0	38·33	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 2 webs $15'' \times \frac{1}{2}''$ and flange plate $2'\ 3'' \times \frac{1}{2}''$	41·5 ,,
Bottom Chord,	*Ab* = *bc*	—	82·8	82·8	13·80	2 Bars $15'' \times \frac{5}{8}''$ with stiffeners of T's $6'' \times 3'' \times \frac{3}{8}'' \times 11{\cdot}08$ lbs.	14·375 nett.
,,	*cd*	—	147·2	147·2	24·53	2 Bars $15'' \times \frac{5}{8}''$ and 2 Bars $15'' \times \frac{1}{2}''$ - - -	25·875 ,,
,,	*de*	—	193·2	193·2	32·20	2 Bars $15'' \times \frac{5}{8}''$ and 4 Bars $15'' \times \frac{1}{2}''$ - - -	37·375 ,,
,,	*ef*	—	220·8	220·8	36·80	2 Bars $15'' \times \frac{5}{8}''$ and 4 Bars $15'' \times \frac{1}{2}''$ - - -	37·375 ,,
Web Verticals,	B*b*	—	18·4	18·4	3·07	4 Angles, $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$ weighing 7·81 lbs. per foot each angle	3·188 ,,
,,	C*c*	49·24	3·24	52·48	8·75	4 Angles, $4\frac{1}{2}'' \times 3\frac{1}{2}'' \times {\cdot}425''$ weighing 10·94 lbs. per foot each angle	12·87 gross.
,,	D*d*	34·08	6·48	40·56	6·76	4 Angles, $4'' \times 3\frac{1}{2}'' \times {\cdot}425''$ weighing 10·22 lbs. per foot each angle	12·00 ,,
,,	E*e*	20·00	10·80	30·8	5·13	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times {\cdot}425''$ weighing 9·5 lbs. per foot each angle	11·18 ,,
,,	F*f*	10·80	3·80	14·6	2·43	4 Angles, $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$ weighing 7·81 lbs. per foot each angle	9·19 ,,
Web Diagonals,	AB	117·00	—	117·0	19·50	4 Angles, $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, 2 webs $15'' \times \frac{1}{2}''$ and 1 flange $2'\ 3'' \times \frac{1}{2}''$	41·5 ,,
,,	B*c*	1·53	92·60	94·13	15·69	2 Plates $14'' \times \frac{3}{4}''$ after deducting 4 rivet holes $\frac{7}{8}''$	15·75 nett.
,,	C*d*	4·58	69·63	74·21	12·37	2 Plates $11'' \times \frac{3}{4}''$ after deducting 3 rivet holes $\frac{7}{8}''$	12·56 ,,
,,	D*e*	9·16	48·19	57·35	9·56	2 Plates $10'' \times \frac{11}{16}''$ after deducting 3 rivet holes $\frac{7}{8}''$	10·14 ,,
,,	E*f*	15·27	28·28	43·55	7·26	2 Plates $8'' \times \frac{5}{8}''$ after deducting 2 rivet holes $\frac{7}{8}''$	7·97 ,,
,,	*e*F	5·37	15·27	20·64	3·44	2 Plates $6'' \times \frac{1}{2}''$ after deducting 2 rivet holes $\frac{7}{8}''$	4·25 ,,

Fig. 4



The ratio of dead load to live load is as 76 : 108, or approximately 2 to 3. Allowing a working stress of 7 tons per square inch for dead load and 5 tons for live load, the working stress for combined loads $= \dfrac{7 \times 2 + 5 \times 3}{2 + 3} = 5{\cdot}8$, say 6 tons per square inch.

Section of Top Chord and End Diagonal.

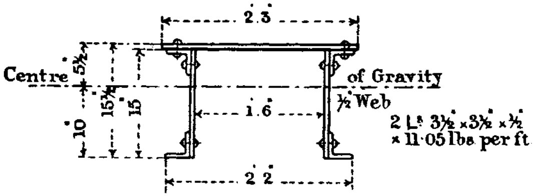


Fig. 5.

Centre of Gravity of Section.

The areas of the several members are as follow:

Flange plate	....	$= 13\frac{1}{2}$	sq. inches.
Web plate,	$15'' \times \frac{1}{2} = 7\frac{1}{2} \times 2$	$= 15$	,,
Two angles,	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2} = 6\frac{1}{2} \times 2$	$= 13$	,,
	Total, 14 sq. inches,	$41\frac{1}{2}$	,,

The centre of gravity of each of the built-up web members is evidently $7\frac{1}{2}$ inches from the bottom and that of the flange plate $15\frac{1}{4}$ inches.

Let c denote the distance of the centre of gravity of the whole section from the bottom.

Then $41\cdot5 \times c = 14 \times 7\frac{1}{2} \times 2 + 13\frac{1}{2} \times 15\frac{1}{4}$,

$$41\cdot5c = 415\cdot875,$$

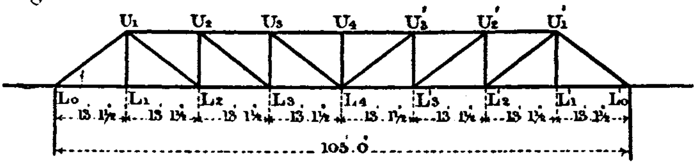
and $c = 415\cdot875 \div 41\cdot5 = 10\cdot02$ inches, say 10 inches.

Full details of the construction of this bridge are given in Plates 10 and 10A.

EXAMPLE No. 9.

Burma Railways (Metre Gauge)—100 feet clear span Pony Truss.

Fig. 1



Trusses.

105′ 0″ span centres of bearings (8 panels × 13′ 1½″).
10′ 0″ effective depth.
15′ 8″ width from centre to centre of main girders.

Loading.

		Tons.
Dead Load.	Steel superstructure, ...	60·8
	Permanent way, ...	6·5
	Footways, ...	5·0
	Total for 2 trusses, ...	72·3

Wind. On loaded span at 1·5 tons per 100 sq. ft.
On train area ... = 13·13 tons.
On bridge .... = 10·87 „
24·0 „

On empty span at 2·5 tons per 100 sq. ft.
On bridge area ... = 24 tons.

Live Load.

For chord stresses = 166·3 tons.

Impact calculated by the formula $I = \frac{300}{L+300}S$.

Column formula:

$$P = 8\left(0{\cdot}95 - 0{\cdot}003\frac{l}{r}\right)$$ tons with a max. P of 6·8 tons.

S = stress due to train load considered as at rest in the position which gives the maximum stress in the member under consideration.

L = the length in feet of that portion of the span which the train has to traverse to reach that position from the point where it first began to produce stress in the member.

I = amount to be added to S to allow for 'impact.'

P = stress allowed in tons per square inch.

l = length of member in inches, centre to centre of connections.

r = least radius of gyration in inches.

Stringers.

Span 13′ 1½″.

			Tons.
Loading.	Live $\frac{38{\cdot}2}{2}$		= 19·1
	Impact (·958)		= 18·3
	Dead	Steel	= 0·47
		Pt. way	= 0·4
			38·27

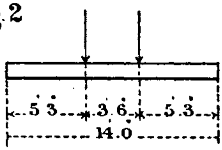
Bending moment $= 754$ inch tons.

Resistance required $= \frac{754}{8} = 94{\cdot}3$ inch units.

Resistance of rolled steel beam $18'' \times 7'' \times 75$ lbs.
$= 127{\cdot}7$ inch units.

Cross Girders $13'\ 1\frac{1}{2}''$ *apart.*

Fig. 2



Effective span $14'\ 0''$.

Depth over all, $30''$; effective depth $= 27''$.

Cross Girder.

Loading. At each stringer point:

		Tons.
Live load	$= 29{\cdot}6 \div 2 =$	$14{\cdot}8$
Impact ($\cdot 92$)	$=$	$13{\cdot}6$
Dead load, own weight ...	$=$	$0{\cdot}38$
„ from stringer	$=$	$0{\cdot}87$
		$29{\cdot}65$

Then stress at centre of girder

$$= \frac{29{\cdot}65 \times 5{\cdot}25}{2{\cdot}25} = 69{\cdot}2 \text{ tons.}$$

Section required .... $= 69{\cdot}2 \div 8 = 8{\cdot}7$ sq. inch net.

Section given $= \frac{1}{8}$ of web $(30 \times \frac{3}{8}) = 1{\cdot}41$ sq. inch.

$2 \angle s\ 4 \times 4 \times \frac{9}{16}\ (-\frac{7}{8}) = 7{\cdot}38$ „

$8{\cdot}79$ „

Shear. $\frac{29{\cdot}65}{5} = 5{\cdot}93$ sq. inch required.

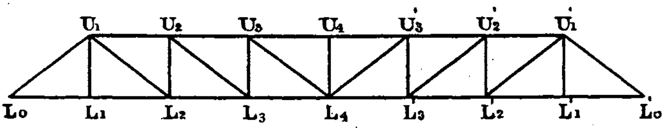
Web $30'' \times \frac{3}{8} = 11{\cdot}25$ sq. inch given.

FIGURES FOR

Piece.	Impact Co-eff.	Dead Load.	Live Load.	Impact.	Wind.	Maxima.		l
						No Wind.	Plus Wind.	
U_1U_2	·741	+35·9	+81·8	+60·6	—	+178·3	—	158
U_2U_3	·741	+45·0	+102	+75·8	—	+222·8	—	158
U_3U_4	·741	+47·8	+109	+80·7	—	+237·5	—	158
L_0L_1	·741	−20·9	−48	−35·5	+ 8·7	−104·4	− 95·7	—
L_1L_2	·741	−20·9	−48	−35·5	− 8·7 or +15	−104·4	−113·1	—
L_2L_3	·741	−35·9	−81·8	−60·6	−15 or +18·8	−178·3	−193·3	—
L_3L_4	·741	−45·0	−102	−75·8	−18·8 or +20	−222·8	−241·6	—
U_1L_1	·92	− 4·55	−14·8	−13·5	—	− 32·9	—	—
U_2L_2	·8	+ 6·85	+21·6	+17·3	—	+ 45·8	—	108
U_3L_3	·833	+ 2·28	+14·9	+12·4	—	+ 29·6	—	108
U_4L_4	—	nil.	nil.	—	—	—	—	—
$U_3'L_3'$	·909	+ 2·28	− 4·7	− 4·3	—	− 6·7	—	—
U_1L_0	·741	+26·4	+62·5	+46·4	—	+135·3	—	198
U_1L_2	·769	−18·9	−48·5	−37·4	—	−104·8	—	—
U_2L_3	·8	−11·3	−35·8	−28·7	—	− 75·8	—	—
U_3L_4	·833	− 3·8	−24·5	−20·4	—	− 48·7	—	—
$U_3'L_4$	·87	− 3·8	+14·8	+13·0	—	+ 24·0	—	198
$U_2'L_3'$	·909	−11·3	+ 7·5	+ 7·0	—	+ 3·2	—	198
$U_1'L_2'$	·95	−18·9	+ 2·6	+ 2·5	No Reversal.			

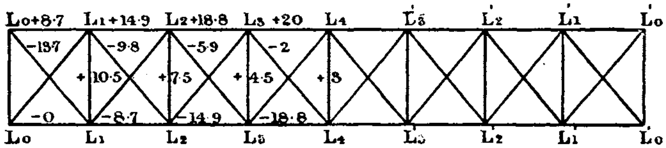
ONE TRUSS.

r	$\frac{l}{r}$	P.	Square Inches required.	Make up.	Square Inches given.
4	40	6·64	26·9	2 Chs. 10″ × 3½″ × 23·6 lbs. = 13·85 sq. in. 1 Plate 30″ × ½″ = 15·00 ,,	28·85
4	40	6·64	33·6	As above = 28·85 ,, +2 Plates 8″ × 7/16″ = 7·00 ,,	35·85
4	40	6·64	35·8	Do. do. do.	35·85
—	—	8	Net 13·1	4 Ls. 6″ × 3½″ × ⅜″ (less 1½″) = 11·44 ,, 1 Plate (20⅝″ − 4″) × ⅜″ = 6·23 ,,	Net 17·67
—	—	8	Net 13·1	Do. do. do.	Net 17·67
—	—	8	Net 22·3	As above = 17·67 ,, +2 Plates (12½″ − 2½″) × ⅜″ = 7·50 ,,	Net 25·17
—	—	8	Net 27·9	4 Ls. 6″ × 6″ × ⅜″ (− 2″) = 14·44 ,, 2 Plates (12½″ − 2½″) × ⅜″ = 7·50 ,, 1 Plate (20⅝″ − 4″) × ⅜″ = 6·23 ,,	Net 28·17
—	—	8	Net 4·1	4 Ls. 4″ × 3″ × ⅜″ (− 1½″) - - - -	Net 7·68
2	54	6·3	7·3	4 Ls. 4″ × 3″ × ⅜″ - - - - - -	9·92
2	54	6·3	4·7 + ·42	Do. do. do.	9·92
—	—	—	—	Do. do. do.	9·92
—	—	8	·84	Add $\frac{\cdot 84}{2} = \cdot 42$ sq. in. to U_3L_3 post - -	—
4	50	6·4	21·2	Same as U_1U_2 (above) - - - - -	28·85
—	—	8	Net 13·1	4 Ls. 6″ × 3½″ × ½″ (− 1¾″) - - -	Net 14·52
—	—	8	9·5 Net ·3	4 Ls. 6″ × 3½″ × 7/16″ (− 1¾″) - - -	Net 12·8
—	—	8	6·1 Net 2·2	4 Ls. 6″ × 3½″ × ⅜″ (− 1¾″) - - -	Net 11·08
2·5	80	5·68	4·25	Add $\frac{4\cdot 25}{2} = 2\cdot 125$ sq. in. to U_3L_4 Tie	—
2·5	80	5·68	0·6	Add $\frac{\cdot 6}{2} = \cdot 3$ sq. in. to U_2L_3 Tie	—



Lateral Bracing.

Wind = 24 tons (span empty or loaded).



For all diagonals, 1 ∠ $3\frac{1}{2} \times 3 \times \frac{1}{2}$ $(-1\frac{1}{4}) = 2·38$ sq. inch net.

Live Load Shears.

L_0L_1	L_1L_2	L_2L_3	L_3L_4	$L_4L'_3$	$L'_3L'_2$	$L'_2L'_1$	Panel
105	90	75	60	45	30	15	Feet loaded for Max.
$\frac{105\times7}{2\times8}$ 1·647	$\frac{90\times6}{2\times8}$ 1739	$\frac{75\times5}{2\times8}$ 1·856	$\frac{60\times4}{2\times8}$ 1·971	$\frac{45\times8}{2\times8}$ 2·122	$\frac{30\times2}{2\times8}$ 2·425	$\frac{15\times1}{2\times8}$ 3·334	Vert. Shears 2 Trusses
37·8	29·3	21·7	14·76	8·93	4·55	1·56	Do. 1 Truss
62·5	48·5	35·8	24·5	14·8	7·5	2·5	Diagonal 1 Truss

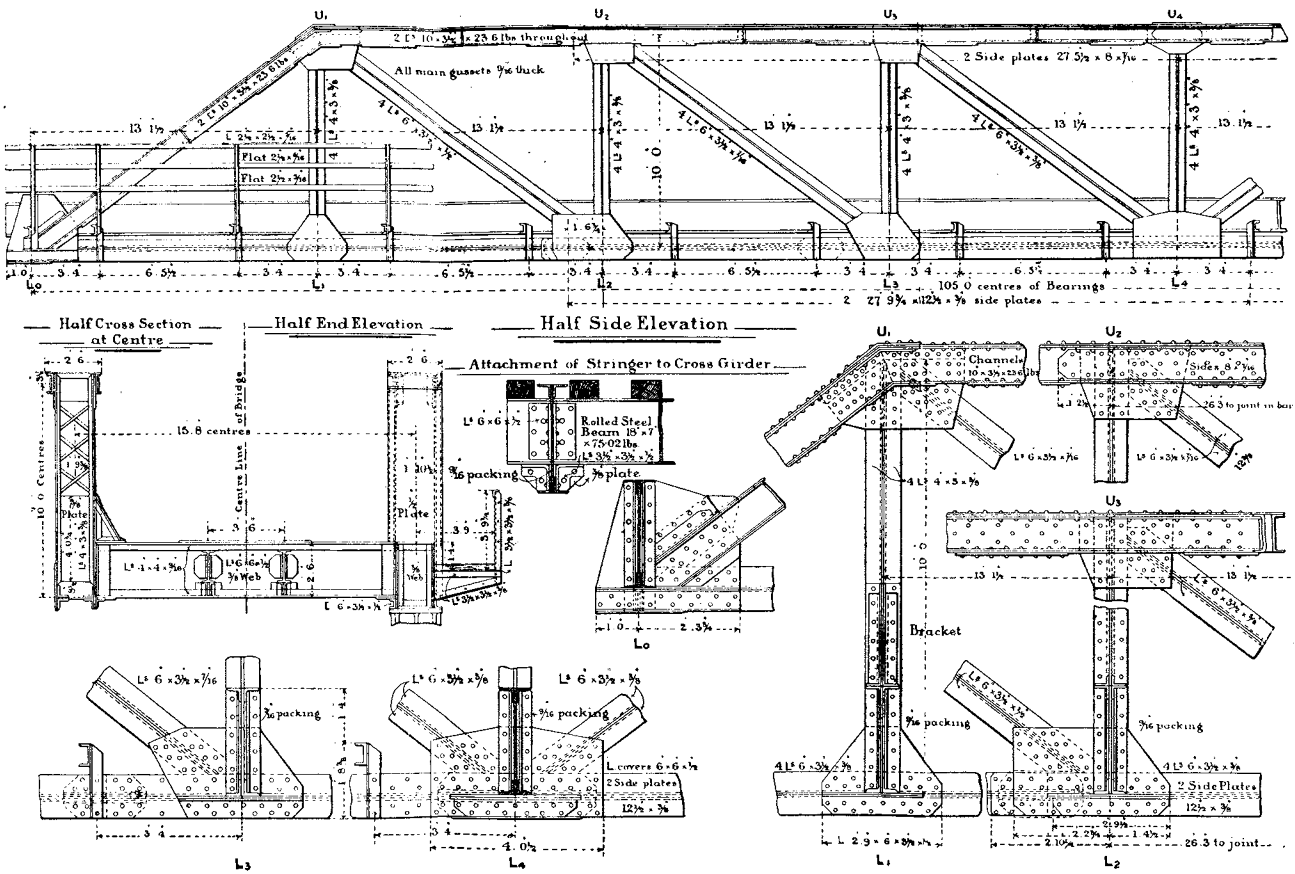
Rivets. Permissible stress, 5 tons per sq. inch for shear.

" " 11 " " bearing.

All permissible stresses for *Max. plus Wind* are 25 per cent. above those for *Max. no Wind.*

The details of the construction of this girder are fully illustrated in Plate XII.

Plate 12.



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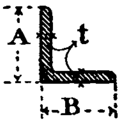
APPENDIX.

British Standard Sections issued by the Engineering Standards Committee.



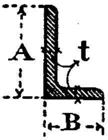
List 1.—Equal Angles.

a = sectional area.

$W = 3{\cdot}4a$ = weight in lbs. per foot run.

Size. A×B.	Standard Thickness. t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
1 × 1	·125	·80	·234
	·250	1·49	·437
1¼ × 1¼	·125	1·02	·299
	·250	1·92	·564
1½ × 1½	·125	1·23	·361
	·250	2·34	·689
1¾ × 1¾	·175	1·98	·583
	·300	3·27	·961
2 × 2	·175	2·28	·670
	·300	3·77	1·110
2¼ × 2¼	·175	2·57	·757
	·300	4·28	1·260
2½ × 2½	·250	4·04	1·187
	·375	5·89	1·733
	·500	7·65	2·249
2¾ × 2¾	·250	4·46	1·312
	·375	6·53	1·921
	·500	8·50	2·499

Size. A×B.	Standard Thickness. t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
3 × 3	·250	4·90	1·440
	·375	7·18	2·111
	·500	9·36	2·752
3½ × 3½	·300	6·84	2·011
	·425	9·50	2·795
	·500	11·05	3·251
4 × 4	·300	7·85	2·310
	·425	10·94	3·219
	·500	12·75	3·749
4½ × 4½	·375	11·00	3·236
	·500	14·46	4·252
5 × 5	·375	12·27	3·610
	·500	16·15	4·750
6 × 6	·450	17·68	5·201
	·625	24·18	7·112
7 × 7	·500	22·97	6·755
	·675	30·60	8·999
8 × 8	·550	28·89	8·497
	·750	38·89	11·437



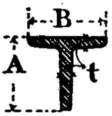
List 2.—Unequal Angles.

a = sectional area.

W = 3·4a weight in lbs. per foot run.

Size. A×B.	Standard Thickness t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
1¼ × 1	·125	·90	·265
	·250	1·70	·500
1½ × 1¼	·125	1·11	·327
	·250	2·12	·624
¾ × 1½	·175	1·83	·539
	·300	3·01	·886
2 × 1½	·175	1·98	·583
	·300	3·27	·961
2½ × 2	·175	2·57	·757
	·300	4·28	1·260
3 × 2	·250	4·04	1·187
	·375	5·89	1·733
	·500	7·65	2·249
3 × 2½	·250	4·46	1·312
	·375	6·53	1·921
	·500	8·50	2·499
3½ × 2½	·250	4·90	1·440
	·375	7·18	2·112
	·500	9·36	2·752
3½ × 3	·250	5·31	1·563
	·375	7·81	2·298
	·500	10·20	3·001
4 × 2½	·250	5·31	1·563
	·375	7·81	2·298
	·500	10·20	3·001
4 × 3	·300	6·84	2·011
	·425	9·50	2·795
	·500	11·05	3·251
4 × 3½	·300	7·34	2·159
	·425	10·22	3·006
	·500	11·90	3·499
4½ × 3	·300	7·34	2·159
	·425	10·22	3·006
	·500	11·90	3·499

Size. A×B.	Standard Thickness. t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
4½ × 3½	·300	7·85	2·309
	·425	10·94	3·219
	·500	12·75	3·749
5 × 3	·300	7·85	2·309
	·425	10·94	3·219
	·500	12·75	3·749
5 × 3½	·375	10·37	3·050
	·500	13·61	4·003
5 × 4	·375	11·00	3·236
	·500	14·46	4·252
5½ × 3	·375	10·37	3·050
	·500	13·61	4·003
5½ × 3½	·375	11·00	3·236
	·500	14·46	4·252
6 × 3½	·375	11·64	3·424
	·500	15·31	4·502
6 × 4	·375	12·27	3·610
	·500	16·15	4·750
6½ × 3½	·375	12·27	3·610
	·500	16·15	4·750
6½ × 4	·525	17·81	5·237
6½ × 4½	·550	19·54	5·746
7 × 3½	·525	17·81	5·237
7 × 4	·550	19·54	5·746
8 × 3½	·575	21·37	6·285
8 × 4	·625	24·18	7·112
9 × 4	·650	27·30	8·029
10 × 4	·675	30·60	8·999



List 3.—T Bars.

a = sectional area.
W = 3·4a = weight in lbs. per foot run.

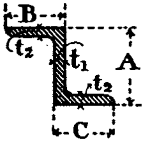
Size. B×A.	Standard Thickness. t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
1 × 1	·125	·82	·240
	·187	1·17	·344
1¼ × 1¼	·125	1·03	·303
	·187	1·49	·438
1½ × 1½	·187	1·81	·531
	·250	2·35	·692
1¾ × 1¾	·187	2·14	·629
	·250	2·79	·820
1½ × 2	·250	2·79	·820
	·312	3·40	1·001
2 × 2	·250	3·22	·947
	·312	3·94	1·159
	·375	4·64	1·366
2¼ × 2¼	·250	3·64	1·071
	·312	4·47	1·314
	·375	5·28	1·553
2½ × 2½	·250	4·07	1·197
	·312	5·00	1·471
	·375	5·92	1·741
3 × 2	·312	5·01	1·472
	·375	5·93	1·743
3 × 2½	·312	5·53	1·627
	·375	6·56	1·929

Size. B×A.	Standard Thickness. t.	Weight per Foot. W.	Sectional Area. a.
Inches.	Inches.	Lbs.	Sq. Ins.
3 × 3	·312	6·08	1·788
	·375	7·21	2·121
	·437	8·30	2·441
3 × 4	·375	8·48	2·494
	·500	11·07	3·256
3½ × 3½	·375	8·49	2·496
	·437	9·78	2·878
	·500	11·08	3·258
4 × 3	·375	8·49	2·498
	·500	11·08	3·260
4 × 4	·375	9·77	2·872
	·500	12·78	3·758
4 × 5	·375	11·06	3·253
	·500	14·50	4·264
5 × 3	·375	9·78	2·875
	·500	12·79	3·762
5 × 3½	·500	13·66	4·018
5 × 4	·500	14·51	4·268
6 × 3	·375	11·08	3·260
	·500	14·53	4·272
6 × 4	·500	16·22	4·771
7 × 3½	·500	17·08	5·023

List 4.—Channels.

a = sectional area.
$W = 3{\cdot}4a$ = weight in lbs. per foot run.

Size.	Standard Thickness.		Weight per Foot.	Sectional Area.
A×B.	t_1.	t_2.	W.	a.
Inches.	Inches.	Inches.	Lbs.	Square Inches.
3 × 1½	·250	·312	5·27	1·549
3½ × 2	·250	·312	6·75	1·986
4 × 2	·250	·375	7·96	2·341
5 × 2½	·312	·375	10·98	3·230
6 × 2½	·312	·375	12·04	3·542
6 × 3	·312	·437	14·49	4·261
6 × 3	·375	·475	16·29	4·791
6 × 3½	·375	·475	17·90	5·266
7 × 3	·375	·475	17·56	5·166
7 × 3½	·400	·500	20·23	5·950
8 × 2½	·312	·437	15·12	4·448
8 × 3	·375	·500	19·30	5·675
8 × 3½	·425	·525	22·72	6·682
8 × 4	·450	·550	25·73	7·569
9 × 3	·375	·437	19·37	5·696
9 × 3½	·375	·500	22·27	6·550
9 × 3½	·450	·550	25·39	7·469
9 × 4	·475	·575	28·55	8·396
10 × 3½	·375	·500	23·55	6·925
10 × 3½	·475	·575	28·21	8·296
10 × 4	·475	·575	30·16	8·871
11 × 3½	·475	·575	29·82	8·771
11 × 4	·500	·600	33·22	9·771
12 × 3½	·375	·500	26·10	7·675
12 × 3½	·500	·600	32·88	9·671
12 × 4	·525	·625	36·47	10·727
15 × 4	·525	·630	41·94	12·334



List 5.—Z Bars.

a = sectional area.
W = 3·4a weight in lbs. per foot run.

SIZE.	Standard Thickness.		Weight per Foot.	Sectional Area.
A×B×C.	t_1.	t_2.	W.	a.
Inches.	Inches.	Inches.	Lbs.	Square Inches.
3 × 2½ × 3	·300	·400	9·81	2·884
4 × 2½ × 3	·325	·425	11·53	3·392
5 × 3 × 3	·350	·450	14·17	4·169
6 × 3½ × 3½	·375	·475	17·88	5·258
7 × 3½ × 3½	·400	·500	20·22	5·948
8 × 3½ × 3½	·425	·525	22·68	6·670
9 × 3½ × 3½	·450	·550	25·33	7·449
10 × 3½ × 3½	·475	·575	28·16	8·283

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