$$\int_0^{\pi} [\varphi(x) - P_n(x)]^2 dx \leq \vartheta_1^n \int_0^{\pi} [\varphi(x)]^2 dx.$$

This proves the closure of the set $\cos \mu_k x$.

As a special case, let $\lambda_k = k-1$, $C = 2/\pi$. Then D can be any number less than $\sqrt{\frac{1}{3\pi^2} + \frac{9}{4}} - \frac{3}{2}\cdot$ In particular, *if*

$$\nu < \sqrt{\frac{1}{3\pi^2} + \frac{9}{4}} - \frac{3}{2}$$

the sequence

$$\cos \nu x, \cos (1 + \nu)x, \cos (2 + \nu)x, \ldots.$$

is closed over $(0, \pi)$.

[1] The terms at the beginning may have to be modified slightly because $F_k(u)$ is to vanish for negative u. This is allowed for in the following estimate.

ON A TYPE OF LORENTZ TRANSFORMATIONS

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1. Under a general Lorentz transformation we understand a linear transformation on the variables x, y, z, t which leaves the form

$$x^2 + y^2 + z^2 - t^2 \qquad (1)$$

invariant. Usually one type of special Lorentz transformations is considered corresponding to the case in which only t and *one* of the other variables is affected. The importance of such transformations in the Theory of Relativity is very well known. In what follows another special type of Lorentz transformations is considered, which so far as I know has not been studied, and which seems to have important applications in the study of radiation.

2. We arrive at the desired transformations in a simple way if we look for linear transformations which leave (1) invariant and, at the same time, do not change the vector 1, 0, 0, 1 and the plane $x = t$, $y = 0$. The last two conditions give four relations on the 16 coefficients of the general linear homogeneous transformation in addition to the 10 relations which express the condition that (1) remains invariant. The transformation

which we want to write down depends, therefore, on two parameters, and if we take for these two parameters the coefficients of y and z in the expression for x' we find easily that, after adjusting the directions on the y and z axes, the transformation assumes the form

$$\begin{array}{rcl} x' &=& (1-{}^1/_2(a^2+b^2))x + ay + bz + {}^1/_2(a^2+b^2)t \\ y' &=& -ax + y + at \\ z' &=& -bx + z + bt \\ t' &=& -{}^1/_2(a^2+b^2)x + ay + bz + (1+{}^1/_2(a^2+b^2))t \end{array} \qquad (2)$$

3. From these formulas we can deduce formulas for point transformations of the x, y, z, t space which do not change the form

$$(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2 - (t_1-t_2)^2, \qquad (3)$$

the plane $x = t$, $y = 0$, and the vector 1, 0, 0, 1. They are

$$\begin{array}{rcl} x' &=& (1-{}^1/_2(a^2+b^2))x + ay + bz + {}^1/_2(a^2+b^2)t + c \\ y' &=& -ax + y + at \\ z' &=& -bx + z + bt \\ t' &=& -{}^1/_2(a^2+b^2)x + ay + bz + (1+{}^1/_2(a^2+b^2))t + c. \end{array} \qquad (4)$$

If we ask when a point x, y, z, t can be transformed into a point x', y', z', t' by a transformation (4) we find that a necessary condition is

$$x'-t' = x-t; \qquad (5)$$

and this condition is also sufficient if $x \neq t$. If we ask what happens in case $x = t$, i.e., when a point x, y, z, x can be transformed into a point x', y', z', x' we find as a necessary and sufficient condition

$$y' = y, \qquad z' = z. \qquad (6)$$

The analogies with and the differences from the more familiar helicoidal motions in Euclidean space are obvious.

4. From the point of view of geometry the situation with respect to types of special Lorentz transformations may be characterized as follows. As in the case of Euclidean space (form $x^2 + y^2 + z^2 + t^2$ with four pluses) in the pseudo-Euclidean space (form (1)) a rotation always leaves two absolutely perpendicular planes in their positions (i.e., rotates them in themselves). But whereas in the Euclidean space there exists only one type of absolutely perpendicular planes—planes which intersect in a point, there are two types of absolutely perpendicular planes in pseudo-Euclidean space: outside of the "general case" in which the two planes have only one point in common there exists "a singular case" in which the two planes have a line in common, viz., a line of zero direction which is perpendicular to itself. To this singular case belongs the rotation given by the formulas (2). More general cases may be obtained by combining (2) with rotations in the x, t and the y, z planes. The two types of absolutely perpendicular

planes have been discussed by the author about two years ago;[1] it was pointed out at that time that, as a consequence, there exist two types of antisymmetric linear vector functions, and since antisymmetric linear vector functions are to be considered as infinitesimal rotations the existence of two types of rotations follows.

5. In conclusion a possible application of the transformations (4) in the theory of light might be mentioned. On the emission theory we have to consider a ray as given by a vector of square zero ("light vector") which is freely movable in the line in which it is contained, just as a particle is given by a time-like vector (four-dimensional momentum) freely movable along its line. If we want to determine the field associated with a light quantum or photon, we will want, of course, to use the same method which determines the (gravitational and electromagnetic) field associated with an electron, i.e., we will seek a field which is not changed by transformations which do not affect the light vector, and these can be deduced from (4).

[1] New York, *Trans. Amer. Math. Soc.*, **27**, January, 1925, p. 113.

CYCLICLY CONNECTED CONTINUOUS CURVES[1]

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A continuous curve M will be said to be *cyclicly connected* if and only if every two points of M lie together on some simple closed curve which is a subset of M.

The points A and B of a continuum M are said to be separated in M by the point X of M provided it is true that $M-X$ is the sum of two mutually separated sets S_1 and S_2 containing A and B, respectively. The point P of a continuous curve M is an end-point of M if and only if it is true that no simple continuous arc of M has P as one of its interior points. I have shown[2] that this definition of an end-point of a continuous curve is equivalent to the one given by Wilder.[3]

In this paper it is to be understood that the point sets considered lie in a Euclidean space of two dimensions.

THEOREM 1. *In order that a continuous curve M should be cyclicly connected it is necessary and sufficient that it should have no cut*[4] *point.*

Proof. The condition is sufficient. Let M denote a continuous curve having no cut point. Then, by a theorem due to H. M. Gehman,[5] M can have no end-point. But I have shown[2] that every continuous curve M is the sum of its cut points, end-points, and points belonging to simple