Aerodynamic Theory

A General Review of Progress

Under a Grant of the Guggenheim Fund
for the Promotion of Aeronautics

William Frederick Durand
Editor-in-Chief

Volume VI

Div. P · Airplane as a Whole · W. F. Durand
Div. Q · Aerodynamics of Airships · Max M. Munk
Div. R · Performance of Airships · K. Arnstein and W. Klemperer
Div. S · Hydrodynamics of Boats and Floats · E. G. Barrillon
Div. T · Aerodynamics of Cooling · H. L. Dryden

With 127 Figures
and 2 Plates

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GENERAL PREFACE

During the active life of the Guggenheim Fund for the Promotion of Aeronautics, provision was made for the preparation of a series of monographs on the general subject of Aerodynamic Theory. It was recognized that in its highly specialized form, as developed during the past twenty-five years, there was nowhere to be found a fairly comprehensive exposition of this theory, both general and in its more important applications to the problems of aeronautic design. The preparation and publication of a series of monographs on the various phases of this subject seemed, therefore, a timely undertaking, representing, as it is intended to do, a general review of progress during the past quarter century, and thus covering substantially the period since flight in heavier than air machines became an assured fact.

Such a present taking of stock should also be of value and of interest as furnishing a point of departure from which progress during coming decades may be measured.

But the chief purpose held in view in this project has been to provide for the student and for the aeronautic designer a reasonably adequate presentation of background theory. No attempt has been made to cover the domains of design itself or of construction. Important as these are, they lie quite aside from the purpose of the present work.

In order the better to suit the work to this main purpose, the first volume is largely taken up with material dealing with special mathematical topics and with fluid mechanics. The purpose of this material is to furnish, close at hand, brief treatments of special mathematical topics which, as a rule, are not usually included in the curricula of engineering and technical courses and thus to furnish to the reader, at least some elementary notions of various mathematical methods and resources, of which much use is made in the development of aerodynamic theory. The same material should also be acceptable to many who from long disuse may have lost facility in such methods and who may thus, close at hand, find the means of refreshing the memory regarding these various matters.

The treatment of the subject of Fluid Mechanics has been developed in relatively extended form since the texts usually available to the technical student are lacking in the developments more especially of interest to the student of aerodynamic theory. The more elementary treatment by the General Editor is intended to be read easily by the average technical graduate with some help from the topics comprised in Division A. The more advanced treatment by Dr. Munk will call for some familiarity with space vector analysis and with more advanced mathematical methods, but will commend itself to more advanced students by the elegance of such methods and by the generality and importance of the results reached through this generalized three-dimensional treatment.

In order to place in its proper setting this entire development during the past quarter century, a historical sketch has been prepared by Professor Giacomelli whose careful and extended researches have resulted in a historical document which will especially interest and commend itself to the study of all those who are interested in the story of the gradual evolution of the ideas which have finally culminated in the developments which furnish the main material for the present work.

The remaining volumes of the work are intended to include the general subjects of: The aerodynamics of perfect fluids; The modifications due to viscosity and compressibility; Experiment and research, equipment and methods; Applied airfoil theory with analysis and discussion of the most important experimental results; The non-lifting system of the airplane; The air propeller; Influence of the propeller on the remainder of the structure; The dynamics of the airplane; Performance, prediction and analysis; General view of airplane as comprising four interacting and related systems; Airships, aerodynamics and performance; Hydrodynamics of boats and floats; and the Aerodynamics of cooling.

Individual reference will be made to these various divisions of the work, each in its place, and they need not, therefore, be referred to in detail at this point.

Certain general features of the work editorially may be noted as follows:

1. Symbols. No attempt has been made to maintain, in the treatment of the various Divisions and topics, an absolutely uniform system of notation. This was found to be quite impracticable.

Notation, to a large extent, is peculiar to the special subject under treatment and must be adjusted thereto. Furthermore, beyond a few symbols, there is no generally accepted system of notation even in any one country. For the few important items covered by the recommendations of the National Advisory Committee for Aeronautics, symbols have been employed accordingly. Otherwise, each author has developed his system of symbols in accordance with his peculiar needs.

At the head of each Division, however, will be found a table giving the most frequently employed symbols with their meaning. Symbols in general are explained or defined when first introduced.

2. General Plan of Construction. The work as a whole is made up of *Divisions*, each one dealing with a special topic or phase of the general subject. These are designated by letters of the alphabet in accordance with the table on a following page.

The Divisions are then divided into chapters and the chapters into sections and occasionally subsections. The Chapters are designated by Roman numerals and the Sections by numbers in bold face.

The Chapter is made the unit for the numbering of sections and the section for the numbering of equations. The latter are given a double number in parenthesis, thus (13.6) of which the number at the left of the point designates the section and that on the right the serial number of the equation in that section.

Each page carries at the top, the chapter and section numbers.

W. F. Durand

Stanford University, California
January, 1934.

GENERAL LIST OF DIVISIONS WITH AUTHORS

Volume I.

A. Mathematical Aids
W. F. DURAND — Professor (Emeritus) of Mechanical Engineering, Stanford University, Calif., Member of the National Advisory Committee for Aeronautics.

B. Fluid Mechanics, Part I
W. F. DURAND

C. Fluid Mechanics, Part II
MAX M. MUNK — Lecturer in Aerodynamics at the Catholic University of America, Washington, D. C., and Technical Editor of the "Aero Digest".

D. Historical Sketch
R. GIACOMELLI — Lecturer in History of Mechanics at the University of Rome, Italy, and Editor of "L'Aerotecnica".
with the collaboration of
E. PISTOLESI — Professor of Mechanics at the Royal School of Engineering at Pisa, Italy, and Editor-in-Chief of "L'Aerotecnica".

Volume II.

E. General Aerodynamic Theory—Perfect Fluids
TH. VON KÁRMÁN — Director of the Guggenheim Aeronautics Laboratory, California Institute of Technology, Pasadena, Calif., and formerly Director of the Aerodynamic Institute. Aachen, Germany.
J. M. BURGERS — Professor of Aero- and Hydrodynamics at the Technische Hoogeschool at Delft, Holland.

Volume III.

F. The Theory of Single Burbling
C. WITOSZYŃSKI — Professor of Aerodynamics at the Warsaw Polytechnical School and Director of the Warsaw Aerodynamic Institute, Poland.
M. J. THOMPSON — Assistant Professor of Aeronautical Engineering at the University of Michigan, Ann Arbor, Mich.

G. The Mechanics of Viscous Fluids
L. PRANDTL — Professor in Applied Mechanics at the University of Göttingen, Germany, and Director of the Kaiser Wilhelm Institute for Fluid Research.

H. The Mechanics of Compressible Fluids
G. I. TAYLOR — Yarrow Research Professor of the Royal Society, Fellow of Trinity College, Cambridge, England.
J. W. MACCOLL — Research Officer, Department of External Ballistics, Ordnance Committee, Woolwich, England.

I. Experimental Methods—Wind Tunnels
A. TOUSSAINT — Director of the Aerodynamic Laboratory, Saint-Cyr-l'École, France.
E. JACOBS — Associate Aeronautical Engineer, in charge of the National Advisory Committee for Aeronautics' variable-density wind tunnel, Langley Field, Virginia.

Volume IV.

J. Applied Airfoil Theory

A. Betz — Professor at the University and Director of the Aerodynamic Research Institute at Göttingen, Germany.

K. Airplane Body (Non-Lifting System) Drag and Influence on Lifting System

C. Wieselsberger — Professor of Aerodynamics and Director of the Aerodynamic Institute, Technische Hochschule, Aachen, Germany.

L. Airplane Propellers

H. Glauert [1] — Past Fellow of Trinity College, Cambridge, England; Principal Scientific Officer at the Royal Aircraft Establishment, Farnborough.

M. Influence of the Propeller on other Parts of the Airplane Structure

C. Koning — Rijks-Studiedienst voor de Luchtvaart, Amsterdam, Holland.

Volume V.

N. Dynamics of the Airplane

B. Melvill Jones — Professor of Aeronautical Engineering in the University of Cambridge, England, Member of the Aeronautical Research Committee of Great Britain.

O. Airplane Performance

L. V. Kerber — Former Chief Aerodynamics Branch Materiel Division, U. S. Army Air Corps, and former Chief, Engineering Section Aeronautics Branch, Department of Commerce.

Volume VI.

P. Airplane as a Whole—General View of Mutual Interactions Among Constituent Systems

W. F. Durand [2] — Professor (Emeritus) of Mechanical Engineering, Stanford University, Calif., Member of the National Advisory Committee for Aeronautics.

Q. Aerodynamics of Airships

Max M. Munk — Lecturer in Aerodynamics at the Catholic University of America, Washington, D. C., and Technical Editor of the "Aero Digest".

R. Performance of Airships

K. Arnstein — Chief Engineer of the Goodyear Zeppelin Company, Akron, Ohio.

W. Klemperer — Research Engineer of the Goodyear Zeppelin Company, Akron, Ohio.

S. Hydrodynamics of Boats and Floats

E. G. Barrillon — Director of the Naval Experimental Tank, Paris, France.

T. Aerodynamics of Cooling

H. L. Dryden — Physicist in the United States Bureau of Standards, Chief of the Aerodynamics Section, Washington, D. C.

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[1] Deceased August 4, 1934.

[2] In the original plan, it was expected that this Division would be prepared by Professor M. Panetti of the R. Scuola di Ingegneria of Turin. Unfortunately, at the last, Professor Panetti found himself unable to give the needed time for this work, and in order not to delay publication, the General Editor has undertaken to prepare a brief treatment of the subject.

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DIVISION Q

AERODYNAMICS OF AIRSHIPS

By **Max M. Munk,**

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DIVISION R

PERFORMANCE OF AIRSHIPS

By **K. Arnstein,**

Chief Engineer of the Goodyear Zeppelin Company, Akron, Ohio

and **W. Klemperer,**

Research Engineer of the Goodyear Zeppelin Company, Akron, Ohio

DIVISION S

HYDRODYNAMICS OF BOATS AND FLOATS

By **E. G. BARRILLON,**

Director of the Naval Experimental Tank, Paris, France

DIVISION T

AERODYNAMICS OF COOLING

By **H. L. Dryden,**

Physicist in the United States Bureau of Standards, Chief of the Aerodynamics Section, Washington, D.C.

NOTATION

The following table comprises a list of the principal notations employed in the present Volume. Notations not listed are either so well understood as to render mention unnecessary, or are only rarely employed and are explained as introduced. Where occasionally a symbol is employed with more than one meaning, the local context will make the significance clear.

DIVISION Q

a	Semi-longitudinal axis
b	Semi-transverse axis
γ	Radius of curvature
x	Length along longitudinal axis
S	Area or surface, usually area of section
α	Angle between axis and direction of motion—angle of attack
φ	Angle of yaw, **9**
U	Axial velocity
V	Velocity in general
v	Lateral component velocity
W	Resultant velocity, **6**
ω	Angular velocity, **10**
F	Force
L	Lift
M	Moment
P	Pressure on surface
c_1, c_2, c_3	Special correction factors, **7**
k_1	Inertia factor for axial motion
k_2	Inertia factor for lateral motion
k'	Inertia factor for rotation
η	Special inertia factor, **2**, **8**

DIVISION R

x	Distance along longitudinal axis
b	Transverse force breadth III **2**
h	Change in metacentric height
R	Radius of circular path
A_D	Drag area II **5**
S	Area, usually of cross section
Q	Volume
α	Angle of attack
ε	Slope of ships path III **1**
V	Speed
L	Lift
D	Drag
F	Lateral air force on ship
M	Mass
P	Power
C_L	Coefficient of lift

C_D	Drag coefficient
k_1	Longitudinal inertia factor II 4
k_2	Transverse inertia factor III 2
η	Propeller efficiency
E	Propulsive efficiency II 5
f	Fuel consumption per unit of power
R	Reynolds number
ϱ	Density of air
T	Temperature, absolute
t	Time

DIVISION S

A	Half wave height, III 2
b	Width, usually of float
D	Diameter of propeller
h	Pitch of propeller
L	Length
A, S	Area or surface
σ	Area or surface, III 7
i	Angle of incidence or slope, or inclination between planes representing wings and hull bottom, III 7
θ	Angle of inclination, usually to water surface
V	Speed
B	Lift due to buoyancy of water
H	Hydrodynamic and resultant force, II 7 Sometimes also hydrodynamic force coefficient, III 5
M	Mass
N, T	Normal and tangential components of force
Q	Torque of propeller
R	Resistance, also aerodynamic coefficient in equation Force $= R V^2$
T	Thrust of propeller
W	Weight
α, β	Special coefficients III 6, 8, 11 also sometimes special forms of expression, III 8
K	Coefficient in general
K_1	Coefficient of aerodynamic lift, III 8
K_2	Coefficient of aerodynamic drag, III 8
K'	Coefficient of hydrodynamic lift, III 8
K''	Coefficient of hydrodynamic drag, III 8
δ	Weight of unit volume of water, III 8
ϱ	Density of air
G	Center of gravity
ר	Hebrew *resch,* used for Reech-Froude ratio $V/\sqrt{g A}$

DIVISION T

x, y, z	Coordinates along axes of X, Y, Z
x, r, φ	Cylindrical coordinates (Fig. 3)
δ	Thickness of boundary layer VIII 1
ε	Apparent or eddy viscosity III 2 Also thickness of thin layer adjacent to wall V 2
l	Mixing Length III 4
L	Length
R	Radius of cylinder or pipe

S	Area or surface	
u, v, w	Component velocities along X, Y, Z	
U or V	Velocity of flow in general	
G	Average mass velocity over a cross-section	
M	Momentum	
p	Pressure, or stress	
F_x	Skin friction force per unit area	
τ	Frictional shearing stress	III 3
c	Specific heat	
k	Conductivity for heat flow	
q	Heat transfer per unit area	
h	Heat transfer divided by temperature difference	
Q	Quantity of heat	
β	Eddy conductivity	III 2
β	Coefficient of thermal expansion	IV 4
S	Rate of heat generation or disappearance	V 3
ψ	Stream function	
φ, ψ	Functional symbols	VII 3
μ	Coefficient of viscosity	
ν	Kinematic viscosity	
g	Acceleration due to gravity	
ϱ	Density of air	
σ	Put for ratio $\mu c/k$	
T	Absolute temperature	
	Temperature	
θ	Time	

DIVISION P

DIVISION P

AIRPLANE AS A WHOLE
GENERAL VIEW OF MUTUAL INTERACTIONS[1] AMONG CONSTITUENT SYSTEMS

By

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PREFACE

The Editor regrets most sincerely to announce that the other engagements of Professor Panetti of the R. Scuola die Ingegneria di Torino, Turin, Italy, have not permitted him the time to prepare the manuscript for the present Division, as was originally planned. Pressed near the close of the period of publication by the need of some treatment of this topic, and in order to fill out the original schedule of Divisions and of subject matter to be treated, the Editor has undertaken to give a brief treatment of the subject which, it is hoped, however, may aid the reader in gaining a more comprehensive view of the airplane as a whole and of its performance as the resultant of a very considerable manifold of actions, interactions and reactions.

In the present Division, the airplane is viewed as a complex of four interacting systems—the lifting system, the non-lifting system, the propulsive system and the control system. These four systems may and do interact mutually, thus modifying in various ways the basic effect or purpose of each of these systems considered as isolated from the others. In all there are twelve such interactions, some of which, however, are of small or negligible importance.

Several of these interactions have been discussed in mathematical terms in other Divisions of this Series and it is the purpose in the present Division to give rather in a descriptive and non-mathematical way, a general view of this entire manifold of interactions, with suitable reference to other Divisions for more complete discussion of the more important in mathematical terms.

Such a bird's eye view, as it were, of these various actions and reactions seems desirable in a series of monographs of the character of the present series, and the Editor only regrets the need of any departure from the original plan regarding the authorship.

[1] See Journées Scientifiques et Techniques de Mécanique des Fluides, Vol. 1, p. 189, Paris 1935.

Also with special reference to sections 4, 5, 9, 11 see Division O, pp. 37—41.

Introductory. We may view the airplane as a complex of four systems as follows:

1) The lifting system.
2) The non-lifting system.
3) The propulsive system.
4) The control system.

The basic aerodynamic effect or purpose of each one of these systems will be in some measure influenced or modified by each one of the other systems. There will be, therefore, twelve such effects or influences as we may choose to call them. Not all of these effects are of equal importance, but it will be of interest to list them all, in order that they may be viewed as a whole and examined, each with reference to its place in the picture of an airplane as performing under the aggregate of this complex of disturbing actions and reactions.

These disturbing inflences are, therefore, as follows:

1) Influence of the lifting system on the non-lifting system.
2) Influence of the non-lifting system on the lifting system.
3) Influence of the lifting system on the propulsive system.
4) Influence of the propulsive system on the lifting system.
5) Influence of the lifting system on the control system.
6) Influence of the control system on the lifting system.
7 Influence of the non-lifting system on the propulsive system.
8) Influence of the propulsive system on the non-lifting system.
9) Influence of the non-lifting system on the control system.
10) Influence of the control system on the non-lifting system.
11) Influence of the propulsive system on the control system.
12) Influence of the control system on the propulsive system.

We proceed, then, with the examination of these in order.

1. Influence of the Lifting System on the Non-Lifting System. The lifting system is represented by the wings and the non-lifting system primarily by the fuselage, to which may be added such items as the landing gear (when not retractable), engine nacelles in multi-engined planes, struts, guy wires, etc. The fuselage also may and usually does contribute something to the lift and the stabilizer likewise contributes vertical forces, though often opposed to wing lift for purposes of longitudinal stability. However, for our present purpose it will be sufficient to consider the influence of the wings on the fuselage.

The purpose of the fuselage is, of course, to house a power plant, operating personnel, passengers and useful load, and to make connection between the wings and the empennage or otherwise to furnish a mounting for the latter. None of these is directly aerodynamic in character and the chief aerodynamic result of the presence of the fuselage is the production of so-called parasitic resistance or drag. At the same time, as noted above, the fuselage may, and at large angles of attack will, give an element of lift, functioning as a poorly formed airfoil of aspect ratio much less than unity. However our major interest in the present inquiry will relate to the influence of the wings on the production of fuselage drag.

The lift produced by the wings is associated with a circulation about them, with the trailing vortices streaming down the wake and producing the well known induced "downwash" or downward component in the resultant airflow over the plane and changing the geometrical angle of attack α by the so-called induced angle φ or α_i (see Division E I 12 and III, Part C).

The circulation about the wing itself will furthermore introduce another factor, especially influential at the leading edge where it tends to produce an upward component in the line of the airflow to the wing. The general character of the lines of airflow in approaching a wing as shown by photography and as indicated by diagram may be seen by reference to Division E I, Figs. 7, 10, 12.

The extent to which the airflow over the wings as influenced by circulation and downwash will influence the airflow to the fuselage, especially in respect to direction, will naturally depend on the location of the fuselage relative to the wings. In the general case, however, some degree of such influence may be expected and, to the extent to which it exists, it will enter as a more or less influential factor in modifying the effect of obliquity due to the attitude of the plane under the conditions of flight controlling at the moment.

Thus, in actual flight, at any one loading, there will be only one speed and one attitude of plane at which the axis of the fuselage will lie strictly in the line of flight. At all other speeds, the attitude will be such as to make some angle of obliquity between this axis and the line of flight.

The combination of these obliquities, of flow and of attitude, will in general, result in an obliquity of airflow relative to the direction for minimum fuselage drag, with increase in the size of the turbulent wake and with the result of an increase of drag, more or less pronounced as the resulting angle of obliquity is large or small. The same obliquity relative to the airflow will likewise influence such lift as the fuselage may give—in general, an increase of fuselage lift with increasing angle of wing attack, up to some angle presumably approaching the burble point for the wings.

In addition to these more or less obvious forms of reaction, there will result on the body of the airplane a positive lift resulting from the development of a positive circulation about the body due to the existence of the circulation about the wings. Where the wing extends continuously across from tip to tip over the airplane body, the wing circulation must in part extend around the body itself with the result of a positive lift. Where the wing is discontinuous, this action will be less pronounced, but even here there may be some lateral extension of the wing circulation in such a way as to at least partially inclose the body of the plane.

We have thus, in summary, first an attitude of the plane relative to the line of flight, dependent on the wings in that such attitude is necessary to enable the wings to realize the lift under the actual conditions of load; second, some modification of the resulting lines of airflow to the fuselage, due to the influence of the downwash and circulation about the wings; and third, some actual increase in the circulation about the fuselage, representing, in a sense, an extension of that about the wings. The combination of obliquities will, as noted, produce some increase in the fuselage drag while the increment of circulation will produce an increment of fuselage lift.

The amount of the increase in drag will be small unless the obliquity of the axis of the fuselage to the line of flight becomes large, in which case it may become serious. The increment of lift due to wing circulation will be relatively small, depending on the arrangement of wings and fuselage.

2. Influence of the Non-Lifting System on the Lifting System. For our present purposes, the non-lifting system may be represented by the fuselage, to which may be added such structures as engine nacelles in the case of multi-engined planes. On the other hand the lifting system is represented by the wings, so that the present question reduces primarily to the influence of the fuselage on the wings.

We may first refer to Division K III, wherein this subject is approached from the mathematical standpoint and results are given based on two methods of treatment as follows:

(1) Representation of the fuselage by an indefinitely long cylindrical body with wings attached.

(2) Treatment of the combination of wings and fuselage (or wings and nacelles) as a generalized form of wing with abrupt change of profile at the location of the fuselage or engine nacelles. The distribution of lift over the span is then developed by suitable mathematical procedures.

Method 1 admits of a certain simplification in the theoretical treatment and furnishes results of definite interest and significance. In general it is shown that there is produced what is called an "additional stream flow", conditioned by the body, and which will produce a downward component of velocity on the wing. It thus results that the body will cause a change in the effective angle of incidence of the wing and likewise a supplementary drag for the same, the character and value of which will depend on the relative locations of body and wing. Division K should be referred to for further details.

Method 2 permits of treatment by several different mathematical procedures of which three are given in brief abstract. This mode of approach to the problem shows, as might be anticipated, an abrupt decrease in lift over that part of the span represented by the body or by engine nacelles. It shows also what may be termed the spread of this influence over the rest of the wing, resulting in a general decrease of lift as compared with that for a wing alone of the same total span (see Fig. 1[1]). This departure of the lift distribution from that for the wing alone causes also an increase in the induced drag, the magnitude of which is, however, relatively small. See Division K, Fig. 45. Attention may be called to the considerable amount of experimental information on this subject, especially as regards the results of fillets between body and wings and the best relative location of engine nacelles and wings[2].

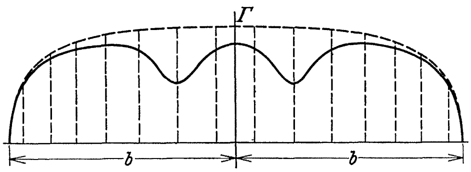

Fig. 1. Effect of nacelles on distribution of lift.

It may be desirable to give some further discussion of this general topic from a slightly different viewpoint and without direct reference to mathematical procedures. This will permit the development of a somewhat more complete picture of the subject as a whole, reaching however, the same general conclusions as above.

To this end we shall find it convenient to consider the question in two parts, depending on whether the wing is continuous across the span from tip to tip or is interrupted by the fuselage. In the latter case it may be considered either as a continuous wing structure with an extreme and abrupt change of form in the central portion, or as two half wings, with attachment at one end to the fuselage and with the other end free.

We take first the case of a continuous wing extending across the span above the fuselage, and recall that such a wing realizes lift by reason of its translation through the air combined with a circulation flow around the wing section. These two types of flow give a resultant

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[1] Repeated from Division K, Fig. 44.

[2] Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, I. Lief., p. 118, 1925.

Muttray, H., Untersuchungen über die Beeinflussung des Tragflügels eines Tiefdeckers durch den Rumpf. Luftfahrtforschung, Bd. 2, Heft 2, 1928.

Gough, M. N., The Effect of Fillets Between Wings and Fuselage on the Drag and Propulsive Efficiency of an Airplane, U.S. N.A.C.A. Technical Note No. 299.

Klein, A. L., Effect of Fillets on Wing-Fuselage Interference, Transactions Am. Soc. Mech. Engrs., 1934.

Wood, Donald, H., Tests of Nacelle-Propeller Combinations I, II, III, U.S. N.A.C.A. Technical Reports Nos. 415, 436, 462; 1932, 1933.

air flow about the wing and we have now to inquire as to the extent to which such flow will be influenced or modified by the presence of the fuselage lying below its middle portion and of breadth some five to ten per cent of the wing span.

Stated in this way it is clear that this influence should not be of serious amount. Its major effect will be confined to a small part of the span near the center, and with usual dimensions, this effect will be relatively small in magnitude. There will be a tendency toward a slight compression of the flow between the fuselage and the central part of the wing, combined with a tendency for the flow to spread obliquely as the air travels along the under side of the wing from the leading to the trailing edge. Compression of the lines of flow will result in higher velocities, the result of which will depend on the form and attitude of the under surface of the wing. Insofar as the velocity is increased, the pressure will be reduced (Bernoulli's law) but to the extent the flow is deflected downward, the upward dynamic reaction will be increased. We have here two opposing effects, the resultant of which will depend on the special circumstances of the case and no general conclusion can be drawn. Obliquely spreading flow may likewise cause a disturbance over a width somewhat greater than that of the fuselage itself, and of a character to reduce the over pressure which would otherwise result from an uninterrupted flow. Viewed otherwise, it would appear that the effect of this crowding of the lines of flow may have the effect of decreasing the circulation velocity below the wing and to the extent to which this might result and for the parts so affected, the lift would be correspondingly reduced.

On the whole, however, with normal dimensions, it is clear that this influence, compared with the lift of the wing as a whole, will be small though perhaps not of vanishing importance.

With certain types of design however, notably with some of the high wing monoplane types, the wing is structurally continuous but with a much reduced if not vanishing clearance between it and the fuselage. In such cases these effects will be much more pronounced and may seriously affect the lift of the under surface of that part of the wing nearest the fuselage. On the other hand, the upper surface of the wing will still be effective, so that the result as a whole may not involve a serious loss in the total lift.

It is clear, therefore, that in any case, viewed in comparison with a continuous wing with unimpeded flow, the near presence of a body such as the fuselage will result in some distortion and change of flow, all of which will react unfavorably on the lift of the wing as a whole; and that the magnitude of such influence will depend in primary degree on the amount of clearance between the wing and the fuselage body.

At the same time such disturbances to the flow over the wing may likewise affect the drag, both form and induced; the first by an increase in the turbulent wake and the energy carried away in it, and the second as a result of relatively abrupt changes in lift distribution and in the resulting system of trailing vortices.

Turning now to the case of a wing interrupted by the fuselage and comparing it with a single wing of the extreme span from tip to tip, it is shown in Division K III 2, as indeed may naturally be expected, that at normal angles of attack the abrupt change in virtual wing form at the center will result in a loss in lift over this part of the span, and furthermore, that this general disturbance will cause a measurable loss over the remainder of the span. At the same time the abrupt change in lift distribution will also cause an increase in the induced drag as noted above. On the other hand it may be noted that at high angles of attack and with special fuselage forms, the combined lift of wings and fuselage may be more than for an uninterrupted wing of the same over all span. In this connection special attention should be given to the influence of the geometrical form at the abrupt transition from wing to fuselage. Such a transition of form, if relatively sharp angled, will result in the formation of turbulence in the angle, spreading outward on the wing and creating generally a disturbance in the normal flow over the wing in this vicinity. The consequences here depend further in marked degree on the character of the flow through the angle between the wing and the fuselage—whether divergent or convergent. In the former case there will be high probability of separation of flow with increase in drag; in the latter case this condition should not develop and this particular increment of drag will be avoided. Naturally the low wing monoplane will be more subject to the consequences of a divergent flow than will the high wing type. The results of such conditions will be, in general, a decrease of lift and an increase in drag.

However, generous and carefully designed fillet forms connecting wings and fuselage are found to reduce in marked degree the prejudicial results due to these conditions[1].

If now we view the lifting system as consisting of two half wings, one projecting on either side of the fuselage, we have a pair of wings, of which one end is free and the other shielded by the fuselage side. We shall therefore have, for each wing, the normal spilling of the air around the free end, while the fuselage form will act in some degree as a shield for the inner or root end of each wing.

In addition, there will be, as discussed above, turbulence and general disturbance to the flow about the root of each wing due to the more or less abrupt transition in form from wing to fuselage.

[1] See references on p. 5.

We now ask, with what form or porportion of wing shall we compare these two half wings in order to discuss the question of the influence of the fuselage on the lift of such a wing system. It will be of interest to consider two such ideal wing forms: (1) A continuous wing of span equal to the combined span of the two wings excluding the fuselage. (2) Two wings of span each equal to the span of the wing structure on one side of the fuselage, but with free ends. This will be, in effect, two wings each of half the span in (1) and hence with half the aspect ratio.

Taking these in order, we shall have, with the actual structure, and excluding the fuselage entirely, a close approach to (1) except for the turbulence and disturbance to the flow at and near the root of each half wing. The shielding due to the fuselage side should much reduce the loss due to spilling or in other words it should give, over the root of the wing, a close approach to two-dimensional flow. There remains, however, the loss due to turbulence and disturbance of flow as already noted. In addition we must remember that, as we have already seen, the disturbance to the flow caused by the presence of the fuselage will extend in some degree over the entire wing span and the present picture is, therefore, the same as that presented by Fig. 1, except that we now exclude from consideration the marked loss in the center over that part of the total span represented by the fuselage itself. There remains then the loss over the actual wings proper plus that due to turbulence caused at the roots of the wings. The latter, as noted, can be much reduced by careful filleting, but the former will persist, and over all, as compared with a single wing of combined wing span, there will result a definite loss in lift together with some increase in drag.

Taking now the second ideal, we have to compare on each side of the plane, two half wings, one of which has both ends free while the other has one end shielded by the fuselage side, with the resultant turbulence formation. The shielding considered by itself, compelling as it will a close approach to two-dimensional flow near the root of the wing, will be favorable to the lift. It will give, in effect, a lift distribution holding up to a considerable value at the wing root, instead of falling to zero as in the case of a free wing tip. There will be, however, some general decrease of lift over the wing due to what may be termed the "spread" in the effect of the fuselage; see Fig. 1.

There will be likewise some further loss in lift and increase in drag due to turbulence at the root of the wing as already discussed. Definite experimental measure seems to be lacking regarding the values of these effects, favorable and unfavorable, but with a well filleted wing junction with the fuselage, the beneficial effect should definitely outweigh the loss due to turbulence, and with a wing structure of this character, the lift should be definitely greater than for two separate wings, each of the span of the half-wing on one side of the fuselage, and with free ends. This would mean in effect, two wings each of approximately half the aspect ratio of the single wing as a whole.

Again, to the lift of the two half-wings as here considered, must be added the lift due to the fuselage, whatever it may be, usually small at low angles of wing incidence, but larger with increase of this angle.

On the whole, therefore, it appears that if the wing be considered as extending across the entire span from wing tip to wing tip, the presence of the fuselage, for small angles of attack, will result in a loss of lift, while for large angles, the combined lift of wings and fuselage may be greater than for a single wing of the same over all span. Likewise if we consider the wing as composed of the two half structures on either side of the fuselage, and compare this with a single wing of the same aggregate span, there will likewise result a loss in lift. But if we compare this latter structure with two wings with free ends, each of the span of the half-wing of the actual plane, the result will involve effects favorable and unfavorable, but with the balance, in any normal case of a well filleted junction, definitely on the favorable side.

In this connection attention may be called to the fact that other things the same, the two wings of half span would have double the induced drag of the single wing, thus giving a further advantage to the actual construction in comparison with this particular combination.

3. Influence of the Lifting System on the Propulsive System. The propulsive system in the simple case is represented by the propeller located at the nose of the fuselage. Propeller performance as based on the simpler theory and as estimated from the results of model or even full scale tests in aerodynamic laboratories, is considered relative to the case of motion in the direction of the shaft. That is, propeller traction (thrust) is considered as acting along the line of the shaft, and the relative motion of propeller and air is assumed to lie along the same line. It is shown, however, in Division L[1] that obliquity of flow of the air to the propeller has the effect of producing lateral forces on the propeller as well as other marked effects on its performance. The production of lateral forces on the propeller and hence on the shaft will have the effect of giving a resultant propeller force oblique to the line of the shaft and likewise in general, oblique to the direction of flight.

[1] See VIII 7 and XII 5; also:

Clarke, T. W. K., Effect of Side Wind on a Propeller, Br. A.R.C. R. and M. 80, 1913.

Bramwell, F. H., Relf, E. F., and Bryant, L. W., Experiments to Determine the Lateral Force on a Propeller in a Side Wind, Br. A.R.C. R. and M. 123, 1914.

Harris, R. G., Forces on a Propeller Due to Sideslip, Br. A.R.C. R. and M. 427, 1918.

Misztal, Franz, Zur Frage der schräg angeblasenen Propeller, Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, Heft 11, p. 5, 1932.

In the case of the actual propeller in flight we have four directions to consider; (1) the direction of flight, (2) the direction of the shaft of the propeller, (3) the direction of the flow of air to the propeller, and (4) the direction of the resultant force on the propeller shaft and hence on the plane. In the general case, neither (2), (3) nor (4) will be the same as (1). The direction of the shaft will vary with every change in the angle of attack (quite aside from the rapid changes in maneuvers) and this will mean generally that with every change of speed the line of the shaft will change relative to the line of flight.

Regarding the line of airflow to the propeller, we remember that, due to the action of the wings on the direction of the airflow over the plane (see **1**) the lines of flow to the propeller will suffer more or less deflection (normally upward) as compared with the line of flight. This deflection will vary, furthermore, with the attitude of the plane, greater as the angle of attack is greater. We shall thus have, in general, a variable angle between the line of airflow to the propeller and the line of flight. This combined with the variation in the angle between the shaft and the line of flight will result in an angle between the shaft and the line of airflow to the propeller, subject to more or less irregular variation, and in the general case differing from zero.

This complex of obliquities may, therefore, be considered as due to the fact that the lifting system, in order to maintain a lift L equal to the weight W at varying speeds, must assume varying attitudes in flight; and in this sense these various consequences on the propulsive system may be considered as due to the characteristics of the lifting system.

It must, therefore, be accepted that the propeller normally does not find itself in the simple condition of operation usually assumed, and in particular that it must in general accept a flow of air with some degree of obliquity relative both to the line of its shaft and to the direction of flight. This will result, as we have seen, in the production of lateral forces on the shaft and in a line of action of the total force on the propeller oblique to the line of flight. The obliquity of the flow to the propeller will itself result in some loss of efficiency and the obliquity of the total resultant propeller force to the line of flight will cause a further loss in propulsive effect. Adequate experimental data bearing on these losses do not seem to be available, but it may be safely assumed that such loss will not be serious until the angles of obliquity approach values larger than those in usual practice. However in the case of a plane making long flights, in particular for those periods of time for which the angle of attack may be relatively high, these sources of propulsive loss will be operative in some degree and corresponding allowance should be made for them.

In addition to these influences traceable primarily to obliquity of airflow and shaft direction, there may be, especially with wing mounted engines, a further influence due to the wing itself by way of a slowing up of the airflow to the propeller, and similar to that of the fuselage as discussed in detail in 7, though smaller in amount. Reference to 7 may, therefore, be made for further details regarding this particular feature of the total reaction on the propeller.

In an extended examination of this same general problem by Betz[1] the conclusion is reached that the action of a propeller in the presence of a wing should be represented by a vector diagram including not only forces axially on the propeller but a vertical component also, equal in magnitude to the increment of lift on the wing (see 4). For some further details of this analysis, see Division L VIII 7.

Another possible source of influence may be found in the partial arrest by the wings of the rotary component in the wake flowing from the propeller. The effect of this rotary component on the wings is referred to in detail in 4. Inversely the presence of the wings in the wake may be reacted back through the rotary component on the propeller, affecting more directly the torque in some small degree.

4. Influence of Propulsive System on the Lifting System. The character and relative magnitude of this influence have been dealt with in mathematical detail in Division M and to which reference should be made.

The reaction between propeller and wing results primarily as a consequence of the wake which the propeller drives aft and in which the characteristics of the flow differ from those for the flow over the wings otherwise, in the following particulars. (1) It has an accelerated velocity aft. (2) It has a velocity of rotation—*i. e.* a rotary component of the total velocity. (3) The turbulence will be more pronounced than in the airflow generally over the wing. We have, however, to remember that this wake extends over only a part of the span of the wing—larger naturally in the case of a multi-engined plane than for a single propulsive unit.

We may first consider briefly and without mathematical detail the more obvious consequences of these three characteristics of the wake as noted above.

First with regard to the wake itself, it should be noted that as a result of the difference in the conditions within and without the wake, there will exist a surface or zone of discontinuity separating the cylindrical wake from the remainder of the airflow. The major influence of the propeller, or of the wake which it generates, will, therefore, be manifest on the parts of the plane lying within this wake. We cannot, however, assume that the wake is entirely without influence on the parts lying outside its boundaries. As shown in Division M the complete mathematical analysis of the problem requires the recognition of certain

[1] Betz, A., Der Wirkungsgradbegriff beim Propeller, Zeitschr. f. Flugtechnik u. Motorl. **19**, 171, 1928.

effects outside the wake boundary, even though their intensity may rapidly decrease outside the boundary limits.

Passing these over for the moment, however, we may restrict ourselves to the more obvious effects due to the three characteristics of the motion in the wake as noted above.

First with regard to the increased axial velocity in the wake. This will have the effect of producing an increased lift over the part of the wing lying directly in the wake, measured approximately by the square of the ratio of the two velocities, within and without the wake. At the same time this increased lift will be obtained at the price of an increased frictional drag over this part of the wing, practically in the same ratio. The increase of lift will also entail a corresponding increase of induced drag[1]. The net result is, therefore, an increase in lift and an increase in drag over this part of the wing, substantially as though it were moving with a speed of advance $(V + u)$ instead of V, where V is the speed of advance of the plane relative to the outside air and u is the speed of the wake to the rear.

Taking next the velocity of rotation, it is evident from elementary mechanics, that the torque which the engine exerts in turning the propeller must have its equal in the production of angular momentum in the medium acted on by propeller—*i. e.* in the air. The external evidence of this angular momentum is then found in the angular velocity imparted to the air in the wake in consequence of which the actual air particles stream backward down the wake in helical paths, forming ideally a sort of twisted rope of air constituting the wake proper and separated from the outlying body of air by the surface of discontinuity to which reference has previously been made.

In the usual case, however, the flow in the wake will not be permitted to form any such coherent ideal helical stream, but will be more or less broken up and diverted by parts of the structure lying directly in its path. Thus in the case of a single engined plane with propeller at the nose of the fuselage, the latter will lie directly in the propeller wake which must, in consequence, divide and pass around the fuselage with the assemblage of wings still farther to the rear. Similarly with multi-engined planes, the wake of the wing propellers will be obstructed by some form of engine nacelle and also by the leading edge of the wing itself, causing usually a complete separation of the wake into two parts, one passing above and the other below the wing.

Nevertheless, the air as it leaves the propeller will have this angular component of velocity and corresponding angular momentum and wherever it impinges on other parts of the structure will produce results accordingly. Thus on the wings, for the part directly within the wake, this angular component may be viewed either as a component velocity

[1] See Division E I (11.4).

impinging on the wings, above on one side and below on the other, or as a component of the total velocity increasing the angle of attack on one side and decreasing it on the other. In either case the result is the same, with an increase of lift on one side and a decrease on the other, thus producing a rolling moment about the longitudinal axis.

Obviously these two effects of increase and decrease will nearly balance, leaving a negligible effect on the lift as a whole. So far, therefore, as the rotary component of the velocity in the wake is concerned, we are left with the production of a rolling moment produced by the upflow and downflow components acting respectively on the under and upper sides of that part of the wing structure lying within the wake boundary, or otherwise, due to a difference in lift on the two sides caused by changes plus and minus in the effective angle of attack on that part of the wing lying within the slipstream, due to the opposite directions of the rotary component of flow on the two sides.

It may be noted in this connection that the engine torque transmitted through the engine frame to the structure of the plane will produce a reaction torque tending to rotate the plane as a whole in a sense opposite to that of the propeller. This is, in fact, the reactive torque, the equivalent of which is found in the production of angular momentum in the wake as noted previously. But the sense of rotation of the air in the wake is the same as that of the propeller and hence the sense of the rolling moment produced by the action of the wake on the wings will be in the same sense. Hence the two moments, one due to engine torque and the other to angular rotation in the wake will be opposite in sense and will thus tend to balance. Actually, however, the moment due to torque will greatly exceed in magnitude that due to angular rotation in the wake, leaving as a net result a distinct rolling moment in the sense opposite to that of the propeller, the existence of which must be recognized by suitable adjustments in the "rigging" of the plane. (Aileron tabs, differential aileron control, etc.)

Passing next to the effect of increased turbulence in the wake, it may be noted that the general subject of varying turbulence in an airstream and its effect on the force reactions on a body immersed in such a stream, is involved in a number of the interactions considered in the present Division, and for this reason certain general statements regarding the subject may be introduced at this point[1].

It has long been known that the behavior of a fluid flowing along the surfaces of a solid body is profoundly affected by the previous history of the fluid as regards the presence or absence of turbulence. Thus in the early experiments on the flow through pipes by Osborne Reynolds and others, it was found that the value of the parameter

[1] See Division G **19**; also Division I, Part 2, p. 329 *et seq.*

lv/ν (the so-called Reynolds number) at which the transition from laminar to turbulent flow occurs, was directly dependent on the condition of the fluid entering the pipe. With sharp edges at the pipe entrance, or in the usual case where some initial turbulence is present, the value of this parameter is known to be about 2,000; while with rounded entrance and by avoidance of initial turbulence, this value may rise to perhaps ten times this figure. This immediate effect is traceable to the more or less abrupt change from laminar to turbulent flow.

In the case of bodies placed in an indefinite fluid stream, in which the flow is always in some degree turbulent (as is the case for all conditions with which we are concerned) the results, first with respect to drag, are much more complex in character, depending on the form of the body and on the Reynolds number and with possibly opposite effects on the two components, "form" drag and "skin friction" drag.

For bodies of bluff form, for example a sphere or an airfoil or wing or body of similar form at a high angle of incidence, the form drag, represented by an eddying turbulent wake, will vary with the size of this wake and with the stream of energy which it represents. That is, the larger the wake the greater the form drag. But the formation of a wake of this character is due to the failure of the stream flow, and in particular the boundary layer, to follow around the form of the body and close in about its after part. Dependent on the form and surface texture of the body and on the Reynolds number there is a tendency of the flow to separate from the body thus forming a surface of separation or boundary, streaming to the rear, and within which the fluid is in a condition of mixed eddying turbulence forming a so-called "dead water" or "dead air" wake.

Now any condition which will tend to retard this separation, or enable the flow to more completely encompass the body will result in a decreased size of this eddying wake and a correspondingly reduced form drag.

But it is known that increase of turbulence in the boundary layer, or generally in the air flowing about the body has precisely this effect and hence we have the general result that with highly turbulent air the boundary layer more effectively resists separation and so in special cases, the resulting form drag is less than with air less turbulent.

A further effect, and of even greater inportance, results with forms intended to produce lift—airfoils, wings, symmetrical sections as used for control surfaces, etc. With such forms, as is known, the lift increases continuously with the angle of incidence, up to some value (the so-called burble point) at which separation develops and the lift suddenly decreases. However, the effect of increasing turbulence in the air flowing about the wing has the effect of postponing such flow separation to higher and higher angles and thus of enabling the wing to carry its normal law of lift increase to greater angles of attack and thus to higher values of the lift than with air of lesser turbulence.

Turning now to the effect of increasing turbulence on skin friction drag, it is known that, in general, greater turbulence in the boundary layer will increase this form of drag. This may be assumed to be due to a more energetic action between highly turbulent air and the surface texture of a body, than with air less turbulent. As a result, where the parasite drag is primarily "frictional" and in only small or negligible degree "form" (as with well streamlined bodies such as airship forms, airfoils and wings under small angles of attack, etc.) the drag will in general increase with increasing turbulence.

In attempting to assess the influence of turbulence, therefore, these two possibly opposite effects must be held in mind. For such forms as are used for wings and control surfaces, the result at small or moderate angles of attack will be an increase of drag with increasing turbulence—small in amount—with no sensible influence on the lift. And then as extreme angles of lift are reached, a marked increase in lift and decrease in drag as compared with values for less turbulent air.

It may be noted here that by the phrase "varying turbulence" we mean varying over the range readily producible by laboratory methods. The air in wind tunnels and as generally used in laboratory research is characterized by rather high turbulence, which may, however, be increased by suitable means (wire screens, etc.). Air in the open atmosphere, as met with in ordinary flying, is normally much less turbulent, but as it meets the various parts of an airplane its turbulence may be much increased by influences as noted, and with results generally as indicated by the preceding discussion[1].

In general, regarding the influence of turbulence, it may be said that its effects are produced primarily by way of its influence on the conditions (form, surface texture, Reynolds number) under which separation of the flow takes place. According as separation of the flow takes place earlier or later, so will the size of the turbulent wake vary and so will the relative areas of contact between the surface of the body and boundary layer flow on the one hand or dead air contact on the other, vary, all with resulting effects on lift and on form and skin friction drag.

We have therefore in summary for the effect of the propeller on the wings, a generally increased value of the force reactions over the

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[1] Dryden, H. L., and Kuethe, A. M., The Effect of Turbulence in Wind Tunnel Measurements, U.S. N.A.C.A. Technical Report No. 342, 1930.

Millikan, C. B., and Klein, A. L., The Effect of Turbulence, Aircraft Engineering, August 1933.

Relf, E. F.: Results From the Compressed Air Tunnel, The Journal of the Royal Aeronautical Society, January 1935.

See also Division G **19**.

part of the wings lying within the wake, comprising an increase of both lift and drag, and the production of a rolling moment. The first of these is due primarily to the increase in axial velocity while the latter is due to the component of rotation which characterizes the motion of the air in the wake.

Regarding the effect of turbulence, it cannot be assumed that, under ordinary conditions of flight, increase of turbulence in the propeller wake will count as a factor of practical importance. It seems possible, however, that at extreme angles and when approaching the burble point, the influence of such turbulence, over that part of the wing on which it is effective, will be to delay the separation of the flow and aid the wing in carrying on to a somewhat greater angle of attack and higher lift than might be otherwise possible.

At this point also, mention may be made of a further influence on the phenomenon of separation[1] due to the possible influence of the propeller in modifying the angle of attack on the wing. Under given conditions otherwise, separation begins at a definite angle of attack which depends primarily on the geometrical form of the section. If then the effective angle of attack is increased, separation will be hastened; if the angle is decreased, it will be retarded. Depending on the combination of angles to which reference has been made above, the action of the propeller may, therefore, through the influence of the wake on the angle of attack, hasten or retard the beginning of separation of the flow from the wing. Normally, the result will lie on the side of retardation of separation, affecting primarily, of course, that part of the wing directly exposed to its influence.

A further influence on separation is to be found in the negative pressure gradient in the propeller wake. It is known that the increment of axial velocity in the wake is subject to a continued increase from its value just behind the propeller disc to twice this value at a great distance down the wake (∞ in strict theory). This will mean a continued acceleration in the wake with a negative pressure gradient along the line of the flow (Bernoulli's law). Now it is known that the phenomena of separation are intimately associated with the development of a positive gradient of pressure in the boundary layer, against which the flow cannot make headway and separation results[2]. Any influence, therefore, which will tend to relieve this condition by supplying a counter tendency, will, to that extent, tend to reduce the tendency to separation otherwise existing.

The results thus far stated are relatively obvious. The detailed analysis developed in Division M, and to which reference should be

[1] See Division M II **13**.

[2] See Division G **15**; Division I, Part 2, I **3**.

made, shows many modifications of this relatively simple picture. Thus by the vortex theory of the propeller[1], the propeller wake may be viewed as a stream of twisted vortex filaments, while again the surface of discontinuity bounding the wake may be viewed as a cylindrical vortex sheet. As may be readily seen, the possible interactions between the wing vortex system and such propeller systems would present a problem of very great complexity. It is also clear that in this broader view, the influence of the propeller wake will be no longer confined to the parts of the structure with which it comes into immediate contact, but will extend broadly to the whole flow system about the plane, such outside effects, however, being relatively small in comparison with those within the wake itself. Broadly this view relates the results of the interaction to changes in the circulation about the wing and to changes in the axial and vertical velocities of the airflow to the wing.

In the analysis of Division M the final results are expressed as due to the four major parts of the complex whole:

(1) The direct influence of the change in longitudinal velocity in the wake.

(2) The direct influence of the vertical component velocity in the wake.

(3) The indirect influence of the slipstream boundary.

(4) The direct influence of the slipstream boundary.

Again each of these is considered with reference to its effect both within and without the slipstream boundary, thus giving eight possible constituent elements of the whole, though in particular cases, certain of these may become zero.

Added to these influences, susceptible in some degree to theoretical treatment, there will be some further influence due to turbulence as referred to at an earlier point.

For further details, reference should be made to Division M. These brief references to the mathematical aspects of the problem have been made only to indicate how vastly more complicated the problem is than might at first sight appear. The more extended view furnished by such analysis shows that while the intensity of the effects of the propeller wake falls off rapidly outside the wake boundary, these effects as a whole, notably on the wings, are still of significance and must be considered in any measurably complete picture of the problem. These conclusions are also borne out by a limited field of experimental results to which reference is made in Division M.

5. Influence of the Lifting System on the Control System. As we have already seen, the action of the lifting system, significant for present purposes, is the production of lift through the combination of velocity and circulation, the latter for a given lift varying inversely as the speed.

[1] See Division L VI.

The effect of this circulation is to produce the familiar downflow velocity, the result of which is to modify by the induced angle α_i, the entire picture of the flow of air over the plane.

In addition to this general effect due to downwash, the direction of flow of the air leaving the following edge of the plane will be further modified in some degree, and we thus find the control system, working in a flow of air with generally a definite downward component as compared with the direction of airflow at a great distance. In general, therefore, the tail control surfaces must be so set for what may be called their zero position, as to take due account of these changes in the direction of airflow.

In addition to this change of direction, the presence of the wing will further induce some degree of turbulence in the air leaving it and meeting the tail surfaces. This effect will, therefore, conjoin with that due to the same general cause resulting from the presence of the fuselage and the action of the propeller, as referred to later in **9** and **11**.

However, as noted in **4**, we can hardly expect any pronounced effect on the control surfaces due to turbulence, except in positions of large or extreme movement from neutral. Under such conditions, as noted for the wings in **4**, it may be expected that some beneficial effect of the turbulence might result.

These remarks on the influence of turbulence refer more directly to what is sometimes called "fine grained" turbulence, such as is normally found within the boundary layer itself. On the other hand, the aerodynamic behavoir of a body of wing or control surface form, placed within a flow of relatively large sized eddies such as constitute in greater part a "dead air" or turbulent wake, is entirely unreliable, especially as regards the "lift" forces which it may be intended to develop. Such a condition may, to some extent develop as between wings and horizontal control surfaces depending on proportions and form and angle of attack. To the extent, therefore, to which such surfaces find themselves within a turbulent air wake (wake shadow) so will they become in relative degree ineffective and unreliable. In the extreme case, control may be lost and a spin may develop with resulting crash, unless suitable altitude is available for recovery[1].

A further influence on the control surfaces causing a loss of force reaction is due to the effect of skin frction on the wings. This slows up the velocity of the air in the boundary layer which then passes on forming the boundary layer wake and contacting the control surfaces. Since such force reactions vary with the square of the velocity, it is not difficult to see that this loss may be very considerable quantitatively and must be allowed for in design.

[1] See Division J III **13**; also Division O VIII.

On the whole, therefore, the normal effect of the wings on the tail control surfaces is threefold: first, by causing a change in the general direction of the airflow acting on these surfaces and to which they must be adjusted in the so-called rigging of the plane; second, an increase in the turbulence of the air flowing over the control surfaces, the effect of which may be, in positions of extreme throw (elevator or rudder), an increase of force reaction or perhaps rather, the saving or postponing of a loss of force reaction under possibly critical conditions of flight; and finally a loss in force reaction due to the effect of skin friction on the wings.

The above discussion refers primarily to the tail surfaces as representing that part of the control system located at the tail of the fuselage and hence entirely separated from the wings themselves. In addition to the tail structure, the ailerons, directly attached to the wings, form a second important feature of the control system. The general character of the influence of the wings on the ailerons may be inferred from the preceding remarks regarding the elevator or rudder. In the normal position they form simply a part of the wings themselves. In the operative position, they receive the air as it flows to them from the wing structure forward, modified as this may be by the circulation and downwash immediately at the rear of the wing. At low angles of attack or for such angles as permit a smooth continuous flow over both upper and lower wing surfaces to contact with the ailerons, the forces on the two ailerons, up and down, for equal angles of throw from mean position, will be practically the same. At large angles of wing attack, however, and especially near the burble point, the aileron turned upward may find itself in a turbulent eddying wake and for reasons noted above, subject to a much reduced force reaction per unit area than under normal conditions of flow. These conditions will result in a decrease of effectiveness of the aileron so situated, a decrease which must be recognized in connection with the use of the ailerons at or near the stalling attitude of the plane.

6. Influence of the Control System on the Lifting System. The control system is represented by the elevator, rudder and ailerons. In the neutral position the influence of these on the wings is small if not negligible. In such position the rudder forms simply an extension of the fin, the elevator an extension of the stabilizer, while the ailerons form extensions of the wings. Under these conditions the elevator together with the stabilizer will form a surface adapted to develop a vertical force, depending on the effective angle of incidence in which it finds itself relative to the air in the wake of the wings and fuselage. For purposes of stability the setting of the stabilizer will be such as to give, with the wings themselves, a combination stable longitudinally. The primary function of these parts of the control system is thus to provide a moment offsetting the naturally unstable moment of the wing by itself, rather than to develop lift as such.

However it is clear that, located as the stabilizer and elevator are, relatively far downstream from the wing, they can have no significant effect on the circulation about the wing, which, with the density and speed determine the amount of the lift on the wing itself. With the elevator thrown out of neutral position, as in the execution of a maneuver, the same general conditions prevail. The aerodynamic conditions about a body in relative motion with the air are highly sensitive to what takes place upstream from the body, but practically insensitive to conditions at any sensible distance downstream. We may conclude, therefore, that so far as the elevator and stabilizer are concerned, their influence on the wing lift is negligible.

Taking next the rudder, it will be clear that the same general conclusions hold here as for the elevator. Whether the rudder is in neutral or displaced position, it is evident that it can have no sensible influence on wing lift.

Coming now to the ailerons; these, when in the neutral position, form a part of the wing proper and thus constitute a part of the wing system. It is only when they are thrown out of neutral position that the question of their influence on the wing lift becomes of importance.

In such case, one aileron will be thrown up and the other down. The wing with the aileron thrown upward suffers immediately a loss of a part of its normal lifting surface. The oblique extension of the aileron upward, will furthermore, for that part of the wing over which it extends, disturb in more or less marked degree, the flow of air over the back of the wing and the circulation around it. In general, these results will all tend in the direction of a decrease in the lift over this part of the wing span, thus favoring the moment about the longitudinal axis which this movement of the ailerons is intended to produce. On the other hand, the oblique extension of the aileron downward, will, for that part of the wing over which it extends, produce a disturbance in the flow tending normally toward an increase in the lift on that part of the wing span and thus again aiding the moment which the aileron movement is intended to produce. This action of a depressed aileron is analogous but somewhat less in extent, than that due to the various forms of flaps which are now taking so important a place in connection with airplane control, especially in the way of a reduction of landing speeds through an increase in lift connected with a sharp increase in drag. While, therefore, the effect of displaced ailerons is to produce a change in the lift, the effects on the two sides of the wing will be opposite in direction and will usually nearly balance, so that the effect on the lift as a whole is not usually of importance.

We reach, therefore, the general conclusion that so far as the lift as a whole is concerned, the influence of the control system, whether in neutral or displaced position, is small or negligible.

However, it may not be out of place to note, with reference to the ailerons, that while their influence on the lift as a whole is small or negligible, the same is not the case with regard to the drag. Lift cannot be realized without drag, and while lift is the basic function of the wings, drag is an unavoidable attendant circumstance. In this respect, the ailerons, displaced from neutral position, will produce a definite influence on the drag. Thus the increase of lift associated with the "down" aileron will give an increment of induced drag, while the possible increase of turbulent wake following such aileron may increase also the form drag. On the other hand the reduced lift caused by the "up" aileron will cause a decrease of induced drag while the possible increase of turbulent wake will have the same effect as for the down aileron. The result will be a difference in drag giving a yawing moment opposing the turn for which the aileron shift is intended to provide the proper rolling moment, and thus requiring a further aileron movement than would otherwise be necessary.

The same increase of form drag for the plane as a whole may also be noted for the rudder or elevator displaced from neutral. Here, of course, the influence is not on the lifting system direct, but rather appears as a circumstance inseparable from the realization of the particular function required of these elements of the control system.

7. Influence of the Non-Lifting System on the Propulsive System[1]. The typical combination presenting this particular reaction is that of a propeller located at the nose of the fuselage, as in all single engined planes and in many multi-engined planes. This combination presents, in effect, the problem of an obstruction located just behind the propeller. To examine the effect of such an obstruction we have first to note that as the obstruction is more or less abrupt and large in transverse dimension, the lines of stream flow approaching the plane will spread, and the relative velocity will decrease in comparison with that of the air at outlying points. This will have the effect of slowing up the air passing through the propeller disk, and this, for a given value of N (the revolutions) will give an increased value of the thrust. In a free stream the form of the thrust coefficient curve corresponding to the equation

$$T = C_T \varrho n^2 D^4$$

shows increasing values of the coefficient for decreasing values of the

[1] For a general discussion of the subject matter of this and the following section, including also to some extent the subject matter of **3** and **4**, reference should be made to Division L VIII. The chapter includes mathematical analysis, formulae and methods for quantitative estimate of results and references to sources of experimental data.

parameter v/nD. But a decrease in the value of the velocity of flow of the air to the propeller will, for the same revolutions, decrease the value of v/nD and hence will give a higher value of C_T and with given n a higher value of T. This, of course, is fully borne out by experiment. In a wind tunnel, for example, with the revolutions held constant, there will be some speed of the air for which the value of T will be zero (speed for zero thrust). At higher speeds a negative value of T is developed, but we are not here concerned with this phase of operation. Starting then from the speed of zero thrust, the value of the thrust will continuously increase with decreasing wind speed, reaching its maximum value (aside from a reversal of the flow) when the tunnel fan is stopped and the air approaching the propeller has only the speed due to the propeller itself acting as a fan.

The simple and direct result of an obstruction such as the nose of a fuselage located directly in the rear of the propeller is, therefore, to give, for a fixed value of the revolutions, an increased value of the thrust. If now the efficiency of the propeller is measured by the product of thrust by speed through the air divided by power delivered by the engine to the propeller, there will be an apparent gain in efficiency since, with other things the same, bringing of the obstruction to a point just behind the propeller has resulted in an increase in the actual thrust developed by the propeller and hence apparently an increase in the numerator of the fraction defining the efficiency. This gain in efficiency however, is only apparent as will appear in 8 when we come to the reverse action of the propeller on the fuselage. These two reactions must in fact be considered together in order to obtain a correct view of the over-all result of this particular pair—the action of the fuselage on the propeller and that of the propeller on the fuselage. Without further discussion of the present phase of the matter, we pass, therefore, to the reverse action in the following section.

8. Influence of the Propulsive System on the Non-Lifting System[1]. As in the preceding section, the typical combination here is a propeller located at the nose of a fuselage. The immediate action of the propeller is to send to the rear a wake of velocity relatively greater than that of the outlying body of air, and such a stream of air parting and passing around the fuselage will give a definitely higher velocity of flow than for the case of like motion without the action of the propeller. If then we compare the case of a fuselage towed through the air with no propeller at the nose, with the actual case of a fuselage drawn through the air by the action of a propeller located just ahead of the forward end, we shall have a definitely higher velocity of flow over and about the fuselage in the latter than in the former case. This will result in an increase of

[1] See footnote to **7**.

resistance to motion, or in other words in an increase in the fuselage drag. It may be also noted that this increase in drag will be due both to the increase in the axial velocity of flow and also to the rotary component in the wake with the resultant disturbance to the lines of flow otherwise and the possible production of an increase in the turbulent wake.

If now we combine in one picture the results for the preceding and the present sections, we have a propeller in a location where it is able to develop an actual increase of thrust for a given number of revolutions, but at the expense of an increase of drag on the fuselage behind it. Comparing these results with those for the same propeller in a free stream and a fuselage in motion without the action of a propeller at its nose, we see that if we credit the propeller with the extra thrust developed, we must likewise charge against it the additional drag developed on the fuselage. The net result will be, therefore, the thrust which the propeller would develop in a free stream (taken as equal to the drag of the airplane as a whole at the given speed) plus the extra thrust developed by reaction with the fuselage and minus the additional drag developed on the latter. This result, as the net or useful thrust taken with the speed and with the shaft power to the propeller, will then give the over-all efficiency of the propeller including the mutual reactions between itself and the fuselage.

In such case, experiment shows that, at least for normal forms, the over-all result, over the operative range of values of v/nD, is a loss in efficiency. The combination of the propeller and the fuselage at close quarters is less efficient than if each could operate independent of all influence from the other.

In order to obtain a more detailed view of these mutual reactions over the whole range of values of v/nD, we may conveniently start again with the influence of the fuselage on the value of v/nD for zero thrust (see Fig. 2).

From what has been said above, and as shown quantitatively by experiment[1], the presence of an obstruction results in shifting to the right (to a larger value) on the axis of v/nD, the value of the latter for zero thrust. This condition is readily seen to follow as a result of slowing down the column of air actually operative on the propeller as compared with the air flowing at a distance. For any value of the wind velocity as based on the latter, the air column operative on the propeller will be slowed down, the thrust generally will be increased and the value of v/nD for zero thrust will be increased. It will result that the curve for thrust or thrust coefficient for the combination of propeller and fuselage, as compared with that for the propeller alone, will start at a larger value

[1] U.S. N.A.C.A. Technical Report No. 220, 1926.

of v/nD and, near the start must lie above that for the propeller alone (see Fig. 2).

However, as the slip becomes greater with decreasing values of v/nD, the gain in net thrust decreases and ultimately the two curves meet and cross. For the conditions covered by the investigation indicated by the reference, this point of crossing was not far from the value of v/nD for best efficiency. Beyond this value the curve for net thrust lies below that for the propeller alone, showing for this range of operative values of v/nD a loss in net thrust for the combination as compared with the propeller alone.

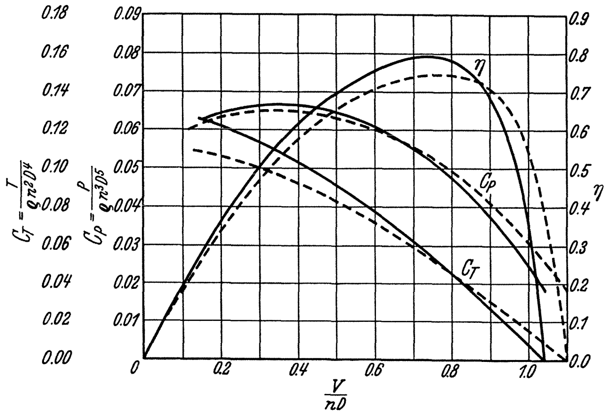

Fig. 2.

It will be noted, however, that the range of values of v/nD, for which there is a gain in net thrust, lies quite outside that for normal operation. The values of v/nD lying between that for best efficiency and that for zero thrust are values for small and decreasing values of the slip and, for a given thrust, would require a prohibitive diameter of propeller. The range of practically operative values of v/nD lies normally near and below that for best efficiency and for this range there is, for the combination, a loss in net thrust.

Similarly as for the thrust, the torque and hence the shaft power coefficient for the combination is increased for large values of v/nD and decreased for small values, with a crossing point apparently at a smaller value of v/nD than for the thrust (see Fig. 2).

In consequence of these relative changes in the values of the thrust and power coefficients, it results that, for very large values of v/nD, there is an actual gain in propulsive efficiency for the combination as compared with the propeller alone, while for moderate and smaller values (over the operative range) there is a loss in propulsive efficiency. This possible gain in propulsive efficiency is readily seen by taking the extreme case of the value for zero thrust with the propeller alone. Under these conditions, the thrust is zero while the torque will have a definite positive value and the efficiency is, therefore, zero. Now with the combination, for the same value of v/nD, the thrust is no longer zero but becomes positive and with a positive value of the torque gives a value of the efficiency greater than zero.

The two curves of efficiency will thus cross and the point of crossing, for the conditions of the reference, was at a value of v/nD somewhat greater than that for the best value on either curve.

Again the best value of the efficiency for the combination is less than that for the propeller alone and generally this relation will hold over the entire range of values of v/nD from that for the point of crossing of the two curves, indefinitely to the left, or toward smaller values of v/nD.

Thus, finally, over the range of practical values of v/nD, the net result for the combination of propeller and fuselage will be a loss in propulsive efficiency. The amount of such loss will obviously depend on many circumstances, but again for the conditions of the reference, the indicated values of the loss ranged from 2 to 5 per cent for normal operative values of v/nD.

The typical combination assumed for the preceding discussion has been taken as a propeller and a fuselage. Evidently the same general principles apply and the same general results will follow for the combination of a propeller and an engine nacelle in the case of multi-engined planes, or for any similar combination of a propeller with an obstruction placed near and in its direct wake.

In addition to the above noted primary sources of reaction between the propeller and the fuselage, mention may be made of a further small increment of drag which may arise as a result of the decreasing pressure gradient in the propeller wake, as referred to above in 4.

9. Influence of the Non-Lifting System on the Control System. For our present purposes, the control system may be considered as represented by (1) the tail surfaces forming the empennage, and (2) the ailerons. The first is located at the rear and in the direct wake of the fuselage; the second is located on the wings and at a distance on either side of the fuselage.

The chief effects of the presence of the fuselage, so far as we are here concerned, are in the disturbance to the direct flow of air to and past the plane, in the production of some degree of turbulence in the air approaching the tail surfaces, and in the effects due to skin friction.

In free air, some turbulence may in general be expected; but the presence of the fuselage will increase this in degree and we may, therefore, expect that the control surfaces forming the empennage must normally be met by air with a relatively high degree of turbulence. The actual condition in this respect, will, furthermore, be due, not only to the presence of the fuselage, but also in still greater degree to the action of the propeller located at the nose of the fuselage. The actual condition at the empennage is, therefore, due to these two agencies in combination, of which, however, at the present moment, we direct attention more especially to the part due to the fuselage.

The result which we are now considering is, therefore, simply that of turbulent air flowing over the control surfaces as compared with air relatively free from turbulence. The general effect of such condition in the air, as we have already seen in 4, will be negligible for small or moderate angles of throw of the control surfaces. Under large angles of throw, however, the effect will tend to retard conditions of airflow separation, and as earlier noted, permit effective service of the control system at angles of throw perhaps somewhat greater than would otherwise be possible.

On the other hand the general disturbance of the flow to these surfaces due to the presence of the fuselage, will tend to produce prejudicial effects on the aerodynamic force reactions upon them. The skin friction will slow up the boundary layer flow, the same as for the flow over the wings referred to in 5, and with a similar result in reducing the force reactions on the control surfaces over which such flow passes. The extreme of such disturbances may result in placing these surfaces in a dead air turbulent fuselage wake "shadow" with more or less complete loss of effective operation. The consequences of such partial or complete loss of control, as the leading factor in the development of spins, lends special importance to effects of this character[1].

On the ailerons, situated as they are, well outside the direct influence of the fuselage, the action here considered will obviously be negligible and need not be further considered.

10. Influence of the Control System on the Non-Lifting System. In more specific terms, this is a question of the influence of the elevator, rudder or ailerons on the fuselage. Put in these terms, it is clear that such influence must be small or negligible. The elevator and rudder are located at the tail end of the fuselage and the ailerons are on the wings usually far removed from the central body of the plane. Remembering then that the aerodynamic result of the fuselage, or of the non-lifting system generally, is the development of drag, it seems clear that the elements of the control system can have no sensible influence on the conditions which determine the amount of such drag and we may, therefore, dismiss this part of the subject without further consideration.

On the other hand reference has been already made to the addition to the total drag which may be due to the elements of the control system when displaced from neutral position. While, therefore, they can have no sensible influence on the fuselage drag as such, they will, when in operative position, cause a definite addition to the drag as a whole.

[1] See Division J III 13; also Division O VIII.

11. Influence of the Propulsive System on the Control System[1]. In **5** and **9** reference was made to the influence of the wings and fuselage on the control surfaces by way of a disturbance to the airflow with generation of turbulence. The propeller, located at the nose of the fuselage, will augment in marked degree these same causes of disturbance at the tail surfaces, and in addition will give an increase in the axial velocity as well as a rotary component in the flow of air to and over the tail surfaces.

No further discussion is required regarding the influence due to turbulence other than to note that the actual final result at the tail surfaces will be the combination of the influences discussed in **5** and **9** and in the present section, and of which the part due to the propeller is presumably the greater of the two.

It may be noted, however, that cases have been known where the dead air wake caused by a propeller hub of unusually large size has had the effect of seriously reducing or even practically destroying the effective operation of the tail surfaces operative in such wake. See also similar condition noted in **9**.

With regard to the increase of the axial component in the propeller wake, this will, in itself, give rise to an increase in all force reactions on the surfaces over which it flows, while at the same time, in combination with the downwash velocity, it will have the effect of decreasing the angle of attack on the horizontal tail surfaces and likewise the rate of change of this angle, or otherwise, of varying this angle according as the increment of axial velocity varies with varying conditions of operation. Actually the condition is still more complex since we have here to consider three directions—the line of flight, the downwash direction, and the line of the shaft. These three directions with the actual values of the several velocities referred to above will determine the final angle of attack and velocity of flow to and over the horizontal tail surfaces.

Likewise for the fin and rudder, the increase in axial velocity will have the effect of modifying the angle of attack and the rate of change of this angle, with corresponding results on its effect in connection with maneuvers and flight stability.

Then we must further include the rotary component of the velocity in the propeller wake. It is clear that this will have the direct effect of modifying the angle of attack for the entire system of tail surfaces from the value which it would have under flow without rotation. For the horizontal surfaces which are located on either side of the center line, the result will be an increase on one side and a decrease on the other, thus introducing a condition of unbalance on the two sides with a resultant moment tending to rotate the plane about the longitudinal axis. This is the same action as that produced by the propeller on the wings and noted more especially in **3**.

[1] See Division M II **15**, **16**.

Likewise on the fin and rudder, usually located for the most part, or wholly, in the upper part of the propeller wake, the action will be a change in the angle of attack, to the right or left according to the direction of rotation of the propeller. This again will have a tendency toward rotation or at least lateral movement, similar to the action on the wings and horizontal surfaces, but naturally less in amount.

Attention may be directed to the remarks in **4** regarding the rolling moment due to the engine torque applied directly on the plane, and to the fact that the moment on the wings due to the rotation in the propeller wake is opposite to it in sense. The same observation applies here in regard to the rolling moment resulting from the action of the wake on the control surfaces. As a whole and as noted in **4** these moments due to the wake will only in part, and normally in small part, balance the moment due to the torque, and provision for such balance must be made otherwise as indicated in **4**.

Finally it may be noted that the increase of axial velocity in the propeller wake will give rise to two opposite effects on the force reactions on the horizontal tail surfaces. The increase in velocity will, in itself, tend to increase these forces and the effects on the angle of attack, as noted above, will usually tend to decrease them, with the final result a compromise between the two.

In order to simplify the discussion thus far, the presence of the wing and fuselage in the propeller wake, ahead of the tail surfaces has been ignored and the effects noted above are those which might be expected with a skeleton plane comprising only propeller and tail planes. Actually, the presence of the wings and fuselage will complicate these conditions in marked degree. Also the location of the tail surfaces relative to the wake, as in the case of multi-engined planes, will furnish controlling conditions with regard to interferences of the type here discussed. The discussions of the preceding sections will serve to indicate the general character of these intermediate influences, but the actual results will depend in primary degree on the details of proportion and arrangement in the individual case.

It is obvious, since the functions of the tail surfaces relate directly to problems of control and maneuverability, that changes in the forces on these surfaces due to changes in angle of attack or velocity of airflow, or both in combination, will have a marked influence on all matters relating to stability and control. Here again, while the trend of individual causes can be clearly enough seen, in combination, the problem becomes very complex and will depend on the balance of interacting conditions resulting from the particular circumstances of the case.

Some discussion of these matters will be found in Divisions J III **9**, **13**; IV, Part B; M II **15**, **16** and N VIII and to which reference may be made.

12. Influence of the Control System on the Propulsive System. When in straight horizontal flight, the various items of the control system are normally in a mean or neutral position and have little influence on the flow of air over the plane and in particular little or no influence on the flow of air to the propeller or on the conditions in the wake. Furthermore, considering the main control items as located in the empennage, these are so far behind the propeller in all cases of normal design that even when thrown to an extreme angle (elevator or rudder) they can have no influence on the flow of air to the propeller itself; neither can there be any influence on the propeller wake which could reach forward to the vicinity of the propeller itself.

While in the extreme and exact sense it may be true that any disturbance from the normal neutral position of the controls will produce a change affecting the entire flow of air to and around the plane as a whole, it is very clear that, for practical purposes, such influence estimated with reference to the propeller must be entirely negligible and may, therefore, be dismissed without further consideration.

13. General Summary of Interferences. With a somewhat different manner of grouping, the more important features of the various interactions and interferences discussed in the preceding sections may be summarized as follows:

The wings will undergo interference: (1) By way of increase of lift and increase of drag, both friction and induced, due to the increase of velocity in the propeller wake. (2) Some small decrease in lift due to the reaction of the wing vortex system with the fuselage, resulting in a downward component of velocity in the air flow to the wing with decrease in the angle of attack[1]. (3) By way of a further decrease in lift and possible increase in drag, due to the production by the fuselage of a general disturbance in the airflow over the wing, such effect extending with decreasing amount from the fuselage outward to the tips of the wings. (4) Some increase in induced drag due to abrupt changes in lift distribution due to causes noted in the preceding number. (5) By way of local disturbance to the airflow and increased turblence generated about the junction of the wings and fuselage, resulting in some loss of lift and possibly a marked increase in drag. This entire effect may be much reduced by the fitting of generous fillets at the junction of wings and fuselage. (6) By way of a rolling moment due to partial arrest of the rotary component in the propeller wake, such moment balancing in some part the counter moment due to the engine torque. (7) At extreme angles of attack (at or near the burble point) a possible beneficial effect due to the action of increased turbulence in the propeller wake aiding in some degree to delay the separation of the flow and consequent sudden

[1] See **1, 2.**

loss of lift. (8) Possible delay of separation of flow due to negative pressure gradient in the propeller wake and to decrease of angle of attack due to increase of axial component of airflow in the wake. (9) From the ailerons considered as part of the control system, causing, when in operative position, a decrease of lift on one side and an increase on the other, thus producing a rolling moment on the plane in accordance with their intended purpose. (10) The increase of drag and the yawing moment due to the ailerons when in operative position may also be noted at this point:

The fuselage as representing the non-lifting system will be subject to interference: (1) By way of a change in the direction of flow to and across the fuselage due to the action of the wings and resulting usually in some increase of turbulent wake and hence increase in drag, and at the same time with some effect plus or minus on the lift which the fuselage may incidentally furnish. (2) By way of some small increment of lift due to reaction with the wings and representing a partial extension of the wing vortex system about the fuselage[1]. (3) Increase of skin drag and turbulence effects due to action of the propeller, both by way of the increased axial and the rotary components of velocity in the propeller wake. (4) Some small increment of drag due to negative pressure gradient in the propeller wake.

The propulsive system, or briefly the propeller will be subject to interference: (1) By way of a slowing down of the inflow air velocity due to the retarding action of the fuselage and wings (or nacelle and wings) immediately behind it, thus producing an increase of thrust, other conditions the same. This increase of thrust, we remember, is, in the general case, more than counterbalanced by an increase of fuselage and wing drag due to the influence of the increased axial velocity in the propeller wake. (2) By way of the influence of the wings in disturbing the direction of the airflow thus causing an inflow to the propeller in a direction oblique to the axis and resulting in the development of lateral forces and some loss in propeller efficiency. (3) By way of obliquity between the line of action of the total force acting on the propeller and the line of flight, resulting in a loss of propulsive effect. (4) By way of a reflection, so to speak, backward from the wings to the propeller, due to the partial arrest of the rotary component in the propeller wake velocity, producing some small increase in torque.

The control system will be subject to interference, **A**: As regards the stabilizer and elevator: (1) By way of a deflection of the air due to the form of the wing and to the downwash component of the airflow, requiring an initial setting of these surfaces in recognition of such effects. (2) By way of a decrease in the force reaction on the control surfaces due to the decrement of axial velocity in the friction wake of fuselage

[1] See **1, 2**.

and wing. (3) By way of increased turbulence due to the combined influence of propeller, wings and fuselage—usually not of serious importance at small or moderate angles of incidence on these surfaces, and possibly of some help in delaying separation of the flow around the elevator when at extreme throw. (4) By way of a decrease in the effective angle of attack on the horizontal surfaces due to the increment in axial velocity in the propeller wake, varying with varying conditions of propeller operation, and with consequent effects on longitudinal stability and control. (5) By way of a further change in the effective angle of attack at different attitudes of the plane and consequent varying obliquity of the propeller shaft with the line of flight. (6) By way of an increase in the force reaction on the control surfaces due to the increment of the axial velocity factor in the propeller wake [(4), (5) and (6) must be considered together with reference to the final result on the force reactions]. (7) By way of a rolling moment due to the rotary component of the velocity in the propeller wake and to which the initial setting must be suitably adjusted. **B**: As regards the fin and rudder: (8) By way of increased velocity in the propeller wake with general disturbance of flow, influence on angle of attack and increased turbulence, all due to the same general causes as for the horizontal surfaces. (9) By way of a lateral force or rolling moment due to the rotary component in the propeller wake, the same as for the horizontal surfaces. (10) Note should also be made of the possibility of a serious or even almost total loss of useful effect on the tail control surfaces in case they are operative in a "dead air" wake due either to the fuselage or an oversized propeller hub or some combination of the two. The same condition may also sometimes develop in maneuvers where the elevator might find itself in the wake of the fin and rudder or *vice versa* the rudder in the wake of the stabilizer. **C**: As regards the ailerons: (11) Small influence due to turbulent flow over the wings at small or moderate angles of wing attack and small aileron displacement, and possibly some advantage in retarding separation of flow when at extreme angles of aileron throw. (12) Possible serious loss of useful effect for the aileron deflected upward when main wing is at extreme angle of attack at or near the burble point, thus placing aileron in the relatively dead or eddying air in the wake of the wing. (13) To the extent to which the ailerons may be subject to influence from the propeller wake, the rotary component in the latter will still further modify the line of airflow to the ailerons, with opposite effects according as the vertical component velocity in question is upward or downward. (14) Added to these effects plus and minus on the direction of flow, there will be a small rolling moment due to the same rotary component in the wake acting on the ailerons, less however, when displaced from neutral position than when in neutral and forming a part of the wing system proper.

DIVISION Q

AERODYNAMICS OF AIRSHIPS

By

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PREFACE

Airship design leans more heavily on aerodynamic theory than does airplane design. Individual airships are much larger and more expensive than airplanes; the completed airship structure can much less readily be modified after its completion, so that the trial and error method is practically not as available for airships as for airplanes; furthermore, there is available comparatively much less experience from earlier airships because not many have been built, and even wind tunnel tests, although they have always been diligently undertaken, carry less persuasion in consequence of the larger scale effect and the larger sensitivity to such effect and to other doubt-inviting factors. All this is indicative of the need of a further development of airship aerodynamics as a foundation for further progress in the construction of large airships. Moreover, since airship design draws on the whole domain of aerodynamics and since special airship aerodynamics should contain as its most notable problem the full analysis of airship drag, it seems quite possible that from airship theory may some day come forward such fundamental progress in aerodynamics as shall revolutionize our technique of air travel.

Airship design involves, as a special field, the investigation of air forces brought into existence by the motion through the air of large, bulky, streamlined solids. The theory of the influence of air friction on these forces, in spite of strenuous efforts, has not yet been developed to a satisfactory status and has not been included in the treatment of the present Division. For this aspect of the general problem, the reader is referred to Division G. The present Division deals only with the theoretical motion of a perfect fluid, and constitutes an application of the principles and results developed in Division C.

The author presents herewith the results of an effort to organize airship aerodynamics along certain well defined logical steps, leading to a unitary, complete and convenient system of mathematical procedure. During the last decade this system has been received and used in the mathematical computations for the design of large airships built during that period. It is hoped that it may thus constitute a permanent nucleus for the development of applied airship aerodynamics.

The basic subdivisions for such foundation for an applied theory are as follows: (A) The resultant or integral aerodynamic effect of the entire airship structure is approximated by a superposition of the air forces on the bare hull, deduced from the laws of classical hydrodynamics, and of the air forces on the fin and control surfaces, assumed to follow the laws of modern airfoil theory. (B) The local distribution of the air forces along the axis and the pressure distribution is computed on the basis of a large elongation of the airship hull, thus reducing the actual three-dimensional flow around the hull to a superposition of two-dimensional flows. (C) The errors introduced by these assumptions are taken care of by the introduction of constant correction multipliers. (D) A three-dimensional flow for a mathematically simple shape is used for the computation of the pressure distribution over the bow region. (E) The general results of strict theory, valid for certain mathematically simple shapes, are generalized by means of engineering rules to cover practical shapes.

In studying the developments of the present Division, the reader will find it helpful to keep in mind these successive steps or stages, as guides or connecting links between the successive sections.

1. Introduction. The aerodynamic theory of airships deals with the loads imposed on the structural system of airships by the air forces, and with the problem of stability and the required fin areas. As a basic assumption the theory assumes the substitution of a perfect, non-viscous fluid for air, and for this reason fails to be of use for the computation of the performance, since solids moving in a perfect fluid experience no resistance. The actual resistance of airship hulls, while not indeed zero, is, however, surprisingly small relative to their bulk, and arises almost entirely from the direct action of viscous forces.

The present treatment is based chiefly on the theory and the solutions discussed in Division C.

The exact results comprised in that Division are confined to a very small number of shapes, all of great mathematical simplicity. It is the object of the present section to discuss the application of these results to airship shapes empirically given. This must be carried out through approximations in accordance with the usual procedure when applying the results of rigorous mathematical methods to the problems of nature. Indeed, improvements in the mathematical methods would be of little further use. The main source of the disagreement between the computed air forces and pressures and the actual ones is not the lack of better mathematical methods, but the incorrectness of the physical assumptions, especially the neglect of the viscosity. In view of the discrepancy between computation and observation caused by the viscosity of the air, the methods discussed in Division C are exact enough, and are furthermore sufficient for most practical purposes.

We proceed then with some discussion of approximate methods for the computation of the numerical values of such aerodynamic quantities as we shall need.

2. Area of Apparent Mass. As the first of these problems we shall take the question of the area of apparent additional mass in two-dimensional flow. In Division B VII 4 it is shown that the area of the apparent additional mass of a circle is equal to the area of this circle itself. The area of apparent additional mass of an ellipse moving in the direction of one of its principal axes is equal to the area of the circle on the diameter at right angles to the direction of motion. This rule includes the circle as a special case, and also the straight line as an ellipse of zero thickness. As a generalization of this theorem, we propose to apply this rule to any shape reasonably resembling an ellipse. The area of apparent additional mass of an airship cross section is accordingly equal to the circle over its height for horizontal transverse motion or components of motion, and to the circle over its width for vertical transverse motions or components of motion. The corresponding inertia factor will be denoted by η, and it must not be supposed that the error committed in adopting this rule is necessarily large for sections that are pear shaped or even with a sharp corner, since it is strictly true even for a straight line. For the common circular cross section

$$\eta = 1 \tag{2.1}$$

For other cross sections

$$\eta = \frac{\pi c^2}{4S} \tag{2.2}$$

where S denotes the area of the cross section and c its largest width or height at right angles to the motion considered, which motion is supposed to be in a principal direction, or in a direction of symmetry or at right angles to it. For lateral motion in other directions, the motion must be split up into components parallel to principal directions and the apparent masses computed separately for the components, as explained in Division C III. With a circle, every diameter is a principal direction.

3. Volume of Apparent Axial Mass. For an estimate of the volume of apparent additional axial mass of an airship hull, there are available in the exact theory the volume of apparent mass for two individual sources (Division C V 5), and the results obtained for the ellipsoid of revolution (Division C VII 5). Both results agree sufficiently well to indicate that with elongated airship hulls the volume of apparent additional mass for axial motion is only a fraction of the volume of the hull itself. This is not surprising as there are no large velocities produced by the axial motion of airship hulls except near the bow and stern. The magnitude of this volume depends much on the shape of the ends of the hull. Blunt ends have larger volumes of apparent mass than well tapered ones of the same volume. Two single sources give [see Division C V (5.5)]

$$K = \frac{2}{3} \pi b^3 \tag{3.1}$$

where b denotes the largest radius of the open body form produced by one source, which is a little larger than the largest radius of the shape equivalent to two sources with equal but opposite strength. The same value of the volume of the apparent additional mass is found for an ellipsoid of revolution with an elongation ratio a/b of about 4.5. In this case, however, b is actually the largest radius, and a the semi axis. With increasing elongation, the coefficient of additional apparent volume of the ellipsoid decreases towards zero, but only very slowly. The apparent volume for a shape equivalent to two concentrated sources approaches a fixed value [that given by (3.1)] if the maximum diameter is kept constant.

The elongation ratio of modern airship hulls is not very different from the ratio $a/b = 5$. The front end is a little blunter than the bow of the equivalent elliptical shape, the rear end somewhat more sharply tapered. Since the volume of apparent additional mass is only a fraction of the hull's own volume, there is no urgent necessity for its precise value. It is therefore exact enough for practical purposes either to apply (3.1) indiscriminately to all hull shapes or to use the apparent mass of the equivalent ellipse more exactly defined in the next section.

4. Lateral Motion. The inertia factor for the transverse motions of airship hulls depends to a large extent on the shape of their cross section. For an estimate, this influence can be taken into account by applying as a correction factor the inertia factor η of the cross section. This is given by (2.2) and with geometrically similar cross sections, η is constant along the entire axis. If the cross sections vary in shape as well as in size, the inertia factor of the hull for transverse motion can be computed by summing up the effects of all its portions by an integration along the axis. In most cases, the assumption of a constant inertia factor η, equal to its average value, is exact enough.

Aside from the influence of the cross section, the inertia factor for transverse motion depends on the elongation of the hull and the distribution of the cross sections along the axis, and is estimated exact enough by introducing the inertia factor $\bar{k}_2$ for an equivalent ellipsoid of revolution. This, however, can be chosen in different ways. We prefer the ellipsoid of revolution with the same area of meridian cross section S' as the hull, and the same length, L. The elongation ratio of the equivalent ellipsoid of revolution is accordingly

$$a/b = \pi L^2/4 S' \tag{4.1}$$

Denoting the inertia factor for lateral motion of this ellipsoid by $\bar{k}_2$, the inertia factor of the hull for lateral motion will then be assumed to be:

$$k_2 = \eta \, \bar{k}_2 \tag{4.2}$$

See Fig. 1 for general diagram of ellipsoid coefficients.

5. Difference of the Inertia Factors. The resultant unstable moment of the hull is seen by Division C III (4.6) to be proportional to the difference $(k_2 - k_1)$ of the coefficients of the additional mass laterally and axially. For circular or nearly circular cross sections (the most common case) it seems most appropriate to take for this difference directly the value of $(k_2 - k_1)$ of the equivalent ellipsoid of revolution. For other cross sections, those, for instance, that are distinctly elliptical, it is necessary to compute k_2 and k_1 separately and then take the difference.

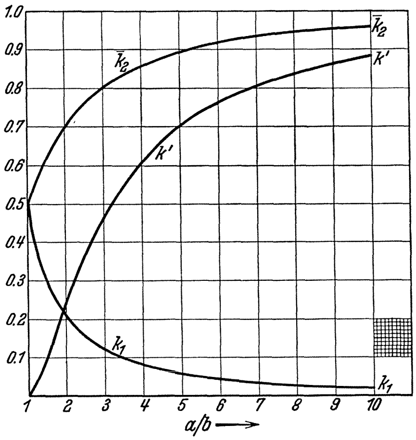

Fig. 1. Inertia coefficients of an ellipsoid moving in a fluid. k_1 axial; $\bar{k}_2$ transverse; k' rotation.

The same rules apply to the estimation of the inertia factor for rotation about an axis passing through the center of gravity of the volume of the hull, at right angles to the main axis. For circular cross sections, it will be exact enough to take the factor k' of the equivalent ellipsoid. For other cases, this factor, then denoted by $\bar{k}'$, must be multiplied by η, the inertia factor of the cross section, or by its average value over the length. Thus

$$k' = \eta \, \bar{k}' \tag{5.1}$$

where η refers to the direction in which the cross sections are moving when rotating.

6. Nose Pressure. The knowledge of the pressure distribution near the bow of airships is of particular importance, especially for ships of the semi-rigid or non-rigid type, because the pressure there reaches its maximum and minimum values, and thus determines the rigidity and strength of the bow stiffening to be provided.

Experience has shown that it is sufficiently exact for practical purposes to substitute, for the computation of this pressure distribution over the bow portion of the hull, an elliptical shell, and to compute the pressure distribution over this shell in accordance with Division C VII 6. Hulls with axial symmetry or nearly so are of chief importance, and we discuss therefore this case first. The generalization to the case of the ellipsoid with three unequal axes is not very different and offers no fundamental difficulties.

The equivalent ellipsoid of revolution used for the computation of the nose pressure is not identical with the equivalent ellipsoid of **3**, which represented the best approximation to the entire hull. It is rather chosen so as to represent the best approximation to the form of the bow only. The first requirement for this purpose is an agreement between the radius of curvature r at the front of the actual hull and of the equivalent ellipsoid. Should there be a small protruding cone or any other like formation at the front of the hull, as for mooring, it is necessary to determine the radius of curvature for the faired bow, treating the protrusion as a local attachment not virtually affecting the pressure distribution over any large area.

With this radius of curvature decided on, there remains only one more variable to be chosen. There are several possibilities, and we prefer to assume the minor axis of the ellipsoid to be equal to the maximum diameter of the hull. It is known that the radius of curvature of an ellipse at the extremity of its major axis $2a$ is b^2/a and at the extremity of its minor axis $2b$ is a^2/b. We have thus given or assumed, b and $r = b^2/a$. We have then $a = b^2/r$, the half length of the equivalent ellipsoid. Then the elongation ratio to be used in the computation of the inertia factors $= a/b = b/r$.

Since the point of maximum pressure occurs on the meridian we consider the motion symmetrical about the meridian plane. The tangent at the point of greatest pressure is at right angles to maximum velocity W of the flow, and the tangent at the point of greatest suction is parallel to the same direction. The magnitude and direction of the maximum velocity W is found by adding vectorially the axial component $u = U \cos \alpha$ multiplied by $(k_1 + 1)$ and the lateral component $v = U \sin \alpha$ multiplied by $(k_2 + 1)$. See Division C VII **12**. For very large values of the elongation ratio, the former factor, $k_1 + 1$ approaches 1 and the latter, 2. The angle between the maximum velocity and the axis is therefore larger than the angle of attack and for very elongated shapes approaches twice its value, provided the angle of attack is small. With a moderate elongation ratio of the bow, the small angle of attack is increased in the ratio $(1 + k_2)/(1 + k_1)$.

At the point where the normal to the surface is parallel to this direction, the relative velocity between air and hull is zero, and the air pressure exceeds the pressure of the undisturbed air at large distance from the airship by $\varrho U^2/2$ where U denotes the velocity of motion. See Division C II **4**. At the points where the tangent to the surface is parallel to the maximum velocity, the air velocity relative to the hull is equal to this maximum velocity

$$U\sqrt{\cos^2 \alpha \, (1 + k_1)^2 + \sin^2 \alpha \, (1 + k_2)^2}$$

and the pressure has its minimum, being smaller than the maximum pressure by the amount:

$$(1/2)\,\varrho\,U^2\,[\cos^2\alpha\,(1+k_1)^2+\sin^2\alpha\,(1+k_2)^2]$$

and hence smaller than the pressure in the undisturbed atmosphere by the amount

$$(1/2)\,\varrho\,U^2\,[\cos^2\alpha\,(1+k_1)^2+\sin^2\alpha\,(1+k_2)^2-1]$$

At all other points, the velocity is a component of the maximum velocity, and the pressure is computed from it in the same way as from the maximum velocity.

We see that the maximum pressure depends on the magnitude of the velocity only, in conjunction with the air density ϱ, and this theoretical value is realized by observation with surprising exactness. The minimum pressure, that is at the largest suction, depends besides on the shape. With bows that are ellipsoids of revolution, there exists relative to the maximum pressure, a theoretical limit to the largest sub-pressure $4\,U^2\,(\varrho/2)$, and obtained only if both the angle of attack and the elongation ratio are very large. If the hull moves strictly sideways, the theoretical greatest sub-pressure measured from the pressure at a great distance, is three times the dynamic pressure $U^2\,(\varrho/2)$, and measured from the maximum pressure, four times the dynamic pressure. In the extreme case, however, the sub-pressure actually realized would fall considerably short of the theoretical value. For small angles of attack, the theory gives quite good agreement with experience. Measured from the maximum pressure, the largest subpressure is then equal to the dynamic pressure, multiplied by $(\cos^2\alpha+4\sin^2\alpha)$ for very large elongation ratios and by $\left(\cos^2\alpha\,(1+k_1)^2+\sin^2\alpha\,(1+k_2)^2\right)$ for moderate ones.

On the other meridian sections, the points of maximum velocity and hence of minimum pressure are situated along an intersection of the ellipsoid with a plane, and hence, in a side view of the plane of symmetry, the points of minimum pressure occupy a straight line connecting the two points of minimum pressure on the meridian in the plane of symmetry. The minimum pressure is constant along this line. If the bow only is drawn, only one point of minimum pressure appears in the drawing of the meridian of symmetry. The line of minimum pressure in the side view can then be obtained approximately by connecting this point of minimum pressure with the center of curvature.

With hulls the cross sections of which are not circular, the method remains substantially the same for motions parallel to a plane of symmetry. The bow has then to be approximated by an ellipsoid with three unequal axes, and in most cases it is exact enough to compute the inertia factors by (4.2) rather than by evaluating the elliptical integrals of Division C VIII (3.9).

For unsymmetrical motion, the same procedure has to be applied to the two components of motion parallel to the principal planes through the axis, and the results superposed. The theorem referring to the curve of minimum pressure stands also for this most general case and likewise the mode of computing the minimum pressure from the maximum velocity. This maximum velocity is then, however, not necessarily parallel to the motion.

7. Stability. Spindle shaped solids moving in a perfect fluid parallel to their axis are in an unstable equilibrium, and tend to turn crosswise to the motion. This follows from the fact that the additional apparent mass for lateral motion of spindles is much larger than for axial motion. This theoretical result is borne out by experience for the motions of such solids through air. Bare airship hulls are unmaneuverable, and bare spindle shaped arrows have been known since time immemorial to fly unsatisfactorily. The remedy has likewise been known since before the dawn of history—the spindle is provided with fins near its rear end, flexible feathers for arrows, and more substantial ones for airship hulls.

Applying the theory of three-dimensional irrotational motion to spindles with fins does not give any explanation of the beneficial effect of the fins. The apparent mass is increased for lateral motion but not for axial motion, making things apparently worse. The effect of the fins can, however, be explained by going a step farther, and employing the wing theory. Solids with sharp trailing edges, such as fins, give rise to a motion of the air which is by no means irrotational throughout the entire space, but involves circulation and the development of forces perpendicular to the plane of the fin. With plane thin surfaces, in particular, there is in consequence produced an air force the magnitude of which is theoretically

$$L = \frac{\varrho V^2}{2} S \frac{\alpha}{1 + 2\,S/b^2} \tag{7.1}$$

where S denotes the area of the surface, b its lateral extension, α its small angle of attack, V the velocity of motion and ϱ the mass density of the air. The air force has the direction $\alpha\left[1 - \frac{1}{1 + 2\,S/b^2}\right]$ with respect to the motion. The center of action of the resultant air force is at 25 per cent of the chord from the leading edge. There is besides a resistance in consequence of the viscosity of the air.

The stabilizing action of the fins is the effect of this air force, the so-called lift. The fins tend to turn the hull into a direction parallel to the motion, counteracting thereby the natural instability.

The correct prediction of the fin area required for a hull is of high importance, as it is difficult to make any change after the ship has been completed. Fortunately, there seems to be a large margin of fin size, within which the stability is satisfactory, for nearly all airships built have been found to possess satisfactory stability on their first flight. Nevertheless is it very desirable to keep the fin size as close to the minimum as possible, for this not only diminishes the drag of the fins, but their weight and the weight of the structure necessary to support them reduce the useful load. Every source of information regarding the necessary fin size is therefore of definite value to the airship designer.

In addition to theory, the designer has the known experience with earlier ships as a guide in the selection of the fin dimensions, and it is of importance to interpret correctly this experience as between hulls of different shape. This interpretation is carried out by the use of some selected formula. A variety of such formulae are in use, leading to a corresponding variation in the results. To these, theory may add another at least as to its general form. The fin area is expressed by means of its ratio to some area computed from the dimensions of the hull, thus eliminating differences of hull shape, and obtaining results that can be directly compared with each other. Different standard areas are used for this purpose—usually the two-thirds power of the hull volume or the area of the meridian plane. These two areas by no means change in the same ratio if the hull shape changes, and hence the use of a fixed relation to the one or the other of two such areas will lead to different results for the fin area.

The theory of infinitely elongated hulls suggests a third standard area—the area of maximum cross section. It would even be better to employ the average cross section, that is the volume of the hull divided by its length. The ratio of these two areas is not very different for hull shapes actually in use, and therefore it would make little difference which of the two is used. Since the moment of instability of the hull with large elongation is proportional to its volume, and since (other things being equal) the stabilizing moment of the fins proportional to the area multiplied by their distance from the center of gravity, it would follow that the fin area should be proportional to the volume divided by a length—that is, to the mean cross section. But the ratio between the mean cross section and the maximum cross section is very nearly constant in modern airship construction and therefore so far as ratios are concerned, the maximum cross section may be properly and consistently used. Taking the maximum cross section then as a standard area, the next step would be the introduction of several correction factors to take care of the influences of the other elements. The rational formula suggested is therefore of the form

$$\text{Fin area} = c_1\, c_2\, c_3\, S_{max}\,. \tag{7.2}$$

where the different factors c are determined by the secondary dimensions.

There are unfortunately two effects, regarding which little is known, and this makes it useless to push the classical theory too far. In the first place, the instability of the hull is smaller than as indicated by theory, since its rear portions produce some stabilizing forces even when fins are absent. On the other hand, a portion of the fins is surrounded by air which has flown along the surface of the hull, and while doing so has, to a certain extent, equalized, under the influence of friction forces, its velocity with that of the hull, so that the relative velocity between the fin and the air is diminished. This effect diminishes the stabilizing moment and would of itself require larger fin areas. The two effects to a large extent seem to neutralize each other, but exact amounts are unknown and for this reason an exact prediction of the necessary fin area is impossible in the present state of our knowledge.

Of the factors c in (7.2), the correction factor for the elongation ratio of the hull,

$$c_1 = k_2 - k_1 \qquad (7.3)$$

and the factor for the aspect ratio of the fins,

$$c_2 = \frac{1}{1 + 2\,S/b^2} \qquad (7.4)$$

These are the most important ones. In the latter equation, (7.4), S is not the fin area proper but the entire area between the outer edges and including the projection of the portion of the hull between the fins.

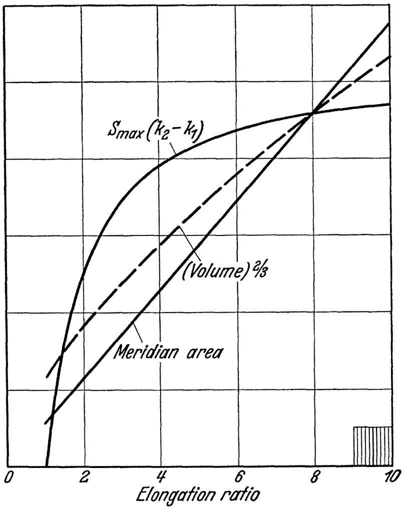

Fig. 2.

It is now interesting to compare the outcome of this method, referring the fin area to the cross section, multiplied by $(k_2 - k_1)$ with the other two methods mentioned. We illustrate the comparison by plotting against the elongation ratio of ellipsoids the three areas, meridian area, $(\text{Volume})^{2/3}$ and the proposed area $S_{max.}$ $(k_2 - k_1)$. These three areas are multiplied by certain constant factors so chosen as to make them agree for the elongation ratio 8. The divergence for other values of the elongation ratio is shown in Fig. 2.

8. Lateral Forces in Straight Motion. The moment of instability

$$(k_2 - k_1)\ \text{Vol.}\,\frac{\varrho\, V^2}{2}\, sin\, 2\,\alpha \qquad (8.1)$$

of the air forces acting on the airship hull is produced by the variation of the pressure over the entire hull. As an intermediate step between dealing with the resultant moment only and with the entire pressure distribution, it is possible to make positive statements regarding the air forces which seem to act at different points of the axis, being the resultants of the pressure distribution around zones of the hull surfaces corresponding to the elements of the axis. These intermediate resultants lend themselves readily to the computations of the bending moments of the hull as a whole, and are therefore of direct and important use for the computation of the necessary structural strength.

According to Division C VI (4.5) very elongated surfaces of revolution when moving in a straight line with a velocity V at a small angle of attack relative to the long axis, experience lateral forces the magnitude of which, per unit length of the axis, is

$$P = \frac{\varrho V^2}{2} \frac{dS}{dx} \sin 2\alpha \tag{8.2}$$

where S denotes the circular cross section. If the cross section is not circular, the distribution of the lateral forces will be

$$P = \eta \frac{\varrho V^2}{2} \frac{dS}{dx} \sin 2\alpha \tag{8.3}$$

where η denotes the inertia factor of the cross section for a symmetrical inclination. A formula of the same form applies to the case of attack in the plane of an unsymmetrical inclination, but in the latter case the forces are not necessarily in the plane of attack.

Equations similar to (8.2) or (8,3) apply further, if the axis of the spindle shaped hull is slightly bent. In this case, the angle of attack α is considered variable and must be included under the differentiation, so that the equations become

$$P = \frac{\varrho V^2}{2} \frac{d}{dx} (S \sin 2\alpha) \tag{8.4}$$

and

$$P = \frac{\varrho V^2}{2} \frac{d}{dx} (\eta S \sin 2\alpha) \tag{8.5}$$

We proceed to the case where the elongation of the hull is not extremely large, but only moderate. The lateral forces along the axis are then modified, and there occur further couples distributed continuously along the axis. These couples arise from the conicity of the hull. If the meridian is inclined under a finite angle toward the axis, the pressure difference on opposite ends of a diameter not only gives rise to lateral components, but also to axial components of the air force, the former giving the lateral forces and the latter in the first place a couple and besides, axial force components. The couple can be computed from the assumption that the pressure around the zone is proportional to the distance from a diameter, in accordance with Division C VI (4.6). This computation gives

$$M = \frac{dp}{dy} \frac{dS}{dx} \frac{S}{2\pi}$$

where dp/dy denotes the apparent pressure gradient at the points of the surface. These couples diminish the resultant couple of the entire hull produced by the lateral forces.

In practice, these couples, continuously distributed along the axis, together with the axial forces, are often neglected, and the lateral forces are computed by a modification of (8.3), multiplying these lateral forces by a constant multiplier of such magnitude as to bring the added effect of all lateral forces in agreement with (8.1).

The constant multiplier is seen to be $(k_2 - k_1)$ and hence the usual formula for the computation of the lateral air forces is

$$P = \frac{\varrho V^2}{2} \frac{dS}{dx} (k_2 - k_1) \sin 2\alpha \tag{8.6}$$

for circular cross sections, and

$$P = \frac{\varrho V^2}{2} \frac{dS}{dx} \eta (k_2 - k_1) \sin 2\alpha \tag{8.7}$$

for other cross sections.

These lateral forces have a resulting couple, but their resultant force is zero.

9. Lateral Forces in Curved Motion. If the motion is not straight, the distribution of the lateral forces is different from that indicated by (8.6).

We begin with the hull moving uniformly along a circular path. The entire reultant air force is then given by Division C III (3.2) and (3.3). If the motion is steady, there is a resultant force passing through the center of rotation, located in general outside the hull. This gives rise to a moment of the air pressures with respect to the center of gravity of the volume of the hull, approximately equal to the couple occurring in straight motion under the same angle of attack at the center of gravity and with same velocity. The distribution of the lateral forces giving rise to the same moment is, however, entirely different in the two cases of straight and circular path. The computation of the distribution in both cases is, however, based on the same principle.

Suppose the hull to turn at an angle of yaw equal to φ, in a circular path of radius r. The momentum of each layer of air transverse to the axis is now

$$\eta v \varrho S \, dx \tag{9.1}$$

where ηS denotes the area of apparent mass of the cross section, v the lateral component of motion between hull and air, and dx the element of the axis. The transverse velocity is now variable, and is composed of the constant portion $V \sin \varphi$ produced by the yaw and the variable portion $V (x/r) \cos \varphi$ produced by the turning with radius r. The value $x = 0$ may represent the center of the hull. Hence the time rate of change of the momentum per unit length, or in other words the transverse force per unit length is

$$\frac{\varrho V^2}{2} \sin 2\varphi \frac{dS}{dx} + \frac{\varrho V^2}{r} \cos 2\varphi \left(S + x \frac{dS}{dx}\right) \tag{9.2}$$

or otherwise written

$$d\,F = d\,x\left[\frac{\varrho\,V^2}{2}\sin 2\,\varphi\,\frac{d\,S}{d\,x} + \frac{\varrho\,V^2}{r}\cos^2\varphi\,S + \frac{\varrho\,V^2}{r}\cos^2\varphi\cdot x\,\frac{d\,S}{d\,x}\right] \tag{9.3}$$

The first term agrees with the lateral forces on the hull flying straight at an angle of pitch φ. The direction of this lateral force is opposite at the two ends, and gives rise to an unstable moment. Ships in actual maneuver have the hull at its center portion turned inward when flying in a turn. Then the lateral force represented by the first term of (9.3) is directed inward near the bow and outward near the stern. The sum of the second and third terms in (9.3) gives neither a resultant force nor a resultant moment. The second term by itself gives a lateral force, being in magnitude and distribution almost equal to the lateral component of the centrifugal force of the displaced air, but reversed. This latter becomes distinctly apparent at the cylindrical portion of the ship, where the other two terms are zero. The front portion of the cylindrical part of the hull moves toward the center of the turn and the rear portion moves away from it. The inward momentum of the flow has to change into an outward momentum requiring an outward force acting on the air and giving rise to an inward force reacting on the hull.

The third term in (9.3) represents forces concentrated near the two ends, and their sum in magnitude and direction is equal to the lateral component of the centrifugal force of the displaced air. They are directed outward.

Hulls only moderately elongated have resultant forces the distribution and magnitude of which are different from those given by (8.2) and (9.3). The assumption of the layers remaining plane is more accurate near the middle of the hull than near the ends, and in consequence the transverse forces, diminished in some degree over the entire length, are diminished to a greater extent at the ends than near the center, when compared with the very elongated hull. In practice, it is usually exact enough to assume the same law of distribution for each single term of the equations, and to reduce the lateral forces by means of constant multipliers. These factors should be chosen different for the different terms of (9.3). The first term represents the forces giving the resultant moment proportional to $(k_2 - k_1)$ and hence it is logical to diminish this term by multiplying it by $(k_2 - k_1)$. The second and third term take care of the momenta of the air flowing transverse with a velocity proportional to the distance from the center. Moments of inertia enter in connection with the turning motion and hence it seems reasonable to affect the terms by the factor k', the ratio of the apparent additional moment of inertia to the moment of inertia of the displaced air.

The transverse component of the centrifugal force produced by the air taken along by the hull due to its longitudinal motion is neglected. It is concentrated near the ends, but its magnitude is small.

The distribution of the lateral force on an airship hull, turning at an angle of yaw φ with the velocity V and the radius r is, as the result of the preceding discussion (see Fig. 3)

$$d F = d x \left[(k_2 - k_1) \frac{\varrho V^2}{2} \sin 2 \varphi \frac{d S}{d x} + \right.$$
$$\left. + k' \frac{\varrho V^2}{r} \cos^2 \varphi \, S + k' \frac{\varrho V^2}{r} \cos^2 \varphi \, x \frac{d S}{d x} \right] \tag{9.4}$$

This expression does not include the air forces on the fins, and refers to circular cross sections. With sections otherwise, the factor η, the inertia factor of the cross section, must be included in the formula.

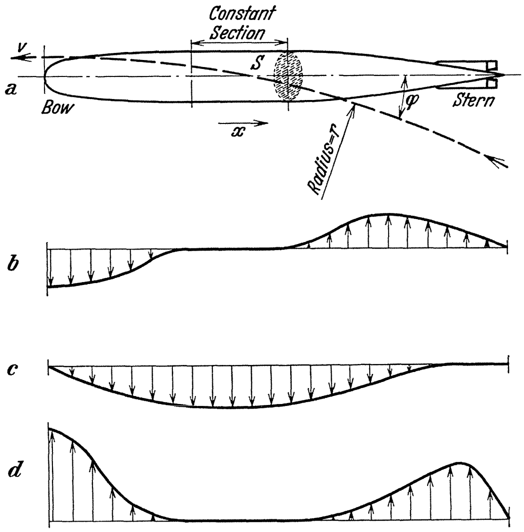

Fig. 3. Airship in curved flight and forces developed [see (9.4)]. a. Attitude of airship in relation to path. b. Curve showing 1st term of (9.4)—the same as in straight flight under pitch. c. Curve showing 2nd term of (9.4)—"negative centrifugal force". d. Curve showing 3rd term of (9.4).

The case of the air moving laterally with different velocities relative to the axis is equivalent to the motion of an elongated hull with bent axis, so that the angle of attack is in conformity with the relative motion between the air and the hull at all points. General formulae are not possible, but the principle used is the same as in the preceding case, and the computation depends on the assumed distribution of the air motion. It may be remarked that in the region of increasing cross section, that is in front, the reaction from the change of the momentum tends to move the hull into the direction of the moving air, so that after some time the relative velocity between hull and air can be expected to be decreased. However, this argument should be used with great caution, as the same reaction increases the angle of attack and may thereby neutralize to a great extent the beneficial effect just mentioned.

10. Lift of the Airship. The cross force on an airship, such as for instance the lift, can be computed to a first approximation from the preceding discussion, from the condition that the lift of an airship moving with constant translational velocity under a small angle of inclination of the axis is equal to the air force on its tail surface necessary to neutralize the unstable moment of the hull. This fin force is supposed to act at the fin, at the distance a from the center of the ship, and is supposed to counteract the moment exactly, although in practice a small "static" instability is admissible and, in consequence of "dynamic" effects, does not actually render the motion unstable.

The unstable moment was seen to be

$$M = (k_2 - k_1)\,\text{Vol.}\,\frac{\varrho\,V^2}{2}\,sin\,2\,\alpha \tag{10.1}$$

This gives the fin force

$$F = (k_2 - k_1)\,\frac{\text{Vol.}}{a}\,\frac{\varrho\,V^2}{2}\,sin\,2\,\alpha \tag{10.2}$$

where α denotes the angle of attack.

In practice, this equation is generally used the other way around. The resultant air force is then not computed from the angle of attack of the airship, but on the contrary this force is assumed in the form of excess or insufficient buoyancy, and the angle of attack necessary to create an equivalent air force is computed by means of (10.2).

This case of an airship with constant translational velocity with its nose turned up or down, producing thereby negative or positive lift, is closely allied to the case of an airship flying along a circular path while producing air forces substantially side-ways. In that case the resultant air force on the hull does not exactly reduce to zero, for even with constant tangential velocity there comes into effect a centrifugal force of the air mass virtually accompanying the ship, and this centrifugal force, together with the centrifugal force of the ship structure, including the air and gas enclosed therein, must be overcome and neutralized by the side force on the tail surfaces. Generally speaking, the strict definition of lift or side force and of drag breaks down as soon as there is no longer a translational velocity to the direction of which these components of the resultant air force can be referred. Different points of the airship move in different directions, and there is no longer a standard direction to which the drag and lift can be naturally referred. In its stead we have then a center of rotation about which the airship is turning at the instant considered. In case of a steady turn, it follows from considerations of energy, that the resultant moment with respect to that center must be zero, and hence the resultant air force must pass through that center. Such resultant air force passing through this center is therefore equivalent to the lift, and the resultant moment about the center represents the drag.

The resultant air force on a steadily turning hull is at right angles to the momentum of the air flow caused by the hull. Its magnitude is equal to the product of this momentum and of the angular velocity of the hull, or, more generally, to the vector cross product of the momentum and the angular velocity. In actual cases, the lateral component of the airship velocity is only small, and the apparent additional mass for axial motion is likewise small. From this it follows that the resultant air force is generally much smaller than the actual centrifugal force of the airship structure, and for many purposes can be neglected in comparison therewith.

If this is done, the centrifugal force of the airship just supported by its buoyancy assumes the magnitude

$$P = \varrho \text{ Vol. } \omega^2 r \tag{10.3}$$

where ω the angular velocity is equal to V/r, and hence the centrifugal force assumes the form,

$$P = \varrho \text{ Vol. } V^2/r \tag{10.4}$$

The "volume" in this expression should be increased by the apparent additional volume, if a more exact value is desired. Equating (10.2) and (10.4) gives the angle of yaw required to turn the ship at a large radius of turn,

$$\alpha = \varphi = \frac{a}{r\,(k_2 - k_1)} \tag{10.5}$$

This angle of attack is to be measured at the center of the hull. It is supposed to be small. Taking into account the apparent centrifugal force of the air leads to a slightly larger value.

These results permit of an interesting conclusion in the ideal case of a very elongated hull where, moreover, the distances from the center of the hull to the two ends are equal. We compute as follows the local angle of attack at the bow. Its lateral motion due to rotation is $a\,\omega$, directed toward the center of turn. This is equivalent to an angle of attack α such that $V \sin \alpha = V\alpha = a\omega$

and hence

$$\alpha = \frac{a\,\omega}{V} = \frac{a}{r} \tag{10.6}$$

This angle is directed opposite to the angle (10.5), for in that case the hull center was flying inward relative to the tangent of the circle of turn, so that the motion relative to the air is outward from the center of turn. Considering further that with a very elongated hull the expression $(k_2 - k_1)$ in (10.5) approaches the value one, it results that the angle (10.6) becomes sensibly equal in magnitude and opposite in direction to the angle (10.5). It is thus seen that the bow has the angle of attack zero; it has no lateral motion relative to the air, but heads directly into the wind in circular motion as well as along straight paths. This relation varies somewhat for ships with moderate elongation and different distances from the center to the two ends, but not in great degree. It can be generally stated that airships in circular flight head into the wind.

11. Conclusion. As the chief result of the foregoing investigation, it follows that in straight motion the lateral air forces acting on the airship hull are distributed proportional to the rate of change of the cross section. This gives a convenient and fundamentally sound specification for the loading assumptions for the structural computation.

The largest bending moment of the hull results proportional to its volume and to the square of the maximum velocity. All other factors can be included in a safety factor or load factor, obtained from experience and judgment. In the absence of any generally recognized method of computing the various air loads in detail from the motion of the ship, the specification of such a safety factor is preferable to the attempt to reach the same result through the specification of fictitious motions, either of the airship or of the air.

DIVISION R

PERFORMANCE OF AIRSHIPS

By

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Akron, Ohio Akron, Ohio

PREFACE

In spite of the widespread interest in airships, actual experience in their construction and operation is accessible only to a rather limited number of engineers, so that it is not surprising that scientific literature regarding the aerodynamic performance characteristics of airships is scanty and mostly scattered through disconnected monographs, cumbersome to collect and not readily available to the student. It was therefore felt that the General Editor's desire to include a comprehensive survey of this knowledge in the present series of volumes is eagerly shared by all aeronautical students interested in lighter-than-air craft. The present authors accepted the invitation to contribute to this work with a full realization that they can do only partial justice to the presentation of such a vast and intricate subject.

They wish to acknowledge the valuable contributions to the present Division made by Mr. Herman Richard Liebert, Project Engineer and Mr. Thomas A. Knowles, Development Engineer, both of the Goodyear Zeppelin Corporation; the former who has contributed from his many years of experience in Project Work and Performance Prediction and from his familiarity with the associated problems; the latter to whom we are indebted for contributions involving navigational and operational problems, and for assistance in the systematic and didactic presentation of the subject matter and the compilation of bibliographical references.

If the experience accumulated by the Goodyear Zeppelin Corporation has been drawn upon profusely, it is only a natural result of the authors' association with this organization. However, a conscious effort has been made to base the treatment on as broad a foundation of international experience as was accessible. The frequent references to the literature are intended to help the student who is interested in more detailed information.

Much aerodynamic knowledge has been acquired through flight experience and research, and in this respect especial acknowledgment is due the "Luftschiffbau Zeppelin" of Friedrichshafen, Germany, whose pioneering of airships since 1900 forms a background for the bulk of our present operational knowledge. In America there have been three active agencies operating airships; the U.S. Navy making important contributions to the development of means for the handling of large rigid airships and smaller pressure airships, including the metal clad type; the U.S. Army developing and operating semi-rigid and non-rigid types; and Goodyear with its widespread commercial blimp operations providing opportunities for research and experiment.

Unfortunately there is no space here to give proper credit to all those who have made important contributions and advancements in the theory, construction, operation and testing of lighter-than-air craft. Many of their names will be found in the literature quoted especially in the beginning of the Division.

In attempting to make the present Division a self-contained unit, it has occasionally been necessary to briefly touch upon matters which have been more fully discussed in previous divisions, notably the one immediately preceding, but it is hoped that this will be pardoned in the interests of coherent presentation. The subject matter has been classified, as clearly as possible, into six chapters.

The first chapter is devoted to a brief discussion of Aerostatics, a subject which is not commonly thought of as a branch of Aerodynamic Theory. It was included here as far as it was thought helpful for a proper approach to many of the aerodynamic problems of airships. This chapter is not, however, a complete text on aerostatics. For instance free ballooning is not covered, even in spite of its interesting aerodynamic aspects.

The second chapter deals with the propulsion of the ship and with the accompanying aerodynamic performance in axial motion. Since the drag of airships is not embraced by the so-called classical aerodynamic theory, this chapter leans heavily on non-classical and empirical methods of approach and the subject is still largely dependent on experimental evidence and on its evaluation by scientific methods.

In the third chapter, the airship is studied in the condition of disturbed buoyancy equilibrium, where the ship flies with dynamic lift. In this condition there are many parallels with the wing lift of airplanes; the difference is mostly due to the lack of geometrical resemblance between an airship hull and an efficient wing.

The fourth chapter is devoted to departures from straight flight, such as turning, deliberate application of controls and disturbed flight through turbulent air. These conditions also have many aspects in common with airplanes, the main difference being the relatively large length of an airship.

The fifth chapter contains some of the aerodynamic problems arising with the mooring and ground handling of airships. These problems are peculiar to lighter-than-air craft.

The sixth and concluding chapter lists some of the outstanding problems and indicates the directions in which the expected future advancement of our knowledge appears to lie.

An effort has been made to deal only with the aerodynamic aspects of airship problems and in many instances the complications introduced by other vital considerations have been deliberately left out of the discussion. After all, no aircraft can be built to satisfy aerodynamic considerations alone, and while these considerations are of great importance, the practical engineer must also have in mind adequate strength and safety and the economy and practicability of construction and operation. For this reason the following pages do not, by any means, constitute a text on airship design.

CHAPTER I

BUOYANCY

1. Buoyancy Equilibrium and Its Maintenance. The flight of an airship differs from that of an airplane inasmuch as it floats in the air by virtue of the fact that the buoyant force due to the displaced air compensates the weight of the structure plus the light gas with which it is inflated. Buoyancy always acts as a vertical force and therefore the vertical equilibrium between lift and weight can be accomplished independently of the axial equilibrium between thrust and drag. Once an airship is in buoyancy equilibrium it can be nosed up and will climb without losing speed along its air path, or it can be nosed down without picking up speed. In fact, when the atmospheric temperature gradient is that actually accompanying the expansion or compression of the buoyant gas with altitude (where no radiation is present to offset the temperature balance between the ship's interior and the surrounding atmosphere), the ship can be made to rise and descend at will without incurring any change of buoyancy equilibrium since the gas and air inside will expand or contract and cool or warm up at the appropriate rate to offset the variation in the density of the air encountered. The ship will simply fly where pointed, and the purely aerodynamic problems of pitching and yawing are essentially the same. This condition will naturally prevail only while the gas containers (gas cells, balloons) are only partially filled, *i. e.*, below the pressure height[1]. In this condition the hull space not occupied by gas is filled with air either at the same pressure as the outside air, as in rigid airships having a load-carrying skeleton framework, or at a slight over-pressure maintained by blowers or by the velocity head of flight[2] and, possibly augmented

[1] UPSON and CHANDLER, Free and Captive Balloons, Ronald Press.

[2] FRITSCHE, C. B., The Metalclad Airship, Journal of the Royal Aeronautical Society, September 1931.

by the propeller slipstream, as in non-rigid or pressure airships. Upon ascending to pressure height (which can be increased by initially inflating to a lesser degree at the expense of useful load or *vice versa*) air is expelled and the gas containers become taut. Further climbing would result in expelling buoyant gas from the automatic safety valves and in making the ship "heavy", *i. e.*, heavier than the displaced air. Other possible causes of heaviness in flight are overloads from rain, snow or ice, and loads deliberately taken aboard, such as airplanes. Heaviness may also arise from cooling of the gas or flying into warmer air. On the other hand, the airship can become lighter than air by loss of load or ballast, by the combustion of fuel, or by heating. In either condition the unbalanced forces must be carried aerodynamically. Under the buoyancy equilibrium condition, however, the ship is "just as light as air".

When the atmospheric temperature gradient differs from that for actual gas expansion, the buoyancy equilibrium is disturbed by climbing or descending. When the temperature drops less with altitude (standard air temperature gradient is 3.566^0 F. per 1000 feet), or if the air is even warmer above than below, *i. e.*, if a temperature inversion exists, then the ship finds a definite zone of stability; descending lower it would become light, rising higher it would become heavy. Flying in such a zone is smooth, but landing through it requires pulling the ship down or releasing gas. On the other hand, where the air is colder above than according to the equilibrium gradient, both the air aggregate and the ship itself are statically unstable; altitude control, and especially landing, then require greater watchfulness. The allowances which the altitude navigator makes must be based upon a knowledge of the stationary temperature field through measurements obtained aboard—for instance by lowering a temperature indicator into the strata below—or received from ground stations by radio or signals. The theory involved is straightforward thermodynamics and need not be here reproduced[1]. Slide rules have been designed to facilitate routine computation of airship buoyancy problems[2].

The vertical temperature gradient in the atmosphere is intimately linked with the humidity of the air because moisture condensing acts as a powerful heat reservoir. The same is true of the humidity of the gas in the ship.

In a descent, when the gas volume shrinks, the gas becomes relatively drier and, as far as heat transfer through the cell walls may be neglected, an adiabatic exponent would govern the simultaneous changes of volume, pressure and temperature. This adiabatic exponent is larger

[1] HUMPHREYS, W. J., Physics of the Air, Chap. II, 1929.

[2] WEAVER, E. R. and PICKERING, S. F., An Airship Slide Rule, U.S. N.A.C.A. Report No. 160, 1923.

for monatomic helium than the 1.4 valid for diatomic gases such as hydrogen, air, etc. Pure helium has $k = C_p/C_v = 1.667$. Actually, due to impurities (air and water vapor) it may vary from 1.65 to 1.58.

In a rapid ascent the same exponents hold only if the dew point is not reached in the gas. Unless super-saturation occurs, the moisture will then condense and fog forms in the gas. This moisture eventually drains off and drips to the bottom of the cells. Thence it will not immediately re-evaporate. Therefore in an ensuing ascent the gas may soon

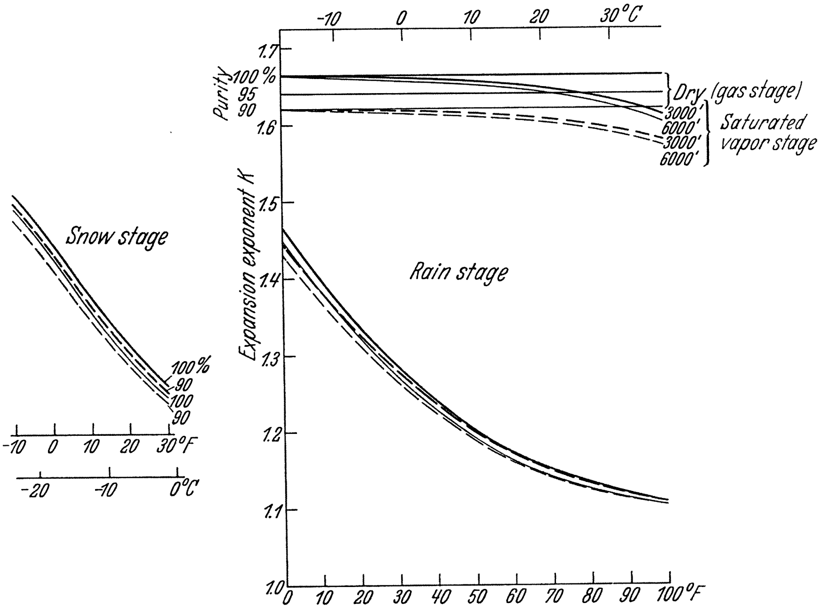

Fig. 1. Exponent for expansion of helium as influenced by temperature and purity.
Heavy full line . . . 100 % pure at 3000 ft.
Light full line. . . . 100 % pure at 6000 ft.
Heavy broken line 90 % pure at 3000 ft.
Light broken line . 90 % pure at 6000 ft.

reacquire a high exponent and retain it on subsequent descents. It is therefore possible to dry the gas by a deliberate ascent.

In such a first "wet" ascent the effective expansion exponent is lowered by condensation. This influence has been calculated for helium and found to depend largely on temperature and to some extent on gas purity and absolute pressure. The results of these calculations are reproduced in Fig. 1 for both the "rain" stage and the (problematic) "snow" stage.

In the preceding paragraphs only such rapid changes of altitude have been considered as preclude appreciable heat transfer between the gas and the air outside. However, in any slow change of altitude and under normal conditions of level flight, a continuous exchange of heat between the ship's interior and the exterior may occur due to four causes: 1) Ventilation with its bodily exchange of appreciable quantities of air drawn into and expelled from the space in the ship not filled with gas, especially emphasized when the ship is descending. 2) Conduction of heat from the gas and air through the outer cover into the boundary layer of the outside air flow, and *vice versa.* 3) Radiation or insolation. 4) Artificial super-heating.

Certain of these causes produce temperature differences between the inside and outside, while others tend to reduce any that may exist. These influences do not lend themselves readily to a generalized theoretical treatment because they depend largely on design details such as the provisions made for, and the effectiveness of, ventilation hoods, scoops and screens. As to the conduction of heat from the outer covering to the air, a theory, although not yet explicitly available, can probably be advanced. Indications are, however, that it makes little difference of what material the outer cover is made. In any case the value of such a theory for practical purposes would be quite limited because the interior phenomena, *viz.* the convection currents in the gas cells and the insulating action of the air space between outer cover and inner gas cells (where such provision is made) practically defy theoretical treatment. Yet it is these factors which, at high flying speeds, largely govern the rate of heat transfer. Therefore, calculations of thermodynamic buoyancy disturbances rely mainly on actual experience gained in the operation of similar ships. In sunny weather insolation may heat the ship to from 10^0 to 35^0 F. above outside temperature[1], causing a surplus lift of the order of ten tons with a large ship starting out without superheat and not quite fully inflated. Temperature lag, adiabatic cooling, evaporation of moisture and radiation from the ship may lower the temperature as much as 10^0 F. below the surrounding atmosphere. The superheat is measured by thermometers inside and outside, or by differential thermometers. However, since it is impracticable to cover the huge bulk of the ship with such instruments, it may well occur that the true average superheat of the ship as a whole will differ from that measured at particular thermometer locations[2]. It also often varies from bow to stern. Superheat in the dead air space in the ship gives a net gain in lift and may, on large ships under not uncommon circumstances, amount to a ton. Superheat of the buoyant gas is, of course, more effective in lift gain but is accompanied by unwelcome expansion of the gas cells and thus a reduction of the pressure height or service ceiling.

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[1] Richmond and Scott, Effect of Meteorological Conditions on Airships, Journal R.A.S., March 1924.

[2] Bassus, K. and Schmauss, A., Zeitschr. f. Flugtechnik u. Motorl., December 16, 1911; Bassus, K., Zeitschr. f. Flugtechnik u. Motorl., March 30, 1912; Captain Gluud and von Soden, Zeitschr. f. Flugtechnik u. Motorl., Heft 7, 1912; Stern, J., Zeitschr. f. Flugtechnik u. Motorl., October 17, 1914 and October 30, 1915; Hovgaard, W., Journal of Mathematics and Physics II., December 4, 1923.

Artificial superheating in flight, using the exhaust and radiator heat of the engines, is probably of limited effectiveness but feasible to a moderate extent. Naturally the structural and mechanical complications of such a system must be weighed against the simplicity of dynamic lift for temporary periods. Artificial superheating before a take off, from energy sources ashore, is quite feasible[1].

A large rigid airship is seldom in exact buoyancy equilibrium. Its condition may be disturbed not only by the thermodynamic variations in air and gas density, but also by changes in the dead weight of the airship and the loads aboard. In the rigid airship these changes are often so small in comparison with the dynamic lift available under way that they do not hinder normal operation of the airship and pass unnoticed. However, under certain circumstances they may accumulate and their influence may gradually make itself felt in the elevator control of the ship.

Large departures from the equilibrium condition have decided disadvantages. They increase the drag (thereby penalizing either the flight speed or the fuel consumption) and the range of available control angles may be seriously reduced. The decrease in the airspeed, if serious, may so reduce the available dynamic lift that the airship "stalls" and rises or falls not unlike a free balloon[2].

The normal changes in loading due to rain, snow, ice, water absorbed by or evaporated from the fabric, flight superheat, and atmospheric changes, can be readily compensated for statically by jettisoning portions of the reserve ballast or by valving off lifting gas always carried aboard large rigid airships.

The most serious problem generally encountered is that connected with the loss of weight due to the consumption of liquid fuel in the airship engines[3]. The airship tends to become lighter by the weight of the fuel consumed. This condition, however, does not arise with a gaseous fuel of unit specific gravity or when burning the proper proportions of liquid fuels and fuel gases lighter than air[4].

The original method of intermittently restoring equilibrium on liquid-fuel airships was to climb to pressure height and to valve buoyant gas. With the advent of helium inflation this process became too expensive and alternative methods were found. The most commonly accepted method of keeping the loading sensibly constant and thus doing away

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[1] Upson, Ralph, H., U.S. Patent 1,096,578, 1913; Blakemore, T. L., Artificial Payload for Coming Commercial Airships, Aeronautical Engineering (Am. Soc. Mech'l Engrs.), Apr.-June, 1933.

[2] Klemperer, W., The Stalling of Airships, Journal of the Institute of the Aeronautical Sciences, July 1934.

[3] Arnstein, Karl, Developments in Lighter-than-Aircraft, S.A.E. Journal, May 1929; Teed, Maj. P. L., Airship Propulsion Methods, Aircraft Engineering, December 1929.

[4] Lempertz, E., Flüssige und gasförmige Brennstoffe im Luftschiffbetrieb, Luftfahrt No. 6, pp. 82—83, Berlin 1927.

with the need for valving is one which owes much of its development to the U.S. Navy, *viz.*, the recovery of the water of combustion from the exhaust gases[1]. Other methods studied and tried with varying degrees of success are rain troughs, surface water pick-ups, and dessicating agents for atmospheric air and for exhaust gases.

Of recent years considerable thought has been given to a combination of the fireproof features of the helium inflated airship with the freedom of valving associated with the hydrogen inflated airship. Hydrogen amounting to a nominal percentage of the total inflation would be carried in special ballonets surrounded by the helium and would be valved freely for the maintenance of equilibrium. If one does not consider the careful use of small quantities of hydrogen as an increased fire hazard, this means of maintaining equilibrium appears attractive, particularly since the added hydrogen lift may compensate for the installation weight.

The large supply of liquid ballast always available in water recovery airships is a decided asset from the operational standpoint and it is very probable that the hydrogen ballast airships of the future will carry partial water recovery or ballast gaining equipment.

The normal method of deliberate valving is to climb slowly above the original pressure height so that gas escapes through all automatic valves without upsetting the trim of the airship.

The total volume of gas which has to be valved when a ship is driven deliberately, or accidentally above pressure height, is readily computed by combining the thermodynamic equation of the expanding gas with the barometric formula expressing the vertical gradient as equal to the ambient air weight density. Thus, for any rate of ascent a, there will be discharged, in unit time, a volume Q, such that

$$Q = \frac{a\,V}{R\,T\,k}$$

all in consistent units, where V is the volume of the cell, R and T the gas constant and ambient (abs.) air temperature at pressure height, and k is the effective adiabatic (or polytropic) expansion exponent already discussed[2].

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[1] Barr, G., On Obtaining Ballast on an Airship During Flight by Means of Water from the Motor Exhaust, Br. A.R.C. R. and M. 234, p. 519, 1915—16; Crocco, G. A., Replacing the Weight of Materials Consumed on Airships, U.S. N.A.C.A. Technical Memorandum No. 211, 1923; Kohr, R. F., Condensation of Water from Engine Exhausts for Airship Ballasting, Bull. Bureau of Standards T-293, August 1925; Cave-Brown-Cave, T. R., Condensation of Exhaust Gases for Water Recovery, Journal of the Royal Aeronautical Society, January 1926; Burgess, C. P., Water Recovery Apparatus for Airships, Trans. Am. Soc. Mech'l Engrs. AER. 54, 11, 83, 1932.

[2] Bird, W. G., Atmospheric Humidity and the Static Lift of Airships, Jr. R. Aero Society, November, 1931.

If, in any design, the valve area A and contraction coefficient ψ are known and the permissible overpressure between the inside and outside of the valve is prescribed as p, then the "permissible rate of ascent" can be expressed as

$$a = \frac{\psi A}{V} \sqrt{p} \cdot k R T \sqrt{\frac{2g}{\gamma}}$$

where $\gamma =$ the weight density of the gas under the conditions prevailing at pressure height.

While the ship rises but stays below the pressure height, air must be vented out to make room for the gas to expand. In rigid ships not depending on internal pressure for form, this is done through ventilation orifices or screens. In pressure ships it is generally done through automatic over-pressure valves. Here again the amount to which the pressure will actually build up over the valve relief pressure setting in a rapid climb depends on details of the design and the aerodynamics of the outlets[1]. For instance, if the valves are exposed to the wash of the outside air flow, they may open under a different pressure in flight than at rest, because the air flow about the ship at the location of the valves or about the valve or parts of it may cause a "false" pressure or suction.

The flow of the escaping air or gas around the valve rim may also cause the opening characteristics (*i. e.*, the correlation between pressure and gap) to deviate from the equivalent characteristic under static loads. This influence is apt to become important since it is quite difficult to build a light valve of large diameter that will be quite tight up to a pressure of the order of an inch of water and open wide under a small increase of pressure without having much hysteresis in closing. Furthermore, chimney draft effects in vertical ducts filling themselves with the escaping gas may cause suction and the valve may fail to close at the correct pressure[2].

The impossibilitity of making the gas containers absolutely gas tight entails the necessity of measuring the permeability. Usually the rate of diffusion is slow so that very sensitive measuring methods must be resorted to[3]. The conventional theory of this diffusion is based

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[1] KLEMPERER, W., Handbook of Experimental Physics, Vol. 4, Part 3, pp. 145—147.

[2] SCHERZ, W.: Luftfahrt, Vols. 22 and 23, 1927.

[3] FRENZEL, W., Die Gasdurchlässigkeit gummierter Ballonstoffe, Zeitschr. f. Flugtechnik u. Motorl., October 17, November 14, 1914; SHAKESPEAR, G. A., Br. A.R.C. R. and M. 317, 1917; R. and M. 447, 1918; ELWORTHY and MURRAY, Permeability of Balloon Fabrics to Hydrogen and Helium. Trans. Royal Soc. Canada 13, 37—45, 1919; EDWARDS, J. D., Determination of Permeability of Balloon Fabrics, Bureau of Standards, Tech. Paper No. 113; Journal Ind. and Eng'g Chemistry, 11, 966, 1919; NIEDNER, Zeitschr. f. Flugtechnik u. Motorl., p. 173, 1922; UPSON, R. H., Free and Captive Balloons, Ronald Press 1926; DAYNES, H. A., Gas Analysis by Measurements of Thermal Conductivity. Cambridge Univ. Press, 1933; also Br. A.R.C. R. and M. 360, 435, 504, 516, 614, 622, 640 by various investigators.

on the concept of the diffusion of gaseous media through pervious membranes and assumes the rate of diffusion of gas out and air in as inversely proportional to the square roots of their absolute densities and independent of the pressure drop. In reality pressure may have an indirect influence as the mechanical stress may cause minute seams to open in varying degrees. Comparative experiments with helium and hydrogen have not consistently confirmed the law that the diffusion of these two gases through fabric is always in inverse proportion to the square roots of their densities. There are marked differences which vary from material to material and with ageing, and therefore these observations lend weight to the theory that the diffusion process may also involve chemical or molecular interactions.

2. The Bulkhead Problem and Aerostatic Stability. Large airships of both the rigid and pressure types must be equipped with bulkheads in order that the buoyant gas may be confined to certain portions of the hull and not surge to the ends of the ship during inclinations. An airship without transverse partitions might be likened to a surface ship carrying a large liquid load unrestrained by bulkheads. It is a matter of practical experience that a surface vessel so loaded is unmanageable in a sea-way.

In general, airship bulkheads must possess a reasonable degree of tautness and stiffness and there are a number of bulkhead systems which have given acceptable solutions to the problem. The type most commonly and successfully used in rigid airships is a taut wire system in the plane of the main frames. Such bulkheads have been successfully built both with and without a central support. The central support helps to reduce the bulging of the bulkhead and its influence upon the static stability. An initially slack bulkhead can be stiffened by deliberately maintaining pressure differences between adjacent cells. Naatz[1] has proposed cylindrical slack bulkheads which may not need a differential pressure.

The stiffness and spacing of the transverse bulkheads may noticeably influence the metacentric height of an airship, and this influence has a bearing upon how low the center of gravity must be kept beneath the center of buoyancy in order to ensure sufficient static stability.

The elasticity of the bulkheads separating individual gas cells, and the flabby nature of the cell bottoms, allows some surging of the gas. A slack bulkhead would suffer under the disadvantage that its large bulge under inclination would reduce the metacentric height, especially at small angles, and might have an undesirable influence upon altitude steering.

The aerostatic stability of airships is governed by the locus of the metacenter and is measured by the metacentric height in a manner

[1] NAATZ, H., Neuere Forschungen im Luftschiffbau, Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt, 1923.

somewhat similar to its nautical parallel which is dealt with in textbooks on naval architecture[1]. The change of shape of the gasbags with shifting bottom level is, indeed, the inverted analogue to the behavior of liquid ballast in sea-going vessels.

The lateral and longitudinal metacentric heights may be different. Lateral stability against rolling is necessary to prevent heeling; longitudinal (trim) stability is desirable for airships, but need not be large.

The longitudinal shifting of the center of buoyancy of a partially full gas cell due to surging of the cell bottom can be computed from the geometry of the cell. In most naval vessels, where the side walls are nearly vertical, the reduction of metacentric height due to the presence of a free liquid ballast surface is simply the quotient of the equatorial moment of inertia of the mobile surface by the volume of the container. In an airship of circular cross-section the same relation would hold for the gas level only if the cell were but half inflated. For any reasonable degree of inflation the reduction of metacentric height is much smaller. The following table is an aid to such calculations and is based on the geometry of the cylinder intersected by a slant surface:

Fullness	93	94	95	96	97	98	99	100%
h	.040	.038	.036	.033	.030	.026	.018	0

where h is a non-dimensional quantity valid for unit radius and unit length of the cell. The actual reduction $-h_1$ of metacentric height due to the surging of the bottom of any cell of length L and radius R is then,

$$-h_1 = h\frac{L^2}{R}$$

For decidedly conical cells such as occur toward the extremities of a rigid ship, an exact solution is more complicated but the secondary effects from the bow and stern largely cancel each other.

The additional metacentric height reductions due to surging of bulkheads depend on the elastic mechanism of the latter. Plane, taut, radial wire net type bulkheads cause a change of metacentric height for a full cell of length L and bulkhead radius r of approximately,

$$-h_2 = \frac{5\pi}{24}\frac{r^3 L}{n T}\beta$$

where β is the buoyancy (weight density of air—weight density of gas),
n is the number of radial wires present, and
T their (average) initial tension.

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[1] JOHOW-FOERSTER, Hilfsbuch für den Schiffbau, Vol. 1, Chap. 4, Berlin: Julius Springer, 1920; HOVGAARD, WILLIAM, General Design of Warships; PEABODY, C. H., Naval Architecture; HILLHOUSE, PERCY, A., Ship Stability and Trim; NIEDERMAIR, J. C., Stability of Ships after Damage, Soc. Nav. Arch. and Mar. Eng., New York, 1932.

(an initially slack diaphragm type of bulkhead would make $h_2 = \infty$ but if tension builds up under bulging, stability may be regained at small inclinations).

The metacentric height reduction of a whole ship is a weighted average of the reductions due to the individual cells, weighted in proportion to the cell volumes.

In actually computing the resultant metacentric height it must be borne in mind that not only the mentioned corrections, but also the actual loci of the centers of buoyancy and gravity, vary with fullness and altitude.

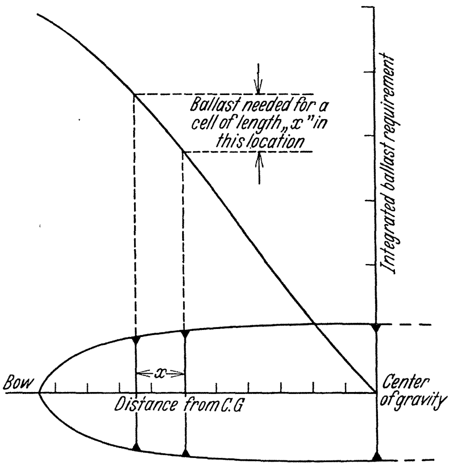

Fig. 2. Curve of integrated ballast requirement.

The inclination angle which gives equilibrium against pitching, or "trim angle" as it is called, can be effectively controlled by the shifting of loads such as fuel, ballast and, in an emergency, personnel and freight. Accidentally, the trim can be upset by such occurrences as the tearing of gas cells forward or aft, or the accidental loss of ballast or other loads. For such cases it is necessary to provide distant-controlled devices for the quick release of an equivalent amount of ballast or gas respectively, lest the ship get out of control with consequent danger of stranding if a low flying ship becomes suddenly heavy.

The hazard of a deflated cell must be provided for in the design. The supply of ballast aboard must be adequate for the restoration of trim and equilibrium in an emergency. The proper ballast requirements for any given airship shape may be obtained by the integration of the lift and deflation moments for axial slices lying between the limits of the given cell spacings[1]. Alternatively the spacing corresponding to any chosen amount of trim ballast may be determined by stepping off the allowable amount of ballast as an ordinate on a plot of integrated ballast requirement versus distance from the center of gravity (see Fig. 2).

3. Aerostatic Performance. The aerostatic performance is expressed in terms of attainable ceiling and intimately linked with the loading and gas capacity of the airship[2].

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[1] VERDUZIO, RODOLFO, U.S. N.A.C.A. Technical Memorandum No. 285; WEISS, G., Schütte-Lanz Airship Projects After the War, U.S. N.A.C.A. Technical Memorandum No. 335.

[2] BURGESS, C. P., Airship Design, Ronald Press, New York 1927.

The flight ceiling or service pressure height of an airship is determined by the volumetric percentage fullness to which the gas cells are inflated at the take-off with any given load. For "standard atmosphere" conditions[1] the ceiling for any sea level fullness at take-off can be read from the standard air density curve, the ratio of inflation in per cent of total inflation being inversely proportional to air densities at the respective heights. In case the take-off field is itself at considerable altitude, the maximum possible fullness at start is the ratio of the standard fullness for the desired ceiling to the standard fullness for the field altitude. Correction for variation of ceiling due to conditions varying in temperature or barometer from standard are made according to the well known laws of thermodynamics. For instance: A ship inflated with helium taking off from a field at 1100 feet elevation above sea level with a ground temperature of 50° F. and 28.90 inch local barometric pressure and no super-heat will have to cross a mountain range at 6,000 feet altitude under weather conditions expected to be equivalent to a rise of 0.6 in. Hg. in the sea level barometer and when the temperature is 25° F. at the altitude. What should be the initial inflation in order to prevent loss of gas enroute? Answer, 90 per cent.

The absolute ceiling or maximum pressure height at which the airship can still float is governed by the specific buoyancy of the gas and by the minimum inflation necessary to float the airship when stripped of all removable loads. The only weight then remaining to be lifted is the "dead weight". The dead weight referred to the total gas capacity (for instance, lbs. per cubic foot) is a structural constant. Its ratio to the specific lift will vary with the latter but will again become a constant if the specific lift[2] of a standard gas in standard air is used. This standardized value of the ratio is frequently used as a comparative weight criterion.

For high altitude performance it is desirable to reduce the dead weight to the lowest possible value. It is of interest to study the variations in dead weight for airships of a common type built to corresponding strength and speed requirements but differing in displacement. The methods of computing the dead weights for such a family of airships has been discussed elsewhere[3]. It may here suffice to note that modern practice still shows a trend toward decreasing values of the dead weight

[1] See Division B X, or, Diehl, W. S., Standard Atmosphere, U.S. N.A.C.A. Report No. 218, 1925.

[2] Havill, Lt. Comdr. Clinton, H., Helium Tables, U.S. N.A.C.A. Technical Note No. 276, p. 15, 1928.

[3] Hunsaker, J. C., Journal of the Royal Aeronautical Society, July 1920; Nobile, U., U.S. N.A.C.A. Technical Note No. 63; Crocco, G. A., U.S. N.A.C.A. Technical Notes Nos. 80 and 274; Lewitt, E. H., The Rigid Airship, Pitman and Sons, London 1925; Burgess, C. P., Airship Design, New York 1927.

per unit volume displacement if existing ships are to be further increased in size[1].

If for any given design specification[2] the dead weight of the airship parts performing individual functions can be expressed in terms proportional (factor k_n) to various powers n of linear ship dimensions l then the total dead weight W would be

$$W = \Sigma \, k_n \, l^n$$

The optimum size for least dead weight per unit volume is indicated by the condition $\Sigma \, (n - 3) \, k_n \, l^{n-4} = 0$

If the parts varying with the power n make up a portion w_n of the whole W, then the optimum condition is expressed by

$$\Sigma \, (n - 3) \, w_n = 0$$

If this critical sum is not 0, its sign indicates that the ship is above (+) or below (—) the optimum size in this respect.

The influence of size on the dead weight criterion is small beyond the capacities of the largest ships built to date. Variations in structural design or speed and strength requirements may have a greater influence. The dead weight of an airship is for instance modified by the choice of the frame spacing or of the number of main frames, bulkheads and cell ends required. An airship with a large frame spacing will have a lower dead weight and good altitude performance but for commercial operations may be under a serious handicap due to the large amounts of trim ballast which must be carried to meet the potential emergency of a deflated cell.

It is possible to compute and plot the variations in dead weight and ballast requirement for a series of alternative frame spacings. With existing airship types there is a definite spacing at which the sum of the dead weight and the required ballast is a minimum (and the remaining lift is a maximum). Such a spacing would give the most efficient airship for commercial purposes, although this optimum ship would not necessarily have the best altitude performance otherwise obtainable.

For airships equipped with ballast gaining equipment, a better criterion is the sum of the dead weight and the weight required for the apparatus (plus weight value of the drag) needed to recover or gain the desired amount of ballast. Since, on flights of normal duration, the conventional ballast gaining equipment gains many times its own weight in ballast, the ballast term has comparatively little influence and the larger frame spacings prove to be most efficient for airships of this type. Naturally also, they have good altitude performance.

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[1] Arnstein, Karl, Some Design Aspects of the Rigid Airship, Trans. Am. Soc. Mech'l Engrs. p. 385, 1934; Ebner, Hans, Der heutige Stand des Luftschiffbaus, etc., Zeitschr. f. Flugtechnik u. Motorl. No. 12 (24. Jahrg., 1933).

[2] Fulton, Comdr. Garland, Some Features of a Modern Airship, Trans. Soc. Nav. Arch. and. Mar. Eng., Vol. 39, 1931.

In airships which normally maintain equilibrium by valving buoyant gas, the amount of ballast required is frequently reduced by flying the airship in a "light" condition; *i. e.*, with a reserve of buoyancy against the contingency of a deflated cell. The higher drag in the "light" (pitched) condition requires a larger fuel load for the accomplishment of a given mission. The criterion in this case is then the sum of the dead weight, the modified ballast load and the added fuel load.

In the measurement of aerostatic performance, chief concern is given to the determination of total lift in what is called the "100 per cent weigh-off". The airship is filled completely full, possibly to a slight over-pressure, and an inventory taken of the loading while the ship, kept almost in buoyancy equilibrium, rests on dynamometers. The dead weight of the airship is obtained by subtracting the known variable loads from the computed total lift. The total volume of the airship is computed from a consideration of the geometry of the cells and the mean specific lift of the buoyant gas. The determination of the mean specific lift involves the accurate measurement of the magnitude and distribution of the temperature, pressure, and humidity of the air and the lifting gas. The density, or chemical composition and purity of the lifting gas, can also be accurately measured by any of the following methods[1].

The density may be determined by measuring the gas pressure difference against air at different levels[2]. Another method consists in taking gas samples in small vessels of accurately known volume and weighing the latter on a sensitive analytical balance before and after the sample gas has been replaced by air of known pressure, or after rebalancing the same vessel with air of suitably reduced pressure[3]. Still another is the Bunsen method of timing the efflux of a gas sample and an air sample through a small orifice (Schillings, Edwards, Simmance)[4].

A portable interferometer has been developed by C. Zeiss to compare gas densities by the interference fringes of two branches of coherent light rays sent through the two samples. The velocity of sound has been proposed as means of indicating gas density[5]. Organ pipes and sonic resonator devices have been tried, also centrifugal compressors measuring speed and pressure generated.

The most common method of chemical analysis of pure helium is that of absorbing all impurities over refrigerated charcoal. The purity

[1] Austerweil, G., Die angewandte Chemie in der Luftfahrt; Oldenbourg 1914; Niedner, Zeitschr. f. Flugtechnik u. Motorl., p. 172, 1922; Bird, W. G., Journal of the Royal Aeronautical Society, November 1931.

[2] Barr, G., Br. A.R.C. R. and M. 88, 1912.

[3] Edwards, J. D., Bureau of Standards, T. 89; Bethuys, G., La Technique Aéronautique, March 15, 1911.

[4] Edwards, J. D., Bureau of Standards, T. 94, p. 359.

[5] Bird, W. G., Journal of the Royal Aeronautical Society, November, 1931.

of dried specimens can be accurately measured by hot wire instruments based on the high heat conductivity of helium and hydrogen (Engelhardt)[1]. Electrostatic measurements have also been tried on helium.

In warm climates the atmospheric humidity has a noticeable influence on the buoyancy of the airship. Not only has moist air less carrying capacity than the heavier dry air, but the moisture also penetrates through the fabric of gas cells and diaphragms into the gas and it may also be absorbed or adsorbed by the fabrics[2].

Many of the buoyancy problems apply to aerostats in general, since an airship, deprived of its motive power, becomes a free balloon. When an airship is in motion through the air, aerodynamic reactions arise which superimpose themselves on the independent aerostatic forces and in some respects are of predominant influence.

CHAPTER II
PROPULSION

1. Axial Motion. When the ship flies straight in buoyancy equilibrium, whether climbing, descending, or maintaining its altitude, the flow about the ship has axial symmetry with respect to the ship's axis, provided the ship's hull is built as a streamlined body of revolution. Rigid airships are usually polygonal in cross section, but the longitudinal or meridional edges follow the natural stream-lines and the number of polygon sides is usually sufficiently great to give a flow departing from axial symmetry only to a negligible degree. The cross sections of non-rigid airships are seldom perfect circles due to the loadings of gas pressure, fabric weight, and car suspensions[3]. When the latter consist of internal rigging or longitudinal curtains, the bag may have a decidedly notched appearance. It is also often the practice to build airship bags with lobed cross sections in order to reduce the maximum radius of curvature and hence the stresses in the fabric due to gas pressure. Although the pear-shaped and multilobed cross sections are not as favorable with regard to the ratio of surface area to volume, they have proved quite acceptable in service.

Airships having elliptical cross sections and therefore lacking flow symmetry have been frequently proposed. The elliptical shape, with

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[1] Daynes, H. A., Gas Analysis by Measurements of Thermal Conductivity, Cambridge, 1933.

[2] Bird, *loc. cit.*; Kline, G. M., Moisture Relations of Aircraft Fabrics, Bureau of Standards Jour. Research. 14, 67, 1935; Houston, D. F., Effective of Protective Coatings on the absorption of Moisture by Gelatine Latex Gas-Cell Fabrics, Bureau of Stan'ards Jour Research 15, 163, 1935. .

[3] Haas and Dietzius, The Stret hing of the Fabric and the Deformation of the Envelope in Non-rigid Balloons, U.S. N.A.C.A. Report No. 16; Blakemore, T. L. and Pagon, W. W., Pressure Airships, Ronald Press, New York 1927; Evans, F. G., The Cross Section of the Semi-rigid Airship, Journal of the Royal Aeronautical Society, August 1930.

the major axis vertical, offers a good approximation to the catenary needed for the transfer of the loads in the vertical plane. Placing the major axis horizontally has been advocated for many years[1], for the reasons that the aerodynamic lift would be considerably improved and the lateral forces incurred in ground handling somewhat reduced. Up to the present time considerations of simplicity and cost have so far precluded the construction of both elliptical types. If such cross sections are built in the future, attention must be given to differences in the aerodynamic force and stability characteristics arising through the lack of symmetry[2].

The surface of airship hulls is usually of fabric, either rubberized (non-rigid ships) or doped (rigid-ships). It has also been demonstrated that very thin sheet metal can be employed as an outer skin[3].

As the airship moves through the air with a flight velocity V, its hull surface is swept by an air stream the velocity of which varies along the longitudinal generatrices. Over the greater part of the ship's length this local air stream velocity is greater than V because the air displaced by the bow must flow around the hull to fill in the space at the stern, and this flow adds itself to the flight speed V. The energy vested in this velocity increment is intimately linked with the phenomenon of apparent additional longitudinal mass[4]. Toward the bow and tail where the surface elements form large angles with the direction of flight, the velocity of the airflow is reduced below flight speed. In between there are two zones, one forward and one aft, where the stream velocity is approximately the same as that of flight. Since the variations in surface velocities amount to many per cent[5] (Fig. 3), it may become of importance to account for them not only in determining the proper pitch of propellers[6] but also in locating airspeed meters[7]. Since the velocity disturbance decreases with distance from the ship, and since with various maneuvers the zone of zero speed increment at the hull proper travels sensitively fore and aft, and since there are marked changes in the magnitude of the disturbance at any fixed station, it is often preferred to suspend an airspeed head at least a ship's radius away from the hull[8].

[1] Burney, Comdr. Sir C. D. Bart, The World, the Air, and the Future, Knopf, London 1929.

[2] Jones, R., Williams, D. H. and Bell, A. H., Experiments on Model of a Rigid Airship of New Design, Br. A.R.C. R. and M. 802.

[3] Fritsche, C. B., *loc. cit.*

[4] See Division C III 2 and VII 7; Lamb, H., Hydrodynamics, 5th Edition, p. 116; Munk, M. M., The Computation of the Apparent Mass of Dirigibles, Journal of the Aeronautical Sciences, Vol. 2, No. 3, May 1935.

[5] Jones, R. and Bell, A., Br. A.R.C. R. and M. 1169, Table 13, 1929.

[6] Wieselsberger, C., Zeitschr. f. Flugtechnik u. Motorl., October 25, 1913.

[7] Stapfer, P., Bulletin Technique 57, March 1929, Ser. Tech. et. Indus. de L'Aéronautique.

[8] Eaton, H. N., Aircraft Instruments, p. 95, Ronald Press.

The velocity distribution about a given body of revolution and the corresponding pressure distribution can be computed by substituting a suitable system of sources and sinks or doublets[1], or in certain cases by the substitution of an equivalent ellipsoid[2]. It has been found that potential flow calculations and the corresponding pressure distributions agree with model[3] and full scale[4] experiments except for the extreme tail region. However if a model is not negligibly small as compared to the tunnel diameter, corrections for the wind restraint or expansion, in a closed or open tunnel respectively, become necessary. A knowledge of the pressure distribution is necessary for the design of the nose stiffening and the design and arrangement of valves, louvers, and other

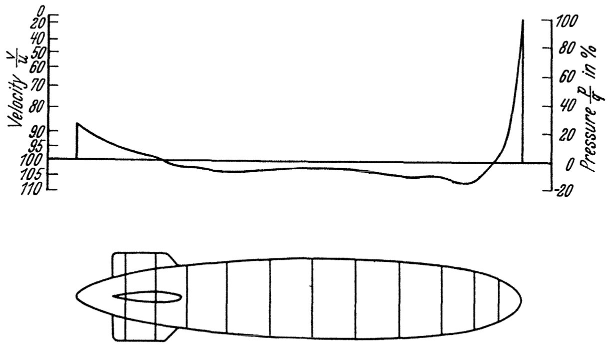

Fig. 3. Curve showing typical distribution of pressure and velocity.

ventilating equiment. The hull contour must be maintained against differences between inside and outside pressures. While the outside pressures vary decidedly over the ship's length, the interior air spaces are generally interconnected and the inside pressure remains substantially constant[5].

The potential flow from sources and sinks gives a faithful picture only at and beyond a certain distance from the hull skin. In the immediate vicinity of the skin, frictional forces act and form a boundary

[1] FUHRMANN, G., Theoretische und experimentelle Untersuchungen an Ballonmodellen, Jahrbuch V der Motorluftschiffahrts-Studiengesellschaft, 1911—12; KÁRMÁN, TH. v., Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Heft 6, pp. 3—17, Aachen, 1927 (U.S. N.A.C.A. Technical Memorandum No. 574); SMITH, R. H., Longitudinal Potential Flow about an Arbitrary Body of Revolution with Application to the Airship *Akron*, Journal of the Aeronautical Sciences, Vol. 3, No. 1, September 1935; KAPLAN, C., Potential Flow About Elongated Bodies of Revolution, U.S. N.A.C.A. Technical Report No. 516, 1935.

[2] See Division C VII 5.

[3] JONES, R., and BELL, A. H., Br. A.R.C. R. and M. 801.

[4] RICHMOND, V. C., Br. A.R.C. R. and M. 1044.

[5] STAPFER, P., *loc. cit.*

layer of air dragged along in greater or less degree. As the approximate dimensions of this boundary can be estimated, it is possible to correct for it in a second approximation by substituting such a system of sources and sinks as will produce, not the actual geometric shape of the hull, but that of the hull plus a "representative" boundary layer shell equivalent in momentum to that of the air dragged along. By selecting sinks slightly less powerful than the sources, the wake of the ship can be represented. The momentum of this additional flow can be interpreted in terms of another contribution to the longitudinal virtual mass which approximate calculations have indicated to be of the order of two per cent of the ship's mass[1].

The momentum imparted to the flow in the boundary layer is the reason for the major part of the drag of the airship and therefore the cause of the consumption of the power required to drive it forward.

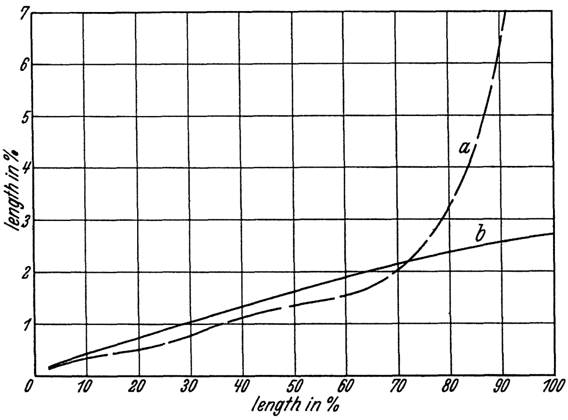

Fig. 4. Curves showing thickness of turbulent boundary layer. (a) Airship model. (b) Flat plate in turbulent flow at same Reynolds number.

The actual texture of the flow in the boundary layer shell and the velocity profile therein, is of much significance. It has a direct influence upon the parasite resistance of any small devices or accessories which may protrude from the hull, and also upon their operations, as in the case of radiators and other heat exchange apparatus, windmills for auxiliary power generation, ventilator hoods and aerodynamic measuring instruments.

The theory of the frictional boundary layer along curved surfaces has been greatly advanced recently[2]. Dr. C. B. Millikan[3] has combined the assumption of a law governing the friction velocity profile on a flat wall with the continuity requirement for elongated bodies of revolution such as airship hulls. He is able, in this manner, to account for the experimentally confirmed fact that in the bow region, where the

[1] SMITH, R. H., *loc. cit.*

[2] WILCKEN, H., Ing.-Arch., pp. 357—376, September 1930; FEDIAEVSKY, C. C., The Boundary Layer and the Drag of a Body of Revolution at Large Reynolds Number, Journal of the Aeronautical Sciences, Vol. 3, No. 1, September 1935; also see Division G.

[3] MILLIKAN, C. B., Transactions of the A.S.M.E., 1932. Also Phil. Mag. VII, p. 655, 1929.

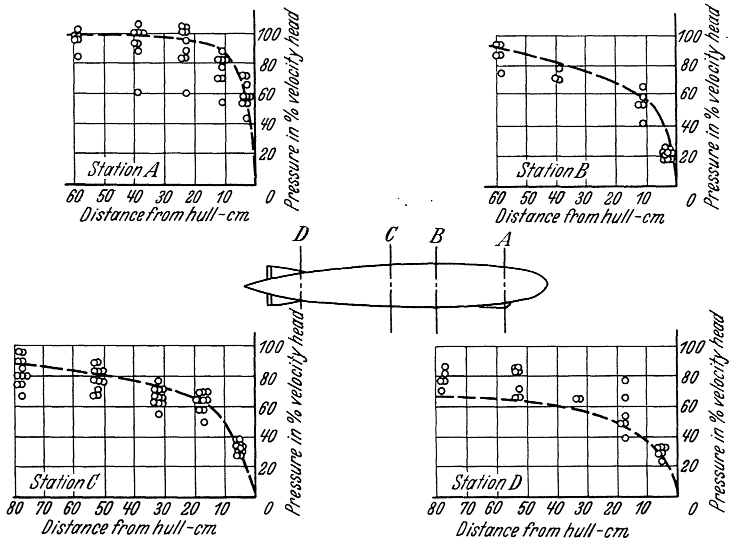

Fig. 5. Curves showing results of experiments on boundary layer velocity. Abscissae, distances outward from surface of hull. Ordinates, pressures in per cent of velocity head.

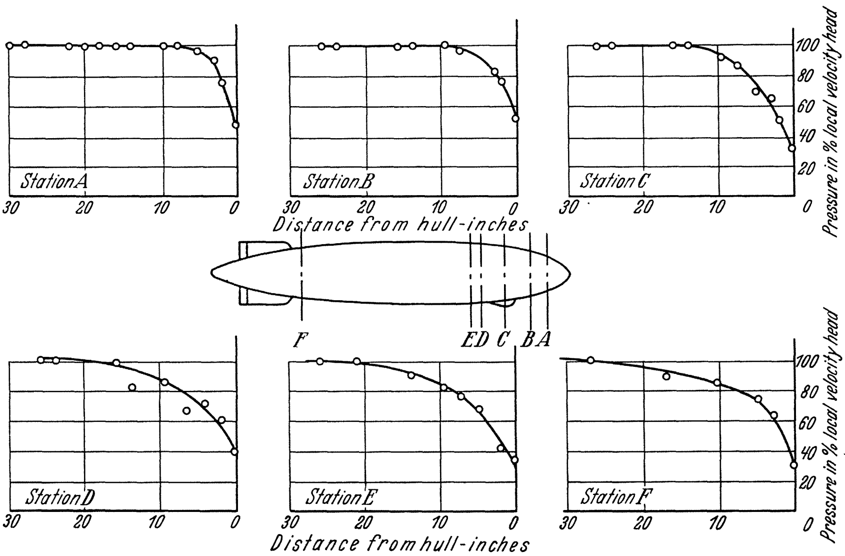

Fig. 6. Curves showing results of experiments on boundary layer velocity. Abscissae, distances outward from surface of hull. Ordinates, pressures in per cent of velocity head. Reynolds number $V\sqrt[3]{\text{Vol.}}/\nu = 635{,}000$.

streamlines locally converge outside the surface, the boundary layer grows much slower, and in the tail region where they locally diverge, it grows much faster than along a flat plate (Fig. 4).

The velocity profile has been experimentally determined in model size for the airship LZ-126 (U.S.S. Los Angeles)[1], for the U.S.S. Akron[2], for models of R-101[3], and for two stream-line bodies[4].

While in the middle body of the ship, the boundary layer follows quite well a fractional power law such as that given by the exponent 1/7, in the forebody there is a tendency for a "fuller" profile and in the rear for a "leaner" one.

In full size there have been a number of scattered experiments of limited scope made on German, English, and American ships and the results of some of these as yet unpublished experiments are here reproduced as Figs. 5 and 6. It must be kept in mind that full size experiments are much handicapped by the difficulties of averaging the fluctuating pressures measured by Pitot tubes in the turbulent zone, and last but not least, by the uncertainty regarding the ship's buoyancy equilbirium and maneuvers. In the presence of dynamic lift the boundary layer is compressed on the attacked side and expanded on the opposite side.

Although the experimental points shown were obtained under various conditions of dynamic lift, they may be considered as in general agreement with the quantitative theory for boundary layer thickness. In the middle body the boundary layer appears to have a thickness similar to that which would develop on a smooth flat plate at the equivalent Reynolds' number in terms of distance downstream from the leading edge or bow. The full scale boundary layer is definitely thinner than would correspond to geometric similarity with the boundary layers found on models. This reduction is reasonably in accord with recent assumptions of the influence of Reynolds' number on boundary layer thickness.

2. Resistance of Hull. The knowledge of the drag of an airship is of principal importance for the prediction in the design stage of its speed or power requirement. The most important single item of drag is that of the huge hull, although through careful streamlining it has become possible to reduce this drag to such a low figure that the sum of the drag contributions of the inevitable appendages and accessories of the ship, though themselves much smaller in size, may be of equal order of magnitude. The problem of aerodynamic improvement in hull shape would seem to resolve itself into finding the form of least drag for a given volume which latter dictates the gross lift obtainable.

Designers have not yet standardized on any particular shape or form of airship hull as "the best" for all purposes. It is commonly

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[1] Klemperer, W., Windkanalversuche an einem Zeppelin-Luftschiff-Modell, Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen 1932.

[2] Freeman, H. B., U.S. N.A.C.A. Report No. 430, 1932.

[3] Jones, R., and Bell, A., Br. A.R.C. R. and M. 1169, 1929; Simmons, L. F. G., Br. A.R.C. R. and M. 1268, 1929.

[4] Ower and Hutton, Br. A.R.C. R. and M. 1271.

understood, however, that a smooth meridional curve preserving continuity up to derivatives of the second order, all along from stem to stern is desirable. Several simple mathematically defined curves, chosen for individual portions of the hull, are often combined for the sake of simplicity in mathematical calculations of such items as the buoyancy and pitching moments of various compartments when empty or partly deflated. If cleverly done, so as to match inclination and curvatures at the junction points, only negligible increase of drag may be incurred and the procedure justified. On the other hand, some investigators have tried to develop formulae from which an entire meridional curve of a good shape smooth in all derivatives can be developed[1]. Such formulae, especially when based on relatively simple source and sink concepts, may have practical advantages in the design office, but to what degree they can insure low drag for a given volume beyond securing smoothness, is problematical. However, there seems to be general agreement that the bow may, to good advantage, be somewhat blunter than an ellipsoid, although if mooring equipment requires a conical nose, no serious harm is done by such form. The insertion of a short cylindrical midship section does not seem to appreciably harm an otherwise good continuously curved shape. The curvature, usually decreasing from bow to master section is usually increased again toward the stern. This latter change, however, should be very easy and gentle. To what degree the tail end may be cut off more or less bluntly without serious harm is a matter of some uncertainty[2].

The question of the best fineness ratio (Diameter to Length) cannot be decided in a general way either. The history of airship design shows uncertain tendencies alternating between fuller and slenderer forms. Many ships, however, may have become more slender than their designers wished, either because they were to fit into available hangars, or because they were subsequently lengthened after some service in order to increase their useful load for more ambitious journeys. As the very slender form implies more surface per volume, it must cope with more frictional drag, whereas the more plump form introduces more severe curvature of the stream-lines, thus giving rise to more rapid growth of boundary layer in the rear and earlier separation of flow. In general there is but little to choose, as far as drag is concerned, from $L/D = 4$ to 8. The optimum is broad with reference to this ratio so that structural considerations which depend upon the details of the design may govern the choice. As a rule non-rigid airships are advantageously made more plump, rigid ships built of annular frames and longitudinals, more slender.

[1] Milarch, Zeitschr. f. Flugtechnik u. Motorl., June 14, 1928 and August 28, 1929; Cox, H. R., Journal of the Royal Aeronautical Society, p. 800, September 1929.

[2] Abbott, I., U.S. N.A.C.A. Report No. 451, 1932.

While with many other vehicles the ratio of resistance against motion to the gross weight carried represents the "frictional coefficient" or "gliding angle" which constitutes a measure of the degree of mechanical perfection, with airships this measure depends greatly upon both size and speed.

If the air resistance were always proportional to the exposed area and the velocity head, the refinement of any shape would be truly reflected by a drag coefficient C_D referred to unit velocity head and to the two thirds power of the volume according to the formula

$$D = C_D \frac{\varrho V^2}{2} Q^{2/3} \tag{2.1}$$ [1]

where D = drag, C_D = drag coefficient, ϱ = density of air, V = speed, Q = volume

The 2/3 power of the volume is preferable to the master section area commonly adopted in airplane fuselage aerodynamics, since the best shape for housing a given volume is not necessarily the same as that providing the best fairing for a given master section. The former is more slender than the latter. In some scientific publications a drag coefficient is determined by reference to the hull surface exposed.

The drag coefficient is, of course, not a true constant, but depends on the Reynolds number R for the ship's size and speed. Reynolds numbers are usually referred either to the cube root of the volume or to the length of the ship. Reynolds numbers of large rigid airships at commercial speeds are of the order of 10^8 to 10^9 and from ten to several hundred times larger than can at present be obtained in model experiments in wind tunnels. Insofar as the hull drag is essentially skin friction, its mechanism may vary sensitively with change of Reynolds number. Therefore the extrapolation from the value of the resistance for any known limited range of R to much higher ranges is quite uncertain, and even if data are available for one type of hull shape, it would be quite unsafe to presume similar relations for other shapes. For very large models tested in atmospheric tunnels giving a value of $L \times V$ greater than 100 m.^{2} per second, as well as in the moderate and high compression range of the N.A.C.A. variable density wind tunnel[2], and for full size airships of slender stream-line shape, a steady drop of the resistance coefficient varying at a rate somewhere between $R^{-0.17}$ and $R^{-0.08}$ has often been observed. A ship may thus have as little as half the drag coefficient shown by its model tests.

All this appears quite reasonable in the light of modern theories of the variations of turbulent boundary layer friction drag, postulating

[1] Burgess, C. P., Airship Design, New York 1927; also U.S. N.A.C.A. Technical Note No. 194.

[2] Abbott, I. H., U.S. N.A.C.A. Report No. 451.

either an exponent of -0.20[1] or more recently a logarithmic law[2] which expresses a lesser influence than an exponent of -0.20 with increasing Reynolds number.

Tests seem to indicate that the more slender the ship's form the more beneficial the "scale effect" to be expected. The more plump form apparently gives rise to an element of the drag due to actual flow detachment[3] at the tail, the magnitude of which would more nearly follow a velocity square law. Small scale model tests are severely handicapped and many show freak drag coefficients quite unsuitable for such extrapolation.

While there has been a great deal of airship model testing in various wind tunnels[4] many discrepancies were noted in the early data. In 1920 an international program[5] was instigated for the testing of two small airship models in many laboratories throughout the world. Even the results of these tests showed wide variations proving that there were obscuring influences due to the air flow in these laboratories or to the experimental technique employed.

Since that time knowledge has been greatly advanced and it appears that there are six major phenomena which are apt to obscure comparative model test results unless their influence is carefully determined and proper corrections made for them.

The first of these phenomena is the presence of a pressure gradient dp/dx in most closed wind tunnels. This causes an axial buoyancy of the order of $Q\,dp/dx$ or more accurately, $K_x\, Q\, dp/dx$[6], by which the measured drag appears too high. Where the pressure gradient is not constant along the region occupied by a long model, or in open jet tunnels where it is usually confined to a small region near the jet entrance nozzle, the product $Q\,dp/dx$ is more logically replaced by $\int_0^L S\,dp$ where S is the cross sectional area of the model at the station where, in its absence, the pressure p would prevail. This integration can be readily carried out as indicated in Fig. 7 especially if the gradient pressures vary in proportion with the tunnel velocity head without change in characteristics.

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1 Kármán, Th. v., Zeitschrift für angewandte Mathematik und Mechanik, Vol. 4, p. 1, 1921; Freeman, H. B., U.S. N.A.C.A. Report No. 430, 1932.

2 Kármán, Th. v., Journal of the Aeronautical Sciences, Vol. 1, January 1934, and Proc. 3rd Int. Congress for Applied Mechanics, Stockholm 1930, and Werft, Reederei, Hafen, April 22, 1928.

3 Gruschwitz, E., Über den Ablösungsvorgang in der turbulenten Reibungsschicht, Zeitschr. f. Flugtechnik u. Motorl. Vol. 23, No. 11, 1932. (U.S. N.A.C.A. Technical Memorandum No. 699.)

4 See Bibliography.

5 International Trials, Br. A.R.C. R. and M. 954, May 1925.

6 Glauert, H., Br. A.R.C. R. and M. 1158 and 1159; Taylor, G. I., Br. A.R.C. R. and M. 1166; Zahm, A. F., U.S. N.A.C.A. Technical Note No. 23; Munk, M. M., U.S. N.A.C.A. Technical Report No. 114, 1921.

The second argument concerns the measurement of the effective velocity head of the test[1]. In an open jet tunnel obviously the difference between the total dynamic head and the static pressure prevailing in the experiment chamber surrounding the jet is a representative measure for the effective velocity head although not necessarily exactly identical with that of free flight conditions. In a closed tunnel, however, the velocity head varies over the entire space surrounding the model, so that a definition of speed measurement becomes necessary, or corrections may have to be computed if the tunnel speed is measured at an arbitrary place in the tunnel and referred to a different standard station. The different degree of flow constraint offered by the open and by the closed tunnel make it doubtful if the drag of any one model should be expected to appear the same in both tunnels. In the open tunnel a given model may act like a fatter one in free air; in the closed tunnel like a slimmer one in free air.

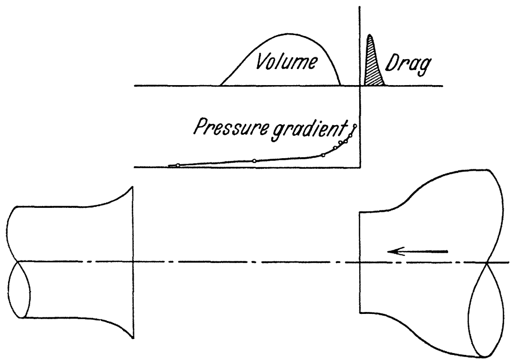

Fig. 7. Correction for pressure gradient.

The third phenomenon apt to obscure model drag test results is the tare drag and flow interference caused by the suspension system connecting the model to the balances. The determination of the net drag of airship models by wind tunnel experiment requires great experimental skill. The forces are small and it is often difficult to keep the suspension tare drag sufficiently low to prevent the final value resulting as the difference between two large quantities. Where the model is suspended by wires, it is necessary to exercise great care to avoid flow obstructions or disturbances at the attachment points without at the same time introducing mechanical constraint. In addition the tare drag area of the wires may vary with the tunnel speed.

The earlier method of determining the tare drag by means of a test with all suspension members doubled is usually less accurate than the dummy method in which the model is independently suspended in place while the original suspension system is alone acting on the balance without contact with the model. Spindle or jig suspensions are very treacherous unless the utmost care is taken to avoid mutual influence of flow. Even if only a tail spindle protrudes from the model, its anchorage on a strut or rig downstream may cause sufficient stagnation

[1] Lamb, H., Br. A.R.C. R. and M. 1010; Lock, C. N., Br. A.R.C. R. and M. 1275.

of the flow upstream to obscure the delicate drag of the model, as was discovered at the Langley Memorial Laboratory[1].

The fourth obscuring influence is inherent in small Reynolds number experiments where a large part of the boundary layer[2] is laminar while with increasing Reynolds number the transition point from laminar to turbulent boundary layer creeps forward on the model until turbulent flow prevails throughout, as it undoubtedly does in full size. Jones[3] has demonstrated that the peculiar and seemingly erratic drop and rise of the drag coefficient observed in many small scale tests can be readily interpreted as due to such a travel of the inception of turbulence. He also demonstrated that the magnitude of the lowest observed drag values fits well a theoretical laminar friction of a flat plate of similar extension as the exposed surface and that for higher Reynolds number the theoretical turbulent friction of an equivalent flat plate is approached. Millikan[4] has refined this picture very much by reconciling these measured drags with those to be computed for a body of revolution retaining from the flat plate theory merely the $1/7^{\text{th}}$ power profile law. The next step along this line is the substitution of the Kármán-Prandtl[5] logarithmic profile law[6], with possibly the introduction of the influence of the surface taper and curvature upon this profile law.

The fifth obscuring influence is the turbulence inherent in the tunnel. It has a bearing on the prevalence of turbulence in the boundary layer. In more turbulent air, naturally more of the layer is turbulent than in smooth air at the same Reynolds number. This has been demonstrated by introducing artificial turbulence into the tunnel air either by annular protuberances placed on the bow of the model or by wire screens placed upstream[7]. It has therefore been suggested that airship model tests, if they must be done at Reynolds numbers insufficiently large to ensure

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[1] ABBOT, I., U.S. N.A.C.A. Report No. 394, 1931.

[2] ZAHM, A. F., U.S. N.A.C.A. Report No. 139.

[3] JONES, B. M., Br. A.R.C. R. and M. 1199; OWER and HUTTON, Br. A.R.C. R. and M. 1271 and 1409.

[4] MILLIKAN, C. B., Transactions of the A.S.M.E., 1932; FREEMAN, H. B., U.S. N.A.C.A. Report No. 430.

[5] PRANDTL, L., Neuere Ergebnisse der Turbulenzforschung, Z. V.D.I., Vol. 77, No. 5, April 28, 1933 (U.S. N.A.C.A. Technical Memorandum No. 720); KÁRMÁN, TH. v., Turbulence and Skin Friction, Journal of the Aeronautical Sciences, Vol. 1, January 1934.

[6] MOORE, N. B., Application of Kármân's Logarithmic Law to Prediction of Airship Hull Drag, Journal of the Aeronautical Sciences, Vol. 2, No. 1, January 1935 and The Boundary Layer and Skin Friction for a Figure of Revolution at Large Reynolds Numbers, Daniel Guggenheim Airship Institute, Akron, Ohio, Publication No. 2, 1935.

[7] RELF, E. F., and LAVENDER, T., Br. A.R.C. R. and M. 597; LYON, H. M., Br. A.R.C. R. and M. 1511; PRANDTL, L., Der Luftwiderstand von Kugeln. K. Gesellschaften zu Göttingen, Math.-physikalische Klasse, 1914; LYON, H. M., The Drag of Streamline Bodies, Aircraft Engineering, September 1934.

essentially turbulent boundary layer, be made with the air stream rendered artificially turbulent and this turbulence measured by the sphere drag[1] or other methods. However, Jones questions the adequacy of this artifice.

The last of the six factors to be considered is the smoothness of the surface. While some investigators have found large variations of drag with surface conditions, others have found practically none. These differences may be due to different turbulence regimes[2]. Smooth wax-polished model surfaces seem to give the lowest and most consistent drag results. In full size the skin of both metal clad and well doped or rubberized fabric covered airships can probably be considered as aerodynamically "smooth"[3].

For unusually rough hulls the theory of friction on rough surfaces would apply. Th v. Kármán[4] has shown that in order to be aerodynamically smooth the hull of airships should not have a roughness exceeding .03 to .04 mm over the greater part of their length, the very bow being the most sensitive. Well doped taut fabric and thin sheet metal under pressure are smooth within this specification. However, in actual service, fabric may flap when not taut and metal sheet may be wrinkled and studded with rivet heads. To what degree such surface irregularities may influence the mechanism of impulse transmission in the boundary layer is still problematical.

In summarizing it may be said that while an injudicious application of wind tunnel test drag measurements to a full size project can be quantitatively and qualitatively grossly misleading, the prediction of full size drag need not necessarily depend solely on a digest of actual flight experience and service performance. On the contrary careful model tests at suitable Reynolds number[5] under controlled turbulence and surface conditions are quite apt to reveal the degree of perfection of a proposed shape with respect to skin friction and pressure drag. For good shapes the full size drag can then be calculated with a satisfactory degree of reliability, confirmed by actual flight performances of ships built.

Theoretically it is interesting to compare the measured drag with the impulse left in the wake[6] which is a large portion of the whole; and

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[1] DRYDEN, H. L., and KUETHE, A. M., U.S. N.A.C.A. Reports Nos. 342 and 392.

[2] WIESELSBERGER, C., Experiments on Model Balloons and the Resistance of Various Kinds of Surfaces, Zeitschr. f. Flugtechnik u. Motorl., September, 1915; ABBOTT, I. H., Airship Model Tests in the Variable Density Wind Tunnel, U.S. N.A.C.A. Report No. 394.

[3] GEHMAN, S. D., and MALLORY, G. C., Skin Friction of Various Surfaces, Journal of the Franklin Institute, Vol. 216, No. 3, 1933.

[4] KÁRMÁN, TH. v., Turbulence and Skin Friction, Journal of the Aeronautical Sciences, Vol. 1, 1934.

[5] OWER, E., and HUTTON, C. T., Br. A.R.C. R. and M. 1409.

[6] BETZ, A., Zeitschr. f. Flugtechnik u. Motorl., p. 42, 1925; SCHRENK, M., Luftfahrtforschung, Vol. 2, Heft 1, 1928.

with the pressure drag from normal pressure measurements by integration over the projected area elements which, with efficient shapes, is a very small portion of the whole[1].

3. Resistance of Accessories. It has not yet been found feasible to house all of the equipment of an airship within the streamlined hull, although there prevails a decided tendency to eliminate more and more of the outside appendages and protuberances. So long as individual propellers are disposed about the ship there are outriggers or complete power cars outside which require individual fairing[2]. The control car accommodating the navigating crew is usually located in the forward lower part of the hull. An empennage is carried at the tail. Furthermore, ground handling and mooring attachments usually protrude from the hull. Last but not least there are items of equipment such as radiators and other devices for the exchange of heat, and likewise hoods and vents for the intake or expulsion of ventilation air and again certain navigational instruments, all of which depend upon exposure to the outside air for their proper operation.

Evidently the drag of such protuberances and appendages can be calculated from model experiments made on much larger scale than for the ship as a whole. In all of this, allowance must of course be made for any local excess or deficiency of airspeed due to the potential flow or the boundary layer about the ship.

Occasionally, relatively small protuberances, especially on the forebody on a model tested at low Reynolds number, have shown an apparent influence by way of an increase of drag far beyond any normal expectation based on the drag of the protuberance itself. That similar freak influences would occur in full dimension seems rather doubtful in the light of experiments made with artificial "spoilers" on full size ships and in high Reynolds number model tests[3].

The drag of nacelles or power cars can be estimated from experiments on similar objects occuring in heavier than air design provided allowance is made, if necessary, for the proximity to the airship hull which acts as a mirror surface of the wash of the propeller slip stream, and if need be, of the flow through such a car containing a radiator vent, or the like.

Undoubtedly the most favorable location, from the viewpoint of drag, for a radiator, water condenser, or other heat exchange device,

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[1] Fuhrmann, G., Jahrbuch V der Motorluftschiffahrts-Studiengesellschaft, 1911—12; Fage, A., and Stern, W. J., Br. A.R.C. R. and M. 107, 1914; Richmond, V. C., Airship Research and Experiment, Journal of the Royal Aeronautical Society, October 1926; Stapfer, P., Bulletin Technique 57, Ser. Tech. et Indus. de L'Aéronautique, March 1929; Freeman, H. B. U.S. N.A.C.A. Report No. 443, 1933.

[2] Arnstein, K., Some Design Aspects of the Rigid Airship, Trans. Am. Soc. Mech'l Engr's p. 385, 1934.

[3] Abbott, I. H., U.S. N.A.C.A. Report No. 451.

would be just inside the hull so that the skin of the ship, the friction of which is inevitable, could be utilized as a heat radiating surface without adding parasite resistance. The area required in such case is, of course, much larger, since the heat must traverse the entire boundary layer of the hull. However in view of the fact that the heat transfer varies with a fractional power of the velocity head, the handicap is not necessarily insurmountable. Just how far the finning of a heat exchange apparatus should protrude into the ship's boundary layer is a matter of design compromise into which considerations of space available, weight, complication and maintenance enter, aside from the mere question of drag.

The drag of the fins of an airship can be computed from model experimental data with a similar degree of accuracy as in the case of airplanes, excepting only that their huge size renders a beneficial scale effect of frictional resistance of importance. For conservative estimates, however, it is well to add a certain average of induced drag to the form drag of fins and control surfaces, because in flight they are, for reasons to be explained later, almost continuously under some attack—a condition which entails the development of induced drag along with the forces of control. The more stable the ship the less the allowance required in this respect.

A sample list of accessories drag is given in Table 1 for three typical rigid airships as estimated under certain experimental conditions, without water recovery apparatus. The accuracy of any analysis of the gross drag of an airship into various parts is naturally dependent upon the

TABLE 1.

Estimated Drag Area Breakdown Without Water Recovery	Bodensee		U.S.S. Los Angeles		U.S.S. Macon	
	Sq. M.	Sq. Ft.	Sq. M.	Sq. Ft.	Sq. M.	Sq. Ft.
A. Bare Hull	9.4	101	21.8	235	39.0	420
B. Fins and Rudders	2.5	27	4.9	53	14.0	151
C. Wing Power Cars or Outrigger Gears, Their Suspension, Ladders, Struts, Hoods, Radiators, Exhaust Mufflers	2.8	30	6.8	73	10.7	115
D. Rear Power Car with Handling Rails and Bumpers	2.4	26	2.2	23.5		
E. Control Car or Passenger Car with Handling Rails and Bumpers . .	2.4	26	4.5	48	2.8	30
F. Miscellaneous Protrusions-Mooring Mast Equipment, Hoods, etc. . .	0.5	5	0.8	8.5	1.8	19
	20.0	215	41.0	441	68.3	735
$(\text{Volume})^{2/3}$	790	8500	1845	19852	3528	37978
Resistance Coefficient C_D	.025		.022		.019	

availability of uniformly accurate data on the contributions of all these parts. By attributing different degrees of importance to the various component data derived from indirect evidence, conclusions can be shifted somewhat[1].

4. Experimental Determination of Drag. The combined drag of the hull and of the accessories—the gross drag—enters into speed and performance computations. Measurement of this gross drag may be attempted by direct experiment full size. It would be interesting to measure this drag directly by towing from another airship. So far this has not yet been accomplished, but undoubtedly will be some day. Measuring the thrust of the propellers would also furnish a measure of the drag. However, consideration must be given to the force reactions due to the presence of the propeller wake impinging on part of the structure. An analysis of the problem has been given by Durand[2]. Successful thrust dynamometers to be inserted between shaft and propeller hub have been constructed[3] and it would be only a matter of carrying out such a test program to obtain exhaustive data. However, the costs and elaborate preparations necessary have thus far prevented such a test. Thrust measurements on one of the five propellers of the U.S.S. Los Angeles were made by the Zeppelin Company and served to confirm the resistance estimates under various operating conditions and engine combinations. However, the experimental error multiplied by 5 and the uncertainty regarding the degree to which the five propellers could be considered identical and equally loaded, limit the accuracy of the conclusions.

There is, however, an indirect method available for the determination of the gross drag, the so-called "deceleration" or "coasting" test[4]. The ship is flown at its top speed and then suddenly on signal all engines are stopped. The ship gradually slows down and the deceleration process is recorded by suitable airspeed meters. The underlying theory of the evaluation of the deceleration records is based on the equilibrium between the aerodynamic drag and the inertia force.

$$D = -M\,(1 + k_1)\,\frac{dV}{dt} \tag{4.1}$$

where M is the ship's mass, k_1 the contribution of the virtual longitudinal mass due to potential flow and boundary layer[5], and V the velocity

[1] HAVILL, LT. C. H., The Drag of Airships, U.S. N.A.C.A. Technical Note No. 247, and 248.

[2] DURAND, W. F., U.S. N.A.C.A. Report No. 235, pp. 3—5, 1926.

[3] GOVE, W. D., and GREEN, M. E., U.S. N.A.C.A. Report No. 252, 1927; SEEWALD, F., Zeitschr. f. Flugtechnik u. Motorl. **13**, 1931.

[4] VON SODEN and DORNIER, Die Bestimmung des Schiffswiderstandes durch den Fahrtversuch, Zeitschr. f. Flugtechnik u. Motorl., No. 19, 1911; MUNK, M., The Drag of Zeppelin Airships, U.S. N.A.C.A. Technical Report No. 117; STAPFER, P.. *loc. cit.*; BURGESS, C. P., Airship Design, New York 1927.

[5] U.S. N.A.C.A. Technical Report No. 164.

at the time t. If the ship is in buoyancy equilibrium, $M = \varrho Q$ and if the drag is expressed as in (2.1) we have immediately

$$C_D = -2\,Q^{1/3}\,(1 + k_1)\,\frac{d\,V}{V^2 d\,t}$$

However, since $-d\,V/V^2 = d\,(1/V)$ this becomes,

$$C_D = 2\,Q^{1/3}\,(1 + k_1)\,\frac{d}{d\,t}\left(\frac{1}{V}\right)$$

If then $1/V$ is plotted against time, the slope of the curve is indicative of the drag coefficient. Figure 8 shows a sample record of an original coasting test, and Fig. 9 its evaluation in terms of $1/V$. In some instances the curve of $1/V$ versus t appears quite straight, thus revealing no variation of the drag coefficient with speed. In other tests the curve appears concave as though indicating the regular "scale effect" of turbulent friction. Others invite interpretation as a broken line, as though two distinctly different slopes and drag coefficients prevail above and below a critical speed[1]. Again others show more erratic behavoir. However, all such detailed conclusions must be taken with due reserve considering the serious experimental difficulties which attend tests of this nature. The lag in the speed recorders, the influence of gusts and slight pitching or yawing of the ship, the drag due to rudder and especially to elevators, as well as the time required to bring the propellers to a stop often prevent experiments on even the same ship in the same flight repeated after only a short interval or with different measuring

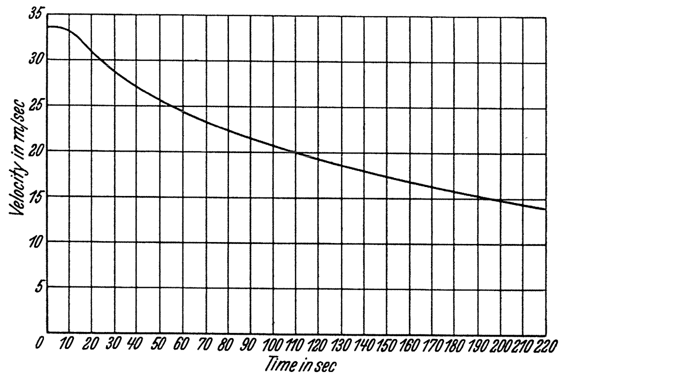

Fig. 8. Deceleration test, velocity on time.

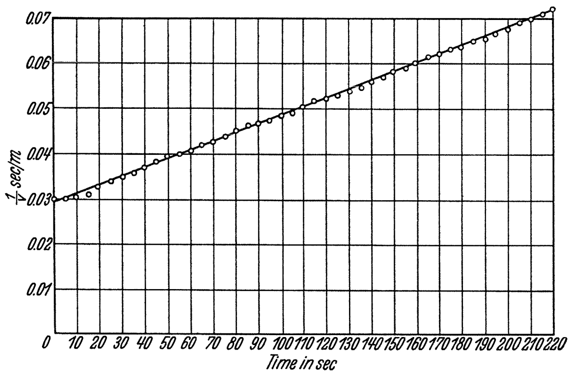

Fig. 9. Deceleration test $1/V$ on time.

[1] Munk, M., U.S. N.A.C.A. Report No. 117, 1921; Defrance, S. J., and Burgess, C. P., U.S. N.A.C.A. Report No. 318, Figs. 4, 8, 12, 1929; Thompson, F. L., and Kirschbaum, H. W., U.S. N.A.C.A. Report No. 397, Fig. 5, 1931.

instruments, from giving duplicate results[1]. For any accurate evaluation, automatic records of elevator angle, ship's inclination and altitude are indispensable. Proper correction for the drag due to pitch, elevator and propellers and the influence of possible lightness or heaviness of the ship have been found to straighten out the $1/V$ curves very remarkably in deceleration tests made in calm air, as shown in Figs. 10 and 11, taken from a typical experiment on a large rigid airship.

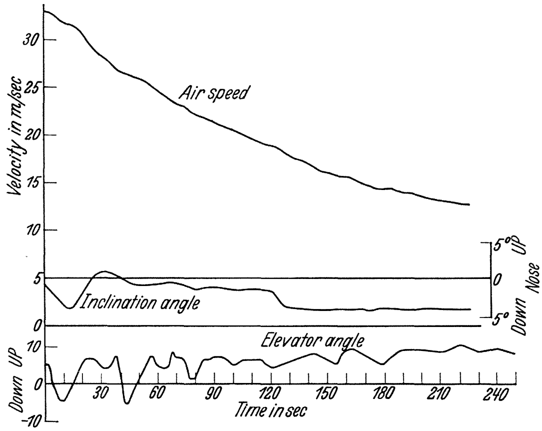

Fig. 10. Deceleration test, velocity on time with records of inclination (pitch) and elevator angles.

In evaluating coasting tests the drag of the dead (or idling) propellers must be separately determined and subtracted from the experimental result in order to obtain the ship's own drag[2].

If the ship was not in perfect buoyancy equilibrium at the time the coasting test was run, a correction for the induced drag of the dynamic lift (L) or dip ($-L$) and for the difference between the ship's actual mass and that of the air displaced, *viz.*, $\pm L/g$, is required.

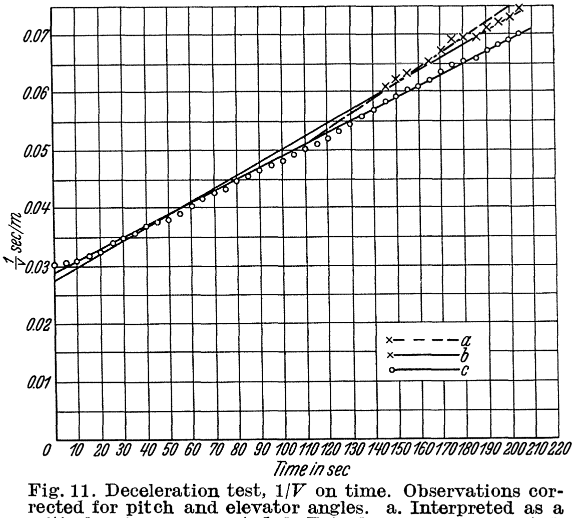

Fig. 11. Deceleration test, $1/V$ on time. Observations corrected for pitch and elevator angles. a. Interpreted as a critical region, uncorrected. b. Faired average, uncorrected. c. Corrected for pitch and elevator action.

5. Propulsive Efficiency. Speed Performance. The power required to drive a ship of known resistance characteristics at a given maximum air speed V is obviously[3],

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[1] U.S. N.A.C.A. Report N. 397, Figs. 4 and 10.

[2] Hartman, E. P., U.S. N.A.C.A. Report No. 464.

[3] Jaray, P., Studien zur Entwicklung der Luftfahrzeuge, Zeitschr. f. Flugtechnik u. Motorl. Heft 11, 1920.

$$
\left.\begin{aligned}
P &= \frac{\varrho V^3 Q^{2/3}}{2\eta} C_D \\
\text{or} \qquad P &= \frac{\varrho V^3 A_D}{2\eta}
\end{aligned}\right\} \tag{5.1}
$$

where the drag area $A_D = C_D Q^{2/3}$ and $\eta =$ propeller efficiency. In horsepower and foot, pound, second units, this becomes

$$
P = \frac{\varrho V^3 A_D}{1100\eta} \tag{5.2}
$$

In order to obtain maximum speed from a given engine power it is necessary to select a propeller type which will reach its highest efficiency and the maximum permissible engine speed at that top velocity.

If in cruising, all engines are throttled to some fraction of maximum power, the propeller efficiency changes only slightly because the engine speed automatically varies nearly proportional to the forward speed, thus leaving the value of (V/nD) (see Division L) nearly constant. This would be exactly so if the drag area A_D were independent of the Reynolds number or of the velocity V; but even scale effect alters this relation so slightly that one may speak of the "effective pitch" of the propeller or of its slip against the zero thrust pitch as a measure of the resistance condition of the ship and any change in this condition will manifest itself in a change of the effective pitch or slope of the V/rpm line. It is significant that the effective pitch or the airspeed made good for any given rpm is independent of the air density.

If in cruising some of the engines are shut down and the others made to do all the work, the disk loading of these propellers will be increased. This will result in an added drag of the dead propellers, an increase of slip, a decrease in the value of V/nD and a loss in efficiency. Adjustable pitch propellers have some advantages in such case as they will permit of better adaptation of the motor speed to the increased loading.

If the power absorbed by the propellers is known (it can be determined by measuring the torque and the shaft speed) then the observation of the air speed made good will give the "propulsive" efficiency E, which is defined as the quotient of the propeller efficiency by the gross drag coefficient.

$$
E = \frac{\eta}{C_D} = \frac{\varrho V^3 Q^{2/3}}{2P} \quad \text{(in consistent units).}
$$

This is a non-dimensional number indicative of the degree of engineering success.

The efficiency of airship propellers is somewhat limited by the low pitch diameter ratio unavoidable for economic air speeds. Low speed motors and geared down propeller drives are therefore preferable from the viewpoint of efficiency alone, although the resulting large propeller diameters and increased weight and resistances of the rigging call for compromise values.

The conventional location of propellers is nearly amidships, somewhat distributed so as to avoid overlapping slipstreams. However, considerations of structural advantage may warrant a departure from this arrangement and invite tandem arrangements despite the contingent loss of propeller efficiency. The pitch of the rear propellers is, of course, made larger than that of the forward ones, and the sense of rotation is best alternated. Placing a propeller in the potential flow about the ship's hull should not cause a first order influence upon its efficiency inasmuch as the potential flow part of the inflow velocity is theoretically recovered from the slipstream in the form of pressure.

The measurement of the air speed of an airship is necessary, in service for navigation of the craft and in tests for the determination of the speed performance. An accurate measurement of the air speed is by no means an easy task. Either anemometric instruments such as cup-anemometers or spinning windmills which are directly responsive to air speed, or aerodynamic instruments such as Pitot tubes[1], Venturis, bridled windmills, pressure plates, etc., which really measure velocity head depending on air density, are commonly in use for this purpose. When the measuring elements are carried close to the ship due allowance must be made for the influence of potential flow (possibly modified by the presence of dynamic lift); when trailing on a long line payed out, they may swing and actually travel more air miles than the ship; also they may be trailing in a stratum of air where the density and the wind may appreciably differ from that at the ship's altitude. It has been a time-honored, though problematic, practice to check air speed measurements against ground speed measurements. While holding the engine speed and altitude constant the ship is made to fly three "legs" in different directions over a well surveyed ground area, crabbing if necessary, to follow straight landmarks between easily distinguished terminal marks. Thus, by clocking the time for each leg of known length and laying out the three ground speed vectors from an origin according to the known azimuths, a circumscribed speed circle is constructed whose center displacement from the origin is interpreted as the average wind vector and whose radius is the air speed. It is not uncommon to find discrepancies as high as 3%, and even 5%, between the vectorial average of the ground speed and of the speed indicated by calibrated air speed meters, even after allowances are made for the inherent instrumental errors already mentioned. Additional errors may occasionally creep in from the following sources: In trying to follow a marked ground track or, in more marked degree, in trying to approach a distant landmark without a guide track, the ship will occasionally swing off her course or pursue a more or less sinusoidal path. The same is true in the vertical plane for the ship will

[1] von Soden, A. F., and Dornier, C., Zeitschr. f. Flugtechnik u. Motorl., October 14, 1911.

occasionally gain or lose altitude. If compass course and inclination or altitude are carefully recorded, the distance flown can be corrected inversely according to the average cosine of the deviation of the actual from the average path. Furthermore, the wind in the trial region may change between runs. Pilot balloon ascensions made over the flight area and repeated throughout the test duration may reveal such changes and furnish clues for corrections. When the balloon runs are simultaneously observed from two theodolites so as to give accurate altitude references, the actual variation of the wind speed with altitude is determined. If the indicated average air speed, or the wind speed, vary slightly from leg to leg, a correction can be worked out by successive approximations.

Instead of the conventional ground speed vector triangle method, an abbreviated and quicker method of making a ground speed trial on two legs becomes feasible over a country where parallel landmarks are available; for instance in certain mid-western States of America where the roads are parallel and located at intervals of 1 or 5 miles. Here the ship simply flies over the area, steered on a compass course determined at right angles to the parallel landmarks. The pilot need not follow any definite track on the ground, but merely tries to maintain the average course by the compass. Since he does not have to mind his wind drift he can hold his course much better than if he had to also pass over definite points. By clocking the passage over the border lines the "crossing" speed is determined and if necessary corrected for mean course error cosine. The procedure is then repeated in the opposite direction. Provided that the wind did not change in the meantime, the algebraic average of the two opposite crossing speeds gives directly the air speed made good, as the wind speed components here cancel out. By running a third or a fourth leg and by optically measuring drift angles, ground speed, etc., additional checks are obtained.

The practical navigator often determines his ground speed and drift by means of optical drifts sights, by observation of the ship's shadow against landmarks, or, over water, against dropped "drift bombs".

6. Fuel Economy, Range. The range of action or distance which the craft can travel without refueling depends not only on its propulsive efficiency, but also on the amount of fuel carried, on the fuel economy, on the air speed made good, on the air density and on the wind. The fuel consumed not only per hour, but also per air mile travelled, decreases with reduced air speed in more marked degree than with airplanes. At modest speeds, large modern airships could stay out weeks and travel to the farthest point on the globe. The computation of the actual range for given initial conditions is a straight forward problem provided the propeller efficiency for various engine combinations and the fuel consumptions for various engine speeds and air densities are known. As with most motors the fuel consumed per hp hr. increases with throttling, investigation will be needed in each particular case to determine whether reduced speed cruising is more economical by throttling all motors, or by shutting some of them down, depending on whether the reduced fuel economy or the loss in propeller efficiency is the more important factor.

In the development of larger and faster airships it has been mandatory that parasite drag be reduced to a bare minimum in order that the desired speeds might be realized without the expenditure of prohibitive amounts of power. Certain matters, such as the removal of unnecessary protuberances and the cowling of those few which remained, were logical steps in the development of design technique and the modern airship now closely approaches a smooth streamlined body of revolution.

While some parts of an airship's propulsive system may be completely cowled, it is necessary that the propellers and the heat dissipating surfaces come into direct contact with the air. The decision as to how much of the auxiliary propulsive equipment should also be placed in an exposed position is one which can be decided from consideration of the relation of the differences in installed dead weight to the differences in drag, as reflected in the extra weight of fuel needed for a given cruising distance, or the extra power plant weight needed for a given design top speed.

Since the horsepower required for the propulsion of airships increases nearly with the cube of speed, the contribution of the propulsive system to the total drag is appreciable on high speed airships. Thus, changes in the weight and drag of the propulsive units become of great importance. As a matter of fact, for speeds now considered, the cleaning up of the design by the installation of inside power plants instead of outboard power cars, may so reduce the power requirement as to offset the higher specific weight per hp inherent with inside power plants[1]. The design top speed at which the dead weight of an airship equipped with inside power plants will be equivalent to that of a similar design incorporating outside power plants is given by

$$v = \sqrt[3]{\frac{1100}{\varrho} \, \frac{w_0 E_i - w_i E_0}{w_0 d_i - w_i d_0}}$$

where w_0 and w_i are the specific weights of the outside and inside power plants respectively (lbs/hp),

E_0 and E_i are the drive efficiencies of the outside and inside power plants respectively,

d_0 and d_i are the specific drags of the outside and inside power plants respectively (sq. ft./hp).

[1] Arnstein, Karl, Some Design Aspects of the Rigid Airship, Trans. Am. Soc. Mech'l Engr's p. 385, 1934.

For the case of comparable drive efficiencies, the airship having the smallest total drag area will have the lowest fuel consumption at any given air speed. Even when the design speed is not sufficiently high to warrant an inside installation purely from considerations of deadweight, the reduced fuel load required for the accomplishment of a given mission frequently makes such installations desirable from the standpoint of lift available for payload. The hours of flight needed to overcome an initial weight handicap are given by the quotient of the total excess power plant weight (inside-outside) by the hourly reduction in fuel consumption. Expressed with the foregoing symbols, the hours required to equalize the weights and loadings are given by

$$h = \frac{1}{f}\left(\frac{v_m}{v_c}\right)^3 \frac{\dfrac{w_i}{1100\,E_i - d_i\,\varrho\,v_m^3} - \dfrac{w_0}{1100\,E_0 - d_0\,\varrho\,v_m^3}}{\dfrac{1}{1100\,E_0 - d_0\,\varrho\,v_c^3} - \dfrac{1}{1100\,E_i - d_i\,\varrho\,v_c^3}}$$

where v_m and v_c are the maximum and cruising speeds respectively, and f is the specific fuel consumption of the engines (lbs/hp-hr) at the chosen cruising speed.

Such methods apply to comparisons of weight savings with the corresponding drag and power influences in passing upon the merits of many other auxiliaries such as radiators, water recovery apparatus, and propeller gearing.

The science of navigation of airships is, of course, replete with interesting theoretical problems such as the best course and airspeed to reach a certain destination with the largest margin of fuel reserve under given or predicted conditions of locally variable winds or cyclones along the route[1]; or again, how best to circumnavigate unfavorable weather conditions or cope with unforeseen wind changes en route. However, these problems are not strictly of an aerodynamic nature and can, therefore, not be pursued here in detail. It may only be mentioned that their solution sometimes differs for airships and airplanes due to the fact that

[1] Scott, G. H., Airship Piloting, Journal of the Royal Aeronautical Society, February 1921; Munk, M., U.S. N.A.C.A. Technical Note No. 89, 1922; Klemperer, W., Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, Vol. II, p. 31, 1922; Scott, G. H., and Richmond, V. C., Effect of Meteorological Conditions on Airships, Journal of the Royal Aeronautical Society, March 1924; Bleistein, W., Effect of Speed on Economy of Airship Traffic, U.S. N.A.C.A. Technical Memorandum No. 302; Seilkopf, H., Die Wetterberatung der Amerikafahrt des LZ-126, Zeitschr. f. Flugtechnik u. Motorl., Heft 6, 1925; Silvester, N. L., The Use of Barometric Charts in the Navigation of Airships, Journal of the Royal Aeronautical Society, January 1927; Wagner, F., Der Einfluß des Windes auf die Reisegeschwindigkeit von Luftfahrzeugen, Annalen der Hydrographie und maritimen Meteorologie, Heft 12, 1927; Zermelo, E., Z.A.M.M., Vol. II, p. 124, 1931; von Mises, R., Fluglehre, 1933. See also Br. A.R.C. R. and M. 281, 389, 521 and 637.

the energy consumed per air mile depends mainly on the air speed with the one and mainly on the gradually diminishing fuel load with the other.

The fuel load that can be taken aboard depends, of course, on the useful lift and on the size of crew and payload to be carried. The former again depends on the buoyancy conditions of the air at the start (temperature and barometric pressure) and on the flight altitude to be reached. In the case of ships burning gaseous fuel the ceiling increases as fuel is consumed, so that high altitudes can be easily reached during the later stages of a flight, when the bulk of the gaseous fuel has been spent. The same is true of ships burning liquid fuel without ballast recovery, when much of the fuel load has been spent and the equivalent amount of the lifting gas has been valved.

The relations between speed, altitude and power requirement are different for the airship and the airplane since with the former the lift remains independent of density up to the ceiling and in addition there is no need to fly faster than at certain minimum air speeds or flight attitudes in order to remain aloft. As we have already seen, the horsepower required for any given airship speed varies directly with the air density. The air density decreases with altitude according to the formula

$$\varrho = \varrho_0 \frac{P - (3/8)\, e}{P_0} \frac{T_0}{T}$$

where ϱ is the density at an altitude where the pressure is P and the temperature T (abs.) and ϱ_0 is the density of dry air at a standard pressure P_0 and temperature T_0. The pressure of aqueous vapor e at the altitude is frequently neglected in which case the density, and hence the drag, is assumed to vary directly with the pressure and inversely with the absolute temperature.

The power output of the engines at any given altitude is given by the formula[1]

$$b\,h\,p = b\,h\,p_0 \left[\frac{P}{P_0}\left(\frac{T_0}{T}\right)^{1/2}\left(1 + \frac{\lambda - \lambda n}{n}\right) - \frac{\lambda - \lambda n}{n}\right]$$

where n is the mechanical efficiency at sea level and λ is the ratio of mechanical friction to friction horsepower at sea level.

If the mechanical efficiency variation is neglected, the power output would be assumed to vary directly with the pressure but inversely with the square root of the absolute temperature.

A comparison of these approximate expressions for power required and power available indicates that with unsupercharged engines the maximum speeds attainable are proportional to the sixth root of the absolute temperatures. Hence, an airship can attain its highest speed at high temperatures. With supercharged engines, the highest air speeds

[1] Gove, W. D., Variation in Engine Power with Altitude, etc., U.S. N.A.C.A. Technical Report No. 295. Also see Br. A.R.C. R. and M. 462, 960, 961 and 1099.

would be obtained at the maximum altitude at which the supercharger can deliver air to the engine at sea level pressure.

For any given cruising speed, however, the power required falls off directly with the density, and for an engine with a constant specific fuel consumption (lbs/hp-hr) the total fuel consumed per hour at a given air speed would also vary directly with the density. Actually the specific fuel consumption does not remain exactly constant with altitude but rather increases. However, this increase is small, particularly so in good airship engines (Fig. 12) and diminishes only slightly the potential saving in fuel inherent in cruising at higher altitudes or lower densities.

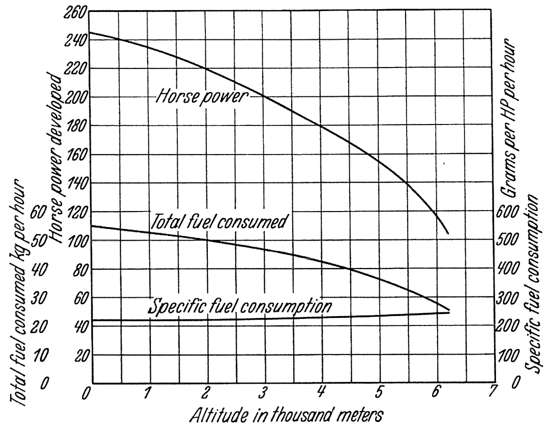

Fig. 12. Fuel characteristics of an airship engine.

The conclusion is that for economical operation, airships should be flown as near their ceiling as possible; *i. e.*, with due regard of course to the vertical structure of the winds en route and other operational considerations. In this respect the helium inflated airship, with no valving and a constant ceiling, may be at a disadvantage when compared with fuel gas and valving airships where the ceiling progressively increases as fuel is consumed and where the mean fuel consumption for the voyage may sometimes be appreciably reduced by flying at a higher average altitude. However, this comparison is tempered somewhat by considerations of meteorological conditions which may dictate the optimum flying altitude more predominantly than mechanical conditions.

It follows that there are differences between the fuel consumptions and operating economies of various types of airships. As a rule they all have a common merit however, in that their transport efficiency improves with increasing size. This transport efficiency is defined, for a given speed, by the quotient,

$$\frac{\text{Payload} \times \text{Distance Carried}}{\text{Energy Expended}}$$

and is expressed[1] by the formula

$$T = K\,(1 - d)\,\frac{\sqrt[3]{Q}\,(\alpha - \psi)\,E}{(V^2/2g)\,C_D} - f\,D$$

[1] Arnstein, Karl, The Development of Large Commercial Rigid Airships, A.S.M.E. Trans., January-April 1928.

where K = Ratio of gas capacity to gross volume,
d = Relative density of lifting gas with reference to air,
Q = Volume in cubic feet,
α = Ratio of loads other than deadweight to gross load,
ψ = Ratio of service loads to gross load,
$(\alpha - \psi)$ = Ratio of combined pay load and fuel load to gross load,
f = Specific fuel consumption (lbs per ft lb of work produced),
D = Flight distance in feet,
$V^2/2g$ = Velocity head in feet,
C_D = Drag coefficient (per Vol.$^{2/3}$),
E = Efficiency of propulsion.

The general statement of the improvement of the transport efficiency with size may be demonstrated by an analysis of the factors involved. Of these factors d, D, and $V^2/2g$ are inherently independent of size and f and E are essentially so. K tends to increase slightly with size because the waste space devoted to corridors, quarters and insulation space may not need to be increased in proportion to the gas volume.

The useful load ratio α tends to increase as the ratio $(1 - \alpha)$ (the deadweight ratio) tends to decrease with increasing size, at least for modern airships[1]. The service load ratio ψ tends to decrease with size, since the crew need not necessarily be increased in number proportional to size, if at all, once a sufficient number of pilots and mechanics are provided. The decrease of the drag coefficient with size due to the influence of Reynolds number has already been discussed[2].

In addition to the benefits derived from these variables the term $\sqrt[3]{Q}$ represents a factor directly proportional to size.

The range of an airship can be somewhat increased by carrying a part of the initial load dynamically[3].

CHAPTER III

DYNAMIC LIFT

1. Flight with Dynamic Lift. When an airship is propelled at an angle of attack, lift forces are created in a similar manner as by the wing of an airplane. It is true that the airship's shape as a wing is very poor and its aspect ratio extremely small; but the size of the exposed surfaces is so great that trememdous aerodynamic force components at right angles to the flight path can be evoked. A part of this "dynamic lift" is produced by the hull of the airship proper (not embraced by the classical treatment of the flow about the ship neglecting friction and

[1] See footnote reference 1, p. 62.

[2] See Division I, Part, II 4.

[3] BLAKEMORE, T. L., Artificial Payload for Coming Commercial Airships, A.S.M.E. Buffalo, June 1932.

circulation) and the rest by the fins and control surfaces which in appearance resemble stubby airplane wings and function to some degree as such.

Dynamic lift (upward) is resorted to whenever the ship becomes "heavier than air" or "heavy" as it is called in airship parlance. This may happen in various ways accidentally, or it may be brought about deliberately. Precipitation in the form of rain, snow or ice on the surface of a large ship may result in an added load of several tons. Running into a layer of warmer air will make the ship heavy due to the lag of the gas inside the ship in assuming temperature equilibrium. These are usually temporary conditions. Loss of buoyant gas through accidental injury of gas cells or in consequence of climbing above pressure height with resultant valving causes a permanent loss of buoyancy. On the other hand an overload may be taken aboard deliberately in the form of mail, passengers, or airplanes. In all these cases the ship flies "heavy", up by the nose at an angle of pitch which must be the larger the less the airspeed. In a similar manner a ship may become "light" and must be flown down by the nose (at a negative angle of pitch) when for instance load or ballast is dropped, or when radiation "superheats" the gas and air inside the ship, or again when liquid fuel is consumed.

It is the practice to avoid these conditions in any marked degree. Well planned navigation will usually succeed in anticipating their causes and in meeting them at least part way. However, they may occur on short notice or they may be accepted deliberately, and in consequence a study of their aerodynamic aspects assumes a definite importance. Transport economy is, of course, reduced by the induced drag accompanying the production of dynamic lift and it is easily seen that, if the voyage is long enough, the fuel consumed to overcome this induced drag might outweigh the increase of useful load so carried. However, for ships burning liquid fuel, which gradually become lighter as fuel is consumed, there would be a distinct advantage in taking off heavy and accepting the drawback of the induced drag for a short while, until equilibrium is regained. For instance if the "overload" at any time t is L, the time rate of change of L is given by

$$\frac{dL}{dt} = -fP \tag{1.1}$$

where f is the fuel consumption per unit of power and time and P is the instantaneous value of the motor horsepower. Assuming as a first approximation that the induced drag is proportional to the square of the lift and that the equivalent "wing aspect ratio" of the ship, so to speak, can be expressed by some "equivalent span", s, which may differ somewhat from the ship's maximum diameter, then the additional power which must be spent in excess of that due to the normal drag of the ship in equilibrium would be: $P_1 = \dfrac{L^2 V}{q \pi s^2 E}$

where L = lift produced,
q = velocity head = $(1/2)\,\varrho\, V^2$,
s = equivalent span, *i. e.*, the span of a wing (assuming elliptical distribution of the lift) which would give rise to the same drag increase and which must be known from experimental data,
E = propeller and drive efficiency.

Then referring to II (5.1), equation (1.1) takes the form:

$$\frac{dL}{dt} = -\frac{fV}{E}\left(qA_D + \frac{L^2}{q\pi s^2}\right)$$

This can be readily integrated and furnishes the gain in range $S = \int V dt$ for L from L_0 to 0 in the form

$$S = \frac{Es}{f}\sqrt{\frac{\pi}{A_D}}\tan^{-1}\left(\frac{L_0}{qs\sqrt{\pi A_D}}\right)$$

or

$$S = \frac{E}{f}\sqrt{\frac{A'}{A_D}}\tan^{-1}\left(\frac{L_0}{q\sqrt{A'A_D}}\right)$$

where $A' = \pi s^2$ the "influence" area. Without the induced drag the range would have been $$S_0 = \frac{E}{f}\frac{L_0}{qA_D} \qquad (1.2)^1$$

The reduction of the range due to the induced drag can be expressed by the series $1 - \frac{1}{3}\frac{L^2}{q^2 A'A_D} + \frac{1}{5}\frac{L^4}{q^4(A'A_D)^2} - \ldots$ The reduction becomes a noticeable percentage only for large overloads. The problem of carrying these during the take-off is a serious one. However, by taking off with artificial superheat secured from a heat source ashore it is possible to start with considerable overload provided the route does not require a high ceiling at the beginning of the flight and before the gas has cooled down. The possibility of airships with heavy overloads taking off like airplanes has been demonstrated with small ships. However, the idea of a combination airship-airplane, which so frequently fascinates inventors, would seem to have only very limited possibilities, unless means may be found for providing a very large wing spread which moreover must admit of folding in close to the body of the ship. Otherwise problems of housing would be complicated beyond any conceivable advantage to be gained.

The dynamic lift of airships is limited by the power available in a way similar to that of airplanes. There is an optimum combination of angle of attack and speed for which the maximum load can be carried

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[1] This simple example omits many important considerations. In reality such factors as the increase of drag at a rate greater than as the square of the lift, the loss of propeller efficiency with increased thrust and with the angularity of the inflow, the arbitrariness of maintaining the same forward speed to be made good, and the possibility of cutting out part of the resistance of water recovery apparatus while not required, all have their practical influence.

with given power which is indicated by the maximum value of C_L^3/C_D^2 in a manner similar to that for the same problem with the airplane. This maximum carrying capacity would be attained at the angle of pitch for which the "induced" drag is three times the parasite drag[1], thus

$$C_L = \sqrt{3\pi\lambda C_D}$$

where λ = the equivalent "aspect ratio" which would have the same induced drag characteristic. Naturally C_L and C_D must be expressed with reference to the same area, for instance $Q^{2/3}$. In reality the maximum lift is much less than would appear from the λ valid for small angles of attack, because the validity of the parabolic induced drag law does not extend to sufficiently high angles, *i. e.*, λ is not constant.

Beyond the angle of pitch corresponding to the condition of maximum lift for given power looms the "stall"[2]. If the dynamic lift were proportional to the angle of attack up to the stalling angle, the latter would be

$$\alpha^* = \sqrt{\frac{C_D}{d\,C_L/d\,\alpha}}$$

and the stalling speed would be $\sqrt[3]{1/4}$ (63%) of the top speed attainable under the same power in the absence of dynamic lift.

In reality the power available drops with the reduction of speed. Rather, it is the engine torque which remains essentially unaltered. Insofar as the actual propeller thrust T available at any speed V is approached by a parabola, $T = T_0 - C_T\, v^2\, (\varrho/2)\, Q^{2/3}$, the power drop expresses itself in the form of an additional drag which makes the formula for the stalling angle α^* (in radians)

$$C_L = (C_D + C_T)\, d\,C_L\, d\,\alpha$$

or

$$\alpha^* = \sqrt{\frac{C_D + C_T}{d\,C_L/d\,\alpha}}$$

and the stalling speed in level flight would become $\sqrt{1/2} = 71\%$ of the top speed attained with the same engine throttle position in the absence of dynamic lift.

Similar to the airplane, the approach to the stall is essentially governed by the aerodynamic attack, and associated with a definite stalling angle of attack (pitch). However the overload that can be carried at this angle of attack depends on the slope ε of the ship's path. In a climb, less overload can be carried. The difference is in first approximation.

$$\frac{\triangle L}{L} = -\frac{\varepsilon}{(\alpha + \varepsilon) + (C_D + C_T)/C_L}$$

When a "heavy" ship, not yet heavy enough in level flight to approach a stall, is made to climb, it may prematurely stall; similarly a light ship,

[1] KLEMPERER, W., Zeitschr. f. Flugtechnik u. Motorl., p, 78, 1922.

[2] KLEMPERER, W., Stalling of Airships, Journal of the Aeronautical Sciences, p. 113, July 1934.

when it is made to descend. On the other hand when a heavy ship is allowed to descend or a light ship allowed to rise, an imminent stall is (temporarily) averted.

It is interesting to note that a heavy ship carrying its overload dynamically, when actually nosed down, ($-\varepsilon/\alpha > 1$) can "glide" and thereby pick up speed exactly as an airplane can. When light, and flying with dynamic down-dip it will "glide up" and pick up speed in so doing when permitted to nose up, unless the added drag of the elevator predominates.

On ships of conventional design, dynamic lift is associated with unwelcome stresses and demands upon controllability. This is due to the manner in which dynamic lift distributes itself unevenly over the length of the ship—a large part at the bow and a considerable amount at the stern, the two not necessarily in equilibrium about the center of buoyancy. In order to appreciate this it is convenient to consider the dynamic lifts of the hull and of the empennage separately as well as their mutual interference.

2. Dynamic Lift of the Hull. In a non-viscous fluid an elongated body (of volume Q) such as an airship hull moving at an acute angle of attack (α) between its longitudinal axis and its path would experience no force such as dynamic lift, but only an unstable deviating moment $(k_2 - k_1)\, Q \cdot q \sin 2\alpha$ [1] where $(k_2 - k_1)$ denotes the difference of the virtual mass coefficients for the transverse and axial flow components and q the velocity head. This moment tends to increase the angle of attack and is largely concentrated on the bow and stern parts of the ship, the components acting there in opposite directions. For the detailed distribution of these transverse forces along the axis of the ship a first approximation is given in Division Q [equations (8.6), (8.7)]

$$b = (k_2 - k_1)\, \frac{dA}{dx} \sin 2\alpha \qquad (2.1)^{2}$$

In wind tunnel tests the pitching moment weighed on the balances appears from 15 to 30 per cent smaller than this, and much less concentrated at the nose, especially for ships having a blunt bow. This is due to the fact that where the taper is pronounced, the equivalence between adjacent length elements and cylindrical slices acting upon the flow independent of each other is no longer valid. For an ellipsoid of

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[1] See Division C III 4; and Q 8.
MUNK, M., U.S. N.A.C.A. Report No. 184, 1923.

[2] We may for present convenience call b the "transverse force breadth" because it would indicate the local breadth or width of a pail filled with water (or any other liquid) to the height equivalent to the velocity head q, in order to produce the same load distribution over the ship considered as a beam, as the actual air forces would. The total transverse force or the dynamic lift would then be $L = q\int_{\text{bow}}^{\text{stern}} b\, dx$.

revolution for which the exact pressure distribution is known[1] the integration around any conical slice at a station where the local taper angle between the tangent of the generatrix and the axis is τ and the local radius or ordinate to the generatrix is r, has the value

$$b = r\,\pi\,sin\,2\,\alpha\,sin\,2\,\tau$$

This is equivalent to the substitution of the variable $cos^2\,\tau$ for the constant $k_2 - k_1$ and even for ships whose bow is somewhat blunter than an ellipsoid gives a much better approximation, as pressure distribution measurements both on wind tunnel models and on ships in flight have shown[2].

A closer investigation and digest of wind tunnel results may require the introduction of corrections for the influences of the finite wind stream dimensions in the laboratory. In the open jet tunnel, at the jet boundary, the pressure influence due to the model is offset, so that there, actual velocity increments have faded out. In the closed tunnel there can be no radial velocity component at the tunnel walls in spite of the presence of the model, which at such distance in a free stream would, in most cases, still give rise to such a component.

For various reasons some designers prefer to choose a hull shape which is expressed by a relatively simple formula for the cross sectional area (S) in terms of the abscissa station (x) rather than for the ordinate r of the generatrix. For contours of this class it is sometimes convenient to express the transverse force breadth in terms of S and $S' = dS/dx$.

This is done by: $b = \dfrac{sin\,2\,\alpha}{1/S' + S'/4\,\pi\,S} = \dfrac{S'\,sin\,2\,\alpha}{1 + S'^2/4\,\pi\,S}$

In order to accurately determine the theoretical distribution of the transverse force breadth for any given shape of hull, recourse may be had to methods given by v. Kármán[3] or Kaplan[4] and by Lotz[5] of which the principal features are as follows:

A system of sources is determined and so distributed along the ship's axis or its hull surface as to represent the shape for the axial component

[1] See Division C VII 5.

LAMB, H., Hydrodynamics, 5th Edition, p. 132; UPSON, R. H., and KLIKOFF, W. A., U.S. N.A.C.A. Report No. 405.

[2] U.S. N.A.C.A. Technical Reports Nos. 223, 324, 405 and 443; JONES, R., Br. A.R.C. R. and M. 1061, 1927; BURGESS, C. P., Airship Design, New York 1927; KLEMPERER, W., Windkanalversuche an einem Zeppelin-Luftschiffmodell, Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen 1932.

[3] KÁRMÁN, TH. v., Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Heft 6, p. 1—17, 1927.

[4] KAPLAN, C., Potential Flow About Elongated Bodies of Revolution, U.S. N.A.C.A. Technical Report No. 516.

[5] LOTZ, I., U.S. N.A.C.A. Technical Memorandum No. 675, Calculation of Potential Flow Past Airship Bodies in Yaw. From Ingenieur-Archiv, Vol. II, 1931.

flow and upon these is superimposed a system of doublets in such manner as to maintain the hull form against the cross component flow.

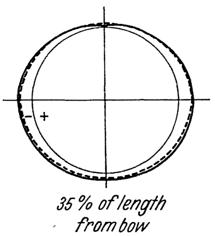

Fig. 13a. Pressure distribution (circumferential) on airship model at pitch angle of 12°. Full lines, calculated values. Dotted lines, measured values.

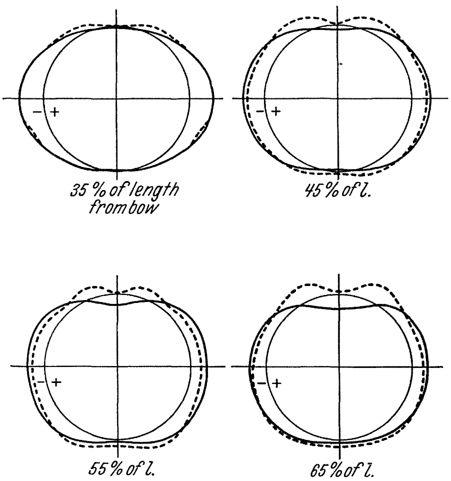

Fig. 13b. Pressure distribution (circumferential) on airship model at pitch angle of 18°. Full lines, calculated values. Dotted lines, measured values.

The pressures may then be computed and integrated around successive slices or zones from station to station, and the longitudinal distribution of transverse force thus determined.

For ships that are not very slender, the pitching moment is slightly smaller than the integral of the moments of the transverse forces, *viz.*, $q\int b\,x\,dx$ because the longitudinal components of the pressure contribute a restoring moment. Theoretically this reduction should amount to $-2\pi q r^2 \sin 2\alpha \sin^2 \tau$ so that the total moment at any station is reduced in the proportion $1:(1 - r/x \tan\tau)$. Again expressed in terms of cross section area only, this zonal moment correction is

$$dM = \left(\frac{2\pi x - S'}{2\pi/S' + S'/2S}\right) q \sin 2\alpha \, dx$$

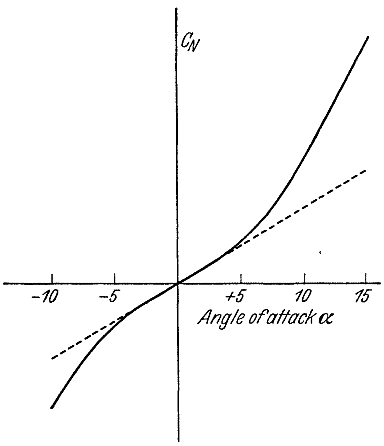

Fig. 14. Diagram showing typical increase of dynamic lift with angle of attack.

However, in reality, in model size as well as full size, the superposition of axial and transverse potential flows gives a faithful picture only in the front windward and midship region as can be readily visualized.

When flying at an angle of pitch one longitudinal will be to the leeward; with a heavy ship the top one, with a light ship the bottom one. In this region and with actual fluids, the stream-lines will be unable to close in behind, and in consequence the pressures will depart from those for a purely potential flow. The skin friction imparts vorticity to the flow and the trailing vortices form the counter part of a circulation which builds up mainly aft of the master section. Pressure distribution experiments on models show that in the rear part of the hull the negative forces (due to defect of pressure) fall considerably short of theoretical values. Figures 13a and 13b show a comparison between calculated pressure distributions and those measured on a wind tunnel model[1]. This pressure deficiency is one of the causes of the difference between the theoretical moment of the hull and that weighed on the wind tunnel balances. It accompanies the development of a lift force. Th. v. Kármán[2] has begun a theoretical treatment of this hull lift adducing plausible assumptions concerning the shedding of circulation.

For higher angles, both pressure distribution and model balance measurements indicate a quicker increase than in the ratio of the sine of the angle of pitch (see Fig. 14). It would therefore appear that the phenomenon of the detachment of vortices on the lee side of an inclined streamlined body is controlled by a sensitive mechanism and that the area subject to it gradually expands upstream, both forward and circumferentially as the angle of attack is increased.

It is reasonable to expect that more insight into the mechanism of the lift of the hull or of the deviation of the pressure distribution from potential flow may be gained from a study of the vorticity in the wake of the acutely attacked hull. An elaborate study of this nature has been begun by Harrington[3]. A survey of the velocity vector field in the wake reveals the presence of two vortex systems trailing downstream through the wake and showing many traits in common with the tip vortices of wings. Figure 15 is a typical example of the results of Harrington's measurements. It is a picture of the transverse velocity components in a section of the wake 20 cm. behind the tail end of an ellipsoid of 99 cm. length and 16.5 cm. diameter attacked at an angle of 21.5^0 at an air speed of 22.3 m./sec.

To what degree the analogy of model and full size laws of hull lift are obscured by scale effect and turbulence is a question needing still further study.

[1] Freeman, H., Pressure Distribution Measurements on the Hull and Fins of a 1/40 Scale Model of the U.S.S. *Akron,* U.S. N.A.C.A. Technical Report No. 443.

[2] Kármán, Th. v., Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Heft 6, p. 1—17, 1927.

[3] Harrington, R. P., An Attack on the Origin of Lift of an Elongated Body, the Daniel Guggenheim Airship Institute, Akron, Ohio, Publication No. 2, 1935.

At very large angles of attack, wind tunnel tests on conventional airship model sizes are likely to run into scale effect troubles as indicated by experiments on round and elliptical cylinders of such width and ellipticity as would correspond to the slant section of an airship parallel to the plane of flow at very high incidence.

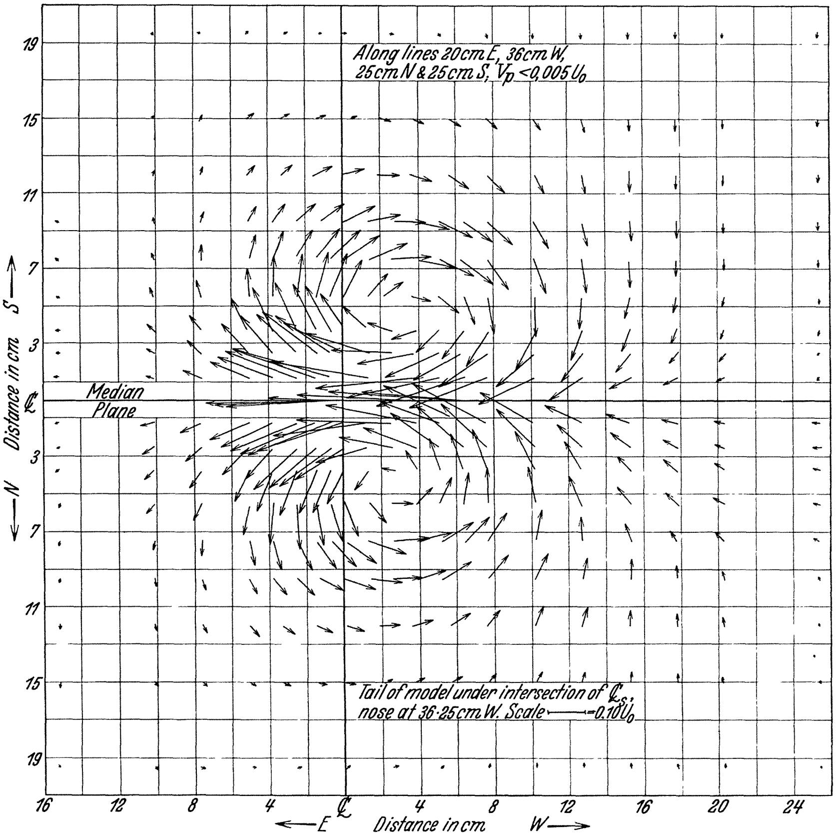

Fig. 15. Velocity field in the wake of an airship.
Induced velocity V_p in plane normal to U_0 located 20 cm. Behind ellipsoid at $\alpha = 21.5^0$

$V_p^2 = v^2 + w^2$ Length of model = 99 cm.
$U_0 = 22.3$ m./s. Fineness Ratio = 6
Scale: |———| = 0.10 U_0.

The dynamic lift characteristics are also somewhat influenced by details of the form—whether round or polygonal or heart or pear shaped; likewise by unsymmetrical arrangements of form such as a pronounced keel structure or other features on the under side of the ship. In such cases the lift may not be zero for zero angle of attack.

The drag D of the bare hull (and indeed also of the ship with empennage) increases with the angle of pitch α, very approximately as the product of lift and *tan* α or in other words the axial component T is, within wide angle limits, unaffected and the action of oblique attack is essentially the evocation of a force N normal to the ship's axis[1].

$$N = L \cos\alpha + D \sin\alpha$$
$$T = D \cos\alpha - L \sin\alpha$$

3. Lift Due to Fins. In order to neutralize the inherent directional instability of the elongated streamlined hull, airships are equipped with tail empennages in manner similar to an arrow. The action of these fins can, in first approximation, be approached by the airplane wing theory. They are airfoils, usually of either flat or biconvex symmetrical airfoil section, mostly tapered toward the rim. Their aerodynamic properties are somewhat difficult to compute and predict in terms of the classical wing theory because of five important secondary influences.

(1) Their shape is usually, for engineering reasons, long, rather than wide, so that in terms of wing theory their aspect ratio is extraordinarily low. Therefore the spill over the edge becomes an important rather than negligible factor. The whole fin is a wing tip rather than a wing.

(2) The part of the hull between opposite fins is usually so large that its size and shape have an important influence upon the flow about and the forces exerted upon the fins.

(3) The angle of attack of the fins is influenced in marked degree by the induced "downwash" which trails off the preceding parts of the ship's hull. The magnitude of this downwash will further vary over the span of the fin.

(4) The presence of fins when the ship is under an angle of attack influences again the pressures on the rear part of the hull, not only between and to the rear of the fins, but also considerably forward of them.

(5) The roots of the fins are in a region of diminished velocity within the boundary layer of the hull.

It is, of course, conceivable to develop a specific method for introducing all these influences properly into a fin theory. For instance the presence of the hull between the fins can be accounted for by the substitution of a fictitious system of sources and sinks or doublets in its place, as is done in v. Kármán's method for dealing with monoplane wings rooted on a fuselage[2]. However, if accurate representation of the actual facts is attempted, any such procedure suffers from the well known difficulties, attending the necessity of preserving the actual fore and aft distribution of lift, and, for the present, the problematic points regarding the generation of lift by the stern of the hull and its attendant downwash.

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[1] NAATZ, H., Neuere Forschungen im Luftschiffbau, Jahrbuch der Wiss. Ges. f. Luftfahrt, 1923; ZAHM, A. F., SMITH, R. H., and LOUDEN, F. A., U.S. N.A.C.A. Technical Report No. 215, 1925. Also U.S. N.A.C.A. Technical Reports Nos. 394 and 432.

[2] LENNERTZ, J., Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Vol. 8, pp. 1—30, 1928.

Many efforts have therefore been made to secure reliable experimental data. Full size experiments are very difficult and very expensive and have been limited largely to pressure distribution measurements. Many of the results are of limited value because of the extreme difficulties of measuring simultaneously the pressures at a sufficiently large number of distant orifice points while the aerodynamic condition of the ship is steady, though departing in marked degree from the simple condition of straight flight equilibrium. There is the further requirement that all parameters of the flight condition must be accurately determined. On the other hand most of the model tests suffer from uncertainty regarding the possible scale "effect". The larger the Reynolds number of the experiment the more valuable the results may appear. The least angular irregularity of flow in a wind tunnel when varying along the length of the experimental section may cause a first order error in the pitching moment measured on a long airship model, whereas with short airplane models, the corresponding error may appear negligibly small. In cases of large models, corrections for tunnel or jet wall influences upon induced drag and effective angle of attack as well as downwash may become in order, as with airplane models. In a closed tunnel the determination of the effective wind speed in the tunnel, as it is increasingly obstructed at larger angles of attack, deserves attention.

Practical experience has shown that a wide variety of fin forms and arrangements may be reasonably satisfactory and there are evidently a great number of variable parameters which may enter into any detailed appreciation of the actual aerodynamic characteristics of an airship empennage. A first approximation to the lift on a fin may be taken on the basis of the conventional airplane wing theory, [see Division E IV (2.15)].

$$L = \frac{2\,\pi\, q\, S\, \alpha}{1 + 2\, S/b^2}$$

where S is the fin area, b its (effective) span, q the velocity head and α the angle of attack. According to the more trustworthy among model tests in wind tunnels (and probably in a similar manner full scale), the actual stabilizing empennage force, as indicated by the difference of the lift with and without fins, is of the theoretical order of magnitude for very small angles of attack only, whereas for angles of practical interest and importance the force is much greater. Much of the surplus is of course borne by the part of the hull between the roots of the fins and even ahead of them. This share can be measured by pressure distribution experiments[1], an example of which is presented in Fig. 16. The

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[1] FREEMAN, H., Pressure Distribution Measurements on the Hull and Fins of a 1/40 Scale Model of the U.S.S. *Akron*, U.S. N.A.C.A. Technical Report No. 443; KLEMPERER, W., Windkanalversuche an einem Zeppelin-Luftschiffmodell, Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Vol. 12.

center of this stabilizing force is therefore not actually at the theoretical quarter chord point (a point not easily defined with fins whose leading edge gradually merges into the rim), but it may be farther forward when computed by dividing the difference in the stabilizing moments with and without fins by the difference of lift with and without fins. For the airship model of the U.S.S. Los Angeles to which Fig. 16 referred, this leverage is of the order of 78 m. from the center of buoyancy whereas the fins extend about 75 m. to 97-$^1/_2$ m. aft of this point. That neither the stabilizing force nor the stabilizing moment due to the fin appear to be even approximately proportional to the angle of attack may perhaps be regarded as an indication that this part of the hull which, between the fins, has the form of a well rounded body and thus does not offer a definite trailing edge, begins to build up its own contribution to the force only when higher angles of attack are reached.

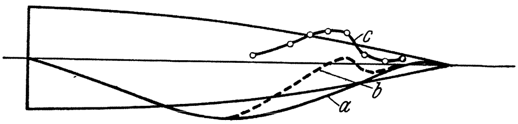

Fig. 16. Distribution of pressure about the empennage. (a) Zonal force integrated from typical pressure distribution without empennage. (b) Zonal force integrated from typical pressure distribution with empennage. (c) Difference due to empennage.

Of the innumerable varieties of fin forms proposed or used on airships, only the major features can be here indicated. For details, reference must be made to the general literature of this subject[1]. Flat fins produce slightly greater forces than fins built up of thick sections tapering from root to tip. The latter, however, offer structural and engineering advantages and are under certain circumstances preferred. Larger aspect ratio of a fin of otherwise fixed shape and location increases its action per unit area[2]. Changing the shape mainly influences the location of the center of action and the pressure distribution. More pronounced leading edge and receding rim moves the center of action forward; a more slanting leading edge gradually flaring into the rim moves it aft. The pressure distribution[3] is similar to that of wing tips. Most of the force is concentrated along the rim. Pressure distribution near the rim is influenced by an angle of yaw simultaneously present with an angle of pitch. Some

[1] Br. A.R.C. R. and M. 714, 799, 802, 1168; U.S. N.A.C.A. Technical Reports Nos. 215, 394, 432; Naatz, H., Neuere Forschungen im Luftschiffbau, Jahrbuch der Wiss. Ges. f. Luftfahrt 1923; Rizzo, Frank, A Study of Static Stability of Airships, U.S. N.A.C.A. Technical Note No. 204; Richmond, V. C., Airship Research and Experiment, Journal of the Royal Aeronautical Society, October 1926.

[2] This is not always an advantage. For slender ships it may be preferable, both structurally and from a weight saving standpoint, to accept a larger fin area if the forces are better distributed over a greater root length.

[3] Jones, R., and Bell, H., Br. A.R.C. R. and M. 1169, 1928; Br. A.R.C. R. and M. 808, 811; U.S. N.A.C.A. Technical Reports Nos. 223, 324, 443; Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Heft 12, 1932.

sample pressure distributions are represented in Fig. 17. For further details reference may be made to the publications here noted. Moving a given set of fins farther forward or aft will increase or reduce respectively the fin forces but within certain limits may scarcely change the stabilizing moment. The most conventional form of empennage is an essentially symmetrical cruciform set of two pairs of fins, one upper and lower in the keel plane for directional stabilization and the other, port and starboard, for stabilization against pitching. Many other arrangements such as box frames, ring shapes[1] or more than four radial fins[2] have also been tried. Problems of slant attack and mutual shielding from tip-spilling as well as biplane influences come into question in connection with these arrangements.

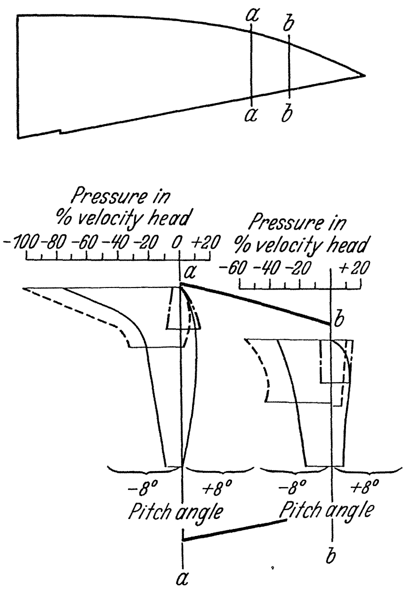

Fig. 17. Distribution of pressure on fin. Dotted line, + 6° angle of yaw. Full line, 0° angle of yaw. Dot-dash line, — 6° angle of yaw.

It is not necessary (nor held desirable by many) to make the fins so large that the moment about the C.B.[3] of the empennaged ship shall be stable for all angles or even for moderate angles of attack, without the aid of the movable control surfaces or even with their aid. Floating without air speed, there is usually a certain small aerostatic stability due to a positive metacentric height. However the elasticity of bulkheads and the floating of lower gas cell levels permit some surging of gas and reduce the metacentric height to less than the value indicated by the level difference of the centers of buoyancy and gravity. As air speed is acquired, an additional (dynamic) metacentric height comes into play, subtractive when the aerodynamical moment is unstable. This dynamic term can, as H. R. Liebert has proposed, be expressed in terms of the velocity height $h = v^2/2g$, *viz.*, $H^* = 2\,(k_2 - k_1)\,h\,(m/m_0)$ where m/m_0 is the ratio of the aerodynamic

[1] DÜRR, L., 25 Jahre Zeppelin-Luftschiffbau, V.D.I.-Verlag, 1924; KÁRMÁN, TH. v., Wind Tunnel Tests on a 1/75 Model of Goodyear Zeppelin Airship ZRS4 with Normal and Ring Tail Surfaces, Report No. 105 of the Guggenheim Aeronautics Laboratory, California Institute of Technology.

[2] UPSON, R., Metalclad Rigid Airship Development, Journal of the Society of Automotive Engineers, February 1926; ABBOTT, I., U.S. N.A.C.A. Technical Report No. 451.

[3] Center of Buoyancy.

moments of the actual empennaged ship to the theoretical moment of the bare hull.

Finally there is always the expedient of shifting ballast so that equilibrium can be established. Just how much fin area is desirable for flying with dynamic lift is therefore largely dependent on navigational problems and on the mechanical and control apparatus provided aboard. It may be mentioned as significant, however, that, with large rigid airships, the first sign of growing heavier usually appears as a tendency to become tail-heavy so that the ship must fly nose up in order to maintain altitude, but with the need of "down elevator" to hold the ship in this attitude. With the ship growing light, corresponding indications, reverse in character, appear. In small nonrigid airships this phenomenon is rarely observed.

4. Dynamic Lift Experiments. The experimental determination of the dynamic lift characteristics[1] of the complete ship, full size, is a very delicate problem. Aside from the difficulty of correctly measuring and averaging the observed angles of pitch and with airspeeds continuously fluctuating as they are, the exact amount of lightness or heaviness is a very elusive quantity. Theoretically the test program is simple, as follows.

(a) Weigh off to make sure that buoyancy equilibrium is established and then either valve a measured quantity of gas or better, drop a measured amount of ballast and determine a set of corresponding pairs of values of air speed and angle of pitch for which the ship will neither rise nor fall;

or otherwise:

(b) First valve a suitable amount of gas and then go through the above measurements and at last see how much ballast must be released in order to reestablish equilibrium. The latter method, especially when valving automatically by deliberately overclimbing the pressure height provides a check when the air density at the ceiling is observed.

An accurate record of elevator angles and of the ship's inclination oscillations must be kept during the experiments because the elevator contributes a considerable amount to the dynamic lift. Corrections required for variations of temperatures inside and outside, for fuel consumed and weights shifted during the time of the tests, render the procedure less simple. This and the reluctance of deliberately putting the ship through the ordeal are the main reasons for the scantiness of data available. Some are compiled in Table 2[2]. When high dynamic

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[1] Regarding model experimentation:

Frazer, R. A., Bateman, H., Measurement of Normal Force and Pitching Moment of Rigid Airship R 33, Br. A.R.C. R. and M. 815; Zahm, A. F., Smith, R. M., and Louden, F. A., U.S. N.A.C.A. Technical Report No. 215, 1925; Jones, R., Br. A.R.C. R. and M. 1168, 1927.

[2] Burgess, C. P., Airship Design, New York 1927.

TABLE 2. *Some Dynamic Lift Experiments, Abstracted.*

Experiment	Dynamic Lift kg.	Pitch Angle Degrees	Elevator Angle Degrees	Air-Speed m./sec.	Temperature °C	Barometric Pressure mm. Hg	Altitude by aneroid m.
British R-33,	1100	— .8	8 up	23.	8.1		760
of 55000 m.³	1100	— .2	7 up	24.3	8.3		780
nom. capacity	1000	— 1.5	16 up	14.5	8.0		740
Trial Flight	1150	— 2.8	9 up	15.5	11.1		780
on May 23, 1921,	1100	— 1.7	6 up	22.9	11.7		765
from Br. A.R.C.							
R. and M. 815	2050	— 3.1	8 up	22.4	11.7		780
	1950	— 9.5	3 down	14.7	12.9		785
	1950	— 8.1	0	14.7	12.9		760
	1900	— 4.1	6 up	22.9	13.8		795
	2700	— 6.2	6 up	21.2	14.0		815
	2800	—10.0	3 down	16.2	14.0		790
U.S.S. Los-Angeles (LZ 126)	1700	~ +2	2 up	25	11.5	685	895
of 70,000 m.³ nom. capacity,	2600	+3 to 4	7 up	25	10.1	670	1030
Trial Flight overLake Constance,	3300	+4 to 5.5	11 up	24	9.3	664	1150
Germany, on Sept. 11, 1924	4000	+6 to 7	12 up	22	8.7	652.5	1300
	4800	+8 to 10	8 to 10 up	20	8.1	645.5	1400

loading occurs unexpectedly in practical navigation, the conditions are usually unfavorable for scientific investigations with neither time nor personnel available.

Attempts have been made to develop instruments to indicate currently the magnitude of the dynamic lift of a ship. Such instruments can be based upon the differences of pressures or airstream velocities prevailing on strategic stations above and below the ship's bow. A calibration must be obtained either from dynamic lift tests or from model tests.

Airships are known to have carried huge loads dynamically on various occasions. Thus the Graf Zeppelin was drenched by a torrential rain upon her start from Brazil in 1930. The rain-load thus carried was estimated to be of the order of five tons. The U.S.S. *Akron* once went through severe winter storms and collected 18,000 pounds of ice on her hull. She continued on her mission which lasted fifty-six more flight hours[1]. The U.S.S. *Macon* on part of a transcontinental trip carried 30,000 pounds by dynamic lift.

[1] ARNSTEIN, K., Über einige Luftschiffprobleme, Zeitschr. f. Flugtechnik u. Motorl., Heft 1, 1933.

It is not without interest that, at a given speed, small ships can carry a larger dynamic lift in proportion to their gross aerostatic lift because the former is proportional to the square of the linear dimensions and the latter to the cube. However, larger ships are usually faster, and loads due to rain and sleet are also proportional to the square of the linear dimensions, so that the proportion does not vary very widely.

CHAPTER IV

MANEUVERING

1. Curvilinear Flight. In the horizontal plane there never occurs a condition similar to that in the vertical plane in steady flight under an angle of pitch. An airship cannot proceed straight at an angle of yaw. It differs in this respect from the airplane which can sideslip. On the airship, whose lift is always vertical, there would be nothing to balance the lateral forces arising from flight at an angle of yaw; the ship would turn. Curved flight in a vertical plane is also theoretically possible, but in practice is confined to short arcs, as airships do not loop the loop.

In steady curved flight the centrifugal force must be balanced by aerodynamic forces acting inwards. Consequently, airships turn under an angle of yaw and are attacked from the outside of their turning circles. The condition of attack in a turn is as depicted in Fig. 18. Due to the considerable length of airships, even in comparison with the radius of curvature in a turn, the angle of yaw varies noticeably from stem to stern. In the region of the empennage it is almost twice as large as it is amidships. The large yaw angle at the fins is the essential factor making the empennage so powerful in curved flight.

The turning performance of airships has repeatedly been measured in flight trials[1]. In such tests the helm is put over to a measured angle and the ship allowed to turn. It will first react by swinging the tail outward to build up an angle of yaw and start along a spiral course which may, by the time the flight direction has changed something like 90°, attain approximately the terminal turning radius. The radius can be measured by any of the following methods: In the simplest method the airspeed is measured by a reliable instrument (itself unaffected by turning and yawing) and the angular velocity can be measured either by clocking the times when the ship in a sighting device appears parallel to

[1] Br. A.R.C. R. and M. 537, 631, 668, 675, 713, 716, 749, 779, 780, 782, 811, 812; U.S. N.A.C.A. Technical Reports Nos. 208, 333; FAIRBANKS, K. J., Pressure Distribution on the Nose of an Airship in Circling Flight, U.S. N.A.C.A. Technical Note No. 224, 1925; VON OREL, E., Bildmessung und Luftbildmeßwesen, No. 2, 1929; LACMANN, O., and BLOCK, W., D.V.L. Report, Photogrammetrische Lage- und Geschwindigkeitsbestimmung des Luftschiffs LZ 127, etc., Zeitschr. f. Flugtechnik u. Motorl., Vol. 11, 1930; WIEN-HARMS, Handbuch der Experimental-Physik, Vol. IV, 3, 1930.

identified ground objects of known azimuth, or by timing a compass swing (corrections should be made for vertical magnetic field component error and lag). In more elaborate methods, in which the yaw angles or the station of zero yaw are also obtained, the path of the ship and its relative attitude are photographed, either in a camera obscura or by stereophotography from two synchronized ground stations or by photographing the ground from aboard. In all these cases allowance must be made for wind drift which distorts the flight path into a cycloid. Gusts enhance the difficulties of measurements. The local yaw angle at any station on the ship has also been measured by a suspended yaw meter, which, however, unless far enough away from the ship's hull and fins, may be influenced by the flow around them. It was early discovered that the region of zero yaw is usually near the nose and that it does not travel perceptibly for different curvatures. Pressure distribution experiments in curved flight show fair symmetry about the nose, the presence of lateral forces amidships, and significant pressure differences on the fins.

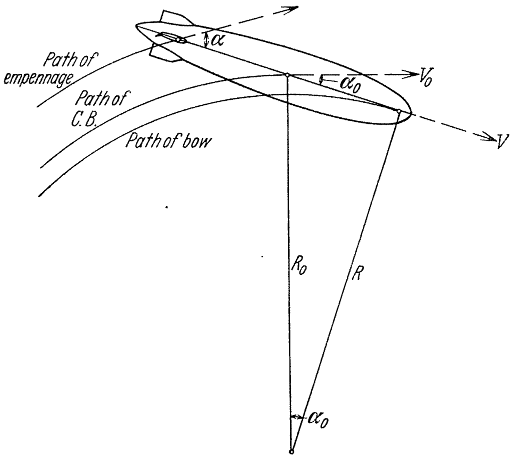

Fig. 18. Conditions of air attack on an airship in a turn.

As a rule an airship is considered satisfactorily maneuverable when its rudder will enable it to turn on a radius of approximately four times the ship's length.

The aerodynamic forces prevailing in curvilinear flight and the distribution of these forces are discussed in Division Q, where references are quoted. The centrifugal force arising in the turn is slightly larger than that of the mechanical mass of the ship. The increment is essentially due to the longitudinal virtual mass[1], and to the momentum of the boundary layer air. The distribution of forces can, in first approximation for the hull, be assumed as according to the potential flow theory of rotation and angle of attack combined[2]. However, for any more accurate appreciation of the part played by the generation of circulation along

[1] $(k_1 \cos^2 \alpha + k_2 \sin^2 \alpha)\, \varrho\, Q\, V^2/R$, according to MAX MUNK, U.S. N.A.C.A. Technical Report No. 184 and Technical Note No. 196; also P. FRANK and TH. v. KÁRMÁN in Riemann-Weber's Differential and Integralgleichungen der Physik, Vieweg, 1927; British writers in various A.R.C. R. and M. have proposed different expressions.

[2] Br. A.R.C. R. and M. 1061; U.S. N.A.C.A. Technical Report No. 405.

the ship's stern, its downwash upon the empennage and the actual empennage forces to be derived from any particular design, recourse must be had to experimental data.

Apparently the most direct imitation of curved flight on model scale consists of carrying a model around a circular path on a whirling arm. This experiment was performed in Italy in water (where however, questions of wave formation arise) and in England in 1925 on an ellipsoid (prolate spheroid) in air. The latter series of tests[1], although confined to a velocity of 12 m. per sec. and made on rather small scale (diameter of ellipsoid = 6″ and length = 24″) and not with the same coordination of yaw and curvature as that which characterizes a turning airship, are of fundamental interest. While it must be realized that these delicate measurements, perhaps obscured by the influence of the air dragged around in the room by the whirling arm, can claim only a limited accuracy, yet they seem to clearly indicate the presence of a resultant aerodynamic centripetal force due to yaw and another force due to curvature at zero yaw.

Fig. 19. Cross-sectional shape of U.S.S. Los Angeles.

Th. Troller has built a new whirling arm laboratory at the Guggenheim Airship Institute at Akron, Ohio[2]. With this modern equipment it should be possible to attain Reynolds numbers much higher than were ever attained before. This laboratory has begun large scale experiments exploring into the conditions of curvilinear flight of airships.

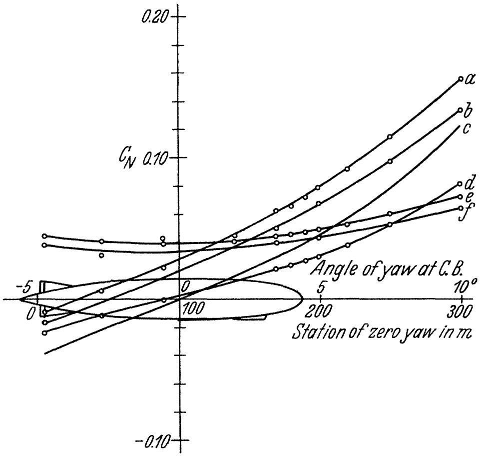

Fig. 20. Curves showing lateral force coefficients of curved model. a. Normal force with fins. b. Normal force with fins and screen. c. Normal force straight model with fins. d. Normal force without fins. e. Drag without screen. f. Drag with screen.

In 1924, at the Luftschiffbau-Zeppelin Werft, W. Klemperer tested a model of the airship LZ-126 (the U.S.S. Los Angeles) constructed with a curved axis[3] in an attempt to thus reproduce steady turn condi-

[1] Jones, R., Br. A.R.C. R. and M. 1061, December 1925.

[2] Troller, T. H., The New Whirling Arm, Journal of Aeronautical Sciences, 1, 4, 1934; also Daniel Guggenheim Airship Institute, Publication No. 3, 1935.

[3] A British experiment on a curved model which, however, was made with a different purpose, is reported in Br. A.R.C. R. and M. 104, 1913.

tions in the wind tunnel. These experiments are quoted here in some detail because the results have not as yet been published elsewhere. The model was built to a scale 1:75 and the axis was curved to a radius of 15.6 m. representing a turning radius of 1168 m. in flight, which would correspond to a moderate rudder maneuver. The cross section is a regular polygon except at the bottom, where the 11th chords on both sides are extended until they meet, thus giving a form of keel, as shown in Fig. 19. The test results, as obtained in the open jet wind tunnel in Friedrichshafen at wind speeds ranging from 25 to 46 m. per sec., with and without fins are given in Figs. 20 to 23. Figure 20 shows the lateral force coefficients (forces per unit velocity head and volume $^{2/3}$) plotted against the angle of yaw at center of buoyancy and, by reference to the ship's contour, also against the position of the station of zero yaw to which each angle of model incidence corresponded. Figure 21 is a similar plot of the yawing moment coefficients (moments referred to unit velocity head and unit volume). Figure 22 depicts the apparent leverage of the fins expressed as the quotient of moment and force differences with and without fins. Figure 23 plots the "lateral force breadth" in terms of unit $\sqrt[3]{\text{volume}}$ as it is found distributed along the bare hull by zonal integration of many pressure measurements for a typical location of the station of zero yaw, *viz.*, at frame No. 180, *i. e.* 7.5 m. aft of the ship's nose. As will be seen, the lateral force and unstable moment of the bare hull due to curvature only, vary moderately from the corresponding influences due to straight yaw. Part of the variation may, of course, be ascribed to the keel, and the hump in the lateral force breadth to the control car. The aerodynamic influence of curvature upon the empennage is quite strong, even stronger than in proportion to the variation of local angle of yaw at the fins.

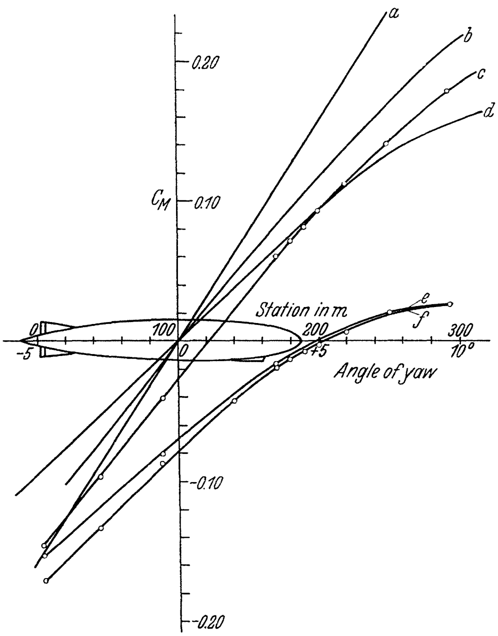

Fig. 21. Curves showing lateral moment coefficients of curved model. a. Theoretical. b. Straight model bare hull. c. Curved model with fins. d. Straight model with fins. e. Curved model with fins and with screen. f. Curved model with fins and without screen.

The analogy between the curved model and circular flight is, of course, somewhat strained. In the model the lengths of the surfaces exposed to the air stream are slightly different on the two sides. In full scale, the air in the actual boundary layer is subjected to centrifugal force and furthermore, the tail swings on a larger radius than the bow and is thus exposed to a higher air speed.

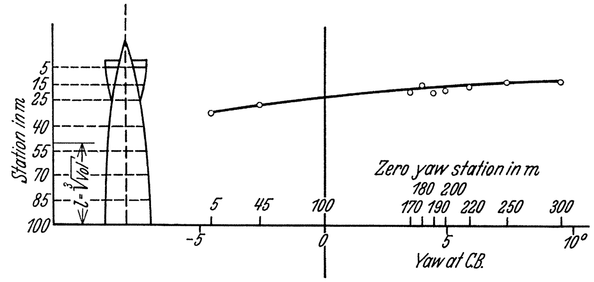

Fig. 22. Curve showing locus of line of action of fins on curved model.

The latter effect can be roughly allowed for by increasing and correcting the observed empennage forces in proportion to the calculated velocity heads. In the Luftschiffbau Zeppelin experiments, an attempt

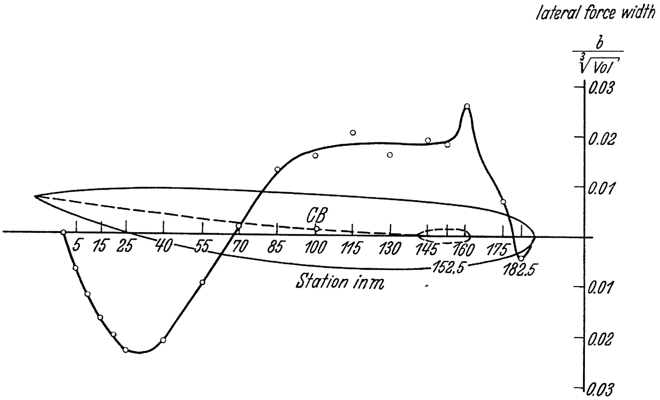

Fig. 23. Curve showing lateral force width [see III (2.1)]. Example for station 180 m at zero yaw.

was also made to imitate this condition by superimposing a velocity gradient upon the wind tunnel jet by means of a wire screen. The dotted curves in Fig. 22 and 23 refer to this condition. The velocity gradient produced in the experiment was, however, less than desired. The presence of the wire screen may also have changed the turbulence in the airstream[1].

[1] GOURJIENKO, Method of Curved Models and its Application to the Study of Curvilinear Flight of Airships, Trans. of the Central Aero-Hydrodynamic Institute, Moscow, 1934.

In 1930—1932, R. H. Smith independently studied the aerodynamics of curvilinear flight by means of curved models at the Massachusetts Institute of Technology, Cambridge, Mass. He tested several models of the U.S.S. Shenandoah with regular polygonal cross section and also a curved ellipsoid. His experiments indicate that the curved model method not only checks results by the older oscillation method (to be described later) but also confirms the usual theoretical assumption that the forces due to rotation are practically proportional to path curvature up to as sharp a curve as can be flown by airships under full rudder. The proportionality factors (rotary derivatives), however, appear to be appreciably influenced by, and not independent of, the simultaneously prevailing angle of yaw (see Fig. 24). This latter phenomenon appears more emphasized on Professor Smith's tests than in the Zeppelin tests. His researches have not yet been published but are quoted here by permission.

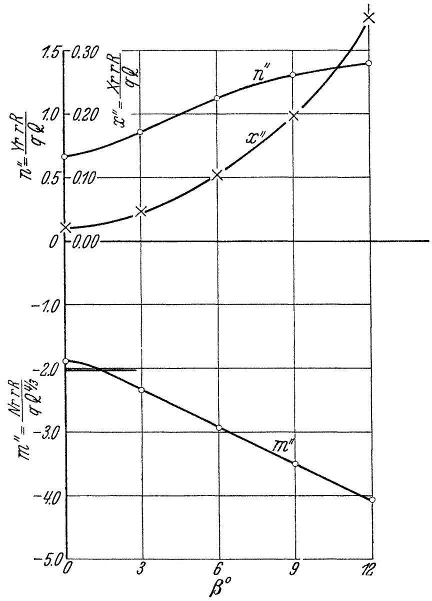

Fig. 24. Non-dimensional coefficients for Yr, Xr, Nr, *vs* yaw at zero rudder for U.S.S. Shenandoah from curved models.

Another less direct method for the determination by model experiments of one of the rotary derivatives, namely the "damping moment" due to rotation (*i. e.* curvature of path), was developed in England by way of the "oscillation" test[1]. The model was suspended elastically in the wind tunnel with its axis parallel to the wind tunnel axis in elastic equilibrium, but so that it could oscillate in yaw about its center of buoyancy. While the wind tunnel was in operation, the model was artificially deflected in yaw and left to oscillate, while observations of the rate of decay of the yaw amplitude were made. The theory underlying the evaluation of this experiment assumes that the aerodynamic moment has one component which is porportional to the angle of yaw (α) and the square of the speed, whereas the other is proportional to the product of the angular velocity of rotation and the air speed itself, while the whole must be equal to the product of the actual moment of inertia of the model, J, (including the virtual inertia) and angular acceleration, *viz.*

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[1] Bairstow, L.,, and Maclachlan, L., Br. A.R.C. R. and M. 78, 1913; Simmons L. F. G., and Bateman, H., Br. A.R.C. R. and M. 665, 1920; Zahm, A. F., Smith, R. H., and Louden, F. A., U.S. N.A.C.A. Technical Report No. 215, 1925.

$$J\ddot{\alpha} = \varrho V^2 Q (k_2 - k_1)\alpha + VA\dot{\alpha}$$

by assuming damped harmonic oscillation $\alpha = \alpha_0 e^{-\mu t} \cos \omega t$ and substituting the derivatives it is readily seen from the $\sin \omega t$ terms that

$$A = -\frac{2 J \mu}{V}$$

Theoretically, it is easily shown that the constant A thus measured by observing the rate of decay μ, is nothing other than

$$A = \varrho a^2 S \frac{d C_L}{d \alpha}$$

where ϱ is the air density, a the lever of the fins, S their area and $dC_L/d\alpha$ their aerodynamic effectiveness including the share of the hull.

A forced oscillation method[1] has also been developed and applied for the study of airplane rotary derivatives. It should be applicable to airships in principle.

Oscillation experiments are naturally delicate and the results must be corrected for the inevitable damping decrement of the apparatus without wind. The objection that the model is exposed to the wake of an unsteady motion and to varying angles of yaw has also been raised. Yet the results obtained are of a similar order of magnitude to those given by other tests. The extension of the oscillation method to determine, for instance, the lift due to rotation by additional tests with the model suspended eccentrically would hardly seem to give promise of results with acceptable accuracy.

An additional drag arises in circular flight and the air speed decreases to a lower terminal velocity when turning at a set position of motor throttle. This drag is due partly to the tangential components of the aerodynamic forces acting on hull and empennage and it is further aggravated by the accompanying loss in propeller efficiency due to increased disk loading and yaw angle prevailing at the propellers.

The bending moment due to turning[2] which stresses the ship as a beam in bending can be computed for any station x_1 along the axis by integrating from either end (preferably from the nose) to that station, the moments due to the aerodynamic transverse or zonal forces and subtracting the integrated moment of the centrifugal forces of the part of the ship on the same side of the frame or station under examination. To a first approximation these centrifugal forces can usually be assumed distributed as the ship's cross sectional area S.

$$\text{Bending Moment} = \int_0^{x_1} (x_1 - x)\left(\frac{dF}{dx} - \frac{\varrho S V^2}{R}\right) dx$$

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[1] SIMMONS, L. F. G., Br. A.R.C. R. and M. 711, January 1921; RELF, E. F., LAVENDER, T., and OWER, E., Br. A.R.C. R. and M. 809, September 1921.

[2] U.S. N.A.C.A. Technical Reports Nos. 115, 323, 325.

which expression can be simplified owing to the relations between dF/dx, the radius R, the angle of yaw, and the velocity V.

2. Dynamic Stability. Dynamic stability is defined as the quality under which the ship, after having accidentally deviated from an attitude of equilibrium of forces and moments (be this straight flight, axial or with an angle of pitch, or a steady turn) will return to this attitude by its own inherent aerodynamic and inertia reactions in such a manner that the disturbance eventually disappears, either in the form of diminishing oscillations or aperiodically[1]. If, on the contrary, the disturbance should grow by way of increasing oscillations or by way of a continuous movement toward a new attitude, the original attitude was dynamically unstable. It is, of course, conceivable that any particular situation may be dynamically stable for disturbances of a certain kind or magnitude and unstable for others. In fact such is actually the case with airships whose aerodynamic reactions for increasing angles of attack increase first with an increasing rate and later with a decreasing rate.

However, by confining the theory to very small deviations, neglect of the higher terms becomes legitimate and one can apply the well known doctrine of "small oscillations" in order to derive the "stability criteria". Since 1920, English writers on the subject have developed this idea by writing down the differential equations for the lateral force and yawing moment, assigning to each of the aerodynamical forces and moments a share or component "due to yaw"[2], and another "due to rotation"[2].

If the derivatives of these component forces F and moments M with yaw α and angular velocity ω are known, *viz.*,

$$F_\alpha = \frac{\partial F}{\partial \alpha}, \quad F_\omega = \frac{\partial F}{\partial \omega}, \quad M_\alpha = \frac{\partial M}{\partial \alpha}, \quad M_\omega = \frac{\partial M}{\partial \omega},$$

then evidently for a small displacement α from equilibrium and under a small angular velocity ω, the two simultaneous differential equations determining the d'Alembert reactions will read

$$\begin{aligned} m V\,[(1+k_1)\,\omega - (1+k_2)\,\dot{\alpha}] &= F_\alpha\,\alpha + F_\omega\,\omega \\ J\,(1+k')\,\dot{\omega} &= M_\alpha\,\alpha + M_\omega\,\omega \end{aligned}$$

where m is the mass and J the moment of inertia, k_1, k_2, and k' the coefficients of longitudinal, transverse and polar additional apparent mass due to the flow about the ship[3] and V the velocity of flight. The lateral acceleration appears split up into two terms, one, $-m\,(1+k_2)\,V\dot{\alpha}$ accounting for the actual reduction of the angle of yaw through lateral

[1] U.S. N.A.C.A. Technical Report No. 212 and Br. A.R.C. R. and M. 257, 602, 751; BURGESS, C. P., Airship Design, New York 1927.

[2] The British A.R.C. R. and M. authors express them in terms of the transverse velocity component (v in yaw or w in pitch) and of the angular velocity (q in pitch or r in yaw).

[3] U.S. N.A.C.A. Technical Reports Nos. 164, 323.

yielding of the ship to the force, and the other, $m(1+k_1)\omega V$ the centrifugal force.

Since the derivatives F and M as defined are dimensional, depending on size and speed, one may prefer to transform the expressions with the help of non-dimensional coefficients depending on geometric properties only. Assuming that the aerodynamic properties of the ship under question had been determined by any one of the methods previously discussed, coefficients of force and moment could have been derived referring to unit $Q^{2/3}$ and unit Q, (volume of the ship) respectively, and to unit velocity head q in each case. The slope of the curve of these coefficients plotted against yaw α, near $\alpha =$ zero, would then constitute the non-dimensional derivative. Calling this n' for the normal force and m' for the moment, we have $F = n'ql^2$ and $M = m'ql^3$ where l stands for $\sqrt[3]{Q}$. Theoretically m' should be $2(k_2 - k_1)$ and $n' = 0$. In reality m' is a little less, and n' is of the order of $1/2$.

In order to express rotation also by an angular measure of curvature, let us introduce the "curvature angle" ζ under which some unit length (for instance l) appears from the center of the turn: that is, $R\zeta = l$ where R is the radius of the turn. Then we shall have the relation $\omega/V = \zeta/l$. For instance, in the Zeppelin experiments with the curved model, ζ was $= 0.0367$ ($= 2^0 9'$). By virtue of the proportionality concept accepted for small curvatures, non-dimensional rotary derivatives n'' and m'' are found by dividing the coefficients measured for zero yaw (or the difference between the curved and the straight model coefficients at any particular angle of yaw) by the value of $\zeta = 0.0367$. They result for that particular model approximately

$$n'' = + \ .9$$
$$m'' = - 2.5$$

If now we realize that for buoyancy equilibrium $m = \varrho l^3$ and since q stands for $(\varrho/2)V^2$, the density ϱ and most of the dimensions l cancel. To make everything non-dimensional, one may introduce the time unit $\tau = l/V$ during which the ship travels a distance equal to $\sqrt[3]{Q}$ and express the moment of inertia by means of a relative radius of gyration j in terms of l which is to include the hydrodynamic or virtual inertia so that $J(1+k') = \varrho l^5 j^2$. Then writing k_x for $1 + k_1$ and k_y for $1 + k_2$, the two equations are reduced to

$$-2\,k_y\,\tau\,\dot{\alpha} = n'\,\alpha + (n'' - 2\,k_x)\zeta \qquad (2.1)$$

$$2\,j^2\,\tau\,\dot{\zeta} = m'\,\alpha + m''\,\zeta \qquad (2.2)$$

From (2.2) we have: $$\alpha = \frac{2\,j^2\,\tau\,\dot{\zeta} - m''\,\zeta}{m'}$$

whence $$\dot{\alpha} = \frac{2\,j^2\,\tau\,\ddot{\zeta} - m''\,\dot{\zeta}}{m'}$$

Substitution of this value in (2.1) then gives

$$4\,k_y j^2 \tau^2 \ddot{\zeta} + 2\,(j^2 n' - k_y m'')\,\tau\,\dot{\zeta} - (2\,k_x m' + n' m'' - m' n'')\,\zeta = 0 \qquad (2.3)$$

This is solved by assuming $\zeta = \zeta_1\,e^{\lambda_1 t} + \zeta_2\,e^{\lambda_2 t}$

where λ_1, λ_2 are the roots of the equation $A\lambda^2 + B\lambda + C = 0$ and the coefficients are taken from the above (2.3) interpreted as

$$A\ddot{\zeta} + B\dot{\zeta} + C = 0\,.$$

Since both $A = 4\,k_y\,j^2\,\tau^2$ and $B = 2\,(j^2\,n' - k_y\,m'')\,\tau$ are always positive, the criterion of stability, *viz.*, for the decrease of ζ, is simply that C must be positive, or, as it is most conveniently expressed

$$D = m' + \frac{n'\,m'' - m'\,n''}{2\,k_x} \leqq 0$$

In the case of the model of the U.S.S. Los Angeles this criterion is approximately zero, the ship appears practically indifferent, *viz.* $n' = 0.5$, $m' = 1.04$, $n'' = 0.9$, $m'' = -2.5$ and $D = 0$ within the accuracy of the determination. If it were not for the damping factor n''[1] and especially m'', the aerodynamic moment about the C. B. due to yaw, m', would indeed have to be zero or negative (restoring). Since with tail fins m'' is always negative, this is not necessary. The mechanism by which an airship with $m' > 0$ flies straight is somewhat equivalent to the phenomenon of a bicycle riding straight above a critical speed. It is as though the angle of yaw at the fins were mechanically compelled to grow (about twice the yaw of the ship) whenever the empennage is called upon, and as though curvature helped out with an invisible rudder always moved the right way. H. R. Liebert has proposed to visualize the influence of this type of dynamic stability as another dynamic metacentric height which can be expressed in our terminology as

$$H_d = -h\left(m' + m''\frac{l}{a}\right)$$

where h is the velocity height and a the distance from the C. B. to the station of zero yaw (the center of "swing").

3. Control Maneuvers. From this state of affairs it is evident that a ship which is even aerodynamically slightly unstable can be held on a practically straight course by a watchful helmsman although when left alone it will turn to port or starboard as the first incidental disturbance may dictate. As an example a diagram is shown in Fig. 25, of the path curvature as a function of rudder inclinations of an English ship as well as for the U.S.S. Los Angeles, the latter, however, while in a condition in which it had less dynamical stability than it later had under service conditions. It is seen that a definite curve is flown with

[1] R. Jones and other British authors having had no convenient method for determining n'', or the force due to rotation, substituted for it the empirical knowledge that for most ships the point of zero yaw is located ahead of the C. B. about 0.9 times the distance of the center of the fins aft of the same point.

the rudder neutral and there is a zone of angles in which the rudder seems to have a sort of hysteresis when reversing the rudder under a condition of turn. The reason for the steady rate of turn with rudder neutral being small, is that actually the aerodynamic forces increase more rapidly than in linear proportion to yaw. In terms of the theory of small deviations, this is equivalent to an increase of the derivatives and thence stability with increasing curvature. Thus, by taking the n', m', n'', m'' not at zero values of α and ζ but at definite small values, a theory of dynamical stability in the turn can be evolved. In order, however, to follow more in detail the consequences of any disturbance or, for that matter, of any definite sequence of control maneuvers, step by step methods may be resorted to[1].

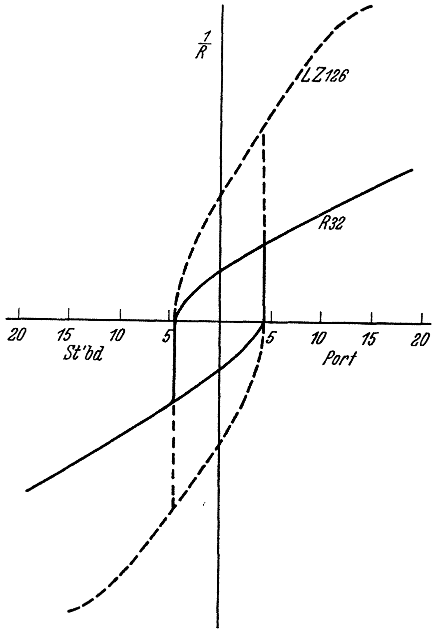

Fig. 25. Curves showing path curvature ($1/R$) as dependent on rudder angles.

The conventional, but not the only, way of controlling airships is by stern control surfaces hinged to the tail fins. Various unconventional types of controls have also been tried. Some early airships, like submarines, had bow control surfaces. Recently bow elevators were made the subject of extensive research both in flight and in wind tunnels by the Goodyear Zeppelin Corporation[2]. These experiments have demonstrated the following: It is possible, under certain circumstances, to steer an airship by the bow and it appears that this method of control has a tendency to involve less undulation of path than stern control, although when compared on terms of equal area and incidence angle, bow control surfaces appear less effective than stern control surfaces. Bow and stern controls, when applied simultaneously, afford a more powerful means to pitch or swing the ship than either alone. This would have certain advantages in clearing obstacles after the take-off and in counteracting gusts, but the additional drag, weight, and mechanism present difficulties which must be considered in connection with the question of supplementary bow control. In order to avoid their

[1] Changeux, P., Dynamique du Dirigeable, Librairie Aéronautique, Paris, Br. A.R.C. R. and M. 781, 1401.

[2] Unpublished as yet.

acting as fixed fins exposed to gusts ahead of the center of buoyancy, bow control surfaces have been successfully arranged "floating". In this condition, if alertly used to counteract gusts, they may tend to alleviate the bending moments imposed on the ship by the gusts whereas similar application of stern controls may enhance them. Bow controls cast a shadow or wake upon tail fins under certain circumstances. The aerodynamics of bow control surfaces are influenced by an induced angle of attack generated by the potential flow around the bow of the ship, especially when the ship flies at an angle of pitch.

The problem of balancing the control surfaces of an airship in order to keep the hinge moments small is of great technical importance. The aerodynamics involved, however, are closely related to the corresponding problems arising in heavier-than-air craft controls.

From the preceding, it will be appreciated why it requires skill and experience to steer an airship steadily, and also why the problem of automatic control sensitized for instance from compasses or gyrostatic apparatus is replete with difficulties. Such a device may indeed work quite satisfactorily in calm weather, once the desired flight attitude and course are attained and disturbances are checked within very small amplitudes. In gusty weather, however, or once a larger deviation is incurred "hunting" is likely to develop. Evidently, an aerodynamic element must be introduced into the control mechanism which, in effect, is equivalent to an increased response to yaw (or pitch) and curvature (*i. e.* increased dynamic stability) before a servo control sensitized from independent parameters can be successfully employed[1].

When the rudders or elevators are suddenly put over, equilibrium is disturbed and non-uniform motion ensues. The balancing force being usually applied at the stern, the ship pivots at a point well forward and the inertia forces are more concentrated near the source of the disturbance. If the unbalanced force F has a lever arm a aft of the C. B., then the pivot about which the ensuing motion can be viewed as an accelerated rotation is located at a distance

$$\xi = \frac{1+k'}{1+k_2} \cdot \frac{i^2}{a} = \frac{j^2}{k_y a}$$

ahead of the C. B., where i is the ship's radius of gyration for rotation about a transverse axis and k' and k_2 are again the virtual mass coefficients (including the shares of the fins). The angular acceleration is,

$$\dot{\omega} = \frac{F\,a}{Q\,i^2\,(1+k')}$$

Likewise the bending moment[2] due to this motion, at any station x_1 is

$$M = \varrho\,\dot{\omega} \int_{x=0}^{x=x_1} (x - x_1)\,(x - \xi)\,(1+k)\,S\,d\,x$$

[1] Crocco, G. A., I Timoni Automatici nei Dirigibili, Rendiconti delle Esperienze e degli Studi etc. Roma. Tip. Acc. dei Lincei 1912.

[2] Bairstow, L., The Aerodynamic Loading of Airships, Br. A.R.C. R. and M. 794.

where k may be some constant (or better a variable) to allow for the (influence of the conicity on) virtual inertia. Integrating from the nose to the station of interest has the advantage of avoiding the uncertainties of the force distribution about the stern. Figure 26 shows typical bending moment and shear diagrams for sudden application of a rudder force.

The answer to the question of the magnitude of a suddenly applied rudder force can, of course, be sought from aerodynamic experiments with models having movable control surfaces. More or less sudden maneuvers are possible from various initial conditions of equilibrium, and not only from straight flight. The most violent maneuver appears to be a rapid reversal of the helm (rudder or elevator); because then the free force is that corresponding to the whole difference of the rudder angles before and after the maneuver, and furthermore, the new bending moments are added to the bending moments already attending the steady turn. However, as a rule, in a sharp steady turn of any duration, the additional drag has already slowed down the ship somewhat, so that the free rudder force will be less than if travelling at full speed. The apparently worst condition in calm air is, therefore, a sudden complete reversal of the controls immediately after starting a sharp turn or dive. Some experimental data of interest are given by Richmond and by Burgess[1].

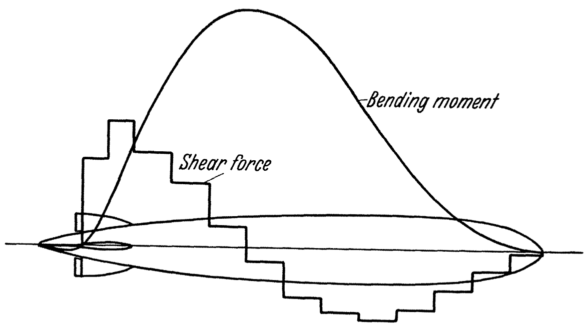

Fig. 26. Bending moment and shear diagram for sudden application of rudder force.

In a vertical reversal maneuver (checking a climb or dive) the stresses due to the more or less sudden application of the controls may occur in addition to the stresses due to carrying some overload (or excess buoyancy) by dynamic lift. In this case the ship's speed is, however, usually diminished owing to the additional drag accompanying the dynamic lift.

Combined elevator and rudder maneuvers, although hardly more severe for the ship than as corresponding to the vectorial resultant free force, may nevertheless cause increased concentrations of fin pressures along the windward fin edges.

4. Gusts. Whereas deliberate steering maneuvers normally produce unbalanced forces at the tail, gusts attack the ship first at the bow,

[1] Richmond, Lt. Col. V. C., Br. A.R.C. R. and M. 1044, 1926; Burgess, C. P., U.S. N.A.C.A. Technical Report No. 325.

when the ship runs into the gust zone, presumably with its own flying speed V. It is conceivable that the seat of the atmospheric irregularity may not always be travelling steadily with the average wind. If the disturbance is propagated through air[1] or if it does not partake of the atmospheric wind as for instance in a mountainous region, its front may be met slantwise or the speed of the gust relative to the ship may differ from the air speed of the latter. Refinements of the more simple theory, in order to take care of such influences will be obvious. An interesting study is afforded by the basic fictitious problem of the ship running with its air speed V into a "cross wind" zone. This picture has parallel applications on the one hand in yaw, as for example adjacent layers of wind of different direction or velocity, especially in mountain passes; and on the other hand, in pitch, for example, the entrance into a vertical stream of air such as may be caused by vertical deflection of wind through mountain ranges or by convection currents and thunderstorms. Vertical velocities of 3 to 6 m. per sec. are often encountered and velocities up to 10 m. per sec. and more have occasionally deliberately been hunted up, and utilized in the soaring flight of gliders and sailplanes[2].

This picture of the "cross wind"[3] zone introduces no difficulties of concept in regard to fluid continuity, but it leaves unexplained how the hydrodynamic equilibrium of the fluid shear force present is achieved, and leaves somewhat in doubt the character of the distribution of aerodynamic forces with the ship partly within and partly without the zone of disturbance. Evidently, if such conditions occur, the angle of attack will differ over the two parts of the ship and there is some question as to what degree the influence of the new angle of attack may precede the passing of the border line.

In a first approximation treatment, the zonal aerodynamic force "breadths" occurring at any station x along the ship may perhaps be assumed to be those which would obtain if the whole ship were flying at the angle of yaw (or pitch if a vertical gust is to be studied) actually

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[1] ECKENER, H., Modern Zeppelin Airships, Journal of the Royal Aeronautical Society, June 1925; WATTENDORF, F. L., Preliminary Investigations of Atmospheric Turbulence; and KUETHE, A. M., Some Feature of Atmospheric Turbulence and the Passage of Fronts, both in Daniel Guggenheim Airship Institute, Publication No. 2, 1933; REICHELDERFER, F. W., Meteorological Aspects of Airship Operations, Daniel Guggenheim Airship Institute, Akron Ohio, Publication No. 3, 1935.

[2] KUSSNER, H. G., Zeitschr. f. Flugtechnik u. Motorl., Vol. 19, pp. 579—582, 1931; and LANGE, K. O., Zeitschr. f. Flugtechnik u. Motorl., Vol. 17, pp. 513—519, 1931.

[3] It may be noted that a gust of given velocity amplitude may be expected to exert the greatest disturbing force not when striking exactly at right angles to the ship's axis, but at an angle with a slight head-on component. However, even for the angle giving the maximum product of lateral and resultant axial relative wind component, the force increment is but small.

prevailing locally at that station[1]. The aerodynamic force due to rotation (if any) is assumed superimposed independently. Several alternative variations of the problem have been studied[1], for instance:

(a) The border of the "gust" can be assumed infinitely sharp, a jump from cross wind zero to v.

(b) A gradual increase from zero to v over a certain distance at a constant rate $dv/dx = b$ (constant vorticity).

(c) An exponential law of increase of the type $v = v_0 (1 - e^{-rx})$.

(d) A sine wave $v = v_0 \sin x/a$.

In these expressions, v is the velocity of the gust and b or r indicate its degree of "sharpness". Various conditions may then be assumed regarding the course of the ship; for instance:

(1) The helmsman is able to hold the ship straight on its compass course. This eliminates rotation and all the complications associated therewith, but introduces unknown tail forces. It is true that they need not be known in order to compute the motion of the ship and the bending moments in the most important parts, but the question may still be asked: Is it possible to realize such a condition? Is the rudder system capable of producing the required forces?

(2) The helmsman holds the rudder straight and lets the ship deviate as conditions may determine. In some respects this is a simpler plan, but the computations are more complicated.

(3) The helmsman uses the controls either in an attempt to counteract the gust in some partial measure or he may also deliberately or accidentally enhance the deviation caused by the gust.

The procedure for any of these assumptions is to first write down the equations of motion *i. e.* for the lateral force (assuming a horizontal gust) and (in case 2) the yawing moment in terms of the aerodynamic and inertia forces for the ship as a whole, and (in case 3) including the control forces and moments as a function of time. As far as a first order of approximation is permissible, *i. e.*, as far as the aerodynamic reaction can be expressed as a sum of one component proportional to the angle of attack, and another independent component proportional to path curvature, the lateral and rotary accelerations can be brought into the form

$$\frac{d\eta}{d\xi} = a_1 \eta + b_1 \zeta + \frac{w}{u} f_1(\xi)$$

and

$$\frac{d\zeta}{d\xi} = a_2 \eta + b_2 \zeta + \frac{w}{u} f_2(\xi)$$

[1] MUNK, M. M., The Aerodynamic Forces on Airship Hulls, U.S. N.A.C.A. Technical Report No. 184, 1924; KLEMPERER, W., Airships in Gusts, Daniel Guggenheim Airship Institute, Publication No. 3, 1935.

C. P. BURGESS in U.S. N.A.C.A. Technical Report No. 204 and D. H. WILLIAMS and A. R. COLLAR in Br. A.R.C. R. and M. 1401 accepted a more drastic simplification, *viz.*, the substitution over the whole ship of the force distribution it would have under the angle of yaw or pitch which exists momentarily at the center of buoyancy.

where the variables are expressed by $\eta = v/u$ which is the ratio of lateral to forward velocity (tangent of the angle of yaw at the center of buoyancy before the passage of the gust border over this station) and $\zeta = \sqrt[3]{Q}/R$ the characteristic angle of path curvature introduced before. These variable flight parameters are expressed as functions of $\xi = x/L = ut/L$, *viz.*, the portion (x) of the ship's length (L) which has just penetrated into the gust, u being the flight velocity, t the time since the nose struck the gust border, and w the lateral velocity of the gust. The coefficients contain the inertia characteristics and aerodynamic derivatives. The functions $f(\xi)$ encompass the aerodynamic action of the lateral attack on the bow from the nose to the transient station to which the gust has just progressed. The above equations can be split into two explicit

equations, $$\frac{d^2\eta}{d\xi^2} + A_1 \frac{d\eta}{d\xi} + B_1 \eta = F_1(\xi)$$

and $$\frac{d^2\zeta}{d\xi^2} + A_2 \frac{d\zeta}{d\xi} + B_2 \zeta = F_2(\xi)$$

In this form they can be solved for the angle of yaw at the C. B. (with respect to the undisturbed air) and for the curvature of path, as functions of time. Having these, the bending moments and if the need be the shear forces for any station can be computed as a function of time. Their maxima, when and wherever they may occur, are the data desired by the stress analyst.

The actual integration of the equations of motion can be performed by graphical or step-by-step methods. For ships whose forebody shape can be expressed by analytical formulae there is a chance of integrating by calculation.

Considerable simplification is afforded by an assumption of zero dynamic stability. In this case the factors B vanish and a solution can be obtained in the form.

$$\frac{d\eta}{d\xi} = -e^{-A_1\xi} \int_0^{\xi} e^{A_1\xi} F(\xi)\, d\xi$$

and similarly for $d\zeta/d\xi$.

Since the bending moments are the differences between moments of aerodynamic and inertia forces, all of which are crowded toward the ends, it is important that too drastic simplifications should not be applied in these regions. This and other refinements somewhat complicate the integration.

The following is an example calculated for a ship project equipped with a hypothetical empennage which would give it zero dynamical stability. This stability and the aerodynamical characteristics were arbitrarily assumed to remain constant while the ship would be exposed to a fictitious sharp-bordered gust of cross-path velocity w. Figure 27 shows how the lateral drift η and path curvature ζ build up as the ship penetrates into the gust zone and Fig. 28 depicts the deflected path the ship assumes while the helmsman does nothing to parry the gust. Figure 29 shows how, under certain assumptions concerning mass distribution, the bending moments at several reference stations ξ_f would build up and die down again as the gust border sweeps over the ship. The ordinates of the angles of Fig. 27 are here referred to unit ratio $\beta = w/u$ of assumed gust velocity to ship speed. For any particular gust or ship speed, they would have to be multiplied by β. Similarly the bending moment coefficient is referred to unit $\varrho u w Q$.

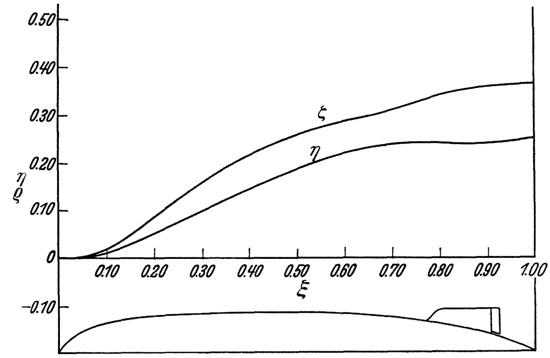

Fig. 27. Characteristics of motion in a sharp bordered gust.

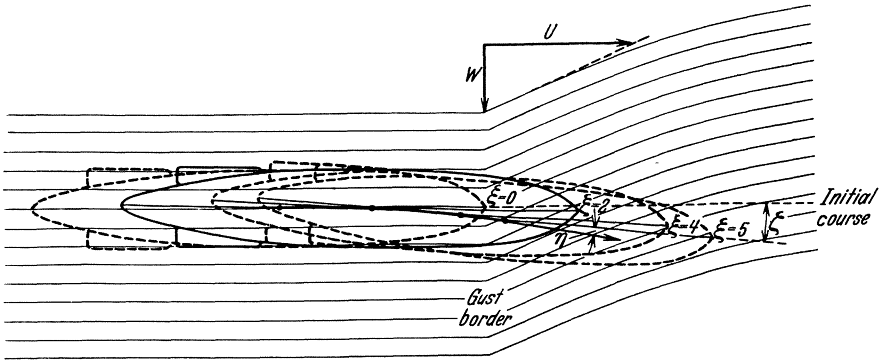

Fig. 28. Deflected path as airship enters a gust.

The examples here given were naturally based on some particular assumptions of the ship's aerodynamical and rotary characteristics. A ship of different empennage for instance would be expected to behave differently.

For any gradual increase of the cross wind to its full velocity w, the deviations and moments are obviously less than for a sharp-bordered gust.

It is also of interest to study the flight of an airship through a field of a moving vortex, which can be interpreted as a particularly simple and well defined form of a "gust". Investigations of this problem were undertaken by Poggi[1] and Oswald[2]. The forces and moments experienced by an ellipsoid exposed to a flow consisting of a straight component and a vortex located on the "port or starboard beam" or "dead ahead" were thus determined. These assumptions are equivalent to the conditions prevailing in the flight past the side of a tornado or toward one. As these calculations are based on classical fluid concepts, they still leave the lateral force on the hull and all actions of an empennage out of consideration. To take these and also deliberate rudder action into account, would constitute the next step toward a study of the actual dynamics of the ensuing motion.

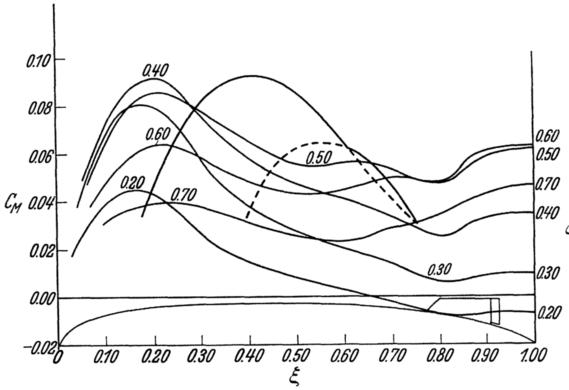

Fig. 29. Bending moments at several stations in relation to sweep of gust border along the ship.

CHAPTER V

MOORING AND GROUND HANDLING

1. Mast Mooring. Whenever it becomes necessary to anchor an airship in the open, it is desirable to hold it headed into the wind as this is the attitude imparting the lowest stresses. Provisions are therefore made to let the ship swing to follow any changes of wind direction in weathercock fashion. Evidently it is most advantageous for this purpose, to anchor the ship to a mast or tower at a point as far forward as possible, *viz.*, at the nose. This has the additional advantage of flow symmetry, thus minimizing any pitching and rolling tendencies in fluctuating winds. If the derivative of the aerodynamic moment about the mooring point due to yaw is zero or negative, the ship will ride stably at zero yaw. In the previous notation this condition is expressed by $(m' - n' a) = 0$ where $m' = \partial C_m/\partial \alpha$ and $n' = \partial C_n/\partial \alpha$, the moment

[1] Poggi, Azioni aerodinamiche su di una ellissoide di rotazione rivestito da un vortice, etc. Estratto d'Atti della Pont. Accademia delle scienze, nuovi Lincei Anno LXXXIV, December 21, 1930, Rome.

[2] Oswald, W. B., The Transverse Force Distribution on Ellipsoidal and Nearly Ellipsoidal Bodies Moving in an Arbitrary Potential Flow. Thesis, Calif. Institute of Technology, 1932; von Kármán, Th., Some Aerodynamic Problems of Airships, Daniel Guggenheim Airship Institute, Publication No. 1, 1933; Tollmien, W., The Motion of Ellipsoidal Bodies Through Curved Streams, Daniel Guggenheim Airship Institute, Publication No. 2, 1935.

and normal force derivatives due to yaw of the empennaged ship, expressed in terms of $\sqrt[3]{Q}$, with a the lever arm from the bow anchor point to the C. B. For small non-rigid ships with large empennages, this condition is usually amply fulfilled. In moderate winds they can even be moored satisfactorily at a "breast plate" some distance aft of the nose, thus permitting lower and simpler masts. Large rigid ships can be satisfactorily moored at the nose even though $(m' - n'a)$ may have a slight positive value at zero yaw. In this case they will ride at a small angle of yaw leaning on the wind to port or starboard as chance may determine and occasionally swaying over gently from one side to the other. These angles are, of course, the ones where the curve $(C_m - aC_n)$ plotted against the angle of yaw α crosses the zero lines under a negative slope as indicated in Fig. 30. Under suitable conditions of trim an angular deviation may also develop in the vertical plane, the position of equilibrium being determined by the balance between the out of trim moment and the aerodynamic forces due to the wind. These conditions can be duplicated and studied in wind tunnel experiments[1]. In such conditions of mooring, there is of course, a small transverse force exerted by the mooring device upon the nose of the ship.

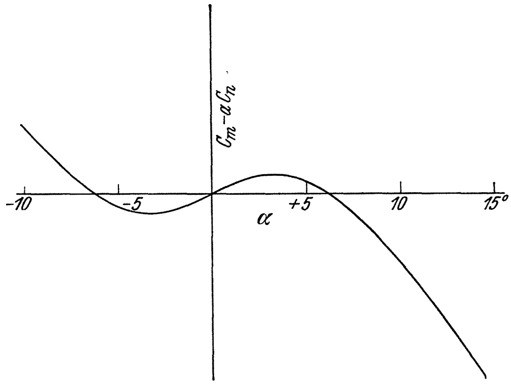

Fig. 30. Curve of $C_m - aC_n$ on angle of yaw.

The merits of various types of mooring and handling equipment have been frequently discussed in the technical literature[2]. Higher masts permit greater clearance between the ship and the ground or the water (in case of a floating mast such as the U.S.S. Patoka). However,

[1] KLEMPERER, W., Windkanalversuche an einem Zeppelin-Luftschiffmodell, Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, 12.

[2] BUTCHER, F. L. C., Airship Mooring and Handling, Journal of the Royal Aeronautical Society, February 1921; RICHMOND, V. C., Airship Research and Experiment, Journal of the Royal Aeronautical Society, October 1926; SCOTT, MAJ. G. H., The Handling and Mooring of Airships, Journal of the Royal Aeronautical Society, November 1929; BLEISTEIN, W., Zeitschr. f. Flugtechnik u. Motorl., pp. 362—365, 1929; ROSENDAHL, LT. COMDR. C. E., Up Ship, p. 109, 1931 (Dodd Mead and Co.). Mooring Masts and Landing Trucks for Airships, Journal, Society of Automotive Engineers, July 1929; BOLSTER, LT. C. M., Mechanical Equipment for Handling Large Rigid Airships, Aeronautical Engineering (A.S.M.E. New York) No. 3, 1933.

low masts are considerably less expensive than high masts, and have the further advantage that they permit of securing the tail to a circular track on the ground, so as to prevent any vertical movement of the ship. A special type of the low mooring mast is the mobile variety. It is ordinarily intended for ground transport, but may occasionally be called upon for temporary mooring services.

In gusty weather with capricious wind shifts, the ship may fail to follow quickly like a weather vane. Due to the great inertia of the ship, there may be a lag in turning and large angles of yaw and large lateral forces may thus occur temporarily. The dynamics of this motion can be dealt with in similar manner as in free flight, only that the motion is confined to rotation about the nose anchor point. Theoretical difficulties attending the arbitrary assumptions regarding the structure and the mechanism of the "gust" still remain.

2. Cable Mooring. There have been various proposals of methods for mooring airships in the air by means of cable systems and many of these methods have been tried[1]. In the cable type of mooring, the airship is attached to the apex of a three wire "pyramid" and is flown light or is pitched upward so that the airship lift keeps the cable system taut and creates the effect of a "virtual" mast. This type of mooring apparently originated with the Italians in 1908[2] but received more extended use in England. It was this type of mooring which was used for the R-34 at Long Island after her trans-Atlantic flight in 1919.

In addition to their full scale experimental work with this three wire system, the English undertook laboratory experiments with this and other forms of "free" wire systems[3]. The dynamics are essentially the same as with a solid mast but for the increased elasticity and the chance that the ship's bow may acquire excessive momentum due to the inevitable sag in the cables or to occasional slackness in one cable or another should the surplus lift be insufficient to keep them taut in some phase of oscillation.

A more complicated dynamical problem is offered when the ship is moored by one cable only, like a kite balloon[4]. This condition can occur while hauling the ship up to a mooring mast by means of a nose cable, should the side guy lines which are supposed to hold taut during

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[1] KRELL, O., Zeitschr. f. Flugtechnik u. Motorl., pp. 401—438, 1928 (also U.S. N.A.C.A. Technical Memoranda 512 and 513, 1929); MASTERMAN, AIR COMMODORE E. A. D., The Evolution of Mooring and Handling Devices for Airships in Air Annual of the British Empire, 1930.

[2] CROCCO, G. A., Translations appearing in U.S. N.A.C.A. Technical Memorandum No. 283, 1923.

[3] FRAZER, R. A., Note on the Mooring of Airships by "Free" Wire Systems, Journal of the Royal Aeronautical Society, February 1921.

[4] BAIRSTOW, L., The Stability of Kite Balloons: Mathematical Investigation, Br. A.R.C. R. and M. 208, 1915.

the maneuver go slack. While the mooring cable is long or slack, the ship can take advantage of the dynamic stability in free flight without much influence upon the forces transmitted by the mooring cable until the latter snaps taut. If, however, the mooring cable has already been hauled in to a length comparable to the ship's length, any yawing of the ship and resultant lateral drift will produce large changes in the angle the cable makes with the wind. The multiple degree of freedom may give rise to complicated coupled oscillations. These, by the way, can be readily studied by model experiments in a water channel.

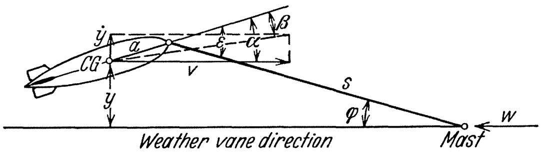

Fig. 31.

If the oscillations are small and confined to one plane and, if an infinitely thin, massless cable is assumed, they can be described by the following primitive theory. With notations and a general arrangement as given below and sketched in Fig. 31, the equations of motion, in terms of accelerations and d'Alembert reactions, are:

$$[k_y\, \ddot{y} - (k_y - k_x)\, U\, \dot{\alpha}] = \frac{U^2}{2\,l}\,[n'\beta + n''\,\zeta - C_D\,\varepsilon] \tag{2.1}$$

$$j^2\, l^2 \ddot{\alpha} = \frac{U^2}{2}\,[m'\,\beta + m''\,\zeta - C_D\,\varepsilon\, a/l] \tag{2.2}$$

The notations aside from those already introduced are,

$l^3 = Q$ = volume of ship,

$j\,l = \sqrt{\frac{J\,k_p}{Q\,\varrho}}$ = radius of gyration of the ship about a master section diameter (including virtual mass),

y = lateral displacement of ship's center of gravity from the weather vane direction,

α = azimuth angle between ship's axis and absolute wind direction (weather vane direction),

β = angle of attack between ship's axis and relative wind,

ε = angle between cable and ship's axis,

U = wind velocity,

C_D = drag coefficient,

s = length of mooring cable,

a = distance from bow to center of gravity.

After elimination of ζ, ε, γ by recourse to the geometrical relations

$$\zeta = l\,\dot{\alpha}/U$$

$$\varepsilon = \frac{y + (a + s)\,\alpha}{s}$$

$$\beta = \alpha - \dot{y}/U$$

two simultaneous equations of the type

$$A_1 \ddot{y} + B_1 \dot{y} + C_1 y \qquad + E_1 \dot{\alpha} + F_1 \alpha = 0 \tag{2.3}$$

$$B_2 \dot{y} + C_2 y + D_2 \ddot{\alpha} + E_2 \dot{\alpha} + F_2 \alpha = 0 \tag{2.4}$$

are obtained.

The equations can be readily solved; the stability criterion for the gradual dying out of the oscillations is expressed by five coefficients, a_0 to a_4, which must all be positive and obey a relation

$$a_2 > \frac{a_1 a_4}{a_3} + \frac{a_3 a_0}{a_1} \tag{2.5}$$

These coefficients a_0, a_1, a_2, a_3, a_4 are derived from the coefficients $A, B, C, D, E, F,$ of the equations (2.3) and (2.4). Inserting the respective original values they become

$$\left.\begin{aligned} a_4 &= 2\, k_y j^2 \\ a_3 &= j\, n' - k_y m'' \\ a_2 &= C_D \left[k_y \frac{a}{l} \left(1 + \frac{a}{s}\right) + \frac{j^2 l}{s} \right] - k_y D \\ a_1 &= -\, C_D \left[m_b' + \frac{m_b'' l - 2\,(k_y - k_x)\, a}{s} \right] \\ a_0 &= -\, \frac{C_D m_b' l}{s} \end{aligned}\right\} \tag{2.6}$$

For the sake of abbreviation and better visualization, we have introduced here the derivatives about the bow attachment point; *viz.*,

$$m_b' = m' - n'\, a/l$$

and

$$m_b'' = m'' - n''\, a/l$$

and the criterion of dynamic stability in free flight

$$D = m' + \frac{n' m'' - m' n''}{2\, k_x}$$

The most significant of the conditions for stability of the mooring lines is expressed by a_0 which dominates all others for short lengths of line. It postulates that for stable flight on a mooring line a ship would have to be so empennaged that even at small angles of yaw, the aerodynamic resultant force passes back of the bow mooring point (m_b' negative). This is easily fulfilled with small airships but has not always been completely fulfilled with large rigid airships. However, the undamped oscillation component which results from this slight deficiency is of very slow period and small. It is kept in bounds by the more rapid increase of the stabilizing forces with increasing angles of attack. The role which the dynamic stability criterion D plays (in a_2) becomes less and less important as the cable length is decreased and it vanishes for $s = 0$ when the ship is finally anchored by her nose. A small dynamic instability ($D \gtrsim 0$) is tolerable. Even with a very long cable a positive D need only meet the condition $D < C_D a/l$ in order to still be compatible with theoretic stability. In reality, of course, variations in cable pull and possible vertical forces due to lack of exact buoyancy equilibrium complicate the picture. The other requirements a_1, a_3, a_4 are always fulfilled, the two former due to the large negative values of the rotary moment derivatives.

The condition (2.5) is not easily visualized. It is usually also fulfilled for any cable length if conditions are such that a_0 and a_2 are positive. Artificial increase of drag (C_D) increases stability in mooring. The case of a streamlined, finned body, towed from aircraft is a related problem. It had been studied by Glauert[1].

3. Ground Transport. The ideal solution of housing or "docking" huge airships, from a mere aerodynamic standpoint, is the turntable hangar which can be turned into the wind direction so that the ship can, after landing, be walked into the shed by man or machine power without broadside exposure to the wind. Such hangars were built in Germany[2] during the war and were successfully operated. The first Zeppelin hangar was built as a floating structure on Lake Constance. The drawback of this type is the additional cost, the original investment required for any type of stationary airship hangar being already large. Where rotatable hangars are not available, stationary hangars or "docks" built with the axis parallel to the "prevailing" direction of moderate winds furnish the next best solution. The ship is then landed on the field into the hands of a landing party or by attachment to a stationary or mobile landing mast and then "walked" into the hangar. Occasionally it will occur that the "prevailing" wind does not prevail and the ship while being maneuvered into the dock has to pass through some phase of slant or broadside exposure to a cross hangar wind. The motion as a rule is slow so that the forces and stresses can be studied as though the ship was held stationary and without regard to inertia forces. There are three distinct aerodynamic problems arising in this connection[3].

(1) The aerodynamic forces due to very large angles of yaw of the airship, including attack broadside and from astern, and quartering.

(2) The influence of the close proximity of the ground.

(3) The wind obstruction formed by the hangar itself and its doors.

The aerodynamic force (and its distribution) normal to the ship's axis at a very large angle of yaw does not as yet lend itself well to a purely theoretical treatment. As a first approximation it may perhaps be assumed roughly proportional to the projected area, including fins and all appendages, with a drag coefficient in the vicinity of unity, both

[1] Glauert, H., Br. A.R.C. R. and M. 1312, 1930.

[2] Christians, Anlage und Betrieb von Luftschiffhäfen, Oldenbourg, 1914; Engberding, Luftschiff und Luftschiffahrt, V.D.I.-Verlag, 1928; Krell, O., Zeitschr. f. Flugtechnik u. Motorl., pp. 401—438, 1928; also U.S. N.A.C.A. Technical Memorandum Nos 512—513, 1929.

[3] Arnstein, K., and Klemperer, W., Aerodynamic Problems in Connection with Ground Handling and Docking of Airships, A.S.M.E., 1934.

for the near cylindrical sections and for the sharp edged empennage region. In view of the uncertainty regarding the other factors, including the wind speed to be assumed, this may suffice for many general purposes. Upon closer scrutiny it would appear that the results reached in this way may quite exaggerate the actual condition. This is for the reason that, for Reynolds numbers above about 300,000, the Kármán vortex street mechanism of the drag of cylindrical bodies breaks down and gives way to a vortex system of much smaller size which drops the drag coefficient to less than half its value for smaller Reynolds' number though not necessarily to a constant value. Model experiments of sufficient size or speed to reach into the Reynolds' range above 10^7, such as would correspond to ships of 30 to 40 m. diameter in wind speeds of the order of 5 to 10 m. per sec, have not yet been published. The unfortunate feature of most model tests thus far is the fact that it has been just barely possible[1] to reach into the above critical Reynolds' region, whereas at the bow and stern, where the diameter is smaller, conditions would remain in the sub-critical region so that the results obtained may be freakish and uncertain. The least little irregularity of the surface may cause the second type of flow to originate sooner on one side than the other, resulting in irregular pitching moments and lift even under symmetrical attack.

Even if the aerodynamic forces and their distribution about the airship under broadside attack in free air were well understood, there would remain the influence of the proximity of the ground. This makes itself felt in two ways. First, it constitutes an aerodynamic mirror surface forcing the near-by stream-lines to follow it, thereby tending to produce some acceleration below the ship. An attraction to ground might thus be expected. By integrating the pressures produced upon the ground surface by a series of repeatedly reflected two-dimensional doublets[2], the attraction between a cylinder and the mirror surfaces can be computed for hypothetical potential flow. In a first approximation by means of a single doublet and mirror image, representing a somewhat flattened hull cross-section, an attraction or negative lift can be calculated

$$\frac{dL}{dx} = -q\pi\frac{R^4}{h^3}$$

per unit length of the ship where q is the velocity head of the wind, h the height of the ship's axis above the ground, and R the radius of a cross-section equivalent to the doublet without its image. Furthermore the wake shed by a cylinder attacked broadside may be simulated by an unbalanced source-sink combination (with source stronger than sink)

[1] Even though the critical speed may be slightly lowered by making the model polygonal in cross section rather than round.

[2] MÜLLER, W., Z.A.M.M., Vol. 9, No. 3, p. 200, 1929; Z.A.M.M., Vol. 11, p. 231, 1931; LAGALLY, Z.A.M.M., Vol. 9, No. 4, p. 299, 1929.

supplemented by its mirror images. In this manner one finds the attraction by the ground reduced and correlated with the drag.

An investigation into the forces on a large cylinder near a ground dummy was recently made at the Daniel Guggenheim Airship Institute, Akron, Ohio, by Th. Troller and F. D. Knoblock who measured the pressure distribution and the force reactions under special precautions, excluding disturbances from the ends of the cylinders[1]. Their results indicated for the section of such a cylinder (two-dimensional flow) that, during the transition from under-critical to above-critical flow regime, the stagnation point on the windward side travels from some 70^0 ahead of the point nearest the ground to 80^0, while the zero over-pressure point travels from about 30^0 to 44^0 ahead of the same location. The force between ground and cylinder is a repulsion in the completely under-critical flow as well as in the completely over-critical flow, whereas during some phase of the transition, when the suction peak is more pronounced in the interspace, the force is a pronounced attraction.

In reality it is possible that circulation builds up around an airship held close to the ground against a side wind. In fact such circulation is favored by the presence of wind friction on the ground. It is conducive to a repulsion or positive lift which is likely to outweigh the attraction postulated in its absence from potential flow. H. v. Sanden[2] has calculated this lift for a symmetrical profile exposed to a wind, the velocity U, of which, increases with height h, at a rate $U' = dU/dh$ and has shown that the lift is proportional to $Q \varrho U U'$, *viz.*; the volume, density, velocity and velocity gradient for any given profile.

Troller and Knoblock also measured the forces on their cylinder above a dummy ground board in the presence of an artificially produced velocity gradient simulating that present in a natural wind under certain conditions. In this particular case they found only repulsion; *i. e.*, positive lift forces.

In the practice of docking airships, especially when maneuvering to the leeward of the dock, it is also important to be on the watch for the presence of huge eddies trailing off from the building and its open doors. The "shelter" or wake region of these doors, and of artificially erected screens, has often been regarded as beneficial and may be so for small ships under certain circumstances. However, actual ground handling experience and extensive wind tunnel experimentation with model ships, docks and screens[3] demonstrate that there may be treacherous whirls

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[1] Knoblock, F. D., and Troller, Th., Tests on the Effect of Sidewind on the Ground-Handling of Airships, Daniel Guggenheim Airship Institute, Publication No. 2, 1935.

[2] Sanden, H. v., Über den Auftrieb im natürlichen Winde, Zeitschrift für Mathematik und Physik, No. 3, pp. 225—245, 1913.

[3] Jones, R., and Levy, H., Br. A.R.C. R. and M. 338, 1917; Nayler, J. L., and Woodford, F. G., Br. A.R.C. R. and M. 428, 1917.

with vertical as well as horizontal and slant axes and high velocities originating in the spillover regions. Experiments in depicting the flow with streamers[1] indicate quite clearly the difficulties of dealing with this problem by mathematical formulae. On the other hand, there is a trend toward minimizing these field disturbances by a smooth design of the dock building and especially by the use of doors which exert a minimum of aerodynamic influence when opened. These thoughts were controlling in the design of the airship dock at Akron, Ohio (Fig. 32). When the ground handling of an airship into and out of the leeward door of a dock is interfered with by aerodynamic effects in the wake of the building the windward doors are sometimes partly opened, thus permitting some direct passage of air through the dock.

CHAPTER VI

OUTSTANDING PROBLEMS

It would be going far beyond the intended scope of the present work if any attempt were made to expose all of the respects in which aerodynamic (and certain aerostatic) phenomena have a bearing on the design of airships. They are so intimately interwoven with structural, mechanical, and navigational considerations that a design handbook would be required to do justice to the outstanding problems in this domain. If, nevertheless, a few such problems are mentioned here, it is done with a full appreciation of the arbitrary character of the selection, and in the thought that they may furnish some suggestions to students and investigators interested in carrying forward the boundaries of our knowledge on this subject.

Man's knowledge of an art can never be complete and there is no exception in the case of Airship Design. The airship designer and the airship operator are continually facing problems[2] which must be solved if airships are to be continually improved. As in all arts the major difficulties must either be solved or skillfully circumvented and the frequency of recurrence of a problem, or the difficulty of dismissing it from practical design considerations determines its lasting importance.

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[1] Krell, O., Zeitschr. f. Flugtechnik u. Motorl., pp. 401—438, 1928 (also U.S. N.A.C.A. Technical Memoranda Nos. 512 and 513, 1929).

[2] Richmond, V. C., Airship Research and Experiment, Journal of the Royal Aeronautical Society, October 1926; Scott, G. H., Research Problem in Airship Development, Journal of the Royal Aeronautical Society, April 1926; Fulton, G., Airship Progress and Airship Problems, Journal of Americal Society of Naval Engineers, February 1929; Arnstein, K., Fulton, G., Hunsaker, J., and Kármán, Th. v., Daniel Guggenheim Airship Institute, Publication No. 1, 1933; Munk, M. M., On the Problems of Progressive Airship Research, Daniel Guggenheim Airship Institute, Publication No. 3, 1935.

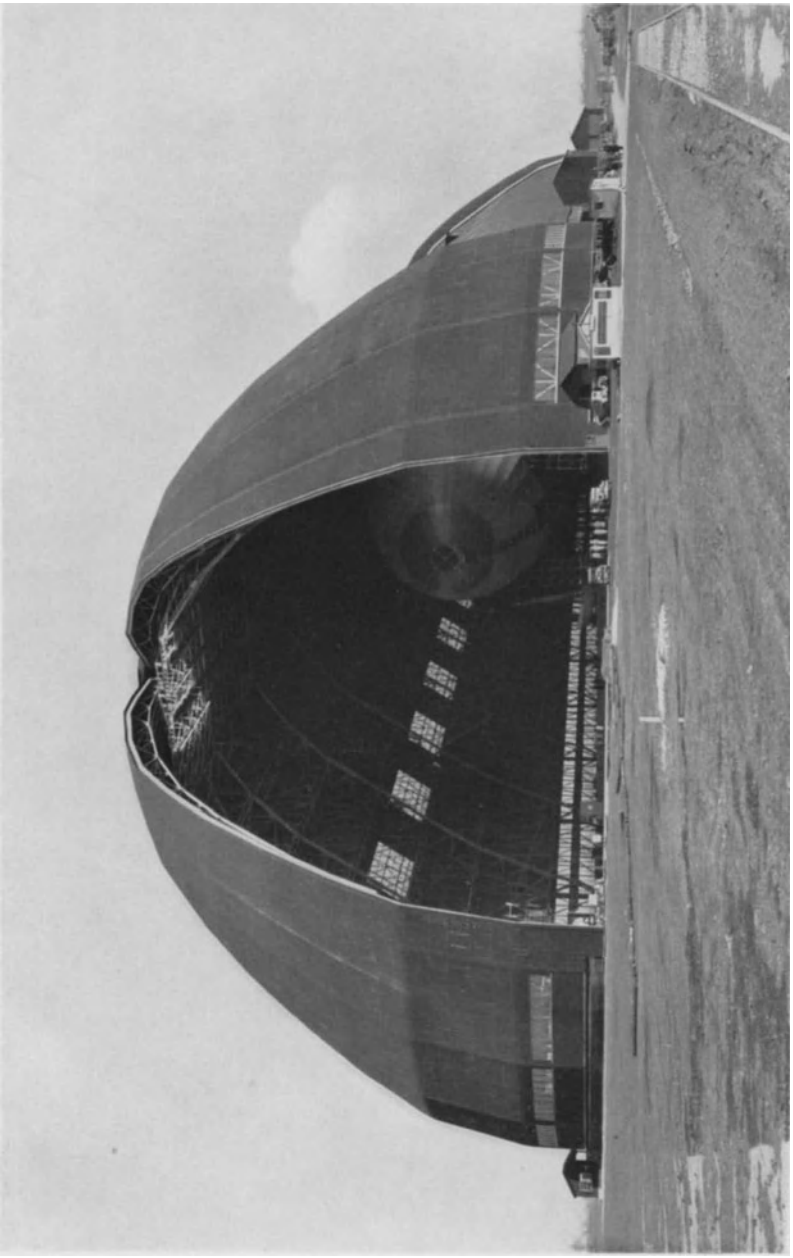

Fig. 32. Modern Airship Dock.

In this connection the acceptance of a *routine method* for the application of aerodynamic data to design procedure and especially to the stressing schedule or assignment of safety factors for various loading assumptions, such as is already the practice with heavier than air craft, must still be regarded as an outstanding problem with lighter than air craft. In fact the appreciative judgment and balanced valuation of partly conflicting consequences of theoretical loading assumptions constitutes one of the most difficult problems confronting airship designers. If, therefore, thumb rules—rules expressing in general terms of a few parameters, such as size and speed and experience coefficients, the "practical" loading assumptions for such problems as the required nose stiffening, an equivalent bow force representing gusts likely to be encountered, a standard bow force to represent average mooring stresses, or even a good "all around" maximum bending moment to cover all possible events—if such rules are sometimes found in the literature of the subject they must be taken for what they were intended, *viz.*, an attempt to evaluate the order of magnitude found in successful designs. Any more searching or responsible procedure will inquire into the manifold aerostatic and aerodynamic variables and their individual as well as combined influences upon the stability, controllability, strength and safety of the ship as a whole and in all its vital and essential parts.

The large variety of possibilities has been a barrier to airship research. For instance in spite of the amount of theoretical and experimental research work which has been done on the aerodynamics of airship hulls, there is still a need for dependable information on the distribution of the air forces over the hull and empennage during certain conceivable flight conditions. Up to date it is a fact that designers have not always obtained as much full flight experimental data as was to be desired.

Difficulties lie in the need for a full *synchronization of all flight data* from all observation points, and for the record, along with the observed data, of such flight characteristics as airspeed, rudder and elevator positions, together with the ship's instantaneous position, attitude, rotation and acceleration. All of these flight observations and the desired test maneuvers must be executed with due regard for the safety of the ship and generally with the stipulation that they shall place no restrictions upon normal operation.

One of the basic problems, therefore, is the continued development of equipment and of experimental technique which will aid in obtaining this greatly needed flight information without undue interference with service operation, unless airships can be made especially available for an extensive program of flight testing.

The research required runs through the whole range of experimental technique from the accurate weighing of a complete 100 ton airship to the delicate measurement of structural stresses in an enormous number of members under conditions of flight through turbulent air of measured characteristics. The importance of the various design factors in determining the overall strength and the efficiency of the airship is known to the designer and he uses this knowledge to guide his research, keeping the goal of a well balanced airship design always foremost in his mind. In many cases over-caution regarding safety against one contingency may invite increased liability with regard to another[1].

The development of a complete *theory of dynamic lift* on airship hulls and fins under small, moderate, and large angles of attack with and without proximity to the ground, would appear welcome in order to give theoretical confirmation and interpretation to the results of delicate experiments.

The same is true regarding the theory of *air forces produced by gusts* although here progress will equally depend on the furtherance of elaborate experimental research into the nature and texture of gusts such as they actually occur.

Step-by-step investigations into what happens when a ship whether in flight, moored or towed, is *struck by a gust* of any assumed type, but with due regard to the variability of the dynamic stability characteristics with the varying phases of motion, should give valuable results. Such studies offer many interesting problems although their value may seem somewhat limited to the specific case and ship under investigation.

An extension of the *theory of airship drag* and of the velocity field in the boundary layer around the airship hull, perhaps including considerations of propulsion and the most advantageous placement from a drag standpoint of radiators and heat transfer apparatus, is a promising field of research especially in the light of the advances of knowledge recently made regarding the nature and laws of the impulse transmission in the boundary layer in general.

The problem of *retaining buoyancy equilibrium* despite the consumption of large quantities of fuel has been attacked in many ways. The recovery of ballast in flight has been tried by the chemical and hygroscopic collection of water from the air, the collection of materials and substances in the exhaust gases, the catching of rain water and the lifting of ballast from the surface. In ships built to date the most successful solution has been the recovery of water from the engine exhausts but it has been replete with problems since the structural and thermal efficiency requirements are antagonistic to weight and drag economy.

The "Graf Zeppelin" has avoided this problem, eliminating it at its source by the consumption of a *gaseous fuel* having a density about equal to that of air. The "space efficiency" of fuel gas ships is excellent but

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[1] ARNSTEIN, KARL, Über einige Luftschiffprobleme, Zeitschr. f. Flugtechnik u. Motorl., January 14, 1933.

there may be offsetting disadvantages in the added fire hazard; also the weight of the suspensions and fuel ballonets and the surging must be allowed for. Ballast is not likely to be available in as generous a quantity as on a ship burning liquid fuel and equipped with exhaust water recovery apparatus.

In assisting the ship to rise when heavy or to land when light, *tiltable* propeller arrangements have shown considerable merit. Since some forward speed may exist in combination with vertical thrust, a suitable theory for the determination of the action and stresses on propellers under large angles of slant inflow becomes of interest, as with travelling helicopters.

Similarly studies of the practice of decelerating from flight speed when making an approach for landing by *reversing the propellers*, requires a knowledge of the theory of a propeller travelling at negative velocity and (unless reversible blades are resorted to) with attack on the back of the blade.

The problem of the best shape of the ship for lowest drag per volume housed may appear to be fairly well in hand. However, it is not possible to predict the extent to which the art of streamlining may be changed by some success in the *removal or repulsion of the boundary layer*. The problems of disposing of the dead air and of handling the appreciable volumes involved without incurring more loss in ducts and fans than the gain may be worth, constitute, however, possible stumbling blocks.

Similarly both theory and experiment may find an interesting field in an investigation directed toward the realization of *steering control* through means other than the conventional movable surfaces[1].

Technical improvement and simplification of the *technique for landing and housing* large airships will always be appreciated and may undoubtedly be helped by improvements in existing aerodynamic theories. Advanced knowledge along these lines cannot help but result in safer and more economical operations.

The original method of landing and housing rigid airships was on the water. Since then the trend of the development has been toward landing and housing them on land. However, *water landings* have occasionally again come to the fore. The merits of various suggestions made in connection with water landing depend upon the proper appraisal of the additional hydrodynamical problems. These arise with the necessity of holding and hauling the ship down and horizontally in winds and waves.

Gas for Airships. The inexplosive inert gas helium offers such evident advantages of safety over the inflammable gas hydrogen, that some means to offset its higher cost and lesser buoyancy would be greeted as most welcome. Whether, in some degree, this can eventually be achieved by mixtures of gases or otherwise is not directly a matter of

[1] Munk, M. M., On Problems of Progressive Airship Research, Daniel Guggenheim Airship Institute, Publication No. 3, 1935.

aerodynamics, but inasmuch as the retention of the gases introduces aerostatic problems, it may properly receive present mention.

The enumeration of such problems could be continued at great length, and as rapidly as the problems are properly stated, airship inventors will conceive new means and devices for overcoming them. If the historical record of the past is an index to the future, the application of these new inventions will introduce further problems. What is really needed is less invention and more application of the time proven principles which have already been successfully demonstrated. After all, the airship is not an abstract thing; it is a man made vehicle operating in an earthly medium and its functioning is governed by nature's physical laws. There is probably no more fitting thought with which to close this brief discussion of airship problems than Count Ferdinand von Zeppelin's motto,

"The forces of nature cannot be eliminated
but they may be balanced one against the other."

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DIVISION S
HYDRODYNAMICS OF BOATS AND FLOATS

By

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Paris

EDITOR'S PREFACE

Any record of progress in aerodynamic theory would be quite incomplete without some reference to marine aircraft—the seaplane and the flying boat. The performance of these craft involves two distinct phases, on the surface of the water and in the air, the one merging into the other at the moment of take-off from the water, or of returning to the same. When flying, the basic problems of marine aircraft are similar to those of land craft except as they may be modified in detail by necessary changes in form and proportion resulting in relative differences in the location of the center of gravity and in the thrust line of the propeller.

For these reasons, the treatment of the present Division is limited to what be termed the "marine" phases of performance. In Chapter I a general view is given of the special conditions imposed on a seaplane, both in repose on the water and during the periods of take-off from the water and return to repose on the same. Special note is made of the significance of the *step* and of the part which it plays in facilitating take-off from the surface of the water.

Then follows Chapter II dealing in further detail with the varying phases during take-off from and return to the water, and under varying conditions of wind and sea. This is followed in Chapter III by a study of theory exemplified by diagrams and dealing analytically and geometrically with this same general range of phenomena. Then follow chapters dealing with certain differences between airplanes and seaplanes with reference to the aerial portions; differences and analogies between forms for hydroplanes and for seaplanes; calculations of displacement and of stability, and varying conditions affecting the latter; rules of extrapolation; tests in a marine tank on reduced scale models; strength, and the present gaps between theory and practice.

The diagrams, in addition to furnishing illustrations of the text, give many forms of graphical construction adapted to the treatment of special problems arising in connection with the study of seaplanes or flying boats and their design.

Tabular matter is also furnished as an aid in following some of the forms of computation, such as those relating to displacement and the location of the metacenter. An extended table is also given based on the Dornier designs and proportions and intended to indicate the possibilities and limitations in the direction of increased size.

With the material presented in this Division supplemented according to wish or need by reference to the literature of the subject as given in the text and Bibliography, the reader should be able to obtain a clear view of the major problems presented by the hydrodynamic phases of marine aircraft and of their relation to the field of aerodynamic theory in its broader aspects.

W. F. Durand.

CHAPTER I

HISTORICAL, DIFFERENT TYPES OF SEAPLANES PRINCIPLE OF THE RAMUS STEP

1. Introductory. A *seaplane* is a mechanical construction, capable as is the airplane, of maintaining itself in flight with a weight greater than that of the air displaced and furthermore capable of resting on the water, of rising from the water, and of alighting upon the water. The history of the seaplane is relatively brief, dating back only to 1910 or 1912. Following the pioneer achievements, however, progress was rapid, and since 1918 seaplanes have been looped and otherwise stunted.

From the first there have been seaplanes with floats and seaplanes with boats (flying boats); in the first the floats contain neither mechanism nor personnel; in the second, the boat may contain both mechanism and personnel.

Intermediate types have furthermore been realized or projected, forming almost a continuous series of forms in which are found successively, the seaplane with boat hull of considerable length carrying the ensemble of tail surfaces, the seaplane with one or two relatively short boat hulls in which the tail surfaces are at some distance from the boat, the seaplane with relatively short boat hull or hulls directly carrying an ensemble of fuselage and tail surfaces, and finally seaplanes with one or several floats separate from the ensemble of fuselage and tail surfaces.

To this classification based on the longitudinal aspect, there might be added another based on the transverse distribution and showing various forms intermediate between a central float associated with two lateral auxiliary floats, three equal floats or again two lateral floats only.

In a seaplane, there is an aerial sustaining structure, a part which may be immersed in the water and a propulsive unit. In the development of the seaplane, progress thus far has depended primarily upon the advance in the aerial structure and in the propulsive unit. These in turn have been made possible through the great development in land planes and through the many "sporting" competitions permitting the progressive improvement of this type of construction. From this condition, it has resulted that the part immersed in the water has, for a considerable time, suffered relative neglect and has been considered as a technical detail of secondary importance. The result of this has been the development of a series of seaplane constructions not having altogether satisfactory nautical qualities.

It is only during the past ten years that adequate recognition has been given to the need of a study of seaplane floats and boats carried out with the same care as in the case of ship design on a large scale. It is now, however, well recognized that the study of the nautical parts of a seaplane should be carried out with regard to both the hydrodynamic and the aerodynamic problems involved; the former in order to satisfy the many conditions imposed from this point of view and the latter in order that when in the passive state, in the air, the floats or the boat shall prejudice in minimum degree the aerodynamic qualities of the construction as a whole, either from the point of view of sustentation or propulsion.

2. Conditions Imposed on Seaplanes. Beyond the conditions imposed by the act of breaking clear of the water, flight and return to the surface of the water, a seaplane must meet other conditions regarding the state of repose on the water, either smooth or rough, conditions relative to movement on the surface of the water either smooth or rough, and either by its own power or by towing.

The conditions imposed by the state of rest on the water are those of any floating body. With smooth water, the examination of these conditions requires only an acquaintance with general laws of the equilibrium and the stability of floating bodies, and of the resistance offered to the wind by the superstructure.

The conditions imposed in connection with movement on the surface of the water, either under power or by towing, permit of study by methods already well known for boats of dimensions similar to the boats or floats forming the nautical part of seaplane construction.

The conditions imposed by flight are the same as those for an airplane carrying a heavy load, at the same time bulky and placed well below the axes of the propellers.

In a summary examination of the general subject, the features characteristic of the seaplane will then relate to the special phenomena which present themselves during the periods of transition from water to air or inversely—periods of takeoff from the surface of the water and of return to the same.

Special Conditions During Take Off From and Return to the Water. In the period of take-off, the seaplane passes progressively from a condition in which the buoyant force of the water just equals the weight, to a condition in which all hydrodynamic forces vanish; in other words, the static and dynamic forces due to the water are progressively replaced by forces due to air in relative motion acting on the surfaces of the structure. The seaplane cannot rise unless this relative motion is sufficiently rapid to furnish complete sustentation. It must therefore acquire, while still in contact with the water, a speed sufficient for sustentation and this speed is high in relation to the dimensions of the boat hull or floats; that is, the coefficient ר[1] $= V/\sqrt{g\Lambda}$ is relatively high.

The known facts regarding the resistance of floats of normal ship or boat form show immediately that, in order to realize such speeds with such forms, far greater power would be required than could be installed in a structure of this character. It has, therefore, been possible to meet the necessary conditions only through the use of a special underwater form characterized by the presence of the *step.*

Step. This step is formed as an abrupt discontinuity or jog in the vertical direction on the lower part of the hull form, such that the part forward is more deeply immersed than the part aft. The step is of advantage only in case the boat is moving at a speed characterized by a high value of the coefficient ר. For such speeds it has been known for nearly a century that a general decrease in the displacement of the boat form will result. The movement resulting in this decrease of displacement comprises a change in the angle of inclination of the hull to the horizon together with an elevation of the center of gravity with reference to the water level.

The search for forms with maximum lift has led to the application of the step to racing boats, hence called "hydroplanes" and which have been, in fact, the forerunners of the hull form for seaplane structures. In a hydroplane in motion, the water does not act by static pressure alone, but also by a vertical component dynamic force analogous to that of the air on the wings of an airplane.

In the period of gradually increasing speed on the water, the seaplane moves first as an ordinary boat and then as a hydroplane. There is, therefore, a period of decreasing displacement and a period of hydroplaning. During the period of decreasing displacement the wings of a seaplane provide a relatively small sustaining force.

The *period of hydroplaning* is that during which the seaplane moves over the surface of the water almost without displacement. The speed increases continuously, the wings provide in consequence a sustaining force continuously greater, the plane lifts further and the resistance due to the water decreases accordingly.

[1] The Hebrew letter *resch.*

The instant of take-off corresponds to the condition in which the air sustaining forces have become sufficient to render further support from the water unnecessary. The plane, from this moment, behaves as an ordinary airplane. During the period of return to the water, analogous phases, though not exactly inverse in character and order, are found.

During these two phases characteristic of the seaplane, numerous problems present themselves: first the study of the forces both aerodynamic and hydrodynamic to which the structure is subjected, the study of the changes which may be effected in these forces through various maneuvers, the study of the regime of the engine, and finally the study of the local forces between the water and the surface of the hull. The problem as a whole requires then studies of very diverse character, among which we shall here chiefly consider those of a hydrodynamic nature concerned with the take-off.

The first question relates to the sustentation which may be expected from diverse forms of underwater body for floats or boats. The order of magnitude of this sustentation can be estimated, as well as the resistance to motion in the case of a hydroplane. This will serve to show the difference between the propulsive resistance of a hydroplane and that of the usual boat or ship form.

3. Principle of Ramus. Let us consider, with Froude, a plane moving with a speed V and resting on a free water surface, making with the latter a small angle θ. This plane is subjected to a system of forces of which the components normal and tangential may be expressed (as to order of quantity) as follows:

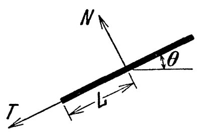


Fig. 1.

$$N = 55.35 \; L\, b\, V^2\, \theta$$
$$T = 0.186\, L\, b\, V^2.$$

L being the length in contact with the water and b the breadth of the surface, Fig. 1. The vertical and horizontal components are then, taking $\cos\theta = 1$ and $\sin\theta = \theta$, as follows:

$$F_v = N - T\,\theta$$
$$F_h = N\theta + T.$$

These should equal, respectively, the weight of the structure and the thrust of the propeller, when the regime of hydroplaning is established.

The most favorable conditions will then be found when the ratio of thrust to weight is minimum. This ratio is

$$\frac{N\,\theta + T}{N - T\,\theta} = \frac{55.35\,\theta^2 + 0.186}{55.35\,\theta - 0.186\,\theta} \tag{3.1}$$

This varies as $55.35\,\theta + 0.186/\theta$, and is minimum for $\theta = \sqrt{.00336} = .058 = 3^0.32$. For this value of θ the ratio of thrust to weight, by substitution in (3.1), is found to be 0.120. We must therefore anticipate a resistance of the order of 120 kg. per ton of displacement.

With normal ship forms a resistance of the order of 50 kg. per ton of displacement is only encountered with the very highest nautical speeds—speeds at the limit of possibility for modern marine construction. From this it is easily seen that lifting planes would be of small value in connection with ordinary ship propulsion. Attempts in this direction have been entirely without useful result. But if very high values of the speed ratio ℸ must be realized, the seaplane here has the advantage that for its wings the ratio of thrust to weight does not increase with the speed as in the case of marine craft.

Furthermore, the figure of 120 kg. per ton found above by a very rough approximation is less than that actually obtained for hydroplanes (138 kg.) while for seaplanes the figure rises to the vicinity of 250 kg. per ton. The principal cause of this larger figure for seaplanes than for hydroplanes is found in the larger angle of attack for the former. In the seaplane the angle of attack is approximately three times the angle for optimum conditions.

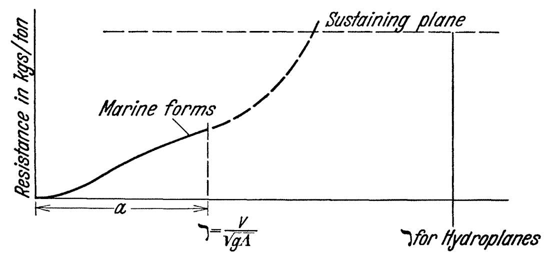


Fig. 2. Range of values for speed ratio $V/\sqrt{gA}$. a denotes approximate limit for Destroyer forms.

Figure 2 shows schematically the difference between the problem of a theoretical hydroplane and an actual ship and shows why the high speed required for the hydroplane phase of the seaplane cannot be realized through the use of normal marine forms.

It is to be here noted that in examining the conditions for hydroplaning, steady conditions have been assumed, that is to say, the plane is supposed to be in motion at uniform velocity. Between the period of displacement reduction and of flight, is found this phase of hydroplaning which, *a priori*, seems difficult to bring into harmony with the conditions of uniform motion.

Under steady motion conditions, the system of waves formed by the boat, and of which the continued development forms an important element of the resistance, accompanies the boat with the same speed as that of the boat itself. In a fast take-off with a seaplane, however, the plane must overrun the system of waves. There must, therefore, be expended not alone the energy required to extend a wave system previously formed, but also the energy required for the continued generation of a new system.

The procedure to be followed in the study of these problems may be found, as in most of the problems of practical hydrodynamics, in three different directions—pure theory, experiments with full scale structures, experiments with reduced scale models. Each of these methods has its limitations. Pure theory is thus far unable to give numerical values with the degree of precision needful for actual construction; experiments with full scale structures is time consuming, costly and sometimes dangerous, experiments with models may lie under the suspicion of not correctly representing all the conditions of the full scale structure.

4. Floatability and Stability at Rest. In a seaplane the necessity of floating on the water at rest imposes the provision of a certain water-tight volume and the condition of stability imposes the realization of a certain moment of inertia of the waterplane area.

If this moment is insufficient it can be augmented by the use of outrigger floats, thus preventing a plane unstable in the upright position from exceeding an inclination of a few degrees.

In flight the water-tight hull required for flotation augments the resistance and should therefore be given a form suitable for small resistance in the air. If the necessary volume is formed by a single hull its moment of inertia with reference to the longitudinal axis will be less than that with reference to a transverse axis. To meet this condition the single hull may be replaced by an ensemble of several hulls.

In a case of two hulls the arrangement is called a twin float or catamaran. A still larger number of hulls may furthermore be considered. Mixed arrangements are found where use is made of a boat hull with outrigger floats or boat hull with fin structures.

As a general rule for the same useful volume and moment of inertia of waterplane, the resistance in flight is diminished by the multiplication of units; on the other hand the weight of the structure is increased for the two reasons that the total exterior surface increases with their number and the weight of the connections with the wings is also greater.

This last consideration is, however, not of great importance under smooth water conditions, but becomes important if one considers either a seaplane resting on rough water or the forces which develop in connection with non-symmetrical take-off or return. The need of protecting the propellers and carburetors from the effects of spray, furthermore, necessitates placing the wing structure and the engines at a considerable height above the hull, and this results in a considerable increase in the weight of the necessary connections. Furthermore, in the case of a seaplane on rough water it is necessary to take account of the inertia of the entire mass of the structure with relation to the longitudinal axis passing through the center of gravity. It is then seen that the forces developed in rolling are strongly augmented by the multiplication of hull elements.

A second effect of the increase in total weight of the hull structure is the lowering of the center of gravity of the structure as a whole. In flight this effect would be unfavorable for the two reasons that if the engine slows down the plane will be in danger of losing speed rather than of beginning a glide and because in descending to alight on the water the pilot may be compelled to keep his engine in operation and to reach the condition of hydroplaning with a speed endangering the strength of the hull. Final mention may be made of secondary effects due to the fact that in a seaplane the general center of gravity is lower than the center of drift in flight. With the present limitation of the study of the seaplane to conditions other than those of flight, it will suffice to have mentioned these different points in order to show that the study of a seaplane cannot really be limited to a study of the phase of take-off from and return to the surface of the water. Rather the nature of the problem imposes the need of a study of all conditions through which the seaplane may pass; in such study contradictory requirements are met with, since in flight it would be desirable to raise the center of gravity, while on the water it would be of interest to lower it.

5. History of the Step. The effect of a step on the action of the hull of a boat is so fundamental in relation to the performance of a seaplane, that it may be of interest to recall the fact that the first observations dealing with the phenomena of the decrease of immersion through dynamic means go back to John Russell in 1834 and that in 1852 a patent relating to this subject was taken out by Apsey. When Ramus proposed the use of the step with ordinary ship forms, Froude was able to show that no advantage would result for the types of construction practicable at that time. In 1872 the ratio of horsepower to weight of machinery was about 7 horsepower per ton and it was impossible to foresee the tremendous advance which, in less than sixty years, has raised this figure to 3,000. With the decreased weight of machinery, the speed ratio $\urcorner$ has risen continuously. In 1881 Raoul Pictet and Ader made certain tests and in 1894, the Forban, under test, showed clearly the phenomena of decrease of immersion through the action of dynamic forces. Further steps before the realization of the seaplane itself, were taken by de Bonnemaison, by de Lambert and by Tellier. In 1905 Forlanini succeeded in lifting a boat completely from the water at a speed of 70 km/hr by the use of immersed sustaining planes and in 1906 Crocco and Ricaldoni realized absolute hydroplaning. Six years later, the seaplane had achieved take-off from the water and flight.

CHAPTER II

DESCRIPTION OF THE DIFFERENT PHASES OF TAKE-OFF FROM AND RETURN TO THE WATER

1. Normal Take-Off. The simplest case is that in which the water is absolutely calm and the air without wind. We assume first in these conditions: the engine in operation, the first effect of the thrust of the propeller is to cause the plane to nose down. Progressively the plane gathers speed and then begins of itself to rise forward. In planes with a single step and chiefly according to the relative positions of the center of gravity and of the step, the amplitude of the tilt nose-up may reach a limit of equilibrium or on the other hand pass through a maximum. For planes with several steps it may happen that the upward and backward tilting will increase continuously up to the take-off. In case the tilting is limited, the plane would continue to move on the water as an ordinary boat without mounting on the step. It will be necessary then for the pilot to use his controls to ease the plane and bring it upon the step. When the tilting passes a maximum and then descends to a lower value it is said that the plane passes of itself onto the step. In this condition the plane experiences, so far as the water is concerned, a relatively low resistance. It continues to hydroplane with a speed continuously increasing, and receives from the air a sustentation larger and larger, resulting in a decrease of the disturbance on the free surface of the water and in consequence a diminution of hydrodynamic resistance.

During this period the attitude varies very little. The speed increases continuously up to the point where the speed being sufficient, the air sustentation equals the weight, following which the plane rises. The duration of the period of hydroplaning may be diminished by increasing the upward tilting of the plane by means of the elevator in order to augment the lift. This manoeuver, however, must be carried out with caution in order to avoid the loss of speed which would cause the plane to fall back immediately upon the water.

2. Varying Conditions. The take-off which has just been described is of normal type such as would obtain in calm air with a plane of known type with motor power generously proportioned and under normal loading conditions. Several particular cases may be noted due to the non-fulfillment of one or another of these conditions.

Supposing always the sea calm, the succession of phenomena which has been given as normal may be interrupted either because some one of the various phases does not properly develop or because it may continue for too long a period of time or because secondary difficulties may present themselves.

If the entrance is too short the plane may tilt downward excessively. If on the other hand the entrance is too blunt the period of mounting upon the step may become too long and the same conditions may result with insufficient wing surface.

Certain planes before mounting on the step undergo a certain pitching movement. It has been assumed that this movement is caused by too flat a bottom or a step too far aft. If the under surface is too much developed forward, the float may leave the water at a sharp angle of upward tilt and at a speed insufficient to secure aerodynamic sustentation.

The plane then falls back upon the water, loses speed, may dip a wing-float, or turn on the side.

It may happen during the period of hydroplaning that the plane may develop a series of jumps more and more marked. The plane is then said to "porpoise". This defect is attributed to a position of the step too near the front. The jumps in this case are in the attitude nose high. Another type of jumping without marked inclination is attributed to too large an under surface area in relation to the surface of the wings or to an angle of incidence of the bottom of the boat too large in relation to the incidence of the wing.

Again it may happen that the waves produced by the boat driven back by the wind may by their blows damage the propeller or drown the carburetors. These defects are still further augmented when the sea is not entirely calm or in the case of any considerable wind or when these two conditions are united.

3. Take-Off With Bad Weather. The circumstances which may develop with bad weather are extremely variable and upon them must depend the particular maneuvers to follow. Here still more than in take-off under conditions of calm, the tactics to be followed depend largely on the skill of the pilot and no general rule can be formulated. For planes of moderate dimensions when the wave system is well formed and the wind is not strong, pilots usually endeavor to take off across the wind, maintaining the plane on the same wave crest, thus avoiding shock from the succession of waves. For larger planes this maneuver may result in the immersion of one of the wing floats, thus preventing the necessary acceleration. Take-off in a direction perpendicular to the wave crests is then necessary. In actual practice there can be no standard maneuver and it must be left to the quick judgment of the pilot to determine in each case, in accordance with the state of the sea, the best plan to follow. In general, with a strong choppy sea with wind approximating strength 4, a seaplane should take off with the least practicable throwing of spray. High waves with little wind may prevent a plane from taking off with full load, but a direct head wind alone is on the whole favorable. On the whole it is desirable that planes should take off across the waves. Planes with skimmers or with large floats near the boat hull have an advantage from this viewpoint. On the contrary, planes with two floats widely separated, cannot, in general take off across the waves.

4. Normal Alighting. With the plane descending normally, the pilot may propose to alight on the water tangentially, as in a normal landing on the ground. Phases the inverse of those in take-off will then present themselves. The pilot uses his elevator, the plane comes in contact with the water, first toward the stern, either extreme aft or on the step. The latter is preferable because it avoids a severe pivoting with the possibility of a rebound due to the resistance of the water. The speed decreases, the plane continues to skim with decreasing speed until, no longer supported by the dynamic reaction of the water, it settles rather abruptly into the water and returns to the conditions of an ordinary high speed boat. In general it is desirable to alight in an attitude nose high, rather marked, in order to touch the water at low speed and seaplanes should be so designed as to avoid any severe return to the attitude tail high.

5. Diverse Conditions. During the period of first contact with the water, violent shocks may result with changes of attitude on the water. These variations of attitude are rapid when the center of gravity is too far longitudinally from the step, either forward or aft. If the step is too far forward, the plane rises forward as soon as it touches the water and then may ricochet, perhaps several times, with possible loss of speed. If, on the contrary, the step is too far aft, the plane, on contact with the water, rises at the tail; this motion is followed by fore and aft rocking movements, very trying to the structure of the hull. Rebounds from the water on alighting are of the same two kinds previously considered and arise from the same causes.

6. Alighting on Rough Water. The difficulties here are similar to those indicated for take-off. The greatest danger is in an attitude insufficiently nose high. In such case the hull of the boat may receive from the water a blow of force sufficient to throw it again high in the air with danger of fall with loss of control. The danger in the inverse case is that, meeting with insufficient repulse from the water, the plane may plunge under with fatal consequences.

7. Graphical Representation. A somewhat greater precision can be introduced into the description of these phenomena by the use of a graphical representation in which it is assumed that for each speed, the plane is under steady conditions. A horizontal speed axis is then laid off and for each speed it is assumed that the plane is in equilibrium under the effect of its weight, aerodynamic forces, hydrodynamic forces, and the trust T of the propeller. Inertia forces are neglected. The weight W of the plane is known and applied at the center of gravity. The remaining forces depend on the speed, both with regard to their magnitude and their location.

For the aerodynamic forces, the influence of change of inclination is neglected and these forces, considered as depending on speed alone, are taken as equal to:

$R_y V^2$ in the vertical direction

$R_x V^2$ in the horizontal direction

For the hydrodynamic forces, account must be taken of the fact that these depend not only on the speed but on the attitude of the plane relative to the indefinite plane of the water surface. It is assumed, then, that these forces may be decomposed into a vertical component corresponding to the displacement of the immersed part and an inclined force due to the form of the immersed surfaces, the latter having again two components

H_y in the vertical direction

H_x in the horizontal direction

For the equilibrium of the plane, it is necessary that the general resultant of this system of forces and the general resultant moment shall both be zero. With the two components of the general resultant, we thus obtain three relations, Fig. 3,

$$R_y V^2 + B + H_y - W = 0$$
$$R_x V^2 + H_x - T = 0$$
$$(b R_x V^2 - a R_y V^2) + B u - (\beta H_x + \alpha H_y) - T \delta = 0$$

In these equations and with reference to G, the center of gravity, α and β are the coordinates of the step, a and b those of the point of application of the aerodynamic forces, δ the distance of the propeller shaft from G, and u the horizontal distance of the force of buoyancy B from G.

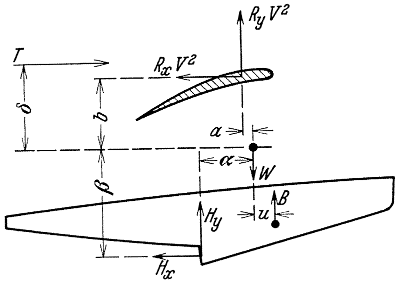


Fig. 3.

To represent graphically these three equations we may employ three diagrams in which may be plotted as ordinates respectively, the vertical components, the horizontal components, and the moments relative to the center of gravity. The axis of abscissae in these three diagrams is an axis of speed V. In order to establish the diagram of horizontal forces, we must have the law of variation of the thrust of the propeller as a function of the speed. We shall therefore first examine this law.

8. The Thrust of a Propeller Under Constant Torque With Variable Speed. The problem of a propeller under constant torque Q presents itself during the period of take-off of a seaplane. The reason for this condition is as follows:

For a seaplane engine the curve of power as a function of revolutions with full throttle and given altitude is nearly a straight line passing through the origin and in order to pass through the critical period of take-off, the pilot runs with full throttle. For a theoretical examination it is then sufficient to assume that the thrust given by the propeller of a seaplane will be determined by the condition necessary to give at any speed a constant torque. The problem is thus reduced to the determination of the law of thrust as a function of speed with constant torque.

To solve this problem use may be made of the results given by systematic experiments on propellers, experiments which give as a function of nD/V the values of T/n^2D^4 and Q/n^2D^5, D being the diameter of the propeller.

For a given propeller a series of values of nD/V (or of V/nh) is taken, h being the pitch of the propeller. To each of these values corresponds a value of T/n^2 and a value of Q/n^2, given by the experimental results, and since Q is known, or assumed, there results the value of n corresponding to each value of V/nh and since T/n^2 is known, the value of T follows.

The computation is developed as shown in the table, the first three columns being given by experimental results *(proportional figures).*

$\frac{V}{nh}$	$\frac{T}{n^2}$	$\frac{Q}{n^2}$	n^2	n	V	T
0.9	11	26	386	621	559	42
0.8	33	40	250	500	400	82
0.7	54	50	200	447	312	108
0.6	73	56	179	423	254	131

It remains to trace the curve connecting T and V.

When the pitch ratio is moderate or low (for example, 0.7) the result approaches an inclined straight line as in Fig. 4. When the pitch ratio is relatively high (for example, 1.3) the result shows a curve descending at first slowly and with a horizontal tangent at the start, as in Fig. 5. In general the thrust decreases with increase of speed and the values

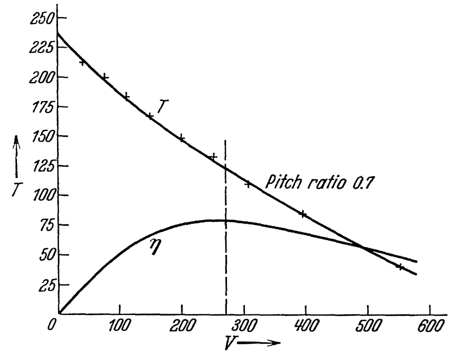


Fig. 4. Thrust and efficiency of propeller under constant torque: pitch ratio = 0.7.

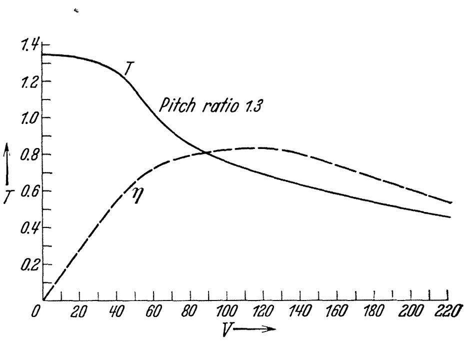


Fig. 5. Thrust and efficiency of propeller under constant torque: pitch ratio = 1.3. Ordinate scale in terms of $V = 60$ as unit.

of T for $V = 0$ and of V for $T = 0$ can be readily determined, at least to the general order of quantity.

The diagrams show two such curves for pitch ratios 0.7 and 1.3. For pitch ratios still higher it may result that the curves for T will rise at the start. This case, however, is only found with racing machines. The diagrams also give in addition the values of the efficiency η.

9. Determination of a Curve of Take-Off. Having in hand now the values of the thrust T as a function of the speed, three graphs may be established. For the first, Fig. 6, we trace a horizontal $W W$, giving the value of the ordinate W equal to the weight of the plane. This ordinate must be equal to the sum of the vertical forces due to the air and the water. The curve representing the change in the force due to the air will be a parabola if the coefficient R_y is constant, that is to say, if the wings have a constant angle of attack, if they are always in the same position with regard to the surface of the water, and if they are non-deformable as a whole and not influenced by the propeller.

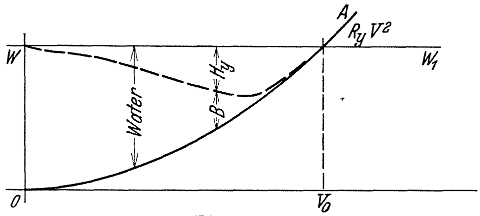


Fig. 6.

In reality the curve of air sustentation will have a form nearly parabolic which we may trace as $R_y V^2$. This parabola and the straight line $W W$ intersect at a point of which the abscissa V_0 is the speed for which the air sustentation will equal the weight. This is the minimum speed for flight at sea level.

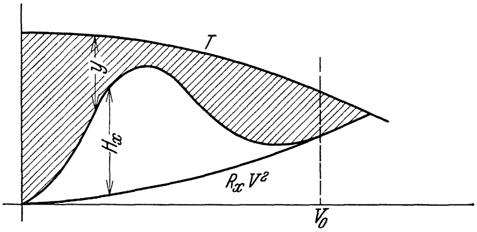


Fig. 7. Curves of propeller thrust and air resistance. The hatched area gives ordinates of water resistance plus the thrust available for acceleration.

The vertical acceleration being neglected for speeds between 0 an V_0, the difference between the ordinates of the straight line and of the parabola will represent the sustentation due to the water. This may be divided into the two parts, one due to static upward thrust and the other due to dynamic reactions. We know only that for $V = 0$ the static upward thrust is exactly W and that for $V = V_0$ this force vanishes.

On the second diagram (Fig. 7) let us trace the curve T representing the horizontal thrust of the propeller and the parabola $R_x V^2$ representing the air resistance of the plane. The difference of the ordinates of these two curves will represent for each speed the sum of the water resistance of the hull and of the thrust available to overcome horizontal inertia. The water resistance is, of course, 0 for $V = 0$ and for $V = V_0$.

We can form an idea of the character of these relations by experiments upon models. The resistance passes through a maximum R_m for a certain value V_m of the speed. We shall suppose here that the curve of total ordinates $H_x + R_x V^2$ does not intersect the curve T. There remains therefore for each speed an overbalance of thrust and in consequence the plane undergoes a horizontal acceleration.

On the third graphic, Fig. 8, we represent the moments with relation to the center of gravity, laying off above the axis moments tending to lift the bow and below the axis moments tending to lift the tail. The propeller gives rise to a moment tending to lift the tail, varying with the thrust of the propeller and represented by the curve $T\delta$. The air resistances give rise to a moment tending to lift the head, varying with the speed, with the distance between the center of gravity and the center of air resistance on the wings and with the increase of resistance of the parts emerging from the water. This moment will vary over all with the square of the speed and for $V = V_0$ will become equal and opposite to the moment due to the propeller. The moment of the resistance due to the water varies likewise in consequence of changes of speed and changes of position of the resultant lift force due to the water. It is an easy matter to determine the location of this force in the two conditions of rest and take-off.

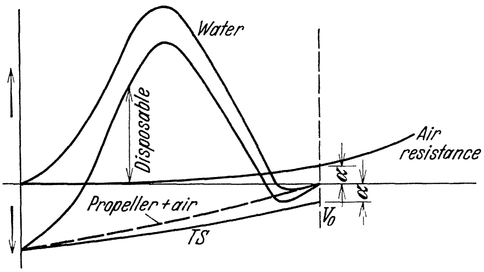


Fig. 8. Moments of forces and resistances during take off.

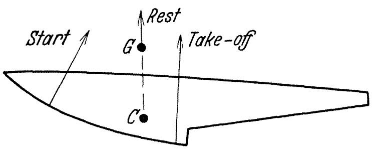


Fig. 9.

The moment, at first zero, begins with a direction tending to raise the bow (see Fig. 9) and with a considerable magnitude because both the lever arm and the resistance are large (although the speed is relatively low). At the end of the take-off the moment is of the reverse sign and tends to depress the bow, at the same time small in value because both the lever arm and the resistance are small (although the speed is large). These conditions develop as a consequence of the progressive reduction of the immersed part of the boat and of the consequent diminution of the sustentation required from the water.

In adding together with their signs the three moments considered, a curve is obtained giving the total acting moments. These moments must be balanced either by the action of the controls or by an inertia couple or by a moment of hydrostatic stability—that is to say, by a variation of the inclination of the plane as a function of the time.

It may be remarked that the curve of available moment is completed by a portion of the vertical axis passing through the origin representing the progressive growth of the moment tending to depress the bow between the two conditions of the plane—on the water with the propeller at rest and the propeller in motion at the instant of departure.

The second graphic indicates the facility of take-off and flight in relation to the power of the motor-propeller unit.

The third graphic indicates the difficulty which may be experienced in relation to the effort demanded of the pilot in his use of the controls.

10. Duration and Distance of Take-Off. The second chart provides for the determination of the duration of the take-off period. For this purpose, it is sufficient to evaluate the cross-hatched area, see Fig. 10, between the curves of water resistance and propeller thrust less air resistance. Hermann has indicated an elegant method for effecting this integration as follows:

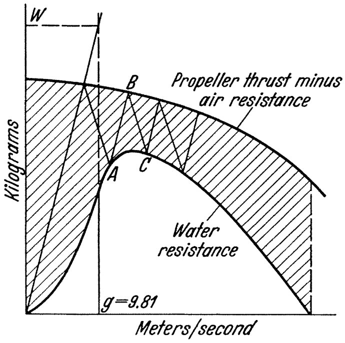


Fig. 10.

On the axis of W lay off the point W representing the weight of the plane. On the axis of speed in meters per second (or feet) lay down the point 9.81 (or 32.2). From the point determined by these two coordinates, draw a line to the origin. Then from the point of intersection with the upper curve, continue to draw between the two curves a series of intercepts having the same slope as the original line. The number of these intercepts will then equal the number of seconds required for the take-off.

Let us consider the course of events for the period during which the speed changes from V_1 to V_2, Fig. 11.

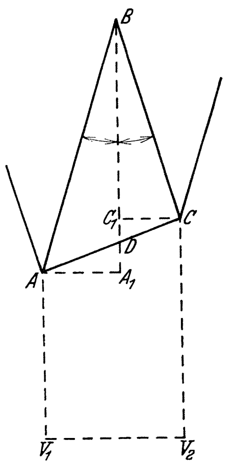


Fig. 11.

We may assume that in the change from speed V_1 to speed V_2 the average accelerating force is BD. Then the construction gives

$$\frac{BD - DC_1}{CC_1} = \frac{W}{g}$$

$$\frac{BD + DA_1}{AA_1} = \frac{W}{g}$$

Adding, $$\frac{BD}{CC_1} + \frac{BD}{AA_1} = \frac{2\,W}{g}$$

Again since CC_1 and AA_1 will, on the average differ but little from $(V_2 - V_1)/2$, we have

$$\frac{4\,BD}{V_2 - V_1} = \frac{2\,W}{g}$$

Furthermore, the mass of the plane, W/g, will move according to the law $$L = V_1 t + \frac{1}{2}\gamma t^2$$

where $$\gamma = g\frac{BD}{W} = \frac{V_2 - V_1}{2}$$

whence $$L = V_1 t + \frac{1}{4}(V_2 - V_1)t^2$$

$$V = V_1 + \frac{1}{2}(V_2 - V_1)t$$

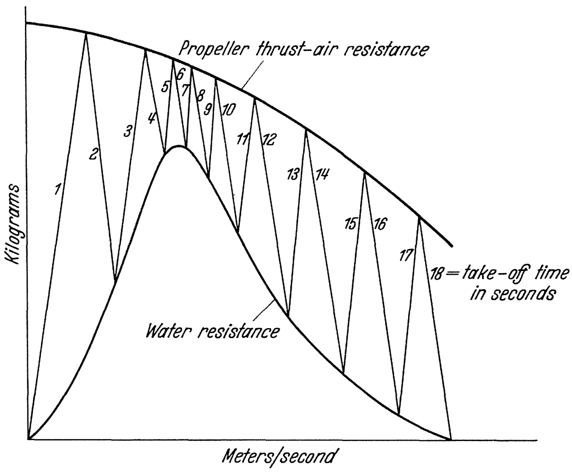


Fig. 12.

For $t = 2$, this gives $V = V_2$ showing that on this assumption, the time required to pass from the abscissa of A to the abscissa of C is two seconds.

On the scale of speeds, the abscissae of the summits ABC will then give the speeds at the end of one second, two seconds, etc. Each of these speeds continues for one second. By adding these abscissae, then, the sum will give approximately the distance run during the period of take-off. In the example given, Fig. 12, the total distance run was 143.3 m. and the duration was eighteen seconds.

CHAPTER III

DISCUSSION OF THE GENERAL PHENOMENA ARISING DURING THE PERIOD OF TAKE-OFF

1. Introductory. We have seen that the seaplane, during its period of contact with the water, passes through two phases of navigation, very different in character. At low speeds, it behaves like an ordinary boat. At high speeds it skims the surface. The instant which separates the first mode from the second is that when the water ceases to follow the hull clear to the after part; that is, the instant when there is suddenly formed a space free from water aft of the step. If we assume that aft of the step the free surface is not smooth, as in a train of waves, the question arises as to whether it becomes possible to predict at what speed this separation from the water will take place.

If this wave train surface is tangent to the bottom of the boat, the problem becomes that of finding under what conditions a wave train can be determined under a given direction and speed at a specified point of immersion.

We examine first the plane during the phase of normal boat movement, then the prevision of the moment of change from one phase to the other and finally, the phase of hydroplane movement.

In the resistance to movement of a seaplane during the first phase, there are a certain number of points in common with those met with in a study of the resistance of a boat of normal form.

It is known that a boat formed body in normal movement at the surface of the water gives rise to changes of level of diverse characters, forming systems or trains of waves. The conditions of propagation of such trains of waves depend on the force of gravity.

If we consider two ship-shaped forms, geometrically similar, and similarly placed with reference to the general water level, these various wave systems will not be similar nor similarly placed with regard to the ship nor with regard to the general water level, unless the Reech-Froude speed-length ratio, $\neg = V/\sqrt{g\Lambda}$, has the same value for both forms.

Theoretical studies of such trains of waves have been made for the case of movement in two dimensions, that is to say, for what may be called cylindrical propagation with right line crests at right angles to the direction of movement; and also for the case of movement in three dimensions; that is, for the general phenomena of changes of level in a free liquid surface with the forms of the crests arbitrarily specified.

For the case of two dimensions, the theory gives a satisfactory solution even when the changes of level are of finite amplitude.

For the case of three dimensions, the theory gives a solution valid only for very small changes in level. In very brief form, these two theories may be summarized as follows:

Two-dimensional Waves. The form of the free surface is a cylinder with trochoidal right section. Between the velocity and the length from crest to crest L is the relation,

$$\frac{V}{\sqrt{gL}} = \frac{1}{\sqrt{2\pi}} = 0.4$$

where V = velocity in meters per second,
L = length in meters,
g = 9.81 meters per second per second.

Such a system of waves cannot be used directly as representative of the transverse system formed by a ship shaped body, because they are periodic and continuous as a system from $-\infty$ to $+\infty$ while transverse waves as actually produced by a body of cylindrical form approach asymptotically the general level of the water at infinity.

The wave pattern and system formed by a ship is due to the progressive invasion of tranquil water by the system considered.

Such an invasion can be studied theoretically in the single case where the amplitude is small; but it may be assumed that the trochoidal system gives an approximate representation of the actual conditions about a body in movement on the water.

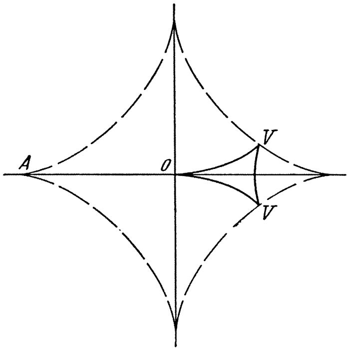


Fig. 13. Three-dimensional wave.

Three-dimensional System. Attention is directed to the form which the crests should take, considering a very small obstacle moving on the surface of the water and capable of creating all possible manner of wave systems, subject only to the condition that the system of crests shall be at rest relative to the obstacle; that is to say, that they accompany the obstacle in its movement.

If V is the speed of the movement, the speed of a train of waves moving in a direction inclined at an angle θ to the movement, will be $V \cos \theta$. To every value of θ will correspond a determinate wave length (assuming $L \sim V^2 \cos^2 \theta / K$). It is then readily shown that the equation to the line of the n^{th} crest from the origin is

$$y \sin \theta - x \cos \theta = \frac{n V^2 \cos^2 \theta}{K^2}$$

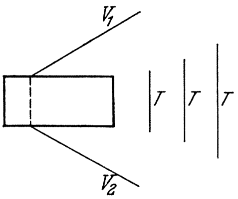


Fig. 14. Divergent and transverse wave system.

This is in the form

$$y \sin \theta - x \cos \theta = N \cos^2 \theta$$

It may be shown by well known methods that the envelop of these lines for N constant and θ variable is the arc of a curve which is, itself, the involute of a hypocycloid of four cusps.

Such a wave $O V V$, Fig. 13, created by the movement of a point O in the direction OA is readily observed with all sorts of water craft. It comprises a part $V O V$ called the divergent system together with the part $V V$ called the transverse system.

In the case of a seaplane in the gliding phase, these systems are reduced to their most simple form—two divergent waves $V_1 V_2$ and a transverse system $T T T$, see Fig. 14.

2. Trochoidal Waves. The equations (λ an auxiliary variable)

$$\left.\begin{aligned} x &= \lambda - A \sin \frac{2\pi}{L} \lambda \\ z &= \frac{\pi A^2}{L} + A \cos \frac{2\pi}{L} \lambda \end{aligned}\right\} \qquad (2.1)$$

represent the free surface of a trochoidal wave formed on a liquid originally at rest with its free surface in the plane $z = 0$. The vertical height is $2A$, the wave length L, the location of the center of the generating circle is at a height of $\pi A^2/L$ above the plane of still water, and the origin is at the point on $z = 0$ underneath the crest.

Let us now determine such a trochoid by the condition that at a given point at a depth h below the plane of repose $(z = -h)$, the slope of the surface shall be i. That is,

$$\frac{dz}{dx} = \tan i$$

To write this condition we take the derivatives $dx/d\lambda$ and $dz/d\lambda$ and then by division find the derivative dz/dx, thus:

$$-\frac{dz}{dx} = \frac{(2\pi A/L) \sin 2\pi\lambda/L}{1-(2\pi A/L)\cos 2\pi\lambda/L} = \frac{(2\pi/L)\sqrt{A^2-(z-\pi A^2/L)^2}}{1+(2\pi/L)(\pi A^2/L-2)} \tag{2.2}$$

The condition may then be written

$$-\tan i = \frac{2\pi\sqrt{A^2-(-h-\pi A^2/L)^2}}{L+2\pi(\pi A^2/L+h)} \tag{2.3}$$

If the velocity of the trochoidal wave is fixed, the length will be fixed likewise by the relation $L = (2\pi/g)\, V^2$ and in consequence (2.3) will determine A as a function of the given quantities i, h, and the velocity V.

No solution is acceptable unless the value found for A is less than $L/2\pi$, for otherwise the trochoid would have double points.

If i is very small, the equation for A gives

$$A = h + \frac{\pi A^2}{L}$$

The limiting case will be reached if this condition is satisfied for $A = L/2\pi$; that is, if $h = L/4\pi$. This is equivalent to the relation between h and the speed V, $V = \sqrt{2gh}$.

For speeds less than $\sqrt{2gh}$ there will be no trochoidal free surface tangent to the obstacle.

If i is not very small, the condition that $L/2\pi$ shall be a solution is found by making $A = L/2\pi$ in the equation

$$\tan^2 i\left[L+2\pi\left(\frac{\pi A^2}{L}+h\right)\right]^2 - 4\pi^2\left[A^2-\left(h+\frac{\pi A^2}{L}\right)^2\right] = 0$$

which gives

$$\frac{3}{4}L^2(1-3\tan^2 i) - 2\pi hL(1+3\tan^2 i) - 4\pi^2h^2(1+\tan^2 i) = 0$$

Let us apply this to an obstacle defined by

$$\tan i = 0.14$$

$$h = 3.45 \text{ cm.}$$

The equation in L becomes

$$L^2 - 32.6\,L - 680 = 0$$

An equation which has only a single positive root—a result holding for all cases of present interest for which $(1 - 3\,tan^2\, i)$ is positive. This root is $L = 47$ cm. The corresponding velocity

$$V = \sqrt{\frac{g\,L}{2\,\pi}} = 85.65 \text{ cm./sec}$$

and

$$A = \frac{L}{2\,\pi} = 7.48 \text{ cm.}$$

The tangent trochoid will then appear in the actual case when the speed of the boat reaches 86 cm./sec.

The horizontal distance from the first crest to the obstacle is found thus:

We first seek the distance $\alpha\alpha_1$, Fig. 15, between two points on the same level, assuming the curve to be practically trochoidal between these two points and hence symmetrical about a vertical through the point distant $L/2$ from the crest. The known value of z in (2.1) will give the angle $(2\,\pi\lambda/L)$. The length L being known, this will give λ and this will give x_1 for the point α_1. Then from the diagram it is clear that

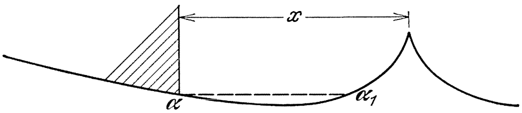


Fig. 15.

$$\frac{\alpha\,\alpha_1}{2} = \frac{L}{2} - x_1 \quad \text{or} \quad \alpha\,\alpha_1 = L - 2\,x_1$$

Thus for the preceding numerical case we find $\pi A^2/L = 3.74$ and $cos\; 2\pi\lambda/L = -.961$ or $2\pi\lambda/L = 164^0$. Then $\lambda = (164/360)\; L = 21.4$. Putting this in (1) we find $x_1 = 19.35$. Whence

$$\alpha\alpha_1 = 47 - 38.7 = 8.3 \text{ cm.}$$

The distance from the crest to the obstacle will then be

$$x_1 + \alpha\alpha_1 = 27.65 \text{ cm.}$$

The distance from the given immersion to the bottom of the wave hollow is $7.48 - (3.74 + 3.45) = .29$ cm.

3. Appearance of the Stern Wave. If we assume a plane with constant inclination to the horizontal and a depth of immersion relative to still water likewise constant, then over a considerable range of speeds, we should expect, *a priori*, to determine a point of brusque variation in the curve of resistance on speed, when the speed is reached where the wave at the stern begins to form clear of the boat. However, no such sudden change is found and it must be assumed that before the appearance of the stern wave, lining in with the obstacle, there is a brief period during which the wave exists though drowned, as it were, by a mass of entrained water.

4. Experiments on the Separation of the Stern Wave. These experiments, to the present time, are few in number, but sufficient to show that the order of magnitude of the speeds at which the wave separates at the stern is that given by the theory thus briefly summarized.

The question of the direction of movement of the water as it issues aft of the plane gliding surface is perhaps of no great importance for a seaplane full size. On the other hand it merits study for the case of a structure of model size. The separation aft of the step being of fundamental importance, it is necessary to make sure that the conditions of separation for the model are similar to those for the planes' actual size. This will give a lower limit to the length of the model.

In the case of models of water gliders with step, the experimental channels with travelling bridge usually employ models of small dimensions. The value of the speed-length ratio should in fact, be the same for the model and for the full-scale object. The maximum speed of the bridge is limited by the character of the installation, and in order to reach a value sufficiently high, it may become necessary to reduce the length. In such case, with very small models, it is found that the reduction of resistance secured for full scale boat by the use of the step, is not always found on the model. The explanation is that for the same value of $\urcorner$ the water separates from the hull of the full scale boat, but does not separate in the case of a very small model. It would appear that there is an effect of capillarity of which due note should be taken.

From the view point of agreement with theory, it is equally useful to know if, at the point of separation, the velocity is in the plane tangent to the surface of the body. Regarding this point, few observations seem to have been made. In a sketch found in a paper of Sottorf[1] a separation is shown aft of a thin gliding plane with a sharp break in direction. It seems rather in point of fact, that while the velocity is not in a direction tangent to the surface of the body it differs but little from it, the stern wave forming a hollow aft of the body, having an initial tangent a little less inclined to the horizon than the lower surface of the body itself.

5. Study of Take-Off From Diagrams. The detailed study of the three equations of equilibrium of the seaplane, as above, gives rise to certain difficulties, the character of which must be noted. Even with simple cases it is impossible to write down explicitly the various coefficients figuring in these equations, because they all depend upon the variables.

The thrust of the propeller, for example, depends at the same time on the speed, on its position with reference to the water and to the wings; co-efficients R_x, R_y, H_x, H_y depend on the same elements and likewise on the instantaneous orientation of the wings, and similarly for the others. It is possible, however, to determine certain things graphically regarding the conditions of equilbrium, and these diagrams have the

[1] Werft, Reederei, Hafen 1929.

advantage of showing the character and relation of the various difficulties.

Any position in general of the seaplane is here defined with reference to the general still water level by the aid of two parameters, of which one, θ, relates to its orientation; for example inclination of the wings to the horizontal, and the other to the immersion ($N—n_1$), the number of vanes immersed in the first diagram, or x, length of plane immersed in the second diagram. Before treating the three equations of equilibrium it is necessary to know the values and positions of the resistances, both aerodynamic and hydrodynamic, as functions of the speed and of the two geometrical parameters [θ and $(N — n_1)$, or θ and x]. The first diagram is arranged in such manner that the equation of moments can be neglected. It is further assumed that the forces—thrust of the propeller, weight, aerodynamic and hydrodynamic resultants, all converge in the same point, the two latter having the same position. The problem is then one of constant incidence and the variable θ does not enter.

6. First Diagram. For a diagram thus constituted, the equations of equilibrium, neglecting the vertical forces of inertia, will be, see Fig. 16.

$$f(V) = \alpha n_1 V^2 + 800\,\alpha\,(N - n_1)\,V^2 + M\frac{dV}{dt} \tag{6.1}$$

$$Mg = \beta n_1 V^2 + 800\,\beta\,(N - n_1)\,V^2 \tag{6.2}$$

The coefficient 800 recognizes the fact that the density of water is about 800 times that of air. The second equation shows that to each value of V there corresponds a determinate immersion characterized by the value of n_1. The immersion is independent of the function $f(V)$ which determines the law of thrust of the propeller. We then eliminate n_1 between the two equations, thus obtaining

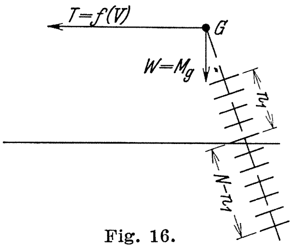


Fig. 16.

$$f(V) = \frac{\alpha}{\beta} Mg + M\frac{dV}{dt}$$

An equation giving the law of variation of V as a function of t. This equation does not contain the factor 800.

The resistance $\quad R = \alpha n_1 V^2 + 800\,\alpha\,(N - n_1)\,V^2$

is in effect constant and equal to $(\alpha/\beta)\,Mg$. The plane regulates itself. It is immersed at each speed in such manner as to satisfy this condition of the constancy of the resistance.

For each law determined for $f(V)$ there will be a law of variation of speed as a function of the time. If $f(V)$ is constant we have dV/dt constant and a uniform accelerated motion. If $f(V) = A - BV$ it would be necessary to integrate

$$A - BV = \frac{\alpha}{\beta} Mg + M\frac{dV}{dt} \tag{6.3}$$

which gives

$$
\left.\begin{aligned}
V &= \frac{1}{B}\left[A - \frac{\alpha}{\beta} Mg - e^{-\frac{B}{M}t}\left(A - \frac{\alpha}{\beta} Mg\right)\right] = \\
&= \frac{1}{B}\left[A - \frac{\alpha}{\beta} Mg\right]\left[1 - e^{-\frac{B}{M}t}\right]
\end{aligned}\right\} \tag{6.4}
$$

and supposing V equals 0 for t equals 0

$$
t = -\frac{M}{B} \log \left[\frac{A - \frac{\alpha}{\beta} Mg - BV}{A - \frac{\alpha}{\beta} Mg}\right] = \frac{M}{B} \log \left[\frac{A - \frac{\alpha}{\beta} Mg}{A - \frac{\alpha}{\beta} Mg - BV}\right] \tag{6.5}
$$

The speed of take-off will then be given by (6.2) when $n_1 = N$ or

$$
V = \left(\frac{Mg}{\beta N}\right)^{1/2}
$$

and the time duration will be given by putting this value of V into (6.5). As will be noted, the factor 800 disappears from the final result. An analogous calculation can be carried out for other laws of variation of the thrust of the propeller. For example, assuming that this thrust varies as:

$$
A \cos \frac{\pi}{2} \frac{V}{V_0}
$$

where V_0 is the speed for which the thrust is 0, it will be necessary to consider the integral

$$
\int \frac{dx}{\cos x - \alpha}
$$

Here we may place x equals $2 \tan^{-1} u$ and reduce to a form directly integrable.

7. Second Diagram. Here the ensemble of the wings (wings and tail surfaces) is assimilated to a single plane of surface σ Fig. 17. The hull is assimilated to a sustaining plane of width b, of which the immersed length is x. The surface σ is inclined at an angle i relative to the surface S. The instantaneous position is defined by x and θ. The equations of vertical forces and moments are

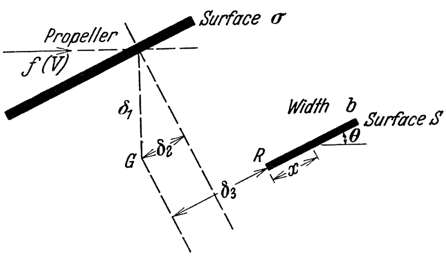


Fig. 17.

$$
Mg = \varrho \sigma V^2 (\theta + i) + 800 \varrho b x V^2 \theta
$$

$$
f(V)\, \delta_1 = \varrho \sigma V^2 (\theta + i)\, \delta_2 + 800 \varrho b x V^2 \theta\, (\delta_3 + 2x/3)
$$

Eliminating θ we have the relation between x and V expressed in the form

$$
u = \frac{Mg - \varrho \sigma V^2 i}{f(V)\, \delta_1 - \varrho \sigma V^2 i \delta_2} = \frac{\sigma + 800 b x}{\sigma \delta_2 + 800 b x [\delta_3 + 2x/3]} \tag{7.1}
$$

Is it then possible to have in the neighborhood of x equals 0 (take-off), V decreasing with increase of x? That is to say,

$$
\frac{dV}{dx} < 0.
$$

For x equals 0 we have

$$Mg - \varrho\sigma V^2 i = \varrho\sigma V^2\theta$$
$$f(V)\,\delta_1 - \varrho\sigma V^2 i\delta_2 = \varrho\sigma V^2\theta\,\delta_2$$

We then form the two derivatives du/dV and du/dx. The former will take its sign from

$$-2\varrho\sigma i\,[f(V)\delta_1 - \varrho\sigma V^2 i\delta_2]\,V - (Mg - \varrho\sigma V^2 i)\,[f'(V)\delta_1 - 2V\varrho\sigma i\delta_2]$$

Then for $x = 0$ with the above relations this reduces to $-f'(V)\,\delta_1$. Then since $f'(V)$ is always negative, the sign of δ_1 will evidently here control.

Similarly the derivative du/dx takes its sign from:

$$[\sigma\,\delta_2 + 800\,b\,x\,(2\,x/3 + \delta_3)]\,800\,b - (\sigma + 800\,b\,x)\,[(4/3)\,800\,b\,x + 800\,\delta_3\,b]$$

For $x = 0$ this reduces to $(\delta_2 - \delta_3)$. Hence the derivative dV/dx will have the sign of $(\delta_2 - \delta_3)/\delta_1$ and the condition sought will be satisfied if $\delta_3 > \delta_2$.

Numerical Example Given:

$Mg = 2400$ $\qquad b = 1.5$
$\sigma = 50$ $\qquad i = 0$
$\delta_1 = 1$ $\qquad f(V) = 2500 - 154\,V$ (propeller equation)
$\delta_2 = \delta_3 = 0.4$

The relation (7.1) reduces to

$$f(V) = 2400\,\frac{80\,x^2 + 48\,x + 2}{120\,x + 5}$$

For each value of x we have a value of $f(V)$ and consequently V. We may then calculate $\varrho\theta$ by the equation of vertical forces, giving

$$\varrho\theta = \frac{Mg}{(\sigma + 1200\,x)\,V^2}$$

Now taking the resistance to advance of the part in the air as proportional to $V^2\theta\sigma$, it is seen to follow from the equation for vertical forces (since ϱ and σ are both constants) that $V^2\theta\sigma$ is also proportional to $Mg/(\sigma + 1200\,x)$. Also we take the resistance of the hydroplane proportional to $V^2\theta x b \times 800$, that is, to $V^2\theta x \times 1200$. Since $\sigma = 50$, the ratio of these two resistance values is seen to be $24\,x$.

We may write, therefore,

$$\text{Resistance (air)} = k'\,\frac{Mg\cdot\sigma}{\sigma + 1200\,x} = k'\,\frac{50\,Mg}{\sigma + 1200\,x}$$
$$\text{Resistance (water)} = k'\,\frac{1200\,Mg\cdot x}{\sigma + 1200\,x}$$

Whence:

$$\text{Resistance (Total)} = k'\,Mg\,\frac{\sigma}{\sigma + 1200\,x} + \frac{1200\,x}{x + 1200\,x}$$

The results of the calculation are given in the following table, and are shown in Figs. 18a and 18b. The above analysis only applies during the period of hydroplaning. The parts of the curves in dotted line indicate the period for which the wing and float cannot yet be represented by the combination of two planes. It will be noted from the formulae and also

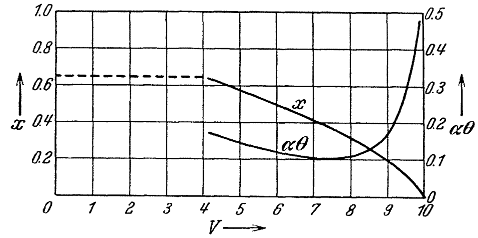


Fig. 18a. Diagram showing values of x and $\alpha\theta$ on speed.

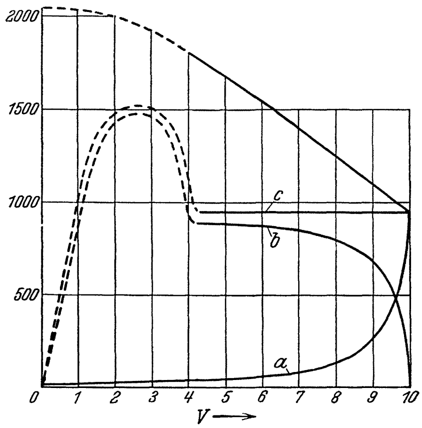


Fig. 18b. Diagram showing resistance on speed. a) Resistance of the air. b) Resistance of the water. c) Total resistance.

from the values in the table that, under the special assumptions made and for the period during which they are valid, the total resistance remains constant.

x [1]	$f(V)$ [2]	V [3]	$\sigma + 1200$ [4]	$\alpha\,\theta$ [5]	$\frac{47500}{\sigma + 1200\,x}$ [6]	Resistance (water) [7]	Resistance (total) [8]
0	960	10	50	0.480	950	0	950
0.1	1073	9.27	170	0.1645	279.4	670.5	950
0.2	1225	8.28	290	0.121	163.8	786.2	950
0.3	1381	7.26	410	0.111	115.9	834.1	950
0.4	1540	6.24	530	0.1166	89.65	860.35	950
0.5	1698	5.20	650	0.1366	73.1	876.91	950
0.6	1858	4.17	770	0.1794	61.7	888.3	950
0.7	2017	3.14	890	0.2741	53.5	896.6	950
0.8	2177	2.1	1010	0.539	47.1	902.9	950
0.9	2336	1.06	1130	1.876	42.1	907.9	950
1	2496	0.03	1250		38	912	950

[1] Values of the variable x.

[2] Values of $f(V) = 2400\,\frac{80\,x^2 + 48\,x + 2}{5 + 120\,x}$.

[3] Values of V deduced from $V = \frac{2500 - f(V)}{154}$.

[4] Values of $\sigma + 1200\,x = 50 + 1200\,x$.

[5] Values of $\alpha\,\theta = \frac{2400}{(\sigma + 1200\,x)\,V^2}$.

[6] Resistance of the part in the air $= \frac{47500}{\sigma + 1200\,x}$.

[7] Resistance of the part immersed $= (6) \times 24\,x$.

[8] Total resistance $= (6) + (7)$.

This second diagram is evidently applicable only for values of θ less than those for which the tail of the plane barely touches the water. The curve of θ as a function of V is, for the period of hydroplaning, not without some similarity with the curves obtained experimentally by the National Advisory Committee for Aeronautics[1].

The principal use of this diagram is to show the relation between the characteristics of the wings and of the propeller on the one hand with those of the hull on the other, and that it is impossible to treat separately a study of these different parts of the structure as a whole.

8. Third Diagram. Determination of Elements Relative to the Critical Speed. We assume for purposes of study a schematic plane formed by a prismatic float and a wing, Fig. 19. Let

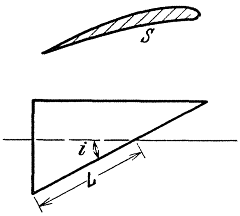


Fig. 19.

W = Weight of the plane in tons,
L = Length at rest of the immersed body along the slope in meters,
S = Surface of the wing in square meters,
V_0 = Minimum speed over the ground in kilometers per hour,
b = Width of the float in meters,
δ = Weight of unit volume of water.

When the quantities W, L, b, are given the slope i of the bottom follows from the condition of equilibrium at rest. We here assume i small and $\cos i$ = sensibly 1. We have then,

$$W = \frac{1}{2} \delta\, b\, L^2 \sin i \qquad (8.1)$$

We shall not consider at present variations of the inclination during the take-off. While in movement the immersed length, l, will be variable between the values L at the start and 0 when the speed reaches V_0. The force of buoyancy will be a function of l alone and taken equal to

$$\frac{l^2}{L^2} W$$

The hydrodynamic sustentation will be taken proportional to the surface of support $b\,l$ to the square of the speed V^2 and to $\sin i$. We may write then, $\frac{1}{2} \delta\, K' b\, l\, V^2 \sin i = \frac{K' l\, W}{L^2} V^2$ [see (8.1)]

where K' is a coefficient of hydrodynamic sustentation.

The resistance to movement due to this sustentation will be sensibly proportional to the sustentation in the ratio $\sin i$, which, putting in $\sin i$ from (8.1), will give

$$K'' \frac{l}{b} \frac{W^2}{L^4} V^2 \quad \text{where} \quad K'' = 2\, K'/\delta$$

[1] Crowley, J. W., Jr., and Ronan, K. M., Characteristics of a Single Float Seaplane During Take-off, U.S. N.A.C.A. Technical Report No. 209, 1925. — Characteristics of a Boat Type Seaplane During Take-off, U.S. N.A.C.A. Technical Report No. 226, 1926.

At the speed V the aerodynamic sustentation will be $K_1 S V^2$, and the air resistance will be $K_2 S V^2$. The condition of equilibrium in flight at the moment of take off gives

$$W = K_1 S V_0^2$$

and consequently the air sustentation at the speed V due to the wings will be

$$W \frac{V^2}{V_0^2}$$

The equations of equilibrium will then be

$$W = W \frac{l^2}{L^2} + K' l \frac{W}{L^2} V^2 + W \frac{V^2}{V_0^2}$$

$$R = K'' \frac{l}{b} \frac{W^2}{L^4} V^2 + K_2 S V^2$$

This system of two equations determines the two unknowns l (position of the float with regard to the water) and R (resistance to movement) as functions of the speed. Eliminating l we shall have the law of resistance to movement as a function of speed. We have first for l

$$l = \frac{R - K_2 S V^2}{K'' W^2 V^2} b L^4$$

and the law of resistance to motion as a function of V is given by the relation between R and V.

$$1 = \frac{1}{L^2} \left[\frac{R - K_2 S V^2}{K'' W^2 V^2} b L^4 \right]^2 + K' \frac{V^2}{L^2} \left[\frac{R - K_2 S V^2}{K'' W^2 V^2} b L^4 \right] + \frac{V^2}{V_0^2}$$

The part of the resistance due to the hull alone in the series of conditions through which the plane passes is $R_1 = R - K_2 V^2 S$ and the law of variation of R_1 as a function of V is

$$1 = \frac{1}{L^2} \left[\frac{R_1 b L^4}{K'' W^2 V^2} \right]^2 + K' \frac{V^2}{L^2} \left[\frac{R_1 b L^4}{K'' W^2 V^2} \right] + \frac{V^2}{V_0^2} \tag{8.2}$$

In this relation S, the area of wing surface, does not directly enter, but only indirectly, through the agency of V_0. We then place

$$x = \frac{V^2}{V_0^2} \quad (x \text{ lies between 0 and 1})$$

$$y = \frac{R_1 L^4}{W^2}$$

$$A = \frac{b}{L K'' V_0^2}$$

$$B = \frac{K'}{K''} \frac{b}{L^2}$$

The discussion of the relation between R_1 and V is then reduced to a discussion of the variations of y as a function of x when y and x are related by the equation

$$A^2 y^2 + B x^2 y + x^3 - x^2 = 0$$

This represents a cubic of the general form shown in Fig. 20. It will have a maximum value for y between $x = 0$ and 1. Let us seek the coordinates of this maximum which will give us the value of V^2/V_0^2 for which $R_1 L^4/W^2$ is maximum and in consequence the value of V for which R_1 will be a maximum. The cubic passes through the points $x = 0$, $y = 0$, and $x = 1$, $y = 0$. We desire then the coordinates of y maximum for x between 0 and 1. To this end we utilize a substitute conic having with the cubic a point in common at the origin, and a point in common at $x = 1$, $y = 0$. By putting $y = mx$ it is readily shown that the tangent of the slope at the origin is $y' = 1/A$.

We next form the two derivatives y' and y'' thus:

$$2A^2yy' + 2Bxy + Bx^2y' + 3x^2 - 2x = 0$$

$$2A^2yy'' + 2A^2y'^2 + 2Bxy' + 2By + 2Bxy' + Bx^2y'' + 6x - 2 = 0$$

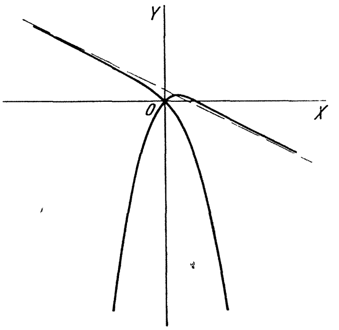


Fig. 20.

At the point $y = 0$, $x = 1$ these give

$$y' = -\frac{1}{B} \qquad y'' = -\frac{2A^2}{B^3}$$

We thus have five conditions—two points, two slopes and one second derivative. This will determine a general conic passing through the origin with five coefficients. It is known from analytical geometry that if $y = m_1x + b_1$ and $y = m_2x + b_2$ are any two lines, then, λ being a constant coefficient, the equation

$$\lambda y^2 + (y - m_1x - b_1)(y - m_2x - b_2) = 0$$

will give a conic crossing X at the same points as these lines and having at these points the same slope as the lines. Hence in the present case the equation to a conic passing through the origin and the point $y = 0$, $x = 1$ and having the same slope at these points as the cubic will be

$$\lambda y^2 + (Ay - x)(By + x - 1) = 0$$

We have now to apply the further condition for y''. To this end we form y' and y'' from this equation thus

$$2\lambda yy' + (Ay' - 1)[By + (x - 1)] + (Ay - x)[By' + 1] = 0$$

$$2\lambda yy'' + 2\lambda y'^2 + (Ay - x)By'' = 0$$

Whence putting in the values of y' and y'' for the point $y = 0$, $x = 1$ we find $\lambda = -A^2$. The conic sought is therefore

$$-A^2y^2 + (Ay - x)[By + (x - 1)] = 0$$

We then seek the maximum using the equation giving y' and putting $y' = 0$. This gives

$$-[By + (x - 1)] + Ay - x = 0$$

Combining this equation with that of the conic we have

$$-A^2y^2 + (Ay - x)^2 = 0$$

or

$$y = \frac{x}{2A}$$

Then replacing y by its value in the equation of condition $y' = 0$

$$-\frac{Bx}{2A} - x + 1 + \frac{x}{2} - x = 0$$

whence

$$x = \frac{2A}{3A+B}$$

$$y = \frac{1}{3A+B}$$

Returning now to the initial data and calling V_m the critical speed, and R_m the resistance of the hull for this speed we have

$$\frac{V_m^2}{V_0^2} = \frac{2}{3 + K' V_0^2/L}$$

$$R_m = \frac{W^2}{b L^3} \frac{K'' V_0^2}{3 + K' V_0^2/L}$$

If we consider the case in which we impose the weight W, the speed of take-off V_0 and length L of the part forward of the step, V_m will be independent of b, and R_m will decrease as b increases. This furthermore is readily seen from (8.2) which gives $R_1 b$ as a function of V.

These latter formulae may be written in such manner as to bring out the speed in relation to the square root of the dimensions. Thus, put

$$\frac{V_0}{\sqrt{L}} = \alpha \quad \text{and} \quad \frac{V_0}{\sqrt{b}} = \beta$$

Then

$$\frac{V_m^2}{V_0^2} = \frac{2}{3 + K' \alpha^2}$$

$$\frac{R_m}{W} = \frac{K'' W}{L^3} \cdot \frac{\beta^2}{3 + K' \alpha^2}$$

Under this form it is easy to determine the values of K' and K'' utilizing the results obtained by experiment either model or full scale.

$\frac{V_m}{V_0}$ is usually not far from 0.4

$\frac{W}{L^3}$ is in general not far from 0.050

Thus $\quad W = 2.4\ T$ for $L = 3.6$ m.

α is usually not far from 53

Thus $\quad V_0 = 100$ km./h. for $L = 3.60$ m.

β is usually not far from 83

Thus $\quad V_0 = 100$ km./h. for $b = 1.45$ m.

This gives $3 + K'\alpha^2 = 12.5$ and K' for $\alpha = 52.7$ has the value 0.00343.

Again R_m/W is not far from 0.2 and hence

$$0.2 = K'' \cdot 0.0503 \cdot \frac{6960}{12.5}$$

whence

$$K'' = 0.00715$$

It is interesting to trace the curve of total resistance as a function of the speed. To this end we examine again the cubic in x, y.

We have here

$$A = \frac{b}{L K'' V_0^2} = .00563 = 6 \cdot 10^{-3} \text{ app.}$$

$$B = \frac{K' b}{K'' L^2} = 0.054$$

The cubic is then

$$36 \cdot 10^{-6} y^2 + 0.054\, x^2 y + x^3 - x^2 = 0$$

This is readily traced by using polar coordinates after having put $y = 100\, y_1$. This gives

$$0.36\, y_1^2 + 5.4\, x^2 y_1 + x^3 - x^2 = 0$$

whence
$$r = \frac{\cos^2\theta - 0.36 \sin^2\theta}{5.4 \sin\theta \cos^2\theta + \cos^3\theta}$$

This curve having been traced, it is only necessary to change the distribution of the abscissae in order to have the curve of resistance as a function of the speed. Furthermore, the curve gives immediately the law of variation of l as a function of the speed, Fig. 21. In effect l is proportional to y/x.

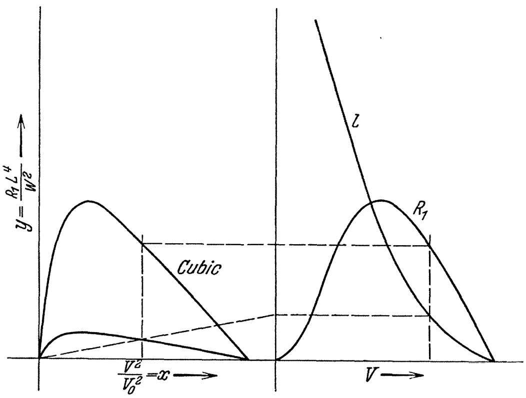


Fig. 21.

By the use of this method it appears that with the hyperbola which has been employed, the abscissae of the summit is less than that for the summit of the cubic, and that the summit of the hyperbola is higher than that of the cubic. If instead of using the hyperbola having three features in common with the cubic for $x = 1$, and two features for $x = 0$, use were made of a hyperbola having two points in common for $x = 0$, and three points for $x = 1$, the approximation would be somewhat better because the maximum is always nearer to $x = 0$ than to $x = 1$. The calculation in this case presents no difficulty but gives rise to formulae less simple than with the other substitution.

The formulae thus developed for the determination of the elements of the critical speed admit of diverse applications. In the first place they determine the length of immersed keel at the moment of maximum resistance, and thus furnish an indication regarding the minimum length to be given to the straight part of the keel before its upward curvature forward. Again they indicate the manner in which the critical speed varies with varying conditions. In a paper by Herrmann[1] there may be found for a single form of float the variations of elements V_m for varying speeds of take-off and for varying weight. It there appears that for the same weight and for varying speeds of take-off there will be found between R_m, V_m, and V_0 the following relations:

V_0 km./h.	V_m m./s.	R_m kgs.	$\sim V_m/V_0$	R_m/V_m^2
70	8.7	665	1.24	8.8
85	9.5	705	1.11	7.8
100	10.0	770	1.00	7.7

It thus appears, as indicated by the formula

$$\frac{V_m^2}{V_0^2} = \frac{2}{3 + K' V_0^2/L}$$

that as V_0 increases, V_m increases but with a decreasing ratio V_m/V_0.

The formula for R_m would indicate the ratio $R_m/V_m^2 =$ const. This, however, is not exactly verified.

It is shown in the same paper that for different total weights the values of the speed of take-off, the maximum speed and the maximum resistance are as follows:

W kgs.	V_0 m./s.	V_m m./s.	R_m	$\sim \dfrac{R_m}{W^2 V_m^2}$
3000	22.0	8.8	887	127
2600	20.5	8.1	700	158
2200	18.5	7.9	512	170
1800	17.0	8.1	295	138

The formula would give for this case $V_m =$ constant and $R_m/W^2 V_m^2 =$ constant. The agreement is not altogether satisfactory, but this result must be expected from a diagram as simple as the one which we have considered.

The practical application of the relation R_1 inversely proportional to b when the weight of the plane, the wing surface and the motor-propeller unit are fixed, is limited by the fact that an increase of b entails an increase in the weight of the hull and in its air resistance. The influence of the increase in air resistance can be neglected in most practical cases. It may be noted further that by the use of an under structure extending beyond the width of hull proper, the value of b may be increased without any corresponding departure from the general fineness of line. However, it is necessary to examine the effect of an increase in hull weight on the general economy of the structure as a whole.

In the comparison of different solutions involving the same general data, due account must be taken of the general schedule of design. The total weight W will represent weight of hull, weight of plane structure, weight of motor-propeller unit, weight of personnel and navigational equipment and useful load. Other elements remaining the same, any increase in hull weight will diminish in like amount the useful load and it might result in enlarging the value of b, starting from an initial design,

[1] Herrmann, H., Seaplane Floats and Hulls, Berichte und Abhandlungen der Wissenschaftlichen Gesellschaft für Luftfahrt, December 1926.

that a point would be reached where the increase in hull weight would equal the original useful load, thus resulting in a structure of no value as an agency of transport.

The problem might, however, present itself in an entirely different manner if, instead of an agency of transport, the design should be for racing purposes and without limiting conditions otherwise regarding useful load. We shall limit the present discussion to the case where the plane is to be of value as an agency of transport.

In order to permit of a simple calculation, let us assume as part of the initial data, a useful load eW and as width a value b. It may be assumed that the weight of hull structure as due to change in b alone will vary as b^2. As a matter of fact the evaluation of this variation should be based in part on an examination of distribution of weight in order to determine the part of the weight pertaining to the forward under structure, and in part on a consideration of the strength of the materials employed in order to determine the increase in local load at the instant of alighting on the water and during the hydroplaning period.

If, in the original design the weight of hull structure is βW, we may assume then, as a result of an increase in b, a weight of $\beta W\ (b_1^2/b^2)$. The combined weight, hull structure plus useful load, would in the original design for $(eW + \beta W)$ become, following the change, $e_1 W + \beta\ (b_1^2/b^2)\, W$ and the limit of b_1 will be reached when $e_1 = 0$. This gives

$$(e + \beta)\, W = \beta \frac{b_1^2}{b^2}\, W$$

or

$$(e + \beta) = \beta \frac{b_1^2}{b^2}$$

$$\frac{b_1}{b} = \sqrt{\frac{e+\beta}{\beta}}$$

If, for example, for the original design we have $\beta = 0.20$ and $e = 0.40$, the limit of b_1/b will be $\sqrt{0.60/0.20} = 1.732$.

Studies of this character make possible an estimate of the sacrifice to accept regarding useful load and the purpose of improving the conditions of take-off.

The problem may be approached from a different point of view by considering a reduction of resistance at the critical point, not for the purpose of improving the conditions of take-off, but in order to increase the load with which take-off can be realized. In this case the resistance at the critical point will be held as a constant, and we associate an increase in b with an increase in total weight as bearing upon the weight of hull structure and on the useful load.

The resistance R varying as W^2/b, it appears that, in this case, W^2/b should be held constant.

Let x denote the ratio b_1/b. Then the weight carried by the design as modified will be $W\sqrt{x}$. For the original design we have

$$W = C + U + D$$

where $C =$ weight of hull structure, U the useful load, and D the remainder of the total weight.

If the weight of hull structure varies with b according to an index m, we shall have, for the modified design

$$W\sqrt{x} = C\,x^m + U_1 + D$$

Comparing the two equations the gain in useful load is

$$G = U_1 - U = W\left(\sqrt{x} - 1\right) - C\,(x^m - 1)$$

From this we have

$$\frac{dG}{dx} = \frac{W}{2\sqrt{x}} - m\,C\,x^{m-1}$$

The limit beyond which increase of b would be useless is then given by $dG/dx = 0$.

If, in the original design there is no purpose in an increase of b, the above value of $dG/dx = 0$ with $x = 1$ will give

$$\frac{W}{2} = m\,C$$

This gives, for the ratio of weight of hull structure to total weight,

$$\beta = \frac{1}{2m}$$

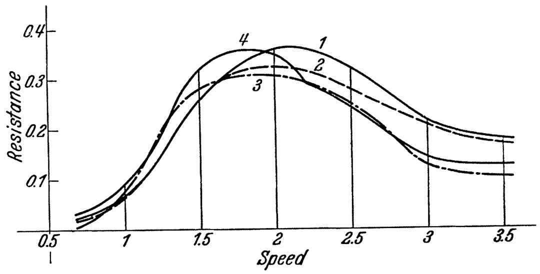


Fig. 22. Resistance as a function of speed for various models:
1 width 0.12 m., weight 1.750 kg.
2 width 0.14 m., weight 1.750 kg.
3 width 0.16 m., weight 1.750 kg.
4 width 0.18 m., weight 1.750 kg.
Incidence 9°, adjustment 11, see Fig. 47.
The centers of gravity are all at the same distance from the step. Scales, proportional figures.

If, as above, we assume the weight of hull structure to vary as b^2, it will then be advantageous to increase b whenever

$$\beta < 0.25$$

These considerations are limited in application by the fact that an increase in b carries with it a decrease in the ratio L/b, and during the period preceding that of hydroplaning, when the hull is moving through the water as an ordinary boat, its resistance may be disproportionately increased. In such case, forms may be found giving rise to a different maximum. An example of this is given, showing on curve 4, Fig. 22, corresponding to the largest model, a maximum different from that which we have considered and which will find its explanation in the study of the fourth curve sheet.

9. Fourth Diagram. Hydroplaning at a Constant Angle of Inclination. The theory of Wm. Froude regarding hydroplaning is briefly as follows.

Let A (Fig. 23) be the surface bearing upon the water (in square feet) and consequently A/b the length of water bearing, θ the angle of inclination to the horizontal, and V the speed in knots.

Froude assumes that the plane is subject to water forces as follows:

(1) A normal force:

$$F_1 = 3\,A\,V^2 \sin\theta \text{ (pounds)}$$

This gives a vertical component upward $3\,A\,V^2 \sin\theta \cos\theta$ and a horizontal resistance component $3\,A\,V^2 \sin\theta$.

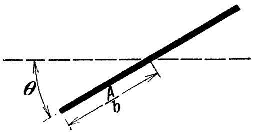


Fig. 23.

(2) A tangential force:

$$F_2 = 0.01\,A\,(V\cos\theta)^2 \text{ (pounds)}$$

This gives a vertical component downward $0.01\,A\,(V\cos\theta)^2 \sin\theta$ and a horizontal resistance component $0.01\,A(V\cos\theta)^2 \cos\theta$.

The sum of the vertical components must equal the weight W, thus

$$W = 3\,A\,V^2 \sin\theta\cos\theta - 0.01\,A\,V^2 \cos^2\theta\sin\theta \tag{9.1}$$

The sum of the horizontal components, under steady conditions, must equal the resistance R. Thus

$$R = 3\,A\,V^2 \sin^2\theta + 0.01\,A\,V^2\cos^3\theta \tag{9.2}$$

In these values for W and R, the speed V enters only in the product $A\,V^2$ and hence the ratio R/W is a function solely of θ. Thus

$$\frac{R}{W} = \frac{3\sin^2\theta + 0.01\cos^3\theta}{3\sin\theta\cos\theta - 0.01\cos^2\theta\sin\theta}$$

For θ small we have

$$\frac{R}{W} = \frac{300\,\theta^2 + 1}{300\,\theta - \theta} = \frac{1}{299}\left(300\,\theta + \frac{1}{\theta}\right)$$

Finding the condition for a minimum of this expression we have

$$\theta = 0.0577 = 3^0.3$$

In (9.1) for a given value of θ it is clear that $A\,V^2$ must remain constant and hence the same in (9.2). It follows according to this theory that the resistance is independent of the speed. That is, A will decrease in the same ratio as V^2 increases and $A\,V^2$ remain constant. If, on the other hand we should introduce a supplementary term proportional to the square of the speed such as $B\,V^2$, recognizing for example the existence of parasitic resistance due to immersed parts, the equations will take the form

$$W = 3\,A\,V^2\sin\theta\cos\theta - 0.01\,A\,V^2\cos^2\theta\sin\theta \tag{9.3}$$

$$R = 3\,A\,V^2\sin^2\theta + 0.01\,A\,V^2\cos^3\theta + B\,V^2 \tag{9.4}$$

In this case the speed does not appear alone in the term $A\,V^2$. Assuming W known and constant, (9.3) and (9.4) give the values of the two unknowns A and R as functions of speed. In order to find the law of resistance (R as a function of V) A must be eliminated between the two equations.

Solving the first equation for A and putting this value in the second, we have
$$R = \frac{(3\,sin^2\,\theta + 0.01\,cos^3\,\theta)\,W}{3\,sin\,\theta\,cos\,\theta - 0.01\,cos^2\,\theta\,sin\,\theta} + B\,V^2$$

At constant speed, the value of θ for minimum R is the same as that found above. In both these cases the resistance is not zero at zero speed. The reason for this insufficiency in the theory arises from the assumption that the hydrodynamic components in V^2 alone equilibrate the weight of the plane, neglecting the part taken by the forces of buoyancy.

In the present state of hydrodynamic science it is not possible to give an exact evaluation of buoyant forces in the case of a body in motion at the surface of separation of two fluids. Even for the case of a body in motion wholly immersed in a fluid there exist solutions only in very special cases. All that can be said with assurance is that in the case where $V = 0$, the vertical buoyant force is equal to the weight of the volume of fluid displaced by the body, relative to the general fluid level,

Fig. 24.

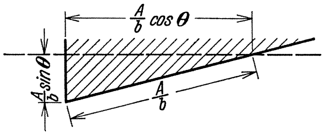


Fig. 25.

and if in movement the volume so displaced is zero, there exists a movement possible at which the buoyant force is zero. A value of the buoyant force satisfying these two limiting conditions will be given by assuming that during the motion the body is subject to two vertical forces, one as assumed by Froude and the other a force of buoyancy measured by the volume of the structure below the general water level plane.

This definition is evidently not correct for the intermediate phases of the movement, since the stern wave may separate from the hull and the addition of a hull volume such as α (see Fig. 24) does not really change the vertical forces in operation while it does change the value as calculated.

In the case of an ordinary ship, the introduction of this supplementary upward force will have no effect on the result. We may, however, admit it by assumption, as a working hypothesis for the present case of floats with step. Instead of representing the float in longitudinal section by a straight inclined line, let us assume it to be limited aft by a vertical line, see Fig. 25. The two equations then become:
$$W = 3\,A\,V^2\,sin\,\theta\,cos\,\theta - 0.01\,A\,V^2\,cos^2\,\theta\,sin\,\theta + 0.5\,\delta\,\frac{A^2}{b}\,sin\,\theta\,cos\,\theta$$
where δ is the weight in pounds of a cubic foot of water.
$$R = 3\,A\,V^2\,sin^2\,\theta + 0.01\,A\,V^2\,cos^3\,\theta$$

This system of two equations gives for each value of the speed V, the values of the two unknowns A and R. We shall have then R as a function of V by eliminating A between the two equations. This gives

$$W = \frac{[3 \sin\theta \cos\theta - 0.01 \cos^2\theta \sin\theta]\, R}{3 \sin^2\theta + 0.01 \cos^3\theta} + \frac{0.5\,(\delta/b) \sin\theta \cos\theta\, R^2}{V^4\,[3 \sin^2\theta + 0.01 \cos^3\theta]^2}$$

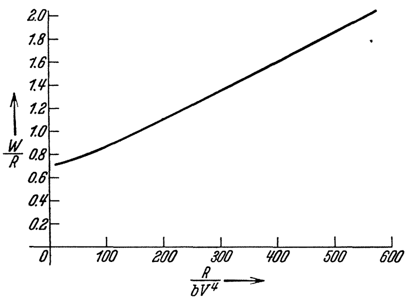


Fig. 26.

Such is the relation giving the resistance R as a function of the speed V. In this form, it is seen that for $V = 0$, $R = 0$, and further, that the relation corresponding to Froude's analysis is correct for $V = \infty$. The Froude theory alone then gives results correct only at the limit $V = \infty$.

In this equation the resistance R results as an implicit function of V. The law of resistance can, however, be represented by two simple methods which are directly indicated by the form of the relation between R and V.

TABLE 1. *Froude's Experiments on the Ramus Step*[2].

Speed	V^4	Resistance	Proportional Values $\frac{1}{R}$	Proportional Values $\frac{R}{V^4}$
3	81	0.475	2.12	586
4	256	0.74	1.35	291
5	625	0.97	1.03	156
6	1300	1.17	0.856	91.0
7	2400	1.24	0.805	52.0
		1.28	0.780	53.1
8	4100	1.28	0.780	31.6
		1.35	0.740	33.2
9	6560	1.30	0.770	19.9
		1.40	0.713	21.3
10	10000	1.30	0.770	13.0
		1.45	0.690	14.8
11	14700	1.50	0.668	10.1
12	20700	1.60	0.627	7.7

If we plot the values of $R/b\,V^4$ as abscissae and those of W/R as ordinates, the points should fall on a straight line. If reference be made to the experiments of W. Froude as reported by Johns[1], it will be seen that points plotted and representing the general results of his experiments, do fall nearly on a straight line. See Table 1 and Fig. 26.

Another form of graphical representation useful for this discussion is given by taking $x = V^4$ and $y = R$. Let us examine the relation between R and V, assuming θ small. We have

$$W = \frac{299\,R}{300\,\theta + \frac{1}{\theta}} + \frac{R^2}{V^4}\,\frac{\delta}{2\,b}\,\frac{\theta}{(3\,\theta^2 + 0.01)^2}$$

Replacing δ by its numerical value and putting $y = R$ and $x = V^4$ we have

$$\frac{W}{299} = \frac{y}{300\,\theta + \frac{1}{\theta}} + 1040\,\frac{\theta}{b}\,\frac{y^2}{x}\,\frac{1}{(300\,\theta^2 + 1)^2} \qquad (9.5)$$

[1] Engineering, Vol. 110, London, September 24, 1920.

[2] The double values in this table refer to two conditions of the model, the larger when dragging along a body of dead water and the smaller when over-riding and free of such dead water.

This equation represents a hyperbola of which the horizontal asymptote is given by
$$y = \frac{W}{299}\left(300\,\theta + \frac{1}{\theta}\right)$$
variable with θ and having a minimum value for $\theta = 0.0577$ (Fig. 27).

Let us examine the relation between R and V for a numerical case

$$W = 299$$
$$b = 30$$
$$\theta = 0.1$$
$$300\,\theta + 1/\theta = 40$$
$$1 = \frac{R}{40} + 3.47\frac{R^2}{V^4} \text{ [Eq. (9.5)]}$$

or
$$V^4 = \frac{138.8\,R^2}{40 - R}$$

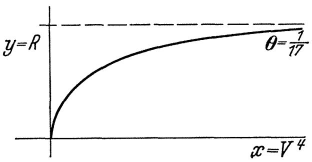


Fig. 27.

This gives the curve shown in Fig. 28. Actual experience gives curves of this same general character.

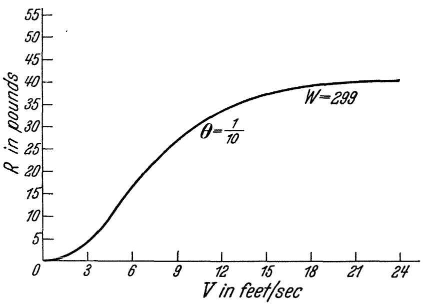


Fig. 28.

The case of a single inclined plane is indeed rather far removed from the form of an actual seaplane under-body. The forward part of the seaplane, in fact, is at a definitely steeper angle than the surface aft near the step and the analogy would be made more complete by the use of two planes in series with different inclinations.

In order to simplify the problem we assume a vertical plane B forming the stem and a plane moderately inclined forming the under surface, Fig. 29. Two cases will then develop, according as the vertical plane is or is not immersed. The second case (A variable with speed) is identical with the case examined above.

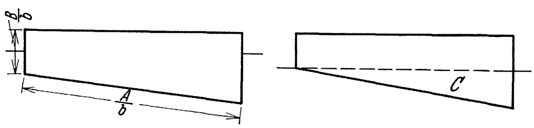


Fig. 29.

In the first case, A will be a constant and B will be the variable to eliminate. The angle of incidence for hydroplaning is here considered always the same.

A position of the float or boat with reference to the general water level is defined by the surface B. The resistance may then be taken in the form
$$R = \alpha A\,V^2 + \beta B\,V^2$$
while the equilibrium of the vertical forces will give
$$W = \delta\left[C + \frac{B}{b}\cdot\frac{A}{b}\cos\theta\right] + K'\,A\,V^2$$
where C is the volume of the immersed part when $B = 0$. B is essentially

positive. If B were negative, the conditions would revert to those of the preceding case. For $B = 0$ the two cases coincide. In these two equations B is an unknown and A a constant. We may then find the law of R as a function of V by eliminating B. This will give

$$R = \left[\alpha A + \frac{\beta b^2}{A \cos\theta}\left(\frac{W}{\delta} - C - \frac{K' A V^2}{\delta}\right)\right] V^2$$

which is of the form $\quad R = A_1 V^2 - A_2 V^4$

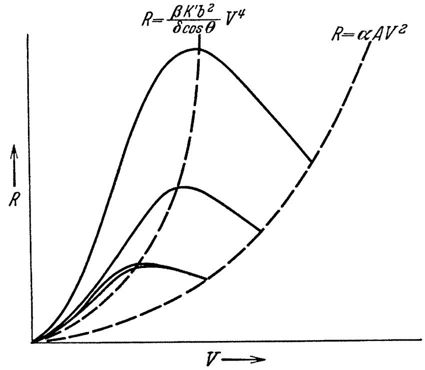


Fig. 30.

This formula is only applicable between $V = 0$ and the value for which $B = 0$, that is the value given by:

$$V^2 = \frac{W - \delta C}{K' A}$$

The value of R for this point will be

$$R = \alpha A V^2 = \alpha\left(\frac{W - \delta C}{K'}\right)$$

If we give to W all possible values, the ensemble of the end points will be on the curve given by eliminating W between these last two equations. This gives, of course, the same equation as above

$$R = \alpha A V^2$$

The end points are then on a parabola (see Fig. 30).

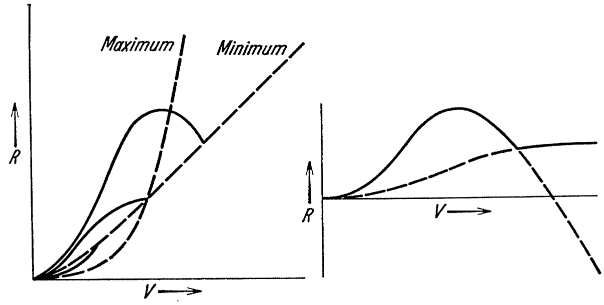


Fig. 31.

The maximum values of R will be given by the condition

$$\frac{dR}{dV} = 0 = 2 A_1 V - 4 A_2 V^3$$

whence $V^2 = A_1/2A_2$ and putting this in the value for R we find:

$$R = \frac{A_1^2}{4 A_2}$$

and using again the value of $V^2 = A_1/2A_2$, this becomes

$$R = A_2 V^4$$

If then we return to the expanded form of R for the value of A_2, we have

$$R = \frac{\beta K' b^2}{\delta \cos \theta} V^4$$

The results thus developed have been obtained by the aid of a doubtful working hypothesis. It is therefore necessary to check, by the aid of model experiments, in order to determine whether the results indicated by the formulae agree in satisfactory measure with those given by experiment. We give, therefore, as follows, certain results of model experiment.

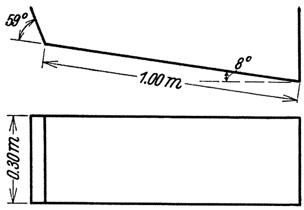


Fig. 32.

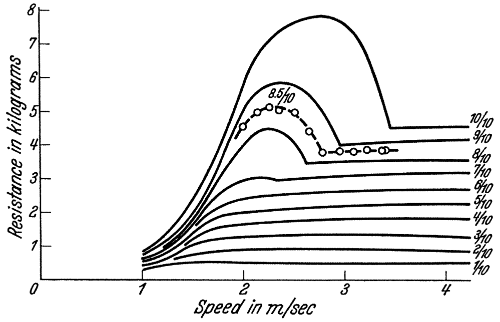


Fig. 33. Curves of resistance on speed.

10. Measures Taken on a Model Seaplane. Hydroplaning at Constant Angle of Incidence. The model Fig. 32 comprised a plane surface inclined 8^0 to the horizontal, together with a forward plane inclined 59^0 to the plane. The length of the first plane was one meter, the width 0.3 m. In these experiments while the model was compelled to maintain a constant attitude relative to the horizontal, it was free to move vertically. The model was run successively with a load of 24 kgs. (noted as 10/10) then with loads of 9/10, 8/10 and thus to the lightest loads. There was made also a similar series of runs with the intermediate series of loads, 9.5/10, 8.5/10 . . .

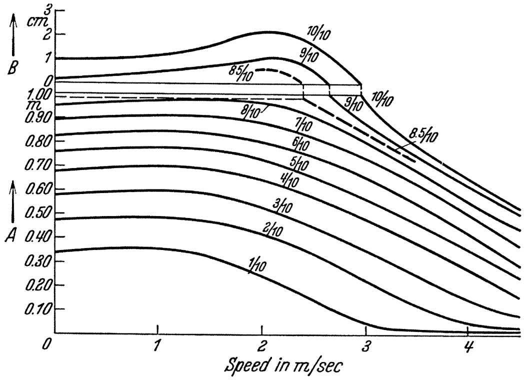


Fig. 34. Curves of location of model on speed.

Each run gave three curves. The first, Fig. 33, is the curve of resistance R as a function of the speed V. The two other curves, Fig. 34, relate to the position of the model relative to the undisturbed water level. One of these latter, B, gives the length of the forward plane below the undisturbed water level, and the second, A, the length of the lower plane below the same level. The curves A and B are given as functions of V.

In addition to these three sets of curves, three other curves are given, placed on the same sheet corresponding to the run with loading 9/10 (see Fig. 35). A comparison of this character brings to light a certain number of phenomena of a general nature.

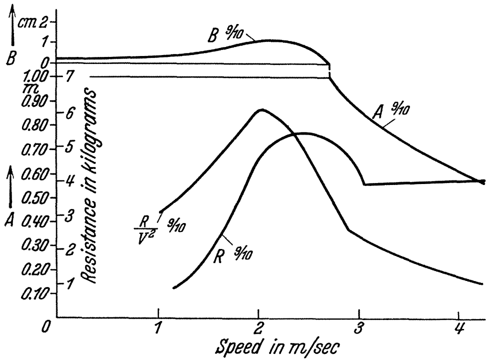


Fig. 35. Curves for loading 9/10.

(a) At low speeds for models starting with B not zero, the immersion increases; that is, B increases at the start.

(b) At low speeds for models starting with the edge between A and B out of water, A increases at the start.

(c) For models with B not zero, the maximum of resistance coincides with the maximum immersion.

(d) For the same models, the appearance of a resistance nearly constant corresponds to a progressive diminution of A.

(e) If the curve R/V^2 is traced, the maximum of this curve and its break, correspond with the maximum of B and with the disappearance of B.

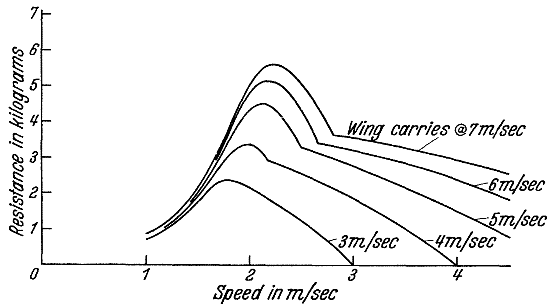


Fig. 36. Curves for resistance (water) assuming aerodynamic lift varying with the square of the speed.

(f) The general character of the curves of R is closely similar to that indicated by theory, but numerical verification is rendered difficult by the fact that, at the start, for example, B increases, a feature not included in the approximate theory above developed.

(g) By the aid of these curves, it is easy to obtain curves of variation of the resistance as a function of V when there exists an aerodynamic sustentation proportional to the square of the speed. If V_0 is the speed for which the aerodynamic sustentation is equal to the weight W of the plane, then for any speed x, the corresponding sustentation will be $W\,(x/V_0)^2$ and the weight carried by the water will be

$$W - W\left(\frac{x}{V_0}\right)^2 = W\left[1 - \left(\frac{x}{V_0}\right)^2\right]$$

The curve for $[1 - (x/V_0)^2]$ will then give the point on the abscissa x. The results of this method of representation are shown in Fig. 36.

Verduzio has given a simple method of showing in graphic form the values of the buoyant force required from the water wherein the incidence of the wings is variable. This graphic gives the values of the ratio, *sustentation from water* to the *square of the speed* wherein the incidence of the wings and the speed are known. It is assumed that the aerodynamic sustentation is of the form $L = \lambda i V^2$. If then the weight is W we have:

$$B = W - \lambda i V^2$$

where B = sustentation due to the water and L = that due to the air. This gives

$$\frac{B}{V^2} = \frac{W}{V^2} - \lambda i$$

These are straight lines for V = constant, each line for a particular value of V, Fig. 37. The ordinate at the origin is W/V^2. The point of intersection with the axis of abscissae gives the incidence corresponding to the speed of flight at altitude zero. In order to trace these lines, it is then sufficient to know, for a single speed, the value of i for flight at sea level.

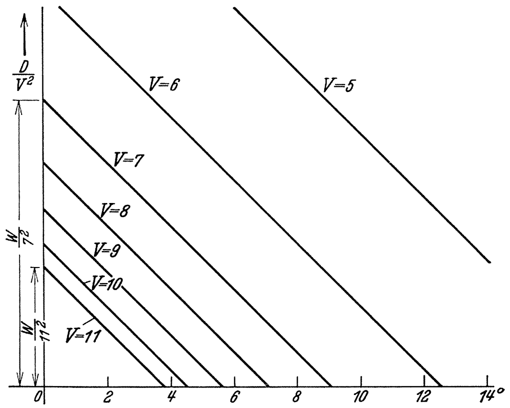


Fig. 37. Buoyant force required from water with varying incidence of the wings and varying speed.

11. Influence of the Angle of Incidence in Hydroplaning. If we consider singly the states of a seaplane at which the hydroplane regimen is established, it may be assumed that the resistance is given as a function of the weight carried p and of the angle of incidence, in the general form,

$$R = \alpha p \theta$$

Taking the angle of incidence of the wings, i, as variable, we may put this in the form,

$$R = \alpha p (e - i)$$

for the law of the resistance under all conditions of incidence and of weight carried. The wing data gives furthermore, the value of the aerodynamic sustentation as a function of speed and of the angle of incidence. This will have the form $\beta V^2 i$.

The weight carried by the reaction on the boat or float will then be

$$p = W - \beta V^2 i$$

and in consequence

$$R = \alpha (W - \beta V^2 i)(e - i)$$

This formula provides, for each value of the speed, for the study of the variation of R as a function of i. To represent these results graphically we employ the method given by Verduzio[1]. The abscissae represent the values of the wing incidence i and the ordinates y represent the values of $(R + R')/V^2$, R' being the resistance of the wings which may be taken in the form $K V^2 (i - i_1)^2$. We may then propose to trace the curves corresponding to a constant speed. We have thus

$$y = \frac{R + R'}{V^2} = \alpha \left(\frac{W}{V^2} - \beta i \right) (e - i) + K (i - i_1)^2$$

The curves corresponding to constant V will then be parabolas. The term in i^2 having a positive coefficient, the curves are concave upward. The value of the incidence i for which y is minimum is given by

$$i_m = \frac{\alpha W/V^2 + \alpha \beta e + 2 K i_1}{2 (\alpha \beta + K)}$$

The value of i_m will then decrease with increasing speed. The ordinate of the summits is found to be

$$y = - \frac{[\alpha W/V^2 + \alpha \beta e + 2 K i_1]^2}{4 (\alpha \beta + K)} + \frac{\alpha W}{V^2} e + K i_1^2$$

The locus of the summits is found by eliminating V between the values of y and of i. This gives likewise a parabola. These results are all in accord with those obtained graphically by Verduzio.

These results are only applicable to the period of hydroplaning. They show that, during this period, in order to reduce the power required to a minimum, the angle of incidence must slightly decrease as the speed increases.

During the critical period of mounting upon the step, the resistance of the air is small in comparison with that of the water and likewise the aerodynamic sustentation is weak. In consequence, during this period, there is no need of bringing in the incidence i of the wings.

It will be noted that thus far there has been no reference to changes of attitude due to the effect of the propeller. The influence due to the longitudinal stability has thus far been left entirely out of account.

This brief and approximate examination has provided means for a determination of the general character of the phenomena involved but can scarcely provide for the calculation of the more exact results required by the constructor. Independently of the fact that we have assumed the angle of incidence constant, a certain number of accessory phenomena have been completely neglected. These latter may be grouped under: phenomena entering in the two-dimensional problem, and, phenomena entering only in the three dimensional problem.

The first of these are: the immersion of the model at low speeds, and the formation of transverse waves (complicated by vortex phenomena).

[1] Hydrodynamic Congress, Innsbruck, 1922.

The second of these are: the divergence of fluid filaments on the sides of the body, the formation of divergent waves and friction of the lateral faces. Nevertheless, this preliminary examination has provided means for a determination of the general character of the conditions affecting the take-off of a seaplane.

12. Complete Study During the Hydroplane Period. The complete study of the plane during the hydroplane phase may be attacked by theory, by model experiment, and by full scale test. The method by theory with our present means hardly seems capable of giving useful results in cases other than where the flow is two-dimensional; that is to say, in the case where the width of the plane in the direction perpendicular to the line of movement is infinite. The method by model test has been employed by several experimenters. The most complete published results are those of Sottorf, which relate to the dynamometric examination of a model in movement under steady conditions.

Tests at full scale have thus far not given important results, principally because in the case of a full scale seaplane the phenomena due to the sustentation surface are confused with those due to parts of the hull aft of the step.

Before any analysis, observation on a model is necessary in order to permit the isolation of the diverse phenomena. We shall suppose that the situation is represented by a plane with small inclination to the horizontal and in uniform translation at high speed. Under these conditions there results a deformation of the free surface of the water similar to that caused by the movement of any body whatever. The free surface of the water takes a form which accompanies the body; that is to say, a form which observed from the body itself appears at rest. This may be expressed by saying that the speed of the system of waves formed by the body is equal to the speed of translation of the body itself.

Observed in longitudinal vertical section this deformation comprises a mass of water B elevated above the general level. This elevated mass forward seems to join with the horizon as an asymptote. At the stern there is formed a cavity, again joining ultimately with the general level of the water by the system of waves V (see Fig. 38). According to circumstances the separation of this system V may develop according to the form V_2, leaving the surface at the stern tangentially with concavity upward, according to form V_3 with concavity upward but not tangent to the under surface, or again according to form V_1 with concavity downward and not tangent to the under surface.

For an obstacle limited laterally, the observable portion of these wave forms is limited by the return of the water which the body has displaced laterally.

In plane form the deformation of the free surface is as follows. The water touches the plane following a surface nearly rectangular, limited forward by an arc pertaining to the elevated mass B. The material body having its lateral faces vertical, these faces are not in contact with the water (see Fig. 39). The free surface of the water is hollowed and

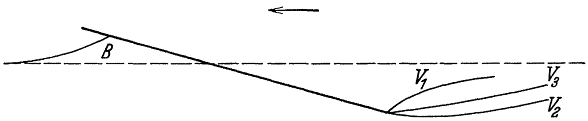


Fig. 38.

joins ultimately with the divergent waves W having their origin in B. The region at the stern is formed by a sort of basin, of which the bottom has the form nearly of a cylinder with horizontal generatrix, and the sides R are formed by the water returning along the longitudinal axis. At J the return of the water from both sides forms a sort of jet, Fig. 40.

Fig. 39.

The nature of the movement in contact with the body may be studied experimentally through the form of the curve B, through the form of the trajectories along the plane and through the value of the local pressure. The form of the curve B is readily observed through the use of a plane of glass. It is thus shown that this curve is convex forward. Its form, however, cannot be defined with any great precision because in this region steady conditions have not yet been fully established.

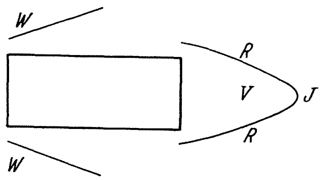


Fig. 40.

The form of the trajectories on the under surface of the body may be observed by the use of a plane of brass in which are fixed small steel rivets flush with the surface. Traces of rust may then be clearly observed

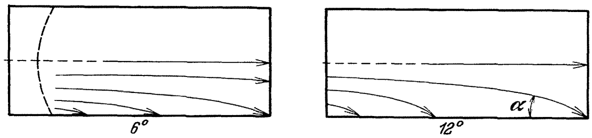


Fig. 41.

showing that a part of the water which has traversed the region B follows the body through to the after edge, while another portion escapes laterally, making an angle α with the direction of the lateral edges, Fig. 41. This angle α increases with the inclination of the plane to the horizon. For a plane inclined at an angle of 12^0 and a speed of two meters per second this angle reaches 60^0 (length of plane 58 cm. and width 14.4 cm.).

The values of the local pressure may be observed by the aid of manometers connecting with small holes pierced in the plane. Results of such measurements have been published by Sottorf[1].

These results relate to three planes having characteristics as shown in the following table.

Width cm.	Length below original level cm.	Wetted length cm.	Inclination	Speed m./sec.	Vertical component kgs.
30	17.5	24.5	8^0	6	18
30	37.0	45.0	6^0	6	18
30	80.0	87.0	4^0	6	18

The local pressure reaches its maximum value in region B and then decreases continuously to the after edge. At this edge Sottorf found for the first plane a positive pressure, and for the other two the pressure zero. It is a simple matter to pass from a knowledge of the pressure at a given point and of its immersion relative to the horizon, to a value of the local velocity. To this end it is only necessary to apply the Bernoulli principle. It thus appears that if at the after edge, which is below the general level, the pressure is equal to that of the atmosphere, the velocity at this point will be greater than the velocity of translation of the body. Applying the Bernoulli equation to the ensemble of the points for which measurements had been taken, Sottorf found for the three planes average velocities as follows:

5.57 m./sec., 5.81 m./sec., 5.92 m./sec.

A knowledge of these values is useful for the determination of that part of the resistance due to friction.

The form of the basin shaped cavity aft of the plane is important in regard to the practical determination of the form to be given to a seaplane aft of the step.

13. Zone of Water Contact Above General Level. Let us consider the part of the liquid forming the elevated mass forward and lying between the undisturbed level of the water and the forward portion of the plane. In consequence of the elevation of this mass above the general level a portion of the surface of the plane above this level is in contact with the water. It has the form indicated in Fig. 42. For a body with fixed attitude relative to the horizontal the dimension σ is a function of the velocity only. As the body is completely defined by b, λ and i, we should then have σ as a function of V, b, λ, i.

In each series of Sottorf's experiments V and b were constant. Hence σ is given as a function of λ, i.

[1] Werft, Reederei, Hafen 1929.

If we plot the values of σ as a function of λ for each series and place the curves according to the values of i, we shall obtain an ensemble of curves on which we may trace the lines for constant i.

It is also necessary to recognize the fact that the sweep of the air will modify the geometry of these phenomena. Experiments with the plane at rest are necessary in order to take account of the importance of this effect.

In a general way, however, it appears that this zone of contact depends principally on λ and i, but in a small degree upon the speed. In consequence, this zone will not be in relation with the system of longitudinal waves, but only with the flow of water laterally.

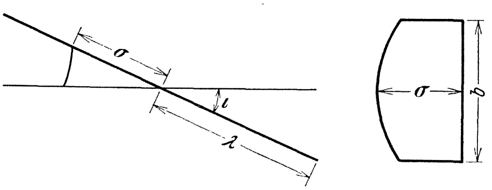


Fig. 42.

In order to definitely clear up this question it is necessary to run the model at all possible speeds, first without a forward air screen, and second with such screen. In such case, there develops an experimental difficulty which arises from the effect of the air on the region forward of the body. Let us consider the three following experimental arrangements relative to the same body (see Fig. 43).

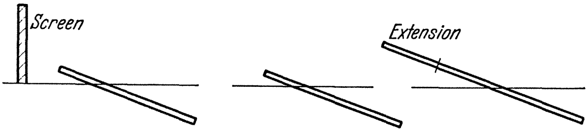


Fig. 43.

The results obtained in these three cases are different, and we must therefore conclude that the velocity of the body relative to the air has an influence on this zone of contact above the general level. This may have an effect of considerable importance on the position of the resultant. Sottorf found in certain cases the resultant passing through this zone.

If this zone were neglected the problem could then be treated by a method derived from that of Bobyleff by considering the body as the half of the Bobyleff body and imposing *a priori* the speed in the wake by the Bernoulli condition.

14. Study of Two Planes in Tandem with Constant Incidence. The curves marked x Fig. 44 give the resistance of a group of two bodies in tandem with an intervening space x. For all these bodies the slope of the under surface is 8^0 and the transverse width 30 cm. The load carried by the group is the same as in the preceding case, 9.6 kg.

The curve $x = \infty$ corresponds to the infinite separation of the two bodies. Its ordinates are therefore double those of the curve corresponding to a single body with load 4.8 kg.

On the curve $x = 80$ cm. it is seen that in comparison with the curve for $x = \infty$ the resistance of the group may be greater than the sum of the individual values for the two bodies. This arises from the fact that the wave produced by the forward body is projected with force on the after body, and meets it between its vertical walls. This situation develops in special degree for the case marked $V = 1.80$ m./sec. The association of the two bodies in this case is unfavorable, and the same condition holds through to $V = 2.4$ m./sec. where there is equivalence. For higher speeds the association becomes favorable and the resistance of the tandem combination passes through a minimum at about $V = 4.3$ m./sec.

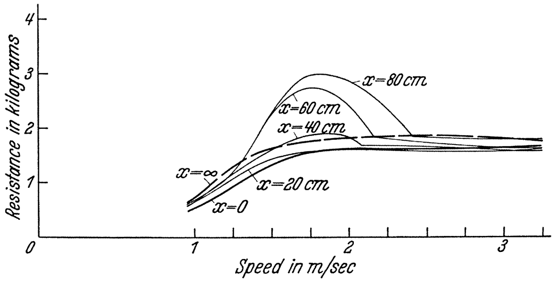


Fig. 44. Curves of resistance on speed for two planes in tandem, with varying distance between the planes.

For spacings larger than 30 cm. the same phenomena are found, with, however, maxima less and less definitely marked. The speeds of minimum resistance are related to the value x, as shown in the following table.

x	V m./sec.
80	4.30
70	4.18
60	3.87
50	3.75
40	3.60
30	3.25
20	2.70

The phenomena described here are susceptible of practical application for the prediction of the resistance of an ensemble of bodies. The description of these phenomena has been given solely for the purpose of showing the complexity of the actions intervening in the period during which the after part of a seaplane trails along in the wave formed by the portion forward of the step. The study of these phenomena through theory alone seems scarcely possible, especially because it would be necessary to take into account the closure of the water upon the after portion of the first step.

The experimental study of these phenomena presents as a principal difficulty the very great number of parameters. The present example is in effect relative to a single width and a single incidence. If such a study were to be undertaken systematically with a view to practical application, the number of parameters to be introduced would be six: the two values of the incidence, the two widths, a variable such as x giving the ratio between the longitudinal separation of the two bodies and the transverse width of one of them, and finally the weight of the ensemble. In such a study it would be further necessary to measure not alone the resistances to movement which have been mentioned here, but likewise the locations of their resultants. The extent of this program serves to explain why thus far such a study could be undertaken only in very special cases. In practice, such researches can only be carried on as variants of practical solutions, and the method by the use of models is the only one permitting in this way the improvement of a given initial design.

15. Comparison of Seaplane Under Water Forms with Variable Width. In these researches the following elements remain the same:

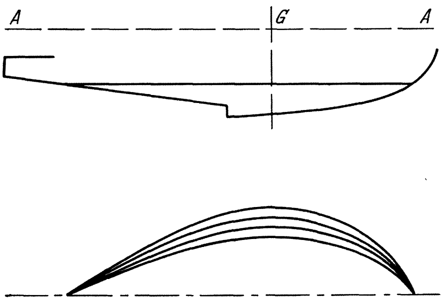


Fig. 45.

The wings and their characteristics, the weight of the model, the position of the wings with regard to the step, and the position of the center of gravity with relation to the step.

All the models have as geometrical form for the bottom, the same cylinder with horizontal generatrix following the form of the keel with step. The cylinder is limited by a vertical cylinder different for each model. All these vertical cylinders, however, are similar; that is to say, for the same transverse section the widths are in the same ratio as the values of b. As all of the models have the same weight, isolated they would not float at the same immersion, nor in the same attitude. The wider models have the reference line AA, Fig. 45, elevated toward the front. In effect the under water body being enlarged will take a lesser immersion for the same weight, and if the attitude of the model should remain unchanged the center of buoyancy would be displaced forward where the water lines are wider than aft.

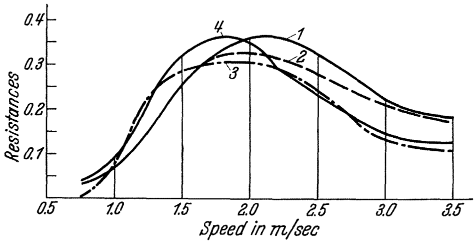


Fig. 46. Curves of resistance on speed for varying widths of model.

As the center of gravity is in the same longitudinal position for all cases, the model would tend to rise forward.

In consequence, at the start, the different models are more inclined upward at the bow as they are wider. In the case of the widest model the value of the inclination at the start is 9 mm. for a length of 900 mm. The value of the inclination at the start for the narrowest model is 0.

The values of b increase by successive equal values, the narrowest model having a width b, the widest model, No. 4, having a width 1.5 b. The constant weight was 1.75 kg. The resistance of these models plotted on speed is shown in Fig. 46.

16. Tests on Model with Varying Longitudinal Location of Wing. Figure 47 shows the resistance of a seaplane together with wing and suspension bars for a constant speed and a constant angle of incidence. There are altogether eleven tests differing with regard to the fixation of the model lengthwise with reference to the wing. The figure shows for each test the position of the model and its resistance. For Test No. 1, the wing being very far forward, the model remains supported on the water only by the part aft of the step, and the attitude is inclined upward in marked degree. For Tests 2, 3, 4, 5, the inclination decreases, but the general situation is the same. From 1 to 5 the resistance likewise decreases. For Test No. 6 the step begins to carry and the plane touches the water at two points. The resistance continues to diminish as well as the angle of inclination. For Tests 7, 8, 9, the step carries alone. The resistance still further diminishes, and for No. 9 the resistance is a minimum and the inclination negligible. For Test No. 10 the inclination begins again to increase as well as the resistance; the plane begins to lack longitudinal stability. For Test No. 11 the plane porpoises, the shift has gone too far. The minimum of resistance for these conditions of test was found for position No. 9. The distance between two consecutive positions of wing and model was 1/94 of the length of the model. The angle of incidence in all cases is 6^0 and the speed is 4.5 meters per second.

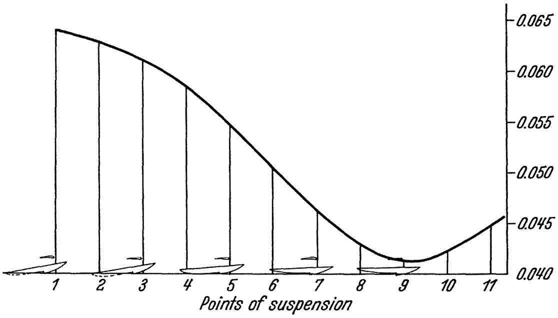


Fig. 47. Diagram showing effect of varying longitudinal location of wing.

CHAPTER IV

DIFFERENCES BETWEEN AIRPLANES AND SEAPLANES WITH REFERENCE TO THE AERIAL PORTIONS

In flight the general condition of equilibrium for any plane is the convergence of the three forces, weight, propeller thrust, aerodynamic reactions. The principal differences between airplanes and seaplanes is found in the distance between the general center of gravity and the propeller shaft. In the airplane this distance is small, and if the center of gravity is above the propeller axis the thrust of the propeller gives a couple tending to elevate the nose. In consequence, in case of a slowing of the motor the plane tends to go down by the nose, which permits it to pick up in speed.

With the seaplane, on the contrary, this distance is large, and the center of gravity is below the axis of the propeller, and in consequence the center of gravity is aft of the aerodynamic resultant R (wings and tail assembly). If, in flight, the plane should rise by the nose, due to any external cause, the resistance to movement increases, and in consequence the speed decreases. This results in an increase of thrust of the propeller, and in consequence an increase in the couple tending to turn the nose down, thus correcting the incipient perturbation. On the other hand, if in flight the motor weakens and stops, the plane rises forward since the opposing couple of the propeller decreases or becomes nothing. This involves a hazard of undue loss of speed. This defect can be corrected by an inclination of the axis of the propeller relative to the plane of the tail assembly in such fashion that in horizontal flight the propeller axis has an incidence positive, and the tail assembly an incidence negative.

The wash of the propeller gives then a downward reaction on the tail assembly and in consequence a moment tending to throw the nose of the plane upward. With a plane thus planned, the center of gravity should be carried forward in order to obtain proper centering in flight. If now the motor weakens, two effects are produced: one due to the decrease in propeller thrust—the effect tending to throw the nose upward; the other due to the decrease in the reaction of the tail assembly—the effect tending to throw the nose downward. In this manner compensation may be realized between these two effects. These consequences of a lack of equilibrium are especially important in small planes for which the distance between the center of gravity and the axis of the propeller is most important.

The study of the differences between airplane and seaplane should be completed by an examination of the stability of route in flight and of the transverse stability in flight and in turning.

(a) Regarding the stability of route, consideration must be given to the relative positions of the general center of gravity and the overall center of air drift. The latter should be located aft of the center of gravity.

The forward part of the boat or float of a seaplane being larger than the corresponding part of an airplane, the natural method of compensation would be found in a larger extent of the vertical drift-surface for the seaplane than for the airplane.

The vertical position of the center of gravity in relation to the center of drift is likewise different in the two forms. In the seaplane the center of drift may be still lower than the center of gravity of the structure as a whole, the result of which is a tendency to incline transversely in the direction of the drift.

(b) For stability among waves, consideration must be given on the longitudinal plan, to the position of the center of drift with reference to the axis of rolling and to the axis of yawing. In order that a seaplane may, at the same time, have stability of route and stability under the action of the waves, the center of air drift should be found in the angle hatched, see Fig. 48. This can only be realized by raising the general vertical drift-surface.

(c) In turning, the rudder carried by the tail of the plane must necessarily be above the water, and in consequence above the axis of rolling. The secondary effects due to pitching and rolling motions caused by turns, will then be more important in the seaplane than in the airplane.

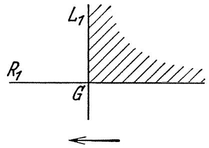


Fig. 48.

These differences between the seaplane and the airplane are not of basic importance with reference to the hydrodynamic study of the problem. There is need, however, to note briefly these relations in order to call to notice the sense in which advantage should be taken of such liberty of change as may develop from the hydrodynamic study, in order to improve the quality of the seaplane in flight.

CHAPTER V

DIFFERENCES AND ANALOGIES BETWEEN FORMS FOR HYDROPLANES AND FOR SEAPLANES

1. Introductory. The differences between seaplanes and ordinary boats have already been noted in the course of this study. They result chiefly from the difference in the values of the speed length ratio $\daleth = V/\sqrt{gA}$ in the two cases. Ships of normal form show values of this ratio less than 0.6. For example, even for a destroyer at 50 knots for a length of 100 m. at the water line, we should have $\daleth = 0.82$. On the contrary, for seaplanes, the speeds at the instant of maximum resistance are not far from 50 km./h. for lengths varying from 25 to 5 m. at the water line, giving values of the speed-length ratio between 0.89 and 2.0. Still higher values are to be met with for the period between the moment of maximum resistance and that of take off. The hull of a seaplane boat or float and that of an ordinary ship are then in conditions markedly different as regards the movements caused by their run over or at the free surface of a fluid. Nevertheless, there exist certain similarities between the conditions of operation of these two forms of hydro-glider. For very small boats, there have, indeed, been obtained values of the speed length ratio of 3.5 but these small boats are not suited for navigation in a troubled sea, and consequently this comparison is without significance. In order to realize more practical conditions, we may consider certain small boats described by Sir William

Thornycroft[1]. These boats have a length of about 17 m. for a speed of 40.5 knots, giving a value of the speed-length ratio of 1.6. These values give conditions similar to those for the hull of a seaplane, both as regards relative speed and the conditions for sustained movement in rough water. Table 2 gives a comparison between a boat of this character and a large modern seaplane.

TABLE 2.

	Boat	Seaplane
Ratio length to beam	5	5.5
Ratio $\frac{\text{distance of step to bow}}{\text{length}}$	0.5	0.67
Ratio $\frac{\text{distance of step to bow}}{\text{beam}}$	2.5	3.7
Ratio $\frac{\text{distance center of gravity from bow}}{\text{length}}$	0.54	0.57
Ratio $\frac{1000\ \text{volume}}{\text{length}^3}$	2.45	6.85
Angle of keel forward of the step with the water level	2^0	2^0
Break of the keel at the step	4^0	8^0

Length is here taken as length at the water level.

Although the two forms present the same general characters—a step, a sharp angle forward of the step, and similar forms of section for the forward body—the ratio length to beam is greater for the seaplane, which has, therefore, a somewhat finer form. For this there are several reasons.

In the first place, the boat in movement should be stable transversely due to its beam alone, while the seaplane is stabilized by the effect of its wings. Baker has indicated that if the length is less than 8.5 times the beam, the seaplane will be stable in movement[2].

Again, the boat is intended normally to move as a hydroplane, and for this case there is reason for an increase of beam, since the boat will have less to fear from jumps clear of the water due to an over development of hydroplane surface.

If instead of the ratio length to beam we consider the ratio 1000 volume $\div$ length^3, it is seen that from this point of view, the seaplane is much fuller in form than the boat. This difference results from the need of limiting the length of the hull of the seaplane in order to make possible the necessary changes in longitudinal attitude by a reasonable effort

[1] THORNYCROFT, SIR J. E., and LIEUT. BREMNER, Coastal Motorboats ("C.M.B."), Their Design and Service During the War, Transactions of the Institution of Naval Architects, Vol. 65, pp. 32—43, London, March 1923.

[2] BAKER, G. S., Flying Boats—the Form and Dimensions of their Hull, Engineering, Vol. 109, pp. 323—327, London, March 5, 1920.

on the controls. This condition is without force for the boat, but is imperative for the seaplane. A further reason is found in the need for an adequate stability of route.

In the seaplane, the length forward of the step should be the least compatible with the realization of a sufficient hydroplane area and of seaworthy qualities generally.

The longitudinal position of the step is very different in the two cases. On the seaplane the step is near the vertical containing the center of gravity. This is necessary in order to readily bring the plane upon the step. For the boat, on the contrary, the normal attitude is that of resting on two surfaces. The center of gravity is therefore, at nearly mid-distance between the center of pressure of the forward surface and that of the after surface, these surfaces being nearly of the same area and having each the optimum angle, determined in taking account of the speed of the water over the surface at each point. The advance of the center of gravity forward is here favorable to the speed but unfavorable from the point of view of seaworthy qualities.

The difference in the break of the keel arises from need, with the seaplane, to disengage completely the after part from the wave formed aft of the step—a condition which does not exist for the boat. The nearer we approach the stern the more the form of the hydro-glider differs from that of the seaplane.

Finally, if we examine, at the state of rest, the silhouette of the part above water, we shall find that the seaplane is much higher above the water and much more elongated aft. The first difference is due to the need, for the seaplane when alighting, of an approach to a rough water surface at a rather brusque angle, while the hydro-glider, always supported by the water, continuously adapts itself to the changes of the level on the free surface. The second difference is due to the conditions regarding the lever arm necessary for the movable air surfaces serving as controls for the seaplane.

If we should turn the comparison to the case of a racing hydroplane rather than a hydroplane boat, the differences would become more pronounced and would relate on the one hand, to the ratio between the resistance and the weight carried, and on the other, to the form of the transverse sections, which would no longer need to be limited at the bottom by a line rising from the mid-length toward each end fore and aft.

In the series of relative speeds where the extremes are occupied by the racing hydroplane and the ordinary boat, the seaplane occupies an intermediate position in which there should be combined a low value of the resistance and seaworthy qualities in good degree.

Historically the form of the seaplane has undoubtedly developed out of the hydro-glider. Practically it is a compromise among conflicting conditions as indeed are all projects in the art of naval architecture. In the case of the seaplane, the necessary nautical qualities must be paid for by a less favorable ratio between the resistance and the weight.

In order to properly understand the progressive evolution leading from the ship form to that of the seaplane hull, it is necessary to note the conditions in which the introduction of a step may be favorable or unfavorable. Dynamometric measures show within what limits of the speed-length ratio any given case may be found, but do not indicate the mechanism of the phenomenon.

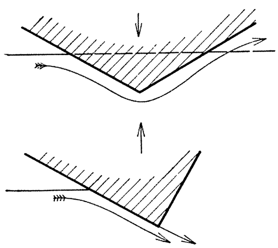


Fig. 49.

Theory can give to this subject some indication as follows:

Consider an obstacle, motionless in a current of water. The theorem of momentum indicates that if the water flows about the obstacle and leaves it with a speed and vertical component directed upward, the obstacle will be subject to a reaction from the water, directed downward; that is, it will tend toward a greater immersion. On the other hand, if the water leaves the obstacle with a velocity and a vertical component directed downward, the body will be subject to a reaction from the water, with vertical component directed upward; that is, to a force of sustentation Fig. 49. The theoretical problem would, then, be much advanced if we could determine the actual conditions under which separation of the plane from the water and take-off occurs.

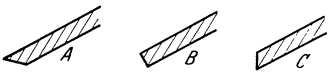


Fig. 50.

2. Experiments on Bevelled Planes. The importance of a brusque separation of the water at the after end of the body is readily shown by observations made in the experimental channel with planes having, either at the after end or on the side, a bevelled edge. For the same weight carried, the resistance is much greater with a following edge similar to A, Fig. 50 rather than to B or C. The same result is found, but in less marked degree, with regard to the lateral edges.

3. The Immersion of Ship-Formed Models and the Emergence of Seaplane Models. The consideration of an obstacle having a position fixed with reference to the level of undisturbed water is important from the point of view of a prevision of results by theory.

If observations are made on a model thus held in a fixed position in a current of water it will be found that for low values of the velocity, the model is subject, from the water to a force directed downward. The same effect is noted with models having one or two degrees of freedom as usually employed in naval experimental tanks. In such case, for low values of the speeds, the upward component is less than with the model at rest, and in consequence, the model settles lower; that is in movement, it takes a position lower than when at rest. If the speed increases, the phenomena for the free model become complicated in consequence of the change of attitude. However, in general the center of gravity of a ship under way is lower than when at rest.

These conditions do not hold for hydro-gliding planes, which beyond a certain speed, rise continuously with reference to the undisturbed water level.

If for a body of given form, fixed attitude and constant speed, the values of the horizontal and vertical components of the total water reaction for all immersions measured relative to the undisturbed level are plotted in curve form, each relative to the value of the immersion, we then have two curves H and V, Fig. 51. The curve H passes through the origin and the curve V below the origin. This is due to the fact that even for a weight zero, a body in horizontal motion will take a finite immersion as soon as it touches the liquid. The curve showing the ratio H/V will then have the form of a hyperbola. It is interesting to compare this curve V to that obtained from the same measurements made on a body at rest relative to the water. This new curve is well known to naval architects under the name of "displacement curve". It gives the total upward force at each immersion, in each case exactly equal to the weight producing that immersion. This curve naturally passes through the origin of coordinates.

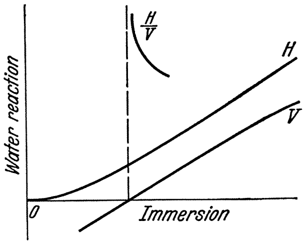


Fig. 51.

CHAPTER VI

CALCULATIONS OF DISPLACEMENT AND OF STABILITY

1. Displacement and Stability of the Seaplane at Rest. A floating body is in equilibrium at the surface of an undisturbed liquid when the hydrostatic pressures over the wetted surface have a single resultant, equal to the weight of the body, directed upward and through the center of gravity. The well known principles of hydrostatics show that this resultant is equal to the weight of the displaced liquid, or to the product of density of the liquid by the volume displaced by the body, and that it passes through the center of figure of this displaced volume.

For a seaplane, the problem for the conditions of equilibrium is usually presented in the following manner. Given the external form of the hull, the total weight of the plane and the position of its center of gravity, it is required to determine the immersion and the attitude of the plane at rest on the water.

The search for a position of equilibrium is, in consequence, a search for a water-plane section through the hull such that the volume cut off shall have a given value and that the perpendicular to this plane through the center of figure of this volume shall pass through a point fixed in advance (center of gravity of the structure). The solution of this problem depends on the manner in which the form of the hull is defined.

2. Determination of Form. As with ship lines, the form of the hull of a seaplane comprises a series of transverse sections, a series of horizontal or water-plane sections, and a series of longitudinal vertical sections (bow and buttock lines). A base line is taken passing through the lowest point of the step, AB on the various plans, and parallel to the water plane FL, Fig. 52. The lines of a seaplane hull differ from those for ships of normal form in two principal points. First the transverse sections are not equally spaced. This is due to the need for a specially accurate definition of the form in regions of rapid change and especially near the step or steps. If, for example, there are two steps, there will be a section at each step and there will be three distances, unequal in the general case, lying forward of, between and aft of these points. The second difference is due to the fact that the hull of a seaplane comprises longitudinal edges, and in consequence the lines will show a discontinuity whenever the plane cuts through such an edge.

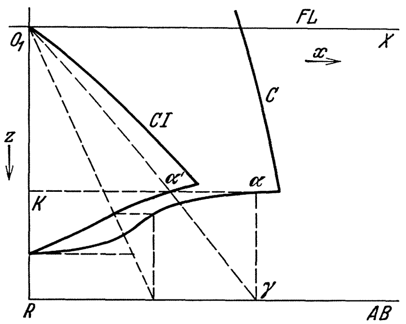


Fig. 52. Sections of form of seaplane body.

For these reasons, it is not possible to fully apply to the lines of a seaplane hull the methods commonly employed for normal ship forms. In particular the corrections at terminal points cannot be realized in the calculations for seaplanes. Constructors of seaplanes having, however, at the start followed the general practice in vogue for ship forms, the methods commonly employed are naturally based on the classic methods of naval architecture. On the contrary, the method here outlined is based on the special conditions of the actual problem. It is a planimetric method using simply a planimeter for areas rather than the more complex integrator giving areas, static moments and moments of inertia. The simple measurements of area may also be made without instrument and with almost equal rapidity by laying out the various diagrams and plans on cross-section paper.

The results required are first:

(a) The volume of hull below any given water-plane.

(b) The location of the center of figure of this volume, both longitudinally and vertically.

To obtain (a) and (b, vertical) there is traced at each section an auxiliary curve which may be called the section integral. The sections and section integrals are then measured by planimeter giving at each section the area and static moment about a reference axis. These areas and moments are then plotted on an axis of length and the curves drawn. These latter curves are then planimetered giving the volume and the moment of volume, and hence the center of volume relative to the reference water-plane. In order to find the longitudinal location of the center of volume the curve of longitudinal moments of section areas must be prepared and then planimetered in similar manner as for the sections.

We may now explain in the necessary detail the character of the various curves. The curve of section integral is found for any given section by the following construction, see Fig. 52. With each point α of the section, there is associated a point α' obtained by drawing $\alpha\alpha'$ and $\alpha\gamma$ respectively parallel and perpendicular to the water-plane and then drawing $O_1\gamma$. It is seen that, putting $\alpha K = x$, $\alpha' K = x'$ and $O_1 K = z$, we have from similar triangles,

$$x' = \frac{z\,x}{O_1 R}$$

The area of a curve with points found in this way is:

$$\int x'\,d\,z = \frac{1}{O_1 R} \int z\,x\,d\,z$$

which is the static moment of the section area about $O_1 X$, divided by $O_1 R$. This area multiplied by $O_1 R$ will then give the static moment of the section about $O_1 X$ (see Table 3).

In the same manner we may find the moment of inertia about $O_1 X$ by tracing a second integral curve. We should have thus

$$x'' = \frac{x' z}{O_1 R} = \frac{z^2 x}{(O_1 R)^2}$$

whence $$\int x''\,d\,z = \frac{1}{(O_1 R)^2} \int z^2\,x\,d\,z$$

The area of the curve of x'' multiplied by $(O_1 R)^2$ will therefore give the moment of inertia of the section about $O_1 R$.

The tracing of the section integral and the two measurements of area need occupy no more than five to ten minutes each, even when using section paper and without the use of a mechanical planimeter. With the latter the procedure is a little more rapid. In all, the number of sections being in the neighborhood of 20, this entire program can be carried through in about two hours.

TABLE 3. *Operations on the Curves of Section Areas.*

Sections	Water-line 1			Water-line 2		
	C_1	C_1I	M_1	C_2	C_2I	M_2
6	19.5	3.0	24.0	3.3	1.0	8.0
7	26.0	5.3	42.4	8.0	2.7	21.6
8	33.8	7.0	56.0	13.2	4.5	34.0
9	42.4	9.5	76.0	20.2	7.0	56.0
10	52.3	13.8	110.4	27.8	10.6	84.8
11	89.2	36.7	293.6	62.0	33.2	265.6
12	85.0	32.2	257.6	57.8	29.0	232.0
13	79.2	28.7	229.6	52.3	25.0	200.0
14	66.3	21.2	169.6	40.3	17.6	140.8
15	39.4	8.6	68.8	17.0	5.8	46.4
16	11.5	1.0	8.0	17.0	5.8	46.4

Depth $O_1R = 8$
Column headed C_1, areas of sections for water-plane 1
Column headed C_1I areas of section integrals as in Fig. 52
Column headed M, moments of section areas = col. $C_1I \times O_1R$
Similarly for water-plane 2

It remains to trace on length the curves of section areas and the curves of section moments vertically and longitudinally, and then to planimeter these two curves. This will occupy about one hour, after which we have the volume and the location of the center of volume. Operating on two water-planes in succession, the total time required is about four hours and we have at our disposal two volumes and two centers of volume. It may be noted that the same section integrals will serve for the two water-plane locations, the area being properly limited in the second case.

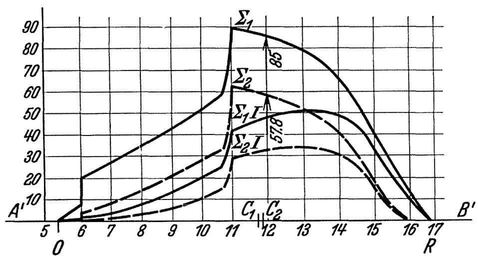


Fig. 53. Curves of section area and section moment on length.

The longest part of the work is that required for tracing the section integrals. This part of the program may be omitted when the forms employed are very similar to those already known, in which case the location of the center of volume may be fixed by a simple application of ratios.

It is evident that the curve of section areas will give the volume and the curve of longitudinal moments of these areas (see Fig. 53) will give the static moment longitudinally and thence by division the longitudinal location of the center of volume (see Table 4).

TABLE 4. Determination of the Center of Buoyancy, Longitudinally and Vertically for Water-Planes 1 and 2.

Curves relate to one-half body form.

Longitudinal (see Fig. 53).

Scales: Abscissae 1/6, Ordinates 1/10.

Area of curve Σ_1 = curve of section areas = 82.3 (cm.)^2
Area of curve $\Sigma_1 I$ = curve of section moments = 43.5 (cm.)^2
Area of curve Σ_2 = curve of section areas = 46.8 (cm.)^2
Area of curve $\Sigma_2 I$ = curve of section moments = 25.3 (cm.)^2
Distance O_1R = 17.65 cm.

$\text{Vol.}_1 = 82.3 \times 10 \times 6 \times 2 = 9870\ (\text{cm.})^3 = 9.87\ (\text{dm.})^3$

$\text{Vol.}_2 = 46.8 \times 10 \times 6 \times 2 = 5616\ (\text{cm.})^3 = 5.62\ (\text{dm.})^3$

Abscissa of center of buoyancy $x_1 = \dfrac{43.5 \times 17.65 \times 6}{82.3} = 56$ cm.

Abscissa of center of buoyancy $x_2 = \dfrac{25.3 \times 17.65 \times 6}{46.8} = 57.2$ cm.

Vertical (see Fig. 54).

Scales: Areas 1/10, Moments 1/80.

Area of curve Σ_1 as above = 82.3 (cm.)^2
Area of curve M_1 = curve of section integrals (Fig. 52) = 25.0 (cm.)^2
Area of curve Σ_2 as above = 46.8 (cm.)^2
Area of curve M_2 = curve of section integrals (Fig. 52) = 20.8 (cm.)^2

Ordinate of center of buoyancy $y_1 = \dfrac{25 \times 8}{82.3} = 2.43$ cm.

Ordinate of center of buoyancy $y_2 = \dfrac{20.8 \times 8}{46.8} = 3.55$ cm.

Similarly, the moments of the sections, as in Fig. 52, laid down as in Fig. 54 will give the moment of volume about the reference water-plane and hence the vertical location of the center of volume (see Table 4).

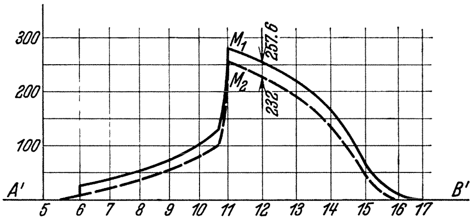


Fig. 54. Curves of section integral on length.

3. Interpolation of Volume. We have thus found, for two locations of the water-plane, the centers of volume both vertically and longitudinally. These are located on the longitudinal plan. One of the planes gives a volume greater than that specified, the other a volume smaller. By simple proportion, we may then locate, parallel to the base line, the water-plane and the volume specified, and also in like manner its center of figure.

This volume must, however, meet the further condition of equilibrium. To determine this condition we must find, for changing inclination with constant volume, the envelope of the water-planes and the locus of the resulting center of volume. To this end we must compute a radius of curvature $R = I/V$ and the derivative value $r = \triangle I/\triangle V$.

To this end we make use of the moments of inertia of the areas of the water-planes.

These moments must be taken about a gravity axis. Table 5 gives the form of calculation for the center of gravity of a water-plane area and for its moment of inertia about a transverse axis through this point. For the first, use is made of the curve FI and for the second, of the curve FII, Fig. 55. Since the moment of inertia directly resulting from this operation will not be about a gravity axis, use is made of the well known relation:

$$I_G = I - h^2 S$$

Where S is area and h is the distance between the first and the gravity axes. This correction, as shown in Table 5, thus gives the values needed, and thence R_1, R_2 and by interpolation R_v corresponding to C_v and r.

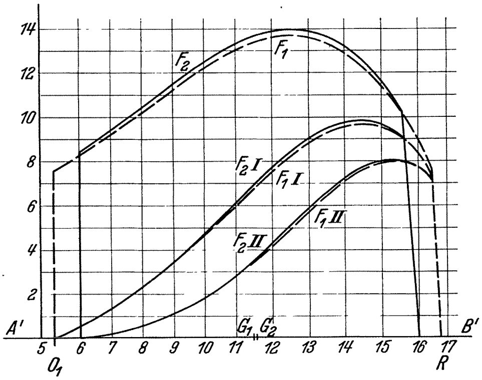


Fig. 55. Curves for two water-planes.

Table 5. *Calculation of the Center of Gravity of the Water-Plane Areas.*

Scales: Abscissae 1/6, Ordinates 1/4.

Planimetry $1/2\ WP_1 = 198.5$ sq. cm.

$$1/2\ WP_1I = 100.4 \quad G_1 = d_1 = \frac{100.4 \times 17.65}{198.5} \times 6 = 53.58 \text{ cm.}$$

$$1/2\ WP_2 = 175$$

$$1/2\ WP_2I = 89.5 \quad G_2 = d_2 = \frac{89.5 \times 17.65}{175} \times 6 = 54.18 \text{ cm.}$$

$$d_1^2 = 0.287 \text{ m.}^2 \qquad d_2^2 = 0.2935 \text{ m.}^2$$

Calculation of r and R.

Planimetry of $WP_1II = 0.0063$ sq. m.

$WP_2II = 0.00545$ sq. m.

$O_1R = 0.1765$ m.

Inertia of WP_1 in relation to $O_1 = 0.0063 \times 17.65^2 \times 6^3 \times 2 = 0.0847$

Inertia of WP_2 in relation to $O_1 = 0.00545 \times 0.1765^2 \times 6^3 \times 2 = 0.0733$

$$I_G = I_{01} - d^2 S$$

$$S\ WP_1 = 0.1946 \times 2 \times 6 = 0.2335 \text{ sq. m.}$$

$$S\ WP_2 = 0.1773 \times 2 \times 6 = 0.213 \text{ sq. m.}$$

$$I_{1G} = 0.0847 - 0.287 \times 0.2335 = 0.0177$$

$$I_{2G} = 0.0740 - 0.2935 \times 0.213 = 0.0113$$

$$r = \frac{\triangle L}{\triangle V} = \frac{0.0064}{0.00425} = 1.505 \text{ m.}$$

$$V \text{ given} = 0.00774$$

$$I_V = 0.0145$$

$$R = \frac{I}{V} = \frac{0.0145}{0.00774} = 1.873$$

These calculations depend on the following geometrical properties which are demonstrated in any treatise of theoretical naval architecture.

(1) For small inclinations about a transverse axis, the envelope of the water-lines is a circle of radius $r = \triangle I/\triangle V$.

(2) Under these conditions the buoyant force passes through a point (sensibly stationary) called the metacenter. The distance from this point to the center of volume is given by I/V. The line joining the metacenter to the center of volume is $\perp$ to the water-line.

(3) The locus of the center of volume is the arc of a circle with the metacenter as center and I/V as radius.

For the actual problem we may then locate on the plan the center of volume, C_v, the center of gravity of the structure G and the metacenter M. The line GM will then be $\perp$ to the water-line. A line drawn $\perp$ to GM and tangent to the circle with $r = \triangle I/\triangle V$ as radius will then give the water-line sought, Fig. 56.

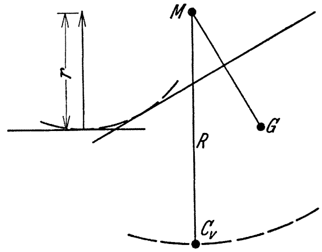


Fig. 56.

On the longitudinal section, Fig. 57, will be found in addition the centers of gravity of water-planes and the centers of volume for a series of other water-plane locations, with curves drawn through, showing the movement of these points with varying immersion. These curves show the character of the discontinuities which are to be anticipated in such cases.

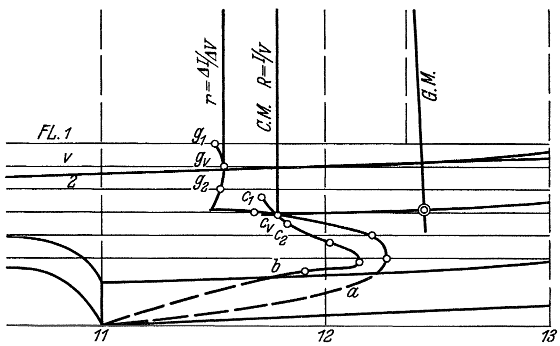


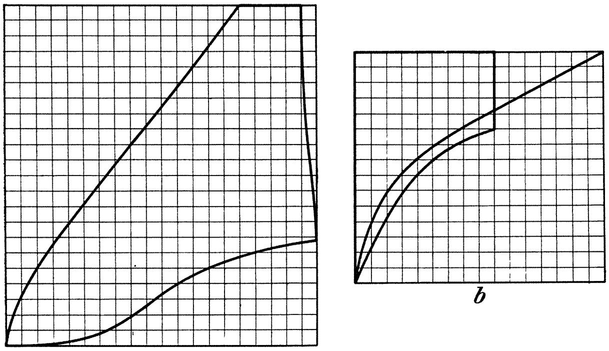
Fig. 57. Centers of figure: curve a of buoyancy, curve b of water-plane area.

4. Bonjean Scales. A procedure due to Bonjean is very useful for calculations of this character in the case where certain compartments of the hull may be invaded by water. In such case it is preferable to use methods of numerical integration.

For each section, the half widths are read for a series of equally spaced water-lines. We then write on each line (see Fig. 58) the half width (Column 1 of the table) and the sum to each line from the bottom upward, putting the results on the right (Column 2). These values, to suitable scale, are then laid off as abscissae giving a curve showing the section area from the base line up to any water-line at choice.

Each section is treated in this manner. If the water-line is drawn in, the corresponding abscissae at each water-line intersection will give the values for a curve of section areas on length. This form of calculation is useful for the determination of the heights to be given to water tight compartments.

In the application of this method to seaplane hulls, care must be taken with reference to the unequal spacing of the sections. In finding volume, for example, each section area must be used with the length which it represents, and then these parts summed.



a b

Fig. 58. Curves for Bonjean Scales.

Bonjean Scales.

Areas	Totals	Areas	Totals
19.0	374.6	9.0	79.7
19.0	355.6	9.0	70.7
19.1	336.6	9.0	61.7
19.1	317.5	9.0	52.7
19.1	298.4	9.0	43.7
19.2	279.3	9.0	34.7
19.3	260.1	6.6	25.7
19.4	240.8	5.0	19.1
19.5	221.4	4.0	14.1
19.5	201.9	3.1	10.1
19.6	182.4	2.5	7.0
19.7	162.8	1.9	4.5
19.8	143.1	1.3	2.6
19.9	123.3	0.9	1.3
19.9	103.4	0.4	0.4
20.0	83.5	0.0	0.0
15.3	63.5		
12.8	48.2		
11.0	35.4		
9.8	24.4		
8.2	14.6		
6.4	6.4		
0.0	0.0		

CHAPTER VII

STABILITY UNDER VARYING CONDITIONS

1. Stability when Drifting with Motors Stopped. A seaplane assumed floating on the water with the motors stopped, will be subject to the action of the wind and waves. The effect of these actions will be different according to the orientation of the wind and that of the wave crests in relation to the longitudinal axis of the plane.

Let us first consider the effect of a head wind acting alone. The wind tends to push the plane sternward, which will, in consequence, pick up sternway. The water opposes this motion and develops thus a hydrodynamic resistance increasing with the speed until it equals the air force. At this moment there is equality between these two forces, but they will not be exactly opposed, and hence will form a couple. The nose of the plane rises, therefore, and a position is taken where the moment of longitudinal stability equals that due to the aero- and hydrodynamic forces.

To study this condition it is necessary to examine what happens when a small perturbation is introduced, and especially the effect produced by a change in the direction of the wind. Under these conditions, the combined air effects give a force sensibly horizontal and passing through the forward third of the part of the hull above water. This is the point $\triangle A$, the center of air drift, Fig. 59.

The plane receives, the same as before, a reaction from the water and this reaction passes sensibly through the after third of the immersed part of the hull. This results from the fact that the forces from the water are the same as though the water were in movement with reference to the plane, the current flowing from stern to bow. This is the center of water drift.

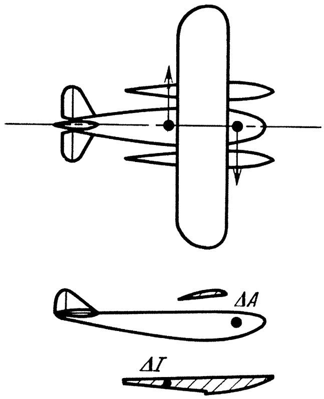


Fig. 59.

The situation of equilibrium with head to wind will not be stable unless the couple due to air and water forces tends to turn back the head into the wind, that is, if the center of air drift is aft of the center of water drift.

To realize this condition it is necessary to carry the center of air drift aft by the addition of a vertical surface toward the stern. With reference to the problem of stability under drift, the seaplane should be studied by model with reference to its parts above water (wings and non-immersed hull). These tests will serve to determine the vertical location of the air forces. The model should then be studied in a seaplane channel under horizontal traction by means of a cord attached at a suitable height at varying positions in length. In this way the location farthest from the stern is found for which the model will move in the direction of the thread. This gives the longitudinal position of the point $\triangle I$, Fig. 59.

When the point $\triangle I$ of the seaplane is aft of the point $\triangle A$ the plane tends of itself to turn head to wind. The effect of wind coming from the side and from aft is found in the same manner. For the case of a wind on the beam it will be necessary to carry out a calculation similar to that for the determination of the stability of a ship under sail—a procedure well known to the naval architect[1].

2. Stability of Route with Motors Running. This problem is treated in the same manner as for ships and as set forth in text books of naval architecture. In this case the resultant force on the plane is directed forward. The center of water drift is placed toward the forward third of the immersed body.

[1] Gagnotto, Luigi, Étude sur l'action du vent lateral sur les navires, *Bulletin de l'Association Technique Maritime et Aéronautique*, Vol. 33, pp. 53—74, Paris, 1929.

In the case of ships, the stability of route is estimated by comparing the positions of the general center of gravity and the center of water drift. The latter being always forward of the former, there is, properly speaking, no stability of route. There is nothing to oppose a continued turn initiated by any external cause. It is well known that the stability of route in such cases can be increased by the increase of immersed area toward the stern. This method may likewise be employed with seaplanes. It may be remarked finally that the increase of stability of route stands in opposition to the improvement of aptitude for evolutions. Too large a stability of route should therefore not be sought.

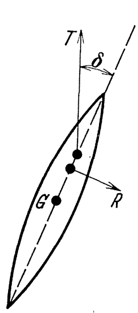


Fig. 60.

3. Stability under Tow. Differing from the conditions for the preceding cases, the problem of a seaplane under tow brings in an external force of which the direction is variable with reference to the plane. The length of the tow line is also a factor in this case. If the tow line is very short, it may be seen that the situation would develop as though the towing force passed through a fixed point of the plane. If this point of attachment is forward of the forward drift center there is stability.

In the case where the tow line is long, this condition is still necessary, but the plane no longer moves in a straight line. With a very long tow line we may assume that the direction of the line is constant. Under the influence of a small deviation δ of the bow, the water resistance will give a considerable transverse force R, which, relative to the center of gravity G, will have a moment greater than that of the traction T, Fig. 60. The angle δ will then increase, thus increasing the moment of T until an angle of yaw is reached where the two moments are equal. Due to the turning inertia, the bow will pass beyond this angle until brought back by the increasing value of the traction moment. The plane will thus have an oscillating movement about its mean path.

CHAPTER VIII

RULES OF EXTRAPOLATION

1. Introductory. The rules of extrapolation represent in a general way the collection of procedures making possible the determination of the principal characteristics of a design without passing through all the detailed calculations necessary for the design itself. Such rules are useful when there are a large number of variants. Rules of extrapolation should not be confused with rules drawn from the law of similitude. The latter relate to details while the former relate rather to the project as a whole. While such rules depend upon general considerations of mechanical similitude they include as well other diverse phenomena.

In order that rules of extrapolation may be established on a suitable foundation it is needful on the one hand that the number of previous cases be large and relative to varied characteristics; and on the other hand, that the project in hand be not too far removed in characteristics from those which have served as a foundation for the formulae employed. It will be understood, therefore, that in a branch of structural technique relatively young, as in the case of seaplane design, extrapolation is relatively difficult.

As an example, we may cite the proportions given by Rumpler. These propositions are based upon exact mechanical principles, but the formulae employed are based upon previous designs too far removed from the cases to which they are applied. Rumpler starts with the following considerations.

The principal difficulty in the increase of the dimensions of a design depends, for planes of present form, upon the fact that with such increase the useful load continually decreases. This results from an increase in the weight of construction due to the concentration of load in the central portion. By avoiding such concentration a percentage of useful weight will be maintained, the same for large planes as for small. This decentralization leads Rumpler to a design which is schematically the lateral juxtaposition of a series of small planes. This decomposition applies to the power plants as well as to the boats or floats. In order to justify such a type of construction Rumpler proposes to show that the limitation imposed by the weight of the wing structure of present designs is nearly realized. The foundation of these considerations is then a problem of extrapolation.

2. Rumpler's Method of Extrapolation. An initial design is assumed as follows:

Wing surface	S
Total Weight, W	2,000 kgs.
Weight of wing structure	300 kgs.
Residual Weight	1,000 kgs.
Total	1,300 kgs.
Useful Load	700 kgs.

It is then assumed that the design is increased in linear ratio n, holding constant the loading per unit area of wing surface. We have then

Wing surface $\Sigma = S\,n^2$

Total weight $Sn^2 \times W/S = 2000\,n^2$

Weight of wings $300\,(0.8\,n^3 + 0.2\,n^2) = 240\,n^3 + 60\,n^2$

Other weight (50 per cent of total) $= 1000\,n^2$

Remainder = useful load = $2000\,n^2 - (240\,n^3 + 60\,n^2 + 1000\,n^2)$

$$= 940\,n^2 - 240\,n^3$$

The latter vanishes for $n = 3.92$

Under these conditions we should have

Wing surface $S \times (3.92)^2 = 15.37\,S$

Total weight $2000 \times (3.92)^2 = 30{,}740$ kgs.

For varying values of n the various elements considered by Rumpler may be computed, holding constant the total weight at 2000 kgs. For such a design the load per square meter would be about 53 kgs. The type design should then have

$$S = \frac{2000}{53} = 37.6 \text{ m.}^2$$

Consequently, for other designs:

$$\text{Wing surface} = 37.6 \times n^2$$

$$\text{Weight} = 2000 \times n^2$$

$$\frac{\text{Useful load}}{\text{Total Weight}} = \frac{940\,n^2 - 240\,n^3}{2000\,n^2} = 0.47 - 0.12\,n$$

The law of linear decrease indicated by Rumpler, and giving for the type design a ratio of 0.35 ($n = 1$) gives a ratio 0 for a limit design of 30,740 kgs.

Discussion. These results are based upon three hypotheses:

(1) The weight carried per square meter of wing area is constant (53 kgs.).

(2) Half of the weight carried is allotted to weights other than wing structure and useful load.

(3) The weight of wing construction per square meter of surface Σ is

$$\frac{240\,n^3 + 60\,n^2}{S\,n^2} = \frac{240\,n + 60}{37.6}$$

With $n = \sqrt{\Sigma/37.6}$, and an aspect ratio of 6, this weight of wing structure per unit area may be expressed as a function of the span by using the relation $\Sigma = 0.167\,b^2$.

The weight of wing per square meter as a function of span is then

$$\frac{240\,b\sqrt{0.167/37.6} + 60}{37.6} = 0.426\,b + 1.6$$

This weight of wing structure per unit area in the hypothesis is then a linear function of the span.

For $b = 10$ m. weight per unit area $= 5.86$ kg./m.²

$b = 50$ m. weight per unit area $= 22.90$ kg./m.²

We may now examine Rumpler's rules in the light of actual realization.

(1) Constant Load per Unit Area of Wing. This is inexact, see Fig. 61, giving the relation between R prediction and D realization.

(2) Half of the weight carried is allotted to weights other than wing and useful load. The Dornier designs show a distribution of weight carried as follows:

Total Weight. . . .	670	2860	6030	14100	51500
Weight of Wings . .	108	535	767	1899	7476
Useful load	80	962	2401	6411	25838
Sum.	188	1497	3168	8310	33314
Ratio to Total Weight	0.280	0.523	0.526	0.590	0.646

Instead of the constant ratio 0.5, it is seen that the useful load per unit of wing area continuously increases. Here again the relation between R and D as in Fig. 61 is shown in Figs. 62 and 63.

(3) The weight of the wing structure per unit area of wing $= 0.426b + 1.6$. This is likewise inexact; the relation is rather *Weight of wing structure per unit area of wing* $= 0.25b + 5$. See Fig. 64, Curve R' where again R and R' are according to these two formulae while D gives actual results.

Figures 63 and 64 show the comparison between the prediction R and the realization D.

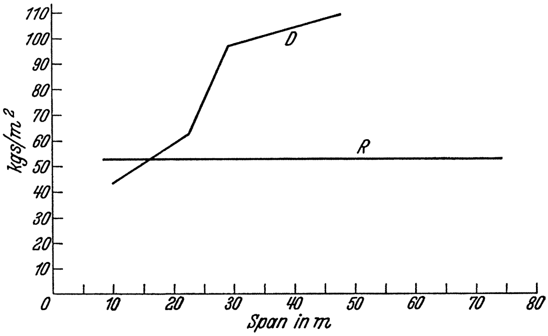


Fig. 61.

If we assume the following rules:

(1) $\dfrac{\text{Total Weight}}{\text{Wing Area}} = \text{constant} = 110$, for example.

(2) Useful load plus weight of wing structure equals one-half total weight.

(3) Weight carried per unit area of wing equals $5 + b/4$ we shall have

$$\text{Useful load} + S\left(5 + \frac{b}{4}\right) = 55\,S$$

The useful load will then become 0 for $5 + \dfrac{b}{4} = 55$ or $b = 200$.

By utilizing the most recent designs of Dornier, the extrapolation would lead to a construction X of 88 m. wing span with 36,000 hp., and of which the wing characteristics are given in Table 6 in comparison with the five Dornier designs indicated *Do.* and the two seaplanes $n = 1$ and $n = 3.92$ to which reference has been made above.

For the design X, the power would be 36,000 hp. and the useful load 110,000 kg., comprising 150 passengers and combustible for some ten hours flight.

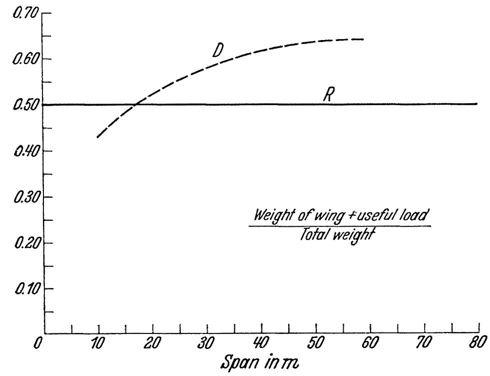


Fig. 62.

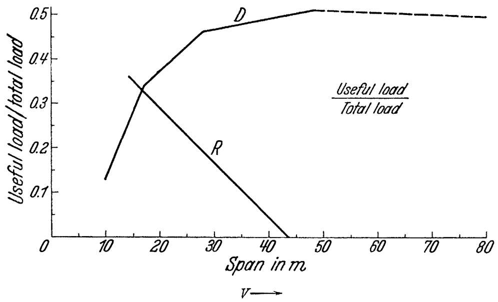


Fig. 63.

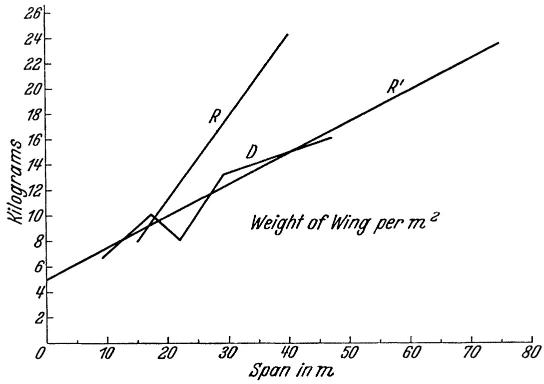


Fig. 64.

TABLE 6.

Type		Span b Meters	Wing Area Σ square meters	$10^3 \frac{\Sigma}{b^2}$	Total Load kg.	$\frac{\text{Total Load}}{\Sigma}$	Wt. Wings	$\frac{\text{Wt. Wings}}{\Sigma}$	Useful Load	$\frac{\text{Useful Load}}{\text{Total Load}}$	$\frac{\text{Useful Load}}{\text{Total Load—Wt. Wings}}$
	Do.	9.80	15.51	162	670.	43.1	108	6.97	80	0.119	0.143
$n = 1.00$		15.00	37.60	167	2000.	53.0	300	8.00	700	0.350	
	Do.	17.50	52.96	172	2860.	54.2	535	10.12	962	0.336	0.414
	Do.	22.50	95.25	190	6030.	63.4	767	8.05	2401	0.400	0.456
	Do.	28.60	143.86	178	14100.	98.0	1899	13.20	6411	0.457	0.526
$n = 3.92$		59.00	580.00	167	30740.	53.0	15318	26.50	0		
	Do.	48.00	467.70	203	51500.	110	7476	15.97	25838	0.503	0.585
	x	88.00	1781.00	230	240451.	135	44500	25.00	110451	0.460	0.564

To sum up, there is certainly a difficulty in the fact that sustentation increases with the square of the span while the weight increases with the cube.

However, the limit placed by Rumpler is too narrow. It may be remarked that there is a ratio nearly constant between the total weight and the weight empty. A simple rule such as

$$\frac{\text{Weight of wing structure}}{\text{Wing area}} = \frac{1}{6}\,\frac{\text{Total weight}}{\text{Wing area}}$$

combined with the assumption of weight varying as the cube of the span, and the supporting surface as the square would lead to a weight of wing construction per unit area proportioned to the span. However, the rule $a + cb$ is more in conformity with actual realization. Table 7 gives the numerical elements which have served as basis for the preceding discussion[1].

TABLE 7. *Weight in Kilograms.*

Span, meters	9.80	17.50	22.50	28.60	48.00	88.00
Wing	108.2	535.10	767.5	1898.9	7475.8	44500
Structure — (Wing and Hull)	67.8	203.0	497.7	878.6	2473.9	11500
Hull	106.7	416.0	1006.6	2023.3	7235.3	34000
Power Plant	194.1	491.1	995.5	2350.8	7143.5	34000
Motor and Control Accessories	33.6	93.0	121.9	217.1	613.7	3000
Crew	80.0	160.0	240.0	320.0	720.0	3000
Useful Load, Gasoline and Oil	80.0	962.0	2401.0	6411.0	25838.0	110451
Total Weight	670.4	2860.2	6030.2	14099.7	51500.2	240451
Horse Power	80	450	900	2000	6000	36000

CHAPTER IX

TESTS ON REDUCED SCALE MODELS

1. Introductory. The impossibility of deducing satisfactory predictions from theory alone, or by the extrapolation from results obtained on actual designs, has lead in the study of seaplane boats or floats, as in many other like problems, to the use of reduced scale models.

In the general technique of the use of models, it is not customary to represent all the minute peculiarities of form of the full scale object. The important question then arises regarding the characteristics which may be neglected, which are those which must be reproduced, and at what desirable scale. These same questions arise in connection with tests of models for ships, and it is known that for the study of resistance to uniform motion, the following conditions must be realized:

Geometrical similitude of form.

Weight of model proportional to the cube of linear dimensions.

[1] DORNIER, C., Notes on a Family of Similar Flying Boats, Journal of the Royal Aeronautical Society, Vol. 32, pp. 981—1020, London, December, 1928.

Fixed relative position of the center of gravity longitudinally.

Speed proportional to the square root of linear dimensions.

It is known also that corrections are necessary to allow for the influence of friction.

Likewise in tests of ordinary ship forms, whenever study is made of a non-permanent movement (tests of turning, oscillation among waves, etc.) it is necessary to realize in addition similitude of inertia. For model tests of seaplanes the question arises as to the need of realizing similitude of inertia as well as center of gravity. The question results from the fact that the models are usually small and not easily realized with suitable weight, and that in consequence it is not always easy to dispose of the weight needed to realize similitude of inertia.

The realization of the correct weight has lead in certain cases to ingenious measures; such, for example, as models of paper[1] but more frequently the excessive weight of the model is corrected by a counter weight acting over a pulley. This causes no trouble for steady movement, but may lead to discussion in the case of models for which it is desired to observe "porpoising". Efforts have therefore been made to realize inertia similitude about a transverse axis passing through the center of gravity.

In our present ignorance of hydrodynamic forces acting on a solid in irregular motion, it is not easy to give decisive arguments proving the necessity of employing models fulfilling the condition of inertia similitude.

In the larger number of the tests carried out thus far, the condition of inertia similitude not having been realized, it has seemed necessary to examine this point experimentally. To this end a model was made with rather heavy weight, a considerable proportion of which was adjustable relative to the center of gravity.

Within the limits between which the moment of inertia of an actual seaplane may be varied, the variations obtained on the model by changing the moment of inertia have been found small.

This question may therefore be considered at the present time as of secondary importance. It is, however, needful to note the point for it may later become of importance.

2. Models with Fixed Incidence. The study of a model with constant incidence is made in general by running the model with free angular movement at a series of speeds, and then over again the same series after a change in the distribution of the weight on the model, or in the forces due to the suspension. From the results of these various tests plotted in suitable curves, points corresponding to a given incidence can be collected.

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[1] DENNY, SIR ARCHIBALD, Transactions Institution Naval Architects, p. 194, 1915.

A more rapid method makes use of a system of articulated parallelograms, as in Fig. 65 which maintains the model with a fixed incidence. The various experimental results used in the present work have been found by means of a mounting of this character.

3. Tests with Unloading Proportional to the Square of the Velocity. Tests are often carried out in which the model is subject to a vertical force variable with the speed, and intended to represent the sustentation due to a wing. Various mechanical dispositions may be employed to this end. Thus on a balance a weight may be placed for each run corresponding to the speed for this run. Otherwise, by the use of a suitable device, the unloading may be readily effected by subjecting the model to the centrifugal force of a rotating counterweight properly connected to the driving mechanism of the carriage.

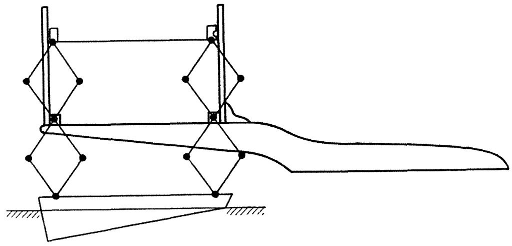

Fig. 65. Mounting for model tests with fixed incidence.

In the case where unloading should be proportional to the square of the speed the special regulation for each run may be avoided by utilizing the procedure suggested by Sottorf and employed in the Hamburg experimental canal[1]. This procedure is based on the use of a resisting body studied by Kempf, and giving at the same time good stability under tow, and a resistance proportional to the square of the speed. This body has schematically the form of a cone with circular base, a portion of the cone being removed as indicated in the diagram Fig. 66. The tow line is fixed on the generatrix opposite x. The displacement of the arc S with reference to the axis O permits a change of ratio between the resistance of the body under tow and the force of sustentation on the model. By the installation of an electric control for the displacement of S by the model itself the effect of variations of incidence on the sustentation due to the wings may be represented. An advantage of this arrangement is the possibility of locating the body under tow at a considerable distance from the model itself in order to avoid all effects of interference.

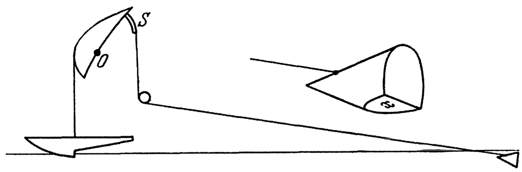

Fig. 66. Mounting for simulating hydrodynamic lift.

[1] Werft, Reederei, Hafen, 1929.

4. Tests with Free Incidence. The apparatus for the test of seaplanes with a representation of the effect due to a wing in the air by the use of an immersed wing[1] is composed of a double Roberval balance carried by the platform P, Fig. 67. The platform is movable vertically through the action of an electric motor driving four vertical screws V. The platform which carries the observer and the dynamometer carriage can thus be placed correctly relative to the water. And in this manner the vertical location of the model may be properly adjusted. The supporting arms of the double balance A, as well as their connection with the platform, are represented in the diagram. The forward balance B is connected by an articulation to the model H. This point of connection α may be varied from one test to another, but remains the same throughout a series of runs.

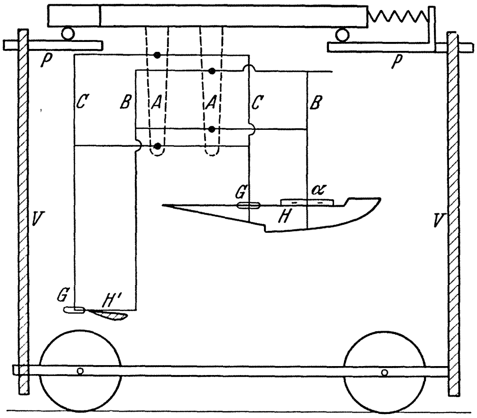

Fig. 67. Mounting for model tests with free incidence.

The forward balance is likewise connected to the forward part of the wing H'. The only difference for the after balance is that the connection to the model H or to the wing H' is made by means of a slide G. The model H is placed in its normal position. The model H' is, on the other hand, mounted reversed.

It is thus seen that when the platform is in motion relative to the water, the wing H' will be subject to a force directed downward, which, by the effect of the articulated parallelograms, will subject the model H to a force directed upward similar to the aerial sustentation due to the wing of an actual seaplane.

The wing H' naturally is not made to the same scale as the model H, since it is acted upon by water, while the model of the actual plane is in the air. We should therefore expect the wing H' to be made to a scale about 800 times smaller than the model H, and this indeed would be necessary if the wing H' were geometrically similar to the wing of the plane itself. Such a model wing would be practically very difficult to realize, and in consequence use is made of a wing very much reduced in span, and but slightly in chord. With the mounting, as indicated, the model H is free to incline, but at the same time it gives a like inclination to the wing H'.

[1] Commission Permanente d'Etudes aéronautiques, February 1927.

The apparatus and model being adjusted, a test consists in a series or runs carried out at uniformly spaced speeds and repeated three times under the following conditions.

In the first series everything is mounted as in the diagram. For each speed the resistance is measured and the position and attitude of the model are noted.

In the second series the model H is removed and instead of leaving the two balances free they are fixed to the supports A in the position taken for the preceding tests at each value of the speed. This second series gives, therefore, the resistance due to wing H' and supporting bars.

In the third series the wing is removed and runs are made giving the resistance due to suspension alone. These data will give then the resistance to which the model is subjected under the various conditions through which it passes between rest and the speed of take off.

Reference may again be made to Fig. 47 in which the results of tests with such a form of balance are given.

The runs differ only by the relative position of wing and hull indicated experimentally by the changing of the point α from one run to another. If the position of the wing is far forward, the model decreases its displacement, but the stern drags in the water. The wing being progressively moved aft, the stern drags less and less in the water, then the model rests on both the stern and the step, and then on the step only. Finally, if the wing is placed too far aft, the model begins to porpoise. If it is desired to study the same model with the same wing in the same longitudinal position, but with incidence variable relative to the hull, it is only necessary to change the length of the vertical connecting links. To this end the after links C are formed of two pieces permitting suitable adjustment in overall length.

5. Channel for Flowing Water. The apparatus used for measures in a current of water is formed of a balance with oscillating movement and an articulated parallelogram. The scale pan I is connected to the parallelogram. The scale pan II is connected to the oscillating bar B, which, with the suspension link and the model, is free to turn about the point Ω, Fig. 68.

With reference to Ω the model may assume four positions, 1, 2, 3, 4, designated respectively as left short, right short, left long, and right long. Four measures are made, one for each of these positions, and in each one the frame C supporting the apparatus is displaced in such manner that the model, for the four measures, shall be in the same position relative to the water.

For a given measurement a weight is placed in scale pan I equal to the vertical component V, which it is desired to obtain. Equilibrium is then reestablished by changing a part of the weight from pan I to pan II. The measure in each of the positions is the value of the weight contained in the pan II at the moment of equilibrium. Before each measurement the apparatus is brought into equilibrium out of the water. There is then no other compensating measure to be made. If w_1, w_2, w_3, w_4 are the measurements made, H and V the values of the horizontal and vertical reactions of the water, d the distance of the resultant R with regard to a reference point on the model, λ_1, λ_2 the two lengths of the vertical bars ($\lambda_2 > \lambda_1$), β the displacement of the vertical bar between the measurements right and left, π the length of the lever arm of the scale, it is seen that

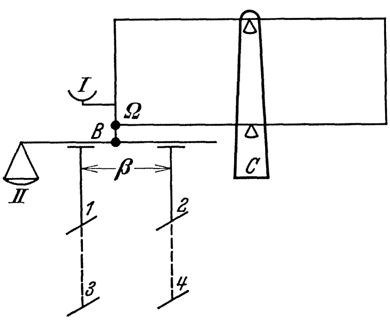

Fig. 68.

$$\pi (w_2 - w_1) = V\beta$$
$$\pi (w_4 - w_3) = V\beta$$
$$\pi (w_3 - w_1) = H (\lambda_2 - \lambda_1)$$
$$\pi (w_4 - w_2) = H (\lambda_2 - \lambda_1)$$

There are various verifications in these measures. Thus we should find $w_2 - w_1 = w_4 - w_3$ or otherwise, $w_3 - w_1 = w_4 - w_2$. Furthermore the two values found for V should be equal to the weight placed initially on the scale I.

The relations written above give immediately V and H and in consequence $R = \sqrt{V^2 + H^2}$ and the inclination $\alpha = tan^{-1} H/V$ of the resultant to the horizontal.

Since anyone of these measures will give the moment of this resultant, we have four determinations of the moment Rd and hence four determinations of d. The part of these measurements requiring the longest time being the balancing of the apparatus, it is preferable to make successively all the measurements No. 1 for a series of increasing values of V at the constant inclination of the model (or otherwise with V constant and i increasing), and then similarly all the measurements No. 2, No. 3, and No. 4. In this manner, with only four adjustments of the balance, the entire series of measurements are made permitting the complete tracing of curves of H, V, and moment for V variable and i constant (or otherwise i constant and V variable).

CHAPTER X

STRENGTH OF SEAPLANE HULL

For the same power and useful load the weight of a seaplane is greater than for a land machine. This increase is due chiefly to the hull structure which not only is much heavier than the normal landing gear, but in addition increases the resistance to movement and consequently the weight of power plant, all without any gain in sustentation.

This difference between the seaplane and the land machine was especially marked with the smaller sizes employed in the earlier years. For this there are several reasons: the dimensions of the human body remaining the same, the space and weight required for personnel is proportionally more important on a small machine. Furthermore the requirement of transverse stability for the seaplane afloat necessitated the addition of wing floats, thus accentuating still further the prejudicial effects due to the hull structure; and finally the necessity of a strength of construction suited to withstand the effects of the waves, leads, as in the case of ordinary boats, to a proportional weight of hull the greater as the structure is smaller.

In proportion as these constructions increase in overall dimension, these defects will become less and less important and would disappear if the dimensions became sufficient to permit of locating within the thickness of the wings all passenger accomodations. There would then remain to the disadvantage of the seaplane only the excess of weight of the hull in relation to the landing gear. It is furthermore by no means certain that for very large constructions, the need of landing on the sea will neccesitate weight greater than that of a corresponding landing gear for a land machine, and it is even possible that a large augmentation of useful load in relation to total load may be realized only by a seaplane design, for which the difficulties due to landing on the ground do not exist.

Under present conditions, it is, in any case, necessary to reduce to the possible minimum the weight of hull structure, always with due regard to the conditions which develop at take-off from and return to the surface of the water. Studies of the local surface loading under these conditions have been made both by model and full scale[1].

The conclusions drawn from these two studies are similar. The highest pressures develop in the vicinity of the step, immediately forward. These maximum pressures decrease rapidly with approach to the bow, but less rapidly along the line of the keel itself. The maximum pressure on the step is of the order of 400 g./cm.^{2}. Similar differences are found at the after step. Absolute values depend on the speed of contact with the water and on the fining out of the form.

These hydrodynamic pressures result in forces which must be balanced by the strength of the different parts of the structure. The role of the hull, considered as a structure calculated to resist these forces, is then

[1] Baker, G. S., Flying Boats—the Form and Dimensions of their Hull, Engineering, Vol. 109, pp. 323—327, London, March 5, 1920.

Thompson, F. L., Water Pressure Distribution on a Seaplane Float, U.S. N.A.C.A. Technical Report No. 290, 1928.

to transmit and diffuse throughout the structure all local loads, operating thus by its mass and by the work required for its deformation.

The influence of an increase in size in a seaplane hull upon its weight, is difficult to determine at the present time, chiefly because the rules of design and factors of safety imposed on constructors are not uniform.

In some cases a load proportional to the weight of the plane is distributed over the bottom; in others, the assumption is made of a load constant per unit of area. By the first rule, an increase in the width will increase the distances over which these loads must be transported but will not augment the loads themselves. With the second rule, both the loads and the distances will be increased.

CHAPTER XI

GAPS BETWEEN THEORY AND PRACTICE

1. Introductory. It certainly would be very advantageous if we could have available a method permitting the use of the general equations of rational mechanics, to predict the results to be expected in service for any form of seaplane hull as defined by its plans. This objective has indeed attracted serious attention and it is useful to examine the degree of approach thus far realized and whether it may be possible to approach still more closely to this ideal.

Let us assume, for the moment, that we have a solution of the following problem—given a form of seaplane hull with a free water surface, the position of the hull being defined with reference to the plane of the undisturbed water level. To this body is given a motion of uniform rectilinear translation. It is required to determine the forces to which the surface of the hull is subject from the liquid.

The solution of this problem would be very useful for ship construction in normal forms, always assuming that the time required for the various calculations would not exceed a reasonable duration. It is certain that such a solution would permit a deep penetration into the mechanism of the production of fluid resistance and would furnish important indications regarding the search for forms of least resistance. But it is likewise certain that this problem has not as yet been solved for a ship form given *a priori*, and that the solutions obtained in special cases (and which have given occasion for remarkable experimental verification) involve numerical calculations of such a length that their practical application cannot be considered by the practitioner and, what is still more important, do not furnish, regarding the means for the improvement of a form, indications superior to those already furnished by experience direct.

Since, then, for ordinary ship forms such theoretical guides as we have at our disposal do not permit, for a ship in motion, a determination of the changes of position relative to the undisturbed water level, we should not expect, in the case of a seaplane hull for which the changes of incidence are of such great importance, to find in the present theories of rational mechanics, any adequate reply to the questions presented in practice. The difficulty is, in fact, still larger in the latter case and for a number of reasons. First, it is no longer a matter of continuous form, but rather of a form discontinuous longitudinally (step) and discontinuous transversely (abrupt angles). Furthermore, it is no longer a problem of a body subject to its weight and a single propulsive thrust and reacting fluid resistance, but of a body subject to external forces depending on the speed both in direction and magnitude and also on its attitude relative to the horizontal. Furthermore, it is not sure *a priori*, that a solution even if obtained, for uniform translation would be applicable to accelerated movement, either with or without pitching.

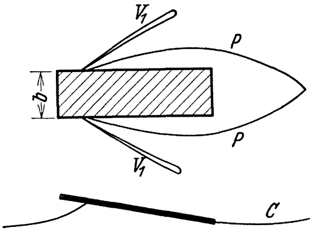

Fig. 69.

The hydrodynamic problem of the seaplane, in all its complexity, must then be considered as lying beyond our present means for theoretical treatment and it becomes then necessary to ask whether simplified solutions will be susceptible of useful applications.

The first simplification to be considered is that of considering uniform translation only. The most important phase in separation and take-off is that of maximum resistance, during which the horizontal acceleration is certainly very small. Such a simplification would, therefore, be acceptable.

A second simplification would consist in the substitution of a two-dimensional problem for one of three dimensions. This would be realized by assuming the hull defined by its vertical axial longitudinal section and generated by an indefinite right line perpendicular to this plane of section. In order to justify this simplification the following reasons might be adduced. The wave resistance is due to the formation of changes of level in the free liquid surface; we see first the diverging system V, Fig. 69, and then a hollow with walls P closing itself behind the body and from the bottom rising according to the form of a cylinder C with transverse horizontal generatrix. If then the principal part of the resistance is due to the formation of this wave form C, we shall have an approach possibly sufficient through a study of this formation as a problem in two dimensions.

Experiment furnishes an answer with reasonable assurance, that this is not the case, and that the part of the resistance due to the transverse wave form is not the principal term in the total value. This conclusion is based on the analysis made by Sottorf of his own experiments, permitting, by the aid of certain hypotheses, the separation of the total into three parts, termed by him, the part due to friction, the induced resistance, and the wave-making resistance. In this analysis, the induced resistance corresponds to the formation of the transverse wave and the wave-making resistance to what we have here called resistance due to the divergent wave system.

2. Sottorf's Analysis[1]. The three elements of the resistance are as noted above. The resistance due to friction having been calculated, subtraction from the total, as furnished by experiment, gives the sum of the induced and wave-making resistances. In order to separate these two, the former is calculated according to the following hypothesis.

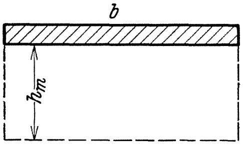

Fig. 70.

It is assumed that the mass of liquid acted upon is a prism $b \times h_m$ where b is the breadth of the body in contact with the water and h_m is unknown, Fig. 70. The mass acted on per second is then $\varrho b h_m V$, ϱ being the density and V the speed. The sustentation is then equal to the time rate of change of momentum in the vertical direction. This gives

$$L = \varrho\, b\, h_m V \cdot V_m \sin\alpha$$

$V_m \sin\alpha$ being the mean vertical component of the velocity of the water in contact with the body surface. This gives:

$$h_m = \frac{L}{\varrho\, b\, V \cdot V_m \sin\alpha} \tag{2.1}$$

Each experiment gives then a value for h_m. For example, with a value of $L = 18$ kgs. $= 18{,}000$ gms., $V = 6$ m./sec. $= 600$ cm./sec., $b = 30$ cm. and $\varrho = 1/981$ (whence $L/b\varrho V = 981$) the following results were found.

Test No.	Inclination of Under Water Surface	Horizontal Length cm.	V_m cm./s.	$V_m \sin\alpha$ cm./s.	h_m cm.
1	4^0	80.0	592	41.3	23.8
2	6^0	37.0	581	60.7	16.2
3	8^0	17.5	557	77.5	12.6

The resulting values of h_m are then as given in the last column of the table.

The kinetic energy expended per second is equal to the induced resistance R_i multiplied by the mean speed V_m. This energy again is equal to half the product of the mass acted on per second by the square of the speed $V_m \sin\alpha$.

Equating these two values we have

$$R_i V = \frac{1}{2} \varrho\, h_m\, b\, V\, (V_m \sin\alpha)^2$$

[1] Werft, Reederei, Hafen 1929.

In this equation if we put the value of h_m in (2.1) we have

$$R_i V = \frac{1}{2} L V_m \sin \alpha$$

Substituting the proper numerical values, values of R_i are found as in the following table.

Test No.	Resistance Due to Friction	Induced Resistance	Wave-Making Resistance	Total
1	1410	619	811	2840
2	708	912	1026	2646
3	352	1167	1374	2893

In a general way the sum of the induced and wave-making resistance equals $L \tan \alpha$. Furthermore, the induced resistance is nearly equal to $(L/2) \sin \alpha$; and V_m/V differs but little from unity. Hence the induced resistance will approximately be half the sum of induced resistance plus the wave-making. That is, the induced resistance would be approximately equal to the wave-making, whatever the ratio of width to length.

It should be noted that in this calculation no account is taken of the lateral deviation of the liquid filaments in contact with the body. This in effect, enters only indirectly in connection with the measurement of the mean velocity.

Comparing these conditions of resistance with those for airplane wings, Sottorf notes that for the conditions of minimum resistance, the resistance of a hydro-gliding plane is divided into three parts of the same order of magnitude and that the part due to friction is much greater than for the aerial wing. The question remains if a part of the induced resistance is not due to the formation of the divergent wave system. An opinion on this point must be based upon an examination of the system of waves formed by a gliding plane.

Although it is recognized that the two-dimensional problem will surely not serve by itself to give a satisfactory solution for the problem in three dimensions, it would be premature to conclude that the two-dimensional solution is without interest. It might be, for example, that the part due to the divergent wave system is constant, in which case some advantage could be drawn from the two-dimensional solution.

To examine this question, let us take three floats with widths 10 cm., 20 cm., 30 cm., with bottom inclined at an angle of 8^0 and moving at a speed of 5 m./sec. The floats are run at an incidence as chosen, and the weights are as 1, 2, 3. We shall then find resistances which are sensibly in arithmetical progression—that is, on a straight line when plotted as in Fig. 71. This line projected back to $b = 0$ gives an ordinate as shown which may be considered as the resistance due to the formation of the divergent wave system independent of the width b of the body. On this assumption, the remainder is due to friction and to the resistance due to the formation of the transverse wave system. The resistance due to friction being nearly proportional to b, the resistance due to the transverse wave system should also be proportional to b.

But it might otherwise be assumed that the resistance due to the divergent system comprises two terms, one a constant and one proportional to the width. To decide this question very exact examination would be required of the divergent system for a series of forms differing only in width and to the present time, no such examination has been made.

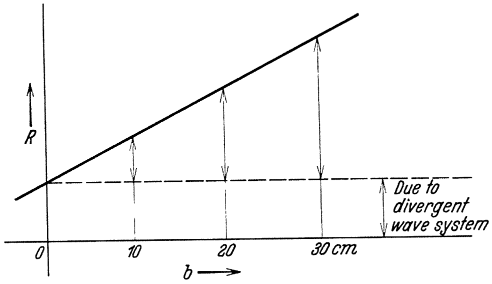

Fig. 71.

In the mean time, it may be agreed that a solution of the two-dimensional problem is very much to be desired, a solution which does not seem to lie beyond the possibilities of present mathematical analysis; and further to be desired likewise, is the solution for a case of three dimensions with a gliding plane.

With the solution of these two problems, there would be furnished a sound foundation for the discussion of experimental results. The solution of problems only slightly more complicated, as that of the comparison of different forms of transverse section, problems, the solution of which would be of immediate practical use, seem at present to be beyond the reach of theoretical treatment.

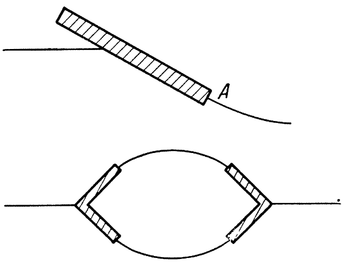

Fig. 72.

It is the same for the problems relating not alone to the forward gliding plane, but also to the part of the hull located aft of the step. Nevertheless, for these cases, the solution of the first problem noted, would permit, already, by a knowledge of the form of the free surface aft of the step, to determine *a priori*, the incidence to be given to the region aft which is to rest upon this free surface during the period preceding that of gliding on the step alone.

As regards the first of these fundamental problems, that of two dimensions, it may be approached in various ways—by seeking a solution in a similar manner to that employed by Lord Kelvin, especially in his paper "On the Front and Rear of a Free Procession"[1], or again by the method given by Lamb[2] for a circular obstacle deeply immersed.

This method of approach seems likely to give a useful result only in the case of a body of small dimensions.

Another method of approach to this problem would be analogous to that employed in the study of diversion dams, only introducing the gravity field for the purpose of determining, by Bernoulli's equation, the local velocity at the point of issue A, Fig. 72. The remainder of the problem would consist in finding a suitable hydrodynamic field, either by a suitable distribution of sources and sinks, or by assimilation to a problem with a surface of discontinuity. This general method of attack has the advantage of permitting the consideration of the case of an obstacle of finite dimensions, but would require, the same as for the case of the dam, an experimental verification in order to justify the hypotheses employed.

CONCLUSION

In the preceding pages, we have held especially in view the possibilities of approach between hydrodynamic theory and the phenomena observed with a seaplane during the period of contact with the water. In consequence, purely technical considerations have been entirely passed by and no note has been made of procedures permitting the avoidance of the maximum resistances due to the water, such as launching by catapult, independent towage, etc. All matters relating to purely structural problems, to economic considerations and to the future of the seaplane have also been entirely passed by.

It is nearly the same with regard to studies relative to the analysis of the results obtained from full scale experiments on actual seaplanes, from studies made on actual hull structures without the wing equipment, and from studies made on models. In this group of studies, theory alone is an insufficient guide and technical procedure takes therefrom only those general principles well known under the name of "rules of kinematic similitude".

At the present time, the principal difficulty in connecting theory with technical procedure is certainly the lack of results furnished by theory alone. The time when theory will be far enough advanced to permit the prediction of the forces to which a hull would be subjected in varying movement does not seem near. On the other hand, we may hope that theory will succeed in giving such results for the case of uniform horizontal translation.

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[1] Kelvin, Lord (Sir William Thompson), "Mathematical and Physical Papers", Vol. IV, p. 307, Cambridge 1882—1911.

[2] Lamb, Horace, Ann. di Matematica, 1913.

Thus far, in order to deal with these forces and with the general behavoir of the plane, we are compelled to use diagrams and procedures in which hydrodynamic and hydrostatic forces are mingled and which are introduced by assimilation with the principle of Archimedes. It is part of the role of theory to show how, when the speed varies, the principle of Archimedes (strictly valid only for $V = 0$) may be brought, as a limiting case into the general equations for laws of fluid motion. At the present time, the vertical force on an immersed body in movement is given by theory only for the case where the immersion is very great[1].

Until theory has overcome this obstacle, we shall, in this domain as in that of ship design, be reduced to the necessity of direct observation, either full scale or on models, and to methods of analysis and interpretation of these measurements based upon subdivisions of the total resistance which find no justification in theory.

In procedures of this character, however, theory will have an important role, especially as a guide to experimental research. At the present time the best method of reducing the gap between theory and practice seems to lie in the direction of a study of models leading progressively from actual forms of seaplane hulls to thin plane surfaces inclined to the surface of the water. Thus, for example, a study of the hull itself, then the hull bounded by a horizontal cylindrical bottom and with vertical cylindrical sides, then the latter limited to the part forward of the step, and finally the thin plane alone. In the present work, several experimental results of this general character have been given. In this manner the most important steps in any practical problem are made evident; but general conclusions can only be reached after studies of this character have been applied to a great number of forms.

Questions of stability, so important for the seaplane, may be approached from two different points of view. We may attack the problem in its overall aspect, in which case theory should furnish a solution relative to non-permanent motion. In this direction, we have thus far no useful result. Or otherwise we may assume *a priori* that in irregular movement, the reactions from the water are the same as for permanent tangential movement; in other words, that a knowledge of the instantaneous movement of the solid will permit a determination of reactions from the water, or otherwise that the latter depend only on the instantaneous velocities.

In the case of a seaplane of which the longitudinal vertical plane remains fixed in space, each instantaneous state is defined by five parameters: two parameters of position (one giving the position of a point in the plane with reference to the undisturbed water level, and the other to fix the attitude) and then three parameters of instantaneous velocity (horizontal translation, vertical translation and rotation). By reasons

[1] Havelock, Proceedings Royal Society 1929.

of symmetry, the reactions from the water will, in this case, become reduced to a single force depending, then, on these five parameters.

In a similar manner as in the case of normal ship forms as dealt with in theoretical naval architecture, the consideration of states of equilibrium leads to a definition of metacenters. With the normal ship form and considering only longitudinal inclinations, there is a single metacenter, because the force of buoyancy is one of fixed magnitude and its location for varying inclination with constant displacement depends on one parameter only, the angle of inclination. Here, on the contrary, giving proper significance to the hydrodynamic forces in play, there will be five metacenters corresponding each to the change in a single one of these five parameters, the other four remaining constant. Furthermore, to each metacenter there will correspond two parameters of distribution—one for changes of intensity, the other for changes of direction. The conditions of a small change in state will therefore require for specification fifteen quantities, these being the partial derivatives of F_x, F_y and M with regard to these five parameter variables.

The facility of application of the metacentric method in the case of a floating body arises (other than for the reasons already noted) from the fact that the external force is always perpendicular to the water-plane and turns exactly with the inclination. There is, then, no need of considering a parameter for varying intensity, nor for varying inclination, and the knowledge of the single position of the metacenter defines the location of the known value of the buoyant force for angular positions near any given initial position of the body.

The application of the metacentric method to the hull of a seaplane, on the contrary, presents a very much more complicated problem. There are at present available only a few practical examples insufficient as a foundation for theory.

Happily, however, notable simplifications in practice are possible. Thus at the start the case may be restricted to a dependence of the force on three parameters—horizontal speed, vertical component force, and inclination. We then have three metacenters corresponding respectively to changes in speed, in load and in inclination. Experiment will then show that often certain metacenters are far separated or that the parameter of distribution of the intensities is of secondary importance. To forward these questions, the study of metacenters by model experiment is the only way which seems at present possible.

It is to be foreseen, therefore, that still for a long time, the study of the marine parts of a seaplane must be based principally on model tests the same as in the case of normal ship design. For the latter, in fact, notwithstanding a condition of theory relatively much farther advanced, it is still by way of experiment on reduced scale models, that projects are carried to realization.

APPENDIX

At the third International Congress of Applied Mechanics (Stockholm 1930) two papers were presented, bearing upon the subject of the Hydrodynamics of Boats and Floats and of which brief abstracts may be presented as follows:

The first paper, by Professor Herbert Wagner of Charlottenburg, relates to the theoretical computation of the loads supported by a V shaped bottom under shock contact with the water. In this paper the problem is considered under two dimensions in a vertical transverse plane, neglecting the effect of weight. It is then necessary to find a correct solution of nonpermanent flow.

Fig. 73.

The author considers first an angle of the V very small, and computes the force and the distribution of pressures neglecting terms of the same order of magnitude as the angle. The energy expended in the shock is found in the energy of the water thrown laterally with high velocity. There results local pressures of high intensity on a restricted zone, and these pressures are not sensibly decreased by the elasticity of the structure of the hull.

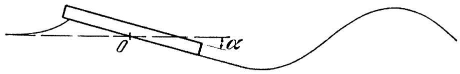

Fig. 74.

The author then considers a finite angle of the V shaped form and calculates for various cases the distribution of the pressure. Although these numerical computations are too long for practical application, the paper is to be especially noted not only by reason of the method employed, but also for the hope which it gives for the ultimate treatment of a problem which thus far has shown itself beyond the reach of theoretical methods.

The second paper, by Pavlenko of Leningrad deals with the theoretical computation of the loads supported by a plane surface in hydroplaning. The problem is considered again in two dimensions in the vertical longitudinal plane, and treated by the method proposed by Lord Kelvin and Lamb[1].

Let us place the origin of coordinates at the point O where the plane of quiet water at infinity meets the plane. The author then arrives, for the free surface, at a half plane passing through O forward of the obstacle, and a sinusoidal wave as it leaves the obstacle (see Fig. 73). He considers furthermore the development L of the plane in the direction of movement as small. It is very doubtful with these assumptions, whether the phenomena can be correctly represented, because no account is taken of the super-elevation of the water forward of the obstacle

[1] Lamb, H., "Hydrodynamics", 4th ed., p. 387, Cambridge, 1924.

(see Fig. 74). The conclusion of the paper gives an expression of the resistance for high velocities under the form

$$R = \frac{\varrho\, V^4 \tan^2 \alpha}{4\, g}$$

It is readily seen that this expression might have been obtained by computing simply the height of the sinusoidal change of level satisfying the two conditions:

No. 1. At the altitude zero having the inclination α to the plane of the horizon.

No. 2. Having a speed equal to that of the obstacle.

The height of the change of level being known, the energy contained in the wave, or otherwise the energy expended in its formation, may be deduced immediately, and from this the resistance to the movement.

The problem of the theoretical determination of the form of free surface before and behind the obstacle still remains therefore to be solved.

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DIVISION T

AERODYNAMICS OF COOLING

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PREFACE

The purpose of this Division is to present a critical review of the fundamental aspects of the aerodynamics of cooling. Neither the theoretical nor the experimental aspects of this subject have been sufficiently developed to permit a logical presentation starting from a few basic assumptions and progressing to the mathematical solution of simple illustrative problems. Instead, it is found necessary to review the progress made along several independent lines of attack, to expose our ignorance of the essential characteristics of eddying flow, and to call attention to many implicit assumptions which are generally made to secure simple formulae. In addition there is a considerable body of theoretical knowledge not yet expressed in precise mathematical form, relating principally to the factors controlling the transition from laminar to eddying flow, which is of sufficient importance to record.

The point of view adopted is that of the physicist, rather than that of the engineer. The engineer will look in vain for design data and no attempt has been made to make exhaustive presentation of experimental results. The experimental data presented have been selected for their illustrative value in respect to basic phenomena.

The subject of heat transfer by forced convection has until recently been studied principally by investigators with training in heat flow, who have had little contact with recent developments in aerodynamics. This situation is rapidly being remedied. It is hoped that the present treatment, written from an aerodynamic background, may prove stimulating and helpful to those interested in the field of heat transfer.

CHAPTER I

FUNDAMENTAL IDEAS

1. Temperature, Heat Energy. Heat is a form of energy associated with irregular, random motions of molecules. Generally we are concerned only with changes in the heat energy of bodies or with the flow of heat from one body to another. The direction of the flow of heat energy between two bodies is determined by that physical property to which the name "temperature" has been given. Experiment has shown that the energy required to increase the temperature of a given body from a temperature t_1 to a higher temperature t_2 is independent of the source of the energy.

To give a number to temperature, some definite property of a definite body, which can be measured and which changes with the temperature, is selected, for example, the apparent volume of a quantity of mercury in a glass bulb with a fine stem. Then two standard thermal conditions are chosen, such as the freezing point and the boiling point of water at an atmospheric pressure of 76 cm. Hg. To these conditions are assigned arbitrarily selected values of the temperature, for example, 0 and 100 respectively on the Centigrade scale, 32 and 212 on the Fahrenheit scale. The selected physical property of the selected body is measured in the standard thermal conditions and in the condition for which the temperature is desired. The value of the temperature is determined by simple proportion. The relation between the Centigrade and Fahrenheit scales is obviously $t^0_F = 32 + 1.8\, t^0_C$.

Heat energy can be measured in the ordinary mechanical units such as ergs or foot-pounds, but since heat effects are not as a rule produced by direct mechanical processes, it is not always convenient to do so. It is more common to choose as a unit the amount of energy required to raise the temperature of a unit mass of water by a known amount. Three such units in general use may be mentioned: the mean gram-calorie (cal.), one one-hundredth of the energy required to raise the temperature of one gram of water from 0^0 C to 100^0 C; the mean British Thermal Unit (B.T.U.), one one-hundred-and-eightieth of the energy required to raise the temperature of one pound of water from 32^0 F to 212^0 F; and the mean Centigrade Heat Unit (C.H.U.), one one-hundredth of the energy required to raise the temperature of one pound of water from 0^0 C to 100^0 C. The ratio of these units to the mechanical unit is known as the mechanical equivalent of heat. It has been determined by experiment with the results given in the following table of equivalents:

1 cal. $= 4.186 \times 10^7$ ergs $= 3.968 \times 10^{-3}$ B.T.U. $= 2.205 \times 10^{-3}$ C.H.U. $= 3.087$ foot-pounds.

1 B.T.U. $= 1054.8 \times 10^7$ ergs $= 252$ cal. $= 0.5556$ C.H.U. $= 778.7$ foot-pounds.

1 C.H.U. $= 1898.7 \times 10^7$ ergs $= 1.8$ B.T.U. $= 453.6$ cal. $= 1401.7$ foot-pounds.

The energy required to raise the temperature of a given mass of water one degree varies somewhat with the temperature, and the calorie has sometimes been defined for a particular 1^0 interval, say 15^0 C to 16^0 C. For engineering purposes this variation can be neglected. In view of the difficulty of accurately measuring the mechanical equivalent of heat in terms of any particular definition of the calorie, it has been suggested by some authorities that a value be arbitrarily selected for the mechanical equivalent and the calorie be defined as standing in this ratio to the mechanical unit. Thus the mean calorie would be defined as 4.186×10^7 ergs, the mean British Thermal Unit as 1054.8×10^7 ergs, etc.

The energy required to raise the temperature of a unit mass of any substance through 1^0 is called the specific heat. Obviously if either Calories, British Thermal Units, or Centigrade Heat Units are used in measuring the heat energy, the specific heat of water (average between freezing and boiling point) is unity; if mechanical units are used, the specific heat of water is equal numerically to the mechanical equivalent of heat. The specific heat is in general a function of the temperature, although for water and many other substances the variation is so small that it is usually neglected in engineering computations.

2. Transmission of Heat. Heat energy may be transferred from place to place by an actual motion of heated particles or portions of a fluid, a process termed convection. If the motion of the fluid is the result of differences in density resulting from temperature differences and the action of gravity, the process is known as natural convection. If the motion is produced by mechanical means, the process is that of forced convection, the only method of transmission to be discussed in any detail in the present Division.

Heat energy may also be transferred from one part of a body to another part of the same body, or from one body to another in physical contact with it, without appreciable motion of the particles of the body. This is the process of conduction. Even in natural and forced convection, the heat energy is transferred from the boundaries of solids to the fluid by this process. If we are dealing with the flow of heat energy in one direction, s, in a homogenous substance, the rate of flow of heat energy $dQ/d\theta$, θ being the time, is proportional to the temperature gradient dt/ds and to the area of cross section S at right angles to the direction of the heat flow.

Thus
$$\frac{dQ}{d\theta} = -kS\frac{dt}{ds} \tag{2.1}$$

the minus sign being introduced since the energy flows from a region of high to a region of low temperature. The coefficient k is known as the conductivity. It generally varies with the temperature, although in engineering computations it is usually regarded as a constant. For conduction in a steady state, $dQ/d\theta = Q/\theta =$ constant.

It is found that heat energy is also transferred from a hot body to a colder body even when there are no material bodies between. This process is called radiation and the energy is believed to be transferred by the same mechanism which conveys the energy of light or electromagnetic waves. The practical aspects of the transfer of heat by radiation are discussed in Chapter III of Heat Transmission by William H. McAdams (McGraw-Hill, 1933).

In nearly all actual cases of heat transmission, more than one process is involved. The separation of the total transfer into these component processes is often difficult, but only in this way can a logical basis of calculation be developed, since the laws governing the three processes differ considerably.

3. Laminar and Eddying Flow[1]. The fundamental aspects of the dynamics of fluids are treated at length in other volumes of this series. Nevertheless it seems advisable to review the fact that there exist in nature two radically different types of flow, commonly designated as laminar and turbulent. Whether one or the other type occurs depends among other things on the turbulence initially present in the on-coming fluid stream. It is necessary to distinguish between this "initial" turbulence and the "fully-developed" turbulent motion. For this reason the author prefers the use of a special term "eddying flow" as a synonym of fully-developed turbulent flow.

By laminar flow is meant a flow in layers, the transfer of momentum between layers being by molecular motions whose effect is integrated in the viscosity. The laminar flow satisfies the equations of Stokes for the flow of a viscous fluid. The laminar flow is not always a steady flow, fluctuations corresponding to a wavering of the laminae about a mean position often being observed. The components of the fluctuations of the speed at any point are not correlated and there is no resultant transfer of momentum by the fluctuations.

In eddying flow there is an additional transfer of momentum by the motion of small masses which is exhibited as rapid fluctuations of the velocity at any point. The components of the fluctuations show correlations. The mean motion does not satisfy the equation of Stokes.

For geometrically similar arrangements and a given initial turbulence, the transition from laminar to eddying flow is determined by the Reynolds number, which equals a reference speed times a reference length fixing the scale, divided by the kinematic viscosity. The Reynolds number at which transition begins is called the critical Reynolds number. Below the critical value, the flow is laminar. The critical Reynolds number is decreased if the initial turbulence is increased. There is in fact a functional relation between the critical Reynolds number and the initial turbulence.

The two radically different types of flow are found in practically all aerodynamic problems. In many cases, both types of flow are present in different parts of the field of flow, leading to complication in integrated

[1] See also Division G.

quantities such as the total friction or the total heat transfer. The theory of heat transfer will be treated separately for the laminar and the eddying flow.

4. Physical Properties of Air. For ready reference, there are listed below the specific heat, conductivity, viscosity, density, and kinematic viscosity of air at several temperatures. The values are taken directly or interpolated between those given in the International Critical Tables.

Specific Heat, c, of Air at Constant Pressure (1 atmosphere).

Temperature		ergs $(\text{gram})^{-1}$ $(^0\text{C})^{-1}$	cal. $(\text{gram})^{-1}$ $(^0\text{C})^{-1}$ or B.T.U. $(\text{lb})^{-1}$ $(^0\text{F})^{-1}$ or C.H.U. $(\text{lb})^{-1}$ $(^0\text{C})^{-1}$
^{0}C	^{0}F		
0	32	1.004×10^7	0.2398
100	212	1.006×10^7	.2403
200	392	1.010×10^7	.2413
400	752	1.017×10^7	.2430
600	1112	1.034×10^7	.2470

Conductivity of Air[1]*, k.*

Temperature		ergs $(\text{sec.})^{-1}$ $(\text{cm.})^{-2}$ $[^0\text{C}\ (\text{cm.})^{-1}]^{-1}$	cal. $(\text{sec.})^{-1}$ $(\text{cm.})^{-2}$ $[^0\text{C}\ (\text{cm.})^{-1}]^{-1}$	B.T.U. $(\text{sec.})^{-1}$ $(\text{ft.})^{-2}$ $[^0\text{F}\ (\text{ft.})^{-1}]^{-1}$ or C.H.U. $(\text{sec.})^{-1}$ $(\text{ft.})^{-2}$ $[^0\text{C}\ (\text{ft.})^{-1}]^{-1}$
^{0}C	^{0}F			
0	32	2230	5.33×10^{-5}	3.57×10^{-6}
50	122	2540	6.08×10^{-5}	4.07×10^{-6}
100	212	2850	6.82×10^{-5}	4.57×10^{-6}
150	302	3120	7.46×10^{-5}	5.00×10^{-6}
212.5	414.5	3460	8.26×10^{-5}	5.53×10^{-6}
430.6	807.1	5750	13.75×10^{-5}	9.21×10^{-6}
539	1002.2	8180	19.56×10^{-5}	13.09×10^{-6}

Viscosity of Air, μ.

Temperature		dyne $(\text{cm.})^{-2}$ sec.	pound $(\text{ft.})^{-2}$ sec.
^{0}C	^{0}F		
0	32	1.709×10^{-4}	3.569×10^{-7}
50	122	1.951×10^{-4}	4.075×10^{-7}
100	212	2.175×10^{-4}	4.543×10^{-7}
150	302	2.385×10^{-4}	4.981×10^{-7}
200	392	2.582×10^{-4}	5.393×10^{-7}
250	482	2.770×10^{-4}	5.785×10^{-7}
300	572	2.946×10^{-4}	6.153×10^{-7}
350	662	3.113×10^{-4}	6.502×10^{-7}
400	752	3.277×10^{-4}	6.844×10^{-7}
450	842	3.433×10^{-4}	7.170×10^{-7}
500	932	3.583×10^{-4}	7.483×10^{-7}

[1] According to I.C.T. the accuracy of the experimental values is probably not better than 7 per cent.

Density of Air, ϱ, at 760 mm. Hg. Pressure.

Temperature		grams (cm.)$^{-3}$	pounds (sec.)^2 (ft.)$^{-4}$
^{0}C	^{0}F		
0	32	1.293×10^{-3}	2.509×10^{-3}
50	122	1.093×10^{-3}	2.121×10^{-3}
100	212	$.946 \times 10^{-3}$	1.836×10^{-3}
150	302	$.834 \times 10^{-3}$	1.619×10^{-3}
200	392	$.746 \times 10^{-3}$	1.448×10^{-3}
250	482	$.675 \times 10^{-3}$	1.310×10^{-3}
300	572	$.616 \times 10^{-3}$	1.195×10^{-3}
350	662	$.567 \times 10^{-3}$	1.099×10^{-3}
400	752	$.525 \times 10^{-3}$	1.018×10^{-3}
450	842	$.488 \times 10^{-3}$	$.947 \times 10^{-3}$
500	932	$.457 \times 10^{-3}$	$.886 \times 10^{-3}$

Kinematic Viscosity, $\nu = \mu/\varrho$, of Air at 760 mm. Hg. Pressure.

Temperature		cm.^{2}/sec.	ft.^{2}/sec.	Temperature		cm.^{2}/sec.	ft.^{2}/sec.
^{0}C	^{0}F			^{0}C	^{0}F		
0	32	0.1322	1.423×10^{-4}	300	572	.4784	5.149×10^{-4}
50	122	.1785	1.921×10^{-4}	350	662	.5496	5.916×10^{-4}
100	212	.2298	2.474×10^{-4}	400	752	.6246	6.723×10^{-4}
150	302	.2859	3.077×10^{-4}	450	842	.7034	7.571×10^{-4}
200	392	.3460	3.724×10^{-4}	500	932	.7846	8.446×10^{-4}
250	482	.4103	4.416×10^{-4}				

The Dimensionless Ratio $\sigma = c\mu/k$ at 760 mm. Hg. Pressure[1].

^{0}C	^{0}F	$\sigma = c\,\mu/k$	^{0}C	^{0}F	$\sigma = c\,\mu/k$
0	32	0.769	300	572	.719
100	212	.760	400	752	.621
200	392	.767	500	932	.521

CHAPTER II

THEORY OF HEAT TRANSFER IN LAMINAR FLOW

1. General Problem. Let us imagine a solid A (Fig. 1), with boundary surface B, which contains a steady source of heat and which is placed in an air stream of speed V. The speed V is assumed to be constant both in magnitude and direction at a great distance from the body. When equilibrium is reached, there will be a steady flow of heat energy by conduction through the solid to the boundary B, thence by conduction to the air close to the boundary, and finally by convection to the air at remote distances. In general the temperature and the heat flow will not be uniform along the boundary B. The final state of affairs

[1] The large decrease at 400 and 500^0 C is in all probability due to inaccurate values of the conductivity.

is fully determinate but depends on the thermal properties of the solid as well as on its external shape.

If now one attempts to divide the problem into two parts, the first dealing with the flow of heat within the solid A and the second with the flow of heat in the fluid, the result is complication. For the temperature gradients and the heat transfer within the solid depend on the rate of loss of heat at the boundary B and this is dependent on the processes occurring in the fluid. Likewise the flow of the fluid and the heat transfer within it depend on the boundary conditions, namely, the distribution of temperature over B which is not known until the first problem is solved.

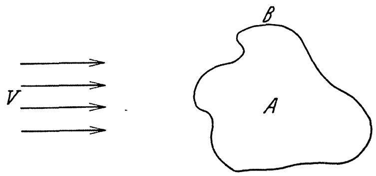

Fig. 1. General problem of forced convection.

The difficulty is analogous to that encountered in the simpler problem of determining the speed of rotation when a fan is placed on the shaft of a particular motor. Nature solves the problem of determining the speed for which the characteristics of motor and fan agree very simply and quickly. But to predict in advance of the experiment requires a knowledge of the speed of the motor with all possible torque loads and a knowledge of the torque of the fan at all possible speeds of rotation.

Likewise in the heat transmission problem, one would have to know the flow of heat within the solid for all possible rates of heat loss at the boundary and the flow of heat within the fluid for all possible rates of supply at the boundary. Then those solutions would be selected for which the boundary conditions agree, that is, for which the rate of loss from the solid equals rate of supply to the fluid at every point of B.

This fully general problem seems beyond our powers of analytical solution for some time to come. The usual procedure is either to assume that the conductivity of the material in A is so high that the temperature throughout is substantially uniform (for a given heat flow there must be a gradient inversely proportional to the conductivity, which is small if the conductivity is large); or to assume some simple distribution of temperature or heat flow based on observations in experiments on similar bodies. It must be borne in mind, however, when dealing either with experimental data or theoretical solutions that the application to any new technical problem must be studied in the light of the more general problem just outlined.

2. Equations of Motion of the Fluid. We restrict the general problem by assuming that the boundary conditions are known either in the form of the distribution of temperature or of the rate of heat flow over the boundaries. We shall likewise consider only problems of the steady state. With these simplifications we proceed to write down the equations of motion of the fluid and in the next section the equations for the flow of heat in the fluid.

By reference to other divisions of this work[1] it will be found that the equations for the steady motion of a viscous fluid are as follows:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=g_x-\frac{1}{\varrho}\left(\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}+\frac{\partial p_{xz}}{\partial z}\right) \tag{2.1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=g_y-\frac{1}{\varrho}\left(\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yy}}{\partial y}+\frac{\partial p_{yz}}{\partial z}\right) \tag{2.2}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=g_z-\frac{1}{\varrho}\left(\frac{\partial p_{xz}}{\partial x}+\frac{\partial p_{yz}}{\partial y}+\frac{\partial p_{zz}}{\partial z}\right) \tag{2.3}$$

where u, v, w are the components of the velocity at any point (x, y, z) along three Cartesian axes of reference, ϱ the density, g_x, g_y, and g_z the components of the acceleration of gravity along the three axes and p_{xx}, p_{yy}, p_{zz}, p_{xy}, p_{xz} and p_{yz} are the components of a stress tensor defined by the equations

$$\begin{aligned}
p_{xx}&=p+2\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\\
p_{yy}&=p+2\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\\
p_{zz}&=p+2\mu\frac{\partial w}{\partial z}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\\
p_{xy}&=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\\
p_{xz}&=\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\\
p_{yz}&=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)
\end{aligned}$$

where p is the pressure and μ is the viscosity.

To these equations must be added the equation of continuity

$$\frac{\partial(\varrho u)}{\partial x}+\frac{\partial(\varrho v)}{\partial y}+\frac{\partial(\varrho w)}{\partial z}=0 \tag{2.4}$$

and the equation of state of the fluid

$$f_1(p, \varrho, t)=0 \tag{2.5}$$

where t is the temperature.

If we regard the fluid as incompressible so that ϱ may be considered independent of p and if the departures from Boyle's law may be neglected, equation (2.5) becomes

$$\varrho=\frac{A}{T} \tag{2.6}$$

where T is the absolute temperature and A is a constant.

Equations (2.1), (2.2) and (2.3) are essentially the equations of Stokes. The density ϱ and the viscosity μ are, however, not constant but vary from place to place as determined by the temperature. Consequently, the reduction to the form commonly given is not permissible.

[1] See Division G, also Bulletin U.S. National Research Council No. 84, Washington D.C. 1932.

3. Equation for the Flow of Heat. The equation for the flow of heat may be developed by considering the conservation of energy as applied to a small element of volume. Radiation of heat from the element and the generation of heat by friction within the element are assumed to be negligible.

Consider first the conduction of heat into the Cartesian volume element $dxdydz$ of Fig. 2. If t is the temperature at the center of the face $ABCD$, the gradient normal to the face is $\partial t/\partial x$ and the rate of loss of heat by conduction across the face is $k\,(\partial t/\partial x)\,dydz$. At the face $EFGH$ the rate of increase of heat by conduction will be

$$\left[k\frac{\partial t}{\partial x}+\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)dx\right]dy\,dz$$

Hence the net rate of increase of heat by conduction for these two faces is $\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)dxdydz$. By similar considerations for the other four faces the total rate of increase by conduction is found to be

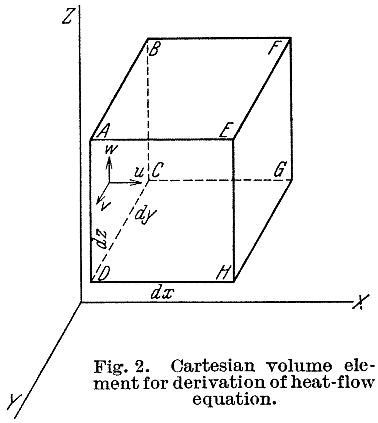

Fig. 2. Cartesian volume element for derivation of heat-flow equation.

$$\left[\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial t}{\partial z}\right)\right]dx\,dy\,dz$$

Consider next the effect of convection. The fluid entering through the face $ABCD$ at temperature t leaves the face $EFGH$ with temperature $t+\frac{\partial t}{\partial x}dx$. The mass entering per unit time is $\varrho udydz$. The increase in heat energy per unit time is $\varrho udydz\,c\,\frac{\partial t}{\partial x}\,dx$ where c is the specific heat. By similar considerations for the other faces, the total rate of increase in heat energy within the volume element by convection is

$$\varrho c\left(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}\right)dx\,dy\,dz$$

This rate of increase must equal that supplied by conduction to satisfy the principle of conservation of energy. Hence the temperature satisfies the equation

$$\varrho c\left(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial t}{\partial z}\right) \tag{3.1}$$

If the heating due to pressure changes is neglected, c is the specific heat at constant pressure.

To take account of the variation of k, c, and μ with temperature, we must add the equations expressing these relations

$$k=f_2\,(t) \tag{3.2}$$

$$c=f_3\,(t) \tag{3.3}$$

$$\mu=f_4\,(t) \tag{3.4}$$

4. Common Assumptions to Facilitate Solution. The determination of the temperature distribution and flow of heat within the fluid requires the solution of the nine equations (2.1), (2.2), (2.3), (2.4), (2.6), (3.1), (3.2), (3.3), and (3.4) in the nine variables u, v, w, p, t, ϱ, k, c, and μ.

Little progress has been made in the solution of this formidable problem. The attention of many workers in the field of heat transmission has been centered on (3.1), usually with the simplification that ϱ, c and k are assumed to be independent of the temperature. The dynamic problem of the motion of the fluid is disposed of by simple assumptions, such as that the velocity or the mass flow is everywhere constant or that the mass flow varies linearly with distance from the boundary. These convenient assumptions seem so far from anything observed in the actual flow of fluids that the solutions can only be regarded as mathematical exercises. Those readers who are interested will find valuable summaries of these solutions in papers by Drew[1] and by Lévêque[2].

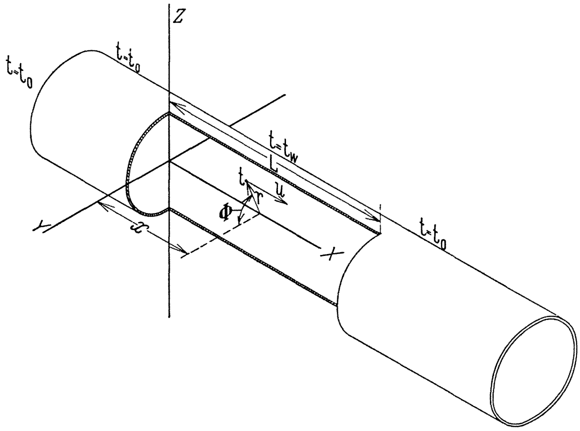

Fig. 3. Cylindrical coordinates for flow in a pipe.

A somewhat more advanced but bold assumption utilizes the results of computations of the isothermal flow of the fluid to obtain values of u, v, and w to be used in (3.1). It is perhaps worth while to illustrate this procedure by Graetz's solution for the flow of heat from the wall of a pipe to a fluid flowing through it in laminar flow, *i. e.* below the critical Reynolds number.

5. Laminar Flow in a Pipe. It is most convenient for this problem to use cylindrical coordinates x, r, and φ as shown in Fig. 3, the x axis coinciding with the axis of the pipe. By direct transformation of (3.1) we obtain

$$\left.\begin{aligned}\varrho c\left(u\frac{\partial t}{\partial x}+\dot r\frac{\partial t}{\partial r}+\dot\varphi\frac{\partial t}{\partial\varphi}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial r}\left(k\frac{\partial t}{\partial r}\right)+\\+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(k\frac{\partial t}{\partial\varphi}\right)+\frac{k}{r}\frac{\partial t}{\partial r}\end{aligned}\right\}\qquad(5.1)$$

where $\dot r$ and $r\dot\varphi$ are the radial and circumferential components of the velocity.

[1] Trans. Am. Inst. Chem. Engineers, Vol. 26, p. 26, 1931.

[2] Annales des Mines, Vol. 13, pp. 201, 305 and 381, 1928.

Assuming symmetry about the axis of the pipe, which means either that the pipe is vertical or that the convection currents are negligible, $\partial t/\partial \varphi$ is zero and (5.1) becomes

$$\varrho c \left(u \frac{\partial t}{\partial x} + \dot{r} \frac{\partial t}{\partial r}\right) = \frac{\partial}{\partial x}\left(k \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial r}\left(k \frac{\partial t}{\partial r}\right) + \frac{k}{r}\frac{\partial t}{\partial r} \tag{5.2}$$

The velocities u and $\dot{r}$ are assumed from the flow in the isothermal case, namely, $\dot{r} = 0$, $\varrho u = 2\,G\,(1 - r^2/R^2)$ where G is the average mass velocity over a cross section and R is the radius of the cylindrical pipe. Setting $r/R = y$, (5.2) reduces to

$$2\,c\,G\,(1 - y^2)\frac{\partial t}{\partial x} = \frac{\partial}{\partial x}\left(k \frac{\partial t}{\partial x}\right) + \frac{1}{R^2}\frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right) + \frac{k}{y\,R^2}\frac{\partial t}{\partial y} \tag{5.3}$$

Graetz made the additional assumptions that $\frac{\partial}{\partial x}\left(k \frac{\partial t}{\partial x}\right)$ could be neglected in comparison with $\frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right)$ and that k and c are independent of the temperature. With these assumptions, (5.3) becomes

$$(1 - y^2)\frac{\partial t}{\partial x} = A\left(\frac{\partial^2 t}{\partial y^2} + \frac{1}{y}\frac{\partial t}{\partial y}\right) \tag{5.4}$$

where $A = k/2\,c\,G\,R^2$, a constant.

As boundary conditions, the pipe wall was assumed to be heated to a constant temperature t_w from $x = 0$ to $x = L$, and the initial temperature of the fluid was assumed constant and equal to t_0.

The type solution of equation (5.4) is of the form

$$\frac{t - t_w}{t_0 - t_w} = \sum_m B_m\, e^{-a_m A\, x} f_m\,(y, a_m) \tag{5.5}$$

where B_m and a_m are constants to be determined and $f_m\,(y, a_m)$ satisfies the differential equation

$$\frac{d^2 f_m}{d\,y^2} + \frac{1}{y}\frac{d\,f_m}{d\,y} + a_m\,(1 - y^2)\,f_m = 0 \tag{5.6}$$

The constant a_m is a root of $f_m\,(1, a_m) = 0$.

Equation (5.6) characterizes certain functions related to Bessel functions which must be evaluated by development in series. According to Drew's revision of Graetz's computation, the average rate of heat flow per unit surface area of the pipe, q_{av}, is given by the formula

$$q_{av} = \frac{c\,G\,R}{2\,L}(t_w - t_0)\,[1 - 8\,P_2\,(A\,L)] \tag{5.7}$$

where P_2 is the convergent infinite series

$$P_2 = 0.10238\,e^{-7.3136\,A\,L} + 0.01220\,e^{-44.61\,A\,L} + \text{etc.}$$

If we consider a very long heated section of pipe, it is obvious that the temperature of the fluid will approach the temperature of the wall of the pipe and the rate of transfer of heat will approach zero. Thus in (5.7), as L increases, the series P_2 approaches zero and because of the L in the denominator, q_{av} approaches zero.

The local rate of transfer of heat q_x at a distance x from the beginning of the heated section is given by the expression

$$q_x = \frac{2\,k\,(t_w - t_0)}{R}\,P_1\,(A\,x) \tag{5.8}$$

where P_1 is the convergent infinite series

$$P_1 = 0.74877\,e^{-7.3136\,A\,x} + 0.544\,e^{-44.61\,A\,x} + \text{etc.}$$

The results are often expressed in terms of a heat transfer number defined in terms of the average temperature of the fluid at the section x. Graetz and Lévêque used an average temperature $t_{a\,v}$ defined by the relation

$$t_{a\,v} - t_0 = \frac{\int_0^1 c\,\varrho\,u\,(t - t_0)\,2\,\pi\,y\,d\,y}{\int_0^1 c\,\varrho\,u\,2\,\pi\,y\,d\,y} \tag{5.9}$$

It is found that $\quad t_w - t_{a\,v} = 8\,(t_w - t_0)\,P_2\,(A\,x)$ $\quad$ (5.10)

The local heat transfer number h_x, defined as $q_x/(t_w - t_{a\,v})$ is given by the relation

$$h_x = \frac{k}{4\,R}\,\frac{P_1\,(A\,x)}{P_2\,(A\,x)} \tag{5.11}$$

Nusselt uses an average temperature $t'_{a\,v}$ defined by the relation

$$t'_{av} - t_0 = \frac{1}{\pi}\int_0^1 (t - t_0)\,2\,\pi\,y\,d\,y \tag{5.12}$$

and obtains a corresponding h'_x

$$h'_x = \frac{k}{2\,R}\,\frac{P_1\,(A\,x)}{N_1\,(A\,x)} \tag{5.13}$$

where $N_1 = 0.14525\,e^{-7.3136\,A\,x} + 0.0334\,e^{-44.61\,A\,x} + \text{etc.}$

The two formulations correspond to two different experimental procedures for determining the average temperature. Equation (5.9) gives the "mixing cup" temperature found by thoroughly mixing the fluid in a special chamber and measuring the temperature of the mixture. Equation (5.12) gives the average temperature found by making a traverse of the pipe with a thermocouple.

When $A\,x$ is greater than 0.2, h_x and h'_x are constant within 0.1 per cent and are given by the formulae

$$h_x = 1.83\,\frac{k}{R} \tag{5.14}$$

$$h'_x = 2.58\,\frac{k}{R} \tag{5.15}$$

Thus while q_x approaches zero as the length of the heated section is increased, the heat transfer number approaches a constant value.

This brief summary gives some idea of the mathematical complications even after many simplifying assumptions have been made.

It must be stated here that experimentally the heat flow is greater than that given by (5.7) by as much as 100 per cent for certain values of $A\,L$. This discrepancy will be discussed more fully in Chapter VII.

6. Laminar Flow in a Two-Dimensional Boundary Layer. When a fluid having a small coefficient of viscosity (for example, air) flows past a solid, the flow at a considerable distance approximates a potential flow and the effect of viscosity is confined to a thin boundary layer at the surface of the body. In a similar way, if the thermal conductivity of the fluid is small, the temperature gradient is large only in a thin layer at the surface of the body. Under these circumstances the general equations permit of certain simplifications which will be discussed for the case of two-dimensional flow. The effects of natural convection and the internal heating of the fluid by viscous friction will be neglected. Likewise the density, viscosity, thermal conductivity, and specific heat will be considered constant. In other words, the equations of motion for the isothermal case will be used to obtain values of the velocities for use in the equation for heat flow.

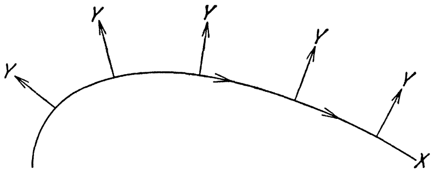

Fig. 4. Coordinates for flow in a boundary layer.

The assumption that a boundary layer exists means that we shall be interested in distances from the surface of the order of magnitude of some small distance ε which may be regarded as some conveniently defined "thickness" of the boundary layer. Actually there is no sharp boundary, but various exact definitions can be made. At present we will be concerned only with orders of magnitude. The X axis (Fig. 4) will be selected as the curvilinear boundary line of a section of the two-dimensional body, the radius of curvature of which is assumed to be sufficiently large in comparison with the thickness of the boundary layer that the effect of the curvature need not be considered. The Y axis will be taken as perpendicular to the X axis.

The simplifications which are made in the general equations for the case of a thin boundary layer are usually introduced by a consideration of orders of magnitude of terms in the general equations. The procedure cannot be considered in any sense as a mathematical proof. The justification for dropping certain terms must come ultimately from the experimental results. Nevertheless the discussion of order of magnitudes is instructive.

Two quantities are to be understood to be of the same order of magnitude when they do not differ by a factor of more than say ten, and of a different order when they differ by a factor of 100 or more. In dealing with quantities which vary throughout the layer, we are interested in maximum values since we desire to know what terms can be dropped. A difficulty arises when dealing with derivatives, since mathematically the value of a derivative is not related to the absolute values of the variables. Thus let us consider u to become of order of magnitude of 1 for x of order of 1 and y of the order of ε. The slope $\partial u/\partial y$ mathematically may lie anywhere between $-\infty$ and $+\infty$, but assuming no sharp discontinuities, we expect in physical problems a slope of the same order of magnitude (in the sense defined above) as the total change in u divided by the total change in y. That is we expect absolute values of $\partial u/\partial y$ at any point to lie between 0 and perhaps ten times the mean value of $\partial u/\partial y$.

Considering u to become of order of magnitude 1 for y of order of ε $\partial u/\partial y$ is of the order of $1/\varepsilon$ and $\partial^2 u/\partial y^2$ of the order of $1/\varepsilon^2$, while $\partial u/\partial x$ and $\partial^2 u/\partial x^2$ will remain of the order of 1. From the equation of continuity (2.4) which becomes, with the assumptions which have been made,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6.1}$$

$\partial v/\partial y$ is of the order of 1 and hence v is of the order of ε and $\partial^2 v/\partial y^2$ of the order $1/\varepsilon$.

Equations (2.1), (2.2), (2.3) reduce for the present case to the following, where the orders of magnitude of the maxima of the several terms are indicated:

$$\left.\begin{array}{ccccc} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} & = & \dfrac{\mu}{\varrho}\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) & - & \dfrac{1}{\varrho}\dfrac{\partial p}{\partial x} \\ 1,\ 1 \qquad \varepsilon,\ \dfrac{1}{\varepsilon} & & 1, \qquad \dfrac{1}{\varepsilon^2} & & 1 \end{array}\right\} \tag{6.2}$$

$$\left.\begin{array}{ccccc} u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} & = & \dfrac{\mu}{\varrho}\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) & - & \dfrac{1}{\varrho}\dfrac{\partial p}{\partial y} \\ 1,\ \varepsilon \qquad \varepsilon,\ 1 & & \varepsilon, \qquad \dfrac{1}{\varepsilon} & & \end{array}\right\} \tag{6.3}$$

From (6.2) it follows that ε^2 must be of the order of μ/ϱ; for if it is greater, the terms involving viscosity drop out when ε is considered small with respect to 1, and if it is less, the second term on the right becomes infinite. Equation (6.2) becomes, when μ/ϱ is assumed of the order of ε^2 and terms of the order of ε are omitted,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\varrho}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\varrho}\frac{\partial p}{\partial x} \tag{6.4}$$

Equation (6.3) then states that $\partial p/\partial y$ is of the order of ε and hence that the pressure may be regarded as independent of y. It therefore equals the pressure in the potential flow outside the boundary layer or the pressure on the boundary.

Equation (3.1) becomes for the two-dimensional case

$$\varrho c\left(u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial t}{\partial y}\right) \tag{6.5}$$

or with constant k and c

$$\varrho c\left(u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y}\right) = k\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right) \tag{6.6}$$

We cannot necessarily conclude that the thermal boundary layer is of the same order of magnitude as ε and therefore we assume that the

y in (6.6) is of order ε_1. It follows that $\partial t/\partial y$ will be of order $1/\varepsilon_1$ and $\partial^2 t/\partial y^2$ of order $1/\varepsilon_1^2$ while $\partial t/\partial x$ and $\partial^2 t/\partial x^2$ are of order 1. It is readily seen that ε_1^2 must be of the order of $k/\varrho c$, in which case (6.6) reduces to

$$\varrho c \left(u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} \right) = k \frac{\partial^2 t}{\partial y^2} \tag{6.7}$$

If $k/\varrho c$ and μ/ϱ are of the same order of magnitude, ε and ε_1 are of the same order and equations (6.1), (6.4) and (6.7) constitute the equations for heat transfer in a laminar boundary layer. For air μ/ϱ is about 0.70 to 0.77 $k/\varrho c$ and hence the thermal and dynamic boundary layers are of the same order of magnitude.

7. Laminar Flow along a Thin Flat Plate. As an application of the boundary layer equations developed in the last section, we shall consider the heat transfer from a thin flat plate placed in an air stream of uniform velocity U_0, the plate being parallel to the flow and maintained at a constant temperature t_p. The temperature of the air stream at a distance from the plate is assumed constant and is designated t_0. Since the air stream is of uniform velocity, $\partial p/\partial x$ is zero. The three equations are therefore

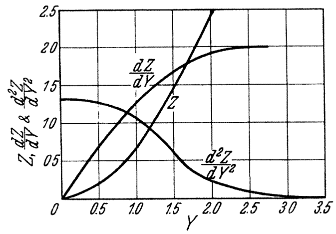

Fig. 5. Blasius' function $Z(Y)$ for flow near skin-friction plate.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\varrho} \frac{\partial^2 u}{\partial y^2} \tag{7.2}$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \frac{k}{\varrho c} \frac{\partial^2 t}{\partial y^2} \tag{7.3}$$

To satisfy (7.1), set $\quad u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x} \qquad (7.4)$

and introduce new variables

$$Y = \frac{1}{2} \sqrt{\frac{U_0 \varrho}{\mu}} \frac{y}{\sqrt{x}} \tag{7.5}$$

$$\psi = \sqrt{\frac{U_0 \mu x}{\varrho}} Z(Y) \tag{7.6}$$

$$t = t_p - (t_p - t_0)\, \theta(Y) \tag{7.7}$$

whence $\quad u = \frac{1}{2} U_0 \frac{dZ}{dY}; \quad v = \frac{1}{2} \sqrt{\frac{U_0 \mu}{\varrho x}} \left(Y \frac{dZ}{dY} - Z \right)$

The boundary conditions $t = t_p$, $u = v = 0$ at $y = 0$ and $t = t_0$, $u = U_0$ at $y = \infty$ become $\quad Z = 0,\ Z' = 0,\ \theta = 0$ at $Y = 0$

and $\quad Z' = 2,\ \theta = 1 \quad$ at $Y = \infty$

The partial differential equations reduce to total differential equations

$$\frac{d^3 Z}{d Y^3} + Z \frac{d^2 Z}{d Y^2} = 0 \tag{7.8}$$

and
$$\frac{d^2\theta}{dY^2} + \sigma Z \frac{d\theta}{dY} = 0 \tag{7.9}$$
where σ is the dimensionless ratio $\mu c/k$.

The function $Z(Y)$ was computed by Blasius by development in series. Figure 5 shows a graph of the function and its derivatives.

Assuming Z as known, the solution of (7.9) can be obtained by quadrature as
$$\theta = \alpha(\sigma) \int_0^Y e^{-\sigma \int_0^Y Z\,dY} dY \tag{7.10}$$
where
$$\alpha(\sigma) = \frac{1}{\int_0^\infty e^{-\sigma \int_0^Y Z\,dY} dY} \tag{7.11}$$

The temperature gradient
$$\frac{dt}{dy} = \frac{\partial t}{\partial \theta}\frac{\partial \theta}{\partial Y}\frac{\partial Y}{\partial y} = -\frac{1}{2}\sqrt{\frac{U_0 \varrho}{\mu x}}\frac{\partial \theta}{\partial Y}(t_p - t_0).$$

But $\partial\theta/\partial Y = \alpha(\sigma)\, e^{-\sigma \int_0^Y Z\,dy}$ and at the surface of the plate $(d\theta/\partial Y)_{Y=y=0} = \alpha(\sigma)$ and hence
$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = -\frac{1}{2}\alpha(\sigma)\sqrt{\frac{U_0 \varrho}{\mu x}}(t_p - t_0) \tag{7.12}$$
Hence the heat transfer per unit area, q_x, is given by
$$q_x = \frac{k}{2}\sqrt{\frac{U_0 \varrho}{\mu x}}\,\alpha(\sigma)\,(t_p - t_0) \tag{7.13}$$
This is the local rate of flow per unit area and is a function of x. The average for a plate of length L is given by
$$q_{av} = \frac{1}{L}\int_0^L q_x\,dx = \frac{k}{L}\sqrt{\frac{U_0 \varrho L}{\mu}}\,\alpha(\sigma)\,(t_p - t_0) \tag{7.14}$$
E. Pohlhausen found that α was approximately equal to $0.664\sqrt[3]{\sigma}$ for σ between 0.6 and 15.0. He also gives graphs of equation (7.10)[1] for various values of σ.

It may be noted that for $\sigma = 1$, the distribution of dZ/dY (hence u) and θ are identical. For air $\sigma = 0.70$ to 0.77, and the difference between the two distributions is not large, but for liquids the difference is very considerable, the thermal boundary layer being much thinner than the dynamic boundary layer.

CHAPTER III

THEORY OF HEAT TRANSFER IN EDDYING FLOW

1. Reynolds Theory of Eddying Flow. We shall now consider the theory of eddying flow, and we shall assume that the entire field of flow

[1] Pohlhausen, E., Zeitschrift für angewandte Mathematik und Mechanik, Vol. 1, p. 115, 1921.

is of this type. At any point the average speed and temperature are constant but the instantaneous values fluctuate with time. Reynolds regarded the flow as consisting of a mean flow and a superposed fluctuating motion. The possibility of clearly distinguishing the two rests upon the possibility of choosing time and space intervals, over which averages are to be taken to obtain the mean motion, which are large in comparison with the period and wave length of the fluctuations but which may be regarded as small in comparison to times and distances which are of significance in the mean motion.

We write $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, $t = \bar{t} + t'$ where the bars relate to the mean motion and the primes to the fluctuations. The fluctuations of ϱ, μ, c and k will be neglected. The above values of u, v, w and t are substituted in the equations of laminar flow and mean values taken. The rules for taking these means were formulated by Reynolds. Thus the mean value of a quantity which has already been averaged is not changed by taking the new mean. The averaging obeys the distributive law. Symbolically, if a and b are two of the quantities for which mean values are to be taken and if the bars are used to denote mean values, $\bar{\bar{a}} = \bar{a}$, $\overline{\bar{a}\, b} = \bar{a}\, \bar{b}$, $\overline{a + b} = \bar{a} + \bar{b}$, $\overline{\partial a/\partial x} = (\partial/\partial x)\, \bar{a}$, etc.

By definition $\overline{u'} = 0$, $\overline{v'} = 0$, $\overline{w'} = 0$, $\overline{t'} = 0$ and hence $\overline{u u} = \bar{u}\bar{u} + \overline{u'u'}$, $\overline{uv} = \bar{u}\bar{v} + \overline{u'v'}$, $\overline{ut} = \bar{u}\bar{t} + \overline{u't'}$, etc.

When the above expressions for u, v, w and t are introduced in the general equations of motion (2.1), (2.2), (2.3), (2.4) and (3.1) of Chapter II with the terms occurring in non-steady flow added, mean values taken, and account taken of the fact that u, v, w and u', v', w' satisfy the equation of continuity, we obtain

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} = g_x - \frac{1}{\varrho}\left(\frac{\partial \bar{p}_{xx}}{\partial x} + \frac{\partial \bar{p}_{xy}}{\partial y} + \frac{\partial \bar{p}_{xz}}{\partial z}\right) \tag{1.1}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z} = g_y - \frac{1}{\varrho}\left(\frac{\partial \bar{p}_{xy}}{\partial x} + \frac{\partial \bar{p}_{yy}}{\partial y} + \frac{\partial \bar{p}_{yz}}{\partial z}\right) \tag{1.2}$$

$$\bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z} = g_z - \frac{1}{\varrho}\left(\frac{\partial \bar{p}_{xz}}{\partial x} + \frac{\partial \bar{p}_{yz}}{\partial y} + \frac{\partial \bar{p}_{zz}}{\partial z}\right) \tag{1.3}$$

where
$$\bar{p}_{xx} = p + 2\mu\frac{\partial \bar{u}}{\partial x} - \frac{2}{3}\mu\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z}\right) - \varrho\,\overline{u'u'}$$

$$\bar{p}_{yy} = p + 2\mu\frac{\partial \bar{v}}{\partial y} - \frac{2}{3}\mu\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z}\right) - \varrho\,\overline{v'v'}$$

$$\bar{p}_{zz} = p + 2\mu\frac{\partial \bar{w}}{\partial z} - \frac{2}{3}\mu\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z}\right) - \varrho\,\overline{w'w'}$$

$$\bar{p}_{xy} = \mu\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right) - \varrho\,\overline{u'v'}$$

$$\bar{p}_{xz} = \mu\left(\frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z}\right) - \varrho\,\overline{u'w'}$$

$$\bar{p}_{yz} = \mu\left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y}\right) - \varrho\,\overline{v'w'}$$

$$\frac{\partial(\varrho\bar{u})}{\partial x} + \frac{\partial(\varrho\bar{v})}{\partial y} + \frac{\partial(\varrho\bar{w})}{\partial z} = 0 \tag{1.4}$$

$$\left.\begin{aligned}\varrho\, c\left(\bar{u}\,\frac{\partial \bar{t}}{\partial x}+\bar{v}\,\frac{\partial \bar{t}}{\partial x}+\bar{w}\,\frac{\partial \bar{t}}{\partial z}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial \bar{t}}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \bar{t}}{\partial y}\right)+\\+\frac{\partial}{\partial z}\left(k\frac{\partial \bar{t}}{\partial z}\right)-c\,\frac{\partial(\varrho\overline{u' t'})}{\partial x}-c\,\frac{\partial(\varrho\overline{v' t'})}{\partial y}-c\,\frac{\partial(\varrho\overline{w' t'})}{\partial z}\end{aligned}\right\} \qquad (1.5)$$

Equations (1.1), (1.2), (1.3) differ from the corresponding equations (2.1), (2.2) and (2.3) of Chapter II only in the presence of the derivatives of the six "eddy stresses"

$$-\varrho\overline{u'u'},\quad -\varrho\overline{u'v'},\quad -\varrho\overline{u'w'},\quad -\varrho\overline{v'v'},\quad -\varrho\overline{v'w'},\quad \text{and}\quad -\varrho\overline{w'w'}$$

Equation (1.4) is the same as (2.4) of Chapter II. Equation (1.5) differs from (3.1) of Chapter II in the presence of the derivatives of the "eddy heat transfer" terms which may be written

$$-c\varrho\overline{u't'},\quad -c\varrho\overline{v't'},\quad \text{and}\quad -c\varrho\overline{w't'}$$

if c is regarded as independent of x, y, z.

If the fluctuations u', v', w', t' were perfectly random, $\overline{u'v'}$, $\overline{u'w'}$, $\overline{v'w'}$, $\overline{u't'}$, $\overline{v't'}$, $\overline{w't'}$ would be zero, *i. e.* the eddy shearing stresses and the eddy heat transfer would be zero. There would remain only the eddy normal stresses $\varrho\overline{u'u'}$, $\varrho\overline{v'v'}$, $\varrho\overline{w'w'}$, which generally are quite insignificant in comparison with the pressure. The essential feature of eddying flow is then the correlation between the fluctuations of the several components of the velocity at any point.

The eddy shearing stresses show a certain parallelism with the viscous shearing stress. In laminar flow, the fluctuations are molecular fluctuations. The speeds u, v, w, are the mean speeds of many molecules. The effect of the molecular motions appears in the "smoothed" equations of motion in the form of the viscosity coefficient, and in the "smoothed" heat convection equation as the conductivity. These coefficients are functions of the temperature.

Reynolds general theory shows the influence of the fluctuations on the mean motion but does not give any information as to the fluctuations themselves. Little progress has been made on a fundamental theory of the fluctuations. However by more or less plausible theoretical considerations combined with a judicious use of empirical results obtained by experiment, notable advances have been made. The more important of these semi-empirical methods are (1) the use of the concept of eddy viscosity by Reynolds, Boussinesq, Stanton, Richardson, G. I. Taylor and many others; (2) the use of the concept of mixing length with momentum transfer by Prandtl; (3) the use of the concept of mixing length with vorticity transfer by G. I. Taylor; and (4) the use of the principle of similarity by von Kármán.

2. The Concept of Eddy Viscosity. Equations (1.1), (1.2) and (1.3) may be made to exhibit a formal similarity with II (2.1), (2.2) and (2.3) by defining an "apparent" or eddy viscosity which is assumed to satisfy simultaneously all of the following equations

$$-\varrho\,\overline{u'u'} = 2\,\varepsilon\frac{\partial\bar{u}}{\partial x} - \frac{2}{3}\,\varepsilon\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial z}\right) \tag{2.1}$$

$$-\varrho\,\overline{u'v'} = \varepsilon\left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{v}}{\partial x}\right) \tag{2.2}$$

$$-\varrho\,\overline{u'w'} = \varepsilon\left(\frac{\partial\bar{v}}{\partial z} + \frac{\partial\bar{w}}{\partial y}\right) \tag{2.3}$$

$$-\varrho\,\overline{v'v'} = 2\,\varepsilon\frac{\partial\bar{v}}{\partial y} - \frac{2}{3}\,\varepsilon\left(\frac{\partial u}{\partial x} + \frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial z}\right) \tag{2.4}$$

$$-\varrho\,\overline{v'w'} = \varepsilon\left(\frac{\partial\bar{w}}{\partial x} + \frac{\partial\bar{u}}{\partial z}\right) \tag{2.5}$$

$$-\varrho\,\overline{w'w'} = 2\,\varepsilon\frac{\partial\bar{w}}{\partial z} - \frac{2}{3}\,\varepsilon\left(\frac{\partial u}{\partial x} + \frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial z}\right) \tag{2.6}$$

On introducing the above relations in (1.1), (1.2) and (1.3), equations identical with those for a viscous fluid are obtained except that the viscosity μ is replaced by $\mu + \varepsilon$. It must be noted however that ε varies from place to place and is a function of the mean motion (not merely of the temperature). Moreover, the use of the concept of eddy viscosity implies certain relations between u', v' and w' at a given point which are expressed in the equations (2.1) to (2.6) inclusive.

Attempts have been made to use a vector quantity ε_x, ε_y, ε_z by some investigators in place of the scalar ε, especially in meteorological problems.

A similar "eddy conductivity" β may be introduced in (1.5) by writing

$$-c\,\varrho\,\overline{u't'} = \beta\frac{\partial\bar{t}}{\partial x} \tag{2.7}$$

$$-c\,\varrho\,\overline{v't'} = \beta\frac{\partial\bar{t}}{\partial y} \tag{2.8}$$

$$-c\,\varrho\,\overline{w't'} = \beta\frac{\partial\bar{t}}{\partial z} \tag{2.9}$$

Here again (2.7), (2.8) and (2.9) involve further assumptions as to the relations between the fluctuations. The introduction of eddy viscosity and eddy conductivity is not a mere introduction of two new symbols, but the introduction of a physical hypothesis as to the relations between the fluctuations of the several components of the speed, the temperature at any point, the mean motion and the mean temperature gradient.

The eddy viscosity ε and the eddy conductivity β are usually large in comparison with μ and k, and hence μ and k are generally neglected in problems of eddying flow. The relation between β and ε cannot be determined without some more definite picture of the underlying mechanism. The rather obvious assumption that $\beta = k\varepsilon/\mu$ does not seem to have been often used. The more usual assumption is that $\beta = c\varepsilon$, based on considerations given in 4 of this Chapter. If $k/\mu c = 1$, which is nearly true for gases, the two assumptions are equivalent.

In spite of the formal simplification, we are still not in a position to solve either the dynamical or the thermal problem. The variation of ε and β from place to place must first be known. Some solutions of the thermal problem have been given by computing from experimental data for the isothermal flow, using one of the relations between β and ε given above. An example is the computation of heat transfer in eddying flow in a pipe by Latzko[1].

3. Eddying Flow in a Pipe. We shall consider a section of a long cylindrical pipe of radius R, diameter D, far from the entrance and suppose the speed to be sufficiently high that the flow is eddying throughout and the final speed distribution has been attained. The air will be assumed to enter the pipe at a uniform temperature t_0, and the pipe wall will be assumed to be heated to a uniform temperature t_w for a distance L. The system of cylindrical coordinates of Fig. 3 will be used as in II 5, symmetry will be assumed, $\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right)$ will be neglected in comparison with $\frac{\partial}{\partial r}\left(k\frac{\partial t}{\partial r}\right)$, and k will be replaced by β. The equation of heat flow then becomes

$$\varrho\, c\, u\frac{\partial t}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(\beta r\frac{\partial t}{\partial r}\right) \tag{3.1}$$

This equation may be derived directly by considering the heat flow in a short cylinder of radius r concentric with the pipe, as shown by Latzko.

The computation of Latzko was made on the basis of the older formulation of the empirical laws of the isothermal eddying flow in which a power law was used. This representation is in good agreement with the experimental data for Reynolds numbers between 3000 and 100,000. Denoting the average velocity by $\bar{u}$, the empirical power law for the velocity distribution as used by Latzko is

$$u = \frac{8}{7}\bar{u}\left(1 - \frac{r^2}{R^2}\right)^{1/7} \tag{3.2}$$

The frictional shearing stress τ_0 at the wall is given by the formula of Blasius

$$\tau_0 = 0.03955\,\varrho\,\bar{u}^2\left(\frac{\bar{u}\,D\,\varrho}{\mu}\right)^{-1/4} \tag{3.3}$$

The shearing stress at any distance r is related to the velocity gradient $\partial u/\partial r$ by the formula developed by von Kármán

$$\tau = \frac{7}{8.82}\,\tau_0^{3/7}\,\varrho^{3/7}\,\mu^{1/7}\left(\frac{R^2 - r^2}{2R}\right)^{6/7}\frac{\partial u}{\partial r} \tag{3.4}$$

From these expressions the eddy viscosity ε is readily computed to be

$$\varepsilon = 0.199\,\bar{u}^{3/4}\,\varrho^{3/4}\,\mu^{1/4}\,D^{-3/28}\left(\frac{R^2 - r^2}{2R}\right)^{6/7} \qquad (3.5)^{2}$$

[1] Zeitschrift für angewandte Mathematik und Mechanik, Vol. 1, p. 269, 1921.
[2] In (3.5) $\overline{u}^{3/4}$ signifies mean u with index 3/4 and similarly in (3.6) and (3.7).

Assuming that the eddy conductivity β equals $k\varepsilon/\mu$, β is given by the expression

$$\beta = 0.199\, k\, \bar{u}^{3/4} \varrho^{3/4} \mu^{-3/4} D^{-3/28} \left(\frac{R^2 - r^2}{2R}\right)^{6/7} \tag{3.6}$$

Introducing the value of u from (3.2) and β from (3.6) in (3.1) and setting

$$A = \frac{0.199\, k}{\frac{8}{7}\, \varrho^{1/4} c\, \bar{u}^{1/4} \mu^{3/4} D^{3/28}} \tag{3.7}$$

we find

$$r\left(1 - \frac{r^2}{R^2}\right)^{1/7} \frac{\partial t}{\partial x} = A \frac{\partial}{\partial r}\left[\left(\frac{R^2 - r^2}{2R}\right)^{6/7} r \frac{\partial t}{\partial r}\right] \tag{3.8}$$

As in II 5 the type solution is

$$\frac{t - t_w}{t_0 - t_w} = \sum_m B_m\, e^{-a_m A x} f_m(r, a_m) \tag{3.9}$$

where $f_m(r, a_m)$ satisfies the ordinary differential equation

$$\frac{d}{dr}\left[r\left(\frac{R^2 - r^2}{2R}\right)^{6/7} \frac{d f_m}{dr}\right] = -a_m f_m r \left(1 - \frac{r^2}{R^2}\right)^{1/7} \tag{3.10}$$

or placing $\left(1 - \frac{r^2}{R^2}\right)^{1/7} = y$ and $49\left(\frac{R}{2}\right)^{8/7} a_m = \omega$

$$\frac{d}{dy}\left[(1 - y^7)\frac{d f_m}{dy}\right] = -\omega\, y^7 f_m \tag{3.11}$$

This again is a differential equation which describes a class of functions which are not readily computed. Solutions satisfying the boundary conditions $f_m = 0$ at $y = 0$ (equivalent to $t = t_w$ at $r = R$) and df_m/dy finite at $y = 1$ (equivalent to $\partial t/\partial r = 0$ at $r = 0$) are obtained only for certain values of ω.

The approximate solution found by Latzko is as follows:

$$\left.\begin{aligned} \frac{t - t_w}{t_0 - t_w} &= 1.129\, e^{-a_1(x/D)} (0.9544\, y - 0.0212\, y^3 + 0.0668\, y^5) \\ &\quad - 0.180\, e^{-a_2(x/D)} (-0.7472\, y - 4.275\, y^3 + 6.022\, y^5) \\ &\quad + 0.048\, e^{-a_3(x/D)} (20.34\, y - 54.80\, y^3 + 35.47\, y^5) \end{aligned}\right\} \tag{3.12}$$

where

$$a_1 = 0.151\left(\frac{\mu}{\bar{u} D \varrho}\right)^{1/4} \frac{k}{c\mu};\quad a_2 = 2.844\left(\frac{\mu}{\bar{u} D \varrho}\right)^{1/4} \frac{k}{c\mu};\quad a_3 = 29.42\left(\frac{\mu}{\bar{u} D \varrho}\right)^{1/4} \frac{k}{c\mu}$$

Latzko placed $k/c\mu = 1$, making the assumed relation $\beta = c\varepsilon$, rather than $\beta/k = \varepsilon/\mu$.

The rate of heat transfer q_x per unit area from the wall of the pipe can be computed from the relation

$$q_x = \left(\beta \frac{dt}{dr}\right)_{r = R} = -\left[\frac{2r}{7R^2}\left(1 - \frac{r^2}{R^2}\right)^{-6/7} \beta \frac{dt}{dy}\right]_{y = 0}$$

There is obtained

$$\left.\begin{aligned} q_x = 0.0346 \frac{k}{D}\left(\frac{\bar{u} D \varrho}{\mu}\right)^{3/4} (t_w - t_0)\, [1.078\, e^{-a_1(x/D)} + \\ + 0.1345\, e^{-a_2(x/D)} + 0.976\, e^{-a_3(x/D)}] \end{aligned}\right\} \tag{3.13}$$

The average rate q_{av} for the heated section of length L is defined as

$$\left.\begin{aligned} q_{av} = \frac{1}{L}\int_0^L q_x\,d\,x = 0.247\,c\,\mu\left(\frac{\bar{u}\,D\,\varrho}{\mu}\right)\frac{(t_w - t_0)}{L}\,[1.0113 - e^{-a_1(L/D)} - \\ - 0.0066\,e^{-a_2(L/D)} - 0.0047\,e^{-a_3(L/D)}] \end{aligned}\right\} \quad (3.14)$$

The mean temperature difference $(t_w - t_m)$ between the pipe wall and the fluid may be computed from the relation

$$(t_w - t_m) = \frac{1}{\pi R^2}\int_0^R (t_w - t)\,2\,\pi\,r\,d\,r$$

whence

$$\frac{t_w - t_m}{t_w - t_0} = 0.970\,e^{-a_1(x/D)} + 0.024\,e^{-a_2(x/D)} + 0.006\,e^{-a_3(x/D)} \quad (3.15)$$

The heat transfer coefficient h_x is defined by Latzko as equal to $q_x/(t_w - t_m)$. We find from (3.13) and (3.15)

$$h_x = 0.0346\,\frac{k}{D}\left(\frac{\bar{u}\,D\,\varrho}{\mu}\right)^{3/4}\frac{1.078\,e^{-a_1(x/D)} + 0.1345\,e^{-a_2(x/D)} + 0.976\,e^{-a_3(x/D)}}{0.970\,e^{-a_1(x/D)} + 0.024\,e^{-a_2(x/D)} + 0.006\,e^{-a_3(x/D)}} \quad (3.16)$$

As x increases, h_x approaches a minimum value

$$h_{min} = 0.03846\,\frac{k}{D}\left(\frac{\bar{u}\,D\,\varrho}{\mu}\right)^{3/4} \quad (3.17)$$

Since Latzko assumed $k/c\mu = 1$, he obtained instead of (3.17)

$$h_{min} = 0.03846\,\bar{u}\,\varrho\,c\left(\frac{\mu}{\bar{u}\,D\,\varrho}\right)^{1/4} \quad (3.18)$$

The experimental results of Nusselt are in better agreement with (3.18) than (3.17) so that the assumption $\beta = c\varepsilon$ is apparently better than $\beta = k\varepsilon/\mu$.

4. The Concept of Mixing Length. The first progress toward a theory relating the eddy viscosity to the mean motion was made by Prandtl, who introduced the concept of mixing length. The mixing length plays the same part in the theory of the fluctuations as the mean free path plays in the kinetic theory of gases. It is the length of path described by a small mass of fluid before it loses its individuality by mixing with neighboring masses. The isolation of small masses and the mixing length itself can be conceived as existing only in a statistical sense.

Considering for simplicity the case of two-dimensional flow of an incompressible fluid in the direction of the X axis with a velocity gradient du/dy perpendicular to the direction of flow, we imagine a small mass of air having a lateral motion of its own of magnitude v' travelling through a lateral distance l before it mixes with its surroundings. If the x component of its speed is equal to the mean x component at the place from which it started, the difference between the speed of this mass and the mean speed $\bar{u}$ in its new position is to a first approximation $-l\,(d\bar{u}/dy)$. We can thus set the average fluctuation u' proportional to $-\, l\, (d\bar{u}/dy)$. The lateral motion can be imagined to arise as the result of collisions of fluid masses with different velocities u' and can therefore be placed proportional to u' and hence to $l\,(d\bar{u}/dy)$. The eddy shearing stress $\tau' = -\varrho\,\overline{u'v'}$ is accordingly proportional to $\varrho l^2 (d\bar{u}/dy)^2$. Since l is still unknown, the various constants of proportionality and the correlation factor involved in the product $-\overline{u'v'}$ may be included in l^2. Since τ' must change sign with $d\bar{u}/dy$, we write

$$\tau' = \varrho\, l^2 \left|\frac{d\bar{u}}{dy}\right| \frac{d\bar{u}}{dy} = \varepsilon \frac{d\bar{u}}{dy} \tag{4.1}$$

The eddy viscosity on this theory is equal to $\varrho l^2 |d\bar{u}/dy|$, where the mixing length l varies from point to point.

At first sight nothing has been gained, since an assumption as to the variation of ε from place to place has merely been replaced by an assumption as to the variation of l from place to place. However, experiment shows that at large Reynolds numbers the mixing length is practically independent of the magnitude of the velocity and simple assumptions as to spatial distribution give reasonably accurate results. Moreover, suitable assumptions can often be found by dimensional reasoning. For example, near a wall, it seems clear that the mixing length must be proportional to the distance from the wall.

In the Prandtl theory the initial velocity of the fluid mass remains constant during the "life" of the mass. In effect this assumption implies a neglect of the effect of pressure fluctuations. Any other property of the fluid such as color or temperature will also be exchanged between two fluid layers by the motion of the small fluid masses.

Thus if we consider the "eddy heat transfer" $-c\varrho\,\overline{v't'}$, we should have v' and t' as proportional to $l\,(d\bar{u}/dy)$ and $-\,l\,(d\bar{t}/dy)$. The "eddy heat transfer" may thus be assumed to be proportional to $c\varrho l^2 \dfrac{d\bar{u}}{dy}\dfrac{d\bar{t}}{dy}$. Here again the constants of proportionality and the correlation factor between v' and t' may be incorporated in the unknown l. This l is usually assumed to be the same as the mixing length involved in the transfer of shearing stress, in which case the eddy conductivity β is equal to $c\varrho l^2 \left|\dfrac{d\bar{u}}{dy}\right|$ which equals $c\varepsilon$.

The identity of the two mixing lengths is usually founded on the identity of the mechanism of transfer, a given mass being responsible for bringing about a certain u' and t' which are in phase and hence have the same correlation with v' and the same factor of proportionality with $d\bar{u}/dy$ and $d\bar{t}/dy$. It has been found experimentally that this identity does not hold in all cases, and the reason has been sought in the neglect of the pressure fluctuations or in the neglect of the rotational motion of the fluid masses.

The mixing length concept can be applied also to three-dimensional problems. The stress tensor is of the form given in (2.1) to (2.6) inclusive with

$$\left.\begin{aligned}\varepsilon^2 = \varrho^2 l^4 \Big[& \left(\frac{d\bar{w}}{dy} + \frac{d\bar{v}}{dz}\right)^2 + \left(\frac{d\bar{u}}{dz} + \frac{d\bar{w}}{dx}\right)^2 + \left(\frac{d\bar{v}}{dx} + \frac{d\bar{u}}{dy}\right)^2 \\ & - 4\frac{d\bar{v}}{dy}\frac{d\bar{w}}{dz} - 4\frac{d\bar{u}}{dx}\frac{d\bar{w}}{dz} - 4\frac{d\bar{u}}{dx}\frac{d\bar{v}}{dy}\Big]\end{aligned}\right\} \tag{4.2}$$

The applications of the Prandtl theory have been largely to the dynamical problem of the isothermal eddying motion. Solutions of heat transfer problems have been of the type described in the preceding section, using the relation $\beta = c\varepsilon = c\varrho l^2 \left|\frac{d\bar{u}}{dy}\right|$, the values of l and of $d\bar{u}/dy$ being taken from the dynamical results for the isothermal case.

A similar theory of eddy stresses and eddy conductivity has been advanced by G. I. Taylor with the difference that the vorticity remains constant during the motion of a fluid mass, the momentum being modified in its journey by changes in pressure.

Consider again the two-dimensional flow of an incompressible fluid along the x axis, and neglect viscosity. The equation of motion is

$$\frac{du}{d\theta} + u\frac{du}{dx} + v\frac{du}{dy} = -\frac{1}{\varrho}\frac{dp}{dx} \tag{4.3}$$

and writing $\eta = \frac{1}{2}\left(\frac{du}{dy} - \frac{dv}{dx}\right)$ for the vorticity, (4.3) becomes

$$-\frac{d}{dx}\left(\frac{p}{\varrho} + \frac{1}{2}u^2 + \frac{1}{2}v^2\right) = \frac{du}{d\theta} + 2v\eta \tag{4.4}$$

Taking mean values and supposing η is on the average independent of x, (4.4) becomes

$$-\frac{1}{\varrho}\frac{d\bar{p}}{dx} = 2\overline{v'\eta'} \tag{4.5}$$

If now the vorticity η is conveyed without change as is the momentum in Prandtl's theory, the correlation between v and η arises in the same manner as the correlation between v and u in Prandtl's theory. The vorticity of the mean motion is $1/2\,\frac{d\bar{u}}{dy}$ and hence the fluctuation η' is proportional to $-l\frac{d}{dy}\left(\frac{1}{2}\frac{d\bar{u}}{dy}\right)$ and the mean value

$$2\overline{v'\eta'} = -2l^2\frac{d\bar{u}}{dy}\frac{d}{dy}\left(\frac{1}{2}\frac{d\bar{u}}{dy}\right) = -l^2\frac{d^2\bar{u}}{dy^2}\frac{d\bar{u}}{dy}$$

Hence (4.5) becomes

$$\frac{1}{\varrho}\frac{d\bar{p}}{dx} = l^2\frac{d^2\bar{u}}{dy^2}\frac{d\bar{u}}{dy} \tag{4.6}$$

Considering the equilibrium of a small prism of the fluid, the term on the left may be recognized as $\frac{1}{\varrho}\frac{d\tau}{dy}$, where τ is the shearing stress.

Therefore

$$\tau = \tau_{y=0} + \int_0^y \varrho\, l^2 \frac{d^2\bar{u}}{dy^2}\frac{d\bar{u}}{dy}\,dy \tag{4.7}$$

The rate M at which momentum is communicated to a unit volume of the fluid is

$$\text{according to Prandtl} \quad M = \varrho \frac{d}{d y}\left[l^2 \left|\frac{d \bar{u}}{d y}\right| \frac{d \bar{u}}{d y}\right] \tag{4.8}$$

$$\text{according to Taylor} \quad M = \varrho\, l^2 \left|\frac{d \bar{u}}{d y}\right| \frac{d^2 \bar{u}}{d y^2} \tag{4.9}$$

On both theories the rate of heat transfer Q per unit area perpendicular to y is

$$Q = c\, \varrho\, l^2 \left|\frac{d \bar{u}}{d y}\right| \frac{d \bar{t}}{d y} \tag{4.10}$$

For the particular case where l is independent of y, we find

$$\text{according to Prandtl} \quad M = 2\, \varrho\, l^2 \frac{d \bar{u}}{d y} \frac{d^2 \bar{u}}{d y^2}$$

$$\text{according to Taylor} \quad M = \varrho\, l^2 \frac{d \bar{u}}{d y} \frac{d^2 \bar{u}}{d y^2}$$

The generalization of Taylor's theory to three dimensions leads to quite complicated equations. Taylor proved however that Prandtl's momentum theory applies when u', the x component of the fluctuation, is zero.

It is clear from these considerations that the ratio between the eddy conductivity and the eddy viscosity is not constant but depends on the circumstances of the motion. Ruden has attempted to give the physical reasons for this fact[1]. He points out that a mass of fluid in a flow with a velocity gradient can maintain its individuality only if it rotates with an angular velocity $\frac{1}{2} \frac{d \bar{u}}{d y}$, and that without any translatory motion, there will be an exchange of properties of the fluid other than momentum, for example heat energy because of the rotational motion. The effect of the rotation will be negligible if the size of the fluid mass is small in comparison with the mixing length. In general, the effect may vary through wide limits.

The Taylor theory likewise does not make possible the direct computation of the mixing length. Solutions of heat transfer problems follow the same lines as in the preceding section, except for the different relations between β and ε.

5. Von Kármán's Principle of Similarity. A further advance in the theory of eddying flow was made by von Kármán by means of the principle of similarity. It was assumed that the fluctuations were similar to each other throughout the field of flow so that the conditions of flow in the neighborhood of two points differ only by a multiplicative factor of the magnitude of the fluctuations and by a length characteristic. The theory has been outlined in detail only for two-dimensional flow.

Let us suppose the average flow to be a parallel flow in the direction of the x axis and that the axis is chosen through the point to be

[1] Die Naturwissenschaften, Vol. 21, p. 375, 1933.

investigated. We then develop the average velocity near the point in powers of y

$$u = U_0 + U_0' y + U_0'' \frac{y^2}{2} + \dots \tag{5.1}$$

If the stream function of the fluctuating motion is designated $\psi'(x, y)$ the stream function ψ of the instantaneous motion will be

$$\psi = U_0 y + U_0' \frac{y^2}{2} + U_0'' \frac{y^3}{6} \dots + \psi' \tag{5.2}$$

We assume that the fluctuations have only a small extension in the y direction so that the development to the term in U_0'' is sufficiently accurate and we assume that the fluctuations may be regarded as a field of flow superposed on and carried along by the fundamental flow. The assumption of similarity means that if we set

$$\left.\begin{aligned} x &= l\,\xi \\ y &= l\,\eta \\ \psi' &= A f(\xi, \eta) \end{aligned}\right\} \tag{5.3}$$

only l and A are dependent on the particular point under investigation, that is on U_0' and U_0'', and $f(\xi, \eta)$ is independent of position in the field.

Th. von Kármán obtains the conditions for similarity by actually writing down the equation of motion, but by dimensional analysis it is immediately obvious that if l and A depend only on U_0' and U_0'', $U_0'' l / U_0'$ is constant, and $U_0''^2 A / U_0'^3$ is constant. Similarly, if the shearing stress τ depends only on ϱ, U_0' and U_0'', $\tau U_0''^2 / \varrho U_0'^4$ is constant. Likewise, if the rate of transfer of heat q per unit area depends only on the product ϱc [ϱ and c occurring only in this combination in the differential equation (1.5)], U_0', U_0'', and the temperature gradient t_0' in the mean flow, $q U_0''^2 / \varrho c t_0' U_0'^3$ are constant.

We may write, therefore,

$$l = \frac{a U_0'}{U_0''} \tag{5.4}$$

$$A = \frac{b U_0'^3}{U_0''^2} = b' l^2 U_0' \tag{5.5}$$

$$\tau = \frac{d \varrho U_0'^4}{U_0''^2} = d' \varrho l^2 U_0'^2 \tag{5.6}$$

$$q = \frac{e \varrho c t_0' U_0'^3}{U_0''^2} = e' \varrho c l^2 U_0' t_0' \tag{5.7}$$

where a, b, b', d, d', e, e' are constants.

Since l, the length characteristic, has an undetermined factor, any one of the constants of proportionality may be chosen to be unity. von Kármán selected $d' = 1$ defining l such that

$$\tau = \varrho l^2 U_0'^2 \tag{5.8}$$

The result (5.8) is identical with Prandtl's result (4.1) and l is commonly spoken of as the mixing length in this theory also. The notable advance is the equation (5.4) which relates the mixing length to the mean flow.

The eddy viscosity ε is given by the same expression as before

$\varepsilon = \varrho l^2 U_0'$ or in the old notation,

$$\varepsilon = \varrho\, l^2 \frac{\partial \bar{u}}{\partial y}$$

The eddy conductivity β is not however $c\varepsilon$ but $e'c\varepsilon$ where e' is an undetermined constant. The principle of similarity therefore yields the result that the eddy conductivity is proportional to but not equal to $c\varepsilon$.

It is interesting to note that Taylor's theory of the transport of vorticity also leads to a ratio between β and ε which depends on the mean flow. By (4.7)

$$\varepsilon = \frac{\tau_{y=0} + \int\limits_0^y \varrho\, l^2 (d^2 \bar{u}/d y^2)(d\bar{u}/d y)\, d y}{d\bar{u}/d y} \tag{5.9}$$

whereas $\beta = c\varrho l^2 |d\bar{u}/dy|$. Hence if $y = 0$ is chosen at $\tau = 0$,

$$\frac{\beta}{\varepsilon} = \frac{c\,\varrho\, l^2 \left|\dfrac{d\bar{u}}{dy}\right| \dfrac{d\bar{u}}{dy}}{\int\limits_0^y \varrho\, l^2 \dfrac{d^2\bar{u}}{d y^2}\dfrac{d\bar{u}}{d y}\, d y} \tag{5.10}$$

This ratio obviously depends on the variation of l, $d\bar{u}/dy$, and $d^2\bar{u}/dy^2$ with y. If l is independent of y, $\beta = 2c\varepsilon$. In general it appears from Taylor's formula that β/ε may be a function of y. The principle of similarity on the other hand leads to a ratio which is independent of y, β/ε being constant for a given flow.

The theory of von Kármán has not been extended to the case of three-dimensional flow. A rigorous computation was given only for the simple case of parallel flow, the results for the flow in pipes being obtained partly by a combination of plausible inferences based on the analogy between boundary layer thickness and radius of the pipe and partly by dimensional reasoning.

6. Present Status. The present position of the theory of heat transfer in eddying flow is very unsatisfactory. It appears quite certain that the exchange of quantities having scalar properties such as heat energy does not always obey the same law as the exchange of a quantity having vector properties such as momentum, the mixing lengths being different. The available experimental data indicate that near a wall the mixing lengths are the same for heat transfer and momentum transfer, whereas in the wake of a heated body the mixing length for the heat transfer is $\sqrt{2}$ times that for the momentum transfer (the eddy conductivity being twice the product of specific heat and eddy viscosity).

The laborious computations of heat transfer based on the formulae recently developed by von Kármán for the isothermal flow in pipes and near a wall have not yet been made. These formulae for the velocity distribution and the skin friction are in very good agreement with the experimental data.

CHAPTER IV

DIMENSIONAL ANALYSIS

1. Introductory. As in many other fields of technical physics, where the differential equations governing the phenomena are such that their solution is impractical if not altogether impossible, the method of dimensional analysis has proved a valuable tool in the treatment of heat-transfer problems. The method is particularly useful in the planning of experiments, and in the interpretation of experimental data.

The application of the theory of dimensions to problems in mechanics has been discussed at some length in Division A IV. Hence a brief review of the essentials and illustrations of the application to heat transfer problems will suffice for present purposes.

2. The Π Theorem. The methods of dimensional analysis depend on the fact that all of the terms of any correct and complete physical equation must have the same dimensions. Expressed in another way, the form of such an equation must be independent of the size of the units involved in the various terms of the equation, since the equation describes a relation which exists quite independently of the units selected to measure the several quantities.

By means of this principle it may be shown that any equation

$$F\,(Q_1,\ Q_2, \ldots . Q_n) = 0 \tag{2.1}$$

describing a relation among the n different kinds of quantity Q_1, Q_2, ... Q_n is always reducible to the form

$$f\,(\Pi_1,\ \Pi_2,\ \ldots .\ \Pi_{n-k}) = 0 \tag{2.2}$$

in which each of the variables Π represents a dimensionless product of the form

$$\Pi = Q_1^a,\ Q_2^b\ \ldots .\ Q_n^n \tag{2.3}$$

where k is the number of independent fundamental units required to specify the units of the n kinds of quantitiy and f is some unknown function to be found by experiment. If there are n separate kinds of quantities but more than one quantity of each kind, for example, a number of lengths all the quantities of any one kind may be represented by specifying a single one of that kind and the ratios r', r'' .... of the others to this one. Equation (2.2) then becomes

$$f\,(\Pi_1,\ \Pi_2,\ \ldots .\ \Pi_{n-k},\ r',\ r''\ \ldots .) = 0 \tag{2.4}$$

This is the basic theorem, usually called the Π theorem.

Equation (2.4) may be solved for one of the Π's, say Π_1 to give

$$\Pi_1 = \varphi(\Pi_2, \ldots. \Pi_{n-k}, r', r'' \ldots.) \tag{2.5}$$

The form of the function φ cannot be obtained except by experiment. In particular, the theory of dimensions does not yield the result sometimes attributed to it

$$\Pi_1 = \varphi_1(\Pi_2)\,\varphi_2(\Pi_3) \ldots.\ \varphi_{n-k-l}(\Pi_{n-k})\,\varphi'(r')\ \varphi''(r'') \ldots. \tag{2.6}$$

The validity of (2.6) in which the variables are separated can be established only as a result of experiment, not as a result of dimensional reasoning. Similarly, the approximation of the function φ by power laws can be established only on an experimental basis. The theory of dimensions alone does not permit us to go beyond (2.5).

The accuracy of the results of dimensional analysis is that of the original list of physical quantities which are involved in the problem. One cannot determine by the theory of dimensions what physical quantities are of importance in any given problem, but only that, *if* certain quantities are of importance, the relation must be of the form (2.5). The accuracy of the result must be tested in the last analysis by experiment.

3. Fundamental Units. In problems in mechanics three independent fundamental units suffice, the units of other physical quantities being expressed in terms of the three fundamental units. In heat-transfer problems it is necessary to add one additional fundamental unit, which may be conveniently the unit of temperature difference. Some writers add a unit of heat energy as a fundamental unit, which is permissible when there is no conversion of heat into mechanical energy and *vice versa,* or in any case if the mechanical equivalent of heat is added as one of the quantities on which the result depends. There is an essential difference between the unit of temperature difference and the unit of heat energy, for whereas a unit of heat energy can readily be derived from the mechanical units of mass, length, and time, the unit of temperature cannot be derived from mechanical units without a further act of arbitrary choice, the choice of the fixed points and of the numerical value to be assigned to the difference between the temperatures of the fixed points (see I **1**).

While any convenient number of fundamental units may be used as discussed by Buckingham[1] if proper account is taken of all the known relations between the units, a great deal of confusion is avoided by the use of the customary four fundamental units, mass, length, time, and temperature difference.

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[1] Notes on the Method of Dimensions, Phil. Mag., Vol. 42, p. 696, 1921.

In terms of these fundamental units, we may write down the several quantities with which we are concerned in the problem of heat transfer by convection and their dimensions:

Quantity	Dimensions
Mass — Fundamental	M
Length — Fundamental	L
Time — Fundamental	θ
Temperature — Fundamental	d
Area	L^2
Density	ML^{-3}
Viscosity	$ML^{-1}\theta^{-1}$
Force	$ML\theta^{-2}$
Pressure	$ML^{-1}\theta^{-2}$
Energy, Heat or Mechanical	$ML^2\theta^{-2}$
Power, or Rate of Flow of Heat Energy	$ML^2\theta^{-3}$
Velocity	$L\theta^{-1}$
Acceleration	$L\theta^{-2}$
Specific Heat (per unit mass)	$L^2\theta^{-2}d^{-1}$
Thermal Conductivity	$ML\theta^{-3}d^{-1}$
Coefficient of Thermal Expansion	d^{-1}
Coefficient of Heat Transfer, *i. e.* Rate of flow of heat energy per unit area divided by a temperature difference	$M\theta^{-3}d^{-1}$

4. Application to Convective Heat Transfer. Let us consider a simple idealized case of heat transfer in apparatus of one particular design so that the only quantity needed for specifying the apparatus is some linear dimension D. Let us suppose further that the density ϱ, viscosity μ, specific heat c, thermal conductivity k, and coefficient of thermal expansion β are constant, hence independent of the temperature. Let the temperature of the heated section be uniform and equal to t. Let the mean speed at a given section of the apparatus (or in some problems the speed at a great distance) be V and the mean temperature of the fluid (or in some problems the temperature at a great distance) be t_0. Suppose the temperature differences are sufficiently low that the heat transfer depends only on the difference $t - t_0$ and not on t and t_0 separately. For the present, we will not neglect the natural convection so that β, the coefficient of thermal expansion, and g, the acceleration of gravity, are considered of importance. The rate of flow q of heat energy per unit area is then determined by the quantities D, V, ϱ, μ, c, k, β, g, $(t - t_0)$.

There is then a relation of the form

$$f\,[q,\ D,\ V,\ \varrho,\ \mu,\ c,\ k,\ \beta,\ g,\ (t - t_0)] = 0 \qquad (4.1)$$

Since there are ten kinds of quantities and four fundamental units, there are six Π's. These may be built up in a great many different ways. By custom some of the non-dimensional quantities which can be formed bear special names as given in the following table:

Name	Symbol	Definition
Nusselt Number	N	$gD/k\,(t-t_0)$
Reynolds Number	R	$D\,V\,\varrho/\mu$
Prandtl Number[1]	σ	$c\,\mu/k$
Grashof Number	Gr	$gD^3\beta\,\varrho^2\,(t-t_0)/\mu^2$

There are two additional independent Π's which may be taken as $\beta\,(t-t_0)$ and $k\,(t-t_0)/\mu\,V^2$. Thus by the Π theorem (4.1), may be written

$$N = \varphi\left[R, \sigma, Gr, \frac{k\,(t-t_0)}{\mu\,V^2}, \beta\,(t-t_0)\right] \tag{4.2}$$

Consider first the case of natural convection, for which $V = 0$. The quantities R and $k\,(t-t_0)/\mu\,V^2$ are then absent and (4.2) becomes

$$N = \varphi'\,[\sigma, Gr, \beta\,(t-t_0)] \tag{4.3}$$

The quantity $\beta\,(t-t_0)$ is a measure of the volume changes in the fluid and may have considerable influence in special problems such as in the loss of heat from a hot wire of very small diameter. Generally, however, the expansion effects will be negligible except insofar as they alter the weight per unit volume and so set up gravity currents. In this case, β will occur only in combination with g in the Grashof number and $\beta\,(t-t_0)$ may be omitted.

Returning to forced convection, let us consider the case in which natural convection currents are negligible so that β and g have no influence on the heat transfer. The Grashof number and $\beta\,(t-t_0)$ are then absent and we have

$$N = \varphi''\left[R, \sigma, \frac{k\,(t-t_0)}{\mu\,V^2}\right] \tag{4.4}$$

This is as far as one can go by dimensional reasoning alone. There are two ways of proceeding to obtain the result commonly given. One is to introduce the assumption that there is no conversion of mechanical work into heat, *i.e.* that the heating due to viscous friction is negligible. Then the ratio $k\,(t-t_0)/\mu\,V^2$, which expresses the ratio of the flow of heat to the energy dissipated by viscous friction, can have no effect. An alternative procedure is to utilize the experimental fact that q is proportional to $t-t_0$ in which case $t-t_0$ cannot enter on the right hand side of (4.4). Either procedure leads to the result, $N = \varphi'''\,(R, \sigma)$ or

$$\frac{q\,D}{k\,(t-t_0)} = \varphi'''\left(\frac{D\,V\,\varrho}{\mu}, \frac{c\,\mu}{k}\right) \tag{4.5}$$

This equation could also be derived by using a unit of heat energy as an additional fundamental unit, but such a procedure involves the tacit assumption that there is no conversion of mechanical energy into heat, the same assumption as introduced above. It should be noted

[1] The reciprocal $k/c\,\mu$ is called the Stanton Number.

that the same assumption was introduced in Chapter II before writing down the differential equations.

5. Effect of Variation of Properties of the Fluid with Temperature. Equation (4.5) was derived for a fluid for which μ, ϱ, c, and k are independent of the temperature. For actual fluids, there is generally considerable variation of some or all of these quantities with the temperature so that the question immediately arises as to the values which should be used in (4.5). If the several temperature coefficients, for example β, the expansion coefficient, are included in the list of quantities governing the phenomena, one finds additional non-dimensional Π's, as in (4.2), of the general form $\beta\,(t - t_0)$. A general formula with such a large number of Π's is of little practical use.

This dilemma is usually met by the proposal to use "average" values, often without a clear definition of what is meant by average value. It is generally recognized that this average should be something more than the simple average throughout a volume defined by the integral $\iiint \varrho\, dx dy dz$ taken over the volume, divided by the volume. A more common proposal is to weight the local values according to the local temperature

$$\mu_{av} = \frac{\iiint \mu\,(t - t_0)\, dx\, dy\, dz}{\iiint (t - t_0)\, dx\, dy\, dz}$$

What is really desired are the values of ϱ, c, μ, and k for a fluid with properties independent of the temperature which would give the same heat transfer as the real fluid with properties which are dependent on the temperature. To define these "average" values with precision would require the solution of the complete problem.

If the range of variation is not large, the various types of averages will not differ greatly, and the temperature weighted average will be sufficiently accurate.

6. Modification by Introduction of Experimental Data. Equation (4.5) is the result of dimensional analysis plus two assumptions, (1) that the effects of natural convection are negligible and (2) that the heating due to viscous friction is negligible. By introducing other assumptions or information obtained from experiment, simpler formulae are obtained. Thus if the effect of viscosity is considered to be negligible (4.5) reduces to

$$\frac{q D}{k\,(t - t_0)} = \varphi'''' \left(\frac{D V c \varrho}{k}\right) \tag{6.1}$$

The combination $D V c \varrho / k$ is known as the Péclet number. This form of equation is that used by Boussinesq and deduced by Rayleigh from the principle of similitude.

The same equation may be obtained if the Prandtl number $c\mu/k$ is constant, as it approximately is for gases. For if $c\mu/k$ is constant it may be omitted and one may use either the Péclet number or the

Reynolds number as the argument of the function on the right hand side of the equation.

If now we find by experiment that q varies as V^n, it is obvious that

$$\varphi''''\left(\frac{D\,V\,c\,\varrho}{k}\right) = \left(\frac{D\,V\,c\,\varrho}{k}\right)^n \tag{6.2}$$

In a number of analyses of experimental data, power laws of this character have been used. The exponent n usually varies a little with the range of Péclet numbers under investigation. The function φ''' in (4.5) is often approximated by a product of powers of the Reynolds number and the Prandtl number. It should be remembered however that the power law and the separation of the function of two variables into the product of two functions each of a single variable do not result from dimensional theory alone. These specific results are approximations found experimentally to apply over certain ranges of the two variables.

CHAPTER V

THE ANALOGY BETWEEN HEAT TRANSFER AND SKIN FRICTION

1. The Reynolds Formulation. Another general method of attack on the heat transfer problem is by means of analogy between heat transfer and skin friction. The earliest formulation of the analogy was given by Reynolds for the flow in a pipe. He assumed that the mechanism for the transfer of heat was the same as the mechanism for the transfer of momentum, and concluded that the ratio of the momentum lost by skin friction between two cross sections distance $d\,x$ apart to the total momentum of the fluid is the same as the ratio of the heat actually supplied (by convection only) to the fluid between these sections and the heat which would have been supplied if the whole of the fluid flowing through the pipe had been carried up to the surface between the two sections.

Let us consider the case of fluid of specific heat c flowing with mean velocity V through a cylindrical pipe of radius R whose walls are maintained at constant temperature t_w. Let G be the mass of fluid passing through the pipe per second, t_{av} the average temperature between the two sections $d\,x$ apart, $dt/d\,x$ the temperature gradient, $d\,p/d\,x$ the pressure gradient. Then Reynolds' conclusion is that

$$\frac{-\pi\,R^2\,(d\,p/d\,x)}{G\,V} = \frac{G\,c\,(d\,t/d\,x)}{G\,c\,(t_w - t_{av})} \tag{1.1}$$

The rate of heat flow q_x per unit area at the surface of the pipe is given by the relation

$$2\,\pi\,R\,q_x = G\,c\,\frac{d\,t}{d\,x} \tag{1.2}$$

The heat transfer number h_x, which equals $q_x/(t_w - t_{av})$ is therefore

$$h_x = \frac{G\,c}{2\,\pi\,R\,(t_w - t_{av})}\,\frac{d\,t}{d\,x} \tag{1.3}$$

The skin friction F_x per unit area is given in terms of the pressure drop by the relation

$$2\pi R F_x = \pi R^2 \frac{dp}{dx} \tag{1.4}$$

The coefficient of skin friction, c_f is defined by the relation

$$F_x = c_f \frac{1}{2} \varrho V^2 \tag{1.5}$$

whence

$$c_f = \frac{R\,(dp/dx)}{\varrho V^2} \tag{1.6}$$

Substituting (1.6) and (1.3) in (1.1) we obtain

$$h_x = \frac{1}{2} \varrho c V c_f = \frac{1}{2} c G c_f \tag{1.7}$$

This is one form of the Reynolds equation. In terms of F and q_x

$$\frac{F_x}{V} = \frac{q_x}{c\,(t_w - t_{av})} \tag{1.8}$$

which is the form originally given by Reynolds.

Although derived for flow in a pipe, it is often assumed that the same relation applies to any flow for which the thermal and dynamic boundary conditions correspond.

Reynolds pointed out that ultimately it is by conductivity that the heat passes from the walls of the pipe to the fluid so that k should enter into the result.

In the derivation of the Reynolds equation, no hypothesis is made as to whether the flow is laminar or eddying. If the molecular motions in the laminar flow are similar to the molar motions in the eddying flow, the relation between heat transfer and momentum transfer will be the same in each case.

Let us examine the solution given in II 5 for laminar flow in a pipe. It is of course obvious that there is no relation between friction and heat transfer when the pipe is heated for only a short section. A relation can exist only at some distance from the beginning of the heated section, where the heat transfer number h_x is constant. For this case we found $h_x = 1.83\,(k/R)$. By simple transformation of the Poiseuille-Hagen formula,

$$c_f = \frac{8\mu}{\varrho V R} \tag{1.9}$$

By (1.7) we would expect from (1.9), that the heat transfer number would be equal to $4\mu c/R$ which is 2.185 $(\mu c/k)$ times the correct value.

Similarly for the laminar flow along a plate. From II (7.13), we have

$$h_x = \frac{k}{2} \alpha \left(\frac{c\mu}{k}\right) \sqrt{\frac{U_0 \varrho}{\mu x}}$$

By the Blasius formula for skin friction

$$c_f = 0.664 \sqrt{\frac{\mu}{U_0 \varrho x}} \tag{1.10}$$

By (1.7) we would expect from (1.10) that h_x would be equal to $0.332\, c\mu \sqrt{\frac{U_0 \varrho}{\mu x}}$ which is $\frac{0.664}{\alpha\,(c\mu/k)} \frac{c\mu}{k}$ times the correct value. Since $\alpha\,(c\mu/k)$

is approximately equal to 0.664 $(c\mu/k)^{1/3}$, the ratio of the predicted to the correct value is $(c\mu/k)^{2/3}$.

It is obvious that the formula (1.7) does not apply to laminar flow, that the ratio of the predicted to the correct value is a function of $c\mu/k$, and that this function is different for different geometrical arrangements.

If we examine the solution of III 3 for eddying flow in a pipe, we find that the Reynolds equation (1.8) is approximately though not exactly satisfied.

2. The Prandtl-Taylor Formulation. As experimental technique developed, it became clear that the flow through a pipe above the critical Reynolds number was not of the eddying type throughout the entire flow, but there was an approximation to laminar flow in a thin film adjacent to the walls of the pipe. Let us consider the hypothetical case of a thin layer of thickness ε adjacent to the wall of the pipe. Let the temperature at the outer boundary of the layer be t_1, the velocity V_1, and suppose that the layer is so thin that the distributions of velocity and temperature are linear. Then

$$F_x = \mu \frac{V_1}{\varepsilon} \tag{2.1}$$

$$q_x = \frac{k\,(t_w - t_1)}{\varepsilon} \tag{2.2}$$

For the turbulent core, the Reynolds equation (1.8) gives

$$\frac{F_x}{V - V_1} = \frac{q_x}{c\,(t_1 - t_{av})} \tag{2.3}$$

By eliminating t_1 and ε from (2.1), (2.2) and (2.3) we obtain

$$\frac{q_x}{F_x} = \frac{c\,(t_w - t_{av})}{V}\left(\frac{1}{1 + (V_1/V)\,(\mu\, c/k - 1)}\right) \tag{2.4}$$

This equation is usually known as the Prandtl-Taylor formulation of the heat transfer friction analogy. Before the equation can be used, V_1/V must be determined. Using the same power law formulation as was used in III 3, we may proceed as follows. From III (3.2)

$$\frac{\partial u}{\partial r} = -\frac{16}{49}\frac{\bar{u}\, r}{R^2}\left(1 - \frac{r^2}{R^2}\right)^{-6/7} = -\frac{16}{49}\frac{\bar{u}\, r}{R^2}\left(\frac{7}{8}\frac{u}{\bar{u}}\right)^{-6} \tag{2.5}$$

Corresponding to our previous assumption of a thin layer in which the flow is laminar, we assume that $-\mu\,(\partial u/\partial r)$ taken near the wall is equal to the shearing stress at the wall given by III (3.3). Hence setting

$$r = R \qquad 0.7276\, \frac{\bar{u}}{R}\, \mu \left(\frac{u}{\bar{u}}\right)^{-6} = 0.03955\, \varrho\, \bar{u}^2 \left(\frac{\bar{u}\, D\, \varrho}{\mu}\right)^{-1/4}$$

whence

$$\frac{u}{\bar{u}} = 1.82\left(\frac{\bar{u}\, D\, \varrho}{\mu}\right)^{-1/8} = 1.67\left(\frac{\bar{u}\, R\, \varrho}{\mu}\right)^{-1/8} \tag{2.6}$$

In the notation of the present section

$$\frac{V_1}{V} = 1.67\left(\frac{V R\, \varrho}{\mu}\right)^{-1/8} \tag{2.7}$$

Prandtl with a slightly different expression for the velocity distribution obtained the value 1.6 for the numerical coefficient, but remarks that its value is very uncertain, since actually there is no sharp transition between the laminar and eddying regions. He suggests a value 1.0 or 1.1 as best fitting the experimental data.

In this Division we are particularly interested in the flow of air, for which $c\mu/k$ is about 0.7. The difference between (2.4) and (1.8) is in this case not very large.

3. General Remarks. The analogy between skin friction and heat transfer has been discussed most extensively for the problem of flow in pipes. The results apply also to certain other cases, as for example the boundary layer flow near a skin friction plate, but the analogy is not universally applicable. To gain an insight into the conditions to which the analogy applies, let us consider the relations at some point very near to a heated surface (temperature t_w) past which a fluid stream flows. We suppose for simplicity that the flow is two-dimensional. If the speed and temperature at the point are u_1 and t_1 respectively and if the point is distant ε from the surface, we have the relations

$$F_x = \frac{\mu u_1}{\varepsilon} \quad \text{and} \quad q_x = \frac{k(t_w - t_1)}{\varepsilon}$$

whence eliminating ε,

$$\frac{F_x}{q_x} = \frac{\mu u_1}{k(t_w - t_1)} \tag{3.1}$$

Assuming that the effects of heating due to viscous friction and of natural convection are negligible, and that c, μ and k are independent of the temperature, we have by dimensional reasoning

$$\frac{u_1}{V} = f_1\left(\frac{c\mu}{k}, \frac{VD\varrho}{\mu}\right) \tag{3.2}$$

and

$$\frac{t_w - t_1}{t_w - t_0} = f_2\left(\frac{c\mu}{k}, \frac{VD\varrho}{\mu}\right) \tag{3.3}$$

where V, D and t_0 are reference speed, reference length, and reference temperature, respectively.

Substituting (3.2) and (3.3) in (3.1)

$$\frac{F_x}{q_x} = \frac{V}{c(t_w - t_0)} \frac{(c\mu/k) f_1}{f_2} \tag{3.4}$$

To obtain the Reynolds equation, we must have

$$\frac{c\mu}{k} f_1 = f_2$$

or returning to u_1 and t_1

$$\frac{u_1}{V} = \frac{k}{c\mu}\left(\frac{t_w - t_1}{t_w - t_0}\right)$$

The equations governing the distribution of u/V and $\frac{k}{c\mu}\frac{(t_w - t)}{(t_w - t_0)}$ must be identical *for points near the surface.* Since the flow is believed to be laminar very near the surface and the point may be selected as close as one pleases to the surface, we may use the equations for laminar flow in a boundary layer, namely

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\varrho} \frac{\partial^2 u}{\partial y^2} - \frac{1}{\varrho} \frac{\partial p}{\partial x} \tag{3.5}$$

$$\varrho c \left(u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} \right) = k \frac{\partial^2 t}{\partial y^2} \tag{3.6}$$

Placing $u/V = X$ and $(k/c\mu)(t_w - t)/(t_w - t_0) = Y$, (3.5) and (3.6) become

$$X \frac{\partial X}{\partial x} + \frac{v}{V} \frac{\partial X}{\partial y} = \frac{\mu}{\varrho V} \frac{\partial^2 X}{\partial y^2} - \frac{1}{\varrho V^2} \frac{\partial p}{\partial x} \tag{3.7}$$

$$X \frac{\partial Y}{\partial x} + \frac{v}{V} \frac{\partial Y}{\partial y} = \frac{k}{\varrho c V} \frac{\partial^2 Y}{\partial y^2} \tag{3.8}$$

The distributions of X and Y will be similar, provided $c\mu/k = 1$ and $dp/dx = 0$. Accordingly these are the conditions for which the Reynolds equation is valid.

Equation (3.6) and hence (3.8) were derived on the assumption that there was no source of heat within the fluid. If heat were generated at the rate S per unit volume, the term S would appear on the right hand side of (3.6) and the term $-\frac{k}{c\mu} \cdot \frac{S}{\varrho c V (t_w - t_0)}$ on the right hand side of (3.8). The distributions of X and Y would then be similar provided $k/c\mu = 1$ and $\frac{1}{V} \frac{\partial p}{\partial x} = \frac{S}{c(t_w - t_0)}$. If $t_w > t_0$, and $\partial p/\partial x$ is negative, S is negative and represents a sink. Suppose then that we consider a hypothetical case in which we have a source of heat at the wall of the pipe and a sink of strength S per unit volume extending throughout the pipe, where

$$S = \frac{c(t_w - t_0)}{V} \frac{\partial p}{\partial x} \tag{3.9}$$

Such a picture in which the heat generated equalled that absorbed by the sink would be a more or less reasonable approximation to the eddying flow in a pipe for which the temperature is approximately uniform over the central core. As suggested by Prandtl we can use (3.9) to compute the heat flow from the wall approximately. The rate of flow of heat q_x per unit area is related to S by the expression $2\pi R q_x = \pi R^2 S$ or $q_x = (R/2) S$. Since the skin friction $F_x = \frac{R}{2} \frac{\partial p}{\partial x}$, (3.9) becomes

$$q_x = \frac{c(t_w - t_0)}{V} F_x$$

From the nature of the approximate picture, t_0 would be selected as the average temperature and V as the average speed.

To conclude, the Reynolds analogy can be shown to be valid only for the case of flow near a skin friction plate of a fluid for which $c\mu/k$ is equal to unity. The application of the Reynolds equation or the Prandtl-Taylor equation to other cases is generally speaking, without other than empirical justification. Neither equation applies to flows in which separation occurs, as for example, the flow outside a cylindrical pipe normal to the wind.

CHAPTER VI

HEAT TRANSFER FROM A SKIN FRICTION PLATE

1. Introductory. In Chapters II and III we have reviewed that part of the theory of the aerodynamics of cooling which is capable of more or less orderly mathematical development. We have treated certain illustrative problems, usually with simplifying assumptions and always, with the assumption that the flow is either everywhere laminar or everywhere eddying. In practice, both types exist in different parts of the field of flow, and the conditions which determine the transition between the two types of flow have been the subject of much investigation, both theoretical and experimental. Some knowledge has been accumulated, but in a form not susceptible as yet of exact mathemetical treatment. This information is of the greatest importance in its bearing on many practical problems of heat transfer.

The distinction between laminar and eddying flow is usually approached on the basis of Reynolds' original experiments on the flow in pipes, corresponding to the historical development of the subject. In some respects, however, the two-dimensional flow of a fluid near a thin flat plate with sharp symmetrical leading edge, the plate being set parallel to the flow, illustrates the basic phenomena somewhat better, and furnishes a more logical beginning. Such a plate is usually known as a "skin-friction" plate.

The heat transfer from a skin friction plate is of considerable interest in aeronautics since the ratio of heat transfer to drag for such a surface represents an ideal maximum which cannot be exceeded. The performance of the plate sets the standard of excellence.

We shall discuss at some length both the dynamical and thermal problem. Experimental data on heat transfer from skin friction plates are not very plentiful, but because of the possibility of applying the Reynolds formula [see V (1.8)], the data on skin friction may be used to compute the heat transfer when dealing with the flow of air. The chief interest, however, is not the data on the total heat transfer or total friction, but the development of certain basic phenomena illustrated in the distribution of speed and temperature near a heated plate. It will be necessary to review certain information which is presented in greater detail in other divisions of this work.

2. Distribution of Speed in Isothermal Flow. The pioneer measurements of the distribution of speed in isothermal flow near a skin friction plate were made by J. M. Burgers and B. C. van der Hegge Zijnen in

1924 with the aid of a hot-wire anemometer. The results are presented in the dissertation of van der Hegge Zijnen in the form of numerous tables and curves giving the observed speeds at several hundred points, whose x and y coordinates with respect to the leading edge of the plate are tabulated, for five speeds of the approaching air stream. Fortunately, it is possible to utilize the methods of dimensional analysis to devise a method of plotting which enables one to obtain a general view of the hundreds of measurements. The speed V at any point (x, y) at distance x from the leading edge and at distance y

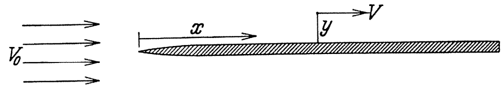

Fig. 6. Coordinates for skin-friction plate.

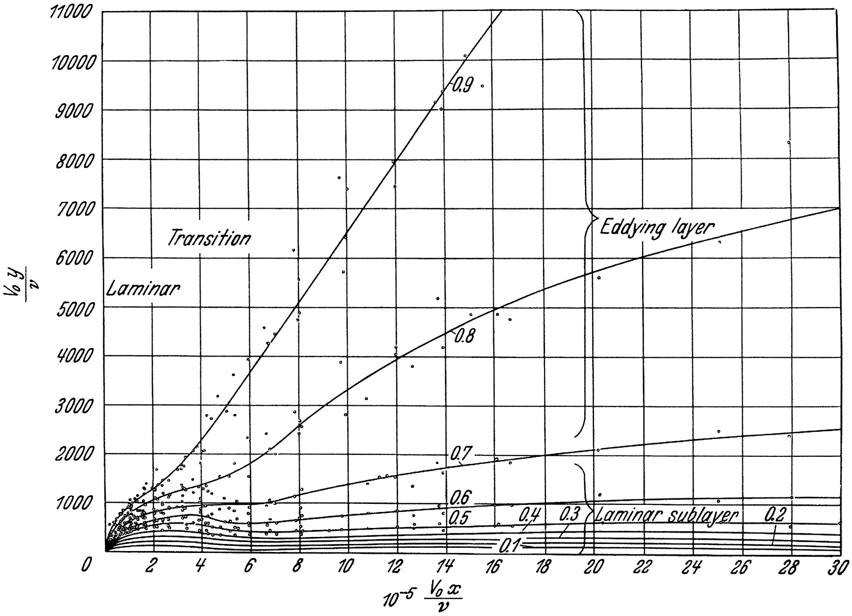

Fig. 7. Van der Hegge Zijnen's measurements of speed distribution near skin-friction plate. See Fig. 6 for x and y. V_0 is speed of free stream. ν is kinematic viscosity of the fluid. Numbers on contours are values of $\frac{V}{V_0}$, where V is the local speed.

from the plate (Fig. 6) is a function of the speed V_0 of the approaching air stream, of the density ϱ and the viscosity μ of the fluid, and of x and y. By the methods of Chapter IV we find at once that

$$\frac{V}{V_0} = \left(\frac{V_0 x \varrho}{\mu}, \frac{V_0 y \varrho}{\mu}\right)$$

The results can therefore be represented by a three-dimensional model, or more conveniently by a contour diagram of the three-dimensional model.

A contour diagram of this type for the measurements of van der Hegge Zijnen is given in Fig. 7. The contours are for values of V/V_0

in steps of 0.1, the corresponding x and y being found by interpolation in the original tables, from which $V_0 x/\nu$ ($\nu = \mu/\varrho$) and $V_0 y/\nu$ were computed. For convenience the scale of $V_0 y/\nu$ has been magnified 200 times. If one wishes to think in terms of x and y, we may imagine a flow of air at a speed of 200 ft./sec., in which case the numbers along the abscissae represent distances in inches and each square along the ordinates, a distance of one thousandth of an inch. Or at a speed of 20 ft./sec., the numbers along the abscissae are in tens of inches and each square along the ordinates a distance of one hundredth of an inch.

The diagram contains data for five speeds and in general the results are very consistent. The deviations correspond to about 0.02 in V/V_0 or 0.005 inches on the average in y. When examined on a large scale there are certain systematic differences between the results at different speeds, which are to be ascribed to the influence of a slight pressure gradient in the air stream in which the measurements were made.

Near the leading edge the contour lines are approximately parabolic in shape and correspond approximately to the theoretical result of Blasius for laminar flow (see II 7). For this reason, the flow in this part of the field is labeled "laminar".

At a $V_0 x/\nu$ of about 300,000, the contours for small values of V/V_0 approach the axis of abscissae, indicating an increasing speed along the plate while the contours for large values bend away from the axis, indicating a rapid thickening of the layer. The process continues over the range from 300,000 to about 500,000, a region usually designated as the "transition" region.

There follows a different type of speed distribution which resembles very closely that found in eddying flow in pipes. In the part marked "eddying" layer, there is at any x a logarithmic relation between V and y. The relations are different near the wall, a region commonly termed the "laminar sub-layer" because the distribution resembles that in the laminar layer. It should be noted that the laminar sub-layer accounts for only a small part of the thickness of the layer but for two-thirds of the fall in speed.

The contour for $V/V_0 = 1$ is not shown, for the reason that V approaches V_0 asymptotically. Various unambiguous procedures can be used to define the "thickness" of the layer of fluid affected by the presence of the plate. We may perhaps think of the distribution as approximated by some specific mathematical expression, and the thickness δ as the value of y which, substituted in that expression, gives $V = V_0$. Or we may make use of the "reduced thickness" δ^* defined by the relation

$$\delta^* = \frac{1}{V_0} \int_0^\infty (V_0 - V)\, d y$$

3. The Concept of Initial Turbulence. In air streams, especially those produced by artificial means as in wind tunnels, the motion is never absolutely steady, and there are always present small ripples or fluctuations which do not usually exceed a few per cent of the average speed. It is difficult to believe that the presence of these fluctuations, usually of frequencies of the order of 20 to perhaps 1000 per second, could play any part in determining the nature of the flow around an object placed in the stream. Yet it has been found experimentally that these fluctuations exert a comparatively large influence in many cases. The basic effect in all these cases is believed to be the effect on the transition from laminar to eddying flow in the boundary layer, which is best illustrated by experiments on flow near a skin friction plate.

Before proceeding with these experiments, it is desirable to review briefly the methods by which this property of air streams, the "initial turbulence", commonly designated simply the turbulence, is evaluated. It is possible to measure directly the mean fluctuation of the speed at any point with time by means of a special form of hot-wire anemometer, with a wire of small diameter, an amplifier, an electrical network to compensate for lag of the wire, and an alternating-current milliammeter. The speed fluctuation is converted into an alternating electric current whose intensity is measured. The turbulence may then be defined as the ratio of the average fluctuation to the mean speed and is usually expressed as a percentage.

Another method of comparing the turbulence of different air streams is by observation of the resistance of spheres or by measurements of the pressure differences between holes at the front and rear of a sphere.

These methods have been compared and both have been found to give a suitable basis for the correlation of data on the effect of turbulence on aerodynamic properties as measured in wind tunnels. A summary of the status of the knowledge of the effects of turbulence as of December 1934 is given in a paper by the author[1]. Experiments completed since the publication of that summary show that some length characteristic as well as an intensity characteristic is required to characterize the effect of the fluctuations on the air resistance of spheres. The method by which the turbulence should be numerically evaluated is therefore still in process of development. This difficulty of measurement and the absence of a satisfactory theory does not alter the fact of the existence of this property of an airstream which has a profound effect on the transition from laminar to eddying flow.

4. The Effect of Turbulence on the Transition from Laminar to Eddying Flow. The qualitative nature of the effect of turbulence was demonstrated by van der Hegge Zijnen by placing a wire screen ahead of his plate to

[1] Journal of the Aeronautical Sciences, p. 67, April 1934.

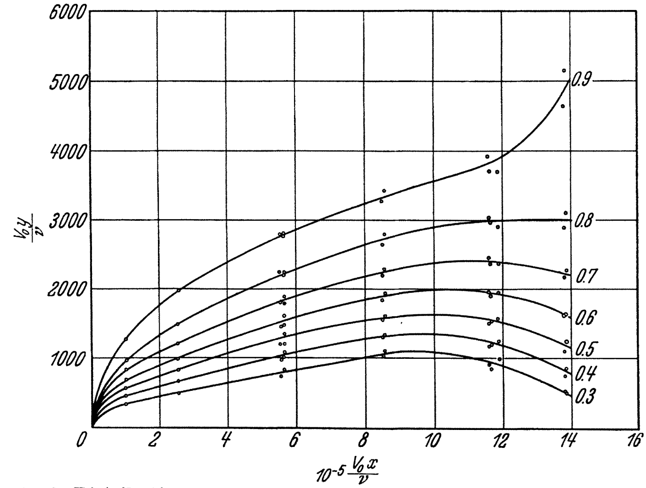

Fig. 8. Distribution of speed near skin-friction plate, turbulence 0.5 per cent Measurements at National Bureau of Standards.

produce greater turbulence. The transition then occurred for $V_0 x/\nu$ of about 100,000. To give some idea of the possible magnitude of the effect, the results of some experiments at the National Bureau of Standards are shown in Figs. 8 and 9. These are contour diagrams similar to Fig. 7. The results of Fig. 8 were obtained in a wind tunnel having a turbulence of 0.5 per cent as measured by the hot-wire method. The transition occurs at a value of $V_0 x/\nu$ of about 1,100,000. For a given speed, the transition begins at a distance from the leading edge nearly four times that found by van der Hegge Zijnen or by Hansen at Aachen.

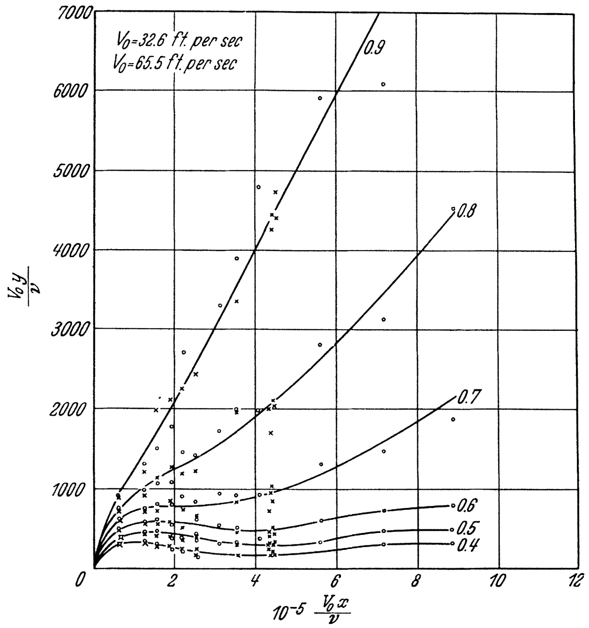

Fig. 9. Distribution of speed near skin-friction plate, turbulence 3.0 per cent.

The results of Fig. 9 were obtained by introducing a screen in the wind tunnel ahead of the plate which increased the turbulence as measured by the hot-wire method to 3.0 per cent. The transition occurs very much earlier, namely at $V_0 x/\nu$ of about 100,000.

These experiments may be summarized as follows. So long as the turbulence of the air stream is unchanged, the transition occurs at a fixed value of the Reynolds number formed from the distance to the leading edge and the speed in the free air stream. If, however, the turbulence is increased, the Reynolds number for transition is decreased and *vice versa*. There is thus a functional relation between the Reynolds number for transition and the turbulence of the air stream, which has not yet been experimentally determined.

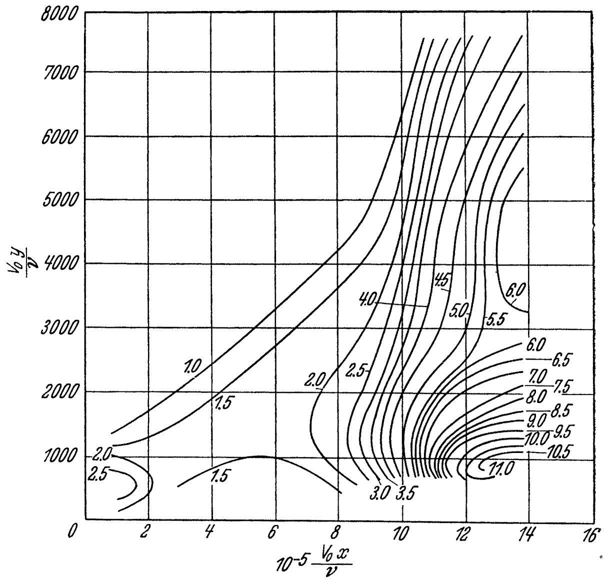

Fig. 10. Distribution of fluctuations of speed corresponding to Fig. 8. Numbers on contours are values of $\frac{100\triangle V}{V_0}$ where $\triangle V$ is the root-mean-square fluctuation of the local speed. In the free stream $\frac{100\triangle V}{V_0} = 0.5$.

5. Distinction Between Laminar and Eddying Flow. The distinction between laminar and eddying flow, made on the basis of the type of distribution of mean speed, is often extended to suggest that the laminar flow is a steady flow (since that is assumed in the theory of laminar flow) and that the eddying flow is not steady, except on the average. The same hot-wire equipment used to measure the turbulence of the air stream can be used to measure the fluctuations near the plate. Contour diagrams of the fluctuations corresponding to Figs. 8 and 9 are given in Figs. 10 and 11. It is seen that the fluctuations in the laminar part of the layer are three times as large as those in the free stream. Large fluctuations develop in the transition region, but these do not persist in the eddying flow.

The distribution of mean speed indicates a gradual transition. Actually, however, the transition is sudden as shown by the oscillographic records of the fluctuations in Fig. 12 (Plate II). It will be noted that the fluctuations are much more rapid when the layer is eddying than when it is laminar. Likewise, the records in the transition region show a sudden and intermittent alteration of the flow from laminar to eddying, occurring at infrequent intervals near the beginning of the transition region and at more and more frequent intervals as the end of the transition region is approached. From another point of view, the point of transition wanders back and forth erratically and this wandering is responsible for the very large fluctuations in the transition region. These observations are entirely analogous to Reynolds observations of the behaviour of bands of color in the flow of water through a pipe.

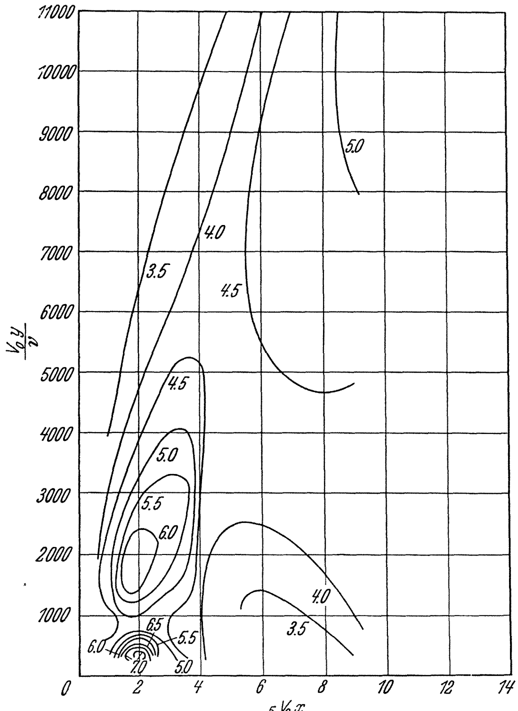

Fig. 11. Distribution of fluctuations of speed corresponding to Fig. 9. Numbers on contours are values of $\frac{100\,\triangle V}{V_0}$ where $\triangle V$ is the root-mean-square fluctuation of the local speed. In the free stream $\frac{100\,\triangle V}{V_0} = 3.0$.

As emphasized in I 3, the true distinction between laminar and eddying flow must rest not on the presence or absence of fluctuations with time, but on the presence or absence of correlation between the three components of the fluctuations of the speed at a point. The values in Figs. 10 and 11 represent only one component of the fluctuations, approximately that parallel to the plate.

6. Effect of Heat Transfer on the Velocity Field. Comparatively little information is available as to the effect of the heating of the plate on the velocity field. Éliás[1] found that the change in speed at any point did not exceed 2 or 3 per cent for a temperature rise of about 35° C. For this temperature range the kinematic viscosity changes by about 20 per cent. Éliás apparently uses the value corresponding to the arithmetic mean of the plate and air temperatures.

When the results of Éliás are plotted in the manner described in 2, there is evidence of a slightly earlier transition for the heated plate. The effect is not very large, however, and may be neglected.

[1] Zeitschrift für angewandte Mathematik und Mechanik, Vol. 9, p. 434, 1929.

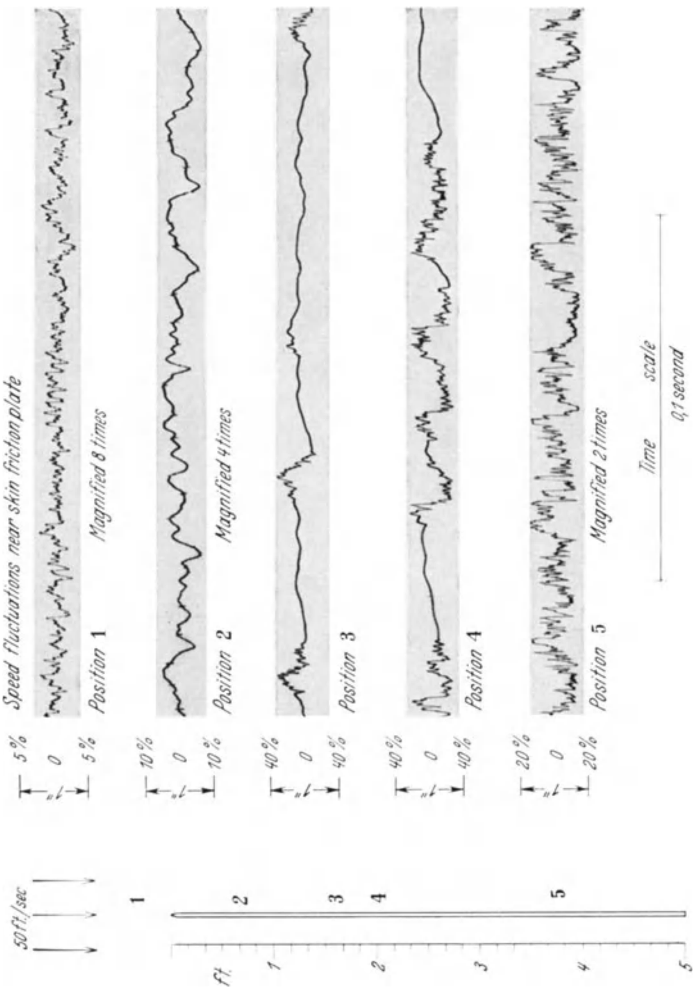

Fig. 12. Oscillographic records of fluctuations of speed at several points near skin-friction plate.

7. Distribution of Temperature. An extensive series of measurements on the temperature distribution near a heated plate in an air stream was made by Éliás. These results may be plotted in the same manner as the measurements of speed, using x and y Reynolds numbers as abscissae and ordinates, and contours of equal temperatures instead of

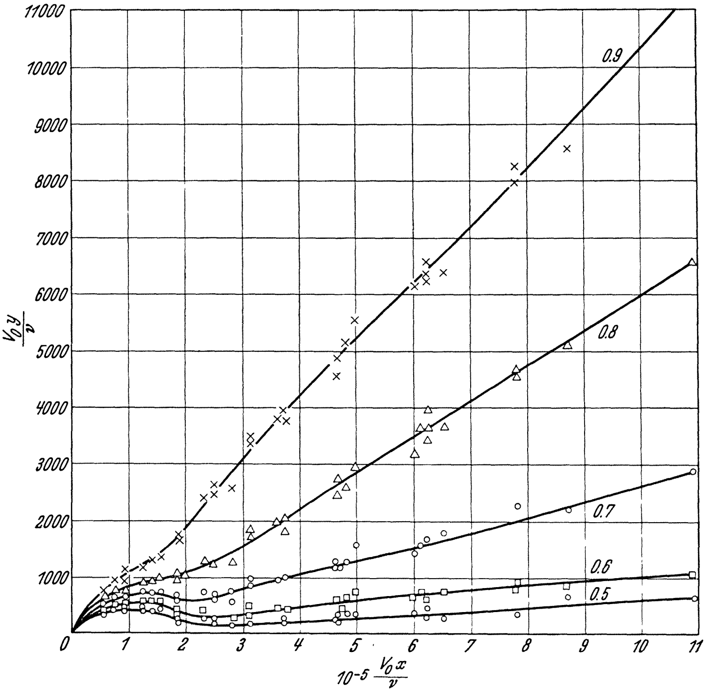

Fig. 13. Distribution of temperature near heated skin-friction plate according to Éliás' measurements. Numbers on contours are $\frac{t_w - t}{t_w - t_0}$, where t_0 is the temperature of the air stream at a distance, t_w is the temperature of the plate, and t is the temperature at points along the contour.

contours of equal speeds. To secure a closer correspondence, the quantity $\frac{t_w - t}{t_w - t_0}$ is used, where t_w is the temperature of the plate, t_0 the temperature of the air stream at a considerable distance from the plate, and t the temperature at points along the contour.

It was not practicable in the experiments of Éliás to heat the plate to a constant temperature, beginning at the sharp leading edge. As a consequence, the thermal boundary layer began at a point 10 cm. behind the leading edge of the plate, at which the dynamic boundary layer begins. The boundary conditions for the speed and temperature were accordingly not quite the same. At distances of 30 cm. and more from the leading edge, the effect of this difference in the boundary conditions appears to have disappeared. We shall therefore confine our attention to the measurements at distances of 30 cm. or more from the leading edge of the plate.

Figures 13 and 14 show the distribution of temperature and the corresponding distribution of speed which was also measured by Éliás. The general similarity of the two diagrams is remarkable. In the laminar region the deviation is in the direction required by the theory of II 7, for $c\mu/k < 1$, *i. e.* the thermal boundary layer is thicker than the dynamic

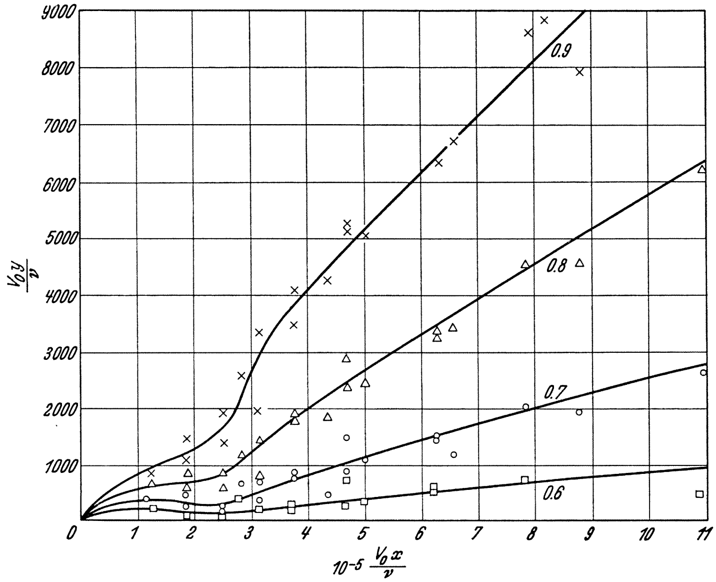

Fig. 14. Distribution of speed near heated skin-friction plate according to Éliás' measurements. See Fig. 6 for x and y. V_0 is speed of free stream. ν is kinematic viscosity of the fluid. Numbers on contours are values of $\frac{V}{V_0}$, where V is the local speed.

boundary layer. In the eddying region, the distributions are substantially identical.

The results of Éliás are applicable to flows for which the turbulence is the same as that present in the wind tunnel at Aachen. It is to be inferred that, corresponding to the speed distributions of Figs. 8 and 9, there are temperature distributions which resemble the speed distributions as Fig. 13 resembles Fig. 14. In general terms, all of the discussion of 2—5 may be applied to the heat transfer problem with temperature substituted for speed. To anticipate a generalization, measurements of heat transfer in air streams, for which no measurement of turbulence has been made, cannot be applied to predict the heat transfer in other air streams, without risk of serious error.

8. Heat Transfer from Skin Friction Plate. It should be clear from the previous discussion that the relations between total skin friction or total heat transfer, the properties of the fluid, and the speed, are likely to be very complex, and that the turbulence must be considered as a variable of primary importance. The skin friction problem is treated in another division of this work. Because of the small practical interest, the heat transfer from plates has not often been experimentally studied.

Éliás' results for the laminar region are in fair agreement with the Pohlhausen formula [II (7.13)], and that formula may be safely used to compute the local heat transfer coefficients in the laminar region. The loss by natural convection and radiation should be investigated, if the speed of the forced air stream is low.

For the eddying region, Latzko by the methods of III 3 obtained the following result for q_x, the local rate of transfer of heat per unit area

$$q_x = 0.0285\,(t_w - t_0)\,V_0\,\varrho\,c\left(\frac{\mu}{V_0\,\varrho\,x}\right)^{1/5} \tag{8.1}$$

which Éliás finds in fair agreement with his results if x is measured from the beginning of the heated section.

The power law formulation of the skin friction theory has been superceded by the logarithmic law

$$\frac{1}{c_f^{1/2}} = 1.7 + 4.26\,log_{10}\,(R_x\,c_f) \tag{8.2}$$

where c_f is the local skin friction coefficient and $R_x = V_0\,\varrho\,x/\mu$. For air flow, the local rate of transfer of heat may be estimated from V (1.7)

$$q_x = h_x\,(t_w - t_0) = \frac{1}{2}\,\varrho\,c\,V_0\,c_f\,(t_w - t_0) \tag{8.3}$$

A comparison of (8.1) and (8.3) is given in the following table:

R_x	c_f	$\frac{q_x}{\varrho\,c\,V_0\,(t_w - t_0)}$ by (8.1)	$\frac{q_x}{\varrho\,c\,V_0\,(t_w - t_0)}$ by (8.3)
200000	0.00480	0.00240	0.00248
500000	.00402	.00201	.00207
1000000	.00357	.00179	.00180
2000000	.00317	.00159	.00157
5000000	.00274	.00137	.00130
10000000	.00245	.00123	.00113
20000000	.00222	.00111	.00099
50000000	.00196	.00098	.00082
100000000	.00179	.00090	.00072
200000000	.00163	.00082	.00062
500000000	.00145	.00073	.00052
1000000000	.00135	.00068	.00045

The power law formulation is a good approximation for Reynolds numbers not exceeding 5,000,000. The experimental results of Éliás extend only to 1,000,000. Formula (8.3) is the more reliable for extrapolation.

To compute the heat transfer from a plate, the Reynolds number of transition and the approximate limits of the transition region must first be estimated. The local rates of transfer in the laminar and eddying regions are then to be computed, and a reasonable transition curve assumed in the transition region. By graphical integration of these local rates of transfer, the total heat transfer may be obtained.

9. Effect of Pressure Gradients. The theoretical treatment of the flow near a skin friction plate is based on the assumption that the pressure gradient along the plate is zero. Both experiment and theory show that for laminar flow the effect of even a very small pressure gradient is quite large, changing the shape of the speed distribution curve, the thickness of the boundary layer, and the local skin friction coefficient. This factor must be considered in the heat flow problem as well.

The existence of a small pressure gradient greatly modifies the Reynolds number of transition for a given turbulence. There is some evidence that a transition Reynolds number defined in terms of the "reduced" thickness (see 2) is less affected by pressure gradients.

There is also evidence that the distribution of speed in an eddying boundary layer depends to some extent on the pressure gradient.

These effects arise naturally in the solution of the boundary layer equations. They are mentioned here only because of the sensitiveness of the boundary layer flow to small pressure gradients.

CHAPTER VII

HEAT TRANSFER FROM A PIPE TO A FLUID STREAM WITHIN THE PIPE

1. Flow near the Entrance. The flow near the entrance of a pipe is analogous to the flow near the front of a skin friction plate. In fact, the plate may be considered as a limiting case of a pipe of infinite radius. Just as the front edge of the plate must be sharp and symmetrical to avoid disturbances which would be similar in effect to increased initial turbulence, the entrance of the tube must be rounded to avoid entrance eddies and the formation of a vena contracta. The fluid enters the pipe with approximately uniform velocity and a boundary layer develops along the wall of the pipe. When however the thickness of the boundary layer equals the radius of the pipe, the process cannot continue further. If the Reynolds number formed from the radius of the pipe and the speed at the center of the pipe does not exceed a critical value determined by the turbulence of the incoming fluid, the flow will remain laminar. If, however, the critical Reynolds number is exceeded, a transition to eddying flow will occur.

The pipe flow differs from the flow about a plate in that the average speed over a cross section must remain constant, the same quantity of fluid passing each cross section, and a pressure gradient is necessarily present. Nevertheless, the phenomena are very similar, and the critical Reynolds numbers are of the same order of magnitude, if that for the plate is expressed in terms of the thickness of the boundary layer rather than in terms of distance from the leading edge. It is possible to have in the entrance region of the pipe laminar flow near the mouth, transition further downstream, and eddying flow as the final type.

Studies of the entrance conditions for a flow which remains laminar throughout have been made by Nikuradse, and approximate theories have been given by Boussinesq[1] and by Schiller[2]. Neither theory is in exact accord with the results of experiments. The final distribution, $u = 2\,\bar{u}\,(1 - y^2/R^2)$, where $\bar{u}$ is the average speed, y the radial distance at which the speed is u and R the radius, is approached asymptotically. The value of x_1, the distance from the entrance for which the actual distribution differs from the parabolic distribution by one per cent, is given by the relation

$$\frac{x_1}{R} = 0.26\,\frac{\bar{u}\,R}{\nu} \tag{1.1}$$

If we consider a tube 1/4 inch in diameter through which air flows at an average speed of 20 feet per second, the entrance length is about 40 inches. The tubes of most aircraft radiators, for example, are not sufficiently long to get out of the entrance length flow.

The entrance length for eddying flow is materially shorter than that for laminar flow and is less dependent on Reynolds number. The estimates range from 20 to 200 radii.

In the entrance length, it is necessary to have a larger pressure gradient than that which is found some distance from the entrance, in order to provide for the increased kinetic energy of the final distribution as compared with that of the initial uniform distribution of speed. In addition, it is probable that the friction is greater. Observations of this increased pressure gradient in short tubes, for example in radiator tubes, have occasionally been interpreted as indicating the presence of eddying flow. Such an interpretation is not always correct.

2. Heat Transfer in the Entrance Length. No accurate measurements of heat transfer in the entrance length of a pipe are known. It is possible to make approximate computations following the methods used by Schiller and Latzko, but the results have not as yet been tested by experiment. The general nature of the results is obvious without computation. At the entrance, both the velocity and temperature gradients are very large, infinite in fact if the velocity and temperature at the entrance are uniform across the section. The gradients decrease rapidly. Hence the local rate of heat transfer per unit area is very high at the entrance but decreases rapidly. If a transition to eddying flow occurs in the entrance section, the gradients of speed and temperature at the wall increase, and hence the local rate of heat transfer increases. Beyond the transition region, the local rate again falls. When the boundary layers coalesce at the center of the pipe at the end of the entrance length,

[1] Comptes rendus, Vol. 113, pp. 9 and 49, 1891.

[2] Zeitschrift für angewandte Mathematik und Mechanik, Vol. 2, p. 96, 1922.

the temperature at the center increases and the average temperature of the fluid increases at much faster rate than in the entrance section. The temperature difference between wall and fluid decreases and hence the local rate of heat transfer decreases. The rate per unit temperature difference, however, approaches a constant value asymptotically. Unfortunately we cannot as yet make accurate computations for this most important practical case of flow in the entrance length, illustrated by oil coolers and water radiators.

3. Heat Transfer for a Short Section of an Infinitely Long Pipe. A problem which has received much attention is that of the transfer of heat from a short section of an infinitely long pipe which is maintained at a uniform temperature. The flow may be either of the laminar or eddying type. The transition type of flow cannot usually be maintained in a very long pipe, although there may be an intermittent alternation between the two types of flow.

The theoretical formulae have already been given in II 5 for laminar flow and in III 3 for eddying flow. In each case, the local rate of transfer is infinite at the beginning of the heated section and decreases to a limiting value as the distance from the beginning of the heated section increases. The formulae are as follows:

Laminar case: $$h'_x \frac{2R}{k} = f_1\left(\frac{k}{c\mu}\,\frac{\mu}{2R\varrho\bar{u}}\,\frac{x}{2R}\right) \tag{3.1}$$

Eddying case: $$h'_x \frac{2R}{k}\left(\frac{2R\varrho\bar{u}}{\mu}\right)^{-3/4} = f_2\left[\frac{k}{c\mu}\left(\frac{\mu}{2R\varrho\bar{u}}\right)^{1/4}\frac{x}{2R}\right] \tag{3.2}$$

where R is the radius of the pipe, x is the distance along the pipe from the beginning of the heated section, k, μ and ϱ are the thermal conductivity, viscosity, and density of the fluid, $\bar{u}$ is the average speed defined by $\frac{2}{R^2}\int_0^R ur\,dr$ and h'_x is the local heat transfer number based on the difference between the temperature t_w of the wall and the average temperature t_{av} of the fluid, defined by

$$t_w - t_{av} = \frac{2}{R^2}\int_0^R (t_w - t)\,r\,dr$$

The local rate of heat transfer, q_x, per unit area is given by the relation $q_x = h'_x\,(t_w - t_{av})$.

For large values of $x/2R$ the formulae approach the following values:

Laminar case: $$\frac{h'_x 2R}{k} = 5.16 \tag{3.3}$$

Eddying case: $$h'_x \frac{2R}{k}\left(\frac{2R\varrho\bar{u}}{\mu}\right)^{-3/4} = 0.03846 \tag{3.4}$$

It is not practicable to measure these local coefficients experimentally. The experimental data usually consist of data on the average rate of transfer of heat per unit area for the whole heated section. It is better then to return to II (5.7) and III (3.14), which may be written as follows:

$$\text{Laminar case:} \quad \frac{2\, q_{av}\, x}{c\, \varrho\, \bar{u}\, R\, (t_w - t_0)} = \psi\left(\frac{k}{c\,\mu}\, \frac{\mu}{2\, R\, \varrho\, \bar{u}}\, \frac{x}{2\, R}\right) \tag{3.5}$$

$$\text{Eddying case:} \quad \frac{2\, q_{av}\, x}{c\, \varrho\, \bar{u}\, R\, (t_w - t_0)} = \varphi\left[\frac{k}{c\,\mu}\left(\frac{\mu}{2\, R\, \varrho\, \bar{u}}\right)^{1/4} \frac{x}{2\, R}\right] \tag{3.6}$$

The left hand members of these equations have a simple physical interpretation. For, if we call t_2 the average temperature with which

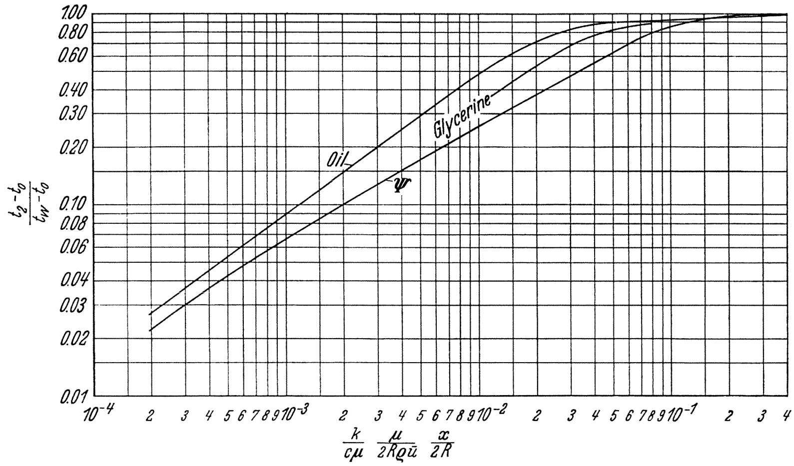

Fig. 15. Heat transfer from a short heated length of an infinitely long pipe to a fluid in laminar flow. t_2 is the average "mixing-cup" temperature of the fluid as it leaves the heated section, the wall of which is at temperature t_w. t_0 is the initial temperature of the fluid. k, c, μ, and ϱ are the thermal conductivity, specific heat, viscosity, and density of the fluid. R is the radius of the pipe, x the length of the heated section, $\bar{u}$ the average speed of the fluid. Curve ψ is the theoretical curve, natural convection negligible. Curves marked oil and and glycerine are experimental curves for those fluids.

the fluid leaves the heated section as determined by the mixing cup method; *i. e.*

$$t_2 - t_0 = \frac{2}{c\, \varrho\, \bar{u}\, R^2} \int_0^R c\, \varrho\, u\, (t - t_0)\, r\, d r$$

the total heat energy transferred to the fluid is

$$c\, \varrho\, \bar{u}\, (t_2 - t_0)\, \pi\, R^2$$

and hence

$$q_{av} = \frac{c\, \varrho\, \bar{u}\, (t_2 - t_0)\, \pi\, R^2}{2\, \pi\, R\, x} = \frac{c\, \varrho\, \bar{u}\, (t_2 - t_0)\, R}{2 x} \tag{3.7}$$

Hence the left hand members of (3.5) and (3.6) reduce to the simple temperature ratio $\frac{t_2 - t_0}{t_w - t_0}$, which represents the actual temperature rise divided by the maximum possible rise. The values of the functions ψ and φ are shown in Figs. 15 and 16.

The theoretical equations were derived for a constant temperature along the heated section, such as is approximated when the pipe wall is steam heated. There are no reliable data on the transfer of heat to air in laminar flow in a pipe. There is plotted in Fig. 15 the empirical curve for transfer of heat to oil flowing in a heated pipe given by Drew, Hogan, and Mc.Adams[1]. It is seen that the actual transfer is greater than that given by (3.5). The nature of the departures suggests that the discrepancy may be due to the failure of the theory to consider the effects of convection currents in promoting a more thorough mixing

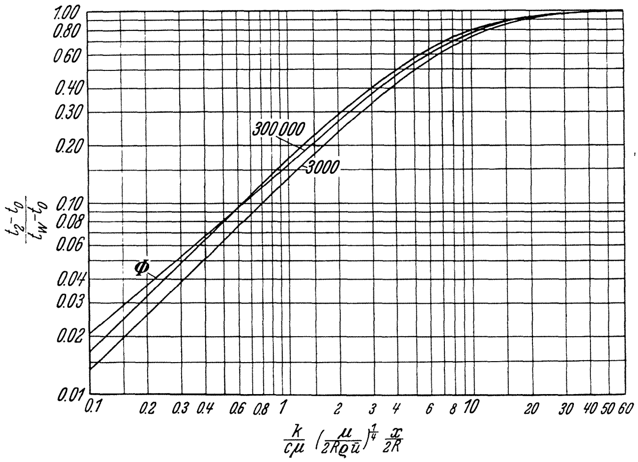

Fig. 16. Heat transfer from a short heated length of an infinitely long pipe to a fluid in eddying flow. For symbols, see Fig. 15. Curve φ is the theoretical curve. Curves marked 3,000 and 300,000 are from McAdams empirical equation representing experimental results for those values of $\frac{2R\bar{u}\varrho}{\mu}$.

of the fluid. Since the Grashof number is somewhat larger for air than for the oil on which the empirical curve is based, it is probable that a curve for air would show a somewhat greater rate of transfer. An upper limit is set by the fact that $\frac{t_2-t_0}{t_w-t_0}$ cannot exceed unity.

Some additional evidence that the discrepancy between theory and experiment is due to the effects of natural convection is provided by the experiments of Drew on glycerine[2] in which the Grashof number was about one one-hundredth that in the experiments on oil shown in Fig. 15. The results for glycerine lie closer to the theoretical curve.

For eddying flow McAdams concludes that the best empirical equation for representing the experimental results is as follows:

$$h\,\frac{2R}{k} = 0.0225\left(\frac{2R\varrho\bar{u}}{\mu}\right)^{0.8}\left(\frac{c\mu}{k}\right)^{0.4} \tag{3.8}$$

[1] Trans. Am. Inst. Chem. Eng., Vol. 26, p. 81, 1931.
[2] Trans. Am. Inst. Chem. Eng., Vol. 27, p. 171, 1931.

The heat transfer number h in this equation is based on an average temperature difference $\triangle t_m$ which is the logarithmic mean of the "mixing cup" temperature of the incoming and outgoing fluid streams. By simple transformations, (3.8) may be written in a form parallel to (3.6) as

$$\frac{2\,q_{av}\,x}{c\,\varrho\,\bar{u}\,R\,(t_w - t_0)} = \frac{t_2 - t_0}{t_w - t_0} = 0.09\left(\frac{2\,R\,\varrho\,\bar{u}}{\mu}\right)^{-0.2}\left(\frac{c\,\mu}{k}\right)^{-0.6}\frac{x}{2\,R}\,\frac{\triangle t_m}{t_w - t_0} \qquad (3.9)$$

If we write for $\triangle t_m$ its equivalent value

$$\frac{(t_2 - t_0)}{\log_e \dfrac{t_w - t_0}{t_w - t_2}}$$

and set $0.09\left(\dfrac{2\,R\,\varrho\,\bar{u}}{\mu}\right)^{-0.2}\left(\dfrac{c\,\mu}{k}\right)^{-0.6}\dfrac{x}{2R} = A$, it is easily seen that

$$\frac{t_2 - t_0}{t_w - t_0} = \frac{e^A - 1}{e^A} \qquad (3.10)$$

Since the various quantities do not enter with the same exponents as in (3.6), a general comparison is difficult. If we assume $c\mu/k$ equal to 1, we can plot in Fig. 16 the values from (3.10) for the extreme Reynolds numbers covered by the experiments (3,000 and 300,000). This has been done, and we see that (3.6) and (3.10) agree within about 15 per cent,

when $$\frac{k}{c\,\mu}\left(\frac{\mu}{2\,R\,\varrho\,\bar{u}}\right)^{1/4}\frac{x}{2\,R}$$

is greater than 1.

4. Remarks on Comparisons with Experimental Data. In the general case we may have to consider the heat transfer in a pipe in which the beginning of the heated section falls at some point within the entrance length. From general considerations we may infer that the local coefficients will be intermediate between those appropriate to the case where the heated section begins at the entrance of the pipe and those appropriate to a short section of an infinitely long pipe. These local coefficients cannot, however, be measured with precision and only overall average values are determined. These average coefficients are functions of the turbulence of the incoming fluid, of the ratio of the length of the heated section to the diameter of the tube, and of the ratio of the length of the unheated entrance to the diameter as well as of the Reynolds number and Prandtl number.

The experimental data available are of a most unsystematic nature as regards these first three important variables and most of the published summaries ignore them. Sufficient data of a systematic nature are not available to determine the magnitude of their influence.

The expression of the data in terms of a heat-transfer number should not mislead those unfamiliar with the subject to infer that the heat-transfer for cases in which the wall temperature varies can be computed from experimental data or theoretical formulae applicable to a constant wall temperature. The local heat-transfer number depends not only on the local wall temperature but also on the conditions elsewhere, especially upstream.

It should be noted that even in the infinitely long pipe, the local rate of transfer of heat varies considerably from upstream to downstream end of the heated length. When the upstream end of the heated section is not sharply defined, disturbing end effects are to be expected. It is not legitimate to determine the average rate of transfer from a limited portion of the heated length.

One frequently finds in the literature the logarithmic mean temperature difference. This type of mean value is correct only if the local rate of transfer is constant. It therefore has no legitimate claim to exactness in the case of heat transfer in a pipe, since the local rate is not constant but varies considerably.

These remarks give some idea of the clear distinctions that must be made and the care that must be taken in the comparison of experimental data from various sources with each other or with theoretical formulae. There is need of a carefully planned experimental investigation in which the influence of the many variables is recognized, and their effects are experimentally determined.

CHAPTER VIII

HEAT TRANSFER FROM CYLINDERS IMMERSED IN A FLUID STREAM

1. Dynamic Boundary Layer. In order to complete the picture of that body of theoretical knowledge not susceptible of mathematical treatment, but which is of importance in heat transfer problems, the final chapter of this Division will be devoted to the heat transfer from circular cylinders immersed in a fluid stream with axes at right angles to the flow. The purpose is primarily to illustrate the influence of pressure gradients and the phenomenon of separation rather than to cover exhaustively the experimental data on cylinders.

The nature of the flow about a cylinder depends on the value of the Reynolds number $V_0 D/\nu$, V_0 being the speed of the fluid stream at a distance from the cylinder, D the diameter and ν the kinematic viscosity. When the Reynolds number is very small, say of the order of 1, the fluid closes in completely behind the cylinder and the flow is everywhere of a laminar character. At a Reynolds number of about 3, a stationary eddy pair develops behind the cylinder. As the Reynolds number is increased, the eddies move away from the cylinder and become unstable. At a Reynolds number of 100, eddies form periodically in the wake, arranging themselves in the well known Kármán vortex street. Over the forward part of the cylinder the flow is still of a laminar character. The fluid stream however does not close in behind the cylinder but separates from the surface. There is evidence that the flow remains of a laminar character for some distance beyond the separation point before eddying motion develops. When the Reynolds number is as large as 30,000, the flow remains laminar up to the point of separation but becomes eddying almost immediately afterward. When the Reynolds number reaches values of the order of 200,000 or more, the flow becomes eddying before separation and the process of separation is delayed, the drag coefficient falling rapidly. The exact values at which the transition to eddying flow occurs in the wake after separation or in the flow ahead of the point of separation depend on the turbulence of the approaching stream.

We shall for the present confine our attention to the flow at values of the Reynolds number between 30,000 and 200,000, for which the drag coefficient is constant. There is a well defined thin boundary layer in which the flow is laminar up to the point of separation, and the flow in the separated boundary layer becomes eddying immediately after separation. The flow outside the boundary layer but not in the wake may be regarded as without viscous friction. The speed just outside the boundary layer increases from zero at the upstream stagnation point to a maximum of about 1.55 times the speed of the approaching stream. The boundary layer is accordingly subjected to a pressure gradient in the direction of flow, associated with the increasing speed. The presence of this pressure gradient has a profound influence on the development of the boundary layer, which is illustrated in the following table. This table gives certain data on the boundary layer flow about a cylinder (taken from Technical Report No. 497 of the National Advisory Committee for Aeronautics) and the thickness which would have been found if there were no pressure gradient, *i. e.* if the speed remained equal to the speed of the approaching stream. R denotes the Reynolds number, D the diameter, x the distance along the surface from the upstream stagnation point, V the local speed in the potential flow just outside the boundary layer, δ the "thickness" as defined by a particular power-series approximation to the speed-distribution curve within the boundary layer, and θ the azimuthal angle.

θ	$\frac{x}{D}$	$\frac{V}{V_0}$	$R\frac{\delta^2}{D^2}$ (cylinder)	$R\frac{\delta^2}{D^2}$ pressure drop absent, $V/V_0=1$
0	0	0	1.125	0
10^0	0.0872	0.307	1.140	2.745
20^0	.1744	.633	1.233	5.490
30^0	.2616	.928	1.377	8.236
40^0	.3488	1.175	1.500	10.981
50^0	.4360	1.384	1.731	13.726
60^0	.5232	1.522	2.340	16.471
65^0	.5668	1.549	2.880	17.844
69^0	.6015	1.554	3.600	18.942

It is obvious that the pressure drop reduces the thickness of the boundary layer very materially, (except near the stagnation point) and hence as shown in many experiments[1], increases the skin friction by a factor of two or more as compared with that on a thin flat plate set parallel to the flow.

2. Thermal Boundary Layer. From the equations for the flow in a laminar boundary layer [see II (6.1), (6.4) and (6.7)], it is seen that there is no term in the equation for the flow of heat corresponding to the pressure drop term in the dynamic equations. Thus the pressure drop affects the growth of the thermal boundary layer only indirectly through changes of u and v. While the mathematical solution of this problem has not been worked out in detail, it is perhaps obvious that the thermal boundary layer will develop substantially as it would along a plane surface, *i. e.*, thickening at a fairly rapid rate with consequent decrease in temperature gradient at the surface and in the local rate of transfer of heat, with increasing distance from the upstream stagnation point. By contrast, the dynamic layer remains of nearly constant thickness for some distance and because of the increasing value of V, the skin friction increases with increasing distance from the nose.

This general picture applies to a cylinder whose external surface is maintained at a constant temperature and not to the case where a small part of the cylindrical wall is heated. Many experimental studies have been made on the loss of heat from a heated strip on a cylinder as a function of the azimuthal angle. While these studies are of interest in exploring the nature of the air flow and because of their bearing on certain theoretical investigations, they give no information on the local rate of transfer of heat from a cylinder whose entire outer surface is maintained at a constant temperature.

3. The Phenomenon of Separation. A well-defined boundary layer is observed adjacent to the surface of the cylinder only over the forward part of the cylinder to an angular distance of about 70^0 from the upstream stagnation point. In this vicinity the flow separates from the surface and immediately behind the point of separation it may be demonstrated by smoke that the air near the surface is moving in a direction opposite to that of the main stream. Separation occurs only when the pressure increases in the downstream direction. The process is usually described in the following way. The particles near the wall are dragged along by the friction of the neighboring faster moving particles but are retarded by the pressure. As the layer thickens, the retarding effect predominates and this finally causes a reversal of the flow near the surface. The reversal of flow, on account of the

[1] For example Br. A.R.C. R. and M. 1369.

consequent accumulation of fluid, separates the flow from the surface[1].

When the Reynolds number of the cylinder is increased above about 200,000, a transition from laminar to eddying flow occurs within the boundary layer before separation and the point of separation moves to a larger azimuthal angle. In the eddying flow there is a more thorough mixing of the air particles, and the driving action of the outer layers on the fluid near the surface is greater. The fluid near the surface can accordingly proceed farther against the adverse pressure gradient.

An exact description of the flow near the surface in the separated region is not available. It is known that the average pressure is substantially uniform and that the velocity fluctuates both in magnitude and direction within comparatively wide limits. The average speed is rather low.

4. Local Rate of Heat Transfer as a Function of Azimuthal Angle. Drew and Ryan[2] describe some measurements made under their direction by Paltz and Starr on the average rates of heat transfer from longitudinal strips of a cylinder, the wall of which was maintained at a uniform temperature. The Reynolds number was 39,600, *i. e.*, within the range in which the boundary layer flow is laminar up to the point of separation and becomes eddying immediately after separation. The cylinder was 11.3 diameters long but spanned the air stream so that the flow was approximately two-dimensional. Each strip was 20° wide. The approximate ratios of the local rates to the average rate are given in the table below:

Azimuthal angle at center of strip	$\dfrac{\text{Local rate per unit area}}{\text{Average rate per unit area}}$
10°	1.39
30°	1.29
50°	1.12
70°	.83
90°	.54
110°	.64
130°	.88
150°	1.02
170°	1.29

By contrast with the behavior of the local skin friction, the local rate of transfer of heat decreases up to and beyond the separation point. In the region of separated flow the local rate of transfer of heat increases and is practically as great at the rear as at the front of the cylinder. The significance of this increase is not definitely known.

These results have been indirectly checked by Lohrisch by diffusion experiments, utilizing the similarity of the differential equations governing

[1] See also Division G **15, 16.**

[2] Trans. Am. Inst. Chem. Eng., Vol. 26, p. 118, 1931.

diffusion and heat transfer. It is obvious that there is no immediate correlation between heat transfer and skin friction because of the effect of the pressure gradient in the dynamic problem. This effect is not present in the diffusion or heat transfer problem.

5. Average Rate of Heat Transfer. Many summaries of the experimental data on the average rate of heat transfer from cylinders in approximately two-dimensional flow (long cylinders extending entirely across the air stream) have been published. Empirical equations, usually power law approximations, are found to be useful representations of the data over limited ranges of Reynolds numbers. McAdams, for example, suggests

$$\frac{q_{av} D}{k\,(t_w - t_0)} = 0.45 + 0.33 \left(\frac{V_0 D}{\nu}\right)^{0.56} \tag{5.1}$$

This equation is satisfactory for engineering computations when the Reynolds number is between 1 and say 100,000. Extrapolation is probably safe to the point where the boundary layer becomes eddying before separation.

It should be noted that results on cylinders in two-dimensional flow may not be applicable to short cylinders, where the fluid flows around the ends. Experimental data are not available to indicate the magnitude of the end effects.

6. Concluding Remarks. It is perhaps clear that while it is not yet possible to compute either the air flow or the heat transfer for a heated body such as a cylinder, we can recognize certain basic phenomena which are encountered in the flow about bodies of any shape for certain values of the Reynolds number. Thus the establishment of thin boundary layers may be expected over the upstream portions. The flow below a certain critical Reynolds number is laminar. A transition to eddying flow occurs when the Reynolds number reaches a critical value which is dependent on the initial turbulence. It is generally believed but not completely proved that if the thickness of the boundary layer is used as the characteristic length in the Reynolds number, the critical Reynolds number will depend only on the initial turbulence, not on the pressure gradient, shape of the body, etc. With either laminar or eddying flow, the boundary layer will under certain circumstances separate from the surface. It has not yet been possible to devise a universally applicable criterion to indicate when separation will occur.

The use of the concepts of boundary layer, laminar and eddying flow, transition, initial turbulence, separation, has been illustrated by the experimental data for skin friction plates, flow in pipes, and flow about cylinders. The same concepts may be applied to the description of the air flow and the study of the heat transfer in other problems.

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