Frontispiece

Observation on *Polaris* for Azimuth

PRACTICAL ASTRONOMY

A TEXTBOOK FOR ENGINEERING SCHOOLS

AND

A MANUAL OF FIELD METHODS

BY

GEORGE L. HOSMER

Associate Professor of Geodesy, Massachusetts Institute of Technology

THIRD EDITION

NEW YORK

JOHN WILEY & SONS, INC.

LONDON: CHAPMAN & HALL, LIMITED

1925

PREFACE

THE purpose of this volume is to furnish a text in Practical Astronomy especially adapted to the needs of civil-engineering students who can devote but little time to the subject, and who are not likely to take up advanced study of Astronomy. The text deals chiefly with the class of observations which can be made with surveying instruments, the methods applicable to astronomical and geodetic instruments being treated but briefly. It has been the author's intention to produce a book which is intermediate between the text-book written for the student of Astronomy or Geodesy and the short chapter on the subject generally given in text-books on Surveying. The subject has therefore been treated from the standpoint of the engineer, who is interested chiefly in obtaining results, and those refinements have been omitted which are beyond the requirements of the work which can be performed with the engineer's transit. This has led to the introduction of some rather crude mathematical processes, but it is hoped that these are presented in such a way as to aid the student in gaining a clearer conception of the problem without conveying wrong notions as to when such short-cut methods can properly be applied. The elementary principles have been treated rather elaborately but with a view to making these principles clear rather than to the introduction of refinements. Much space has been devoted to the Measurement of Time because this subject seems to cause the student more difficulty than any other branch of Practical Astronomy. The attempt has been made to arrange the text so that it will be a convenient reference book for the engineer who is doing field work.

For convenience in arranging a shorter course those subjects which are most elementary are printed in large type. The matter printed in smaller type may be included in a longer course and will be found convenient for reference in field practice, particularly that contained in Chapters X to XIII.

The author desires to acknowledge his indebtedness to those who have assisted in the preparation of this book, especially to Professor A. G. Robbins and Mr. J. W. Howard of the Massachusetts Institute of Technology and to Mr. F. C. Starr of the George Washington University for valuable suggestions and criticisms of the manuscript.

G. L. H.

BOSTON, *June*, 1910.

PREFACE TO THE THIRD EDITION

THE adoption of Civil Time in the American Ephemeris and Nautical Almanac in place of Astronomical Time (in effect in 1925) necessitated a complete revision of this book. Advantage has been taken of this opportunity to introduce several improvements, among which may be mentioned: the change of the notation to agree with that now in use in the principal textbooks and government publications, a revision of the chapter on the different kinds of time, simpler proofs of the refraction and parallax formulæ, the extension of the article on interpolation to include two and three variables, the discussion of errors by means of differentiation of the trigonometric formulæ, the introduction of valuable material from Serial 166, U. S. Coast and Geodetic Survey, a table of convergence of the meridians, and several new illustrations. In the chapter on Nautical Astronomy, which has been re-written, the method of Marcq Saint-Hilaire and the new tables (H. O. 201 and 203) for laying down Sumner lines are briefly explained. An appendix on Spherical Trigonometry is added for convenience of reference. The size of the book has been reduced to make it convenient for field use. This has been done without reducing the size of the type.

In this book an attempt has been made to emphasize the great importance to the engineer of using the true meridian and true azimuth as the basis for all kinds of surveys; the chapter on Observations for Azimuth is therefore the most important one from the engineering standpoint. In this new edition the chapter has been enlarged by the addition of tables, illustrative examples and methods of observing.

Thanks are due to Messrs. C. L. Berger & Sons for the use of electrotypes, and to Professor Owen B. French of George Washington University (formerly of the U. S. Coast and Geodetic Survey) for valuable suggestions and criticisms. The author desires to thank those who have sent notices of errors discovered in the book and asks their continued coöperation.

G. L. H.

CAMBRIDGE, MASS., *June, 1924.*

CONTENTS

CHAPTER I

The Celestial Sphere — Real and Apparent Motions

CHAPTER II

Definitions — Points and Circles of Reference

CHAPTER III

Systems of Coördinates on the Sphere

CHAPTER IV

Relation between Coördinates

CHAPTER V

Measurement of Time

CHAPTER VI

The American Ephemeris and Nautical Almanac — Star Catalogues — Interpolation

CHAPTER VII

The Earth's Figure — Corrections to Observed Altitudes

CHAPTER VIII

Description of Instruments — Observing

CHAPTER IX

The Constellations

CHAPTER X

Observations for Latitude

CHAPTER XI

Observations for Determining the Time

CHAPTER XII

Observations for Longitude

CHAPTER XIII

Observations for Azimuth

CHAPTER XIV

Nautical Astronomy

TABLES

PRACTICAL ASTRONOMY

CHAPTER I

THE CELESTIAL SPHERE — REAL AND APPARENT MOTIONS

1. Practical Astronomy.

Practical Astronomy treats of the theory and use of astronomical instruments and the methods of computing the results obtained by observation. The part of the subject which is of especial importance to the surveyor is that which deals with the methods of locating points on the earth's surface and of orienting the lines of a survey, and includes the determination of (1) latitude, (2) time, (3) longitude, and (4) azimuth. In solving these problems the observer makes measurements of the directions of the sun, moon, stars, and other heavenly bodies; he is not concerned with the distances of these objects, with their actual motions in space, nor with their physical characteristics, but simply regards them as a number of visible objects of known positions from which he can make his measurements.

2. The Celestial Sphere.

Since it is only the directions of these objects that are required in practical astronomy, it is found convenient to regard all heavenly bodies as being situated on the surface of a sphere whose radius is infinite and whose centre is at the eye of the observer. The apparent position of any object on the sphere is found by imagining a line drawn from the eye to the object, and prolonging it until it pierces the sphere. For example, the apparent position of S_1 on the sphere (Fig. 1) is at S_1', which is supposed to be at an infinite distance from C; the position of S_2 is S_2', etc. By means of this imaginary sphere all problems involving the angular distances between points, and angles between planes through the centre of the sphere, may readily be solved by applying the formulæ of spherical trigonometry. This device is not only convenient for mathematical purposes, but it is perfectly consistent with what we see, because all celestial objects are so far away that they appear to the eye to be at the same distance, and consequently on the surface of a great sphere.

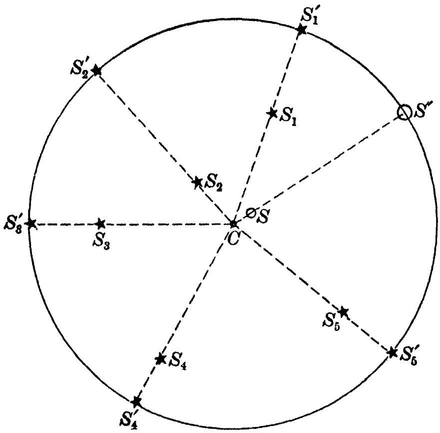


FIG. 1. APPARENT POSITIONS ON THE SPHERE

From the definition it will be apparent that each observer sees a different celestial sphere, but this causes no actual inconvenience, for distances between points on the earth's surface are so short when compared with astronomical distances that they are practically zero except for the nearer bodies in the solar system. This may be better understood from the statement that if the entire solar system be represented as occupying a field one mile in diameter the nearest star would be about 5000 miles away on the same scale; furthermore the earth's diameter is but a minute fraction of the distance across the solar system, the ratio being about 8000 miles to 5,600,000,000 miles,* or one 700,000th part of this distance.

* The diameter of Neptune's orbit.

Since the radius of the celestial sphere is infinite, all of the lines in a system of parallels will pierce the sphere in the same point, and parallel planes at any finite distance apart will cut the sphere in the same great circle. This must be kept constantly in mind when representing the sphere by means of a sketch, in which minute errors will necessarily appear to be very large. The student should become accustomed to thinking of the appearance of the sphere both from the inside and from an outside point of view. It is usually easier to understand the spherical problems by studying a small globe, but when celestial objects are actually observed they are necessarily seen from a point inside the sphere.

3. Apparent Motion of the Celestial Sphere.

If a person watches the stars for several hours he will see that they appear to rise in the east and to set in the west, and that their paths are arcs of circles. By facing to the north (in the northern hemisphere) it will be found that the circles are smaller and all appear to be concentric about a certain point in the sky called the **pole**; if a star were exactly at this point it would have no apparent motion. In other words, the whole celestial sphere appears to be rotating about an axis. This apparent rotation is found to be due simply to the actual rotation of the earth about its axis (from west to east) in the opposite direction to that in which the stars appear to move.*

4. Motions of the Planets.

If an observer were to view the solar system from a point far outside, looking from the north toward the south, he would see that all of the planets (including the earth) revolve about the sun in elliptical orbits which are nearly circular, the direction of the motion being *counter-clockwise* or left-handed rotation.

* This apparent rotation may be easily demonstrated by taking a photograph of the stars near the pole, exposing the plate for several hours. The result is a series of concentric arcs all subtending the same angle. If the camera is pointed southward and high enough to photograph stars near the equator the star trails appear as straight lines.

He would also see that the earth rotates on its axis, once per day, in a counter-clockwise direction. The moon revolves around the earth in an orbit which is not so nearly circular, but the motion is in the same (left-handed) direction. The

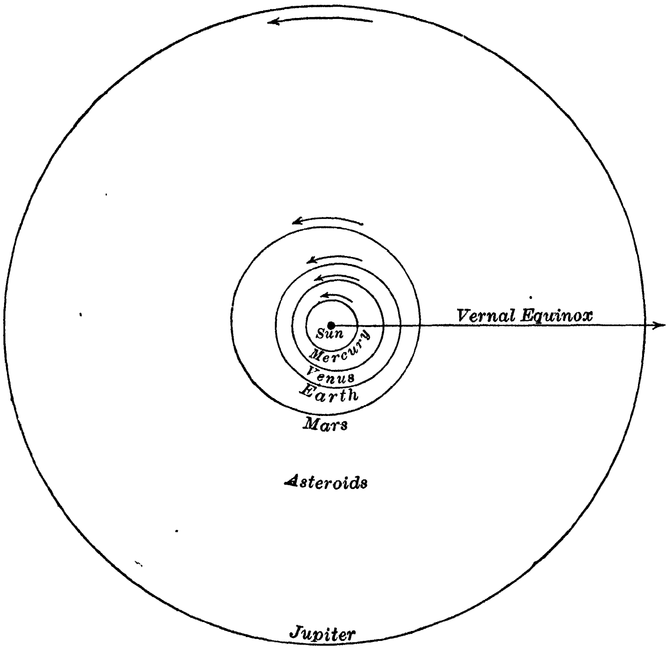


FIG. 2. DIAGRAM OF THE SOLAR SYSTEM WITHIN THE ORBIT OF SATURN

apparent motions resulting from these actual motions are as follows: The whole celestial sphere, carrying with it all the stars, sun, moon, and planets, appears to rotate about the earth's axis once per day in a *clockwise* (right-handed) direction. The stars change their positions so slowly that they appear to be fixed in position on the sphere, whereas all objects within the solar system rapidly change their apparent positions among the stars. For this reason the stars are called **fixed stars** to distinguish them from the planets; the latter, while closely resembling the stars in appearance, are really of an entirely different character. The sun appears to move slowly eastward among the stars at the rate of about 1° per day, and to make one revolution around the earth

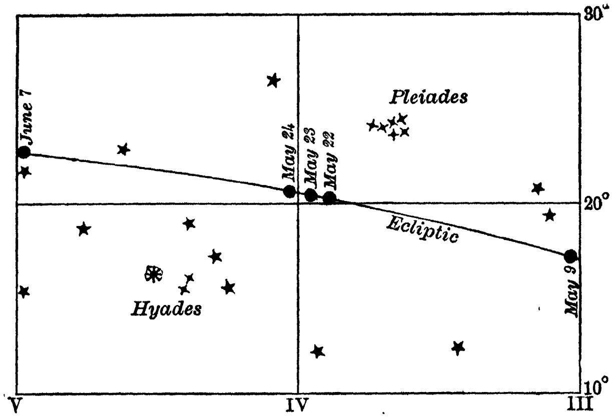


FIG. 3a. SUN'S APPARENT POSITION AT GREENWICH NOON ON MAY 22, 23, AND 24, 1910

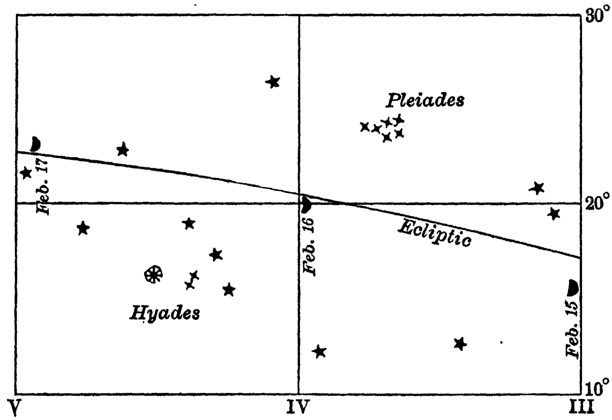


FIG. 3b. MOON'S APPARENT POSITION AT 14^h ON FEB. 15, 16, AND 17, 1910

in just one year. The moon also travels eastward among the stars, but at a much faster rate; it moves an amount equal to its own diameter in about an hour, and completes one revolution in a lunar month. Figs. 3a and 3b show the daily motions of the sun and moon respectively, as indicated by their plotted positions when passing through the constellation *Taurus*. It should be observed that the motion of the moon eastward among the stars is an actual motion, not merely an apparent one like that of the sun. The planets all move eastward among the stars, but since we ourselves are on a moving object the motion we see is a combination of the real motions of the planets around

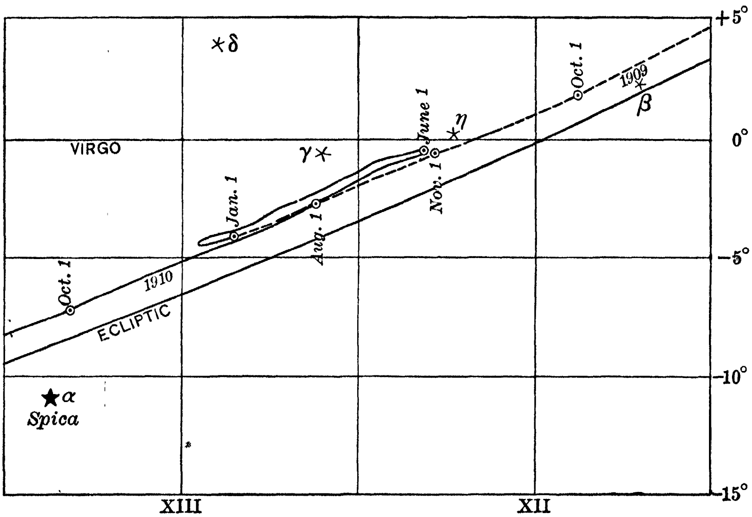


FIG. 4. APPARENT PATH OF JUPITER FROM OCT., 1909 TO OCT., 1910.

the sun and an apparent motion caused by the earth's revolution around the sun; the planets consequently appear at certain times to move westward (i.e., backward), or to **retrograde.** Fig. 4 shows the loop in the apparent path of the planet Jupiter caused by the earth's motion around the sun. It will be seen that the apparent motion of the planet was *direct* except from January to June, 1910, when it had a *retrograde* motion.

5. Meaning of Terms East and West.

In astronomy the terms " east " and " west " cannot be taken to mean the same as they do when dealing with directions in one plane. In plane surveying "east" and "west" may be considered to mean the directions perpendicular to the meridian plane. If a person at Greenwich (England) and another person at the 180° meridian should both point due east, they would actually be pointing to opposite points of the sky. In Fig. 5 all four of the arrows are pointing east at the places shown. It will be seen from this figure that the terms "east" and "west" must therefore be taken to mean *directions of rotation.*

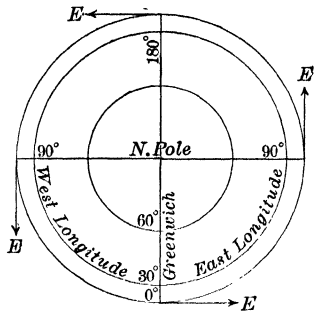


FIG. 5. ARROWS ALL POINT EASTWARD

6. The Earth's Orbital Motion. — The Seasons.

The earth moves eastward around the sun once a year in an orbit which lies (very nearly) in one plane and whose form is that

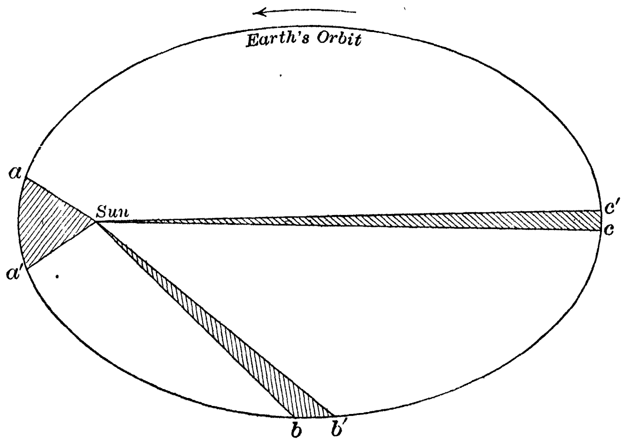


FIG. 6. THE EARTH'S ORBITAL MOTION

of an ellipse, the sun being at one of the foci. Since the earth is maintained in its position by the force of gravitation, it moves, as a consequence, at such a speed in each part of its path that the line joining the earth and sun moves over equal areas in equal times. In Fig. 6 all of the shaded areas are equal and the arcs *aa′*, *bb′*, *cc′* represent the distances passed over in the same number of days.*

The axis of rotation of the earth is inclined to the plane of the orbit at an angle of about 66°½, that is, the plane of the earth's equator is inclined at an angle of about 23°½ to the plane of the orbit. This latter angle is known as the **obliquity of the ecliptic.** (See Chapter II.) The direction of the earth's axis of rotation is nearly constant and it therefore points nearly to the same place in the sky year after year.

The changes in the seasons are a direct result of the inclination of the axis and of the fact that the axis remains nearly parallel

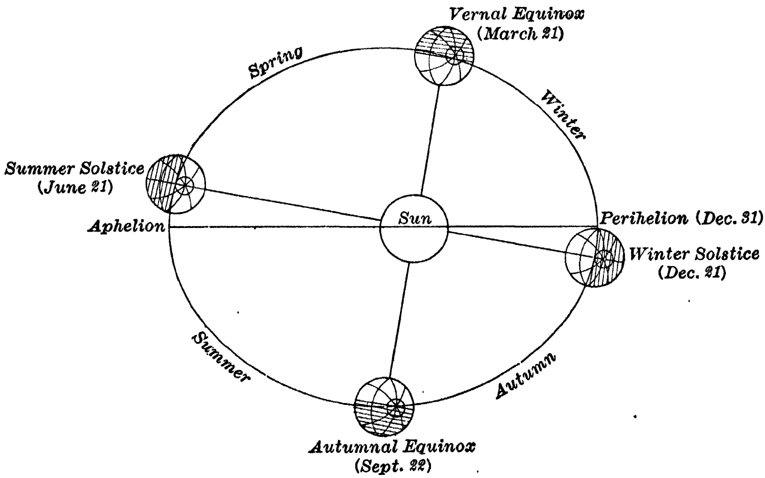


FIG. 7. THE SEASONS

to itself. When the earth is in that part of the orbit where the northern end of the axis is pointed away from the sun (Fig. 7) it is winter in the northern hemisphere. The sun appears to be

* The eccentricity of the ellipse shown in Fig. 6 is exaggerated for the sake of clearness; the earth's orbit is in reality much more nearly circular, the variation in the earth's distance from the sun being only about three per cent.

farthest south about Dec. 21, and at this time the days are shortest and the nights are longest. When the earth is in this position, a plane through the axis and perpendicular to the plane of the orbit will pass through the sun. About ten days later the earth passes the end of the major axis of the ellipse and is at its point of nearest approach to the sun, or **perihelion.** Although the earth is really nearer to the sun in winter than in summer, this has but a small effect upon the seasons; the chief reasons why it is colder in winter are that the day is shorter and the rays of sunlight strike the surface of the ground more obliquely. The sun appears to be farthest north about June 22, at which time summer begins in the northern hemisphere and the days are longest and the nights shortest. When the earth passes the other end of the major axis of the ellipse it is farthest from the sun, or at **aphelion.** On March 21 the sun is in the plane of the earth's equator and day and night are of equal length at all places on the earth (Fig. 7). On Sept. 22 the sun is again in the plane of the equator and day and night are everywhere equal. These two times are called the **equinoxes** (vernal and autumnal), and the points in the sky where the sun's centre appears to be at these two dates are called the **equinoctial points,** or more commonly the **equinoxes.**

7. The Sun's Apparent Position at Different Seasons. The apparent positions of the sun on the celestial sphere corresponding to these different positions of the earth are shown in Fig. 8. As a result of the sun's apparent eastward motion from day to day along a path which is inclined to the equator, the angular distance of the sun from the equator is continually changing. Half of the year it is north of the equator and half of the year it is south. On June 22 the sun is in its most northerly position and is visible more than half the day to a person in the northern hemisphere (*J*, Fig. 8). On Dec. 21 it is farthest south of the equator and is visible less than half the day (*D*, Fig. 8). In between these two extremes it moves back and forth across the equator, passing it about March 21 and Sept. 22 each year.

The apparent motion of the sun is therefore a *helical* motion about the axis, that is, the sun, instead of following the path which would be followed by a fixed star, gradually increases or decreases its angular distance from the pole at the same time that it revolves once a day around the earth. The sun's motion eastward on the celestial sphere, due to the earth's orbital motion,

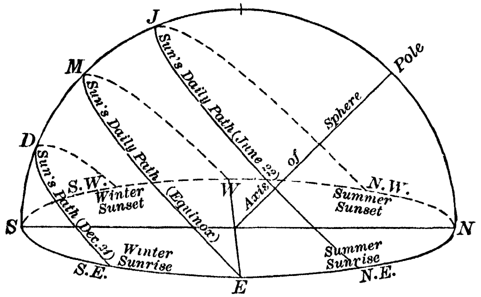


FIG. 8. SUN'S APPARENT POSITION AT DIFFERENT SEASONS

is not noticed until the sun's position is carefully observed with reference to the stars. If a record is kept for a year showing which constellations are visible in the east soon after sunset, it will be found that these change from month to month, and at the end of a year the one first seen will again appear in the east, showing that the sun has apparently made the circuit of the heavens in an eastward direction

8. Precession and Nutation.

While the direction of the earth's rotation axis is so nearly constant that no change is observed during short periods of time, there is in reality a very slow progressive change in its direction. This change is due to the fact that the earth is not quite spherical in form but is *spheroidal,* and there is in consequence a ring of matter around the equator upon which the sun and the moon exert a force of attraction which tends to pull the plane of the equator into coincidence with the plane of the orbit. But since the earth is rotating with a high velocity and resists this attraction, the actual effect is not to change permanently the inclination of the equator to the orbit, but *first* to cause the earth's axis to describe a cone about an axis perpendicular to the orbit, and *second* to cause the inclination of the axis to go through certain periodic changes (see Fig. 9). The movement of the axis in a conical surface causes the line of intersection of the equator and the plane of the orbit to revolve slowly westward, the pole itself always moving directly toward the vernal equinox. This causes the vernal equinox, *V*, to move westward in the sky, and hence the sun crosses the equator each spring earlier than it would otherwise; this is known as the

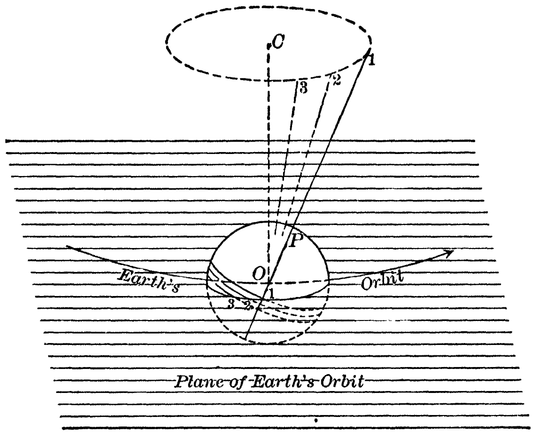


FIG. 9. PRECESSION OF THE EQUINOXES

precession of the equinoxes. In Fig. 9 the pole occupies successively the positions *1*, *2* and *3*, which causes the point *V* to occupy points *1*, *2* and *3*. This motion is but 50″.2 per year, and it therefore requires about 25,800 years for the pole to make one complete revolution. The force causing the precession is not quite constant, and the motion of the equinoctial points is therefore not perfectly uniform but has a small periodic variation. In addition to this periodic change in the rate of the precession there is also a slight periodic change in the obliquity, called **Nutation.** The maximum value of the nutation is about 9″; the period is about 19 years. The phenomenon of precession is clearly illustrated by means of the apparatus called the gyroscope. As a result of the precessional movement of the axis all of the stars gradually change their positions *with reference to the plane of the equator* and the position of the equinox. The stars themselves have but a very slight angular motion, this apparent change in position being due almost entirely to the change in the positions of the circles of reference.

9. Aberration of Light.

Another apparent displacement of the stars, due to the earth's motion, is that known as **aberration.** On account of the rapid motion of the earth through space, the direction in which a star is seen by an observer is a result of the combined velocities of the observer and of light from the star. The star always appears to be slightly displaced in the direction in which the observer is actually moving. In Fig. 10, if light moves from C to B in the same length of time that the observer moves from A to B, then C would appear to be in the direction AC. This

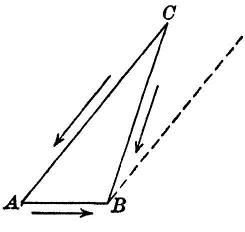


FIG. 10

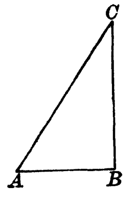


FIG. 11

may be more clearly understood by using the familiar illustration of the falling raindrop. If a raindrop is falling vertically, CB, Fig. 11, and while it is falling a person moves from A to B, then, considering only the two motions, it appears to the person that the raindrop has moved toward him in the direction CA. If a tube is to be held in such a way that the raindrop shall pass through it without touching the sides, it must be held at the inclination of AC. The apparent displacement of a star due to the observer's motion is similar to the change in the apparent direction of the raindrop.

There are two kinds of aberration, **annual** and **diurnal.** Annual aberration is that produced by the earth's motion in its orbit and is the same for all observers. Diurnal aberration is due to the earth's daily rotation about its axis, and is different in different latitudes, because the speed of a point on the earth's surface is greatest at the equator and diminishes toward the pole.

If v represents the velocity of the earth in its orbit and V the velocity of light, then when CB is at right angles to AB the displacement is a maximum and

$$\tan \alpha_0 = \frac{v}{V},$$

where α_0 is the angular displacement and is called the "constant of aberration." Its value is about 20.″5. If CB is not perpendicular to AB, then

$$\sin \alpha = \frac{v}{V} \sin A$$

or approximately

$$\tan \alpha = \sin \alpha = \frac{v}{V} \sin B,$$

where α is the angular displacement and B is the angle ABC.

CHAPTER II

DEFINITIONS—POINTS AND CIRCLES OF REFERENCE

10. The following astronomical terms are in common use and are necessary in defining the positions of celestial objects on the sphere by means of spherical coördinates.

Vertical Line.

A **vertical line** at any point on the earth's surface is the direc tion of gravity at that point, and is shown by the plumb lin or indirectly by means of the spirit level (*OZ*, Fig. 12).

Zenith — Nadir.

If the vertical at any point be prolonged upward it will pier the sphere at a point called the **Zenith** (*Z*, Fig. 12). This poi is of great importance because it is the point on the sphere whi indicates the position of the observer on the earth's surface The point where the vertical prolonged downward pierces th sphere is called the **Nadir** (*N'*, Fig. 12).

Horizon.

The **horizon** is the great circle on the celestial sphere cut by a plane through the centre of the earth perpendicular to the vertical (*NESW*, Fig. 12). The horizon is everywhere 90° from the zenith and the nadir. It is evident that a plane through the observer perpendicular to the vertical cuts the sphere in this same great circle. The **visible horizon** is the circle where the sea and sky seem to meet. Projected onto the sphere it is a small circle below the true horizon and parallel to it. Its distance below the true horizon depends upon the height of the observer's eye above the surface of the water.

Vertical Circles.

Vertical Circles are great circles passing through the zenith and nadir. They all cut the horizon at right angles (*HZJ*, Fig. 12).

Almucantars.

Parallels of altitude, or **almucantars,** are small circles parallel to the horizon (*DFG*, Fig. 12).

Poles.

If the earth's axis of rotation be produced indefinitely it will pierce the sphere in two points called the celestial **poles** (*PP′* Fig. 12).

Equator.

The **celestial equator** is a great circle of the celestial sphere cut by a plane through the centre of the earth perpendicular to

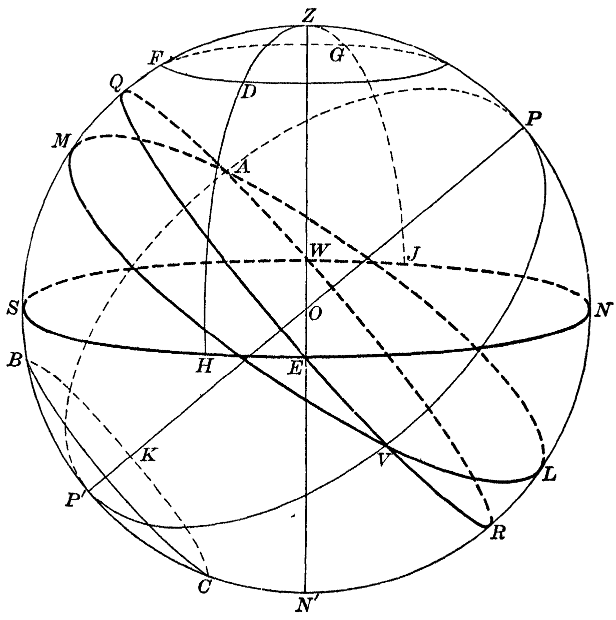


FIG. 12. THE CELESTIAL SPHERE

the axis of rotation (*QWRE*, Fig. 12). It is everywhere 90° from the poles. A parallel plane through the observer cuts the sphere in the same circle.

Hour Circles.

Hour Circles are great circles passing through the north and south celestial poles (*PVP′*, Fig. 12).

The 6-hour circle is the hour circle whose plane is perpendicular to that of the meridian.

Parallels of Declination.

Small circles parallel to the plane of the equator are called **parallels of declination** (*BKC*, Fig. 12).

Meridian.

The **meridian** is the great circle passing through the zenith and the poles (*SZPL*, Fig. 12). It is at once an hour circle and a

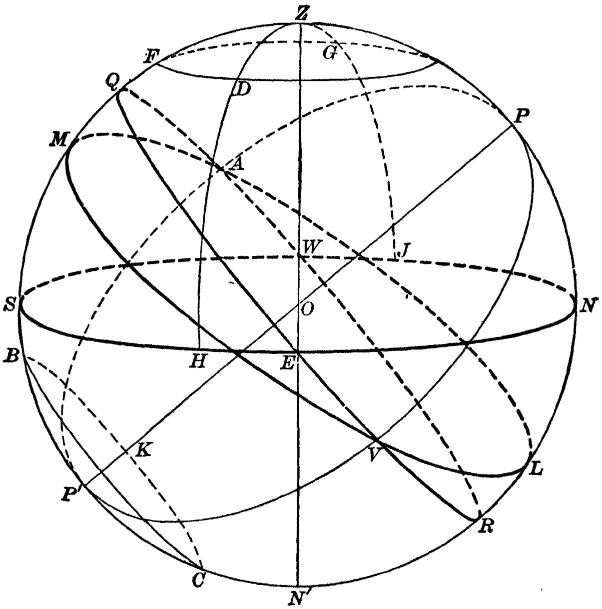


Fig. 12. The Celestial Sphere

vertical circle. It is evident that different observers will in general have different meridians. The meridian cuts the horizon in the **north and south points** (*N*, *S*, Fig. 12). The intersection of the plane of the meridian with the horizontal plane through the observer is the **meridian line** used in plane surveying.

Prime Vertical.

The **prime vertical** is the vertical circle whose plane is perpendicular to the plane of the meridian (*EZW*, Fig. 12). It cuts the horizon in the **east** and **west points** (*E*, *W*, Fig. 12).

Ecliptic.

The **ecliptic** is the great circle on the celestial sphere which the sun's centre appears to describe during one year (*AMVL*, Fig. 12). Its plane is the plane of the earth's orbit; it is inclined to the plane of the equator at an angle of about 23° 27′, called the **obliquity of the ecliptic.**

Equinoxes.

The points of intersection of the ecliptic and the equator are called the **equinoctial points** or simply the **equinoxes.** That intersection at which the sun appears to cross the equator when going from the south side to the north side is called the **Vernal Equinox,** or sometimes the **First Point of Aries** (*V*, Fig. 12). The sun reaches this point about March 21. The other intersection is called the **Autumnal Equinox** (*A*, Fig. 12).

Solstices.

The points on the ecliptic midway between the equinoxes are called the winter and summer **solstices.**

Questions

1. What imaginary circles on the earth's surface correspond to hour circles? To parallels of declination? To vertical circles?

2. What are the widths of the torrid, temperate and arctic zones and how are they determined?

CHAPTER III

SYSTEMS OF COÖRDINATES ON THE SPHERE

11. Spherical Coördinates.

The direction of a point in space may be defined by means of two spherical coördinates, that is, by two angular distances measured on a sphere along arcs of two great circles which cut each other at right angles. Suppose that it is desired to locate C (Fig. 13) with reference to the plane OAB and the line

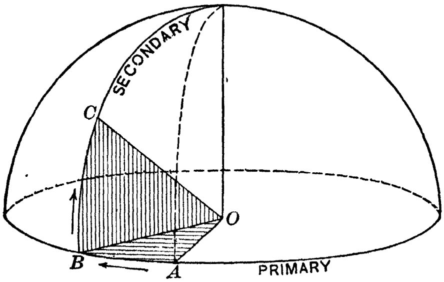


FIG. 13. SPHERICAL COÖRDINATES

OA, O **being the origin** of coördinates. Pass a plane OBC through OC perpendicular to OAB; these planes will intersect in the line OB. The two angles which fix the position of C, or the spherical coördinates, are BOC and AOB. These may be regarded as the angles at the centre of the sphere or as the arcs BC and AB. In every system of spherical coördinates the two coördinates are measured, one on a great circle called the **primary**, and the other on one of a system of great circles at right angles to the primary called **secondaries**. There are an infinite number of secondaries, each passing through the two poles of the primary. The coördinate measured from the primary is an arc of a secondary circle; the coördinate measured between the secondary circles is an arc of the primary.

12. Horizon System.

In this system the primary circle is the horizon and the secondaries are vertical circles, or circles passing through the zenith and nadir. The first coördinate of a point is its angular distance above the horizon, measured on a vertical circle; this is called the **Altitude.** The complement of the altitude is called the **Zenith distance.** The second coördinate is the angular distance on the horizon between the meridian and the vertical circle through the point; this is called the **Azimuth.** Azimuth may be reckoned either from the north or the south point and in either direction, like bearings in surveying, but the custom is to reckon it from the south point right-handed from 0° to 360° except for stars near the pole, in which case it is more convenient to reckon

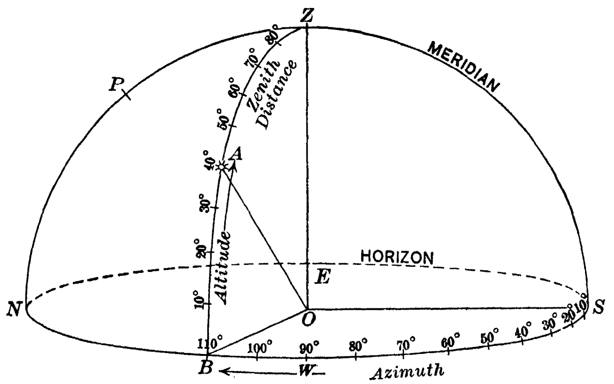


FIG. 14. THE HORIZON SYSTEM

from the north, and either to the east or to the west. In Fig. 14 the altitude of the star *A* is *BA*; its azimuth is *SB*.

13. The Equator Systems.

The circles of reference in this system are the equator and great circles through the poles, or hour circles. The first coördinate of a point is its angular distance north or south of the equator, measured on an hour circle; it is called the **Declination.** Declinations are considered positive when north of the equator, negative when south. The complement of the declination is called the **Polar Distance.** The second coördinate of the point is the arc of the equator between the vernal equinox and the foot of the hour circle through the point; it is called **Right Ascension.** Right ascension is measured from the equinox *eastward* to the hour circle through the point in question; it may be measured in degrees, minutes, and seconds of arc, or in hours, minutes, and

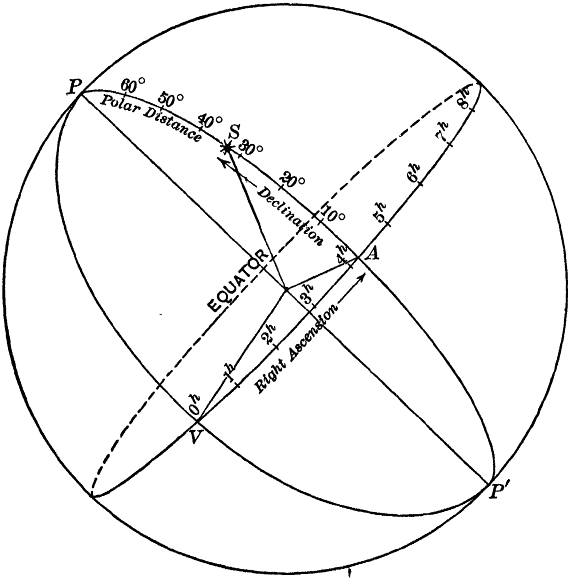


FIG. 15. THE EQUATOR SYSTEM

seconds of time. In Fig. 15 the declination of the star S is AS; the right ascension is VA.

Instead of locating a point by means of declination and right ascension it is sometimes more convenient to use declination and **Hour Angle.** The hour angle of a point is the arc of the equator between the observer's meridian and the hour circle through the point. It is measured from the meridian *westward* (clockwise) from 0^h to 24^h or from 0° to 360°. In Fig. 16 the declination of the star S is AS (negative); the hour angle is

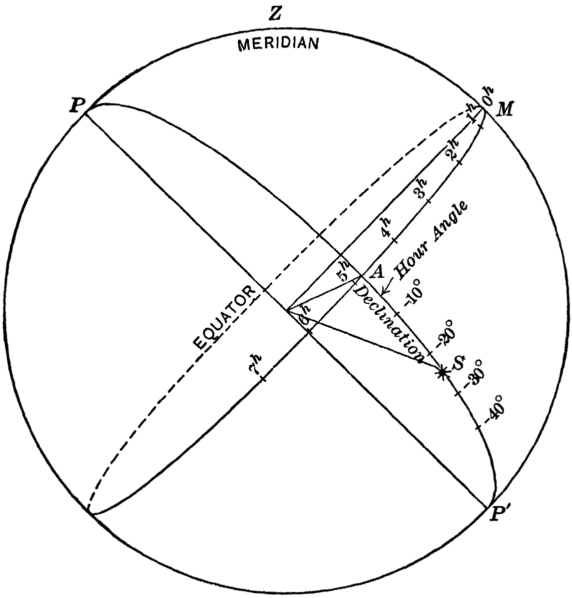


FIG. 16. HOUR ANGLE AND DECLINATION

MA. For the measurement of time the hour angle may be counted from the upper or the lower branch of the meridian.

These three systems are shown in the following table.

Name.	Primary.	Secondaries.	Origin of Coördinates.	1st coörd.	2nd coörd.
Horizon System	Horizon	Vert. Circles	South point.	Altitude	Azimuth
Equator Systems {	Equator	Hour Circles	Vernal Equinox.	Declin.	Rt. Ascen.
	"	" "	Intersection of Meridian and Equator.	"	Hour Angle

14. There is another system which is employed in some branches of astronomy but will not be used in this book. The coördinates are called **celestial latitude** and **celestial longitude;** the primary circle is the ecliptic. Celestial latitude is measured from the ecliptic just as declination is measured from the equator. Celestial longitude is measured eastward along the ecliptic from the equinox, just as right ascension is measured eastward along the equator. The student should be careful not to confuse celestial latitude and longitude with terrestrial latitude and longitude. The latter are the ones used in the problems discussed in this book.

15. Coördinates of the Observer.

The observer's position is located by means of his **latitude** and **longitude.** The latitude, which on the earth's surface is the angular distance of the observer north or south of the equator, may be defined astronomically as the declination of the observer's zenith. In Fig. 17, the terrestrial latitude is the arc *EO*,

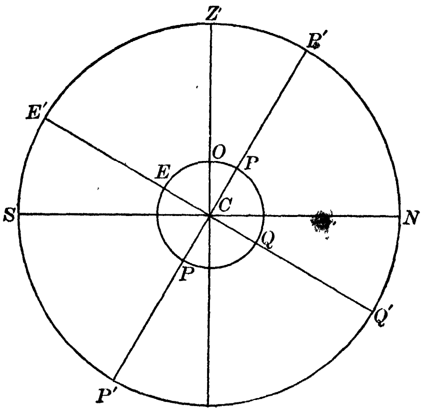


FIG. 17. THE OBSERVER'S LATITUDE

EQ being the equator and *O* the observer. The point *Z* is the observer's zenith, so that the latitude on the sphere is the arc *E'Z*, which evidently will contain the same number of degrees as *EO*. The complement of the latitude is called the **Co-latitude.**

The terrestrial longitude of the observer is the arc of the equator between the primary meridian (usually that of Greenwich) and the meridian of the observer. On the celestial sphere the longitude would be the arc of the celestial equator contained between two hour circles whose planes are the planes of the two terrestrial meridians.

16. Relation between the Two Systems of Coördinates.

In studying the relation between different points and circles on the sphere it may be convenient to imagine that the celestial sphere consists of two spherical shells, one within the other.

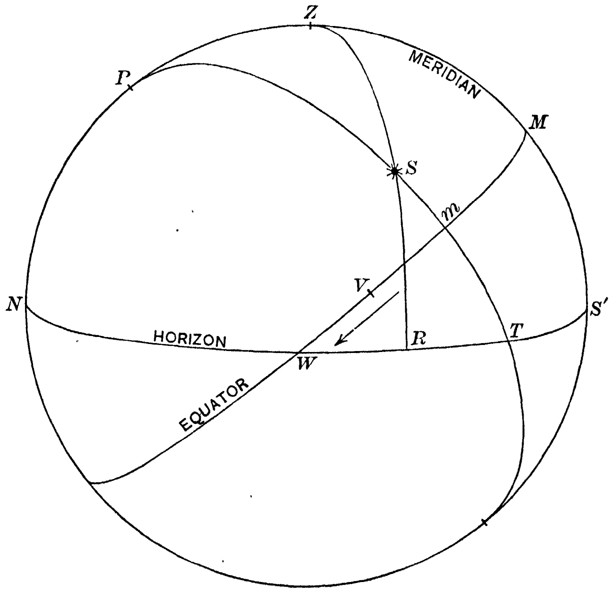


FIG. 18. THE SPHERE SEEN FROM THE OUTSIDE

The outer one carries upon its surface the ecliptic, equinoxes, poles, equator, hour circles and all of the stars, the sun, the moon and the planets. On the inner sphere are the zenith, horizon, vertical circles, poles, equator, hour circles, and the meridian. The earth's daily rotation causes the inner sphere to revolve, while the outer sphere is motionless, or, regarding only the apparent motion, the outer sphere revolves once per day on its axis, while the inner sphere appears to be motionless. It is evident that the coördinates of a fixed star in the first equatorial system (Declination and Right Ascension) are practically always the same, whereas the coördinates in the horizon system are continually changing. It will also be seen that in the first equatorial system the coördinates are independent of the observer's position, but in the horizon system they are entirely dependent upon his position. In the second equatorial system one coördinate is independent of the observer, while the other (hour angle) is not. In making up catalogues of the positions of the stars it is necessary to use right ascensions and declinations in defining these positions. When making observations

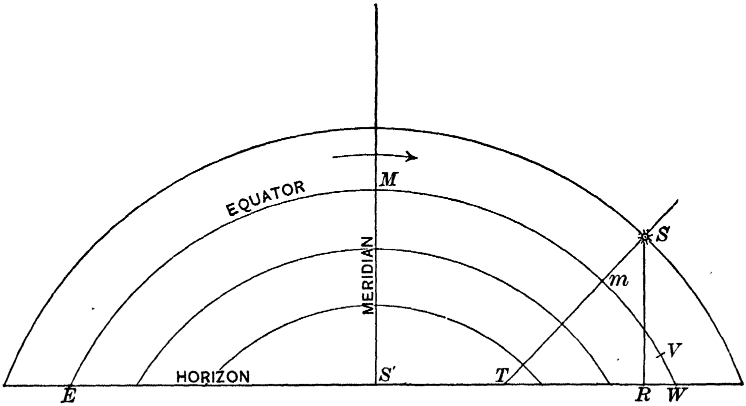


FIG. 19. PORTION OF THE SPHERE SEEN FROM THE EARTH (LOOKING SOUTH)

with instruments it is usually simpler to measure coördinates in the horizon system. Therefore it is necessary to be able to compute the coördinates of one system from those of another. The mathematical relations between the spherical coördinates are discussed in Chapter IV.

Figs. 18, 19, and 20 show three different views of the celestial sphere with which the student should be familiar. Fig. 18 is the sphere as seen from the outside and is the view best adapted to showing problems in spherical trigonometry. The star S has the altitude RS, azimuth $S'R$, hour angle Mm, right ascension Vm, and declination mS; the meridian is ZMS'. Fig. 19 shows a portion of the sphere as seen by an observer looking southward; the points are indicated by the same letters as in Fig. 18. Fig. 20

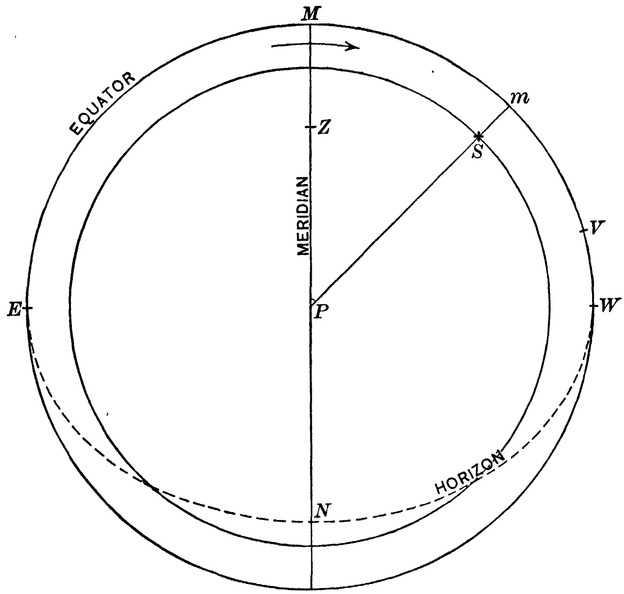


FIG. 20. THE SPHERE PROJECTED ONTO THE PLANE OF THE EQUATOR

shows the same points projected on the plane of the equator. In this view of the sphere the angles at the pole (i.e., the angles between hour circles) are shown their true size, and it is therefore a convenient diagram to use when dealing with right ascension and hour angles.

Questions and Problems

1. What coördinates on the sphere correspond to latitude and longitude on the earth's surface?

2. Make a sketch of the sphere and plot the position of a star having an altitude of 20° and an azimuth of 250°. Locate a star whose hour angle is 16^h and whose declination is $-10°$. Locate a star whose right ascension is 9^h and whose declination is N. 30°.

3. If a star is on the equator and also on the horizon, what is its azimuth? Its altitude? Its hour angle? Its declination?

4. What changes take place in the azimuth and altitude of a star during twenty-four hours?

5. What changes take place in the right ascension and declination of the observer's zenith during a day?

6. A person in latitude 40° N. observes a star, in the west, whose declination is 5° N. In what order will the star pass the following three circles; (*a*) the 6^h circle, (*b*) the horizon, (*c*) the prime vertical?

CHAPTER IV

RELATION BETWEEN COÖRDINATES

17. Relation between Altitude of Pole and Latitude of Observer.

In Fig. 21, *SZN* represents the observer's meridian; let *P* be the celestial pole, *Z* the zenith, *E* the point of intersection of the meridian and the equator, and *N* and *S* the north and south points of the horizon. By the definitions, *CZ* (vertical) is perpendicular to *SN* (horizon) and *CP* (axis) is perpendicular to *EC* (equator).

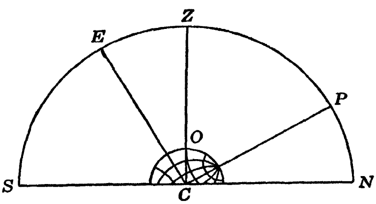


FIG. 21.

Therefore the arc *PN* = arc *EZ*. By the definitions *EZ* is the declination of the zenith, or the latitude, and *PN* is the altitude of the celestial pole. Hence *the altitude of the pole is always equal to the latitude of the observer.* The same relation may be seen from Fig. 22, in which *NP* is the north pole of the earth, *OH* is the plane of the horizon, the observer being at *O*, *EQ* is the equator, and *OP′* is a line parallel to *C-NP* and consequently points to the celestial pole. It may readily be shown that *ECO*, the observer's latitude, equals

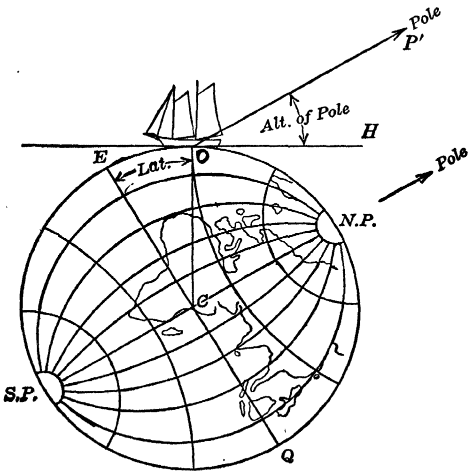


FIG. 22

HOP′, the altitude of the celestial pole. A person at the equator would see the north celestial pole in the north point of his horizon and the south celestial pole in the south point of his horizon. If he travelled northward the north pole would appear to rise, its altitude being always equal to his latitude, while the south pole would immediately go below his horizon. When the traveller reached the north pole of the earth the north celestial pole would be vertically over his head.

To a person at the equator all stars would appear to move vertically at the times of rising and setting, and all stars would be above the horizon 12^h and below 12^h during one revolution

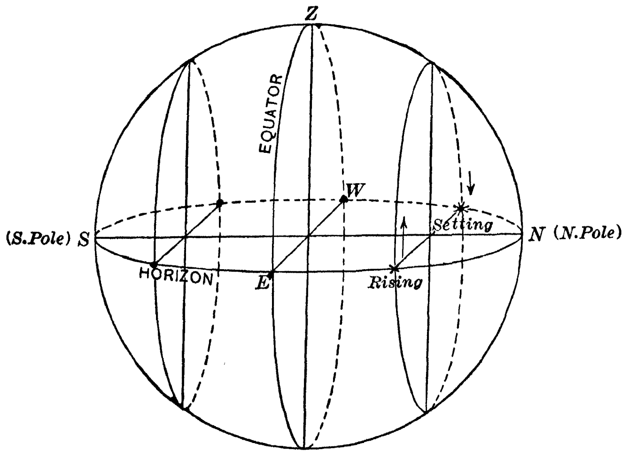


Fig. 23. The Right Sphere

Appearance of Sphere to Observer at Earth's Equator.

of the sphere. All stars in both hemispheres would be above the horizon at some time every day. (Fig. 23.)

If a person were at the earth's pole the celestial equator would coincide with his horizon, and all stars in the northern hemisphere would appear to travel around in circles parallel to the horizon; they would be visible for 24^h a day, and their altitudes would not change. The stars in the southern hemisphere would never be visible. The word *north* would cease to have its usual meaning, and *south* might mean any horizontal direction. The longitude of a point on the earth and its azimuth from the Greenwich meridian would then be the same. (Fig. 24.)

At all points between these two extreme latitudes the equator cuts the horizon obliquely. A star on the equator will be above

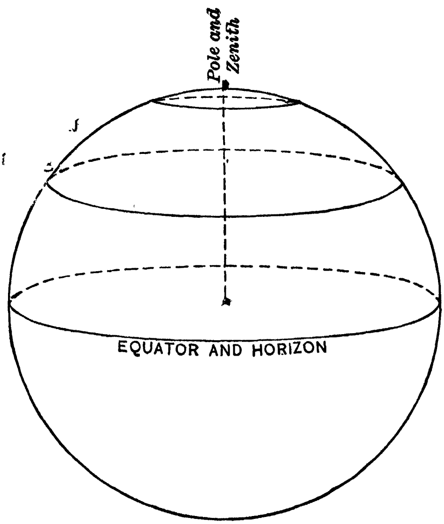


FIG. 24. THE PARALLEL SPHERE

Appearance of Sphere to Observer at Earth's Pole

the horizon half the time and below half the time. A star north of the equator will (to a person in the northern hemisphere) be above the horizon more than half of the day; a star south of the equator will be above the horizon less than half of the day. If the north polar distance of a star is less than the observer's north latitude, the whole of the star's diurnal circle is above the horizon, and the star will therefore remain above the horizon all of the time. It is called in this case a **circumpolar star** (Fig. 25). The south circumpolar stars are those whose south polar distances are less than the latitude; they are never visible to an observer in the northern hemisphere. If the observer travels north until he is beyond the arctic circle, latitude 66° 33′ north, then the sun becomes a circumpolar at the time of the summer solstice. At noon the sun would be at its maximum altitude; at midnight it would be at its minimum altitude but would still be above the horizon. This is called the "midnight sun."

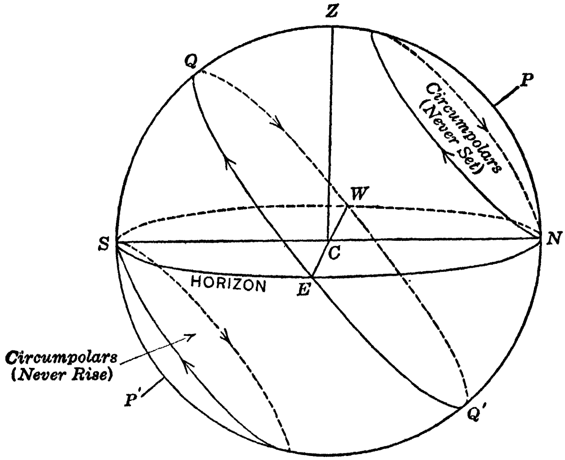


FIG. 25. CIRCUMPOLAR STARS

18. Relation between Latitude of Observer, and the Declination and Altitude of a Point on the Meridian.

The relation between the latitude of the observer and the declination and altitude of a point on the observer's meridian may be seen by referring to Fig. 26. Let A be any point on the meridian, such as a star or the centre of the sun, moon, or a planet, located south of the zenith but north of the equator; then

$EZ = \phi$, the latitude*
$EA = \delta$, the declination
$SA = h$, the meridian altitude
$ZA = \zeta$, the meridian zenith distance.

* The Greek alphabet is given on p. 242.

From the figure it is evident that

$$\phi = \zeta + \delta. \quad [1]$$

If A is south of the equator δ becomes negative, but the same equation applies in this case provided the quantities are given

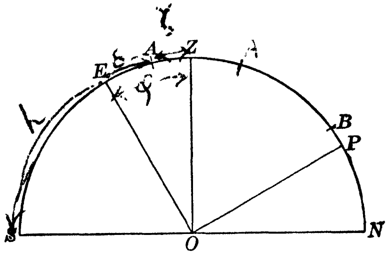


FIG. 26. STAR ON THE MERIDIAN

their proper signs. If A is north of the zenith we should have $\phi = \delta - \zeta$ [2]; but if we regard ζ as negative when north of the zenith and positive when south of the zenith, then equation [1] covers all cases. When the point is below the pole the same formula might be employed by counting the declination beyond 90°. In such cases it is usually simpler to employ the polar distance, p, instead of the declination.

If the star is north of the zenith but above the pole, as at B, then since $p = 90° - \delta$,

$$\phi = h - p. \quad [3]$$

If B were below the pole we should have

$$\phi = h + p. \quad [4]$$

19. The Astronomical Triangle.

By joining the pole, zenith, and any star S on the sphere by arcs of great circles we obtain a triangle from which the relation existing among the spherical coördinates may be obtained. This triangle is so frequently employed in astronomy and navigation that it is called the "astronomical triangle" or the "*PZS* triangle." In Fig. 27 the arc PZ is the complement of the latitude, or co-latitude; arc ZS is the zenith distance or complement of the altitude; arc PS is the polar distance or complement of the declination; the angle at P is the hour angle of the star if S is west of the meridian, or 360° minus the hour angle if S is east of the meridian; and Z is the azimuth of S (from the north point), or 360° minus the azimuth, according as S is west or east of the meridian. The angle at S is called the parallactic angle. If any three parts of this triangle are known the other three may be calculated. The fundamental formulæ of spherical trigonometry are (see p. 257)

$$\cos a = \cos b \cos c + \sin b \sin c \cos A, \qquad [5]$$
$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A, \qquad [6]$$
$$\sin a \sin B = \sin b \sin A. \qquad [7]$$

If we put $A = t$, $B = S$, $C = Z$, $a = 90° - h$, $b = 90° - \phi$, $c = 90° - \delta$, then these three equations become

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t \qquad [8]$$
$$\cos h \cos S = \sin \phi \cos \delta - \cos \phi \sin \delta \cos t \qquad [9]$$
$$\cos h \sin S = \cos \phi \sin t. \qquad [10]$$

If $A = t$, $B = Z$, $C = S$, $a = 90° - h$, $b = 90° - \delta$, $c = 90° - \phi$, then the [6] and [7] become

$$\cos h \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos t \qquad [11]$$
$$\cos h \sin Z = \cos \delta \sin t. \qquad [12]$$

If $A = Z, B = S, C = t, a = 90° - \delta, b = 90° - \phi, c = 90° - h$, then

$$\sin \delta = \sin \phi \sin h + \cos \phi \cos h \cos Z \qquad [13]$$
$$\cos \delta \cos S = \sin \phi \cos h - \cos \phi \sin h \cos Z \qquad [14]$$
$$\cos \delta \sin S = \cos \phi \sin Z. \qquad [15]$$

If $A = Z, B = t, C = S, a = 90° - \delta, b = 90° - h, c = 90° - \phi$, then

$$\cos \delta \cos t = \sin h \cos \phi - \cos h \sin \phi \cos Z. \qquad [16]$$

Other forms may be derived, but those given above will suffice for all cases occurring in the following chapters.

The problems arising most commonly in the practice of surveying and navigation are:

1. Given the declination, latitude, and altitude, to find the azimuth and the hour angle.

2. Given the declination, latitude, and hour angle, to find the azimuth and the altitude.

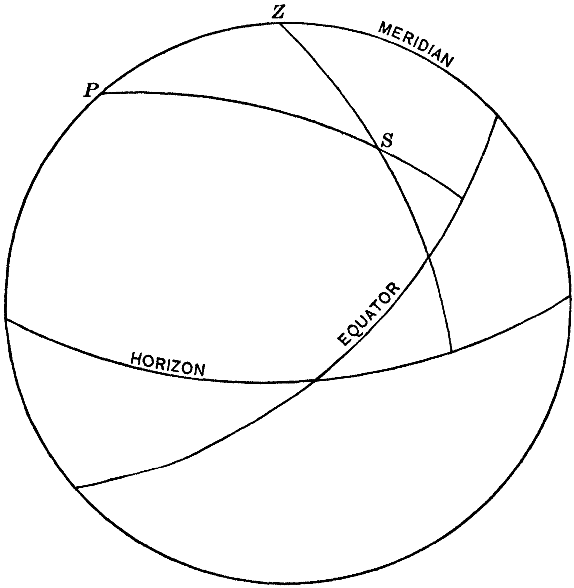


FIG. 27. THE ASTRONOMICAL TRIANGLE

In following formulæ let

$$t = \text{hour angle}$$
$$Z = \text{azimuth*}$$
$$h = \text{altitude}$$

* The trigonometric formulæ give the interior angle of the triangle, and consequently the azimuth from the north point, unless the form of the equation is changed so as to give the exterior angle.

ζ = zenith distance
δ = declination
p = polar distance
ϕ = latitude

and $s = \frac{1}{2}(\phi + h + p)$.

For computing t any of the following formulæ may be used.

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\cos s \sin (s - h)}{\cos \phi \sin p}} \qquad [17]$$

$$\cos \tfrac{1}{2} t = \sqrt{\frac{\cos (s - p) \sin (s - \phi)}{\cos \phi \sin p}} \qquad [18]$$

$$\tan \tfrac{1}{2} t = \sqrt{\frac{\cos s \sin (s - h)}{\cos (s - p) \sin (s - \phi)}} \qquad [19]$$

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta} \qquad [20]$$

$$\cos t = \frac{\sin h}{\cos \phi \cos \delta} - \tan \phi \tan \delta \qquad [20a]$$

$$\text{vers}\ t = \frac{\cos (\phi - \delta) - \sin h}{\cos \phi \cos \delta}. \qquad [21]$$

For computing the azimuth, Z, from the north point either toward the east or the west, we have

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - h) \sin (s - \phi)}{\cos \phi \cos h}} \qquad [22]$$

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\cos s \cos (s - p)}{\cos \phi \cos h}} \qquad [23]$$

$$\tan \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}} \qquad [24]$$

$$\cos Z = \frac{\sin \delta - \sin \phi \sin h}{\cos \phi \cos h} \qquad [25]$$

$$\cos Z = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h \qquad [25a]$$

$$\text{vers } Z = \frac{\cos (\phi - h) - \sin \delta}{\cos \phi \cos h}. \qquad [26]$$

Only slight changes are necessary to adapt these to the direct computation of Z_s from the south point of the horizon. For example, formulæ [24], [25] and [26] would take the forms

$$\cot \tfrac{1}{2} Z_s = \sqrt{\frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}} \qquad [27]$$

$$\cos Z_s = \frac{\sin \phi \sin h - \sin \delta}{\cos \phi \cos h} \qquad [28]$$

$$\text{vers } Z_s = \frac{\cos (\phi + h) + \sin \delta}{\cos \phi \cos h}. \qquad [29]$$

While any of these formulæ may be used to determine the angle sought, the choice of formulæ should depend somewhat upon the precision with which the angle is defined by the function. If the angle is quite small it is more accurately found through its sine than through its cosine; for an angle near 90° the reverse is true. The tangent, however, on account of its rapid variation, always gives the angle more precisely than either the sine or the cosine. It will be observed that some of the formulæ require the use of both logarithmic and natural functions. This causes no particular inconvenience in ordinary 5-place computations because engineer's field and office tables almost invariably contain both logarithmic and natural functions. If 7-place logarithmic tables are being used the other formulæ will be preferred.

The altitude of an object may be found from the formulæ

$$\sin h = \cos (\phi - \delta) - 2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2} t \qquad [30]$$

or

$$\sin h = \cos (\phi - \delta) - \cos \phi \cos \delta \text{ vers } t, \qquad [30a]$$

which may be derived from Equa. [8].

If the declination, hour angle, and altitude are given, the azimuth is found by

$$\sin Z = \sin t \cos \delta \sec h. \qquad [31]$$

For computing the azimuth of a star near the pole when the hour angle is known the following formula is frequently used:

$$\tan Z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}. \qquad [32]$$

This equation may be derived by dividing [12] by [11] and then dividing by $\cos \delta$.

Body on the Horizon.

Given the latitude and declination, find the hour angle and azimuth when the object is on the horizon. If in Equa. [8] and [13] we put $h = 0$, we have

$$\cos t = -\tan \delta \tan \phi \qquad [33]$$

and

$$\cos Z = \sin \delta \sec \phi. \qquad [34]$$

These formulæ may be used to compute the time of sunrise or sunset, and the sun's bearing at these times.

Greatest Elongation.

A special case of the *PZS* triangle which is of great practical importance occurs when a star which culminates north of the zenith is at its *greatest elongation.* When in this position the azimuth of the star is a maximum and its diurnal circle is tangent to the vertical circle through the star; the triangle is therefore right-angled at the point *S* (Fig. 28). The formulæ for the hour angle and azimuth are

$$\cos t = \tan \phi \cot \delta \qquad [35]$$

and

$$\sin Z = \sin p \sec \phi, \qquad [36]$$

from which the time of elongation and the bearing of the star may be found. (See Art. 97.)

20. Relation between Right Ascension and Hour Angle.

In order to understand the relation between the right ascension and the hour angle of a point, we may think of the equa-

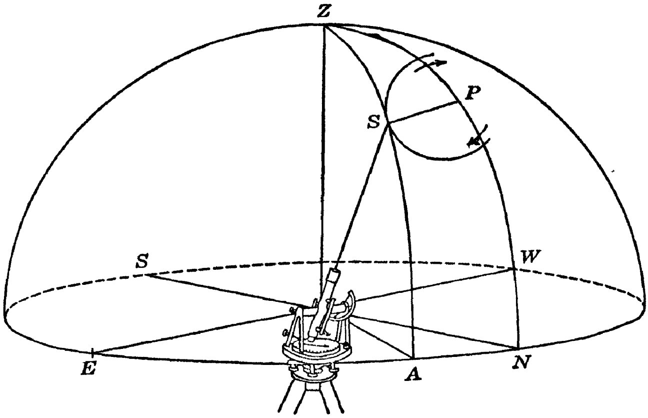


FIG. 28. STAR AT GREATEST ELONGATION (EAST).

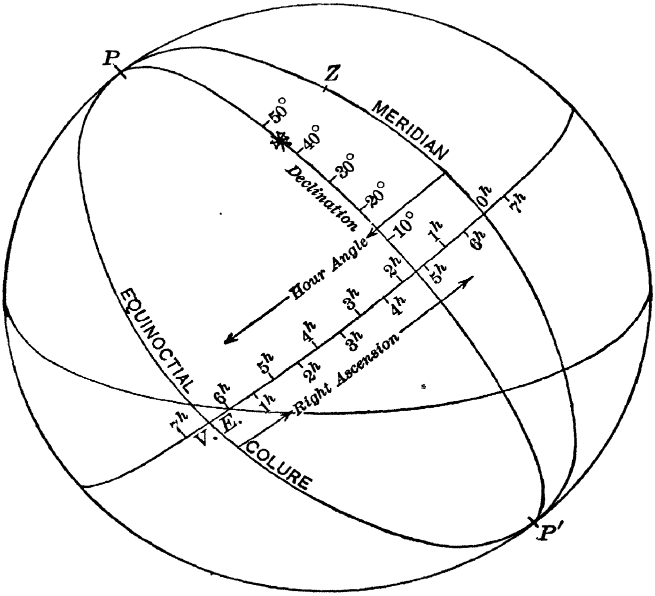


FIG. 29. RIGHT ASCENSION AND HOUR ANGLE

tor on the outer sphere as graduated into hours, minutes and seconds of right ascension, zero being at the equinox and the numbers increasing toward the east. The equator on the inner sphere is graduated for hour angles, the zero being at the observer's meridian and the numbers increasing toward the west. (See Fig. 29.) As the outer sphere turns, the hour marks on the right ascension scale will pass the meridian in the order of the numbers. The number opposite the meridian at any instant

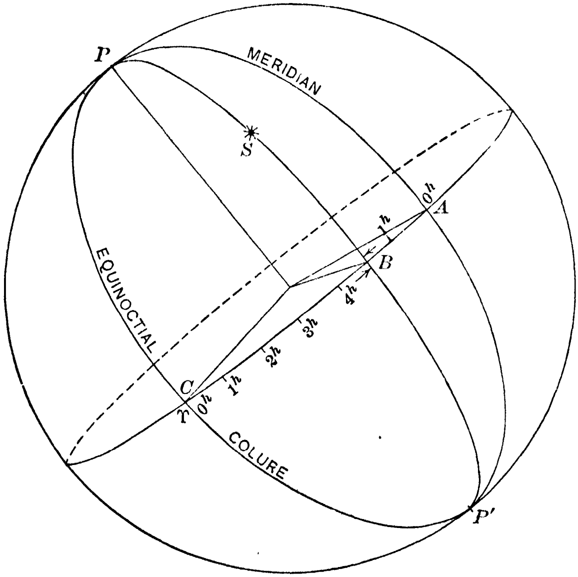


FIG. 30

shows how far the sphere has turned since the equinox was on the meridian. If we read the hour angle scale opposite the equinox, we obtain exactly the same number of hours. This number of hours (or angle) may be considered as either the right ascension of the meridian or the hour angle of the equinox. In Fig. 30 the star S has an hour angle equal to AB and a right ascension CB. The sum of these two angles is AC, or the hour angle of the equinox. The same relation will be found to hold true for all positions of S. The general relation existing between these coördinates is, then,

Hour angle of Equinox = Hour angle of Star + Right Ascension of Star.

Questions and Problems

1. What is the greatest north declination a star may have and pass the meridian to the south of the zenith?
2. What angle does the plane of the equator make with the horizon?
3. In what latitudes can the sun be overhead?
4. What is the altitude of the sun at noon in Boston (42° 21′ N.) on December 22?
5. What are the greatest and least angles made by the ecliptic with the horizon at Boston?
6. In what latitudes is *Vega* (Decl. = 38° 42′ N.) a circumpolar star?
7. Make a sketch of the celestial sphere like Fig. 12 corresponding to a latitude of 20° south and the instant when the vernal equinox is on the eastern horizon.
8. Derive formula [36].
9. Compute the hour angle of *Vega* when it is rising in latitude 40° North.
10. Compute the time of sunrise on June 22, in latitude 40° N.

CHAPTER V

MEASUREMENT OF TIME

21. The Earth's Rotation.

The measurement of intervals of time is made to depend upon the period of the earth's rotation on its axis. Although the period of rotation is not absolutely invariable, yet the variations are exceedingly small, and the rotation is assumed to be uniform. The most natural unit of time for ordinary purposes is the *solar day*, or the time corresponding to one rotation of the earth with respect to the sun's direction. On account of the motion of the earth around the sun once a year the direction of this reference line is continually changing with reference to the directions of fixed stars, and the length of the solar day is not the true time of one rotation of the earth. In some kinds of astronomical work it is more convenient to employ a unit of time based upon this true time of one rotation, namely, *sidereal time* (or star time).

22. Transit or Culmination.

Every point on the celestial sphere crosses the plane of the meridian of an observer twice during one revolution of the sphere. The instant when any point on the celestial sphere is on the meridian of an observer is called the time of *transit*, or *culmination*, of that point over that meridian. When it is on that half of the meridian containing the zenith, it is called the upper transit; when it is on the other half it is called the lower transit. Except in the case of stars near the elevated pole the upper transit is the only one visible to the observer; hence when the transit of a star is mentioned the upper transit will be understood unless the contrary is stated.

23. Sidereal Day.

The sidereal day is the interval of time between two successive upper transits of the vernal equinox over the same meridian. If the equinox were fixed in position the sidereal day as thus defined would be the true rotation period with reference to the fixed stars, but since the equinox has a slow (and variable) westward motion caused by the precessional movement of the axis (see Art. 8) the actual interval between two transits of the equinox differs about $0^s.01$ of time from the true time of one rotation. The sidereal day actually used in practice, however, is the one previously defined and not the true rotation period. This causes no inconvenience because sidereal days are not used for reckoning long periods of time, dates always being given in solar days, so this error never becomes large. The sidereal day is divided into 24 hours and each hour is subdivided into 60 minutes, and each minute into 60 seconds. When the vernal equinox is at upper transit it is 0^h, or the beginning of the sidereal day. This may be called "sidereal noon."

24. Sidereal Time.

The sidereal time at a given meridian at any specified instant is equal to the hour angle of the vernal equinox measured from the upper half of that meridian. It is therefore a measure of the angle through which the earth has rotated since the equinox was on the meridian, and shows at once the position of the sphere at this instant with respect to the observer's meridian.

25. Solar Day.

A solar day is the interval of time between two successive *lower* transits of the sun's centre over the same meridian. The lower transit is chosen in order that the date may change at midnight. The solar day is divided into 24 hours, and each hour is divided into 60 minutes, and each minute into 60 seconds. When the centre of the sun is on the upper side of the meridian (upper transit) it is *noon*. When it is on the lower side it is *midnight*. The instant of midnight is taken as 0^h, or the beginning of the civil day.

26. Solar Time.

The solar time at any instant is equal to the hour angle of the sun's centre plus 180° or 12 hours; in other words it is the hour angle counted from the lower transit. It is the angle through which the earth has rotated, with respect to the sun's direction, since midnight, and measures the time interval that has elapsed.

Since the earth revolves around the sun in an elliptical orbit in accordance with the law of gravitation, the apparent angular motion of the sun is not uniform, and the days are therefore of different length at different seasons. In former times when sun dials were considered sufficiently accurate for measuring time, this lack of uniformity was unimportant. Under modern conditions, which demand accurate measurement of time by the use of clocks and chronometers, an invariable unit of time is essential. The time ordinarily employed is that kept by a fictitious point called the "mean sun," which is imagined to move at a uniform rate along the equator,* its rate of motion being such that it makes one apparent revolution around the earth in the same time as the actual sun, that is, in one year. The fictitious sun is so placed that on the whole it precedes the true sun as much as it follows it. The time indicated by the position of the mean sun is called *mean solar time*. The time indicated by the position of the real sun is called *apparent solar time* and is the time shown by a sun dial, or the time obtained by direct instrumental observation of the sun's position. Mean time cannot, of course, be observed directly, but must be derived by computation.

27. Equation of Time.

The difference between mean time and apparent time at any instant is called the *equation of time* and depends upon how much the real sun is ahead of or behind its average position. It is given in ordinary almanacs as "sun fast" or "sun slow." The amount of this difference varies from about $-14m$ to $+16m$.

* This statement is true in a general way, but the motion is not strictly uniform because the motion of the equinox itself is variable. The angle from the equinox to the "mean sun" at any instant is the sun's "mean longitude" (along the ecliptic) plus small periodic terms.

The exact interval is given in the American Ephemeris and in the (small) Nautical Almanac for specified times each day.

This difference between the two kinds of time is due to several causes, the chief of which are (1) the inequality of the earth's angular motion in its orbit, and (2) the fact that the real sun moves in the plane of the ecliptic and the mean sun in the plane of the equator, and equal arcs on the ecliptic do not correspond to equal arcs in the equator, or equal angles at the pole.

In the winter, when the earth is nearest the sun, the rate of angular motion about the sun is greater than in the summer (see Art. 6). The sun will then appear to move eastward in the sky at a faster rate than in summer, and its daily revolution about

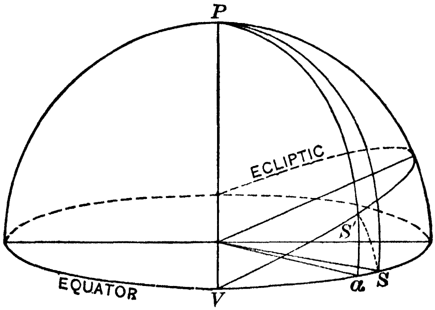


FIG. 31

the earth will therefore be slower. This delays the instant of apparent noon, making the solar day longer than the average, and therefore a sun dial will "lose time." About April 1 the sun is moving at its average rate and the sun dial ceases to lose time; from this date until about July 1 the sun dial gains on mean time, making up what it lost between Jan. 1 and April 1. During the other half of the year the process is reversed; the sun dial gains from July 1 to Oct. 1 and loses from Oct. 1 to Jan. 1. The maximum difference due to this cause alone is about 8 minutes, either + or −.

The second cause of the equation of time is illustrated in Fig. 31. Assume that point S' (sometimes called the "first mean sun ") moves uniformly along the ecliptic at the average rate of the actual sun; the time as indicated by this point will evidently not be affected by the eccentricity of the orbit. If the *mean sun*, *S* (also called the " second mean sun "), starts at *V*, the vernal equinox, at the same instant that *S′* starts, then the arcs

TABLE A. EQUATION OF TIME FOR 1910.

	1st.	10th.	20th.	30th.
January	− 3^m 26^s	− 7^m 27^s	− 11^m 02^s	− 13^m 22^s
February	− 13 41	− 14 24	− 13 59	....
March	− 12 38	− 10 36	− 7 48	− 4 45
April	− 4 08	− 1 31	+ 0 58	+ 2 47
May	+ 2 55	+ 3 42	+ 3 42	+ 2 48
June	+ 2 31	+ 0 57	− 1 08	− 3 15
July	− 3 27	− 5 01	− 6 06	− 6 16
August	− 6 11	− 5 19	− 3 26	− 0 46
September	− 0 09	+ 2 48	+ 6 20	+ 9 46
October	+ 10 05	+ 12 45	+ 15 01	+ 16 13
November	+ 16 18	+ 16 02	+ 14 26	+ 11 28
December	+ 11 06	+ 7 23	+ 2 36	− 2 21

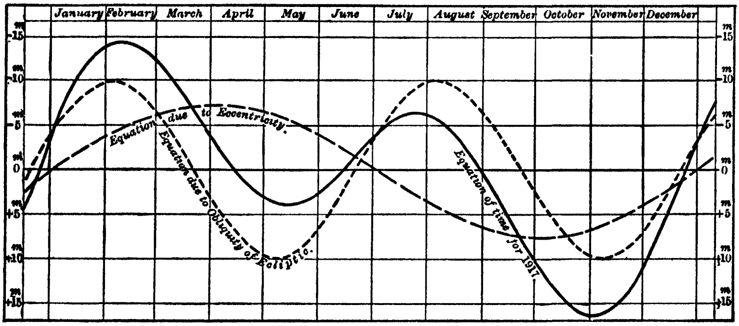


FIG. 32. CORRECTION TO MEAN TIME (TO GET APPARENT TIME)

VS and *VS′* are equal, since both points are moving at the same rate. By drawing hour circles through these two points it will be seen that these hour circles do not coincide unless the points *S* and *S′* happen to be at the equinoxes or at the solstices. Since *S* and *S′* are not, in general, on the same hour circle they will not cross the meridian at the same instant, the difference in time being represented by the arc *aS*. The maximum length of *aS* is about 10 minutes of time, and may be either + or −. The combined effect of these two causes, or the *equation of time*, is shown in Table A and (graphically) in Fig. 32.

28. Conversion of Mean Time into Apparent Time and *vice versa*.

Mean time may be converted into apparent time by adding algebraically the equation of time for the instant. The value of the equation of time is given in the American Ephemeris for 0^h civil time (midnight) at Greenwich each day, together with the proper algebraic sign. For any other time it must be found by adding or subtracting the amount by which the equation has increased or diminished since midnight. This correction is obtained by multiplying the hours of the Greenwich Civil Time by the variation per hour.

Example. Find the apparent time at Greenwich when the mean time (Civil) is $14^h\ 30^m$ on Oct. 28, 1925. The equation of time at 0^h Greenwich Civil Time is $+16^m\ 05^s.00$; the variation per hour is $+0^s.218$. (The values are numerically increasing.) The corrected equation of time at $14^h\ 30^m$ is therefore $+16^m\ 05^s.00 + 14^h.5 \times 0^s.218 = +16^m\ 05^s.00 + 3^s.16 = 16^m\ 08^s.16$. The Greenwich Apparent Time is $14^h\ 30^m + 16^m\ 08^s.16 = 14^h\ 46^m\ 08^s.16$.

When converting apparent time into mean time we may proceed in either of two ways. Since apparent time is given and the equation is tabulated for mean time it is first necessary to find the mean time with sufficient accuracy to enable us to take out the correct equation of time.

Example. The Gr. Apparent Time is $14^h\ 46^m\ 08^s.16$ on Oct. 28, 1925; find the Gr. Civil Time. Subtracting the approximate equation $(+16^m\ 05^s.00)$ we obtain $14^h\ 30^m\ 03^s$ for the approximate Gr. Civil Time. The corrected equation is therefore $+16^m\ 05^s.00 + 0^s.218 \times 14^h.5 = +16^m\ 08^s.16$ and the Gr. Civil Time is $14^h\ 30^m\ 00^s.00$.

If preferred the Ephemeris of the sun for the meridian of Washington (following the star lists) may be used. The equation for Washington Apparent noon Oct. 28, 1925, is $-16^m\ 08^s.49$; varia. per hour $= -0^s.196$. Since the longitude of Washington is $5^h\ 08^m\ 15^s.78$ west, the Washington Apparent Time corresponding to Greenwich Apparent Time $14^h\,46^m\,08^s.16$ is $9^h\,38^m\,52^s.38$. The equation for this instant is $-16^m\,08^s.49 + 0.196 \times 2^h.35 = -16^m\,08^s.03$. This fails to check the equation derived above ($+16^m\,08^s.16$) because the method of interpolation is imperfect. If a more accurate interpolation formula is used the results check to hundredths.

29. Astronomical Time — Civil Time.

Previous to 1925 the time used in the Ephemeris was Astronomical Time, in which 0^h occurred at the instant of noon, the hours being counted continuously up to 24^h. In this system the date changed at noon, so that in the afternoon the Astronomical and Civil dates agreed but in the forenoon they differed one day. For example: 7^h P.M. of Jan. 3 would be 7^h Jan. 3 in astronomical time; but 3^h A.M. of May 11 would be 15^h, May 10, when expressed in astronomical time.

Beginning with the issue for 1925 the time used in the Ephemeris is designated as Civil Time, the hours being counted from midnight to midnight. The dates therefore change at midnight, as in ordinary civil time, the only difference being that in the 24-hour system the afternoon hours are greater than 12.

For ordinary purposes we prefer to divide the day into halves and to count from two zero points; from midnight to noon is called A.M. (ante meridiem), and from noon to midnight is called P.M. (post meridiem). When consulting the Ephemeris or the Nautical Almanac it is necessary to add 12^h to the P.M. hours before looking up corresponding quantities. The data found opposite 3^h are for 3^h A.M.; those opposite 15^h are for 3^h P.M.

30. Relation between Longitude and Time.

The hour angle of the sun, counted from the lower meridian of any place, is the solar time at that meridian, and will be apparent or mean according to which sun is being considered. The hour angle of the sun from the (lower) meridian of Greenwich is the corresponding Greenwich solar time. The difference between the two times, or hour angles, is the longitude of the place east or west of Greenwich, and expressed either in degrees or in hours according as the hour angles are in degrees or in hours. Similarly, the difference between the local solar times of any two places at a given instant is their difference in longitude in hours, minutes, and seconds. In Fig. 33, $A'AC$ is the Greenwich solar time or the hour angle of the sun from A'; $B'BC$ is the time at P or the hour angle of the sun from B'. The difference $A'B'$, or AB, is the longitude of P west of Greenwich.

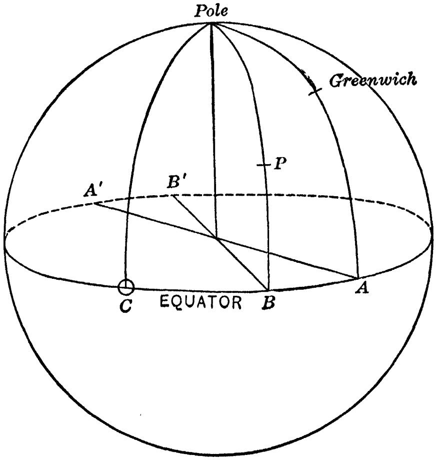


FIG. 33

It should be observed that the reasoning is exactly the same whether C represents the true sun or the fictitious sun. The same result would be found if C were to represent the vernal equinox. In this case the arc AC would be the hour angle of the equinox, or the Greenwich Sidereal Time. BC would be the Local Sidereal Time at P and AB would be the difference in longitude. The measurement of longitude differences is therefore independent of the kind of time used, provided the times compared are of the *same* kind.

The truth of the preceding may be more readily seen by noticing that the difference in the two sidereal times, at meridian A and meridian B, is the interval of sidereal time during which a star would appear to travel from A to B. Since the star requires 24 sidereal hours to travel from A to A again, the time interval AB bears the same relation to 24 sidereal hours that the longitude difference bears to 360°. The difference in the mean solar times at A and B is the number of solar hours that the mean sun would require to travel from A to B; but since the sun requires 24 solar hours to go from A to A again, the time interval from A to B bears the same ratio to 24 solar hours that the longitude difference bears to 360°. The difference in longitude is correctly given when either time is used, provided the same kind of time is used for both places.

To Change from Greenwich Time to Local Time or from Local Time to Greenwich Time.

The method of changing from Greenwich to local time (and the reverse) is illustrated by the following examples. Remember that the more easterly place will have the later time.

Example 1. The Greenwich Civil Time is $19^h\ 40^m\ 10^s.0$. Required the civil time at a meridian $4^h\ 50^m\ 21^s.0$ West.

$$\begin{aligned} \text{Gr. Civ. T.} &= 19^h\ 40^m\ 10^s.0 \\ \text{Long. West} &= \underline{\ \ 4^h\ 50^m\ 21^s.0} \\ \text{Loc. Civ. T.} &= 14^h\ 49^m\ 49^s.0 \\ &= \ \ 2^h\ 49^m\ 49^s.0 \text{ P.M.} \end{aligned}$$

Example 2. The Greenwich Civil Time is $3^h\ 00^m$. Required the local civil time at a place whose longitude is $8^h\ 00^m$ West. In this instance the time at the place is 8^h earlier than 3^h, that is it is 5^h before midnight of the preceding day, or 19^h. This may also be obtained by adding 24^h to the given 3^h before subtracting the longitude difference.

$$\begin{aligned} \text{Gr. Civ. T.} &= 27^h\ 00^m \\ \text{Long. West} &= \underline{\ \ 8^h\ \ \ \ \ \ } \\ \text{Loc. Civ. T.} &= 19^h\ 00^m \\ &= \ \ 7^h\ 00^m \text{ P.M.} \end{aligned}$$

Example 3. The Greenwich Civil Time is $20^h\ 00^m$. What is the time at a place 3^h east of Greenwich?

$$\begin{aligned} \text{Gr. Civ. T.} &= 20^h\ 00^m \\ \text{Long. East} &= \underline{\ \ 3^h\ 00^m} \\ \text{Loc. Civ. T.} &= 23^h\ 00^m \\ &= 11^h\ 00^m \text{ P.M.} \end{aligned}$$

31. Relation between Hours and Degrees.

Since a circle may be divided either into 24^h or into $360°$, the relation between these two units is constant.

Since $$24^h = 360°,$$
we have $$1^h = 15°,$$
$$1^m = 15',$$
$$1^s = 15''.$$

Dividing the second equation by 15 we have
$$4^m = 1°;$$
also $$4^s = 1'.$$

By means of these two sets of equivalents, hours may be converted into degrees, and degrees into hours without writing down the intermediate steps. If it is desired to state the process as a rule it may be done as follows: To convert degrees into hours, divide the degrees by 15 and call the result hours; multiply the remainder by 4 and call the result minutes; divide the minutes (of an angle) by 15 and call the result minutes (of time); multiply the remainder by 4 and call it seconds; divide the seconds (of angle) by 15 and call the result seconds (of time).

Example. Convert $47° 17' 35''$ into hours, minutes and seconds.

$$\begin{aligned} 47° &= 45° + 2° = 3^h\ 08^m \\ 17' &= 15' + 2' = \quad 01^m\ 08^s \\ 35'' &= 30'' + 5'' = \quad\quad 02^s.33 \\ \text{Result} &= 3^h\ 09^m\ 10^s.33 \end{aligned}$$

To convert hours into degrees, reverse this process.

Example. Convert $6^h\ 35^m\ 51^s$ into degrees, minutes, and seconds.

$$\begin{aligned} 6^h &= 90° \\ 35^m &= 32^m + 3^m = 8°\ 45' \\ 51^s &= 48^s + 3^s = \quad 12'\ 45'' \\ \text{Result} &= 98°\ 57'\ 45'' \end{aligned}$$

One should be careful to use *m* and *s* for the minutes and seconds corresponding to hours, and ′, ″ for the minutes and seconds corresponding to degrees.

It should be observed that the relation $15^\circ \doteq 1^h$ is quite independent of the length of time which has elapsed. A star requires one sidereal hour to increase its hour angle 15°; the sun requires one solar hour to increase its hour angle 15°. In the sense in which the term is used here 1^h means primarily an angle, not an absolute interval of time. It becomes an absolute interval of time only when a particular kind of time is specified.

32. Standard Time.

From the definition of mean solar time it will be seen that at any given instant the solar times at two places will differ by an amount equal to their difference in longitude expressed in hours, minutes, and seconds. Before 1883 it was customary in this country for each large city or town to use the mean solar time of a meridian passing through that place, and for the smaller towns in that vicinity to adopt the same time. Before railroad travel became extensive this change of time from one place to another caused no great difficulty, but with the increased amount of railroad and telegraph business these frequent and irregular changes of time became so inconvenient and confusing that in 1883 a uniform system of time was adopted. The country is divided into time belts, each one theoretically 15° wide. These are known as the Eastern, Central, Mountain, and Pacific time belts. All places within these belts use the mean local time of the 75°, 90°, 105°, and 120° meridians respectively. The time of the 60° meridian is called Atlantic time and is used in the Eastern part of Canada. The actual positions of the dividing lines between these time belts depend partly upon the location of the large cities and the points at which the railway companies change their time. The lines shown in Fig. 34 are in accordance with the decisions of the Interstate Commerce Commission in 1918. Wherever the change of time occurs the amount of the change is always exactly one hour. The minutes and seconds of all clocks are the same as those of the Greenwich clock. When it is noon at Greenwich it is 7^h A.M. Eastern time, 6^h A.M. Central time, 5^h A.M. Mountain time, and 4^h A.M. Pacific time.

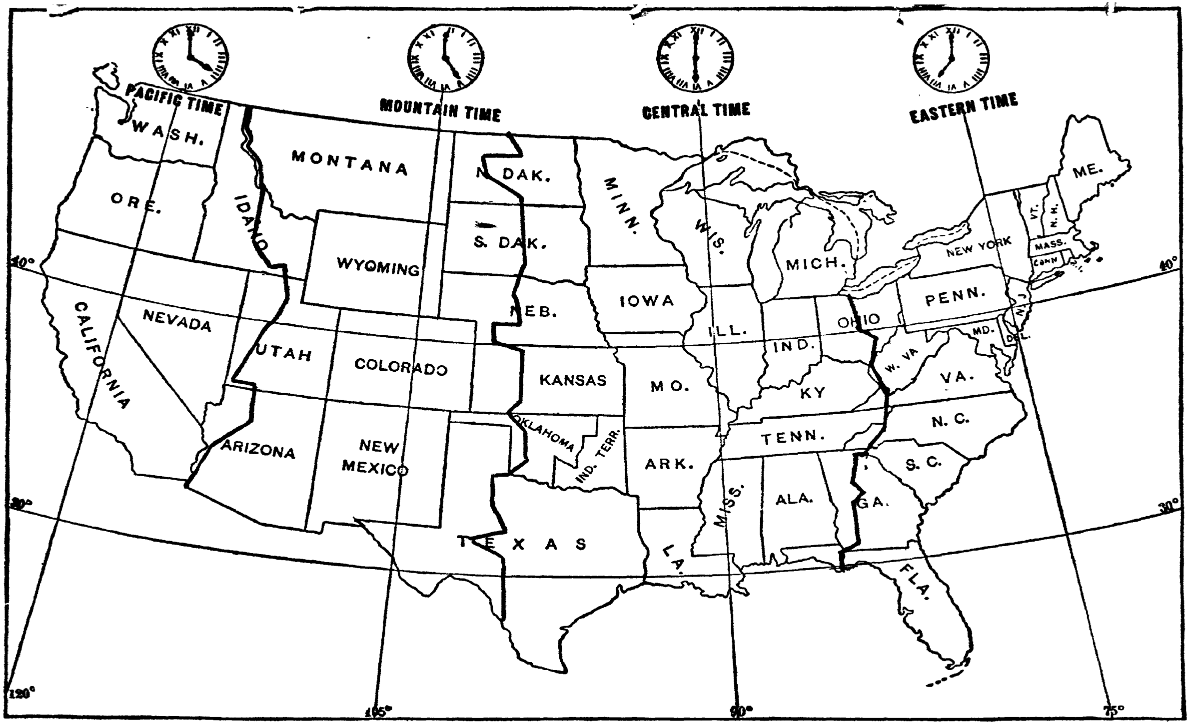


FIG. 34. MAP SHOWING THE STANDARD TIME BELTS IN THE UNITED STATES
(The clocks show the time corresponding to the instant of Greenwich mean noon.)

Standard time is now in use in the principal countries of the world; in most cases the systems of standard time are based on the meridian of Greenwich.

Daylight Saving time for any locality is the time of a belt that lies one hour to the east of the place in question. If, for example, in the Eastern States the clocks are set to agree with those of the Atlantic time belt (60° meridian west) this is designated as daylight saving time in the Eastern time belt.

To Change from Local to Standard Time or the Contrary.

The change from local to standard time, or the contrary, is made by expressing the difference in longitude between the given meridian and the standard meridian in units of time and adding or subtracting this correction, remembering that the farther west a place is the earlier it is in the day at the given instant of time.

Example 1. Find the standard time at a place 71° West of Greenwich when the local time is $4^h\ 20^m\ 00^s$ P.M. In longitude 71° the standard time would be that of the 75° meridian. The difference in longitude is $4° = 16^m$. Since the standard meridian is west of the 71° meridian the time there is 16^m earlier than the local time. The standard time is therefore $4^h\ 04^m\ 00^s$ P.M.

Example 2. Find the local time at a place 91° West of Greenwich when the Central Standard time is $9^h\ 00^m\ 00^s$ A.M. The difference in longitude is $1° = 4^m$. Since the place is west of the 90° meridian the local time is earlier. The local time is therefore $8^h\ 56^m\ 00^s$ A.M.

33. Relation between Sidereal Time, Right Ascension, and Hour Angle of any Point at a Given Instant.

In Fig. 35 the hour angle of the equinox, or local sidereal time, at the meridian of P, is the arc AV. The hour angle of the star S at the meridian of P is the arc AB. The right ascension of the star S is the arc VB. It is evident from the figure that

$$AV = VB + AB$$

or

$$S = \alpha + t \qquad [37]$$

where S = the sidereal time at P, α = the right ascension and t = the hour angle of the star. This relation is a general one and will be found to hold true for all positions, except that it will be necessary to add 24^h to the actual sidereal time when the sum of α and t exceed 24^h. For instance, if the hour angle is 10^h and the right ascension is 20^h the sum is 30^h, so that the actual sidereal time is 6^h. When the sidereal time and the right ascension are given and the hour angle is required we must first add 24^h (if necessary) to the sidereal time ($24^h + 6^h = 30^h$) before subtracting the 20^h right ascension, to obtain the hour angle 10^h. If, however, it is preferred to compute the hour angle in a direct manner the result is the same. When the right ascension

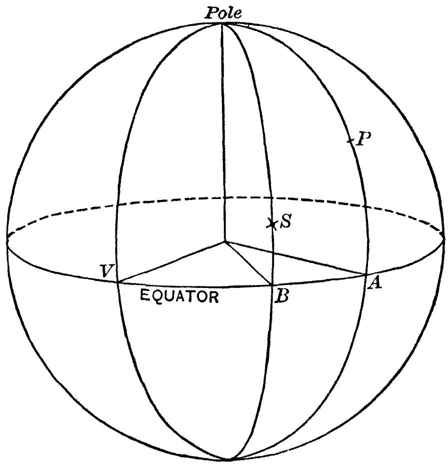


FIG. 35

is 20^h the angle from V westward to the point must be $24^h - 20 = 4^h$. This 4^h added to the 6^h sidereal time gives 10^h for the hour angle as before.

34. Star on the Meridian.

When a star is on any meridian the hour angle of the star at that meridian becomes 0^h. The sidereal time at the place then becomes numerically equal to the right ascension of the star. This is of great practical importance because one of the best methods of determining the time is by observing transits of stars over the plane of the meridian. The sidereal time thus becomes known at once when a star of known right ascension is on the meridian.

35. Mean Solar and Sidereal Intervals of Time.

It has already been stated that on account of the earth's orbital motion the sun has an apparent eastward motion among the stars of nearly 1° per day. This eastward motion of the sun makes the intervals between the sun's transits greater by nearly

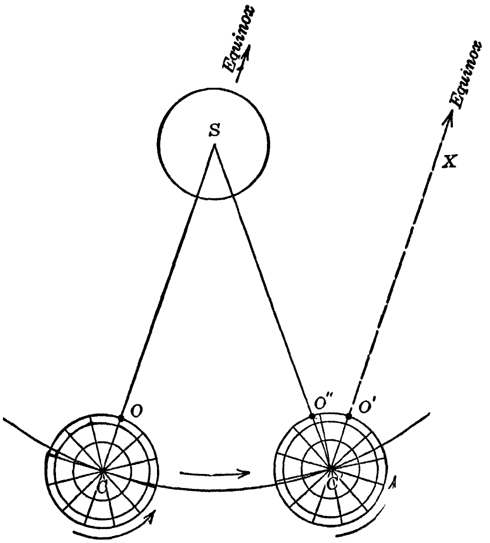


FIG. 36

4^m than the interval between the transits of the equinox, that is, the solar day is nearly 4^m longer than the sidereal day. In Fig. 36, let C and C' represent the positions of the earth on two consecutive days. When the observer is at O it is noon at his meridian. After the earth makes one complete rotation (with reference to a fixed star) the observer will be at O', and the sidereal time will be exactly the same as it was the day before when he was at O. But the sun's direction is now $C'O''$, so the earth must turn through an additional degree (nearly) until the sun is again on this observer's meridian. This will require nearly 4^m

additional time. Since each kind of day is subdivided into hours, minutes and seconds, all of these units in solar time will be proportionally larger than the corresponding units of sidereal time. If two clocks, one regulated to mean solar time and the other to sidereal time, were started at the same instant, both reading 0^h, the sidereal clock would immediately begin to gain on the solar clock, the gain being exactly proportional to the time elapsed, that is, about 10^s per hour, or more nearly $3^m\ 56^s$ per day.

In Fig. 36 C and C' may be taken to represent the earth's position at the date of the equinox and any subsequent date. The angle CSC' will then represent that angle through which the earth has revolved in the interval since March 22, and the angle $SC'X$ (always equal to CSC') represents the accumulated difference between solar and sidereal time since March 22. This angle is, of course, equal to the sun's right ascension. The angle $SC'X$ becomes 24^h or $360°$ when the angle CSC' becomes $360°$; in other words, at the end of one year the sidereal clock has gained exactly one day.

This fact enables us to establish the exact relation between the two time units. It is known that the tropical year (equinox to equinox) contains 365.2422 mean solar days. Since the number of sidereal days is one greater we have

$$366.2422 \text{ sidereal days} = 365.2422 \text{ solar days},$$

or $$1 \text{ sidereal day} = 0.99726957 \text{ solar days}, \qquad [38]$$

and $$1 \text{ solar day} = 1.00273791 \text{ sidereal days}. \qquad [39]$$

Equations [38] and [39] may be written

$$24^h \text{ sidereal time} = (24^h - 3^m\ 55^s.909) \text{ mean solar time}. \qquad [40]$$

$$24^h \text{ mean solar time} = (24^h + 3^m\ 56^s.555) \text{ sidereal time}. \qquad [41]$$

These equations may be put into more convenient form for computation by expressing the difference in time as a correction to be applied to any interval of time to change it from one unit to the other. If I_m is a mean solar interval and I_s the corresponding number of sidereal units, then

$$I_s = I_m + 0.00273791 \times I_m \qquad [42]$$

and

$$I_m = I_s - 0.00273043 \times I_s. \qquad [43]$$

These give $+9^s.8565$ and $-9^s.8296$ as the corresponding corrections for one hour of solar and sidereal time respectively. Tables II and III (pp. 227–8) were constructed by multiplying different values of I_m and I_s by the constants in Equa. [42] and [43]. More extended tables (II and III) will be found in the Ephemeris.

Example 1. Assuming that a sidereal chronometer and a solar clock start together at a zero reading, what will be the reading of the solar clock when the sidereal chronometer reads $9^h\ 23^m\ 51^s.0$? From Table II, opposite 9^h, is the correction $-1^m\ 28^s.466$; opposite 23^m and in the 4th column is $-3^s.768$, and opposite 51^s and in the last column is $0^s.139$. The sum of these three partial corrections is $-1^m\ 32^s.373$; $9^h\ 23^m\ 51^s.0 - 1^m\ 32^s.373 = 9^h\ 22^m\ 18^s.627$, the reading of the solar clock.

Example 2. Reduce $7^h\ 10^m$ in solar time units to the corresponding interval in sidereal time units. In Table III the correction for 7^h is $+1^m\ 08^s.995$; for 10^m it is $+1^s.643$. The sum, $1^m\ 10^s.638$, added to $7^h\ 10^m$ gives $7^h\ 11^m\ 10^s.638$ of sidereal time.

It should be remembered that the conversion of time discussed above concerns the change of a short interval of time from one kind of unit to another, and is like changing a distance from yards to metres. When changing a long interval of time such, for example, as finding the local sidereal time on Aug. 1, when the local solar time is 10^h A.M., we make use of the total accumulated difference between the two times since March 22, which is the same thing as the right ascension of the mean sun.

36. Approximate Corrections.

Since both corrections are nearly equal to 10^s per hour, or 4^m per day, we may use these as rough approximations. For a still closer correction we may allow 10^s per hour and then deduct 1^s for each 6^h in the interval. The correction for 6^h would then be $6 \times 10^s - 1^s = 59^s$. The error of this correction is but

$0^s.023$ per hour for solar time and $0^s.004$ per hour for sidereal time.

37. Relation between Sidereal Time and Mean Solar Time at any Instant.

If in Fig. 35, Art. 37, the point B is taken to represent the *mean sun*, then equation [37] becomes

$$S = \alpha_s + t_s \qquad [44]$$

in which α_s and t_s are the right ascension and hour angle of the mean sun at the instant considered. If the civil time is represented by T then $t_s = T + 12^h$, and

$$S = \alpha_s + T + 12^h \qquad [45]$$

which enables us to find sidereal time when civil time is given and vice versa. If the equation is written

$$S - T = \alpha_s + 12^h, \qquad [46]$$

then, since the value of α_s does not depend upon the time at any place but only upon the absolute instant of time considered, it is evident that the difference between sidereal time and civil time at any instant is the same for all places on the earth. The values of S and T will be different at different meridians, but the difference, $S - T$, is the same for all places at the given instant.

In order that Equa. [45] shall hold true it is essential that α_s and T shall refer to the same position of the sun, that is, to the same absolute instant of time. The right ascension of the mean sun obtained from the Ephemeris is its value of 0^h of Greenwich Civil Time. To reduce this right ascension to its value at the desired instant it is necessary to increase it by a correction equal to the product of the hourly increase in the right ascension times the number of hours elapsed since midnight, that is, by the number of hours in the Greenwich Civil Time (T). The hourly increase in the right ascension of the mean sun is constant and equal to $+9^s.8565$ per solar hour. This is the same quantity that was tabulated as the "reduction from solar to sidereal units of time" and is given in Table III. The difference between solar and sidereal time is due to the fact that the sun's right ascension increases, hence the two are numerically the same. It is not necessary in practice to multiply the above constant by the hours of the civil time, but the correction may be looked up at once in Table III. Similarly, Table II furnishes at once the correction to the right ascension for any number of sidereal hours. Equation [45] will not hold true, therefore, until the above correction to α_s has been made, and this correction may be regarded either as the increase in the right ascension or as the change from solar to sidereal time, or the contrary.

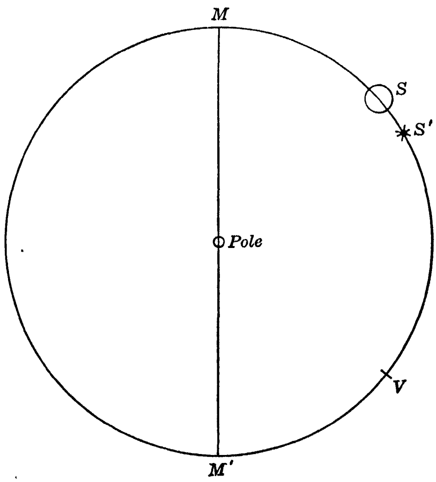


FIG. 37

Suppose that the sun S (Fig. 37) and a star S' passed the meridian opposite M at the same instant, and that at the civil time T it is desired to compute the corresponding sidereal time. Since the sun is apparently moving at a slower rate than the star, it will describe the arc $M'MS$ ($= T$) while the star describes the arc $M'MS'$. The arc SS' represents the gain of sidereal time on mean time during the mean time interval T. But S'

is the position of the sun at 0^h and VS' is the right ascension (α_s) at 0^h. The required right ascension is VS (or α_s at time T) so α_s at 0^h must be increased by the amount SS', or the correction from Table III corresponding to T hours.

The right ascension of the mean sun is given in the Ephemeris as "sidereal time of 0^h G. C. T." or "right ascension of the mean sun $+ 12^h$." For convenience we may write Equa. [45] in the form

$$S = (\alpha_s + 12^h) + T \qquad [45a]$$

in which it is understood that a correction (Table III) is to be added to reduce the interval T to sidereal units.

If the student has difficulty in understanding the process indicated by Equa. [45a] it may be helpful to remember that all the quantities represented are really angles, and may be expressed in degrees, minutes, and seconds. If all three parts, the sun's right ascension $+ 12^h$, the hour angle of the mean sun from the lower meridian (T), and the increase in the sun's right ascension since midnight (Table III), are expressed as angles then it is not difficult to see that the hour angle of the equinox is the sum of these three parts.

Another view of it is that the actual sidereal time interval from the transit of the equinox over the upper meridian to the transit of the "mean sun" over the lower meridian (midnight) is $\alpha_s + 12^h$; to obtain the sidereal time (since the upper transit of the equinox) we must add to this the sidereal time interval since midnight, which is the mean time interval since midnight plus the correction in Table III.

Example 1. To find the Greenwich Sidereal Time corresponding to the Greenwich Civil Time $9^h\ 00^m\ 00^s$ on Jan. 7, 1925. The "right ascension of the mean sun $+ 12^h$" for Jan. 7, 1925, is $7^h\ 04^m\ 09^s.74$. The correction in Table III for 9^h is $+1^m\ 28^s.71$. The sidereal time is then found as follows:

$$\begin{aligned}(\alpha_s + 12^h) \text{ at } 0^h &= 7^h\ 04^m\ 09^s.74\\ T &= 9\ \ 00\ \ 00\\ \text{Table III} &= \underline{\quad\ \ 1\ \ 28\ .71}\\ S &= 16^h\ 05^m\ 38^s.45\end{aligned}$$

If it is desired to find the civil time T when the sidereal time S is given, the equation is

$$T = S - (\alpha_s + 12^h). \qquad [45b]$$

In this instance it is not possible to correct the right ascension at once for the change since 0^h, for that is not yet known. It is possible, however, to find the number of sidereal hours since midnight, for this results directly from the subtraction of the tabulated value of $(\alpha_s + 12^h)$ from S. T is therefore found by subtracting from this last result the corresponding correction in Table II.

Example 2. If the Greenwich sidereal time $16^h\,05^m\,38^s.45$ had been given, to find the civil time, we should first subtract from S the tabulated value of $\alpha_s + 12^h$, obtaining the sidereal interval of time since midnight. This interval less the correction in Table II is the civil time, T.

S =	$16^h\,05^m\,38^s.45$
$(\alpha_s + 12^h)$ at 0^h =	7 04 09 .74
Sidereal interval =	$9^h\,01^m\,28^s.71$

From Table II we find

for 9^h =	$1^m\,28^s.466$
for 1^m =	.164
for $28^s.71$ =	.078
total corr. =	$1^m\,28^s.708$

Subtracting this from the above sidereal interval we have

$$T = 9^h\,00^m\,00^s.$$

Example 3. If the time given is that for a meridian other than that of Greenwich the corresponding Greenwich time may be found at once (Art. 30) and the problem solved as before. Suppose that the civil time is 11^h at a place 60° (= 4^h) west of Greenwich and the date is May 1, 1925. The right ascension $+12^h = 14^h\,33^m\,36^s.86$. Then,

Local Civil Time =	$11^h\,00^m\,00^s$
Add Longitude W. =	4 00 00
Gr. Civil Time =	$15^h\,00^m\,00^s$
$(\alpha_s + 12^h)$ at 0^h =	14 33 36 .86
Table III =	2 27 .85
Gr. Sid. Time =	$29^h\,36^m\,04^s.71$
Subtract Long. W. =	4^h
Loc. Sid. Time =	$25^h\,36^m\,04^s.71$
=	$1^h\,36^m\,04^s.71$

Example 4. If the local sidereal time had been given, to find the local civil time the computation would be as follows:

Local Sidereal Time =	$1^h\ 36^m\ 04^s.71$
Add Longitude W. =	4
Greenwich Sidereal Time =	$5^h\ 36^m\ 04^s.71$ (add 24^h)
$(\alpha_s + 12^h)$ at 0^h =	14 33 36 .86
Sidereal Interval =	$15^h\ 02^m\ 27^s.85$
Table II =	2 27 .85
Greenwich Civil Time =	$15^h\ 00^m\ 00^s$
Subtract Longitude W. =	4
Local Civil Time =	$11^h\ 00^m\ 00^s$

Example 5. Alternative method. The same result may be obtained by applying to the tabulated $\alpha_s + 12^h$ a correction to reduce it to its value at 0^h of local civil time. The time interval between 0^h at Greenwich and 0^h at the given place is equal to the number of hours in the longitude, in this case 4 solar hours. In Table III we find for 4^h the correction $+39^s.426$. The value of $(\alpha_s + 12^h)$ at 0^h local time is $14^h\ 33^m\ 36^s.86 + 39^s.43 = 14^h\ 34^m\ 16^s.29$. (If the longitude is east this correction is subtractive). The remainder of the computation is as follows:

Local Civil Time =	$11^h\ 00^m\ 00^s$
$(\alpha_s + 12^h)$ at 0^h (local) =	14 34 16 .29
Table III =	1 48 .42
Local Sidereal Time =	$25^h\ 36^m\ 04^s.71$
=	$1^h\ 36^m\ 04^s.71$

Conversely,

Local Sidereal Time =	$1^h\ 36^m\ 04^s.71$ (add 24^h)
$(\alpha_s + 12^h)$ at 0^h (local) =	14 34 16 .29
Sidereal interval =	$11^h\ 01^m\ 48^s.42$
Table II =	01 48 .42
Local Civil Time =	$11^h\ 00^m\ 00^s.00$

38. The Date Line.

If a person were to start at Greenwich at the instant of noon and travel westward at the rate of about 600 miles per hour, *i.e.*, rapidly enough to keep the sun always on his own meridian, he would arrive at Greenwich 24 hours later, but his own (local) time would not have changed at all; it would have remained *noon* all the time. His date would therefore not agree with that kept at Greenwich but would be a day behind it. When travelling westward at a slower rate the same thing happens except that it takes place in a longer interval of time. The traveller has to set his watch back a little every day in order to keep it regulated to the meridian at which his noon occurs. As a consequence, after he has circumnavigated the globe, his watch has recorded one day less than it has actually run, and his calendar is one day behind that of a person who remained at Greenwich. If the traveller goes east he has to set his watch ahead every day, and, after circumnavigating the globe, his calendar is one day ahead of what it should be. In order to avoid these discrepancies in dates it has been agreed to change the date when crossing the 180° meridian from Greenwich. Whenever a ship crosses the 180° meridian, going westward, a day is omitted from the calendar; when going eastward, a day is repeated. As a matter of practice the change is made at the midnight occurring nearest the 180° meridian. For example, a steamer leaving Yokohama July 16th at noon passed the 180° meridian about 4 P.M. of the 22d. At midnight, when the date was to be changed, the calendar was set back one day. Her log therefore shows two days dated Monday, July 22. She arrived at San Francisco on Aug. 1 at noon, having taken 17 days for the trip.

The *international date line* actually used does not follow the 180° meridian in all places, but deviates so as to avoid separating the Aleutian Islands, and in the South Pacific Ocean it passes east of several groups of islands so as not to change the date formerly used in these islands.

39. The Calendar.

Previous to the time of Julius Cæsar the calendar was based upon the *lunar month*, and, as this resulted in a continual change in the dates at which the seasons occurred, the calendar was frequently changed in an arbitrary manner in order to keep the seasons in their places. This resulted in extreme confusion in the dates. In the year 45 B.C., Julius Cæsar reformed the calendar and introduced one based on a year of $365\frac{1}{4}$ days, since called the Julian Calendar. The $\frac{1}{4}$ day was provided for by making the ordinary year contain 365 days, but every fourth year, called *leap year*, was given 366 days. The extra day was added to February in such years as were divisible by 4.

Since the year actually contains $365^d\ 5^h\ 48^m\ 46^s$, this difference of $11^m\ 14^s$ caused a gradual change in the dates at which the seasons occurred. After many centuries the difference had accumulated to about 10 days. In order to rectify this error Pope Gregory XIII, in 1582, ordered that the calendar should be corrected by dropping ten days and that future dates should be computed by omitting the 366th day in those leap years which occurred in century years not divisible by 400; that is, such years as 1700, 1800, 1900 should not be counted as leap years.

This change was at once adopted by the Catholic nations. In England it was not adopted until 1752, at which time the error had accumulated to 11 days. Up to that time the legal year had begun on March 25, and the dates were reckoned according to the Julian Calendar. When consulting records referring to dates previous to 1752 it is necessary to determine whether they are dated according to "Old Style" or "New Style." The date March 5, 1740, would now be written March 16, 1741. "Double dating," such as 1740–1, is frequently used to avoid ambiguity.

Questions and Problems

1. If a sun dial shows the time to be 9^h A.M. on May 1, 1925, at a place in longitude 71° West what is the corresponding Eastern Standard Time? The corrected equation of time is $+2^m\ 56^s$.

2. When it is apparent noon on Oct. 1, 1925, at a place in longitude 76° West what is the Eastern Standard Time? The corrected equation of time is $+10^m\ 17^s$.

3. Make a design for a horizontal sun dial for a place whose latitude is 42° 21′ N. The gnomon *ad* (Fig. 38), or line which casts the shadow on the horizontal plane, must be parallel to the earth's rotation axis; the angle which the gnomon makes with the horizontal plane therefore equals the latitude. The shadow lines for the hours (X, XI, XII, I, II, etc.) are found by passing planes through the gnomon and finding where they cut the horizontal plane of the dial. The vertical plane *adb* coincides with the meridian and therefore is the noon (XII^h) line. The other planes make, with the vertical plane, angles equal to some multiple of 15°. In finding the trace *dc* of one of these planes on the dial it should be observed that the

foot of the gnomon, d, is a point common to all such traces. In order to find another point c on any trace, or shadow line, pass a plane abc through some point a on the gnomon and perpendicular to it. This plane (the plane of the equator) will cut an east and west line ce on the dial. If a line be drawn in this plane making an angle of $n \times 15°$ with the meridian plane, it will cut ce at a point c which is on the shadow line. Joining c with the foot of the gnomon gives the required line.

In making a design for a sun dial it must be remembered that the west edge of the gnomon casts the shadow in the forenoon and the east edge in the afternoon; there will be of course two noon lines, and the two halves of the diagram will be symmetrical and separated from each other by the thickness of the gnomon. The

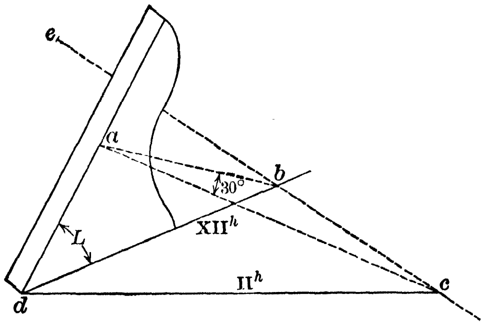


FIG. 38

dial may be placed in position by levelling the horizontal surface and then computing the watch time of apparent noon and turning the dial so that the shadow is on the XII^h line at the calculated time.

Prove that the horizontal angle bdc is given by the relation

$$\tan bdc = \tan t \sin \phi,$$

in which t is the sun's hour angle and ϕ is the latitude.

4. Prove that the difference in longitude of two points is independent of the kind of time used, by selecting two points at which the solar time differs by say 3^h, and then converting the solar time at each place into sidereal time.

5. On Jan. 20, 1925, the Eastern Standard Time at a certain instant is $7^h 30^m$ P.M. (Civ. T. $19^h 30^m$). What is the local sidereal time at this instant at a place in longitude 72° 10′ West? (Right ascension of Mean Sun + 12^h at 0^h G. C. T. = $7^h 55^m 25^s.0$.)

6. At a place in longitude 87° 30′ West the local sidereal time is found to be $19^h 13^m 10^s.5$ on Sept. 30, 1925. What is the Central Standard Time at this instant? (The right ascension of Mean Sun + 12^h at 0^h G. C. T. = $0^h 32^m 53^s.2$.)

7. If a vessel leaves San Francisco on July 16 and makes the trip in 17 days, on what date will she arrive at Yokohama?

CHAPTER VI

THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC — STAR CATALOGUES — INTERPOLATION

40. The Ephemeris.

In discussing the problems of the previous chapters it has been assumed that the right ascensions and declinations of the celestial objects and the various other data mentioned are known to the computer. These data consist of results calculated from observations made with large instruments at the astronomical observatories, and are published by the Government (Navy Dept.) in the American Ephemeris and Nautical Almanac. This may be obtained a year or two in advance from the Superintendent of Documents, Washington, D.C., price one dollar, It contains the coördinates of the sun, moon, planets, and stars, as well as the semidiameters, parallaxes, the equation of time, and other necessary data.

It should be observed that the quantities given in the Almanac vary with the time and are therefore computed for equidistant intervals of solar time at some assumed meridian, usually that of the Greenwich (England) Observatory.

The Ephemeris is divided into three principal parts. Part I contains the data for the sun, moon, and planets, at stated hours of Greenwich Civil Time, usually at 0^h (midnight), or the beginning of the Civil Day. (Previous to 1925 such data were given for Greenwich Mean Noon.) Part II contains the lists of star places, the data being referred to the meridian of the U. S. Naval Observatory at Washington ($5^h\ 08^m\ 15^s.78$ west of Greenwich); the instant being that of transit. Part III con-

* Similar publications by other governments are: The Nautical Almanac (Great Britain), Berliner Astronomisches Jahrbuch (Germany), Connaissance des Temps (France), and Almanaque Nautico (Spain).

tains data needed for the prediction of eclipses, occultations, etc. At the end of the volume will be found a series of tables of particular value to the surveyor.

There is also published a smaller volume entitled The American Nautical Almanac* which contains data for the sun, moon, and stars referred to the meridian of Greenwich. The arrangement of the tables is somewhat different from that given in the Ephemeris. This almanac is intended primarily for the use of navigators.

Whenever the value of a coördinate, or other quantity, is given in the Ephemeris, it is stated for a particular instant of Greenwich (or Washington) time, and the rate of change, or variation per hour, of the quantity is given for the same instant. These rates of change are the differential coefficients of the tabulated functions. If the value of the quantity is desired for any other instant it is essential that the Greenwich time for that instant be known. The accuracy with which this time must be known will depend upon how rapidly the coördinate is varying. If the time given is local time it must be converted into Greenwich time as explained in Chapter V.

On p. 67 is a sample page taken from the Ephemeris for 1925. On p. 69 are given portions of the table of "mean places" of stars, both circumpolars and others. On pp. 70 and 71 are extracts from the tables of "apparent places" in which the coördinates are given for every day for close circumpolars and for every 10 days for other stars. The precession of the equinoxes causes the right ascension of close circumpolar stars to vary much more rapidly and more irregularly than for stars nearer the equator; the coördinates are therefore given at more frequent intervals. On p. 72 are extracts from the Nautical Almanac and The Washington Tables of the Ephemeris for 1925.

In Part II of the Ephemeris will be found a table entitled

* Sold by the Superintendent of Documents, Washington, D. C., for 15 cents.

SUN, 1925

For 0^h Greenwich Civil Time

Date	Day of the Week	Apparent Right Ascension	Var. per Hour	Apparent Declination	Var. per Hour	Semi-diameter	Hor. Par.	Equation of Time. App. — Mean	Var. per Hour	Sidereal Time. Right Ascension of Mean Sun. $+12^h$
		h m s	s	° ′ ″	″	′ ″	″	m s	s	h m s
Jan. 1	Th	18 43 51 25	11 049	−23 3 51.8	+11 57	16 17.80	8.95	− 3 20 85	−1 193	6 40 30.40
2	Fr	18 48 16 27	11 035	22 59 0 4	12 72	16 17 91	8 95	3 49 32	1.179	6 44 26.95
3	Sa	18 52 40 94	11.020	22 53 41 4	13 86	16 17 91	8 95	4 17.43	1.163	6 48 23.51
4	Su	18 57 5 22	11 003	22 47 55 0	15 00	16 17 91	8 95	4 45 15	1.147	6 52 20.07
5	Mo	19 1 29 09	10 986	22 41 41 5	16 13	16 17 91	8 95	5 12 47	1.129	6 56 16.62
6	Tu	19 5 52 53	10 967	−22 35 1 1	+17.25	16 17.90	8.95	− 5 39 35	−1.110	7 0 13.18
7	We	19 10 15 50	10 947	22 27 53 9	18.35	16 17 88	8.95	6 5 77	1.091	7 4 9.74
8	Th	19 14 37 99	10 927	22 20 20 1	19 46	16 17.86	8.95	6 31 70	1.070	7 8 6.30
9	Fr	19 18 59 97	10.905	22 12 19 9	20 55	16 17.83	8.95	6 57.12	1.048	7 12 2.85
10	Sa	19 23 21 41	10.882	22 3 53 6	21.63	16 17.79	8.95	7 22 01	1.025	7 15 59.41
11	Su	19 27 42 31	10.859	−21 55 1 5	+22 71	16 17.75	8.95	− 7 46 34	−1.002	7 19 55.97
12	Mo	19 32 2 63	10.834	21 45 43 7	23 77	16 17 71	8 95	8 10 10	0 978	7 23 52.52
13	Tu	19 36 22 35	10.809	21 36 0 6	24 82	16 17 65	8.95	8 33.27	0 952	7 27 49.08
14	We	19 40 41 46	10.783	21 25 52 4	25.86	16 17 59	8 95	8 55 82	0.927	7 31 45.64
15	Th	19 44 59.94	10.757	21 15 19 4	26 89	16 17 53	8.95	9 17 75	0 900	7 35 42.20
16	Fr	19 49 17 77	10.729	−21 4 21 9	+27 90	16 17 45	8.95	− 9 39 03	−0 873	7 39 38.75
17	Sa	19 53 34 94	10.702	20 53 0 2	28 90	16 17 38	8 95	9 59.65	0 845	7 43 35.31
18	Su	19 57 51 45	10.673	20 41 14 6	29 90	16 17 29	8 94	10 19 59	0 817	7 47 31.87
19	Mo	20 2 7.26	10 644	20 29 5 4	30 87	16 17 21	8 94	10 38 83	0.787	7 51 28.42
20	Tu	20 6 22 35	10 614	20 16 33 0	31 83	16 17 12	8.94	10 57 37	0.758	7 55 24.98
21	We	20 10 36.73	10 584	−20 3 37 7	+32 77	16 17 02	8.94	−11 15 20	−0 727	7 59 21.53
22	Th	20 14 50.37	10 553	19 50 19 9	33 70	16 16 92	8.94	11 32 28	0 696	8 3 18.09
23	Fr	20 19 3.26	10 521	19 36 39 9	34 62	16 16.82	8.94	11 48.61	0 665	8 7 14.65
24	Sa	20 23 15.37	10 488	19 22 38 0	35 53	16 16 71	8.94	12 4 17	0.632	8 11 11.20
25	Su	20 27 26.70	10 456	19 8 14 7	36 41	16 16 60	8.94	12 18 95	0.599	8 15 7.76
26	Mo	20 31 37 24	10 422	−18 53 30 3	+37 28	16 16 49	8 94	−12 32 93	−0 565	8 19 4.31
27	Tu	20 35 46 96	10.388	18 38 25.2	38 13	16 16 37	8 94	12 46 10	0 532	8 23 0.87
28	We	20 39 55.87	10.354	18 22 59.9	38.97	16 16 25	8.93	12 58.45	0.497	8 26 57.42
29	Th	20 44 3 95	10.319	18 7 14.7	39.79	16 16 13	8.93	13 9.97	0.463	8 30 53.98
30	Fr	20 48 11 19	10.284	17 51 10.0	40 60	16 16 00	8.93	13 20 66	0 428	8 34 50.54
31	Sa	20 52 17.60	10.249	−17 34 46.3	+41 38	16 15.87	8.93	−13 30.51	−0.392	8 38 47.09
Feb. 1	Su	20 56 23 16	10.214	17 18 3.9	42.15	16 15.74	8.93	13 39 51	0.358	8 42 43.65
2	Mo	21 0 27.89	10.179	17 1 3.2	42 90	16 15.60	8.93	13 47.68	0.323	8 46 40.20
3	Tu	21 4 31 78	10.145	16 43 44.6	43 64	16 15.45	8.93	13 55 01	0 288	8 50 36.76
4	We	21 8 34 83	10.110	16 26 8.5	44.36	16 15.31	8.93	14 1.51	0.253	8 54 33.31
5	Th	21 12 37 04	10.075	−16 8 15.5	+45 06	16 15.15	8.92	−14 7.18	−0.219	8 58 29.87
6	Fr	21 16 38 43	10.041	15 50 5.8	45 75	16 14.99	8.92	14 12 02	0.185	9 2 26.42
7	Sa	21 20 39.01	10.007	15 31 39.8	46.41	16 14.83	8.92	14 16 04	0.150	9 6 22.98
8	Su	21 24 38 77	9.974	15 12 58.1	47.06	16 14.66	8.92	14 19 25	0.117	9 10 19.53
9	Mo	21 28 37 74	9.940	14 54 0.9	47.70	16 14.49	8.92	14 21 66	0.084	9 14 16.08
10	Tu	21 32 35.92	9.907	−14 34 48.7	+48.32	16 14.31	8.92	−14 23 28	−0.051	9 18 12.64
11	We	21 36 33.31	9 875	14 15 21.9	48.91	16 14.12	8.92	14 24 12	−0.019	9 22 9.19
12	Th	21 40 29.93	9.844	13 55 41.0	49.50	16 13.93	8.91	14 24 19	+0.013	9 26 5.75
13	Fr	21 44 25.80	9 813	13 35 46.2	50.07	16 13.74	8.91	14 23.51	0.044	9 30 2.30
14	Sa	21 48 20.94	9.782	13 15 38 0	50.61	16 13.54	8.91	14 22.09	0.075	9 33 58.86
15	Su	21 52 15 34	9.752	−12 55 16.8	+51.15	16 13.34	8.91	−14 19 94	+0.105	9 37 55.41
16	Mo	21 56 9.02	9.722	−12 34 43.0	+51.66	16 13.13	8.91	−14 17.07	+0.134	9 41 51.96

NOTE. — 0^h Greenwich Civil Time is twelve hours before Greenwich Mean Noon of the same date.

"Moon Culminations." This table gives the data required in determining longitude by observing meridian transits of the moon. (See Art. 94.)

The tables at the end of the Ephemeris, already referred to, include:

Table I.	For finding the Latitude by an observed Altitude of Polaris.
Table II.	Sidereal into Mean Solar Time.
Table III.	Mean Solar into Sidereal Time.
Table IV.	Azimuth of Polaris at All Hour Angles.
Table V.	Azimuth of Polaris at Elongation.
Table V*a*.	For reducing to Elongation observations made near Elongation.
Table VI.	For finding, by observation, when Polaris passes the Meridian.
Table VII.	Time of Upper Culmination, Elongation, etc., and other tables.

41. Star Catalogues.

Whenever it becomes necessary to observe stars which are not included in the list given in the Ephemeris, their positions must be taken from one of the star catalogues. These catalogues give the mean place of each star at some epoch, such as the beginning of the year 1890, or 1900, together with the necessary data for reducing it to the mean place for any other year. The mean place of a star is that obtained by referring it to the mean equinox at the beginning of the year, that is, the position it would occupy if its place were not affected by the small periodic terms of the precession.

The year employed in such reductions is that known as the Besselian fictitious year. It begins when the sun's mean longitude (arc of the ecliptic) is 280°, that is when the right ascension of the mean sun is $18^h\ 40^m$, which occurs about January 1. After the catalogued position of the star has been brought up to the mean place at the beginning of the given year, it must still be reduced to its "apparent place," for the exact date of the ob-

MEAN PLACES OF TEN-DAY STARS, 1925

FOR JANUARY 0^d.654, WASHINGTON CIVIL TIME

Name of Star	Magnitude	Spectrum	Right Ascension	Annual Variation	Annual P.M.	Declination	Annual Variation	Annual P.M.
			h m s	s	s	° ′ ″	″	″
33 Piscium	4.7	Ko	0 1 29.826	+3.0713	−.0006	− 6 7 37.68	+20.135	+0.091
α Andromedæ (*Alpheratz*)	2.2	Aop	0 4 30.419	3.0979	+.0107	+28 40 35.02	19.878	−0.163
β Cassiopeiæ	2.4	F5	0 5 9.934	3.1906	+.0681	+58 44 10.16	19.859	−0.180
ε Phœnicis	3.9	Ko	0 5 36.485	3.0484	+.0096	−46 9 40.94	19.846	−0.193
22 Andromedæ	5.1	Fo	0 6 25.013	3.1131	+.0021	+45 39 17.71	20.032	−0.004
γ Pegasi	2.9	B2	0 9 22.290	+3.0875	+.0003	+14 46 0.00	+20.018	−0.010
σ Andromedæ	4.5	A2	0 14 24.284	3.1302	−.0044	+36 22 10.01	19.958	−0.017
ι Ceti	3.8	Ko	0 15 36.413	3.0567	−.0013	− 9 14 22.41	19.969	−0.030
ζ Tucanæ	4.3	F8	0 16 10.633	3.1411	+.2734	−65 18 54.60	21.167	+1.172
44 Piscium	6.0	G5	0 21 33.433	3.0747	−.0014	+ 1 31 27.67	19.933	−0.023
β Hydri	2.9	Go	0 21 50.154	+3.1856	+.6947	−77 40 35.90	+20.272	+0.319
α Phœnicis	2.4	Ko	0 22 34.886	2.9702	+.0187	−42 42 47.83	19.544	−0.403
12 Ceti	6.0	K5	0 26 12.693	3.0623	+.0011	− 4 22 17.32	19.913	0.000
13 Ceti†	5.2	Go	0 31 23.214	3.0873	+.0273	− 4 0 19.60	19.840	−0.017
ζ Cassiopeiæ	3.7	B2	0 32 47.044	3.3338	+.0036	+53 29 3.76	19.833	−0.007
π Andromedæ	4.4	B3	0 32 52.213	+3.2002	+.0019	+33 18 24.20	+19.839	0.000
ε Andromedæ	4.5	G5	0 34 35.279	3.1665	−.0172	+28 54 17.07	19.563	−0.254
δ Andromedæ	3.5	Ko	0 35 18.777	3.2044	+.0110	+30 27 2.27	19.710	−0.097
α Cassiopeiæ (*Schedir*)†	*var.*	Ko	0 36 14.395	3.3926	+.0063	+56 7 34.54	19.763	−0.032
μ Phœnicis	4.6	Ko	0 37 46.988	2.8371	−.0046	−46 29 49.23	19.741	−0.032

13 Ceti, dup., $5^m.5$, $6^m.2$, 0″.3.

α Cassiop., var. irreg., $2^m.2$, $2^m.8$.

MEAN PLACES OF CIRCUMPOLAR STARS, 1925

FOR JANUARY 0^d.654, WASHINGTON CIVIL TIME

Name of Star	Magnitude	Spectrum	Right Ascension	Annual Variation	Annual P.M.	Declination	Annual Variation	Annual P.M.
43 H. Cephei	4.5	Ko	0 58 11.014	+ 7.7785	+.0737	+85 51 20.58	+19.398	−0.004
α Ursæ Min. (*Polaris*)†	2.1	F8	1 34 13.588	+31.1184	+.1528	+88 54 11.12	+18.376	+0.001
4 G. Octantis	5.6	Ko	1 41 32.636	− 3.6690	+.0086	−85 8 56.44	+18.137	+0.028
Groombridge 750	6.7	F8	4 12 24.065	+17.7589	+.0132	+85 21 23.96	+ 9.111	+0.042
Groombridge 944	6.4	Ko	5 37 43.075	+18.8025	+.0130	+85 9 46.65	+ 1.942	−0.004
31 G. Mensæ	6.2	Ao	5 44 41.365	−11.6658	−.0118	−84 49 36.02	+ 1.425	+0.087
ζ Mensæ	5.6	A2	6 46 18.940	− 4.9567	−.0035	−80 44 9.91	− 3.941	+0.082
51 H. Cephei	5.3	Ma	7 5 56.472	+28.9317	−.0582	+87 10 10.26	− 5.722	−0.034
7 G. Octantis	6.4	F5	7 13 37.284	−20.4769	−.0145	−86 54 58.20	− 6.323	+0.006
25 H. Camelopardalis	5.1	Mb	7 15 24.635	+12.7709	+.0131	+82 33 38.50	− 6.524	− 0.047
Groombridge 1119	7.0	Ao	8 23 37.298	+57.3310	−.0376	+88 51 28.61	−11.738	+0.018
ζ Octantis	5.4	A3	9 7 52.155	− 8.2911	−.1153	−85 21 54.58	−14.608	+0.044
1 H. Draconis	4.6	Ko	9 26 31.760	+ 8.7249	−.0059	+81 39 35.81	−15.743	−0.027
ζ Chamæleontis	5.2	B3	9 36 8.988	− 1.6820	−.0121	−80 36 16.52	−16.205	+0.019
30 H. Camelopardalis	5.3	F5	10 22 5.096	+ 7.4985	−.0460	+82 56 28.35	−18.234	+0.009
η Octantis	6.3	Ao	10 59 52.260	− 0.3914	−.0578	−84 11 25.52	−19.363	−0.005
Bradley 1672	6.3	Fo	12 14 31.648	+ 0.4296	−.0702	+88 6 56.51	−19.946	+0.058
ι Octantis	5.4	Ko	12 46 55.237	+ 6.0474	+.0368	−84 42 59.21	−19.602	+0.024
32 H. Camelop. *seq.*†	5.3	A2	12 48 34.024	+ 0.4586	−.0183	+83 49 13.82	−19.580	+0.016
κ Octantis	5.6	A2	13 28 28.100	+ 9.2533	−.0770	−85 24 11.14	−18.594	−0.024

α Ursæ Min., star 9^m, 18″ s. pr.

32 H. Camelop., star $5^m.8$, 21″.6 n. pr.

APPARENT PLACES OF STARS, 1925

CIRCUMPOLAR STARS

For the Upper Transit at Washington

43 H. Cephei. Mag. 4.5			α Ursæ Minoris (*Polaris*) Mag. 2.1			4 G. Octantis Mag. 5.6			Groombridge 750 Mag. 6.7			Groombridge 944 Mag. 6.4		
Wash.C.T.	Right Ascension	Declination	Wash.C.T.	Right Ascension	Declination	Wash.C.T.	Right Ascension	Declination	Wash.C.T.	Right Ascension	Declination	Wash.C.T.	Right Ascension	Declination
	h m	° ′		*h m*	° ′		*h m*	° ′		*h m*	° ′		*h m*	° ′
Jan.	0 58	+85 51	Jan.	1 34	+88 54	Jan.	1 41	−85 9	Jan.	4 12	+85 21	Jan.	5 37	+85 9
	s	″		*s*	″		*s*	″		*s*	″		*s*	″
0 8	17.25	34 50	0 8	46 55	23 79	0 8	32 96	23 31	0.9	36.14	27 50	0 9	54 86	44 32
1.8	16.93	34.55	1 8	45 42	23.91	1 8	32.71	23 36	1 9	36 00	27 78	1.9	54 82	44 64
2.8	16 65	34 59	2 8	44 34	24 01	2 8	32 44	23 41	2 9	35 86	28 04	2 9	54.78	44.93
3.8	16.36	34 62	3 8	43 30	24 10	3 8	32 17	23 46	3 9	35 72	28 29	3 9	54 73	45 23
4 8	16 09	34 65	4 8	42 28	24 20	4 8	31 89	23.50	4 9	35 58	28 53	4 9	54 69	45 53
5 7	15 82	34 70	5 8	41 26	24 30	5 8	31 59	23 50	5 9	35 46	28.78	5.9	54.66	45 83
6 7	15 55	34 74	6 8	40 25	24 41	6 8	31 31	23 50	6 9	35.34	29 05	6 9	54 63	46 14
7 7	15 27	34 79	7.8	39 21	24 51	7 8	31 04	23.49	7.9	35 22	29 31	7 9	54.60	46.45
8.7	14.99	34.83	8 8	38 14	24 61	8 8	30 77	23 45	8 9	35 10	29 57	8 9	54 57	46 76
9 7	14 69	34.88	9 8	37 03	24 71	9 8	30.50	23 40	9 9	34 97	29 86	9 9	54 53	47 08
10.7	14 38	34 93	10 8	35 87	24 81	10.8	30 25	23.34	10 9	34 82	30.15	10 9	54 48	47 41
11.7	14.06	34 96	11.8	34 66	24 90	11 8	30 01	23.27	11.9	34.65	30 43	11 9	54 43	47 75
12.7	13 73	34 97	12 8	33 41	24 98	12 8	29 76	23.22	12.9	34.48	30 70	12.9	54 35	48 09
13.7	13 39	34 97	13 8	32 14	25 04	13 8	29 53	23 17	13 9	34 28	30 97	13 9	54 25	48 43
14 7	13 06	34 93	14 7	30 86	25 09	14.8	29 29	23 12	14 9	34 08	31 22	14 9	54 14	48 76
15 7	12 73	34 89	15 7	29 60	25 11	15 8	29.04	23 08	15 9	33 85	31 46	15 9	54 03	49.06
16 7	12 42	34 84	16 7	28 40	25 12	16 7	28 78	23.05	16.9	33 65	31 67	16 9	53 90	49.35
17 7	12 13	34 79	17 7	27 27	25 12	17.7	28 51	23 01	17 8	33 45	31 87	17 9	53.77	49 62
18.7	11.85	34 72	18 7	26 21	25 13	18 7	28 23	22 96	18 8	33 26	32 05	18 9	53 67	49.89
19.7	11.59	34 67	19 7	25 19	25 13	19 7	27.93	22 87	19 8	33 09	32 23	19 9	53.58	50.13
20.7	11 33	34.64	20 7	24 21	25 14	20 7	27 64	22 78	20 8	32 92	32 43	20.9	53.49	50.37
21 7	11 07	34.63	21.7	23 19	25 18	21 7	27 35	22 65	21.8	32.77	32 64	21 9	53.41	50 65
22 7	10.81	34 61	22.7	22 11	25 21	22 7	27.08	22 52	22 8	32 61	32 86	22 9	53 34	50.93
23.7	10 53	34.58	23.7	20 96	25 22	23 7	26.83	22 37	23 8	32 44	33.10	23 9	53 26	51.24
24 7	10 21	34 55	24.7	19.74	25 24	24 7	26.58	22 21	24 8	32 25	33 33	24.9	53 16	51.55
25 7	9 89	34 50	25 7	18.47	25 25	25 7	26.36	22.06	25 8	32.02	33 56	25 9	53 03	51.86
26 7	9 56	34 41	26 7	17 19	25 24	26.7	26.13	21.91	26 8	31.79	33 76	26 9	52 89	52.16
27.7	9 23	34 31	27.7	15 91	25 20	27 7	25 89	21 78	27 8	31.54	33 95	27.9	52.71	52 44
28.7	8 91	34 19	28 7	14.68	25.14	28 7	25.65	21 67	28 8	31 27	34.11	28.9	52 53	52 69
29.7	8 63	34 05	29.7	13 52	25.06	29.7	25.40	21.56	29.8	31.02	34 26	29.9	52 35	52 94
30.7	8.35	33 90	30 7	12 42	24 98	30 7	25 13	21 43	30 8	30.78	34.39	30.9	52.17	53 17
31.7	8.10	33 76	31.7	11 37	24 89	31 7	24.86	21 30	31.8	30.55	34 49	31.9	52.00	53.37
13.85	+13.81		52.42	+52.41		11.84	−11.80		12.36	+12.31		11.86	+11.82	
0*h* 58*m* 11*s*.014			1*h* 34*m* 13*s*.588			1*h* 41*m* 32*s*.636			4*h* 12*m* 24*s*.065			5*h* 37*m* 43*s*.075		
+85° 51′ 20″.58			+88° 54′ 11″.12			−85° 8′ 56″.44			+85° 21′ 23″.96			+85° 9′ 46″.65		

NOTE. — 0*h* Washington Civil Time is twelve hours before Washington Mean Noon of the same date.

APPARENT PLACES OF STARS, 1925

For The Upper Transit at Washington

Washington Civil Time	33 Piscium Mag. 4.7				α Andromedæ (*Alpheratz*) Mag. 2 2				β Cassiopeiæ Mag. 2.4				ε Phœnicis Mag. 3.9			
	Right Ascension		Declination		Right Ascension		Declination		Right Ascension		Declination		Right Ascension		Declination	
	h m 0 1 *s*		° ′ − 6 7 ″		*h m* 0 4 *s*		° ′ +28 40 ″		*h m* 0 5 *s*		° ′ +58 43 ″		*h m* 0 5 *s*		° ′ −46 9 ″	
Jan. 0 7	28.841	112	45 99	61	29.734	148	38.63	97	9.620	326	81 85	80	35 010	199	61.39	32
10.7	28 729	104	46 60	53	29 586	142	37 66	124	9 294	316	81 05	131	34.811	183	61.07	79
20 7	28.625	93	47 13	36	29 444	130	36 42	143	8 978	290	79.74	178	34.628	161	60 28	122
30.6	28 532	78	47.49	22	29 314	110	34.99	158	8 688	252	77 96	217	34 467	134	59 06	163
Feb. 9 6	28 454	55	47 71	2	29 204	86	33 41	168	8 436	204	75 79	247	34 333	101	57 43	200
19.6	28.399	32	47.73	14	29.118	55	31 73	167	8 232	142	73 32	268	34.232	63	55.43	231
Mar. 1 6	28 367	1	47 59	39	29 063	17	30.06	160	8 090	75	70 64	276	34.169	22	53.12	258
11 5	28 368	34	47 20	62	29 046	24	28.46	146	8 015	2	67 88	274	34 147	26	50.54	280
21.5	28 402	71	46 58	83	29 070	69	27 00	123	8 017	81	65 14	259	34.173	75	47 74	295
31 5	28 473	110	45 75	111	29 139	116	25 77	96	8.098	159	62.55	236	34 248	126	44.79	307
Apr. 10.5	28.583	150	44 64	133	29.255	163	24.81	61	8 257	238	60 19	203	34 374	178	41 72	309
20 4	28 733	188	43.31	152	29 418	207	24 20	25	8 495	308	58 16	162	34.552	228	38.63	305
30 4	28 921	224	41 79	173	29 625	246	23 95	14	8 803	370	56 54	114	34 780	276	35.58	297
May 10 4	29 145	253	40.06	188	29 871	280	24 09	54	9 173	421	55 40	64	35.056	318	32.61	282
20.3	29 398	280	38.18	198	30.151	308	24.63	91	9 594	462	54 76	12	35.374	354	29.79	257
30 3	29 678	299	36.20	204	30 459	326	25.54	128	10.056	487	54.64	42	35 728	381	27.22	230
June 9 3	·29 977	308	34 16	204	30 785	335	26.82	161	10 543	498	55 06	94	36.109	400	24 92	194
19 3	30 285	311	32 12	200	31.120	337	28 43	188	11 041	495	56 00	142	36.509	407	22 98	155
29 2	30 596	305	30 12	188	31 457	328	30.31	212	11 536	482	57.42	187	36 916	402	21 43	111
July 9 2	30 901	291	28 24	174	31 785	311	32 43	229	12.018	453	59.29	228	37.318	391	20.32	65
19 2	31 192	271	26.50	158	32.096	288	34.72	241	12 471	416	61.57	263	37 709	365	19 67	17
29.2	31.463	244	24 92	131	32 384	256	37 13	247	12.887	369	64 20	291	38.074	333	19.50	30
Aug. 8.1	31 707	212	23.61	107	32.640	222	39.60	248	13.256	315	67.11	313	38 407	290	19 80	77
18 1	31 919	174	22.54	82	32 862	182	42 08	244	13.571	256	70.24	329	38 697	241	20.57	121
28.1	32.093	138	21 72	51	33.044	143	44.52	235	13 827	192	73.53	336	38.938	186	21.78	158
Sept. 7 0	32.231	96	21.21	27	33.187	101	46 87	221	14 019	130	76.89	338	39 124	131	23 36	190
17.0	32 327	61	20 96	2	33 288	60	49.08	203	14 149	66	80.27	334	39 255	73	25.26	216
27 0	32 388	22	20.94	22	33 348	23	51 11	184	14 215	4	83 61	321	39.328	18	27 42	230
Oct. 7 0	32 410	8	21 16	43	33 371	13	52 95	159	14 219	57	86.82	301	39 346	35	29.72	236
16 9	32 402	40	21.59	58	33 358	43	54 54	135	14 162	112	89.83	277	39.311	83	32.08	233
26 9	32.362	60	22.17	66	33 315	71	55 89	107	14.050	162	92.60	245	39 228	122	34 41	218
Nov. 5.9	32.302	82	22 83	77	33 244	94	56 96	77	13 888	208	95.05	207	39.106	156	36.59	197
15.9	32 220	93	23 60	80	33.150	114	57 73	47	13 680	248	97.12	164	38 950	180	38.56	166
25 8	32 127	106	24 40	82	33.036	128	58 20	15	13.432	280	98.76	115	38.770	197	40.22	129
Dec. 5.8	32 021	113	25.22	79	32.908	138	58 35	18	13.152	306	99.91	64	38.573	205	41.51	87
15.8	31.908	113	26 01	73	32.770	145	58 17	48	12.846	321	100 55	10	38 368	208	42.38	42
25 7	31 795	111	26.74	65	32.625	148	57 69	79	12 525	327	100 65	45	38.160	202	42.80	6
35 7	31.684		27.39		32.477		56.90		12.198		100 20		37.958		42.74	
Mean Place	29.826		37.68		30.419		35.02		9 934		70.16		36 485		40.94	
Sec δ, Tan δ	1.006		−0.107		1.140		+0.547		1 927		+1.647		1 444		−1.041	
$D_{\bar{\psi}}\alpha$, $D_{\omega}\alpha$	+0.061		+0.007		+0.061		−0.036		+0.062		−0.110		+0.060		+0.069	
$D_{\psi}\delta$, $D_{\omega}\delta$	+0.40		+0.01		+0.40		+0.02		+0.40		+0.02		+0.40		+0.03	

Note. — 0^h Washington Civil Time is twelve hours before Washington Mean Noon of the same date.

SUN, JANUARY 1925

G. C. T.	Sun's Decl.	Equation of Time	Sun's Decl.	Equation of Time	Sun's Decl.	Equation of Time	Sun's Decl.	Equation of Time
	Thursday 1		Monday 5		Friday 9		Tuesday 13	
h	° ′	m s	° ′	m s	° ′	m s	° ′	m s
0	−23 3.9	−3 20.9	−22 41 7	−5 12.5	−22 12 3	−6 57.1	−21 36.0	−8 33.3
2	23 3.5	3 23.2	22 41.2	5 14.7	22 11.6	6 59.2	21 35.2	8 35.2
4	23 3.1	3 25.6	22 40.6	5 17.0	22 11.0	7 1.3	21 34.4	8 37.1
6	23 2.7	3 28.0	22 40.1	5 19.2	22 10.3	7 3.4	21 33.5	8 39.0
8	23 2 3	3 30.4	22 39.5	5 21.5	22 9 6	7 5.5	21 32 7	8 40.9
10	23 1.9	3 32.8	22 39 0	5 23.7	22 8.9	7 7.6	21 31.8	8 42.7
12	23 1.5	3 35 1	22 38 4	5 26.0	22 8.2	7 9.6	21 31.0	8 44.6
14	23 1.1	3 37.5	22 37.8	5 28 2	22 7.5	7 11.7	21 30 1	8 46.5
16	23 0 7	3 39 9	22 37.3	5 30.4	22 6 8	7 13.8	21 29 3	8 48.4
18	23 0.3	3 42 2	22 36 7	5 32.7	22 6 0	7 15.8	21 28.4	8 50.2
20	22 59.8	3 44.6	22 36 2	5 34.9	22 5.3	7 17.9	21 27 6	8 52.1
22	22 59 4	3 47.0	22 35.6	5 37 1	22 4 6	7 20.0	21 26 7	8 54.0
H. D.	0.2	1.2	0 3	1 1	0 4	1 0	0 4	0.9
	Friday 2		Tuesday 6		Saturday 10		Wednesday 14	
0	−22 59.0	−3 49.3	−22 35 0	−5 39 4	−22 3.9	−7 22 0	−21 25 9	−8 55.8
2	22 58.6	3 51 7	22 34.4	5 41 6	22 3 2	7 24 1	21 25.0	8 57 7
4	22 58.1	3 54.0	22 33 9	5 43 8	22 2 4	7 26.1	21 24.1	8 59.5
6	22 57.7	3 56.4	22 33 3	5 46 0	22 1.7	7 28.2	21 23.3	9 1.4
8	22 57.3	3 58 7	22 32 7	5 48.2	22 1.0	7 30 2	21 22.4	9 3.2
10	22 56.8	4 1 1	22 32 1	5 50 4	22 0.3	7 32 2	21 21 5	9 5.0
12	22 56.4	4 3.4	22 31 5	5 52 6	21 59.5	7 34 2	21 20 7	9 6.9
14	22 56.0	4 5.8	22 30 9	5 54.8	21 58.8	7 36.3	21 19 8	9 8.7
16	22 55 5	4 8 1	22 30 3	5 57 0	21 58.0	7 38 3	21 18 9	9 10.5
18	22 55 1	4 10 4	22 29 7	5 59 2	21 57 3	7 40 3	21 18.0	9 12.3
20	22 54 6	4 12.8	22 29.1	6 1.4	21 56 5	7 42.3	21 17 1	9 14.1
22	22 54.2	4 15.1	22 28 5	6 3.6	21 55.8	7 44 3	21 16.2	9 16.0
H. D.	0.2	1.2	0 3	1.1	0.4	1 0	0.4	0.9

NOTE. — The Equation of Time is to be applied to the G. C. T. in accordance with the sign as given.

0^h Greenwich Civil Time is twelve hours before Greenwich Mean Noon of the same date.

SUN, 1925

FOR WASHINGTON APPARENT NOON

Date	Apparent Right Ascension	Var. per Hr.	Apparent Declina-nation	Var per Hr.	Equation of Time. Mean — App.	Var. per Hr.	Semi-diam-eter	S. T. of Sem. Pass. Merid.	Sidereal Time of 0^h Civil Time
	h m s	s	° ′ ″	″	m s	s	′ ″	m s	h m s
Jan. 1	18 47 1.20	11.043	−23 0 25.6	+12 40	+ 3 41.29	+1.183	16 17.90	1 11.04	6 41 21.04
2	18 51 26.06	11.027	22 55 14.3	13 54	4 9.51	1 168	16 17.91	1 11.00	6 45 17.59
3	18 55 50 54	11.011	22 49 35.7	14.68	4 37.36	1.152	16 17.91	1 10.95	6 49 14.15
4	19 0 14.62	10.994	22 43 29.8	15.81	5 4.80	1.135	16 17.91	1 10.90	6 53 10.71
5	19 4 38.26	10.976	22 36 56.8	16.94	5 31.81	1.116	16 17.90	1 10.84	6 57 7.26
6	19 9 1.45	10.957	−22 29 57.0	+18.05	+ 5 58 37	+1.097	16 17.89	1 10.78	7 1 3 82
7	19 13 24.16	10.936	22 22 30.5	19.16	6 24.45	1.076	16 17.87	1 10.71	7 5 0 38
8	19 17 46.36	10.914	22 14 37.6	20.25	6 50.02	1.054	16 17.83	1 10.64	7 8 56.94
9	19 22 8.03	10.892	22 6 18.5	21.34	7 15.07	1.032	16 17.80	1 10.57	7 12 53.49
10	19 26 29.15	10.868	21 57 33.5	22.41	7 39.56	1.009	16 17.76	1 10.49	7 16 50.05
11	19 30 49.70	10.844	−21 48 22.7	+23.48	+ 8 3.49	+0.985	16 17.72	1 10.41	7 20 46.61
12	19 35 9.67	10.819	21 38 46.4	24 54	8 26.83	0.960	16 17.67	1 10.33	7 24 43 16
13	19 39 .29.02	10.793	21 28 45 0	25.58	8 49.57	0.934	16 17.61	1 10.24	7 28 39 72
14	19 43 47.74	10.767	21 18 18 8	26.61	9 11 68	0.908	16 17.55	1 10.15	7 32 36.28
15	19 48 5.82	10.740	21 7 28 0	27.63	9 33.15	0.881	16 17.48	1 10.06	7 36 32.84
16	19 52 23.25	10.712	−20 56 12.9	+28.63	+ 9 53.96	+0.853	16 17.40	1 9.97	7 40 29.39
17	19 56 40.00	10.684	20 44 33.8	29.62	10 14.09	0.825	16 17.32	1 9.88	7 44 25.95
18	20 0 56.06	10.655	20 32 31.0	30.60	10 33.54	0.796	16 17.23	1 9.78	7 48 22.51
19	20 5 11.41	10.625	20 20 4.9	31.56	10 52.29	0.766	16 17.14	1 9.68	7 52 19.06
20	20 9 26.04	10.594	20 7 15 8	32.52	11 10.32	0.736	16 17.05	1 9.57	7 56 15.62

NOTE. — For mean time interval of semidiameter passing meridian, subtract $0^s.19$ from the sidereal interval.

0^h Washington Civil Time is twelve hours before Washington Mean Noon of the same date.

servation, by employing formulæ and tables given for the purpose in Part II of the Ephemeris.

There are many star catalogues, some containing the positions of a very large number of stars, but determined with rather inferior accuracy; others contain a relatively small number of stars, but whose places are determined with the greatest accuracy. Among the best of these latter may be mentioned the Greenwich ten-year (and other) catalogues, and Boss' Catalogue of 6188 stars for the epoch 1900. (Washington, 1910.)

For time and longitude observations, the list given in the Ephemeris is sufficient, but for special kinds of work where the observer has but a limited choice of positions, such as finding latitude by Talcott's method, many other stars must be observed.

42. Interpolation.

When taking data from the Ephemeris corresponding to any given instant of Greenwich Civil Time, it will generally be necessary to interpolate between the tabulated values of the function. The usual method of interpolating, in trigonometric tables, for instance, consists in assuming that the function varies uniformly between two successive values in the table, and, if applied to the Ephemeris, consists in giving the next preceding tabulated value an increase (or decrease) directly proportional to the time elapsed since the tabulated Greenwich time. If the function is represented graphically, it will be seen that this process places the computed point on a *chord* of the function curve.

Since, however, the " variation per hour," or differential coefficient of the function, is given opposite each value of the function it is simpler to employ this quantity as the rate of change of the function and to multiply it by the time elapsed. An inspection of the diagram (Fig. 39) will show that this is also a more accurate method than the former, provided we always work from the nearer tabulated value; when the differential coefficient is used the computed point lies on the *tangent* line, and the curve is nearer to the tangent than to the chord for any distance which is less than half the interval between tabulated values.

To illustrate these methods of interpolating let it be assumed that it is required to compute the sun's declination at 21^h Greenwich Civil Time, Feb. 1, 1925. The tabulated values for 0^h (midnight) on Feb. 1, and Feb. 2, are as follows:

	Sun's declination	Variation per hour
Feb. 1, 0^h	$-17° 18' 03''.9$	$+42''.15$
Feb. 2, 0^h	$-17\ 01\ 03\ .2$	$+42\ .90$

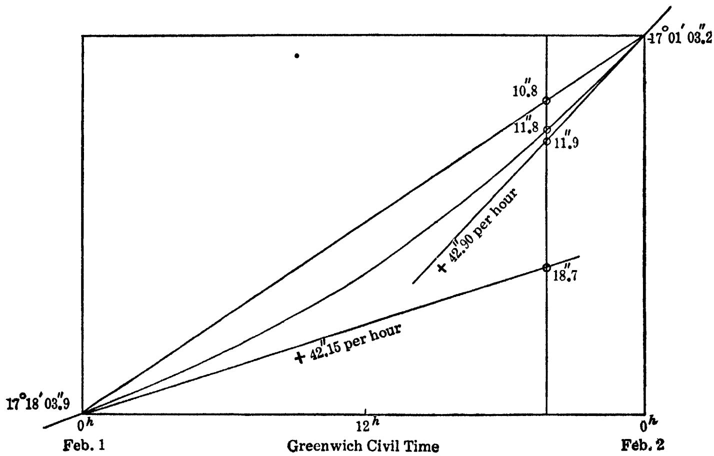


FIG. 39

The given time, 21^h, is nearer to midnight of Feb. 2 than it is to midnight of Feb. 1, so we must correct the value $-17° 01' 03''.2$ by subtracting (algebraically) a correction equal to $+42''.90$ multiplied by 3^h, giving $-17° 03' 11''.9$. If we work from the value for 0^h, Feb. 1, we obtain $-17° 18' 03''.9 - 42''.15 \times 21 = -17° 03' 18''.7$. For the sake of comparison let us interpolate directly between the two tabulated values. This gives $-17° 01' 03''.2 + \frac{8}{24} \times 17' 00''.7 = -17° 03' 10''.8$. These three values are shown on the tangents and chord respectively in Fig. 39. It is clear that the first method gives a point nearer to the function curve than either of the others.

Whenever these methods are insufficient, as might be the case when the tabular intervals are long, or the variations in the " varia. per hour " are rapid, it is possible to make a closer approximation by interpolating between the given values of the differential coefficients to obtain a more accurate value of the rate of change for the particular interval employed. If we imagine a parabola (Fig. 40) with its axis vertical and so placed that it passes through the two given points, C and C', of the function curve and has the same slope at these points, then it is

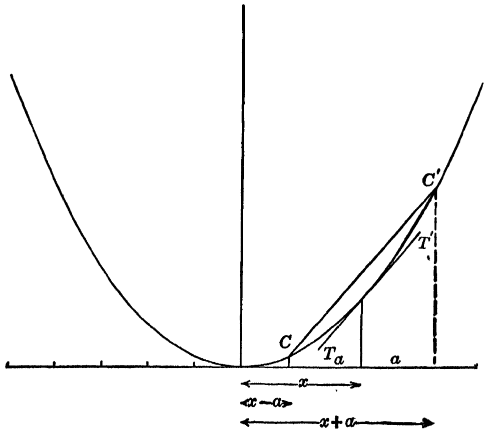


FIG. 40. PARABOLA $x^2 = ky$

obvious that this parabola must lie very close to the true curve at all points between the tabulated values. By the following process we may find a point exactly on the *parabola* and consequently close to the true value. The second differential coefficient of the equation of the parabola is constant, and the slope (dy/dx) may therefore be found for any desired point by simple interpolation between the given values of dy/dx. If we determine the value of dy/dx for a point whose abscissa is *half way* between the tabulated time and the required time we obtain the slope of a tangent line, TT', which is also the slope of a chord of the parabola extending from the point representing the tabulated value to the point representing the desired value; for it may be proved that for this particular parabola such a chord is exactly parallel to the tangent (slope) so found. By finding the value of the "varia. per hour" corresponding to the *middle* of the time interval over which we are interpolating, and employing this in place of the given "varia. per hour" we place our point exactly on the parabola, which must therefore be close to the true point on the function curve. In the preceding example this interpolation would be carried out as follows:

From 0^h Feb. 2 back to 21^h Feb. 1 is 3^h and the time at the middle of this interval is $22^h\,30^m$. Interpolating between $+42''.90$ and $+42''.15$ we find that the rate of change for 22^h 30^m is $+42''.90 - \frac{1\frac{1}{2}}{24} \times 0''.75 = +42''.86$.

The declination is therefore

$$-17^\circ\,01'\,03''.2 - 3 \times 42''.86 = 17^\circ\,03'\,11''.8.$$

This is the most accurate of the four values obtained.

As another example let it be required to find the right ascension of the moon at $9^h\,40^m$ on May 18, 1925. The Ephemeris gives the following data.

Green. Civ. Time	Rt. Asc.	Var. per Min.
9^h	$0^h\,29^m\,59^s.56$	2.0548
10^h	0 32 02 .80	2.0531

The Gr. Civ. Time at the middle of the interval from 9^h to $9^h\,40^m$ is $9^h\,20^m$, or one-third the way from the first to the second tabulated value. The interpolated "variation per minute" for this instant is 2.0542, one-third the way from 2.0548 to 2.0531. The correction to the right ascension at 9^h is $40^m \times 2^s.0542 = 82^s.168$ and the corrected right ascension is therefore $0^h\,31^m\,21^s.73$. If we interpolate from the right ascension at 10^h using a "var. per min." which is one-sixth the way from 2.0531 to 2.0548 we obtain the same result.

43. Double Interpolation.

When the tabulated quantity is a function of two or more variables the interpolation presents greater difficulties. If the tabular intervals are not large, and they never are in a well-planned table, the interpolation may be carried out as follows. Starting from the nearest tabulated value, determine the change in the function produced by each variable separately and apply these corrections to the tabulated value. For example in Table F, p. 203, we find the following:

p sin *t*

H. A.	1925	1930
$1^h\ 52^m$	30′.9	30′.2
$1^h\ 56^m$	31 .9	31 .2

Suppose that we require the value of p sin t for the year 1927 and for the hour angle $1^h\ 53^m.5$. We may consider that the value 30′.9 is increased because the hour angle increases and is decreased by the change of 2 years in the date, and that these two changes are independent. The increase due to the $1^m.5$ increase in hour angle is $\frac{1.5}{4.0} \times 1'.0 = 0'.38$. The decrease due to the change in date is $\frac{2}{5} \times 0'.7 = 0'.28$. The corrected value is $30'.9 + 0'.38 - 0'.28 = 31'.0$.

In a similar manner the tabulated quantity may be corrected for three variations.

Example. Suppose that it is desired to take from the tables of the sun's azimuth (H. O. No. 71) the azimuth corresponding to declination $+11°\ 30'$ and hour angle (apparent time from noon) $3^h\ 02^m$, the latitude being 42° 20′ N. From the page for latitude 42° we find

Declination

	11°	12°
$3^h\ 10^m$	112° 39′	111° 45′
$3^h\ 00^m$	114° 56′	114° 01′

and from page for latitude 43° we find

Declination

	11°	12°
$3^h\ 10^m$	113° 22′	112° 29′
$3^h\ 00^m$	115° 41′	114° 48′

Selecting 114° 56′ as the value from which to start, we correct for the three variations as follows:

For latitude 42°, decrease in 10^m time = 2° 17′; decrease for 2^m time = 27′.4. For latitude 42°, decrease for 1° of declination = 55′; decrease for 30′ of declination = 27′.5. For $3^h\ 00^m$ increase for 1° of latitude = 45′; increase for 20′ of latitude = 15′.

The corrected value is

$$114° 56' - 27'.4 - 27'.5 + 15' = 114° 16'.1$$

For more general interpolation formulæ the student is referred to Chauvenet's Spherical and Practical Astronomy, Doolittle's Practical Astronomy, Hayford's Geodetic Astronomy, and Rice's Theory and Practice of Interpolation.

Questions and Problems

1. Compute the sun's apparent declination when the local civil time is 15^h (3^h P.M.) Jan. 15, 1925, at a place 82° 10′ West of Greenwich (see p. 67).

2. Compute the right ascension of the mean sun $+12^h$ at local 0^h Jan. 10, 1925, at a place 96° 10′ West of Greenwich (see p. 67).

3. Compute the equation of time for local apparent noon Jan. 30, 1925, at a place 71° 06′ West of Greenwich.

4. Compute the apparent right ascension of the sun at G. C. T. 10^h on Jan. 10, 1925, by the four different methods explained in Art. 42.

5. In Table F, p. 203, find by double interpolation the value of p' sin t for $t = 8^h\ 42^m.5$ and for 1926.

CHAPTER VII

THE EARTH'S FIGURE — CORRECTIONS TO OBSERVED ALTITUDES

44. The Earth's Figure.

The form of the earth's surface is approximately that of an ellipsoid of revolution whose shortest axis is the axis of revolution. The actual figure departs slightly from that of the ellipsoid but this difference is relatively small and may be neglected in astronomical observations of the character considered in this book. Each meridian may therefore be regarded as an ellipse, and the equator and the parallels of latitude as perfect circles. In fact the earth may, without appreciable error, be regarded as a sphere in such problems as arise in navigation and in field astronomy with small instruments. The semi-major axis of the meridian ellipse, or radius of the equator on the Clarke (1866) Spheroid, used as the datum for Geodetic Surveys in the United States, is 3963.27 statute miles, and the semi-minor (polar) axis is 3949.83 miles in length. This difference of about 13 miles, or about one three-hundredth part, would only be noticeable in precise work. The length of 1° of latitude at the equator is 68.703 miles; at the pole it is 69.407 miles. The length of 1° of the equator is 69.172 miles. The radius of a sphere having the same volume as the ellipsoid is about 3958.9 miles. On the Hayford (1909) spheroid the semi-major axis is 3963.34 miles and the semi-minor is 3949.99 miles.

In locating points on the earth's surface by means of spherical coördinates there are three kinds of latitude to be considered. The latitude as found by direct astronomical observation is dependent upon the direction of gravity as indicated by the spirit levels of the instrument; this is distinguished as the *astronomical latitude*. It is the angle which the vertical or plumb line makes with the plane of the equator. The *geodetic latitude* is that shown by the direction of the normal to the surface of the spheroid, or ellipsoid. It differs at each place from the astronomical latitude by a small amount which, on the average, is about 3″, but occasionally is as great as 30″. This discrepancy is known as the "local deflection of the plumb line," or the "station error"; it is a direct measure of the departure of the actual surface from that of an ellipsoid. Evidently the geodetic latitude cannot be observed directly but must be derived by calculation. If a line is drawn from any point on the surface to the center of the earth the angle which this line makes with the plane of the equator is called the *geocentric latitude.* In Fig. 41 AD is normal to the surface of the spheroid, and the angle ABE is the geodetic latitude. The plumb line, or line of gravity, at this place would coincide closely with AB, say AB', and the angle it makes ($AB'E$) with the equator is the astronomical latitude of A. The angle ACE is the geocentric latitude. The difference between the geocentric and geodetic latitudes is the angle BAC, called the *angle of the vertical*, or the

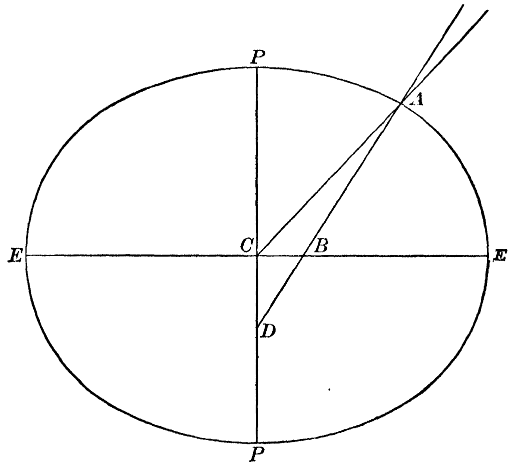


FIG. 41

reduction of latitude. The geocentric latitude is always less than the geodetic by an amount which varies from 0° 11′ 30″ in latitude 45° to 0° at the equator and at the poles. Whenever observations are made at any point on the earth's surface it becomes necessary to reduce the measured values to the corresponding values at the earth's centre before they can be combined with other data referred to the centre. In making this reduction the geocentric latitude must be employed if great exactness in the results is demanded. For the observations of the character treated in the following chapters it will be sufficiently accurate to regard the earth as a sphere when making such reductions.

45. The Parallax Correction.

The coördinates of celestial objects as given in the Ephemeris are referred to the centre of the earth, whereas the coördinates obtained by direct observation are necessarily measured from a point on the surface and hence must be reduced to the centre. The case of most frequent occurrence in practice is that in which the altitude (or the zenith distance) of an object is observed and the geocentric altitude (or zenith distance) is desired. For all objects except the moon the distance from the earth is so great that it is sufficiently accurate to regard the earth as a *sphere*, and even for the moon the error involved is not large when compared with the errors of measurement with small instruments.

In Fig. 42 the angle ZOS is the observed zenith distance, and S_1OS is the observed altitude; ZCS is the true (geocentric) zenith distance, and HCS is the true altitude. The object therefore appears to be lower in the sky when seen from O than it does when seen from C. This apparent displacement of the object on the celestial sphere is called *parallax*. The effect of parallax is to decrease the altitude of the object. If the effect of the spheroidal form of the earth is considered it is seen that the azimuth of the body is also affected, but this small error will not be considered here. In the figure it is seen that the difference in the directions of the lines OS and CS is equal to the small angle OSC, the *parallax correction.* When the object is vertically overhead points C, O and S are in a straight line and the angle is zero; when S is on the horizon (at S_1) the angle OS_1C has its maximum value, and is known as the *horizontal parallax.*

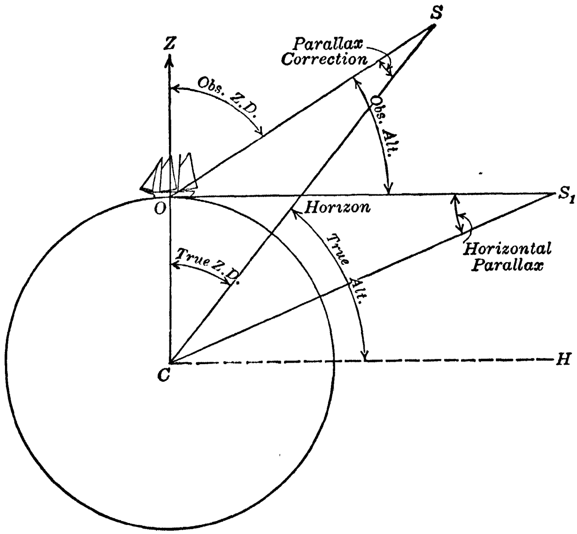


FIG. 42

In the triangle OCS, the angle at O may be considered known, since either the altitude or the zenith distance has been observed. The distance OC is the semidiameter of the earth (about 3959 statute miles), and CS is the distance from the centre of the earth to the centre of the object and is known for bodies in the solar system. To obtain S we solve this triangle by the law of sines, obtaining

$$\sin S = \sin ZOS \times \frac{OC}{CS}. \qquad [47]$$

From the right triangle OS_1C we see that

$$\sin S_1 = \frac{OC}{CS_1}. \qquad [48]$$

The angle S_1, or *horizontal parallax*, is given in the Ephemeris for each object; we may therefore write,

$$\sin S = \sin S_1 \sin ZOS \qquad [49]$$

or

$$\sin S = \sin S_1 \cos h. \qquad [50]$$

At this point it is to be observed that S and S_1 are very small angles, about 9″ for the sun and only 1° for the moon. We may therefore make a substitution of the angles themselves (in radians) for their sines, since these are very nearly the same.* This gives

$$S \text{ (rad)} = S_1 \text{ (rad)} \times \cos h. \qquad [51]$$

To convert these angles expressed in radians into angles expressed in seconds† we substitute

$$S \text{ (rad)} = S'' \times .000004848 \ldots$$

and

$$S_1 \text{ (rad)} = S_1'' \times .000004848 \ldots,$$

the result being

$$S'' = S_1'' \cos h, \qquad [52]$$

that is

$$\text{parallax correction} = \text{horizontal parallax} \times \cos h. \qquad [53]$$

---

* To show the error involved in this assumption express the sine as a series,

$$\sin x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$$

Since we have assumed that $\sin x = x$ the error is approximately equal to the next term, $\frac{x^3}{6}$. For $x = 1°$ the series is

$$\sin 1° = 0.0174533 - 0.0000009 + 0.0000000.$$

The error is therefore 9 in the seventh place of decimals and corresponds to about 0″.18. For angles less than 1° the error would be much smaller than this since the term varies as the cube of the angle.

If, as is frequently done, the cosine of a small angle is replaced by 1, the error is that of the small terms of the series

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4} - \ldots.$$

For 1° this series becomes

$$\cos 1° = 1 - 0.0001523+.$$

The error therefore corresponds to an angle of 31″.42, much larger than for the first series.

† To reduce radians to seconds we may divide by arc 1″ (= 0.000004848137) or multiply by 206264.8.

As an example of the application of Equa. [53] let us compute the parallax correction of the sun on May 1, 1925, when at an apparent altitude of 50°. From the Ephemeris the horizontal parallax is found to be 8″.73. The correction is therefore

$$8''.73 \times \cos 50° = 5''.61$$

and the true altitude is 50° 00′ 05″.61.

Table IV (*A*) gives approximate values of this correction for the sun.

46. The Refraction Correction.

Astronomical refraction is the apparent displacement of a celestial object due to the bending of the rays of light from the object as they pass through the atmosphere. The angular amount of this displacement is the refraction correction. On account of the greater density of the atmosphere in the lower portion the ray is bent into a curve, which is convex upward, and

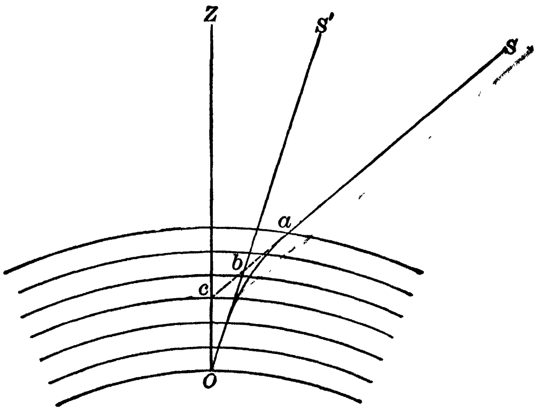


FIG. 43

more sharply curved in the lower portion. In Fig. 43 the light from the star *S* is curved from *a* down to *O*, and the observer at *O* sees the light apparently coming from *S*′, along the line *bO*. The star seems to him to be higher in the sky than it really is. The difference between the direction of *S* and the direction of

S' is the correction which must be applied either to the apparent zenith distance or to the apparent altitude to obtain the true zenith distance or the true altitude. A complete formula for the refraction correction for any altitude, any temperature, and any pressure, is rather complicated. For observations with a small transit a simple formula will answer provided its limitations are understood. The simplest method of deriving such a formula is to consider that the refraction takes place at the upper limit of the atmosphere just as it would at the upper surface of a plate of glass. This does not represent the facts but its use may be justified on the ground that the total amount of refraction is the same as though it did happen this way. In Fig. 44 light from the star S is bent at O' so that it assumes the direction $O'O$ and the observer sees the star apparently at S'. $ZO'S$ $(= \zeta')$ is the true zenith distance, $ZO'S'$ $(= \zeta)$ is the apparent zenith distance, and $SO'S'$ $(= r)$ is the refraction correction; then, from the figure,

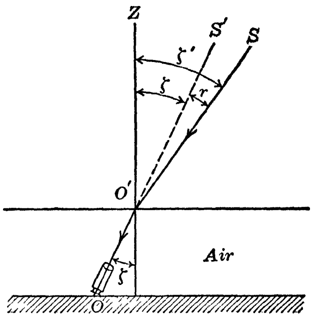


FIG. 44

$$\zeta' = \zeta + r. \qquad [54]$$

Whenever a ray of light passes from a rare to a dense medium (in this case from vacuum into air) the bending takes place according to the law

$$\sin \zeta' = n \sin \zeta, \qquad [55]$$

where n is the index of refraction. For air this may be taken as 1.00029. Substituting [54] in [55]

$$\sin (\zeta + r) = n \sin \zeta. \qquad [56]$$

Expanding the first member,

$$\sin \zeta \cos r + \cos \zeta \sin r = n \sin \zeta. \qquad [57]$$

Since r is a small angle, never greater than about 0° 34′, we may write with small error (see note, p. 83).

$$\sin r = r$$

and

$$\cos r = 1$$

whence

$$\sin \zeta + r \cos \zeta = n \sin \zeta \qquad [58]$$

from which

$$r = (n - 1) \tan \zeta \qquad [59]$$

being in radians.

To reduce r to minutes we divide by arc 1′(= 0.0002909 . . .). The final value of r is therefore approximately

$$\underset{(\text{min})}{r} = \frac{.00029}{.00029} \tan \zeta \qquad [60]$$

$$= \tan \zeta \qquad [61]$$

$$= \cot h. \qquad [62]$$

This formula is simple and convenient but must not be regarded as showing the true law of refraction. The correction varies nearly as the tangent of ζ from the zenith down to about $\zeta = 80°$ ($h = 10°$), beyond which the formula is quite inaccurate. The extent to which the formula departs from the true refraction may be judged by a comparison with Table I, which gives the values as calculated by a more accurate formula for a temperature of 50° F. and pressure 29.5 inches.

As an example of the use of this formula [62] and Table I suppose that the lower edge of the sun has a (measured) altitude of 31° 30′. By formula [62] the value of r is 1′.63, or 1′ 38″. The corrected altitude is therefore 31° 28′ 22″. By Table I the correction is 1′ 33″, and the true altitude is 31° 28′ 27″. This difference of 5″ is not very important in observations made with an engineer's transit. Table I, or any good refraction table, should be used when possible; the formula may be used when a table of tangents is available and no refraction table is at hand. For altitudes lower than 10° the formula should not be considered reliable. More accurate refraction tables may be found in any of the text books on Astronomy to which reference has been made (p. 78). Table VIII, p. 233, gives the refraction and parallax corrections for the sun.

As an aid in remembering the approximate amount of the refraction it may be noted that at the zenith the refraction is 0; at 45° it is 1′; at the horizon it is about 0° 34′, or a little larger than the sun's angular diameter. As a consequence of the fact that the horizontal refraction is 34′ while the sun's diameter is 32′, the entire disc of the sun is still visible (apparently above the horizon) after it has actually set.

47. Semidiameters.

The discs of the sun and the moon are sensibly circular, and their angular semidiameters are given for each day in the Ephemeris. Since a measurement may be taken more accurately to the edge, or limb, of the disc than to the centre, the altitude of the centre is usually obtained by measuring the altitude of the upper or lower edge and applying a correction equal to the angular semidiameter. The angular semidiameter as seen by the observer may differ from the tabulated value for two reasons. When the object is above the horizon it is nearer to the observer than it is to the centre of the earth, and the angular semidiameter is therefore larger than that stated in the Ephemeris. When the object is in the zenith it is about 4000 miles nearer the observer than when it is in the horizon. The moon is about 240,000 miles distant from the earth, so that its apparent semidiameter is increased by about one sixtieth part or about 16″.

Refraction is greater for a lower altitude than for a higher altitude; the lower edge of the sun (or the moon) is always apparently lifted more than the upper edge. This causes an apparent contraction of the vertical diameter. This is most noticeable when the sun or the moon is on the horizon, at which time it appears elliptical in form. This contraction of the vertical diameter has no effect on an observed altitude, however, because the refraction correction applied is that corresponding to the altitude of the edge observed; but the contraction must be allowed for when the angular distance is measured (with the sextant) between the moon's limb and the sun, a star, or a planet. The approximate angular semidiameter of the sun on the first day of each month is given in Table IV (*B*).

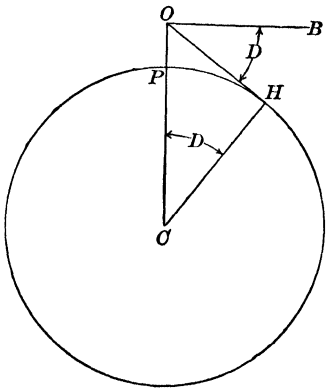


FIG. 45

48. Dip of the Sea Horizon.

If altitudes are measured above the sea horizon, as when observing on board ship with a sextant, the measured altitude must be diminished by the angular dip of the sea horizon below the true horizon. In Fig. 45 suppose the observer to be at *O*; the true horizon is *OB* and the sea horizon is *OH*. Let $OP = h$, the height of the observer's eye above the water surface, expressed in feet; $PC = R$, the radius of the earth, regarded as a sphere; and *D*, the angle of dip. Then from the triangle *OCH*,

$$\cos D = \frac{R}{R+h}. \qquad [63]$$

Replacing $\cos D$ by its series $1 - \frac{D^2}{2} + \ . \ . \ . \ -$ and neglecting terms in powers higher than the second, we have,

$$\frac{D^2}{2} = \frac{h}{R+h}.$$

Since *h* is small compared with *R* this may be written

$$\frac{D^2}{2} = \frac{h}{R}$$

or

$$\underset{\text{(rad)}}{D} = \sqrt{\frac{2h}{R}}.$$

Replacing *R* by its value in feet (20,884,000) and dividing by arc 1′ (= .0002909), to reduce *D* to minutes,

$$D = \frac{1}{\sqrt{\frac{R}{2}} \times \text{arc } 1'} \times \sqrt{h}$$

$$= 1.064 \sqrt{h}. \qquad [64]$$

This shows the amount of the dip with no allowance for refraction. But the horizon itself is apparently lifted by refraction and the dip which affects an observed altitude is therefore less than that given by [64]. If the coefficient 1.064 is arbitrarily taken as unity the formula is much nearer the truth and is very simple, although the dip is still somewhat too large. It then becomes

$$D' = \sqrt{h \text{ ft.}} \qquad [65]$$

that is, the dip in minutes equals the square root of the height in feet. Table IV (*C*), based upon a more accurate formula, will be seen to give smaller values.

49. Sequence of Corrections.

Strictly speaking, the corrections to the observed altitude must be made in the following order: (1) Instrumental corrections; (2) dip (if made at sea); (3) refraction; (4) semidiameter; (5) parallax. In practice, however, it is seldom necessary to follow this order exactly. The parallax correction for the sun will not be appreciably different for the altitude of the centre than it will for the altitude of the upper or lower edge; if the altitude is low, however, it is important to employ the refraction correction corresponding to the edge observed, because this may be sensibly different from that for the centre. In navigation it is customary to combine all the corrections, except the first, into a single correction given in a table whose arguments are the "height of eye," and "observed altitude." (See Bowditch, American Practical Navigator, Table 46.)

Problems

1. Compute the sun's mean horizontal parallax. The sun's mean distance is 92,900,000 miles; for the earth's radius see Art. 44. Compute the sun's parallax at an altitude of 60°

2. Compute the moon's mean horizontal parallax. The moon's mean distance is 238,800 miles; for the earth's radius see Art. 44. Compute the moon's parallax at an altitude of 45°.

3. If the altitude of the sun's centre is 21° 10′ what is the parallax correction? the corrected altitude?

4. If the observed altitude of a star is 15° 30′ what is the refraction correction? the corrected altitude?

5. If the observed altitude of the lower edge of the sun is 27° 41′ on May 1st what is the true central altitude, corrected for refraction, parallax, and semidiameter?

6. The altitude of the sun's lower limb is observed at sea, Dec. 1, and found to be 18° 24′ 20″. The index correction of the sextant is +1′ 20″. The height of eye is 30 feet. Compute the true altitude of the centre.

CHAPTER VIII

DESCRIPTION OF INSTRUMENTS

50. The Engineer's Transit.

The engineer's transit is an instrument for measuring horizontal and vertical angles. For the purpose of discussing the theory of the instrument it may be regarded as a telescopic line of sight having motion about two axes at right angles to each other, one vertical, the other horizontal. The line of sight is determined by the optical centre of the object glass and the intersection of two cross hairs* placed in its principal focus. The vertical axis of the instrument coincides with the axes of two spindles, one inside the other, each of which is attached to a horizontal circular plate. The lower plate carries a graduated circle for measuring horizontal angles; the upper plate has two verniers, on opposite sides, for reading angles on the circle. On the top of the upper plate are two uprights, or standards, supporting the horizontal axis to which the telescope is attached and about which it rotates. At one end of the horizontal axis is a vertical arc, or a circle, and on the standard is a vernier, in contact with the circle, for reading the angles. The plates and the horizontal axis are provided with clamps and slow-motion screws to control the motion. On the upper plate are two spirit levels for levelling the instrument, or, in other words, for making the vertical axis coincide with the direction of gravity.

The whole instrument may be made to turn in a horizontal plane by a motion about the vertical axis, and the telescope may be made to move in a vertical plane by a motion about the horizontal axis. By means of a combination of these two

* Also called wires or threads; they are either made of spider threads, or platinum wires, or are lines ruled upon glass.

motions, vertical and horizontal, the line of sight may be made to point in any desired direction. The motion of the line of sight in a horizontal plane is measured by the angle passed over by the index of the vernier along the graduated horizontal circle. The angular motion in a vertical plane is measured by the angle on the vertical arc indicated by the vernier attached to the standard. The direction of the horizon is defined by means of a long spirit level attached to the telescope. When the bubble is central the line of sight should lie in the plane of the horizon. To be in perfect adjustment, (1) the axis of each spirit level * should be in a plane at right angles to the vertical axis; (2) the horizontal axis should be at right angles to the vertical axis; (3) the line of sight should be at right angles to the horizontal axis; (4) the axis of the telescope level should be parallel to the line of sight, and (5) the vernier of the vertical arc should read zero when the bubble is in the centre of the level tube attached to the telescope. When the plate levels are brought to the centres of their tubes, and the lower plate is so turned that the vernier reads 0° when the telescope points south, then the vernier readings of the horizontal plate and the vertical arc for any position of the telescope are coördinates of the horizon system (Art. 12). If the horizontal circles are clamped in any position and the telescope is moved through a complete revolution, the line of sight describes a vertical circle on the celestial sphere. If the telescope is clamped at any altitude and the instrument turned about the vertical axis, the line of sight describes a cone and traces out on the sphere a parallel of altitude.

51. Elimination of Errors.

It is usually more difficult to measure an altitude accurately with the transit than to measure a horizontal angle. While the precision of horizontal angles may be increased by means of repetitions, in measuring altitudes the precision cannot be

* The axis of a level may be defined as a line tangent to the curve of the glass tube at the point on the scale taken as the zero point, or at the centre of the tube.

increased by repeating the angles, owing to the construction of the instrument. The vertical arc usually has but one vernier, so that the eccentricity cannot be eliminated, and this vernier often does not read as closely as the horizontal vernier. One of the errors, which is likely to be large, but which may be eliminated readily, is that known as the **index error.** The measured altitude of an object may differ from the true reading for two reasons: first, the zero of the vernier may not coincide with the zero of the circle when the telescope bubble is in the centre of its tube; second, the line of sight may not be horizontal when the bubble is in the centre of the tube. The first part of this error can be corrected by simply noting the vernier reading when the bubble is central, and applying this as a correction to the measured altitude. To eliminate the second part of the error the altitude may be measured twice, once from the point on the horizon directly beneath the object observed, and again from the opposite point of the horizon. In other words, the instrument may be reversed (180°) about its vertical axis and the vertical circle read in each position while the horizontal cross hair of the telescope is sighting the object. The mean of the two readings is free from the error in the sight line. Evidently this method is practicable only with an instrument having a complete vertical circle. If the reversal is made in this manner the error due to non-adjustment of the vernier is eliminated at the same time, so that it is unnecessary to make a special determination of it as described above. If the circle is graduated in one direction, it will be necessary to subtract the second reading from 180° and then take the mean between this result and the first altitude. In the preceding description it is assumed that the plate levels remain central during the reversal of the instrument, indicating that the vertical axis is truly vertical. If this is not the case, the instrument should be relevelled before the second altitude is measured, the difference in the two altitude readings in this case including all three errors. If it is not desirable to relevel, the error of inclination of the vertical axis may still be eliminated by reading the vernier of the vertical circle in each of the two positions when the telescope bubble is central, and applying these corrections separately. With an instrument provided with a vertical arc only, it is essential that the axis of the telescope bubble be made parallel to the line of sight, and that the vertical axis be made truly vertical. To make the axis vertical without adjusting the levels themselves, bring both bubbles to the centres of their tubes, turn the instrument 180° in azimuth, and then bring each bubble *half way* back to the centre by means of the levelling screws. When the axis is truly vertical, each bubble should remain in the same part of its tube in all azimuths. The axis may always be made vertical by means of the long bubble on the telescope; this is done by setting it over one pair of levelling screws and centring it by means of the tangent screw on the standard; the telescope is then turned 180° about the vertical axis, and if the bubble moves from the centre of its tube it is brought half way back by means of the tangent screw, and then centred by means of the levelling screws. This process should be repeated to test the accuracy of the levelling; the telescope is then turned at right angles to the first position and the whole process repeated. This method should always be used when the greatest precision is desired, because the telescope bubble is much more sensitive than the plate bubbles.

If the line of sight is not at right angles to the horizontal axis, or if the horizontal axis is not perpendicular to the vertical axis, the errors due to these two causes may be eliminated by combining two sets of measurements, one in each position of the instrument. If a horizontal angle is measured with the vertical circle on the observer's right, and the same angle again observed with the circle on his left, the mean of these two angles is free from both these errors, because the two positions of the horizontal axis are placed symmetrically about a true horizontal line,* and

* Strictly speaking, they are placed symmetrically about a perpendicular to the vertical axis.

the two directions of the sight line are situated symmetrically about a true perpendicular to the rotation axis of the telescope. If the horizontal axis is not perpendicular to the vertical axis the line of sight describes a plane which is inclined to the true vertical plane. In this case the sight line will not pass through the zenith, and both horizontal and vertical angles will be in error. In instruments intended for precise work a striding level is provided, which may be set on the pivots of the horizontal axis. This enables the observer to level the axis or to measure its inclination without reference to the plate bubbles. The striding level should be used in both the direct and the reversed position and the mean of the two results used in order to eliminate the errors of adjustment of the striding level itself. If the line of sight is not perpendicular to the horizontal axis it will describe a cone whose axis is the horizontal axis of the instrument. The line of sight will in general not pass through the zenith, even though the horizontal axis be in perfect adjustment. The instrument must either be used in two positions, or else the cross hairs must be adjusted. Except in large transits it is not usually practicable to determine the amount of the error and allow for it.

52. Attachments to the Engineer's Transit. — Reflector. When making star observations with the transit it is necessary to make some arrangement for illuminating the field of view. Some transits are provided with a special shade tube into which is fitted a mirror set at an angle of 45° and with the central portion removed. By means of a lantern or a flash light held at one side of the telescope light is reflected down the tube. The cross hairs appear as dark lines against the bright field. The stars can be seen through the opening in the centre of the mirror. If no special shade tube is provided, it is a simple matter to make a substitute, either from a piece of bright tin or by fastening a piece of tracing cloth or oiled paper over the objective. A hole about $\frac{3}{4}$ inch in diameter should be cut out, so that the light from the star may enter the lens. If cloth or paper is used, the flash light must be held so that the light is diffused in such a way as to make the cross hairs visible, but so as not to shine into the observer's eyes.

53. Prismatic Eyepiece.

When altitudes greater than about 55° to 60° are to be measured, it is necessary to attach to the eyepiece a totally reflecting prism which reflects the rays at right angles to the sight line. By means of this attachment altitudes as great as 75° can be measured. In making observations on the sun it must be remembered that the prism inverts the image, so that with a transit having an erecting eyepiece with the prism attached the apparent lower limb is the true upper limb; the positions of the right and left limbs are not affected by the prism.

54. Sun Glass.

In making observations on the sun it is necessary to cover the eyepiece with a piece of dark glass to protect the eye from the sunlight while observing. The sun glass should not be placed in front of the objective. If no shade is provided with the instrument, sun observations may be made by holding a piece of paper behind the eyepiece so that the sun's image is thrown upon it. By drawing out the eyepiece tube and varying the distance at which the paper is held, the images of the sun and the cross hairs may be sharply focussed. By means of this device an observation may be quite accurately made after a little practice.

55. The Portable Astronomical Transit.

The astronomical transit differs from the surveyor's transit chiefly in size and the manner of support. The diameter of the object glass may be from 2 to 4 inches, and the focal length from 24 to 48 inches. The instrument is mounted on a brick or a concrete pier and may be approximately levelled by means of foot screws. The older instruments were provided with several vertical threads (usually 5 or 11) in order to increase the number of observations that could be made on one star. These were spaced about ½′ to 1′ apart, so that an equatorial star would require from 2^s to 4^s to move from one thread to the next. The more recent transits are provided with the "transit micrometer"; in this pattern there is but a single vertical thread, which the observer sets on the moving star as it enters the field of view, and keeps it on the star continuously by turning the micrometer screw, until it has passed beyond the range of observation. The passage of the thread across certain fixed points in the field is recorded electrically. This is equivalent

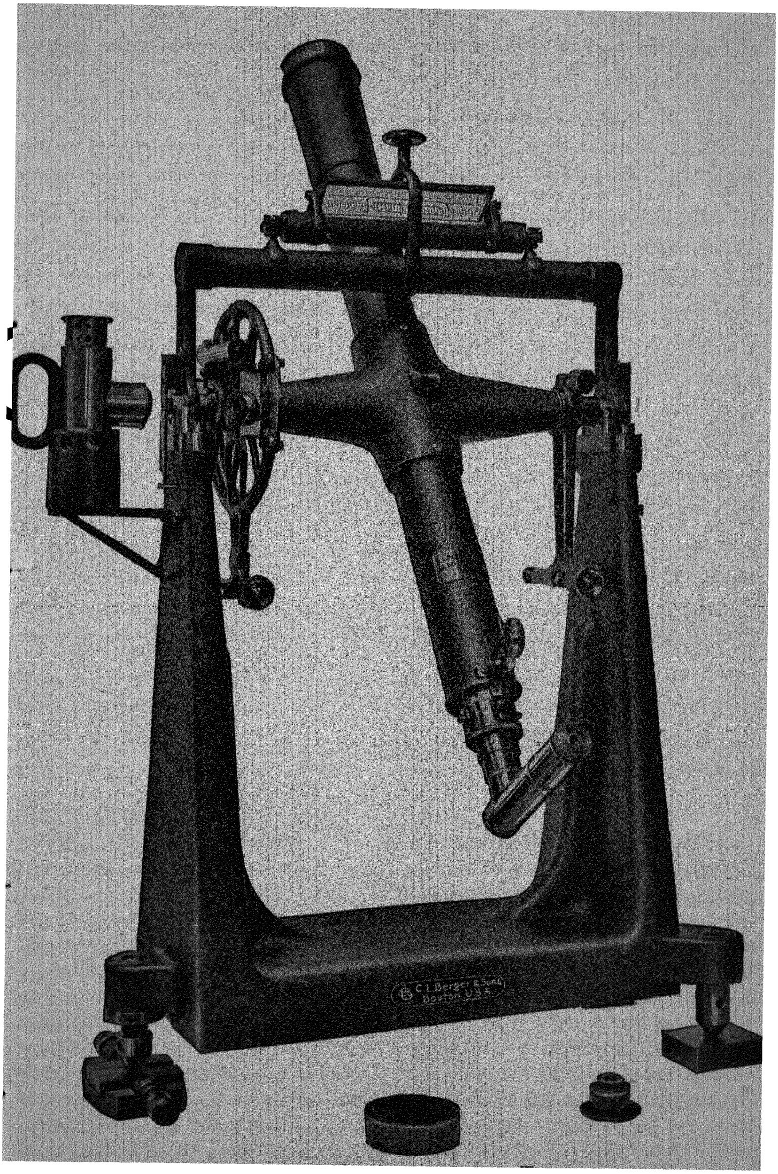


FIG. 46. PORTABLE ASTRONOMICAL TRANSIT
From the catalogue of C. L. Berger & Sons

to observations on 20 vertical threads. The field is illuminated by electric lights which are placed near the ends of the axis. The axis is perforated and a mirror placed at the centre to reflect the light toward the eyepiece. The motion of the telescope in altitude is controlled by means of a clamp and a tangent screw. The azimuth motion is usually very small, just sufficient to permit of adjustment into the plane of the meridian. The axis is levelled or its inclination is measured by means of a sensitive striding level applied to the pivots. The larger transits are provided with a reversing apparatus.

The transit is used chiefly in the plane of the meridian for the direct determination of sidereal time by star transits. It may, however, be used in any vertical plane, and for either time or latitude observations. The principal part of the work consists in the determination of the instrumental errors and in calculating the corrections. The transit is in adjustment when the central thread is in a plane through the optical centre perpendicular to the horizontal axis, and the vertical threads are parallel to this plane. For observations of meridian transits this plane must coincide with the plane of the meridian and the horizontal axis must be truly horizontal.

The chief errors to be determined and allowed for are (1) the azimuth, or deviation of the plane of collimation from the true meridian plane; (2) the inclination of the horizontal axis to the horizon; and (3) collimation error, or error in the sight line. Corrections are also applied for diurnal aberration of light, for the rate of the timepiece, and for the inequality of the pivots. The corrections to reduce an observed time to the true time of transit over the meridian are given by formulæ [66], [67], and [68]. These corrections would apply equally well to observations made with an engineer's transit, and are of value to the surveyor chiefly in showing him the relative magnitudes of the errors in different positions of the objects observed. This may aid him in selecting stars even though no corrections are actually applied for these errors.

The expressions for the corrections to any star are

$$\text{Azimuth correction} = a \cos h \sec \delta \qquad [66]$$

$$\text{Level correction} = b \sin h \sec \delta \qquad [67]$$

$$\text{Collimation correction} = c \sec \delta \qquad [68]$$

in which a, b, and c are the constant errors in azimuth, level, and collimation, respectively, expressed in *seconds of time*, and h is the altitude and δ the declination of the star. If the zenith distance is used instead of the altitude $\cos h$ and $\sin h$ should be replaced by $\sin \zeta$ and $\cos \zeta$ respectively. These formulæ may be easily derived from spherical triangles. Formula [66] shows that for a star near the zenith the azimuth correction will be small, even if a is large, because $\cos h$ is nearly zero. Formula [67] shows that the level correction for a zenith star will be larger than for a low star because $\sin h$ for the former is nearly unity. The azimuth error a is found by comparing the results obtained from stars which culminate north of the zenith with those obtained from south stars; if the plane of the instrument lies to the east of south; stars south of the zenith will transit too early and those north of the zenith will transit too late. From the observed times the angle may be computed. The level error b is measured directly with the striding level, making readings of both ends of the bubble, first in the direct, then in the reversed positions, the angular value of one level division being known. The collimation error, c, is found by comparing the results obtained with the axis in the direct position with the results obtained with the axis in the reversed (end-for-end) position.

TABLE B. ERROR IN OBSERVED TIME OF TRANSIT (IN SECONDS OF TIME) WHERE a, b OR $c = 1'$.

Altitude (for Inclination Error)	Declinations.									Altitude (for Azimuth Error)
h	0°	10°	20°	30°	40°	50°	60°	70°	80°	h
0°	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	90°
10	0.7	0.7	0.8	0.8	0.9	1.1	1.4	2.0	4.0	80
20	1.4	1.4	1.4	1.6	1.8	2.1	2.7	4.0	7.9	70
30	2.0	2.0	2.1	2.3	2.6	3.1	4.0	5.8	11.5	60
40	2.6	2.6	2.7	3.0	3.4	4.0	5.2	7.5	14.8	50
50	3.1	3.1	3.3	3.6	4.0	4.8	6.1	9.0	17.6	40
60	3.5	3.5	3.7	4.0	4.5	5.4	6.9	10.1	19.9	30
70	3.8	3.8	4.0	4.4	4.9	5.8	7.5	11.0	21.6	20
80	3.9	4.0	4.2	4.6	5.2	6.1	7.9	11.5	22.7	10
90	4.0	4.1	4.2	4.6	5.2	6.2	8.0	11.7	23.0	0

Note. — Use the bottom line for the collimation error.

From the preceding equations Table B has been computed. It is assumed that the collimation plane is $1'$, or 4^s, out of the meridian ($a = 4^s$); that the axis is inclined $1'$, or 4^s, to the horizon ($b = 4^s$); and that the sight line is $1'$, or 4^s, to the right or left of its true position ($c = 4^s$). An examination of the table will show that for low stars the azimuth corrections are large and the level corrections are small, while for high stars the reverse is true. As an illustration of the use of this table, suppose that the latitude is 42°, and the star's declination is +30°; and that $a = 1'$ (4^s) and $b = 2'$ (8^s). The altitude of the star = 90° − (42° − 30°) = 78°. The azimuth correction is therefore $1^s.0$ and the level correction is $2 \times 4^s.6 = 9^s.2$. If the line of sight were $\frac{1}{4}'$ (or 1^s) in error the (collimation) correction would be $1^s.2$. This shows that with a transit set closely in the meridian but with a large possible error in the inclination of the axis, low stars will give better results than high stars. This is likely to be the case with a surveyor's transit. If, however, the inclination of the axis can be accurately measured but the adjustment into the plane of the meridian is difficult, then the high stars will be preferable. This is the condition more likely to prevail with the larger astronomical transits. For the complete theory of the transit see Chauvenet's Spherical and Practical Astronomy, Vol. II; the methods employed by the U. S. Coast and Geodetic Survey are given in Special Publication, No. 14.

56. The Sextant.

The sextant is an instrument for measuring the angular distance between two objects, the angle always lying in the plane through the two objects and the eye of the observer. It is particularly useful at sea because it does not require a steady support like the transit. It consists of a frame carrying a graduated arc, *AB*, Fig. 47, about 60° long, and two mirrors *I* and *H*, the first one movable, the second one fixed. At the centre of the arc, *I*, is a pivot on which swings an arm *IV*,

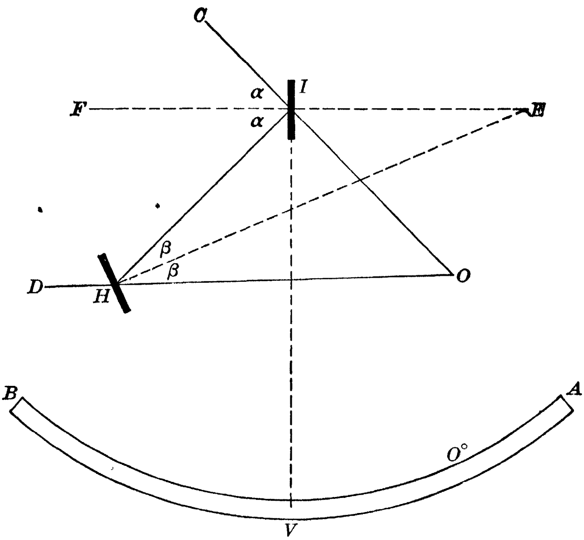


FIG. 47

6 to 8 inches long. This arm carries a vernier *V* for reading the angles on the arc *AB*. Upon this arm is placed the index glass *I*. At *H* is the horizon glass. Both of these mirrors are set so that their planes are perpendicular to the plane of the arc *AB*, and so that when the vernier reads 0° the mirrors are parallel. The half of the mirror *H* which is farthest from the frame is unsilvered, so that objects may be viewed directly through the glass. In the silvered portion other objects may be seen by reflection from the mirror *I* to the mirror *H* and thence to

point O. At a point near O (on the line HO) is a telescope of low power for viewing the objects. Between the two mirrors and also to the left of H are colored shade glasses to be used when making observations on the sun. The principle of the instrument is as follows: — A ray of light coming from an object at C is reflected by the mirror I to H, where it is again reflected to O. The observer sees the image of C in apparent coincidence with the object at D. The arc is so graduated that the reading

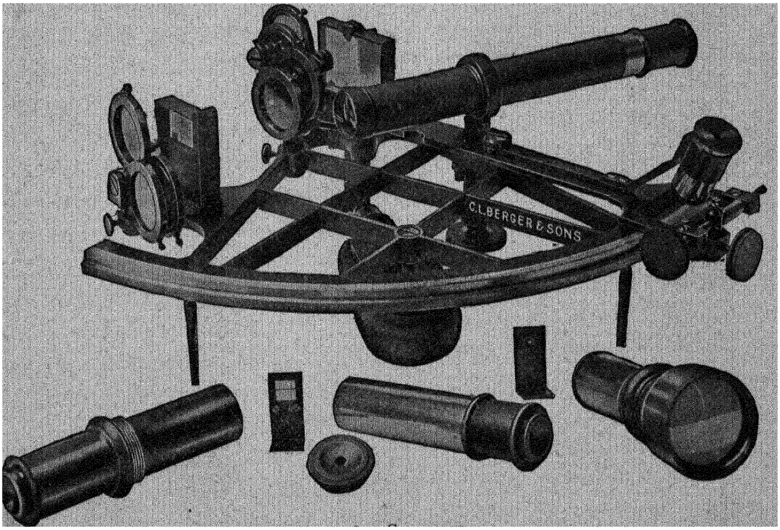


FIG. 48. SEXTANT

of the vernier gives directly the angle between OC and OD. Drawing the perpendiculars FE and HE to the planes of the two mirrors, it is seen that the angle between the mirrors is $\alpha - \beta$. Prolonging CI and DH to meet at O, it is seen that the angle between the two objects is $2\alpha - 2\beta$. The angle between the mirrors is therefore half the angle between the objects that appear to coincide. In order that the true angle may be read directly from the arc each half degree is numbered as though it were a degree. It will be seen that the position of the vertex O is variable, but since all objects observed are at great distances the errors caused by changes in the position of O are always negligible in astronomical observations.

The sextant is in adjustment when, (1) both mirrors are perpendicular to the plane of the arc; (2) the line of sight of the telescope is parallel to the plane of the arc; and (3) the vernier reads 0° when the mirrors are parallel to each other. If the vernier does not read 0° when the double reflected image of a point coincides with the object as seen directly, the index correction may be determined and applied as follows. Set the vernier to read about 30′ and place the shades in position for sun observations. When the sun is sighted through the telescope two images will be seen with their edges nearly in contact. This contact should be made as nearly perfect as possible and the vernier reading recorded. This should be repeated several times to increase the accuracy. Then set the vernier about 30′ on the opposite side of the zero point and repeat the whole operation, the reflected image of the sun now being on the opposite side of the direct image. If the shade glasses are of different colors the contacts can be more precisely made. Half the difference of the two (average) readings is the index correction. If the reading *off* the arc was the greater, the correction is to be *added* to all readings of the vernier; if the greater reading was *on* the arc, the correction must be *subtracted*.

In measuring an altitude of the sun above the sea horizon the observer directs the telescope to the point on the horizon vertically under the sun and then moves the index arm until the reflected image of the sun comes into view. The sea horizon can be seen through the plain glass and the sun is seen in the mirror. The sun's lower limb is then set in contact with the horizon line. In order to be certain that the angle is measured to the point vertically beneath the sun, the instrument is tipped slowly right and left, causing the sun's image to describe an arc. This arc should be just tangent to the horizon. If at any point the sun's limb goes below the horizon the altitude measured is too great. The vernier reading corrected for index error and dip is the apparent altitude of the lower limb above the true horizon.

57. Artificial Horizon.

When altitudes are to be measured on land the visible horizon cannot be used, and the **artificial horizon** must be used instead. The surface of any heavy liquid, like mercury, molasses, or heavy oil, may be used for this purpose. When the liquid is placed in a basin and allowed to come to rest, the surface is perfectly level, and in this surface the reflected image of the sun may be seen, the image appearing as far below the horizon as the sun is above it. Another convenient form of horizon consists of a piece of black glass, with plane surfaces, mounted on a frame supported by levelling screws. This horizon is brought

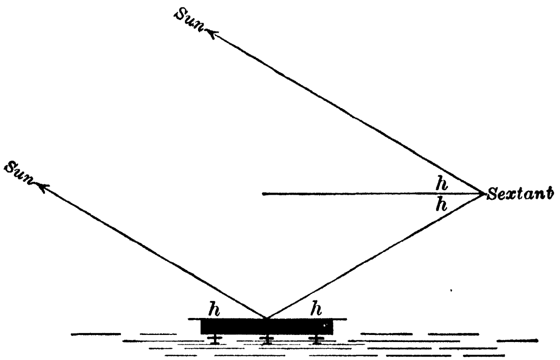


FIG. 49. ARTIFICIAL HORIZON

into position by placing a spirit level on the glass surface and levelling alternately in two positions at right angles to each other. This form of horizon is not so accurate as the mercury surface but is often more convenient. The principle of the artificial horizon may be seen from Fig. 49. Since the image seen in the horizon is as far below the true horizon as the sun is above it, the angle between the two is $2h$. In measuring this angle the observer points his telescope toward the artificial horizon and then brings the reflected sun down into the field of view by means of the index arm. By placing the apparent lower limb of the reflected sun in contact with the apparent upper limb of the image seen in the mercury surface, the angle measured is twice the altitude of the sun's lower limb. The two points in contact are really images of the same point. If the telescope inverts the image, this statement applies to the upper limb. The index correction must be applied before the angle is divided by 2 to obtain the altitude. In using the mercury horizon care must be taken to protect it from the wind; otherwise small waves on the mercury surface will blur and distort the image. The horizon is usually provided with a roof-shaped cover having glass windows, but unless the glass has parallel faces this introduces an error into the result. A good substitute for the glass cover is one made of fine mosquito netting. This will break the force of the wind if it is not blowing hard, and does not introduce errors into the measurement.

58. Chronometer.

The chronometer is simply an accurately constructed watch with a special form of escapement. Chronometers may be regulated for either sidereal or mean time. The beat is usually a half second. Those designed to register the time on chronographs are arranged to break an electric circuit at the end of every second or every two seconds. The 60th second is distinguished either by the omission of the break at the previous second, or by an extra break, according to the construction of the chronometer. Chronometers are usually hung in gimbals to keep them level at all times; this is invariably done when they are taken to sea. It is important that the temperature of the chronometer should be kept as nearly uniform as possible, because fluctuation in temperature is the greatest source of error.

Two chronometers of the same kind cannot be directly compared with great accuracy, $0^s.1$ or $0^s.2$ being about as close as the difference can be estimated. But a sidereal and a solar chronometer can easily be compared within a few hundredths of a second. On account of the gain of the sidereal on the solar chronometer, the beats of the two will coincide once in about every $3^m\ 03^s$. If the two are compared at the instant when the beats are apparently coincident, then it is only necessary to note the seconds and half seconds, as there are no fractions to be estimated. By making several comparisons and reducing them to some common instant of time it is readily seen that the comparison is correct within a few hundredths of a second. The accuracy of the comparison depends upon the fact that the ear can detect a much smaller interval between the two beats than can possibly be estimated when comparing two chronometers whose beats do not coincide.

59. Chronograph.

The chronograph is an instrument for recording the time kept by a chronometer and also any observations the times of which it is desired to determine. The paper on which the record is made is wrapped around a cylinder which is revolved by a clock mechanism at a uniform rate, usually once per minute. The pen which makes the record is placed on the armature of an electromagnet which is mounted on a carriage drawn horizontally by a long screw turned by the same mechanism. The mark made by the pen runs spirally around the drum, which results in a series of straight parallel lines when the paper is laid flat. The chronometer is connected electrically with the electromagnet and records the seconds by making notches in the line corresponding to the breaks in the circuit, one centimeter being equivalent to one second. Observations are recorded by the observer by pressing a telegraph key, which also breaks (or makes) the chronograph circuit and makes a mark on the record sheet. If the transit micrometer is used the passage of the vertical thread over fixed points in the field is automatically recorded on the chronograph. The circuit with which the transit micrometer is connected operates on the " make " instead of the " break " circuit. When the paper is laid flat the time appears as a linear distance on the sheet and may be scaled off directly with a scale graduated to fit the spacing of the minutes and seconds of the chronograph.

60. The Zenith Telescope.

The Zenith Telescope is an instrument designed for making observations for latitude by a special method known as the Harrebow-Talcott method. The instrument consists of a telescope attached to one end of a short horizontal axis which is mounted on the top of a vertical axis. About these two axes the telescope has motions in the vertical and horizontal planes like a transit. A counterpoise is placed at the other end of the horizontal axis to balance the instrument. The essential parts of the instrument are (1) a micrometer placed in the focal plane of the eyepiece for measuring small angles in the vertical plane, and (2) a sensitive spirit level attached to the vernier arm of a small vertical circle on the telescope tube for measuring small changes in the inclination of the telescope. The telescope is ordinarily used in the plane of the meridian, but may be used in any vertical plane.

The zenith telescope is put in adjustment by placing the line of sight in a plane perpendicular to the horizontal axis, the micrometer thread horizontal, the horizontal axis perpendicular to the vertical axis, and the base levels in planes perpendicular to the vertical axis. For placing the line of sight in the plane of the meridian there are two adjustable stops which must be so placed and clamped that the telescope may be quickly turned about the vertical axis from the north to the south meridian, or vice versa, and clamped in the plane of the meridian.

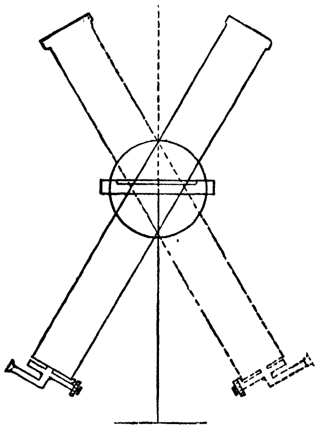


FIG. 50. THE ZENITH TELESCOPE

The observations consist in measuring with the micrometer the difference in zenith distance of two stars, one north of the zenith, one south of it, which culminate within a few minutes of each other, and in reading the scale readings of the ends of the bubble on the latitude level. The two stars must have zenith distances such that they pass the meridian within the range of the micrometer.

A diagram of the instrument in the two positions is shown in Fig. 50. The inclination of the telescope to the latitude level is not changed during the observation. Any change in the inclination of the telescope to the vertical is measured by the latitude level and may be allowed for in the calculation. The principle involved in this method may be seen from Fig. 51. From the zenith distance of the star S_s the latitude would be

$$\phi = \delta_s + \zeta_s$$

and from the star S_n

$$\phi = \delta_n - \zeta_n.$$

The mean of the two gives

$$\phi = \frac{\delta_s + \delta_n}{2} + \frac{\zeta_s - \zeta_n}{2} \qquad [69]$$

showing that the latitude is the mean of the two declinations corrected by a small angle equal to half the difference of the zenith distances. The declinations are furnished by the star catalogues, and the difference of zenith distance is measured accurately with the micrometer. The complete formula would include a term for the level correction and one for the small differential refraction. This method gives the most precise latitudes that can be determined with a field instrument.

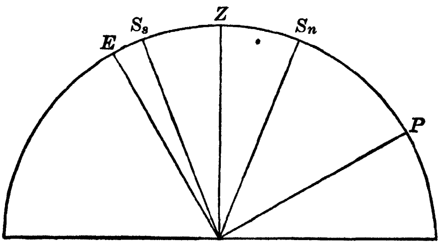


FIG. 51

It is possible for the surveyor to employ this same principle if his transit is provided with a gradienter screw and an accurate level. The gradienter screw takes the place of the micrometer. A level may be attached to the end of the horizontal axis and made to do the work of a latitude level.

61. Suggestions about Observing with Small Instruments. The instrument used for making such observations as are described in this book will usually be either the engineer's transit or the sextant. In using the transit care must be taken to give the tripod a firm support. If the ground is shaky three pegs may be driven and the points of the tripods set in depressions in the top of the pegs. It is well to set the transit in position some time before the observations are to be begun; this allows the instrument to assume the temperature of the air and the tripod legs to come to a firm bearing on the ground. The observer should handle the instrument with great care, particularly during night observations, when the instrument is likely to be accidentally disturbed. In reading angles at night it is important to hold the light in such a position that the graduations on the circle are plainly visible and may be viewed along the lines of graduation, not obliquely. By changing the position of the flash light and the position of the eye it will be found that the reading varies by larger amounts than would be expected when reading in the daylight. Care should be taken not to touch the graduated silver circles, as they soon become tarnished. If a lantern is used it should be held so as to heat the instrument as little as possible, and so as not to shine into the observer's eyes. Time may be saved and mistakes avoided if the program of observations is laid out beforehand, so that the observer knows just what is to be done and the proper order of the different steps. The observations should be arranged so as to eliminate instrumental errors by reversing the instrument; but if this is not practicable, then the instrument must be put in good adjustment. The index correction should be determined and applied, unless it can be eliminated by the method of observing.

In observations for time it will often be necessary to use an ordinary watch. If there are two observers, one can read the time while the other makes the observations. If a chronometer is used, one observer may easily do the work of both, and at the same time increase the accuracy. In making observations by this method (called the "eye and ear method") the observer looks at the chronometer, notes the reading at some instant, say at the beginning of some minute, and, listening to the half-second beats, carries along the count mentally and without looking at the chronometer. In this way he can note the second and estimate the fraction without taking his attention from the star and cross hair. After making his observation he may check his count by again looking at the chronometer to see if the two agree. After a little practice this method can be used easily and accurately. In using a watch it is possible for one observer to make the observations and also note the time, but it cannot be done with any such precision as with the chronometer, because on account of the rapidity of the ticks (5 per second), the observer cannot count the seconds mentally. The observer must in this case look quickly at his watch and make an allowance, if it appears necessary, for the time lost in looking up and taking the reading.

62. Errors in Horizontal Angles.

When measuring horizontal angles with a transit, such, for example, as in determining the azimuth of a line from the pole-star, any error in the position of the sight line, or any inclination of the horizontal axis will be found to produce a large error in the result, on account of the high altitude of the star. In ordinary surveying these errors are so small that they are neglected, but in astronomical work they must either be eliminated or determined and allowed for in the calculations.

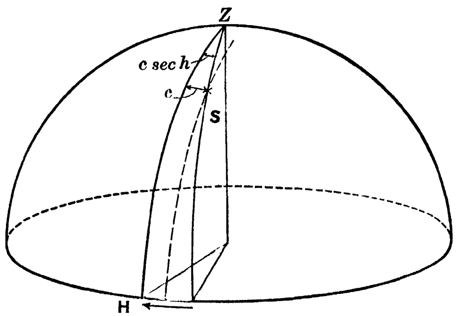


FIG. 52. LINE OF SIGHT IN ERROR (CROSS-HAIR OUT)

In Fig. 52 ZH is the circle traced out by the true collimation axis, and the dotted circle is that traced by the actual line of sight, which is in error by the small angle c. The effect of this on the direction of a star S is the angle SZH.

In Fig. 53 the vertical axis is not truly vertical, but is inclined by the angle i owing to poor levelling. This produces an error in the direction of point P which is equal to the angle $HZ'P$. If the vertical axis is truly vertical but the horizontal axis is inclined to the horizon by the angle i, owing to lack of adjustment, the error in the direction of the point (S) is the same in amount and equal to the angle H_1ZH_2 in Fig. 54.

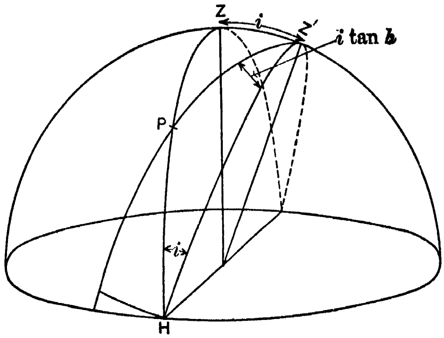


Fig. 53. Plate Levels Adjusted — Bubbles not Centred

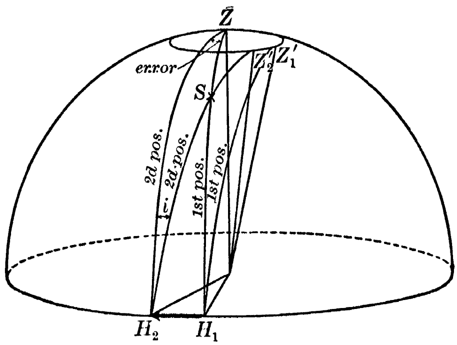


Fig. 54. Plate Levels Correct — Horizontal Axis Out of Adjustment

Problems

1. Show that if the sight line makes an angle c with the perpendicular to the horizontal axis (Fig. 52) the horizontal angle between two points is in error by the angle

$$c \sec h' - c \sec h'',$$

where h' and h'' are the altitudes of the two points.

2. Show that if the horizontal axis is inclined to the horizon by the angle i (Figs. 53 and 54) the effect upon the azimuth of the sight line is $i \tan h$, and that an angle is in error by

$$i (\tan h' - \tan h''),$$

where h' and h'' are the altitudes of the points.

CHAPTER IX

THE CONSTELLATIONS

63. The Constellations.

A study of the constellations is not really a part of the subject of Practical Astronomy, and in much of the routine work of observing it would be of comparatively little value, since the stars used can be identified by means of their coördinates and a knowledge of their positions in the constellations is not essential. If an observer has placed his transit in the meridian and knows approximately his latitude and the local time, he can identify stars crossing the meridian by means of the times and the altitudes at which they culminate. But in making occasional observations with small instruments, and where much of the astronomical data is not known to the observer at the time, some knowledge of the stars is necessary. When a surveyor is beginning a series of observations in a new place and has no accurate knowledge of his position nor the position of the celestial sphere at the moment, he must be able to identify certain stars in order to make approximate determinations of the quantities sought.

64. Method of Naming Stars.

The whole sky is divided in an arbitrary manner into irregular areas, all of the stars in any one area being called a **constellation** and given a special name. The individual stars in any constellation are usually distinguished by a name, a Greek letter,* or a number. The letters are usually assigned in the order of brightness of the stars, α being the brightest, β the next, and so on. A star is named by stating first its letter and then the name of the constellation in the (Latin) genitive form. For instance,

---

* The Greek alphabet is given on p. 190.

in the constellation *Ursa Minor* the star α is called α *Ursæ Minoris;* the star *Vega* in the constellation *Lyra* is called α *Lyræ.* When two stars are very close together and have been given the same letter, they are often distinguished by the numbers 1, 2, etc., written above the letter, as, for example, α^2 *Capricorni,* meaning that the star passes the meridian after α^1 *Capricorni.*

65. Magnitudes.

The brightness of stars is shown on a numerical scale by their **magnitudes.** A star having a magnitude 1 is brighter than one having a magnitude 2. On the scale of magnitudes in use a few of the brightest stars have fractional or negative magnitudes. Stars of the fifth magnitude are visible to the naked eye only under favorable conditions. Below the fifth magnitude a telescope is usually necessary to render the star visible.

66. Constellations Near the Pole.

The stars of the greatest importance to the surveyor are those near the pole. In the northern hemisphere the pole is marked by a second-magnitude star, called the *polestar, Polaris,* or α *Ursæ Minoris,* which is about 1° 06′ distant from the pole at the present time (1925). This distance is now decreasing at the rate of about one-third of a minute per year, so that for several centuries this star will be close to the celestial north pole. On the same side of the pole as *Polaris,* but much farther from it, is a constellation called *Cassiopeia,* the five brightest stars of which form a rather unsymmetrical letter W (Fig. 55). The lower left-hand star of this constellation, the one at the bottom of the first stroke of the W, is called δ, and is of importance to the surveyor because it is very nearly on the hour circle passing through *Polaris* and the pole; in other words its right ascension is nearly the same as that of *Polaris.* On the opposite side of the pole from *Cassiopeia* is *Ursa Major,* or the great dipper, a rather conspicuous constellation. The star ζ, which is at the bend in the dipper handle, is also nearly on the same hour circle as *Polaris* and δ *Cassiopeiæ.* If a line be drawn on the sphere between δ *Cassiopeiæ* and ζ *Ursæ Majoris*, it will pass nearly through *Polaris* and the pole, and will show at once the position of *Polaris* in its diurnal circle. The two stars in the bowl of the great dipper on the side farthest from the handle are in a line which, if prolonged, would pass near to *Polaris*. These stars are therefore called the **pointers** and may be used to find the polestar. There is no other star near *Polaris* which is likely to be confused with it. Another star which should be remembered is β *Cassiopeiæ*, the one at the upper right-hand corner of the W. Its right ascension is very nearly 0^h and therefore the hour circle through it passes nearly through the equinox. It is possible then, by simply glancing at β *Cassiopeiæ* and the polestar, to estimate approximately the local sidereal time. When β *Cassiopeiæ* is vertically above the polestar it is nearly 0^h sidereal time; when the star is below the polestar it is 12^h sidereal time; half way between these positions, left and right, it is 6^h and 18^h, respectively. In intermediate positions the hour angle of the star (= sidereal time) may be roughly estimated.

67. Constellations Near the Equator.

The principal constellations within 45° of the equator are shown in Figs. 56 to 58. Hour circles are drawn for each hour of R. A. and parallels for each 10° of declination. The approximate declination and right ascension of a star may be obtained by scaling the coördinates from the chart. The position of the ecliptic, or sun's path in the sky, is shown as a curved line. The moon and the planets are always found near this circle because the planes of their orbits have only a small inclination to the earth's orbit. A belt extending about 8° each side of the ecliptic is called the **Zodiac,** and all the members of the solar system will always be found within this belt. The constellations along this belt, and which have given the names to the twelve "signs of the Zodiac," are *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Pisces*. These constellations were named many centuries ago, and the

names have been retained, both for the constellations themselves and also for the positions in the ecliptic which they occupied at that time. But on account of the continuous westward motion of the equinox, the "signs" no longer correspond to the constellations of the same name. For example, the *sign of Aries* extends from the equinoctial point to a point on the ecliptic 30° eastward, but the constellation actually occupying this space at present is *Pisces*. In Figs. 56 to 58 the constellations are shown as seen by an observer on the earth, not as they would appear on a celestial globe. On account of the form of projection used in these maps there is some distortion, but if the observer faces south and holds the page up at an altitude equal to his co-latitude, the map represents the constellations very nearly as they will appear to him. The portion of the map to be used in any month is that marked with the name of the month at the top; for example, the stars under the word "February" are those passing the meridian in the middle of February at about 9 P.M. For other hours in the evening the stars on the meridian will be those at a corresponding distance right or left, according as the time is earlier or later than 9 P.M. The approximate right ascension of a point on the meridian may be found at any time as follows: First compute the R. A. of the sun by allowing 2^h per month, or more nearly 4^m per day for every day since March 23, remembering that the R. A. of the sun is always increasing. Add this R. A. + 12^h to the local civil time and the result is the sidereal time or right ascension of a star on the meridian.

Example. On October 10 the R. A. of the sun is $6 \times 2^h + 17 \times 4^m = 13^h\ 08^m$. The R. A. of sun + 12^h is $25^h\ 08^m$, or $1^h\ 08^m$. At 9^h P.M. the local civil time is 21^h. $1^h\ 08^m + 21^h = 22^h\ 08^m$. A star having a R. A. of $22^h\ 08^m$ would therefore be close to the meridian at 9 P.M.

Fig. 59 shows the stars about the south celestial pole. There is no bright star near the south pole, so that the convenient methods of determining the meridian by observations on the *polestar* are not practicable in the southern hemisphere.

68. The Planets.

In using the star maps, the student should be on the lookout for planets. These cannot be placed on the maps because their positions are rapidly changing. If a bright star is seen near the ecliptic, and its position does not correspond to that of a star on the map, it is a planet. The planet *Venus* is very bright and is never very far east or west of the sun; it will therefore be seen a little before sunrise or a little after sunset. *Mars*, *Jupiter*, and *Saturn* move in orbits which are outside of that of the earth and therefore appear to us to make a complete circuit of the heavens. *Mars* makes one revolution around the sun in 1 year 10 months, *Jupiter* in about 12 years, and *Saturn* in 29½ years. *Jupiter* is the brightest, and when looked at through a small telescope shows a disc like that of the full moon; four satellites can usually be seen lying nearly in a straight line. *Saturn* is not as large as *Jupiter*, but in a telescope of moderate power its rings can be distinguished; in a low-power telescope the planet appears to be elliptical in form. *Mars* is reddish in color and shows a disc.

CHAPTER X

OBSERVATIONS FOR LATITUDE

IN this chapter and the three immediately following are given the more common methods of determining latitude, time, longitude, and azimuth with small instruments. Those which are simple and direct are printed in large type, and may be used for a short course in the subject. Following these are given, in smaller type, several methods which, although less simple, are very useful to the engineer; these methods require a knowledge of other data which the engineer must obtain by observation, and are therefore better adapted to a more extended course of study.

69. Latitude by a Circumpolar Star at Culmination. This method may be used with any circumpolar star, but *Polaris* is the best one to use, when it is practicable to do so, because it is of the second magnitude, while all of the other close circumpolars are quite faint. The observation consists in measuring the altitude of the star when it is a maximum or a minimum, or, in other words, when it is on the observer's meridian. This altitude may be obtained by trial, and it is not necessary to know the exact instant when the star is on the meridian. The approximate time when the star is at culmination may be obtained from Table V or by Art. 34 and Equa. [45]. It is not necessary to know the time with accuracy, but it will save unnecessary waiting if the time is known approximately. In the absence of any definite knowledge of the time of culmination, the position of the pole star with respect to the meridian may be estimated by noting the positions of the constellations. When δ *Cassiopeiæ* is directly above or below *Polaris* the latter is at upper or lower culmination. The observation should be begun some time before one of these positions is reached. The horizontal cross hair of the transit should be set on the star* and the motion of the star followed by means of the tangent screw of the horizontal axis. When the desired maximum or minimum is reached the vertical arc is read. The index correction should then be determined. If the instrument has a complete vertical circle and the time of culmination is known approximately, it will be well to eliminate instrumental errors by taking a second altitude with the instrument reversed, provided that neither observation is made more than 4^m or 5^m from the time of culmination. If the star is a faint one, and therefore difficult to find, it may be necessary to compute its approximate altitude (using the best known value for the latitude) and set off this altitude on the vertical arc. The star may be found by moving the telescope slowly right and left until the star comes into the field of view. *Polaris* can usually be found in this manner some time before dark, when it cannot be seen with the unaided eye. It is especially important to focus the telescope carefully before attempting to find the star, for the slightest error of focus may render the star invisible. The focus may be adjusted by looking at a distant terrestrial object or, better still, by sighting at the moon or at a planet if one is visible. If observations are to be made frequently with a surveyor's transit, it is well to have a reference mark scratched on the telescope tube, so that the objective may be set at once at the proper focus.

The latitude is computed from Equa. [3] or [4], p. 31. The true altitude h is derived from the reading of the vertical circle by applying the index correction with proper sign and then subtracting the refraction correction (Table I). The polar distance is found by taking from the Ephemeris (Table of Circumpolar Stars) the *apparent declination* of the star and subtracting this from 90°.

* The image of a star would be practically a point of light in a perfect telescope, but, owing to the imperfections in the corrections for spherical and chromatic aberration, the image is irregular in shape and has an appreciable width. The image of the star should be bisected with the horizontal cross hair.

Example 1.

Observed altitude of *Polaris* at upper culmination = 43° 37′; index correction = +30″; declination = +88° 44′ 35″.

Vertical circle	= 43° 37′ 00″
Index correction	= +30
Observed altitude	= 43 37 30
Refraction correction	= 1 00
True altitude	= 43 36 30
Polar distance	= 1 15 25
Latitude	= 42° 21′ 05″

Since the vertical circle reads only to 1′ the resulting value for the latitude must be considered as reliable only to the nearest 1′.

Example 2.

Observed altitude of 51 *Cephei* at lower culmination = 39° 33′ 30″; index correction = 0″; declination = + 87° 11′ 25″.

Observed altitude	= 39° 33′ 30″
Refraction correction	= 1 09
True altitude	= 39 32 21
Polar distance	= 2 48 35
Latitude	= 42° 20′ 56″

70. Latitude by Altitude of Sun at Noon.

The altitude of the sun at noon (meridian passage) may be determined by placing the line of sight of the transit in the plane of the meridian and observing the altitude of the upper or lower limb of the sun when it is on the vertical cross hair. The watch time at which the sun will pass the meridian may be computed by converting 12^h local apparent time into Standard or local mean time (whichever is used) as shown in Arts. 28 and 32. Usually the direction of the meridian is not known, so the maximum altitude of the sun is observed and assumed to be the same as the meridian altitude. On account of the sun's changing declination the maximum altitude is not quite the same as the meridian altitude; the difference is quite small, however, usually a fraction of a second, and may be entirely neglected for observations made with the engineer's transit or the sextant. The maximum altitude of the upper or lower limb is found by trial, the horizontal cross hair being kept tangent to the limb as long as it continues to rise. When the observed limb begins to drop below the cross hair the altitude is read from the vertical arc and the index correction is determined. The true altitude of the centre of the sun is then found by applying the corrections for index error, refraction, semidiameter, and parallax. In order to compute the latitude it is necessary to know the sun's declination at the instant the altitude was taken. If the longitude of the place is known the sun's declination may be corrected as follows: If the Greenwich Time or the Standard Time is noted at the instant of the observation the number of hours since 0^h Gr. Civ. Time is known at once. If the time has not been observed it may be derived from the known longitude of the place. Since the sun is on the meridian the local apparent time is 12^h. Adding the longitude we obtain the Gr. App. Time. This is converted into Gr. Civil Time by subtracting the equation of time. The declination is then corrected by an amount equal to the "variation per hour" multiplied by the hours of the Gr. Civ. Time. The time need not be computed with great accuracy since an error of 1^m will never cause an error greater than $1''$ in the computed declination. The latitude is computed by applying equation [1] or its equivalent.

Example 1. Observed maximum altitude of the sun's lower limb, Jan. 15, 1925, $= 26° 15'$ (sun south of zenith); index correction, $+1'$; longitude $71° 06'$ W.; sun's declination Jan. 15 at 0^h Greenwich Civil Time $= -21° 15' 19''.4$; variation per hour, $+26''.89$; Jan. 16, $-21° 04' 21''.9$; variation per hour, $+27''.90$; equation of time, $-9^m 17^s$; semidiameter, $16' 17''.53$.

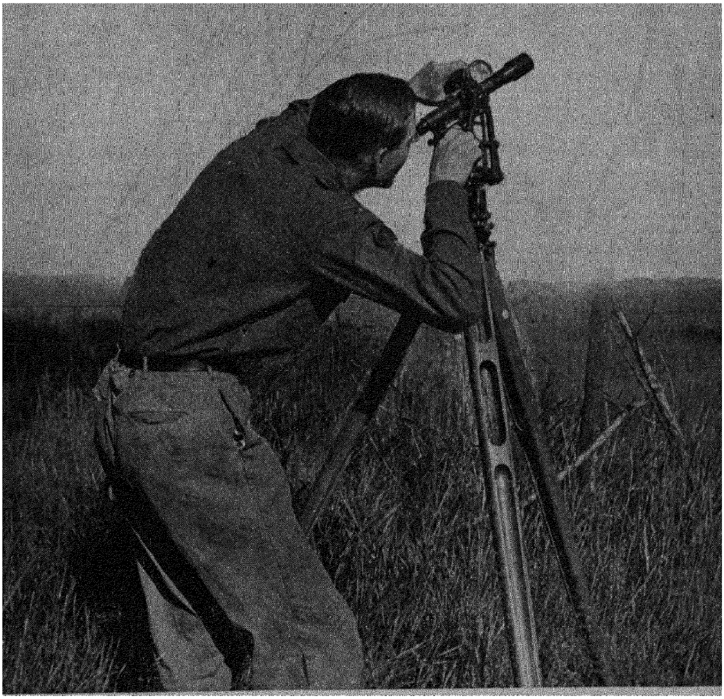
Observed altitude	= $26° 15'$
Index correction	= $+1'$
	$26° 16'$
Refraction	-1.9
	$26° 14'.1$
Semidiameter	$+16.3$
	$26° 30'.4$
Parallax	$+.1$
	$26° 30'.5$
Declination	$-21\ 07.7$
Co-latitude	$47° 38'.2$
Latitude	$42° 21'.8$

Loc. App. Time	=	12^h
Longitude	=	$4^h 44^m 24^s$
Gr. App. Time	=	$16^h 44^m 24^s$
Equa. Time	=	$-9\ 17$
Gr. Civ. Time	=	$16^h 53^m 41^s$
Decl. at 0^h	=	$-21° 04' 21''.9$
$+27''.90 \times 7^h.1$	=	$3\ 18.1$
Corrected Decl.	=	$-21° 07' 40''.0$

Example 2. Observed maximum altitude of sun's lower limb June 1, 1925 $= 44° 48' 30''$ bearing *north;* index correction $= 0''$; Gr. Civil Time $= 14^h 50^m 12^s$,

declination of sun at 0^h, G. C. T., = $+21° 57' 13''.7$; variation per hour, $+21''.11$; semidiameter, $15' 48''.05$.

Observed altitude	44° 48′ 30″	Decl. at 0^h	= +21° 57′ 13″.7
Refraction	−57	$+21''.11 \times 14^h.84$ =	+5 13 .3
	44° 47′ 33″	Corrected Decl.	+22° 02′ 27″.0
Semidiameter	15 48		
h	45° 03′ 21″		
$\zeta = 90° - h$	44 56 39		
δ	+22 02 27		
ϕ	22° 54′ 12″ South		



71. By the Meridian Altitude of a Southern* Star.

The latitude may be found from the observed maximum altitude of a star which culminates south of the zenith, by the method of the preceding article, except that the parallax and

* The observer is assumed to be in the northern hemisphere.

semidiameter corrections become zero, and that it is not necessary to note the time of the observation, since the declination of the star changes so slowly. In measuring the altitude the star's image is bisected with the horizontal cross hair, and the maximum found by trial as when observing on the sun. For the method of finding the time at which a star will pass the meridian see Art. 76.

Example. Observed meridian altitude of θ *Serpentis* = 51° 45′; index correction = 0; declination of star = +4° 05′ 11″.

Observed altitude of θ *Serpentis*	=	51° 45′ 00″
Refraction correction	=	−45
		51° 44′ 15″
Declination of star	=	+ 4 05 11
Co-latitude	=	47° 39′ 04″
Latitude	=	42 20 56

Constant errors in the measured altitudes may be eliminated by combining the results obtained from circumpolar stars with those from southern stars. An error which makes the latitude too great in one case will make it too small by the same amount in the other case.

72. Altitudes Near the Meridian.

If altitudes of the sun or a star are taken near the meridian they may be reduced to the meridian altitude provided the latitude and the times are known with sufficient accuracy. To derive the formula for making the reduction to the meridian we employ Equa. [8], p. 32.

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \qquad [8]$$

This is equivalent to

$$\sin h = \cos (\phi - \delta) - \cos \phi \cos \delta \text{ vers } t \qquad [70]$$

or

$$\sin h = \cos (\phi - \delta) - \cos \phi \cos \delta \; 2 \sin^2 \frac{t}{2} \cdot \qquad [71]$$

Denoting by h_m the meridian altitude, $90° - (\phi - \delta)$, the equations become

$$\sin h_m = \sin h + \cos \phi \cos \delta \text{ vers } t \qquad [72]$$

$$\sin h_m = \sin h + \cos \phi \cos \delta \; 2 \sin^2 \frac{t}{2} \cdot \qquad [73]$$

If the time is noted when the altitude is measured the value of t may be computed, provided the error of the timepiece is known. With an approximate value of ϕ the second term may be computed and the meridian altitude h_m found through its sine. If the latitude computed from h_m differs much from the preliminary value a second computation should be made, using the new value for the latitude. These equations are exact in form and may be used even when t is large. The method may be employed when the meridian observation cannot be obtained.

Example. Observed double altitude of sun's lower limb Jan. 28, 1910, with sextant and artificial horizon.

	Double Altitude
	56° 44′ 40″
	49 00
	52 40
Mean	56° 48′ 47″
I. C.	+30
	2)56° 49′ 17″
	28° 24′ 38″
Refr. and par.	1 38
	28° 23′ 00″
Semidiameter	+16 16
h =	28° 39′ 16″

	Watch
	$11^h\ 15^m\ 25^s$
	16 22
	17 10
	$11^h\ 16^m\ 19^s$
Watch corr.	+1 19
E. S. T. of observ.	$11^h\ 17^m\ 38^s$
E. S. T. of app. noon	11 57 21
Hour angle =	$39^m\ 43^s$
t =	9° 55′ 45″

log cos ϕ	= 9.86763
log cos δ	= 9.97745
log vers t	= 8.17546
log corr.	= 8.02054
corr.	= .01048
nat. sin h	= .47953
nat. sin h_m	= .49001
h_m	= 29° 20′ 29″
ζ	= 60 39 31
δ	= 18 18 20
ϕ	= 42° 21′ 11″ N.

Assumed latitude	42° 30′
Declination	−18° 18′ 20″
Loc. app. noon	$12^h\ 00^m\ 00^s$
Equa. time	−13 03
	$12^h\ 13^m\ 03^s$
Long. diff.	15 42
E. S. T. of noon	$11^h\ 57^m\ 21^s$

Note: A recomputation of the latitude, using this value, changes the result to 42° 21′ 04″ N.

When the observations are taken within a few minutes of meridian passage the following method, taken from Serial No. 166, U. S. Coast and Geodetic Survey, may be employed for reducing the observations to the meridian. This method makes it possible to utilize all of the observations taken during a period of 20 minutes and gives a more accurate result than would be obtained from a single meridian altitude.

From Equa. [73]

$$\sin h_m - \sin h = 2 \cos \phi \cos \delta \sin^2 \frac{t}{2}\,. \qquad [74]$$

By trigonometry,

$$\sin h_m - \sin h = 2 \cos \tfrac{1}{2} (h_m + h) \sin \tfrac{1}{2} (h_m - h)$$

therefore

$$\sin \tfrac{1}{2} (h_m - h) = \cos \phi \cos \delta \sin^2 \frac{t}{2} \sec \tfrac{1}{2} (h_m + h) \qquad [75]$$

since $h_m - h$ is small, we may replace $\sin \frac{1}{2} (h_m - h)$ by $\frac{1}{2} (h_m - h) \sin 1''$; and also replace $\frac{1}{2} (h_m + h)$ by $h = 90° - \zeta$.

Then [75] becomes

$$h_m - h = \cos \phi \cos \delta \frac{2 \sin^2 \frac{t}{2}}{\sin 1''} \operatorname{cosec} \zeta$$

or

$$h_m = h + \cos \phi \cos \delta \operatorname{cosec} \zeta \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}\,. \qquad [76]$$

Placing $A = \cos\phi \cos\delta \operatorname{cosec} \zeta$

and $$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}.$$

Then $$h_m = h + Am. \qquad [77]$$

The latitude is then found by

$$\phi = \delta + \zeta.$$

Values of m will be found in Table X and values of A in Table IX. The errors involved in this method become appreciable when the value of t is more than 10 minutes of time.

The observations should be begun about 10 minutes before local apparent noon (or meridian passage, if a star is being observed) and continued until about 10 minutes after noon. The chronometer time or watch time of noon should be computed beforehand by the methods as explained in Chapter V. In the example given on p. 123 the chronometer was known to be $27^h\ 20^m$ slow of local civil time, and the equation of time was $-5^m\ 53^s$. The chronometer time of noon was therefore $12^h + 5^m\ 53^s - 27^m\ 20^s = 11^h\ 38^m\ 33^s$.

The values of t are found by subtracting the chronometer time of noon from the observed times. The value of m is taken from Table X for each value of t. A is taken from Table IX for approximate values of ϕ, δ and ζ. The values of the correction Am are added to the corresponding observed altitudes. The mean of all of the reduced altitudes, corrected for refraction and parallax, is the true meridian altitude of the centre.

73. Latitude by Altitude of Polaris when the Time is Known.

The latitude may be found conveniently from an observed altitude of *Polaris* taken at any time provided the error of the timepiece is approximately known. *Polaris* is but a little more than a degree from the pole and small errors in the time have a relatively small effect upon the result. It is advisable to take several altitudes in quick succession and note the time at each pointing on the star. Unless the observations extend over a long period, say more than 10 minutes of time, it will be sufficiently accurate to take the mean of the altitudes and the mean of the times and treat this as the result of a single observation. If the transit has a complete vertical circle, half the altitudes may be taken with the telescope in the direct position, half in the reversed position. The index correction should be carefully determined.

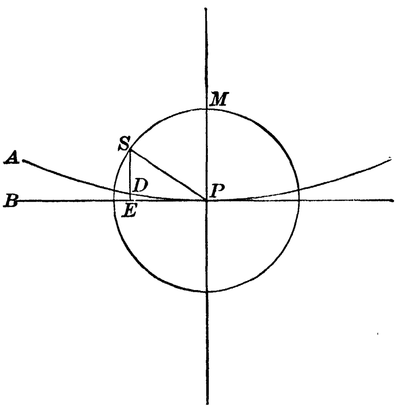


FIG. 60

The hour angle (t) of the star must be computed for the instant of the observation. This is done according to the methods given in Chapter V. In the following example the watch is set to Eastern Standard Time. This is first converted into local civil time (from the known longitude) and then into local

Example.

OBSERVATIONS OF SUN FOR LATITUDE

Station, Smyrna Mills, Me. Date, Friday, August 5, 1910
Theodolite of mag'r No. 20. Observer, H. E. McComb
Chronometer No. 245. Temperature, 24° C.

Sun's limb.	V. C.	Chronometer time.	Vertical circle A.	B.	Mean.
U	*R*	11^h 30^m 04^s	61° 14′ 00″	13′ 30″	61° 13′ 45″
L	*L*	11 31 16	119 23 00	20 00	60 38 30
L	*L*	11 33 14	119 22 30	19 30	60 39 00
U	*R*	11 34 38	61 16 30	15 30	61 16 00
U	*R*	11 36 36	61 17 00	15 30	61 16 15
L	*L*	11 37 34	119 21 30	19 00	60 39 45
L	*L*	11 39 32	119 21 30	19 00	60 39 45
U	*R*	11 40 33	61 17 30	16 00	61 16 45
U	*R*	11 42 46	61 16 30	15 00	61 15 45
L	*L*	11 43 30	119 22 30	20 00	60 38 45
			Obs'd. max. alt.		60 58 15
			R & *P*		−27
			h		60 57 48
			ζ		29 02 12
			δ		17 06 18
			ϕ		46 08 30

COMPUTATION OF LATITUDE FROM CIRCUMMERIDIAN ALTITUDES OF SUN

Station, Smyrna Mills, Me. Date, August 5, 1910.

Chron. correction on L. M. T. $^h+27^m$ 20^s
Local mean time of app. noon 12 05 53
Chron. time of apparent noon 11 38 33

t	*m*	*A*	*Am*	Reduced h. of sun's limb.	Reduced h. of sun.
-8^m 29^s	141″	1.36	192″	61° 16′ 57″	
−7 17	104		141	60 40 51	60° 58′ 54″
−5 19	56		76	60 40 16	
−3 55	30		41	61 16 41	60 58 28
−1 57	8		11	61 16 26	
−0 59	2		3	60 39 48	60 58 07
+0 59	2		3	60 39 48	
+2 00	8		11	61 16 56	60 58 22
+4 13	35		48	61 16 33	
+4 57	48		65	60 39 50	60 58 12
				Mean	60 58 25
				R. & P.	−27
				h	60 57 58
				ζ	29 02 02
				δ	17 06 19
				ϕ	46 08 21

sidereal time (see Art. 37). The hour angle, t, is the difference between the sidereal time and the star's right ascension.

The latitude is computed by the formula

$$\phi = h - p \cos t + \tfrac{1}{2} p^2 \sin^2 t \tan h \sin 1'' \qquad [78]$$

the polar distance, p, being in seconds. For the derivation of this formula see Chauvenet, Spherical and Practical Astronomy, Vol. I, p. 253.

In Fig. 60 P is the pole, S the star, MS the hour angle, and PDA the parallel of altitude through the pole. The point D is therefore at the same altitude as the pole. The term $p \cos t$ is approximately the distance from S to E, a point on the 6-hour circle PB. The distance desired is SD, the diffeernce between the altitude of S and the altitude of the pole. The last term of the formula represents very nearly this distance DE. When S is above the pole DE diminishes SE; when S is below the pole it increases it.

Example 1.

Observed altitudes of *Polaris*, Jan. 9, 1907

	Watch		Altitudes
	$6^h\ 49^m\ 26^s$		$43°\ 28'\ 30''$
	51 45		28 30
	54 14		28 00
	56 45		28 00
Mean	$6^h\ 53^m\ 02^s.5$	Mean	$43°\ 28'\ 15''$

Index correction, $-1'\ 00''$; $p = 1°\ 11'\ 09'' = 4269''$; t is found from the observed watch times to be $13°\ 50'.7$.*

log p	= 3.63033	log const.	= 4.3845
log cos t	= 9.98719	log p^2	= 7.2607
log (p cos t)	= 3.61752	log $\sin^2 t$	= 8.7578
p cos t	= $-4145''.0$	log tan h	= 6.9768
			0.3798
		last term	= $+2''.4$

Observed alt.	= $43°\ 28'\ 15''$
Index corr.	= $-1\ 00$
Refraction	= $-1\ 00$
	$43°\ 26'\ 15''$
1st and 2nd terms	1 09 03
Latitude	$42°\ 17'\ 12''$ N.

This computation may be greatly shortened by the use of Table I of the Ephemeris, or Table I of the Nautical Almanac. In the Ephemeris the total correction to the altitude is tabulated for every 3^m of hour angle and for every $10''$ of declination. In the Almanac the correction is given for every 10^m of local sidereal time.

Example 2.

The observed altitude of Polaris on March 10, 1925 = $42°\ 20'$; Watch time = $8^h\ 49^m\ 30^s$ P.M.; watch 30^s slow of E. S. T. Long. $71°\ 10'$ W. Index correction, $+1'$. Declination of *Polaris*, $+88°\ 54'\ 18''$; right ascension, $1^h\ 33^m\ 35^s.6$

* If the error of the watch is known the sidereal time may be found by the methods explained in Chap. V. For methods of finding the sidereal time by direct observation see Chap. XI.

Watch	$8^h\ 49^m\ 30^s$
Error	30
E. S. T.	$8^h\ 50^m\ 00^s$ P.M.
Civ. Time	= 20 50 00
Dif. Long.	15 20
Loc. Civ. Time	= $21^h\ 05^m\ 20^s$
Table III	= 3 27 .9
Sun's R. A. $+12^h$	= 11 08 36 .1
Table III (Long.)	46 .8
	$32^h\ 18^m\ 10^s.8$
	24
Loc. Sid. Time	= $8^h\ 18^m\ 10^s.8$
Rt. Asc. Star	= 1 33 35 .6
Hour Angle of *Polaris*	$6^h\ 44^m\ 35^s.2$

Observed alt.	42° 20′
I. C.	+1
Refr.	−1 .0
h	42° 20′.0
Corr., Table I	+13 .0
Latitude	42° 33′.0

Note: From the Almanac the correction for Loc. Sid. Time $8^h\ 18^m\ 10^s.8$ is +13′.0. From the Ephemeris the correction for hour angle $6^h\ 44^m\ 35^s.2$ and declination +88° 54′ 18″ is +13′ 12″. The latter is more accurate.

Example 3. Observed altitude of *Polaris* May 5, 1925 = 41° 10′ at Gr. Civ. Time $23^h\ 50^m$. Longitude 5^h West.

Gr. Civ. Time	$23^h\ 50^m\ 00^s$.
Table III	3 54 .91
R. A. $+12^h$	14 49 23 .08
	$38^h\ 43^m\ 17^s.99$
	24
Greenwich Sidereal Time	$14^h\ 43^m\ 17^s.99$
	5
Loc. Sid. Time	$9^h\ 43^m\ 17^s.99$
R. A. Polaris	1 33 30 .68
Hour Angle, t,	$8^h\ 09^m\ 47^s.31$
Observed Altitude	41° 10′ 00″
Refraction	−1 05
	41° 08′ 55″
Correction, Table I (Eph.)	+35 58
Correction, Table Ia (Eph.)	−5
Latitude	41° 44′ 48″ N.

74. Precise Latitudes — Harrebow-Talcott Method.

The most precise method of determining latitude is that known as the "Harrebow-Talcott" Method, in which the zenith telescope is employed. Two stars are selected, one of which will culminate north of the observer's zenith, the other south, and whose zenith distances differ by only a few minutes of angle. For convenience the right ascensions should differ by only a few minutes of time, say 5^m to 10^m. The approximate latitude must be known in advance, that is, within 1′ or 2′, in order that the stars may be selected. This may be determined with the zenith telescope, using the method of Art. 71. It will usually be necessary to consult the star catalogues in order to find a sufficient number of pairs which fulfill the necessary conditions as to difference of zenith distance and difference of right ascension.

If the first star is to culminate south of the zenith the telescope is turned until the stop indicates that it is in the plane of the meridian, on the south side, and then clamped in this position. The mean of the two zenith distances is then set on the finder circle and the telescope tipped until the bubble of the latitude level is in the centre of its tube. When the star appears in the field it is bisected with the micrometer wire; at the instant of passing the vertical wire, that is, at culmination, the bisection is perfected. The scale readings of the ends of the bubble of the latitude level are read immediately, then the micrometer is read and the readir recorded. The chronometer should also be read at the instant of culminatior order to verify the setting of the instrument in the meridian.

The telescope is then turned to the north side of the meridian (as indicated by the stop) and the observations repeated on the other star. Great care should be taken not to disturb the relation between the telescope and the latitude level. The tangent screw should not be touched during observation on a pair.

When the observations have been completed the latitude may be computed by the formula

$$\phi = \tfrac{1}{2}(\delta_s + \delta_n) + \tfrac{1}{2}(m_s - m_n) \times R + \tfrac{1}{2}(l_s + l_n) + \tfrac{1}{2}(r_s - r_n) \qquad [79]$$

in which m_s, m_n, are the micrometer readings, R the value of 1 division of the micrometer, l_s, l_n, the level corrections, positive when the north reading is tl larger, and r_s, r_n, the refraction corrections. Another correction would be requir in case the observation is taken when the star is not exactly on the meridian.

In order to determine the latitude with the precision required in geodetic operations it is necessary to observe as many pairs as is possible during one night (say 10 to 20 pairs). In some cases observations are made on more than one night in order to secure the necessary accuracy. By this method the latitude may readily be determined within 0''.10 (or less) of the true latitude, that is, with an error of 10 feet or less on the earth's surface.

Questions and Problems

1. Observed maximum altitude of the sun's lower limb, April 27, 1925 = 61° 28', bearing *South*. Index correction = +30''. The Eastern Standard Time is $11^h\ 42^m$ A.M. The sun's declination April 27 at 0^h Gr. Civ. T. = +13° 35' 51''.3; the varia. per hour is +48''.19; April 28, +13° 55' 01''.0; varia. per hour, +47''.62; the semidiameter is 15' 55''.03. Compute the latitude.

2. Observed maximum altitude of the sun's lower limb Dec. 5, 1925 = 30° 10', bearing *South*. Longitude = 73° W. Equation of time = $+9^m\ 22^s$. Sun's declination Dec. 5 at 0^h Gr. Civ. T. = −22° 16' 54''.0; varia. per hour = −19''.85; Dec. 6, −22° 24' 37''.5; varia. per hour, −18''.77; semidiameter, 16' 15''.84. Compute the latitude.

3. The noon altitude of the sun's lower limb, observed at sea Oct. 1, 1925 = 40° 30' 20'', bearing *South*. Height of eye, 30 feet. The longitude is 35° 10' W. Equation of time = $+10^m\ 03^s.56$. Sun's declination Oct. 1 at 0^h Gr. Civ. T. = −2° 53' 38''.2; varia. per hour = −58''.28; on Oct. 2, −3° 16' 56''.1; varia. per hour, −58''.20; semidiameter = 16' 00''.57. Compute the latitude.

4. The observed meridian altitude of *δ Crateris* = 33° 24', bearing *South*; index correction, +30''; declination of star = −14° 17' 37''. Compute the latitude.

5. Observed (ex. meridian) altitude of *α Ceti* at $3^h\ 08^m\ 49^s$ local sidereal time = 51° 21'; index correction = −1'; the right ascension of *α Ceti* = $2^h\ 57^m\ 24^s.0$ declination = +3° 43' 22''. Compute the latitude.

6. Observed altitude of *Polaris*, 41° 41' 30''; chronometer time, $9^h\ 44^m\ 38^s.5$ (local sidereal); chronometer correction = -34^s. The right ascension of Polaris = $1^h\ 25^m\ 42^s$; the declination = +88° 49' 29''. Compute the latitude.

7. Show by a sketch the following three points: 1. *Polaris* at greatest elongation; 2. *Polaris* on the 6-hour circle; 3. *Polaris* at the same altitude as the pole. (See Art. 73, p. 122, and Fig. 28, p. 37.)

8. Draw a sketch (like Fig. 19) showing why the sun's maximum altitude is not the same as the meridian altitude.

CHAPTER XI

OBSERVATIONS FOR DETERMINING THE TIME

75. Observation for Local Time.

Observations for determining the local time at any place at any instant usually consist in finding the error of a timepiece on the kind of time which it is supposed to keep. To find the solar time it is necessary to determine the hour angle of the sun's centre. To find the sidereal time the hour angle of the vernal equinox must be measured. In some cases these quantities cannot be measured directly, so it is often necessary to measure other coördinates and to calculate the desired hour angle from these measurements. The **chronometer correction** or **watch correction** is the amount to be added algebraically to the reading of the timepiece to give the true time at the instant. It is positive when the chronometer is slow, negative when it is fast. The **rate** is the amount the timepiece gains or loses per day; it is positive when it is losing, negative when it is gaining.

76. Time by Transit of a Star.

The most direct and simple means of determining time is by observing transits of stars across the meridian. If the line of sight of a transit be placed so as to revolve in the plane of the meridian, and the instant observed when some known star passes the vertical cross hair, then the local sidereal time at this instant is the same as the right ascension of the star given in the Ephemeris for the date. The difference between the observed chronometer time T and the right ascension α is the chronometer correction ΔT,

or

$$\Delta T = \alpha - T. \qquad [86]$$

If the chronometer keeps mean solar time it is only necessary to convert the true sidereal time α into mean solar time by

Equa. [45], and the difference between the observed and computed times is the chronometer correction.

The transit should be set up and the vertical cross hair sighted on a meridian mark previously established. If the instrument is in adjustment the sight line will then swing in the plane of the meridian. It is important that the horizontal axis should be accurately levelled; the plate level which is parallel to this axis should be adjusted and centred carefully, or else a striding level should be used. Any errors in the adjustment will be eliminated if the instrument is used in both the direct and reversed positions, provided the altitudes of the stars observed in the two positions are equal. It is usually possible to select stars whose altitudes are so nearly equal that the elimination of errors will be nearly complete.

In order to find the star which is to be observed, its approximate altitude should be computed beforehand and set off on the vertical arc. (See Equa. [1].) In making this computation the refraction correction may be omitted, since it is not usually necessary to know the altitude closer than 5 or 10 minutes. It is also convenient to know beforehand the approximate time at which the star will culminate, in order to be prepared for the observation. If the approximate error of the watch is already known, then the watch time of transit may be computed (Equa. [45]) and the appearance of the star in the field looked for a little in advance of this time. If the data from the Ephemeris are not at hand the computation may be made, with sufficient accuracy for finding the star, by the following method: Compute the sun's R. A. by multiplying 4^m by the number of days since March 22. Take the star's R. A. from any list of stars or a star map. The star's R. A. minus the (sun's R. A. + 12^h) will be the mean local time within perhaps 2^m or 3^m. This may be reduced to Standard Time by the method explained in Art. 32. In the surveyor's transit the field of view is usually about $1°$, so the star will be seen about 2^m before it reaches the vertical cross hair. Near culmination the star's path is so nearly horizontal that it will appear to coincide with the horizontal cross hair from one side of the field to the other. When the star passes the vertical cross hair the time should be noted as accurately as possible. A stop watch will sometimes be found convenient in field observations with the surveyor's transit. When a chronometer is used the "eye and ear method" is the best. (See Art. 61.) If it is desired to determine the latitude from this same star, the observer has only to set the horizontal cross hair on the star immediately after making the time observation, and the reading of the vertical arc will give the star's apparent altitude at culmination. (See Art. 71.)

The computation of the watch correction consists in finding the true time at which the star should transit and comparing it with the observed watch time. If a sidereal watch or chronometer is used the error may be found at once since the star's right ascension is the local sidereal time. If civil time is desired, the true sidereal time must be converted into local civil time, or into Standard Time, whichever is desired.

Transit observations for the determination of time can be much more accurately made in low than in high latitudes. Near the pole the conditions are very unfavorable.

Example.

Observed the transit of α *Hydræ* on April 5, 1925, in longitude $5^h\ 20^m$ west. Observed watch time (approx. Eastern Standard Time) = $8^h\ 48^m\ 24^s$ P.M. or $20^h\ 48^m\ 24^s$ Civil Time. The right ascension of α *Hydræ* for this date is $9^h\ 23^m\ 54^s.84$; the R. A. of the mean sun $+12^h$ is $12^h\ 51^m\ 06^s.48$ at 0^h Gr. Civ. T. From Table III the correction for $5^h\ 20^m$ is $+52^s.57$.

Rt. Asc. of *Hydræ* $+24^h$	=	$33^h\ 23^m\ 54^s.84$
Corrected R. A. sun $+12^h$	=	12 51 59 .05
Sid. int. since M'n't.	=	$20^h\ 31^m\ 55^s.79$
Table II	=	−3 21 .82
Local Civil Time	=	$20^h\ 28^m\ 33^s.97$
Red. to 75° merid.	=	20 00 .00
Eastern (Civ.) Time	=	$20^h\ 48^m\ 33^s.97$
Watch	=	20 48 24
Watch correction	=	$+9^s.97$ (slow)

77. Observations with Astronomical Transit.

The method previously described for the small transit is the same in principle as that used with the larger astronomical transits for determining sidereal time. The chief difference is in the precision with which the observations are made and the corrections which have to be applied to allow for instrumental errors. The number of observations on each star is increased by using several vertical threads or by employing the transit micrometer. These are recorded on the electric chronograph and the times may be scaled off with great accuracy.

When the transit is to be used for time determination it is set on a concrete or brick pier, levelled approximately, and turned into the plane of the meridian as nearly as this is known. The collimation is tested by sighting the middle thread at a fixed point, then reversing the axis, end for end, and noting whether the thread is still on the point. The diaphragm should be moved until the object is sighted in both positions. The threads may be made vertical by moving the telescope slowly up and down and noting whether a fixed point remains on the middle thread. The adjustment is made by rotating the diaphragm. To adjust the line of sight (middle thread) into the meridian plane the axis is first levelled by means of the striding level, and an observation taken on a star crossing the meridian near the zenith. This star will cross the middle thread at nearly the correct time even if the instrument is not closely in the meridian. From this observation the error of the chronometer may be obtained within perhaps 2 or 3 seconds. The chronometer time of transit of a circumpolar star is then computed. When this time is indicated by the chronometer the instrument is turned (by the azimuth adjustment screw) until the middle thread is on the circumpolar star. To test the adjustment this process is repeated, the result being a closer value of the chronometer error and a closer setting of the transit into the plane of the meridian. Before observations are begun the axis is re-levelled carefully.

The usual list of stars for time observations of great accuracy would include twelve stars, preferably near the zenith, six to be observed with the "Clamp east," six with "Clamp west." This division into two groups is for the purpose of determining the collimation constant, c. In each group of six stars, three should be north of the zenith and three south. From the discrepancies between the results of these two groups the constant a may be found for each half set. Sometimes a is found by including one slow (circumpolar) star in each half set, its observed time being compared with that of the "time stars," that is those near the zenith. The inclination of the horizontal axis, b, is found by means of the striding level. The observed times are scaled from the chronograph sheet for all observations, and the mean of all threads taken for each star. This mean is then corrected for azimuth error by adding the quantity

$$a \sin \zeta \sec \delta. \qquad [66]$$

The error resulting from the inclination of the axis to the horizon is corrected by adding

$$b \cos \zeta \sec \delta \qquad [67]$$

and finally the collimation error is allowed for by adding

$$c \sec \delta. \qquad [68]$$

Small corrections for the changing error of the chronometer and for the effect of diurnal aberration of light are also added. The final corrected time of transit is subtracted from the right ascension of the star, the result being the chronometer correction on local sidereal time. The mean of all of the results will usually give the time within a few hundredths of a second.

78. Selecting Stars for Transit Observations.

Before the observations are begun the observer should prepare a list of stars suitable for transit observations. This list should include the name or number of the star, its magnitude, the approximate time of culmination, and its meridian altitude or its zenith distance. The right ascensions of consecutive stars in the list should differ by sufficient intervals to give the observer time to make and record an observation and prepare for the next one. The stars used for determining time should be those which have a rapid diurnal motion, that is, stars near the equator; slowly moving stars are not suitable for time determinations. Very faint stars should not be selected unless the telescope is of high power and good definition; those smaller than the fifth magnitude are rather difficult to observe with a small transit, especially as it is difficult to reduce the amount of light used for illuminating the field of view. The selection of stars will also be governed somewhat by a consideration of the effect of the different instrumental errors. An inspection of Table B, p. 99, will show that for stars near the zenith the azimuth error is zero, while the inclination error is a maximum; for stars near the horizon the azimuth error is a maximum and the inclination error is zero. If the azimuth of the instrument is uncertain and the inclination can be accurately determined, then stars having high altitudes should be preferred. On the other hand, if the level parallel to the axis is not a sensitive one and is in poor adjustment, and if the sight line can be placed accurately in the meridian, which is usually the case with a surveyor's transit, then low stars will give the more accurate results. With the surveyor's transit the choice of stars is somewhat limited, however, because it is not practicable to sight the telescope at much greater altitudes than about 70°

with the use of the prismatic eyepiece and 55° or 60° without this attachment.

Following is a sample list of stars selected for observations in a place whose latitude is 42° 22′ N., longitude, 71° 06′ W., date, May 5, 1925; the hours, between 8^h and 9^h P.M., Eastern Standard Time. The limiting altitudes chosen are 10° and 65°. The "sidereal time of 0^h Greenwich Civil Time," or "Right ascension of the mean sun $+12^h$," is $14^h\ 49^m\ 23^s.08$. The local civil time corresponding to 20^h E. S. T. is $20^h\ 15^m\ 36^s$. The local sidereal time is therefore $20^h\ 15^m\ 36^s + 14^h\ 49^m\ 23^s$ + a correction from Table III (which may be neglected for the present purpose) giving about $11^h\ 05^m$ for the right ascension of a star on the meridian at 8^h P.M. Eastern time.

The co-latitude is 47° 38′, the meridian altitude of a star on the equator. For altitudes of 10° and 65° this gives for the limiting declinations +17° 22′ and −37° 38′ respectively.

In the table of "Mean places" of Ten-Day Stars (1925) the following stars will be found. The complete list contains some 800 stars. In the following list many stars between those given have been intentionally omitted, as indicated by the dotted lines.

Star	Mag.	Rt. Asc.	Decl.
....	....	....	....
α Crateris........	4.2	$10^h\ 56^m\ 07^s.105$	−17° 53′ 57″.53
d Leonis...... ..	5.0	10 56 41 .272	+ 4 01 13 .67
....	....	....	....
β Crateris.........	4.5	11 07 58 .012	−22 24 58 .47
δ Leonis....... .	2.6	11 10 07 .379	+20 56 05 .33
....	....	....	....
π Centauri........	4.3	11 17 34 .823	−54 04 47 .36
....	....	....	....
λ Draconis........	4.1	11 26 58 .349	+69 44 42 .71
ξ Hydræ....... ..	3.7	11 29 18 .582	−31 26 33 .36
....	....	....	....
π Chameleontis....	5.7	11 34 09 .389	−75 28 52 .97
3 Draconis........	5.5	11 38 18 .337	+67 09 36 .24
ζ Crateris.........	4.9	11 40 57 .535	−17 56 01 .41
....	....	....	....
γ Corvi...........	2.8	12 11 56 .763	−17 07 31 .85
....	....	....	....

In this list there are three stars, *β Crateris*, *ξ Hydræ*, and *ζ Crateris*, whose declinations and right ascensions fall within the required limits. There are 13 others which could be observed but were omitted in the above list to save space. After selecting the stars to be observed the approximate watch time of transit of the first star should be computed. The times of the other stars may be estimated with sufficient accuracy by means of the differences in the right ascension. The watch times will differ by almost exactly the difference of right ascension. The altitudes (or the zenith distances) should be computed to the nearest minute. This partial list would then appear as follows:

Star	Mag.	Approx. E. S. T.	Approx. Alt.
β Crateris	4.5	$19^h\ 58^m\ 49^s$	25° 13′
ξ Hydræ	3.7	20 20 09	16 11
ζ Crateris	4.9	20 31 48	29 42

In searching for stars the right ascension should be examined first. As the stars are arranged in the list in the order of increasing right ascension it is only necessary to find the right ascension for the time of beginning the observations and then follow down the list. Next check off those stars whose declinations fall within the limits that have been fixed. Finally note the magnitudes and see if any are so small as to make the star an undesirable one to observe.

When the stars have been selected, look in the table of "Apparent Places of stars" to obtain the right ascension and declination for the date. These may be obtained by simple interpolation between the values given for every 10 days. The mean places given in the preceding table may be in error for any particular date by several seconds. With the correct right ascensions the exact time of transit may be calculated as previously explained.

79. Time by Transit of Sun.

The apparent solar time may be determined directly by observing the watch times when the west and the east limbs of the sun cross the meridian. The mean of the two readings is the watch time of the instant of Local Apparent Noon, or 12^h apparent time. This 12^h is to be converted into Local Civil Time and then into Standard Time. If only one edge of the sun's disc can be observed the time of transit of the centre may be found by adding or subtracting the "time of semidiameter passing the meridian." This is given in the Ephemeris for Washington Apparent Noon. The tabulated values are in sidereal time, but may be reduced to mean time by subtracting $0^s.18$ or $0^s.19$ as indicated in a footnote.

Example.

The time of transit of the sun over the meridian 71° 06′ W. is to be observed March 2, 1925.

Local Apparent Time	=	$12^h\ 00^m\ 00^s$	G. C. T. = $16^h.95$		
Equation of Time	=	$-12\quad 19\ .93$			
Local Civil Time	=	$12^h\ 12^m\ 19^s.93$	Equa. of Time at 0^h	=	$12^m\ 16^s.29$
Longitude diff.	=	$15\quad 36\ .$	$0^s.517 \times 7^h.05$	=	$3\ .64$
Eastern Standard Time	=	$11^h\ 56^m\ 43^s.93$	Corrected Equa. of T.	=	$12^m\ 19^s.93$

The observed time of the west and east limbs are $11^h\ 55^m\ 47^s$ and $11^h\ 57^m\ 56^s$ respectively. The mean of these is $11^h\ 56^m\ 51^s.5$; the watch is therefore $7^s.6$ fast. The time of the semidiameter passing the meridian is $1^m\ 05^s.16$. If the second observation had been lost the watch time of transit of the centre would be $11^h\ 55^m\ 47^s + 1^m\ 05^s.16 = 11^h\ 56^m\ 52^s.16$, and the resulting watch correction would be $-8^s.2$.

80. Time by an Altitude of the Sun.

The apparent solar time may be determined by measuring the altitude of the sun when it is not near the meridian, and then solving the *PZS* triangle for the angle at the pole, which is the hour angle of the sun east or west of the meridian. The west hour angle of the sun is the local apparent time. The observation is made by measuring several altitudes in quick succession and noting the corresponding instants of time. The mean of the observed altitudes is assumed to correspond to the mean of the observed times, that is, the curvature of the path of the sun is neglected. The error caused by neglecting the correction for curvature is very small provided the sun is not near the meridian and the series of observations extends over but a few minutes' time, say 10^m. The measurement of altitude must of course be made to the upper or the lower limb and a correction applied for the semidiameter. The observations may be made in two sets, half the altitudes being taken on the upper limb and half on the lower limb, in which case no semidiameter correction is required. The telescope should be reversed between the two sets if the instrument has a complete vertical circle. The mean of the altitudes must be corrected for index error, refraction, and parallax, and for semidiameter if but one limb is observed. The declination at 0^h Gr. Civ. Time is to be corrected by adding the "variation per hour" multiplied by the number of hours in the Greenwich civil time. If the watch used is keeping Standard time the Greenwich time is found at once (see Art. 32). If the watch is not more than 2^m or 3^m in error the resulting error in the declination will not exceed $2''$ or $3''$, which is usually negligible in observations with small instruments. If the Standard time is not known but the longitude is known then the Greenwich time could be computed if the local time were known. Since the local time is the quantity sought the only way of obtaining it is first to compute the hour angle (t) using an approximate value of the declination. From this result an approximate value of the Greenwich civil time may be computed. The declination may now be computed more accurately. A re-computation of the hour angle (t), using this new value of the declination, may be considered final unless the declination used the first time was very much in error.

In order to compute the hour angle the latitude of the place must be known. This may be obtained from a reliable map or may be observed by the methods of Chapter X. The precision with which the latitude must be known depends upon how precisely the altitudes are to be read and also upon the time at which the observation is made. When the sun is near the prime vertical the effect of an error in the latitude is small.

The value of the hour angle is computed by applying any of the formulæ for t in Art. 19. This hour angle is converted into hours, minutes and seconds; if the sun is west of the meridian this is the local apparent time P.M., but if the sun is east of the meridian this time interval is to be subtracted from 12^h to obtain the local apparent time. This apparent time is then converted into mean (civil) time by subtracting the (corrected) equation of time. The local time is then converted into Standard time by means of the longitude difference. The difference between the computed time and the time read on the watch is the watch correction. This observation is often combined with the observation on the sun for azimuth, the watch readings and altitude readings being common to both.

Example.

Nov. 28, 1925. Lat. 42° 22′; Long. 71° 06′.

		Watch (E. S. T.)
Lower limb	14° 41′	$8^h\ 39^m\ 42^s$ A.M.
(Tel. dir.)	15 00	8 42 19
Upper limb	15 55	8 45 34
(Tel. rev.)	16 08	8 47 34
Mean	15° 26′.0	$8^h\ 43^m\ 47^s.2$ E. S. T. (Approx.)
Refraction and parallax	3 .3	5
h	15° 22′.7	$13^h\ 43^m\ 47^s.2$ Gr. Civ. T. (Approx.)

$\phi = 42°\ 22'$
$h = 15\ \ 22.7$
$p = 111\ \ 17.2$
$2s = 169°\ 01'.9$

Decl. at 0^h = $-21°\ 10'\ 58''.8$
$-27''.14 \times 13^h.7$ = $-6\ \ 11.8$
$\delta = -21°\ 17'\ 10''.6$
$p = 111\ \ 17\ \ 10.6$

$s = 84°\ 30'.9$ log cos 8.98039
$s - h = 69\ \ 08.2$ log sin 9.97055
$s - \phi = 42\ \ 08.9$ log csc 0.17324
$s - p = -26\ \ 46.3$ log sec 0.04924
2)9.17342

Eq. of t. at 0^h = $+12^m\ 14^s.56$
$0^s.828 \times 13^h.7$ = 11.34
Eq. of t. $+12^m\ 03^s.22$

$$\log \tan \frac{t}{2} = 9.58671$$

$$\frac{t}{2} = 21°\ 06'\ 43''$$

$$t = 42°\ 13'\ 26''$$
$$= 2^h\ 48^m\ 53^s.7$$

L. A. T. =	$9^h\ 11^m\ 06^s.3$
Eq. of t. =	$+12\ \ 03\ .2$
Loc. Civ. T. =	$8^h\ 59^m\ 03^s.1$
Long. diff. =	$15\ \ 36\ .$
Eastern Standard Time =	$8^h\ 43^m\ 27^s.1$
Watch =	$8\ \ 43\ \ 47\ .2$
Watch fast	$20^s.1$

The most favorable conditions for an accurate determination of time by this method are when the sun is on the prime vertical and the observer is on the equator. When the sun is east or west it is rising or falling at its most rapid rate and an error in the altitude produces less error in the calculated hour angle than the same error would produce if the sun were near the meridian. The nearer the observer is to the equator the greater is the inclination of the sun's path to the horizon, and consequently the greater its rise or fall per second of time. If the observer were at the equator and the declination zero the sun would rise or fall 1′ in 4^s of time. In the preceding example the rise is 1′ in about 8^s of time. When the observer is near the pole the method is practically useless.

Observations on the sun when it is very close to the horizon should be avoided, however, even when the sun is near the prime vertical, because the errors in the tabulated refraction correction due to variations in temperature and pressure of the air are likely to be large. Observations should not be made when the altitude is less than 10° if it can be avoided.

81. Time by the Altitude of a Star.

The method of the preceding article may be applied equally well to an observation on a star. In this case the parallax and semidiameter corrections are zero. If the star is west of the meridian the computed hour angle is the star's true hour angle; if the star is east of the meridian the computed hour angle must be subtracted from 24^h. The sidereal time is then found by adding the right ascension of the star to its hour angle. If mean time is desired the sidereal time thus found is to be converted into mean solar time by Art. 37. Since it is easy to select stars in almost any position it is desirable to eliminate errors in the measured altitudes by taking two observations, one on a star which is nearly due east, the other on one about due west. The mean of these two results will be nearly free from instrumental errors, and also from errors in the assumed value of the observer's latitude. If a planet is used it will be necessary to know the Gr. Civ. Time with sufficient accuracy for correcting the right ascension and declination.

Example.

Observed altitude of *α Boötis* (*Arcturus*) on Apr. 15, 1925 = 40° 10′ (east). Watch, $8^h\ 54^m\ 20^s$ P.M. Latitude = 42° 18′ N., Long. = 71° 18′ W. Rt. Asc. *α Boötis*, $14^h\ 12^m\ 15^s.6$; decl. +19° 34′ 14″. Rt. Asc. Mean Sun $+12^h = 13^h\ 30^m\ 32^s.01$.

$$\begin{array}{lr} \text{Obs. alt.} & 40^\circ\ 10' \\ \text{Refr.} & -1'.1 \\ h = & 40^\circ\ 08'.9 \end{array}$$

$$\begin{array}{rlll} \phi = & 42^\circ\ 18'.0 & \log\sec & 0.13098 \\ h = & 40\ \ 08\ .9 & & \\ p = & 70\ \ 25\ .8 & \log\csc & 0.02584 \\ & 2)\,152\ \ 52\ .7 & & \\ s = & 76^\circ\ 26'.3 & \log\cos & 9.37013 \\ s - h = & 36\ \ 17\ .4 & \log\sin & 9.77223 \\ & & & 2\,)\,9.29918 \end{array}$$

$$\log\sin\frac{t}{2} = 9.64959$$

$$\frac{t}{2} = 26^\circ\ 30'\ 15''$$

$$t = 53^\circ\ 00'\ 30''\ \text{(east)}$$

$$= 3^h\ 32^m\ 02^s\ \text{(east)}$$

Rt. Asc. of star	=	$14^h\ 12^m\ 15^s.6$
Loc. Sid. T.	=	$10^h\ 40^m\ 13^s.6$
Long. W.	=	4 45 12 .
Gr. Sid. T.	=	$15^h\ 25^m\ 25^s.6$
R. A. Sun $+12^h$	=	13 30 32 .0
		$1^h\ 54^m\ 53^s.6$
Table II		18 .8
Gr. Civil T.	=	$1^h\ 54^m\ 34^s.8$
		5
Eastern. Stand. Time	=	$20^h\ 54^m\ 34^s.8$
	=	8 54 34 .8 P.M.
Watch		8 54 20 P.M.
Watch		$14^s.8$ slow

82. Effect of Errors in Altitude and Latitude.

In order to determine the exact effect upon t of any error in the altitude h let us differentiate equation [8] with respect to h, the quantities ϕ and δ being regarded as constant.

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \qquad [8]$$

Differentiating,

$$\cos h = 0 - \cos \phi \cos \delta \sin t \frac{dt}{dh}$$

$$\frac{dt}{dh} = - \frac{\cos h}{\cos \phi \cos \delta \sin t}$$

$$= - \frac{1}{\cos \phi \sin Z} \text{ by Equa. [12].} \qquad [81]$$

An inspection of this equation shows that when $Z = 90°$ or $270°$ $\sin Z$ is a maximum and $\frac{dt}{dh}$ a minimum for any given value of ϕ. It also shows that the smaller the latitude, the greater is its cosine and consequently the smaller the value of $\frac{dt}{dh}$. The most favorable position of the body is therefore on the prime vertical. The negative sign shows that the hour angle decreases as the altitude increases. When Z is zero (body on meridian) the value of $\frac{dt}{dh}$ is infinite and t cannot be found from the observed altitude.

The effect of an error in the latitude may be found by differentiating [8] with respect to ϕ. The result is

$$0 = \cos \phi \sin \delta + \cos \delta \left(- \cos \phi \sin t \frac{dt}{d\phi} - \cos t \sin \phi\right)$$

$$\cos \phi \cos \delta \sin t \frac{dt}{d\phi} = \cos \phi \sin \delta - \sin \phi \cos \delta \cos t$$

$$= \cos h \cos Z \text{ by [11]}$$

$$\therefore \frac{dt}{d\phi} = \frac{\cos h \cos Z}{\cos \phi \cos \delta \sin t}$$

$$= \frac{\cos Z}{\sin Z \cos \phi} \text{ by [12]}$$

$$= \frac{1}{\cos \phi \tan Z}. \qquad [82]$$

This shows that when $Z = 90°$ or $270°$ an error in ϕ has no effect on t, since $\frac{dt}{d\phi} = 0$. In other words, the most favorable position of the object is on the prime vertical. It also shows that the method is most accurate when the observer is on the equator.

83. Time by Transit of Star over Vertical Circle through Polaris.*
In making observations by this method the line of sight of the telescope is set in the vertical plane through *Polaris* at any (observed) instant of time, and the time of transit of some southern star across this plane is observed immediately afterward; the correction for reducing the star's right ascension to the true sidereal time of the observation is then computed and added to the right ascension. The advantages of the method are that the direction of the meridian does not have to be established before time observations can be begun, and that the interval which must elapse between the two observed times is so small that errors due to the instability of the instrument are reduced to a minimum.

The method of making the observation is as follows: Set up the instrument and level carefully; sight the vertical cross hair on *Polaris* (and clamp) and note and record the watch reading; then revolve the telescope about the horizontal axis, being careful not to disturb its azimuth; set off on the vertical arc the altitude of some southern star (called the *time-star*) which will transit about 4^m or 5^m later; note the instant when this star passes the vertical cross hair. It will be of assistance in making the calculations if the altitude of each star is measured immediately after the time has been observed. The altitude of the time-star at the instant of observation will be so nearly equal to its meridian altitude that no special computation is necessary beyond what is required for ordinary transit observations. If the times of meridian transit are calculated beforehand the actual times of transit may be estimated with sufficient accuracy by noting the position of *Polaris* with respect to the meridian. If *Polaris* is near its elongation then the azimuth of the sight line will be a maximum. In latitude 40° the azimuth of *Polaris* for 1925 is about 1° 26′; a star on the equator would then pass the vertical cross hair nearly 4^m later than the computed time if *Polaris* is at eastern elongation (see Table B, p. 99). If *Polaris* is near western elongation the star will transit earlier by this amount. In order to eliminate errors in the adjustment of the instrument, observations should be made in the erect and inverted positions of the telescope and the two results combined. A new setting should be made on *Polaris* just before each observation on a time-star.

* For a complete discussion of this method see a paper by Professor George O. James, in the Jour. Assoc. Eng. Soc., Vol. XXXVII, No. 2; also Popular Astronomy, No. 172. A method applicable to larger instruments is given by Professor Frederick H. Sears, in Bulletin No. 5, Laws Observatory, University of Missouri.

In order to deduce an expression for the difference in time between the meridian transit and the observed transit let α and α_0 be the right ascensions of the stars, S and S_0 the sidereal times of transit over the cross hair, t and t_0 the hour angles of the stars, the subscripts referring to *Polaris*. Then by Equa. 37, p. 52,

$$t = S - \alpha$$

and

$$t_0 = S_0 - \alpha_0.$$

Subtracting,

$$t_0 - t = (\alpha - \alpha_0) - (S - S_0). \quad [83]$$

The quantity $S - S_0$ is the observed interval of time between the two observations expressed in sidereal units. If a mean time chronometer or watch is used the interval must be increased by the amount of the correction in Table III. Equa. [83] may then be written

$$t_0 - t = (\alpha - \alpha_0) - (T - T_0) - C \quad [84]$$

where T and T_0 are the actual watch readings and C is the correction from Table III to convert this interval into sidereal time.

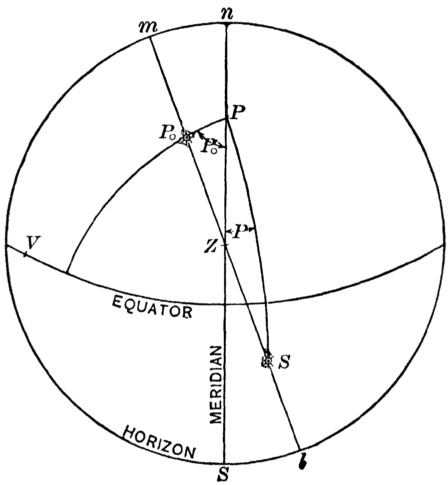


FIG. 61

In Fig. 61 let P_0 be the position of *Polaris* when it is observed; P, the celestial north pole; Z, the zenith of the observer; and S, the time star in the position in which it is observed. It should be noticed that when S is passing the cross hair, *Polaris* is not in the position P_0, but has moved westward (about P) by an angle equal to the (sidereal) interval between the two observations. Let p_0 be the polar distance of *Polaris*; ζ and ζ_0, the zenith distances of the two stars; and h and h_0 their altitudes.

Then in the triangle P_0PS,

$$\frac{\sin S}{\sin P_0PS} = \frac{\sin p_0}{\sin P_0S},$$

or

$$\sin S = \sin P_0PS \sin p_0 \operatorname{cosec} (\zeta + \zeta_0)$$
$$= \sin (t_0 - t) \sin p_0 \operatorname{cosec} (h + h_0). \quad [85]$$

In triangle PZS,

$$\frac{\sin (-t)}{\sin S} = \frac{\sin \zeta}{\cos \phi}$$

or

$$\sin (-t) = \sin S \cos h \sec \phi. \quad [86]$$

Substituting the value of sin S in Equa. [85],

$$\sin(-t) = \sin p_0 \sin(t_0 - t) \operatorname{cosec}(h + h_0) \cos h \sec \phi. \qquad [87]$$

Since t and p_0 are small the angles may be substituted for their sines, and

$$-t = p_0 \sin(t_0 - t) \operatorname{cosec}(h + h_0) \cos h \sec \phi. \qquad [88]$$

If the altitudes h and h_0 have not been measured the factor cos h may be replaced by $\sin(\phi - \delta)$ and $\operatorname{cosec}(h + h_0)$ may be replaced by $\sec(\delta - c)$ with an error of only a few hundredths of a second, δ being the declination of the time star and c the correction in Table I in the Ephemeris or the Almanac.

In this method the latitude ϕ is supposed to be known. If it is not known, then the altitudes of the stars must be measured and ϕ computed. It will usually be accurate enough to assume that the observed altitude of the time star is the same as the meridian altitude, and apply Equa. [1]; otherwise a correction may be made by formula [77]. The latitude may also be found from the altitude of the polestar, using the method of Art. 73.

After the value of t (in seconds of time*) has been computed it is added to the right ascension of the time star to obtain the local sidereal time of the observation on this star. This sidereal time may then be converted into local civil time and then into standard time and the watch correction obtained.

If it is desired to find the azimuth of the line of sight this may be done by computing a in the formula

$$a = t \sec h \cos \delta. \qquad [89]$$

The above method is applicable to transit observations made with small instruments. For the large astronomical transit a more refined method of making the reductions should be used.

Example.

Observation of *o Virginis* over Vertical Circle through *Polaris;* latitude, 42° 21′ N.; longitude $4^h\,44^m\,18^s.3$ W.; date, May 8, 1906.

Observed time on *Polaris*	$8^h\,35^m\,58^s$
Observed time on *o Virginis*	8 39 43
Diff.	$3^m\,45^s$

α	$12^h\,00^m\,26^s.3$
α_0	1 24 35 .4
$\alpha - \alpha_0 =$	$10^h\,35^m\,50^s.9$
$T - T_0$	3 45 .0
Table III	0 .6
$t_0 - t$	$10^h\,32^m\,05^s.3$
	$= 158°\,01'.3$

$\phi =$	42° 21′
$\delta =$	+9 15
$\phi - \delta =$	33° 06′

$\delta =$	+9° 15′
$c =$	+1 06 .5
$\delta - c =$	8° 08′.5

p_0	$= 71'.85$
$\log p_0$	= 1.8564
$\log \sin(t_0 - t)$	= 9.5732
$\log \sec(\delta - c)$	= 0.0044
$\log \sin(\phi - \delta)$	= 9.7373
$\log \sec \phi$	= 0.1313
$\log 4$	= 0.6021
$\log t$	= 1.9047
t	$= -80^s.30$
	$= -1^m\,20^s.3$

* The factor 4 has been introduced in the following example in order to reduce minutes of angle to seconds of time.

The true sidereal time may now be found by subtracting $1^m\ 20^s.3$ from the right ascension of *o Virginis*, the result being as follows:

$$\begin{aligned} \alpha &= 12^h\ 00^m\ 26^s.3 \\ t &= \underline{\quad -1\ \ 20\ .3} \\ S &= 11^h\ 59^m\ 06^s.0 \end{aligned}$$

The local civil time corresponding to this instant of sidereal time for the date is $20^h\ 55^m\ 14^s.5$. The corresponding Eastern Standard time is $20^h\ 39^m\ 32^s.8$, or $8^h\ 39^m\ 32^s.8$ P.M. The difference between this and the watch time, $8^h\ 39^m\ 43^s$, shows that the watch was $10^s.2$ fast.

84. Time by Equal Altitudes of a Star.

If the altitude of a star is observed when it is east of the meridian at a certain altitude, and the same altitude of the same star again observed when the star is west of the meridian, then the mean of the two observed times is the watch reading for the instant of transit of the star. It is not necessary to know the actual value of the altitude employed, but it is essential that the two altitudes should be equal. The disadvantage of the method is that the interval between the two observations is inconveniently long.

85. Time by Two Stars at Equal Altitudes.

In this method the sidereal time is determined by observing when two stars have equal altitudes, one star being east of the meridian and the other west. If the two stars have the same declination then the mean of the two right ascensions is the sidereal time at the instant the two stars have the same altitude. As it is not practicable to find pairs of stars having exactly the same declination it is necessary to choose pairs whose declinations differ as little as possible and to introduce a correction for the effect of this difference upon the sidereal time. It is not possible to observe both stars directly with a transit at the instant when their altitudes are equal; it is necessary, therefore, to observe first one star at a certain altitude and to note the time, and then to observe the other star at the same altitude and again note the time. The advantage of this method is that the actual value of the altitude is not used in the computations; any errors in the altitude due either to lack of adjustment of the transit or to abnormal refraction are therefore eliminated from the result, provided the two altitudes are made equal. In preparing to make the observations it is well to compute beforehand the approximate time of equal altitudes and to observe the first star two or three minutes before the computed time. In this way the interval between the observations may be kept conveniently small. It is immaterial whether the east star is observed first or the west star first, provided the proper change is made in the computation. If one star is faint it is well to observe the bright one first; the faint star may then be more easily found by knowing the time at which it should pass the horizontal cross hair. The interval by which the second observation follows the time of equal altitudes is nearly the same as the interval between the first observation and the time of equal altitudes. It is evident that in the application of this method the observer must be able to identify the stars he is to observe. A star map is of great assistance in making these observations.

The observation is made by setting the horizontal cross hair a little above the easterly star 2^m or 3^m before the time of equal altitudes, and noting the instant when the star passes the horizontal cross hair. Before the star crosses the hair the clamp to the horizontal axis should be set firmly, and the plate bubble which is perpendicular to the horizontal axis should be centred. When the first observation has been made and recorded the telescope is then turned toward the westerly star, care being taken not to alter the inclination of the telescope, and the time when the star passes the horizontal cross hair is observed and recorded. It is well to note the altitude, but this is not ordinarily used in making the reduction. If the time of equal altitudes is not known, then both stars should be bright ones that are easily found in the telescope. The observer may measure an approximate altitude of first one and then the other, until they are at so nearly the same altitude that both can be brought into the field without changing the inclination of the telescope. The altitude of the east star may then be observed at once and the observation on the west star will follow by only a few minutes. If it is desired to observe the west star first, it must be observed at an altitude which is greater than when the east star is observed first. In this case the cross hair is set a little below the star.

In Fig. 62 let *nesw* represent the horizon, Z the zenith, P the pole, S_e the easterly star, and S_w the westerly star. Let t_e and t_w be the hour angle of S_e and S_w, and let HS_eS_w be an almucantar, or circle of equal altitudes.

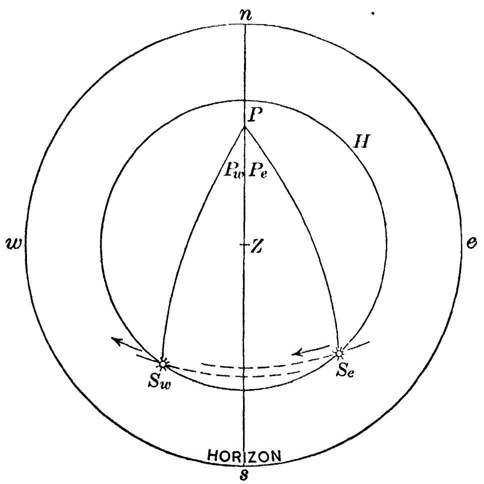


FIG. 62

From Equa. [37], for the two stars S_e and S_w, the sidereal time is

$$S = \alpha_w + t_w$$
$$S = \alpha_e - t_e.^*$$

Taking the mean value of S,

$$S = \frac{\alpha_w + \alpha_e}{2} + \frac{t_w - t_e}{2}, \quad [92]$$

from which it is seen that the true sidereal time equals the mean right ascension corrected by half the difference in the hour angles. To derive the equation for correcting the mean right ascension so as to obtain the true sidereal time let the fundamental equation

$$\sin h = \sin \delta \sin \phi + \cos \delta \cos \phi \cos t \quad [8]$$

* t_e is here taken as the actual value of the hour angle east of the meridian.

be differentiated regarding δ and t as the only variables, then there results

$$0 = \sin\phi\cos\delta - \cos\delta\cos\phi\sin t\frac{dt}{d\delta} - \cos\phi\cos t\sin\delta, \qquad [90]$$

from which may be obtained

$$\frac{dt}{d\delta} = \frac{\tan\phi}{\sin t} - \frac{\tan\delta}{\tan t} \qquad [91]$$

If the difference in the declination is small, $d\delta$ may be replaced by $\frac{1}{2}(\delta_w - \delta_e)$, in which case dt will be the resulting change in the hour angle, or $\frac{1}{2}(t_w - t_e)$.

The equation for the sidereal time then becomes

$$S = \frac{\alpha_w + \alpha_e}{2} + \frac{\delta_w - \delta_e}{2}\left[\frac{\tan\phi}{\sin t} - \frac{\tan\delta}{\tan t}\right], \qquad [92]$$

in which $(\delta_w - \delta_e)$ must be expressed in seconds of time. δ may be taken as the mean of δ_e and δ_w. The value of t would be the mean of t_e and t_w if the two stars were observed at the same instant, but since there is an appreciable interval between the two times t must be found by

$$t = \frac{\alpha_e - \alpha_w}{2} + \frac{T_w - T_e}{2}, \qquad [93]$$

T_w and T_e being the actual watch readings.

LIST FOR OBSERVING BY EQUAL ALTITUDES

Lat., 42° 21′ N. Long., $4^h\ 44^m\ 18^s$ W. Date, Apr. 30, 1912.

Stars.	Magn.	Sidereal time of equal altitudes.	Eastern time of equal altitudes.	Observed times.
α Corona Borealis	2.3	$10^h\ 28^m$	$7^h\ 38^m$	
β Tauri	1.8			
α Boötis	0.2	10 37	7 47	
ζ Geminorum	4			
α Boötis	0.2	10 48	7 58	
δ Geminorum	3.5			
ρ Boötis	3.6	11 00	8 10	
α² Geminorum	1.9			
π Hydræ	3.5	11 10	8 20	
ρ Argus	2.9			
β Herculis	2.8	11 19	8 29	
η Geminorum	3.5			
α Serpentis	2.7	11 35	8 45	
α Canis Minoris	0.5			
β Herculis	2.8	11 51	9 01	
δ Geminorum	3.5			
α Serpentis	2.7	12 02	9 12	
β Cancri	3.8			
α Serpentis	2.7	12 11	9 21	
ε Hydræ	3.5			
β Libræ	2.9	12 20	9 30	
α Hydræ	2.1			
β Herculis	2.8	12 32	9 42	
γ Cancri	4.9			

If the west star is observed first, then the last term becomes a negative quantity. Strictly speaking this last term should be converted into sidereal units, but the effect upon the result is usually very small. In regard to the sign of the correction to the mean right ascension it should be observed that if the west star has the greater declination the time of equal altitudes is later than that indicated by the mean right ascension. In selecting stars for the observation the members of a pair should differ in right ascension by 6 to 8 hours, or more, according to the declinations. Stars above the equator should have a longer interval between them than those below the equator. On account of the approximations made in deriving the formula the declinations should differ as little as possible. If the declinations do not differ by more than about 5°, however, the result will usually be close enough for observations made with the engineer's transit. From the extensive star list now given in the American Ephemeris it is not difficult to select a sufficient number of pairs at any time for making an accurate determination of the local time. On page 141 is a short list taken from the American Ephemeris and arranged for making an observation on April 30, 1912.

Following is an example of an observation for time by the method of equal altitudes.

Example.

Lat., 42° 21′ N. Long., $4^h\ 44^m\ 18^s$ W. Date, Dec. 14, 1905.

Star.	Rt. Asc.	Decl.	Watch.
α Ceti (E)	$2^h\ 57^m\ 22^s.1$	$+3°\ 43'\ 69''.1$	$T_e\ 5^h\ 18^m\ 00^s$
δ Aquilæ (W)	19 20 43 .6	+2 55 44 .0	T_w 5 22 13
Mean	$23^h\ 09^m\ 02^s.8$	$+3°\ 19'\ 56''.6$	$5^h\ 20^m\ 06^s.5$
Diff.	7 36 38 .5	2)−0 48 25 .1	04 13
$T_w - T_e$	4 13 .7		
	2) $7^h\ 40^m\ 52^s.1$	$\frac{\delta_w - \delta_e}{2} = -24'\ 12''.6$	
	$t = 3^h\ 50^m\ 26^s.1$	$= -96^s.84$	
	$= 57°\ 36'\ 31''.5$		

Mean R. A. $= 23^h\ 09^m\ 02^s.8$
Corr. $=\ -01\ \ 41\ .0$

Sid. Time $= 23^h\ 07^m\ 21^s.8$

The local civil time corresponding to this is

$17^h\ 35^m\ 43^s.4$
Long. diff. 15 42 .0

Eastern time $17^h\ 20^m\ 01^s.4$
$=$ 5 20 01 .4 P.M.
Watch reading 5 20 06 .5

Watch fast $5^s.1$

$\log \frac{\delta_w - \delta_e}{2} = 1.9861$ (n) $\quad$ 1.9861 (n)
$\log \tan \phi = 9.9598$ $\quad \log \tan \delta = 8.7650$
$\log \csc t = 0.0735$ $\quad \log \cot t = 9.8024$

$= 2.0194$ (n) $\quad$ 0.5535 (n)
$-\ 104^s.6$ $\quad -\ 3^s.6$
$-\ 3\ .6$

Corr. $= -\ 101^s.0 = -1^m\ 41^s.0$

86. Formula [91] may be madė practically exact by means of the following device. Applying Equa. [8] to each star separately and subtracting one result from the other we obtain the equation*

$$\sin \Delta t = \frac{\tan \phi \tan \Delta\delta}{\sin t} - \frac{\tan \delta \tan \Delta\delta}{\tan t} + \frac{\tan \delta \tan \Delta\delta}{\tan t} \operatorname{vers} \Delta t, \qquad [94]$$

TABLE C. CORRECTIONS TO BE ADDED TO $\Delta\delta$ AND Δt.

(Equa. [94], Art. 86.)

Arc or sine.	Correction to $\Delta\delta$.	Correction to Δt.	Arc or sine.	Correction to $\Delta\delta$.	Correction to Δt.
s	s	s	s	s	s
100	0.00	0.00	800	0.90	0.45
200	0.01	0.01	850	1.08	0.54
300	0.05	0.02	900	1.29	0.64
400	0.11	0.06	950	1.51	0.76
500	0.22	0.11	1000	1.77	0.88
600	0.38	0.19	1050	2.05	1.02
650	0.48	0.24	1100	2.35	1.17
700	0.60	0.30	1150	2.69	1.34
750	0.74	0.37	1200	3.06	1.52

TABLE D. CORRECTION TO BE ADDED TO Δt†

(Equa. [94], Art. 86)

	Δt (in seconds of time).									
2d term.	100^s	200^s	300^s	400^s	500^s	600^s	700^s	800^s	900^s	1000^s
s	s	s	s	s	s	s	s	s	s	s
100	0.00	0.01	0.02	0.04	0.07	0.10	0.13	0.17	0.21	0.26
200	0.01	0.02	0.05	0.08	0.13	0.19	0.26	0.34	0.43	0.53
300	0.01	0.03	0.07	0.13	0.20	0.29	0.39	0.51	0.64	0.79
400	0.01	0.04	0.10	0.17	0.26	0.38	0.52	0.68	0.86	1.06
500	0.01	0.05	0.12	0.21	0.33	0.48	0.65	0.85	1.07	1.32
600	0.02	0.06	0.14	0.25	0.40	0.57	0.78	1.02	1.28	1.59
700	0.02	0.07	0.17	0.30	0.46	0.67	0.91	1.18	1.50	1.85
800	0.02	0.08	0.19	0.34	0.53	0.76	1.04	1.35	1.71	2.11
900	0.02	0.10	0.21	0.38	0.59	0.86	1.17	1.52	1.93	2.38
1000	0.03	0.11	0.24	0.42	0.66	0.95	1.30	1.69	2.14	2.64
1100	0.03	0.12	0.26	0.47	0.73	1.05	1.42	1.86	2.36	2.91
1200	0.03	0.13	0.29	0.51	0.79	1.14	1.55	2.03	2.57	3.17

† The algebraic sign of this term is always opposite to that of the second term.

Chauvenet, *Spherical and Practical Astronomy*, Vol. I, p. 199.

where $\Delta\delta$ is half the difference in the declinations and Δt is the correction to the mean right ascension. If sin Δt and tan $\Delta\delta$ are replaced by their arcs and the third term dropped, this reduces to Equa. [91], except that $\Delta\delta$ and Δt are finite differences instead of infinitesimals. In order to compensate for the errors thus produced let $\Delta\delta$ be increased by a quantity equal to the difference between the arc and the tangent (Table C); and let a correction be added to the sum of the first two terms to allow for the difference between the arc and sine of Δt (Table C). With the approximate value of Δt thus obtained the third term of the series may be taken from Table D. By this means the precision of the computed result may be increased, and the limits of $\Delta\delta$ may therefore be extended without increasing the errors arising from the approximations.

Example.

Compute the time of equal altitudes of α *Boötis* and ι *Geminorum* on Jan. 1, 1912, in latitude 42° 21′. R. A. α *Boötis* = $14^h\ 11^m\ 37^s.98$; decl. = +19° 38′ 15″.2. R. A. ι *Geminorum* = $7^h\ 20^m\ 16^s.85$; decl. = +27° 58′ 30″.8.

$$\begin{array}{rl} & 14^h\ 11^m\ 37^s.98 \\ & 7\quad 20\quad 16\ .85 \\ \hline 2) & 6^h\ 51^m\ 21^s.13 \\ \hline & 3^h\ 25^m\ 40^s.56 \\ t = & 51^\circ\ 25'\ 08''.4 \end{array}$$

$$\begin{array}{rl} & 27^\circ\ 58'\ 30''.8 \\ & 19\quad 38\quad 15\ .2 \\ \hline 2) & 8^\circ\ 20'\ 15''.6 \\ \hline \Delta\delta = & 4^\circ\ 10'\ 07''.8 \\ = & 1000^s.52 \\ \text{Corr., Table C} = & 1\ .77 \\ \hline \Delta\delta = & 1002^s.29 \end{array}$$

$$\begin{array}{rl} \log \Delta\delta = & 3.000993 \\ \log \tan \phi = & 9.959769 \\ \log \csc t = & 0.106945 \\ \hline & 3.067707 \\ \text{1st term} = & 1168^s.71 \\ \text{2d term} = & -352\ .76 \\ \hline \Delta t\ \text{(approx.)} = & 815^s.95 \\ \text{Corr., Table C} = & +\quad .48 \\ \text{Corr., Table D} = & +\quad .63 \\ \hline \Delta t = & +817^s.06 \\ = & +\ 13^m\ 37^s.06 \\ \text{Mean R. A.} = & 10\quad 45\quad 57\ .42 \\ \hline \text{Sid. Time of Equal Alt.} = & 10^h\ 59^m\ 34^s.48 \end{array}$$

$$\begin{array}{rl} \log \Delta\delta = & 3.00099 \\ \log \tan \delta = & 9.64462 \\ \log \cot t = & 9.90187 \\ \hline & 2.54748 \\ \text{2d term} = & -352.76 \end{array}$$

For refined observations the inclination of the vertical axis should be measured with a spirit level and a correction applied to the observed time. With the engineer's transit the only practicable way of doing this is by means of the plate-level which is parallel to the plane of motion of the telescope. If both ends of this level are read at each observation, O denoting the reading of the object end and E the eye end of the bubble, then the change in the inclination is expressed by

$$i = \Big((O - E) - (O' - E')\Big) \times \frac{d}{2},$$

where d is the angular value of one scale division in seconds of arc. The correction to the mean watch reading is

$$\text{Corr.} = \frac{i}{30 \sin S \cos \delta} = \frac{i}{30 \cos \phi \sin Z},$$

in which S may be taken from the Azimuth* tables or Z may be found from the measured horizontal angle between the stars. If the west star is observed at a higher altitude than the east star (bubble nearer objective), the correction must be added to the mean watch reading. If it is applied to the mean of the right ascensions the algebraic sign must be reversed.

87. The correction to the mean right ascension of the two stars may be conveniently found by the following method, provided the calculation of the parallactic angle, S in the PZS triangle, can be avoided by the use of tables. Publication No. 120 of the U. S. Hydrographic Office gives values of the azimuth angle for every whole degree of latitude and declination and for every 10^m of hour angle. The parallactic angle may be obtained from these tables (by interpolation) by interchanging the latitude and the declination, that is, by looking up the declination at the head of the page and the latitude in the line marked " Declination." For latitudes under 23° it will be necessary to use Publication No. 71

In taking out the angle the table should be entered with the next less whole degree of latitude and of declination and the next less 10^m of hour angle, and the corresponding tabular angle written down; the proportional parts for minutes of latitude, of declination, and of hour angle are then taken out and added algebraically to the first angle. The result may be made more accurate by working from the nearest tabular numbers instead of the next less. The instructions given in Pub. 120 for taking out the angle when the latitude and declination are of opposite sign should be modified as follows. Enter the table with the **supplement** of the hour angle, the latitude and declination being interchanged as before, and the tabular angle is the value of S sought.

Suppose that two stars have equal declinations and that at a certain instant their altitudes are equal, A being east of the meridian and B west of the meridian. If the declination of B is increased so that the star occupies the position C, then the star must increase its hour angle by a certain amount x in order to be again on the almucantar through B. Half of the angle x is the desired correction. In Fig. 63 BC is the increase in declination; BD is the almucantar through A, B and D; and CD is the arc of the parallel of declination through which the star must move in order to reach BD. The arcs BD and CD are not arcs of great circles, and the triangle BCD is not strictly a spherical triangle, but it may be shown that the error is usually negligible in observations made with the engineer's transit if BCD is computed as a spherical triangle or even as a plane triangle. The angle ZBP is the angle S and DBC is $90° - S$. The length of the arc CD is then $BC \cot S$, or $(\delta_w - \delta_e) \cot S$. The angle at P is the same as the arc $C'D'$ and equals $CD \sec \delta$. If $(\delta_w - \delta_e)$ is expressed in minutes of arc and the cor-

* See Arts. 87 and 122 for the method of using these tables.

rection is to be in seconds of time, then, remembering that the correction is half the angle x,

$$\text{Correction} = 2\,(\delta_w - \delta_e) \cot S \sec \delta. \qquad [95]$$

δ should be taken as the mean of the two declinations, and the hour angle, used in finding S, is half the difference in right ascension corrected for half the watch interval.

The trigonometric formula for determining the correction for equal altitudes is

$$\tan \frac{\Delta t}{2} = \sin \frac{\Delta \delta}{2} \cot \tfrac{1}{2}\,(S_1 + S_2) \sec \tfrac{1}{2}\,(\delta_1 + \delta_2). \qquad [96]$$

By substituting arcs for the sine and tangent this reduces to the equation given above, except that the mean of S_1 and S_2 is not exactly the same as the value of S obtained by using the mean of the hour angles.

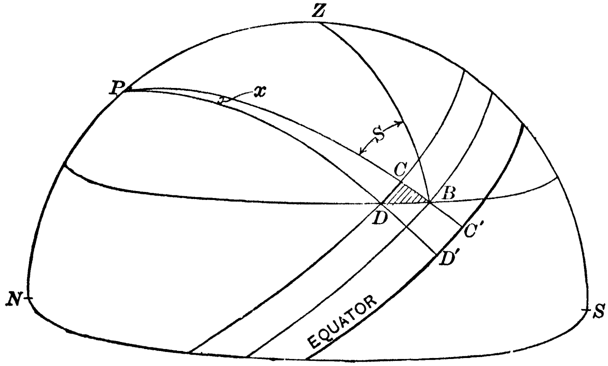


FIG. 63

The example on p. 146 worked by this method is as follows. From the azimuth tables, using a declination of 42°, latitude 3°, and hour angle $3^h\,50^m$, the approximate value of S is 44° 05′. Then from the tabular differences, —

Correction for 21′ decl. = −22′
Correction for 20′ lat. = +07
Correction for 26^s h. a. = +02

The corrected value of S is therefore 43° 52′.

$$\begin{aligned} 2\,(\delta_w - \delta_e) = -96'.84 \quad \log &= 1.9861\ (n) \\ \log \cot S &= 0.0172 \\ \log \sec \delta &= 0.0007 \\ \log \text{corr.} &= 2.0040\ (n) \\ \log \text{corr.} &= -100^s.9 \end{aligned}$$

This solution is sufficiently accurate for observations made with the engineer's transit, provided the difference in the declinations of the two stars is not greater than about 5° and the other conditions are favorable. For larger instruments and for refined work this formula is not sufficiently exact.

The equal-altitude method, like all of the preceding methods, gives more precise results in low than in high latitudes.

88. Rating a Watch by Transit of a Star over a Range.

If the time of transit of a fixed* star across some well-defined range can be observed, the rate of a watch may be quite accurately determined without knowing its actual error. The disappearance of the star behind a building or other object when the eye is placed at some definite point will serve the purpose. The star will pass the range at the same instant of sidereal time every day. If the watch keeps sidereal time, then its reading should be the same each day at the time of the star's transit over the range. If the watch keeps mean time it will lose $3^m\ 55^s.91$ per sidereal day, so that the readings on successive days will be less by this amount. If, then, the passage of the star be observed on a certain night, the time of transit on any subsequent night is computed by multiplying $3^m\ 55^s.91$ by the number of days intervening and subtracting this correction from the observed time. The difference between the observed and computed times divided by the number of days is the daily gain or loss. After a few weeks the star will cross the range in daylight, and it will be necessary before this occurs to transfer to another star which transits later in the same evening. In this way the observations may be carried on indefinitely.

89. Time Service.

The Standard Time used in the United States is determined by means of star transits at the U. S. Naval Observatory (Georgetown Heights) and is sent out to all parts of the country east of the Rocky Mountains by means of electric signals transmitted over the lines of the telegraph companies and is relayed from the Arlington and Annapolis radio stations. For the territory west of the Rocky Mountains the time is determined at the Mare Island Navy Yard.

The error of the standard (sidereal) clock is determined about 15 times per month by transits of 6 to 12 stars over the meridian. Two instruments are employed as a check, one a large (6 inch)

* A planet should not be used for this observation.

transit, the other a small one which may be reversed in the middle of a set of observations on a star.

When signals are to be sent out the sending clocks are compared with the sidereal clock by means of a chronograph, the two clocks recording simultaneously. After the error of the sending clock is determined the clock is " set " correct by means of an automatic device which accelerates or retards its rate for a short time until a chronographic comparison shows that it is correct. The sending clock makes the signals through a relay directly onto the wires both for the wire and the wireless signals from Arlington and Annapolis.

In order to test and keep record of the errors in these signals they are received and recorded on a chronograph at the observatory. Thus the error of the sending clock and the error of the signal are on record and may be obtained for use in accurate work. The error of the time signal is rarely as much as a tenth of a second.

The " noon " signal is sent out each day at 12^h Eastern Standard Time, the series of signals beginning at $11^h\ 55^m$ and ending at 12^h. This signal may be heard on the sounder at any telegraph office or railroad station. The sounder gives a click once per second. The end of each minute is shown by the omission of the 55th to 59th seconds inclusive, except for the noon signal, which is preceded by a silent interval of 10 seconds. A similar signal is sent out at 10^h P.M. Eastern Standard Time and is usually relayed by radio stations so that it may be heard on an ordinary receiving set.

Questions and Problems

1. Compute the Eastern Standard time of the transit of *Regulus* (*α Leonis*) over the meridian 71° 06′.0 west of Greenwich on March 3, 1925. The right ascension of the star is $10^h\ 04^m\ 23^s.8$. The right ascension of the mean sun $+12^h =$ $10^h\ 41^m\ 00^s.26$ at 0^h of March 3, G. C. T.

2. At what time (E. S. T.) will the centre of the sun be on the meridian on Apr. 1, 1925 in longitude 71° 06′.0 W.? Equa. of time at 0^h G. C. T. Apr. 1 = $-4^m\ 12^s.47$; varia. per hour = $+0^s.755$.

3. Compute the error of the watch from the data given in prob. 5, p. 207.

4. Compute the error of the watch from the data given in prob. 6, p. 208.

5. Compute the sidereal time of transit of δ *Capricorni* over the vertical circle through *Polaris* on Oct. 26, 1906. Latitude = 42° 18′.5; longitude = $4^h\,45^m\,07^s$. Observed watch time of transit of *Polaris* = $7^h\,10^m\,20^s$; of δ *Capricorni* = $7^h\,13^m\,28^s$, Eastern Time. The declination of *Polaris* = +88° 48′ 31″.3; right ascension = $1^h\,26^m\,37^s.9$; declination of δ *Capricorni* = −16° 33′ 02″.8; right ascension = $21^h\,41^m\,53^s.3$. $c = -39'.3$. [Right ascension of mean sun $+12^h$ at time of observation = $2^h\,17^m\,48^s.3$.] Compute the error of the watch.

6. Time observation on May 3, 1907, in latitude 42° 21′.0 N., longitude $4^h\,44^m\,18^s.0$ W. Observed transit of *Polaris* at $7^h\,16^m\,17^s.0$; of μ *Hydræ* at $7^h\,18^m\,50^s.5$. Declination of *Polaris* = +88° 48′ 28″.3; right ascension = $1^h\,24^m\,50^s.2$. Declination of μ *Hydræ* = −16° 21′ 53″.2; right ascension = $10^h\,21^m\,36^s.1$. $c = +50'.1$. [Right ascension of mean sun $+12^h$ at time of observation = $14^h\,42^m\,58^s.03$.] Compute the sidereal time of transit of μ *Hydræ* over the vertical circle through *Polaris* and also the error of the watch in Standard time.

7. Observation for time by equal altitudes, Dec. 8, 1904.

	Right Ascension	Declination	Watch
α *Tauri* (E)	$4^h\,30^m\,29^s.01$	+16° 18′ 59″.9	$7^h\,34^m\,56^s$
α *Pegasi* (W)	22 59 61 .12	+14 41 43 .7	7 39 45

Lat. = 42° 28′.0 N.; long. = $4^h\,44^m\,15^s.0$. [Right ascension of mean sun $+12^h$ at instant of observation = $5^h\,48^m\,44^s.41$.] Compute the sidereal time and the error of the watch.

8. Observation for time by equal altitudes, Oct. 13, 1906.

	Right Ascension	Declination	Watch
ν *Ophinchi* (W)	$17^h\,53^m\,52^s.15$	−9° 45′ 34″.6	$7^h\,13^m\,49^s$
ι *Ceti* (E)	0 14 40 .99	−9 20 25 .7	7 28 25

Lat. = 42° 18′.0; long. = $4^h\,45^m\,06^s.8$ W. [Right ascension of mean sun $+12^h$ at instant of observation = $1^h\,26^m\,34^s.29$.] Compute the sidereal time and the error of the watch.

CHAPTER XII

OBSERVATIONS FOR LONGITUDE

90. Method of Measuring Longitude.

The measurement of the difference in longitude of two places depends upon a comparison of the local times of the places at the same absolute instant of time. One important method is that in which the timepiece is carried from one station to the other and its error on local time determined in each place. The most precise method, however, and the one chiefly used in geodetic work, is the telegraphic method, in which the local times are compared by means of electric signals sent through a telegraph line. Other methods, most of them of inferior accuracy, are those which depend upon a determination of the moon's position (moon culminations, eclipses, occultations) and upon eclipses of Jupiter's satellites, and those in which terrestrial signals are employed.

91. Longitude by Transportation of Timepiece.

In this method the error of the watch or chronometer with reference to the first meridian is found by observing the local time at the first station. The rate of the timepiece should be determined by making another observation at the same place at a later date. The timepiece is then carried to the second station and its error determined with reference to this meridian. If the watch runs perfectly the two watch corrections will differ by just the difference in longitude. Assume that the first observation is made at the easterly station and the second at the westerly station. To correct for rate, let r be the daily rate in seconds, + when losing − when gaining, c the watch correction at the east station, c' the watch correction at the west station, d the number of days between the observations, and T the watch reading at the second observation. Then the difference in the longitude is found as follows:

$$\text{Local time at W. station} = T + c'$$
$$\text{Local time at E. station} = T + c + dr$$
$$\text{Diff. in time} = \text{Diff. in Long.} = c + dr - c'. \qquad [97]$$

The same result will be obtained if the stations are occupied in the reverse order.

If the error of a mean-time chronometer or watch is found by star observations, it is necessary to know the longitudes accurately enough to correct the sun's right ascension. If a sidereal chronometer is used and its error found on local sidereal time this correction is rendered unnecessary.

In order to obtain a check on the rate of the timepiece the observer should, if possible, return to the first station and again determine the local time. If the rate is uniform the error in its determination will be eliminated by taking the mean of the results. This method is not as accurate as the telegraphic method, but if several chronometers are used and several round trips between stations are made it will give good results. It is useful at sea and in exploration surveys.

Example.

Observations for local mean time at meridian A indicate that the watch is $15^m\ 40^s$ slow. At a point B, west of A, the watch is found to be $14^m\ 10^s$ slow on local mean time. The watch is known to be gaining 8^s per day. The second observation is made 48 hours after the first. The difference in longitude is therefore

$$+15^m\ 40^s - 2 \times 8^s - 14^m\ 10^s = 1^m\ 14^s.$$

The meridian B is therefore $1^m\ 14^s$ or $18'\ 30''$ west of meridian A.

92. Longitude by the Electric Telegraph.

In the telegraphic method the local sidereal time is accurately determined by star transits observed at each of the stations. The observations are made with large portable transits and are recorded on chronographs which are connected with break-circuit chronometers. The stars observed are selected in such a manner as to permit determining the instrumental errors so that the effect of these errors may be eliminated from the results. The stars are divided into two groups.

Half the number are observed with the axis in one position and the other half with the axis reversed. This determines the error of the sight line. In each half set some of the stars are north of the zenith and some south. The differences in times of transit of these two groups measures the azimuth error. The inclination error is measured with the striding level. (See Arts. 55 and 77.)

After the corrections to the two chronometers have been accurately determined the two chronographs are switched into the main-line circuit and signals sent either by making or breaking the circuit a number of times by the use of a telegraph key. These signals are recorded on both chronographs. In order to eliminate the error due to the time of transmission of the signal,* the signals are sent first in the direction E to W and then in the direction W to E. The mean of the two results is free from the error provided it is constant during the interval. The personal errors of the observers are now nearly eliminated by the use of the impersonal transit micrometer, instead of by exchange of observers, as was formerly done. After all of the observations have been corrected for azimuth, collimation and level, and the error of the chronometer on local sidereal time is known, each signal sent over the main line will be found to correspond to a certain instant of sidereal time at the east station and a different instant of sidereal time at the west station. The difference between the two is the difference in longitude expressed in time units.

In the more recent work (since 1922) the longitude of a station is determined with reference to Washington by receiving the time signal by radio. This cuts the cost of the work in half since there is but one station to be occupied.

By the telegraph method a longitude difference may be determined with an error of about $0^s.01$ or about 10 feet on the earth's surface.

93. Longitude by Time Signals.

If it is desired to obtain an approximate longitude for any purpose this may be done in a simple manner provided the observer is able to obtain the standard time at some telegraph office or railroad station, or by radio, as given by the noon signal or the 10^h P.M. signal. He may determine his local mean time by any of the preceding methods (Chapter XI). The difference between the local time and the standard time by telegraph or radio is the correction to be applied to the longitude of the standard meridian to obtain the longitude of the observer.

Example.

Altitude of sun, $27° 44' 35''$; latitude, $42° 22'$ N.; declination, $19° 00' 09''$ N., equation of time, $+3^m 48^s.8$; watch reading, $4^h 18^m 13^s.8$. From these data the local mean time is found to be $4^h 33^m 43^s.9$, making the watch $15^m 30^s.1$ slow. By comparison with the telegraph signal at noon the watch is found to be 6^s fast of Eastern Standard Time. The longitude is then computed as follows:—

$$\begin{aligned} \text{Correction to L. M. T.} &= +15^m\, 30^s.1 \\ \text{Correction to E. S. T.} &= \underline{-00 \;\; 06\,.0} \\ \text{Difference in Longitude} &= \;\;\; 15^m\, 36^s.1 \\ &= 3°\, 54'\, 01''.5 \\ \text{Longitude} = 75° - 3°\, 54'.0 &= 71°\, 06'.0 \text{ West} \end{aligned}$$

* In a test made in 1905 it was found that the time signal sent from Washington reached Lick Observatory, Mt. Hamilton, Cal., in $0^s.05$.

94. Longitude by Transit of the Moon.

A method which is adapted to use with the surveyor's transit and which, although not precise, may be useful on exploration surveys, is that of determining the moon's right ascension by observing its transit over the meridian. The right ascension of the moon's centre is tabulated in the Ephemeris for every hour of Greenwich Civil Time; hence if the right ascension can be determined, the Greenwich Civil Time becomes known. A comparison of this with local time gives the longitude.

The observation consists in placing the instrument in the plane of the meridian and noting the time of transit of the moon's bright limb and also of several stars whose declinations are nearly the same as that of the moon. The table of "Moon Culminations" in the Ephemeris shows which limb (I or II) may be observed. (See note on p. 159.) The observed interval of time between the moon's transit and a star's transit (reduced to sidereal time if necessary) added to or subtracted from the star's right ascension gives the right ascension of the moon's limb. A value of the right ascension is obtained from each star and the mean value used. To obtain the right ascension of the centre of the moon it is necessary to apply to the right ascension of the edge a correction, taken from the Ephemeris, called "sidereal time of semidiameter passing meridian." In computing this correction the increase in right ascension during this short interval has been allowed for, so the result is not the right ascension of the centre at the instant of transit of the limb, but at the instant of transit of the centre. If the west limb was observed this correction must be added; if the east limb, it must be subtracted. The result is the right ascension of the centre at the instant of transit, which is also the *local sidereal time* at that instant. Then the Greenwich Civil Time corresponding to this instant is found by interpolation in the table giving the moon's right ascension for every hour. To obtain the Greenwich Civil Time by simple interpolation find the next less right ascension in the table and the "varia. per min." on the same line; subtract the tabular right ascension from the given right ascension (obtained from the observation) and divide this difference by the "varia. per min." The result is the number of minutes (and decimals of minutes) to be added to the tabulated hour of Greenwich Civil Time. If the "varia. per min." is varying rapidly it will be more accurate to interpolate as follows: Interpolate between the two values of the "varia. per min." to obtain a "varia. per min." which corresponds to the *middle* of the interval over which the interpolation is carried. In observations made with a surveyor's transit this refinement is seldom necessary.

In order to compare the Greenwich time with the local time it is necessary to convert the Greenwich Civil Time into the corresponding instant of Greenwich Sidereal Time. The difference between this and the local sidereal time is the longitude from Greenwich.

In preparing for observations of the moon's transit the Ephemeris should be consulted (Table of Moon Culminations) to see whether an observation can be made and to find the approximate time of transit. The time of transit may be obtained either from the Washington civil time of transit or from the Greenwich civil time in the first part of the Ephemeris. The tabular time must be corrected for longitude. The apparent altitude of the moon should be computed and allowance made for parallax. The moon's parallax is so large that the moon would not be in the field of view if this correction were neglected. The horizontal parallax multiplied by the cosine of the altitude is the required correction. The moon will appear lower than it would if seen from the centre of the earth. The correction is therefore subtractive.

Since the moon increases its right ascension about 2^s in every 1^m of time it is evident that any error in determining the right ascension will produce an error about thirty times as great in the longitude, so that this method cannot be made to give very precise results. It has, however, one great advantage. If for any reason, such as an accident to the timepiece, a knowledge of the Greenwich time is completely lost, it is still possible by this method to recover the Greenwich time with a fair degree of accuracy.

Following is an example of an observation for longitude by the method of moon culminations made with an engineer's transit.

Example.

Observed transit of moon's west limb July 30, 1925, for longitude. Watch time of transit moon's west limb, $7^h\ 27^m\ 14^s$; transit of σ *Scorpii*, $7^h\ 29^m\ 20^s$.

σ *Scorpii*	$7^h\ 29^m\ 20^s$
Moon, I	7 27 14
Interval	$2^m\ 06^s$
Table III	0 .3
Sid. int.	$2^m\ 06^s.3$
Rt. Asc. σ *Scorpii*	16 16 39 .50
Rt. Asc. Moon's limb	$16^h\ 14^m\ 33^s.20$
Time of s. d. passing	1 10 .36
Rt. Asc. centre	$16^h\ 15^m\ 43^s.56$ = local sidereal time

From Ephemeris

July 31	Gr. Civ. T.	Rt. Asc. Moon	Varia. per Min.
	0^h	$16^h\ 14^m\ 37^s.54$	2.3986
	1	16 17 01 .64	2.4049
		$2^m\ 24^s.10$	63

$16^h\ 15^m\ 43^s.56$
16 14 37 .54

$1^m\ 06^s.02$ = $66^s.02$ log 1.81968

Interpolated varia. per min. = 2.4000 log 0.38021

1.43947

$$\frac{33.0}{144.1} \times 63 = 14$$

$27^m.509$
= 27 $30^s.5$

G. C. T.	=	$0^h\ 27^m\ 30^s.50$
$\alpha_s + 12^h$	=	20 32 23 .49
Table III		04 .52
Gr. Sid. T.		$20^h\ 59^m\ 58^s.51$
Loc. Sid. T.		16 15 43 .56
Longitude		$4^h\ 44^m\ 14^s.95$ W. = 71° 03′ 45″ W.

NOTE. It has already been stated that the moon moves eastward on the celestial sphere at the rate of about 13° per day; as a result of this motion the time of meridian passage occurs about 51^m later (on the average) each day. On account of the eccentricity of its orbit, however, the actual retardation may vary considerably from the mean. The moon's orbit is inclined at an angle of about 5° 08′ to the plane of the earth's orbit. The line of intersection of these two planes rotates in a similar manner to that described under the precession of the equinoxes, except that its period is only 19 years. The moon's maximum declination, therefore, varies from 23° 27′ + 5° 08′ to 23° 27′ − 5° 08′, that is, from 28° 35′ to 18° 19′,

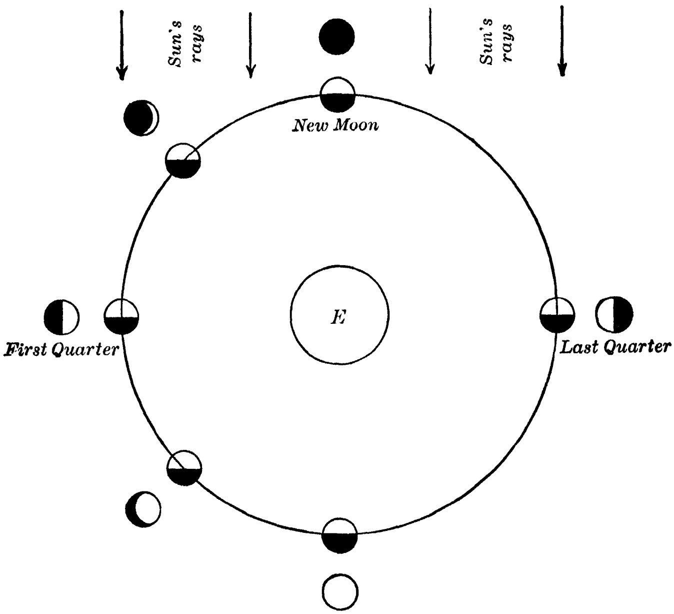


FIG. 64. THE MOON'S PHASES

according to the relative position of the plane of the moon's orbit and the plane of the equator. The rapid changes in the relative position of the sun, moon, and earth, and the consequent changes in the amount of the moon's surface that is visible from the earth, cause the moon to present the different aspects known as the moon's **phases**. Fig. 64 shows the relative positions of the three bodies at several different times in the month. The appearance of the moon as seen from the earth is shown by the figures around the outside of the diagram.

It may easily be seen from the diagram that at the time of first quarter the moon will cross the meridian at about 6 P.M.; at full moon it will transit at midnight; and at last quarter it will transit at about 6 A.M. Although the part of the illuminated hemisphere which can be seen from the earth is continually changing, the part of the moon's surface that is turned toward the earth is always the same, because the moon makes but one rotation on its axis in one lunar month. Nearly half of the moon's surface is never seen from the earth.

Questions and Problems

1. Compute the longitude from the following observed transits: (θ *Aquarii*, $5^h\ 16^m\ 04^s$ P.M.; π *Aquarii*, $5^h\ 24^m\ 40^s$; moon's west limb, $5^h\ 32^m\ 27^s$; λ *Aquarii*, $5^h\ 51^m\ 47^s$. Right ascensions; θ *Aquarii*, $22^h\ 11^m\ 27^s.6$; π *Aquarii*, $22^h\ 20^m\ 04^s.6$; λ *Aquarii*, $22^h\ 47^m\ 18^s.3$. The sidereal time of semi-diameter passing meridian $= 60^s.3$. At Gr. Civ. T. 22^h the moon's right ascension was $22^h\ 27^m\ 53^s.3$. The varia. per min. $= 1^s.9800$. The right ascension of the mean sun $+12^h = 4^h\ 36^m\ 29^s.7$.

2. Which limb of the moon can be observed for longitude by meridian transit if the observation is taken in the morning?

3. At about what time (local civil) will the moon transit when it is at first quarter?

4. The sun's corrected altitude is $57°\ 15'\ 36''$; latitude, $42°\ 22'$ N.; corrected declination, $18°\ 58'.6$ N.; corrected equation of time, $+3^m\ 49^s.0$; watch reading, $1^h\ 29^m\ 08^s$, P.M. Error of watch on Eastern Standard time by noon signal is -10^s (fast). Compute the longitude.

5. On April 2, 1925 the transit of ζ *Hydræ* is observed; watch reading $7^h\ 52^m\ 31^s$ P.M. At 10^h P.M. E. S. T. the radio signal shows that the watch is 3^s fast. The right ascension of the sun $+12^h$ at 0^h G. C. T. April 2, 1925, is $12^h\ 39^m\ 16^s.82$; on April 3, it is $12^h\ 43^m\ 13^s.38$. Compute the longitude.

CHAPTER XIII

OBSERVATIONS FOR AZIMUTH

95. Determination of Azimuth.

The determination of the azimuth of a line or of the direction of the true meridian is of frequent occurrence in the practice of the surveyor and is probably the most important to him of all the astronomical observations. In geodetic surveys, in which triangulation stations are located by means of their latitudes and longitudes, the precise determination of astronomical position is of as great importance as the orientation; but in general engineering practice, in topographical work, etc., the azimuth observation is the one that is most frequently required.

Too much stress cannot be laid on the desirability of employing the true meridian and true azimuths for all kinds of surveys. The use of the magnetic meridian or of an arbitrary reference line may save a little trouble at the time but is likely to lay up trouble for the future. As surveys are extended and connected and as lines are re-surveyed the importance of using the true meridian becomes greater and greater.

96. Azimuth Mark.

When an observation is made at night it is frequently inconvenient or impossible to sight directly at the object whose azimuth is to be determined; it is necessary in such cases to determine the azimuth of a special *azimuth mark*, which can be seen both at night and in the day, and then to measure the angle between this mark and the first object during the day. The azimuth mark usually consists of a lamp or a lantern placed inside a box having a small hole cut in the side through which the light may be seen. The size of the opening will vary with the distance, power of telescope, etc.; for accurate work it should subtend an angle not much greater than $0''.5$ to $1''.0$. If possible the mark should be placed so far away that the focus of the telescope will not have to be altered when changing from the star to the mark. For a large telescope of high power the distance should be a mile or so, but for surveyor's transits it may be much less; and in fact the topography around the station may be such that it is impossible to place the mark as far away as is desirable.

97. Azimuth of Polaris at Greatest Elongation.

The simplest method of determining the direction of the meridian with accuracy is by means of an observation of the

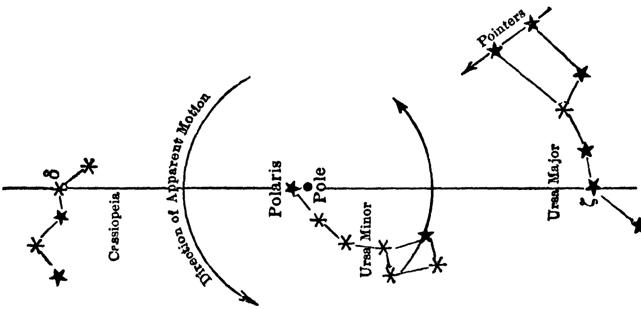


FIG. 65. CONSTELLATIONS NEAR THE NORTH POLE. POLARIS AT WESTERN ELONGATION

polestar, or any other close circumpolar, when it is at its greatest elongation. (See Art. 19, p. 36.) The appearance of the constellations at the time of this observation on Polaris may be seen by referring to the star map (Fig. 55) and Fig. 65. When the *polestar* is west of the pole the Great Dipper is on the right and Cassiopeia on the left. The exact time of elongation may be found by computing the sidereal time when the star is at elongation and changing this into local civil time and then into standard time, by the methods of Arts. 37 and 32.

To find the sidereal time of elongation first compute the hour angle (t_e) by Equa. [35] and express it in hours, minutes and seconds. If western elongation is desired, t_e is the hour angle;

if eastern elongation is desired, $24^h - t_e$ is the hour angle. The sidereal time is then found by adding the hour angle to the right ascension. An average value for t_e for *Polaris* for latitudes between 30° and 50° is about $5^h\ 56^m$ of sidereal time, or $5^h\ 55^m$ of mean time; this is sufficiently accurate for a rough estimate of the time of elongation, and as it changes but little from year to year and in different latitudes, it may be used instead of the exact value for many purposes. Approximate values of the times of elongation of *Polaris* may be taken from Table V.

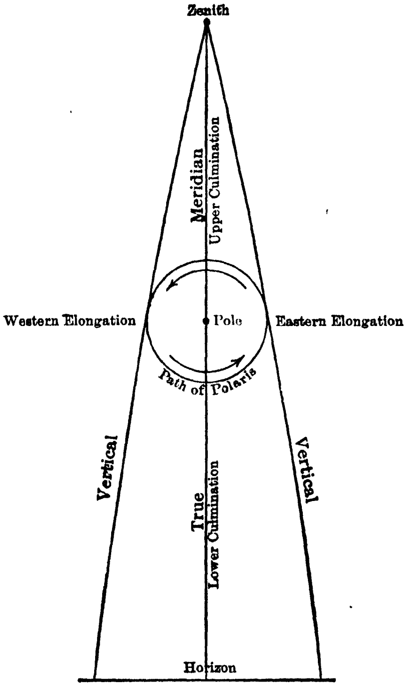


FIG. 66

Example. Find the Eastern Standard time of western elongation of *Polaris* on April 25, 1925, in latitude 42° 22′ N., longitude 71° 06′ W. The right ascension of *Polaris* is $1^h\ 33^m\ 27^s.15$; the declination is $+88°\ 54'\ 04''.54$. The right ascension of the mean sun $+12^h$ at 0^h G. C. T. $= 14^h\ 09^m\ 57^s.54$; corrected for longitude it is $14^h\ 10^m\ 44^s.26$.

$\log \tan \phi = 9.96002$	$t_e = 5^h\ 55^m\ 57^s.45$	$S = 7^h\ 29^m\ 22^s.60$
$\log \tan \delta = 1.71717$	$\alpha = 1\ \ 33\ \ 27\ .15$	$\alpha_s + 12^h = 14\ \ 10\ \ 44\ .26$
$\log \cos t_e = 8.24285$	$S = 7^h\ 29^m\ 22^s.60$	Sid. int. $= 17^h\ 18^m\ 38^s.34$
$t_e = 88°\ 59'\ 51''.7$		Table II $= 2\ \ 50\ .16$
$= 5^h\ 55^m\ 55^s.45$		Loc. Civ. T. $= 17^h\ 15^m\ 48^s.18$
		Long. diff. $= 15\ \ 36\ .00$
		E. S. T. (civil) $= 17^h\ 00^m\ 12^s.18$

The transit should be set in position half an hour or so before elongation. The star should be bisected with the vertical cross hair and, as it moves out toward its greatest elongation, its motion followed by means of the tangent screw of the upper or lower plate. Near the time of elongation the star will appear to move almost vertically, so that no motion in azimuth can be detected for five minutes or so before or after elongation. About 5^m before elongation, centre the plate levels, set the vertical hair carefully on the star, lower the telescope without disturbing its azimuth, and set a mark carefully in line at a distance of several hundred feet north of the transit. Reverse the telescope, re-centre the levels if necessary, bisect the star again, and set another point beside the first one. If there are errors of adjustment (line of collimation and horizontal axis) the two points will not coincide; the mean of the two results is the true point. The angle between the meridian and the line to the stake (star's azimuth) is found by the equation

$$\sin Z_n = \sin p \sec \phi \qquad [36]$$

where Z_n is the azimuth from the north (toward the east or the west); p, the polar distance of the star; and ϕ, the latitude of the place. The polar distance may be obtained by taking the declination from the Ephemeris and subtracting it from 90°; or, it may be taken from Table E if an error of 30″ is permissible The latitude (ϕ) may be taken from a map or found by observation. (Chap. X.) The latitude does not have to be known with great precision; a differentiation of [36] will show that an error of 1′ in ϕ causes an error of only about 1″ in Z_n for *Polaris* for latitudes within the United States.

TABLE E

MEAN POLAR DISTANCE OF POLARIS

Year	Mean Polar Distance	Year	Mean Polar Distance
1924	1° 06′ 07″.3	1930	1° 04′ 17″.2
1925	1 05 48 .9	1931	1 03 59 .0
1926	1 05 30 .5	1932	1 03 40 .7
1927	1 05 12 .2	1933	1 03 22 .5
1928	1 04 53 .8	1934	1 03 04 .3
1929	1 04 35 .5	1935	1 02 46 .1

The above method is general and may be applied to any circumpolar star. For *Polaris*, whose polar distance in 1925 is about $1° 06'$, it is usually accurate enough to use the approximate formula

$$Z_n'' = p'' \sec \phi \qquad [98]$$

in which Z_n'' and p'' are expressed in seconds of arc.

This computed angle (Z_n) may be laid off in the proper direction by means of a transit (preferably by daylight), using the method of repetitions, or with a tape, by measuring a perpendicular offset calculated from the measured distance to the stake and the star's azimuth. The result will be the true north-and-south line.

It is often desirable to measure the horizontal angle between the star at elongation and some fixed point, instead of marking the meridian itself. On account of the slow change in the azimuth there is ample time to measure several repetitions before the error in azimuth amounts to more than $1''$ or $2''$. In latitude $40°$ the azimuth changes about $1'$ in half an hour before or after elongation; the change in azimuth varies nearly as the square of the time interval from elongation. The errors of adjustment of the transit will be eliminated if half the angles are taken with the telescope erect, and half with the telescope inverted. The plate levels should be re-centred for each position of the instrument before the measurements are begun and while the telescope is pointing toward the star.

Example.

Compute the azimuth of *Polaris* at greatest elongation on April 25, 1925, in latitude $42° 22'$ N. The declination of *Polaris* is $+88° 54' 04''.54$. Polar distance, p, is $1° 05' 55''.46 = 3955''.46$.

By formula [36]	By formula [98]
$\log \sin p = 8.28272$	$\log p'' = 3.59720$
$\log \sec \phi = 0.13145$	$\log \sec \phi = 0.13145$
$\log \sin Z_n = 8.41417$	$\log Z_n'' = 3.72865$
$Z_n = 1° 29' 13''.6$	$Z_n'' = 5353''.6$
	$= 1° 29' 13''.6$

If the mark set in line with the star is 630.0 feet away from the transit the perpendicular offset to the meridian is calculated as follows:

log 630.00	=	2.79934
log tan Z_n	=	8.41432
log offset	=	1.21366
offset	=	16.355 ft.

98. Observations Near Elongation. If observations are made on a close circumpolar star within a few minutes of elongation the azimuth of the star at the instant of pointing may be reduced to its value at elongation if the time of the observation is known. The formula for computing the correction is

$$C = 112.5 \times 3600 \times \sin 1'' \times \tan Z_e \times (T - T_e)^2 \quad [99]$$

in which Z_e is the azimuth at greatest elongation, T is the observed time and T_e the time of elongation. $T - T_e$ must be expressed in sidereal minutes. The correction is in seconds of angle. Values of this correction are given in Table Va in the Ephemeris for each minute up to 25, or in Table VI of this book.

Example.

The horizontal angle from a mark to the right to *Polaris* is 2° 37′ 30″, the watch reading $6^h\ 28^m\ 00^s$ when the star was sighted. The watch time of western elongation is $6^h\ 04^m\ 00^s$. The azimuth of *Polaris* at elongation is 1° 37′ 48″. The correction corresponding to a 24^m mean time interval, or a $24^m\ 04^s$ sidereal interval, is 32″. The horizontal angle from the mark to the elongation position of the star is 2° 37′ 30″ − 32″ = 2° 36′ 58″. The bearing of the mark is the sum of this and the azimuth at elongation, or 2° 36′ 58″ + 1° 37′ 48″ = 4° 14′ 46″. The bearing is therefore N. 4° 14′ 46″ W.

99. Azimuth by Elongations in the Southern Hemisphere. The method described in the preceding article may be applied to stars near the south pole, but since there are no bright stars within about 20° of the pole the observation is not quite so simple and the results are somewhat less accurate. As the polar distance increases the altitude of the star at elongation increases and the diurnal motion becomes more rapid. The increase in altitude causes greater inconvenience in making the pointings and also magnifies the effect of instrumental errors. On account of the rapid motion of the star it is important to know beforehand both the time at which elongation will occur and the altitude of the star at this instant.

The time of elongation is computed as explained for *Polaris* in Art. 91. The altitude may be found by the formula

$$\sin h = \frac{\sin \phi}{\sin \delta} = \sin \phi \sec p.$$

There is usually time enough to reverse the transit and make one observation in each position of the axis, without serious error, if the first is taken when the star is 10′ to 15′ below eastern elongation, or the same amount above western elongation.

Example.

Mean observed horizontal angle between mark and α *Triang. Austr.* at Eastern elongation May 31, 1920 = 35° 10′ 30″. (Mark E. of star.) Decl. α *Triang. Austr.* = −68° 53′ 11″; right ascension = $16^h\,40^m\,18^s.5$. Lat. = −34° 35′ (S.); Long. = 58° 25′ W.

The time and altitude are computed as follows:

$$\begin{aligned} \log \tan \phi &= 9.83849 \\ \log \tan \delta &= \underline{0.41326} \\ \log \cos t_e &= 9.42523 \\ 360^\circ - t_e &= 74^\circ\ 33'.6 \\ 24^h - t_e &= 4^h\ 58^m\ 14^s.4 \\ t_e &= 19\ \ 01\ \ 45\ .6 \\ \alpha &= \underline{16\ \ 40\ \ 18\ .5} \\ S &= 11^h\ 42^m\ 04^s.1 \end{aligned}$$

$$\begin{aligned} \log \sin \phi &= 9.75405 \\ \log \sin \delta &= \underline{9.96982} \\ \log \sin h &= 9.78423 \\ h &= 37^\circ\ 28'.7 \end{aligned}$$

The local civil time corresponding to $11^h\,42^m\,04^s.1$ is $19^h\,05^m\,32^s.5$.

For the azimuth of the star and the resulting bearing of the mark we have

$$\begin{aligned} \log \cos \delta &= 9.55657 \\ \log \cos \phi &= \underline{9.91556} \\ \log \sin Z &= 9.64101 \\ Z &= 25^\circ\ 56'.8 \text{ (East of South)} \\ \text{Measured angle} &= \underline{35\ \ 10\ .5} \\ \text{Bearing of mark S } &61^\circ\ 07'.3 \text{ E} \end{aligned}$$

100. Azimuth by an Altitude of the Sun.

In order to determine the azimuth of a line by means of an observation on the sun the instrument should be set up over one of the points marking the line and carefully levelled. The plate vernier is first set at 0° and the vertical cross hair sighted on the other point marking the line. The colored shade glass is then screwed on to the eyepiece, the upper clamp loosened, and the telescope turned toward the sun. The sun's disc should be sharply focussed before beginning the observations. In making the pointings on the sun great care should be taken not to mistake one of the stadia hairs for the middle hair. If the observation is to be made, say, in the forenoon (in the northern hemisphere), first set the cross hairs so that the vertical hair is tangent to the right edge of the sun and the horizontal hair cuts off a small segment at the lower edge of the disc. (Fig. 67.)* The arrow in the figure shows the direction of the sun's apparent motion. Since the sun is now rising it will in a few seconds be tangent to the horizontal hair. It is only necessary to follow the right edge by means of the upper plate tangent screw until both cross hairs are tangent. At this instant, stop following the sun's motion and note the time. If it is desired to determine the time accurately, so that the watch correction may be found from this same observation, it can be read more closely by a second observer. Both the horizontal and the vertical circles are read, and both angles and the time are recorded. The same observation may be repeated three or four times to increase the accuracy. The instrument should then be reversed and the set of observations repeated, except that the horizontal cross hair is set tangent to the upper edge of the sun and the vertical cross hair cuts a segment from the left edge (Fig. 68). The same number of pointings should

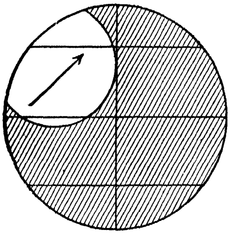


FIG. 67. POSITION OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION (A.M. Observation in Northern Hemisphere.)

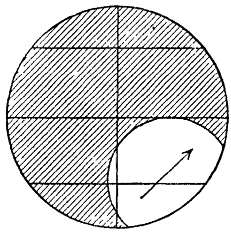


FIG. 68. POSITION OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION (A.M. Observation in Northern Hemisphere.)

* In the diagram only a portion of the sun's disc is visible; in a telescope of low power the entire disc can be seen.

be taken in each position of the instrument. After the pointings on the sun are completed the telescope should be turned to the

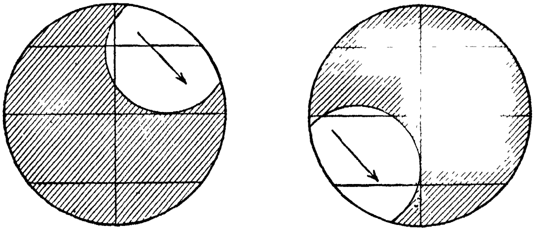
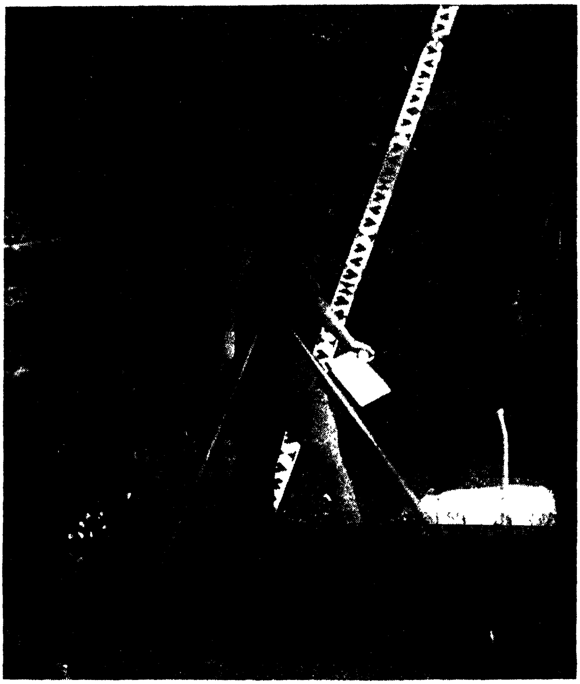


FIG. 69. POSITIONS OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION (P.M. Observation in Northern Hemisphere.)



mark again and the vernier reading checked. If the transit has a vertical arc only, the telescope cannot be used in the reversed position and the index correction must therefore be determined. If the observation is to be made in the afternoon the positions will be those indicated in Fig. 69.*

In computing the azimuth it is customary to neglect the curvature of the sun's path during the short interval between the first and last pointings, unless the series extends over a longer period than is usually required to make such observations. If the observation is taken near noon the curvature is greater than when it is taken near the prime vertical. The mean of the altitudes and the mean of the horizontal angles are assumed to correspond to the position of the sun's centre at the instant shown by the mean watch reading. The mean altitude reading corrected for refraction and parallax is the true altitude of the sun's centre. The azimuth is then computed by any one of the formulæ [22] to [29]. The resulting azimuth combined with the mean horizontal circle reading gives the azimuth of the mark. Five-place logarithmic tables will give the azimuth within 5″ to 10″, which is as great a degree of precision as can be expected in this method.

Example.

Observation on Sun for Azimuth and Time. Lat., 42° 29′.5 N.; long., 71° 07′.5 W. Date, May 25, 1925.

	Hor. Circle (ver. A.)		Vert. Circle		Watch (E. S. T.)
Mark	0° 00′				
L & L limbs	67 54		43° 35′		2^h 58^m 00^s P.M.
	68 11		43 20		2 59 21
	68 26		43 08		3 00 33
(instrument reversed)					
R & U limbs	69 25		43 25		3 01 53
	69 39		43 12		3 03 05
	69 52		43 00		3 04 10
Mean	68° 54′.5		43° 16′.7		3^h 01^m $10^s.3$
Mark	0° 00′	R & P	−0 .9		12
Hor. angle, mark to sun	68° 54′.5	I. C.	+1.0	Civ. T.	15^h 01^m $10^s.3$
		$h =$	43° 16′.8		5
				G. C. T.	20^h 01^m $10^s.3$

* It should be kept in mind that if the instrument has an inverting eyepiece the direction of the sun's apparent motion is reversed. If a prism is attached to the eyepiece, the upper and lower limbs of the sun are apparently interchanged, but the right and left limbs are not.

Equa. [28]	nat	log
nat sin δ =	.35786	
log sin ϕ =		9.82962
log sin h =		9.83605
sin ϕ sin h =	.46310	9.66567
numerator =	.10524	
log " =		9.02218
log sec ϕ =		0.13231
log sec h =		0.13787
log cos Z_s =		9.29236
Z_s =		78° 41'.7
Hor. angle =		68 54 .5
Azimuth of mark		9° 47'.2

Sun's decl. at 0^h	=	+20° 48' 55".8
+27".60 × $20^h.02$	=	+9 12 .6
	δ =	+20° 58' 08".4

If it is desired to compute the time from the same observation it may be found by formula [12], by [19], or by those derived on p. 174. The resulting Eastern standard time is $3^h 00^m 42^s.7$, making the watch $27^s.6$ fast. (The equation of time at 20^h Gr. Civ. T. is $+3^m 14^s.7$.)

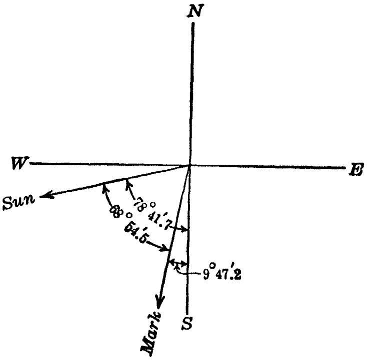


FIG. 70

If for any reason only one limb of the sun has been observed, the azimuth observed may be reduced to the centre of the sun by applying the correction s sec h, where s is the semidiameter and h is the altitude of the centre.

The following examples and explanations are taken from Serial 166, U. S. Coast and Geodetic Survey, and illustrate the method of observing for azimuth and longitude with a small theodolite as practised on magnetic surveys.

Having leveled and adjusted the theodolite and selected a suitable azimuth mark, a well-defined object nearly in the horizon and more than 100 yards distant, the azimuth observations are made in the following order, as shown in the sample set given on pages 172 and 173.

Point on the mark with vertical circle to the right of the telescope (V.C.R.) and read the horizontal circle, verniers A and B. Reverse the circle, invert the telescope and point on the mark again, this time with vertical circle left (V.C.L.). Place the colored glass in position on the eyepiece and point on the sun with vertical circle left, bringing the horizontal and vertical cross wires tangent to the sun's disc. At the moment when both cross wires are tangent note the time by the chronometer. If an appreciable interval is required to look from the eyepiece to the face of the chronometer, the observer should count the half-seconds which elapse and deduct the amount from the actual chronometer reading. The horizontal and vertical circles are then read and recorded. A second pointing on the sun follows, using the same limbs as before. The alidade is then turned 180° and the telescope inverted and two more pointings are made, but with the cross wires tangent to the limbs of the sun opposite to those used before reversal. This completes a set of observations. A second set usually follows immediately, but with the order of the pointings reversed, ending up with two pointings on the mark. Between the two sets the instrument should be releveled if necessary.

Form 266

OBSERVATIONS OF SUN FOR AZIMUTH AND TIME*

Station, Smyrna Mills, Me.
Theodolite of mag'r No. 20.
Mark, Flagpole on school building.
Chronometer, 245.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Temperature, 20° C.

Sun's limb	V. C.	Chronometer time	Horizontal circle			Vertical circle		
			A	B	Mean	A	B	Mean
			° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
	L	Mark	124 43 40	43 50	124 43 45			
	R		304 43 40	43 40	304 43 40			
					124 43 42			
		h m s						
⊙	R	8 25 54	155 12 30	12 50	155 12 40	41 10 30	11 30	41 11 00
⊙	R	27 56	155 40 40	41 00	155 40 50	41 31 00	31 30	41 31 15
⊙	L	30 03	337 00 10	00 20	337 00 15	138 47 00	45 30	41 13 45
⊙	L	32 06	337 28 20	28 30	337 28 25	138 27 00	25 30	41 33 45
		8 28 59.8			336 20 32			41 22 26
								− 57
⊙	L	8 33 45	337 51 00	51 20	337 51 10	138 10 30	09 00	41 50 15
⊙	L	35 59	338 24 30	24 50	338 24 40	137 48 30	47 00	42 12 15
⊙	R	38 20	158 10 00	10 20	158 10 10	43 10 30	11 30	43 11 00
⊙	R	40 37	158 41 50	42 10	158 42 00	43 31 30	32 30	43 32 00
		8 37 10 2			338 17 00			42 41 23
								−54
	R	Mark	304 43 40	43 50	304 43 45			
	L		124 43 20	43 40	124 43 30			
					124 43 38			

* From U. S. Coast and Geodetic Survey Serial 166.

The chronometer and circle readings for the four pointings of a set are combined to get mean values for the subsequent computation. When the vertical circle is graduated from zero to 360°, the readings with vertical circle right give the apparent altitude of one limb of the sun, while those with vertical circle left must be subtracted from 180° to get the apparent altitude of the other limb. The mean of the four pointings gives the apparent altitude of the sun's center. This must be corrected for refraction and parallax to get the true altitude.

Form 266

OBSERVATIONS OF SUN FOR AZIMUTH AND TIME*

Station, Smyrna Mills, Me.
Theodolite of mag'r No. 20.
Mark, Flagpole on school building.
Chronometer, 245.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Temperature, 21° C.

Sun's limb	V. C.	Chronometer time	Horizontal circle A	Horizontal circle B	Horizontal circle Mean	Vertical circle A	Vertical circle B	Vertical circle Mean
			° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
	R	Mark	280 45 00	45 20	280 45 10			
	L		100 45 20	45 40	100 45 30			
					280 45 20			
		h m s						
⊙⌟	L	3 12 38	96 35 50	36 20	96 36 05	143 03 00	00 00	36 58 30
⊙⌝	L	14 38	97 01 20	01 50	97 01 35	143 23 00	20 00	36 38 30
⌜⊙	R	16 44	278 04 10	04 30	278 04 20	36 53 30	53 00	36 53 15
⌞⊙	R	18 46	278 29 00	29 20	278 29 10	36 33 00	32 00	36 32 30
		3 15 41.5			277 32 48			36 45 41
								− 1 08
⌜⊙	R	3 20 12	278 47 20	47 40	278 47 30	36 18 00	17 30	36 17 45
⌞⊙	R	22 12	279 12 00	12 20	279 12 10	35 58 00	57 30	35 57 45
⊙⌝	L	23 50	98 57 40	58 10	98 57 55	144 56 30	53 30	35 05 00
⊙⌟	L	25 50	99 22 40	23 10	99 22 55	145 17 00	14 00	34 44 30
		3 23 01.0			279 05 08			35 31 15
								−1 11
	L	Mark	100 45 20	45 40	100 45 30			
	R		280 45 20	45 30	280 45 25			
					280 45 28			

* From U. S. Coast and Geodetic Survey Serial 166.

It is important to test the accuracy of the observations as soon as they have been completed, so that additional sets may be made if necessary. This may be done by comparing the mean of the first and fourth pointings of a set with the mean of the second and third, or by comparing the rate of change in the altitude and azimuth of the sun between the first and second pointings, the third and fourth, fourth and fifth, fifth and sixth, and seventh and eighth. For the period of 15 or 20 minutes required for two sets of observations the rate of motion of the sun does not change much.

COMPUTATION

From formula [24] we have

$$\cot^2 \frac{Z_s}{2} = \sec s \sec (s - p) \sin (s - \phi) \sin (s - h)$$

and from [19],

$$\tan \frac{t}{2} = \sqrt{\left(\cos s \sin (s - h) \csc (s - \phi) \sec (s - p)\right)}$$

$$= \sqrt{\left(\frac{\cos s \cos (s - p)}{\sin (s - \phi) \sin (s - h)} \cdot \frac{\sin^2 (s - h)}{\cos^2 (s - p)}\right)}$$

$$= \tan \frac{Z_s}{2} \sin (s - h) \sec (s - p).$$

The angle Z_s is the azimuth of the sun from the south point, east if in the morning, west if in the afternoon.

The form of computation is shown in the following example, for the sets of observations at Smyrna Mills, Me., given above.

The different steps of the computation are most conveniently made in the following order:

Enter the corrected altitude, mean readings of the horizontal circle for the pointings on the sun and on the mark, and the chronometer time for each set of observations in their proper places. Enter the value of latitude obtained from the latitude observations or other source. Compute the chronometer correction on standard time for the time of each set of observations from the comparisons with telegraphic time signals. Unless the chronometer has a large rate its correction may be taken the same for two contiguous sets of observations. Compute the Greenwich time of observation for each set, and find from the

Form 269

COMPUTATION OF AZIMUTH AND LONGITUDE*

Station, Smyrna Mills, Me.

Date	Aug.	Aug. 5	Aug. 5	Aug. 5
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
h	41 21 29	42 40 29	36 44 33	35 30 04
ϕ	46 08 21	46 08 21	46 08 21	46 08 21
p	72 51 34	72 51 39	72 56 08	72 56 12
$2s$	160 21 24	161 40 29	155 49 02	154 34 37
s	80 10 42	80 50 14	77 54 31	77 17 18
$s - p$	7 19 08	7 58 35	4 58 23	4 21 06
$s - h$	38 49 13	38 09 45	41 09 58	41 47 14
$s - \phi$	34 02 21	34 41 53	31 46 10	31 08 57
log sec s	0.76807	0 79795	0.67887	0.65749
" sec $(s - p)$	0 00355	0 00422	0 00164	0 00125
" sin $(s - h)$	9 79718	9 79091	9.81839	9 82371
" sin $(s - \phi)$	9.74800	9 75530	9 72140	9.71372
" $\text{ctn}^2 \frac{1}{2} Zs$	0 31680	0 34838	0.22030	0.19617
" $\text{ctn} \frac{1}{2} Zs$	0 15840	0.17419	0 11015	0.09808
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Z from South	69 33 04	67 36 43	75 37 17	77 10 09
Circle reads	336 20 32	338 17 00	277 32 48	279 05 08
S. Mer. "	45 53 36	45 53 43	201 55 31	201 54 59
Mark "	124 43 42	124 43 38	280 45 20	280 45 28
Azimuth of Mark	78 50 06	78 49 55	78 49 49	78 50 29
Mean	78 50 05			
log sec $(s - p)$ sin $(s - h)$	9 80073	9 79513	9.82003	9.82496
" $\tan \frac{1}{2} t$	9.64233	9 62094	9.70988	9 72688
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
t in arc	47 23 24	45 20 52	54 17 25	56 07 55
	h m s	h m s	h m s	h m s
	−3 09 33 6	−3 01 23.5	3 37 09 7	3 44 31 7
E	+ 5 53.4	+ 5 53 3	+ 5 51 8	+ 5 51 8
Local time	8 56 19 8	9 04 29.8	3 43 01.5	3 50 23.5
Chron. time	8 28 59.8	8 37 10.2	3 15 41.5	3 23 01.0
Δt on local time	+ 27 20.0	+ 27 19 6	+ 27 20.0	+ 27 22.5
Δt on 75 merid. time	− 5.8	− 5 8	− 5 8	− 5 8
$\Delta\lambda$	−27 25 8	−27 25.4	−27 25.8	−27 28.3
Mean	−27 26.3=	− 6°51′.6	λ = 68° 08′.4	

* From U. S. Coast and Geodetic Survey Serial 166.

American Ephemeris, or the Nautical Almanac, the sun's polar distance and the equation of time for that time as previously explained. The succeeding steps require little explanation. As the horizontal circles of theodolites are with few exceptions graduated clockwise, and as the sun is east of south in the morning and west of south in the afternoon, it follows that in order to find the horizontal circle reading of the south point, the azimuth of the sun must be added to the circle reading of the sun for the morning observations and subtracted from it for the afternoon observations. The horizontal circle reading of the south point subtracted from the mark reading gives the azimuth of the mark, counted from south around by west from 0 to 360°.

For the computation of t, the logarithms of sec $(s - p)$ and sin $(s - h)$ are found in the azimuth computation and their sum can be written down in its proper place. From that must be subtracted log ctn $\frac{1}{2} Z_s$ to find log tan $\frac{1}{2} t$. The corresponding value of t is the time before or after apparent noon. If in the case of the morning observations ctn $\frac{1}{2} t_1$ be substituted for tan $\frac{1}{2} t$, t_1 will be counted from midnight. The difference between the chronometer correction on local mean time and the correction on standard time is the difference in longitude between the standard meridian and the place of observation.

101. Observations in the Southern Hemisphere.

In making observations on the sun for azimuth in the southern hemisphere (latitude greater than declination) the pointings would be made on the left and lower limbs and on the right and upper limbs in the forenoon, and on the right and lower and on the left and upper limbs in the afternoon, as indicated in Fig. 71.

If the instrument has no vertical and horizontal hairs but has cross hairs of the X pattern the sun's image may be placed in any two symmetrical positions instead of those indicated above.

The same formulæ used for the northern hemisphere may be adapted to the southern hemisphere either by considering the latitude ϕ as negative and employing the regular forms, or by taking ϕ as positive, and using the *south polar distance* instead of the north polar distance when employing Equa. [24]; the resulting azimuth in this case will be that measured from the south point of the meridian.

As an illustration of an observation made in the southern hemisphere the following observation is worked out by two methods. On April 24, 1901 (P.M.), the mean altitude of the sun is 22° 12′ 30″; the corrected declination is 12° 40′ 30″ N.; latitude, 0° 41′ 52″ S.; mean horizontal angle from mark, toward

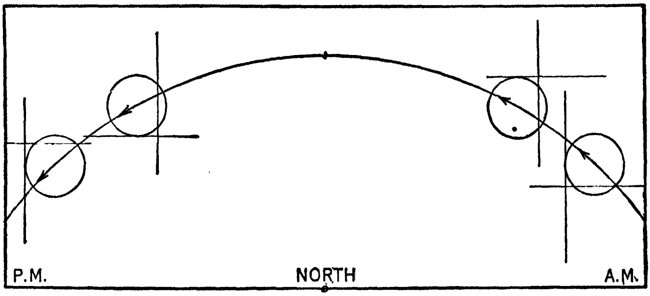


FIG. 71

the left, to sun = 75° 53′ 30″. Employing formula [24] the computation is as follows:

ϕ	−0° 41′ 52″	
h	22 12 30	
p	77 19 30	
$2s$	98° 50′ 08″	
s	49 25 04	log sec 0.18673
$s-\phi$	50 06 56	log sin 9.88498
$s-h$	27 12 34	log sin 9.66015
$s-p$	−27 54 26	log sec 0.05369
		2) 9.78555

log tan $\frac{1}{2}Z$ = 9.89278
$\frac{1}{2}Z$ = 37° 59′ 53″
Z = N 75 59 46 W
Hor. angle = 75 53 30
True bearing of mark = N 0° 06′ 16″ W

If in formula [24] we had used the south polar distance, 102° 40′ 30″, and considered the latitude as positive the result would have been the azimuth of the sun from the south point or S 104° 00′ 14″ W.

If formula [25] is employed we may take ϕ positive, reverse the sign of δ and obtain the bearing of the sun from the south.

nat sin δ =	−0.21942
log sin ϕ =	8.08558
log sin h =	9.57746
sum =	7.66304
nat sin ϕ sin h =	0.00460
numerator =	0.22402
log numerator =	9.35029 n
log sec ϕ =	0.00003
log sec h =	0.03348
log cos Z_s =	9.38380 n
Z_s =	S 104° 00′ 15″ W
Measured angle	75 53 30
True Bearing of Mark =	S 179° 53′ 45″ W
or	N 0° 06′ 15″ W

In this case it would have been quite as simple to solve [25] in its original form, obtaining the bearing from the north point. If the south latitude is greater than the sun's declination (say, lat. 40° S., decl. 20° S.) then the method used in the example would be preferable.

102. Most Favorable Conditions for Accuracy.

From an inspection of the spherical triangle Pole — Zenith — Sun, it may be inferred that the nearer the sun (or other observed body) is to the observer's meridian the less favorable are the conditions for an accurate determination of azimuth from a measured altitude. At the instant of noon the azimuth becomes indeterminate. Also, as the observer approaches the pole the accuracy diminishes, and when he is at the pole the azimuth is indeterminate.

To find from the equations the error in Z due to an error in h differentiate Equa. [13], regarding h as the independent variable; the result is

$$0 = \sin\phi\cos h + \cos\phi\left(-\cos h\sin Z\frac{dZ}{dh} - \cos Z\sin h\right)$$

or,
$$\cos\phi\cos h\sin Z\frac{dZ}{dh} = \sin\phi\cos h - \cos\phi\cos Z\sin h$$
$$= \cos\delta\cos S \quad \text{by [14]}$$

$$\therefore \frac{dZ}{dh} = \frac{\cos\delta \cos S}{\cos\phi \cos h \sin Z}, \text{ which by [15]}$$

$$= \frac{\cos S}{\sin S \cos h},$$

$$= \frac{1}{\cos h \tan S}. \qquad [102]$$

If the declination of the body is greater than the latitude (and in the same hemisphere) there will be an elongation, and at this point the angle S (at the sun or star) will be 90°; the error dZ will therefore be zero. For objects whose declinations are such that an elongation is possible it is clear that this is the most favorable position for an accurate determination of azimuth since an error in altitude has no effect upon Z.

If the declination is less than the latitude, or is in the opposite hemisphere, the most favorable position will depend partly upon S, partly upon h. From Equa. [15] it is seen that the maximum value of S occurs simultaneously with the maximum value of Z, that is, when the body is on the prime vertical ($Z = 90°$ or 270°). To determine the influence of h suppose that there are two positions of the object, one north of the prime vertical and one south of it, such that the angle S is the same for the two. The minimum error (dZ) will then occur where $\cos h$ is greatest; this corresponds to the value of h which is least, and therefore, on the side of the prime vertical toward the pole. The exact position of the body for greatest accuracy could be found for any particular case by differentiating the above expression and placing it equal to zero.

To find the error in the azimuth due to an error in latitude differentiate [13] with respect to $d\phi$. This gives

$$0 = \sin h \cos\phi + \cos h\left(-\cos\phi \sin Z \frac{dZ}{d\phi} - \cos Z \sin\phi\right)$$

or

$$\cos h \cos\phi \sin Z \frac{dZ}{d\phi} = \sin h \cos\phi - \cos h \cos Z \sin\phi$$

$$= \cos\delta \cos t \quad \text{by [16]}$$

$$\therefore \frac{dZ}{d\phi} = \frac{\cos \delta \cos t}{\cos h \cos \phi \sin Z}$$

$$= \frac{\sin Z \cos t}{\sin t \cos \phi \sin Z} \qquad \text{by [12]}$$

$$= \frac{1}{\tan t \cos \phi} \qquad [103]$$

From this equation it is evident that the least error in Z due to an error in ϕ will occur when the object is on the 6-hour circle ($t = 90°$).

Combining the two results it is clear that observations on an object which is in the region between the 6-hour circle and the prime vertical will give results slightly better than elsewhere; observations on the body when on the other side of the prime vertical will, however, be almost as accurate. The most important matter so far as the spherical triangle is concerned is to avoid observations when the body is near the meridian.

The above discussion refers to the trigonometric conditions only. Another condition of great importance is the atmospheric refraction near the horizon. An altitude observed when the body is within 10° of the horizon is subject to large uncertainties in the refraction correction, because this correction varies with temperature and pressure and the observer often does not know what the actual conditions are. This error may be greater than the error of the spherical triangle. When the two requirements are in conflict it will often be better to observe the sun nearer to the meridian than would ordinarily be advisable, rather than to take the observation when the sun is too low for good observing. In winter in high latitudes the interval of time during which an observation may be made is rather limited so that it is not possible to observe very near the prime vertical. The only remedy is to obtain the altitude and the latitude with greater accuracy if this is possible.

103. Azimuth by an Altitude of a Star near the Prime Vertical. The method described in the preceding article applies equally well to an observation on a star, except that the star's image is bisected with both cross hairs and the parallax and semi-diameter corrections become zero. The declination of the star changes so little during one day that it may be regarded as constant, and consequently the time of the observation is not required. Errors in the altitude and the latitude may be partially eliminated by combining two observations, one on a star about due east and the other on one about due west.

Example.

Mean altitude of *Regulus* (bearing east) on Feb. 11, 1908, is 17° 36′.8. Latitude, 42° 21′ N. The right ascension is $10^h\ 03^m\ 29^s.1$ and the declination is +12° 24′ 57″. Compute the azimuth and the hour angle.

	$\phi =$ 42° 21′	log sec = 0.13133
	$h =$ 17 33 .8	log sec = 0.02073
cos .90788	$\phi - h =$ 24° 47′.2	
sin .21502	$\delta =$ +12 25 .0	
$c - s$ .69286		log = 9.84065
		log vers Z_n = 9.99271
		Z_n = 89° 02′.8

The star's bearing is therefore N 89° 02′.8 E.

To obtain the time we may employ formula [12].

log sin Z_n =	9.99994
log cos h =	9.97927
log sec δ =	0.01028
log sin t =	9.98949
t =	−77° 26′.7
=	$5^h\ 09^m\ 46^s.7$ (east)
Right ascension	10 03 29 .1
Sidereal time =	$4^h\ 53^m\ 42^s.4$

104. Azimuth Observation on a Circumpolar Star at any Hour Angle. The most precise determination of azimuth may be made by measuring the horizontal angle between a circumpolar star and an azimuth mark, the hour angle of the star at each pointing being known. If the sidereal time is determined accurately, by any of the methods given in Chapter XI, the hour angle of the star may be found at once by Equa. [37] and the azimuth of the star at the instant may be computed. Since the close circumpolar stars move very slowly and errors in the observed times will have a small effect upon the computed azimuth, it is evident that only such stars should be used if precise results are sought. The advantage of observing the star at any hour angle, rather than at elongation, is that the number of observations may be increased indefinitely and greater accuracy thereby secured.

The angles may be measured either with a repeating instrument (like the engineer's transit) or with a direction instrument in which the circles are read with great precision by means of micrometer microscopes. For refined work the instrument should be provided with a sensitive striding level. If there is no striding level provided with the instrument* the plate level which is parallel to the horizontal axis should be a sensitive one and should be kept well adjusted. At all places in the United States the celestial pole is at such high altitudes that errors in the adjustment of the horizontal axis and of the sight line have a comparatively large effect upon the results.

The star chosen for this observation should be one of the close circumpolar stars given in the circumpolar list in the Ephemeris. (See Fig. 72.) *Polaris* is the only bright star in this group and should be used in preference to the others when it is practicable to do so. If the time is uncertain and *Polaris* is near the meridian, in which case the computed azimuth would be uncertain, it is better to use 51 *Cephei*,† because this star would then be near its elongation and comparatively large errors in the time would have but little effect upon the computed azimuth. If a repeating theodolite or an ordinary transit is used the observations consist in repeating the angle between the star and the mark a certain number of times and then reversing the instrument and making another set containing the same number of repetitions. Since the star is continually changing its azimuth it is necessary to read and record the time of each pointing on the star with the vertical cross hair. The altitude of the star should be measured just before and again just after each half-set so that its altitude for any desired instant may be obtained by simple interpolation. If the instrument has no striding level the cross-level on the plate should be recentred before the second half-set is begun. If a striding level is used the inclination of the axis may be measured, while the telescope is pointing toward the star, by reading both ends of the bubble, with the level first in the direct position and then in the reversed position.

In computing the results the azimuth of the star might be computed for each of the observed times and the mean of these azimuths combined with the mean

---

* The error due to inclination of the axis may be eliminated by taking half of the observations direct and half on the image of the star reflected in a basin of mercury.

† 51 *Cephei* may be found by first pointing on *Polaris* and then changing the altitude and the azimuth by an amount which will bring 51 *Cephei* into the field. The difference in altitude and in azimuth may be obtained with sufficient accuracy by holding Fig. 72 so that *Polaris* is in its true position with reference to the meridian (as indicated by the position of δ *Cassiopeiæ*) and then estimating the difference in altitude and the difference in azimuth. It should be remembered that the distance of 51 *Cephei* east or west of *Polaris* has nearly the same ratio to the difference in azimuth that the polar distance of *Polaris* has to its azimuth at elongation, i.e., 1 to sec ϕ.

of the measured horizontal angles. The labor involved in this process is so great, however, that the common practice is first to compute the azimuth corresponding to the *mean* of the observed times, and then to correct this result for the effect of the curvature of the star's path, i.e., by the difference between the mean azimuth and the azimuth at the mean of the times.

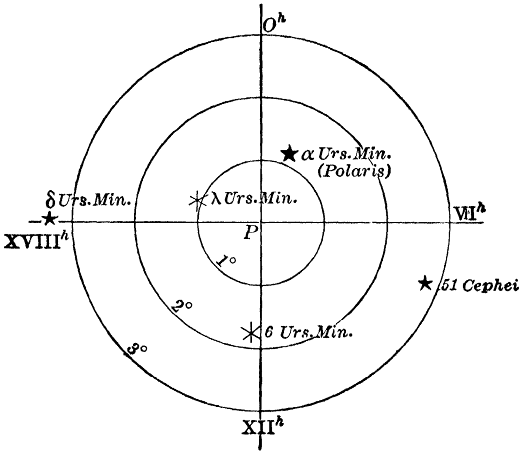


FIG. 72

For a precise computation of the azimuth of the star formula [32] may be used,

$$\tan Z_n = -\frac{\sin t}{\cos\phi \tan\delta - \sin\phi\cos t} \qquad [32]$$

the azimuth being counted from the north toward the east.

A second form may be obtained by dividing the numerator and denominator by $\cos\phi\tan\delta$, giving

$$\tan Z_n = -\frac{\cot\delta\sec\phi\sin t}{1 - \cot\delta\tan\phi\cos t}. \qquad [104]$$

If $\cot\delta\tan\phi\cos t$ is denoted by a, then

$$\tan Z_n = -\cot\delta\sec\phi\sin t\frac{1}{1-a}. \qquad [105]$$

If values of $\log\frac{1}{1-a}$ are tabulated for different values of $\log a$ the use of this third form will be found more rapid than the others. Such tables will be found in Special Publication No. 14, U. S. Coast and Geodetic Survey.*

For a less precise value of the azimuth the following formula will be found convenient;

$$Z = p\sin t\sec h \qquad [106]$$

* Sold by the Superintendent of Documents, Washington, D. C., for 35 cents.

in which Z and p are both in seconds or both in minutes of angle. The error due to substituting the arcs for sines is very small. The precision of the computed azimuth depends largely upon the precision with which h can be measured. If the vertical arc of the transit cannot be relied upon it will be better to use formula [32].

105. The Curvature Correction.

If the azimuth of the star corresponding to the mean of the observed times has been computed it is necessary to apply a curvature correction to this result to obtain the mean of all the azimuths corresponding to the separate hour angles. The curvature correction may be computed by the formula

$$\tan Z_n \frac{1}{n} \sum \frac{2 \sin^2 \frac{\tau}{2}}{\sin 1''} \qquad [107]$$

in which n is the number of pointings on the star in the set and τ for each observation is the difference (in sidereal time) between the observed time and the mean of the times for the set. The interval τ is tabulated as a time interval for convenience, but is taken as an angle when computing the tabular number. The sign of this correction always decreases the angle between the star and pole.

Values of $\frac{2 \sin^2 \frac{\tau}{2}}{\sin 1''}$ are given in Table X.

The curvature correction may also be computed by the formula

$$-\tan Z_0\, [0.2930] \frac{1}{n} \Sigma\, (T - T_0)^2 \qquad [107a]$$

in which the quantity in brackets is a logarithm; $\Sigma(T - T_0)^2$ is the sum of the squares of the time intervals in (sidereal) minutes. This correction should be subtracted from the azimuth Z_0 calculated for the mean of the observed times. If it is preferred to express the time interval in seconds the logarithm becomes [6.73672]. The curvature correction is very small when the star is near the meridian; near elongation it is a maximum.

106. The Level Correction.

The inclination of the horizontal axis should be measured by the striding level, w and e being the readings of the west and east ends of the bubble in one position of the level, and w' and e' the readings after reversal of the level. The level correction is then

$$= \frac{d}{4} \left\{ (w + w') - (e + e') \right\} \tan h \qquad [108]$$

if the graduations are numbered in both directions from the middle, or

$$= \frac{d}{4} \left\{ (w - w') + (e + e') \right\} \tan h \qquad [109]$$

if the graduations are numbered continuously in one direction. In this formula the primed letters refer to the readings taken when the level "zéro" is west. In both formulæ d is the angular value of a level division and h is the altitude of the star.

If the azimuth mark is not in the horizon a similar correction must be applied to the readings on the mark. Ordinarily this correction is negligible.

When applying this correction it should be observed that when the west end of the axis is too high the instrument has to be turned too far west (left) when pointing at the star. The correction must therefore be added to the measured angle if the mark is west of the star; in other words the reading on a circle numbered clockwise must be increased. If the correction is applied to the computed azimuth of the mark the sign must be reversed.

107. Diurnal Aberration.

If a precise azimuth is required a correction should be applied for the effect of diurnal aberration, or the apparent displacement of the star due to the earth's rotation. The observer is being carried directly toward the east point of the horizon with a velocity depending upon his latitude. The displacement will therefore be in a plane through the observer, the east point, and the star. The amount of the correction is given by

$$0''.32 \cos \phi \cos Z_n \sec h \qquad [110]$$

The product of $\cos \phi$ and $\sec h$ is always nearly unity for a close circumpolar and $\cos Z$ is also nearly unity. The correction therefore varies but little from $0''.32$. Since the star is displaced toward the east the correction to the star's azimuth is positive.

108. Observations and Computations.

In the examples which follow, the first illustrates a method appropriate for small surveyor's transits. The time is determined by the altitude of a star near the prime vertical and the azimuth of *Polaris* is computed by formula [106]. Corrections for curvature, inclination and aberration are omitted.

In the second example the time was determined somewhat more precisely and a larger number of repetitions was used. The instrument was an 8-inch repeater reading to $10''$.

The third and fourth examples are taken from the U. S. Coast and Geodetic Survey Spec. Publ. No. 14, and illustrates the methods used by that Survey where the most precise results are required for geodetic purposes.

Example 1

Observed altitudes of *Regulus* (east), Feb. 11, 1908, in lat. $42° 21'$.

Altitude	Watch
$17° 05'$	$7^h 12^m 16^s$
17 31	14 31
17 49	16 07
18 02	17 20

The right ascension of *Regulus* is $10^h 03^m 29^s.1$; the declination is $+12° 24' 57''$. From these data the sidereal time corresponding to the mean watch reading ($7^h 15^m 03^s.5$) is found to be $4^h 53^m 42^s.7$.

Observed horizontal angles from azimuth mark to *Polaris*.

(Mark east of north.)

Telescope Direct		Time of pointing on *Polaris*
Mark	0° 00′	$7^h\ 20^m\ 38^s$
		23 00
Third repetition	201° 48′	23 56
Mean =	67° 16′.0	$7^h\ 22^m\ 31^s.3$
Telescope Reversed		
Mark =	0° 00′	$7^h\ 27^m\ 09^s$
		28 17
Third Repetition	201° 54′	29 21
Mean =	67° 18′.0	$7^h\ 28^m\ 15^s.7$

Altitude of *Polaris* at $7^h\ 20^m\ 38^s$ = 43° 03′
Altitude of *Polaris* at 7 29 21 = 43 01
Mean watch reading for *Polaris* = $7^h\ 25^m\ 23^s.5$
Corresponding sidereal time = 5 04 04 .4
Right Ascension of *Polaris* = 1 25 32 .3
Hour-angle of *Polaris* = 3 38 32 .1
t = 54° 38′

p = 4251
log p = 3.62849
log sin t = 9.91141
log sec h = 0.13611

log azimuth = 3.67601
azimuth = 4743″
= 1° 19′.0
Mean angle = 67 17 .0

Mark East of North = 65° 58′ 0

Example 2.

RECORD OF TIME OBSERVATIONS

Polaris: — Chronometer, $12^h\ 09^m\ 31^s.5$; alt., 41° 15′ 40″
ε Corvi: — Chronometer, 12 13 37 .5; alt., 25 34 00

Polaris: R. A. = $1^h\ 25^m\ 51^s.1$; decl. = + 88° 49′ 24″.8
ε Corvi: R. A. = $12^h\ 5^m\ 30^s.5$; decl. = − 22° 07′ 21″.0

	Chronometer	R. A.	Decl.
α Serpentis (E)	$12^h\ 24^m\ 15^s.7$	$15^h\ 39^m\ 51^s.6$	+ 6° 42′ 20″.7
ε Hydræ: (W)	12 18 32 .0	8 42 00 .5	+ 6 44 58 .9

(Lat. = 42° 21′ 00″ N.; Long. = $4^h\ 44^m\ 18^s.0$ W.)

From these observations the chronometer is found to be $10^m\ 22^s.1$ fast.

Example 2 (*continued*)

RECORD OF AZIMUTH OBSERVATIONS

Instrument (B. & B. No. 3441) at South Meridian Mark. Boston, May 16, 1910.
(One division of level = 15″.0.)

Object.	Pos. of tel.	No. of rep.	Chronometer.	Horizontal circle. Vernier A.	B.	Level readings and angles.
Polaris..	Direct		$11^h\ 24^m\ 35^s.0$	0° 00′ 00″	00″	W E 7.0 3.9 5.8 5.1 12.8 9.0
			27 15.0			9.0
			28 31.5			3.8
			30 00.0			Corr. = 12″.5
			31 20.5			Alt. *Polaris* at $11^h\ 34^m\ 20^s.5$ = 41° 20′ 30″
			32 27.0			Alt. *Polaris* at $11^h\ 51^m\ 04^s.0$ = 41° 18′ 40″
Mark...		6		*39° 33′ 30″	30″	Mean horizontal angle = 66° 35′ 35″.0
Polaris..	Reversed		11 42 45.5	39° 33′ 30″	30″	W E 5.1 5.8 3.3 7.6
			44 09.0			8.4 13.4
			45 15.0			8.4
			46 29.5			5.0
			47 25.0			Corr. = 16″.5
			48 54.5			
Mark...		6		*78° 27′ 30″	20″	Mean horizontal angle = 66° 28′ 59″.2 Alt. *Polaris* at $12^h\ 09^m\ 31^s.5$ = 41° 15′ 40″

* Passed 360°.

Example 2 (*continued*)

COMPUTATION OF AZIMUTH

Mean of Observed times	=	11^h	37^m	$25^s.6$
Chronometer correction	=	−	10	22 .1
Sidereal time	=	11	27	03 .5
R. A. of *Polaris*	=	1	25	51 .1
Hour Angle of *Polaris*	=	10	01	12 .4

$t = 150° \ 18' \ 06''$

$\log \cos \phi = 9.868670$

$\log \tan \delta = 1.687490$

$\log \cos \phi \tan \delta = 1.556160$

$\cos \phi \tan \delta = 35.9882$

$\log \sin \phi = 9.82844$

$\log \cos t = 9.93884$

$\log \sin \phi \cos t = 9.76728$

$\sin \phi \cos t = .5852$

denominator $= 36.5734$

$\log \sin t = 9.694985$

log denom. $= 1.563165$

$\log \tan Z = 8.131820$

$Z = 0° \ 46' \ 34''.2$

Curvature correction	=			2.1
Azimuth of star	=	0	46	32 .1
Measured angle, first half	=	66°	35′	35″.0
Level correction	=			−12 .5
Corrected angle	=	66	35	22 .5
Measured angle, second half	=	66	28	59 .2
Level correction	=			+16 .5
Corrected angle	=	66	29	15 .7
Mark east of star	=	66	32	19 .1
Mark east of North	=	65°	45′	47″.0

109. Meridian by Polaris at Culmination.

The following method is given in Lalande's Astronomy and was practiced by Andrew Ellicott, in 1785, on the Ohio and Pennsylvania boundary survey. The direction of the meridian is determined by noting the instant when *Polaris* and some

Example 3

RECORD — AZIMUTH BY REPETITIONS.

[Station, Kahatchee △. State, Alabama. Date, June 6, 1898. Observer, O. B. F. Instrument, 10-inch Gambey No. 63. Star, Polaris.]

[One division striding level = 2.″67.]

Objects.	Chr. time on star.	Pos. of tel.	Repetitions.	Level readings. W.	E.	Circle readings. °	′	A ″	B ″	Mean.	Angle.
	h m s										° ′ ″
Mark......		D	0			178	03	22.5	20	21.2	
Star.......	14 46 30		1	4.5	10.7						
				9.2	5.9						
	49 08		2								
	52 51	D	3	9.6	5.6						
				5.2	10.0						
	56 10	R	4	11.3	4.0						
				7.8	7.4						
Set No. 5..	14 59 12		5								
	15 01 55	R	6	8.7	6.6	100	16	20	20	20	72 57 50.2
				11.9	3.4						
	14 54 17.7			68.2	53.6						
					+ 14.6						
Star.......	15 04 44	R	1	11.9	3.4						
				8.5	6.8						
	07 18		2								
	09 54	R	3	7.9	7.3						
				11.2	4.1						
Set No. 6..	14 15	D	4	9.0	6.1						
				5.9	9.6						
	16 14		5								
	15 18 24		6	5.9	9.6						
				9.1	6.2						
Mark.....		D				177	27	00	00	00	72 51 46.7
	15 11 48.2			69.4	53.1						
					+16.3						

COMPUTATION — AZIMUTH BY REPETITIONS

[Kahatchee, Ala. ϕ = 33° 13′ 40″.33.]

Date, 1898, set	June 6 5	June 6 6
Chronometer reading	14 54 17.7	15 11 48.2
Chronometer correction	−31.1	−31.1
Sidereal time	14 53 46.6	15 11 17.1
α of Polaris	1 21 20.3	1 21 20.3
t of Polaris (time)	13 32 26.3	13 49 56.8
t of Polaris (arc)	203° 06′ 34″.5	207° 29′ 12″.0
δ of Polaris	88 45 46.9	
log cot δ	8.33430	8.33430
log tan ϕ	9.81629	9.81629
log cos t	9.96367n	9.94798n
log a (to five places)	8.11426n	8.09857n
log cot δ	8.334305	8.334305
log sec ϕ	0.077535	0.077535
log sin t	9.593830n	9.664211n
log $\frac{1}{1-a}$	9.994387	9.994584
log (−tan A) (to 6 places)	8.000057n	8.070635n
A = Azimuth of Polaris, from north *	0° 34′ 22″.8	0° 40′ 26″.8
	m s ″	m s ″
τ and $\frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}$	7 47.7 119.3	7 04.2 98.1
	5 09.7 52.3	4 30.2 39.8
	1 26.7 4.1	1 54.2 7.1
	1 52.3 6.9	2 26.8 11.8
	4 54.3 47.2	4 25.8 38.5
	7 37.3 114.0	6 35.8 85.4
Sum	343.8	280.7
Mean	57.3	46.8
log $\frac{1}{n}\sum\frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}$	1.758	1.670
log (curvature corr.)	9.758	9.741
Curvature correction	−0.6	−0.6
Altitude of Polaris = h	32° 07′	
$\frac{d}{4}$ tan h = level factor	0.419	0.419
Inclination	+3.6	+4.1
Level correction	−1″.5	−1″.7
Angle, star — mark	72 57 50.2	72 51 46.7
Corrected angle	72 57 48.7	72 51 45.0
Corrected azimuth of star *	0 34 22.2	0 40 26.2
Azimuth of mark E of N	73 32 10.9	73 32 11.2
	180 00 00.0	180 00 00.0
Azimuth of mark (Clockwise from south)	253 32 10.9	253 32 11.2

To the mean result from the above computation must be applied corrections for diurnal aberration and eccentricity (if any) of Mark. Carry times and angles to tenths of seconds only.

*** Minus if west of north.**

Example 4

HORIZONTAL DIRECTIONS

[Station, Sears, Tex. (Triangulation Station). Observer, W. Bowie. Instrument, Theodolite 168. Date, Dec. 22, 1908.]

Position.	Objects observed.	Time.	Tel. D or R.	Mic.	Backward.			For-ward.	Mean.	Mean D and R.	Direc-tion.	Remarks.
		h m			°	′	″	″			″	
1	Morrison..	8 19	D	*A*	0	0	35	35				1 division of the
				B			41	41				striding level
				C			36	34	37.0			= 4″.194
			R	*A*	180	00	36	35				
				B			32	31				
				C			35	34	33.8	35.4	00.0	
	Buzzard..		D	*A*	53	30	43	42				
				B			41	42				
				C			34	33	39.2			
			R	*A*	233	30	39	37				
				B			34	32				
				C			38	38	36.3	37.8	02.4	
	Allen.....		D	*A*	170	14	61	62				
				B			57	55				
				C			61	59	59.2			
			R	*A*	350	14	50	49				
				B			63	60				
				C			53	53	54.7	57.0	21.6	
	Polaris....		D	*A*	252	01	54	53				W E
	h m s			*B*			54	53				9.3 28.0
	1 48 35.5			*C*			51	51	52.7			27.7 9.1
	1 51 06.0											
												18.4 − 0.5 18.9
	1 49 50.8											
			R	*A*	72	01	09	09				24.9 6.3
				B			02	01				13.0 31.7
				C			10	08	06.5	29.6		
												11.9 −13.5 25.4
												− 7.0

COMPUTATION OF AZIMUTH, DIRECTION METHOD.

[Station, Sears, Tex. Chronometer, sidereal 1769. $\phi = 32° 33' 31''$
Instrument, theodolite 168. Observer, W. Bowie.]

Date, 1908, position	Dec. 22, 1	2	3	4
Chronometer reading	1 49 50.8	2 01 33.0	2 16 31.0	2 43 28.8
Chronometer correction	− 4 37.5	− 4 37.5	− 4 37.4	− 4 37.3
Sidereal time	1 45 13.3	1 56 55.5	2 11 53.6	2 38 51.5
α of Polaris	1 26 41.9	1 26 41.9	1 26 41.8	1 26 41.8
t of Polaris (time)	0 18 31.4	0 30 13.6	0 45 11.8	1 12 09.7
t of Polaris (arc)	4° 37′ 51″.0	7° 33′ 24″.0	11° 17′ 57″.0	18° 02′ 25″.5
δ of Polaris	88 49 27.4			
log cot δ	8.31224	8.31224	8.31224	8.31224
log tan ϕ	9.80517	9.80517	9.80517	9.80517
log cos t	9.99858	9.99621	9.99150	9.97811
log a (to five places)	8.11599	8.11362	8.10891	8.09552
log cot δ	8.312243	8.312243	8.312243	8.312243
log sec ϕ	0.074254	0.074254	0.074254	0.074254
log sin t	8.907064	9.118948	9.292105	9.490924
$\log \frac{1}{1-a}$	0.005710	0.005679	0.005618	0.005445
log (−tan A) (to 6 places)	7.299271	7.511124	7.684220	7.882866
A = Azimuth of Polaris, from north*	0 06 50.8	0 11 09.2	0 16 36.9	0 26 15.0
Difference in time between D. and R.	m s 2 30	m s 2 00	m s 3 18	m s 1 38
Curvature correction	0	0	0	0
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Altitude of Polaris = h	33 46	33 46	33 46	33 46
$\frac{d}{4}$ tan h = level factor	0.701	0.701	0.701	0.701
Inclination †	−7.0	−7.2	−7.0	−1.8
Level correction	−4.9	−5.0	−4.9	−1.3
Circle reads on Polaris	252 01 29.6	86 58 11.2	281 54 27.0	116 45 48.6
Corrected reading on Polaris	252 01 24.7	86 58 06.2	281 54 22.1	116 45 47.3
Circle reads on mark	170 14 57.0	5 15 58.2	200 17 42.4	35 18 45.4
Difference, mark — Polaris	278 13 32.3	278 17 52.0	278 23 20.3	278 32 58.1
Corrected azimuth of Polaris, from north *	0 06 50.8	0 11 09.2	0 16 36.9	0 26 15.0
	180 00 00.0	180 00 00.0	180 00 00.0	180 00 00.0
Azimuth of Allen (Clockwise from South)	98 06 41.5	98 06 42.8	98 06 43.4	98 06 43.1

To the mean result from the above computation must be applied corrections for diurnal aberration and eccentricity (if any) of Mark.

Carry times and angles to tenths of seconds only.

* Minus, if west of north.

† The values shown in this line are actually four times the inclination of the horizontal axis in terms of level divisions.

other star are in the same vertical plane, and then waiting a certain interval of time, depending upon the date and the star observed, when *Polaris* will be in the meridian. At this instant *Polaris* is sighted and its direction then marked on the ground by means of stakes. The stars selected for this observation should be near the hour circle through the polestar; that is, their right ascensions should be nearly equal to that of the polestar, or else nearly 12^h greater. The stars best adapted for this purpose at the present time are δ *Cassiopeiæ* and ζ *Ursæ Majoris*.

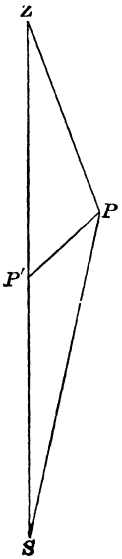


FIG. 73

The interval of time between the instant when the star is vertically above or beneath *Polaris* and the instant when the latter is in the meridian is computed as follows: In Fig. 73 P is the pole, P' is *Polaris*, S is the other star (δ *Cassiopeiæ*) and Z is the zenith. At the time when S is vertically under P', $ZP'S$ is a vertical circle. The angle desired is ZPP', the hour angle of *Polaris*. PP' and PS, the polar distances of the stars, are known quantities; $P'PS$ is the difference in right ascension, and may be obtained from the Ephemeris. The triangle $P'PS$ may therefore be solved for the angle at P'. Subtracting this from 180° gives the angle $ZP'P$; PP' is known, and PZ is the colatitude of the observer. The triangle $ZP'P$ may then be solved for ZPP', the desired angle. Subtracting ZPP' from 180° or 12^h gives the sidereal interval of time which must elapse between the two observations. The angle SPP' and the side PP' are so small that the usual formulæ may be simplified, by replacing sines by arcs, without appreciably diminishing the accuracy of the result. A similar solution may be made for the upper culmination of δ *Cassiopeiæ* or for the two positions of the star ζ *Ursæ Majoris*, which is on the opposite side of the pole from *Polaris*. The above solution, using the right ascensions and declinations for the date, gives the exact interval required; but for many purposes it is sufficient to use a time interval calculated from the mean places of the stars and for a mean latitude of the United States. The time interval for the star δ *Cassiopeiæ* for the year 1910 is $6^m.1$ and for 1920 it is about $12^m.3$. For the star ζ *Ursæ Majoris* the time interval for the year 1910 is approximately $6^m.7$, while for 1920 it is $11^m.3$. Beginning with the issue for 1910 the American Ephemeris and Nautical Almanac gives values of these intervals, at the end of the volume, for different latitudes and for different dates. Within the limits of the United States it will generally be necessary to observe on δ *Cassiopeiæ* when *Polaris* is at lower culmination and on ζ *Ursæ Majoris* when *Polaris* is at upper culmination.

The determination of the instant when the two stars are in the same vertical plane is necessarily approximate, since there is some delay in changing the telescope from one star to the other. The motion of *Polaris* is so slow, however, that a very fair degree of accuracy may be obtained by first sighting on *Polaris*, then pointing the telescope to the altitude of the other star (say δ *Cassiopeiæ*) and waiting until it appears in the field; when δ *Cassiopeiæ* is seen, sight again at *Polaris* to allow for its motion since the first pointing, turn the telescope again to δ *Cassiopeiæ* and observe the instant when it crosses the vertical hair. The motion of the polestar during this short interval may safely be neglected. The tabular interval of time corrected to date must be added to the watch reading. When this computed time arrives, the cross hair is to be set accurately on *Polaris* and then the telescope lowered in this vertical plane and a mark set in line with the cross hairs. The change in the azimuth of *Polaris* in 1^m of time is not far from half a minute of angle, so that an error of a few seconds in the time of sighting at *Polaris* has but little effect upon the result. It is evident that the actual error of the watch on local time has no effect whatever upon the result, because the only requirement is that the *interval* should be correctly measured.

110. Azimuth by Equal Altitudes of a Star.

The meridian may be found in a very simple manner by means of two equal altitudes of a star, one east of the meridian and one west. This method has the advantage that the coördinates of the star are not required, so that the Almanac or other table is not necessary The method is inconvenient because it requires two observations at night several hours apart. It is of special value to surveyors in the southern hemisphere, where there is no bright star near the pole. The star to be used should be approaching the meridian (in the evening) and about 3^h or 4^h from it. The altitude should be a convenient one for measuring with the transit, and the star should be one that can be identified with certainty 6^h or 8^h later. Care should be taken to use a star which will reach the same altitude on the opposite side of the meridian before daylight interferes with the observation. In the

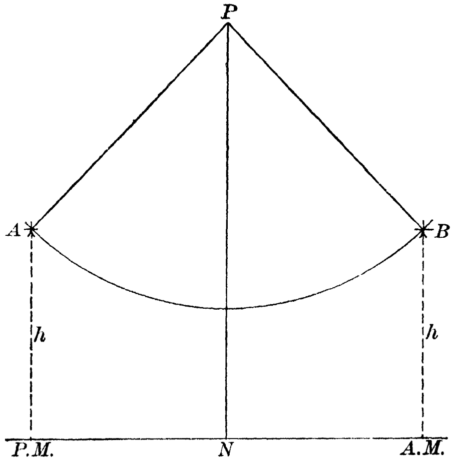


FIG. 74

northern hemisphere one of the stars in *Cassiopeia* might be used. The position at the first (evening) observation would then be at A in Fig. 74. The star should be bisected with both cross hairs and the altitude read and recorded. A note or a sketch should be made showing which star is used. The direction of the star should be marked on the ground, or else the horizontal angle measured from some reference mark to the position of the star at the time of the observation. When the star is approaching the same altitude on the opposite side of the meridian (at B) the telescope should be set at exactly the same altitude as was read at the first observation. When the star comes into the field it is bisected with the vertical cross hair and followed in azimuth until it reaches the horizontal hair. The motion in azimuth should be stopped at this instant. Another point is then set on the ground (at same distance from the transit as the first) or else another angle is turned to the same reference mark. The bisector of the angle between the two directions is the meridian line through the transit. It will usually be found more practicable to turn angles from a fixed mark to the star than to set stakes. The accuracy of the result may be increased by observing the star at several different altitudes and using the mean value of the horizontal angles. In this method the index correction (or that part of it due to non-adjustment) is eliminated, since it is the same for both observations. The refraction error is also eliminated, provided it is the same at the two observations. Error in the adjustment of the horizontal axis and the line of sight will be eliminated if the first half of the set is taken with the telescope direct and the second half with the telescope reversed. With a transit provided with a vertical arc (180°) this cannot be done. Care should be taken to re-level the plates just before the observation is begun; the levelling should not, of course, be done between the pointing on the mark and the pointing on the star, but may be done whenever the lower clamp is loose.

111. Observation for Meridian by Equal Altitudes of the Sun in the Forenoon and in the Afternoon.

This observation consists in measuring the horizontal angle between the mark and the sun when it has a certain altitude in the forenoon and measuring the angle again to the sun when it has an equal altitude in the afternoon. Since the sun's declination will change during the interval, the mean of the two angles will not be the true angle between the meridian and the mark, but will require a small correction. The angle between the south point of the meridian and the point midway between the two directions of the sun is given by the equation

$$\text{Correction} = \frac{\frac{1}{2}d}{\cos\phi \sin t}, \qquad [111]$$

in which d is the hourly change in declination multiplied by the number of hours elapsed between the two observations, ϕ is the latitude, and t is the hour angle of the sun, or approximately half the elapsed interval of time. The correction depends upon the change in the declination, not upon its absolute value, so that the hourly change may be taken with sufficient accuracy from the Almanac for any year for the corresponding date.

VARIATION PER HOUR IN SUN'S DECLINATION

(1925)

Day of month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	+12″	+42″	+57″	+58″	+46″	+21″	− 9″	−37″	−54″	−58″	−49″	−24″
5	16	45	58	57	43	17	13	40	55	58	46	20
10	22	48	59	56	40	12	18	43	57	57	43	14
15	27	51	59	54	36	7	23	46	58	56	39	9
20	32	54	59	52	32	+2	27	49	58	54	35	−3
25	36	+56	59	49	28	−3	32	51	59	52	30	+3
30	+41		+58	+46	+23	−8	−36	−53	−58	−50	−25	+9

In making the observation the instrument is set up at one end of the line whose azimuth is to be determined, and the plate vernier set at 0°. The vertical cross hair is set on the mark and the lower clamp tightened. The sun glass is then put in position, the upper clamp loosened, and the telescope pointed at the sun. It is not necessary to observe on both edges of the sun, but is sufficient to sight, say, the lower limb at both observations, and to sight the vertical cross hair on the opposite limb in the afternoon from that used in the forenoon. The horizontal hair is therefore set on the *lower* limb and the vertical cross hair on the *left* limb. When the instrument is in this position the time should be noted as accurately as possible. The altitude and the horizontal angle are both read. In the afternoon the instrument is set up at the same point, and the same observation is made, except that the vertical hair is now sighted on the *right* limb; the horizontal hair is set on the *lower* limb as before. A few minutes before the sun reaches an altitude equal to that observed in the morning the vertical arc is set to read exactly the same altitude as was read at the first observation. As the sun's altitude decreases the vertical hair is kept tangent to the right limb until the lower edge of the sun is in contact with the horizontal hair. At this instant the time is again noted accurately; the horizontal angle is then read. The mean of the two circle readings, corrected for the effect of change in declination, is the angle from the mark to the south point of the horizon. The algebraic sign of the correction is determined from the fact that if the sun is going north the mean of the two vernier readings lies to the west of the south point, and vice versa. The precision of the result may be increased by taking several forenoon observations in succession and corresponding observations in the afternoon.

Example.

Lat. 42° 18′ N. Apr. 19, 1906.

A.M. Observations.

Reading on Mark, 0° 00′ 00″

U & L limbs { Alt., 24° 58′; Hor. Angle, 357° 14′ 15″; Time, $7^h\ 19^m\ 30^s$

½ elapsed time = $4^h\ 26^m\ 22^s$

t = 66° 35′ 30″

log sin t = 9.96270

log cos ϕ = 9.86902

9.83172

log 230″.9 = 2.36342

2.53170

Corr. = 340″.2

P.M. Observations.

Reading on Mark, 0° 00′ 00″

U & R limbs { Alt., 24° 58′; Hor. Angle, 162° 28′ 00″; Time, $4^h\ 12^m\ 15^s$

Incr. in decl. = + 52″ × 4^h.44′

= +230″.9

Mean Circle Reading = 79° 51′ 08″

Correction = 5 40

True Angle = S 79° 45′ 28″ E.

Azimuth = 280° 14′ 32″

112. Azimuth of Sun near Noon.

The azimuth of the sun near noon may be determined by means of Equa. [30], provided the local apparent time is known or can be computed. If the longitude and the watch correction on Standard Time are known within one or two seconds the local apparent time may be readily calculated. This method may be useful when it is desired to obtain a meridian during the middle of the day, because the other methods are not then applicable.

If, for example, an observation has been made in the forenoon from which a reliable watch correction may be computed, then this correction may be used in the azimuth computation for the observation near noon; or if the Standard Time can be obtained accurately by the noon signal and the longitude can be obtained from a map within about 500 feet, the local apparent time may be found with sufficient accuracy. This method is not usually convenient in midsummer, on account of the high altitude of the sun, but if the altitude is not greater than about 50° the method may be used without difficulty. The observations are made exactly as in Art. 93, except that the *time* of each pointing is determined more precisely; the accuracy of the result depends very largely upon the accuracy with which the hour angle of the sun can be computed, and great care must therefore be used in determining the time. The observed watch reading is corrected for the known error of the watch, and is then converted into local apparent time. The local apparent time converted into degrees is the angle at the pole, t. The azimuth is then found by the formula

$$\sin Z = \sin t \sec h \cos \delta. \qquad [12]$$

Errors in the time and the longitude produce large errors in Z, so this method should not be used unless both can be determined with certainty.

Example.

Observation on the sun for azimuth.

Lat. 42° 21'. Long. $4^h\ 44^m\ 18^s$ W. Date, Feb. 5, 1910.

Hor. Circle	Vert. Circle		Watch.
Mark, 0° 00'			(30^s fast)
app. L & L limbs, 29 01	31° 49'		$11^h\ 43^m\ 22^s$
app. U & R limbs, 28 39	31 16		11 44 20
Mean, 28° 50'	31° 32'.5		$11^h\ 43^m\ 51^s$
	Refr., 1 .6	Watch corr. =	−30
	h = 31° 30'.9	E. S. T. =	$11^h\ 43^m\ 21^s$
			15 42
		L. M. T. =	$11^h\ 59^m\ 03^s$
	$\delta = -16° 02' 32.2''$	Eq. t. =	14 09 .1
		L. A. T. =	$11^h\ 44^m\ 53^s.9$
		t =	15 06 .1
	Eq. t. = $14^m\ 09^s.07$	=	3° 46'.5
		log sin t =	8.81847
		log cos δ =	9.98275
		log sec h =	0.06930
		log sin Z =	8.87052
		Z =	4° 15'.4
		Hor. Circle =	28 50
		Azimuth =	S 33° 05'.4 E
		=	326° 54'.6

113. Meridian by the Sun at the Instant of Noon.*

If the error of the watch can be determined within about one second, and the sun's declination is such that the noon altitude is not too high for convenient observing and accurate results, the following method of determining the meridian will often prove useful. Before beginning the observation the watch time of local apparent noon should be computed and carefully checked. If the centre of the sun can be sighted accurately with the vertical hair at this time the line of sight will be in the meridian, pointing to the south if the observer is in the northern hemisphere. As this is not usually practicable the vernier reading for the south point of the horizon may be found as follows: Set the "A" vernier to read 0°, sight on a reference mark (such as a point on the line whose bearing is to be found) and clamp the lower motion. Loosen the upper clamp and, about 10^m before noon, set the vernier so that the vertical hair is a little in advance of the west edge of the sun's disc. Read the watch as each limb passes the vertical hair and note the vernier reading. Then set the vernier so that the line of sight is nearly in the meridian and repeat the observation. It is best to make a third set as soon as possible after the second to be used as a check.

The mean of the two watch readings in each observation is the reading for the centre of the sun. This may be checked roughly by reading also the time when the sun's disc appears to be bisected. From the first and second vernier readings dad the corresponding watch times compute the motion of the sun in azimuth per second of time. Then from the second watch reading and the watch time of apparent noon, the difference of which is the interval before or after noon, compute the correction to the second vernier reading to give the reading for the meridian. The third set of observations may be used in conjunction with the first to check the preceding computations, by computing the second vernier reading from the observed time and comparing with the actual reading. The reading for the south point may also be computed by using the first and third or the second and third observations.

The accuracy of the results depends upon the accuracy with which the error of the watch may be obtained, upon the reliability of the watch, and the accuracy of the longitude, obtained from a map or by observation. When the sun is high the sights are more difficult to take and the sun's motion in azimuth is rapid so that an error in the watch time of noon produces a larger error in the result than when it has a low altitude at noon. The method is therefore more likely to prove satisfactory in winter than in summer. In winter the conditions for a determination of meridian from a morning or afternoon altitude of the sun are not favorable, so that this method may be used as a substitute. The method is more likely to give satisfactory results when the observer is able to obtain daily comparisons of his watch with the time signal so that the reliability of the watch time may be judged.

---

* The method described in this article was given by Mr. T. P. Perkins, Engineering Dept., Boston & Maine R. R., in the *Engineering News*, March 31, 1904.

Example.

On Jan. 1, 1925, in latitude 42° 22′ N., longitude 71° 05′.6 W., the "A" vernier is set at 0° and cross hair sighted at mark; the vernier is then set to read 42° 40′ (to the right). The observed times of transit of the west and east edges of the sun over the vertical hair are $11^h\ 36^m\ 39^s$ and $11^h\ 39^m\ 01^s$. The vernier is then set at 45° 04′.5; the times of transit are $11^h\ 46^m\ 10^s$ and $11^h\ 48^m\ 31^s$. As a check the vernier is set on 45° 35′, the times of transit being $11^h\ 48^m\ 08^s$ and $11^h\ 50^m\ 31^s$. The watch is 13^s slow of Eastern time. Find the true bearing of the mark from the transit.

Local Apparent Time	= $12^h\ 00^m\ 00^s$
Equation of Time	= $3\ \ 40.8$
Local Civil Time	= $12^h\ 03^m\ 40^s.8$
Longitude difference	= $15\ \ 37.6$
Eastern Standard Time	= $11^h\ 48^m\ 03^s.2$
Watch slow	13
Watch time of apparent noon	= $11^h\ 47^m\ 50^s.2$

Interval, 1st obs. to 2nd obs.	= $9^m\ 30^s.5$	= $570^s.5$
Interval, 2nd obs. to noon	=	$29^s.7$
Diff. in vernier readings	= $2°\ 24'.5$	= $144'.5$

The correction (x) to the 2nd reading (45° 04′.5) is found by the proportion

$$x : 144.5 = 29.7 : 570.5$$
$$\therefore\ x = 7'.5$$

The vernier reading for the meridian is therefore 45° 04′.5 + 7′.5 = 45° 12′ making the bearing of the mark S 45° 12′ E.

114. Approximate Azimuth of Polaris when the Time is Known.

If the error of the watch is known within half a minute or so, the azimuth of *Polaris* may be computed to the nearest minute, that is, with sufficient precision for the purpose of checking the angles of a traverse. The horizontal angle between *Polaris* and a reference mark should be measured and the watch time of the pointing on the star noted. It is desirable to measure also the altitude of *Polaris* at the instant, although this is not absolutely necessary. A convenient time to make this observation is just before dark, when both the star and the cross hairs can be seen without using artificial light. The program of observation would be: 1. Set on 0° and sight the mark, clamping the lower clamp. 2. Set both the vertical and the horizontal cross hairs on the star and note the time. 3. Read the horizontal and the vertical angles. 4. Record all three readings. The method of repetition may be employed if desired.

If the American Ephemeris is at hand the azimuth of *Polaris* may be taken out at once from Table IV when its hour angle and the latitude are known. The watch time of the observation should be converted into local sidereal time and the hour angle of the star computed. The azimuth may be found by double interpolation in Table IV (Ephem.)

Example.

On May 18, 1925, the angle from a reference mark (clockwise) to *Polaris* was 36° 10′ 30″; watch time $8^h\ 10^m\ 20^s$ P.M.; watch slow 10^s; altitude 41° 21′.5; latitude 42° 22′ N.; longitude 71° 06′ W. Find the azimuth of the mark. (See Fig. 75.)

First Solution

Watch reading	$8^h\ 10^m\ 20^s$ P.M.
Watch correction	+10
Eastern Time	$8^h\ 10^m\ 30^s$ P.M.
	15 36
Local Time	$8^h\ 26^m\ 06^s$ P.M.
Loc. Civ. T.	20 26 06
Table III	3 21 .4
Table III (Long.)	46 .7
$\alpha_s + 12^h$	15 40 38 .3
	$36^h\ 10^m\ 52^s.4$
Loc. Sid. T.	12 10 52 .4
α, *Polaris*	1 33 38 .6
t, *Polaris*	$10^h\ 37^m\ 13^s.8$

From Table IV, Ephemeris, azimuth = 0° 30′.9 West
Measured horizontal angle = 36 10 .5
Bearing of mark = N 36° 41′.4 W.

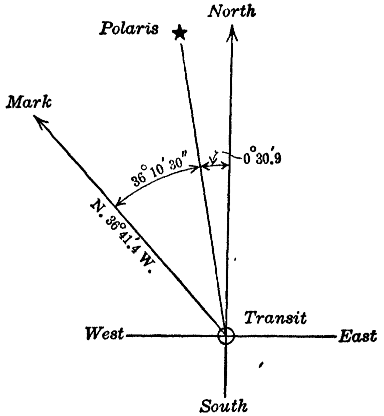


FIG. 75

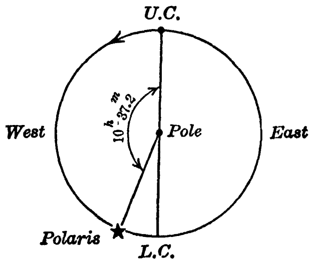


FIG. 76

If the Ephemeris is not at hand the azimuth may be found from the tables on pp. 203 and 204 of this volume. The watch time of the observation is corrected for the known error of the watch and then converted into local time. From Table V the local civil time of the upper culmination of *Polaris* may be found. The difference between the two is the star's hour angle in mean solar units. This should be converted into sidereal units by adding 10^s for each hour in the interval. (Fig. 76.) It should be observed that if the time of upper culmination is less than the observed time the difference is the hour angle measured toward the west, and the star is therefore west of the meridian if this hour angle is less than 12^h. If the time of upper culmination is greater than the observed time the difference is the hour angle measured toward the east (or 24^h — the true hour angle) and the star is therefore east of the meridian if this angle is less than 12^h.

To obtain the azimuth from Tables F and G we use the formula

$$Z' = p' \sin t \sec h. \qquad [106]$$

Table F gives values of $p' \sin t$ for the years 1925, 1930, and 1935, and for every 4^m (or 1°) of hour angle. To multiply by sec h, enter Table G with the value of $p' \sin t$ at the top and the altitude (h) at the side. The number in the table is to be added to $p' \sin t$, to obtain the azimuth Z'.

It is evident that the result might be obtained conveniently by using the time of lower culmination when the hour angle (from U. C.) is nearer to 12^h than to 0^h.

If the altitude of the star has not been measured it may be estimated from the known latitude by inspecting Figs. 65 and 72 and estimating how much *Polaris* is above or below the pole at the time of the observation. This correction to the altitude may also be obtained from Table I in the Ephemeris if the hour angle of the star is known.

As an illustration of the use of these tables we will work out the example given on p. 201.

Second Solution

Observed time	$8^h\ 10^m\ 20^s$ P.M.
Watch correction	+10
Eastern time	$8^h\ 10^m\ 30^s$ P.M.
	15 36
Local time	$8^h\ 26^m\ 06^s$ P.M.
Loc. Civ. T.	20 26 .1
Upper culmin.	9 50 .7
	$10^h\ 35^m.4$
$10^s \times 10^h.6$	1 .8
Hour angle	$10^h\ 37^m.2$

Table V, U. C., May 15,	$10^h\ 02^m.1$
Corr. for 3 days	11 .8
May 18	$9^h\ 50^m.3$
Corr. for 1925	+0 .2
Corr. for long.	+0 .2
Upper culmin.	$9^h\ 50^m.7$

From Table F, $p' \sin t$	=	0° 23′.2
From Table G, corr.	=	7 .7
Azimuth	=	0° 30′.9
Measured horizontal angle	=	36 10 .5
Bearing of mark	=	N. 36° 41′.4 W.

Since the observation just described does not have to be made at any particular time it is usually possible to arrange to sight *Polaris* during twilight when terrestrial objects may still be seen distinctly and no illumination of the field of the telescope is necessary. In order to find the star quickly before dark the telescope should be focussed upon a very distant object and then elevated at an angle equal to the star's altitude as nearly as this can be judged. It will be of assistance in

TABLE F

Values of $p \sin t$ for Polaris (in minutes)

t	1925	1930	1935	*t*	*t*	1925	1930	1935	*t*
h m				*h m*	*h m*				*h m*
0 0	0.0	0 0	0.0	12 00	3 00	46.5	45.4	44 4	9 00
4	1.1	1.1	1 1	11 56	04	47.3	46.2	45 2	56
8	2 3	2 2	2 2	52	08	48.1	47.0	45 9	52
12	3 4	3.4	3 3	48	12	48.8	47 7	46 6	48
0 16	4.6	4.5	4.4	11 44	3 16	49 6	48.4	47 4	8 44
20	5.7	5 6	5 5	40	20	50.4	49 2	48.1	40
24	6 8	6 7	6.6	36	24	51.2	49 9	48 8	36
28	8.0	7 8	7.6	32	28	51.9	50 6	49 5	32
0 32	9 1	8 9	8.7	11 28	3 32	52.6	51 3	50 1	8 28
36	10.3	10.1	9 8	24	36	53.3	52.0	50.8	24
40	11.4	11.2	10 9	20	40	53 9	52.6	51.4	20
44	12.5	12 3	12.0	16	44	54 6	53 3	52 0	16
0 48	13 7	13.4	13.0	11 12	3 48	55 2	53 9	52.6	8 12
52	14.8	14 4	14 1	08	52	55 8	54 5	53 2	08
56	15 9	15.5	15.2	04	56	56 4	55.0	53.8	04
1 00	17.0	16.6	16.2	11 00	4 00	57.0	55 6	54 4	8 00
1 04	18 1	17 7	17.3	10 56	4 04	57.6	56 2	54 9	7 56
08	19.2	18.8	18 3	52	08	58 1	56 7	55 4	52
12	20 3	19 9	19 4	48	12	58 6	57.2	55 9	48
16	21.4	20.9	20 4	44	16	59 2	57.8	56.4	44
1 20	22.5	22 0	21.5	10 40	4 20	59.6	58.2	56.9	7 40
24	23 5	23 0	22.5	36	24	60.1	58.7	57.3	36
28	24 6	24 1	23.5	32	28	60.6	59.2	57.8	32
32	25 7	25 1	24 5	28	32	61 0	59.6	58 2	28
1 36	26.7	26 1	25 5	10 24	4 36	61 4	60 0	58.6	7 24
40	27.8	27 2	26.5	20	40	61.8	60.4	59.0	20
44	28 8	28 2	27.5	16	44	62.2	60.8	59.3	16
48	29.9	29.2	28.5	12	48	62.6	61 1	59.7	12
1 52	30.9	30 2	29.5	10 08	4 52	63.0	61.4	60 0	7 08
56	31 9	31.2	30 4	04	56	63.3	61.8	60.3	04
2 00	32.9	32 1	31.4	10 00	5 00	63.5	62.0	60.6	7 00
04	33.9	33 1	32.3	9 56	04	63 8	62 3	60.9	6 56
2 08	34.9	34.1	33.3	9 52	5 08	64.1	62.6	61.2	6 52
12	35.8	35 0	34 2	48	12	64.3	62.9	61.4	48
16	36.8	35 9	35.1	44	16	64.6	63.1	61.6	44
20	37.7	36 8	36 0	40	20	64.8	63.3	61.8	40
2 24	38.7	37.8	36.9	9 36	5 24	65.0	63.4	62 0	6 36
28	39.6	38.7	37.8	32	28	65.2	63.6	62.2	32
32	40.5	39.6	38.6	28	32	65.3	63.8	62.3	28
36	41.4	40.4	39.5	24	36	65.5	63.9	62.4	24
2 40	42.3	41.3	40.4	9 20	5 40	65.6	64.0	62.5	6 20
44	43 2	42.2	41.2	16	44	65.6	64.1	62.6	16
48	44.0	43.0	42.0	12	48	65.7	64.2	62.7	12
52	44.9	43.8	42.8	08	52	65.8	64.2	62.7	08
2 56	45.7	44.6	43.6	9 04	5 56	65.8	64.2	62.8	6 04
3 00	46.5	45.4	44.4	9 00	6 00	65.8	64.3	62 8	6 00

TABLE G. — CORRECTION FOR ALTITUDE

Alt.	*p* sin t						Proportional Parts								
	10′	20′	30′	40′	50′	60′	1′	2′	3′	4′	5′	6′	7′	8′	9′
15°	0′.4	0′.7	1′.1	1′.4	1′.8	2′.1	0′.0	0′.1	0′.1	0′.1	0′.2	0′.2	0′.2	0′.3	0′.3
18	0.5	1.0	1.5	2.1	2.6	3.1	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5
21	0.7	1.4	2.1	2.8	3.6	4.3	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.6
24	0.9	1.9	2.8	3.8	4.7	5.7	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
27	1.2	2.4	3.7	4.9	6.1	7.3	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
30	1.5	3.1	4.6	6.2	7.7	9.3	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4
31	1.7	3.3	5.0	6.7	8.3	10.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5
32	1.8	3.6	5.4	7.2	9.0	10.8	0.2	0.4	0.5	0.7	0.9	1.1	1.3	1.4	1.6
33	1.9	3.8	5.8	7.7	9.6	11.5	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.5	1.7
34	2.1	4.1	6.2	8.2	10.3	12.4	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.9
35	2.2	4.4	6.6	8.8	11.0	13.2	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.8	2.0
36	2.4	4.7	7.1	9.4	11.8	14.2	0.2	0.5	0.7	0.9	1.2	1.4	1.7	1.9	2.1
37	2.5	5.0	7.6	10.1	12.6	15.1	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3
38	2.7	5.4	8.1	10.8	13.5	16.1	0.3	0.5	0.8	1.1	1.3	1.6	1.9	2.1	2.4
39	2.9	5.7	8.6	11.5	14.3	17.2	0.3	0.6	0.9	1.1	1.4	1.7	2.0	2.3	2.6
40	3.1	6.1	9.2	12.2	15.3	18.3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7
40–30	3.2	6.3	9.5	12.6	15.8	18.9	0.3	0.6	1.0	1.3	1.6	1.9	2.2	2.5	2.8
41	3.3	6.5	9.8	13.0	16.3	19.5	0.3	0.7	1.0	1.3	1.6	2.0	2.3	2.6	2.9
41–30	3.4	6.7	10.1	13.4	16.8	20.1	0.3	0.7	1.0	1.4	1.7	2.0	2.4	2.7	3.0
42	3.5	6.9	10.4	13.8	17.3	20.7	0.3	0.7	1.0	1.4	1.7	2.1	2.4	2.8	3.1
42–30	3.6	7.1	10.7	14.2	17.8	21.3	0.4	0.7	1.1	1.4	1.8	2.2	2.5	2.9	3.2
43	3.7	7.3	11.0	14.7	18.4	22.0	0.4	0.7	1.1	1.5	1.8	2.2	2.6	2.9	3.3
43–30	3.8	7.6	11.4	15.1	18.9	22.7	0.4	0.8	1.1	1.5	1.9	2.3	2.7	3.0	3.4
44	3.9	7.8	11.7	15.6	19.5	23.4	0.4	0.8	1.2	1.6	2.0	2.3	2.7	3.1	3.5
44–30	4.0	8.0	12.1	16.1	20.1	24.1	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
45	4.1	8.3	12.4	16.6	20.7	24.9	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.3	3.7
45–30	4.3	8.5	12.8	17.1	21.3	25.6	0.4	0.8	1.3	1.7	2.1	2.6	3.0	3.4	3.8
46	4.4	8.8	13.2	17.6	22.0	26.4	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	3.9
46–30	4.5	9.0	13.6	18.1	22.6	27.2	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1
47	4.7	9.3	14.0	18.7	23.3	28.0	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.7	4.2
47–30	4.8	9.6	14.4	19.2	24.0	28.8	0.5	1.0	1.4	1.9	2.4	2.9	3.4	3.8	4.3
48	4.9	9.9	14.8	19.8	24.7	29.7	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.4
48–30	5.1	10.2	15.3	20.4	25.5	30.5	0.5	1.0	1.5	2.0	2.6	3.1	3.6	4.1	4.6
49	5.2	10.5	15.7	21.0	26.2	31.5	0.5	1.0	1.6	2.1	2.6	3.1	3.7	4.1	4.7
49–30	5.4	10.8	16.2	21.6	27.0	32.4	0.5	1.0	1.6	2.2	2.7	3.2	3.8	4.3	4.9
50	5.6	11.1	16.7	22.2	27.8	33.3	0.6	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0
50–30	5.7	11.4	17.2	22.9	28.6	34.3	0.6	1.1	1.7	2.3	2.9	3.4	4.0	4.6	5.2
51	5.9	11.8	17.7	23.6	29.5	35.3	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3
51–30	6.1	12.1	18.2	24.3	30.3	36.4	0.6	1.2	1.8	2.4	3.0	3.6	4.3	4.9	5.5
52	6.2	12.5	18.7	25.0	31.2	37.5	0.6	1.2	1.9	2.5	3.1	3.7	4.4	5.0	5.6
52–30	6.4	12.8	19.3	25.7	32.1	38.6	0.6	1.3	1.9	2.6	3.2	3.9	4.5	5.1	5.8
53	6.6	13.2	19.8	26.5	33.1	39.7	0.7	1.3	2.0	2.6	3.3	4.0	4.6	5.3	6.0
53–30	6.8	13.6	20.4	27.2	34.1	40.9	0.7	1.4	2.0	2.7	3.4	4.1	4.8	5.4	6.1
54	7.0	14.0	21.0	28.1	35.1	42.1	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3
54–30	7.2	14.4	21.7	28.9	36.1	43.3	0.7	1.4	2.2	2.9	3.6	4.3	5.0	5.8	6.5
55	7.4	14.9	22.3	29.7	37.2	44.6	0.7	1.5	2.2	3.0	3.7	4.5	5.2	5.9	6.7

finding the horizontal direction of the star if its magnetic bearing is estimated and the telescope turned until the compass needle indicates this bearing. If there is so much light that the star proves difficult to find it is well to move the telescope very slowly right and left. The star may often be seen when it is in apparent motion, while it might remain unnoticed if the telescope were motionless.

115. Azimuth from Horizontal Angle between ***Polaris*** **and** ***β Ursæ Minoris.**** In order to avoid the necessity for determining the time, which is often the chief difficulty with the preceding methods, the azimuth of *Polaris* may be derived from the measured horizontal angle between it and some other star, such as *β Ursæ Minoris*. If the horizontal angle between the two stars is measured and the latitude is known the azimuths of the stars may be calculated.

The observation consists in sighting at a mark with the vernier reading 0°, then sighting *Polaris* and reading the vernier, and finally sighting at *β Ursæ Minoris* and reading the vernier. The difference between the two vernier readings is the difference in azimuth of the stars (neglecting the slight change in the azimuth of *Polaris* during the interval). From an inspection of the table the corresponding azimuth of *Polaris* may be found. This azimuth combined with the vernier reading for *Polaris* is the azimuth of the mark.

The following example is taken from the publication† referred to:

FIELD RECORD

Simultaneous Observations on α and *β Ursæ Minoris* for Azimuth
(α observed first)

Date: Friday, Nov. 9, 1923, P.M. Latitude 37° 57′ 15″ N.

Telescope Direct

Point Sighted	A Vernier	Angle between α and β	Angle between α and mark	Time Interval
Polaris	00° 00′ 00″	13° 48′ 00″		
β Urs. Min.	346 12 00		13° 57′ 00″	24s
Mark	346 03 00			
	Telescope Inverted			
Polaris	00° 00′ 00″	13° 38′ 00″		
β Urs. Min.	346 22 00		13° 56′ 00″	62s
Mark	346 04 00			
		2)27° 26′ 00″	2)27° 53′ 00″	2)86
		13° 43′ 00″	13° 56′ 30″	43s

* This method and the necessary tables were published by C. E. Bardsley, Rolla, Mo., 1924.

† School of Mines and Metallurgy, University of Missouri; Technical Bulletin, Vol. 7, No. 2.

COMPUTATION RECORD

Selected Values for Interpolation from Table I

No.	Latitude 37°		*Latitude 37° 57′ 15″*		Latitude 38°	
	Az. α	Angle between α and β	*Az. α*	*Angle between α and β*	Az. α	Angle between α and β
133	0° 41′.9	13° 46′.1	*0° 42′.38* *(0 41 .51)*	*13°53′.06* *(13 43 .00)*	0° 42′.4	13° 53′.4
134	0 38 .7	13 09 .3	*0 39 .18*	*13 15 .88*	0 39 .2	13 16 .2

Interpolated Azimuth of *Polaris* Table I = 0°41′.51	=	0° 41′ 31″
Correction for declination	=	+05
Correction for right ascension	=	+25
Polaris East of North	=	0° 42′ 01″
Angle *Polaris* to Mark	=	13 56 30
Mark West of North	=	13° 14′ 29″
		180 00 00
Azimuth of Mark from South	=	166° 45′ 31″

The time interval should ordinarily be kept within one minute, so that the observations are as nearly simultaneous as possible. If, however, the order of the pointings is reversed in the second half set the error due to this time interval is nearly eliminated. Double pointings might be made on β *Ursæ Minoris*, one before and one after that on *Polaris*, from which the simultaneous reading might be interpolated.

116. Convergence of the Meridians.

Whenever observations for azimuth are made at two different points of a survey for the purpose of verifying the angular measurements, the convergence of the meridians at the two places will be appreciable if the difference of their longitudes is large. At the equator the two meridians are parallel, regardless of their difference of longitude; at the pole the convergence is the same as the difference in longitude. It may easily be shown that the convergence always equals the difference in longitude multiplied by the sine of the latitude. If the two places are in different latitudes the middle latitude should be used. Table VII was computed according to this formula, the angular convergence in seconds of angle being given for each degree of latitude and for each 1000 feet of distance along the parallel of latitude.

Whenever it is desired to check the measured angles of a traverse between two stations at which azimuths have been observed the latitude differences and departure differences should be computed for each line and the total difference in departure of the two azimuth stations obtained. Then in the column containing the number of thousands of feet in this departure and on a line with the latitude will be found the angular convergence of the meridians. The convergence for numbers not in the table may be found by combining those that are given. For instance, that for 66,500 feet, in lat. 40°, may be found by adding together 10 times the angle for 6000, the angle for 6000, and one-tenth the angle for 5000. The result is 549″.3, the correction to be applied to the second observed azimuth to refer the line to the first meridian.

Example.

Assume that at Station 1 (lat. 40°) the azimuth of the line 1 to 2 is found to be 82° 15′ 20″, and the survey proceeds in a general southwesterly direction to station 21, at which point the azimuth of 21 to 20 is found by observation to be 269° 10′ 00″. The calculation of the survey shows that 21 is 3100 feet south and 15,690 feet west of 1. From the table the convergence (by parts) for 15,690 feet is 2′ 09″.7. Therefore if the direction of 21 − 20 is to be referred to the meridian at 1 this correction should be added to the observed azimuth, giving 269° 12′ 09″.7. The difference between the observed azimuth at 1 and the corrected azimuth at 21 (−180°) is 6° 56′ 49″.7, the total deflection, or change in azimuth, that should be shown by the measured angles if there were no error in the field work.

Problems

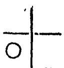
1. Compute the approximate Eastern Standard Time of the eastern elongation of *Polaris* on Sept. 10. The right ascension of the star is $1^h\ 35^m\ 32^s.4$. For the approximate right ascension of the mean sun at any date and the hour angle of *Polaris* at elongation see Arts. 76 and 97.

2. Compute the exact Eastern Standard Time of the eastern elongation of *Polaris* on March 7, 1925. The right ascension of the star is $1^h\ 33^m\ 38^s.00$; the declination is +88° 54′ 19″.18; the latitude of the place is 42° 21′.5 N. The right ascension of the mean sun $+12^h$ on March 7, at 0^h Gr. Civ. T. is $10^h\ 56^m\ 46^s.47$.

3. Compute the azimuth of *Polaris* at elongation from the data of Problem 2.

4. Compute the local time of eastern elongation of α^2 *Centauri* on April 1, 1925, in latitude 20° South. Compute the altitude and azimuth of the star at elongation. The right ascension of the star is $14^h\ 34^m\ 32^s.65$; its declination is −60° 31′ 26″.12. The right ascension of the mean sun $+12^h$ at 0^h G. C. T. is $12^h\ 35^m\ 20^s.27$.

5. Compute the azimuth of the mark from the following observations on the sun, May 25, 1925.

	Ver. A.	Alt.	Watch (E. S. T.)
Mark	0°		
	71° 01′	40° 46′	$3^h\ 13^m\ 33^s$ P.M.
	71 16	40 33	3 14 50
	71 28	40 22	3 15 50
(telescope reversed)			
	72° 21′	40° 42′	$3^h\ 16^m\ 50^s$
	72 32	40 32	3 17 48
	72 41	40 23	3 18 36
Mark	0°		

Lat. = 42° 29′.5 N.; long. = 71° 07′.5 W. I. C. = 0″. Declination at G. C. T. 0^h = +20° 48′ 55″.8; varia. per hour = +27″.60. Equa. of time = +3^m 19^s.48; varia. per hour = −0^s.230.

6. Compute the azimuth of the mark from the following observations on the sun, May 25, 1925.

	Ver. A.	Alt.	Watch (E. S. T.)
Mark	0°		
	76° 25′	35° 28′	3^h 42^m 37^s P.M.
	76 39	35 13	3 43 57
	76 49	35 02	3 44 55
(telescope reversed)			
	77° 37′	35° 25′	3^h 45^m 50^s
	77 49	35 12	3 46 55
	78 00	35 00	3 47 58

Lat. = 42° 29′.5 N.; long. = 71° 07′.5 W. I. C. = 0′. Declination at G. C. T. 0^h = +20° 48′ 55″.8; varia. per hour = +27″.60. Equa. of time = +3^m 19^s.48; varia. per hour = −0^s.230.

7. On March 2, 1925, in latitude 42° 01′ N., longitude 71° 07′ W., the horizontal angle is turned clockwise from a mark to the sun with the following results: Left and lower limbs; hor. circle, 53° 56′; altitude, 40° 31′; watch, 11^h 58^m 50^s. Right and upper limbs; hor. circle, 55° 09′; altitude, 41° 02′; watch 12^h 00^m 20^s. Watch is 3^s fast of E. S. T. The declination of the sun at 0^h G. C. T. = −7° 28′ 40″.8; varia. per hour, +57″.06. Equa. of time, −12^m 28^s.45; varia. per hour +0^s.496. Compute the azimuth of the mark.

8. On March 2, 1925 vernier A is set at 0° and telescope pointed at a mark. Vernier A is then set to read (clockwise) 50° 01′; west edge of sun passed at 11^h 44^m 43^s and east edge at 11^h 46^m 53^s, by watch. Vernier is next set at 53° 18′; west edge of sun passed at 11^h 54^m 44^s and east edge at 11^h 56^m 55^s. Watch is 3^s fast of Eastern Standard Time. The latitude is 42° 01′ N., longitude is 71° 07′ west. The equation of time at 0^h G. C. T. March 2, 1925 is −12^m 28^s.45; varia. per hour, +0^s.496. Compute the azimuth of the mark.

9. The transit is at sta. B; vernier reads 0° when sighting on sta. A. At 8^h 00^m P.M. (E. S. T.) *Polaris* is sighted; alt. = 41° 25′. Horizontal angle 113° 30′ (clockwise) to star. Date, May 8, 1926. Compute the bearing of B. A.

10. Prove that the horizontal angle between the centre of the sun and the right or left limb is s sec h where s is the apparent angular semidiameter and h is the apparent altitude.

11. Prove that the level correction (Art. 106, p. 184) is i tan h where i is the inclination as given by the level.

12. Why could not Equa. [106], p. 183, be used in place of Equa. [31], p. 36, in the method of Art. 112?

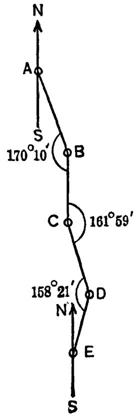
13. If there is an error of 4^s in the assumed value of the watch correction and an azimuth is determined by the method of Art. 112, (near noon) what would be the

relative effect of this error when the sun is on the equator and when it is 23° south? Assume that the latitude is 45° N. (See Table B, p. 99.)

14. At station A on Aug. 5, 1925, about 5^h P.M. a sun observation is made to obtain the bearing of AB. The corrected altitude is 22° 29′, the latitude is 42° 29′ N., the corrected declination is 16° 56′ N., and the horizontal angle from B, clockwise, to the sun is 102° 42′.

After running southward to station E an observation is made on *Polaris*, the watch time being $8^h\ 50^m$ P.M. (E. S. T.). The altitude is 42° 05′ and the horizontal angle from sta. D toward the *left* to *Polaris* is 2° 09′. The longitude is approximately 71° 30′ W.

Find the error in the angles of the survey.



15. May 8, 1925, in lat. 42° 22′ N., long. 71° 06′ W, transit at sta. 1, 0° on sta. 2. Horizontal angle clockwise to sun, L and L limbs, 183° 52′, alt. 45° 21′, watch $3^h\ 35^m\ 00^s$; horizontal angle to sun, U and R limbs, 183° 25′, alt. 44° 37′, watch $3^h\ 36^m\ 15^s$. Index correction −0′.5. Corrected declination, +16° 59′.0 (N).

May 8, 1925, transit at sta. 2, 0° on sta. 1. Horizontal angle (clockwise) to *Polaris* 113° 30′; alt. 41° 25′; watch $8^h\ 00^m$ E. S. T. Compute the bearing of 1 − 2 from each observation.

16. With the transit at station 21, on June 7, 1924, in latitude 42° 29′.5 N., longitude, 71° 07′.5 W., the following sights are taken on the sun, the reference mark being station 22;

	Hor. Circle.	Vert. Arc.	Watch (E. S. T.)
Mark	0°		
Sun, L and L	155° 43′	42° 03′	$3^h\ 14^m\ 20^s$ P.M.
Sun, L and L	155 55	41 52	3 15 27
Sun, U and R	156 50	42 11	3 16 30
Sun, U and R	157 00	42 02	3 17 22

Index correction = +1′. Sun's declination (corrected) = +22° 47′.3.

The deflection angle at sta. 22 is 5° 26′ R; at 23 it is 7° 36′ L; at 24 it is 12° 11′ R.

At sta. 24 an observation is taken on *Polaris*; 0° on sta. 25; first angle, at $7^h\ 02^m\ 05^s$ E. S. T., 252° 44′ (clockwise); second repetition, at $7^h\ 04^m\ 25^s$, 145° 29′; third repetition, at $7^h\ 06^m\ 40^s$, 38° 14′. The altitude is 41° 27′. Station 24 is 2800 feet east of station 21.

Compute the error in the traverse angles between stations 21 and 25, assuming that there is no error in the observations on the sun and *Polaris*.

17. Differentiate the formula

$$\sin Z = \sin p \sec \phi$$

to obtain $\frac{dZ}{d\phi}$ and $\frac{dZ}{dp}$ and from these compute the error in Z produced by an error of 1′ in ϕ or an error of 1″ in δ.

18. Observation on sun Oct. 21, 1925, for azimuth.

	Hor. Angle.	Altitude	Watch (E. S. T.)
Mark	0°		
Sun, L and R	15° 55′	10° 26′	7^h 10^m 30^s A.M.
Sun, L and R	16 06	10 35	7 11 22′
Sun, U and L	15 46	11 18	7 12 30
Sun, U and L	15 57	11 28	7 13 32
Mark	0°		

Index correction to altitude = +1′.

Declination at 0^h, G. C. T., Oct. 21 = −10° 26′ 02″.5; varia. per hour = −53″.78. Latitude = 42° 29′.5; longitude = 71° 07′.5 W. Equa. of time at 0^h = $+15^m$ 11^s.21; varia. per hour, +.417. Compute the azimuth of the mark

CHAPTER XIV

NAUTICAL ASTRONOMY

117. Observations at Sea.

The problems of determining a ship's position at sea and the bearing of a celestial object at any time are based upon exactly the same principles as the surveyor's problems of determining his position on land and the azimuth of a line of a survey. The method of making the observations, however, is different, since the use of instruments requiring a stable support, such as the transit and the artificial horizon, is not practicable at sea. The sextant does not require a stable support and is well adapted to making observations at sea. Since the sextant can be used only to measure the angle between two visible points, it is necessary to measure all altitudes from the sea-horizon and to make the proper correction for dip.

Determination of Latitude at Sea

118. Latitude by Noon Altitude of Sun.

The determination of latitude by measuring the maximum altitude of the sun's lower limb at noon is made in exactly the same way as described in Art. 70. The observation should be begun a little before local apparent noon and altitudes measured in quick succession until the maximum is reached. In measuring an altitude above the sea-horizon the observer should bring the sun's image down until the lower limb appears to be in contact with the horizon line. The sextant should then be tipped by rotating right and left about the axis of the telescope so as to make the sun's image describe an arc; if the lower limb of the sun drops below the horizon at any point, the measured altitude is too great, and the index arm should be moved until the sun's image is just tangent to the horizon when at the lowest point of the arc. (Fig. 77.) This method is illustrated by the following example.

Example.

Observed altitude of sun's lower limb 69° 21′ 30″, bearing north. Index correction = −1′ 10″; height of eye = 18 feet; corrected sun's declination = +9° 00′ 26″ (N). The approximate latitude and longitude are 11° 30′ S, 15° 00′ W. The corrections for dip, refraction, parallax and semidiameter may be taken out separately; in practice the whole correction is taken from Bowditch, American Practical Navigation, Table 46. The latitude is computed by formula [1]. If the sun is bearing N the zenith distance is marked S, and vice versa. The zenith distance and the declination are then added if both are N or both are S, but subtracted if one is N and one is S; the latitude will have the same name (N or S) as the greater of the two.

Observed alt.	69° 21′ 30″		Tab. 46	+11′ 28″
Correction	+10 18		I. C.	+1 10
True altitude	69° 31′ 48″			+10′ 18″
Zenith distance	20 28 12	S		
Declination	9 00 26	N		
Latitude	11° 27′ 46″	S		

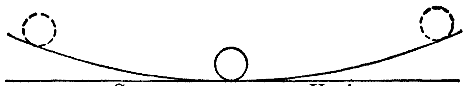


FIG. 77

119. Latitude by Ex-Meridian Altitude.

If for any reason the altitude is not taken precisely at noon the latitude may be found from an altitude taken near noon provided the time is known. If the interval from noon is not over 25 minutes the correction may be taken from Tables 26 and 27, Bowditch. For a longer interval of time formula [30*a*] should be used. When using Table 26, look up the declination at the top of the page and the latitude at the side; the tabular number (*a*) is the variation of the altitude in one minute from meridian passage. To use Table 27, find this number (*a*) at the side and the number of minutes (*t*) before or after noon at the top; the tabular number is the required correction, at^2.

Example 1.

The observed altitude of the sun's lower limb Jan. 1, 1925 is 26° 10′ 30″ bearing south; chronometer time, $15^h\ 30^m\ 10^s$; chronometer 15^s fast. Height of eye 18 feet; I. C. = 0″. The declination is −23° 00′.8; equa. of time is $-3^m\ 39^s.3$. Lat. by dead reckoning, 40° 40′ N; long. by dead reckoning, 50° 02′ 30″.

Chron.	$15^h\ 30^m\ 10^s$
Corr.	-15
G. C. T.	$15^h\ 29^m\ 55^s$
Equa.	$-3\ \ 39.3$
G. A. T.	$15^h\ 26^m\ 15^s.7$
Long.	$3\ \ 20\ \ 10$
L. A. T.	$12^h\ 06^m\ 05^s.7$

Table 46,	$+10'\ 19''$
I. C.	00
Corr.	$+10'\ 19''$

$$\left.\begin{array}{l}\text{Table 26,}\\ \text{Lat. } 41^\circ\\ \text{Dec. } -23^\circ\end{array}\right\} a = 1''.5$$

$$\left.\begin{array}{l}\text{Table 27}\\ a = 1''.5\\ t = 6^m.1\end{array}\right\} at^2 = 54''$$

Obs. alt. =	$26^\circ\ 10'\ 30''$	
Corr. =	$+10\ \ 19$	
True Alt. =	$26^\circ\ 20'\ 49''$	
at^2 =	54	
h =	$26^\circ\ 21'\ 43''$	
Zenith Dist. =	$63\ \ 38\ \ 17$	N
Declination =	$23\ \ 00\ \ 48$	S
Latitude =	$40^\circ\ 37'\ 29''$	N

Example 2.

Observed altitude of sun's lower limb Jan. 20, 1910, = 20° 05′ (south); I. C. = 0′; G. A. T. $1^h\ 35^m\ 28^s$; lat. by D. R. = 49° 20′ N.; long. by D. R. 16° 19′ W.; height of eye, 16 feet; corrected declination, 20° 14′ 27″ S. Find the latitude. If this is solved by Equa. [30*a*] the resulting latitude is 49° 11′ N.

Determination of Longitude at Sea

120. By the Greenwich Time and the Sun's Altitude.

The longitude of the ship may be found by measuring the sun's altitude, calculating the local time, and comparing this with the Greenwich time as shown by the chronometer. The error of the chronometer on Greenwich Civil Time and its rate of gain or loss must be known. The error of the chronometer may be checked at sea by the radio time signals. In solving the triangle for the sun's hour angle the latitude of the ship and the declination of the sun are required, as well as the observed altitude. The latitude used is that obtained from the last preceding observation brought up to the time of the present observation by allowing for the run of the ship during the interval. This is the latitude "by dead reckoning." On account of the uncertainty of this (D. R.) latitude it is important to make the observation when the sun is near the prime vertical. The formula usually employed is a modified form of Equa. [17] (see also p. 259).

The same method may be applied to a star or a planet. In this case the longitude is obtained from the sidereal time. As the observation is ordinarily computed the Gr. Civ. T. is converted into Gr. Sid. T. and the hour angle of the star at Greenwich then computed. The solution of the pole — zenith — star triangle gives directly the hour angle of the star at the ship's meridian. The difference between the two hour angles is the longitude.

Example.

Observed altitude of sun's lower limb on Aug. 8, 1925 (P.M.), = 32° 06′ 30″; chronometer $20^h\,37^m\,40^s$; chronometer correction, $-1^m\,30^s$; index correction, +1′ 00″. Height of eye 12 feet. Lat. by D. R., 44° 47′ N. Sun's declination at 20^h, G. C. T., +16° 07′.9; H. D., −0′.7. Equa. of time at 20^h, $-5^m\,33^s.1$; H. D. $+0^s.3$.

Chron.	$20^h\,37^m\,40^s$	Decl. 20^h	+16° 07′.9
C. C.	−1 30	−0.7 × 0.6	−.4
G. C. T.	$20^h\,36^m\,10^s$	Decl.	+16° 07′.5
Eq.	−5 32.9		
G. A. T.	$20^h\,30^m\,37^s.1$	Eq. t. 20 h	$-5^m\,33^s.1$
		+0.3 × 0.6	+.2

Lat.	44° 47′	log sec 0.14888	Eq. t.	$-5^m\,32^s.9$
Alt.	32 18 30″	log csc 0.01743		
pol. dist.	73 52 30	log cos 9.39909	Obs. alt.	32° 06′ 30″
	2)150 58	log sin 9.83520	I. C.	+1 00
			Tab. 46	+11 00
half sum	75 29	log hav t 9.40060	true alt.	32° 18′ 30″

half sum−alt. 43 10 30	t	$4^h\,00^m\,48^s.7$ (Bowditch, Table 45)
	L. A. T.	16 00 48 .7
	G. A. T.	20 30 37 .1
	Long. =	$4^h\,29^m\,48^s.4$
	=	67° 27′.1 W.

Determination of Azimuth at Sea

121. Azimuth of the Sun at a Given Time.

For determining the error of the compass and for other purposes it is frequently necessary to know the sun's azimuth at an observed instant of time. The azimuth may be computed by any formula giving the value of Z when t, ϕ and δ are known. In practice it is not usually necessary to calculate Z, but its value may be taken from tables. Publication No. 71 of the U. S. Hydrographic Office gives azimuths of the sun for every 1° of latitude and of declination and every 10^m of hour angle. Burdwood's and Davis's tables may be used for the same purpose. For finding the azimuth of a star or any object whose declination is greater than 23° Publication No. 120 may be used.

Example.

As an illustration of the method of using No. 71 suppose that we require the sun's azimuth in latitude 42° 01′ N, declination 22° 47′ S, and hour angle, or local apparent time, $9^h\ 25^m\ 20^s$ A.M. Under lat. 42° N and declination 22° S, hour angle $9^h\ 20^m$ we find the azimuth N 141° 40′ E. The corresponding azimuth for lat. 43° is 141° 50′, that is 10′ greater. The azimuth for lat. 42°, decl. 23° and hour angle $9^h\ 20^m$ is 142° 11′, or 31′ greater. For lat. 42°, decl. 22°, and hour angle $9^h\ 30^m$ the azimuth is 143° 47′, or 2° 07′ greater. The first azimuth, 141° 40′ must be increased by a proportional part of each one of these variations. The desired azimuth is therefore

$$141°\ 40' + \frac{1}{60} \times 10' + \frac{47}{60} \times 31' + \frac{5 \cdot 3}{10} \times 127 = 143°\ 12'.$$

The azimuth is N 143° 12′ E or S 36° 48′ E.

If at the time stated ($9^h\ 25^m 18^s$) the compass bearing of the sun were S 17° E, the total error of the compass would be 19° 48′, the north end of the compass being west of true north. If the "variation of the compass" per chart is 24° **W**, the deviation of the compass is 24° − 19° 48′ = 4° 12′ **E.**

Determination of Position by Means of Sumner Lines

122. Sumner's Method of Determining a Ship's Position.*

If the declination of the sun and the Greenwich Apparent Time are known at any instant, these two coördinates are the latitude and longitude respectively of a point on the earth's surface which is vertically under the sun's centre and which may be called the "sub-solar" point. If an observer were at the sub-solar point he would have the sun in his zenith. If he were located 1° from this point, in any direction, the sun's zenith distance would be 1°; if he were 2° away, the zenith distance would be 2°. It is evident, then, that if an observer measures an altitude of the sun he locates himself on the circumference of a circle whose centre is the sub-solar point and whose radius (in degrees) is the zenith distance of the sun. This circle could be drawn on a globe by first plotting the position of the sub-solar point by means of its coördinates, and

* This method was first described by Captain Sumner in 1843.

then setting a pair of dividers to subtend an arc equal to the zenith distance (by means of a graduated circle on the globe) and describing a circle about the sub-solar point as a centre. The observer is somewhere on this circle because all positions on the earth where the sun has this measured altitude are located on this same circle. This is called a **circle of position**, and any portion of it a **line of position** or a **Sumner line.**

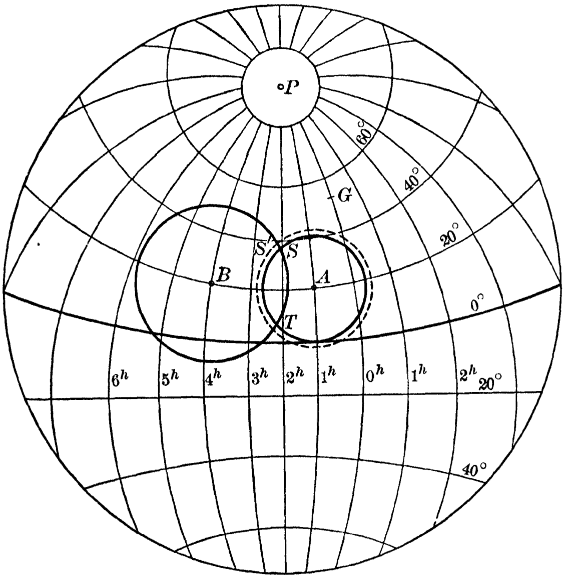


FIG. 78

Suppose that at Greenwich Apparent Time 1^h the sun is observed to have a zenith distance of 20°, the declination being 20° N. The sub-solar point is then at A, Fig. 78, and the observer is somewhere on the circle described about A with a radius 20°. If he waits until the G. A. T. is 4^h and again observes the sun, obtaining 30° for his zenith distance, he locates himself on the circle whose centre is B, the coördinates being 4^h and (say) 20° 02′ N, and the radius of which is 30°. If the ship's position has not changed between the observations it is either at S or at T; in practice there is no difficulty in deciding which is the correct point, on account of their great distance apart. A knowledge of the sun's bearing also shows which portion of the circle contains the point. If, however, the ship has changed its position since the first observation, it is necessary to allow for its "run" as follows. Assuming that the ship has sailed directly away from the sun, say a distance of 60 miles or 1°, then, if the first observation had been made while the ship was in the second position, the point A would be the same, but the radius of the circle would be 21°, locating the ship on the dotted circle. The true position of the ship at the second observation is, therefore, at the intersection S'. If the vessel does not actually sail directly away from or directly toward the sun it is a simple matter to calculate the increase or decrease in radius due to the change in the observer's zenith.

This is in principle Sumner's method of locating a ship. In practice the circles would seldom have as short radii as those in Fig. 78; small circles were chosen only for convenience in illustrating the method. On account of the long radius of the circle of position only a small portion of this circle can be shown on an ordinary chart; in fact, the portion which it is necessary to use is generally so short that the curvature is negligible and the line of position appears on the chart as a straight line. In order to plot a Sumner line on the chart, two latitudes may be assumed between which the actual latitude is supposed to lie; and from these, the known declination, the observed altitude, and the chronometer reading, two longitudes may be computed (Art. 120), one for each of the assumed latitudes. This gives the coördinates of two points on the line of position by means of which it may be plotted on the chart. Another observation may be made a few hours later and the new line plotted in a similar manner. In order to allow for the change in the radius of the circle due to the ship's run between observations, it is only necessary to move the first position line parallel to itself in the direction of the ship's course and a distance equal to the ship's run. In Fig. 79, AB is a line obtained from a 9 A.M observation on the sun and by assuming the latitudes 42° and 43°. A second observation is made at 2 P.M.; between 9^h and 2^h the ship has sailed S 75° W, 67′.* Plotting this run on the chart so as to move any point on AB, such as x, in the direction S 75° W and a distance of 67′, the new position line for the first

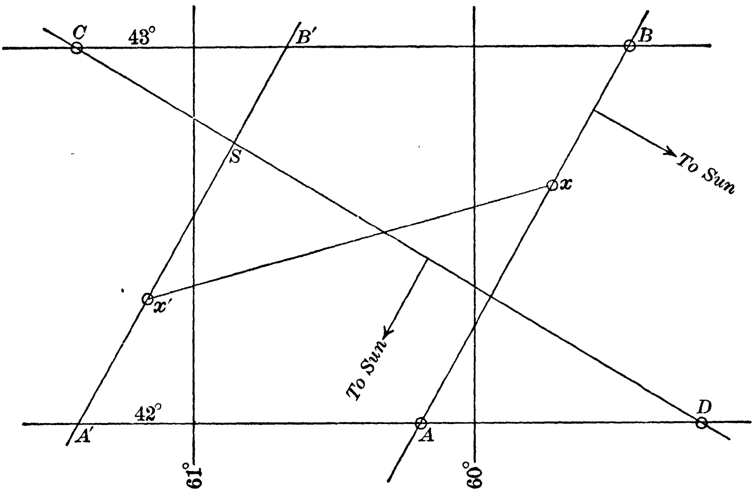


FIG. 79

observation is $A'B'$. The P.M. line of position is located by assuming the same latitudes, 42° and 43°, the result being the line CD. The point of intersection S is the position of the ship at the time of the second observation. Since the bearing of the Sumner line is always at right angles to the bearing of the sun, it is evident that the line might be plotted from one latitude and one longitude instead of two. If the assumed latitude and the calculated longitude are plotted and a line drawn through the point at right angles to the direction of the sun (as shown by

* The nautical mile (6080.20 feet) is assumed to be equal to an arc of 1′ of a Great Circle on anypart of the earth's surface.

the azimuth tables) the result is the Sumner line; the ship is somewhere on this line. The two-point method of laying down the line really gives a point on the chord and the one-point method gives a point on the tangent to the circle of position. The second method is the one usually employed for the plotting the lines of position.

One of the great advantages of this method is that even if only one observation can be taken it may be utilized to locate the ship along a (nearly) straight line; and this is often of great value. For example, if the first position line is found to pass directly through some point of danger, then the navigator knows the bearing of the point, although he does not know his distance from it; but with the single observation he is able to avoid the danger. In case it is a point which it is desired to reach, the true course which the ship should steer is at once known.

123. Position by Computation.

The latitude and longitude of the point of intersection of the position lines may be calculated more precisely than they can be taken from the chart. When the first altitude is taken a latitude is assumed which is near to the true latitude (usually the D. R. lat.), and a longitude is calculated. The azimuth of the sun is taken out of the table for the same lat. and hour angle. From the run of the ship between the first and second observations the change in lat. and change in long. are calculated, usually by the traverse tables. (Tables 1 and 2, Bowditch). These differences are applied as corrections to the assumed lat. and calculated long. This places the ship on the corrected Sumner line (corresponding to $A'B'$, Fig. 79). When the second observation is made this *corrected* latitude is used in computing the new longitude. The result of two such observations is shown in Fig. 80. Point A is the first position; A' is the position of A after correcting for the run of the ship; B is the position obtained from the second observation using the latitude of A'. The distance $A'B$ is therefore the discrepancy in the longitudes, owing to the fact that a wrong latitude has been chosen, and is the base of a triangle the vertex of which is C, the true position of the ship. The base angles A' and B are the same as the azimuths of the sun at the times of the two observations. If we drop a perpendicular from C to $A'B$, forming two right triangles, then

$$Bd = Cd \cot Z_2$$
$$A'd = Cd \cot Z_1$$

or

$$\Delta p_2 = \Delta\phi \cot Z_2$$
$$\Delta p_1 = \Delta\phi \cot Z_1$$

where $\Delta\phi$ is the error in latitude and Δp the difference in departure. In order to express *Bd* and *A'd* as differences in longitude ($\Delta\lambda$) it is necessary to introduce the factor sec ϕ, giving,

$$\left.\begin{aligned}\Delta\lambda_2 &= \Delta\phi \sec\phi \cot Z_2 \\ \Delta\lambda_1 &= \Delta\phi \sec\phi \cot Z_1\end{aligned}\right\} \qquad [107]$$

These coefficients of $\Delta\phi$ are called "longitude factors" and may be taken from Bowditch, Table 47. These formulæ may also be obtained by differentiation.

To find $\Delta\phi$, the correction to the latitude, the distance $A'B = \Delta\lambda_1 + \Delta\lambda_2$ is known, the factors sec ϕ cot Z are calculated or taken from the table, and then $\Delta\phi$ is found by

$$\Delta\phi = \frac{A'B}{\sec\phi \cot Z_1 + \sec\phi \cot Z_2}. \qquad [108]$$

Having found $\Delta\phi$, the corrections $\Delta\lambda_1$, $\Delta\lambda_2$, are computed from [107].

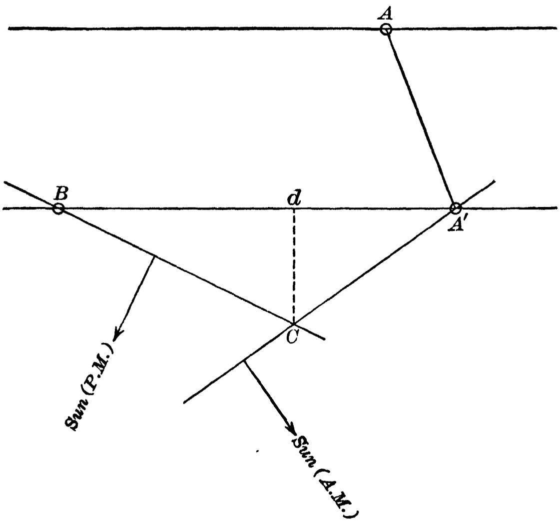


FIG. 80

If one of the observations is taken in the forenoon and one in the afternoon the denominator of [108] is the sum of the factors; if both are on the same side of the meridian the denominator is the difference between the factors. The difference between the two azimuths should not be less than 30° for good results. When the angle is small the position will be more accurately found by computation than by plotting. If two objects can be observed at the same time and their bearings differ by 30° or more the position is found at once, because there is no run of the ship to be allowed for. This observation might be made on the sun and the moon, or on two bright stars or planets. It should be observed that the accuracy of the resulting longitude depends entirely upon the accuracy of the chronometer, just as in the method of Art. 120.

Example.

Location of Ship by Sumner's Method.

On Jan. 4, 1910 at Greenwich Civil Time $13^h\ 12^m\ 33^s$ the observed altitude of the sun is 15° 53′ 30″; index correction = 0″; height of eye, 36 feet; lat. by D. R. 42° 00′ N.

At Gr. Civ. T. $18^h\ 05^m\ 31^s$ the observed altitude of the sun is 17° 33′ 30″; index correction 0″; height of eye, 36 feet. The run between the observations was N 89° W, 45 miles.

First Observation

G. C. T.	$13^h\ 12^m\ 33^s$	Observed alt.	15° 53′ 30″	Declination	−22° 47′ 04″
Equa.	−4 51	Table 46	+7 11	Polar dist.	112° 47′ 04″
G. A. T.	$13^h\ 07^m\ 42^s$	true alt.	16° 00′ 41″	Equa. t.	$-4^m\ 51^s.2$

alt.	16° 00′.7		
lat.	42 00	sec	0.12893
p. d.	112 47.1	csc	0.03528
	2)170° 47′.8		
half sum	85° 23′.9	cos	8.90433
remainder	69 23 .2	sin	9.97127
		log hav. t	9.03981

	t	$2^h\ 34^m\ 40^s$
Sun's Az. S 36° 48′ E	L. A. T. =	9 25 20
Az. factor 1.80	G. A. T. =	13 07 42
	Long.	$3^h\ 42^m\ 22^s$
		= 55° 35′ 30″ W.

Lat.	42° 00′ N	Long.	55° 35′.5 W.
run	0 .8 N	run	1 00 .7 W.
Cor'd. Lat.	42° 00′.8 N	Cor'd. Long.	56° 36′.2 W.

Second Observation

G. C. T.	$18^h\ 05^m\ 31^s$	Observed alt.	17° 33′ 30″	Declination	−22° 45′ 50″
Equa.	−4 56 .8	Table 46	+7 31	p. d.	112 45 50
G. A. T.	$18^h\ 00^m\ 35^s.2$	true alt.	17° 41′ 01″	Equa. t.	$-4^m\ 56^s.8$

alt.	17° 41′.0		
lat.	42 00 .8	sec	0.12902
p. d.	112 45 .8	csc	0.03522
	2)172 27 .6		
half sum	86° 13′.8	cos	8.81790
remainder	68° 32′.8	sin	9.96882
	log hav. t		8.95096

Sun's Az. A 33° 30′ W	t	$2^h\ 19^m\ 07^s$
Az. factor 2.03	L. A. T.	14 19 07
	G. A. T.	18 00 35
	Long.	$3^h\ 41^m\ 28^s$
		= 55° 22′ W.

Cor'd Long. = 56° 36'.2   19'.4 × 1.80 = 34'.9, corr. to 1st Long.
2d Long. = 55 22   19'.4 × 2.03 = 39 .3, corr. to 2d Long.

Diff.   1° 14'.2 = 74'.2

1st Long.	56° 36'.2	2d Long.,	55° 22'
Corr.	34 .9	corr.	39 .3
	56° 01'.3		56° 01'.3

$$\frac{74'.2}{1.80 + 2.03} = 19'.4, \text{ corr. to lat.}$$

∴ Lat. = 42° 20'.2 N   ∴ Long. = 56° 01'.3 W.

124. Method of Marcq St. Hilaire.

Instead of solving the triangle for the angle at the pole, as explained in the preceding article, we may assume a latitude and a longitude, near to the true position and calculate the *altitude* of the observed body. If the assumed position does not happen to lie on the Sumner line the computed altitude will not be the same as the observed altitude. The difference in minutes between the two altitudes is the distance in miles from the assumed position to the Sumner line. If the observed altitude is the *greater* then the assumed point should be moved *toward* the sun by the amount of the altitude difference. A line through this point perpendicular to the sun's direction is the true position line. It is now customary to work up all observation by this method except those taken when the sun is exactly on the meridian or close to the prime vertical. The former may be worked up for latitude as explained in Art. 118. The latter may be advantageously worked as a " time sight " or " chronometer sight " as in Art. 120.

The formula for calculating the altitude is

Hav. zen. dist. = hav. (Lat. ∽ Decl.) + cos Lat. cos Decl. hav. (hour angle)

in which (Lat. ∽ Decl.) is the difference between Lat. and Decl. when they have the same sign, but their sum if they have different signs. The altitude is 90° minus the zenith distance. To illustrate this method the first observation on p. 221 will be worked out. If we assume Lat. = 42° 00' N, and Long. = 56° 30' W, the hour angle (t) is computed as follows:

G. C. T.	$13^h\ 12^m\ 33^s$		
Long.	3 46 00		
L. C. T.	$9^h\ 26^m\ 33^s$		
Equa. t.	−4 51 .2		
L. A. T.	$9^h\ 21^m\ 41^s.8$	log hav.	9.05922
Lat.	42° 00'	log cos	9.87107
Decl.	−22 47 .1	log cos	9.96471
		log	8.89500
		number	.07852
Lat. Decl.	64° 47'.1	nat. hav.	.28699
Zen. dist.	74 23 .7	nat. hav.	.36551
Calc. alt.	15 36. 3		
Obs. alt.	16 00 .7		
Alt. diff.	24 .4 toward sun		
Sun's az. S	36° 48' E.		

From the point in lat. 42° 00′ N, long. 56° 30′ W, draw a line in direction S 36° 48′ E. On this line lay off 24.4 miles (1′ of lat. = 1 naut. mile) toward the sun. Through this last point draw a line in direction S 53° 12′ W. This is the required position line.

125. Altitude and Azimuth Tables — Plotting Charts.

To facilitate the graphical determination of position the Hydrographic Office publishes two sets of tables containing solutions of triangles, and a series of charts designed especially for rapid plotting of lines of position.

The table designated as H. O. 201 gives simultaneous altitudes and azimuths of the sun (or any body whose declination is less than 24°) for each whole degree of latitude and declination and each 10^m of hour angle. Since it is immaterial what point is assumed for the purpose of calculation, provided it is not too far from the true position, interpolation for latitude and hour angle may be avoided by taking the nearest whole degree for the latitude, and a longitude which corresponds to an hour angle that is in the table, that is, some even 10^m. By interpolating for the minutes of the declination the altitude and azimuth are readily taken from the table. The difference between the altitude from the table (calculated h) and the observed altitude is the altitude difference to be laid off from the assumed position, *toward* the sun if the observed altitude is the *greater*. To work out the example of Art. 124 by this table we should enter with lat. = 42°, hour angle $2^h 40^m$ and decl. −23°. Interpolating for the 13′ difference in declination, the corresponding altitude is 15° 24′.7 and the azimuth is N 142°.1 E. The longitude corresponding to an hour angle of $2^h 40^m$ ($9^h 40^m$ L. A. T.) is $3^h 47^m 42^s$ or 56° 55′.5 W. If we plot this point (42° N, 56° 55′.5 W) and then lay off 36′.0 toward the sun (N 142.1 E) we should find a position on the same Sumner line as that obtained in Art. 124. Small variations in the azimuth will occur when the assumed position is changed. The different portions of the Sumner line will not coincide exactly in direction because they are tangents to a circle.

The table designated as H. O. 203 gives the hour angle and the azimuth for every whole degree of latitude, altitude, and declination. In this table the declinations extend to 27°. When using this table we assume an altitude which is a whole degree but not far from the observed altitude. It is necessary to interpolate, as before, for the minutes of declination; this is easily done by the use of the rates of change per minute which are tabulated with the hour angle and the azimuth. In working out the preceding example by the use of H. O. 203 we might use lat. 42° N, alt. 16°, decl. −22° 47′.1 S. The resulting hour angle is $2^h 34^m 50^s.6$ (or L. A. T. $9^h 25^m 09^s.4$) and the azimuth is 143°.1. The longitude corresponding to this hour angle is 55° 38′ W. Plotting this position (42°N., 55° 38′ W) and laying off 0′.7 (the difference between the observed alt. 16°00′.7 and the tabular alt. 16°) toward the sun we obtain another point on the same position line.

The charts designed for plotting these lines show each whole degree of latitude and longitude.* The longitude degrees are 4 inches wide and the latitude degrees

* No. 3000, sheet 7, extends from 35° N to 40° N; sheet 8 extends from 40° N to 45° N; etc.

proportionally greater. On certain meridians and parallels are scales of minutes for each degree. The minutes on the latitude scale serve also as a scale of nautical miles for laying off the altitude differences. A compass circle is provided for laying off the azimuths. A pair of dividers, a parallel ruler and a pencil are all the instruments needed for plotting the lines.

TABLES

TABLE I. MEAN REFRACTION.

Barometer, 29.5 inches. Thermometer, 50° F.

App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.
0° 00′	33′ 51″	10° 00′	5′ 13″	20° 00′	2′ 36″	35° 00′	1′ 21″
30	28 11	30	4 59	30	2 32	36 00	1 18
1 00	23 51	11 00	4 46	21 00	2 28	37 00	1 16
30	20 33	30	4 34	30	2 24	38 00	1 13
2 00	17 55	12 00	4 22	22 00	2 20	40 00	1 08
30	15 49	30	4 12	30	2 17	42 00	1 03
3 00	14 07	13 00	4 02	23 00	2 14	44 00	0 59
30	12 42	30	3 54	30	2 11	46 00	0 55
4 00	11 31	14 00	3 45	24 00	2 08	48 00	0 51
30	10 32	30	3 37	30	2 05	50 00	0 48
5 00	9 40	15 00	3 30	25 00	2 02	52 00	0 45
30	8 56	30	3 23	26 00	1 57	54 00	0 41
6 00	8 19	16 00	3 17	27 00	1 52	56 00	0 38
30	7 45	30	3 10	28 00	1 47	58 00	0 36
7 00	7 15	17 00	3 05	29 00	1 43	60 00	0 33
30	6 49	30	2 59	30 00	1 39	65 00	0 27
8 00	6 26	18 00	2 54	31 00	1 35	70 00	0 21
30	6 05	30	2 49	32 00	1 31	75 00	0 15
9 00	5 46	19 00	2 44	33 00	1 28	80 00	0 10
30	5 29	30	2 40	34 00	1 24	85 00	0 05
10 00	5 13	20 00	2 36	35 00	1 21	90 00	0 00

TABLE II. FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

(Increase in Sun's Right Ascension for Sidereal h. m. s.)

Mean Time = Sidereal Time − C'.

Sid. Hrs.	Corr.	Sid. Min.	Corr.	Sid. Min.	Corr.	Sid. Sec.	Corr.	Sid. Sec.	Corr.
	m s		s		s		s		s
1	0 9.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 8.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 7.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 6.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

TABLE III. FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

(Increase in Sun's Right Ascension for Solar h. m. s.)

Sidereal Time = Mean Time + *C*.

Mean Hrs.	Corr.	Mean Min.	Corr.	Mean Min.	Corr.	Mean Sec.	Corr.	Mean Sec.	Corr.
	m s		s		s		s		s
1	0 9.856	1	0.164	31	5.093	1	0.003	31	0.085
2	0 19.713	2	0.329	32	5.257	2	0.005	32	0.088
3	0 29.569	3	0.493	33	5.421	3	0.008	33	0.090
4	0 39.426	4	0.657	34	5.585	4	0.011	34	0.093
5	0 49.282	5	0.821	35	5.750	5	0.014	35	0.096
6	0 59.139	6	0.986	36	5.914	6	0.016	36	0.099
7	1 8.995	7	1.150	37	6.078	7	0.019	37	0.101
8	1 18.852	8	1.314	38	6.242	8	0.022	38	0.104
9	1 28.708	9	1.478	39	6.407	9	0.025	39	0.107
10	1 38.565	10	1.643	40	6.571	10	0.027	40	0.110
11	1 48.421	11	1.807	41	6.735	11	0.030	41	0.112
12	1 58.278	12	1.971	42	6.900	12	0.033	42	0.115
13	2 8.134	13	2.136	43	7.064	13	0.036	43	0.118
14	2 17.991	14	2.300	44	7.228	14	0.038	44	0.120
15	2 27.847	15	2.464	45	7.392	15	0.041	45	0.123
16	2 37.704	16	2.628	46	7.557	16	0.044	46	0.126
17	2 47.560	17	2.793	47	7.721	17	0.047	47	0.129
18	2 57.417	18	2.957	48	7.885	18	0.049	48	0.131
19	3 7.273	19	3.121	49	8.049	19	0.052	49	0.134
20	3 17.129	20	3.285	50	8.214	20	0.055	50	0.137
21	3 26.986	21	3.450	51	8.378	21	0.057	51	0.140
22	3 36.842	22	3.614	52	8.542	22	0.060	52	0.142
23	3 46.699	23	3.778	53	8.707	23	0.063	53	0.145
24	3 56.555	24	3.943	54	8.871	24	0.066	54	0.148
		25	4.107	55	9.035	25	0.068	55	0.151
		26	4.271	56	9.199	26	0.071	56	0.153
		27	4.435	57	9.364	27	0.074	57	0.156
		28	4.600	58	9.528	28	0.077	58	0.160
		29	4.764	59	9.692	29	0.079	59	0.162
		30	4.928	60	9.856	30	0.082	60	0.164

TABLE IV.

PARALLAX — SEMIDIAMETER — DIP.

(A) Sun's parallax.

Sun's altitude.	Sun's parallax.
0°	9″
10	9
20	8
30	8
40	7
50	6
60	4
70	3
80	2
90	0

(B) Sun's semidiameter.

Date.	Semidiameter.
Jan. 1	16′ 18″
Feb. 1	16 16
Mar. 1	16 10
Apr. 1	16 02
May 1	15 54
June 1	15 48
July 1	15 46
Aug. 1	15 47
Sept. 1	15 53
Oct. 1	16 01
Nov. 1	16 09
Dec. 1	16 15

(C) Dip of the sea horizon.

Height of eye in feet.	Dip of sea horizon.
1	0′ 59″
2	1 23
3	1 42
4	1 58
5	2 11
6	2 24
7	2 36
8	2 46
9	2 56
10	3 06
11	3 15
12	3 24
13	3 32
14	3 40
15	3 48
16	3 55
17	4 02
18	4 09
19	4 16
20	4 23
21	4 29
22	4 36
23	4 42
24	4 48
25	4 54
26	5 00
27	5 06
28	5 11
29	5 17
30	5 22
35	5 48
40	6 12
45	6 36
50	6 56
55	7 16
60	7 35
65	7 54
70	8 12
75	8 29
80	8 46
85	9 02
90	9 18
95	9 33
100	9 48

TABLE V

LOCAL CIVIL TIME OF THE CULMINATIONS AND ELONGATIONS OF POLARIS IN THE YEAR 1922

(Latitude, 40° N.; longitude, 90° or 6^h west of Greenwich)

Civil date 1922	East elongation		Upper culmination		West elongation		Lower culmination	
	h	*m*	*h*	*m*	*h*	*m*	*h*	*m*
January 1	12	54.7	18	50.0	0	49 2	6	51.9
January 15	11	59.3	17	54.7	23	50.0	5	56.6
February 1	10	52.2	16	47.5	22	42.8	4	49.4
February 15	9	56.9	15	52.2	21	47.5	3	54.2
March 1	9	01.7	14	57.0	20	52.3	2	58.9
March 15	8	06.5	14	01.8	19	57.1	2	03.7
April 1	6	59.6	12	54.9	18	50.2	0	56.8
April 15	6	04.5	11	59.8	17	55.1	0 23	01.7 57.8
May 1	5	01.6	10	56.9	16	62.2	22	55.0
May 15	4	06.8	10	02.1	15	57.4	22	00.1
June 1	3	00.1	8	55.4	14	50.7	20	53.5
June 15	2	05.3	8	00.6	13	55.9	19	58.6
July 1	1	02.7	6	58.0	12	53.3	18	56.0
July 15	0	07.9	6	03.2	11	58.5	18	01.3
August 1	22	57.5	4	56.7	10.	52.0	16	54.7
August 15	22	02.7	4	01.9	9	57.2	15	59.9
September 1	20	56.1	2	55.3	8	50.6	14	53.3
September 15	20	01.2	2	00.4	7	55.7	13	58.5
October 1	18	58.4	0	57.6	6	52.9	12	55.7
October 15	18	03.4	0 23	02.8 58.7	5	57.9	12	00.7
November 1	16	56.6	22	51.9	4	51.1	10	53.8
November 15	16	01.5	21	56.8	3	56.0	9	58.7
December 1	14	58.5	20	53.8	2	53.0	8	55.7
December 15	14	03.2	19	58.5	1	57 7	8	00.5

A. *To refer the tabular values to years other than* 1922.

For year 1923	add	$1^m.4$
1924	add	2 .8 up to Mar. 1.
	subtract	1 .1 on and after Mar. 1
1925	add	0 .2
1926	add	1 .5
1927	add	2 .7
1928	add	4 .1 up to Mar. 1
	add	0 .2 on and after Mar. 1
1929	add	1 .6
1930	add	3 .1
1931	add	4 .5
1932	add	5 .9 up to Mar. 1
	add	2 .0 on and after Mar. 1

B. *To refer to any calendar day other than the first and fifteenth of each month* SUBTRACT *the quantities below from the tabular quantity for the* PRECEDING DATE.

Day of month.	Minutes.	No. of days elapsed.	Day of month.	Minutes.	No. of days elapsed.
2 or 16	3.9	1	10 or 24	35.3	9
3 17	7.8	2	11 25	39.2	10
4 18	11.8	3	12 26	43.1	11
5 19	15.7	4	13 27	47.0	12
6 20	19.6	5	14 28	51.0	13
7 21	23.5	6	29	54.9	14
8 22	27.4	7	30	58.8	15
9 23	31.4	8	31	62.7	16

C. *To refer the table to Standard Time:*

(*a*) ADD to the tabular quantities four minutes for every degree of longitude the place is west of the Standard meridian and SUBTRACT when the place is east of the Standard meridian.

(*b*) Times given in the table are A.M., if less than 12^h; those greater than 12^h are P.M.

D. *To refer to any other than the tabular latitude between the limits of* 10° *and* 50° *north:* ADD to the time of west elongation $0^m.10$ for every degree south of 40° and SUBTRACT from the time of west elongation $0^m.16$ for every degree north of 40°. Reverse these operations for correcting times of east elongation.

E. *To refer to any other than the tabular longitude:* ADD $0^m.16$ for each 15° east of the ninetieth meridian and SUBTRACT $0^m.16$ for each 15° west of the ninetieth meridian.

TABLE VI

FOR REDUCING TO ELONGATION OBSERVATIONS MADE NEAR ELONGATION

Time* \ Azimuth at Elon	1° 0′	1° 10′	1° 20	1° 30′	1° 40′	1° 50′	2° 0′	2° 10′	Azimuth at Elon / Time*
m	″	″	″	″	″	″	″	″	m
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
1	0.0	0.0	0.0	+0.1	+0.1	+0.1	+0.1	+0.1	1
2	+0.1	+0.2	+0.2	0.2	0.2	0.3	0.3	0.3	2
3	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.7	3
4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	4
5	+0.9	+1.0	+1.1	+1.3	+1.4	+1.6	+1.7	+1.9	5
6	1.2	1.4	1.6	1.8	2.1	2.3	2.5	2.7	6
7	1.7	2.0	2.2	2.5	2.8	3.1	3.4	3.7	7
8	2.2	2.6	2.9	3.3	3.7	4.0	4.4	4.8	8
9	2.8	3.2	3.7	4.2	4.6	5.1	5.6	6.0	9
10	+3.4	+4.0	+4.6	+5.1	+5.7	+6.3	+6.9	+7.4	10
11	4.1	4.8	5.5	6.2	6.9	7.6	8.3	9.0	11
12	4.9	5.8	6.6	7.4	8.2	9.0	9.9	10.7	12
13	5.8	6.8	7.7	8.7	9.7	10.6	11.6	12.6	13
14	6.7	7.8	9.0	10.1	11.2	12.3	13.4	14.6	14
15	+7.7	+9.0	+10.3	+11.6	+12.8	+14.1	+15.4	+16.7	15
16	8.8	10.2	11.7	13.2	14.6	16.1	17.5	19.0	16
17	9.9	11.5	13.2	14.9	16.5	18.2	19.8	21.5	17
18	11.1	12.9	14.8	16.7	18.5	20.4	22.2	24.1	18
19	12.4	14.4	16.5	18.6	20.6	22.7	24.7	26.8	19

* Sidereal time from elongation.

TABLE VII

CONVERGENCE IN SECONDS FOR EACH 1000 FEET ON THE PARALLEL

Lat.	Distance (East or West)								
φ	1000	2000	3000	4000	5000	6000	7000	8000	9000
°	″	″	″	″	″	″	″	″	″
20	3.51	7.01	10.51	14.02	17.52	21.03	24.53	28.04	31.54
21	3.78	7.57	11.35	15.13	18.91	22.69	26.48	30.26	34.04
22	3.98	7.96	11.94	15.92	19.90	23.88	27.86	31.84	35.83
23	4.18	8.36	12.55	16.73	20.91	25.09	29.27	33.46	37.64
24	4.39	8.77	13.16	17.54	21.93	26.32	30.70	35.09	39.47
25	4.59	9.19	13.78	18.37	22.97	27.56	32 15	36.75	41.34
26	4.80	9.61	14.42	19.22	24.02	28.83	33.63	38.44	43.24
27	5.02	10.04	15.06	20.08	25.10	30.11	35.13	40.15	45.17
28	5.24	10.48	15.71	20.95	26.19	31.42	36.66	41.90	47.13
29	5.46	10.92	16.38	21.84	27.30	32.76	38.22	43.68	49.14
30	5.69	11.37	17.06	22.74	28.43	34.12	39.80	45.49	51.17
31	5.92	11.83	17.75	23.67	29.59	35.51	41.42	47.34	53.26
32	6.16	12.31	18.46	24.62	30.77	36.92	43.08	49.23	55.38
33	6.39	12.78	19.17	25.57	31.96	38.36	44.75	51.15	57.54
34	6.64	13.29	19.92	26.57	33.21	39.85	46.49	53.13	59.77
35	6.89	13.79	20.68	27.58	34.47	41.37	48.26	55.15	62.05
36	7.15	14.31	21.46	28.61	35.77	42.92	50.07	57.22	64.38
37	7.42	14.84	22.26	29.67	37.09	44.51	51.93	59.35	66.77
38	7.69	15.38	23.08	30.77	38.46	46.15	53.84	61.53	69.22
39	7.97	15.95	23.92	31.89	39.86	47.83	55.80	63.77	71.74
40	8.26	16.52	24.78	33.04	41.30	49.56	57.82	66.08	74.34
41	8.55	17.11	25.67	34.22	42.78	51.33	59.89	68.45	77.00
42	8.86	17.72	26.58	35.45	44.31	53.17	62.03	70.89	79.76
43	9.18	18.36	27.53	36.71	45.89	55.06	64.24	73.42	82.60
44	9.50	19.01	28.51	38.01	47.52	57.02	66.52	76.02	85.53
45	9.84	19.68	29.52	39.36	49.20	59.04	68.88	78.72	88.56
46	10.19	20.38	30.57	40.76	50.95	61.13	71.32	81.51	91.70
47	10.55	21.10	31.65	42.20	52.76	63.31	73.86	84.41	94.96
48	10.93	21.85	32.78	43.71	54.63	65.56	76.49	87.41	98.34
49	11.32	22.63	33.95	45.27	56.59	67.90	79.22	90.54	101.85
50	11.72	23.45	35.17	46.89	58.62	70.34	82.06	93.78	105.51

TABLE VIII

Correction for Parallax and Refraction to be Subtracted from Observed Altitude of the Sun

Apparent altitude	Temperature, centigrade −10°	0°	+10°	+20°	+30°	+40°	+50°	Apparent altitude
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
0	36 36	35 15	34 00	32 50	31 45	30 45	29 50	0
1	26 10	25 12	24 18	23 28	22 42	22 00	21 20	1
2	19 35	18 51	18 11	17 34	16 59	16 26	15 57	2
3	15 20	14 46	14 15	13 46	13 18	12 53	12 30	3
4	12 31	12 03	11 37	11 13	10 50	10 30	10 11	4
5	10 29	10 05	9 44	9 24	9 05	8 48	8 32	5
6	8 59	8 38	8 20	8 03	7 47	7 32	7 18	6
7	7 49	7 31	7 15	7 00	6 46	6 33	6 21	7
8	6 55	6 39	6 25	6 12	5 59	5 48	5 37	8
9	6 11	5 57	5 44	5 32	5 21	5 11	5 01	9
10	5 34	5 22	5 10	4 59	4 49	4 39	4 30	10
11	5 04	4 52	4 42	4 32	4 23	4 14	4 06	11
12	4 39	4 29	4 19	4 10	4 01	3 53	3 46	12
13	4 17	4 07	3 58	3 50	3 42	3 35	3 28	13
14	3 58	3 49	3 41	3 33	3 26	3 19	3 13	14
15	3 42	3 34	3 26	3 19	3 12	3 06	3 00	15
16	3 27	3 19	3 12	3 05	2 59	2 53	2 47	16
17	3 14	3 07	3 00	2 54	2 48	2 42	2 37	17
18	3 02	2 55	2 49	2 43	2 37	2 32	2 27	18
19	2 52	2 45	2 39	2 33	2 28	2 23	2 19	19
20	2 42	2 36	2 30	2 25	2 20	2 15	2 11	20
21	2 33	2 27	2 22	2 17	2 12	2 08	2 04	21
22	2 26	2 20	2 15	2 10	2 06	2 02	1 58	22
23	2 18	2 13	2 08	2 03	1 59	1 55	1 51	23
24	2 12	2 07	2 02	1 58	1 54	1 50	1 46	24
25	2 05	2 00	1 56	1 52	1 48	1 44	1 41	25
26	2 00	1 55	1 51	1 47	1 43	1 39	1 36	26
27	1 55	1 50	1 46	1 42	1 38	1 35	1 32	27
28	1 49	1 45	1 41	1 37	1 34	1 31	1 28	28
29	1 45	1 41	1 37	1 33	1 30	1 27	1 24	29
30	1 41	1 37	1 33	1 30	1 26	1 23	1 21	30
32	1 33	1 29	1 26	1 23	1 19	1 17	1 15	32
34	1 26	1 22	1 19	1 16	1 13	1 11	1 09	34
36	1 19	1 16	1 13	1 10	1 08	1 05	1 03	36
38	1 13	1 10	1 07	1 04	1 02	1 00	0 58	38
40	1 08	1 05	1 02	1 00	0 58	0 56	0 54	40
42	1 03	1 00	0 58	0 56	0 54	0 52	0 50	42
44	0 59	0 56	0 54	0 52	0 50	0 48	0 47	44
46	0 54	0 52	0 50	0 48	0 46	0 45	0 43	46
48	0 51	0 49	0 47	0 45	0 44	0 42	0 41	48
50	0 47	0 45	0 43	0 41	0 40	0 38	0 37	50
55	0 39	0 37	0 36	0 35	0 33	0 32	0 31	55
60	0 32	0 30	0 29	0 28	0 27	0 26	0 25	60
65	0 25	0 24	0 23	0 22	0 21	0 20	0 20	65
70	0 20	0 19	0 18	0 17	0 17	0 16	0 15	70
75	0 14	0 14	0 13	0 12	0 12	0 12	0 11	75
80	0 10	0 09	0 09	0 09	0 08	0 08	0 08	80
85	0 04	0 04	0 04	0 04	0 04	0 04	0 03	85
90	0 00	0 00	0 00	0 00	0 00	0 00	0 00	90

TABLE IX

Latitude from Circum-meridian Altitudes of the Sun

$[A = \cos \delta \cos \phi \operatorname{cosec} \zeta]$

ϕ \ ζ	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	ζ / ϕ
0	9.51	8.14	7.12	6.31	5.67	5.15	4.70	4.33	4.01	3.74	3.49	3.27	3.08	0
1	9.53	8.16	7.13	6.33	5.69	5.16	4.72	4.35	4.03	3.75	3.50	3.29	3.09	1
2	9.54	8.17	7.14	6.34	5.70	5.17	4.73	4.36	4.04	3.76	3.52	3.30	3.11	2
3	9.54	8.17	7.15	6.35	5.71	5.18	4.74	4.37	4.05	3.77	3.53	3.31	3.12	3
4	9.54	8.17	7.15	6.35	5.71	5.19	4.75	4.38	4.06	3.78	3.54	3.32	3.13	4
5	9.53	8.17	7.15	6.35	5.71	5.19	4.76	4.39	4.07	3.79	3.55	3.33	3.14	5
6	9.51	8.16	7.14	6.35	5.71	5.19	4.76	4.39	4.07	3.80	3.55	3.34	3.15	6
7	9.49	8.14	7.13	6.34	5.71	5.19	4.76	4.39	4.07	3.80	3.56	3.34	3.15	7
8	9.47	8.12	7.12	6.33	5.70	5.18	4.75	4.39	4.07	3.80	3.56	3.35	3.16	8
9	9.44	8.10	7.10	6.31	5.69	5.17	4.74	4.38	4.07	3.80	3.56	3.35	3.16	9
10	9.41	8.07	7.07	6.29	5.67	5.16	4.73	4.37	4.06	3.79	3.55	3.34	3.16	10
11	9.36	8.04	7.04	6.27	5.65	5.14	4.72	4.36	4.05	3.78	3.55	3.34	3.15	11
12	9.31	8.00	7.01	6.24	5.63	5.13	4.70	4.35	4.04	3.77	3.54	3.33	3.15	12
13	9.25	7.95	6.97	6.21	5.60	5.10	4.68	4.33	4.03	3.76	3.53	3.32	3.14	13
14	9.19	7.90	6.93	6.18	5.57	5.08	4.66	4.31	4.01	3.75	3.52	3.31	3.13	14
15	9.13	7.85	6.89	6.14	5.54	5.05	4.64	4.29	3.99	3.73	3.50	3.30	3.12	15
16	9.06	7.79	6.84	6.10	5.51	5.02	4.61	4.27	3.97	3.71	3.49	3.29	3.10	16
17	8.98	7.73	6.79	6.05	5.47	4.98	4.58	4.24	3.95	3.69	3.47	3.27	3.09	17
18	8.90	7.66	6.73	6.00	5.42	4.95	4.55	4.21	3.92	3.67	3.45	3.25	3.08	18
19	8.81	7.59	6.67	5.95	5.38	4.91	4.51	4.18	3.89	3.64	3.43	3.23	3.06	19
20	8.72	7.51	6.60	5.90	5.33	4.86	4.47	4.15	3.86	3.62	3.40	3.21	3.04	20
21	8.63	7.43	6.54	5.84	5.28	4.82	4.43	4.11	3.83	3.59	3.37	3.19	3.02	21
22	8.53	7.35	6.46	5.78	5.22	4.77	4.39	4.07	3.80	3.56	3.34	3.16	2.99	22
23	8.42	7.26	6.39	5.71	5.16	4.72	4.35	4.03	3.76	3.52	3.31	3.13	2.97	23
24	8.31	7.17	6.31	5.64	5.10	4.66	4.30	3.99	3.72	3.49	3.28	3.10	2.94	24
25	8.20	7.07	6.23	5.57	5.04	4.61	4.25	3.94	3.68	3.45	3.25	3.07	2.91	25
26	8.08	6.97	6.14	5.49	4.98	4.55	4.19	3.89	3.63	3.41	3.21	3.04	2.88	26
27	7.96	6.87	6.05	5.42	4.91	4.49	4.14	3.84	3.59	3.37	3.17	3.00	2.85	27
28	7.83	6.76	5.96	5.34	4.84	4.43	4.08	3.79	3.54	3.32	3.13	2.96	2.81	28
29	7.71	6.65	5.87	5.25	4.76	4.36	4.02	3.74	3.49	3.28	3.09	2.93	2.78	29
30		6.54	5.77	5.17	4.69	4.29	3.96	3.68	3.44	3.23	3.05	2.89	2.74	30
31			5.67	5.08	4.61	4.22	3.90	3.62	3.39	3.18	3.00	2.85	2.70	31
32				4.99	4.53	4.15	3.83	3.56	3.33	3.13	2.96	2.80	2.66	32
33					4.45	4.07	3.77	3.50	3.28	3.08	2.91	2.76	2.62	33
34						4.00	3.70	3.44	3.22	3.03	2.86	2.71	2.58	34
35							3.63	3.38	3.16	2.97	2.81	2.66	2.54	35
36								3.31	3.10	2.92	2.76	2.62	2.49	36
37									3.04	2.86	2.70	2.57	2.44	37
38										2.80	2.65	2.52	2.40	38
39											2.60	2.46	2.35	39
40												2.41	2.30	40
41													2.25	41

TABLE IX (*Continued*)

Latitude from Circum-meridian Altitudes of the Sun

φ \ ζ	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	ζ / φ
0°	2.90	2.75	2.61	2.48	2.36									0°
1	2.92	2.76	2.62	2.49	2.37	2.26								1
2	2.94	2.78	2.64	2.51	2.39	2.28	2.18							2
3	2.95	2.79	2.65	2.52	2.40	2.29	2.19	2.10						3
4	2.96	2.80	2.66	2.53	2.41	2.30	2.20	2.11	2.02					4
5	2.97	2.81	2.67	2.54	2.42	2.32	2.21	2.12	2.03	1.95				5
6	2.98	2.82	2.68	2.55	2.43	2.33	2.22	2.13	2.04	1.96	1.89			6
7	2.98	2.82	2.69	2.56	2.44	2.33	2.23	2.14	2.05	1.97	1.90	1.83		7
8	2.99	2.83	2.69	2.56	2.45	2.34	2.24	2.15	2.06	1.98	1.91	1.84	1.77	8
9	2.99	2.83	2.70	2.57	2.45	2.35	2.25	2.15	2.06	1.99	1.92	1.84	1.78	9
10	2.99	2.84	2.70	2.57	2.46	2.35	2.25	2.16	2.07	2.00	1.92	1.85	1.79	10
11	2.99	2.83	2.70	2.57	2.46	2.35	2.25	2.16	2.08	2.00	1.93	1.86	1.79	11
12	2.98	2.83	2.70	2.57	2.46	2.35	2.26	2.17	2.08	2.00	1.93	1.86	1.80	12
13	2.98	2.82	2.69	2.57	2.46	2.35	2.26	2.17	2.08	2.00	1.93	1.86	1.80	13
14	2.97	2.81	2.69	2.56	2.45	2.35	2.25	2.17	2.08	2.01	1.93	1.87	1.80	14
15	2.96	2.81	2.68	2.56	2.45	2.35	2.25	2.16	2.08	2.00	1.93	1.87	1.80	15
16	2.95	2.80	2.67	2.55	2.44	2.34	2.25	2.16	2.08	2.00	1.93	1.87	1.80	16
17	2.94	2.79	2.66	2.54	2.43	2.33	2.24	2.15	2.07	2.00	1.93	1.86	1.80	17
18	2.92	2.78	2.65	2.53	2.42	2.33	2.23	2.15	2.06	2.00	1.93	1.86	1.80	18
19	2.90	2.76	2.64	2.52	2.41	2.32	2.22	2.14	2.06	1.99	1.92	1.85	1.80	19
20	2.89	2.75	2.62	2.51	2.40	2.30	2.21	2.13	2.05	1.98	1.92	1.85	1.79	20
21	2.87	2.73	2.61	2.49	2.39	2.29	2.20	2.12	2.04	1.97	1.91	1.84	1.79	21
22	2.84	2.71	2.59	2.48	2.37	2.28	2.19	2.11	2.03	1.96	1.90	1.84	1.78	22
23	2.82	2.69	2.57	2.46	2.36	2.26	2.18	2.10	2.02	1.95	1.89	1.83	1.77	23
24	2.80	2.66	2.55	2.44	2.34	2.25	2.16	2.08	2.01	1.94	1.88	1.82	1.76	24
25	2.77	2.64	2.52	2.42	2.32	2.23	2.14	2.07	2.00	1.93	1.87	1.81	1.75	25
26	2.74	2.61	2.50	2.39	2.30	2.21	2.13	2.05	1.98	1.91	1.85	1.79	1.74	26
27	2.71	2.59	2.47	2.37	2.27	2.19	2.11	2.03	1.96	1.90	1.84	1.78	1.73	27
28	2.68	2.56	2.44	2.34	2.25	2.17	2.09	2.01	1.95	1.88	1.82	1.77	1.71	28
29	2.65	2.53	2.42	2.32	2.23	2.14	2.06	1.99	1.93	1.86	1.80	1.75	1.70	29
30	2.61	2.49	2.39	2.29	2.20	2.12	2.04	1.97	1.91	1.84	1.79	1.73	1.68	30
31	2.58	2.46	2.36	2.26	2.17	2.09	2.02	1.95	1.88	1.82	1.77	1.71	1.66	31
32	2.54	2.43	2.32	2.23	2.14	2.06	1.99	1.92	1.86	1.80	1.75	1.69	1.65	32
33	2.50	2.39	2.29	2.20	2.11	2.04	1.97	1.90	1.84	1.78	1.73	1.67	1.63	33
34	2.46	2.35	2.25	2.17	2.08	2.01	1.94	1.87	1.81	1.76	1.70	1.65	1.61	34
35	2.42	2.31	2.22	2.13	2.05	1.98	1.91	1.85	1.79	1.73	1.68	1.63	1.59	35
36	2.38	2.27	2.18	2.10	2.02	1.95	1.88	1.82	1.76	1.71	1.66	1.61	1.56	36
37	2.33	2.23	2.14	2.06	1.98	1.91	1.85	1.79	1.73	1.68	1.63	1.58	1.54	37
38	2.29	2.19	2.10	2.02	1.95	1.88	1.82	1.76	1.70	1.65	1.60	1.56	1.52	38
39	2.24	2.15	2.06	1.98	1.91	1.85	1.78	1.73	1.67	1.63	1.58	1.54	1.49	39
40	2.20	2.10	2.02	1.94	1.88	1.81	1.75	1.70	1.64	1.60	1.55	1.51	1.47	40
42	2.10	2.01	1.94	1.86	1.80	1.74	1.68	1.63	1.58	1.54	1.49	1.45	1.42	42
44			1.85	1.78	1.72	1.66	1.61	1.56	1.51	1.47	1.43	1.40	1.36	44
46					1.64	1.58	1.53	1.49	1.45	1.41	1.37	1.34	1.30	46
48							1.46	1.41	1.38	1.34	1.31	1.27	1.24	48
50									1.30	1.27	1.24	1.21	1.18	50

TABLE IX (*Continued*)

Latitude from Circum-meridian Altitudes of the Sun

ζ / φ	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	ζ / φ
9°	1.72													9°
10	1.72	1.66												10
11	1.73	1.67	1.62											11
12	1.73	1.68	1.62	1.57										12
13	1.74	1.68	1.63	1.58	1.53									13
14	1.74	1.68	1.63	1.58	1.53	1.48								14
15	1.74	1.69	1.63	1.58	1.53	1.49	1.44							15
16	1.74	1.69	1.63	1.58	1.53	1.49	1.45	1.41						16
17	1.74	1.69	1.64	1.59	1.53	1.49	1.45	1.41	1.37					17
18	1.74	1.69	1.63	1.59	1.53	1.49	1.45	1.41	1.37	1.33				18
19	1.74	1.68	1.63	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30			19
20	1.73	1.68	1.63	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27		20
21	1.73	1.68	1.63	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.24	21
22	1.72	1.67	1.62	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.24	22
23	1.72	1.66	1.62	1.57	1.53	1.48	1.44	1.41	1.37	1.33	1.30	1.27	1.24	23
24	1.71	1.66	1.61	1.57	1.52	1.48	1.44	1.41	1.37	1.33	1.30	1.27	1.24	24
25	1.70	1.65	1.60	1.56	1.51	1.47	1.43	1.40	1.36	1.33	1.30	1.26	1.23	25
26	1.69	1.64	1.59	1.55	1.51	1.47	1.43	1.39	1.36	1.32	1.29	1.26	1.23	26
27	1.68	1.63	1.58	1.54	1.50	1.46	1.42	1.38	1.35	1.32	1.29	1.26	1.23	27
28	1.66	1.62	1.57	1.53	1.49	1.45	1.41	1.38	1.34	1.32	1.28	1.25	1.22	28
29	1.65	1.60	1.56	1.52	1.48	1.44	1.40	1.37	1.34	1.31	1.27	1.24	1.22	29
30	1.63	1.59	1.55	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.27	1.24	1.21	30
31	1.62	1.57	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.29	1.26	1.23	1.20	31
32	1.60	1.56	1.52	1.48	1.44	1.40	1.37	1.34	1.31	1.28	1.25	1.22	1.19	32
33	1.58	1.54	1.50	1.46	1.42	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.18	33
34	1.56	1.52	1.48	1.45	1.41	1.38	1.34	1.31	1.28	1.25	1.23	1.20	1.18	34
35	1.54	1.50	1.47	1.43	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.19	1.16	35
36	1.52	1.48	1.45	1.41	1.38	1.34	1.31	1.28	1.26	1.23	1.20	1.18	1.15	36
37	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.19	1.17	1.14	37
38	1.48	1.44	1.41	1.37	1.34	1.31	1.28	1.25	1.23	1.20	1.17	1.15	1.13	38
39	1.46	1.42	1.38	1.35	1.32	1.29	1.26	1.24	1.21	1.18	1.16	1.14	1.11	39
40	1.43	1.40	1.36	1.33	1.30	1.27	1.24	1.22	1.19	1.17	1.14	1.12	1.10	40
42	1.38	1.35	1.32	1.29	1.26	1.23	1.20	1.18	1.16	1.13	1.11	1.09	1.07	42
44	1.33	1.30	1.27	1.24	1.21	1.19	1.16	1.14	1.12	1.09	1.07	1.05	1.04	44
46	1.27	1.24	1.22	1.19	1.16	1.14	1.12	1.10	1.07	1.05	1.04	1.02	1.00	46
48	1.21	1.19	1.16	1.14	1.11	1.09	1.07	1.05	1.03	1.01	.99	.98	.96	48
50	1.15	1.13	1.10	1.08	1.06	1.04	1.02	1.00	.98	.97	.95	.94	.92	50
55	1.00	.98	.96	.94	.92	.91	.89	.88	.86	.85	.84	.82	.81	55
60						.76	.75	.74	.73	.72	.71	.70	.69	60
65											.58	.57	.57	65

TABLE IX (*Continued*)

Latitude from Circum-meridian Altitudes of the Sun

ζ / φ	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	ζ / φ
22°	1.21													22°
23	1.21	1.18												23
24	1.21	1.18	1.15											24
25	1.20	1.18	1.15	1.12										25
26	1.20	1.17	1.15	1 12	1.10									26
27	1.20	1 17	1.14	1.12	1.10	1.07								27
28	1.19	1 17	1.14	1 12	1 09	1.07	1 05							28
29	1.19	1.16	1.14	1.11	1.09	1 07	1 04	1.02						29
30	1 18	1 16	1 13	1 11	1 08	1.06	1 04	1.02	1.00					30
31	1.18	1 15	1.13	1.10	1.08	1.06	1.04	1.02	1.00	0.98				31
32	1.17	1 14	1.12	1 10	1 07	1.05	1.03	1.01	.99	.97	0.95			32
33	1 16	1.14	1.11	1 09	1 07	1 05	1.03	1.01	.99	.97	.95	0.93		33
34	1.15	1.13	1.10	1 08	1.06	1.04	1.02	1.00	.98	.96	.94	.93	0.91	34
35	1.14	1 12	1 10	1 07	1 05	1.03	1.01	.99	.98	.96	.94	92	.91	35
36	1.13	1 11	1.09	1 07	1.05	1.03	1 01	.99	97	.95	93	.92	.90	36
37	1 12	1.10	1 08	1.06	1.04	1.02	1.00	.98	.96	.94	.93	.91	.90	37
38	1.11	1 08	1.06	1.04	1.02	1.01	.99	.97	.95	.94	.92	.90	.89	38
39	1.09	1 07	1.05	1 03	1.01	1 00	.98	.96	.94	.93	.91	.90	.88	39
40	1 08	1 06	1.04	1 02	1 00	.98	.97	.95	.93	.92	.90	.89	.87	40
42	1.05	1 03	1.01	.99	98	96	.94	.93	.91	.90	.88	.87	.86	42
44	1 02	1 00	.98	.97	95	.93	.92	.90	.89	.88	.86	.85	.84	44
46	.98	.97	.95	.93	.92	.90	.89	.88	.86	.85	.84	.82	.81	46
48	.94	93	.92	.90	.89	.87	.86	.85	.83	.82	.81	.80	.79	48
50	.91	.89	.88	.86	.85	.84	.83	.82	.80	.79	.78	.77	.76	50
55	.80	.79	.78	.77	.76	.75	.74	.73	.72	.71	.70	.69	.68	55
60	.68	.67	.67	.66	65	.64	.64	.63	.62	.61	.61	.60	.60	60
65	.56	.56	.55	.54	.54	.53	.53	.52	.52	.51	.51	.50	.50	65
70	.. ...	....	43	.43	42	.42	.42	.41	.41	.41	.40	.40	.40	70

ζ / φ	58°	59°	60°	61°	62°	63°	65°	67°	69°	71°	73°	78°	83°	ζ / φ
35	0.89													35
36	.88	0.87												36
37	.88	.86	0.85											37
38	.87	.86	.84	0.83										38
39	.87	.85	.84	.82	0.81									39
40	.86	.84	.83	.82	.80	0.79								40
42	.84	.83	.82	.80	.79	.78	0 75							42
44	.82	.81	.80	.79	.78	.76	.74	0.72						44
46	.80	.79	.78	.77	.76	.75	.72	.70	0.69					46
48	.78	.77	.76	.75	.74	.73	.71	.69	.67	0.65				48
50	.75	.74	.73	.72	.71	.70	.69	.67	.65	.63	0.62			50
55	.68	.67	.66	.65	.64	.64	.62	.61	.60	.58	0.57	0.54		55
60	.59	.58	.58	.57	.57	.56	.55	.54	.53	.52	.51	.49	0.46	60
65	.49	.49	.48	.48	.48	.47	.47	.46	.45	.44	.43	.42	.40	65
70	.39	.39	.39	.39	.38	.38	.38	.37	.37	.36	.36	.35	.34	70

TABLE X

Values of $m = \frac{2 \sin^2 \frac{1}{2} \tau}{\sin 1''}$

τ	0m	1m	2m	3m	4m	5m	6m	7m	8m
s	"	"	"	"	"	"	"	"	"
0	0.00	1 96	7.85	17.67	31 42	49.09	70 68	96 20	125.65
1	0 00	2 03	7 98	17.87	31 68	49.41	71.07	96 66	126.17
2	0 00	2.10	8 12	18 07	31 94	49 74	71.47	97.12	126.70
3	0.00	2 16	8.25	18 27	32 20	50 07	71 86	97 58	127.22
4	0.01	2 23	8.39	18 47	32 47	50.40	72.26	98.04	127 75
5	0.01	2 31	8 52	18 67	32 74	50.73	72 66	98 50	128.28
6	0.02	2 38	8.66	18.87	33.01	51.07	73 06	98.97	128.81
7	0.02	2.45	8 80	19 07	33 27	51.40	73.46	99 43	129.34
8	0 03	2 52	8 94	19 28	33 54	51 74	73.86	99 90	129.87
9	0 04	2 60	9.08	19 48	33 81	52 07	74 26	100 37	130.40
10	0 05	2 67	9.22	19 69	34.09	52 41	74.66	100.84	130 94
11	0 06	2 75	9 36	19 90	34 36	52 75	75 06	101 31	131 47
12	0.08	2 83	9 50	20 11	34 64	53 09	75 47	101 78	132.01
13	0 09	2 91	9 64	20.32	34 91	53 43	75 88	102.25	132 55
14	0.11	2 99	9.79	20 53	35 19	53 77	76 29	102 72	133 09
15	0 12	3 07	9 94	20 74	35.46	54.11	76 69	103 20	133 63
16	0.14	3 15	10 09	20 95	35 74	54 46	77 10	103 67	134 17
17	0 16	3 23	10 24	21 16	36 02	54 80	77.51	104.15	134 71
18	0 18	3 32	10 39	21 38	36 30	55.15	77 93	104 63	135 25
19	0 21	3 40	10.54	21.60	36 58	55 50	78.34	105 10	135 80
20	0.22	3 49	10.69	21.82	36 87	55 84	78 75	105 58	136 34
21	0 24	3 58	10 84	22.03	37 15	56.19	79 16	106.06	136 88
22	0 26	3 67	11.00	22.25	37 44	56 55	79 58	106.55	137.43
23	0 28	3 76	11.15	22 47	37 72	56 90	80 00	107.03	137.98
24	0.32	3.85	11 31	22.70	38.01	57 25	80.42	107.51	138 53
25	0.34	3 94	11 47	22 92	38 30	57 60	80.84	107.99	139.08
26	0 37	4 03	11 63	23 14	38 59	57.96	81 26	108 48	139 63
27	0.40	4 12	11 79	23 37	38 88	58.32	81 68	108.97	140.18
28	0 43	4.22	11.95	23 60	39 17	58.68	82 10	109.46	140.74
29	0 46	4.32	12 11	23.82	39.46	59 03	82 52	109 95	141.29
30	0.49	4 42	12.27	24 05	39 76	59 40	82 95	110 44	141.85
31	0 52	4 52	12 43	24 28	40 05	59 75	83 38	110 93	142 40
32	0 56	4 62	12.60	24 51	40.35	60.11	83 81	111 43	142.96
33	0 59	4.72	12.76	24.74	40.65	60 47	84 23	111.92	143 52
34	0.63	4 82	12.93	24 98	40 95	60 84	84.66	112.41	144 08
35	0 67	4 92	13.10	25.21	41 25	61 20	85.09	112.90	144.64
36	0.71	5 03	13 27	25 45	41.55	61 57	85 52	113.40	145.20
37	0 75	5 13	13 44	25 68	41 85	61 94	85 95	113.90	145.76
38	0 79	5 24	13.62	25 92	42.15	62 31	86.39	114 40	146.33
39	0 83	5.34	13 79	26.16	42.45	62.68	86.82	114 90	146 89
40	0 87	5.45	13 96	26.40	42.76	63 05	87 26	115 40	147 46
41	0.91	5 56	14 13	26 64	43.06	63 42	87.70	115.90	148.03
42	0.96	5 67	14 31	26 88	43.37	63.79	88 14	116 40	148 60
43	1.01	5 78	14 49	27 12	43.68	64.16	88.57	116.90	149.17
44	1.06	5 90	14 67	27 37	43.99	64.54	89.01	117.41	149.74
45	1 10	6 01	14 85	27.61	44.30	64 91	89 45	117 92	150 31
46	1 15	6 13	15 03	27.86	44 61	65.29	89.89	118.43	150.88
47	1 20	6 24	15 21	28.10	44 92	65.67	90 33	118.94	151.45
48	1.26	6 36	15.39	28.35	45.24	66 05	90.78	119 45	152 03
49	1.31	6.48	15.57	28.60	45.55	66.43	91.23	119.96	152.61
50	1.36	6.60	15.76	28.85	45.87	66.81	91.68	120.47	153.19
51	1.42	6.72	15 95	29 10	46.18	67.19	92.12	120 98	153.77
52	1 48	6.84	16.14	29 36	46.50	67.58	92.57	121.49	154.35
53	1.53	6.96	16.32	29.61	46.82	67.96	93.02	122.01	154.93
54	1.59	7.09	16.51	29.86	47.14	68.35	93.47	122 53	155.51
55	1.65	7.21	16.70	30.12	47.46	68.73	93.92	123 05	156.09
56	1.71	7.34	16.89	30.38	47.79	69.12	94 38	123.57	156.67
57	1.77	7.46	17.08	30.64	48.11	69.51	94.83	124.09	157.25
58	1.83	7.60	17.28	30.90	48.43	69.90	95.29	124.61	157.84
59	1.89	7.72	17.47	31.16	48.76	70.29	95 74	125.13	158.43

TABLE X (*Continued*)

$$\frac{2 \sin^2 \frac{1}{2} \tau}{\sin 1''}$$

τ	9^m	10^m	11^m	12^m	13^m	14^m	15^m	16^m
s	"	"	"	"	"	"	"	"
0	159.02	196.32	237.54	282 68	331.74	384 74	441.63	502.46
1	159.61	196.97	238.26	283 47	332.59	385 65	442 62	503.50
2	160.20	197.63	238 98	284 26	333.44	386.56	443.60	504 55
3	160 80	198.28	239.70	285 04	334.29	387 48	444 58	505 60
4	161.39	198.94	240.42	285.83	335.15	388.40	445.56	506.65
5	161.98	199 60	241 14	286.62	336 00	389 32	446 55	507.70
6	162.58	200 26	241 87	287.41	336.86	390 24	447.54	508.76
7	163.17	200 92	242 60	288.20	337.72	391 16	448.53	509.81
8	163 77	201 59	243.33	289.00	338 58	392 09	449.51	510.86
9	164.37	202 25	244 06	289.79	339.44	393.01	450.50	511 92
10	164 97	202.92	244.79	290.58	340.30	393 94	451.50	512.98
11	165.57	203 58	245.52	291.38	341.16	394 86	452 49	514 03
12	166.17	204 25	245 25	292.18	342 02	395 79	453 48	515.09
13	166 77	204 92	246.98	292.98	342.88	396.72	454 48	516.15
14	167.37	205.59	247 72	293.78	343.75	397 65	455.47	517.21
15	167 97	206.26	248.45	294.58	344 62	398.58	456.47	518.27
16	168 58	206.93	249.19	295 38	345.49	399 52	457 47	519.34
17	169.19	207.60	249.93	296.18	346 36	400.45	458.47	520.40
18	169.80	208.27	250.67	296 99	347 23	401 38	459 47	521.47
19	170 41	208 94	251 41	297 79	348.10	402 32	460.47	522.53
20	171.02	209.62	252.15	298 60	348 97	403 26	461.47	523.60
21	171 63	210 30	252.89	299.40	349.84	404.20	462.48	524.67
22	172 24	210 98	253 63	300 21	350 71	405 14	463.48	525.74
23	172 85	211 66	254 37	301 02	351 58	406 08	464.48	526.81
24	173.47	212 34	255.12	301.83	352 46	407.02	465.49	527.89
25	174.08	213.02	255.87	302.64	353 34	407.96	466.50	528.96
26	174 70	213.70	256 62	303.46	354 22	408 90	467.51	530.03
27	175 32	214 38	257.37	304 27	355.10	409 84	468.52	531 11
28	175 94	215 07	258 12	305 09	355.98	410 79	469.53	532.18
29	176.56	215.75	258.87	305.90	356.86	411 73	470.54	533.26
30	177.18	216 44	259.62	306.72	357 74	412.68	471.55	534.33
31	177 80	217.12	260.37	307.54	358.62	413 63	472.57	535.41
32	178 43	217 81	261 12	308.36	359 51	414 59	473.58	536.50
33	179 05	218.50	261 88	309 18	360.39	415.54	474.60	537.58
34	179.68	219.19	262.64	310.00	361.28	416.49	475.62	538.67
35	180.30	219 88	263.39	310 82	362.17	417.44	476.64	539.75
36	180 93	220 58	264 15	311.65	363.07	418.40	477.65	540.83
37	181.56	221.27	264.91	312.47	363.96	419 35	478 67	541.91
38	182.19	221.97	265.68	313 30	364.85	420.31	479.70	543.00
39	182.82	222.66	266.44	314.12	365.75	421.27	480.72	544.09
40	183 46	223.36	267.20	314.95	366.64	422.23	481.74	545.18
41	184.09	224.06	267 96	315.78	367.53	423.19	482.77	546.27
42	184.72	224.76	268.73	316.61	368.42	424.15	483.79	547.36
43	185.35	225.46	269.49	317.44	369.31	425.11	484.82	548.45
44	185.99	226.16	270 26	318.27	370.21	426.07	485.85	549.55
45	186.63	226.86	271.02	319.10	371.11	427.04	486.88	550.64
46	187.27	227.57	271.79	319.94	372.01	428.01	487.91	551.73
47	187.91	228.27	272 56	320 78	372 92	428.97	488.94	552.83
48	188.55	228.98	273.34	321 62	373.82	429.93	489.97	553.93
49	189.19	229.68	274.11	322.45	374.72	430.90	491.01	555.03
50	189.83	230.39	274.88	323.29	375.62	431.87	492.05	556.13
51	190.47	231 10	275.65	324.13	376.52	432.84	493.08	557.24
52	191.12	231.81	276.43	324.97	377.43	433.82	494.12	558.34
53	191.76	232.52	277.20	325.81	378.34	434.79	495.15	559.44
54	192.41	233.24	277.98	326.66	379.26	435.76	496.19	560.55
55	193.06	233.95	278.76	327 50	380.17	436.73	497.23	561.65
56	193.71	234.67	279.55	328.35	381.08	437.71	498.28	562.76
57	194.36	235.38	280.33	329.19	381.99	438.69	499.32	563.87
58	195.01	236.10	281.12	330.04	382.90	439.67	500.37	564.98
59	195.66	236.82	281.90	330 89	383 82	440.65	501 41	566.08

OBSERVATION ON SUN FOR AZIMUTH

of line from to

Lat................ Long............... Date 192

Object	Horizontal circle	Vertical circle	Watch
Mark	....		......M.
			h m s
☉	....	....	....
☉	....	....	....
☉	....	....	....
☉	....	....	....
Mark	....	Mean	Mean
Mean reading on Sun		I. C.	
Hor. angle, Mark to Sun		refr.	5*h*
		h	G. C. T.

logs. Decl. at 0^h G. C. T. =

nat sin δ = ...

log sin ϕ = ... corr., x =

log sin h = ... δ =

sin ϕ sin h = ...

sin δ − sin ϕ sin h = ... $\cos Z = \dfrac{\sin \delta - \sin \phi \sin h}{\cos \phi \cos h}$

log numerator = ...

log sec ϕ = ...

log sec h = ...

log cos Z = ...

Z = ...

Hor. angle = ...

Azimuth of line = ...

OBSERVATION ON SUN FOR AZIMUTH

of Line from.............to..........

A.M. P.M.	Date,	192

Object	Hor. Circle	Vert. Circle	Watch
Mark	° ′	° ′	h m s
☉			
☉			
☉			
☉			
Mark		Mean	Mean

Mean, ☉ I. C.

Hor. Ang. Refr. 5^h

Mk. to ☉

Alt. ° ′ G. C. T.

° ′

Lat....................log sec.................

Alt....................log sec.................

Nat. Cos..............Sum..................

Nat. Sin...............Decl..................

Sum.. log...................

log vers Z_s....................

° ′

Z_s = ...

Hor. Ang. = ...

Azimuth...................to.................. ° ′

GREEK ALPHABET

Letters.	Name.	Letters.	Name.
Α, α,	Alpha	Ν, ν,	Nu
Β, β,	Beta	Ξ, ξ,	Xi
Γ, γ,	Gamma	Ο, ο,	Omicron
Δ, δ,	Delta	Π, π,	Pi
Ε, ε,	Epsilon	Ρ, ρ,	Rho
Ζ, ζ,	Zeta	Σ, σ, ς,	Sigma
Η, η,	Eta	Τ, τ,	Tau
Θ, θ, ϑ,	Theta	Υ, υ,	Upsilon
Ι, ι,	Iota	Φ, φ,	Phi
Κ, κ,	Kappa	Χ, χ,	Chi
Λ, λ,	Lambda	Ψ, ψ,	Psi
Μ, μ,	Mu	Ω, ω,	Omega

ABBREVIATIONS USED IN THIS BOOK

♈ or V = vernal equinox
R. A. or α = right ascension
δ = declination
p = polar distance
h = altitude
ζ = zenith distance
Az. or Z = azimuth
t = hour angle
ϕ = latitude
λ = longitude
Sid. or S = sidereal time
Eq. T. = equation of time
G. C. T. = Greenwich Civil Time
U. C. = upper culmination
L. C. = lower culmination
I. C. = index correction
ref. or r = refraction
par. or p = parallax
s. d. or s = semidiameter

APPENDIX A

THE TIDES

The Tides.

The engineer may occasionally be called upon to determine the height of mean sea level or of mean low water as a datum for levelling or for soundings. The exact determination of these heights requires a long series of observations, but an approximate determination, sufficiently accurate for many purposes, may be made by means of a few observations. In order to make these observations in such a way as to secure the best results the engineer should understand the general theory of the tides.

Definitions.

The periodic rise and fall of the surface of the ocean, caused by the moon's and the sun's attraction, is called the **tide.** The word " tide " is sometimes applied to the horizontal movement of the water (**tidal currents**), but in the following discussion it will be used only to designate the vertical movement. When the water is rising it is called **flood tide**; when it is falling it is called **ebb tide.** The maximum height is called **high water**; the minimum is called **low water.** The difference between the two is called the **range** of tide.

Cause of the Tides.

The principal cause of the tide is the difference in the force of attraction exerted by the moon upon different parts of the earth. Since the force of attraction varies inversely as the square of the distance, the portion of the earth's surface nearest the moon is attracted with a greater force than the central portion, and the latter is attracted more powerfully than the portion farthest from the moon. If the earth and moon were at rest the surface of the water beneath the moon would be elevated as shown in Fig. 81 at A. And since the attraction at B is the least, the water surface will also be elevated at this point. The same forces which tend to elevate the surface at A and B tend to depress it at C and D. If the earth were set rotating, an observer at any point O, Fig. 72, would be carried through two high and two low tides each day, the approximate interval between the high and the low tides being about $6\frac{1}{4}$ hours. This explanation shows what would happen if the tide were developed while the two bodies were at rest; but, owing to the high velocity of the earth's rotation, the shallowness of the water, and the interference of continents, the actual

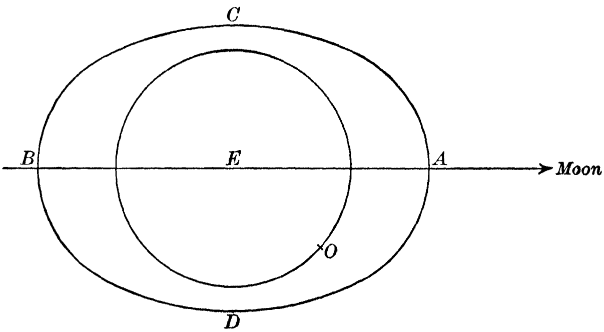


FIG. 81

tide is very complex. If the earth's surface were covered with water, and the earth were at rest, the water surface at high tide would be about two feet above the surface at low tide. The interference of continents, however, sometimes forces the tidal wave into a narrow, or shallow, channel, producing a range of tide of fifty feet or more, as in the Bay of Fundy.

The sun's attraction also produces a tide like the moon's, but considerably smaller. The sun's mass is much greater than the moon's but on account of its greater distance the ratio of the tide-producing forces is only about 2 to 5. The tide actually observed, then, is a combination of the sun's and the moon's tides.

Effect of the Moon's Phase.

When the moon and the sun are acting along the same line, at new or full moon, the tides are higher than usual and are called **spring tides.** When the moon is at quadrature (first or last quarter), the sun's and the moon's tides partially neutralize each other and the range of tide is less than usual; these are called **neap tides.**

Effect of Change in Moon's Declination.

When the moon is on the equator the two successive high tides are of nearly the same height. When the moon is north

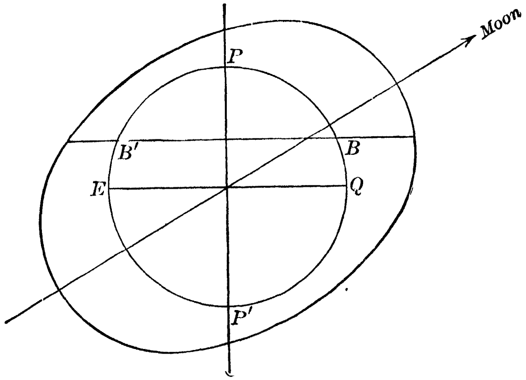


FIG. 82

or south of the equator the two differ in height, as is shown in Fig. 82. At point *B* under the moon it is high water, and the depth is greater than the average. At *B'*, where it will again be high water about 12^h later, the depth is less than the average. This is known as the **diurnal inequality.** At the points *E* and *Q*, on the equator, the two tides are equal.

Effect of the Moon's Change in Distance.

On account of the large eccentricity of the moon's orbit the tide-raising force varies considerably during the month. The actual distance of the moon varies about 13 per cent, and as a result the tides are about 20 per cent greater when the moon is nearest the earth, at perigee, than they are when the moon is farthest, at apogee.

Priming and Lagging of the Tides.

On the days of new and full moon the high tide at any place follows the moon's meridian passage by a certain interval of time, depending upon the place, which is called the **establishment of the port.** For a few days after new or full moon the crest of the combined tidal wave is west of the moon's tide and high water occurs earlier than usual. This is called the **priming** of the tide. For a few days before new or full moon the crest is east of the moon's tide and the time of high water is delayed. This is called **lagging** of the tide.

All of these variations are shown in Fig. 83, which was constructed by plotting the predicted times and heights from the U. S. Coast Survey Tide Tables and joining these points by straight lines. It will be seen that at the time of new and full moon the range of tide is greater than at the first and last quarters; at the points where the moon is farthest north or south of the equator (shown by *N*, *S*,) the diurnal inequality is quite marked, whereas at the points where the moon is on the equator (*E*) there is no inequality; at perigee (*P*) the range is much greater than at apogee (*A*).

Effect of Wind and Atmospheric Pressure.

The actual height and time of a high tide may differ considerably from the normal values at any place, owing to the weather conditions. If the barometric pressure is great the surface is depressed, and *vice versa.* When the wind blows steadily into a bay or harbor the water is piled up and the height of the tide is increased. The time of high water is delayed because the water continues to flow in after the true time of high water has passed; the maximum does not occur until the ebb and the effect of wind are balanced.

Observation of the Tides.

In order to determine the elevation of mean sea level, or, more properly speaking, of mean half-tide, it is only necessary to observe, by means of a graduated staff, the height of high and low water for a number of days, the number depending upon

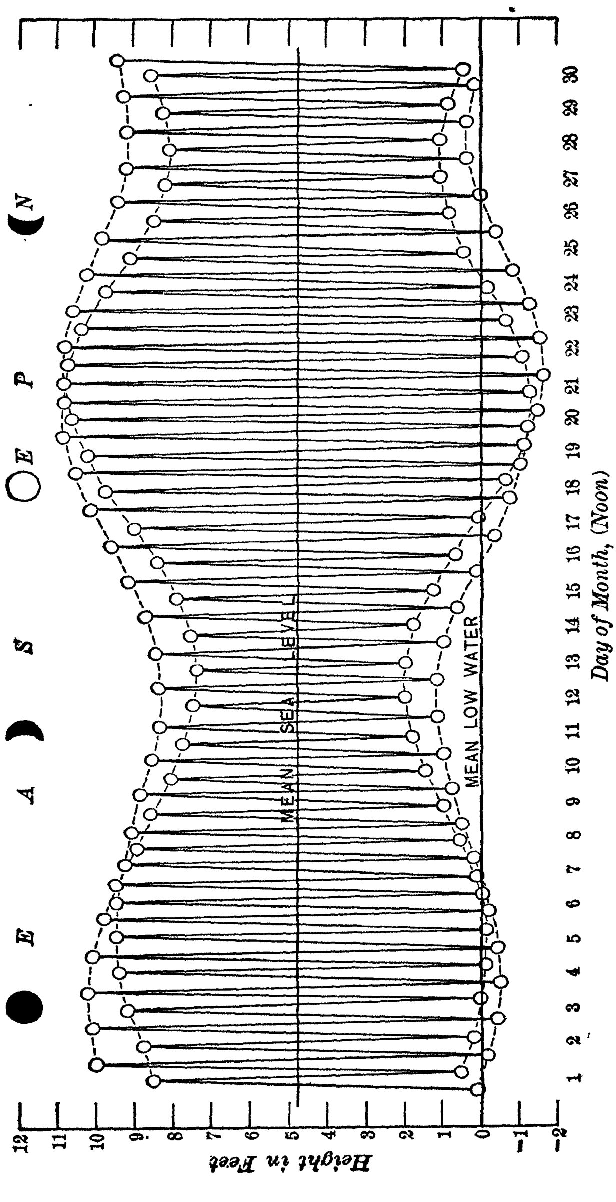


FIG. 83. DIAGRAM OF PREDICTED TIDES AT BOSTON, MASS., SEPTEMBER, 1910

the accuracy desired, and to take the mean of the gauge readings. If the height of the zero point of the scale is referred to some bench mark, by means of a line of levels, the height of the bench mark above mean sea level may be computed. In order to take into account all of the small variations in the tides it would be necessary to carry on the observations for a series of years; a very fair approximation may be obtained, however, in one lunar month, and a rough result, close enough for many purposes, may be obtained in a few days.

Tide Gauges.

If an elaborate series of observations is to be made, the self-registering tide gauge is the best one to use. This consists of a float, which is enclosed in a vertical wooden box and which rises and falls with the tide. A cord is attached to the float and is connected by means of a reducing mechanism with the pen of a recording apparatus. The record sheet is wrapped about a cylinder, which is revolved by means of clockwork. As the tide rises and falls the float rises and falls in the box and the pen traces out the tide curve on a reduced scale. The scale of heights is found by taking occasional readings on a staff gauge which is set up near the float box and referred to a permanent bench mark. The time scale is found by means of reference marks made on the sheet at known times.

When only a few observations are to be made the staff gauge is the simplest to construct and to use. It consists of a vertical graduated staff fastened securely in place, and at such a height that the elevation of the water surface may be read on the graduated scale at any time. Where the water is comparatively still the height may be read directly on the scale; but where there are currents or waves the construction must be modified. If a current is running rapidly by the gauge but the surface does not fluctuate rapidly, the ripple caused by the water striking the gauge may be avoided by fastening wooden strips on the sides so as to deflect the current at a slight angle. The horizontal cross section of such a gauge is shown in

Fig. 84. If there are waves on the surface of the water the height will vary so rapidly that accurate readings cannot be made. In order to avoid this difficulty a glass tube about $\frac{3}{4}$ inch in diameter is placed between two wooden strips (Fig. 85), one of which is used for the graduated scale. The water enters the glass tube and stands at the height of the water surface outside. In order to check sudden variations in height the water is allowed to enter this tube only through a very small tube (1^{mm} inside diameter) placed in a cork or rubber stopper at the lower end of the large tube. The water can rise in the tube rapidly enough to show the general level of the water surface, but small waves have practically no effect upon the reading. For convenience the gauge is made in sections about three feet long. These may be placed end to end and the large tubes connected by means of the smaller ones passing through the stoppers. In order to read the gauge at a distance it is convenient to have a narrow strip of red painted on the back of the tube or else blown into the glass.* Above the water surface this strip shows its true size, but below the surface, owing to the refraction of light by the water, the strip appears several times its true width, making it easy to distinguish the dividing line. Such a gauge may be read from a considerable distance by means of a transit telescope or field glasses.

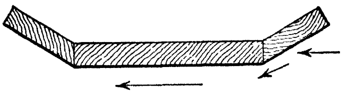


FIG. 84

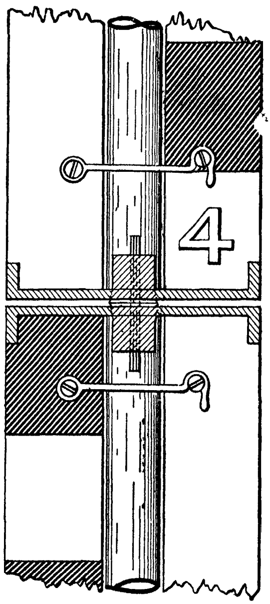


FIG. 85

* Tubes of this sort are manufactured for use in water gauges of steam boilers.

Location of Gauge.

The spot chosen for setting up the gauge should be near the open sea, where the true range of tide will be obtained. It should be somewhat sheltered, if possible, against heavy seas. The depth of the water and the position of the gauge should be such that even at extremely low or extremely high tides the water will stand at some height on the scale.

Making the Observations.

The maximum and minimum scale readings at the times of high and low tides should be observed, together with the times at which they occur. The observations of scale readings should be begun some thirty minutes before the predicted time of high or low water, and continued, at intervals of about 5^m, until a little while after the maximum or minimum is reached. The height of the water surface sometimes fluctuates at the time high or low tide, so that the first maximum or minimum reached may not be the true time of high or low water. In order to determine whether the tides are normal the force and direction of the wind and the barometric pressure may be noted.

Reducing the Observations.

If the gauge readings vary so that it is difficult to determine by inspection where the maximum or minimum occurred, the observations may be plotted, taking the times as abscissæ and gauge readings as ordinates. A smooth curve drawn through the points so as to eliminate accidental errors will show the position of the maximum or minimum point. (Figs. 86a and 86b.) When all of the observations have been worked up in this way the mean of all of the high-water and low-water readings may be taken as the scale reading for mean half-tide. There should of course be as many high-water readings as low-water readings. If the mean half-tide must be determined from a very limited number of observations, these should be combined in pairs in such a way that the diurnal inequality does not introduce an error. In Fig. 87 it will be seen that the mean of *a* and *b*, or the mean of *c* and *d*, or *e* and *f*, will give nearly the mean half-tide; but if *b* and *c*, or *d* and *e*, are combined, the mean is in

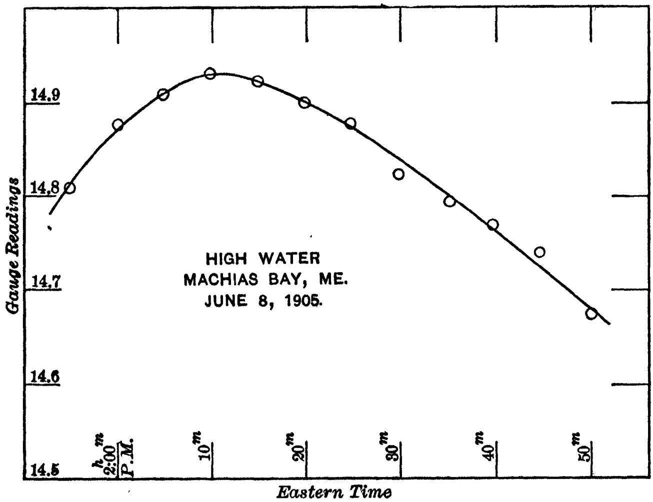


Fig. 86a

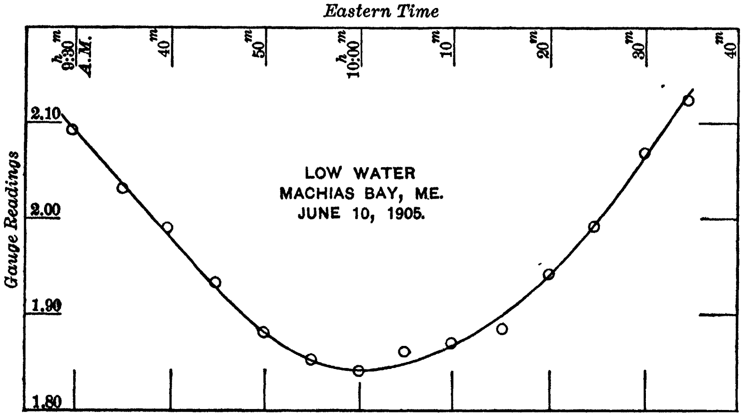


Fig. 86b

one case too small and in the other case too great. The proper selection of tides may be made by examining the predicted heights and times given in the tables issued by the U. S. Coast and Geodetic Survey. By examining the predicted heights the exact relation may be found between mean sea level and the mean half-tide as computed from the predicted heights corresponding to those tides actually observed. The difference between these two may be applied as a correction to the mean of the observed tides to obtain mean sea level. For example, suppose that the predicted heights at a port near the place of observation indicate that the mean of *a*, *b*, *c*, *d*, *e*, and *f* is 0.2 ft.

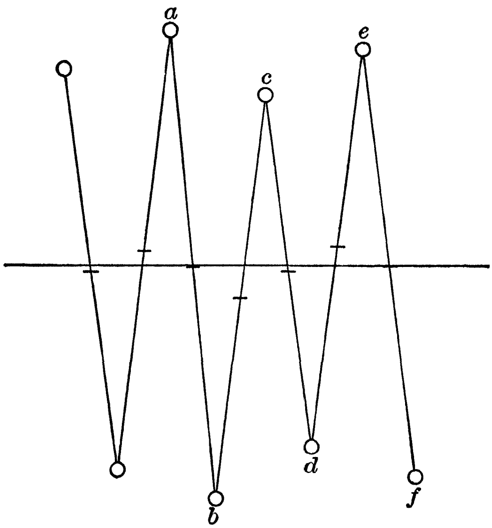


FIG. 87

below mean sea level. Then if these six tides are observed and the results averaged, a correction of 0.2 ft. should be added to the mean of the six heights in order to obtain mean sea level.

Prediction of Tides.

Since the local conditions have such a great influence in determining the tides at any one place, the prediction of the times and heights of high and low water for that place must be based upon a long series of observations made at the same point. *Tide Tables* giving predicted tides for one year are published annually by the United States Coast and Geodetic Survey; these tables give the times and heights of high and low water for the principal ports of the United States, and also for many foreign ports. The method of using these tables is explained in a note at the foot of each page. A brief statement of the theory of tides is given in the Introduction.

The approximate time of high water at any place may be computed from the time of the moon's meridian passage, provided we know the average interval between the moon's transit and the following high water, i.e., the "establishment of the port." The mean time of the moon's transit over the meridian of Greenwich is given in the Nautical Almanac for each day, together with the change per hour of longitude. The local time of transit is computed by adding to the tabular time the hourly change multiplied by the number of hours in the west longitude; this result, added to the establishment of the port, gives the approximate time of high water. The result is nearly correct at the times of new and full moon, but at other times is subject to a few minutes variation.

APPENDIX B

SPHERICAL TRIGONOMETRY

The formulæ derived in the following pages are those most frequently used in engineering field practice and in navigation. Many of the usual formulæ of spherical trigonometry are purposely omitted. It is not intended that this appendix shall serve as a general text book on spherical trigonometry, but merely that it should supplement that part of the preceding text which deals with spherical astronomy.

A spherical triangle is a triangle formed by arcs of great circles. If from the vertices of the triangle straight lines are drawn to the centre of the sphere there is formed at this point a triedral angle (solid angle) the three face angles of which are measured by the corresponding sides of the spherical triangle, and the three diedral angles (edge angles) of which are equal to the corresponding spherical angles. For any triedral angle a spherical triangle may be formed, by assuming that the center of the sphere is at the vertex of the angle, and assigning any arbitrary value to the radius. The three faces (planes) cut out arcs of great circles which form the sides of the triangle. The solution of the spherical triangle is really at the same time the solution of the solid angle since the six parts of one equal the six corresponding parts of the other. Any three lines passing through a common point define a triedral angle. For example, the earth's axis of rotation, the plumb line at any place on the surface of the earth, and a line in the direction of the sun's centre, may be conceived to intersect at the earth's centre. The relation among the three face angles and the three edge angles of this triedral angle may be calculated by the formulæ of spherical trigonometry. The sphere employed, however, is merely an imaginary one.

The fundamental formulæ mentioned on p. 32 may be derived by applying the principles of analytic geometry to the spherical triangle. In Fig. 88 the radius of the sphere is assumed to be unity. If a perpendicular CP be dropped from C to the XY plane, and a line CP' be drawn from C perpendicular to OX

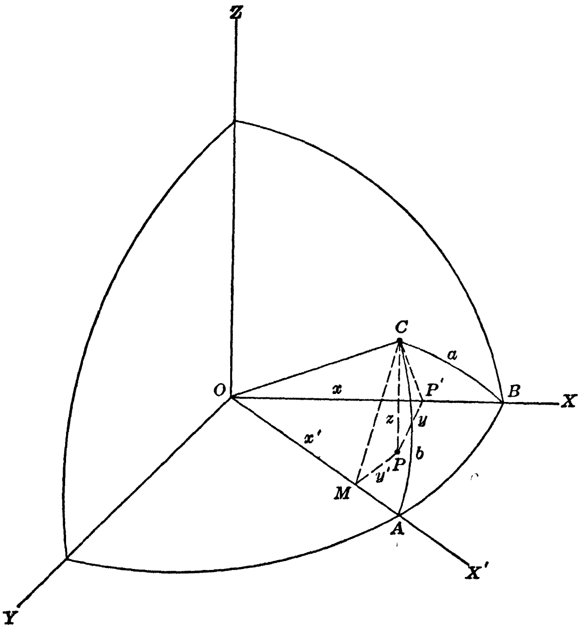


FIG. 88

then x and y may be expressed in terms of the parts of the spherical triangle as follows: —

$$x = \cos a$$
$$y = \sin a \cos B$$
$$z = \sin a \sin B.$$

If we change to a new axis $\overline{OX'}$, CM being drawn perpendicular to OX', then we have (y' being negative in this figure)

$$x' = \cos b$$
$$y' = -\sin b \cos A$$
$$z' = \sin b \sin A.$$

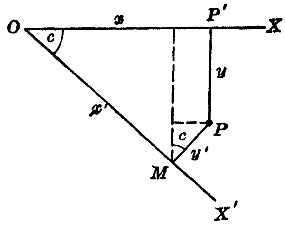


FIG. 89

From Fig. 89, the formulæ for transformation are

$$x = x' \cos c - y' \sin c$$
$$y = x' \sin c + y' \cos c$$
$$z = z'.$$

By substitution,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad (1)$$
$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \quad (2)$$
$$\sin a \sin B = \sin b \sin A \quad (3)$$

Corresponding formulæ may be written for angles B and C.

By employing the principle of the polar triangle, namely, that the angle of a triangle and the opposite side of its polar triangle are supplements, we may write three sets of formulæ like (1), (2) and (3) in which each small letter is replaced by a large letter and each large letter replaced by a small letter. For example, the first two equations would be

$$\cos A = \cos B \cos C + \sin B \sin C \cos a \quad (a)$$
$$\sin A \cos b = \cos B \sin C - \sin B \cos C \cos a. \quad (b)$$

There will also be two other sets of equations for the angles B and C.

Equation (1) may be regarded as the fundamental formula of spherical trigonometry because all of the others may be derived from it. All problems may be solved by means of (1), although not always so conveniently as with other special forms.

Solving for A, Equa. (1) may be written

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \quad (1a)$$

in which form it may be used to find any angle A when the three sides are known. See Equa. [20] and [20a], and [25] and [25a], pp. 34 and 35. If each side of the equation is subtracted from unity we have

$$1 - \cos A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}$$

or

$$\text{vers } A = \frac{\cos (b - c) - \cos a}{\sin b \sin c} \qquad (2)$$

or

$$2 \sin^2 \frac{A}{2} = \frac{2 \sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c}. \qquad (3)$$

Dividing (3) by 2 and denoting $\sin^2 \frac{A}{2}$ by "hav." the "haversine," or half versed-sine, we may write

$$\text{hav. } A = \frac{\sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c}. \qquad (4)$$

From (2) we may derive [21] and [26] by substituting $A = t$, $b = 90° - \delta$, $c = 90° - \phi$, and $a = 90° - h$; or $A = Z$, $b = 90° - h$, $c = 90° - \phi$, and $a = 90° - \delta$.

By putting $s' = \frac{1}{2}(a + b + c)$, (3) becomes

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s' - b) \sin (s' - c)}{\sin b \sin c}}. \qquad (5)$$

If we add each member of (1a) to unity we may derive by a similar process

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s' \sin (s' - a)}{\sin b \sin c}}. \qquad (6)$$

Dividing (5) by (6) we have

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s' - b) \sin (s' - c)}{\sin s' \sin (s' - a)}}. \qquad (7)$$

Formulæ [17], [22], [18], [23], [19] and [24] may be derived from (5), (6), (7) respectively or, more readily, from the intermediate forms, like (3), by putting $s = \frac{1}{2}(\phi + h + p)$ and

$A = t$ or Z. For example, if in (3) we put $A = t$, $a = 90° - h$, $b = 90° - \phi$, and $c = p$, then

$$\sin^2\frac{t}{2} = \frac{\sin\frac{1}{2}(90° - h + 90° - \phi - p)\sin\frac{1}{2}(90° - h - 90° + \phi + p)}{\cos\phi\sin p}$$

$$= \frac{\sin\frac{1}{2}(180° - (\phi + h + p)\sin\frac{1}{2}(\phi - h + p)}{\cos\phi\sin p}.$$

If $s = \frac{1}{2}(\phi + h + p)$ then

$$\sin^2\frac{t}{2} = \frac{\cos s\sin(s - h)}{\cos\phi\sin p}$$

from which we have [17].

Formula (4) or [17] may be written

$$\text{hav.}\, t = \frac{\cos s\sin(s - h)}{\cos\phi\sin p} \qquad (8)$$

the usual form (in navigation) for the calculation of the hour angle from an observed altitude of an object.

For the purpose of calculating the great-circle distance between two points on the earth's surface formula (1) may be put in the form

$$\text{hav. (dist.)} = \text{hav.}\,(\phi_A \sim \phi_B) + \cos\phi_A\cos\phi_B\,\text{hav.}\,\Delta\lambda \qquad (9)$$

in which ϕ_A, ϕ_B are the latitudes of two points on the earth's surface and $\Delta\lambda$ their difference in longitude. $\phi_A \sim \phi_B$ means the difference between the latitudes if they are both N or both S; the sum if they are in opposite hemispheres.

Formula (9) is derived by substituting the co-latitudes for b and c and $\Delta\lambda$ for A in (1) which gives

$$\cos\text{(dist.)} = \cos(90° - \phi_A)\cos(90° - \phi_B) + \sin(90° - \phi_A)\sin(90° - \phi_B)\cos\Delta\lambda.$$

If we add and subtract $\cos\phi_A\cos\phi_B$ in the right-hand member we obtain,

$$\begin{aligned}\cos\text{ (dist.)} &= \sin\phi_A \sin\phi_B + \cos\phi_A\cos\phi_B - \cos\phi_A\cos\phi_B \\ &\quad + \cos\phi_A\cos\phi_B\cos\Delta\lambda \\ &= \cos(\phi_A \sim \phi_B) - \cos\phi_A\cos\phi_B(1-\cos\Delta\lambda) \\ &= \cos(\phi_A \sim \phi_B) - \cos\phi_A\cos\phi_B \text{ vers } \Delta\lambda.\end{aligned}$$

Subtracting both members from unity

$$1 - \cos\text{ (dist.)} = 1 - \cos(\phi_A \sim \phi_B) + \cos\phi_A\cos\phi_B\text{ vers }\Delta\lambda$$

or $$\text{vers (dist.)} = \text{vers }(\phi_A \sim \phi_B) + \cos\phi_A\cos\phi_B\text{ vers }\Delta\lambda.$$

Dividing by 2

$$\text{hav. (dist.)} = \text{hav. }(\phi_A \sim \phi_B) + \cos\phi_A\cos\phi_B\text{ hav. }\Delta\lambda. \qquad (9)$$

Note: A table of natural and logarithmic haversines may be found in Bowditch, *American Practical Navigator.* (Table 45.)

The same formula may be applied to the calculation of the zenith distance of an object. In this case it is written

$$\text{Hav. }\zeta = \text{hav. }(\phi \sim \delta) + \cos\phi\cos\phi\text{ hav. }t. \qquad (10)$$

This is the formula usually employed in the method of Marcq Saint Hilaire.

Right Triangles

By writing formulæ (1), (2), (3), (*a*) and (*b*) in terms of the three parts and placing $C = 90°$, we may obtain the following ten right triangle formulæ.

$$\cos c = \cos a \cos b$$

$$\sin A = \frac{\sin a}{\sin c} \qquad \sin B = \frac{\sin b}{\sin c}$$

$$\cos A = \frac{\tan b}{\tan c} \qquad \cos B = \frac{\tan a}{\tan c}$$

$$\tan A = \frac{\tan a}{\sin b} \qquad \tan B = \frac{\tan b}{\sin a} \qquad (11)$$

$$\sin A = \frac{\cos B}{\cos b} \qquad \sin B = \frac{\cos A}{\cos a}$$

$$\cos c = \cot A \cot B.$$

These are readily remembered from their similarity to the corresponding formulæ of plane triangles.

To solve a right triangle select the three formulæ which involve the two given parts and one of the three parts to be found. To check the results select the formula involving the three parts just computed. The computed values should satisfy this equation.

Radians — Degrees, Minutes, and Seconds.

If the length of an arc is divided by the radius it expresses the central angle in radians. The number obtained is the corresponding length of arc on a circle whose radius is unity. The unit of measurement of angles in this system is the radius of the circle, that is, an angle of 1 is an angle whose arc equals the radius, and therefore contains about $57^\circ.3$.

Since the ratio of the semi-circumference to the radius is π, there are π radians in 180° of the circumference.* The conversion of angles from degrees into radians (or π measure, or arc-measure) is effected by multiplying by the ratio of these two.

$$\text{Angle in degrees} = \text{angle in radians} \times \frac{180^\circ}{\pi}$$

and $$\text{angle in radians} = \text{angle in degrees} \times \frac{\pi}{180^\circ}.$$

To convert an angle in radians into minutes multiply by $\dfrac{180 \times 60}{\pi}$ $= 3437'.77$; or divided by $\dfrac{\pi}{180 \times 60} = .0002909$. This latter number, the arc $1'$, is nearly equal to $\sin 1'$ or $\tan 1'$.

To convert an angle expressed in radians into seconds multiply by $\dfrac{180 \times 60 \times 60}{\pi} = 206264.8$; or, divide by the reciprocal, .00000,48481,36811, the arc $1''$; this number is identical with $\sin 1''$ or $\tan 1''$ for 16 decimal places.

* $\pi = 3.14159, 26535$.

Area of a Spherical Triangle.

In text books on geometry it is shown that "the area of a spherical triangle equals its spherical excess times the area of the tri-rectangular triangle," the right angle being the unit of angles. If Δ represents the area of any spherical triangle, whose angles are A, B, and C, then

$$\Delta = \frac{A + B + C - 180^\circ}{90^\circ} \times \frac{4\,\pi R^2}{8}$$

or

$$\Delta = \frac{(A + B + C - 180^\circ)\,\pi R^2}{180^\circ}$$

$$= \frac{e^\circ \pi R^2}{180^\circ}, \text{ if } e^\circ \text{ is the spherical excess in degrees}$$

or

$$= \frac{e'' \pi R^2}{180^\circ \times 60' \times 60''}, \quad \text{if } e'' \text{ is the spherical excess in seconds.} \qquad (12)$$

Spherical Excess.

If Equa. (12) is solved for e'' we have

$$e'' = \frac{\Delta}{R^2} \times \frac{180 \times 60 \times 60}{\pi}$$

$$= \frac{\Delta}{R^2} \times 206264.8, \quad \text{the constant being the number of seconds in one radian}$$

$$= \frac{\Delta}{R^2 \text{ arc } 1''}. \qquad (13)$$

Note: Arc $1'' = .00000,48481,36811$; it is the reciprocal of the above constant, and is the length of the arc which subtends an angle of $1''$ when the radius is unity.

Solid Angles.

Any solid angle* may be measured by an area on the surface of the sphere in the same manner that plane angles are measured by arcs on the circumference of a circle. The extent of the opening between the planes of a triedral angle is proportional to

* A solid angle is one formed by the intersection of any number of planes in a common point. The triedral angle is a special case of the solid angle.

the area of the corresponding spherical triangle, or in other words proportional to the spherical excess of that triangle. This is true not only of triangles, but also of spherical polygons and spherical areas formed by circles (sectors). The unit of measurement of the solid angle is the *steradian*. A unit (plane) angle is one whose arc is equal in length to the radius of the circle; that is, it intercepts an arc whose length is R. The steradian is a solid angle which intercepts on the surface of the sphere an area equal to R^2; or it intercepts on the sphere of unit radius an area equal to unity. Just as the plane angle (in radians) when multiplied by the radius gives the length of arc, so the solid angle or the spherical excess (in radians) when multiplied by R^2 gives the area of the spherical triangle. To obtain a more definite idea of the size of this angle we may compute the length of arc from the centre to the circumference of a small circle having an area equal to R^2 (or 1 on a sphere of unit radius). This comes out about $32° 46'+$. The spherical area enclosed by the parallel of latitude $57° 14'$ corresponds to one steradian in the angle of the cone whose apex is at the centre of the globe.

Functions of Angles near 0° or 90°.

When obtaining from tables the values of sines or tangents of small angles (or angles near to 180°) and cosines or cotangents of angles near to 90° there is some difficulty encountered in the interpolation, on account of the rapid rate of change of the logarithms. In practice these values are often found by approximate methods which enable us to avoid the use of second differences in the interpolation. There are two assumptions which may be made, which result in two methods of obtaining the log functions.

For sines, we may assume that

$$\sin x = x'' \times \sin 1''$$

or

$$\log \sin x = \log x'' + \log \sin 1''.$$

The log sin $1'' = 4.685\ 5749$.

This method is accurate for very small angles; the limiting value of the angle for which the sine may be so computed depends entirely upon the accuracy demanded, that is, upon the number of places required.

Example. Find the value of log sin 0° 10′ 25″ from a 5-place table.

$$\begin{aligned} \log \sin 1'' &= 4.68557 \\ \log 625'' &= \underline{2.79588} \\ \log \sin x &= 7.48145 \end{aligned}$$

The result is correct to five figures.

A similar assumption may be made for tangents of small angles or for cosines and cotangents of angles near 90°.

For angles slightly larger than those for which the preceding method would be employed, we may assume that

$$\frac{\sin (A + a'')}{A + a''} = \frac{\sin A}{A}.$$

The ratio of $\frac{\sin A}{A''}$ changes slowly and is therefore very nearly the same for both members of the equation. We may therefore compute the log sin $(A + a'')$ by the equation,

$$\log \sin (A + a'') = \log (A'' + a'') + \log \frac{\sin A}{A''}$$

in which A'' signifies that the angle must be reduced to seconds.

The latter logarithm is given in many tables in the margin of the page. It may be computed for any number which is stated in the table. This method is more accurate than the former.

Example.

Find the value of log sin 2° 01′ 30″ in a five-place table.

2° 01′ 30″ = 7290″. In the marginal table is found opposite "S" the logarithm 4.68548 which is the difference between log sin A and log A. If this is not given it may be computed by taking from the table the nearest log sin, say log sin 2° 01′ = 8.54642 and subtracting from it the log of 7260″ = 3.86094. The result is 4.68548.

Then

$$\log \frac{\sin A}{A} = 4.68548$$
$$\log 7290'' = \underline{3.86273}$$
$$\log \sin x = 8.54821$$

This result is correct to five figures.

INDEX