THEORETICAL RELATIONS IN THE INTERFEROMETRY OF SMALL ANGLES[1]

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In addition to the sides of the ray parallelogram (base, b) and the radius of rotation R, we shall have to consider the following angles or angular increment: $\Delta\alpha$ the angular rotation of the paired mirrors, $\Delta\theta$ the corresponding angular displacement of the fringes, ΔN the linear displacement of the micrometer mirror (in a direction normal to its face), and $\Delta\varphi$ the angle subtended by two consecutive fringes. If n is the order of the fringe we may write

$$\Delta\varphi = \Delta\theta/\Delta n \tag{1}$$

Moreover if i is the angle of incidence (45°) of the impinging beams at the mirrors and λ the wave length in question,

$$2 \cos i \, \Delta N/\Delta n = \lambda, \tag{2}$$

or on substitution,

$$\Delta\theta/\Delta N = \Delta\varphi/(\Delta N/\Delta n) = 2 \cos i\text{-}\Delta\varphi/\lambda \tag{3}$$

Again if s is the angle at the apex of the distance triangle on the base b,

$$\Delta s = 2\Delta\alpha \tag{4}$$

and

$$\Delta\alpha = \Delta N \cos i/R \tag{5}$$

and since the distance $d = b/2\Delta s = b/2\Delta\alpha$, from (5)

$$d = bR/2\Delta N \cos i = F/\Delta N \tag{6}$$

so that the sensitiveness is from (6),

$$\delta d = (2d^2 \cos i/bR) \, \delta\,(\Delta N) = \delta\,(\Delta N)/Fd^2 \tag{7}$$

If μ is the index of refraction, e the effective thickness of the plates, i.e., the difference of effective thickness of the two half silvered plates through which the beams pass, we may write as in the colors of thin plates (since the respective beams pass each plate but once)

$$n\lambda = e\mu \cos r - 2N \cos i \tag{8}$$

if r is the angle of refraction corresponding to the incidence, i. If n, i, r, alone vary while e, μ, λ, N, are fixed and since $\sin i = \mu \sin r$, i.e., if the eye travels through the field of the telescope from left to right,

$$\Delta\varphi = di/dn = \lambda/(2N \sin i - e \tan r \cos i) \tag{9}$$

so that $\Delta\varphi$ depends inversely on e and N.

When the spectrum ellipses are centered, $N = N_c$, a condition necessary for the occurrence of achromatic fringes, where,

$$2N_c = e\ (\mu \cos r + 2B/\lambda^2 \cos r)/\cos i \tag{10}$$

if $\lambda d\mu/d\lambda = 2B/\lambda^2$ is adequate. Equation (10) may now be inserted in equation (9) and the coefficient of e, viz., the long parenthesis containing circular functions evaluated for $i = 45°$, $\lambda = 6 \times 10^{-5}$, $\mu = 1.55$, $2B/\lambda^2 = .026$. Its value is slightly greater than 1. Hence we may write approximately but with much greater convenience

$$\Delta\varphi = \lambda/e \tag{11}$$

and we thus obtain the breadth of the fringes for different values of the parameter e, roughly. It would, of course, be easy to compute the accurate value of i. This equation placed in the above equations (3), (5), (6) gives in succession,

$$\Delta\theta = \frac{2\Delta\varphi\ \Delta N \cos i}{\lambda} = 2\frac{\cos i}{e}\Delta N \tag{12}$$

$$\Delta\theta = \frac{2R}{\lambda}\Delta\varphi\Delta\alpha = \frac{2R}{e}\Delta\alpha \tag{13}$$

$$d = \frac{bR}{2\Delta N \cos i} = \frac{bR}{e\Delta\theta} \tag{14}$$

so that the measurement of the long distance d depends ultimately on the area, $2bR$, of the ray parallelogram, the differential thickness of paired glass plates, e, and the displacement $\Delta\theta$ of the achromatic fringes.

From equation (14) we obtain the sensitiveness by differentiation, or

$$\delta d = \frac{d^2 e}{bR}\delta\,(\Delta\theta) \tag{15}$$

Let the angle $\Delta\theta$ or its variation be measured in a telescope of length L, and provided with an ocular micrometer, so that the angle $\Delta\theta = x/L$, x being the linear magnitude measured on this micrometer. Hence,

$$\delta d = \frac{d^2 e}{bRL}\delta x \tag{16}$$

and if we introduce moderate estimates, d = kilometer = 10^5 cm., $e = 10^{-2}$ cm., $b = 200$ cm., $R = 10$ cm., $L = 50$ cm., $\delta x = 10^{-2}$ cm., then $\delta d = 10$ cm., or d should be measurable to 10 cm. at a kilometer, so far as the interferometer only is concerned. It may be noticed that in equations (11) to (14), e is variable and equal to $\lambda/\Delta\varphi$. If the plate halfsilvers traversed by the interfering beams are not equally thick optic plate (or in general), a full compensation is secured by rotating the mirror nearest the telescope, provided it is the thinner of the pair. In such a case the system of mirrors will not be parallel, however, when the fringes are infinite in size (circles) and the zero position of the micrometer must be independently found, as one of the constants of the apparatus.

[1] See these PROCEEDINGS, 3, June, 1917, 412, 432, 436.
[2] From a Report to the Carnegie Institution, of Washington, D. C.

INTER-PERIODIC CORRELATION IN THE EGG PRODUCTION OF THE DOMESTIC FOWL

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In no animal organism except man has the investigation of the interrelationship of the various morphological and physiological characteristics of the individual by means of the modern methods of statistical analysis applied to large masses of quantitatively recorded observations been carried out on a more extensive scale than in the domestic fowl, to which Pearl and Surface, and others associated with them, have devoted their attention for a number of years.

Notwithstanding the many problems dealt with in this series of investigations, our knowledge of the interrelationships of characters of economic importance in this organism is still far from complete. Data from breeds different from those used by Pearl and Surface are particularly needed for purposes of comparison.

In an earlier paper[1] we discussed the correlation between the concentration of yellow pigment in the somatic tissues and egg production in the White Leghorn fowl. In the present investigation we have considered the correlations between the egg production of various periods.

The intensity of the correlation between the number of eggs laid in the several individual months and the total egg production of the year as a whole is shown by the solid dots in figure 1. A curve in excellent agreement was found for the previous year.

Economically these coefficients are of the greatest importance since they make possible the selection of groups of birds of high annual egg production from the trap nest records of individual months. Biologically they are in some degree spurious because of the fact that the cor-