This last theorem is of interest in the method developed in recent years by Wilczynski for dealing with questions in projective differential geometry. In fact, the coefficients of the transformed system of differential equations are what he has generally called seminvariants of the original system; the theorem affords a means for calculating these seminvariants in a purely mechanical way. In Wilczynski's method, the geometric problem becomes the study of a completely integrable system of the kind we have been considering.

The results outlined above have been developed at length in a memoir which is to appear in the *Transactions of the American Mathematical Society*.

SYSTEMATIC MOTION AMONG STARS OF THE HELIUM TYPE

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Several investigators using different methods and different material have shown beyond a doubt that the stars evidence a preference for motion in two opposite directions in the sky. This does not mean that all stars move in one of the two directions, but that there is a stronger tendency for stars to move in the favored directions than in any other.

In such investigations the helium, or B-type, stars have presented considerable difficulties, since their motions are small, and since as a class they are situated at a great distance from the sun. It has seemed desirable, therefore, to devise a method whereby the preference of motion among the helium stars might be determined with some degree of confidence.

In the first place the zone in which all the helium stars lie was mapped off into twelve arbitrary divisions. For each division means were taken of the amount of proper motion in the two co-ordinates right ascension and declination, and these mean values were then subtracted from each proper motion. Thus the center of the velocity-figure was obtained. The rectangular co-ordinates were converted into polar co-ordinates and arranged in the order of their position-angles from the north pole. Then for thirty degree groups, 0°–30°, 10°–40°, etc., means were taken of the position-angles, and sums of the amount of proper motion. With the mean position-angles as abscissae and the sums of proper motion as ordinates, the results were plotted and smooth curves drawn to represent them. Figure 1 shows how well the observations can be fitted by mooth curves. It will also be noted that there is more than one maximum, though in the cases of the curves 5 and 10 only one maximum is real. That is, in the two cases cited, a slightly different grouping of the data would produce great changes in the curves with the exception of one maximum which is real.

Fig. 1

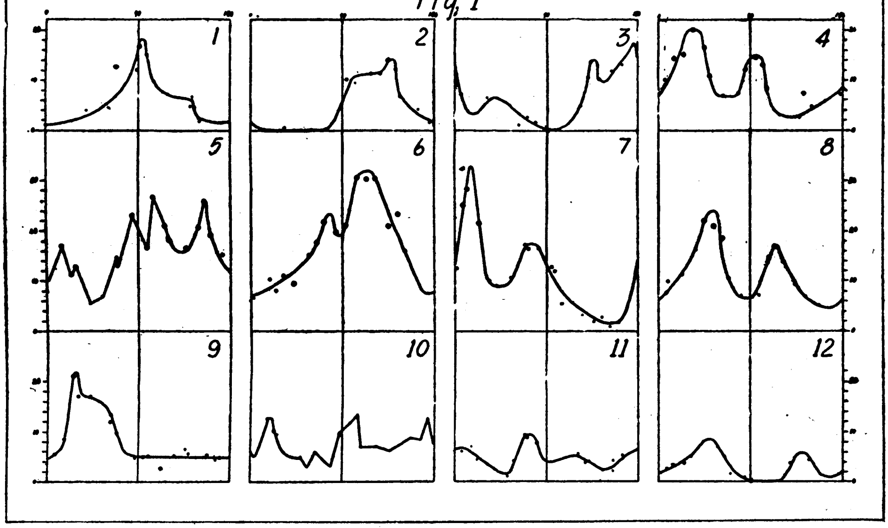

Fig. 2

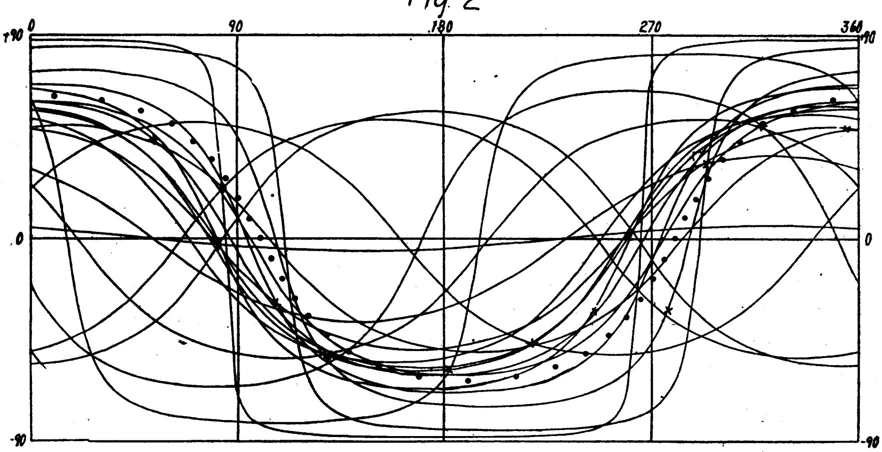

Figure 2 gives a graphic representation of the results. The crosses mark the positions of the centers of the areas treated. From these centers great circles were drawn in the direction obtained by reading off the position-angles of the maxima from the curves of figure 1. The figure then shows the paths along which there is a strong tendency for stars of the helium type to move. The black dots trace the path of the Milky Way. The paths sharply define a plane which is that of the stars used instead of the plane of the Milky Way, but in addition there are considerable tendencies of motion which carry the helium stars into other regions. In some cases this tendency seems to be the greater of the two. In the care of area 4 there is no tendency for a preference for motion in the Milky Way.

To summarize the conclusions drawn from the investigation of the systematic motions of the helium stars, there appears to be a strong tendency for these stars to move in their own plane, which should therefore be preserved, at least until the next step in the star's evolution. As a matter of fact the A-type stars, supposedly representing the next stage in evolution, exhibit a strong tendency to crowd toward this plane. But there are likewise strong tendencies for the stars of helium type to depart from the plane, so that the tendency for the stars to spread in every direction, so clearly manifest in advanced stages in the evolution of a star, has its birth in the helium stage of evolution. There is apparently nothing systematic in the motions directed away from the plane of the stars.

THE ABUNDANCE OF THE ELEMENTS IN RELATION TO THE HYDROGEN-HELIUM STRUCTURE OF THE ATOMS

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According to the theory already presented in a number of papers[1] the atoms of all the 91 elements of our ordinary system heavier than hydrogen are built up as intra-atomic (not chemical) compounds of hydrogen. The first of these 91 elements, helium, is the second in the system, and therefore has the atomic number 2. It has an atomic weight of 4.00, and may be considered to be composed of 4 hydrogen atoms. The element of atomic number 3, lithium, has an atomic weight of about 7. Now it has been found that in general among the elements of low atomic weight, the elements of even atomic number, beginning with helium, seem to be built up from helium atoms, and therefore may be said to have the general formula $n\mathrm{He}'$, where the prime is added to indicate that these elements are intra-atomic, not chemical, compounds. The odd numbered elements, beginning with lithium, seem in general to have the formula $n\mathrm{He}'+\mathrm{H}_3'$. Thus the elements seem to fall into two series which may be called the *even* and the *odd* series, or the