very largely on the absolutely brighter stars. A real test of the luminosity curve and the auxiliary laws can only be obtained if we use values of H_c at least as large as 14. Thus the Wolf proper-motion stars might be used to advantage were it not for the fact that the value of the systematic correction to be applied to Wolf's magnitudes is rather uncertain. There are indications, however, that with any reasonable value for this correction, the agreement between theoretical and observational curves will remain bad in this case, there being an observed excess of stars with large values of H.

[1] *Mt. Wilson Contr.*, No. 188.

[2] *Ann. Harvard Coll. Obs.*, **85,** No. 5, p. 103.

ATMOSPHERIC PULSATION OF THE CEPHEID VARIABLE, η AQUILAE[1]

By W. Carl Rufus

Observatory of the University of Michigan

Communicated, April 22, 1924

A new method of analyzing variable radial velocity data by a systematic study of line displacements from different elements and at different levels of a star's atmosphere has been developed and applied. The usual method of selecting lines giving consistent displacements has masked an effect here tentatively called atmospheric pulsation. A difference in the velocities from the hydrogen and the helium lines in the star,[2] B.D. + 56° 2617, led to the idea of isolating the velocities of high-level elements. Application was made in the case of χ Aurigæ by isolating the hydrogen lines and a systematic distribution of the hydrogen velocity residuals with respect to phase was found.[3] Based upon the isolation of velocities at three levels in the star's atmosphere the method[4] adapted to the problem of Cepheid variation was first applied by Mr. J. A. Aldrich in a study of S. Sagittæ.[5] The method is also applicable to long-period and irregular variables, to secular and periodic changes in peculiar spectra, and to other astrophysical problems.

The Method as Applied in η Aquilae.—Velocity-difference curves correlated with the light variation and other periodic changes characterize the method here applied. The radial velocities of concentric layers of the star's atmosphere are determined separately by isolating certain elements, and by grouping lines of the spectrum originating at assumed levels based upon the determination of their heights in the sun by St. John[6] and by Mitchell.[7] The approach of two layers indicates compression of the intervening gases and recession indicates expansion. Resulting changes in temperature, density, radiation, and absorption, may be correlated with the light variation and changes of spectrum to explain many of the anomalous characteristics of Cepheid variation. Inasmuch as spectroscopic work is limited to surface radiations characterized by atmospheric effects, a new method of analysis to determine changes of atmospheric conditions should yield results of fundamental significance.

The Velocity Curves.—Forty-two plates of η Aquilæ made with the single-prism spectrograph of the Observatory of the University of Michigan were used. The mean velocity curve from lines of all levels has an amplitude of 40 km. The chief feature is a "hump," not obtained by Wright,[8]

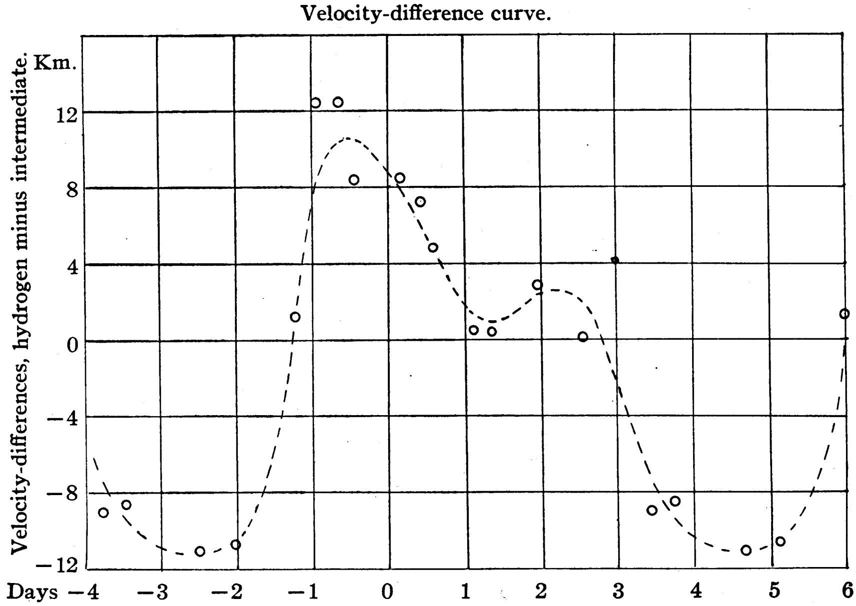

FIGURE 1

Positive values indicate compression and negative values indicate expansion of the atmospheric gases between the two effective levels.

which occurs about one day, (one-seventh of the period), after the "Still-stand" of the light curve by Wylie.[9] This irregularity resembles the typical "hump" of a Cepheid velocity curve, which Curtis represents quite satisfactorily in some cases by adding a single oscillation to an elliptical curve.[10] Velocity curves for high, intermediate, and low levels, and separate curves for hydrogen and strontium, were also formed. The secondary feature is more prominent in the high and the low level curves, and is greatly emphasized in the case of the isolated elements, but is lacking in the intermediate level. This feature of the velocity curves of η Aquilæ occurs at the time of greatest radial expansion of the star as a whole, on the basis of the pulsation theory, with a lag of phase of the upper layer. The difference in form and phase of these curves indicates that the ordinary velocity curve of a Cepheid is not definite, and the "elements of an orbit" depend upon the selection of lines.

The Velocity-Difference Curves.—The differential method, automatically eliminating plate errors, may be expected to increase the degree of accuracy of these curves. High-minus-intermediate, intermediate-minus-low, and hydrogen-minus-intermediate curves were formed. Positive differences represent compression. In general, compression of the atmosphere occurs after maximum compression of the star as a whole, which accounts for the retardation of the light maximum in accordance with the suggestion of Eddington that the retardation, not determined analytically, must occur in the non-adiabatic region of the star.[11] With sufficient data it is probable that the pulsations may be traced through the successive layers and Eddington's "gusts of radiation" may be observed. The hydrogen-minus-intermediate curve (Figure 1) has an amplitude greater than 20 km., which exceeds one-half the amplitude of the velocity curve itself. Special attention is called to the similarity of this velocity-difference curve to Wylie's photometric light curve,[9] and to a spectral variation curve based upon Shapley's data.[12] The intermediate-minus-low curve resembles the reverse of the light curve with synchronous secondary features. *The "humps" of the velocity-difference curves are synchronous with the "Stillstand" of the light curve,* which is not true in the case of the velocity curves. The "Stillstand" of the light variation seems to be due to a stage of comparative rest in the atmosphere of the star.

[1] Presented in part to Section D of the American Association for the Advancement of Science, December 1923; and in part to the American Astronomical Society, December 1923. *Pop. Astron.*, **32**, 1924 (228).

[2] In January 1920 the author discovered this spectroscopic binary, announced in *Astroph. J.*, **51**, 1920 (252), and made the following record:

Variable radial velocity of star B.D. +56° 2617 discovered Jan. 14, 1920. The plates on hand show a range for the hydrogen lines from +92.6 to —65.2, nearly 160 km. The H and K lines appear to be nearly stationary, only 8 km. range in the set of five plates measured, with a displacement averaging —25 km. *The helium lines seem to indicate a much smaller range of displacement than the hydrogen.* Further investigation is desirable.

[3] Rufus, W. Carl, *J. Roy. Astron. Soc. Can.*, **14**, 1920 (139).

[4] Rufus, W. Carl, "Atmospheric Pulsation of Cepheids, A Method of Attack," presented to the American Astronomical Society, September 1923. *Pop Astron.*, **32**, 1924 (22).

[5] Aldrich, J. A., "A Study of S Sagittæ," a thesis under the direction of Professor R. H. Curtiss, to be published by the Observatory of the University of Michigan. The isolation of the velocities from the hydrogen lines was proposed to Mr. Aldrich by the writer and systematic motion with reference to the metallic elements was found. The isolation of the velocities from lines of intermediate level was then proposed to Mr.

Aldrich by Professor Curtiss. Mr. Aldrich's results were submitted to members of the faculty of the University of Michigan in May 1923 and were presented to the American Astronomical Society in December 1923. *Pop. Astron.*, **32**, 1924 (218).

[6] St. John, Charles E., *Astroph. J.*, **37**, 1913 (322) and **38**, 1913 (341).

[7] Mitchell, S. A., *Ibid.*, **38**, 1913 (407).

[8] Wright, W. H., *Ibid.*, **9**, 1899 (62).

[9] Wylie, C. C., *Ibid.*, **56**, 1922 (225).

[10] Curtis, Heber D., These PROCEEDINGS, **9**, 1923 (187).

[11] Eddington, A. S., *Mon. Not. Roy. Astron. Soc. London*, **79**, 1919 (181).

[12] Shapley, H., *Astroph. J.*, **44**, 1916 (287).

ON PARAMETRIC REPRESENTATIONS OF CONTINUOUS SURFACES

BY B. DE KERÉKJÁRTÓ

PRINCETON UNIVERSITY

Communicated, April 23, 1924

1° The following theorem on parametric representations of continuous curves has been proved by Fréchet:[1]

Given the functions

$$x = f(u),\ y = g(u), \dots,\ z = h(u)$$

continuous and uniform in the interval $0 \leqq u \leqq 1$ which thus represent a continuous curve in euclidian space with the coördinates $x, y, \dots, z$, it is possible to represent the curve by a change of the parameter

$$u = u(t),\quad 0 \leqq t \leqq 1$$

in such a way that the functions

$$x = f(u(t)) = F(t),\ y = G(t), \dots,\ z = H(t)$$

are not simultaneously constant in any interval of t.

Let us take indeed for

$$t = t(u)$$

a continuous uniform function which is monotonically increasing from $t(0) = 0$ to $t(1) = 1$ except in those intervals in which all the functions $f(u), g(u), \dots, h(u)$ are simultaneously constant and in which the function $t(u)$ also should be constant.[1]

2° We want to investigate the analogous question for the case of continuous surfaces that is to say for continuous uniform images of surfaces, e.g. for those of the sphere.[2]