THEORETICAL ELEMENTS

OF

ELECTRICAL ENGINEERING

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BY

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THIRD EDITION

THOROUGHLY REVISED AND GREATLY CORRECTED

ANY

PREFACE.

THE first part of the following volume originated from a series of University lectures which I once promised to deliver. This part can, to a certain extent, be considered as an introduction to my work on "Theory and Calculation of Alternating Current Phenomena," leading up very gradually from the ordinary sine wave representation of the alternating current to the graphical representation by polar coördinates, from there to rectangular components of polar vectors, and ultimately to the symbolic representation by the complex quantity. The present work is, however, broader in its scope, in so far as it comprises the fundamental principles not only of alternating, but also of direct currents.

The second part is a series of monographs of the more important electrical apparatus, alternating as well as direct current. It is, in a certain respect, supplementary to "Alternating Current Phenomena." While in the latter work I have presented the general principles of alternating current phenomena, in the present volume I intended to give a specific discussion of the particular features of individual apparatus. In consequence thereof, this part of the book is somewhat less theoretical, and more descriptive, my intention being to present the most important electrical apparatus in all their characteristic features as regard to external and internal structure, action under normal and abnormal conditions, individually and in connection with other apparatus, etc.

I have restricted the work to those apparatus which experience has shown as of practical importance, and give only those theories and methods which an extended experience in the design and operation has shown as of practical utility. I consider this the more desirable as, especially of late years, electrical literature has been haunted by so many theories (for instance of the induction machine) which are incorrect, or too complicated for use, or valueless in practical application. In the class last mentioned are most of the graphical methods, which, while they may give an approximate insight in the inter-relation of phenomena, fail entirely in engineering practice owing to the great difference in the magnitudes of the vectors in the same diagram, and to the synthetic method of graphical representation, which generally require one to start with the quantity which the diagram is intended to determine.

I originally intended to add a chapter on Rectifying Apparatus, as arc light machines and alternating current rectifiers, but had to postpone this, due to the incomplete state of the theory of these apparatus.

The same notation has been used as in the Third Edition of "Alternating Current Phenomena," that is, vector quantities denoted by dotted capitals. The same classification and nomenclature have been used as given by the report of the Standardizing Committee of the American Institute of Electrical Engineers.

PREFACE TO THE THIRD EDITION.

NEARLY eight years have elapsed since the appearance of the second edition, during which time the book has been reprinted without change, and a revision, therefore, became greatly desired.

It was gratifying, however, to find that none of the contents of the former edition had to be dropped as superseded or antiquated. However, very much new material had to be added. During these eight years the electrical industry has progressed at least as rapidly as in any previous period, and apparatus and phenomena which at the time of the second edition were of theoretical interest only, or of no interest at all, have now assumed great industrial importance, as for instance the single-phase commutator motor, the control of commutation by commutating poles, etc.

Besides rewriting and enlarging numerous paragraphs throughout the text, a number of new sections and chapters have been added, on alternating-current railway motors, on the control of commutation by commutating poles ("interpoles"), on converter heating and output, on converters with variable ratio of conversion ("split-pole converters"), on three-wire generators and converters, short-circuit currents of alternators, stability and regulation of induction motors, induction generators, etc.

In conformity with the arrangement used in my other books, the paragraphs of the text have been numbered for easier reference and convenience.

When reading the book, or using it as text-book, it is recommended:

After reading the first or general part of the present volume, to read through the first 17 chapters of "Theory and Calculation of Alternating Current Phenomena," omitting, however, the mathematical investigations as far as not absolutely required for the understanding of the text, and then to take up the study of the second part of the present volume, which deals with special apparatus. When reading this second part, it is recommended to parallel its study with the reading of the chapter of "Alternating Current Phenomena" which deals with the same subject in a different manner. In this way a clear insight into the nature and behavior of apparatus will be imparted.

Where time is limited, a large part of the mathematical discussion may be skipped and in that way a general review of the material gained.

Great thanks are due to the technical staff of the McGraw-Hill Book Company, which has spared no effort to produce the third edition in as perfect and systematic a manner as possible, and to the numerous engineers who have greatly assisted me by pointing out typographical and other errors in the previous edition.

CHARLES PROTEUS STEINMETZ.

SCHENECTADY, *September*, 1909.

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PART I.

GENERAL THEORY.

1. MAGNETISM AND ELECTRIC CURRENT.

1. A magnet pole attracting (or repelling) another magnet pole of equal strength at unit distance with unit force* is called a *unit magnet pole.*

The space surrounding a magnet pole is called a *magnetic field of force*, or *magnetic field.*

The magnetic field at unit distance from a unit magnet pole is called a *unit magnetic field*, and is represented by one line of magnetic force (or shortly "one line") per sq. cm., and from a unit magnet pole thus issue a total of 4π lines of magnetic force.

The total number of lines of force issuing from a magnet pole is called its *magnetic flux.*

The magnetic flux Φ of a magnet pole of strength m is,

$$\Phi = 4\pi m.$$

At the distance l_r from a magnet pole of strength m, and therefore of flux $\Phi = 4\pi m$, assuming a uniform distribution in all directions, the magnetic field has the intensity,

$$\mathcal{H} = \frac{\Phi}{4\pi l_r^2} = \frac{m}{l_r^2},$$

since the Φ lines issuing from the pole distribute over the area of a sphere of radius l_r, that is the area $4\pi l_r^2$.

A magnetic field of intensity $\mathcal{H}$ exerts upon a magnet pole of strength m the force,

$$m\mathcal{H}.$$

Thus two magnet poles of strengths m_1 and m_2, and distance l_r from each other, exert upon each other the force,

$$\frac{m_1 m_2}{l_r^2}.$$

* That is, at one centimeter distance with such force as to give to the mass of one gram the acceleration of one centimeter per second.

2. Electric currents produce magnetic fields also; that is, the space surrounding the conductor carrying an electric current is a magnetic field, which appears and disappears and varies with the current producing it, and is indeed an essential part of the phenomenon called an electric current.

Thus an electric current represents a *magnetomotive force* (m.m.f.).

The magnetic field of a straight conductor, whose return conductor is so far distant as not to affect the field, consists of lines of force surrounding the conductor in concentric circles. The intensity of this magnetic field is directly proportional to the current strength and inversely proportional to the distance from the conductor.

Since the lines of force of the magnetic field produced by an electric current return into themselves, the magnetic field is a *magnetic circuit*. Since an electric current, at least a steady current, can exist only in a closed circuit, electricity flows in an *electric circuit*. The magnetic circuit produced by an electric current surrounds the electric circuit through which the electricity flows, and inversely. That is, the electric circuit and the magnetic circuit are *interlinked* with each other.

Unit current in an electric circuit is the current which produces in a magnetic circuit of unit length the field intensity 4π, that is, produces as many lines of force per square centimeter as issue from a unit magnet pole.

In unit distance from an electric conductor carrying unit current, that is in a magnetic circuit of length 2π, the field intensity is $\frac{4\pi}{2\pi} = 2$, and in the distance 2 the field intensity is unity; that is, unit current is the current which, in a straight conductor, whose return conductor is so far distant as not to affect its magnetic field, produces unit field intensity in distance 2 from the conductor.

One tenth of unit current is the practical unit, called *one ampere*.

3. One ampere in an electric circuit or turn, that is, one ampere-turn, thus produces in a magnetic circuit of unit length the field intensity, 0.4π, and in a magnetic circuit of length

l the field intensity $\frac{0.4\pi}{l}$, and $\mathfrak{F}$ ampere-turns produce in a magnetic circuit of length l the field intensity:

$$\mathcal{H} = \frac{0.4\pi\mathfrak{F}}{l} \text{ lines of force per sq. cm.}$$

regardless whether the $\mathfrak{F}$ ampere-turns are due to $\mathfrak{F}$ amperes in a single turn, or one ampere in $\mathfrak{F}$ turns, or $\frac{\mathfrak{F}}{n}$ amperes in n turns.

$\mathfrak{F}$, that is, the product of amperes and turns, is called *magnetomotive force* (m.m.f.).

The m.m.f. per unit length of magnetic circuit, or ratio:

$$\mathfrak{K} = \frac{\text{m.m.f.}}{\text{length of magnetic circuit}}$$

is called the *magnetizing force*.

Hence, m.m.f. is expressed in *ampere-turns;* magnetizing force in *ampere-turns per centimeter* (or in practice frequently ampere-turns per inch), field intensity in lines of magnetic force per square centimeter.

At the distance l_r from the conductor of a loop or circuit of $\mathfrak{F}$ ampere-turns, whose return conductor is so far distant as not to affect the field, assuming the m.m.f. $= \mathfrak{F}$, since the length of the magnetic circuit $= 2\pi l_r$, we obtain as the magnetizing force,

$$\mathfrak{K} = \frac{\mathfrak{F}}{2\pi l_r},$$

and as the field intensity,

$$\mathcal{H} = 0.4\pi\mathfrak{K} = \frac{0.2\mathfrak{F}}{l_r}.$$

4. The magnetic field of an electric circuit consisting of two parallel conductors (or any number of conductors, in a polyphase system), as the two wires of a transmission line, can be considered as the superposition of the separate fields of the conductors (consisting of concentric circles). Thus, if there are I amperes in a circuit consisting of two parallel conductors (conductor and return conductor), at the distance l_1 from the

first and l_2 from the second conductor, the respective field intensities are,

$$\mathcal{H}_1 = \frac{0.2\, I}{l_1},$$

and

$$\mathcal{H}_2 = \frac{0.2\, I}{l_2},$$

and the resultant field intensity, if τ = angle between the direcions of the two fields,

$$\begin{aligned}\mathcal{H} &= \sqrt{\mathcal{H}_1^{\,2} + \mathcal{H}_2^{\,2} + 2\, \mathcal{H}_1 \mathcal{H}_2 \cos \tau},\\ &= \frac{0.2\, I}{l_1 l_2} \sqrt{l_1^2 + l_2^2 + 2\, l_1 l_2 \cos \tau}.\end{aligned}$$

In the plane of the conductors, where the two fields are in the same, or opposite direction, the resultant field intensity is,

$$\mathcal{H} = \frac{0.2\, I\, (l_1 \pm l_2)}{l_1 l_2},$$

where the plus sign applies to the space between, the minus sign the space outside of the conductors.

The resultant field of a circuit of parallel conductors consists of excentric circles, interlinked with the conductors, and crowded together in the space between the conductors.

The magnetic field in the interior of a spiral (solenoid, helix, coil) carrying an electric current, consists of straight lines.

5. If a conductor is coiled in a spiral of l centimenter axial length of spiral, and N turns, thus $n = \frac{N}{l}$ turns per centimeter length of spiral, and I = current, in amperes, in the conductor, the m.m.f. of the spiral is

$$\mathcal{F} = IN,$$

and the magnetizing force in the middle of the spiral, assuming the latter of very great length,

$$\mathfrak{K} = nI = \frac{N}{l} I,$$

thus the field intensity in the middle of the spiral or solenoid,

$$\begin{aligned}\mathcal{H} &= 0.4\, \pi \mathfrak{K}\\ &= 0.4\, \pi n I.\end{aligned}$$

Strictly this is true only in the middle part of a spiral of such length that the m.m.f. consumed by the external or magnetic return circuit of the spiral is negligible compared with the m.m.f. consumed by the magnetic circuit in the interior of the spiral, or in an endless spiral, that is a spiral whose axis curves back into itself, as a spiral whose axis is curved in a circle.

Magnetomotive force $\mathfrak{F}$ applies to the total magnetic circuit, or part of the magnetic circuit. It is measured in ampere-turns.

Magnetizing force $\mathfrak{H}$ is the m.m.f. per unit length of magnetic circuit. It is measured in ampere-turns per centimeter.

Field intensity $\mathcal{H}$ is the number of lines of force per square centimeter.

If l = length of the magnetic circuit or a part of the magnetic circuit,

$$\mathfrak{F} = l\mathfrak{H}, \qquad \mathfrak{H} = \frac{\mathfrak{F}}{l},$$

$$\mathcal{H} = 0.4\,\pi\mathfrak{H} \qquad \mathfrak{H} = \frac{\mathcal{H}}{0.4\,\pi},$$

$$= 1.257\,\mathfrak{H} \qquad \mathfrak{H} = 0.796\,\mathcal{H}.$$

6. The preceding applies only to magnetic fields in air or other unmagnetic materials.

If the medium in which the magnetic field is established is a "magnetic material," the number of lines of force per square centimeter is different and usually many times greater. (Slightly less in diamagnetic materials.)

The ratio of the number of lines of force in a medium, to the number of lines of force which the same magnetizing force would produce in air (or rather in a vacuum), is called the *permeability* or magnetic conductivity μ of the medium.

The number of lines of force per square centimeter in a magnetic medium is called the *magnetic induction* $\mathcal{B}$. The number of lines of force produced by the same magnetizing force in air is called the *field intensity* $\mathcal{H}$.

In air, magnetic induction $\mathcal{B}$ and field intensity $\mathcal{H}$ are equal.

As a rule, the magnetizing force in a magnetic circuit is changed by the introduction of a magnetic material, due to the change of distribution of the magnetic flux.

The permeability of air $= 1$ and is constant.

The permeability of iron and other magnetic materials varies with the magnetizing force between a little above 1 and values as high as 6000 in soft iron.

The magnetizing force $\mathfrak{F}$ in a medium of permeability μ produces the field intensity $\mathcal{H} = 0.4\ \pi\mathfrak{F}$ and the magnetic induction $\mathcal{B} = 0.4\ \pi\mu\mathfrak{F}$.

EXAMPLES.

7. (1.) A pull of 2 grams at 4 cm. radius is required to hold a horizontal bar magnet 12 cm. in length, pivoted at its center, in a position at right angles to the magnetic meridian. What is the intensity of the poles of the magnet, and the number of lines of magnetic force issuing from each pole, if the horizontal intensity of the terrestrial magnetic field $\mathcal{H} = 0.2$, and the acceleration of gravity $= 980$?

The distance between the poles of the bar magnet may be assumed as five-sixths of its length.

Let $m =$ intensity of magnet pole. $l_r = 5$ is the radius on which the terrestrial magnetism acts.

Thus $2\ m\mathcal{H}l_r = 2\ m =$ torque exerted by the terrestrial magnetism.

2 grams weight $= 2 \times 980 = 1960$ units of force. These at 4 cm. radius give the torque $4 \times 1960 = 7840\ g - \text{cm}$.

Hence $2\ m = 7840$.

$m = 3920$ is the strength of each magnet pole and

$\Phi = 4\ \pi m = 49{,}000$, the number of lines of force issuing from each pole.

8. (2.) A conductor carrying 100 amperes runs in the direction of the magnetic meridian. What position will a compass needle assume, when held below the conductor at a distance of 50 cm., if the intensity of the terrestrial magnetic field is 0.2?

The intensity of the magnetic field of 100 amperes 50 cm. from the conductor, is $\mathcal{H} = \dfrac{0.2\ I}{l_r} = 0.2 \times \dfrac{100}{50} = 0.4$, the direction is at right angles to the conductor, that is at right angles to the terrestrial magnetic field.

If τ = angle between compass needle and the north pole of the magnetic meridian, l = length of needle, m = intensity of its magnet pole, the torque of the terrestrial magnetism is $\mathcal{H}ml \sin \tau = 0.2\, ml \sin \tau$, the torque of the current is

$$\mathcal{H}ml \cos \tau = \frac{0.2\, Iml \cos \tau}{l_r} = 0.4\, ml \cos \tau.$$

In equilibrium, $0.2\, ml \sin \tau = 0.4\, ml \cos \tau$, or $\tan \tau = 2$, $\tau = 63.4°$.

9. (3.) What is the total magnetic flux per $l = 1000$ m. length, passing between the conductors of a long distance transmission

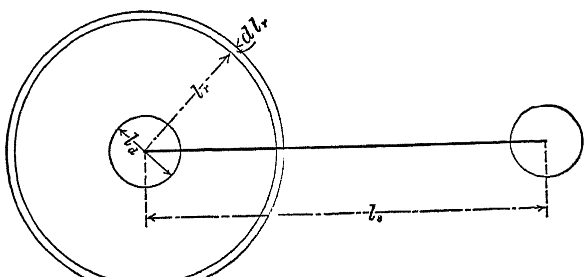


Fig. 1. Diagram of Transmission Line for Inductance Calculation.

line carrying I amperes of current, if $l_d = 0.82$ cm. is the diameter of the conductors (No. 0 B. & S.), $l_s = 45$ cm. the spacing or distance between them?

At distance l_r from the center of one of the conductors (Fig. 1), the length of the magnetic circuit surrounding this conductor is $2\pi l_r$, the m.m.f., I ampere turns; thus the magnetizing force $\mathfrak{K} = \frac{I}{2\pi l_r}$, and the field intensity $\mathcal{H} = 0.4\pi \mathfrak{K} = \frac{0.2\, I}{l_r}$, and the flux in the zone dl_r is $d\Phi = \frac{0.2\, I l dl_r}{l_r}$, and the total flux from the surface of the conductor to the next conductor is,

$$\Phi = \int_{\frac{l_d}{2}}^{l_s} \frac{0.2\, I l dl_r}{l_r} =$$

$$0.2\, Il \Big[\log_\epsilon l_r\Big]_{\frac{l_d}{2}}^{l_s} = 0.2\, Il \log_\epsilon \frac{2\, l_s}{l_d}.$$

The same flux is produced by the return conductor in the same direction, thus the total flux passing between the transmission wires is,

$$2\,\Phi = 0.4\,Il \log_\epsilon \frac{2\,l}{l_d}$$

or per 1000 m. = 10^5 cm. length,

$$2\,\Phi = 0.4 \times 10^5\,I \log_\epsilon \frac{90}{0.82} = 0.4 \times 10^5 \times 4.70\,I = 0.188 \times 10^6\,I,$$

or 0.188 I megalines or millions of lines per line of 1000 m. of which 0.094 I megalines surround each of the two conductors.

10. (4.) In an alternator each pole has to carry 6.4 millions of lines, or 6.4 megalines magnetic flux. How many ampere-turns per pole are required to produce this flux, if the magnetic circuit in the armature of laminated iron has the cross section of 930 sq. cm. and the length of 15 cm., the air-gap between stationary field poles and revolving armature is 0.95 cm. in length and 1200 sq. cm. in section, the field-pole is 26.3 cm. in length and 1075 sq. cm. in section, and is of laminated iron, and the outside return circuit or yoke has a length per pole of 20 cm. and 2250 sq. cm. section, and is of cast iron?

The magnetic densities are; $\mathfrak{B}_1 = 6880$ in the armature, $\mathfrak{B}_2 = 5340$ in the air-gap, $\mathfrak{B}_3 = 5950$ in the field-pole, and $\mathfrak{B}_4 = 2850$ in the yoke. The permeability of sheet iron is $\mu_1 = 2550$ at $\mathfrak{B}_1 = 6880$, $\mu_3 = 2380$ at $\mathfrak{B}_3 = 5950$. The permeability of cast iron is $\mu_4 = 280$ at $\mathfrak{B}_4 = 2850$. Thus the field intensity $\left(\mathcal{H} = \frac{\mathfrak{B}}{\mu}\right)$ is, $\mathcal{H}_1 = 2.7$, $\mathcal{H}_2 = 5340$, $\mathcal{H}_3 = 2.5$, $\mathcal{H}_4 = 10.2$.

The magnetizing force $\left(\mathfrak{H} = \frac{\mathcal{H}}{0.4\,\pi}\right)$ is, $\mathfrak{H}_1 = 2.15$, $\mathfrak{H}_2 = 4250$, $\mathfrak{H}_3 = 1.99$, $\mathfrak{H}_4 = 8.13$ ampere-turns per cm. Thus the m.m.f. ($\mathfrak{F} = \mathfrak{H}l$) is, $\mathfrak{F}_1 = 32$, $\mathfrak{F}_2 = 4040$, $\mathfrak{F}_3 = 52$, $\mathfrak{F}_4 = 163$, or the total m.m.f. per pole is

$$\mathfrak{F} = \mathfrak{F}_1 + \mathfrak{F}_2 + \mathfrak{F}_3 + \mathfrak{F}_4 = 4290 \text{ ampere-turns.}$$

The permeability μ of magnetic materials varies with the density $\bar{\mathfrak{B}}$, thus tables or curves have to be used for these quantities. Such curves are usually made out for density $\mathfrak{B}$ and magnetizing force $\mathcal{H}$, so that the magnetizing force $\mathcal{H}$ corresponding to the density $\mathcal{B}$ can be derived directly from the curve. Such a set of curves is given in Fig. 2.

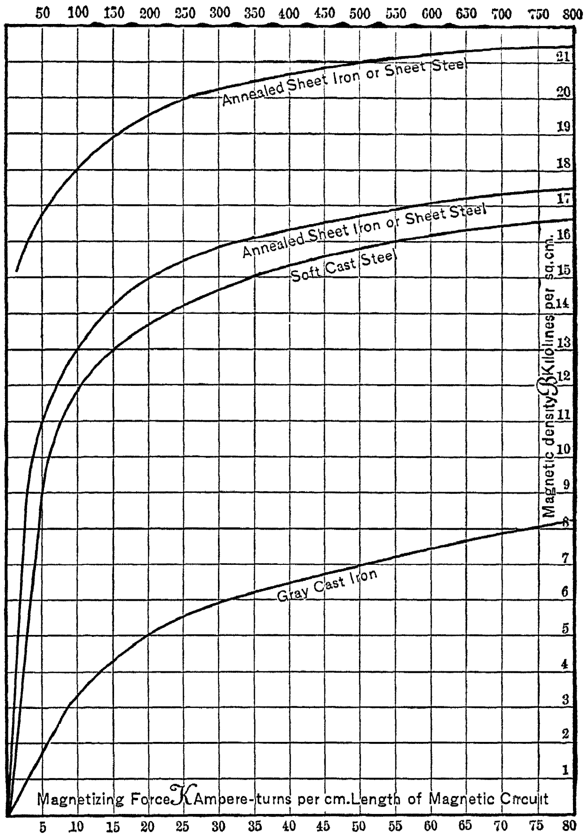


Fig. 2. Magnetization Curves of Various Irons.

2. MAGNETISM AND E.M.F.

11. In an electric conductor moving relatively to a magnetic field, an e.m.f. is generated proportional to the rate of cutting of the lines of magnetic force by the conductor.

Unit e.m.f. is the e.m.f. generated in a conductor cutting one line of magnetic force per second.

10^8 times unit e.m.f. is the practical unit, called the *volt.*

Coiling the conductor n fold increases the e.m.f. n fold, by cutting each line of magnetic force n times.

In a closed electric circuit the e.m.f. produces an *electric current.*

The ratio of e.m.f. to electric current produced thereby is called the *resistance* of the electric circuit.

Unit resistance is the resistance of a circuit in which unit e.m.f. produces unit current.

10^9 times unit resistance is the practical unit, called the *ohm.*

The ohm is the resistance of a circuit, in which one volt produces one ampere.

The resistance per unit length and unit section of a conductor is called its resistivity, ρ.

The resistivity ρ is a constant of the material, varying with the temperature.

The resistance r of a conductor of length l, area of section A, and resistivity ρ is $r = \frac{l\rho}{A}$.

12. If the current in the electric circuit changes, starts, or stops, the corresponding change of the magnetic field of the current generates an e.m.f. in the conductor carrying the current, which is called the *e.m.f. of self-induction.*

If the e.m.f. in an electric circuit moving relatively to a magnetic field produces a current in the circuit, the magnetic field produced by this current is called its *magnetic reaction.*

The fundamental law of self-induction and magnetic reaction is, that these effects take place in such a direction as to oppose their cause (Lentz's law).

Thus the e.m.f. of self-induction during an increase of current is in the opposite direction, during a decrease of current in the same direction as the e.m.f. producing the current.

The magnetic reaction of the current produced in a circuit moving out of a magnetic field is in the same direction, in a circuit moving into a magnetic field in opposite direction to the magnetic field.

Essentially, this law is nothing but a conclusion from the law of conservation of energy.

EXAMPLES.

13. (1.) An electromagnet is placed so that one pole surrounds the other pole cylindrically as shown in section in Fig. 3, and a copper cylinder revolves between these poles at 3000 rev. per min. What is the e.m.f. generated between the ends of this cylinder, if the magnetic flux of the electromagnet is $\Phi = 25$ megalines?

During each revolution the copper cylinder cuts 25 megalines. It makes 50 rev. per sec. Thus it cuts $50 \times 25 \times 10^6 = 12.5 \times 10^8$

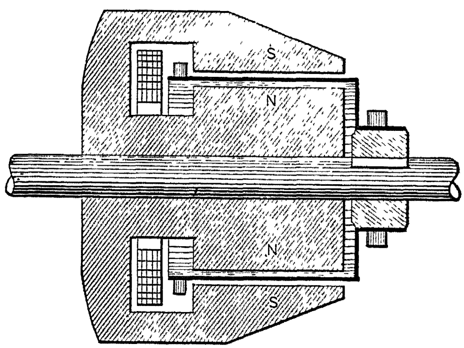


Fig. 3. Unipolar Generator.

lines of magnetic flux per second. Hence the generated e.m.f. is $E = 12.5$ volts.

Such a machine is called a "unipolar," or more properly a "nonpolar" or an "acyclic" generator.

14. (2.) The field spools of the 20-pole alternator in Section 1, Example 4, are wound each with 616 turns of wire No. 7 (B. & S.), 0.106 sq. cm. in cross section and 160 cm. mean length of turn. The 20 spools are connected in series. How many amperes and how many volts are required for the excitation of this alternator field, if the resistivity of copper is 1.8×10^{-6} ohms per cm.^{3} *

Since 616 turns on each field spool are used, and 4280 ampere-turns required, the current is $\frac{4280}{616} = 6.95$ amperes.

* Cm.^{3} refers to a cube whose side is one centimeter, and should not be confused with cu. cm.

The resistance of 20 spools of 616 turns of 160 cm. length, 0.106 sq. cm. section, and 1.8×10^{-6} resistivity is,

$$\frac{20 \times 616 \times 160 \times 1.8 \times 10^{-6}}{0.106} = 33.2 \text{ ohms},$$

and the e.m.f. required $6.95 \times 33.2 = 230$ volts.

3. GENERATION OF E.M.F.

15. A closed conductor, convolution or turn, revolving in a magnetic field, passes during each revolution through two positions of maximum inclosure of lines of magnetic force *A* in Fig. 4, and two postions of zero inclosure of lines of magnetic force *B* in Fig. 4.

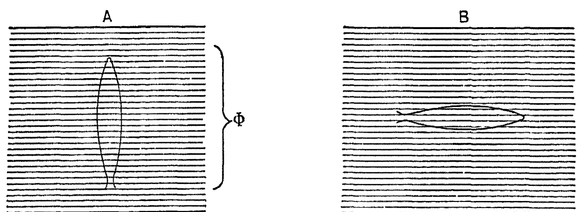


Fig. 4. Generation of e.m.f.

Thus it cuts during each revolution four times the lines of force inclosed in the position of maximum inclosure.

If $\Phi =$ the maximum number of lines of flux inclosed by the conductor, $f =$ the frequency in revolutions per second or cycles, and $n =$ number of convolutions or turns of the conductor, the lines of force cut per second by the conductor, and thus the *average generated e.m.f.* is,

$$E = 4\, fn\Phi \text{ absolute units,}$$
$$= 4\, fn\Phi\, 10^{-8} \text{ volts.}$$

If f is given in hundreds of cycles, Φ in megalines,

$$E = 4\, fn\Phi \text{ volts.}$$

If a coil revolves with uniform velocity through a uniform magnetic field, the magnetism inclosed by the coil at any instant is,

$$\Phi \cos \tau$$

where Φ = the maximum magnetism inclosed by the coil and τ = angle between coil and its position of maximum inclosure of magnetism.

The e.m.f. generated in the coil, which varies with the rate of cutting or change of $\Phi \cos \tau$, is thus,

$$e = E_0 \sin \tau,$$

where E_0 is the maximum value of e.m.f., which takes place for $\tau = 90°$, or at the position of zero inclosure of magnetic flux, since in this position the rate of cutting is greatest.

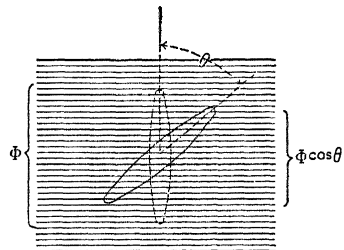


Fig. 5. Generation of e.m.f. by Rotation.

Since avg. $(\sin \tau) = \frac{2}{\pi}$, the average generated e.m.f. is,

$$E = \frac{2}{\pi} E_0.$$

Since, however, we found above that,

$$E = 4\, fn\Phi \text{ is the average generated e.m.f.,}$$

it follows that

$$E_0 = 2\, \pi fn\Phi \text{ is the } \textit{maximum}, \text{ and}$$
$$e = 2\, \pi fn\Phi \sin \tau \text{ the } \textit{instantaneous generated e.m.f.}$$

The interval between like poles forms 360 electrical-space degrees, and in the two-pole model these are identical with the mechanical-space degrees. With uniform rotation, the space angle, τ, is proportional to time. Time angles are designated by θ, and with uniform rotation $\theta = \tau$, τ being measured in electrical-space degrees.

The period of a complete cycle is 360 time degrees, or 2π or $\frac{1}{f}$ seconds. In the two-pole model the period of a cycle is that of one complete revolution, and in a $2\, n_p$-pole machine, $\frac{1}{n_p}$ of that of one revolution.

Thus,
$$\theta = 2\, \pi ft$$
$$e = 2\, \pi fn\Phi \sin 2\, \pi ft.$$

If the time is not counted from the moment of maximum inclosure of magnetic flux, but t_1 = the time at this moment, we have

$$e = 2\pi f n \Phi \sin 2\pi f (t - t_1)$$

or,

$$e = 2\pi f n \Phi \sin (\theta - \theta_1).$$

Where $\theta_1 = 2\pi f t_1$ is the angle at which the position of maximum inclosure of magnetic flux takes place, and is called its phase.

These e.m.fs. are alternating.

If at the moment of reversal of the e.m.f. the connections between the coil and the external circuit are reversed, the e.m.f. in the external circuit is pulsating between zero and E_0, but has the same average value E.

If a number of coils connected in series follow each other successively in their rotation through the magnetic field, as the armature coils of a direct-current machine, and the connections of each coil with the external circuit are reversed at the moment of reversal of its e.m.f., their pulsating e.m.fs. superimposed in the external circuit make a more or less steady or continuous external e.m.f.

The average value of this e.m.f. is the sum of the average values of the e.m.fs. of the individual coils.

Thus in a direct-current machine, if Φ = maximum flux inclosed per turn, n = total number of turns in series from commutator brush to brush, and f = frequency of rotation through the magnetic field.

$E = 4 fn\Phi$ = generated e.m.f. (Φ in megalines, f in hundreds of cycles per second).

This is the *formula of the direct-current generator.*

EXAMPLES.

17. (1.) A circular wire coil of 200 turns and 40 cm. mean diameter is revolved around a vertical axis. What is the horizontal intensity of the magnetic field of the earth, if at a speed of 900 revolutions per minute the average e.m.f. generated in the coil is 0.028 volts?

The mean area of the coil is $\frac{40^2 \pi}{4} = 1255$ sq. cm., thus the terrestrial flux inclosed is 1255 $\mathcal{H}$, and at 900 revolutions per minute or 15 revolutions per second, this flux is cut $4 \times 15 = 60$ times per second by each turn, or $200 \times 60 = 12{,}000$ times by the coil. Thus the total number of lines of magnetic force cut by the conductor per second is $12{,}000 \times 1255\,\mathcal{H} = 0.151 \times 10^8$ $\mathcal{H}$, and the average generated e.m.f. is 0.151 $\mathcal{H}$ volts. Since this is $= 0.028$ volts, $\mathcal{H} = 0.186$.

18. (2.) In a 550-volt direct-current machine of 8 poles and drum armature, running at 500 rev. per min., the average voltage per commutator segment shall not exceed 11, each armature coil shall contain one turn only, and the number of commutator segments per pole shall be divisible by 3, so as to use the machine as three-phase converter. What is the magnetic flux per field-pole?

550 volts at 11 volts per commutator segment gives 50, or as next integer divisible by 3, $n = 51$ segments or turns per pole.

8 poles give 4 cycles per revolution, 500 revs. per min. gives $500/60 = 8.33$ revs. per sec. Thus the frequency is, $f = 4 \times 8.33$ $= 33.3$ cycles per sec.

The generated e.m.f. is $E = 550$ volts, thus by the formula of direct-current generator,

$$E = 4\,fn\Phi,$$

or,

$$550 = 4 \times 0.333 \times 51\,\Phi,$$

$$\Phi = 8.1 \text{ megalines per pole.}$$

19. (3.) What is the e.m.f. generated in a single turn of a 20-pole alternator running at 200 rev. per min., through a magnetic field of 6.4 megalines per pole?

The frequency is $f = \dfrac{20 \times 200}{2 \times 60} = 33.3$ cycles.

$$e = E_0 \sin \tau,$$
$$E_0 = 2\,\pi fn\Phi,$$
$$\Phi = 6.4,$$
$$n = 1,$$
$$f = 0.333.$$

Thus,

$$E_0 = 2\,\pi \times 0.333 \times 6.4 = 13.4 \text{ volts maximum, or}$$
$$e = 13.4 \sin \theta.$$

4. POWER AND EFFECTIVE VALUES.

20. The power of the continuous e.m.f., E producing continuous current I is $P = EI$.

The e.m.f. consumed by resistance r is $E_1 = Ir$, thus the power consumed by resistance r is $P = I^2r$.

Either $E_1 = E$, then the total power in the circuit is consumed by the resistance, or $E_1 < E$, then only a part of the power is consumed by the resistance, the remainder by some counter e.m.f., $E - E_1$.

If an alternating current $i = I_0 \sin \theta$ passes through a resistance r, the power consumed by the resistance is,

$$i^2r = I_0^2 r \sin^2 \theta = \frac{I_0^2 r}{2}(1 - \cos 2\theta),$$

thus varies with twice the frequency of the current, between zero and $I_0^2 r$.

The average power consumed by resistance r is,

$$\text{avg. } (i^2r) = \frac{I_0^2 r}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 r$$

since avg. $(\cos) = 0$.

Thus the alternating current $i = I_0 \sin \theta$ consumes in a resistance r the same effect as a continuous current of intensity

$$I = \frac{I_0}{\sqrt{2}}.$$

The value $I = \frac{I_0}{\sqrt{2}}$ is called the *effective value* of the alternating current $i = I_0 \sin \theta$; since it gives the same effect.

Analogously $E = \frac{E_0}{\sqrt{2}}$ is the effective value of the alternating e.m.f., $e = E_0 \sin \theta$.

Since $E_0 = 2\pi f n \Phi$, it follows that

$$E = \sqrt{2}\, \pi f n \Phi$$
$$= 4.44\, f n \Phi,$$

is the *effective alternating e.m.f.*, generated in a coil of n turns rotating at a frequency of f (in hundreds of cycles per second) through a magnetic field of Φ megalines of force.

This is the *formula of the alternating-current generator.*

21. The formula of the direct-current generator,

$$E = 4 fn\Phi,$$

holds even if the e.m.fs. generated in the individual turns are not sine waves, since it is the average generated e.m.f.

The formula of the alternating current generator,

$$E = \sqrt{2}\,\pi fn\Phi,$$

does not hold if the waves are not sine waves, since the ratios of average to maximum and of maximum to effective e.m.f. are changed.

If the variation of magnetic flux is not sinusoidal, the effective generated alternating e.m.f. is,

$$E = \gamma\sqrt{2}\,\pi fn\Phi.$$

γ is called the *form factor* of the wave, and depends upon its shape, that is the distribution of the magnetic flux in the magnetic field.

Frequently *form factor* is defined as the ratio of the effective to the average value. This definition is undesirable since it gives for the sine wave, which is always considered the standard wave, a value differing from one.

EXAMPLES.

22. (1.) In a star connected 20-pole three-phase machine, revolving at 33.3 cycles or 200 rev. per min., the magnetic flux per pole is 6.4 megalines. The armature contains one slot per pole and phase, and each slot contains 36 conductors. All these conductors are connected in series. What is the effective e.m.f. per circuit, and what the effective e.m.f. between the terminals of the machine?

Twenty slots of 36 conductors give 720 conductors, or 360 turns in series. Thus the effective e.m.f. is,

$$\begin{aligned} E_1 &= \sqrt{2}\,\pi fn\Phi \\ &= 4.44 \times 0.333 \times 360 \times 6.4 \\ &= 3400 \text{ volts per circuit.} \end{aligned}$$

The e.m.f. between the terminals of a star connected three-phase machine is the resultant of the e.m.fs. of the two phases, which differ by 60° degrees, and is thus 2 sin 60° $= \sqrt{3}$ times that of one phase, thus,

$$E = E_1 \sqrt{3}$$
$$= 5900 \text{ volts effective.}$$

23. (2.) The conductor of the machine has a section of 0 22 sq. cm. and a mean length of 240 cm. per turn. At a resistivity (resistance per unit section and unit length) of copper of $\rho = 1.8 \times 10^{-6}$, what is the e.m.f. consumed in the machine by the resistance, and what the power consumed at 450 kw. output?

450 kw. output is 150,000 watts per phase or circuit, thus the current $I = \frac{150,000}{3400} = 44.2$ amperes effective.

The resistance of 360 turns of 240 cm. length, 0.22 sq. cm. section and 1.8×10^{-6} resistivity, is

$$r = \frac{360 \times 240 \times 1.8 \times 10^{-6}}{0.22} = 0.71 \text{ ohms per circuit.}$$

44.2 amp. $\times$ 0.71 ohms gives 31.5 volts per circuit and $(44.2)^2 \times 0.71 = 1400$ watts per circuit, or a total of $3 \times 1400 = 4200$ watts loss.

24. (3.) What is the self-inductance per wire of a three-phase line of 14 miles length consisting of three wires No. 0 ($l_d = 0.82$ cm.), 45 cm. apart, transmitting the output of this 450 kw. 5900-volt three-phase machine?

450 kw. at 5900 volts gives 44.2 amp. per line. 44.2 amp. effective gives $44.2 \sqrt{2} = 62.5$ amp. maximum.

14 miles = 22,400 m. The magnetic flux produced by I amperes in 1000 m. of a transmission line of 2 wires 45 cm. apart and 0.82 cm. diameter was found in paragraph 1, example 3, as $2\,\Phi = 0.188 \times 10^6\, I$, or $\Phi = 0.094 \times 10^6\, I$ for each wire.

Thus at 22,300 m. and 62.5 amp. maximum, the flux per wire is

$$\Phi = 22.3 \times 62.5 \times 0.094 \times 10^6 = 131 \text{ megalines.}$$

Hence the generated e.m.f., effective value, at 33.3 cycles is,

$$E = \sqrt{2}\, \pi f \Phi$$
$$= 4.44 \times 0.333 \times 131$$
$$= 193 \text{ volts per line;}$$

the maximum value is,

$$E_0 = E \times \sqrt{2} = 273 \text{ volts per line;}$$

and the instantaneous value,

$$e = E_0 \sin(\theta - \theta_1) = 273 \sin(\theta - \theta_1);$$

or, since $\theta = 2\pi f t = 210\, t$, we have,

$$e = 273 \sin 210\,(t - t_1).$$

25. (4.) What is the form factor (*a*) of the e.m.f. generated in a single conductor of a direct-current machine having 80 per cent pole arc and negligible spread of the magnetic flux at the pole corners, and (*b*) what is the form factor of the voltage between two collector rings connected to diametrical points of the armature of such a machine?

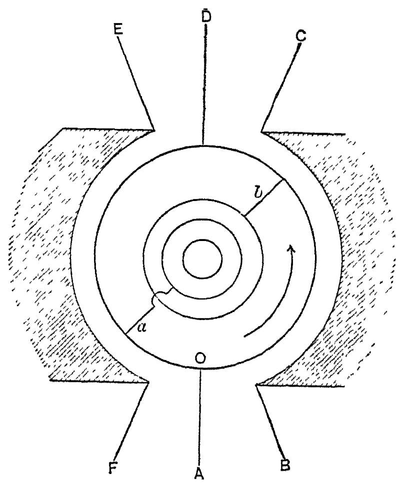


Fig. 6. Diagram of Bipolar Generator.

(*a*.) In a conductor during the motion from position *A*, shown in Fig. 6, to position *B*, no e.m.f. is generated; from position *B* to *C* a constant e.m.f. *e* is generated, from *C* to *E* again no e.m.f., from *E* to *F* a constant e.m.f. $-e$,

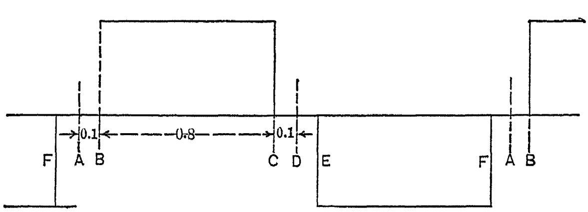


Fig. 7. E.m.f. of a Single Conductor, Direct-Current Machine 80 per cent Pole Arc.

and from *F* to *A* again zero e.m.f. The e.m.f. wave thus is as shown in Fig. 7.

The average e.m.f. is

$$e_1 = 0.8\, e;$$

hence, with this average e.m.f., if it were a sine wave, the maximum e.m.f. would be

$$e_2 = \frac{\pi}{2} e_1 = 0.4\,\pi e,$$

and the effective e.m.f. would be

$$e_3 = \frac{e_2}{\sqrt{2}} = \frac{0.4\,\pi e}{\sqrt{2}}.$$

The actual square of the e.m.f. is e^2 for 80 per cent and zero for 20 per cent of the period, and the average or mean square thus is

$$0.8\, e^2,$$

and therefore the actual effective value,

$$e_4 = e\sqrt{0.8}.$$

The form factor γ, or the ratio of the actual effective value e_4 to the effective value e_3 of a sine wave of the same mean value and thus the same magnetic flux, then is

$$\gamma = \frac{e_4}{e_3} = \frac{\sqrt{10}}{\pi} = 1.006;$$

that is, practically unity.

(*b*.) While the collector leads a, b move from the position F, C, as shown in Fig. 6, to B, E, constant voltage E exists between them, the conductors which leave the field at C being replaced by the conductors entering the field at B. During the motion of the leads a, b from B, E to C, F, the voltage steadily decreases,

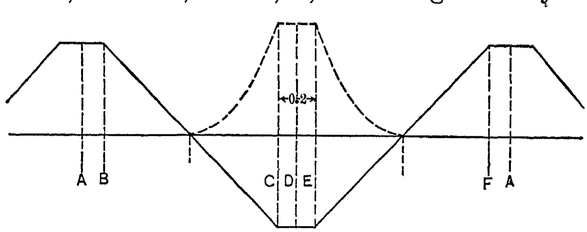


Fig. 8. E.m.f. between two Collector Rings connected to Diametrical Points of the Armature of a Bipolar Machine having 80 per cent Pole Arc.

reverses, and rises again, to $-E$, as the conductors entering the field at E have an e.m.f. opposite to that of the conductors leaving at C. Thus the voltage wave is, as shown by Fig. 8, triangular, with the top cut off for 20 per cent of the half wave.

Then the average e.m.f. is

$$e_1 = 0.2\,E + 2 \times \frac{0.4\,E}{2} = 0.6\,E.$$

The maximum value of a sine wave of this average value is

$$e_2 = \frac{\pi}{2} e_1 = 0.3\,\pi E,$$

and the effective value corresponding thereto is

$$e_3 = \frac{e_2}{\sqrt{2}} = \frac{0.3\,\pi E}{\sqrt{2}}.$$

The actual voltage square is E^2 for 20 per cent of the time, and rising on a parabolic curve from 0 to E^2 during 40 per cent of the time, as shown in dotted lines in Fig. 8.

The area of a parabolic curve is width times one-third of height, or

$$\frac{0.4\,E^2}{3},$$

hence, the mean square of voltage is

$$0.2\,E^2 + 2 \times \frac{0.4\,E^2}{3} = \frac{1.4\,E^2}{3},$$

and the actual effective voltage is

$$e_4 = E\sqrt{\frac{1.4}{3}};$$

hence, the form factor is

$$\gamma = \frac{e_4}{e_3} = \frac{1}{\pi}\sqrt{\frac{280}{27}} = 1.025,$$

or, 2.5 per cent higher than with a sine wave.

5. SELF-INDUCTANCE AND MUTUAL INDUCTANCE.

26. The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in the circuit is called the *inductance* of the circuit.

The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in a second electric circuit is called the *mutual inductance* of the second upon the first circuit. It is equal to the mutual inductance of the first upon the second circuit, as will be seen, and thus is called the mutual inductance between the two circuits.

The number of interlinkages of an electric circuit with the lines of magnetic flux produced by unit current in this circuit and not interlinked with a second circuit is called the *self-inductance* of the circuit.

If i = current in a circuit of n turns, Φ = flux produced thereby and interlinked with the circuit, $n\Phi$ is the total number of interlinkages, and $L = \frac{n\Phi}{i}$ the inductance of the circuit.

If Φ is proportional to the current i and the number of turns n,

$$\Phi = \frac{ni}{\mathcal{R}}, \text{ and } L = \frac{n^2}{\mathcal{R}} \text{ the inductance.}$$

$\mathcal{R}$ is called the *reluctance* and ni the m.m.f. of the magnetic circuit.

In magnetic circuits the reluctance $\mathcal{R}$ has a position similar to that of resistance r in electric circuits.

The reluctance $\mathcal{R}$, and therefore the inductance, is not constant in circuits containing magnetic materials, such as iron, etc.

If $\mathcal{R}_1$ is the reluctance of a magnetic circuit interlinked with two electric circuits of n_1 and n_2 turns respectively, the flux produced by unit current in the first circuit and interlinked with the second circuit is $\frac{n_1}{\mathcal{R}_1}$ and the mutual inductance of the first upon the second circuit is $M = \frac{n_1 n_2}{\mathcal{R}_1}$, that is, equal to the mutual inductance of the second circuit upon the first circuit, as stated above.

If no flux leaks between the two circuits, that is, if all flux is interlinked with both circuits, and L_1 = inductance of the first, L_2 = inductance of the second circuit, and M = mutual inductance, then

$$M^2 = L_1 L_2.$$

If flux leaks between the two circuits, then $M^2 < L_1 L_2$.

In this case the total flux produced by the first circuit consists of a part interlinked with the second circuit also, the mutual inductance, and a part passing between the two circuits, that is, interlinked with the first circuit only, its self-inductance.

27. Thus, if L_1 and L_2 are the inductances of the two circuits, $\frac{L_1}{n_1}$ and $\frac{L_2}{n_2}$ is the total flux produced by unit current in the first and second circuit respectively.

Of the flux $\frac{L_1}{n_1}$ a part $\frac{L_{s_1}}{n_1}$ is interlinked with the first circuit only, L_{s_1} being its leakage inductance, and a part $\frac{M}{n_2}$ interlinked with the second circuit also, M being the mutual inductance and $\frac{L_1}{n_1} = \frac{L_{s_1}}{n_1} + \frac{M}{n_2}$.

Thus, if

L_1 and L_2 = inductance,
L_{s_1} and L_{s_2} = self-inductance,
M = mutual inductance of two circuits of n and n_2 turns respectively, we have

$$\frac{L_1}{n_1} = \frac{L_{s_1}}{n_1} + \frac{M}{n_2} \qquad \frac{L_2}{n_2} = \frac{L_{s_2}}{n_2} + \frac{M}{n_1},$$

or

$$L_1 = L_{s_1} + \frac{n_1}{n_2} M \qquad L_2 = L_{s_2} + \frac{n_2}{n_1} M,$$

or

$$M^2 = (L_1 - L_{s_1})(L_2 - L_{s_2}).$$

The practical unit of inductance is 10^9 times the absolute unit or 10^8 times the number of interlinkages per ampere (since 1 amp. = 0.1 unit current), and is called the *henry* (h); 0.001 of it is called the *milhenry* (mh.).

The number of interlinkages of i amperes in a circuit of L henry inductance is $iL\ 10^8$ lines of force turns, and thus the e.m.f. generated by a change of current di in time dt is

$$e = -\frac{di}{dt} L\, 10^8 \text{ absolute units}$$
$$= -\frac{di}{dt} L \text{ volts.}$$

A change of current of one ampere per second in the circuit of one henry inductance generates one volt.

EXAMPLES.

28. (1.) What is the inductance of the field of a 20-pole alternator, if the 20 field spools are connected in series, each spool contains 616 turns, and 6.95 amperes produces 6.4 megalines per pole?

The total number of turns of all 20 spools is $20 \times 616 = 12{,}320$. Each is interlinked with 6.4×10^6 lines, thus the total number of interlinkages at 6.95 amperes is $12{,}320 \times 6.4 \times 10^6 = 78 \times 10^9$.

6.95 amperes = 0.695 absolute units, hence the number of interlinkages per unit current, or the inductance, is

$$\frac{78 \times 10^9}{0.695} = 112 \times 10^9 = 112 \text{ henrys}.$$

29. (2.) What is the mutual inductance between an alternating transmission line and a telephone wire carried for 10 miles below and 1.20 m. distant from the one, 1.50 m. distant from the other conductor of the alternating line? and what is the e.m.f. generated in the telephone wire, if the alternating circuit carries 100 amperes at 60 cycles?

The mutual inductance between the telephone wire and the electric circuit is the magnetic flux produced by unit current in the telephone wire and interlinked with the alternating circuit, that is, that part of the magnetic flux produced by unit current in the telephone wire, which passes between the distances of 1.20 and 1.50 m.

At the distance l_x from the telephone wire the length of magnetic circuit is $2\pi l_x$. The magnetizing force $\mathcal{H} = \frac{I}{2\pi l_x}$ if I = current in telephone wire in amperes, and the field intensity $\mathcal{K} = 0.4\pi\mathcal{H} = \frac{0.2\,I}{l_x}$, and the flux in the zone dl_x is

$$d\Phi = \frac{0.2\,Il}{l_x} dl_x.$$

$$l = 10 \text{ miles} = 1610 \times 10^3 \text{ cm}.$$

thus,

$$\Phi = \int_{120}^{150} \frac{0.2\,Il}{l_x} dl_x$$

$$= 322 \times 10^3\, I \log_\epsilon \frac{150}{120} = 72\, I\, 10^3;$$

or, $72\, I\, 10^3$ interlinkages, hence, for $I = 10$, or one absolute unit,

thus, $M = 72 \times 10^4$ absolute units, $= 72 \times 10^{-5}$ henrys $=$ 0.72 mh.

100 amperes effective or 141.4 amperes maximum or 14.14 absolute units of current in the transmission line produces a maximum flux interlinked with the telephone line of $14.14 \times 0.72 \times 10^{-3} \times 10^9 = 10.2$ megalines. Thus the e.m.f. generated at 60 cycles is

$$E = 4.44 \times 0.6 \times 10.2 = 27.3 \text{ volts effective.}$$

6. SELF-INDUCTANCE OF CONTINUOUS-CURRENT CIRCUITS.

30. Self-inductance makes itself felt in continuous-current circuits only in starting and stopping or in general when the current changes in value.

Starting of Current. If r = resistance, L = inductance of circuit, E = continuous e.m.f. impressed upon circuit, i = current in circuit at time t after impressing e.m.f. E, and di the increase of current during time moment dt, then the increase of magnetic interlinkages during time dt is

$$Ldi,$$

and the e.m.f. generated thereby is

$$e_1 = -L\frac{di}{dt}.$$

By Lentz's law it is negative, since it is opposite to the impressed e.m.f., its cause.

Thus the e.m.f. acting in this moment upon the circuit is

$$E + e_1 = E - L\frac{di}{dt},$$

and the current is

$$i = \frac{E + e_1}{r} = \frac{E - L\dfrac{di}{dt}}{r};$$

or, transposing,

$$-\frac{rdt}{L} = \frac{di}{i - \dfrac{E}{r}},$$

the integral of which is

$$-\frac{rt}{L} = \log_\epsilon\left(i - \frac{E}{r}\right) - \log_\epsilon c,$$

where $-\log_\epsilon c$ = integration constant.
This reduces to

$$i = \frac{E}{r} + c\varepsilon^{-\frac{rt}{L}}$$

at $t = 0$, $i = 0$, and thus

$$-\frac{E}{r} = c.$$

Substituting this value, the current is

$$i = \frac{E}{r}\left(1 - \varepsilon^{-\frac{rt}{L}}\right),$$

and the e.m.f. of inductance is

$$e_1 = ir - E = -E\varepsilon^{-\frac{rt}{L}}.$$

At $t = \infty$,

$$i_0 = \frac{E}{r}, \qquad e_1 = 0.$$

Substituting these values,

$$i = i_0\left(1 - \varepsilon^{-\frac{rt}{L}}\right)$$

and

$$e_1 = -ri_0\varepsilon^{-\frac{rt}{L}}.$$

The expression $u = \frac{r}{L}$ is called the "*time constant of the circuit.*"*

* The name *time constant* dates back to the early days of telegraphy, where it was applied to the ratio: $\frac{L}{r}$, that is, the reciprocal of what is above denoted as time constant. This quantity which had gradually come into disuse, again became of importance when investigating transient electric phenomena, and in this work it was found more convenient to denote the value $\frac{r}{L}$ as time constant, since this value appears as one term of the more general time constant of the electric circuit: $\left(\frac{r}{L} + \frac{g}{C}\right)$. (Theory and Calculation of Transient Electric Phenomena and Oscillations, Section IV).

Substituted in the foregoing equation this gives

$$i = \frac{E}{r}\left(1 - \varepsilon^{-ut}\right)$$

and

$$e_1 = - E\varepsilon^{-ut}.$$

At $t = \frac{1}{u}$

$$e_1 = -\frac{E}{r} = - 0.368\, E.$$

31. *Stopping of Current.* In a circuit of inductance L and resistance r, let a current $i_0 = \frac{E}{r}$ be produced by the impressed e.m.f. E, and this e.m.f. E be withdrawn and the circuit closed through a resistance r_1.

Let the current be i at the time t after withdrawal of the e.m.f. E and the change of current during time moment dt be di. di is negative, that is, the current decreases.

The decrease of magnetic interlinkages during moment dt is

$$Ldi.$$

Thus the e.m.f. generated thereby is

$$e_1 = - L\frac{di}{dt}.$$

It is negative since di is negative, and e_1 must be positive, that is, in the same direction as E, to maintain the current or oppose the decrease of current, its cause.

Then the current is

$$i = \frac{e_1}{r + r_1} = - \frac{L}{r + r_1}\frac{di}{dt};$$

or, transposing,

$$-\frac{r + r_1}{L}dt = \frac{di}{i},$$

the integral of which is

$$-\frac{r + r_1}{L}t = \log_\epsilon i - \log_\epsilon c,$$

where $- \log_\epsilon c =$ integration constant.

This reduces to $$i = c\varepsilon^{-\frac{r+r_1}{L}t},$$

for $$t = 0, \qquad i_0 = \frac{E}{r} = c.$$

Substituting this value, the current is

$$i = \frac{E}{r}\varepsilon^{-\frac{(r+r_1)t}{L}},$$

and the generated e.m.f. is

$$e_1 = i(r + r_1) = E\frac{r + r_1}{r}\varepsilon^{-\frac{(r+r_1)t}{L}}.$$

Substituting $i_0 = \frac{E}{r}$, the current is

$$i = i_0\varepsilon^{-\frac{r+r_1}{L}t},$$

and the generated e.m.f. is

$$e_1 = i_0(r + r_1)\varepsilon^{-\frac{r+r_1}{L}t}.$$

At $t = 0$,

$$e_1 = E\frac{r + r_1}{r};$$

that is, the generated e.m.f. is increased over the previously impressed e.m.f. in the same ratio as the resistance is increased.

When $r_1 = 0$, that is, when in withdrawing the impressed e.m.f. E the circuit is short-circuited,

$$i = \frac{E}{r}\varepsilon^{-\frac{rt}{L}} = i_0\varepsilon^{-\frac{rt}{L}} \text{ the current, and}$$

$$e_1 = E\,\varepsilon^{-\frac{rt}{L}} = i_0 r\varepsilon^{-\frac{rt}{L}} \text{ the generated e.m.f.}$$

In this case, at $t = 0$, $e_1 = E$, that is, the e.m.f. does not rise.

In the case $r_1 = \infty$, that is, if in withdrawing the e.m.f. E, the circuit is broken, we have $t = 0$ and $e_1 = \infty$, that is, the e.m.f. rises infinitely.

The greater r_1, the higher is the generated e.m.f. e_1, the faster, however, do e_1 and i decrease.

If $r_1 = r$, we have at $t = 0$,

$$e_{11} = 2E, \qquad i = i_0,$$

and $$e_{11} - i_0 r = E;$$

that is, if the external resistance r_1 equals the internal resistance r, at the moment of withdrawal of the e.m.f. E the terminal voltage is E.

The effect of the e.m.f. of inductance in stopping the current at the time t is

$$ie_1 = i_0^2 (r + r_1) \varepsilon^{-2\frac{r+r_1}{L}t};$$

thus the total energy of the generated e.m.f.

$$W = \int_0^\infty ie_1 dt$$

$$= i_0^2 (r + r_1)\left[\varepsilon^{-2\frac{r+r_1}{L}t}\right]_0^\infty \left(-\frac{L}{2(r + r_1)}\right) = \frac{i_0^2 L}{2};$$

that is, the energy stored as magnetism in a circuit of current i_0 and inductance L is

$$W = \frac{i_0^2 L}{2},$$

which is independent both of the resistance r of the circuit and the resistance r_1 inserted in breaking the circuit. This energy has to be expended in stopping the current.

EXAMPLES.

32. (1.) In the alternator field in Section 1, Example 4, Section 2, Example 2, and Section 5, Example 1, how long a time after impressing the required e.m.f. $E = 230$ volts will it take for the field to reach (*a*) $\frac{1}{2}$ strength, (*b*) $\frac{9}{10}$ strength?

(2.) If 500 volts are impressed upon the field of this alternator, and a non-inductive resistance inserted in series so as to give the required exciting current of 6.95 amperes, how long after impressing the e.m.f. $E = 500$ volts will it take for the field to reach (*a*) $\frac{1}{2}$ strength, (*b*) $\frac{9}{10}$ strength, (*c*) and what is the resistance required in the rheostat?

(3.) If 500 volts are impressed upon the field of this alternator without insertion of resistance, how long will it take for the field to reach full strength?

(4.) With full field strength what is the energy stored as magnetism?

(1.) The resistance of the alternator field is 33.2 ohms (Section 2, Example 2), the inductance 112 h. (Section 5, Example 1), the impressed e.m.f. is $E = 230$, the final value of current $i_0 = \frac{E}{r} = 6.95$ amperes. Thus the current at time t, is

$$i = i_0 \left(1 - \varepsilon^{-\frac{rt}{L}}\right)$$
$$= 6.95\ (1 - \varepsilon^{-0.296\,t}).$$

(*a.*) $\frac{1}{2}$ strength $i = \frac{i_0}{2}$, hence $(1 - \varepsilon^{-0.296\,t}) = 0.5$.

$\varepsilon^{-0.296t} = 0.5$, $-0.296\,t \log \varepsilon = \log 0.5$, $t = \frac{-\log 0.5}{0.296 \log \varepsilon}$, and $t =$ 2.34 seconds.

(*b.*) $\frac{9}{10}$ strength: $i = 0.9\ i_0$, hence $(1 - \varepsilon^{-0.296\,t}) = 0.9$, and $t = 7.8$ seconds

(2.) To get $i_0 = 6.95$ amperes, with $E = 500$ volts, a resistance $r = \frac{500}{6.95} = 72$ ohms, and thus a rheostat having a resistance of $72 - 33.2 = 38.8$ ohms is required.

We then have

$$i = i_0 \left(1 - \varepsilon^{-\frac{rt}{L}}\right)$$
$$= 6.95\ (1 - \varepsilon^{-0.643\,t}).$$

(*a.*) $i = \frac{i_0}{2}$, after $t = 1.08$ seconds.

(*b.*) $i = 0.9\ i_0$, after $t = 3.6$ seconds.

(3.) Impressing $E = 500$ volts upon a circuit of $r = 33.2$, $L = 112$, gives

$$i = \frac{E}{r}\left(1 - \varepsilon^{-\frac{rt}{L}}\right)$$
$$= 15.1\ (1 - \varepsilon^{-0.296\,t}).$$

$i = 6.95$, or full field strength, gives

$$6.95 = 15.1\ (1 - \varepsilon^{-0.296\,t}).$$
$$1 - \varepsilon^{-0.296\,t} = 0.46$$
and $t = 2.08$ seconds.

(4.) The stored energy is

$$\frac{i_0^2 L}{2} = \frac{6.95^2 \times 112}{2} = 2720 \text{ watt-seconds or joules}$$
$$= 2000 \text{ foot-pounds.}$$

(1 joule = 0.736 foot-pounds.)

Thus in case (3), where the field reaches full strength in 2.08 seconds, the average power input is $\frac{2000}{2.08} = 960$ foot-pounds per second, = 1.75 hp.

In breaking the field circuit of this alternator, 2000 foot-pounds of energy have to be dissipated in the spark, etc.

33. (5.) A coil of resistance $r = 0.002$ ohm and inductance $L = 0.005$ milhenry, carrying current $I = 90$ amperes, is short-circuited.

(*a.*) What is the equation of the current after short circuit?

(*b.*) In what time has the current decreased to 0.1, its initial value?

(*a.*) $i = I \varepsilon^{-\frac{rt}{L}}$
$= 90\, \varepsilon^{-400\, t}$.

(*b.*) $i = 0.1\, I$, $\varepsilon^{-400t} = 0.1$, after $t = 0.00576$ second.

(6.) When short-circuiting the coil in Example 5, an e.m.f. $E = 1$ volt is inserted in the circuit of this coil, in opposite direction to the current.

(*a.*) What is equation of the current?

(*b.*) After what time does the current become zero?

(*c.*) After what time does the current reverse to its initial value in opposite direction?

(*d.*) What impressed e.m.f. is required to make the current die out in $\frac{1}{2000}$ second?

(*e.*) What impressed e.m.f. E is required to reverse the current in $\frac{1}{1000}$ second?

(*a.*) If e.m.f. $-E$ is inserted, and at time t the current is denoted by i, we have

$$e_1 = -L\frac{di}{dt}, \text{ the generated e.m.f.;}$$

Thus, $$-E + e_1 = -E - L\frac{di}{dt}, \text{ the total e.m.f.};$$

and

$$i = \frac{-E + e_1}{r} = -\frac{E}{r} - \frac{L}{r}\frac{di}{dt}, \text{ the current};$$

Transposing,

$$-\frac{r}{L}dt = \frac{di}{\frac{E}{r} + i},$$

and integrating,

$$-\frac{rt}{L} = \log_\epsilon\left(\frac{E}{r} + i\right) - \log_\epsilon c,$$

where $-\log_\epsilon c$ = integration constant.

At $t = 0$, $i = I$, thus $c = I + \frac{E}{r}$;

Substituting,

$$i = \left(I + \frac{E}{r}\right)\epsilon^{-\frac{rt}{L}} - \frac{E}{r},$$

$$i = 590\,\epsilon^{-400\,t} - 500.$$

(*b.*) $i = 0$, $\epsilon^{-400\,t} = 0.85$, after $t = 0.000405$ second.

(*c.*) $i = -I = -90$, $\epsilon^{-400\,t} = 0.694$, after $t = 0.00091$ second

(*d.*) If $i = 0$ at $t = 0.0005$, then

$$0 = (90 + 500\,E)\,\epsilon^{-0.2} - 500\,E,$$

$$E = \frac{0.18}{\epsilon^{0.2} - 1} = 0.81 \text{ volt.}$$

(*e.*) If $i = -I = -90$ at $t = 0.001$, then

$$-90 = (90 + 500\,E)\,\epsilon^{-0.4} - 500\,E,$$

$$E = \frac{0.18\,(1 + \epsilon^{-0.4})}{1 - \epsilon^{-0.4}} = 0.91 \text{ volt.}$$

7. INDUCTANCE IN ALTERNATING-CURRENT CIRCUITS

34. An alternating current $i = I_0 \sin 2\,\pi f t$ or $i = I_0$ sin can be represented graphically in rectangular coordinates by curved line as shown in Fig. 9, with the instantaneous value i as ordinates and the time t, or the arc of the angle correspond ing to the time, $\theta = 2\,\pi f t$, as abscissas, counting the time from th zero value of the rising wave as zero point.

If the zero value of current is not chosen as zero point of time, the wave is represented by

$$i = I_0 \sin 2\pi f(t - t'),$$

or

$$i = I_0 \sin (\theta - \theta'),$$

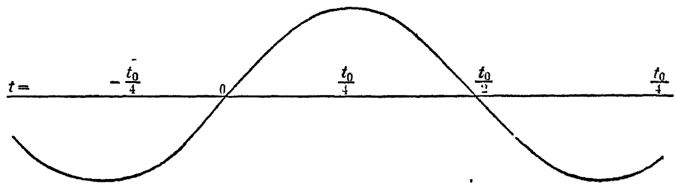


Fig. 9. Alternating Sine Wave.

where t' and θ' are respectively the time and the corresponding angle at which the current reaches its zero value in the ascendant.

If such a sine wave of alternating current $i = I_0 \sin 2\pi ft$ or $i = I_0 \sin \theta$ passes through a circuit of resistance r and inductance L, the magnetic flux produced by the current and thus its interlinkages with the current, $iL = I_0 L \sin \theta$, vary in a wave line similar also to that of the current, as shown in Fig. 10 as Φ.

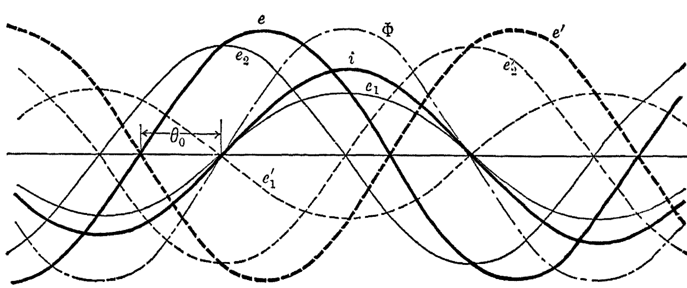


Fig. 10. Self-Induction Effects produced by an Alternating Sine Wave of Current.

The e.m.f. generated hereby is proportional to the change of iL, and is thus a maximum where iL changes most rapidly, or at its zero point, and zero where iL is a maximum, and according to Lentz's law it is positive during falling and negative during rising current. Thus this generated e.m.f. is a wave following the wave of current by the time $t = \frac{t_0}{4}$, where t_0 is time of one complete period, $= \frac{1}{f}$, or by the time angle $\theta = 90°$.

This e.m.f. is called the *counter e.m.f. of inductance.* It is

$$e_2' = -L\frac{di}{dt}$$

$$= -2\,\pi f L I_0 \cos 2\,\pi f t.$$

It is shown in dotted line in Fig. 10 as e_2'.

The quantity $2\,\pi f L$ is called the *inductive reactance* of the circuit, and denoted by x. It is of the nature of a resistance, and expressed in ohms. If L is given in 10^9 absolute units or henrys, x appears in ohms.

The counter e.m.f. of inductance of the current, $i = I_0 \sin 2\,\pi f t = I_0 \sin \theta$, of effective value

$$I = \frac{I_0}{\sqrt{2}}, \text{ is}$$

$$e_2' = -xI_0 \cos 2\,\pi f t = -xI_0 \cos \theta,$$

having a maximum value of xI_0 and an effective value of

$$E_2 = \frac{xI_0}{\sqrt{2}} = xI;$$

that is, the effective value of the counter e.m.f. of inductance equals the reactance, x, times the effective value of the current, I, and lags 90 time degrees, or a quarter period, behind the current.

35. By the counter e.m.f. of inductance,

$$e_2' = -xI_0 \cos \theta,$$

which is generated by the change in flux due to the passage of the current $i = I_0 \sin \theta$ through the circuit of reactance x, an equal but opposite e.m.f.

$$e_2 = xI_0 \cos \theta$$

is consumed, and thus has to be impressed upon the circuit. This e.m.f. is called the *e.m.f. consumed by inductance.* It is 90 time degrees, or a quarter period, ahead of the current, and shown in Fig. 10 as a drawn line e_2.

Thus we have to distinguish between counter e.m.f. of inductance 90 time degrees lagging, and e.m.f. consumed by inductance 90 time degrees leading.

These e.m.fs. stand in the same relation as action and reaction in mechanics. They are shown in Fig. 10 as e_2' and as e_2.

The e.m.f. consumed by the resistance r of the circuit is proportional to the current,

$$e_1 = ri = rI_0 \sin \theta,$$

and in phase therewith, that is, reaches its maximum and its zero value at the same time as the current i, as shown by drawn line e_1 in Fig. 10.

Its effective value is $E_1 = rI$.

The resistance can also be represented by a (fictitious) counter e.m.f.,

$$e_1' = -rI_0 \sin \theta,$$

opposite in phase to the current, shown as e_1' in dotted line in Fig. 10.

The counter e.m.f. of resistance and the e.m.f. consumed by resistance have the same relation to each other as the counter e.m.f. of inductance and the e.m.f. consumed by inductance or inductive reactance.

36. If an alternating current $i = I_0 \sin \theta$ of effective value $I = \frac{I_0}{\sqrt{2}}$ exists in a circuit of resistance r and inductance L, that is, of reactance $x = 2\pi fL$, we have to distinguish:

E.m.f. consumed by resistance, $e_1 = rI_0 \sin \theta$, of effective value $E_1 = rI$, and in phase with the current.

Counter e.m.f. of resistance, $e_1' = -rI_0 \sin \theta$, of effective value $E_1 = rI$, and in opposition or 180 time degrees displaced from the current.

E.m.f. consumed by reactance, $e_2 = xI_0 \cos \theta$, of effective value $E_2 = xI$, and leading the current by 90 time degrees or a quarter period.

Counter e.m.f. of reactance, $e_2' = -xI_0 \cos \theta$, of effective value $E_2' = xI$, and lagging 90 time degrees or a quarter period behind the current.

The e.m.fs. consumed by resistance and by reactance are the e.m.fs. which have to be impressed upon the circuit to overcome the counter e.m.fs. of resistance and of reactance.

Thus, the total counter e.m.f. of the circuit is

$$e' = e_1' + e_2' = -I_0 (r \sin \theta + x \cos \theta),$$

and the total impressed e.m.f., or e.m.f. consumed by the circuit, is

$$e = e_1 + e_2 = I_0 (r \sin \theta + x \cos \theta).$$

Substituting

$$\frac{x}{r} = \tan \theta_0 \text{ and}$$

$$\sqrt{r^2 + x^2} = z,$$

it follows that

$$x = z \sin \theta_0, \qquad r = z \cos \theta_0,$$

and we have as the total impressed e.m.f.

$$e = zI_0 \sin (\theta + \theta_0),$$

shown by heavy drawn line e in Fig. 10, and total counter e.m.f.

$$e' = - zI_0 \sin (\theta + \theta_0),$$

shown by heavy dotted line e' in Fig. 10, both of effective value

$$e = zI.$$

For $\theta = - \theta_0$, $e = 0$, that is, the zero value of e is ahead of the zero value of current by the time angle θ_0, or the current lags behind the impressed e.m.f. by the angle θ_0.

θ_0 is called the *angle of time lag* of the current, and $z = \sqrt{r^2 + x^2}$ the *impedance* of the circuit. e is called the e.m.f. consumed by impedance, e' the counter e.m.f. of impedance.

Since $E_1 = rI$ is the e.m.f. consumed by resistance,

$E_2 = xI$ is the e.m.f. consumed by reactance,

and $E = zI = \sqrt{r^2 + x^2}\, I$ is the e.m.f. consumed by impedance,

we have

$$E = \sqrt{E_1^2 + E_2^2}, \text{ the total e.m.f.}$$

and

$$E_1 = E \cos \theta_0,$$

$$E_2 = E \sin \theta_0, \text{ its components.}$$

The tangent of the angle of lag is

$$\tan \theta = \frac{x}{r} = \frac{2 \pi f L}{r},$$

and the time constant of the circuit is

$$\frac{r}{L} = 2 \pi f \cot \theta_0.$$

The total e.m.f., e, impressed upon the circuit consists of two components, one, e_1, in phase with the current, the other one, e_2, in quadrature with the current.

Their effective values are

$$E, E \cos \theta_0, E \sin \theta_0.$$

EXAMPLES.

37. (1.) What is the reactance per wire of a transmission line of length l, if l_d = diameter of the wire, l_s = spacing of the wires, and f = frequency?

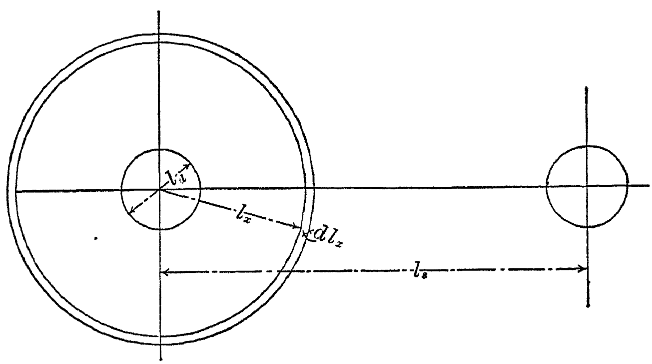


Fig. 11. Diagram for Calculation of Inductance between two Parallel Conductors.

If I = current, in absolute units, in one wire of the transmission line, the m.m.f. is I; thus the magnetizing force in a zone dl_x at distance l_x from center of wire (Fig. 11) is $\mathfrak{H} = \frac{I}{2\pi l_x}$ and the field intensity in this zone is $\mathcal{H} = 4\pi \mathfrak{H} = 2\frac{I}{l_x}$. Thus the magnetic flux in this zone is

$$d\Phi = \mathcal{H}\, l dl_x = \frac{2\, Il dl_x}{l_x};$$

hence, the total magnetic flux between the wire and the return wire is

$$\Phi = \int_{\frac{l_d}{2}}^{l_s} d\Phi = 2\, Il \int_{\frac{l_d}{2}}^{l_s} \frac{dl_x}{l_x} = 2\, Il \log_\epsilon \frac{2\, l_s}{l_d},$$

neglecting the flux inside the transmission wire.

The inductance is

$$L = \frac{\Phi}{I} = 2\,l \log_\epsilon \frac{2\,l_s}{l_d} \text{ absolute units}$$

$$= 2\,l \log_\epsilon \frac{2\,l_s}{l_d}\,10^{-9} \text{ henrys,}$$

and the reactance $x = 2\,\pi f L = 4\,\pi f l \log_\epsilon \frac{2\,l_s}{l_d}$, in absolute units;

or $$x = 4\,\pi f l \log_\epsilon \frac{2\,l_s}{l_d}\,10^{-9}, \text{ in ohms.}$$

38. (2.) The voltage at the receiving end of a 33.3-cycle threephase transmission line 14 miles in length shall be 5500 between the lines. The line consists of three wires, No. 0 B. & S. (l_d = 0.82 cm.), 18 in. (45 cm.) apart, of resistivity $\rho = 1.8 \times 10^{-6}$.

(*a.*) What is the resistance, the reactance, and the impedance per line, and the voltage consumed thereby at 44 amperes?

(*b.*) What is the generator voltage between lines at 44 amperes to a non-inductive load?

(*c.*) What is the generator voltage between lines at 44 amperes to a load circuit of 45 time degrees lag?

(*d.*) What is the generator voltage between lines at 44 amperes to a load circuit of 45 time degrees lead?

Here l = 14 miles = $14 \times 1.6 \times 10^5 = 2.23 \times 10^6$ cm.

l_d = 0.82 cm.

Hence the cross section, A = 0.528 sq. cm.

(*a.*) Resistance per line, $r = \rho \frac{l}{A} = \frac{1.8 \times 10^{-6} \times 2.23 \times 10^6}{0.528}$ = 7.60 ohms.

Reactance per line, $x = 4\,\pi f l \log_\epsilon \frac{2\,l_s}{l_d} \times 10^{-9} = 4\,\pi \times 33.3 \times 2.23 \times 10^6 \times \log_\epsilon 110 \times 10^{-9}$ = 4.35 ohms.

The impedance per line, $z = \sqrt{r^2 + x^2}$ = 8.76 ohms. Thus if I = 44 amperes per line,

the e.m.f. consumed by resistance is $E_1 = rI$ = 334 volts,
the e.m.f. consumed by reactance is $E_2 = xI$ = 192 volts,
and the e.m.f. consumed by impedance is $E_3 = zI$ = 385 volts.

(*b.*) 5500 volts between lines at receiving circuit give $\frac{5500}{\sqrt{3}} =$ 3170 volts between line and neutral or zero point (Fig. 12), or per line, corresponding to a maximum voltage of $3170\sqrt{2} =$ 4500 volts. 44 amperes effective per line gives a maximum value of $44\sqrt{2} = 62$ amperes.

Denoting the current by $i = 62 \sin \theta$, the voltage per line at the receiving end with non-inductive load is $e = 4500 \sin \theta$.

The e.m.f. consumed by resistance, in phase with the current, of effective value 334, and maximum value $334\sqrt{2} = 472$, is

$$e_1 = 472 \sin \theta.$$

The e.m.f. consumed by reactance, 90 time degrees ahead of the current, of effective value 192, and maximum value $192\sqrt{2} = 272$, is

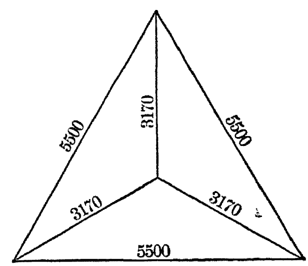


Fig. 12. Voltage Diagram for a Three-Phase Circuit.

$$e_2 = 272 \cos \theta.$$

Thus the total voltage required per line at the generator end of the line is

$$\begin{aligned} e_0 = e + e_1 + e_2 &= (4500 + 472) \sin \theta + 272 \cos \theta \\ &= 4972 \sin \theta + 272 \cos \theta. \end{aligned}$$

Denoting $\frac{272}{4972} = \tan \theta_0$, we have

$$\sin \theta_0 = \frac{\tan \theta_0}{\sqrt{1 + \tan^2 \theta_0}} = \frac{272}{4980}.$$

$$\cos \theta_0 = \frac{1}{\sqrt{1 + \tan^2 \theta_0}} = \frac{4972}{4980}.$$

Hence, $$\begin{aligned} e_0 &= 4980 (\sin \theta \cos \theta_0 + \cos \theta \sin \theta_0) \\ &= 4980 \sin (\theta + \theta_0). \end{aligned}$$

Thus θ_0 is the lag of the current behind the e.m.f. at the generator end of the line, = 3.2 time degrees, and 4980 the maximum voltage per line at the generator end; thus $E_0 = \frac{4980}{\sqrt{2}}$ $= 3520$, the effective voltage per line, and $3520 \sqrt{3} = 6100$, the effective voltage between the lines at the generator.

(*c*.) If the current

$$i = 62 \sin \theta$$

lags in time 45 degrees behind the e.m.f. at the receiving end of the line, this e.m.f. is expressed by

$$e = 4500 \sin (\theta + 45) = 3170 (\sin \theta + \cos \theta);$$

that is, it leads the current by 45 time degrees, or is zero at $\theta = -45$ time degrees.

The e.m.f. consumed by resistance and by reactance being the same as in (*b*), the generator voltage per line is

$$e_0 = e + e_1 + e_2 = 3642 \sin \theta + 3442 \cos \theta.$$

Denoting $\frac{3442}{3642} = \tan \theta_0$, we have

$$e_0 = 5011 \sin (\theta + \theta_0).$$

Thus θ_0, the time angle of lag of the current behind the generator e.m.f., is 43 degrees, and 5011 the maximum voltage; hence 3550 the effective voltage per line, and $3550 \sqrt{3} = 6160$ the effective voltage between lines at the generator.

(*d*.) If the current $i = 62 \sin \theta$ leads the e.m.f. by 45 time degrees, the e.m.f. at the receiving end is

$$\begin{aligned} e &= 4500 \sin (\theta - 45) \\ &= 3170 (\sin \theta - \cos \theta). \end{aligned}$$

Thus at the generator end

$$e_0 = e + e_1 + e_2 = 3642 \sin \theta - 2898 \cos \theta.$$

Denoting $\frac{2898}{3642} = \tan \theta_0$, it is

$$e_0 = 4654 \sin (\theta - \theta_0).$$

Thus θ_0, the time angle of lead at the generator, is 39 degrees, and 4654 the maximum voltage; hence 3290 the effective voltage per line and 5710 the effective voltage between lines at the generator.

8. POWER IN ALTERNATING-CURRENT CIRCUITS.

39. The power consumed by alternating current $i = I_0 \sin \theta$, of effective value $I = \frac{I_0}{\sqrt{2}}$, in a circuit of resistance r and reactance $x = 2 \pi f L$, is

$$p = ei,$$

where $e = zI_0 \sin (\theta + \theta_0)$ is the impressed e.m.f., consisting of the components

$e_1 = rI_0 \sin \theta$, the e.m.f. consumed by resistance

and $e_2 = xI_0 \cos \theta$. the e.m.f. consumed by reactance.

$z = \sqrt{r^2 + x^2}$ is the impedance and $\tan \theta_0 = \frac{x}{r}$ the time-phase angle of the circuit; thus the power is

$$\begin{aligned} p &= zI_0^2 \sin \theta \sin (\theta + \theta_0) \\ &= \frac{zI_0^2}{2} (\cos \theta_0 - \cos (2 \theta + \theta_0)) \\ &= zI^2 (\cos \theta_0 - \cos (2 \theta + \theta_0)). \end{aligned}$$

Since the average $\cos (2 \theta + \theta_0) =$ zero, the average power is

$$\begin{aligned} P &= zI^2 \cos \theta_0 \\ &= rI^2 = E_1 I; \end{aligned}$$

that is, the power in the circuit is that consumed by the resistance, and independent of the reactance.

Reactance or self-inductance consumes no power, and the e.m.f. of self-inductance is a *wattless e.m.f.*, while the e.m.f. of resistance is a *power e.m.f.*

The wattless e.m.f. is in quadrature, the power e.m.f. in phase with the current.

In general, if $\theta =$ angle of time-phase displacement between the resultant e.m.f. and the resultant current of the circuit, $I =$ current, $E =$ impressed e.m.f., consisting of two components, one, $E_1 = E \cos \theta$, in phase with the current, the other, $E_2 = E \sin \theta$, in quadrature with the current, the power in the circuit is $IE_1 = IE \cos \theta$, and the e.m.f. in phase with the current $E_1 = E \cos \theta$ is a power e.m.f., the e.m.f. in quadrature with the current $E_2 = E \sin \theta$ a wattless or *reactive* e.m.f.

40. Thus we have to distinguish *power e.m.f.* and *wattless* or *reactive e.m.f.*, or power component of e.m.f., in phase with the current and wattless or reactive component of e.m.f., in quadrature with the current.

Any e.m.f. can be considered as consisting of two components, one, the power component, e_1, in phase with the current, and the other, the reactive component, e_2, in quadrature with the current. The sum of instantaneous values of the two components is the total e.m.f.

$$e = e_1 + e_2.$$

If E, E_1, E_2 are the respective effective values, we have

$$E = \sqrt{E_1^2 + E_2^2}, \text{ since}$$
$$E_1 = E \cos \theta,$$
$$E_2 = E \sin \theta,$$

where θ = time-phase angle between current and e.m.f.

Analogously, a current I due to an impressed e.m.f. E with a time-phase angle θ can be considered as consisting of two component currents,

$I_1 = I \cos \theta$, the *power* component of the current, and
$I_2 = I \sin \theta$, the *wattless* or *reactive* component of the current.

The sum of instantaneous values of the power and reactive components of the current equals the instantaneous value of the total current,

$$i_1 + i_2 = i,$$

while their effective values have the relation

$$I = \sqrt{I_1^2 + I_2^2}.$$

Thus an alternating current can be resolved in two components, the power component, in phase with the e.m.f., and the wattless or reactive component, in quadrature with the e.m.f.

An alternating e.m.f. can be resolved in two components, the power component, in phase with the current and the wattless or reactive component, in quadrature with the current.

The power in the circuit is the current times the e.m.f. times the cosine of the time phase angle, or is the power component of the current times the total e.m.f., or the power component of the e.m.f. times the total current.

EXAMPLES.

41. (1.) What is the power received over the transmission line in Section 7, Example 2, the power lost in the line, the power put into the line, and the efficiency of transmission with non-inductive load, with 45-time-degree lagging load and 45-degree leading load?

The power received per line with non-inductive load is $P = EI = 3170 \times 44 = 139$ kw.

With a load of 45 time degrees phase displacement, $P = EI \cos 45° = 98$ kw.

The power lost per line $P_1 = I^2R = 44^2 \times 7.6 = 14.7$ kw.

Thus the input into the line $P_0 = P + P_1 = 151.7$ kw. at non-inductive load,

and $\qquad = 111.7$ kw. at load of 45 degrees phase displacement.

The efficiency with non-inductive load is

$$\frac{P}{P_0} = 1 - \frac{14.7}{151.7} = 90.3 \text{ per cent,}$$

and with a load of 45 degrees phase displacement is

$$\frac{P}{P_0} = 1 - \frac{14.7}{111.7} = 86.8 \text{ per cent.}$$

The total output is $3\,P = 411$ kw. and 291 kw., respectively.
The total input $3\,P_0 = 451.1$ kw. and 335.1 kw., respectively.

9. POLAR COORDINATES.

42. In polar coordinates, alternating waves are represented, with the instantaneous values as radius vectors, and the time as an angle, counting left-handed or counter clockwise, and one revolution or 360 degrees representing one complete period.

The sine wave of alternating current $i = I_0 \sin \theta$ is represented by a circle (Fig. 13) with the vertical axis as diameter, equal in length $\overline{OI_0}$ to the maximum value I_0, and shown as heavy drawn circle.

The e.m.f. consumed by inductance, $e_2 = xI_0 \cos \theta$, is represented by a circle with diameter $\overline{OE_2}$ in horizontal direction to the right, and equal in length to the maximum value, xI_0.

Analogously, the counter e.m.f. of self-inductance E_2' is represented by a circle $\overline{OE_2'}$, in Fig. 13; the e.m.f. consumed by resistance r by circle $\overline{OE_1}$ of a diameter $= E_1 = rI_0$, and the counter e.m.f. of resistance E_1' by circle $\overline{OE_1'}$.

The counter e.m.f. of impedance is represented by circle $\overline{OE'}$ of a diameter equal in length to E', and lagging $180 - \theta_0$ behind the diameter of the current circle. This circle passes through the points E_1' and E_2', since at the moment $\theta = 180$ degrees,

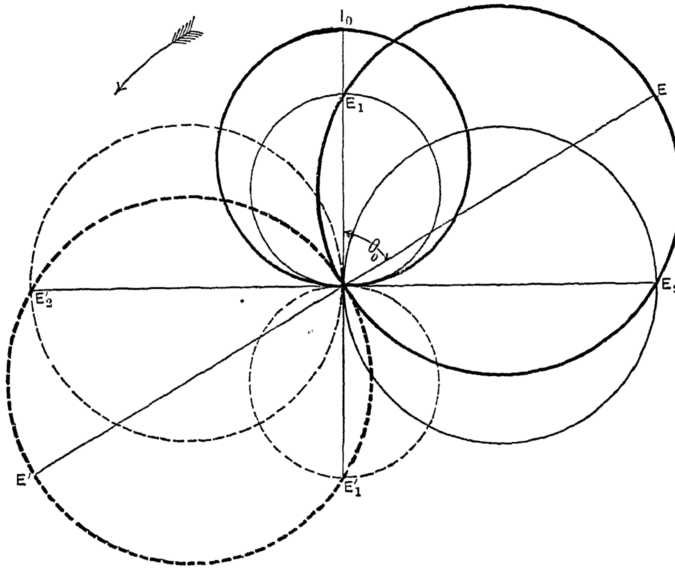


Fig. 13. Sine waves represented in Polar Coordinates.

$e_1' = 0$, and thus the counter e.m.f. of impedance equals the counter e.m.f. of reactance $e' = e_2'$, and at $\theta = 270$ degrees, $e_2' = 0$, and the counter e.m.f. of impedance equals the counter e.m.f. of resistance $e' = e_1'$.

The e.m.f. consumed by impedance, or the impressed e.m.f., is represented by circle $\overline{OE}$ having a diameter equal in length to E, and leading the diameter of the current circle by the angle θ_0. This circle passes through the points E_1 and E_2.

An alternating wave is determined by the length and direction of the diameter of its polar circle. The length is the maximum value or intensity of the wave, the direction the phase of the maximum value, generally called the phase of the wave.

43. Usually alternating waves are represented in polar coordinates by mere vectors, the diameters of their polar circles, and the circles omitted, as in Fig. 14.

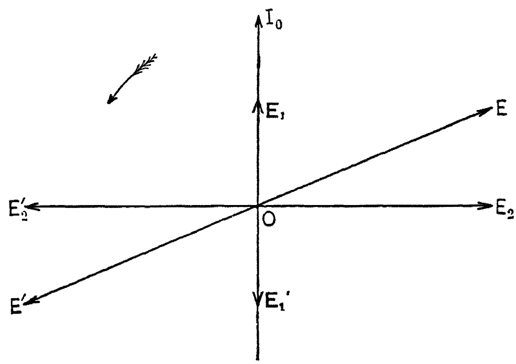


Fig. 14. Vector Diagram.

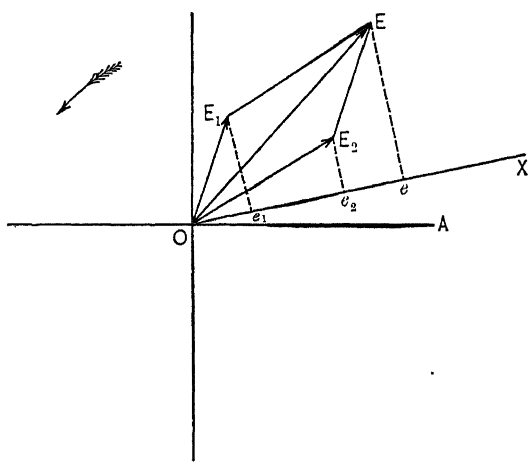


Fig. 15. Vector Diagram of two e.m.fs. Acting in the same Circuit.

Two e.m.fs., e_1 and e_2, acting in the same circuit, give a resultant e.m.f. e equal to the sum of their instantaneous values. In polar coordinates e_1 and e_2 are represented in intensity and in phase by two vectors, OE_1 and OE_2, Fig. 15. The instantaneous values in any direction $\overline{OX}$ are the projections $\overline{Oe_1}$, $\overline{Oe_2}$ of $\overline{OE_1}$ and $\overline{OE_2}$ upon this direction.

Since the sum of the projections of the sides of a parallelogram is equal to the projection of the diagonal, the sum of the projections $\overline{Oe_1}$ and $\overline{Oe_2}$ equals the projection $\overline{Oe}$ of $\overline{OE}$, the diagonal of the parallelogram with $\overline{OE_1}$ and $\overline{OE_2}$ as sides, and $\overline{OE}$ is thus the diameter of the circle of resultant e.m.f.; that is, in polar coordinates alternating sine waves of e.m.f., current, etc., are combined and resolved by the parallelogram or polygon of sine waves.

Since the effective values are proportional to the maximum values, the former are generally used as the length of vector of the alternating wave. In this case the instantaneous values are given by a circle with $\sqrt{2}$ times the vector as diameter.

44. As phase of the first quantity considered, as in the above instance the current, any direction can be chosen. The further quantities are determined thereby in direction or phase.

In polar coordinates, as phase of the current, etc., is here and in the following understood the time or the angle of its vector, that is, the time of its *maximum* value, and a current of phase zero would thus be denoted analytically by $i = I_0 \cos \theta$.

The zero vector $\overline{OA}$ is generally chosen for the most frequently used quantity or reference quantity, as for the current, if a number of e.m.fs. are considered in a circuit of the same current, or for the e.m.f., if a number of currents are produced by the same e.m.f., or for the generated e.m.f. in apparatus such as transformers and induction motors, synchronous apparatus, etc.

With the current as zero vector, all horizontal components of e.m.f. are power components, all vertical components are reactive components.

With the e.m.f. as zero vector, all horizontal components of current are power components, all vertical components of current are reactive components.

By measurement from the polar diagram numerical values can hardly ever be derived with sufficient accuracy, since the magnitudes of the different quantities used in the same diagram are usually by far too different, and the polar diagram is therefore useful only as basis for trigonometrical or other calculation, and to give an insight into the mutual relation of the different quantities, and even then great care has to be taken to distinguish between the two equal but opposite vectors, counter e.m.f. and e.m.f. consumed by the counter e.m.f., as explained before.

45. In the polar coordinates described in the preceding, and used throughout this book, the angle represents the time, and is counted positive in left-handed or counter-clockwise rotation, with the instantaneous values of the periodic function as radii, so that the periodic function is represented by a closed curve, and one revolution or 360 degrees as one period. This "*time diagram*" is the polar coordinate system universally used in other sciences to represent periodic phenomena, as the cosmic motions in astronomy, and even the choice of counter-clockwise as positive rotation is retained from the custom of astronomy, the rotation of the earth being such.

In the time diagram, the sine wave is given by a circle, and this circle of instantaneous values of the sine wave is represented, in size or position, by its diameter. That is, the position of this diameter denotes the time, t, or angle, $\theta = 2\pi f t$, at which the sine wave reaches its maximum value, and the length of this diameter denotes the intensity of the maximum value.

If then, in polar coordinate representation, Fig. 16, $\overline{OE}$ denotes an e.m.f., $\overline{OI}$ a current, this means that the maximum value of e.m.f. equals $\overline{OE}$, and is reached at the time, t_1, represented by angle $AOE = \theta_1 = 2\pi f t_1$. The current in this diagram then has a maximum value equal to $\overline{OI}$, and this maximum value is reached at the time, t_2, represented by angle $AOI = \theta_2 = 2\pi f t_2$.

If then angle $\theta_2 > \theta_1$, this means that the current reaches its maximum value later than the e.m.f., that is, the current in Fig. 16 lags behind the e.m.f., by the angle $EOI = \theta_2 - \theta_1 = 2\pi f (t_2 - t_1)$, or by the time $t_2 - t_1$.

Frequently in electrical engineering another system of polar coordinates is used, the so-called "*crank diagram.*" In this, sine waves of alternating currents and e.m.fs. are represented as projections of a revolving vector upon the horizontal. That is, a vector, equal in length to the maximum of the alternating wave, revolves at uniform speed so as to make a complete revolution per period, and the projections of this revolving vector upon the horizontal then denote the instantaneous values of the wave.

Obviously, by the crank diagram only sine waves can be represented, while the time diagram permits the representation of any wave shape, and therefore is preferable.

Let, for instance, $\overline{OI}$ represent in length the maximum value of current, $i = I \cos (\theta - \theta_2)$. Assume, then, a vector, $\overline{OI}$, to

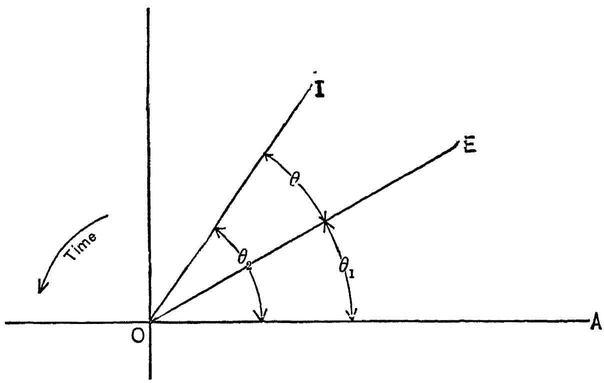


Fig. 16. Representation of Current and e.m.f. by Polar Coordinates.

revolve, left-handed or in positive direction, so that it makes a complete revolution during each cycle or period. If then at a certain moment of time, this vector stands in position $\overline{OI}_1$ (Fig. 17), the projection, $\overline{OA}_1$, of $\overline{OI}_1$ on $\overline{OA}$ represents the instantaneous value of the current at this moment. At a later moment $\overline{OI}$ has moved farther, to $\overline{OI}_2$, and the projection, $\overline{OA}_2$, of $\overline{OI}_2$ on $\overline{OA}$ is the instantaneous value. The diagram thus shows the instantaneous condition of the sine waves. Each sine wave reaches the maximum at the moment when its revolving vector, $\overline{OI}$, passes the horizontal.

If Fig. 18 represents the crank diagram of an e.m.f., $\overline{OE}$, and a current, $\overline{OI}$, and if angle $AOE > AOI$, this means that the e.m.f., $\overline{OE}$, is ahead of the current, $\overline{OI}$, passes during the revolution the zero line or line of maximum intensity, $\overline{OA}$, earlier than the current, or leads; that is, the current lags behind the e.m.f. The same Fig. 18 considered as polar diagram would mean that the current leads the e.m.f.; that is, the maximum value of current, $\overline{OI}$, occurs at a smaller angle, AOI, that is, at an earlier time, than the maximum value of the e.m.f., $\overline{OE}$.

46. In the crank diagram, the first quantity therefore can be put in any position. For instance, the current, $\overline{OI}$, in Fig. 18, could be drawn in position $\overline{OI}$, Fig. 19. The e.m.f. then being

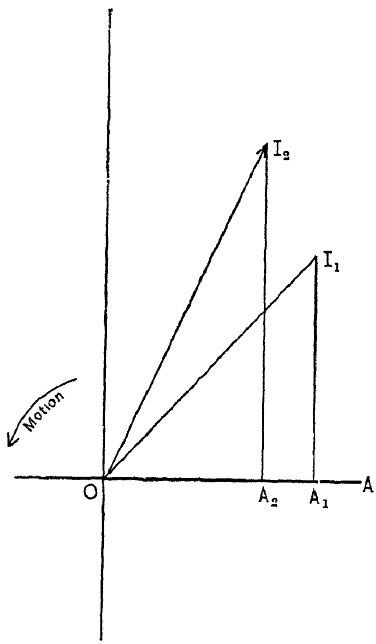


Fig. 17. Crank Diagram showing instantaneous values.

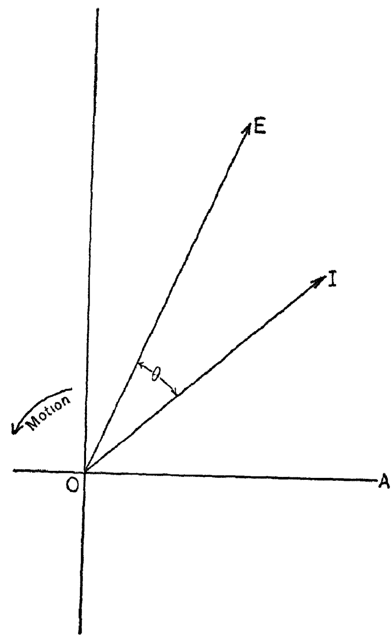


Fig. 18. Crank Diagram of an e.m.f. and Current.

ahead of the current by angle $EOI = \theta$ would come into the position $\overline{OE}$, Fig. 19.

A polar diagram, Fig. 16, with the current, $\overline{OI}$, lagging behind the e.m.f., $\overline{OE}$, by the angle, θ, thus considered as crank diagram would represent the current leading the e.m.f. by the angle, θ, and a crank diagram, Fig. 18 or 19, with the current lagging behind the e.m.f. by the angle, θ, would as polar diagram represent a current leading the e.m.f. by the angle, θ.

The main difference in appearance between the crank diagram and the polar diagram therefore is that, with the same direction of rotation, lag in the one diagram is represented in the same manner as lead in the other diagram, and inversely. Or, a representation by the crank diagram looks like a representation by the polar diagram, with reversed direction of rotation, and vice versa. Or, the one diagram is the image of the other and can be transformed into it by reversing right and left, or top and bottom. Therefore the crank diagram, Fig. 19, is the image of the polar diagram, Fig. 16.

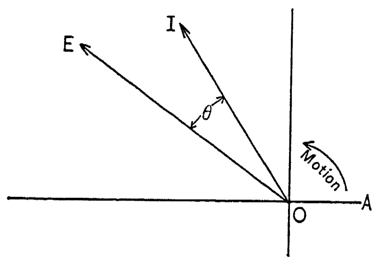


Fig. 19. Crank Diagram.

Since the time diagram, in which the position of the vector represents its phase, that is, the moment of its maximum value, is used in all other sciences, and also is preferable in electrical engineering, it will be exclusively used in the following, the positive direction being represented as counter-clockwise.

EXAMPLES.

47. In a three-phase long-distance transmission line, the voltage between lines at the receiving end shall be 5000 at no load, 5500 at full load of 44 amperes power component, and proportional at intermediary values of the power component of the current; that is, the voltage at the receiving end shall increase proportional to the load. At three-quarters load the current shall be in phase with the e.m.f. at the receiving end. The generator excitation however and thus the (nominal) generated e.m.f. of the generator shall be maintained constant at all loads, and the voltage regulation effected by producing lagging or leading currents with a synchronous motor in the receiving circuit. The line has a resistance $r_1 = 7.6$ ohms and a reactance $x_1 = 4.35$ ohms per wire, the generator is star connected, the resistance per circuit being $r_2 = 0.71$, and the (synchronous) reactance is $x_2 = 25$ ohms. What must be the wattless or

reactive component of the current, and therefore the total current and its phase relation at no load, one-quarter load, one-half load, three-quarters load, and full load, and what will be the terminal voltage of the generator under these conditions?

The total resistance of the line and generator is $r = r_1 + r_2 = 8.31$ ohms; the total reactance, $x = x_1 + x_2 = 29.35$ ohms.

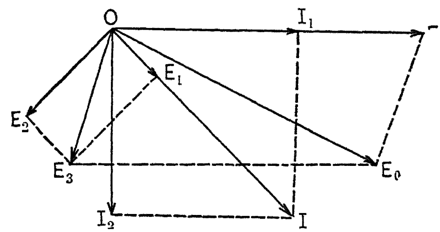


Fig. 20. Polar Diagram of e.m.f. and Current in Transmission Line. Current Leading.

Let, in the polar diagram, Fig. 20 or 21, $\overline{OE} = E$ represent the voltage at the receiving end of the line, $\overline{OI_1} = I_1$ the power component of the current corresponding to the load, in phase with $\overline{OE}$, and $\overline{OI_2} = I_2$ the reactive component of the current

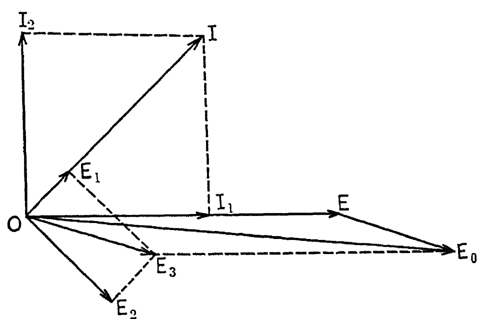


Fig. 21. Polar Diagram of e.m.f. and Current in Transmission Line. Current Lagging.

in quadrature with $\overline{OE}$, shown leading in Fig. 20, lagging in Fig. 21.

We then have total current $I = \overline{OI}$.

Thus the e.m.f. consumed by resistance, $\overline{OE_1} = rI$, is in phase with I, the e.m.f. consumed by reactance, $\overline{OE_2} = xI$, is 90 degrees ahead of I, and their resultant is $\overline{OE_3}$, the e.m.f. consumed by impedance.

$\overline{OE_3}$ combined with $\overline{OE}$, the receiver voltage, gives the generator voltage $\overline{OE_0}$.

Resolving all e.m.fs. and currents into components in phase and in quadrature with the received voltage E, we have

	Phase Component.	Quadrature Component.
Current	I_1	I_2
E.m.f. at receiving end of line, $E =$	E	0
E.m.f. consumed by resistance, $E_1 =$	rI_1	rI_2
E.m.f. consumed by reactance, $E_2 =$	xI_2	$-xI_1$
Thus total e.m.f. or generator voltage,		
$E_0 = E + E_1 + E_2 =$	$E + rI_1 + xI_2$	$rI_2 - xI_1$

Herein the reactive lagging component of current is assumed as positive, the leading as negative.

The generator e.m.f. thus consists of two components, which give the resultant value

$$E_0 = \sqrt{(E + rI_1 + xI_2)^2 + (rI_2 - xI_1)^2};$$

substituting numerical values, this becomes

$$E_0 = \sqrt{(E + 8.31\, I_1 + 29.35\, I_2)^2 + (8.31\, I_2 - 29.35\, I_1)^2};$$

at three-quarters load,

$$E = \frac{5375}{\sqrt{3}} = 3090 \text{ volts per circuit,}$$

$$I_1 = 33, \qquad I_2 = 0, \text{ thus}$$

$$E_0 = \sqrt{(3090 + 8.31 \times 33)^2 + (29.35 \times 33)^2} = 3520 \text{ volts}$$

per line or $3520 \times \sqrt{3} = 6100$ volts between lines as (nominal) generated e.m.f. of generator.

Substituting these values, we have

$$3520 = \sqrt{(E + 8.31\, I_1 + 29.35\, I_2)^2 + (8.31\, I_2 - 29.35\, I_1)^2}.$$

The voltage between the lines at the receiving end shall be:

	No Load.	¼ Load.	½ Load.	¾ Load.	Full Load.
Voltage between lines,	5000	5125	5250	5375	5500
Thus, voltage per line, $\div \sqrt{3}$, $E =$	2880	2950	3020	3090	3160

The power components of current per line, $I_1 =$ 0 11 22 33 44

Herefrom we get by substituting in the above equation

	No Load.	¼ Load.	½ Load.	¾ Load.	Full Load.
Reactive component of current, $I_2 =$	21.6	16.2	9.2	0	−9.7
hence, the total current, $I = \sqrt{I_1^2 + I_2^2} =$	21.6	19.6	23.9	33.0	45.05
and the power factor, $\frac{I_1}{I} = \cos\theta =$	0	56.0	92.0	100.0	97.7
the lag of the current, $\theta =$	90°	61°	23°	0°	−11.5°

the generator terminal voltage per line is

$$
\begin{aligned}
E' &= \sqrt{(E + r_1 I_1 + x_1 I_2)^2 + (r_1 I_2 - x_1 I_1)^2} \\
&= \sqrt{(E + 7.6\, I_1 + 4.35\, I_2)^2 + (7.6\, I_2 - 4.35\, I_1)^2}
\end{aligned}
$$

thus:

	No Load.	¼ Load.	½ Load.	¾ Load.	Full Load
Per line, $E' =$	2980	3106	3228	3344	3463
Between lines, $E'\sqrt{3} =$	5200	5400	5600	5800	6000

Therefore at constant excitation the generator voltage rises with the load, and is approximately proportional thereto.

10. HYSTERESIS AND EFFECTIVE RESISTANCE.

48. If an alternating current $\overline{OI} = I$, in Fig. 22, exists in a circuit of reactance $x = 2\,\pi f L$ and of negligible resistance, the magnetic flux produced by the current, $\overline{O\Phi} = \Phi$, is in phase with the current, and the e.m.f. generated by this flux, or counter e.m.f. of self-inductance, $\overline{OE'''} = E''' = xI$, lags 90 degrees behind the current. The e.m.f. consumed by self-inductance or impressed e.m.f. $\overline{OE''} = E'' = xI$ is thus 90 degrees ahead of the current.

Inversely, if the e.m.f. $\overline{OE''} = E''$ is impressed upon a circuit of reactance $x = 2\,\pi f L$ and of negligible resistance, the current $\overline{OI} = I = \frac{E''}{x}$ lags 90 degrees behind the impressed e.m.f.

This current is called the *exciting* or magnetizing current of the magnetic circuit, and is wattless.

If the magnetic circuit contains iron or other magnetic material, energy is consumed in the magnetic circuit by a frictional resistance of the material against a change of magnetism, which is called *molecular magnetic friction.*

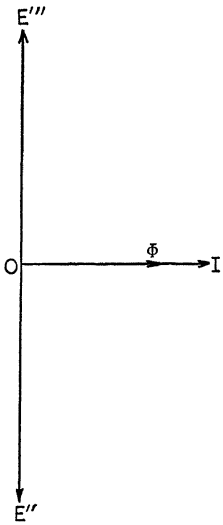


Fig. 22. Phase Relations of Magnetizing Current, Flux and Self-Inductive e.m.f.

If the alternating current is the only available source of energy in the magnetic circuit, the expenditure of energy by molecular magnetic friction appears as a lag of the magnetism behind the m.m.f. of the current, that is, as *magnetic hysteresis*, and can be measured thereby.

Magnetic hysteresis is, however, a distinctly different phenomenon from molecular magnetic friction, and can be more or less eliminated, as for instance by mechanical vibration, or can be increased, without changing the molecular magnetic friction.

49. In consequence of magnetic hysteresis, if an alternating e.m.f. $\overline{OE''} = E''$ is impressed upon a circuit of negligible resistance, the exciting current, or current producing the magnetism, in this circuit is not a wattless current, or current of 90 degrees lag, as in Fig. 22, but lags less than 90 degrees, by an angle $90 - \alpha$, as shown by $\overline{OI} = I$ in Fig. 23.

Since the magnetism $\overline{O\Phi} = \Phi$ is in quadrature with the e.m.f. E'' due to it, angle α is the phase difference between the magnetism and the m.m.f., or the lead of the m.m.f., that is, the exciting current, before the magnetism. It is called the *angle of hysteretic lead.*

In this case the *exciting current* $\overline{OI} = I$ can be resolved in two components, the *magnetizing current* $\overline{OI_2} = I_2$, in phase with the magnetism $\overline{O\Phi} = \Phi$, that is, in quadrature with the e.m.f. $\overline{OE''} = E''$, and thus wattless, and the *magnetic power component of the current* or the *hysteresis current* $\overline{OI_1} = I_1$, in phase with the e.m.f. $OE'' = E''$, or in quadrature with the magnetism $\overline{O\Phi} = \Phi$.

Magnetizing current and hysteresis current are the two components of the exciting current.

If the circuit contains besides the reactance $x = 2\pi fL$, a resistance r, the e.m.f. $\overline{OE''} = E''$ in the preceding Figs. 22 and 23 is not the impressed e.m.f., but the e.m.f. consumed by self-inductance or reactance, and has to be combined, Figs, 24 and 25, with the e.m.f. consumed by the resistance, $\overline{OE'} = E' = Ir$, to get the impressed e.m.f. $\overline{OE} = E$.

Due to the hysteretic lead α, the lag of the current is less in Figs. 23 and 25, a circuit expending energy in molecular magnetic friction, than in Figs. 22 and 24, a hysteresisless circuit.

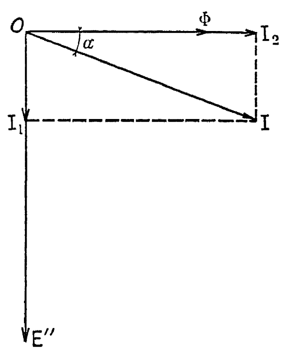


Fig. 23. Angle of Hysteretic Lead.

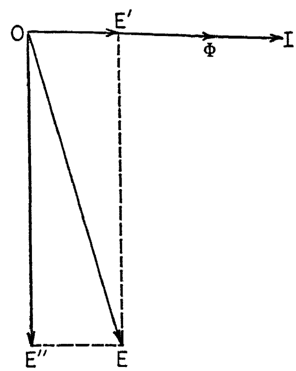


Fig. 24. Effective Resistance on Phase Relation of Impressed e.m.f in a Hysteresisless Circuit.

As seen in Fig. 25, in a circuit whose ohmic resistance is not negligible, the hysteresis current and the magnetizing current are not in phase and in quadrature respectively with the impressed e.m.f., but with the counter e.m.f. of inductance or e.m.f. consumed by inductance.

Obviously the magnetizing current is not quite wattless, since energy is consumed by this current in the ohmic resistance of the circuit.

Resolving, in Fig. 26, the impressed e.m.f. $\overline{OE} = E$ into two components, $\overline{OE_1} = E_1$ in phase, and $\overline{OE_2} = E_2$ in quadrature with the current $\overline{OI} = I$, the power component of the e.m.f.,

$\overline{OE_1} = E_1$, is greater than $E' = Ir$, and the reactive component $\overline{OE_2} = E_2$ is less than $E'' = Ix$.

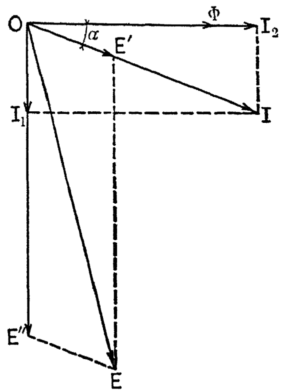


Fig. 25. Effective Resistance on Phase Relation of Impressed e.m.f. in a Circuit having Hysteresis.

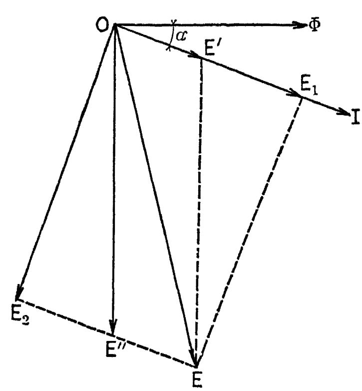


Fig. 26. Impressed e.m.f. Resolved into Components in Phase and in Quadrature with the Exciting Current.

The value $r' = \frac{E_1}{I} = \frac{\text{power e.m.f.}}{\text{total current}}$ is called the *effective resistance*, and the value $x' = \frac{E_2}{I} = \frac{\text{wattless e.m.f.}}{\text{total current}}$ is called the *apparent* or *effective reactance* of the circuit.

50. Due to the loss of energy by hysteresis (eddy currents, etc.), the effective resistance differs from, and is greater than, the ohmic resistance, and the apparent reactance is less than the true or inductive reactance.

The loss of energy by molecular magnetic friction per cubic centimeter and cycle of magnetism is approximately

$$W = \eta \mathcal{B}^{1.6},$$

where $\mathcal{B}$ = the magnetic flux density, in lines per sq. cm.

W = energy, in absolute units or ergs per cycle ($= 10^{-7}$ watt-seconds or joules), and η is called the coefficient of hysteresis.

In soft annealed sheet iron or sheet steel, η varies from 0.75×10^{-3} to 2.5×10^{-3}, and can in average, for good material, be assumed as 2.00×10^{-3}.

The loss of power in the volume, V, at flux density $\mathfrak{B}$ and frequency f, is thus

$$P = Vf\eta\mathfrak{B}^{1.6} \times 10^{-7}\text{, in watts,}$$

and, if I = the exciting current, the hysteretic effective resistance is

$$r'' = \frac{P}{I^2} = Vf\eta\, 10^{-7}\frac{\mathfrak{B}^{1.6}}{I^2}.$$

If the flux density, $\mathfrak{B}$, is proportional to the current, I, substituting for $\mathfrak{B}$, and introducing the constant k, we have

$$r'' = \frac{kf}{I^{0.4}},$$

that is, the effective hysteretic resistance is inversely proportional to the 0.4 power of the current, and directly proportional to the frequency.

51. Besides hysteresis, eddy or Foucault currents contribute to the effective resistance.

Since at constant frequency the Foucault currents are proportional to the magnetism producing them, and thus approximately proportional to the current, the loss of power by Foucault currents is proportional to the square of the current, the same as the ohmic loss, that is, the effective resistance due to Foucault currents is approximately constant at constant frequency, while that of hysteresis decreases slowly with the current.

Since the Foucault currents are proportional to the frequency, their effective resistance varies with the square of the frequency, while that of hysteresis varies only proportionally to the frequency.

The total effective resistance of an alternating-current circuit increases with the frequency, but is approximately constant, within a limited range, at constant frequency, decreasing somewhat with the increase of magnetism.

EXAMPLES.

52. A reactive coil shall give 100 volts e.m.f. of self-inductance at 10 amperes and 60 cycles. The electric circuit consists of 200 turns (No. 8 B. & S.) (= 0.013 sq. in.) of 16 in. mean

length of turn. The magnetic circuit has a section of 6 sq. in. and a mean length of 18 in. of iron of hysteresis coefficient $\eta = 2.5 \times 10^{-3}$. An air gap is interposed in the magnetic circuit, of a section of 10 sq. in. (allowing for spread), to get the desired reactance.

How long must the air gap be, and what is the resistance, the reactance, the effective resistance, the effective impedance, and the power-factor of the reactive coil?

The coil contains 200 turns each 16 in. in length and 0.013 sq. in. in cross section. Taking the resistivity of copper as 1.8×10^{-6}, the resistance is

$$r_1 = \frac{1.8 \times 10^{-6} \times 200 \times 16}{0.013 \times 2.54} = 0.175 \text{ ohm},$$

where 2.54 is the factor for converting inches to centimeters. (1 inch = 2.54 cm.)

Writing $E = 100$ volts generated, $f = 60$ cycles per second, and $n = 200$ turns, the maximum magnetic flux is given by $E = 4.44\ fn\Phi$; or, $100 = 4.44 \times 0.6 \times 200\,\Phi$, and $\Phi = 0.188$ megaline.

This gives in an air gap of 10 sq. in. a maximum density $\mathcal{B} = 18{,}800$ lines per sq. in., or 2920 lines per sq. cm.

Ten amperes in 200 turns give 2000 ampere-turns effective or $\mathfrak{F} = 2830$ ampere-turns maximum.

Neglecting the ampere-turns required by the iron part of the magnetic circuit as relatively very small, 2830 ampere-turns have to be consumed by the air gap of density $\mathcal{B} = 2920$.

Since

$$\mathcal{B} = \frac{4\,\pi\mathfrak{F}}{10\,l},$$

the length of the air gap has to be

$$l = \frac{4\,\pi\,\mathfrak{F}}{10\,\mathcal{B}} = \frac{4\,\pi \times 2830}{10 \times 2920} = 1.22 \text{ cm., or } 0.48 \text{ in.}$$

With a cross section of 6 sq. in. and a mean length of 18 in., the volume of the iron is 108 cu. in., or 1770 cu. cm.

The density in the iron, $\mathcal{B}_1 = \dfrac{188{,}000}{6} = 31{,}330$ lines per sq. in., or 4850 lines per sq. cm.

With an hysteresis coefficient $\eta = 2.5 \times 10^{-3}$, and density $\mathfrak{B}_1 = 4850$, the loss of energy per cycle per cu. cm. is

$$\begin{aligned} W &= \eta \mathfrak{B}_1^{1.6} \\ &= 2.5 \times 10^{-3} \times 4850^{1.6} \\ &= 1980 \text{ ergs,} \end{aligned}$$

and the hysteresis loss at $f = 60$ cycles and the volume $V = 1770$ is thus

$$\begin{aligned} P &= 60 \times 1770 \times 1980 \text{ ergs per sec.} \\ &= 21.0 \text{ watts,} \end{aligned}$$

which at 10 amperes represent an effective hysteretic resistance,

$$r_2 = \frac{21.0}{10^2} = 0.21 \text{ ohm.}$$

Hence the total effective resistance of the reactive coil is

$$r = r_1 + r_2 = 0.175 + 0.21 = 0.385 \text{ ohm;}$$

the effective reactance is

$$x = \frac{E}{I} = 10 \text{ ohms;}$$

the impedance is

$$z = 10.01 \text{ ohms;}$$

the power-factor is

$$\cos\theta = \frac{r}{z} = 3.8 \text{ per cent;}$$

the total apparent power of the reactive coil is

$$I^2 z = 1001 \text{ volt-amperes,}$$

and the loss of power,

$$I^2 r = 38 \text{ watts.}$$

11. CAPACITY AND CONDENSERS.

53. The charge of an electric condenser is proportional to the impressed voltage, that is, potential difference at its terminals, and to its capacity.

A condenser is said to have unit capacity if unit current existing for one second produces unit difference of potential at its terminals.

The practical unit of capacity is that of a condenser in which one ampere during one second produces one volt difference of potential.

The practical unit of capacity equals 10^{-9} absolute units. It is called a farad.

One farad is an extremely large capacity, and therefore one millionth of one farad, called microfarad, mf., is commonly used.

If an alternating e.m.f. is impressed upon a condenser, the charge of the condenser varies proportionally to the e.m.f., and thus there is current to the condenser during rising and from the condenser during decreasing e.m.f., as shown in Fig. 27.

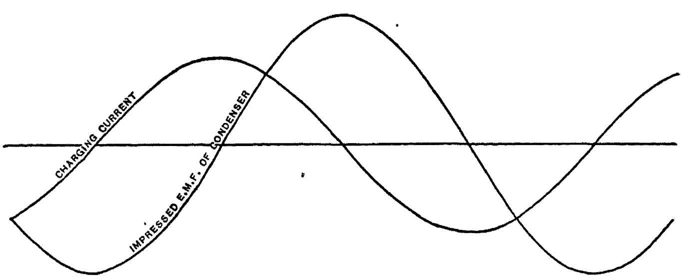


Fig. 27. Charging Current of a Condenser across which an Alternating e.m.f. is Impressed.

That is, the current consumed by the condenser leads the impressed e.m.f. by 90 time degrees, or a quarter of a period.

Denoting f as frequency and E as effective alternating e.m.f. impressed upon a condenser of C mf. capacity, the condenser is charged and discharged twice during each cycle, and the time of one complete charge or discharge is therefore $\frac{1}{4f}$.

Since $E\sqrt{2}$ is the maximum voltage impressed upon the condenser, an average of $CE\sqrt{2}\,10^{-6}$ amperes would have to exist during one second to charge the condenser to this voltage, and to charge it in $\frac{1}{4f}$ seconds an average current of $4fCE\sqrt{2}\,10^{-6}$ amperes is required.

Since $$\frac{\text{effective current}}{\text{average current}} = \frac{\pi}{2\sqrt{2}},$$

the effective current $I = 2\pi fCE\,10^{-6}$; that is, at an impressed e.m.f. of E effective volts and frequency f, a condenser of C mf. capacity consumes a current of

$$I = 2\pi fCE\,10^{-6} \text{ amperes effective,}$$

which current leads the terminal voltage by 90 time degrees or a quarter period.

Transposing, the e.m.f. of the condenser is

$$E = \frac{10^6 I}{2\pi fC} = x_0 I.$$

The value $x_0 = \dfrac{10^6}{2\pi fC}$ is called the *condensive reactance* of the condenser.

Due to the energy loss in the condenser by dielectric hysteresis, the current leads the e.m.f. by somewhat less than 90 time degrees, and can be resolved into a *wattless charging current* and a *dielectric hysteresis current*, which latter, however, is generally so small as to be negligible.

54. The capacity of one wire of a transmission line is

$$C = \frac{1.11 \times 10^{-6} \times l}{2 \log_\epsilon \dfrac{2\,l_s}{l_d}}, \text{ in mf.,}$$

where l_d = diameter of wire, cm.; l_s = distance of wire from return wire, cm.; l = length of wire, cm., and 1.11×10^{-6} = reduction coefficient from electrostatic units to mf.

The logarithm is the natural logarithm; thus in common logarithms, since $\log_\epsilon a = 2.303 \log_{10} a$, the capacity is

$$C = \frac{0.24 \times 10^{-6} \times l}{\log_{10} \dfrac{2\,l_s}{l_d}}, \text{ in mf.}$$

The derivation of this equation must be omitted here.

The charging current of a line wire is thus

$$I = 2\pi fCE\,10^{-6},$$

where f = the frequency, in cycles per second, E = the difference of potential, effective, between the line and the neutral (E = $\frac{1}{2}$ line voltage in a single-phase, or four-wire quarter-phase system, $\dfrac{1}{\sqrt{3}}$ line voltage, or Y voltage, in a three-phase system).

EXAMPLES.

55. In the transmission line discussed in the examples in 37, 38, 41 and 47, what is the charging current of the line at 6000 volts between lines, at 33.3 cycles? How many volt-amperes does it represent, and what percentage of the full load current of 44 amperes is it?

The length of the line is, per wire, $l = 2.23 \times 10^6$ cm.
The distance between wires, $l_s = 45$ cm.
The diameter of transmission wire, $l_d = 0.82$ cm.

Thus the capacity, per wire, is

$$C = \frac{0.24 \times 10^{-6}\, l}{\log_{10} \frac{2\, l_s}{l_d}} = 0.26 \text{ mf.}$$

The frequency is $f = 33.3$,
The voltage between lines, 6000.

Thus per line, or between line and neutral point,

$$E = \frac{6000}{\sqrt{3}} = 3460 \text{ volts}\cdot$$

hence, the charging current per line is

$$\begin{aligned} I_0 &= 2\,\pi f C E\, 10^{-6} \\ &= 0.19 \text{ amperes,} \end{aligned}$$

or 0.43 per cent of full-load current;

that is, negligible in its influence on the transmission voltage.

The volt-ampere input of the transmission is,

$$\begin{aligned} 3\, I_0 E &= 2000 \\ &= 2.0 \text{ kv-amp.} \end{aligned}$$

12. IMPEDANCE OF TRANSMISSION LINES.

56. Let r = resistance; $x = 2\,\pi f L$ = the reactance of a transmission line; E_0 = the alternating e.m.f. impressed upon the line; I = the line current; E = the e.m.f. at receiving end of the line, and θ = the angle of lag of current I behind e.m.f. E.

$\theta < 0$ thus denotes leading, $\theta > 0$ lagging current, and $\theta = 0$ a non-inductive receiver circuit.

The capacity of the transmission line shall be considered as negligible.

Assuming the phase of the current $\overline{OI} = I$ as zero in the polar diagram, Fig. 28, the e.m.f. E is represented by vector $\overline{OE}$, ahead of $\overline{OI}$ by angle θ. The e.m.f. consumed by resistance r is $\overline{OE_1} = E_1 = Ir$ in phase with the current, and the e.m.f. consumed by reactance x is $\overline{OE_2} = E_2 = Ix$, 90 time degrees ahead of the current; thus the total e.m.f. consumed by the line, or e.m.f. consumed by impedance, is the resultant $\overline{OE_3}$ of $\overline{OE_1}$ and $\overline{OE_2}$, and is $E_3 = Iz$.

Combining $\overline{OE_3}$ and $\overline{OE}$ gives $\overline{OE_0}$, the e.m.f. impressed upon the line.

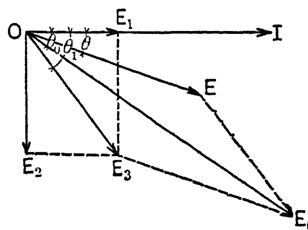


Fig. 28. Polar Diagram of Current and e.m.fs. in a Transmission Line Assuming Zero Capacity.

Denoting $\tan \theta_1 = \frac{x}{r}$ the time angle of lag of the line impedance, it is, trigonometrically,

$$\overline{OE_0}^2 = \overline{OE}^2 + \overline{EE_0}^2 - 2\,\overline{OE} \times \overline{EE_0} \cos OEE_0.$$

Since

$$\overline{EE_0} = \overline{OE_3} = Iz,$$
$$OEE_0 = 180 - \theta_1 + \theta,$$

we have

$$E_0^2 = E^2 + I^2z^2 + 2\,EIz \cos(\theta_1 - \theta)$$
$$= (E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2},$$

and

$$E_0 = \sqrt{(E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2}},$$

and the drop of voltage in the line,

$$E_0 - E = \sqrt{(E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2}} - E.$$

57. That is, the voltage E_0 required at the sending end of a line of resistance r and reactance x, delivering current I at voltage E, and the voltage drop in the line, do not depend upon current and line constants only, but depend also upon the angle of time-phase displacement of the current delivered over the line.

If $\theta = 0$, that is, non-inductive receiving circuit,

$$E_0 = \sqrt{(E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1}{2}}\,;$$

that is, less than $E + Iz$, and thus the line drop is less than Iz.

If $\theta = \theta_1$, E_0 is a maximum, $= E + Iz$, and the line drop is the impedance voltage.

With decreasing θ, E_0 decreases, and becomes $= E$; that is, no drop of voltage takes place in the line at a certain negative

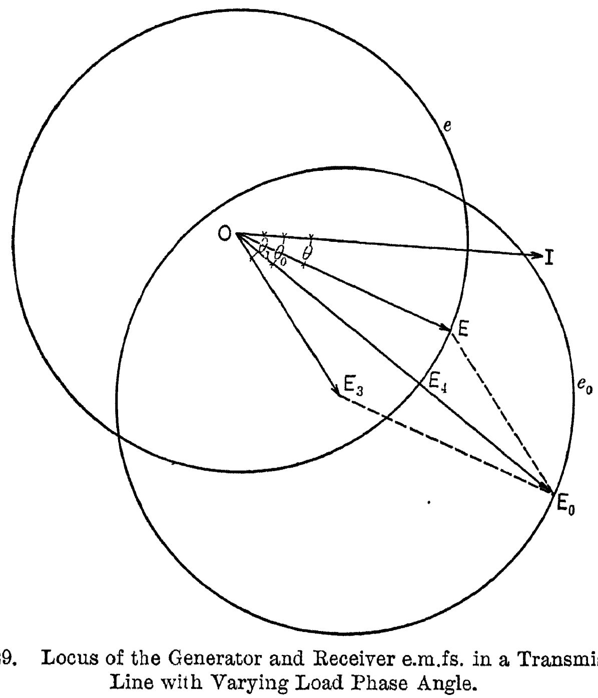


Fig. 29. Locus of the Generator and Receiver e.m.fs. in a Transmission Line with Varying Load Phase Angle.

value of θ which depends not only on z and θ_1 but on E and I. Beyond this value of θ, E_0 becomes smaller than E; that is, a rise of voltage takes place in the line, due to its reactance. This can be seen best graphically:

Choosing the current vector $\overline{OI}$ as the horizontal axis, for the same e.m.f. E received, but different phase angles θ, all vectors $\overline{OE}$ lie on a circle e with O as center. Fig. 29. Vector $\overline{OE}_3$ is constant for a given line and given current I.

Since $E_3E_0 = \overline{OE}$ = constant, E_0 lies on a circle e_0 with E_3 as center and $\overline{OE} = E$ as radius.

To construct the diagram for angle θ, $\overline{OE}$ is drawn at the angle θ with $\overline{OI}$, and $\overline{EE_0}$ parallel to $\overline{OE_3}$.

The distance E_4E_0 between the two circles on vector $\overline{OE_0}$ is the drop of voltage (or rise of voltage) in the line.

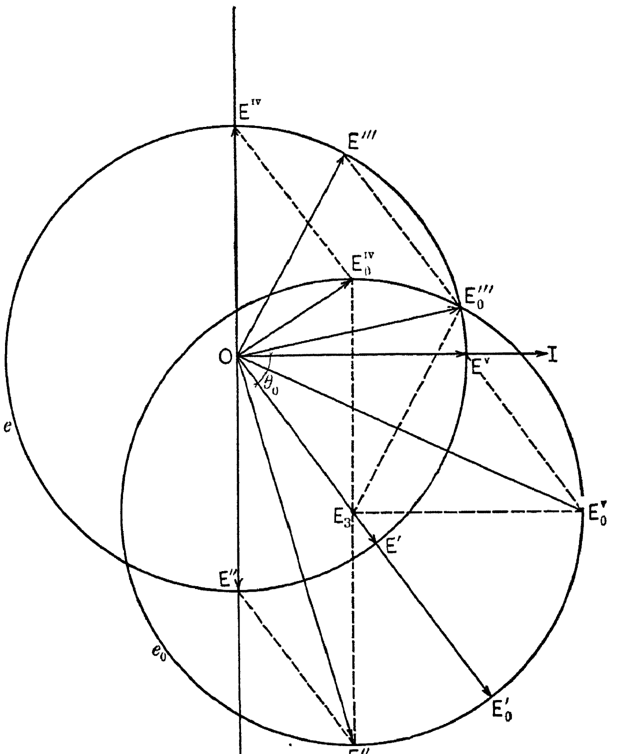


Fig. 30. Locus of the Generator and Receiver e.m.fs. in a Transmission Line with Varying Load Phase Angle.

As seen in Fig. 30, E_0 is maximum in the direction $\overline{OE_3}$ as $\overline{OE_0'}$, that is for $\theta = \theta_0$, and is less for greater as well, $\overline{OE_0''}$, as smaller angles θ. It is $= E$ in the direction $\overline{OE_0'''}$, in which case $\theta < 0$, and minimum in the direction $\overline{OE_0^{IV}}$.

The values of E corresponding to the generator voltages E_0', E_0'', E_0''', E_0^{IV} are shown by the points E' E'' E''' E^{IV}

respectively. The voltages E_0'' and E_0^{IV} correspond to a wattless receiver circuit E'' and E^{IV}. For non-inductive receiver circuit $\overline{OE}_{\text{V}}$ the generator voltage is $\overline{OE_0}^{\text{V}}$.

58. That is, in an inductive transmission line the drop of voltage is maximum and equal to Iz if the phase angle θ of the receiving circuit equals the phase angle θ_0 of the line. The drop of voltage in the line decreases with increasing difference between the phase angles of line and receiving circuit. It becomes zero if the phase angle of the receiving circuit reaches a certain negative value (leading current). In this case no drop of voltage takes place in the line. If the current in the receiving circuit leads more than this value a rise of voltage takes place in the line. Thus by varying phase angle θ of the receiving circuit the drop of voltage in a transmission line with current I can be made anything between Iz and a certain negative value. Or inversely the same drop of voltage can be produced for different values of the current I by varying the phase angle.

Thus, if means are provided to vary the phase angle of the receiving circuit, by producing lagging and leading currents at will (as can be done by synchronous motors or converters) the voltage at the receiving circuit can be maintained constant within a certain range irrespective of the load and generator voltage.

In Fig. 31 let $\overline{OE} = E$, the receiving voltage; I, the power component of the line current; thus $\overline{OE_3} = E_3 = Iz$, the e.m.f. consumed by the power component of the current in the impedance. This e.m.f. consists of the e.m.f. consumed by resistance $\overline{OE_1}$ and the e.m.f. consumed by reactance $\overline{OE_2}$.

Reactive components of the current are represented in the diagram in the direction $\overline{OA}$ when lagging and $\overline{OB}$ when leading. The e.m.f. consumed by these reactive components of the current in the impedance is thus in the direction e_3', perpendicular to $\overline{OE_3}$. Combining $\overline{OE_3}$ and $\overline{OE}$ gives the e.m.f. $\overline{OE_4}$ which would be required for non-inductive load. If E_0 is the generator voltage, E_0 lies on a circle e_0 with $\overline{OE_0}$ as radius. Thus drawing $\overline{E_4E_0}$ parallel to e_3' gives $\overline{OE_0}$, the generator voltage; $OE_3' = \overline{E_4E_0}$, the e.m.f. consumed in the impedance by the reactive component of the current; and as proportional thereto, $\overline{OI'} = I'$ the reactive current required to give at generator voltage E_0 and power current I the receiver voltage E. This reactive current I', lags behind E_3' by less than 90 and more than zero degrees.

59. In calculating numerical values, we can proceed either trigonometrically as in the preceding, or algebraically by resolv-

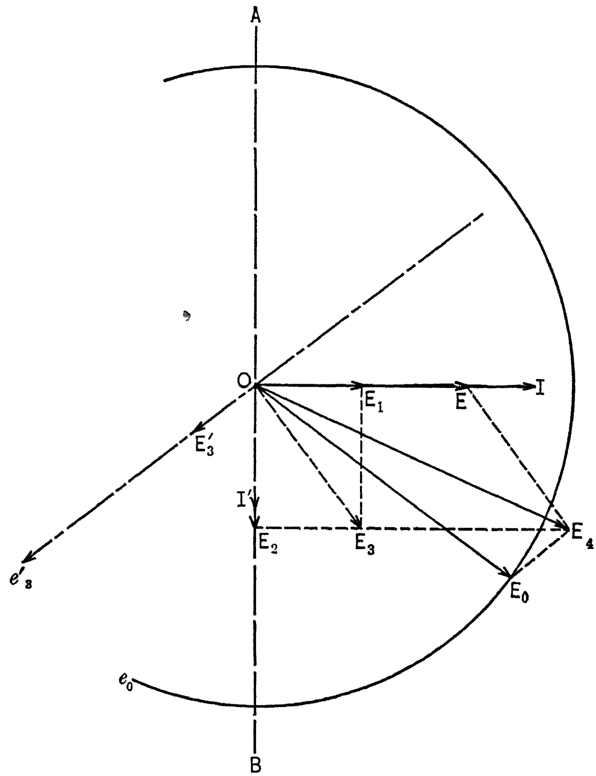


Fig. 31. Regulation Diagram for Transmission Line.

ing all sine waves into two rectangular components; for instance, a horizontal and a vertical component, in the same way as in mechanics when combining forces.

Let the horizontal components be counted positive towards the right, negative towards the left, and the vertical components positive upwards, negative downwards.

Assuming the receiving voltage as zero line or positive horizontal line, the power current I is the horizontal, the wattless current I' the vertical component of the current. The e.m.f. consumed in resistance by the power current I is a horizontal component, and that consumed in resistance by the reactive current I' a vertical component, and the inverse is true of the e.m.f. consumed in reactance.

We have thus, as seen from Fig. 31.

	Horizontal Component.	Vertical Component.
Receiver voltage, E,	$+E$	0
Power current, I,	$+I$	0
Reactive current, I',	0	$\pm I'$
E.m.f. consumed in resistance r by the power current, Ir,	$+Ir$	0
E.m.f. consumed in resistance r by the reactive current, $I'r$,	0	$\pm I'r$
E.m.f. consumed in reactance x by the power current, Ix,	0	$-Ix$
E.m.f. consumed in reactance x by the reactive current, $I'x$,	$\pm I'x$	0
Thus, total e.m.f. required, or impressed e.m.f., E_0,	$E + Ir \pm I'x$	$\pm I'r - Ix$;

hence, combined,

$$E_0 = \sqrt{(E + Ir \pm I'x)^2 + (\pm I'r - Ix)^2};$$

or, expanded,

$$E_0 = \sqrt{E^2 + 2E(Ir \pm I'x) + (I^2 + I'^2)z^2}.$$

From this equation I' can be calculated; that is, the reactive current found which is required to give E_0 and E at energy current I.

The lag of the total current in the receiver circuit behind the receiver voltage is

$$\tan\theta = \frac{I'}{I}.$$

The lead of the generator voltage ahead of the receiver voltage is

$$\tan\theta_1 = \frac{\text{vertical component of } E_0}{\text{horizontal component of } E_0}$$

$$= \frac{\pm I'r - Ix}{E + Ir \pm I'x},$$

and the lag of the total current behind the generator voltage is

$$\theta_0 = \theta + \theta_1.$$

As seen, by resolving into rectangular components the phase angles are directly determined from these components.

The resistance voltage is the same component as the current to which it refers.

The reactance voltage is a component 90 time degrees ahead of the current.

The same investigation as made here on long-distance transmission applies also to distribution lines, reactive coils, transformers, or any other apparatus containing resistance and reactance inserted in series into an alternating-current circuit.

EXAMPLES.

60. (1.) An induction motor has 2000 volts impressed upon its terminals; the current and the power-factor, that is, the cosine of the angle of lag, are given as functions of the output in Fig. 32.

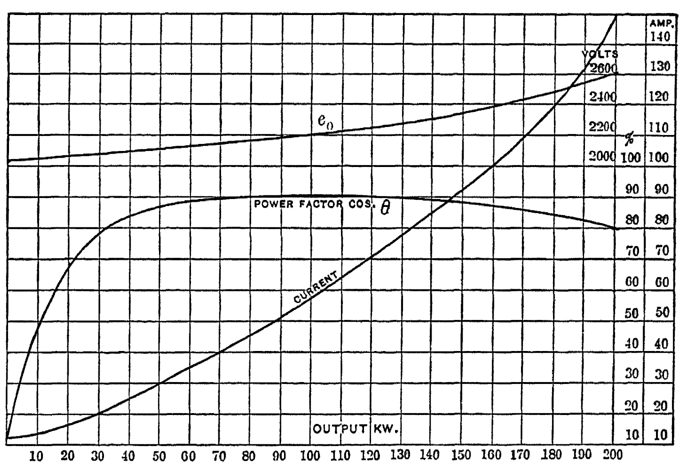


Fig. 32. Characteristics of Induction Motor and Variation of Generator e.m f. Necessary to Maintain Constant the e.m.f. Impressed upon the Motor.

The induction motor is supplied over a line of resistance $r = 2.0$ and reactance $x = 4.0$.

(a.) How must the generator voltage e_0 be varied to maintain constant voltage $e = 2000$ at the motor terminals, and (*b.*) At constant generator voltage $e_0 = 2300$, how will the voltage at the motor terminals vary?

We have

$$e_0 = \sqrt{(e + iz)^2 - 4\,eiz \sin^2 \frac{\theta_1 - \theta}{2}}. \qquad e = 2000.$$

$$z = \sqrt{r^2 + x^2} = 4.472.$$

$$\tan \theta_1 = \frac{x}{r} = 2. \qquad \theta_1 = 63.4°.$$

$$\cos \theta = \text{power-factor}.$$

Taking i from Fig. 32 and substituting, gives (*a*) the values of e_0 for $e = 2000$, which are recorded in the table, and plotted in Fig. 32;

(*b.*) The terminal voltage of the motor is $e = 2000$, the current i, the output P, at generator voltage e_0. Thus at generator voltage $e_0' = 2300$, the terminal voltage of the motor is

$$e' = \frac{2300}{e_0} e = \frac{2300}{e_0} 2000;$$

the current is

$$i' = \frac{2300}{e_0} i$$

and the power is

$$P' = \left(\frac{2300}{e_0}\right)^2 P.$$

The values of e', i', P' are recorded in the second part of the table under (*b*) and plotted in Fig. 33.

(*a*) At $e = 2000$,			Thus, e_0.	(*b*) Hence, at $e_0 = 2300$,		
Output, P = kw.	Current, i.	Lag, θ		Output, P'.	Current, i'.	Voltage, e'.
0	12.0	84.3°	2048	0.	13.45	2240
5	12.6	72.6°	2055	6.25	14.05	2234
10	13.5	62.6°	2060	12.4	15 00	2230
15	14.8	54.6°	2065	18.6	16.4	2220
20	16.3	47.9°	2071	24.4	18.0	2216
30	20.0	37.8°	2084	36.3	22.0	2200
40	25.0	32.8°	2093	48.0	27.5	2198
50	30.0	29.0°	2110	59.5	32.7	2180
69	40.0	26.3°	2146	78.5	42.8	2160
102	60.0	24.5°	2216	110.2	62.6	2080
132	80.0	25.8°	2294	131.0	79.5	1990
160	100.0	28.4°	2382	149.0	96.4	1928
180	120.0	31.8°	2476	156.5	111.5	1860
200	150.0	36.9°	2618	155.0	132.0	1760

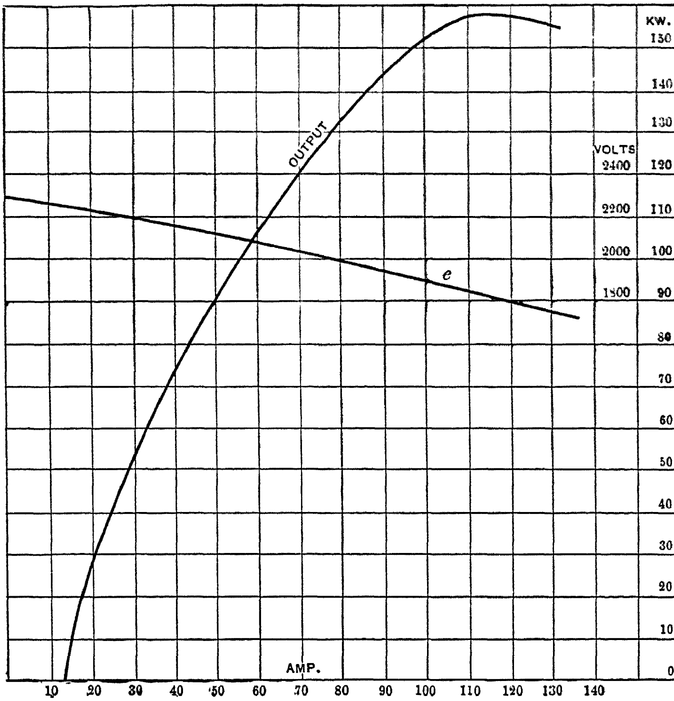


Fig. 33. Characteristics of Induction Motor, Constant Generator e.m.f.

61. (2.) Over a line of resistance $r = 2.0$ and reactance $x = 6.0$ power is supplied to a receiving circuit at a constant voltage of $e = 2000$. How must the voltage at the beginning of the line, or generator voltage, e_0, be varied if at no load the receiving circuit consumes a reactive current of $i_2 = 20$ amperes, this reactive current decreases with the increase of load, that is, of power current i_1, becomes $i_2 = 0$ at $i_1 = 50$ amperes, and then as leading current increases again at the same rate?

The reactive current,

$$i_2 = 20 \text{ at } i_1 = 0,$$
$$i_2 = 0 \text{ at } i_1 = 50,$$

and can be represented by

$$i_2 = \left(1 - \frac{i_1}{50}\right) 20 = 20 - 0.4\, i_1$$

the general equation of the transmission line is

$$e_0 = \sqrt{(e + i_1 r + i_2 x)^2 + (i_2 r - i_1 x)^2}$$
$$= \sqrt{(2000 + 2 i_1 + 6 i_2)^2 + (2 i_2 - 6 i_1)^2};$$

hence, substituting the value of i_2,

$$e_0 = \sqrt{(2120 - 0.4 i_1)^2 + (40 - 6.8 i_1)^2}$$
$$= \sqrt{4,496,000 + 46.4 i_1^2 - 2240 i_1}.$$

Substituting successive numerical values for i_1 gives the values recorded in the following table and plotted in Fig. 34.

i_1.	e_0.
0	2120
20	2114
40	2116
60	2126
80	2148
100	2176
120	2213
140	2256
160	2308
180	2365
200	2430

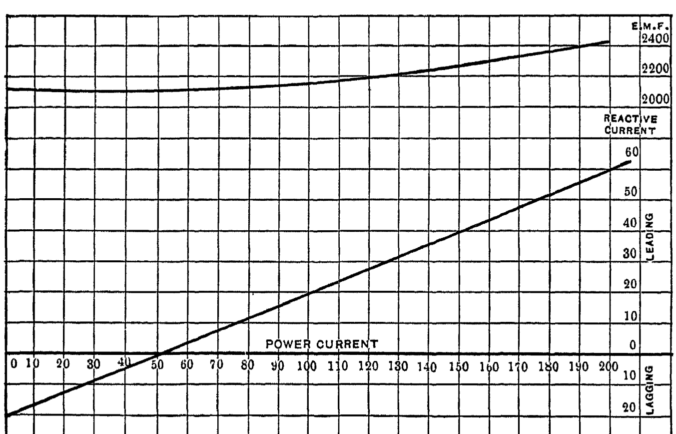


Fig. 34. Variation of Generator e.m.f. Necessary to Maintain Constant Receiver Voltage if the Reactive Component of Receiver Current varies proportional to the Change of Power Component of the Current.

13. ALTERNATING-CURRENT TRANSFORMER.

62. The alternating-current transformer consists of one magnetic circuit interlinked with two electric circuits, the primary circuit which receives energy, and the secondary circuit which delivers energy.

Let r_1 = resistance, $x_1 = 2\pi f L_{s_2}$ = self-inductive or leakage reactance of secondary circuit,

r_0 = resistance, $x_0 = 2\pi f L_{s_1}$ = self-inductive or leakage reactance of primary circuit,

where L_{s_2} and L_{s_1} refer to that magnetic flux which is interlinked with the one but not with the other circuit.

Let a = ratio of $\frac{\text{secondary}}{\text{primary}}$ turns (ratio of transformation).

An alternating e.m.f. E_0 impressed upon the primary electric circuit causes a current, which produces a magnetic flux Φ interlinked with primary and secondary circuits. This flux Φ generates e.m.fs. E_1 and E_i in secondary and in primary circuit, which are to each other as the ratio of turns, thus $E_i = \frac{E_1}{a}$.

Let E = secondary terminal voltage, I_1 = secondary current, θ_1 = lag of current I_1 behind terminal voltage E (where $\theta_1 < 0$ denotes leading current).

Denoting then in Fig. 35 by a vector $\overline{OE} = E$ the secondary terminal voltage, $\overline{OI_1} = I_1$ is the secondary current lagging by the angle $EOI = \theta_1$.

The e.m.f. consumed by the secondary resistance r_1 is $\overline{OE_1'} = E_1' = I_1 r_1$ in phase with I_1.

The e.m.f. consumed by the secondary reactance x_1 is $\overline{OE_1''} = E_1'' = I_1 x_1$, 90 time degrees ahead of I_1. Thus the e.m.f. consumed by the secondary impedance $z_1 = \sqrt{r_1^2 + x_1^2}$ is the resultant of $\overline{OE_1'}$ and $\overline{OE_1''}$, or $\overline{OE_1'''} = E_1''' = I_1 z_1$.

$\overline{OE_1'''}$ combined with the terminal voltage $\overline{OE} = E$ gives the secondary e.m.f. $\overline{OE_1} = E_1$.

Proportional thereto by the ratio of turns and in phase therewith is the e.m.f. generated in the primary $\overline{OE_i} = E_i$ where $E_i = \frac{E_1}{a}$.

To generate e.m.f. E_1 and E_i, the magnetic flux $\overline{O\Phi} = \Phi$ is required, 90 time degrees ahead of $\overline{OE}_1$ and $\overline{OE}_i$. To produce flux Φ the m.m.f. of $\mathfrak{F}$ ampere-turns is required, as determined from the dimensions of the magnetic circuit, and thus the primary current I_{00}, represented by vector $\overline{OI}_{00}$, leading $\overline{O\Phi}$ by the angle α.

Since the total m.m.f. of the transformer is given by the primary exciting current I_{00}, there must be a component of

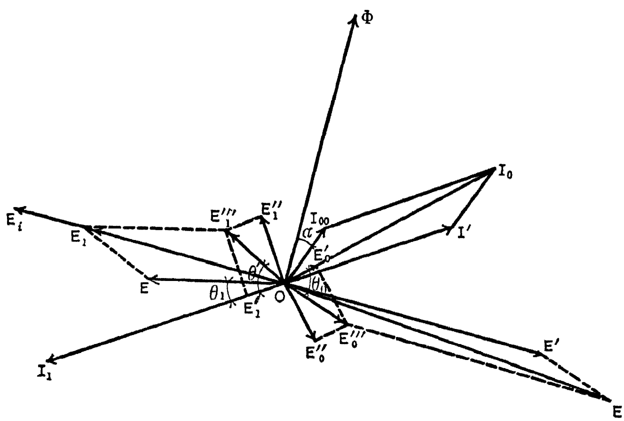


Fig. 35. Vector Diagram of e.m.fs. and Currents in a Transformer.

primary current I', corresponding to the secondary current I_1, which may be called the primary load current, and which is opposite thereto and of the same m.m.f.; that is, of the intensity $I' = aI_1$, thus represented by vector $\overline{OI'} = I' = aI_1$.

$\overline{OI}_{00}$, the primary exciting current, and the primary load current $\overline{OI'}$, or component of primary current corresponding to the secondary current, combined, give the total primary current $\overline{OI}_0 = I_0$.

The e.m.f. consumed by resistance in the primary is $\overline{OE}_0' = E_0' = I_0 r_0$ in phase with I_0.

The e.m.f. consumed by the primary reactance is $\overline{OE}_0'' = E_0'' = I_0 x_0$, 90 time degrees ahead of $\overline{OI}_0$.

$\overline{OE}_0'$ and $\overline{OE}_0''$ combined gives $\overline{OE}_0'''$, the e.m.f. consumed by the primary impedance.

Equal and opposite to the primary counter-generated e.m.f. $\overline{OE_i}$ is the component of primary e.m.f., $\overline{OE'}$, consumed thereby.

$\overline{OE'}$ combined with $\overline{OE_0'''}$ gives $\overline{OE_0} = E_0$, the primary impressed e.m.f., and angle $\theta_0 = E_0OI_0$, the phase angle of the primary circuit.

Figs. 36, 37, and 38 give the polar diagrams for $\theta_1 = 45°$ or lagging current, $\theta_1 =$ zero or non-inductive circuit, and $\theta =$ $- 45°$ or leading current.

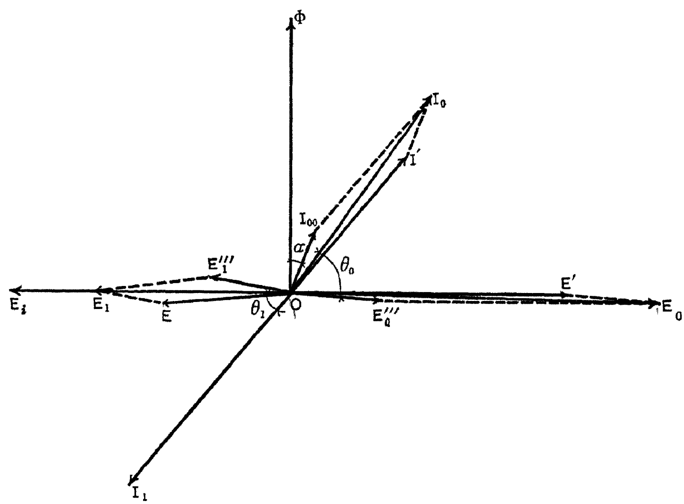


Fig. 36. Vector Diagram of Transformer with Lagging Load Current.

63. As seen, the primary impressed e.m.f. E_0 required to produce the same secondary terminal voltage E at the same current I_1 is larger with lagging or inductive and smaller with leading current than on a non-inductive secondary circuit; or, inversely, at the same secondary current I_1 the secondary terminal voltage E with lagging current is less and with leading current more than with non-inductive secondary circuit, at the same primary impressed e.m.f. E_0.

The calculation of numerical values is not practicable by measurement from the diagram, since the magnitudes of the different quantities are too different, E_1': E_1'': E_1: E_0 being frequently in the proportion 1 : 10: 100: 2000.

Trigonometrically, the calculation is thus:

In triangle OEE_1, Fig. 35, writing

$$\tan \theta' = \frac{x_1}{r_1},$$

we have,

$$\overline{OE_1}^2 = \overline{OE}^2 + \overline{EE}^2_1 - 2\, \overline{OE}\, \overline{EE_1} \cos OEE_1;$$

also,

$$\overline{EE}_1 = I_1 z_1$$
$$\measuredangle OEE_1 = 180 - \theta' + \theta_1,$$

hence,

$$E_1^2 = E^2 + I_1^2 z_1^2 + 2\, E I_1 z_1 \cos(\theta' - \theta_1).$$

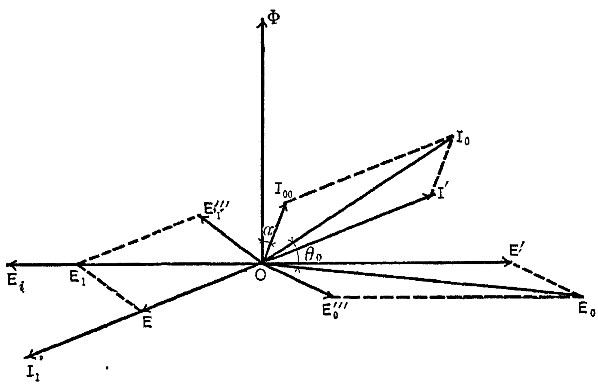


Fig. 37. Vector Diagram of Transformer with Non-Inductive Loading.

This gives the secondary e.m.f., E_1, and therefrom the primary counter-generated e.m.f.

$$E_i = \frac{E_1}{a}.$$

In triangle EOE_1 we have

$$\sin E_1OE \div \sin E_1EO = \overline{EE}_1 \div \overline{E_1O}$$

thus, writing

$$\measuredangle E_1OE = \theta'',$$

we have

$$\sin \theta'' \div \sin(\theta' - \theta_1) = I_1 z \div E_1,$$

wherefrom we get

$$\measuredangle \theta'', \text{ and } \measuredangle E_1OI_1 = \theta = \theta_1 + \theta'',$$

the phase displacement between secondary current and secondary e.m.f.

In triangle $OI_{00}I_0$ we have

$$\overline{OI_0}^2 = \overline{OI_{00}}^2 + \overline{I_{00}I_0}^2 - 2\,\overline{OI_{00}}\,\overline{I_{00}I_0}\cos OI_{00}I_0,$$

since

$$\measuredangle\, E_1O\phi = 90°,$$
$$\measuredangle\, OI_{00}I_0 = 90 + \theta + \alpha,$$

and

$$\overline{I_{00}I_0} = I' = aI_1,$$
$$\overline{OI_{00}} = I_{00} = \text{exciting current},$$

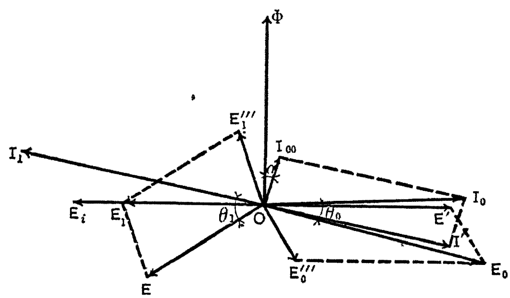


Fig. 38. Vector Diagram of Transformer with Leading Load Current.

calculated from the dimensions of the magnetic circuit. Thus the primary current is

$$I_0^2 = I_{00}^2 + a^2I_1^2 + 2\,aI_1I_{00}\sin(\theta + \alpha).$$

In triangle $OI_{00}I_0$ we have

$$\sin I_{00}OI_0 \div \sin OI_{00}I_0 = \overline{I_{00}I_0} \div \overline{OI_0};$$

writing

$$\measuredangle\, I_{00}OI_0 = \theta''$$

this becomes

$$\sin\theta_0'' \div \sin(\theta + \alpha) = aI_1 \div I_0;$$

therefrom we get θ_0'', and thus

$$\measuredangle\, E'OI_0 = \theta_2 = 90° - \alpha - \theta_0''.$$

In triangle $OE'E_0$ we have

$$\overline{OE_0}^2 = \overline{OE'}^2 + \overline{E'E_0}^2 - 2\,\overline{OE'}\,\overline{E'E_0}\cos \overline{OE'}\,E_0;$$

writing

$$\tan \theta_0' = \frac{x_0}{r_0},$$

we have

$$\measuredangle\, OE'E_0 = 180° - \theta' + \theta_2,$$

$$\overline{OE'} = E_i = \frac{E_1}{a},$$

$$\overline{E'E_0} = I_0 z_0;$$

thus the impressed e.m.f. is

$$E_0^2 = \frac{E_1^2}{a^2} + I_0^2 z_0^2 + \frac{2\,E_1 I_0 z_0}{a} \cos (\theta_0' - \theta_2).$$

In triangle $OE'E_0$

$$\sin E'OE_0 \div \sin OE'E_0 = \overline{E'E_0} \div OE_0;$$

thus, writing

$$\measuredangle\, E'OE_0 = \theta_1'',$$

we have

$$\sin \theta_1'' \div \sin (\theta_0' - \theta_2) = I_0 z_0 \div E_0;$$

herefrom we get $\measuredangle\, \theta_1''$, and

$$\measuredangle\, \theta_0 = \theta_2 + \theta_1'',$$

the phase displacement between primary current and impressed e.m.f.

As seen, the trigonometric method of transformer calculation is rather complicated.

64. Somewhat simpler is the algebraic method of resolving into rectangular components.

Considering first the secondary circuit, of current I_1 lagging behind the terminal voltage E by angle θ_1.

The terminal voltage E has the components $E \cos \theta_1$ in phase, $E \sin \theta_1$ in quadrature with and ahead of the current I_1.

The e.m.f. consumed by resistance r_1, $I_1 r_1$, is in phase.

The e.m.f. consumed by reactance x_1, $I_1 x_1$, is in quadrature ahead of I_1.

Thus the secondary e.m.f. has the components

$E \cos \theta_1 + I_1 r_1$ in phase,

$E \sin \theta_1 + I_1 x_1$ in quadrature ahead of the current I_1, and the total value,

$$E_1 = \sqrt{(E \cos \theta_1 + I_1 r_1)^2 + (E \sin \theta_1 + I_1 x_1)^2},$$

and the tangent of the phase angle of the secondary circuit is

$$\tan\theta = \frac{E\sin\theta_1 + I_1x_1}{E\cos\theta_1 + I_1r_1}.$$

Resolving all quantities into components in phase and in quadrature with the secondary e.m.f. E_1, or in horizontal and in vertical components, choosing the magnetism or mutual flux as vertical axis, and denoting the direction to the right and upwards as positive, to the left and downwards as negative, we have

	Horizontal Component.	Vertical Component.
Secondary current, I_1,	$-I_1\cos\theta$	$-I_1\sin\theta$
Secondary e.m.f., E_1,	$-E_1$	0
Primary counter-generated e.m.f., $E_i = \frac{E_1}{a}$,	$-\frac{E_1}{a}$	0
Primary e.m.f. consumed thereby, $E' = -E_i$,	$+\frac{E_1}{a}$	0
Primary load current, $I' = -aI_1$,	$+aI_1\cos\theta$	$+aI_1\sin\theta$
Magnetic flux, Φ,	0	Φ
Primary exciting current, I_{00}, consisting of core loss current,	$I_{00}\sin\alpha$	
Magnetizing current,		$I_{00}\cos\alpha$

Hence, total primary current, I_0,

Horizontal Component.	Vertical Component.
$aI_1\cos\theta_1 + I_{00}\sin\alpha$	$aI_1\sin\theta_1 + I_{00}\cos\alpha$

E.m.f. consumed by primary resistance r_0, $E_0' = I_0r_0$ in phase with I_0,

Horizontal Component.	Vertical Component.
$r_0aI_1\cos\theta + r_0I_{00}\sin\alpha$	$r_0aI_1\sin\theta + r_0I_{00}\cos\alpha$

E.m.f. consumed by primary reactance x_0, $E_0 = I_0x_0$, 90° ahead of I_0,

Horizontal Component.	Vertical Component.
$x_0aI_1\sin\theta + x_0I_{00}\cos\alpha$	$-x_0aI_1\cos\theta - x_0I_{00}\sin\alpha$

E.m.f. consumed by primary generated e.m.f., $E' = \frac{E_1}{a}$ horizontal.

The total primary impressed e.m.f., E_0,

HORIZONTAL COMPONENT.

$$\frac{E_1}{a} + aI_1 (r_0 \cos\theta + x_0 \sin\theta) + I_{00} (r_0 \sin\alpha + x_0 \cos\alpha).$$

VERTICAL COMPONENT.

$$aI_1 (r_0 \sin\theta - x_0 \cos\theta) + I_{00} (r_0 \cos\alpha - x_0 \sin\alpha),$$

or writing $\tan\theta_0' = \frac{x_0}{r_0}$,

since

$$\sqrt{r_0^2 + x_0^2} = z_0, \ \sin\theta_0' = \frac{x_0}{z_0}, \text{ and } \cos\theta_0' = \frac{r_0}{z_0}.$$

Substituting this value, the horizontal component of E_0 is

$$\frac{E_1}{a} + az_0 I_1 \cos(\theta - \theta_0') + z_0 I_{00} \sin(\alpha + \theta_0');$$

the vertical component of E_0 is

$$az_0 I_1 \sin(\theta - \theta_0') + z_0 I_{00} \cos(\alpha + \theta_0'),$$

and, the total primary impressed e.m.f. is

$$E_0 = \sqrt{\left[\frac{E_1}{a} + az_0 I_1 \cos(\theta - \theta_0') + z_0 I_{00} \sin(\alpha + \theta_0')\right] + \left[az_0 I_1 \sin(\theta - \theta_0') + z_0 I_{00} \cos(\alpha + \theta_0')\right]^2}$$

$$= \frac{E_1}{a}\sqrt{1 + \frac{2a^2 z_0 I_1}{E_1}\cos(\theta - \theta_0') + \frac{2az_0 I_{00}}{E_1}\sin(\alpha + \theta_0') + \frac{a^4 z_0^2 I_1^2}{E_1^2} + \frac{a^2 z_0^2 I_{00}^2}{E_1^2} + \frac{2a^3 z_0^2 I_1 I_{00}}{E_1^2}\sin(\theta + \alpha)}.$$

Combining the two components, the total primary current is

$$I_0 = \sqrt{(aI_1 \cos\theta + I_{00} \sin\alpha)^2 + (aI_1 \sin\theta + I_{00} \cos\alpha)}$$

$$= aI_1 \sqrt{1 + \frac{2 I_{00}}{aI_1} \sin(\theta + \alpha) + \frac{I_{00}^2}{a^2 I_1^2}}.$$

Since the tangent of the phase angle is the ratio of vertical component to horizontal component, we have, primary e.m.f. phase,

$$\tan\theta' = \frac{az_0 I_1 \sin(\theta - \theta_0') + z_0 I_{00} \cos(\alpha + \theta_0')}{\frac{E_1}{a} + az_0 I_1 \cos(\theta - \theta_0') + z_0 I_{00} \sin(\alpha - \theta_0')}$$

primary current phase,

$$\tan\theta'' = \frac{aI_1 \sin\theta + I_{00} \cos\alpha}{aI_1 \cos\theta + I_{00} \sin\alpha},$$

and lag of primary current behind impressed e.m.f.,

$$\theta_0 = \theta'' - \theta'$$

EXAMPLES.

65. (1.) In a 20-kw. transformer the ratio of turns is 20 ÷ **1**, and 100 volts is produced at the secondary terminals at full load. What is the primary current at full load, and the regulation, that is, the rise of secondary voltage from full load to no load, at constant primary voltage, and what is this primary voltage?

(*a*) at non-inductive secondary load,
(*b*) with 60 degrees time lag in the external secondary circuit,
(*c*) with 60 degrees time lead in the external secondary circuit.

The exciting current is 0.5 ampere, the core loss 600 watts, the primary resistance 2 ohms, the primary reactance 5 ohms, the secondary resistance 0.004 ohm, the secondary reactance 0.01 ohm.

Exciting current and core loss may be assumed as constant.

600 watts at 2000 volts gives 0.3 ampere core loss current, hence $\sqrt{0.5^2 - 3^2} = 0.4$ ampere magnetizing current.

We have thus

$$r_0 = 2 \qquad r_1 = 0.004 \qquad I_{e0} \cos \alpha = 0.3 \qquad a = 0.05$$
$$x_0 = 5 \qquad x_1 = 0.01 \qquad I_{00} \sin \alpha = 0.4$$
$$I_{00} = 0.5$$

1. Secondary current as horizontal axis:

	Non-inductive, $\theta_1 = 0$.		Lag, $\theta_1 = +60°$.		Lead, $\theta_1 = -60°$.	
	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
Secondary current, I_1	200	0	200	0	200	0
Secondary terminal voltage, E	100	0	50	−86.6	50	+86.6
Resistance voltage, I_1r_1	0.8	0	0.8	0	0.8	0
Reactance voltage, I_1x_1	0	−2.0	0	−2.0	0	−2.0
Secondary e.m.f., E_1	100.8	−2.0	50.8	−88.6	50.8	+84.6
Secondary e.m.f., total	100.80		102.13		98.68	
tan θ	+0.0198		+ 1.745		−1.665	
θ	+1.1°		+ 60.2°		−59.0°	

2. Magnetic flux as vertical axis:

	Non-inductive, $\theta_1=0$.		Lag, $\theta_1=+60°$.		Lead, $\theta_1=-60°$.	
	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
Secondary generated e.m.f., E	−100.80	0	−102.13	0	− 98.68	0
Secondary current, I_1	−200	− 4	− 99.4	172.8	−103	+171.4
Primary load current, $I'=-aI_1$	+ 10	+ 0.2	+ 4.97	+ 8.64	+ 5.15	− 8.57
Primary exciting current, I_{00}	0.3	0.4	0.3	0.4	0.3	0.4
Total primary current, I_0	+ 10.3	+ 0.6	+ 5.27	+ 9.04	+ 5.45	− 8.17
Primary resistance, voltage, I_0r_0	20.6	1.2	10.54	18.08	10.90	− 16.34
Primary reactance, voltage, I_0x_0	3.0	−51.3	45.20	− 26.35	− 40.85	− 27.25
E.m.f. consumed by primary counter e.m.f., $\frac{-E_1}{a}$	2016	0	2042.6	0	1973.6	0
Total primary impressed e.m.f., E_0	2039.6	−50.1	2098.34	− 8.27	1943.65	− 43.59

Hence,

	Non-inductive $\theta_1=0$.	Lag, $\theta_1=+60°$.	Lead, $\theta_1=-60°$.
Resultant E_0	2040.1	2098.3	1944.2
Resultant I_0	10.32	10.47	9.82
Phase of E_0	−1.4°	− .2°	− 1.2°
Phase of I_0	+3.3°	+59 8°	−56.3°
Primary lag, θ_0	+4.7°	+60.0°	−55.1°
Regulation $\frac{E_0}{2000}$	1.02005	1.04915	0.9721
Drop of voltage, per cent	2.005	4.915	− 2.79
Change of phase, $\theta_0-\theta_1$	4.7°	0	4.9°

14. RECTANGULAR COORDINATES.

66. The polar diagram of sine waves gives the best insight into the mutual relations of alternating currents and e.m.fs.

For numerical calculation from the polar diagram either the trigonometric method or the method of rectangular components is used.

The method of rectangular components, as explained in the above paragraphs, is usually simpler and more convenient than the trigonometric method.

In the method of rectangular components it is desirable to distinguish the two components from each other and from the resultant or total value by their notation.

To distinguish the components from the resultant, small letters are used for the components, capitals for the resultant. Thus in the transformer diagram of Section 13 the secondary current I_1 has the horizontal component $i_1 = -I_1 \cos \theta_1$, and the vertical component $i_1' = -I_1 \sin \theta_1$.

To distinguish horizontal and vertical components from each other, either different types of letters can be used, or indices, or a prefix or coefficient.

Different types of letters are inconvenient, indices distinguishing the components undesirable, since indices are reserved for distinguishing different e.m.fs., currents, etc., from each other.

Thus the most convenient way is the addition of a prefix or coefficient to one of the components, and as such the letter j is commonly used with the vertical component.

Thus the secondary current in the transformer diagram, Section 13, can be written

$$i_1 + ji_2 = I_1 \cos \theta_1 + jI_1 \sin \theta_1. \tag{1}$$

This method offers the further advantage that the two components can be written side by side, with the plus sign between them, since the addition of the prefix j distinguishes the value ji_2 or $jI_1 \sin \theta_1$ as vertical component from the horizontal component i_1 or $I_1 \cos \theta_1$.

$$I_1 = i_1 + ji_2 \tag{2}$$

thus means that I_1 consists of a horizontal component i_1 and a vertical component i_2, and the plus sign signifies that i_1 and i_2 are combined by the parallelogram of sine waves.

The secondary e.m.f. of the transformer in Section 13, Fig. 35, is written in this manner, $\underset{\cdot}{E}_1 = -e_1$, that is, it has the horizontal component $-e_1$ and no vertical component.

The primary generated e.m.f. is

$$\underset{\cdot}{E}_1 = \frac{-e_1}{a}, \tag{3}$$

and the e.m.f. consumed thereby

$$\underset{\cdot}{E}' = +\frac{e_1}{a}. \tag{4}$$

The secondary current is

$$\underset{\cdot}{I}_1 = -i_1 - ji_2, \tag{5}$$

where

$$i_1 = I_1 \cos\theta_1, \quad i_2 = I_1 \sin\theta_1, \tag{6}$$

and the primary load current corresponding thereto is

$$\underset{\cdot}{I}' = -a\underset{\cdot}{I}_1 = ai_1 + jai_2. \tag{7}$$

The primary exciting current,

$$\underset{\cdot}{I}_{00} = h + jg, \tag{8}$$

where $h = I_{00} \sin\alpha$ is the hysteresis current, $g = I_{00} \cos\alpha$ the reactive magnetizing current.

Thus the total primary current is

$$\underset{\cdot}{I}_0 = \underset{\cdot}{I}' + \underset{\cdot}{I}_{00} = (ai_1 + h) + j(ai_2 + g). \tag{9}$$

The e.m.f. consumed by primary resistance r_0 is

$$r_0\underset{\cdot}{I}_0 = r_0(ai_1 + h) + jr_0(ai_2 + g). \tag{10}$$

The horizontal component of primary current $(ai_1 + h)$ gives as e.m.f. consumed by reactance x_0 a negative vertical component, denoted by $-jx_0(ai_1 + h)$. The vertical component of primary current $j(ai_2 + g)$ gives as e.m.f. consumed by reactance x_0 a positive horizontal component, denoted by $x_0(ai_2 + g)$.

Thus the total e.m.f. consumed by primary reactance x_0 is

$$x_0(ai_2 + g) - jx_0(ai_1 + h), \tag{11}$$

and the total e.m.f. consumed by primary impedance is

$$r_0(ai_1 + h) + x_0(ai_2 + g) + j[r_0(ai_2 + g) - x_0(ai_1 + h)]. \tag{12}$$

Thus, to get from the current the e.m.f. consumed in reactance x_0 by the horizontal component of current, the coefficient

$-j$ has to be added; in the vertical component the coefficient j omitted; or, we can say the reactance is denoted by $-jx_0$ for the horizontal and by $\frac{x_0}{j}$ for the vertical component of current. In other words, if $\dot{I} = i + ji'$ is a current, x the reactance of its circuit, the e.m.f. consumed by the reactance is

$$-jxi + xi' = xi' - jxi.$$

67. If instead of omitting j in deriving the reactance e.m.f. for the vertical component of current, we would add $-j$ also (as done when deriving the reactance e.m.f. for the horizontal component of current), we get the reactance e.m.f.

$$-jxi - j^2xi',$$

which gives the correct value $-jxi + xi'$, if

$$j^2 = -1; \tag{13}$$

that is, we can say, in deriving the e.m.f. consumed by reactance, x, from the current, we multiply the current by $-jx$, and substitute $j^2 = -1$.

By defining, and substituting, $j^2 = -1$, $-jx$ can thus be called the reactance in the representation in rectangular coordinates and $r - jx$ the impedance,

The primary impedance voltage of the transformer in the preceding could thus be derived directly by multiplying the current,

$$\dot{I}_0 = (ai_1 + h) + j(ai_2 + g), \quad (9)$$

by the impedance,

$$Z_0 = r_0 - jx_0,$$

which gives

$$\begin{aligned} \dot{E}_0' = Z_0\dot{I}_0 &= (r_0 - jx_0)\,[(ai_1 + h) + j(ai_2 + g)] \\ &= r_0(ai_1 + h) + jr_0(ai_2 + g) - jx_0(ai_1 + h) - j^2x_0(ai_2 + g), \end{aligned}$$

and substituting $j^2 = -1$,

$$\dot{E}_0' = [r_0(ai_1 + h) + x_0(ai_2 + g)] + j[r_0(ai_2 + g) - x_0(ai_1 + h)], \tag{14}$$

and the total primary impressed e.m.f. is thus

$$\begin{aligned} \dot{E}_0 &= \dot{E}' + \dot{E}_0' \\ &= \left[\frac{e_1}{a} + r_0(ai_1 + h) + x_0(ai_2 + g)\right] + j\left[r_0(ai_2 + g) - x_0(ai + h)\right]. \end{aligned} \tag{15}$$

68. Such an expression in rectangular coordinates as

$$\dot{I} = i + ji' \tag{16}$$

represents not only the current strength but also its phase.

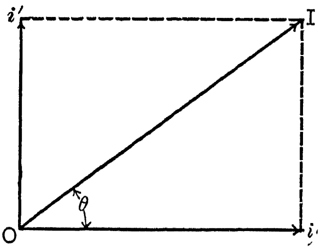


Fig. 39. Magnitude and Phase in Rectangular Coordinates.

It means, in Fig. 39, that the total current $\overline{OI}$ has the two rectangular components, the horizontal component $I \cos \theta = i$ and the vertical component $I \sin \theta = i'$.

Thus,

$$\tan \theta = \frac{i'}{i}; \tag{17}$$

that is, the tangent function of the phase angle is the vertical component divided by the horizontal component, or the term with prefix j divided by the term without j.

The total current intensity is obviously

$$I = \sqrt{i^2 + i'^2}. \tag{18}$$

The capital letter $\dot{I}$ in the symbolic expression $\dot{I} = i + ji'$ thus represents more than the I used in the preceding for total current, etc., and gives not only the intensity but also the phase. It is thus necessary to distinguish by the type of the latter the capital letters denoting the resultant current in symbolic expression (that is, giving intensity and phase) from the capital letters giving merely the intensity regardless of phase; that is,

$$\dot{I} = i + ji'$$

denotes a current of intensity

$$I = \sqrt{i^2 + i'^2},$$

and phase

$$\tan \theta = \frac{i'}{i}.$$

In the following, dotted italics will be used for the symbolic expressions and plain italics for the absolute values of alternating waves.

In the same way $z = \sqrt{r^2 + x^2}$ is denoted in symbolic representation of its rectangular components by

$$\dot{Z} = r - jx. \tag{19}$$

When using the symbolic expression of rectangular coordinates it is necessary ultimately to reduce to common expressions.

Thus in the above discussed transformer the symbolic expression of primary impressed e.m.f.

$$E_0=\left[\frac{e_1}{a}+r_0(ai_1+h)+x_0(ai_2+g)\right]+j\left[r_0(ai_2+g)-x_0(ai_1+h)\right] \tag{15}$$

means that the primary impressed e.m.f. has the intensity

$$E_0=\sqrt{\left[\frac{e_1}{a}+r_0(ai_1+h)+x_0(ai_2+g)\right]^2+\left[r_0(ai_2+g)-x_0(ai_1+h)\right]^2}, \tag{20}$$

and the phase

$$\tan\theta_0=\frac{r_0(ai_2+g)-x_0(ai_1+h)}{\frac{e_1}{a}+r_0(ai_1+h)+x_0(ai_2+g)}. \tag{21}$$

This symbolism of rectangular components is the quickest and simplest method of dealing with alternating-current phenomena, and is in many more complicated cases the only method which can solve the problem at all, and therefore the reader must become fully familiar with this method.

EXAMPLES.

69. (1.) In a 20-kw. transformer the ratio of turns is 20 : 1, and 100 volts are required at the secondary terminals at full load. What is the primary current, the primary impressed e.m.f., and the primary lag,

(*a*) at non-inductive load, $\theta_1=0$;

(*b*) with $\theta_1=60$ degrees time lag in the external secondary circuit;

(*c*) with $\theta_1=-60$ degrees time lead in the external secondary circuit?

The exciting current is $I_{00}'=0.3+0.4j$ ampere, at $e=2000$ volts impressed, or rather, primary counter-generated e.m.f.

The primary impedance, $Z_0=2-5j$ ohms.

The secondary impedance, $Z_1=0.004-0.01j$ ohm.

We have, in symbolic expression, choosing the secondary current I_1 as real axis the results calculated in tabulated form on page 88.

	Non-inductive.	60° Lag.	60° Lead.
Secondary current, $\dot{I}_1$=	200	200	200
Secondary impedance voltage, $\dot{E}' = I_1 Z_1$= ...	$0.8-2j$	$0.8-2j$	$0\ 8-2j$
Secondary terminal voltage, $\dot{E}=100\ (\cos\theta_1 - j\sin\theta_1)$=	100	$50-86.6j$	$50+86\ 6j$
Thus, secondary counter-generated e.m.f., $\dot{E}_1=\dot{E}+\dot{E}_1'$=	$100.8-2j$	$50.8-88.6j$	$50.8+84.6j$
Primary counter-generated e.m.f., $\dot{E}_i=20\ \dot{E}_1$=	$2016-40j$	$1016-1772j$	$1016+1692j$
Primary load current' $= I\dfrac{\dot{I}_1}{20}$ = ...	10	10	10
Primary exciting current, at $e=2000$ volts impressed, $\dot{I}_{00}$=	$0.3+0.4j$	$0.3+0.4j$	$0.3+0.4j$
Thus, at primary counter-generated e.m.f. $\dot{E}_i$ the exciting current is $\dfrac{\dot{E}_i\dot{I}_{00}}{2000}$ = ...	$\dfrac{(0.3+0.4j)\ (2016-40j)}{2000}$	$\dfrac{(0.3+0.4j)\ (1016-1772j)}{2000}$	$\dfrac{(0.3+0.4j)\ (1016+1692j)}{2000}$
Hence, expanded, $\dot{I}_{00}$=	$0.310+397j$	$0.507-0\ 063j$	$-0.186+0.407j$
Total primary current, $\dot{I}_0=\dot{I}_{00}+\dot{I}'$	$10.31+0.397j$	$10.507-0.063j$	$9\ 814+407j$
Primary impedance voltage, $\dot{E}_0'=\dot{I}_0 Z_0$=	$(2-5j)\ (10.31+0.397j)$	$(2-5j)\ (10.507-0.063j)$	$(2-5j)\ (9.814+0.407j)$
Hence, expanded, $\dot{E}_0'$=	$22.6-50.76j$	$20.7-52.66j$	$21.66-48\ 26j$
Thus, primary impressed e.m.f., $\dot{E}_0 = \dot{E}_i + \dot{E}_0'$=	$2038.6-90.8j$	$1036.7-1824.7j$	$1037.7+1643.7j$
Hence, primary e.m.f. phase, $\tan\theta^1$=	$-\dfrac{90.8}{2038.6}$	$-\dfrac{1824.7}{1036.7}$	$+\dfrac{1643\ 7}{1037\ 7}$
θ'=	$-2.6°$	$-60.4°$	$+57\ 7°$
Primary current phase, $\tan\theta''$=	$+\dfrac{0.397}{10\ 31}$	$-\dfrac{0\ 063}{10.507}$	$+\dfrac{0\ 407}{9\ 814}$
θ''=	$+2.2°$	$-0.4°$	$+2\ 4°$
Primary lag, $\theta_0=\theta''-\theta'$=	$+4.8°$	$+60.0°$	$-55\ 3°$
And, reduced, primary impressed e.m.f., E_0=	$\sqrt{2038.6^2+90.8^2}=2041$	$\sqrt{1036.7^2+1824.7^2}=2099$	$\sqrt{1037.7^2+1643.7^2}=1943$
Primary current, I_0	$\sqrt{10.31^2+0.397^2}=10.32$	$\sqrt{10.507^2+0.063^2}=10\ 51$	$\sqrt{9\ 814^2+0.407^2}=9.82$

We then have	$Z=r=0.5$	$Z=0.3-0.4j$	$Z=0.3+0.4j$
Secondary current $I_1=\frac{e}{Z}$	$=\frac{e}{0.5}$	$\frac{e}{0.3-0.4j}$	$\frac{e}{0.3+0.4j}$
Expanded by the associate term of the denominator, and substitute, $j^2=-1$, $I_1=$	$2e$ $=2e$	$\frac{e\,(0.3+0.4j)}{(0.3-0\ 4j)\,(0.3+0.4j)}$ $=4e\,(0.3+0.4j)$	$\frac{e\,(0\ 3-0.4j)}{(0.3+0.4j)\,(0.3-0\ 4j)}$ $=4e\,(0.3-0.4j)$
Secondary impedance voltage, $E_1'=I_1Z_1=$	$2e\,(0.004-0.01j)$ $=e\,(0.008-0.02j)$	$4e(0\ 3+0.4j)(0.004-0\ 01j)$ $=e\,(0.0208-0.0056j)$	$4e(3-4j)(0.004-0\ 01j)$ $=e\,(-0\ 0112-0.0184j)$
Secondary terminal voltage, $E=e-E_1'=$	$e\,(0.992+0.02j)$	$e\,(0.9792+0.0056j)$	$e\,(1.0112+0.0184j)$
Or, reduced, $E=$	$e\sqrt{0.992^2+0.02^2}$ $=0.992\,e$	$e\sqrt{0.9792^2+0.0056^2}$ $=0.9792\,e$	$e\sqrt{1.011^2+0.0184^2}$ $=1\ 0114\,e$
Primary counter-generated e.m.f., $E_i=$	$20\,e$	$20\,e$	$20\,e$
Primary load current, $I'=\frac{1}{20}I_1=$	$0.1\,e$	$0.2\,e\,(0.3+0\ 4j)$	$0.2\,e\,(0\ 3-0\ 4j)$
Primary exciting current, $I_{00}=EY=$	$e\,(3+4j)\,10^{-3}$	$e\,(3+4j)\,10^{-3}$	$e\,(0.3+0.4j)\,10^{\ 2}$
Thus, total primary current $I_0=I'+I_{00}=$	$e\,(0.103+0.004j)$	$e\,(0.063+0.084j)$	$e\,(0.063-0.076j)$
Primary impedance voltage, $E_0'=Z_0I_0=$	$e\,(0.103+0.004j)\,(2-5j)$	$e\,(0.063+0.084j)\,(2-5j)$	$e\,(0.063-0.076j)\,(2-5j)$
Expanded=	$e\,(0.226-505j)$	$e\,(0.546-0.147j)$	$e\,(-0.254-0.467j)$
Thus, primary impressed e.m.f., $E_0=E_i+E_0'=$	$e\,(20.226-0.505j)$	$e\,(20\ 546-0.147j)$	$e\,(19.746-0.467j)$
Or, reduced, $e_0=$	$e\sqrt{20.226^2+0.505^2}$ $=20.23\,e$	$e\sqrt{(20.564^2+0.147^2}$ $=20.55\,e$	$e\sqrt{19.746^2+0.467^2}$ $=19.75\,e$
Or, e=	$\frac{e_0}{20.23}$	$\frac{e_0}{20\ 55}$	$\frac{e_0}{19.75}$
Since $e_0=2000$, $e=$	98 85	97.32	101.25
Substituting e gives			
Secondary current, $I_1=$	197.7	$116.8+155.6j$	$121.8-162j$
Reduced, $I_1=$	197.7	194.6	202.5
Secondary terminal voltage, $E_1=$	$98.1+2j$	$95.3+0.54j$	$102.4+1.86j$
Reduced, $E_1=$	98.1	95 3	102.4
Primary current, $I_0=$	$10.18+.004j$	$6.13+8.17j$	$6.38-7\ 70j$
Reduced, $I_0=$	10 18	10.22	10.00

70. (2.) $e_0 = 2000$ volts are impressed upon the primary circuit of a transformer of ratio of turns 20:1. The primary impedance is $Z_0 = 2 - 5j$, the secondary impedance, $Z_1 = 0.004 - 0.01\ j$, and the exciting current at $e' = 2000$ volts counter-generated e.m.f. is $\dot{I}_{00} = 0.3 + 0.4\ j$; thus the exciting admittance, $Y = \frac{I_{00}'}{e'} = (0.15 + 0.2\ j)\ 10^{-3}$.

What is the secondary current and secondary terminal voltage and the primary current if the total impedance of the secondary circuit (internal impedance plus external load) consists of

(*a*) resistance,
$Z = r = 0.5$ — non-inductive circuit.

(*b*) impedance,
$Z = r - jx = 0.3 - 0.4\ j$ — inductive circuit.

(*c*) impedance,
$Z = r - jx = 0.3 + 0.4\ j$ — anti-inductive circuit.

Let e = secondary e.m.f.,

assumed as real axis in symbolic expression, and carrying out the calculation in tabulated form, on page 89:

71. (3.) A transmission line of impedance $Z = r - jx = 20 - 50\ j$ ohms feeds a receiving circuit. At the receiving end an apparatus is connected which produces reactive lagging or leading currents at will (synchronous machine); 12,000 volts are impressed upon the line. How much lagging and leading currents respectively must be produced at the receiving end of the line to get 10,000 volts (*a*) at no load, (*b*) at 50 amperes power current as load, (*c*) at 100 amperes power current as load?

Let e = 10,000 = e.m.f. received at end of line, i_1 = power current, and i_2 = reactive lagging current; then

$$\dot{I} = i_1 + ji_2 = \text{total line current.}$$

The voltage at the generator end of the line is then

$$\begin{aligned} \dot{E}_0 &= e + Z\dot{I} \\ &= e + (r - jx)(i_1 + ji_2) \\ &= (e + ri_1 + xi_2) + j(ri_2 - xi_1) \\ &= (10{,}000 + 20\,i_1 + 50\,i_2) + j(20\,i_2 - 50\,i_1); \end{aligned}$$

or, reduced,

$$E_0 = \sqrt{(e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2};$$

thus, since $E_0 = 12{,}000$,

$$12{,}000 = \sqrt{(10{,}000 + 20\,i_1 + 50\,i_2)^2 + (20\,i_2 - 50\,i_1)^2}.$$

(*a.*) At no load $i_1 = 0$, and

$$12{,}000 = \sqrt{(10{,}000 + 50\,i_2)^2 + 400\,i_2^2};$$

hence,

$i_2 = +\,39.5$ amp., reactive lagging current, $\underset{\cdot}{I} = +\,39.5\,j$.

(*b.*) At half load $i_1 = 50$, and

$$12{,}000 = \sqrt{(11{,}000 + 50\,i_2)^2 + (20\,i_2 - 2500)^2};$$

hence,

$i_2 = +\,16$ amp., lagging current, $\underset{\cdot}{I} = 50 + 16\,j$.

(*c.*) At full load $i_1 = 100$, and

$$12{,}000 = \sqrt{(12{,}000 + 50\,i_2)^2 + (20\,i \;- 5000)^2};$$

hence,

$i_2 = -\,27.13$ amp., leading current, $\underset{\cdot}{I} = 100 - 27.13\,j$.

15. LOAD CHARACTERISTIC OF TRANSMISSION LINE.

72. The load characteristic of a transmission line is the curve of volts and watts at the receiving end of the line as function of the amperes, and at constant e.m.f. impressed upon the generator end of the line.

Let r = resistance, x = reactance of the line. Its impedance $z = \sqrt{r^2 + x^2}$ can be denoted symbolically by

$$Z = r - jx.$$

Let E_0 = e.m.f. impressed upon the line.

Choosing the e.m.f. at the end of the line as horizontal component in the polar diagram, it can be denoted by $\underset{\cdot}{E} = e$.

At non-inductive load the line current is in phase with the e.m.f. e, thus denoted by $\underset{\cdot}{I} = i$.

The e.m.f. consumed by the line impedance $Z = r - jx$ is

$$\begin{aligned} \underset{\cdot}{E}_1 &= Z\underset{\cdot}{I} = (r - jx)\,i \\ &= ri - jxi. \end{aligned} \qquad (1)$$

Thus the impressed voltage,

$$\dot{E}_0 = \dot{E} + \dot{E}_1 = e + ri - jxi. \tag{2}$$

or, reduced,

$$E_0 = \sqrt{(e + ri)^2 + x^2i^2}, \tag{3}$$

and

$$e = \sqrt{E_0^2 - x^2i^2} - ri, \text{ the e.m.f.} \tag{4}$$

$$P = ei = i\sqrt{E_0^2 - x^2i^2} - ri^2, \tag{5}$$

the power received at end of the line.

The curve of e.m.f. e is an arc of an ellipse.

With open circuit $i = 0$: $e = E_0$ and $P = 0$, as is to be expected.

At short-circuit, $e = 0$, $0 = \sqrt{E_0^2 - x^2i^2} - ri$, and

$$i = \frac{E_0}{\sqrt{r^2 + x^2}} = \frac{E_0}{z}; \tag{6}$$

that is, the maximum line current which can be established with a non-inductive receiver circuit and negligible line capacity.

73. The condition of maximum power delivered over the line is

$$\frac{dP}{di} = 0; \tag{7}$$

that is,

$$\sqrt{E_0^2 - x^2i^2} + \frac{\frac{1}{2}i(-2x^2i)}{\sqrt{E_0^2 - x^2i^2}} - 2ri = 0;$$

substituting, 2:

$$\sqrt{E_0^2 - x^2i^2} = e + ri,$$

and expanding, gives

$$\begin{aligned} e^2 &= (r^2 + x^2)\, i^2 \\ &= z^2i^2; \end{aligned} \tag{8}$$

hence,

$$e = zi, \quad \text{and} \quad \frac{e}{i} = z. \tag{9}$$

$\frac{e}{i} = r_1$ is the resistance or effective resistance of the receiving circuit; that is, the maximum power is delivered into a non-inductive receiving circuit over an inductive line upon which is impressed a constant e.m.f., if the resistance of the receiving circuit equals the impedance of the line, $r_1 = z$.

In this case the total impedance of the system is

$$Z_0 = Z + r_1 = r + z - jx, \tag{10}$$

or,

$$z_0 = \sqrt{(r + z)^2 + x^2}. \tag{11}$$

Thus the current is

$$i_1 = \frac{E_0}{z_0} = \frac{E_0}{\sqrt{(r + z)^2 + x^2}}, \tag{12}$$

and the power transmitted is

$$P_1 = i_1^2 r_1 = \frac{E_0^2 z}{(r + z)^2 + x^2}$$

$$= \frac{E_0^2}{2(r + z)}; \tag{13}$$

that is, the maximum power which can be transmitted over a line of resistance r and reactance x is the square of the impressed e.m.f. divided by twice the sum of resistance and impedance of the line.

At $x = 0$, this gives the common formula,

$$P_1 = \frac{E_0^2}{4r}. \tag{14}$$

Inductive Load.

74. With an inductive receiving circuit of lag angle θ, or power factor $p = \cos\theta$, and inductance factor $q = \sin\theta$, at e.m.f. $\dot{E} = e$ at receiving circuit, the current is denoted by

$$\dot{I} = I(p + jq); \tag{15}$$

thus the e.m.f. consumed by the line impedance $Z = r - jx$ is

$$\begin{aligned}\dot{E}_1 &= Z\dot{I} = I(p + jq)(r - jx) \\ &= I[(rp + xq) + j(rq - xp)],\end{aligned}$$

and the generator voltage is

$$\begin{aligned}\dot{E}_0 &= \dot{E} + \dot{E}_1 \\ &= [e + I(rp + xq)] + jI(rq - xp);\end{aligned} \tag{16}$$

or, reduced,

$$E_0 = \sqrt{[e + I(rp + xq)]^2 + I^2(rq - xp)^2}, \qquad (17)$$

and

$$e = \sqrt{E_0^2 - I^2(rq - xp)^2} - I(rp + xq). \qquad (18)$$

The power received is the e.m.f. times the power component of the current; thus

$$P = eIp$$
$$= Ip\sqrt{E_0^2 - I^2(rq - xp)^2} - I^2p(rp + xq). \qquad (19)$$

The curve of e.m.f., e, as function of the current I is again an arc of an ellipse.

At short-circuit $e = 0$; thus, substituted,

$$I = \frac{E_0}{z}, \qquad (20)$$

the same value as with non-inductive load, as is obvious.

75. The condition of maximum output delivered over the line is

$$\frac{dP}{dI} = 0; \qquad (21)$$

that is, differentiated,

$$\sqrt{E_0^2 - I^2(rq - xp)^2} = e + I(rp + xq); \qquad (22)$$

substituting and expanding,

$$e^2 = I^2(r^2 + x^2)$$
$$= I^2z^2;$$
$$e = Iz;$$

or

$$\frac{e}{I} = z. \qquad (23)$$

$z_1 = \frac{e}{I}$ is the impedance of the receiving circuit; that is, the power received in an inductive circuit over an inductive line is a maximum if the impedance of the receiving circuit, z_1, equals the impedance of the line, z.

In this case the impedance of the receiving circuit is

$$Z_1 = z(p - jq), \qquad (24)$$

and the total impedance of the system is

$$\begin{aligned} Z_0 &= Z + Z_1 \\ &= r - jx + z\,(p - jq) \\ &= (r + pz) - j\,(x + qz). \end{aligned}$$

Thus, the current is

$$I_1 = \frac{E_0}{\sqrt{(r + pz)^2 + (x + qz)^2}}, \tag{25}$$

and the power is

$$P_1 = I_1^2 zp = \frac{E_0^2 zp}{(r + pz)^2 + (x + qz)^2}$$

$$= \frac{E_0^2 p}{2\,(z + rp + xq)}. \tag{26}$$

EXAMPLES.

76. (1.) 12,000 volts are impressed upon a transmission line of impedance $Z = r - jx = 20 - 50j$. How do the voltage and the output in the receiving circuit vary with the current with non-inductive load?

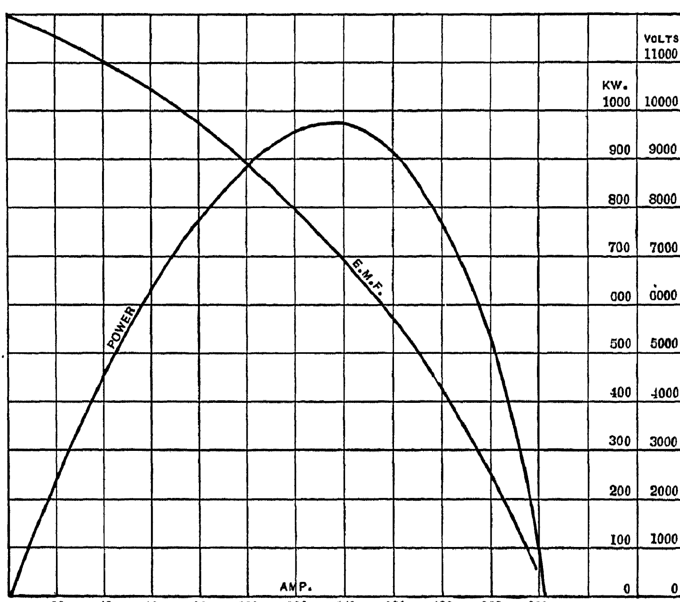


Fig. 40. Non-reactive Load Characteristics of a Transmission Line. Constant Impressed e.m.f.

Let e = voltage at the receiving end of the line, i = current: thus = ei = power received. The voltage impressed upon the line is then

$$\dot{E}_0 = e + Zi$$
$$= e + ri - jxi;$$

or, reduced,

$$E_0 = \sqrt{(e + ri)^2 + x^2 i^2}.$$

Since $E_0 = 12{,}000$,

$$12{,}000 = \sqrt{(e + ri)^2 + x^2 i^2} = \sqrt{(e + 20\,i)^2 + 2500\,i^2},$$
$$e = \sqrt{12{,}000^2 - x^2 i^2} - ri = \sqrt{12{,}000^2 - 2500\,i^2} - 20\,i.$$

The maximum current for $e = 0$ is

$$0 = \sqrt{12{,}000^2 - 2{,}500\,i^2} - 20\,i;$$

thus,

$$i = 223.$$

Substituting for i gives the values plotted in Fig. 40.

i.	e.	$p = ei$.
0	12,000	0
20	11,500	230×10^3
40	11,000	440×10^3
60	10,400	624×10^3
80	9,700	776×10^3
100	8,900	890×10^3
120	8,000	960×10^3
140	6,940	971×10^3
160	5,750	920×10^3
180	4,340	784×10^3
200	2,630	526×10^3
220	400	88×10^3
223	0	0

16. PHASE CONTROL ON TRANSMISSION LINES.

77. If in the receiving circuit of an inductive transmission line the phase relation can be changed, the drop of voltage in the line can be maintained constant at varying loads or even decreased with increasing load; that is, at constant generator voltage the transmission can be compounded for constant voltage at the receiving end, or even over-compounded for a voltage increasing with the load.

1. Compounding of Transmission Lines for Constant Voltage.

Let r = resistance, x = reactance of the transmission line, e_0 = voltage impressed upon the beginning of the line, e = voltage received at the end of ~~end~~ line.

Let i = power current in the receiving circuit; that is, $P = ei$ = transmitted power, and i_1 = reactive current produced in the system for controlling the voltage. i_1 shall be considered positive as lagging, negative as leading current.

Then the total current, in symbolic representation, is

$$\underset{\cdot}{I} = i + ji_1;$$

the line impedance is

$$Z = r - jx,$$

and thus the e.m.f. consumed by the line impedance is

$$\begin{aligned} \underset{\cdot}{E}_1 = Z\underset{\cdot}{I} &= (r - jx)(i + ji_1) \\ &= ri + jri_1 - jxi - j^2xi_1; \end{aligned}$$

and substituting $j^2 = -1$,

$$\underset{\cdot}{E}_1 = (ri + xi_1) + j(ri_1 - xi).$$

Hence the voltage impressed upon the line

$$\begin{aligned} \underset{\cdot}{E}_0 &= e + \underset{\cdot}{E}_1 \\ &= (e + ri + xi_1) + j(ri_1 - xi); \qquad (1) \end{aligned}$$

or, reduced,

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}. \qquad (2)$$

If in this equation e and e_0 are constant, i_1, the reactive component of the current, is given as a function of the power component current i and thus of the load ei.

Hence either e_0 and e can be chosen, or one of the e.m.fs. e_0 or e and the reactive current i_1 corresponding to a given power current i.

78. If $i_1 = 0$ with $i = 0$, and e is assumed as given, $e_0 = e$. Thus,

$$e = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2};$$

$$2e(ri + xi_1) + (r^2 + x^2)(i^2 - i_1^2) = 0.$$

From this equation it follows that

$$\dot{i}_1 = -\frac{ex \pm \sqrt{e^2x^2 - 2\,eriz^2 - \dot{i}^2z^4}}{z^2}. \tag{3}$$

Thus, the reactive current $\dot{i}_1$ must be varied by this equation to maintain constant voltage $e = e_0$ irrespective of the load $e\dot{i}$.

As seen, in this equation, $\dot{i}_1$ must always be negative, that is, the current leading.

$\dot{i}_1$ becomes impossible if the term under the square root becomes negative, that is, at the value

$$e^2x^2 - 2\,eriz^2 - \dot{i}^2z^4 = 0;$$

or,

$$\dot{i} = \frac{e\,(z - r)}{z^2}. \tag{4}$$

At this point the power transmitted is

$$P = e\dot{i} = \frac{e^2\,(z - r)}{z^2}. \tag{5}$$

This is the maximum power which can be transmitted without drop of voltage in the line with a power current $\dot{i} = \frac{e\,(z - r)}{z^2}$.

The reactive current corresponding hereto, since the square root becomes zero, is

$$\dot{i}_1 = -\frac{ex}{z^2}; \tag{6}$$

thus the ratio of reactive to power current, or the tangent of the phase angle of the receiving circuit, is

$$\tan\theta_1 = \frac{\dot{i}_1}{\dot{i}} = -\frac{x}{z - r}. \tag{7}$$

A larger amount of power is transmitted if e_0 is chosen $> e$, a smaller amount of power if $e_0 < e$.

In the latter case $\dot{i}_1$ is always leading; in the former case $\dot{i}_1$ is lagging at no load, becomes zero at some intermediate load, and leading at higher load.

79. If the line impedance $Z = r - jx$ and the received voltage e is given, and the power current $\dot{i}_0$ at which the reactive

current shall be zero, the voltage at the generator end of the line is determined hereby from the equation (2):

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2},$$

by substituting $i_1 = 0$, $i = i_0$,

$$e_0 = \sqrt{(e + ri_0)^2 + x^2 i_0^2}. \qquad (8)$$

Substituting this value in the general equation (2):

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}$$

gives

$$(e + ri_0)^2 + x^2 i_0^2 = (e + ri + xi_1)^2 + (ri_1 - xi)^2 \qquad (9)$$

as equation between i and i_1.

If at constant generator voltage e_0,
at no load

$$\left.\begin{array}{ll} & i = 0,\ e = e_0,\ i_1 = i_0', \\ \text{and at the load} & \\ & i = 1_0,\ e = e_0,\ i_1 = 0 \end{array}\right\} \qquad (10)$$

it is, substituted:
no load,

$$e_0 = \sqrt{(e_0 + xq)^2 + r^2 i_0'^2}, \qquad (11)$$

load i_0,

$$e_0 = \sqrt{(e_0 + ri_0)^2 + x^2 i_0^2}. \qquad (12)$$

Thus,

$$(e_0 + xi_0') + x^2 i_0' = (e_0 + ri_0)^2 + x^2 i_0^2;$$

or, expanded,

$$i_0' (r^2 + x^2) + 2\, i_0' x e_0 = i_0^2 (r^2 + x^2) + 2\, i_0 r e_0. \qquad (13)$$

This equation gives i_0' as function of i_0, e_0, r, x.

If now the reactive current i_1 varies as linear function of the power current i, as in case of compounding by rotary converter with shunt and series field, it is

$$i_1 = \frac{(i_0 - i)}{i_0} i_0'. \qquad (14)$$

Substituting this value in the general equation

$$(e_0 + ri_0)^2 + x^2 i_0^2 = (e + ri + xi_1)^2 + (ri_1 - xi)^2$$

gives e as function of i; that is, gives the voltage at the receiving end as function of the load, at constant voltage e_0 at the generating end, and $e = e_0$ for no load,

$$i = 0, \ i_1 = i_0',$$

and $e = e_0$ for the load,

$$i = i_0, \ i_1 = 0.$$

Between $i = 0$ and $i = i_0$, $e > e_0$, and the current is lagging. Above $i = i_0$, $e < e_0$, and the current is leading.

By the reaction of the variation of e from e_0 upon the receiving apparatus producing reactive current i_1, and by magnetic saturation in the receiving apparatus, the deviation of e from e_0 is reduced, that is, the regulation improved.

2. Over-compounding on Transmission Lines.

80. The impressed voltage at the generator end of the line was found in the preceding,

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}. \quad (2)$$

If the voltage at the end of the line e shall rise proportionally to the power current i, then

$$e = e_1 + ai; \tag{15}$$

thus,

$$e_0 = \sqrt{[e_1 + (a + r)\,i + xi_1]^2 + (ri_1 - xi)^2}, \tag{16}$$

and herefrom in the same way as in the preceding we get the characteristic curve of the transmission.

If $e_0 = e_1$, $i_1 = 0$ at no load, and is leading at load. If $e_0 < e_1$, i_1 is always leading, the maximum output is less than before.

If $e_0 > e_1$, i_1 is lagging at no load, becomes zero at some intermediate load, and leading at higher load. The maximum output is greater than at $e_0 = e_1$.

The greater a, the less is the maximum output at the same e_0 and e_1.

The greater e_0, the greater is the maximum output at the same e_1 and a, but the greater at the same time the lagging current (or less the leading current) at no load.

EXAMPLES.

81. (1.) A constant voltage of e_0 is impressed upon a transmission line of impedance $Z = r - jx = 10 - 20j$. The voltage at the receiving end shall be 10,000 at no load as well as at full load of 75 amp. power current. The reactive current in the receiving circuit is raised proportionally to the load, so as to be lagging at no load, zero at full load or 75 amp., and leading beyond this. What voltage e_0 has to be impressed upon the line, and what is the voltage e at the receiving end at $\frac{1}{3}$, $\frac{2}{3}$, and $1\frac{1}{3}$ load?

Let $\dot{I} = i_1 + ji_2 =$ current, $\dot{E} = e$ voltage in receiving circuit. The generator voltage is then

$$\begin{aligned} \dot{E}_0 &= e + Z\dot{I} \\ &= e + (r - jx)(i_1 + ji_2) \\ &= (e + ri_1 + xi_2) + j(ri_2 - xi_1) \\ &= (e + 10\,i_1 + 20\,i_2) + j(10\,i_2 - 20\,i_1); \end{aligned}$$

or, reduced,

$$\begin{aligned} e_0^2 &= (e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2; \\ &= (e + 10\,i_1 + 20\,i_2)^2 + (10\,i_2 - 20\,i_1)^2. \end{aligned}$$

When

$$i_1 = 75, i_2 = 0, e = 10{,}000;$$

substituting these values,

$$e_0^2 = 10{,}750^2 + 1500^2 = 117.81 \times 10^6;$$

hence,

$$e_0 = 10{,}860 \text{ volts is the generator voltage.}$$

When

$$i_1 = 0,\ e = 10{,}000,\ e_0 = 10{,}860,\ \text{let } i_2 = i;$$

these values substituted give

$$\begin{aligned} 117.81 \times 10^6 &= (10{,}000 + 20\,i)^2 + 100\,i^2 \\ &= 100 \times 10^6 + 400\,i \times 10^3 + 500\,i^2, \end{aligned}$$

or,

$$i = 44.525 - 1.25\,i^2\,10^{-3};$$

this equation is best solved by approximation, and then gives

$$i = p = 42.3 \text{ amp. reactive lagging current at no load.}$$

Since

$$e_0^2 = (e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2,$$

it follows that

$$e = \sqrt{e_0^2 - (ri_2 - xi_1)^2} - (ri_1 + xi_2);$$

or,

$$e = \sqrt{117.81 \times 10^6 - (10\,i_2 - 20\,i_1)^2} - (10\,i_1 + 20\,i_2).$$

Substituting herein the values of i_1 and i_2 gives e.

i_1.	i_2.	e.
0	42.3	10,000
25	28.2	10,038
50	14.1	10,038
75	0	10,000
100	−14.1	9,922
125	−28.2	9,803

82. (2.) A constant voltage e_0 is impressed upon a transmission line of impedance $Z = r - jx = 10 - 10\,j$. The voltage at the receiving end shall be 10,000 at no load as well as at full load of 100 amp. power current. At full load the total current shall be in phase with the e.m.f. at the receiving end, and at no load a lagging current of 50 amp. is permitted. How much additional reactance x_0 is to be inserted, what must be the generator voltage e_0, and what will be the voltage e at the receiving end at ½ load and at 1½ load, if the reactive current varies proportionally with the load?

Let x_0 = additional reactance inserted in circuit.

Let $\dot{I} = i_1 + ji_2$ = current.

Then

$$e_0^2 = (e + ri_1 + x_1 i_2)^2 + (ri_2 - x_1 i_1)^2 = (e + 10\,i_1 + x_1 i_2)^2 + (10\,i_2 - x_1 i_1)^2,$$

where

$x_1 = x + x_0$ = total reactance of circuit between e and e_0.

At no load,

$$i_1 = 0, \; i_2 = 50, \; e = 10{,}000;$$

thus, substituting,

$$e_0^2 = (10{,}000 + 50\,x_1)^2 + 250{,}000.$$

At full load,

$$i_1 = 100,\ i_2 = 0,\ e = 10{,}000;$$

thus, substituting,

$$e_0^2 = 121 \times 10^6 + 10{,}000\, x_1^2.$$

Combining these gives

$$(10{,}000 + 50\, x_1)^2 + 250{,}000 = 121 \times 10^6 + 10{,}000\, x_1^2;$$

hence,

$$\begin{aligned} x_1 &= 66.5 \pm 40.8 \\ &= 107.3 \text{ or } 25.7; \end{aligned}$$

thus

$$x_0 = x_1 - x = 97.3, \text{ or } 15.7 \text{ ohms additional reactance.}$$

Substituting

$$x_1 = 25.7$$

gives

$$e_0^2 = (e + 10\, i_1 + 25.7\, i_2)^2 + (10\, i_2 - 25.7\, i_1)^2,$$

but at full load

$$i_1 = 100,\ i_2 = 0,\ e = 10{,}000,$$

which values substituted give

$$\begin{aligned} e_0^2 &= 121 \times 10^6 + 6.605 \times 10^6 = 127.605 \times 10^6, \\ e_0 &= 11{,}300, \text{ generator voltage.} \end{aligned}$$

Since

$$e = \sqrt{e_0^2 - (10\, i_2 - 25.7\, i_1)^2} - (10\, i_1 + 25.7\, i_2),$$

it follows that

$$e = \sqrt{127.605 \times 10^6 - (10\, i_2 - 25.7\, i_1)^2} - (10\, i_1 + 25.7\, i_2).$$

Substituting for i_1 and i_2 gives e.

i_1.	i_2.	e.
0	50	10,000
50	25	10,105
100	0	10,000
150	−25	9,658

83. (3.) In a circuit whose voltage e_0 fluctuates by 20 per cent between 1800 and 2200 volts, a synchronous motor of internal impedance $Z_0 = r_0 - jx_0 = 0.5 - 5j$ is connected through a reactive coil of impedance $Z_1 = r_1 - jx_1 = 0.5 - 10j$ and run light, as compensator (that is, generator of reactive currents). How will the voltage at the synchronous motor terminals e_1, at constant excitation, that is, constant counter e.m.f. $e = 2000$, vary as function of e_0 at no load and at a load of $i = 100$ amp. power current, and what will be the reactive current in the synchronous motor?

Let $\dot{I} = i_1 + ji_2 =$ current in receiving circuit of voltage e_1. Of this current $\dot{I}$, ji_2 is taken by the synchronous motor of counter e.m.f. e, and thus

$$\begin{aligned} \dot{E}_1 &= e + Z_0 ji_2 \\ &= e + x_0 i_2 + jr_0 i_2; \end{aligned}$$

or, reduced,

$$e_1^2 = (e + x_0 i_2)^2 + r_0^2 i_2^2.$$

In the supply circuit the voltage is

$$\begin{aligned} \dot{E}_0 &= \dot{E}_1 + \dot{I} Z_1 \\ &= e + x_0 i_2 + jr_0 i_2 + (i_1 + ji_2)(r_1 - jx_1) \\ &= [e + r_1 i_1 + (x_0 + x_1) i_2] + j[(r_0 + r_1) i_2 - x_1 i_1]; \end{aligned}$$

or, reduced,

$$e_0^2 = [e + r_1 i_1 + (x_0 + x_1) i_2]^2 + [(r_0 + r_1) i_2 - x_1 i_1]^2.$$

Substituting in the equations for e_1^2 and e_0^2 the above values of r_0 and x_0: at no load, $i_1 = 0$, we have

$$e_1^2 = (e + 5\,i_2)^2 + 0.25\,i_2^2 \text{ and } e_0^2 = (e + 15\,i_2)^2 + i_2^2;$$

at full load, $i_1 = 100$, we have

$$\begin{aligned} e_1^2 &= (e + 5\,i_2)^2 + 0.25\,i_2^2, \\ e_0^2 &= (e + 50 + 15\,i_2)^2 + (i_2 - 1000)^2, \end{aligned}$$

and at no load, $i_1 = 0$, substituting $e = 2000$, we have

$$\begin{aligned} e_1^2 &= (2000 + 5\,i_2)^2 + 0.25\,i_2^2, \\ e_0^2 &= (2000 + 15\,i_2)^2 + i_2^2; \end{aligned}$$

at full load, $i_1 = 100$, we have

$$e_1^2 = (2000 + 5\,i_2)^2 + 0.25\,i_2^2,$$
$$e_0^2 = (2050 + 15\,i_2)^2 + (i_2 - 1000)^2.$$

Substituting herein e_0 = successively 1800, 1900, 2000, 2100, 2200, gives values of i_2, which, substituted in the equation for e_1^2, give the corresponding values of e_1 as recorded in the following table.

As seen, in the local circuit controlled by the synchronous compensator, and separated by reactance from the main circuit of fluctuating voltage, the fluctuations of voltage appear in a greatly reduced magnitude only, and could be entirely eliminated by varying the excitation of the synchronous compensator.

$e = 2{,}000$.				
e_0.	No load, $i_1 = 0$,		Full load, $i_1 = 100$,	
	i_2.	e_1.	i_2.	e_1.
1,800	−13.3	1,937	−39	1,810
1,900	− 6.7	1,965	−30.1	1,850
2,000	0	2,000	−22	1,885
2,100	+ 6.7	2,035	−13.5	1,935
2,200	+13.3	2,074	− 6.5	1,970

17. IMPEDANCE AND ADMITTANCE.

84. In direct-current circuits the most important law is Ohm's law,

$$i = \frac{e}{r}, \quad \text{or } e = ir, \quad \text{or } r = \frac{e}{i},$$

where e is the e.m.f. impressed upon resistance r to produce current i therein.

Since in alternating-current circuits a current i through a resistance r may produce additional e.m.fs. therein, when applying Ohm's law, $i = \frac{e}{r}$ to alternating current circuits, e is the total e.m.f. resulting from the impressed e.m.f. and all e.m.fs. produced by the current i in the circuit.

Such counter e.m.fs. may be due to inductance, as self-inductance, or mutual inductance, to capacity, chemical polarization, etc.

The counter e.m.f. of self-induction, or e.m.f. generated by the magnetic field produced by the alternating current i, is represented by a quantity of the same dimensions as resistance, and measured in ohms: reactance x. The e.m.f. consumed by reactance x is in quadrature with the current, that consumed by resistance r in phase with the current.

Reactance and resistance combined give the impedance,

$$z = \sqrt{r^2 + x^2};$$

or, in symbolic or vector representation,

$$Z = r - jx.$$

In general in an alternating-current circuit of current i, the e.m.f. e can be resolved in two components, a power component e_1 in phase with the current, and a wattless or reactive component e_2 in quadrature with the current.

The quantity

$$\frac{e_1}{i} = \frac{\text{power e.m.f., or e.m.f. in phase with the current}}{\text{current}} = r_1$$

is called the *effective resistance*.

The quantity

$$\frac{e_2}{i} = \frac{\text{reactive e.m.f., or e.m.f. in quadrature with the current}}{\text{current}} = x_1$$

is called the *effective reactance* of the circuit.

And the quantity

$$z_1 = \sqrt{r_1^2 + x^2}$$

or, in symbolic representation,

$$Z_1 = r_1 - jx_1$$

is the impedance of the circuit.

If power is consumed in the circuit only by the ohmic resistance r, and counter e.m.f. produced only by self-inductance, the effective resistance r_1 is the true or ohmic resistance r, and the effective reactance x_1 is the true or inductive reactance x.

By means of the terms effective resistance, effective reactance, and impedance, Ohm's law can be expressed in alternating-current circuits in the form

$$i = \frac{e}{z_1} = \frac{e}{\sqrt{r_1^2 + x_1^2}}; \tag{1}$$

or, $$e = iz_1 = i\sqrt{r_1^2 + x_1^2}; \tag{2}$$

or, $$z_1 = \sqrt{r_1^2 + x_1^2} = \frac{e}{i}; \tag{3}$$

or, in symbolic or vector representation,

$$\dot{I} = \frac{\dot{E}}{Z_1} = \frac{\dot{E}}{r_1 - jx}; \tag{4}$$

or, $$\dot{E} = \dot{I}Z_1 = \dot{I}(r_1 - jx_1); \tag{5}$$

or, $$Z_1 = r_1 - jx_1 = \frac{\dot{E}}{\dot{I}}. \tag{6}$$

In this latter form Ohm's law expresses not only the intensity but also the phase relation of the quantities; thus

$$e_1 = ir_1 = \text{power component of e.m.f.,}$$
$$e_2 = ix_1 = \text{reactive component of e.m.f.}$$

85. Instead of the term impedance $z = \frac{e}{i}$ with its components, the resistance and reactance, its reciprocal can be introduced,

$$\frac{i}{e} = \frac{1}{z},$$

which is called the *admittance.*

The components of the admittance are called the *conductance* and *susceptance.*

Resolving the current i into a power component i_1 in phase with the e.m.f. and a wattless component i_2 in quadrature with the e.m.f., the quantity

$$\frac{i_1}{e} = \frac{\text{power current, or current in phase with e.m.f.}}{\text{e.m.f.}} = g$$

is called the *conductance.*

The quantity

$$\frac{i_2}{e} = \frac{\text{reactive current, or current in quadrature with e.m.f.}}{\text{e.m.f.}} = b$$

is called the *susceptance* of the circuit.

The conductance represents the current in phase with the e.m.f. or power current, the susceptance the current in quadrature with the e.m.f. or reactive current.

Conductance g and susceptance b combined give the admittance

$$y = \sqrt{g^2 + b^2}; \tag{7}$$

or, in symbolic or vector representation,

$$Y = g + jb. \tag{8}$$

Thus Ohm's law can also be written in the form

$$i = ey = e\sqrt{g^2 + b^2}; \tag{9}$$

or,

$$e = \frac{i}{y} = \frac{i}{\sqrt{g^2 + b^2}}; \tag{10}$$

or,

$$y = \sqrt{g^2 + b^2} = \frac{i}{e}; \tag{11}$$

or, in symbolic or vector representation,

$$\underset{\cdot}{I} = \underset{\cdot}{E}Y = \underset{\cdot}{E}(g + jb); \tag{12}$$

or,

$$\underset{\cdot}{E} = \frac{\underset{\cdot}{I}}{Y} = \frac{\underset{\cdot}{I}}{g + jb}; \tag{13}$$

or,

$$Y = g + jb = \frac{\underset{\cdot}{I}}{\underset{\cdot}{E}}. \tag{14}$$

and $\quad i_1 = eg =$ power component of current,

$\quad\quad\; i_2 = eb =$ reactive component of current.

86. According to circumstances, sometimes the use of the terms impedance, resistance, reactance, sometimes the use of the terms admittance, conductance, susceptance, is more convenient.

Since, in a number of series-connected circuits, the total e.m.f., in symbolic representation, is the sum of the individual e.m.fs., it follows that in a number of series-connected circuits the total impedance, in symbolic expression, is the sum of the impedances of the individual circuits connected in series.

Since, in a number of parallel-connected circuits, the total current, in symbolic representation, is the sum of the individual currents, it follows that in a number of parallel-connected circuits the total admittance, in symbolic expression, is the sum of the admittances of the individual circuits connected in parallel.

Thus in series connection the use of the term impedance, in parallel connection the use of the term admittance, is generally more convenient.

Since in symbolic representation

$$Y = \frac{1}{Z}; \tag{15}$$

or,

$$ZY = 1; \tag{16}$$

that is,

$$(r - jx)(g + jb) = 1; \tag{17}$$

it follows that

$$(rg + xb) + j(rb - xg) = 1;$$

that is

$$rg + zb = 1,$$
$$rb - xg = 0.$$

Thus,

$$r = \frac{g}{g^2 + b^2} = \frac{g}{y^2}, \tag{18}$$

$$x = \frac{b}{g^2 + b^2} = \frac{b}{y^2}, \tag{19}$$

$$g = \frac{r}{r^2 + x^2} = \frac{r}{z^2}, \tag{20}$$

$$b = \frac{x}{r^2 + x^2} = \frac{x}{z^2}; \tag{21}$$

or, in absolute values,

$$y = \frac{1}{z}, \tag{22}$$

$$zy = 1, \tag{23}$$

$$(r^2 + x^2)(g^2 + b^2) = 1. \tag{24}$$

Thereby the admittance with its components, the conductance and susceptance, can be calculated from the impedance and its components, the resistance and reactance, and inversely.

If $x = 0$, $z = r$ and $g = \frac{1}{r}$, that is, g is the reciprocal of the resistance in a non-inductive circuit; not so, however, in an inductive circuit.

EXAMPLES.

87. (1.) In a quarter-phase induction motor having an impressed e.m.f. $e = 110$ volts per phase, the current is $I_0 = i_1 + ji_2 = 100 + 100j$ at standstill, the torque $= D_0$.

The two phases are connected in series in a single-phase circuit of e.m.f. $e = 220$, and one phase shunted by a condenser of 1 ohm capacity reactance.

What is the starting torque D of the motor under these conditions, compared with D_0, the torque on a quarter-phase circuit, and what the relative torque per volt-ampere imput, if the torque is proportional to the product of the e.m.fs. impressed upon the two circuits and the sine of the angle of phase displacement between them?

In the quarter-phase motor the torque is

$$D_0 = ae^2 = 12{,}100\, a,$$

where a is a constant. The volt-ampere input is

$$P_{a_0} = 2\, e \sqrt{i_1^2 + i_2^2} = 31{,}200;$$

hence, the "apparent torque efficiency," or torque per volt-ampere input,

$$\eta_0 = \frac{D_0}{P_{a_0}} = 0.388\, a.$$

The admittance per motor circuit is

$$Y = \frac{I}{e} = 0.91 + 0.91\, j,$$

the impedance is

$$Z = \frac{e}{I} = \frac{110}{100 + 100\, j} = \frac{110\ (100 - 100\, j)}{(100 + 100\, j)\ (100 - 100\, j)} = 0.55 - 0.55\, j,$$

the admittance of the condenser is

$$Y_0 = -j;$$

thus, the joint admittance of the circuit shunted by the condenser is

$$\begin{aligned} Y_1 &= Y + Y_0 = 0.91 + 0.91\, j - j \\ &= 0.91 - 0.09\, j; \end{aligned}$$

its impedance is

$$Z_1 = \frac{1}{Y_1} = \frac{1}{0.91 - 0.09\, j} = \frac{0.91 + 0.09\, j}{0.91^2 + 0.09^2} = 1.09 + 0.11\, j,$$

and the total impedance of the two circuits in series is

$$\begin{aligned} Z_2 &= Z + Z_1 \\ &= 0.55 - 0.55\, j + 1.09 + 0.11\, j \\ &= 1.64 - 0.44\, j. \end{aligned}$$

Hence, the current, at impressed e.m.f. $e = 220$,

$$\dot{I} = i_1 + ji_2 = \frac{e}{Z_2} = \frac{220}{1.64 - 0.44\,j} = \frac{220\,(1.64 + 0.44\,j)}{1.64^2 + 0.44^2}$$
$$= 125 + 33.5\,j;$$

or, reduced,

$$I = \sqrt{125^2 + 33.5^2}$$
$$= 129.4 \text{ amp.}$$

Thus, the volt-ampere input,

$$P_a = eI = 220 \times 129.4$$
$$= 28{,}470.$$

The e.m.fs. acting upon the two motor circuits respectively are

$$\dot{E}_1 = \dot{I}Z_1 = (125 + 33.5\,j)\ (1.09 + 0.11\,j) = 132.8 + 50.4\,j$$

and

$$\dot{E}' = \dot{I}Z = (125 + 33.5\,j)\,(0.55 - 0.55\,j) = 87.2 - 50.4\,j.$$

Thus, the tangents of their phase angles are

$$\tan\theta_1 = +\frac{50.4}{132.8} = +\,0.30;\ \text{hence, } \theta_1 = +\,21°;$$

$$\tan\theta' = -\frac{50.4}{87.2} = -\,0.579;\ \text{hence, } \theta' = -\,30°;$$

and the phase difference,

$$\theta = \theta_1 - \theta' = 51°.$$

The absolute values of these e.m.fs. are

$$e_1 = \sqrt{132.8 + 50.4^2} = 141.5$$

and

$$e' = \sqrt{87.2^2 - 50.4^2} = 100.7;$$

thus, the torque is

$$D = ae_1e'\sin\theta$$
$$= 11{,}100\,a;$$

and the apparent torque efficiency is

$$\eta_t = \frac{D}{P_a}\frac{11{,}100\,a}{28{,}470} = 0.39\,a.$$

Hence, comparing this with the quarter-phase motor, the relative torque is

$$\frac{D}{D_0} = \frac{11{,}100\,a}{12{,}100\,a} = 0.92,$$

and the relative torque per volt-ampere, or relative apparent torque efficiency, is

$$\frac{\eta t}{\eta t_0} = \frac{0.39\,a}{0.388\,a} = 1.005.$$

88. (2.) At constant field excitation, corresponding to a nominal generated e.m.f. $e_0 = 12{,}000$, a generator of synchronous impedance $Z_0 = r_0 - jx_0 = 0.6 - 60j$ feeds over a transmission line of impedance $Z_1 = r_1 - jx_1 = 12 - 18\,j$, and of capacity susceptance 0.003, a non-inductive receiving circuit. How will the voltage at the receiving end, e, and the voltage at the generator terminals, e_1, vary with the load if the line capacity is represented by a condenser shunted across the middle of the line?

Let $\dot{I} = i$ = current in receiving circuit, in phase with the e.m.f., $\dot{E} = e$.

The voltage in the middle of the line is

$$\begin{aligned}\dot{E}_2 &= \dot{E} + \frac{Z_1}{2}\dot{I}\\ &= e + 6\,i - 9\,ij.\end{aligned}$$

The capacity susceptance of the line is, in symbolic expression, $Y = -\,0.003\,j$; thus the charging current is

$$\begin{aligned}\dot{I}_2 = \dot{E}_2 Y &= -\,0.003\,j\,(e + 6\,i - 9\,ij)\\ &= -\,0.027\,i - j\,(0.003\,e + 0.018\,i),\end{aligned}$$

and the total current is

$$\dot{I}_1 = \dot{I} + \dot{I}_2 = 0.973\,i - j\,(0.003\,e + 0.018\,i).$$

Thus, the voltage at the generator end of the line is

$$\begin{aligned}\dot{E}_1 &= \dot{E}_2 + \frac{Z_1}{2}\dot{I}_1\\ &= e + 6\,i - 9\,ij + (6 - 9\,j)\,[0.973\,i - j\,(0.003\,e + 0.018\,i)]\\ &= (0.973\,e + 11.68\,i) - j\,(17.87\,i + 0.018\,e),\end{aligned}$$

and the nominal generated e.m.f. of the generator is

$$\begin{aligned}\dot{E}_0 &= \dot{E}_1 + Z_0\dot{I}_1\\ &= (0.973\,e + 11.68\,i) - j\,(17.87\,i + 0.018\,e) + (0.6 - 60\,j)\\ &\qquad [0.973\,i - j\,(0.003\,e + 0.018\,i)]\\ &= (0.793\,e + 11.18\,i) - j\,(76.26\,i + 0.02\,e);\end{aligned}$$

or, reduced, and $e_0 = 12{,}000$ substituted,

$$e_0^2 = 144 \times 10^6 = (0.793\,e + 11.18\,i)^2 + (76.26\,i + 0.02\,e)^2;$$

thus,

$$e^2 + 33\,ei + 9450\,i^2 = 229 \times 10^6,$$

$$e = -16.5\,i + \sqrt{229 \times 10^6 - 9178\,i^2},$$

and

$$e_1 = \sqrt{(0.973\,e + 11.68\,i)^2 + (17.87\,i + 0.018\,e)^2};$$

at

$$i = 0,\ e = 15{,}133,\ e_1 = 14{,}700;$$

at

$$e = 0,\ i = 155.6,\ e_1 = 3327.$$

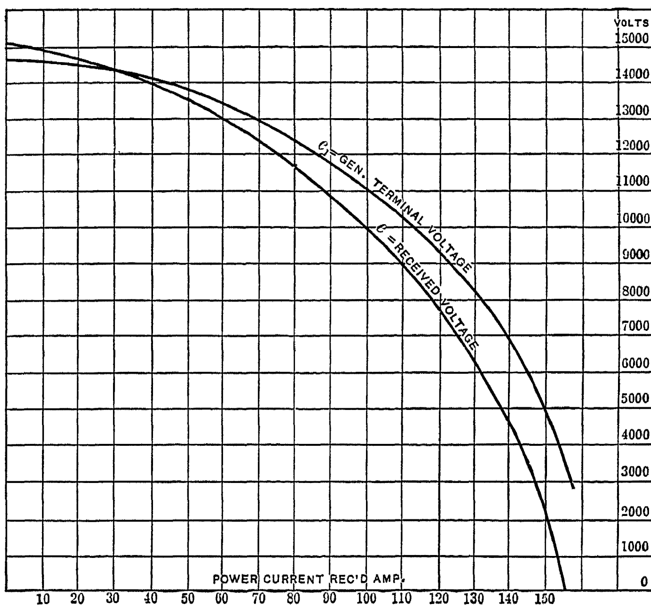


Fig. 41. Reactive Load Characteristics of a Transmission Line fed by a Synchronous Generator with Constant Field Excitation.

Substituting different values for i gives

i	$e.$	$e_1.$	$i.$	$e.$	$e_1.$
0	15,133	14,700	100	10,050	11,100
25	14,488	14,400	125	7,188	8,800
50	13,525	13,800	150	2,325	4,840
75	12,063	12,730	155.6	0	3,327

which values are plotted in Fig. 41.

18. EQUIVALENT SINE WAVES.

89. In the preceding chapters, alternating waves have been assumed and considered as sine waves.

The general alternating wave is, however, never completely, frequently not even approximately, a sine wave.

A sine wave having the same effective value, that is, the same square root of mean squares of instantaneous values, as a general alternating wave, is called its corresponding "equivalent sine wave." It represents the same effect as the general wave.

With two alternating waves of different shapes, the phase relation or angle of lag is indefinite. Their equivalent sine waves, however, have a definite phase relation, that which gives the same effect as the general wave, that is, the same mean (ei).

Hence if e = e.m.f. and i = current of a general alternating wave, their equivalent sine waves are defined by

$$e_0 = \sqrt{\text{mean } (e^2)},$$
$$i_0 = \sqrt{\text{mean } (i^2)};$$

and the power is

$$p_0 = e_0 i_0 \cos e_0 i_0 = \text{mean } (ei);$$

thus,

$$\cos e_0 i_0 = \frac{\text{mean } (ei)}{\sqrt{\text{mean } (e^2)}\sqrt{\text{mean } (i^2)}}.$$

Since by definition the equivalent sine waves of the general alternating waves have the same effective value or intensity and the same power or effect, it follows that in regard to intensity and effect the general alternating waves can be represented by their equivalent sine waves.

Considering in the preceding the alternating currents as equivalent sine waves representing general alternating waves, the investigation becomes applicable to any alternating circuit irrespective of the wave shape.

The use of the terms reactance, impedance, etc., implies that a wave is a sine wave or represented by an equivalent sine wave.

Practically all measuring instruments of alternating waves (with exception of instantaneous methods) as ammeters, voltmeters, wattmeters, etc., give not general alternating waves but their corresponding equivalent sine waves.

EXAMPLES.

90. In a 25-cycle alternating-current transformer, at 1000 volts primary impressed e.m.f., of a wave shape as shown in Fig. 42 and Table I, the number of primary turns is 500, the length of the magnetic circuit 50 cm., and its section shall be chosen so as to give a maximum density $\mathcal{B} = 15{,}000$.

At this density the hysteretic cycle is as shown in Fig. 43 and Table II.

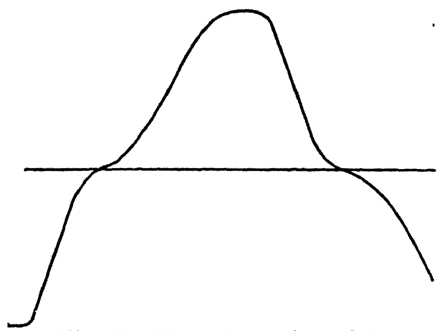


Fig. 42. Wave-shape of e.m.f. in Example 90.

What is the shape of current wave, and what the equivalent sine waves of e.m.f., magnetism, and current?

The calculation is carried out in attached table.

TABLE II.

$\mathcal{H}$	
0	± 8,000
2	+10,400− 2,500
4	+11,700+ 5,800
6	+12,400+ 9,300
8	+13,000+11,200
10	+13,500+12,400
12	+13,900+13,200
14	+14,200+13,800
16	+14,500+14,300
18	+14,800+14,700
20	+15,000

In column (1) are given the degrees, in column (2) the relative values of instantaneous e.m.fs., e corresponding thereto, as taken from Fig. 42.

Column (3) gives the squares of e. Their sum is 24,939; thus the mean square, $\frac{24,939}{18} = 1385.5$, and the effective value,

$$e' = \sqrt{1385.5} = 37.22.$$

Since the effective value of impressed e.m.f. is = 1000, the instantaneous values are $e_0 = e\frac{1000}{37.22}$ as given in column (4).

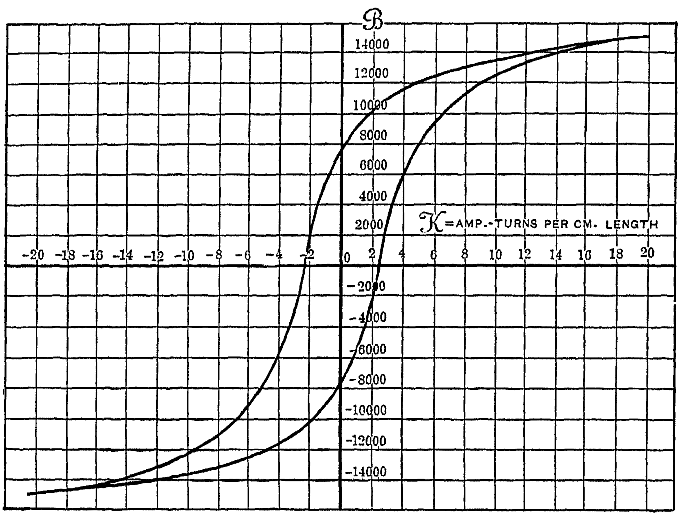


Fig. 43. Hysteretic Cycle in Example 90.

Since the e.m.f. e_0 is proportional to the rate of change of magnetic flux, that is, to the differential coefficient of $\mathfrak{B}$, $\mathfrak{B}$ is proportional to the integral of the e.m.f., that is, to Σe_0 plus an integration constant. Σe_0 is given in column (5), and the integration constant follows from the condition that $\mathfrak{B}$ at 180° must be equal, but opposite in sign, to $\mathfrak{B}$ at 0°. The integration constant is, therefore,

$$-\tfrac{1}{2}\sum_0^{180} e_0 = -\frac{14,648}{2} = -7324,$$

and by subtracting 7324 from the values in column (5) the values of $\mathfrak{B}'$ of column (6) are found as the relative instantaneous values of magnetic flux density.

TABLE I.

(1) Degrees.	(2) e.	(3) e^2.	(4) $e\frac{1000}{37 \cdot 22} = e_0$.	(5) Σe_0.	(6) $\Sigma e_0 - 7,324 = \mathfrak{B}'$.	(7) $\mathfrak{B}' \frac{15000}{7324} = \mathfrak{B}$.	(8) From Hysteresis cycle, $\mathfrak{H}$.	(9) $50\ \mathfrak{H} = \mathfrak{F}$.	(10) $\frac{\mathfrak{F}}{500} = i$.	(11) i^2.	(12) $p = ie_0$.
0	0	0	0	0	−7,324	−15,000	−20	−1,000	−2.00	4.00	0
10	1	1	27	27	−7,296	−14,950	−19 5	− 975	−1.95	3.80	− 53
20	3.5	12	94	121	−7,203	−14,800	−18	− 900	−1.80	3.24	− 169
30	8	64	216	337	−6,987	−14,300	−14 3	− 715	−1.43	2.04	− 308
40	14	196	377	714	−6,610	−13,550	−10	− 500	−1.00	1.00	− 377
50	22	484	591	1,305	−6,019	−12,350	− 5.5	− 275	− .55	.30	− 325
60	31	961	835	2,140	−5,184	−10,600	− 2.3	− 115	− .23	.05	− 191
70	41	1,681	1,100	3,240	−4,084	− 8,370	− .2	− 10	− .02	.00	− 22
80	50	2,500	1,345	4,585	−2,739	− 5,600	+ 1.0	+ 50	+ .10	.01	+ 134
90	55	3,025	1,480	6,065	−1,259	− 2,580	+ 1 9	+ 95	+ .19	.04	+ 281
100	57	3,249	1,535	7,600	+ 276	+ 570	+ 2 6	+ 130	+ .26	.07	+ 398
110	58	3,364	1,560	9,160	+1,836	+ 3,550	+ 3 3	+ 165	+ .33	.11	+ 514
120	58	3,364	1,560	10,720	+3,396	+ 6,970	+ 4.5	+ 225	+ .45	.20	+ 700
130	56	3,136	1,508	12,228	+4,904	+10,050	+ 6.6	+ 330	+ .66	.44	+ 995
140	43	1,849	1,155	13,383	+6,059	+12,400	+10.0	+ 500	+1.00	1.00	+1,155
150	29	841	780	14,163	+6,839	+14,000	+14 2	+ 710	+1.42	2.02	+1,108
160	14	196	377	14,540	+7,216	+14,800	+18.8	+ 940	+1 88	3.53	+ 710
170	4	16	108	14,648	+7,324	+15,000	+20.0	+1,000	+2 00	4.09	+ 216
180	0	0	0	14,648	+7,324	+15,000					
		$\Sigma = 24,939$								$\Sigma = 25.85$	$\Sigma = 4,766$

$$\text{mean } e^2 = \frac{24939}{18} = 1385.5$$

$$e' = \text{eff. } e = \sqrt{1385.5} = 37.22.$$

$$\text{mean } i^2 = \frac{25\ 85}{18} = 1.436$$

$$i' = \text{eff. } i = \sqrt{1.436} = 1.198$$

$$p' = \text{mean } p = \frac{4766}{18} = 264.8$$

Since the maximum magnetic flux density is 15,000 the instantaneous values are $\mathfrak{B} = \mathfrak{B}' \frac{15{,}000}{7324}$, plotted in column (7).

From the hysteresis cycle in Fig. 43 are taken the values of magnetizing force $\mathfrak{H}$, corresponding to magnetic flux density $\mathfrak{B}$. They are recorded in column (8), and in column (9) the instantaneous values of m.m.f. $\mathfrak{F} = l\mathfrak{H}$, where $l = 50 =$ length of magnetic circuit.

$i = \frac{\mathfrak{F}}{n}$, where $n = 500 =$ number of turns of the electric circuit, gives thus the exciting current in column (10).

Column (11) gives the squares of the exciting current, i^2. Their sum is 25.85; thus the mean square, $\frac{25.85}{18} = 1.436$, and the effective value of exciting current, $i' = \sqrt{1.436} = 1.198$ amp.

Column (12) gives the instantaneous values of power, $p = ie_0$. Their sum is 4766; thus the mean power, $p' = \frac{4766}{18} = 264.8$.

Since $$p' = i'e_0' \cos \theta,$$

where e_0' and i' are the equivalent sine waves of e.m.f. and of current respectively, and θ their phase displacement, substituting these numerical values of p', e', and i', we have

$$264.8 = 1000 \times 1.198 \cos \theta \cdot$$

hence,

$$\cos \theta = 0.221,$$
$$\theta = 77.2°,$$

and the angle of hysteretic advance of phase,

$$\alpha = 90° - \theta = 12.8°.$$

The hysteresis current is then

$$i' \cos \theta = 0.265,$$

and the magnetizing current,

$$i' \sin \theta \doteq 1.165.$$

Adding the instantaneous values of e.m.f. e_0 in column (4) gives 14,648; thus the mean value, $\frac{14,648}{18} = 813.8$. Since the effective value is 1000, the mean value of a sine wave would be $1000 \frac{2\sqrt{2}}{\pi} = 904$; hence the form factor is

$$\gamma = \frac{904}{813.8} = 11.1.$$

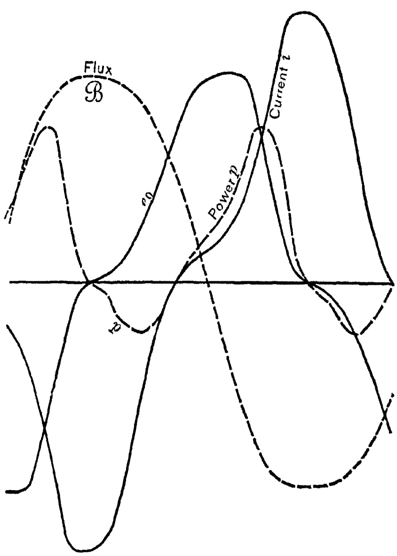


Fig. 44. Waves of Exciting Current. Power and Flux Density Corresponding to e.m.f. in Fig. 42 and Hysteretic Cycle in Fig. 43.

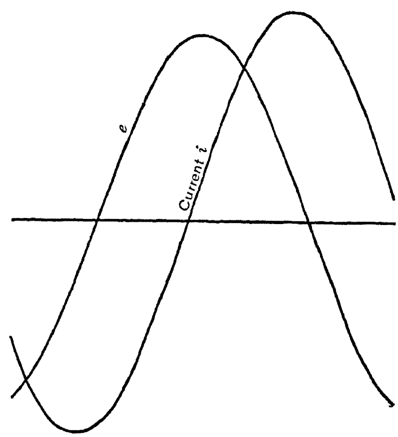


Fig. 45. Corresponding Sine Waves for e.m.f. and Exciting Current in Fig. 44.

Adding the instantaneous values of current i in column (10), irrespective of their sign, gives 17.17; thus the mean value, $\frac{17.17}{18} = 0.954$. Since the effective value $= 1.198$, the form factor is

$$\gamma = \frac{1.198}{0.954} \frac{2\sqrt{2}}{\pi} = 1.12.$$

The instantaneous values, of e.m.f. e_0, current, i, flux density $\mathfrak{B}$ and power p are plotted in Fig. 44, their corresponding sine waves in Fig. 45.

PART II.

SPECIAL APPARATUS.

INTRODUCTION.

1. By the direction of the energy transmitted, electric machines have been divided into generators and motors. By the character of the electric power they have been distinguished as direct-current and as alternating-current apparatus.

With the advance of electrical engineering, however, these subdivisions have become unsatisfactory and insufficient.

The division into generators and motors is not based on any characteristic feature of the apparatus, and is thus not rational. Practically any electric generator can be used as motor, and conversely, and frequently one and the same machine is used for either purpose. Where a difference is made in the construction, it is either only quantitative, as, for instance, in synchronous motors a much higher armature reaction is used than in synchronous generators, or it is in minor features, as direct-current motors usually have only one field winding, either shunt or series, while in generators frequently a compound field is employed. Furthermore, apparatus have been introduced which are neither motors nor generators, as the synchronous machine producing wattless lagging or leading current, etc., and the different types of converters.

The subdivision into direct-current and alternating-current apparatus is unsatisfactory, since it includes in the same class apparatus of entirely different character, as the induction motor and the alternating-current generator, or the constant-potential commutating machine and the rectifying arc light machine.

Thus the following classification, based on the characteristic features of the apparatus, has been adopted by the A. I. E. E. Standardizing Committee, and is used in the following discussion. It refers only to the apparatus transforming between electric and electric and between electric and mechanical power.

1st. *Commutating machines*, consisting of a magnetic field and a closed-coil armature, connected with a multi-segmental commutator.

2d. *Synchronous machines*, consisting of a undirectional magnetic field and an armature revolving relatively to the magnetic field at a velocity synchronous with the frequency of the alternating-current circuit connected thereto.

3d. *Rectifying apparatus;* that is, apparatus reversing the direction of an alternating current synchronously with the frequency.

4th. *Induction machines*, consisting of an alternating magnetic circuit or circuits interlinked with two electric circuits or sets of circuits moving with regard to each other.

5th. *Stationary induction apparatus*, consisting of a magnetic circuit interlinked with one or more electric circuits.

6th. *Electrostatic and electrolytic apparatus* as condensers and polarization cells.

Apparatus changing from one to a different form of electric energy have been defined as:

A. *Transformers*, when using magnetism, and as

B. *Converters*, when using mechanical momentum as intermediary form of energy.

The transformers as a rule are stationary, the converters rotary apparatus. Motor-generators transforming from electrical over mechanical to electric power by two separate machines, and dynamotors, in which these two machines are combined in the same structure, are not included under converters.

2. (1.) *Direct-current commutating machines* as *generators* are usually built to produce constant potential for railway, incandescent lighting, and general distribution. Only rarely they are designed for approximately constant power for electro-metallurgical work, or approximately constant current for series incandescent or arc lighting. As *motors* commutating machines give approximately constant speed, — shunt motors, — or large starting torque, — series motors.

When inserted in series in a circuit, and controlled so as to give an e.m.f. varying with the conditions of load on the system, these machines are "*boosters*," and are generators when raising the voltage, and motors when lowering it.

Commutating machines may be used as *direct-current converters* by transforming power from one side to the other side of a three-wire system.

Alternating-current commutating machines are mainly used as single-phase railway motors.

(2.) While in commutating machines the magnetic field is almost always stationary and the armature rotating, *synchronous machines* were built with stationary field and revolving armature, or with stationary armature and revolving field, or as inductor machines with stationary armature and stationary field winding but revolving magnetic circuit. Generally now the revolving field type is used.

By the number and character of the alternating circuits connected to them they are single-phase or polyphase machines. As *generators* they comprise practically all single-phase and polyphase alternating-current generators; as motors a very important class of apparatus, the *synchronous motors*, which are usually preferred for large powers, especially where frequent starting and considerable starting torque are not needed. Synchronous machines may be used as *compensators* to produce wattless current, leading by over-excitation, lagging by under-excitation, or may be used as *phase converters* by operating a polyphase synchronous motor by one pair of terminals from a single-phase circuit. The most important class of converters, however, are the synchronous commutating machines, to which, therefore, a special chapter will be devoted in the following.

Inserted in series to another synchronous machine, and rigidly connected thereto, synchronous machines are also occasionally used as boosters.

Synchronous commutating machines contain a unidirectional magnetic field and a closed circuit armature connected simultaneously to a segmental direct-current commutator and by collector rings to an alternating circuit, generally a polyphase system. Thus these machines can either receive alternating and yield direct current power as synchronous converters or simply "*converters*," or receive direct and yield alternating current power as *inverted converters*, or driven by mechanical power yield alternating and direct current as *double-current generators*.

Or they can combine motor and generator action with their converter action. Thus a common combination is a synchronous converter supplying a certain amount of mechanical power as a synchronous motor.

(3.) *Rectifying machines* are apparatus which by a synchronously revolving rectifying commutator send the successive half waves of an alternating single-phase or polyphase circuit in the same direction into the receiving circuit. The most important class of such apparatus are the *open-coil arc light machines,* which generate the rectified e.m.f. at approximately constant current, in a star-connected three-phase armature in the Thomson-Houston, as quarter-phase e.m.f. in the Brush arc-light machine.

(4.) *Induction machines* are generally used as *motors,* polyphase or single-phase. In this case they run at practically constant speed, slowing down slightly with increasing load. As *generators* the frequency of the e.m.f. supplied by them differs from and is lower than the frequency of rotation, but their operation depends upon the phase relation of the external circuit. As *phase converters* induction machines can be used in the same manner as synchronous machines. Another important use besides as motors is, however, as *frequency converters,* by changing from an impressed primary polyphase system to a secondary polyphase system of different frequency. In this case, when lowering the frequency, mechanical energy is also produced; when raising the frequency, mechanical energy is consumed.

(5.) The most important *stationary induction apparatus* is the *transformer,* consisting of two electric circuits interlinked with the same magnetic circuit. When using the same or part of the same electric circuit for primary and secondary, the transformer is called an *auto-transformer* or *compensator.* When inserted in series into a circuit, and arranged to vary the e.m.f., the transformer is called *potential regulator* or booster. The variation of secondary e.m.f. may be secured by varying the relative number of primary and secondary turns, or by varying the mutual inductance between primary and secondary circuit, either electrically or magnetically. The stationary induction apparatus with one electric circuit are used for producing wattless lagging currents, as *reactive* or *choking coils.*

(6.) *Condensers* and *polarization cells* produce wattless leading currents, the latter, however, usually at a low efficiency, while the efficiency of the condenser is extremely high, frequently above 99 per cent; that is, the loss of power is less than 1 per cent of the apparent volt-ampere input.

Unipolar, or, more correctly, *non-polar,* or *acyclic machines* are apparatus in which a conductor cuts a continuous magnetic field at a uniform rate. They have become of industrial importance with the development of high-speed prime movers.

Regarding apparatus transforming between electric energy and forms of energy differing from electric or mechanical energy: The transformation between electrical and chemical energy is represented by the primary and secondary battery and the electrolytic cell; the transformation between electrical and heat energy by the thermopile and the electric heater or electric furnace; the transformation between electrical and light energy by the incandescent and arc lamps.

A. SYNCHRONOUS MACHINES.

I. General.

3. The most important class of alternating-current apparatus consists of the synchronous machines. They comprise the alternating-current generators, single-phase and polyphase, the synchronous motors, the phase compensators, the synchronous boosters and the exciters of induction generators, that is, synchronous machines producing wattless lagging or leading currents, and the converters. Since the latter combine features of the commutating machines with those of the synchronous machines they will be considered separately.

In the synchronous machines the terminal voltage and the generated e.m.f. are in synchronism with, that is, of the same frequency as, the speed of rotation.

These machines consist of an armature, in which e.m.f. is generated by the rotation relatively to a magnetic field, and a continuous magnetic field, excited either by direct current, or by the reaction of displaced phase armature currents, or by permanent magnetism.

The formula for the e.m.f. generated in synchronous machines, commonly called alternators, is

$$E = \sqrt{2}\,\pi f n \Phi = 4.44\, f n \Phi,$$

where n is the number of armature turns in series interlinked with the magnetic flux Φ (in megalines per pole), f the frequency of rotation (in hundreds of cycles per second), E the e.m.f. generated in the armature turns.

This formula assumes a sine wave of e.m.f. If the e.m.f. wave differs from sine shape, the e.m.f. is

$$E = 4.44\, \gamma f n \Phi,$$

where γ = form factor of the wave, or $\frac{2\sqrt{2}}{\pi}$ times ratio of effective to mean value of wave, that is, the ratio of the effective value of the generated e.m.f. to that of a sine wave generated by the same magnetic flux at the same frequency.

The form factor γ depends upon the wave shape of the generated e.m.f. The wave shape of e.m.f. generated in a single conductor on the armature surface is identical with that of the distribution of magnetic flux at the armature surface and will be discussed more fully in the chapter on commutating machines. The wave of total e.m.f. is the sum of the waves of e.m.f. in the individual conductors, added in their proper phase relation, as corresponding to their relative positions on the armature surface.

4. In a Y or star-connected three-phase machine, if E_0 = e.m.f. per circuit, or Y or star e.m.f., $E = E_0 \sqrt{3}$ is the e.m.f. between terminals or Δ (delta) or ring e.m.f., since two e.m.fs. displaced by 60 degrees are connected in series between terminals ($\sqrt{3} = 2 \cos 30°$).

In a Δ-connected three-phase machine, the e.m.f. per circuit is the e.m.f. between the terminals, or Δ e.m.f.

In a Y-connected three-phase machine, the current per circuit is the current issuing from each terminal, or the line current, or Y current.

In a Δ-connected three-phase machine, if I_0 = current per circuit, or Δ current, the current issuing from each terminal, or the line or Y current, is

$$I = I_0 \sqrt{3}.$$

Thus in a three-phase system, Δ current and e.m.f., and Y current and e.m.f. (or ring and start current and e.m.f. respectively), are to be distinguished. They stand in the proportion $1 \div \sqrt{3}$.

As a rule, when speaking of current and of e.m.f. in a three-phase system, under current the Y current or current per line, and under e.m.f. the Δ e.m.f. or e.m.f. between lines is understood.

5. While the voltage wave of a single conductor has the same shape as the distribution of the magnetic flux at the armature circumference and so may differ considerably from a sine, that is, contains pronounced higher harmonics, the terminal voltage is the resultant of the waves of many conductors, and, especially with a distributed armature winding, shows the higher harmonics in a much reduced degree; that is, the resultant

is nearer sine shape, and some harmonics may be entirely eliminated in the terminal voltage wave, though they may appear in the voltage wave of a single conductor. Thus, for instance, in a three-phase Y-connected machine, the voltage per circuit, or Y voltage, may contain a third harmonic and multiples thereof, while in the voltage between the terminals this third harmonic is eliminated. The voltage between the terminals is the resultant of two Y voltages, displaced from each other by 60 degrees. Sixty degrees for the fundamental, however, is $3 \times 60° = 180°$, or opposition for the third harmonic; that is, the third harmonics in those two Y voltages, which combine to the delta or terminal voltage, are opposite, and so neutralize each other.

Even in a single turn, harmonics existing in the magnetic field and thus in the single conductor can be eliminated by fractional pitch. Thus, if the pitch of the armature turn is not 180 degrees, but less by $\frac{1}{n}$, the e.m.fs. generated in the two conductors of a single turn are not exactly in phase, but differ by $\frac{1}{n}$ of a half wave for the fundamental, and thus a whole half wave for the nth harmonic, so that their nth harmonics are in opposition and thus cancel. Fractional pitch winding of a "pitch deficiency" of $\frac{1}{n}$ thus eliminates the nth harmonic; for instance, with 80 per cent pitch, the fifth harmonic cannot exist.

In this manner higher harmonics of the e.m.f. wave can be reduced or entirely eliminated, though in general, with a distributed winding, the wave shape is sufficiently close to sine shape without special precaution being taken in the design.

II. Electromotive Forces.

6. In a synchronous machine we have to distinguish between terminal voltage E, real generated e.m.f. E_1, virtual generated e.m.f. E_2, and nominal generated e.m.f. E_0.

The real generated e.m.f. E_1 is the e.m.f. generated in the alternator armature turns by the resultant magnetic flux, or magnetic flux interlinked with them, that is, by the magnetic flux passing through the armature core. It is equal to the terminal voltage plus the e.m.f. consumed by the resistance of the armature, these two e.m.fs. being taken in their proper phase relation; thus

$$\dot{E}_1 = \dot{E} + \dot{I}r,$$

where I = current in armature, r = effective resistance.

The virtual generated e.m.f. E_2 is the e.m.f. which would be generated by the flux produced by the field poles, or flux corresponding to the resultant m.m.f., that is, the resultant of the m.m.fs. of field excitation and of armature reaction. Since the magnetic flux produced by the armature, or flux of armature self-inductance, combines with the field flux to the resultant flux, the flux produced by the field poles does not pass through the armature completely, and the virtual e.m.f. and the real generated e.m.f. differ from each other by the e.m.f. of armature self-inductance; but the virtual generated e.m.f., as well as the e.m.f. generated in the armature by self-inductance, have no real and independent existence, but are merely fictitious components of the real or resultant generated e.m.f. E_1.

The virtual generated e.m.f. is

$$\dot{E}_2 = \dot{E}_1 - j\dot{I}x,$$

where x is the self-inductive armature reactance, and the e.m.f. consumed by self-inductance Ix is to be combined with the real generated e.m.f. E_1 in the proper phase relation.

7. The nominal generated e.m.f. E_0 is the e.m.f. which would be generated by the field excitation if there were neither self-inductance nor armature reaction, and the saturation were the same as corresponds to the real generated e.m.f. It thus does not correspond to any magnetic flux, and has no existence at all, but is merely a fictitious quantity, which, however, is very useful for the investigation of alternators by allowing the combination of armature reaction and self-inductance into a single effect by a (fictitious) self-inductance or synchronous reactance x_0. The nominal generated e.m.f. would be the terminal voltage with open circuit and load excitation if the saturation curve were a straight line.

The synchronous reactance x_0 is thus a quantity combining armature reaction and self-inductance of the alternator. It is the only quantity which can easily be determined by experiment by running the alternator on short-circuit with excited field. If in this case I_0 = current, P_0 = loss of power in the armature coils, E_0 = e.m.f. corresponding to the field excitation at open circuit, $\frac{E_0}{I_0} = z_0$ is the synchronous impedance, $\frac{P_0}{I_0^2} = r_0$ is the effective resistance (ohmic resistance plus load losses), and $x_0 = \sqrt{z_0^2 - r_0^2}$ the synchronous reactance.

In this feature lies the importance of the term "nominal generated e.m.f." E_0,

$$\dot{E}_0 = \dot{E}_1 - j\dot{I}x_0,$$

the terms being combined in their proper phase relation. In a polyphase machine, these considerations apply to each of the machine circuits individually.

III. Armature Reaction.

8. The magnetic flux in the field of an alternator under load is produced by the resultant m.m.f. of the field exciting current and of the armature current. It depends upon the phase relation of the armature current. The e.m.f. generated by the field

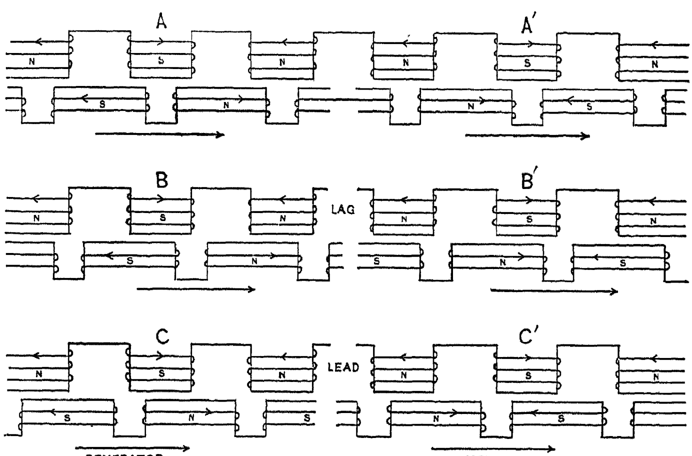


Fig. 46. Model for Study of Armature Reaction. Armature Coils in Position of Maximum Current.

exciting current or the nominal generated e.m.f. reaches a maximum when the armature coil faces the position midway between the field poles, as shown in Fig. 46, A and A'. Thus, if the armature current is in phase with the nominal generated e.m.f., it reaches its maximum in the same position A, A' of armature coil as the nominal generated e.m.f., and thus magnetizes the preceding, demagnetizes the following magnet pole (in the direction of rotation) in an alternating-current generator A; magnetizes the following and demagnetizes the preceding magnet pole in a synchronous motor A' (since in a generator the rotation is against, in a synchronous motor with the magnetic attractions and repulsions between field and armature). In this case the armature current neither magnetizes nor demagnetizes the field as a whole, but magnetizes the one side, demagnetizes the other side of each field pole, and thus merely distorts the magnetic field.

9. If the armature current lags behind the nominal generated e.m.f., it reaches its maximum in a position where the armature coil already faces the next magnetic pole, as shown in Fig. 46, B and B', and thus demagnetizes the field in a generator B, magnetizes it in a synchronous motor B'.

If the armature current leads the nominal generated e.m.f., it reaches its maximum in an earlier position, while the armature coil still partly faces the preceding magnet pole, as shown in Fig. 46, C and C', and thus magnetizes the field in a generator, Fig. 46, C, and demagnetizes it in a synchronous motor C'.

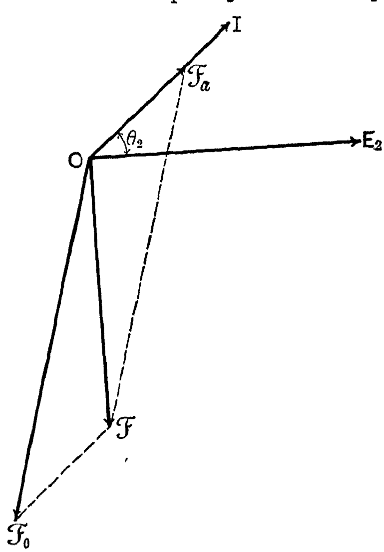


Fig. 47. Diagram of m.m.fs. in Loaded Generator.

With non-inductive load, or with the current in phase with the terminal voltage of an alternating-current generator, the current lags behind the nominal generated e.m.f., due to armature reaction and self-inductance, and thus partly demagnetizes; that is, the voltage is lower under load than at no load with the same field excitation. In other words, lagging current demagnetizes and leading current magnetizes the field of an alternating-current generator, while the opposite is the case with a synchronous motor.

10. In Fig. 47 let $\overline{OF} = \mathfrak{F}$ = resultant m.m.f. of field exci-

tation and armature current (the m.m.f. of the field excitation being alternating with regard to the armature coil, due to its rotation) and θ_2 the lag of the current I behind the virtual e.m.f. E_2 generated by the resultant m.m.f.

The virtual e.m.f. E_2 lags in time 90 degrees behind the resultant flux of $\overline{O\mathfrak{F}}$, and is thus represented by $\overline{OE_2}$ in Fig. 47, and the m.m.f. of the armature current $\mathfrak{F}_a$ by $\overline{O\mathfrak{F}_a}$, lagging by angle θ_2 behind $\overline{OE_2}$. The resultant m.m.f. $\overline{O\mathfrak{F}}$ is the diagonal of the parallelogram with the component m.m.fs. $\overline{O\mathfrak{F}_a}$ = armature m.m.f. and $\overline{O\mathfrak{F}_0}$ = total impressed m.m.f. or field excitation, as sides, and from this construction $\overline{O\mathfrak{F}_0}$ is found. $\overline{O\mathfrak{F}_0}$ is thus the position of the field pole with regard to the armature. It is trigonometrically,

$$\mathfrak{F}_0 = \sqrt{\mathfrak{F}^2 + \mathfrak{F}_a^2 + 2\,\mathfrak{F}\mathfrak{F}_a \sin\theta_2}.$$

If I = current per armature turn in amperes effective, n = number of turns per pole in a single-phase alternator, the armature reaction is $\mathfrak{F}_a = nI$ ampere-turns effective, and is pulsating between zero and $nI\sqrt{2}$.

In a quarter-phase alternator with n turns per pole and phase in series and I amperes effective per turn, the armature reaction per phase is nI amperes-turns effective and $nI\sqrt{2}$ ampere-turns maximum. The two phases magnetize in quadrature, in phase and in space. Thus, at the time t, corresponding to angle θ after the maximum of the first phase, the m.m.f. in the direction by angle θ behind the direction of the magnetization of the first phase is $nI\sqrt{2}\cos^2\theta$. The m.m.f. of the second phase is $nI\sqrt{2}\sin^2\theta$; thus the total m.m.f. or the armature reaction $\mathfrak{F}_a = nI\sqrt{2}$, and is constant in intensity, but revolves synchronously with regard to the armature; that is, it is stationary with regard to the field.

In a three-phaser of n turns in series per pole and phase and I amperes effective per turn, the m.m.f. of each phase is $nI\sqrt{2}$ ampere-turns maximum; thus at angle θ in position and angle θ in time behind the maximum of one phase;

The m.m.f. of this phase is

$$nI\sqrt{2}\cos^2\theta.$$

The m.m.f. of the second phase is

$$nI\sqrt{2}\cos^2(\theta + 120) = nI\sqrt{2}\,(-0.5\cos\theta - 0.5\sqrt{3}\sin\theta)^2.$$

The m.m.f. of the third phase is

$$nI\sqrt{2}\cos^2(\theta + 240) = nI\sqrt{2}\,(-0.5\cos\theta + 0.5\sqrt{3}\sin\theta)^2.$$

Thus the total m.m.f. or armature reaction,

$$\mathfrak{F}_a = nI\sqrt{2}\,(\cos^2\theta + 0.25\cos^2\theta + 0.75\sin^2\theta + 0.25\cos^2\theta + 0.75\sin^2\theta) = 1.5\,nI\sqrt{2},$$

constant in intensity, but revolving synchronously with regard to the armature; that is, stationary with regard to the field. These values of armature reaction correspond strictly only to the case where all conductors of the same phase are massed together in one slot. If the conductors of each phase are distributed over a greater part of the armature surface, the values of armature reaction have to be multiplied by the average cosine of the total angle of spread of each phase.

11. The single-phase machine thus differs from the polyphase machine: in the latter, on balanced load, the armature reaction is constant, while in the single-phase machine the armature reaction and thereby the resultant m.m.f. of field and armature is pulsating. The pulsation of the resultant m.m.f. of the single-phase machine causes a pulsation of its magnetic field under load, of double frequency, which generates a third harmonic of e.m.f. in the armature conductors. In machines of high armature reaction, as steam turbine driven single-phase alternators, the pulsation of the magnetic field may be sufficient to cause serious energy losses and heating by eddy currents, and thus has to be checked. This is usually done by a squirrel-cage induction machine winding in the field pole faces, or by short-circuited conductors laid in the pole faces in electrical space quadrature to the field coils. In these conductors, secondary currents of double frequency are produced which equalize the resultant m.m.f. of the machine.

IV. Self-Inductance.

12. The effect of self-inductance is similar to that of armature reaction, and depends upon the phase relation in the same manner.

If E_1 = real generated voltage, θ_1 = lag of current behind generated voltage E_1, the magnetic flux produced by the armature current I is in phase with the current, and thus the counter e.m.f. of self-inductance is in quadrature behind the current, and therefore the e.m.f. consumed by self-inductance is in quadrature ahead of the current. Thus in Fig. 48, denoting $\overline{OE}_1 = E_1$ the generated e.m.f., the current is $\overline{OI} = I$, lagging θ_1 behind $\overline{OE}_1$, the e.m.f. consumed by self-inductance $\overline{OE}_1''$, is 90 degrees ahead of the current, and the virtual generated e.m.f. E_2, is the resultant of $\overline{OE}_1$ and $\overline{OE}_1''$. As seen, the diagram of e.m.fs. of self-inductance is similar to the diagram of m.m.fs. of armature reaction.

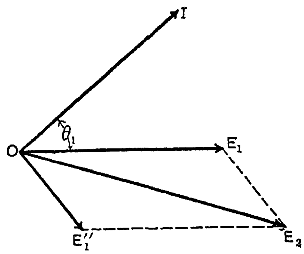


Fig. 48. Diagram of e.m.fs. in Loaded Generator.

13. From this diagram we get the effect of load and phase relation upon the e.m.f. of an alternating-current generator.

Let E = terminal voltage per machine circuit,
I = current per machine circuit,
and θ = lag of the current behind the terminal voltage.

Let r = resistance,
x = reactance of the alternator armature.

Then, in the polar diagram, Fig. 49,

$\overline{OE} = E$, the terminal voltage, assumed as zero vector.
$\overline{OI} = I$, the current, lagging by the angle $EOI = \theta$.

The e.m.f. consumed by resistance is $\overline{OE}_1' = Ir$ in phase with $\overline{OI}$.

The e.m.f. consumed by reactance is $\overline{OE}_2' = Ix$, 90 degrees ahead of $\overline{OI}$.

The real generated e.m.f. is found by combining $\overline{OE}$ and $\overline{OE}_1'$ to

$$\overline{OE}_1 = E_1.$$

The virtual generated e.m.f. is $\overline{OE}_1$ and $\overline{OE}_2'$ combined to

$$\overline{OE}_2 = E_2.$$

The m.m.f. required to produce this e.m.f. E_2 is $\overline{O\mathfrak{F}} = \mathfrak{F}$, 90 deg. ahead of $\overline{OE_2}$. It is the resultant of the armature m.m.f. or armature reaction and of the impressed m.m.f. or field excitation. The armature m.m.f. is in phase with the current I, and is nI in a single-phase machine, $n\sqrt{2}\,I$ in a quarter-phase machine, $1.5\sqrt{2}\,nI$ in a three-phase machine, if n = number of armature turns per pole and phase. The m.m.f. of armature reaction is represented in the diagram by $\overline{O\mathfrak{F}_a} = \mathfrak{F}_a$ in phase

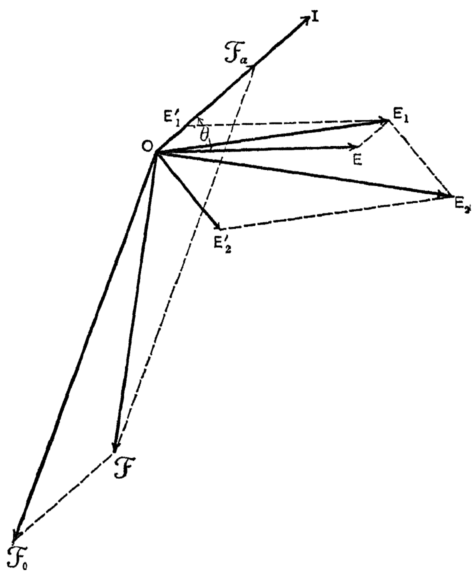


Fig. 49. Diagram Showing Combined Effect of Armature Reaction and Armature Self-Inductance.

with $\overline{OI}$, and the impressed m.m.f. or field excitation $\overline{O\mathfrak{F}_0} = \mathfrak{F}_0$ is the side of a parallelogram with $\overline{O\mathfrak{F}}$ as diagonal and $\overline{O\mathfrak{F}_a}$ as other side; or, the m.m.f. consumed by armature reaction is represented by $\overline{O\mathfrak{F}_a'} = \mathfrak{F}_a$ in opposition to $\overline{OI}$. Combining $\overline{O\mathfrak{F}_a'}$ and $\overline{O\mathfrak{F}}$ gives $\overline{O\mathfrak{F}_0} = \mathfrak{F}_0$, the field excitation.

In Figs. 50, 51, 52 are drawn the diagrams for θ = zero or non-inductive load, θ = 60 degrees, or 60 degrees lag (inductive load of power-factor 0.50), and $\theta = -60$ deg., or 60 deg. lead (anti-inductive load of power-factor 0.50).

Thus it is seen that with the same terminal voltage E a much higher field excitation, $\mathfrak{F}_0$, is required with inductive load than with non-inductive load, while with anti-inductive

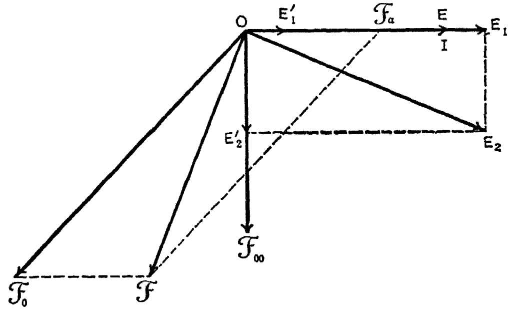


Fig. 50. Diagram of Generator e.m.fs. and m.m.fs. for Non-Reactive Load

load a much lower field excitation is required. With open circuit the field excitation required to produce the terminal voltage E would be $\frac{E}{E_0}\mathfrak{F} = \mathfrak{F}_{00}$, or less than the field excitation $\mathfrak{F}_0$ with non-inductive load.

Inversely, with constant field excitation, the voltage of an alternator drops with non-inductive load, drops much more with inductive load, and drops less, or even rises, with anti-inductive load.

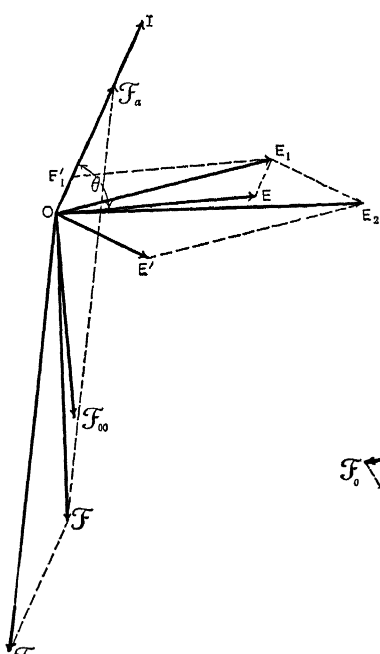


Fig. 51. Diagram of Generator e.m.fs. and m.m.fs. for Lagging Reactive Load. Power Factor 0.50.

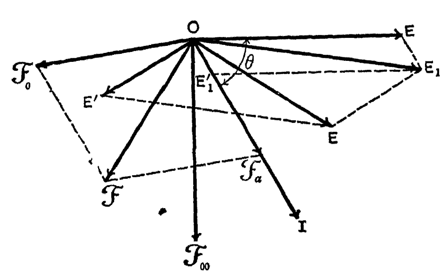


Fig. 52. Diagram of Generator e.m.fs. and m.m.fs. for Leading Reactive Load. Power Factor 0.50.

V. Synchronous Reactance.

14. In general, both effects, armature self-inductance and armature reaction, can be combined by the term "synchronous reactance."

In a polyphase machine, the synchronous reactance is different, and lower, with one phase only loaded, as "single-phase synchronous reactance," than with all phases uniformly loaded, as "polyphase synchronous reactance." The resultant armature reaction of all phases of the polyphase machine is higher than that with the same current in one phase only, and so also the self-inductive flux, as resultant flux of several phases, and thus represents a higher synchronous reactance.

Let r = effective resistance,

x_0 = synchronous reactance of armature, as discussed in Section II.

Let E = terminal voltage,

I = current,

θ = angle of lag of the current behind the terminal voltage.

It is in polar diagram, Fig. 53,

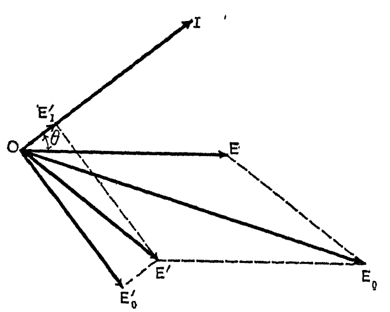


Fig. 53. Diagram Showing Effect of Synchronous Reactance.

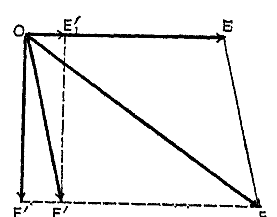


Fig 54. Diagram of Generator e.m.fs. Showing Effect of Synchronous Reactance with Non-Reactive Load.

$\overline{OE} = E$ = terminal voltage assumed as zero vector. $\overline{OI} = I$ = current lagging by the angle $EOI = \theta$ behind the terminal voltage.

$\overline{OE_1'} = Ir$ is the e.m.f. consumed by resistance, in phase with $\overline{OI}$, and $\overline{OE_0'} = Ix_0$ the e.m.f. consumed by the synchronous reactance, 90 degrees ahead of the current $\overline{OI}$.

$\overline{OE_1'}$ and $\overline{OE_0'}$ combined give $\overline{OE'} = E'$ the e.m.f. consumed by the synchronous impedance.

Combining $\overline{OE_1'}$, $\overline{OE_0'}$, $\overline{OE}$ gives the nominal generated e.m.f. $\overline{OE_0} = E_0$, corresponding to the field excitation $\mathfrak{F}_0$.

In Figs. 54, 55, 56, are shown the diagrams for $\theta = 0$ or non-inductive load, $\theta = 60$ degrees lag or inductive load, and $\theta = -60$ degrees or anti-inductive load.

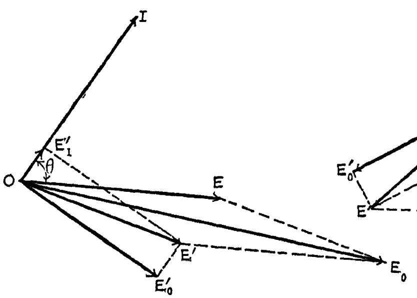


Fig. 55. Diagram of Generator e.m.fs. Showing Effect of Synchronous Reactance with Lagging Reactive Load. $\theta = 60$ degrees.

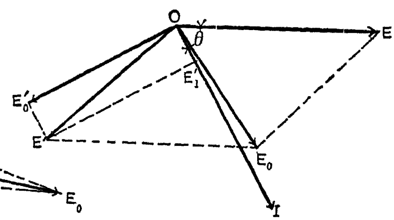


Fig. 56. Diagram of Generator e.m.fs. Showing Effect of Synchronous Reactance with Leading Reactive Load. $\theta = -60$ degrees.

Resolving all e.m.fs. into components in phase and in quadrature with the current, or into power and reactive components, in symbolic expression we have:

the terminal voltage $\underset{\cdot}{E} = E \cos \theta - jE \sin \theta$;

the e.m.f. consumed by resistance, $\underset{\cdot}{E}_1' = ir$;

the e.m.f. consumed by synchronous reactance, $\underset{\cdot}{E}_0' = -jix_0$, and the nominal generated e.m.f.,

$$\underset{\cdot}{E}_0 = \underset{\cdot}{E} + \underset{\cdot}{E}_1' + \underset{\cdot}{E}_0' = (E \cos \theta + ir) - j\,(E \sin \theta + ix_0);$$

or, since

$$\cos \theta = p = \text{power factor of the load} \left(= \frac{\text{power current}}{\text{total current}}\right)$$

and

$$q = \sqrt{1 - p^2} = \sin \theta = \text{inductance factor of the load}$$
$$\left(= \frac{\text{wattless current}}{\text{total current}}\right),$$

it is

$$\dot{E}_0 = (Ep + ir) - j(Eq + ix_0),$$

or, in absolute values,

$$E_0 = \sqrt{(Ep + ir)^2 + (Eq + ix_0)^2};$$

hence,

$$E = \sqrt{E_0^2 - i^2 (x_0 p - rq)^2} - i (rp + x_0 q).$$

The power delivered by the alternator into the external circuit is

$$P = iEp;$$

that is, the current times the power component of the terminal voltage.

The electric power produced in the alternator armature is

$$P_0 = i (Ep + ir);$$

that is, the current times the power component of the nominal generated e.m.f., or, what is the same thing, the current times the power component of the real generated e.m.f.

VI. Characteristic Curves of Alternating Current Generator.

15. In Fig. 57 are shown, at constant terminal voltage E, the values of nominal generated e.m.f. E_0, and thus of field excitation $\mathfrak{F}_0$, with the current I as abscissas and for the three conditions,

1. Non-inductive load, $p = 1$, $q = 0$.
2. Inductive load of $\theta = 60$ degrees lag, $p = 0.5$, $q = 0.866$.
3. Anti-inductive load of $-\theta = 60$ degrees lead, $p = 0.5$, $q = -0.866$.

The values $r = 0.1$, $x_0 = 5$, $E = 1000$, are assumed. These curves are called the *compounding curves of the synchronous generator.*

In Fig. 58 are shown, at constant nominal generated e.m.f. E_0, that is, at constant field excitation $\mathfrak{F}_0$, the values of terminal voltage E with the current I as abscissas and for the same resistance and synchronous reactance $r = 0.1$, $x_0 = 5$, for the three different conditions,

1. Non-inductive load, $p = 1$, $q = 0$, $E_0 = 1127$.
2. Inductive load of 60 degrees lag,
   $p = 0.5$, $q = 0.866$, $E_0 = 1458$.
3. Anti-inductive load of 60 degrees lead,
   $p = 0.5$, $q = -0.866$, $E_0 = 628$.

The values of E_0 (and thus of $\mathfrak{F}_0$) are assumed so as to give $E = 1000$ at $I = 100$. These curves are called the *regulation curves* of the alternator, or the *field characteristics* of the synchronous generator.

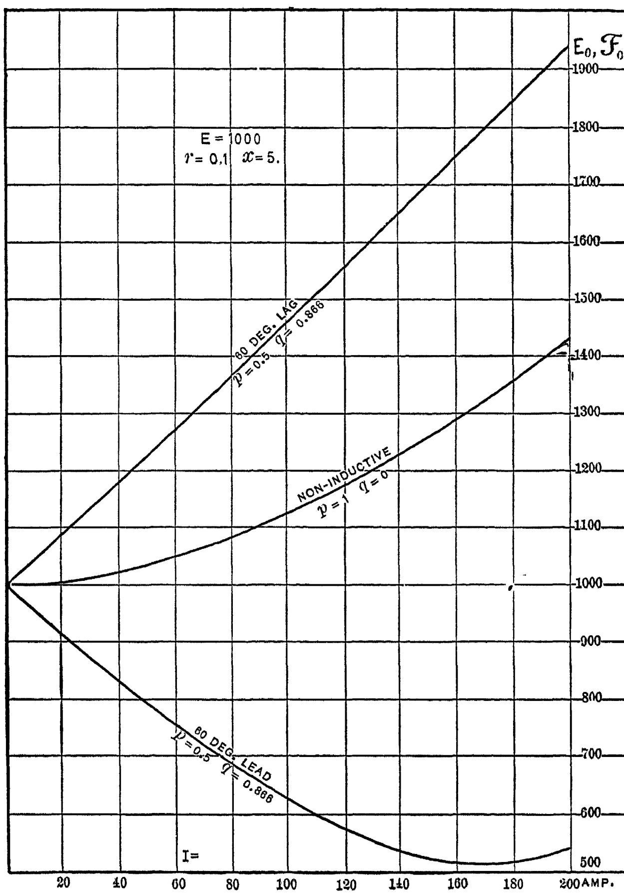


Fig. 57. Synchronous Generator Compounding Curves.

In Fig. 59 are shown the *load curves* of the machine, with the current I as abscissas and the watts output as ordinates corresponding to the same three conditions as Fig. 58. From the field characteristics of the alternator are derived the open-circuit voltage of 1127 at full non-inductive load excitation, which

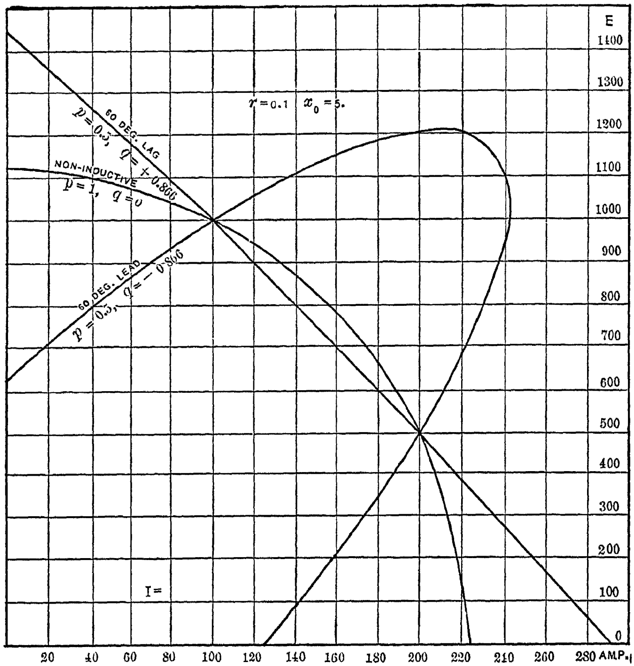


Fig. 58. Synchronous Generator Regulation Curves.

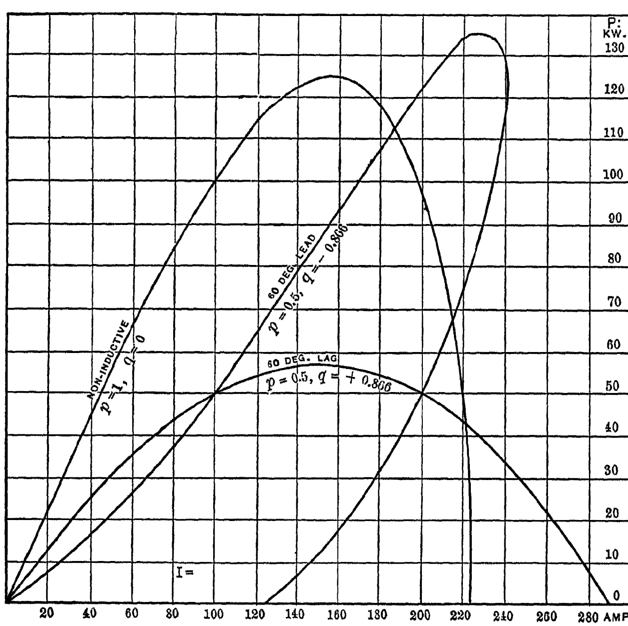


Fig. 59. Synchronous Generator Load Curves.

is 1.127 times full-load voltage; the short-circuit current 225 at full non-inductive load excitation, which is 2.25 times full-load current; and the maximum output, 124 kw., at full non-inductive load excitation, which is 1.24 times rated output, at 775 volts and 160 amperes. It depends upon the point on the field characteristic at which the alternator works, whether it tends to regulate for, that is, maintains, constant voltage, or constant current, or constant power, approximately.

VII. Synchronous Motor.

16. As seen in the preceding, in an alternating-current generator the field excitation required for a given terminal voltage and current depends upon the phase relation of the external circuit or the load. Inversely, in a synchronous motor the phase relation of the current into the armature at a given terminal voltage depends upon the field excitation and the load.

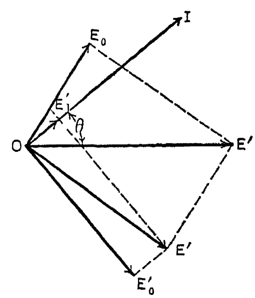


Fig. 60. Polar Diagram of Synchronous Motor.

Thus, if E = terminal voltage or impressed e.m.f., I = current, θ = lag of current behind impressed e.m.f. in a synchronous motor of resistance r and synchronous reactance x_0, the polar diagram is as follows, Fig. 60.

$\overline{OE} = E$ is the terminal voltage assumed as zero vector. The current $\overline{OI} = I$ lags by the angle $EOI = \theta$.

The e.m.f. consumed by resistance, is $\overline{OE}_1' = Ir$. The e.m.f. consumed by synchronous reactance, $\overline{OE}_0' = Ix_0$. Thus, combining $\overline{OE}_1'$ and $\overline{OE}_0'$ gives $\overline{OE'}$, the e.m.f. consumed by the synchronous impedance. The e.m.f. consumed by the synchronous impedance $\overline{OE'}$ and the e.m.f. consumed by the nominal generated or counter e.m.f. of the synchronous motor $\overline{OE}_0$, combined, give the impressed e.m.f. $\overline{OE}$. Hence $\overline{OE}_0$ is one side of a parallelogram, with $\overline{OE'}$ as the other side, and $\overline{OE}$ as diagonal. $\overline{OE}_{00}$, (not shown) equal and opposite $\overline{OE}_0$, would thus be the nominal counter-generated e.m.f. of the synchronous motor.

In Figs. 61 to 63 are shown the polar diagrams of the synchronous motor for $\theta = 0$ deg., $\theta = 60$ deg., $\theta = -60$ deg. It is seen that the field excitation has to be higher with leading and lower with lagging current in a synchronous motor, while the opposite is the case in an alternating-current generator.

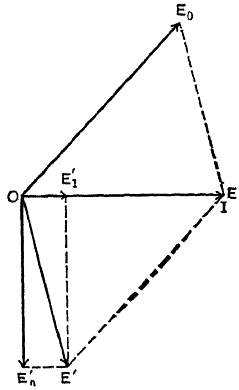


Fig. 61. Polar Diagram of Synchronous Motor. $\theta = 0$.

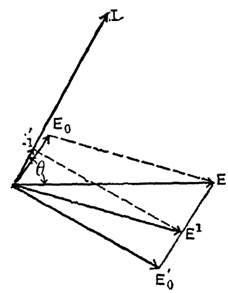


Fig. 62. Polar Diagram of Synchronous Motor. $\theta = 60$ deg.

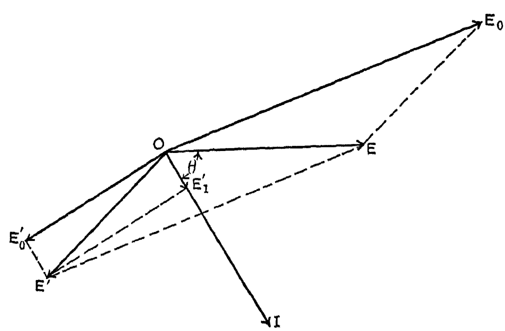


Fig. 63. Polar Diagram of Synchronous Motor. $\theta = -60$ degrees.

In symbolic representation, by resolving all e.m.fs. into power components in phase with the current and wattless components in quadrature with the current i, we have:

the terminal voltage, $\underset{\cdot}{E} = E \cos \theta - jE \sin \theta = Ep - jEq$;
the e.m.f. consumed by resistance, $\underset{\cdot}{E}_1{}' = ir$,

and the e.m.f. consumed by synchronous reactance, $\underset{\cdot}{E}_0{}' = -jix_0$.

Thus the e.m.f. consumed by the nominal counter-generated e.m.f. is

$$\dot{E}_0 = \dot{E} - \dot{E}_1' - \dot{E}_0' = (E\cos\theta - ir) - j\,(E\sin\theta - ix_0)$$
$$= (Ep - ir) - j\,(Eq - ix_0);$$

or, in absolute values,

$$E_0 = \sqrt{(E\cos\theta - ir)^2 + (E\sin\theta - ix_0)^2}$$
$$= \sqrt{(Ep - ir)^2 + (Eq - ix_0)^2};$$

hence,

$$E = i\,(rp + x_0 q) \pm \sqrt{E_0^{\,2} - i^2\,(x_0 p - rq)^2}.$$

The power consumed by the synchronous motor is

$$P = iEp;$$

that is, the current times the power component of the impressed e.m.f.

The mechanical power delivered by the synchronous motor armature is

$$P_0 = i\,(Ep - ir);$$

that is, the current times the power component of the nominal counter-generated e.m.f. Obviously to get the available mechanical power, the power consumed by mechanical friction and by molecular magnetic friction or hysteresis, and the power of field excitation, have to be subtracted from this value P_0.

VIII. Characteristic Curves of Synchronous Motor.

17. In Fig. 64 are shown, at constant impressed e.m.f. E, the nominal counter-generated e.m.f. E_0 and thus the field excitation $\mathfrak{F}_0$ required,

1. At no phase displacement, $\theta = 0$, or for the condition of minimum input;
2. For $\theta = +60$, or 60 deg. lag: $p = 0.5$, $q = +0.866$, and
3. For $\theta = -60$, or 60 deg. lead: $p = 0.5$, $q = -0.866$;

with the current I as abscissas, the constants being

$$r = 0.1,\ x_0 = 5, \text{ and } E = 1000.$$

These curves are called the *compounding curves of the synchronous motor.*

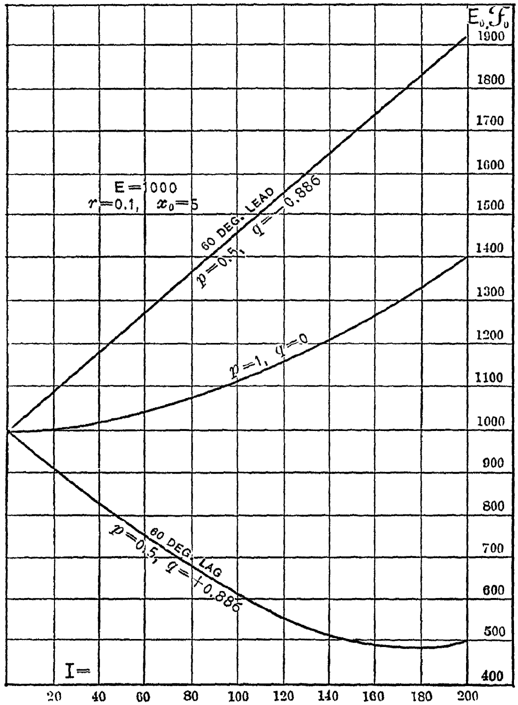


Fig. 64. Synchronous Motor Compounding Curves.

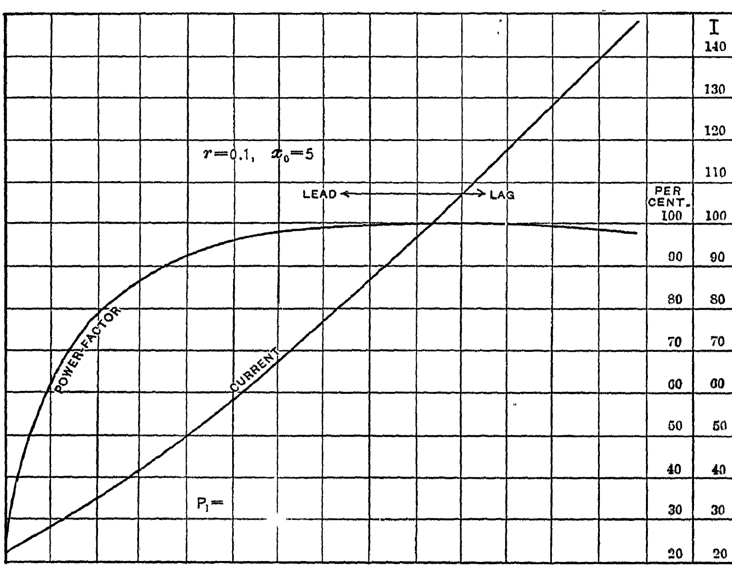


Fig. 65. Synchronous Motor Load Characteristics.

In Fig. 65 are shown, with the power output $P_1 = i\,(Ep - ir) -$ (iron loss and friction) as abscissas, and the same constants $r = 0.1$, $x_0 = 5$, $E = 1000$, and constant field excitation $\mathcal{F}_0$; that is, constant nominal counter-generated e.m.f. $E_0 = 1109$ (corresponding to $p = 1$, $q = 0$ at $I = 100$), the values of current I and power-factor p. As iron loss is assumed 3000 watts, as friction 2000 watts. Such curves are called *load characteristics of the synchronous motor.*

18. In Fig. 66 are shown, with constant power output, $P_0 = i\,(Ep - ir)$, and the same constants, $r = 0.1$, $x_0 = 5$, $E = 1000$,

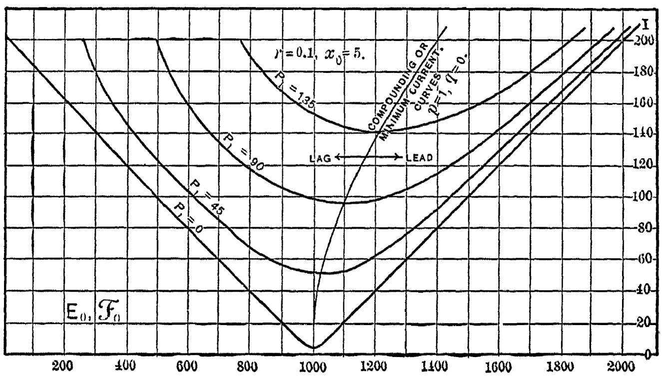


Fig. 66. Synchronous Motor Phase Characteristics.

and with the nominal counter-generated voltage E_0, that is field excitation $\mathcal{F}_0$, as abscissas, the values of current I for the four conditions,

$P_0 =$ 5 kw., or $P_1 =$ 0, or no load,
$P_0 =$ 50 kw., or $P_1 =$ 45 kw., or half load,
$P_0 =$ 95 kw., or $P_1 =$ 90 kw., or full load,
$P_0 =$ 140 kw., or $P_1 =$ 135 kw., or 150 per cent of load.

Such curves are called *phase characteristics of the synchronous motor.*

We have

$$P_0 = iEp - i^2 r.$$

Hence,

$$p = \frac{P_0 + i^2 r}{iE}, \quad q = \sqrt{1 - p^2}.$$

$$E_0 = \sqrt{(Ep - ir)^2 + (Eq - ix_0)^2}.$$

Similar phase characteristics exist also for the synchronous generator, but are of less interest. It is seen that on each of the four phase characteristics a certain field excitation gives minimum current, a lesser excitation gives lagging current, a greater excitation leading current. The higher the synchronous reactance x_0, and thus the armature reaction of the synchronous motor, the flatter are the phase characteristics; that is, the less sensitive is the synchronous motor for a change of field excitation or of impressed e.m.f. Thus a relatively high armature reaction is desirable in a synchronous motor to secure stability, that is, independence of minor fluctuations of impressed voltage or of field excitation.

19. The theoretical maximum output of the synchronous motor, or the load at which it drops out of step, at constant impressed voltage and frequency is, even with very high armature reaction, usually far beyond the heating limits of the machine. The actual maximum output depends on the drop of terminal voltage due to the increase of current, and on the steadiness or uniformity of the impressed frequency, thus upon the individual conditions of operation, but is as a rule far above full load.

Hence, by varying the field excitation of the synchronous motor the current can be made leading or lagging at will, and the synchronous motor thus offers the simplest means of producing out of phase or wattless currents for controlling the voltage in transmission lines, compensating for wattless currents of induction motors, etc. Synchronous machines used merely for supplying wattless currents, that is, synchronous motors or generators running light, with over-excited or under-excited field, are called synchronous compensators. They can be used as exciters for induction generators, as compensators for the reactive lagging currents of induction motors, etc. Sometimes they are called "rotary condensers" or "dynamic condensers" when used only for producing leading currents.

IX. Magnetic Characteristic or Saturation Curve.

20. The dependence of the generated e.m.f., or terminal voltage at open circuit, upon the field excitation, is called the *magnetic characteristic*, or *saturation curve*, of the synchronous machine It has the same general shape as the curve of magnetic flux density, consisting of a straight part below saturation, a bend or

knee, and a saturated part beyond the knee. Generally the change from the unsaturated to the over-saturated portion of the curve is more gradual, thus the knee is less pronounced in the magnetic characteristic of the synchronous machines, since the different parts of the magnetic circuit approach saturation successively.

The dependence of the terminal voltage upon the field excitation, at constant full-load current through the armature into a non-inductive circuit, is called the *load saturation curve* of the synchronous machine. It is a curve approximately parallel to the no-load saturation curve, but starting at a definite value of field

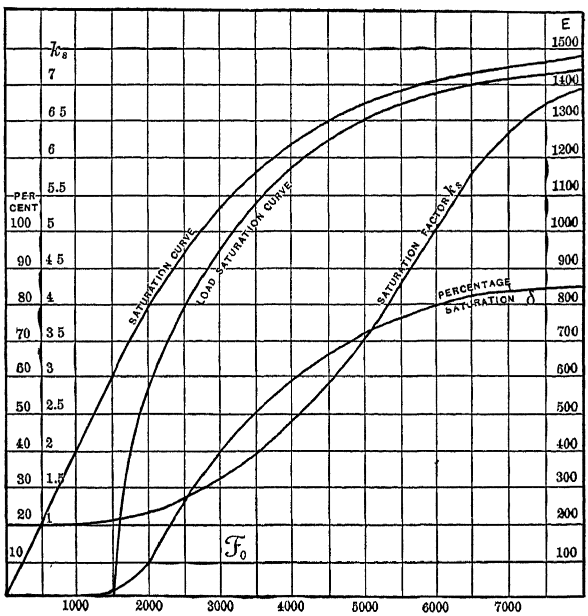


Fig. 67. Synchronous Generator Magnetic Characteristics.

excitation for zero terminal voltage, the field excitation required to maintain full-load current through the armature against its synchronous impedance.

The ratio
$$\frac{d\mathfrak{F}}{\mathfrak{F}} \div \frac{dE}{E}$$
is called the *saturation factor* k_s of the machine. It gives the

ratio of the proportional change of field excitation required for a change of voltage. The quantity $\sigma = 1 - \frac{1}{k_s}$ is called the *percentage saturation* of the machine, as it shows the approach to saturation.

In Fig. 67 is shown the magnetic characteristic or no-load saturation curve of a synchronous generator, the load saturation curve and the no-load saturation factor, assuming $E = 1000$, $I = 100$ as full-load values.

In the preceding the characteristic curves of synchronous machines were discussed under the assumption that the saturation curve is a straight line; that is, the synchronous machines working below saturation.

21. The effect of saturation on the characteristic curves of synchronous machines is as follows: The compounding curve is impaired by saturation; that is, a greater change of field excitation is required with changes of load. Under load the magnetic density in the armature corresponds to the true generated e.m.f. E_1, the magnetic density in the field to the virtual generated e.m.f. E_2. Both, especially the latter, are higher than the no-load e.m.f. or terminal voltage E of the generator, and thus a greater increase of field excitation is required in the presence of saturation than in the absence thereof. In addition thereto, due to the counter m.m.f. of the armature current, the magnetic stray field, that is, that magnetic flux which leaks from field pole to field pole through the air, increases under load, especially with inductive load where the armature m.m.f. directly opposes the field, and thus a still further increase of density is required in the field magnetic circuit under load. In consequence thereof, at high saturation the load saturation curve differs more from the no-load saturation curve than corresponds to the synchronous impedance of the machine.

The regulation becomes better by saturation; that is, the increase of voltage from full load to no load at constant field excitation is reduced, the voltage being limited by saturation. Owing to the greater difference of field excitation between no load and full load in the case of magnetic saturation, the improvement in regulation is somewhat, and in certain cases entirely, offset.

X. Efficiency and Losses.

22. Besides the above described curves the efficiency curves are of interest. The efficiency of alternators and synchronous motor is usually so high that a direct determination by measuring the mechanical power and the electric power is less reliable than the method of adding the losses, and the latter is therefore commonly used.

The losses consist of the following: the resistance loss in the armature; the resistance loss in the field circuit; the hysteresis and eddy current losses in the magnetic circuit; the friction and

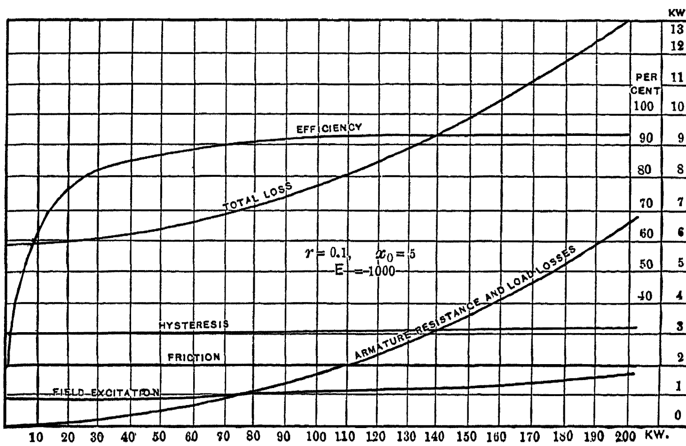


Fig. 68. Synchronous Generator, Efficiency and Losses.

windage losses, and eventually load losses, that is, losses due to eddy currents and hysteresis produced by the load current in the armature.

The resistance loss in the armature is proportional to the square of the current, I.

The resistance loss in the field circuit is proportional to the square of the field excitation current, that is, the square of the nominal generated or counter-generated e.m.f., E_0.

The hysteresis loss is proportional to the 1.6th power of the real generated e.m.f., $E_1 = E \pm Ir$.

The eddy current loss is usually proportional to the square of the generated e.m.f., E_1.

The friction and windage loss is assumed as constant.

The load losses vary more or less proportionally to the square of the current in the armature, and should be small with proper design. They can be represented by an "effective" armature resistance.

Assuming in the preceding example a friction loss of 2000 watts; an iron loss of 3000 watts, at the generated e.m.f. $E_1 = 1000$; a resistance loss in the field circuit of 800 watts, at $E_c = 1000$, and a load loss at full load of 600 watts.

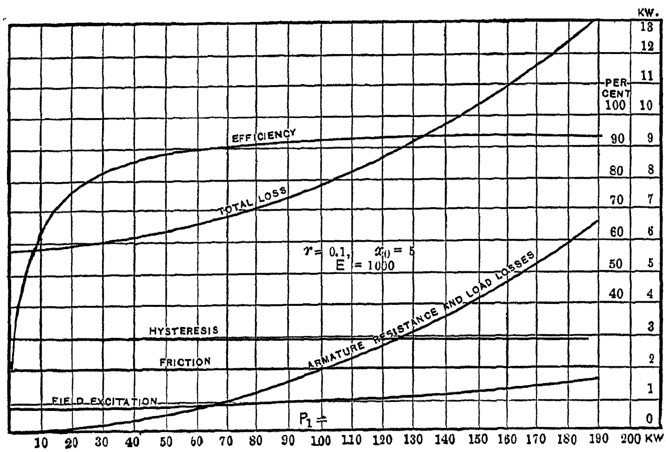


Fig. 69. Synchronous Motor Efficiency and Losses.

The loss curves and efficiency curves are plotted in Fig. 68 for the generator, with the current output at non-inductive load or $\theta = 0$ as abscissas, and in Fig. 69 for the synchronous motor, with the mechanical power output as abscissas.

XI. Unbalancing of Polyphase Synchronous Machines.

23. The preceding discussion applies to polyphase as well as single-phase machines. In polyphase machines the nominal generated e.m.fs. or nominal counter-generated e.m.fs. are necessarily the same in all phases (or bear a constant relation to each other). Thus in a polyphase generator, if the current or the phase relation of the current is different in the different branches, the terminal voltage must become different also, more or less. This is called the unbalancing of the polyphase generator. It is

due to different load or load of different inductance factor in the different branches.

Inversely, in a polyphase synchronous motor, if the terminal voltages of the different branches are unequal, due to an unbalancing of the polyphase circuit, the synchronous motor takes more current or lagging current from the branch of higher voltage, and thereby reduces its voltage, and takes less current or leading current* from the branch of lower voltage, or even returns current into this branch, and thus raises its voltage. Hence a synchronous motor tends to restore the balance of an unbalanced polyphase system; that is, it reduces the unbalancing of a polyphase circuit caused by an unequal distribution or unequal phase relation of the load on the different branches. To a less degree the induction motor possesses the same property.

XII. Starting of Synchronous Motors.

24. In starting, an essential difference exists between the single-phase and the polyphase synchronous motor, in so far as the former is not self-starting but has to be brought to complete synchronism, or in step with the generator, by external means before it can develop torque, while the polyphase synchronous motor starts from rest and runs up to synchronism with more or less torque.

In starting, the field excitation of the polyphase synchronous motor must be zero or very low.

The starting torque is due to the magnetic attraction of the armature currents upon the remanent magnetism left in the field poles by the currents of the preceding phase, or the eddy currents produced therein.

Let Fig. 70 represent the magnetic circuit of a polyphase synchronous motor. The m.m.f. of the polyphase armature currents acting upon the successive projections or teeth of the armature, 1, 2, 3, etc., reaches a maximum in them successively; that is, the armature is the seat of a m.m.f. rotating synchronously in the direction of the arrow *A*. The magnetism in the face of the field pole opposite to the armature projections lags behind the m.m.f., due to hysteresis and eddy currents, and thus

* Since with lower impressed voltage the current is leading, with higher impressed voltage lagging, in a synchronous motor.

is still remanent, while the m.m.f. of the projection 1 decreases, and is attracted by the rising m.m.f. of projection 2, etc., or, in other words, while the maximum m.m.f., in the armature has a position *a*, the maximum magnetism in the field pole face still has the position *b*, and is thus attracted towards *a*, causing the field to revolve in the direction of the arrow *A* (or with a stationary field, the armature to revolve in the opposite direction *B*).

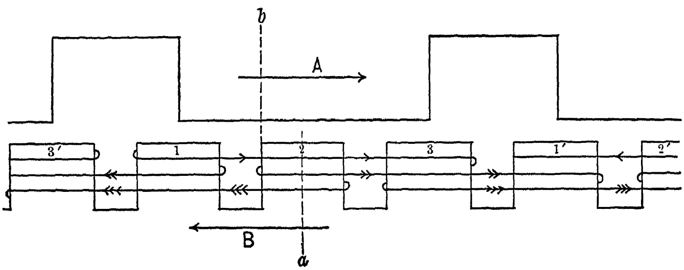


Fig. 70. Magnetic Circuit of a Polyphase Synchronous Motor.

Lamination of the field poles reduces the starting torque caused by eddy currents in the field poles, but increases that caused by remanent magnetism or hysteresis, due to the higher permeability of the field poles. Thus the torque per volt-ampere input is approximately the same in either case, but with laminated poles the impressed voltage required in starting is higher and the current lower than with solid field poles. In either case, at full impressed e.m.f. the starting current of a synchronous motor is large, since in the absence of a counter e.m.f. the total impressed e.m.f. has to be consumed by the impedance of the armature circuit. Since the starting torque of the synchronous motor is due to the magnetic flux produced by the alternating armature currents, or the armature reaction, synchronous motors of high armature reaction are superior in starting torque.

XIII. Parallel Operation.

25. Any alternator can be operated in parallel, or synchronized with any other alternator. A single-phase machine can be synchronized with one phase of a polyphase machine, or a quarter-phase machine operated in parallel with a three-phase machine by synchronizing one phase of the former with one phase of the latter. Since alternators in parallel must be in step with each other and have the same terminal voltage, the condition of satisfactory parallel operation is that the frequency of the machines is identically the same, and the field excitation such as would give the same terminal voltage. If this is not the case, there will be cross currents between the alternators in a local circuit; that is, the alternators are not without current at no load, and their currents under load are not of the same phase and proportional to their respective capacities. The cross currents between alternators when operated in parallel can be wattless currents or power currents.

If the frequencies of two alternators are identically the same, but the field excitation not such as would give equal terminal voltage when operated in parallel, there is a local current between the two machines which is wattless and leading or magnetizing in the machine of lower field excitation, lagging or demagnetizing in the machine of higher field excitation. At load this wattless current is superimposed upon the currents from the machines into the external circuit. In consequence thereof the current in the machine of higher field excitation is lagging behind the current in the external circuit, the current in the machine of lower field excitation leads the current in the external circuit. The currents in the two machines are thus out of phase with each other, and their sum larger than the joint current, or current in the external circuit. Since it is the armature reaction of leading or lagging current which makes up the difference between the impressed field excitation and the field excitation required to give equal terminal voltage, it follows that the lower the armature reaction, that is, the closer the regulation of the machines, the more sensitive they are for inequalities or variations of field excitation. Thus, too low armature reaction is undesirable for parallel operation.

With identical machines the changes in field excitation required for changes of load must be the same. With machines of different compounding curves the changes of field excitation for varying load must be different, and such as correspond to their respective compounding curves, if wattless currents shall be avoided. With machines of reasonable armature reaction the wattless cross currents are small even with relatively great inequality of field excitation. Machines of high armature reaction have been operated in parallel under circumstances where one machine was entirely without field excitation, while the other carried twice its normal field excitation, with wattless currents, however, of the same magnitude as full-load current.

XIV. Division of Load in Parallel Operation.

26. Much more important than equality of terminal voltage before synchronizing is equality of frequency. Inequality of frequency, or rather a tendency to inequality of frequency (since by necessity the machines hold each other in step or at equal frequency), causes cross currents which transfer energy from the machine whose driving power tends to accelerate to the machine whose driving power tends to slow down, and thus relieves the latter by increasing the load on the former. Thus these cross currents are power currents, and cause at no load or light load the one machine to drive the other as synchronous motor, while under load the result is that the machines do not share the load in proportion to their respective capacities.

The speed of the prime mover, as steam engine or turbine, changes with the load. The frequency of alternators driven thereby must be the same when in parallel. Thus their respective loads are such as to give the same speed of the prime mover (or rather a speed corresponding to the same frequency). Hence the division of load between alternators connected to independent prime movers depends almost exclusively upon the speed regulation of the prime movers. To make alternators divide the load in proportion to their capacities, the speed regulation of their prime movers must be the same; that is, the engines or turbines must drop in speed from no load to full load by the same percentage and in the same manner.

If the regulation of the prime movers is not the same, the load is not divided proportionally between the alternators, but the alternator connected to the prime mover of closer speed regulation takes more than its share of the load under heavy loads, and less under light loads. Thus, too close speed regulation of prime movers is not desirable in parallel operation of alternators.

XV. Fluctuating Cross Currents in Parallel Operation.

27. In alternators operated from independent prime movers, it is not sufficient that the average frequency corresponding to the average speed of the prime movers be the same, but still more important that the frequency be the same at any instant, that is, that the frequency (and thus the speed of the prime mover) be constant. In rotary prime movers, as turbines or electric motors, this is usually the case; but with reciprocating machines, as steam engines, the torque and thus the speed of rotation rises and falls periodically during each revolution, with the frequency of the engine impulses. The alternator connected with the engine will thus not have uniform frequency, but a frequency which pulsates, that is, rises and falls. The amplitude of this pulsation depends upon the design of the engine, the momentum of its fly-wheel, and the action of the engine governor.

If two alternators directly connected to equal steam engines are synchronized so that the moments of maximum frequency coincide, there will be no energy cross currents between the machines, but the frequency of the whole system rises and falls periodically. In this case the engines are said to be synchronized. The parallel operation of the alternators is satisfactory in this case provided that the pulsations of engine speeds are of the same size and duration; but apparatus requiring constant frequency, as synchronous motors and especially rotary converters, when operated from such a system, will give a reduced maximum output, due to periodic cross currents between the generators of fluctuating frequency and the synchronous motors of constant frequency, and in an extreme case the voltage of the whole system will be caused to fluctuate periodically. Even with small fluctuations of engine speed the unsteadiness of current due to this source is noticeable in synchronous motors and synchronous converters.

If the alternators happen to be synchronized in such a position that the moment of maximum speed of the one coincides with the moment of minimum speed of the other, alternately the one and then the other alternator will run ahead, and thus there will be a pulsating power cross current between the alternators, transferring power from the leading to the lagging machine, that is, alternately from the one to the other, and inversely, with the frequency of the engine impulses. These pulsating cross currents are the most undesirable, since they tend to make the voltage fluctuate and to tear the alternators out of synchronism with each other, especially when the conditions are favorable to a cumulative increase of this effect by what may be called mechanical resonance (hunting) of the engine governors, etc. They depend upon the synchronous impedance of the alternators and upon their phase difference, that is, the number of poles and the fluctuation of speed, and are specially objectionable when operating synchronous apparatus in the system.

28. Thus, for instance, if two 80-pole alternators are directly connected to single-cylinder engines of 1 per cent speed variation per revolution, twice during each revolution the speed will rise, and fall twice; and consequently the speed of each alternator will be above average speed during a quarter revolution. Since the maximum speed is $\frac{1}{2}$ per cent above average, the mean speed during the quarter revolution of high speed is $\frac{1}{4}$ per cent above average speed, and by passing over 20 poles the armature of the machine will during this time run ahead of its mean position by $\frac{1}{4}$ per cent of 20 or $\frac{1}{20}$ pole, that is, $\frac{180}{20} =$ 9 electrical space degrees. If the armature of the other alternator at this moment is behind its average position by 9 electrical space degrees, the phase displacement between the alternator e.m.fs. is 18 electrical time degrees; that is, the alternator e.m.fs. are represented by $\overline{OE_1}$ and $\overline{OE_2}$ in Fig. 71, and when running in parallel the e.m.f. $\overline{OE'} = \overline{E_1E_2}$ is short-circuited through the synchronous impedance of the two alternators.

Since $E' = \overline{OE_1} = 2\,E_1 \sin 9$ deg. the maximum cross current is

$$I' = \frac{E_1 \sin 9 \text{ deg.}}{z_0} = \frac{0.156\,E_1}{z_0} = 0.156\,I_0,$$

where $I_0 = \frac{E_1}{z_0}$ = short-circuit current of the alternator at full-load excitation. Thus, if the short-circuit current of the alternator is only twice full-load current, the cross current is 31.2 per cent of full-load current. If the short-circuit current is 6 times full-load current, the cross current is 93.6 per cent of full-load current or practically equal to full-load current. Thus the smaller the armature reaction, or the better the regulation, the larger are the pulsating cross currents between the alternators, due to the inequality of the rate of rotation of the prime movers. Hence for satisfactory parallel operation of alternators connected to steam engines, a certain amount of armature reaction is desirable and very close regulation undesirable.

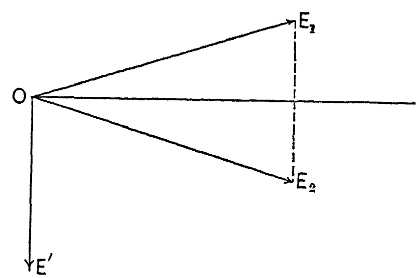


Fig. 71. Phase Displacement between Alternators to be Synchronized.

By the transfer of energy between the machines the pulsations of frequency, and thus the cross currents, are reduced somewhat. Very high armature reaction is objectionable also, since it reduces the synchronizing power, that is, the tendency of the machines to hold each other in step, by reducing the energy transfer between the machines. As seen herefrom, the problem of parallel operation of alternators is almost entirely a problem of the regulation of their prime movers, especially steam engines, but no electrical problem at all.

With alternators driven by gas engines, the problem of parallel operation is made more difficult by the more jerky nature of the gas engine impulse. In such machines, therefore, squirrel-cage windings in the field pole faces are commonly used, to assist synchronizing by the currents induced in this short circuited winding, on the principle of the induction machine.

From Fig. 71 it is seen that the e.m.f. $\overline{OE'}$ or $\overline{E_1E_2}$, which causes the cross current between two alternators in parallel connection, if their e.m.fs. $\overline{OE_1}$ and $\overline{OE_2}$ are out of phase, is approximately in quadrature with the e.m.fs. $\overline{OE_1}$ and $\overline{OE_2}$ of the machines, if these latter two e.m.fs. are equal to each other. The cross current between the machines lags behind the e.m.f. producing it, $\overline{OE'}$, by the angle ω, where $\tan \omega = \frac{x_0}{r_0}$, and $x_0 =$ reactance, r_0 = effective resistance of alternator armature. The energy component of this cross current, or component in phase with $\overline{OE'}$, is thus in quadrature with the machine voltages

$\overline{OE}_1$ and $\overline{OE}_2$, that is, transfers no power between them. The power transfer or equalization of load between the two machines takes place by the wattless or reactive component of cross current, that is, the component which is in quadrature with OE', and thus in phase with one and in opposition with the other of the machine e.m.fs. $\overline{OE}_1$ and $\overline{OE}_2$.

29. Hence, machines without reactance would have no synchronizing power, or could not be operated in parallel. The theoretical maximum synchronizing power exists if the reactance equals the resistance: $x_0 = r_0$. This condition, however, cannot be realized, and if realized would give a dangerously high synchronizing power and cross current. In practice, x_0 is always very much greater than r_0, and the cross current thus practically in quadrature with $\overline{OE}'$, that is, in phase (or opposition) with the machine voltages, and is consequently an energy-transfer current.

If, however, alternators are operated in parallel over a circuit of appreciable resistance, as two stations at some distance from each other are synchronized, especially if the resistance between the stations is non-inductive, as underground cables, with alternators of very low reactance, as turbo alternators, the synchronizing power may be insufficient. In this case, reactance has to be inserted between the stations, to lag the cross current and thereby make it a power transferring or synchronizing current.

If, however, the machine voltages $\overline{OE}_1$ and $\overline{OE}_2$ are different in value but approximately in phase with each other, the voltage causing cross currents, $\overline{E_1E}_2$, is in phase with the machine voltages and the cross currents thus in quadrature with the machine voltages $\overline{OE}_1$ and $\overline{OE}_2$, and hence do not transfer energy, but are wattless. In one machine the cross current is a lagging or demagnetizing, and in the other a leading or magnetizing, current.

Hence two kinds of cross currents may exist in parallel operation of alternators, — currents transferring power between the machines, due to phase displacement between their e.m.fs., and wattless currents transferring magnetization between the machines, due to a difference of their induced e.m.fs.

In compound-wound alternators, that is, alternators in which the field excitation is increased with the load by means of a series field excited by the rectified alternating current, it is almost, but not quite, as necessary as in direct-current machines, when operating in parallel, to connect all the series fields in parallel by equalizers of negligible resistance, for the same reason, — to insure proper division of current between machines.

XVI. High-Frequency Cross Currents between Synchronous Machines.

30. If several synchronous machines of different wave shapes are connected into the same circuit, cross currents exist between the machines of frequencies which are odd multiples of the circuit frequency, that is, higher harmonics thereof. The machines may be two or more generators in the same or in different stations, of wave shapes containing higher harmonics of different order, intensity or phase, or synchronous motors or converters of wave shapes different from that of the system to which they are connected.

The intensity of these cross currents is the difference of the corresponding harmonics of the machines divided by the impedance between the machines. This impedance includes the self-inductive reactance of the machine armatures. The reactance obviously is that at the frequency of the harmonic, that is, if x = reactance at fundamental frequency, it is nx for the nth harmonic.

In most cases these cross currents are very small and negligible, with machines of distributed armature winding, the intensity of the harmonic is low, that is, the voltage nearly a sine wave, and with machines of massed armature winding, as unitooth alternators, the reactance is high. These cross currents thus usually are noticeable only at no load, and when adjusting the field excitation of the machines for minimum current. Thus in a synchronous motor or converter, at no load, the minimum current, reached by adjusting the field, while small compared with full-load current, may be several times larger than the minimum point of the "V" curve in Fig. 59, that is, the value of the energy current supplying the losses in the machine.

It is only in the parallel operation of very large high-speed machines (steam turbine driven alternators) of high armature reaction and very low armature self-induction that such high frequency cross currents may require consideration, and even then only in three-phase Y-connected generators with grounded neutral, as cross currents between the neutrals of the machines. In a three-phase machine, the voltage between the terminals, or delta voltage, contains no third harmonic or its multiple, as the third harmonics of the Y voltage neutralize in the delta voltage, and such a machine, with a terminal voltage of almost sine shape, may contain a considerable third harmonic in the Y voltage. As the three Y voltages of the three-phase system are 120 deg. apart in phase, their third harmonics are 3×120 deg. $= 360$ deg. apart, or in phase with each other, from the main terminals to the neutral, and by connecting the neutrals of two three-phase machines of different third harmonics with each other, as by grounding the neutrals, a cross current flows between the machines over the neutral, which may reach very high values. Even in machines of the same wave shape, such a triple frequency current appears between the machines over the neutral, when by a difference in field excitation a difference in the phase of the third harmonic is produced. It therefore is undesirable to ground or connect together, without any resistance, the neutrals of three-phase machines, but in systems of grounded neutral either the neutral should be grounded through separate resistances or grounded only in one machine.

XVII. Short-Circuit Currents of Alternators.

31. The short-circuit current of an alternator at full load excitation usually is from three to five times full-load current. It is

$$I_0 = \frac{E_0}{z_0},$$

where E_0 = nominal generated e.m.f., z_0 = synchronous impedance of alternator, representing the combined effect of armature reaction and armature self-inductance.

In the first moment after short-circuiting, however, the current frequently is many times larger than the permanent short-circuit current, that is,

$$I_1 = \frac{E_0}{z},$$

where z = self-inductive impedance of the alternator.

That is, in the first moment after short-circuiting the polyphase alternator the armature current is limited only by the armature self-inductance, and not by the armature reaction, and some appreciable time — occasionally several seconds — elapses before the armature reaction becomes effective.

At short-circuit, the magnetic field flux is greatly reduced by the demagnetizing action of the armature current, and the generated e.m.f. thereby reduced from the nominal value E_0 to the virtual value E_2; the latter is consumed by the armature self-inductive impedance z, or self-inductive reactance, which is practically the same in most cases.

The armature self-inductance is instantaneous, since the magnetic field rises simultaneously with the armature current which produces it; armature reaction, however, requires an appreciable time to reduce the magnetic flux from the open-circuit value to the much lower short-circuit value, since the magnetic field flux is surrounded by the field exciting coils, which act as a short-circuited secondary opposing a rapid change of field flux; that is, in the moment when the short-circuit current starts it begins to demagnetize the field, and the magnetic field flux therefore begins to decrease; in decreasing, however, it generates an e.m.f. in the field coils, which opposes the change of field flux, that is, increases the field current so as momentarily to maintain the full field flux against the demagnetizing action of the armature reaction. In the first moment the armature current thus rises to the value given by the e.m.f. generated by the full field flux, while the field current rises, frequently to many times its normal value (hence, if circuit breakers are in the field circuit, they may open the circuit). Gradually the field flux decreases, and with it decrease the field current and the armature current to their normal values, at a rate depending on the resistance and the inductance of the field exciting circuit. The decrease in value of the field flux will be the more rapid the higher the resistance of the field circuit, the slower the higher the inductance, that is, the greater the magnetic flux of the machine. Thus, the momentary short-circuit current of the machine can be made to decrease somewhat more rapidly by increasing the resistance of the field circuit, that is, wasting exciting power in the field rheostat.

In the very first moment the short-circuit current waves are unsymmetrical, as they must simultaneously start from zero in all phases and gradually approach their symmetrical appearance, i.e., in a three-phase machine three currents displaced by 120 deg. Hereby the field current is made pulsating, with normal or synchronous frequency, that is, with the same frequency as the armature current. This full frequency pulsation gradually dies out and the field current becomes constant with a polyphase short-circuit, while with a single-phase short-circuit it remains pulsating with double frequency, due to the pulsating armature reaction. In a polyphase short-circuit this full frequency pulsation due to the unsymmetrical starting of the currents is independent of the point of the wave at which the short-circuit starts, since the resultant asymmetry of all the polyphase currents is the same regardless of the point of the wave at which the circuit is closed. In a single-phase short-circuit, however, the full frequency pulsation depends on the point of the wave at which the circuit is closed, and is absent if the circuit is closed at that moment at which the short-circuit current would pass through zero.

The momentary short-circuit current of an alternator thus represents one of the few cases in which armature self-inductance and armature reaction do not act in the same manner, and the synchronous reactance can be split into two components, thus, $x_0 = x + x'$, where x = self-inductive reactance, which is due to a true self-inductance, and x' = effective reactance of armature reaction, which is not instantaneous.

32. In machines of high self-inductance and low armature reaction, as high frequency alternators, this momentary increase of short-circuit current over its normal value is negligible, and moderate in machines in which armature reaction and self-inductance are of the same magnitude, as large modern multipolar low-speed alternators. In large high-speed alternators of high armature reaction and low self-inductance, as steam turbine alternators, the momentary short-circuit current may exceed the permanent value ten or more times. With such large currents magnetic saturation of the self-inductive armature circuit still further reduces the reactance x, that is, increases the current, and in such cases the mechanical shock on the generator becomes so enormous that it is necessary to reduce the momentary short-circuit current

by inserting self-inductance, that is, reactance coils into the generator leads.

In view of the excessive momentary short-circuit current, it may be desirable that automatic circuit breakers on such systems have a time limit, so as to keep the circuit closed until the short-circuit current has somewhat decreased.

33. In single-phase machines, and in polyphase machines in case of a short-circuit on one phase only, the armature reaction is pulsating, and the field current in the first moment after the short-circuit therefore pulsates, with double frequency, and remains pulsating even after the permanent condition has been reached. The double frequency pulsation of the field current in case of a single-phase short-circuit generates in the armature a third harmonic of e.m.f. The short-circuit current wave becomes greatly distorted thereby, showing the saw-tooth shape characteristics of the third harmonic, and in a polyphase machine on single-phase short-circuit, in the phase in quadrature with the short-circuited phase, a very high voltage appears, which is greatly distorted by the third harmonic and may reach several times the value of the open-circuit voltage. Thus, with a single-phase short-circuit on a polyphase system, destructive voltages may appear in the open-circuited phase, of saw-tooth wave shape.

Upon this double frequency pulsation of the field current during a single-phase short-circuit the transient full frequency pulsation resulting from the unsymmetrical start of the armature current is superimposed and thus causes a difference in the intensity of successive waves of the double frequency pulsation, which gradually disappears with the dying out of the transient full frequency pulsation, and depends upon the point of the wave at which the short-circuit is closed, and thus is absent, and the double frequency pulsation symmetrical, if the circuit is closed at the moment when the short-circuit current should be zero.

34. As illustration is shown, in Fig. 72, the oscillogram of one phase of the three-phase short-circuit of a three-phase turbo-alternator, giving the unsymmetrical start of the armature currents and the full frequency pulsation of the field current.

In Fig. 73 is shown a single-phase short-circuit of the same

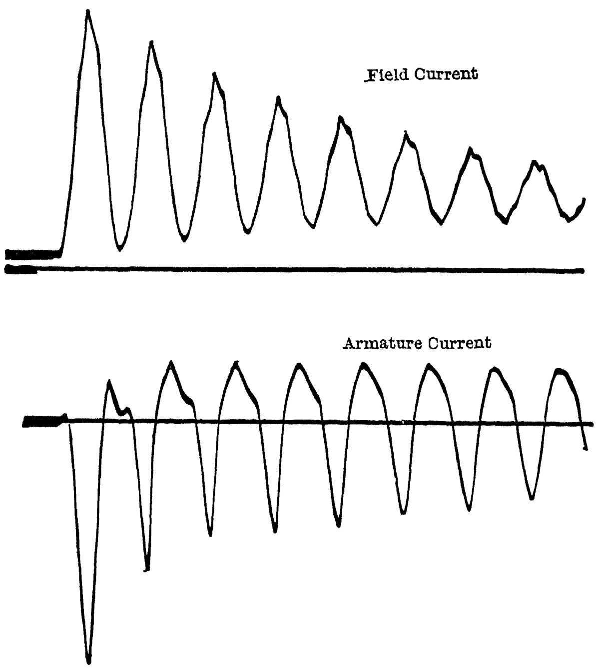


Fig. 72. Three-phase Short-circuit Current in a Turbo-alternator.

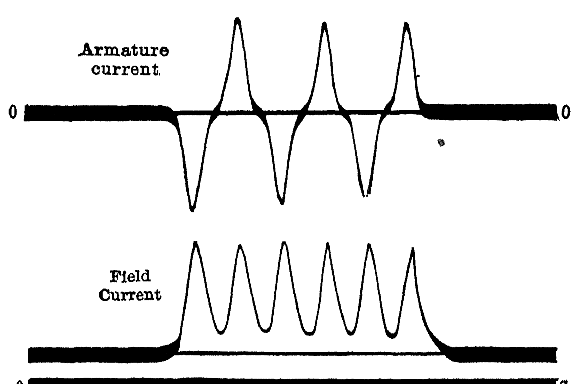


Fig. 73. Single-phase Short-circuit Current in a Three-phase Turbo-alternator.

machine, in which the circuit is closed at the zero value of the current; the current wave therefore is symmetrical, and the field current shows only the double frequency pulsation due to the single-phase armature reaction.

In Fig. 74 is shown another single-phase short-circuit, in which the armature current wave starts unsymmetrical, thus giving a

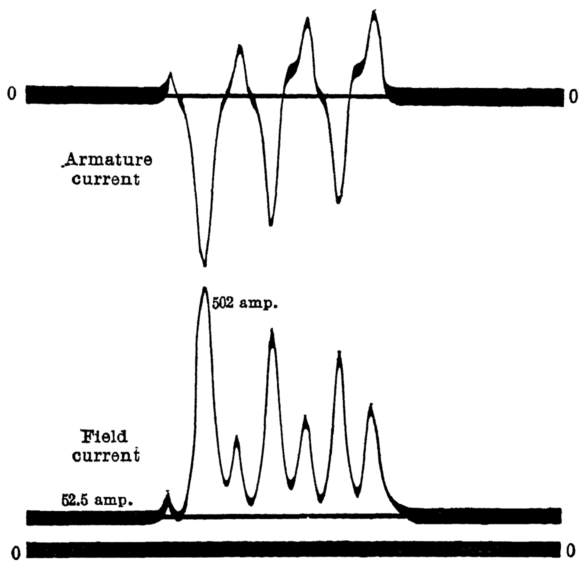


Fig. 74. Single-phase Short-circuit Current in a Three-phase Turbo-alternator.

transient full frequency term in the field current. Thus in the double frequency pulsation of the field current at first large and small waves alternate, but the successive waves gradually become equal with the dying out of the full frequency term.

For further discussion, and the theoretical investigation of momentary short-circuit currents, see "Theory and Calculation of Transient Electric Phenomena and Oscillations," Part I, Chapters XI and XII.

B. DIRECT-CURRENT COMMUTATING MACHINES.

I. General.

35. Commutating machines are characterized by the combination of a continuously excited magnet field with a closed circuit armature connected to a segmental commutator. According to their use, they can be divided into direct-current generators which transform mechanical power into electric power, direct-current motors which transform electric power into mechanical power, and direct-current converters which transform electric power into a different form of electric power. Since the most important class of the latter are the synchronous converters, which combine features of the synchronous machines with those of the commutating machines, they shall be treated in a separate chapter.

By the excitation of their magnet fields, commutating machines are divided into magneto machines, in which the field consists of permanent magnets; separately excited machines; shunt machines, in which the field is excited by an electric circuit shunted across the machine terminals, and thus receives a small branch current at full machine voltage, as shown diagrammatically in Fig. 75; series machines, in which the electric field circuit is connected in series with the armature, and thus receives the full machine current at low voltage (Fig. 76); and compound machines, excited by a combination of shunt and series field (Fig. 77). In compound machines the two windings can magnetize either in the same direction (cumulative compounding) or in opposite directions (differential compounding). Differential compounding has been used for constant-speed motors. Magneto machines are used only for very small sizes.

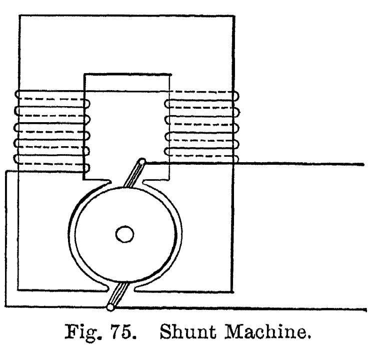


Fig. 75. Shunt Machine.

36. By the number of poles commutating machines are divided into bipolar and multipolar machines. Bipolar machines are used only in small sizes. By the construction of the armature, commutating machines are divided into smooth-core machines and iron-clad or "toothed" armature machines. In the smooth-

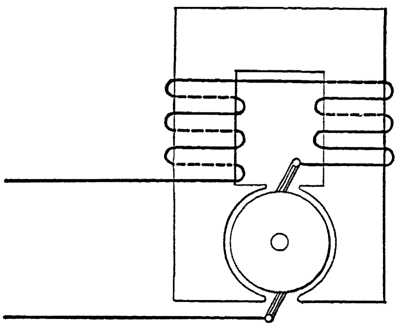


Fig. 76. Series Machine.

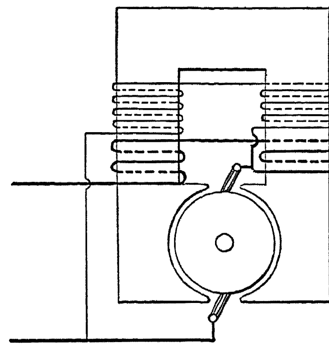


Fig. 77. Compound Machine.

core machine the armature winding is arranged on the surface of a laminated iron core. In the iron-clad machine the armature winding is sunk into slots. The iron-clad type has the advantage of greater mechanical strength, but the disadvantage of higher self-inductance in commutation, and thus requires high-resistance, carbon or graphite, commutator brushes. The iron-clad type has the advantage of lesser magnetic stray field, due to the shorter gap between field pole and armature iron, and of lesser magnet distortion under load, and thus can with carbon brushes be operated with constant position of brushes at all loads. In consequence thereof, for large multipolar machines the iron-clad type of armature is best adapted; the smooth-core type is hardly ever used nowadays.

Either of these types can be drum wound or ring wound. The drum winding has the advantage of lesser self-inductance and lesser distortion of the magnetic field, and is generally less difficult to construct and thus mostly preferred. By the armature winding, commutating machines are divided into multiple-wound and series-wound machines. The difference between multiple and series armature winding, and their modifications, can best be shown by diagram.

II. Armature Winding.

37. Fig. 78 shows a six-pole multiple ring winding, and Fig. 79 a six-polar multiple drum winding. As seen, the armature coils are connected progressively all around the armature in closed circuit, and the connections between adjacent armature coils lead to the commutator. Such an armature winding has as many circuits in multiple, and requires as many sets of commutator brushes, as poles. Thirty-six coils are shown in Figs. 78 and 79, connected to 36 commutator segments, and the two sides of each coil distinguished by drawn and dotted lines. In a drum-wound machine, usually the one side of all coils forms the upper and the other side the lower layer of the armature winding.

Fig. 80 shows a six-pole series drum winding with 36 slots and 36 commutator segments. In the series winding the circuit passes from one armature coil, not to the next adjacent armature coil as in the multiple winding, but first through all the armature coils having the same relative position with regard to the magnet poles of the same polarity, and then to the armature coil next adjacent to the first coil. That is, all armature coils having the same or approximately the same relative position to poles of equal polarity form one set of integral coils. Thus the series winding has only two circuits in multiple, and requires two sets of brushes only, but can be operated also with as many sets of brushes as poles, or any intermediate number of sets of brushes. In Fig. 80, a series winding in which the number of armature coils is divisible by the number of poles, the commutator segments have to be cross connected. Therefore this form of series winding is hardly ever used. The usual form of series winding is the winding shown by Fig. 81. This figure shows a six-polar armature having 35 coils and 35 commutator segments. In consequence thereof the armature coils under corresponding poles which are connected in series are slightly displaced from each other, so that after passing around all corresponding poles the winding leads symmetrically into the coil adjacent to the first armature coil. Hereby the necessity of commutator cross connections is avoided, and the winding is perfectly symmetrical. With this form of series winding, which is mostly used, the number of armature coils must be chosen to follow certain rules.

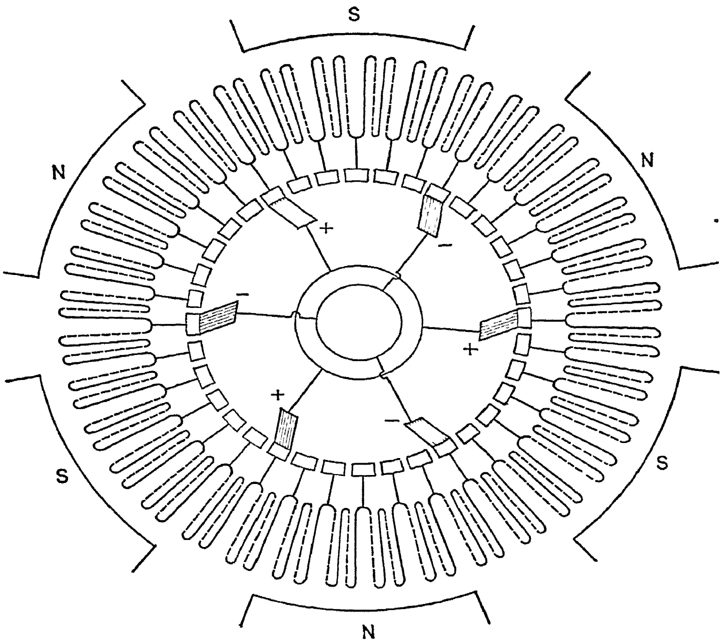


Fig. 78. Multiple Ring Armature Winding.

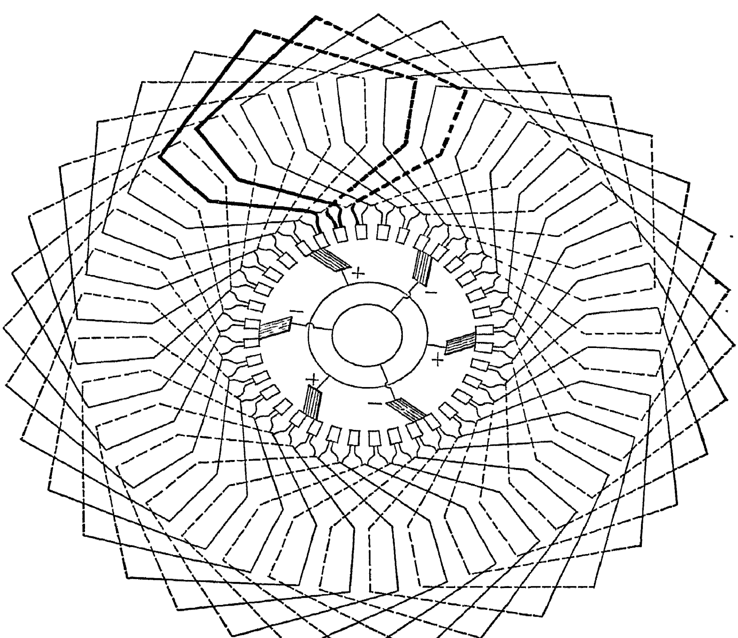


Fig. 79. Multiple Drum Full Pitch Winding.

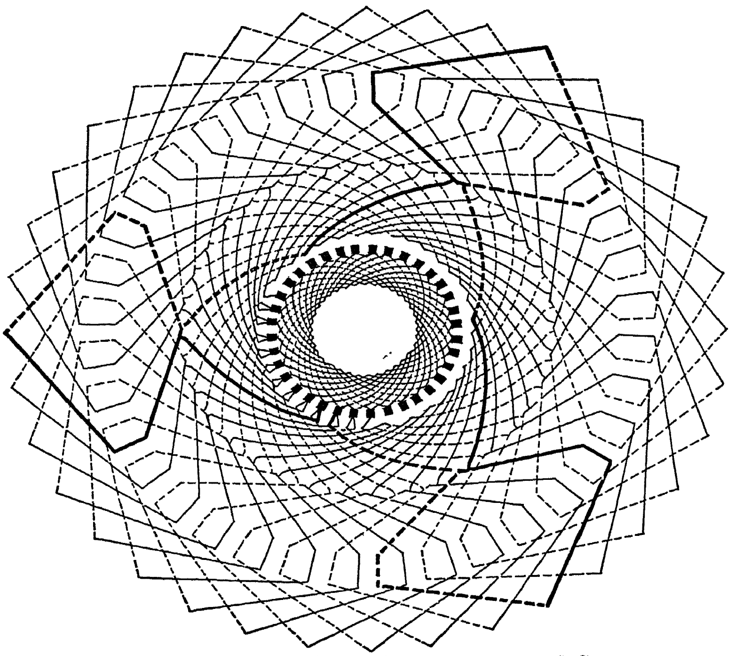


Fig. 80. Series Drum Winding with Cross-connected Commutator.

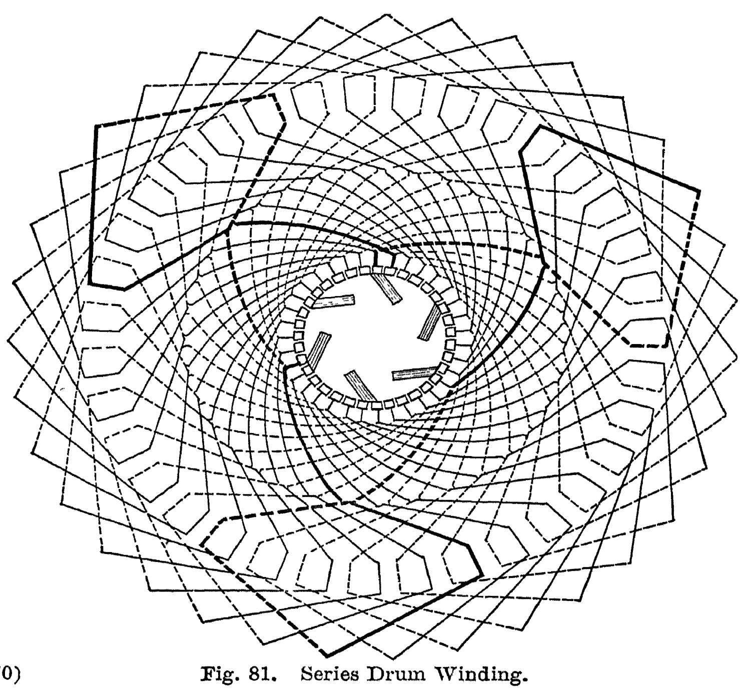


Fig. 81. Series Drum Winding.

Generally the number of coils is one less or one more than a multiple of the number of poles.

All these windings are closed-circuit windings; that is, starting at any point, and following the armature conductor, the circuit returns into itself after passing all e.m.fs. twice in opposite direction (thereby avoiding short-circuit). An instance of an open-coil winding is shown in Fig. 82, a series-connected three-phase

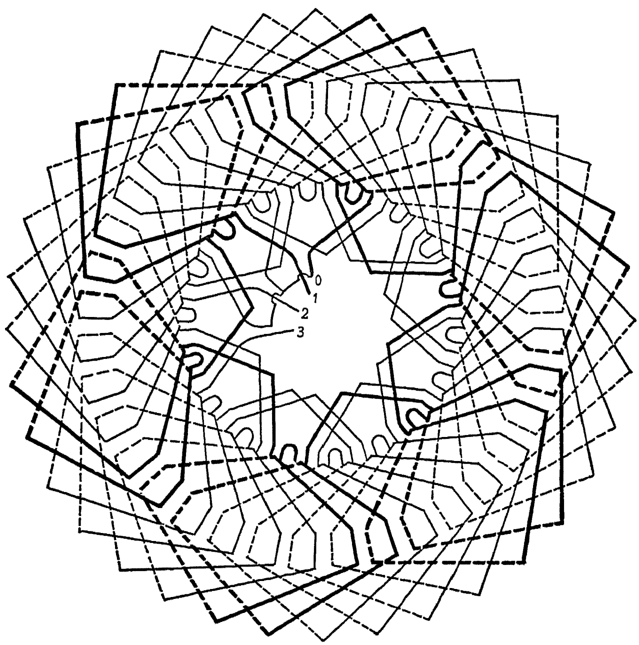


Fig. 82. Open-circuit Three-phase Series Drum Winding.

star winding similar to that used in the Thomson-Houston arc machine. Such open-coil windings, however, cannot be used in commutating machines. They are generally preferred in synchronous and in induction machines.

38. By leaving space between adjacent coils of these windings a second winding can be laid in between. The second winding can either be entirely independent from the first winding, that is, each of the two windings closed upon itself, or after passing through the first winding the circuit enters the second winding, and after passing through the second winding it reenters the first winding. In the first case the winding is called a double spiral

winding (or multiple spiral winding if more than two windings are used), in the latter case a double reentrant winding (or multiple reentrant winding). In the double spiral winding the number of coils must be even; in the double reentrant winding, odd.

Multiple spiral and multiple reentrant windings can be either multiple or series wound; that is, each spiral can consist either of a multiple or of a series winding. Fig. 83 shows a double

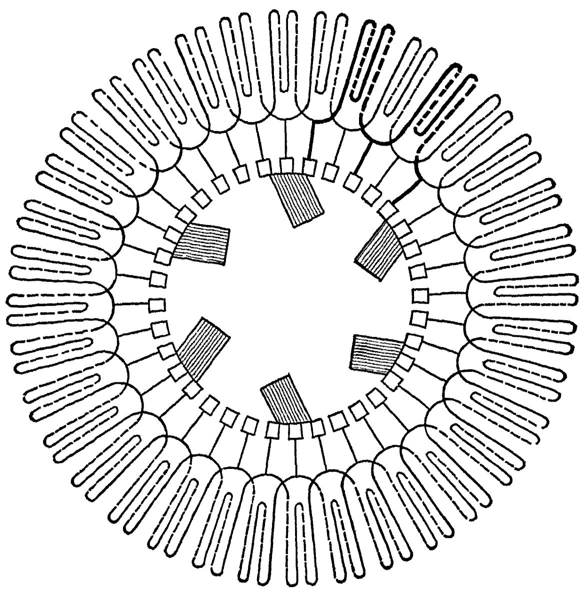


Fig. 83. Multiple Double Spiral Ring Winding.

spiral multiple ring winding, Fig. 84 a double spiral multiple drum winding, Fig. 85 a double reentrant multiple drum winding. As seen in the double spiral or double reentrant multiple winding, twice as many circuits as poles are in multiple. Thus such windings are mostly used for large low-voltage machines.

39. A distinction is frequently made between lap winding and wave winding. These are, however, not different types; but the wave winding is merely a constructive modification of the series drum winding with single-turn coil, as seen by comparing Figs. 86 and 87. Fig. 86 shows a part of a series drum winding developed. Coils C_1 and C_2, having corresponding

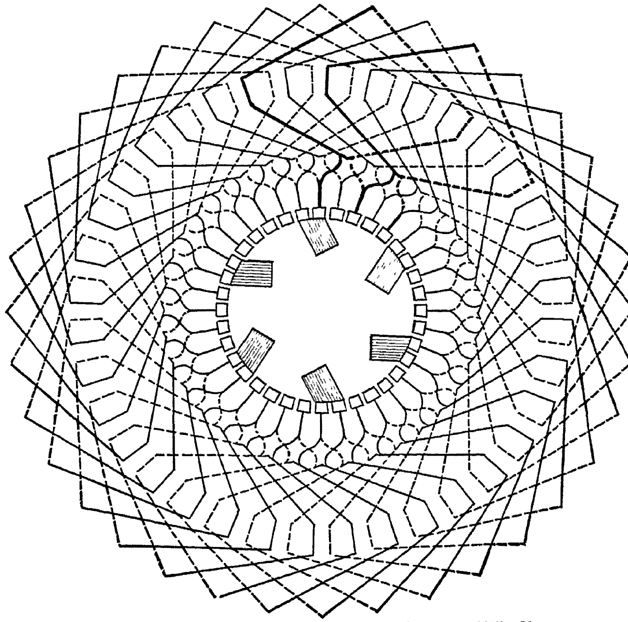


Fig. 84. Multiple Double Spiral Full Pitch Winding.

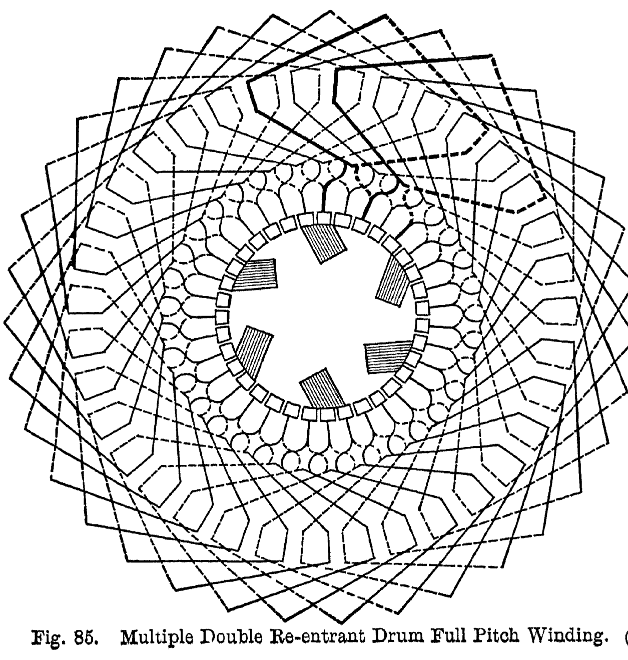


Fig. 85. Multiple Double Re-entrant Drum Full Pitch Winding.

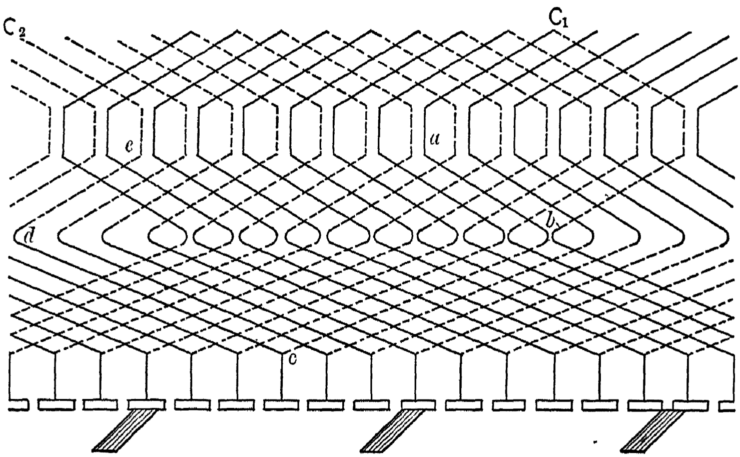


Fig. 86. Series Lap Winding.

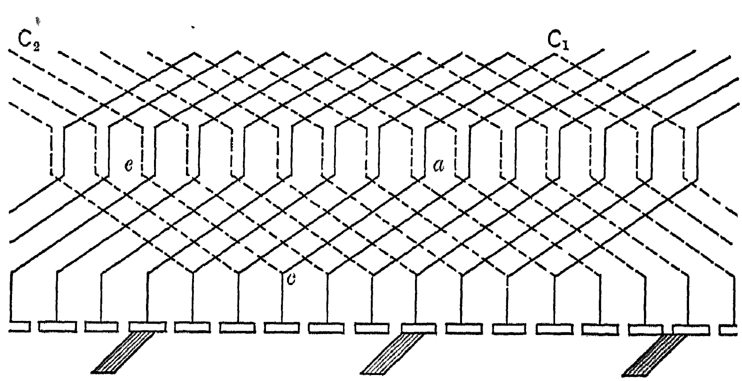


Fig. 87. Wave Winding.

positions under poles of equal polarity, are joined in series. Thus the end connection *ab* of coil C_1 connects by cross connection *bc* and *cd* to the end connection *de* of coil C_2. If the armature coils consist of a single turn only, as in Fig. 86, and thus are open at *b* and *d*, the end connection and the cross connection can be combined by passing from *a* in coil C_1 directly to *c* and from *c*

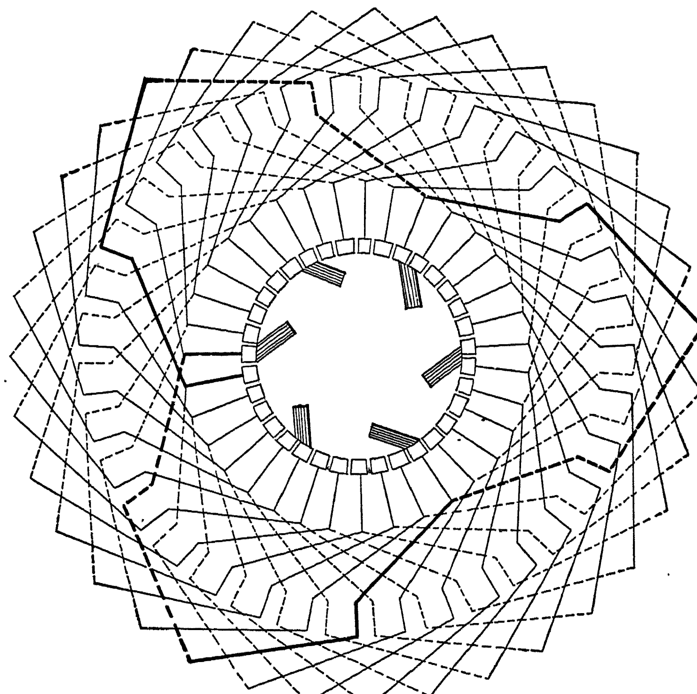


Fig. 88. Series Drum Wave Winding.

directly to *e* in coil C_2; that is, the circuit *abcde* is replaced by *ace*. This has the effect that the coils are apparently open at one side.

Such a winding has been called a wave winding. Only series windings with a single turn per coil can be arranged as wave windings, while windings with several turns per coil must necessarily be lap or coil windings. In Fig. 88 is shown a series drum winding with 35 coils and commutator segments, and a single turn per coil arranged as wave winding. This winding may be compared with the 35-coil series drum winding in Fig. 81.

40. Drum winding can be divided into full-pitch and fractional-pitch windings. In the full-pitch winding the spread of the coil covers the pitch of one pole; that is, each coil covers one-sixth of the armature circumference in a six-pole machine, etc. In a fractional-pitch winding it covers less or more.

Series drum windings without cross-connected commutator in which thus the number of coils is not divisible by the number of

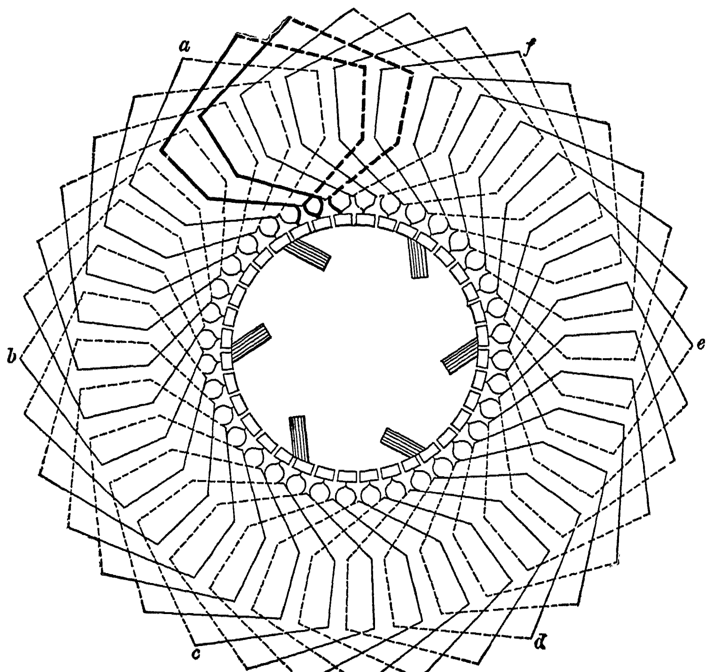


Fig. 89. Multiple Drum Five-sixth Fractional Pitch Winding.

poles are necessarily always slightly fractional pitch; but generally the expression "fractional-pitch winding" is used only for windings in which the coil covers one or several teeth less than correspond to the pole pitch. Thus the multiple drum winding in Fig. 79 would be a fractional-pitch winding if the coils spread over only four or five teeth instead of over six. As five-sixths fractional-pitch multiple drum winding it is shown in Fig. 89.

Fractional-pitch windings have the advantage of shorter end connections and less self-inductance in commutation, since commutation of corresponding coils under different poles does not take place in the same, but in different, slots, and the flux of self-inductance in commutation is thus more subdivided. Fig. 89 shows the multiple drum winding of Fig. 79 as a fractional-pitch winding with five teeth spread, or five-sixths pitch. During commutation the coils *a b c d e f* commutate simultaneously. In Fig. 79 these coils lie by twos in the same slots, in Fig. 89 they lie in separate slots. Thus, in the former case the flux of self-inductance interlinked with the commutated coil is due to two coils; that is, twice that in the latter case. Fractional-pitch windings, however, have the disadvantage of reducing the width of the neutral zone, or zone without generated e.m.f. between the poles, in which commutation takes place, since the one side of the coil enters or leaves the field before the other. Therefore, in commutating machines it is seldom that a pitch is used that falls short of full pitch by more than one or two teeth, while in induction and synchronous machines occasionally as low a pitch as 50 per cent is used, and two-thirds pitch is frequently employed.

For special purposes, as in single-phase commutator motors (see section C), fractional-pitch windings are sometimes used.

41. Series windings find their foremost application in machines with small currents, or small machines in which it is desirable to have as few circuits as possible in multiple, and in machines in which it is desirable to use only two sets of brushes, as in smaller railway motors. In multipolar machines with many sets of brushes a series winding is liable to give selective commutation; that is, the current does not divide evenly between the sets of brushes of equal polarity.

Multiple windings are used for machines of large currents, thus generally for large machines, and in large low-voltage machines the still greater subdivision of circuits afforded by the multiple spiral and the multiple reentrant winding is resorted to.

To resume, then, armature windings can be subdivided into

(*a*) Ring and drum windings.

(*b*.) Closed-circuit and open-circuit windings. Only the former can be used for commutating machines.

(*c*.) Multiple and series windings.

(*d*.) Single-spiral, multiple-spiral, and multiple-reentrant windings. Either of these can be multiple or series windings.

(*e*.) Full-pitch and fractional-pitch windings.

III. Generated E.M.FS.

42. The formula for the generation of e.m.f. in a direct-current machine, as discussed in the preceding, is

$$e = 4\,fn\Phi,$$

where e = generated e.m.f., f = frequency = number of pairs of poles $\times$ hundreds of rev. per sec., n = number of turns in series between brushes, and Φ = magnetic flux passing through the armature per pole, in megalines.

In ring-wound machines, Φ is one-half the flux per field pole, since the flux divides in the armature into two circuits, and each armature turn incloses only half the flux per field pole. In ring-wound armatures, however, each armature turn has only one conductor lying on the armature surface, or face conductor, while in a drum-wound machine each turn has two face conductors. Thus, with the same number of face conductors — that is, the same armature surface — the same frequency, and the same flux per field pole, the same e.m.f. is generated in the ring-wound as in the drum-wound armature.

The number of turns in series between brushes, n, is one-half the total number of armature turns in a series-wound armature, $\frac{1}{p}$ the total number of armature turns in a single-spiral multiple-wound armature with p poles. It is one-half as many in a double-spiral or double-reentrant, one-third as many in a triple-spiral winding, etc.

By this formula, from frequency, series turns, and magnetic flux the e.m.f. is found, or inversely, from generated e.m.f., frequency, and series turns the magnetic flux per field pole is calculated:

$$\Phi = \frac{e}{4\,fn}.$$

From magnetic flux, and section and lengths of the different parts of the magnetic circuit, the densities and the ampere-turns required to produce these densities are derived, and as the sum of the ampere-turns required by the different parts of the magnetic circuit, the total ampere-turns excitation per field pole is found, which is required for generating the desired e.m.f.

Since the formula for the generation of direct-current e.m.f. is independent of the distribution of the magnetic flux, or its wave shape, the total magnetic flux, and thus the ampere-turns required therefor, are independent also of the distribution of magnetic flux at the armature surface. The latter is of importance, however, regarding armature reaction and commutation.

IV. Distribution of Magnetic Flux.

43. The distribution of magnetic flux in the air gap or at the armature surface can be calculated approximately by assuming the density at any point of the armature surface as proportional to the m.m.f. acting thereon, and inversely proportional to the

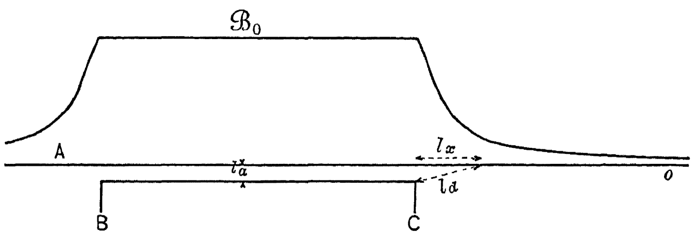


Fig. 90. Distribution of Magnetic Flux under a Single Pole.

nearest distance from a field pole. Thus, if $\mathfrak{F}_0$ = ampere-turns acting upon the air gap between armature and field pole, l_a = length of air gap, from iron to iron, the density under the magnet pole, that is, in the range BC of Fig. 90, is

$$\mathcal{B}_0 = \frac{4\,\pi\mathfrak{F}_0}{10\,l_a}.$$

At a point having the distance l_x from the end of the field pole on the armature surface, the distance from the next field pole is $l_d = \sqrt{l_a^2 + l_x^2}$, and the density thus,

$$\mathcal{B}_0 = \frac{4\,\pi\mathfrak{F}_0}{10\,\sqrt{l_a^2 + l_x^2}}.$$

Herefrom the distribution of magnetic flux is calculated and plotted in Fig. 90, for a single pole BC, along the armature surface A, for the length of air gap $l_a = 1$, and such a m.m.f. as to give $\mathcal{B}_0 = 8000$ under the field pole; that is, for $\mathcal{H}_0 = 6400$ or $\mathcal{H}_0 = 8000$.

Around the surface of the direct-current machine armature, alternate poles follow each other. Thus the m.m.f. is constant only under each field pole, but decreases in the space between the field poles, from C to E in Fig. 91, from full value at C to

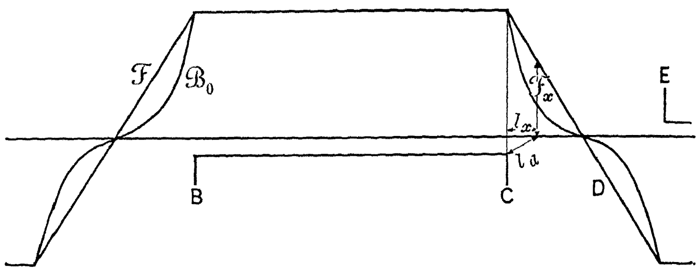


Fig. 91. Distribution of Magnetic Force and Flux at No Load.

full value in opposite direction at E. The point D midway between C and E, at which the m.m.f. of the field equals zero, is called the "neutral." The distribution of m.m.f. of field

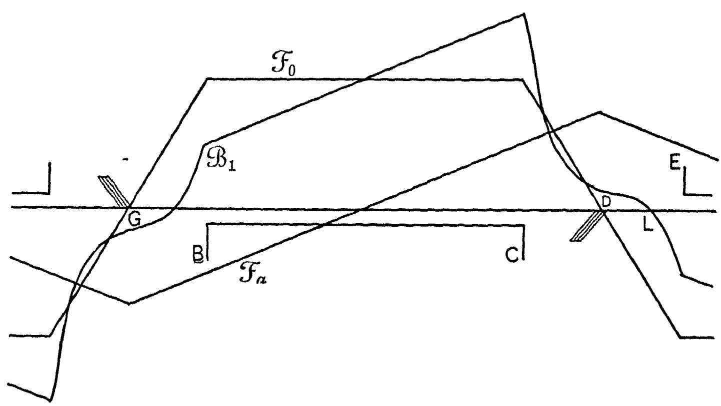


Fig. 92. Distribution of Flux with Current in the Armature.

excitation is thus given by the line $\mathcal{F}$ in Fig. 91. The distribution of magnetic flux as shown in Fig. 91 by $\mathcal{B}_0$ is derived by the formula

$$\mathcal{B} = \frac{4\pi\mathcal{F}}{10\, l_d}$$

where

$$l_d = \sqrt{l_a^2 + l_x^2}.$$

This distribution of magnetic flux applies only to the no-load condition. Under load, that is, if the armature carries current, the distribution of flux is changed by the m.m.f. of the armature current, or armature reaction.

44. Assuming the brushes set at the middle points between adjacent poles, D and G, Fig. 92, the m.m.f. of the armature is maximum at the point connected with the commutator brushes, in this case at the points D and G, and gradually decreases from full value at D to equal but opposite value at G, as shown by the line $\mathfrak{F}_a$ in Fig. 92, while the line $\mathfrak{F}_0$ gives the field m.m.f. or impressed m.m.f.

If n = number of turns in series between brushes per pole, i = current per turn, the armature reaction is $\mathfrak{F}_a = ni$ ampere-turns. Adding $\mathfrak{F}_a$ and $\mathfrak{F}_0$ gives the resultant m.m.f. $\mathfrak{F}$, and therefrom the magnetic distribution:

$$\mathfrak{B} = \frac{4\,\pi\mathfrak{F}}{10\,l_d}.$$

The latter is shown as line $\mathfrak{B}_1$ in Fig. 92.

With the brushes set midway between adjacent field poles, the armature m.m.f. is additive on one side and subtractive on the other side of the center of the field pole. Thus the magnetic intensity is increased on one side and decreased on the other. The total m.m.f., however, and thus, neglecting saturation, the total flux entering the armature, are not changed. Thus, armature reaction, with the brushes midway between adjacent field poles, acts distorting upon the field, but neither magnetizes nor demagnetizes, if the field is below saturation.

The distortion of the magnetic field takes place by the armature ampere-turns beneath the pole, or from B to C. Thus, if τ = pole arc, that is, the angle covered by pole face (two poles or one complete period being denoted by 360 degrees), the distorting ampere-turns of the armature reaction are: $\frac{\tau\mathfrak{F}_a}{180}$.

As seen, in the assumed instance, Fig. 92, where $\mathfrak{F}_a = \frac{3\,\mathfrak{F}_0}{4}$, the m.m.f. at the two opposite pole corners, and thus the magnetic densities, stand in the proportion 1 to 3. As seen, the generated e.m.f. is not changed by the armature reaction, with the brushes set midway between the field poles, except by the small amount corresponding to the flux entering beyond D and G, that is, shifted beyond the position of brushes. At D, however, the flux still enters the armature, depending in intensity upon the armature reaction; and thus with considerable armature reaction the brushes when set at this point are liable to spark by short-circuiting an active e.m.f. Therefore, under load, the brushes are shifted towards the following pole, that is, towards the direction in which the zero point of magnetic flux has been shifted by the armature reaction.

45. In Fig. 93, the brushes are assumed as shifted to the corner of the next pole, E respectively B. In consequence thereof, the subtractive range of the armature m.m.f. is larger

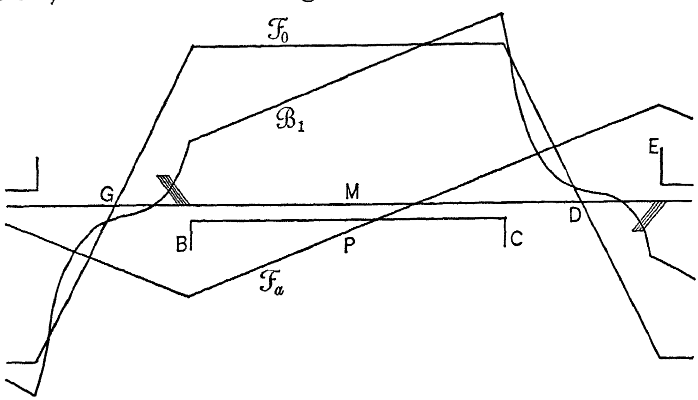


Fig. 93. Distribution of Flux with Current in the Armature and Brushes shifted from the Magnetic Neutral.

than the additive, and the resultant m.m.f. $\mathfrak{F} = \mathfrak{F}_0 + \mathfrak{F}_a$ is decreased; that is, with shifted brushes the armature reaction demagnetizes the field. The demagnetizing armature ampere-turns are $PM = \frac{GB}{GM} \mathfrak{F}_a$. That is, if τ_1 = angle of shift of brushes or angle of lead (= GB in Fig. 93), assuming the pitch of two poles = 360 degrees, the demagnetizing component of armature reaction is $\frac{2 \tau_1 \mathfrak{F}_a}{180}$, the distorting component is $\frac{\tau \mathfrak{F}_a}{180}$, where τ = pole arc.

Thus, with shifted brushes the field excitation has to be increased under load to maintain the same total resultant m.m.f., that is, the same total flux and generated e.m.f. Hence, in Fig. 93 the field excitation $\mathfrak{F}_0$ has been assumed by $\frac{2\tau_1 \mathfrak{F}_a}{180} = \frac{\mathfrak{F}}{3}$ larger than in the previous figures, and the magnetic distribution $\mathfrak{B}_1$ plotted for these values.

V. Effect of Saturation on Magnetic Distribution.

46. The preceding discussion of Figs. 90 to 93 omits the effect of saturation. That is, the assumption is made that the magnetic materials near the air gap, as pole face and armature teeth, are so far below saturation that at the demagnetized pole corner the magnetic density decreases, at the strengthened pole corner increases, proportionally to the m.m.f.

The distribution of m.m.f. obviously is not affected by saturation, but the distribution of magnetic flux is greatly changed thereby. To investigate the effect of saturation, in Figs. 94 to 97 the assumption has been made that the air gap is reduced to one-half its previous value, $l_a = 0.5$, thus consuming only one-half as many ampere-turns, and the other half of the ampere-turns are consumed by saturation of the armature teeth. The length of armature teeth is assumed as 3.2, and the space filled by the teeth is assumed as consisting of one-third of iron and two-thirds of non-magnetic material (armature slots, ventilating ducts, insulation between laminations, etc.).

In Figs. 94, 95, 96, 97, curves are plotted corresponding to those in Figs. 90, 91, 92, and 93. As seen, the spread of magnetic flux at the pole corners is greatly increased, but farther away from the field poles the magnetic distribution is not changed.

47. The magnetizing, or rather demagnetizing, effect of the load with shifted brushes is not changed. The distorting effect of the load is, however, very greatly decreased, to a small percentage of its previous value, and the magnetic field under the field pole is very nearly uniform under load.

The reason is: Even a very large increase of m.m.f. does not much increase the density, the ampere-turns being consumed by saturation of the iron, and even with a large decrease of m.m.f. the density is not decreased much, since with a small decrease of density the ampere-turns consumed by the saturation of the iron become available for the air gap.

Thus, while in Fig. 93 the densities at the center and the two pole corners of the field pole are 8000, 12,000, and 4000, with the saturated structure in Fig. 97 they are 8000, 9040, and 6550.

At or near the theoretical neutral, however, the saturation has no effect.

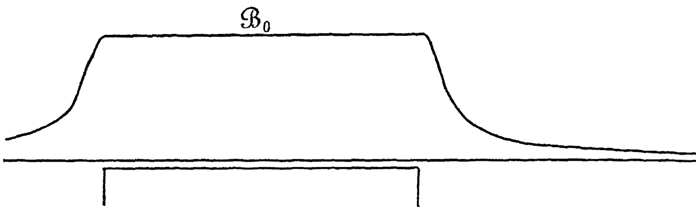


Fig. 94. Flux Distribution under a Single Pole.

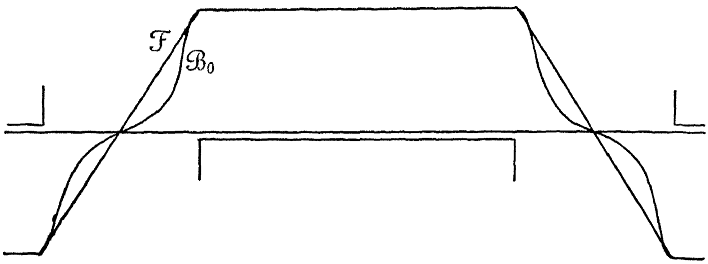


Fig. 95. Distribution of Flux and m.m.f. at No Load.

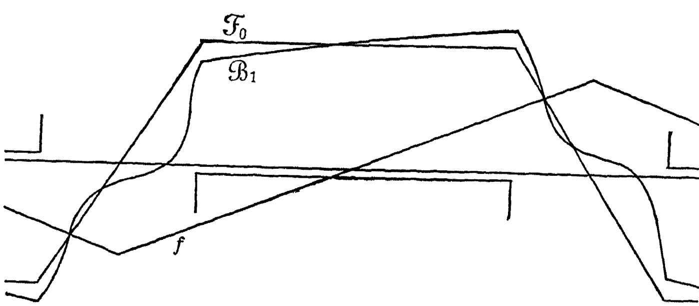


Fig. 96. Distribution of Flux and m.m.f. at Load, with Brushes at Neutral.

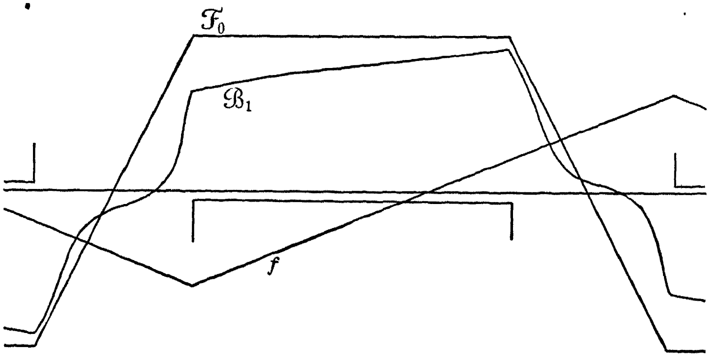


Fig. 97. Distribution of Flux and m.m.f. at Load, with Brushes shifted to next Pole Corner.

That is, saturation of the armature teeth affords a means of reducing the distortion of the magnetic field, or the shifting of flux at the pole corners, and is thus advantageous for machines which shall operate over a wide range of load with fixed position of brushes, if the brushes are shifted near to the next following pole corner.

It offers no direct advantage, however, for machines commutating with the brushes midway between the field poles, as converters.

An effect similar to saturation in the armature teeth is produced by saturation of the field pole face, or more particularly, saturation of the pole corners of the field.

VI. Effect of Commutating Poles.

48. With the commutator brushes of a generator set midway between the field poles, as in Fig. 92, the m.m.f. of armature reaction produces a magnetic field at the brushes. The e.m.f. generated by the rotation of the armature through this field opposes the reversal of the current in the short-circuited armature coil under the brush, and thus impairs commutation. If therefore the commutation constants of the machines are not abnormally good, — high field strength, low armature reaction, low self-inductance and frequency of commutation, — the machine does not commutate satisfactorily under load, with the brushes midway between the field poles, and the brushes have to be shifted to the edge of the next field poles, as shown in Fig. 93, until the fringe of the magnetic flux of the field poles reverses the armature reaction and so generates an e.m.f. in the armature coil, which reverses the current and thus acts as commutating flux. The commutating e.m.f. and therefore the commutating flux should be proportional to the current which is to be reversed, that is, to the load. The magnetic flux of the field pole of a shunt or compound machine, however, decreases with increasing load at the pole corners towards which the brushes are shifted, by the demagnetizing action of the armature reaction, and the shift of brushes therefore has to be increased with the load, from nothing at no load. At overload, the pole corners towards which the brushes are shifted may become so far weakened that even under the pole not sufficient reversing e.m.f. is generated, and

satisfactory commutation ceases, that is, the sparking limit is reached.

In general, however, varying the brush shift with the load is not permissible, and with rapidly fluctuating load not feasible, and therefore the brushes are set permanently at a mean shift. In this case, however, instead of increasing proportionally with the load, the commutating field is maximum at no load, and gradually decreases with increase of load, and is correct only at one particular load. At constant shift of the brushes, the commutation of the constant potential machine, direct-current generator or motor, is best at a certain load, and usually becomes poorer at lighter or heavier loads, and ultimately becomes bad by inductive sparks due to insufficient commutating flux. In machines in which very good commutating constants cannot be secured, as in large high-speed machines, (steam turbine driven direct-current generators), this may lead to bad sparking or even flashing over at sudden overloads as well as when throwing off full load.

49. This has led to the development of the *commutating pole*, also called *interpole*, that is, a narrow magnetic pole located between the main poles at the point of the armature surface, at which commutation occurs, and excited so as to produce a commutating flux proportional to the load, and thus giving the required commutating field at all loads. Such machines then give no inductive sparking, but regarding commutation are limited in overload capacity only by the current density under the brush.

Such commutating poles are excited by series coils, that is, coils connected in series with the armature and having a number of effective turns higher than the number of effective series turns per armature pole, so that at the position of the brushes the m.m.f. of the commutating pole overpowers and reverses the m.m.f. of the armature, and produces a commutating m.m.f. equal to the product of the armature current and difference of commutating turns and armature turns, and thereby produces a commutating flux proportional to the load, as long as the magnetic flux in the commutating poles does not reach too high magnetic saturation.

In Fig. 98 is shown the distribution of m.m.f. around the circumference of the armature, and in Fig. 99 the distribution of magnetic flux calculated in the manner as described in paragraphs 46 and 47. *M* represents the main poles, *C* the commutating poles. A main field excitation $\mathfrak{F}_0$ is assumed of 10,000 ampere-turns per pole, and an armature reaction $\mathfrak{F}_a$ of 6000

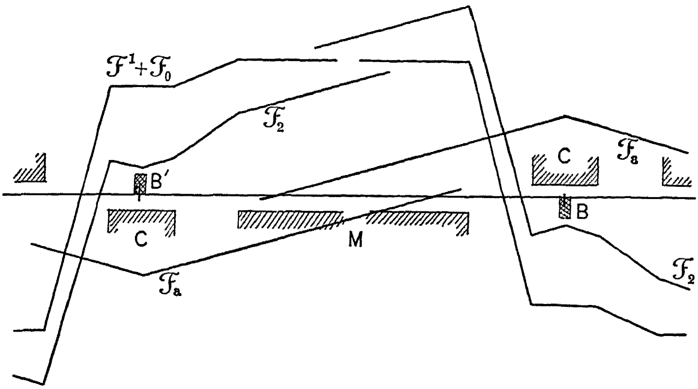


Fig. 98. Magnetive Force Distribution with Commutating Pole.

ampere-turns per pole. Choosing then 8000 ampere-turns per commutating pole $\mathfrak{F}'$, leaves 2000 ampere-turns as resultant commutating m.m.f. at full load, half as much at half load, etc. The resultant m.m.f. of the main field $\mathfrak{F}_0$, the armature $\mathfrak{F}_a$, and

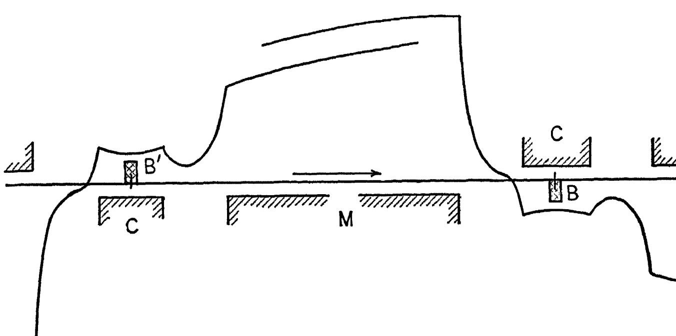


Fig. 99. Magnetic Flux Distribution with Commutating Pole.

the commutating pole $\mathfrak{F}'$ is represented in Fig. 98, by $\mathfrak{F}_2$, and the flux produced by it is shown in Fig. 99. As seen, with the commutator brushes midway between the field poles, that is, in the center of the commutating pole, a commutating flux proportional to the armature current enters the armature at the brush B and B', and is cut by the revolving armature during commutation.

The use of the commutating pole or interpole thus permits controlling the commutation, with fixed brush position midway between the field poles, and commutating poles therefore are extensively used in larger machines, especially of the high-speed type.

The commutating pole makes the commutation independent of the main field strength, and therefore permits the machines to operate with equally good commutation over a wide voltage range, and at low voltage, that is, low field strength, as required for instance in boosters, etc.

50. With multiple-wound armatures, at least one commutating pole for every pair of main poles is required, while with a series-wound armature a single commutating pole would be sufficient for all the sets of armature brushes, if of sufficient strength. In general, however, as many commutating poles as main poles are used.

With the position of the brushes at the neutral, as is the case when using commutating poles, the armature reaction has no demagnetizing component, and the only drop of voltage at load is that due to the armature resistance drop and the distortion of the main field, which at saturation produces a decrease of the total flux, as shown in Fig. 96.

As is seen in Fig. 99, the magnetic flux of the commutating pole is not symmetrical, but the spread of flux is greater at the side of the main pole of the same polarity. As result thereof, the total magnetic flux is slightly increased by the commutating poles; that is, the two halves of the commutating flux on the two sides of the brush do not quite neutralize, and the commutating flux thus exerts a slight compounding action, that is, tends to raise the voltage. This can be still further increased by shifting the brushes slightly back and thus giving a magnetizing component of armature reaction. This can be done without affecting commutation as long as the brushes still remain under the commutating pole. In this manner a compounding or even a slight over-compounding can be produced without a series winding on the main field poles, or, inversely, by shifting the brushes slightly forward, a demagnetizing component of armature reaction can be introduced.

In operating machines with commutating poles in multiple, care therefore must be taken not to have the compounding action of the commutating poles interfere with the distribution of load; for this purpose an equalizer connection may be used between the commutating pole windings of the different machines, and the commutating windings treated in the same way as series coils on the main poles, that is, equalized between the different machines to insure division of load.

51. The advantage of the commutating pole over the shift of brushes to the edge of the next field pole, in constant potential machines, — shunt or compound wound, — thus is that the commutating flux of the former has the right intensity at all loads, while that of the latter is right only at one particular load, too high below, too low above that load. In series-wound machines, that is, machines in which the main field is excited in series with the armature, and thus varies in strength with the armature current, armature reaction and field excitation are always proportional to each other, and the distribution of magnetic flux at the armature circumference therefore always has the same shape, and its intensity is proportional to the current, except as far as saturation limits it. As the result thereof, shifting the brushes to the edge of the field poles, as in Fig. 93, brings them in a field which is proportional to the armature current and thus has the proper intensity as a commutating field. Therefore with series-wound machines commutating poles are not necessary for good commutation, but the shifting of the brushes gives the same result. However, in cases where the direction of rotation frequently reverses, as in railway motors, the direction of the shift of brushes has to be reversed with the reversal of rotation. In railway motors this cannot be done without objectionable complication, therefore the brushes have to be set midway, and the use of the magnetic flux at the edge of the next pole, as commutating flux, is not feasible. In this case a commutating pole is used, to give, without mechanical shifting of the brushes, the same effect which a brush shift would give. Therefore in railway motors, especially when wound for high voltage, as 1000 to 1500 volts, a commutating pole is sometimes used. This commutating pole, having a series winding just like the main pole, changes proportionally with the main pole. When reversing the direction of rotation, however, the armature

and the commutating poles are reversed, while the main poles remain unchanged, or the main poles are reversed, while the armature and the commutating poles remain unchanged; that is, the separate commutating pole becomes necessary because during the reversal of rotation it has to be treated differently from the main pole.

VII. Effect of Slots on Magnetic Flux.

52. With slotted armatures the pole face density opposite the armature slots is less than that opposite the armature teeth, due to the greater distance of the air path in the former case. Thus, with the passage of the armature slots across the field pole a local pulsation of the magnetic flux in the pole face is produced, which, while harmless with laminated field pole faces, generates eddy currents in solid pole pieces. The frequency of this pulsation is extremely high, and thus the energy loss due to eddy currents in the pole faces may be considerable, even with pulsations of small amplitude. If S = peripheral speed of the armature in centimeters per second, l_p = pitch of armature slot (that is, width of one slot and one tooth at armature surface), the frequency is $f_1 = \frac{S}{l_p}$. Or, if f = frequency of machine, n = number of armature slots per pair of poles, $f_1 = nf$.

For instance, $f = 33.3$, $n = 51$, thus $f_1 = 1700$.

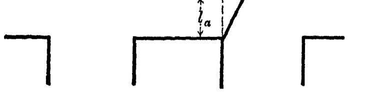


Fig. 100.—Effect of Slots on Flux Distribution.

Under the assumption, width of slots equals width of teeth = 2 × width of air gap, the distribution of magnetic flux at the pole face is plotted in Fig. 100.

The drop of density opposite each slot consists of two curved branches equal to those in Fig. 90, that is, calculated by

$$\mathcal{B} = \frac{\mathcal{F}}{\sqrt{l_a^2 + l_x^2}}.$$

The average flux is 7525; that is, by cutting half the armature surface away by slots of a width equal to twice the length of air gap, the total flux under the field pole is reduced only in the proportion 8000 to 7525, or about 6 per cent.

The flux $\mathfrak{B}$ pulsating between 8000 and 5700 is equivalent to a uniform flux $\mathfrak{B}_1 = 7525$ superposed with an alternating flux $\mathfrak{B}_0$, shown in Fig. 101, with a maximum of 475 and a minimum of 1825. This alternating flux $\mathfrak{B}_0$ can, as regards production of

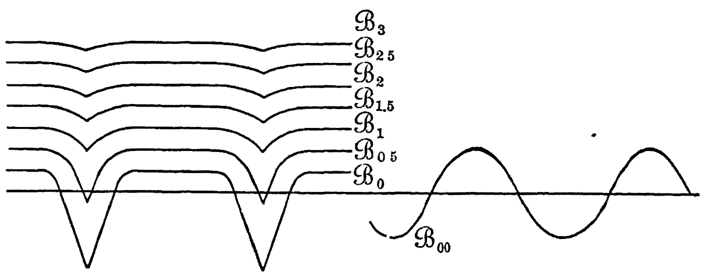


Fig. 101. Effect of Slots on Flux Distribution.

eddy currents, be replaced by the equivalent sine wave $\mathfrak{B}_{00}$, that is, a sine wave having the same effective value (or square root of mean square). The effective value is 718.

The pulsation of magnetic flux farther in the interior of the field-pole face can be approximated by drawing curves equidistant from $\mathfrak{B}_0$. Thus the curves $\mathfrak{B}_{0.5}$, $\mathfrak{B}_1$, $\mathfrak{B}_{1.5}$, $\mathfrak{B}_2$, $\mathfrak{B}_{2.5}$, and $\mathfrak{B}_3$ are drawn equidistant from $\mathfrak{B}_0$ in the relative distances 0.5, 1, 1.5, 2, 2.5, and 3 (where $l_a = 1$ is the length of air gap). They give the effective values:

$\mathfrak{B}_0$	$\mathfrak{B}_{.5}$	$\mathfrak{B}_1$	$\mathfrak{B}_{1.5}$	$\mathfrak{B}_{2.5}$	$\mathfrak{B}_{2.5}$	$\mathfrak{B}_3$
718	373	184	119	91	69	57

That is, the pulsation of magnetic flux rapidly disappears towards the interior of the magnet pole, and still more rapidly the energy loss by eddy currents, which is proportional to the square of the magnetic density.

53. In calculating the effect of eddy currents, the magnetizing effect of eddy currents may be neglected (which tends to reduce the pulsation of magnetism); this gives the upper limit of loss.

Let $\mathfrak{B}$ = effective density of the alternating magnetic flux,
S = peripheral speed of armature in centimeters per second, and
l = length of pole face along armature.

The e.m.f. generated in the pole face is then

$$e = Sl\mathfrak{B} \times 10^{-8},$$

and the current in a strip of thickness Δl and one centimeter width,

$$\Delta i = \frac{e\Delta l}{\rho l} = \frac{Sl\mathfrak{B}\Delta l\, 10^{-8}}{\rho l} = \frac{S\mathfrak{B}\Delta l\, 10^{-8}}{\rho}.$$

where ρ = resistivity of the material.

Thus the effect of eddy currents in this strip is

$$\Delta p = e\Delta i = \frac{S^2 l \mathfrak{B}^2 \Delta l\, 10^{-16}}{\rho}.$$

or per cu. cm.

$$p = \frac{S^2 \mathfrak{B}^2\, 10^{-16}}{\rho};$$

that is, proportional to the square of the effective value of magnetic pulsation, the square of peripheral speed, and inversely proportional to the resistivity.

Thus, assuming for instance,

$$S = 2000,$$
$$\rho = 20 \times 10^{-6}, \text{ for cast steel},$$
$$\rho = 100 \times 10^{-6}, \text{ for cast iron}.$$

we have in the above example,

At Distance from Pole Face.	$\mathfrak{B}$.	p	
		Cast Steel.	Cast Iron.
0	718	10.3	2.06
$\frac{l_a}{2}$	373	2.78	0.56
l_a	184	0.677	0.135
$\frac{3\,l_a}{2}$	119	0.283	0.057
$2\,l_a$	91	0.166	0.033
$\frac{5\,l_a}{2}$	69	0.095	0.019
$3\,l_a$	57	0.065	0.013

VIII. Armature Reaction.

54. At no load, that is, with no current in the armature circuit, the magnetic field of the commutating machine is symmetrical with regard to the field poles.

Thus the density at the armature surface is zero at the point or in the range midway between adjacent field poles. This point, or range, is called the "neutral" point or "neutral" range of the commutating machine.

Under load the armature current represents a m.m.f. acting in the direction from commutator brush to commutator brush of opposite polarity, that is, in quadrature with the field m.m.f. if the brushes stand midway between the field poles; or shifted against the quadrature position by the same angle by which the commutator brushes are shifted, which angle is called the angle of lead.

If n = turns in series between brushes per pole, and i = current per turn, the m.m.f. of the armature is $\mathfrak{F}_a = ni$ per pole. Or, if n_0 = total number of turns on the armature, n_c = number of turns or circuits in multiple, $2 n_p$ = number of poles, and i_0 = total armature current, the m.m.f. of the armature per pole is $\mathfrak{F}_a = \frac{n_0 i_0}{2 n_p n_c}$. This m.m.f. is called the armature reaction of the continuous-current machine.

Since the armature turns are distributed over the total pitch of pole, that is, a space of the armature surface representing 180 deg., the resultant armature reaction is found by multiplying $\mathfrak{F}_a$ with the average $\cos \left\{ \begin{array}{c} +90 \\ -90 \end{array} \right. = \frac{2}{\pi}$, and is thus

$$\mathfrak{F}_{a_0} = \frac{2\,\mathfrak{F}_a}{\pi} = \frac{2\,ni}{\pi}.$$

When comparing the armature reaction of commutating machines with other types of machines, as synchronous machines, etc., the resultant armature reaction $\mathfrak{F}_{a_0} = \frac{2\,\mathfrak{F}_a}{\pi}$ has to be used. In discussing commutating machines proper, however, the value $\mathfrak{F}_a = ni$ is usually considered as the armature reaction.

55. The armature reaction of the commutating machine has a distorting and a magnetizing or demagnetizing action upon the magnetic field. The armature ampere-turns beneath the field poles have a distorting action as discussed under "Magnetic Distribution" in the preceding paragraphs. The armature ampere-turns between the field poles have no effect upon the resultant field if the brushes stand at the neutral; but if the brushes are shifted, the armature ampere-turns inclosed by twice the angle of lead of the brushes have a demagnetizing action.

Thus, if τ = pole arc as fraction of pole pitch, τ_1 = shift of brushes as fraction of pole pitch, $\mathfrak{F}_a$ the m.m.f. of armature reaction, and $\mathfrak{F}_0$ the m.m.f. of field excitation per pole, the demagnetizing component of armature reaction is $\tau_1\mathfrak{F}_a$, the distorting component of armature reaction is $\tau\mathfrak{F}_a$, and the magnetic density at the strengthened pole corner thus corresponds to the m.m.f. $\mathfrak{F}_0 + \frac{\tau\mathfrak{F}_a}{2}$ at the weakened field corner to the m.m.f. $\mathfrak{F}_0 - \frac{\tau\mathfrak{F}_a}{2}$.

IX. Saturation Curves.

56. As *saturation curve* or *magnetic characteristic* of the commutating machine is understood the curve giving the generated voltage, or terminal voltage at open circuit and normal speed, as function of the ampere-turns per pole field excitation.

Such curves are of the shape shown in Fig. 102 as A. Owing to the remanent magnetism or hysteresis of the iron part of the magnetic circuit, the saturation curve taken with decreasing

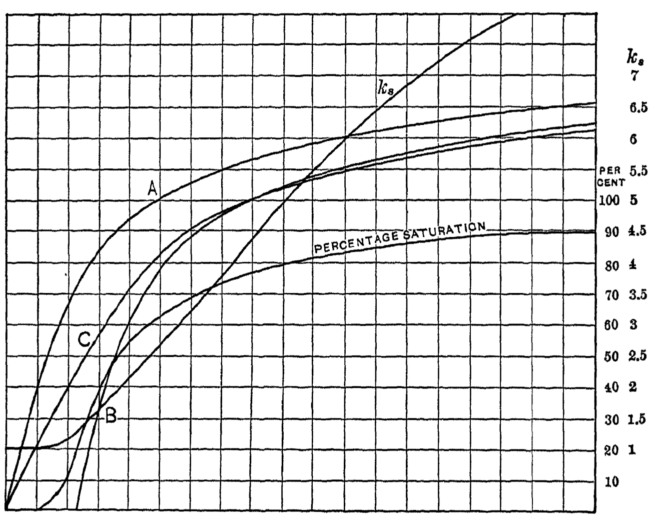


Fig. 102. Saturation Characteristics.

field excitation usually does not coincide with that taken with increasing field excitation, but is higher, and by gradually first increasing the field excitation from zero to maximum and then decreasing again, the looped curve in Fig. 103 is derived, giving as average saturation curve the curve shown in Fig. 102 as *A* and as central curve in Fig. 103.

Direct-current generators are usually operated at a point of the saturation curve above the bend, that is, at a point where the terminal voltage increases considerably less than proportionally to the field excitation. This is necessary in self-exciting direct-current generators to secure stability.

The ratio

$$\frac{\text{increase of field excitation}}{\text{total field excitation}} \div \frac{\text{corresponding increase of voltage}}{\text{total voltage}},$$

that is,

$$\frac{d\mathfrak{F}_0}{\mathfrak{F}_0} \div \frac{de}{e},$$

is called saturation factor k_s, and is plotted in Fig. 102. It is the ratio of a small percentage increase in field excitation to a corre-

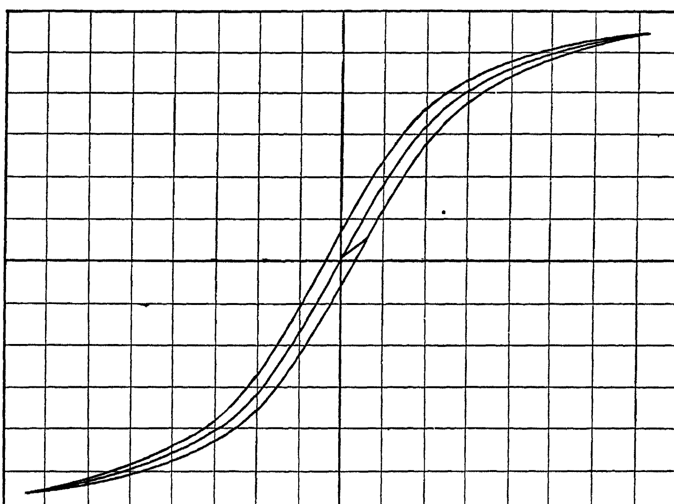


Fig. 103. Saturation Curves.

sponding percentage increase in voltage thereby produced. The quantity $1 - \frac{1}{k_s}$ is called the percentage saturation of the machine, as it shows the approach of the machine field to magnetic saturation.

57. Of considerable importance also are curves giving the terminal voltage as function of the field excitation at load.

Such curves are called *load saturation curves*, and can be *constant current load saturation curve*, that is, terminal voltage as function of field ampere-turns at constant full-load current through the armature, and *constant resistance load saturation curve*, that is, terminal voltage as function of field ampere-turns if the machine circuit is closed through a constant resistance giving full-load current at full-load terminal voltage.

A constant-current load saturation curve is shown as B, and a constant resistance load saturation curve as C in Fig. 102.

X. Compounding.

58. In the direct-current generator the field excitation required to maintain constant terminal voltage has to be increased with the load. A curve giving the field excitation in ampere-turns per pole, as function of the load in amperes, at constant terminal voltage, is called the *compounding curve* of the machine.

The increase of field excitation required with load is due to:

1. The internal resistance of the machine, which consumes e.m.f. proportional to the current, so that the generated e.m.f., and thus the field m.m.f. corresponding thereto, has to be greater under load. If k_r = resistance drop in the machine as fraction of terminal voltage, $= \frac{ir}{e}$, the generated e.m.f. at load has to be $e\,(1 + k_r)$, and if $\mathfrak{F}_0$ = no-load field excitation, and k_s = saturation coefficient, the field excitation required to produce the e.m.f. $e\,(1 + k_r)$ is $\mathfrak{F}_0\,(1 + k_s k_r)$; thus an additional excitation of $k_s k_r \mathfrak{F}_0$ is required at load, due to the armature resistance.

2. The demagnetizing effect of the ampere-turns armature reaction of the angle of shift of brushes τ_1 requires an increase of field excitation by $\tau_1 \mathfrak{F}_a$. (Section VII.)

3. The distorting effect of armature reaction does not change the total m.m.f. producing the magnetic flux. If, however, magnetic saturation is reached or approached in a part of the magnetic circuit adjoining the air gap, the increase of magnetic density at the strengthened pole corner is less than the decrease at the weakened pole corner, and thus the total magnetic flux with the same total m.m.f. is reduced, and to produce the same

total magnetic flux an increased total m.m.f., that is, increase of field excitation, is required. This increase depends upon the saturation of the magnetic circuit adjacent to the armature conductors.

4. The magnetic stray field of the machine, that is, that part of the magnetic flux which passes from field pole to field pole without entering the armature, usually increases with the load. This stray field is proportional to the difference of magnetic potential between field poles; that is, at no-load it is proportional to the ampere-turns m.m.f. consumed in air gap, armature teeth, and armature core. Under load, with the same generated e.m.f., that is, the same magnetic flux passing through the armature core, the difference of magnetic potential between adjacent field poles is increased by the counter m.m.f. of the armature and by saturation. Since this magnetic stray flux passes through field poles and yoke, the magnetic density therein is increased and the field excitation correspondingly, especially if the magnetic density in field poles and yoke is near saturation. This increase of field strength required by the increase of density in the external magnetic circuit, due to the increase of magnetic stray field, depends upon the shape of the magnetic circuit, the armature reaction, and the saturation of the external magnetic circuit.

Curves giving, with the amperes output as abscissas, the ampere-turns per pole field excitation required to increase the voltage proportionally to the current, are called *over-compounding* curves. In the increase of field excitation required for over-compounding, the effects of magnetic saturation are still more marked.

XI. Characteristic Curves.

59. The *field characteristic* or *regulation curve*, that is, curve giving the terminal voltage as function of the current output at constant field excitation, is of less importance in commutating machines than in synchronous machines, since commutating machines are usually not operated with separate and constant excitation, and the use of the series field affords a convenient means of changing the field excitation proportionally to the load. The curve giving the terminal voltage as function of current output, in a compound-wound machine, at constant resistance in the shunt field, and constant adjustment of the series field, is, how-

ever, of importance as regulation curve of the direct-current generator. This curve would be a straight line except for the effect of saturation, etc., as discussed above.

XII. Efficiency and Losses.

60. The losses in a commutating machine which have to be considered when deriving the efficiency by adding the individual losses are:

1. Loss in the resistance of the armature, the commutator leads, brush contacts and brushes, in the shunt field and the series field with their rheostats.
2. Hysteresis and eddy currents in the iron at a voltage equal to the terminal voltage, plus resistance drop in a generator, or minus resistance drop in a motor.
3. Eddy currents in the armature conductors when large and not protected.
4. Friction of bearings, of brushes on the commutator, and windage.
5. Load losses, due to the increase of hysteresis and of eddy currents under load, caused by the change of the magnetic distribution, as local increase of magnetic density and of stray field.

The friction of the brushes and the loss in the contact resistance of the brushes are frequently quite considerable, especially with low-voltage machines.

Constant or approximately constant losses are: friction of bearings and of commutator brushes, and windage; hysteresis and eddy current losses; and shunt field excitation. Losses increasing with the load, and proportional or approximately proportional to the square of the current: armature resistance losses; series field resistance losses; brush contact resistance losses; and the so-called "load-losses," which, however, are usually small in commutating machines.

XIII. Commutation.

61. The most important problem connected with commutating machines is that of commutation.

Fig. 104 represents diagrammatically a commutating machine. The e.m.f. generated in an armature coil A is zero with this coil at or near the position of the commutator brush B_1. It rises and reaches a maximum about midway between two adjacent sets of brushes, B_1 and B_2, at C, and then decreases again, reaching zero at or about B_2, and then repeats the same change in opposite direction. The current in armature coil A, however, is constant during the motion of the coil from B_1 to B_2. While the coil A passes the brush B_2, however, the current in the coil

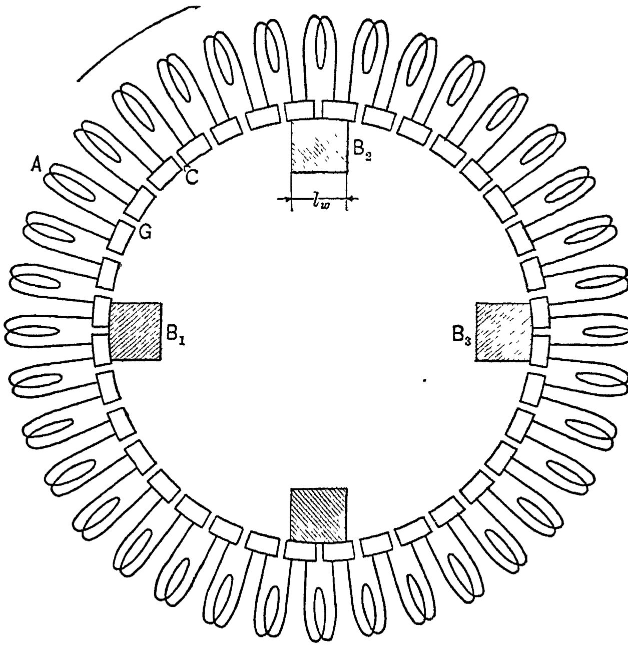


Fig. 104. Diagram for the Study of Commutation.

A reverses, and then remains constant again in opposite direction during the motion from B_2 to B_3. Thus, while the armature coils of a commutating machine are the seat of a system of polyphase e.m.fs. having as many phases as coils, the current in these coils is constant, reversing successively.

62. The reversal of current in coil A takes place while the gap G between the two adjacent commutator segments between which the coil A is connected, passes the brush B_2. Thus, if l_w = width of brushes, S = peripheral speed of commutator per second in the same measure in which l_w is given, as in inches per second if l_w is given in inches, $t_0 = \frac{l_w}{S}$ is the time during which the current in A reverses. Thus, considering the reversal as a single alternation, t_0 is a half period, and thus $f_0 = \frac{1}{2\,t_0} = \frac{S}{2\,l_w}$ is the *frequency of commutation;* hence, if L = inductance of the armature coil A, the e.m.f. generated in the armature coil during commutation is $e_0 = 2\,\pi f_0 L i_0$, where i_0 = current reversed, and the energy which has to be dissipated during commutation is $i_0{}^2 L$.

The frequency of commutation is very much higher than the frequency of synchronous machines, and averages from 300 to 1000 cycles per second, or more.

63. In reality, however, the changes of current during commutation are not sinusoidal, but a complex exponential function, and the resistance of the commutated circuit enters the problem as an important factor. In the moment when the gap G of the armature coil A reaches the brush B_2, the coil A is short-circuited by the brush and the current i_0 in the coil begins to die out, or rather to change at a rate depending upon the internal resistance and the inductance of the coil A and the e.m.f. generated in the coil by the magnetic flux of armature reaction and by the field magnetic flux. The higher the internal resistance the faster is the change of current, and the higher the inductance the slower the current changes. Thus two cases have to be distinguished:

1. No magnetic flux enters the armature at the position of the brushes, that is, no e.m.f. is generated in the armature coil under commutation, except that of its own self-inductance. In this case the commutation is entirely determined by the inductance and resistance of the armature coil A, and is called *resistance commutation.*

2. Commutation takes place in an active magnetic field; that is, in the armature during commutation an e.m.f. is generated by its rotation through a magnetic field. This magnetic field may be the magnetic field of armature reaction, or the reverse magnetic field of a commutating pole, or the fringe of the main field of the machine, into which the brushes are shifted. In this case the commutation depends upon the inductance and the resistance of the armature coil and the e.m.f. generated therein by the main magnetic field, and if this magnetic field is a commutating field, is called *voltage commutation.*

In either case the resistance of the brushes and their contact may either be negligible, as usually the case with copper brushes, or it may be of the same or a higher magnitude than the internal resistance of the armature coil A. The latter is usually the case with carbon or graphite brushes.

In the former case the resistance of the short-circuit of armature coil A under commutation is approximately constant; in the latter case it varies from infinity in the moment of beginning commutation down to minimum, and then up again to infinity at the end of commutation.

64. (*a*.) Negligible resistance of brush and brush contact.

This is more or less approximately the case with copper brushes.

Let

i_0 = current,

L = inductance,

r = resistance of armature coil,

$t_0 = \frac{l_w}{S}$ = time of commutation,

and $-e$ = e.m.f. generated in the armature coil by its rotation through the magnetic field, where e is negative for the magnetic field of armature reaction and positive for the commutating field.

Denoting the current in the coil A at time t after beginning of commutation by i, the e.m.f. of self-inductance is

$$e_1 = -L\frac{di}{dt}.$$

Thus the total e.m.f. acting in coil A,

$$-e + e_1 = -e - L\frac{di}{dt},$$

and the current is

$$i = \frac{-e + e_1}{r} = -\frac{e}{r} - \frac{L}{r}\frac{di}{dt}.$$

Transposing, this expression becomes

$$-\frac{rdt}{L} = \frac{di}{\frac{e}{r} + i},$$

the integral of which is

$$-\frac{rt}{L} = \log_\epsilon\left(\frac{e}{r} + i\right) - \log_\epsilon c,$$

where $\log_\epsilon c$ = integration constant.

Since at $t = 0$, $i = i_0$, we have

$$\log_\epsilon c = \log\left(\frac{e}{r} + i_0\right),$$

therefore

$$c = \left(\frac{e}{r} + i_0\right), \quad \text{and} \quad i = \left(\frac{e}{r} + i_0\right)\varepsilon^{-\frac{r}{L}t} - \frac{e}{r};$$

and, at the end of commutation, or, $t = t_0$,

$$i_1 = \left(\frac{e}{r} + i_0\right)\varepsilon^{-\frac{r}{L}t_0} - \frac{e}{r}.$$

For perfect commutation,

$$i_1 = - i_0;$$

that is, the current at the end of commutation must have reversed and reached its full value in opposite direction.

Substituting in this last equation the value $i_1 = i_0$ from the preceding equation, and transforming, we have

$$\varepsilon^{-\frac{r}{L}t_0} = \frac{\frac{e}{r} - i_0}{\frac{e}{r} + i_0}$$

taking the logarithms of both terms,

$$\frac{r}{L}t_0 = \log_\epsilon \frac{\frac{e}{r} + i_0}{\frac{e}{r} - i_0};$$

and, solving the exponential equation for e, we obtain

$$e = ri_0 \frac{1 + \varepsilon^{-\frac{r}{L}t_0}}{1 - \varepsilon^{-\frac{r}{L}t_0}}.$$

It is evident that the inequality $e > i_0 r$ must be true, otherwise perfect commutation is not possible.

If

$$e = 0,$$

we have

$$i = i_0 \varepsilon^{-\frac{r}{L}t_0};$$

that is, the current never reverses, but merely dies out more or less, and in the moment when the gap G of the armature coil leaves the brush B the current therein has to rise suddenly to full intensity in opposite direction. This being impossible, due to the inductance of the coil, the current forms an arc from the brush across the commutator surface for a length of time depending upon the inductance of the armature coil.

Therefore, with low-resistance brushes, resistance commutation is not permissible except with machines of extremely low armature inductance, that is, armature inductance so low that the magnetic energy $\frac{i_0^2 L}{2}$, which appears as spark in this case, is harmless.

Voltage commutation is feasible with low-resistance brushes, but requires a commutating e.m.f. e proportional to current i_0; that is, requires shifting of brushes proportionally to the load, or a commutating pole.

In the preceding, the e.m.f. e has been assumed constant during the commutation. In reality it varies somewhat, usually increasing with the approach of the commutated coil to a denser field. It is not possible to consider this variation in general, and e is thus to be considered the average value during commutation.

65. (*b.*) High-resistance brush contact.

Fig. 105 represents a brush B commutating armature coil A.

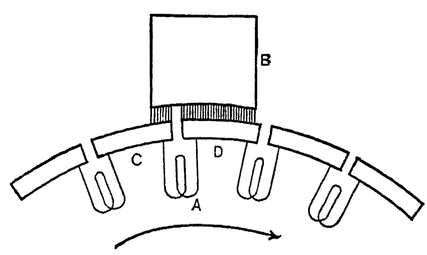


Fig. 105. Brush Commutating Coil A.

Let r_0 = contact resistance of the brush, that is, resistance from the brush to the commutator surface over the total bearing surface of the brushes. The resistance of the commutated circuit is thus internal resistance of the armature coil r plus the resistance from C to B plus the resistance from B to D.

Thus, if t_0 = time of commutation, at the time t after the beginning of the commutation, the resistance from C to B is $\frac{t_0 r_0}{t}$ and from B to D is $\frac{t_0 r_0}{t_0 - t}$; thus, the total resistance of the commutated coil is

$$R = r + \frac{t_0 r_0}{t} + \frac{t_0 r_0}{t_0 - t} = r + \frac{t_0^2 r_0}{t\,(t_0 - t)}.$$

If i_0 = current in coil A before commutation, the total current into the armature from brush B is $2\,i_0$. Thus, if i = current in commutated coil, the current from B to $D = i_0 + i$, the current from B to $C = i_0 - i$.

Hence, the difference of potential from D to C is

$$\frac{t_0 r_0}{t_0 - t}(i_0 + i) - \frac{t_0 r_0}{t}(i_0 - i).$$

The e.m.f. acting in coil A is

$$-e - \frac{Ldi}{dt},$$

and herefrom the difference of potential from D to C is

$$-e - L\frac{di}{dt} - ir;$$

hence,

$$-e - L\frac{di}{dt} - ir = \frac{t_0 r_0}{t_0 - t}(i_0 + i) - \frac{t_0 r_0}{t}(i_0 - i);$$

or, transposing,

$$\frac{Ldi}{dt} + e + ir + \frac{t_0 r_0 i_0 (2\,t - t_0)}{t\,(t_0 - t)} + \frac{t_0^2 r_0 i}{t\,(t_0 - t)} = 0.$$

$$L\frac{di}{dt} + e + i\left(r + \frac{r_0 t_0^2}{t\,(t_0 - t)}\right) + \frac{r_0 t_0 i_0\,(2\,t - t_0)}{t\,(t_0 - t)} = 0.$$

The further solution of this general problem becomes difficult, but even without integrating this differential equation a number of important conclusions can be derived.

Obviously the commutation is correct and thus sparkless, if the current entering over the brush shifts from segment to segment in direct proportion to the motion of the gap between adjacent segments across the brush, that is, if the current density is uniform all over the contact surface of the brush. This means that the current i in the short-circuited coil varies from $+\,i_0$ to $-\,i_0$ as a linear function of the time. In this case it can be represented by

$$i = i_0\frac{t_0 - 2\,t}{t_0};$$

thus,

$$\frac{di}{dt} = -\frac{2\,i_0}{t_0}.$$

Substituting this value in the general differential equation gives, after some transformation,

$$\frac{e}{i_0} t_0 + r\,(t_0 - 2\,t) - 2\,L = 0;$$

or,

$$e = i_0 \left\{ \frac{2\,L}{t_0} - r \left(1 - 2\,\frac{t}{t_0}\right) \right\},$$

which gives at the beginning of commutation, $t = 0$,

$$e_1 = i_0 \left(\frac{2\,L}{t_0} - r \right);$$

at the end of commutation, $t = t_0$,

$$e_2 = i_0 \left(\frac{2\,L}{t_0} + r \right);$$

that is, even with high-resistance brushes, for perfect commutation, voltage commutation is necessary, and the e.m.f. e impressed upon the commutated coil must increase during commutation from e_1 to e_2, by the above equation. This e.m.f. is proportional to the current i_0, but is independent of the brush resistance r_0.

Resistance Commutation.

66. Herefrom it follows that resistance commutation cannot be perfect, but that at the contact with the segment that leaves the brush the current density must be higher than the average. Let k_i = ratio of actual current density at the moment of leaving the brush to average current density of brush contact, and considering only the end of commutation, as the most important moment, we have

$$i = i_0 \frac{(2\,k_i - 1)\,t_0 - 2\,k_i t}{t_0}.$$

For $t = t_0 - t^1$ this gives

$$i = -\,i_0 + 2\,k_i \frac{t^1}{t_0}\,i_0,$$

while uniform current density would require

$$i = -\,i_0 + 2\,\frac{t^1}{t_0}\,i_0.$$

The general differential equation of resistance commutation, $e = 0$, is

$$L \frac{di}{dt} + i \left(r + \frac{r_0 t_0^2}{t (t_0 - t)} \right) + \frac{r_0 t_0 i_0 (2t - t_0)}{t (t_0 - t)} = 0.$$

Substituting in this equation the value of i from the foregoing equation, expanding and cancelling $t_0 - t$, we obtain

$$2 r_0 t_0^2 (k_i - 1) + r t t_0 (2 k_i - 1) - 2 k_i r t^2 - 2 k_i L t = 0;$$

hence,

$$k_i = \frac{t_0 (2 r_0 t_0 + r t)}{2 (r_0 t_0^2 + r t t_0 - r t^2 - L t)},$$

and for $t = t_0$,

$$k_i = \frac{t_0 (2 r_0 + r)}{2 (r_0 t_0 - L)} = 1 + \frac{L + \frac{r}{2} t_0}{r_0 t_0 - L};$$

that is, k_i is always > 1.

The smaller L and the larger r_0, the smaller is k_i; that is, the nearer it is to 1, the condition of perfect commutation, and the better is the commutation.

Sparkless commutation is impossible for very large values of k_i, that is, when L approaches $r_0 t_0$, or when r_0 is not much larger than $\frac{L}{t_0}$. For this reason, in machines in which L cannot be made small, r is sometimes made large by inserting resistors in the leads between the armature and the commutator, so-called "resistance" or "preventive" leads as used in alternating-current commutator motors.

XIV. Types of Commutating Machines.

67. By the methods of excitation, commutating machines are subdivided into magneto, separately excited, shunt, series, and compound machines. Magneto machines and separately excited machines are very similar in their characteristics. In either, the field excitation is of constant, or approximately constant, impressed m.m.f. Magneto machines, however, are little used, except for very small sizes.

By the direction of energy transformation, commutating machines are subdivided into generators and motors.

Of foremost importance in discussing the different types of machines is the saturation curve or magnetic characteristic; that is, a curve relating terminal voltage at constant speed to ampere-turns per pole field excitation, at open circuit. Such a curve is shown as A in Figs. 106 and 107. It has the same general shape as the magnetic flux density curve, except that the

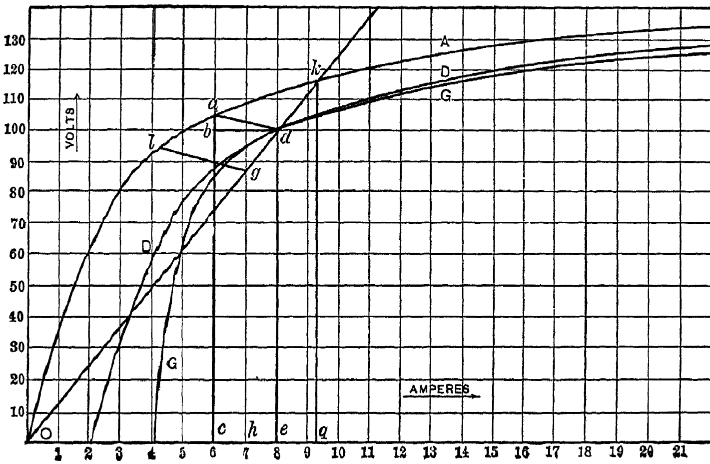


Fig. 106. Generator Saturation Curves.

knee or bend is less sharp, due to the different parts of the magnetic circuit reaching saturation successively.

Thus, in order to generate voltage ac the field excitation oc is required. Subtracting from ac in a generator, Fig. 106, or adding in a motor, Fig. 107, the value $ab = ir$, the voltage consumed by the resistance of the armature, commutator, etc., gives the terminal voltage bc at current i, and adding to oc the value $ce = bd = iq =$ armature reaction, or rather field excitation required to overcome the armature reaction, gives the field excitation oe required to produce the terminal voltage de at current i. The armature reaction iq, corresponding to current i, is calculated as discussed before, and q may be called the *coefficient of armature reaction.*

68. Such a curve, D, shown in Fig. 106 for a generator, and in Fig. 107 for a motor, and giving the terminal voltage de at current i, corresponding to the field excitation oe, is called a *load saturation curve.* Its points are respectively distant from the corresponding points of the no-load saturation curve A a constant distance equal to ad, measured parallel thereto.

Curves D are plotted under the assumption that the armature reaction is constant. Frequently, however, at lower voltage the armature reaction, or rather the increase of excitation required to overcome the armature reaction iq, increases, since with

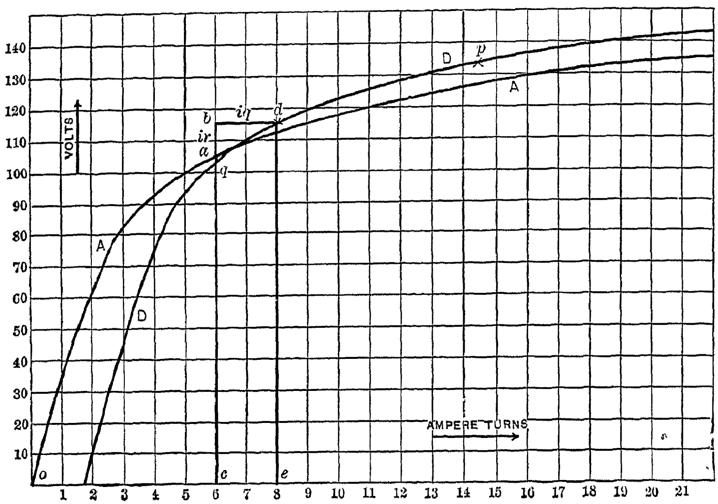


Fig. 107.—Motor Saturation Curves.

voltage commutation at lower voltage, and thus weaker field strength, the brushes have to be shifted more to secure sparkless commutation, and thus the demagnetizing effect of the angle of lead increases. At higher voltage iq usually increases also, due to increase of magnetic saturation under load, caused by the increased stray field. Thus, the load saturation curve of the continuous-current generator more or less deviates from the theoretical shape D towards a shape shown as G.

A. Generators.

Separately excited and Magneto Generator.

69. In a separately excited or magneto machine, that is, a machine with constant field excitation $\mathfrak{F}_0$, a *demagnetization curve* can be plotted from the magnetization or saturation curve A in Fig. 106. At current i, the resultant m.m.f. of the machine is $\mathfrak{F}_0 - iq$, and the generated voltage corresponds thereto by the saturation curve A in Fig. 107. Thus, in Fig. 108 a demagnetization curve A is plotted with the current $ob = i$ as abscissas and the generated e.m.f. ab as ordinates, under the

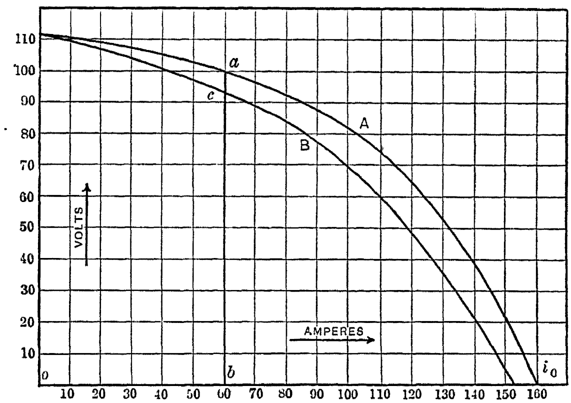


Fig. 108. Separately Excited or Magneto-generator Demagnetization Curve and Load Characteristic with Constant Shift of Brushes.

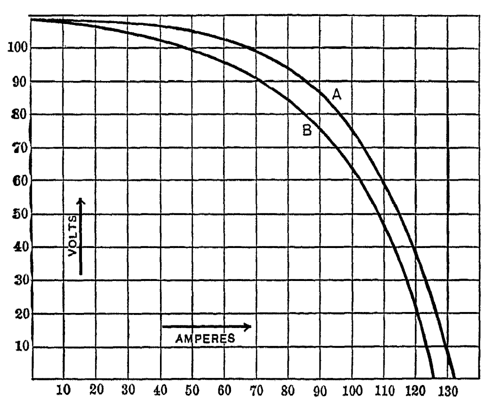


Fig. 109. Separately Excited or Magneto-generator Demagnetization Curve and Load Characteristic with Variable Shift of Brushes.

assumption of constant coefficient of armature reaction q, that is, corresponding to curve D in Fig. 106. This curve becomes zero at the current i_0, which makes $i_0q = \mathfrak{F}_0$. Subtracting from curve A in Fig. 39 the drop of voltage in the armature and commutator resistance, $ac = ir$, gives the external characteristic

B of the machine as generator, or the curve relating the terminal voltage to the current.

In Fig. 109 the same curves are shown under the assumption that the armature reaction varies with the voltage in the way as represented by curve *G* in Fig. 106.

In a separately excited or magneto motor at constant speed the external characteristic would lie as much above the demagnetization curve *A* as it lies below in a generator in Fig. 108, and at constant voltage the speed would vary inversely proportional hereto.

Shunt Generator.

70. The external or load characteristic of the shunt generator is plotted in Fig. 110 with the current as abscissas and the terminal voltage as ordinates, as *A* for constant coefficient of

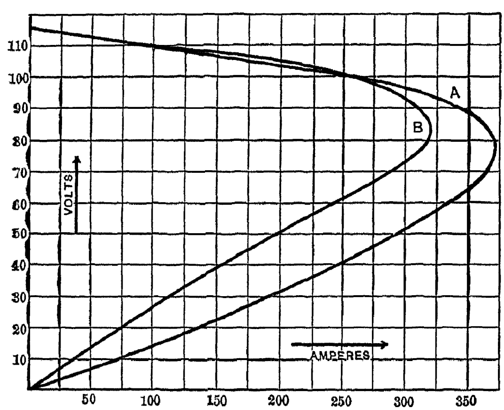


Fig. 110.—Shunt Generator Load Characteristic.

armature reaction, and as *B* for a coefficient of armature reaction varying with the voltage in the way as shown in *G*, Fig. 106. The construction of these curves is as follows:

In Fig. 106, *og* is the straight line giving the field excitation *oh* as function of the terminal voltage *hg* (the former obviously being proportional to the latter in the shunt machine). The open-circuit or no-load voltage of the machine is then *kq*.

Drawing *gl* parallel to *da* (assuming constant coefficient of armature reaction, or parallel to the hypothenuse of the triangle *iq*, *ir* at voltage *og*, when assuming variable armature reaction), then the current which gives voltage gh is proportional to gl, that is, $i : i_0 = gl : da$, where i_0 is the current at the voltage de.

As seen from Fig. 110, a maximum value of current exists which is less if the brushes are shifted than at constant position of brushes.

From the load characteristic of the shunt generator the resistance characteristic is plotted in Fig. 111; that is, the dependence of the terminal voltage upon the external resistance $R = \frac{\text{terminal voltage}}{\text{current}}$. Curve A in Fig. 111 corresponds to constant, curve B to varying armature reaction. It is seen that at a certain definite resistance the voltage becomes zero, and for lower resistance the machine cannot generate but loses its excitation.

The variation of the terminal voltage of the shunt generator with the speed at constant field resistance is shown in Fig. 112, at no load as A, and at constant current i as B. These curves are derived from the preceding ones. They show that below a certain speed, which is much higher at load than at no load, the machine cannot generate. The lower part of curve B is unstable and cannot be realized.

Series Generator.

71. In the series generator the field excitation is proportional to the current i, and the saturation curve A in Fig. 113 can thus be plotted with the current i as abscissas. Subtracting $ab = ir$, the resistance drop, from the voltage, and adding $bd = iq$, the armature reaction, gives a load saturation curve or external characteristic B of the series generator. The terminal voltage is zero at no load or open circuit, increases with the load, reaches a maximum value at a certain current, and then decreases again and reaches zero at a certain maximum current, the current of short-circuit.

Curve B is plotted with constant coefficient of armature reaction q. Assuming the brushes to be shifted with the load and proportionally to the load, gives curves C, D, and E, which are higher at light load, but fall off faster at high load. A still further shift of brushes near the maximum current value even overturns the curve as shown in F. Curves E and F correspond

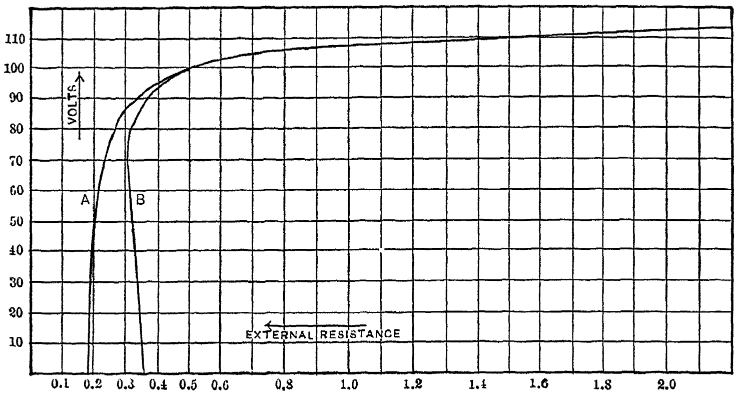


Fig. 111. Shunt Generator Resistance Characteristic.

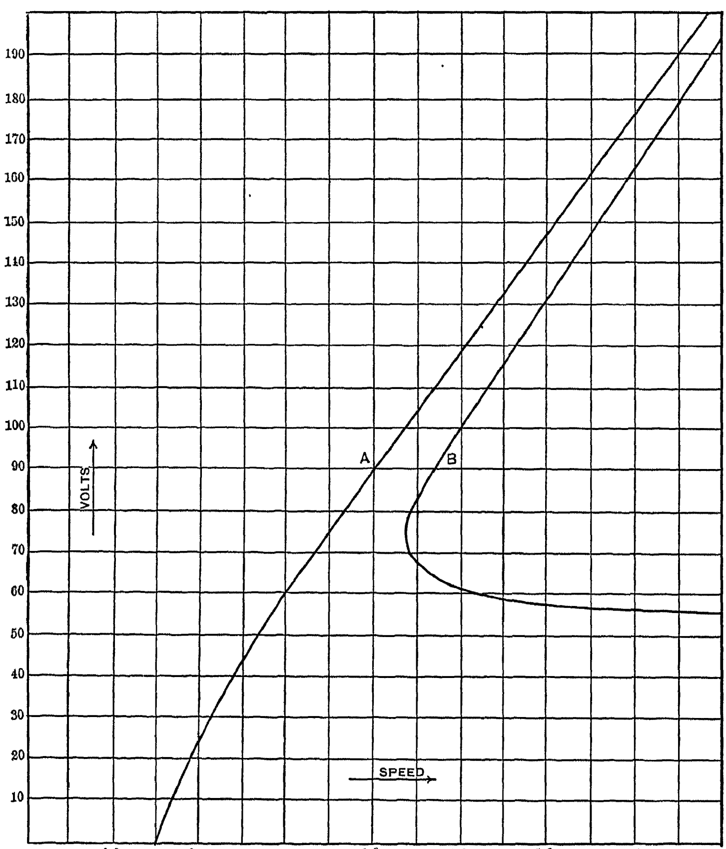


Fig. 112. Shunt Generator Speed Characteristic at Constant Field Circuit Resistance.

to a very great shift of brushes, and an armature demagnetizing effect of the same magnitude as the field excitation, as realized in arc-light machines, in which the last part of the curve is used to secure inherent regulation for constant current.

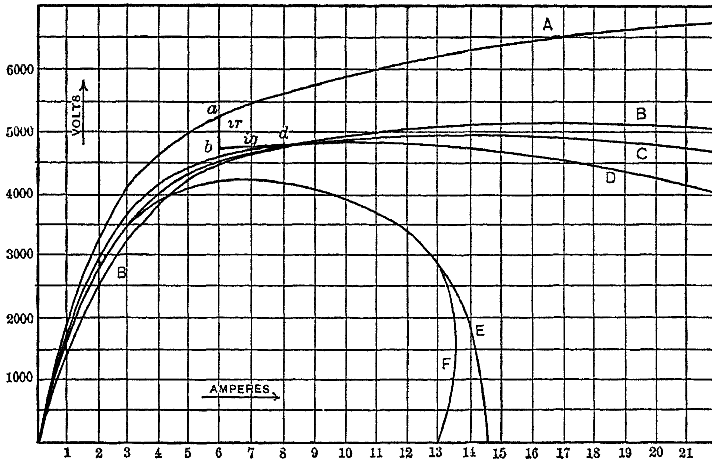


Fig. 113. Series Generator Saturation Curve and Load Characteristic.

The resistance characteristic, that is the dependence of the current and of the terminal voltage of the series generator upon the external resistance, are constructed from Fig. 113 and plotted in Fig. 114.

B_1 and B_2 in Fig. 114 are terminal volts and amperes corresponding to curve B in Fig. 113, E_1, E_2, and F_2 volts and amperes corresponding to curves E and F in Fig. 113.

Above a certain external resistance the series generator loses its excitation, while the shunt generator loses its excitation below a certain external resistance.

Compound Generator.

72. The saturation curve or magnetic characteristic A, and the load saturation curves D and G of the compound generator, are shown in Fig. 115 with the ampere-turns of the shunt field as abscissas. A is the same curve as in Fig. 106, while D and G in Fig. 115 are the corresponding curves of Fig. 106 shifted to the left by the distance iq_0, the m.m.f. of ampere-turns of the series field.

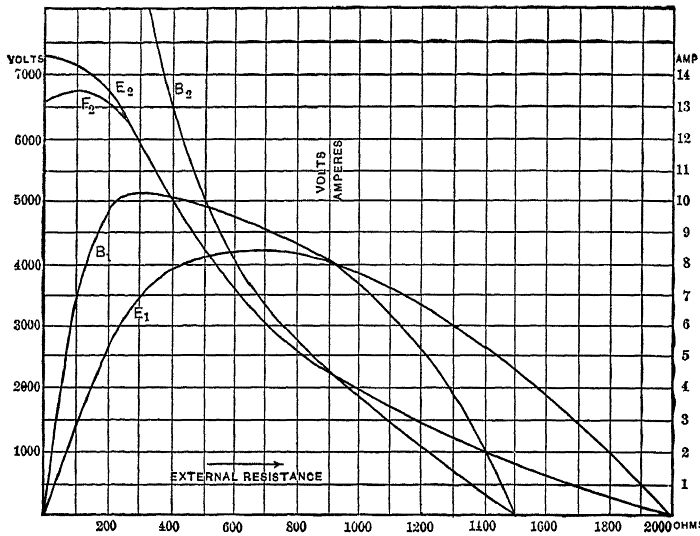


Fig. 114. Series Generator Resistance Characteristic.

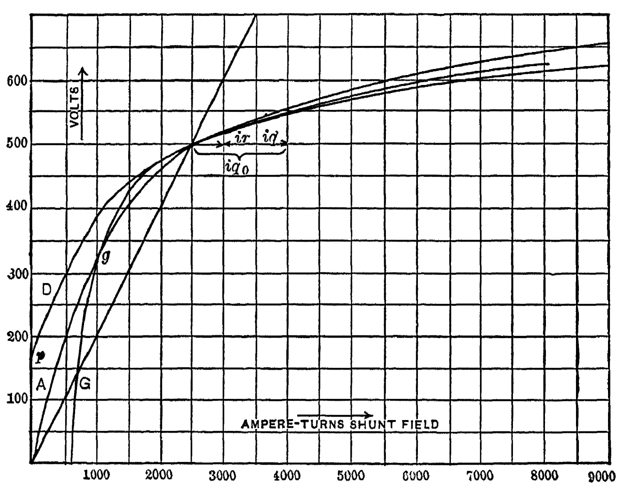


Fig. 115. Compound Generator Saturation Curve.

At constant position of brushes the compound generator, when adjusted for the same voltage at no load and at full load, under-compounds at higher and over-compounds at lower voltage, and even at open circuit of the shunt field gives still a voltage *op* as series generator. When shifting the brushes under load, at lower voltage a second point g is reached where the machine compounds correctly, and below this point the machine under-compounds and loses its excitation when the shunt field decreases below a certain value; that is, it does not excite itself as series generator.

B. Motors.

Shunt Motor.

73. Three speed characteristics of the shunt motor at constant impressed e.m.f. e are shown in Fig. 116 as A, P, Q, corresponding to the points d, p, q of the motor load saturation curve, Fig. 107. Their derivation is as follows: At constant impressed

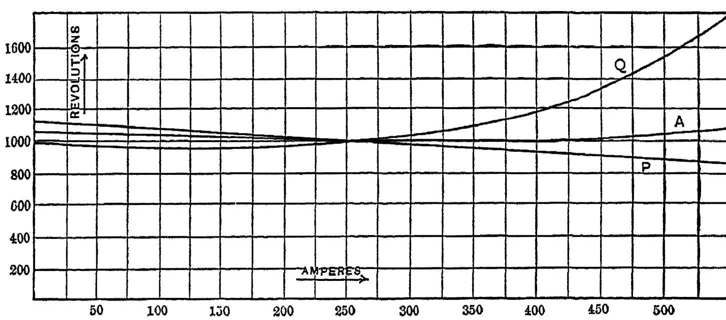


Fig. 116. Shunt Motor Speed Curves, Constant Impressed e.m.f.

e.m.f. e the field excitation is constant and equals $\mathfrak{F}_0$, and at current i the generated e.m.f. must be $e - ir$. The resultant field excitation is $\mathfrak{F}_0 - iq$, and corresponding hereto at constant speed the generated e.m.f. taken from saturation curve A in Fig. 107 is e_1. Since it must be $e - ir$, the speed is changed in the proportion $\frac{e - ir}{e_1}$.

At a certain voltage the speed is very nearly constant, the demagnetizing effect of armature reaction counteracting the effect of armature resistance. At higher voltage the speed falls, at lower voltage it rises with increasing current.

In Fig. 117 is shown the speed characteristic of the shunt motor as function of the impressed voltage at constant output, that is, constant product, current times generated e.m.f. If i = current and P = constant output, the generated e.m.f.

must be approximately $e_1 = \frac{P}{i}$, and thus the terminal voltage $e = e_1 + ir$. Proportional hereto is the field excitation $\mathfrak{F}_0$. The resultant m.m.f. of the field is thus $\mathfrak{F} = \mathfrak{F}_0 - iq$, and corresponding thereto from curve A in Fig. 108 is derived the e.m.f. e_0 which would be generated at constant speed by the m.m.f. $\mathfrak{F}$.

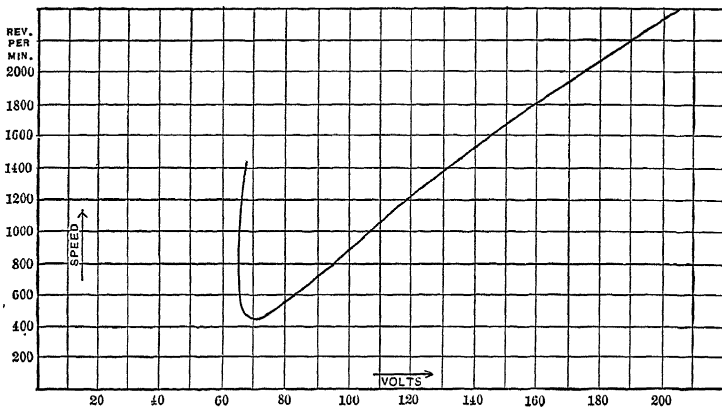


Fig. 117. Shunt Motor Speed Curve, Variable Impressed e.m.f.

Since, however, the generated e.m.f. must be e_1, the speed is changed in the proportion $\frac{e_1}{e_0}$.

The speed rises with increasing and falls with decreasing impressed e.m.f. Still further decreasing the impressed e.m.f., the speed reaches a minimum and then increases again, but the conditions become unstable.

Series Motor.

74. The speed characteristic of the series motor is shown in Fig. 118 at constant impressed e.m.f. e. A is the saturation curve of the series machine, with the current as abscissas and at constant speed. At current i, the generated e.m.f. must be $e - ir$, and the speed is thus $\frac{e - ir}{e_1}$ times that, for which curve A is plotted, where e_1 = e.m.f. taken from saturation curve A. This speed curve corresponds to a constant position of brushes midway between the field poles, as generally used in railway motors and other series motors. If the brushes have a constant

shift or are shifted proportionally to the load, instead of the saturation curve A in Fig. 118 a curve is to be used corresponding to the position of brushes, that is, derived by adding to the

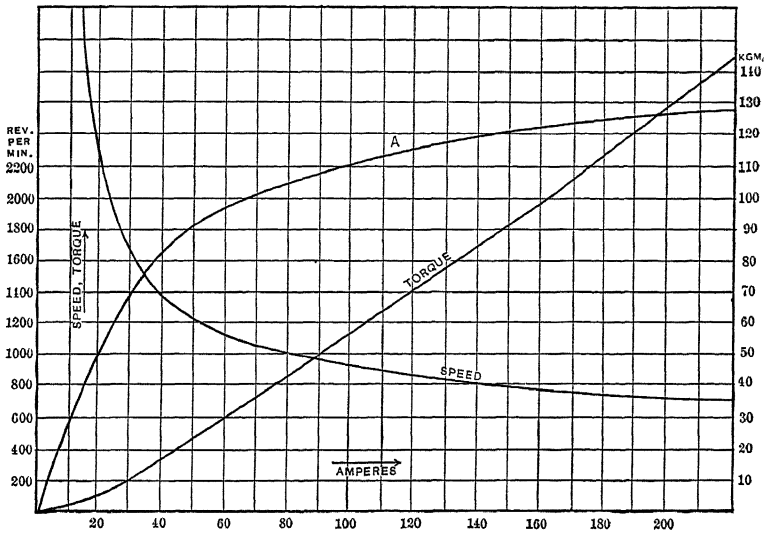


Fig. 118. Series Motor Speed Curve.

abscissas of A the values iq, the demagnetizing effect of armature reaction.

The torque of the series motor is shown also in Fig. 118, derived as proportional to $A \times i$, that is, current $\times$ magnetic flux.

Compound Motors.

75. Compound motors can be built with cumulative compounding and with differential compounding.

Cumulative compounding is used to a considerable extent, as in elevator motors, etc., to secure economy of current in starting and at high loads at the sacrifice of speed regulation; that is, a compound motor with cumulative series field stands in its speed and torque characteristic intermediate between the shunt motor and the series motor.

Differential compounding is used to secure constancy of speed with varying load, but to a small extent only, since the speed regulation of a shunt motor can be made sufficiently close, as was shown in the preceding.

Conclusion.

76. The preceding discussion of commutating machine types can obviously be only very general, showing the main characteristics of the curves, while the individual curves can be modified to a considerable extent by suitable design of the different parts of the machine when required to derive certain results, as, for instance, to extend the constant-current part of the series generator; or to derive a wide range of voltage at stability, that is, beyond the bend of the saturation curve in the shunt generator; or to utilize the range of the shunt generator load characteristic at the maximum current point for constant-current regulation; or to secure constancy of speed in a shunt motor at varying impressed e.m.f., etc.

The use of the commutating machine as direct-current converter has been omitted from the preceding discussion. By means of one or more alternating-current compensators or auto-transformers, connected to the armature by collector rings, the commutating machine can be used to double or halve the voltage, or convert from one side of a three-wire system to the other side and, in general, to supply a three-wire Edison system from a single generator. Since, however, the direct-current converter and three-wire generator exhibit many features similar to those of the synchronous converter, as regards the absence of armature reaction, the reduced armature heating, etc., they will be discussed as an appendix to the synchronous converter

C. ALTERNATING-CURRENT COMMUTATING MACHINES.

I. General.

Alternating-current commutating machines have so far become of industrial importance mainly as motors of the series or varying-speed type, for single-phase railroading, and to some extent also as constant-speed motors for cases as elevators or hoists, where efficient acceleration under heavy torque is necessary. As generators, they would be of advantage for the generation of very low frequency, since in this case synchronous machines are uneconomical, due to their very low speed, resultant from the low frequency.

The direction of rotation of a direct-current motor, whether shunt or series motor, remains the same at a reversal of the impressed e.m.f., as in this case the current in the armature circuit and the current in the field circuit and so the field magnetism both reverse. Theoretically, a direct-current motor therefore could be operated on an alternating impressed e.m.f. provided that the magnetic circuit of the motor is laminated, so as to follow the alternations of magnetism without serious loss of power, and that precautions are taken to have the field reverse simultaneously with the armature. If the reversal of field magnetism should occur later than the reversal of armature current, during the time after the armature current has reversed, but before the field has reversed, the motor torque would be in opposite direction and thus subtract; that is, the field magnetism of the alternating-current motor must be in phase with the armature current, or nearly so. This is inherently the case with the series type of motor, in which the same current traverses field coils and armature windings.

Since in the alternating-current transformer the primary and secondary currents and the primary voltage and the secondary voltage are proportional to each other, the different circuits of the alternating-current commutator motor may be connected with each other directly (in shunt or in series, according to the type of the motor) or inductively, with the interposition of a transformer, and for this purpose either a separate transformer may be used or the transformer feature embodied in the motor, as in the so-called repulsion type of motors. This gives to the alternating-current commutator motor a far greater variety of connections than possessed by the direct-current motor.

While in its general principle of operation the alternating-current commutator motor is identical with the direct-current motor, in the relative proportioning of the parts a great difference exists. In the direct-current motor, voltage is consumed by the counter e.m.f. of rotation, which represents the power output of the motor, and by the resistance, which represents the power loss. In addition thereto, in the alternating-current motor voltage is consumed by the inductance, which is wattless or reactive and therefore causes a lag of current behind the voltage, that is, a lowering of the power-factor. While in the direct-current motor good design requires the combination of a strong field and a relatively weak armature, so as to reduce the armature reaction on the field to a minimum, in the design of the alternating-current motor considerations of power-factor predominate; that is, to secure low self-inductance and therewith a high power-factor, the combination of a strong armature and a weak field is required, and necessitates the use of methods to eliminate the harmful effects of high armature reaction.

As so far only the varying-speed single-phase commutator motor has found an extensive use as railway motor, this type of motor will mainly be treated in the following, and the other types discussed in the concluding paragraphs.

II. Power-Factor.

In the commutating machine the magnetic field flux generates the e.m.f. in the revolving armature conductors, which gives the motor output; the armature reaction, that is, the magnetic flux produced by the armature current, distorts and weakens the field, and requires a shifting of the brushes to avoid sparking due to the short-circuit current under the commutator brushes, and where the brushes cannot be shifted, as in a reversible motor, this necessitates the use of a strong field and weak armature to keep down the magnetic flux at the brushes. In the alternating-current motor the magnetic field flux generates in the armature conductors by their rotation the e.m.f. which does the work of the motor, but, as the field flux is alternating, it also generates in the field conductors an e.m.f. of self-inductance, which is not useful but wattless, and therefore harmful in lowering the power-factor, hence must be kept as low as possible.

This e.m.f. of self-inductance of the field, e_0, is proportional to the field strength Φ, to the number of field turns n_0, and to the frequency f of the impressed e.m.f.

$$e_0 = 2\,\pi f n_0 \Phi\, 10^{-8}, \tag{1}$$

while the useful e.m.f. generated by the field in the armature conductors, or "e.m.f. of rotation," e, is proportional to the field strength Φ, to the number of armature turns n_1, and to the frequency of rotation of the armature, f_0,

$$e = 2\,\pi f_0 n_1 \Phi\, 10^{-8}. \tag{2}$$

This later e.m.f., e, is in phase with the magnetic flux Φ, and so with the current i, in the series motor, that is, is a power e.m.f., while the e.m.f. of self-inductance, e_0, is wattless, or in quadrature with the current, and the angle of lag of the motor current thus is given by

$$\tan\theta = \frac{e_0}{e + ir}, \tag{3}$$

where ir = voltage consumed by the motor resistance. Or approximately, since ir is small compared with e (except at very low speed),

$$\tan\theta = \frac{e_0}{e}, \tag{4}$$

and, substituting herein (1) and (2),

$$\tan\theta = \frac{f}{f_0}\frac{n_0}{n_1}. \tag{5}$$

Small angle of lag and therewith good power-factor therefore require high values of f_0 and n_1 and low values of f and n_0.

High f_0 requires high motor speeds and as large number of poles as possible. Low f means low impressed frequency; therefore 25 cycles is generally the highest frequency considered for large commutating motors.

High n_1 and low n_0 means high armature reaction and low field excitation, that is, just the opposite conditions from that required for good commutator motor design.

Assuming synchronism, $f_0 = f$, as average motor speed,—750 revolutions with a 4-pole 25-cycle motor,—an armature reaction n_1 equal to the field excitation n_0 would then give $\tan \theta = 1$ $\theta = 45$ deg., or 70.7 per cent power-factor; that is, with an armature reaction beyond the limits of good motor design, the power-factor is still too low for use.

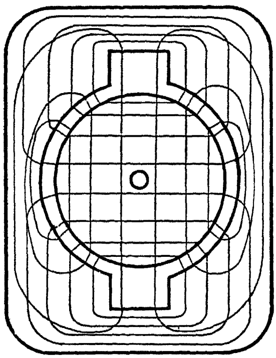


Fig. 119. Distribution of Main Field and Field of Armature Reaction.

The armature, however, also has a self-inductance; that is, the magnetic flux produced by the armature current as shown diagrammatically in Fig. 119 generates a reactive e.m.f. in the armature conductors, which again lowers the power-factor. While this armature self-inductance is low with small number of armature turns, it becomes considerable when the number of armature turns n_1 is large compared with the field turns n_0.

Let $\mathcal{R}_0$ = field reluctance, that is, reluctance of the magnetic field circuit, and $\mathcal{R}_1 = \frac{\mathcal{R}_0}{b}$ = the armature reluctance, that is, $b = \frac{\mathcal{R}_0}{\mathcal{R}_1}$ = ratio of reluctances of the armature and the field magnetic circuit; then, neglecting magnetic saturation, the field flux is

$$\Phi = \frac{n_0 i}{\mathcal{R}_0},$$

the armature flux is

$$\Phi_1 = \frac{n_1 i}{\mathcal{R}_1} = \frac{n_1 b i}{\mathcal{R}_0} = \frac{n_1}{n_0} b \Phi, \tag{6}$$

and the e.m.f. of self-inductance of the armature circuit is

$$e_1 = 2\pi f n_1 \Phi_1 10^{-8}$$
$$= 2\pi f \frac{n_1^2}{n_0} b \Phi 10^{-8}; \tag{7}$$

hence, the total e.m.f. of self-inductance of the motor, or wattless e.m.f., by (1) and (7) is

$$e_0 + e_1 = 2\pi f \Phi 10^{-8} \left(\frac{n_0^2 + b n_1^2}{n_0}\right), \tag{8}$$

and the angle of lag θ is given by

$$\tan\theta = \frac{e_0 + e_1}{e}$$

$$= \frac{f}{f_0}\frac{n_0^2 + bn_1^2}{n_0 n_1}; \tag{9}$$

or, denoting the ratio of armature turns to field turns by

$$q = \frac{n_1}{n_0},$$

$$\tan\theta = \frac{f}{f_0}\left(\frac{1}{q} + bq\right), \tag{10}$$

and this is a minimum; that is, the power-factor a maximum, for

$$\frac{d}{dq}\{\tan\theta\} = 0,$$

or,

$$q_0 = \frac{1}{\sqrt{b}}, \tag{11}$$

and the maximum power-factor of the motor is then given by

$$\tan\theta_0 = \frac{f}{f_0}\frac{2}{\sqrt{b}}. \tag{12}$$

Therefore the greater b is the higher the power-factor that can be reached by proportioning field and armature so that $\frac{n_1}{n_0} = q_0 = \frac{1}{\sqrt{b}}$.

Since b is the ratio of armature reluctance to field reluctance, good power-factor thus requires as high an armature reluctance and as low a field reluctance as possible; that is, as good a magnetic field circuit and poor magnetic armature circuit as feasible. This leads to the use of the smallest air gaps between field and armature which are mechanically permissible. With an air gap of 0.10 to 0.15 in. as the smallest safe value in railway work, b cannot well be made larger than about 4.

Assuming, then, $b = 4$, gives $q = 2$, that is, twice as many armature turns as field turns; $n_1 = 2\,n_0$.

The angle of lag in this case is, by (12), at synchronism: $f_0 = f$,

$$\tan\theta_0 = 1,$$

giving a power-factor of 70.7 per cent.

It follows herefrom that it is not possible, with a mechanically safe construction, at 25 cycles to get a good power-factor at moderate speed, from a straight series motor, even if such a design as discussed above were not inoperative, due to excessive distortion and therefore destructive sparking.

Thus it becomes necessary in the single-phase commutator motor to reduce the magnetic flux of armature reaction, that is, increase the effective magnetic reluctance of the armature far beyond the value of the true magnetic reluctance. This is accomplished by the compensating winding devised by Eickemeyer, by surrounding the armature with a stationary winding closely adjacent and parallel to the armature winding, and energized by a current in opposite direction to the armature current, and of the same m.m.f., that is, the same number of ampere-turns, as the armature winding.

Every single-phase commutator motor thus comprises a field winding F, an armature winding A, and a compensating winding C, usually located in the pole faces of the field, as shown in Figs. 120 and 121.

The compensating winding C is either connected in series (but in reversed direction) with the armature winding, and then has

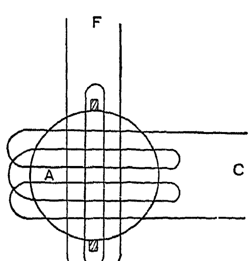


Fig. 120. Circuits of Single-Phase Commutator Motor.

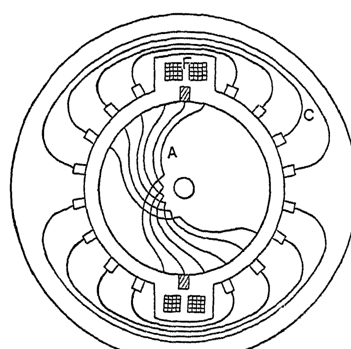


Fig. 121. Massed Field Winding and Distributed Compensating Winding.

the same number of effective turns, or it is short-circuited upon itself, thus acting as a short-circuited secondary with the armature winding as primary, or the compensating winding is energized by the supply current, and the armature short-circuited as secondary. The first case gives the conductively compensated series motor, the second case the inductively compensated series motor, the third case the repulsion motor.

In the first case, by giving the compensating winding more turns than the armature, over-compensation, by giving it less turns, under-compensation, is produced. In the other two cases always complete (or practically complete) compensation results, irrespective of the number of turns of the winding, as primary and secondary currents of a transformer always are opposite in direction, and of the same m.m.f. (approximately).

With a compensating winding C of equal and opposite m.m.f. to the armature winding A, the resultant armature reaction is zero, and the field distortion, therefore, disappears; that is, the ratio of the armature turns to field turns has no direct effect on the commutation, but high armature turns and low field turns can be used. The armature self-inductance is reduced from that corresponding to the armature magnetic flux Φ_1 in Fig. 119 to that corresponding to the magnetic leakage flux, that is, the magnetic flux passing between armature turns and compensating turns, or the "slot inductance," which is small, especially if relatively shallow armature slots and compensating slots are used.

The compensating winding, or the "cross field," thus fulfils the twofold purpose of reducing the armature self-inductance to that of the leakage flux, and of neutralizing the armature reaction and thereby permitting the use of very high armature ampere-turns. Theoretically, that is, if there were no leakage flux, both purposes would be accomplished by the same number of ampere-turns of the compensating winding, equal to the ampere-turns of the armature proper. Practically, this is approximately the case even where the magnetic leakage flux exists, except in the case of motors of very small pole pitch and small number of armature and compensating slots per pole, as is the case with high-frequency commutator motors. In the latter case, minimum self-inductance occurs at a number of compensating ampere-turns lower than the number of armature ampere-turns, that is, at under-compensation.

The main purpose of the compensating winding thus is to decrease the armature self-inductance; that is, increase the effective armature reluctance and thereby its ratio to the field reluctance, b, and thus permit the use of a much higher ratio, $q = \frac{n_1}{n_0}$, before maximum power-factor is reached, and thereby a higher power-factor.

Even with compensating winding, with increasing q, ultimately a point is reached where the armature self-inductance equals the field self-inductance, and beyond this the power-factor again decreases. It becomes possible, however, by the use of the compensating winding, to reach, with a mechanically good design, values of b as high as 16 to 20.

Assuming $b = 16$ gives, substituted in (11) and (12),

$$q = 4;$$

that is, four times as many armature turns as field turns, $n_1 = 4\,n_0$ and

$$\tan \theta_0 = \frac{f}{2 f_0};$$

hence, at synchronism,

$$f_0 = f : \tan \theta_0 = 0.5, \text{ or } 89 \text{ per cent power-factor.}$$

At double synchronism, which about represents maximum motor speed at 25 cycles,

$$f_0 = 2f : \tan \theta_0 = 0.25, \text{ or } 98 \text{ per cent power-factor;}$$

that is, very good power-factors can be reached in the single-phase commutator motor by the use of a compensating winding, far higher than are possible with the same air gap in polyphase induction motors.

III. Field Winding and Compensating Winding.

The purpose of the field winding is to produce the maximum magnetic flux Φ with the minimum number of turns n_0. This requires as large a magnetic section, especially at the air gap, as possible. Hence, a massed field winding with definite polar projections of as great pole arc as feasible, as shown in Fig. 121, gives a better power-factor than a distributed field winding.

The compensating winding must be as closely adjacent to the armature winding as possible, so as to give minimum leakage flux between armature conductors and compensating conductors, and therefore is a distributed winding, located in the field pole faces, as shown in Fig. 121.

The armature winding is distributed over the whole circumference of the armature, but the compensating winding only in the field pole faces. With the same ampere-turns in armature and compensating winding, their resultant ampere-turns are equal and opposite, and therefore neutralize, but locally the two windings do not neutralize, due to the difference in the distribution curves of their m.m.fs. The m.m.f. of the field winding is constant over the pole faces, and from one pole corner to the next pole corner reverses in direction, as shown diagrammatically by F in Fig. 122, which is the development of Fig. 121. The m.m.f. of the armature is a maximum at the brushes, mid-

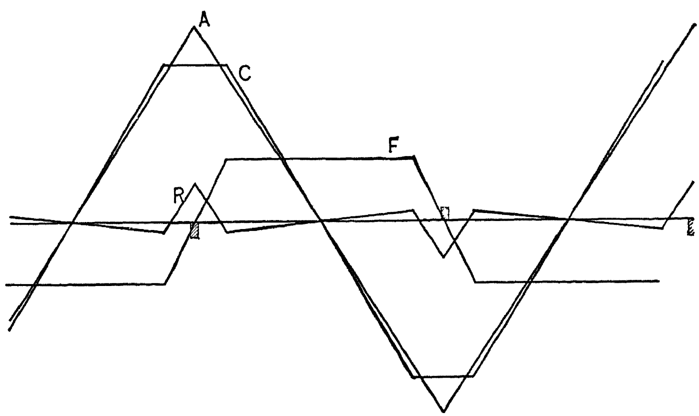


Fig. 122. Distribution of m.m.f. in Compensated Motor.

way between the field poles, as shown by A in Fig. 122, and from there decreases to zero in the center of the field pole. The m.m.f. of the compensating winding, however, is constant in the space from pole corner to pole corner, as shown by C in Fig. 122, and since the total m.m.f. of the compensating winding equals that of the armature, the armature m.m.f. is higher at the brushes, the compensating m.m.f. higher in front of the field poles, as shown by curve R in Fig. 122, which is the difference between A and C; that is, with complete compensation of the resultant armature and compensating winding, locally under-compensation exists at the brushes, over-compensation in front of the field poles. The local under-compensated armature reaction at the brushes generates an e.m.f. in the coil short-circuited under the brush, and therewith a short-circuit current of commutation and sparking. In the conductively compensated motor, this can be avoided by over-compensation, that is, raising the flat top of the compensating m.m.f. to the maximum armature m.m.f., but this results in a lowering of the power-factor, due to the self-inductive flux of over-compensation, and therefore is undesirable.

To get complete compensation even locally requires the compensating winding to give the same distribution curve as the armature winding, or inversely. The former is accomplished by distributing the compensating winding around the entire circumference of the armature, as shown in Fig. 123. This, however, results in bringing the field coils further away from the armature surface, and so increases the magnetic stray flux of the field winding, that is, the magnetic flux, which passes through the field coils, and there produces a reactive voltage of self-inductance, but does not pass through the armature conductors, and so does not work; that is, it lowers the power-factor, just as over-compensation would do. The distribution curve of the armature winding can, however, be made equal to that of the compensating winding, and therewith local complete compensation secured, by using a fractional pitch armature winding of a pitch equal to the pole arc. In this case, in the space between the pole corners, the currents are in opposite direction in the upper and the lower layer of conductors in each armature slot, as shown in Fig. 124, and thus neutralize magnetically; that is, the armature reaction extends only over the space of the armature circumference covered by the pole arc, where it is neutralized by the compensating winding in the pole face.

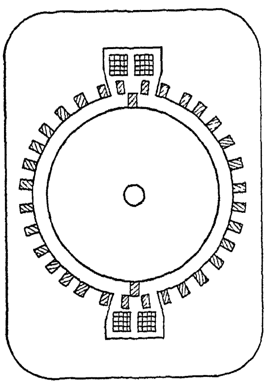


Fig. 123. Completely Distributed Compensating Winding.

To produce complete compensation even locally, without impairing the power-factor, therefore, requires a fractional-pitch armature winding, of a pitch equal to the field pole arc, or some equivalent arrangement.

Historically, the first compensated single-phase commutator motors, built about twenty years ago, were Prof. Elihu Thomson's repulsion motors. In these the field winding and compensating winding were massed together in a single coil, as shown diagrammatically in Fig. 125. Repulsion motors are still occasionally built in which field and compensating coils are combined in a single distributed winding, as shown in Fig. 126. Soon after the first repulsion motor, conductively and induc-

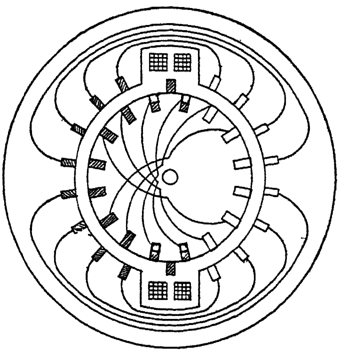


Fig. 124. Fractional Pitch Armature Winding.

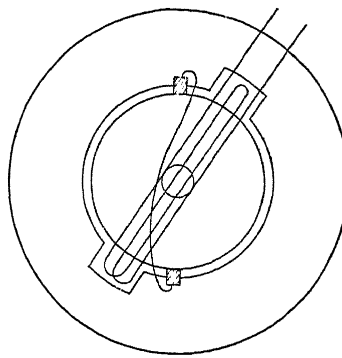


Fig. 125. Repulsion Motor with Massed Winding.

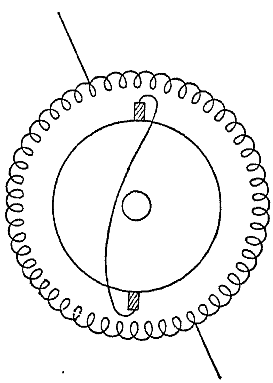


Fig. 126. Repulsion Motor with Distributed Winding.

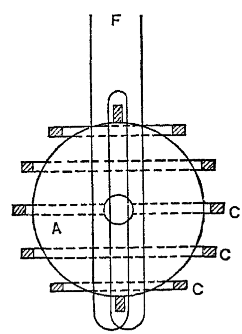


Fig. 127. Eickemeyer Inductively Compensated Series Motor.

tively compensated series motors were built by Eickemeyer, with a massed field winding and a separate compensating winding, or cross coil, either as single coil or turn or distributed in a number of coils or turns, as shown diagrammatically in Fig. 127, and by W. Stanley.

For railway motors, separate field coils and compensating coils are always used, the former as massed, the latter as distributed winding, since in reversing the direction of rotation either the field winding alone must be reversed or armature and compensating winding are reversed while the field winding remains unchanged.

IV. Types of Varying-Speed Single-Phase Commutator Motors.

The armature and compensating windings are in inductive relations to each other. In the single-phase commutator motor with series characteristic, armature and compensating windings therefore can be connected in series with each other, or the supply voltage impressed upon the one, the other closed upon itself as secondary circuit, or a part of the supply voltage impressed upon the one, and another part upon the other circuit, and in either of these cases the field winding may be connected in series either to the compensating winding or to the armature winding. This gives the motor types, denoting the armature by A, the compensating winding by C, and the field winding by F.

	Primary.	Secondary.	
1.	$A+C+F$	...	Conductively compensated series motor. Fig. 128.
2.	$A+F$	C	Inductively compensated series motor. Fig. 129.
3.	A	$C+F$	Inductively compensated series motor with secondary excitation, or inverted repulsion motor. Fig. 130.
4.	$C+F$	A	Repulsion motor. Fig. 131.
5.	C	$A+F$	Repulsion motor with secondary excitation. Fig. 132.
6.	$A+F, C$	...	Series repulsion motors. Figs. 133, 134.
7.	$A, C+F$	...	

Since in all these motor types all three circuits are connected directly or inductively in series with each other, they all have the same general characteristics as the direct-current series motor; that is, a speed which increases with a decrease of load, and a torque per ampere input which increases with increase

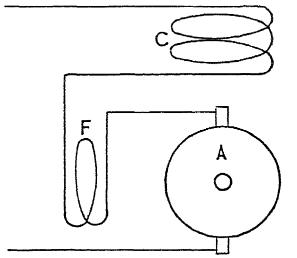


Fig. 128.

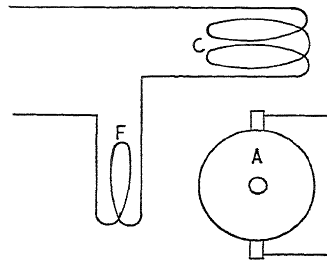


Fig 131.

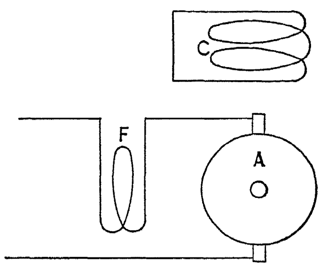


Fig. 129.

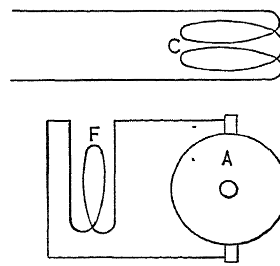


Fig. 132.

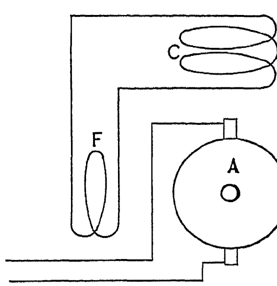


Fig. 130.

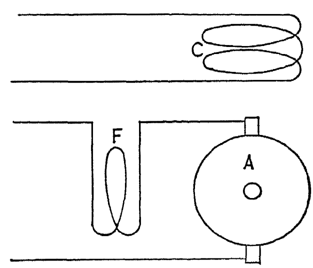


Fig. 133.

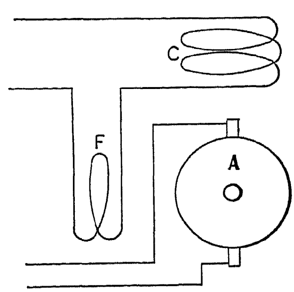


Fig. 134.

Figs. 128–134. Types of Alternating Current Commutating Motors.

of current, and therefore with decrease of speed, and the different motor types differ from each other only by their commutation as affected by the presence or absence of a magnetic flux at the brushes, and indirectly thereby in their efficiency as affected by commutation losses.

In the conductively compensated series motor, by the choice of the ratio of armature and compensating turns, over-compensation, complete compensation, or under-compensation can be produced. In all the other types, armature and compensating windings are in inductive relation, and the compensation therefore approximately complete.

A second series of motors of the same varying speed characteristics results by replacing the stationary field coils by armature excitation, that is, introducing the current, either directly or by transformer, into the armature by means of a second set of brushes at right angles to the main brushes. Such motors are used to some extent abroad. They have the disadvantage of requiring two sets of brushes, but the advantage that their power-factor can be controlled and above synchronism even leading current produced. Fig. 135 shows diagrammatically such a motor, as designed by Winter-Eichberg-Latour, the so-called compensated repulsion motor. In this case compensated means compensated for power-factor.

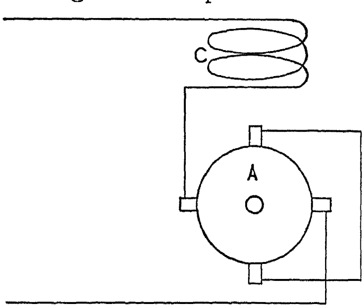


Fig. 135. Type of Alternating Current Commutating Motor.

The voltage which can be used in the motor armature is limited by the commutator: the voltage per commutator segment is limited by the problem of sparkless commutation, the number of commutator segments from brush to brush is limited by mechanical consideration of commutator speed and width of segments. In those motor types in which the supply current traverses the armature, the supply voltage is thus limited to values even lower than in the direct-current motor, while in the repulsion motor (4 and 5), in which the armature is the secondary circuit, the armature voltage is independent of the supply voltage, so can be chosen to suit the requirements of commutation, while the motor can be built for any supply voltage for which the stator can economically be insulated.

Alternating-current motors as well as direct-current series motors can be controlled by series parallel connection of two or more motors. Further control, as in starting, with direct-current motors is carried out by rheostat, while with alternating-current motors potential control, that is, a change of supply voltage by transformer or auto-transformer, offers a more efficient method of control. By changing from one motor type to another motor type, potential control can be used in alternating-current motors without any change of supply voltage, by appropriately choosing the ratio of turns of primary and secondary circuit. For instance, with an armature wound for half the voltage and thus twice the current as the compensating winding $\left(\text{ratio of turns } \frac{n_2}{n_1} = 2\right)$, a change of connection from type 3 to type 2, or from type 5 to type 4, results in doubling the field current and therewith the field strength. A change of distribution of voltage between the two circuits, in types 6 and 7, with A and C wound for different voltages, gives the same effect as a change of supply voltage, and therefore is used for motor control.

In those motor types in which a transformation of power occurs between compensating winding C and armature winding A, a transformer flux exists in the direction of the brushes, that is, at right angles to the field flux. In general, therefore, the single-phase commutator motor contains two magnetic fluxes in quadrature position with each other, the main flux or field flux Φ, in the direction of the axis of the field coils, or at right angles to the armature brushes, and the quadrature flux, or transformer flux, or commutating flux, Φ_1, in line with the armature brushes, or in the direction of the axis of the compensating winding, that is, at right angles (electrical) with the field flux.

The field flux Φ depends upon and is in phase with the field current, except as far as it is modified by the magnetic action of the short-circuit current in the armature coil under the commutator brushes.

In the conductively compensated series motor, 1, the quadrature flux is zero at complete compensation, and in the direction of the armature reaction with under-compensation, in opposition to the armature reaction at over-compensation, but in either case in phase with the current and so approximately with the field.

In the other motor types, whatever quadrature flux exists is not in phase with the main flux, but as transformer flux is due to the resultant m.m.f. of primary and secondary circuit.

In a transformer with non-inductive or nearly non-inductive secondary circuit, the magnetic flux is nearly 90 deg. in time phase behind the primary current, a little over 90 deg. ahead of the secondary current, as shown in transformer diagram, Fig. 136.

In a transformer with inductive secondary, the magnetic flux is less than 90 deg. behind the primary current, more than 90 deg. ahead of the secondary current, the more so the higher is the inductivity of the secondary circuit, as shown by the transformer diagram, Fig. 137.

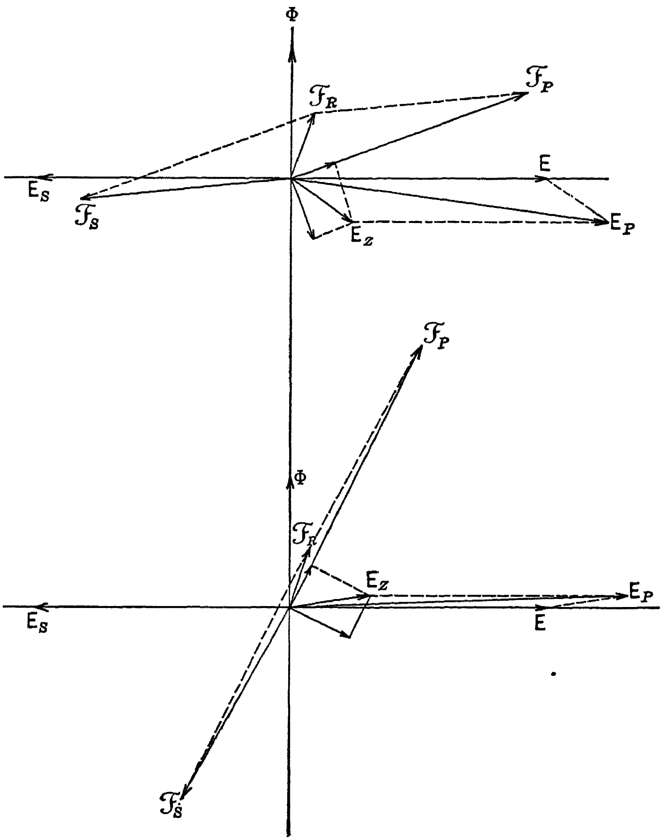
Herefrom it follows that—

In the inductively compensated series motor, 2, the quadrature flux is very small and practically negligible, as very little voltage is consumed in the low impedance of the secondary circuit C; whatever flux there is, lags behind the main flux.

In the inductively compensated series motor with secondary excitation, or inverted repulsion motor, 3, the quadrature flux Φ_1 is quite large, as a considerable voltage is required for the field excitation, especially at moderate speeds and therefore high currents, and this flux Φ_1 lags behind the field flux Φ, but this lag is very much less than 90 deg., since the secondary circuit is highly inductive; the motor field thus corresponding to the conditions of the transformer diagram, Fig. 137. As result hereof, the commutation of this type of motor is very good, flux Φ_1 having the proper phase and intensity required for a commutating flux, as will be seen later, but the power-factor is poor.

In the repulsion motor, 4, the quadrature flux is very considerable, since all the voltage consumed by the rotation of the armature is induced in it by transformation from the compensating winding, and this quadrature flux Φ_1 lags nearly 90 deg. behind the main flux Φ, since the secondary circuit is nearly non-inductive, especially at speed.

In the repulsion motor with secondary excitation, 5, the quadrature flux Φ_1 is also very large, and practically constant, corresponding to the impressed e.m.f., but lags considerably less than 90 deg. behind the main flux Φ, the secondary circuit being inductive, since it contains the field coil F. The lag of the flux Φ_1 increases with increasing speed, since with increasing speed



Figs. 136–137. Transformer Diagram, Non-Inductive and Inductive Load.

the e.m.f. of rotation of the armature increases, the e.m.f. of self-inductance of the field decreases, due to the decrease of current, and the circuit thus becomes less inductive.

The series repulsion motors 6 and 7, give the same phase relation of the quadrature flux Φ_1 as the repulsion motors, 5 and 6, but the intensity of the quadrature flux Φ_1 is the less the smaller the part of the supply voltage which is impressed upon the compensating winding.

V. Commutation.

In the commutator motor, the current in each armature coil or turn reverses during its passage under the brush. In the armature coil, while short-circuited by the commutator brush, the current must die out to zero and then increase again to its original value in opposite direction. The resistance of the armature coil and brush contact accelerates, the self-inductance retards the dying out of the current, and the former thus assists, the latter impairs commutation. If an e.m.f. is generated in the armature coil by its rotation while short-circuited by the commutator brush, this e.m.f. opposes commutation, that is, retards the dying out of the current, if due to the magnetic flux of armature reaction, and assists commutation by reversing the armature current, if due to the magnetic flux of over-compensation, that is, a magnetic flux in opposition to the armature reaction.

Therefore, in the direct-current commutator motor with high field strength and low armature reaction, that is, of negligible magnetic flux of armature reaction, fair commutation is produced with the brushes set midway between the field poles, — that is, in the position where the armature coil which is being commutated encloses the full field flux and therefore cuts no flux and has no generated e.m.f., — by using high-resistance carbon brushes, as the resistance of the brush contact, increasing when the armature coil begins to leave the brush, tends to reverse the current. Such "resistance commutation" obviously cannot be perfect; perfect commutation, however, is produced by impressing upon the motor armature at right angles to the main field, that is, in the position of the commutator brushes, a magnetic field opposite to that of the armature reaction and proportional to the armature current. Such a field is produced by over-compensation or by the use of a commutating pole or interpole.

As seen in the foregoing, in the direct-current motor the counter e.m.f. of self-inductance of commutation opposes the reversal of current in the armature coil under the commutator brush, and this can be mitigated in its effect by the use of high-resistance brushes, and overcome by the commutating field of over-compensation. In addition hereto, however, in the alternating-current commutator motor an e.m.f. is generated in the coil short-circuited under the brush, by the alternation of the magnetic flux, and this e.m.f., which does not exist in the direct-current motor, makes the problem of commutation of the alternating-current motor far more difficult. In the position of commutation no e.m.f. is generated in the armature coil by its rotation through the magnetic field, as in this position the coil encloses the maximum field flux; but as this magnetic flux is alternating, in this position the e.m.f. generated by the alternation of the flux enclosed by the coil is a maximum. This "e.m.f. of alternation" lags in time 90 deg. behind the magnetic flux which generates it, is proportional to the magnetic flux and to the frequency, but is independent of the speed, hence exists also at standstill, while the "e.m.f. of rotation"—which is a maximum in the position of the armature coil midway between the brushes, or parallel to the field flux — is in phase with the field flux and proportional thereto and to the speed, but independent of the frequency. In the alternating-current commutator motor, no position therefore exists in which the armature coil is free from a generated e.m.f., but in the position parallel to the field, or midway between the brushes, the e.m.f. of rotation, in phase with the field flux, is a maximum, while the e.m.f. of alternation is zero, and in the position under the commutator brush, or enclosing the total field flux, the e.m.f. of alternation, in electrical space quadrature with the field flux, is a maximum, the e.m.f. of rotation absent, while in any other position of the armature coil its generated e.m.f. has a component due to the rotation — a power e.m.f. — and a component due to the alternation — a reactive e.m.f. The armature coils of an alternating-current commutator motor, therefore, are the seat of a system of polyphase e.m.fs., and at synchronism the polyphase e.m.fs. generated in all armature coils are equal, above synchronism the e.m.f. of rotation is greater, while below synchronism the e.m.f. of alternation is greater, and in the latter case the brushes thus stand at that point of the commutator where the voltage between commutator segments is a maximum. This e.m.f. of alternation, short-circuited by the armature coil in the position of commutation, if not controlled, causes a short-circuit current of excessive value, and therewith destructive sparking; hence, in the alternating-current commutator motor it is necessary to provide means to control the short-circuit current under the commutator brushes, which results from the alternating character of the magnetic flux, and which does not exist in the direct-current motor; that is, in the alternating-current motor the armature coil under the brush is in the position of a short-circuited secondary, with the field coil as primary of a transformer; and as in a transformer primary and secondary ampere-turns are approximately equal, if n_0 = number of field turns per pole and i = field current, the current in a single armature turn, when short-circuited by the commutator brush, tends to become $i_0 = n_0 i$, that is, many times full-load current; and as this current is in opposition, approximately, to the field current, it would demagnetize the field; that is, the motor field vanishes, or drops far down, and the motor thus loses its torque. Especially is this the case at the moment of starting; at speed, the short-circuit current is somewhat reduced by the self-inductance of the armature turn. That is, during the short time during which the armature turn or coil is short-circuited by the brush the short-circuit current cannot rise to its full value, if the speed is considerable, but it is still sufficient to cause destructive sparking.

Various means have been proposed and tried to mitigate or eliminate the harmful effect of this short-circuit current, as high resistance or high reactance introduced into the armature coil during commutation, or an opposing e.m.f. either from the outside, or by a commutating field.

High-resistance brush contact, produced by the use of very narrow carbon brushes of high resistivity, while greatly improving the commutation and limiting the short-circuit current so that it does not seriously demagnetize the field and thus cause the motor to lose its torque, is not sufficient, for the reason that the resistance of the brush contact is not high enough and also is not constant. The brush contact resistance is not of the nature of an ohmic resistance, but more of the nature of a counter e.m.f.; that is, for large currents the potential drop at the brushes becomes approximately constant, as seen from the volt-ampere characteristics of different brushes given in Figs. 138 and 139. Fig. 138 gives the voltage consumed by the brush contact of a copper brush, with the current density as abscissas, while Fig. 139 gives the voltage consumed by a high-resistance carbon brush, with the current density in the brush as abscissas. It is seen that such a resistance, which decreases approximately inversely proportional to the increase of current, fails in limiting the current just at the moment where it is most required, that is, at high currents.

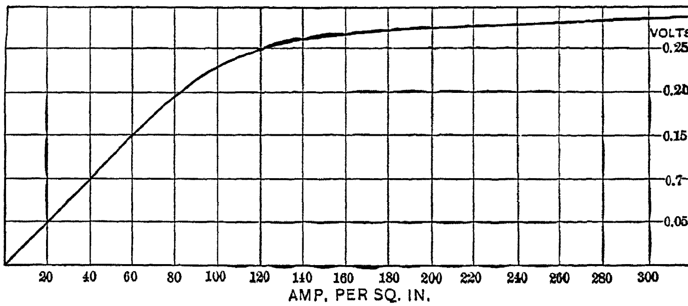


Fig. 138. E.m.f. Consumed at Contact of Copper Brush.

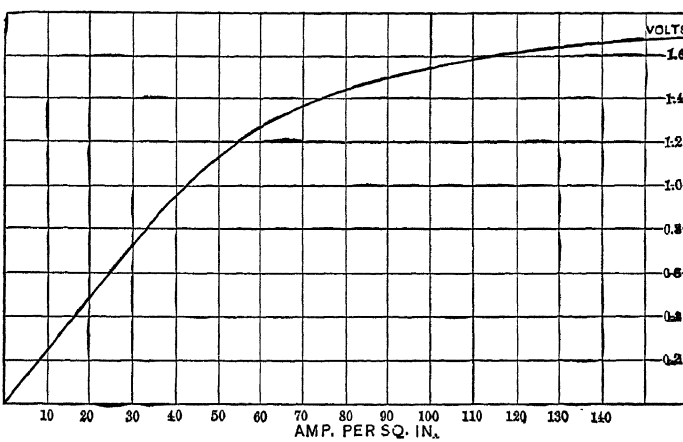


Fig. 139. E.m.f. Consumed at Contact of High-resistance Carbon Brush.

Commutator Leads.

Good results have been reached by the use of metallic resistances in the leads between the armature and the commutator. As shown diagrammatically in Fig. 140 as C, each commutator segment connects to the armature A by a high non-inductive resistance B, and thus two such resistances are always in the circuit of the armature coil short-circuited under the brush, but also one or two in series with the armature main circuit, from brush to brush. While considerable power may therefore be consumed in these high-resistance leads, nevertheless the efficiency of the motor is greatly increased by their use; that is, the reduction in the loss of power at the commutator by the reduction of the short-circuit current usually is far greater than the waste of

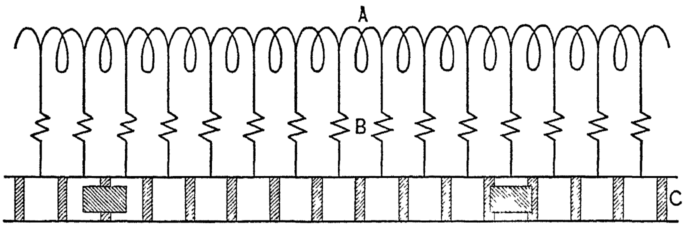


Fig. 140. Commutation with Resistance Leads.

power in the resistance leads. To have any appreciable effect, the resistance of the commutator lead must be far higher than that of the armature coil to which it connects. Of the e.m.f. of rotation, that is, the useful generated e.m.f., the armature resistance consumes only a very small part, a few per cent only. The e.m.f. of alternation is of the same magnitude as the e.m.f. of rotation,—higher below, lower above synchronism. With a short-circuit current equal to full-load current, the resistance of the short-circuit coil would consume only a small part of the generated e.m.f. of alternation, and to consume the total e.m.f. the short-circuit current therefore would have to be about as many times larger than the normal armature current as the useful generated e.m.f. of the motor is larger than the resistance drop in the armature. Long before this value of short-circuit current is reached the magnetic field would have disappeared by the demagnetizing force of the short-circuit current, that is, the motor would have lost its torque.

The ratio of the maximum e.m.f. of alternation e_1 at the brushes to the maximum e.m.f. of rotation e_0 midway between the brushes, is the ratio of frequency of alternation, f, to frequency of rotation, f_0, or $e_1 \div e_0 = f \div f_0$. The mean value of the e.m.f. of rotation, per armature coil, then is $\frac{2}{\pi} e_0$, and, assuming $\frac{1}{p}$

as the fraction of the generated voltage consumed by the armature resistance, and r_0 = resistance per armature coil, the voltage consumed by the resistance of the armature coil at normal current i would be

$$ir = \frac{1}{p}\frac{2}{\pi}e_0. \tag{13}$$

If then r_0 is the total resistance of the short-circuit under the brush, armature coil, brush contact, and resistance leads required to limit the short-circuit current to q times the armature current, the total voltage of alternation is consumed by current $i_0 = qi$, in resistance r_0, if

$$qir_0 = e_1; \tag{14}$$

hence, dividing,

$$\frac{r}{qr_0} = \frac{1}{p}\frac{2}{\pi}\frac{e_0}{e_1}$$

$$= \frac{1}{p}\frac{2}{\pi}\frac{f_0}{f}, \tag{15}$$

and

$$r_0 = \frac{\pi p f}{2\,q f_0} r; \tag{16}$$

hence, if at synchronism, $f_0 = f$, the resistance drop in the armature is 5 per cent, or $p = \frac{1}{0.05} = 20$, and the short-circuit current should be limited to twice the armature current, $q = 2$,

$$r_0 = 15.7\,r;$$

the resistance of the two leads and brush contact thus is $2\,r_1 = r_0 - r = 14.7\,r$, and each resistance lead and brush contact thus about 7 times the resistance of the armature coil to which it connects.

The current i_0 in the resistance lead, however, is twice the armature current i, and the heat produced in the resistance lead, $i_0^2 r_1$, therefore is about 30 times that produced in the armature coil. The space available for the resistance lead, however, is less than that available for the armature coil.

It is obvious herefrom that it is not feasible to build these resistance leads so that each lead can dissipate continuously, or even for any appreciable time, without rapid self-destruction, the heat produced in it while in circuit.

When the motor is revolving, even very slowly, this is not necessary, since each resistance lead is only a very short time in circuit, during the moment when the armature coils connecting to it are short-circuited by the brushes; that is, if n_1 = number of armature turns from brush to brush, the lead is only $\frac{2}{n_1}$ of the time in circuit, and though excessive current densities in materials of high resistivity are used, the heating is moderate. In starting the motor, however, if it does not start instantly, the current continues to flow through the same resistance leads, and thus they are overheated and destroyed if the motor does not start promptly. Hence care has to be taken not to have such motors stalled for any appreciable time with voltage on.

The most serious objection to the use of high-resistance leads, therefore, is their liability to self-destruction by heating if the motor fails to start immediately, as for instance when putting the voltage on the motor before the brakes are released, as is done when starting on a steep up-grade to keep the train from starting to run back.

Thus the advantages of resistance commutator leads are the improvement in commutation resulting from the reduced short-circuit current, and the absence of a series demagnetizing effect on the field at the moment of starting, which would result from an excessive short-circuit current under the brush, and such leads are therefore extensively used; their disadvantage, however, is that when they are used the motor must be sure to start immediately by the application of voltage, otherwise they are liable to be destroyed.

It is obvious that even with high-resistance commutator leads the commutation of the motor cannot be as good as that of the motor on direct-current supply; that is, such an alternating-current motor inherently is more or less inferior in commutation to the direct-current motor, and to compensate for this effect far more favorable constants must be chosen in the motor design than permissible with a direct-current motor, that is, a lower voltage per commutator segment and lower magnetic flux per pole, hence a lower supply voltage on the armature, and thus a larger armature current and therewith a larger commutator, etc.

The insertion of reactance instead of resistance in the leads connecting the commutator segments with the armature coils of the single-phase motor also has been proposed and used for limiting the short-circuit current under the commutator brush.

Reactance has the advantage over resistance, that the voltage consumed by it is wattless and therefore produces no serious heating and reactive leads of low resistance thus are not liable to self-destruction by heating if the motor fails to start immediately.

On account of the limited space available in the railway motor considerable difficulty, however, is found in designing sufficiently high reactances which do not saturate and thus decrease at larger currents.

At speed, reactance in the armature coils is very objectionable in retarding the reversal of current, and indeed one of the most important problems in the design of commutating machines is to give the armature coils the lowest possible reactance. Therefore the insertion of reactance in the motor leads interferes seriously with the commutation of the motor at speed, and thus requires the use of a suitable commutating or reversing flux, that is, a magnetic field at the commutator brushes of sufficient strength to reverse the current, against the self-inductance of the armature coil, by means of an e.m.f. generated in the armature coil by its rotation. This commutating flux thus must be in phase with the main current, that is, a flux of over-compensation. Reactive leads require the use of a commutating flux of over-compensation to give fair commutation at speed.

Counter e.m.fs. in Commutated Coil.

Theoretically, the correct way of eliminating the destructive effect of the short-circuit current under the commutator brush resulting from the e.m.f. of alternation of the main flux would be to neutralize the e.m.f. of alternation by an equal but opposite e.m.f. inserted into the armature coil or generated therein. Practically, however, at least with most motor types, considerable difficulty is met in producing such a neutralizing e.m.f. of the proper intensity as well as phase. Since the alternating current has not only an intensity but also a phase displacement, with an alternating-current motor the production of commutating flux or commutating voltage is more difficult than with direct-current motors in which the intensity is the only variable.

By introducing an external e.m.f. into the short-circuited coil under the brush it is not possible entirely to neutralize its e.m.f. of alternation, but simply to reduce it to one-half. Several such arrangements were developed in the early days by Eickemeyer, for instance the arrangement shown in Fig. 141, which represents the development of a commutator. The commutator consists of alternate live segments S and dead segments S', that is, segments not connected to armature coils, and shown shaded in Fig. 141. Two sets of brushes on the commutator, the one, B_1, ahead in position from the other, B_2, by one commutator segment, and connected to the first by a coil N, containing an e.m.f. equal in phase, but half in intensity, and opposite, to the e.m.f. of alternation of the armature coil; that is, if the armature coil contains a single turn, coil N is a half turn located in the main field space; if the armature coil A contains m turns, $\frac{m}{2}$ turns in the main field space are used in coil N. The dead segments S' are cut between the brushes B_1 and B_2, so as not to short-circuit between the brushes.

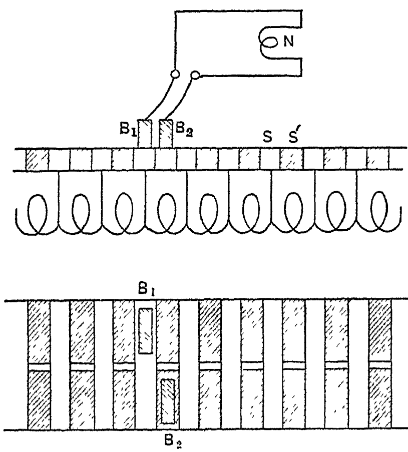


Fig. 141. Commutation with External e.m.f.

In this manner, during the motion of the brush over the commutator, as shown by Fig. 142 in its successive steps, in position.

1 there is current through brush B_1;
2 there is current through both brushes B_1 and B_2, and the armature coil A is closed by the counter e.m.f. of coil N, that is, the difference $A - N$ is short-circuited;
3 there is current through brush B_2;
4 there is current through both brushes B_1 and B_2, and the coil N is short-circuited;
5 the current enters again by brush B_1;

thus alternately the coil N of half the voltage of the armature coil A, or the difference between A and N is short-circuited, that is, the short-circuit current reduced to one-half.

Complete elimination of the short-circuit current can be produced by generating in the armature coil an opposing e.m.f. This e.m.f. of neutralization, however, cannot be generated by the alternation of the magnetic flux through the coil, as this

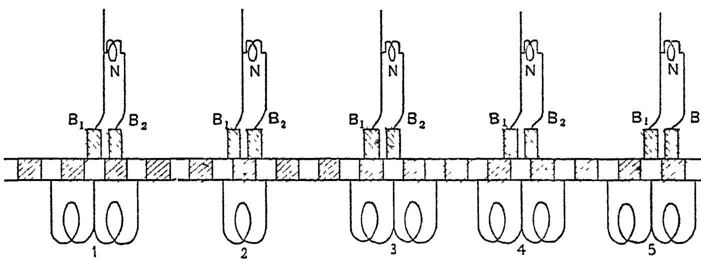


Fig. 142. Commutation by External e.m.f.

would require a flux equal but opposite to the full field flux traversing the coil, and thus destroy the main field of the motor. The neutralizing e.m.f., therefore, must be generated by the rotation of the armature through the commutating field, and thus can occur only at speed; that is, neutralization of the short-circuit current is possible only when the motor is revolving, but not while at rest.

The e.m.f. of alternation in the armature coil short-circuited under the commutator brush is proportional to the main field Φ, to the frequency f, and is in time quadrature with the main field, being generated by its rate of change; hence, it can be represented by

$$e_0 = 2\,\pi f \Phi\, 10^{-8}\, j. \tag{17}$$

The e.m.f. e_1 generated by the rotation of the armature coil through a commutating field Φ' is, however, in time phase with the field which produces it; and since e_1 must be equal and in phase with e_0 to neutralize it, the commutating field Φ', therefore, must be in time phase with e_0, hence in time quadrature with Φ; that is, the commutating field Φ' of the motor must be in time quadrature with the main field Φ to generate a neutralizing voltage e_1 of the proper phase to oppose the e.m.f. of alternation in the short-circuited coil. This e.m.f. e_1 is proportional to its generating field Φ', and to the speed, or frequency of rotation, f_0, hence is

$$e_1 = 2\,\pi f_0 \Phi'\, 10^{-8}, \tag{18}$$

and from $e_1 = e_0$ it then follows that

$$\Phi' = j\Phi \frac{f}{f_0}; \tag{19}$$

that is, the commutating field of the single-phase motor must be in time quadrature behind and proportional to the main field, proportionl to the frequency and inversely proportional to the speed; hence, at synchronism, $f_0 = f$, the commutation field equals the main field in intensity, and, being displaced therefrom in quadrature both in time and in space, the motor thus must have a uniform rotating field, just as the induction motor.

Above synchronism, $f_0 > f$, the commutating field Φ' is less than the main field; below synchronism, however, $f_0 < f$, the commutating field must be greater than the main field to give complete compensation. It obviously is not feasible to increase the commutating field much beyond the main field, as this would require an increase of the iron section of the motor beyond that required to do the work, that is, to carry the main field flux. At standstill Φ' should be infinitely large, that is, compensation is not possible.

Hence, by the use of a commutating field in time and space quadrature, in the single-phase motor the short-circuit current under the commutator brushes resulting from the e.m.f. of alternation can be entirely eliminated at and above synchronism, and more or less reduced below synchronism, the more the nearer the speed is to synchronism, but no effect can be produced at standstill. In such a motor either some further method, as resistance leads, must be used to take care of the short-circuit current at standstill, or the motor designed so that its commutator can carry the short-circuit current for the small fraction of time when the motor is at standstill or running at very low speed.

The main field Φ of the series motor is approximately inversely proportional to the speed f_0, since the product of speed and field strength, $f_0\Phi$, is proportional to the e.m.f. of rotation, or useful e.m.f. of the motor, hence, neglecting losses and phase displacements, to the impressed e.m.f., that is, constant. Substituting therefore $\Phi = \frac{f}{f_0}\Phi_0$, where Φ_0 = main field at synchronism, into equation (19),

$$\Phi' = j\Phi_0 \left(\frac{f}{f_0}\right)^2; \tag{20}$$

that is, the commutating field is inversely proportional to the square of the speed; for instance, at double synchronism it should be one-quarter as high as at synchronism, etc.

Of the quadrature field Φ' only that part is needed for commutation which enters and leaves the armature at the position of the brushes; that is, instead of producing a quadrature field Φ' in accordance with equation (20), and distributed around the armature periphery in the same manner as the main field Φ, but in quadrature position thereto, a local commutating field may be used at the brushes, and produced by a commutating pole or commutating coil, as shown diagrammatically in Fig. 143 as K_1 and K. The excitation of this commutating coil K then would have to be such as to give a magnetic air-gap density $\mathfrak{B}'$, relative to that of the main field, $\mathfrak{B}$, by the same equations (19) and (20):

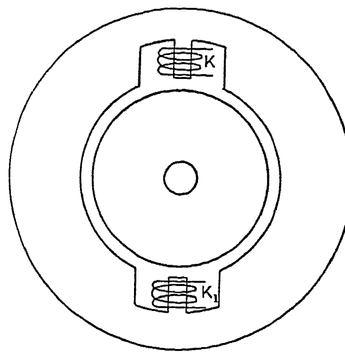


Fig. 143. Commutation with Commutating Poles.

$$\left.\begin{aligned} \mathfrak{B}' &= j\mathfrak{B}\frac{f}{f_0} \\ &= j\mathfrak{B}_0\left(\frac{f}{f_0}\right)^2. \end{aligned}\right\} \tag{21}$$

As the alternating flux of a magnetic circuit is proportional to the voltage which it consumes, that is, to the voltage impressed upon the magnetizing coil, and lags nearly 90 deg. behind it, the magnetic flux of the commutating poles K can be produced by energizing these poles by an e.m.f. e, which is varied with the speed of the motor, by equation

$$e = e_0\left(\frac{f}{f_0}\right)^2, \tag{22}$$

where e_0 is its proper value at synchronism.

Since $\mathfrak{B}'$ lags 90 deg. behind its supply voltage e, and also lags 90 deg. behind $\mathfrak{B}$, by equation (2), and so behind the supply current and, approximately, the supply e.m.f. of the motor, the voltage, e, required for the excitation of the commutating poles is approximately in phase with the supply voltage of the motor; that is, a part thereof can be used, and is varied with the speed of the motor.

Perfect commutation, however, requires not merely the elimination of the short-circuit current under the brush, but requires a reversal of the load current in the armature coil during its passage under the commutator brush. To reverse the current, an e.m.f. is required proportional but opposite to the current and therefore with the main field; hence, to produce a reversing e.m.f. in the armature coil under the commutator brush a second commutating field is required, in phase with the main field and approximately proportional thereto.

The commutating field required by a single-phase commutator motor to give perfect commutation thus consists of a component in quadrature with the main field, or the neutralizing component, which eliminates the short-circuit current under the brush, and a component in phase with the main field, or the reversing component, which reverses the main current in the armature coil under the brush; and the resultant commutating field thus must lag behind the main field, and so approximately behind the supply voltage, by somewhat less than 90 deg., and have an intensity varying approximately inversely proportional to the square of the speed of the motor.

Of the different motor types discussed under IV, the series motors, 1 and 2, have no quadrature field, and therefore can be made to commutate satisfactorily only by the use of commutator leads, or by the addition of separate commutating poles. The inverted repulsion motor, 3, has a quadrature field, which decreases with increase of speed, and therefore gives a better commutation than the series motors, though not perfect, as the quadrature field does not have quite the right intensity.

The repulsion motors, 4 and 5, have a quadrature field, lagging nearly 90 deg. behind the main field, and thus give good commutation at those speeds at which the quadrature field has the right intensity for commutation. However, in the repulsion motor with secondary excitation, 5, the quadrature field is constant and independent of the speed, as constant supply voltage is impressed upon the commutating winding C, which produces the quadrature field, and in the direct repulsion motor, 4, the quadrature field increases with the speed, as the voltage consumed by the main field F decreases, and that left for the compensating winding C thus increases with the speed, while to give proper commutating flux it should decrease with the square of the speed. It thus follows that the commutation of the repulsion motors improves with increase of speed, up to that speed where the quadrature field is just right for commutating field,—which is about at synchronism,—but above this speed the commutation rapidly becomes poorer, due to the quadrature field being far in excess of that required for commutating.

In the series repulsion motors, 6 and 7, a quadrature field also exists, just as in the repulsion motors, but this quadrature field depends upon that part of the total voltage which is impressed upon the commutating winding C, and thus can be varied by varying the distribution of supply voltage between the two circuits; hence, in this type of motor, the commutating flux can be maintained through all (higher) speeds by impressing the total voltage upon the compensating circuit and short-circuiting the armature circuit for all speeds up to that at which the required commutating flux has decreased to the quadrature flux given by the motor, and from this speed upwards only a part of the supply voltage, inversely proportional (approximately) to the square of the speed, is impressed upon the compensating circuit, the rest shifted over to the armature circuit. The difference between 6 and 7 is that in 6 the armature circuit is more inductive, and the quadrature flux therefore lags less behind the main flux than in 7, and by thus using more or less of the field coil in the armature circuit its inductivity can be varied, and therewith the phase displacement of the quadrature flux against the main flux adjusted from nearly 90 deg. lag to considerably less lag, hence not only the proper intensity but also the exact phase of the required commutating flux produced.

As seen herefrom, the difference between the different motor types of IV is essentially found in their different actions regarding commutation.

It follows herefrom that by the selection of the motor type quadrature fluxes Φ_1 can be impressed upon the motor, as commutating flux, of intensities and phase displacements against the main flux Φ, varying over a considerable range.

VI. Motor Characteristics.

The single-phase commutator motor of varying speed or series characteristic comprises three circuits, the armature, the compensating winding, and the field winding, which are connected in series with each other, directly or indirectly.

The impressed e.m.f. or supply voltage of the motor then consists of the components:

1. The e.m.f. of rotation e_1, or voltage generated in the armature conductors by their rotation through the magnetic field Φ. This voltage is in phase with the field Φ and therefore approximately with the current i, that is, is power e.m.f., and is the voltage which does the useful work of the motor. It is proportional to the speed or frequency of rotation f_0, to the field strength Φ, and to the number of effective armature turns n_1,

$$e_1 = 2\,\pi f_0 n_1 \Phi\, 10^{-8}. \tag{23}$$

The number of effective armature turns n_1, with a distributed winding, is the projection of all the turns on their resultant direction. With a full-pitch winding of n series turns from brush to brush, the effective number of turns thus is

$$n_1 = m\,[avg \cos]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \frac{2}{\pi} m. \tag{24}$$

With a fractional pitch winding of the pitch of τ degrees, the effective number of turns is

$$n_1 = m\frac{\tau}{\pi}[avg\ \cos]_{-\frac{\tau}{2}}^{+\frac{\tau}{2}} = \frac{2}{\pi} m \sin\frac{\tau}{2}. \tag{25}$$

2. The e.m.f. of alternation of the field, e_0, that is, the voltage generated in the field turns by the alternation of the magnetic flux Φ, produced by them and thus enclosed by them. This voltage is in quadrature with the field flux Φ and thus approximately with the current $\dot{I}$, is proportional to the frequency of the impressed voltage f, to the field strength Φ, and to the number of field turns n_0.

$$e_0 = -\,2\,j\pi f n_0 \Phi\, 10^{-8}. \tag{26}$$

3. The impedance voltage of the motor,

$$e' = \dot{I}Z \tag{27}$$

and

$$Z = r - jx,$$

where r = total effective resistance of field coils, armature with commutator and brushes, and compensating winding, x = total self-inductive reactance, that is, reactance of the leakage flux of armature and compensating winding — or the stray flux passing locally between the armature and the compensating conductors — plus the self-inductive reactance of the field, that is, the reactance due to the stray field or flux passing between field coils and armature.

In addition hereto, x comprises the reactance due to the quadrature magnetic flux of incomplete compensation or over-compensation, that is, the voltage generated hy the quadrature flux Φ' in the difference between armature and compensating conductors, $n_1 - n_2$ or $n_2 - n_1$.

Therefore the total supply voltage E of the motor is

$$\begin{aligned} \dot{E} &= e_1 + e_0 + e' \\ &= 2\pi f_0 n_1 \Phi\, 10^{-8} - 2j\pi f n_1 \Phi\, 10^{-8} + (r - jx)\dot{I}. \end{aligned} \tag{28}$$

Let, then, R = magnetic reluctance of field circuit, thus $\Phi = \frac{n_0 \dot{I}}{R}$ = the magnetic field flux, when assuming this flux as in phase with the excitation $\dot{I}$, and denoting

$$\frac{2\pi f n_0^2 10^{-8}}{R} = x_0 \tag{30}$$

as the effective reactance of field inductance, corresponding to the e.m.f. of alternation,

$$\left.\begin{aligned} S &= \frac{f_0}{f} = \text{ratio of speed to frequency, or speed as fraction of synchronism,} \\ c &= \frac{n_1}{n_0} = \text{ratio of effective armature turns to field turns;} \end{aligned}\right\} \tag{31}$$

substituting (30) and (31) in (28),

$$\begin{aligned} \dot{E} &= cSx_0\dot{I} - jx_0\dot{I} + (r - jx)\dot{I} \\ &= [(r + cSx_0) - j(x + x_0)]\dot{I}; \end{aligned} \tag{32}$$

or,

$$\dot{I} = \frac{\dot{E}}{(r + cSx_0) - j(x + x_0)}, \tag{33}$$

and, in absolute values,

$$i = \frac{e}{\sqrt{(r + cSx_0)^2 + (x + x_0)^2}}. \tag{34}$$

The power-factor is given by

$$\tan\theta = \frac{x + x_0}{r + cSx}. \tag{35}$$

The useful work of the motor is done by the e.m.f. of rotation,

$$\dot{E}_1 = cSx_0\dot{I},$$

and, since this e.m.f. $\dot{E}_1$ is in phase with the current $\dot{I}$, the useful work, or the motor output (inclusive friction, etc.), is

$$P = E_1 I = cSx_0 i^2$$
$$= \frac{cSx_0 e^2}{(r + cSx_0)^2 + (x + x_0)^2}, \tag{36}$$

and the torque of the motor is

$$D = \frac{P}{S} = cx_0 i^2$$
$$= \frac{cx_0 e^2}{(r + cSx_0)^2 + (x + x_0)^2}. \tag{37}$$

For instance, let

$$e = 200 \text{ volts}, \quad c = \frac{n_1}{n_0} = 4,$$
$$Z = r - jx = 0.02 - 0.06\,j, \quad x_0 = 0.08;$$

then

$$i = \frac{10{,}000}{\sqrt{(1 + 16\,S)^2 + 49}} \text{ in amp.},$$
$$\cot\,\theta = \frac{1 + 16\,S}{7},$$
$$P = \frac{32{,}000\,S}{(1 + 16\,S)^2 + 49}, \text{ in kw.},$$
$$D = \frac{32{,}000}{(1 + 16\,S)^2 + 49} \text{ in syn. kw.}$$

The behavior of the motor at different speeds is best shown by plotting i, $p = \cos\,\theta$, P and D as ordinates with the speed S as abscissas, as shown in Fig. 144.

In railway practice, by a survival of the practice of former times, usually the constants are plotted with the current I as abscissas, as shown in Fig. 145, though obviously this arrangement does not as well illustrate the behavior of the motor.

Graphically, by starting with the current I as zero axis $\overline{OI}$, the motor diagram is plotted in Fig. 146.

The voltage consumed by the resistance r is $\overline{OE_r} = ir$, in phase with $\overline{OI}$; the voltage consumed by the reactance x is $\overline{OE_x} = ix$, and 90 deg. ahead of $\overline{OI}$. $\overline{OE_r}$ and $\overline{OE_x}$ combine to the voltage consumed by the motor impedance, $\overline{OE'} = iz$.

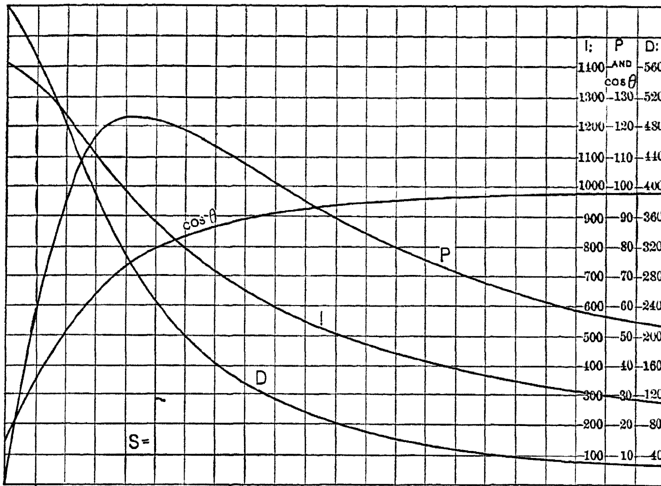


Fig. 144. Single-Phase Commutator Motor Speed Characteristics.

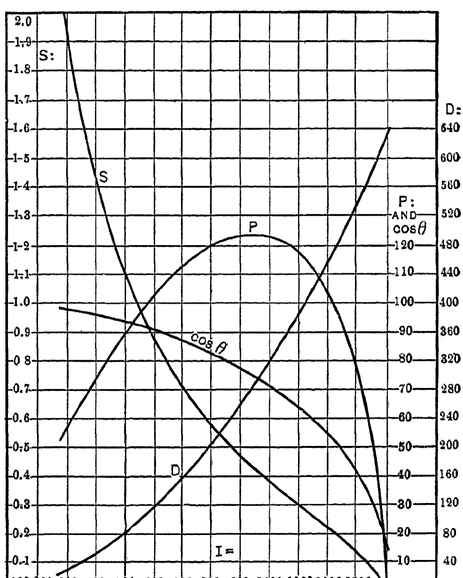


Fig. 145. Single-Phase Commutator Motor Current Characteristics.

Combining $\overline{OE'} = iz$, $\overline{OE_1} = e_1$, and $\overline{OE_0} = e_0$ thus gives the terminal voltage $\overline{OE} = e$ of the motor, and the phase angle $EOI = \theta$.

In this diagram, and in the preceding approximate calculation, the magnetic flux Φ has been assumed in phase with the current I.

In reality, however, the equivalent sine wave of magnetic flux Φ lags behind the equivalent sine wave of exciting current I

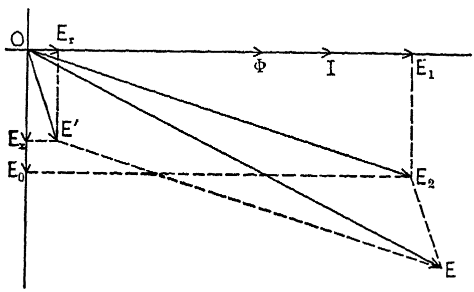


Fig. 146. Single-Phase Commutator Motor Vector Diagram.

by the angle of hysteresis lag, and still further by the power consumed by eddy currents, and, especially in the commutator motor, by the power consumed in the short-circuit current under the brushes, and the vector $\overline{O\Phi}$ therefore is behind the current vector $\overline{OI}$ by an angle θ_a, which is small in a motor in which the short-circuit current under the brushes is eliminated and the eddy currents are negligible, but may reach considerable values in the motor of poor commutation.

Assuming then, in Fig. 147, $\overline{O\Phi}$ lagging behind $\overline{OI}$ by angle θ_a, $\overline{OE_1}$ is in phase with $\overline{O\Phi}$, hence lagging behind $\overline{OI}$; that is, the e.m.f. of rotation is not entirely a power e.m.f., but contains a wattless lagging component. The e.m.f. of alternation, $\overline{OE_0}$, is 90 deg. ahead of $\overline{O\Phi}$, hence less than 90 deg. ahead of $\overline{OI}$, and therefore contains a power component representing the power consumed by hysteresis, eddy currents, and the short-circuit current under the brushes.

Completing now the diagram, it is seen that the phase angle θ is reduced, that is, the power-factor of the motor increased by the increased loss of power, but is far greater than corresponding thereto. It is the result of the lag of the e.m.f. of rotation, which produces a lagging e.m.f. component, partially compensating for the leading e.m.f. consumed by self-inductance, a lag of the e.m.f. being equivalent to a lead of the current.

As the result of this feature of a lag of the magnetic flux Φ, by producing a lagging e.m.f. of rotation and thus compensating

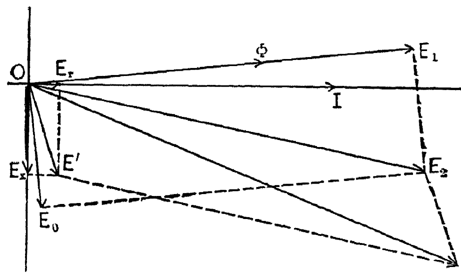


Fig. 147. Single-Phase Commutator Motor Diagram with Phase Displacement between Flux and Current.

for the lag of current by self-inductance, single-phase motors having poor commutation usually have better power-factors, and improvement in commutation, by eliminating or reducing the short-circuit current under the brush, usually causes a slight decrease in the power-factor, by bringing the magnetic flux Φ more nearly in phase with the current I.

Inversely, by increasing the lag of the magnetic flux Φ, the phase angle can be decreased and the power-factor improved. Such a shift of the magnetic flux Φ behind the supply current i can be produced by dividing the current i into components i' and i'', and using the lagging component for field excitation. This is done most conveniently by shunting the field by a non-inductive resistance. Let r_0 be the non-inductive resistance in shunt with the field winding, of reactance $x_0 + x_1$, where x_1 is that part of the self-inductive reactance x due to the field coils. The current i' in the field is lagging 90 deg. behind the current i'' in a non-inductive resistance, and the two currents have the ratio $\frac{i'}{i''} = \frac{r_0}{x_0 + x_1}$; hence, dividing the total current $\overline{OI}$ in this proportion into the two quadrature components $\overline{OI'}$ and $\overline{OI''}$, in Fig. 148, gives the magnetic flux $\overline{O\Phi}$ in phase with $\overline{OI'}$, and so lagging behind $\overline{OI}$, and then the e.m.f. of rotation is $\overline{OE_1}$, the e.m.f. of alternation $\overline{OE_0}$, and combining $\overline{OE_1}$, $\overline{OE_0}$, and $\overline{OE'}$

gives the impressed e.m.f. $\overline{OE}$, nearer in phase to $\overline{OI}$ than with $\overline{O\Phi}$ in phase with $\overline{OI}$.

In this manner, if the e.m.fs. of self-inductance are not too large, unity power-factor can be produced, as shown in Fig. 149.

Let $\overline{OI}$ = total current, $\overline{OE'}$ = impedance voltage of the motor, $\overline{OE}$ = impressed e.m.f. or supply voltage, and assumed in phase with $\overline{OI}$. $\overline{OE}$ then must be the resultant of $\overline{OE'}$ and of $\overline{OE_2}$, the voltage of rotation plus that of alternation, and resolving therefore $\overline{OE_2}$ into two components $\overline{OE_1}$ and $\overline{OE_0}$, in quadrature with each other, and proportional respectively to the e.m.f. of rotation and the e.m.f. of alternation, gives the magnetic flux $\overline{O\Phi}$ in phase with the e.m.f. of rotation $\overline{OE_1}$, and the component of current in the field $\overline{OI'}$ and in the non-inductive resistance $\overline{OI''}$, in phase and in quadrature respectively with $O\Phi$, which combined make up the total current. The projection of the e.m.f. of rotation $\overline{OE_1}$ on $\overline{OI}$ then is the power component of the e.m.f., which does the work of the motor, and the quadrature projection of $\overline{OE_1}$ is the compensating component of the e.m.f. of rotation, which neutralizes the wattless component of the e.m.f. of self-inductance.

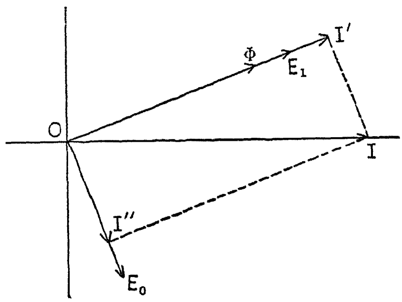


Fig. 148. Single-Phase Commutator Motor Improvement of Power-Factor by Introduction of lagging e.m.f. of Rotation.

Obviously such a compensation involves some loss of power in the non-inductive resistance r_0 shunting the field coils, and as the power-factor of the motor usually is sufficiently high, such compensation is rarely needed.

In motors in which some of the circuits are connected in series with the others the diagram is essentially the same, except that a phase displacement exists between the secondary and the primary current. The secondary current I_1 of the transformer lags behind the primary current I_0 slightly less than 180 deg.; that is, considered in opposite direction, the secondary current leads the primary by a small angle θ_0, and in the motors with secondary excitation the field flux Φ, being in phase with the field current I_1 (or lagging by angle θ_α behind it), thus leads the primary current I_0 by angle θ_0 (or angle $\theta_0 - \theta_\alpha$). As a lag of the magnetic flux Φ increases, and a lead thus decreases the power-factor, motors with secondary field excitation usually have a slightly lower power-factor than motors with primary field

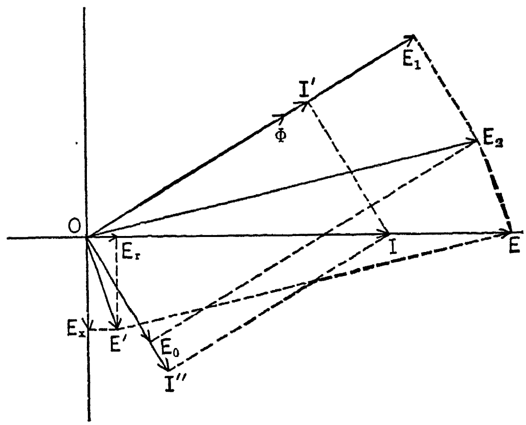


Fig. 149. Single-Phase Commutator Motor. Unity Power-Factor Produced by Lagging e.m.f. of Rotation.

excitation, and therefore, where desired, the power-factor may be improved by shunting the field with a non-inductive resistance r_0. Thus for instance, if in Fig. 150 $\overline{OI_0}$ = primary current, $\overline{OI_1}$ = secondary current, $\overline{OE_1}$, in phase with $\overline{OI_1}$, is the e.m.f. of rotation, in the case of the secondary field excitation, and $\overline{OE_0}$, in quadrature ahead of $\overline{OI_1}$, is the e.m.f. of alternation, while $\overline{OE'}$ is the impedance voltage, and $\overline{OE_1}$, $\overline{OE_0}$ and $\overline{OE'}$ combined give the supply voltage $\overline{OE}$, and $EOI = \theta$ the angle of lag.

Shunting the field by a non-inductive resistance r_0, and thus resolving the secondary current $\overline{OI_1}$ into the components $\overline{OI_1'}$ in the field and $\overline{OI_1''}$ in the non-inductive resistance, gives the diagram Fig. 151, where $\theta_\alpha = I_1'O\Phi$ = angle of lag of magnetic field.

The action of the commutator in an alternating-current motor, in permitting compensation for phase displacement and thus allowing a control of the power-factor, is very interesting and important, and can also be used in other types of machines, as induction motors and alternators, by supplying these machines with a commutator for phase control.

A lag of the current is the same as a lead of the e.m.f., and inversely a leading current inserted into a circuit has the same effect as a lagging e.m.f. inserted. The commutator, however, produces an e.m.f. in phase with the current. Exciting the field by a lagging current in the field, a lagging e.m.f. of rotation is

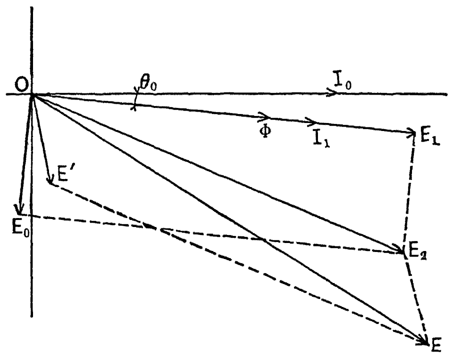


Fig. 150. Single-Phase Commutator Motor Diagram with Secondary Excitation.

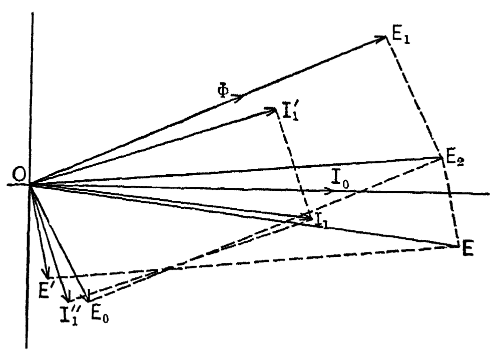


Fig. 151. Single-Phase Commutator Motor with Secondary Excitation Power-Factor Improved by Shunting Field Winding with Non-Inductive Circuit.

produced which is equivalent to a leading current. As it is easy to produce a lagging current by self-inductance, the commutator thus affords an easy means of producing the equivalent of a leading current. Therefore the alternating-current commutator is one of the important methods of compensating for lagging currents. Other methods are the use of electrostatic or electrolytic condensers and of overexcited synchronous machines.

Based on this principle, a number of designs of induction motors and other apparatus have been developed, using the commutator for neutralizing the lagging magnetizing current and the lag caused by self-inductance, and thereby producing unity power-factor or even leading currents. So far, however, none of them has come into extended use.

This feature, however, explains the very high power-factors feasible in single-phase commutator motors even with considerable air gaps, far larger than feasible in induction motors.

VII. Efficiency and Losses.

The losses in single-phase commutator motors are essentially the same as in other types of machines:

(*a.*) Friction losses, —air friction or windage, bearing friction and commutator brush friction, and also gear losses or other mechanical transmission losses.

(*b.*) Core losses, as hysteresis and eddy currents. These are of two classes, — the alternating core loss, due to the alternation of the magnetic flux in the main field, quadrature field, and armature; and the rotating core loss, due to the rotation of the armature through the magnetic field. The former depends upon the frequency, the latter upon the speed.

(*c.*) Commutation losses, as the power consumed by the short-circuit current under the brush, by arcing and sparking, where such exists.

(*d.*) i^2r losses in the motor circuits,—the field coils, the compensating winding, the armature and the brush contact resistance.

(*e.*) Load losses, mainly represented by an effective resistance, that is, an increase of the total effective resistance of the motor beyond the ohmic resistance.

Driving the motor by mechanical power and with no voltage on the motor gives the friction and the windage losses, exclusive of commutator friction, if the brushes are lifted off the commutator, inclusive, if the brushes are on the commutator. Energizing now the field by an alternating current of the rated frequency, with the commutator brushes off, adds the core losses to the friction losses; the increase of the driving power then measures the rotating core loss, while a wattmeter in the field exciting circuit measures the alternating core loss.

Thus the alternating core loss is supplied by the impressed electric power, the rotating core loss by the mechanical driving power.

Putting now the brushes down on the commutator adds the commutation losses.

The ohmic resistance gives the i^2r losses, and the difference between the ohmic resistance and the effective resistance, calculated from wattmeter readings with alternating current in the motor circuits at rest and with the field unexcited, represents the load losses.

However, the different losses so derived have to be corrected for their mutual effect. For instance, the commutation losses are increased by the current in the armature; the load losses are less with the field excited than without, etc.; so that this method of separately determining the losses can give only an estimate of their general magnitude, but the exact determination of the efficiency is best carried out by measuring electric input and mechanical output.

VIII. Discussion of Motor Types.

Varying-speed single-phase commutator motors can be divided into two classes, namely, compensated series motors and repulsion motors. In the former, the main supply current is through the armature, while in the latter the armature is closed upon itself as secondary circuit, with the compensating winding as primary or supply circuit. As the result hereof the repulsion motors contain a transformer flux, in quadrature position to the main flux, and lagging behind it, while in the series motors no such lagging quadrature flux exists, but in quadrature position to the main flux, the flux either is zero — complete compensation — or in phase with the main flux — over- or under-compensation.

A. Compensated Series Motors.

Series motors give the best power-factors, with the exception of those motors in which by increasing the lag of the field flux a compensation for power-factor is produced, as discussed in V. The commutation of the series motor, however, is equally poor at all speeds, due to the absence of any commutating flux, and with the exception of very small sizes such motors therefore are inoperative without the use of either resistance leads or commutating poles. With high-resistance leads, however, fair operation is secured, though obviously not of the same class with that of the direct-current motor; with commutating poles or coils producing a local quadrature flux at the brushes good results have been produced abroad.

Of the two types of compensation, conductive compensation, 1, with the compensating winding connected in series with the armature, and inductive compensation, 2, with the compensated winding short-circuited upon itself, inductive compensation necessarily is always complete or practically complete compensation, while with conductive compensation a reversing flux can be produced at the brushes by over-compensation, and the commutation thus somewhat improved, especially at speed, at the sacrifice, however, of the power-factor, which is lowered by the increased self-inductance of the compensating winding. On the short-circuit current under the brushes, due to the e.m.f. of alternation, such over-compensation obviously has no helpful effect. Inductive compensation has the advantage that the compensating winding is not connected with the supply circuit, can be made of very low voltage, or even of individually short-circuited turns, and therefore larger conductors and less insulation used, which results in an economy of space, and therewith an increased output for the same size of motor. Therefore inductive compensation is preferable where it can be used. It is not permissible, however, in motors which are required to operate also on direct current, since with direct-current supply no induction takes place and therefore the compensation fails, and with the high ratio of armature turns to field turns, without compensation, the field distortion is altogether too large to give satisfactory commutation, except in small motors.

The inductively compensated series motor with secondary excitation, or inverted repulsion motor, 3, takes an intermediary position between the series motors and the repulsion motors; it is a series motor in so far as the armature is in the main supply circuit, but magnetically it has repulsion motor characteristics, that is, contains a lagging quadrature flux. As the field excitation consumes considerable voltage, when supplied from the compensating winding as secondary circuit, considerable voltage must be generated in this winding, thus giving a corresponding transformer flux. With increasing speed and therewith decreasing current, the voltage consumed by the field coils decreases, and therewith the transformer flux which generates this voltage. Therefore the inverted repulsion motor contains a transformer flux which has approximately the intensity and the phase required for commutation; it lags behind the main flux, but less than 90 deg., thus contains a component in phase with the main flux, as reversing flux, and decreases with increase of speed. Therefore the commutation of the inverted repulsion motor is very good, far superior to the ordinary series motor, and it can be operated without resistance leads; it has, however, the serious objection of a poor power-factor, resulting from the lead of the field flux against the armature current, due to the secondary excitation, as discussed in V. To make such a motor satisfactory in power-factor requires a non-inductive shunt across the field, and thereby a waste of power. For this reason it has not come into commercial use.

B. Repulsion Motors.

Repulsion motors are characterized by a lagging quadrature flux, which transfers the power from the compensating winding to the armature. At standstill, and at very low speeds, repulsion motors and series motors are equally unsatisfactory in commutation; while, however, in the series motors the commutation remains bad (except when using commutating devices), in the repulsion motors with increasing speed the commutation rapidly improves, and becomes perfect near synchronism. As the result hereof, under average railway conditions a much inferior commutation can be allowed in repulsion motors at very low speeds than in series motors, since in the former the period of poor commutation lasts only a very short time. While, therefore, series motors cannot be satisfactorily operated without resistance leads (or commutating poles), in repulsion motors resistance leads are not necessary and not used, and the excessive current density under the brushes in the moment of starting permitted, as it lasts too short a time to cause damage to the commutator.

As the transformer field of the repulsion motor is approximately constant, while the proper commutating field should decrease with the square of the speed, above synchronism the transformer field is too large for commutation, and at speeds considerably above synchronism — 50 per cent and more —the repulsion motor becomes inoperative because of excessive sparking. At synchronism, the magnetic field of the repulsion motor is a rotating field, like that of the polyphase induction motor.

Where, therefore, speeds far above synchronism are required, the repulsion motor cannot be used; but where synchronous speed is not much exceeded the repulsion motor is preferred because of its superior commutation. Thus when using a commutator as auxiliary device for starting single-phase induction motors the repulsion motor type is used. For high frequencies, as 60 cycles, where peripheral speed forbids synchronism being greatly exceeded, the repulsion motor is the only type to be seriously considered.

Repulsion motors also may be built with primary and secondary excitation. The latter usually gives a better commutation, because of the lesser lag of the transformer flux, and therewith a greater in-phase component, that is, greater reversing flux, especially at high speeds. Secondary excitation, however, gives a slightly lower power-factor.

A combination of the repulsion motor and series motor types is the series repulsion motor, 6 and 7. In this only a part of the supply voltage is impressed upon the compensating winding and thus transformed to the armature, while the rest of the supply voltage is impressed directly upon the armature, just as in the series motor. As result thereof the transformer flux of the series repulsion motor is less than that of the repulsion motor, in the same proportion in which the voltage impressed upon the compensating winding is less than the total supply voltage. Such a motor, therefore, reaches equality of the transformer flux with the commutating flux, and gives perfect commutation at a higher speed than the repulsion motor, that is, above synchronism. With the total supply voltage impressed upon the compensating winding, the transformer flux equals the commutating flux at synchronism. At n times synchronous speed the commutating flux should be $\frac{1}{n^2}$ of what it is at synchronism, and by impressing $\frac{1}{n^2}$ of the supply voltage upon the compensating winding, the rest on the armature, the transformer flux is reduced to $\frac{1}{n^2}$ of its value, that is, made equal to the required commutating flux at n times synchronism.

In the series repulsion motor, by thus gradually shifting the supply voltage from the compensating winding to the armature and thereby reducing the transformer flux, it can be maintained equal to the required commutating flux at all speeds from synchronism upwards; that is, the series repulsion motor arrangement permits maintaining the perfect commutation, which the repulsion motor has near synchronism, for all higher speeds.

With regard to construction, no essential difference exists between the different motor types, and any of the types can be operated equally well on direct current by connecting all three circuits in series. In general, the motor types having primary and secondary circuits, as the repulsion and the series repulsion motors, give a greater flexibility, as they permit winding the circuits for different voltages, that is, introducing a ratio of transformation between primary and secondary circuit. Shifting one motor element from primary to secondary, or inversely, then gives the equivalent of a change of voltage or change of turns. Thus a repulsion motor in which the stator is wound for a higher voltage, that is, with more turns, than the rotor or armature, when connecting all the circuits in series for direct-current operation, gives a direct-current motor having a greater field excitation compared with the armature reaction, that is, the stronger field which is desirable for direct-current operation but not permissible with alternating current.

In general, the constructive differences between motor types are mainly differences in connection of the three circuits. For instance, let F = field circuit, A = armature circuit, C = compensating circuit, T = supply transformer, R = resistance used in starting and at very low speeds. Connecting, in Fig. 152, the armature A between field F and compensating winding C. With switch 0 open the starting resistance is in circuit; closing switch 0 short-circuits the starting resistance and gives the running conditions of the motor.

With all the other switches open the motor is a conductively compensated series motor.

Closing 1 gives the inductively compensated series motor.

Closing 2 gives the repulsion motor with primary excitation.

Closing 3 gives the repulsion motor with secondary excitation.

Closing 4 or 5 or 6 or 7 gives the successive speed steps of the series repulsion motor with armature excitation.

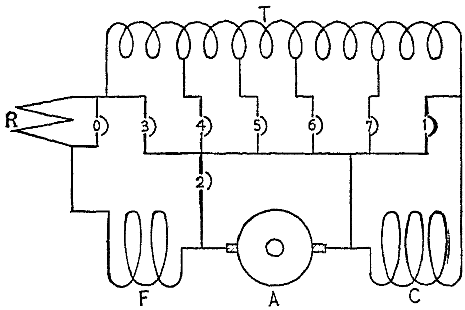


Fig. 152. Alternating-Current Commutator Motor Arranged to Operate either as Series or Repulsion Motor.

Connecting, in Fig. 153, the field F between armature A and compensating winding C, the resistance R is again controlled by switch 0.

All other switches open gives the conductively compensated series motor.

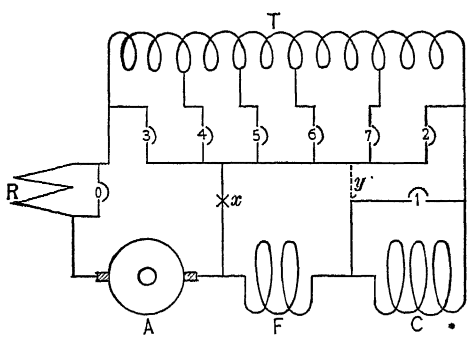


Fig. 153. Alternating-Current Commutator Motor Arranged to Operate either as Series or Repulsion Motor.

Switch 1 closed gives the inductively compensated series motor.

Switch 2 closed gives the inductively compensated series motor with secondary excitation, or inverted repulsion motor.

Switch 3 closed gives the repulsion motor with primary excitation.

Switches 4 to 7 give the different speed steps of the series repulsion motor with primary excitation.

Opening the connnection at x and closing at y (as shown in dotted line), the steps 3 to 7 give respectively the repulsion motor with secondary excitation and the successive steps of the series repulsion motor with armature excitation.

Still further combinations can be produced in this manner, as for instance, in Fig. 152, by closing 2 and 4, but leaving 0 open, the field F is connected across a constant potential supply, in series with resistance R, while the armature also receives constant voltage, and the motor then approaches a finite speed, that is, has shunt motor characteristic, and in starting, the main field F and the quadrature field AC are displaced in phase, so give a rotating or polyphase field (unsymmetrical).

To discuss all these motor types with their in some instances very interesting characteristics obviously is not feasible. In general, they can all be classified under series motor, repulsion motor, shunt motor, and polyphase induction motor, and combinations thereof.

IX. Other Commutator Motors.

Most of the development on single-phase commutator motors has taken place in the direction of varying speed motors for railway service. In other directions commutators have been applied to alternating-current motors and such motors developed —

(*a*.) For limited speed, or of the shunt motor type, that is, motors of similar characteristic as the single-phase railway motor, except that the speed does not indefinitely increase with decreasing load but approaches a finite no-load value. Several types of such motors have been developed, as stationary motors for elevators, variable-speed machinery etc., usually of the single-phase type.

By impressing constant voltage upon the field the magnetic field flux is constant, and the speed thus reaches a finite limiting value at which the e.m.f. of rotation of the armature through the constant field flux consumes the impressed voltage of the armature. By changing the voltage supply to the field different speeds can be produced, that is, an adjustable speed motor. The main problem in the design of such motors is to get the field excitation in phase with the armature current and thus produce a good power-factor.

(*b*.) Adjustable-speed polyphase induction motors. In the secondary of the polyphase induction motor an e.m.f. is generated which, at constant impressed e.m.f. and therefore approximately constant flux, is proportional to the slip from synchronism. With short-circuited secondary the motor closely approaches synchronism. Inserting resistance into the secondary reduces the speed by the voltage consumed in the secondary. As this is proportional to the current and thus to the load, the speed control of the polyphase induction motor by resistance in the secondary gives a speed which varies with the load, just as the speed control of a direct-current motor by resistance in the armature circuit; hence, the speed is not constant, and the operation at lower speeds inefficient. Inserting, however, a constant voltage into the secondary of the induction motor the speed is decreased if this voltage is in opposition, and is increased if this voltage is in the same direction as the secondary generated e.m.f., and in this manner a speed control can be produced. If c = voltage inserted into the secondary, as fraction of the voltage which would be induced in it at full frequency by the rotating field, then the polyphase induction motor approaches at no load and runs at load near to the speed $(1 - c)$ or $(1 + c)$ times synchronism, depending upon the direction of the inserted voltage.

Such a voltage inserted into the induction motor secondary must, however, have the frequency of the motor secondary currents, that is, of slip, and therefore can be derived from the full frequency supply circuit only by a commutator revolving with the secondary. If cf is the frequency of slip, then $(1 - c) f$ is the frequency of rotation, and thus the frequency of commutation, and at frequency f impressed upon the commutator the effective frequency of the commutated current is $f - (1 - c) f = cf$, or the frequency of slip, as required.

Thus the commutator affords a means of inserting voltage into the secondary of induction motors and thus varying its speed.

However, while these commutated currents in their resultant give the effect of the frequency of slip, they actually consist of sections of waves of full frequency, that is, meet the full stationary impedance in the rotor secondary, and not the very much lower impedance of the low-frequency currents in the ordinary induction motor.

If therefore the brushes on the commutator are set so that the inserted voltage is in phase with the voltage generated in the secondary, the power-factor of the motor is very poor. Shifting the brushes, by a phase displacement between the generated and the inserted voltage, the secondary currents can be made to lead, and thereby compensate for the lag due to self-inductance and unity power-factor produced. This, however, is the case only at one definite load, and at all other loads either over-compensation or under-compensation takes place, resulting in poor power-factor, either lagging or leading. Such a polyphase adjustable-speed motor thus requires shifting of the brushes with the load or other adjustment, to maintain reasonable power-factor, and for this reason has not been used.

(*c*) Power-factor compensation. The production of an alternating magnetic flux requires wattless or reactive volt-amperes, which are proportional to the frequency. Exciting an induction motor not by the stationary primary but by the revolving secondary, which has the much lower frequency of slip, reduces the volt-amperes excitation in the proportion of full frequency to frequency of slip, that is, to practically nothing. This can be done by feeding the exciting current into the secondary by commutator. If the secondary contains no other winding but that connected to the commutator, the exciting current from the commutator still meets full open magnetic circuit inductance, and the motor thus gives a poor power-factor. If, however, in addition to the exciting winding, fed by the commutator, a permanently short-circuited winding is used, as a squirrel-cage winding, the exciting impedance of the former is reduced to practically nothing by the short-circuit winding coincident with it, and so by over-excitation unity power-factor or even leading current can be produced. The presence of the short-circuited winding, however, excludes this method from speed control, and such a motor (Heyland motor) runs near synchronism just as the ordinary induction motor, differing merely by the power-factor.

This method of excitation by feeding the alternating current through a commutator into the rotor has been used very successfully abroad in the so-called "compensated repulsion motor" of Winter-Eichberg. This motor differs from the ordinary repulsion motor merely by the field coil F in Fig. 154 being replaced by a set of exciting brushes G in Fig. 155, at right angles to the main brushes of the armature, that is, located so that the m.m.f. of the current between the brushes G magnetizes in the

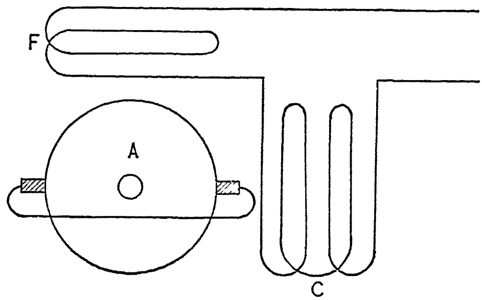


Fig. 154. Plain Repulsion Motor.

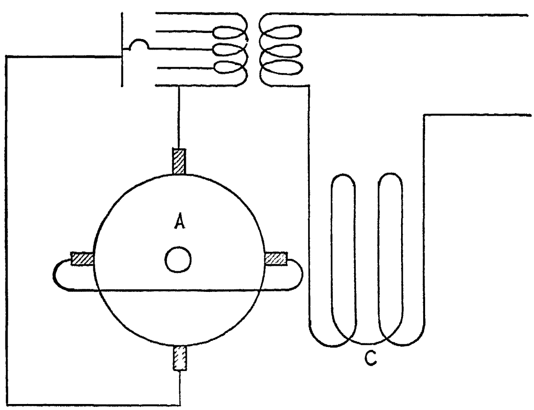


Fig. 155. Winter-Eichberg Motor.

same direction as the field coils F in Fig. 154. Usually the exciting brushes are supplied by a transformer or compensator, so as to vary the excitation and thereby the speed.

This arrangement then lowers the e.m.f. of self-inductance of field excitation of the motor from that corresponding to full frequency in the ordinary repulsion motor to that of the frequency of slip, hence to a negative value above synchronism; so that hereby a compensation for lagging current can be produced above synchronism, and unity power-factor or even leading currents produced.

D. SYNCHRONOUS CONVERTERS.

I. General.

110. For long-distance transmission, and to a certain extent also for distribution, alternating currents, either polyphase or single-phase, are extensively used. For many applications, however, as especially for electrolytic work, direct currents are required, and are usually preferred also for electrical railroading and for low-tension distribution on the Edison three-wire system. Thus, where power is derived from an alternating system, transforming devices are required to convert from alternating to direct current. This can be done either by a direct-current generator driven by an alternating synchronous or induction motor, or by a single machine consuming alternating and producing direct current in one and the same armature. Such a machine is called a converter, and combines, to a certain extent, the features of a direct-current generator and an alternating synchronous motor, differing, however, from either in other features.

Since in the converter the alternating and the direct current are in the same armature conductors, their e.m.fs. stand in a definite relation to each other, which is such that in practically all cases step-down transformers are necessary to generate the required alternating voltage.

Comparing thus the converter with the combination of synchronous or induction motor and direct-current generator, the converter requires step-down transformers, the synchronous motor, if the alternating line voltage is considerably above 10,000 volts, generally requires step-down transformers also, with voltages of 1000 to 10,000 volts, however, usually the synchronous motor and frequently the induction motor can be wound directly for the line voltage and stationary transformers saved. Thus on the one side we have two machines with or generally without stationary transformers, on the other side a single machine with transformers.

Regarding the reliability of operation and first cost, obviously a single machine is preferable.

Regarding efficiency, it is sufficient to compare the converter with the synchronous-motor-direct-current-generator set, since the induction motor is inherently less efficient than the synchronous motor. The efficiency of stationary transformers of large size varies from 97 per cent to 98 per cent, with an average of 97.5 per cent. That of converters or of synchronous motors varies between 91 per cent and 95 per cent, with 93 per cent as average, and that of the direct-current generator between 90 per cent and 94 per cent, with 92 per cent as average. Thus the converter with its step-down transformers will give an average efficiency of 90.7 per cent, a direct-current generator driven by synchronous motor with step-down transformers an efficiency of 83.4 per cent, without step-down transformers an efficiency of 85.6 per cent. Hence the converter is more efficient.

Mechanically the converter has the advantage that no transfer of mechanical energy takes place, since the torque consumed by the generation of the direct current and the torque produced by the alternating current are applied at the same armature conductors, while in a direct-current generator driven by a synchronous motor the power has to be transmitted mechanically through the shaft.

II. Ratio of e.m.fs. and of Currents.

111. In its structure the synchronous converter consists of a closed-circuit armature, revolving in a direct-current excited field, and connected to a segmental commutator as well as to collector rings. Structurally it thus differs from a direct-current machine by the addition of the collector rings, from certain (now very little used) forms of synchronous machines by the addition of the segmental commutator.

In consequence hereof, regarding types of armature windings and of field windings, etc., the same rule applies to the converter as to all commutating machines, except that in the converter the total number of armature coils with a series-wound armature, and the number of armature coils per pair of poles with a multiple-wound armature, must be divisible by the number of phases.

Regarding the wave-shape of the alternating counter-generated e.m.f., similar considerations apply as for a synchronous machine with closed-circuit armature; that is, the generated e.m.f. usually approximates a sine wave, due to the multi-tooth distributed winding.

Thus, in the following, only those features will be discussed in which the synchronous converter differs from the commutating machines and synchronous machines treated in the preceding chapters.

Fig. 156 represents diagrammatically the commutator of a direct-current machine with the armature coils A connected to adjacent commutator bars. The brushes are B_1B_2, and the field poles F_1F_2.

If now two oppositely located points a_1a_2 of the commutator are connected with two collector rings D_1D_2, it is obvious that the e.m.f. between these points a_1a_2, and thus between the collector rings D_1D_2, will be a maximum in the moment when the points a_1a_2 coincide with the brushes B_1B_2, and is in this moment equal to the direct voltage E of this machine. While the points a_1a_2 move away from this position, the difference of potential between a_1 and a_2 decreases and becomes zero in the moment where a_1a_2 coincide with the direction of the field poles F_1F_2. In this moment the difference in potential between a_1 and a_2 reverses and then increases again, reaching equality with E, but in opposite direction, when a_1 and a_2 coincide with the brushes B_2 and B_1; that is, between the collector rings D_1 and D_2 an alternating voltage is produced whose maximum value equals the direct-current electromotive force E, and which makes a complete period for every revolution of the machine (in a bipolar converter, or n_p periods per revolution in a machine of $2\,n_p$ poles).

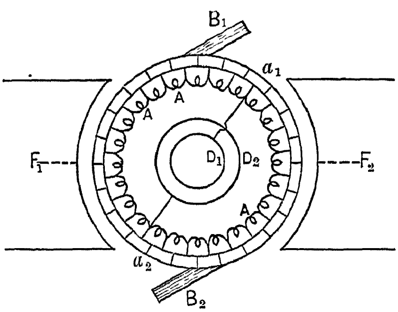


Fig. 156. Single-Phase Converter Commutator.

Hence, this alternating e.m.f. is

$$e = E \sin 2\,\pi f t,$$

where f = frequency of rotation, E = e.m.f. between brushes of the machine; thus, the effective value of the alternating e.m.f. is

$$E_1 = \frac{E}{\sqrt{2}}.$$

112. That is, a direct-current machine produces between two collector rings connected with two opposite points of the commutator an alternating e.m.f. of $\frac{1}{\sqrt{2}} \times$ the direct-current voltage, at a frequency equal to the frequency of rotation. Since every alternating-current generator is reversible, such a direct-current machine with two collector rings, when supplied with an alternating e.m.f. of $\frac{1}{\sqrt{2}} \times$ the direct-current voltage at the frequency of rotation, will run as synchronous motor, or if at the same time generating direct current, as synchronous converter.

Since, neglecting losses and phase displacement, the output of the direct-current side must be equal to the input of the alternating-current side, and the alternating voltage in the single-phase converter is $\frac{1}{\sqrt{2}} \times E$, the alternating current must be $= \sqrt{2} \times I$, where I = direct-current output.

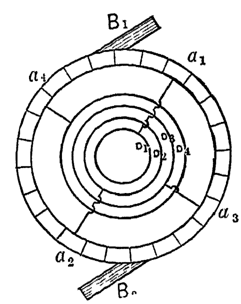


Fig. 157. Four-Phase Converter Commutator.

If now the commutator is connected to a further pair of collector rings, $D_3 D_4$ (Fig. 157), at the points a_3 and a_4 midway between a_1 and a_2, it is obvious that between D_3 and D_4 an alternating voltage of the same frequency and intensity will be produced as between D_1 and D_2, but in quadrature therewith, since at the moment where a_3 and a_4 coincide with the brushes $B_1 B_2$ and thus receive the maximum difference of potential, a_1 and a_2 are at zero points of potential.

Thus connecting four equidistant points a_1, a_2, a_3, a_4 of the direct-current generator to four collector rings D_1, D_2, D_3, D_4, gives a four-phase converter, of the e.m.f.

$$E_1 = \frac{1}{\sqrt{2}} E \text{ per phase.}$$

The current per phase is (neglecting losses and phase displacement)

$$I_1 = \frac{I}{\sqrt{2}},$$

since the alternating power, $2\,E_1 I_1$, must equal the direct-current power, EI.

Connecting three equidistant points of the commutator to three collector rings as in Fig. 158 gives a three-phase converter.

113. In Fig. 159 the three e.m.fs. between the three collector rings and the neutral point of the three-phase system (or Y voltages) are represented by the vectors $\overline{OE_1}$, $\overline{OE_2}$, $\overline{OE_3}$, thus

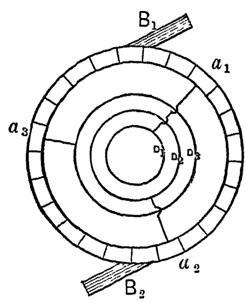


Fig. 158. Three-Phase Synchronous Converter.

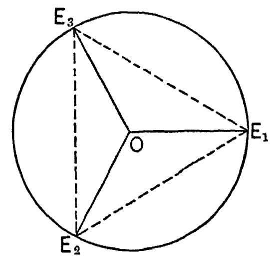


Fig. 159. E.m.f. Diagram of Three-Phase Converter.

the e.m.f. between the collector rings or the delta voltages by vectors $\overline{E_1E_2}$, $\overline{E_2E_3}$, and $\overline{E_3E_1}$. The e.m.f. $\overline{OE_1}$ is, however, nothing but half the e.m.f. E_1 in Fig. 156, of the single-phase converter, that is, $= \frac{E}{2\sqrt{2}}$. Hence the Y voltage, or voltage between collector ring and neutral point or center of the three-phase voltage triangle, is

$$E_1 = \frac{E}{2\sqrt{2}},$$

and thus the delta voltage is

$$E' = E_1\sqrt{3} = \frac{E\sqrt{3}}{2\sqrt{2}} = 0.612\,E.$$

Since the total three-phase power $3\,I_1E_1$ equals the total continuous-current power IE, it is

$$I_1 = \frac{IE}{3\,E_1} = \frac{2\sqrt{2}}{3} I = .943\,I.$$

In general, in an n-phase converter, or converter in which n equidistant points of the commutator (in a bipolar machine, or n equidistant points per pair of poles in a multipolar machine with multiple-wound armature), are connected to n collector rings, the voltage between any collector ring and the common neutral, or star voltage, is

$$E_1 = \frac{E}{2\sqrt{2}},$$

consequently the voltage between two adjacent collector rings, or ring voltage, is

$$E' = 2\,E_1 \sin\frac{\pi}{n} = \frac{E \sin\frac{\pi}{n}}{\sqrt{2}},$$

since $\frac{2\pi}{n}$ is the angular displacement between two adjacent collector rings, and herefrom the current per line, or star current, is found as

$$I_1 = \frac{2\sqrt{2}\,I}{n},$$

and the current from line to line, or from collector ring to adjacent collector ring, or ring current, is

$$I' = \frac{\sqrt{2}\,I}{n \sin\frac{\pi}{n}}.$$

114. As seen in the preceding, in the single-phase converter consisting of a closed-circuit armature tapped at two equidistant points to the two collector rings, the alternating voltage is $\frac{1}{\sqrt{2}}$ times the direct-current voltage, and the alternating-current $\sqrt{2}$ times the direct current. While such an arrangement of the single-phase converter is the simplest, requiring only two collector rings, it is undesirable especially for larger machines, on account of the great total and especially local I^2r heating in the armature conductors, as will be shown in the following, and due to the waste of e.m.f., since in the circuit from collector ring to collector ring the e.m.fs. generated in the coils next to the leads are wholly or almost wholly opposite to each other.

The arrangement which I have called the *two-circuit single-*

phase converter, and which is diagrammatically shown in Fig. 160, is therefore preferable. The step-down transformer T contains two independent secondary coils A and B, of which, one, A, feeds into the armature over conductor rings D_1D_2 and leads a_1a_2, the other, B, over collector rings D_3D_4 and leads a_3a_4,

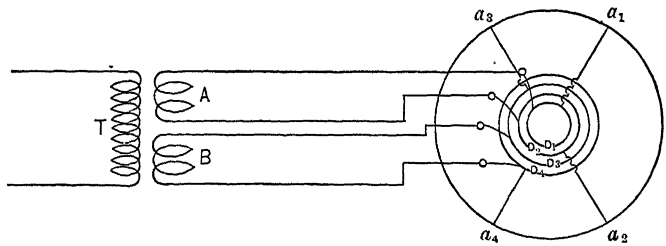


Fig. 160. Two-Circuit Single-Phase Converter.

so that the two circuits a_1a_2 and a_3a_4 are in phase with each other, and each spreads over 120 deg. arc instead of 180 deg. arc as in the single-circuit single-phase converter.

In consequence thereof, in the two-circuit single-phase converter the alternating counter-generated e.m.f. bears to the continuous current e.m.f. the same relation as in the three-phase converter, that is,

$$E_1 = \frac{\sqrt{3}}{2\sqrt{2}} E = 0.612\,E,$$

and from the equality of alternating- and direct-current power,

$$2\,I_1E_1 = IE,$$

it follows that each of the two single-phase supply currents is

$$I' = \frac{\sqrt{2}}{\sqrt{3}} I = 0.817\,I.$$

It is seen that in this arrangement one-third of the armature, from a_1 to a_3 and from a_2 to a_4, carries the direct current only, the other two-thirds, from a_1 to a_2 and from a_3 to a_4, the differential current.

A six-phase converter is usually fed from a three-phase system by three transformers or one three-phase transformer. These transformers can either have each one secondary coil only of twice the star or Y voltage, $= \frac{E}{\sqrt{2}}$, which connects with its two terminals two collector rings leading to two opposite points of the armature, or each of the stepdown transformers contains two independent secondary coils, and each of the two sets of secondary coils is connected in three-phase delta or Y, but the one set of coils reversed with regard to each other, thus giving two three-phase systems which join to a six-phase system.

For further arrangements of six-phase transformation, see "Theory and Calculation of Alternating-Current Phenomena," fourth edition, Chapter XXXVI.

The table below gives, with the direct-current voltage and direct current as unit, the alternating voltages and currents of the different converters.

	Direct Current.	Single-Circuit Single-phase	Two-Circuit Single-phase.	Three-phase.	Four-phase.	Six-phase.	Twelve-phase.	n-phase
Volts between collector ring and neutral point ..		$\frac{1}{2\sqrt{2}}$ =0.354	$\frac{1}{2\sqrt{2}}$ =0.354	$\frac{1}{2\sqrt{2}}$ =0 354	$\frac{1}{2\sqrt{2}}$ =0 354	$\frac{1}{2\sqrt{2}}$ =0 354	$\frac{1}{2\sqrt{2}}$ =0 354	$\frac{1}{2\sqrt{2}}$ =0 354
Volts between adjacent collector rings......	1.0	$\frac{1}{\sqrt{2}}$ =0 707	$\frac{\sqrt{3}}{2\sqrt{2}}$ =0 612	$\frac{\sqrt{3}}{2\sqrt{2}}$ =0.612	$\frac{1}{2}$=0 5	$\frac{1}{2\sqrt{2}}$ =0.354	0 183	$\frac{\sin\frac{\pi}{n}}{\sqrt{2}}$
Amperes per line .	1.0	$\sqrt{2}$ =1.414	$\frac{\sqrt{2}}{\sqrt{3}}$ =0 817	$\frac{2\sqrt{2}}{3}$ =0 943	$\frac{1}{\sqrt{2}}$ =0.707	$\frac{\sqrt{2}}{3}$ =0.472	0 236	$\frac{2\sqrt{2}}{n}$
Amperes between adjacent lines .		$\sqrt{2}$ =1 414	$\frac{\sqrt{2}}{\sqrt{3}}$ =0.817	$\frac{2\sqrt{2}}{3\sqrt{3}}$ =0.545	$\frac{1}{2}$=0.5	$\frac{\sqrt{2}}{3}$ =0.472	0 455	$\left\{\frac{\sqrt{2}}{n}\times \sin\frac{\pi}{n}\right.$

These currents give only the power component of alternating current corresponding to the direct-current output. Added thereto is the current required to supply the losses in the machine, that is, to rotate it, and the wattless component if a phase displacement is produced in the converter.

III. Variation of the Ratio of Electromotive Forces.

115. The preceding ratios of e.m.fs. apply strictly only to the generated e.m.fs. and that under the assumption of a sine wave of alternating generated e.m.f.

The latter is usually a sufficiently close approximation, since the armature of the converter is a multitooth structure, that is, contains a distributed winding.

The ratio between the difference of potential at the commutator brushes and that at the collector rings of the converter usually differs somewhat from the theoretical ratio, due to the e.m.f. consumed in the converter armature, and in machines converting from alternating to continuous current, also due to the shape of the impressed wave.

When converting from alternating to direct current, under load the difference of potential at the commutator brushes is less than the generated direct e.m.f., and the counter-generated alternating e.m.f. less than the impressed, due to the voltage consumed by the armature resistance.

If the current in the converter is in phase with the impressed e.m.f., armature self-inductance has little effect, but reduces the counter-generated alternating e.m.f. below the impressed with a lagging and raises it with a leading current, in the same way as in a synchronous motor.

Thus in general the ratio of voltages varies somewhat with the load and with the phase relation, and with constant impressed alternating e.m.f. the difference of potential at the commutator brushes decreases with increasing load, decreases with decreasing excitation (lag), and increases with increasing excitation (lead).

When converting from direct to alternating current the reverse is the case.

The direct-current voltage stands in definite proportion only to the maximum value of the alternating voltage (being equal to twice the maximum star voltage), but to the effective value (or value read by voltmeter) only in so far as the latter depends upon the former, being $= \frac{1}{\sqrt{2}}$ maximum value with a sine wave.

Thus with an impressed wave of e.m.f. giving a different ratio of maximum to effective value, the ratio between direct and alternating voltage is changed in the same proportion as the ratio of maximum to effective; thus, for instance, with a flat-topped wave of impressed e.m.f., the maximum value of alternating impressed e.m.f., and thus the direct voltage depending thereupon, are lower than with a sine wave of the same effective value, while with a peaked wave of impressed e.m.f. they are higher, by as much as 10 per cent in extreme cases.

In determining the wave shape of impressed e.m.f. at the converter terminals, not only the wave of generated generator e.m.f., but also that of the converter counter-generated e.m.f., may be instrumental. Thus, with a converter connected directly to a generating system of very large capacity, the impressed e.m.f. wave will be practically identical with the generator wave, while at the terminals of a converter connected to the generator over long lines with reactive coils or inductive regulators interposed, the wave of impressed e.m.f. may be so far modified by that of the counter e.m.f. of the converter as to resemble the latter much more than the generator wave, and thereby the ratio of conversion may be quite different from that corresponding to the generator wave.

Furthermore, for instance, in three-phase converters fed by ring or delta connected transformers, the star e.m.f. at the converter terminals, which determines the direct voltage, may differ from the star e.m.f. impressed by the generator, by containing different third and ninth harmonics, which cancel when compounding the star voltages to the delta voltage, and give identical delta voltages, as required.

Hence, the ratios of e.m.fs. given in Section II have to be corrected by the drop of voltage in the armature, and have to be multiplied by a factor which is $\sqrt{2}$ times the ratio of effective to maximum value of impressed wave of star e.m.f. ($\sqrt{2}$ being the ratio of maximum to effective of the sine wave on which the ratios in Section II were based), that is, by the "form factor" of the e.m.f. wave.

With an impressed wave differing from sine shape, there is a current of higher frequency, but generally of negligible magnitude, through the converter armature, due to the difference between impressed and counter e.m.f. wave.

IV. Armature Current and Heating.

116. The current in the armature conductors of a converter is the difference between the alternating-current input and the direct-current output.

In Fig. 161, a_1, a_2 are two adjacent leads connected with the collector rings D_1, D_2 in an n-phase converter. The alternating e.m.f. between a_1 and a_2, and thus the power component of the alternating current in the armature section between a_1 and a_2, will reach a maximum when this section is midway between the brushes B_1 and B_2, as shown in Fig. 161.

The direct current in every armature coil reverses at the moment when the coil passes under brush B_1 or B_2, and is thus a rectangular alternating current as shown in Fig. 162 as I. At the moment when the power component of the alternating current is a maximum, an armature coil d midway between two adjacent alternating leads a_1 and a_2 is midway between the brushes B_1 and B_2, as in Fig. 161, and is thus in the middle of its rectangular continuous-current wave, and consequently in this coil the power component of the alternating current and the rectangular direct current are in phase with each other, but opposite, as shown in Fig. 162 as I_1 and I, and the actual current is their difference, as shown in Fig. 163.

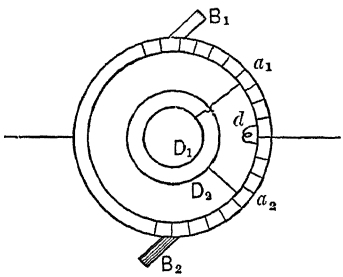


Fig. 161. Diagram for Study of Armature Heating in Synchronous Converters.

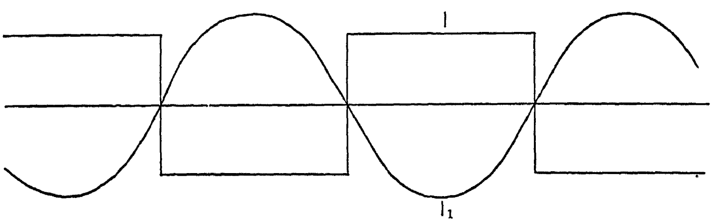


Fig. 162. Direct Current and Alternating Current in Armature Coil d, Fig. 161.

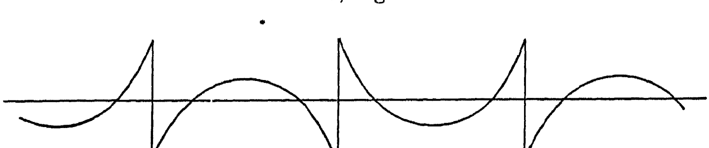


Fig. 163. Resultant Current in Coil d, Fig. 161.

In successive armature coils the direct current reverses successively; that is, the rectangular currents in successive armature coils are successively displaced in phase from each other; and since the alternating current is the same in the whole section a_1 a_2, and in phase with the rectangular current in the coil d, it becomes more and more out of phase with the rectangular current when passing from coil d towards a_1 or a_2, as shown in Figs. 164 to 167, until the maximum phase displacement between alternating and rectangular current is reached at the alternating leads a_1 and a_2, and is equal to $\frac{\pi}{n}$.

117. Thus, if E = direct voltage, and I = direct current, in an armature coil displaced by angle τ from the position d, midway between two adjacent leads of the n-phase converter, the direct current is $\frac{I}{2}$ for the half period from 0 to π, and the alternating current is

$$\sqrt{2}\, I' \sin(\theta - \tau),$$

where

$$I' = \frac{I\sqrt{2}}{n \sin \frac{\pi}{n}}$$

is the effective value of the alternating current. Thus, the actual current in this armature coil is

$$\begin{aligned} i_0 &= \sqrt{2}\, I' \sin(\theta - \tau) - \frac{I}{2} \\ &= \frac{I}{2}\left\{\frac{4 \sin(\theta - \tau)}{n \sin \frac{\pi}{n}} - 1\right\}. \end{aligned}$$

In a double-current generator, instead of the minus sign, a plus sign would connect the alternating and the direct current in the parenthesis

The effective value of the resultant converter current thus is:

$$\begin{aligned} I_0 = \sqrt{\frac{1}{\pi}\int_0^\pi i_0^2 d\theta} &= \frac{I}{2}\sqrt{\frac{1}{\pi}\int_0^\pi \left\{\frac{4 \sin(\theta - \tau)}{n \sin \frac{\pi}{n}} - 1\right\}^2 d\theta} \\ &= \frac{I}{2}\sqrt{\frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \tau}{n\pi \sin \frac{\pi}{n}}}. \end{aligned}$$

Since $\frac{I}{2}$ is the current in the armature coil of a direct-current

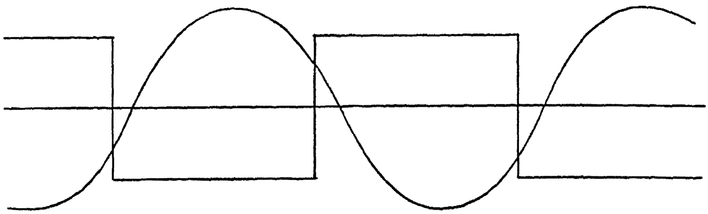


Fig. 164. Alternating Current and Direct Current in Coil between d and a_1 or a_2, Fig. 161.

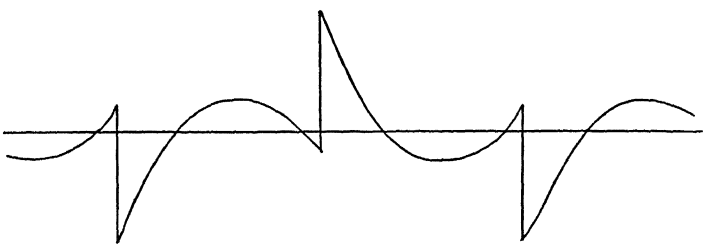


Fig. 165. Resultant of Currents Given in Fig. 164.

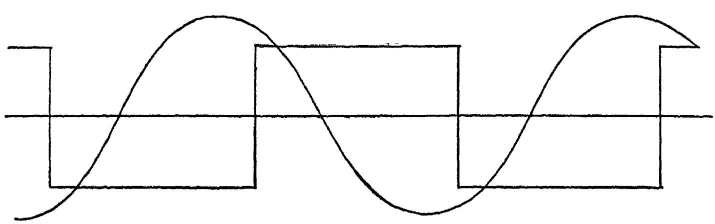


Fig. 166. Alternating Current and Direct Current in Coil between d and a_1 or a_2, Fig. 161.

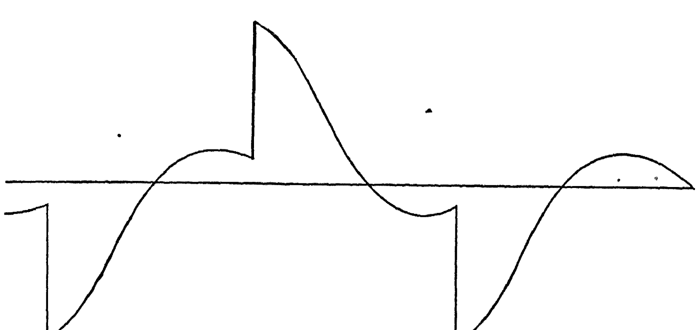


Fig. 167. Resultant of Currents Shown in 166.

generator of the same output, we have

$$\gamma_\tau = \left[\frac{I_0}{\frac{I}{2}}\right]^2 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \tau}{n\pi \sin \frac{\pi}{n}},$$

the ratio of the power loss in the armature coil resistance of the converter to that of the direct-current generator of the same output, and thus the ratio of coil heating.

This ratio is a maximum at the position of the alternating leads, $\tau = \frac{\pi}{n}$, and is

$$\gamma_m = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \frac{\pi}{n}}{n\pi \sin \frac{\pi}{n}}.$$

It is a minimum for a coil midway between adjacent alternating leads, $\tau = 0$, and is

$$\gamma_0 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16}{n\pi \sin \frac{\pi}{n}}.$$

Integrating over τ from 0 (coil d) to $\frac{\pi}{n}$, that is, over the whole phase or section $a_1\ a_2$, we have

$$\Gamma = \frac{n}{\pi}\int_0^{\frac{\pi}{n}} \gamma_\tau d\tau = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16}{\pi^2},$$

the ratio of the total power loss in the armature resistance of an n-phase converter to that of the same machine as direct-current generator at the same output, or the relative armature heating.

Thus, to get the same loss in the armature conductors, and consequently the same heating of the armature, the current in the converter, and thus its output, can be increased in the proportion $\frac{1}{\sqrt{\Gamma}}$ over that of the direct-current generator.

The calculation for the two-circuit single-phase converter is somewhat different; since in this in one-third of the armature the I^2r loss is that of the direct-current output, and only in the other two-thirds — or an arc $\frac{2\pi}{3}$ — is there alternating current. Thus in an armature coil displaced by angle τ from the center of this latter section the resultant current is

$$i_0 = \sqrt{2}\, I' \sin(\theta - \tau) - \frac{I}{2}$$
$$= \frac{I}{2}\left\{\frac{4}{\sqrt{3}} \sin(\theta - \tau) - 1\right\},$$

giving the effective value

$$I_0 = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_0^2\, d\theta} = \frac{I}{2}\sqrt{\frac{11}{3} - \frac{16}{\pi\sqrt{3}} \cos\tau};$$

thus, the relative heating is

$$\gamma_\tau = \left(\frac{I_0}{\frac{I}{2}}\right)^2 = \frac{11}{3} - \frac{16}{\pi\sqrt{3}} \cos\tau,$$

with the minimum value at $\tau = 0$, it is

$$\gamma_0 = \frac{11}{3} - \frac{16}{\pi\sqrt{3}} = 0.70,$$

and with the maximum value at $\tau = \frac{\pi}{3}$, it is

$$\gamma_m = \frac{11}{3} - \frac{8}{\pi\sqrt{3}} = 2.18;$$

the average current heating in two-thirds of the armature is

$$\Gamma_1 = \frac{3}{\pi}\int^{\frac{\pi}{3}} \gamma_\tau\, d_\tau = \frac{11}{3} - \frac{48}{\pi^2\sqrt{3}} \sin\frac{\pi}{3}$$
$$= \frac{11}{3} - \frac{24}{\pi^2} = 1.236;$$

in the remaining third of the armature, $\Gamma_2 = 1$, thus the average is

$$\Gamma = \frac{2\,\Gamma_1 + \Gamma_2}{3}$$
$$= 1.151,$$

and therefore the rating is

$$\frac{1}{\sqrt{\Gamma}} = 0.93.$$

By substituting for n in the general equations of current heating and rating based thereon, numerical values, we get the following table:

Type.	Direct-current generator.	Single-circuit single-phase	Two-circuit single-phase.	Three-phase.	Four-phase.	Six-phase.	Twelve-phase.	∞-phase.
n	....	2	2	3	4	6	12	
γ_0	1 00	0 45	0 70	0.225	.20	0.19	0.187	0 187
γ_m	1.00	3 00	2 18	1.20	.73	0 42	0.24	0 187
Γ	1.00	1 37	1.157	0 555	37	0 26	0 20	0 187
Rating (by mean arm. heating),	1.00	0 85	0 93	1.34	1 64	1.96	2 24	2.31

As seen, in the two-circuit single-phase converter the armature heating is less, and more uniformly distributed, than in the single-circuit single-phase converter.

118. A very great gain is made in the output by changing from three-phase to six-phase, but relatively little by still further increasing the number of phases.

In these values, the small power component of current supplying the losses in the converter has been neglected.

These values apply only to the case where the alternating current is in phase with the supply voltage, that is, for unity power-factor of supply. If, however, the current lags, or leads, by the time angle θ, then the alternating current and direct current are not in opposition in the armature coil d midway between adjacent leads, Fig. 161, and the resultant current is a minimum and of the shape shown in Fig. 163, at a point of the armature winding displaced from mid position d by angle $\tau = \theta$. At the leads the displacement between alternating current and direct current then is not $\frac{\pi}{n}$, but $\frac{\pi}{n} + \theta$ at the one, $\frac{\pi}{n} - \theta$ at the other lead, and thus at the other side of the same lead. The resultant current is thus increased at the one, decreased at the other lead, and the heating changed accordingly.

For instance, in a quarter-phase converter at zero phase displacement, the resultant current at the lead would be as shown in Fig. 168, $\frac{\pi}{n} = 45$ deg., while at 30 deg. lag the resultant currents in the two coils adjacent to the commutator lead are displaced respectively by $\frac{\pi}{n} + \theta = 75$ deg. and by $\frac{\pi}{n} - \theta = 15$ deg., and so of very different shape, as shown by Figs. 169 and 170, giving very different local heating. Phase displacement thus increases the heating at the one, decreases it at the other side of each commutator lead.

Let again,

I = direct current per commutator brush.

The effective value of the alternating power current in the armature winding, or ring current, corresponding thereto, is

$$I' = \frac{I\sqrt{2}}{n \sin \frac{\pi}{n}}.$$

Let pI' = total power current, allowing for the losses of power in the converter; qI' = reactive current in the converter, assumed as positive when lagging, as negative when leading, and sI' = total current, where $s = \sqrt{p^2 + q^2}$ is the ratio of total current to the load current, that is, power current corresponding to the direct-current output, and $\frac{q}{p} = \tan \theta$ is the time lag of the supply current; p is a quantity slightly larger than 1, by the losses in the converter, or slightly smaller than 1 in an inverted converter.

The actual current in an armature coil displaced in position by angle τ from the middle position d between the adjacent collector leads, then, is

$$i_0 = \sqrt{2}\, I' \{p \sin(\beta - \tau) - q \cos(\beta - \tau)\} - \frac{I}{2}$$

$$= \frac{I}{2}\left\{\frac{4}{n \sin \frac{\pi}{n}} [p \sin(\beta - \tau) - q \cos(\beta - \tau)] - 1\right\},$$

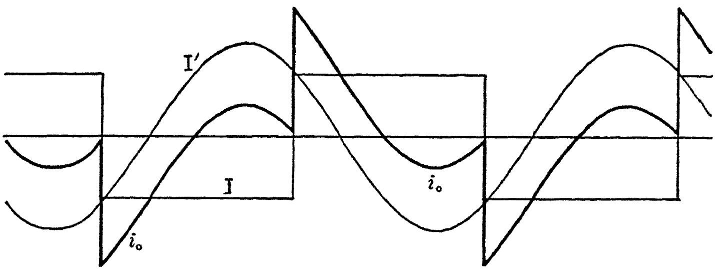


Fig. 168. Quarter-Phase Converter Unity Power Factor, Armature Current at Collector Lead.

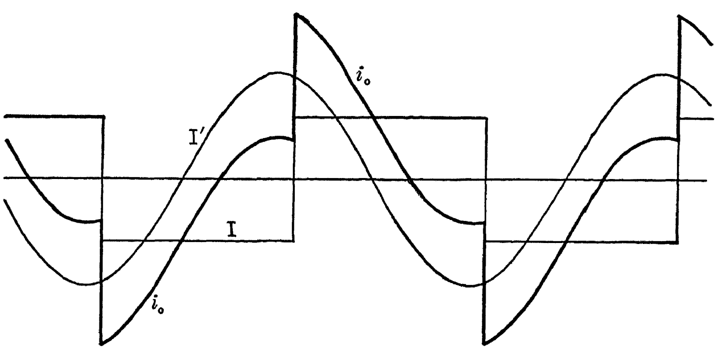


Fig. 169. Quarter-Phase Converter Phase Displacement 30 Degrees, Armature Current at Collector Lead.

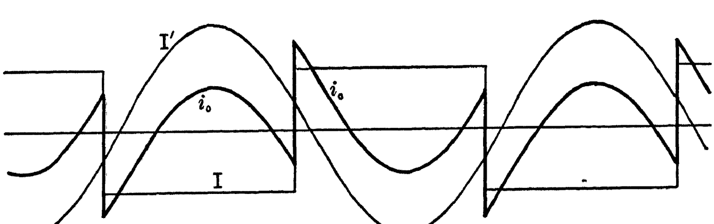


Fig. 170. Quarter-Phase Converter Phase Displacement 30 Degrees, Armature Current at Collector Lead.

and, therefore, its effective value is

$$I_0 = \sqrt{\frac{1}{\pi}\int_0^\pi i_0^2 d\beta}$$

$$= \frac{I}{2}\sqrt{1 + \frac{8(p^2+q^2)}{n^2\sin^2\frac{\pi}{n}} - \frac{16\,(p\cos\tau + q\sin\tau)}{\pi n\sin\frac{\pi}{n}}}$$

$$= \frac{I}{2}\sqrt{1 + \frac{8\,s^2}{n^2\sin^2\frac{\pi}{n}} - \frac{16\,s\cos(\tau-\theta)}{\pi n\sin\frac{\pi}{n}}},$$

and herefrom the relative heating in an armature coil displaced by angle τ from the middle between adjacent commutator leads:

$$\gamma_\tau = 1 + \frac{8\,s^2}{n^2\sin^2\frac{\pi}{n}} - \frac{16\,s\cos(\tau-\theta)}{\pi n\sin\frac{\pi}{n}};$$

this gives at the leads, or for $\tau = \mp\frac{\pi}{n}$,

$$\gamma_m = 1 + \frac{8\,s^2}{n^2\sin^2\frac{\pi}{n}} - \frac{16\,s\cos\left(\frac{\pi}{n}+\theta\right)}{\pi n\sin\frac{\pi}{n}},$$

$$\gamma_m' = 1 + \frac{8\,s^2}{n^2\sin^2\frac{\pi}{n}} - \frac{16\,s\cos\left(\frac{\pi}{n}-\theta\right)}{\pi n\sin\frac{\pi}{n}}.$$

Averaging from $-\frac{\pi}{n}$ to $+\frac{\pi}{n}$ gives the mean current-heating of the converter armature.

$$\Gamma = \frac{2n}{\pi}\int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} \gamma_\tau\, d\tau$$

$$= 1 + \frac{8\,s^2}{n^2\sin^2\frac{\pi}{n}} - \frac{8\,s\left[\sin\left(\frac{\pi}{n}+\theta\right) + \sin\left(\frac{\pi}{n}-\theta\right)\right]}{\pi^2\sin\frac{\pi}{n}}$$

$$= 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,s \cos\theta}{\pi^2}$$

$$= 1 + \frac{8\,(p^2 + q^2)}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,p}{\pi^2}.$$

119. This gives for

Three-phase, $n = 3$:

$$\gamma_\tau = 1 + 1.185\,s^2 - 1.955\,s \cos(\tau - \theta),$$
$$\gamma_m = 1 + 1.185\,s^2 - 1.955\,s \cos(60 \pm \theta),$$
$$\Gamma = 1 + 1.185\,s^2 - 1.620\,p.$$

Quarter-phase, $n = 4$:

$$\gamma_\tau = 1 + s^2 - 1.795\,s \cos(\tau - \theta),$$
$$\gamma_m = 1 + s^2 - 1.795\,s \cos(45 \pm \theta),$$
$$\Gamma = 1 + s^2 - 1.620\,p.$$

Six-phase, $n = 6$:

$$\gamma_\tau = 1 + 0.889\,s^2 - 1.695\,s \cos(\tau - \theta),$$
$$\gamma_m = 1 + 0.889\,s^2 - 1.695\,s \cos(30 + \theta),$$
$$\Gamma = 1 + 0.889\,s^2 - 1.62\,p,$$

∞-phase, $n = \infty$:

$$\gamma_\tau = \gamma_m = \Gamma = 1 + 0.810\,s^2 - 1.62\,s \cos\theta$$
$$= 1 + 0.810\,s^2 - 1.62\,p.$$

Choosing $p = 1.04$, that is, assuming 4 per cent loss in friction and windage, core loss and field excitation, — the i^2r loss of the armature is not included in p, as it is represented by a drop of direct-current voltage below that corresponding to the alternating voltage, and not by an increase of the alternating current over that corresponding to the direct current — we get, for different phase angles from $\theta = 0$ deg. to $\theta = 60$ deg., the values given below:

$\theta =$	0	10	20	30	40	50	60
$s = \frac{p}{\cos\theta} =$	1.04	1.056	1.108	1.20	1.36	1.62	2.08
$q = s \sin\theta = \frac{\text{react. cur.}}{\text{power cur.}} =$	0	0 184	0.379	0.60	0.876	1.24	1.80
$\tan\theta =$	0	0.176	0.364	0.577	0.839	1.192	1.732
Three-phase:							
$\gamma_m =$	1.26	1.62	2.08	2.70	3.65	5 19	8.16
$\gamma_m' =$		1.00	0.80	0 68	0.70	0.99	2.06
$\Gamma =$	0.60	0.64	0.77	1.02	1.51	2.43	4.45

θ =	0	10	20	30	40	50	60
Quarter-phase:							
γ_m =	0·76	1.02	1.39	1.88	2.64	3.87	6.30
γ_m' =	0·76	0.55	0.43	0.38	0.42	0.73	1.71
Γ =	0.40	0.43	0.54	0.75	1.16	1.94	3.64
Six-phase:							
γ_m =	0.44	0.62	0.88	1.27	1.86	2.85	4.85
γ_m' =	0.44	0.31	0.24	0.25	0.38	0.75	1.79
Γ =	0.28	0.31	0.41	0.60	0.97	1.65	3.17
∞-phase:							
$\gamma_m = \gamma_m' = \Gamma$ =	0.20	0.22	0.32	0.49	0.82	1.45	2.82

120. The values are shown graphically in Figs. 171 and 172, with $\tan\theta = \frac{\text{reactive current}}{\text{energy current}}$ as abscissas, and γ_m as ordinates in Fig. 171, Γ as ordinates in Fig. 172.

As seen, with increasing phase displacement, irrespectively whether lag or lead, the average as well as the maximum armature heating very greatly increases. This shows the necessity of keeping the power-factor near unity at full load and overload, and when applied to phase control of the voltage by converter, means that the shunt field of the converter should be adjusted so as to give a considerable lagging current at no load, so that the current comes into phase with the voltage at about full load. It therefore is very objectionable in this case to adjust the converter for minimum current at no load, as occasionally done by ignorant engineers, since such wrong adjustment would give considerable leading current at load, and therewith unnecessary armature heating.

It must be considered, however, that above values are referred to the direct-current output, and with increase of phase angle the alternating-current input, at the same output, increases, and the heating increases with the square of the current. Thus at 60 deg. lag or lead, the power-factor is 0.5, and the alternating-current input thus twice as great as at unity power-factor, corresponding to four times the heating. It is interesting therefore to refer the armature heating to the alternating-current input, that is, compare the heating of the converter with that of a synchronous motor of the same alternating-current input. This is given by

$$\Gamma_1 = \frac{\Gamma_2}{s}$$

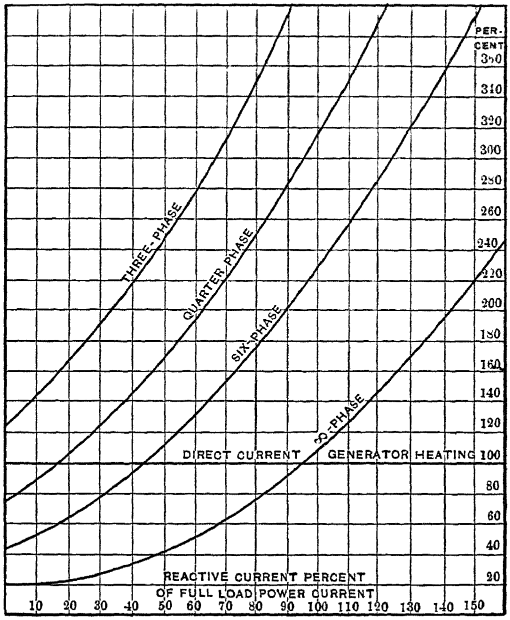


Fig. 171. Maximum I^2r Heating in Converter Armature Coil Expressed in Per Cent of Direct-Current Generator I^2r Heating.

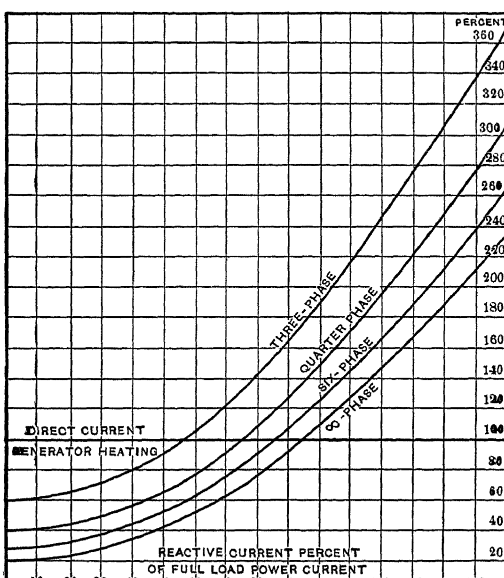


Fig. 172. Average I^2r Heating in Converter Armature Expressed in Per Cent of Direct-Current Generator I^2r Heating.

and, for $p = 1.04$, gives the following values:

$\theta =$	0	10	20	30	40	50	60
$\tan\theta =$	0	0.176	0.364	0.577	0.839	1.192	14.32
Three-phase:							
$\Gamma_1 =$	0.555	0.57	0.63	0.71	0.82	0.93	1.03
Quarter-phase:							
$\Gamma_1 =$	0.37	0.385	0.44	0.52	0.63	0.74	0.84
Six-phase:							
$\Gamma_1 =$	0.26	0.28	0.335	0.42	0.52	0.63	0.73
∞-phase:							
$\Gamma_1 =$	0.185	0.197	0.26	0.34	0.44	0.55	0.65

It is seen that, compared with the total alternating-current input, the armature heating increases much less with increasing phase displacement, and is almost always much lower than the heating of the same machine at the same input and phase angle, when running a synchronous motor, as shown in Fig. 173.

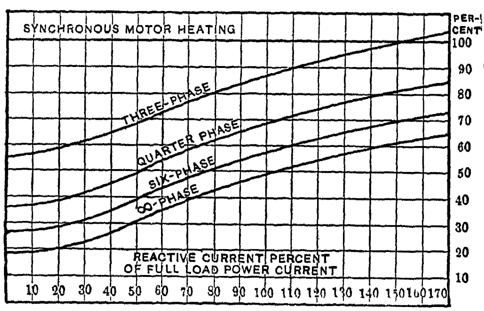


Fig. 173. Average I^2r Heating in Converter Armature Expressed in Per Cent of Synchronous Motor I^2r Heating at the Same Power-Factor

V. Armature Reaction.

121. The armature reaction of the polyphase converter is the resultant of the armature reactions of the machine as direct-current generator and as synchronous motor. If the commutator brushes are set at right angles to the field poles or without lead or lag, as is usually done in converters, the direct-current armature reaction consists in a polarization in quadrature behind the field magnetism. The armature reaction due to the power component of the alternating current in a synchro-

nous motor consists of a polarization in quadrature ahead of the field magnetism, which is opposite to the armature reaction as direct-current generator.

Let m = total number of turns on the bipolar armature or per pair of poles of an n-phase converter, I = direct current, then the number of turns in series between the brushes $= \frac{m}{2}$, hence the total armature ampere-turns, or polarization, $= \frac{mI}{2}$. Since, however, these ampere-turns are not unidirectional, but distributed over the whole surface of the armature, their resultant is

$$\mathfrak{F} = \frac{mI}{2} \text{ avg. } \cos \left\{ \begin{array}{l} +\frac{\pi}{2} \\ -\frac{\pi}{2} \end{array} \right.$$

and, since

$$\text{avg. } \cos \left\{ \begin{array}{l} +\frac{\pi}{2} \\ -\frac{\pi}{2} \end{array} \right. = \frac{2}{\pi},$$

we have $\mathfrak{F} = \frac{mI}{\pi}$ = direct-current polarization of the converter (or direct-current generator) armature.

In an n-phase converter the number of turns per phase $= \frac{m}{n}$. The current per phase, or current between two adjacent leads (ring current), is

$$I' = \frac{\sqrt{2}\, I}{n \sin \frac{\pi}{n}};$$

hence, the ampere-turns per phase,

$$\frac{mI'}{n} = \frac{\sqrt{2}\, mI}{n^2 \sin \frac{\pi}{n}}.$$

These ampere-turns are distributed over $\frac{1}{n}$ of the circumference of the armature, and their resultant is thus

$$\mathfrak{F}_1 = \frac{mI'}{n} \text{ avg. } \cos \left\{ \begin{array}{l} +\frac{\pi}{n} \\ -\frac{\pi}{n} \end{array} \right. ,$$

and, since

$$\text{avg. } \cos \left\{ \begin{array}{l} +\frac{\pi}{n} \\ -\frac{\pi}{n} \end{array} \right. = \frac{n}{\pi} \sin \frac{\pi}{n},$$

we have

$$\mathfrak{F}_1 = \frac{\sqrt{2}\, mI}{\pi n} = \text{resultant polarization,}$$

in effective ampere-turns of one phase of the converter.

The resultant m.m.f. of n equal m.m.fs. of effective value of $\mathfrak{F}_1$, thus maximum value of $\mathfrak{F}_1\sqrt{2}$, acting under equal angles $\frac{2\pi}{n}$, and displaced in phase from each other by $\frac{1}{n}$ of a period, or phase angle $\frac{2\pi}{n}$, is found thus:

Let $\mathfrak{F}_i = \mathfrak{F}_1\sqrt{2}\sin\left(\theta - \frac{2i\pi}{n}\right)$ = one of the m.m.fs. of phase angle $\theta = \frac{2i\pi}{n}$, acting in the direction $\tau = \frac{2i\pi}{n}$; that is, the zero point of one of the m.m.fs. $\mathfrak{F}_1$ is taken as zero point of time θ, and the direction of this m.m.f. as zero point of direction τ.

The resultant m.m.f. in any direction τ is thus

$$\begin{aligned}
\mathfrak{F} &= \sum_1^n{}_i\, \mathfrak{F}\cos\left(\tau + \frac{2i\pi}{n}\right)\\
&= \mathfrak{F}_1\sqrt{2}\sum_1^n{}_i \sin\left(\theta - \frac{2i\pi}{n}\right)\cos\left(\tau - \frac{2i\pi}{n}\right)\\
&= \frac{\mathfrak{F}_1\sqrt{2}}{2}\sum_1^n{}_i\left\{\sin\left(\theta + \tau - \frac{4i\pi}{n}\right) + \sin(\theta - \tau)\right\}\\
&= \frac{\mathfrak{F}_1\sqrt{2}}{2}\left\{\sum_1^n{}_i \sin\left(\theta + \tau - \frac{4i\pi}{n}\right) + n\sin(\theta - \tau)\right\},
\end{aligned}$$

and, since

$$\sum_1^n{}_i \sin\left(\theta + \tau - \frac{4i\pi}{n}\right) = 0,$$

we have

$$\mathfrak{F} = \frac{n\mathfrak{F}_1\sqrt{2}}{2}\sin(\theta - \tau);$$

that is, the resultant m.m.f. in any direction τ has the phase

$$\theta = \tau,$$

and the intensity,

$$\mathfrak{F}' = \frac{n\mathfrak{F}_1\sqrt{2}}{2},$$

thus revolves in space with uniform velocity and constant intensity, in synchronism with the frequency of the alternating current.

Since in the converter,

$$\mathfrak{F}_1 = \frac{\sqrt{2}mI}{\pi n},$$

we have

$$\mathfrak{F}' = \frac{mI}{\pi},$$

the resultant m.m.f. of the power component of the alternating current in the n-phase converter.

This m.m.f. revolves synchronously in the armature of the converter; and since the armature rotates at synchronism, the resultant m.m.f. stands still in space, or, with regard to the field poles, in opposition to the direct-current polarization. Since it is equal thereto, it follows that the resultant armature reactions of the direct current and of the corresponding power component of the alternating current in the synchronous converter are equal and opposite, thus neutralize each other, and the resultant armature polarization equals zero. The same is obviously the case in an inverted converter, that is, a machine changing from direct to alternating current.

122. The conditions in a single-phase converter are different, however. At the moment when the alternating current = 0, the full direct-current reaction exists. At the moment when the alternating current is a maximum, the reaction is the difference between that of the alternating and of the direct current; and since the maximum alternating current in the single-phase converter equals twice the direct current, at this moment the resultant armature reaction is equal but opposite to the direct-current reaction.

Hence, the armature reaction oscillates with twice the frequency of the alternating current, and with full intensity, and since it is in quadrature with the field excitation, tends to shift the magnetic flux rapidly across the field poles, and thereby tends to cause sparking and power losses. This oscillating reaction is, however, reduced by the damping effect of the magnetic field structure. It is somewhat less in the two-circuit single-phase converter.

Since in consequence hereof the commutation of the single-phase converter is not as good as that of the polyphase converter, in the former usually voltage commutation has to be resorted to; that is, a commutating pole used, or the brushes shifted from the position mid-way between the field poles; and in the latter case the continuous-current ampere-turns inclosed by twice the angle of lead of the brushes act as a demagnetizing armature reaction, and require a corresponding increase of the field excitation under load.

123. Since the resultant main armature reactions neutralize each other in the polyphase converter, there remain only —

1. The armature reaction due to the small power component of current required to rotate the machine, that is, to cover the internal losses of power, which is in quadrature with the field excitation or distorting, but of negligible magnitude.

2. The armature reaction due to the wattless component of alternating current where such exists.

3. An effect of oscillating nature, which may be called a higher harmonic of armature reaction.

The direct current, as rectangular alternating current in the armature, changes in phase from coil to coil, while the alternating current is the same in a whole section of the armature between adjacent leads.

Thus while the resultant reactions neutralize, a local effect remains which in its relation to the magnetic field oscillates with a period equal to the time of motion of the armature through the angle between adjacent alternating leads; that is, double frequency in a single-phase converter (in which it is equal in magnitude to the direct-current reaction, and is the oscillating armature reaction discussed above), sextuple frequency in a three-phase converter, and quadruple frequency in a four-phase converter.

The amplitude of this oscillation in a polyphase converter is small, and its influence upon the magnetic field is usually negligible, due to the damping effect of the field spools, which act like a short-circuited winding for an oscillation of magnetism.

A polyphase converter on unbalanced circuit can be considered as a combination of a balanced polyphase and a single-phase converter; and since even single-phase converters operate quite satisfactorily, the effect of unbalanced circuits on the polyphase converter is comparatively small, within reasonable limits.

Since the armature reaction of the direct current and of the alternating current in the converter neutralize each other, no change of field excitation is required in the converter with changes of load.

Furthermore, while in a direct-current generator the armature reaction at given field strength is limited by the distortion of the field caused thereby, this limitation does not exist in a converter; and a much greater armature reaction can be safely used in converters than in direct-current generators, the distortion being absent in the former.

The practical limit of overload capacity of a converter is usually far higher than in a direct-current generator, since the armature heating is relatively small, and since the distortion of field, which causes sparking on the commutator under overloads in a direct-current generator, is absent in a converter.

The theoretical limit of overload — that is, the overload at which the converter as synchronous motor drops out of step and comes to a standstill — is usually far beyond reach at steady frequency and constant impressed alternating voltage, while on an alternating circuit of pulsating frequency or drooping voltage it obviously depends upon the amplitude and period of the pulsation of frequency or on the drop of voltage.

VI. Reactive Currents and Compounding.

124. Since the polarization due to the power component of the alternating current as synchronous motor is in quadrature ahead of the field magnetization, the polarization or magnetizing effect of the lagging component of alternating current is in phase, that of the leading component of alternating current in opposition to the field magnetization; that is, in the converter no magnetic distortion exists, and no armature reaction at all if the current is in phase with the impressed e.m.f., while the armature reaction is demagnetizing with a leading and magnetizing with a lagging current.

Thus if the alternating current is lagging, the field excitation at the same impressed e.m.f. has to be lower, and if the alternating current is leading, the field excitation has to be higher, than required with the alternating current in phase with the e.m.f. Inversely, by raising the field excitation a leading current, or by lowering it a lagging current, can be produced in a converter (and in a synchronous motor).

Since the alternating current can be made magnetizing or demagnetizing according to the field excitation, at constant impressed alternating voltage, the field excitation of the converter can be varied through a wide range without noticeably affecting the voltage at the commutator brushes; and in converters of high armature reaction and relatively weak field, full load and overload can be carried by the machine without any field excitation whatever, that is, by exciting the field by armature reaction by the lagging alternating current. Such converters without field excitation, or reaction converters, must always run with more or less lagging current, that is, give the same reaction on the line as induction motors, which, as known, are far more objectionable than synchronous motors in their reaction on the alternating system, and therefore they are no longer used.

Conversely, however, at constant impressed alternating voltage the direct-current voltage of a converter cannot be varied by varying the field excitation (except by the very small amount due to the change of the ratio of conversion), but a change of field excitation merely produces wattless currents, lagging or magnetizing with a decrease, leading or demagnetizing with an increase of field excitation. Thus to vary the continuous-current voltage of a converter usually the impressed alternating voltage has to be varied. This can be done either by potential regulator or compensator, that is, transformers of variable ratio of transformation, or by the effect of wattless currents on self-inductance. The latter method is especially suited for converters, due to their ability of producing wattless currents by change of field excitation.

The e.m.f. of self-inductance lags 90 deg. behind the current; thus, if the current is lagging 90 deg. behind the impressed e.m.f., the e.m.f. of self-inductance is 180 deg. behind, or in opposition to, the impressed e.m.f., and thus reduces it. If the current is 90 deg. ahead of the e.m.f., the e.m.f. of self-inductance is in phase with the impressed e.m.f., thus adds itself thereto and raises it. Therefore, if self-inductance is inserted into the lines between converter and constant-potential generator, and a wattless lagging current is produced by the converter by a decrease of its field excitation, the e.m.f. of self-inductance of this lagging current in the line lowers the alternating impressed voltage at the converter and thus its direct-current voltage; and if a wattless leading current is produced by the converter by an increase of its field excitation, the e.m.f. of self-inductance of this leading current raises the impressed alternating voltage at the converter and thus its direct-current voltage.

125. In this manner, by self-inductance in the lines leading to the converter, its voltage can be varied by a change of field excitation, or conversely its voltage maintained constant at constant generator voltage or even constant generator excitation, with increasing load and thus increasing resistance drop in the line; or the voltage can even be increased with increasing load, that is, the system over-compounded.

The change of field excitation of the converter with changes of load can be made automatic by the combination of shunt and series field, and in this manner a converter can be compounded or even over-compounded similarly to a direct-current generator. While the effect is the same, the action, however, is different; and the compounding takes place not in the machine as with a direct-current generator, but in the alternating lines leading to the machine, in which self-inductance becomes essential.

As the reactance of the transmission line is rarely sufficient to give phase control over a wide range without excessive reactive currents, it is customary, especially at 25 cycles, to insert reactive coils into the leads between the converter and its step-down transformers, in those cases in which automatic phase control by converter series fields is desired, as in power transmission for suburban and interurban railways, etc. Usually these reactive coils are designed to give at full-load current a reactance voltage equal to about 15 per cent of the converter supply voltage, and therefore capable of taking care of about 10 per cent line drop at good power factors.

VII. Variable Ratio Converters ("Split Pole") Converters.

126. With a sine wave of alternating voltage, and the commutator brushes set at the magnetic neutral, that is, at right angles to the resultant magnetic flux, the direct voltage of a converter is constant at constant impressed alternating voltage. It equals the maximum value of the alternating voltage between

two diametrically opposite points of the commutator, or "diametrical voltage," and the diametrical voltage is twice the voltage between alternating lead and neutral, or star or Y voltage of the polyphase system.

A change of the direct voltage, at constant impressed alternating voltage, can be produced —

Either by changing the position angle between the commutator brushes and the resultant magnetic flux, so that the direct voltage between the brushes is not the maximum diametrical alternating voltage but only a part thereof,

Or by changing the maximum diametrical alternating voltage, at constant effective impressed voltage, by wave-shape distortion by the superposition of higher harmonics.

In the former case, only a reduction of the direct voltage below the normal value can be produced, while in the latter case an increase as well as a reduction can be produced, an increase if the higher harmonics are in phase, and a reduction if the higher harmonics are in opposition to the fundamental wave of the diametrical or Y voltage.

A. Variable Ratio by a Change of the Position Angle between Commutator Brushes and Resultant Magnetic Flux.

127. Let, in the commutating machine shown diametrically in Fig. 174, the potential difference, or alternating voltage between one point a of the armature winding and the neutral 0 (that is, the Y voltage, or half the diametrical voltage), be represented by the sine wave, Fig. 176. This potential difference is a maximum, e, when a stands at the magnetic neutral, at A or B.

If, therefore, the brushes are located at the magnetic neutral, A and B, the voltage between the brushes is the potential difference between A and B, or twice the maximum Y voltage, $2\,e$, as indicated in Fig. 176. If now the brushes are shifted by an angle τ to position C and D, Fig. 175, the direct voltage between the brushes is the potential difference between C and D, or $2\,e \cos \tau$ with a sine wave. Thus, by shifting the brushes from the position A, B, at right angles with the magnetic flux, to the position E, F, in line with the magnetic flux, any direct voltage between 2 e and 0 can be produced, with the same wave of alternating voltage a.

As seen, this variation of direct voltage between its maximum value and zero, at constant impressed alternating voltage, is independent of the wave shape, and thus can be produced whether the alternating voltage is a sine wave or any other wave.

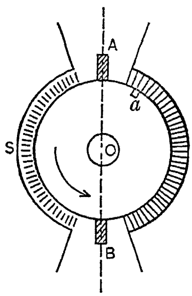


Fig. 174. Diagram of Commutating Machine with Brushes in the Magnetic Neutral.

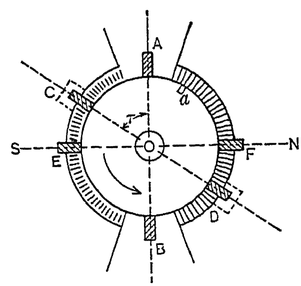


Fig. 175. E.m.f. Variation by Shifting the Brushes.

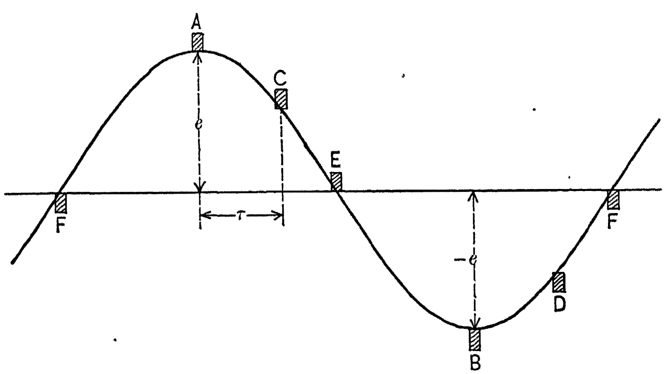


Fig. 176. Sine Wave of e.m.f.

It is obvious that, instead of shifting the brushes on the commutator, the magnetic field poles may be shifted, in the opposite direction, by the same angle, as shown in Fig. 177, A, B, C.

Instead of mechanically shifting the field poles, they can be shifted electrically, by having each field pole consist of a number of sections, and successively reversing the polarity of these sections, as shown in Fig. 178, A, B, C, D.

Instead of having a large number of field pole sections, obviously two sections are sufficient, and the same gradual change can be brought about by not merely reversing the sections but reducing the excitation down to zero and bringing it up again in opposite direction, as shown in Fig. 179, *A*, *B*, *C*, *D*, *E*.

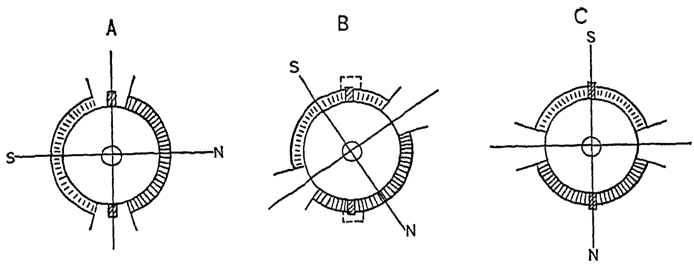


Fig. 177. E.m.f. Variation by Mechanically Shifting the Poles.

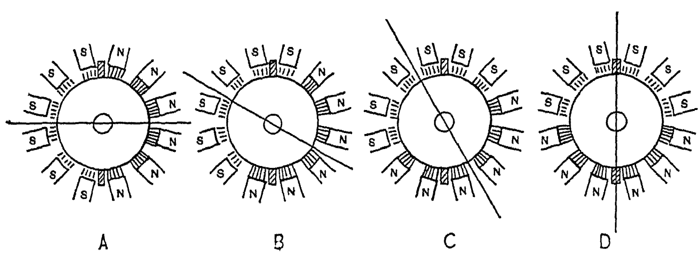


Fig. 178. E.m.f. Variation by Electrically Shifting the Poles.

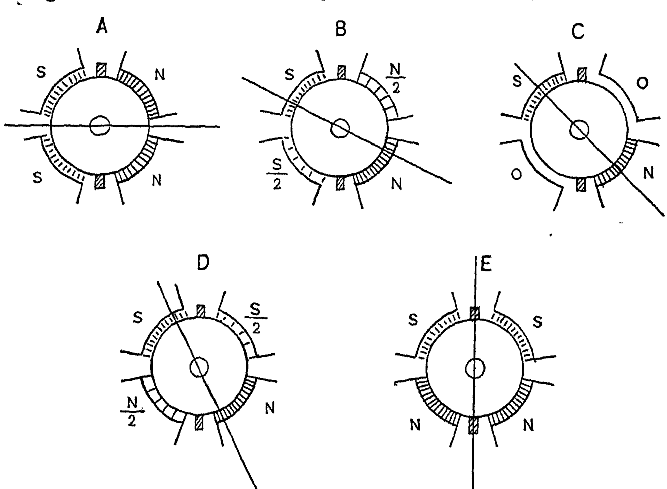


Fig. 179. E.m.f. Variation by Shifting Flux Distribution.

In this case, when reducing one section in polarity, the other section must be increased by approximately the same amount, to maintain the same alternating voltage.

When changing the direct voltage by mechanically shifting the brushes, as soon as the brushes come under the field pole faces, self-inductive sparking on the commutator would result if the iron of the field poles were not kept away from the brush position by having a slot in the field poles, as indicated in dotted line in Fig. 175 and Fig. 177, *B*. With the arrangement in Figs. 175 and 177, this is not feasible mechanically, and these arrangements are, therefore, unsuitable. It is feasible, how-

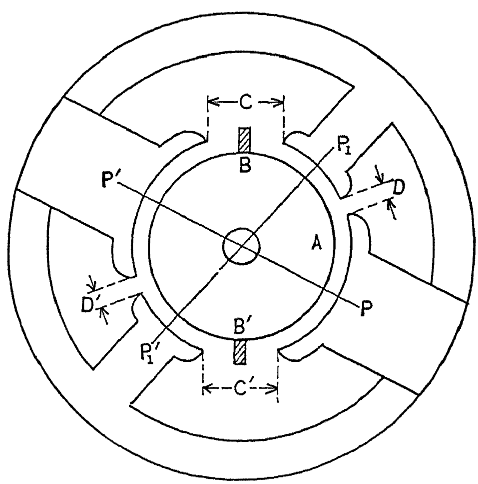


Fig. 180. Variable Ratio or Split-Pole Converter.

ever, as shown in Figs. 178 and 179, that is, when shifting the resultant magnetic flux electrically, to leave a commutating space between the polar projections of the field at the brushes, as shown in Fig. 179, and thus secure as good commutation as in any other commutating machine.

Such a variable-ratio converter, then, comprises an armature *A*, Fig. 180, with the brushes *B*, *B′* in fixed position and field poles *P*, *P′* separated by interpolar spaces *C*, *C′* of such width as required for commutation. Each field pole consists of two parts, *P* and P_1, usually of different relative size, separated by a narrow space, *DD′*, and provided with independent windings. By varying, then, the relative excitation of the two polar sections P and P_1 an effective shift of the resultant field flux and a corresponding change of the direct voltage is produced.

As this method of voltage variation does not depend upon the wave shape, by the design of the field pole faces and the pitch of the armature winding the alternating voltage wave can be made as near a sine wave as desired. Usually not much attention is paid hereto, as experience shows that the usual distributed winding of the commutating machine gives a sufficiently close approach to sine shape.

Armature Reaction and Commutation.

128. With the brushes in quadrature position to the resultant magnetic flux, and at normal voltage ratio, the direct-current generator armature reaction of the converter equals the synchronous motor armature reaction of the power component of the alternating current, and at unity power-factor the converter thus has no resultant armature reaction, while with a lagging or leading current it has the magnetizing or demagnetizing reaction of the wattless component of the current.

If by a shift of the resultant flux from quadrature position with the brushes, by angle τ, the direct voltage is reduced by factor $\cos \tau$, the direct current and therewith the direct-current armature reaction are increased, by factor $\frac{1}{\cos \tau}$, as by the law of conservation of energy the direct-current output must equal the alternating-current input (neglecting losses). The direct-current armature reaction $\mathfrak{F}$ therefore ceases to be equal to the armature reaction of the alternating energy current $\mathfrak{F}_0$, but is greater by factor $\frac{1}{\cos \tau}$:

$$\mathfrak{F} = \frac{\mathfrak{F}_0}{\cos \tau}.$$

The alternating-current armature reaction $\mathfrak{F}_0$, at no phase displacement, is in quadrature position with the magnetic flux. The direct-current armature reaction $\mathfrak{F}$, however, appears in the position of the brushes, or shifted against quadrature position by angle τ; that is, the direct-current armature reaction is not in opposition to the alternating-current armature reaction, but differs therefrom by angle τ, and so can be resolved into two components, a component in opposition to the alternating-current armature reaction $\mathfrak{F}_0$, that is, in quadrature position with the resultant magnetic flux,

$$\mathfrak{F}'' = \mathfrak{F} \cos \tau = \mathfrak{F}_0,$$

that is, equal and opposite to the alternating-current armature reaction, and thus neutralizing the same; and a component in quadrature position with the alternating-current armature reaction $\mathfrak{F}_0$, or in phase with the resultant magnetic flux, that is, magnetizing or demagnetizing,

$$\mathfrak{F}' = \mathfrak{F} \sin \tau = \mathfrak{F}_0 \tan \tau;$$

that is, in the variable-ratio converter the alternating-current armature reaction at unity power-factor is neutralized by a component of the direct-current armature reaction, but a resultant armature reaction $\mathfrak{F}'$ remains, in the direction of the resultant magnetic field, that is, shifted by angle $(90 - \tau)$ against the position of brushes. This armature reaction is magnetizing or demagnetizing, depending on the direction of the shift of the field τ.

It can be resolved into two components, one at right angles with the brushes,

$$\mathfrak{F}_1' = \mathfrak{F}' \cos \tau = \mathfrak{F}_0 \sin \tau,$$

and one, in line with the brushes,

$$\mathfrak{F}_2' = \mathfrak{F}' \sin \tau = \mathfrak{F} \sin^2\tau = \mathfrak{F}_0 \sin \tau \tan \tau,$$

as shown diagrammatically in Figs. 181 and 182.

There exists thus a resultant armature reaction in the direction of the brushes, and thus harmful for commutation, just as in the direct-current generator, except that this armature reaction in the direction of the brushes is only $\mathfrak{F}_2' = \mathfrak{F} \sin^2\tau$, that is, $\sin^2\tau$ of the value of that of a direct-current generator.

The value of $\mathfrak{F}_2'$ can also be derived directly, as the difference between the direct-current armature reaction $\mathfrak{F}$ and the component of the alternating-current armature reaction, in the direction of the brushes, $\mathfrak{F}_0 \cos \tau$, that is,

$$\mathfrak{F}_2' = \mathfrak{F} - \mathfrak{F}_0 \cos \tau = \mathfrak{F}(1 - \cos^2\tau) = \mathfrak{F} \sin^2\tau = \mathfrak{F}_0 \sin \tau \tan \tau.$$

129. The shift of the resultant magnetic flux, by angle τ, gives a component of the m.m.f. of field excitation $\mathfrak{F}_f'' = \mathfrak{F}_f \sin \tau$,

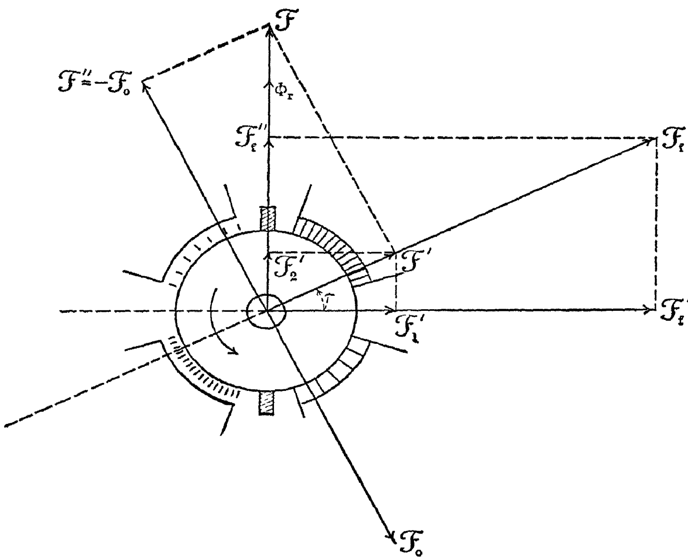


Fig. 181. Diagram of m.m.fs. in Split-Pole Converter.

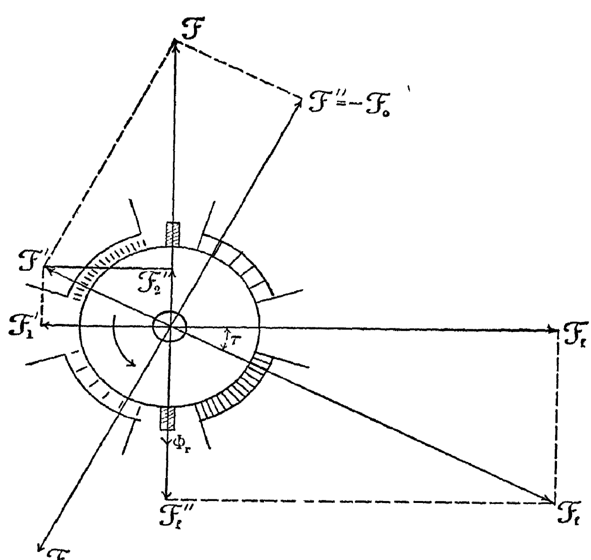


Fig. 182. Diagram of m.m.fs. in Split-Pole Converter.

(where $\mathcal{F}_f$ = m.m.f. of field excitation), in the direction of the commutator brushes, and either in the direction of armature reaction, thus interfering with commutation, or in opposition to the armature reaction, thus improving commutation.

If the magnetic flux is shifted in the direction of armature rotation, that is, that section of the field pole weakened towards which the armature moves, as in Fig. 181, the component $\mathcal{F}_f''$ of field excitation at the brushes is in the same direction as the armature reaction, $\mathcal{F}_2'$, thus adds itself thereto and impairs the commutation, and such a converter is hardly operative. In this case the component of armature reaction, $\mathcal{F}'$, in the direction of the field flux is magnetizing.

If the magnetic flux is shifted in opposite direction to the armature reaction, that is, that section of the field pole weakened which the armature conductor leaves, as in Fig. 182, the component $\mathcal{F}_f''$ of field excitation at the brushes is in opposite direction to the armature reaction $\mathcal{F}_2'$, therefore reverses it, if sufficiently large, and gives a commutating or reversing flux Φ_r, that is, improves commutation so that this arrangement is used in such converters. In this case, however, the component of armature reaction, $\mathcal{F}'$, in the direction of the field flux is demagnetizing, and with increasing load the field excitation has to be increased by $\mathcal{F}'$ to maintain constant flux. Such a converter thus requires compounding, as by a series field, to take care of the demagnetizing armature reaction.

If the alternating current is not in phase with the field, but lags or leads, the armature reaction of the lagging or leading component of current superimposes upon the resultant armature reaction $\mathcal{F}'$, and increases it — with lagging current in Fig. 181, leading current in Fig. 182 — or decreases it — with lagging current in Fig. 182, leading current in Fig. 181 —, and with lag of the alternating current, by phase angle $\theta = \tau$, under the conditions of Fig. 182, the total resultant armature reaction vanishes, that is, the lagging component of synchronous motor armature reaction compensates for the component of the direct-current reaction, which is not compensated by the armature reaction of the power component of the alternating current. It is interesting to note that in this case, in regard to heating, output based thereon, etc., the converter equals that of one of normal voltage ratio.

B. Variable Ratio by Change of Wave Shape of the Y Voltage.

130. If in the converter shown diagrammatically in Fig. 183 the magnetic flux disposition and the pitch of the armature winding are such that the potential difference between the point a of the armature and the neutral 0, or the Y voltage, is a sine wave, Fig. 184 A, then the voltage ratio is normal. Assume, however, that the voltage curve a differs from sine shape by the superposition of some higher harmonics: the third harmonic in Figs. 184 B and C; the fifth harmonic in Figs. 184 D and E. If, then, these higher harmonics are in phase with the fundamental, that is, their maxima coincide, as in Figs. 184 B and D, they increase the maximum of the alternating voltage, and thereby the direct voltage; and if these harmonics are in opposition to the fundamental, as in Figs. 184 C and E, they decrease the maximum alternating and thereby the direct voltage, without appreciably affecting the effective value of the alternating voltage. For instance, a higher harmonic of 30 per cent of the fundamental increases or decreases the direct voltage by 30 per cent, but varies the effective alternating voltage only by $\sqrt{1 + 0.3^2} = 1.044$, or 4.4 per cent.

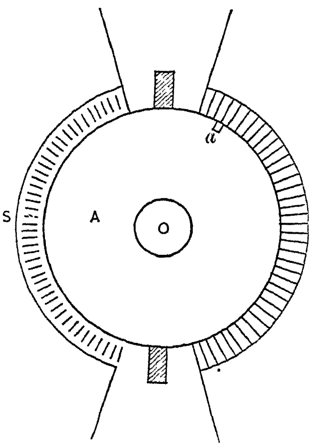


Fig. 183. Variable Ratio Converter by Changing Wave Shape of the Y e.m.f.

The superposition of higher harmonics thus offers a means of increasing as well as decreasing the direct voltage, at constant alternating voltage, and without shifting the angle between the brush position and resultant magnetic flux.

Since, however, the terminal voltage of the converter does not only depend on the generated e.m.f. of the converter, but also on that of the generator, and is a resultant of the two e.m.fs. in approximately inverse proportion to the impedances from the converter terminals to the two respective generated e.m.fs., for varying the converter ratio only such higher harmonics can be used which may exist in the Y voltage without appearing in the converter terminal voltage or supply voltage.

In general, in an n-phase system an nth harmonic existing in the star or Y voltage does not appear in the ring or delta

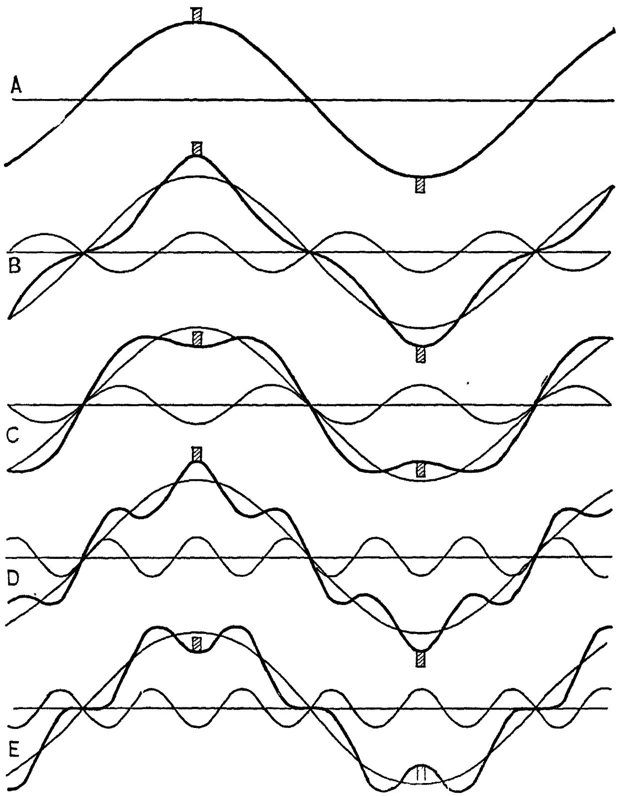


Fig. 184. Superposition of Harmonics to Change the e.m.f. Ratio.

voltage, as the ring voltage is the combination of two star voltages displaced in phase by $\frac{180}{n}$ degrees for the fundamental, and thus by 180 degrees, or in opposition, for the nth harmonic.

Thus, in a three-phase system, the third harmonic can be introduced into the Y voltage of the converter, as in Figs. 184 B and C, without affecting or appearing in the delta voltage, so can be used for varying the direct-current voltage, while the fifth harmonic cannot be used in this way, but would reappear and

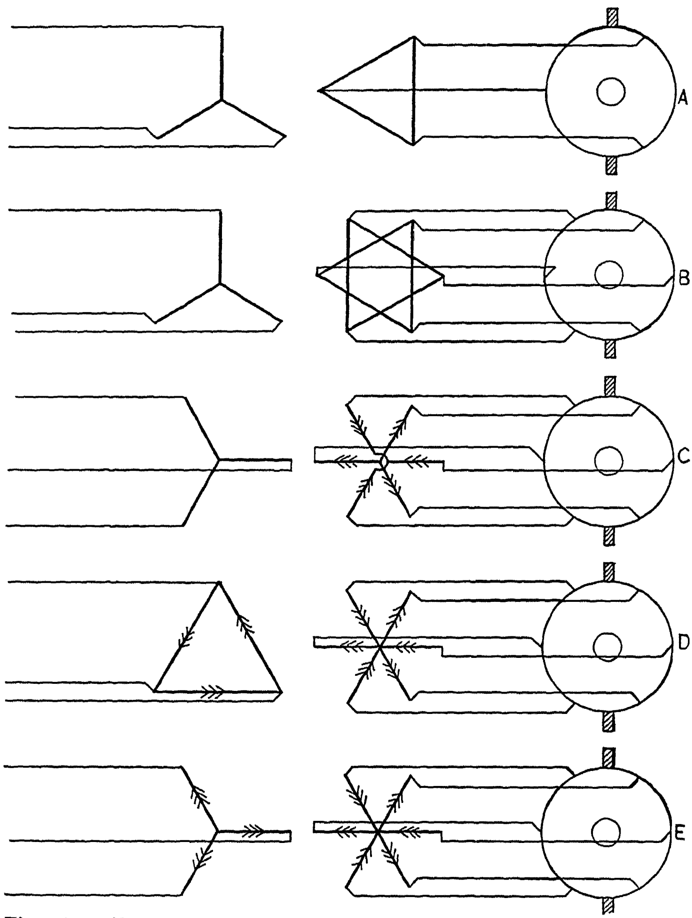


Fig. 185. Transformer Connections for Varying the e.m.f. Ratio by Superposition of the Third Harmonic.

cause a short-circuit current in the supply voltage, hence should be made sufficiently small to be harmless.

131. The third harmonic thus can be used for varying the direct voltage in the three-phase converter diagrammatically shown in Fig. 185 A, and also in the six-phase converter with double-delta connection, as shown in Fig. 185 B, or double-*Y* connection, as shown in Fig. 185 C, since this consists of two separate three-phase triangles of voltage supply, and neither of them contains the third harmonic. In such a six-phase con-

verter with double-Y connection, Fig. 185 C, the two neutrals, however, must not be connected together, as the third harmonic voltage exists between the neutrals. In the six-phase converter with diametrical connections, the third harmonic of the Y voltage appears in the terminal voltage, as the diametrical voltage is twice the Y voltage. In such a converter, if the primaries of the supply transformers are connected in delta, as in Fig. 185 D, the third harmonic is short-circuited in the primary voltage triangle, and thus produces excessive currents, which cause heating and interfere with the voltage regulation, therefore, this arrangement is not permissible. If, however, the primaries are connected in Y, as in Fig. 185 E, and either three separate single-phase transformers, or a three-phase transformer with three independent magnetic circuits, is used, as in Fig. 186, the triple frequency

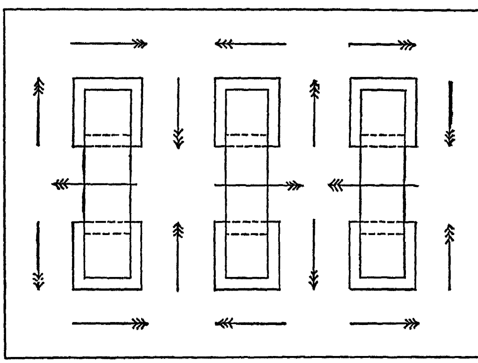


Fig. 186. Shell Type Transformer.

voltages in the primary are in phase with each other between the line and the neutral, and thus, with isolated neutral, cannot produce any current. With a three-phase transformer as shown in Fig. 187, that is, in which the magnetic circuit of the third harmonic is open, triple frequency currents can exist in the secondary and this arrangement therefore is not satisfactory.

In two-phase converters, higher harmonics can be used for regulation only if the transformers are connected in such a manner that the regulating harmonic, which appears in the converter terminal voltage, does not appear in the transformer terminals, that is, by the connection analogous to Figs. 185 E and 186.

Since the direct-voltage regulation of a three-phase or six-phase converter of this type is produced by the third harmonic, the problem is to design the magnetic circuit of the converter so as to produce the maximum third harmonic, the minimum fifth and seventh harmonics.

If q = interpolar space, thus $(1 - q)$ = pole arc, as fraction of pitch, the wave shape of the voltage generated between the

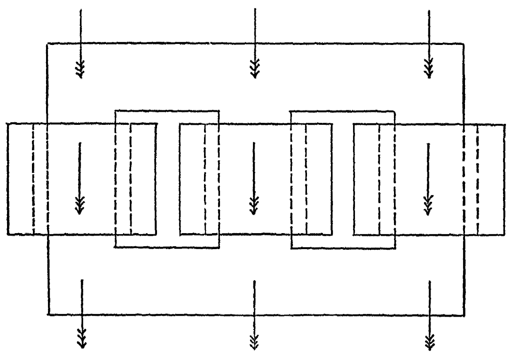


Fig. 187. Core Type Transformer.

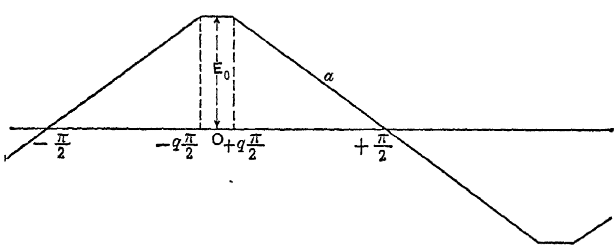


Fig. 188. Y e.m.f. Wave.

point a of a full-pitch distributed winding — as generally used for commutating machines — and the neutral, or the induced Y voltage of the system is a triangle with the top cut off for distance q, as shown in Fig. 188, when neglecting magnetic spread at the pole corners.

If then e_0 = voltage generated per armature turn while in front of the field pole (which is proportional to the magnetic density in the air gap), m = series turns from brush to brush, the maximum voltage of the wave shown in Fig. 188 is

$$E_0 = me_0 (1 - q);$$

developed into a Fourier series, this gives, as the equation of the voltage wave a, Fig. 188,

$$e = \frac{8\,E_0}{(1-q)\pi^2}\sum_1^{\infty}{}^n \frac{\cos\frac{(2n-1)q\pi}{2}}{(2\,n-1)^2}\cos(2n-1)\,\theta;$$

or, substituting for E_0, and denoting

$$A = \frac{8\,me_0}{\pi^2},$$

$$e = A\sum_1^{\infty}{}^n \frac{\cos\frac{2\,n-1}{2}q\pi}{(2\,n-1)^2}\cos(2\,n-1)\,\theta$$

$$= A\left\{\cos q\frac{\pi}{2}\cos\theta + \frac{1}{9}\cos 3\,q\frac{\pi}{2}\cos 3\,\theta + \frac{1}{25}\cos 5\,q\frac{\pi}{2}\cos 5\,\theta + \frac{1}{49}\cos 7\,q\frac{\pi}{2}\cos 7\,\theta + \right\}.$$

Thus the third harmonic is a positive maximum for $q = 0$, or 100 per cent pole arc, and a negative maximum for $q = \frac{2}{3}$, or 33.3 per cent pole arc.

For maximum direct voltage, q should therefore be made as small, that is, the pole arc as large, as commutation permits. In general, the minimum permissible value of q is about 0.15 to 0.20.

The fifth harmonic vanishes for $q = 0.20$ and $q = 0.60$, and the seventh harmonic for $q = 0.143$, 0.429, and 0.714.

For small values of q, the sum of the fifth and seventh harmonics is a minimum for about $q = 0.18$, or 82 per cent pole arc. Then for $q = 0.18$, or 82 per cent pole arc,

$$\begin{aligned} e_1 &= A\,\{0.960\cos\theta + 0.0736\cos 3\,\theta + 0.0062\cos 5\,\theta \\ &\qquad - 0.0081\cos 7\,\theta + \ldots\} \\ &= 0.960\,A\,\{\cos\theta + 0.0766\cos 3\,\theta + 0.0065\cos 5\,\theta \\ &\qquad - 0.0084\cos 7\,\theta + \ldots\}; \end{aligned}$$

that is, the third harmonic is less than 8 per cent, so that not much voltage rise can be produced in this manner, while the fifth and seventh harmonics together are only 1.3 per cent, thus negligible.

132. Better results are given by reversing or at least lowering the flux in the center of the field pole. Thus, dividing the pole

face into three equal sections, the middle section, of 27 per cent pole arc, gives the voltage curve $q = 0.73$ thus,

$$e_2 = A \{0.411 \cos \theta - 0.1062 \cos 3\theta + 0.0342 \cos 5\theta - 0.0035 \cos 7\theta \ldots\}$$
$$= 0.411 A \{\cos \theta - 0.258 \cos 3\theta + 0.083 \cos 5\theta - 0.0085 \cos 7\theta \ldots\}.$$

The voltage curves given by reducing the pole center to one-half intensity, to zero, reversing it to half intensity, to full intensity, and to such intensity that the fundamental disappears, then are given by:

Center part of pole density.		
(1)	full, $e = e_1$	$=0.960 A \{\cos \theta + 0.077 \cos 3\theta + 0.0065 \cos 5\theta - 0.0084 \cos 7\theta \ldots\}$
(2)	0.5, $e = e_1 - 0.5 e_2$	$=0.755 A \{\cos \theta + 0.168 \cos 3\theta - 0.0144 \cos 5\theta - 0.0085 \cos 7\theta \ldots\}$
(3)	0, $e = e_1 - e_2$	$=0.549 A \{\cos \theta + 0.328 \cos 3\theta - 0.053 \cos 5\theta - 0.084 \cos 7\theta \ldots\}$
(4)	-0.5 $e = e_1 - 1.5 e_2$	$=0.344 A \{\cos \theta + 0.680 \cos 3\theta - 0.131 \cos 5\theta - 0.0084 \cos 7\theta \ldots\}$
(5)	$-$ full, $e = e_1 - 2 e^2$	$=0.138 A \{\cos \theta + 2.07 \cos 3\theta - 0.45 \cos 5\theta - 0.008 \cos 7\theta \ldots\}$
(6)	-1.17, $e = e_1 - 2.34 e_2$	$=0.322 A \{\cos 3\theta - 0.227 \cos 5\theta \ldots\}.$

It is interesting to note that in the last case the fundamental frequency disappears and the machine is a generator of triple frequency, that is, produces or consumes a frequency equal to three times synchronous frequency. In this case the seventh harmonic also disappears, and only the fifth is appreciable, but could be greatly reduced by a different kind of pole arc. From above table follows:

	(1)	(2)	(3)	(4)	(5)	(6)	normal
Maximum fundamental alternating volts...	0.960	0.755	0.549	0.344	0.138	0	0.960
Direct volts.....	1.033	0.883	0.743	0.578	0.423	0.322	0.960

133. It is seen that a considerable increase of direct voltage beyond the normal ratio involves a sacrifice of output, due to the decrease or reversal of a part of the magnetic flux, whereby the air-gap section is not fully utilized. Thus it is not advisable to go too far in this direction.

By the superposition of the third harmonic upon the fundamental wave of the Y voltage, in a converter with three sections per pole, thus an increase of direct voltage over its normal voltage can be produced by lowering the excitation of the middle section and raising that of the outside sections of the field pole, and also inversely a decrease of the direct voltage below its normal value by raising the excitation of the middle section and decreasing that of the outside sections of the field poles; that is, in the latter case making the magnetic flux distribution at the armature periphery peaked, in the former case by making the flux distribution flat topped or even double peaked.

Armature Reaction and Commutation.

134. In such a split-pole converter let p equal ratio of direct voltage to that voltage which it would have, with the same alternating impressed voltage, at normal voltage ratio, where $p > 1$ represents an over-normal, $p < 1$ a subnormal direct voltage. The direct current, and thereby the direct-current armature reaction, then is changed from the value which it would have at normal voltage ratio, by the factor $\frac{1}{p}$, as the product of direct volts and amperes must be the same as at normal voltage ratio, being equal to the alternating power input minus losses.

With unity power-factor, the direct-current armature reaction $\mathfrak{F}$ in a converter of normal voltage ratio is equal and opposite, and thus neutralized by the alternating-current armature reaction $\mathfrak{F}_0$, and at a change of voltage ratio from normal, by factor p, and thus change of direct current by factor $\frac{1}{p}\cdot$ The direct-current armature reaction thus is

$$\mathfrak{F} = \frac{\mathfrak{F}}{p}$$

hence, leaves an uncompensated resultant.

As the alternating-current armature reaction at unity power-factor is in quadrature with the magnetic flux, and the direct-current armature reaction in line with the brushes, and with this type of converter the brushes stand at the magnetic neutral, that is, at right angles to the magnetic flux, the two armature reactions are in the same direction in opposition with each other, and thus leave the resultant, in the direction of the commutator brushes,

$$\begin{aligned} \mathfrak{F}' &= \mathfrak{F} - \mathfrak{F}_0 \\ &= \mathfrak{F}_0 (1 - p). \end{aligned}$$

The converter thus has an armature reaction proportional to the deviation of the voltage ratio from normal.

135. If $p > 1$, or over-normal direct voltage, the armature reaction is negative, or motor reaction, and the magnetic flux produced by it at the commutator brushes thus a commutating flux. If $p < 1$, or subnormal direct voltage, the armature reaction is positive, that is, the same as in a direct-current generator, but less in intensity, and thus the magnetic flux of armature reaction tends to impair commutation. In a direct-current generator, by shifting the brushes to the edge of the field poles, the field flux is used as reversing flux to give commutation. In this converter, however, decrease of direct voltage is produced by lowering the outside sections of the field poles, and the edge of the field may not have a sufficient flux density to give commutation, with a considerable decrease of voltage below normal, and thus a separate commutating pole is required. Preferably this type of converter should be used only for raising the voltage, for lowering the voltage the other type, which operates by a shift of the resultant flux, and so gives a component of the main field flux as commutating flux, should be used, or a combination of both types.

With a polar construction consisting of three sections, this can be done by having the middle section at low, the outside sections at high excitation for maximum voltage, and, to decrease the voltage, raise the excitation of the center section, but instead of lowering both outside sections, leave the section in the direction of the armature rotation unchanged, while lowering the other outside section twice as much, and thus produce, in addition to the change of wave shape, a shift of the flux, as represented by the scheme Fig. 189.

Magnetic Density.

Pole section	1	2	3	1′	2′	3′
Max. voltage	$+\mathcal{B}$	0	$+\mathcal{B}$	$-\mathcal{B}$	0	$-\mathcal{B}$
	$+\frac{2}{3}\mathcal{B}$	$+\frac{1}{3}\mathcal{B}$	$+\mathcal{B}$	$-\frac{2}{3}\mathcal{B}$	$-\frac{1}{3}\mathcal{B}$	$-\mathcal{B}$
	$+\frac{1}{3}\mathcal{B}$	$+\frac{2}{3}\mathcal{B}$	$+\mathcal{B}$	$-\frac{1}{3}\mathcal{B}$	$-\frac{2}{3}\mathcal{B}$	$-\mathcal{B}$
	0	$+\mathcal{B}$	$+\mathcal{B}$	0	$-\mathcal{B}$	$-\mathcal{B}$
Min. voltage	$-\frac{1}{3}\mathcal{B}$	$+1\frac{1}{3}\mathcal{B}$	$+\mathcal{B}$	$+\frac{1}{3}\mathcal{B}$	$-1\frac{1}{3}\mathcal{B}$	$-\mathcal{B}$

Where the required voltage range above normal is not greater than can be produced by the third harmonic of a large pole arc

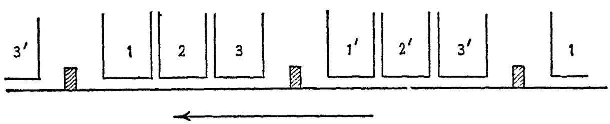


Fig. 189. Three-Section Pole for Variable Ratio Converter.

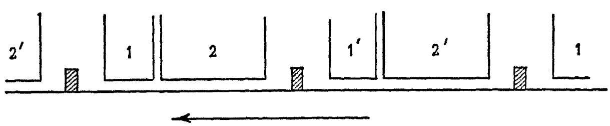


Fig. 190. Two-Section Pole for Variable Ratio Converter.

with uniform density, this combination of voltage regulation by both methods can be carried out with two sections of the field poles, of which the one (towards which the armature moves) is greater than the other, as shown in Fig. 190, and the variation then is as follows:

Magnetic Density.

Pole section	1	2	1′	2′
Max. voltage	$+\mathcal{B}$	$+\mathcal{B}$	$-\mathcal{B}$	$-\mathcal{B}$
	$+\frac{1}{2}\mathcal{B}$	$+1\frac{1}{4}\mathcal{B}$	$-\frac{1}{2}\mathcal{B}$	$-1\frac{1}{4}\mathcal{B}$
	0	$1\frac{1}{2}\mathcal{B}$	0	$-1\frac{1}{2}\mathcal{B}$
Min. voltage	$-\frac{1}{2}\mathcal{B}$	$+1\frac{3}{4}\mathcal{B}$	$+\frac{1}{2}\mathcal{B}$	$-1\frac{3}{4}\mathcal{B}$

Heating and Rating.

136. The distribution of current in the armature conductors of the variable-ratio converter, the wave form of the actual or differential current in the conductors, and the effect of the wattless current thereon, are determined in the same manner as in the standard converter, and from them are calculated the local heating in the individual armature turns and the mean armature heating.

In an n-phase converter of normal voltage ratio, let $E_0 =$ direct voltage; I_0 = direct current; E^0 = alternating voltage between adjacent collector rings (ring voltage), and I^0 = alternating current between adjacent collector rings (ring current); then, as seen in the preceding,

$$E^0 = \frac{E_0 \sin \frac{\pi}{n}}{\sqrt{2}}, \tag{1}$$

and as by the law of conservation of energy, the output must equal the input, when neglecting losses,

$$I^0 = \frac{I_0 \sqrt{2}}{n \sin \frac{\pi}{n}}, \tag{2}$$

where I^0 is the power component of the current corresponding to the direct-current output.

The voltage ratio of a converter can be varied —

(*a.*) By the superposition of a third harmonic upon the star voltage, or diametrical voltage, which does not appear in the ring voltage, or voltage between the collector rings of the converter.

(*b.*) By shifting the direction of the magnetic flux.

(*a*) can be used for raising the direct voltage as well as for lowering it, but is used almost always for the former purpose, since when using this method for lowering the direct voltage commutation is impaired.

(*b*) can be used only for lowering the direct voltage.

It is possible, by proportioning the relative amounts by which the two methods contribute to the regulation of the voltage, to maintain a proper commutating field at the brushes for all loads and voltages. Where, however, this is not done, the brushes are shifted to the edge of the next field pole, and into the fringe of its field, thus deriving the commutating field.

137. In such a variable-ratio converter let, then, t = intensity of the third harmonic, or rather of that component of it which is in line with the direct-current brushes, and thus does the voltage regulation, as fraction of the fundamental wave. t is chosen as positive if the third harmonic increases the maximum of the fundamental wave (wide pole arc) and thus raises the direct voltage, and negative when lowering the maximum of the fundamental and therewith the direct voltage (narrow pole arc).

p_l = loss of power in the converter, which is supplied by the current (friction and core loss) as fraction of the alternating input (assumed as 4 per cent in the numerical example).

τ_b = angle of brush shift on the commutator, counted positive in the direction of rotation.

θ_1 = angle of time lag of the alternating current (thus negative for lead).

τ_a = angle of shift of the resultant field from the position at right angles to the mechanical neutral (or middle between the pole corners of main poles and auxiliary poles), counted positive in the direction opposite to the direction of armature rotation, that is, positive in that direction in which the field flux has been shifted to get good commutation, as discussed in the preceding article.

Due to the third harmonic t, and the angle of shift of the field flux, τ_a, the voltage ratio differs from the normal by the factor

$$(1 + t) \cos \tau_a,$$

and the ring voltage of the converter thus is

$$E = \frac{E^0}{(1 + t) \cos \tau_a}; \tag{3}$$

hence, by (1),

$$E = \frac{E_0 \sin \frac{\pi}{n}}{\sqrt{2}\,(1 + t) \cos \tau_a} \tag{4}$$

and the power component of the ring current corresponding to the direct-current output thus is, when negelecting losses, from (2),

$$\begin{aligned} I' &= I^0 (1 + t) \cos \tau_a \\ &= \frac{I_0 \sqrt{2}\,(1 + t) \cos \tau_a}{n \sin \frac{\pi}{n}}. \end{aligned} \tag{5}$$

Due to the loss p_l in the converter, this current is increased by $(1 + p_l)$ in a direct converter, or decreased by the factor $(1 - p_l)$ in an inverted converter.

The *power component* of the alternating current thus is

$$I_1 = I' (1 + p_l)$$
$$= I_0 \frac{\sqrt{2}\,(1 + t)\,(1 + p_l) \cos \tau_a}{n \sin \frac{\pi}{n}}, \tag{6}$$

where p_l may be considered as negative in an inverted converter.

With the angle of lag θ_1, the *reactive component* of the current is

$$I_2 = I_1 \tan \theta_1,$$

and the *total alternating ring current* is

$$I = \frac{I_1}{\cos \theta_1}$$
$$= \frac{I_0 \sqrt{2}\,(1 + t)\,(1 + p_l) \cos \tau_a}{n \sin \frac{\pi}{n} \cos \theta_1}, \tag{7}$$

or, introducing for simplicity the abbreviation

$$k = \frac{(1 + t)\,(1 + p_l) \cos \tau_a}{\cos \theta_1}, \tag{8}$$

it is

$$I = \frac{I_0 k \sqrt{2}}{n \sin \frac{\pi}{n}}. \tag{9}$$

138. Let, in Fig. 191, $\overline{A'OA}$ represent the center line of the magnetic field structure.

The resultant magnetic field flux $\overline{O\Phi}$ then leads $\overline{OA}$ by angle $\Phi OA = \tau_a$.

The resultant m.m.f. of the alternating power current I_1 is $\overline{OI_1}$, at right angles to $\overline{O\Phi}$, and the resultant m.m.f. of the alternating reactive current I_2 is $\overline{OI_2}$, in opposition to $\overline{O\Phi}$, while the total alternating current I is $\overline{OI}$, lagging by angle θ_1 behind $\overline{OI_1}$.

The m.m.f. of direct-current armature reaction is in the direction of the brushes, thus lagging by angle τ_b behind the position $\overline{OB}$, where $BOA = 90$ deg., and given by $\overline{OI_0}$.

The angle by which the direct-current m.m.f. $\overline{OI_0}$ lags in space behind the total alternating m.m.f. $\overline{OI}$ thus is, by Fig. 191,

$$\tau_0 = \theta_1 - \tau_a - \tau_b. \tag{10}$$

If the alternating m.m.f. in a converter coincides with the direct-current m.m.f., the alternating current and the direct current are in phase with each other in the armature coil midway

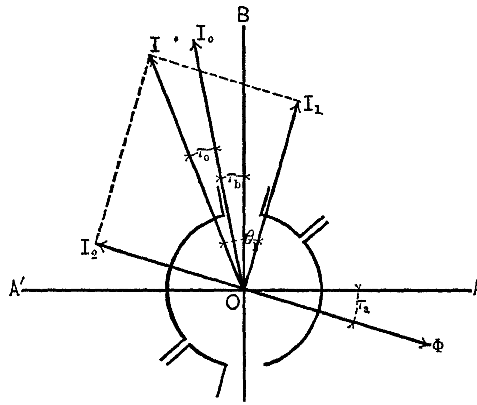


Fig. 191. Diagram of Variable Ratio Converter.

between adjacent collector rings, and the current heating thus a minimum in this coil.

Due to the lag in space, by angle τ_0, of the direct-current m.m.f. behind the alternating current m.m.f., the reversal of the

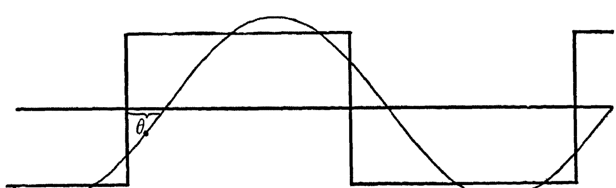


Fig. 192. Alternating and Direct Current in a Coil Midway between Adjacent Collector Leads.

direct current is reached in time before the reversal of the alternating current in the armature coil; that is, the alternating current lags behind the direct current by angle $\theta_0 = \tau_0$ in the armature coil midway between adjacent collector leads, as shown by Fig. 192, and in an armature coil displaced by angle τ from the middle position between adjacent collector leads the alternating current thus lags behind the direct current by angle $(\tau + \theta_0)$, where τ is counted positive in the direction of armature rotation (Fig. 193).

The alternating current in armature coil τ thus can be expressed by

$$i = I\sqrt{2}\sin(\theta - \tau - \theta_0); \tag{11}$$

hence, substituting (9),

$$i = \frac{2 I_0 k}{n \sin \frac{\pi}{n}} \sin(\theta - \tau - \theta_0), \tag{12}$$

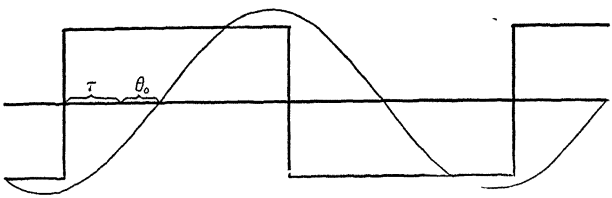


Fig. 193. Alternating and Direct Current in a Coil at the Angle τ from the Middle Position.

and as the direct current in this armature coil is $\frac{I_0}{2}$, and opposite to the alternating current i, the *resultant current* in the armature coil τ is

$$i_0 = i - \frac{I_0}{2}$$

$$= \frac{I_0}{2}\left\{\frac{4k}{n \sin \frac{\pi}{n}} \sin(\theta - \tau - \theta_0) - 1\right\}, \tag{13}$$

and the ratio of heating, of the resultant current i_0 compared with the current $\frac{I_0}{2}$ of the same machine as direct-current generator of the same output, thus is

$$\frac{i_0^2}{\left(\frac{I_0}{2}\right)^2} = \left\{\frac{4k}{n \sin \frac{\pi}{n}} \sin(\theta - \tau - \theta_0) - 1\right\}^2. \tag{14}$$

Averaging (14) over one half-wave gives the relative heating of the armature coil τ as

$$\gamma_\tau = \frac{1}{\pi}\int_0^\pi \frac{i_0^2}{\left(\frac{I_0}{2}\right)^2} d\theta = \frac{1}{\pi}\int_0^\pi \left\{\frac{4k}{n \sin \frac{\pi}{n}} \sin(\theta - \tau - \theta_0) - 1\right\}^2 d\theta. \tag{15}$$

Integrated, this gives

$$\gamma_\tau = \frac{8\,k^2}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16\,k \cos(\tau + \theta_0)}{\pi n \sin \frac{\pi}{n}}. \qquad (16)$$

139. Herefrom follows the local heating in any armature coil τ, in the coils adjacent to the leads by substituting $\tau = \pm \frac{\pi}{n}$, and also follows the average armature heating by averaging γ_τ from $\tau = -\frac{\pi}{n}$ to $\tau = +\frac{\pi}{n}$.

The average armature heating of the n-phase converter therefore is

$$\Gamma = \frac{2\,n}{\pi} \int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} \gamma_\tau\, d_\tau,$$

or, integrated,

$$\Gamma = \frac{8\,k^2}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16\,k \cos \theta_0}{\pi^2}. \qquad (17)$$

This is the same expression as found for the average armature heating of a converter of normal voltage ratio, when operating with an angle of lag θ_1 of the alternating current, where k denotes the ratio of the total alternating current to the alternating power current corresponding to the direct-current output.

In an n-phase variable ratio converter (split-pole converter), the average armature heating thus is given by

$$\Gamma = \frac{8\,k^2}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16\,k \cos \theta_0}{\pi^2}, \qquad (18)$$

where

$$k = \frac{(1+t)\,(1+p_l) \cos \tau_a}{\cos \theta_1}, \qquad (8)$$

$$\theta_0 = \theta_1 - \tau_a - \tau_b; \qquad (10)$$

and t = ratio of third harmonic to fundamental alternating voltage wave; p_l = ratio of loss to output; θ_1 = angle of lag of alternating current; τ_a = angle of shift of the resultant magnetic field in opposition to the armature rotation, and τ_b = angle of shift of the brushes in the direction of the armature rotation.

140. For a *three-phase converter*, equation (18) gives ($n = 3$)

$$\left.\begin{aligned}\Gamma &= \frac{32\,k^2}{27} + 1 - 1.621\,k\cos\theta_0\\ &= 1.185\,k^2 + 1 - 1.621\,k\cos\theta_0.\end{aligned}\right\}\qquad(19)$$

For a six-phase converter, equation (18) gives ($n = 6$)

$$\left.\begin{aligned}\Gamma &= \frac{8\,k^2}{9} + 1 - 1.621\,k\cos\theta_0\\ &= 0.889\,k^2 + 1 - 1.621\,k\cos\theta_0.\end{aligned}\right\}\qquad(20)$$

For a converter of normal voltage ratio,

$$t = 0, \qquad \tau_a = 0,$$

using no brush shift,

$$\tau_b = 0;$$

when neglecting the losses,

$$p_l = 0,$$

it is

$$k = \frac{1}{\cos\theta_1},$$

$$\theta_0 = \theta_1,$$

and equations (19) and (20) assume the form:

Three-phase:

$$\Gamma = \frac{1.185}{\cos^2\theta_1} - 0.621.$$

Six-phase:

$$\Gamma = \frac{0.889}{\cos^2\theta_1} - 0.621.$$

The equation (18) is the most general equation of the relative heating of the synchronous converter, including phase displacement θ_1, losses p_l, shift of brushes τ_b, shift of the resultant magnetic flux τ_a, and the third harmonic t.

While in a converter of standard or normal ratio the armature heating is a minimum for unity power-factor, this is not in general the case, but the heating may be considerably less at same lagging current, more at leading current, than at unity power-factor, and inversely.

141. It is interesting therefore to determine under which conditions of phase displacement the armature heating is a minimum so as to use these conditions as far as possible and avoid conditions differing very greatly therefrom, as in the latter case the armature heating may become excessive.

Substituting for k and θ_0 from equations (8) and (10) into equation (18) gives

$$\Gamma = 1 + \frac{8\,(1+t)^2\,(1+p_l)^2\cos^2\tau_a}{n^2\sin^2\frac{\pi}{n}\cos^2\theta_1} - \frac{16\,(1+t)\,(1+p_l)\cos\tau_a\cos(\theta_1-\tau_a-\tau_b)}{\pi^2\cos\theta_1}. \qquad (19)$$

Substituting

$$\frac{n}{\pi}\sin\frac{\pi}{n} = m, \quad (20)$$

which is a constant of the converter type, and is for a three-phase converter, $m_3 = 0.744$; for a six-phase converter, $m_6 = 0.955$ and rearranging, gives

$$\begin{aligned}\Gamma = 1 &+ \frac{8}{\pi^2}\frac{(1+t)^2\,(1+p_l)^2\cos^2\tau_a}{m^2} \\ &- \frac{16}{\pi^2}(1+t)\,(1+p_l)\cos\tau_a\cos(\tau_a+\tau_b) \\ &+ \frac{8}{\pi^2}\frac{(1+t)^2\,(1+p_l)^2\cos^2\tau_a}{m^2}\tan^2\theta_1 \\ &- \frac{16}{\pi^2}(1+t)\,(1+p_l)\cos\tau_a\sin(\tau_a+\tau_b)\tan\theta_1. \qquad (21)\end{aligned}$$

Γ is a minimum for the value θ_1 of the phase displacement, given by

$$\frac{d\Gamma}{d\tan\theta_1} = 0,$$

and this gives, differentiated,

$$\tan\theta_2 = \frac{m^2\sin(\tau_a+\tau_b)}{(1+t)\,(1+p_l)\cos\tau_a}. \qquad (22)$$

Equation (22) gives the phase angle θ_2, for which, at given τ_a, τ_b, t and p_l, the armature heating becomes a minimum.

Neglecting the losses p_l, if the brushes are not shifted, $\tau_b = 0$, and no third harmonic exists, $t = 0$,

$$\tan\theta_2' = m^2\tan\tau_a,$$

where $m^2 = 0.544$ for a three-phase, 0.912 for a six-phase converter.

For a six-phase converter it thus is approximately $\theta_2' = \tau_a$, that is, the heating of the armature is a minimum if the alternating current lags by the same angle (or nearly the same angle) as the magnetic flux is shifted for voltage regulation.

From equation (22) it follows that energy losses in the converter reduce the lag θ_2 required for minimum heating; brush shift increases the required lag; a third harmonic, t, decreases the required lag if additional, and increases it if subtractive.

Substituting (22) into (21) gives the minimum armature heating of the converter, which can be produced by choosing the proper phase angle θ_2 for the alternating current. It is then, after some transpositions,

$$\Gamma_0 = 1 + \frac{8}{\pi^2}\left\{\left[\frac{(1+t)(1+p_l)\cos\tau_a}{m}\right]^2 - 2(1+t)(1+p_l)\right.$$

$$\left.\cos\tau_a\cos(\tau_a+\tau_b) - m^2\sin^2(\tau_a+\tau_b)\right\}$$

$$= 1 - \frac{8\,m^2}{\pi^2}\left\{1 - \left[\frac{(1+t)(1+p_l)\cos\tau_a}{m^2} - \cos(\tau_a+\tau_b)\right]^2\right\} \quad (23)$$

The term Γ_0 contains the constants t, p_l, τ_a, τ_b only in the square under the bracket and thus becomes a minimum if this square vanishes, that is, if between the quantities t, p_l, τ_a, τ_b such relations exists that

$$\frac{(1+t)(1+p_l)\cos\tau_a}{m^2} - \cos(\tau_a+\tau_b) = 0. \quad (24)$$

142. Of the quantities t, p_l, τ_a, τ_b; p_l and τ_b are determined by the machine design. t and τ_a, however, are equivalent to each other, that is, the voltage regulation can be accomplished either by the flux shift τ_a or by the third harmonic t, or by both, and in the latter case can be divided between τ_a and t so as to give any desired relations between them.

Equation (24) gives

$$t = 1 - \frac{m^2\cos(\tau_a+\tau_b)}{(1+p_l\cos\tau_a)}, \quad (25)$$

and by choosing the third harmonic t as function of the angle of flux shift τ_a, by equation (25), the converter heating becomes a minimum, and is

$$\Gamma_0^0 = 1 - \frac{8\,m^2}{\pi^2}; \quad (26)$$

hence, $\Gamma_0{}^0 = 0.551$ for a three-phase converter, (27)

$\Gamma_0{}^0 = 0.261$ for a six-phase converter. (28)

Substituting (25) into (22) gives

$$\tan \theta_2 = \tan (\tau_a + \tau_b);$$

hence,

$$\theta_2 = \tau_a + \tau_b; \tag{29}$$

or, in other words, the converter gives minimum heating $\Gamma_0{}^0$ if the angle of lag θ_2 equals the sum of the angle of flux shift τ_a and of brush shift τ_b.

It follows herefrom that, regardless of the losses, p_l, of the brush shift τ_b, and of the amount of voltage regulation required, that is, at normal voltage ratio as well as any other ratio, the same minimum converter heating $\Gamma_0{}^0$ can be secured by dividing the voltage regulation between the angle of flux shift τ_a and the third harmonic t in the manner as given by equation (25), and operating at a phase angle between alternating current and voltage equal to the sum of the angles of flux shift τ_a and of brush shift τ_b; that is, the heating of the split-pole converter can be made the same as that of the standard converter of normal voltage ratio.

Choosing $p_l = 0.04$, or 4 per cent loss of current, equation (25) gives, for the three-phase and for the six-phase converter:

(*a*) no brush shift ($\tau_b = 0$):

$$\left.\begin{aligned} t_3{}^0 &= 0.467, \\ t_6{}^0 &= 0.123; \end{aligned}\right\} \tag{30}$$

that is, in the three-phase converter this would require a third harmonic of 46.7 per cent, which is hardly feasible; in the six-phase converter it requires a third harmonic of 12.3 per cent, which is quite feasible.

(*b*) 20 deg. brush shift ($\tau_b = 20$):

$$\left.\begin{aligned} t_3{}^0 &= 1 - 0.533 \frac{\cos (\tau_a + \tau_b)}{\cos \tau_a}, \\ t_6{}^0 &= 1 - 0.877 \frac{\cos (\tau_a + \tau_b)}{\cos \tau_a}; \end{aligned}\right\} \tag{31}$$

for $\tau_a = 0$, or no flux shift, this gives

$$\left.\begin{aligned} t_3{}^{00} &= 0.500, \\ t_6{}^{00} &= 0.176. \end{aligned}\right\} \tag{32}$$

Since $\dfrac{\cos (\tau_a + \tau_b)}{\cos \tau_a} < 1$ for brush shift in the direction of armature rotation, it follows that shifting the brushes increases the third harmonic required to carry out the voltage regulation without increase of converter heating, and thus is undesirable.

It is seen that the third harmonic, t, does not change much with the flux shift τ_a, but remains approximately constant, and positive, that is, voltage raising.

It follows herefrom that the most economical arrangement regarding converter heating is to use in the six-phase converter a third harmonic of about 17 per cent to 18 per cent for raising the voltage (that is, a very large pole arc), and then do the regulation by shifting the flux, by the angle τ_a, without greatly reducing the third harmonic, that is, keep a wide pole arc excited.

As in a three-phase converter the required third harmonic is impracticably high, it follows that for variable voltage ratio the six-phase converter is preferable, because its armature heating can be maintained nearer the theoretical minimum by proportioning t and τ_a.

VIII. Starting.

143. The polyphase converter is self-starting from rest; that is, when connected across the polyphase circuit it starts, accelerates, and runs up to complete synchronism. The e.m.f. between the commutator brushes is alternating in starting, with the frequency of slip below synchronism. Thus a direct-current voltmeter or incandescent lamps connected across the commutator brushes indicate by their beats the approach of the converter to synchronism. When starting, the field circuit of the converter has to be opened or at least greatly weakened. The starting of the polyphase converter is largely a hysteresis effect and entirely so in machines with laminated field poles, while in machines with solid magnet poles or with a short-circuited winding (squirrel-cage) in the field poles, secondary currents in the latter contribute to the starting torque, but at the same time reduce the magnetic starting flux by their demagnetizing effect. The torque is produced by the attraction between the alternating currents of the successive phases upon the remanent magnetism and secondary currents produced by the preceding phase. It is necessarily comparatively weak, and from full-load to twice full-load current at from one-third to one-half of full voltage is required to start from rest without load. For larger converters,

low-voltage taps on the transformers are used to give the lower starting voltage.

While an induction motor can never reach exact synchronism, but must even at no-load slip slightly to produce the friction torque, the converter or synchronous motor reaches exact synchronism, due to the difference of the magnetic reluctance in the direction of the field poles and in the direction in electrical quadrature thereto; that is, the field structure acts like a shuttle armature and the polar projections catch with the rotating magnet poles in the armature, in a similar way as an induction motor armature with a single short-circuited coil (synchronous induction motor, reaction machine) drops into step. Obviously, the single-phase converter is not self-starting.

At the moment of starting, the field circuit of the converter is in the position of a secondary to the armature circuit as primary; and since in general the number of field turns is very much larger than the number of armature turns, excessive e.m.fs. may be generated in the field circuit, reaching frequently 4000 to 6000 volts, which have to be taken care of by some means, as by breaking the field circuit into sections. As soon as synchronism is reached, which usually takes from a few seconds to a minute or more, and is seen by the appearance of continuous voltage at the commutator brushes, the field circuit is closed and the load put on the converter. Obviously, while starting, the direct-current side of the converter must be open-circuited, since the e.m.f. between commutator brushes is alternating until synchronism is reached.

When starting from the alternating side, the converter can drop into synchronism at either polarity; but its polarity can be reversed by strongly exciting the field in the right direction by some outside source, as another converter, etc., or by momentarily opening the circuit and thereby letting the converter slip one pole.

Since when starting from the alternating side the converter requires a very large and, at the same time, lagging current, it is occasionally preferable to start it from the direct-current side as direct-current motor. This can be done when connected to storage battery or direct-current generator. When feeding into a direct-current system together with other converters or converter stations, all but the first converter can be started from the continuous current side by means of rheostats inserted into the armature circuit.

To avoid the necessity of synchronizing the converter, by phase lamps, with the alternating system in case of starting by direct current (which operation may be difficult where the direct voltage fluctuates, owing to heavy fluctuations of load, as railway systems), it is frequently preferable to run the converter up to or beyond synchronism by direct current, then cut off from the direct current, open the field circuit and connect it to the alternating system, thus bringing it into step by alternating current.

If starting from the alternating side is to be avoided, and direct current not always available, as when starting the first converter, a small induction motor (of less poles than the converter) is used as starting motor.

IX. Inverted Converters.

145. Converters may be used to change either from alternating to direct current or as inverted converters from direct to alternating current. While the former use is by far the more frequent, sometimes inverted converters are desirable. Thus in low-tension direct-current systems an outlying district may be supplied by converting from direct to alternating, transmitting as alternating, and then reconverting to direct current. Or in a station containing direct-current generators for short-distance supply and alternators for long-distance supply, the converter may be used as the connecting link to shift the load from the direct to the alternating generators, or inversely, and thus be operated either way according to the distribution of load on the system. Or inverted operation may be used in emergencies to produce alternating current.

When converting from alternating to direct current, the speed of the converter is rigidly fixed by the frequency, and cannot be varied by its field excitation, the variation of the latter merely changing the phase relation of the alternating current. When converting, however, from direct to alternating current as the only source of alternating current, that is, not running in multiple with engine- or turbine-driven alternating-current generators, the speed of the converter as direct-current motor depends upon the field strength, thus it increases with decreasing and decreases with increasing field strength. As alternating-current generator, however, the field strength depends upon the intensity and phase relation of the alternating current, lagging current reducing the field strength and thus increasing speed and frequency, and leading current increasing the field strength and thus decreasing speed and frequency.

Thus, if a load of lagging current is put on an inverted converter, as, for instance, by starting an induction motor or another converter thereby from the alternating side, the demagnetizing effect of the alternating current reduces the field strength and causes the converter to increase in speed and frequency. An increase of frequency, however, may increase the lag of the current, and thus its demagnetizing effect, and thereby still further increase the speed, so that the acceleration may become so rapid as to be beyond control by the field rheostat and endanger the machine. Hence inverted converters have to be carefully watched, especially when starting other converters from them; and some absolutely positive device is necessary to cut the inverted converter off the circuit entirely as soon as its speed exceeds the danger limit. The relatively safest arrangement is separate excitation of the inverted converter by an exciter mechanically driven thereby, since an increase of speed increases the exciter voltage at a still higher rate, and thereby the excitation of the converter, and thus tends to check its speed.

This danger of racing does not exist if the inverted converter operates in parallel with alternating generators, provided that the latter and their prime movers are of such size that they cannot be carried away in speed by the converter. In an inverted converter running in parallel with alternators the speed is not changed by the field excitation, but a change of the latter merely changes the phase relation of the alternating current supplied by the converter; that is, the converter receives power from the direct-current system, and supplies power into the alternating-current system, but at the same time receives wattless current from the alternating system, lagging at under-excitation, leading at over-excitation, and can in the same way as an ordinary converter or synchronous motor be used to compensate for wattless currents in other parts of the alternating system, or to regulate the voltage by phase control.

X. Frequency.

146. While converters can be designed for any frequency, the use of high frequency, as 60 cycles, imposes such severe limitations on the design, especially that of the commutator, as to make the high-frequency converter inferior to the low-frequency or 25-cycle converter.

The commutator surface moves the distance from brush to next brush, or the commutator pitch, during one-half cycle, that is, $\frac{1}{50}$ second with a 25-cycle, $\frac{1}{120}$ second with a 60-cycle converter. The peripheral speed of the commutator, however, is limited by mechanical, electrical, and thermal considerations,— centrifugal forces, loss of power by brush friction, and heating caused thereby. The limitation of peripheral speed limits the commutator pitch. Within this pitch must be included as many commutator segments as necessary to take care of the voltage from brush to brush, and these segments must have a width sufficient for mechanical strength. With the smaller pitch required for high frequency, this may become impossible, and the limits of conservative design thus may have to be exceeded.

In a converter, due to the absence of armature reaction and field distortion, a higher voltage per commutator segment can be allowed than in a direct-current generator. Assuming 15 volts as limit of conservative design would give for a 600-volt converter 40 segments from brush to brush. Allowing 0.25 inch for segment and insulation, as minimum conservative value, 40 segments give a pitch of 10 inches. Estimating 4200 feet per minute as conservative limit of commutator speed gives 70 feet or 840 inches peripheral speed per second, and with 10 inches pitch this gives 84 half cycles, or 42 cycles, as limit of the frequency, permitting conservative commutator design.

At 60 cycles higher voltage per segment, narrower segments and higher commutator speeds thus become necessary than represent best design, and the 60-cycle converter does not permit as conservative commutator design, especially at higher voltage, as a low-frequency converter, and a lower self-inductance of commutation thus must be aimed at than permissible in a 25-cycle converter, the more so as the frequency of commutation (half the number of commutator segments per pole times frequency of rotation) necessarily is higher in the 60-cycle converter.

Thus shallower armature slots become necessary at the higher frequency.

Somewhat similar considerations also apply to the armature construction: the peripheral speed of the armature, even if chosen higher for the 60-cycle converter, limits the pitch per pole at the armature circumference, and thereby the ampere conductors per pole and thus the armature reaction, the more so as shallower slots are necessary. The 60-cycle converter cannot be built with anything like the same armature reaction as is feasible at lower frequency. On the armature reaction, however, very largely depends the stability of a synchronous motor or converter, and machines of low armature reaction tend far more to surging and pulsation of current and voltage than machines of high armature reaction.

The 60-cycle converter therefore cannot be made as stable and capable of taking care of violent fluctuations of load and of excessive overloads as 25-cycle converters can, and in this respect must remain inferior to the lower-frequency machine, though under reasonably favorable conditions regarding variations of load, variations of supply voltage, and overload they can be built to give good service.

It is this inherent inferiority of the 60-cycle converter which has largely been instrumental in introducing 25 cycles as the frequency of electric power generation and distribution.

XI. Double-Current Generators.

147. Similar in appearance to the converter, which changes from alternating to direct current, and to the inverted converter, which changes from direct to alternating current, is the double-current generator; that is, a machine driven by mechanical power and producing direct current as well as alternating current from the same armature, which is connected to commutator and collector rings in the same way as in the converter. Obviously the use of the double-current generator is limited to those sizes and speeds at which a good direct-current generator can be built with the same number of poles as a good alternator; that is, low-frequency machines of large output and relatively high speed; while high-frequency low-speed double-current generators are undesirable.

The essential difference between double-current generator and converter is, however, that in the former the direct current and the alternating current are not in opposition as in the latter, but in the same direction, and the resultant armature polarization thus the sum of the armature polarization of the direct current and of the alternating current.

Since at the same output and the same field strength the armature polarization of the direct current and that of the alternating current are the same, it follows that the resultant armature polarization of the double-current generator is proportional to the load regardless of the proportion in which this load is distributed between the alternating- and direct-current sides. The heating of the armature due to its resistance depends upon the sum of the two currents, that is, upon the total load on the machine. Hence, the output of the double-current generator is limited by the current heating of the armature and by the field distortion due to the armature reaction, in the same way as in a direct-current generator or alternator, and is consequently much less than that of a converter.

In double-current generators, owing to the existence of armature reaction and consequent field distortion, the commutator brushes are more or less shifted against the neutral, and the direction of the continuous-current armature polarization is thus shifted against the neutral by the same angle as the brushes. The direction of the alternating-current armature polarization, however, is shifted against the neutral by the angle of phase displacement of the alternating current. In consequence thereof, the reactions upon the field of the two parts of the armature polarization, that due to the continuous current and that due to the alternating current, are usually different. The reaction on the field of the direct-current load can be overcome by a series field. The reaction on the field of the alternating-current load when feeding converters can be compensated for by a change of phase relation, by means of a series field on the converter, with self-inductance in the alternating lines, or reactive coils at the converters.

Thus, a double-current generator feeding on the alternating side converters can be considered as a direct-current generator in which a part of the commutator, with a corresponding part of the series field, is separated from the generator and located at a distance, connected by alternating leads to the generator. Obviously, automatic compounding of a double-current generator is feasible only if the phase relation of the alternating current changes from lag at no load to lead at load, in the same way as produced by a compounded converter. Otherwise, rheostatic control of the generator is necessary. This is, for instance, the case if the voltage of the double-current generator has to be varied to suit the conditions of its direct-current load, and the voltage of the converter at the end of the alternating lines varied to suit the conditions of load at the receiving end, independent of the voltage at the double-current generator, by means of alternating potential regulators or compensators.

Compared with the direct-current generator, the field of the double-current generator must be such as to give a much greater stability of voltage, owing to the strong demagnetizing effect which may be exerted by lagging currents on the alternating side, and may cause the machine to lose its excitation altogether. For this reason it is frequently preferable to excite double-current generators separately.

XII. Conclusion.

148. Of the types of machines, converter, inverted converter, and double-current generator, sundry combinations can be devised with each other and with synchronous motors, alternators, direct-current motors and generators. Thus, for instance, a converter can be used to supply a certain amount of mechanical power as synchronous motor. In this case the alternating current is increased beyond the value corresponding to the direct current by the amount of current giving the mechanical power, and the armature reactions do not neutralize each other, but the reaction of the alternating current exceeds that of the direct current by the amount corresponding to the mechanical load. In the same way the current heating of the armature is increased. An inverted converter can also be used to supply some mechanical power. Either arrangement, however, while quite feasible, has the disadvantage of interfering with automatic control of voltage by compounding.

Double-current generators can be used to supply more power into the alternating circuit than is given by their prime mover, by receiving power from the direct-current side. In this case a part of the alternating power is generated from mechanical power, and the other converted from direct-current power, and the machine combines the features of an alternator with those of an inverted converter. Conversely, when supplying direct-current power and receiving mechanical power from the prime mover and electric power from the alternating system, the double-current generator combines the features of a direct-current generator and a converter. In either case the armature reaction, etc., are the sum of those corresponding to the two types of machines combined.

149. A combination of the converter with the direct-current generator is represented by the so-called "*motor converter*," which consists of the concatenation of a commutating machine with an induction machine.

If the secondary of an induction machine is connected to a second induction or synchronous machine on the same shaft, and of the same number of poles, the combination runs at half synchronous speed, and the first induction machine as frequency converter supplies half of its power as electric power of half frequency to the second machine, and changes the other half as motor into mechanical power, driving the second machine as generator. (Or, if the two machines have different number of poles, or are connected to run at different speeds, the division of power is at a different but constant ratio.) Using thus a double-current generator as second machine, it receives half of its power mechanically, by the induction machine as motor, and the other half electrically, by the induction machine as frequency converter. Such a machine, then, is intermediate between a converter and a direct-current generator, having an armature reaction equal to half that of a direct-current generator.

Such motor converters are occasionally used on high-frequency systems, as their commutating component is of half frequency, and thus affords a better commutator design than a high-frequency converter. They are necessarily much larger than standard converters, but are smaller than motor generator sets, as half the power is converted in either machine. One advantage of this type of machine for phase control is that it requires no additional reactive coils, as the induction machine affords sufficient reactance. The motor converter, however, is a synchronous machine, that is, it cannot replace induction motor generator sets at the end of very long transmission lines, where synchronous machines tend to surging.

The use of the converter to change from alternating to alternating of a different phase, as, for instance, when using a quarter-phase converter to receive power by one pair of its collector rings from a single-phase circuit and supplying from its other pair of collector rings the other phase of a quarter-phase system, or a three-phase converter on a single-phase system supplying the third wire of a three-phase system from its third collector ring, is outside the scope of this treatise, and is, moreover, of very little importance, since induction or synchronous motors are superior in this respect.

Appendix.

XIII. Direct-Current Converter.

150. If n equidistant pairs of diametrically opposite points of a commutating machine armature are connected to the ends of n compensators or auto-transformers, that is, electric circuits interlinked with a magnetic circuit, and the centers of these compensators connected with each other to a neutral point as shown diagrammatically in Fig. 194 for $n = 3$, this neutral is equidis-

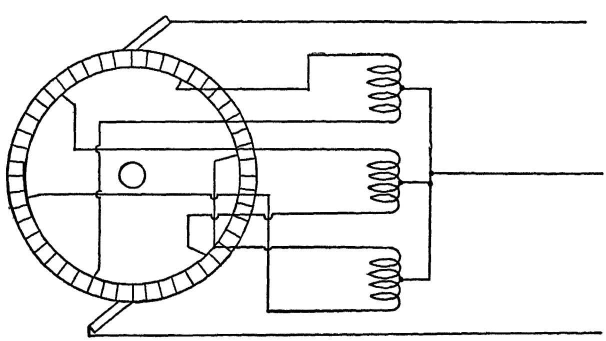


Fig. 194. Diagram of Direct-Current Converter.

tant in potential from the two sets of commutator brushes, and such a machine can be used as continuous-current converter, to transform in the ratio of potentials 1 : 2 or 2 : 1 or 1 : 1, in the latter case transforming power from one side of a three-wire system to the other side.

Obviously either the n compensators can be stationary and connected to the armature by $2n$ collector rings, or the compensators rotated with the armature and their common neutral connected to the external circuit by one collector ring.

The distribution of potential and of current in such a direct-current converter is shown in Fig. 195 for $n = 2$, that is, two compensators in quadrature.

With the voltage $2e$ between the outside conductors of the system, the voltage between the neutral and outside conductor is $\pm e$, that on each of the $2n$ compensator sections is

$$e \sin\left(\theta - \theta_0 - \frac{\pi k}{n}\right), \qquad k = 0, 1, 2 \ldots 2n - 1.$$

Neglecting losses in the converter and the compensator, the currents in the two sets of commutator brushes are equal and of the same direction, that is, both outgoing or both incoming, and

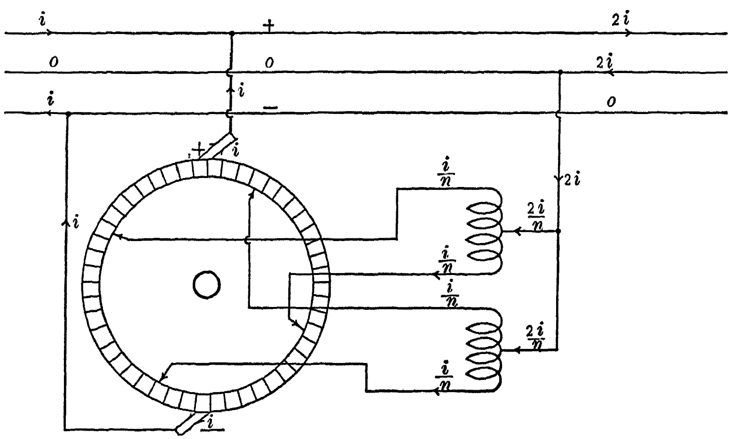


Fig. 195. Distribution of e.m.f. and Current in Direct-Current Converter.

opposite to the current in the neutral; that is, two equal currents i enter the commutator brushes and issue as current $2i$ from the neutral, or inversely.

From the law of conservation of energy it follows that the current $2i$ entering from the neutral divides in $2n$ equal and constant branches of direct current, $\frac{i}{n}$, in the $2n$ compensator sections, and hence enters the armature, to issue as current i from each of the commutator brushes.

In reality the current in each compensator section is

$$\frac{i}{n} + i_0 \sqrt{2} \cos \left(\theta - \theta_0 - \frac{\pi k}{n} + \alpha \right),$$

where i_0 is the exciting current of the magnetic circuit of the compensator, and α the angle of hysteretic advance of phase. At the commutator the current on the motor side is larger than the current on the generator side, by the amount required to cover the losses of power in converter and compensator.

In Fig. 195 the positive side of the system is generator, the negative side motor. This machine can be considered as receiving the current i at the voltage e from the negative side of the system, and transforming it into current i at voltage e on the positive side of the system, or it can be considered as receiving current i at voltage $2e$ from the system, and transforming it into current $2i$ at the voltage e on the positive side of the system, or of receiving current $2i$ at voltage e from the negative side, and returning current i at voltage $2e$. In either case the direct-current converter produces a difference of power of $2ie$ between the two sides of the three-wire system.

The armature reaction of the currents from the generator side of the converter is equal but opposite to the armature reaction of the corresponding currents entering the motor side, and the motor and generator armature reactions thus neutralize each other, as in the synchronous converter; that is, the resultant armature reaction of the continuous-current converter is practically zero, or the only remaining armature reaction is that corresponding to the relatively small current required to rotate the machine, that is, to supply the internal losses in the same. The armature reaction of the current supplying the electric power transformed into mechanical power obviously also remains, if the machine is used simultaneously as motor, as for driving a booster connected into the system to produce a difference between the voltages of the two sides, or the armature reaction of the currents generated from mechanical power if the machine is driven as generator.

151. While the currents in the armature coils are more or less sine waves in the alternator, rectangular reversed currents in the direct-current generator or motor, and distorted triple-frequency currents in the synchronous converter, the currents in the armature coils of the direct-current converter are approximately triangular double-frequency waves.

Let Fig. 196 represent a development of a direct-current converter with brushes B_1 and B_2, and C one compensator receiving current $2\,i$ from the neutral. Consider first an armature coil a_1

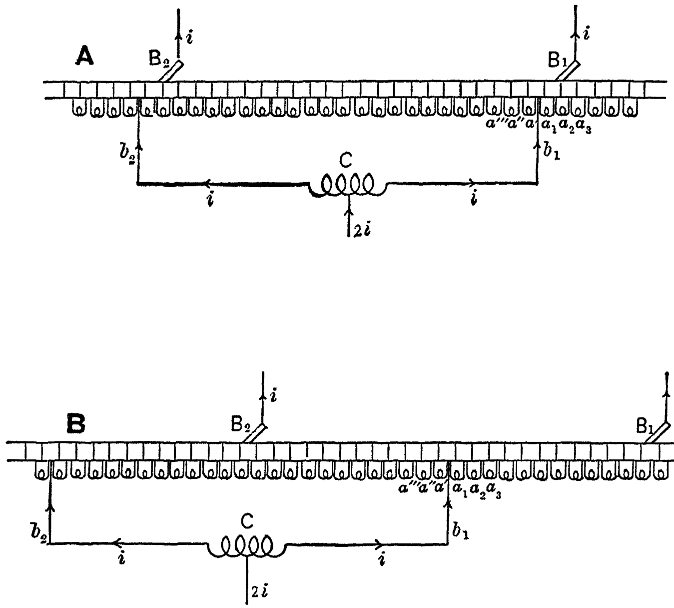


Fig. 196. Development of a Direct-Current Converter.

adjacent and behind (in the direction of rotation) a compensator lead b_1. In the moment when compensator leads b_1 b_2 coincide with the brushes B_1 B_2 the current i directly enters the brushes and coil a_1 is without current. In the next moment (Fig. 196*A*) the total current i from b_1 passes coil a_1 to brush B_1, while there is yet practically no current from b_1 over coils a' a'', etc., to brush B_2. But with the forward motion of the armature less and less of the current from b_1 passes through a_1 a_2, etc., to brush B_1 and more over a' a'', etc., to brush B_2, until in the position of a_1 midway between b_1 and b_2 (Fig. 196*B*), one-half of the current from b_1 passes a_1 a_2, etc., to B_1, the other half a' a'', etc., to B_2. With the further rotation the current in a_1 grows less and becomes zero when b_1 coincides with B_2, or half a cycle after its coincidence with B_1. That is, the current in coil a_1 approximately has the triangular form shown as i_1 in Fig. 197, changing twice per period from 0 to i. It is shown negative, since it is against the direction of rotation of the armature. In the same way we see that the current in the coil a', adjacent ahead of the lead b_1, has a shape shown as i' in Fig. 197. The current in coil

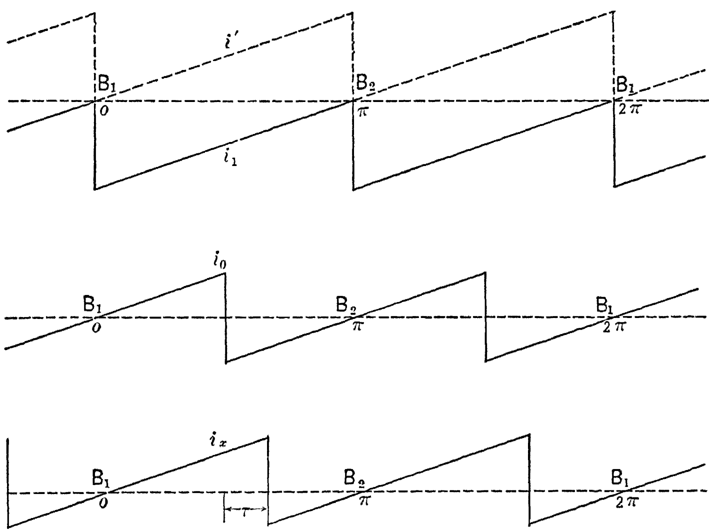


Fig. 197. Current in the Various Coils of a Direct-Current Converter.

a_0 midway between two commutator leads has the form i_0, and in general the current in any armature coil a_x, distant by angle τ from the midway position a_0, has the form i_x, Fig. 197.

All the currents become zero at the moment when the compensator leads b_1 b_2 coincide with the brushes B_1 B_2, and change by i at the moment when their respective coils pass a commutator brush. Thus the lines A and A' in Fig. 198 with zero values at B_1 B_2, the position of brushes, represent the currents in the individual armature coils. The current changes from A to A' at the moment $\theta = \tau$ when the respective armature coil passes the brush, twice per period. Due to the inductance of the armature coils, which opposes the change of current, the current waves are not perfectly triangular, but differ somewhat therefrom.

With n compensators, each compensator lead carries the current $\frac{i}{n}$, which passes through the armature coils as triangular current,

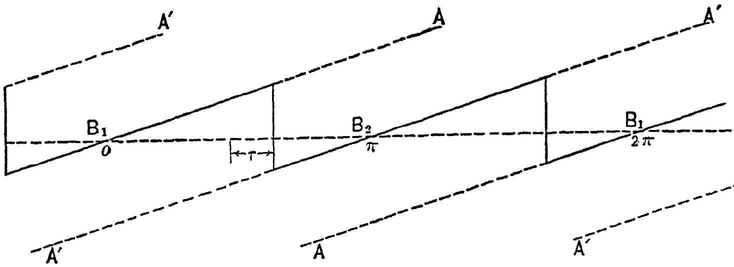


Fig. 198. Current in Individual Coils of a Direct-Current Converter with One Compensator.

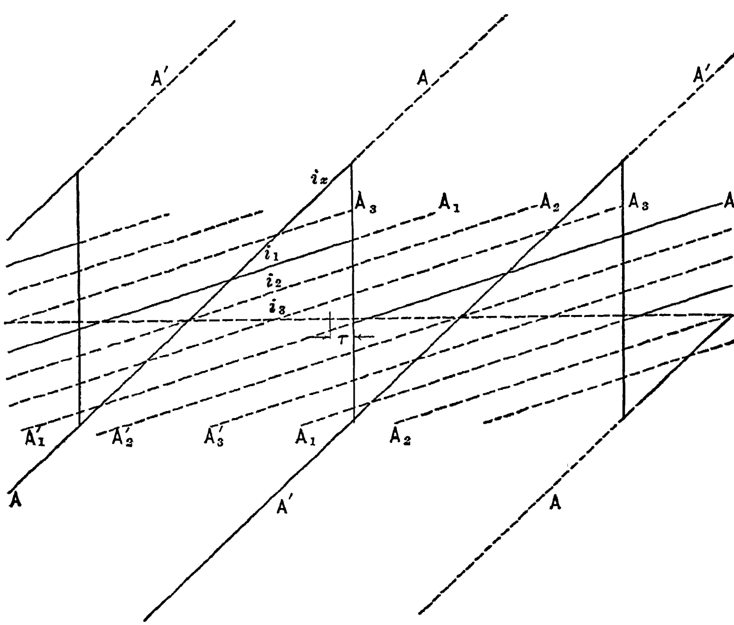


Fig. 199. Current in a Single Coil of a Direct-Current Converter with Three Compensators.

changing by $\frac{i}{n}$ in the moment the armature coil passes a commutator brush. This current passes the zero value in the moment the compensator lead coincides with a brush. Thus, the different currents of n compensators which are superposed in an armature coil a_x have the shape shown in Fig. 199 for $n = 3$. That is, each compensator gives a set of slanting lines A_1A_1', A_2A_2', A_3A_3', and all the branch currents i_1, i_2, i_3, superposed, give a resultant current i_x, which changes by i in the moment the coil passes the brush. i_x varies between the extreme values $\frac{i}{2}(2p-1)$ and $\frac{i}{2}(2p+1)$, if the armature coil is displaced from the midway position between two adjacent compensator leads by angle τ, and $p = \frac{\tau}{\pi}$. p varies between $-\frac{1}{2n}$ and $+\frac{1}{2n}$.

Thus the current in an armature coil in position $p = \frac{\tau}{\pi}$ can be denoted in the range from p to $1 + p$, or τ to $\pi + \tau$, by

$$i_x = \frac{i}{2}(2x - 1),$$

where

$$x = \frac{\theta}{\pi}.$$

The effective value of this current is

$$I = \sqrt{\int_p^{p+1} i_x^2 dx}$$

$$= \frac{i}{2}\sqrt{\tfrac{1}{3} + 4p^2}.$$

Since in the same machine as direct-current generator at voltage $2e$ and current i, the current per armature coil is $\frac{i}{2}$, the ratio of current is

$$\frac{I}{\frac{i}{2}} = \sqrt{\tfrac{1}{3} + 4p^2},$$

and thus the relative I^2r loss or the heat developed in the armature coil,

$$\gamma = \left(\frac{I}{\frac{i}{2}}\right)^2 = \tfrac{1}{3} + 4p^2,$$

with a minimum,

$$p = 0, \quad \gamma_0 = \tfrac{1}{3},$$

and a maximum,

$$p = \frac{1}{2n}.$$

$$\gamma_m = \frac{1}{3} + \frac{1}{n^2} = \frac{3 + n^2}{3n^2}.$$

The mean heating or I^2r of the armature is found by integrating over γ from

$$p = -\frac{1}{2n} \text{ to } p = +\frac{1}{2n},$$

as

$$\Gamma = n \int_{-\frac{1}{2n}}^{+\frac{1}{2n}} \gamma dp$$

$$= \frac{1}{3} + \frac{1}{3n^2} = \frac{1 + n^2}{3n^2}.$$

This gives the following table, for the direct-current converter, of minimum current heating, γ_0, in the coil midway between adjacent commutator leads, maximum current heating, γ_m, in the coil adjacent to the commutator lead, mean current heating, Γ, and rating as based on mean current heating in the armature, $\frac{1}{\sqrt{\Gamma}}$:

DIRECT-CURRENT CONVERTER I^2r RATING

No. of compensators, n=	d. c gen.	1	2	3	4	n	∞
Minimum current heating $p=0$, γ_0=	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Maximum current heating, $p=+\frac{1}{n}$, γm=	1	$\frac{4}{3}$	$\frac{7}{12}$	$\frac{4}{9}$	$\frac{19}{48}$	$\frac{1}{3}+\frac{1}{n^2}$	$\frac{1}{3}$
Mean current heating, Γ=	1	$\frac{2}{3}$	$\frac{5}{12}$	$\frac{10}{27}$	$\frac{17}{48}$	$\frac{1}{3}+\frac{1}{3n^2}$	$\frac{1}{3}$
Rating, $\frac{1}{\sqrt{\Gamma}}$ =	1	1.225	1.549	1.643	1.681	$\sqrt{\frac{3n^2}{1+n^2}}$	1.732

As seen, the output of the direct-current converter is greater than that of the same machine as generator. Using more than three compensators offers very little advantage, and the difference between three and two compensators is comparatively small, also, but the difference between two and one compensator, especially regarding the local armature heating, is considerable, so that for most practical purposes a two-compensator converter would be preferable.

The number of compensators used in the direct-current converter has a similar effect regarding current distribution, heating, etc., as the number of phases in the synchronous converter.

Obviously these relative outputs given in above table refer to the armature heating only. Regarding commutation, the total current at the brushes is the same in the converter as in the generator, the only advantage of the former being the better commutation due to the absence of armature reaction.

The limit of output set by armature reaction and corresponding field excitation in a motor or generator obviously does not exist at all in a converter. It follows herefrom that a direct-current motor or generator does not give the most advantageous direct-current converter, but that in the direct-current converter just as in the synchronous converter, it is preferable to proportion the parts differently, in accordance with above discussion, as, for instance, to use less conductor section, a greater number of conductors in series per pole, etc.

XIV. Three-Wire Generator and Converter.

152. A machine based upon the principle of the direct-current converter is frequently used to supply a three-wire direct-current distribution system (Edison system). This machine may be a single generator or synchronous converter, which is designed for the voltage between the outside conductors of the circuit (the positive and the negative conductor), 220 to 280 volts, while the middle conductor of the system, or neutral conductor, is connected to the generator by compensator and collector rings, or, in the case of a synchronous converter, is connected to the neutral of the step-up transformers, and the latter thus used as compensators.

A three-wire generator thus is a combination of a direct-current generator and a direct-current converter, and a three-wire converter is a combination of a synchronous converter and a direct-current converter. Such a three-wire machine has the advantage over two separate machines, connected to the two sides of the three-wire direct-current system, of combining two smaller machines into one of twice the size, and thus higher

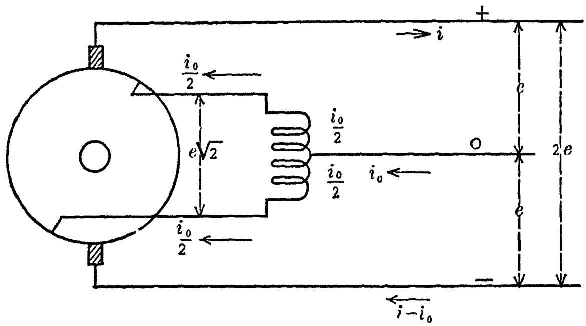


Fig. 200. Three-Wire Machine with Single Compensator.

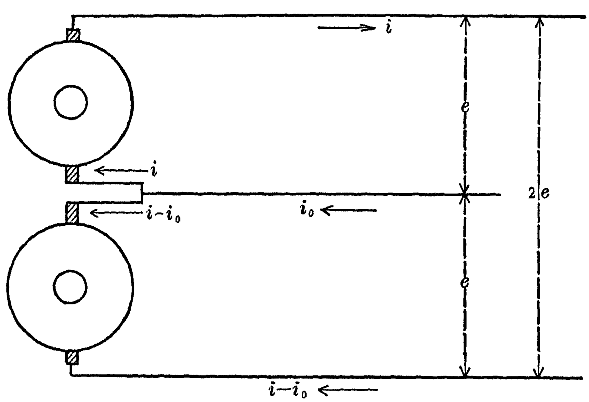


Fig. 201. Three-Wire System with Two Machines.

space- and operation-economy and lower cost, and has the further advantage that only half as large current is commutated as by the use of two separate machines; that is, the positive brush of the machine on the negative and the negative brush of the machine on the positive side of the system are saved, as seen by the diagrammatic sketch of the machine in Fig. 200 and the two separate two-wire machines in Fig. 201. The use of three-wire 220-volt machines on three-wire direct-current systems thus has practically displaced that of two separate 110-volt machines.

A. Three-Wire Direct-Current Generator.

153. In such machines, either only one compensator or autotransformer is used for deriving the neutral, as shown diagrammatically in Fig. 200, or two compensators in quadrature, as shown in Fig. 202, but rarely more.

As the efficiency of conversion of a direct-current converter with two compensators in quadrature (Fig. 202) is higher than that of a direct-current converter with single compensator (Fig.

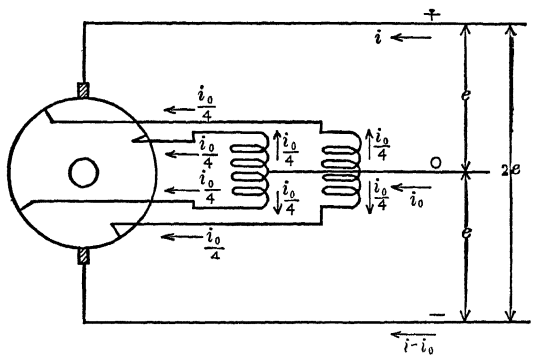


Fig. 202. Three-Wire Machine with Two Compensators.

200), it is preferable to use two (or even more) compensators where a large amount of power is to be converted, that is, where a very great unbalancing between the two sides of the three-wire system may occur, or one side may be practically unloaded while the other is overloaded. Where, however, the load is fairly distributed between the two sides of the system, that is, the neutral current (which is the difference between the currents on the two sides of the system) is small and so only a small part of the generator power is converted from one side to the other, and the efficiency of this conversion thus of negligible influence on the heating and the output of the machine, a single compensator is preferable because of its simplicity. In three-wire distribution systems the latter is practically always the case, that is, the load fairly balanced and the neutral current small.

The size of the compensators depends upon the amount of unbalanced power, that is, the maximum difference between the load on the two sides of the three-wire system, and thus equals the product of neutral current i_0 and voltage e between neutral and outside conductor; that is, in the three-wire system of voltage e per circuit, voltage $2e$ between the outside conductors, and maximum current i in the outside conductors, the generator power rating is

$$p = 2\,ei.$$

Let now i_0 = maximum unbalanced current in the neutral — usually not exceeding 10 to 20 per cent of i and using a single compensator, connected diametrically across the armature, Fig. 200, the maximum of the alternating voltage which it receives is $2e$, and its effective voltage therefore $e\sqrt{2}$. As the neutral current i_0 divides when entering the compensator, the current in the compensating winding is $\frac{i_0}{2}$ (neglecting the small exciting current), and the volt-ampere capacity of the compensator thus is

$$p_0 = \frac{ei_0}{\sqrt{2}},$$

and

$$\frac{p_0}{p} = \frac{1}{2\sqrt{2}}\,\frac{i_0}{i}$$

$$= 0.354\,\frac{i_0}{i}.$$

Even with the neutral current equal to the current in the outside conductor, or the one side of the system fully loaded, the other not loaded, the compensator thus would have only 35.4 per cent of the volt-ampere capacity of the generator, and as a compensator of ratio 1 ÷ 1 is half the size of a transformer of the same volt-ampere capacity, in this case the compensator has, approximately, the size of a transformer of 17.7 per cent of the size of the generator.

With the maximum unbalancing of 20 per cent, or $\frac{i_0}{i} = 0.2$, the compensator thus has 7 per cent of the volt-ampere capacity of the generator, or the size of a transformer of only 3.5 per cent of the generator capacity, that is, is very small, and this method is therefore the most convenient for deriving the neutral of a three-wire distribution system.

When using n compensators, obviously each has $\frac{1}{n}$ of the size which a single compensator would have.

The disadvantage of the three-wire generator over two separate generators is that a three-wire generator can only divide the voltage in two equal parts, that is, the two sides of the system have the same voltage at the generator. The use of two separate generators, however, permits the production of a higher voltage on one side of the system than on the other, and thus takes care of the greater line drop on the more evenly loaded side. Even in the case, however, where a voltage difference between the two sides of the system is desired for controlling feeder drops, it can more economically be given by a separate booster in the neutral, as such a booster would require only a capacity equal to the neutral current times half the desired voltage difference between the two sides, and with 20 per cent neutral current and 10 per cent voltage difference between the two sides, thus would have only one per cent of the size of the generator.

B. Three-Wire Converter.

154. In a converter feeding a three-wire direct-current system the neutral can be derived by connection to the transformer neutral. Even in this case, however, frequently a separate compensator is used, connected across a pair of collector rings of the converter, since, as seen above, with the moderate unbalancing usually existing, such a compensator is very small.

When connecting the direct-current neutral to the transformer neutral it is necessary to use such a connection that the transformer can operate as compensator, that is, that the direct current in each transformer divides into two branches of equal m.m.f., otherwise the direct-current produces a unidirectional magnetization in the transformer, which superimposed upon the magnetic cycle raises the magnetic induction beyond saturation, and thus causes excessive exciting current and heating, except when very small.

For instance, with Y connection of the transformers supplying a three-phase converter, Fig. 203, each transformer secondary receives one-third of the neutral current, and if this current is not very small and comparable with the exciting current of the transformer — which can rarely be — the magnetic density in the transformer rises beyond saturation by this unidirectional m.m.f. This connection thus is in general not permissible for deriving the neutral.

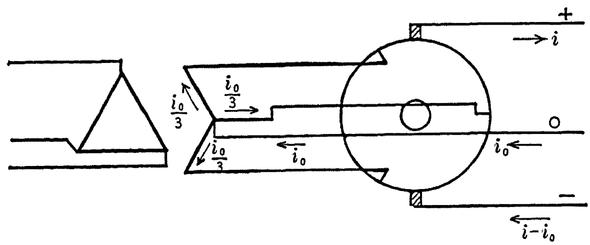


Fig. 203. Neutral of Y-Connected Transformers Connected to Neutral of Three-Wire System Supplied from a Three-Phase Converter.

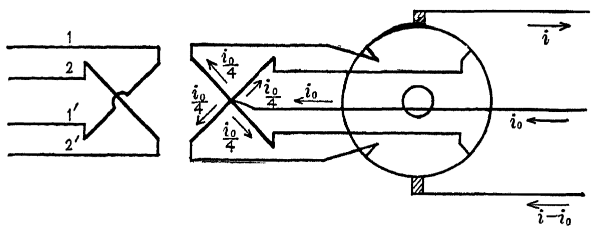


Fig. 204. Quarter-Phase Converter with Transformer Neutral Connected to Direct-Current Neutral.

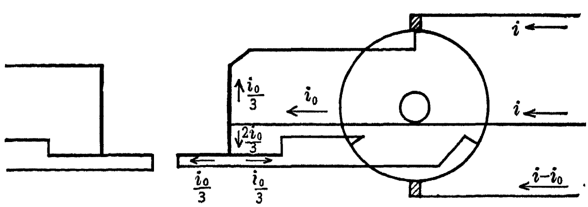


Fig. 205. Three-Phase Converter with Neutral of the T-Connected Transformers as Direct-Current Neutral.

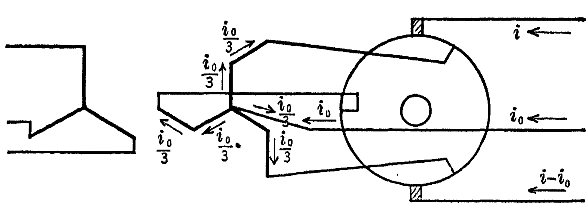


Fig. 206. Three-Phase Converter with Transformer Neutral Connected to Direct-Current Neutral.

In a quarter-phase converter, as shown in Fig. 204, the transformer neutral can be used as direct-current neutral, since in each transformer the direct current divides into two equal branches, which magnetize in opposite direction, and so neutralize.

The T connection, Fig. 205, can be used for three-phase converters with the neutral derived from a point at one-third the height of the teaser transformer, since the m.m.fs. of the direct current i_0 balance in the transformers, as seen in Fig. 205.

Delta connection on three-phase and double delta on six-phase converters cannot be used, as it has no neutral, but in this case a separate compensator is required.

The diagrammatical connections of transformers can, however, be used on six-phase converters, and the connection shown in Fig. 206, which has two coils on each transformer, connected to different phases, on three-phase converters.

E. INDUCTION MACHINES.

I. General.

155. The direction of rotation of a direct-current motor, whether shunt- or series-wound, is independent of the direction of the current supplied thereto; that is, when reversing the current in a direct-current motor the direction of rotation remains the same. Thus theoretically any continuous-current motor should operate also with alternating currents. Obviously in this case not only the armature but also the magnetic field of the motor must be thoroughly laminated to exclude eddy currents, and care taken that the currents in the field and armature circuits reverse simultaneously. Obviously the simplest way of fulfilling the latter condition is to connect the field and armature circuits in series as alternating-current series motor. Such motors are used to a considerable extent, mainly for railroading. Their disadvantage for many purposes is the use of a commutator, and also that their speed is not constant but depends upon the load.

The shunt motor on an alternating-current circuit has the objection that in the armature winding the current should be power current, thus in phase with the e.m.f., while in the field winding the current is lagging nearly 90 deg., as magnetizing current. Thus field and armature would be out of phase with each other. To overcome this objection either there is inserted in series with the field circuit a condenser of such capacity as to bring the current back into phase with the voltage (Stanley), or the field may be excited from a separate e.m.f. differing 90 deg. in phase from that supplied to the armature. The former arrangement has the disadvantage of requiring almost perfect constancy of frequency, and therefore is not practicable. In the latter arrangement the armature winding of the motor is fed by one, the field winding by the other phase of a quarter-phase system, and thus the current in the armature brought approximately into phase with the magnetic flux of the field.

Such an arrangement obviously loads the two phases of the system unsymmetrically, the one with the armature power current, the other with the lagging field current. To balance the system two such motors may be used simultaneously and combined in one structure, the one receiving power current from the first, magnetizing current from the second phase, the second motor receiving magnetizing current from the first and power current from the second phase.

The objection that the use of the commutator, in an alternating-current motor tends to vicious sparking and therefore greatly limits the design, can be entirely overcome by utilizing the alternating feature of the current; that is, instead of leading the current into the armature by commutator and brushes, producing it therein by electromagnetic induction, by closing the armature conductors upon themselves and surrounding the armature by a primary coil at right angles to the field exciting coil.

Such motors have been built, consisting of two structures each containing a magnetizing circuit acted upon by one phase and a primary power circuit acting upon a closed-circuit armature as secondary and excited by the other phase of a quarter-phase system (Stanley motor).

Going still a step further, the two structures can be combined into one by having each of the two coils fulfill the double function of magnetizing the field and producing currents in the secondary which are acted upon by the magnetization produced by the other phase.

Obviously, instead of two phases in quadrature any number of phases can be used.

This leads us by gradual steps of development from the continuous-current shunt motor to the alternating-current polyphase induction motor.

In its general behavior the alternating-current induction motor is therefore analogous to the continuous-current shunt motor. Like the shunt motor, it operates at approximately constant magnetic density. It runs at fairly constant speed, slowing down gradually with increasing load. The main difference is that in the induction motor the current in the secondary does not pass through a system of brushes, as in the continuous-current shunt motor, but is produced in the secondary as the short-circuited secondary of a transformer; and in consequence thereof the primary circuit of the induction motor fulfills the double function of an exciting circuit corresponding to the field circuit of the continuous-current machine and a primary circuit producing a secondary current in the secondary by electromagnetic induction.

156. Since in the secondary of the induction motor the currents are produced by induction from the primary impressed currents, the induction motor in its electromagnetic features is essentially a transformer; that is, it consists of a magnetic circuit or magnetic circuits interlinked with two electric circuits or sets of circuits, the primary and the secondary circuits. The difference between transformer and induction motor is that in the former the secondary is fixed regarding the primary, and the electric energy in the secondary is made use of, while in the latter the secondary is movable regarding the primary, and the mechanical force acting between primary and secondary is used. In consequence thereof the frequency of the currents in the secondary of the induction motor differs from, and as a rule is very much lower than, that of the currents impressed upon the primary, and thus the ratio of e.m.fs. generated in primary and in secondary is not the ratio of their respective turns, but is the ratio of the product of turns and frequency.

Taking due consideration of this difference of frequency between primary and secondary, the theoretical investigation of the induction motor corresponds to that of the stationary transformer. The transformer feature of the induction motor predominates to such an extent that in theoretical investigation the induction motor is best treated as a transformer, and the electrical output of the transformer corresponds to the mechanical output of the induction motor.

The secondary of the motor consists of two or more circuits displaced in phase from each other so as to offer a closed secondary to the primary circuits, irrespective of the relative motion. The primary consists of one or several circuits.

In consequence of the relative motion of the primary and secondary, the magnetic circuit of the induction motor must be arranged so that the secondary while revolving does not leave the magnetic field of force. That means, the magnetic field of force must be of constant intensity in all directions, or, in other words, the component of magnetic flux in any direction in space be of the same or approximately the same intensity but differing in phase. Such a magnetic field can either be considered as the superposition of two magnetic fields of equal intensity in quadrature in time and space, or it can be represented theoretically by a revolving magnetic flux of constant intensity, or rotating field, or simply treated as alternating magnetic flux of the same intensity in every direction.

157. The operation of the induction motor thus can also be considered as due to the action of a rotating magnetic field upon a system of short-circuited conductors. In the motor field or primary, usually the stator, by a system of polyphase impressed e.m.fs. or by the combination of a single-phase impressed e.m.f. and the reaction of the currents produced in the secondary, a rotating magnetic field is produced. This rotating field produces currents in the short-circuited armature or secondary winding, usually the rotor, and by its action on these currents drags along the secondary conductors, and thus speeds up the armature and tends to bring it up to synchronism, that is, to the same speed as the rotating field, at which speed the secondary currents would disappear by the armature conductors moving together with the rotating field, and thus cutting no lines of force. The secondary therefore slips in speed behind the speed of the rotating field by as much as is required to produce the secondary currents and give the torque necessary to carry the load. The slip of the induction motor thus increases with increase of load, and is approximately proportional thereto. Inversely, if the secondary is driven at a higher speed than that of the rotating field, the field drags the armature conductors back, that is, consumes mechanical torque, and the machine then acts as a brake or induction generator.

In the polyphase induction motor this magnetic field is produced by a number of electric circuits relatively displaced in space, and excited by currents having the same displacement in phase as the exciting coils have in space.

In the monocyclic motor one of the two superimposed quadrature fields is excited by the primary power circuit, the other by the magnetizing or teaser circuit.

In the single-phase motor one of the two superimposed magnetic quadrature fields is excited by the primary electric circuit, the other by the secondary currents carried into quadrature position by the rotation of the secondary. In either case, at or near synchronism the magnetic fields are practically identical.

The transformer feature being predominant, in theoretical investigations of induction motors it is generally preferable to start therefrom.

The characteristics of the transformer are independent of the ratio of transformation, other things being equal; that is, doubling the number of turns for instance, and at the same time reducing their cross-section to one-half, leaves the efficiency, regulation, etc., of the transformer unchanged. In the same way, in the induction motor it is unessential what the ratio of primary to secondary turns is, or, in other words, the secondary circuit can be wound for any suitable number of turns, provided the same total copper cross-section is used. In consequence hereof the secondary circuit is mostly wound with one or two bars per slot, to get maximum amount of copper, that is, minimum resistance of secondary.

The general characteristics of the induction motor being independent of the ratio of turns, it is for theoretical considerations simpler to assume the secondary motor circuits reduced to the same number of turns and phases as the primary, or of the ratio of transformation 1 to 1, by multiplying all secondary currents and dividing all secondary e.m.fs. by the ratio of turns, multiplying all secondary impedances and dividing all secondary admittances by the square of the ratio of turns, etc.

Thus in the following under secondary current, e.m.f., impedance, etc., shall always be understood their values reduced to the primary, or corresponding to a ratio of turns 1 to 1, and the same number of secondary as primary phases, although in practice a ratio 1 to 1 will hardly ever be used, as not fulfilling the condition of uniform effective reluctance desirable in the starting of the induction motor.

II. Polyphase Induction Motor.

1. Introduction.

158. The typical induction motor is the polyphase motor. By gradual development from the direct-current shunt motor we arrive at the polyphase induction motor.

The magnetic field of any induction motor, whether supplied by polyphase, monocyclic, or single-phase e.m.f., is at normal condition of operation, that is, near synchronism, a polyphase field. Thus to a certain extent all induction motors can be called polyphase machines. When supplied with a polyphase system of e.m.fs. the internal reactions of the induction motor are simplest and only those of a transformer with moving secondary, while in the single-phase induction motor at the same time a phase transformation occurs, the second or magnetizing phase being produced from the impressed phase of e.m.f. by the rotation of the motor, which carries the secondary currents into quadrature position with the primary current.

The polyphase induction motor of the three-phase or quarter-phase type is the one most commonly used, while single-phase motors have found a more limited application only, and especially for smaller powers.

Thus in the following more particularly the polyphase induction machine shall be treated, and the single-phase type discussed only in so far as it differs from the typical polyphase machine.

2. Calculation.

159. In the polyphase induction motor,

Let

$Y = g + jb$ = primary exciting admittance, or admittance of the primary circuit with open secondary circuit;

that is,

ge = magnetic power current,

be = wattless magnetizing current,

where e = counter-generated e.m.f. of the motor; $Z_0 = r_0 - jx_0$ = primary self-inductive impedance, and $Z_1 = r_1 - jx_1$ = secondary self-inductive impedance, reduced to the primary by the ratio of turns.*

All these quantities refer to one primary circuit and one corresponding secondary circuit. Thus in a three-phase induction motor the total power, etc., is three times that of one circuit, in the quarter-phase motor with three-phase armature $1\frac{1}{2}$ of the three secondary circuits are to be considered as corresponding to each of the two primary circuits, etc.

Let e = primary counter-generated e.m.f., or e.m.f. generated in the primary circuit by the flux interlinked with primary and

* The self-inductive reactance refers to that flux which surrounds one of the electric circuits only, without being interlinked with the other circuits.

secondary (mutual induction); s = slip, with the primary frequency as unit; that is, $s = 0$ denoting synchronous rotation, $s = 1$ standstill of the motor.

We then have

$1 - s$ = speed of the motor secondary as fraction of synchronous speed,

sf = frequency of the secondary currents,

where f = frequency impressed upon the primary:

hence, se = e.m.f. generated in the secondary.

The actual impedance of the secondary circuit at the frequency sf is

$$Z_1^s = r_1 - jsx_1;$$

hence, the secondary current is

$$I_1 = \frac{se}{Z_1^s} = \frac{se}{r_1 - jsx_1} = e\left(\frac{sr_1}{r_1^2 + s^2x_1^2} + j\frac{s^2x_1}{r_1^2 + s^2x_1^2}\right) = e\,(a_1 + ja_2),$$

where

$$a_1 = \frac{sr_1}{r_1^2 + s^2x_1^2}, \qquad a_2 = \frac{s^2x_1}{r_1^2 + s^2x_1^2};$$

the primary exciting current is

$$I_{00} = eY = e\,[g + jb],$$

and the total primary current is

$$I_0 = e\,[(a_1 + g) + j\,(a_2 + b)] = e\,(b_1 + jb_2),$$

where

$$b_1 = a_1 + g, \qquad b_2 = a_2 + b.$$

The e.m.f. consumed in the primary circuit by the impedance Z_0 is $I_0 Z_0$, the counter-generated e.m.f. is e, hence, the primary terminal voltage is

$$E_0 = e + I_0 Z_0 = e\,[1 + (b_1 + jb_2)\,(r_0 - jx_0)] = e\,(c_1 + jc_2),$$

where,

$$c_1 = 1 + r_0 b_1 + x_0 b_2 \text{ and } c_2 = r_0 b_2 - x_0 b_1.$$

Eliminating complex quantities, we have

$$E_0 = e\sqrt{c_1^2 + c_2^2},$$

hence, the counter-generated e.m.f. of motor,

$$e = \frac{E_0}{\sqrt{c_1^2 + c_2^2}},$$

where E_0 = impressed e.m.f., absolute value.

Substituting this value in the equations of I_1, I_{00}, I_0, etc., gives the complex expressions of currents and e.m.fs., and eliminating the imaginary quantities we have the primary current,

$$I_0 = e\sqrt{b_1^2 + b_2^2}, \text{ etc.}$$

The torque of the polyphase induction motor (or any other motor or generator) is proportional to the product of the mutual magnetic flux and the component of ampere-turns of the secondary, which is in phase with the magnetic flux in time, but in quadrature therewith in direction or space. Since the generated e.m.f. is proportional to the mutual magnetic flux and the number of turns, but in quadrature thereto in time, the torque of the induction motor is proportional also to the product of the generated e.m.f. and the component of secondary current in quadrature therewith in time and in space.

Since $I_1 = e\,(a_1 + ja_2)$ is the secondary current corresponding to the generated e.m.f. e, the secondary current in the quadrature position thereto in space, that is, corresponding to the e.m.f. je, is

$$jI_1 = e\,(-a_2 + ja_1),$$

and a_1e is the component of this current in quadrature in time with the e.m.f. e.

Thus the torque is proportional to $e \times a_1 e$, or

$$\begin{aligned} D &= e^2 a_1 \\ &= \frac{e^2 r_1 s}{r_1^2 + s^2 x_1^2} = \frac{E_0^2 r_1 s}{(c_1^2 + c_2^2)(r_1^2 + s^2 x_1^2)}. \end{aligned}$$

This value D is in its dimension a *power*, and it is the power which the torque of the motor would develop at synchronous speed.

160. In induction motors, and in general motors which have a definite limiting speed, it is preferable to give the torque in the form of the power developed at the limiting speed, in this case synchronism, as "synchronous watts," since thereby it is made independent of the individual conditions of the motor, as its number of poles, frequency, etc., and made comparable with the power input, etc. It is obvious that when given in synchronous watts, the maximum possible value of torque which could be reached, if there were no losses in the motor, equals the power input. Thus, in an induction motor with 9000 watts power

input, a torque of 7000 synchronous watts means that $\frac{7}{9}$ of the maximum theoretically possible torque is realized, while the statement, "a torque of 30 lb. at one foot radius," would be meaningless without knowing the number of poles and the frequency. Thus, the denotation of the torque in synchronous watts is the most general, and preferably used in induction motors.

Since the theoretical maximum possible torque equals the power input, the ratio

$$\frac{\text{torque in synchronous watts output}}{\text{power input}},$$

that is,

$$\frac{\text{actual torque}}{\text{maximum possible torque}},$$

is called the *torque efficiency* of the motor, analogous to the power efficiency or

$$\frac{\text{power output}}{\text{power input}};$$

that is,

$$\frac{\text{power output}}{\text{maximum possible power output}}.$$

Analogously

$$\frac{\text{torque in synchronous watts}}{\text{volt-amperes input}}$$

is called the *apparent torque efficiency.*

The definition of these quantities, which are of importance in judging induction motors, are thus:

The "*efficiency*" or "*power efficiency*" is the ratio of the true mechanical output of the motor to the output which it would give at the same power input if there were no internal losses in the motor.

The "*apparent efficiency*" or "*apparent power efficiency*" is the ratio of the mechanical output of the motor to the output which it would give at the same volt-ampere input if there were neither internal losses nor phase displacement in the motor.

The "*torque efficiency*" is the ratio of the torque of the motor to the torque which it would give at the same power input if there were no internal losses in the motor.

The "*apparent torque efficiency*" is the ratio of the torque of the motor to the torque which it would give at the same volt-ampere input if there were neither internal losses nor phase displacement in the motor.

The torque efficiencies are of special interest in starting where the power efficiencies are necessarily zero, but it nevertheless is of importance to find how much torque per watt or per volt-ampere input is given by the motor.

Since $D = e^2 a_1$ is the power developed by the motor torque at synchronism, the power developed at the speed of $(1 - s) \times$ synchronism, or the actual power output of the motor, is

$$\begin{aligned} P &= (1 - s) D \\ &= e^2 a_1 (1 - s) \\ &= \frac{e^2 r_1 s (1 - s)}{r_1^2 + s^2 x_1^2}. \end{aligned}$$

The output P includes friction, windage, etc.; thus, the net mechanical output is $P -$ friction, etc. Since, however, friction, etc., depend upon the mechanical construction of the individual motor and its use, it cannot be included in a general formula. P is thus the mechanical output, and D the torque developed at the armature conductors.

The primary current

$$\dot{I}_0 = e (b_1 + j b_2)$$

has the quadrature components eb_1 and eb_2.

The primary impressed e.m.f.

$$\dot{E}_0 = e (c_1 + j c_2)$$

has the quadrature components ec_1 and ec_2.

Since the components eb_1 and ec_2, and eb_2 and ec_1, respectively, are in quadrature with each other, and thus represent no power, the power input of the primary circuit is

$$eb_1 \times ec_1 + eb_2 \times ec_2,$$

or

$$P_0 = e^2 (b_1 c_1 + b_2 c_2).$$

The volt-amperes or apparent input is obviously,

$$\begin{aligned} P_a &= I_0 E_0 \\ &= e^2 \sqrt{(b_1^2 + b_2^2)(c_1^2 + c_2^2)}. \end{aligned}$$

161. These equations can be greatly simplified by neglecting the exciting current of the motors, and approximate values of

current, torque, power, etc., derived thereby, which are sufficiently accurate for preliminary investigations of the motor at speeds sufficiently below synchronism to make the total motor current large compared with the exciting current.

In this case the *primary current* equals the secondary current, that is,

$$I_0 = I_1 = \frac{se}{Z^s} = e\,(a_1 + ja_2),$$

where

$$a_1 = \frac{sr_1}{r_1^2 + s^2x_1^2}, \text{ etc.,}$$

and

$$\begin{aligned} E_0 &= e + Z_0 I_0 \\ &= e\left\{1 + \frac{sZ_0}{Z_1^s}\right\} = \frac{e\,(Z_1^s + sZ_0)}{Z_1^s} \\ &= e\,\frac{(r_1 + sr_0) - js\,(x_1 + x_0)}{r_1 - jsx_1}, \end{aligned}$$

and, in absolute values,

$$e_0 = e\,\frac{\sqrt{(r_1 + sr_0)^2 + s^2(x_1 + x_0^2)}}{\sqrt{r_1^2 + s^2x_1^2}};$$

hence,

$$e = \frac{e_0\sqrt{r_1^2 + s^2x_1^2}}{\sqrt{(r_1 + sr_0)^2 + s^2(x_1 + x_0)^2}},$$

and the *torque*, in synchronous watts, is

$$D = e^2a_1 = \frac{se^2r_1}{r_1^2 + s^2x_1^2};$$

hence, substituting for e,

$$D = \frac{se_0^2r_1}{(r_1 + sr_0)^2 + s^2\,(x_1 + x_0)^2},$$

and the *power* is

$$P = \frac{s\,(1 - s)\,e_0^2r_1}{(r_1 + sr_0)^2 + s^2(x_1 + x_0)^2}.$$

If the additional resistance r is inserted into the armature circuit, and the total armature resistance thus becomes $r_1 + r$, instead of r_1, substituting $(r_1 + r)$ in above equations we have

$$D = \frac{se_0^2\,(r_1 + r)}{(r_1 + r + sr_0)^2 + s^2(x_1 + x_0)^2}$$

and

$$P = \frac{s\,(1 - s)\,e_0^2\,(r_1 + r)}{(r_1 + r + sr_0)^2 + s^2(x_1 + x)^2}, \text{ etc.}$$

Neglecting also the primary self-inductive impedance, $Z_0 = r_0 - jx_0$, which sometimes can be done as first approximation, especially at large values of r, these equations become

$$D = \frac{se_0^2\,(r_1+r)}{(r_1+r)^2+s^2x_1^2},$$

$$P = s\,\frac{(1-s)\,e_0^2\,(r_1+r)}{(r_1+r)^2+s^2x_1^2}, \text{ etc.}$$

162. Since the counter-generated e.m.f. e (and thus the impressed e.m.f. E_0) enters in the equation of current, magnetism, etc., as a simple factor, in the equations of torque, power input and output, and volt-ampere input as square, and cancels in the equation of efficiency, power-factor, etc., it follows that the current, magnetic flux, etc., of an induction motor are proportional to the impressed e.m.f.; the torque, power output, power input, and volt-ampere input are proportional to the square of the impressed e.m.f., and the torque and power efficiencies and the power-factor are independent of the impressed voltage.

In reality, however, a slight decrease of efficiency and power-factor occurs at higher impressed voltages, due to the increase of resistance caused by the increasing temperature of the motor and due to the approach to magnetic saturation, and a slight decrease of efficiency occurs at lower voltages when including in the efficiency the loss of power by friction, since this is independent of the output and thus at lower voltage, that is, lesser output, a larger percentage of the output, so that the efficiencies and the power-factor can be considered as independent of the impressed voltage, and the torque and power proportional to the square thereof only approximately, but sufficiently close for most purposes.

3. Load and Speed Curves.

163. The calculation of the induction motor characteristics is most conveniently carried out in tabulated form by means of above-given equations as follows:

Let $Z_0 = r_0 - jx_0 = 0.1 - 0.3\,j$ = primary self-inductive impedance.

$Z_1 = r_1 - jx_1 = 0.1 - 0.3\,j$ = secondary self-inductive impedance reduced to primary.

$Y = g + jb = 0.01 + 0.1\,j$ = primary exciting admittance.

$E_0 = 110$ volts = primary impressed e.m.f.

It is then, per phase,

s.	$m = r_1^2 + s^2 x_1^2$.	$a_1 = \frac{s r_1}{m}$	$a_2 = \frac{s^2 x_1}{m}$	$b_1 = a_1 + g$.	$b_2 = a_2 + b$.	$c_1 = 1 + r_0 b_1 + x_0 b_2$	$c_2 = r_0 b_2 - x_0 b_1$.	$c = \sqrt{c_1^2 + c_2^2}$.	$e = \frac{E_0}{c}$	$g = \sqrt{b_1^2 + b_2^2}$.	$I = eg$.
0	0 0100	0	0	0 01	0.10	1 031	+0 007	1 031	106.6	0 1010	10.8
0 01	0.0100	0 100	0.003	0.11	0.103	1.042	−0.023	1.042	105 7	0.1507	15.9
0.02	0 0100	0.200	0.012	0.21	0.112	1.055	−0.052	1.056	104.3	0.238	24.8
0.05	0 0102	0.490	0.073	0.50	0 173	1.102	−0.133	1.110	99 2	0.522	51.8
0 1	0 0109	0.920	0 276	0.93	0 376	1.206	−0 241	1.230	89.5	1.003	89 7
0.15	0 0120	1.25	0.563	1.26	0 663	1.325	−0.308	1.360	80.9	1.424	115
0.2	0 0136	1.47	0.883	1.48	0.983	1.443	−0.354	1.485	74.2	1 777	132
0 3	0 0181	1.66	1 49	1.67	1.50	1.617	−0.351	1.654	66 6	2.245	149
0.5	0.0325	1.54	2.31	1.55	2.41	1.878	−0.224	1.891	58.2	2 865	167
1.0	0.1000	1.00	3.00	1.01	3 10	2.031	+0.007	2.031	54.1	3.261	176

s.	e^2.	$D = e^2 a_1$.	$P = (1-s)D$	$P_a = E_0 I$.	$p = b_1 c_1 + b_2 c_2$.	$P_0 = e^2 p$.	*eff.* $= \frac{P}{P_0}$	*app. eff.* $= \frac{P}{P_a}$	*pow. fac.* $= \frac{P_0}{P_a}$
0	11,360	0	0	1.19	0 011	0.125	0	0	10.5
0.01	11,170	1.117	1.106	1.75	0.112	1.249	88.5	63 2	71 5
0.02	10,880	2 176	2.133	2.73	0.216	2.350	91.0	78.3	86.2
0.05	9,840	4.82	4.58	5 70	0.528	5.20	88 3	80 5	91.3
0.1	8,010	7.38	6.64	9 87	1.030	8.25	80.7	67.3	83 5
0.15	6,540	8 20	6.97	12 65	1.466	9.60	72 5	55.0	76 0
0.2	5,510	8.10	6 48	14.52	1.782	9.80	66 0	44.6	67.5
0.3	4,440	7.36	5.15	16 4	2.154	9.55	53.8	31 5	58.3
0.5	3,390	5.23	2.61	18 4	2 370	8 04	32 3	14.2	43.8
1.0	2,930	2.93	0	19 4	2.072	6 08	0	0	31.3

Diagrammatically it is most instructive in judging about an induction motor to plot from the preceding calculation —

1st. The *load curves*, that is, with the load or power output as abscissas, the values of speed (as a fraction of synchronism), of current input, power-factor, efficiency, apparent efficiency, and torque.

2d. The *speed curves*, that is, with the speed, as a fraction of synchronism, as abscissas, the values of torque, current input, power-factor, torque efficiency, and apparent torque efficiency.

The load curves are most instructive for the range of speed near synchronism, that is, the normal operating conditions of the motor, while the speed curves characterize the behavior of the motor at any speed.

In Fig. 207 are plotted the load curves, and in Fig. 208 the speed curves of a typical polyphase induction motor of moderate size, having the following constants: $e_0 = 110$; $Y = 0.01 - 0.1\,j$; $Z_1 = 0.1 - 0.3\,j$, and $Z_0 = 0.1 - 0.3\,j$.

As sample of a poor motor of high resistance and high admittance or exciting current are plotted in Fig. 209 the load curves

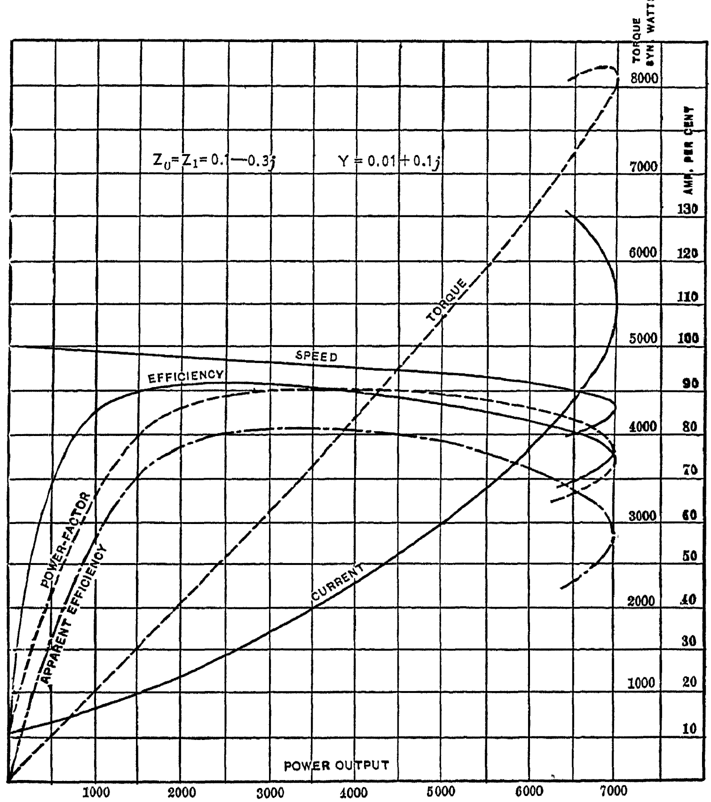


Fig. 207. Induction Motor Load Curves.

of a motor having the following constants: $e_0 = 110$; $Y = 0.04 + 0.4\,j$; $Z_1 = 0.3 - 0.3\,j$, and $Z_0 = 0.3 - 0.3\,j$, showing the overturn of the power-factor curve frequently met in poor motors.

164. The shape of the characteristic motor curves depends entirely on the three complex constants, Y, Z_1, and Z_0, but is essentially independent of the impressed voltage.

Thus a change of the admittance Y has no effect on the char-

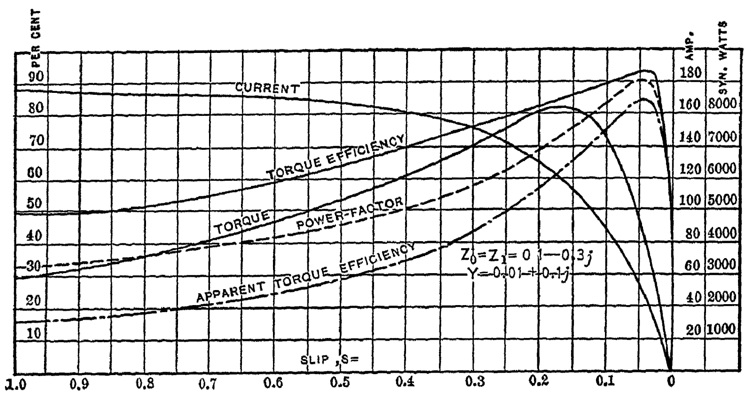


Fig. 208. Induction Motor Speed Curves.

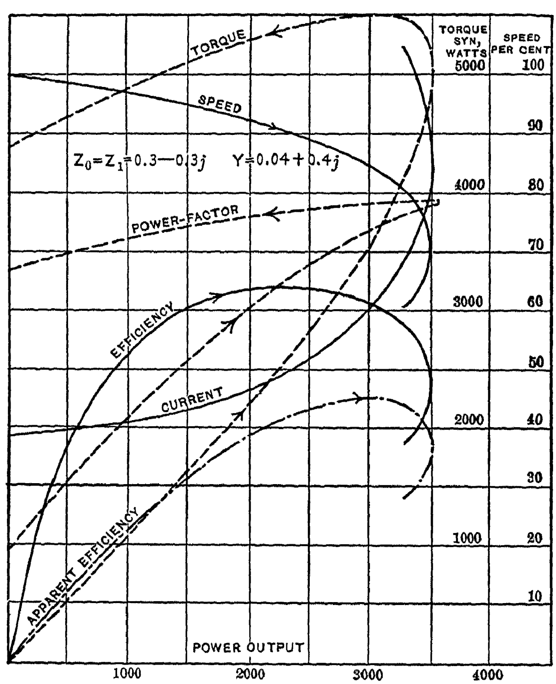


Fig. 209. Load Curves of Poor Induction Motor.

acteristic curves, provided that the impedances Z_1 and Z_0 are changed inversely proportional thereto, such a change merely representing the effect of a change of impressed voltage. A moderate change of one of the impedances has relatively little effect on the motor characteristics, provided that the other impedance changes so that the sum $Z_1 + Z_0$ remains constant, and thus the motor can be characterized by its total internal impedance, that is,

$$Z = Z_1 + Z_0;$$

or

$$r - jx = (r_1 + r_0) - j\,(x_1 + x_0).$$

Thus the characteristic behavior of the induction motor depends upon two complex imaginary constants, Y and Z, or four real constants, g, b, r, x, the same terms which characterize the stationary alternating-current transformer on non-inductive load.

Instead of conductance g, susceptance b, resistance r, and reactance x, as characteristic constants may be chosen: the absolute exciting admittance $y = \sqrt{g^2 + b^2}$; the absolute self-inductive impedance $z = \sqrt{r^2 + x^2}$; the power-factor of admittance $\beta = g/y$, and the power-factor of impedance $\alpha = r/z$.

165. If the admittance y is reduced n-fold and the impedance z increased n-fold, with the e.m.f. $\sqrt{n}E_0$ impressed upon the motor, the speed, torque, power input and output, volt-ampere input and excitation, power-factor, efficiencies, etc., of the motor, that is, all its characteristic features, remain the same, as seen from above given equations, and since a change of impressed e.m.f. does not change the characteristics, it follows that a change of admittance and of impedance does not change the characteristics of the motor provided the product $\gamma = yz$ remains the same.

Thus the induction motor is characterized by three constants only:

The product of exciting admittance and self-inductive impedance $\gamma = yz$, which may be called the characteristic constant of the motor.

The power-factor of primary admittance $\beta = \frac{g}{y}$.

The power-factor of self-inductive impedance $\alpha = \frac{r}{z}$.

All these three quantities are absolute numbers.

The physical meaning of the characteristic constant or the product of the exciting admittance and impedance is the following:

If I_{00} = exciting current and I_{10} = starting current, we have, approximately,

$$y = \frac{I_{00}}{E_0},$$

$$z = \frac{E_0}{I_{10}},$$

$$\gamma = yz = \frac{I_{00}}{I_{10}}.$$

The characteristic constant of the induction motor $\gamma = yz$ is the ratio of exciting current to starting current or current at standstill.

At given impressed e.m.f., the exciting current I_{00} is inversely proportional to the mutual inductance of primary and secondary circuit. The starting current I_{10} is inversely proportional to the sum of the self-inductance of primary and secondary circuit.

Thus the characteristic constant $\gamma = yz$ is approximately the ratio of total self-inductance to mutual inductance of the motor circuits; that is, the ratio of the flux interlinked with only one circuit, primary or secondary, to the flux interlinked with both circuits, primary and secondary, or the ratio of the waste or leakage flux to the useful flux. The importance of this quantity is evident.

4. Effect of Armature Resistance and Starting.

166. The secondary or armature resistance r_1 enters the equation of secondary current thus:

$$I_1 = \frac{se}{r_1 - jsx_1} = e\left(\frac{sr_1}{r_1^2 + s^2x_1^2} + j\,\frac{s^2x_1}{r_1^2 + s^2x_1^2}\right) = e\,(a_1 + ja_2),$$

and the further equations only indirectly in so far as r_1 is contained in a_1 and a_2.

Increasing the armature resistance n-fold, to nr_1, we get at an n-fold increased slip ns,

$$I_1 = \frac{nse}{nr_1 - jnsx_1} = \frac{se}{r_1 - jsx};$$

that is, the same value, and thus the same values for e, I_0, D, P_0, P_a, while the power is decreased from $P = (1 - s) D$ to $P = (1 - ns) D$, and the efficiency and apparent efficiency are correspondingly reduced. The power-factor is not changed; hence, an increase of armature resistance r_1 produces a proportional increase of slip s, and thereby corresponding decrease of power output, efficiency and apparent efficiency, but does not change the torque, power input, current, power-factor, and the torque efficiencies.

Thus the insertion of resistance in the armature or secondary of the induction motor offers a means of reducing the speed corresponding to a given torque, and thereby the desired torque can be produced at any speed below that corresponding to short-circuited armature or secondary without changing the input or current.

Hence, given the speed curve of a short-circuited motor, the speed curve with resistance inserted in the armature can be derived therefrom directly by increasing the slip in proportion to the increased resistance.

This is done in Fig. 210, in which are shown the speed curves of the motor Figs. 207 and 208, between standstill and synchronism, for —

Short-circuited armature, $r_1 = 0.1$ (same as Fig. 208).

0.15 ohm additional resistance per circuit inserted in armature, $r_1 = 0.25$, that is, 2.5 times increased slip.

0.5 ohm additional resistance inserted in the armature, $r_1 = 0.6$, that is, 6 times increased slip.

1.5 ohm additional resistance inserted in the armature, $r_1 = 1.6$, that is, 16 times increased slip.

The corresponding current curves are shown on the same sheet.

With short-circuited secondary the maximum torque of 8250 synchronous watts is reached at 16 per cent slip. The starting torque is 2950 synchronous watts, and the starting current 176 amperes.

With armature resistance $r_1 = 0.25$, the same maximum torque is reached at 40 per cent slip, the starting torque is increased to 6050 synchronous watts, and the starting current decreased to 160 amperes.

With the secondary resistance $r_1 = 0.6$, the maximum torque of 8250 synchronous watts approximately takes place in starting, and the starting current is decreased to 124 amperes.

With armature resistance $r_1 = 1.6$, the starting torque is below the maximum, 5620 synchronous watts, and the starting current is only 64 amperes.

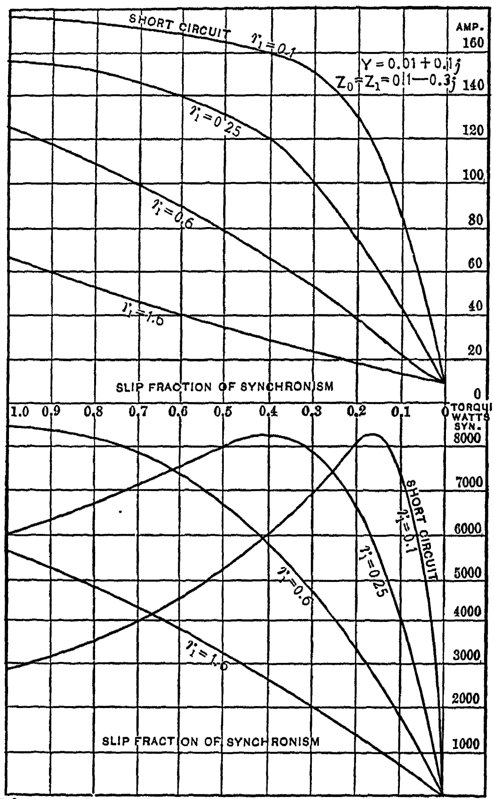


Fig. 210. **Induction Motor Speed, Torque and Current Curves.**

In the two latter cases the lower or unstable branch of the torque curve has altogether disappeared, and the motor speed is stable over the whole range; the motor starts with the maximum torque which it can reach, and with increasing speed, torque and current decrease; that is, the motor has the characteristic of the direct-current series motor, except that its maximum speed is limited by synchronism.

167. It follows herefrom that high secondary resistance, while very objectionable in running near synchronism, is advantageous in starting or running at very low speed, by reducing the current input and increasing the torque.

In starting we have

$$s = 1.$$

Substituting this value in the equations of subsection 2 gives the starting torque, starting current, etc., of the polyphase induction motor.

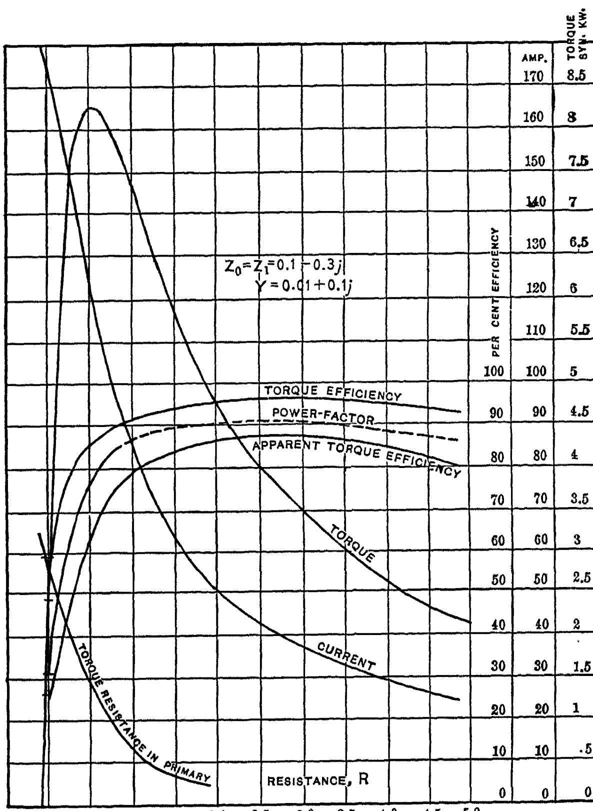


Fig. 211. Induction Motor Starting Torque with Resistance in the Secondary.

In Fig. 211 are shown for the motor in Figs. 207, 208, and 210 the values of starting torque, current, power-factor, torque efficiency, and apparent torque efficiency for various values of the secondary motor resistance, from $r_1 = 0.1$, the internal resistance of the motor, or $R = 0$ additional resistance to $r_1 = 5.1$ or $R = 5$ ohms additional resistance. The best values of torque efficiency are found beyond the maximum torque point.

The same Fig. 211 also shows the torque with resistance inserted into the primary circuit.

The insertion of reactance, either in the primary or in the secondary, is just as unsatisfactory as the insertion of resistance in the primary circuit.

Capacity inserted in the secondary very greatly increases the torque within the narrow range of capacity corresponding to resonance with the internal reactance of the motor, and the torque which can be produced in this way is far in excess of the maximum torque of the motor when running or when starting with resistance in the secondary.

But even at its best value, the torque efficiency available with capacity in the secondary is far below that available with resistance.

For further discussion of the polyphase induction motor, see Transactions American Institute of Electrical Engineers, 1897, page 175, and "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

III. Single-phase Induction Motor.

1. Introduction.

168. In the polyphase motor a number of secondary coils displaced in position from each other are acted upon by a number of primary coils displaced in position and excited by e.m.fs. displaced in phase from each other by the same angle as the displacement of position of the coils.

In the single-phase induction motor a system of secondary circuits is acted upon by one primary coil (or system of primary coils connected in series or in parallel) excited by a single alternating current.

A number of secondary circuits displaced in position must be used so as to offer to the primary circuit a short-circuited secondary in any position of the armature. If only one secondary coil is used, the motor is a synchronous induction motor, and belongs to the class of reaction machines.

A single-phase induction motor will not start from rest, but when started in either direction will accelerate with increasing torque and approach synchronism.

When running at or very near synchronism, the magnetic field of the single-phase induction motor is practically identical with that of a polyphase motor, that is, can be represented by the theory of the rotating field. Thus, in a turn wound under angle τ to the primary winding of the single-phase induction motor, at synchronism an e.m.f. is generated equal to that generated in a turn of the primary winding, but differing therefrom by angle $\theta = \tau$ in time phase.

In a polyphase motor the magnetic flux in any direction is due to the resultant m.m.f. of primary and of secondary currents, in the same way as in a transformer. The same is the case in the direction of the axis of the exciting coil of the single-phase induction motor. In the direction at right angles to the axis of the exciting coil, however, the magnetic flux is due to the m.m.f. of the secondary currents alone, no primary e.m.f. acting in this direction.

Consequently, in the polyphase motor running synchronously, that is, doing no work whatever, the secondary becomes currentless, and the primary current is the exciting current of the motor only. In the single-phase induction motor, even when running light, the secondary still carries the exciting current of the magnetic flux in quadrature with the axis of the primary exciting coil. Since this flux has essentially the same intensity as the flux in the direction of the axis of the primary exciting coil, the current in the armature of the single-phase induction motor running light, and therefore also the primary current corresponding thereto, has the same m.m.f., that is, the same intensity, as the primary exciting current, and the total primary current of the single-phase induction motor running light is thus twice the exciting current, that is, it is the exciting current of the main magnetic flux plus the current producing in the secondary the exciting current of the cross magnetic flux. In reality it is slightly less, especially in small motors, due to the drop of voltage in the self-inductive impedance and the drop of quadrature magnetic flux below the impressed primary magnetic flux caused thereby. In the secondary at synchronism this secondary exciting current is a current of twice the primary frequency; at any other speed it is of a frequency equal to speed (in cycles) plus synchronism.

Thus, if in a quarter-phase motor running light one phase is open-circuited, the current in the other phase doubles. If in the three-phase motor two phases are open-circuited, the current in the third phase trebles, since the resultant m.m.f. of a three-phase machine is 1.5 times that of one phase. In consequence thereof, the total volt-ampere input of the motor remains the same and at the same magnetic density, or the same impressed e.m.f., all induction motors, single-phase as well as polyphase, consume approximately the same volt-ampere input, and the same power input for excitation, and give the same distribution of magnetic flux.

169. Since the maximum output of a single-phase motor at the same impressed e.m.f. is considerably less than that of a polyphase motor, it follows therefrom that the relative exciting current in the single-phase motor must be larger.

The cause of this cross magnetization in the single-phase induction motor near synchronism is that the secondary armature currents lag 90 deg. behind the magnetism, and are carried by the synchronous rotation 90 deg. in space before reaching their maximum, thus giving the same magnetic effect as a quarter-phase e.m.f. impressed upon the primary system in quadrature position with the main coil. Hence they can be eliminated by impressing a magnetizing quadrature e.m.f. upon an auxiliary motor circuit, as is done in the monocyclic motor.

Below synchronism, the secondary currents are carried less than 90 deg., and thus the cross magnetization due to them is correspondingly reduced, and becomes zero at standstill.

The torque is proportional to the power component of the armature currents times the intensity of magnetic flux in quadrature position thereto.

In the single-phase induction motor, the armature power currents $I_1' = ea_1$ can exist only coaxially with the primary coil, since this is the only position in which corresponding primary currents can exist. The magnetic flux in quadrature position is proportional to the component e carried in quadrature, or approximately to $(1 - s)\, e$, and the torque is thus

$$D = (1 - s)\, eI' = (1 - s)\, e^2 a_1,$$

thus decreases much faster with decreasing speed, and becomes zero at standstill. The power is then

$$P = (1-s)^2 eI' = (1-s)^2 e^2 a_1.$$

Since in the single-phase motor only one primary circuit but a multiplicity of secondary circuits exist, all secondary circuits are to be considered as corresponding to the same primary circuit, and thus the joint impedance of all secondary circuits must be used as the secondary impedance, at least at or near synchronism. Thus, if the armature has a quarter-phase winding of impedance Z_1 per circuit, the resultant secondary impedance is $\frac{Z_1}{2}$, if it contains a three-phase winding of impedance Z_1 per circuit, the resultant secondary impedance is $\frac{Z_1}{3}$.

In consequence hereof the resultant secondary impedance of a single-phase motor is less in comparison with the primary impedance than in the polyphase motor. Since the drop of speed under load depends upon the secondary resistance, in the single-phase induction motor the drop in speed at load is generally less than in the polyphase motor; that is, the single-phase induction motor has a greater constancy of speed than the polyphase induction motor, but just as the polyphase induction motor, it can never reach complete synchronism, but slips below synchronism, approximately in proportion to the speed.

The further calculation of the single-phase induction motor is identical with that of the polyphase induction motor, as given in the previous chapter.

In general, no special motors are used for single-phase circuits, but polyphase motors adapted thereto. An induction motor with only one primary winding could not be started by a phase-splitting device, and would necessarily be started by external means. A polyphase motor, as for instance a three-phase motor operating single-phase, by having two of its terminals connected to the single-phase mains, is just as satisfactory a single-phase motor as one built with only one primary winding. The only difference is that in the latter case a part of the circumference of the primary structure is left without winding, while in the polyphase motor this part contains windings also, which, however, are not used, or are not effective when running as single-phase motor, but are necessary when starting by means of displaced e.m.fs. Thus, in a three-phase motor operating from single-phase mains, in starting, the third terminal is connected to a phase-displacing device, giving to the motor the cross magnetization in quadrature to the axis of the primary coil, which at speed is produced by the rotation of the secondary currents, and which is necessary for producing the torque by its action upon the secondary power currents.

Thus the investigation of the single-phase induction motor resolves itself into the investigation of the polyphase motor operating on single-phase circuits.

2. Load and Speed Curves.

170. Comparing thus a three-phase motor of exciting admittance per circuit $Y = g + jb$ and self-inductive impedances $Z_0 = r_0 - jx_0$ and $Z_1 = r_1 - jx_1$ per circuit with the same motor operating as single-phase motor from one pair of terminals, the single-phase exciting admittance is $Y' = 3\,Y$ (so as to give the same volt-amperes excitation $3eY$), the primary self-inductive impedance is the same, $Z_0 = r_0 - jx_0$; the secondary self-inductive impedance single-phase, however, is only $Z_1' = \frac{Z_1}{3}$, since all three secondary circuits correspond to the same primary circuit, and thus the total impedance single-phase is $Z' = Z_0 + \frac{Z_1}{3}$, while that of the three-phase motor is $Z = Z_0 + Z_1$.

Assuming approximately $Z_0 = Z_1$, we have

$$Z' = \frac{2\,Z}{3}.$$

Thus, in absolute value,

$$Y' = 3\,Y,$$
$$Z' = \tfrac{2}{3}\,Z, \text{ and}$$
$$\gamma' = 2\,\gamma;$$

that is, the characteristic constant of a motor running single-phase is twice what it is running three-phase, or polyphase in general; hence, the ratio of exciting current to current at standstill, or of waste flux to useful flux, is doubled by changing from polyphase to single-phase.

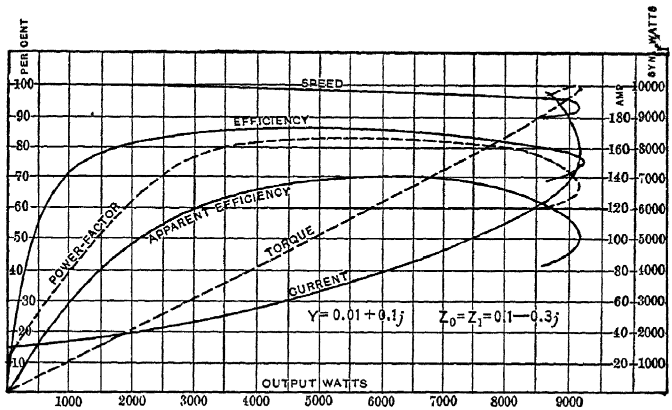


Fig. 212. Three-Phase Induction Motor on Single-Phase Circuit, Load Curves.

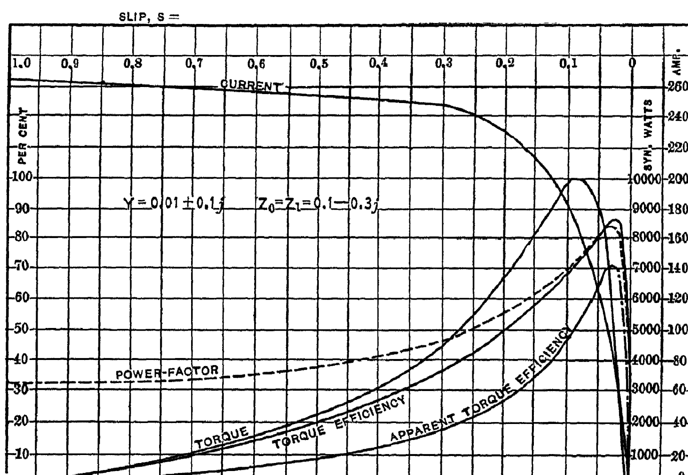


Fig. 213. Three-Phase Induction Motor on Single-Phase Circuit, Speed Curves.

This explains the inferiority of the single-phase motor compared with the polyphase motor.

As a rule, an average polyphase motor makes a poor single-phase motor, and a good single-phase motor must be an excellent polyphase motor.

As instances are shown in Figs. 212 and 213 the load curves and speed curves of the three-phase motor of which the curves of one circuit are given in Figs. 207 and 208, having the following constants:

THREE PHASE.	$e_0 = 110$.	SINGLE PHASE.
$Y = 0.01 + 0.1\,j$,		$Y = 0.03 + 0.3\,j$,
$Z_0 = 0.1 - 0.3\,j$,		$Z_0 = 0.1 - 0.3\,j$,
$Z_1 = 0.1 - 0.3\,j$,		$Z_1 = 0.033 - 0.1\,j$,
Thus $\gamma = 6.36$.		Thus, $\gamma = 12.72$.

It is of interest to compare Fig. 212 with Fig. 207 and to note the lesser drop of speed (due to the relatively lower secondary resistance) and lower power-factor and efficiencies, especially at light load. The maximum output is reduced from $3 \times 7000 = 21,000$ in the three-phase motor to 9100 watts in the single-phase motor.

Since, however, the internal losses are less in the single-phase motor, it can be operated at from 25 to 30 per cent higher magnetic density than the same motor polyphase, and in this case its output is from two-thirds to three-quarters that of the polyphase motor.

171. The preceding discussion of the single-phase induction motor is approximate, and correct only at or near synchronism, where the magnetic field is practically a uniformly rotating field of constant intensity, that is, the quadrature flux produced by the armature magnetization equal to the main magnetic flux produced by the impressed e.m.f.

If an accurate calculation of the motor at intermediate speed and at standstill is required, the changes of effective exciting admittance and of secondary impedance, due to the decrease of the quadrature flux, have to be considered.

At synchronism the total exciting admittance gives the m.m.f. of main flux and auxiliary flux, while at standstill the quadrature flux has disappeared or decreased to that given by the starting device, and thus the total exciting admittance has decreased to one-half of its synchronous value, or one-half plus the exciting admittance of the starting flux.

The effective secondary impedance at synchronism is the joint impedance of all secondary circuits; at standstill, however, only the joint impedance of the projection of the secondary coils on the direction of the main flux, that is, twice as large as at synchronism. In other words, from standstill to synchronism the effective secondary impedance gradually decreases to one-half its standstill value at synchronism.

For fuller discussion hereof the reader must be referred to my second paper on the Single-phase Induction Motor, Transactions A. I. E. E., 1900, page 37.

The torque in Fig. 213 obviously slopes towards zero at standstill. The effect of resistance inserted in the secondary of the single-phase motor is similar to that in the polyphase motor in so far as an increase of resistance lowers the speed at which the maximum torque takes place. While, however, in the polyphase motor the maximum torque remains the same, and merely shifts towards lower speed with the increase of resistance, in the single-phase motor the maximum torque decreases proportionally to the speed at which the maximum torque point occurs, due to the factor $(1 - s)$ entering the equation of the torque,

$$D = e^2 a_1 (1 - s).$$

Thus, in Fig. 214 are given the values of torque of the single-phase motor for the same conditions and the same motor of which the speed curves polyphase are given in Fig. 210.

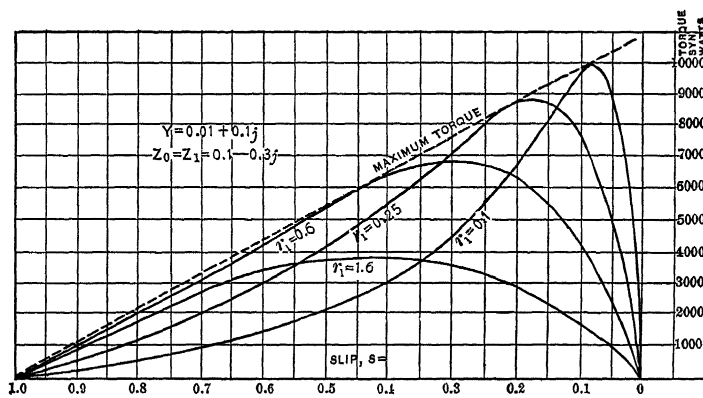


Fig. 214. Three-Phase Induction Motor on Single-Phase Circuit, Torque Curves.

The maximum value of torque which can be reached at any speed lies on the tangent drawn from the origin onto the torque curve for $r_1 = 0.1$ or short-circuited secondary. At low speeds the torque of the single-phase motor is greatly increased by the insertion of secondary resistance, just as in the polyphase motor.

3. Starting Devices of Single-phase Motors.

172. At standstill, the single-phase induction motor has no starting torque, since the line of polarization due to the secondary currents coincides with the axis of magnetic flux impressed by the primary circuit. Only when revolving is torque produced, due to the axis of secondary polarization being shifted by the rotation, against the axis of magnetism, until at or near synchronism it is in quadrature therewith, and the magnetic disposition thus identical with that of the polyphase induction motor.

Leaving out of consideration starting by mechanical means and starting by converting the motor into a series or shunt motor, that is, by passing the alternating current by means of commutator and brushes through both elements of the motor, the following methods of starting single-phase motors are left:

1st. Shifting of the axis of armature or secondary polarization against the axis of generating magnetism.

2d. Shifting the axis of magnetism, that is, producing a magnetic flux displaced in position from the flux producing the armature currents.

The first method requires a secondary system which is unsymmetrical in regard to the primary, and thus, since the secondary is movable, requires means of changing the secondary circuit, such as commutator brushes short-circuiting secondary coils in the position of effective torque, and open-circuiting them in the position of opposing torque.

Thus this method leads to the repulsion motor, which is a commutator motor also.

With the commutatorless induction motor, or motor with permanently closed armature circuits, all starting devices consist in establishing an auxiliary magnetic flux in phase with the secondary currents in time, and in quadrature with the line of secondary polarization in space. They consist in producing a component of magnetic flux in quadrature in space with the primary magnetic flux producing the secondary currents, and in phase with the latter, that is, in time quadrature with the primary magnetic flux.

Thus, if

$\mathfrak{F}_p$ = polarization due to the secondary currents,

Φ_a = auxiliary magnetic flux,

θ = phase displacement in time between Φ_a and Φ_p,

and

τ = phase displacement in space between Φ_a and Φ_p,

the torque is

$$D = \mathfrak{F}_p \Phi_a \sin \tau \cos \theta.$$

In general the starting torque, apparent torque efficiency, etc., of the single-phase induction motor with any of these devices are given in per cent of the corresponding values of the same motor with polyphase magnetic flux, that is, with a magnetic system consisting of two equal magnetic fluxes in quadrature in time and space.

173. The infinite variety of arrangements proposed for starting single-phase induction motors can be grouped into three classes.

1. *Phase-Splitting Devices.* The primary system is composed of two or more circuits displaced from each other in position, and combined with impedances of different inductance factors so as to produce a phase displacement between them.

When using two motor circuits, they can either be connected in series between the single-phase mains, and shunted with impedances of different inductance factors, as, for instance, a condensance and an inductance, or they can be connected in shunt between the single-phase mains but in series with impedances of different inductance factors. Obviously the impedances used for displacing the phase of the exciting coils can either be external or internal, as represented by high-resistance winding in one coil of the motor, etc.

In this class belongs the use of the transformer as a phase-splitting device by inserting a transformer primary in series with one motor circuit in the main line and connecting the other motor circuit to the secondary of the transformer, or by feeding one of the motor circuits directly from the mains and the other from the secondary of a transformer connected across the mains with its primary. In either case it is, respectively, the internal impedance, or internal admittance, of the transformer which is combined with one of the motor circuits for displacing its phase, and thus this arrangement becomes most effective by using transformers of high internal impedance or admittance as constant power transformers or open magnetic circuit transformers.

2. *Inductive Devices.* The motor is excited by the combination of two or more circuits which are in inductive relation to each other. This mutual induction between the motor circuits can take place either outside of the motor in a separate phase-splitting device or in the motor proper.

In the first case the simplest form is the divided circuit whose branches are inductively related to each other by passing around the same magnetic circuit external to the motor.

In the second case the simplest form is the combination of a primary exciting coil and a short-circuited secondary coil on the primary member of the motor, or a secondary coil closed by an impedance.

In this class belong the shading coil and the accelerating coil.

3. *Monocyclic Starting Devices.* An essentially wattless e.m.f. of displaced phase is produced outside of the motor, and used to energize a cross magnetic circuit of the motor, either directly by a special teaser coil on the motor, or indirectly by combining this wattless e.m.f. with the main e.m.f. and thereby deriving a system of e.m.fs. of approximately three-phase or any other relation. In this case the primary system of the motor is supplied essentially by a polyphase system of e.m.fs. with a single-phase flow of energy, a system which I have called "monocyclic."

The wattless quadrature e.m.f. is generally produced by connecting two impedances of different inductance factors in series between the single-phase mains, and joining the connection between the two impedances to the third terminal of a three-phase induction motor, which is connected with its other two terminals to the single-phase lines, as shown diagrammatically in Fig. 215, for a conductance a and an inductive susceptance ja.

This starting device, when using an inductance and a condensance of proper size, can be made to give an apparent starting torque efficiency superior to that of the polyphase induction motor. Usually a resistance and an inductance are used, which, though not giving the same starting torque efficiency as available by the use of a condensance, have the advantage of greater simplicity and reliability. After starting, the impedances are disconnected.

For a complete discussion and theoretical investigation of the different starting devices, the reader must be referred to the paper on the single-phase induction motor, "American Institute of Electrical Engineers' Transactions, February, 1898."

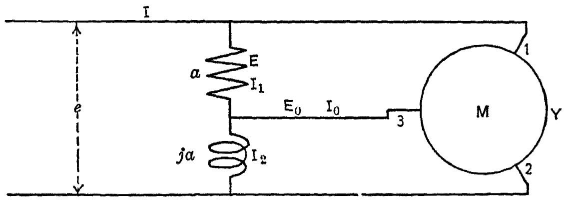


Fig. 215. Connections for Starting Single-Phase Motor.

174. The use of the resistance-inductance, or monocyclic, starting device with three-phase wound induction motor will be discussed somewhat more explicitly as the only method not using condensers which has found extensive commercial application. It gives relatively the best starting torque and torque efficiencies.

In Fig. 215, *M* represents a three-phase induction motor of which two terminals, 1 and 2, are connected to single-phase mains, and the terminal 3 to the common connection of a conductance a $\left(\text{that is, a resistance} \frac{1}{a}\right)$ and an equal susceptance ja $\left(\text{thus a reactance} - \frac{j}{a}\right)$ connected in series across the mains.

Let $Y = g + jb$ = total admittance of motor between terminals 1 and 2 while at rest. We then have $\frac{4}{3} Y$ = total admittance from terminal 3 to terminals 1 and 2, regardless of whether the motor is delta- or Y-wound.

If e = e.m.f. in the single-phase mains and $\dot{E}$ = difference of potential across conductance a of the starting device, then we have the current in a as $\quad \dot{I}_1 = \dot{E}a,$
and the e.m.f. across ja as $\quad e - \dot{E};$
thus, the current in ja is

$$\dot{I}_2 = ja\,(e - \dot{E}),$$

and the current in the cross magnetizing motor circuit from 3 to 1, 2 is

$$\dot{I}_0 = \dot{I}_1 - \dot{I}_2 = \dot{E}a - ja\,(e - \dot{E}).$$

The e.m.f. E_0 of the cross magnetizing circuit is, as may be seen from the diagram of e.m.fs., which form a triangle with e, E and $e - E$ as sides,

$$E_0 = E - (e - E) = 2E - e,$$

and since

$$I_0 = \tfrac{4}{3} Y E_0,$$

we have

$$Ea - ja(e - E) = \tfrac{4}{3} Y (2E - e).$$

This expression solved for E becomes

$$E = e \frac{3ja - 4Y}{3a + 3ja - 8Y},$$

which from the foregoing value of E_0 gives

$$E_0 = \frac{3ea(j - 1)}{3a + 3ja - 8Y};$$

or, substituting

$$Y = g + jb,$$

expanding, and multiplying both numerator and denominator by

$$(3a - 8g) - j(3a - 8b),$$

gives
$$E_0 = ea \frac{\frac{8}{3}(g - b) + j\left(2a - \frac{8(g + b)}{3}\right)}{(a - \frac{8}{3}g)^2 + (a - \frac{8}{3}b)^2},$$

and the imaginary component thereof, or e.m.f. in quadrature to e in time and in space, is

$$E_0^j = -jea \frac{2a - \frac{8}{3}(g + b)}{(a - \frac{8}{3}g)^2 + (a - \frac{8}{3}b)^2}.$$

In the same motor on a three-phase circuit this quadrature e.m.f. is the altitude of the equilateral triangle with e as sides, thus $= je\frac{\sqrt{3}}{2}$, and since the starting torque of the motor is proportional to this quadrature e.m.f., the relative starting torque of the monocyclic starting device, or the ratio of starting torque of the motor with monocyclic starting device to that of the same motor on three-phase circuit, is

$$D' = \frac{E_0^j}{j\frac{e\sqrt{3}}{2}} = \frac{2a}{\sqrt{3}} \frac{2a - \frac{8}{3}(g - b)}{(a - \frac{8}{3}g)^2 + (a - \frac{8}{3}b)^2}.$$

A starting device which has been extensively used is the condenser in the tertiary circuit. In its usual form it can be considered as a modification of the monocyclic starting device, by using a condensance as the one impedance and making the other impedance infinite, that is, omitting it. It thus comprises a three-phase induction motor, in which two terminals are connected to the single-phase supply and the third terminal and one of the main terminals to a condenser. Usually the condenser is left in circuit after starting, and made of such size that its leading current compensates for the lagging magnetizing-current of the motor, and the motor thus gives approximately unity power-factor.

For further discussion of this subject the reader is referred to the paper on "Single-phase Induction Motors," mentioned above, and to the "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

4. Acceleration with Starting Device.

175. The torque of the single-phase induction motor (without a starting device) is proportional to the product of main flux, or magnetic flux produced by the primary impressed e.m.f., and the speed. Thus it is the same as in the polyphase motor at or very near synchronism, but falls off with decreasing speed and becomes zero at standstill.

To produce a starting torque, a device has to be used to impress an auxiliary magnetic flux upon the motor, in quadrature with the main flux in time and in space, and the starting torque is proportional to this auxiliary or quadrature flux. During acceleration or at intermediate speed the torque of the motor is the resultant of the main torque, or torque produced by the primary main flux, and the auxiliary torque produced by the auxiliary quadrature or starting flux. In general, this resultant torque is not the sum of main and auxiliary torque, but less, due to the interaction between the motor and the starting device.

All the starting devices depend more or less upon the total admittance of the motor and its power-factor. With increasing speed, however, the total admittance of the motor decreases and its power-factor increases, and an auxiliary torque device suited for the admittance of the motor at standstill will not be suited for the changed admittance at speed.

The currents produced in the secondary by the main or primary magnetic flux are carried by the rotation of the motor more or less into quadrature position, and thus produce the quadrature flux giving the main torque as discussed before.

This quadrature component of the main flux generates an e.m.f. in the auxiliary circuit of the starting device, and thus changes the distribution of currents and e.m.fs. in the starting device. The circuits of the starting device then contain, besides the motor admittance and external admittance, an active counter e.m.f., changing with the speed. Inversely, the currents produced by the counter e.m.f. of the motor in the auxiliary circuit react upon the counter e.m.f., that is, upon the quadrature component or main flux, and change it.

Thus during acceleration we have to consider —

1. The effect of the change of total motor admittance and its power-factor upon the starting device.

2. The effect of the counter e.m.f. of the motor upon the starting device and the effect of the starting device upon the counter e.m.f. of the motor.

1. The total motor admittance and its power-factor change very much during acceleration in motors with short-circuited low-resistance secondary. In such motors the admittance at rest is very large and its power-factor low, and with increasing speed the admittance decreases and its power-factor increases greatly. In motors with short-circuited high-resistance secondary the admittance also decreases greatly during acceleration, but its power-factor changes less, being already high at standstill. Thus the starting device will be affected less. Such motors, however, are inefficient at speed. In motors with variable secondary resistance the admittance and its power-factor can be maintained constant during acceleration by decreasing the resistance of the secondary circuit in correspondence with the increasing counter e.m.f. Hence, in such motors the starting device is not thrown out of adjustment by the changing admittance during acceleration.

In the phase-splitting devices, and still more in the inductive devices, the starting torque depends upon the internal or motor admittance, and is thus essentially affected by the change of admittance during acceleration, and by the appearance of a counter e.m.f. during acceleration, which throws the starting device out of its proper adjustment, so that frequently while a considerable torque exists at standstill, this torque becomes zero and then reverses at some intermediate speed, and the motor, while starting with fair torque, is not able to run up to speed with the starting device in circuit. Especially is this the case where capacity is used in the starting device. With the monocyclic starting device this effect is small in any case and absent when a condenser is used in the tertiary circuit, and therefore the latter may advantageously be left in the circuit at speed.

IV. Regulation and Stability.

1. Load and Stability.

176. At constant voltage and constant frequency the torque of the polyphase induction motor is a maximum at some definite speed and decreases with increase of speed over that corresponding to the maximum torque, to zero at synchronism; it also decreases with decrease of speed from that at the maximum torque point, to a minimum at standstill, the starting torque. This maximum torque point shifts towards lower speed with increase of the resistance in the secondary circuit, and the starting torque thereby increases. Without additional resistance inserted in the secondary circuit the maximum torque point, however, lies at fairly high speed not very far below synchronism, 10 to 20 per cent below synchronism with smaller motors of good efficiency. Any value of torque between the starting torque and the maximum torque is reached at two different speeds. Thus in a three-phase motor having the following constants: impressed e.m.f., $e_0 = 110$ volts; exciting admittance, $Y = 0.01 + 0.1\,j$; primary impedance, $Z_0 = 0.1 - 0.3\ j$, and secondary impedance, $Z_1 = 0.1 - 0.3\ j$, the torque of 5.5 synchronous kilowatts is reached at 54 per cent of synchronism and also at the speed of 94 per cent of synchronism, as seen in Fig. 216.

When connected to a load requiring a constant torque, irrespective of the speed, as when pumping water against a constant head by reciprocating pumps, the motor thus could carry the load at two different speeds, the two points of intersection of the horizontal line L in Fig. 216, which represents the torque consumed by the load and the motor torque curve D. Of these two points d and c, the lower one, d, represents unstable conditions of operation; that is, the motor cannot operate at this speed, but either stops or runs up to the higher speed point c, at which stability is reached. At the lower speed d a momentary decrease of speed, as by a small pulsation of voltage, load,

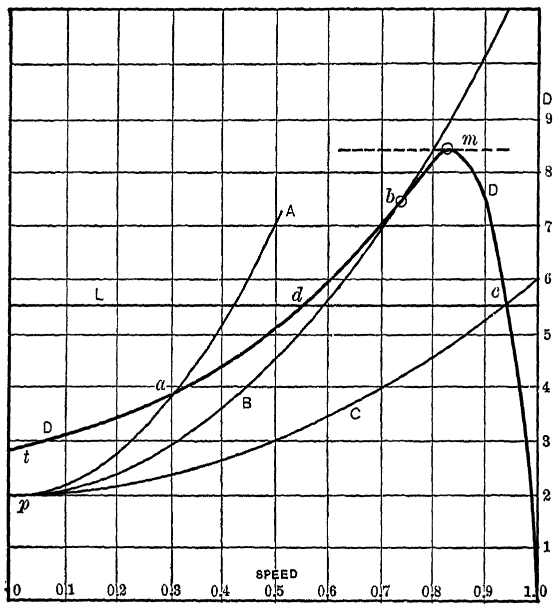


Fig. 216. Speed Torque Characteristics of Induction Motor and Load for Determination of the Stability Point.

etc., decreases the motor torque D below the torque L required by the load, thus causes the motor to slow down, but in doing so its torque still further decreases, and it slows down still more, loses more torque, etc., until it comes to a standstill. Inversely, a momentary increase of speed increases the motor torque D beyond the torque L consumed by the load, and thereby causes an acceleration, that is, an increase of speed. This increase of speed, however, increases the motor torque and thereby the speed still further, and so on, and the motor increases in speed up to the point c, where the motor torque D again becomes equal to the torque consumed by the load. A momentary increase of speed beyond c decreases the motor torque D, and thus limits itself, and inversely a momentary decrease of speed below c increases the motor torque D beyond L, thus accelerates and recovers the speed; that is, at c the motor speed is stable.

With a load requiring constant torque the induction motor thus is unstable at speeds below that of the maximum torque point, but stable above it; that is, the motor curve consists of two branches, an unstable branch, from standstill t to the maximum torque point m, and a stable branch, from the maximum torque point m to synchronism.

177. It must be realized, however, that this instability of the lower branch of the induction motor speed curve is a function of the nature of the load, and as described above applies only to a load requiring a constant torque L. Such a load the motor could not start (except by increasing the motor torque at low speeds by resistance in the secondary), but when brought up to a speed above d would carry the load at speed c in Fig. 216.

If, however, the load on the motor is such as to require a torque which increases with the square of the speed, as shown by curve C in Fig. 216, that is, consists of a constant part p (friction of bearings, etc.) and a quadratic part, as when driving a ship's propeller or driving a centrifugal pump, then the induction motor is stable over the entire range of speed, from standstill to synchronism. The motor then starts, with the load represented by curve C, and runs up to speed c. At a higher load, represented by curve B, the motor runs up to speed b, and with excessive overload, curve A, the motor would run up to low speed, point a, only, but no overload of such nature would stop the motor, but merely reduce its speed, and inversely, it would always start, but at excessive overloads run at low speed only. Thus in this case no unstable branch of the motor curve exists, but it is stable over the entire range.

With a load requiring a torque which increases proportionally to the speed, as shown by C in Fig. 217, that is, which consists of a constant part p and a part proportional to the speed, as when driving a direct-current generator at constant excitation, connected to a constant resistance as load — as a lighting system — the motor always starts, regardless of the load, — provided that the constant part of the torque, p, is less than the starting torque. With moderate load C the motor runs up to a speed c near synchronism. With very heavy load A the motor starts, but runs up to a low speed only. Especially interesting is the case of an intermediary load as represented by line B in Fig. 217. B intersects the motor torque curve D in three points, b_1, b_2, b_3; that is, three speeds exist at which the motor gives the torque required by the load,—24 per cent, 60 per cent, and 88 per cent of synchronism. The speeds b_1 and b_3 are stable, the speed b_2 unstable. Thus, with this load the motor starts from standstill, but does not run up to a speed near synchronism, but accelerates only to speed b_1, and keeps revolving at this low speed

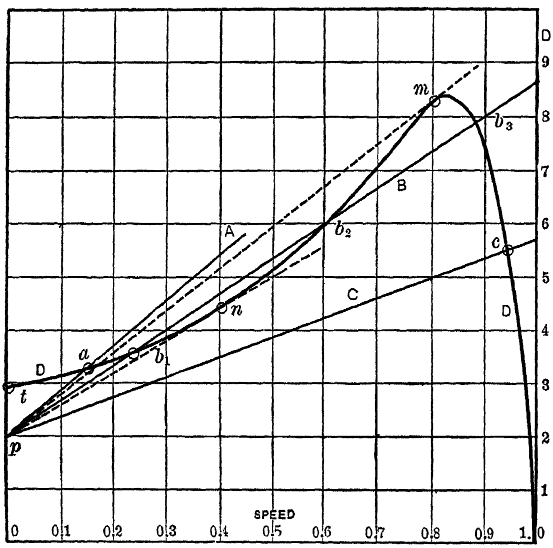


Fig. 217. Speed Torque Characteristics of Induction Motor and Load for Determination of the Stability Point.

(and a correspondingly very large current). If, however, the load is taken off and the motor allowed to run up to synchronism or near to it, and the load then put on, the motor slows down only to speed b_3, and carries the load at this high speed; hence, the motor can revolve continuously at two different speeds, b_1 and b_3, and either of these speeds is stable; that is, a momentary increase of speed decreases the motor torque below that required by the load, and thus limits itself, and inversely a decrease of motor speed increases its torque beyond that corresponding to the load, and thus restores the speed. At the intermediary speed, b_2, the conditions are unstable, and a momentary increase of speed causes the motor to accelerate up to speed b_3, a momentary decrease of speed from b_2 causes the motor to slow down to speed b_1, where it becomes stable again. In the speed range between b_2 and b_3 the motor thus accelerates up to b_3, in the speed range between b_2 and b_1 it slows down to b_1.

For this character of load, the induction motor speed curve D thus has two stable branches, a lower one, from standstill t to the point n, and an upper one, from point m to synchronism, where m and n are the points of contact of the tangents from the required starting torque p on-to the motor curve D; these two stable branches are separated by the unstable branch, from n to m, on which the motor cannot operate.

178. The question of stability of motor speed thus is a function not only of the motor speed curve but also of the character of the load in its relation to the motor speed curve, and if the change of motor torque with the change of speed is less than the change of the torque required by the load, the condition is stable, otherwise it is unstable; that is, it must be $\frac{dD}{dS} < \frac{dL}{dS}$ to give stability, where L is the torque required by the load at speed S.

Occasionally on polyphase induction motors on a load as represented in Fig. 217 this phenomenon is observed in the form that the motor can start the load but cannot bring it up to speed. More frequently, however, it is observed on single-phase induction motors in which the maximum torque is nearer to synchronism, with some forms of starting devices which decrease in their effect with increasing speed and thus give motor speed characteristics of forms similar to Fig. 218. With a torque speed curve as shown in Fig. 218, even at a load requiring constant torque, three speed points may exist of which the middle one is unstable. In polyphase synchronous motors and converters, when starting by alternating current, that is, as induction machines, the phenomenon is frequently observed that the machine starts at moderate voltage, but does not run up to synchronism, but stops at an intermediary speed, in the neighborhood of half speed, and a considerable increase of voltage, and thereby of motor torque, is required to bring the machine beyond the dead point, or rather "dead range," of speed and make it run up to synchronism. In this case, however, the phenomenon is complicated by the effects due to varying magnetic reluctance (magnetic locking), inductor machine effect, etc.

Instability of such character as here described occurs in electric circuits in many instances, of which the most typical is the electric arc in a constant-potential supply. It occurs whenever the effect produced by any cause increases the cause and thereby becomes cumulative. When dealing with energy, obviously

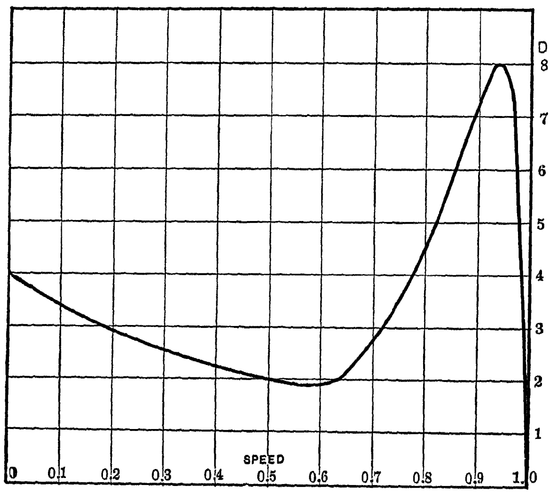


Fig. 218. Speed Torque Characteristic of Single-Phase Induction Motor.

the effect must always be in opposition to the cause (Lenz's Law), as result of the law of conservation of energy. When dealing with other phenomena, however, as the speed-torque relation or the volt-ampere relation, etc., instability due to the effect assisting the cause, intensifying it, and thus becoming cumulative, may exist, and frequently does exist, and causes either indefinite increase or decrease, or surging or hunting.

2. Voltage Regulation and Output.

179. Load and speed curves of induction motors are usually calculated and plotted for constant supply voltage at the motor terminals. In practice, however, this condition usually is only approximately fulfilled, and due to the drop of voltage in the step-down transformers feeding the motor, in the secondary

and the primary supply lines, etc., the voltage at the motor terminals drops more or less with increase of load. Thus, if the voltage at the primary terminals of the motor transformer is constant, and such as to give the rated motor voltage at full load, at no load the voltage at the motor terminals is higher, but at overload lower by the voltage drop in the internal impedance of the transformers. If the voltage is kept constant in the center of distribution, the drop of voltage in the line adds itself to the impedance drop in the transformers, and the motor supply voltage thus varies still more between no load and overload.

With a drop of voltage in the supply circuit between the point of constant potential and the motor terminals, assuming the circuit such as to give the rated motor voltage at full load, the voltage at no load and thus the exciting current is higher, the voltage at overload and thus the maximum output and maximum torque of the motor, and also the motor impedance current, that is, current consumed by the motor at standstill, and thereby the starting torque of the motor, are lower than on a constant-potential supply. Hereby then the margin of overload capacity of the motor is reduced, and the characteristic constant of the motor, or the ratio of exciting current to short-circuit current, is increased, that is, the motor characteristic made inferior to that given at constant voltage supply, the more so the higher the voltage drop in the supply circuit.

Assuming then a three-phase motor having the following constants: primary exciting admittance, $Y = 0.01 - 0.1\ j$; primary self-inductive impedance, $Z_0 = 0.1 - 0.3\ j$; secondary self-inductive impedance, $Z_1 = 0.1 - 0.3\ j$; supply voltage, $e_0 = 110$ volts, and rated output, 5000 watts per phase.

Assuming this motor to be operated —

1. By transformers of about 2 per cent resistance and 4 per cent reactance voltage, that is, transformers of good regulation, with constant voltage at the transformer terminals.

2. By transformers of about 2 per cent resistance and 15 per cent reactance voltage, that is, very poorly regulating transformers, at constant supply voltage at the transformer primaries.

3. With constant voltage at the generator terminals, and about 8 per cent resistance, 40 per cent reactance voltage in line and transformers between generator and motor.

This gives, in complex quantities, the impedance between the motor terminals and the constant voltage supply:

$$\begin{aligned} &1.\ Z = 0.04 - 0.08\,j, \\ &2.\ Z = 0.04 - 0.3\,j, \\ &3.\ Z = 0.16 - 0.8\,j. \end{aligned}$$

It is assumed that the constant supply voltage is such as to give 110 volts at the motor terminals at full load.

The load and speed curves of the motor, when operating under these conditions, that is, with the impedance Z in series between the motor terminals and the constant voltage supply e_1, then can be calculated from the motor characteristics at constant terminal voltage e_0 as follows:

At slip s and constant terminal voltage e_0 the current in the motor is i_0, its power-factor $p = \cos\theta$. The effective or equivalent impedance of the motor at this slip then is $z^0 = \frac{e_0}{i_0}$, and, in complex quantities, $Z^0 = \frac{e_0}{i_0}(\cos\theta - j\sin\theta)$, and the total impedance, including that of transformers and line, thus is

$$Z_1 = Z^0 + Z = \left(\frac{e_0}{i_0}\cos\theta + r\right) - j\left(\frac{e_0}{i_0}\sin\theta + x\right),$$

or, in absolute values,

$$z_1 = \sqrt{\left(\frac{e_0}{i_0}\cos\theta + r\right)^2 + \left(\frac{e_0}{i_0}\sin\theta + x\right)^2},$$

and, at the supply voltage e_1, the current thus is

$$i_1 = \frac{e_1}{z_1},$$

and the voltage at the motor terminals is

$$e_0' = z^0 i_1 = \frac{z^0}{z_1} e_1.$$

If e_0 is the voltage required at the motor terminals at full load, and i_0^0 the current, z_1^0 the total impedance at full load, it is

$$i_0^0 = \frac{e_1}{z_1^0};$$

hence, the required constant supply voltage is

$$e_1 = z_1^0 i_0^0,$$

and the speed and torque curves of the motor under this condition then are derived from those at constant supply voltage e_0 by multiplying all voltages and currents by the factor $\frac{e_0'}{e_0}$, that is, by the ratio of the actual terminal voltage to the full-load terminal voltage, and the torque and power by multiplying with the square of this ratio, while the power-factors and the efficiencies obviously remain unchanged.

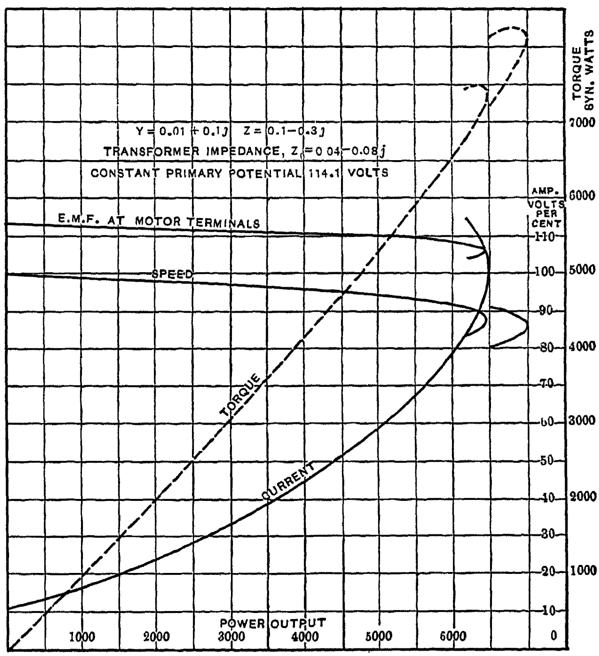


Fig. 219. Induction Motor Load Curves Corresponding to 110 Volts at Motor Terminals at 5000 Watts Load.

In this manner, in the three cases assumed in the preceding, the load curves are calculated, and are plotted in Figs. 219, 220, and 221.

180. It is seen that, even with transformers of good regulation, Fig. 219, the maximum torque and the maximum power are appreciably reduced. The values corresponding to constant terminal voltage are shown, for the part of the curves near maximum torque and maximum power, in Figs. 219, 220, and 221.

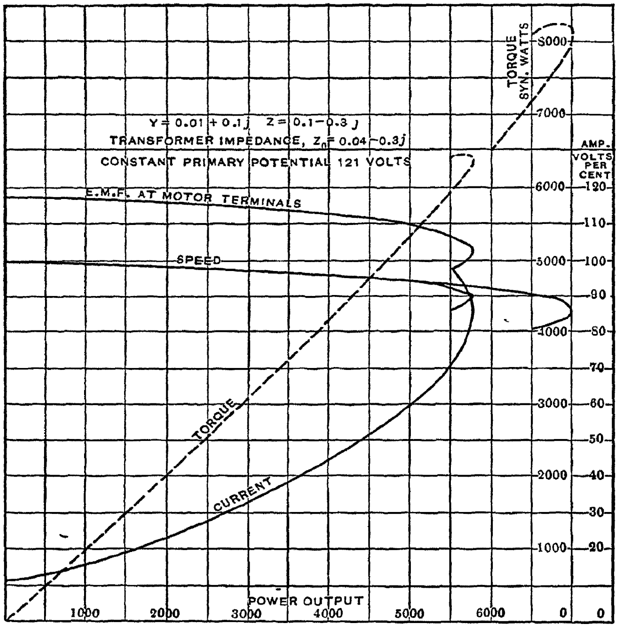


Fig. 220. Induction Motor Load Curves Corresponding to 110 Volts at Motor Terminals at 5000 Watts Load.

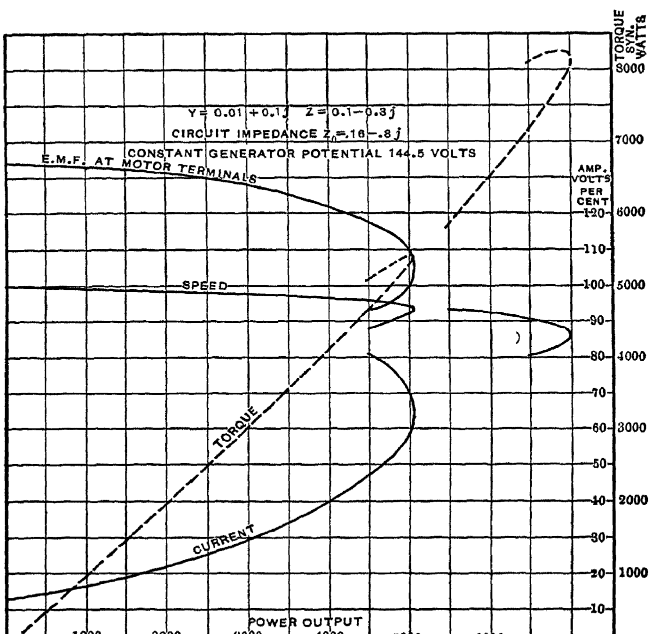


Fig. 221. Induction Motor Load Curves Corresponding to 110 Volts at Motor Terminals at 5000 Watts Load.

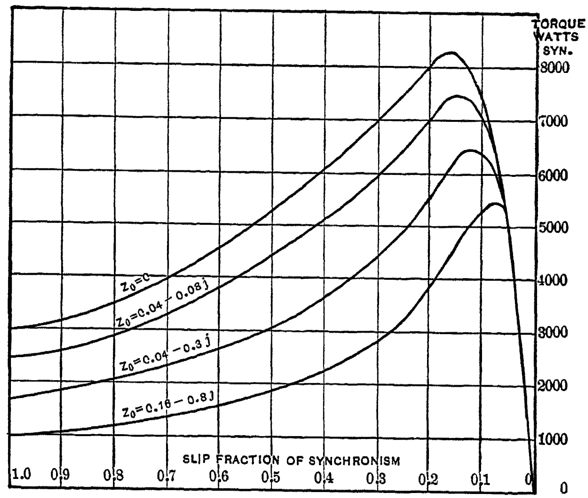


Fig. 222. Induction Motor Speed Torque Characteristics with Short-Circuited Secondary.

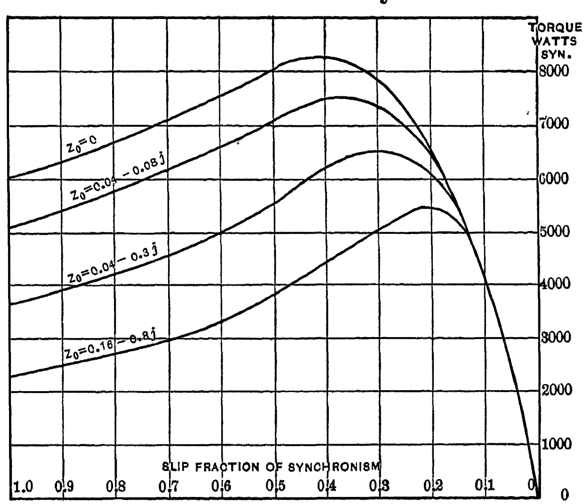


Fig. 223. Induction Motor Speed Torque Characteristics with a Resistance of 0.15 Ohm in Secondary Circuit.

In Figs. 222, 223, 224, and 225 are given the speed-torque curves of the motor, for constant terminal voltage, $Z = 0$, and the three cases above discussed; in Fig. 222 for short-circuited secondaries, or running condition; in Fig. 223 for 0.15 ohm; in Fig. 224 for 0.5 ohm; and in Fig. 225 for 1.5 ohms additional resistance inserted in the armature. As seen, the line and transformer impedance very appreciably lowers the torque, and especially the starting torque, which, with short-circuited arma-

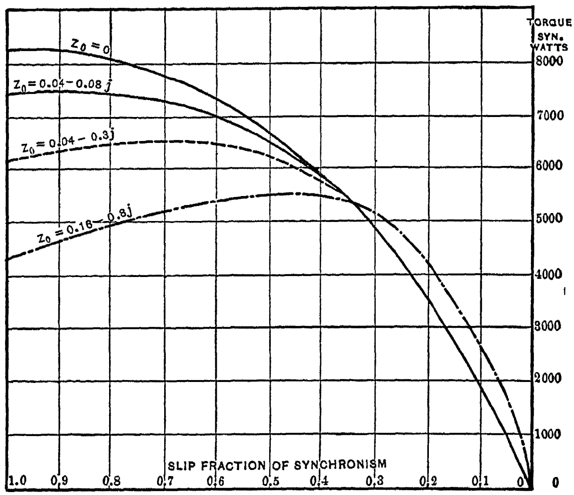


Fig. 224. Induction Motor Speed Torque Characteristics with a Resistance of 0.5 Ohm in Secondary Circuit.

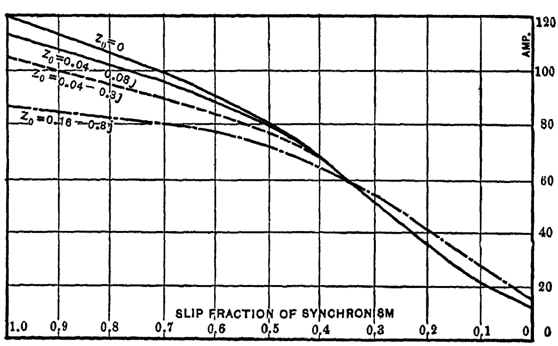


Fig. 225. Induction Motor Speed Current Characteristics with a Resistance of 1.5 Ohms in Secondary Circuit.

ture, in the case 3 drops to about one-third the value given at constant supply voltage.

It is interesting to note that in Fig. 224, with a secondary resistance giving maximum torque in starting, at constant terminal voltage, with high impedance in the supply, the starting

TABLE I. — VOLTAGE REGULATION AND OUTPUT.

$Y = 0.01 + 0.1\,j. \quad Z = 0.1 - 0.3\,j.$

Line and transformer impedance, z_0	E.m.f. at motor terminals.				Power,		Torque,		Starting torque.			Current.				Per cent exciting current of current at $\frac{3}{4}$ maximum torque.	Characteristic constant, $\gamma = \frac{\text{Excit.}}{\text{Start.}}$
	Open circuit.	No load.	5000 watts output.	Standstill.	Max. watts.	Per cent of 5000 watts.	Max. syn. watts.	Per cent of 5300 watts.				Exciting.	Starting.				
							$r_1 = 0.1$		$r_1 = 0.1$	$r = 0.25$	$r_1 = 0.6$		$r_1 = 0.1$	$r_1 = 0.25$	$r_1 = 0.6$		
0	110	110	110	110	7000	1.40	8250	1.555	2950	6050	8250	10.7	176	160	120	17.4	6.1
$0.04 - 0.08\,j$	114.1	113.3	110	99.5	6450	1.29	7500	1.415	2420	5100	7450	11.0	159	147	114	20.0	6.9
$0.04 - 0.3\,j$	121	118	110	82	5780	1.156	6460	1.22	1635	3650	6150	11.45	131	124	104	24.7	8.75
$0.16 - 0.8\,j$	144.5	134	110	63	5070	1.014	5450	1.03	965	2260	4400	13.1	101	97	88	35.8	13.0
0	110	110	......	110	7000	1.40	8250	1.555	2950	6050	8250	......	176	......	......	....	......
$0.04 - 0.08\,j$	110	109	......	95.5	5970	1.194	6940	1.315	2250	4730	6920	......	153	......	......	....	......
$0.04 - 0.3\,j$	110	107.5	......	74.6	4780	0.956	5330	1.005	1360	3030	5100	......	119	......	......	....	......
$0.16 - 0.8\,j$	110	102	......	48	2940	0.588	3170	0.60	560	1310	2550	......	77	......	......	....	......

torque drops so much that the maximum torque is shifted to about half synchronism.

In induction motors, especially at overloads and in starting, it therefore is important to have as low impedance as possible between the point of constant voltage and the motor terminals.

In Table I the numerical values of maximum power, maximum torque, starting torque, exciting current and starting current are given for above motor, at constant terminal voltage and for the three values of impedance in the supply lines, for such supply voltage as to give the rated motor voltage of 110 volts at full load and for 110 volts supply, voltage. In the first case, maximum power and torque drop down to their full-load values with the highest line impedance, and far below full-load values in the latter case.

3. Frequency Pulsation.

181. If the frequency of the voltage supply pulsates with sufficient rapidity that the motor speed cannot appreciably follow the pulsations of frequency, the motor current and torque also pulsate; that is, if the frequency pulsates by the fraction p above and below the normal, at the average slip, s, the actual slip pulsates between $s + p$ and $s - p$, and motor current and torque pulsate between the values corresponding to the slips $s + p$ and $s - p$. If then the average slip $s < p$, at minimum frequency, the actual slip $s - p$ becomes negative; that is, the motor momentarily generates and returns energy.

As instance are shown, in Fig. 226, the values of current and of torque for maximum and minimum frequency, and for the average frequency, for $p = 0.025$, that is, 2.5 per cent pulsation of frequency from the average. As seen, the pulsation of current is moderate until synchronism is approached, but becomes very large near synchronism, and from slip $s = 0.025$ up to synchronism the average current remains practically constant, thus at synchronism is very much higher than the current at constant frequency. The average torque also drops somewhat below the torque corresponding to constant frequency, as shown in the upper part of Fig. 226.

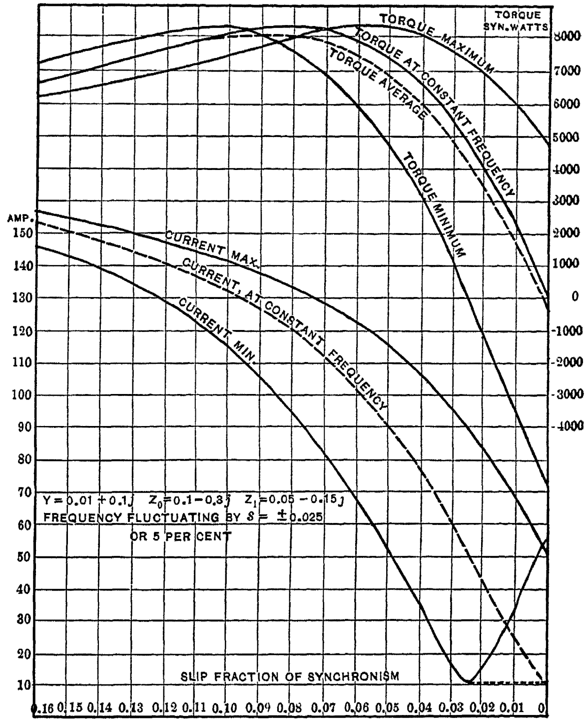


Fig. 226. Induction Motor Pulsation of Frequency.

4. Generator Regulation and Stability.

182. If the voltage at the induction motor terminals decreases with increase of load, the maximum torque and output are decreased the more the greater the drop of voltage. But even if the voltage at the induction motor terminals is maintained constant, the maximum torque and power may be reduced essentially, in a manner depending on the rapidity with which the voltage regulation at changes of load is effected by the generator or potential regulator, which maintains constancy of voltage, and the rapidity with which the motor speed can change, that is, the mechanical momentum of the motor and its load.

This instability of the motor, produced by the generator regulation, may be discussed for the case of a load requiring constant torque at all load, though the corresponding phenomenon may exist at all classes of load, as discussed under 1.

The torque curve of the induction motor at constant terminal voltage consists of two branches, a stable branch, from the maximum torque point to synchronism, and an unstable branch, that is, a branch at which the motor cannot operate on a load requiring constant torque, from standstill to maximum torque. With increasing slip s the current i in the motor increases. If then D = torque of the motor, $\frac{dD}{di}$ is positive on the stable, negative on the unstable branch of the motor curve, and this rate of change of the torque, with change of current, expressed as fraction of the current, is

$$k_s = \frac{1}{D}\frac{dD}{di};$$

it may be called the *stability coefficient* of the motor.

If k_s is positive, an increase of i, caused by an increase of slip s, that is, by a decrease of speed, increases the torque D, and thereby checks the decrease of speed, and inversely, that is, the motor is stable.

If, however, k_s is negative, an increase of i causes a decrease of D, thereby a decrease of speed, and thus further increase of i and decrease of D; that is, the motor slows down with increasing rapidity, or inversely, with a decrease of i, accelerates with increasing rapidity, that is, is unstable.

For the motor used as illustration in the preceding, of the constants $e = 110$ volts; $Y = 0.01 - 0.1\,j$; $Z_0 = 0.1 - 0.3\,j$, $Z_1 = 0.1 - 0.3\,j$, the stability curve is shown, together with speed, current, and torque, in Fig. 227, as function of the output. As seen, the stability coefficient k_s is very high for light load, decreases first rapidly and then slowly, until an output of 7000 watts is approached, and then rapidly drops below zero; that is, the motor becomes unstable and drops out of step, and speed, torque, and current change abruptly, as indicated by the arrows in Fig. 227.

The stability coefficient k_s characterizes the behavior of the motor regarding its load-carrying capacity. Obviously, if the terminal voltage of the motor is not constant, but drops with the load, as discussed in 1, a different stability coefficient results; which intersects the zero line at a different and lower torque.

183. If the induction motor is supplied with constant terminal voltage from a generator of close inherent voltage regulation and of a size very large compared with the motor, over a supply circuit of negligible impedance, so that a sudden change of motor current cannot produce even a momentary tendency of change of the terminal voltage of the motor, the stability curve k_s

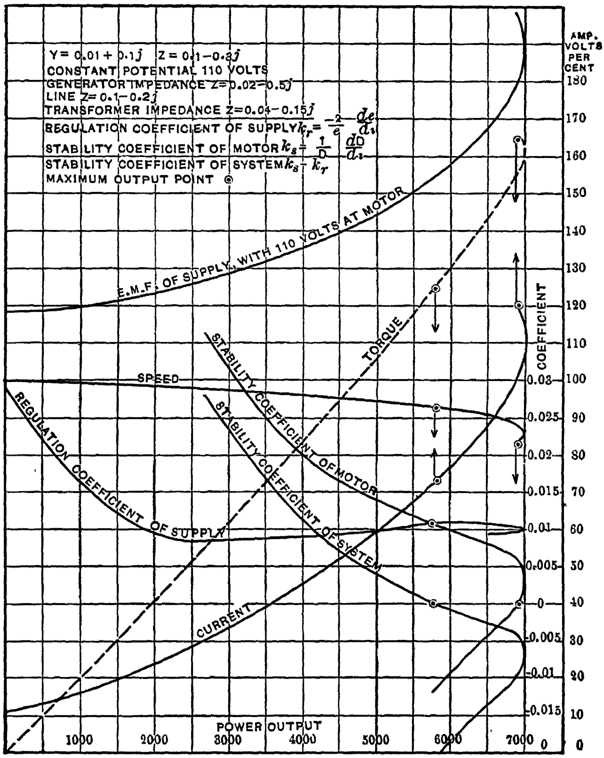


Fig. 227. Induction Motor Load Curves.

of Fig. 227 gives the performance of the motor. If, however, at a change of load and thus of motor current the regulation of the supply voltage to constancy at the motor terminals requires a finite time, even if this time is very short, the maximum output of the motor is reduced thereby, the more so the more rapidly the motor speed can change.

Assuming the voltage control at the motor terminals effected by hand regulation of the generator or the potential regulator in the circuit supplying the motor, or by any other method which is slower than the rate at which the motor speed can adjust itself to a change of load, then, even if the supply voltage at the motor terminals is kept constant, for a momentary fluctation of motor speed and current, the supply voltage momentarily varies, and with regard to its stability the motor corresponds not to the condition of constant supply voltage but to a supply voltage which varies with the current, hence the limit of stability is reached at a lower value of motor torque.

At constant slip s the motor torque D is proportional to the square of the impressed e.m.f. e^2. If by a variation of slip caused by a fluctuation of load the motor current i varies by di, if the terminal voltage e remains constant the motor torque D varies by the fraction $k_s = \frac{1}{D}\frac{dD}{di}$, or the stability coefficient of the motor. If, however, by the variation of current di the impressed e.m.f. e of the motor varies, the motor torque D, being proportional to e^2, still further changes, proportional to the change e^2, that is, by the fraction $k_r = \frac{1}{e^2}\frac{de^2}{di} = \frac{2}{e}\frac{de}{di}$, and the total change of motor torque resultant from a change di of the current i thus is $k_0 = k_s + k_r$.

Hence, if a momentary fluctuation of current causes a momentary fluctuation of voltage, the stability coefficient of the motor is changed from k_s to $k_0 = k_s + k_r$, and as k_r is negative, the voltage e decreases with increase of current i, the stability coefficient of the system is reduced by the effect of voltage regulation of the supply, k_r, and k_r thus can be called the *regulation coefficient of the system.*

$k_r = \frac{2}{e}\frac{de}{di}$ thus represents the change of torque produced by the momentary voltage change resulting from a current change di in the system; hence, is essentially a characteristic of the supply system and its regulation, but depends upon the motor only in so far as $\frac{de}{di}$ depends upon the power-factor of the load.

In Fig. 227 is shown the regulation coefficient $-k_r$ of the supply system of the motor, at 110 volts maintained constant at the motor terminals, and an impedance $Z = 0.16 - 0.8\,j$ between motor terminals and supply e.m.f. As seen, the regulation coefficient of the system drops from a maximum of about 0.03, at no load, down to about 0.01, and remains constant at this latter value, over a very wide range.

The resultant stability coefficient, or *stability coefficient of the system* of motor and supply, $k_0 = k_s + k_r$, as shown in Fig. 227, thus drops from very high values at light load down to zero at the load at which the curves k_s and k_r in Fig. 227 intersect, or at 5800 kw., and there become negative; that is, the motor drops out of step, although still far below its maximum torque point, as indicated by the arrows in Fig. 227.

Thus, at constant voltage maintained at the motor terminals by some regulating mechanism which is slower in its action than the retardation of a motor speed change by its mechanical momentum, the motor behaves up to 5800 watts output in exactly the same manner as if its terminals were connected directly to an unlimited source of constant voltage supply, but at this point, where the slip is only 7 per cent in the present instance, the motor suddenly drops out of step without previous warning, and comes to a standstill, while at inherently constant terminal voltage the motor would continue to operate up to 7000 watts output, and drop out of step at 8250 synchronous watts torque at 16 per cent slip.

By this phenomenon the maximum torque of the motor thus is reduced from 8250 to 6300 synchronous watts, or by nearly 25 per cent.

184. If the voltage regulation of the supply system is more rapid than the speed change of the motor as retarded by the momentum of motor and load, the regulation coefficient of the system as regards to the motor obviously is zero, and the motor thus gives the normal maximum output and torque. If the regulation of the supply voltage, that is, the recovery of the terminal voltage of the motor with a change of current, occurs at about the same rate as the speed of the motor can change with a change of load, then the maximum output as limited by the stability coefficient of the system is intermediate between the minimum value of 6300 synchronous watts and its normal value of 8250 synchronous watts. The more rapid the recovery of the voltage and the larger the momentum of motor and load, the less is the motor output impaired by this phenomenon of instability. Thus, the loss of stability is greatest with hand

regulation, less with automatic control by potential regulator, the more so the more rapidly the regulator works; it is very little with compounded alternators, and absent where the motor terminal voltage remains constant without any control by practically unlimited generator capacity and absence of voltage drop between generator and motor.

Comparing the stability coefficient k_s of the motor load and the stability coefficient k_0 of the entire system under the assumed conditions of operation of Fig. 227, it is seen that the former intersects the zero line very steeply, that is, the stability remains high until very close to the maximum torque point, and the motor thus can be loaded up close to its maximum torque without impairment of stability. The curve k_0, however, intersects the zero line under a sharp angle, that is, long before the limit of stability is reached in this case the stability of the system has dropped so close to zero that the motor may drop out of step by some momentary pulsation. Thus, in the case of instability due to the regulation of the system, the maximum output point, as found by test, is not definite and sharply defined, but the stability gradually decreases to zero, and during this decrease the motor drops out at some point. Experimentally the difference between the dropping out by approach to the limits of stability of the motor proper and that of the system of supply is very marked by the indefiniteness of the latter.

In testing induction motors it thus is necessary to guard against this phenomenon by raising the voltage beyond normal before every increase of load, and then gradually decrease the voltages again to normal.

A serious reduction of the overload capacity of the motor, due to the regulation of the system, obviously occurs only at very high impedance of the supply circuit; with moderate impedance the curve k_r is much lower, and the intersection between k_r and k_s occurs still on the steep part of k_s, and the output thus is not materially decreased, but merely the stability somewhat reduced when approaching maximum output.

This phenomenon of the impairment of stability of the induction motor by the regulation of the supply voltage is of practical importance, as similar phenomena occur in many instances. Thus, with synchronous motors and converters the regulation of the supply system exerts a similar effect on the overload capacity, and reduces the maximum output so that the motor drops out of step, or starts surging, due to the approach to the stability limit of the entire system. In this case, with synchronous motors and converters, increase of their field excitation frequently restores their steadiness by producing leading currents and thereby increasing the power-carrying capacity of the supply system, while with surging caused by instability of the synchronous motor the leading currents produced by increase of field excitation increase the surging, and lowering the field excitation tends towards steadiness.

V. Induction Generator.

1. Introduction.

185. In the range of slip from $s = 0$ to $s = 1$, that is, from synchronism to standstill, torque, power output, and power input of the induction machine are positive, and the machine thus acts as a motor, as discussed before.

Substituting, however, in the equations in paragraph 1 for s values > 1, corresponding to backward rotation of the machine, the power input remains positive, the torque also remains positive, that is, in the same direction as for $s < 1$, but since the speed $(1 - s)$ becomes negative or in opposite direction, the power output is negative, that is, the torque in opposite direction to the speed. In this case the machine consumes electrical energy in its primary and mechanical energy by a torque opposing the rotation, thus acting as brake.

The total power, electrical as well as mechanical, is consumed by internal losses of the motor. Since, however, with large slip in a low-resistance motor, the torque and power are small, the braking power of the induction machine at backward rotation is, as a rule, not considerable, excepting when using high resistance in the armature circuit.

Substituting for s negative values, corresponding to a speed above synchronism, torque and power output and power input become negative, and a load curve can be plotted for the induction generator which is very similar, but the negative counterpart of the induction motor load curve. It is for the machine shown as motor in Fig. 207, given as Fig. 228, while Fig. 229

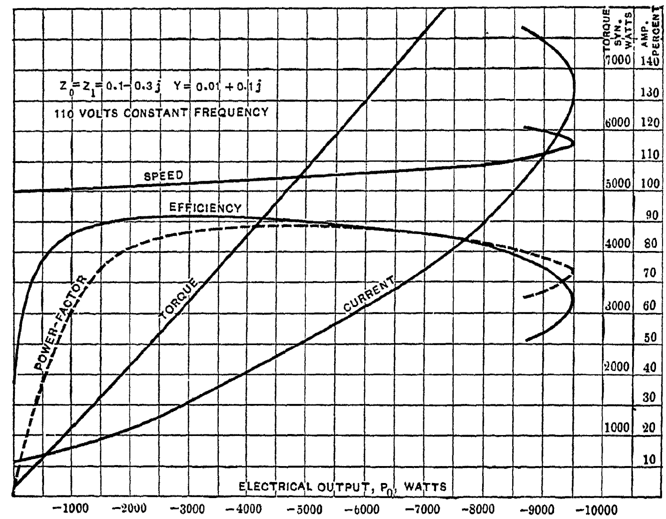


Fig. 228. Induction Generator Load Curves.

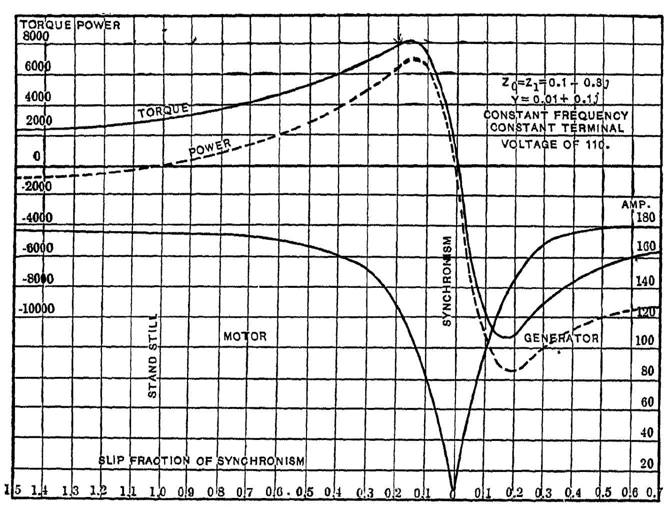


Fig. 229. Induction Machine Speed Curves.

gives the complete speed curve of this machine from $s = 1.5$ to $s = -1$.

The generator part of the curve, for $s < 0$, is of the same character as the motor part, $s > 0$, but the maximum torque and maximum output of the machine as generator are greater than as motor.

Thus an induction motor when speeded up above synchronism acts as a powerful brake by returning energy into the lines, and the maximum braking effort and also the maximum electric power returned by the machine will be greater than the maximum motor torque or output.

2. Constant-Speed Induction or Asynchronous Generator.

186. The curves in Fig. 229 are calculated at constant frequency f, and thus to vary the output of the machine as generator the speed has to be increased. This condition may be

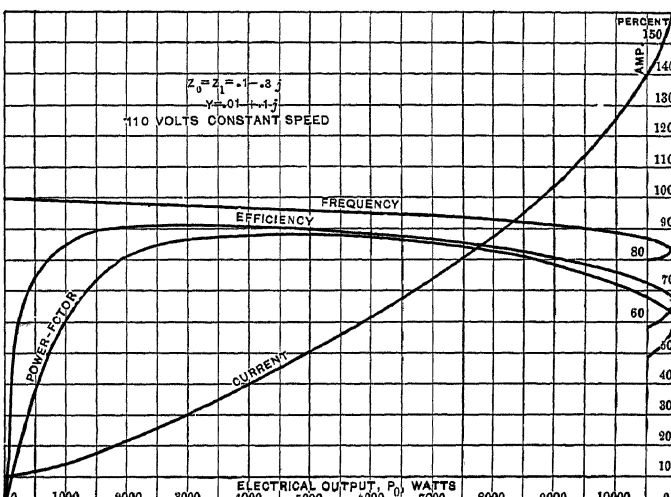


Fig. 230. Induction Generator Load Curves.

realized in case of induction generators running in parallel with synchronous generators under conditions where it is desirable that the former should take as much load as its driving power permits; as, for instance, if the induction generator is driven by a water power while the synchronous generator is driven by a steam engine. In this case the control of speed would be effected on the synchronous generator, and the induction generator be without speed-controlling devices, running up beyond synchronous speed as much as required to consume the power supplied to it.

Conversely, however, if an induction machine is driven a constant speed and connected to a suitable circuit as load, the frequency given by the machine will not be synchronous with the speed, or constant at all loads, but decreases with increasing load from practically synchronism at no load, and thus for the induction generator at constant speed a load curve can be constructed as shown in Fig. 230, giving the decrease of frequency with increasing load in the same manner as the speed of the induction motor at constant frequency decreases with the load. In the calculation of these induction generator curves for constant speed the change of frequency with the load has obviously to be considered, that is, in the equations the reactance x_0 has to be replaced by the reactance $x_0 (1 - s)$, otherwise the equations remain the same.

3. Power-Factor of Induction Generator.

187. The induction generator differs essentially from a synchronous alternator (that is, a machine in which an armature revolves relatively through a constant or continuous magnetic field) by having a power-factor requiring leading current; that is, in the synchronous alternator the phase relation between current and terminal voltage depends entirely upon the external circuit, and according to the nature of the circuit connected to the synchronous alternator the current can lag or lead the terminal voltage or be in phase therewith. In the induction or asynchronous generator, however, the current must lead the terminal voltage by the angle corresponding to the load and voltage of the machine, or, in other words, the phase relation between current and voltage in the external circuit must be such as required by the induction generator at that particular load.

Induction generators can operate only on circuits with leading current or circuits of negative effective reactance.

In Fig. 231 are given for the constant-speed induction generator in Fig. 230 as function of the impedance of the external circuit $z = \frac{e_0}{i_0}$ as abscissas (where e_0 = terminal voltage, i_0 = current in external circuit), the leading power-factor $p = \cos \theta$ required in the load, the inductance factor $q = \sin \theta$, and the frequency.

Hence, when connected to a circuit of impedance z this induction generator can operate only if the power-factor of its circuit is p; and if this is the case the voltage is indefinite, that is, the

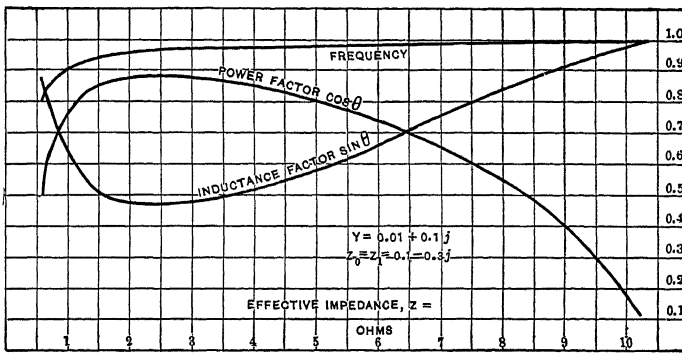


Fig. 231. Three-phase Induction Generator Power Factor and Inductance Factor of External Circuit.

circuit unstable, even neglecting the impossibility of securing exact equality of the power-factor of the external circuit with that of the induction generator.

Two possibilities thus exist with such an induction generator circuit.

1st. The power-factor of the external circuit is constant and independent of the voltage, as when the external circuit consists of resistances, inductances, and capacities.

In this case if the power-factor of the external circuit is higher than that of the induction generator, that is, the leading current less, the induction generator fails to excite and generate. If the power-factor of the external circuit is lower than that of the induction generator, the latter excites and its voltage rises until by saturation of its magnetic circuit and the consequent increase of exciting admittance, that is, decrease of internal power-factor, its power-factor has fallen to equality with that of the external circuit

In this respect the induction generator acts like the direct-current shunt generator, and gives load characteristics very similar to those of the direct-current shunt generator as discussed in B; that is, it becomes stable only at saturation, but loses its excitation and thus drops its load as soon as the voltage falls below saturation.

Since, however, the field of the induction generator is alternating, it is usually not feasible to run at saturation, due to excessive hysteresis losses, except for very low frequencies.

2d. The power-factor of the external circuit depends upon the voltage impressed upon it.

This, for instance, is the case if the circuit consists of a synchronous motor or contains synchronous motors or synchronous converters.

In the synchronous motor the current is in phase with the impressed e.m.f. if the impressed e.m.f. equals the counter e.m.f. of the motor plus the internal loss of voltage. It is leading if the impressed e.m.f. is less, and lagging if the impressed e.m.f. is more. Thus when connecting an induction generator with a synchronous motor, at constant field excitation of the latter the voltage of the induction generator rises until it is as much below the counter e.m.f. of the synchronous motor as required to give the leading current corresponding to the power-factor of the generator. Thus a system consisting of a constant-speed induction generator and a synchronous motor at constant field excitation is absolutely stable. At constant field excitation of the synchronous motor, at no load the synchronous motor runs practically at synchronism with the induction generator, with a terminal voltage slightly below the counter e.m.f. of the synchronous motor. With increase of load the frequency and thus the speed of the synchronous motor drops, due to the slip of frequency in the induction generator, and the voltage drops, due to the increase of leading current required and the decrease of counter e.m.f. caused by the decrease of frequency.

By increasing the field excitation of the synchronous motor with increase of load, obviously the voltage of the generator can be maintained constant, or even increased with the load.

When running from an induction generator, a synchronous motor gives a load curve very similar to the load curve of an induction motor running from a synchronous generator; that is, a magnetizing current at no load and a speed gradually decreasing with the increase of load up to a maximum output point, at which the speed curve bends sharply down, the current curve upward, and the motor drops out of step.

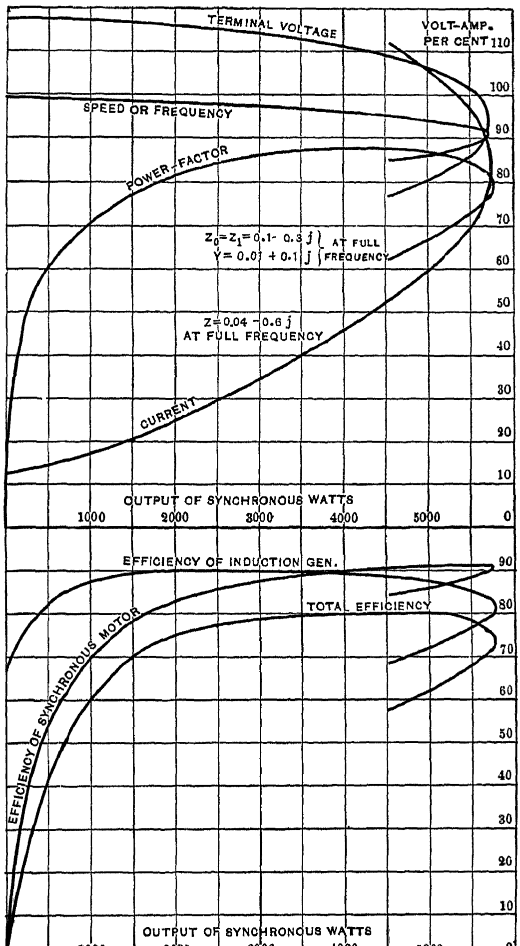


Fig. 232. Induction Generator and Synchronous Motor Load Curves.

The current, however, in the case of the synchronous motor operated from an induction generator is leading, while it is lagging in an induction motor operated from a synchronous generator. In either case it demagnetizes the synchronous machine and magnetizes the induction machine, that is, the synchronous machine supplies magnetization to the induction machine.

In Fig. 232 is shown the load curve of a synchronous motor operated from the induction generator in Fig. 230.

In Fig. 233 is shown the load curve of an over-compounded synchronous converter operated from an induction generator,

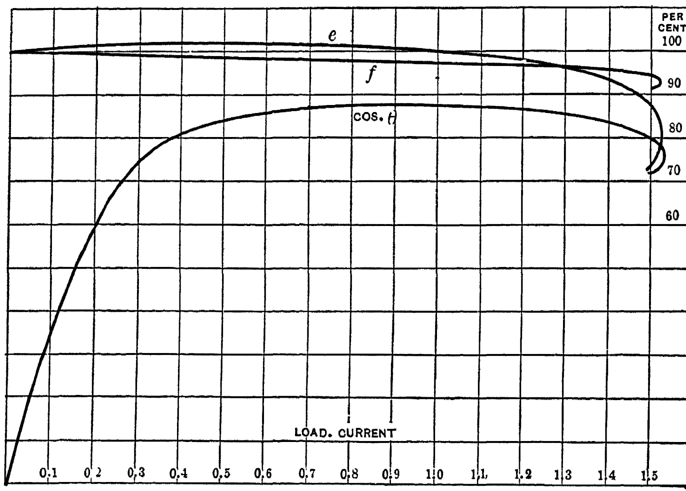


Fig. 233. Induction Generator and Synchronous Converter Phase Control, no Line Impedance.

the over-compounding being such as to give approximately constant terminal voltage *e*.

188. Obviously when operating a self-exciting synchronous converter from an induction generator the system is unstable also if both machines are below magnetic saturation, since in this case in both machines the generated e.m.f. is proportional to the field excitation and the field excitation proportional to the voltage; that is, with an unsaturated induction generator the synchronous converter operated therefrom must have its magnetic field excited to a density above the bend of the saturation curve.

Since the induction generator requires for its operation a circuit with leading current varying with the load in the manner determined by the internal constants of the motor, to make an induction or asynchronous generator suitable for operation on a general alternating-current circuit, it is necessary to have a synchronous machine as exciter in the circuit consuming leading current, that is, supplying the required lagging or magnetizing current to the induction generator; and in this case the voltage of the system is controlled by the field excitation of the synchronous machine, that is, its counter e.m.f. Either a synchronous motor of suitable size running light can be used herefor as exciter of the induction generator, or the exciting current of the induction generator may be derived from synchronous motors or converters in the same system, or from synchronous alternating-current generators operated in parallel with the induction generator, in which latter case, however, these currents can be said to come from the synchronous alternator as lagging currents. Electrostatic condensers, as an underground cable system, may also be used for excitation, but in this case besides the condensers a synchronous machine is required to secure stability.

The induction machine may thus be considered as *consuming* a lagging reactive magnetizing current at all speeds, and *consuming* a power current below synchronism, as motor, *supplying* a power current (that is, consuming a negative power current) above synchronism, as generator.

Therefore, induction generators are best suited for circuits which normally carry leading currents, as synchronous motor and synchronous converter circuits, but less suitable for circuits with lagging currents, since in the latter case an additional synchronous machine is required, giving all the lagging currents of the system plus the induction generator exciting current.

Obviously, when running induction generators in parallel with a synchronous alternator no synchronizing is required, but the induction generator takes a load corresponding to the excess of its speed over synchronism, or conversely, if the driving power behind the induction generator is limited, no speed regulation is required, but the induction generator runs at a speed exceeding synchronism by the amount required to consume the driving power.

The foregoing consideration obviously applies to the polyphase induction generator as well as to the single-phase induction generator, the latter, however, requiring a larger exciter in consequence of its lower power-factor. Therefore, even in a single-phase induction generator, preferably polyphase excitation is used, that is, the induction machine and its synchronous exciter wound as polyphase machines, but the load connected to one phase only of the induction machine. The curves shown in the preceding apply to the machine as polyphase generator.

The effect of resistance in the secondary is essentially the same in the induction generator as in the induction motor. An increase of resistance increases the slip, that is, requires an increase of speed at the same torque, current, and output, and thus correspondingly lowers the efficiency.

Induction generators have been proposed and used to some extent for high-speed prime movers, as steam turbines, since their squirrel-cage rotor appears mechanically better suited for very high speeds than the revolving field of the synchronous generator.

The foremost use of induction generators will probably be for collecting small water powers in one large system, due to the far greater simplicity, reliability, and cheapness of a small induction generator station feeding into a big system compared with a small synchronous generator station. The induction generator station requires only the hydraulic turbine, the induction machine, and the step-up transformer, but does not even require a turbine governor, and so needs practically no attention, as the control of voltage, speed, and frequency takes place by a synchronous generator or motor main station, which collects the power while the individual induction generator stations feed into the system as much power as the available water happens to supply.

The synchronous induction motor, comprising a single-phase or polyphase primary and a single-phase secondary, tends to drop into synchronism and then operates essentially as reaction machine. A number of types of synchronous induction generators have been devised, either with commutator for excitation or without commutator and with excitation by low-frequency synchronous or commutating machine, in the armature, or by high-frequency excitation. For particulars regarding these very interesting machines, see "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

VI. Induction Booster.

189. In the induction machine, at a given slip s, current and terminal voltage are proportional to each other and of constant phase relation, and their ratio is a constant. Thus when connected in an alternating-current circuit, whether in shunt or in series, and held at a speed giving a constant and definite slip s, either positive or negative, the induction machine acts like a constant impedance.

The apparent impedance and its components, the apparent resistance and apparent reactance represented by the induction machine, vary with the slip. At synchronism apparent impedance, resistance, and reactance are a maximum. They decrease with increasing positive slip. With increasing negative slip the apparent impedance and reactance decrease also, the apparent resistance decreases to zero and then increases again in negative direction as shown in Fig. 234, which gives the apparent impedance, resistance, and reactance of the machine shown in Figs. 207 and 208, etc., with the speed as abscissas.

The cause is that the power current is in opposition to the terminal voltage above synchronism, and thereby the induction machine behaves as an impedance of negative resistance, that is, adding a power e.m.f. into the circuit proportional to the current.

As may be seen herefrom, the induction machine when inserted in series in an alternating-current circuit can be used as a booster, that is, as an apparatus to generate and insert in the circuit an e.m.f. proportional to the current, and the amount of the boosting effect can be varied by varying the speed, that is, the slip at which the induction machine is revolving. Above synchronism the induction machine boosts, that is, raises the voltage; below synchronism it lowers the voltage; in either case also adding an out-of-phase e.m.f. due to its reactance. The greater the slip, either positive or negative, the less is the apparent resistance, positive or negative, of the induction machine.

The effect of resistance inserted in the secondary of the induction booster is similar to that in the other applications of the induction machine; that is, it increases the slip required for a certain value of apparent resistance, thereby lowering the efficiency of the apparatus, but at the same time making it less dependent upon minor variations of speed; that is, requires a lesser constancy of slip, and thus of speed and frequency, to give a steady boosting effect.

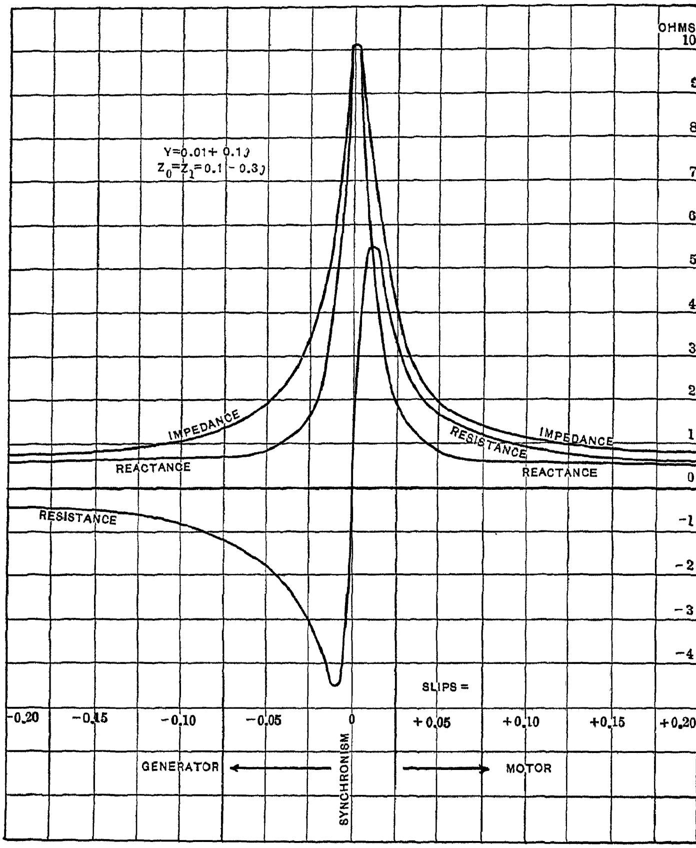


Fig. 234. Effective Impedance of Three-Phase Induction Machine.

VII. Phase Converter.

190. It may be seen from the preceding that the induction machine can operate equally well as motor, below synchronism, and as generator, above synchronism.

In the single-phase induction machine the motor or generator action occurs in one primary circuit only, but in the direction in quadrature to the primary circuit there is a mere magnetizing current either in the secondary, in the single-phase motor proper, or in an auxiliary field-circuit, in the monocyclic motor.

The motor and generator action can occur, however, simultaneously in the same machine, some of the primary circuits acting as motor, others as generator circuits. Thus, if one of the two circuits of a quarter-phase induction machine is connected to a single-phase system, in the second circuit an e.m.f. is generated in quadrature with and equal to the generated e.m.f. in the first circuit; and this e.m.f. can thus be utilized to produce currents which, with currents taken from the primary single-phase mains, give a quarter-phase system. Or, in a three-phase motor connected with two of its terminals to a single-phase system, from the third terminal an e.m.f. can be derived which, with the single-phase system feeding the induction machine, combines to a three-phase system. The induction machine in this application represents a phase converter.

The phase converter obviously combines the features of a single-phase induction motor with those of a double transformer, transformation occurring from the primary or motor circuit to the secondary or armature, and from the secondary to the tertiary or generator circuit.

Thus, in a quarter-phase motor connected to single-phase mains with one of its circuits, if

$Y = g + jb$ = primary polyphase exciting admittance,

$Z_0 = r_0 - jx_0$ = self-inductive impedance per primary or tertiary circuit,

$Z_1 = r_1 - jx_1$ = resultant single-phase self-inductive impedance of secondary circuits.

Let

e = e.m.f. generated by the mutual flux and

$Z = r - jx$ = impedance of the external circuit supplied by the phase converter as generator of second phase.

We then have

$\dot{I} = \dfrac{e}{Z+Z_0}$ = current of second phase produced by phase converter,

$\dot{E} = \dot{I}Z = \dfrac{eZ}{Z+Z_0} = \dfrac{e}{1+\dfrac{Z_0}{Z}}$ = terminal voltage at generator circuit of phase converter.

The current in the secondary of the phase converter is then

$$I_1 = I + I' + I'',$$

where

$$I = \text{load current} = \frac{e}{Z+Z_0},$$

$$I' = eY = \text{exciting current of quadrature magnetic flux},$$

$$I' = \frac{es}{r_1 - jsx_1} = \text{current required to revolve the machine},$$

and the primary current is

$$I_0 = I_1 + I',$$

where

$$I' = eY = \text{exciting current of main magnetic flux.}$$

From these currents the e.m.fs. are derived in a similar manner as in the induction motor or generator.

Due to the internal losses in the phase converter, the e.m.fs. of the two circuits, the motor and generator circuits, are practically in quadrature with each other and equal only at no load, but shift out of phase and become more unequal with increase of load, the unbalancing depending upon the constants of the phase converter.

It is obvious that the induction machine is used as phase converter only to change single-phase to polyphase, since a change from one polyphase system to another polyphase system can be effected by stationary transformers. A change from single-phase to polyphase, however, requires a storage of energy, since the power arrives as single-phase pulsating, and leaves as steady polyphase flow, and the momentum of the revolving phase converter secondary stores and returns the energy.

With increasing load on the generator circuit of the phase converter its slip increases, but less than with the same load as mechanical output from the machine as induction motor.

An application of the phase converter is made in single-phase motors by closing the tertiary or generator circuit by a condenser of suitable capacity, thereby generating the exciting current of the motor in the tertiary circuit.

The primary circuit is thereby relieved of the exciting current of the motor, the efficiency essentially increased, and the power-factor of the single-phase motor with condenser in tertiary circuit becomes practically unity over the whole range of load.

At the same time, since the condenser current is derived by double transformation in the multitooth structure of the induction machine, which has a practically uniform magnetic field, irrespective of the shape of the primary impressed e.m.f. wave, the application of the condenser becomes feasible irrespective of the wave shape of the generator.

Usually the tertiary circuit in this case is arranged on an angle of 60 deg. with the primary circuit, and in starting a powerful torque is thereby developed, with a torque efficiency superior to any other single-phase motor starting device, and when combined with inductive reactance in a second tertiary circuit, the apparent starting torque efficiency can be made even to exceed that of the polyphase induction motor (see page 385).

For further discussion hereof, see A. I. E. E. Transactions, 1900, p. 37.

VIII. Frequency Converter or General Alternating-Current Transformer.

191. The e.m.fs. generated in the secondary of the induction machine are of the frequency of slip, that is, synchronism minus speed, thus of lower frequency than the impressed e.m.f. in the range from standstill to double synchronism; of higher frequency outside of this range.

Thus, by opening the secondary circuits of the induction machine and connecting them to an external or consumer's circuit, the induction machine can be used to transform from one frequency to another, as frequency converter.

It lowers the frequency with the secondary running at a speed between standstill and double synchronism, and raises the frequency with the secondary either driven backward or above double synchronism.

Obviously, the frequency converter can at the same time change the e.m.f. by using a suitable number of primary and secondary turns, and can change the phases of the system by having a secondary wound for a different number of phases from the primary, as, for instance, convert from three-phase 6000-volts 25-cycles to quarter-phase 2500-volts 62.5-cycles.

Thus, a frequency converter can be called a "general alternating-current transformer."

For its theoretical discussion and calculation, see "Theory and Calculation of Alternating-Current Phenomena."

The action and the equations of the general alternating-current transformer are essentially those of the stationary alternating-current transformer, except that the ratio of secondary to primary generated e.m.f. is not the ratio of turns but the ratio of the product of turns and frequency, while the ratio of secondary current and primary load current (that is, total primary current minus primary exciting current) is the inverse ratio of turns.

The ratio of the products of generated e.m.f. and current, that is, the ratio of electric power generated in the secondary, to electric power consumed in the primary (less excitation), is thus not unity but is the ratio of secondary to primary frequency.

Hence, when lowering the frequency with the secondary revolving at a speed between standstill and synchronism, the secondary output is less than the primary input, and the difference is transformed into mechanical work; that is, the machine acts at the same time as induction motor, and when used in this manner is usually connected to a synchronous or induction generator feeding preferably into the secondary circuit (to avoid double transformation of its output) or to a synchronous converter, which transforms the mechanical power of the frequency converter into electrical power.

When raising the frequency by backward rotation, the secondary output is greater than the primary input (or rather the electric power generated in the secondary greater than the primary power consumed by the generated e.m.f.), and the difference is to be supplied by mechanical power by driving the frequency changer backward by synchronous or induction motor, preferably connected to the primary circuit, or by any other motor device.

Above synchronism the ratio of secondary output to primary input becomes negative; that is, the induction machine generates power in the primary as well as in the secondary, the primary power at the impressed frequency, the secondary power at the frequency of slip, and thus requires mechanical driving power.

The secondary power and frequency are less than the primary below double synchronism, more above double synchronism, and are equal at double synchronism, so that at double synchronism the primary and secondary may be connected in multiple or in series and the machine is then a double synchronous alternator further discussed in the "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

As far as its transformer action is concerned, the frequency converter is an open magnetic circuit transformer, that is, a transformer of relatively high magnetizing current. It combines therewith, however, the action of an induction motor or generator. Excluding the case of over-synchronous rotation, it is approximately (that is, neglecting internal losses) electrical input ÷ electrical output ÷ mechanical output = primary frequency ÷ secondary frequency ÷ speed or primary minus secondary frequency; that is, the mechanical output is negative when increasing the frequency by backward rotation.

Such frequency converters are to a certain extent in commercial use, and have the advantage over the motor-generator plant of requiring an amount of apparatus equal only to the output, while the motor-generator set requires machinery equal to twice the output.

An application of the frequency converter when lowering the frequency is made in concatenation or tandem control of induction machines, as described in the next section. In this case the first motor, or all the motors except the last of the series are in reality frequency converters.

IX. Concatenation of Induction Motors.

192. In the secondary of the induction motor an e.m.f. is generated of the frequency of slip. Thus connecting the secondary circuit of the induction motor to the primary of a second induction motor, the latter is fed by a frequency equal to the slip of the first motor, and reaches its synchronism at the frequency of slip of the first motor, the first motor then acting as frequency converter for the second motor.

If, then, two equal induction motors are rigidly connected together and thus caused to revolve at the same speed, the speed of the second motor, which is the slip s of the first motor at no load, equals the speed of the first motor: $s = 1 - s$, and thus

$s = 0.5$. That is, a pair of induction motors connected this way in tandem or in concatenation, that is, "chain connection," as commonly called, or in cascade, as called abroad, tends to approach $s = 0.5$, or half synchronism, at no load, slipping below this speed under load; that is, concatenation of two motors reduces their synchronous speed to one-half, and thus offer as means to operate induction motors at one-half speed.

In general, if a number of induction machines are connected in tandem, that is, the secondary of each motor feeding the primary of the next motor, the secondary of this last motor being short-circuited, the sum of the speeds of all motors tends towards synchronism, and with all motors connected together so as to revolve at the same speed the system operates at $\frac{1}{n}$ synchronous speed, when n = number of motors. If the two induction motors on the same shaft have a different number of poles, they synchronize at some other speed below synchronism, or if connected differentially, they synchronize at some speed above synchronism.

Assuming the ratio of turns of primary and secondary as 1:1, with two equal induction motors in concatenation at standstill, the frequency and the e.m.f. impressed upon the second motor, neglecting the drop of e.m.f. in the internal impedance of the first motor, equal those of the first motor. With increasing speed, the frequency and the e.m.f. impressed upon the second motor decrease proportionally to each other, and thus the magnetic flux and the magnetic density in the second motor, and its exciting current, remain constant and equal to those of the first motor, neglecting internal losses; that is, when connected in concatenation the magnetic density, current input, etc., and thus the torque developed by the second motor, are approximately equal to those of the first motor, being less because of the internal losses in the first motor.

Hence, the motors in concatenation share the work in approximately equal portions, and the second motor utilizes the power which without the use of a second motor at less than one-half synchronous speed would have to be wasted in the secondary resistance; that is, theoretically concatenation doubles the torque and output for a given current, or power input into the motor system. In reality the gain is somewhat less, due to the second motor not being quite equal to a non-inductive resistance for the secondary of the first motor, and due to the drop of voltage in the internal impedance of the first motor, etc.

At one-half synchronism, that is, the limiting speed of the concatenated couple, the current input in the first motor equals its exciting current plus the transformed exciting current of the second motor, that is, equals twice the exciting current.

193. Hence, comparing the concatenated couple with a single motor, the primary exciting admittance is doubled. The total impedance, primary plus secondary, is that of both motors, that is, doubled also, and the characteristic constant of the concatenated couple is thus four times that of a single motor, but the speed reduced to one-half.

Comparing the concatenated couple with a single motor rewound for twice the number of poles, that is, one-half speed also, such rewinding does not change the self-inductive impedance, but quadruples the exciting admittance, since one-half as many turns per pole have to produce the same flux in one-half the pole arc, that is, with twice the density. Thus the characteristic constant is increased fourfold also. It follows herefrom that the characteristic constant of the concatenated couple is that of one motor rewound for twice the number of poles.

The slip under load, however, is less in the concatenated couple than in the motor with twice the number of poles, being due to only one-quarter the internal impedance, the secondary impedance of the second motor only, and thus the efficiency is slightly higher.

Two motors coupled in concatenation are in the range from standstill to one-half synchronism approximately equivalent to one motor of twice the admittance, three times the primary impedance, and the same secondary impedance as each of the two motors, or more nearly 2.8 times the primary and 1.2 times the secondary impedance of one motor. Such a motor is called the *equivalent motor.*

194. The calculation of the characteristic curve of the concatenated motor system is similar to, but more complex than, that of the single motor. Starting from the generated e.m.f. e of the second motor, reduced to full frequency, we work up to the impressed e.m.f. of the first motor e_0, by taking due consideration of the proper frequencies of the different circuits. Herefor the

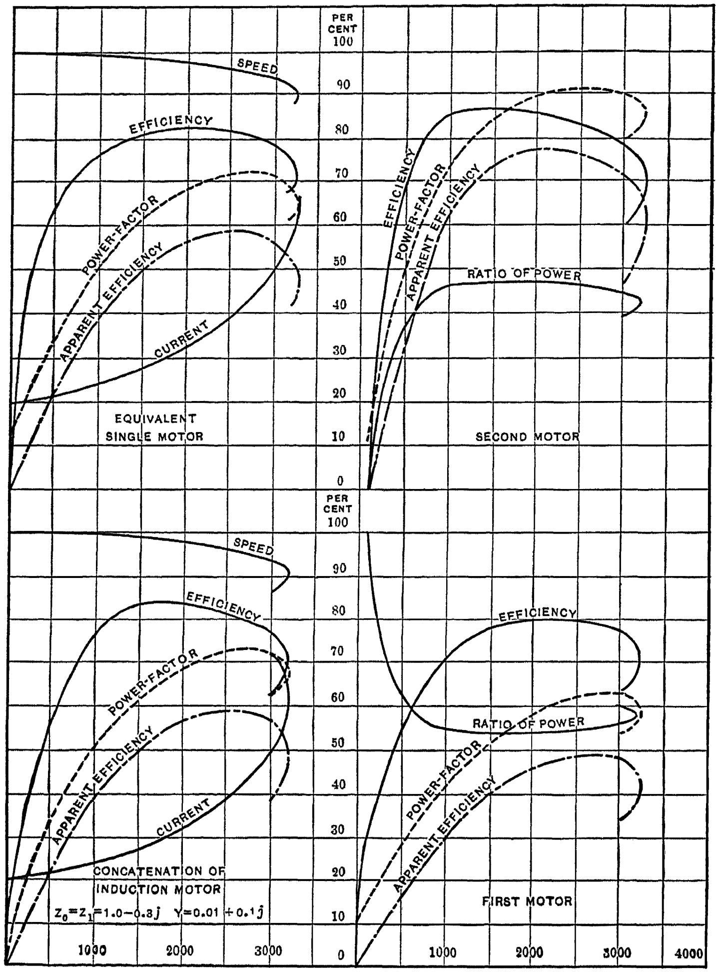


Fig. 235. Comparison of Concatenated Motors with a Single Motor of Double the Number of Poles.

reader must be referred to "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

The load curves of the pair of three-phase motors of the same constants as the motor in Figs. 207 and 208 are given in Fig. 235, the complete speed curve in Fig. 236.

Fig. 235 shows the load curve of the total couple, of the two individual motors, and of the equivalent motor.

As seen from the speed curve, the torque from standstill to one-half synchronism has the same shape as the torque curve of a single motor between standstill and synchronism. At one-half synchronism the torque reverses and becomes negative. It reverses again at about two-thirds synchronism, and is positive between about two-thirds synchronism and synchronism, zero at synchronism, and negative beyond synchronism.

Thus, with a concatenated couple, two ranges of positive torque and power as induction motor exist, one from standstill

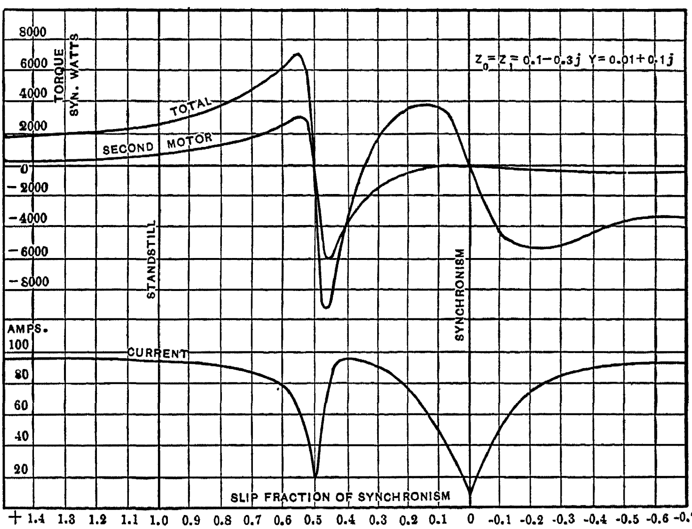


Fig. 236. Concatenation of Induction Motors, Speed Curves.

to half synchronism, the other from about two-thirds synchronism to synchronism.

In the ranges from one-half synchronism to about two-thirds synchronism, and beyond synchronism, the torque is negative, that is, the couple acts as generator.

The insertion of resistance in the secondary of the second motor has in the range from standstill to half synchronism the same effect as in a single induction motor, that is, shifts the maximum torque point towards lower speed without changing its value. Beyond half synchronism, however, resistance in the secondary lengthens the generator part of the curve, and makes the second motor part of the curve more or less disappear, as seen in Fig. 236, which gives the speed curves of the same motor as Fig. 235, with resistance in circuit in the secondary of the second motor.

The main advantages of concatenation are obviously the ability of operating at two different speeds, the increased torque and power efficiency below half speed, and the generator or braking

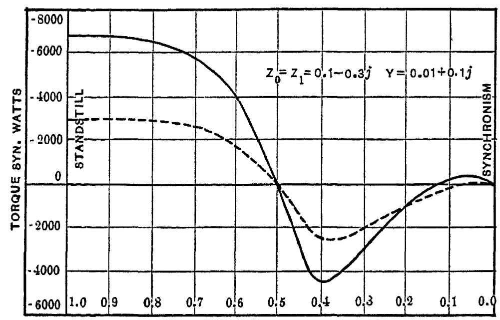


Fig. 237. Concatenation of Induction Motors Speed Curve with Resistance in the Secondary Circuit.

action between half speed and synchronism, and such concatenation is therefore used to some extent in three-phase railway motor equipments, while for stationary motors usually a change of the number of poles by reconnecting the primary winding through a suitable switch is preferred where several speeds are desired, as it requires only one motor.

X. Synchronizing Induction Motors.

195. Occasionally two or more induction motors are operated in parallel on the same load, as for instance in three-phase railroading, or when securing several speeds by concatenation. In this case the secondaries of the induction motors may be connected in multiple and a single rheostat used for starting and speed control. Thus, when using two motors in concatenation for speeds from standstill to half synchronism, from half synchronism to full speed, the motors may also be operated on a single rheostat by connecting their secondaries in parallel. As in parallel connection the frequency of the secondaries must be the same, and the secondary frequency equals the slip, it follows that the motors in this case must

operate at the same slip, that is, at the same frequency of rotation, or in synchronism with each other. If the connection of the induction motors to the load is such that they cannot operate in exact step with each other, obviously separate resistances must be used in the motor secondaries, so as to allow different slips. When rigidly connecting the two motors with each other, it is essential to take care that the motor secondaries have exactly the same relative position to their primaries so as to be in phase with each other, just as would be necessary when operating two alternators in parallel with each other when rigidly connected to the same shaft or when driven by synchronous motors from the same supply. As in the induction motor secondary an e.m.f. of definite frequency, that of slip, is generated by its rotation through the revolving motor field, the induction motor secondary is an alternating-current generator, which is short-circuited at speed and loaded by the starting rheostat during acceleration, and the problem of operating two induction motors with their secondaries connected in parallel on the same external resistance is thus the same as that of operating two alternators in parallel. In general, therefore, it is undesirable to rigidly connect induction motor secondaries mechanically if they are electrically connected in parallel, but it is preferable to have their mechanical connection sufficiently flexible, as by belting etc., so that the motors can drop into exact step with each other and maintain step by their synchronizing power.

It is of interest, then, to examine the synchronizing power of two induction motors which are connected in multiple with their secondaries on the same rheostat and operated from the same primary impressed e.m.f.

Assume two equal induction motors with their primaries connected to the same voltage supply and with their secondaries connected in multiple with each other to a common resistance r, and neglecting for simplicity the exciting current and the voltage drop in the impedance of the motor primaries as not materially affecting the synchronizing power.

Let $Z_1 = r_1 - jx_1$ = secondary self-inductive impedance at full frequency; s = slip of the two motors, as fraction of synchronism; e_0 = absolute value of impressed e.m.f. and thus, when neglecting the primary impedance, of the e.m.f. generated in the primary by the rotating field.

If then the two motor-secondaries are out of phase with each other by angle 2τ, and the secondary of the motor 1 is behind in the direction of rotation and the secondary of the motor 2 ahead of the average position by angle τ, then

$$\dot{E}_1 = se_0 (\cos \tau - j \sin \tau) = \text{secondary generated e.m.f. of the first motor,} \tag{1}$$

$$\dot{E}_2 = se_0 (\cos \tau + j \sin \tau) = \text{secondary generated e.m.f. of the second motor.} \tag{2}$$

And if $\dot{I}_1$ = current coming from the first, $\dot{I}_2$ = current coming from the second motor secondary, the total current, or current in the external resistance, r, is

$$\dot{I} = \dot{I}_1 + \dot{I}_2; \tag{3}$$

it is then, in the circuit comprising the first motor secondary and the rheostat r,

$$\dot{E}_1 - \dot{I}_1 Z - \dot{I} r = 0, \tag{4}$$

in the circuit comprising the second motor secondary and the rheostat r,

$$\dot{E}_2 - \dot{I}_2 Z - \dot{I} r = 0, \tag{5}$$

where $Z = r_1 - jsx_1$;

substituting (3) into (4) and (5) and rearranging gives

$$\dot{E}_1 - \dot{I}_1 (Z + r) - \dot{I}_2 r = 0,$$
$$\dot{E}_2 - \dot{I}_1 r - \dot{I}_2 (Z + r) = 0.$$

These two equations added and subtracted give

$$\dot{E}_1 + \dot{E}_2 - (\dot{I}_1 + \dot{I}_2)(Z + 2r) = 0,$$
$$\dot{E}_1 - \dot{E}_2 - (\dot{I}_1 - \dot{I}_2) Z = 0;$$

hence,

$$\dot{I}_1 + \dot{I}_2 = \frac{\dot{E}_1 + \dot{E}_2}{Z + 2r}$$

and

$$\dot{I}_1 - \dot{I}_2 = \frac{\dot{E}_1 - \dot{E}_2}{Z}. \tag{6}$$

Substituting for convenience and abbreviation,

$$\frac{1}{Z + 2r} = Y = g + jb,$$
$$\frac{1}{Z} = Y_1 = g_1 + jb_1, \tag{7}$$

into equation (6) and substituting (1) and (2) into (6), gives

$$\dot{I}_1 + \dot{I}_2 = 2\, se_0 Y \cos \tau,$$

$$\dot{I}_1 - \dot{I}_2 = -2\, jse_0 Y_1 \sin \tau; \tag{8}$$

hence,

$$\dot{I}_2^1 = se_0 \{Y \cos \tau \pm jY_1 \sin \tau\} \tag{9}$$

is the current in the secondary circuit of the motor, and therefore also the primary load current, that is, the primary current corresponding to the secondary current, and thus, when neglecting the exciting current, also the primary motor current, where the upper sign corresponds to the first, or lagging, the lower sign to the second, or leading, motor.

Substituting in (9) for Y and Y_1 gives

$$\dot{I}_2^1 = se_0 \{(g \cos \tau \pm b_1 \sin \tau) + j\,(b \cos \tau \mp g_1 \sin \tau)\}, \tag{10}$$

the primary e.m.f. corresponding hereto is

$$\dot{E}_2^1 = e_0 \{\cos \tau \mp j \sin \tau\}, \tag{11}$$

where again the upper sign corresponds to the first, the lower to the second motor.

The power consumed by the current $\dot{I}_2^1$ with the e.m.f. $\dot{E}_2^1$ is the sum of the products of the horizontal components, and of the vertical components, that is, of the real components and of the imaginary components of these two quantities (as a horizontal component of one does not represent any power with a vertical component of the other quantity, being in quadrature therewith).

$$P_2^1 = | \dot{E}_2^1\, \dot{I}_2^1 |,$$

where the brackets denote that the sum of the product of the corresponding parts of the two quantities is taken.

As discussed in the preceding, the *torque* of an induction motor, in synchronous watts, equals the power consumed by the primary counter e.m.f.; that is,

$$D_2^1 = P_2^1,$$

and substituting (10) and (11) this gives

$$D_2^1 = se_0^2 \{\cos \tau\,(g \cos \tau \pm b_1 \sin \tau) \mp \sin \tau\,(b \cos \tau \mp g \sin \tau)\}$$

$$= se_0^2 \left\{\frac{g_1 + g}{2} - \frac{g_1 - g}{2} \cos 2\,\tau \pm \frac{b_1 - b}{2} \sin 2\,\tau\right\}, \tag{12}$$

and herefrom follows the motor output or power, by multiplying with $(1 - s)$.

The sum of the torques of both motors, or the *total torque*, is

$$2\,D_t = D_1 + D_2 = se_0^2\,\{(g_1 + g) - (g_1 - g)\cos 2\,\tau\}. \qquad (13)$$

The difference of the torque of both motors, or the *synchronizing torque*, is

$$2\,D_s = se_0^2\,(b_1 - b)\sin 2\,\tau, \qquad (14)$$

where, by (7),

$$\left.\begin{aligned} g_1 &= \frac{r_1}{m_1}, & g &= \frac{r_1 + 2\,r}{m}, \\ b_1 &= \frac{sx_1}{m_1}, & b &= \frac{sx_1}{m}. \\ m_1 &= r_1^2 + s^2x_1^2, & m &= (r_1 + 2\,r)^2 + s^2x_1^2, \end{aligned}\right\} \qquad (15)$$

In these equations primary exciting current and primary impedance are neglected. The primary impedance can be introduced in the equations, by substituting $(r_1 + sr_0)$ for r_1, and $(x_1 + x_0)$ for x_1, in the expression of m_1 and m, and in this case only the exciting current is neglected, and the results are sufficiently accurate for most purposes, except for values of speed very close to synchronism, where the motor current is appreciably increased by the exciting current. It is then

$$\left.\begin{aligned} m_1 &= (r_1 + rs_0)^2 + s^2\,(x_1 + x_0)^2, \\ m &= (r_1 + sr_0 + 2\,r)^2 + s^2\,(x_1 + x_0)^2; \end{aligned}\right\} \qquad (16)$$

all the other equations remain the same.

From (15) and (16) follows

$$\frac{b_1 - b}{2} = \frac{2\,srx_1\,(r_1 + sr_0 + r)}{mm_1}, \qquad (17)$$

hence, is always positive.

196. $(b_1 - b)$ is always positive, that is, the synchronizing torque is positive in the first or lagging motor, and negative in the second or loading motor; that is, the motor which lags in position behind gives more power and thus accelerates, while the motor which is ahead in position gives less power and thus drops back. Hence, the two motor armatures pull each other into step, if thrown together out of phase, just like two alternators.

The synchronizing torque (14) is zero if $\tau = 0$, as obvious, as for $\tau = 0$ both motors are in step with each other. The synchronizing torque also is zero if $\tau = 90$ deg., that is, the two motor armatures are in opposition. The position of opposition is unstable, however, and the motors cannot operate in opposition, that is, for $\tau = 90$ deg., or with the one motor secondary short-circuiting the other; in this position, any decrease of τ below 90 deg. produces a synchronizing torque which pulls the motors together, to $\tau = 0$, or in step. Just as with alternators, there thus exist two positions of zero synchronizing power,—with the motors in step, that is, their secondaries in parallel and in phase, and with the motors in opposition, that is, their secondaries in opposition,—and the former position is stable, the latter unstable, and the motors thus drop into and retain the former position, that is, operate in step with each other, within the limits of their synchronizing power.

If the starting rheostat is short-circuited, or $r = 0$, it is, by (15), $b_1 = b$, and the synchronizing power vanishes, as is obvious, since in this case the motor secondaries are short-circuited and thus independent of each other in their frequency and speed.

With parallel connection of induction motor armatures a synchronizing power thus is exerted between the motors as long as any appreciable resistance exists in the external circuit, and the motors thus tend to keep in step until the common starting resistance is short-circuited and the motors thereby become independent, the synchronizing torque vanishes, and the motors can slip against each other without interference by cross currents.

Since the term $\frac{b_1 - b}{2}$ contains the slip, s, as factor, the synchronizing torque decreases with increasing approach to synchronous speed.

For $\tau = 0$, or with the motors in step with each other, it is, by (12), (15), and (16),

$$D_2^1 = se_0^2 g = \frac{se_0^2 (r_1 + 2r)}{(r_1 + sr_0 + 2r)^2 + s^2 (x_1 + x_0)^2}; \qquad (18)$$

that is, the same value as found for a single motor in paragraph 161. (As the resistance r is common to both motors, for each motor it enters as $2r$.)

For $\tau = 90$ deg., or the unstable positions of the motors, it is

$$D_2^1 = se_0^2 g_1 = \frac{se_0^2 r_1}{(r_1 + sr_0)^2 + s^2 (x_1 + x_0)^2}; \qquad (19)$$

that is, the same value as the motor would give with short-circuited armature. This is to be expected, as the two motor armatures short-circuit each other.

The synchronizing torque is a maximum for $\tau = 45$ deg., and is, by (14), (15), and (16),

$$D_s = se_0^2 \frac{b_1 - b}{2}. \tag{20}$$

As instances are shown, in Fig. 238, the motor torque, from equation (18), and the maximum synchronizing torque, from

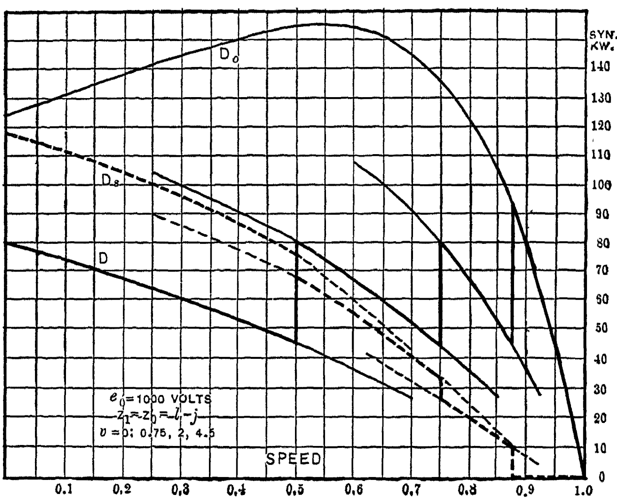


Fig. 238. Synchronizing Induction Motor, Motor Torque and Synchronizing Torque.

equation (20), for a motor of 5 per cent drop of speed at full load and very high overload capacity (a maximum power nearly 2.5 times and a maximum torque somewhat over three times the rated value), that is, of low reactance, as can be produced at low frequency, and is desirable for intermittent service, hence of the constants

$$Z_1 = Z_0 = 1 - j,$$
$$Y = 0.005 + 0.02\, j,$$
$$e_0 = 1000 \text{ volts},$$

for the values of additional resistance inserted into the armatures,

$$r = 0;\ 0.75;\ 2;\ 4.5,$$

giving the values

$$g_1 = \frac{1}{m_1}, \qquad g = \frac{1 + 2r}{m},$$

$$b_1 = \frac{2s}{m_1}, \qquad b = \frac{sx_1}{m},$$

$$m_1 = (1 + s)^2 4 s^2, \qquad m = (1 + s + 2r)^2 + 4 s^2.$$

As seen, in this instance the synchronizing torque is higher than the motor torque up to half speed, slightly below the motor torque between half speed and three-quarters speed, but above three-quarters speed rapidly drops, due to the approach to synchronism, and becomes zero when the last starting resistance is cut out.

XI. Self-Exciting Induction Machines.

197. In addition to the short-circuited secondary winding, in which by the rotating primary magnet field the secondary currents of the frequency of slip are produced, which do the work of the induction machine, the secondary member may be supplied with a closed coil winding connected to a commutator and brushes in the same manner as in continuous-current or commutating machines, except that with the three-phase system three sets of brushes displaced from each other by 120° deg., in a quarter-phase system four sets of brushes displaced by 90 deg. in position, are used.

Supplying these brushes with currents of the impressed frequency, either directly from the primary impressed e.m.f. or by step-down transformer or compensator, these full-frequency currents are commutated so as to give in the motor armature a resultant polarization of the frequency of slip, that is, of constant relative position to the system of revolving primary magnet poles. The intensity of this secondary polarization depends upon the voltage impressed upon the commutator, its relative position with the primary magnet field upon the position of the brushes. Setting the brushes so that the secondary polarization is in line with the primary magnet field it supplies excitation for the latter so that the primary magnetizing current is decreased, and by increasing the commutated current can be made to disappear or even to reverse, that is, the primary current of the induction machine becomes leading. Hence, by varying the impressed e.m.f. at the commutator, the primary current can be changed from lag to lead, and gives a V-shaped characteristic curve similar to that of the synchronous motor in Fig. 66, with exciting e.m.f. as abscissas and primary current as ordinates.

By shifting the brushes from the position of coincidence of secondary polarization with the primary magnet field, the secondary polarization can be resolved into two components, one in phase with the primary field giving the excitation of the machine, and one in quadrature therewith, which produces or consumes power in the machine in the same manner as in any commutating machine, and thereby is of no further interest.

Instead of using two secondary windings, one winding can be used connected to a commutator and shunted by resistance connected between the commutator segments. In this case, which, however, has the disadvantage over the double winding that the commutated circuit is of very low voltage, one and the same winding fulfills the function of exciting winding by carrying the impressed commutated exciting current and of energy winding by carrying the secondary current corresponding to the work of the machine.

The advantage of this method of excitation by commutator on the secondary is that the exciting current of the induction machine is in a circuit coincident with the short-circuited secondary, and its self-inductance is thereby reduced from the open-circuit impedance of the machine $\left(\frac{1}{Y}\right)$ to the short-circuited impedance (Z_1), and the volt-amperes excitation reduced in the same proportion, and the power-factor correspondingly improved, and by over-excitation leading currents can be produced. The disadvantage, however, is the addition of commutator and brushes, and thereby the loss of the main advantage offered by the induction machine over other forms of machines.

While the currents supplied by the commutator to the exciting winding give the same resultant m.m.f. as currents of the frequency of slip, they are not low-frequency currents, but have a shape of the character shown in Fig. 239, which represents the (commutated) exciting current of a three-phase motor at five per cent slip. The self-inductance of these exciting currents, therefore, is not the self-inductance corresponding to the low frequency of slip, but that of full frequency; that is, the self-inductance is the same whether the current is supplied directly by permanent connections or through a commutator.

The advantage resulting from such an excitation of the secondary member by commutator is not due to the lowering of the self-inductance of excitation to the frequency of slip, but due to the lowering of the self-inductance by mutual inductance,

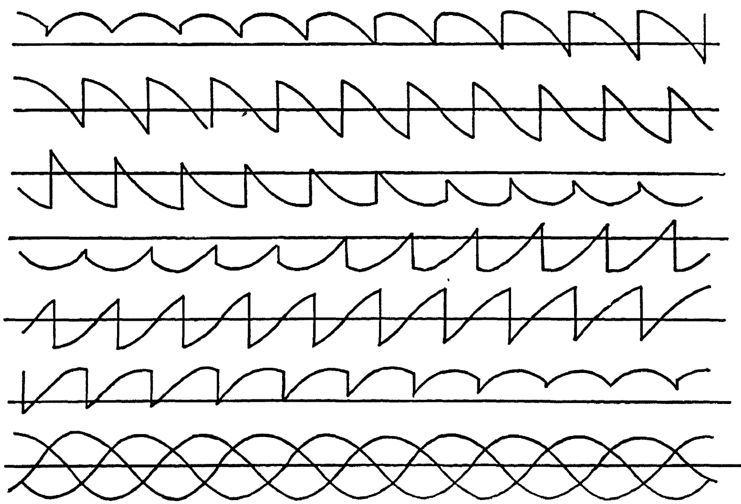


Fig. 239. Self-Excited Induction Machine, Current Supplied by Commutator to the Exciting Winding.

resulting from the coincidence of the exciting winding with the short-circuited winding, and the existence of a short-circuited secondary winding or its equivalent is therefore essential. Without it the machine is a mere alternating commutator machine.

Regarding the theoretical investigation of the self-excitation of induction machines, see "Theory and Calculation of Alternating-Current Phenomena," fourth edition.

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