ACOUSTICS AND ARCHITECTURE

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BY

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PREFACE

The last fifteen years have seen a rapidly growing interest, both scientific and popular, in the subject of acoustics. The discovery of the thermionic effect and the resulting development of the vacuum tube have made possible the amplification and measurement of minute alternating currents, giving to physicists a powerful new device for the quantitative study of acoustical phenomena. As a result, there have followed remarkable developments in the arts of communication and of the recording and reproduction of sound. These have led to a demand for increased knowledge of the principles underlying the control of sound, a demand which has been augmented by the necessity of minimizing the noise resulting from the ever increasing mechanization of all our activities.

Thus it happens that acoustical problems have come to claim the attention of a large group of engineers and technicians. Many of these have had to pick up most of their knowledge of acoustics as they went along. Even today, most colleges and technical schools give only scant instruction in the subject. Further, the fundamental work of Professor Wallace Sabine has placed upon the architect the necessity of providing proper acoustic conditions in any auditorium which he may design. Some knowledge of the behavior of sound in rooms has thus become a necessary part of the architect's equipment.

It is with the needs of this rather large group of possible readers in mind that the subject is here presented. No one can be more conscious than is the author of the lack of scientific elegance in this presentation. Thus, for example, the treatment of simple harmonic motion and the development of the wave equation in Chap. II would be much more neatly handled for the mathematical reader by the use of the differential equation of the motion of a particle under the action of an elastic force. The only excuse for the treatment given is the hope that it may help the non-mathematical reader to visualize more clearly the dynamic properties of a wave and its propagation in a medium.

In further extenuation of this fault, one may plead the inherent difficulties of a strictly logical approach to the problem of waves within a three-dimensional space whose dimensions are not great in comparison with the wave length. Thus, in Chap. III, conditions in the steady state are considered from the wave point of view; while in Chap. IV, we ignore the wave characteristics in order to handle the problem of the building up and decay of sound in rooms. The theory of reverberation is based upon certain simplifying assumptions. An understanding of these assumptions and the degree to which they are realized in practical cases should lead to a more adequate appreciation of the precision of the solution reached.

No attempt has been made to present a full account of all the researches that have been made in this field in very recent years. Valuable contributions to our knowledge of the subject are being made by physicists abroad, particularly in England and Germany. If undue prominence seems to be given to the results of work done in this country and particularly to that of the Riverbank Laboratories, the author can only plead that this is the work about which he knows most. Perhaps no small part of his real motive in writing a book has been to give permanent form to those portions of his researches which in his more confident moments he feels are worthy of thus preserving.

Grateful recognition is made of the kindness of numerous authors in supplying reprints of their papers. It is also a pleasure to acknowledge the painstaking assistance of Miss Cora Jensen and Mr. C. A. Anderson of the staff of the Riverbank Laboratories in the preparation of the manuscript and drawings for the text.

In conclusion, the author would state that whatever is worth while in the following pages is dedicated to his friend Colonel George Fabyan, whose generous support and unfailing interest in the solution of acoustical problems have made the writing of those pages possible.

P. E. S.

RIVERBANK LABORATORIES,
GENEVA, ILLINOIS,
July, 1932.

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AND
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CHAPTER I

INTRODUCTION

Historical.

Next to mechanics, acoustics is the oldest branch of physics.

Ideas of the nature of heat, light, and electricity have undergone profound changes in the course of the experimental and theoretical development of modern physics. Quite on the contrary, however, the true nature of sound as a wave motion, propagated in the air by virtue of its elastic properties, has been clearly discerned from the very beginning. Thus Galileo in speaking of the ratio of a musical interval says: "I assert that the ratio of a musical interval is determined by the ratio of their frequencies, that is, by the number of pulses of air waves which strike the tympanum of the ear causing it also to vibrate with the same frequency." In the "Principia," Newton states: "When pulses are propagated through a fluid, every particle oscillates with a very small motion and is accelerated and retarded by the same law as an oscillating pendulum." Thus we have a mental picture of a sound wave traveling through the air, each particle performing a to-and-fro motion, this motion being transmitted from particle to particle as the wave advances. On the theoretical side, the study of sound considered as the physical cause of the sensation of hearing is thus a branch of the much larger study of the mechanics of solids and fluids.

Branches of Acoustics.

On the physical side, acoustics naturally divides itself into three parts: (1) the study of vibrating bodies including solids and partially inclosed fluids; (2) the propagation of vibratory energy through elastic fluids; and (3) the study of the mechanism of the organ of perception by means of which the vibratory motion of the fluid medium is able to induce nerve stimuli. There is still another branch of acoustics, which involves not only the purely physical properties of sound but also the physiological and psychological aspects of the subject as well as the study of sound in its relation to music and speech.

Of the three divisions of purely physical acoustics, the study of the laws of vibrating bodies has, up until the last twenty-five years, received by far the greatest attention of physicists. The problems of vibrating strings, of thin membranes, of plates, and of air columns have all claimed the attention of the best mathematical minds. A list of the outstanding names in the field would include those of Huygens, Newton, Fourier, Poisson, Laplace, Lagrange, Kirchhoff, Helmholtz, and Rayleigh, on the mathematical side of the subject. Galileo, Chladni, Savart, Lissajous, Melde, Kundt, Tyndall, and Koenig are some who have made notable experimental contributions to the study of the vibrations of bodies. The problems of the vibrations of strings, bars, thin membranes, plates, and air columns have all been solved theoretically with more or less completeness, and the theoretical solutions, in part, experimentally verified. It should be pointed out that in acoustics, the agreement between the theory and experiment is less exact than in any other branch of physics. This is due partly to the fact that in many cases it is impossible to set up experimental conditions in keeping with the assumptions made in deriving the theoretical solution. Moreover, the theoretical solution of a general problem may be obtained in mathematical expressions whose numerical values can be arrived at only approximately.

Velocity of Sound.

Turning from the question of the motion of the vibrating body at which sound originates, it is essential to know the changes taking place in the medium through which this energy is propagated. The first problem is to determine the velocity with which sound travels. The theoretical solution of the problem was given by Newton in 1687. Starting with the assumption that the motion of the individual particle of air is one of pure vibration and that this motion is transmitted with a definite velocity from particle to particle, he deduced the law that the speed of travel of a disturbance through a solid, liquid, or gaseous medium is numerically equal to the square root of the ratio of the volume elasticity to the density of the medium. The volume elasticity of a substance is a measure of the resistance which the substance offers to compression or dilatation. Suppose, for example, that we have a given volume V of air under a given pressure and that a small change of pressure δP is produced. A small change of volume δV will result. The ratio of the change of pressure to the change of volume per unit volume gives us the measure of the elasticity, the so-called "coefficient of elasticity" of the air

$$\epsilon = \delta P \div \frac{\delta V}{V}$$

Boyle's law states a common property of all gases, namely, that if the temperature of a fixed mass of gas remains constant, the volume will be inversely proportional to the pressure. This is the law of the isothermal expansion and contraction of gases. It is easy to show that under the isothermal condition, the elasticity of a gas at any pressure is numerically equal to that pressure; so that Newton's law for the velocity c of propagation of sound in air becomes

$$c = \sqrt{\frac{\text{pressure}}{\text{density}}} = \sqrt{\frac{P}{\rho}}$$

The pressure and density must of course be expressed in absolute units. The density of air at 0° C. and a pressure

76 cm. of mercury is 0.001293 g. per cubic centimeter. A pressure of one atmosphere equals $76 \times 13.6 \times 980 = 1{,}012{,}930$ dynes per square centimeter; and by the Newton formula the value of c should be

$$c = \sqrt{\frac{1{,}012{,}930}{0.001293}} = 27{,}990 \text{ cm./sec.} = 918.0 \text{ ft./sec.}$$

The experimentally determined value of c is about 18 per cent greater than this theoretical value given by the Newton formula. This disagreement between theory and experiment was explained in 1816, by Laplace, who pointed out that the condition of constant temperature under which Boyle's law holds is not that which exists in the rapidly alternating compressions and rarefactions of the medium that are set up by the vibrations of sound. It is a matter of common experience that if a volume of gas be suddenly compressed, its temperature rises. This rise of temperature makes necessary a greater pressure to produce a given volume reduction than is necessary if the compression takes place slowly, allowing time for the heat of compression to be conducted away by the walls of the containing vessel or to other parts of the gas. In other words, the elasticity of air for the rapid variations of pressure in a sound wave is greater than for the slow isothermal changes assumed in Boyle's law. Laplace showed that the elasticity for the rapid changes with no heat transfer (adiabatic compression and rarefaction) is γ times the isothermal elasticity where γ is the ratio of the specific heat of the medium at constant pressure to the specific heat at constant volume. The experimentally determined value of this quantity for air is 1.40, so that the Laplace correction of the Newton formula gives at 76 cm. pressure and 0° C.

$$c = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.40 \times 1{,}012{,}930}{0.001293}} = 33{,}120 \text{ cm./sec.}$$
$$= 1{,}086.2 \text{ ft./sec.} \quad (1)$$

Table I gives the results of some of the better known measurements of the velocity of sound.

Other determinations have been made, all in close agreement with the values shown in Table I, so that it may be said that the velocity of sound in free atmosphere is known with a fairly high degree of accuracy. The weight of all the experimental evidence is to the effect that this velocity is independent of the pitch, quality, and intensity of the sound over a wide range of variation in these properties.

Table I.—Speed of Sound in Open Air at 0° C.

Velocity		Authority
m./sec.	ft./sec.	
331 20	1,086 7	Bureau des Longitudes, 1822
331 40	1,087.3	Regnault, 1864
331.57	1,087 5	Szathmari, 1877
330 13	1,082 8	Blaikley, 1884
331 67	1,087.8	Blaikley, 1884
331 36	1,087 1	Violle, 1900
331 30	1,087 0	Hebb, 1904

Effect of Temperature.

Equation (1) shows that the velocity of sound depends only upon the ratio of the elasticity and density of the transmitting medium. This implies that the velocity in free air is independent of the pressure, since a change in pressure produces a proportional change in density, leaving the ratio of pressure to density unchanged. On the other hand, since the density of air is inversely proportional to the temperature measured on an absolute scale, it follows that the velocity will increase with rising temperature. The velocity of sound c_t at the centigrade temperature t is given by the formula

$$c_t = 331.2\sqrt{1 + \frac{t}{273}} \qquad (2a)$$

or, if temperature is expressed on the Fahrenheit scale,

$$c_t = 331.2\sqrt{1 + \frac{t - 32}{491}} \qquad (2b)$$

A simpler though only approximate formula for the velocity of sound between 0° and 20° C. is

$$c_t = 331.2 + 0.60t$$

Velocity of Sound in Water.

As an illustration of the application of the fundamental equation for the velocity of sound in a liquid medium, we may compute the velocity of sound in fresh water. The compressibility of water is defined as the change of volume per unit volume for a unit change of pressure. For water at pressures less than 25 atmospheres, the compressibility as defined above is approximately 5×10^{-11} c.c. per c.c. per dyne per sq. cm. The coefficient of elasticity as defined above is the reciprocal of this quantity or 2×10^{10}. The density of water is approximately unity, so that the velocity of sound in water is

$$c_w = \sqrt{\frac{e}{\rho}} = \sqrt{\frac{2 \times 10^{10}}{1}} = 141{,}400 \text{ cm./sec.}$$

Colladon and Sturm, in 1826, found experimentally a velocity of 1,435 m. per second in the fresh water of Lake Geneva at a temperature of 8° C. Recent work by the U. S. Coast and Geodetic Survey gives values of sound in sea water ranging from 1,445 to more than 1,500 m. per second at temperatures ranging from 0° to 22° C. for depths as great as 100 m. Here, as in the case of air, the difference between the isothermal and adiabatic compressibility tends to make the computed less than the measured theoretical value of the velocity.[1]

The velocity of sound in water is thus approximately four times as great as the velocity in air, although water has a density almost eighty times that of air. This is due to the much greater elasticity of water.

Propagation of Sound in Open Air.

Although the theory of propagation of sound in a homogeneous medium is simple, yet the application of this theory to numerous phenomena of the transmission of sound in the

[1] It is important to have a clear idea of the meaning of the term "elasticity" as defined above. In popular thinking, there is frequently encountered a confusion between the terms "elasticity" and "compressibility."

free atmosphere has proved extremely difficult. For example, if we assume a source of sound of small area set up in the open air away from all reflecting surfaces, we should expect the energy to spread in spherical waves with the source of sound as the center. At a distance r from the source, the total energy from the source passes through the surface of a sphere of radius r, a total area of $4\pi r^2$. If E is the energy generated per second at the source, then the energy passing through a unit surface of the sphere would be $E/4\pi r^2$; that is, the intensity, defined as the energy per second through a unit area of the wave front, decreases as

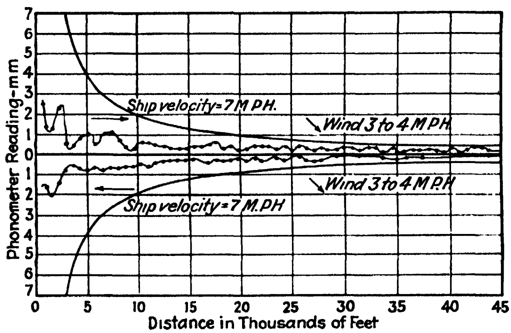


FIG. 1.—Variation of amplitude of sound in open air with distance from source. (*After L. V. King.*)

the square of the distance r increases. This is the well-known inverse-square law of variation of intensity with distance from the source, stated in all the elementary textbooks on the subject. As a matter of fact, search of the literature fails to reveal any experimental verification of this law so frequently invoked in acoustical measurements. The difficulty comes in realizing experimentally the conditions of "no reflection" and a "homogeneous medium." Out of doors, reflection from the ground disturbs the ideal

Elasticity is a measure of the ability of a substance to resist compression. In this sense, solids and liquids are more elastic because less compressible than gases.

condition, and moving air currents and temperature variations nullify our assumption of homogeneity of the medium. Indoors, reflections from walls, floor, and ceiling of the room result in a distribution of intensity in which usually there is little or no correlation between the intensity and the distance from the source.

Figure 1 is taken from a report of an investigation on the propagation of sound in free atmosphere made by Professor Louis V. King at Father Point, Quebec, in 1913.[1] A Webster phonometer was used to measure the intensity

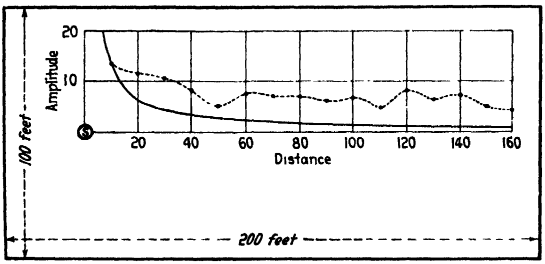


FIG. 2.—Variation of amplitude of sound in an enclosure with distance from source.

of sound at varying distances from a diaphone fog signal. The solid lines indicate what the phonometer readings should be, assuming the inverse-square law to hold. The observed readings are shown by the lighter curves. Clearly, the law does not hold under the conditions prevailing at the time of these measurements.

Indoors, the departure from the law is quite as marked. Figure 2 gives the results of measurements made in a large armory with the Webster phonometer using an organ pipe blown at constant pressure as the source. Here the heavy curve gives what the phonometer readings would have been on the assumption of an intensity decreasing as the square of the distance increases. The measured values are shown on the broken curve. There is obviously little correlation between the two. The actual intensity does not fall off with increasing distance from the source

[1] *Phil. Trans. Roy. Soc. London*, Ser. A, vol. 218, pp. 211–293.

nearly so rapidly as would be the case if the intensity were simply that of a train of spherical waves proceeding from a source; and we note that the intensity may actually increase as we go away from the source.

Acoustic Properties of Inclosures.

The measurements presented in Fig. 2 indicate that the behavior of sound within an inclosure cannot, in general, be profitably dealt with from the standpoint of progressive waves in a medium. The study of this behavior constitutes the subject matter of the first part of "Architectural Acoustics," namely, the acoustic properties of audience rooms. One may draw the obvious inference from Fig. 2 that, within an inclosed space bounded by sound-reflecting surfaces, the intensity at any point is the sum of two distinct components: (1) that due to the sound coming directly from the source, which may be considered to decrease with increasing distance from the source according to the inverse-square law; and (2) that which results from sound that has been reflected from the boundaries of the inclosure. From the practical point of view, the problem of auditorium acoustics is to provide conditions such that sound originating in one portion of the room shall be easily and naturally heard throughout the room. It follows then that the study of the subject of the acoustic properties of rooms involves an analysis of the effects that may be produced by the reflected portion of the total sound intensity upon the audibility and intelligibility of the direct portion.

Search of the literature reveals that practically no systematic scientific study of this problem was made prior to the year 1900. In Winkelmann's *Handbuch der Physik*, one entire volume of which is devoted to acoustics, only three pages are given to the acoustics of buildings, with only six references to scientific papers on the subject. On the architectural side, we find numerous references to the subject, beginning with the classic work on architecture by Vitruvius ("De Architectura"). In these references, we find, for the most part, only opinion, based on more or less superficial observation. Nowhere is there evidence either of a thoroughgoing analysis of the problem or of any attempt at its scientific solution.

In 1900, there appeared in the *American Architect* a series of articles by Wallace C. Sabine—at that time an instructor in physics at Harvard University—giving an analysis of the conditions necessary to secure good hearing in an auditorium. This was the first study ever made of the problem by scientific methods. The state of knowledge on the subject at that time can best be shown by quoting an introductory paragraph from the first of these papers.[1]

No one can appreciate the condition of architectural acoustics—the science of sound as applied to buildings—who has not with a pressing case in hand sought through the scattered literature for some safe guidance. Responsibility in a large and irretrievable expenditure of money compels a careful consideration and emphasizes the meagerness and inconsistency of the current suggestions. Thus the most definite and often-repeated statements are such as the following: that the dimensions of a room should be in the ratio 2:3:5 or, according to some writers, 1:1:2, and others, 2:3:4. It is probable that the basis of these suggestions is the ratios of the harmonic intervals in music, but the connection is untraced and remote. Moreover, such advice is rather difficult to apply; should one measure the length to the back or to the front of the galleries, to the back or front of the stage recess? Few rooms have a flat roof—where should the height be measured? One writer, who had seen the Mormon Temple, recommended that all auditoriums be elliptical. Sanders Theater is by far the best auditorium in Cambridge and is semicircular in general shape but with a recess that makes it almost anything; and, on the other hand, the lecture room in the Fogg Art Museum is also semicircular, indeed was modeled after Sanders Theater, and it was the worst. But Sanders Theater is in wood and the Fogg lecture room is plaster on tile; one seizes on this only to be immediately reminded that Sayles Hall in Providence is largely lined with wood and is bad. Curiously enough, each suggestion is advanced as if it alone were sufficient. As examples of remedies may be cited the placing of vases about the room for the sake of resonance, wrongly supposed to have been the object of the vases in Greek theaters, and the stretching of wires, even now a frequent though useless device.

[1] Sabine, Wallace C., "Collected Papers on Acoustics," Harvard University Press, p. 1. Cambridge 1922.

In a succeeding paragraph, Sabine states very succinctly the necessary and sufficient conditions for securing good hearing conditions in any room. He says:

> In order that hearing may be good in any auditorium, it is necessary that the sound should be sufficiently loud; that the simultaneous components of a complex sound should maintain their proper relative intensities; and that the successive sounds in rapidly moving articulation, either of speech or music, should be clear and distinct, free from each other and from extraneous noises. These three are the necessary, as they are the entirely sufficient, conditions for good hearing. The architectural problem is, correspondingly, threefold, and in this introductory paper an attempt will be made to sketch and define briefly the subject on this basis of classification. Within the three fields thus defined is comprised without exception the whole of acoustics.

Very clearly, Sabine puts the problem of securing good acoustics largely as a matter of eliminating causes of acoustical difficulties rather than as one of improving hearing conditions by positive devices. Increasing knowledge gained by quantitative observations and experiment during the thirty years since the above paragraph was written confirms the correctness of this point of view. It is to be said that during the twenty-five years which Sabine himself devoted to this subject, his investigations were directed along the lines here suggested. Most of the work of others since his time has been guided by his pioneer work in this field. In the succeeding chapters, we shall undertake to present the subject of sound in buildings from his point of view, including only so much of acoustics in general as is necessary to a clear understanding of the problems in the special field.

CHAPTER II

NATURE AND PROPERTIES OF SOUND

We may define sound either as the sensation produced by the stimulation of the auditory nerve, or we may define it as the physical cause of that stimulus. For the present purpose, we shall adopt the latter and define sound as the undulatory movement of the air or of any other elastic medium, a movement which, acting upon the auditory mechanism, is capable of producing the sensation of hearing. An undulatory or wave motion of a medium consists of the rapid to-and-fro movement of the individual particles, this motion being transmitted at a definite speed which is determined by the ratio of the elasticity to the density of the medium.

Simple Harmonic Motion.

For a clear understanding of the origin and propagation of vibrational energy through a medium, let us consider in detail, though in an elementary way, the ideal simple case of the transfer of a simple harmonic motion (S.H.M.) as a plane wave in a medium. Simple harmonic motion may be defined as the projection of uniform circular motion upon a diameter of the circular path.

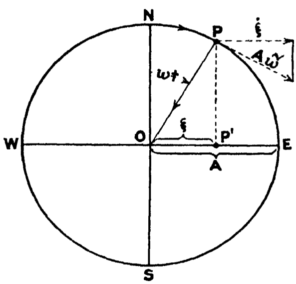


FIG. 3.—Relation of simple harmonic motion to uniform circular motion.

Thus if the particle P (Fig. 3) is moving with a constant speed upon the circumference of a circle of radius A, and P' is moving upon a horizontal diameter so that the vertical line through P always passes through P', then the motion of P' is simple harmonic motion.

If the angular velocity of P is ω radians per second, and time is measured from the instant that P passes through N, and P', moving to the right, passes through O, then the displacement ξ of P' is given by the equation

$$\xi = A \sin \omega t \tag{3}$$

The position of P' as well as its direction of motion is given by the value of the angle ωt. This angle is called the phase angle and is a measure of the phase of the motion of P'. Thus when the phase angle is 90 deg., $\pi/2$ radians, P' is at the point of maximum excursion to the right. When the phase angle is 180 deg., π radians, P' is in its undisplaced position, moving to the left. When the phase is 360 deg., or 2π radians, P' is again in its undisplaced position moving to the right. The instantaneous velocity of P' is the component of the velocity of P parallel to the motion of P' or, as is easily seen from the figure,

$$\xi = A\omega \cos \omega t = A\omega \sin \left(\omega t + \frac{\pi}{2}\right) \tag{4}$$

In simple harmonic motion, the velocity is 90 deg. in phase in advance of the displacement.

Now the acceleration of the particle P moving with a uniform speed s, on the circumference of a circle of radius A, is $s^2/A = (A\omega)^2/A = A\omega^2$. Since the tangential speed is constant, this acceleration must be always toward the center of the circle. The acceleration of the particle P' is the horizontal component of the acceleration of P, namely, $-A\omega^2 \sin \omega t$. Let m be the mass of the particle P', $\ddot{\xi}$ its acceleration, and F_x the force which produces its motion. Then by the second law of motion

$$F_x = m\ddot{\xi} = -mA\omega^2 \sin \omega t = mA\omega^2 \sin (\omega t + \pi) \tag{5}$$

Comparing Eqs. (3) and (5), it appears that the force is directly proportional to the displacement but of opposite sign. Thus if P' is displaced toward the right, it is acted upon by a force toward the left, which increases as the displacement increases. The force F_x always acts to restore the particle to its undisplaced position, and its magnitude is directly proportional to the displacement.

Now the restoring forces called into play when any elastic body is subjected to strain are of just this type. Therefore, it follows that a particle moving under the action of an elastic restoring force will perform a simple harmonic motion. Further, it can be shown that the free movement of any body under the action of elastic forces only can be expressed as the resultant of a series of simple harmonic motions.

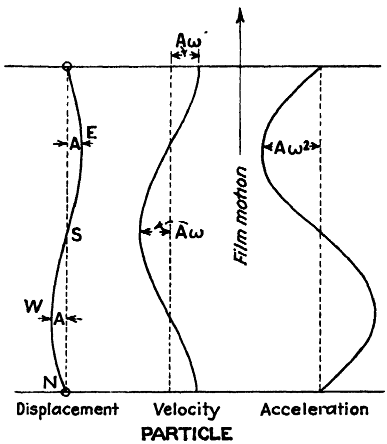


Fig. 4.—Simple harmonic motion projected on a uniformly moving film.

Suppose now that the motion of P' is recorded on a film moving with uniform speed at right angles to the vibration. The trace of the motion will be a sine curve. For this reason simple harmonic motion is spoken of as sinusoidal motion. If at the same time we could devise mechanisms that would record particle velocity and particle acceleration, the traces of these magnitudes on the moving film would be shown as in Fig. 4.

The maximum excursion on one side of the undisplaced position, the distance A, is the amplitude of the vibration. The number of complete to-and-fro excursions per second is the frequency f of vibration. Since one complete to-and-fro movement of P' occurs for each complete rotation of P, corresponding to 2π radians of angular motion, then

$$\omega = 2\pi f$$

Energy of Simple Harmonic Motion.

It is evident that, in the ideal case we have assumed, the kinetic energy of the particle P' is a maximum at O, the position of zero displacement and maximum velocity. Here the velocity is the same as that of P, namely ωA, and the kinetic energy is

$$U_{\text{max.}} = \tfrac{1}{2}m\omega^2 A^2 = 2\pi^2 m f^2 A^2 \qquad (6)$$

At the maximum excursion, when the particle is momentarily stationary, the kinetic energy is zero. The total energy here is potential. At intermediate points the sum of the potential and kinetic energies is constant and equal to $\tfrac{1}{2}m\omega^2 A^2$. The average kinetic as well as the average potential energy throughout the cycle is one-half the maximum or $\tfrac{1}{4}m\omega^2 A^2$ or $\pi m f^2 A^2$.[1] The total energy, kinetic and potential, of a vibrating particle equals $\tfrac{1}{2}m\omega^2 A^2$ or $2\pi^2 m f^2 A^2$.

Wave Motion.

Having considered the motion of the individual particle, let us trace the propagation of this motion from particle to particle as a plane wave, that is, a wave in which all particles of the medium in any given plane at right angles to the direction of propagation have the same phase. Let the wave be generated by the rapid to-and-fro movement of the piston moving in parallel ways and driven with simple harmonic motion by the "disk-pin-and-slot" mechanism indicated in Fig. 5*a*. The horizontal component of the uniform circular motion of the disk is transmitted to the piston by the pin, which slides up and down in the vertical slot in the piston head as the disk revolves. We may consider that the disturbance is kept from spreading

[1] The mathematical proof of this statement is not difficult. The interested reader with an elementary knowledge of the calculus may easily derive the proof for himself.

laterally by having the propagating medium confined in a tube so long that we need not consider what happens at the open end. Represent the undisturbed condition of the air by 41 equally spaced particles. Let the distance from 0 to 40 be the distance the disturbance travels during a complete vibration of a single particle.[1] This distance is

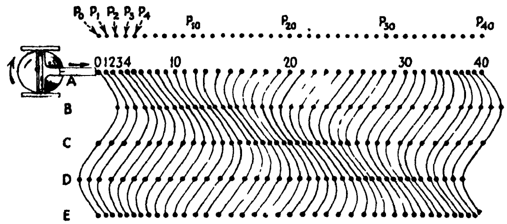


FIG. 5a.

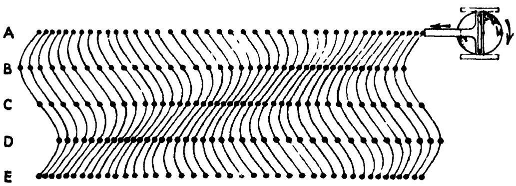


FIG. 5b.

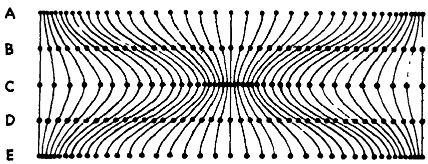


FIG. 6.

FIG. 5a.—Compressional plane wave moving to the right.
FIG. 5b.—Compressional plane wave reflected to the left.
FIG. 6.—Stationary wave resultant of Figs. 5a and 5b.

called the wave length of the wave motion and is denoted by the Greek letter λ.

The line *A* shows the positions of each of the particles at the instant that P_0, the first particle, having made a complete vibration, is in its equilibrium position and is

[1] According to the conception of the kinetic theory of gases, the molecules of a gas are in a state of thermal agitation, and the pressure which the gas exerts is due to this random motion of its molecules. Here for the purpose of our illustration we shall consider stationary molecules held in place by elastic restraints.

moving to the right. The first member of the family of sine curves shown is the trace on a moving film of the motion of P_0. P_1 having performed only thirty-nine-fortieths of a vibration is, at this instant, displaced slightly to the left. Curve 1 represents its motion during the interval that P_0 performs the motion shown by curve *O*. The phase difference between the motions of two adjacent particles is $2\pi/40$ radians, or 9 deg. P_{20} is 180 deg. in phase behind P_0 and is in its undisplaced position and moving to the left. P_{40} is 2π radians or 360 deg. behind P_0, and the motions of the two particles coincide, P_0 having performed one more complete vibration than P_{40}.

Lines *B*, *C*, *D*, *E* (Fig. 5*a*) give the positions of the 41 particles ¼, ½, ¾, and 1 period respectively later than those shown at *A*.

Types of Wave Motion.

It will be noted that the motion of the particles is in the line of propagation of the disturbance. This type of motion is called a compressional wave, and, as appears, such a wave consists of alternate condensations and rarefactions of the medium. This is the only kind of wave that can be propagated through a gas. In *solids*, the particle motion may be at right angles to the direction of travel, and the wave would consist of alternate crests and troughs and is spoken of as a transverse wave. As a matter of fact, in general when any portion of a solid is disturbed, both compressional and transverse waves result and the motion becomes extremely complicated. A wave traveling through the body of a liquid is of the compressional type. At the free surface of a *liquid*, waves occur in which the particles move in closed loops under the combined action of gravity and surface tension.

Equation of Wave Motion.

Equation (3) gives the displacement of a vibrating particle in terms of the time, measured from the instant the particle passed through its undisplaced position, and of the amplitude and frequency of vibration. The equation of a wave gives the displacement of any particle in terms of the time and the coordinates that determine the undisplaced position of the particle. In the case of a plane wave, since all the particles in a given plane perpendicular to the direction of propagation have the same phase, the distance of this plane from the origin is sufficient to fix the phase of the particle's motion relative to that of a particle located at the origin. Call this distance x.

In Fig. 5a, consider the motion of a particle at a distance x from the origin P_0. Let c be the velocity with which the disturbance travels, or the velocity of sound. Then the time required for the motion at P_0 to be transmitted the distance x is x/c. The particle at x will repeat the motion of that at P_0, x/c sec. later. The equation therefore for the particle at x, referred to the time when P_0 is in its neutral position, is

$$\xi = A \sin \omega\left(t - \frac{x}{c}\right) = A \sin 2\pi f\left(t - \frac{x}{c}\right) \qquad (7a)$$

Similarly the velocity of the particle at x is

$$\dot{\xi} = A\omega \cos \omega\left(t - \frac{x}{c}\right) = 2\pi A f \cos 2\pi f\left(t - \frac{x}{c}\right) \qquad (7b)$$

and the acceleration

$$\ddot{\xi} = -A\omega^2 \sin \omega\left(t - \frac{x}{c}\right) = -4\pi^2 A f^2 \sin 2\pi f\left(t - \frac{x}{c}\right) \qquad (7c)$$

Equation (7a) is the equation of a plane compressional wave of simple-harmonic type traveling to the right, and from it the displacement of any particle at any time may be deduced. In the present instance we have assumed a simple harmonic motion of the particle at the origin. We might have given this particle any complicated motion whatsoever, and, in an elastic medium, this motion would be transmitted, so that each particle would perform this same motion with a phase retardation of $\omega x/c$ radians. In other words, the reasoning given applies to the general case of a disturbance of any type set up in an elastic medium in which the velocity of propagation is independent of the frequency of vibration.

Wave Length and Frequency.

Now the wave length of the sound has been defined as the distance the disturbance travels during a complete vibration of a single particle, for example, the distance P_0 to P_{40} (Fig. 5*a*). The phase difference between two particles one wave length apart is 2π radians. Letting $x = \lambda$, we have

$$2\pi f \frac{\lambda}{c} = 2\pi$$

$$f\lambda = c \tag{8}$$

This important relationship makes it possible to compute the frequency of vibration, the wave length, or the velocity of sound if the two other quantities are known. Since c, the velocity of sound in air at any temperature, is known, Eq. (8) gives the wave length of sound of any given frequency. Table I of Appendix A gives the frequencies and wave lengths in air at 20° C. (68° F.) of the tones of one octave of the tempered and physical scales.

The frequencies and wave lengths given are for the first octave above middle C (C_3). To obtain the frequencies of tones in the second octave above middle C we should multiply the frequencies given in the table by 2. In the octave above this we should multiply by 4, and in the next octave by 8, etc. For the octaves below middle C we should divide by 2, 4, 8, etc.

Density and Pressure Changes in a Compressional Wave.

In the preceding paragraphs, we have followed the progressive change in the motion of the individual particles. Figure 5*a* also indicates the changes that occur in the configuration of the particles with reference to each other. Initially, there is a crowding together of the particles at P_0, with a corresponding separation at P_{20}. One-quarter of a period later, the maximum condensation is at P_{10}, and the rarefaction is at P_{30}. At the end of a complete period, there is again a condensation at P_0. A wave length therefore as defined above includes one complete condensation and one rarefaction of the medium.

The condensation, denoted by the letter s, is defined as the ratio of the increment of density to the undisturbed density:

$$s = \frac{\delta\rho}{\rho}$$

Thus if the density of the undisturbed air is 1.293 g: per liter and that in the condensation phase of a particular sound wave is 1.294 g. per liter, the maximum condensation is 1/1,293. It is apparent that a condensation results from the fact that the displacement of each particle at any instant is slightly different from that of an adjacent particle. Referring once more to Fig. 5*a*, one sees that if all the particles were displaced by equal amounts at the same time there would be no variation in the spacing of the particles, that is, no variation in the density. It can be easily shown that in a plane wave the condensation is equal to minus the rate per unit distance at which the displacement varies from particle to particle. Expressed mathematically,

$$s = \frac{\delta\rho}{\rho} = -\frac{\partial\xi}{\partial x} \tag{9a}$$

Differentiating Eq. (7*a*) with regard to x, we have

$$s = \frac{2\pi Af}{c}\cos 2\pi f\left(t - \frac{x}{c}\right) = \frac{\dot{\xi}}{c} \tag{9b}$$

We thus arrive at the interesting relation that the condensation in a progressive wave is equal to the ratio of the particle velocity to the wave velocity. Further, it appears that the condensation is in phase with the velocity and $\pi/2$ radians in advance of the particle displacement.

At constant temperature, the density of a gas is directly proportional to the pressure. As was indicated in Chap. I, for the rapid alternations of pressure in a sound wave, the temperature rises in compression and the pressure increases more rapidly than the density, so that the fractional change in pressure equals γ (1.40) times the fractional change in density. Whence we have

$$\frac{\delta P}{P} = \frac{\gamma \delta \rho}{\rho} = \gamma \frac{\dot{\xi}}{c} \tag{10}$$

The maximum pressure increment, which may be called the pressure amplitude, is therefore $2\pi\gamma PAf/c$.

Energy in a Compressional Wave.

We have seen that the total energy, kinetic and potential, of a particle of mass m vibrating with a frequency f and amplitude A is $2\pi^2 mA^2f^2$. If there are N of these particles per cubic centimeter, the total energy in a cubic centimeter is $2Nm\pi^2A^2f^2$. The product Nm is the weight per cubic centimeter of the medium, or the density. The total energy per cubic centimeter is, therefore, $2\pi^2\rho f^2A^2$, of which, on the average, half is potential and half kinetic.

The term "intensity of sound" may be used in two ways: either as the energy per unit volume of the medium or as the energy transmitted per second through a unit section perpendicular to the direction of propagation. The former would more properly be spoken of as the "energy density," and the latter as the "energy flux." We shall denote the energy density by symbol I and the energy flux by symbol J. If the energy is being transmitted with a velocity of c cm. per second, then the energy passing in 1 sec. through 1 sq. cm. is c times the energy per cubic centimeter, or

$$J = cI = 2\pi^2\rho A^2f^2c \tag{11}$$

Equation (11) and the expression for the pressure amplitude given above may be combined to give a simple relationship between pressure change and flux intensity.

$$\frac{\delta P^2_{\text{max.}}}{J} = \frac{4\pi^2\gamma^2P^2A^2f^2}{2\pi^2\rho A^2f^2c^3} = \frac{2\gamma^2P^2}{\rho c^3}$$

Using the relationship $c = \sqrt{\gamma P/\rho}$, we have

$$J = \frac{1}{2}\frac{\delta P^2_{\text{max}}}{\rho c} = \frac{1}{2}\frac{\delta P^2_{\text{max.}}}{\sqrt{\gamma P\rho}} \tag{12}$$

Now it can be shown that the average value of the square of δP over one complete period is one-half the square of its maximum value. Hence, if we denote the square root of the mean square value of the pressure increment by p, Eq. (12) may be put in the very simple form

$$J = \frac{p^2}{\rho c} = \frac{p^2}{\sqrt{\gamma P \rho}} = \frac{p^2}{\sqrt{\epsilon\rho}} = \frac{p^2}{r} \tag{13}$$

in which

$$r = \sqrt{\epsilon\rho} \text{ and } \epsilon = \gamma P.$$

The expression $\sqrt{\epsilon\rho}$ has been called the "acoustic resistance" of the medium. Table II of Appendix A gives the values of c and r for various media.

Comparison of Eq. (13) with the familiar expression for the power expended in an electric circuit suggests the reason for calling the expression r, the acoustic resistance of a medium. It will be recalled that the power W expended in a circuit whose electrical resistance is R is given by the expression

$$W = \frac{E^2}{R}$$

where E is the electromotive force (e.m.f.) applied to the circuit. In the analogy between the electrical transfer of power and the passage of acoustical energy through a medium E, the e.m.f. corresponds to the effective pressure increment, and the electrical resistance to r, the "acoustic resistance" of the medium. The analogue of the electrical current is the root-mean-square (r.m.s.) value of the particle velocity.

The mathematical treatment of acoustical problems from the standpoint of the analogous electrical case is largely due to Professor A. G. Webster,[1] who introduced the term "acoustical impedance" to include both the resistance and reactance of a body of air in his study of the behavior of horns. For an extension of the idea and its application to various problems, the reader is referred to Chap. IV

[1] *Proc. Nat. Acad. Sci.*, vol. 5, p. 275, 1919.

of Crandall's "Theory of Vibrating Systems and Sound" and to the recently published "Acoustics" by Stewart and Lindsay.

Equation (13) gives flux intensity in terms of the r.-m.-s. pressure change and the physical constants of the medium only. It will be noted that it does not involve the frequency of vibration. This leads to the very important fact that if we have an instrument that will measure the pressure changes, the flux intensities of sounds of different frequencies may be compared directly from the readings of such an instrument. For this reason, instruments which record the pressure changes in sound waves are to be preferred to those giving the amplitude.

Temperature Changes in a Sound Wave.

We have seen that to account for the measured velocity of sound, it is necessary to assume that the pressure and density changes in the air take place adiabatically, that is, without transfer of heat from one portion of the medium to another. This means that at any point in a sound wave there is a periodic variation of temperature, a slight rise above the normal when a condensation is at the point in question, and a corresponding fall in the rarefaction. The relation between the temperature and the pressure in an adiabatic change is given by the relation

$$\frac{P + \delta P}{P} = \left[\frac{\theta + \delta\theta}{\theta}\right]^{\frac{\gamma}{\gamma - 1}} \tag{14}$$

where γ is the ratio of the specific heats of the gas, equal to 1.40, and θ is the temperature on the absolute scale.

Now $\delta\theta/\theta$ will in any case be a very small quantity, and the numerical value of $\frac{\gamma}{\gamma - 1}$ is 3.44. Expanding the second member of (14) by the binomial theorem and neglecting the higher powers of $\delta\theta/\theta$ we have

$$\frac{\delta P}{P} = 3.44\frac{d\theta}{\theta} \tag{15}$$

Obviously the temperature fluctuations in a sound wave are extremely small—too small, in fact, to be measurable; but the fact of thermal changes is of importance when we come to consider the mechanism of absorption of sound by porous bodies.

Numerical Values.

The qualitative relationships between the various phenomena that constitute a sound wave having been dealt with, it is next of interest to consider the order of magnitude of the quantities with which we are concerned. All values must of course be expressed in absolute units. For this purpose we shall start with the pressure changes in a sound of moderate intensity. At the middle of the musical range, 512 vibs./sec., the r.m.s. pressure increment of a sound of comfortable loudness would be of the order of 5 bars (dynes per square centimeter), approximately five-millionths of an atmosphere. From Eq. (13) the flux intensity J for this pressure is found to be $p^2 \div 41.5 = 0.6$ erg per second per square centimeter or 0.06 microwatt per square centimeter. From the relation that $J = \frac{1}{2}r\dot{\xi}^2_{\text{max.}}$ we can compute the maximum particle velocity, which is found to be 0.17 cm. per second. This maximum velocity is $2\pi fA$, and the amplitude of vibration is therefore 0.000053 cm. A moment's consideration of the minuteness of the physical quantities involved in the phenomena of sound suggests the difficulties that are to be encountered in the direct experimental determination of these quantities and why precise direct acoustical measurements are so difficult to make. As a matter of fact, it has been only since the development of the vacuum tube as a means of amplifying very minute electrical currents that quantitative work on many of the problems of sound has been possible.

Complex Sounds.

In the preceding sections we have dealt with the case of sound generated by a body vibrating with simple harmonic motion. The tone produced by such a source is known as *pure tone*, and the most familiar example is that produced by a tuning fork mounted upon a resonator. The phonographic record of such a tone would be a sinusoidal curve, as pictured in Fig. 3. If we examine records of ordinary musical sounds or speech, we shall find that instead of the simple sinusoidal curves produced by pure tones, the traces are periodic, but the form is in general extremely complicated. Figure 7 is an oscillograph record of the sound of a vibrating piano string, and it will be noted that it consists of a repetition of a single pattern. The movement of the air particles that produces this record is obviously not the simple harmonic motion considered above.

However, it is possible to give a mathematical expression to a curve of this character by means of Fourier's theorem.[1]

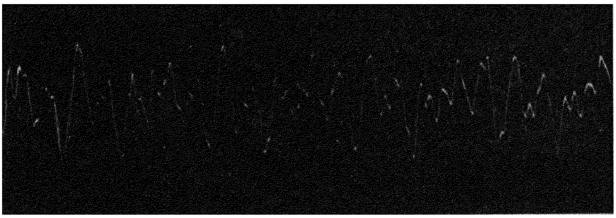


FIG. 7.—Oscillogram of sound from a piano string. (*Courtesy of William Braid White.*)

In general, musical tones are produced by the vibrations either of strings, as in the piano and the violin, or by those of air columns, as in the organ and orchestral wind instruments. When a string or air column is excited so as to produce sound, it will vibrate as a whole and also in segments which are aliquot parts of the whole. The vibration as a whole produces the lowest or fundamental tone, and the partial vibrations give a series of tones whose frequencies are integral numbers 2, 3, 4, etc., times this fundamental frequency. The motion of the air particles in the sound thus produced will be complex, and the gist of Fourier's theorem is that such a complex motion may be accurately expressed by a series of sine (or cosine) terms of suitable

[1] FOURIER, J. B. J., "La Théorie Analytique de la Chaleur," Paris, 1822; CARSE and SHEARER, "Fourier's Analysis," London, 1915.

amplitude and phase. Thus the displacement of a point on the vibrating body at any time t is given by an equation of the form

$$\xi = A_1 \sin(\omega t + \varphi_1) + A_2 \sin(2\omega t + \varphi_2) + A_3 \sin(3\omega t + \varphi_3) +, \text{etc.}$$

That property of musical sounds which makes the difference between the sounds of two different musical instruments producing tones of the same pitch is called "quality" or "timbre," and its physical basis lies in the relative amplitudes of the simple harmonic components of the two complex sounds.

Harmonic Analysis and Synthesis.

The harmonic analysis of a periodic curve consists in the determination of the amplitude and phase of each member of the series of simple harmonic components. This may be done mathematically, but the process is tedious. Various machines have been devised for the purpose, the best known being that of Henrici, in 1894, based on the rolling sphere integrator. This and other mechanical devices are described in full in Professor Dayton C. Miller's book, "The Science of Musical Sounds,"[1] to which the reader is referred for further details.

The reverse process, of drawing a periodic curve from its harmonic components, is called "harmonic synthesis." A machine for this purpose consists essentially of a series of elements each of which will describe a simple harmonic motion and means by which these motions may be combined into a single resultant motion which is recorded graphically. Perhaps the simplest mechanical means of producing S.H.M. is that indicated in Fig. 5, in which a pin carried off center by a revolving disk imparts the component of its motion in a single direction to a member free to move back and forth in this direction and in no other.

In his book, Professor Miller describes and illustrates numerous mechanical devices for harmonic analysis and

[1] The Macmillan Company, 1916.

synthesis including the 30 element synthesizer of his own construction. In Fig. 8 are shown the essential features of

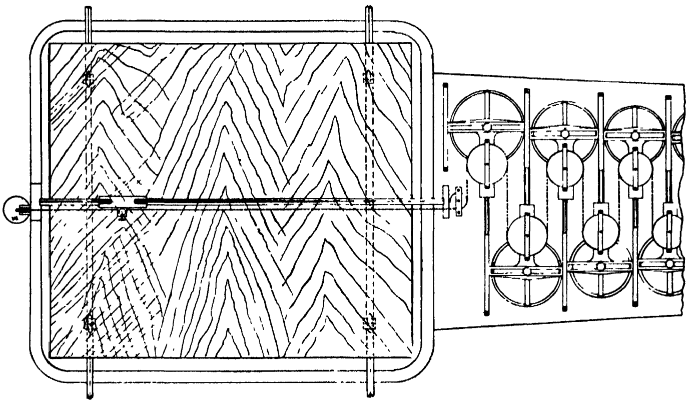


FIG. 8a.—Disc, pin and slot, and chain mechanism of the Riverbank harmonic synthesizer.

the 40-element machine built by Mr. B. E. Eisenhour of the Riverbank Laboratories.

The rotating disks are driven by a common driving shaft, carrying a series of 40 helical gears. Each driving gear

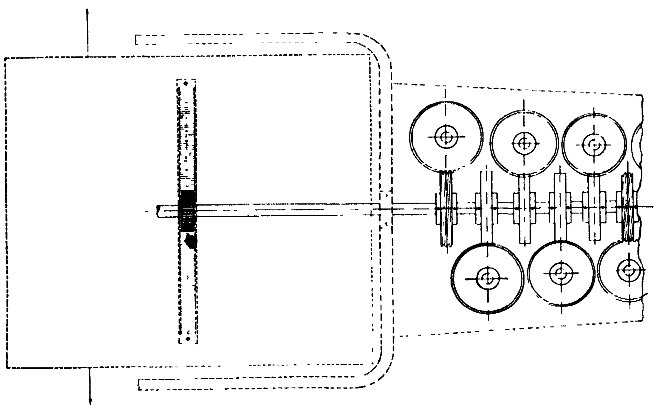


FIG. 8b.—Helical gear drive of the Riverbank synthesizer

engages a gear mounted on a vertical shaft on which is also mounted one of the rotating disks. The gear ratio for the first element is 40:1, while that for the fortieth is 1:1, so that for 40 rotations of the main shaft the disks revolve 1, 2, 3, 4 . . . 40 times respectively. Each disk carries a pin which moves back and forth in a slot cut in a sliding member free to move in parallel ways. Each of these sliding members carries a nicely mounted pulley. The amplitude of motion of each sliding element can be adjusted by the amount to which the driving pin is set off center, and this is measured to 0.01 cm. by means of a scale and vernier engraved on the surface of the disk. The phase of the starting position of each disk is indicated on a circular

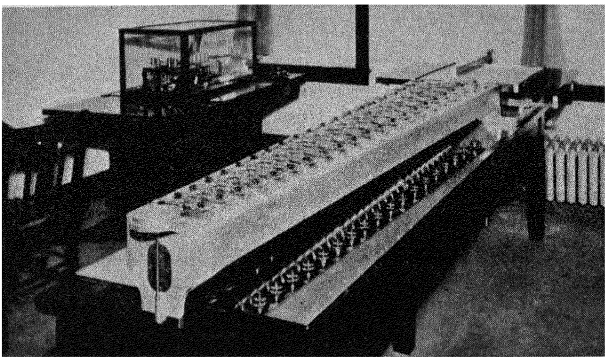


FIG. 9.—Forty-element harmonic synthesizer of the Riverbank Laboratories.

scale engraved on the periphery by reference to a fixed line on the instrument. It is thus possible to set to any desired values both the amplitude and the phase of each of the 40 sliding elements.

The combined motion of all the sliding members is transferred to the recording pen, by means of an inextensible chronometer fusee chain, threaded back and forth around the pulleys and attached to the pen carriage. The back-and-forth motion of each element thus transmits to the pen an amplitude equal to twice that of the element. The pen motion is recorded on the paper mounted on a traveling table which is driven at right angles to the motion of the pen carriage by the main shaft by means of a rack-andpinion arrangement. The chain is kept tight by means of weights suspended over pulleys at each end of the synthesizer. An ingenious arrangement allows continuous adjustment of the length of table travel for 40 revolutions of the main shaft over a range of from 10 to 80 cm., so that a wave of any length within these limits may be drawn.

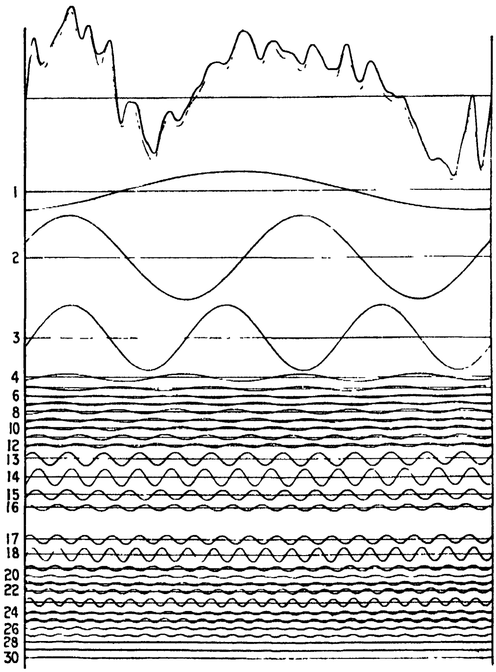


FIG. 10.—Analysis and re-synthesis of 30 elements of complex sound wave by harmonic synthesizer. Re-synthesis of 40 elements gives original curve.

With this instrument we are able to draw mechanically any wave form, which may be expressed by an equation of the form

$$\xi = \sum_{k=1}^{k=40} A_k \sin (k\omega t + \varphi_k)$$

Moreover, the instrument may be used as an analyzer to determine the amplitudes and phases of a Fourier series of

40 terms that will be an approximate representation of any given wave form as shown by Dr. F. W. Kranz.[1]

Figure 10 shows the simple harmonic components of an oscillograph record of the vowel sound $\bar{o}$, spoken by a masculine voice, as determined by Kranz's method of analysis. A resynthesis of these components reproduces the original wave form with an exactness that leaves no doubt as to the reliability of the instrument or the correctness of the analysis. The method thus affords a means of studying and expressing quantitatively the quality or *timbre* of musical sounds.

Properties of Musical Sounds.

A musical sound as distinguished from noise is characterized by having a fairly definite pitch and quality, sustained for an appreciable length of time. Pitch is expressed quantitatively by specifying the lowest frequency of vibrations of the sounding body. Quality is expressed by giving the relative amplitudes or intensities of the simple harmonic components into which the complex tone may be analyzed. In the recorded motion of the sounding body or of the air itself, the length of the wave is the criterion of the pitch of the sound, while the shape of the wave determines the quality. The record of a musical sound consists of a definite pattern repeated at regular intervals.

Noise is sound without definite pitch characteristics and an indeterminate quality. On the physical side, the line of distinction between musical sounds and noises is not sharp. Many sounds ordinarily classified as noises will upon careful examination be found to have fairly definite pitches. Thus striking a block of wood with a hammer would commonly be said to make a noise. But if we assemble a series of blocks of wood of the proper lengths, we find that the tones of the musical scale can be produced by striking them, and we have the xylophone, though whether the xylophone is really a musical instrument is perhaps a matter of opinion.

[1] *Jour. Franklin Inst.*, pp. 245–262, August, 1927.

Speech sounds are a mixture of musical sounds and noises. In the vowels, the musical characteristics predominate, although both pitch and quality vary rapidly. The consonant sounds are noises that begin and terminate the sounds of the vowels. In singing and intoning, the pitch of each vowel is sustained for a considerable length of time, and only the definite pitches of the musical scale are produced—that is, in good singing.

For the most recent and complete treatment of this subject the reader should consult Dr. Harvey Fletcher's book on "Speech and Hearing" and Sir Richard Paget's "Human Speech."

Summary.

Starting with the case of a body performing simple harmonic motion, we have considered the propagation of this motion as a plane wave in an elastic medium. To visualize the physical changes that take place when a sound wave travels through the air, we fix our attention on a single small region in the transmitting medium. We see each particle performing a to-and-fro motion through its undisplaced position similar to that of the sounding body. Accompanying this periodic motion is a corresponding change in its distance from adjacent particles, resulting in changes in the density of the medium and consequently a pressure which oscillates about the normal pressure. Accompanying these pressure changes are corresponding slight changes in the temperature.

Viewing the progress of the plane wave through the air, we see all the foregoing changes advancing from particle to particle with a velocity equal to the square root of the ratio of the elasticity to the density. In time, each set of conditions is repeated at any point in the medium once in each cycle. In space, the conditions prevailing at any instant at a given point are duplicated at points distant from it by any integral number of wave lengths.

We have also seen that any complex periodic motion may be closely approximated by a series of simple harmonic motions whose relative frequencies are in the order of 1, 2, 3, 4, etc., and whose amplitudes and phase may be determined by a Fourier analysis of the curve showing the complex motion. The derivation of the relationship for a single S.H.M. may be considered as applying to the separate components of the complex sound. In other words, the propagation of a disturbance of any type in a medium in which the velocity is independent of the frequency will take place as in the simple harmonic case treated. The assumption of a plane wave, while simplifying the mathematical treatment, does not alter the physical picture. For a more general and a more rigorous mathematical treatment, standard treatises such as Lord Rayleigh's "Theory of Sound" or Lamb's "Dynamic Theory of Sound" should be consulted. Among recent texts, "Vibrating Systems and Sound," by Crandall; "A Textbook on Sound," by A. B. Wood; "Acoustics," by Stewart and Lindsay, will be found helpful.

CHAPTER III

SUSTAINED SOUND IN AN INCLOSURE

In the preceding chapter, we have considered the phenomena occurring in a progressive plane wave, that is, a wave in which any particle of the medium repeats the movement of any other particle with a time lag between the two motions of x/c. Within an inclosure, sound reflection occurs at the bounding surfaces, so that the motion of any particle in the inclosure is the resultant of the motion due to the direct wave and to waves that have suffered one or more reflections. The three-dimensional case is complicated, and the general mathematical solution of the problem of the distribution of pressures and velocities throughout the space has not yet been effected. This distribution within a room is called the "sound pattern" or the "interference pattern," and the variation of intensity from point to point within rooms with reflecting walls is one of the chief sources of difficulty in making indoor acoustical measurements.

Stationary Wave in a Tube.

We may clarify our ideas as to the general sort of thing taking place with sound in an inclosed space by a detailed elementary study of the one-dimensional case of a plane wave within a tube closed at one end by a perfectly reflecting surface, that is, a surface at which none of the energy of the wave is dissipated or transmitted to the stopping barrier in the process of reflection. This condition is equivalent to saying that there is no motion of the barrier and correspondingly no motion of the air molecules directly adjacent to it.

By Newton's third law of motion, the force of the reaction of the reflecting surface is exactly equal in magnitude and opposite in direction to the force under which the vibrating air particle adjacent to it moves. In other words, the reflected motion is the same as would be imparted to stationary particles by a simple harmonic motion generator 180 deg. out of phase with the motion in the direct wave. This reaction generator is indicated in Fig. 5*b*. Its motion is given by the equation

$$\xi = -A \sin \omega t \tag{16}$$

Considered alone, this motion gives rise to a reflected wave, and the equation for the displacement in the reflected wave is

$$\xi = -A \sin \omega\left(t + \frac{x}{c}\right) \tag{17}$$

It is to be noted that the sign of x/c is positive, since the reflected wave is advancing in the opposite direction from that in which x increases. The progress of the reflected wave considered alone is shown in Fig. 5*b*.

The resultant particle displacement due to the superposition of both the direct and the reflected waves is

$$\xi_{d+r} = A\left[\sin \omega\left(t - \frac{x}{c}\right) - \sin \omega\left(t + \frac{x}{c}\right)\right] = -2A\left[\cos \omega t \sin \frac{\omega x}{c}\right] \tag{18}$$

In Fig. 6, the particle positions at quarter-period intervals are shown, each displacement being the resultant of the two displacements due to the direct and reflected waves shown in Figs. 5*a* and 5*b* respectively. One notes immediately that particles P_0, P_{20}, and P_{40} remain stationary throughout the whole cycle, while particles P_{10} and P_{30} have a maximum amplitude of $2A$. The first are the nodes of the "stationary wave" spaced at intervals of half a wave length. The second are the antinodes. At the nodes, there is a maximum variation with time in the condensation and hence in the pressure; while at the antinodes, there is no variation in the pressure. (Note that the distance between particles 9 and 11 is constant.) In a stationary wave, the nodes are points of no motion and maximum pressure variation; while at the internodes, there is maximum motion and zero pressure change. It is to be observed further that within the half wave length between the nodes, all the particles move together, while corresponding particles on opposite sides of a node are at any instant equally displaced but moving in opposite directions.

It is easy to see, both from the concept of the stationary wave as the resultant of two waves of equal amplitudes moving in opposite directions and also from the fact that all the particles between nodes move together, that there is no transfer of energy from particle to particle in either direction, so that the energy flux in a stationary wave is zero.

We may derive all these facts from consideration of Eq. 18. Thus for a given value of t, the displacement varies as the sine of $\omega x/c$, being a maximum at the points for which $\omega x/c = 2\pi x/\lambda = \pi/2, 3\pi/2, 5\pi/2$, etc., that is, at points for which $x = \lambda/4, 3\lambda/4, 5\lambda/4$, etc. The displacement is always zero for all points at which $\sin \omega x/c = 0$, that is, at values of $x = 0, \lambda/2, \lambda, 3\lambda/2$, etc.

The particle velocity is obtained by differentiating Eq. (18) with respect to the time

$$\dot{\xi} = 2A\omega \sin \omega t \sin \frac{\omega x}{c} \tag{19}$$

while the condensation is given by differentiating with respect to x

$$s = -\frac{\partial \xi}{\partial x} = 2A\frac{\omega}{c} \cos \omega t \cos \frac{\omega x}{c} \tag{20}$$

The kinetic-energy density

$$U' = \frac{1}{2}\rho\dot{\xi}^2 = 2\rho A^2\omega^2 \sin^2 \omega t \sin^2 \frac{\omega x}{c}$$

$$= 8\rho\pi^2 A^2 f^2 \sin^2 \omega t \sin^2 \frac{\omega x}{c}$$

The maximum kinetic-energy density is at the midpoint between the nodes for values of $t = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}$, etc., times the period of one vibration, that is, where $\sin \omega t$ and sin

$\frac{\omega x}{c}$ are both unity; hence

$$U'_{\text{max.}} = 8\pi\rho A^2 f^2$$

This, it will be noted, is four times the maximum kinetic energy in the direct progressive wave, a result which is to be expected, since the amplitude of the stationary wave at this point is twice that of the direct wave and the energy is proportional to the square of the amplitude. However, the kinetic-energy density of the stationary wave averaged over an entire wave length may be shown to be only twice the energy of the direct wave.

Total kinetic energy per wave length equals

$$2\rho A^2\omega^2 \sin^2 \omega t \int_0^\lambda \sin^2 \frac{\omega x}{c}\, dx = A^2\rho\lambda\omega^2 \sin^2 \omega t \qquad (21)$$

The kinetic-energy density, averaged over a wave length, is $A^2\rho\omega^2 \sin^2 \omega t$. The maximum kinetic energy exists in the medium at the times when $\omega t = \pi/4,\ 3\pi/4,\ 5\pi/4$ and $\sin \omega t$ is unity. At these times, its value is $\rho A^2\omega^2$ or twice the energy density of the direct wave.

The results of the foregoing considerations may be summarized in the following tables of the analytical expressions for the various quantities involved in the progressive and stationary waves pictured in Figs. 5*a* and 6.

TABLE II

	Progressive wave	Stationary wave
Wave equation	$\xi = A \sin \omega\left(t - \frac{x}{c}\right)$	$\xi = -2A \cos \omega t \sin \frac{\omega x}{c}$
Particle velocity	$\dot{\xi} = A\omega \cos \omega\left(t - \frac{x}{c}\right)$	$\dot{\xi} = 2A\omega \sin \omega t \sin \frac{\omega x}{c}$
Particle acceleration	$\ddot{\xi} = -A\omega^2 \sin \omega\left(t - \frac{x}{c}\right)$	$\ddot{\xi} = 2A\omega^2 \cos \omega t \sin \frac{\omega x}{c}$
Condensation	$s = A\frac{\omega}{c} \cos \omega\left(t - \frac{x}{c}\right)$	$s = -2A\frac{\omega}{c} \cos \omega t \cos \frac{\omega x}{c}$
Pressure	$\delta P = \gamma PA\frac{\omega}{c} \cos \omega\left(t - \frac{x}{c}\right)$	$\delta P = 2\gamma PA\frac{\omega}{c} \cos \omega t \cos \frac{\omega x}{c}$
Energy density	$I = \frac{1}{2}\rho A^2\omega^2$	$I = \rho A^2\omega^2$
Energy flux	$J = \frac{1}{2}\rho A^2\omega^2 c$	$J = 0$

It should be said that the form of the expressions for the various quantities considered depends upon the convention adopted as to sign and phase of the motion at the origin. The convention here used is such that in the condensation phase the particle motion coincides with the direction of propagation.

Vibrations of Air Columns.

Referring again to Fig. 6, we note that the particles at P_0 and at P_{20} under the action of the direct and reflected waves are at all times stationary. Accordingly, after the actual source has performed two complete vibrations, giving a complete wave down the tube to the reflecting end and back, thus setting up the column vibration, we may suppose the source removed and a rigid wall placed at the point P_0 (or P_{20}) which will reflect the particle motion, and, in the absence of dissipative forces, the air column as a whole will continue its longitudinal vibration indefinitely just as does a plucked string or a struck tuning fork. It is plain, therefore, that the term "stationary wave" is something of a misnomer. What we have is the compressional vibration of the air column as a whole. Since the length of the vibrating column is one-half or any integral number of halves of the wave length, it appears that a given column of air closed at both ends may vibrate with a series of frequencies, 1, 2, 3, 4, etc., times the lowest frequency, that at which the length of the column is one-half the wave length.

Algebraically, if m is any integer, f a possible frequency of vibration, and l the length of the inclosed column of air,

$$l = \frac{1}{2}m\lambda = \frac{1}{2}m\frac{c}{f}$$

$$f = \frac{mc}{2l} \tag{22}$$

The series of frequencies obtained by giving successive integral values 1, 2, 3, 4, etc., to m are the natural frequencies or the resonant frequencies of the air column.

The point will be further considered in connection with the resonance in rooms.

Closed Tube with Absorbent Ends.

In the foregoing discussion of the standing wave set up by the reflection of a plain wave, we have assumed that none of the vibrational energy is dissipated in the process of reflection and also that there is no dissipation of energy in the passage of sound along the tube.

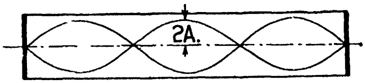


FIG. 11.—Stationary wave in a tube with reflecting ends.

In Fig. 11, the maximum displacement of the particles is pictured at right angles to the direction of propagation, and the envelope of the excursions of the particles is shown. Suppose now that the ideal perfectly reflecting barrier is replaced by one at which a part of the incident energy is absorbed, so that the amplitude of the reflected wave is not A but kA, k being less than unity.

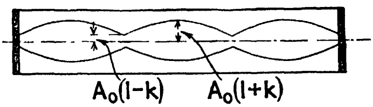


FIG. 12.—Stationary wave in a tube with absorbent ends.

Referring to Eq. 18, we may write in the case of an absorbent barrier

$$\xi_{(d+r)} = A\left[\sin \omega\left(t - \frac{x}{c}\right) - k \sin \omega\left(t + \frac{x}{c}\right)\right] =$$

$$A\left[(1 - k) \sin \omega t \cos \frac{\omega x}{c} - (1 + k) \cos \omega t \sin \frac{\omega x}{c}\right] \qquad (23a)$$

$$= -2kA \cos \omega t \sin \frac{\omega x}{c} + A(1 - k) \sin \omega\left(t - \frac{x}{c}\right) \qquad (23b)$$

At the internodes, $\cos \frac{\omega x}{c}$ is zero, and $\sin \frac{\omega x}{c} = 1$; hence at these points the displacement amplitude is $A(1 + k)$, while at the nodes $\cos \frac{\omega x}{c} = 1$, and $\sin \frac{\omega x}{c} = 0$, and the amplitude is $A(1 - k)$.

We note from the form of Eq. (23*b*) that the particle displacement due to the direct and reflected waves represents a progressive wave of amplitude $A(1 - k)$ superimposed upon a stationary wave of amplitude $2kA$. Thus the state of affairs set up in a tube with partially absorbing ends is not a true stationary wave, since there is a transfer of energy in the direct portion which represents the energy absorbed at the end of the tube.

The foregoing is in a very elementary way the basis of the so-called "stationary-wave method" of measuring the sound-absorption coefficients of materials, first proposed and used by H. O. Taylor[1] and adopted in modified form at the Bureau of Standards and the National Physical Laboratory. The development of the theory of the method from this point on will be given in Chap. VI.

Intensity Pattern in a Room. Resonance.

We have considered somewhat at length the one-dimensional case of a sound wave in a tube with reflecting ends. If we extend the two dimensions which are at right angles to the axis of the tube, the tube becomes a room, and the character of the standing wave system becomes much more complicated. We no longer assume that the particle motion occurs in a single direction parallel to the axis of the tube. As a result, the simple distribution of nodes and loops in the one-dimensional case gives place to an intricate pattern of sound intensities, a pattern which may be radically altered by even slight changes in the position of reflecting surfaces in the room and of the source of sound. The single series of natural or resonance frequencies obtained for the one-dimensional case by putting integral values 1, 2, 3, etc., for m in Eq. (22) is replaced by a trebly infinite series with the number of possible modes of vibration greatly increased. The simplest three-dimensional case would be that of the cube. For the sake of comparison, the first five natural frequencies of a tube 28 ft.

[1] TAYLOR, H. O., *Phys. Rev.*, vol. 2, p. 270, 1913.

long and of a room 28 by 28 by 28 ft., assuming a velocity of sound of 1,120 ft. per second, are given.[1]

Tube	Room
20	20
40	28.3
80	34.7
160	40
320	44 6

The term "resonance" is somewhat loosely used to mean the vibrational response of an elastic body to any periòdic driving force. Thus, the enhancement of sound produced by the body of a violin or the sounding board of a piano over the sound produced by the string vibrating alone is usually spoken of as "resonance." With strict nomenclature we should use the term "forced vibration," reserving the term "resonance" to apply to the enhanced response of a vibrating body to a periodic driving force whose frequency coincides with the frequency of one of the natural modes of vibration of the vibrating body. We shall use the term in this latter sense.

It is apparent that resonance in a room of ordinary dimensions, that is, the pronounced response of the inclosed volume of air to a tone of one particular frequency corresponding to one of its natural modes of vibration, cannot be very marked, since the frequencies of the possible modes of vibration are so close together, and that moreover these frequencies for the lower terms of the series are very low in actual rooms of moderate size. In small rooms, resonance may frequently be observed, but the phenomenon is not an important factor in the acoustic properties of rooms in general.[2]

Survey of Intensity Pattern.

The mathematical solution of the problem of the distribution of sound intensities even in the simple case of a rec-

[1] The theoretical treatment of the problem of the vibration in a rectangular chamber with reflecting walls is given in Rayleigh's "Theory of Sound," vol. 2, p. 69, Macmillan & Co., Ltd., 1896.

[2] An interesting study of the effects of resonance in small rooms is reported by V. O. Knudsen in the July, 1932 number of the *Journal of the Acoustical Society of America.*

tangular room, has not, to the writer's knowledge, yet been effected. There are clearly two distinct problems: first, assuming that there is a sustained source of sound within the room, in which case the solution would depend upon the location of the source and the degree to which its motion is affected by the reaction of the resulting stationary wave on the source; and second, assuming that the stationary wave has been set up and the source then stopped. Stopping the source produces a sudden shift from one condition to the other, a fact which accounts for the frequently observed phenomenon of a sudden rise of intensity at certain places in a reverberant room with the stopping of the source of sound.

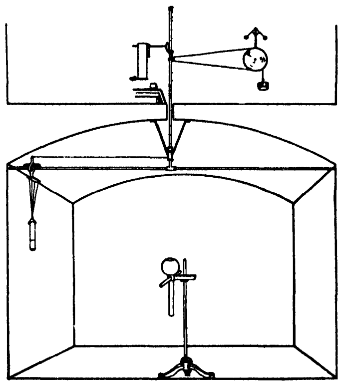


FIG. 13.—W. C. Sabine's experimental arrangement for studying distribution of sound intensity in a room.

In 1910, Professor Wallace Sabine made an elaborate series of experimental investigations of the sound pattern in the constant-temperature room of the Jefferson Physical Laboratory. This room is wholly of brick, rectangular in plan, 23.2 by 16.2 ft., with a cylindrical ceiling. The axis of curvature of the ceiling arch is at the floor level. The height of the room is 9.75 ft. at the sides and 12 ft. at the center of the arch. A detailed account of these experiments has never been given, although in a paper published in March, 1912, Professor Sabine gave the general results of this study and stated that the subject would be fully treated in a forthcoming paper "now about ready for publication." This paper never appeared, probably for the reason that with his passion for perfection, Professor Sabine felt that the work was still incomplete and awaited opportunity for further study. From his notes of the period it is possible to give the experimental details of the investigation, and the importance of the results in the light which they throw on the distribution of sound within an inclosure is offered as a justification for so doing.

Figure 13 shows the experimental arrangement. The source of sound was an electrically driven tuning fork, 248 vibs./sec., associated with a Helmholtz resonator mounted at a height of 4.2 ft. above the floor. The amplitude of vibration of the tuning fork was measured by projecting the image of an illuminated point on the fork on to a distant scale. An interrupted current of the same frequency as that of the fork was supplied from a direct-current source interrupted by a second fork of this same frequency. This current was controlled so as to give any desired amplitude of the fork by means of a rheostat in the circuit.

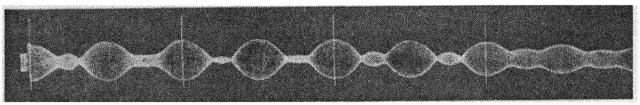


FIG. 14.—Record of sound amplitude at different points in a room.

The intensity pattern was explored by means of a traveling telephone receiver associated with a resonator mounted on a horizontal arm, supported by a vertical shaft, which was driven at a uniform speed by means of a weight-driven chronograph drum belted to the vertical shaft. The exploring telephone was supported by a carriage which was pulled along the horizontal shaft by a cord wound up on a fixed sleeve around the vertical shaft as the latter turned. The exploring telephone thus described a spiral path in the room in a horizontal plane, the pitch of the spiral being the circumference of the fixed sleeve.

The current generated in the exploring telephone passed through the silvered fiber of an Einthoven string dynamometer, the magnified image of which was focused by means of a magnifying optical system upon a moving film. This film was driven by a shaft geared to the rotating shaft bearing the exploring telephone which also carried a finger that opened a light shutter on to the moving film at each revolution of the vertical shaft, so as to mark the position f the exploring telephone corresponding to any point on he film.

Figure 14 is a reproduction of one of the films. Distances along the film are proportional to distances along the piral. The width of the light band produced by the image

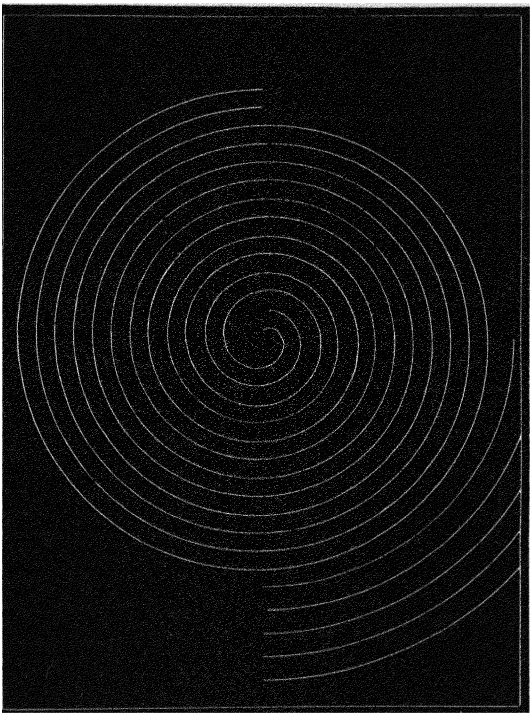


FIG. 15.—Spiral paths followed in acoustical survey of a room.

of the dynamometer string gives the relative sound mplitudes at points along the spiral. Employing two parallel spirals of the same pitch gave a fairly fine-grained urvey of the sound-amplitude pattern. These spiral paths are shown on Fig. 15, while Fig. 16 shows a map of a ection of the room with the amplitudes noted at various points of the exploring spirals. In Fig. 17 we have the results expressed by a map of what may be called "isophonic" lines or lines of equal intensities after the manner of contour lines of a topographic map.

The distribution shown is that in a single horizontal plane. To map out the sound field in a vertical plane, one

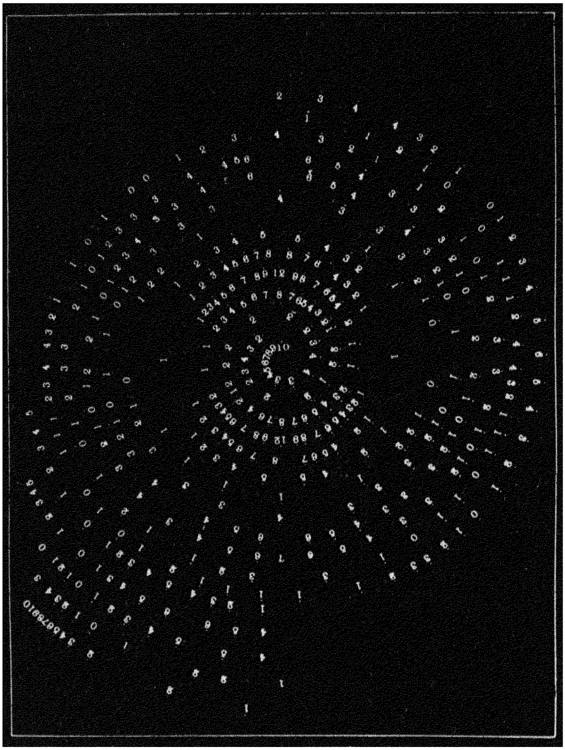


Fig. 16.—Relative amplitudes of sound in a room with a steady source.

may work from spiral records taken in a series of horizontal planes. Figure 18 is a series of records made in planes at four different levels above the mouth of the source resonator. The upper series shows the sound pattern with no absorbent material present, and the lower series shows it when the floor of the room is covered with absorbent material. It is obvious that the distribution in three dimensions does not admit of easy representation either graphically or mathematically and that the intensity at any point of the room is a function of the position of the source and of every reflecting surface in the room. For these reasons, intensity measurements made within rooms are extremely difficult of interpretation. The point will be further considered in Chap. VI in connection with the problem of the reaction of the room on the source of sound.

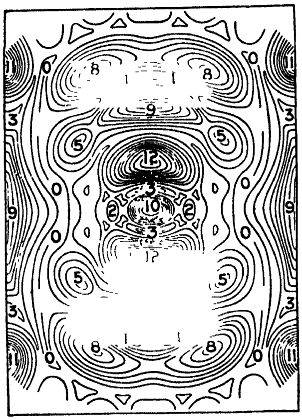


FIG. 17.—Distribution of sound amplitudes at points in a single horizontal plane.

We are now in a position to visualize the condition that exists in a room in which a constant source of sound is operative. We have seen that by actual measurements the intensity of the sound does not fall off with the distance from the source according to the inverse-square law but that the intensity is much more nearly uniform than would be predicted from this law. At any point and at any time, there is added to the direct sound from the source the combined effect of waves emitted earlier, which arrive, after a series of reflections from the bounding surfaces, simultaneously at the given point. The intensity at any point is thus the resultant of a large number of separate waves and varies in a most complicated manner from point to point. It is an interesting experience to walk about an empty room in which a pure tone is being produced and note this point-to-point variation. It is not slight. With a fairly powerful source, movement of only a foot or so or even a few inches, with a high-pitched sound, will change the intensity from a very faint to a very intense sound.

Moreover, this distribution shifts with changes in the positions of objects within the room and with any shift in the pitch of the sound. One may easily note the effect of the movement of a second person in an empty room by the shift produced in the intensity pattern. Fortunately, however, this very complicated phenomenon is a rather infrequent source of acoustical difficulty, since most of the sounds in both music and speech are not prolonged in

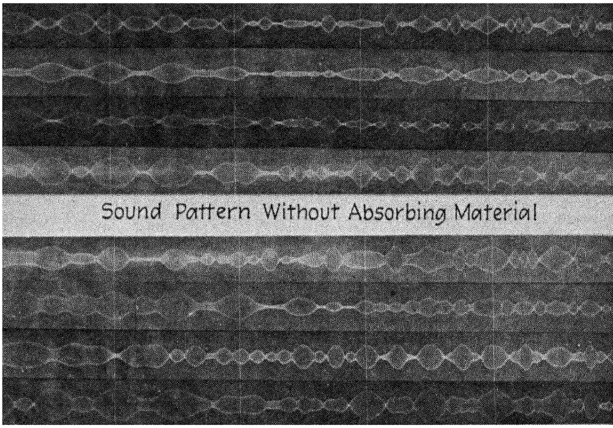


FIG. 18.—Sound patterns at four different levels. Upper figure is for the empty room, lower figure, with entire floor covered with sound-absorbent material.

time or constant in pitch, so that in listening to music or speech in ordinary audience rooms, it is the direct sound which plays the predominating role. Only when one comes to make instrumental measurements of the intensity of pure sustained tones of constant pitch do interference phenomena become troublesome. Here the difficulties of quantitative determination are almost insuperable. It is for this reason that progress in the scientific treatment of acoustical properties of rooms has been made by a method which, on its face, seems almost primitive, namely, the reverberation method, which will be considered in the next chapter.

CHAPTER IV

REVERBERATION (THEORETICAL)

In Chap. III, we have noted that with a sustained source of sound within an inclosure, there is set up an intensity pattern in which the intensity of the sound energy varies from point to point in a complicated manner and that in the absence of dissipative forces, the vibrational energy of the air within the inclosure tends to persist after the source has ceased. This prolongation of sound within an inclosed space is the familiar phenomenon of reverberation which has come to be recognized as the most important single factor in the acoustic properties of audience rooms.

Reverberation in a Tube.

We shall now proceed to the consideration of this phenomenon, first in the one-dimensional case of a plane wave within a tube and then in the more complicated three-dimensional case of sound within a room. In this consideration, the variation of sound intensity from point to point mentioned in the preceding chapter will be ignored, and it will be assumed that there is a uniform average intensity throughout the inclosure. It will further be assumed that the dissipation of the energy occurs wholly at the bounding surface of the inclosure and that dissipation throughout the volume of the inclosure is negligibly small, in comparison with the absorption at the boundaries. We shall first, following Sabine, consider that the dissipation of acoustic energy takes place continuously and develop the so-called reverberation formula on this assumption and then give the analysis recently presented by Norris, Eyring, and others, treating both the growth and the decay of sound within an inclosed space as discontinuous processes.

Growth of Sound Intensity in a Tube.

In Fig. 19, we represent a tube of length l and cross section S with absorbent ends, whose coefficient of absorption is α, and with perfectly reflecting walls. At one end we set up a source of sound which sends into the tube E units of sound energy per second. For simplicity we shall

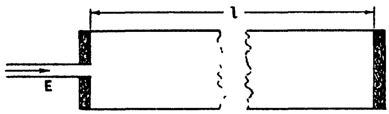


FIG. 19.—Tube with partially absorbent ends and source at one end.

assume that this is in the form of a plane wave train. The energy density of the direct wave that has undergone no reflections will be E/c.

Let

Δt = time required for sound to travel length of tube
m = number of reflections per second $= c/l = 1/\Delta t$

We desire to find the total energy in the tube $t(= n\Delta t)$ sec. after the source was started. In the interval between reflections, the source emits $\frac{El}{c}$ units of sound energy. The total energy in the tube therefore at the end of any interval $n\Delta t$ is this energy plus the residues from those portions of the sound which have undergone 1, 2, 3 . . . $n - 1$ reflections respectively. Of the sound reflected once, $\frac{El}{c}(1 - \alpha)$ units remain. From the twice-reflected sound, there is $\frac{El}{c}(1 - \alpha)^2$; and of the sound that has been reflected $(n - 1)$ times, the residue is $\frac{El}{c}(1 - \alpha)^{n-1}$. Summing up the entire series, we have

$$\text{Total energy} = \frac{El}{c}[1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 \cdots (1 - \alpha)^{n-1}]$$

Note that in the analysis given, we have tacitly assumed that the sound is emitted discontinuously in instantaneous puffs or quanta of El/c units each and that it is absorbed instantaneously and discontinuously at the two ends of the tube. In Fig. 20, the broken line shows the building up of the intensity in the tube according to this analysis, for a value of $\alpha = 0.10$.

If we assume that n is very large—that is, if sound is produced for a long time—we may take the limiting value

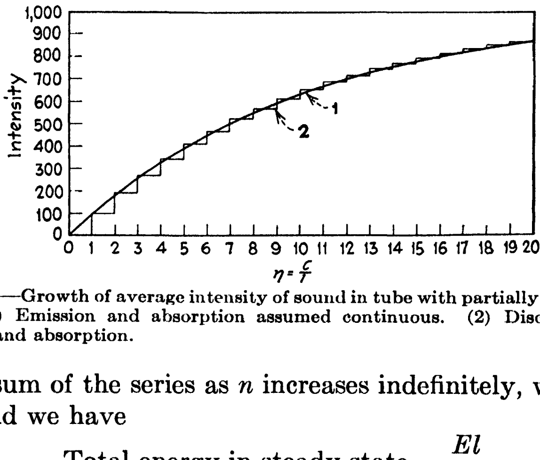


FIG. 20.—Growth of average intensity of sound in tube with partially absorbent ends. (1) Emission and absorption assumed continuous. (2) Discontinuous emission and absorption.

of the sum of the series as n increases indefinitely, which is $1/\alpha$, and we have

$$\text{Total energy in steady state} = \frac{El}{c\alpha}$$

and for the steady-state energy density we have this total energy divided by the volume, $El/c\alpha V$.

We may arrive at the same result for the average value of the steady-state intensity by assuming that the processes of emission and absorption are both continuous.

The rate at which the intensity increases due to emission from the source is E/V. αI is the energy absorbed at each reflection from an end of the tube, and $m\alpha I$ is the energy absorbed per second. Call the net rate of intensity change dI/dt. The net rate of change of intensity is given by the equation

$$\frac{dI}{dt} = \frac{E}{V} - m\alpha I = \frac{E}{V} - \frac{c}{l}\alpha I \tag{24}$$

As I increases, the energy absorbed each second increases and approaches a final steady state in which absorption and emission take place at the same rate, and hence the change of intensity becomes vanishingly small, as time goes on. Call this final steady-state intensity I_1; then

$$\frac{E}{V} - \frac{c\alpha I_1}{l} = 0$$

whence

$$I_1 = \frac{El}{c\alpha V}$$

Equation (24) is easily solved by integration. The solution gives the familiar form

$$I = I_1\left(1 - e^{-\frac{\alpha ct}{l}}\right) \tag{25}$$

or, expressed logarithmically,

$$\log_e \frac{I_1}{I_1 - I} = \frac{\alpha ct}{l} \tag{26}$$

The continuous line in Fig. 20 shows the development of the average intensity upon the assumption of a continuous emission and absorption of sound energy at the source and the absorbing boundaries respectively. The broken line represents the state of affairs assuming that the sound is emitted instantaneously and absorbed instantaneously at the end of each interval. The height of each step represents the difference between the total energy emitted during each interval and the total energy absorbed. The energy absorbed increases with time, since we have assumed that it is a constant fraction of the intensity. We note that the broken line and the curve both approach asymptotically the final steady-state value $I_1 = El/\alpha cV$. Further, it is also to be observed that neither the broken line nor the curve represents the actual condition of growth of intensity in the tube. The important point is that both approximations give the same value for the final steady state after the source has been operating for a long time.

Decrease of Intensity. *Sabine's Treatment.*

After the sound has been fed into the tube for a time long enough for the intensity to reach the final steady state, let us assume that the source is stopped. Consider first the case assuming that the absorption of energy at the ends of the tube takes place continuously. Then in Eq. (24), $E = 0$, and we have, if T is the time measured from the moment of cut-off,

$$\frac{\alpha c I}{l} = -\frac{dI}{dT} \tag{27}$$

with the initial condition that $I = I_1 = El/\alpha c V$, when $T = 0$. The solution of (27) gives

$$I = I_1 e^{-m\alpha T} = I_1 e^{-\frac{\alpha c T}{l}} \tag{28a}$$

or, in the logarithmic form,

$$\log_e \frac{I_1}{I} = \frac{\alpha c T}{l} \tag{28b}$$

It is well to keep in mind the assumptions made in the derivation of (28*a*) and (28*b*), namely, that we have an average uniform intensity throughout the tube at the instant that the source ceases and that, although the dissipation of energy takes place only at the absorbing surfaces placed at the ends of the tube, yet the rate of decay of this average intensity at any time is the same at all points in the tube. With these assumptions, (28*a*) and (28*b*) tell us that during the decay process, in any given time interval, the average intensity decreases to a constant fraction of its value at the beginning of this interval.

Now the "reverberation time" of a room was defined by Sabine as the time required for the average intensity of sound in the room to decrease to one millionth of its initial value. Denote this by T_0, and we have for the tube, considered as a room, $\frac{\alpha c T_0}{l} = \log_e 1{,}000{,}000 = 13.8$

or

$$T_0 = \frac{13.8 l}{c\alpha} \tag{29}$$

That is, the reverberation time for a tube varies inversely as the absorption coefficient of the absorbing surface and directly as the length of the tube. When we come to extend the analysis to the three-dimensional case, we shall see what other factors enter into this quantity.

Decay Assumed Discontinuous.

The foregoing is essentially the method of treatment, given by Wallace C. Sabine, of the problem of the growth and decay of sound in a room, applied to the simple one-dimensional case. Before proceeding to the more general three-dimensional problem, we shall apply the analysis given by Schuster and Waetzmann,[1] Eyring,[2] and Norris[3] to the case of the tube in order to bring out the essential difference in the assumptions made and in the final equa-

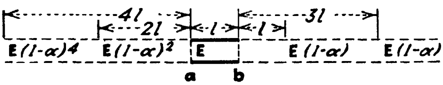


FIG. 21.—Tube with end reflection replaced by image sources.

tions obtained. We shall follow Eyring in method, though not in detail, in this section. This analysis is based on the assumption that image sources may replace the reflecting and absorbing walls in calculating the rate of decay of sound in a room.

Suppose that sound originating at a point P is reflected from a surface S. The state of affairs in the reflected sound is the same as though the reflecting surface were removed and a second source were set up at the point P', which is the optical image of the point P formed by the surface S, acting as a mirror. Hence, for the purposes of this analysis, we may think of the length of the tube simply as a segment without boundaries of an infinite tube, with a series of image sources that will give the same distribution of energy in the segment as is given by reflections in the actual tube.

[1] *Ann. Physik*, vol. 1, pp. 671–695, March, 1929.
[2] *Jour. Acous. Soc. Amer.*, vol. 1, No. 2, p. 217.
[3] *Ibid.*, p. 174.

The energy in the actual tube due to the first reflection is the same as would be produced in the imagined segment by a source whose output is $(1 - \alpha)E$ located at a distance l to the right of b (Fig. 21). That due to the second reflection may be thought of as coming from a second image source of power $E(1 - \alpha)^2$ located at a distance $2l$ from a. The third reflection contributes the same energy as would be contributed by a source $E(1 - \alpha)^3$ located a distance $3l$ to the right of b; and so on. The number of images will correspond to the number of reflections which we think of as contributing to the total energy in the tube.

In the building-up process, we think of all the image sources as being set up at the instant the sound is turned on. Their contribution to the energy in l, however, will be recorded in each case only after a lapse of time sufficient for sound to travel the distance from the particular image source to the boundaries of the segment. We thus have the discontinuous building-up process shown in the broken line of Fig. 20. The final steady state then would be that produced by the original source E and an infinite series of mage sources and would be represented by the equation

$$I_1 = \frac{El}{cV}[1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 \cdots (1 - \alpha)^\infty] = \frac{El}{c\alpha V}$$

Now when this steady state has been reached, the source of sound is stopped. All the image sources are stopped simultaneously, but the stoppage of any image source will be recorded in the segment only after a time interval sufficient for the last sound in the space between the image source and the nearer boundary of the tube to reach the latter. Thus the intensity measured at any point in the tube will decrease discontinuously. For example, suppose we measure the instantaneous value at the point b, l meters from the source. When the source is stopped, the intensity at b will remain the constant steady-state intensity I_1 for the time l/c required for the end of the train of waves from E to travel the length of the tube. At the end of this interval, the recording instrument at b will note the absence of the contribution from E, and at the same time it will note the drop due to the cessation of $E_1 = E(1 - \alpha)$, which is equally distant from b. No further change will be recorded at this point until after the lapse of a second interval $2l/c$, at which time the effect of

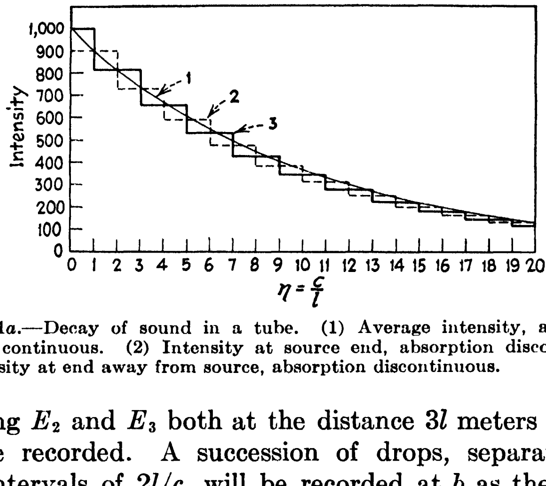


FIG. 21*a*.—Decay of sound in a tube. (1) Average intensity, absorption assumed continuous. (2) Intensity at source end, absorption discontinuous. (3) Intensity at end away from source, absorption discontinuous.

stopping E_2 and E_3 both at the distance $3l$ meters from b, will be recorded. A succession of drops, separated by time intervals of $2l/c$, will be recorded at b as the sound intensity in the segment decreases, as pictured in the solid line 3 of Fig. 21*a*.

If instead of setting up our measuring instrument at b, the most distant point from the source, we had observed the intensity at a, near the source, the first drop would have been recorded at the instant the source was turned off, and the history of the decay would be that shown by the broken line 2 of Fig. 21*a*.

If now we could set up at the mid-point of the tube an instrument which by some magic could record the average intensity in the tube, its readings would be represented by the series of diagonals of the rectangles formed by the solid and dotted lines of Fig. 21*a*. The reading of this instrument at the exact end of any time $T = nl/c$ is

$$I = I_1 - \frac{El}{cV}[1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 \dots (1 - \alpha)^{n-1}]$$

The sum of the series for a finite number of terms is

$$\frac{1 - (1 - \alpha)^n}{1 - (1 - \alpha)} = \frac{1 - (1 - \alpha)^n}{\alpha}$$

and

$$I = I_1 - \frac{El}{c\alpha V}(1 - (1 - \alpha)^n) = I(1 - \alpha)^n$$

and

$$\frac{I}{I_1} = (1 - \alpha)^n \tag{30}$$

Taking the logarithm of both sides of Eq. (30), we have $\log_e \frac{I_1}{I} = -n \log_e (1 - \alpha)$ or

$$\log_e I_1 - \log_e I = -\frac{cT}{l} \log_e (1 - \alpha) \tag{31}$$

If, as in Eq. (29), we substitute for I_1 and I the values of 1,000,000 and 1 respectively, we have

$$-\frac{cT_0}{l} \log_e (1 - \alpha) = 13.8$$

$$T_0 = -\frac{13.8l}{c \log_e (1 - \alpha)} \tag{32}$$

Comparison of the Two Methods.

We note that the analysis based upon the picture of the decay of sound as a continuous process and that considering it as a step-by-step process led to two different expressions for the reverberation time. The first contains the term α where the second has the term $-\log_e (1 - \alpha)$. Now for small values of α the difference between α and $-\log_e (1 - \alpha)$ is not great, as is shown in the following table, but increases with increasing values of α. In this same table we have given the number of reflections necessary to reduce the intensity in the ratio of 1,000,000:1 and the times, assuming

TABLE III

α	$-\log_e (1 - \alpha)$	Percentage difference	n for $\frac{I_1}{I} = 10^6$	$T_0 = 13.8\frac{K}{\alpha}$	$T_0 = -\frac{13.8K}{\log_e (1 - \alpha)}$
0.01	0 010	....	1,380	10 00	10.00
0.02	0.020	....	690	5 00	5.00
0.05	0.051	2.0	271	2.00	1.96
0.10	0 105	5 0	131	1.00	0.95
0 15	0.163	8 7	85	0 67	0.61
0.20	0 223	11 5	62	0 50	0.45
0 25	0 288	15.2	48	0.40	0 35
0 30	0.357	19 0	39	0.33	0 28
0 40	0.511	27.8	27	0 25	0 195
0 50	0 693	38 6	20	0 20	0.145

that the time is 10 sec., when the absorption coefficient of the ends of the tube is 0.01, first using the Sabine formula and second using the later formula developed by Eyring and others.

We note in the example chosen, that while the percentage difference in the values of T_0 computed by the two formulas increases with increasing values of the assumed coefficient of absorption, yet the absolute difference in T_0 does not increase.

From the theoretical point of view, it should be said that the later treatment is logically more rigorous in the particular example chosen. In this instance, we are dealing with a special case in which the sound travels only back and forth along a given line. Reflections occur at definite fixed intervals of time, so that, in both the growth and decay processes, the rate of change of intensity must alter abruptly at the end of each of these intervals. Further, we note that in the assumption of a continuously varying rate of building up and decay, we also assume that the influence of the absorbing surface affects the intensity of a train of waves both before and after reflection. Thus when the source starts, the total energy actually in the tube increases linearly with the time, since the absorbing process does not begin until the end of the first interval. In setting down the differential equation, however, we assume that absorption begins at the instant the sound starts. We arrive at the same final steady state by the two methods of analysis only by assuming that the source operates so long that the number of terms in the series may be considered as infinite. But in the decay, we are concerned with the decrease of intensity in a limited time, and n is a finite number. The divergence in the results of the final equations increases with increasing values of α and corresponding decrease in n.

Growth of Sound in a Room: Steady State.

In the case of the tube, we have assumed that the sound travels only back and forth as a plane wave along the tube, so that the path between reflections is l, the length of the tube. To take the more general case of sound in a room, we assume that the sound is emitted in the form of a spherical wave traveling in all directions from the source, that it strikes the various bounding surfaces of the room at all angles of incidence, and that hence after a comparatively small number of reflections the various portions of the initial sound will be traveling in all directions. The distance traveled between reflections will not be any definite length. Its maximum value will be the distance between the two most remote points of the room, while its minimum value approaches zero. To form a picture of the two-dimensional case, one may imagine a billiard ball shot at random on a table and note the varying distance it will travel between its successive impacts with the cushions. If we think of the billiard ball as making a large number of these impacts and take the average distance traveled, we shall have a quantity corresponding to what has been called the "mean free path" of a sound element in a room. We can extend the picture of the building-up process in a tube, where the mean free path is the length of the tube, to the three-dimensional case, simply by substituting for l, the length of the tube, p, the mean free path of a sound element in the room, and for α the coefficient of the ends, α_a, the average coefficient of the boundaries. In the case of a room, then, with a volume V, a source of sound emitting E units of sound energy per second, and an average coefficient of absorption of α_a, we should have by analogy the average steady-state intensity

$$I_1 = \frac{Ep}{c\alpha_a V} \tag{33a}$$

where

$$\alpha_a = \frac{\alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3 + \text{etc.}}{S}$$

α_1, α_2, α_3, etc., are the absorption coefficients of the surfaces whose areas are S_1, S_2, S_3, etc.

S is the total area of the bounding surfaces of the room, and the summation of the products in the numerator includes all the surfaces at which sound is reflected. This sum has been called the "total absorbing power" of the room and is denoted by the letter a. Hence

$$\alpha_a = \frac{a}{S}$$

and Eq. (33a) becomes

$$I_1 = \frac{E}{V} \cdot \frac{Sp}{ac} \tag{33b}$$

A further simplification is effected if we can express p, the mean free path, in terms of measurable quantities. As a result of his earlier experiments, Sabine arrived at a tentative value for p of $0.62V^{1/3}$. He recognized that this expression does not take account of the fact that the mean free path will depend upon the shape as well as the size of the room and, subsequently, as a result of experiment put his equations into a form in which p is involved in another constant k. Franklin first showed in a theoretical derivation of Sabine's reverberation equation[1] that

$$p = \frac{4V}{S}$$

[1] For the derivation of this relationship, see Franklin, *Phys. Rev.*, vol. 16, p. 372, 1903; Jaeger, *Wiener Akad. Ber. Math-Nature Klasse*, Bd. 120, Abt. II*a*, 1911; Eckhardt, *Jour. Franklin Inst.*, vol. 195, pp. 799–814, June, 1923; Buckingham, *Bur. Standards Sci. Paper* 506, pp. 456–460.

Putting this value of p in Eq. (33b), we have

$$I_1 = \frac{4E}{ac} \tag{33c}$$

This expression for the steady-state intensity has been derived by analogy from the case of sound in the tube. Since the result of the analysis for the tube is the same regardless of whether we treat the growth of the sound as a continuous process or as a series of steps, Eq. (33c) gives the final intensity on either hypothesis.

Decay of Intensity in a Room. *Sabine's Treatment.*

We proceed to derive the equation for the intensity at any time in the decay process in terms of the time, measured from the instant of cut-off, in a manner quite similar to that used in the case of the tube. On the average, the number of reflections that will occur in each second is c/p. At each reflection, the intensity I is decreased on the average by the fraction $\alpha_a I = \frac{a}{S}I$. The rate of change per second is then $\frac{c}{p}\frac{a}{S}I$ or

$$\frac{dI}{dt} = -\frac{acI}{Sp} = -\frac{acI}{4V}$$

$$\frac{dI}{I} = -\frac{ac}{4V}dT$$

Integrating, and using the initial condition that $I = I_1$ when $T = 0$, we have

$$\log_e \frac{I_1}{I} = \frac{acT}{4V} \tag{34a}$$

or, in the exponential form,

$$I = I_1 e^{\frac{-acT}{4V}} \tag{34b}$$

For the reverberation time T_0, the time required for the sound to decrease from an intensity of 1,000,000 to an intensity of 1, we have

$$\log_e \frac{1{,}000{,}000}{1} = \frac{acT_0}{4V}$$

$$T_0 = \frac{13.8 \times 4V}{ac} = \frac{KV}{a} \qquad (35)$$

Taking the velocity of sound as 342 m. per second, we have K equal to 0.162, when a is expressed in square meters and V in cubic meters (or 0.0495 in English units). Sabine's experimental value is 0.164, a surprisingly close agreement when one considers the difficulties of precise quantitative determination under the conditions in which he worked.

Process Assumed Discontinuous.

We shall consider next the dying away of sound in a room, picturing the process from the image point of view. In the case of a room, where p, the mean free path, is a statistical mean of a number of actual paths, ranging in magnitude from the distance between the two most remote points of the room to zero, the picture of the decay taking place in discrete steps is not so easy to visualize as in the case of the tube already considered, where the distance between reflections is definitely the length of the tube. In his analysis, Eyring assumes that the image sources which replace the reflecting walls can be located in discrete zones, surfaces of concentric spheres, whose radii are p, $2p$, $3p$, etc., from the source. Then p/c is the time interval between the arrival of the sound from any two successive zones. The energy supplied by the source E in this interval is $E\Delta t = Ep/c$ and by the first zone of images is $\frac{Ep}{c}(1 - \alpha_a)$. That by the second zone is $\frac{Ep}{c} \cdot (1 - \alpha_a)^2$. Hence the total energy in the steady state

$$\frac{Ep}{c}[1 + (1 - \alpha_a) + (1 - \alpha_a)^2 \cdot \cdot \cdot (1 - \alpha_a)^\infty] = \frac{Ep}{c\alpha_a} \qquad (36)$$

and

$$I_1 = \frac{Ep}{c\alpha_a V}$$

We then consider that the source is stopped and with it all the sources in the image zones. If our point of observation is taken near the source, the average intensity will drop by an amount Ep/cV at the instant the source stops, and there will be drops of $\frac{Ep}{cV}(1 - \alpha_a)$, $\frac{Ep}{cV}(1 - \alpha_a)^2$, etc., at the beginning of each succeeding p/c interval. The total diminution in the intensity at the end of the time $T = np/c$ will be

$$\frac{Ep}{cV}[1 + (1 - \alpha_a) + (1 - \alpha_a)^2 + (1 - \alpha_a)^3 + \cdots (1 - \alpha_a)^{n-1}]$$

and we have

$$I = I_1 - \frac{Ep}{cV}\left(\frac{1 - (1 - \alpha_a)^n}{1 - (1 - \alpha_a)}\right) = I_1(1 - \alpha)^n = I_1(1 - \alpha_a)^{\frac{cT}{p}}$$

Taking the logarithm of both sides of this equation, we have

$$\log_e \frac{I_1}{I} = -\frac{cT}{p} \log_e (1 - \alpha_a) = -\frac{cST}{4V} \log_e (1 - \alpha_a) \quad (37)$$

For the reverberation time $T = T_0$, $I_1/I = 1{,}000{,}000$ we have

$$T_0 = -\frac{13.8 \times 4V}{cS \log_e (1 - \alpha_a)} = -\frac{KV}{S \log_e \left(1 - \frac{a}{S}\right)} \quad (38)$$

Comparing (35) and (38), we note, as in the case of the tube, that the final equations arrived at by the two analyses differ only in the fact that for a in the Sabine formula we have $-S \log_e \left(1 - \frac{a}{S}\right)$ in the later formula. We have also seen that for values of $\alpha_a = a/S$, less than 0.10 the computed values of the reverberation time will be the same by the two formulas. We have also seen that while the percentage difference increases with increasing values of the average coefficient, yet due to the decrease in the absolute value of the reverberation time the numerical difference does not increase proportionately.

Eyring's argument in favor of the new formula is much more detailed and carefully elaborated than the foregoing, but one has the suspicion that as applied to the general case it may not be any more rigorous. Certainly in the case of an ordered distribution of direction and intensity of sound in an inclosure, as would be the condition, for example, of sound in a tube, or from a source located at the center of a sphere, the assumptions of a diffuse distribution and a continuously changing rate of decay do not hold. But in a room of irregular shape, where the individual paths between reflections vary widely, replacing the average reflections by average image sources all located at a fixed distance from the point of observation introduces a discontinuity which does not exist in actuality. It is true that the earlier analysis implicitly assumes that each element of an absorbing surface effects a reduction in any particular train of waves both before and after reflection, whereas the later treatment recognizes that, viewed from an element of the reflecting surface, there will be a constant-energy flow toward the surface from any given direction during the time required for sound to travel the distance from the preceding point of reflection in this direction. The latter reasoning, however, is valid, on the assumption that the actual paths of sound from all directions may be replaced by the mean free path. The point at issue seems to be whether or not the introduction of the idea of the mean free path makes valid the concept of the decay as a continuous process and the use of the differential equation as a mathematical expression of it. For "dead" rooms where n, the total number of reflections in the time T_0, is small the older treatment is not rigorous. On the other hand, there is the question whether, in such a case, the assumption of a mean free path is compatible with the fact of a small number of reflections.

In this dilemma, we shall in the succeeding treatment adhere to the older point of view, since the existing values of the absorption coefficients have been obtained using the Sabine equations, and the criterion of acoustical excellence has been established on the basis of Sabine coefficients and the $0.05V/a$ formula. We can, if desired, shift to the later point of view by substituting for a in the older expression $-S \log_e\left(1 - \frac{a}{S}\right)$. It must be remembered, however, that the absorption coefficients of materials now extant are all derived on the older theory, so that to shift would involve a recalculation of all absorption coefficients based on the later formula.

Experimental Determination of Reverberation Time.

We have developed a simple usable formula for computing the reverberation time T_0 from the values of volume and absorbing power. We next have to consider the question of the experimental measurement of this quantity. To do this directly we should need some means of direct determination of the average intensity of the sound in a room at two instants of time during the decay process and of precise measurement of the intervening time. We have already seen the inequality that exists in the intensities at different points due to interference, a phenomenon that in the theoretical treatment we have ignored, by assuming an average uniform intensity throughout the room. Moreover, there has not as yet been developed any simple portable apparatus by which the required intensity measurement can be made.[1] We are thus forced to indirect methods for experimental determination.

The method employed by Sabine involved the minimum audible intensity or the threshold of hearing, for the lower of the two intensities used in defining reverberation time.

[1] Very recently, apparatus has been devised for making direct measurements of this character. In the *Jour. Soc. Mot. Pict. Eng.*, vol. 16, No. 3, p. 302, Mr. V. A. Schlenker describes a truck-mounted acoustical laboratory for studying the acoustic properties of motion-picture theaters, in which an oscillograph is used for recording the decay of sound within a room. The equipment required is elaborate. The spark chronograph of E. C. Wente described in Chap. VI has also been used for this purpose.

The absolute value of this quantity varies with the pitch of the sound and is by no means the same at any given pitch for two observers or, indeed, for the two ears of any one observer. Fortunately, it does remain fairly constant over long periods of time for a given observer, so that with proper precautions it may be used in quantitative work. Another quantity which Sabine assumed could be considered as constant under different room conditions is the quantity E, the energy emitted per second by an organ pipe blown at a definite wind pressure. We note that neither E, the power of the source, nor i, the intensity at the threshold of hearing, is known independently, but we may put our equations into a form that will involve only their ratio.

We shall, in the following, denote by T_1 the time that sound from a source of output E, which has been sustained until the steady state I_1 has been reached, remains audible after the source is stopped. Let i be the threshold intensity (ergs per cubic meter) for a given observer. Then, by Eq. (34*a*), we have

$$\log_e \frac{I_1}{i} = \frac{acT_1}{4V}$$

Putting in the value of I_1 given by Eq. (33*b*), we have

$$T_1 = \frac{4V}{ac} \log_e \frac{4E}{aci} \qquad (39)$$

T_1 is a directly measurable quantity. The same mechanism that stops the pipe may simultaneously start the timing device, which, in turn, the observer stops at the moment he judges the decaying sound to be inaudible. The quickness and simplicity of the operation allow observations to be made easily, so that by averaging a large number of such observations made at different parts of the room, the differences due to interference and the personal error of a single observation may be made reasonably small. However, in Eq. (39), there are two unknown quantities, a and E/i. Obviously, some means of determining one or the other of them independently of Eq.

(39) is necessary. Thus, if the output of a given source of sound is known in terms of the observer's threshold intensity, the measured value of the duration of audible sound from this source within a room allows us to compute the total absorbing power of the room.

The extremely ingenious method which Sabine employed for experimentally determining one of the two unknown quantities of Eq. (39) with no apparatus other than the unaided ear, a timing device, and organ pipes will be considered in the next chapter.

CHAPTER V

REVERBERATION (EXPERIMENTAL)

In the preceding chapter, we have treated the question of the growth and decay of sound within a room from the theoretical point of view. Starting with the simple one-dimensional case, in which visualization of the process is easy, we proceeded by analogy to the three-dimensional case and derived the relations between the various quantities involved.

A connected account of Professor Sabine's series of experiments, on which he based the theory of reverberation, should prove helpful to the reader who is interested in something more than a theoretical knowledge of the subject.

In the first place, it is well to remind ourselves of the situation which confronted the investigator in the field of architectural acoustics at the time this work was begun nearly forty years ago. In undertaking any problem, the first thing the research student does is to go through the literature of the subject to find what has already been done and what methods of measurement are available. In Sabine's case, this first part of the program was easy. Aside from one or two small treatises dealing with the problem purely from the observational side and an occasional reference to the acoustic properties of rooms in the general texts on acoustics, there were no signposts to point out the direction which the proposed research should follow. It was a new and untouched field for research. To offset the attractiveness of such a field, however, there were no known means available for direct measurement of sound intensity. Tuning forks and organ pipes were about the only possible sources of sound. These do not lend themselves readily to variation in pitch or intensity. Study of the problem thus involved the invention of a wholly new technique of experimental procedure.

Reverberation and Absorbing Power.

The immediate occasion for the research was the necessity of correcting the very poor acoustic properties of the

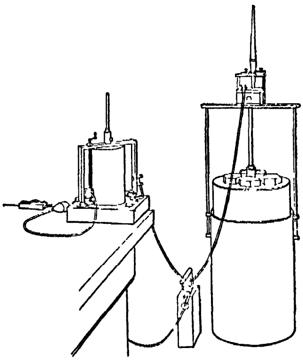


FIG. 22.—W. C. Sabine's original single-pipe apparatus for reverberation measurements.

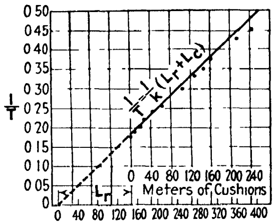


FIG. 23.—Reciprocal of reverberation time, constant source, plotted against length of seat cushion introduced.

then new lecture room of the Fogg Art Museum. This room was semicircular in plan, not very different in general design from Sanders Theater, a much larger room, but one which was acoustically excellent.

To even casual observation, one outstanding difference between the two rooms was in the matter of the interior finish and furnishing. There was a considerable area of wood paneling in Sanders Theater. Aisles were carpeted and the pews were furnished with heavy cushions. Reverberation in the smaller room of the Fogg Art Museum was markedly greater.

The first thing to try, therefore, was the effect of the seat cushions upon the reverberation in the new lecture hall.

The apparatus was simple. An organ pipe blown by air from the constant-pressure tank shown in Fig. 22 was the source of sound. The time of stopping the source and the time at which the reverberant sound ceased to be audible to the observer were recorded on a chronograph. Seat cushions were introduced and the duration of the audible sound with varying lengths of seat cushion present was measured. The relation found between these two variables is shown on the graph (Fig. 23), on which is plotted not T but $1/T$ as a function of the length of cushions.

$$\frac{1}{T} = \frac{1}{k}(L_r + L_c) \tag{40}$$

where k is a constant, the reciprocal of the slope of the straight line; L_c is the length of seat cushions brought in; and L_r is obviously the length of seat cushion that is the equivalent in absorbing power of the walls and contents of the room before any cushions were introduced. If we include in the term a_c the total absorbing power of the room and its contents, measured in meters of seat cushion, the expression assumes the now familiar form

$$a_cT = k_c \tag{41}$$

The numerical value of a_c and consequently of k_c is determined by the number which finally is to be attached to the absorbing power of one meter of cushions as measured by its effect on the reverberation time.

We note that the experimental points of Fig. 23 show a tendency to fall further and further from the straight line as the total absorbing power is increased. So carefully was the work done that Sabine concluded that this departure was more than could be ascribed to errors of experiment. The significance which he attached to it will be considered later. For the time being we shall retain Eq. (41) as an approximate statement of the facts so far adduced.

Reverberation and Volume.

The next step was to extend the experiment to a number of different rooms, first to establish the generality of the ation found for the single room and, also, to ascertain at meaning is to be ascribed to the constant k. Accord- ;ly the laborious task of performing the seat-cushion ɔeriment in a number of different rooms of different ıpes and sizes was undertaken. These ranged from a all committee room with a volume of 65 cu. m. to a ›ater seating 1,500 persons and having a volume of ·00 cu. m.

[n order to secure the necessary conditions of quiet, the rk had to be done in the early hours of the morning. e mere physical job of handling the large number of ,t cushions necessary for the experiments in the larger ›ms was a formidable task. There was, however, no ıer way. The results of this work well repaid the effort. ey showed first that the approximate constancy of the ›duct of absorbing power and time held for all the rooms which the experiment was tried. More important l, it appeared that the values of this product for different ›ms are directly proportional to their volumes. Equa- n (41) may therefore be written in the form

$$a_c T = K_c V, \tag{42}$$

ere K_c is a new constant, approximately the same for rooms. Again its numerical value will depend upon the nber which is to be attached to the absorbing power of : cushions.

en-window Unit of Absorbing Power.

Sabine recognized that in order to give K_c something ıer than a purely empirical significance, it was necessary ɜxpress the total absorbing power of rooms in some more ıdamental and reproducible unit than meters of a ·ticular kind of seat cushion. He proposed, therefore, treat the area presented by an open window as a surface which all the incident sound energy is transmitted to ,side space with none returned to the room. In other rds, at this stage of the investigation the open window ɜ to be considered as an ideal perfectly absorbing surface with a coefficient of absorption of unity. A comparison between the absorbing power of window openings and seat cushions as measured by the change in the reverberation time was accordingly made in a room having seven windows, each 1.10 m. wide. The width of the openings was successively 0.20, 0.40, and 0.80 m. The experiment showed that within the limits of error of observation the increase in absorbing power was proportional to the width of the openings. It should be noted, in passing, that in this experiment the increase in absorbing power was secured by a number of comparatively small openings. Later experiments have shown that had the increased absorption been secured by increasing the dimensions of a single opening, the results would have been quite different. That Professor Sabine[1] recognized this possibility is shown by his statement "that, *at least for moderate breadths*, the absorbing power of open windows as of cushions is accurately proportional to the area."

Experiments comparing seat cushions and open windows in several rooms gave the absorbing power of the cushion as 0.80 times that of an equal area of open window.

This figure gave him the data for evaluating the constant K, using a square meter of open window as his unit of absorbing power simply by multiplying the parameter K_c as obtained by experiment with cushions by the factor 0.80. This operation yielded the equation

$$aT = 0.171V \tag{43}$$

Here T is the duration of sound produced by the particular organ pipe used in these experiments, a gemshorn pipe one octave above middle C, 512 vibs./sec., blown at a constant pressure. V is the volume of the room in cubic meters, and a is the total absorbing power in square meters of open window, measured as outlined above.

Logarithmic Decay of Sound in a Room.

At this point in his study of reverberation, Sabine was confronted with two problems. The first was to give

[1] "Collected Papers on Acoustics," p. 23.

a coherent theoretical treatment of the phenomenon that would lead to the interesting experimental fact expressed in Eq. (43). The second was to explain the departure from strict linearity in the relation between the reciprocal of the observed time of reverberation, using a presumably constant source, and the total absorbing power. Obviously Eq. (43) does not involve the acoustical power of the source. It is equally evident that the time required after the source has ceased for the sound to decrease to the threshold intensity should depend upon the initial intensity, and this, in turn, upon the sound energy emitted per second by the source. The next step then in the investiga-

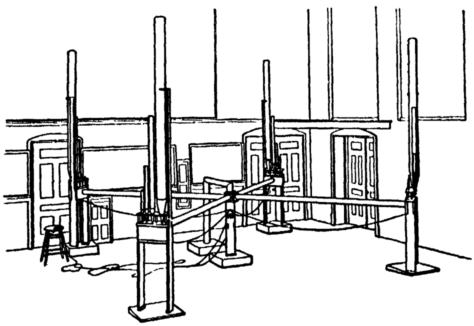


FIG. 24.—W. C. Sabine's four-organ apparatus.

tion is to find the relation between the acoustical power of the source and the duration of audible sound. Here, again, the experimental procedure was simple and direct. The apparatus is shown in Fig. 24. Four small organs were set up in a large reverberant room. They were spaced at a distance of 5 m. from each other so that the output of one might not be affected by the close proximity of another pipe speaking at the same time. They were supplied with air pressure from a common source. Each pipe was controlled by an electropneumatic valve, and the electric circuits were arranged so that each pipe could be made to speak alone or in all possible combinations with any or all of the remaining pipes. The time between stopping the pipe and the moment at which it ceased to be audible was recorded on a chronograph placed in an adjoining room. By sounding each pipe alone and then in all possible combinations with the remaining pipes, allowance was made for slight inequalities in the acoustic powers of the individual pipes.

The results of one experiment of this sort were as follows:[1]

$t_1 = 8.69$	$t_2 - t_1 = 0.45$
$t_2 = 9.14$	$t_3 - t_1 = 0.67$
$t_3 = 9.36$	$t_4 - t_1 = 0.86$
$t_4 = 9.55$	$t_4 - t_3 = 0.19$
	$t_3 - t_2 = 0.22$

The fact that the difference in time for one and two pipes is very nearly one-half of the difference for one and four pipes suggests that the difference in times is proportional to the logarithm of the ratio of the number of pipes, a relation that is clearly brought out by graph 1 of Fig. 25.

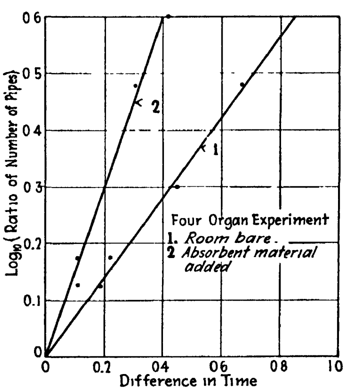


FIG. 25.—Results of the four-organ experiment.

Let us now assume that the average intensity of sound in the room, that is, the average sound energy per cubic meter set up by n pipes, is n times the average intensity produced by a single pipe. We may write

$$\log_e I_n - \log_e I_1 = \log_e \frac{I_n}{I_1} = A(t_n - t_1) \qquad (44)$$

wherein A is the slope of the line in Fig. 25. Its numerical value in the example given is 1.59.

Suppose, now, that we have a pipe of such minute power that the average intensity which it sets up is i, the minimum audible intensity. The sound from such a pipe would, of

[1] "Collected Papers on Acoustics," p. 36.

course, cease to be audible the instant the pipe stops. Therefore T for such a pipe is zero. If the relation given in Eq. (44) is a general one, then we may write

$$\log_e \frac{I_1}{i} = A(T_1 - 0) \tag{45a}$$

In other words, the time required for sound of any given initial intensity to decrease to the threshold of audibility is proportional to the logarithm of the initial intensity measured in terms of the threshold intensity.

The next step in the investigation was to relate the quantity A, which is the change per second in the natural logarithm of the intensity of the reverberant sound, to the total absorbing power of the room. To do this, a large quantity of a highly absorbent felt was installed upon the walls of the room, and the four-organ experiment was repeated. The following results were obtained:

$t'_1 = 3.65$	$t'_2 - t'_1 = 0.20$
$t'_2 = 3.85$	$t'_3 - t'_1 = 0.31$
$t'_3 = 3.96$	$t'_4 - t'_1 = 0.42$
$t'_4 = 4.07$	$t'_4 - t'_3 = 0.11$
	$t'_3 - t_2'' = 0.11$

Plotting all the possible relations between the difference in times and the ratio of the number of pipes, we have the straight line 2, in Fig. 25. The slope of this line is 3.41.

It is apparent that the logarithmic decrement A in the intensity increases as the absorbing power of the room increases. Now the experiment with the seat cushions gave the result that the product of absorbing power and time for a given room is *nearly* constant. The results of the four-organ experiments give the same sort of approximate relation between the logarithmic decrement and the time. For

$$1.59 \times 8.69 = 13.80 = \log_e \frac{I}{i}$$

$$3.41 \times 3.65 = 12.45 = \log_e \frac{I'}{i}$$

Solving for I and I', we have

$$I = 1{,}000{,}000i, \text{ and } I' = 250{,}000i.$$

We have here the explanation of the departure from a constant value of the product $a \times T$ in the experiments with seat cushions. It lies in the fact that with a source that generates sound energy at a constant rate, independently of room conditions, the level to which the intensity (sound energy per cubic meter) rises is less in the absorbent than in the reverberant room. The presence of absorbent material thus acts in two ways to reduce the time of reverberation from a fixed source: first, by increasing the rate of decay and, second, by lowering the level of the steady-state intensity. The complete theory must take account of both of these effects.

Total Absorbing Power and the Logarithmic Decrement.

It still remains to connect the quantities a, the total absorbing power of the room in the equation $aT = 0.171V$, and A, the change per second in the natural logarithm of the decaying sound in the equation $AT_1 = \log_e \frac{I_1}{i}$. This latter equation suggests that the rate at which the intensity decreases is proportional to the intensity at that instant, that is,

$$\frac{dI}{dt} = -AI$$

Integrating, we have

$$-\log_e I + C = AT.$$

When T is 0, that is, at the moment of cut-off of the source, $I = I_1$. So that $-\log_e I_1 + C = 0$ whence

$$\log_e I_1 - \log_e I = AT$$

If $T = T_1$, the time required for the reverberant sound to decrease to the threshold intensity i, we have

$$AT_1 = \log_e \frac{I_1}{i} \qquad (45b)$$

Thus A, the slope of the straight line in the four-organ experiment, is the instantaneous rate of change of intensity per unit intensity, when no sound is being produced in the room.

Now a, the total absorbing power of a room in the reverberation equation, is the area of perfectly absorbing surface (open window) that would, in an ideal room that is otherwise perfectly reflecting, produce the same rate of decay as that of the actual room. If we divide this area by S, the total area of the exposed surfaces in the room, we have the average absorption coefficient of these surfaces or the fraction by which the intensity of the sound is decreased at each reflection. Call this average coefficient α_a. Then

$\alpha_a = \frac{a}{S}$ = change of intensity per unit intensity per reflection

A = change of intensity per unit intensity per second

Then $A = m\alpha_a$ where m is the average number of reflections per second which any single element of the sound undergoes in its passage back and forth across the room during the decay process. In terms of the mean free path, already referred to in Chap. IV,

$$A = \frac{c}{p}\alpha_a = \frac{c}{p} \cdot \frac{a}{S} \qquad (46)$$

We may now express the results of the four-organ experiment in terms of the total absorbing power as measured by the cushion and open-window experiments. We have

$$AT_1 = \frac{a}{S} \cdot \frac{c}{p}T_1 = \log_e \frac{I_1}{i}$$

whence

$$aT_1 = \frac{Sp}{c} \log_e \frac{I_1}{i} \qquad (47)$$

Power of Source.

The experimentally determined value of A makes it possible to express the acoustical power of the source in terms of the threshold intensity.

Let E be the number of units of sound energy supplied by the source to the room in each second. The rate then at which the sound energy density I is increased by E is E/V. The rate at which the energy decreases due to absorption is AI. The net rate of increase while the source is operating is therefore given by the equation

$$\frac{dI}{dt} = \frac{E}{V} - AI$$

The steady-state intensity I_1 is that at which the rates of absorption and emission are just balanced, so that when $I = I_1$, $\frac{dI}{dt} = 0$, and we have $\frac{E}{V} - AI_1 = 0$, whence

$$I_1 = \frac{E}{VA}$$

and

$$AT_1 = \log_e \frac{I_1}{i} = \log_e \frac{E}{VAi} \tag{48}$$

and

$$\log_e \frac{E}{i} = AT_1 + \log_e VA$$

Approximate Value of Mean Free Path.

Equations (43) and (46) make it possible to arrive at an approximate experimental value of the mean free path. To a first approximation

$$p = \frac{ac}{AS} = \frac{0.171cV}{AST_1} \tag{49}$$

By performing the four-organ-pipe experiment in two rooms of the same relative dimensions but of different volumes, Sabine arrived at a tentative value of $p = 0.62V^{1/3}$. This clearly is not an exact expression, since it is apparent that p, the average distance between reflections, must depend upon the shape as well as the volume. For this reason, it was desirable to put the expression for p in a form which includes this fact. This was done by a more exact experimental determination of the constant K making use of the results of the four-organ experiment.

Precise Evaluation of the Constant K.

The experiment with the seat cushions and open windows led to the approximate relation $aT_1 = 0.171V$, while the four-organ experiment gave the equation $aT_1 = \frac{Sp}{c} \log_e \frac{I_1}{i}$.

The complete solution of the problem then involves an exact correlation of the expressions $0.171V$ and $\frac{Sp}{c} \log_e \frac{I_1}{i}$.

We note immediately that since p is proportional to $V^{1/3}$ and the surface S is proportional to $V^{2/3}$, their product must be proportional to V itself. We may therefore write Eq. (47) in the form

$$aT_1 = kV \log_e \frac{I_1}{i} = kV \log_e \left[\frac{ESp}{Vaci}\right] \tag{50}$$

In the rooms in which his experiments had been conducted, the steady-state intensities set up by the particular pipe employed were of the order of $10^6 \times i$. Sabine therefore adopted this as a standard intensity, and upon the assumption of this fixed steady-state intensity, the results of the four-organ experiment reduce to the form given by the cushion experiments, namely,

$$aT_0 = kV \log_e 10^6 = k \times 13.8V$$

where T_0 is the time required for a sound of initial intensity $10^6 \times i$ to decrease to the threshold intensity, and k is a new constant whose precise value is desired.

One is compelled to admire the skill with which Sabine handled the various approximate relations in order to arrive at as precise a value as possible for his fundamental constant. His procedure was as follows: From the four-organ-pipe experiment he derived the value of E, the acoustic power of his organ pipe. He then measured the times with this pipe as a source in a number of rooms of widely varying sizes and shapes, with windows first closed and then open. He assumed that the open windows produced the same change in reverberation as would a perfectly absorbing surface of equal area.

Let w equal the open-window area, and T'_1 the time under the open-window condition. Then

$$(a + w)T'_1 = kV \log_e \left[\frac{ESp}{ciV(a + w)}\right] \qquad (51)$$

Dividing (50) by (51) gives

$$\frac{aT_1}{(a + w)T'_1} = \frac{\log_e \left[\frac{ESp}{ciVa}\right]}{\log_e \left[\frac{ESp}{ciV(a + w)}\right]} = \frac{\log_e \frac{I_1}{i}}{\log_e \frac{I'_1}{i}}. \qquad (52)$$

In this equation we have two quantities a and p, both of which are known approximately. $a = 0.171V/T$ and $p = 0.62V^{1/3}$ are both approximations. In the right-hand member of (52), these quantities are involved only in logarithmic expressions, so that slight departures from their true values will not materially affect the numerical value of the right-hand member of the equation. Evaluating the right-hand member of (52) in this way, the equation was solved for a. Using this value of a, Eq. (50) may be solved for k. The constant K of Sabine's simple reverberation equation is $13.8k$.

The following are the data and the results of the experiment for the precise determination of K.

TABLE IV

Places of experiment	V	I_1/i	w	K
Lobby, Fogg Art Museum:				
1 pipe	96	8,800,000	1.86	0 159
16 pipes	96	67,000,000	1.86	0.164
Jefferson Laboratory:				
Room 15	202	1,000,000	5 10	0.169
Room 1	1,630	390,000	12.0	0 167
Room 41	1,960	300,000	14.6	0.161
				Ave. 0.164

Experimental Value of Mean Free Path.

Equation (47) and Sabine's experimental value for K enable us to arrive at an expression for p, the mean free path in terms of the volume and bounding surface of an inclosure. If in Eq. (47) I/i is put equal to 1,000,000, we have

$$aT_0 = \frac{Sp}{c} \log_e (1{,}000{,}000) = 13.8\frac{Sp}{c} = 0.164V$$

With a velocity of sound at 20° C. of 342 m. per second, this gives

$$p = \frac{4.06V}{S} \tag{53}$$

The theoretical derivation of the relation $p = 4V/S$ is given in Appendix *B*. The close agreement between the experimental and theoretical relations furnishes abundant evidence of the validity of the general theory of reverberation as we have it today. We shall hereafter use the value of $K = 0.162$, corresponding to the theoretically derived value of p, where V is expressed in cubic meters and a is expressed in square meters of perfectly absorbing surface. If these quantities are expressed in English units, the reverberation equation becomes

$$aT_0 = 0.0494V$$

Complete Reverberation Equation.

In Chap. IV, the picture of the phenomenon of reverberation was drawn from the simple one-dimensional case of sound in a tube and extended by analogy to the three-dimensional case of sound within a room, considering the building up and decay of the sound. In the preceding paragraphs, we have followed the experimental work, as separate processes, by which the fundamental principles were established and the necessary constants were evaluated. We shall now proceed to the formal derivation of a single equation giving the relation between all the quantities involved.

The underlying assumptions are as follows:

1. The acoustic energy generated per second by the source is constant and is not influenced by the reaction of sound already in the room.

2. The sustained operation of the source sets up a final steady-state intensity. In this steady state, the sound energy in the room is assumed to be "diffuse"; that is, its average energy density is the same throughout the room, and all directions of energy flow at any point are equally probable.

3. In the steady state, the total energy per second generated at the source equals the total energy dissipated per second by absorption at the boundaries. Dissipation of energy throughout the volume of the room is assumed to be negligibly small.

4. The time rate of change of average intensity at any instant is directly proportional to the intensity at that instant, provided the source is not operating.

5. At any given surface whose dimensions are large in comparison with the wave length, a definite fraction of the energy of the incident diffuse sound is not returned by reflection to the room. This fraction is a function of the pitch of the sound, but we shall assume it to be independent of the intensity. It is the absorption coefficient of the surface.

6. The absorbing power of a surface is the product of the absorption coefficient and the area. The "total absorbing power of the room" is the sum of the absorbing powers of all of the exposed surfaces in the room.

While the source is operating, the rate of change of energy density is the difference between the rates per unit volume of emission and absorption or

$$\frac{dI}{dt} = \frac{E}{V} - AI$$

Remembering that in the steady state, the rate of change of intensity $dI/dt = 0$ and that therefore $AI_1 = E/V$, we may write

$$\frac{dI}{dt} = A(I_1 - I)$$

Integrating and supplying the constant of integration, we have

$$I_t = I_1(1 - e^{-At}) = \frac{E}{AV}(1 - e^{-At}) \qquad (54)$$

If after the source has been operating for a time t it is suddenly stopped, the sound begins to die away. During this stage $E = 0$, and, denoting the time measured from the instant of cut-off by T, we have for the decay process

$$\frac{dI}{dT} = -AI$$

Integrating, and supplying the constant of integration from the fact that at the moment of cut-off $T = 0$, $I_T = I_t$, we have

$$I_T = I_t e^{-AT}$$

and

$$\frac{I_t}{I_T} = e^{AT} \text{ or } \frac{E(1 - e^{-At})}{AVI_T} = e^{AT}$$

Taking the logarithm of both sides, we have

$$AT = \log_e \left[\frac{E}{AVI_T}(1 - e^{-At})\right] \qquad (55a)$$

In order not to complicate matters unduly, let us simply call T the time from cut-off required for sound to decrease to the threshold intensity i. Putting $I_T = i$, (55a) then is written

$$AT = \log_e \left(\frac{E}{AVi}(1 - e^{-At})\right) \qquad (55b)$$

This may be expressed in terms of the total absorbing power a, by the relations already deduced, namely, $A = ac/Sp = ac/4V$,

whence we have $\frac{acT}{4V} = \log_e \frac{4E}{aci}\left(1 - e^{\frac{act}{4V}}\right)$

and

$$aT = \frac{4V}{c} \log_e \left\{\frac{4E}{aci}\left(1 - e^{\frac{-act}{4V}}\right)\right\} \qquad (56)$$

Here t is the time interval during which the source speaks, while T is the period from cut-off to the instant at which the sound becomes inaudible

The expression $1 - e^{\frac{-act}{4V}}$ is the fraction of the final steady intensity which the intensity of the sound in the building-up process attains in the time t. The greater the ratio of absorbing power to volume the shorter the time required to reach a given fraction of the steady state. As t increases indefinitely, $e^{\frac{-act}{4V}}$ approaches zero, and I_t approaches I_1, the steady-state intensity. If T_1 equals the time required for the intensity to decrease from this steady state to the threshold, we have

$$aT_1 = \frac{4V}{c} \log_e \left(\frac{4E}{aci}\right). \tag{57}$$

which reduces to the Sabine equation $aT_0 = 0.162V$, if $4E/aci$ is set equal to 10^6.

The whole history of the building-up and decay of sound in a room is shown graphically in Fig. 26.[1] It must be

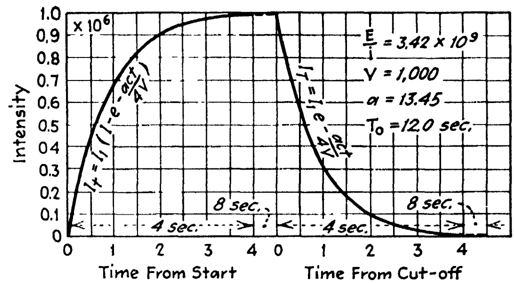


FIG. 26.—Statistical growth and decay of sound in a room—absorption assumed continuous.

remembered that what is here shown is the theoretical value of the average intensity (sound-energy density). In both the building-up and decay processes, the actual intensity at any point fluctuates widely as the interference pattern referred to in Chap. III shifts in an altogether undetermined way. Figure 27 is an oscillograph record of the decay of

[1] For an instructive series of curves, showing many of the implications of the general reverberation equation as related to the acoustic properties of rooms, the reader is referred to an article by E. A. Eckhardt, *Jour. Franklin Inst.*, vol. 195, p. 799, 1923.

sound in a room, kindly furnished by Mr. Vesper A. Schlenker.[1]

Extension of Reverberation Principles to Other Physical Phenomena.

Reference should be made here to an extremely interesting theoretical paper by Dr. M. J. O. Strutt,[2] in which from

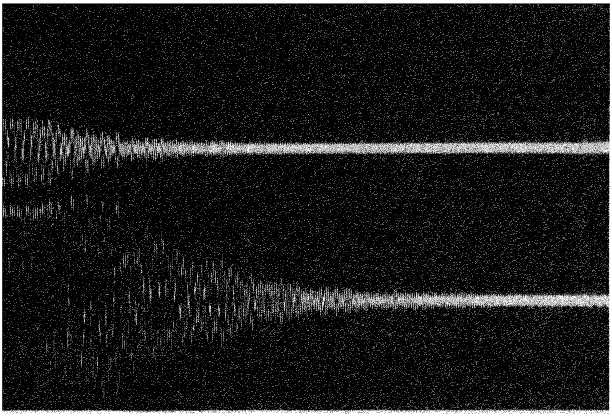


FIG. 27.—Oscillogram of decay of sound at a single point in a room. (*Courtesy of V. A. Schlenker.*)

the most general hydrodynamic considerations he deduced Sabine's law. This he states as follows:

> The duration of residual sound in large rooms measured by the time required for the intensity to decrease to 1/1,000,000 of the steady-state intensity is proportional to the volume of the room over the total absorbing power but does not depend upon the shape of the room, the places of source, and experimenter, while the frequency does not change much after the source has stopped.

He shows that the law holds even without the assumption of a diffuse distribution in the steady state. Further, he shows that the law does not hold unless the frequency of the source is much higher than the lowest resonance fre-

[1] A Truck-mounted Laboratory, *Jour. Soc. Mot. Pict. Eng.*, vol. 16, No. 3, p. 302.

[2] *Phil. Mag.*, vol. 8, pp. 236–250, 1929.

quencies of the room, that is, unless the dimensions of the room are considerably greater than the wave length of the sound. This has an important bearing upon the problem of the measurement of sound absorption coefficients by small scale methods.

It is also shown that a similar law holds for other physical phenomena, as, for example, radiation within a closed space and the theory of specific heats of solid bodies.

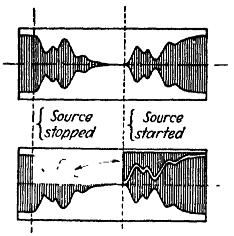


FIG. 28.—Oscillograms showing that the building up and dying out of sound are complementary. (*After Strutt.*)

Perhaps the most interesting fact brought out by Strutt is the proof that the amplitude at any point in the room at any time in the building-up process is complementary to the corresponding amplitude in the decay process; that is, at a given point, the amplitude at the time t measured from the moment of starting plus the amplitude at the time T measured from stopping the source after the steady state has been reached equals the amplitude in the steady state. This is shown in Fig. 28 taken from Strutt's paper. That this is true for the average intensities follows from the fact that in the building-up process, $I_1 - I_t$ decreases according to the same law as that followed by I_T in the decay process. That it should be true for building-up and decaying intensities at every point of the room is a fact not brought out by the elementary treatment here given.

Summary.

In the present chapter we have followed Sabine's attack on the problem of reverberation from the experimental side to the point of a precise evaluation of the constant in his well-known equation. This equation has been shown to be a special case of the more general relation given in Eq. (56). He expressed all the relations with which we are concerned in terms of this constant K with correction terms to take care of departures from the assumption of a steady-state intensity of $10^6 \times i$. The simplicity of Sabine's equation makes it extremely useful in practical applications of the theory. It is well, however, to hold in mind the rather special assumptions involved in its derivation.

CHAPTER VI

MEASUREMENT OF ABSORPTION COEFFICIENTS

In order to apply the theory of reverberation developed in Chaps. IV and V to practical problems of the control of this phenomenon, it is necessary to have information as to the sound-absorptive properties of the various materials that enter into the finished interiors of rooms. These properties are best expressed quantitatively by the numerical values of the sound-absorption coefficients of materials. In this chapter, we shall be concerned with the precise significance of this term and the various methods employed for its determination.

Two Meanings of Absorption Coefficient.

In the study of the one-dimensional case of sound in a tube, α (the absorption coefficient) was defined as the fraction of itself by which the incident energy is reduced at each reflection from the end of the tube. Here we were dealing with a train of plane waves, incident at right angles to the absorbing surface. If I_d is the energy density in the direct train and I_r that in the reflected train, then

$$\alpha = \frac{I_d - I_r}{I_d} = 1 - \frac{I_r}{I_d} \tag{58a}$$

In going to the three-dimensional case, diffuse sound was assumed to replace the plane unidirectional waves in the tube. In a diffuse distribution, all angles of incidence from 0 to 90 deg. are assumed to be equally probable. Now it is quite possible that the fraction of the energy absorbed at each reflection will depend upon the angle at which the wave strikes the reflecting surface, so that the absorption coefficient of a given material measured for normal incidence may be quite different from that obtained by reverberation methods. It is not easy to subject the question to direct experiment. In view of the fact that there is not a very close agreement between measurements made by the two methods, we shall for the sake of clarity refer to coefficients obtained by measurements in tubes as "stationary-wave coefficients." Coefficients obtained by reverberation methods we shall call "reverberation coefficients."

Measurement of Stationary-wave Coefficients. Theory.

In Eq. (23*a*), Chap. III, let the origin be at the absorbing surface. Then the equation for the displacement at any point at a distance x from this surface due to both the direct and reflected waves is

$$\xi_{d+r} = A_0\left[\sin \omega\left(t + \frac{x}{c}\right) - k \sin \omega\left(t - \frac{x}{c}\right)\right] =$$

$$A_0\left[(1 - k) \sin \omega t \cos \frac{\omega x}{c} + (1 + k) \cos \omega t \sin \frac{\omega x}{c}\right] \quad (58b)$$

If we elect to express the condition in the tube in terms of the pressure, we have

$$dP_{(d+r)} = \frac{\gamma P_0 \omega A_0}{c}\left[(1 - k) \sin \omega t \sin \frac{\omega x}{c} + (1 + k) \cos \omega t \cos \frac{\omega x}{c}\right] \quad (58c)$$

For values of $\omega x/c = 0, \pi, 2\pi$, etc., $dP_{\text{max.}} = B(1 + k)$, while for the values of $\omega x/c = \pi/2, 3\pi/2$, etc., $dP_{\text{max.}} = B(1 - k)$, where B is a constant.

Let M be the value of dP_{max} at the pressure internodes

Let N be the value of dP_{max} at the pressure nodes

$$M = B\ (1 + k)$$
$$N = B\ (1 - k)$$

Adding,

$$M + N = 2B$$

Subtracting,

$$M - N = 2kB$$

whence

$$k = \frac{M - N}{M + N}$$

For a fixed frequency, the intensity is proportional to the square of the amplitude. Then

$$\frac{I_r}{I_d} = k^2$$

By definition,

$$\alpha = 1 - \frac{I_r}{I_d}, = 1 - k^2 = 1 - \left(\frac{M - N}{M + N}\right)^2$$

or

$$\alpha = \frac{4MN}{(M + N)^2} \qquad (59)$$

The absorption coefficient then can be determined by the use of any device the readings of which are proportional to the alternating pressure in the stationary wave. It is obvious that the absolute value of the pressures need not be known, since the value of α depends only upon the relative values of the pressures at the maxima and minima. One notes further that the percentage error in α is almost the same as the percentage error in N, the relative pressure at the minimum. For materials which are only slightly absorbent N will be small, and for a given absolute error the percentage error will be large. For this reason the stationary-wave method is not precise for the measurement of small absorption coefficients. It is also to be observed that a velocity recording device may be used in the place of one whose readings are proportional to the pressure. M and N would then correspond to velocity maxima and minima respectively.

The foregoing analysis assumes that there is no change of phase at reflection from the absorbent surface. A more detailed analysis shows[1] that there is a change of phase upon reflection. The effect of this, however, is simply to shift the maxima and minima along the tube.[2] If, in the experimental procedure, one locates the exact position of the maxima and minima by trial and measures the pressure at these points, no error due to phase change is introduced.

[1] Paris, E. T., *Proc. Phys. Soc. London*, vol. 39, No. 4, pp. 269–295, 1927.

[2] Davis and Evans, *Proc. Roy. Soc.*, Ser. A, vol. 127, pp. 89–110, 1930.

Dissipation along the Tube.

Eckhardt and Chrisler[1] at the Bureau of Standards found that due to dissipation of energy along the tube there was a continuous increase in the values of both the maxima and minima with increasing distance from the closed end. Thus the oncoming wave diminishes in amplitude as it approaches the reflecting surface, while the reflected wave diminishes in amplitude as it recedes from the reflecting surface. Taking account of this effect, Eckhardt and Chrisler give the expression

$$\alpha = 1 - \left[\frac{M_1 - N_1 + \dfrac{N_2 - N_1}{2}}{M_1 + N_1 - \dfrac{N_2 - N_1}{2}} \right]^2 \tag{60}$$

$N_2 - N_1$ is the difference between the pressures measured at two successive minima.

Davis and Evans give this correction term in somewhat different form. Assuming that as the wave passes along the tube the amplitude decreases, owing to dissipation at the walls of the tube, according to the law

$$A_x = A_0 e^{-\Delta x}$$

the values for the *n*th maximum and minimum respectively are

$$M_n = M + \tfrac{1}{2}(n - 1)\Delta\lambda N \tag{61}$$

$$N_n = N + \frac{(2n - 1)}{4}\Delta\lambda M \tag{62}$$

In order to make the necessary correction, a highly reflecting surface was placed in the closed end of the tube, and successive maxima and minima were measured. From these data, the value of Δ was computed, which in Eqs. (61) and (62) gave the values for M and N to be used in Eq. (59).

It is to be noted that since N is small in comparison with M, the correction in the former will have the greater

[1] *Bur. Standards Sci. Paper* 526, 1926.

effect upon the measured value of α. For precision of results therefore it is desirable to have Δ, the attenuation coefficient of the tube, as small as possible. This condition is secured by using as large a tube as possible, with smooth, rigid, and massive walls. The size of the tube that can be used is limited, however, since sharply defined maxima and minima are difficult to obtain in tubes of large diameter. Davis and Evans found that with a pipe 30 cm. in diameter, radial vibrations may be set up for frequencies greater than 1,290 cycles per second. This of course would vitiate the assumption of a standing-wave system parallel to the axis of the tube, limiting the frequencies at which absorption coefficients can be measured in a tube of this size.

Standing-wave Method (Experimental).

H. O. Taylor[1] was the first to use the stationary-wave method of measuring absorption coefficients in a way to yield results that would be comparable with those obtained by reverberation methods.[2] The foregoing analysis is essentially that given by him. For a source of sound he used an organ pipe the tone of which was freed from harmonics by the use of a series of Quincke tubes, branch resonators, each tuned to the frequency of the particular overtone that was to be filtered out. The tube was of wood, 115 cm. long, with a square section 9 by 9 cm. The end of this tube was closed with a cap, in which the test sample was fitted. A glass tube was used as a probe for exploring the standing-wave system. This was connected with a Rayleigh resonator and delicately suspended disk. The deflections of the latter were taken as a measure of the relative pressures at the mouth of the exploring tube and gave the numerical values of M and N in Eq. (59).

Tube Method at the Bureau of Standards.

Figure (29) illustrates the modification of Taylor's experimental arrangement developed and used for a time

[1] *Phys. Rev.*, vol. 2, p. 270, 1913.

[2] The method was first proposed by Tuma (*Wien. Ber.*, vol. 111, p. 402, 1902), and used by Weisbach (*Ann. Physik*, vol. 33, p. 763, 1910).

at the Bureau of Standards.[1] The source of sound was a loud-speaker, supplied with alternating current from a vacuum-tube oscillator. The standing-wave tube was of brass and was tuned to resonance with the sound produced by the loud-speaker. The exploring tube was terminated by a telephone receiver, and the electrical potential generated in the receiver by the sound was taken as a measure of the pressure in the standing wave. In order to measure

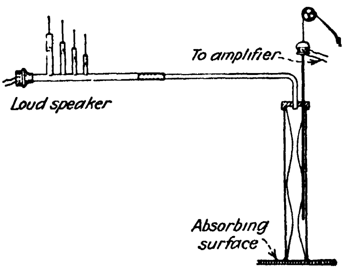


FIG. 29.—Apparatus for measuring absorption coefficients formerly used at the Bureau of Standards.

the e.m.f. generated by the sound, the current from the telephone receiver after amplification and rectification was led into a galvanometer. The deflection of the galvanometer produced by the sound was then duplicated by applying an alternating e.m.f. from a potentiometer whose current supply was taken from the oscillator. The potentiometer reading gave the e.m.f. produced by the

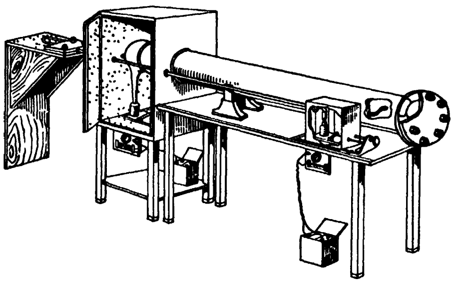


FIG. 30.—Stationary-wave apparatus for absorption measurements used at the National Physical Laboratory, Teddington, England.

sound. These readings at maxima and minima gave the M's and N's of Eq. (59), from which the absorption coefficients of the test samples were computed.

[1] ECKHARDT and CHRISLER, *Bur. Standards Sci. Paper* 526, 1926.

Dr. E. T. Paris,[1] working at the Signals Experimental Establishment at Woolwich, England, has carried out investigations on the absorption of sound by absorbent plasters using the standing-wave method. His intensity measurements were made with a hot-wire microphone.

Work at the National Physical Laboratory.

Some extremely interesting results have been obtained by the stationary-wave method by Davis and Evans, at the National Physical Laboratory at Teddington,[2] England.

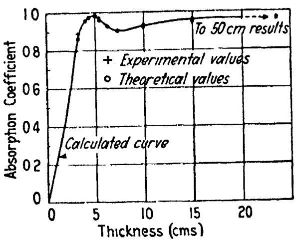


FIG. 31.—Absorption coefficient as a function of the thickness. (*After Davis and Evans.*)

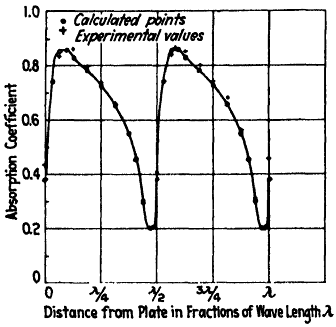


FIG. 32.—Variation in absorption affected by position of test sample in stationary wave.

This apparatus is shown in Fig. 30 and is not essentially different in principle from that already described. Experimental details were worked out with a great deal of care, and tests of the apparatus showed that its behavior was quite consistent with what is to be expected on theoretical grounds. A loud-speaker source of sound was used. The exploring tube was of brass 1.2 cm. in diameter, which communicated outside the stationary-wave tube with a moving-coil loud-speaker movement. The pressure measurements were made by determining the e.m.fs. generated in a manner similar to that employed at the Bureau of Standards.

[1] *Proc. Phys. Soc.*, vol. 39, p. 274, 1927.

[2] DAVIS and EVANS, *Proc. Roy. Soc. London*, Ser. A, vol. 127, 1930.

Among the facts brought out in their investigations was the experimental verification of a prediction made on theoretical grounds by Crandall,[1] that at certain thicknesses an increase of thickness will produce a decrease of absorption. The phenomenon is quite analogous to the selective reflection of light by thin plates with parallel surfaces. In Fig. 31, the close correspondence between the theoretical and experimental results is shown. A second interesting fact is presented by the graph of Fig. 32, in which the absorption coefficient of ½-in. felt is plotted against the distance expressed in fractions of a wave length at which it is mounted from the reflecting end plate of the tube. We

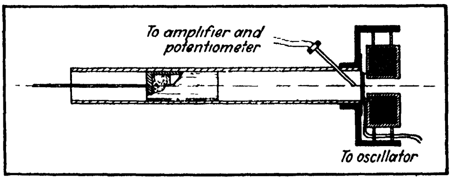


FIG. 33.—Measurement of absorption coefficient by impedance method. (*After Wente.*)

note that the absorption is a minimum at points very close to what would be the pressure internodes and the velocity nodes of the stationary wave if the sample were not present. In other words, the absorption is least at those points where the particle motion is least. The points of maximum absorption are not at what would be the points of maximum motion in the tube if the sample were not present. The presence of the sample changes the velocity distribution in the tube.

The very great increase in the absorption coefficient when the sample is mounted away from the backing plate is to be noted. It naturally suggests the question as to how we shall define the absorption coefficient as determined by standing-wave measurements. Its value obviously depends upon the position of the sample in the standing-wave pattern. As ordinarily measured, with the sample mounted

[1] "Theory of Vibrating Systems and Sound," D. Van Nostrand Company, p. 195

at the closed end of the tube, the measured value is that for the sample placed at a pressure node, almost the minimum. We shall consider this question further when we come to compare standing-wave and reverberation coefficients.

Absorption Measurement by Acoustic Impedance Method. This method was devised by E. C. Wente of the Bell Telephone Laboratories.[1] The analysis is based upon the analogy between particle velocity and pressure in the standing-wave system and the current and voltage in an electrical-transmission line. Acoustic impedance is defined as the ratio of pressure to velocity. The experimental

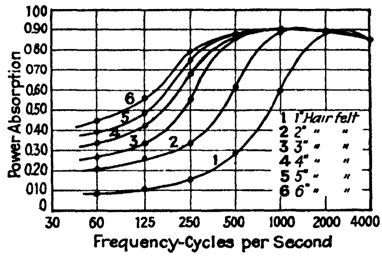


FIG. 34.—Absorption coefficients of felt by impedance method. (*After Wente.*)

arrangement is indicated in Fig. 33. The tube was 3-in. internal diameter Shelby tubing, 9 ft. long, with ¼-in. walls. The test sample was mounted on the head of a nicely fitting piston which could be moved back and forth along the axis of the tube. The source of sound was a heavy diaphragm 2⅞ in. in diameter, to which was attached the driving coil lying in a radial magnetic field. The annular gap between the diaphragm and the interior of the tube was filled with a flexible piece of leather.

Instead of measuring the maximum and minimum pressures at points in a tube of fixed length, the pressure at a point near the source, driven at constant amplitude, is measured for different tube lengths. Wente's analysis

[1] *Bell System Tech. Jour.*, vol. 7, pp. 1–10, 1928.

gives for the absorption coefficient in terms of the maximum and minimum pressures measured near the source

$$\alpha = \frac{\sqrt{M_s N_s}}{M_s + 2\sqrt{M_s N_s} + N_s} \tag{63}$$

The pressures were determined by measuring with an alternating-current potentiometer the voltages set up in a telephone transmitter, connected by means of a short tube, to a point in the large tube near the source. Figure 34 shows the absorption coefficients of felt of various thicknesses as measured by this method. Figure 35 shows the effect on the absorption produced by different degrees of

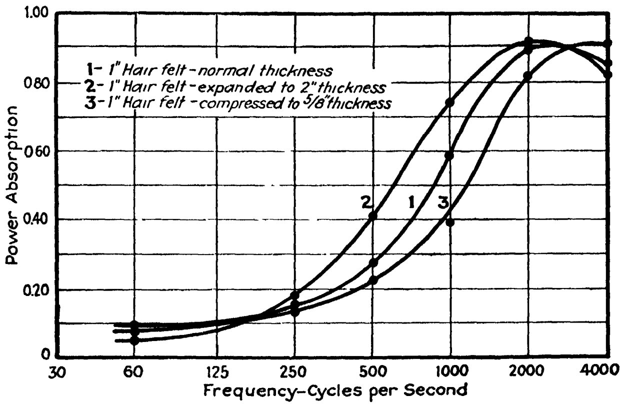


FIG. 35.—Effect of compression of hair felt on absorption coefficients.

packing of the hair felt. The three curves were all obtained upon the same sample of material but with different degrees of packing. They are of a great deal of significance in the light which they throw upon the discrepancy of the results obtained by different observers in the measurement of absorption coefficients of materials of this sort. Both thickness and degree of packing produce very large differences in the absorption coefficients, so that it is safe to say that differences in the figures quoted by different authorities are due in part at least to actual differences in the samples tested but listed under the same description.

TABLE V.—STATIONARY-WAVE COEFFICIENTS OF MATERIALS

Materials	1		2		3		4		5		Observer
	Frequency	Coefficient	Frequency	Coefficient	Frequency	Coefficient	Frequency	Coefficient	Frequency	Coefficient	
Felt:											
1.3 cm.	250	0 03	500	0 10	800	0 26	1,000	0.35	1,200	0 46	Davis and Evans
1.9 cm.	250	0 07	500	0.21	800	0.41	1,000	0.43	1,200	0.63	Davis and Evans
2.5 cm.	297	0 33	..	....	...	....	1,095	0.62	2,190	0.64	Bur. Standards
2.5 cm.	250	0 08	500	0.19	...	....	1,000	0.35	1,200	0.45	Davis and Evans
2.5 cm.	250	0.15	500	0 28	800	0 36	1,000	0.60	2,000	0 90	Wente
5.0 cm.	250	0.33	500	0.60		...	1,000	0.88	2,000	0.90	Wente
Acoustic tile:											
2.5 cm.	..	.....	569	0 30	...	....	1,095	0.43	2,190	0.38	Bur. Standards
2 5 cm.	250	0.11	500	0.18	..	...	1,000	0.48	2,000	0.76	Davis and Evans
Celotex:											
B	297	0.18	...	....	...	....	1,095	0.40	2,190	0.73	Bur. Standards
B	250	0.12	500	0.23	..	...	1,000	0 45	1,200	0.53	Davis and Evans
BB.	250	0.19	500	0.37	800	0 56	1,000	0.61	1,200	0.69	Davis and Evans
Standard celotex.	...	..	500	0.04	800	0 06	1,000	0.08	1,200	0.12	Davis and Evans
Standard celotex.	297	0 058	..	....	...	....	1,095	0.056	2,190	0 07	Bur. Standards

Comparison of Standing-wave Coefficients by Different Observers.

In Table V are given values of the absorption coefficients of a number of materials which have been measured by the investigators mentioned, using some form of the standing-wave method.

For values on other materials reference may be made to the papers cited. Unfortunately there has not been any standard practice in the choice of test frequencies. Moreover, in certain cases all the conditions that affect the absorption coefficients are not specified, so that comparison of exact values is not possible. The materials here given were selected as probably being sufficiently alike under the different tests to warrant comparison with each other and also with the results of measurements made by reverberation methods. Inspecting the table, one notes rather wide differences between the results obtained by different observers in cases where a fair measure of agreement is to be expected.

On the whole, it has to be said that while the stationary-wave method has the advantage of being applicable to measurements on small samples and, under a certain fixed condition, is capable of giving results that are of relative significance, yet it is questionable whether coefficients of absorption so measured should be used in the application of the reverberation theory.

Reverberation Coefficient: Definition.

We shall define the reverberation coefficient of a surface in terms of the quantities used in the reverberation theory developed in Chaps. IV and V. We can define it best in terms of the total absorbing power of a room, and this in turn can be best defined in terms of A, the rate of decay. A is defined by the equation

$$\frac{dI}{dt} = -AI$$

and a in turn by the relation

$$a = \frac{4AV}{c}$$

We shall define the absorption coefficient of a given surface as its contribution per unit area to the *total absorbing power* (as just defined) of a room. If, therefore, α_1, α_2, α_3, etc., be the reverberation coefficients of the exposed surfaces whose areas are s_1, s_2, s_3, etc., then

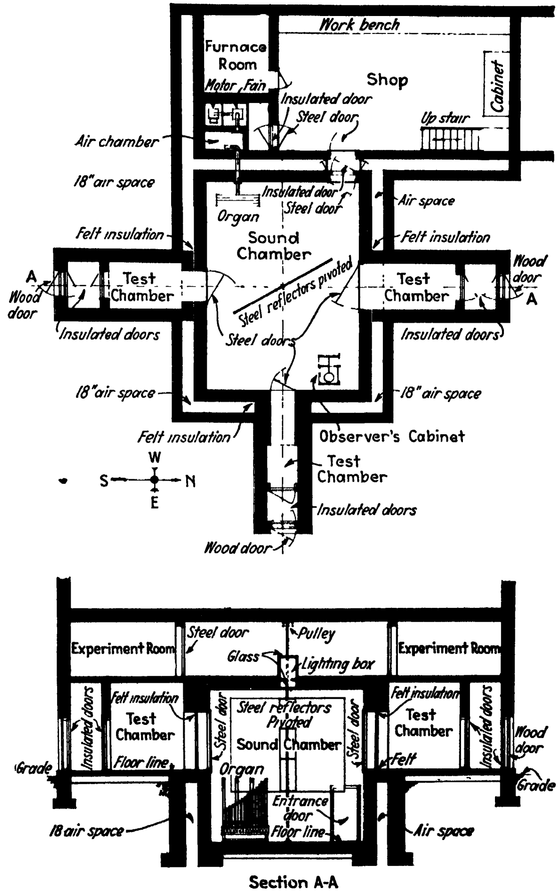
$$a = \alpha_1 s_1 + \alpha_2 s_2 + \alpha_3 s_3 + \cdots$$

where the summation includes all the surfaces in the room exposed to the sound. As will appear, this definition conforms to the usual practice in reverberation measurements of absorption coefficients and is based on the assumptions made in the reverberation theory.

Sound Chamber.

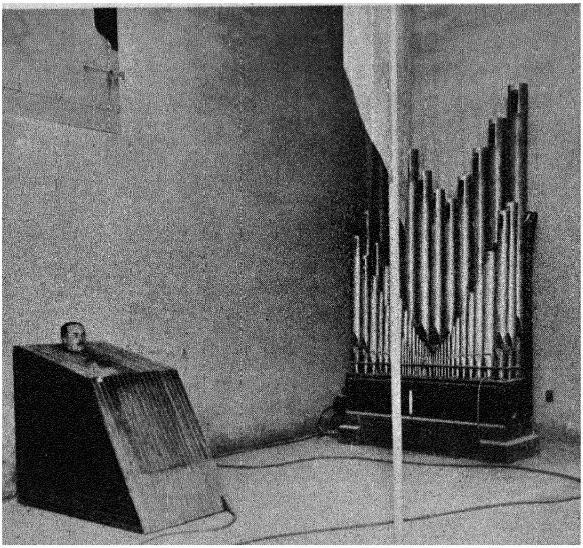
Any empty room with highly reflecting walls and a sufficiently long period of reverberation may be used as a sound chamber. The calibration of a sound chamber amounts simply to determining the total absorbing power of the room in its standard condition for tones covering the desired frequency range. This standard condition should be reproducible at will. For this reason, whatever furnishing it may have in the way of apparatus and the like should be kept fixed in position and should be as non-absorbent as possible. If methods depending upon the threshold of audibility are employed, the room should be free from extraneous sounds. In any event, the sound level from outside sources should be below the threshold of response of the apparatus used for recording sound intensities. The ideal condition would be an isolated structure in a quiet place, from which other activities are excluded. If the sound chamber is a part of another building, it should be designed so as to be free from noises that originate elsewhere. The first room of this sort to be built is that at the Riverbank Laboratories, designed by Professor Wallace Sabine shortly before his death and built for him by Colonel

George Fabyan. It was fully described in the *American Architect* of July 30, 1919, but for those readers to whom that article may not be accessible, the plan and section of the room are here shown.



Plan and section of Riverbank sound chamber

The dimensions of the room are 27 ft. by 19 ft. by 19 ft. 10 in., and the volume is 10,100 cu. ft. (286 cu. m.). To diminish the inequality of distribution of intensity due to interference, large steel reflectors mounted on a vertical



View of sound chamber of the Riverbank Laboratories.

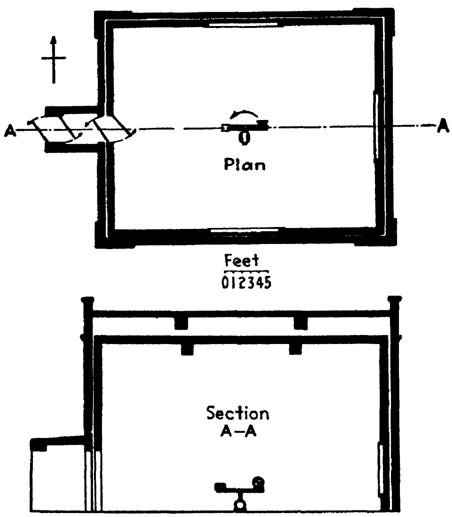


FIG. 36.—Plan and section of sound chamber at Bureau of Standards.

shaft are rotated noiselessly during the course of the observations. The source of sound usually employed is a 73-pipe organ unit, with provisions for air supply at constant pressure to the pipes. A loud-speaker operated by vacuum-tube oscillator and amplifier is also used as a sound source.

Other sound chambers have been subsequently built in this country, notably those at the University of Michigan and the University of California at Los Angeles. The plan and section of the chamber recently built at the Bureau of Standards[1] are shown in Fig. 36. The dimensions of this room are 25 by 30 by 20 ft., and the double walls are of brick 8 in. thick with 4-in. intervening air space. In order to reduce the effect of the interference pattern, the source of sound is moved on a rotating arm, approximately 2 ft. long, during the course of the observations.

Sound-chamber Methods: Constant Source.

Essentially any sound-chamber method of measuring the reverberation coefficients of a material is based upon measuring the change in total absorbing power produced by the introduction of the material into a room whose total absorbing power without the material is known. If a and a' be the total absorbing power of the room, first empty and then with an area of s square units of absorbing material introduced, then $(a' - a)/s$ is the increase in absorbing power per square unit effected by the material. If the test material replaces a surface of the empty room whose absorption coefficient is α_1, then the absorption coefficient of the material in question is

$$\alpha = \alpha_1 + \frac{a' - a}{s} \tag{64}$$

The equations of Chap. V suggest various ways in which a and a' may be measured, either by measurements of time of decay or by measurements of intensities. The procedure developed by Professor Sabine which has been followed

[1] *Bur. Standards Res. Paper* 242.

for the most part by investigators since his time is as follows:

1. The value of a, the absorbing power of the empty room, is determined by means of the four-organ experiment or some modification thereof, in which the times for sources of known relative powers are measured.

2. From the known value of the absorbing power of the empty room and the time required for the reverberant sound from a given source to decrease to the threshold of a given observer, the ratio E/i for this particular source and observer can be computed by Eq. (39).

3. E/i being known, and assuming that E, the acoustic output of the source, is not influenced by altering the absorbing power of the room, Eq. (39) is evoked to determine a' from the measured value of T', the measured time when the sample is present. Since Eq. (39) contains both a' and $\log a'$, its solution for a has to be effected by a method of successive approximations. As a matter of convenience, therefore, it is better to compute the values of T' for various values of a' and plot the value of a' as a function of T'. The values of a' for any value of T' are then read from this curve.

Calibration of Sound Chamber: Four-organ Method.

Illustrating the method of calibration and the measurement of absorption coefficients outlined above, the procedure followed at the Riverbank Laboratories will be given somewhat in detail.[1] Four small organs, each provided with six C pipes, 128 to 4,096 vibs./sec., operated by electropneumatic action, were set up in the sound chamber. These were operated from a keyboard in the observer's cabinet, wired so that each pipe of a given pitch could be made to speak singly or in any combination with the other pipes of the same pitch. Air pressure was supplied from the organ blower outside the room. The pressure was controlled by throttling the air supply so that the speaking

[1] *Jour. Franklin Inst.*, vol. 207, No. 3, p. 341, 1929.

pressure was the same whether one or four pipes were speaking. The absorbing power of the sound chamber was increased over that of its standard condition by the presence of this apparatus. Before the experiment the four pipes of each pitch were carefully tuned to unison. Slight variations of pitch were found to produce a marked difference in the measured time. As in all sound-chamber experiments, the large steel reflector was kept revolving at the rate of one revolution in two minutes. It was found that even with the reflector in motion, the observed time varied slightly with the observer's position. For this reason timings with each combination of pipes were made in five different positions. At the end of the series of readings, the time for the pipe of the large organ was measured, with the four-organ apparatus first in and then out of the room. This gave the necessary data for evaluating E/i for the pipes of the large organ used as the standard sources of sound and also for determining the absorbing power of the room in its standard condition from the measurements made with the four organs present.

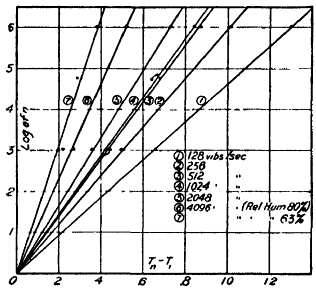


FIG. 37.—Difference in reverberation times as a function of the logarithm of the number of pipes.

Figure 37 gives the results of the four-organ experiment made in 1925. The maximum departure from the straight-line relation called for by the theory is 0.04 sec.

Let a' be the absorbing power at a given frequency of the sound chamber with the four organs present.

$$a' = \frac{4V \log_e n}{c(T_n - T_1)} = \frac{4 \times 286 \times 2.3 \log_{10} n}{342(T_n - T_1)} = 7.7m$$

where m is the slope of the corresponding line of Fig. 37. Computing E/i for the pipe of the large organ and the particular observer, we can compute a for the sound chamber in its standard condition.

TABLE VI.—SOUND-CHAMBER CALIBRATION, FOUR-ORGAN PIPE METHOD

Frequency	T'	T	a'	a	$\log_{10} \frac{E}{i}$
128	12 58	14 70	3.56	3 09	8.31
256	12.42	14 50	4.67	4.03	10.14
512	11.94	14.25	5.38	4.55	11.01
1,024	11.32	12.80	5.58	4.98	10 91
2,048	9.16	9.86	6.34	5.92	10 27
4,096*	5 29	5.63	8.90	8.40	8 98
4,096†	4 86	.....	12.2	....	8 84

* Relative humidity 80 per cent.
† Relative humidity 63 per cent.

Table VI gives the values of absorbing powers of the sound chamber, and $\log_{10} E/i$ for the organ-pipe sources used in measuring absorption coefficients of materials. It is to be noted that i is the threshold of audibility for a given observer. Knowing the absorbing power, E/i for a second observer can be obtained from his timings of sound from the same sources. In this way, the method is made independent of the absolute value of the observer's threshold of hearing. Comparison of the values of E/i for two observers using the same sources of sound is made in Table VII. One notes a marked difference in the threshold of audibility of these two observers, both of whom have what would be considered normal hearing.[1]

TABLE VII.—REVERBERATION TIMES BY TWO OBSERVERS

Frequency . .	128		256		512		1,024		2,048	
Observer. . .	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
Time	14.70	12 59	14.58	13.04	14.25	13.81	12.80	13 24	9.80	9.73
$\log_{10} \frac{E}{i}$. . . .	8.31	7 46	10.14	9.33	11.01	10.75	10.91	11.20	10 27	10.21
Difference . .	−0.85		−0.81		−0.26		+0.29		−0.06	

[1] For the variation in the absolute sensitivity of normal ears see FLETCHER, "Speech and Hearing," p. 132, D. Van Nostrand Company, 1929; KRANZ, F. W., *Phys. Rev.*, vol. 21, No. 5, May, 1923.

Effect of Humidity upon Absorbing Power.

In Table VI, the values of the absorbing power at 4,096 vibs./sec. are given for two values of the relative humidity. It was early noted in the research at the Riverbank Laboratories that the reverberation time at the higher frequencies varied with the relative humidity, being greater when the relative humidity was high. It was at first supposed that this effect was due to surface changes in the walls, possibly an increase in the surface porosity of the plaster as the relative humidity decreased. Experiments showed, however, that changes in reverberation time followed too promptly the decrease in humidity to account for the effect as due to the slow drying out of the walls. Subsequent painting of the walls with an enamel paint did not alter this effect. Hence, it was concluded that the variation of reverberation time with changes in humidity must be due to the effect of water vapor in the air upon the atmospheric absorption of acoustic energy. High-frequency sound is more strongly absorbed in transmission through dry than through moist air.

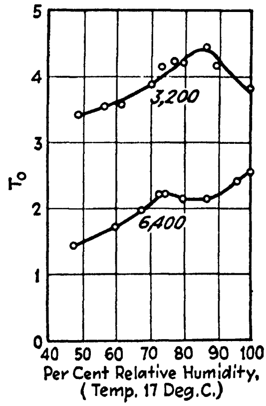


FIG. 38.—Variation of reverberation time with relative humidity. (*After Meyer.*)

Erwin Meyer,[1] working at the Heinrich Hertz Institute, has also noted this effect. Figure 38 shows the relation, as given by Meyer, between reverberation time and relative humidity at 6,400 and 3,200 vibs./sec.

The most recent work on this point has been done by V. O. Knudsen.[2] The curves of Fig. 39*a* taken from his paper show the variation of reverberation times for frequencies from 2,048 to 6,000 vibs./sec. with varying relative humidities. We note that the time increases linearly with

[1] *Zeits. tech. Physik*, No. 7, p. 253, 1930.
[2] *Jour. Acous. Soc. Amer.*, p. 126, July, 1931.

relative humidity up to about 60 per cent. In these experiments the temperature of the wall was lower than that of the air in the room. It was observed that condensation on the walls began at a relative humidity of about 70 to 80 per cent. In other experiments, conducted when the wall temperature was higher than the room temperature and there was no condensation on the walls, the reverberation time increased uniformly with the relative humidity up to more than 90 per cent. Knudsen ascribes the bend in the curves to surface effects which increased the absorption at the walls when moisture collected on them.

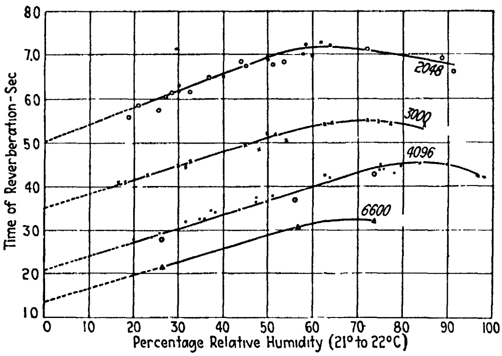


FIG. 39a.—Knudsen's results on variation of reverberation time with changes in relative humidity.

Knudsen further found that when the relative humidity was maintained at 100 per cent and fog appeared in the room, the reverberation times became markedly lower for all frequencies. Thus the reverberation time at 512 vibs./sec. was decreased from 12.65 sec., relative humidity 80 per cent to 6.52 sec., relative humidity 100 per cent with fog present. Knudsen is inclined to ascribe this marked increase in absorption with fog in the air to the presence of moisture on the wall. The author questions this explanation in view of the magnitude of the effect, particularly at the low frequencies. The decrease from 12.65 to 6.52 sec. calls for a doubling of the coefficient of absorption, if we assume the effect to be due only to changes in surface condition. In numerous instances, in the Riverbank sound chamber, moisture due to excessive humidity has collected on floors and walls but without fog in the room. No marked change in the reverberation has been observed in such cases. It would seem more likely that the effect noted when fog is present is due to an increase of atmospheric absorption. The question is of considerable importance both theoretically and practically in atmospheric acoustics.

Making reverberation measurements in two rooms of different volumes but with identical surfaces of painted concrete, Knudsen was able to separate the surface and volume absorption and to measure the attenuation due to

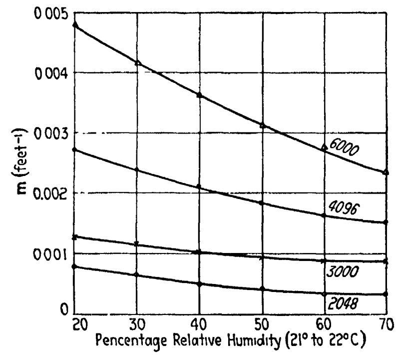


FIG. 39b.—Values of m (see text) as a function of relative humidity. (*After Knudsen.*)

absorption of acoustic energy in the atmosphere. Assume that the intensity of a plane wave in air decreases according to the equation $I = I_0e^{-mct}$. Then if we take account of both the surface and volume absorption, the Sabine formula in English units becomes

$$T_0 = \frac{0.049V}{a + 4mV}$$

or, with the Eyring modification,

$$T_0 = \frac{0.049V}{-S \log_e (1 - \alpha_a) + 4mV}$$

Figure 39*b* gives Knudsen's values of m in English units for different frequencies and different relative humidities. For frequencies below 2,048 vibs./sec. m is so small as to render the expression $4mV$ negligible in comparison with the surface absorption.

Experiments on the effect of moisture on the viscosity of the atmosphere show that while there is a slight decrease in viscosity with increase in relative humidity, the magnitude of the effect is far too small to account for observed changes in atmospheric sound absorption. A greater heat conductivity from the compression to the rarefaction phase for low than for high humidity is a possible explanation. If this be true, then the velocity of sounds of high frequency should be greater in moist than in dry air, since, assuming a heat transfer between compression and rarefaction, the velocity of sound would tend to the lower Newtonian value. The writer knows of no experimental evidence for such a supposition. The point is of considerable theoretical interest and is worthy of further study.

Calibration of Sound Chamber with Loud-speaker.

The development in recent years of the vacuum-tube oscillator and amplifier together with the radio loud-speaker of the electrodynamic or moving-coil type gives a convenient source of sound of variable output, for use in the measurement of sound-absorption coefficients. Experiment shows that the amplitude response of the electrodynamic free edge-cone type of loud-speaker is proportional to the alternating-current input.[1]

Under constant room conditions, therefore, the acoustic-power output is proportional to the square of the input current, whence we may write

$$E = kC^2$$

where k is a constant.

[1] Recent experiments show that this relation holds over only a limited range for most commercial types of dynamic loud speakers.

Equation (57) then may be written

$$aT_1 = \frac{4V}{c} \frac{\log_e kC_1^2}{aci}$$

The equivalent of the four-organ experiment then may be very simply performed by measuring the reverberation time for different measured values of the audiofrequency current input of the loud-speaker source. If T_2 be the measured time with a constant input C_2, we have

$$aT_2 = \frac{4V}{c} \log_e \frac{kC_2^2}{aci}$$

whence

$$a = \frac{4V}{c}\left[\frac{\log_e C_1^2 - \log_e C_2^2}{T_1 - T_2}\right] = 9.2\frac{V}{c}\left[\frac{\log_{10}(C_1^2/C_2^2)}{T_1 - T_2}\right]$$

For the Riverbank sound chamber this becomes

$$a = 15.4 \frac{\log_{10}(C_1/C_2)}{T_1 - T_2} \tag{65}$$

In Fig. 40 the logarithm of the ratio of the current in the loud-speaker to the minimum current employed is plotted

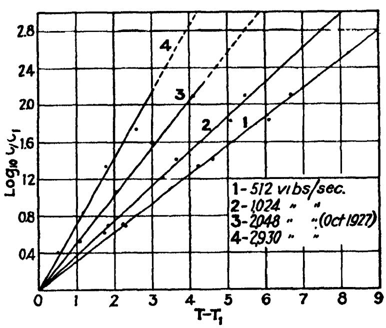


FIG. 40.—Difference in reverberation time as a function of logarithm of loud-speaker current ratio.

against the difference in the corresponding reverberation times.

The expression $\frac{\log_{10}(C_1/C_2)}{T_1 - T_2}$ for each frequency is the slope of the corresponding line in Fig. 40. We note that with the loud-speaker we have a range of roughly 250 to 1 in the currents, corresponding to a variation of about 62,500 to 1 in the intensities, whereas in the experiment with the organ pipes the range of intensities was only 4 to 1. The loud-speaker thus affords a much more precise means of sound-chamber calibration. In Table VIII, the results of the four-organ calibration and of three independent loud-speaker calibrations are summarized.

TABLE VIII.—ABSORBING POWER IN SQUARE METERS OF RIVERBANK SOUND CHAMBER

Frequency	Four organs	Loud-speaker			
			1930		1931*
	1925	1928	Steady tone	Flutter tone	Flutter tone
128	3 09	4.03	....	....	4 85
256	4 03	4.87	4.70	4.68	5 05
512	4.55	4 56	4 70	4.68	5.30
1,024	4 98	5 32	4 73	5.08	5 42
2,048	5 86	7.02	7.22	7 38	7 65

* Room conditions slightly altered from those of 1930.

We note fair agreement between the results with the four organs and the loud-speaker at 512 and 1,024 but considerable difference at the low and high frequencies. It is quite possible that for the lower tones the separation of the four organs was not sufficiently great to fulfill the assumption that the sound emitted by a single pipe is independent of whether or not other pipes are speaking simultaneously. At 2,048 the difference in times between one and two or more pipes was small, so that the possible error in the slope of the lines was considerable, in view of the limited range of intensities available in this means of calibration.

Data are also presented in which a so-called "flutter tone" was employed, that is, a tone the frequency of which is continuously varied over a small range about a mean frequency. The flutter range in these measurements was about 6 per cent above and below the mean frequency, and the flutter frequency about two per second. This expedient serves to reduce the errors in timing due to interference.

Calibration Using the Rayleigh Disk.

A variation of the four-organ method of calibration has been used by Professor F. R. Watson[1] at the University of Illinois. He used the Rayleigh disk as a means of evaluating the relative sound outputs of his sources of sound. For the latter he used a telephone receiver associated with a

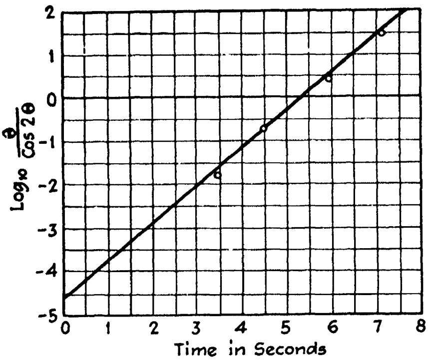


FIG. 41.—Sound-chamber calibration using Rayleigh disk. (*After Watson.*)

Helmholtz resonator. The receiver was driven by current from a vacuum-tube oscillator and amplifier.

The relative outputs of the source for different values of the input current were measured by placing the sound source together with a Rayleigh disk inside a box lined with highly absorbent material. The intensities of the sound for different values of the input current were taken as proportional to $\theta/\cos 2\theta$, where θ is the angle through which the disk turns under the action of the sound against the restoring torque exerted by the fiber suspension. The maximum intensity was 1830 times the minimum intensity used. Figure 41, taken from Watson's paper, shows the

[1] *Univ. Ill. Eng. Exp. Sta. Bull.* 172, 1927.

linear relation between the logarithm of the intensity measured by the Rayleigh disk and the time as measured by the ear. From the slope of this line and the constants of the room the absorbing power was computed. With this datum the absorption coefficients of materials are measured as outlined in the preceding section.

Other Methods of Sound-chamber Calibration.

The ear method of calibration is open to the objections that it is laborious and that it requires a certain amount of training upon the part of the observer to secure consistency in his timings. The objection is also raised that variation

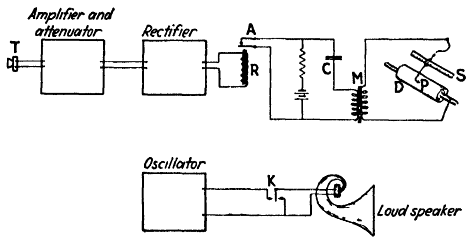


FIG. 42.—Reverberation meter of Wente and Bedell.

in the acuity of the observer's hearing may introduce considerable personal errors. However, the writer's own experience of twelve years' use of this means is that with proper precautions the personal error may be made less than errors due to variation in the actual time of decay due to the fluctuation of the interference pattern, as the reverberant sound dies away. Various instrumental methods either of measuring the total time required for the reverberant sound to die away through a given intensity range or of following and recording the decay process continuously have been proposed and used.

Relay and Chronograph Method.

The apparatus, devised at the Bell Laboratories and described by Wente and Bedell,[1] is shown in Fig. 42. The authors describe the method as consisting of an "electro-

[1] *Jour. Acous. Soc. Amer.*, vol. 1, No. 3, Part 1, p. 422, April, 1930.

acoustical ear of controllable threshold sensitivity." The microphone T serves as a "pick-up" and is connected to a vacuum tube amplifier provided with an attenuator which may control the amplification in definite logarithmic steps. The amplifier is terminated by a double-wave rectifier. The rectified current passes through the receiving windings of a relay, which is constructed so that when the current exceeds a certain value, the armature opens the contact at A, charging the condenser C. When the current falls below a certain value, the armature is released and the condenser discharges through the primary windings of the spark coil M causing a spark to pass to the rotary drum D at P. The drum is rotated at a constant speed and is covered by a sheet of waxed paper on which the passage of the spark leaves a permanent impression. The key k opens the circuit from the oscillator which supplies current to the loud-speaker source, thus cutting off the sound. This key is operated mechanically by a trigger not shown. This trigger is released automatically when the drum is in a given angular position.

The threshold of the instrument is set at a definite value by adjustment of the attenuator. The sound source is started. After the sound in the room has reached a steady state, the trigger of the key k is set and is then automatically released by the rotating drum. When the decaying sound has reached the threshold intensity of the instrument, the relay A operates, causing the spark to jump. The distance on the waxed paper from the cut-off of the sound to the record of the spark gives the time required for the sound to decrease from the steady state to the threshold of the instrument. Call this initial threshold i_1. The threshold of the instrument is then raised to a second value i_2. The point P is shifted on the scale S to the right an amount proportional to the $\log i_2 - \log i_1$, and the operation is repeated; and by repeatedly raising the threshold and shifting the point P, a series of dots giving the times for the reverberant sound to decrease from the steady state to known relative intensities are recorded on the waxed paper. Such a record is shown in Fig. 43. The authors state: "If the decay of a sound at the microphone had been strictly logarithmic, these dots would all lie along a straight line. This ideal will almost never be encountered in practice. We must therefore be content with drawing a line of best fit through these points." From the slope of the line and the peripheral speed of the drum the absorbing power of the room may be obtained, in the manner already described in the four-organ experiment.

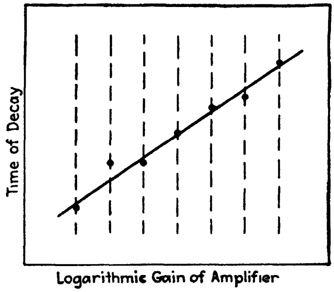


FIG. 43.—Decay of reverberant sound as obtained with reverberation meter.

We note that this method is based on measurement of the times from a fixed steady-state intensity to a threshold, variable in known ratios, whereas the four-organ experiment and its loud-speaker variant measure the time from a variable steady state to a fixed threshold. Properly designed and freed from mechanical and electrical sources of error, the apparatus is not open to the objection of

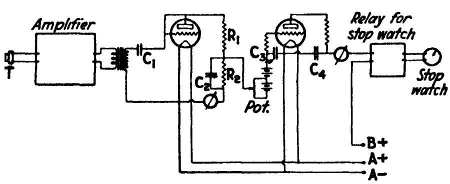


FIG. 44.—Vacuum-tube circuit for automatically recording reverberation times. (*After Meyer.*)

personal error. The uncertainties due to the shifting interference pattern, however, still exist.

Another arrangement for automatically determining the reverberation time, due to Erwin Meyer[1] of the Heinrich Hertz Institute, is shown in Fig. 44. Here the reverberant sound is picked up by a condenser microphone. The

[1] *Zeits. tech. Physik*, vol. 11, No. 7, p. 253, 1930.

current is amplified and rectified, and the potential drop of the rectified current is made to control the grid voltage of a short radio-wave oscillator. In the plate circuit of this oscillator is a sensitive relay which operates the stopping mechanism of a stop watch. As long as the sound is above a given intensity, the negative grid voltage of the oscillating tube is sufficiently great to prevent oscillation. As the reverberant sound dies away, the negative grid bias decreases. When the intensity reaches the given value, oscillations are set up, increasing the plate current, operating the relay, and stopping the watch.

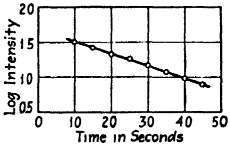


FIG. 45.—Decay of sound intensity, Bureau of Standards sound chamber, oscillograph method.

Oscillograph Methods.

The oscillograph has been employed for recording the actual decay of sound in a room. Experiments using a steady tone have shown that the extreme fluctuations of intensity due to the shifting of the interference pattern give records from which it is extremely difficult to obtain precise quantitative data.[1] Employing a flutter tone and at the same time rotating the source of sound, Chrisler and Snyder[2] found that oscillograms could be made on which the average amplitude of the decaying sound could be drawn as the envelope of the trace made by the oscillograph mirror on the moving film. These envelope curves proved to be logarithmic, and by measuring the ordinates for different times the rate of decay can be obtained. In Fig. 45, the squares of the amplitudes from one

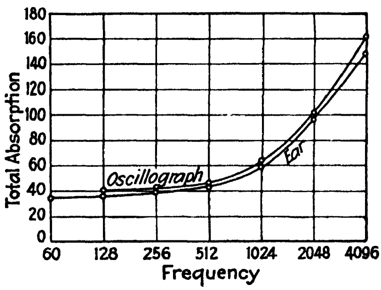


FIG. 46.—Total absorption, Bureau of Standards sound chamber, by the oscillograph and ear measurements.

[1] KNUDSEN, V. O., *Phil. Mag.*, vol. 5, pp. 1240–1257, June, 1928.

[2] CHRISLER, V. L., and W. F. SNYDER, *Bur. Standards. Jour. Res.*, vol. 5, pp. 957–972, October, 1930.

of their curves are plotted against the time. In Fig. 46, the absorbing powers of the sound chamber at the Bureau of Standards as determined both by oscillograph and by the ear method are plotted as a function of the frequency of the sound. In summing up the results of their research with the oscillograph, Chrisler and Snyder state that for

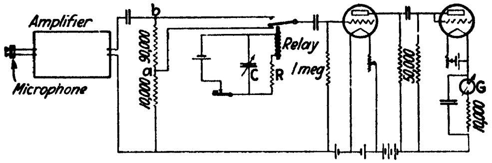


FIG. 47.—Meyer and Just's apparatus for recording decay of sound intensity.

accuracy of results a considerable number of records must be made and that the time required is greater than that required to make measurements by ear. For these reasons the oscillograph method has been abandoned at the Bureau of Standards.

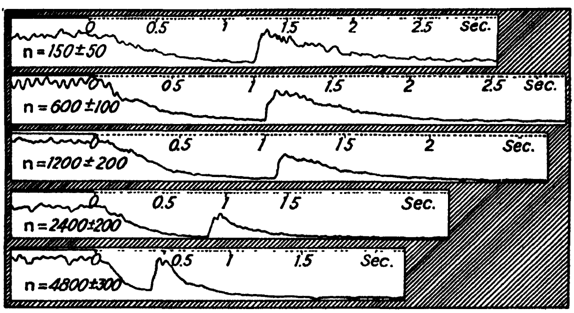


FIG. 48.—Decay of sound intensity recorded with apparatus of Meyer and Just.

A modification of the oscillograph method has been used by E. Meyer and P. Just.[1] Their electrical circuit is shown in Fig. 47. A microphone is placed in the room in which the reverberation time is to be measured. The current is amplified first by a two-stage amplifier. Before the third stage is an automatic device which at the end of a given period increases the amplification by a factor of ten,

[1] *Electr. Nachr-Techn.*, vol. 5, pp. 293–300, 1928.

thus increasing the sensitivity in the same ratio. The amplified current is passed through a vacuum-tube rectifier, in the output of which is a short-period galvanometer. The deflection of the galvanometer is recorded upon a moving film, on which time signals are marked. A series of records from their paper is shown in Fig. 48. The logarithmic decay is computed from these records as indicated in the previous paragraph.

Methods Based on Intensity Measurements.

With a source of acoustic power E, the output of which is independent of room conditions, we have for the steady-state intensity

$$I_1 = \frac{4E}{ac}$$

whence

$$acI_1 = 4E$$

If now an absorbing area be brought into the room, the total absorbing power becomes a', and the steady-state intensity I'_1, so that

$$a'cI'_1 = 4E$$

and

$$a' = \frac{aI_1}{I'_1}$$

Subtracting,

$$a' - a = a\left[\frac{I_1 - I'_1}{I_1}\right] \tag{66}$$

Equation (66) offers an attractively simple method of measuring the absorbing power of the material brought into the room, provided one has means of measuring the average intensity of the sound in the room and knows the value of a, the absorbing power of the room in its standard condition. Professor V. O. Knudsen[1] at the University of California in Los Angeles has used this method of measuring change in absorbing power. The experimental arrange-

[1] *Phil. Mag.*, vol. 5,pp. 1240–1257, June, 1928.

ment is shown in Fig. 49. A loud-speaker source driven by an oscillator and amplifier was used, and in one series of experiments the sound was picked up by four electromagnetic receivers, suitably mounted on a vertical shaft which was rotated with a speed of 40 r.p.m. In other experiments, an electrodynamic type of loud-speaker was substituted, and the sound was received by a single-condenser microphone mounted on a swinging pendulum.

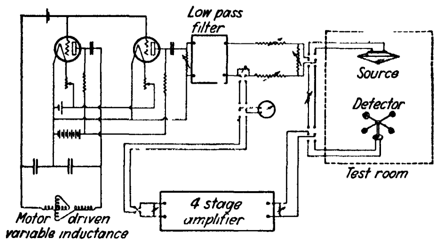


FIG. 49.—Knudsen's apparatus for determining absorbing power by sound-intensity measurements.

Table IX gives results taken and computed from Knudsen's measurement by this method of the absorbing power of a room as more and more absorbing material is brought in.

TABLE IX

Area absorbent material, square feet	Average galvanometer deflection d'	Average intensity	Absorbing power	Coefficient
None	14.25	$5.00 \times 10^7 i$	28.3	
4	13 34	4.68	30 2	0.489
16	11.13	3 91	36 2	0.507
36	9 25	3 25	43.6	0.439
49	8.19	2.88	49 2	0.442
64	6.99	2.46	57 5	0.471
81	6.42	2.25	62.8	0.434
100	5.61	1 97	71.9	0 450
127	5.05	1.77	79.8	0 422
				Ave. 0 457

Calibration of the amplifier showed that the galvanometer deflection was proportional to the square of the voltage input of the amplifier and therefore proportional to the sound-energy density in the room. The values of a' are based upon a value of $a = 28.3$ units, obtained by reverberation measurements in the empty room. Absorbing powers are given in English units.

The mean value of the absorption coefficient of this same material obtained by both the reverberation method and the intensity method is given by the author as 0.433. It is obvious that the precision of the method is no greater than that with which a, the absorbing power of the empty room, can be determined, and this in turn goes back to reverberation methods.

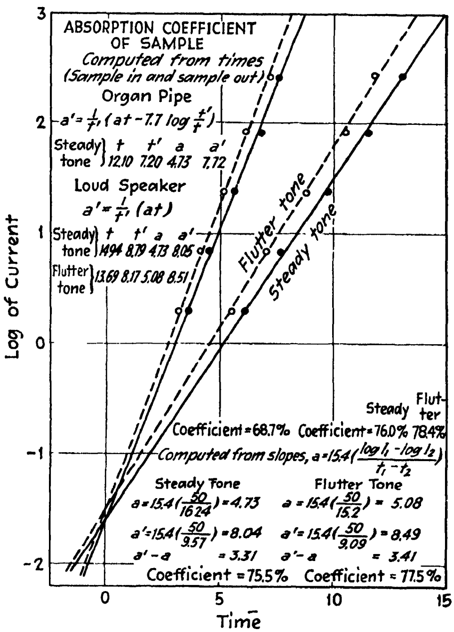


FIG. 50.—Data for determination of absorption coefficients by various methods

Absorption Coefficient Using Source with Varying Output.

With a source of sound whose acoustic output can be varied in measured amounts the absorbing power of the reverberation chamber both in its standard condition and with the absorbent material present can be determined directly. A carefully conducted experiment of this sort serves as a useful check on the validity of the constant-source method and the assumptions made and also shows the degree of precision that may be obtained in reverberation methods of measurement. Figure 50 presents the results of such an experiment. Here the logarithm of the current in milliamperes in the loud-speaker is plotted against the duration of audible sound, first without absorbent material and then with 4.46 sq. m. (48 sq. ft.) of an absorbent material placed in the sound chamber. The data presented afford two independent means of computing the absorption coefficient of the material: (1) From the slopes of the straight lines a and a' may be determined by Eq. (65). Thus

$$a = 15.4m$$

and

$$a' = 15.4m'$$

m, and m' being the slopes of the straight lines representing the experimental points without and with the absorbent material present. (2) Assuming equal acoustic outputs for equal loud-speaker currents, a' may be computed from the values of T and T', the times for any given current value before and after the introduction of the absorbent material, and a, the absorbing power of the empty room. Thus if E be the acoustical power of the source, assumed for the moment to be the same for a given current under the two room conditions, we have

$$aT = 7.70 \log_{10} \frac{4E}{aci} = 7.7 \log_{10} \frac{I_1}{i} \qquad (67a)$$

$$a'T' = 7.70 \log_{10} \frac{4E}{a'ci} = 7.7 \log_{10} \frac{I'_1}{i} \qquad (67b)$$

Whence by eliminating $4E/ci$, we have

$$a' = \frac{1}{T'}\left(aT - 7.7 \log_{10} \frac{a'}{a}\right) \tag{68}$$

We note, however, that if the straight lines of Fig. 50 be extrapolated, their intersection falls on the axis of zero time. This means that equal currents in the loud-speaker set up equal steady-state intensities throughout the room, whereas the assumption of equal acoustical powers for a given current implies a lower intensity in the more absorbent room. We are forced to the conclusion, therefore, that the sound output of a loud-speaker operating at a fixed amplitude is not independent of the room conditions. The data here presented indicate that for a fixed amplitude of the source the output of the speaker is directly proportional to the absorbing power; that is, $E'/a' = E/a$, $I = I'$, and hence

$$a' = \frac{aT}{T'} \tag{69}$$

The values of the absorption coefficients for the material as determined by organ-pipe data and by the two methods from the loud-speaker data are shown in Fig. 50. The computations from the organ-pipe data are based on the assumption that the power of the pipe is constant under altered room conditions. The value of a' is computed from Eq. (69) instead of Eq. (68), since the latter would give different values of a', depending upon the particular current values for which T and T' are taken.

Reaction of Room on the Source.

The foregoing brings up the very important question of the assumption to be made as to what effect on the rate of emission of sound energy from a source of constant amplitude results from altering the absorbing power of the room in which it is placed. On this point Professor Sabine states:[1]

In choosing a source of sound, it has usually been assumed that a source of fixed amplitude is also a source of fixed intensity (power). On

[1] "Collected Papers on Acoustics," Harvard University Press, p. 279, 1922.

the contrary, this is just the sort of source whose emitting power varies with the position in which it is placed in the room. On the other hand, an organ pipe is able within certain limits to adjust itself automatically to the reaction due to the interference system. We may say briefly that the best standard source of sound is one in which the greatest percentage of emitted energy takes the form of sound.

Sabine is here speaking of the effect of shifting the source of sound with reference to the stationary-wave system. In the experiment of the preceding section, the large stèel reflectors already mentioned were kept moving, thus continuously shifting the stationary-wave system. The use of the flutter tone also would preclude a fixed interference pattern, so that if there were a difference in the output of the loud-speaker brought about by the introduction of the absorbent, this difference was due to the change in the total absorbing power of the room. Repeating the experiment for the tones 1,024 and 2,048 vibs./sec. gave the same results, namely, straight lines whose intersection was on the axis of zero times. Earlier experiments,[1] using a different loud-speaker and slightly different electrical arrangements, showed the intersection of the lines at positive values of the time. On the other hand, Chrisler reports[2] a few sets of measurements in which the intersection of the lines was at negative values of the time, indicating that the loud-speaker at constant-current input acts as a source of constant acoustical output independent of room conditions. Existing data are therefore equivocal as to just how a loud-speaker driven at constant amplitude behaves as the absorbing power of the room is altered.

In this connection, the results of Professor Sabine's acoustical survey of a room shown in Fig. 18 of Chap. III are interesting. The upper series shows the amplitudes at points in the empty room. The lower series gives the amplitudes at the same points when the entire floor is covered with hair felt. The amplitude of the source was the same in the two cases. Comparing the two, one notes

[1] *Jour. Franklin Inst.*, vol. 207, p. 341 March, 1929.
[2] *Bur. Standards Res. Paper* 242.

that the introduction of the felt does not materially alter the general distribution of sound intensity. The maxima and minima fall at the same points for the two conditions. Further, the minima in the empty room are for the most part more pronounced than when the absorbent is present, and finally we note that on the whole the amplitude is *less* in the empty than in the felted room. Taking the areas of the figures as measured with a planimeter, we find that the average amplitudes in the empty and felted rooms are in the ratio of 1:1.38, and this for a source in which the measured amplitude is the same in the two cases. One cannot escape the conclusions (1) that in this experiment, at least, covering the entire floor with an absorbent material did not shift the interference pattern in horizontal planes and (2) that the acoustic efficiency of the constant-amplitude source set up in the absorbent room was enough greater than in the empty room to establish a steady-state intensity $1.9(1.38)^2$ times as great. The same output should, on the reverberation theory, produce a steady-state intensity only about one-half as great.

Lack of sufficient data to account for the apparently paradoxical character of these results probably led Professor Sabine to withhold their publication until further experiments could be made. Penciled notations in his notes of the period indicate that further work was contemplated. Repeating the experiment with the tremendously improved facilities now available both with loud-speaker and organ-pipe sources and with steady and flutter tones would be extremely interesting and should throw light on the question in point.

The writer's own analysis of the problem indicates that with a constant shift of the interference pattern, by means of a flutter tone or a moving source or by moving reflectors, the constant-amplitude source should set up the same steady-state intensity under both the absorbent and the non-absorbent room conditions. Operated at constant current, a loud-speaker should thus act as a source whose output in a given room is directly proportional to the absorbing power, while at a constant power input it should operate as a constant-output source. Experimental verification of these conclusions has not yet been attained, so that there is still a degree of uncertainty in the determination of absorption coefficients by the reverberation method using the electrical input as a measure of the sound output of the source.

In Table X are given the values of the absorption coefficients at three different frequencies of a single material, computed in the manners indicated from organ-pipe and

TABLE X

$$\text{Organ pipe, } a' = \frac{1}{t}\left(at - 7.7 \log \frac{a'}{a}\right) \qquad (1)$$

$$\text{Loud-speaker, } a' = \frac{1}{t'}\,(at) \qquad (2)$$

$$\text{Loud-speaker, variable current, } a' = 15.4\left[\frac{\log_{10} C_1 - \log_{10} C_2}{t_1 - t_2}\right] \qquad (3)$$

Frequency	Source	Equation	Tone	Coefficient	Bureau of Standards	Watson	Knudsen
	Pipe	(1)	Steady	64 7			
	Loud speaker	(2)	Steady	61 6			
512	Loud speaker	(2)	Flutter	57 3	61	70	67
	Loud speaker	(3)	Steady	60 9			
	Loud speaker	(3)	Flutter	57 8			
	Pipe	(1)	Steady	68 7			
	Loud speaker	(2)	Steady	76 0			
1,024	Loud speaker	(2)	Flutter	78 4	72	76	74
	Loud speaker	(3)	Steady	75.5			
	Loud speaker	(3)	Flutter	77 5			
	Pipe	(1)	Steady	71 2			
	Loud speaker	(2)	Steady	73 9			
2,048	Loud speaker	(2)	Flutter	79 2	76	76	80
	Loud speaker	(3)	Steady	78 5			
	Loud speaker	(3)	Flutter	79 5			

loud-speaker data obtained in the Riverbank sound chamber. For comparison, figures by the Bureau of Standards, by F. R. Watson and V. O. Knudsen, on the same material are given. The latter were all obtained by the reverberation method. The table gives a very good idea of the order of agreement that is to be expected in measurements of this sort. The fact that the coefficient is obtained by taking the difference between two quantities whose precision of measurement is not great will account for rather large variations in the computed value of the coefficient. Thus in Fig. 50, errors of 1 per cent in the value of a and a' would, if cumulative, make an error of 4.5 per cent in the computed values of the coefficient. The variations that are to be expected in determining a and a', due to interference, are certainly as great as 1 per cent, so that the precision with which absorption coefficients can be measured by existing methods is not great.

Summary.

We have seen that the standing-wave method gives coefficients of absorption for normal incidence only and for samples placed always at a motion node of the standing-wave system. Moreover, with materials whose absorption is due to inelastic flexural vibrations, the small-scale measurements on rigidly mounted samples fail to give the values that are to be expected from extended areas having a degree of flexural motion. On the other hand, reverberation coefficients are deduced on the assumptions made and verified in the reverberation theory as it is applied to the practical problems of architectural acoustics.

We have seen also that all of the reverberation methods now in use go back to the determination of the rate of decay of sound in a reverberation chamber and the effect of the absorbent material on this rate of decay and that in the very nature of the case, the precision of such measurements is not great. The oscillograph method is equally laborious and requires repeated measurements in order to eliminate the error due to the irregularities in the decay curve resulting from interference. Finally, we have noted that there is a certain degree of uncertainty as to the assumptions to be made when we take the electrical input of a telephonic source of sound as a measure of the sound energy which it generates under varying absorbing powers of the room. As will be seen in the succeeding chapter, there are a number of other factors that affect the measured values of reverberation coefficients. All things considered, it has to be stated that before precise agreement on measured values can be attained, arbitrary standards as to methods and conditions of measurement will have to be adopted.

CHAPTER VII

SOUND-ABSORPTION COEFFICIENTS OF MATERIALS

In this chapter, it is proposed to consider the physical properties that affect the sound-absorbing efficiency of materials and the variation of this efficiency with the pitch and quality of the sound. We shall also consider various conditions of test that affect the values of the absorption coefficients of materials as measured by reverberation methods and finally deal with some questions that arise in the practical use of sound absorbents in the correction of acoustical defects and the reduction of noise in rooms.

Physical Properties of Sound Absorbents.

The energy of a train of sound waves in the air resides in the regular oscillations of the molecules. The absorption of this energy can occur only by some process by which these ordered oscillations are converted into the random molecular motion of heat. In other words, the absorption of sound is a dissipative process and occurs only when the vibrational motions are damped by the action of the forces of friction or viscosity. Now experience shows that only materials which are porous or inelastically flexible or compressible absorb sound in any considerable degree. For a material to be highly absorbent, the porosity must consist of intercommunicating channels, which penetrate the surface upon which the sound is incident. Cellular products with unbroken cell walls or with an impervious surface do not show any marked absorptive properties. A simple practical test as to whether a material possesses absorbing efficiency because of its porosity is to attempt to force air into it or through it by pressure. If air cannot be forced into it, it will not show high absorbent properties.

By inelastically flexible and compressible materials we mean those in which the damping force is large in comparison with the elastic forces brought into play when such materials are distorted.

Absorption Due to Porosity.

The theoretical treatment of this problem is beyond the scope of our present purpose. Theoretical treatment is given in papers by Lord Rayleigh, by E. T. Paris, and by Crandall.[1] In an elementary way, it can be said that the absorption coefficient of a porous, non-yielding material will depend upon the following factors: (*a*) the cross section of the pore channels, (*b*) their depth, and (*c*) the ratio of perforated to unperforated area of the surface. Rayleigh's analysis is for normal incidence and leads to the conclusion that the absorption increases approximately as the square root of the frequency. For a given ratio of unperforated to perforated area, the absorption at low frequencies increases, though not linearly with the radius of the pores, considered as cylindrical tubes. If the radius of pores be greater than 0.01 cm., the assumptions made in the theory do not hold. For a coarse-grained structure, the thickness required to produce a given absorption at a given frequency is greater than with a fine-pored material. Crandall[2] has worked out the theoretical coefficients of absorption of an ideal wall of closely packed honeycomb structure (*i.e.*, one in which the ratio of unperforated to perforated area is small), the diameter of the pores being 0.02 cm. The thick-

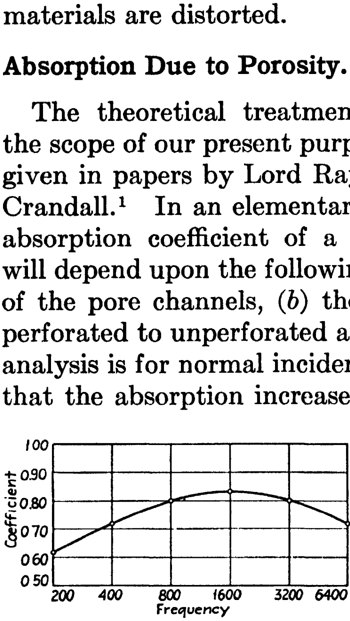


FIG. 51.—Theoretical absorption of a closely packed porous material of great thickness. (*After Crandall.*)

[1] RAYLEIGH, "Theory of Sound," vol. II, pp. 328–333.
———, *Phil. Mag.*, vol. 39, p. 225, 1920.
PARIS, E. T., *Proc. Roy. Soc.*, Ser. A, vol. 115, 1927.
CRANDALL, "Theory of Vibrating Systems and Sound," p. 186, D. Van Nostrand Company, 1926.

[2] CRANDALL, *op. cit.*, p. 189.

ness is assumed great enough to give maximum absorption. His values are plotted in Fig. 51. We note a maximum of absorption at 1,600 vibs./sec. This suggests selective absorption due to resonance; but as Crandall points out, it is "quite accidental, as no resonance phenomenon or selective absorption has been implied" in the problem. On his analysis, we should expect a porous material always to show a maximum of absorption at some frequency, this maximum shifting to lower frequencies as the coarseness of the porosity is increased. Thus, he states, if the cross section of the pores is doubled, the curve shown would be shifted one octave lower. It is to be remembered that a porous wall of great thickness is assumed.

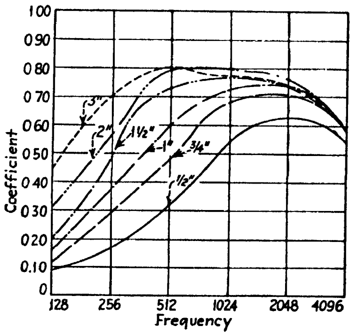


FIG. 52.—Absorption coefficients of asbestos hair felt of different thicknesses.

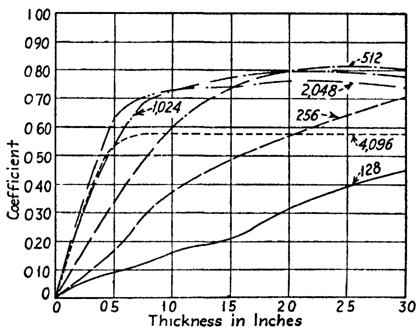


FIG. 52a.—Absorption coefficients of asbestos hair as a function of thickness.

Effect of Thickness of Porous Materials.

For limited thickness, the absorption coefficient of a porous material increases in general with the thickness approaching a maximum value as the thickness is increased.

The curve shown in Fig. 31 shows the absorption by the stationary-wave method of cotton wool as a function of thickness. Davis and Evans state that the maximum absorption there shown occurs at thicknesses of one-quarter of the wave length of sound in the material. In Fig. 52 are shown the reverberation coefficients of an asbestos hair felt of different thicknesses at different frequencies. Figure 53 shows the absorption coefficients of a porous tile as given by the Bureau of Standards. We note, in all cases, a maximum absorption over a frequency range. This

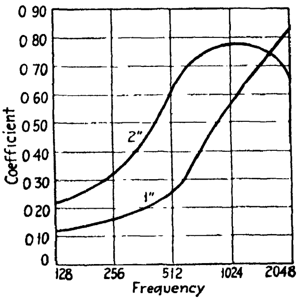


FIG. 53.—Absorption coefficients of porous tile. (*Bureau of Standards.*)

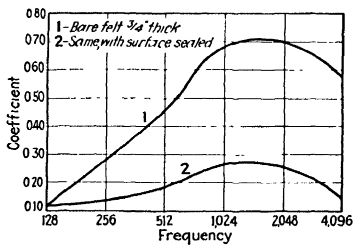


FIG. 54.—Effect of sealing the surface of a porous compressible material.

maximum shifts to lower frequencies as the thickness of the absorbing layer is increased. These facts are in qualitative agreement with the theory of absorption by porous bodies deduced by Rayleigh and Crandall.

In soft, feltlike materials, the absorption, particularly at lower frequencies, is due both to porosity and to inelastic compressibility. The curves of Fig. 54 show the effect of sealing the surface of felt with an impervious membrane. Curve 2 may be assumed to be the absorption due to the compressibility of the material. We note the marked falling off at the higher frequencies where the porosity is the more important factor.

Absorption Due to Flexural Vibrations.

The absorption of sound by fiber boards is due very largely to the inelastic flexural vibration of the material under the alternating pressure. The absorption of sound by wood paneling is of this character. Figure 55 shows the coefficients of pine sheathing 2.0 cm. thick as given by Professor Sabine. We note the irregular character of the curve suggesting that resonance plays an important rôle in absorption by this means. The difference in the mechanics of the absorption of sound by damped flexible materials and absorption due to porosity is shown by comparison of the curves of Fig. 56 with those of felt in

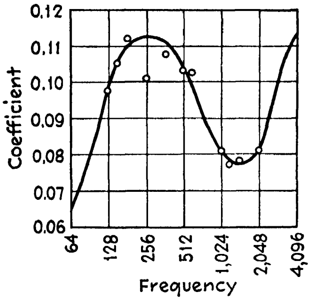


FIG. 55. — Absorption coefficients of pine sheathing 2.0 cm. thick. (*After W. C. Sabine.*)

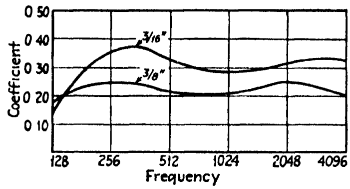


FIG. 56.—Effect of thickness of material whose absorption is due to damped flexural vibrations.

Fig. 52*a*. The former are for a stiff pressed board with a fairly impervious surface made of wood fiber. The lower curve is for a $\frac{3}{8}$-in. thickness, while the upper curve is for the same material $\frac{3}{16}$ in. thick. In contrast to the felt, the thinner more flexible material shows the higher absorption. In absorption due to flexural vibration, the density, stiffness, and damping coefficient of the material affect the absorption coefficient. The mathematical theory of the process has not yet been worked out. The almost uniform value of the coefficients for different frequencies in Fig. 56 indicates the effect of damping in decreasing the effects due to resonance.

Area Effects in Absorption Measurements.

In an investigation conducted in 1922, upon the absorption of impact sounds, it appeared that the increase of absorption of sounds of this character was not strictly proportional to the area of the absorbent surface, introduced into the sound chamber, small samples showing markedly greater absorption per unit area than large samples of the same material. The investigation was extended, using sustained tones, and the same phenomenon was observed as in the case of short impact sounds. In Fig. 57 are shown the apparent absorbing powers per unit area of a highly absorbent hair felt plotted against the area of the test sample.

The same effect under somewhat less ideal conditions was observed in the case of the absorbing power of (trans-

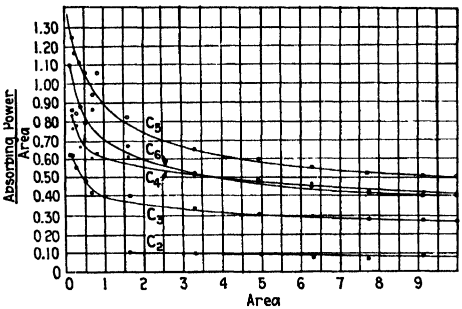


FIG. 57.—Absorbing power per unit area as a function of area.

mission through) an opening. A large window in an empty room 30 by 30 by 10 ft. was fitted with a series of frames so that the area of the opening could be varied, the ratio of dimensions being kept constant. The absorption coefficient for these openings at 512 vibs./sec. varied from 1.10 to 0.80 as the size was increased from 3.68 to 30.2 sq. ft. A doorway 8 by 9 ft. in a room 30 by 30 by 9 ft. showed an apparent coefficient as low as 0.65. (In this case, conditions were complicated by reflection of sound from the ground outside.)

In view of the fact that the earlier measurements of Professor Wallace Sabine were based on the open window as an ideal absorber with an assumed coefficient of 1.00, it was of interest to recompute from his data the values of the absorption coefficients of openings. These data were used by him, assuming a coefficient of 1.00 in the determination of the constant K of the simple reverberation formula. Fortunately the data necessary for these computations are found in his original notes, but, unfortunately for the present purpose, only the total open-window area is given and not the dimensions of the individual openings. The figures are shown in Table XI, where w is the total area of the open windows, and a and a' are computed from the equation

$$aT = \frac{9.2V}{c}\left\{\log_{10}\frac{4E}{aci}\right\}$$

TABLE XI.—ABSORPTION COEFFICIENTS OF OPENINGS (512 VIBS./SEC.)

Room	V	t	t'	a	w	a'	$\frac{a'-a}{w}$
Fogg Art Museum:							
1 pipe..................	96	4.59	3.00	3.99	1 86	5.85	1.00
16 pipes (7.6 × 1 pipe)..	96	5.26	3.43	3.86	1 86	5 80	1.04
Jefferson Physical Laboratory:							
Room 15....	202	1.78	1.66	19.15	1.34	20 5	1.01
Room 15.....	202	1.78	1.56	19.15	2.67	21.7	0.96
Room 15.......... . ..	202	1.78	1.41	19.15	5.28	23.7	0.86
Room 1......	1,630	3.88	3.26	65.0	12.2	76.2	0 92
Room 41..........	1,960	3.41	2 88	87 0	14.9	101 2	0.88

It will be noted that there is a marked variation in the values of the coefficients for the open window. The data for Room 15 show a decrease in the apparent absorbing power as the area of the individual openings is increased, quite in agreement with the results obtained in this laboratory.

One finds an explanation of these facts in the phenomenon of diffraction and the screening effect of an absorbent area upon adjacent areas In the reverberation theory, we assume a random distribution of the direction of propagation of sound energy. Thus, on the average, two-thirds of the energy is being propagated parallel to the surface of the absorbent material. Neglecting diffraction, in such a distribution, only that portion traveling at right angles to the absorbent surface would be absorbed by a very large area of a perfectly absorbent material. Due to diffraction, however, the portion traveling parallel to the surface is also absorbed over the entire area but more strongly at the edges.

The following experiment illustrates this "edge effect." Strips of felt 12 in. wide were laid on the sound-chamber floor, forming a hollow rectangle 8 by 5 ft. (2.44 by 1.53 m.). The increase in the absorbing power due to the sample was measured. The space inside was then filled and the increase produced by the solid rectangle measured. The absorbing power per unit area of the peripheral and central portions is given below:

Area	C_2	C_3	C_4	C_5	C_6
Peripheral	0 12	0 42	0 69	0.88	0.68
Central	0.05	0 27	0 24	0 29	0 30

The screening effect of the edges is obvious and serves to explain the decrease in absorbing power per unit area shown in Fig. 57. In a similar manner, long, narrow samples show more effective absorbing power than equal areas in square form, as shown below:

Area, square meters	Dimensions, centimeters	Coefficient			
		C_3	C_4	C_5	C_6
1	330 × 33	0 62	0 78	1 01	0 73
1	100 × 100	0 42	0.62	0.86	0 70
2	666 × 30	0.53	0 76	0.93	0.71
2	141 × 141	0.37	0.55	0.73	0 59

The fact that small samples show an apparent absorption coefficient greater than unity calls for notice. It is to be remembered that we are here dealing with linear dimensions that are of the order of or even less than the wave length of the sound. The mathematics of the problem involves the same considerations as that of radiation of sound from the open end of an organ pipe or the amplification by a spherical resonator. Diffraction plays an important rôle, and just as in the case of the resonator energy is drawn from the sound field around the resonator to be reradiated, so the presence of the small absorbent sample or small opening affects a portion of the wave front greater than its own area. Qualitatively, the effect is shown by the sound photograph of Fig. 58, which shows a sound pulse reflected

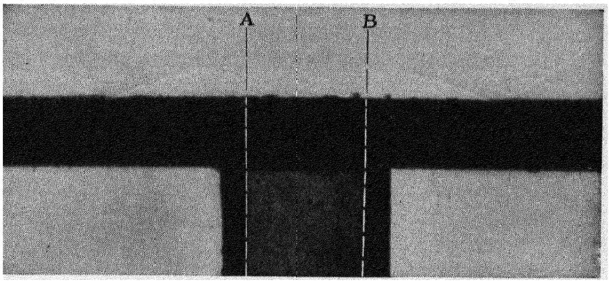


FIG. 58.—Sound pulse incident upon a barrier with an opening. *A-B* marks the limits of the opening.

from the surface of a barrier with an opening. The cross section of the portion cut out of the reflected pulse is clearly greater than that of the opening.

Absorption Coefficients of Small Areas.

Some very interesting results flowing from the phenomena described in the preceding section were brought out in a series of experiments conducted at the Riverbank Laboratories for the Johns-Manville Corporation and reported by Mr. John S. Parkinson.[1]

A fixed area 48 sq. ft. (4.46 sq. m.) of absorbent material was cut up into small units, which were distributed with various spacings and in various patterns. The addition to the total absorbing power of the room was measured by the reverberation method. In Fig. 59, the apparent absorption coefficients of 1-in. hair felt are shown, under the conditions indicated. The explanation of the increase in

[1] *Jour. Acous. Soc. Amer.*, vol. 2, No. 1, pp. 112–122, July, 1930.

absorbing power as the units are separated lies in the screening effect of an absorbent surface on adjacent surfaces. We note that the increase in absorption with separation is a function both of the absorbing efficiency and of the wave length of the sound. The experiment does not permit us to separate the effects of these two factors. We do note, however, that for the two lower frequencies, where the absorption coefficient and the separation measured in wave lengths are both small, the effect of spacing the units is small.

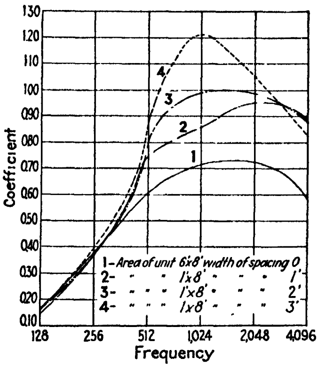


FIG. 59.—Effect of spacing on absorption by small units.

An interesting fact is brought out by the curves of Fig. 60, taken from Parkinson's paper. Here the absorbing power per unit area of the total pattern is plotted as ordinate against the ratio

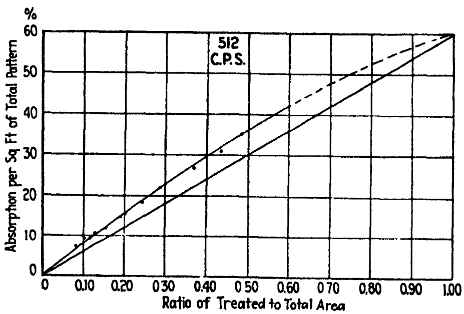


FIG. 60.—Absorbing power per square unit of distributed material plotted as a function of ratio of absorbent area to total area of pattern. (*After Parkinson.*)

of the actual area of felt to the total area over which it is distributed. In any one case, the units were all of one size and uniformly spaced, but the different points represent units ranging in size from 1 by 1 ft. to 2 by 8 ft. spaced at distances of from 1 to 4 ft. Diamondıaped and hexagonal units are also represented. The ıct that points so obtained fall upon a smooth curve shows ıat with a given area of absorbent material cut into units f the same size and uniformly distributed over a given rea, the total absorption is independent of the size and hape of the units and of the particular pattern in which hey are distributed. The difference between the ordinate f the straight line and the curve for any ratio of treated to otal area gives the increase in absorbing power per square oot due to spreading the material.

ffect of Sample Mounting on Absorption Coefficients.

Figure 32 (Chap. VI) shows the very marked increase in bsorption, as measured by the stationary-wave method, f shifting a porous material from a motion node to a

Vibrations per second	Mounted on 1-in. furring	Mounted on 2 by 4-in. studs
128	. . .	0 19
256	0 16	0.14
512	0.22	0.13
1,024	0.20	0.14
2,048	0.16	0 14
4,096	0 15	0 16

ıotion loop of the standing wave. In this case, the change f position of the test sample was of the order of half a wave ɛngth. Professor Sabine measured the effect of mounting -in. hair felt at distances of 2, 4, and 6 in., respectively, rom the wall of the sound chamber. The absorption oefficient at 512 vibs./sec. was increased from 0.57 to 0.67 ʼith an air space of 6 in. between the felt and the wall, with orresponding increases at lower frequencies. At higher requencies the increase was negligibly small. That the ffect of the method of mounting flexible, non-porous naterials may be very pronounced is shown by the above gures for a pressed-vegetable fiber board ½ in. thick. n the first instance, it was nailed loosely on 1-in. furring strips laid on the floor of the sound chamber, while in the second case, it was nailed firmly to a 2 by 4-in. wood-stud construction, with studs set 16 in. apart. The more rigid attachment to the heavier structure, allowing less freedom of motion, accounts for the difference.

In the table of absorption coefficients given in Appendix *C*, we note the marked difference in the absorbing efficiency of draperies resulting from hanging at different distances from the wall and from different amounts of folding.

Effect of Quality of Test Tones.

It is well known that the tones produced by organ pipes are rich in harmonic overtones. In using an organ pipe as a source of sound for absorption measurements, one assumes that the separate component frequencies of the complex tone are absorbed independently and that the absorption coefficient measured is that for the fundamental frequency. It is obvious that if the strength of any given harmonic relative to the fundamental is so great that this particular harmonic persists longer than the fundamental itself in the reverberant sound or if the conditions are such that the rate of decay of this harmonic is less than that of the fundamental under the two conditions of the sound chamber, then the absorption coefficient obtained by reverberation measurements will be that for the frequency of this harmonic rather than for the fundamental. The fact that the reverberation time of a sound chamber decreases as the pitch of the sound is raised makes for purification of the tone as the reverberant sound dies away; that is, the higher-pitched components die out first, leaving the fundamental as the tone for which the times are measured. If this condition exists, and if the assumption of the independent absorption of the harmonic components is correct, then the absorption coefficient obtained from a complex source should be the same as that for a pure tone of the same frequency.

The following data bear evidence on the question of the effect of tone quality upon the measured values of absorption coefficients. The absorption at 512 vibs./sec. of four different materials was measured using seven organ pipes of different tone qualities. These measurements on each material with the different pipes were all made under identical conditions. Three of the pipes were of the open diapason stop, from different manufacturers. The tone qualities may be roughly described as follows:

1. Tibia clausa...........	Stopped wooden pipe, strong fundamental, weak octave and twelfth
2. Melodia.	Open wood pipe, strong fundamental, weak octave and twelfth
3, 4, 5. Open diapason.. .	Strong fundamental and strong octave
6. Gemshorn	Strong fundamental, weak octave, strong higher harmonics
7. Gamba..	Weak fundamental, weak octave, strong higher harmonic

The materials were chosen so as to represent a wide diversity of characteristics as regards the variation of absorption with pitch.

TABLE XII.—COEFFICIENTS AT 512 VIBS./SEC.

Source	1	2	3	4	5	6	7	Loud-speaker
1. ½-in. acoustical plaster...............	0.26	0.28	0.31	0.27	0.30	0.29	0.30	
2. 1-in. wood-fiber tile..	0 36	0.37	0.37	0.36	0.39	0.40	0.33	
3. 1-in. asbestos hair felt	0.52	0 52	0.65	0.57	0.61	0.56	0.52	
4. 1¼-in. perforated fiber tile	0.54	0 55	0 65	0 60	0 61	0 58	0 57	0 62 0.61

It is to be said that for materials 1, 3, and 4 the coefficients for the open diapason tones are higher than for the other pipes. All of these materials have higher coefficients at the octave above 512, so that it seems fair to conclude that the presence of the strong octave in these tones tends to raise the coefficient at 512. The wood-fiber tile, on the other hand, has a nearly uniform coefficient over the whole frequency range, and we note no very marked difference which can be ascribed to the difference in quality. On the whole, it may be said that the presence of strong harmonics in the source of sound will occasion somewhat higher measured coefficients for materials that are decidedly more absorbent at the harmonic than at the fundamental frequency. With the exceptions just noted, the differences shown in Table XII are little if any greater than can be accounted for by experimental error.

Absorbing Power of Individual Objects.

In practical problems of computing the reverberation time in rooms, we must include the addition to the total absorbing power made by separate articles of furniture, such as seats and chairs and more particularly the absorbing power of the audience. The experimental data are best expressed not as absorption coefficients pertaining to the exposed surface but as units of absorbing power contributed to the total by each of the objects in question. Thus, in the case of chairs, for example, we should measure the change in the total absorbing power by bringing a number of chairs into the sound chamber and divide this change by the number.

Absorbing Power of Seats.

The wide variation in the absorbing power of seats of different sorts is shown in Table I (Appendix C), compiled from various sources. It is a difficult matter to specify all of the factors which affect the absorbing power, so that the figures given are to be taken as representative rather than exact. In the table, the absorbing power is expressed in English units, that is, as the equivalent area in square feet of an ideal surface whose coefficient is unity. The values in metric units may be obtained by dividing by 10.76, the number of square feet in a square meter.

Absorbing Power of an Audience.

By far the largest single contributor to the total absorbing power of an auditorium is the audience itself, and for this reason the reverberation will be markedly influenced by this factor. The usual procedure in estimating the total absorbing power of an audience is to find by measurement the absorbing power per person and multiply this by the number of persons. The data generally accepted for the absorbing power per person are those published by Wallace C. Sabine in 1906. Expressed in English units they are as follows:

Frequency	Absorbing Power
128	3.6
256	4.2
512	4.6
1,024	4.7
2,048	4 9
4,096	4.9

The audience on which these measurements were made consisted of 77 women and 105 men, and the measurements were made in the large lecture room of the Jefferson Physical Laboratory. These values are considerably higher than those given by the results of more recent measurements made under more ideal conditions as regards quiet. The effect of the inevitable noise created by this number of persons as well as disturbing sounds from without would tend toward higher values. Moreover, the much lighter clothing, particularly of women, at the present time, is not an inappreciable factor in reducing the coefficient from these earlier figures. The fact that Sabine's auditors were seated in the old ash settees with open backs, shown in his "Collected Papers," thus exposing more of the clothing to the sound, would give higher absorbing powers than are to be expected with an audience seated in chairs or pews with solid, non-absorbent backs. In view of the results already noted on the effect of spacing on the effective absorbing power of materials, it is apparent that the seating area per person also is a factor in the absorbing power of an audience.

The table shown on p. 142 gives some recent data on the measured absorbing power of people.

As will be noted, these figures are considerably lower than those of Sabine. There is another way of treating the absorbing power of an audience, and that is to regard the

TABLE XIII

Audience	128	256	512	1,024	2,048	Area per seat, square feet	Date	Authority
20 women	1 2	1 9	2 7	3 3	3 3	3.9	1931	P. E. S.
28 women	.	2.2	3.4	4 0	3 8	4.6	1920	P. E. S.
20 men	..	1.9	3.2	3.7	3 9	3.9	1931	P. E. S.
15 women .	0 7	1.3	2 3	3.6	4 5		1930	V. L. C.*
15 men. .	1 4	2.2	4 1	5 3	7 2	...	1930	V. L. C.*
Average.	1 1	2 1	3.2	4 0	4 5			

* CHRISLER, V L, *Jour Acous Soc. Amer.*, p. 126, July, 1930.

audience as an absorbing surface and to express the absorbing power per unit area. The following are the results so expressed obtained by the writer compared with the earlier figures.

Audience	256	512	1,024	2,048	Authority
Women, 1920 . . .	0.48	0 74	0 87	0 83	P. E. S.
Women, 1931.	0 48	0.69	0 84	0.84	P. E. S.
Men, 1931.	0.48	0 82	0.95	1.00	P. E. S.
Average	0 48	0 75	0 89	0 89	
Mixed, 1906	0 88	0 96	0 98	0 99	W. C. S.

The materially higher values of the earlier measurements can be accounted for partly by the change of style in clothing and partly by the difference in conditions as regards quiet.

Chrisler[1] gives the following figures for a mixed audience of six men and six women occupying upholstered theater chairs:

Frequency	Absorbing Power
256	3.6
512	4 1
1,024	4.8
2,048	4.2

[1] *Jour. of Acous. Soc. Amer.*, Vol. 2, No. 1, p. 127, July, 1930.

These values are more nearly in accord with those that are universally used in practice.

It is apparent from the foregoing that the sound-absorbing power of an audience is a quantity which depends upon a number of variable and uncontrollable factors and cannot be specified with any great degree of scientific precision. Up to the present time, the universal practice has been to use the values given by Professor Sabine, and the criteria of acoustical excellence are all based on these figures. We shall therefore, in considering the computation of the reverberation time of rooms in Chap. VIII, use his values, even while recognizing that they are higher than those which in scientific accuracy apply to present-day audiences.

Absorption Coefficients of Materials.

The last ten years have seen the commercial development and production of a very large number of materials designed for use as absorbents in reducing the reverberation in auditoriums and the quieting of noise in offices, hospitals and the like. The problems to be solved in the development of such materials are threefold: (1) the reduction of cost of material and application, (2) the production of materials that shall meet the practical requirements of appearance, durability, fireproofness, and cleanliness, and (3) the securing of materials with sufficiently high absorption coefficients to render them useful for acoustical purposes.

In the earliest application of the method, hair felt was extensively used. This was surfaced with fabric of various sorts stretched on furring over the felt. Painting of this fabric in the usual manner was found materially to lessen the absorbing efficiency. The development of better grades of felt mixed with asbestos, the gluing of a perforated, washable membrane directly to the felt, and finally the substitution of thin, perforated-metal plates for the fabric mark the evolution of this form of absorbent treatment. Boards made from sugar-cane fiber show moderately high absorption coefficients. These have been markedly increased by the expedient of increasing their thickness and boring holes at equally spaced intervals. Professor Sabine early developed a porous tile composed of granular particles bonded only at their points of contact, which has found extensive use in churches and other rooms where a tile treatment is applicable. In 1920, the writer began a series of investigations looking to the development of a plaster that should be much more highly absorbent than are the usual plaster surfaces. These investigations have resulted in a practicable commercial product of considerable use. Recently a highly absorbent tile of which the chief ingredient is mineral wool has been extensively used. Materials fabricated from excelsior, flax fiber, wood wool, and short wood fiber are also on the market.

The more widely used of these materials are listed under their trade names in the table of absorption coefficients given in Appendix C. As a result of development research, many manufacturers are effecting increases in the absorption coefficients of their materials, so that the date of tests is quoted in each instance. Earlier published data from the Riverbank Laboratories were based on the four-organ calibration. The figures given in the table are corrected to the more precise values given by loud-speaker methods of calibration.

CHAPTER VIII

REVERBERATION AND THE ACOUSTICS OF ROOMS

Having developed the theory of reverberation and its application to the measurement of the absorption coefficients of materials, we are now in a position to apply the theory to the prediction and control of reverberation. In this chapter, we shall consider first the detailed methods of calculating the reverberation time from the architect's plans for an audience room and, second, the question of the desired reverberation time considered both from the standpoint of the size of the room and also as it depends upon the uses for which the room is intended.

Calculation of Reverberation Time: Rectangular Room. While it is theoretically possible to calculate precisely the reverberation time of any given room from a knowledge of volume and the areas and absorption coefficients of all the absorbing surface, yet in any practical case, certain approximations will have to be made, and the prediction of the reverberation in advance of construction is a matter of enlightened estimate rather than precise calculation. The nature of these approximations will be indicated in the two numerical examples given. Fortunately the limits between which the reverberation time may lie without materially affecting hearing conditions are rather wide, so that such an estimate will be quite close enough for practical purposes.

We shall take as our first example a simple case of a small high-school auditorium, rectangular in plan and section, without balcony. From the architect's plans the following data are secured:

Dimensions, 100 by 50 by 20 ft.
Walls, gypsum plaster on tile, smooth finish
Ceiling, gypsum on metal lath, smooth finish

Stage opening, 30 by 12 ft., velour curtains
Wood floor
Wood-paneled wainscot, 6 ft. high, side and rear walls
700 unupholstered seats.

The common practice is to figure the reverberation for the tone 512 vibs./sec. The absorption coefficients are taken from Table II in Appendix C.

$$\text{Volume} = 100 \times 50 \times 20 = 100{,}000 \text{ cu. ft.}$$

Material	Area	Coefficients	Absorbing power
Plaster on tile	4,140	0 020	82
Plaster on lath	5,000	0 032	160
Wood paneling	1,500	0 10	150
Wood floor	5,000	0 04	200
Stage opening	360	0 44	159
700 seats	..	0 25	175
Total	.		926

In computing the absorption due to the audience, it is the common practice to assume that the additional absorption due to each person is 4.6 minus the absorbing power of the seat which he occupies. We have then the following for different audiences:

Audience	Additional absorption $n(4.6 - 0.25)$	Total absorbing power	$T = 0.05V/a$
None	.....	926	5 40
200	870	1,796	2 78
400	1,740	2,666	1.87
600	2,610	3,536	1 41
700	3,045	3,961	1 26

In passing, the preponderating rôle which the audience plays in the total absorbing power of the room is worth noting. In this case, 77 per cent of the total for the occupied room is represented by the audience. It is apparent that the absorption characteristics of an audience considered as a function of pitch, will in large measure determine the absorption frequency characteristics of audience rooms.

We may for the sake of comparison calculate the reverberation times using the later formula

$$T_0 = \frac{0.05V}{-S \log_e (1 - \alpha)}$$

where α is the average coefficient of all reflecting surfaces, and S is the total surface. In this very simple case, the average coefficient may be obtained by dividing the total absorbing power by the total area of floors, walls, and ceiling. In more complicated problems of rooms with balconies and recesses, arbitrary judgments will have to be made as to just what are to be considered as the bounding surface. In the present example, S = 16,000, and $0.05V/S$ = 0.312. We have the following values:

Audience	$\alpha = \frac{a}{16{,}000}$	$-\log_e (1 - \alpha)$	$T_0 = \frac{0.312}{-\log_e (1 - \alpha)}$
None	0 058	0 0585	5.33
200	0 1136	0 1178	2 65
400	0 1690	0 1845	1.69
600	0 2242	0 2540	1 23
700	0 2520	0 2910	1.07

We shall refer to this difference in the results of computation by the two formulas in considering the question of desirable reverberation times.

Empirical Formula for Absorbing Power.

Estimating the areas of the various surfaces in a room when the design is not simple may be a tedious process. Since, in audience rooms, the contribution to the total absorbing power of the empty room exclusive of the seats is usually only about one-fourth or one-fifth of the total when the room is filled, it is apparent that extreme precision in estimating the empty room absorption is not required. Thus in the example given, an error of 10 per cent in the empty-room absorption would make an error of only 3.4

per cent in the reverberation time for an audience of 400 and 2.3 per cent for the full audience.

The following empirical rule was arrived at by computing from the plans the absorbing powers of some 50 rooms ranging in volume from 50,000 to 1,000,000 cu. ft. Assuming only the usual interior surfaces of masonry walls and ceilings, wood floors, and having seats with an absorbing power of 0.3 unit each, the absorbing power exclusive of carpets, draperies, or other furnishings is given approximately by the relation

$$a \text{ (empty)} = 0.3V^{2/3}$$

Illustrating the use of this empirical formula, we estimate the total absorbing power of the empty room of the preceding example.

Bare room, masonry walls throughout, wood seats, $0.3\sqrt[3]{(100{,}000)^2}$	630
For 1,500 sq. ft. wood-paneling coefficient 0.10 in place of masonry coefficient 0.02, add 1,500 (0.10 − 0.02)	120
Stage opening	159
Total	909

The total of 909 units is quite close enough for practical purposes to the 926 units given by the more detailed estimate. In more complicated cases, the rule makes a very useful shortcut in estimating the empty-room absorbing power. We shall use it in estimating the reverberation time of a theater with balconies.

Reverberation Time in a Theater.

Figure 61 shows the plan and longitudinal section of the new Chicago Civic Opera House. The transverse section is rectangular, so that the room as a whole presents a series of expanding rectangular arches. In figuring the total volume, the volumes of these separate sections were computed, and deductions made for the balcony and box spaces. The necessary data, taken from the plans and preliminary specifications, are as shown on page 150.

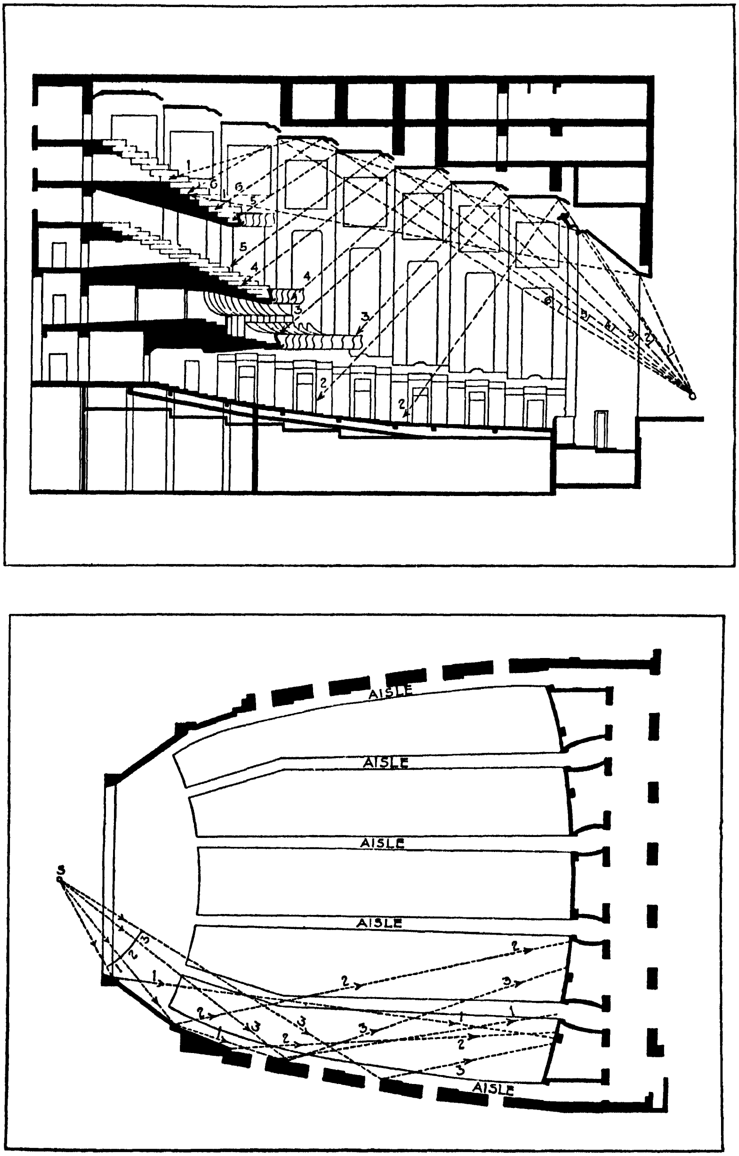


FIG. 61.—Plan and section, Chicago Civic Opera.

Total volume, from curtain line and including spaces under balcony, 842,000 cu. ft.
Walls and ceiling of hard plaster
Floors of cement
All aisles carpeted
Boxes carpeted and lined with plush
Heavy velour draperies in wall panels
3,600 seats, upholstered, seat and back
Velour curtain, 36 by 50 ft.

The figures for the total absorbing power follow:

	Units
1. Absorbing power of bare room, $0.3\sqrt[3]{842{,}000^2}$ (assuming wood seats 0.3)...	2,630
2. Boxes,* 8 by 112 ft. × 1.0	896
3. Stage, 1,800 sq. ft. × 0.44	792
4. Carpets, 3,200 sq. ft. × 0.25	800
5. Seats,† 3,600 × (2.6 − 0.3)...	8,300
6. Wall drapes, 2,400 sq. ft. × 0.44..	1,050
Total absorbing power of empty room...	14,468

$$T_0 = \frac{0.5 \times 842{,}000}{14{,}468} = 2.91 \text{ sec.}$$

* On account of the heavy padding of the boxes, the total opening of the boxes into the main body of the room was considered as an area from which no sound was reflected.

† Seats with absorbing power of 0.3 are assumed in the formula for the bare room; hence the deduction of 0 3 from 2.6, the absorbing power of the seats used.

Upon completion, careful measurements were made to determine the reverberation time. An organ pipe whose acoustic output was measured by timing in the sound chamber was used for this purpose. From the known value of a, the total absorbing power of the sound chamber, the value of E/i for this particular pipe and observer was determined from Eq. 39. With this value known, and the measured value of T_1 in the completed room, the value of the total absorbing power of the latter was computed using the same equation. The total absorbing power thus measured turned out to be 13,830 units, giving for the reverberation time a value of 3.05 sec., as compared with the estimated value of 2.91 sec.

Noting the effect of an audience occupying the more highly absorbent upholstered seats in comparison with the less absorbent chairs of the first example, we have the following reverberation times:

Audience	Added absorption	Total absorption	T_0
None.	. . . . .	14,400	2 93
1,200 × 2.0	2,400*	16,800	2 51
2,400 × 2.0	4,800	19,200	2.19
3,600 × 2.0	7,200	21,600	1 95

* The added absorbing power per person is assumed to be 4 6. The absorbing power of the person less 2 6, the absorbing power of the seat, which he is assumed to replace, is 2 0.

The value of the upholstered chairs in minimizing the effect of the audience upon the reverberation time is well brought out by the two examples chosen.

Allowance for Balcony Recesses.

In the foregoing, we have treated the recessed spaces under the balconies as a part of the main body of the room, contributing both to the volume and to the absorbing power terms of the reverberation equation. We may also consider these spaces as separate rooms coupled to the main body of the auditorium.[1] Assuming, for the moment, that the average coefficient of absorption of the surfaces of the under-balcony spaces is the same as that of the main room, it is apparent that the rate of decay of sound intensity in the former will be greater due to the fact that the mean free path is smaller and the number of reflections per second is correspondingly greater. In a recent paper, Eyring[2] reports the results of some interesting experiments on the reverberation times as measured in an under-balcony space 27 ft. long and 12 ft. deep, with a ceiling height of 11 ft. All the surface in this space except the floor was covered with sound-absorbent material. His measurements showed that at the rear of the space, there were two distinct rates of decay for tones of frequencies above 500 vibs./sec., the more rapid rate taking place during the first part of the decay process, while the slower rate at the end corresponds

[1] For a recent account of experimental research on this question the reader is referred to Reverberation Time in Coupled Rooms, by Carl F. Eyring, *Jour. Acous. Soc. Amer.*, vol. 3, No. 2, p. 181, October, 1931.

[2] *Jour. Soc. Mot. Pict. Eng.*, vol. 15, pp. 528–548, October, 1930.

very closely to that in the main body of the room. At the front, there was only a single rate of decay, except for the highest frequency of 4,000 vibs./sec. This single rate was nearly the same as the slower rate measured in the main body of the room.

The initial higher rate at the rear is thus the rate of decay of sound in the under-balcony space considered as a separate room. During this stage of the decay, the balcony opening feeds some energy back into the main body of the room and hence, looked at from the large space, does not act as an open window. Later on, however, the sound originally under the balcony having been absorbed, energy is fed in through the opening, and the opening behaves more like a perfectly absorbent surface for the main body of the room.

It is apparent that there is no precise universal rule by which allowance can be made for the effect of a recessed space upon the reverberation time. It will depend upon the average absorption coefficient of the recessed portion, the wave length of the sound, and the depth of the recess relative to the dimensions of the opening. A common-sense rule, and one which in the writer's experience works very well in practice, is the following:

Compute the total absorbing power of the space under the balcony. If this is less than the absorption supplied by treating the opening as a totally absorbing surface with a coefficient of unity, then consider this space as contributing to the volume and absorbing power of the room. Otherwise neglect both the volume and the absorbing power of the recessed space and consider the opening into the recessed portion as contributing to the main body of the room, an absorbing power equal to its area.

We shall, as an illustration, estimate the reverberation time of the Civic Opera House treating the balcony openings as perfectly absorbing surfaces. In the space under the balconies, there were 1,150 seats and 1,200 sq. ft. of carpet. These are to be deducted from the figures for the room considered as a whole.

Volume (excluding space under boxes and in the first balcony) 765,000 cu. ft.

	Units
1. Absorbing power of bare room $0.3\sqrt[3]{V^2}$..	2,400
2. Boxes, 8 by 112 ft. × 1.0...	896
3. Stage, 1,800 sq. ft. × 0.44.............	792
4. Carpets, 2,000 sq. ft. × 0.25............	500
5. Seats, 2,450 × (2.6 − 0.3)..	5,650
6. Wall drapes, 2,400 sq. ft. × 0.44.........	1,050
7. Balcony opening, 15 by 112 ft. × 1.0.. ..	1,680
8. Balcony opening, 15 by 114 ft. × 1.0	1,710
Total..	14,678

$$T_0 = \frac{0.05 \times 765{,}000}{14{,}678} = 2.61 \text{ sec.}$$

This we note is decidedly less than the measured value of 3.05 sec. Upon comparison of the data with that considering the under-balcony spaces as part of the main body of the room, it appears that the total absorbing power under the balconies is 3,190 units while the area of the openings is 3,390. According to the rule given above, we should expect the results of the first calculation to agree more nearly with the measured value, as they do. It turns out, in general, that if the depth of the balcony is more than three times the height from floor to ceiling at the front, then calculations based on the "open-window" concept agree more closely with measured values. It also appears that when the space under the balcony has a total absorbing power considerably greater than that of the opening considered as a surface with a coefficient of unity, the results of computing the reverberation times on the two assumptions do not differ materially.

This last follows from the fact that we reduce both the assumed volume and the total absorbing power when we treat the balcony opening as a perfectly absorbing portion of the boundary of the main body of the room. This decrease in both V and a will not materially alter their ratio, which determines the computed reverberation time. The interested reader may satisfy himself on this point by computing the reverberation time for the full-audience condition, treating the balcony opening as a perfectly absorbing surface.

Effect of Reverberation on Hearing.

Referring to the conditions for good hearing in audience rooms, as given by Professor Sabine, we note that the last of these is that "the successive sounds in rapidly moving articulation, either of music or speech, shall be free from each other." The effect of the prolongation of individual sounds by reverberation obviously militates against this requirement for good hearing. For example, the sound of a single spoken syllable may persist as long as 4 sec. in a reverberant room. During this interval, a speaker may utter 16 or more syllables. The overlapping that results will seriously lessen the intelligibility of sustained speech. With music, the effect of excessive reverberation is quite similar to that of playing the piano with the sustaining pedal held down. On the other hand, common experience shows that in heavily damped rooms, speech, while quite distinct, lacks apparent volume, and music is lifeless and dull. The problem to be solved therefore is to find the happy medium between these two extremes. Two lines of attack suggest themselves. By direct experiment one may vary the reverberation time in a single room to what is considered to be the most satisfactory condition and measure, or calculate this time; or one may measure the reverberation time in rooms which have an established reputation for good acoustical properties. The first method was employed by Professor Sabine for piano music in small rooms.[1] His results showed a remarkably precise agreement by a jury of musicians upon a value of 1.08 sec. as the most desirable reverberation time for piano music, for rooms with volumes between 2,600 and 7,400 cu. ft. (74 and 210 cu. m.) His contemplated extension of this investigation to larger rooms and different types of music was never carried out.

[1] "Collected Papers on Acoustics," p. 71.

Reverberation: Speech Articulation, Knudsen's Work.

V. O. Knudsen[1] has made a thoroughgoing investigation of the effect of reverberation upon articulation. The method employed was that used by telephone engineers in testing speech transmission by telephone equipment. The "percentage articulation" of an auditorium is the percentage of the total number of meaningless syllables correctly heard by an average listener in the auditorium. Typical monosyllabic speech sounds are called in groups of three by a speaker. Observers stationed at various parts of the room record what they think they hear. The number of syllables correctly recorded expressed as a percentage of the number spoken is the percentage articulation for the auditorium. The curves of Fig. 62 taken from Knudsen's paper show the percentage articulation in a number of rooms similar in shape and with volumes between 200,000 and 300,000 cu. ft. The lower curve gives the best fit with the observed data, while the upper curve shows the percentage articulation assuming that the level of intensity in all cases was $10^7 \times i$. (It will be recalled that for a source of given acoustical power, the average intensity set up is inversely proportional to the absorbing power.) We note that increasing the reverberation time from 1.0 to 2.0 sec. lowers the percentage articulation from 90 to 86 per cent, while if the reverberation is still further increased to 3.0 sec., the articulation becomes about 80 per cent.

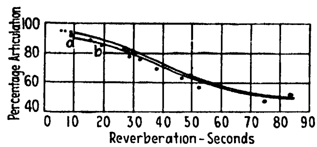


FIG. 62.—Relation between percentage articulation and reverberation time. (*After Knudsen.*)

Reverberation and Intelligibility of Speech.

These figures, however, do not give the real effect of reverberation upon the intelligibility of connected speech. In listening to the latter, the loss of an occasional syllable produces a very slight decrease in the intelligibility of con-

[1] *Jour. Acous. Soc. Amer.*, vol. 1, No. 1, p. 57. 1929.

nected phrases and sentences. The context supplies the meaning. Tests made on this point by Fletcher[1] at the Bell Laboratories show the following relation between the percentage articulation and the intelligibility of connected speech.

Percentage Articulation	Intelligibility, Per Cent
70	98
80	99
90	99+

In these tests, the intelligibility was the percentage of questions correctly understood over telephone equipment, which gave the single-syllable articulation shown. These figures taken with Knudsen's results indicate that reverberation times as great as 3 sec. may not materially affect the intelligibility of connected speech.

Further, it seems fair to say that with a reverberation time of 2 sec. or less in rooms of this size (200,000 to 300,000 cu. ft.) connected speech of sufficient loudness should be quite intelligible. This conclusion should be borne in mind when we come to consider the reverberation time in rooms that are intended for both speech and music. Viewed simply from the standpoint of intelligibility, the requirements of speech do not impose any very precise limitation upon the reverberation time of auditoriums further than that it should be less than 2.0 sec. While in Knudsen's tests the decrease in reverberation time below 2.0 sec. produced measurable increase in syllable articulation, yet the improvement in intelligibility of connected speech is negligibly small.

Reverberation and Music.

In order to arrive at the proper reverberation for rooms intended primarily for music, the procedure has been to secure data on rooms which according to the consensus of musical taste as well as of popular approval are acoustically satisfactory. Obviously both of these are, in the very

[1] "Speech and Hearing," D. Van Nostrand Company, p. 266, 1929.

ture of the case, somewhat difficult to arrive at. With me rare and refreshing exceptions the critical faculties musicians do not extend to the scientific aspects of their t, while public opinion seldom becomes articulate in icing approval.

However, the data given in Table XIV, compiled from rious sources, are for rooms which enjoy established

ABLE XIV.—REVERBERATION TIMES OF ACOUSTICALLY GOOD ROOMS

Auditorium	Volume, cubic feet	Seats	T_0	T_1	Authority
sic Rooms, New England Conservatory	2,600–7,400	.....	1.08	1.07–1.19	W. C. Sabine
ıll auditorium, Moscow	4,450	....	1.06	1.15	Lifshitz
scow Conservatory	90,000	550	1.30	1.19	Lifshitz
sikvereinssaal, Vienna	290,000	1,800	1.62*	1.39	Knudsen
ozig Gewandhaus	363,000	1,560	1.90	1.64	Bagenal
ozig Gewandhaus	363,000	1,560	1.50*	....	Knudsen
demy of Music, Philadelphia	400,000	2,800	1.76	1.49	P. E. Sabine
hestra Hall, Detroit	400,000	2,200	1.44	1.20	P. E. Sabine
demy of Music, Brooklyn	430,000	2,200	1.60	1.31	Tallant
at Theater, Moscow	486,000	2,300	1.55†	1.25	Lifshitz
ton Opera House	500,000	2,350	1.51	1.23	P. E. Sabine
erance Hall, Cleveland	554,000	1,840	1.85	1.51	D. C. Miller
at Hall, Moscow Conservatory	600,000	2,150	2.00‡	1.63	Lifshitz
ıphony Hall, Boston	650,000	2,600	1.93	1.55	P. E. Sabine
negie Hall, New York	737,000	2,600	1.75	1.38	P. E. Sabine
Memorial, Ann Arbor, Michian	795,000	5,000	1.70	1.33	Tallant
tman Theater, Rochester	790,000	3,340	2.08	1.65	Watson
c Opera House, Chicago	842,000	3,600	1.95	1.48	P. E. Sabine
cago Auditorium	925,000	3,640	1.90	1.48	P. E. Sabine

$T_0 = \frac{0.05V}{a}$. $T_1 = \frac{0.0083V}{a}(9.1 - \log_{10} a)$.

* Computed by formula $T_0 = \frac{0.05V}{-S \log_e \left(1 - \frac{a}{S}\right)}$.

† Reverberation considered too low

‡ Reverberation considered too high.

outations for good acoustics. In Fig. 63, the reverberan times computed by the formula $T = 0.05V/a$ are otted as a function of the volume in cubic feet. So otted, these data seem to justify two general statements. rst, the reverberation time for acoustically good rooms ows a general tendency to increase with the volume in bic feet and, second, for rooms of a given volume there is a fairly wide range within which the reverberation time may fall.

In Fig. 64 is shown an empirical curve given by Professor Watson[1] of what he has called the "optimum time of reverberation," also a curve partly empirical and partly theoretical proposed by Lifshitz.[2]

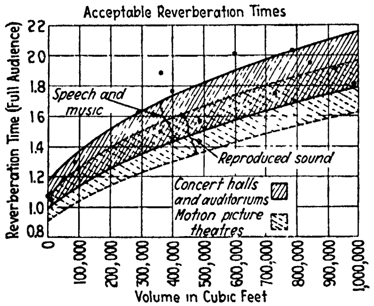


FIG. 63.—Acceptable reverberation times for rooms of different volumes.

Watson's curve is deduced from the computed reverberation times of acoustically good rooms. Lifshitz finds experimental verification for his curve in the judgment of trained musicians as to the best time of reverberation in a small room (4,500 cu. ft.) and the reverberation times of a number of auditoriums in Moscow.

Recently MacNair[3] has undertaken to arrive at a theoretical basis for the increase in desirable reverberation time with the volume of the room by assuming that the loudness of the reverberant sound integrated over the total time of decay shall have a constant value. This implies that both the maximum intensity and the duration of a sound contribute to the magnitude of the psychological impression.

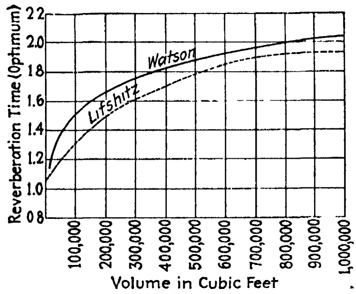


FIG. 64.—Optimum reverberation times proposed by Watson and Lifshitz.

MacNair's theoretical curve for optimum reverberation time does not differ materially from those shown in Figs. 63 and 64, a fact which would seem to give weight to his assumption. Experience in phonograph recording also

[1] *Architecture*, vol. 55, pp. 251–254, May, 1927.
[2] *Phys. Rev.*, vol. 3, pp. 391–394, March, 1925.
[3] *Jour. Acous. Soc. Amer.*, vol. 1, No. 2, p. 242, January 1930.

hows that a certain amount of reverberation gives the effect f increased volume of tone to recorded music, an effect 'hich apparently cannot be obtained by simply raising the ;vel of the recorded intensity.

Optimum Reverberation Time."

The practical question arises as to whether, in the light f present knowledge, given a proposed audience room f given volume, we are justified in assigning a precise alue to the reverberation time in order to realize the best earing conditions. (The author objects strenuously to ıe frequently used term "perfect acoustics.") There ; also the other question as to whether this specified time f reverberation should depend upon the use to which the ɔom is to be put, whether speech or music, and, if the latter, pon the particular type of music contemplated.

The data on music halls given above suggest a negative nswer to the first question, if the emphasis is laid upon the 'ord *precise*.

To draw a curve of best fit of the points shown and say ıat the time given by this curve for an auditorium of any iven volume is the optimum time would be straining for a ;ientific precision which the approximate nature of our stimate of the reverberation time does not warrant. Take ıe best-known case, that of the Leipzig Gewandhaus. abine's estimate of the reverberation time, based on the ıformation available to him, was 2.3 sec. Bagenal,[1] ·om fuller architectural data obtained in the hall itself, stimates the time at 1.9 sec.; while Knudsen, from ɜverberation measurements in the empty room, estimates time of 1.5 sec. for the full-audience condition. Further, ıere is a question as to the precision of musical taste as pplied to different rooms. Several years ago, a questionaire was sent to the leading orchestra conductors in .merica in an attempt to elicit opinions as to the relative

[1] Bagenal, Hope, and Bursar Godwin, *Jour. R.I.B.A.*, vol. 36, pp. 56–763, Sept. 21, 1929.

acoustical merits of American concert halls. Only five[1] of the gentlemen replied.

Rated both by the number of approvals and the unqualified character of the comments of these five, of the rooms for which we have data the Academy of Music (Philadelphia), Carnegie Hall (New York), the Chicago Auditorium, Symphony Hall (Detroit), and Symphony Hall (Boston) may be taken as outstanding examples of satisfactory concert halls. Referring to Fig. 63, we note that all of these, with the exception of the Chicago Auditorium, fail to satisfy our desire for scientific precision by refusing to fall exactly on the average curve. The shaded area of Fig. 63 represents what would seem to be the range covered by the reverberation times of rooms which are acoustically satisfactory for orchestral music. The author suggests the use of the term "acceptable range of reverberation times" in place of "optimum reverberation time," as more nearly in accord with existing facts as we know them. There can be only *one optimum*, and the facts do not warrant us in specifying this with any greater precision than that given by Fig. 63.

Speech and Musical Requirements.

Turning now to the question of discriminating between the requirements for speech and music, we recall that on the basis of articulation tests a reverberation time as great as 2.0 seconds does not materially affect the intelligibility of connected speech. Since the requirements for music call for reverberation times less than this, there appears to be no very strong reason for specifying conditions for rooms intended primarily for speaking that are materially different from those for music. As a practical matter, auditoriums in general are designed for a wide variety of uses, and, except in certain rare instances, a reasonable compromise that will meet all requirements

[1] It is a pleasure to acknowledge the courteous and valuable information supplied by Mr. Frederick Stock, Mr. Leopold Stokowski, Mr. Ossip Gabrilowitsch, Mr. Willem Van Hoogstraten, and Mr. Eric De Lamarter.

is the more rational procedure. The range of reverberation times shown in Fig. 63 represents such a compromise. Thus the Chicago Auditorium and Carnegie Hall have long been considered as excellent rooms for public addresses and for solo performances as well as for orchestral music, while the new Civic Opera House in Chicago, intended primarily for opera, has received no criticism when used for other purposes. Tests conducted after its completion showed that the speech of very mediocre speakers was easily understood in all parts of the room.[1]

European Concert Halls.

V. O. Knudsen[2] gives an interesting and valuable account of some of the more important European concert halls. His findings are that reputations for excellent acoustics are associated with reverberation times lying between 1 and 2 sec., with times between 1.0 and 1.5 sec. more commonly met with than are times in the upper half of the range. The older type of opera house of the conventional horseshoe shape with three or four levels of boxes and galleries all showed relatively low reverberation times, of the order of 1.5 sec. in the empty room. This is to be expected in view of the fact that placing the audience in successive tiers makes for relatively small volumes and large values of the total absorbing power. Custom would account, in part, for the general approval given to the low reverberation times in rooms of this character. The desirability of the usual, no doubt, is also responsible for the general opinion that organ music requires longer reverberation times than orchestral music. Organ music is associated with highly reverberant churches, while the orchestra is usually heard in crowded concert halls.

In the paper referred to, Knudsen gives curves showing reverberation times which he would favor for different

[1] Severance Hall in Cleveland may be cited as a further instance. In the design of this room, primary consideration was given to its use for orchestral music. Its acoustic properties prove to be eminently satisfactory for speech as well.

[2] *Jour. Acous. Soc. Amer.*, vol. 2, No. 4, p. 434, April, 1931.

types of music and for speech. The allowable range which he gives for music halls is considerably greater than that shown in Fig. 63, although the middle of the range coincides very closely with that here given. He proposes to distinguish between desirable reverberation for German opera as contrasted with Italian opera, allowing longer times for the former. He advocates reverberation times for speech, which are on the average 0.25 sec. less than the lowest allowable time for music. These times are considerably lower than those in existing public halls, and their acceptance would call for a marked revision downward from present accepted standards. The adoption of these lower values would necessitate either marked reduction in the ratio of volume to seating capacity or the adoption of the universal practice of artificially deadening public halls. Whether the slight improvement in articulation secured thereby would justify such a revision is in the mind of the author quite problematical.[1]

Formulas for Reverberation Times.

In the foregoing, all computations of reverberation times have been made by the simple formula $T_0 = 0.05V/a$, which gives the time for the continuous decay of sound through an intensity range of 1,000,000 to 1. Since the publication of Eyring's paper giving the more general relation $T = \frac{0.05V}{-S \log_e (1 - \alpha_a)}$, some writers on the subject have preferred to use the latter. As we have seen, this gives lower values for the reverberation time than does the earlier formula, the ratio between the two increasing with the average value of the absorption coefficient. Therefore, there is a considerable difference between the two, particularly for the full-audience conditions. As long as we adhere to a single formula both in setting up our criterion of acoustical excellence on the score of reverberation and

[1] Since the above was written, the author has been informed by letter that Professor Knudsen's recommended reverberation times are based on the Eyring formula. This fact materially lessens the difference between his conclusions and those of the writer.

also in computing the reverberation time of any proposed room, it is immaterial which formula is used. Consistency in the use of one formula or the other is all that is required. In view of the long-established use of the earlier formula and the fact that the criterion of excellence is based upon it, and also because of its greater simplicity, there appears to be no valid reason for changing to the later formula in cases in which we are interested only in providing satisfactory hearing conditions in audience rooms.

Variable Reverberation Times.

Several writers have proposed that, in view of the supposedly different reverberation requirements of different types of music, means of varying the reverberation time of concert halls is desirable. This plan has been employed in sound-recording rooms and radio broadcasting studios. Osswald of Zurich has suggested a scheme whereby the volume term of the reverberation equation may be reduced by lowering movable partitions which would cut off a part of a large room when used by smaller audiences and for lighter forms of music. Knudsen intimates the possible use of suitable shutters in the ceiling with absorptive materials behind the shutters as a quick and easy means of adapting a room to the particular type of music that is to be given. In connection with Osswald's scheme, one must remember that in shutting off a recessed space, we reduce both volume and absorbing power and that such a procedure might raise instead of lower the reverberation time. Knudsen's proposal is open to a serious practical objection in the case of large rooms on the score of the amount of absorbent area that would have to be added to make an appreciable difference in the reverberation time. Take the example given earlier in this chapter. Assume that 2.0 sec. is agreed upon as a proper time for "Tristan and Isolde" and 1.5 sec. for "Rigoletto." The absorbing power would have to be increased from 21,600 to 28,800 units, a difference of 7,200 units. With a material whose coefficient is 0.72 we should need 10,000 sq. ft. of shutter area to effect the change. Even if he were willing to sacrifice all architectural ornament of the ceiling, the architect would be hard put to find in this room sufficient ceiling area that would be available for the shutter treatment. One is inclined to question whether the enhanced enjoyment of the average auditor when listening to "Rigoletto" with the shutters open would warrant the expense of such an installation and the sacrifice of the natural architectural demand for normal ceiling treatment.

Waiving this practical objection, one is inclined to wonder if there is valid ground for the assumption that there is a material difference in reverberation requirements for different types of music. Wagnerian music is associated with the tradition of the rather highly reverberant Wagner Theater in Bayreuth. Italian opera is associated with the much less reverberant opera houses of the horseshoe shape and numerous tiers of balconies and boxes. Therefore, we are apt to conclude that Wagner's music demands long reverberation times, while the more florid melodic music of the Italian school calls for short reverberation periods. Organ music is usually heard in highly reverberant churches and cathedrals, while chamber music is ordinarily produced in smaller, relatively non-reverberant rooms. All of these facts are tremendously important in establishing not musical taste but musical tradition.

On the whole it would appear that the moderate course in the matter of reverberation, as given in Fig. 63, will lead to results that will meet the demands of all forms of music and speech, without imposing any serious special limitations upon the architect's freedom of design or calling for elaborate methods of acoustical treatment.

Reverberation Time with a Standard Sound Source.

In 1924, the author[1] proposed a formula for the calculation of the reverberation time based on the assumption of a fixed acoustic output of the source, instead of a fixed value of the steady-state intensity of $10^6 \times i$. For a source

[1] *Amer. Architect*, vol. 125, pp. 579–586, June 18, 1924.

of given acoustic output E, the steady-state intensity is given by the equation

$$I_1 = \frac{4E}{ac}$$

so that the steady-state intensity for a given source will vary inversely as the absorbing power. This decrease of steady-state intensity with increasing absorbing power accounts in part for the greater allowable reverberation time in large rooms, so that it would seem that the reverberation in good rooms computed for a fixed source would show less variation with volume than when computed for a fixed steady-state intensity. The acoustical output of the fundamental of an open-diapason organ pipe, pitch 512 vibs./sec., is of the order of 120 microwatts. This does not differ greatly from the power of very loud speech. Taking this as the fixed value of the acoustic output of our sound source, we have, by the reverberation theory,

$$T_1 = \frac{0.0083V}{a}(9.1 - \log_{10} a) \tag{70}$$

The bracketed expression is the logarithm of the steady-state intensity set up by a source of the specified output in a room whose absorbing power is a. When this equals 6.0 (logarithm of 10^6), Eq. (70) reduces to the older formula. The expression log a thus becomes a correction term for the effect of absorbing power on initial intensity. The values of T_1, computed on the foregoing assumptions, are included in Table XIV. We note a considerably smaller spread between the values of the reverberation times for good rooms when computed by Eq. (70) and a less marked tendency toward increase with increasing volume. For rooms with volumes between 100,000 and 1,000,000 cu. ft. we may lay down a very safe working rule that the reverberation time computed by Eq. (70) should lie between 1.2 and 1.6 sec. For smaller rooms and rooms intended primarily for speech the reverberation time should fall in the lower half of the range, and for larger rooms and rooms intended for music it may fall in the upper half.

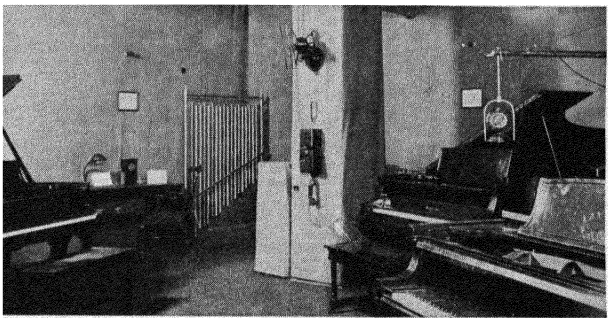
Reverberation for Reproduced Sound.

An important difference between original speech and music and that reproduced in talking motion pictures lies in the fact that the acoustical output of an electrical loud-speaker may be and usually is considerably greater than that of the original source. This raises the average steady-state intensity set up by the loud-speaker source above that which would be produced in the same room by natural sources and hence for a given relation of volume to absorbing power produces actually longer duration of the residual sound. For this reason it has generally been assumed that the desirable computed reverberation should be somewhat less than for theaters or concert halls. S. K. Wolf[1] has measured the reverberation in a large number of rooms that are considered excellent for talking motion pictures and gives a curve of optimum reverberation times. Interestingly enough, his average curve coincides almost precisely with the lower limit of the range given in Fig. 63. Due to the directive effect of the usual types of loud-speaker, echoes from rear walls are sometimes troublesome in motion-picture theaters. What would be an excessive amount of absorption, reverberation alone considered, is sometimes employed to eliminate these echoes. By raising the reproducing level, the dullness of overdamping may be partly eliminated, so that talking-motion-picture houses are seldom criticised on this score even though sometimes considerably overabsorbent. At the present time, there is a tendency to lay many of the sins of poor recording and poor reproduction upon the acoustics of the theater. The author's experience and observation would place the proper reverberation time for motion-picture theaters in the lower half of the range for music and speech, given in Fig. 63, with a possible extension on the low side as shown. Sound recording and reproduction have not yet reached a stage of perfection at which a precise criterion of acoustical excellence for sound-motion-picture theaters can be set up.

[1] *Jour. Soc. Mot. Pict. Eng.*, vol. 45, No. 2, p. 157.

Reverberation in Radio Studios and Sound-recording Rooms.

Early practice in the treatment of broadcasting studios and in phonograph-recording rooms was to make them as "dead" (highly absorbent) as possible. For this purpose, the walls and ceilings were lined throughout with extremely heavy curtains of velour or other fabric, and the floors were heavily carpeted. Reverberation was extremely low, frequently less than 0.5 sec., computed by the $0.05V/a$ formula. The best that can be said for such treatment is that from the point of view of the radio or phonograph

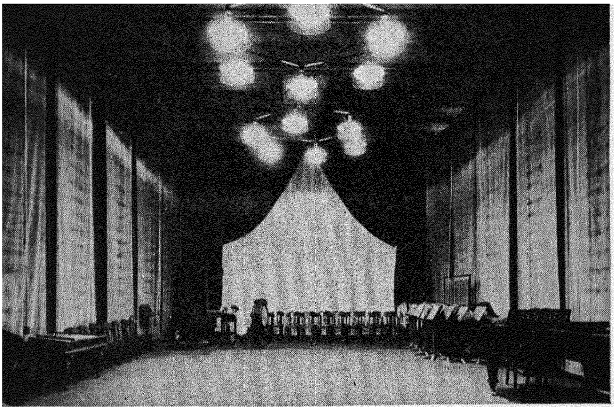


Early radio broadcasting studio. All surfaces were heavily padded with absorbent material.

auditor the effect of the room was entirely negative—one got no impression whatsoever as to the condition under which the original sound was produced. For the performer, however, the psychological effect of this extreme deadening was deadly. It is discouraging to sing or play in a space where the sound is immediately "swallowed up" by an absorbing blanket.

One of the best university choirs gave up studio broadcasting because of the difficulty and frequent failure to keep on the key in singing without accompaniment.

Gradually the tendency in practice toward more reverberant rooms has grown. In 1926, the author, through the courtesy of Station WLS in Chicago, tried the effect on the radio listeners of varying the reverberation time in a small studio, with a volume of about 7,000 cu. ft. The reverberation time could be quickly changed in two successive steps from 0.25 to 0.64 sec. The same short program of music was broadcast under the three-room conditions, and listeners were asked to report their preference. There were 121 replies received. Of these, 16 preferred the least reverberant condition, and 73 the most reverberant condition. It was not possible to carry the reverberation to still longer times. J. P. Maxfield states that in sound



Modern broadcasting studio. Absorption can be varied by proper disposal of draperies.

recording for talking motion pictures, the reverberation time of the recording room should be about three-fourths that of a room of the same size used for audience purposes. This, he states, is due to the fact that in binaural hearing, the two ears give to the listener the power to distinguish between the direct and the reverberant sound and that attention is therefore focused on the former, and the latter is ignored. With a single microphone this attention factor is lacking, with the result that the apparent reverberation is enhanced.

On this point, it may be said that phonograph records of large orchestras are often made in empty theaters.

Music by the Philadelphia Symphony Orchestra is recorded in the empty Academy of Music in Philadelphia, where according to the author's measurements, the measured reverberation time is 2.3 sec. It is easy to note the reverberation in the records, but this does not in any measure detract from the artistic quality or naturalness of the recorded music.

The Sunday afternoon concerts of the New York Philharmonic Orchestra are broadcast usually from Carnegie Hall or the Brooklyn Academy of Music. Here the reverberation of these rooms under the full-audience condition is not noticeably excessive for the radio listener. All these facts considered, it would appear that reverberation times around the lower limit given for audience rooms in general will meet the requirements of phonograph recording and radio broadcasting.

In view of the fact that in sound recording the audience is lacking as a factor in the total absorbing power, it follows that this deficiency will have to be supplied by the liberal use of artificial absorbents. It is at present a mooted question as to just what the frequency characteristics of such absorbents should be. If the pitch characteristics of the absorbing material *do* have an appreciable effect upon the quality of the recorded sound, then these characteristics are more important than in the case of an auditorium. In the latter, the audience constitutes the major portion of the total absorbing power, and its absorbing power considered as a function of pitch will play a preponderant rôle. This rôle is taken by the artificial absorbent in the sound stage. A "straight-line absorption," that is, uniform absorption at all frequencies, has been advocated as the most desirable material for this use. As has already been indicated, porous materials show marked selective absorption when used in moderate thickness. Thus hair felt 1 in. thick is six times as absorbent at high as at low frequencies, and this ratio is 2.5:1 for felt 3 in. thick. Certain fiber boards show almost uniform absorption over the frequency range, but the coefficients are relatively low as compared with those of fibrous material. The nearest approach to a uniform absorption for power over the entire range of which the writer has any knowledge consists of successive layers of relatively thin felt, ½ in. thick, with intervening air space. A rather common current practice in sound-picture stages is the use of 4 in. of mineral wool packed loosely between 2 by 4-in. studs and covered with plain muslin protected by poultry netting. The absorption coefficients of this material are as follows:

Frequency	Coefficient
128	0 46
256	0 61
512	0 82
1,024	0 82
2,048	0 64
4,096	0 60

Here the maximum coefficient is less than twice the minimum, and this seems to be as near straight-line absorption as we are likely to get without building up complicated absorbing structures for the purpose.

Whether or not uniform absorption is necessary to give the greatest illusion of reality is a question which only recording experience can answer. If the effect desired is that of out of doors, it probably is. But for interiors, absorption characteristics which simulate the conditions pictured would seem more likely to create the desired illusion. For a detailed treatment of the subject of reverberation in sound-picture stages the reader is referred to the chapter by J. P. Maxfield in "Recording Sound for Motion Pictures."[1]

[1] McGraw-Hill Book Company, Inc., 1931.

CHAPTER IX

ACOUSTICS IN AUDITORIUM DESIGN

The definition of the term auditorium implies the necessity of providing good hearing conditions in rooms intended for audience purposes. The extreme position that the designer might take would be to subordinate all other considerations to the requirements of good acoustics. In such a case, the shape and size of the room, the contours of walls and ceiling, the interior treatment, both architectural and decorative, would all be determined by what, in the designer's opinion, is dictated by acoustical demands. The result, in all probability, would not be a thing of architectural beauty. Acoustically, it might be satisfying, assuming that the designer has used intelligence and skill in applying the knowledge that is available for the solution of his problem.

Fortunately, good hearing conditions do not impose any very hard and fast demands that have to be met at the sacrifice of other desirable features of design. Rooms are good not through possession of positive virtues so much as through the absence of serious faults. The avoidance of acoustical defects will yield results which experience shows are, in general, quite as satisfying as are attempts to secure acoustical virtues. The present chapter will be devoted to a consideration of features of design that lead to undesired acoustical effects and ways in which they may be avoided.

Defects Due to Curved Shapes.

Figure 65 gives the plan and section of an orchestral concert hall, very acceptable in the matter of reverberation but with certain undesirable effects that are directly traceable to the contours of walls and ceilings. These effects are almost wholly confined to the stage. The conductor of the orchestra states that he finds it hard to secure a satisfactory balance of his instruments and that the musical effect as heard at the conductor's desk is quite different from the same effect heard at points in the audience. The organist states that at his bench at one side and a few feet above the stage floor, it is almost impossible to hear certain instruments at all. A violinist in the front row on the left speaks of the sound of the wood winds on the

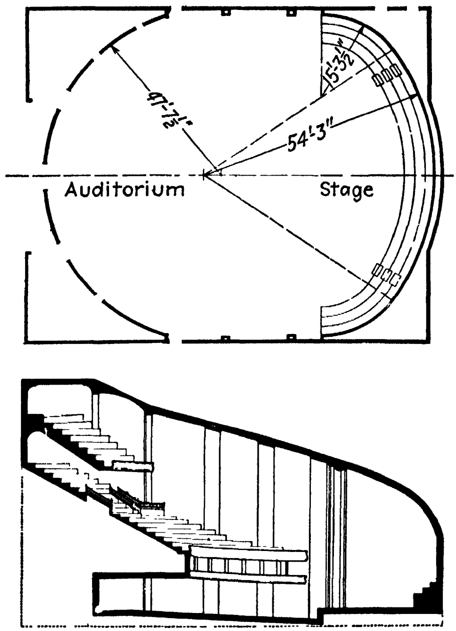


FIG. 65.—Plan and section, Orchestra Hall, Chicago.

right and farther back on the stage as "rolling down on his head from the ceiling." A piano-solo performance heard at a point on the stage is accompanied by what seems like a row of pianos located in the rear of the room. In listening to programs broadcast from this hall, one is very conscious of any noise such as coughing that originates in the audience, as well as an effect of reverberation that is much greater than is experienced in the hall itself. None of these effects is apparent from points in the audience.

The sound photographs of Fig. 66 were made using plaster models of plan and section of the stage. In *A* and *B*, the source was located at a position corresponding to the

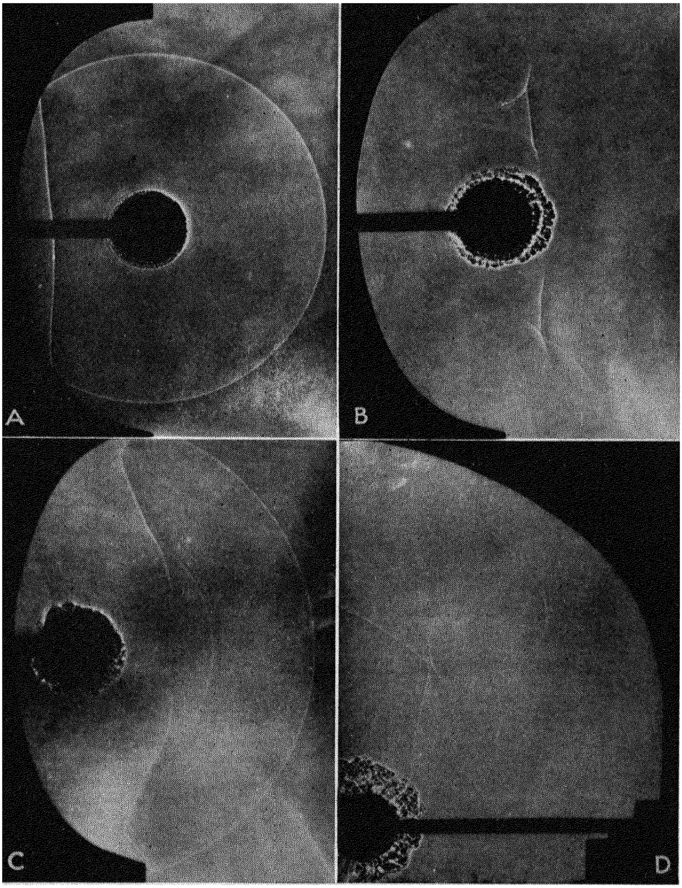


FIG. 66.—Reflection from curved walls, Orchestra Hall stage.

conductor's desk. Referring to the plan drawing, we note that this is about halfway between the rear stage wall and the center of curvature of the mid-portion of this wall. In the optical case, this point is called the principal focus of the concave mirror, and a spherical wave emanating from this point is reflected as a plane wave from the concave surface, a condition shown by the reflected wave in *A*. In *B*, we note that the sound reflected from the more sharply curved portions is brought to two real focuses at the sides of the stage. In *C*, with the source near the back stage wall, the reflected wave front from the main curvature is convex, while the concentrations due to the

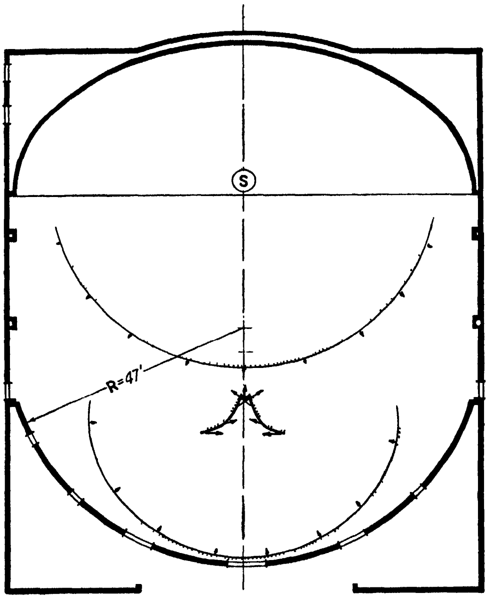


Fig. 67.—Concentration from curved rear wall.

coved portions at the side are farther back than when the source is located at the front of the stage. In "Collected Papers,"[1] Professor Sabine shows the effect of a cylindrical rear stage wall upon the distribution of sound intensity on the stage, with marked maxima and minima, and an intensity variation of 47 fold. The example here cited is somewhat more complicated by the fact that there are two regions of concentration at the sides instead of a single region as in the cylindrical case. But it is apparent that

[1] "Collected Papers on Acoustics," Harvard University Press, p. 167, 1922.

the difficulty of securing a uniformly balanced orchestra at the conductor's desk is due to the exaggerated effects of interference which these concentrations produce. The effect of the ceiling curvature in concentrating the reflected sound is apparent in *D*, which accounts for the difficulty reported by the violinist. This photograph also explains the pronounced effect at the microphone of noise originating

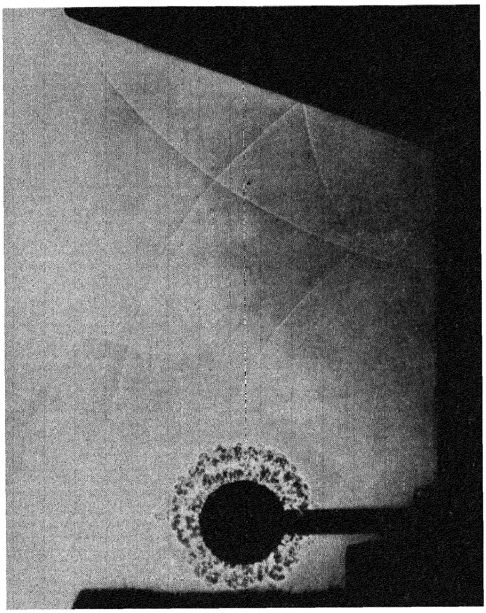


FIG. 68.—Plane ceiling of stage prevents concentration.

in the audience. Such sounds will be concentrated by reflection from the stage wall and ceiling in the region of the microphone. The origin of the echo from the rear wall of the room noted on the stage is shown in Fig. 67. Here the reflected wave was plotted by the well-known Huygens construction. We note the wave reflected from the curved rear wall converging on a region of concentration, from which it will again diverge, giving to the listener on the stage the effect of image sources located in the rear of the room. Again, a second reflection from the back stage wall will refocus it near the front of the stage. The conductor's position thus becomes a sort of acoustical "storm center," at which much of the sound reflected once or twice from the principal bounding surfaces tends to be concentrated. Figure 68 shows the effect of substituting plain surfaces for the curved stage ceiling. One notes that the reflected wave front is convex, thus eliminating the focusing action that is the source of difficulty.

Allowable Curved Shapes.

If we were to generalize from the foregoing, we should immediately lay down the general rule that all curved shapes in wall and ceiling contours of auditoriums should be avoided. This rule is safe but not eminently sane, since there are many good rooms with curved walls and ceilings. Moreover, the application of such a rule would place a serious limitation upon the architectural treatment of auditorium interiors. In the example given, we note that the centers of curvature fall within the room and either near the source of sound or near the auditors. We may borrow from the analogous optical case a formula by which the region of concentration produced by a curved surface may be located. If s is the distance measured along a radius of curvature from the source of sound to the curved reflecting surface, and R the radius of curvature, then the distance x from the surface (measured along a radius) at which the reflected sound will be concentrated will be given by the equation

$$x = \frac{sR}{2s - R} \qquad (71)$$

The rule is only approximate, but it will serve as a means of telling us whether or not a curved surface will produce concentrations in regions that will prove troublesome.

Starting with the source near the concave reflecting surface, $s < \frac{1}{2}R$, x comes out negative, indicating that there will be no concentration within the room. This is the condition pictured in C (Fig. 66). When $s = \frac{1}{2}R$, x

becomes infinite; that is, a plane wave is reflected from the concave surface as shown in *A*. When *s* is greater than ½*R* and less than *R*, *x* comes out greater than *R*, while for any value of *s* greater than *R*, *x* will lie between ½*R* and *R*. When *s* equals *R*, *x* comes out equal to *R*. Sound originating at the center of a concave spherical surface will be focused directly back on itself. This is the condition obtaining in the well-known whispering gallery in the Hall of Statuary in the National Capitol at Washington.[1] There the center of curvature of the spherical segment, comprising a part of the ceiling, falls at nearly head level near the center of the room. A whisper uttered by the

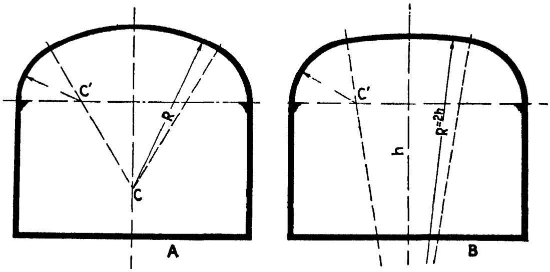


FIG. 69.—*A*. Vaulted ceiling which will produce concentration. *B*. Flattening middle portion of vault will prevent concentration.

guide at a point to one side of the axis of the room is heard with remarkable clearness at a symmetrically located point on the other side of the axis. If the speaker stands at the exact center of curvature of the domed portion of the ceiling, his voice is returned with striking clearness from the ceiling. Such an effect, while an interesting acoustical curiosity, is not a desirable feature of an auditorium.

From the foregoing, one may lay down as a safe working rule that *when concave surfaces are employed, the centers or axes of these curvatures should not fall either near the source of sound or near any portion of the audience.*

Applied to ceiling curvatures this rule dictates a radius of curvature either considerably greater or considerably less than the ceiling height. Thus in Fig. 69, the curved ceiling

[1] See "Collected Papers on Acoustics," p. 259.

shown in *A* would result in concentration of ceiling-reflected sound with inequalities in intensity due to interference enhanced. At *B* is shown a curved ceiling which would be free from such effects. Here the radius of curvature of the central portion is approximately twice the ceiling height. No real focus of the sound reflected from this portion can result. The coved portions at the side have a radius so short that the real focuses fall very close to the ceiling without producing any difficulty for either the performers or the auditors.

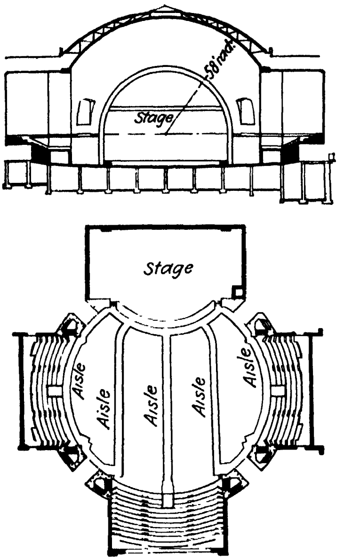


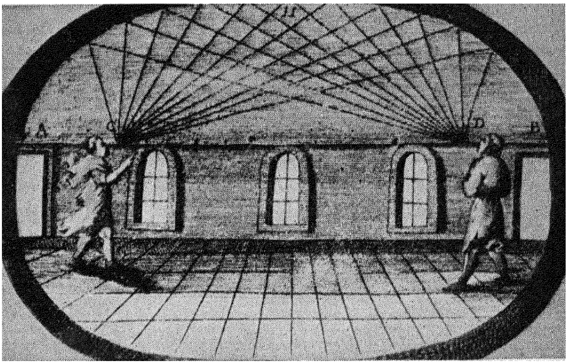
Fig. 70.—Auditorium in which circular plan and spherical dome produce focused echoes.

Figure 70 shows the plan and longitudinal section of a large auditorium, circular in plan, surmounted by a spherical dome the center of curvature of which falls about 15 ft. above the floor level. In this particular room, the general reverberation is well within the limits of good hearing conditions, yet the focused echoes are so disturbing as to render the room almost unusable. It is to be noted that a spherical surface produces much more marked focusing action due to the concentration in two planes, whereas cylindrical vaults give concentrations only in the plane of curvature. Illustrating the local character of defects due to concentrated reflection, it was observed that in the case just cited, speech from the stage was much more intelligible when heard outside the room in the lobby through the open doors than when the listener was within the hall. It is to be said, in passing, that while absorbent treatment of concave surfaces of the curvatures just described may alleviate undesired effects, yet, in general, nothing short of a major operation producing radical alterations in design will effect a complete cure.

In talking-motion-picture houses, the seating lines and rear walls are frequently segments of circles which center at a point near the screen. Owing to the directive action of the loud-speakers the rear-wall reflection not infrequently produces a pronounced and sometimes troublesome echo in the front of the room which absorbent treatment of rear-wall surface will only partially prevent. The main radius of curvature of the back walls in talking-motion-picture houses should be at least twice the distance from the curtain line to the rear of the room. This rule may well be observed in any room intended for public speaking, to save the speaker from an annoying "back slap" when speaking loudly. The semicircular plan of many lecture halls with the speaker's platform placed at the center is particularly unfortunate in this regard, unless, as is the case, for example, in the clinical amphitheater, the seating tiers rise rather sharply from the amphitheater floor. In this case, the absorbent surface of the audience replaces the hard reflecting surface of the rear wall, so that the speaker is spared the concentrated return of the sound of his own voice. In council or legislative chambers, where the semicircular seating plan is desirable for purposes of debate, the plan of the room itself may be semioctagonal rather than semicircular, with panels of absorbent material set in the rear walls to minimize reflection. Rear-wall reflection is almost always acoustically a liability rather than an asset.

Ellipsoidal Shapes: Mormon Tabernacle.

The great Mormon Tabernacle in Salt Lake City has a world-wide reputation for good acoustics, based very largely on the striking whispering-gallery effect, which is daily demonstrated to hundreds of visitors. This phenomenon is treated by Professor Sabine in his chapter on whispering galleries in "Collected Papers," together with an interesting series of sound photographs illustrating the focusing effect of an ellipsoidal surface. As a result of the geometry of the figure, if a sound originates at one focus of an ellipsoid, it will after reflection from the surface all be concentrated at the other focus. In the Tabernacle, these two focuses are respectively near the speaker's desk and at a point near the front of the rear balcony. A pin dropped in a stiff hat at the former is heard with considerable clarity at the latter—a distance of about 175 ft. (Parenthetically it may be said



An old illustration of the concentrated reflection from the inner surface of an ellipsoid. The two figures are at the foci. (*Taken from Neue Hall-und Thon-Kunst, by Athanasius Kircher, published in* 1684.)

that under very quiet conditions, this experiment can be duplicated in almost any large hall.)

Based on this well-known fact and the fact that the Salt Lake tabernacle is roughly elliptical in plan and semi-elliptical in both longitudinal and transverse section, superior acoustical virtues are sometimes ascribed to the ellipsoidal shape. That the phenomenon just described is due to the shape may well be admitted. It does not follow, however, that this shape will always produce desirable acoustical conditions or that even in this case the admittedly good acoustic properties are due solely to the shape. In 1925, through the courtesy of the tabernacle authorities, the writer made measurements in the empty room, which gave a reverberation time of 7.3 sec. for the tone 512 vibs./sec. Mr. Wayne B. Hales, in 1922, made a study of the room. From his unpublished paper the following data are taken:

Length	232 ft.
Width	132 ft.
Height	63.5 ft.
Computed volume	1,242,400 cu. ft.
Estimated seating capacity	8,000

From these data one may compute the reverberation time as 1.5 sec. with an audience of 8,000 and as 1.8 sec. with an audience of 6,000.

One notes here a low reverberation period as a contributing factor to the good acoustic properties of this room.

In 1930, Mr. Hales published a fuller account of his study.[1] Articulation tests showed a percentage articulation of about that which would be expected on the basis of Knudsen's curves for articulation as a function of reverberation. That is to say, there is no apparent improvement in articulation that can be ascribed to the particular shape of this room. Finally he noted echoes in a region where an echo from the central curved portion of the ceiling might be expected.

Taken altogether, the evidence points to the conclusion that apart from the whispering-gallery effect there are no outstanding acoustical features in the Mormon Tabernacle that are to be ascribed to its peculiarity of shape. A short period of reverberation[2] and ceiling curvatures which for the most part do not produce focused echoes are sufficient to account for the desirable properties which it possesses. The elliptical plan is usable, subject to the same limitation as to actual curvatures as are other curved forms.[3]

[1] *Jour. Acous. Soc. Amer.*, vol. 1, pp. 280–292, 1930.

[2] This is due to the small ratio of volume to seating capacity—about 155 cu. ft. per seat. A balcony extending around the entire room and the relatively low ceiling height for the horizontal dimension yields a low value of the volume-absorbing power ratio.

[3] The case of one of the best known and most beautiful of theaters in New York City may be cited as an example of undesirable acoustical results from elliptical contours.

Paraboloidal Shapes: Hill Memorial.

Figure 71 shows the important property of a paraboloidal mirror of reflecting in a parallel beam all rays that originate at the principal focus of the paraboloid. Hence if a source of sound be placed at the focus of an extended paraboloid of revolution, the reflected wave will be a plane wave traveling parallel to the axis. The action is quite analogous to the directive action of a searchlight. This property of the paraboloid has from time to time commended it to designers as an ideal auditorium shape, from the standpoint of acoustics. The best known example in America of a room of this type is the Hill Memorial Auditorium of the University of Michigan, at Ann Arbor. The main-floor plan and section are shown in Fig. 72. A detailed description of the acoustical design is given by Mr. Hugh Tallant, the acoustical consultant, in *The Brickbuilder* of August, 1913. The forward surfaces of the room are paraboloids of revolution, with a common focus near the speaker's position on the platform. The axes of these paraboloids are inclined slightly below the horizontal, at angles such as to give reflections to desired parts of the auditorium. The acoustic diagram is shown in Fig. 73. Care was taken in the design that the once-reflected sound should not arrive at any point in the room at an interval of more than 1/15 sec. after the direct sound. This was taken as the limit within which the reflected sound would serve to reinforce the direct sound rather than produce a perceptible echo. Tallant states that the final drawings were made with sufficient accuracy to permit of scaling the dimensions to within less than an inch. In 1921, the author made a detailed study of this room. The source of sound was set up at the speaker's position on the stage, and measurements of the sound amplitude were made at a large number of

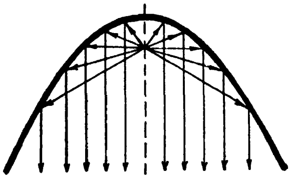


FIG. 71.—Rays originating at the focus of a paraboloid mirror are reflected in a parallel beam.

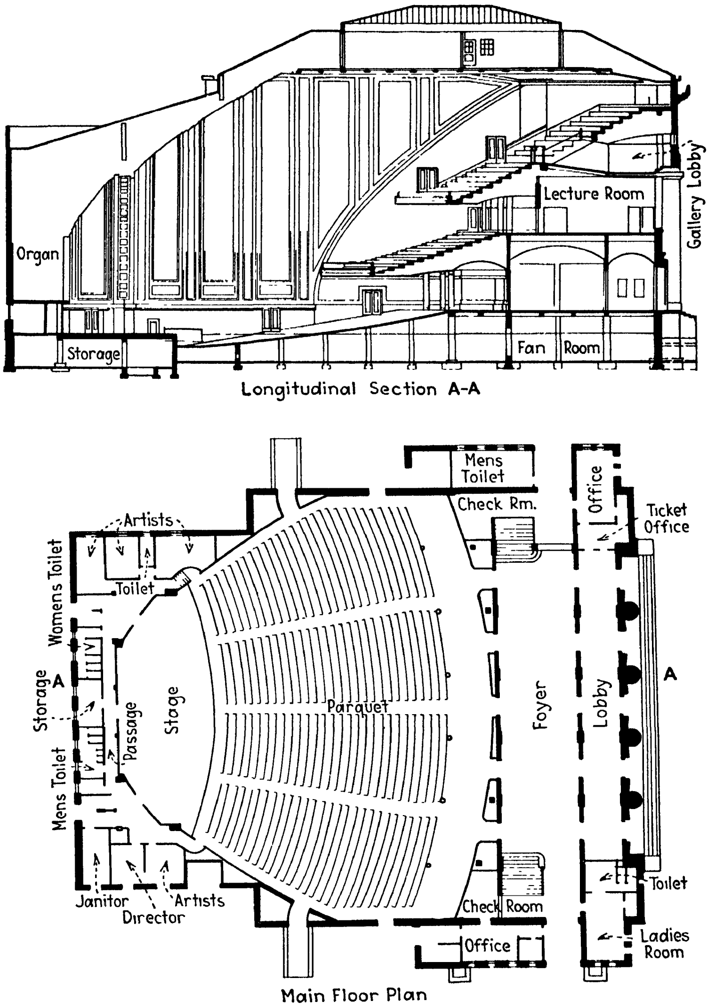


FIG. 72.—Paraboloidal plan and section of the Hill Memorial Auditorium, Ann Arbor, Michigan.

points throughout the room. The intensity proved to be very uniform, with a maximum value at the front of the first balcony and a value throughout the second balcony a trifle greater than on the main floor, a condition which the acoustical design would lead one to expect. This equality of distribution of intensity was markedly less when the source was moved away from the focal point. In the empty room, pronounced echoes were observed on the stage from a source on the stage, but these were not apparent in the main body of the room. The measured rever-

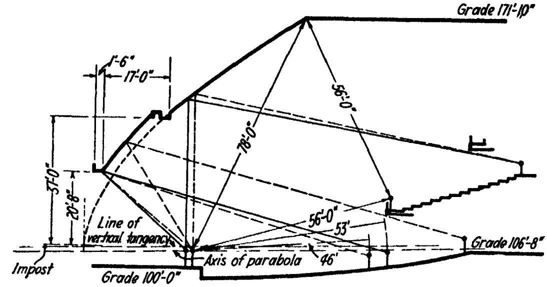


FIG. 73.—Ray reflections from parabolic surface, Hill Memorial Auditorium.

beration time in the empty room was 6.1 instead of 4.0 sec. computed by Mr. Tallant. The difference is doubtless due to the value assumed for the absorbing power of the empty seats. The reverberation for the full audience, figured from the measured empty-room absorbing power, agreed precisely with Tallant's estimate from the plans.

The purpose of the design was skillfully carried out, and the results as far as speaking is concerned fully meet the designer's purpose, namely, to provide an audience room seating 5,000 persons, in which a speaker of moderate voice placed at a definite point upon the stage can be distinctly heard throughout the room.

For orchestral and choral use, however, the stage is somewhat open to the criticisms made on the first example given in this chapter. Only when the source is at the principal focus is the sound reflected in a parallel beam. If the source be located closer to the rear wall than this, the reflected wave front will be convex, while if it is outside the focus, the reflected wave front will be concave. This

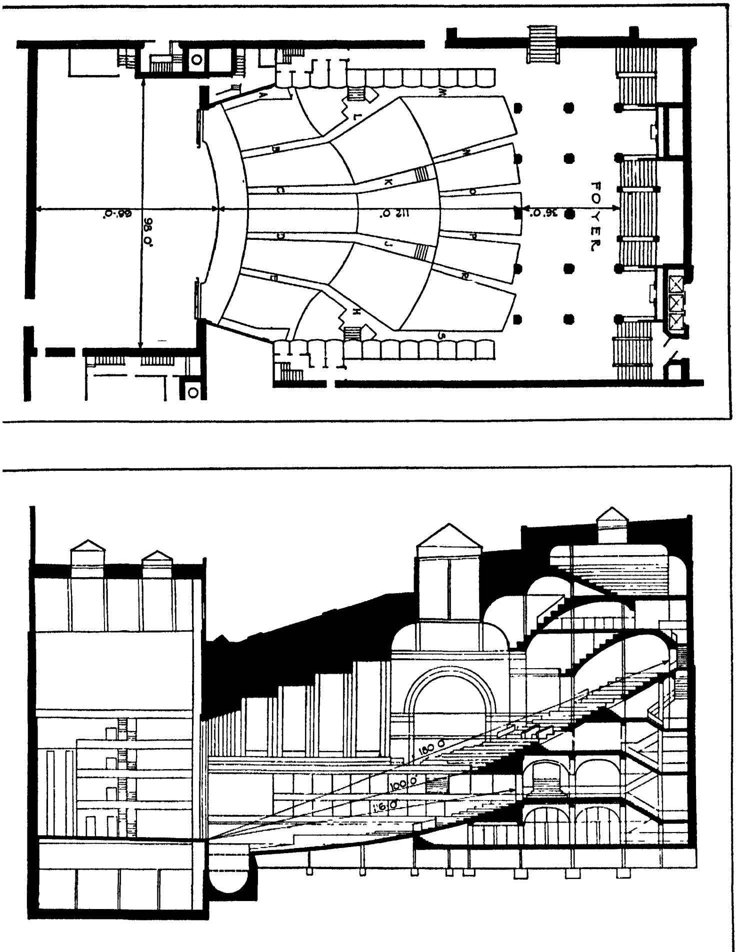


FIG. 74.—Plan and section of Auditorium Theater, Chicago.

fact renders it difficult to secure a uniform balance of orchestral instruments as heard by the conductor. This is particularly true for chorus accompanied by orchestra, with the latter and the soloists placed on an extension of the regular stage.

This serves to indicate the weakness of precise geometrical planning for desired reflections from curved forms. Such planning presupposes a definite fixed position of the source. Departure of the source from this point may materially alter the effects produced. Properly disposed plain surfaces may be made to give the same general effect in directing the reflected sound, without danger of undesired results when the position of the source is changed.

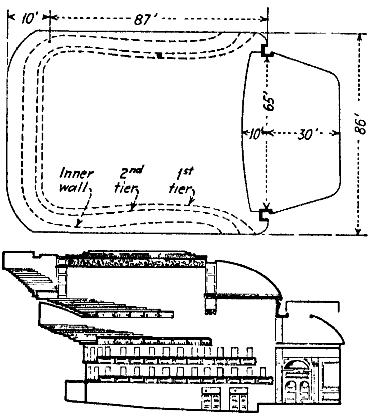


Fig. 75.—Plan and section of Carnegie Hall, New York. (*Courtesy of Johns-Manville Corporation.*)

A second objection lies in the fact that just as sound from a given point on the stage is distributed uniformly by reflection to all parts of the room, so sound or noise originating at any part of the room tends to be focused at this point on the stage. This fact has already been noted in the first example given.

Finally it has to be said that acoustically planned curved shapes are apt to betray their acoustical motivation. The skill of the designer will perhaps be less seriously taxed in evolving acoustical ideas than in the rendering of these ideas in acceptable architectural forms.

By way of comparison and illustrating this point Fig. 74 shows the plan and section of the Chicago Auditorium. By common consent this is recognized as one of the very excellent music rooms of the country. We note, in the longitudinal section, the same general effect of the ceiling rising from the proscenium; while in plan, the plain splays from the stage to the side walls serve the purpose of reflecting sound toward the rear of the room instead of diagonally across it. Figure 75 gives plan and section of Carnegie Hall in New York. Here is little, if any, trace of acoustical purpose, and yet Carnegie Hall is recognized as acoustically very good.

All of which serves to emphasize the point originally made, that the acoustical side of the designer's problem consists more in avoiding sources of difficulty than in producing positive virtues.

Reverberation and Design.

Since reverberation can be controlled by absorbent treatment more or less independently of design, it is all too frequently the practice to ignore the extent to which it may be controlled by proper design. It is true that in most cases any design may be developed without regard to the reverberation, leaving this to be taken care of by absorbent treatment. While this is a possible procedure, it seems not to be the most rational one. Since reverberation is determined by the ratio of volume to absorbing power, it obviously is possible to keep the reverberation in a proposed room down to desired limits quite as effectively by reducing volume as by increasing absorbing power. As has already appeared, the greater portion of the absorbing power of an audience room in which special absorbent treatment is not employed is represented by the audience. It is therefore apparent that, without any considerable area of special absorbents, the ratio of volume of the room to the number of persons in the audience will largely determine the reverberation time.

Table XV gives the ratio of volumes to seating capacities for the halls listed in Table XIV. In none of these are there any considerable areas of special absorbents, other than the carpets and the draperies that constitute the normal interior decorations of such rooms. We note that, with few exceptions, a range of 150 to 250 cu. ft. per person will cover this ratio for these rooms. A similar table prepared for rooms of the theater type with one or two balconies gave values ranging from 150 to 200 cu. ft. per person. One may then give the following as a working rule for the relation of volume to seating capacity:

Volume	Use	Volume per seat
100,000 to 500,000	Speech	150 to 175
	Music	150 to 200
500,000 to 1,000,000	Speech	175 to 200
	Music	200 to 250

TABLE XV.—RATIO OF VOLUME TO SEATING CAPACITY IN ACOUSTICALLY GOOD ROOMS

Auditorium	Time (full audience)	Volume per seat
Moscow Conservatory	1.30	163
Musikvereinssaal	1.62	161
Leipzig Gewandhaus	1.90	232
Academy of Music, Philadelphia	1.76	143
Orchestra Hall, Detroit	1.44	182
Academy of Music, Brooklyn	1.60	195
Great Theater, Moscow	1.55	210
Boston Opera House	1.51	212
Great Hall, Moscow Conservatory	2 00	279
Symphony Hall, Boston	1.93	250
Carnegie Hall	1.75	274
Hill Memorial	1.70	159
Eastman Theater, Rochester	2.08	236
Civic Opera House, Chicago	1.95	234
Chicago Auditorium	1.90	254

If the requirements of design allow ratios of volume to seating capacity as low as the above, then for the full-audience condition additional absorptive treatment will not in general be needed. On the other hand, there are many types of rooms such as high-school auditoriums, court rooms, churches, legislative halls, and council chambers in which the capacity audience is the exception rather than the rule. In all such rooms the total absorbing power should be adjusted to give tolerable reverberation times for the average audiences, rather than the most desirable times for capacity audiences. In such rooms, a compromise must be effected so as to provide tolerable hearing under all audience conditions.

Adjustment for Varying Audience.

For purposes of illustration, we shall take a typical case of a modern high-school auditorium. A room of this sort is ordinarily intended for the regular daily assembly of the school, with probably one-half to two-thirds of the seats occupied. In addition, occasional public gatherings, with addresses, or concerts or student theatrical performances will occupy the room, with audiences from two-thirds to full seating capacity. In general, the construction will be fireproof throughout, with concrete floors, hard plaster walls and ceiling, wood seats without upholstery, and a minimum of absorptive material used in the normal interior finish of the room.

The data for the example chosen, taken from the preliminary plans, are as follows:

Dimension, 127 by 60 by 48 ft. or 366,000 cu. ft.
Floors, cement throughout, linoleum in aisles
Walls, hard plaster on clay tile
Ceiling, low paneled relief, hard plaster on suspended metal lath
Stage opening, velour curtains 36 by 20 ft.
1,600 wood seats, coefficient 0.3

Absorbing Power

	Units
Empty room including seats, $0.3\sqrt[3]{V^2}$	1,480
Stage, 36 by 20 × 0.44	316
Absorbing power of empty room	1,796

Curve 1 (Fig. 76) shows the reverberation times plotted against the number of persons in the audience, assuming the room to be built as indicated in the preliminary design. We note that the ratio of volume to seating capacity is somewhat large—230 cu. ft. per person. This, coupled with the fact that there is a dearth of absorbent materials in the normal furnishings, gives a reverberation time that

is great even with the capacity audience present. In this particular example, it was possible to lower the ceiling height by about 8 ft., giving a volume of 305,000 cu. ft. Curve 2 shows the effect of this alteration in design. This lowers the reverberation time to 1.75 sec. with the maximum audience, a value somewhat greater than the upper limit for a room of this volume shown in Fig. 63. In view of the probably frequent small audience use of the room, the desirability of artificial absorbents is apparent. The question as to how much additional absorption ought to be specified should be answered with the particular uses in mind. At the daily school assembly, 900 to 1,000 pupils were expected to be present. A reverberation time of 2 sec. for a half audience would thus render comfortable hearing for assembly purposes. The total absorbing power necessary to give this time is

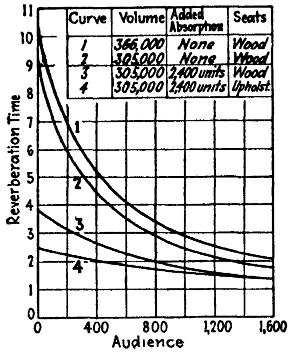


Fig. 76.—Effect of volume, character of seats, size of audience, and added absorption on reverberation time.

$$\frac{0.05 \times 305{,}000}{2} = 7{,}630 \text{ units}$$

Without absorbent treatment the total absorbing power with 800 persons present is as follows:

	Units
Empty room, $\sqrt[3]{V^2}$	1,330
Stage, 36 by 20 × 0.44.	316
800 persons × 4.3	3,440
Total..	5,086

Additional absorption necessary to give reverberation of 2.0 sec. with half audience 7,630 − 5,086 = 2,544 units. Curve 3 gives the reverberation times with this amount of added absorbing power. It is to be noted that with this amount of acoustical treatment, the reverberation time with the capacity audience is 1.4 sec., which is the lower limit for a room of this volume given by Fig. 63, while with a half audience it is not greater than 2.0 sec.—a compromise that should meet all reasonable demands.

An even more satisfactory adjustment can be effected, if upholstered seats be specified. In curve 4, we have the reverberation times under the same conditions as curve 3, except that the seats are upholstered in imitation leather and have an absorbing power of 1.6 per seat instead of the 0.3 unit for the wood seats. The effect of this substitution is shown in the following comparison:

Items of absorption	Absorbing power	
	Wood seats	Upholstered seats
Empty room	1,330	1,330
Stage	316	316
Added absorption	2,544	2,544
Upholstered seats (1,600 × 1.3)	....	2,080
Total absorbing power (without audience)	4,190	6,270

Seated in the wood seats, the audience adds 4.3 units per person and in the upholstered seat 3.0 units per person, so that we have:

Audience	Absorbing power	
	Wood seats	Upholstered seats
None	4,190	6,270
300	5,480	7,170
600	6,770	8,070
900	8,060	8,970
1,200	9,350	9,870
1,600	11,070	11,070

The effect of the upholstered seats in reducing the reverberation for small audience use is apparent. With the upholstered seats and the added absorbing power the reverberation is not excessive with any audience greater than 400 persons.

Choice of Absorbent Treatment.

The area of absorbent surface which is required to give the additional absorbing power desired will be the number of units divided by the absorption coefficient of the material used. Thus with a material whose absorption coefficient is 0.35, the area needed in the preceding example would be 2,466 − 0.35 = 7,000 sq. ft., while with a material twice as absorbent, the required area would be only half as great. As a practical matter, it is ordinarily more convenient to apply absorbent treatment to the ceiling. In the example chosen, the area available for acoustical treatment was approximately 7,000 sq. ft. in the soffits of the ceiling panels. In this case, the application of one of the more highly absorbent of the acoustical plasters with an absorption coefficient between 0.30 and 0.40 would have been a natural means of securing the desired reverberation time. Had the available area been less, then a more highly absorbent material applied over a smaller area would be indicated. In designing an interior in which acoustical treatment is required, knowledge, in advance, of the amount of treatment that will be needed and provision for working this naturally into the decorative scheme is an essential feature of the designer's problem. The choice of materials for sound absorption should be dictated quite as much by their adaptability to the particular problem in hand as by their sound-absorbing efficiencies.

Location of Absorbent Treatment.

As has been noted earlier, sound that has been reflected once or twice will serve the useful purpose of reinforcing the direct sound, provided the path difference between direct and reflected sound is not greater than about 70 ft. producing a time lag not greater than about $\frac{1}{16}$ sec. For this reason, in rooms so large that such reinforcement is desirable, absorbent treatment should not be applied on surfaces that would otherwise give useful reflections. In general, this applies to the forward portions of side walls and ceilings, as well as to the stage itself. Frequently one finds stages hung with heavy draperies of monk's cloth or other fabric. Such an arrangement is particularly bad because of the loss of all reflections from the stage boundaries thus reducing the volume of sound delivered to the auditorium. Further, the recessed portion of the stage acts somewhat as a separate room, and if this space is "dead," the speaker or performer has the sensation of speaking or playing in a padded cell, whereas the reverberation in the auditorium proper may be considerable.

Professor F. R. Watson[1] gives the results of some interesting experiments on the placing of absorbent materials in auditoriums. As a result of these experiments he advocated the practice of deadening the rear portion of audience rooms by the use of highly absorbent materials, leaving the forward portions highly reflecting. Carried to the extreme, in very large rooms this procedure is apt to lead to two rates of decay of the residual sound, the more rapid occurring in the highly damped rear portions. In one or two instances within the author's knowledge, this has led to unsatisfactory hearing in the front of the room, while seats in the rear prove quite satisfactory. As a general rule, the wider distribution of a moderately absorbent material leads to better results than the localized application of a smaller area of a highly absorbent material.

In general, the application of sound-absorbent treatment to ceilings under balconies is not good practice. Properly designed, such ceilings may give useful reflection to the extreme rear seats. Moreover, if the under-balcony space is deep, absorbents placed in this space are relatively ineffective in lowering the general reverberation in the room, since the opening under the balcony acts as a nearly perfectly absorbing surface anyway. Rear-wall treatment under balconies may sometimes be needed to minimize reflection back to the stage.

[1] *Jour. Amer. Inst. Arch.*, July, 1928.

Wood as an Acoustical Material.

There is a long-standing tradition that rooms with a large amount of wood paneling in the interior finish have superior acoustical merits. Very recently Bagenal and Wood published in England a comprehensive treatise on architectural acoustics.[1] These authors strongly advocate the use of wood in auditoriums, particularly those intended for music, on the ground that the resonant quality of wood "improves the tone quality," "brightens the tone." In support of this position, they cite the fact that wood is employed in relatively large areas in many of the better known concert halls of Europe. The most noted example is that of the Leipzig Gewandhaus, in which there is about 5,300 sq. ft. of wood paneling. The acoustic properties of this room have assumed the character of a tradition. Faith in the virtues of the wood paneling is such that its surface is kept carefully cleaned and polished.

The origin of this belief in the virtue of wood is easily accounted for. It is true that wood has been extensively used as an interior finish in the older concert halls. It is also true that the reverberation times in these rooms are not excessive. For the Leipzig Gewandhaus, Bagenal and Wood give 2.0 sec. Knudsen estimates it as low as 1.5 sec. with a full audience of 1,560 persons. It is not impossible that the acoustical excellence which has been ascribed to the use of wood may be due to the usually concomitant fact of a proper reverberation time. Figure 55 shows that the absorption coefficient of wood paneling is high, roughly 0.10 as compared with 0.03 for plaster on tile. Small rooms in which a large proportion of the surface is wood naturally have a much lower reverberation time than similar rooms done in masonry throughout, particularly when empty. Noting this fact, a musician with the piano sounding board and the violin in mind would naturally arrive at the conclusion that the wood finish as such is responsible for the difference.

[1] "Planning for Good Acoustics," Methuen & Co., London, 1931.

That the presence of wood is not an important factor in reducing reverberation is evidenced by the fact that the 5,300 sq. ft. of paneling accounts for only about 5.5 per cent of the total absorbing power of the Leipzig Gewandhaus when the audience is present. Substituting plastered surfaces for the paneled area would not make a perceptible difference in the reverberation time, nor could it change the quality of tone in any perceptible degree, in a room of this size. It is possible that in relatively small rooms, in which the major portion of the bounding surfaces are of a resonant material, a real enhancement of tone might result. In larger rooms and with only limited areas, the author inclines strongly to the belief that the effect is largely psychological.

For the stage floor, and perhaps in a somewhat lesser degree for stage walls in a concert hall, a light wood construction with an air space below would serve to amplify the fundamental tones of cellos and double basses. These instruments are in direct contact with the floor, which would act in a manner quite similar to that of a piano sounding board. This amplification of the deepest tones of an orchestra produces a real and desired effect. One may note the effect by observing the increased volume of tone when a vibrating tuning fork is set on a wood table top.[1]

Orchestra Pit in Opera Houses.

Figure 77 shows the section of the orchestra pit of the Wagner Theater in Bayreuth. This is presented to call attention to the desirability of assigning a less prominent

[1] Of interest on this point is some recent work by Eyring (*Jour. Soc. Mot. Pict. Eng.*, vol. 15, No. 4, p. 532). At points near a wall made of 5⁄16-in. ply-wood panels in a small room, two rates of decay of reverberant sound were observed. The earlier rate, corresponding to the general decay in the room, was followed by a slower rate, apparently of energy, reradiated from the panels. This effect was observed only at two frequencies. Investigation proved that the panels were resonant for sound of these frequencies and that the effect disappeared when the panels were properly nailed to supports at the back.

place to the orchestra in operatic theaters. The wide orchestra space in front of the stage as it exists in many opera houses places the singer at a serious disadvantage both in the matter of distance from the audience and in the fact that he has literally to sing over the orchestra. Wagner's solution of the problem was to place most of the orchestra under the stage, the sound emerging through a restricted opening. Properly designed, with resonant floor and highly reflecting walls and ceiling, ample sound can be projected into the room from an orchestra pit of this

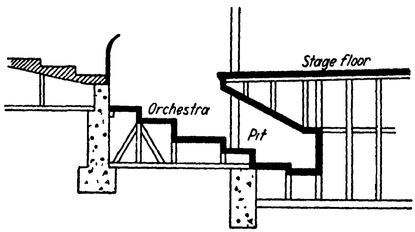


FIG. 77.—Section of orchestra pit of Wagner Theater, Bayreuth.

type. With the usual orchestra pit, the preponderance of orchestra over singers for auditors in the front seats is decidedly objectionable.

Acoustical Design in Churches.

No type of auditorium calls for more careful treatment from the acoustical point of view than does that of the modern evangelical church in America. Puritanism gave to ecclesiastical architecture the New England meeting house, rectangular in plan, often with shallow galleries on the sides and at the rear. With plain walls and ceilings and usually with the height no greater than necessary to give sufficient head room above the galleries, the old New England meeting house presented no acoustical problems. The modern version is frequently quite different. Simplicity of design and treatment is often coupled with horizontal dimensions much greater than are called for by the usual audience requirements and with heights which are correspondingly great. Elimination of the galleries and the substitution of harder, more highly reflecting materials in floors, walls, and ceilings often render the modern church fashioned on New England colonial lines highly unsatisfactory because of excessive reverberation. Frequently the plain ceiling is replaced with a cylindrical vault with an axis of curvature that falls very close to the head level of the audience, with resultant focusing and unequal distribution of intensity due to interference.[1] These difficulties have only to be recognized in order to be avoided. Excessive reverberation may be obviated by use of absorbent materials. A flattened ceiling with side coves will not produce the undesired effects of the cylindrical vault. Both of these can be easily taken care of in the original design. They are extremely difficult to incorporate in a room that has once been completed in a type of architecture whose excellence consists in the simplicity of its lines and the perfect fitness of its details.

The Romanesque revival of the seventies and eighties brought a type of church auditorium which is acoustically excellent, but which has little to commend it as ecclesiastical architecture. Nearly square in plan, with the pulpit and choir placed in one corner (Tallmadge has called this period the "cat-a-corner age") and with the pews circling about this as a center, the design is excellent for producing useful reflections of sound to the audience. Add to this the fact that encircling balconies are frequently employed, giving a low value to the volume per person and hence of the reverberation time, and we have a type of room which is excellent for the clear understanding of speech. This type of auditorium is admirably adapted to a religious service in which liturgy is almost wholly lacking and of which the sermon is the most important feature.

The last twenty-five years, however, have seen a marked tendency toward ritualistic forms of worship throughout the Protestant churches in America. Concurrently with this there has been a growing trend, inspired largely by Bertram Grosvenor Goodhue and Ralph Adams Cram, toward the revival of Gothic architecture even for nonliturgical churches. Now, the Gothic interior, with its great height and volume, its surfaces of stone, and its relatively small number of seats, is of necessity highly reverberant. This is not an undesirable property for a form of service in which the clear understanding of speech is of secondary importance. The reverberation of a great cathedral adds to that sense of awe and mystery which is so essential an element in liturgical worship.

The adaptation of the Gothic interior to a religious service that combines the traditional forms of the Roman church with the evangelical emphasis upon the words of the preacher presents a real problem, which has not as yet had an adequate solution. Apart from reverberation, the cruciform plan is acoustically bad for speech. The usual locations of the pulpit and lectern at the sides of the chancel afford no reinforcement of the speaker's voice by reflection from surfaces back of him. The break caused by the transepts allows no useful reflections from the side walls. Both chancel and transepts produce delayed reflections, that tend to lower the intelligibility of speech. The great length of the nave may occasion a pronounced echo from the rear wall, which combined with the delayed reflections from the chancel and transepts makes hearing particularly bad in the space just back of the crossing. The sound photograph of Fig. 78 show the cause of a part of the difficulty in hearing in this region. Finally the attempt by absorption to adjust the reverberation to meet the demands of both the preacher and the choirmaster usually results in a compromise that is not wholly satisfactory to either.

The chapel of the University of Chicago, designed by Bertram Goodhue, is an outstanding example of a modern Gothic church intended primarily for speaking. The plan is shown in Fig. 79. The volume is approximately 900,000 cu. ft., and 2,200 seats are provided. The ratio of volume to seating capacity is large. In order to reduce reverberation, all wall areas of the nave and transept are plastered with a sound-absorbing plaster with a coefficient of 0.20.

The groined ceiling is done in colored acoustical tile with a coefficient of 0.25. The computed reverberation time for the empty room is 3.6 sec., which is reduced to 2.4 sec. when all the seats are occupied. The reverberation measured in the empty room is 3.7 sec. The rear-wall echo is largely eliminated by the presence of a balcony and, above this, the choir loft in the rear. Speakers who use a speaking voice of only moderate power and a sustained

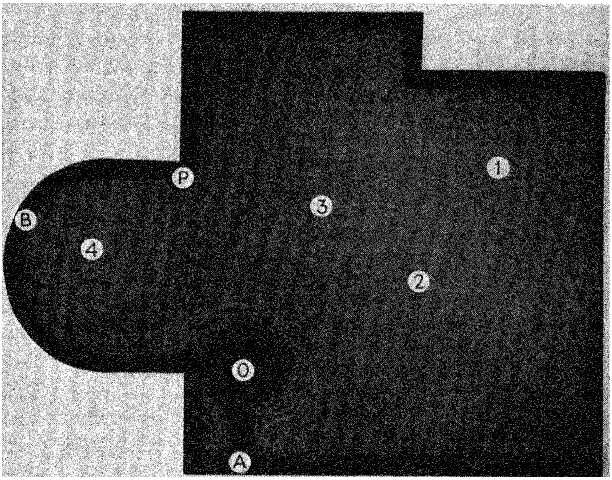


FIG. 78.—Sound pulse in modified cruciform plan. Hearing conditions much better in portion represented by lower half of photograph.
O, Pulpit; 1, direct sound; 2, reflected from side wall *A*; 3, diffracted from *P*; 4, reflected and focused from curved wall of sanctuary.

manner of speaking can be clearly heard throughout the room. Rapid, strongly emphasized speech becomes difficult to understand. Dr. Gilkey, the university minister, has apparently mastered the technique of speaking under the prevailing conditions, as he is generally understood throughout the chapel. In this connection, it may be said that the disquisitional style of speaking, with a fairly uniform level of voice intensity and measured enunciation, is much more easily understood in reverberant rooms than is an oratorical style of preaching. In fact it is highly probable that the practice of intoning the ritualistic service had its origin in part at least in the acoustic demands of highly reverberant medieval churches.

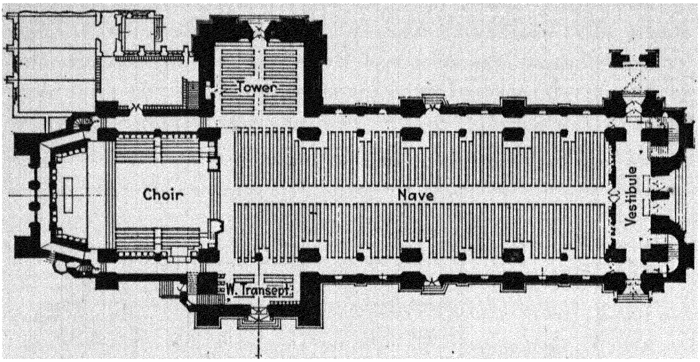


FIG. 79.—Plan of the University of Chicago Chapel.

Interior of the University of Chicago Chapel. (*Bertram Goodhue Associates, Architects.*)

The choirmaster, Mr. Mack Evans, finds the chapel very satisfactory both for his student choir and for the organ. The choir music is mostly in the medieval forms, sung *a capella,* with sustained harmonic rather than rapid melodic passages.

All things considered, it may be fairly said that this room represents a reasonable compromise between the extreme conditions of great reverberation existing in the older churches of cathedral type and proportions and that of low reverberation required for the best understanding of speech. It has to be said that, like all compromises, it fails to satisfy the extremists on both sides. The conditions

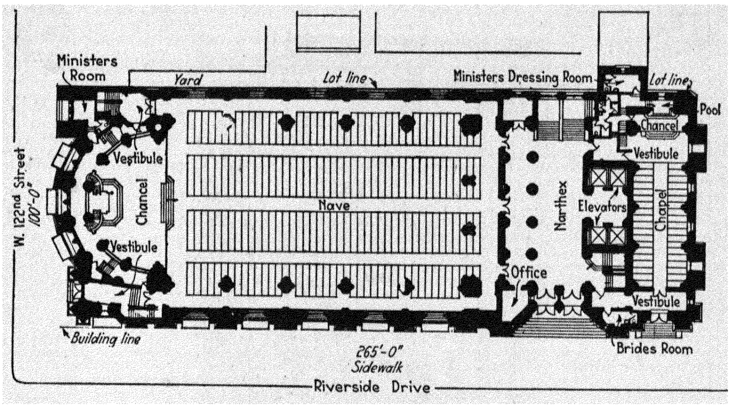


FIG. 80.—Plan of Riverside Church, New York.

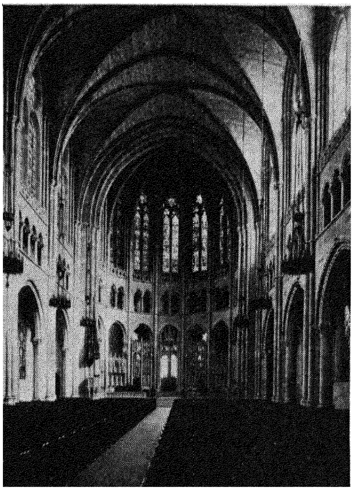
inherent in the cruciform plan, of the long, narrow seating space and consequently great distances between the speaker and the auditor, and the lack of useful reflections are responsible for a considerable part of what difficulty is experienced in the hearing of speech. These difficulties would be greatly magnified were the room as reverberant as it would have been without the extensive use of sound-absorbent materials.

Riverside Church, New York.

This building is by far the most notable example of the adaptation of the Gothic church to the uses of Protestant worship. We note from the plan[1] (Fig. 80) the

[1] Acknowledgment is made of the courtesy of the architects Henry C. Pelton and Allen and Collens, Associates, in supplying the plan and photograph.

rectangular shape, the absence of the transepts, and a comparatively shallow chancel. The height of the nave is very great—104 ft. to the center of the arch, giving a volume for the main cell of the church of more than 1,000,000 cu. ft. With the seating capacity of 2,500, we have over 400 cu. ft. per person. To supply the additional absorption that was obviously needed, the groined ceiling vault of the nave and chancel, the ceiling above the aisles, and all wall surfaces above a 52-ft. level were finished in acoustical tile.



Interior of Riverside Church, New York. (*Henry C. Pelton, Allen & Collens Associates, Architects.*)

The reverberation time measured in the empty room is 3.5 sec., which with the full audience present reduces to 2.5 sec. While the room is still noticeably reverberant even with a capacity audience, yet hearing conditions are good even in the most remote seats. A very successful public-address system has been installed. The author is indebted to Mr. Clifford M. Swan, who was consulted on the various acoustical problems in connection with this church, for the following details of the electrical amplifying system:

Speech from the pulpit is picked up by a microphone in the desk, amplified, and projected by a loud-speaker installed within the tracery of a Gothic spire over the sounding board, directly above the preacher's head. The voices of the choir are picked up by microphones concealed in the tracery of the opposing choir rail and projected from a loud-speaker placed at the back of the triforium gallery in the center of the apse. The voice of the reader at the lectern is also projected from this position. The amplification is operated at as low a level of intensity as is consistent with distinct hearing.

It is worth noting that the success of the amplifying system is largely conditioned upon the reduced reverberation time. Were the reverberation as great as it would have been had ordinary masonry surfaces been used throughout, amplification would have served little in the way of increasing the intelligibility of speech. Loud speaking in a too reverberant room does not diminish the overlapping of the successive elements of speech. The judicious use of amplifying power and the location of the loud-speakers near the original sources also contribute to the success of this scheme. It would appear that with the rapid improvement that has been made in the means for electrical amplification and with highly absorbent masonry materials available, the acoustical difficulties inherent in the Gothic church can be overcome in very large measure.

CHAPTER X

MEASUREMENT AND CONTROL OF NOISE IN BUILDINGS

It has been noted in Chap. II that a musical sound as distinguished from a noise is characterized by having definite and sustained pitch and quality. This does not tell the whole story, however, for a sound of definite pitch and quality is called a noise whenever it happens to be annoying. Thus the hum of an electrical motor or generator has a definite pitch and quality, but neighbors adjacent to a power plant may complain of it as noise. The desirability or the reverse of any given sound seems to be a determining factor in classifying it either as a musical tone or as a noise. Hence the standardization committee of the Acoustical Society of America proposes to define noise as "any unwanted sound." Obviously then, the measurement of a *noise* should logically include some means of evaluating its undesirability. Unfortunately this psychological aspect of noise is not susceptible of quantitative statement. Moreover, neglecting the annoyance factor, the mere sensations of loudness produced by two sound stimuli depend upon other factors than the physical intensity of those stimuli. Therefore to deal with noise in a quantitative way, it is necessary to take account of the characteristics of the human ear as a sound-receiving apparatus, if we want our noise measurements to correspond with the testimony of our hearing sense.

To go into this question of the characteristics of the ear in any detail would quite exceed the scope of this book. For our quantitative knowledge of the subject we are largely indebted to the extensive research at the Bell Telephone Laboratories, and the reader is referred to the comprehensive account of this work given in Fletcher's "Speech and Hearing." For the present purpose certain general facts with regard to hearing will suffice.

Frequency and Intensity Range of Hearing.

There is a wide variation among individuals in the range of frequencies which produce the sensation of tone. In general, however, this range may be said to extend from 20 to 20,000 vibs./sec. Below this range, the alternations of pressure are recognized as separate pulses; while above the upper limit, no sensation of hearing is produced.

The intensity range of response of the ear is enormous. Thus for example, at the frequency 1,024 vibs./sec. the physical intensity of a sound so loud as to be painful is something like 2.5×10^{13} times the least intensity which can be heard at this frequency. One cannot refrain from marveling at the wonder of an instrument that will register the faintest sound and yet is not wrecked by an intensity 25 trillion times as great. Illustrating the extreme sensitivity of the ear to small vibrations, Kranz[1] has calculated that the amplitudes of vibration at the threshold of audibility at higher frequencies are of the order of one-thirtieth of the diameter of a nitrogen molecule and one ten-thousandth of the mean free path of the molecules.

Another calculation shows that the sound pressure at minimum audibility is not greater than the weight of a hair whose length is one-third of its diameter.

The intensity range varies with the frequency. It is greatest for the middle of the frequency range. The data for Table XVI are taken from Fletcher and show the pressure range measured in bars between the faintest and most intense sounds of the various pitches and also the intensity ratio between painfully loud and minimum-audible sound at each pitch.

Decibel Scale.

The enormous range of intensities covered by ordinary auditory experience suggests the desirability of a scale whose readings correspond to ratios rather than to differences of intensity. This suggests a logarithmic scale, since

[1] *Phys. Rev.*, vol. 21, No. 5, May, 1923.

TABLE XVI

Frequency	Pressure range in bars	Intensity ratio, maximum divided by minimum
64	0.12 to 200	1.7×10^{6}
128	0.021 to 630	9.0×10^{8}
256	0.0039 to 2,000	2.6×10^{11}
512	0.0010 to 3,200	1.0×10^{13}
1,024	0.00052 to 2,500	2.5×10^{13}
2,048	0.00041 to 1,000	6.0×10^{12}
4,096	0.00042 to 320	6.4×10^{11}

the logarithm of the ratio of two numbers is the difference between the logarithms of these numbers. The decibel scale is such a scale and is applied to acoustic powers, intensities, and energies and to the mechanical or electrical sources of such power. The unit on such a logarithmic scale is called the bel in honor of Alexander Graham Bell. The difference of level expressed in bels between two intensities is the logarithm of the ratio of these intensities. The intensity level expressed in bels of a given intensity is the number of bels above or below the level of unit intensity or, since the logarithm of unity is zero, simply the logarithm of the intensity. Thus if the microwatt per square centimeter is the unit of intensity, then 1,000 microwatts per square centimeter represents an intensity level of 3 bels or 30 db. ($\log_{10} 1{,}000 = 3$). An intensity of 0.001 microwatt gives an intensity level of -3 bels or -30 db. Two intensities have a difference of level of 1 db. if the difference in their logarithm, that is, the logarithm of their ratio, is 0.1. Now the number whose logarithm is 0.1 is 1.26, so that an increase of 26 per cent in the intensity corresponds to a rise of 1 db. in the intensity level. The table on page 207 shows the relation between intensities and decibel levels above unit intensity:

Obviously, we can express the intensity ratios given in Table XVI as differences of intensity levels expressed in decibels. Thus, for example, at the tone 512 vibs./sec., the intensity ratio between the extremes is 1×10^{13}. The

I	log I	Intensity level, decibels
1.00	0 0	0 0
1.26	0 1	1 0
1.58	0 2	2 0
2.00	0 3	3 0
2.55	0 4	4 0
3 16	0 5	5 0
4 00	0 6	6 0
5 03	0 7	7 0
6 30	0 8	8 0
8 00	0 9	9 0
10 00	1 0	10 0
100 00	2 0	20 0
1,000 00	3.0	30 0
1,000,000 00	6 0	60 0

logarithm of 10^{13} is 13, and the difference in intensity levels between a painfully loud and a barely audible sound of this frequency is 13 bels or 130 db.

Sensation Level.

The term "sensation level" is used to denote the intensity level of any sound above the threshold of audibility of that sound. Thus in the example just given, the sensation level of the painfully loud sound is 130 db. We note that levels expressed in decibels give us not the absolute values of the magnitudes so expressed but simply the logarithms of their ratios to a standard magnitude. The sensation level for a given sound is the intensity level minus the intensity level of the threshold.

Threshold of Intensity Difference.

The decibel scale has a further advantage in addition to its convenience in dealing with the tremendous range of intensities in heard sounds. The Weber-Fechner law in psychology states that the increase in the intensity of a stimulus necessary to produce a barely perceptible increase in the resulting sensation is a constant fraction of the original intensity. Applied to sound, the law implies that any sound intensity must be increased by a constant fraction of itself before the ear perceives an increase of loudness. If ΔI be the minimum increment of intensity that will produce a perceptible difference in loudness, then according to the Weber-Fechner law

$$\frac{\Delta I}{I} = k \text{ (constant)}$$

Hence, if I' and I be any two intensities, one of which is just perceptibly louder than the other, then

$$\log I' - \log I = \text{constant}$$

On the basis of the Weber-Fechner law as thus stated, we could build a scale of loudness, each degree of which is the minimum perceptible difference of loudness. The intensities corresponding to successive degrees on this scale would bear a constant ratio to each other. They would thus form a geometric series, and their logarithms an arithmetical series.

The earlier work by psychologists seemed to establish the general validity of the Weber-Fechner law as applied to hearing. However, more recent work by Knudsen,[1] and subsequently by Riesz at the Bell Laboratories, has shown that the Weber-Fechner law as applied to differences of intensities is only a rough approximation to the facts, that the value of $\Delta I/I$ is not constant over the whole range of intensities. For sensation levels above 50 db. it is nearly constant, but for low intensities at a given pitch it is much greater than at higher levels. Moreover, it has been found that the ability to detect differences of intensities between two sounds of the same pitch depends upon time between the presentations of the two sounds. For these reasons, a scale based on the minimum perceptible difference of intensity would not be a uniform scale. However, over the entire range of intensities the total number of threshold-of-difference steps is approximately the same as the number of decibels in this range, so that on the average 1 db. is the minimum difference of sensation level which the ear can

[1] *Phys. Rev.*, vol. 21, No. 1, January, 1923.

detect. For this reason, the decibel scale conforms in a measure to auditory experience. However, there has not yet been established any quantitative relation between sensation levels in decibels and magnitudes of sensation. That is to say, a sound at a sensation level of 50 db. is not judged twice as loud as one at 25 db.[1]

Reverberation Equation in Decibels.

It will be recalled that the reverberation time of a room has been defined as the time required for the average energy density to decrease to one-millionth of its initial value and also that in the decay process the logarithm of the initial intensity minus the logarithm of the intensity at the time T is a linear function of the time. The logarithm of 1,000,000 is 6, so that the reverberation time is simply the time required for the residual intensity level to decrease by 60 db. Denote the rate of decay in decibels per second by δ; then

$$\delta = \frac{60}{T_0} = \frac{60a}{.05V} = 1{,}200\frac{a}{V}$$

The rate of decay in decibels per second is related to A, the absolute rate of decay, by the relation

$$\delta = 4.35A$$

Recent writers on the subject sometimes express the reverberation characteristic in terms of the rate of decrease in the sensation level. The above relations will be useful in connecting this with the older mode of statement.

Intensity and Sensation Levels Expressed in Sound Pressures.

In Chap. II (page 22), we noted that the flux intensity of sound is given by the relation $J = p^2/r$, where p is the

[1] Very recently work has been done on the quantitative evaluation of the loudness sensation. The findings of different investigators are far from congruent, however. Thus Ham and Parkinson report from experiments with a large number of listeners that a sound is judged half as loud when the intensity level is lowered about 9 db. Laird, experimenting with a smaller group, reports that an intensity level of 80 db. appears to be reduced one-half in loudness when reduced by 23 db. It is still questionable as to just what we mean, if anything, when we say that one sound is half as loud as another.

root mean square of the pressure, and r is the acoustic resistance of the medium. Independently of pitch, the intensity of sound is thus proportional to the square of the pressure, so that intensity measurements are best made by measuring sound pressures. The intensity ratio of two sounds will be the square of this pressure ratio, and the logarithm of the intensity ratio will be twice the logarithm of the pressure ratio. The difference in intensity level in decibels between two sounds whose pressures are p_1 and p_2 is therefore $20 \log_{10} \frac{p_1}{p_2}$.

Loudness of Pure Tones.

"Loudness" refers to the magnitude of the psychological sensation produced by a sound stimulus. The loudness of a sound of a given pitch increases with the intensity of the stimulus. We might express loudness of sound of a given pitch in terms of the number of threshold steps above minimum audibility. As has already been pointed out, such a loudness scale would correspond only roughly to the decibel scale. Moreover, it has been found that two sounds of different pitches, at the same sensation levels, that is, the same number of decibels above their respective thresholds, are not, in general, judged equally loud, so that such a scale will not serve as a means of rating the loudness either of musical tones of different pitches or of noise in which definite pitch characteristics are lacking. It is apparent that in order to speak of *loudness* in quantitative terms, it is necessary to adopt some arbitrary conventional scale, such that two sounds of different characters but judged equally loud would be expressed by the same number and that the relative ratings of the loudness of a series of sounds as judged by the ear would correspond at least qualitatively to the scale readings expressing their loudness.

In "Speech and Hearing," Fletcher has proposed to use as a measure of the loudness of a given musical tone of definite pitch the sensation level of a 1,000-cycle tone which normal ears judge to be of the same loudness as the given tone. This standard of measurement seems to be by way of being generally adopted by various engineering and scientific societies which are interested in the problem. The use of such a scale calls for the experimental loudness matching of a large number of frequencies with the standard frequency at different levels. To allow for individual variation such matching has to be done by a large number of observers. Kingsbury of the Bell Telephone Labora-

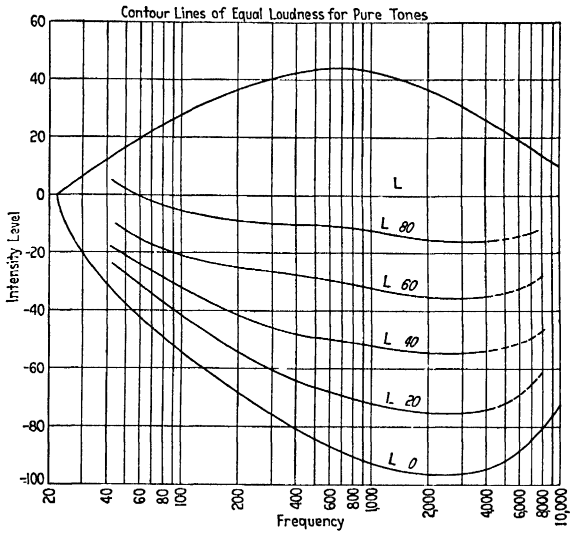


FIG. 81.—Each curved line shows the intensity levels at different frequencies which sound as loud as a 1,000-cycle tone of the indicated intensity level above threshold. (*After Kingsbury.*)

tories has made an investigation of this sort using 22 different observers. The results of this work are given in Fig. 81. Here the zero intensity level is one microwatt per square centimeter. The lines of the figure are called contour lines of equal loudness. The ordinates of any contour line give the intensity levels at the different frequencies which sound equally loud. The loudness level assigned to each contour is the sensation level of the 1,000-cycle tone at the intensity shown on the graph. Illustrating the use of the diagram, take, for example, the contour marked 60. We have for equal loudness the following:

Frequency	Intensity level measured from 1 microwatt/sq. cm.	Intensity, microwatts/sq. cm.
125	−24	0 004
250	−28	0 0016
500	−30	0.001
1,000	−32	0 00063
2,000	−35	0 00032
4,000	−34	0 00040

According to Kingsbury's work these frequencies at the respective intensity levels shown sound as loud as a 1,000-cycle tone 60 db. above threshold.

To the layman this doubtless sounds a bit complicated, but it is typical of the sort of thing to which the physicist is driven whenever he attempts to assign numerical measures to psychological impressions.

Loudness of Noises.

Numerical expression of the loudness of noises becomes even more complicated by virtue of the fact that we have to deal with a medley of sounds of varying and indiscriminate pitches. As we have seen, the sensation of loudness depends both upon the pitch and upon the intensity of sound. Moreover, the annoyance factor of noise may depend upon still other elements which will escape evaluation. The sound pressures produced by noises may be measured, but their relative loudness as judged by the ear will not necessarily follow the same order as these measured pressures. At the present time, the question of evaluating noise levels is the subject of considerable discussion among physicists and engineers. Standardization committees have been appointed by various technical societies, notably the American Institute of Electrical Engineers, the American Society of Mechanical Engineers, and the Acoustical Society of America.

The whole subject of noise measurement is still young, and much research still is needed before we are sure that our quantitative values correspond to the testimony of our ears with regard to the loudness of noises. For example, one might ask whether the ear forms any quantitative judgments at all as to the relative loudness of noise. Further, does the ear rate the noise of 10 vacuum cleaners as ten times that of one? The probable answer to the latter question is in the negative, but merely asking the question serves to show the inherent difficulties in expressing the loudness of noise in a way that will be meaningful in terms of ordinary experience.

Comparison of Noises. Masking Effect.

While the exact measurement of the loudness level of noise is still a matter of considerable uncertainty, yet it is possible to make quantitative comparisons that will

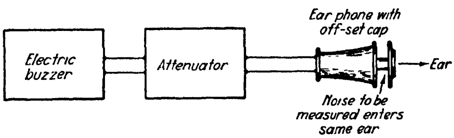


FIG. 82.—Buzzer audiometer.

serve a great many useful purposes. Various types of audiometers, devised primarily for the testing of the acuity of hearing, have been employed. That most commonly used is the Western Electric 3-A audiometer (Fig. 82) of the so-called buzzer type. In this, a magnetic interrupter interrupts an electric current which passes through a resistance network called an attenuator, consisting of parallel- and series-connected resistances arranged to reduce the current by definite fractions of itself.

The attenuator dial is graduated to read the relative sound-output levels of the ear phone in decibels. The zero of the instrument for a given observer is determined by setting the attenuator so as to produce a barely audible sound in a perfectly quiet place. The reading above this zero is the sensation level of the sound produced by this setting for this particular observer.

In comparing noises by this type of instrument, one makes use of the so-called masking effect of one sound upon another. Suppose that a sound is of such intensity as to be barely audible in a quiet place. Then in a noisy place it will not be heard due to the masking effect of the noise. The rise in intensity level which must be effected in the sound before it can be heard in the presence of a given noise is the masking effect of that noise. In using the 3-A audiometer, the instrument is taken to a perfectly quiet place (which, by the way, is seldom easy to find), and the attenuator dial is adjusted so that the buzz is just heard in the receiver by the observer. The difference between this and the setting necessary to produce an audible sound in the receiver in a noisy place is the masking effect of the noise on the sound from the audiometer. Different noises are compared by measuring their masking effects on the noise from the audiometer. Measurements may also be made by matching the unknown noise with that made by the audiometer, but experience shows that judgments of equal loudness are apt to be less precise than are those on masking. No very precise relation can be given between the matching and the masking levels. Galt[1] states that for levels between 20 and 70 db. above the threshold, the matching level is 12 db. higher than the masking level. For very loud noises such as that of the cheering of a large crowd the difference is apparently greater, probably 20 db. for levels of around 100 db.

Tuning-fork Comparison of Noises.

A. H. Davis[2] has proposed and used an extremely simple method of comparing noises by means of tuning forks. The dying away of a sound of a struck tuning fork is very approximately logarithmic, so that time intervals

[1] Galt, R. H., Noise Out of Doors, *Jour. Acous. Soc. Amer.*, vol. 2, No 1, pp. 30–58, July, 1930.

[2] *Nature*, Jan. 11, 1930.

measured during the decay of the fork are proportional to the drop in decibels of the intensity level of the sound. The decibel drop per second can be determined by measuring in a quiet place the times required for the sound of the fork to sink to the masking levels for two different settings of an audiometer. An alternative method of calibration is to measure by means of a microscope with micrometer eyepiece the amplitude of the fork at any two moments in the decay period. Twice the logarithm of the ratio of the two amplitudes divided by the intervening time interval gives the drop of intensity level in bels per second. (The intensity of the sound is proportional to the square of the amplitude of the fork, hence the factor 2.)

In use, one measures in a perfectly quiet place the time required for the sound of the fork to decrease to the threshold. In strict accuracy, the fork should always be struck with the same force. However, considerable variation in the force of the blow will make only slight difference in the time. Measurement is then made of the time required for the sound of the fork to become inaudible in the presence of the noise to be measured. The difference in time between the quiet and noisy conditions multiplied by the number of decibels drop per second gives the masking effect of the noise on the sound of the fork. Different noise levels may be directly compared in this manner. Obviously, the pitch of the fork will make a difference, but with forks between 500 and 1,000 vibs./sec. this difference is less than the fluctuation in levels which commonly occurs in ordinary noises. Figure 83 shows the type of fork designed by Mr. B. E. Eisenhour of the Riverbank Laboratories for noise measurements of this sort. The particular shape

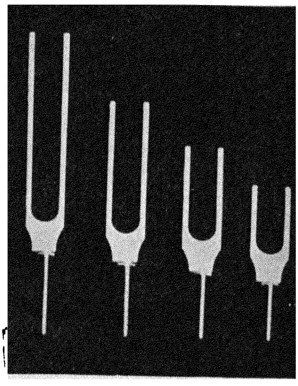


FIG. 83.—Riverbank tuning forks designed for noise measurements.

is such that the damping of the fork is about 2 db. per second and the duration of audible sound is approximately 60 sec. By means of Gradenigo figures etched on the prong of the fork it is possible to measure the time from a fixed amplitude. This makes an extremely simple means of rating noise levels.

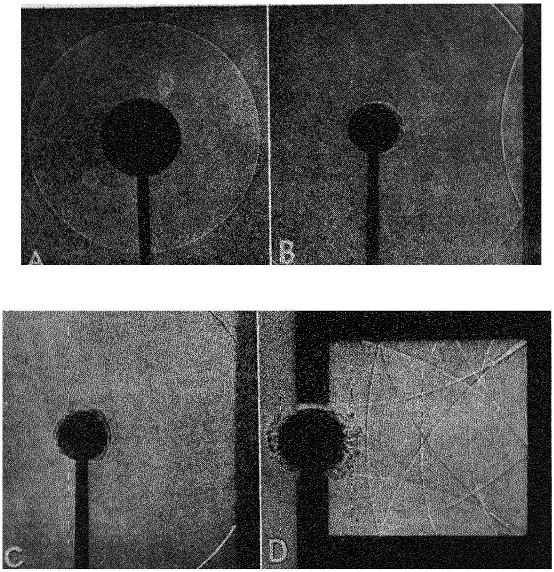


FIG. 84.—A single sound pulse is shown in *A*. In *B*, the pulse is reflected from a hard surface with slight decrease in energy. In *C*, the energy reflected from the absorbent surface is too small to be photographed. *D* shows the repeated reflections inside an inclosure.

Measured Values of Noise Levels.

The most comprehensive survey of the general noise conditions in a large city is that conducted in 1930 by the Noise Abatement Commission under the Department of Public Health of the city of New York. A complete report is published under the title "City Noise," issued by the New York Health Department. In Appendix D are given average values of both indoor and outdoor noise levels above the threshold of hearing as measured in a large number of places and under varying conditions in that survey. Such figures help very materially in relating noise-level values to auditory experience. The value of a decibel rating is shown when we consider that the range of physical intensities is from 1 to 10^{10}.

Reverberation and Noise Level in Rooms.

The sound photographs of Fig. 84 will serve to show qualitatively how the repeated reflection of sound to and fro will increase the general noise level within a room and also the effect of absorbent materials in reducing this

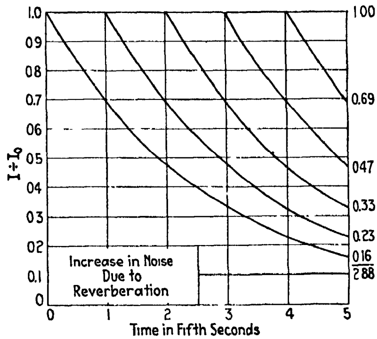


FIG. 85.—Increase in noise due to reverberation.

level. *A* shows the pulse generated by a single impact sound. *B* shows the reflection of this pulse from a hard, highly reflecting wall or ceiling with only a slight diminution of its intensity. *C* shows its dissipation at a highly absorbing surface. In *D* we have the state of affairs within a room after the sound has undergone one or two reflections from non-absorbent surfaces. In a room 30 by 30 ft., *D* would represent conditions 0.05 sec. after the sound was produced. Assume that the absorption coefficient of the bounding surfaces is 0.03 and that the mean time between reflections is 0.05 sec. Then at the end of this time 97 per cent of the energy would still be in the room. At the end of two such intervals, the residual energy would be $(0.97)^2$ or 94.1 per cent of the original. If at the end of 0.1

sec. the impact producing the sound is repeated, then the total sound energy in the room due to the two impacts would be 1.94 times the energy of a single impact. If we imagine the impacts repeated at intervals of 0.1 sec., the sound of each one requiring a considerable length of time to be dissipated, the cumulative effect on the general noise level is easily pictured. The effect of introducing absorbing material in reducing this cumulative action is also obvious.

Figure 85 shows quantitatively the effect of reverberation on the noise produced by the click of a telegraph sounder in the sound chamber of the Riverbank Laboratories. In this room, the sound of a single impact persisted for about 7.5 sec. Experiments showed that the intensity decreased at the rate of about 8 db. per second.

In the figure, it is assumed that the impacts occur at the rate of five per second, and the total sound energy in the room due to one second's operation of the sounder is, as shown, 2.88 times that produced at each impact. Under continuous operation, this would be further increased by the residual sound from impacts produced in preceding seconds.

An analysis similar to that given in Chap. IV for the steady-state intensity set up by a sustained tone is easily made.

Let e = sound energy of a single impact
N = number of impacts per second
α = average coefficient of absorption
p = mean free path = $4V/S$

Then we may treat the source of noise as a sustained source whose acoustic output is Ne. The total energy in the room under sustained operation is

$$\text{Total energy} = \frac{Nep}{c}[1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 \cdots (1 - \alpha)^{n-1}]$$

Making n large, the sum of the series is $1/\alpha$; hence

$$\text{Total energy} = \frac{Nep}{c\alpha} = 4\frac{NeV}{\alpha Sc} = 4\frac{NeV}{ac} \qquad (72)$$

The average energy density of the sound in the room under continued operation is $4Ne/ac$.

It is to be noted that the analysis is based on the assumption that the repeated impact sounds may be treated as a sustained source whose acoustic output is Ne. This assumption holds if the interval between impacts is short compared with the duration of the sound from the impact.

It appears from the foregoing that with a given amount of noise generated in a room, the average intensity due to diffusely reflected sound is inversely proportional to the total absorbing power of the room. Now the noise level is roughly proportional to the logarithm of the intensity. Hence with a given source of sound in a room, the noise level will decrease linearly not with the absorbing power but with the logarithm of the absorbing power. This fact should be borne in mind in considering the amount of quieting in interiors that can be secured by absorbent treatment.

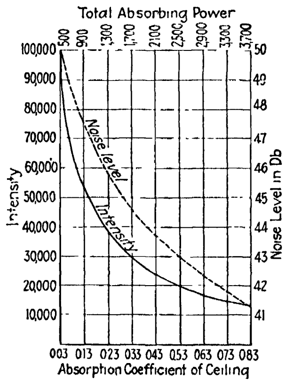


FIG. 86.—Noise level in a room as a function of absorption coefficient of ceiling.

Figure 86 shows the variation of noise intensity and noise level with total absorbing power in a typical case of office quieting. We have assumed an office space 100 by 40 by 10 ft., occupied by 50 typists and initially without any absorbent material other than that of the usual office furniture and the clothing of the occupants. The absorbing power of such a room would normally be about 500 square units. We shall consider that this is initially a fairly noisy office, with a noise level of 50 db. above threshold ($I = 100{,}000i$), and that the ceiling originally has an absorption coefficient of 0.03. The curves show the effect upon the intensity and the noise level of surfacing the 4,000 sq. ft. of ceiling successively with materials of increasingly greater coefficient.

We note here what may be called a law of diminishing returns. Each additional increment of absorption yields a smaller reduction in the intensity and noise level than does the preceding. Thus the first 10-point increase of absorption effects a reduction of 2.5 db.; the last 10-point increase, a reduction of only 0.5 db. Since the threshold of intensity difference is approximately 1 db., it is plain that an increase of from 0.73 to 0.83 in the absorption coefficient of the ceiling treatment would not make a perceptible difference in the noise level, while an increase from 0.63 to 0.83 would effect only a barely perceptible difference.

Measured Reduction Produced by Absorption.

The most important practical use of the reduction of noise by absorbent treatment is in the quieting of business offices and hospitals. The use of absorbent treatment in large business-office units is a matter of common practice in this country, and the universal testimony is in favor of this means of alleviating office noise. There are, however, comparatively few published data on the actual reduction so effected. The writer at various times has made audiometer measurements both before and after the application of absorbent treatment in the same offices. The most carefully conducted of these was under conditions approximately described in the example of the preceding section. The average noise level before treatment, using a Western Electric 3-A audiometer, was 55.3 db. A material with an absorption coefficient of 0.65 was applied to the entire ceiling, and the measured value was reduced to 48 db. The introduction of the absorbent treatment increased the total absorbing power about fivefold. The theoretical reduction in decibels would be $10 \log_{10} 5 = 7.0$ db. as compared with the measured 7.3 db. R. H. Galt[1] has measured, under rather carefully controlled conditions, the effect of absorbent treatment upon the noise level generated

[1] Method and Apparatus for Measuring the Noise Audiogram, *Jour. Acous. Soc. Amer.*, vol. 1, No. 1, pp. 147–157, October, 1929.

by a fixed source of noise. A phonograph record of the noise in a large office was made. The amplified sound from this record was admitted to a test room by way of two windows from a smaller room in which the record was reproduced. The absorbing power of the test room was varied by bringing in absorbent panels. Galt found that a fivefold increase in the total absorbing power reduced the intensity to one-fifth, corresponding to a reduction of 7 db. in the noise level. He found that closing the windows of a tenth-floor office room reduced the noise from the street by approximately this same amount. Perhaps this comparison affords a better practical notion of what is to be expected in the way of office quieting by absorbent treatment than do the numerical values given. Tests conducted by the author for the Western Felt Works showed a reduction of 7 db. in the noise of passing street cars as a result of closing the windows.

Computation of Noise Reduction.

By Eq. (72) we have for the intensity of the noise

$$I = \frac{4Ne}{ac}$$

If the absorbing power of the room is increased to a', then the intensity is reduced to

$$I' = \frac{4Ne}{a'c}$$

whence

$$\frac{I}{I'} = \frac{a'}{a} = \frac{T_0}{T'_0}$$

The reduction in bels is the logarithm of the ratio of the two intensities; hence

$$\text{Reduction } (db) = 10 \log \frac{a'}{a} = 10 \log \frac{T_0}{T_0'}$$

It is apparent that the degree of quieting effected by absorbent treatment depends not upon the absolute value of the added absorbing power but upon its ratio to the original. Adding 2,000 units to a room whose absorbing power is 500 increases the total absorbing power fivefold, giving a noise reduction of 10 log 5 = 7 db. Adding the same number of units to an initial absorbing power of 1,000 units increases the total threefold, and the reduction is 10 log 3 = 4.8 db.

Quieting of Very Loud Noise.

It is evident that the reduction in the noise level secured by the absorbent treatment of a room is the same independently of the original level. Thus if the introduction of sound-absorbing materials into a room reduces the noise level from 50 to 40, let us say, then the same treatment would produce the same reduction—10 db.—if the original noise level were raised to 80 or 90 db. This suggests the comparative ineffectiveness of absorption as a means of quieting excessively noisy spaces. A treatment that would lower the general noise level in an office from 50 to 40 db. would make a marked difference in the working conditions. In its effect on speech, for example, the 40-db. level is 10 db. below the level of quiet speech, and its effect on intelligibility is decidedly less than that of the original 50-db. noise level. In a boiler factory, however, with an original noise level of 100 db. this 10-db. reduction to 90 db. would still necessitate shouting at short range in order to be heard. Moreover, it must be remembered that absorption can affect only the contribution to the general noise level made by the reflected sound. The operator working within a few feet of a very noisy machine where the direct sound is the preponderating factor is not going to be helped very much by sound-absorbent treatment applied to the walls.

While these common-sense considerations are obvious enough, yet one finds them frequently ignored in attempts to achieve the impossible in the way of quieting extremely noisy conditions. The terms "noisy" and "quiet" are relative. A given reduction of the noise level in an office, where the work requires mental concentration on the part of the workers, may change working conditions from noisy to quiet. The same reduction in a boiler factory would produce no such desired results.

Effect of Office Quieting.

In addition to the actual physical reduction of the noise level from the machines in a modern business office, there are certain psychological factors that are operative in promoting the sense of quiet in rooms in which sound reflection is reduced by absorbent treatment. We have noted the effect of reverberation upon the intelligibility of speech. In reverberant rooms, there is therefore the tendency for employees to speak considerably above the ordinary conversational loudness in order to meet this handicap. The effect is cumulative. Employees talk more loudly in order to be understood, thus adding to the sense of noise and confusion. Similarly, the reduction of reverberation operates cumulatively in the opposite sense. Further, there is evidence to indicate that under noisy conditions the attention of the worker is less firmly fixed upon the work in hand. Hence his attention is more easily and frequently distracted, and he has the sense of working under strain. A relief of this strain produces a heightened feeling of well-being which increases the psychological impression of relief from noise. If these and other purely subjective factors could be evaluated, it is altogether probable that they would account for a considerable part of the increased efficiency which office quieting effects.

Precise evaluation of the increased efficiency due to office quieting is attended with some uncertainty. Perhaps the most reliable study of this question is that made in certain departments of the Aetna Life Insurance Company of Hartford, Connecticut, the results of which were reported by Mr. P. B. Griswold of that company to the National Office Management Association, in June, 1930. The study was made in departments in which the employees worked on a bonus system, the bonuses being awarded on the basis of the efficiency of the department as a whole. Records of the efficiency were kept for a year prior to the installation of absorbent treatment and for a year thereafter. All other conditions that might affect efficiency were kept constant over the entire period. The reduction in noise as measured by a 3-A audiometer was from 41.3 to 35.3 db. (These were probably masking levels.) The observed over-all increase in efficiency in three departments was 8.8 per cent. The quieting was found to produce a decrease of 25 per cent in the number of errors made in the department. A large department store reports a reduction in the number of errors in bookkeeping and in monthly statements from 118 to 89 per month—a percentage reduction of 24 per cent.

Physiological Effects of Noise.

In psychological literature, there is a considerable body of information on the effect of noise and music on mental states and processes. A bibliography of the subject is given by Professor Donald A. Laird in the pamphlet "City Noise" already referred to. A paper by Laird on the physiological effects of noise appeared in the *Journal of Industrial Hygiene.*[1] This paper gives the results of measurements on the energy consumption as determined by metabolism tests on typists working under extreme conditions of noise, in a small room (6 by 15 by 9 ft.), with and without absorbent material. Laird states that the absorbent treatment "reduced the heard sound in the room by about 50 per cent." Four typists working at maximum speed were the subject of the experiment. The outstanding result was that the working-energy consumption under the quieter conditions was 52 per cent greater than the consumption during rest, while under the noisier condition it was 71 per cent greater. Further, the tests showed that the slowing up of speed of the various operators was in the order of their relative speeds; that is, the "speedier the worker the more adversely his output is affected by the distraction of

[1] October, 1927.

noise." If fatigue is directly related to oxygen consumption, it would appear that these tests furnish positive evidence of the increase of fatigue due to noise.

Absorption Coefficients for Office Noises.

Due to the nondescript character of noise in general and to the fact that most of the published data on the absorbing efficiencies of material are for musical tones of definite pitch, it is not possible to assign precise values to the noise-absorbing properties of materials. In 1922, the writer made a series of measurements on the relative absorbing efficiencies of various materials for impact sounds, such as the clicks of telegraph sounders and typewriters.[1] The experimental procedure was quite analogous to that used in the measurement of absorption coefficients by the reverberation method for musical tones.

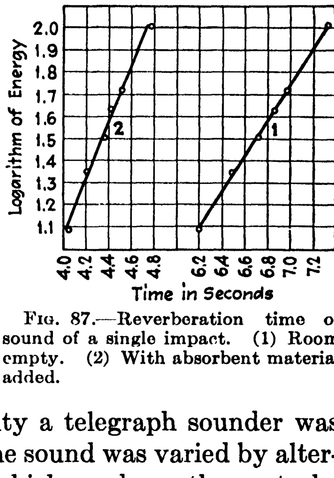


Fig. 87.—Reverberation time of sound of a single impact. (1) Room empty. (2) With absorbent material added.

As a source of variable intensity a telegraph sounder was employed. The intensity of the sound was varied by altering the strength of the spring which produces the upstroke of the sounder bar. In Fig. 87, the logarithm of the energy of the impact of the bar is plotted against the measured reverberation times. We note the straight-line relation between these two quantities, quite in agreement with the results obtained using a musical tone of varying initial intensities. It follows therefore that the sound energy generated by an impact is proportional to the mechanical energy of the impact. From the slope of the straight line, we determine the absorbing power of the empty room, and proceeding in a manner similar to that for musical tones,

[1] Nature and Reduction of Office Noise, *Amer. Architect*, May 24; June 7, 1922.

we may measure the absorption coefficients of materials for this particular noise. The experiment was extended to include other impact sounds including noise of typewriters of different makes. It appeared that the absorbing power of the empty sound chamber was the same as that for musical tones in the range from 1,024 to 2,048 vibs./sec. and that the relative absorbing efficiencies of a number of different materials were the same as their efficiencies for musical tones in this range.

If we include voice sounds as one of the components of office noise, it would appear that the mean coefficient over

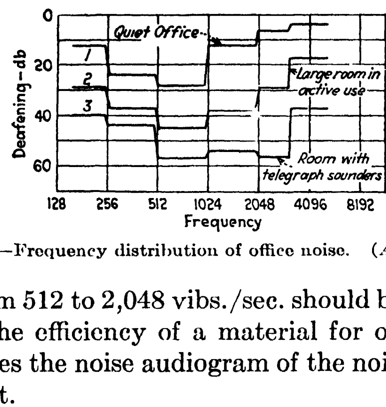


FIG. 88.—Frequency distribution of office noise. (*After Galt.*)

the range from 512 to 2,048 vibs./sec. should be taken as the measure of the efficiency of a material for office quieting. Figure 88 gives the noise audiogram of the noise in offices as given by Galt.

Here we note that the range of maximum deafening for the large office in active use extends from 256 to 2,048, while in the room with telegraph sounders, the maximum occurs in the range 500 to 3,000 vibs./sec.

Quieting of Hospitals.

Nowhere is the necessity for quiet greater than in the hospital, and perhaps no type of building, under the usual conditions, is more apt to be noisy. The accepted notions of sanitation call for walls, floors, and ceiling with hard, non-porous, washable surfaces. The same ideas limit the furnishings of hospital rooms to articles having a minimum of sound absorption. The arrangement of a series of patients' rooms all opening on to a long corridor with highly reflecting walls, ceiling, and floor makes for the easy propagation of sound over an entire floor. Sanitary equipment suggests basins and pans of porcelain or enamel ware that rattle noisily when washed or handled. Modern fireproof steel construction makes the building structure a solid continuous unit through which vibrations set up by pumps or ventilating machinery and elevators are transmitted with amazing facility. In the city, the necessity for open-window ventilation adds outside noises to those of internal origin.

The solution of the problem of noise in hospitals is a matter both of prevention and of correction. Prevention calls for attention to a host of details. The choice of a site, the layout of plans, the selection, location, and installation of machinery so that vibrations will not be transmitted to the main building structure, acoustical isolation of nurseries, delivery rooms, diet kitchens, and service rooms, provision for noiseless floors, quiet plumbing, doors that will not slam, elevators and elevator doors that will operate with a minimum of noise—attention to all of these in the planning of a hospital will go far toward securing the desired degree of quiet. The desirability of a quiet site is obvious. City hospitals, however, must frequently be located in places where the noise of traffic is inevitable. In such cases, a plan in which patients' rooms face an inside court, with corridors and service rooms adjacent to the street, will afford a remarkable degree of shielding from traffic noise. Laundries and heating plants should, whenever possible, be in a separate building. Failing this, location in basement rooms on massive isolated foundations with proper precautions for the insulation of air-borne sounds to the upper floors can be made very effective in preventing the transmission of vibrations. Properly designed spring mountings afford an effective means of reducing structural vibrations that would otherwise be set up by motors and other machinery that have to be installed in the building proper.

In addition to the prevention of noise, there is the possibility of minimizing its effects by absorbent treatment. There has been a somewhat general feeling among those responsible for hospital administration against the use of sound-absorbent materials on grounds of sanitation. It has been assumed that soft or porous materials of plaster, felts, vegetable fiber, and the like are open to objections as harboring and breeding places for germs and bacteria. Hospital walls and ceilings come in for frequent washings and for surface renewals by painting.

The whole question of the applicability of sound-absorbent treatment to the reduction of hospital noises, including the important item of cost of installation and upkeep, has been gone into by Mr. Charles F. Neergaard of New York City. The results of his investigations have been published in a series of papers which should be consulted by those responsible for building and maintenance of a hospital.[1]

A number of acoustical materials were investigated, both as to the viability of bacteria within them and as to the possibilities of adequately disinfecting them. The general conclusion reached was that certain of these materials are well adapted for use in hospitals, that the sanitary hazard is more theoretical than real, and that the additional cost of installation and upkeep is more than compensated for by the alleviation of noise which they afford.

On the strength of these findings, one feels quite safe in urging the use of sound-absorbent treatment on the ceilings and upper side walls of hospital corridors, in diet kitchens, service rooms, and nurseries, as well as in private rooms intended for cases where quiet and freedom from shock are essential parts of the curative régime.

[1] How to Achieve Quiet Surroundings in Hospitals, *Modern Hospital*, vol. 32, Nos. 3 and 4, March and April, 1929; Practical Methods of Making the Hospital Quiet, *Hospital Progress*, March, 1931; Are Acoustical Materials a Menace in the Hospital? *Jour. Acous. Soc. Amer.*, vol. 2, No. 1, July, 1930; Correct Type of Hardware, *Hospital Management*, February, 1931.

Noise from Ventilating Ducts.

A frequent source of annoyance and difficulty in hearing in auditoriums is the noise of ventilating systems. There may be several sources of such noise; the more important are (1) that due to the fluctuations in air pressure as the fan blades pass the lip of the fan housing; (2) noise from the motor or other driving machinery; (3) noise produced by the rush of air through the ducts, and particularly at the ornamental grills covering the duct opening. It is not uncommon to find cases in which complaints are made of poor acoustical conditions, which upon investigation show that the noise level produced by the ventilating system is responsible. The noise from the motor and fan is transmitted in two ways: as mechanical vibrations along the walls of the duct and as air-borne sound inside the ducts. It is good practice to supply a short-length flexible rubber-lined canvas coupling between the fan housing and the duct system. This will obviate the transfer of mechanical vibrations.

Lining the duct walls with sound-absorbent material measurably reduces the air-transmitted noise. Such treatment is more effective in small than in large conduits. Both duct and fan noises, however, increase with the speed of the fan and the velocity of air flow, so that in absorbent-lined ducts, the preference as between low speeds through large ducts and high speeds through small ducts is doubtful.

Larson and Norris[1] have reported the results of a valuable study on the question of noise reduction in ventilating systems. The tests were made on a 30-ft. section of 10 by 10-in. galvanized iron duct made up in units 2 ft. long. Air speeds ranged from about 300 to 5,000 ft. per minute, and fan speeds from 100 to 1,300 r.p.m. The noise level was measured by means of an acoustimeter, with the receiving microphone set up 2 ft. from the duct opening. Two

[1] Some Studies on the Absorption of Noise in Ventilating Ducts, *Jour. Heating, Piping,* and *Air Conditioning, Amer. Soc. Heating Ventilating Eng.*, January, 1931.

types of absorbent lining were used. The absorbent material was a wood-fiber blanket 1 in. thick. In one case it was bare and, in the other, covered with thin, perforated sheet metal. In the lined duct, the outer casing was 12 by 12 in. giving the same section 10 by 10 in. for the air flow through both the lined and the unlined ducts.

The following are some of the more important facts deduced:

1. The perforated metal covering over the absorbent material produced no reduction in the air flow for a given fan speed. The bare absorbent reduced the air flow by about 11 per cent.

2. The reduction of noise was the same with as without the perforated metal covering over the absorbent material.

3. The reductions in decibels in the noise level produced by lining the entire duct are given below.

Air speed, feet per minute	Noise level		Reduction, dbs.
	Unlined	Lined	
1,000	33	23	0
2,000	44	32	12
3,000	54	41	13
4,000	63	48	15
5,000	68	53	15

4. Absorbent lining placed near the inlet end of the duct produces a somewhat greater reduction than the same length at the outlet end. Thus 6 ft. at the intake produced a greater reduction than 12 ft. at the outlet.

5. The authors state that a 41-db. level at the outlet end does not materially affect the hearing in an auditorium. At this level, lining the duct throughout would allow an increase of 75 per cent in the air speed over that which would produce this level in an unlined duct.

6. The noise reduction increases with the length of duct that is lined. The increment in the reduction per unit of lining decreases as the lining already present increases.

These results serve to show the general effect on noise reduction of lining ducts. The reduction effected will also depend upon the size of the duct, decreasing as the cross section increases. Data on this point are lacking. Another source is that due to the rush of air through the grill work covering the opening. This may be expected to increase markedly with the air velocity. All things considered, it would appear that a high-velocity system is apt to produce more noise for a given delivery of air to the room than a low-speed system.

CHAPTER XI

THEORY AND MEASUREMENT OF SOUND TRANSMISSION

Nature of the Problems.

Two distinct problems arise in the study of the transmission of sound from room to room within a building. The first is illustrated by the case of a motor or other electric machinery mounted directly upon the building structure. Due to inevitable imperfections in the bearings and to the periodic character of the torque exerted on the armature, vibrations are set up in the machine. These are transmitted directly to the structure on which it is mounted and hence by conduction through solid structural members to remote parts of the building. These vibrations of the extended surfaces of walls, floors, and ceiling produce sound waves in the air. Consideration of this aspect of sound transmission in buildings will be reserved for a later chapter.

The second problem is the transmission of acoustic energy between adjacent rooms by way of intervening solid partitions, walls, floors, or ceiling. Transmission of the sound of the voice or of a violin from one room to another is a typical case. The transmission of the sound of a piano or of footfalls on a floor to the room below would come under the first type of problem.

Mechanism of Transmission by Walls.

Let us suppose that A and B (Fig. 89) are two adjacent rooms separated by a partition P and that sound is produced by a source S in A. The major portion of the sound energy striking the partition will be reflected back into A, but a small portion of it will appear as sound energy in B. There are three distinct ways in which the transfer of energy from A to B by way of the intervening partition is effected: (1) If *P* is perfectly rigid, compressional air waves in *A* will give rise to similar waves in the solid structure *P*, which in turn will generate air waves in *B*. (2) If *P* is a porous structure such as to allow the passage of air through it, the pressure changes due to the sound in *A* will set up corresponding changes in *B* by way of the pore channels in the partition. A part of the energy entering the pore channels will be dissipated by friction; that is, there will be a loss of energy by absorption in transmission through a porous wall. (3) If *P* is non-porous and not absolutely

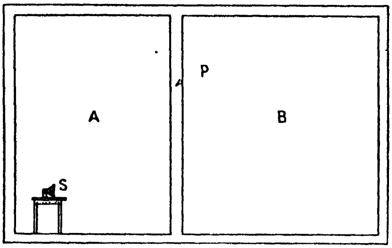


Fig. 89.

rigid, the alternating pressure changes on the surface will set up minute flexural vibrations of the partition, which will in turn set up vibrations in *B*.

Of these three modes of transmission the first is of negligible importance in any practical case, in comparison with the two others. When sound in one medium is incident upon the surface of a second medium, a part of its energy is reflected back into the first medium and part is refracted. The ratio of this refracted portion to the incident energy is equal to the ratio of the acoustic resistances of the first medium to that of the second. From the values of acoustic resistances in various media given in Table II of Appendix *A* it will be seen that for solid materials the acoustic resistance is very high as compared with that of air, so that the sound entering a solid partition is a very small fraction of the incident sound, and hence very little sound is transmitted through solid walls in this manner. Davis and Kaye state that a mahogany board two inches thick, if rigid, would transmit only 20 parts in 1,000,000.[1]

We shall consider the transmission of energy in the two other ways in Chap. XII.

Measurement of Sound Transmission: Reverberation Method.

The first serious attempt at the measurement of the sound transmitted by walls was made by W. C. Sabine prior to 1915. The results of these earliest measure-

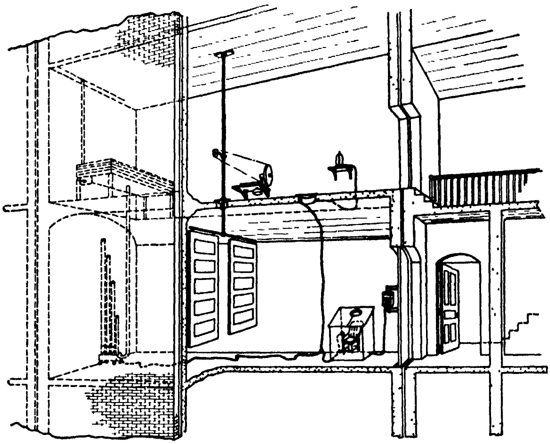


FIG. 90.—W. C. Sabine's experimental arrangement for sound transmission measurements.

ments for a single tone are described in his "Collected Papers." The method used was based on the reverberation theory, already employed in absorption measurements. The experimental arrangements are shown in Fig. 90 taken from the "Collected Papers." Sound was produced by organ pipes located in the constant-temperature room of the Jefferson Physical Laboratory. The test panels were mounted in the doorway constituting the only means of entrance into the room, so that when the panel was in place the experimenter had to be lowered into the room by means

[1] "Acoustics of Buildings," p. 179, George Bell & Sons, 1927.

of a rope through a manhole in the ceiling. The experimental procedure was to measure the time of decay of sound heard in the constant-temperature room and then the time as heard through the test panel in a small vestibule outside. If the average steady-state intensity set up by the source in the larger room is I_1, and I is the intensity t sec. after the source is stopped, then the reverberation equation gives

$$\log_e \frac{I_1}{I} = At = \frac{act}{sp} = \frac{act}{4V}$$

If t_1 be the time required for the intensity in the source room to decrease to the threshold intensity i, then

$$\log_e \frac{I_1}{i} = \frac{act_1}{4V}$$

Let k, the reduction factor of the partition, be the ratio at any moment of the average intensity of the sound in the source room to the intensity at the same moment on the farther side of the partition and close to it. Then when the sound heard through the partition has just reached the threshold intensity, the intensity I in the source room is given by the relation

$$I = ki$$

If a sound of initial intensity I_1 can be heard for t_2 sec. through the test partition, we have

$$\log_e \frac{I_1}{ki} = \frac{act_2}{4V}$$

whence, by subtraction, we have

$$\log_e k = \frac{ac}{4V}(t_1 - t_2)$$

and

$$\log_{10} k = \frac{1}{2.3}\frac{ac}{4V}(t_1 - t_2) \tag{73}$$

The value of the total absorbing power of the source room is known from t_1, the measured reverberation time in the source room and the initial calibration as described in Chap. V.

This method is simple and direct and involves no assumptions save those of the general theory of reverberation.

Using this method, Professor Sabine made measurements upon a considerable number of materials and structural units such as doors and windows of various types. His only paper on the subject of sound transmission, however, gives the results only for hair felt of various thicknesses and a complex wall of alternate layers of sheet iron and felt, and these at only a single frequency. The experiments with felt showed a strictly linear relation between the thickness and the logarithm of k as defined above. The intervention of the World War and Professor Sabine's untimely death just at its close prevented the carrying out of the extensive program of research along this line which he had planned.

Experimental Arrangement at Riverbank Laboratories.

The sound chamber of the Riverbank Laboratories was built primarily to provide for Professor Sabine the facilities for carrying on the research to which reference has just been made. A general description together with detailed drawings have already been given in Chap. VI (page 98). To that description it is necessary to add only the details of construction of the test chambers—rooms corresponding to the small vestibule of the constant-temperature room at Harvard. As will be noted, the sound chamber is built upon a separate foundation and is structurally isolated from the rest of the building as well as from the test chambers. The openings from the two smaller test chambers into the sound chamber are each 3 by 8 ft., while that from the larger is 6 by 8 ft. The test chambers are protected from sounds of outside origin by extremely heavy walls of brick, and entrance is effected by means of two heavy ice-box doors through a small vestibule. Tests on small structural units such as doors and windows are made with these mounted in the smaller opening, screwed securely to heavy wooden frames set in the openings. All cracks are carefully sealed with putty. Leads for electrically operating the organ are supplied to each of the test chambers. Originally, experiments were made to determine the effect of the size and absorbing power of the receiving space upon the measured duration of sound from the sound chamber as heard through a test wall. It was found that both of these factors produced measurable effects, so that the standard practice was adopted of closing off a small space adjacent to the test wall by means of highly absorbent panels. Reverberation in the test rooms was thus rendered negligibly small in comparison with that in the sound chamber. The effect of this will be considered somewhat in detail later. During the twelve years of the laboratory's operation, test conditions have been maintained constant so that all results might be comparable.

All hitherto published results from this laboratory have been based upon the values of absorbing power obtained by the four-organ calibration of the sound chamber. It will be noted in Eq. (73) that the value of the logarithm of k varies directly as the value assigned to a, the absorbing power of the sound chamber. In view of the commercial importance attached to many of the tests, these earlier values have been adhered to for the sake of consistency, even though the work of recent years using a loud-speaker source indicated that for certain frequencies these values were somewhat too low. Correction to these later values for the sound-chamber absorption are easily made, however, and in the results hereinafter given these corrections are applied.

Bureau of Standards Method.

Figure 91 shows the experimental arrangements for sound-transmission measurements at the Bureau of Standards.[1] The source of sound is placed in a small room S, provided with two openings in which the test panels are placed. The source room is structurally isolated from the receiving rooms R_1 and R_2 in the manner shown. The walls

[1] ECKHARDT and CHRISLER, Transmission and Absorption of Sound by Some Building Materials, *Bur. Standards Sci. Paper* 526.

are of concrete 6 in. thick, separated from the receiving room walls by a 3-in. air space. Lighter constructions are prepared as panels and mounted in the ceiling opening, while heavier walls are built directly into the vertical window.

The source, supplied with alternating current from a vacuum-tube oscillator, is mounted on an arm approximately 2 ft. long, which rotates at a speed of about one revolution per second. The frequency of the tone is varied cyclically over a limited range of frequencies. The width of the frequency band can be controlled within limits by varying the capacity of a rotating condenser

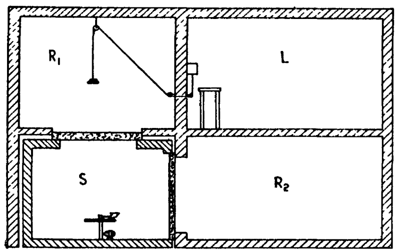


FIG 91.—Room for sound transmission measurements at the Bureau of Standards.

which is associated with the fixed capacity of the oscillator circuit. The frequency variation and the rotation of the source serve to produce a constantly shifting interference pattern in the source room. The intensity measurements are made by means of a long-period galvanometer, which tends to give an averaged value of the intensity at any position of the pick-up device.

To determine relative intensities, a telephone receiver is placed in the position at which the intensity is to be measured. The e. m. f. generated in the receiver windings is taken as proportional to the amplitude of the sound. The alternating current produced, after being amplified and rectified, is passed through a long-period galvanometer, and the deflection noted. Shift is then made from the telephone pick-up to a potentiometer which derives oscillating current from the oscillator which operates the sound source, and the potentiometer is adjusted to give the same deflection of the galvanometer as was produced by the sound. The potentiometer reading is taken as a measure of the sound amplitude, and the relative intensities of two sounds are to each other as the squares of these values. The validity of this method of measurements rests upon the strict proportionality between the amplitude of the sound and the e. m. f. generated in the field windings of the pick-up.

In measuring the reduction of intensity, the procedure is as follows: Without the panel in place, a series of intensity measurements is made at points along a line through the middle point of the opening and perpendicular to its plane. Denote the average values of these readings in the transmitting room by T, and in the receiving room by R. The panel is then placed and the readings repeated. It is found that T is usually increased to $T + t$, let us say, while R is reduced to $R - r$. The apparent ratio of the intensities in the receiving room without and with the panel in place is $R/(R - r)$. But the placing of the panel has increased the intensity in the transmitting room in the ratio of $\frac{T + t}{T}$, so that for the same intensity in the transmitting room under the two conditions the ratio of the intensities in the receiving room is $R(T + t)/T(R - r)$.

This expression Eckhardt and Chrisler have also called the reduction factor. Obviously it is not the same thing as the reduction factor as defined above. $(T + t)/(R - r)$ is the ratio of the intensities in the two rooms with the partition in place, which is the reduction factor k as defined by the writer. The Bureau of Standards reduction factor is therefore kR/T. R/T is the ratio of the intensities in the receiving and transmitting rooms without the partition. This is less than unity, so that values for the reduction factor given by the Bureau of Standards should, presumably, be less than those by the reverberation method as outlined in the preceding section.

Other Methods of Transmission Measurements.

Figure 92 shows the experimental arrangement proposed and used by Professor F. R. Watson[1] for the measurement of sound transmission. An organ pipe was mounted at the focus of a parabolic mirror, which is presumed to direct a beam of sound at oblique incidence upon the test panel mounted in the opening between two rooms. The intensity of the sound was measured by means of a Rayleigh disk and resonator in the receiving room, first without and then with the transmitting panel in the opening. The

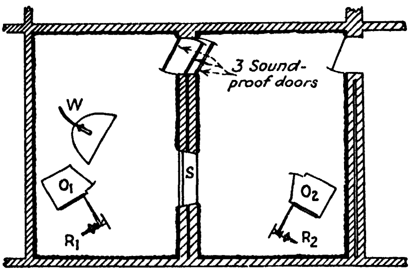


FIG. 92.—Arrangement for sound transmission measurements at the University of Illinois. (*After Watson.*)

relative sound reductions by various partitions were compared by comparing the ratios of the deflections of the Rayleigh disk under the two conditions. Observations were made apparently at only a single point of the sound beam, so that the effect of the presence of the panel on the stationary-wave system in the receiving room was ignored.

A modification of the Watson method has been used at the National Physical Laboratory in England.[2] Two unusually well-insulated basement rooms with double walls and an intervening air space were used for the purpose. The opening in which the test panels were set was 4 by 5 ft. Careful attention was paid to the effects of interference, and precautions taken to eliminate them as far as possible by means of absorbent treatment. The measure-

[1] Univ. Ill., Eng. Exp. Sta. *Bull.* 127, March, 1922.
[2] DAVIS and LITTLER, *Phil. Mag.*, vol. 3, p. 177, 1927; vol. 7, p. 1050, 1929.

ments were made by apparatus not essentially different from that used at the Bureau of Standards. The general set-up is shown in Fig. 93. Some interesting facts were brought out in connection with these measurements. For example, experiments showed that with an open window between the two rooms there was a considerable variation

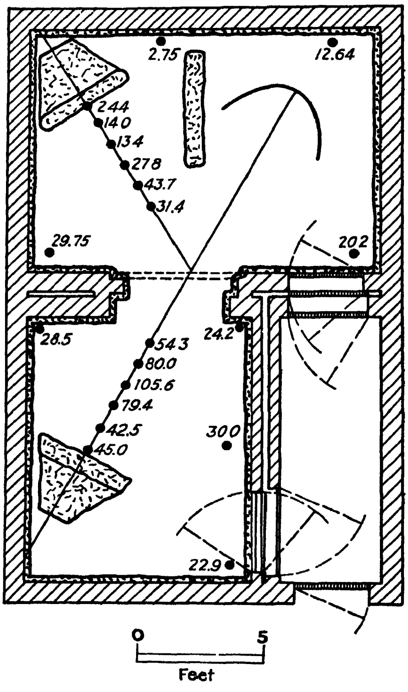


Fig. 93.—National Physical Laboratory rooms for sound transmission measurements. (*After Davis and Littler.*)

along the path of the beam sent out by the parabolic reflector but that the distribution of intensity in the receiving room was practically the same with a felt panel in place as when the sound passed through the unobstructed opening. This implies that the oblique beam is transmitted through the felt unchanged in form. It was further found that even with a 4½-in. brick wall interposed there was a pronounced beam in the receiving room and that the intensity of sound outside this beam was comparatively small. In actual practice, measurements were made at a number of points, and the average values taken.

Davis and Littler used the term "reduction factor" for the ratio between the average intensity along the sound beam in the receiving room without the partition to the intensity at the same points with the partition. This is clearly not quite the same thing as either the Riverbank or Bureau of Standards definition. It differs from both in the fact that it assumes transmission at a single angle of incidence instead of a diffuse distribution of the incident sound. Further, it is assumed that the intensity on the source side is the same regardless of the presence of the partition.

Sound transmission measurements have also been made by Professor H. Kreuger of the Royal Technical University in Stockholm. His experimental arrangement is taken from a paper by Gunnar Heimburger[1] and is shown in Fig. 94. A loud-speaker is set up very close to one face of the test wall, and a telephone receiver is similarly placed on the opposite side. The loud-speaker and the telephone pick-up are both inclosed in absorbent-lined boxes. The current generated in the pick-up is amplified and measured in a manner similar to that employed at the Bureau of Standards. The intensity as recorded by the pick-up when the sound source is directly in front of the receiver divided by the intensity when the two are placed on opposite sides of the test wall is taken as the reduction factor of the wall. It is fairly obvious that this method of measurement, while affording results that will give the relative sound-insulating properties of different partitions,

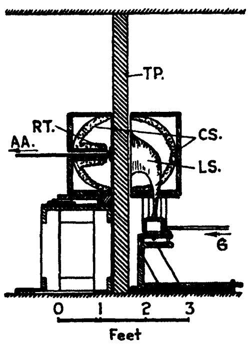


FIG. 94.—Experimental set-up for sound transmission measurements used by Kreuger.

[1] *Amer. Architect.*, vol. 133, pp. 125–128, Jan. 20, 1928.

will not give absolute values that are comparable with values obtained when the alternating pressures are applied over the entire face of the wall or to the sound reduction afforded by the test wall when in actual use.

Here the driving force is applied over a limited area, but the entire partition is set into vibration. The amplitude of this vibration will be very much less and the recorded intensity on the farther side will be correspondingly lower than when the sound pressure is applied to the entire wall as is the case in the preceding methods.

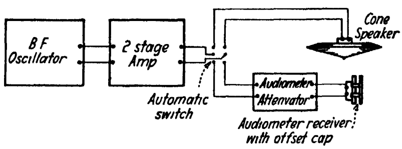


FIG. 95.—Audiometer method of sound transmission measurements. (*After Waterfall.*)

An audiometric method of measurement has been used by Wallace Waterfall.[1] The experimental arrangement is shown in Fig. 95. The sound is produced by a loud-speaker supplied with current from a vacuum-tube oscillator and amplifier. The output of the amplifier is fed into a rotary motor-driven double-pole double-throw switch, which connects it alternately to the loud-speaker and to an attenuator and receiver of a Western Electric 3-A audiometer. The loud-speaker is set up on one side of the test partition, and the attenuator is adjusted so that the tone as heard through the receiver is judged to be equally loud with the sound from the speaker direct. The observer moves to the opposite side of the partition, and a second loudness match is made. The dial of the attenuator being graduated to read decibel difference in the settings on the two sides gives the reduction produced by the wall. The apparatus is portable and has the advantage of being applicable to field tests of walls in actual use. The results of measurements by this method are in very good

[1] *Jour. Acous. Soc. Amer.*, p. 209, January, 1930.

agreement with those obtained by the reverberation method.

Not essentially different in principle is the arrangement used by Meyer and Just at the Heinrich Hertz Institute and shown in Fig. 96. Two matched telephone transmitters are set up on the opposite sides of the test wall. Number 1 on the transmitting side feeds into an attenuator and thence through a rotary double-pole switch into a potentiometer, amplifier, and head set. Number 2, on the farther side, feeds directly into the potentiometer. The attenuator is adjusted for equal loudness of the sounds from

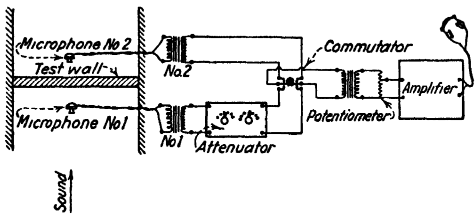


FIG. 96.—Method of sound transmission measurements used by Meyer and Just.

the two transmitters as heard alternately in the head set through the rotating switch. The electrical reduction produced by the attenuator is taken as numerically equal to the acoustical reduction produced by the wall. The reduction as thus measured corresponds closely to the reduction factor as defined under the reverberation method.

Resonance Effects in Sound Transmission.

As has been indicated, sound is transmitted from room to room mainly by virtue of the flexural vibrations set up in the partition by the alternating pressure of the sound waves. The amplitude of such a forced vibration is determined by the mass, flexural elasticity or stiffness, and the frictional damping of the partition as a whole, as well as by the frequency of the sound. A partition set in an opening behaves as an elastic rectangular plate clamped at the edges. Such a plate has its own natural frequencies of vibrations which will depend upon its physical properties of mass, thickness, stiffness and its linear dimensions. Under forcing, the amplitude of the forced vibrations at a given frequency will depend upon the proximity of the forcing frequency to one of the natural frequencies of the plate. For example, it can be shown that a plate of ¼-in. glass 3 by 8 ft. may have 37 different natural modes of vibration and the same number of natural frequencies below 1,000 vib./sec.

Its response to any of these frequencies would theoretically be much greater than to near-by frequencies, and its transmission of sound at these natural frequencies correspondingly greater. As an experimental fact, these effects of resonance do appear in transmission measurements, so that a thorough study of the properties of a single partition would involve measurements at small frequency intervals over the whole sound spectrum. To rate the relative over-all sound-insulating properties of constructions on the basis of tests made at a single frequency would be misleading. Thus, for example, in tests on plaster partitions it was found that, at a particular frequency, a wall 1½ in. thick showed a greater reduction than a similar wall 2½ in. thick, although over the entire tone range the thicker wall showed markedly greater reduction.

For this reason, we can scarcely expect very close agreement in the results of measurements on a given construction at a single frequency under different test conditions. In addition to this variation with slight variations in frequency, most constructions show generally higher reductions for high than for low frequencies. It appears, therefore, that in the quantitative study of sound transmission, it is necessary to make measurements at a considerable number of frequencies and to adopt some standard practice in the distribution of these test tones in the frequency range.

Up to the present time, there has been no standard practice among the different laboratories in which transmission measurements have been made in the selection of test frequencies. At the Riverbank Laboratories, with few exceptions, tests have been made at 17 different frequencies ranging from 128 to 4,096 vibs./sec., with 4 frequencies in each octave from 128 to 1,024, and 2 each in the octaves above this. This choice in the distribution of test tones was made in view of the fact that variation in the reduction with frequency is less marked for high- than for low-pitched sounds and also because in practice the more frequent occurrence of the latter makes them of more importance in the practical problem of sound insulation. At the Bureau of Standards, most of the tests give more importance to frequencies above 1,000, while the total frequency range covered has been from 250 to 3,300. The National Physical Laboratory has used test tones of 300, 500, 700, 1,000, and 1,600. Kreuger employed a series of tones at intervals of 25 cycles ranging from 600 to 1,200 vibs./sec. This lack of uniformity renders comparisons at a single frequency impossible and average values over the whole range scarcely comparable.

Coefficient of Transmission and Transmission Loss.

As has been noted above, different workers in the field of sound transmission have used the term "reduction factor" for quantities which are not identically defined. In any scientific subject, it is highly desirable to employ terms susceptible of precise definition and to use any given term only in its strict sense. The term reduction factor was originally employed by the author as a convenient means of expressing the results of transmission measurements as conducted at the Riverbank Laboratories.[1] Since its subsequent use, with a slightly different significance, by other investigators leaves its precise definition by usage somewhat in doubt, the introduction of another term seems advisable. It was early recognized that the volume and absorbing power of the receiving room had a measurable effect upon the reverberation time as heard through the partition and that therefore the computed value of k would depend

[1] *Amer. Architect*, vol. 118, pp. 102–108, July 28, 1920.

upon the acoustic conditions of the receiving room as well as upon the sound-transmitting properties of the test wall. Accordingly the standard practice was adopted of measuring the time in a small, heavily padded receiving room, with the observer stationed close to the test partition. This procedure minimizes the effect of reflected sound in the receiving room and gives a value of k which is a function only of the transmitting panel.

In 1925, Dr. Edgar Buckingham[1] published a valuable critical paper on the interpretation of sound-transmission measurements, giving a mathematical treatment of the effect of reflection of sound in the receiving room upon the measured intensity. Following his analysis,

Let J_i = energy incident per second per unit of surface of test wall.

J_t = energy per second per unit surface entering receiving room

τ = coefficient of transmission = J_t/J_i

S = area of test wall

I_1 = average sound density in transmitting room

I_2 = average sound density in receiving room

V_1, a_1 = volume and absorbing power of transmitting room

V_2, a_2 = volume and absorbing power of receiving room

Buckingham showed that for a diffuse distribution J_i the incident energy flux is given by the relation

$$J_i = \frac{cI_1}{4}$$

hence,

$$J_t = \tau J_i = \frac{\tau c I_1}{4}$$

and the total energy per second E_2 entering through the entire surface is

$$E_2 = SJ_t = \frac{\tau c I_1 S}{4} \tag{74}$$

[1] *Bur. Standards Sci. Paper* 506.

In the steady state, the wall acts for the receiving room as a sound source whose acoustic output is E_2, and the average steady-state sound density I_2 is given by the equation

$$I_2 = \frac{4E_2}{a_2c} = \frac{4}{a_2c} \cdot \frac{\tau c I_1 S}{4} = \frac{\tau I_1 S}{a_2}$$

whence

$$\frac{1}{\tau} = \frac{I_1 S}{I_2 a_2} \qquad (75)$$

The coefficient of transmission τ is a quantity which pertains alone to the transmitting wall and is quite independent of the acoustic properties of the rooms which it separates. The intensity reductions produced by partitions separating the same two rooms will be directly proportional to the reciprocals of their transmission coefficients, so that $1/\tau$ may be taken as a measure of the sound-insulating merits of a given partition. Knudsen[1] has proposed that the term "transmission loss in decibels" be used to express the sound-insulating properties of partitions and that this be defined by the relation

$$T.\,L.\ \text{(transmission loss)} = 10 \log_{10} \frac{1}{\tau} \qquad (76)$$

This would seem to be a logical procedure. Thus defined and measured, the numerical expression of the degree of sound insulation afforded by partitions does not depend upon conditions outside the partitions themselves. Moreover, this is expressed in units which usage in other branches of acoustics and telephony has made familiar. Illustrating its meaning, if experiment shows that the energy transmitted by a given wall is $1/1000$ of the incident energy $\tau = 0.001$, $\frac{1}{\tau} = 1{,}000$, $\log \frac{1}{\tau} = 3$, and the transmission loss is 30 db. The relation between "reduction factor" as measured and transmission loss as just defined remains to be considered.

[1] *Jour. Acous. Soc. Amer.*, vol. 11, No. 1, p. 129, July, 1930.

Transmission Loss and Reduction Factor.

We note in Eq. (75) that $1/\tau$ is equal to the ratio of the average intensity in the transmitting room to that in the receiving room, multiplied by the expression S/a_2. This equation applies to the steady-state intensity set up while the source is in operation. It would appear therefore that in any method based on direct measurement of the steady-state intensities in two rooms on opposite sides of a partition, the reduction factor, defined as the ratio of the measured value of I_1/I_2, multiplied by S/a_2 gives the value of $1/\tau$. Hence

$$T.L. = 10 \log \left(k \cdot \frac{S}{a_2}\right) = 10\left[\log k + \log \frac{S}{a_2}\right] \qquad (77)$$

Obviously, if the area of the test panel is numerically equal to the total absorbing power of the receiving room, then

$$k = \frac{1}{\tau}$$

In the case of the Riverbank measurements, we have not steady-state intensities but instantaneous relative values of decreasing intensities. This case calls for further analytical consideration, which Buckingham gives. From his analysis, and assuming that the coefficient of transmission is small, so that the amount of energy transmitted from the sound chamber to the test chamber and then back again is too small to have any effect on the intensity in the sound chamber, and that the reverberation time in the test chamber is very small compared with that of the sound chamber, we have

$$\log_{10} \frac{1}{\tau} = \log_{10} k + \log_{10} \frac{S}{a_2} - \log_{10} \left(1 - \frac{a_1 V_2}{a_2 V_1}\right) \qquad (78)$$

Here $\log_{10} k$ is taken as $\frac{a_1 c}{9.2 V_1}(t_1 - t_2)$, the logarithm of the reduction factor of the Riverbank tests.

If the sound chamber is a large room with small absorbing power, and the test chamber is a small room with a relatively large absorbing power, the expression a_1V_2/a_2V_1 is numerically small, and the third term of the right-hand member of Eq. (78) becomes negligibly small. We then have for the transmission loss

$$T.\ L. = 10 \log_{10}\left(\frac{1}{\tau}\right) = 10\left[\log_{10} k + \log_{10} \frac{S}{a_2}\right].$$

The relation between reduction factor and transmission loss is sensibly the same when measured by the reverberation method as when measured by the steady-intensity method.

Using the values of S and a_2 for the Riverbank test chambers, the corrections that must be *added* to 10 log k to give transmission loss in decibels are as follows:

Frequency	Correction
128	2 5
256	1.2
512	0.5
1,024	0.5
2,048	1 3
Average	1 0

This correction is scarcely more than the experimental errors in transmission measurements.

Total Sound Insulation.

It is sometimes desirable to know the over-all reduction of sound between two rooms separated by a dividing structure composed of elements which have different coefficients of transmission.[1]

Suppose that a room is located where the average intensity outside is I_1 and that s_1, s_2, s_3, etc., sq. ft. of the intervening wall have coefficients of τ_1, τ_2, τ_3, etc., respectively.

Let the average intensity of the incident sound be I_1. Then the rate at which sound strikes the bounding wall is $cI_1/4$ per square unit of surface. Calling E_2 the rate at which transmitted sound enters the room, we have

[1] The theoretical treatment that follows is due to Knudsen.

$$E_2 = \frac{cI_1}{4}[s_1\tau_1 + s_2\tau_2 + s_3\tau_3 + \cdots]$$

Then I_2, the intensity inside the room, will be

$$I_2 = \frac{4E_2}{a_2c} = \frac{I_1}{a_2}[s_1\tau_1 + s_2\tau_2 + s_3\tau_3 + \cdots] = \frac{I_1T}{a_2}$$

where T is the total transmittance of the boundaries, and

$$\frac{I_1}{I_2} = \frac{a_2}{T}$$

The reduction of sound level in decibels is

$$10 \log \frac{I_1}{I_2} = 10 \log \frac{a_2}{T} \tag{79}$$

Illustrating by a particular example: Suppose that a hotel room with a total absorbing power of 100 units is separated from an adjacent room by a 2½-in. solid-plaster partition whose area is 150 sq. ft., with a communicating door having an area of 21 sq. ft. The transmission loss through such a wall is about 40 db., and for a solid-oak door 1¾ in. thick, about 25 db.

For wall:

$$40 = 10 \log \frac{1}{\tau},\ \log \frac{1}{\tau} = 4, \text{ and } \tau = 0.0001$$

For door:

$$25 = 10 \log \frac{1}{\tau},\ \log \frac{1}{\tau} = 2.5, \text{ and } \tau = 0.0032$$

For wall and door, total transmittance is

$$T = (150 \times 0.0001) + (21 \times 0.0032) = 0.015 + 0.0672 = 0.082$$

Reduction in decibels between rooms . . . . . . $10 \log \frac{100}{0.082} = 30.9$

If there were no door, the reduction would be 36.8 db. If we substitute for the heavy oak door a light paneled door of veneer, with a transmission loss of 22 db., the reduction becomes 28.2, while with the door open we calculate a reduction of only 6.8 db.

The illustration shows in a striking manner the effect on the over-all reduction of sound between two rooms of introducing even a relatively small area of a structure having a high coefficient of transmission. We may carry it a step further and compute the reduction if we substitute an 8-in. brick wall with a transmission loss of 52 db. for the 2½-in. plaster wall. The comparison of the sound reductions afforded by two walls, one of 8-in. brick and the other of 2½-in. plaster, is given in the following table:

Wall	T.L.	Reduction, in decibels			
		No opening	1¾-in. door T.L. 25	Light veneer door T.L. 22	Opening T.L. = 0
8-in. brick..........	52	47	31.7	28.7	6.8
2½-in. plaster.......	40	36.8	30.9	28.2	6.8

This comparison brings out the fact that any job of sound insulation is little better than the least efficient element in it and explains why attempts at sound insulation so frequently give disappointing results. Thus in the above illustration, we see that to construct a highly sound-insulating partition between two rooms with a connecting doorway is of little use unless we are prepared to close this opening with an efficient door. The marked effect of even very small openings in reducing the sound insulation between two rooms is also explained.

Equation (79) shows that the total absorbing power in the receiving room plays a part in determining the relative intensities of sound in the two rooms. With a given wall separating a room from a given source of sound the general level of sound due to transmission can be reduced by increasing the absorbing power of the room, just as with sound from a source within the room the intensity produced from outside sources is inversely proportional to the total absorbing power. The general procedure then to secure the minimum of noise within an enclosure, from both inside and outside sources, is to use walls giving a high transmission loss or low coefficients of transmission and inner surfaces having high coefficients of absorption.

CHAPTER XII

TRANSMISSION OF SOUND BY WALLS

Owing to the rather wide diversity not only in the methods of test but in the experimental conditions and the choice of test frequencies, it is scarcely possible to present a coordinated account of all the experimental work that has been done in various laboratories on the subject of sound transmission by walls. We shall therefore, in the present chapter, confine ourselves largely to the results of the twelve years' study of the problem carried on at the Riverbank Laboratories, in which a single method has been employed throughout, and where all test conditions have been maintained constant. The study of the problem has been conducted with a threefold purpose in mind: (1) to determine the various physical properties of partitions that affect the transmission of sound and the relative importance of these properties; (2) to make quantitative determination of the degree of acoustical insulation afforded by ordinary wall constructions; and (3) to discover if possible practicable means of increasing acoustic insulation in buildings.

Statement of Results.

Reference has already been made to the fact that due to resonance, the reduction of sound by a given wall may vary markedly with slight variations in pitch. For this reason, tests at a single frequency or at a small number of frequencies distributed throughout the frequency range may be misleading, and difficult to duplicate under slightly altered conditions of test, such, for example, as variations in the size of the test panel. It appears, however, that for most of the constructions studied there is a general similarity in the shape of the frequency-reduction curve, namely, a general increase in the reduction with increasing frequency, so that the average value of the logarithm of the reduction will serve as a quantitative expression of the sound-insulating properties of walls. For any frequency, we shall call ten times the logarithm of the ratio of the intensities on the two sides of a given partition under the conditions described in Chap. XI the "reduction in decibels" produced by the partition or, simply, the "reduction." The "average reduction" is the average of these single-frequency reductions over the six-octave range from 128 to 4,096 vibs./sec., with twice as many test tones in the range from 128 to 1,024 as in the two upper octaves. This average reduction can be expressed as average "transmission loss" as defined by Knudsen by simply adding 1 db.

Doors and Windows.

A door or window may be considered as a single structural unit, through which the transmission of acoustic energy takes place by means of the minute vibrations set up in the structure by the alternating pressure of the incident sound. Therefore the gross mechanical properties of mass, stiffness, and internal friction or damping of these constructions determine the reduction of sound intensity which they afford. Of these three factors it appears that the mass per unit area of the structure considered as a whole is the most important. As a general rule, the heavier types of doors and windows show greater sound-insulating properties.

Table XVII is typical of the more significant of the results obtained on a large number of different types of doors and windows that have been tested. These tests were made on units 3 by 7 ft., sealed tightly, except as otherwise noted, into an opening between the sound chamber and one of the test chambers. Numbers 2, 6, and 7 show the relative ineffectiveness of filling a hollow door with a light, sound-absorbing material. Comparison of Nos. 4 and 5 indicates the order of magnitude of the effect of the usual clearance necessary in hanging a door. Inspection of the figures for the windows suggests that the cross bracing of the sash effects a slight increase in insulation over that of larger unbraced areas of glass. On the whole, it may be said that ordinary door and window constructions cannot be expected to show transmission losses greater than about 30 db.

Sound-proof Doors.

Various types of nominally "sound-proof" doors are now on the market. These are usually of heavy construction, with ingenious devices for closing the clearance cracks.

Table XVII*a* presents the results of measurements made on a number of doors of this type. It is interesting to note the increase in the sound reduction with increasing weight, and this regardless of whether the increased weight is due to the addition of lead or steel sheets incorporated within the door or simply by building a door of heavier construction. The significance of this will be considered in a later section.

TABLE XVII.—SOUND REDUCTION BY DOORS AND WINDOWS

Number	Description	Average reduction, decibels
1	¼-in. steel door	34 7
2	Refrigerator door, 5½-in. yellow pine filled with cork	29.4
3	Solid oak, 1¾ in. thick	25.0
4	Hollow flush door, 1¾ in. thick	26.8
5	No. 4, as normally hung	24.1
6	No. 4, with 2 layers of ½-in. Celotex in hollow space	26.8
7	No. 4 with 1-in. balsam wool in hollow space	26.6
8	Light-veneer paneled door	21.8
9	Two-veneer paneled doors, with 2-in. separation, normally hung in single casement	30.0
10	Window single pane 79 × 30 in. ¼-in. plate glass	26.2
11	Window, 4 panes each 15 × 39 in. ¼-in. plate glass	29.2
12	Window 2 panes each 31 × 39 in. 3/16-in. plate glass	22.8
13	Same as 12, but double glazed, glass set in putty both sides, 1-in. separation	26.6
14	Same as 13, but with glass set in felt	28.9
15	Diamond-shape leaded panes, 3/16-in. glass	28.4
16	Window, 12 panes 10 × 19 in., ⅛-in. glass	24.7

TABLE XVIIa.—REDUCTION BY SOUND-PROOF DOORS*

Material	Thickness, inches	Weight per square foot, pounds	Average reduction, decibels
Wood	2⅝	6.85	30.2
Wood	2⅞	7.00	30.7
Wood	3	7.65	31.5
Wood, steel sheathed	3	9.6	33.0
Wood, steel sheets, inclosed	3	11.6	35.6
Wood, lead sheets, inclosed	3	15.3	37.3

* These data are published with the kind permission of Mr Irving Hamlin of Evanston, Illinois, and of the Compound and Pyrono Door Company of St. Joseph, Michigan, for whom these tests were made.

It is of interest to compute the reduction in the example given at the end of Chap. XI. When a door giving a reduction of 35 db. is placed in the brick wall, in place of the oak door with a reduction of 25 db., we find, upon calculation, that with the 35-db. door the over-all reduction is 41 db.—only 6 db. lower than for the solid wall. With the 25-db. door the over-all reduction is 31.7 db. The comparison is instructive, showing that in any job of sound insulation, improvement is best gained by improvement in the least insulating element.

Porous Materials.

In contrast to impervious septa of glass, wood, and steel, porous materials allow the direct transmission of the pressure changes in the sound waves by way of the pore channels. We should thus expect the sound-transmitting properties of porous materials to differ from those of solid impervious materials. Experiment shows this to be the case. With porous materials, the reduction in transmission increases uniformly with the frequency of the sound. Further, for a given frequency the reduction in decibels increases linearly with the thickness of the material.[1]

[1] For a full account of the study of the transmission of sound by porous materials, see *Amer. Architect*, Sept. 28, Oct. 12, 1921.

Experiments on six different materials showed that, in general, the reduction of sound by porous materials can be expressed by an equation of the form

$$R = 10 \log k = 10 \log (r + qt) \tag{80}$$

where r and q are empirical constants for a given material, and t is the thickness.

Both r and q are functions of the frequency of the sound, which with few exceptions in the materials tested increase as the frequency is raised. In general, the denser the material of the character here considered the greater the value of q. From the practical point of view, the most interesting result of these tests is the fact that each additional unit of thickness gives the same increment in insulating value to a partition composed wholly of a porous material.

Davis and Littler,[1] working at the National Physical Laboratory at Teddington, England, by the method described in Chap. XI made similar tests on a somewhat heavier felt than that tested at the Riverbank Laboratories. For frequencies above 500 vibs./sec. they found a linear relation between the reduction and the number of layers of felt. For frequencies between 250 and 500 vibs./sec. their results indicate that the increase in reduction with increasing thickness is slightly less than is given by Eq. (80). The difference in method and the slightly different definitions of the term "reduction" are such as to explain this difference in results.

Equation (80) suggests that the reduction in transmission by porous materials results from the absorption of acoustic energy in its passage through the porous layers. Each layer absorbs a constant fraction and transmits a constant fraction of the energy which comes to it. Let I_1 be the average intensity throughout the sound chamber, and I_1/r be the intensity in the sound chamber, directly in front of the panel. Let $1/q$ be the fraction of the energy

[1] *Phil. Mag.*, vol. 3, p. 177, 1927.

transmitted by a unit thickness of the material; then $(1/q)^t$ is the fraction transmitted by a thickness t. Hence

$$I_2, \text{ the intensity on the farther side, will be } \frac{I_1}{rq^t},$$

whence

$$\log \frac{I_1}{I_2} = \log (r + qt)$$

and

$$R = 10 \log (r + qt)$$

The altered character of the phenomenon when layers of impervious material are interposed is shown by reference to the curves of Fig. 97. The straight line gives reduction at 512 for thicknesses from 1 to 4 in. of standard hair felt. The upper curve gives the reduction of alternate layers of felt and heavy building paper. We note a marked increase in reduction. Experiments showed that this increase is considerably greater than the measured value of the reduction afforded by the paper alone. We note also that each additional unit of paper and felt does not produce an equal increment in the reduction. In similar experiments with thin sheet iron and felt, Professor Sabine obtained similar results. Commenting on this difference, he states: "The process [in the composite structure] must be regarded not as a sequence of independent steps or a progress of an independent action but as that of a structure which must be considered dynamically as a whole."

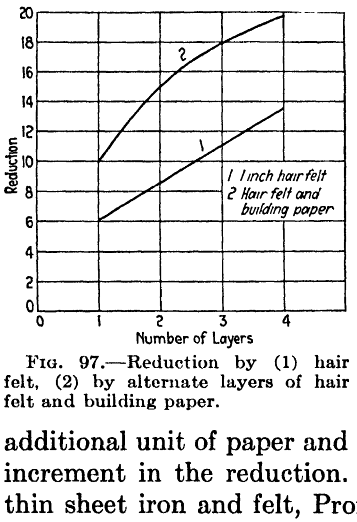


FIG. 97.—Reduction by (1) hair felt, (2) by alternate layers of hair felt and building paper.

Subsequent work disclosed the fact that the average reduction produced by masonry walls over the entire frequency range is almost wholly determined by the mass per unit area of the wall. This suggests the possibility that this may also hold true for the composite structures just considered. Accordingly, in Fig. 98 we have plotted the reduction against the logarithm of the number of layers for the paper and felt, for the sheet iron and air space, and for the sheet iron, felt, and air space. (In Sabine's experiments, ½-in. felt was placed in a 1-in. space between the metal sheets.) We note, in each case, a linear relation between the reduction and the logarithm of the number of layers, which, in any one type of con-

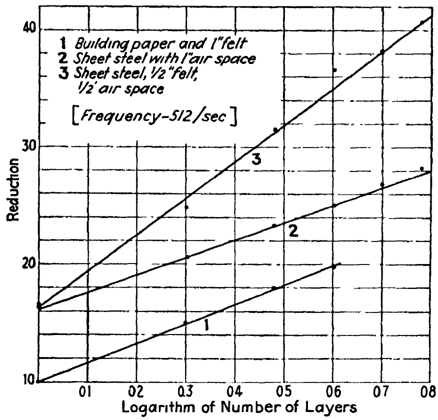


FIG. 98.—Intensity reduction in transmission by composite partitions of steel and felt.

struction, is proportional to the weight. As far as these experiments go, the results lead to the conclusion that the reduction afforded by a wall of this type wherein non-porous layers alternate with porous materials may be expressed by an equation of the form

$$k = \frac{I_1}{I_2} = aw^b$$

where w is the weight per unit area of the composite structure, and a and b are empirical constants whose values are functions of the frequency of the sound and also of the materials and arrangement of the elements of which the wall is built. The values of b for the single frequency 512 vibs./sec. for the three walls are as follows:

Paper and felt	1.67
Steel with air space	1.47
Steel, felt, air space	3.1

We shall have occasion to refer to a similar law when we come to consider the reduction given by masonry walls. It is to be borne in mind that under the conditions of the experiment, we are not dealing with structurally isolated units. The clamping at the edges necessarily causes the entire structure to act as a unit.

The figures given in Table XVIII show the average reductions given by porous materials. As will subsequently appear, one cannot draw conclusions from the results of tests conducted on porous materials alone as to how these will behave when incorporated into an otherwise rigid construction. In such cases, sound reduction is dependent upon the mechanical properties of the structure as a whole, rather than upon the insulating properties of its components.

TABLE XVIII.—REDUCTION OF SOUND BY POROUS PARTITIONS

Number	Material	Average reduction, decibels
17	Hair felt 1 in. thick	7.1
18	Hair felt 2 in. thick	10 5
19	Hair felt 3 in. thick	13.4
20	Hair felt 4 in. thick	16.7
21	3 layers 1-in. felt, alternated with 4 layers building paper	31.7
22	4 layers ½-in. Cabot quilt	22 0
23	4 layers ½-in. Flaxlinum	29.6

Continuous Masonry.

By "continuous masonry" we shall mean single walls, as contrasted with double walls, of clay or gypsum tile, either hollow or solid, of solid plaster laid on metal lath and channels, and of brick or concrete. These include most of the common types of all-masonry partitions. Figure 99 shows the general similarity of the curves obtained by plotting the reduction as a function of the frequency. This similarity in shape justifies our taking the average reduction as a measure of the relative sound-insulating merits of walls in general. The graph for the 4 in. of hair felt is instructive, as showing the much smaller sound-insulating value of a porous material and the essentially different character of the phenomena involved. In Table XIX, the average reduction for 16 partitions of various masonry materials is given, together with the weight per square foot of each finished construction.

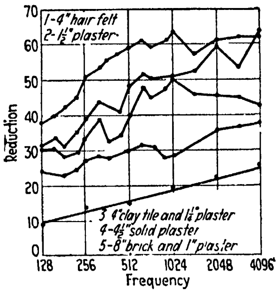


FIG. 99.—Intensity reduction as a function of frequency for homogeneous partitions.

In the column headed "Relative stiffness" are given the steady pressure in pounds per square foot over the entire face of the walls that produce a yielding of 0.01 in. at the middle point of the wall. In each case, the test partition was 6 by 8 ft. built solidly into the opening. Gypsum plaster was used throughout this series of tests.

TABLE XIX.—CONTINUOUS MASONRY WALLS

Number	Construction	Weight per square foot, pounds	Relative stiffness	Average reduction	Log weight
24	2-in. gypsum tile, unplastered.	10.4	.....	25 9	1 02
25	3-in. hollow gypsum tile, unplastered..	11.1	....	27.2	1.05
26	1½-in. plaster on metal lath	13.9	5.8	29 6	1.14
27	3-in. solid gypsum tile, unplastered ...	14.2	.....	31 4	1 15
28	2-in. solid gypsum tile, ½-in. plaster ..	15.0	.....	31 0	1.18
29	4-in. clay tile, unplastered....	17.0	.....	33 2	1 23
30	2-in. solid gypsum tile, ⅞-in. plaster ..	19.6	.....	34 2	1 29
31	2-in. solid gypsum tile, 1¼-in. plaster	21.4	122.0	35 2	1 33
32	4-in. clay tile, ½-in. plaster......... ..	22.0	.. ..	36 3	1.36
33	2½-in. plaster on metal lath..........	23.2	24 6	37 9	1.38
34	3-in. solid gypsum tile, 1¼-in. plaster..	25.4	187.0	39 0	1.41
35	4-in. clay tile, 1-in. plaster...........	27.0	.....	39 8	1 43
36	4-in. clay tile, 1¼-in. plaster..........	28.6	173.0	40 5	1.46
37	3½-in. plaster on metal lath..........	32.5	111.0	41.0	1 51
38	4½-in. plaster on metal lath...	41.8	.....	45 4	1 62
39	8-in. brick, 1-in. plaster.	88.0	....	53.8	1 95

One notes in the table a close correspondence between the weight and the reduction. Figure 100, in which the average reduction is plotted against the logarithm of the weight per square foot of continuous masonry partitions, brings out this correspondence in a striking manner. The

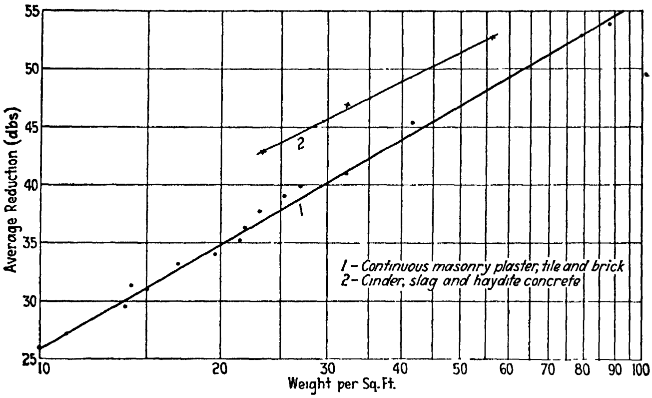


FIG. 100.—Riverbank measurements on (1) plaster, tile, and brick; (2) cinder, slag, and haydite concrete.

equation of the straight line, along which the experimental points lie, is

$$R = 10 \log k = 10(3 \log w - 0.4)$$

This may be thrown into the form

$$k = 0.4w^3 \tag{81}$$

It should be pointed out and clearly borne in mind that Eq. (81) is a purely empirical equation formulating the results of the Riverbank tests on masonry walls. The values of the two constants involved will depend upon the range and distribution of the test tones employed. Thus it is clear from Fig. 99 that if relatively greater weight in averaging were given to the higher frequencies, the average reductions would be greater. Equation (81) does, however, give us a means of estimating the reduction to be expected from any masonry wall of the materials specified in Table XIX and a basis of comparison as to the insulating merits of other constructions and other materials. For example, other things being equal, a special construction weighing 20 lb. per square foot giving an average reduction of 40 db. would have the practical advantage of smaller building load over a 4-in. clay tile partition giving the same reduction but weighing 27 lb. per square foot. Further, having determined the reduction for a particular type of construction, Eq. (81) enables us to state its sound-insulating equivalent in inches of any particular type of solid masonry —brick, let us say. Thus a staggered-wood stud and metal-lath partition weighing 20 lb. per square foot showed a reduction of 44 db., which by Eq. (81) is the same as that for a continuous masonry partition weighing 41 lb. per square foot. Now a brick wall weighs about 120 lb. per cubic foot, so that the staggered-stud wall is the equivalent of 4.1 in. of brick. The use of the staggered stud would thus effect a reduction of 21 lb. per square foot in building load over the brick wall. On the other hand, the over-all thickness of the staggered-stud wall is 7½ in., so that the advantage in decreased weight is paid for in loss of available floor space due to increased thickness of partitions. Obviously, the sound-insulating properties of partition-wall constructions should be considered in connection with other structural advantages or disadvantages of these constructions. The data of Table XIX and the empirical formula derived therefrom make quantitative evaluation of sound insulation possible.

Relative Effects of Stiffness and Mass.

Noting the values of the relative stiffness of the walls listed in Table XIX, we see no apparent correspondence between these values and the sound reductions. This does not necessarily imply that the stiffness plays no part in determining the sound transmitted by walls. At any given frequency, the transmitted sound does undoubtedly depend upon both mass and stiffness. The data presented only show that under the conditions and subject to the limitations of these tests the mass per unit area plays the predominating rôle in determining the average reduction over the entire range of frequencies. Dynamically considered, the problem of the transmission of sound by an impervious septum is that of the forced vibration of a thick plate clamped at its edges. The alternating pressure of the incident sound supplies the driving force, and the response of the partition for a given value of the pressure amplitude at a given frequency will depend upon the mass, stiffness (elastic restoring force), the damping due to internal friction, and also the dimensions of the partition. In the tests considered, the three factors of mass, stiffness, and damping vary from wall to wall, and it is not possible to segregate the effect due to any one of them. The ratio of stiffness to mass determines the series of natural frequencies of the wall, and the transmission for any given frequency will depend upon its proximity to one of these natural frequencies; hence the irregularities to be noted in the graphs of Fig. 99.

It is easy to show mathematically that if we have a partition of a given mass free from elastic restraint and from internal friction, driven by a given alternating pressure uniformly distributed over its face, the energy of vibration of the partition would vary inversely as the square of the frequency and that the ratio of the intensity of the incident sound to the intensity of the transmitted sound would vary directly as the square of the frequency. At any given frequency, the ratio of the intensities would be proportional to the square of the mass per unit area. If we were dealing with the ideal case of a massive wall free from elastic and frictional restraints, the graphs of Fig. 99 would be parallel straight lines, and the exponent of w, in Eq. (81), would be 2 instead of 3. For such an ideal case, the increase with frequency in the reduction by a wall of given weight would be uniformly 6 db. per octave.

The departure of the measured results from the theoretical results, based on the assumption that both the elasticity and the internal friction are negligible in comparison with the effects of inertia, indicates that a complete theoretical solution of the problem of sound transmission must take account of these two other factors. With a homogeneous wall, both the stiffness and the frictional damping will vary as the thickness and, consequently, the weight per unit area are varied. The empirical equation (81) simply expresses the over-all effect of variation in all three factors in the case of continuous masonry walls. There is no obvious theoretical reason for expecting that it would hold in the case of materials such, for example, as glass, wood, or steel, in which the elasticity and damping for a given weight are materially different from those of masonry.

Exceptions to Weight Law for Masonry.

As bearing on this point we may cite the results for walls built from concrete blocks in which relatively light aggregates were employed. Reference is made to the line 2, in Fig. 100. The description of the walls for the three points there shown, taken in the order of their weights, is as follows:

Material	Total thickness, inches	Weight per square foot, pounds	Reduction, decibels
4-in. haydite block, 1-in. plaster	5	23.2	43 0
4-in. cinder concrete block, 1-in. plaster	5	32 3	47.0
8-in. blast-furnace slag blocks, 1-in. plaster.	9	56 0	52.6

For this series we note a fairly linear variation of the reduction with logarithm of the weight per square foot. But the reduction is considerably greater than for tile, plaster, and brick walls of equal weight. The three walls here considered are similar in the fact that the blocks are all made of a coarse angular aggregate bonded with Portland cement. The fact that in insulating value they depart radically from what would be expected from the tests on tile, plaster, and brick serves to emphasize the point already made, that there is not a single numerical relation between weight and sound reduction that will hold for all materials, regardless of their other mechanical properties.

Bureau of Standards Results.

The fact of the predominating part played by the weight per unit area in the reduction of sound produced by walls of masonry material was first stated by the writer in 1923.[1] No generalization beyond the actual facts of experiment was made. Since that time, work in other laboratories has led to the same general conclusion in the case of other

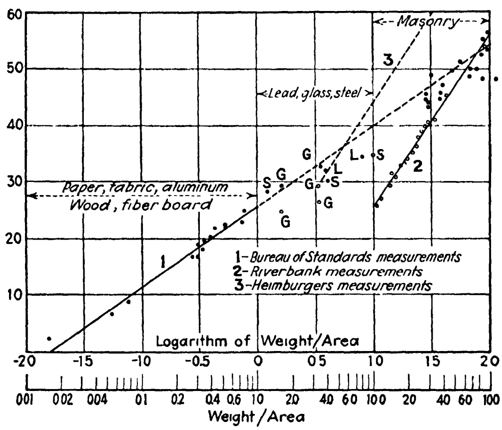


FIG. 101.—Reduction as a function of weight per unit area. Results from different laboratories compared.

homogeneous materials. The straight line 1 (Fig. 101) shows the results of measurements made at the Bureau of Standards[2] on partitions ranging from a single sheet of wrapping paper weighing 0.016 to walls of brick weighing more than 100 lb. per square foot. For comparison, the Riverbank results on masonry varying from 10 to 88 lb.

[1] *Amer. Architect*, July 4, 1923.

[2] CHRISLER, V. L., and W. F. SNYDER, *Bur. Standards Res. Paper* 48, March, 1929.

per square foot and also results for glass, wood, and steel partitions are shown.

We note that from the Bureau of Standards tests the points for the extremely light materials of paper, fabric, aluminum, and fiber board show a linear relation between the average reduction and the logarithm of the weight per unit area. The heavier, stiffer partitions of lead, glass, and steel as well as the massive masonry constructions tested by the Bureau show, in general, lower reductions than are called for by the extrapolation of the straight line for the very light materials.

Taken by themselves, the Bureau of Standards figures for heavy masonry constructions, ranging in weight from 30 to 100 lb. per square foot, do not show any very definite correlation between sound reduction and weight. On the whole, the points fall closer to the Riverbank line for continuous masonry than to the extension of the Bureau of Standards line for light materials.

Figure 101 gives also the results on homogeneous structures ranging in weight from 10 to 50 lb. per square foot, as reported by Heimburger.[1] As was indicated in Chap. XI, in Kreuger's tests, a loud-speaker horn was placed close to the panel and surrounded by a box, and the intensities with and without the test panels intervening were measured. This method will give much higher values of the reduction than would be obtained if the whole face of the panel were exposed to the action of the sound. It is worth noting, however, that the slope of the line representing Kreuger's data is very nearly the same as that for the Riverbank measurements.

Here, again, the data presented in Fig. 101 lead one to doubt whether there is a single numerical formula connecting the weight and the sound reduction for homogeneous structures that will cover all sorts of materials. Equation (81) gives an approximate statement of the relation between weight and reduction for all-masonry constructions. The Bureau of Standards findings on light septa

[1] *Amer. Architect*, vol. 133, pp. 125–128, Jan. 20, 1928

(less than 1 lb. per square foot) can be expressed by a similar equation but with different constants, namely:

$$k = 360w^{1.43} \qquad (82)$$

It should be borne in mind that both Eqs. (81) and (82) are simply approximate generalizations of the results of experiment and apply to particular test conditions.

The data obtained for the sound-proof doors bear interestingly upon this point. Plotting the sound reduction against the weight per square foot, we find again a very close approximation to a straight line. From the equation of this line one gets the relation

$$k = 77.5w^{2.1}$$

as the empirical relation between weight per square foot and sound reduction for these constructions. Here the exponent 2.1 is close to the theoretical value 2.0, derived on the assumption that the mass per unit area is the only variable. In this series of experiments, this condition was very nearly met. The thickness varied only slightly, 2⅝ to 3 in., and the metal was incorporated in the heavier doors in such a way as not materially to increase their structural stiffness, while adding to their weight.

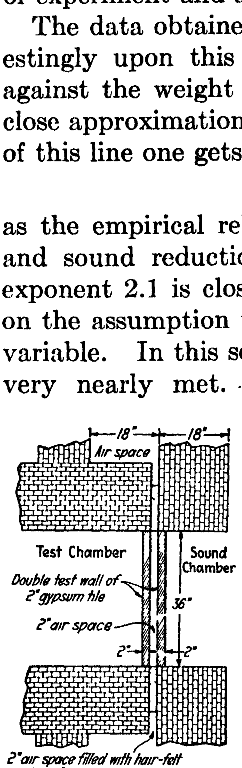


FIG. 102.—Double partitions completely separated.

Lacking any general theoretical formulation, the graphs of Fig. 101 furnish practical information on the degree of sound insulation by homogeneous structures covering a wide variation in physical properties.

Double Walls, Completely Separated.

In practice, it is seldom possible to build two walls entirely separated. They will of necessity be tied together at the edges. The construction of the Riverbank sound chamber and the test chambers is such, however, as to allow two walls to be run up with no structural connection whatsoever (Fig. 102). This arrangement makes it possible to study the ideal case of complete structural separation and also the effect of various degrees of bridging or tying as well as that produced by various kinds of lagging fill between the walls. Figure 103 gives the detailed results of tests on a single wall of 2-in. solid gypsum tile and of two such walls completely separated, with intervening air spaces of 2 in. and 4 in., respectively. One notes that the increased separation increases the insulation for tones up to 1,600 vibs./sec. At higher frequencies, the 2-in.

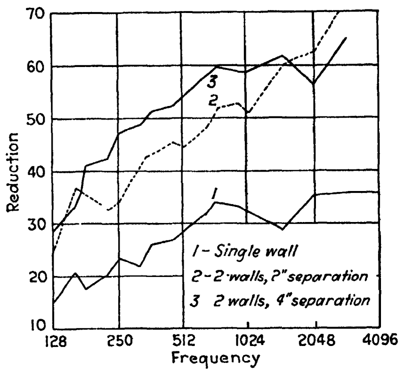


FIG. 103.—Effect of width of air space between structurally isolated partitions.

separation is better. Another series of experiments showed that still further increase in the separation shifts the dip in the curve at 2,048 vibs./sec. to a lower frequency. This fact finds its explanation in the phenomenon of resonance of the enclosed air, so that there is obviously a limit to the increased insulation to be secured by increasing the separation.

Figure 104 shows the effect (*a*) of bridging the air gap with a wood strip running lengthwise in the air space and in contact with both walls and (*b*) of filling the inter-wall space with sawdust. One notes that the unbridged, unfilled space gives the greatest sound reduction and, further, that any damping effect of the fill is more than offset by its bridging effect. Experiments with felt and granulated blast-furnace slag showed the same effect, so that one arrives at the conclusion that *if complete structural separation were possible*, an unfilled air space would be the most effective means of securing sound insulation by means of double partitions. As will appear in a later section,

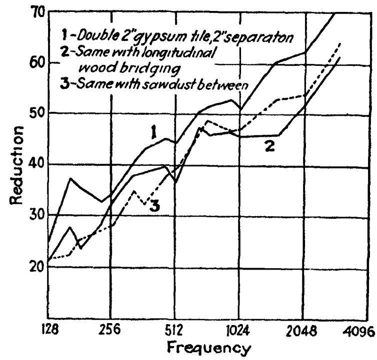


FIG. 104.—Effect of bridging and filling the air space between structurally isolated partitions.

this conclusion does not include cases in which there is a considerable degree of structural tying between the two members of the double construction.

Table XX gives the summarized results of the tests on double walls with complete structural isolation.

TABLE XX.—DOUBLE WALLS COMPLETELY SEPARATED

Number	Specifications	Weight per square foot, pounds	Reduction, decibels	Equivalent masonry, inches
40	Double 2-in. solid-gypsum tile, unplastered, unbridged, 2-in. separation....	20 4	56 0	10 0
41	The same, bridged at middle.. ..	20.4	48 0	5 4
42	The same, filled with sawdust.	23.0	47.7	5.4
43	The same, filled with slag. . .	30 9	48.6	5 7
44	The same, filled with felt..	22.3	55.1	9 3
45	The same as No. 40 but with 4-in. separation..................	20.4	59.0	12.5
46	The same as No. 45 but bridged top and bottom............. .	20.4	53 3	8 3
47	The same as No. 45 but with inner faces lined with 1-in. felt... .	22 3	65 0	

Double Partitions, Partially Connected.

Under this head are included types of double walls in which the two members are tied to about the same degree as would be necessary in ordinary building practice. In this connection, data showing the effect of the width of the air space may be shown. This series of tests was conducted with two single-pane ¼-in. plate glass windows 82 by 34 in. set in one of the sound-chamber openings. Spacing frames of 1-in. poplar to which ½-in. saddler's felt was cemented were used to separate the two windows.

The separation between the windows was increased by increasing the number of spacing frames. It is evident that the experiments did not show the effect of increased air space alone, since a part of the transfer of sound energy is by way of the connection at the edges. However, the results presented in Table XXI show that the spatial separation between double walls does produce a very appreciable effect in increasing sound insulation.

TABLE XXI.—DOUBLE WINDOWS, ¼-IN. PLATE GLASS

Number	Description	Reduction, decibels	Equivalent masonry, inches
48	Sashes in contact	33.2	1.7
49	1½-in. separation	38.6	2.5
50	4½-in. separation	40.1	2.9
51	7½-in. separation	44.2	4.0
52	9½-in. separation	46.3	4.7
53	13½-in. separation	48.2	5.5
54	16-in. separation	48.8	5.8

Experiments in which a solid wood spacer replaced the alternate layers of felt and wood showed practically the same reduction, as shown by the alternate wood and felt, indicating that the increasing insulation with increasing separation is to be ascribed largely to the lower transmission across the wider air space rather than to improved insulation at the edges.

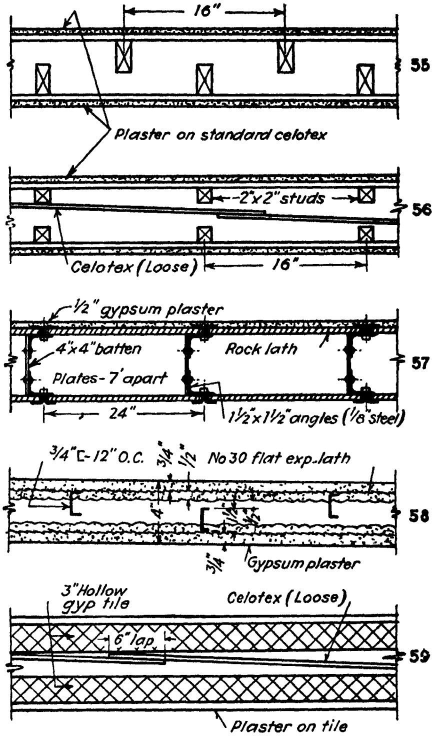


FIG. 105.—Double walls with normal amount of bridging.

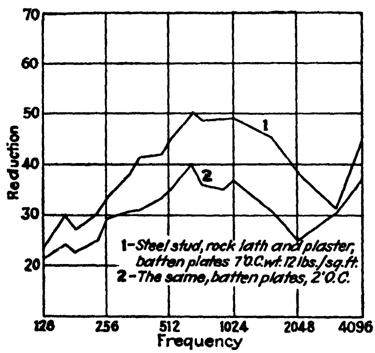


FIG. 106.—Double walls: (1) loosely tied; (2) closely tied.

Figure 105 shows a number of types of double wall construction that have been tested, with results shown in Table XXII.

TABLE XXII.—DOUBLE WALLS, CONNECTED AT THE EDGES

Number	Description	Weight per square foot, pounds	Reduction, decibels	Thickness, inches	Equivalent masonry, inches
55	Staggered 2 × 4-in wood studs:				
	a. Metal lath, ½-in. gypsum plaster	19.8	44.4	7½	4 0
	b. ½-in. Celotex, ½-in. gypsum plaster	13.0	41.6	7½	3.3
56	2 × 2 in. wood studs, set on 6-in. plate. ½-in. gypsum plaster on ½-in. Celotex ½-in. Celotex stood loosely between...	12.2	52.2	8	7 5
57	⅛-in steel studs, rock lath, ½-in. gypsum plaster:				
	a. Batten plates between angles 7 ft. O.C.	12.0	45.3	5¾	4.3
	b. Batten plates 2 ft. O.C	12 0	35.9	5¾	2.1
58	Double metal lath on ¾-in. channels, ¾-in. gypsum plaster:				
	a. Without cross-bracing clips	18 0	49.5	4	6.0
	b. With cross-bracing clips	18.0	39.7	4	2.9
59	Double 3-in. hollow gypsum tile:				
	a. Unplastered, 3-in air space	22 0	42 6	9	3.6
	b. Unplastered 2½-in. air space, ½-in felt	22 6	46 0	9	4.5
	c Same as *b*, with 1-in. plaster.	31 8	48.0	10	5.4
	d. Plastered, ½-in. Celotex in 2-in. air space	32 0	47 8	9	5 4

Figure 106 shows in a striking way the effect of the bridging by the batten plates tying together the two ⅛-in. steel angles forming the steel stud of No. 57. Table XXII indicates the limitation imposed by excessive thickness upon sound insulation by double wall construction. In only one case—that of the unclipped double metal-lath construction—is the double construction thinner than the equivalent masonry. The moral is that, generally speaking, with structural materials one has to pay for sound insulation either in increased thickness using double construction or by increased weight using single construction.

Wood-stud Partitions.

The standard wood-stud construction consists of 2 by 4-in. studs nailed bottom and top to 2 by 4-in. plate and header. One is interested to know the effect on sound insulation of the character of the plaster—whether lime or gypsum—the character of the plaster base—wood lath, metal lath, or fiber boards of various sorts—and finally the effect of filling of different kinds between the studs. Table XXIII gives some information on these points. The plaster was intended to be standard scratch and brown coats, ½- to ⅝-in. total thickness. The weight in each case was determined by weighing samples taken from the wall after the tests were completed. Figure 107 shows the effect of the sawdust fill in the Celotex wall both with and without plaster. The contrast with the earlier case,

TABLE XXIII.—WOOD-STUD WALLS

Number	Plaster material	Plaster base	Weight per square foot, pounds	Reduction, decibels	Equivalent masonry, inches
60	Gypsum	Metal lath	17 4	33 2	1 8
61	No. 60 filled with granulated slag	Metal lath	27.4	41.6	3 4
62	Gypsum	Wood lath	18.0	33.4	1.8
63	Lime	Wood lath	17.4	43 2	3.8
64	None	½-in. Celotex	3 0	26.2	1 0
65	No. 64 filled with sawdust	½-in. Celotex	6.6	34 0	1 8
66	Gypsum	½-in. Celotex	12 0	37 0	2 3
67	No 66 filled with sawdust	½-in. Celotex	15 6	39 8	2 9
68	Gypsum, ⅝ in. thick	½-in. masonite	16 0	43 0	3 8
69	Gypsum	½-in. felt, ⅞-in. furring metal lath	18 0	45 4	4.6

where there was no structural tie between the two members and in which the sawdust filling actually *decreased* the insulation, is instructive. In the wood-stud construction, the two faces are already completely bridged by the studs, so that the addition of the sawdust affords no added bridging effect. Its effect is therefore to add weight and possibly to produce a damping of the structure as a whole. This suggests that the answer to the question as to whether a lagging material will improve insulation depends upon the structural conditions under which the lagging is applied. In Fig. 107, it is interesting to note the general similarity in shape of the four curves and also the fact that the addition of the sawdust makes a greater improvement in the light unplastered wall than in the heavier partition after plastering.

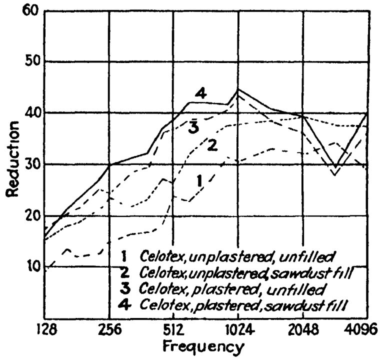


FIG. 107.—Effect of filling wood stud, Celotex, and plaster walls.

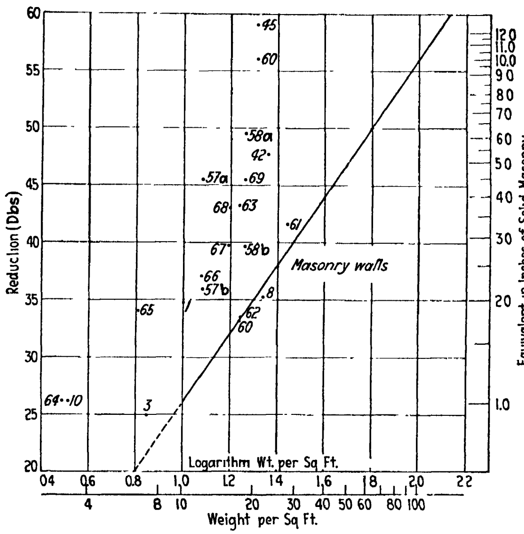


FIG. 108.—Various constructions compared with masonry walls of equal weight.

General Conclusions.

Figure 108 presents graphically what general conclusions seem to be warranted by the investigation so far. In the figure, the vertical scale gives the reduction in decibels; and the horizontal scale, the logarithm of the weight per square foot of the partitions. The numbered points correspond to various partitions described in the preceding text.

1. For continuous masonry of clay and gypsum tile, plaster, and brick the reductions will fall very close to the straight line plotted. Wood-stud construction with gypsum plaster falls on this line (No. 62). Lime plaster on wood studs and gypsum plaster on fiber-board plaster bases on wood studs give somewhat greater reductions than continuous masonry of equal weight (Nos. 63, 68, 66). Glass and steel show greater reductions than masonry of equal weights (Nos. 1 and 10). The superiority of lime over gypsum plaster seems to be confined to wood-stud constructions. The Bureau of Standards reports the results of tests in which lime and gypsum plasters were applied to identical masonry walls of clay tile, gypsum tile, and brick. In each case, two test panels were built as nearly alike as possible, one being finished with lime plaster. In each case, the panel finished with the gypsum plaster showed slightly greater reduction than a similar panel finished with lime plaster. The difference, however, was not sufficiently great to be of any practical importance.[1]

These facts bring out the fact referred to earlier, that the sound insulation afforded by partitions is a matter of structural properties, rather than of the properties of the materials comprising the structure.

2. The reduction afforded by double construction is a matter of the structural and spatial separation of the two units of the double construction (*cf.* Nos. 45, 40, 42, also 58*a* and 58*b*). In double constructions with only slight structural tying, lagging fills completely filling the air space are not advantageous. In hollow construction, where filling appreciably increases the weight of the structure, filling gives increased insulation (compare No. 64 with No. 65, and No. 66 with No. 67). In each case, the increased reduction due to the filling is about what would be expected

[1] See *Bur. Standards Sci. Papers* 526 and 552.

from the increase in weight. It is fairly easy to see that since the filling material is incorporated in the wall, it can have only slight damping effect upon the vibration of the structure as a whole. Following this line of reasoning, the increase in reduction due to filling should be proportional to the logarithm of the ratio of the weight of the filled to the unfilled wall. This relationship is approximately verified in the instances cited. However, the slag filling of the metal lath and plaster wall (Nos. 60 and 61) produces a somewhat greater reduction than can be accounted for by the increased weight, so that the character of the fill may be of some slight importance.

Meyer's Measurements on Simple Partitions.

Since the foregoing was written, an important paper by E. Meyer[1] on sound insulation by simple walls has come to hand.

This paper not only reports the results on sound transmission but also gives measurements of the elasticity and damping of 12 different simple partitions ranging in weight from 0.4 to 93 lb. per square foot. The frequency range covered was from 50 to 4,000 vibs./sec. Actual measurements of the amplitude of vibration of the walls under the action of sound waves were also made. For this purpose, a metal disk, attached to the wall, served as one plate of an air condenser. This condenser constituted a part of the capacity of a high-frequency vacuum-tube oscillator circuit. The vibration of the walls produced a periodic variation in the capacity of this wall plate-fixed plate condenser, a variation which impressed an audiofrequency modulation upon the high-frequency current of the oscillator. These modulated high-frequency currents were rectified as in the ordinary radio receiving set, and the audiofrequency voltage was amplified and measured by means of a vacuum-tube voltmeter. The readings of the latter were translated into amplitudes of wall vibrations

[1] Fundamental Measurements on Sound Insulation by Simple Partition Walls, *Sitzungber. Preuss. Akad. Wis., Phys.-Math. Klasse*, vol. 9, 1931.

by allowing the wall to remain stationary. The fixed plate of the measuring condenser was moved by means of a

TABLE XXIV.—MEYER'S DATA ON VIBRATION OF WALLS

Kind of wall	Thickness, inches	Weight per square foot, pounds	Natural frequency		Damping, db. per sec.	Elasticity modulus, kg./cm.^{2}	Transmission loss, decibels
			Calculated	Measured			
Wood plate	0.2	0.45	7.8	7.0	9.5	110,000	18 5
Sheet iron, stiffened in middle	0 08	3.2	10.5	12.0	5 4	18,600,000	34.0
Rabitzwand	1.0	6.3	16.0	17.0	11.1	42,000	33.0
Pressed straw, plastered	3.5	13 9	24 0	30.0	41.0	4,300	38 0
Schwemmsteinwand, plastered	4 75	25.0	32.0	38.0	54 0	5,600	39.0
Pumice concrete	4 3	26 7	31.0	28.0	16.0	8,100	41.0
Brick wall, plastered	3.5	31.2	27 5	29.0	87.0	13,000	42.0
	6 0	46 5	45 0	48.0	69.0	12,000	43 0
	10.5	93.0	75 0	51 0	48 0	11,000	49.0

micrometer screw. The tube voltmeter reading was thus calibrated in terms of wall movement. The author of the paper claims that amplitudes as small as 10^{-8} cm. can be measured in this way.

The same device was also used to measure the deflection of the walls under constant pressure. From these measurements the moduli of elasticity were computed. Finally, by substituting an oscillograph for the vacuum-tube oscillator, records of the actual vibration of the walls when struck by a hammer were made. From these oscillograms the lowest natural frequency and the damping were obtained.

In Table XXIV, data given by Meyer are tabulated. We note the low natural frequency of these walls, for the most part well below the lower limit of the frequency range of measurements. We note also their relatively high damping, with the exception of the steel and wood. This high damping would account for the fact that the changes in sound reduction with changes in frequency are no more abrupt than experiments prove them to be. It is further to be noted that, excepting the first three walls listed, the moduli of elasticity are not widely different for the different walls. The steel, however, has very much greater elasticity than any of the other walls, and it is to be noted that the steel shows a much greater transmission loss for its weight than do the other materials. This fact has already been observed in the Riverbank tests. We should expect this high elasticity of steel to show an even greater effect than it does were it not for the fact that the great elasticity is accompanied by a relatively low damping.

Meyer's work throws light on the "why" of the facts that the Riverbank and the Bureau of Standards researches have brought out. For example, the fact that the relation between mass and reduction is so definitely shown by the

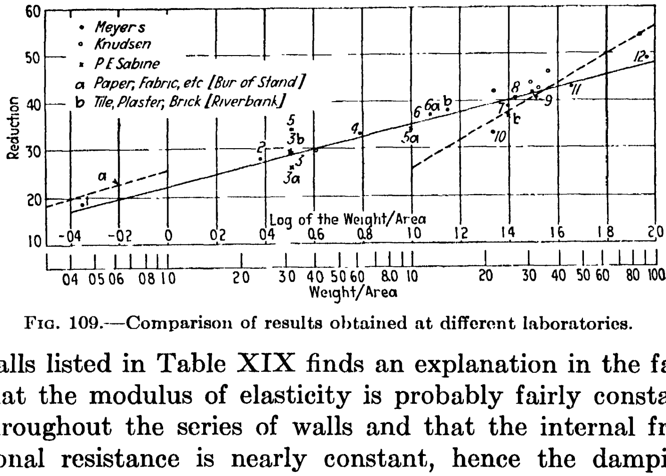


FIG. 109.—Comparison of results obtained at different laboratories.

walls listed in Table XIX finds an explanation in the fact that the modulus of elasticity is probably fairly constant throughout the series of walls and that the internal frictional resistance is nearly constant, hence the damping increases according to a definite law with increasing massiveness of the walls. These facts, together with the fact that the fundamental natural frequency is low in all cases, would leave the predominating rôle in determining the response to forced vibrations to be played by the mass alone.

Further, Meyer's work would seem to show that in the cases of wide variation in the damping and elasticity, this effect of mass alone may be masked by these other factors. Sheet iron is a case in point. The fact that sheet iron only

0.08 in. thick stiffened at the middle shows so high a reduction is quite in agreement with the Riverbank tests on windows, which showed that the cross bracing of the sash effected an appreciable increase in the insulating power.

In Table XXV, Meyer's results are given together with figures on comparable constructions as obtained at the Riverbank Laboratories. These values are plotted in Fig. 109. On the same graph, values given by Knudsen[1] are shown for a number of walls, the characters of which are not specified. For further comparison, the Bureau of Standards line for light septa of paper, fabric, aluminum, and wood as well as the Riverbank line for tile, plaster, and brick are shown.

TABLE XXV

Number	Wall	Weight per square foot, pounds	Transmission loss, decibels	Authority
1	Wood 0.2 in. thick	0.45	18.5	Meyer
2	Plate glass, 0.11 to 0.16 in. thick	2.4	28 0	Meyer
3	Plate glass, 0.27 to 0.31 in. thick	3 2	29 0	Meyer
3*a*	Plate glass, 0.25 in. thick	3.2	27.0	Sabine
3*b*	Plate glass, 0.25 in. thick (cross braced)	3 2	30.0	Sabine
4	*Rabitzwand* 1 in. thick	6 1	33 0	Meyer
5	Sheet iron stiffened	3.2	34.0	Meyer
5*a*	Sheet steel, no bracing	10.0	34.0	Sabine
6*a*	Celotex plastered	12.0	37.0	Sabine
6*b*	Pressed straw plastered	14.0	38 0	Meyer
7	*Schwemmsteinwand* plastered	25.0	39.0	Meyer
8	Pumice concrete, cork interior	27 0	41 0	Meyer
9	3½-in. wall plastered	31.2	42.0	Meyer
10	Brick wall unplastered	22.0	33 0	Meyer
11	6-in. brick wall plastered	46 0	43.0	Meyer
12	10½-in. brick wall plastered	93 0	49.0	Meyer

One notes very fair agreement between Meyer's figures and the Riverbank figures for glass (*cf.* Nos. 3, 3*a*, 3*b*). Moreover, the transmission loss shown by the plastered compressed straw measured by Meyer is quite comparable

[1] *Jour. Acous. Soc. Amer.*, vol. 2, No. 1, p. 133, July, 1930.

to the Riverbank figures for plastered Celotex board of nearly the same weight (*cf.* Nos. 6*a*, 6*b*). With the exception of the 10½-in. brick wall, Meyer's values for brick do not depart very widely from the line for plaster, tile, and brick, shown by the Riverbank measurements. The unplastered brick (No. 10) and the *Schwemmsteinwand* (No. 7) both fall below Meyer's line. On the other hand, the pumice concrete, which falls directly on Meyer's line, is similar in structure to those materials, such as cinder and slag concrete, which according to the Riverbank tests showed higher transmission losses than walls of ordinary masonry of equal weight.

On the whole, Meyer's results taken alone might be thought to point to a single relation between the mass and the sound insulation by simple partitions. Viewed critically and in comparison with the results of other researches on the problem, however, they still leave a question as to the complete generality of any single relation. Obviously there are important exceptions, and for the present at least the answer must await still further investigation. The measurements of the elasticity and damping of structures in connection with sound-transmission measurements is a distinct advantage in the study of the problem.

CHAPTER XIII

MACHINE ISOLATION

In every large modern building, there is usually a certain amount of machine installation. The operation of ventilating, heating, and refrigerating systems and of elevators calls for sources of mechanical power. In many cases, both manufacturing and merchandising activities are carried on under a single roof. It therefore becomes important to confine the noise of machinery to those portions of a building in which it may originate. There are two distinct ways in which machine noise may be propagated to distant parts of a building.

The first is by direct transmission through the air. The second is by the transmission of mechanical vibration through the building structure itself. These vibrations originate either from a lack of perfect mechanical balance in machines or, in the case of electric motors, from the periodic character of the torque on the armature. These vibrations are transmitted through the machine supports to the walls or floor, whence, through structural members, they are conducted to distant parts of the building. The thoroughly unified character of a modern steel structure facilitates, to a marked degree, this transfer of mechanical vibration. In the very nature of the case, good construction provides good conditions for the transfer of vibrations. The solution of the problem therefore lies, first, in the design of quietly operating machines and, second, in providing means of preventing the transfer of vibrations from the machines to the supporting structure.

The reduction of vibration by features of machine design is a purely mechanical problem.[1]

[1] For a full theoretical treatment the reader should consult a recent text on the subject: "Vibration Problems in Engineering," by Professor S.

Related to this problem but differing from it in certain respects is the problem of floor insulation. In hotels and apartment houses, the impact of footfalls and the sound of radios and pianos are frequently transmitted to an annoying degree to the rooms below. Experience shows that the insulation of such noise is most effectively accomplished by modifications of the floor construction. The difficulty arises in reconciling the necessities of good construction with sound-insulation requirements.

Natural Frequency of a Vibrating System.

The usual method of vibration insulation is to mount the machine or other source of vibration upon steel springs, pads of cork, felt, rubber, or other yielding material. The common conception of just how such a mounting reduces the transmission of vibration to the supporting structure is usually put in the statement that the "pad *damps* the vibration of the machine."

As a matter of fact, the damping action is only a part of the story, and, as will appear later, a resilient mounting may under certain conditions actually increase the transmission of vibrational energy. It is only within recent years that an intelligent attack has been made upon the problem in the light of our knowledge of the mechanics of the free and forced vibration of elastically controlled systems.

A complete mathematical treatment of the problem is beyond our present purpose.[1] We shall try only to present as clear a picture as possible of the various mechanical factors involved and a formulation without proof of the relations between these factors. For this purpose, we shall consider the ideally simple case of the motion of a

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Timoshenko, D. Van Nostrand Company, New York, 1928. A selected bibliography is to be found in "Noise and Vibration Engineering," by S. E. Slocum, D. Van Nostrand Company, New York, 1931.

[1] For such a mathematical treatment, the reader should consult any text on the theory of vibration, *e.g.*, Wood, "Textbook of Sound," p. 36 *et seq.*, G. Bell & Sons, London, 1930. Crandall, "Theory of Vibrating Systems and Sound," p. 40, D. Van Nostrand Company, New York, 1926.

system free to move in only one direction, displaced from its equilibrium position, and moving thereafter under the action of the elastic stress set up by the displacement and the frictional forces generated by the motion. Figure 110 represents such an ideal simple system. The mass m is supported by a spring, and its motion is damped by the frictional resistance in a dashpot. Assume that T, the force of compression of the spring, is proportional to the

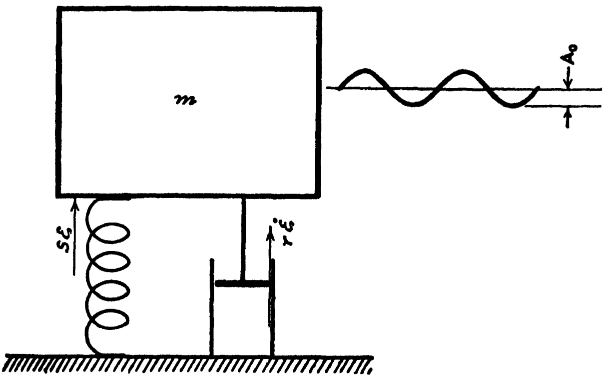


FIG. 110.—The mass m moves under the action of the elastic restoring force of the spring and the frictional resistance in the dash pot.

displacement ξ from the equilibrium position (Hooke's law).

$$T = -s\xi$$

where s is the force in absolute units that will produce a unit extension or compression of the spring. We shall call s the "spring factor."

Assume further that the frictional resistance at any time due to the motion of m is proportional to the velocity at that time and that it opposes the motion. The frictional force called into play by a velocity $\dot{\xi}$ is $r\dot{\xi}$. The force due to the inertia of the mass m moving with an acceleration $\ddot{\xi}$ is $m\ddot{\xi}$. If there is no external applied force, then the force equation for the system is

$$m\ddot{\xi} + r\dot{\xi} + s\xi = 0 \tag{83}$$

In mathematical language, this is a "homogeneous linear differential equation of the second order," and its solutions are well known.[1]

For the present purpose, the most useful form for the solution of Eq. (83) is

$$\xi = A_0 e^{-\frac{rt}{2m}} \sin\left\{t\sqrt{\frac{s}{m} - \frac{r^2}{4m^2}} + \varphi\right\} \tag{84}$$

$$= A_0 e^{-kt} \sin(\omega_1 t + \varphi)$$

where k, called the "damping coefficient," is defined by the equation $k = r/2m$, and ω_1 is defined by the equation

$$\omega_1 = 2\pi f_1 = \sqrt{\frac{s}{m} - \frac{r^2}{4m^2}} \tag{85}$$

φ is an arbitrary constant whose value depends upon the displacement at the moment from which we elect to measure times. f_1 is the frequency of the damped system. If the system is displaced from its equilibrium position and then allowed to move freely, its motion is given by Eq. (84). There are two possible cases. If $r^2/4m^2 > s/m$, *i.e.*, if the damping is large, the expression $\left(\frac{s}{m} - \frac{r^2}{4m^2}\right)$ is negative, and its square root is imaginary. The physical interpretation of this is that in such a case the motion is not periodic, and the system will return slowly to its equilibrium position under the action of the damping force. Under the other possibility, $r^2/4m^2 < s/m$, the system in coming to rest will perform damped oscillations with a frequency of $\frac{1}{2\pi}\sqrt{\frac{s}{m} - \frac{r^2}{4m^2}}$. In the usual dashpot damping, the resistance term is large, thus preventing oscillations. The automobile snubber is designed to increase the frictional

[1] Wood, "Textbook of Sound," p. 34. Mellor, "Higher Mathematics for Students of Chemistry and Physics," p. 404, Longmans, Green & Co., London, 1919.

For the solution in the analogous electric case of the discharge of a condenser through a circuit containing inductance and resistance, see Pierce, "Electric Oscillations and Electric Waves," p. 13, McGraw-Hill Book Company, Inc., New York, 1920.

resistance and thus reduce the oscillations that would otherwise result from the elastic action of the spring.

In any practical case of machine isolation in buildings, the free movement of the machine on a resilient mounting will be represented by the second case, $r^2/4m^2 < s/m$. The motion is called "damped sinusoidal oscillation" and is shown graphically in Fig. 111. The dotted lines show the decrease of amplitude with time due to the action of the damping force.

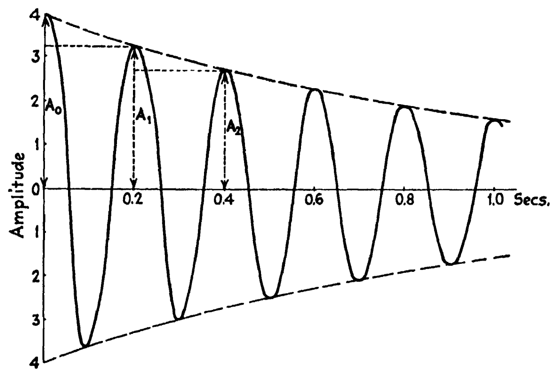


FIG. 111.—Graph of damped sinusoidal oscillation. $A_0/A_1 = A_1/A_2$

Forced Damped Oscillations.

In the foregoing, we have considered the movement when no external force is applied. If impacts are delivered at irregular intervals, the motion following each impact is that described. If, however, the system be subjected to a periodically varying force, then in the steady state it will vibrate with the frequency of the driving force. Thus in alternating-current generators and motors there is a periodic torque of twice the frequency of the alternating current. Rotating parts which are not in perfect balance give rise to periodic forces whose frequency is that of the rotation.

For mathematical treatment, suppose that the impressed force is sinusoidal with a frequency $f = \omega/2\pi$ and that it has a maximum value F_0. Then the motion of the system shown in Fig. 110, under the action of such a force, is given by the equation

$$m\ddot{\xi} + r\dot{\xi} + s\xi = F_0 \sin \omega t \tag{86}$$

The solution of this equation is well known,[1] and the form of the expression for the displacement in the steady state in terms of the constants of Eq. (86) will depend upon the relative magnitudes of m, r, and s. There are three cases:

Case I $\frac{r^2}{4m^2} < \frac{s}{m}$ (small damping)

Case II $\frac{r^2}{4m^2} = \frac{s}{m}$ (critical damping)

Case III $\frac{r^2}{4m^2} > \frac{s}{m}$ (large damping)

Practical problems of machine isolation come under case I, and the particular solution under this condition is given by the equation

$$\xi_{\text{max.}} = A_0 = \frac{F_0}{m}\sqrt{\frac{1}{4k^2\omega^2 + (\omega_0{}^2 - \omega^2)^2}} \tag{87}$$

Here $\omega_0 = s/m$, $k = r/2m$, and $\omega = 2\pi$ times the frequency of the impressed force. It can be easily shown that the natural frequency of the undamped system is given by the relation

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

From Eq. (87) it is apparent that the amplitude of the forced vibration for a given value of the impressed force is a maximum when $\omega = \omega_0$, *i.e.*, when the frequency of the impressed force coincides with the natural frequency of the vibrating system. In this case, Eq. (87) reduces to

$$A_0 = \frac{F_0}{m}\sqrt{\frac{1}{4k^2\omega^2}} = \frac{F_0}{r}$$

This coincidence of the driving frequency with the

[1] WOOD, A. B., "A Textbook of Sound," p. 37.

natural frequency of the vibrating system is the familiar phenomenon of resonance, and Eq. (88) tells us that the amplitude at resonance in the steady state is directly proportional to the amplitude of the driving force and inversely proportional to the coefficient of frictional resistance. It is apparent that when the impressed frequency is close to the resonance frequency, the frictional resistance plays the preponderant rôle in determining the amplitude of vibration set up. If we could set up a system in which there were *no* frictional damping whatsoever, then any periodic force no matter how small operating at the resonance frequency would in time set up vibrations of infinite amplitude. This is the scientific basis for the often repeated popular statement that the proper tone played on a violin would shatter the most massive building. Fortunately for the permanence of our buildings, movements of material bodies always call frictional forces into play. The vibrations set up in the body of an automobile for certain critical motor speeds is a familiar example of the effect of resonance. It frequently occurs that machines which are mounted on the floor slab in steel and concrete construction set up extreme vibrations of the floor. This can be frequently traced to the close proximity of the operating speed of the machine to a natural frequency of the floor supporting it. The tuning of a radio set to the incoming electromagnetic frequency of the sending station is an application of the principle of resonance.

Inspection of Eq. (87) shows that while the frictional damping is most effective in reducing vibrations at or near resonance, yet increasing k decreases the amplitude for all values of the frequency of the impressed force. In general, it may be said therefore that if we are concerned only with reducing the vibration of the machine on its support, the more frictional resistance we can introduce into the machine mounting the better. As we shall see, however, this general statement does not hold true, if we concern ourselves with the transmission of vibration to the supporting floor.

Inertia Damping.

We consider now the question of the effect of mass upon the amplitude of vibration of a system under the action of an impressed periodic force. In the expression for the amplitude given in Eq. (87), it would appear that increasing m will always decrease A_0, since m appears in the denominator of the right-hand member of the equation. It must be remembered, however, that m is involved in both $k(= r/2m)$ and $\omega_0\left(= \sqrt{s/m}\right)$. Putting in these values, Eq. (87) may be thrown into the form

$$A_0 = \frac{F_0}{\sqrt{r^2\omega^2 + (s - m\omega^2)^2}} \tag{88}$$

It is clear that the effect produced on A_0 by increasing m will depend upon whether this increases or lowers the absolute value of the expression $(s - m\omega^2)^2$. If $\omega_0{}^2 = s/m < \omega^2$, then increasing m decreases $(s - m\omega^2)^2$, decreasing the denominator of the fraction and hence increasing the value of A_0. In other words, if the driving frequency is below the natural frequency, increasing the mass and thus lowering the natural frequency brings us nearer to resonance and increases the vibration. If, on the other hand, the driving frequency is *above* the natural frequency, the reverse effect ensues. Added mass is thus effective in reducing vibration only in case the driving frequency is above the natural frequency of the vibrating system. By similar reasoning it follows that under this latter condition, decreased vibration is effected by decreasing the spring factor, *i.e.*, by weakening the supporting spring.

Graphical Representation.

The foregoing discussion will perhaps be clarified by reference to Fig. 112. Here are shown the relative amplitudes of vibration of a machine weighing 1,000 lb. for a fixed value of the amplitude and frequency of the impressed force. These are the familiar resonance curves plotted so as to show their application to the practical problem in hand. Referring to the lower abscissae, we note that starting with a low value of the spring factor, the amplitude increases as s increases up to a certain value and then decreases. The peak value occurs when the relation between the spring factor and the mass of the machine is such that $s/m = \omega^2$. We note further that increased damping decreases the vibration under all conditions but

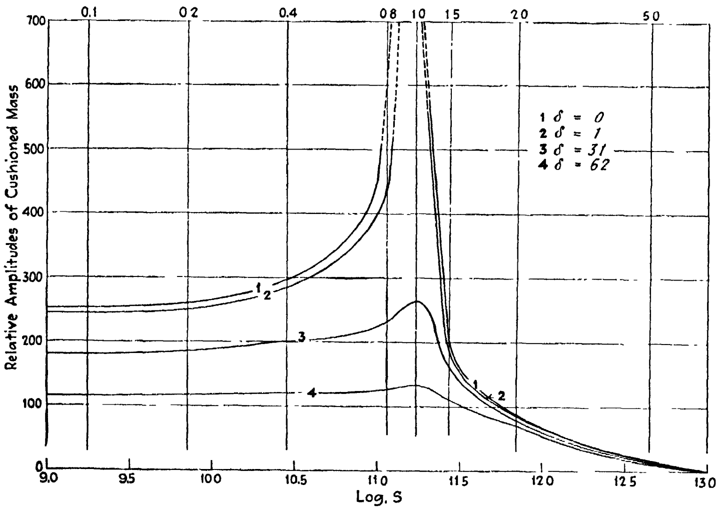


FIG. 112.—Amplitudes of forced vibrations with a fixed driving frequency and varying values of the stiffness of the resilient mounting. The upper abscissae are the ratios of the natural frequencies to the fixed driving frequency $\delta = \pi r/\sqrt{sm} = k/f_0 = A_0/A_1$, in Fig. 111.

that this effect is most marked when the natural frequency is in the neighborhood of the impressed frequency. Finally, it is to be observed that if we are concerned only with reducing the vibration of the machine itself, this can best be done by using a very stiff mounting, making the natural frequency high in comparison with the impressed frequency, *i.e.*, with a rigid mounting. As we shall see subsequently, however, this is the condition which makes for increased transfer of vibrations to the supporting structure.

Transmission of Vibrations.

Let us suppose that the mass m of Fig. 110 is a machine which due to unbalance or some other cause exerts a periodic force on its mounting. Suppose that the maximum value of the force exerted by the machine on the support is F_1 and that this transmits a maximum force F_2 to the floor. The transmissibility τ of the support is defined as F_1/F_2. Soderberg[1] has worked out an expression for the value of τ in terms of the mass of the machine and the spring factor and damping of the support.

A convenient form for the transmissibility of the support is given by the equation

$$\tau = \sqrt{\frac{4k^2\omega^2 + \omega_0^4}{4k^2\omega^2 + (\omega^2 - \omega_0^2)^2}} \tag{89}$$

where $\omega_0 = 2\pi f_0$ and $\omega = 2\pi f$, f_0, and f being the natural frequency and the impressed frequency respectively.

In most cases of design of resilient machine mounting, the effect of frictional damping is small. Neglecting the term $4k^2\omega^2$, Eq. (89) reduces to the simple form

$$\tau = \frac{\omega_0^2}{\omega^2 - \omega_0^2} = \frac{1}{\dfrac{\omega^2}{\omega_0^2} - 1} = \frac{1}{R^2 - 1}$$

where R is the ratio of the impressed to the natural frequency. We note that for all values of R between 0 and $\sqrt{2}$, τ is greater than unity. In the neighborhood of resonance, therefore, the resilient mounting does not reduce but increases the vibratory force on the floor. Figure 113 gives the values of the transmissibility for varying values of the spring factor and for four different values of the logarithmic decrement $\delta = \pi r/\sqrt{sm}$. We note that the transmissibility is less than unity only for low values of the natural frequency $\omega_0 = s/m$. By inspection of Eq. (89) it is seen that, when $\omega_0/\omega = \frac{1}{2}\sqrt{2}$, the value of τ is unity regardless of the damping. This is shown in the common point for all the curves of Fig. 113.

[1] Soderberg, C. R., *Elec. Jour.*, vol. 21, pp. 160–165, January, 1924.

This means that for any resilient mounting to be effective in reducing the transmission of vibration to the supporting structure, the ratio of s/m must be such that the natural frequency is less than 0.7 times the driving frequency. We have already noted that if it is a question of simply reducing the vibration of the machine itself, placing the natural frequency well above the impressed frequency will be effective. This, however, increases the transmission of vibration to the floor, as shown by the curves of Fig. 113.

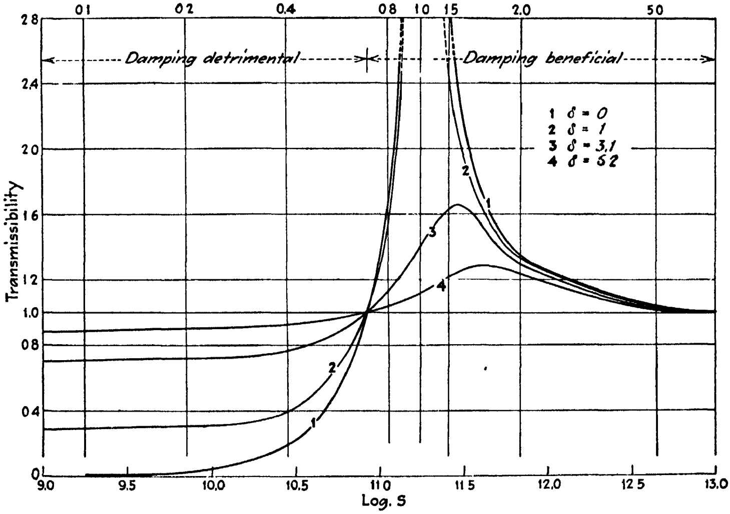


FIG. 113.—Transmissibility of resilient mounting. The upper abscissae are the ratios of the natural frequencies to the driving frequency.

In other words, isolation of the machine has to be secured at the price of increased vibration in the machine itself. The limit of the degree of isolation that can be secured by resilient mounting is thus fixed by the extent to which vibration of the machine on its mounting can be tolerated. If the floor itself were *perfectly* rigid, then a rigid mounting would be best from the point of view both of reduced machine vibration and of reduced transmission to other parts of the building. In general, resilient mounting will be effective in reducing general building vibration only provided the natural frequency of the machine on the resilient mounting is farther below the driving frequency than is the natural frequency of the floor, when loaded with the machine. Obviously, a complete solution of the problem calls for a knowledge of the vibration characteristics of floor constructions. Up to the present, these facts are lacking, so that the effectiveness of a resilient mounting in reducing building vibrations in any particular case is a matter of some uncertainty, even when the machine base is properly designed to produce low values of the transmissibility to the supporting floor.

We shall assume in the following discussion that the natural frequency of the machine on the resilient base is farther below the driving frequency than is the natural frequency of the floor. In such case, the proper procedure for efficient isolation is to provide a mounting of such compliance that the natural frequency is of the order of one-fifth the impressed frequency. Inspection of the curves of Fig. 113 shows that decreasing the natural frequency below this gives a negligible added reduction in the transmission.

Effect of Damping on Transmission.

Inspection of Fig. 112 shows that damping in the support reduces the vibration of the spring mounted body at all frequencies, while Fig. 113 shows that only in the frequency range where the transmissibility is greater than unity does it reduce transmission. It follows, therefore, that when conditions are such as to reduce transmission, damping action is detrimental rather than beneficial. For machines that are operated at constant speed, which is always greater than the resonance speed, the less damping the better. On the other hand, when the operating speed comes near the resonance speed, damping is desirable to prevent excessive vibrations of the mounted machine.

Practical Application. Numerical Examples.

There are two distinct cases of machine isolation that may be considered. The first is that in which the motion of the machine produces non-periodic impacts upon the structure on which it is mounted. The impacts of drop hammers and of paper-folding and paper-cutting machines, in which very great forces are suddenly applied, are cases in point. Under these conditions, a massive machine base mounted upon a resilient pad with a low spring factor and high damping constitutes a system in which the energy of the impact tends to be confined to the mounted machine. In extreme cases, even these measures may be insufficient, so that, in general, massive machinery of this type should be set up only where it is possible to provide separate foundations which are not carried on the structural members of the building.

The more frequent, and hence more important, problem is that of isolating machines in which periodic forces result from the rotation of the moving parts. It appears from the foregoing that the isolation of the vibrations thus set up can be best effected by resilient mounting so designed that the natural frequency of the mounted machine is well below the frequency of the impressed force.

In the practical use of the equation $f_0 = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$, s and m must be expressed in absolute units. In the metric system, s is the force in dynes that produces a deflection of one centimeter, and m is the mass in grams. Engineering data on the compressibility of materials are usually given in graphs on which the force in pounds per square foot is plotted against the deformation in inches. For a given material, the spring factor is roughly proportional to the area of the load-bearing surface and inversely proportional to the thickness.[1] If a force of L lb. per square foot produces a decrease in thickness of d in. in a resilient mounting whose area, A sq. ft., carries a total mass m lb. then

[1] This statement is not even approximately true for a material like rubber, which is practically incompressible when confined. When compressed, the change in thickness results from an increase in area, so that Young's modulus for rubber increases almost linearly with the area of the test sample.

$$f_0 = \frac{1}{2\pi}\sqrt{385\frac{LA}{md}} = 3.13\sqrt{\frac{LA}{md}} \qquad (90)$$

As an example, let us take the case of a motor-driven fan weighing 8,000 lb., with a base 3 by 4 ft. Suppose that a light-density cork is to be used as the isolating medium. For 2-in. thickness, a material of this sort shows a compression of about 0.2 in. under a loading of 5,000 lb. per square foot. If the cork is applied under the entire base of the machine, then we shall have the natural frequency of the machine so mounted

$$f_0 = 3.13\sqrt{\frac{5,000 \times 12}{8,000 \times 0.2}} = 19.1$$

If the vibration to be isolated is that due to the varying torque on the armature produced by a 60-cycle current, the applied frequency is 120 vibs./sec., and the mounting will be effective. If, on the other hand, the motor operates at a speed of 1,800 r.p.m., then the vibration due to any unbalance in the motor will have a frequency of 30 per second. Effectively to isolate this lower-frequency vibration, we should need to lower the natural frequency to about 6 per second. This can be done either by decreasing the load-bearing area of the cork or by increasing its thickness. We can compute the area needed by Eq. (90), assuming that the natural frequency is to be 6 vibs./sec.

$$f_0 = 6 = 3.13\sqrt{\frac{5,000 \times A}{8,000 \times 0.2}}$$

from which we find $A = 1.17$ sq. ft. This gives a loading of about 6,800 lb. per square foot. A larger area of material is perhaps desirable. We can keep the same spring factor by using a larger area of a thicker material. Thus if we double the thickness, we should need twice the area, giving a loading of only 3,400 lb. per square foot and the same transmission.

It is apparent from the foregoing that the successful use of a material like cork or rubber involves a knowledge of the stress-strain characteristics of the material and the frequencies for which isolation is desired. Figure 114 shows the deformation of three qualities of cork used for machine isolation.[1] The lines *AB* and *CD* indicate the

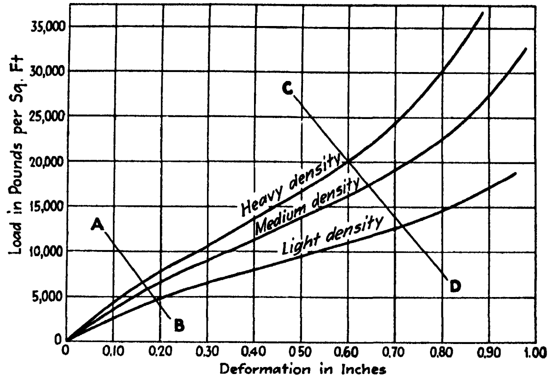


FIG. 114.—Compressibility of three grades of machinery cork. (*Courtesy of Armstrong Cork Co.*)

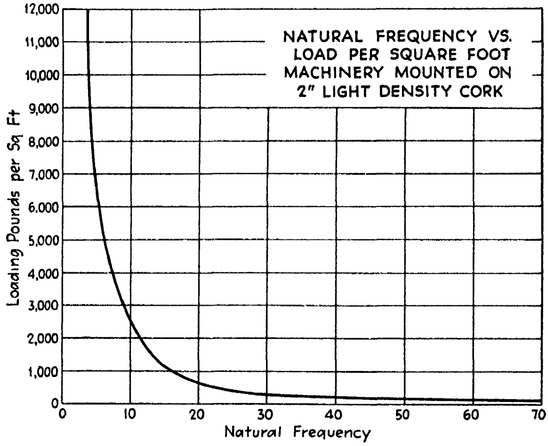


FIG. 115.—Natural frequencies for various loadings on the light density cork of Fig. 114.

limits of loading recommended by the manufacturers. We note that the increase in deformation is not a linear function of loading, indicating that these materials do not

[1] Acknowledgment is made to the Armstrong Cork Company for permission to use these data.

follow Hooke's law for elastic compression. For any particular loading, therefore, we must take the slope of the line at that loading in computing the load per unit deflection.

In Fig. 115, the natural frequencies of machines mounted on the light-density cork are given for values of the load per square foot carried by the cork. The compressibility of cork is known to vary widely with the conditions of manufacture, so that the curves should be taken only as typical. Similar curves for any material may be plotted

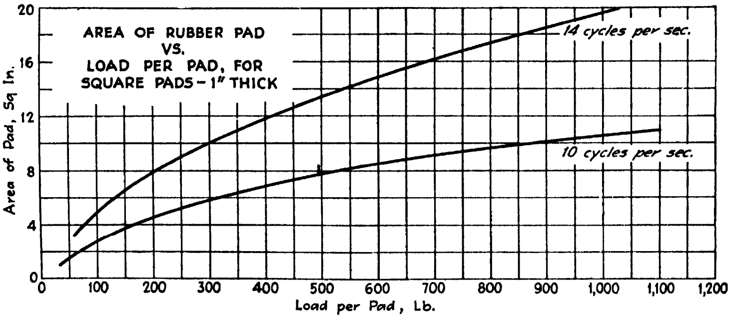


FIG. 116.—Effect of area of rubber pads upon the natural frequency for various loadings. (*Hull and Stewart.*)

using Eq. (90) giving the deformation under varying loadings. For the heavier-density corks shown in Fig. 114, the loading necessary to produce any desired natural frequency would be considerably greater. For 1-in. cork the loading necessary to produce any desired natural frequency would theoretically have to be twice as great as those shown, while for 4-in. material the loading would need to be only half as great.

As has been indicated, when rubber is confined so as not to flow laterally, its stiffness increases. For this reason, rubber in large sheets is much less compressible than when used in smaller units. The same thing is true to a certain degree of cork when bound laterally. To produce a given natural frequency, cork confined laterally by metal bands will require greater loading than when free.

The curves of Fig. 116 are taken from a paper by Hull and Stewart.[1] They show the size and loading of square rubber pads 1 in. thick that must be used to give natural frequencies of 10 and 14 vibs./sec. These pads are of a high-quality rubber containing 90 per cent pure rubber. From the curves we see that the 8,000-pound machine mounted on 10 of these pads each approximately 10 sq. in. will have a natural frequency of 10 vibs./sec.; while if it is mounted on 10 pads each 17 sq. in., the natural frequency is 14 vibs./sec.

Natural Frequency of Spring Mountings.

The computation of the natural frequency of a machine mounted on metal springs is essentially the same as that when the weight is distributed over an area. Suppose it were required to isolate the 8,000-lb. machine of the previous example using springs for which a safe loading is 500 lb. We should thus need to use for the purpose 8,000 ÷ 500 = 16 springs, designed so that each spring with a load of 500 lb. will have a natural frequency of 6 per second. Then we may compute the necessary deflection for this load by the formula

$$6 = 3.13\sqrt{\frac{500}{500d}}$$

from which

$$d = 0.27 \text{ in.}$$

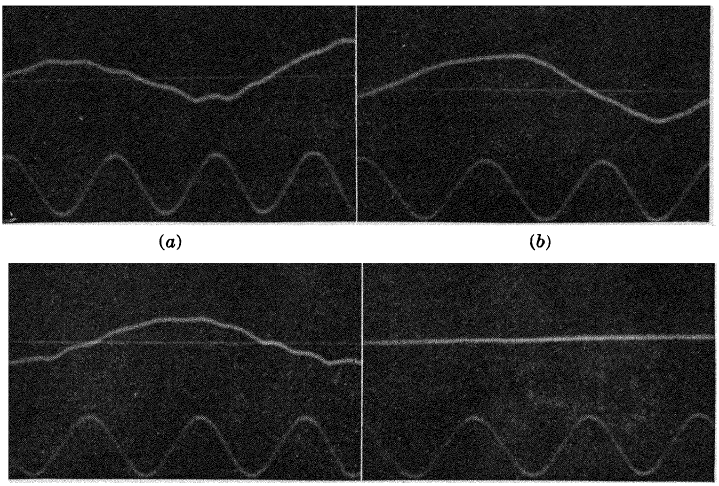
From the known properties of steel, springs may be designed having any desired characteristics over a fairly wide range. From the standpoint of predictability of performance and the control of spring factor to meet any desired condition, spring mounting is advantageous. Because of the relatively low damping, in comparison with organic materials, the amplitude of vibration of the machine and the transmission to the floor are large when the machine is operating at or near the resonance speed. The trans-

[1] Elastic Supports for Isolating Rotating Machinery, *Trans. A. I. E. E.*, vol. 50, pp. 1063–1068, September, 1931.

mission, however, is less when the machine is operating at speeds considerably above resonance.

Results of Experiment.

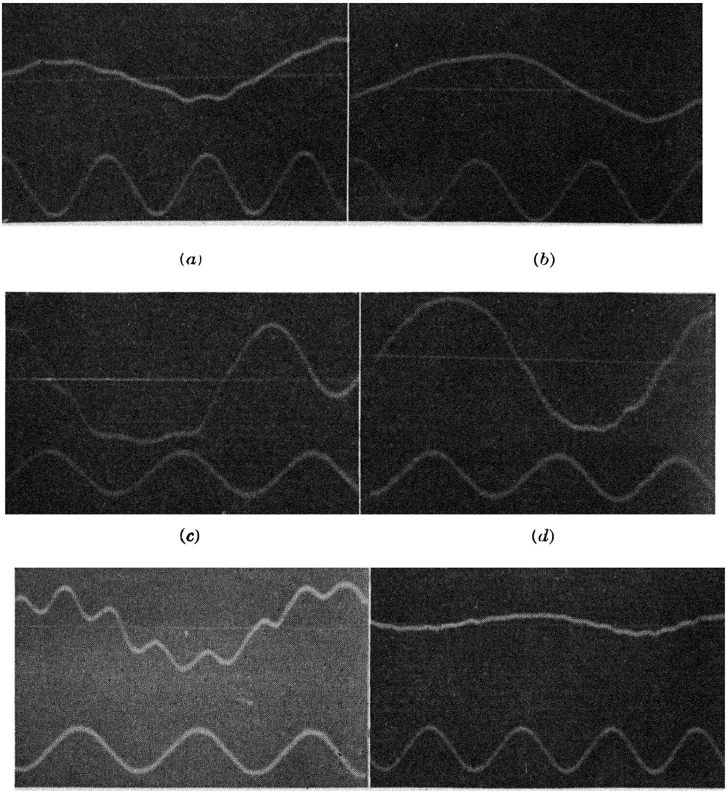
Precise experimental verification of the principles deduced in the foregoing is difficult, due to the uncertainty pertaining to the vibration characteristics of floor construction. We have assumed that the floor on which the machine is mounted is considerably stiffer, *i.e.*, has a higher natural frequency than that of the mounted machine.



(*a*) (*b*) (*c*) (*d*)

FIG. 117.—(*a*) Vibration of machine with solid mounting. (*b*) Vibration of floor with solid mounting. (*c*) Vibration of machine mounted on U. S. G. 500 lb. clip. (*d*) Vibration of floor machine mounted on U. S. G. 500 lb. clip.

The oscillograms of machine and floor vibrations in Figs. 117 and 118 were kindly supplied by the Building Research Laboratory of the United States Gypsum Co. They were obtained by direct electrical recording of the vibration conditions on and under a moderately heavy machine with a certain amount of unbalance, carried by a typical clay-tile arch concrete floor. The operating speed was 1,500 r.p.m. In Fig. 117 are shown the vibration of the machine and floor, first, when the machine is solidly mounted on the floor and then with the machine mounted on springs



(*a*) (*b*)

(*c*) (*d*)

(*e*) (*f*)

FIG. 118.—(*a*) Vibration of machine with solid mounting. (*b*) Vibration of floor with solid mounting. (*c*) Vibration of machine on 1-in. high-density cork; loading 1,000 lb. per square foot. (*d*) Vibration of floor, machine mounted on 1-in. high-density cork; loading 1,000 lb. per square foot. (*e*) Vibration of machine on 2-in. low-density cork; loading 3,000 lb. per square foot. (*f*) Vibration of floor, machine mounted on 2-in. low-density cork; loading 3,000 lb. per square foot.

each of which was designed to carry a load of 500 lb. and to have, so loaded, a natural frequency of 7.5 vibs./sec. Figure 118 shows the necessity of proper loading in order to secure efficient isolation by the use of cork. The middle curves show increased vibration both of the machine and of the floor when 1 in. of heavy-density material loaded to 1,000 lb. per square foot is used. This is probably explained by the fact that on this mounting the natural frequency of the machine approximates that of the floor slab. In the lower curves, the loading on the cork is much nearer what it should be for efficient isolation. From the graph of Fig. 115, we see that the natural frequency of the 2-in. light-density cork loaded 3,000 lb. to the square foot is about 9 per second. This is a trifle more than one-third the driving frequency and is in the region of efficient isolation. These curves show in a strikingly convincing manner the importance of knowing the mechanical properties of the cushioning material and of adjusting the loading and spring factor so as to yield the proper natural frequency. In general, this should be well below the lowest frequency to be isolated, in which case higher frequencies will take care of themselves.

Tests on Floor Vibrations under Newspaper Presses.

In June, 1931, the writer was commissioned to make a study of the vibrations of the large presses and of the floors underneath them and in adjacent parts of the building of the *Chicago Tribune.* For the purpose of this study, a vibration meter was devised consisting of a light telephonic pick-up, associated with a heavy mass the inertia of which held it relatively stationary, while the member placed in contact with the vibrating surface moved. The electrical currents set up by the vibration, after being amplified and rectified, were measured on a sensitive meter. The readings of the meter were standardized by checking the apparatus with sources of known vibrations and were roughly proportional to the square of the amplitude.

In the *Tribune* plant, three different types of press mounting had been employed. In every case, the presses were ultimately carried on the structural steel underneath the reel room at the lower-basement floor level. Still a fourth type had been used in the press room of the *Chicago Daily News*, and permission was kindly granted to make similar measurements there. The four types of mounting are shown in Fig. 119. *A* shows the press columns mounted directly on the structural steel with no

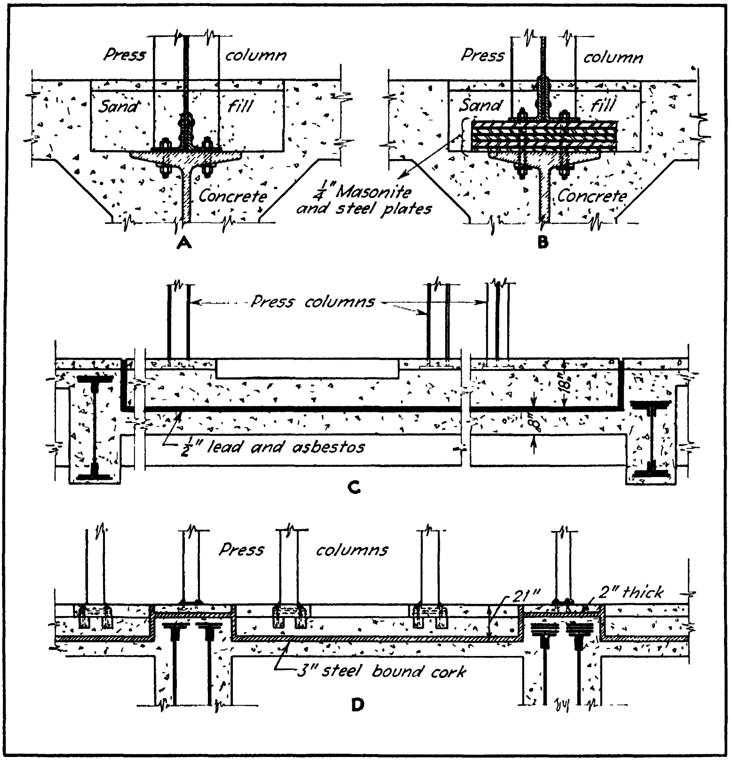


FIG. 119.—Four types of newspaper press mountings studied.

attempt at cushioning. In *B*, the press mounting and floor construction are similar, except that layers of alternating ⅝-in. steel and ¼-in. compressed masonite fiber board are interposed between the footings of the press columns and the structural girders. The loading on these pads was about 13,000 lb. per square foot. *C* shows the press supports carried on an 18-in. reinforced concrete base, floated on a layer of lead and asbestos ½ in. thick. This floated slab is carried on a 7½-in. reinforced concrete bed carried on the girders. *D* is essentially the same as *C*, except that a 3-in. continuous layer of Korfund (a steel-framed cork mat) is interposed between the floated slab and the 12-in. supporting floor. Here the loading was approximately 4,000 lb. per square foot.

Experiment showed that the vibration varied widely for different positions both on the presses themselves and on the supporting structure. Accordingly, several hundred measurements were made in each case, an attempt being made to take measurements at corresponding positions about the different presses. Measurements were made

TABLE XXVI

Test	Insulation	On press	On column footing or floated slab	On reel-room floor	On floor above press room
A .	None	1,375	. . .	262	34
B. .	Masonite and steel	1,450	72	21	2.6
C.. ..	Lead and asbestos	1,000	55	41	9.4
D.....	Steel-bound cork	2,000	412	30	8.0

with the presses running at approximately the same speed, namely, 35,000 to 40,000 papers per hour.

TABLE XXVII

Test	Mounting	Ratio of press vibrations to		
		Footings or supporting slab	Reel-room floor	Floor above press room
A	Direct to structural steel	. . .	5:1	40:1
B	Masonite and steel	20:1	69:1	440:1
C . ..	½-in. asbestos and lead	18:1	24:1	102:1
D ...	3-in. cork	5:1	67:1	250:1

The averages of the measurements made are shown in Table XXVI.

Since the vibrations of the presses themselves vary rather widely, it will be instructive to find the ratio of the press vibrations to the vibration at the other points of measurement. These ratios are shown in Table XXVII.

So many factors besides the single one of insulation affect the vibration set up in the building structure that it is dangerous to draw any general conclusions from these tests. Oscillograph records of the vibrations showed no preponderating single frequency of vibration. The weight and stiffness of the floor structures varied among the different tests, so that it is not safe to ascribe the differences found to the differences in the mountings alone. However, it is apparent from comparison of the ratios in Table XXVII that all of the three attempts at isolation resulted in less building vibration than when the presses were mounted directly on the steel. Conditions in *A* and *B* were nearly the same except for the single fact of the masonite and steel pads *B*. The vibrations of the presses themselves was about the same in the two cases (1,375 and 1,450), so that it would appear that this method of mounting in this particular case reduced the building vibration in about the ratio of 13:1. The loading was high—13,000 lb. per square foot—and the masonite was precompressed so as to carry this load. The steel plates served to give a uniform loading over the surface of the masonite.

Study of the figures for *C* and *D* discloses some interesting facts. In *C* (Table XXVI), with the lead and asbestos, we note that the vibrations of the press and of the floated slab are both low and, further, that there is only slight reduction in going from the slab to the reel-room floor. Comparison with *D*, where vibration of both presses and floated slab was high, indicates that the cushioning action of the ½ in. of lead and asbestos was negligibly small, in comparison with 3 in. of cork. This latter, however, was obtained at the expense of increased press vibration. The loading on the 3 in. of cork was comparatively low. In the light of both theory and experiment, one feels fairly safe in saying that considerably better performance with the cork would have resulted from a much higher loading.

General Conclusions.

It is fairly apparent from what has been presented that successful machine isolation is a problem of mechanical engineering rather than of acoustics. Each case calls for a solution. Success rests more on the intelligence used in analysis of the problem and the adaptation of the proper means of securing the desired end than on the merits of the materials used. Mathematically, the problem is quite analogous to the electrical problem of coupled circuits containing resistance, inductance, and capacity. The mathematical solutions of the latter are already at hand. Their complete application to the case of mechanical vibrations calls for more quantitative data than are at present available. In particular, it is desirable to know the vibration characteristics of standard reinforced floor constructions and the variation of these with weight, thickness, and horizontal dimension. Such data can be obtained partly in the laboratory but more practically by field tests on existing buildings of known construction. Since the problem is of importance to manufacturers of machines, to building owners, to architects, and to structural engineers, it would seem that a cooperative research sponsored by the various groups should be undertaken.

A more detailed theoretical and mathematical treatment of the subject may be found in the following references.

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APPENDIX A

TABLE I.—PITCH AND WAVE LENGTH OF MUSICAL TONES

Tone	International pitch Tempered scale $A = 435$ vib./sec.			Physical pitch Diatonic scale $C_3 = 256$ vib./sec.		
	Frequency	Wave length		Frequency	Wave length	
		Meters	Feet		Meters	Feet
Middle C	258 6	1.328	4 35	256	1.341	4.40
C#	274.0	1 253	4 11			
D	290 3	1.183	3.88	288	1.191	3.91
D#	307.6	1.118	3 66			
E	325 9	1.054	3 46	320	1 071	3 52
F	345.3	0.995	3 26	341.3	1.005	3.31
F#	365.8	0.939	3.08			
G	387 5	0.886	2 91	384	0.894	2 93
G#	410 6	0 836	2.74			
A	435 0	0 789	2 59	426 7	0 806	2 64
A#	460 9	0 745	2 44			
B	488.3	0.703	2 31	480	0 714	2 35
C	517.3	0.664	2.18	512	0.670	2 20

Velocity of sound at 20° C. = 343.33 m /sec. = 1,126.1 ft./sec.

TABLE II.—COEFFICIENTS OF VOLUME ELASTICITY, DENSITY, VELOCITY OF SOUND, AND ACOUSTIC RESISTANCE

Material	ϵ	ρ	c	r
Steel	$19,600 \times 10^8$	7.8	5,010	391×10^4
Cast iron	$9,480 \times 10^8$	7.0	3,650	255×10^4
Brass	$6,370 \times 10^8$	8.4	2,750	232×10^4
Bronze	$3,140 \times 10^8$	8.8	1,890	166×10^4
Lead	588×10^8	11.4	718	82×10^4
Wood:				
Teak	$1,570 \times 10^8$	0.86	4,270	37×10^4
Fir	880×10^8	0.45	4,430	20×10^4
Birch	590×10^8	0 80	2,710	22×10^4
Water	196×10^8	1.0	1,400	14×10^4
Rubber	1×10^8	1 0	100	1×10^4
Air (0° C.)	0.014×10^8	0.00129	330	0.0042×10^4

ϵ = coefficient of volume elasticity in bars
ρ = density, grams per cubic centimeter
c = velocity of sound, meters per second
r = acoustic resistance, grams per centimeters^{-2} seconds^{-1}

APPENDIX B

Mean Free Path within an Inclosure.

Given an inclosed space of volume V and a bounding surface S, in which there is a diffuse distribution of sound of average energy density I: To show that p, the mean free path between reflections at the boundary, of a small portion of a sound wave, is given by the equation

$$p = \frac{4V}{S}$$

A diffuse distribution is one in which the flow of energy through a small section of given cross-sectional area is the same independently of the orientation of the area. For the

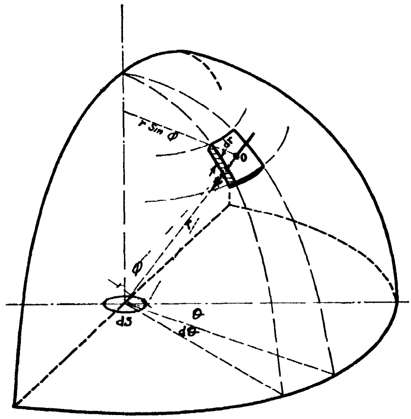

Fig. 1.

purpose of this proof we may consider that the energy is concentrated in unit particles of energy, each traveling with the velocity of sound and moving independently of all the other particles. The number of particles per unit volume is I, and the total energy in the inclosure is VI.

In the proof, we shall first derive an expression for the energy incident per second on a small element dS of the bounding surface and then, by relating this to the mean free path of a single particle, arrive at the desired relation.

In Fig. 1, dV is any small element of volume at a distance r from the element of surface dS. We can locate dV on the surface of a sphere of radius r by assigning to it a colatitude φ and a longitude θ. We shall express its volume in terms of small increments $d\varphi$, $d\theta$, and dr of the three coordinates. From the figure we have

$$dV = r^2 \sin \varphi d\theta dr \tag{1}$$

The number of unit energy particles in this volume is

$$IdV = Ir^2 \sin \varphi d\varphi d\theta dr \tag{2}$$

In view of the diffuse distribution, all the energy contained in dV will pass through the surface of a sphere of radius r. The fraction which will strike dS is given by the ratio of the projection of dS on the surface of this sphere, which is $dS \cos \varphi$, to $4\pi r^2$, the total surface of the sphere. Therefore the energy from dV that strikes dS is

$$\frac{dS \cos \varphi}{4\pi r^2} IdV = \frac{IdS}{4\pi} \sin \varphi \cos \varphi d\varphi d\theta dr \tag{3}$$

Energy leaving dV will reach dS within one second for all values of r less than c, the velocity of sound. Hence the total energy per second that arrives at dS from all directions is given by the summation of the right-hand member of Eq. (3) to include all volume elements similar to dV within a hemisphere of radius c. This summation is given by the definite integral

$$\frac{IdS}{4\pi} \int_0^{\frac{\pi}{2}} \sin \varphi \cos \varphi d\varphi \int_0^{2\pi} d\theta \int_0^c dr = \frac{IcdS}{4} \tag{4}$$

The total energy that is incident per second on a unit area is therefore $IC/4$, and on the entire bounding surface S is $IcS/4$.

Now we can find an expression for this same quantity in terms of the mean free path of our supposed unit energy particles. If p is the average distance traveled between impacts by a single particle, the average number of impacts per second of each particle on some portion of the bounding surface S is c/p. The total number of particles is VI, so that the total number of impacts per second of all the particles on the surface S is VIc/p. By the definition of the unit particle this is the total energy per second incident upon S, so that we have

$$\frac{IcS}{4} = \frac{VIc}{p} \tag{5}$$

or

$$p = \frac{4V}{S} \tag{6}$$

the relation which was to be shown.

APPENDIX C

TABLE I.—ABSORBING POWER OF SEATS

Description	Absorbing power					Authority
	128	256	512	1,024	2,048	
Ash chairs, solid seats, open back	0 15	0.16	0 17	0.18	0 20	W. C. Sabine
Auditorium chairs, solid veneer seat and back	0.15	0.22	0 25	0 28	0.50	P. E. Sabine
		0 19	0 24	0.39	0 38	F. R. Watson
Cushions, vegetable fiber, canvas covered, damask cloth	0 75	1.04	1 45	1 59	1.42	W. C. Sabine
Cushions, hair covered with canvas and plush	0.99	1 13	1 77	1.67	1.37	W. C. Sabine
Cushions, hair covered with canvas and thin leather	1.13	1.27	1.93	1.27	0 73	W. C. Sabine
Cushions, elastic cotton covered with canvas and plush	1.66	1.88	2.04	2.77	1.95	W. C. Sabine
Theater chairs, upholstered seat and back, in imitation leather	....	1.4	1.6	1.7	2.1	F. R. Watson
Theater seats, upholstered in mohair (average for 5 types)	..	3 1	3.0	3 2	3 4	F. R. Watson
Theater seats, upholstered	....	3 4	3 0	3 2	3.7	V. L. Chrisler
Theater seats upholstered	....		2 6	...	.	P. E. Sabine
Steel chairs wood seats, open back	0 10	0 15	0 15	0.14	0 30	P. E. Sabine

TABLE II.—COEFFICIENTS OF ABSORPTION OF MATERIALS[1]

No	Material	Thickness, inches	Coefficients 128	256	512	1,024	2,048	4,096	Date
1	Acoustex, excelsior tile	1	0.14	0.28	0.55	0.77	0.79	0.69	1930
2	Acoustex, excelsior tile.	1⅝	0.20	0.41	0.69	0.86	0.79	0.65	1930
3	Akoustolith tile, fine texture.	⅞	0.09	0.27	0.29	0.50	0.62	0.39	1929
4	Balsam wool, scrim facing, paper backing	1	0.14	0.33	0.50	0.71	0.70	0.60	1929
5	Balsam wool, "Quiet tile" .	1	0.12	0.40	0.60	0.72	0.77	0.62	1931
6	Celotex, Acousti type B . .	⅞		0.33	0.41	0.48	0.52	0.52	1929
7	Celotex, Acousti type BB . . .	1¼	. .	0.47	0.64	0.76	0.69	0.57	1929
8	Celotex standard on 1 in. furring	7/16	. . . .	0.19	0.22	0.21	0.20	0.19	1929
9	Celotex, standard on 2 × 4-in. studs.	7/16	0.27	0.17	0.13	0.15	0.17	0.20	1929
10	Draperies, cotton 10 oz/sq. yd. hung straight in contact with wall.	. . .	0.04	0.05	0.11	0.18	0.30	0.44	1929
11	Draperies, same but weighing 14 oz/sq. yd	. . .	0.06	0.08	0.13	0.23	0.40	0.44	1929
12	Draperies, velour weighing 18 oz/sq yd. hung 4 in from wall	. .	0.09	0.33	0.45	0.52	0.50	0.44	1929
13	Draperies, same hung 8 in. from wall.	. .	0.12	0.36	0.45	0.52	0.50	0.44	1929
14	Draperies, cotton weighing 14 oz/sq. yd draped ⅞		0.04	0.15	0.15	0.28	0.46	0.52	1929
15	Draperies, same draped ¾	. .	0.06	0.28	0.41	0.60	0.66	0.50	1929
16	Draperies, same draped ½ .	. . .	0.10	0.38	0.50	0.85	0.82	0.67	1929
17	Felt, standard cattle hair	1	0.13	0.41	0.56	0.69	0.65	0.49	1929
18	Flaxlinum, semi-stiff fiber board	½	0.13	0.18	0.35	0.60	0.64	0.59	1929
19	Johns-Manville insulating board, vegetable fiber, on 1-in furring	½	0.16	0.24	0.16	0.18	0.20	0.26	1929
20	Masonite, standard wood fiber board on 1-in furring.	½	0.21	0.36	0.34	0.34	0.37	0.46	1929
21	Nashkote A, asbestos hair felt, painted membrane	1	0.16	0.32	0.39	0.50	0.51	0.40	1928
22	Nashkote A, membrane perforated in place . .	1	0.14	0.47	0.78	0.80	0.67	0.46	1928
23	Nashkote B-316, asbestos hair felt, covered with oilcloth, 3/16-in. perforations. .	1	0.13	0.40	0.71	0.79	0.72	0.56	1928
24	Nashkote B-332, same but with 3/32-in. perforations	1	0.16	0.38	0.68	0.85	0.81	0.62	1928
25	Plaster, gypsum on wood lath, rough finish .	½	0.023	0.039	0.039	0.052	0.037	0.035	1929
26	Plaster, same, smooth finish .	⅝	0.029	0.026	0.032	0.041	0.048	0.035	1929
27	Plaster, gypsum on metal lath, smooth finish . . .	⅝	0.020	0.022	0.032	0.039	0.039	0.028	1929
28	Plaster, lime, on wood lath, rough finish . . .	½	0.039	0.056	0.061	0.089	0.054	0.070	1929
29	Plaster, lime, on wood lath, smooth finish. . . .	⅝	0.035	0.033	0.031	0.039	0.023	0.041	1929
30	Plaster, Sabinite on gypsum base	½	0.09	0.19	0.21	0.30	0.42	0.46	1930

[1] Measurements made by timing duration of audible sound from organ-pipe source. Empty-room absorbing power measured by variable source methods (loud-speaker). Riverbank Laboratory tests.

TABLE II.—COEFFICIENTS OF ABSORPTION OF MATERIALS.—(*Continued*)

No.	Material	Thickness, inches	Coefficients 128	256	512	1,024	2,048	4,096	Date
31	Plaster improved Sabinite on gypsum base	½	0.13	0.29	0 37	0 56	0.60	0 58	1931
32	Plaster, Sabinite A, on gypsum base, trowel finish	½	0 13	0.24	0 36	0 50	0 56	0 62	1931
33	Plaster, Sabinite A, on gypsum base, float finish	½	0 14	0 24	0 39	0 56	0 56	0 49	1931
34	Sanacoustic tile, 1¼-in. rock wool, perforated metal	. .	0 20	0 47	0 84	0 90	0 95	0 74	1930
35	Soundex (excelsior tile)	1	0.13	0.36	0 57	0.84	0.74	0 62	1931
36	Soundex (excelsior tile)	1½	0 21	0 49	0 80	0 84	0 74	0 77	1931
37	U. S. Gypsum acoustical tile	½	0 07	0 16	0 48	0 60	0 52	0 56	1931
38	U. S. Gypsum acoustical tile	¾	0 06	0 29	0 62	0 60	0 66	0 56	1931
39	U. S. Gypsum, acoustical tile	1	0 15	0.46	0 67	0.60	0 64	0.56	1931
40	U. S. Gypsum, perforated metal, 1½-in. mineral wool	. . .	0 33	0 74	0.81	0.81	0.69	0 54	1931
41	Westfelt, jute	¼	0 09	0 15	0 20	0 42	0 54	0 55	1930
	Westfelt, jute	½	0 12	0 21	0 33	0 65	0 69	0 55	1930
	Westfelt, jute	1	0 13	0 27	0 50	0 67	0 69	0 65	1930

TABLE III.—ABSORPTION COEFFICIENTS OF MATERIALS BY DIFFERENT AUTHORITIES, USING REVERBERATION METHODS

B. S. = Bureau of Standards F. R. W. = F. R. Watson
B. R. S. = Building Research Station V. O. K. = V. O. Knudsen
W. C. S. = W. C. Sabine C. M. S. = C. M. Swan

No.	Material	Thickness, inches	Coefficients						Authority	Date
			128	256	512	1,024	2,048	4,096		
1	Acoustex (excelsior tile)	1	0.11	0 21	0 53	0.81	0.81	. . .	B. S.	1931
2	Acoustex (excelsior tile)	1½	0.16	0 34	0.75	0.85	0.84	.....	B. S.	1931
	Acoustex, with 6 coats spray paint	1½	0 14	0 30	0 74	0.90	0 85	.....	B. S.	1930
3	Akoustolith, grade D	1	0.08	0.13	0 25	0.54	0 67	.....	B. S.	1930
4	Akoustolith, grade D	2	0 15	0 26	0 59	0 74	0.52	.....	B. S.	1930
5	Arborite, fiber board	.	0.21	0.48	0 34	0.31	0.41	.....	B. S.	1930
6	Balsam wool, quiet tile	1	0.12	0 24	0 62	0.76	0 76	0 83	B. S.	1931
7	Brick wall, painted	18	0 012	0 013	0 017	0 020	0.023	0 025	W. C. S.	1900
8	Brick wall, unpainted	18	0.024	0 025	0 031	0 042	0 049	0 070	W. C. S.	1900
9	Carpet, lined	..	.	.	0.20	. .	. .	.....	W. C. S.	1900
10	Carpet, on concrete	⅜	0 09	0.08	0 21	0 26	0 27	0.37	B. R. S.	
11	Carpet lined with felt	½	0.11	0.14	0.37	0.43	0 27	0.27	B. R. S.	
12	Celotex, Acousti single B	⅝	0.08	0.18	0.48	0.63	0.75	....	B. S.	1931
13	Celotex, Acousti type B	⅞	. . .	0.24	0.47	0 49	0 60	...	F. R. W.	1927
		⅞	0 28	0 32	0.46	0.56	0.61	0.62	V. O. K.	1927
14	Celotex, Acousti double B	1³⁄₁₆	0 15	0 24	0 62	0.76	0.73	.....	B. S.	1931
15	Celotex, Acousti type BB	1¼	. ..	0.36	0 70	0.76	0.76	.....	F. R. W.	1927
16	Celotex, Acousti type BB	1¼	0 37	0.50	0.67	0.74	0.80	0.77	V. O. K.	1927
17	Celotex, Acousti triple B	1¼	0 18	0 33	0 84	0.97	0.76	.....	B. S.	1931
18	Cork-tile floor	.....	0 04	0 03	0 05	0 11	0 07	0.02	B. R. S.	
19	Corkoustic C	1½	0 08	0.14	0 61	0.56	0.64	0 65	B. S.	1931
20	Felt, standard hair	1	0 10	0.23	0 58	0 72	0.66	0.46	W. C. S.	1900
21	Flaxlinum	1	0 09	0 31	0 62	0 77	0 69		B. S.	1930
22	Flaxlinum	1	..	. .	0 49	0 61	0 67	0 66	F. R. W.	1927
23	Grills, ventilating	.....	0 15 to 0 50 depending on opening							
24	Insulite, standard board	½	0.23	0.26	0.28	0.29	0.32	.	V. O. K.	1926
25	Linoleum on concrete	.....	. .	...	0 03	. ..	.		W. C. S.	1900
26	Masonite, standard board	½	0 19	0 25	0.32	0.36	0.36	.	V. O. K.	1928
27	Nashkote A, asbestos hair felt, painted fabric	1	0.12	0 20	0.33	0.33	0.28	0 28	B. S.	1929
28	Nashkote A, perforated after erection	1	0.13	0.26	0 58	0.73	0.77	0.71	B. S.	1929

TABLE III.—ABSORPTION COEFFICIENTS OF MATERIALS BY DIFFERENT AUTHORITIES, USING REVERBERATION METHODS.—(*Continued*)

No.	Material	Thickness, inches	Coefficients						Authority	Date
			128	256	512	1,024	2,048	4,096		
29	Nashkote B-332	1	0.19	0.26	0.51	0.73	0.89	0.77	B. S.	1929
30	Nashkote B-332.....	¾	0.12	0.21	0.40	0.63	0.81	..	B. S.	1929
31	Plaster, gypsum on clay tile ..	.. .	0.013	0.015	0.020	0.028	0.040	0.050	W. C. S.	1900
32	Plaster, Acousticlime	¾	0.17	0.23	0.28	0.36	0.64	.	B. S.	1930
33	Plaster, Akoustolith	¼	0.13	0.21	0.19	0.23	0.33	.	B. S.	1931
34	Plaster, Akoustolith	½	0.21	0.24	0.29	0.33	0.37	0.42	C. M. S.	
35	Plaster, Hachmeister-Lind stippled with pins ½ in. deep..	.	0.16	0.19	0.25	0.36	0.44		B. S.	1930
36	Plaster, Macoustic perforated with pins ½ in. deep.	½	0.06	0.17	0.33	0.56	0.58	.	B. S.	1931
37	Plaster, Reverbolite perforations with large pins..	½	0.07	0.15	0.34	0.47	0.65		B. S.	1930
38	Sabinite A.... .	½	0.19	0.20	0.37	0.60	0.61	0.46	B. S.	1932
39	Plaster, hydraulic Sabinite	½	0.14	0.24	0.27	0.38	0.49		B. S.	1931
40	Rock wool	1	0.35	0.49	0.63	0.80	0.83	.	V. O. K.	1928
41	Rock wool.........	2	0.44	0.59	0.68	0.82	0.84		V. O. K.	1928
42	Rubber carpet on concrete....... .	3/16	0.04	0.04	0.08	0.12	0.03	0.10	B. R. S.	
43	Sanacoustic tile 1¼-in. rock-wool filler perforated metal.		0.17	0.41	0.82	0.94	0.85	.	B. S.	1930
44	Same on 1 3/16-in. furring......		0.19	0.64	0.87	0.87	0.80		B. S.	1931
45	Soundex (excelsior tile)........ .	¾	0.04	0.22	0.45	0.72	0.75	0.65	B. S.	1931
46	T. M. B. fiber tile (excelsior) ..	½	0.07	0.15	0.28	0.51	0.71		B. S.	1931
47	The same. ..	1	0.12	0.27	0.58	0.79	0.80		B. S.	1931
48	The same.... . .	1½	0.17	0.36	0.78	0.85	0.85	. .	B. S.	1931
49	U. S. Gypsum tile (mineral wool).	½	0.09	0.20	0.48	0.64	0.66	.	B. S.	1930
50	The same .	¾	0.13	0.28	0.61	0.73	0.73		B. S.	1930
51	The same	1	0.18	0.38	0.64	0.73	0.73		B. S.	1930
52	Wood paneling .	½	0.10	0.11	0.10	0.08	0.08	0.11	W. C. S.	1900

APPENDIX D

TABLE I.—NOISE DUE TO SPECIFIC SOURCES

Source or description of noise	Minimum, decibels	Average, decibels	Maximum, decibels	Distance source to microphone, feet
Hammering on steel plate, 4 blows per second	...	113	...	2
Riveter as heard near by	94	97	101	35
as heard ordinarily on street		79.5	.	200
Blast of explosives, open-cut digging	.	96	...	50
Subway station underground, noise on platform				
local station: express train passing	88	94	97	15 to 25
local station: local train	85	88.5	91	6 to 30
5 turnstiles, rush hour	78	83	91	3 to 7
Steamship whistle (fairly loud), tests near by	92	93	94	115
tests in 10th-floor office, windows open	59	61	65	1,450
as heard ordinarily on street	47	56	68	2,500
Automobile horn, 34 types directed toward microphone	72	91	102	23
as heard ordinarily on street	57	71.5	85	25 to 100
Elevated train, on open structure, as heard near by	85	89	91	15 to 20
as heard ordinarily on street	70	81.5	91	15 to 75
as heard at 750 ft	51	57.5	63	750
Fire apparatus: siren and bell	.	83	..	100
Bell: fire chief's car		81	.	50
Police whistle, as heard near by	80	82	83	15
as heard ordinarily on street	64	74	83	15 to 75
as heard at 185 ft.	55.5	57.5	62	185
Radio loud-speaker on street	73	79	81	30
Motor truck, not muffled	70	77.5	87	15 to 50
changing gears	68	74	83	15 to 50
as heard ordinarily on street (not including horn)	55.5	73.5	87	15 to 50
Electric street car moving fast	73.5	76.5	77.5	10 to 15
over track crossing	68	74	81	40
as heard ordinarily on street	63	72.5	83	15 to 50
moving slowly	68.5	69.5	70.5	10 to 15
Motor bus changing gears	68	71	75	15 to 50
Automobile squeaking brakes	62	71	76	15 to 50
changing gears	60	70.5	83	15 to 50
exhaust	64	70	74	15 to 50
as heard ordinarily on street (not including horn)	50	65.5	83	15 to 50

TABLE II.—NOISE IN BUILDINGS[1]

Location and Source	Level above Threshold, Decibels
Boiler factory	97
Subway, local station with express passing	95
Noisy factories	85
Very loud radio in home	80
Stenographic room, large office	70
Average of six factories	68
Information booth, large railway station	57
Noisy office or department store	57
Moderate restaurant clatter	50
Average office	47
Noises measured in residence	45
Very quiet radio in home	40
Quiet office	37
Quietest non-residential location	33
Average residence	32
Quietest residence measured	22

[1] Taken from "City Noise," Department of Health, New York City.

APPENDIX E

Within the past year, a considerable amount of research has been done in an attempt to standardize methods of measurement of absorption coefficients. Diverse results obtained by different laboratories on ostensibly the same material have led to considerable confusion in commercial application. Early in 1931, a committee was appointed by the Acoustical Society of America to make an intensive study of the problem with a view to establishing if possible standard procedure in the measurement of absorption coefficients. The work of this committee has taken the form of a cooperative research to determine first the sources of disagreement before making recommendations as to standard practice.

The Bureau of Standards sponsored a program of comparative measurements using a single method and apparatus in different sound chambers. This work was carried on by Mr. V. L. Chrisler and Mr. W. F. Snyder. The apparatus developed and used at the Bureau was transported to the Riverbank Laboratories, the laboratory at the University of Illinois, and the laboratory of the Electrical Research Products, Incorporated, in New York. Measurements were made on each of three identical samples in each of these laboratories. The Bureau of Standards equipment consisted of a moving-coil loud-speaker as a source of sound. The source was rotated as described in Chap. VI. The sound was picked up by a Western Electric condenser microphone. The microphone current was fed into an attenuator, graduated in decibels of current squared, and amplified by a resistance-coupled amplifier. By means of a vacuum-tube trigger circuit and a delicate relay, the amplified current was made to operate a timing device. This device measured automatically the time between the instant of cut-off at the source and the moment at which the relay was released. By varying the attenuation in the pick-up circuit, one measures the times required for the sound in the chamber to decay from the initial steady state to different intensity levels and thus may plot the relative intensities as a function of the time. It consists essentially of an electrical ear whose threshold can be varied in known ratios.

The purpose of the Bureau's work was to determine the degree to which the measured values of coefficients are a function of the room in which the measurements are made.

At the Riverbank Laboratories, a comparison of the results obtained by different methods in the same room has been undertaken. The methods may be summarized as follows:

1. Variable pick-up, constant source.
   a. Loud-speaker, fixed current.
   b. Organ pipe, constant pressure.
2. Variable source (loud-speaker) constant pick-up.
   a. Moving microphone, with electrical timing.
   b. Ear observations (four positions).

3. Constant source (organ pipe).
Constant pick-up, ear (calibrated empty room).

In these measurements, the loud-speaker source was mounted at a fixed position in the ceiling of the room. The organ pipes were stationary. In all cases, the large steel reflectors already described were in motion during all measurements. When the microphone was used, this was mounted on the reflectors. The ear observations were made with the observer inclosed in a wooden cabinet placed successively at four different positions in the room. A detailed account of these experiments cannot be given here, but a summary of the results is presented in Tables I and II.

TABLE I.—ABSORPTION COEFFICIENTS OF THE SAME SAMPLE AS MEASURED IN DIFFERENT ROOMS WITH BUREAU OF STANDARDS EQUIPMENT

Frequency	Laboratory				
	Bureau of Standards	Riverbank	University of Illinois		E.R.P.I
128	0.18	0.14	..		0.18
256	0.36	0.34	0.38	0.43	0.42
512	0.73	0.77	0.75	0.86	0.72
1,024	0.89	0.88	0.87	0.95	0.83
2,048	0.83	0.90	0.87	1.06	0.80
4,096	0.69	0.67	0.74	0.74	0.56

TABLE II.—ABSORPTION COEFFICIENTS OF THE SAME SAMPLE MEASURED IN THE RIVERBANK SOUND CHAMBER, USING DIFFERENT METHODS

Frequency	Method				
	1*a*	1*b*	2*a*	2*b*	3
128	0.15	0.23	0.15	0.19	0.18
256	0.36	0.52	0.39	0.48	0.45
512	0.71	0.69	0.73	0.67	0.63
1,024	0.91	0.86	0.90	0.92	0.69
2,048	0.85	0.74	0.82	0.74	0.71
4,096					

A full report of the Bureau of Standards investigation has not as yet been published, and further study of the sources of disagreement is still in progress at the Riverbank Laboratories. The data of the foregoing table serve, however, to indicate the degree of congruence in results that is to be expected in methods of measurements thus far employed.

Method 1*a* in Table II is the same as the Bureau of Standards method except that in the former, we have a moving pick-up with a stationary source; while in the latter, we have a stationary pick-up with a moving source. The agreement is quite as close as can be expected in measurements of this sort. In Table II, we note rather wide divergence at certain frequencies between the results of ear observations and those in which a microphone was employed. Very recent work at both the Bureau of Standards and the Riverbank Laboratories points to the possibility that in some cases, the rate of decay of reverberant sound is not uniform. If this is true, then the measured value of the coefficient of absorption will depend upon the initial intensity of the sound and the range of intensities over which the time of decay is measured. In such a case, uniformity of results can be obtained only by adoption by mutual agreement of a standard method and carefully specified test conditions. This is the procedure frequently followed where laboratory data have to be employed in practical engineering problems. The measurements of thermal insulation by materials is a case in point. Pending such an establishment of standard methods and specifications, the data on absorption coefficients given in Table II (Appendix C) may be taken as coefficients measured by the method originally devised and used by W. C. Sabine. The data given out by the Bureau of Standards for 1931 and later were obtained by the method outlined in the second paragraph of this section.

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