and the ratio between such quantities, measured for two stars of approximately the same temperature (whose energy background distribution may, therefore, be taken to be approximately the same) should give the most reliable intensity ratio that is at present available. Few such intensities have as yet been measured, and these, for the most part, refer to wide and diffuse lines,[10] like those of the stars of class A.

Summary.—1. The fundamental equation for ionization is applied in comparing the conditions in the atmospheres of stars that differ both in temperature and pressure.

2. The various forms of the ionization curve are reproduced, both for temperature changes at constant pressure, and for pressure changes at constant temperature. The change in the number of effective atoms for a given change of temperature and pressure can be read from either type of curve.

3. The curves for pressure change at constant temperature may be used to derive a ratio in partial electron pressure between the atmospheres of two stars of known temperatures, if the intensity ratio for the corresponding spectrum line is known.

[1] NATIONAL RESEARCH FELLOW.

[2] Fowler and Milne, *M. N. R. A. S.*, **84**, 1924 (499).

[3] Payne, *Harvard Monograph No.* **1**, 1925 (137).

[4] Milne, *M. N. R. A. S.*, **85**, 1925 (739).

[5] Payne, *Harvard Monograph No.* **1**, 1925 (140, 151).

[6] Milne, *M. N. R. A. S.*, **85**, 1925 (783).

[7] Russell, *Harv. Circ.*, **291**, 1926.

[8] Payne, *Harvard Monograph No.* **1**, 1925 (101).

[9] *Ibid.* (138).

[10] Shapley, *Harv. Bull.*, **805**, 1924; Payne and Shapley, *Harv. Reprint*, **28**, 1926.

ON THE SPECTRA OF STARS OF CLASS c F8

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The *c*-stars form a group of decided spectroscopic qualities and distinct physical properties; they have long been regarded as displaying very low values of surface gravity, and accordingly as possessing atmospheres of very low pressure. Their spectra are signalized by the strength of the lines of ionized atoms. The methods described in the previous paper will be applied in the present one to an examination of certain physical implications of the spectroscopic characteristics of the *c*-stars.

In a general study of the *c*-stars that is being undertaken by the writer, the two stars γ Cygni and δ Canis Majoris, which have very similar spectra, have been selected for special analysis. Lines of known series relations are found in the spectra of these stars for the atoms of Fe, Fe^+, Ti, Ti^+, Ca, Ca^+, Sr, Sr^+, Sc, Sc^+, Y, Y^+. Curves of equal temperature are reproduced, for each of these atoms, in figure 1, the assumed temperature being 5600°, which is slightly lower than that for a normal star of class *F8*, to allow for the pronounced *c*-character. The curves are derived from the elementary Fowler-Milne formula, no account being taken of other

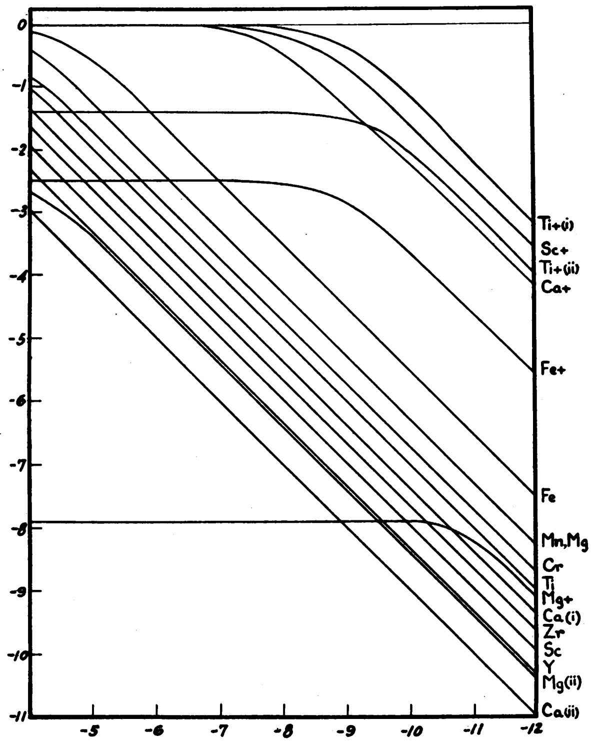

FIGURE 1

Curves of equal temperature, corresponding to the atoms indicated on the right-hand margin, drawn for T = 5600°. Ti + (i), Ca (i), and Mg represent ultimate lines; Ti + (ii), Ca (ii), and Mg (ii) represent lines of excitation potential 1.16, 1.88, and 2.67 volts, respectively.

states of the atom than the one under discussion. The corresponding correction would be determinate if the relative probabilities of the other important states could be measured or deduced.

Several marked characteristics of the spectrum of a *c*-star of class *F8* receive ready interpretation from the diagram. The excess of intensity of all the ionized lines, except those of iron, over that of the lines of the corresponding neutral atom, is at once explained by their far greater values of n_r; the excess of ionized over neutral intensity is greatest for scandium and yttrium, next for titanium, followed closely by calcium and strontium, and is negative for iron. In the absence of accurate measures it is not possible to check this order observationally, but it may be mentioned that while the lines of the ionized atoms of yttrium, scandium, and even lanthanum and zirconium are definitely present, the lines of neutral yttrium, lanthanum and zirconium are altogether absent from the spectrum, those of neutral scandium are very faint; on the other hand the neutral iron lines are intense.

If any quantitative estimates of line intensity were available, it would be possible to obtain an estimate of the partial electron pressure in the atmospheres of stars whose spectra show the lines of the same atom in two or more stages of ionization. The ratio in apparent intensity for the neutral and ionized lines could be used to read directly from the diagram the value of P_e corresponding to that ratio in n_r. The correction for difference of n_r due to difference of level[2] could be applied to the result, and the actual relative values of P_e appropriate to each of the atomic states concerned could be thus directly deduced.

Although it is hoped that significant values of relative intensity will soon be derived[3] it seems to be worthwhile to point out a few qualitative conclusions that can be drawn from material already available. The curves for Fe and Fe^+ cross where log P_e (expressed in atmospheres) is -7. If we assume, for the moment, that the absorption efficiencies of the atoms of Fe and Fe^+ are the same, we may conclude that the value of P_e is greater than 10^{-7} atmospheres for stars, such as the sun, for which the Fe lines are stronger than the Fe^+ lines and would be less than this quantity for stars for which the Fe^+ lines were stronger. The fact that no such stars are found sets a lower limit to the observed stellar values of P_e, at least for second type stars. If the pressure for the Sun is of the order of 10^{-4} atmospheres, the lines of Sc^+, Y^+, Sr^+, and Ca^+ should be stronger than the corresponding lines for the neutral atoms.

The following table gives the values of the intensities of the strongest unblended lines of these and certain other atoms, taken from Rowland's table, which, though it does not supply intensities appropriate to the determination of intensity ratios, is a satisfactory source of intensity differences, at least for the blue and violet portions of the spectrum. The table contains, in successive columns, the atom, the wave-length of the line considered, the corresponding ionization and excitation potentials, the logarithm of the fractional concentration computed for a partial pressure of 10^{-4} atmospheres, the excess (difference between the values of log n_r for neutral and ionized atoms), and the solar intensity, taken from Rowland's table. Italicized values in the third column are estimated ionization potentials, derived from astrophysical and other evidence. They are probably of the right order; but for the present purpose an error of two volts would not be serious, as the important quantity is the excitation potential. The ionization potential of Ti^{+}, marked with an asterisk, was kindly communicated by Professor Russell.

TABLE 1

ATOM	LINE	IONIZATION POTENTIAL	EXCITATION POTENTIAL	LOG n_r (P_e = 10^{-4} ATM.)	EXCESS	SOLAR INTENSITY
Mg	5183.67	7.61	2.67	$\bar{1}.6$	6.5	30
Mg^{+}	4481.33	14.97	8.83	$\bar{8}.1$		..
Ca	4226.73	6.09	0.00	$\bar{2}.7$	1.3	20
Ca^{+}	3933.66	11.82	0.00	0.0		1000
Sr	4607.34	5.67	0.00	$\bar{1}.0$	1.0	1
Sr^{+}	4077.71	10.98	0.00	0.0		8
Ba	5535.53	5.19	0.00	$\bar{2}.2$	1.8	000
Ba^{+}	4554.04	9.96	0.00	0.0		8
Sc	6305.69	5.4	0.00	$\bar{2}.0$	2.0	000
Sc^{+}	4314.09	*13.0*	0.60	0.0		5
Y	4128.32	5.0	0.00	$\bar{3}.7$	2.3	0
Y^{+}	3774.33	*12.5*	0.00	0.0		3
Ti	3998.65	6.5	0.00	$\bar{1}.0$	1.0	4
Ti^{+}	3088.03	13.6*	0.00	0.0	0.4	7
Ti^{+}	4300.05	13.6*	1.16	$\bar{2}.6$		3
V	4379.24	*6.5*	0.28	$\bar{1}.0$	1.0	4
V^{+}	3125.29	*12.5*	0.00	0.0		5
Cr	4254.34	6.72	0.0	$\bar{1}.2$	?	8
Cr^{+}	3124.97	*12.5*	?	?		4
Mn	4033.22	7.41	0.0	$\bar{1}.6$	?	7
Mn^{+}	3442.90	*12.5*	?	?		6
Fe	3859.91	8.05	0.0	$\bar{1}.9$	−2.4	20
Fe^{+}	4923.92	*12.5*	2.88	$\bar{3}.5$		5

It may be remarked that the excess of intensity of the lines of neutral chromium and neutral manganese over those of the corresponding ionized atoms indicates that these ionized lines of unassigned series relations are not ultimate lines; the excitation potential corresponding to the Cr^{+} line is probably greater than that for the Mn^{+} line.

The results for the solar atmosphere only confirm conclusions that are already fairly certain, and are chiefly of use as a check on the standpoint of the present paper. The corresponding results for the atmospheres of the *c*-stars are, however, of more original interest.

It is to be noticed from figure 1 that the difference of computed intensity between the ionized and neutral lines increases, as P_e is reduced, to a limit which occurs at different values of P_e for different elements, varying according to the ionization potential. As the pressure in the atmosphere of a star at a given temperature is reduced, the ionized lines will become progressively more prominent, as compared to the neutral lines, down to a limit of pressure that differs for different elements. Beyond this limit no change in relative intensities of neutral and ionized atoms should occur, although these low pressures (which do not seem to be encountered in practice) could be detected, by comparing the line intensities for different elements, down to 10^{-10} atmospheres.

The *c*-character is displayed in varying degrees by different stars; some which show it most markedly were called *c*-stars by Miss Maury,[4] and others, in which the character is present, but less marked, were called *ac*-stars. It was natural to assume that the condition which is recognized by the *c*-character is more strongly present in the *c*-stars than in the *ac*-stars. The facts now receive the interpretation indicated above; the progression in *c*-character, of which the prominence of the ionized lines is a typical feature, is the progression that would be expected for a series of stars with decreasing pressure in the atmosphere.

Although there are degrees of *c*-character, it should not be too readily assumed that the normal giants, the *ac*-stars, and the *c*-stars form a *continuous* sequence. From the spectra this appears unlikely, but more numerous and more precise measures of line intensity are required before a definite conclusion can be reached.

The following table is in the same form as table 1, and contains corresponding data for a typical *c*-star of class *F8*, such as γ Cygni. The last

TABLE 2

ATOM	LOG n_r	EXCESS	INTENSIFICATION
Mg	$\bar{4}.2$	-3.9	$-$
Mg^+	$\bar{8}.1$		
Ca	$\bar{3}.2$	2.8	+
Ca^+	0.0		
Sr	$\bar{3}.5$	2.5	+
Sr^+	0.0		
Ba	0.0	2.2	+
Ba^+	0.0		
Sc	$\bar{4}.5$	3.5	+
Sc^+	0.0		
Y	$\bar{4}.2$	3.8	+
Y^+	0.0		
Ti	$\bar{3}.5$	2.5	+
Ti^+	0.0	0.9	+
Ti^+	$\bar{2}.6$		
V	$\bar{3}.8$	2.2	+
V^+	0.0		
Cr	$\bar{3}.8$	?	+
Cr^+	?	?	
Mn	$\bar{2}.4$	?	$-$
Mn^+	?		
Fe	$\bar{1}.0$	-1.5	$-$
Fe^+	$\bar{3}.5$		

column, headed "Intensification," indicates the relative behavior of the lines of the ionized and neutral atoms; the plus sign indicates that the ionized lines are the stronger, and the minus sign, that the neutral lines are the more intense. The intensifications contained in this column refer not to definite pairs of lines, as in table 1, but to the general relative behavior of the neutral and ionized lines throughout the spectrum. A value of $10^{-5.5}$ atmospheres is used for P_e, instead of the value 10^{-4} atmospheres, as for the sun. This estimate is adopted from various considerations; if we assume that the masses and temperatures of the stars are equal, and that the partial electron pressures in their atmospheres are proportional to the square root of the surface gravity,[5] it provides a difference of seven and a half magnitudes between the sun and γ Cygni, a value that is certainly not in excess of the truth. The temperatures of γ Cygni and the sun are probably nearly equal; if γ Cygni is more massive than the sun, and the surface gravities are to remain in a given ratio, γ Cygni must be larger than the previous assumption required, and its total luminosity, therefore, still greater than that of the sun. A value of $10^{-5.5}$ is, therefore, *assumed* for present purposes, as the partial pressure in the atmosphere of γ Cygni; later it will be possible to invert the procedure and derive the ratio in P_e from measured differences in line intensity.

The table shows that the well-known features in the spectrum of a *c*-star can be qualitatively accounted for. The stronger line has in every case a numerically smaller value of log n_r, corresponding to a greater fractional concentration of the atom concerned. Moreover, the logarithm of the ratio of ionized to neutral atoms of any one element (contained in the column headed "excess") is greater in every case than it is for the corresponding atoms in table 1, expressing numerically the greater prominence of the ionized lines in the spectrum of the *c*-star. When accurate intensity measures are available, these computed ratios can be put to a numerical test. It is to be remarked that corrections for relative abundance of any element are eliminated by the use of neutral and ionized atoms of the same element in making the numerical application.

All the above conclusions rest upon the assumption either that the formulae in question are applicable at these low pressures, or that the deviations from them affect both neutral and ionized atoms to the same extent. The absorbing efficiencies of the two types of atoms are also assumed to be equal, but this assumption could be replaced by a suitable correction, if the ratio of the absorbing efficiencies were known. The whole of the present paper is designed rather to indicate a method than to present conclusions. It should be possible, with more reliable data than are now available, to measure by means of the ionization curves the relative values of P_e, and hence the ratios in surface gravity, for pairs of stars, and to determine a quantity that is a function only of absolute magnitude and mass, and thus to estimate the absolute luminosities of super-giant stars.

[1] NATIONAL RESEARCH FELLOW.

[2] Payne, *Harvard Monograph No.* 1, 1925 (137).

[3] Payne and Shapley, *Harv. Reprint*, 28, 1926.

[4] Maury, *Harv. Ann.*, 28, 1900 (55).

[5] Milne, *M. N. R. A. S.*, 85, 1925 (783).

THE APPARITION DATES OF THE ANDROMEDE (OR BIELID) METEOR SWARMS

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In a paper reviewing Quetelet's list[1] of meteor shower dates, H. A. Newton[2] says:

"IV. *The December Periods.* There appear to be two epochs in December, each marking a distinct shower, viz.: Dec. 6th–7th, and Dec. 12th. There is no early date corresponding to the first epoch...."

By "early date" he appears to mean dates of the eighteenth century and previous. The "first epoch" seems to mean the time of the shower now known as the Andromede or Bielid shower. This shower is traced by C. P. Olivier[3] back to 1741, when it appeared in early December. Its later apparitions came earlier and earlier in the year, but with a puzzling irregularity in the rate of advance of the date.

In 1925 the question arose as to what was the proper date to suggest to a group of amateurs (the Bond Astronomical Club) for observation of these meteors. Finding the answer somewhat uncertain, the writer was led to apply to the data the method used by Newton, in the same paper, in forming his lists of the ancient shower dates of various swarms. Of his tables Newton says:

"In the following tables are given the historic dates of star showers from Quetelet's list..... They are expressed in the Gregorian calendar, and therefore represent approximately corresponding dates of the tropical year.... To express these dates in a sidereal year there is given at the same time the *corresponding* day (and fraction of a day) of 1850; that is, the time when the earth's longitude in her orbit, measured from a fixed equinox, was the same as on the day of the shower. The following formula was used in the computation.

"Let x be the number of days to be added to the recorded date, expressed in the Gregorian calendar, t the given year of the Christian era, n the