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of Fraunhofer lines**

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The influence of electron captures on the contours of Fraunhofer lines.

By Bengt Strömgren.

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The problem of the central intensities of stellar absorption lines is considered. The discussion is restricted to absorption lines originating from the ground state. The atomic processes of exciting and de-exciting of the upper stationary state corresponding to the line considered are transitions from and to the ground state together with electron captures on the stationary state and photoionisation from it. The corresponding emission coefficients are evaluated. The resulting fundamental equation governing the radiation field in the stellar atmosphere is discussed. The predictions of the theory concerning central intensities of strong absorption lines are compared with observations. It is possible to account for the observational facts qualitatively, and partly also quantitatively.

1. The problem of the central intensities of stellar absorption lines has been considered by several authors in recent years. SCHUSTER's theory of pure scattering when combined with the natural damping expression for the line absorption coefficient by UNSÖLD [1] gave central intensities practically equal to zero. As is well known this result disagrees with observations. Subsequent more refined theories based on the mechanism of pure scattering yielded the same result [cfr. (7)]. SCHWARZSCHILD [2] already, in his classical paper on stellar atmospheres, had revealed the fundamental difference between scattering and true absorption. It was clear from EDDINGTON's discussion of the problem of stellar absorption lines [cfr. (3) and (4)] that a stellar atmosphere, in which absorption was of importance compared with scattering right up to the surface of the atmosphere, would radiate a spectrum with non-zero central intensities. As the discussion of line scattering had shown that *continuous* absorption was negligible compared with line scattering near the center of a strong absorption line, it was natural to consider the effect of *line* absorption in comparison with line scattering. EDDINGTON [3] and MILNE [5] had shown that collisions involving the absorbing atoms gave rise to line absorption. In fact UNSÖLD [6] and WOOLLEY [7] found that collisions were probably responsible for the observed central intensities of the order of magnitude 0.1 of the intensity of the adjacent continuous spectrum. Extensive numerical calculations by PANNEKOEK [8] showed however that collisions have a very small effect only, in the case of strong absorption lines, the reason being that strong absorption lines are formed in very high layers of the atmosphere, where the pressure is so low that the effect of collisions is practically negligible.

ROSSELAND [9] pointed out that fluorescence ought to be of importance in connection with the problem of central intensities. UNSÖLD [10] and PANNEKOEK [11] have discussed the fluorescence coupling between discrete stationary states of the absorbing atoms in stellar atmospheres. It appears that in many important cases this type of coupling has no appreciable effect. There may be cases, however, where it is of importance. It was suggested by the author [12] that fluorescence involving free electrons might explain the observed central intensities, in other words that the light observed in the centers of strong absorption lines was due to emissions following captures of free electrons on the upper stationary state corresponding to the absorption line in question. That the number of such captures is larger than the number of reverse processes (photoionisations) is obvious, because the reduced intensity in the absorption line is directly connected with a defect in the number of atoms in the corresponding upper stationary state. In connection with the problem of central intensities the mechanism of electron captures and subsequent line emission was independently put forward by WOOLLEY [13] in a recent paper. WOOLLEY, using a method due to ROSSELAND [14], gave a quantitative discussion of the fluorescence coupling of the stationary states of the hydrogen atom including those corresponding to free electron. From this discussion WOOLLEY concluded that the observed central intensities of $H\alpha$ and $H\beta$ could be explained with the aid of the mechanism in question.

It is the object of the present paper to show how the mechanism of electron captures can be incorporated in the general theory of stellar absorption lines.

We shall restrict ourselves in the present paper to the case that the effect of fluorescence coupling with other discrete stationary states is negligible (cfr. above). It may be noted, however, that electron captures have an indirect effect on this type of coupling.

2. The fundamental differential equations governing the radiation field in a stellar atmosphere give the variation of intensity with depth in terms of absorption and emission coefficients of the matter interacting with the radiation field. The physical side of the problem considered is the evaluation of absorption and emission coefficients in terms of the properties of stellar matter.

We shall base the following discussion of absorption and emission processes on the EINSTEIN-EHRENFEST picture of naturally broadened discrete stationary states as elaborated by WEISSKOPF and WIGNER [15]. The results are not exact in as far as intensity distribution and polarisation

are not given by the simplified theory. The simplification in question is, however, common to all discussions of stellar absorption lines. Probably the effects neglected are quite small [cfr. (9), p. 127].

We shall restrict the problem by considering only absorption lines originating from the ground state of a neutral atom or an ion. (It is well known, however, that results obtained for such lines have applications to certain other lines also, cfr. p. 253). The expression for the absorption coefficient can be written down immediately for such lines, being practically independent of the unknowns of the problem, viz. the intensities of the radiation field. In fact, the ground state being practically sharp, the absorption coefficient is obtained as the product of the number of atoms in the ground state and an atomic absorption coefficient that depends on the conditions, which the atoms are subject to, only through the damping constant of the absorption line in question. Now the damping constant generally is practically independent of the radiation field, the reason being that the effective mean life-time of the upper state in question is generally only very slightly reduced by transitions induced by the radiation field.

The emission coefficient on the other hand generally depends essentially on the unknown intensities and must be evaluated in terms of these. It is only in the case of thermodynamical or local thermodynamical equilibrium that the emission coefficient is independent (or practically independent) of the radiation field, being given by the **KIRCHHOFF** expression $j_\nu = k_\nu B_\nu(T)$, where k_ν and j_ν are absorption and emission coefficients respectively and $B_\nu(T)$ is the **PLANCK** intensity corresponding to the frequency and the local temperature T .

In order to calculate the emission coefficient we shall consider the number of atoms in a given *element of state* belonging to the broadened upper state corresponding to the absorption line considered. We shall denote the number of atoms in the stationary state k with energies between $E^{(k)}$ and $E^{(k)} + dE$ per unit volume by $N(E^{(k)}) dE$. The central energy of the stationary state k we shall denote by $E_0^{(k)}$. It is then well known [cfr. (15)] that in thermodynamical equilibrium, and also in local thermodynamical equilibrium, the energies of the atoms in the stationary state k are distributed according to a *normal distribution*, as follows:

$$N^{(0)}(E^{(k)}) dE = \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2} N_k^{(0)} \quad (1)$$

with

$$\beta_k = \frac{h}{4\pi} \frac{1}{\tau_k} \quad (2)$$

Here the suffix 0 indicates that the quantity has the value corresponding to thermodynamical equilibrium. Further τ_k denotes the mean life-time of an atom in the stationary state k ; τ_k is the same for all the elements of state belonging to the stationary state k .

It is convenient to introduce $N^{(0)}(E^{(k)})$ as given by (1) as a unit for $N(E^{(k)})$. We therefore define a quantity $\xi(E^{(k)})$ as follows:

$$N(E^{(k)}) = \xi(E^{(k)}) N^{(0)}(E^{(k)}). \quad (3)$$

Now we may write ξ_ν instead of $\xi(E^{(k)})$. It is characteristic of the method adopted in the treatment of the interaction of matter and radiation that BOHR'S frequency relation

$$h\nu = E^{(k)} - E_1 \quad (4)$$

is assumed to hold exactly. Further it is assumed that the ground state is sharp (with energy E_1). Consequently by (4) ν is uniquely determined by $E^{(k)}$ and $E^{(k)}$ by ν .

We now proceed to consider the various processes by which an element of state belonging to the stationary state k is excited and de-excited (the latter term being understood in its broadest sense).

An element of state $(E^{(k)}, E^{(k)} + dE)$ is excited in consequence of transitions from the ground state by which light quanta of frequencies between ν and $\nu + h dE$ are absorbed, where ν is given by (4). The number of such transitions, per unit time and unit volume, is according to WEISSKOPF and WIGNER [cfr. (15)] given by

$$N_1 b_{1k} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2} \cdot \int I_\nu \frac{d\omega}{4\pi}, \quad (5)$$

where N_1 is the number of atoms in the ground state per unit volume and b_{1k} is the EINSTEIN transition probability characteristic of the total number of transitions from the ground state to any element of state belonging to the stationary state k . The atomic constant b_{1k} in particular has the same value for all elements of state belonging to k . The integral (understood to be taken over all directions) denotes the mean intensity J_ν in the frequency given by (4) in terms of $E^{(k)}$. It follows from (5) that when, as is the case in thermodynamical equilibrium, J_ν is practically constant over the spectral line, then a normal distribution (1) results (or rather, if present, is not disturbed) by excitations of the type considered.

Further an element of state $(E^{(k)}, E^{(k)} + dE)$ is excited in consequence of transitions from the ground state due to collisions. The number of such transitions, per unit time and unit volume, is given by

$$N_1 c_{1k} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2}. \quad (6)$$

Here c_{1k} depends on local physical parameters; c_{1k} is the same for all elements of state belonging to k . The form of (6) is inferred from our knowledge that in thermodynamical equilibrium the normal distribution (1) must result by excitations of the kind considered, combined with the fact that the entity of collisions is practically equivalent to the entity of collisions in a certain region in thermodynamical equilibrium (cfr. a recent paper by WEISSKOPF [16], and also for this way of reasoning a paper by MILNE [5]).

Now in a complete theory we ought to consider the analogous transitions from other stationary states than the ground state. According to the scope of the present paper we shall however neglect the fluorescence coupling with the discrete stationary states (cfr. p. 238). We are then left with the transitions from the stationary states corresponding to free electron discussed in the introduction. The number of transitions of this kind to an element of state $(E^{(k)}, E^{(k)} + dE)$, i. e. the number of captures of free electrons on this element of state, per unit time and unit volume, is given by

$$C_{fk} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2}, \quad (7)$$

where C_{fk} (the index f is used to indicate all the states corresponding to free electron) like c_{1k} depends on local physical parameters and is the same for all elements of state belonging to the discrete stationary state k . It is seen that C_{fk} is the *total* number of captures of electrons on the stationary state k , per unit time and unit volume. The conclusion that (7) is the correct form is reached by a reasoning similar to that leading to (6). The entity of captures is practically equivalent to the entity of captures in a certain region in thermodynamical equilibrium, and this again is known to be described by (7) because it must not disturb the normal distribution (1). (It should be noticed in connection with arguments of this kind, that the transition probabilities of the reverse transitions *from* elements of state belonging to k are equal for all these elements of state).

Next we consider the transitions by which the elements of state belonging to k are de-excited, i. e. we consider the reverse transitions of those discussed above.

An element of state $(E^{(k)}, E^{(k)} + dE)$ is de-excited in consequence of transitions to the ground state by which light quanta of frequencies between ν and $\nu + hdE$ are emitted, where ν is given by (4). For the sake of simplicity we shall consider only spontaneous transitions of this kind and neglect transitions induced by the radiation field. It is well known that when the condition $h\nu/kT \gg 1$ is satisfied (and it is in most practical applications), then this procedure is justified. On the other hand it would

present no difficulty to take account of induced emissions if required. We then have according to WEISSKOPF and WIGNER [15] that the number of emission transitions from an element of state $(E^{(k)}, E^{(k)} + dE)$ to the ground state is given by

$$N(E^{(k)}) dE \cdot a_{k1}. \quad (8)$$

Here a_{k1} is the EINSTEIN transition probability measuring the *total* number of transitions from k to the ground state. In particular a_{k1} has the same value for all elements of state belonging to k :

Further an element of state $(E^{(k)}, E^{(k)} + dE)$ is de-excited in consequence of transitions to the ground state due to collisions of the second kind. The number of such transitions, per unit time and unit volume, is given by

$$N(E^{(k)}) dE \cdot c_{k1}, \quad (9)$$

where c_{k1} depends on local physical parameters and has the same value for all elements of state belonging to k .

Finally an element of state $(E^{(k)}, E^{(k)} + dE)$ is de-excited in consequence of transitions to stationary states corresponding to free electron, i. e. by photoionisation. The number of such transitions, per unit time and unit volume, is given by

$$N(E^{(k)}) dE \cdot C_{kf}, \quad (10)$$

where C_{kf} depends on local physical parameters, namely the intensities of the radiation in the frequencies capable of photoionisation from the stationary state k . Again C_{kf} has the same value for all elements of state belonging to k .

We can now obtain an equation for the number of atoms in an element of state $(E^{(k)}, E^{(k)} + dE)$ by writing down the condition of stationarity. The number of transitions to $(E^{(k)}, E^{(k)} + dE)$ obtained by adding the contributions (5), (6) and (7) must be equal to the number of transitions from $(E^{(k)}, E^{(k)} + dE)$ obtained by adding the contributions (8), (9) and (10):

$$\begin{aligned} N_1 b_{1k} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2} J_\nu + N_1 c_{1k} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2} \\ + C_{fk} \cdot \frac{\beta_k}{\pi} \frac{dE}{\beta_k^2 + (E^{(k)} - E_0^{(k)})^2} \\ = N(E^{(k)}) dE \cdot a_{k1} + N(E^{(k)}) dE \cdot c_{k1} + N(E^{(k)}) dE \cdot C_{kf}. \end{aligned} \quad (11)$$

The equation may be written in a simpler form by introducing $\xi(E^{(k)})$ instead of $N(E^{(k)})$ according to (3) and using (1):

$$N_1 b_{1k} \cdot J_\nu + N_1 c_{1k} + C_{fk} = \xi(E^{(k)}) \{N_k^{(0)} a_{k1} + N_k^{(0)} c_{k1} + N_k^{(0)} C_{kf}\}. \quad (12)$$

Here again the suffix o indicates that the value of the quantity in thermodynamical equilibrium at the local temperature and density is meant.

We now make the simplifying assumption that the state of ionisation of the kind of atom considered is the same as in thermodynamical equilibrium at the local temperature and density. The more refined calculations of the state of ionisation by GERASIMOVIČ and PANNEKOEK [17] have in fact shown that in normal stellar atmospheres the error of such an assumption is not large. Further we make the assumption that C_{kf} , the number of photoionisations per unit time and unit volume, has the same value as in thermodynamical equilibrium at the local temperature and density (where the radiation field is a uniform PLANCK radiation field corresponding to the local temperature). This assumption is justified by quite similar calculations (cfr. also Table 3, p. 252).

It should be emphasized, however, at this point, that these assumptions are only justified in the case of a normal stellar atmosphere, and even there the deviations in question are of importance, as we shall see. We shall discuss below what difference it makes in the resulting formulae, when there are deviations from the assumptions made.

With the assumptions made we can immediately write down the relations connecting the coefficients on the left-hand side of equation (12) with those on the right-hand side. Applying (12) to the special case of a region in thermodynamical equilibrium at the local temperature and density we know that each term on the left-hand side is equal to the corresponding term on the right-hand side, because in this case there is *detailed* balancing. We therefore have, remembering that in thermodynamical equilibrium $\xi(E^{(*)})$ is 1 by definition and J_ν is equal to $B_\nu(T)$:

$$\left. \begin{aligned} N_1^{(0)} b_{1k} \cdot B_\nu(T) &= N_k^{(0)} a_{k1}, \\ N_1^{(0)} c_{1k}^{(0)} &= N_k^{(0)} c_{k1}^{(0)}, \\ C_{fk}^{(0)} &= N_k^{(0)} C_{kf}^{(0)}. \end{aligned} \right\} \quad (13)$$

Now by virtue of our assumptions we have

$$\left. \begin{aligned} N_1^{(0)} &= N_1, \\ c_{1k}^{(0)} &= c_{1k}, \quad c_{k1}^{(0)} = c_{k1}, \\ C_{fk}^{(0)} &= C_{fk}, \quad C_{kf}^{(0)} = C_{kf}, \end{aligned} \right\} \quad (14)$$

for the entities, both of collisions and of electron captures, are identical.

From (13) and (14) the relations required are immediately derived:

$$\left. \begin{aligned} N_1 b_{1k} \cdot B_\nu(T) &= N_k^{(0)} a_{k1}, \\ N_1 c_{1k} &= N_k^{(0)} c_{k1}, \\ C_{fk} &= N_k^{(0)} C_{kf}. \end{aligned} \right\} \quad (15)$$

The first equation reduces to the well-known relation between the EINSTEIN coefficients, when BOLTZMANN's formula and PLANCK's formula, simplified in accordance with the assumption $h\nu/kT \gg 1$ into WIEN's formula, are used. The second equation is equivalent to an equation derived by MILNE [5]. The third equation finally is an analogous equation connecting the probabilities of photoionisation and electron capture. Introducing now equations (15) in (12) and dividing both sides by the common factor $N_k^{(0)}$ we obtain the following equation:

$$a_{k1} \frac{J_\nu}{B_\nu(T)} + c_{k1} + C_{kf} = \xi_\nu \{a_{k1} + c_{k1} + C_{kf}\}. \quad (16)$$

Solving for ξ_ν we get

$$\xi_\nu = \frac{a_{k1} \frac{J_\nu}{B_\nu(T)} + c_{k1} + C_{kf}}{a_{k1} + c_{k1} + C_{kf}}. \quad (17)$$

Introducing

$$\varepsilon = \frac{c_{k1} + C_{kf}}{a_{k1} + c_{k1} + C_{kf}}, \quad (18)$$

we can write this in the form

$$\xi_\nu = (1 - \varepsilon) \frac{J_\nu}{B_\nu(T)} + \varepsilon. \quad (19)$$

It is now an easy matter to calculate the emission coefficient j_ν . In a region in thermodynamical equilibrium at the local temperature and density the KIRCHHOFF relation $j_\nu = k_\nu B_\nu(T)$ is valid. Now the contribution to the emission coefficient in a frequency ν due to the emission in the spectral line in question is proportional to the number of atoms in the corresponding element of the stationary state k per unit volume, and further proportional to the emission transition probability from this element of state to the ground state. Neglecting induced emission we have that this transition probability is an atomic constant; the atomic constant has the same value for all elements of state belonging to k . It follows from the definition of ξ_ν [cfr. equation (3)] that the required contribution to the emission coefficient is equal to ξ_ν times the value of the emission coefficient in a region in thermodynamical equilibrium at the local temperature and density.

Introducing now in accordance with the usual notation the coefficient of continuous absorption k_ν and the coefficient of spectral line absorption l_ν corresponding to the absorption line considered, we can write down the expression for the emission coefficient, remembering that for *continuous*

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absorption and emission the KIRCHHOFF relation $j_\nu = k_\nu B_\nu(T)$ gives sufficient accuracy:

$$j_\nu = \xi_\nu l_\nu B_\nu(T) + k_\nu B_\nu(T). \quad (20)$$

With the aid of equation (19) we finally get:

$$j_\nu = (1 - \varepsilon) l_\nu J_\nu + \varepsilon l_\nu B_\nu(T) + k_\nu B_\nu(T). \quad (21)$$

With this expression for the emission coefficient the fundamental differential equation for the intensity becomes in the usual notation:

$$\cos \Theta \frac{dI_\nu}{\rho dx} = (k_\nu + l_\nu) I_\nu - (1 - \varepsilon) l_\nu J_\nu - \varepsilon l_\nu B_\nu(T) - k_\nu B_\nu(T). \quad (22)$$

Formally this equation agrees completely with EDDINGTON's equation for simultaneous absorption and scattering [cfr. (4)]. However our ε has the different meaning defined by (18). We may note that we have by our arguments proved that ε is constant over the region of the spectral line.

If conditions differ from those assumed on p. 243, then equation (18) for ε and the following equations have to be modified. In order that (22) should still be valid, in equation (18) the term C_{kf} in the denominator would have to be multiplied by a correction factor Q , which is equal to: The ratio of the actual number of photoionisations from the *ground* state to the number of photoionisations in the corresponding region in thermodynamical equilibrium, divided by the ratio of the actual number of photoionisations from the *excited* state to the number of photoionisations in the corresponding region in thermodynamical equilibrium. In a dilute radiation field the resulting factor is always greater than 1 (cfr. Table 5, p. 253).

To prove the above statement we introduce ξ -factors, analogous to $\xi(E^{(k)})$ (i. e. ξ_ν) in (3), describing the deviation of the state of ionisation from that in thermodynamical equilibrium at the local temperature and density. Let N_1 and N_{1+} be the number of atoms in the ground states of the stage of ionisation considered and that following it, and N_e the number of free electrons, per unit volume. Again let the suffix 0 denote the corresponding thermodynamical equilibrium values. Then ξ_1 , ξ_{1+} and ξ_e are defined by:

$$\left. \begin{aligned} N_1 &= \xi_1 N_1^{(0)}, \\ N_{1+} &= \xi_{1+} N_{1+}^{(0)}, \\ N_e &= \xi_e N_e^{(0)}. \end{aligned} \right\} \quad (23)$$

These equations are now to be used instead of equations (14) for the transformation of the general equations (13). We shall not in this connection investigate the change in the collision factor equation, because in the applications of the resulting equations to the problem of central intensities the

collision term in (18) is negligible. The first equation (13) can now be written in the form:

$$N_1 b_{1k} \cdot B_\nu(T) = \xi_1 N_k^{(0)} a_{k1}. \quad (24)$$

The coefficient C_{fk} , being equal to the total number of captures of free electrons on the stationary state k per unit time and unit volume [cfr. equation (7)], is derived from the corresponding coefficient $C_{fk}^{(0)}$ for thermodynamical equilibrium at the local temperature and density by multiplication with $\xi_{1+} \cdot \xi_e$:

$$C_{fk} = \xi_{1+} \xi_e C_{fk}^{(0)}. \quad (25)$$

Hence the third equation (13) can be written as:

$$C_{fk} = \xi_{1+} \xi_e \cdot N_k^{(0)} C_{kf}^{(0)} \quad (26)$$

or

$$C_{fk} = \xi_{1+} \xi_e \cdot \frac{C_{kf}^{(0)}}{C_{kf}} \cdot N_k^{(0)} C_{kf}. \quad (27)$$

With the aid of (24) and (27) (and making a change of negligible effect in the collision term cfr. above) we can now transform the stationarity equation (12), dividing by the common factor $N_k^{(0)}$:

$$\xi_1 \left\{ a_{k1} \frac{J_\nu}{B_\nu(T)} + c_{k1} + \frac{\xi_{1+} \xi_e}{\xi_1} \frac{C_{kf}^{(0)}}{C_{kf}} \cdot C_{kf} \right\} = \xi_\nu \{ a_{k1} + c_{k1} + C_{kf} \}. \quad (28)$$

Introducing now the correction factor Q by

$$Q = \frac{\xi_{1+} \xi_e}{\xi_1} \cdot \frac{C_{kf}^{(0)}}{C_{kf}}, \quad (29)$$

we find, solving for ξ_ν :

$$\xi_\nu = \xi_1 \frac{a_{k1} \frac{J_\nu}{B_\nu(T)} + c_{k1} + Q C_{kf}}{a_{k1} + c_{k1} + C_{kf}} \quad (30)$$

instead of (17).

Now the spectral line contributions to absorption and emission coefficient are calculated in the same way as before, by comparison with a region in thermodynamical equilibrium at the local temperature and density. From the first equation (23) it follows that

$$l_\nu = \xi_1 l_\nu^{(0)}. \quad (31)$$

Further we have

$$j'_\nu = \xi_\nu j'_\nu^{(0)} \quad (32)$$

(j'_ν is the spectral line contribution to j_ν), and

$$j'_\nu^{(0)} = l_\nu^{(0)} B_\nu(T). \quad (33)$$

Therefore it follows that

$$j'_\nu = \frac{\xi_\nu}{\xi_1} l_\nu B_\nu(T). \quad (34)$$

This, according to (30), is equal to

$$j'_v = \frac{a_{k1}}{a_{k1} + c_{k1} + C_{kf}} l_v J_v + \frac{e_{k1}}{a_{k1} + c_{k1} + C_{kf}} l_v B_v + \frac{Q C_{kf}}{a_{k1} + c_{k1} + C_{kf}} l_v B_v. \quad (35)$$

Using the same definition as before for ε , i. e.

$$\varepsilon = \frac{c_{k1} + C_{kf}}{a_{k1} + c_{k1} + C_{kf}}$$

we can write the resulting fundamental equation as:

$$\begin{aligned} \cos \Theta \frac{dI_v}{\rho dx} = & (k_v + l_v) I_v - (1 - \varepsilon) l_v J_v - \varepsilon l_v B_v(T) - k_v B_v(T) \\ & - (Q - 1) \frac{C_{kf}}{a_{k1} + c_{k1} + C_{kf}} l_v B_v(T). \end{aligned} \quad (36)$$

For $Q = 1$ equation (36) reduces to (22), as it should. When c_{k1} is negligible compared with C_{kf} , which will be the case in the problem of central intensities, then the last term reduces to $(Q - 1) \varepsilon l_v B_v(T)$, so that we have:

$$\cos \Theta \frac{dI_v}{\rho dx} = (k_v + l_v) I_v - (1 - \varepsilon) l_v J_v - Q \varepsilon l_v B_v(T) - k_v B_v(T). \quad (37)$$

The correction factor Q is given by (29). Now it follows from the theory of ionisation by a radiation field deviating from a uniform PLANCK radiation field [cfr. (17)] that the ionisation in the high level layers of a stellar atmosphere, where the ionisation is determined by the radiation field, is given by

$$\frac{\xi_{1+} \xi_e}{\xi_1} = \frac{C_{1f}}{C_{1f}^{(0)}}, \quad (38)$$

i. e. the ratio of the number of ionisation from the ground state to the number of photoionisations in thermodynamical equilibrium at the local temperature and density. This equation follows immediately by comparison of the number of captures und photoionisations. In deriving (38) it is assumed that the number of photoionisations from the ground state is considerably larger than the number of ionisations from the excited discrete stationary states. This assumption is not quite justified. In order to correct (38) one would have to divide the right-hand side by a certain factor, which is however probably less than 2. (In the case of hydrogen an estimate can easily be made with the aid of a table given by CILLIE [18]). For alkalis and alkaline earths the factor in question is probably smaller than for hydrogen, the continuous absorption coefficient here being probably more increased for the ground state than for the excited states.

From (29) and (38) it is finally found that

$$Q = \frac{\frac{C_{1f}}{C_{1f}^{(0)}}}{\frac{C_{kf}}{C_{kf}^{(0)}}}. \quad (39)$$

This completes the proof of the statement on p. 245. We have preferred to introduce Q in the differential equation rather than change the definition of ε .

In the following section we shall discuss the results obtained, in connection with the problem of the central intensities of stellar absorption lines.

3. We shall now discuss the results concerning the contours of stellar absorption lines that follow from the equations (18) and (22) resp. (18), (37) and (39).

It will suffice for the present purpose to consider the case that the quantities l_ν/k_ν , ε and Q are constant through the atmosphere. It should be emphasized in this connection that, whereas c_{k1} tends to zero with the density (and according to PANNEKOEK is negligible small in the high level layers, where strong absorption lines are formed), C_{kf} is practically constant through the high level layers, because the continuous spectrum responsible for the photoionisations measured by C_{kf} is of practically constant intensity through these layers¹). Here therefore ε is in fact practically independent of depth. It follows from the same reason that Q is also practically independent of depth in the high level layers. With regard to the influence of variations of l_ν/k_ν with depth we may note (cfr. p. 256) that, what really is assumed to be constant in the simplified theory, is the quantity

$$\frac{1 + \varepsilon \frac{l_\nu}{k_\nu}}{1 + \frac{l_\nu}{k_\nu}}.$$

Now near the center of a strong absorption line this ratio is practically equal to ε and so in fact is practically constant through the high level layers.

¹) For the sake of clarity we shall show also directly that the number of captures on the stationary state k per atom in the ground state of the same stage of ionisation as k is independent of the density. Let there be N_1 atoms in the ground state mentioned. The number of captures is proportional to $N_{1+} \cdot N_e$, where N_{1+} stands for the number of atoms in the ground state of the stage of ionisation following that considered before, and N_e is the number of free electrons (all numbers reckoned per unit volume). Hence the required number of captures per atom is proportional to $N_{1+} N_e / N_1$, which depends only on temperature (or the ionising radiation field) and is independent of density.

Further we shall take $B_\nu = B_\nu(T_0)$, where T_0 is the boundary temperature. This is justified, when only the central parts of strong absorption lines are considered, as then only the high level layer B_ν -values are of influence. It is well known that it presents no difficulties to take account of the increase of B_ν with depth, only the results are a little more complicated. In connection with the problem of central intensities discussed here, the approximation gives ample accuracy.

With these assumptions we have the following expression for the ratio r_ν of the intensity at the frequency ν within a spectral line to the intensity in the adjacent continuous spectrum (cfr. the paper by **EDDINGTON** [4] quoted above, see also p. 257):

$$r_\nu = \frac{\frac{2}{3} \sqrt{\lambda_\nu}}{1 + \frac{2}{3} \sqrt{\lambda_\nu}}, \quad (40)$$

where

$$\lambda_\nu = 3 \frac{1 + \varepsilon \eta_\nu}{1 + \eta_\nu} \quad (41)$$

and

$$\eta_\nu = \frac{l_\nu}{k_\nu}. \quad (42)$$

Equation (40) is valid only when the correction factor Q is equal to 1. We shall show in section 5 that when $Q \neq 1$ the expression for r_ν has to be multiplied by a factor which near the center of a strong line is practically equal to Q :

$$r_\nu = \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} \frac{\frac{2}{3} \sqrt{\lambda_\nu}}{1 + \frac{2}{3} \sqrt{\lambda_\nu}}. \quad (43)$$

In the center of a strong absorption line η_ν is very large, so by (41) λ_ν is practically equal to 3ε . It follows from (43) that the central intensity r_{ν_0} (in units of the intensity of the adjacent continuous spectrum) is about equal to

$$r_{\nu_0} \simeq \frac{2}{3} \sqrt{3} \cdot \sqrt{\varepsilon} \cdot Q \quad (44)$$

when r_{ν_0} is not large. If ε is equal to 0.01, then r_{ν_0} is about $0.1 Q$; ε equal to 0.001 gives r_{ν_0} about $0.04 Q$.

It follows that an ε -value of the order of magnitude 0.01 corresponds about to the observed central intensities of strong resonance lines in the solar spectrum. Even a value smaller than 0.001 in some cases would not conflict with observation. The value of r_{ν_0} given by (44) is a minimum value, though the contributions from the neglected $1/\eta_\nu$ -term is probably

very small. Further Q is generally considerably larger than 1, about 3 (cfr. p. 247 and Table 5). Finally recent measures of central intensities of FRAUNHOFER lines by THACKERAY [19], with the monochromator apparatus designed by REDMAN [20] to eliminate scattered light, give central intensities of the order 0.05 for several lines. A table of measured values of central intensities of strong solar absorption resonance lines is given below.

Table 1.

Element	Line	λ in Å	χ volt	χ_k volt	$\cos \theta$	r_{v_0}	Authority
Na	$1^2S_{1/2} - 2^2P_{1/2, 3/2}$	5890	5.12	3.03	1	0.25, 0.23	[23], [24]
		5896			1	0.3	[23]
		5890			0.38	0.25	[23]
		5896			0.38	0.3	[23]
Al	$2^2P_{1/2, 3/2} - 2^2S_{1/2}$	3944	5.96	2.84	1	0.15	[23]
		3962			1	0.11	[23]
		3944			0.21	0.21	[23]
		3962			0.21	0.20	[23]
Ca	$1^1S_0 - 2^1P_1$	4227	6.08	3.16	1	0.03	[19]
Mg ¹⁾	$2^3P_{0, 1, 2} - 1^3S_1$	5167	7.61 (4.91)	2.53	1	0.23	[26]
		5173			1	0.22, 0.16	[26], [25]
		5184			1	0.19, 0.16, 0.20	[26], [25], [24]
		5167			0.238	0.37	[26]
		5173			0.238	0.36	[26]
		5184			0.238	0.34	[26]
Ca ⁺	$1^2S_{1/2} - 2^2P_{1/2, 3/2}$	3934	11.8	8.7	1	0.08, 0.08	[23], [19]
		3968			1	0.08	[23]
		3934			0.21	0.17	[23]
		3968			0.21	0.17	[23]
Sr ⁺	$1^2S_{1/2} - 2^2P_{1/2, 3/2}$	4078	11.0	8.0	1	0.2	[23]
		4216			1	0.3	[23]

It should be noted that on account of the square root occurring in (44) the predicted value of r_{v_0} is rather insensitive to changes in the assumed value of ϵ .

A more exact discussion of the relation between r_v , ϵ and η_v according to equations (40) and (41) can easily be carried out with the aid of the following table. We can in (41) for our purposes safely neglect 1 compared with η_v . Then we have

$$\frac{1}{3} \lambda_v = \frac{1}{\eta_v} + \epsilon.$$

1) Cfr. p. 253: the value of χ in brackets refers to the 3P states.

The table gives $1/\eta_v + \varepsilon$ with argument r_v , computed from this equation and (40), rewritten in the form

$$\frac{1}{3} \lambda_v = \frac{3}{4} \left(\frac{r_v}{1 - r_v} \right)^2.$$

The table shows clearly that at frequencies a little off the center of the line, where r_v has reached a value say three times the central value, r_v is only little influenced by ε . Also, a little off the center the factor

$\frac{1 + \varepsilon \eta_v Q}{1 + \varepsilon \eta_v}$ becomes nearly equal to 1.

Hence electron captures have practically no influence on the wings of FRAUNHOFER lines.

Now according to (18), when the effect of collisions is negligible ε measures the probability that an atom in the stationary state k is ionised (by photoionisation) and so does not jump to the ground state emitting

the corresponding light quantum. Now the transition probabilities required for the theoretical calculation of this quantity, when the atom is placed in a specified field of radiation, f. i. the field of radiation in the high level layers of the sun, are only known from quantum mechanical calculations in the case of COULOMB-field atoms (hydrogen). There is reason to believe, however, that the right order of magnitude is obtained, when the appropriate COULOMB-field values are used for hydrogen-like atoms.

The photoionisation probability C_{kf} for an electron in a COULOMB-field can be calculated with the aid of expressions derived by MILNE [27] and PANNEKOEK [17] (cfr. also CILLIÉ [18]). It is easily found that for an electron (mass m_e) moving in the field of a nuclear charge Ze , in a stationary state k of principal quantum number n (binding energy χ), the photoionisation probability C_{kf} is given by

$$C_{kf} = \frac{512 \pi^5}{3 \sqrt{3}} \frac{m_e c^{10}}{c^3 h^6} \frac{Z^4}{n^5} E i \left(\frac{\chi_k}{k T} \right), \tag{45}$$

where

$$E i(x) = \int_x^\infty \frac{e^{-t}}{t} dt \sim \frac{1}{x} e^{-x} \quad \text{for large } x. \tag{46}$$

Table 2.

r_v	$\frac{1}{\eta_v} + \varepsilon$	r_v	$\frac{1}{\eta_v} + \varepsilon$
0.01	0.000 08	0.10	0.0093
02	000 31	12	014
03	000 7	14	020
04	001 3	16	027
05	002 1	18	036
06	003 1	20	047
07	004 2	22	060
08	005 7	24	075
09	007 3	26	093
10	009 3	28	11
		30	14

Here T is the colour temperature of the ionising radiation. If the ionising radiation is dilute, C_{kf} has to be multiplied by the dilution factor W . In deriving (45) the correction factor g to KRAMERS' expression for the atomic absorption coefficient has been put equal to 1. Further $e^{-\frac{\chi_k}{kT}}$ has been neglected in comparison with 1. The following table gives numerical values of C_{kf} for $Z = 1$, $n = 2$ and a radiation field corresponding to that in the high level layers of the sun ($W = \frac{1}{3}$, $T = 6500^{\circ}$). Values for $W = 1$ and $T = 4900^{\circ}$ are also given.

Numerical values of the photoionisation probability have been calculated by WOOLLEY (l. c. [13]) for hydrogen atoms in the radiation field of the high level layers of the sun. The radiation field was assumed to be dilute PLANCK radiation with the dilution factor $W = \frac{1}{3}$ and $T = 6500^{\circ}$. The following numerical values are taken from WOOLLEYS table.

Table 3.

χ_k volt	C_{kf} ($W = \frac{1}{3}$; $T = 6500^{\circ}$)	C_{kf} ($W = 1$; $T = 4900^{\circ}$)
2	$5 \cdot 10^5$	$4 \cdot 10^5$
3	$6 \cdot 10^4$	$3 \cdot 10^4$
4	$8 \cdot 10^3$	$2 \cdot 10^3$
6	150	10
8	3	0,07

Table 4.

State	χ_k volt	C_{kf}
1 <i>s</i>	13.54	$3.8 \cdot 10^{-2}$
2 <i>s</i>	3.38	$1.4 \cdot 10^5$
2 <i>p</i>	3.38	$0.8 \cdot 10^5$
3 <i>s</i>	1.50	$7.1 \cdot 10^5$
3 <i>p</i>	1.50	$5.2 \cdot 10^5$
3 <i>d</i>	1.50	$2.4 \cdot 10^5$

The values in Table 4 are a little larger than the corresponding values in Table 3, due to the approximations used by WOOLLEY.

Results of exact quantum mechanical calculations of photoionisation probabilities for alkalis and alkaline earths would be very interesting. Somewhat larger values of C_{kf} might perhaps result as a consequence of the interaction of the valency electron with the inner parts of the field of the atom. Great changes are not to be expected however.

The numerical values of C_{kf} given in Tables 3 and 4 should be compared with the numerical values of the transition probabilities a_{k_1} for strong lines in the visual and photographic region, viz. $a_{21} = 6 \cdot 10^7$ for the sodium *D* lines and $1.6 \cdot 10^8$ for the calcium ion lines *K* and *H*. With these numerical values corresponding values of ε ranging from 0.001 to 0.01 result, when the values in Tables 3 and 4 corresponding to strong binding ($\chi_k \geq 4$ volt) are excepted. The central intensities predicted from these values are between $0.04 Q$ and $0.12 Q$.

Numerical values of Q , according to (39), can be computed with the aid of (45). The following table gives Q with the arguments χ (ionisation potential) and λ , the wave length of the resonance line considered ($\chi_k = \chi - hc/\lambda$). A radiation field corresponding to $T_c = 6500^{\circ}$ ($W = \frac{1}{3}$) and a local temperature $T = 4900^{\circ}$ was assumed in the calculations.

The approximate expression

$$Q \simeq e^{(\chi - \chi_k) \left(\frac{1}{kT} - \frac{1}{kT_c} \right)}$$

gives sufficient accuracy in the present case.

Though these calculations can only show the order of magnitude, we

may conclude that by the processes considered we can explain the observed central intensities in the case of solar lines with corresponding upper states with binding energies of a few volts. This means that the central intensities of solar resonance lines of Na, Al, Ca, Ba and Sr are accounted for.

Due to the very great strength of the invisible $L\alpha$ line of hydrogen the BALMER lines presumably behave approximately like lines originating from the ground state (cfr. WOOLLEY, [13]), so that the theory applies to these lines also. Here also the observed central intensities are accounted for.

Finally the theory may also reasonably be applied to the Mg triplet originating from the 2^3P levels, which are comparable with the ground state. The predicted central intensities agree reasonably well with those observed by AZAMBUJA [24], MINNAERT and MULDER [25], and H. H. PLASKETT [26].

For the atoms mentioned the predicted central intensities are in some cases somewhat smaller than the observed ones. The disagreements are perhaps not larger than the uncertainty of the comparison, but the possible influence of fluorescence coupling with discrete stationary states (cfr. p. 238) should be mentioned in this connection.

The photoionisation probability C_{kf} according to (45) and (46) is roughly proportional to $e^{-\frac{\chi_k}{kT}}$, where χ_k is the binding energy of the upper stationary state involved. With lines in the visible and photographic region χ_k is equal to the ionisation potential minus 2—3 volt. This means that for ions like Ca^+ and Sr^+ , with ionisation potentials of 11.8 volt and 11.0 volt, the numerical value of C_{kf} in the solar atmosphere is quite small (cfr. Tables 3 and 4). On the other hand the absorption coefficient is greater on account

Table 5.

λ	4000 Å	5000 Å	6000 Å
χ			
volt			
5	6	4	3
8	6	4	3
12	6	4	3

of the greater effective charge of the central field, in which the electron moves [cfr. the factor Z^4 in equation (45)]. Still for these ions, in the solar atmosphere, C_{kf} is probably so small that the corresponding ε is considerably smaller than 10^{-4} , leaving a central intensity of less than $0.005 Q$. The situation may also be described by the statement that in the solar atmosphere the ionisation of Ca^+ and Sr^+ to Ca^{++} and Sr^{++} on account of the relatively high ionisation potential has not proceeded far enough to yield the sufficient number of captures of free electrons per singly ionised ion.

Clearly in the case of the resonance lines of Ca^+ and Sr^+ a very careful discussion of other possibilities of explaining the central intensities must be carried out. But it is not impossible that the outcome of the discussion might be the conclusion that in the case of Ca^+ and Sr^+ , with their high χ_k/kT , there is, in the high level layers, a deviation from thermodynamical equilibrium ionisation of the type discussed by ADAMS and RUSSELL [28] and UNSÖLD [29], in the sense of excessive ionisation to Ca^{++} and Sr^{++} , a deviation considerably larger than that indicated by the normal Q -factor ($Q \sim 5$) for the lines considered.

For stellar atmospheres of *higher* temperature than the solar atmosphere the C_{kf} -values and ε -values, and so the corresponding central intensities, will be greater for the same lines. The range of temperatures available for comparison of the lines is not very great, however, for with rising temperature the ionisation proceeds in the same measure as C_{kf} and ε increase. Perhaps one may interpret the broad flat absorption lines observed in *B*- and *A*-Stars as lines with a high ε -value due to high photoionisation probability. Other possibilities such as axial rotation (cfr. G. SHAJN and O. STRUVE [30]) must, however, be considered in a discussion of this question.

For low-temperature stellar atmospheres the C_{kf} -values are low, and small central intensities result. This conclusion must be modified, however, if in such stars there is strong deviation from thermodynamical equilibrium ionisation.

4. It is well known that for atoms where the transition probabilities have not been evaluated with the aid of quantum mechanics, certain conclusions of rather exact nature can still be drawn concerning multiplet components of relatively small separation (cfr. FERMI [31]). For such multiplet components the transition probabilities a_{k1} are equal, and the photoionisation probabilities C_{kf} are equal also. Hence the ε -values are equal. Thus equal residual intensities are predicted for multiplet components. As far as the observations go (cfr. v. KLÜBER [21], MINNAERT and

MULDERS [25] and H. H. PLASKETT [26]) this prediction, which was also made from the collision theory and the discrete state fluorescence theory of residual intensities [cfr. (6) and (10)], is confirmed.

5. It appears from the discussion in section 2. that the assumption that ε , Q and B_ν do not vary with depth is probably very near the truth in the high level layers essential in the case of central intensities of strong lines. Also it was mentioned there that variations of η_ν with depth are of very small influence in this case. It may be worth while therefore to consider the variations of the central intensities over the solar disk predicted with the aid of the assumptions about constancy mentioned. The corresponding analytical investigation of (22) has been carried out by MILNE [cfr. also a paper by UNSÖLD (6) and ROSSELAND's exposition in (9)]. The main steps of the analysis are given below.

Introducing the optical depth at the frequency ν by

$$dt_\nu = (k_\nu + l_\nu) \varrho \, dx \tag{47}$$

and using the notation

$$\eta_\nu = \frac{l_\nu}{k_\nu} \tag{48}$$

we can write equation (37) in the form

$$\cos \Theta \frac{dI_\nu}{dt_\nu} = I_\nu - (1 - \varepsilon) \frac{\eta_\nu}{1 + \eta_\nu} J_\nu - \varepsilon \frac{\eta_\nu}{1 + \eta_\nu} Q B_\nu(T) - \frac{1}{1 + \eta_\nu} B_\nu(T). \tag{49}$$

Performing now after EDDINGTON [cfr. (3) and (4)] the operations

$$\int \dots \frac{d\omega}{4\pi} \quad \text{and} \quad \int \dots \cos \Theta \frac{d\omega}{4\pi}$$

and using EDDINGTON's approximation

$$\int I_\nu \cos^2 \Theta \frac{d\omega}{4\pi} = \frac{1}{3} \int I_\nu \frac{d\omega}{4\pi} \tag{50}$$

the following equations are obtained:

$$\frac{dH_\nu}{dt_\nu} = \frac{1 + \varepsilon \eta_\nu}{1 + \eta_\nu} J_\nu - \frac{1 + \varepsilon \eta_\nu Q}{1 + \eta_\nu} B_\nu(T), \tag{51}$$

$$\frac{1}{3} \frac{dJ_\nu}{dt_\nu} = H_\nu, \tag{52}$$

where H_ν is defined by

$$H_\nu = \int I_\nu \cos \Theta \frac{d\omega}{4\pi} \tag{53}$$

and so is equal to the net-flux in the frequency ν divided by 4π .

Eliminating H_ν from (51) and (52) one obtains

$$\frac{d^2 J_\nu}{d t_\nu^2} = 3 \frac{1 + \varepsilon \eta_\nu}{1 + \eta_\nu} J_\nu - 3 \frac{1 + \varepsilon \eta_\nu Q}{1 + \eta_\nu} B_\nu(T). \quad (54)$$

Now taking

$$\lambda_\nu = 3 \frac{1 + \varepsilon \eta_\nu}{1 + \eta_\nu}, \quad (55)$$

and also $\frac{1 + \varepsilon \eta_\nu Q}{1 + \eta_\nu}$ to be constant (cfr. p. 248) and further $B_\nu(T)$ to be constant and equal to $B_\nu(T_0)$ (cfr. p. 249) one arrives at the following general solution of (54):

$$J_\nu - \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} B_\nu(T_0) = \alpha_\nu e^{\sqrt{\lambda_\nu} t_\nu} + \beta_\nu e^{-\sqrt{\lambda_\nu} t_\nu}. \quad (56)$$

The boundary condition for $t_\nu \rightarrow \infty$ at once shows that $\alpha_\nu = 0$. For $t_\nu = 0$ the mean intensity J_ν is assumed to be equal to $2 H_\nu$ after EDDINGTON. This boundary condition determines β_ν when use is made of (52). The particular solution of (54) satisfying the boundary conditions is

$$J_\nu = \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} B_\nu(T_0) \left\{ 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} e^{-\sqrt{\lambda_\nu} t_\nu} \right\}. \quad (57)$$

It may be noticed in passing that this in connection with equations (30) and (18) shows that (the collision term being assumed to be negligible):

$$\frac{\xi_\nu}{\xi_1} = (1 - \varepsilon) \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} \left\{ 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} e^{-\sqrt{\lambda_\nu} t_\nu} \right\} + \varepsilon Q. \quad (58)$$

For $\varepsilon \eta_\nu \gg 1$ this reduces to

$$\frac{\xi_\nu}{\xi_1} = Q \left\{ 1 - (1 - \varepsilon) \frac{1}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} e^{-\sqrt{\lambda_\nu} t_\nu} \right\}. \quad (59)$$

Introducing the expression (57) for J_ν and $B_\nu = B_\nu(T_0)$ in (49) and solving for I_ν , fixing the constant of the solution in the customary way, one finds for the value of $I_\nu(t_\nu, \Theta)$ for $t_\nu = 0$, i. e. the intensity of the radiation leaving the atmosphere at the angle Θ with the normal:

$$I_\nu(0, \Theta) = \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} B_\nu(T_0) \sec \Theta \int_0^\infty e^{-z \sec \Theta} \left[1 - \frac{1}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} e^{-\sqrt{\lambda_\nu} z} + \frac{1}{3} \lambda_\nu \frac{1}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} e^{-\sqrt{\lambda_\nu} z} \right] dz. \quad (60)$$

Performing the integration one obtains (for $Q = 1$, cfr. MILNE [32] and UNSÖLD [10]):

$$I_\nu(0, \Theta) = \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} B_\nu(T_0) \left\{ 1 - \frac{1 - \frac{1}{3} \lambda_\nu}{1 + \frac{2}{3} \sqrt{\lambda_\nu}} \frac{1}{1 + \sqrt{\lambda_\nu} \cos \Theta} \right\}. \quad (61)$$

When, as is the case in the present discussion of central intensities, $\sqrt{\lambda_\nu}$ is small, it may be convenient to expand in powers of $\sqrt{\lambda_\nu}$:

$$I_\nu(0, \Theta) = \frac{1 + \varepsilon \eta_\nu Q}{1 + \varepsilon \eta_\nu} B_\nu(T_0) \left\{ \frac{2}{3} \sqrt{\lambda_\nu} (1 + \frac{3}{2} \cos \Theta) - \lambda_\nu (\frac{1}{3} + \cos \Theta)^2 + \dots \right\}. \quad (62)$$

This expression shows the variation of the absolute intensity in the centre of the line over the solar disk. The leading term is seen to vary as $1 + \frac{3}{2} \cos \Theta$ (cfr. UNSÖLD [6]). The corresponding coefficient of darkening is $\frac{3}{5}$.

In order to get an expression for the variation of the *relative* central intensity r_ν over the disk we may use the approximate expression

$$I_{\nu, \text{(kont)}}(0, \Theta) = \frac{1 - u + u \cos \Theta}{1 - u} B_\nu(T_0) \quad (63)$$

for the intensity in the adjacent continuous spectrum. This expression gives sufficient accuracy for most purposes, when the coefficient of darkening u is determined from observations in the region of the continuous spectrum considered (cfr. f. i. the table given by MILNE after LINDBLAD [33] in Handb. d. Astrophys. III, 1, 149, containing ABBOT'S observational [34], and MILNE'S and LINDBLAD'S theoretical results).

Table 6.

λ in Å	u	$u/(1-u)$
3737	0.83	4.7
4265	0.81	4.2
5062	0.68	2.2
5955	0.59	1.4
6702	0.52	1.1

Numerical values of u are given below. They have been deduced from the observed intensities at $\sin \Theta = 0$ and $\sin \Theta = 0.95$.

From (62) and (63) one finds

$$r_\nu = \frac{1 + \varepsilon \eta_\nu Q}{1 + \eta_\nu} \left\{ \frac{2}{3} \sqrt{\lambda_\nu} \frac{1 + \frac{3}{2} \cos \Theta}{1 + \frac{u}{1-u} \cos \Theta} - \lambda_\nu \frac{(\frac{1}{3} + \cos \Theta)^2}{1 + \frac{u}{1-u} \cos \Theta} + \dots \right\}. \quad (64)$$

Comparing with the numerical values given in Table 3 one sees that in the spectral region around 5900 Ångström, where u is 0.6, the relative central intensity is approximately constant over the solar disk. Here also the formula reduces to (43), section 3, very nearly.

One would therefore expect that the relative central intensity of the sodium *D* lines is nearly constant over the disk. The observations (cfr. Table 1) in fact show that this is the case.

On the other hand one would expect lines in the violet and ultraviolet to show smaller relative central intensities at the center of the solar disk

than near the limb, the darkening being much stronger in the adjacent continuous spectrum than in the center of the absorption line. Here also there is agreement with observations, qualitatively at least (cfr. Table 1).

A more exact discussion is possible with the aid of (61) and numerical values describing the intensity variation of the adjacent continuous spectrum over the solar disk.

One may use two observations of r_{v_0} for different Θ -values to find numerical values of both Q and λ_{v_0} , and thus of both Q and ε . However, it is generally difficult to separate Q and λ_{v_0} . The run of $I_{v_0}(0, \Theta)$ or r_{v_0} with Θ is rather insensitive to changes in Q . What is determined from the observations with relatively great certainty is the product $Q \sqrt{\lambda_v}$. This is illustrated by the following table which shows ε -values computed from observed central intensities in the Mg triplet (cfr. Table 1 and p. 253) at the centre of the solar disk for different *assumed* values of Q , together with values of r_{v_0} for $\cos \Theta = 0.238$ predicted with the aid of Q and the ε -values thus found. There is a considerable range of Q -values leading to a relatively satisfactory fit.

Table 7.

$Q = 3$	$Q = 2$	$Q = 1$
$\cos \Theta = 1$		
$r_{v_0} = 0.21$ (observed value)	$r_{v_0} = 0.21$ (observed value)	$r_{v_0} = 0.21$ (observed value)
$\sqrt{\lambda_v} = 0.15$	$\sqrt{\lambda_v} = 0.24$	$\sqrt{\lambda_v} = 0.65$
$\varepsilon = 0.007$	$\varepsilon = 0.019$	$\varepsilon = 0.14$
$\cos \Theta = 0.238$		
$r_{v_0} = 0.26$ (computed value)	$r_{v_0} = 0.27$ (computed value)	$r_{v_0} = 0.32$ (computed value)
$r_{v_0} = 0.36$ (observed value)	$r_{v_0} = 0.36$ (observed value)	$r_{v_0} = 0.36$ (observed value)

(Mg triplet λ 5167, 5173, 5184, H. H. PLASKETT's observations; $u = 0.67$).

A Q -value between 2 and 3 and a corresponding ε -value about 0.01 appears plausible.

The measurements of the sodium D lines by UNSÖLD [23] in the same way are seen to be compatible with any Q -value larger than about 2.

Finally SCHWARZSCHILD's [2] measurements of the absolute intensities of the K and H lines of Ca^+ , which showed a magnitude difference of 1^m0 between center and limb in the centers of the lines, are compatible with any Q larger than 1. In this connection we may note that, according to the discussion in section 3, there is a possibility that Q is large in the case of these lines.

We may conclude that by the atomic mechanisms considered in the present paper it is possible to account for the observational facts concerning central intensities of strong absorption lines, qualitatively and partly also quantitatively. A more refined comparison between theory and observation will be possible when quantum mechanical calculations of atomic absorption coefficients in the continua of alkalis and alkaline earths are available. For the purpose of a refined comparison it will probably be desirable to work out a theory which takes account of the fluorescence coupling between the discrete stationary states also.

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