systematically from those of normal stars at the same temperature and should, moreover, more nearly represent that temperature. It should be noted that supergiants tend to have lower temperatures than normal stars *of the same spectral class*, and, therefore, a significant comparison of color indices can be made only for a giant and a supergiant which give the same temperature from spectrophotometric measurements to the red of 3900. No data for making such a comparison are known to the writer. Similarity of spectrum is no guide to the selection of a suitable pair.

The effect of the distortion of background on measurements of color index has a bearing on the investigation of the very luminous apparently faint stars in clusters and Magellanic clouds by means of their color indices. Such stars are likely to be *c*-stars with narrow lines, and their color indices may, therefore, differ systematically from those of nearby stars with spectra assigned to the same class. The *c*-character probably also introduces a systematic difference for the classification of these very distant stars of Class *A*, as pointed out by Shapley, by displacing their apparent spectral class towards *F*. It is, therefore, necessary to evaluate the relative contributions of color index and spectral class; discrepancies between spectral class and color temperature might be expected for these distant stars.

[1] Hogg, *Harv. Obs. Bull.* **856**, 1928.

[2] Fowler, *Report on Series in Line Spectra*, **1922.**

[3] Davidovich, *Harv. Obs. Bull.* **846**, 1927.

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[5] Payne, *Harv. Obs. Circ.* **307**, 1927.

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A CASE OF STREAMING IN A VALVE

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Communicated December 28, 1927

For the engineer it is a matter of great importance to know the pressure of water against a trap-valve, a problem which seems not yet to have been treated.

We confine ourselves to a special case of a streaming in two dimensions. It is unnecessary to say that every such streaming is represented by a function $f(z)$ of the complex variable $z = x + iy$. Then let

$$f(z) = \varphi(x, y) + i\psi(x, y) \equiv \varphi + i\psi$$
$$w(z) = f'(z) = \varphi_x - i\varphi_y = \psi_y + i\psi_x. \qquad (1)$$

Note that

$$\oint wdz = \oint (\varphi_x dx + \varphi_y dy) + i\oint(-\varphi_y dx + \varphi_x dy) = \oint \varphi_s ds + i\oint \varphi_n ds = curl\ \varphi + i\ div\ \varphi \tag{2}$$

all integrals being taken around a closed curve.

Now, let us suppose the following streaming in two dimensions: let a fluid rising from the source of the divergence $2q$ move between an immovable infinite wall and a parallel one, also infinite, being translated perpendicularly to itself with the velocity $c = c(t)$. Suppose further the fixed wall is $y=0$ the movable is $y = h$ and the source is at the origin. Then the conditions on the velocity with the components u and v are

$$\begin{array}{ll} v = 0 & \text{when } y = 0 \text{ except at the origin} \\ v = c & \text{when } y = h \\ \oint(udy - vdx) = 2q & \end{array} \tag{3}$$

the integral being taken around a curve including the origin.

Let the fundamental equations in hydrodynamics be written:

$$\gamma\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x},$$
$$\gamma\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}, \tag{4}$$

where γ is the density of the fluid and p the pressure.

Then let $\varphi(x, y)$ be an analytic function of x and y. Then the equations above are fulfilled by putting

$$u = \varphi_x,\ v = \varphi_y,$$
$$p = -\gamma\left(\varphi_t + \frac{1}{2}(u^2 + v^2)\right). \tag{5}$$

Introduce besides φ the stream-function ψ so that

$$w = u - iv,\ f = \varphi + i\psi = \int wdz.$$

With regard to the conditions above, w must be a function regular except at the origin in the whole strip limited by $y = 0$ and $y = h$, and must satisfy the conditions:

w is real for a real variable, except at the origin; w is purely imaginary for a purely imaginary variable; $w = u - ic$ [where $c = c(t)$]; when $z = x + ih, \oint wdz = 2iq$. (6)

The last equation is stronger than the last one in (3), for it requires further the vanishing circulation. To build this function we make use of the Riemann-Schwarz principle of symmetry. Since, on the straight line

$y = h$, w has a constant imaginary component it can be continued over that line so that at opposite points it has conjugate imaginary values. Then we have a function defined in the strip limited by $y = 0$ and $y = 2h$. It can also be continued over that last line, and so on, so that we get a function with the period $2h$. The fundamental functions of such a kind are $\sin \frac{\pi y}{2h}$ and $\cos \frac{\pi y}{2h}$ or, in our case, $\text{sh} \frac{\pi z}{2h}$ and $\text{ch} \frac{\pi z}{2h}$ where $\text{sh}x$ and $\text{ch}x$ are the hyperbolic functions $-i \sin ix$ and $\cos ix$. Since w is odd and infinite at the origin, it must be a function of $\frac{1}{\text{sh}\frac{\pi z}{2h}}$ or of $\text{cth} \frac{\pi z}{2h}$. As

$$\frac{1}{\text{sh} \frac{\pi z}{2h}} = \frac{2h}{\pi z} - \frac{\pi z}{12h} + \ldots$$

$$\text{cth} \frac{\pi z}{2h} = \frac{2h}{\pi z} + \frac{\pi z}{6h} + \ldots$$

we have now the two functions $\frac{q}{2h} \cdot \frac{1}{\text{sh} \frac{\pi z}{2h}}$ and $\frac{q}{2h} \cdot \text{cth} \frac{\pi z}{2h}$ which satisfy the first, second and last of the conditions of (6). But if we put $z = x + ih$ we see that the latter function has a constant imaginary component, viz., zero, so that by putting

$$w = -\frac{c}{h} z + \frac{q}{2h} \text{cth} \frac{\pi z}{2h}$$

we obtain the desired function.

Streamlines.—Let us consider now the streamlines of motion. Since $w = u - iv$, we have

$$u = -\frac{c}{h} x + \frac{q}{2h} \cdot \frac{\text{sh} \frac{\pi x}{h}}{\text{ch} \frac{\pi x}{h} - \cos \frac{\pi y}{h}} \tag{7}$$

$$v = \frac{c}{h} y + \frac{q}{2h} \cdot \frac{\sin \frac{\pi y}{2h}}{\text{ch} \frac{\pi x}{2h} - \cos \frac{\pi y}{h}}. \tag{8}$$

By integration of w and by a convenient choice of the integration's constant we get

$$f = -\frac{1}{2}\frac{c}{h}z^2 + \frac{q}{\pi}\left[\ln \operatorname{sh}\frac{\pi z}{2h} - \frac{i\pi}{2}\right] \tag{9}$$

$$\psi = -\frac{c}{h}xy - \frac{q}{\pi}\operatorname{arc\,ctg}\left(\operatorname{cth}\frac{\pi x}{2h}\,.\,\operatorname{tg}\frac{\pi y}{2h}\right). \tag{10}$$

Here, arc ctg has the same sign as x and varies between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$. The straight line $x = 0$ is the streamline $\psi = 0$. In this manner no streamline $\psi \neq 0$ can meet the straight line $x = 0$ otherly than at the origin. That is, an infinite number of streamlines comes from the source $z = 0$. Since in its neighborhood, $\operatorname{cth}\frac{\pi x}{2h}\,.\,\operatorname{tg}\frac{\pi y}{2h} = \frac{y}{x}$, ψ has, on these streamlines, the value $q\frac{\alpha}{\pi}$ where α is the positively taken angle of the streaming direction and $x = 0$. For these streamlines, therefore, is $|\psi| < \frac{q}{2}$ and ψ has the contrary sign as x.

In the following we consider only positive values of α, the streaming being symmetrical to $x = 0$.

From the equations (7), (8), (10) we draw the conclusions

$$\left.\begin{aligned} u &= -\frac{c}{h}x + \frac{q}{2h}\,.\operatorname{cth}\frac{\pi x}{2h} \\ v &= 0 \end{aligned}\right\}\text{when } y = 0 \tag{11.1}$$

$$\left.\begin{aligned} u &= -\frac{c}{h}x + \frac{q}{2h}\,.\operatorname{th}\frac{\pi x}{2h} \\ v &= c \end{aligned}\right\}\text{when } y = c. \tag{11.2}$$

$$\psi = -\,cx \quad \text{when } y = h \tag{11.3}$$

$$\left.\begin{aligned} u &= 0 \\ v &= \frac{c}{h}y + \frac{q}{2h}\,.\operatorname{ctg}\frac{\pi y}{2h} \end{aligned}\right\}\text{when } x = 0. \tag{11.4}$$

Hence, we have the following results:

The x-axis is a streamline. (12.1)

At the valve the vertical velocity is constantly c. (12.2)

All streamlines coming from the source either meet or do not meet the valve according as $c>0$ or $c<0$, since near the source always $\psi<0$. (12.3)

The y-axis is a streamline. (12.4)

These results are valid for *every* streaming. But for a closer inquiry two cases are to be distinguished: $c<0$, $c>0$.

I. $c<0$. *Case of Shutting Valve.*—In this case we have:

$\psi = -cx$, when $y = h$. (13.1)

Vanishing of u and v is only possible if $x = 0$. Then y is determined by

$2cy + q.\text{ctg}\,\frac{\pi y}{2h} = 0$ (cf. 11.1 and 11.4). (13.2)

$$u = \frac{|c|}{h}x + \frac{q}{2h} \text{ when } x \text{ is very great.} \quad (13.3)$$

$$v = -\frac{|c|}{h}y \text{ when } x \text{ is very great.} \quad (13.4)$$

Should ψ be a constant k it results from (11) that

$$y = h\,.\frac{2k+q}{2|c|x}, \text{ when } x \text{ is very great.} \quad (13.5)$$

From this we draw the conclusions:

Not one streamline coming from the source meets the trap-valve. (14.1)

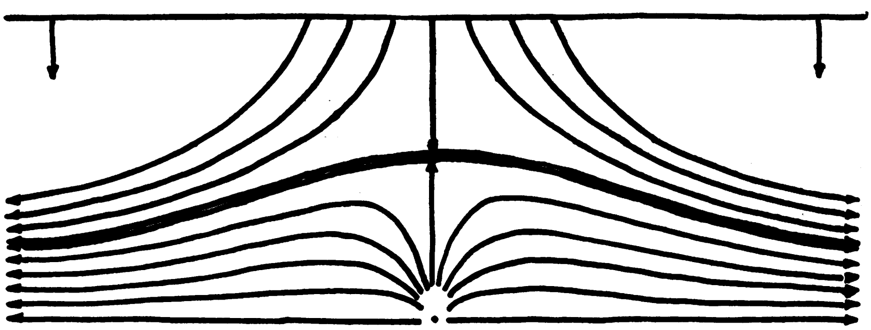

FIGURE 1

There is, on the y-axis, a singular point at which the velocity is zero. At this point water coming from valve meets water rising from the source. (14.2)

The fluid runs always to the right and the more rapidly the greater is the distance from the source. (14.3)

For sufficiently great values of x fluid is streaming towards the fixed wall. (14.4)

All streamlines approach asymptotically the x-axis as does a hyperbola, especially the second streamline $\psi = 0$ which crosses rectangularly the y-axis at the singular point of (14.2). This streamline separates the water of the source from that of the trap-valve. (14.5)

So we have figure 1.

II. $c>0$. *Case of Opening Valve.*

$$v>0 \text{ (cf. 8)} \tag{15.1}$$

$$u = -\frac{c}{h}x + \frac{q}{2h} \text{ when } x \text{ is very great.} \tag{15.2}$$

$$v = \frac{c}{h}y \text{ when } x \text{ is very great.}$$

Should $u = 0$ then must $y = 0$ and $x = \frac{q}{2c}.\text{cth}\,\frac{\pi x}{2h}$. (15.3)

or

$$y = h \text{ and } x = \frac{q}{2c}.\text{th}\frac{x\pi}{2h}.$$

Generally, $u = 0$ for points satisfying the equation

$$\cos\frac{\pi y}{h} = \text{ch}\,\frac{\pi x}{h} - \frac{q}{2cx}\cdot\text{sh}\,\frac{\pi x}{h}. \tag{15.4}$$

Let

$$u = -\frac{c}{h}x + \frac{q}{2h}\cdot\frac{\text{sh}\,\frac{x\pi}{h}}{\text{ch}\,\frac{\pi x}{h} + s} \tag{15.5}$$

then

$$\frac{du}{ds} = -\frac{q}{2h}\cdot\frac{\text{sh}\,\frac{\pi x}{h}}{\left(\text{ch}\,\frac{\pi x}{h} + s\right)^2} < 0.$$

That is to say:

All streamlines rising from the source meet the trap-valve. (16.1)

For sufficiently great values of x water comes running out of the infinite with a velocity growing proportionally to x. (16.2)

On the x-axis, there is always a singular point of vanishing velocity. (16.3)

The two walls are perpendicularly met by the curve

$$y = \frac{h}{\pi}\cdot\text{arc}\cos\left(\text{ch}\,\frac{\pi x}{h} - \frac{q}{2cx}\cdot\text{sh}\,\frac{\pi x}{h}\right)$$

for there $y' = \infty$. At each of their points, the curve is crossed vertically by the streamlines. (16.4)

x being constant and y growing the velocity u decreases! That is, the curve mentioned goes from above to the left downwards to the right. In the part enclosed by the fixed wall and the curve water goes away from the

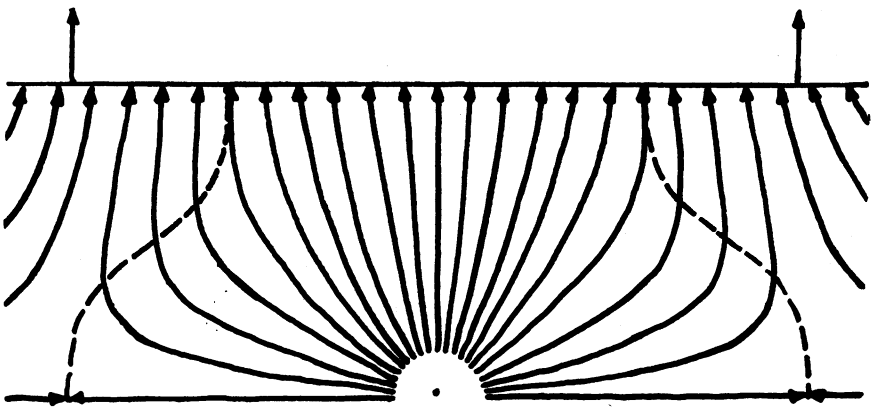

FIGURE 2

middle, in the part enclosed by the movable wall and the curve it runs towards the middle. (16.5)

The second equation of (15.3)

$$x = \frac{q}{2c} \cdot \operatorname{th} \frac{\pi x}{2h}$$

has a solution only if $\frac{\pi q}{4hc} > 1$. For the curve $\xi = \frac{q}{2c} \cdot \operatorname{th} \frac{\pi x}{2h}$ has the gradient $\frac{q\pi}{4hc} \cdot \frac{1}{\operatorname{ch}^2 \frac{\pi x}{2h}} > 0$, that is, at the origin $\frac{\pi q}{4hc}$.

So *two cases* are again to be distinguished

$$(a)\ \frac{\pi q}{4hc} > 1.$$

If $y = h$, u and $\frac{du}{dx}$ are at first positive, later negative (Fig. 2).

$$(b)\ \frac{\pi q}{4hc} < 1.$$

At the trap-valve $y = h$ is always $u < 0$ and $\frac{du}{dx} < 0$.

The curve (Fig. 3) above crosses the y-axis at the point determined by

$$y = \frac{h}{\pi} \cdot \text{arc cos}\left(1 - \frac{\pi q}{2ch}\right).$$

There is still a difficulty in the preceding: One must consider that all the

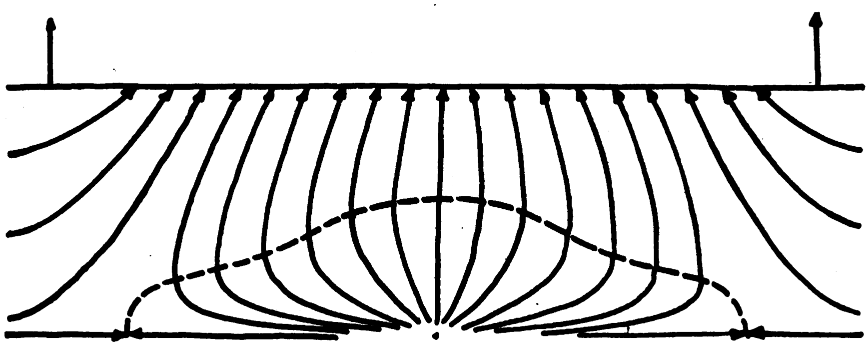

FIGURE 3

streamlines always spoken of are not real; that is, they are not constant, but, the trap-valve moving, they *change* every moment since c and h are functions of t. These real streamlines shall, perhaps, be discussed in a later article.

STUDIES ON BIOCHEMICAL DIFFERENCES BETWEEN SEXES IN MUCORS

5. *Quantitative Determinations of Sugars in* (+) *and* (−) *Races*[1]

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Communicated March 7, 1928

The presence in fungi of some soluble carbohydrates, such as trehalose and glycogen, and of the carbohydrate alcohol mannit, which are seldom found in other plants, and the absence of cane sugar and starch is characteristic of this group of plants. Bourquelot,[2] in a series of experiments, has shown a close relation between trehalose and mannit in various higher fungi. This disaccharide is converted into mannit extremely rapidly during desiccation of the fungus. A number of analyses have also shown that the amount of trehalose is greater at the beginning of fructification and that it decreases or disappears entirely when the fungi have become mature.[3] The reverse was found to be true of the mannit. Most of these tests were made with higher fungi and our present knowledge of the men, with sophomores indeterminate. The following table shows that (with the exception of the sophomores) the undergraduates for whom we have intelligence data (Column *A*) are a representative fraction of those for whom we have survival indices (Column *B*), but that judged by their median percentile (Column *C*) they are not an unselected group of college students:

	A	B	C
Seniors	8	12	60
Juniors	20	23	42
Sophomores	7	31	50
Freshmen	25	29	74

Instead of the decline in the survival index accompanying a decrease in the intelligence of the groups, then, it accompanies an *increase*, and the negative tendency indicated by the correlation is confirmed—partly. The modifier is necessary because by taking three variables in each of two phases (intelligence increasing and decreasing, index increasing or decreasing, correlation positive or negative) eight diagrams may be drawn demonstrating the possibility of any phase of one coexisting with any phase of either of the others.

In summary, by an extension of the sibling method of determining the fertility of stocks backward one more generation, and its application to 214 college men and controls, a relation between fertility and intelligence indicated by an r of -0.3 has been shown; a marked drop in average survival from the older to the younger academic groups (faculty excluded), appearing to indicate a positive relation, is probably due to a local reversal of the expected principle of selection. There are no marked differences between the controls and the lower academic groups, and the faculty group is also not conspicuously different from these. The point deserves emphasis that in none of the groups studied are the stocks on the average anywhere near the level at which continued maintenance is menaced, the average proportion falling below a liberal estimate of that level being only about one-quarter.

ACKNOWLEDGMENT

The problem treated in the article "A Case of Streaming in a Valve" in the April issue of the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES, was indicated to me by Professor Blumenthal of Aachen (Germany) who had already found the results contained in the article, and proposed to me to develop the subject further. Guided by his advice, I refound the results. I regret that, through a misunderstanding, they were offered in my behalf to the PROCEEDINGS.

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