NEW LINES IN THE K SERIES OF X-RAYS

By William Duane

Physical Research Laboratory, Harvard University

Communicated December 10, 1931

This article describes experiments with the K series of x-rays, chiefly those of molybdenum. The lines in the series are put in evidence by the Moseley photographic method. The distance from the slit in front of the target to the reflecting crystal equals that from the crystal to the photographic plate, thus corresponding to the Bragg focusing scheme. An accurate instrument with a worm gear causes the crystal to oscillate at constant speed back and forth over an angular range, which varies from five to sixty minutes of arc in different experiments. The oscillation is about a vertical axis of rotation. The distances from this axis of rotation to the slit in front of the target and to the center of the photographic plate amounts to 4725 mm., producing quite a large dispersion. The β lines in the K series of Mo, shown in the accompanying diagrams, lay 0.88 mm. apart on the plate, corresponding to a difference between the wave-lengths of the β_1 and β_2 lines amounting to 0.00056 Ångström. The measurements by the ionization method of this doublet made some years ago by Allison and Armstrong in our laboratory and published in the *Physical Review* gave practically the same difference of wave-length between the β_1 and β_2 lines, namely, 0.000563 Ångström.

Plate 1 represents the K series lines of molybdenum, the reflecting crystal being calcite, and the 100 planes doing the reflecting. The horizontal ink lines below the spectrum lines represent the excursions produced by the oscillating crystal. The relative blackness of the various lines does not represent the relative intensities, for the times of exposure for the different lines were different from each other. The time of exposure of the α_1 and α_2 lines was 100 minutes. For the β_1 and β_2 lines it amounted to 150 minutes. For the lines of somewhat shorter wave-lengths, including the γ line, the time of exposure amounted to $18^1/_2$ hours. This length of time makes the γ line appear as black as the α_1 line. The γ line is, however, very much less intense than the α_1 line.

A new line appears on this plate a little to the right of the γ line, corresponding, therefore, to a shorter wave-length. Such a line in the K series of Mo has been reported by Leide, but, if the wave-lengths published by him in the *Comptes rendus* are correct, his line must lie further away from the γ line than does the new line shown on Plate 1. As indicated on the other plates here presented, this new line is not a single line but a band.

The critical absorption of the K series of Mo, as carefully determined

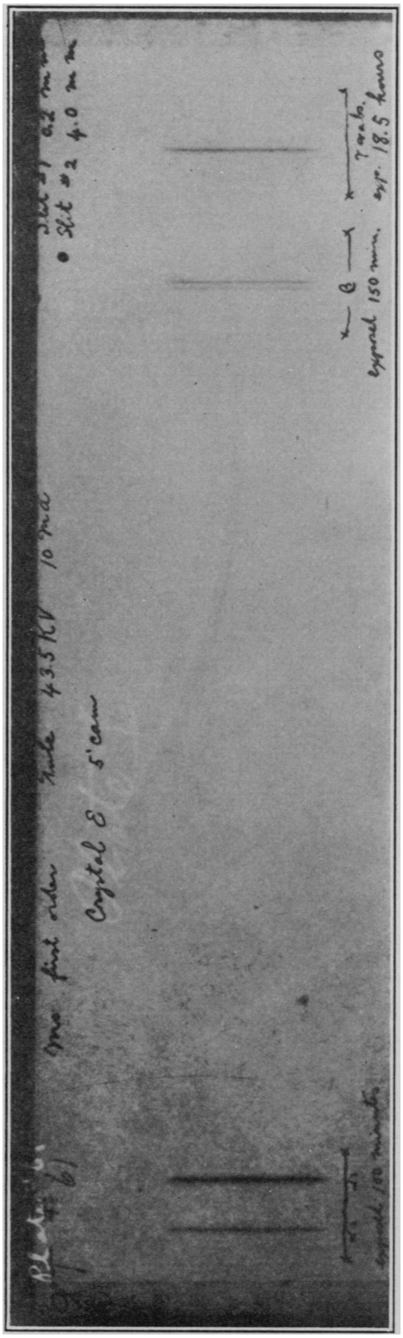


PLATE 1

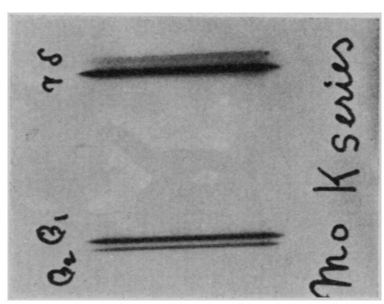


PLATE 2

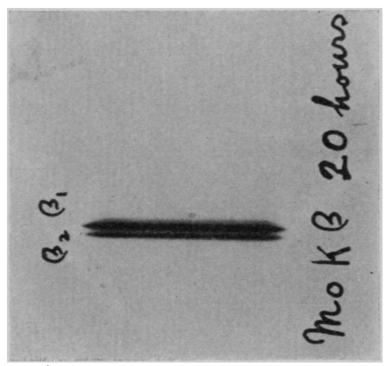


PLATE 3

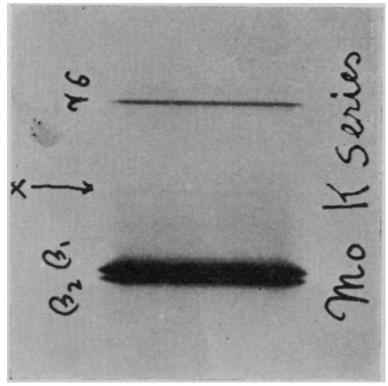


PLATE 4

in several ionization experiments performed in my laboratories, lies to the right of this new band, but very close to it. The new band, therefore, has wave-lengths that are only very slightly longer than that of the critical absorption.

As is well known, a number of interesting theories of x-radiation have been presented in recent years, such as the Bohr theory and amplifications of it. We have also corpuscular theories of radiation, such as those suggested during the first two or three years of this century and later put into such beautiful quantitative form by Einstein in his energy equation. If we use such a theory, we may regard the reflection of the x-rays by the crystal as representing transfers of momenta of radiation corpuscles to the crystal in quanta. We may also regard x-radiation from the point of view of the newer wave conception of de Broglie and the newer wave mechanics.

We now have a number of very interesting *physical* quantitative theories that have been proposed by various individuals. The importance of a physical theory depends (*a*) on the number of actual facts or phenomena that it clearly explains, (*b*) on the number of new laws and phenomena which it predicts and suggests for experimental verification and (*c*) on its clearness and the facility with which it can be used.

We may not, however, regard these theories as representing what, for lack of suitable English expressions, we may call real truth. They represent the ways in which certain types of human minds think, particularly if those minds have been trained in higher mathematics or along the lines of mathematical physics, as it is so well taught in Cambridge University, England. In the past these physical theories have not represented experimentally determined truth accurately and completely; so we must expect not them to in the future. They may represent nature in the sense of idealistic philosophy, an idea so clearly described by Bishop Berkeley in his analytical philosophy, namely, that the material world exists only as conceptions in the minds of human beings.

In this article we will use the theory that represents so many facts, namely, the theory that assumes x-rays to be the quantities of energy that are liberated when electrons fall in toward atomic nuclei.

I presented a preliminary report of the researches described in this article to the American Physical Society at its meeting in New York last February (1931). At that time I suggested that we might regard the new band mentioned above as due, at least in part, to the conductivity electrons falling into K positions made vacant by the electron bombardment of the target. I sent a copy of Plate 2 to Doctors DuMond and Hoyt, which they very kindly photometered for me, using the apparatus in the California Institute of Technology. They made, also, some interesting calculations, assuming that the entire band is due to conductivity electrons distributed according to the Fermi probability laws, using Sommerfeldt's equation (*Physical Review*, Aug. 15, 1931, p. 839). These calculations indicated that the number of conductivity electrons in metallic molybdenum amounted to as many as 5 per atom and that there was a difference of potential amounting to as much as 15 volts. It may be that

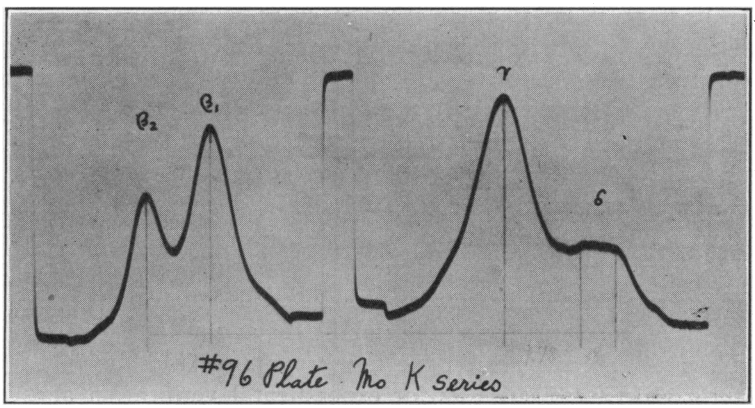
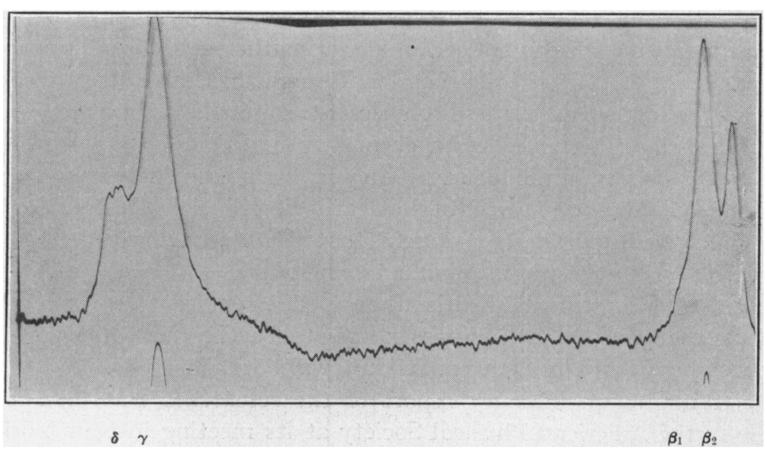


PLATE 5



(The figures in plates 5 and 6 are in mutually reversed order.)

PLATE 6

the band should be regarded as representing in part a real δ line (O $\longrightarrow$ K electrons). This line would correspond to the small hump on the band next to the γ line, and the rest of the band would correspond to the group of lines due to the conductivity electrons. This way of looking at it would give a smaller difference of potential and a smaller number of conductivity electrons, perhaps 2 or 3 per atom. In order to decide these points of detail, we will have to wait until further analyses have been made of this radiation.

Plates 2, 3 and 4 represent the β, γ and δ groups taken with much longer times of exposure. In plate 2 the exposure of the β lines amounted to 10 hours and that of the γ and δ lines to 91 hours. From the breadth of the γ line in various plates one might assume that it is a doublet, which it should be according to certain theories. It looks, also, as if the β_1 line were more than a single line.

Plate 3 contains the β lines produced by about 20 hours' exposure. β_1 is becoming quite fuzzy. There is, also, a very, very faint new line about half-way between the β_1 and γ lines, which is so faint that probably it will be impossible to see it in the reproduction of this plate.

The line is marked X in plate 4. In this plate the β lines correspond to an exposure of $33^1/_2$ hours, whereas the δ and γ lines correspond to only 10 hours' exposure. There is no known x-ray line belonging to any chemical element that could lie in its first-order reflection at the position marked X. In the second-order reflection one of the β lines of cerium might lie at X. The wave-lengths of these cerium lines are not sufficiently well known to enable us to tell whether the X line is a second-order reflection of cerium β_1, or not. If it is cerium β_1, it should be accompanied, theoretically, by another line close to it, cerium β_2. Further, if this is cerium β_1 the two α lines in the K series of cerium should appear in their second-order reflections close to the Mo K α lines. Experiments are in progress to test the radiation for the K α_1 and α_2 lines of cerium reflected in the second order. If they really exist they should be easily detected, and, if they are not detected, the X line cannot be due to the radiation from cerium, but must be a new line.

No lines could appear at X reflected in the 3rd or higher orders, for the constant voltage applied to the x-ray tube came from our storage battery and did not exceed 48. kv. in any experiment. This voltage would not produce lines short enough to be reflected to X in the third order reflection.

The β_1 line of Mo on this plate appears to have a satellite on the short wave-length side of it. One would be inclined to think that the β_1 line is somewhat broader than β_2 on all of the plates, in spite of the fact that it is really twice as intense as β_2.

Plate 5 represents the β, γ and δ lines of Plate 2 photometered by means of the photometer now existing in the Jefferson Laboratory. This instrument was designed by Professors Saunders and Crawford and very kindly loaned to me. The separation of the β_1 and β_2 lines does not appear to be as complete as on Plate 2 when examined by the eye. This, I understand, is usually the case when close lines are photometered. The point is being studied by a new photometer, which has been constructed in our new physical research laboratory. It may be that the curves indicate a number of very close and very weak satellites.

Plate 6 represents the photometer curve kindly made for me at the California Institute of Technology. It corresponds almost exactly to the one obtained in the Jefferson Laboratory. The x-ray lines have been very kindly photometered for me in half a dozen laboratories throughout the United States. None of the curves, however, are superior to those represented in Plates 5 and 6.

A new δ line (O $\longrightarrow$ K electrons) has appeared on some photographs of the K series of tungsten made some time ago in my laboratory by Miss Armstrong, Assistant Professor of Physics at Wellesley College, when she was acting as one of my assistants. The spectra that she photographed also show two β lines as well as the γ and the two α lines of W.

Accurate details of these K series spectra of Mo and W will be published as soon as the photographs can be examined by means of the new photometer I have designed and am having set up in our new physical research laboratory.

It gives me great pleasure to sincerely thank my assistant, Mr. Lanza, who has spent so much time and care in setting up my apparatus and taking photographs.

---

*NOTE ON PERRON'S SOLUTION OF THE DIRICHLET PROBLEM**

BY HASSLER WHITNEY†

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Communicated December 12, 1931

In 1923 Perron[1] gave a method of attacking the Dirichlet Problem, the only real difficulty of which lay in the proof of the fact that his function u was harmonic. Simplifications of this proof were later given by several authors.[2] We give here a short proof involving only Harnack's first convergence theorem and inequality, both of which follow directly from the Poisson integral, and two lemmas from Perron's paper (Hilfssätze I and II). The convergence theorem is: *Let $u_1, u_2 \ldots,$ be a sequence of functions harmonic in the closed region S, and converging uniformly in S to a limit u. Then u is harmonic in S.* The inequality we state in the form: *Let χ be harmonic and $\geqq 0$ within the circle K of radius a and with center P. Then if L is a circle about P of half the radius,*