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ALTERNATING CURRENT ENGINEERING

PRACTICALLY TREATED

BY

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WITH 104 ILLUSTRATIONS

THIRD EDITION

REVISED AND ENLARGED, WITH AN ADDITIONAL CHAPTER
ON
"THE ROTARY CONVERTER"

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PREFACE.

THE directing for many years in practical engineering of graduates from technical colleges of this and other countries, and also young men starting in the electrical business without complete technical training, has impressed upon my mind the necessity for a general treatise on alternating current engineering presented in a practical and compact way, without complex methods of explanation, and free from any matters not bearing directly on purely engineering work. To this end this book has been written, covering, without the use of calculus, an outline of the subjects embraced by alternating current engineering. In order that a proper knowledge of the theory and operation of apparatus may be had, the first part of the book is devoted to elucidating the general laws of magnetism and alternating currents as applied to alternating work. The second part deals directly with modern alternating apparatus, covering in a general way the designing principles and the principles of operation, and in detail the methods of test that have been found to be the best.

Owing to the importance of the rotary converter in modern transmission and distributing systems, a chapter on this subject has been added to this edition.

E. B. RAYMOND.

SCHENECTADY, N.Y.,
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ALTERNATING CURRENT ENGINEERING.

PART I.

CHAPTER I.

MAGNETISM.

MAGNETISM in iron, early known to man, manifests itself in many ways. A piece of iron magnetized will attract another piece of iron; if thrust into a circuit of wire, it will create in that circuit an unbalanced electrical condition. This magnetism is a property that must be measured like anything else, and to this end a certain assumption is made, as follows: —

UNIT OF MAGNETISM.

The unit of magnetism is that amount of magnetism which, if concentrated at a point, will exert a unit of force on a similar amount of magnetism situated at a unit distance away. Thus, a unit of magnetism, 1 centimeter away from another unit, will exert on it a unit's force, that is, 1 dyne. The dyne is the unit of force in the absolute system of units, and is equal approximately to the weight of $\frac{1}{980}$ of a gram, or to $\frac{1}{445000}$ of a pound.

LINES OF FORCE.

To express magnetism still more specifically, a magnetic pole (for instance, the end of a magnet) is assumed to have streaming from it what are called "lines of force," and it is always understood that the strength of the pole is determined by the number of lines of force emanating from it. It is assumed that the unit magnetic pole sends out one line per square centimeter at a unit distance from the pole in all directions. Hence, since the area of a globe 1 cm. in radius is 4π, a unit pole sends out 4π lines. This, then, defines the unit pole, and gives a basis of strength for it. Thus, given the lines per sq. cm., coming out from a pole, at a unit distance, its strength is at once known, as well as how much greater than those of a unit pole are its effects in any way.

The direction of these lines of force is plotted out by the path made by a free north pole with no inertia, if allowed to move by the magnetic force acting upon it. Thus, any source of magnetism is surrounded in space by these imaginary lines of force, and they are conceived so to exist and to be so plotted and computed that their density per sq. cm. at right angles to their path at any point gives the force of magnetism at that point; that is, the dyne's force on a unit pole.

These lines of force are known as flux. Thus, the flux from a magnetic pole is the flow of lines from the pole, and the number of lines per square centimeter is the same as the flux per sq. cm., or the density per sq. cm., or the force in dynes on a unit pole. It is known by the symbol H for an air magnetic circuit.

This flux may be produced in various ways. Iron possessing magnetic characteristics is found in nature. The usual method of producing magnetic flux is by an electromagnet.

THE ELECTROMAGNET.

The electromagnet is a coil of wire, with or without an iron core. Oersted, in 1820, discovered that a wire carrying a current of electricity produced lines of force similar to those produced by natural magnetic iron. The direction of these lines of force was found to be around the wire, as shown in Fig. 1. That is, if a current is passing into this paper perpendicular to it, the lines of force are concentric circles in the plane of the paper, causing a free north pole to move around in the direction of the hands of a clock, always

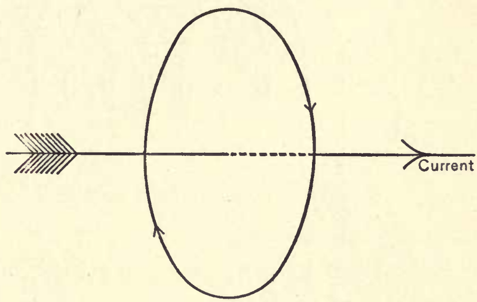

Fig. 1.

returning to the point from which it started. If, therefore, the wire is wound in a helix, as shown in Fig. 2, and a current of electricity passes through it, as shown by arrows, it is evident that the lines of force will have the direction, as shown by the dotted lines, coming out of the helix at *A*, and going in at *B*, thus producing a magnet. A magnet has two poles, so called, one north and one south. The north pole is that which points to the north of the earth, the earth itself being a magnet, and thus attracting it. The other pole is called south. Looking toward the end of a magnet, the pole about which a current of electricity is circulating clockwise is a south pole, and counter-clockwise, a north pole.

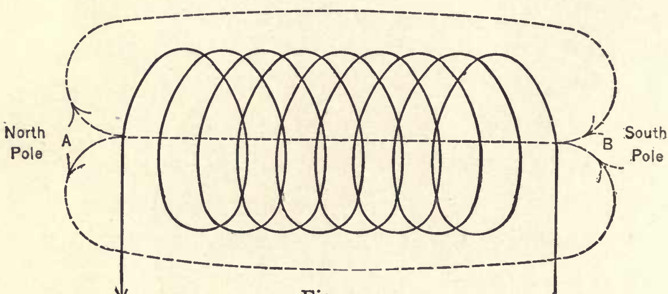

Fig. 2.

Reference to Fig. 2 shows that a north pole will be attracted to *B*, the south end, and a south pole to *A*, the north end, for a north pole moves in the direction of the lines of force; and, as shown in the figures, the direction is towards the south and away from the north pole of the electromagnet.

PRODUCTION OF FLUX.

Let us consider the production of flux. It has been shown that a unit of magnetism sends out lines of force, one per sq. cm. one centimeter away, and thus a total of 4π lines (since the surface of a sphere 1 cm. in radius $= 4\pi$). It has been shown, also, that a current of electricity creates lines of force exactly as a magnet does, having also the same magnetic effects as lines of force from a magnet. These lines of force have been shown to be concentric circles around the current producing them; thus, if a current is flowing in a wire perpendicular to this page, the lines of force are concentric circles in the plane of the page.

A magnetic circuit is the path of lines of force. Therefore the current of electricity, as described, is perpendicular to a magnetic circuit, in which lines of force are circulating, their number being proportional, in air, to the strength of the current flowing, and their density per sq. cm. depending upon their nearness to the wire. Various definitions of the unit of current are given. The following is one that has been suggested by Mr. C. P. Steinmetz: —

The unit current is one which produces in a magnetic circuit as described above, but of unit length, $4\pi/10$ lines of force per sq. cm., or, what is the same thing, produces a force on a unit pole of $4\pi/10$ dynes. Therefore, one ampere flowing into this paper will produce $4\pi/10$ lines of force per sq. cm. in the magnetic circuit, which has a length of 1 cm. in the plane of the paper. If two amperes flowed, $8\pi/10$ lines would be produced. If one ampere flowed, but through two turns, $8\pi/10$ lines would also be produced. The effect of two turns and one ampere is the same as the effect of two amperes and one turn.

By definition, 1 ampere gives a field intensity *in a magnetic circuit of unit length* of $4\pi/10$, at a distance therefor of 1 cm. from the wire carrying current, the field intensity, — that is, force in dynes on a unit pole, — or, what is the same thing, line of force per sq. cm., is $4\pi/10 \div 2\pi$ (since 2π is the length of the magnetic circuit 1 cm. away from the wire), or $\frac{2}{10}$. Hence I amperes produce the field intensity $2I/10$, and if N turns are interlinked with the flux (or force), $2IN/10$ expresses the field intensity. At any distance T the field intensity, which is usually expressed by the letter H, is $2IN/10T$.

Having now obtained an algebraic expression for *field intensity*, let us pass to another relation, that is *magnetizing* FORCE.

IN, the ampere-turns, are called magnetomotive force, since, as has been shown, the flux is proportional to them, also ampere-turns per unit of length of magnetic circuit are termed "magnetizing force," expressed,

$$\text{Magnetizing force} = \frac{\text{magnetomotive force}}{\text{length of magnetic circuit}}.$$

Thus, we have two equations as follows: —

1st, at distance T, as shown above, the field intensity (or lines per sq. cm.) $= \frac{2IN}{10T} = H$ (1)

and, 2d, The magnetizing force, $= \frac{IN}{2\pi T}$. (2)

since, by definition, magnetizing force = ampere-turns per unit length of magnetic circuit, and at any distance T from the wire the magnetic circuit has the length $2\pi T$ (the circumference of the circle of radius T).

From (2) $\quad T = \frac{IN}{(2\pi)\,(\text{magnetizing force})}$.

Substituting in (1), we get

$$\frac{2\,IN}{10} \times \frac{2\pi\,(\text{magnetizing force})}{IN} = \frac{(4\pi)\,(\text{magnetizing force})}{10} = H,$$

or the flux produced per sq. cm., in any air magnetic circuit, is equal to the ampere-turns per unit length of magnetic circuit times $\frac{4\pi}{10} = 1.257$.

This, then, shows the law of production of flux. If total flux is to be considered, instead of flux densities, H, as found, must be multiplied by the area of the magnetic circuit. And, if the medium be other than air, H, as found, must be multiplied by the factor μ, which shows the multiplying power of the medium over air in the production of flux with a given magnetomotive force. The density with other materials than air is designated by the letter B; thus, $B = \mu H$; μ is called permeability; and thus, for air, $\mu = 1$; for other materials, μ may be as high as several thousand.

PERMEABILITY.

Permeability is the name of the value which shows, as compared with air, the multiplying power as regards lines of force, or magnetism of the material composing the magnetic circuit. Thus, iron placed in a given magnetic circuit under magnetomotive force multiplies the lines of force many hundred times over the number obtained with air alone. Flux is generally designated by the letter ϕ, and permeability by the letter μ.

SATURATION.

It can be seen, by inspection of the formula,

$$H = 1.257 \times \text{ampere-turns per unit length},$$

that, whatever the density in a magnetic circuit of air, the ampere-turns required to produce it are proportional to it.

In a magnetic circuit of iron, however, μ is found to be a variable, ultimately decreasing with increasing density. Therefore, the flux is not proportional to ampere-turns, but with increasing values of ampere-turns the flux responds more and more slowly.

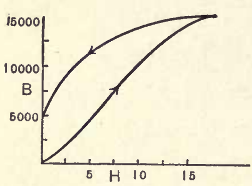

Fig. 3.

It is found in practice inadvisable to run, in the magnetic circuits of steel dynamo machines, much over 14,000 lines per sq. cm., and, in cast iron machines, much over 6,500 lines per sq. cm. The curve showing the relation between magnetizing force, H, and magnetic induction, B, in iron is called a saturation curve, one of which is illustrated in Fig. 3, the magnetizing force being plotted to the right, and the induction upward.

One method of obtaining this curve is to make a helix, as shown in Fig. 2, but having a length about 200 times its diameter. Measure the current flowing in helix, and, on breaking this current, note the throw of the needle of a "ballistic" galvanometer connected to the helix, and calibrated to read the total flux producing its deflection. Then flux equals $\frac{4\pi IN}{10\,l} \times A\mu$. Flux is measured by the galvanometer. l equals length of the solenoid (the length of the return air magnetic circuit being considered 0 on account of having, as compared to the area of the solenoid, an infinite area), $A =$ area of solenoid, $N =$ turns of wire on solenoid, and μ is the quantity desired, which can thus at once be deduced.

Knowing μ, B can be at once calculated, since, as has been shown,

$$B = \mu H \text{ and } H = \frac{4\pi IN}{10\,l}.$$

ELECTROMOTIVE FORCE.

Electromotive force expresses tendency of current to flow. In 1832 Faraday discovered that if a wire were moved to cut across the lines of force of a magnetic field, that an electromotive force would be created. An elaborated arrangement of this method of producing electromotive force (e.m.f.) is the modern dynamo.

For the measure of e.m.f., a practical unit, called the volt, is chosen. The volt is the e.m.f. produced in any electric circuit, if in that circuit 10^8 lines of force are cut per second; or, what is the same thing, in a closed circuit embracing a certain number of lines of force, if these are reduced or increased at the rate

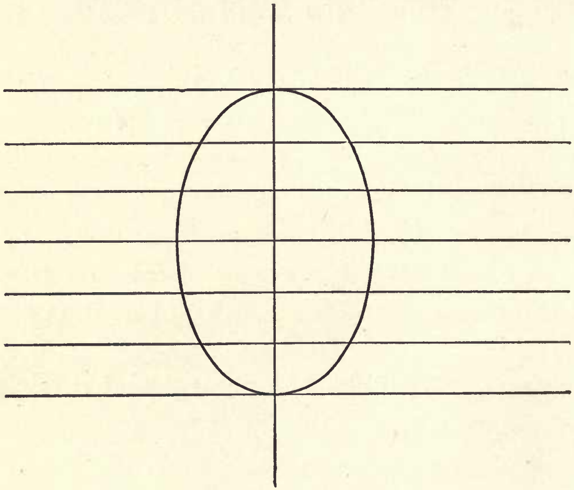

Fig. 4.

of 10^8 per second, a volt is produced. Looking at such a circuit, and along the lines of force, that is, in the direction a free north pole would move under their influence, if the lines of force are increasing, the electromotive force would tend to produce current in the circuit in a counter-clockwise direction. If decreasing, the e.m.f. induced would tend to produce current in the circuit in a clockwise direction.

SINE CURVE.

From this it can be seen that a turn of wire revolving uniformly about its axis in a magnetic field, — see Fig. 4,— will have an e.m.f. induced first in one di-

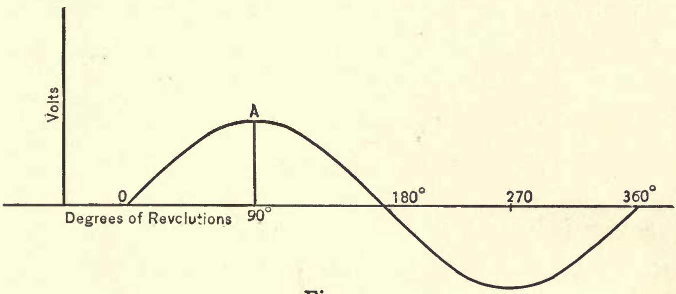

Fig. 5.

rection, and then in the other. In fact, if we plot this e.m.f., it appears as in Fig. 5, starting at 0, at 0°, rising to a maximum at 90°, 0 again at 180°, repeating in the other direction from 180° to 360°. The e.m.f. thus produced is called an alternating e.m.f.

Before discussing this curve of e.m.f., it is necessary to consider certain properties of angles.

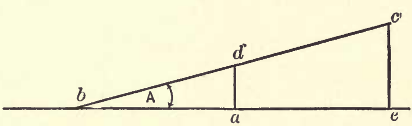

Fig. 6.

Referring to Fig. 6, the lines *cb* and *ab* intersect at *b*, forming with each other an angle *cba*. At any point along the line *bc*, say at *d*, draw a line perpendicular to *ba*, intersecting it at *a*. We then have the right-angled triangle *abd*, with the side *ad* perpendicular to the side *ba*. In such a triangle which can always be completed from the intersection of two lines by dropping a perpendicular from one side to the other, certain properties are given to the angle $A = abd$. First is the property of the angle A called a *sine*. The sine of the angle, A, is the ratio of the side, *ad*, to the side, *bd*, or, what is the same, the ratio of the line *ce* to the line *cb*, etc. Thus, in a right triangle the sine of any acute angle is the ratio of the leg opposite the angle to the hypothenuse of the triangle.

Second is the property of the angle A called a *cosine*. Referring again to Fig. 6, this is the ratio of the line *ab* to the line *bd*, or, what is the same, the ratio of *be* to *bc*. Third, the *tangent* is the ratio of *ad* to *ab*.

Thus the cosine is the ratio of the leg adjacent to the acute angle in question, divided by the hypothenuse of the triangle. These ratios are definite for any angle. For instance, the sine of 30° is $\frac{1}{2}$; the cosine of $30° = .866$.

Once more examining Fig. 5, which represents the voltage produced in a turn of wire revolving uniformly about its axis in a magnetic field, it will be seen — and it is an experimental as well as theoretical fact — that the voltage at any point along the horizontal line is equal to the maximum value at *A* multiplied by the sine of the angle from the o starting point to the point in question. The curve so produced, shown in Fig. 5, is, therefore, called a sine curve, and represents the voltages given by alternating dynamos and delivered to alternating motors. This sine curve possesses certain definite properties.

One property of great importance is a value called its *square root of mean square;* that is, $\sqrt{\text{mean square}}$, which is, as the name implies, the square root of the average of the squares of all the values. This value and its ratio to other values of a sine curve is of great importance. Ordinary measuring instruments, such as voltmeters, ammeters, etc., always record on their scales, and thus give readings which are square root of mean square values.

PHASE AND AMPLITUDE.

The angular displacement from any reference point, usually o, is called the phase. The maximum value at *A*, Fig. 5, is called the amplitude. The zero voltage point is when the coil in the magnet field incloses the maximum number of lines of force, since in revolving, the rate of change of the lines of force, that is, the number of lines of force cut per second, is there zero. By working out the various values of the sine curve, it is found that the square root of mean square value of voltage equals the maximum value at

A, Fig. 5, divided by $\sqrt{2}$, or $\frac{A}{1.414}$. Also, that the average value of e.m.f. $= .637\ A$, that is, $A \div \pi/2$. Hence, the average value of e.m.f. divided by square root of mean square

$$= \frac{.637}{.707} = .90 = \frac{2\sqrt{2}}{\pi}.$$

Thus there is about 10% difference between them.

The formula for the maximum e.m.f., A, thus produced, is, as is shown later, $\frac{2\pi n\phi N}{10^8}$ where $\pi =$ 3.14159, $N =$ number of turns in series inclosing flux, $\phi =$ the number of lines of force inclosed by the coil in its maximum position, $n =$ cycles per second. A cycle is one complete electrical revolution. Thus, in a dynamo of two poles, the cycles equal the revolutions per second; with four poles, the cycles are the number of revolutions $\times \frac{\text{number of poles}}{2}$, thus twice the revolutions, and so on.

Since the square root of the mean square value equals the maximum value divided by $\sqrt{2}$, the formula for the square root of mean square value becomes

$$\frac{2\pi n\phi N}{10^8\sqrt{2}} = \frac{4.44\ n\phi N}{10^8}$$

which is the voltage as read at the terminals of a 2 pole alternator by any commercial voltmeter, when the alternator has a flux of ϕ lines of force from each pole, passing through the winding in which the e.m.f.

is produced. That is, N turns in series all inclosing the lines of force twice each revolution, and finally a speed of n revolutions per second. Where the winding of the armature of a dynamo is of such construction that all the flux does not pass through all the turns of the coils, such, for instance, as a pancake coil like Fig. 7, a proper allowance must be made for the reduced lines of force in the internal turns.

The electromotive force, as just described, is called an alternating e.m.f., or sine curve of e.m.f. There is

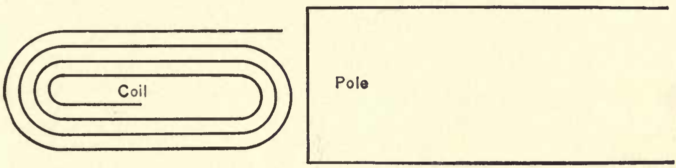

Fig. 7.

another kind, called continuous e.m.f., which instead of rising and falling, as shown in Fig. 4, has a constant value. This e.m.f. is produced by cutting across lines of force as before, but the windings of the dynamos producing it require a commutator with brushes resting on it, instead of collector rings connected to the ends of the winding, as in the case of dynamos producing alternating currents. The segments of the commutator are tapped at regular intervals; thus, as far as the brushes are concerned, the production of e.m.f. is always in a constant direction. This is illustrated in Fig. 8. *A* is the commutator, with segments insulated one from another, and connected to the windings of the revolving part, *B*, called an armature, at regular intervals. *CC* are the brushes resting on the commutator, *D* is the magnet wound with wire

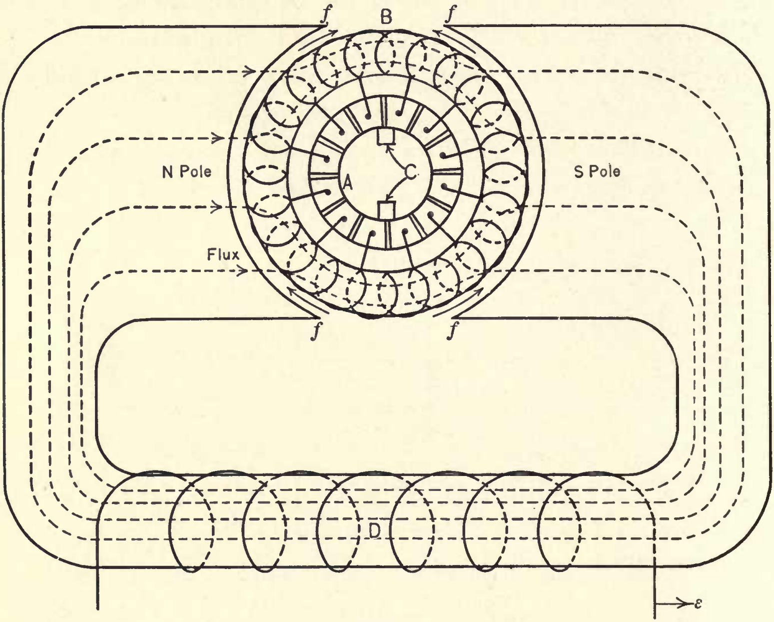

Fig. 8.

to produce the magnetic flux passing through the coils on the armature, *B*. When *B* is rotated clockwise, all the wires on the left of a vertical line passing through the center of the armature are producing an e.m.f. upwards toward *B;* all the wires on the right of this line are doing likewise. This can be determined by the following law: looking along the direction of the lines of force, that is, from the north pole to the south, a coil of wire in which the lines of force are increasing will have an e.m.f. induced in it in a counter-clockwise direction. Thus, if at *B* and at the opposite point, brushes be placed, this e.m.f. becomes available, and is constant in quantity, since, whatever position the armature may take, the number of wires in series between the brushes is the same and in the same position in space, and the e.m.f., as regards the brushes, has a constant direction. In fact, the e.m.f. is the average of the e.m.f. of all the coils of the armature between the brushes; that is, the average value of the sine curve. This is shown to be expressed as follows: volts $= \frac{4\,Nn\phi}{10^8}$ where $\phi =$ magnetic flux passing through the armature, $N =$ number of turns in series between brushes, $n =$ cycles per second.

This relation follows, since four times in each revolution the coils are filled and emptied of flux, and since volts = rate of change of flux $\div\ 10^8$. From this now follows the formula for

$$\text{maximum e.m.f.} = \frac{2\,\pi n\phi N}{10^8}$$

since in a sine curve the

$$\text{maximum value} = \text{average value} \times 1.5708 =$$

$$\text{average} \times \frac{\pi}{2} = \frac{4\,nN\phi}{10^8} \times \frac{\pi}{2} = \frac{2\,\pi n\phi N}{10^8}.$$

As stated, e.m.f. tends to produce a flow of electricity; that is, tends to produce current. A current of electricity is the amount of electricity passing a given point of an electric circuit in a second. Its practical unit is called an ampere. An ampere is that current which, in flowing in a straight conductor of infinite length, will exert a force of $\frac{2}{10}$ of a dyne on a unit pole situated 1 cm. from the wire.

Another definition of the same thing is, that current which in an air magnetic circuit 1 cm. in length will produce a field intensity of $\frac{4\pi}{10}$.

Another definition is, that current which, flowing in a wire bent to a unit radius, will exert on a unit pole at the center, a tenth of a unit force (called a dyne) per unit length of conductor.

As stated, magnetic lines of force are produced by a current flow, just as they exist in a natural magnet, and hence, from the definition of lines of force, exert a force on a unit magnet pole, through which relation the unit current is defined. The quantity of electricity passing each second a given point of a circuit, which gives a flow called unit current, or an ampere, is a coulomb. Thus, an ampere is 1 coulomb per second; and inversely, a coulomb is the amount of electricity which, flowing each second by a point in an electric circuit, gives an ampere.

RESISTANCE.

In any circuit in which an e.m.f. has been produced, a certain current will flow. In some circuits, more will flow than in others. The value which keeps the current down is called, in continuous circuits, resistance. The relation governing the flow of continuous current in circuits is called

OHM'S LAW.

It is as follows:—

$I = \frac{E}{R}$. If $I = 1$ amp. and $E = 1$ volt, the resistance, R, as calculated, is the unit of resistance, and is called an ohm. An ohm, therefore, is that resistance which will in an electric circuit allow an ampere of current to flow under the action of one continuous volt. This resistance is a constant, in a given material, except for the effect of heat.

INCREASE OF RESISTANCE BY HEAT.

If $R^\circ =$ resistance at 0° centigrade, the resistance at any other temperature T is found to be expressed by the following formula:

$$R_T = R_o(1 + \alpha T)$$

where $\alpha =$ a coefficient depending on the material. In copper $\alpha = .0042$. From this it can be seen that a continuous e.m.f. applied to a resistance produces a continuous current, and a variable e.m.f., like the sine curve of e.m.f. shown in Fig. 4, produces a variable current. Thus we may have sine curves of current just as we do of e.m.f.'s, having a square root of mean square value equal to the maximum value divided by $\sqrt{2}$, and having phase, amplitude, etc.

We thus have direct e.m.f.'s and currents and alternating e.m.f.'s and currents.

SELF-INDUCTION.

As has been pointed out, we may have a sine curve of current, just as we do of e.m.f.; that is, a current varying from instant to instant. Since in a circuit lines of force exist in proportion to the current, at each of its different current values there is a different value of the lines of force. Thus, in a circuit of varying current we have a continually varying flux. Therefore, this variation of flux produces in itself an electromotive force. As a matter of fact, by varying the current an e.m.f. is produced, tending to stop the change of current. This characteristic is designated as back e.m.f. due to self-induction. In any circuit the coefficient of self-induction L is equal to the

$$\frac{\text{maximum flux} \times \text{turns}}{\text{amperes} \times 10^8}.$$

This value L is expressed in a unit called the henry. When multiplied by $2\pi n$, ohms are obtained, as shown in the following:

Since square root of mean square volts equals E,

$$(1)\quad \frac{2\pi n N\phi\,(\text{max})}{\sqrt{2} \times 10^8} = E,$$

and

$$(2)\quad L = \frac{\phi\,(\text{max})\,N}{\text{amp. (max)} \times 10^8},$$

from definition.

Readjusting (1) gives

$$(3)\ E = \frac{\phi\ (\text{max})\ N\ \text{amp. (max)}}{\text{amp. (max)} \times 10^8} \times \frac{2\ \pi n}{\sqrt{2}}.$$

Substituting (2) in (3) gives

$$(4)\ E = L \times \text{amp. (max)} \times \frac{2\ \pi n}{\sqrt{2}};$$

but amp. $\frac{(\text{max})}{\sqrt{2}}$ = amp. square root of mean square as read on an ammeter, hence,

$$E \text{ square root of mean square} = 2\ \pi n L I,$$

where I is the square root of mean square current and $2\ \pi n L$ is a value in ohms, as can be seen, since from Ohm's law

$$E = IR, \text{ hence } R \text{ must equal } 2\ \pi n L.$$

Thus the back e.m.f. of self-induction is equal to $2\ \pi n L I$. Since the flux of a simple air circuit is proportional to and coincident with the current, and since, when the current is 0, the back e.m.f. is a maximum (due to the rate of change of current, and therefore of flux being here a maximum), it follows that the e.m.f. of self-induction lags behind the current 90°. Thus the applied e.m.f. is opposed to two factors: first, the e.m.f. drop of resistance = IR, and second, 90° therefrom, the e.m.f. of self-induction = $2\ \pi n L I$. This can be shown in two ways: first, as in Fig. 9; and second, as in Fig. 10. Both express the same thing. In Fig. 9 are shown the current and e.m.f.

at each instant of phase, the current lagging about 70°, the angle of phase being plotted on the horizontal line, and the volts on the vertical. In Fig. 10 the square root of mean square values only are considered, and are plotted by vectors in angular displacement one from another. In diagrams of this kind, the angle that the vector makes with any reference line, usually the horizontal line of the figure, represents the phase of the vector. The length of the vector repre-

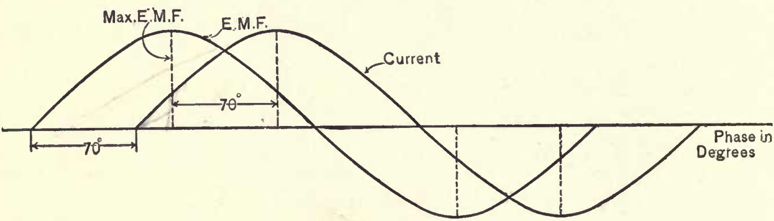

Fig. 9.

sents the maximum value of its sine curve, or, more often, the square root of mean square value, since they bear a definite ratio to each other. Thus, in this style of diagrams, angles represent phase difference, and lengths of line represent maximum or square root of mean square values of the sine curves. A diagram like Fig. 10 is called a polar coördinate diagram. One like Fig. 9 is called a rectangular coördinate diagram. The polar coördinate diagram is the one most used to present the relations of alternating currents, since it offers a most convenient method of combining two or more waves into a single resultant wave, or, conversely, of separating a single wave into its component waves. It is a mathematical fact that sine curves can be combined by combining their vectors, as shown in the polar diagram of Fig. 10. If a line represents the maximum or square root of mean square value of a sine curve of a given frequency, and if its position as related to any reference line represents the phase, the resultant combination sine curve of

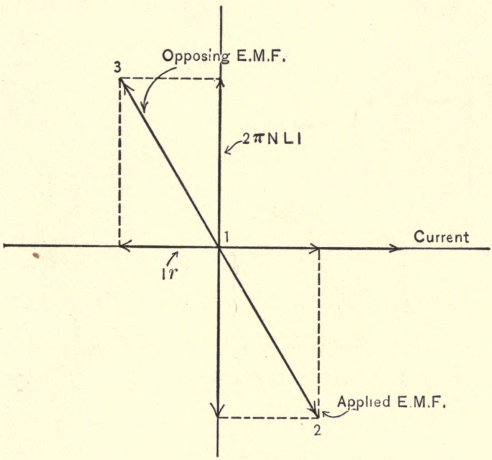

Fig. 10.

it and any other sine curve of the same frequency similarly represented is the diagonal of the parallelogram whose sides are the lines representing the sine curves being combined. Thus, in the case of resistance and inductance opposing an applied e.m.f., the resultant of these opposing values is expressed by the diagonal of the parallelogram whose sides are made up of the resistance drop and inductance drop, as shown in Figs. 10 and 11. The resultant opposing e.m.f., under conditions as described, that is, the IR drop in phase with the current, and the inductance drop $2\pi nLI$ at right angles to the current, is found by plotting the resultant of the two components. Thus, in an alternating circuit, the applied e.m.f. is made up of the "vector sum," that is, the resultant of the various component electromotive forces. The line, 1-2, in Fig. 10, shows the summation of the component e.m.f.'s, IR and $\pi 2nLI$, and the line 1-3, shows the actual opposing resultant e.m.f. It is customary to show the summation. The diagram showing this for the case in Fig. 10 is shown in Fig. 11, which expresses the fact that current lags behind the applied e.m.f. by 70°.

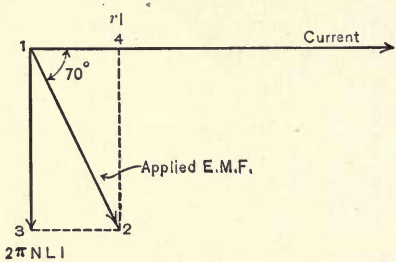

Fig. 11.

Since, by geometry (Fig. 11),

$$\overline{1-2}^2 = \overline{1-4}^2 + \overline{1-3}^2$$

it follows that

$$R^2I^2 + \overline{2\pi nLI}^2 = (\text{applied e.m.f.})^2$$

or

$$\text{applied e.m.f.} = I\sqrt{R^2 + \overline{2\pi nL}^2}.$$

In the alternating current circuit, therefore,

$$\sqrt{R^2 + \overline{2\pi nL}^2}$$

takes the place of resistance alone.

IMPEDANCE.

In alternating circuits this value, square root of the sum of the squares of resistance and the value of $2\pi nL$, which takes the place of resistance, is called impedance. The value $2\pi nL$ is called reactance, and is expressed in ohms when L is expressed in henrys $= \frac{\text{flux} \times \text{turns}}{\text{amp.} \times 10^8}$. We thus have, in alternating circuits, two ohms to consider, one equal to the plain resistance, and the other equal to $2\pi nL$, the two combining at right angles to a value, called impedance. The reciprocal of the impedance is called admittance. Thus the admittance is the current flowing in a circuit with one volt applied. Thus admittance times volts = amperes.

POWER IN INDUCTIVE CIRCUITS.

The power in inductive circuits, when the current lags behind the e.m.f., as shown in Fig. 11, is equal to $E \times I \times$ cosine of the angle of lag, E and I being expressed in square root of mean square values. The sine, cosine, tangent, etc., of all angles have been calculated and tabulated. For instance, the cosine of 70° = .9397; hence the power can at once be determined. In practice, this power can be measured by an instrument called a wattmeter. In vector diagrams, like Fig. 11, the power is the product of the e.m.f., and the current in phase with it or the projection of the current on the e.m.f., which is $E \times I \times$ cosine of angle between them. This is a natural conclusion, since if the lag between E and I is 90°, when the current is a maximum the e.m.f. is 0, and the power 0. Multiplying together all the values of the sine curves representing the e.m.f. and current, and adding the products to get total power, gives the same result; namely, power equals $IE \cos \alpha$ where α equals angle of phase displacement between the maximum values of the two curves.

WATTMETER.

A wattmeter has two coils: one movable, to which a pointer is attached, which shows a deflection on a properly calibrated scale; and the other stationary. The movable coil is in shunt across the voltage of the circuit, and the current flowing in it is in phase with the e.m.f., the coil being made non-inductive by having in series with it a large non-inductive resistance, and hence its magnetism is in phase with the voltage. The stationary coil carries the current flowing into the circuit whose energy is desired, and hence its magnetism is in phase with the current. Thus the force between the two coils, which results from the two magnetisms, $= E \times I \times \cos$, of lag, or the power of the circuit is recorded. In non-inductive circuits the lag is zero; and since the cosine of 0° is 1, the power in watts $= EI$.

746 watts = 1 horse-power.
1 watt-second = 1 joule.

RISE AND FALL OF CURRENT WITH SUDDENLY APPLIED OR WITHDRAWN CONTINUOUS E.M.F.

Not only do we have a back e.m.f. of self-induction in alternating circuits, but also, for a short space of time, a back e.m.f. exists even in continuous current circuits. Take a coil of wire and apply suddenly to it a constant e.m.f. Since the wire has no current in it, and since, owing to the application of the constant e.m.f., it must shortly have current, it follows that the current must rise from o to a value equal $\frac{E}{R}$. In rising, therefore, the lines of force must increase, and by so doing a back e.m.f. must be created, delaying the rise. So also when a constant condition has been reached, if the e.m.f. be suddenly withdrawn, the current must shortly change from $I=\frac{E}{R}$ to o; and during the change an e.m.f. will be created, delaying the change. Thus anywhere, if an attempt be made to change the current, a back e.m.f. is introduced by the change of current, tending to stop the change. This fact has been expressed by what is generally known as

LENZ'S LAW,

which is as follows:

In all cases of magnetic induction, the induced currents have such direction that their reaction tends to stop the motion which produces them.

The speed with which a current rises or falls in a circuit depends upon its resistance and its inductance. The formula for the rise of current can be shown to be as follows:

$$I = I_1\left[1 - \epsilon^{-\frac{tR}{L}}\right].$$

Where I = the current in amperes at any instant t.
I_1 = the final current.
ϵ = the base of the Napierian logarithms = 2.71828.
R = resistance of the circuit in ohms.
and L = the inductance in henrys.

TIME CONSTANT OF A CIRCUIT.

The value of $\frac{L}{R}$ of any circuit is called the time constant of that circuit, and it can be shown that it is equal in seconds to the time it takes for the current to rise to about $\frac{2}{3}$ of its final value.

The formula for the fall of current in a circuit closed by an additional resistance R, at the instant the voltage is withdrawn, is

$$I = I_1\epsilon^{-\frac{R+R_1}{L}t}.$$

ELECTROMAGNETIC ENERGY.

When a circuit is magnetized by a current, it possesses energy, or, in other words, a certain definite number of foot-pounds of work* due to this magnetism.

* WORK: A certain definite number of foot-pounds, and is independent of time.

POWER: Rate of doing work; that is, the foot-pounds per second. Thus in electricity a watt equals a rate and represents power.

A joule represents work.

Therefore, to bring up a magnetic circuit to a certain degree of magnetism requires the expenditure of a certain definite amount of work, depending upon the value of L, the coefficient of self-induction of that circuit. It can be shown that the amount of work expressed in watt-seconds or joules $= \frac{1}{2} LI^2$, when L is expressed in henrys.

The inductance, L, of a circuit is a constant, if the permeability, μ, of the circuit is a constant.

$$\text{For } L = \frac{\phi \times \text{turns}}{\text{amp.} \times 10^8}$$

and ϕ is proportioned to μ; thus L must be proportioned to μ. Therefore, in coils with iron cores, L is a variable, changing with the value of current, just as μ in the iron changes. In practice, when iron composes the magnetic circuit, there is a certain loss in this iron, since it is magnetic, and also electrically conducting; and in dynamo machines and in coils with iron cores this loss is a matter to be most carefully considered. This loss in iron is called hysteretic loss.

HYSTERESIS.

When iron is magnetized, the molecules all set themselves in a certain direction as nearly as they can. If the magnetism is reversed, the molecules all try to turn the other way. The result of this is that when the magnetism is continually changed from north to south, as in the case of a coil of wire with an iron core with a sine wave of e.m.f. applied to it, the molecules, turning back and forth, rub on each other, and produce a loss due to friction upon themselves. This has been carefully investigated by Mr. C. P. Steinmetz in a set of classical experiments, and he finds an expression for it as follows:

Loss of energy in watt-seconds, or joules per $cm.^3$ and cycle of magnetism $= \frac{KB^{1.6}}{10^7}$ where $B =$ flux density per cm., and K is a constant depending upon the quality of the iron.

FOUCAULT CURRENTS.

If the core of one magnet under consideration be solid, the changing flux in it produces currents called Foucault or eddy currents, for the iron is a conductor of electricity as well as of magnetism, and, therefore, a varying flux of magnetism through it produces currents circulating around in the iron. To prevent these currents and the losses due to their presence, the armatures of dynamo machines and cores of alternating current magnets and transformers are made laminated, so as to increase the resistance of the natural paths of the currents. Since the e.m.f. producing them is proportional to the flux, and, therefore, below saturation, to the main current flowing in the winding, and since the loss of energy by a current passing through a resistance is $I \times IR = I^2R$, it follows:

First, that the energy loss from Foucault currents is proportional to the square of the main or inducing current.

Since, also, the e.m.f. producing the Foucault currents is proportional to the cycles

$$\left(\text{note that e.m.f.} = \frac{4.44\, nN\phi}{10^8}\right),$$

and the current appearing on this account is proportional to the e.m.f., it follows:

Second, that Foucault currents are proportional to the square of the cycles. Thus, in our coil and its diagram, as shown in Fig. 11, the component RI in phase with the current has included in the value R, not only the resistance of the coil but also an amount which, when multiplied by I^2, gives the loss due to hysteresis and Foucault currents that may exist.

These Foucault or eddy currents may exist in wires themselves, when carrying alternating currents, the effect being negligible with ordinary frequencies, or with small wires, but important enough to be considered in specific cases when the frequency is high or the size of the wire large.

CAPACITY.

If two conducting plates be set near together, and be connected to an alternating applied e.m.f., a current will flow in and out of the plates just as if the circuit were mechanically complete. The reason for this is that a constant voltage applied to the plates produces a displacement of electricity, the plate, A,

Fig. 12, being abnormally charged, and the plate, *B*, being abnormally discharged, due to the presence of the voltage. (See pages 93–95 for further discussion.)

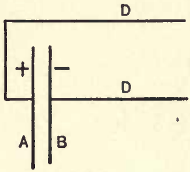

Fig. 12.

If, therefore, the voltage, *E*, instead of being constant, is variable, the charge must readjust itself according to the voltage whether it be plus or minus. Since a sine curve is first + and then —, the charge of electricity will flow back and forth from *A* to *B*, and return, so that in the wires, *DD*, connected to the plates, a current will flow, varying with the applied voltage.

Referring to Fig. 13, the full line sine curve shows the applied voltage, and the dotted line shows the current flowing. During the rising part of the volt-

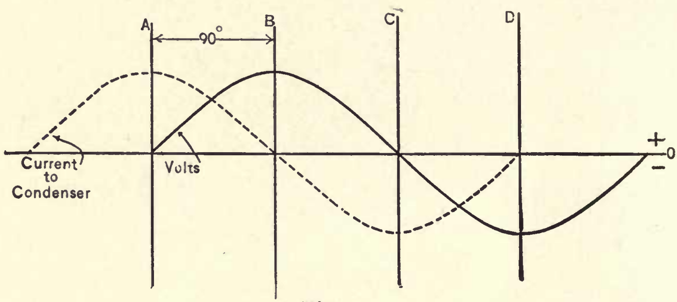

Fig. 13.

age curve, current will naturally flow into the condenser as the volts are getting higher; that is, from *A* to *B*, current will flow into the condenser. At *B*, the voltage remaining constant for an instant, no current will flow, the current becoming 0, as shown.

During the falling part, from B to C, the current will flow out of the condenser as the voltage decreases from B to D. From an examination of Fig. 13, it is apparent that the current leads the e.m.f. by 90°,

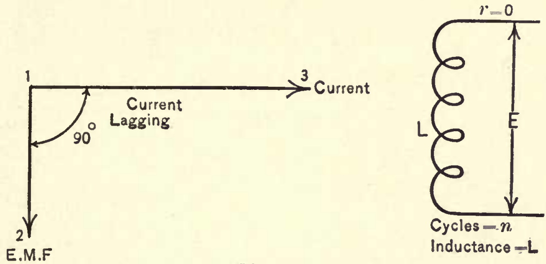

Fig. 14.

instead of lagging behind it, as in the case of voltage applied to inductance. Also, if the applied e.m.f. curve be a sine curve, the resultant current in the condenser will be a sine curve. In condensers there is a loss in resistance due to the flow of current in its plates, also a hysteresis loss in the dielectric, namely,

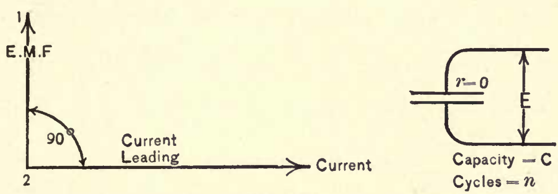

Fig. 15.

insulation space, separating its plates. These losses, however, are usually small, or even negligible.

The two cases of inductance and capacity are shown by polar diagram in the above figures, 14 and 15, where for inductance the inductive reactance $= 2\pi nL$. The capacity reactance as shown below $= \frac{1}{2\pi nC}$ when C is expressed in farads and n in cycles; for, by definition, the

UNIT OF CAPACITY,

called the farad, is such that an ampere for a second flowing into it, as described, will create between its plates a volt difference of potential. If the capacity be over a unit, more current or more time than one second will be required to produce a volt, in proportion to the amount of capacity. To produce more than a volt in a given capacity, more current must flow for a second in proportion to voltage to be produced.

CAPACITY REACTANCE.

Hence, if an applied square root of mean square voltage, $E = \frac{E_{max}}{\sqrt{2}}$, be applied to a condenser of capacity C at a frequency equal to n, twice in each complete cycle the condenser is charged and discharged. Hence the time of one charge equals $\frac{1}{4n}$, and hence the average amperes that must flow to give the voltage $\sqrt{2}E$ must equal $4n\sqrt{2}EC$. Since the square root of mean square amperes $= \frac{\pi}{2\sqrt{2}} \times$ average amperes (which relation can be deduced from a sine curve), it follows that the square root of mean square amperes flowing in the condenser

$$= 4n\sqrt{2}EC\frac{\pi}{2\sqrt{2}},$$

or $$= 2\pi nCE.$$

Therefore $$I = 2\pi nCE$$

or $$I = \frac{E}{\frac{1}{2\pi nC}}.$$

Therefore, by analogy with Ohm's law,

$$(1)\ \frac{1}{2\pi nC} = \text{capacity reactance.}$$

From an inspection of formula (1), it can be seen that the higher the frequency, the more current flows into a condenser, which reverses the case of inductance, where the higher the frequency the less the current.

From an examination of Figs. 14 and 15 it can be seen that if a condenser be put in series with an inductance, the value of which in ohms equals the value of capacity reactance in ohms, a short circuit results unless the circuit also have resistance; the line, 1–2, in Fig. 14, neutralizing the line, 1–2, Fig. 15. In fact, the diagram would look as in Fig. 16.

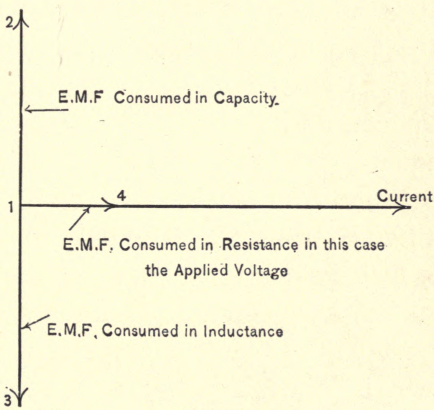

Fig. 16.

As can be seen, 1–2 and 1–3 neutralize each other, and the only factor holding back the current is the resistance of the circuit.

It can also be seen that the values, 1–2 and 1–3, can be very large as compared with the applied voltage, or, what is the same thing, the consumed voltage, 1–4. Thus, where such a condition exists, the voltages, 1–2 and 1–3, may rise to such values that the insulation of the circuit may be punctured. This condition is called resonance. When the values, 1–2 and 1–3, are not exactly equal, the condition is called partial resonance.

VARIATION IN WAVE SHAPE.

Since all dynamo machines do not give exact sine curves of e.m.f., and since in alternating calculations it is necessary that sine curves be considered (since the laws of the variation of values and combinations of sine curves, one with another, are mathematically known, and since measuring instruments record only square root of mean square values), it is desirable to substitute for waves other than sine curves their sine curve equivalents. The only condition to be observed is that the equivalent sine curves have the same square root of mean square value as the irregular curves, and that they are plotted properly in positions one to another. Thus, if in the conditions, as before discussed, we have an irregular wave of electromotive force, applied to a coil of wire, we have at the same time an irregular wave of current flowing. For proper discussion of alternating phenomena, these waves must be replaced by equivalent sine curves, and the relative phase of the two sine curves, that is, the angle between them, must be such that the product of their square root of mean square values and the cosine of the angle of difference of phase between them must equal the mean of all the instantaneous products of volts and amperes in the original waves. For otherwise, in calculating the power from the volts and amperes (power always equals the product of volts and amperes × cosine of angular difference of phase), the true result would not be obtained. The true result is, of course, the average of all the products E and I in the original waves. Likewise the square root of mean square value of each equivalent sine curve must equal the square root of the mean square of corresponding original wave, since commercial measuring instruments record automatically the square root of mean square of any wave.

HARMONICS.

It can be shown that an irregular wave of electromotive force can be resolved into its constituent sine waves. Thus, take the waves shown in Fig. 17. Curve A is the original wave, curve B is the equivalent sine wave curve, and curve C is the sine curve which, if combined with curve B, the equivalent sine curve, would give the original sine wave A, as can be seen by examination of the figure. As shown, curve

C has 3 times the frequency of the original wave or the equivalent wave, and its amplitude is less. In fact, the wave *C* is the third "harmonic" of the fundamental wave *B*, and original waves may be of such a shape that 3d, 5th, 7th, etc., harmonics may appear. It can be shown that even harmonics do not exist. In circuits to which are applied such irregular waves, the conception is correct, and in calculations it is proper to assume in the circuit the fundamental

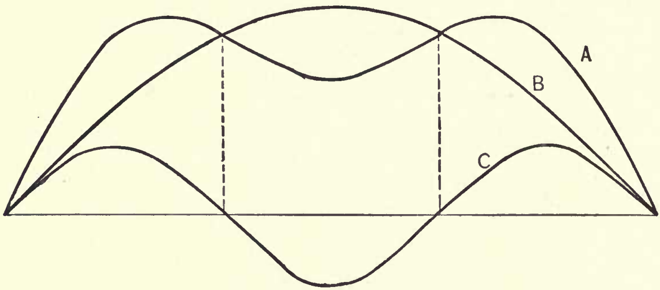

Fig. 17.

equivalent sine curve and simultaneously the harmonic. Hence, when the harmonic is of any considerable amount, it must be taken into account in the calculation of current flowing. Thus, in circuits having an appreciable capacity, the question of resonance, as illustrated in Fig. 16, must be considered, not only for the fundamental frequency and square root of mean square value, but also for the harmonic frequency and its square root of mean square. Since the current flowing into a condenser increases in proportion to the frequency, the harmonic causes increased condenser, and therefore resonance effect, and is not, therefore, at all to be desired.

One case where a peaked wave offers some advantage, is that of transformer iron losses where the core, or hysteresis loss, is less with such an applied e.m.f. wave than with an applied sine wave of the square root of mean square value.

For example, refer to Fig. 18. The curve *A* represents the peaked wave of applied e.m.f., or, what

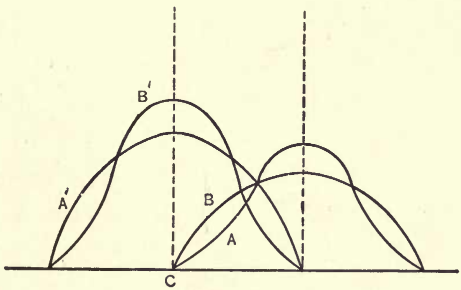

Fig. 18.

is the same thing, the back e.m.f., and the curve *B* represents the sine curve of applied e.m.f., both having the same square root of mean square value. At the point, *C*, in both curves the e.m.f. is increasing at the maximum rate. With the curve, *B*, the rate of increase, as can be seen, is faster than with the curve *A*. Therefore, since the two curves have the same square root of mean square value, the flux, *B'*, the maximum of which occurs at *C*, must be larger and have a greater rate of change in the case of B than in the case of A, whose flux curve is shown as A'. Thus the B flux must vary fast at C, and hence be more peaked than the A flux, A' varying more slowly at C, as shown. Since the loss due to hysteresis is dependent on maximum flux, the loss for the sine curve of applied e.m.f. is greater than for the peaked.

FORM FACTOR.

The inverse ratio of the average of all the values of any wave to the average of the equivalent sine wave is called the form factor of the wave, and gives a rough estimate of the presence of harmonics and effects on core losses of transformers, etc. One source of variation of wave shapes is the effect on a current flowing into an iron circuit. Here the distortion naturally results, owing to the saturation of the iron, and to the fact that to magnetize iron with the current rising requires more ampere-turns than with the current falling, and to the fact that an alternating applied e.m.f. always rises to a positive maximum, then goes to zero, and then to a negative maximum.

Fig. 19 represents the saturation curve of a piece of soft iron carried to a positive maximum, through zero, and then to a negative maximum, that is, just the cycle that an applied sine curve of e.m.f. would produce.

Starting at the point, C, and increasing in a positive direction, the magnetizing force, H (that is, the applied ampere-turns per cm. of magnetic circuit times $\frac{4\pi}{10}$, or, what is the same thing, the flux in an air circuit 1 cm. in cross section and 1 cm. long), we get a flux, as shown by the line, *CA*, reaching a maximum of 15,000 lines per sq. cm. At *A*, the magnetizing source commences to reduce (since a sine wave of current reaches a maximum, and then approaches zero), but it is a fact that now the flux does not reduce along

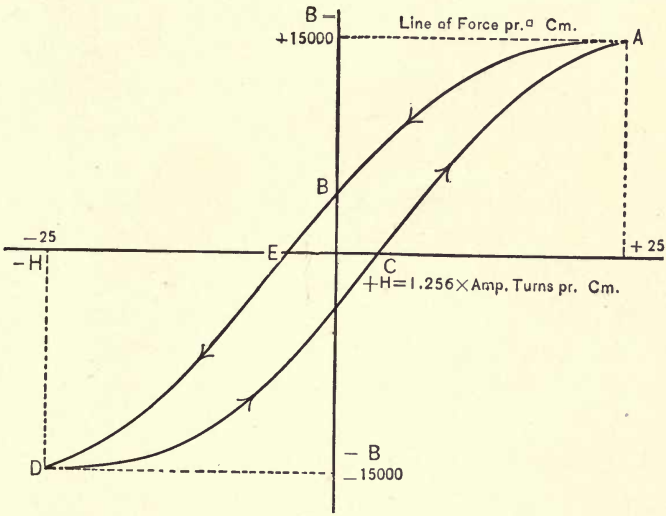

Fig. 19.

the line, *AC*, but along the line, *AB*, and at zero magnetizing force the flux is a considerable positive amount. Not until a negative magnetizing force, $-H$, at *E* is reached, does the flux become zero. This is on account of the fact that the molecules do not turn wholly around to agree with the opposite magnetizing force until a considerable amount of it has been consumed in turning them. Simultaneously with this turning around, we get the loss called hysteretic or "core loss," mentioned before, due to the rubbing of molecules of iron on one another. This curve is called the "hysteresis loop," and it can be shown that the area inclosed by the lines is a measure of the hysteresis loss per cycle. From E onward, the upper half of the wave is repeated, but in a negative direction. We have thus completed a cycle of magnetism. From an examination of Fig. 19, the fact of more magnetizing force being required when the magnetism is increasing than when it is falling, is clearly shown. From this follows the fact that distortion of current wave shape may result with an applied sine curve of e.m.f.; for, take a coil of wire with an iron core, and let a sine wave of e.m.f. be applied to it like a transformer with open-circuited secondary. Assume, in this case, R to be practically negligible, then

$$I = \frac{E}{2\pi n L}, \text{ or } 2\pi n L I = E, \text{ but } 2\pi n L I = \text{e.m.f.}$$

as has been shown; thus the applied e.m.f. practically equals the back e.m.f. at any instant. But the applied e.m.f. is a sine-wave, therefore the back e.m.f. must be a sine curve, and, therefore, the flux must vary in a sine curve, since the coils are stationary and constant in number; but, in accordance with Fig. 19, the production of a sine curve of flux during its rising, requires more magnetizing force (or, what is the same thing, more ampere-turns) than during its falling, hence the current curve producing the sine curve of flux is higher in value than a sine wave during the rising part, and lower during the falling part, and is thus distorted somewhat, depending on the quality of the iron core in the magnet.

POWER FACTOR.

(N.B. In future, unless otherwise stated, E and I equal square root of mean square values.)

In the coil of wire, as shown in Fig. 7, and discussed in Figs. 9 and 10, it has been shown that the current flowing into the coil lagged 70°. It has also been shown that the power delivered into a circuit, as measured in watts (746 watts = 1 h.p.), by a wattmeter, equals $E \times I \times \cos$ of angle between them. This cosine of the angle of lag is called the "power factor." In any right-angled triangle, the cosine of either of the other two angles is the ratio of two sides adjacent to the angle, the longest side being the denominator. Thus, in Fig. 20 the cosine of angle $ABC =$ ratio of $\frac{BC}{AB}$. The sine of the angle is the ratio of the side opposite the angle to the longest side, thus in Fig. 20 the sine of angle $ABC = \frac{AC}{AB}$. Therefore, if BD equals the applied e.m.f., and if AB equals the current flowing and lagging 70°, the power equals

$BD \times AB \times$ cos angle ABD. But since the cos $ABD = \frac{BC}{AB}$, the power equals $BD \times AB \times \frac{BC}{AB} = BD \times BC$, or equals the product of BD and the projection of the current vector upon it. Thus, power is always the product of vectors in phase; that is, coincident with each other. The same thing may be expressed in another manner. The product of the current BA and

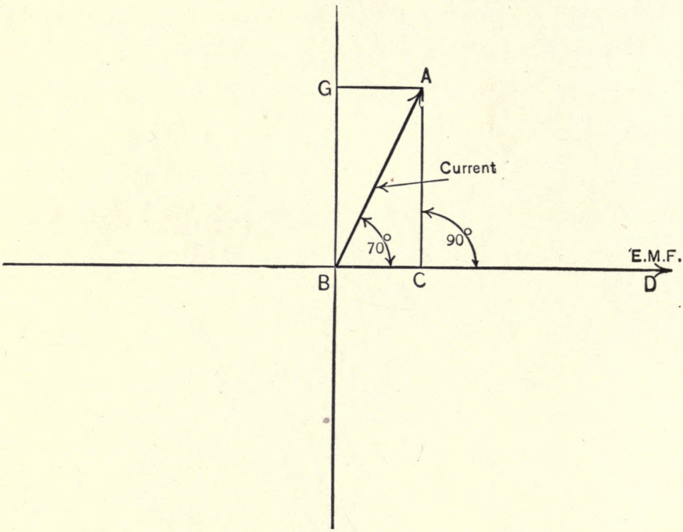

Fig. 20.

the projection of BD on it give the very same value; that is, the power of the circuit. Since the power factor $= \frac{BC}{AB}$ (Fig. 20), and since the real power $= BD \times BC$, that is, the e.m.f. $\times$ the current in phase with it, and since the apparent power $= BD \times AB$, that is, the e.m.f. $\times$ the total current flowing, it follows that power factor $= \frac{BC \times BD}{AB \times BD}$, or equals the real input in watts divided by the apparent input in watts. Thus, the lower the power factor, the greater the component of idle current. Moreover, the power factor equals the real, divided by the apparent, watts, so that the component of current or e.m.f. in phase with the other, and thus representing the energy part, is equal to the e.m.f. or current multiplied by the power factor. Thus, in Fig. 20, the real power $= BD$, the e.m.f., times the power factor times the current, BA, since power factor $= \frac{BC}{BA}$, and the real power $= BD \times BC$ $= BD \times BA \frac{BC}{BA}$.

CHAPTER II.

VARIOUS DIAGRAMS OF ALTERNATING CURRENTS AND E.M.F.'s.

LET us now consider the discussion of a single phase alternating circuit as shown in Fig. 21. The problem is to find the necessary voltage, E, to be given the alternator, A, to transmit the current, I, through the

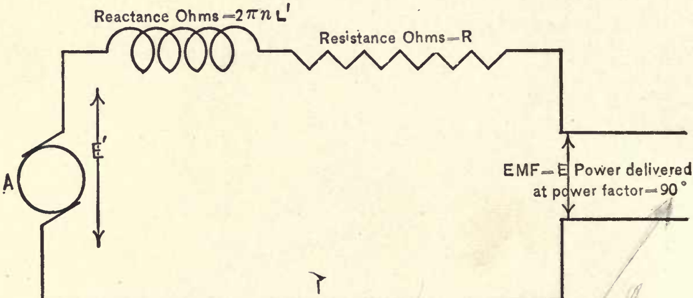

Fig. 21.

reactance ohms, $2\pi nL$, and the resistance ohms, R, and have left a voltage, E, at the point where the power is desired, such power having a power factor of .90. The diagram of the circuit is given in Fig. 22, where

AG = the e.m.f. applied to the load,

AD = current delivered to the load and flowing in the circuit,

$AB =$ volts consumed in resistance,
$= RI$, which is in phase with the current, AD,
$AK = 2\,\pi n L I$ volts consumed in reactance, which is 90° out of phase with the current, AD.

All the above values are the square root of mean square values of the sine waves of current and electromotive forces respectively, and can be combined into single resultants. Also, any vector can be separated into two components at right angles to each other. Combining

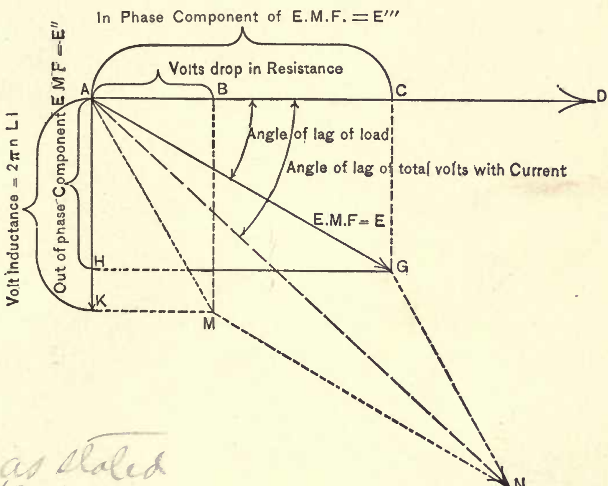

Fig. 22.

AB, the resistance drop, with AK, the inductance drop, gives the diagonal, AM, which is thus

$$I\sqrt{R^2 + \overline{2\pi n L}^2}$$

or the impedance drop. Combining AM with AG, the voltage at load, or receiving end, gives AN, the final resultant, or the voltage required at the alternator for producing existing conditions. It will be noted that the angle of lead of *total* volts with respect to current is greater than the angle of lead of e.m.f. at the load. This is caused by the presence of the inductance in the line. To calculate the line, AN, or the total volts required, first separate the voltage of the load into its two components, one, AC, in phase with the current, AD, and the other, AH, out of phase with the current. AG then equals $\sqrt{\overline{AH}^2 + \overline{AC}^2}$.

We then have, in phase with the current, AD, the following:

AB = volts consumed in resistance = IR,
AC = in phase component of load e.m.f. = E'''
= $AG \times .90$ (.90 being the power factor)

and we have at right angles to the power current :

$AK = 2\,\pi nLI$ = volts consumed in reactance,
AH = out of phase component of load e.m.f. = E''

which from the triangle ACG gives

$$CG = \sqrt{\overline{AG}^2 - \overline{AC}^2} = \sqrt{E^2 - E'''^2}$$

since the square of the long side of a right-angled triangle = sum of squares of other two sides.

The applied voltage $AN = E'$ equals

$$\sqrt{(IR + E''')^2 + (2\,\pi nLI + E'')^2}$$

all the values of which are known. This is the basis of the calculation of the drop in transmission lines, since polyphase lines can be separated into component single phase lines, as will be shown.

EFFECT OF DROP OF VOLTS WITH CHANGE OF POWER FACTOR.

It can readily be seen from Figs. 23 and 24, that the higher the inductance in the line, and the lower the power factor of the load, the higher voltage will be

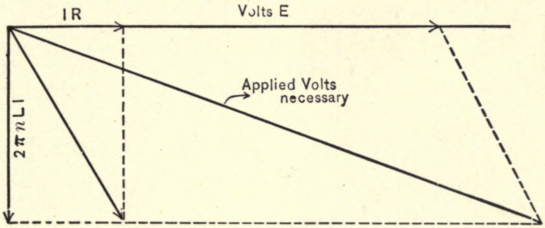

Fig. 23.

required from the alternator. Fig. 23 shows the applied voltage necessary when the power factor of the load is unity, that is, the e.m.f. and current in phase with each other (cosine of 0° is unity), and Fig. 24

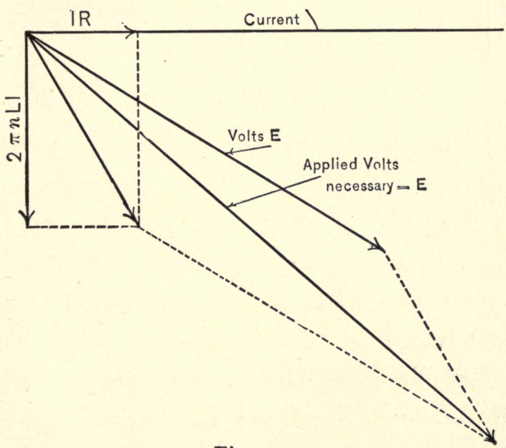

Fig. 24.

shows the applied voltage necessary when the power factor is about .70.

It can be noted that the voltage necessary in Fig. 24 is much more than is necessary in Fig. 23.

Fig. 25 shows the effect of having the power factor low, but with a *leading* current instead of a *lagging*

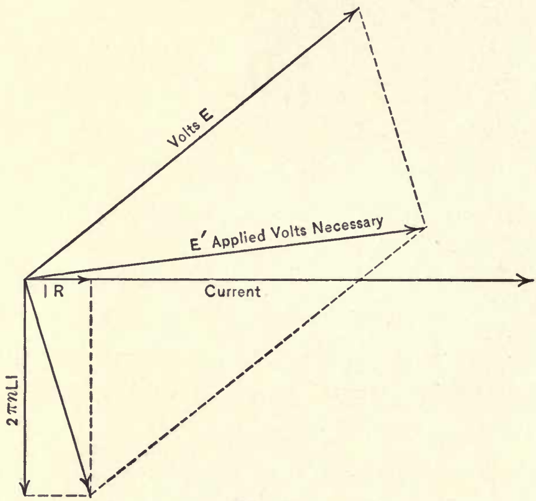

Fig. 25.

current in the load. It can be seen that the voltage E' necessary to produce the volts, E, is actually less than the volts, E. In fact, owing to the leading current, the voltage has increased through the inductance, L. The applied voltage necessary lags behind the current, I, but not as much as the voltage of the load, E, because of the inductance decreasing the lead of the current. Thus, when leading current passes through an inductance, a varying drop of voltage results, depending upon the amount of lead of current. When lagging current passes through an inductance, a drop of voltage results which is greater with greater lag of current and inductance. Thus, inductance and capacity give opposite effects, and all the effects must be combined, as shown in Figs. 21 to 25, by the vector process; that is, by separating voltages or currents, as most convenient, into their two right angle components, or by combining right angle components into resultants.

Fig. 22 shows a single phase transmission line, and the calculation determines the generator voltage for a given line and load condition. If capacity exists in the line, it may usually be considered as shunting from the load a certain amount of current, such current leading the e.m.f. It is usually sufficient to assume the capacity concentrated at a point at the center of the transmission line, the capacity current there being 90° ahead of the voltage at that point. The calculation then consists in determining the voltage necessary at the condenser, to give a certain voltage at the load. Then from this determined voltage determine the voltage required at the alternator, remembering that from the condenser to the alternator the current in the line differs, in value and phase, from the current in the line between the condenser and the load, for the reason, as stated, that the condenser takes off or shunts a certain amount of current from the load. The above discussion concerns a single phase line.

POLYPHASE TRANSMISSION AND RELATIVE WEIGHTS OF COPPER WITH DIFFERENT SYSTEMS.

There are other equally important transmission systems.

The "quarter phase," otherwise called the "two

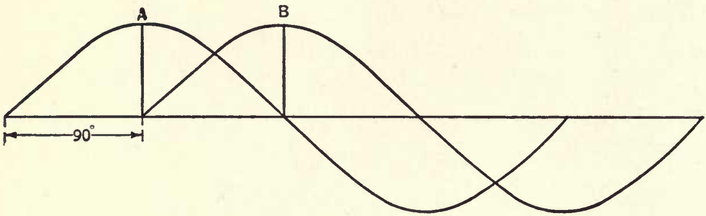

Fig. 26.

phase," system consists of two separate alternators, or two separate windings of the same alternator, furnishing two separate currents. There are either four wires, two for each circuit, or one separate wire for each circuit, and a third wire which acts as a common return.

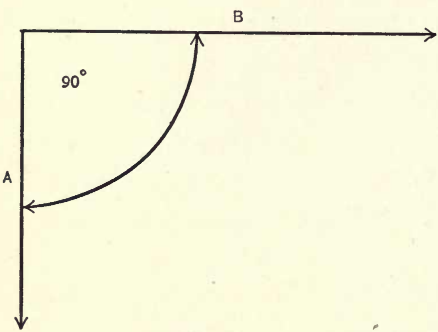

Fig. 27.

The e.m.f.'s from the two windings are 90°, or quarter of a cycle apart, and are as shown in Figs. 26 and 27. Both figures express the same thing, the first in rectangular coördinates, and the second in polar coördinates. Such an arrangement possesses certain notable characteristics.

Of the greatest importance is the fact that the resultant of the e.m.f.'s, or of currents in case the curves represent currents, *A* and *B*, Fig. 26, is always a constant, but, as can be seen, it moves forward in space. In fact, if such currents are passed into coils set in space also 90° apart, and located in a single magnetic circuit with uniform magnetic reluctance in all directions, such, for instance, as a ring of iron with two coils set crosswise, that is, 90° from each other, there will result a constant magnetic field rotating in space at a speed equal to the frequency of the applied current. This is the basis of the modern induction motor, as will later be explained. Such a transmission line is calculated as two single phase circuits, each carrying half the power. It is possible instead of using four wires to use three, letting one wire be the common return. On an examination of Figs. 28 and 29, it will be seen that the com-

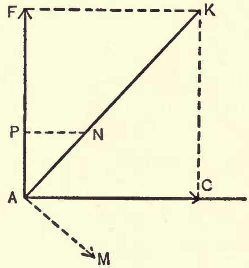

Fig. 28.

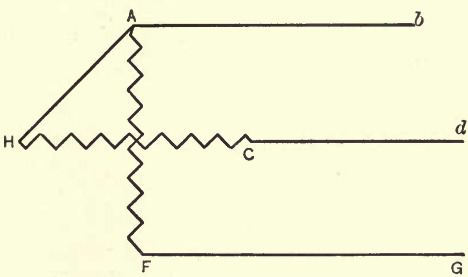

Fig. 29.

mon return will carry 1.414 times the current of the other two when the two phases are equally loaded. In Fig. 29 the wire, AB, will carry this extra current, and A and H are connected together. Fig. 28 shows the calculation. $AF =$ one current, and AC the other. Combining them, as has been shown, their resultant, or the current flowing in the common return, is represented by the diagonal, AK, which equals $\sqrt{2}$ times either side; since the angle $KAC = 45°$, and $AK = AC \times \cos 45° = AC \times \sqrt{2}$. There is an objection to such an arrangement, since the inductance of the common return wire causes a raising of voltage in one phase and a lowering in the other, and thus an unbalancing of voltage in the two circuits, which is undesirable. This effect is shown in Fig. 28, where the line, AM, represents the opposing e.m.f. of self-induction of the current, AK. As can be seen, this value of AM combines with AF, the e.m.f. of one phase, to decrease it, and with AC, the e.m.f. of the other phase, to increase it. In quarter phase transmission, with a given voltage between wires and with four wires used, the weight of wire for a given line drop is the same as for a single phase transmission at the same voltage, as will, of course, be seen. When three wires are used, that is, a common return, a change of weight of copper is effected, as shown below. The basis of comparison in the following discussion is that the delivered voltage between outside wires in the three wire quarter phase circuit shall be the same as the voltage between wires in the single phase circuit. In the single phase non-inductive circuit the current $I = W/E$ where W equals watts transmitted. With a percentage drop of P, the volts drop in the line is PE, and the resistance is PE/I. If l equals length of circuit both ways, the resistance per foot equals PE/lI. In the three wire quarter phase circuit, the voltage between lines equals E, just as in a single phase circuit, but the voltage of each circuit, as can be seen by examination of Fig. 29, is $E/\sqrt{2}$. In this figure the voltage between the outside CD and FG, is $\sqrt{2}$ times the voltage of one circuit, for, referring for a moment to Fig. 28, the resultant of AF and $AC = \sqrt{2} \times AF$. Hence, the current per outside line

$$= \frac{W}{2} \div \frac{E}{\sqrt{2}} = I_1 = \frac{W}{E\sqrt{2}} \text{ or } \frac{I}{\sqrt{2}}$$

For the same percentage drop as in a single phase circuit, the volts drop in line of one circuit $= PE/\sqrt{2}$. Hence, the resistance of outside wire

$$= \frac{PE}{2\sqrt{2}} \div \frac{I}{\sqrt{2}} = \frac{PE}{2I},$$

and the resistance per foot

$$= \frac{PE}{2I} \div \frac{l}{2} = \frac{PE}{lI}.$$

Hence, since $PE/lI =$ resistance per foot of a single phase transmission, we have the same sized wire as in the single phase circuit. In the two circuits of the three wire quarter phase system, therefore, the two outside wires weigh the same as the total wire of the single phase circuit. To calculate the size of the common return circuit, we note the current in it to be equal to $\sqrt{2}$ times current in outside wires, therefore equal to

$$\sqrt{2} \times \frac{I}{\sqrt{2}} = I.$$

The percentage drop equals $\sqrt{2}$ times percentage drop in the outside wires, being multiplied by $\sqrt{2}$, since the current in the return wire is not in phase with the current in the outside circuits. To give the same "in phase" drop as the outside wires, $\sqrt{2}$ times as much drop is permitted in the middle leg, as can be seen by examining Fig. 28. In this figure it can be seen that to produce a certain drop, AP, along the line, AF, a drop, $AN = \sqrt{2}\,(AP)$, is permitted along the line AK. By permitting this extra drop, the current density in the middle wire naturally runs up. If the current density is kept the same in the middle wire as in the outside, a value of ratio 1.457 is obtained, instead of one of 1.50, which is now shown. This drop in the middle wire equals $\frac{PE}{2}$, and the resistance equals

$$\frac{PE}{2} \div I = \frac{PE}{2I}.$$

The resistance per foot equals

$$\frac{PE}{2I} \div \frac{L}{2} = \frac{PE}{LI}.$$

The resistance per foot of the wire in the single phase circuit under comparison $= \frac{PE}{LI}$. That is, the resistance per foot in this common wire of the quarter phase equals the resistance per foot of the single phase line, hence the weight per foot equals the weight per foot of the single phase line or of the outside wires. There is, however, just half the length of this common wire, hence the weight of it compared with the single phase wire $= \frac{1}{2}$. Hence the weight of the total wire quarter phase to single phase $= \frac{1 + \frac{1}{2}}{1} = 1.50$. This arrangement is thus unsuitable for long distance transmission.

However, lest this basis of comparison convey a wrong impression, we will consider the effect upon a given four wire transmission of removing one wire and using one of the remaining wires as the common return. This gives an arrangement like the last, in using three equal wires; but the wires are only half the size, or weight per foot. That is, we have removed one wire out of four, or saved one-quarter of the copper. The loss in the common return is the same as in the two wires that it replaces; that is, the total loss is unaltered. The current in each part of this system is $1/\sqrt{2}$ times the current in the corresponding part of the system before considered, while the voltage is $\sqrt{2}$ times the corresponding voltage, making the power transmitted the same. It is obvious, therefore, that, with a given generator, one quarter of the copper is saved by running three wires, and this would seem to be the natural basis of comparison. But when, as in all well-designed long distance transmissions, the line voltage is already as high as is practicable, if three wires be used in place of four, the generator voltage has to be reduced until the maximum line voltage is the same as with four wires. Hence, the first basis of comparison is the correct one.

Again, it may seem somewhat paradoxical that reducing the density of the current in the common return reduces the total weight of copper, but this means simply that the minimum resistance loss occurs when the copper is so distributed that the current density is everywhere equal. If, therefore, copper be transferred from the outside wires to the common return until the current densities in all three are equal, the loss will be less than it was before; or, for the same loss as before, all the wires may be reduced.

THREE PHASE TRANSMISSION.

There is another method of transmitting power, called the three phase method. Here, instead of the phases being 90° apart, they are 120° apart. Currents 120° apart flowing in a uniform magnetic circuit into coils, also 120° apart, give, as with the quarter phase, a constant rotating magnetic field. In considering this question, examine Figs. 30 and 31. The sine

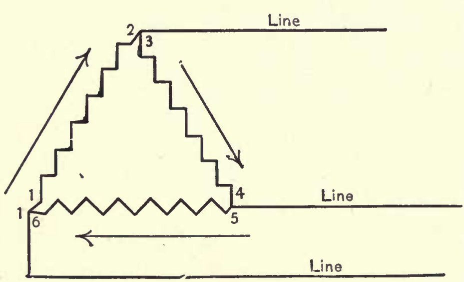

Fig. 30.

curves represented by the arrows 1–2, 3–4, and 5–6 are 120° apart. In Figs. 31 and 32, the phases represented by the arrows are 120° apart, and are combined by the diagram of forces, as in Fig. 32. Thus, in Fig. 32, if 2 be connected to 3, 4 to 5, and 6 to 1, the resultant of 1–2 and 3–4 is 1–7, equal and opposite to 1–6. Hence, in such a connection no current flows in the coils themselves producing such voltages. This connection

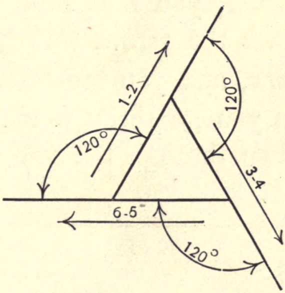

Fig. 31.

is called delta connection, on account of its resemblance to the Greek letter Δ, and the voltage between wires

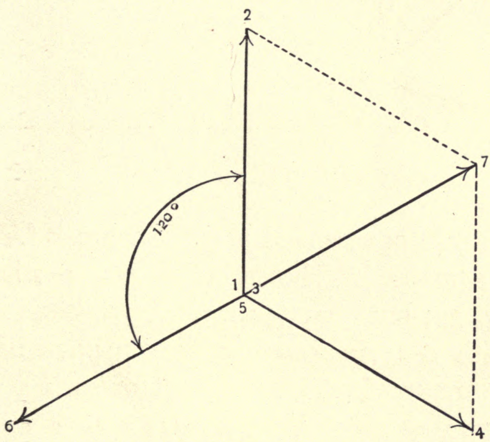

Fig. 32.

coming from such a connection is equal to the voltage of each coil composing the connection. Suppose, instead of connecting the coils producing the voltages, as shown in Figs. 30 and 31, they are connected as shown in Fig. 33. Starting at 1, and following this figure, you find that you pass through the first coil *with* the arrow, through the next coil *against* the arrow, and you are then open-ended at 3. Starting at 3, you go through the coil 3–4 *with* the arrow, and then through the coil 5–6 *against* the arrow, and you

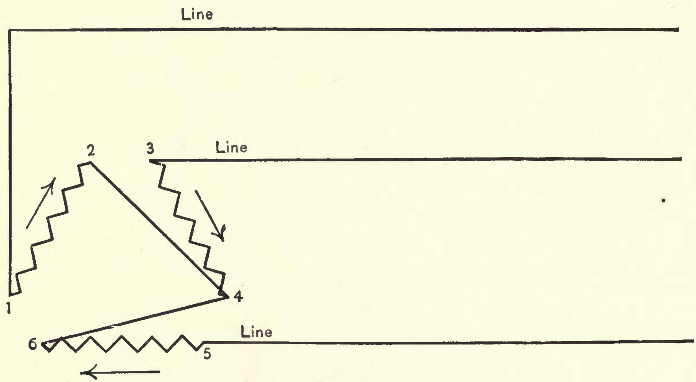

Fig. 33.

are then again open-ended at 5. Such e.m.f.'s or currents are combined as shown in Fig. 34. 1–2 is plotted as in Fig. 32; but now 3–4 must be reversed in combining with 1–2, since in Fig. 33 you passed through the arrow *reversed*. The resultant of such a connection is thus 1–7, which is $\sqrt{3} = 1.732 \times 1 - 2$, as can be shown by geometry. The combination of 3–4 and 5–6 gives 3–8, the arrow, 5–6, being reversed to make the combination, since in passing from 3 to 6, Fig. 33, you go *with* the arrow through 3–4, and *against* it through 6–5. The combination of 5–6 and 1–2 gives 5–9. In this case 1–2 is reversed for the combination, since, in passing from 5 to 1, Fig. 33, you go *with* the arrow from 5 to 6, and *against* it from 2 to 1. Thus, in forming the combination voltages in such polyphase system, it is well to have two diagrams of voltage or currents: one like Fig. 33, to tell which way you pass through the arrows in going from one point to another between which voltage is desired; and the sec-

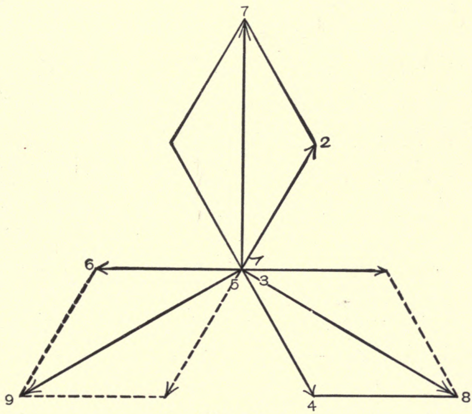

Fig. 34.

ond to show the actual combination, like Fig. 34, telling when to reverse an arrow by reference to the first figure. The connection just described is called a "Y" connection, from its resemblance to the letter Y. From an inspection of Fig. 33, it will be seen that the current in the line is the same as the current in the winding (the lines 1–2, 3–4, 5–6, representing the windings of a three phase alternator when connected Y); also that the voltage between lines is the same between any two, and is equal to $\sqrt{3}$ times the voltage of one phase of the alternator, see Fig. 34, where the resultant of 1–2 and 3–4, etc., is equal to 1–7, which, as can be shown by geometry, $= 1 - 2 \times \sqrt{3}$. The voltage between lines is always called the Δ voltage. With a Δ connected alternator, as in Fig. 30, the voltage between lines equals the voltage of the various phases themselves. The voltage of each phase of a Y connection is called the Y voltage, and it equals the Δ voltage ÷ by $\sqrt{3} = \frac{E \text{ delta}}{\sqrt{3}}$. The current in the lines

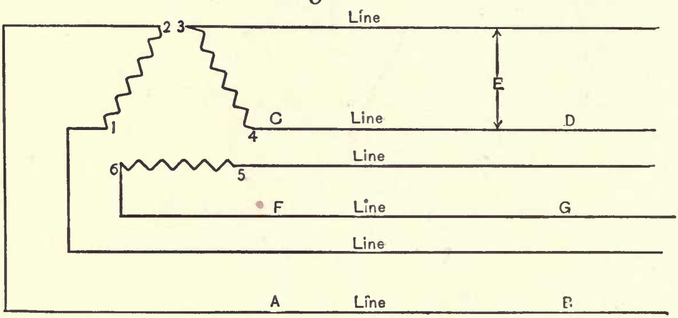

Fig. 35.

is always called the Y current; and the current between lines, that is, the current flowing in the coils Δ connected, as in Fig. 30, is called the Δ current.

Let us imagine, in Fig. 33, that instead of running the lines as shown, six lines are run out, as shown in Fig. 35. We then have three single phase circuits, each of voltage E. Suppose, now, we combine the lines GD, FG, AB. What current will flow in the common line? Looking at Fig. 34, it can be seen that the line currents represented by vectors 1–7, 3–8, and 5–9 combine to equal zero, any one being equal and opposite to the combination of the other two. Thus we can obliterate the lines, CD, FG, AB, in Fig. 35, and connect the points, 4–2 and 6, together, as has been done in Fig. 33. In the latter figure, the voltage between lines becomes $\sqrt{3}\,E$, E being the volts per coil. On this account, the three phase system gives a saving of copper, as compared with a single phase transmission, as follows:

The condition of comparison is that the delivered voltage between lines shall be the same three phase, as it is single phase.

In the single phase non-inductive circuit, the current in the line $= I = \frac{W}{E}$ when $W =$ the total watts transmitted. Thus the resistance of the line for a given percentage drop, P, equals $\frac{PE}{I}$, and if l equals the length of the circuit both ways, the resistance per foot $= \frac{PE}{lI}$. In the three phase circuit the watts per circuit $= \frac{W}{3}$ since there are three circuits. The volts between each line and the neutral of the three phase circuits $= \frac{E}{\sqrt{3}}$, hence

$$\text{the current per line } = \frac{W}{3} \div \frac{E}{\sqrt{3}} = I_1 = \frac{W}{\sqrt{3}E}.$$

Thus,
$$I_1 = \frac{I}{\sqrt{3}}.$$
For the same percentage drop, P, we have a voltage drop of $PE/\sqrt{3}$. Hence

$$\text{a resistance of } \frac{PE}{\sqrt{3}} \div \frac{I}{\sqrt{3}} = \frac{PE}{I},$$

and

$$\text{a resistance per foot of } \frac{PE}{I} \div \frac{l}{2} = \frac{2\,PE}{lI},$$

since here there is no return wire. But, with the single phase circuit, the resistance per foot is $\frac{PE}{lI}$, and in one of the three circuits of the three phase system the resistance per foot equals $\frac{2\,PE}{lI}$; thus twice the resistance per foot, and thus ½ the weight per foot. There is, however, but ½ the length of wire in the single circuit of the three phase system, since the return current is neutralized with the other circuits. Hence, the weight of copper for the single circuit is ¼ the weight of copper for the single phase circuit. But there are three of these single circuits composing the 3 phase circuit, hence the weight of copper is ¾ that of the single phase circuit under comparison. For this reason it is most desirable to use three phase systems for long distance transmission of energy, when the copper item is one of much importance.

SIX PHASE TRANSMISSION.

Another mode of transmission, used particularly between the secondaries of transformers and rotaries, is a six phase transmission. The arrows in Fig. 36 show a six phase relation, and Fig. 37 shows the vector diagram of the e.m.f.'s. As can be seen from these figures, the e.m.f.'s, 1–2, 5–6, and 9–10, form a regular three phase diagram, and the e.m.f.'s, 3–4, 7–8, 11–12, form also a regular three phase diagram, but exactly reversed from the first one. Thus, six phase systems can be formed by two sets of three phases, one set being reversed. This is done in transformers by having each transformer with two independent secondaries. Reversing one set of secondaries gives a six phase circuit, as shown in Fig. 38. The numbering of the ends of the wires in Fig. 38 should be as shown. For a Y connection, the points, 2–8–6–12–10–4 (Fig. 38), should be connected together. (Refer to Fig. 36, and note the rotation "with and against the arrows" for each pair of arrows; also note in Fig. 36 that 7–8 is exactly opposite in phase to 1–2, 9–10 opposite to 3–4, 11–12 opposite to 5–6.) For a Δ connection the points (see again Fig. 36 to show this) 2–3, 4–5, 6–7, 8–9, 10–11, 12–1, in Fig. 38, should be connected together.

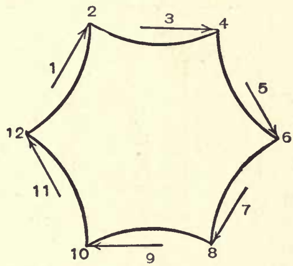

Fig. 36.

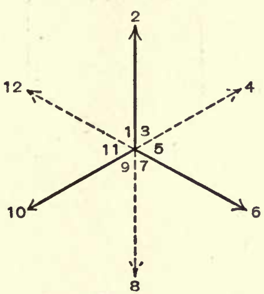

Fig. 37.

There is yet another way to get six phase, with only three secondaries to transformers. It is shown in Fig. 39. On connecting the figures in the circle (which

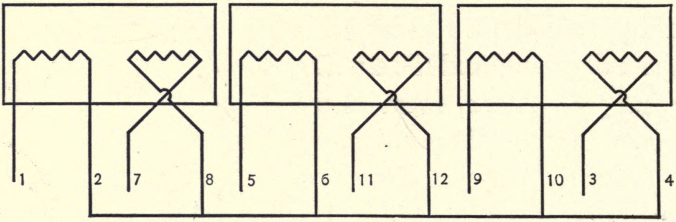

Fig. 38.

correspond to a uniform magnetic circuit) with the ends of the transformer, as marked by numbers, magnetism results, as shown by the arrows, *a–b–c*, which

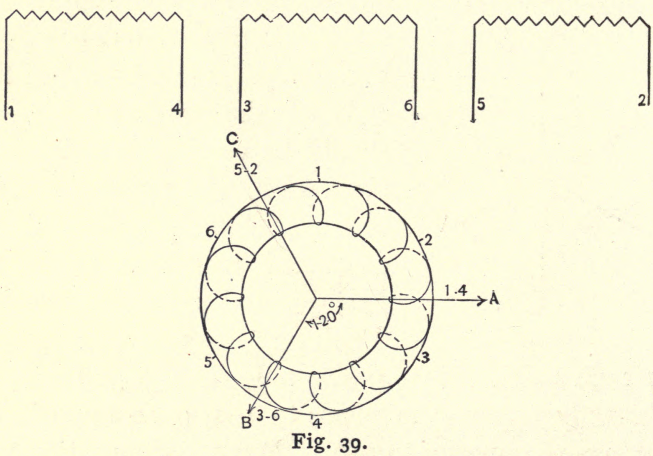

Fig. 39.

are 120° from each other. Any other connection, for instance reversing 5–2, would reverse one of the arrows, and six phase would no longer result. It will be shown under the subject of alternating motors, that in any uniform magnetic circuit, when coils are placed at angles from one another equal to the angle between the phases of the applied e.m.f.'s (or currents), a resultant magnetomotive force is obtained equal to the ampere-turns per coil, $\times \frac{4\pi}{10} \times$ number of phases divided by 2. Thus, in an air magnetic circuit with coils set 120° apart, and each of N turns of wire and three phase currents flowing in the coils, the resultant magnetomotive force to produce the constant resultant revolving flux is equal to

$$\phi = \frac{4\pi IN}{l 10} \frac{3}{2} A\mu .$$

For quarter phase, otherwise called two phase, this value becomes

$$\phi = \frac{4\pi IN \times \frac{2}{2}}{10 l} \times A\mu,$$

and so forth. The above is of importance in calculating fluxes in induction motors and armature reactions in polyphase alternators, or wherever magnetomotive force is produced by polyphase circuits.

SINGLE PHASE FROM THREE PHASE.

Since two sine curves can be combined to produce a third, or resultant, sine curve, a three phase machine can be run as a single phase machine. It is interesting to note the diagram under such conditions. It is shown in Fig. 40. 1–2, 1–3, and 1–4 are the phases 120° apart, 1–5 is the current, its location with respect to the other lines representing its phase, and its length representing its amplitude. Continuing, 1–2 and 1–3 give the resultant, 1–6, assumed to lead the current in this figure by the angle α. The line, 1–7, represents

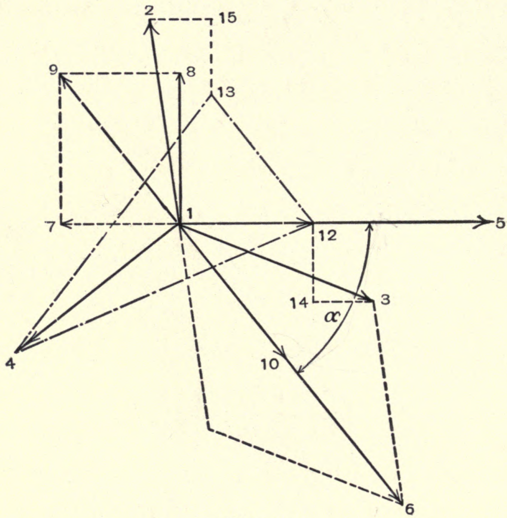

Fig. 40.

the resistance drop e.m.f. IR; the line, 1–8, represents the reactive e.m.f. $2\pi nLI$; and the line, 1–9, represents the total opposing e.m.f. of the impedance drop. The line 1–10 (1–10 = 1–6 minus 1–9) equals the resultant e.m.f., since 1–9 and 1–10 happen to come in line. The original triangle of e.m.f., which was 2–3–4, becomes 4–12–13, thus having one short side and two long.

Thus, loading a three phase circuit unbalanced (in this case, Fig. 40, one phase was loaded), produces unbalanced voltages between the phases. The point, 13 and 12, Fig. 40, are obtained by plotting at the points, 2 and 3, the *IR* drop and inductance drop, halving it between the two vectors, 1–2 and 1–3, since each takes the same current. The line, 3–14, represents the *IR* drop; and the line, 14–12, the inductance drop of the vector, 1–3; and the lines, 2–15 and 15–13, the resistance and inductance drop for the vector, 1–2. It will be noted that 2–15 is drawn in the opposite direction to 3–14 and 15–13 opposite 14–12. This is because, combining 1–3 and 1–2, 1–2 should be reversed, as has been done in obtaining 1–6. Instead, however, of reversing 1–2, we reverse 2–15 and 15–13, which gives the same result. Or, putting it in another way, the line 1–5 represents the outgoing current for the phase 1–3, and thus the current is returning into 1–2, and must be reversed in its effects, since *IR* and $2\pi nLI$ become reversed, or, as stated, 2–15 and 15–13 are plotted for the return current just opposite 3–14 and 14–12 for the outgoing current.

SPLITTING OF PHASE.

It is more difficult to obtain polyphase from single phase, though rough approximations can be obtained by "splitting" the phase. Thus, if alternating current be passed through resistance and inductance in multiple, the phase of the current in the resistance will be different from that in the inductance, and both will differ from the current in the line. This effect is shown in Figs. 41 and 42.

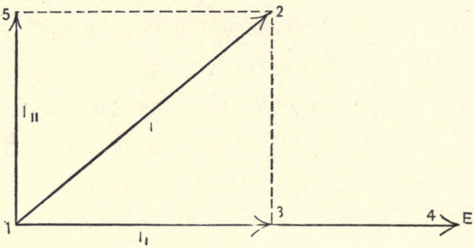

Fig. 41.

Fig. 42 represents an inductance of *L*, connected in multiple with a resistance, *R*, the two having an e.m.f., *E*, applied to them in multiple. In Fig. 41, 1–4 represents the e.m.f., *E*. Since, in simple resistance, the current flowing is always in phase with the e.m.f. applied, the line, 1–3, represents the current, *I*, flowing in the resistance. Since the current flowing into an inductance is 90° behind the e.m.f. applied to it, the line, 1–5, represents the current flowing into the inductance. The resultant of the two currents is represented, in phase and magnitude, by the diagonal of the parallelogram whose sides are composed of the lines 1–5 and 1–3,

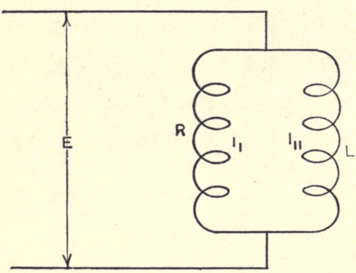

Fig. 42.

or the line 1–2, which, therefore, represents the current flowing in the line. From an examination of Fig. 41, it will be seen that the current in the line (shown by the line 1–2) is leading the current in the inductance, and lagging behind the current in the resistance. Thus, the current in the line is said to split into two components, each of displaced phase; also, as can be seen, the actual sum of the currents, 1–3 and 1–5, is greater than the resultant. By combinations of resistance, inductance, and capacity, currents out of phase can be produced from a single current, and thus a poor approximation to a polyphase circuit be secured.

SURGES IN TRANSMISSION CIRCUITS.

The following method of explaining surges has been used by Mr. A. E. Kennelly: —

It has been stated that the work in joules, that is, watt-seconds (1 watt-second = .737 ft. lbs.), to bring in any circuit the current from the value 0 to I, is equal to $\frac{1}{2} LI^2$ where L is the inductance of the circuit [L equals flux times turns ÷ amp × 10^8]. This work represents the magnetic energy in the system when the current is flowing. If, therefore, the current be suddenly stopped, this energy must expend itself somewhere. Thus, in transmission lines where a current is flowing, a certain amount of energy exists in the form of the magnetic field around the wires of the circuit.

This energy, when released, expends itself by forcing current back and forth through the resistance of the line, if the insulation be not punctured, in which case the energy is otherwise dissipated. The current is forced into the capacity of the line, for a transmission line has capacity and will hold bound a certain definite amount of electricity, just as will two plates set opposite and close to each other. In the case of a transmission line, each wire represents one of the plates of an ordinary condenser, and, where there are more than two wires, of a combination condenser. In this case, the condenser plate, which is the transmission wire itself, is of very small surface per foot, but the length is enormous, thus producing a large surface. The arrangement of the ordinary condenser is to put tinfoil, or its equivalent, on the two sides of some thin insulating material, such as paraffined paper or glass. Each sheet of tinfoil becomes one plate of the condenser. An ordinary condenser might have a hundred of these plates so arranged. Thus a very large surface is obtained, and the plates are very close together, — the two requisites for creating capacity. Transmission lines far apart, and far from the surface of the earth, have, of course, much less capacity than lines near the earth, and near each other. Twin cables for underground work or for submarine work have large capacity, and before current can get through them the capacity must be satisfied, since capacity acts like a shunt circuit, deflecting a certain amount of the current passing through. Thus, submarine telegraphy must necessarily be slower than land telegraphy, since the currents transmitted are small, and the capacity much greater than in overhead wires. We thus have in transmission lines capacity into which the electromagnetic energy, when released by suddenly breaking the circuit, can transfer itself. This amount of energy may be quite large, especially when, because of some accident, the transmission line has become short-circuited and then opened again. Owing to the short circuit, the current becomes very large, — perhaps four or five times that which would flow under normal conditions. The current flowing at the instant of breaking the circuit is the one with which we are concerned. Since we are considering sine waves of current, the value at the instant of breaking may vary from 0 to $\sqrt{2}\,I$, where I is the square root of mean square value; for, as has been shown, the maximum value of a sine curve equals its square root of mean square value multiplied by $\sqrt{2}$. If the break occur when the curve is passing through 0, there is no return of energy, for with 0 current there can be no magnetism. If the break occur when the current is a maximum, a maximum effect occurs, since flux in an air magnetic circuit is directly proportional to current. At intermediate points of break, intermediate effects are produced. Hence it follows that an oil switch by its inherent tendency to open the circuit at zero current will not cause vicious arcing, and consequent high voltage rises in circuits containing capacity and inductance. It is an established fact, that if a condenser of capacity, C, be charged with a voltage, E, the work represented by the charge of electricity is equal to $\frac{1}{2}CE^2$, where E is expressed in volts, C in farads, and work in watt-seconds or joules. If, therefore, an amount of watt-seconds represented by $\frac{1}{2}LI^2$ be suddenly released in a transmission line, it will produce a surge of electricity the energy of which will be represented by $\frac{1}{2}CE^2$, if a line of zero resistance be assumed. There is, under such conditions, a continual surging back and forth of electricity from an electromagnetic form to an electrostatic form. The cycles of this surging may be much higher than in the fundamental circuit, being, perhaps, in the neighborhood of 1,000 cycles per second under certain actual conditions, the voltage rising in corresponding degree. If, instead of an actual break of current, there be only a variation, similar effects are produced, but in a less degree, depending upon the amount of the variation. Thus, breaking or varying a current in a long transmission line is attended by serious danger to insulation.

Since $\frac{1}{2}LI^2$ and $\frac{1}{2}CE^2$ represent the same amount of watt-seconds in a line of 0 resistance, or the same amount of foot-pounds of energy, they can be equated, and then

$$\tfrac{1}{2}LI^2 = \tfrac{1}{2}CE^2$$

$$E = I\sqrt{\frac{L}{C}}.$$

Since the inductance, L, as well as the capacity, C, is proportional to the length, it follows that

$$E = I\sqrt{\frac{l}{c}},$$

where $l =$ inductance coefficient for unit of length,
and $c =$ capacity coefficient for unit of length.

From the inspection of this formula, a few interesting conclusions can be reached. The e.m.f. induced by the break of current, I, is proportional to the square root of the inductance, and inversely proportional to the square root of the capacity. Hence, *up to the amount of resistance where surging occurs*, a transmission line of high inductance and low capacity gives a greater surge voltage, and thus has a greater tendency to puncture its insulation, than a circuit of low inductance and high capacity. Hence, on underground cables, where the ratio of l to c is much lower than in overhead wires, the tendency to puncture, due to surging, is much less. A rough estimate for a sudden break would indicate it to be only about $\frac{1}{15}$ on underground cables. This is fortunate, since the question of insulation on underground cables is a much more serious matter than on overhead wires. When the break is not instantaneous, the induced surge is much less. It has been found in practice and by experiments that ordinary knife-blade switches increase the surge greatly, owing to the chance of their breaking the circuit near the maximum value of the current wave. On the other hand, it has been found that oil switches tend to break the current when it is near zero. Thus oil switches are strongly recommended for such circuits. It can be shown that the induced surge voltage on making a circuit can never exceed twice the applied voltage, whereas the induced surge voltage produced by breaking a current may be many times the natural voltage of the circuit; thus the danger attending a make of a circuit is of far less importance than the break. This surge voltage is of a frequency independent of the fundamental frequency of the circuit, and depends upon the geometry of the circuit. The surge voltage is also an entirely separate phenomenon from that of resonance, which is based upon the fundamental frequency, as has been shown, or upon harmonics which are the result of a distorted wave shape. The frequency of these harmonics, however, is far less than the surge frequency. The latter frequency may be 1,000 to 1,500 cycles per second. The puncturing resulting from surge is at the ends of the circuit, for there, as can be seen, the induced voltage from the release of magnetic energy is greatest.

PART II.

CHAPTER III.

THE TRANSFORMER.

IF an iron core have wound upon it two coils of wire, side by side, or one on top of the other, both coils surround any magnetic flux that may exist in the core, and any change that takes place in that flux will affect both coils simultaneously. Assume that there is an alternating e.m.f. applied to the terminals of one coil, and, for the time being, that the resistance of the coil is negligible. The e.m.f. generated in this coil will be equal to the applied e.m.f. Thus, if the applied e.m.f. follow the sine law, the back e.m.f. in the coil will follow the sine law, and thus, since the turns are stationary, the flux will vary according to a sine law. Since both coils have passing through them the same flux, the e.m.f.'s in them are proportional to their respective turns. Thus, if the coil, to which the e.m.f. is applied, have 1,000 turns, and the other coil 500 turns, and if the e.m.f. applied to the coil of 1,000 turns be 1,000 volts, we will have, assuming the primary resistance to be negligible, an induced back e.m.f. of 1,000 volts in the coil of the 1,000 turns, and an induced voltage of 500 in the coil of 500 turns. The coil to which the e.m.f. is applied (in this case the 1,000 turns coil) is called the "primary," and the coil in which the varying flux produces a terminal e.m.f. (in this case the 500 turns coil) is called the "secondary." Thus, a transformer consists of two coils of wire, one called "primary," and the other called "secondary," mounted on a common magnetic circuit. This is shown diagrammatically in Fig. 43. It is a fact

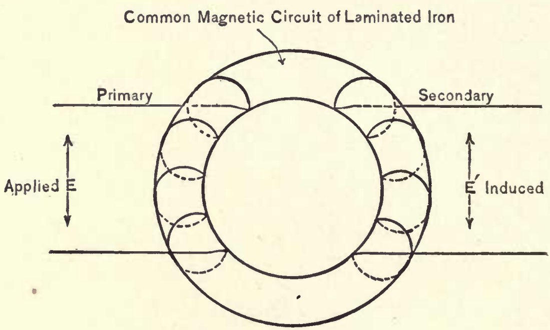

Fig. 43.

that the ratio of voltage between primary and secondary is practically proportional to the turns. Thus, if you wish to transform 1,000 volts to 20 volts, the ratio of turns of secondary to primary should be $\frac{20}{1000}$.

Since the flux in the common magnetic circuit varies in the form of a sine curve, and since a varying flux passing through any metal produces therein currents (usually called Foucault or eddy currents) circulating in closed paths at right angles to the path of the flux, it becomes necessary to laminate the iron core of the transformer, thus introducing in the natural path of these eddy currents resistance, in the form of contact between the sheets of iron, and cutting them down to reasonable magnitudes. Even when the core is carefully laminated with sheets of iron .014 of an inch thick, insulated one sheet from another, some of these currents exist, having a path whose diameter is the thickness of the plate itself. The amount of this loss

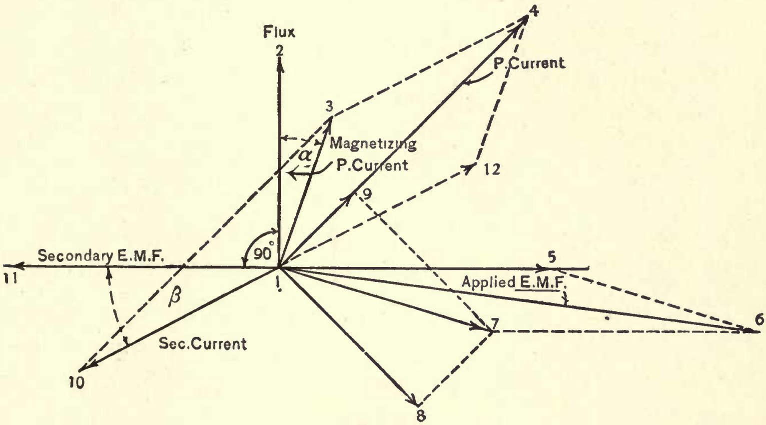

Fig. 44.

is small, however, in a proper construction. The diagram showing the phase and amplitude of the e.m.f.'s and currents of a transformer is shown in Fig. 44. The line 1–2 is drawn, showing the flux. This flux, pulsating with a certain frequency, creates in the iron, as has been shown, a certain loss due to hysteresis and eddy currents. The line, 1–3, is drawn of such value that it will, in the iron circuit under consideration, give the magnetic flux, ϕ. The line also represents the square root of mean square value of the equivalent sine curve of the actual curve of excitation, obtained as has been shown in previous pages. This line, 1–3, is also drawn away from the line 1–2, an angular amount, so that its product with the back e.m.f., 1–5, and the cosine of the angle, 5–1–3, is equal to the sum of the hysteresis and eddy losses in the iron. It has been shown, that energy, in this case the hysteresis and eddy loss, in an alternating circuit is equal to the product of current and e.m.f. and cosine of angle of lag, in this case the angle 3–1–5. The angle, 3–1–2, marked, in Fig. 44, as the angle α, is called the angle of "hysteretic advance." It alone depends upon the magnetic density and upon the quality of the iron, and is independent, of course, of the copper on the transformer, dimensions, etc. The line, 1–3, therefore, represents approximately the equivalent sine curve of excitation current of the transformer, in amplitude and phase, and represents the amount of current that would flow into the transformer, when on open circuit and receiving normal voltage. As has been shown when a flux varying in the form of a sine curve, as in this case, reaches its maximum, it is constant in value for a very short interval of time. When, however, the flux passes through its 0 value, it has the maximum rate of variation. Hence an e.m.f. is generated which is proportional to the rate of change of flux. It is zero when the flux is a maximum, and a maximum when the flux is zero, or, in other words, the maximum generated e.m.f. lags 90° behind the flux. Therefore, in the secondary of the transformer under discussion, the voltage generated by the change of flux is shown in value and phase relation to the flux, ϕ, by the line, 1–11, Fig. 44. The line, 1–10, is drawn to represent in value and phase the current flowing from the secondary of the transformer. In this case the current lags behind the secondary e.m.f. by the angle, β. This current, 1–10, in the secondary represents, in connection with the e.m.f., 1–11, a certain definite amount of energy. Since the source of energy is applied to the primary winding, the equivalent of the secondary energy must appear in the primary. Hence, in the primary, the secondary current, 1–10, appears in the line, 1–12, corrected for the ratio of turns, equal and opposite to 1–10 (since 1–12 is incoming energy, and 1–10 outgoing energy); also, for the same reason, the line, 1–11, the secondary e.m.f., appears in the primary, corrected for the ratio of turns, as 1–5, since 1–5 is e.m.f. applied, and 1–11 is e.m.f. given out. Thus, in the primary, the total current flowing is the resultant of the equivalent secondary current flowing, namely, 1–12, and the magnetizing current, 1–3. This is shown in the diagram as 1–4, which is the total current, in value and phase, flowing into the primary. The line, 1–9, in phase with the current, 1–4, represents the e.m.f. consumed by resistance drop in the primary; the line, 1–8, 90° from the current, as has been shown, represents the e.m.f. consumed by inductance; and thus the line, 1–7, represents the e.m.f. consumed by impedance in the primary. Therefore, the total primary e.m.f. must be the resultant of 1–7 and 1–5, which is the back e.m.f. of the transformer, or, what is the same thing, the secondary e.m.f., reproduced in the primary, allowing properly for ratio of turns. The resultant of 1–7 and 1–5 is the line, 1–6, which thus represents, in value and phase, the primary applied e.m.f. We thus see that in such a transformer the primary e.m.f., secondary e.m.f., primary current, secondary current, and flux, all differ in phase. In practice, when the magnetizing current is small, as well as the resistance and inductance drop, the primary and secondary currents are practically opposite in phase. The same is true of the primary and secondary e.m.f.'s. Moreover, the ratio of e.m.f.'s is practically equal to the ratio of turns of primary and secondary, and the ratio of the currents is the reciprocal of this ratio. Thus, a transformer cannot be used to alter the phase of currents or e.m.f.'s, except by 180°, which can be done, of course, by reversing the terminals of the primary in relation to the secondaries. By an examination of Fig. 44, it is seen that the ratio of voltage under load is not exactly the ratio of turns due to the impedance drop in primary and secondary. In a well-designed transformer, however, this ratio does not vary more than 2% from the turn ratio. Fig. 45 represents diagrammatically the windings of a transformer, with their respective voltages and currents. From the circuit to which the transformer is connected is drawn a magnetizing current, lagging practically 90° behind the applied e.m.f. Thus the use of a transformer draws on the supply of current, if the transformer ratio takes care of the voltage. The action on a line due to the presence of a transformer is shown in Fig. 46, where E equals the primary voltage, $\sqrt{R_1^2 + (2\pi n L_1)^2}$ = primary impedance, $\sqrt{R_2^2 + (2\pi n L_2)^2}$ = secondary impedance, and I represents

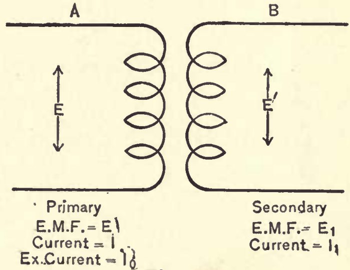

Fig. 45.

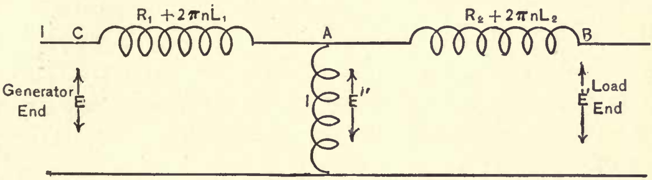

Fig. 46.

the excitation current taken off between primary and secondary. Thus, in calculating a single phase transmission line (polyphase transmission lines can always be calculated by resolving them into their equivalent single phase circuits), in which exists a transformer, the lagging current and impedance of the latter must be considered.

Thus, in Fig. 46, the current in the line from *A* to *C* is different, in value and phase, from the current from *A* to *B*, by the amount the excitation current effects it.

TESTING TRANSFORMERS.

To investigate the existing properties of a given transformer, it is necessary to ascertain:

1. The excitation current.

2. The loss in watts in the iron when running at normal voltage.

3. The resistance of primary and secondary at any known temperature.

4. The inductance of primary and secondary.

5. The regulation, that is, the difference of ratio, between no load and full load.

6. The ability to withstand more than normal voltage between the various windings and the frame of the transformer, as well as between the various windings themselves.

7. The rise in temperature of all parts ultimately reached when operating at normal potential and at normal current output.

8. The rise in temperature after a run of a certain definite time at a certain percentage overload, starting at normal running temperature.

9. Polarity.

(Corresponding terminals on all transformers should be capable of being connected together with no cross currents. Equal voltages 180° apart in phase connected together make a short circuit through their generating windings; equal voltages in phase, give no cross currents whatever.)

10. The amount, pressure, quantity, and temperature of cooling medium used.

11. The watts lost with full load current and transformer short-circuited.

1. The excitation current is read by connecting an ammeter of proper size in the primary circuit, reading the amperes at normal voltage and no load.

2. The loss in watts in the iron of the transformer is found by inserting a wattmeter in the primary circuit, and reading the deflection of same. This gives the watts used up in the iron plus the I^2R of the current flowing. Knowing the resistance R of the circuit, the I^2R can be taken out, and the remainder is the iron loss.

A refinement of this measurement is often advisable. It consists in subtracting not only the I^2R loss of the exciter current, but also the losses in the voltmeter and the pressure coil of the wattmeter. Instead of calculating these losses, they may be read by noting the deflection of the wattmeter with wattmeter connected. The usual connection of instruments, voltmeter and wattmeter, is shown in Fig. 47.

It should be noted that the core loss varies with the wave shape of the alternator supplying the energy, as has been shown, and also varies with the temperature of the transformer, so that both these items should be covered for full core loss data. The wattmeter may be inserted either on the primary or the secondary side, as most convenient.

3. The resistance of primary and secondary, both hot and cold, is measured by passing a small direct current through the coils, and reading the d.c. volts required to do so. Then the resistance is found by Ohm's law $R = EI$. The transformer should have been a long time in the room in which resistance is

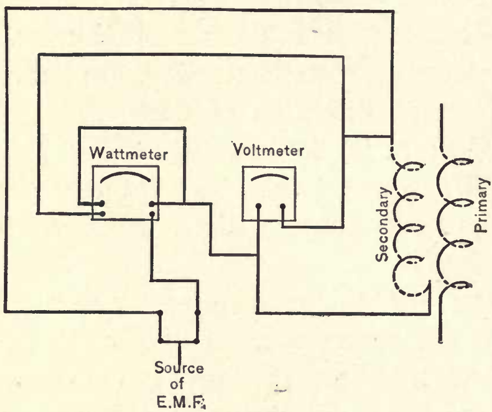

Fig. 47.

taken, so that the room temperature will indicate the transformer temperature, which it is necessary to know, and which must not vary during the measurement.

4. The inductance of primary and secondary is taken by short-circuiting through a proper ammeter the secondary on itself with wire of negligible resistance, then increasing, at normal cycles, the voltage on the primary until full current is flowing through the secondary, recording the voltage required. Then the total

impedance of the transformer, primary plus secondary, equals at the current flowing

$$\sqrt{R^2 + (2\,\pi n L^2)} = \frac{E}{I}.$$

Since E and I are read, and R_1 the total resistance of the transformer, is known, $2\,\pi n L$, or the induction of the total transformer, is calculated.

5. The regulation, that is, the difference of ratio, between no load and full load, cannot be measured well directly, but must be calculated from the values of resistance and inductance as measured.

A sufficient approximation can be found as follows:

Let IR = total resistance drop in the transformer in per cent of rated voltage.

Let $IX = 2\,\pi n L I$ = total inductance drop in the transformer in per cent of rated voltage,

where $X = 2\,\pi n L$ or the inductance in ohms of the transformer.

Let SI = per cent of lagging current flowing into the transformer. (SI equals sum of per cent of magnetizing current and per cent of lagging current in load.)

If the load on the transformer consists of a current which leads the e.m.f. instead of lagging behind it, the value SI equals the difference of the leading component of the load current and the lagging component of the excitation current.

Let MI = per cent of energy current in load.

Let 100 = per cent of voltage at secondary terminals of transformer when under load.

Let E = secondary no load voltage.

Then, as will be shown,

$$E = \sqrt{(100 + SIX + MIR)^2 + (MIX - SIR)^2}.$$

The magnetizing current is equal to the

$$\sqrt{(\text{Exciting current})^2 - \left(\frac{\text{core loss}}{\text{voltage}}\right)^2}.$$

Ordinarily the magnetizing current is about $\frac{3}{4}$ of the exciting current. The above may be made clear by an examination of Fig. 48.

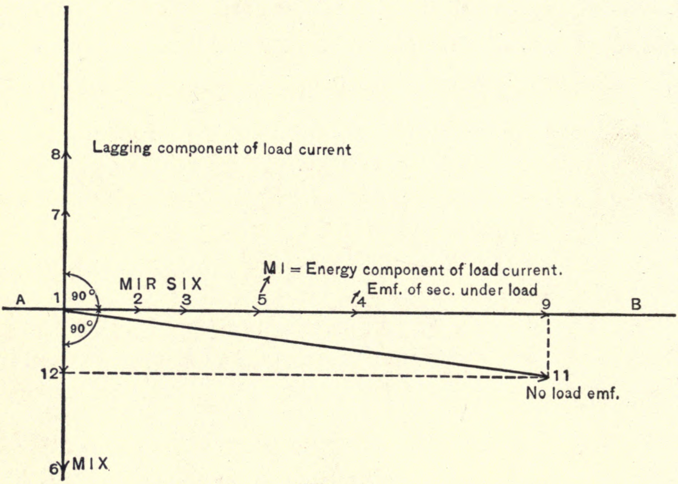

Fig. 48.

The distance, 1–5, equals the energy component of the load current, and is, therefore, drawn in phase with 1–4, the e.m.f. of secondary under load.

The distance, 1–8, equals the lagging component of the load current plus the magnetizing current, which is, therefore, drawn 90° from the energy component.

The lagging component of the load current and the magnetizing current are in phase with each other, and are, therefore, added directly. As quantities to be combined by the parallelogram method approach in phase, the parallelogram becomes narrower until finally, when the quantities coincide in phase, the parallelogram disappears entirely, leaving only a straight line, thereby reducing the combination to simple addition.

The e.m.f. necessary to produce the load terminal e.m.f., 1–4, is the summation, with the phases properly considered, of the terminal e.m.f., 1–4, and all the *IR* and *IX* voltages; that is, all the voltages consumed by resistance and reactance.

The e.m.f.'s that are in phase with the external e.m.f., 1–4, and with the energy component of the current, 1–5, are the *IR* voltage, or resistance drop, which is always in phase with the current, and the e.m.f. consumed by the lagging component of the current in the self-induction; that is, the e.m.f. required to overcome the reactive e.m.f. of this component. The reactive e.m.f. lags 90° behind the current, consequently the e.m.f. required to overcome it leads the current 90°, or is in phase with 1–4 and 1–5.

We thus have 1–4, the e.m.f. of the secondary under load, 1–2, the resistance drop of the energy component, and 1–3, the e.m.f. consumed in the self-induction by the lagging component, 1–8, all in phase with each other and, therefore, capable of being added directly. Their sum is 1–9.

The e.m.f.'s that are at right angles to 1–4 are 1–7, the resistance drop of the lagging component of the current, 1–8, equal to *SIR*, and, therefore, in phase with 1–8, and the e.m.f. consumed in the self-induction by the energy component of the current, 1–5 = *MI*. This e.m.f., being 90° ahead of the energy component of the current, is represented by the vector, 1–6, and is equal to *MIX*. Since 1–6 and 1–7 are in the same phase, that is, shown in the same straight line in the figure, they are combined by adding or subtracting directly: by adding, if both are on the same side of the horizontal reference line, *AB*; by subtracting the lower from the upper, if one is above the line, *AB*, and the other below it, as in this instance. The difference between 1–6 and 1–7 is shown as 1–12, which equals the resultant, out of phase by 90°, component of the e.m.f. The resultant of 1–12 and 1–9, which are 90° apart, is equal to the square root of the sum of their squares; that is, to the line, 1–11, in the figure,

or $$\overline{1-11} = \sqrt{\overline{1-12}^2 + \overline{1-9}^2}.$$

But $$\overline{1-12} = MIX - SIR$$

and $$\overline{1-\ \ 9} = 100 + SIX + MIR,$$

1–4 being assumed 100 for this investigation. Hence, if *E* be the voltage on the secondary at no load,

$$E = \sqrt{(100 + SIX + MIR)^2 + (MIX - SIR)^2}.$$

This formula gives the value of *E* in percentage of the load e.m.f. at the secondary terminals. To derive the value of magnetizing current, we must know the core loss, as measured by wattmeter, and the excitation current, as measured by ammeter. Consider Fig. 49.

In this figure, 1–2 equals the applied e.m.f. of the transformer with secondary open-circuited. As a result of this application of e.m.f., enough current flows into the transformer to produce sufficient magnetism to create a back e.m.f. practically equal to that applied.

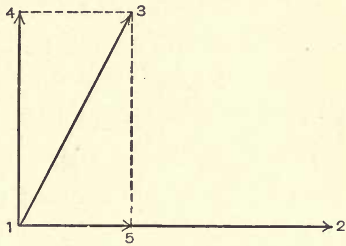

Fig. 49.

This magnetism, however, is attended by a certain loss in the iron, as has been shown, so that the current that flows not only has to produce magnetism, but has to supply, in connection with the applied e.m.f., the energy for this loss. Simple magnetizing current lags always just 90° behind the applied e.m.f., since, as has been shown, the energy given to bring the flux to its maximum is given back again by the decrease of the flux. Also, since from magnetism alone, without attending iron loss, there is no energy loss, the lag must be such that the cosine of the angle of lag must equal 0, for energy equals E times I times cosine of lag. The only angle with a cosine equaling 0 is 90°.

Energy current is always in phase with the e.m.f. Thus, in Fig. 49, the line, 1–5, is drawn to show the value of the current necessary to supply the core loss (1–5 × 1–2 = watts core loss), 1–4 is drawn to show the magnetizing current, and is thus 90° away from 1–2. Thus, the total exciting current flowing is 1–3, the resultant of 1–4 and 1–5. Thus, 1–3 is the exciting current, as read on the ammeter, 1–4 is the magnetizing current calculated for use in the formula cf regulation, and 1–5 is the energy component of the exciting current.

Since watts core loss = 1–2, the e.m.f., times 1–5, the energy component of current, 1–5 must be equal to $\frac{\text{core loss}}{\text{e.m.f.}}$.

Since $\overline{1\text{–}3}^2 = \overline{1\text{–}5}^2 + \overline{5\text{–}3}^2$, or, what is the same thing, $\overline{1\text{–}4}^2 + \overline{1\text{–}5}^2$, it follows that $\overline{1\text{–}4}^2 = \overline{1\text{–}3}^2 - \overline{1\text{–}5}^2$, or

$$\text{Magnetizing current} = \sqrt{(\text{excitation current})^2 - \left(\frac{\text{core loss}}{\text{voltage}}\right)^2}.$$

6. The ability to stand more than normal voltage between the various windings and the frame of the transformer, as well as between the various windings themselves, that is, the dielectric strength, is determined by applying to the secondary of the transformer enough voltage to produce in the primary the desired voltage, say twice normal running voltage, which gives also twice normal voltage per turn in the transformer. A higher frequency than normal must be used, as otherwise too much excitation current will flow from the higher voltage applied and the resulting saturation of the iron. For the insulation test of the windings to frame, an external source of e.m.f. is applied to the

windings and frame of transformer, care being taken, when applying the high potential between primary and secondary or between primary and core and frame, to see that the secondary is connected to core and frame. It is well also to connect primary leads together, and also secondary leads, to make a more uniform potential strain throughout the winding.

The grounding of secondary is necessary, since the potential on the primary is transferred to the secondary by static inductance, as primary and secondary are close together but insulated from each other; and any two bodies insulated from each other and near together have a static electrical relation one to another

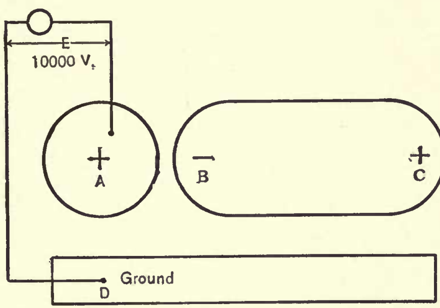

Fig. 50.

such that a potential on one will induce a similar and opposite potential on the other. This principle is shown in Fig. 50.

If 10,000 volts are applied between the ball, *A* (representing the primary winding of a transformer), and the ground, *D* (representing the core and frame of a transformer), there will be a difference of potential, without necessarily current flow, between *A* and *D*. It is an experimental fact that if a ball, *A*, be placed near a ball or round-ended cylinder, *BC*, that the potential at *A*, which produces on *A*, since it is assumed to be insulated in space, a *static* charge, will create or induce on *BC* an equal and opposite charge which is held on the end of *BC* at *B* as near as possible to *A*. At the same time a similar charge, but of opposite polarity to that on the *B* end, is drawn over to the *C* end. The charge at *B* is said to be a bound charge, being held at the end of *BC* by the presence of the charged ball, *A*. The charge on *A*, and hence the charges on *BC*, are proportional to the potential raising of *A*, above the earth or ground as shown. This static charge so induced, is the same electricity that is called *when flowing* current. When still, it is called *static electricity*. This, in the case of a transformer where the primary is represented by the ball, *A*, and the secondary by the ball, *BC*, the application of potential between the earth, namely core and frame, creates a static charge on the windings of the primary, which by static induction produces an opposite charge on the secondary. Referring again to Fig. 50, the charge at *C*, repelled to the other end of the ball, *BC*, which represents the secondary, is the one which tries to go to earth, and which in so doing strains the secondary insulation to frame and core. Unless the secondary is grounded, this free static charge may puncture the secondary insulation, which naturally need not be of a quality to stand the same high potential to ground as the primary.

Finally, by the same means the insulation test should be applied between primary and secondary. It is well to use needle-points at the point of application of high potential, set apart a distance across which the current will jump if the voltage be raised a trifle above the desired amount. This protects the winding from undue strains, caused by accidentally raising the potential more than desired, and especially from a sudden rise of voltage due to the leading capacity current. There is a considerable flow of leading current when the high potential is applied. Since the winding and frame form the plates of a condenser like Fig. 12, this raises the potential through the inductance of the transformer, delivering the high potential, as previously shown in Fig. 25.

7. The temperature of all parts ultimately reached, when operating at normal potential and at normal current output, can be found by loading the transformer on a resistance and applying normal potential. Thermometers should be applied carefully to all parts accessible, such as the outside of frame, windings, core, etc., and should be read every ten minutes or so, in order that the rate of rise of temperature and its ultimate constancy can be demonstrated. In addition, the resistance of the windings should be taken every half hour or so; but the primary alone will do if second-

ary is so low in resistance as to make it difficult to get accurate resistance measurements. This rise of temperature, computed from the increase of resistance, gives an indication of the internal temperatures that a thermometer cannot reach. The rise in temperature by increase of resistance measurements is always found to be greater than by thermometer. In order to read this resistance promptly, without taking up unnecessary time and thus cooling off the transformer, it is necessary to have a double throw switch, so that the

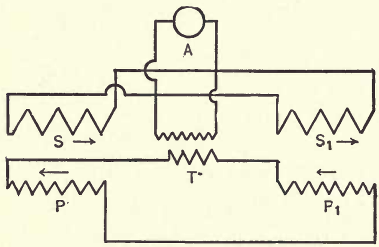

Fig. 51.

alternating current can be taken off, and the direct current, from which the resistance is calculated, applied. When all temperatures and resistances have become constant, the data desired are at hand, as well as the rate of increase of temperature, which is shown by the resistance measurements. Instead of loading a transformer directly as resistance, if two similar transformers are available, a return energy method is advisable, since it saves all the energy necessary to run the transformers, except, of course, the losses. In Fig. 51 is shown such an arrangement, using two single phase transformers with primaries, P and P_1 respectively, and secondaries, S and S_1. In this figure, A is the alternator, producing, at proper frequency, the voltage equal to the secondary voltage of the transformers to be tested. The voltage can be obtained from A, of course, by stepping up or down, in case its voltage is not the same as the secondary voltage of the transformer being tested. The corresponding ends of the secondaries are connected together, as shown. Thus, the two transformers are in multiple on the secondary side, and excited from the secondary side to normal density.

The two primaries are connected together, as shown, but also through the secondary of another small transformer, T. If T were cut out, no current would flow in the primary windings, and only the excitation current in the secondary windings. Cutting, however, transformer T into the primary circuit, with a value of secondary voltage such that it will produce in the primaries of the transformers being tested normal current, gives not only normal primary current, but simultaneously normal secondary current. This voltage must equal the current desired, times the sum of the transformer impedances. Since the secondary is wound around the same magnetic circuit as the primary, and the flux produced by the current flowing in the primary cuts the short-circuited secondary turns, it produces in them opposing, equal, ampere-turns (see Lenz's law). Thus, by this means, we have full excitation and full current flowing, full losses and therefore normal temperature producing conditions, and the only energy drawn from alternator, *A*, is enough to provide the losses of the system. Thus, two 1,000 k. w. transformers can be run at full load with an expenditure of energy of only 40 k. w. if the efficiency of each is 98%.

Referring again to Fig. 51, it is not necessary that a transformer be used at *T*. Another alternator may be used, and in practice it is found that the phase of the alternator may be different from the phase of the exciting currents without any material difference in temperature results. Likewise, if the transformer to be tested be a three phase transformer, or if three transformers are to be tested simultaneously, the exciting current may be applied from a three phase alternator, and the current circulated in the windings by a separate single phase alternator. This makes a most convenient testing arrangement, since transformers are often used in threes, and three phase transformers are fast coming into use.

8. The temperature, after a run of a certain definite time, at a certain per cent over load, say 50% over load for two hours, is taken just as described for the full load test, except that the run must start with the transformer at the temperature resulting from a long run at normal load.

A transformer ought not to rise much over 50° Cent. at normal load, and its regulation should be within 2%, and efficiencies on larger sizes as high as 98% at full load.

9. The polarity test of transformers is an important one. If, for instance, in Fig. 51, the polarity were wrong on one transformer, the operation as described could not be carried out. Instead, with the connections as shown, a short circuit would result. Under such conditions, transformers connected in multiple in accordance with instructions sent out by the manufacturers, would burn out very quickly unless the enormous current flowing were relieved. An excellent method of determining polarity is to pass a direct current into the primary from lead *A* to lead *B*, Fig. 52. Put on the secondary side a d.c. voltmeter, also put in the primary side a d.c. voltmeter. It is desired to show that leads *A′B′* on the secondary are related in their windings properly to the primary; in other words, that when *A* is + to *B* on the primary side, *A′* will be + to *B′* on the secondary side. Breaking the circuit on the primary side, the lines of force set up by the current in the primary suddenly cease. The secondary, being in the same magnetic circuit, has included these same lines, hence their sudden disappearance sets up an e.m.f. in it. The kick of the voltmeter connected to *A′* and *B′*, in its relation to the reading the voltmeter would give with the current flowing and when connected to *A* and *B*, the primary terminals, serves as a polarity criterion.

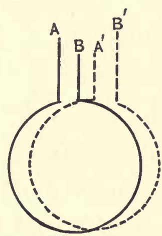

Fig. 52.

If the current flows from A to B, Fig. 52, the voltmeter will show a certain deflection; and the lines of force are coming up through the paper in the figure. If these lines of force suddenly cease by breaking the current, the e.m.f. induced in the secondary from A' to B' is in the same direction; that is, from A' to B'. If, for any reason, the terminals A' to B' were by mistake interchanged, the voltmeter would read backwards, and the mistake would be detected.

10. The measurement of the pressure and rise of temperature of the cooling medium, used with any transformer, is an important matter.

Transformers may keep cool by radiating the heat into the surrounding atmosphere, just as a hot ball will assume the temperature of its surrounding medium. This is heat dissipation by radiation. This radiation is much accelerated if the transformer be immersed in a case filled with oil. The oil coming into contact with the heated parts circulates, because hot oil is lighter than cool. Thus, fresh cooled oil takes the place of the hot, and the hot oil in its motion comes into contact with the outside case, which being the coolest part, takes up the heat rapidly. Thus the oil acts as a conveyor of heat, carrying it where it can be more easily radiated. In addition, proper oil is an excellent insulation; so that in modern practice oil is used extensively for transformers. In recording results of heat runs, the temperature of the oil at various parts is recorded, as well as the amount of oil used.

Another method of assisting the carrying off of the heat of a transformer is to blow air through the coils of the transformer as well as through the crevices of the iron. Ducts are arranged, in assembling the parts, to allow the air to pass through. The air takes up the heat as it circulates, blowers providing the pressure, and new, cooler air takes its place. This is a combination of radiation and convection, or carrying off of heat. In making heat tests on such a transformer, the amount of air and its incoming and outgoing temperature are recorded, as well as the necessary air pressure in coil and iron.

A third method of assisting the carrying off of heat is to place in the oil a coil of pipe through which cool water passes. This cools the oil, and thus the whole transformer. The amount of water, its incoming and outgoing temperature, and the necessary pressure to move the same, is recorded as a part of the heat run.

A fourth method, used particularly when the currents are considerable and the copper is large in cross section, is to have the copper itself a hollow tube, through which water is made to circulate. This brings the water very close to the source of heat generation, and is a most effective way of keeping down temperature.

As a part of the heat run, as in the case of a coil of pipe immersed in oil and carrying a flow of water, the amount of water, its incoming and outgoing temperature, and the necessary pressure to move it is recorded.

11. The watts lost with full load current and transformer short-circuited on itself, just as in taking impedance, are obtained, just as is the measurement of current and watts for the core loss, and the same precautions must be observed. The reading gives the sum of the I^2R of the windings and any eddy currents caused by the flow of current in the windings. What are called load losses in a transformer are approximately obtained by this measurement.

The load loss is the loss over and above the ordinary core loss, and is created by the current in the coils themselves when under full load. This current creates a stray field, as has been shown under the subject of regulation, which, being alternating, induces eddy currents in any conducting material in its path. Moreover, the passage of the current in the copper of the transformer itself, particularly if the copper is of considerable cross section, causes eddy losses in the copper, and this measurement gives the value of these losses very closely.

Finally, the efficiency of the transformer is obtained by dividing the output by the sum of the losses, as measured, plus the output; that is, by dividing the output by the input. Thus,

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + I^2R + \text{Core loss} + \text{Load loss}},$$

bearing in mind that the core loss is at a voltage $= E$ (the applied voltage) $- IM$, the primary impedance drop, and that the I used must include (combined by the vector diagram, as has been shown) the excitation current, if it be of appreciable value.

DESIGN OF TRANSFORMERS.

As has been shown, the voltage produced in a transformer is practically equal to the voltage applied, also the ratio of voltage primary and secondary is practically proportional to the turns. Furthermore, it has been stated that the volts are equal to $\frac{4.44\ \phi Nn}{10^8}$. When ϕ = maximum flux in lines of force, N = turns embracing the flux, n = cycles per second, and volts obtained = square root of mean square volts.

Thus the voltage of a transformer applied to the primary, or, what is prac. .ally the same thing, the voltage produced *in* the primary by the flux pulsating in the winding, is the result, at a given frequency, of the amount of flux and the number of turns embracing that flux. Hence, a large flux and few turns can be used, or a small flux and a large number of turns. A large flux and few turns produce close regulation. This can be seen by an inspection of the formula, shown to be suitable for regulation calculation, namely,

$$E = \sqrt{(100 + SIX + MIR)^2 + (MIX - SIR)^2}.$$

E, the voltage to which the secondary rises when the load is thrown off, becomes nearer to 100, the value in the formula chosen for the secondary e.m.f. when the transformer is under load, the less the values of R and X; and these decrease with the decrease of turns. On the other hand, a large flux means a large amount of iron to produce it, and a large amount of iron carrying a flux pulsating in the form of a sine curve (producing thereby a back e.m.f. practically equal to the applied) has within it a large hysteresis loss, which cuts down the efficiency, while too much iron adds materially to the expense of the transformer. It thus becomes a question of judgment as to the relative weight of copper and iron to cover a given design. Having chosen this relative weight, to give minimum cost consistent with reasonable regulation and temperature, it becomes necessary to place the primary winding in such relation to the secondary that there will be a minimum amount of leakage of lines of force between them. The value, X, of the transformer reactance is equal to $2\pi nL$ and

$$L = \frac{\text{flux} \times \text{turns}}{\text{amp} \times 10^8}.$$

The flux in the above formula is that which circulates around the wire of the primary and does not get into the secondary, thus leaking in between them, added to the flux which circulates around the wire of the secondary and does not get into the primary. This flux is produced by the current which flows through the wire of the primary, or secondary as the case may be, and this increases as the load increases. The ratio of flux to amperes, which, as the formula shows, equals L, stays practically constant, however. It thus becomes important to mix in the primary and secondary windings so as to reduce the path and ampere-turns of the leakage flux, and in practice this is done, a primary coil being first placed in position, then a secondary coil, then a primary, and so forth. Thus, for a given number of turns, a minimum amount of self-induction is obtained. If the primary and secondary were entirely separate and complete in themselves without intermingling, a very high value of X or self-induction, and consequent poor regulation, would be obtained, a grave defect in a transformer. Knowing the applied e.m.f., and having decided upon the turns for trial calculation, the flux to correspond with them is calculated by the formula

$$E = \frac{4.44\,\phi\,Nn}{10^8} \text{ and thus } \phi = \frac{E\,10^8}{4.44 \times Nn},$$

where $N =$ turns, $n =$ cycles, $\phi =$ max. flux, and $E =$ square root of mean square value of volts. Knowing ϕ and the density per sq. cm. of lines of force to run for trial in the iron, the cross section of the iron is known. The length of the iron magnetic circuit is determined by the arrangement of the coils in the iron core. Thus, in the case of a transformer with iron circuit, as shown in Fig. 53, the length A is determined by the

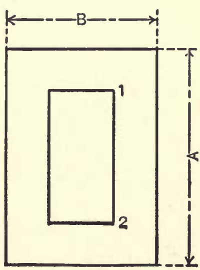

Fig. 53.

number of turns as chosen necessary to be placed between 1 and 2. *A* is long if the turns are wound close to the core and the turns are thus shallow in depth, and is short if the coils are deeper. Too deep coils cause internal heating; too shallow ones reduce the internal heating to a minimum; but the length *A* becomes excessive, so judgment must be used. The length *B* becomes larger the thicker the coils. This thickness of coil being chosen, the length of the magnetic circuit is determined, and thus the weight of iron can be calculated. Knowing the length and cross section of the iron, the magnetizing current which the transformer will take on open circuit can be calculated as follows: As shown, the maximum flux in the transformer

$$\phi = \frac{E\,10^8}{4.44\,Nn}$$

where

$$N = \text{turns},\ n = \text{cycles},\ E = \text{volts}.$$

From the cross section of iron and flux, ϕ, the density is given. The proper density in this case is assumed.

From the curve of magnetization, as shown in Fig. 3, the ampere-turns necessary to produce the flux density to be used is shown. Hence, knowing the turns, which are already chosen to correspond with flux chosen, the amperes are known. Knowing the number of turns and the dimensions of the magnetic circuit, the length of the turns, and hence the resistance of the transformer, may be calculated. Similarly, knowing the dimensions of the magnetic circuit and location of coils, the inductance of the transformer is calculated.

Knowing the amount of iron, the density and cycles of magnetism within it, the hysteresis loss is determined, as has been shown, by the formula:

$$W = \frac{KB^{1.6}}{10^7}$$

$K = vn\eta$ where v = vol of iron.
n = frequency.
η = hysteretic constant.
B = maximum flux density.

joules, or watt-seconds, per cubic centimeter. Hence the loss of energy in watts within the transformer can be calculated, being the sum of the I^2R and the total hysteresis loss. If the magnetizing current be small, the current for calculation of I^2R may be the energy current alone. If the magnetizing current amounts to anything, it must first be combined with the energy current by the parallelogram of forces; considering, in doing this, the lagging magnetizing current component, the energy hysteresis component, and the external load current component.

We thus have, for the given assumption of flux and turns, of iron density and thickness and application of coils, a knowledge, by calculation, of

1st. The regulation.

2d. The total resistance, and hence the loss in watts due to I^2R.

3d. The total inductance.

4th. The total loss in watts due to hysteresis.

5th. The excitation current, or current taken with no load on transformer.

6th. The rise in temperature, since a certain definite rise in temperature will always result from a certain radiation of energy loss per sq. cm. of radiating surface; and since the dimensions of the transformer are determined, the radiating surface, that is, the surface exposed to the air, is known.

Thus, the rise in temperature $= \frac{SW}{A}$.

Where S is a constant, known by experiment, $W =$ total watts lost, that is, $I^2R +$ hysteresis, and $A =$ the area of radiating surface. If oil be used for cooling purposes, if cooling coils of water pipe be immersed in same, or other means of keeping the transformer cool be employed, the constant, S, will naturally be smaller than for natural radiation.

If the regulation, cost, temperature, efficiency and excitation current do not suit, another assumption of flux and turns and iron density may be taken and the constants calculated, until a general satisfactory result is obtained.

Magnetizing curves of various kinds and qualities of iron must be possessed, in order properly to calculate the excitation amperes, and these can be obtained, as shown previously, under the heading of "Saturation," by using a helix with a core composed of iron to be experimented upon, the helix having a length two hundred times its diameter.

THE CONSTANT CURRENT TRANSFORMER.

The transformers just discussed are called multiple transformers, since they are all connected in multiple in the circuit. They transform voltage from a higher to a lower value, or the reverse. This ratio of voltages also is practically constant, with varying load. There is another class of transformer, called the constant current transformer. It is designed to give constant current, irrespective of the external resistance. Such transformers are purposely made with primary and secondary separate, and with a special leakage path for flux, between primary and secondary, so that the regulation may be as poor as possible. Thus any tendency for an increase of current, due to change of resistance in the external circuit, is counterbalanced by the immediate drop of voltage, due to the high self-induction of the transformer. Such transformers can be made, even with the primary stationary with respect to the secondary, to give very fair current regulation, by means of exceedingly poor voltage regulation.

THE SERIES TRANSFORMER.

There is another class of transformer called the series transformer, which transforms currents instead of voltages. Referring to Fig. 54, the series transformer consists of a laminated core, *E*, with the primary winding, *B*, and a secondary winding, *C*, closed through any impedance, *D*. The current, I^1, in the impedance, *D*, always bears a certain definite relation, equal practically to ratio of turns, to the current, *I*, passing through the primary of the transformer. The transformer diagram is similar to the diagram for the multiple transformer

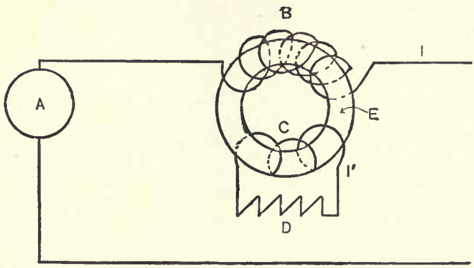

Fig. 54.

shown in Fig. 44. The action of the transformer is as follows:

At the current, *I*, with the secondary open-circuited, the back e.m.f. of the transformer is that due to the flux produced by the current, *I*, flowing through the turns, *B*, Fig. 54. This back e.m.f., therefore, is con-

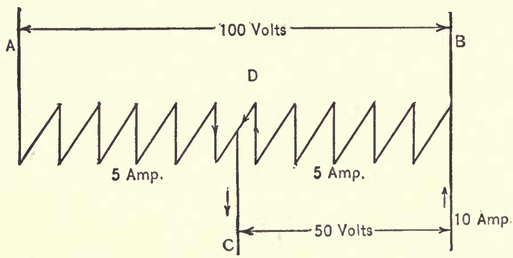

Fig. 55.

siderable, since there are no ampere-turns to oppose the magnetization. Thus, a series transformer on open circuit gives a very high back e.m.f. When the secondary winding, *C*, is closed, however, the current in it opposes the primary, so that the back e.m.f. of the transformer on this account immediately decreases, there now being practically no flux in the core, its value depending upon the value of the impedance *D*. If $D = 0$, the drop in the series transformer is practically 0. If *D* is infinity, that is, open-circuited, the drop in the transformer is a maximum. In between open circuit of the secondary and short circuit, the voltage drop through the transformer takes intermediate values. Thus, a series transformer transforms current, and uses up, in doing so, an amount of voltage depending upon the impedance to which the secondary is connected. The core loss is a variable depending upon the secondary load. At a given load, and thus at a given voltage, upon the primary and secondary, the series transformer diagram is like Fig. 44; but for each value of current or of secondary impedance connection a new diagram is required, since the primary voltage changes.

Series transformers are valuable in transforming currents to lower or higher values, so that they can be read more conveniently on ordinary instruments. Thus, it would be practically impossible to read 10,000 amperes directly on an ammeter. Inserting a 100 to 1 current transformer in the circuit gives a convenient value of current to read. Since the secondary of the series transformer is short-circuited through the ammeter, and since the impedance of the ammeter is negligible, the drop through the series transformer is negligible. Series transformers are often used in compounding alternators when it is desired to get an increasing voltage from the secondary of the transformer with increasing current output from the alternator, the main alternator current passing through the primary of the series transformer.

THE COMPENSATOR.

There is another type of transformer in use, called the compensator, where the secondary winding, instead of being separate from the primary, is a continuation of it. The advantage of such an arrangement is that, for the same energy to be transmitted, the compensator is smaller than the transformer. The disadvantage is that the secondary is electrically connected to the primary, and thus the compensator is not suited for use on high potential circuits. Owing to the reduction in its size as compared with a transformer, and the consequent reduction in its cost, a compensator is most desirable on low potential circuits, in which the secondary voltage is not small as compared with the primary. It is, therefore, usually used for starting induction and synchronous motors.

The principle of the action of a compensator can best be explained by reference to Fig. 55. Assume the primary voltage to be 100, and the secondary to be 50, as shown in the figure. The ratio of turns, primary to secondary, in the compensator will, therefore, be just as in a transformer, 100 to 50. The excitation current flows from *A* to *B*. The current in the secondary circuit, *C*, is, however, the sum of the current from *B* to *D* and from *A* to *D*. Thus, if the external current in the secondary is 10, 5 amperes will be supplied by the secondary winding, and 5 by the primary; the two currents meeting at *D*, at a common potential, and from there flowing into the external secondary circuit. Expressed differently, the ampere-turns in the secondary produced by the current flowing into the external secondary circuit create an equal and opposite value of ampere-turns in the primary winding, as in an ordinary transformer, which ampere-turns can only close themselves through the external circuit, and hence the amperes thus produced in the primary must join the amperes flowing in the same circuit from the secondary, as indicated by the arrows in Fig. 55. Note that the current from *B* to *D*, which is flowing through the winding acting as secondary, is opposite to the current from *A* to *D*, which is flowing through the winding acting as primary, just as in any transformer, and thus the two currents join at *D*. Thus, in the compensator, the secondary supplies only half of the energy transmitted, instead of the whole, as in an ordinary transformer, and the primary only half, instead of the whole, as in an ordinary transformer. The compensator, therefore, needs to be only half as large as a regular transformer transmitting the same energy. If the ratio of voltage between primary and secondary were 3 to 1, instead of

2 to 1, the compensator would be two-thirds as large as a regular transformer, since in that case the secondary would supply $\frac{1}{3}$ the energy, and the primary $\frac{2}{3}$. This ratio of size, of course, neglects excitation currents. The diagram for the compensator voltages and currents is similar to that of the regular transformer, shown in Fig. 44, and the various characteristics indicated for transformers apply also to compensators.

CHAPTER IV.

VARIOUS TRANSFORMER CONNECTIONS AND DISTRIBUTION SYSTEMS.

THE ordinary lighting single phase system delivers the power at 1,040 or 2,080 volts, and transforms it down to 104 by transformers with ratio of 10 or 20 to 1. Thus the voltage at the lamps is low, while the

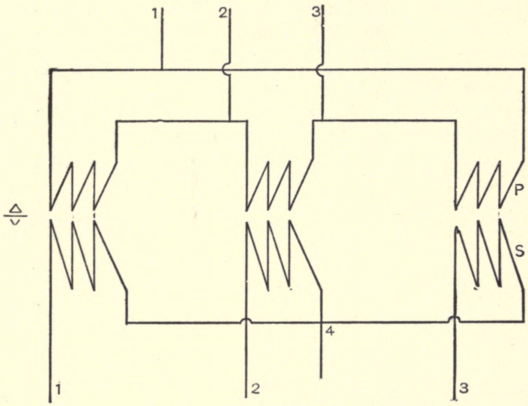

Fig. 56.

voltage on the line is high. Since weight of copper varies as the square of the line voltage, it can be seen what an enormous advantage exists by transformer use in this regard. In three phase systems for power only, the voltage can be generated at a value economical for generator design, say 2,000, and be transformed up to any value that line insulators can safely handle, such as 40,000 volts, for instance. Here a triple advantage is secured: 1st, the saving of copper due to the high voltage; 2d, the saving of copper due to the use of three phase, which is only $\frac{3}{4}$ that for single phase; and 3d, the increase in safety in insulation in the generator, as transformers can be more economically insulated for high voltages than alternators.

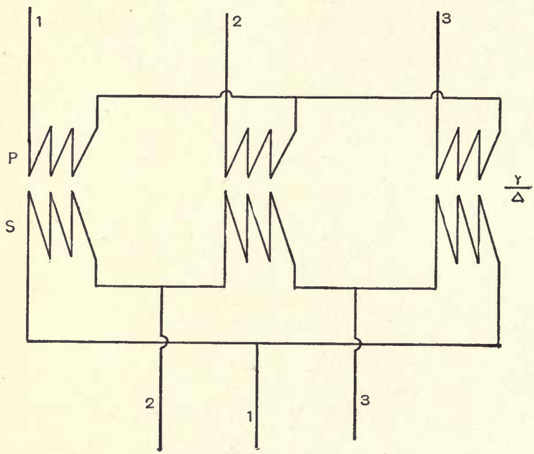

Fig. 57.

Some connections of transformers on three phase circuits are shown in Figs. 56, 57, 58, and 59.

Fig. 56 shows a three phase connection using three transformers when the primaries are connected Δ and the secondaries Y. This gives the ratio $R \times \sqrt{3}$, where $R =$ ratio of transformer itself, and $\sqrt{3}$ is the multiplying factor for the voltage between lines as related to the voltage of the transformer making up the Y connection, as has been shown on p. 62. From the point 4 a neutral wire can be brought out, serving a similar purpose to the neutral wire in an Edison three wire system, it being without current on balanced load.

Fig. 57 shows, for three transformers, the reverse of Fig. 56; the ratio of transformation being $\frac{R}{\sqrt{3}}$ instead of $R\sqrt{3}$.

Fig. 58 shows a △ to △ connection, using, however, only two transformers instead of three. The ratio here is R. With such a connection, three phase is transmitted at the ratio R, but one phase has the impedance of two transformers, and the other two the impedance of one to consider in calculating voltage drop, and thus under load a slight unbalancing may exist. From this it may be seen that if one transformer in a transformer connection like that shown in Fig. 56 gives out, it can be cut out of circuit, and no interruption need exist for long in the service.

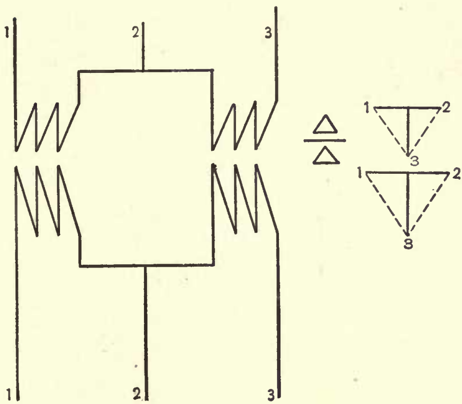

Fig. 58.

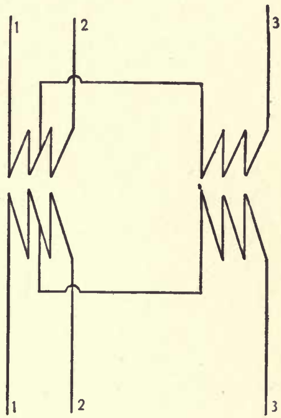

Fig. 59.

Fig. 59 shows a two transformer three phase transformation. The turns on transformer A are related to turns on transformer B as 2 to the $\sqrt{3}$.

Fig. 60 shows a transformer connection, suggested by Mr. Charles Scott, using two transformers, and giving a transformation from three phase to quarter phase. The primary of one transformer is 2–8, and the secondary is 4–5. The primary of the other transformer is 1–3, and the secondary is 6–7. The primary of the first transformer is connected at the center of the primary of the other, as shown at the point 8. The turns on the secondary 5–4 may be $\frac{2}{\sqrt{3}}$ times the turns on secondary 6–7 if 2–8 and 1–3 have the same turns, for the voltage from 2–8 equals, as can be seen from the triangle 1–2–8, 1–2 times sine of the angle 2–1–8, which is 60°, or 1–2 $\frac{\sqrt{3}}{2}$, since sine 60° = $\frac{\sqrt{3}}{2}$. Looking at the diagram, the three phase voltage is applied at the points 1–2 and 3, and the voltage 1–2 or 2–3 can be resolved by the parallelogram of forces into 2–8 and 1–8, or 2–8 and 3–8, at right angles to each other. Thus it is apparent that in the voltage 2–8, one of the components will be less than 1–2, the producing voltage, and also that the voltages, 2–8 and 1–3, will be at

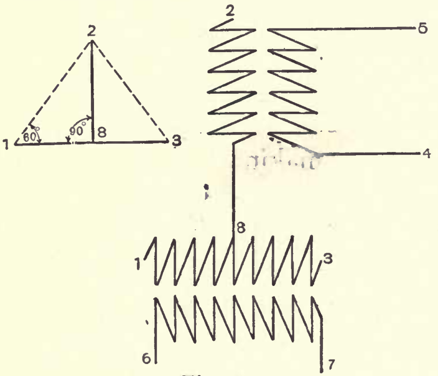

Fig. 60.

right angles to each other, or, in other words, quarter phase. The voltage given by 5–4 is obtained from 2–8 as primary. On the other hand, the voltage given by 6–7 is obtained from 1–3 as primary. Moreover, 1–3 equals 1–2, and is $\frac{2}{\sqrt{3}}$ times as great as 2–8, which produces the voltage in the secondary 5–4. Hence, since the voltage is proportional to turns, and since 5–4 must equal 6–7, to be a proper quarter phase system, it becomes necessary to have the turns in 5–4, $\frac{2}{\sqrt{3}}$ times the number of 6–7, or, what is the same thing, the turns in 2–8 must be less than the turns in 1–3, if the turns in 5–4 are the same in number as in 6–7. This transformation serves also to transform from quarter phase to three phase by making the primary 5–4 and 6–7 and the secondary the points 1–2 and 3. Thus the arrangement consists in the using a component voltage, 2–8, of an original voltage, 1–2, or 2–3, as an actual primary voltage, from which a secondary voltage is obtained.

In general, various transformations of polyphase circuits can be made by combining or transforming components, so that a desired resultant value is obtained in phase and amplitude. Three phase transformers are sometimes built with all the windings on a common magnetic core. This has the advantage of saving in magnetic cross section of iron, as well as saving in frame, or non-magnetic, iron. The latter is selfevident, since a single structure self-contained, as is a transformer, requires less structural iron than it would when divided into three parts. The saving in magnetic material is effected by the fact that in certain parts of the magnetic circuit flux from two different legs of the transformer passes, and the resultant of these two fluxes is equal to either one of them. This is shown by referring to the diagrams, Fig. 61 and Fig. 62. The iron magnetic circuit is shown with coils in place in Fig. 61. In the space shown at *ST* and *UV*, the coil, *A*, joins that from the coil, *B*, but, owing to their phase relation, the vector sum of these fluxes is equal in magnitude to either one of them, while in phase it is midway between them, or in opposition to the flux from the third coil, *C*. For, referring to Fig. 63, it will be seen that the resultant of the fluxes, 1–2 and 1–3, equals 1–7, which is equal in length to either 1–2, 1–3, or 1–6, and is, in phase, midway between 1–2 and 1–3, or opposite to 1–6.

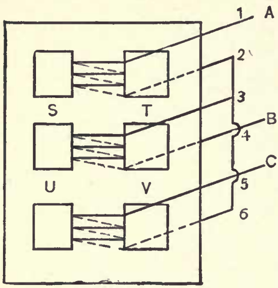

Fig. 61.

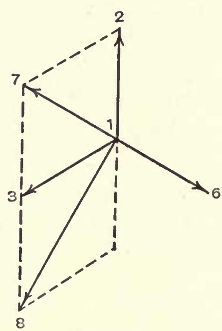

Fig. 62.

The path of this flux is *ST*. If the phase *B* be reversed, the resultant of it and phase *A* is shown in Fig. 62, as the line, 1–8, which is $\sqrt{3} \times$ either component, and thus the flux represented by it would require in its path, *ST*, Fig. 61, a larger cross section. Self-contained three phase transformers are coming quite rapidly into use in connection with rotary converters, whose low a.c. potential requires usually a transformation from the transmission line potential.

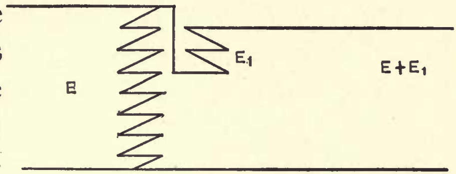

Fig. 63.

The transformer connections for six phase, using three transformers with single secondaries, and also using three transformers with double secondaries, have been shown on pages 65–66.

Transformers can be used for boosters to raise or lower the voltage in a circuit. Fig. 63 shows a single transformer so connected. In this case the voltage *E* is increased by the secondary voltage of the transformer *E*. Fig. 64 shows the connections for a three phase circuit. The voltage E_{11} is increased over the voltage E in this case by $\sqrt{3}\,E_1$. Examine diagram shown in Fig. 65. The lines 2–7,

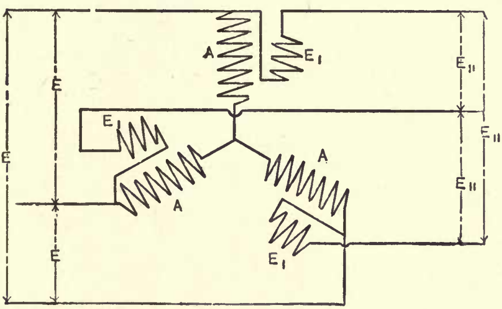

Fig. 64.

7–8, and 8–2 show the voltages, E, 120° apart. The secondary voltages, E_1, shown in Fig. 64, are obtained from the transformers, A, which are connected in Fig. 64, Y, and thus are shown in phase in Fig. 65 by the lines 7–6, 6–8, and 6–2. The secondary voltages, E, from these transformers are, therefore, shown by the lines 1–2, 8–5, 7–9, in Fig. 65, in phase respectively with 2–6, 6–8, and 6–9. The new voltage between lines, resulting from the boosting connection, is shown by the line 1–5. In the triangle 1–2–3, Fig. 65, the side, 1–3 = 1–2 × cosine of the angle 2–1–3, which is 30°. The cosine

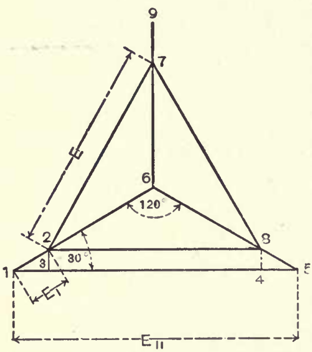

Fig. 65.

of $30° = \frac{\sqrt{3}}{2}$. Therefore, the line

$$1\text{–}3 = 1\text{–}2 \times \frac{\sqrt{3}}{2}.$$

Similarly, the line,

$$4\text{–}5 = 8\text{–}5 \times \frac{\sqrt{3}}{2}.$$

Hence, the line

$$1\text{–}5 = 1\text{–}3 + 3\text{–}4 + 4\text{–}5,$$

or equals

$$2\text{–}8 + 1\text{–}2 \times \sqrt{3}$$

or

$$E_{11} = E + E_1\sqrt{3}.$$

Another approximate and convenient method of getting quarter phase from three phase, or the reverse, is shown in Fig. 66.

The triangle, 1–2–4, shows a regular delta three phase transformer connection, the phases being 1–2, 1–4, and 2–4. The special connection consists in halving the phase 2–4, and putting one-half to the left at 2–3, and the other half to the right at 4–5. The resultant of 2–3 and 1–2 is the third side of the triangle; that is,

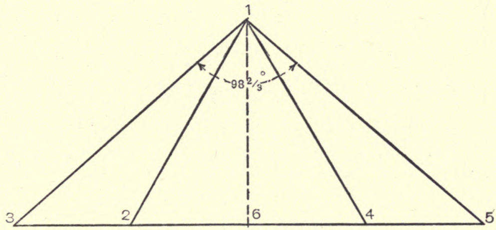

Fig. 66.

1–3. The resultant of 1–4 and 4–5 is 1–5. We then get the two phases 1–3 and 1–5, and they are a trifle over 90° apart; for assume the length of the line 1–2 = 2; then 2–6 = 1, and 2–3 = 1. Therefore 3–6 = 2. Also the angle 1–2–6 = 60°, and the sine of the angle.

$$1\text{–}2\text{–}6 = \frac{1\text{–}6}{1\text{–}2} = \frac{1\text{–}6}{2},$$

or

$$1\text{–}6 = 2 \times \frac{\sqrt{3}}{2},$$

since

$$\text{sine } 60° = \frac{\sqrt{3}}{2}.$$

Therefore,

$$1\text{–}6 = \sqrt{3}.$$

The tangent of the angle $1\text{–}3\text{–}6 = \frac{1\text{–}6}{3\text{–}6}$ = therefore $\frac{\sqrt{3}}{2}$ = .866, but the angle whose tangent is .866 is an angle of 40.67 degrees. Therefore, the angles 1–3–6 and 1–5–6 are angles of 40.67 degrees. But in any triangle the sum of all the angles equals 180°. Hence in the triangle 1–3–5 the angle 3–1–5 must equal 180° – 2 (40.67°) or $98\frac{2}{3}$ degrees.

DISTRIBUTION SYSTEMS.

Various alternating distribution systems are in use. Formerly the single phase system at 1,000 or 2,000 volts, with house-to-house transformers, was the only one used. This was varied, under certain circumstances, by having centers of transformer secondary distribution, with three or five secondary circuits, arrangements being made to cut out transformers not actually needed, thus freeing the generating station from unnecessary core loss.

Polyphase systems are now often used, embracing the advantages of reduced copper loss in the transmission line, ability to operate alternating motors, and ability to obtain desired direct current by rotary, or other a.c. to d.c. converters. This, in turn, means that in the generating station but one kind of generator is required, that is, the polyphase alternator, which in itself is cheaper to build than the single phase, for similar constants of operation. One method of using three phase alternators is to put practically all the lights on one phase, running out the third wire for a power wire, which, in connection with the other two wires, permits the operation of induction motors, synchronous motors, and rotary converters, thus giving a.c. or d.c. power at any desired point with reasonable copper expenditure. This has the advantage that the voltage regulation on the light circuit can be made very fine, even somewhat at the expense of the other circuits, which is desirable, since the main lighting load requires closer regulation than the power load. Various special circuits can be put on the other phases, the potential being regulated by feeder regulators, in case where such load is incandescent lamps, and by automatic transformers, where arc lamps are operated from constant current transformers. This, in connection with the fact that the power load draws equally from all three phases, results in a much nearer balanced load on the generator than might at first be supposed.

Another arrangement is carefully to balance the light load on the three legs of the alternator, so that all three circuits run, and require the same regulation. This is a fair method, and has the advantage of giving a three phase transmission with its consequent saving of line drop, and moreover less heating in the alternator proper, the load of lights and power being equally distributed throughout the winding, instead of two phases combined into one doing more of the work. The disadvantage is, that where the three phases become unbalanced, their voltages become unbalanced, and thus the lights in one of the circuits may be low, and in another high, giving poor light in the one case, and injury to the lamps and shortened life in the other case. This unbalancing can be neutralized by putting in the feeder circuits feeder regulators, which, automatically or by hand regulation, give constancy of voltage. This means, of course, additional apparatus, and some, though slight, departure from the first simplicity, but the results of operation are quite as satisfactory. The former method is preferable when most of the load is a lighting service of a high order, and when the lighting circuits are three wire network. The latter method is preferable when most of the load is power which automatically balances the phases, and thus permits the connecting of lights on any convenient phase. There is a useful modification of the latter method, in which a neutral wire runs either from the alternator, its winding being connected *Y* for the purpose, or from the secondaries of the transmission transformers. This neutral wire carries the current simply to balance the load, and permits the use of transformers with less primary voltage than the voltage of the transmission line, the ratio being $\sqrt{3}$, since the voltage between any two lines in a three phase circuit is $\sqrt{3}$ times the voltage between any line and the neutral. Thus with a 2,300 volt transformer, a line transmission of 4,000 volts can be used, without step-up or step-down line transformers.

For ordinary power transmission for short distances, such as twenty miles, alternators giving up to 13,000 volts are perfectly safe and practical to design. By their use, the step-up transformers at the generating station are dispensed with, and thus an expense is saved. At the receiving end of the line, step-down transformers may be used, which give the desired voltage to rotary converters, for power or lighting purposes. An excellent arrangement is to have at the end of the line motor generator sets, the motor being either induction or a synchronous, and the generator giving the desired d.c. voltage. In the former case, step-down transformers have to be used, since 10,000 volt induction motors are not practical to build. Great stability of operation is obtained with induction motors, since they are practically free from any "hunting" or "pulsating" effects of speed and current, met under certain circumstances in synchronous motors. Induction motors, in starting, also take much less current, disturbing to a less degree the line potential. When synchronous motors are used, on the other hand, no step-down transformers are needed, which is a saving; and at the same time synchronous motors, which are alternators run as motors, are cheaper to build than induction motors. The generators coupled to the motors can be direct current, and can have attached to them simple regulating devices, so that the voltage can be kept perfectly constant, irrespective, within reasonable limits, of the line voltage variation, or speed of generators at generating station. Thus, with motor generator sets, the line voltage variations do not affect the voltage of the generator of the set, so long as the speed of the main generators is constant; whereas, in rotary converters, the voltage of the line at the receiving end is transmitted at the outgoing or direct current end. Such a transmission is highly satisfactory in its operation, and owing to the absence of step-up and step-down transformers, it is also inexpensive.

Where the transmission distance is great, it becomes necessary to step up at the generating station, to get a line potential sufficiently large to make the cost of line copper reasonable. In such circumstances it is proper to choose a generating potential suitable for economic generator design, such as 2,000 volts, and transmit at the potential desired by step-up transformers. Step-down transformers are used at the receiving end of the line, to give proper potential to the motor generator sets, or rotaries. With long transmission lines, the dangers of surging, touched upon in a previous chapter, must be kept in mind. In general, lagging currents in a transmission line are undesirable, for, as has been shown, lagging currents produce more line drop than currents of the same value in phase. When, however, a transmission line is very long, a large leading or charging current may flow, raising the potential of the generating station more than desired. The effects on line drop with varying power factors have been treated in a previous chapter. In such circumstances, lagging current may actually be desired, to neutralize the excessive leading or capacity current.

ALTERNATING CURRENT MOTORS.

The most commonly used motor on alternating circuits is the induction motor. It is usually wound either quarter phase or three phase. For discussion, take the quarter phase motor. In construction it consists of a magnetic circuit, which in any direction has a uniform magnetic reluctance. On this magnetic circuit are placed primary windings, 90° apart. The secondary need not, however, have the same winding phase displacement as the primary, as will be shown later. It is also arranged that the secondary may rotate, either phase of it being able, therefore, to move before either of the primary phases. The revolving part is called the armature, and the stationary the field. In reality, an induction motor, therefore, is nothing but a polyphase transformer, but so arranged mechanically that the secondary can rotate before the primary without occasioning change of magnetic reluctance in the magnetic circuit.

A quarter phase motor in its elements appears as in Fig. 67. One phase is the winding from 4 to 6, in series with the winding from 5 to 3; and the other phase is the winding from 1 to 7, in series with the winding from 8 to 2. Thus the winding between the terminals, 1–2, magnetizes in the direction shown by the arrow, *A*, and the winding from the terminals, 3–4, in the direction shown by the arrow, *B*. To operate the induction motor normally, an alternating e.m.f. is applied between 1 and 2, the magnitude depending upon the winding of the motor; and simultaneously, a similar e.m.f., differing in phase 90° from the first, is applied between 3 and 4. Thus we have coils of the motor, set 90° apart, receiving respectively similar alternating e.m.f.'s, also 90° apart in phase. The effect of this can be seen by referring to Fig. 67. All current entering phase 1–2 magnetizes in the direction of arrow *A*. All current entering phase 3–4 magnetizes in the direction of arrow *B*. From the nature of the applied quarter phase currents, when the first is a maximum the second is zero, and vice-versa, as shown in Fig. 68. When curve 1–2 is a maximum, at *T*, curve

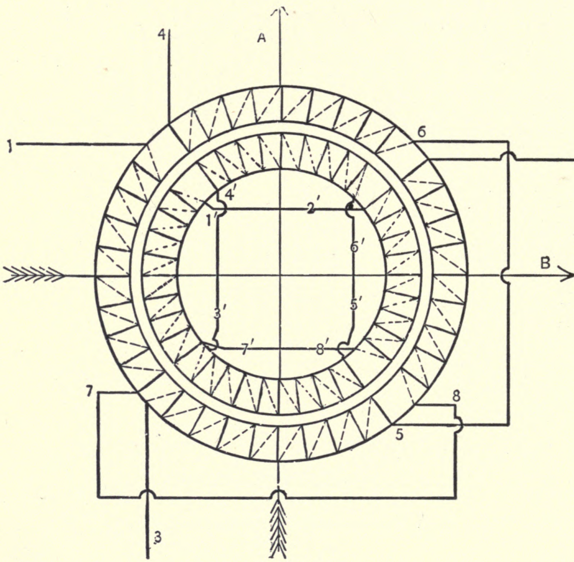

Fig. 67.

3–4 is 0, at *S*; and when curve 3–4 is a maximum, at *V*, curve 1–2 is 0, at *U*.

Thus, referring again to Fig. 67, when 1–2 is a maximum, 3–4 is 0, and hence the direction of magnetism in the magnetic circuit is as shown by arrow *A*. When 1–2 is 0, the direction of magnetism is as shown by arrow *B*. Thus the magnetism has swung around 90°.

Examining Fig. 68, it is seen that when 1–2 is a maximum and decreasing, 3–4 is 0 and increasing, and at any point along the line, *XY*, the vector sum of 1–2 and 3–4, that is their resultant combined at right angles since the coils are set at right angles to each other, is a constant. From *T* to *A*, 1–2 predominates. From *A* to *V*, 3–4 predominates. So, therefore, referring again to Fig. 67, the magnetism starting at direction *A*, with 1–2 a maximum, and still keeping constant in

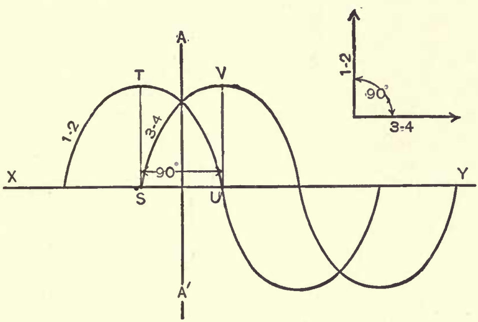

Fig. 68.

value, swings in direction, reaching finally the direction *B*, and from there the cycle is repeated. Half way between *A* and *B*, Fig. 67, which corresponds to the position *AA′*, Fig. 68, each phase contributes half. At the point *V*, Fig. 68, phase 3–4 does all the magnetizing. Thus, we have the phenomenon of a magnetic field rotating in space and passing through its magnetic circuit, so arranged to accommodate it, in any direction. This rotating field is produced by polyphase currents, which are received by coils set in space in angular relation corresponding to the phase relation of the currents.

It is interesting to note the effect of reversing the lines, 1 and 2, Fig. 67; that is, reversing a phase. The effect is shown by referring to Fig. 69. In this figure on *AB* are plotted the incoming phases in a certain direction, positive current, and therefore magnetism,

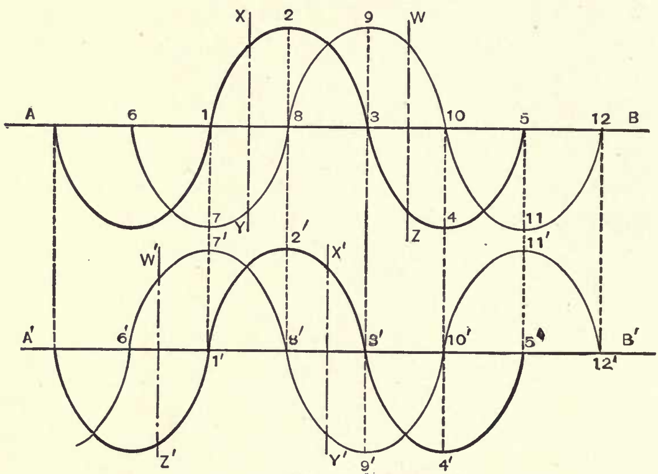

Fig. 69.

being shown above the line, and negative below. On *A′B′* are plotted the incoming phases, but with one of them reversed. In the upper and lower plot, the curves 1–2–3–4–5 and 1′–2′–3′–4′–5′ are drawn the same. In the lower plot, the curve 6′–7′–8′–9′–10′–11′ is reversed. Consider the curves between the lines *XY* and *WZ*, and between *X′Y′* and *W′Z′*, and in the upper plot note that between the lines *XY* and *WZ*, the resultant of the two curves, 1–2–3–4, etc., and 6–7–8–9, etc., is positive or +; that is, above the line. Starting at *XY*, the positive resultant moves to the right until the line *WZ* is reached, when the resultant becomes negative. On the lower plot, the positive resultant lies between *X′Y′* and *W′Z′*, but it can be noted that the resultant moves *to the left* to *W′Z′*, instead of to the right, as in the upper plot. Thus, reversing a phase of a quarter phase motor,—and the same holds true for a three phase motor, — makes the resultant magnetism revolve in the direction opposite to that before the reversal. The clearest conception of the physical actions going on in an induction motor is perhaps derived from a consideration of the actions resulting from the revolving field. As has been shown, in an induction motor there exists an actual revolving magnetism passing every pair of poles (Fig. 67 shows a two pole motor), at the normal frequency of the circuit. This flux passes through the field magnetic circuit, across the air gap and into the armature magnetic circuit. In the armature, as shown in Fig. 67, is placed, similarly to the field, a set of armature windings; in this case set in space one from another exactly as the field coils are. Hence, in the armature winding the revolving flux creates an electromotive force. The armature winding is short-circuited upon itself, hence the current equals the e.m.f. produced by the revolving field $\div \sqrt{R^2_1 + \overline{2\,\pi n L}^2_1}$, where $n =$ the frequency of the revolving field within the armature. At standstill, this frequency is that of the external circuit, — precisely like any stationary transformer. If, however, the armature turns, etc., rotate in the same direction as the revolving field, the frequency of the armature e.m.f. will come down, reaching o when the armature rotates just as fast as the revolving field itself. Under this condition, no e.m.f. is created in the armature, since no lines of force are cut by the armature wires, and no current flows, the conditions being similar to any transformer on open circuit in secondary. Thus, between rest and synchronism, the frequency of the current and the

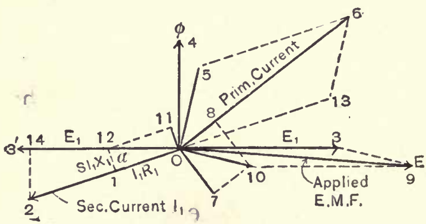

Fig. 70.

amount of the e.m.f. induced in the armature of an induction motor is directly dependent upon the speed. The word "slip" is usually applied to the ratio of the difference between the armature and circuit frequencies, to the circuit frequency, expressed decimally. Thus, at synchronism, the slip is o. At standstill, the slip is 1. The slip, multiplied by the circuit frequency, equals the armature frequency.

The diagram of an induction motor is shown in Fig. 70. From an inspection of the figure, it can be seen that it is practically the same as the vector diagram of an ordinary transformer, with the secondary e.m.f. and secondary inductance reduced in proportion to the slip. Fig. 70 shows the diagram for a certain definite slip. This diagram is perfectly proper, even though the frequency of the secondary is less than that of the primary, for this reason. At any given phase of the primary current, the secondary current bears a constant definite physical relation to it, just as in any transformer where the secondary current is always equal and opposite in its ampere-turns to the primary current. In a stationary secondary, this is understood and expected. With a movable secondary, containing currents at different frequency from the primary, this is not so clear. It must be borne in mind, however, that the frequency of revolution of the induction motor armature, plus the frequency of the secondary current, always equals the primary frequency; so that the revolution of the armature carries around the secondary currents, and, although of different frequency, they always bear a fixed relation to the primary currents, having reactions upon them, as far as phase and frequency are concerned, exactly as if the armature were standing still, with the currents in it at normal frequency. Thus, a vector diagram is perfectly proper under such circumstances. Referring to Fig. 70, the line o–ϕ = the flux which is common to both field and armature, o–5 equals the vector denoting in length and phase the current necessary to produce the flux o–ϕ.

The line, 0–5, is drawn at such an angle with the flux e.m.f., line 0–3, that 0–3 × 0–5 × cos of the angle, 5–0–3, = core loss and friction and windage of the induction motor. In a transformer the angle 4–0–5 is called the angle of hysteretic advance. The vector 0–3 = the e.m.f. produced in the primary windings by the flux, 0–ϕ, pulsating through them. The vector 0–2 = the secondary current = 0–3 × $S \div \sqrt{R_1^2 + (2\,\pi n L_1 S)^2}$. Where S = the slip, as explained, 0–3 = total e.m.f. of each phase of winding of secondary when standing still.

R_1 = resistance of secondary winding per phase.
L_1 = inductance of secondary winding per phase.
n = cycles of applied circuit (thus Sn = secondary cycles) and $\pi = 3.14159$.

It is convenient, in considering induction motors, to consider the secondary as having the same number of turns as the primary, with a ratio thus between them of 1 to 1. Hence, to make a certain secondary winding correspond to a certain primary winding, the resistance and inductance of secondary must be multiplied by the square of the turns. It is convenient to consider also that the secondary has the same number of phase as the primary. Thus, in Fig. 70 the vector 03_1 and 03 equals the e.m.f. per phase in both primary and secondary when standing still; and L, the inductance of primary, equals L_1, the inductance of secondary, when standing still; and R, the resistance of primary per phase, equals R_1, the resistance of secondary per phase. The vectors, 0–11 and 0–1, equal the inductance drop and resistance drop respectively in the secondary, at the slip existing in the diagram. The vector, o–6, composed of the combination of o–5 and the equivalent in the primary of o–2, o–13, is thus the primary current. The e.m.f. consumed by resistance in the primary is shown by the vector, o–8, in phase with the current, and the e.m.f. consumed by inductance is shown by the line, o–7, 90° displaced from the current. Hence o–10, the combination of o–8 and o–7, shows the impedance drop due to the primary current. The vector, o–3, 90° from the flux, o–ϕ, shows the e.m.f. induced in the primary by the flux, ϕ, and hence the total primary voltage is the resultant of this back e.m.f. due to flux and the impedance drop, or the resultant of o–10 and o–3, or the vector o–9. This diagram, Fig. 70, shows the flux relations, that is, the angular time differences in the vectors representing the square root of mean square values, of the e.m.f.'s and currents existing in an induction motor at any definite slip. It can be seen that the excitation current remains practically constant at all loads, and the power factor improves as load is put on, since the magnetizing component of current remains constant, and the energy component grows larger and larger, up to the point where the inductance of the windings themselves enters, which, in a well-designed motor, is at a considerable overload. Thus induction motors on light loads materially lower the power factor of the system on which they operate.

The torque of an induction motor is due to the action of the revolving field upon the currents induced by it in the armature. There are several ways of explaining it.

First, it can be explained in accordance with Lenz's law, which is as follows: Any displacement of the relative positions of a closed circuit and of a current or magnet, develops an induced current, the direction of which is such as would tend to oppose the motion. It will be seen from this that the armature will pull back upon the revolving field, or, what is the same thing, the field will pull the armature forward. The armature will tend to run faster and faster at first, but, as it approaches synchronism, the induced e.m. f., and consequently the current, in the armature become less and less, ceasing altogether at synchronism. At synchronism, therefore, the torque is zero, and, consequently, an induction motor never quite reaches synchronism, but the acceleration ceases when the torque is just sufficient to carry the load. When the motor is running light, the speed, however, is very near synchronism, for a very little slip will induce sufficient current to give a torque that will carry the friction load.

Another explanation considering the armature to be quarter phase, is that the revolving field induces in each of the armature circuits an e.m.f. lagging 90° behind the field itself. That is to say, the phase in each armature coil is 90° behind the phase of the field magnetism at that point, and, the armature coils being set

90° apart in space, the phases of their e.m.f.'s are 90° apart. Near synchronism and at normal load, the frequency in the armature is low, say three per cent of the field frequency, so that the reactive component of e.m.f. is small, and the current in the armature circuit is fairly well in phase with the e.m.f. Hence, there are two armature currents, 90° apart, and each 90° behind the flux producing it. At the instant when the flux through one coil of the armature is a maximum, the current is zero, and is a maximum in the other coil, through which the flux is zero. The coil carrying the maximum current, being 90° from the maximum flux, is in the position to exert a maximum torque. This is shown in Fig. 71.

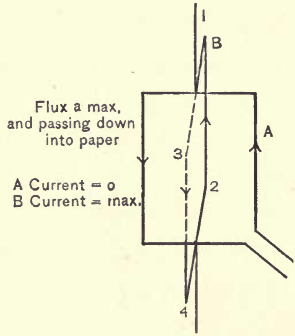

Fig. 71.

The coil, *B*, carrying current, tends to turn into the position, *A*, since the part of the coil from 1 to 2 is urged toward the left, and the part from 3 to 4 is urged toward the right.

A wire, carrying current and located in a magnetic field, is acted upon with a force, across the field, depending upon the strength of the field, length of wire, amount of current and direction of wire as related to direction of field. All of which follows from the definition of unit current, which is: a current flowing in a wire of unit length, bent into a circular arc of unit radius, will act upon a unit pole at the center, with unit force. Action and reaction being equal, the unit pole will act upon the wire with unit force; and it is clear, therefore, that in a uniform magnetic field, each unit length of a straight wire will be acted upon by a force equal, in dynes, to one-tenth the current in amperes times the strength of the magnetic field, the quotient multiplied by the sine of the angle between the wire and the lines of force. Hence, in the case under consideration, there is always a full torque relation between the flux and one or both of the armature circuits. Or, expressed in a more general way, the revolving field induces, in passing by an armature coil, a current within it, which is a maximum when the flux is zero, or, in other words, the current lags 90° behind the flux. Therefore, when the current is a maximum in this coil, the flux that induced the current has passed on 90°, thus being in the most advantageous position for exerting the torque; that is, the position, *B*, Fig. 71. In this particular coil, the torque passed through zero when the current was zero; but if there are other coils, set away in space 90°, 120°, etc., they act in turn, so that a uniform torque is obtained. Thus the flux moves onward at such a rate as to be in proper position to produce torque when the current in the armature coil has reached its maximum value, and induced by the very flux that has revolved onward.

A third explanation of the torque of an induction motor, assuming as before, for convenience, a quarter phase motor, is to consider that one field winding produces in any armature circuit a current, by means of the (imaginary) flux produced by itself alone, not considering the other field winding. The current so produced is thus in a proper torque position, as related to the other field winding, and its (imaginary) flux, not considering the first field winding, since the field coils are 90° apart. This can be illustrated further by considering the instant when the flux from field coil is zero, and armature coil is set in space across the flux as coil *A*, Fig. 71. The current produced by the flux in this armature coil is a maximum, since induced currents by magnetic fields lag 90° behind such fields, and 90° behind zero means maximum.

The other field, 2, is a maximum when field 1 is zero, since they are 90° apart in phase. Hence we have the condition of field 2 being a maximum acting to produce torque on the armature coil in question, which coil is located "edgewise" to field 2, or, what is the same thing, located in a maximum torque position as related to field 2. Thus arises the conception that field 1 produces a current in the armature so placed and related to field 2 as to produce maximum torque, or, in other words, the conception of one field producing the current in the armature for the benefit of the other field for the production of torque.

The actual formulæ and calculation of torque of an induction motor is as follows:

Let ϕ = the maximum value of the revolving flux.
I_1 = armature current in amperes.
p = number of field (and armature) circuits. Thus, 2 for quarter phase, and 3 for three phase.
T = turns in series per circuit of armature and field.
Let L_1 = inductance per circuit of armature in henrys.
L = inductance per circuit of field in henrys.
R_1 = resistance per circuit of armature in ohms.
R = resistance per circuit of field in ohms.

Assume armature and field to have same number of circuits and same turns per circuit. (If different, they can be reduced one to another by ratio of square of turns as far as resistance and inductance are concerned.)

Let n = cycles per second in primary,
and let S = slip of secondary. (At standstill, $S = 1$. At synchronism, $S = 0$.)
Thus the cycles in armature at any slip $S = Sn$.
Let a = the radius of the armature.
Let b = the number of poles of the motor.

Then, owing to the revolving flux, ϕ, the volts created in an armature circuit at any slip, S,

$$E = \frac{2\,\pi TSn\phi}{\sqrt{2}\,10^8}.$$

The current in an armature circuit $= I_1$ and its lag from the induced e.m.f. = the angle α (Fig. 71).

Thus, the electrical power in watts per circuit under these conditions

$$= \frac{2\,\pi TSn\phi I_1 \cos\alpha}{\sqrt{2}\times 10^8}.$$

(This includes all poles if T includes all poles.) But one watt = 10^7 ergs per second.

An erg is a dyne centimeter, and is a measurement of work just as a foot-pound is a measurement of work. Moreover, as so many (33,000) foot-pounds *per minute* equal a horse-power, so a certain number of dyne centimeters, or ergs, per second equal a joule. Thus an erg per second is a fraction of a horse-power. 1 watt = 10^7 ergs per second, and 1 erg per second is, therefore, $\frac{1}{746 \times 10^7}$ horse-power.

Thus, the ergs per second per circuit at a slip S,

$$= \frac{2\,\pi\phi n T I_1 S\, 10^7 \cos\alpha}{\sqrt{2} \times 10^8},$$

or, canceling,

$$= \frac{2\,\pi\phi n T I_1 S \cos\alpha}{10\sqrt{2}}.$$

A certain amount of power is put into the primary, and in the secondary the energy appears in two forms: First, an electric form, which is the power to make the current pass through the wires of the armature, that is I^2R; and second, the mechanical form, which makes the armature rotate. Since both of these are power, they must be derived from the primary.

If there were no drop in speed S (due to the resistance of the armature, the inductance being small at the low frequency), there would appear in the secondary (or armature), in the form of mechanical revolution, all the energy given it from the primary. Because of the loss in the resistance of the armature, only a part appears; that is, $1-S$. Thus, if $S=3\%$, $1-S=97\%$, or 97% of the energy appears in the form of mechanical power, and 3% is lost in the electrical power in the secondary.

Accordingly, the expression

$$\frac{2\,\pi\phi n T I_1 S \cos\alpha}{10\sqrt{2}}$$

equals the ergs per second lost in putting current through the secondary, since the expression $\frac{2\,\pi\phi S n T}{\sqrt{2}\,10^8}$ equals the volts necessary to put the current through; and the factor $I_1 \cos\alpha\, 10^7$, multiplied by this, gives the ergs per second necessary. Enough slip S occurs to produce frequency $S.n$, and thus a voltage to give the current. So the relation between electrical loss in secondary and energy in secondary $=\frac{S}{1-S}$, or energy in secondary $=$ electrical loss $\times \frac{1-S}{S}$.

Thus, the expression for energy of rotation of armature

$$=\frac{2\,\pi\phi n T I_1 S \cos\alpha}{10\sqrt{2}} \cdot \frac{1-S}{S}. \qquad (a.)$$

The peripheral speed of the armature, that is, the linear speed of the conductors, $=2\,\pi\times r\times$ revolutions per second.

The cycles of the primary $=n\times\frac{b}{2}$ where $b=$ number of poles, $n=$ synchronous revolutions per second.

Actual revolutions of armature = synchronous revolutions × $(1-S)$.

Thus,

$$\text{synchronous revolutions} = \frac{\text{actual revolutions}}{1-S}.$$

Substituting in (b),

$$n = \text{actual revolutions} \times b \div (1-S)2,$$

or

$$\text{actual revolutions} = 2n(1-S) \div b.$$

$$\text{Peripheral speed} = 2\pi r \times 2n\frac{(1-S)}{b},$$

or the peripheral speed at 1 cm. radius

$$= \frac{2\pi \times 2n(1-S)}{b}. \qquad (c.)$$

Since energy = ergs = dyne centimeters per sec. = dynes × centimeters per sec., it follows that dynes = ergs ÷ centimeters per second or peripheral speed, shown at (c).

Thus, $\text{Dynes} = (a) \div (c)$

or

$$= \frac{2\pi\phi n T I_1 S 10^7 \cos\alpha\,(1-S)\,b}{\sqrt{2}\,10^8 S\, 2\pi\, 2n(1-S)},$$

or

$$\text{dynes torque} = \frac{\phi}{\sqrt{2}} \cdot \frac{I_1 T}{10} \cdot \frac{p}{2}\, b\cos\alpha. \qquad (d.)$$

But $\dfrac{\phi}{\sqrt{2}}$ = square root of mean square flux,

and $\dfrac{I_1 T p}{10 \times 2}$ = armature magnetomotive force,

and b = number of poles.

Thus, the torque per pole equals the product of effective magnetism with effective armature magnetomotive force. The fact that $\frac{I_1 T}{10} \frac{p}{2}$ equals the magnetomotive force can be shown as follows. In a quarter phase motor, the revolving flux, which has been shown to exist, is a constant at all times. When one phase has its maximum current, the other phase at this instant has in it zero current. Hence the total ampere-turns are those of one phase, or $\frac{2}{2} \times$ one phase, or $\frac{p}{2}$ times one phase.

Now take a three phase motor. Fig. 72 shows the three phase relation.

At all times, as in a quarter phase motor, the revolving field produced is a constant.

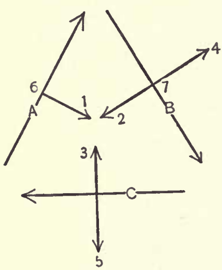

Fig. 72.

Take any instant, for example when the phase, A, is a maximum. By referring to three sine waves 120° apart, Fig. 32, it will be seen that when the current of one phase is a maximum in a certain direction, the currents in the other two phases are opposite in direction, and each one-half of the maximum. Thus, if at any instant A is + and maximum B and C are negative and ½ maximum. Referring to Fig. 72, the three coils are placed 120° apart. That is, the lines connecting the center points, 6,7,8, of the coils, to the center of the figure, are 120° apart. This is the arrangement in an induction motor.

Hence + currents flow as shown by arrows. Thus + currents produce magnetism as shown by arrows

1, 2, and 3 to the right, looking along the arrows. Therefore, if the current is + in A, the magnetism is shown by arrow 1. If − in B, the magnetism is shown by arrow 4. If − in C, by arrow 5.

Combining by the parallelogram of forces the three arrows 1, 4, and 5, the resultant ampere-turns producing magnetism are obtained, since at any instant they are a constant. This combining is shown in Fig. 73, where 0–4, 0–5, and 0–1 show the arrows; 7–4, 8–5, and 6–1 in Fig. 72. From the triangle 0–4–6, Fig. 73, it is seen that 0–6 = ½ of 0–1. Hence the combination of the ampere-turns at the instant chosen is equal to 1½ times the ampere turns of one phase, or equal to $\frac{3}{2}$ × one phase, or $= \frac{p}{2}$ × one phase.

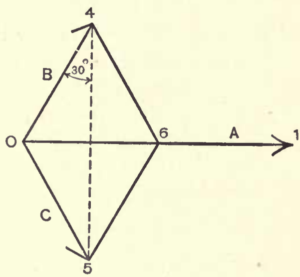

Fig. 73.

Thus it can be shown, for any number of phases, that the resultant ampere-turns of the currents flowing in coils, set apart in space distances corresponding to the phases of the currents, are equal to the number of phases divided by 2, times ampere-turns of one phase. Thus, once more referring to our formula of torque, we see that the factor $\frac{I_1 T}{10} \frac{p}{2}$ means resulting ampere-turns producing magnetism; and hence the conclusion that torque = magnetism per pole, times effective (square root of mean square), armature magnetomotive force. This relation holds true for any motor, direct or alternating.

Referring to Fig. 70, the cosine of the angle $\alpha =$ $\frac{R_1}{\sqrt{R_1^2 + S^2 X_1^2}}$ when $X_1 = 2\ \pi n L_1$, since the angle 12–1–0 is a right angle, and the e.m.f. vector 0–12 is equal to the square root of the sum of the squares of the other two sides; but the side 0–1 $= I_1 R_1$, and the side 1–12 = 0–11 equals $S\ 2\pi n L_1 I_1$ or $S X_1 I_1$. Thus the cosine equals $\frac{0\text{–}1}{0\text{–}12}$ or $\frac{R_1}{\sqrt{R_1^2 + S^2 X_1^2}}$. (*e*.) Also, if the e.m.f. $E =$ the e.m.f. induced through the windings of the primary by the revolving flux, that is, if $e =$ the induced e.m.f., its value in the secondary $= Se$, since the frequency is reduced to Sn in the secondary, by its revolutions. Thus the secondary current in amp = the e.m.f. in volts, divided by the secondary impedance in ohms; or

$$\underset{(\text{amp})}{I_1} = \frac{Se}{\sqrt{R_1^2 + 2\ \pi n S L_1^2}}. \qquad (f.)$$

Also, in volts $$e = \frac{2\ \pi n T \phi}{\sqrt{2}\ 10^8},$$

or $$T\phi = \frac{e\sqrt{2}\ 10^8}{2\ \pi n}. \qquad (g.)$$

Substituting (*e*) (*f*) (*g*) in the formula for torque, namely,

$$\frac{T\phi}{\sqrt{2}} \cdot \frac{I_1}{10} \cdot \frac{p}{2}\, b \cos \alpha,$$

we get torque in dynes at radius of 1 cm.

$$= \frac{Se^2pbR_1 10^8}{4\pi n 10(R_1^2 + \overline{2\pi n S L_1^2})}. \qquad (h.)$$

If expressed in pounds, the formula becomes T (lbs.) at a radius of 1 foot at an armature slip equal to S, equals

$$\frac{Se^2pbR_1 \times 10^8}{4\pi n 10(R_1^2 + \overline{2\pi S n L_1^2})} \times \frac{1}{13563600},$$

or torque in lbs. at 1 foot radius and at the slip equals

$$\frac{Se^2pbR_1}{17.04\, n(R_1^2 + \overline{2\pi S n L_1^2})}$$

where R_1 is expressed in ohms, L_1 in henrys, n in cycles per second, E in volts, and S in per cent slip, and where p = number of circuits, and b = number of poles. This dyne-pound relation is shown as follows:

1 erg = 1 dyne-centimeter;

10^7 ergs per second = 1 watt;

746 watts = 1 horse-power = 550 foot-pounds per second.

Therefore

746×10^7 dynes cm. per second = 550 foot lbs. per second

= $550 \times 12 \times 2.54$ lb. cm. per second

since 2.54 cm. = 1″ and 12″ = 1 foot.

Thus,

$$1 \text{ pound} = \frac{746 \times 10^7}{550 \times 12 \times 2.54} \text{ dynes} = 445{,}000 \text{ dynes.}$$

To express formula (h), using in the numerator, E_0, the e.m.f. applied to the terminals of the motor, instead of e, the e.m.f. due to the flux pulsating in the windings, proceed as follows. From the triangle 0–1–12, Fig. 70, the angle 0–1–12 is a right angle, and thus

$$(0-12)^2 = (12-1)^2 + (0-1)^2,$$

or

$$S^2E^2 = R_1^2I_1^2 + S^2X_1^2I_1^2 \text{ where } X_1 = 2\,\pi n L_1,$$

or

$$E^2 = \frac{R_1^2I_1^2}{S^2} + X_1^2I_1^2. \quad (1.)$$

Also, the applied e.m.f. E_0 = the vector sum (that is an addition properly, considering the phases of the values added) of e and the e.m.f. drop in the primary windings. As of no vital importance in this calculation, omit from consideration the excitation current. Then the secondary current, I_1, appears as the primary current, using up a certain potential due to its passage through the primary winding. Consider all the e.m.f.'s used up in phase with this current I_1. They are, 1st, primary resistance drop I_1R_0, and, 2d, the component $\frac{R_1I_1}{S}$, which, forming one side of a right-angled triangle, in connection with X_1I_1 as the other side, gives the formula (1) above, where the two sides are squared and added, as is done in formula (1).

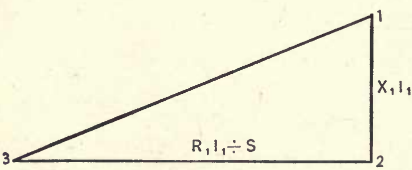

Fig. 74.

This triangle is shown in Fig. 74, the inductive part, X_1I_1, appearing as the vertical side of the triangle, and the non-inductive part, $\frac{R_1I_1}{S}$, appearing as the horizontal side.

In Fig. 74

$$(1-2)^2 + (2-3)^2 = (1-3)^2.$$

Where
$$1-2 = X_1I_1,$$
$$2-3 = \frac{R_1I_1}{S},$$

and
$$(1-3)^2 = E^2 = \frac{R^2{}_1I^2}{S^2} + X_1^2I_1^2,$$

as is expressed in formula (1) above.

Consider next all the e.m.f.'s used up at 90° from this current I_1. They are, 1st, X_0I_1, the primary induction drop ($X_0 = 2\pi nL_0$), and, 2d, X_1I_1, the secondary induction drop. Hence, the total e.m.f. in phase with the current is $\frac{R_1I_1}{S} + I_1R_0$, and the total e.m.f. at right angles to the current is $X_1I_1 + X_0I_1$. Hence the total e.m.f. or E_0 (applied) = the square root of the sum of the squares, just as in Fig. 74 above, or

$$E_0^2 = \left[\frac{R_1I_1}{S} + I_1R_0\right]^2 + [X_1I_1 + X_0I_1]^2,$$

$$E_0^2 = I_1^2\left[\left(\frac{R_1}{S} + R_0\right)^2 + (X_1 + X_0)^2\right],$$

or
$$E_0 = I_1\sqrt{\left[\frac{R_1}{S} + R_0\right]^2 + (X_1 + X_0)^2}.$$

But
$$I_1 = \frac{eS}{\sqrt{R_1^2 + S^2X_1^2}},$$

that is, the secondary e.m.f. eS divided by the secondary impedance

$$\sqrt{R_1^2 + S^2X_1^2}.$$

Hence,

$$E_0 = \frac{eS}{\sqrt{R_1^2 + S^2X_1^2}}\sqrt{\left(\frac{R}{S} + R_0\right)^2 + (X_1 + X_0)^2},$$

or $$E_0 = \frac{e}{\sqrt{R_1^2 + S^2X_1^2}}\sqrt{(R_1 + SR_0)^2 + S^2(X_1 + X_0)^2},$$

or $$e = E_0\frac{\sqrt{R_1^2 + S^2X_1^2}}{\sqrt{(R_1 + SR_0)^2 + S^2(X_1 + X_0)^2}},$$

which establishes the relation between e and E_0. Substituting this value of e in the formula for torque, (h), that is,

$$T = \frac{Se^2pbR_1 10^8}{4\,\pi n 10\,(R_1^2 + 2\,\pi SnL_1^2)},$$

and we get T in dynes at 1 cm.

$$= \frac{SE_0^2pbR_1 10^8}{4\,\pi n\,10\,\{(R_1 + SR_0)^2 + S^2(2\,\pi nL_1 + 2\,\pi nL_0)^2\}},$$

or T in lbs. at 1 foot

$$= \frac{SE_0^2\,pbR_1}{17.04n\,[(R_1 + SR_0)^2 + S^2(2\,\pi L_1 + 2\,\pi nL_0)^2]}.$$

The actual peripheral speed of an induction motor at the radius r,

$$= 2\,\pi r\frac{2\,n(1 - S)}{b}.$$

Thus the peripheral speed at radius 1 cm., in cm. per second

$$= \frac{4\,\pi n\,(1 - S)}{b}.$$

Multiplying the value of torque in dynes at 1 cm. by the speed in cm. per second, gives dyne – cm. per second, or ergs per second. Thus the power in ergs per second

$$= \frac{pR_1E_0^2\,10^8(S)(1 - S)}{10\,[(R_1 + SR_0)^2 + S^2(2\,\pi nL_1 + 2\,\pi nL_0)^2]},$$

or the horse-power = torque in lbs. at 1 foot, times peripheral speed in feet per second ÷ 550 (since 550 foot lbs. per sec. = 1 h. p.).

Therefore

$$\text{H.P.} = \frac{pR_1E_0^2S(1 - S)}{746\,[(R_1 + SR_0)^2 + S^2(2\,\pi nL_1 + 2\,\pi nL_0)^2]}.$$

MAXIMUM TORQUE.

By trial it will be found that the value of S which makes the value of the torque a maximum in the above formula, is

$$\frac{R_1}{\sqrt{R_0^2 + (X_1 + X_0)^2}}.$$

(For mathematical proof, see Steinmetz, Alternating Current Phenomena.)

Substituting this in the formula for torque, in dynes, at 1 cm., above, gives

$$\text{Maximum } T = \frac{E_0^2pb10^8}{8\,\pi n10\,(R_0 + \sqrt{R_0^2 + (X_1 + X_0)^2})},$$

or maximum torque in lbs. at 1 foot

(1) $$= \frac{E_0^2pb}{34.09\,[R_0 + \sqrt{R_02 + (X_1 + X_0)^2}}.$$

MAXIMUM HORSE-POWER.

By trial it will be found that the value of S which makes the value of ergs per second a maximum, in the above formula, is

$$S = \frac{R_1}{R_1 + \sqrt{(R_1 + R_0)^2 + (X^1 + X_0)^2}}.$$

(For mathematical proof, see Steinmetz, Alternating Current Phenomena.)

Substituting this in the formula for power in ergs per second, above, we get maximum ergs per second

$$= \frac{pE_0^2 10^8}{20\left[(R_1 + R_0) + \sqrt{(R_1 + R_0)^2 + (X_1 + X_0)^2}\right]}.$$

Or the maximum output expressed in horse-power

$$(2) \qquad = \frac{pE_0^2}{1492\left[(R_1 + R_0) + \sqrt{R_1 + R_0)^2 + (X_1 + X_0)^2}\right]}.$$

From an inspection of the formula for maximum torque, it will be seen that the factor R_1 does not appear. Hence the conclusion can be drawn that the maximum torque is independent of the secondary resistance. Thus, since the speed of the motor is dependent upon the secondary resistance, a value of R_1 can be chosen so that the maximum torque will occur at starting, thus giving to the motor maximum accelerating power. Since the efficiency of the motor is cut down by this resistance, it is necessary, however, to cut it out on the way up to synchronism, which is ordinarily done by means of switches, either rotating with the armature, which obviates collector rings, or external to the armature, using collector rings. An inspection of the formula for horse-power or maximum horse-power shows the applied e.m.f. in the numerator squared; which indicates that the output of an induction motor is proportional to the square of the applied e.m.f. Thus halving the applied e.m.f. reduces the output of a motor to $\frac{1}{4}$ of its output at full voltage. Hence it is desirable to operate induction motors somewhere near their rated voltage, as well as to design them so that they will have at least 50% overload maximum output to avoid trouble if the voltage should be low.

CALCULATION OF POWER FACTOR, EFFICIENCY, etc.

The output of an induction motor armature equals, as has been shown, $\phi \,.\, TI_1 \,.\, C \,.\, \cos \alpha \,.\, (1-S)$.

When $\phi=$ flux, $TI_1=$ ampere-turns of armature, C $=$ a constant, and $S=$ slip. Thus the output is a product of flux and winding, since I_1 depends directly upon the resistance and inductance of the winding. Hence, the proper design of a certain sized motor necessitates, just as in a transformer, a decision as to the relation between flux and turns, or weight of iron to weight of copper. Having chosen for trial a certain flux, the turns are determined closely, as in a transformer, by the formula

$$E_0 \text{ (per circuit)} = \frac{2\,\pi n \phi T_1}{\sqrt{2} \times 10^8},$$

or $$T_1\text{, the turns per circuit,} = \frac{\sqrt{2} E_0 10^8}{2\,\pi n \phi}.$$

Where $n=$ cycles per second, $\phi=$ flux, $T_1=$ turns.

Having chosen the flux, ϕ, and knowing the proper magnetic density to run in the iron and air gap of the motor, and also the proper current densities in the copper, the physical dimensions of the motor can be determined.

The density to use in the iron and air gap is dependent upon two things:

First, the loss of energy in the iron due to hysteresis, which must be kept down to a proper value to prevent excessive temperatures, since a certain exposed surface, in this case the outside of the motor, can carry away only a certain definite amount of heat for a given temperature rise. At any density, B, the loss due to hysteresis per cubic centimeter per cycle $= KB^{1.6}$, when K is a constant depending upon the quality of the iron used. From this the loss in the iron can be determined. Having chosen the air gap density to use, the excitation current of the motor can be calculated, since, knowing the density in iron and air, the necessary ampere-turns are figured as follows:

The ampere-turns in the iron for a given maximum density, B, are taken, as has been shown, from the regular iron saturation curves. The ampere-turns for air at a given density are covered by the formula: ampere-turns in air per unit (cm.) length $=$ density per sq. cm. $\div$ 1.258 (lengths of gap in induction motors vary from .015″ to .060″). Thus the total ampere-turns $=$ sum of iron and air ampere-turns. But, as has been shown, the total m.m.f. in a polyphase induction motor $=$ ampere-turns per circuit multiplied by the number of circuits $\div$ 2. Hence ampere-turns per circuit $=$ the total calculated as above $\div \frac{p}{2}$.

But the assumed turns are known from the flux-turns, hence the magnetizing current in each circuit is known. This magnetizing current must be kept down to a reasonable figure, so that the power factor of the motor will be proper. 30 to 40% is as high as can be accepted under ordinary conditions. Since the iron loss is known, and since the bearing loss for various-sized motors is known from experimental data, the excitation current per circuit can be calculated. Thus magnetizing current per circuit = ampere-turns per circuit, determined as above ÷ turns (per circuit) = I_{00}.

The energy component of the "running light" current of the motor, namely, flowing at no load, = total watts core plus friction loss ÷ number of circuits, p, and ÷ e per circuit, $= \frac{\text{Core loss}}{pE}$, where E = the e.m.f. produced by the flux pulsating through the windings. Let this be designated by I_{11}. This component of current I_{11} is in phase with e, since it represents energy; and the component I_{00} is 90° displaced from E, since it represents magnetizing amperes and no energy.

In the secondary (referring now to Fig. 70), the e.m.f., E, is represented by the line 0–3′, and the current by the line 0–2. Hence the energy component of the current is represented by the line 0–14, namely, the projection of 0–2 on 0–3. The inductive, e or 90° out of phase, component of the current is the line 2–14.

But $$0\text{–}2 = \frac{Se}{\sqrt{R_1^2 + S^2X_1^2}},$$

and $$0\text{–}14 = 0\text{–}2 \text{ times } \cos \alpha,$$

and $$\operatorname{Cos} \alpha = \frac{0\text{–}1}{0\text{–}12} = \frac{R_1}{\sqrt{R_1^2 + S^2X_1^2}}.$$

Hence 0–14 or the energy component of the secondary current

$$= \frac{Se}{\sqrt{R_1^2 + S^2X_1^2}} \times \frac{R_1}{\sqrt{R_1^2 + S^2X_1^2}} = \frac{SeR_1}{R_1^2 + S^2X_1^2}.$$

The inductive component

$$2\text{–}14 = \sqrt{\overline{0\text{–}2}^2 - \overline{0\text{–}14}^2},$$

since the square of the long side of a right-angled triangle equals the sum of the squares of the other two sides.

Hence $$2\text{–}14 = \sqrt{\frac{S^2e^2}{R_1^2 + S^2X_1^2} - \frac{S^2e^2R_1^2}{(R_1^2 + S^2X_1^2)^2}},$$

or $$2\text{–}14 = \frac{S^2eX_1}{R_1^2 + S^2X_1^2}.$$

We thus have four components of current in the induction motor, all being combined to give as a resultant the primary current. Tabulated, they are as follows:

90° Out of Phase with "E."		In Phase with "E."
		Energy current of core and friction loss.
I_{00} = magnetizing current		or $I_{11} = \frac{\text{Core loss}}{pe}$
or 2–14 (see Fig. 70) $= \frac{S^2eX_1}{R_1^2 + S^2X_1^2}$	and	0–14 (see Fig. 70) $= \frac{SeR_1}{R_1^2 + S^2X^2}.$

Thus, to combine the above, it is only necessary to get the square root of the sum of the squares of the "in phase with e" components with the "out of phase with e" components, or primary current squared

$$= I_0^2 = \left(I_{00} + \frac{S^2 e X_1}{R_1^2 + S^2 X_1^2}\right)^2 + \left(I_{11} + \left(\frac{S e R_1}{R_1^2 + S^2 X^2}\right)\right)^2. \quad (1.)$$

For small values of S up to .03, and thus in an ordinary induction motor going to full load slip, we can neglect terms containing S squared.

Hence, approximately, the primary current squared

$$I_0^2 = I_{00}^2 + \left(I_{11} + \frac{Se}{R_1}\right)^2 \text{ or, } I_0 = \sqrt{I_{00}^2 + \left(I_{11} + \frac{Se}{R_1}\right)^2}. \quad (2.)$$

But, as has been previously shown (page 152),

$$e = E_0 \frac{\sqrt{R_1^2 + S^2 X_1^2}}{\sqrt{(R_1 + SR_0)^2 + S^2(X_1 + X_0)^2}}.$$

Hence, neglecting terms of X^2, we obtain,

$$e = \frac{E_0 R_1}{\sqrt{R_1^2 + 2R_1 S R_0}}. \quad (3.)$$

Hence (3) can be put in place of e in (1) or in (2) above, if desired, though for S up to .03 it is unnecessary for a first approximation. Therefore, in our preliminary design, we have available for calculation of the actual results of efficiency, power factor, output, maximum output, torque, maximum torque, and slip, all the factors necessary. For, having chosen the flux and turns, the densities in iron and copper, the loss in same, the radiating surface, and thus the rise in temperature, are available: so also is I_{00} the magnetizing current. The proper current density to use in the copper is known. This gives the cross section of copper, and the dimensions of the machine give the length and total amount of copper.

I_{11} = the power component of excitation current.
R_0 = the primary resistance.
R_1 = the secondary resistance.
X_0 = the primary reactance.
X_1 = the secondary reactance.

$X_0 + X_1$ are calculated, as in a transformer, by the formula

$$X = 2\,\pi n L = \frac{2\,\pi n \phi_1 T}{I 10^8},$$

where ϕ_1 = the flux that surrounds the wire in the winding itself, not contributing to the production of the e.m.f. in the windings e; in other words, ϕ_1, above, is the lost flux of self-induction. Usually the calculation of $X_0 + X_1$ is difficult, and is made by using constants obtained by testing windings in various-shaped slots, etc. Thus, a design requires considerable experimental data for proper calculation of $X_0 + X_1$.

In addition, knowing the above values of the other quantities for any assumed value of S, the maximum output can be calculated by the formula:

$$\text{Maximum h.p.} = \frac{pE_0^2}{1492\,[(R_1 + R_0) + \sqrt{(R_1 + R_0)^2 + (X_1 + X_0)^2}}.$$

The h.p. at any slip by the formula:

$$\text{h.p.} = \frac{pR_1E_0^2S(1 - S)}{746\,(R_1 + SR_0)^2 + S^2(2\,\pi n L_1 + 2\,\pi n L_0)^2}. \qquad (a.)$$

The current input by the formula, sensibly accurate for values of S up to .03,

$$I_0 = \sqrt{I_{00}^2 + \left(I_{11} + \frac{Se}{R_1}\right)^2}; \qquad (b.)$$

and hence the apparent h.p. input by the formula:

$$\text{apparent h.p. input} = \frac{I_0 E_0 p}{746}. \qquad (c.)$$

This does not mean the real h.p. input, since this current, as shown by formula (b), is not in phase with E_0, and real energy is only the product of e.m.f.'s and currents in phase. The ratio of the real output to the apparent output is called "apparent efficiency," and is equal to

$$\frac{\text{real output}}{\text{apparent input}} = \frac{(a)}{(c)}.$$

The secondary current at any slip S equals

$$I_1 = \frac{Se}{\sqrt{R_1^2 + X_1^2}}. \qquad (d.)$$

The real input = the real output (formula (a)), $+ \dfrac{I_0^2 R_0 p}{746} + \dfrac{I_1^2 R_1 p}{746} +$ total core and friction losses in horse-power. (e.)

Hence the real output ÷ real input = efficiency.

Also the real input ÷ apparent input = the power factor.

Thus, for any assumed ratio between flux and turns, we can calculate efficiency, apparent efficiency, power factor, output, torque, input current, magnetizing current for various values of the slip S. If the

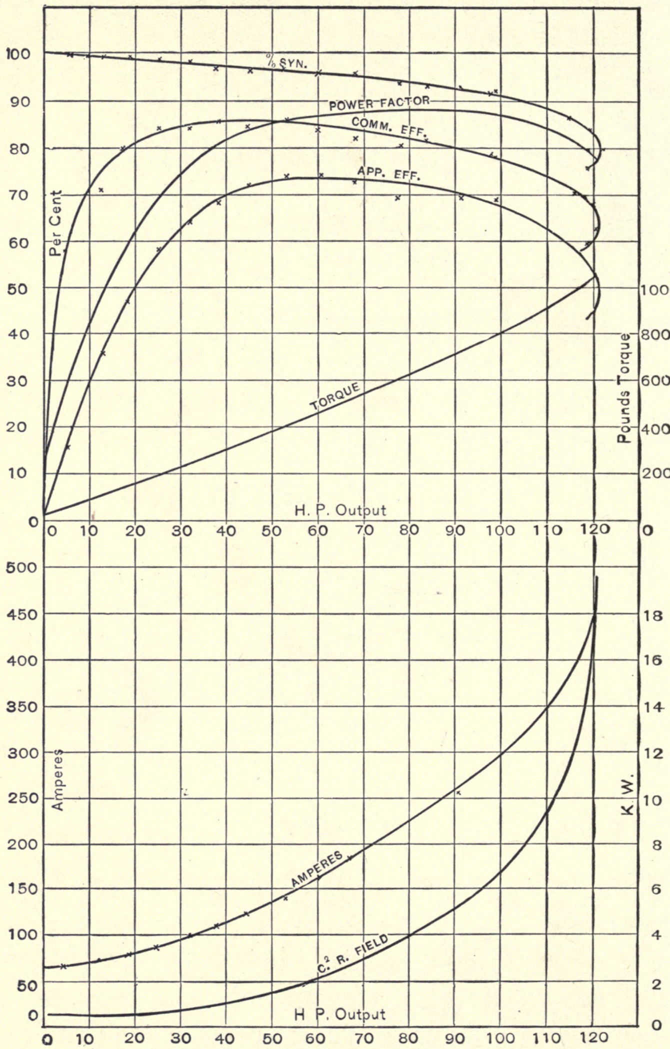

Fig. 75. 10 Pole, 50 H.P., 720 Revolutions, 220 Volt Motor.

ratio of flux and turns does not give the desired values of the above, a change in their relative values can be made until the desired characteristics are obtained. Thus all the constants of an induction motor can be predetermined.

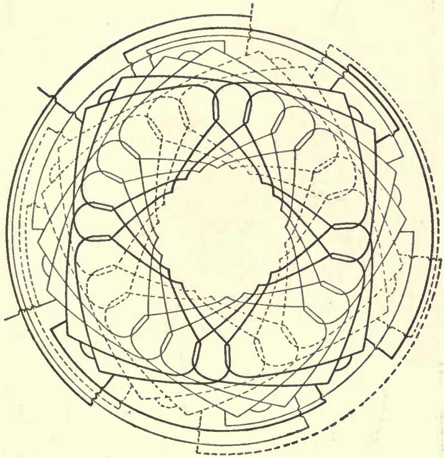

Fig. 76.

The curve of a (50) h.p. motor made as shown is given in Fig. 75, and a diagram of the electrical circuits in Fig. 76.

INDUCTION MOTORS ON SINGLE PHASE CIRCUITS.

If given a start, a polyphase induction motor on a single phase circuit will come up to speed, and, at that point will take load with characteristics similar to the motor on a polyphase circuit. This action of single phase motors on polyphase circuits can be explained as follows:

Refer to Fig. 77. The curve on AB represents the single phase flux curve having a positive maximum at

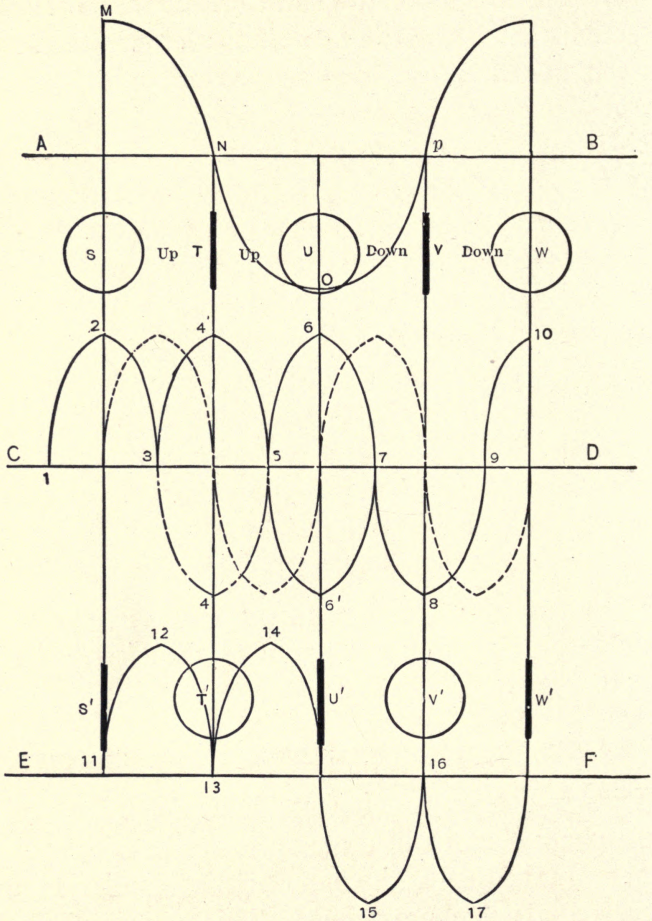

Fig. 77.

M, a zero value at N, and a negative maximum at O, etc.

Consider an armature coil located flatwise in relation to the flux at *S*, when the flux is a positive maximum at *m*. Let the armature turn, by any means, at synchronism. Then when the flux becomes 0, at *n*, the coil has taken an edgewise position, at *T*. When flux is 0, the coil is at *U*, and so on.

The e.m.f. curve of this coil is drawn dotted on the line *CD*, being 0 in value when the flux is at the point *M*, *N*, *O*, *P*, etc. This is evident, since at *S* the coil is carrying maximum flux, the rate of change is zero, and the e.m.f. zero. At *n*, the coil being edgewise at *T*, the e.m.f. would be a maximum if it were flatwise to the flux, but it is now edgewise to the flux, so that the maximum rate of change occurring in the flux at *n* has no effect on the coil at *T*; hence here again the e.m.f. is zero. At position, *U*, the coil is holding maximum flux, and the e.m.f. is again zero; and finally at *V*, the coil being again edgewise, the e.m.f. is zero. Thus the e.m.f. is double the frequency of the main flux and source of supply. Since the frequency is high, and since the armature magnetic circuit is the same as the field magnetic circuit, the current in the armature coil referred to lags practically 90°. This current, therefore, is shown on the line *CD* by the curve 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. In the reaction on the primary, however, the current must be regarded in respect to the direction which it magnetizes. Following the curve 1, 2, 3, 4, 5, etc., and the positions of the coil *S*, *T*, *U*, *V*, etc., note that at *S* the magnetism in the coil is in one direction (say up), and at *U* in the other direction (say down), (since the coil has turned over). Similarly, *U* and *W* bear a similar reversed relation. Hence, the point 4 must be reversed to 4′, to indicate a similar direction of magnetism to the point 2. So also 6 must be reversed to 6′, to correspond to the point 8. Thus, as regards the action of this armature coil upon the primary, the current curve can be considered to be 1, 2, 3, 4′, 5, 6′, 7, 8.

Considering this curve, 1, 2, 3, 4′, 5, 6′, etc., note that at *M* the coil carries maximum current and is in a maximum mutual inductive position to the primary, as indicated at *S*. At *N*, the coil is edgewise, as shown at *T*, and has thus no mutual inductive relation to the primary. At o, the coil shown at *U* is again in a maximum mutual inductive position to the primary. Thus the coil as related to the primary may be considered to have normal frequency, since its maximums coincide as well as its zeros. Moreover, and most important, note that at the points *T* and *V*, when the armature coil magnetizes at right angles to the primary (being then edgewise to the primary, as shown), the current is a maximum, while the primary current is zero. In other words, we have the condition that in a uniform magnetic circuit there are two equal magnetizing forces acting at right angles to each other, 90° apart in time phase. This condition, as has been shown, produces a rotating field; and hence, under such conditions, although a single phase current is applied in the primary, a rotating field is produced, the right angle magnetism being created, as shown, by the armature coil being carried into a 90° position, and the phase of its current in time being also 90° from the primary current. Since the armature winding can be reduced to two coils at right angles to each other, the next coil to be considered as to action on primary, and toward producing any effect on the flux, is shown in its various positions on the line *EF*. Since this coil is displaced 90° from the other already considered, the curves of current in it can be plotted as shown on the line *EF*, 90° displaced from the equivalent curve, 1, 2, 3, 4′, 5, 6′, 7, 8, 9, 10, on *CD*, as shown. Considering now the curve as shown on *EF*, note that at the point *n* (line *AB*, curve), the current in *EF* at 13 is zero; so also at the point *p* on *AB* the current at 16 on *EF* is zero. Hence there is no rotating field effect which would require, 1st, a maximum current on *EF*, when 0 current occurs on *AB*, and, 2d, the coils at the points 13 and 16 to be edgewise, which does not occur. Next as to mutual inductive effect on the primary. At the point 13, when the coil is in an inductive position as related to the primary, the current is zero, and hence the current representing it in the primary is zero. So also at 16. The current in the coil whose current curve is shown on *EF*, acts in conjunction with the current shown on *CD* to produce, as related to the primary, a full frequency armature reaction, having its maximum in a direction at right angles to the primary coil maximum, and occurring at a time 90° lagging in phase, thus producing a true rotating field. We thus from the coil whose curves of current e.m.f., etc., are shown on the line *CD*, have produced a revolving field just as in any polyphase induction motor, as long as the armature rotates at synchronism. As soon as the armature drops below synchronism, the revolving field commences to lose its constancy, since the coil shown on *CD* does not move exactly into a 90° position, as shown at *T* and *V*, but a little behind it. Hence at the points, *T* and *V*, the component is at a slight angle α, and the two right angle components = ampere-turns $\times \cos \alpha$. As the drop in speed becomes more and more, this proper 90° component becomes less and less, until finally, when the armature is at a standstill, the angle $\alpha = 90°$, or $\cos \alpha = 0$, or there is no right angle component. Hence there is no revolving field and no torque. As the armature slows down from synchronism, its conductors are cut by the revolving field at a frequency $= Sn$, when $S =$ slip and $n =$ normal external frequency, and hence torque is produced by the reaction of these slip currents upon the revolving field, as has been explained for polyphase induction motors. Thus, in the armature of the induction motor are superposed upon the double frequency currents shown in Fig. 77 the low frequency armature currents due to the revolving field. Also, since at standstill there is no right angle component, $\cos \alpha = 0$, there is no revolving field and no starting torque.

Hence single phase induction motors need some starting devices. Small motors often are started by hand, larger ones by throwing out of phase the current in a second field circuit set in space off from the regular winding. This throwing out of phase can be accomplished by various means. One way is to shunt around the second field circuit a non-inductive resistance. The current in the winding is then out of phase with the current in the shunt, as shown in Figs. 41 and 42, while the current in the winding is out of phase with the current in the main coil, and hence an approximate revolving field is produced, and the motor starts. The revolving field is not constant, since by such means the 90° difference of phase which would be necessary for a true revolving field cannot be obtained. However, enough phase displacement may be produced to give the motor a start, after which the revolutions take care of the production of the revolving field and polyphase characteristics.

A better method is to shunt the second circuit with a condenser, which throws the current in the winding more out of phase with the currents in the main winding than by shunting with resistance. This has the advantage of improving the power factor, and gives to the condenser a good wave shape, which exists in the induction motor in the extra circuit, thus avoiding troubles due to external harmonics.

TESTING OF INDUCTION MOTORS.

In order to obtain a knowledge of the properties of an induction motor and its suitability for ordinary service, it is necessary to measure the following quantities:

1st, The temperature of its various parts, after a run at full load until temperatures are constant, and after a run at overload for a certain time, starting at normal full load temperatures;

2d, Core loss;

3d, I^2R of windings;

4th, The friction losses;

5th, The excitation and magnetizing currents;

6th, Power factor at various loads;

7th, Real efficiency at various loads;

8th, Apparent efficiency and currents at various loads;

9th, Maximum output;

10th, Starting torque and current at starting;

11th, Torque from rest to synchronism;

12th, Drop in speed at various loads.

First. To ascertain the temperature, it is necessary to load the motor carefully, taking at the end of the run the temperatures of the various parts, such as laminations, field and armature, windings of field and armature, frame, spider and bearings.

Second. To ascertain the core loss, run the motor free for a time, until the bearings attain a normal condition of smoothness, and then read the watts input at various voltages. Plot the results in a curve, as in Fig. 78.

The line *AC* will be obtained. At the point, *A*, the reading of the wattmeters shows the total energy in watts to run the motor, namely, core loss + I^2R + friction. As the voltage becomes lower and lower, the core loss becomes less and less. At the point, *D*, the voltage cannot be lowered further, without reaching the maximum output for that voltage. Extend the curve to the point, *C*, which can always be done accurately, since *D* and *C* are not far apart. Then the loss at $C =$ friction + I^2R. Since the motor is running light, the $I_1^2R_1$ of the rotating part is negligible. Hence I_0^2R of the field can be subtracted, leaving the friction alone. Draw the line, *CE*, parallel to *FB*, and then the distance $AG =$ core loss + I^2R, and hence subtracting I^2R gives the core loss of the motor.

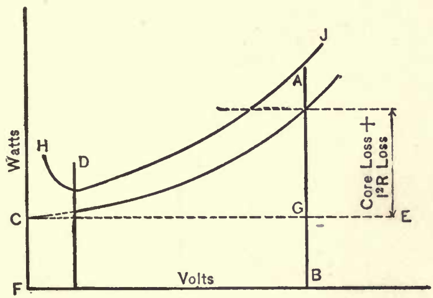

Fig. 78.

The connections of the wattmeter for a single phase motor are as explained in transformer testing. The connections of a quarter phase motor are practically the same as explained, except that a wattmeter is required for each circuit. The connections for a three phase motor are as shown in Fig. 79: the current coil, *A*, of wattmeter, *AB*, being in line 1, its potential coil between 1 and 3, and the current coil of wattmeter, *A'B'*, being in line 2, and its potential coil between 2 and 3. The sum of the two wattmeter read-

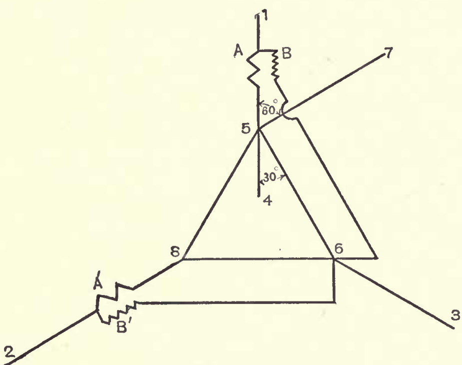

Fig. 79.

ings gives the total watts. When the lag of the current in the line becomes 60°, one of the wattmeters reads 0. This can be seen by referring to Fig. 79. The line, 4–5, represents the phase of the current in one line, which flows through the current coil of the wattmeter, *AB*. This current makes with the line 5–6 (which represents the phase of the voltage, and which gives the magnetism to the voltmeter coil of the wattmeter) an angle of 150°. That is, the angle 1–5–6 = 150°. If the line, 1–5, takes the position, 7–5, namely, 60° lag, the angle it makes with 5–6 = 150 — 60 = 90°. Thus the wattmeter at this point reads 0, since *E.I.* cos 90° = 0, or, physically, the two magnetisms within the wattmeter, one for the current coil and one for the potential coil under the 90° lag condition, produce no torque, and thus no deflection of the wattmeter needle. Beyond a lag of 60°, the wattmeter readings should be subtracted. If desired, the excitation curve of watts for either a three phase or a quarter phase motor can be taken single phase. For as has been shown under the subject of operating polyphase motors on single phase circuits, a normal revolving flux is produced at synchronous rotation; and since, when running light, an almost exactly synchronous rotation exists, the method becomes acceptable and often convenient, particularly as only one wattmeter is necessary.

Third. To ascertain the I^2R of the windings.

The primary resistance can be measured directly, which gives $I_0^2R_0$. To ascertain the secondary resistance, which after a motor is connected up cannot be measured directly, and never can be properly ascertained in a "squirrel cage" winding (consisting of a single set of bars set through the armature slots, and connected together at both ends with copper rings), proceed as follows:

Connect the wattmeter as has been shown in Fig. 79 for core loss, but have the motor standing still and the armature blocked mechanically by some means, so it cannot turn. Take a curve of power input and current for various voltages. A result will be obtained like Fig. 80.

Hence, for any current, the total power used up in primary and secondary is available. This power in a motor as usually designed is principally I^2R of primary and secondary; so that if from its value be subtracted the $I_0^2R_0$ of the primary, which is known by measurement, the remainder is $I_1^2R_1$ of secondary. Hence, $I_0^2R_0$ and $I_1^2R_1$ are determined.

Fourth. To ascertain the friction losses, lay off on the core loss curve the line *CE*, Fig. 78. It is equal to *GB*.

Fifth. To ascertain the excitation and magnetizing current, run the motor light at different voltages, just as was done to obtain core loss. The current curve, *HJ*, Fig. 78, at various voltages, is then obtained. At normal voltage, let the current $= I_{00}$ and the core loss $= K$. Hence, the core loss per circuit $= \frac{K}{p}$ when p $=$ number of circuits. Hence, the energy current per circuit $I_{11} = \frac{K}{pe}$ when $e =$ volts per circuit, as has previously been used. Since the energy component of current is in phase with e, and the purely magnetizing component, I_{111}, is at right angles (90° lagging) with e, the two combine to produce the excitation current, I_{00}. Thus, $I_{00}^2 = I_{11}^2 + I_{111}^2$, from which can be obtained the magnetizing current proper.

Sixth and Seventh. To obtain the power factor and efficiency at various loads.

At any load in horse-power, obtain S, the slip, from the formula which has already been deduced, namely,

$$\text{h.p.} = \frac{PR_1E_0^2S\,(1 - S)}{746\,[(R_1 + SR_0)^2 + S^2(2\,\pi nL_1 + 2\pi\,nL_0)^2]}.$$

In this formula $(2\pi\,nL_1 + 2\pi\,nL_0) =$ the sum of the inductances of primary and secondary.

When taking impedance, results of which are shown in Fig. 80, both watts and amperes are read. Thus, at any current, as read and recorded on curve, Fig. 80, the current input is known and the e.m.f. to produce it. At the point chosen, the current equals

$$\frac{\text{volts}}{\sqrt{(R_0 + R_1)^2 + (X_0 + X_1)^2}}.$$

In this formula, current and e.m.f. are read, R_0 is measured, R_1 is calculated as follows: Section third, page 173, gives a method of getting $I_1^2R_1$ of the secondary, or

$$I_1^2R_1 = a \text{ (as found).} \qquad (1)$$

We have also on page 159, equation (3), a formula for e, or

$$e = \frac{E_0R_1}{\sqrt{R_1^2 + R_1SR_0}}. \qquad (2)$$

A third equation can be assumed correct down to perhaps three-quarters of the break-down point,

$$I_1 = \frac{Se}{R_1}. \qquad (3)$$

From these three equations, R_1 can be obtained. Thus, the procedure on a given induction motor is as follows: At a given desired primary current input, measure the drop in speed, S. Knowing this primary current input from the static impedance reading with watt-meter, the $I_1^2R_1$ can be obtained.

In formula (2) above, insert this same value of S, and the resistance of the primary as measured, and the applied voltage E_0 which of course is known.

In formula (3) above, insert this same S. Substitute in equation (2) the value of e found from (3), and then solve for R_1 using equations (1) and (2) after the substitution is made. The result is,

$$R_1 = \frac{S^2E_0^2 - aSR_0}{a}.$$

Hence, in the formula for horse-power, all terms are known except S, which may then be calculated. Next, knowing S for a given horse-power, insert it in the approximate formula for the primary current,

$$I_0 = \sqrt{I_{00}^2 + \left(I_{11} + \frac{Se}{R_1}\right)^2},$$

where I_0 = primary current.

I_{00} = exciting current.

I_{11} = energy component of exciting current.

R_1 = secondary resistance.

$$S = \text{slip and } E = \frac{E_0R_1}{\sqrt{R_1^2 + 2\,R_1SR_0}}.$$

Knowing the current input I_0 and applied e.m.f., E_0, the apparent input $= \frac{I_0 E_0 p}{746}$ (when $p =$ no. of circuits).

The real input = real output, shown above, plus

$$\left(\frac{I_0^2 R_0 p}{746} + \frac{I_1^2 R_1 P}{746} + \text{total core and friction loss in horse-power.}\right)$$

The items in brackets being the sum of all the losses, copper, iron and friction. The power factor equals the real input ÷ the apparent input, or

$$\frac{E_0 I}{E_0 I_0} = \frac{I}{I_0},$$

or the energy component of current, I, divided by the total current, I_0.

The efficiency $= \frac{\text{real output}}{\text{real input}}$ as shown above.

Eighth. The apparent efficiency and currents at various loads are obtained as just shown, the apparent efficiency being equal to the real output divided by apparent input.

The currents, I_0, at various loads, are obtained as just shown.

Ninth. The maximum output is obtained from the formula deduced previously, maximum output

$$= \frac{p E_0^2}{1492\left[(R_1 + R_0) + \sqrt{(R_1 + R_0)^2 + (X_1 + X_0)^2}\right]},$$

all the terms of which are deduced as just shown from the excitation and impedance curves shown in Figs. 78 and 80.

Tenth. The starting torque and current at starting are deduced by substitution in the starting torque formula previously deduced, the terms of which are obtained from the excitation and impedance curves, Figs. 78 and 80.

The starting current can be read directly from the impedance curve shown in Fig. 80.

Eleventh. The torque from rest to synchronism can be calculated by similar substitutions in the formula for torque previously deduced.

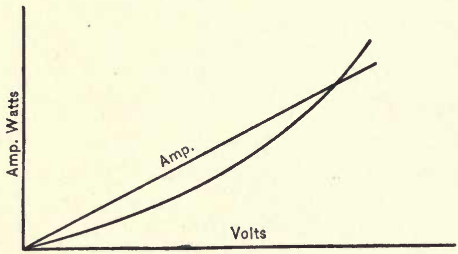

Fig. 80.

Twelfth. The drop in speed at various loads is calculated as shown in sections 6th and 7th.

Thus, from simply an "excitation" curve, Fig. 78, obtained with the motor running light, and from an "impedance" curve, Fig. 80, obtained with the motor standing still, all the actual operating factors of the motor can be figured. This is the most reliable method to follow in testing induction motors. Actual readings of input and output, by Prony brake, can be taken, but any error of wattmeter, an instrument which not particularly accurate under varying power factors, enters directly into the results. Moreover, commercial wattmeters are not built large enough to cover the wide range of motors now built, and the use of multiple current and potential transformers with wattmeters renders their readings practically worthless. Hence, the method of reading the losses directly, giving far greater accuracy in ultimate results of efficiency, power factor, etc., and far greater range of wattmeter application, is much to be preferred.

THE REPULSION MOTOR.

There is another alternating motor coming into prominence, suitable for single phase alternating circuits, and usually known as the repulsion motor. Its general construction consists of an ordinary induction motor field with a direct current armature within, having its brushes shifted forward and the external circuit from the armature closed through a negligible resistance; or, in other words, an armature on short circuit, with its brushes shifted forward. Fig. 81 shows the general arrangement.

The primary winding *A* is similar to an induction motor winding, except that it is single phase. An ordinary three phase winding may be used, with two phases in series, or one phase of a quarter phase winding. Within this field is an ordinary direct current armature, *B*, with a commutator and brushes, 3–4. The + and — brushes are short-circuited through the wire, 3–4, and shifted forward from the line of magnetism, 6–5, by the angle α (about 15°). If at 1–2 a single phase alternating voltage be applied, the motor will start with vigorous torque, and will have speed and torque characteristics similar to that of an ordinary railway motor; that is, on light loads the speed will be high and the torque low, and on heavy loads the speed

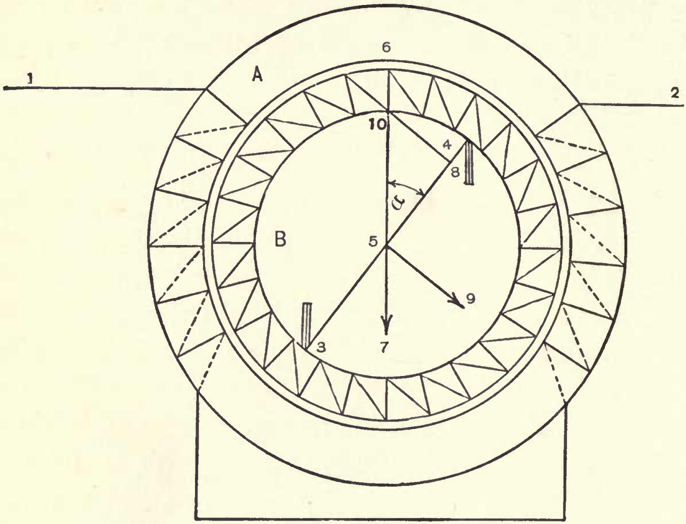

Fig. 81.

low and the torque high; a condition favorable for railway work requiring high starting torque and natural low speed at starting. The cause of the torque in this motor is as follows: When the alternating voltage is applied at 1 and 2, Fig. 81, an alternating flux is produced, which passes through the short-circuited armature, *B*, and induces in it a secondary current, I_1. This current necessarily must pass only between the brushes and through the external short circuit; hence it has in space the direction 3–5–4. Therefore, the secondary current, I_1, would have, if there were no brushes, a value in the normal direction of $I_1 \cos \alpha$, where α = the angle 6–5–4. The flux in the primary is the resultant flux of all the ampere-turns; that is, the resultant of the primary current, I_0, and the component of the secondary current, I_1, acting in a similar direction. This component is $I_1 \cos \alpha$. The back e.m.f. from this flux so produced, added to the drop due to the primary current, I_0, passing through the primary impedance, Z_0, the phase of the various values being properly considered, equals the applied e.m.f., E_0. Thus, with phases properly considered, $(I_0 - I_1 \cos \alpha)$ times turns ÷ total reluctance = flux ϕ producing back e.m.f., E^1, in primary. This is similar to an ordinary transformer or induction motor, where the production of flux, or magnetizing current, I_{00}, is the resultant of I_1 and I_0, the relative phases being properly considered. Hence,

$$E_0 = \frac{4.44 \, \phi n N}{10^8} + I_0 Z_0$$

when ϕ = flux as described, N = turns in series in primary, n = cycles per second, Z_0 = primary impedance. In the secondary or revolving part, the armature current, and hence the armature magnetism, can be only in the direction, 3–4, since, whatever position the armature may take, the brushes remain stationary in space, and thus the direction of the current in space remains 3–4.

To produce alternating magnetism in the direction to give an e.m.f. between brushes, due to the cycles of this magnetism, namely, in the direction, 3–4, the secondary current acts, as a whole, $= I_1$, since it is constrained by the brushes to do this. The primary current, I_0, tends to produce magnetism in the direction, 6–5. Hence, to produce magnetism in the direction, 5–4, only the component, 5–8, of the primary is effective. Thus, the primary effect for producing magnetism in the direction $5–4 = I_0 \cos \alpha$. Hence, 1st, the total ampere-turns producing in the secondary a flux to give an e.m.f. due to its cycles $= I_1 - I_0 \cos \alpha$. The subtraction is subject to the relative time phases of I_0 and I_1; that is, they cannot be subtracted as indicated, but must be subtracted vectorially. Thus, the flux to produce this e.m.f. between brushes $= (I_1 - I_0 \cos \alpha)$ times turns $\div$ reluctance, and hence the e.m.f. due to this flux is

$$E_1 = \frac{4.44\, \phi_1 N n}{10^8}.$$

2d, There is the e.m.f. used up by the passage of the secondary current I_1 through the secondary impedance Z_1. This e.m.f. $= E_{11} = I_1 Z_1$, the usual product of current and impedance.

3d, is the e.m.f. *due to rotation* of the armature in the flux in a direction at right angles to the line of brushes, 3–4. As has been explained and illustrated in Fig. 71, a d.c., e.m.f. is generated in an armature in a direction at right angles to the direction of the flux passing through the armature. Hence, if we can show a flux passing through the repulsion motor armature in a direction, 5–9, at right angles to the direction of brushes, 3–4, there will be generated at right angles to the flux, and hence in the direction of brushes, 3–4, an e.m.f. exactly in phase with the flux. Thus, if this flux be alternating, as in the repulsion motor, the e.m.f. will be alternating, and the frequency of the e.m.f. will be the frequency of the flux, and the amplitude of the e.m.f. so created will depend upon the speed of the armature at the time. This follows naturally, for, if the flux were constant, the e.m.f. would depend upon the speed alone. At a constant speed, the e.m.f. would be a constant, if the flux were constant. If this flux varies, the e.m.f. varies exactly with it. If the form of the flux variation is a sine curve, the e.m.f. at any speed is also a sine curve. Hence, the e.m.f. due to rotation in a repulsion motor is exactly in phase with the flux producing it, instead of lagging 90°, as is the case of an ordinary back e.m.f. due to flux pulsation. At rest, this back e.m.f. of rotation is zero, rising to a greater value as the speed increases. Let us consider all the amperes that go to produce flux in the direction, 5–9. The secondary current, being forced to flow between 3 and 4 by the commutator and brushes, can have no influence whatever in producing flux in the direction, 5–9, at right angles to itself. This leaves only the primary current, I_0. This produces flux in the direction, 10–5, as has been shown. Let 10–5 equal the value of primary ampere-turns. It may be resolved into two components, 10–4 and 5–4, at right angles to each other, 10–5 being equal to the square root of the sum of the squares of the other two sides, 10–4 and 5–4. The ampere-turns represented by the line, 5–4, have no influence in producing flux in the direction 10–4. This leaves the component of primary ampere-turns, 10–4, as alone available for the production of flux at right angles to the direction of the secondary current, which flux, therefore, is the one in conjunction with the secondary current I_1, which produces torque. But, 10–4 = 10–5 times the sine of the angle, 10–5–4, or 10–4 = I_0 sine α. Thus I_0 sine α times turns ÷ reluctance of the circuit through which the flux passes, gives the value of the flux in this direction. This reluctance is uniform in all directions, as in an induction motor. Let this equal ϕ_{11}. Then e.m.f. of rotation

$$E_{111} = \frac{4.44\,\phi_{11}Nn}{10^8} \times \frac{R}{R^1},$$

where ϕ_{11} = the flux as just described, n = normal frequency of the circuit, $\frac{R}{R_1}$ = ratio of rotation of armature to rotation producing normal e.m.f. for the existing flux, n = number of turns in series.

Thus, we have in the secondary the three e.m.f.'s, E_1, E_{11}, E_{111}, acting together in the armature circuit, and when added, proper consideration being given to the relative phases of each to the other, the vector sum must equal zero, since the brushes are short-circuited by an external wire of zero resistance. Another conception is that the e.m.f. of rotation, E_{111}, minus the back e.m.f. of flux pulsation, E_1, gives a resulting e.m.f., which, acting through the impedance of the secondary circuit, is entirely consumed.

Thus, the equation of the secondary is $E_1 + E_{11} + E_{111} = 0$, the various E's being computed as has been shown. Thus the fundamental equations of the repulsion motor are:

(1) $$E_0 = \frac{4.44\,\phi n N}{10^8} + I_0 Z_0,$$

(2) $$0 = \frac{4.44\,\phi_1 n N}{10^8} + \frac{4.44\,\phi_{11} n_1 N}{10^8} + I_1 Z_1,$$

from which all the values of horse-power, torque, etc., may be computed as has been illustrated in the case of

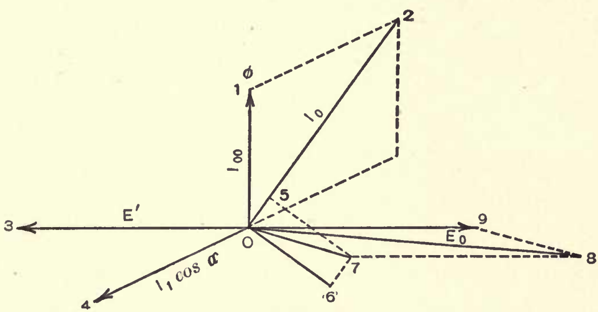

Fig. 82.

the induction motor. The time phases of these various values are shown in the following figures.

First, in Fig. 82 is shown for the primary the phase relations between $I_0 I_{00}$ and the component of secondary current acting in the direction of the primary current, where $I_0 =$ the primary current, $I_{00} =$ the resultant current of I_0 and $I_1 \cos \alpha$, and is thus the current producing the flux ϕ, producing the back e.m.f. in primary $= \frac{4.44\ \phi n N}{10^8}$.

In Fig. 82, o–4 $= I_1 \cos \alpha$ and o–2 $= I_0$, which combines with $I_1 \cos \alpha$ to produce I_{00}, which, in turn, produces the flux ϕ, the angle of hysteretic advance not being here considered. The line, o–3, 90° behind the

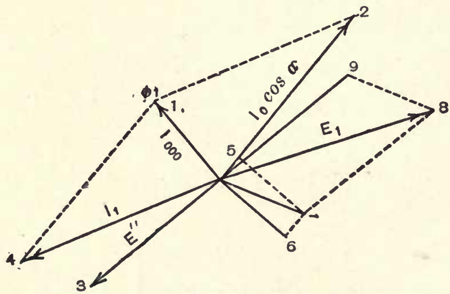

Fig. 83.

flux, ϕ, therefore represents the induced e.m.f. E' in the primary; o–5 = the e.m.f. consumed in the primary due to resistance of primary, and o–6 = e.m.f. consumed by inductance; o–7 = the impedance drop; the combination of o–5 and o–6. Thus the applied e.m.f., E_0, equals the sum of E' and the impedance, o–7, or equals o–8. This Fig. 82, therefore, shows the phase relations of the primary equation, (1).

Figs. 83 and 84 show the phases of the secondary,

Fig. 83 shows the relations of the e.m.f.'s due to the pulsation of the flux through the secondary, and Fig. 84 shows the phase of the e.m.f. due to rotation.

In Fig. 83, $I_{000} = 0\text{–}1$, $0\text{–}4 = I_1$, $0\text{–}2 = I_0 \cos \alpha$, and $0\text{–}3 = E''$, the back e.m.f. in the secondary due to flux pulsation. It should be noted that the phase of ϕ_1, Fig. 83, is different from the phase of ϕ, Fig. 82, since 0–2, Fig. 83, is smaller than 0–2, Fig. 82. This is because 0–2, Fig. 82, is I_0, while 0–2, Fig. 83, is $I_0 \cos \alpha$; the cosine of every angle (except zero) being less than unity.

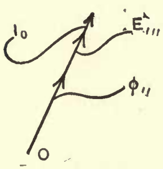

Fig. 84.

Fig. 84 shows the phase relations of the e.m.f. due to rotation. The flux, ϕ_{11}, is in phase with the primary current, and, moreover, with the e.m.f. of rotation. Thus, the three secondary e.m.f.'s are related to each other, as shown in Fig. 85, adding together vectorially to equal zero, as shown in equation (2). The back e.m.f. of rotation is an important feature of the repulsion motor, placing it, in many respects, above all other single phase motors. The phase of this back e.m.f. is the same as that of the primary current, so it tends, in connection with the e.m.f. due to flux pulsation, to make the final resultant e.m.f. in the rotating part move around toward the phase of the primary current. In other words, the secondary current becomes less and less lagging as related to the

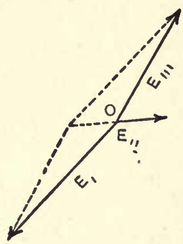

Fig. 85.

primary current, or as related to the induced e.m.f. due to the pulsation of the flux, or the current becomes less and less lagging as related to the applied e.m.f. Under certain conditions of rotation, e.m.f., a leading current condition may be produced in the motor. The particular function of this e.m.f. of rotation is, therefore, to improve the power factor. The larger the value of this e.m.f., the less the lag of current entering the motor, and, consequently, the higher the power factor. It is well adapted for railway and hoisting work. At starting, the power factor is of course that of the natural impedance of the motor standing still (perhaps 40%), but running under load this power factor rises to as high as 96% with reasonable air gap length. Another important feature is the fact that the brushes cannot "flash over," causing "bucking," as in the case of direct current motors, since there can be no voltage between brushes, as they are short-circuited upon themselves. In addition, the repulsion motor will "build up" as an ordinary d.c. series railway motor, and thus can be used as an electric brake. The sparking at the brushes due to the pulsating flux through the coils short-circuited by the brushes themselves can without any difficulty be brought down to a moderate amount by properly taking care of the commutator design, so that finally an excellent high power factor, high torque per ampere, single phrase motor is available. The sparking at the commutator of the repulsion motor is a minimum at synchronous speeds.

THE ALTERNATING CURRENT GENERATOR.

The single phase alternating current generator appears in its elements as in Fig. 86.

The magnetic flux passes through the armature *D* from the pole *N* to the pole *S*. At two points, 1 and 2, taps are taken off to the collector rings, *A* and *B*. From *A* and *B* can be taken off an alternating e.m.f.,

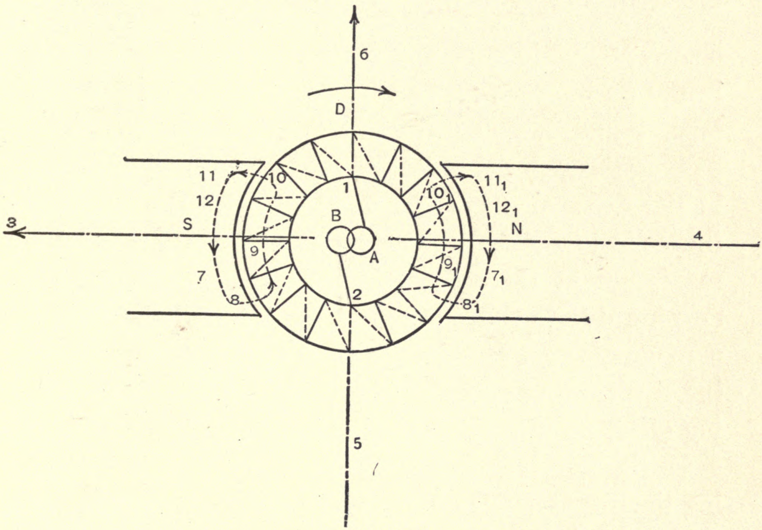

Fig. 86.

approximately in the shape of a sine curve, as shown in Fig. 4. As has been demonstrated, the formula for this e.m.f. is E (square root of mean square) $= \frac{4.44\, N n \phi}{10^8}$. Where $N =$ frequency, $n =$ turns in series between 1 and 2, namely, half the turns on the total armature, shown in Fig. 86, $\phi =$ the number of lines of force coming out of the pole, N, and going into S. If for any reason all the flux does not pass through all the coils in series, a factor K must be introduced, taking into account the turns not containing the full flux. As has been shown, the maximum value of this e.m.f. is $\sqrt{2}$ times the square root of mean square value, and the frequency $N = \frac{\text{poles}}{2} \times$ revolutions per minute $\div$ 60. The maximum e.m.f. occurs when the armature is in the position shown in Fig. 86; the line between the taps, 1 and 2, being at right angles to the line, 3–4, passing through the center of the poles. If the current in the external circuit be in phase with the e.m.f. at the collector, AB, the current is also (very nearly) a maximum at the same instant. Thus the armature current, due to this current in the external circuit, has a magnetizing effect in the direction, 5–6, and hence has a distorting effect on the field, 3–4. This distorting effect requires increased ampere-turns in the field, since the flux is crowded to one side, thereby increasing the density. The reduction of density in the other side partially compensates for this, but there is a preponderance of the effect of the increased density. This is partly due to the fact that the permeability of the iron becomes less as the density increases; but even were the permeability constant, there would still be some extra ampere-turns required. If the armature current, instead of being in phase with the e.m.f., lag behind, in consequence of the inductance of the external circuit, the current will not reach a maximum until the armature has rotated onward somewhat, depending upon the amount of the lag; so that, instead of having a cross magnetizing effect alone, the armature ampere-turns will have a demagnetizing effect as well. This is shown by the arrow, 7–8, Fig. 87. Hence,

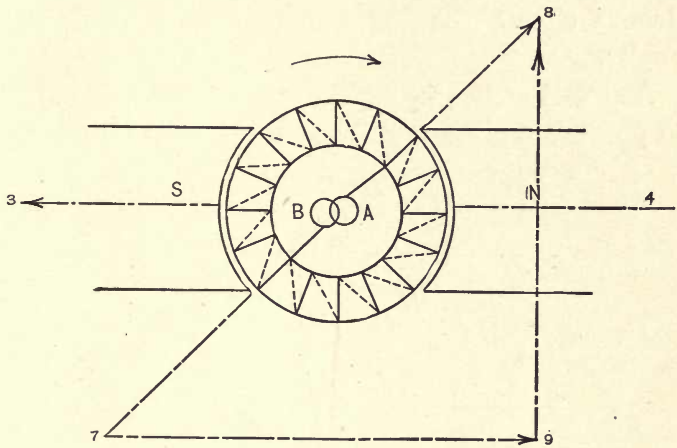

Fig. 87.

the demagnetizing effect is shown by the line, 7–9, and the cross effect by the line, 9–8. If the lag of the current be 90°, the armature will have turned, as shown in Fig. 88, before the current in it is a maximum. As may be seen, the arrow, 1–2, is now exactly opposing the arrow, 3–4, or, in other words, the armature ampere-turns are exactly opposing the field ampere-turns. Since, in any magnetic circuit, the ampere-turns in all parts of the circuit must be summed up, it follows that the demagnetizing ampere-turns must be met by increased ampere-turns in the field, the extra amount depending upon the amount of the lag of the current and upon its amplitude.

This demagnetizing effect of the armature is called armature reaction. On lagging current, a component of the armature current reacts directly. On non-inductive circuit, the armature current reacts at right angles.

A second action, necessitating increased field current to produce a given flux through the poles, is the self-

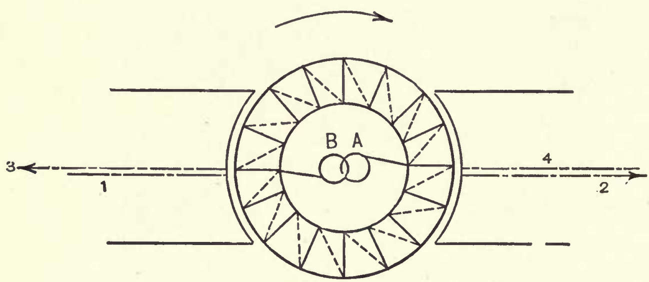

Fig. 88.

induction of the armature. As has been shown, self-induction equals $\frac{\text{flux} \times \text{turns}}{\text{ampere } 10^8}$. The turns, in the case of armature self-induction, are the turns of the armature, n. The flux is that which passes through the armature core, due to the armature ampere-turns as propelling force, which flux, however, does not pass through the field circuit of the main lines of force, but returns on itself, as shown in Fig. 89, by the lines 7–8–9–10–11–12 and 7′–8′–9′–10′–11′–12′. Thus, it is not a useful, but a leakage flux, as in the case of the self-inductive flux in transformers; moreover, it does not exist in the armature as a separate quantity, but exists only in imagination or mathematically, since only one armature flux can exist in the armature. Hence, to meet this flux tendency, the ampere-turns of the spools must be increased, just as in the case of armature reaction. Moreover, since, on non-inductive current, the self-inductive back e.m.f. has a volt reducing effect at right angles, and since, on lagging current, the self-

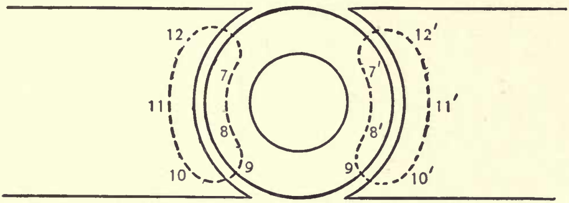

Fig. 89.

inductive back e.m.f. has a volt reducing effect directly opposing, as has been shown in Fig. 10, it follows that the armature reaction effect and the self-inductive effects are similar. Because of this similarity, Mr. C. P. Steinmetz has suggested calling their combined effect "synchronous reactance," treating it as one quantity.

A third cause for field increase, in order to keep a constant voltage at the collector rings, is the resistance drop of the current passing through the armature. Since the armature reaction is met by field ampere-turns, and since also the inductance flux is similarly met, the only increase of actual flux in the armature is that necessary to overcome the IR drop of the armature; and hence, in calculating or measuring core losses of alternators, the flux for any terminal voltage, E, and current, I, equals that to produce the voltage within the winding, that is, the total induced voltage, of $E + IR$.

Having now determined the three causes of reduction of voltage in an alternator under load, it is possible to calculate this specifically as follows:

Let $A =$ field ampere-turns to produce the terminal voltage, E, at no load, the armature running at normal speed. Let $B =$ the ampere-turns in the field to force through the armature when running at normal speed full current, I, when the armature is short-circuited upon itself. In such circumstances, the current in the armature practically lags 90° behind the e.m.f. induced by its rotation, since the resistance of the armature is much less than the inductance; or, in other words, under short circuit, the armature winding itself is an inductive circuit to the e.m.f. induced within it. Or, as has been pointed out above, and shown in Fig. 88, the ampere-turns of the armature reaction as well as the self-inductive effect, combined as "synchronous reaction," directly oppose the field ampere-turns, and hence the latter are a direct measure of the former; which offers a convenient method of determining the value of synchronous reactance from an actual dynamo in test.

The ampere-turns to give proper voltage can now be calculated, being equal to $\sqrt{A^2 + B^2}$ for a non-inductive load.

The relation follows as shown in Fig. 90.

In this figure, 1–3 = the current, I, exactly in phase with the external e.m.f., E, and approximately with the e.m.f. $E + IR$. Hence the ampere-turns to produce the e.m.f. $E + IR$ can be plotted as 1–3 in phase with I. The e.m.f. of synchronous impedance, being practically an inductance, can be plotted at right angles to the current, as has been shown early in this book. Hence, the ampere-turns to produce this synchronous impedance can be plotted as 1–2, Fig. 90.

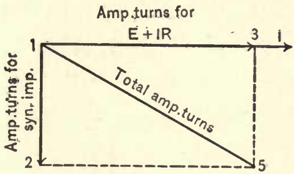

Fig. 90.

The resultant of 1–2 + 1–3 gives, therefore, the resultant ampere-turns. The writer has used this method for many years, the calculation agreeing with actual results from tests very satisfactorily.

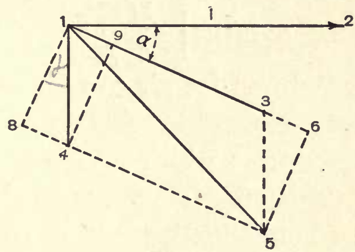

Fig. 91.

Knowing the total ampere-turns, 1–5, Fig. 90, the regulation can be calculated; for if the load be thrown off, and the ampere-turns be left on, a no load voltage E' will result, greater than E, since the ampere-turns are greater than necessary to produce E. The relation

$\frac{E' - E}{E}$ is called the regulation of the alternator. On well-designed machines, this should not be much over 7% on non-inductive load, as illustrated in Fig. 90. To get the no load voltage, E', having determined the full load ampere-turns, the saturation curve of the machine must be known; that is, the curve showing the relation between ampere-turns in the field and volts armature at no load. This curve can be calculated, when the magnetic densities in the field frame, air gap, armature teeth, and armature core are known. From the saturation curve of iron, the ampere-turns at a given density in the various parts can be calculated. (See Fig. 3, and also calculation of exciter current under transformer design.) From the relation between H and ampere-turns $\left[\frac{H}{1.257} = \text{ampere-turns}\right]$, the necessary ampere-turns for air can be calculated. The sum at the various densities, therefore, represents the total ampere-turns from which the complete saturation curve can be calculated. Thus the predetermination of regulation on non-inductive load of the dynamo is determined.

The determination at any load other than non-inductive is figured by embodying the same principles, namely, by keeping the synchronous impedance ampere-turns at right angles to the current. This is illustrated in Fig. 91.

Here the current, I, lags behind the e.m.f., $(E + IR)$, an amount shown by the angle, α. The synchronous impedance must, therefore, be drawn at right angles to

I, or as 1–4. The vector summation of 1–3 and 1–4 equals, therefore, the total ampere-turns required, and it will be noticed that the value is greater than when the current, I, is in phase with $E + IR$. 1–5, Fig. 91, is no longer equal to $\sqrt{\overline{1\text{–}3}^2 + \overline{1\text{–}4}^2}$, as in the case of the non-inductive condition illustrated in Fig. 91, but now equals $\sqrt{\overline{1\text{–}8}^2 + (1\text{–}9 + 9\text{–}3 + 3\text{–}6)^2}$. In Fig. 91, the angle 2–1–3 $= \alpha$, also the angle 8–1–4 $= \alpha$. This follows, since 2–1–4 is a right angle by construction, also 8–1–9 is a right angle. If from each, therefore, we subtract the same angle, namely, the angle, 3–1–4, the remainders must be equal; that is, the angles, 2–1–3 and 8–1–4. Hence, 1–9 $=$ 1–4 sine α, and 3–6 equals 1–9 by construction, hence 9–3 $+$ 3–6 $= E + IR$, also 1–8 $=$ 1–4 cosine α. Therefore,

$$1\text{–}5 = \sqrt{[(1\text{–}4)\cos\alpha]^2 + [(E + IR) + (1\text{–}4)\text{ sine }\alpha]^2},$$

all of which are known, giving in this case, as in the case of a non-inductive load, the resulting ampere-turns from which regulation can be calculated as before. When designing an alternator, the "synchronous reactance" is calculated by multiplying the armature reaction by a constant which depends upon the number of teeth per pole, shape of pole, number of phases of alternator, etc.; determined by the experience and judgment of the designer.

By the methods presented above, we have shown how the ampere-turns for a given voltage, load and power factor can be calculated. Knowing the voltage of the circuit from which the excitation of the alternator is obtained, the resistance of the spools for a given current can be calculated from Ohm's law. Knowing this current, the turns are at once known, being equal to the ampere-turns divided by this current. Knowing the turns, the amount of space necessary to accommodate them can be figured, as the size of the wire is governed by the amount of current, and thus the dimensions of the spool can be obtained.

As in the case of a transformer or an induction motor, it is necessary for the designer of a generator to choose a ratio between the weight of magnetic iron and the weight of copper producing the electromotive force and carrying the necessary current. For any given ratio between copper and iron, and for the various magnetic densities found by experience necessary to give satisfactory results, and for various current densities in the copper found necessary to give reasonable temperatures, the cross sections and lengths of the various parts of the machine can be figured, and thus all the dimensions of the dynamo can be predetermined. If, for a given ratio of copper and iron, the desired regulation cannot be obtained as calculated from the synchronous impedance and saturation curve shown above, another ratio must be chosen until desired results are obtained. In general, the regulation is better with more iron and less copper, since, the more the turns of the armature, the greater the synchronous reactance, and thus the poorer the regulation.

The predetermination of temperature is based on the general law that a given surface can radiate a certain amount of energy per square inch with a certain definite rise resulting. Thus, the outside of a spool can radiate one watt per square inch, and, in so doing, the rise above the room temperature will be about 60 degrees *C*. An armature has a smaller constant. Thus the temperature can be predicted by determining the losses per square inch of radiating surface. These losses are predetermined as has been shown in the case of the transformer and induction motor. They include I^2R of fields from which the field temperature can be calculated, and in the armature include the I^2R and core and eddy losses. In the single phase alternator, the current in the armature passes through zero, at which time there can be no armature reaction. With such machines, therefore, the armature reaction is pulsating; eddy currents are more easily created, and the load losses are somewhat higher than where the armature reaction is constant. By load losses are meant the eddy currents due to the current in the armature which are thus negligible at no load. In the polyphase armature, the reaction is constant at all times, since, as has been shown on pages 145–146, the resultant m.m.f. of coils set in space an angular distance one from another equal to the phase displacement of the currents circulating in them, is constant, and equals the number of phases divided by 2, multiplied by the magnetizing action of one of the coils. Thus, polyphase generators not only have a constant armature reaction, but it is less than that of single phase machines of equal output. Polyphase machines accordingly are cheaper to build, or for the same size give a greater output than single phase machines, so that they are desirable for economical reasons, as well as for the many other reasons already touched upon in the chapter on transmitting systems.

As in the case of transformers, the predetermination of efficiency here consists in summing up the losses, all of which can be calculated as already has been shown.

CHAPTER V.

TESTING ALTERNATORS.

THE tests to be made on a given alternator to determine its various characteristics mentioned above, are as follows:

(1.) Saturation.
(2.) Synchronous reactance.
(3.) Rise of temperature under non-inductive load.
(4.) Rise of temperature under load of power factor less than unity.
(5.) Core losses.
(6.) Load losses.
(7.) Efficiency at various loads and power factors.
(8.) Regulation at various loads and power factors.
(9.) Field characteristic.
(10.) Field characteristic, power factor of load less than unity.
(11.) Field compounding at unity and lower power factors.
(12.) Maximum output at various power factors.
(13.) Insulation resistance when hot.
(14.) Ability to withstand high potential strain on insulation when hot.
(15.) Wave shape of electromotive force, no load and full load.
(16.) Ability to withstand short circuit.
(17.) Noise of operation.
(18.) Mechanical defects.

SATURATION.

To obtain saturation curve, drive the alternator at a constant speed, and record the amperes in the field simultaneously with the volts at the collector rings, the latter being measured by an ordinary commercial portable voltmeter, which measures the square root of the mean square value. Knowing the turns, the ampere-turns are known, and hence, the relation between them and volts can be recorded, or the saturation curve, which appears as in Fig. No. 92.

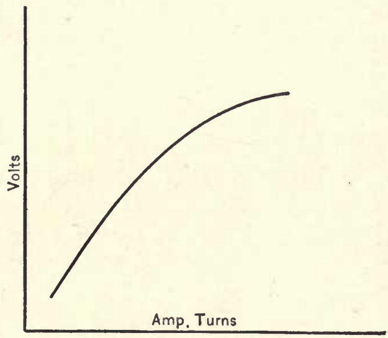

Fig. 92.

This saturation curve enables the designer to check up his predetermination of the summation of the ampere-turns required in the machine, and permits him to verify his various empirical constants, such as the leakage coefficient of the lines of force (that is, the % of lines of force induced in the poles, which do not pass through the armature), and other constants.

SYNCHRONOUS REACTANCE.

As previously stated, this is obtained by running the alternator at proper speed, with its armature short-circuited on itself. Apply enough current in the fields to produce in the short-circuited armature the various currents desired, from a small current up to 50% over normal current.

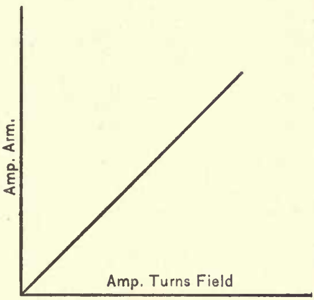

Fig. 93.

Since the current lags a great amount, as related to the electromotive force thus induced in the armature coil, it practically opposes the ampere-turns of the field, as has been shown in the first of this chapter, and thus the latter when read will measure the former. The curve of this synchronous reactance appears as shown in Fig. 93.

RISE OF TEMPERATURE UNDER NON-INDUCTIVE LOAD, AND UNDER LOAD OF POWER FACTOR LESS THAN UNITY.

There are various methods of obtaining this data. The first that naturally suggests itself is to place the load on the alternator, driving it by sufficient mechanical power, and using for load any resistance convenient, such as copper wire, or a tub of water with plates immersed in it and separated by some little distance. While this method is satisfactory for small machines, it becomes impracticable and expensive for machines of large power, such as 2,000 or 3,000 k.w. With such machines a convenient method offers itself, which consists in opposing one group of the poles against another group, and short-circuiting the armature upon itself. If the groups of poles are not opposed, that is, if one of them is reversed in polarity, the amount of current in the field to get full current in the armature is small, and much below the normal field current, since it is a fact that the synchronous reactance ampere-turns are always far less than normal load ampere-turns. If just one-half of the poles are opposed against the other half, any amount of current can be put through the two halves without causing any resultant electromotive force in the armature, since, with one group of poles reversed, the electromotive force generated by it equals and is exactly opposite that generated by the other group. If, however, one of these groups contains fewer poles than the other, there will result an electromotive force generated by the difference and obstructed only by the synchronous reactance of the alternator. Thus, a ratio between the number of poles in the two groups can be chosen so that full current can flow in the spools, and at the same time in the armature. While this method is not an exact imitation of a run under normal conditions on ordinary load, the agreement is quite satisfactory, and the method is an excellent one.

Its error arises from the fact that at the end of one group of poles the polarity is the same as at the end of the other group; or, in other words, two adjacent poles at each end of the groups are of the same polarity, and thus the total core loss is somewhat reduced. An alternator usually has quite a number of poles, so that the percentage error introduced by this is generally negligible. The second source of error consists in the fact that the armature current generated by the resultant magnetism lags considerably, and thus bears a demagnetizing relation to the larger group of poles, since the latter control and produce the armature electromotive force. If this is true of this group, it must follow that the armature current is highly leading as related to the smaller group of poles, and thus has a tendency here to increase the magnetic flux, while, in the larger group, the tendency is to decrease the flux. However, owing to the revolutions, this difference is first on one part of an armature and then another, progressing uniformly around with the rotation, and there results an average effect, which agrees quite closely with the ordinary load results. The method, therefore, gives practically full iron losses and full copper losses throughout, requiring for running only power enough to supply these losses. The run can be made to cover any power factor condition by properly choosing the field current corresponding to the power factor, and always arranging the number of the opposing poles to give the desired armature current.

CORE LOSSES.

In taking core losses on any dynamo, whether direct current or alternating, the method to be followed is practically the same. On a source of power the voltage of which can be changed and controlled at will, connect a motor, preferably a copper brush machine, with armature voltage the same as the power voltage. As to the size of the motor, it is rather difficult to set any absolute rule, but one between 15 and 20% of the size of the alternator should be selected if possible.

There are several conditions which should be fulfilled in taking a core loss. For instance, to avoid much armature reaction in the driving motor, the input in amperes into the motor should never be much over one-half the full load current of the motor when volts of the alternator in test are normal; the field of motor should be normal and held constant throughout the test. The input into the motor when overcoming the friction and windage of the outfit alone should preferably be less than one-half the total input when normal volts are on alternator in test, so that the core loss to be measured will be a prominent percentage of the total input measured in the driving motor.

Having selected the motor, see that the pulleys are so proportioned that they will run it close to the rated speed. It is then belted to the generator with a glued belt if possible. If this is not obtainable, the lacing should be as light as possible. Sometimes it is advisable to use a special splice in the belt, to do away with the jump in the ammeter when the splice passes over the pulley. Belt tension should not be excessive, but just enough to carry the load. Bearings should not heat or change in temperature during the test. If the brushes on the driving motor are always at the electrical neutral, as the load is changed the increase in volts necessary to keep the speed constant is about equal to the *IR* drop in the motor armature and brushes. It is not convenient to keep the brushes at the above position, as this would necessitate a change of the brush position with every change of load. If the brushes are given a shift too much out of the electrical neutral either way, we may find that the volts increase on the driving motor is not proportional to the *IR* drop. A setting must be chosen giving no appreciable variation in applied e.m.f. to keep the speed constant. The motor is supposed to have constant flux during the entire test, so that the volts armature need be increased only to overcome the *IR* drop in brush contact and armature resistance when the load comes on. The belt must be watched to see that there is no slipping. This can be noted by reading the speed of the motor with no field on the alternator in test, then reading the speed again with full field on the alternator. It should be practically the same in both cases.

It is sometimes found that the volts increase out of proportion to the *IR* drop. It is then probable that the brushes are not correctly set, and that their position and the short circuit current under them magnetizes or demagnetizes the field. If carbon brushes are used, this trouble is not so noticeable; but other troubles come from them, which must be guarded against, the principal one being the varying *IR* drop in the brush contact as the load changes. To be independent of this, one brush on each of the two studs of opposite potential can be used for volt reading, and must be insulated from the brush-holder box so that it will carry no current. To this is attached a voltmeter lead, thus giving the actual voltage on the commutator independent of any *IR* drop in brush contact. If carbon brushes be used, the commutator should be run perfectly dry, as any lubrication will change the friction reading. The difference between the input to the driving motor with the field on the alternator and that with the field off it gives the core loss of the alternator. To be independent of the varying I^2R of the driving motor armature and brushes, it should be subtracted from each input reading. Thus core loss $=$ (input with field $- I^2R$) $-$ (input without field $- I_1^2R$) when I and $I_1 =$ the current input under the two conditions. A curve can be plotted, using volts alternator as abscissæ and core loss as ordinates, as shown in Fig. 94. If all points come on a good curve, the

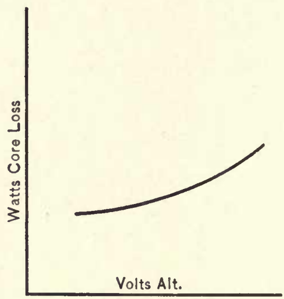

Fig. 94.

belt can be taken off, and the "running light" readings taken on the motor. This should be taken with the brushes in the running position, as well as at neutral point for a check. This reading enables the bearing friction of the alternator to be determined. The resistance of the driving motor armature should be taken before and immediately after the test, while the armature is still at running temperature. Before any reading is taken, the speed must have been constant at least a minute, otherwise one can never be sure that he is not reading energy which is being taken up in acceleration, or the reverse. This is particularly true of large machines. The speed of the driving motor is best controlled by varying the field of the source of power, thus varying the volts across the armature of the driving motor, the field of which is kept exactly constant throughout the test.

LOAD LOSSES.

The only practical method of obtaining the load loss of an alternator is to measure the core loss with the armature short-circuited and carrying the desired current. From this value of core loss subtract the I^2R of the current flowing, and the remainder is an approximation of the load losses. The method of measuring is exactly the same as that employed in measuring the regular core loss. It is necessary, in addition to the regular core loss measurements, to know the resistance of the alternator armature at the start and at the end of the test, so that the I^2R subtracted will be the value actually existing during the test. The amount of flux in the armature to produce the short-circuited current is practically negligible, and thus the true core loss from this cause can be neglected. It is usually the custom, in calculating the efficiency of an alternator, to use one-third of this load loss in the calculation.

EFFICIENCY AT VARIOUS LOADS AND POWER FACTORS.

Having measured the core loss and the load loss, and knowing the resistance of the armature and of the fields at the normal running temperature, and knowing for the load and power factor desired the amperes in the field, the volts armature and the amperes armature, all the factors necessary for the calculations of all the losses are at hand. These losses are as follows:

(1.) I^2R of armature.
(2.) $I_1^2R_1$ of fields.
(3.) Core loss.
(4.) One-third "load" loss.
(5.) Friction losses.

The core loss is that corresponding to the external voltage plus IR of the armature, as has been explained. Let the sum of these losses equal L; let the output in watts equal W; then L plus W equals the total input, and hence the efficiency equals *W* divided by (*L* plus *W*).

REGULATION AT VARIOUS LOADS AND POWER FACTORS.

This is calculated from the synchronous reactance and the saturation curve, as has been explained previously in this chapter. The method of taking the synchronous reactance and saturation curve has also been explained.

FIELD CHARACTERISTIC.

This is the relation between the volts armature and the amperes armature with a constant field. Its usual shape is shown in Fig. 95. It is calculated from the

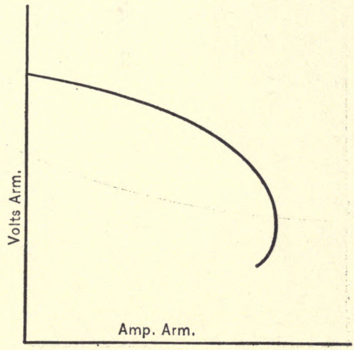

Fig. 95.

synchronous reactance and the saturation curve at the various currents. In reality, it is a plot of the regulation of the alternator between no load and full load and also between intermediate loads and full load. It shows how the voltage varies as the load changes, the field being kept constant.

FIELD CHARACTERISTIC, POWER FACTOR OF LOAD LESS THAN UNITY.

This is exactly similar to the above, except that the regulation should be calculated at the power factor in question.

FIELD COMPOUNDING AT UNITY AND LOWER POWER FACTORS.

This curve appears as in Fig. 96, and records the amperes in the field necessary to keep a constant voltage at the terminals of the alternator with varying load.

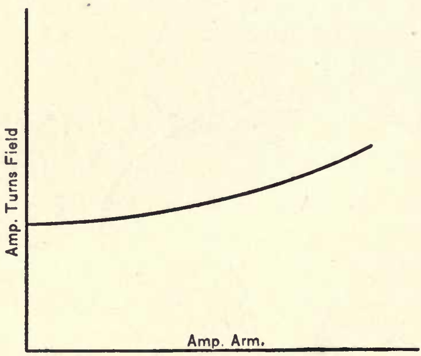

Fig. 96.

As in the case of the field characteristic, this curve is calculated from the saturation and synchronous impedance; the method for doing which has previously been explained.

MAXIMUM OUTPUT AT VARIOUS POWER FACTORS.

This is one particular point on the field characteristic. In calculating the voltage for different loads with the field kept constant, it grows continually lower with increase of load. A point is reached where the product of volts and amperes diminishes instead of increases. At this point the maximum output of the alternator is reached. The lower the power factor, the smaller the maximum output. This naturally follows, since, on low power factor, the voltage of the alternator is diminished for a given load, much more than for unity power factor.

INSULATION RESISTANCE WHEN HOT.

It is desirable to know in any alternator the insulation resistance in ohms of its windings to the metal parts of the machine. A convenient method to use is to place the winding to be measured in series with an ordinary portable voltmeter on a d.c. circuit connecting one terminal of the voltmeter to the winding and the other terminal to the source of e.m.f. The other terminal of the source of voltage is connected to the iron part. This connection is shown in Fig. 97, where A represents the winding, B the iron part, C a voltmeter, and D the source of voltage. If the voltage at D equals V, and if, when connected, the deflection of the voltmeter C is equal to V_1, and if the resistance of the voltmeter C is equal to R, the insulation resistance is equal to

$$\left(\frac{V}{V_1} - 1\right) (R).$$

This formula is deduced as follows:

The voltage of the circuit V is used up by the leakage current passing through the resistance of the voltmeter and insulation resistance. If all the voltage were used up in the resistance of the voltmeter, it would

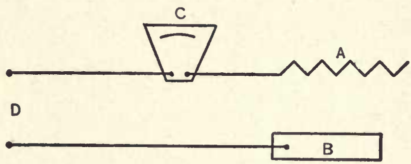

Fig. 97.

show a deflection corresponding to the voltage of the circuit. If the insulation resistance were equal to the resistance of the voltmeter, the drop in voltage through the voltmeter (or its deflection) would be the same as the drop of potential through the insulation resistance; or, expressed differently, the drop through the voltmeter and insulation resistance is in direct proportion to their relative resistances. Thus, assuming the resistance of the voltmeter to be R, and the insulation resistance to be R_1, we have

$$\frac{R}{R + R_1} = \frac{V_1}{V},$$

or

$$R = \frac{V_1R + V_1R_1}{V},$$

or

$$RV = V_1R + V_1R_1,$$

or $$R_1 = \frac{RV - RV_1}{V_1},$$

or $$R_1 = R\left(\frac{V}{V_1} - I\right).$$

ABILITY TO WITHSTAND HIGH POTENTIAL STRAIN ON INSULATION WHEN HOT.

Although the insulation resistance of a generator may appear to be all right, the ability of its insulation to stand puncture from an applied potential greater than normal may not be satisfactory. It may usually be expected that a generator when at normal temperature should be capable of standing an applied alternating sine wave of e.m.f. with its square root of mean square value at least two and one-half times the normal working voltage of the alternator. The method of doing this is to apply such an e.m.f. from a proper source, one terminal of which is attached to the frame of machine, and the other terminal to the winding, the insulation of which is being tested. This voltage should be held on for a minute, during which time no signs of deterioration on the part of the insulation should appear, nor should there be an excessive leakage of current through the insulation. This latter can be read by an ammeter, usually placed on the low tension side of the step-up transformer used for obtaining the high potential. The volts can be read by a static voltmeter, which consists of two disks, one movable as related to the other, and carrying a pointer, both disks being charged statically from the potential. They repel each other, owing to the static charges, causing the deflection of the needle. This deflection can be calibrated and the instrument used as a high potential voltmeter. It is not proper to rely on the ratio of the transformer, since static stress of this source causes a leading current to flow, the winding acting as one plate of a condenser, and the frame as the other. This leading current, passing through the inductance of the transformer, may raise its ratio considerably above the normal. This phenomenon has been pointed out in previous chapters. In certain cases, especially if the contact on the winding is poor, resonance may occur, causing very excessive rise. Thus the static voltmeter is an essential adjunct to such a test. For safety also in determining any excessive rise, needle points should be used as well as the static voltmeter. The voltage which will cause a jump of electricity between needle points set apart various distances, is experimentally known so that such an arrangement serves not only as a protection against too much voltage, but, in the absence of a static voltmeter, suffices satisfactorily for an actual method of determining the voltage applied.

WAVE SHAPE OF E.M.F., NO LOAD AND FULL LOAD.

A proper wave shape is quite essential on all alternating generators, since, if it is poor, harmonics may exist in the line, which, particularly on long lines, may in turn cause resonance and various puncturing troubles. The question of harmonics has been touched upon in a previous chapter. It is desired that the wave shape of alternators should be a true sine curve, as nearly as possible, and these harmonics, which are the component waves of higher frequency, causing their particular troubles, should not exist. Also, as the load comes on the alternator, the wave shape should change as little as possible, which means that the armature reaction, that is, the distorting effect on the field, should be small; and the field strength, or the lines of force per square inch, should be high. The usual method of taking the wave shape is to put on the alternator a mirror galvanometer. The mirror is so light and has so little inertia that it follows the voltage from instant to instant. A spot of light reflected from the mirror on a photographic film moving uniformly forward impresses on the film the exact wave shape, which is thus recorded and made available for inspection.

ABILITY TO WITHSTAND SHORT CIRCUIT.

All alternators are liable, when installed, to suffer a short circuit on the line. For a very brief space of time the voltage holds up under this short circuit, and an immense current is drawn. This voltage is held up by an induced current in the field from the armature current flow, in accordance with Lenz's law. This holding up lasts a time, depending upon the time constant of the field circuit. It should be noted, as has been stated, that the time constant is the number of seconds which it takes in any circuit to bring the current up to approximately two-thirds its final value. This exceedingly heavy current on short circuit has a tendency to distort the armature coils due to the repelling or attractive action of the currents in them, so that a well-designed alternator must stand this strain without mechanical movement of the coils or deflection of the field frame to a serious extent. It is the general practice on modern alternators to make this test as a part of the requirements.

NOISE OF OPERATION.

All alternators, when properly designed, should operate without any buzzing or humming sound. This noise in poor designs may be quite excessive and prove exceedingly disagreeable, causing complaints from people living near stations where they are installed. Sometimes the trouble is due to loose laminations, which can be easily fixed. Sometimes it is due to pure magnetic humming, which requires a change of design for remedy. Sometimes it is due to puffs of air occurring at regular intervals in the openings of the armature construction. This latter trouble can be cured by properly filling such spaces.

MECHANICAL DEFECTS.

All alternators should operate with bearings free from any sign of oil on the outside. The bearings should have a rise of less than 40° C. as an ultimate temperature. The balance of the revolving parts should be perfect; and, in general, in a test of a machine, every detail of this sort must receive most careful consideration.

PARALLEL OPERATION OF ALTERNATORS.

Alternators of various sizes and makes can be operated in parallel just as direct current machines. Certain conditions in reference to such operation, however, have to be observed, and where trouble exists specific remedies can be applied to correct them. In connecting alternators in multiple, it is only necessary that their frequencies and their voltage be alike. When thrown together, each should take its proportion of load, and on no load there should be no exchange of current between them. It is sometimes found, however, that there is a large exchange of current between the alternators when running at no load, and at full load or intermediate loads proper division of work does not occur. The causes of this cross current may be:

(1.) A difference in wave shape of the two alternators.

(2.) A pulsation during each revolution of the prime mover of the alternators.

(3.) A difference in the value of the e.m.f.

The first trouble is usually small, and rarely manifests itself to a serious degree.

The second trouble is the most serious of all. Its action may be seen by reference to Figs. 98 and 99. In Fig. 98 the e.m.f.'s of the two alternators are represented by the vectors 1–2, 1–4. They are drawn side by side, but in reality are exactly superposed on each other, since their supposed e.m.f.'s are exactly in phase. Thus the points, 2, 4, and the points 1 are at the same potential, and no current flows between the alternators.

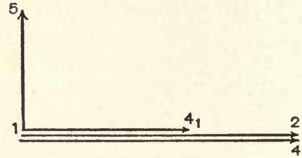

Fig. 98.

If the prime mover of one, however, during a revolution, lags behind or gets ahead of the other, the two e.m.f.'s, instead of being superposed on each other, swing apart, and are represented as shown in Fig. 99. Under these conditions, the points 2, 4, are no longer at the same potential, but have a difference of potential between them which acts through the impedance of the two alternators in series, creating thereby a current equal to the voltage, 2–4, divided by this impedance.

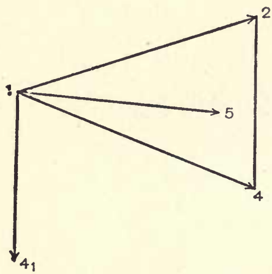

Fig. 99.

In completing the diagram in Fig. 99, 2–4 appears as 1–4_1, since all vectors must go to a common center, as at 1 in this case. Therefore, the free e.m.f. acting through the two impedances as stated, is shown by the vector 1–4_1. This current, resulting from this e.m.f., lags behind it a large amount, since the reactance ohms of the armature are much greater than the resistance; or, in other words, the circuit of the two armatures in series is an inductive circuit. Thus the current flowing from this e.m.f. 1–4_1 is represented by the vector 1–5. An examination of the position of this vector shows it to be well in phase with the vectors 1–2 and 1–4, or, in other words, the current is energy current; and thus the cross current of alternators which swing apart in phase somewhat, because of irregularity

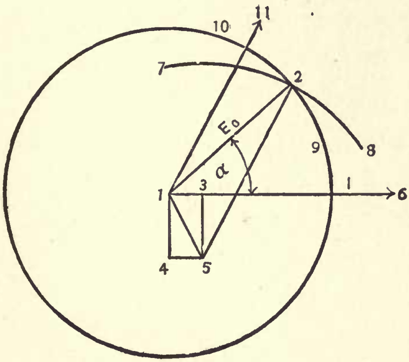

Fig. 100.

in rotation of their prime movers, is an energy current, tending to pull them together again.

Third. When two alternators are properly phased, and their prime movers run without variation or with similar variations, the above effect disappears. When, however, the field of one alternator is weaker than it should be, there is a cross current, even though the phasing of the two alternators may be exact. This cross current is the result of the e.m.f. 1–41, as shown in Fig. 98. The current from 1–41, Fig. 98, just as the current from 1–41, Fig. 100, is lagging, and is shown by the vector 1–5, Fig. 98, about 90° lagging from the e.m.f.'s 1–2, 1–4. Thus, in the case of unequal fields, the cross current is a lagging current, and does not represent energy. Its action, however, is to assist the magnetism of the weaker machine and lower the magnetism of the stronger machine, for the flow of current naturally goes from the higher voltage to the lower; it is a lagging current, as shown in relation to the higher voltage, and thus demagnetizes that alternator; it is a leading current in relation to the lower voltage, and thus magnetizes the other alternator; in fact, enough current flows to make the two voltages alike.

The method of correcting the unequal field is, of course, a matter of rheostat adjustment. The method of correcting the irregularities of the prime mover is an engine problem, and is usually handled by placing suitable dash-pots on the governors of the engines, making them less variable in speed during a single revolution.

THE SYNCHRONOUS MOTOR.

An ordinary alternator, if placed upon a circuit and synchronized with the cycles of the circuit, will operate as a motor, taking energy from the line, and giving out mechanical power. In other words, an alternator can act as a generator or as a motor. When connected and operated as a motor, an alternator is called a synchronous motor.

The diagram showing the phase of the various currents and e.m.f.'s is shown in Fig. 100. In this figure let 1–6 represent in length and phase the current flowing into the synchronous motor; let 1–3 in phase with this current equal the e.m.f. consumed by resistance; let 1–4, 90° away from the current 1–6, represent the e.m.f. consumed by inductance. The resultant of 1–3 and 1–4, that is 1–5, therefore represents the impe-

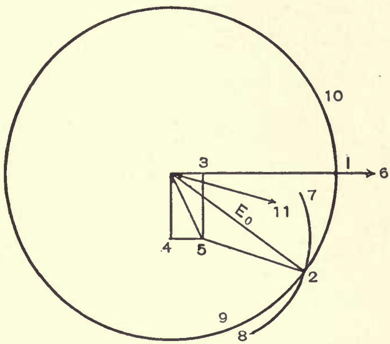

Fig. 101.

dance drop in the armature of the synchronous motor. From 1 as a center, draw the circle 9–10, equal in value to the e.m.f., E_0, applied to the terminals of the synchronous motor. From the point 5 draw another circle 7–8, intersecting the first circle at the point 2. The triangle 1–2–5 then contains all the e.m.f.'s existing in a synchronous motor. Draw 5–2 from 1 as a center, giving the vector 1–11. We now have the three e.m.f.'s

1–2, 1–11, and 1–5, the first being the applied e.m.f. of the synchronous motor, the second being the e.m.f. used up in impedance at the current in question, and the third being the back e.m.f. induced by the revolution of the armature in the field of the synchronous motor. Examination of this figure shows that the current leads the applied e.m.f., E_0, by the angle α; also the back e.m.f. 1–11 is larger than the applied e.m.f.; or, in other words, in order to make the current leading, the back e.m.f. had to be made larger than the applied.

In Fig. 100 the projection of 1–2 on 1–6 (which is the component of E_0 in phase with the current) multiplied by 1–6, the current, equals the total energy input of the motor. Also projection of 1–5 on 1–6, that is 1–3, multiplied by 1–6 equals the I^2R loss, and the projection of 1–11 on 1–6 multiplied by 1–6 equals the energy given out by the synchronous motor.

Fig. 101 shows the effect of reducing in size the back e.m.f. 1–11. The intersection in Fig. 101 is at 2, giving a back e.m.f. of 5–2 or 1–11. Under these conditions the current, 1–6, is lagging behind the applied e.m.f. instead of leading it as in Fig. 100. In other words, reducing the back e.m.f. 1–11 to its new value in Fig. 101 causes a lagging instead of a leading condition.

It should be noted that the back e.m.f. 1–11 is the result not only of the field ampere-turns but the armature ampere-turns, and that where the current in a synchronous motor is a leading current, more ampere-turns in the field are required for a given flux. This exactly reverses the case of generators, where a leading current magnetizes and assists the field ampere-turns.

Therefore, in a synchronous motor the lagging current assists the field in magnetizing it. Thus, by varying the field, the phase of the incoming current as related to the e.m.f. can be altered. A strong field creates a leading input current, a weak field a lagging input current. This feature of synchronous motors makes them particularly useful on lines with low power factors, since, by properly adjusting the field, the current they take may be made leading, neutralizing the lagging effects of the line. In fact, there are installations where synchronous motors are connected to the lines solely for the purpose of altering the phase of the main current flowing into the line, the synchronous motor taking leading or lagging current as desired for the neutralization. Therefore, in respect to power factors, a synchronous motor has an advantage over an induction motor, whose power factor cannot be made even at full load much better than 95%.

In respect to starting torque, however, the synchronous motor is not nearly the equal of the induction motor. The torque of the synchronous motor is zero when it is single phase. If polyphase, there is some torque from the induced currents in the face of the pole pieces or in special pole piece windings sometimes applied, these windings acting like the armature of an induction motor. In addition to this, there is a torque due to remnant magnetism, that left by one phase being acted upon by the current in the next phase; or, in other words, hysteresis serves as a method of torque. In an alternator, it is desirable, as has been pointed out, to have the armature reaction low, so that the demagnetizing effect of it upon the field may be small, thus slightly influencing the voltage with variation of load and power factor. In the synchronous motor, however, this low armature effect is not to be desired. Instead, a high armature reaction, which means a high impedance, is required in order that when the synchronous motor is thrown upon the line so as to give it energy to start, the rush of current will not be excessive. This armature reaction in well-designed synchronous motors is such that the motor will start from rest with about double full load current; thus, if a compensator with the ratio of one-half be used for starting, only full load current will be drawn from the line. Another cause for preferring such a high reaction armature lies in the fact that, if for any reason the field should weaken or be broken, the amount of current flowing into the armature to produce magnetism in place of the weakened or destroyed field would be small as compared with a low reaction armature. Thus, while an alternator and a synchronous motor may be used interchangeably, without armature winding alteration, each performs poorly in the place of the other.

CHAPTER VI.

THE ROTARY CONVERTER.

VOLTAGE RELATIONS.

A ROTARY converter consists essentially of a direct current generator, which is driven by alternating currents applied to collector rings, which are connected in turn to proper points on the winding. This driving from alternating currents is an action similar to that of a synchronous motor explained in the previous chapter. Thus a rotary converter is on the one side a synchronous motor and on the other side a direct current generator. Thus any direct current generator can be made into a converter. Unlike a d.c. machine, however, the voltage taken from the commutator brushes depends upon the a.c. voltage applied to the collectors and is practically independent of the strength of the field current. To determine the ratio of the d.c. voltage to the a.c. consider Fig. 102, which shows diagrammatically a three phase rotary of two poles. The d.c. brushes rest at the points 1 and 2, and thus at the instant shown are connected to the windings at these points. The three a.c. collector rings are at the same instant connected to the windings at the points 1, 5, and 4, at three equidistant points (i.e. 120°

from each other). With machines with more poles each pair would be connected, as in Fig. 102, and the various corresponding pairs connected in multiple.

Since the counter e.m.f. equals practically the applied, the ratio of the e.m.f.'s. 1–4 : 1–5 and 5–4 to 1–2 gives the ratio at no load of the applied a.c. e.m.f. to the d.c. e.m.f. Also if the d.c. brushes rested on the commutator at 1 and 2, and if the windings were at these

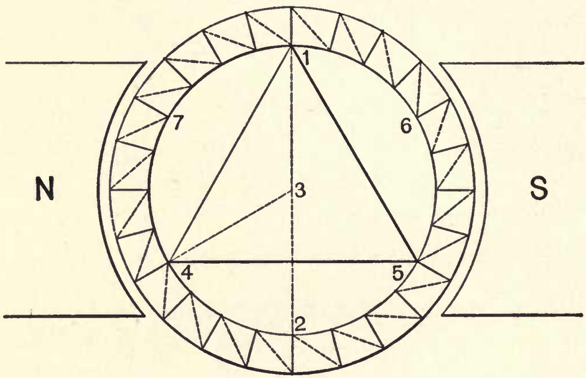

Fig 102

Fig. 102.

points connected to two collector rings, and to them an alternating a.c. voltage were applied, then at the instant when the points 1 and 2 passed beneath the brushes the a.c. and d.c. voltages would be exactly alike, the a.c. and d.c. brushes being connected for an instant together. But at this point the a.c. voltage generated by the rotation of the rotary (or what is practically the same thing, the applied a.c. e.m.f.) is a maximum. Therefore, the d.c. voltage can be regarded to be equal to the applied single phase a.c. voltage or twice the voltage from one collector to the neutral point (point 3 in the figure). The line 1–3 can, therefore, represent the maximum applied voltage, from the point 1 to which one lead is connected to an imaginary neutral point 3 at the center. Since the triangle 1–4–5 has equal sides and the point 3 is at the center, the angle 3–1–4 = 30°, each angle of the triangle 1–4–5 being 60°.

Thus $$1 - 4 = 2\,[(1 - 3) \times \text{cosine } 30°],$$

but $$\text{cosine } 30° = \frac{\sqrt{3}}{2}.$$

Thus $$1 - 4 = \sqrt{3} \times (1 - 3) \text{ or } = \frac{\sqrt{3}}{2} \times (1 - 2),$$

but $1 - 2 =$ the direct current voltage

and $1 - 4 =$ the maximum of the a.c. voltage *between lines.*

Thus the $$\frac{\text{d.c. voltage}}{\text{max. value of the a.c. voltage}} = \frac{2}{\sqrt{3}}$$

or

$$\frac{\text{d.c. voltage}}{\text{sq. root of mean sq. value of the a.c. voltage}} = \frac{2\sqrt{2}}{\sqrt{3}}$$

since the square root of mean square voltage of an alternating wave = maximum value of the voltage ÷ $\sqrt{2}$. Thus a.c. voltage of a three phase rotary = d.c. voltage ×.612.

CURRENT RELATIONS.

This ratio changes with the load on the rotary due to the impedance drop in the rotary windings. The whole range, however, is only a few per cent. Since one side of a rotary gives out energy in the form of energy current and the other side takes in energy in the form of energy current, and since both these currents flow in the same windings, it follows that they must tend to neutralize each other. It can be shown that due to this neutralization, the I^2R loss in the winding of a three phase rotary, running without lagging or leading current, is 58.5 % of the loss when running as a d.c. generator. Hence, in designing rotaries, advantage is taken of this fact, the amount of the armature copper is cut down. Moreover the slots into which the armature bars are wound can be made less deep and less wide, cutting down the self-induction and tendency to spark at the commutator. Thus a three phase rotary is smaller for the same output than a d.c. generator. Another important feature of a rotary is the fact that there is practically no armature reaction, that is, the effect as shown in connection with alternators on page 189 practically disappears. Owing to the presence of so little armature reaction, the need of shifting the brushes forward to reverse the current under the brush during the act of commutation disappears, and rotaries can run with the brushes at the neutral point. If in Fig. 102 we add three more collector rings to the windings at 6, 7, and 2, half way between the other taps, we get a *six phase* machine with six collectors. A distinct advantage arises from this, for we get not only the neutralization of armature reaction, but a greater neutralization of the I^2R loss of the armature windings than with a three phase rotary. It can be shown that in a six phase rotary the I^2R loss = only 26.7% of that in the machine running as a d.c. generator. A further advantage exists:—in a three phase generator the neutralization of currents occurs more perfectly as we depart from the taps where the leads from the collectors connect to the windings. At the taps the full a.c. current is carried, hence the clips which fasten the leads from the collectors to the windings at the point where they enter the windings have to be of large carrying capacity—not always easy to arrange. With a six phase rotary each lead carries just half as much current as in a three phase, with its attendant advantage. Naturally in the care of a six phase machine there are three more collectors to attend to, but this is not a serious matter. Large rotaries like 1000 k.w. and over and those of heavy currents are six phase, gaining on clips as well as in extra armature neutralization of I^2R loss.

Rotaries are usually three or six phase, though they can be made single or quarter phase. A polyphase rotary will start just as well as a polyphase synchronous motor. A single phase will not start, and its windings can be shown to have 47.5% *more* I^2R loss than an equivalent d.c. generator. Hence they are never used unless absolutely necessary. With a three phase rotary the input $= E \times I \times \sqrt{3}$ and equals, assuming 100% efficiency, the output $E'I'$ where E = the alternating e.m.f. between lines and I the a.c. current, E' = the d.c. voltage, and I' the d.c. current. But we have shown that

$$E = E' \times .612. \quad \text{Thus } EI\sqrt{3} = \frac{EI'}{.612}$$

or
$$I = I' \div [.612 \times \sqrt{3}] = .94I'$$

Thus assuming an efficiency of the rotary of 95% the a.c. and d.c. currents as read on commercial instruments are about the same for a three phase rotary. With a six phase rotary the a.c. current is one-half the d.c. As has been explained on pages 222, 223, 224, in connection with a synchronous motor, the phase of the incoming current in a rotary is regulated by the strength of the field current. A weak field means that enough lagging armature current will flow to make up the deficiency of magnetization since the back e.m.f. must equal the applied. Also if the field current be increased the entering armature currents will be leading, giving as before the desired back e.m.f. Thus the armature current automatically takes the place of the field excitation. Advantage of this fact is taken when placing rotaries on long lines where a considerable drop of potential results with load. The rotary is provided with a series field through which passes the d.c. current. As the load comes on the rotary the field gets stronger, and hence the entering a.c. current becomes more and more leading. If the line has not much inductance some artificial inductance is inserted just before the current enters the collector rings. The result of passing leading currents through inductance is to raise the voltage as is illustrated in Figs. 23, 24, and 25, pages 49 and 50. Hence the load on the rotary raises the voltage automatically on the a.c. end and hence, of course, on the d.c. end, so that the rotary compounds just like a regular d.c. generator as the load increases. It should be noted that unless a rotary runs at minimum input, i.e., without leading or lagging current, the neutralization of I^2R loss in the armature does not occur, as has been discussed. The lagging or leading component is unneutralized and hence adds its heating to that already existing. A 30% lagging current would result in a total I^2R loss of 69% of a d.c. machine instead of 58.5%. Rotaries may be run in parallel just like d.c. machines.

HUNTING OF CONVERTERS.

Under some conditions of the source of power, such as a lack of regularity in the rotation of the dynamo, or even when this feature is satisfactory, hunting sometimes occurs. This is a mechanical swinging back and forth of the armature, with a frequency depending upon the weight of the armature, strength of field, etc. This swinging is accompanied by a varying of the d.c.

voltage and entering a.c. current. It may gradually increase until the rotary actually will drop out of step and stop with serious flashing of the brushes. To stop hunting bridges or dampeners are fastened to the poles in which eddy currents are induced by the motion, stopping it (an application of Lenz's law, page 27). Hunting occurs far more seriously with leading current than with minimum input or lagging current.

STARTING OF CONVERTERS.

When starting, the voltage on the d.c. end is alternating, becoming direct when synchronism is reached. The polarity of a given brush holder may be + or − when reaching synchronism, but can be forced to the desired polarity by sending current through the field. If the right direction, the polarity remains the same, if opposite the polarity reverses and the armature *drops back* a pole, doing this suddenly and actually in space but without any serious flashing at the brushes, so that starting from the a.c. end a rotary can be made any desired polarity. Often to avoid the rush of a.c. current necessary to start, a rotary is started from the d.c. end and phased with the a.c. source of power, just as two alternators are phased together before being thrown in multiple. Rotary converters can be run inverted, the energy being taken in on the d.c. end and given out on the a.c. In this case a runaway occurs if a sudden lagging current is drawn, since then lagging current demagnetizes the field, and being a d.c. motor nothing but increase of speed can create the proper counter e.m.f. To avoid "runaways" due to this cause rotaries are usually fitted with *speed limit switches*, which automatically cut off the power when the speed becomes excessive. Since "reversal" of d.c. energy can come under certain conditions, even with regular rotary operation when in multiple with other sources of power, all rotaries are so fitted out to render them safe under all conditions.

TESTING OF CONVERTERS.

In testing rotaries the line of procedure is similar to that outlined for alternators on page 200. The losses include, however, the I^2R losses of brushes, both a.c. and d.c. as well as friction of them. The coefficient of friction of carbon brushes for ordinary normal conditions on commutators is two-tenths, and the contact resistance varies from .026 ohms per sq. inch at 40 amperes per sq. inch current density in the face of the brush to .042 ohms at 15 amperes per sq. inch. The current density resistance curve appears as in Fig. 103. The resistance of the carbon itself as compared with the contact resistance is negligible. Copper brushes have about one-tenth the contact resistance of carbon. The coefficient of friction is about the same. The ratio of voltages with different loads should be found and a trial should be made of commutation with these loads. A well designed rotary should stand double load without shift of brushes and without sparking at the brushes. A trial should be made for "pulsation" by running on an artificial line of considerable inductance. In carrying out this test it is best to put about

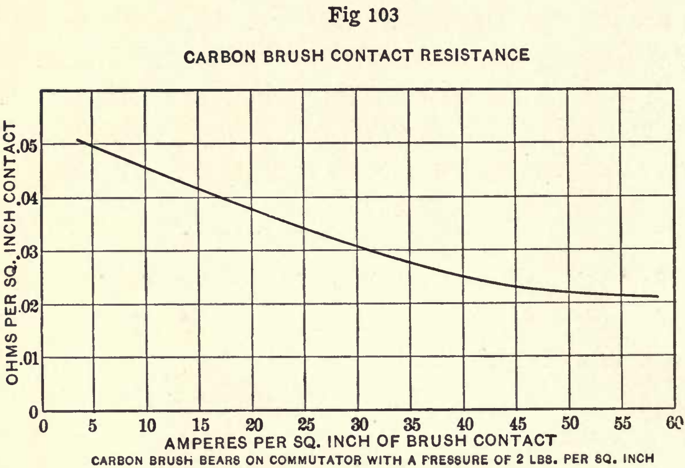

Fig. 103.

15% inductance or resistance drop between two rotaries. If surging or hunting weakness exists, the armatures will soon commence to swing accompanied by surging of the current, voltage, etc.

A starting test should be made from the a.c. end, which is done by gradually raising the applied voltage with field current broken till the rotary starts and then, holding this voltage constant, till synchronism is reached. Some machines will start properly but will "stick" at half synchronism, which is as serious as not starting at all. A phase characteristic at full load and no load should be taken, which shows the a.c. current for varying currents in the field windings. Such a curve at no load is shown in Fig. 104. The field current 4–2 gives minimum input at 2. A weak field gives a large current input and so does a strong field. In taking this curve the a.c. voltage should be kept constant and the d.c. voltage read.

Since rotaries are necessarily of comparatively low voltage they are used with transformers. Pages 64–5

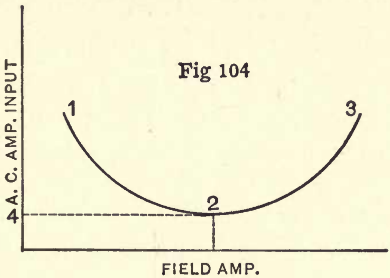

Fig. 104.

show a six phase transformer connection and Chapter IV various three phase transformer connections. Rotaries are usually used on transmission circuits. The main generating station has perhaps 13,000 volt alternators. From them run three phase lines, and at any point desired are tapped off from them line circuits running to transformers, secondaries of which are connected to the rotaries. The d.c. end of the rotaries furnish d.c. power of any usual voltage. Thus by their means d.c. power is available with the advantages of a polyphase a.c. transmission. The efficiency of rotaries is higher than d.c. or a.c. generators due to the exceedingly low I^2R loss of armatures, as has been explained. It is not uncommon to get 96% on a three phase rotary. With their constancy of brush position, high efficiency and low cost per k.w., with great overload capacity in commutation as well as heating, they are a most desirable piece of apparatus. Sometimes a motor-generator set composed of a synchronous motor driving a d.c. generator is used instead of rotaries. While the efficiency of such a set is less than a rotary and its cost greater, it is often used on account of its greater flexibility in voltage control and independence of line conditions as to voltage regulation on the d.c. end.

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