

thermo-electric action made years ago with no reference to the problem of electronic emission.

As to the value of the constant A' , usually written A and often called a "universal constant," I predicted¹ a year or two ago that it would prove to be different in different metals. Du Bridge says, regarding this matter, "On computing the value of A from carefully taken emission data on several out-gassed platinum specimens . . . values 200 times or more as great as the theoretical were obtained," the "theoretical" value being that suggested by Richardson and Dushman and confirmed, approximately, by Dushman's study of tungsten, tantalum and molybdenum.

¹ *Proc. Nat. Acad. Sci.*, 13, 56, 1927.

² This is not a gratuitous or a new assumption. See my note on the Temperature Relations of Photo-Electric Emission and Thermionic Emission, *Ibid.*, 12, 486, 1926.

³ *Ibid.*, 13, 325, 1927.

THE QUANTUM LAWS AND THE UNCERTAINTY PRINCIPLE OF HEISENBERG

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Communicated January 11, 1929

In preceding papers¹ we have shown how the second and third laws of thermodynamics may be deduced from a single statistical principle which implied that the properties of a system can be described by assuming a finite number of possible states. This number was called Ω . It is our purpose in this paper to justify and to interpret this assumption, to restate the fundamental laws upon which quantum theory is based, and to show how this restatement leads directly to the uncertainty principle of Heisenberg.²

In our previous work, although we endeavored to say nothing that was contrary to the new mechanics, we did not use its phraseology. Indeed, for our statistical purpose, it made no difference whether we considered the number of ways in which certain particles may be distributed among the cells of a phase space, or the number of ways in which the same number of particles with the same energy may be assigned the various discrete solutions of the mechanical equations. The same value is obtained for Ω , for it has been shown by Schrödinger³ that between any given limits the number of such solutions coincides with the number of cells formerly employed, in the simple cases which have so far been investigated.

Whether this is always true is a question that must be reopened when in the next paper we discuss the electronic states of an atom.

The methods that we have been discussing imply either that every system can occur only in certain definite and fully quantized states, or that a larger number, possibly an infinite continuum of states, is possible; but that within a certain range, that is, within one "cell," these states are for some fundamental reason inherently indistinguishable from one another. These two views seem at first sight to exhaust all the possibilities consistent with a finite Ω . There is, nevertheless, a third view, which is the one we are going to develop here. According to this view, while it is quite permissible to account for the properties of a system by assuming an infinity of states, all distinct from one another, yet it is *sufficient* in order to account for the whole *behavior* of a system to postulate a minimum number of exactly defined states. This minimum number is Ω .

A Restatement of the Quantum Laws.—According to the original law of Bohr, a system in a definite state B with energy ϵ_B changes to the definite state A with energy ϵ_A , emitting, in case the process is one involving light, the energy $h\nu = \epsilon_B - \epsilon_A$. However, we know that this law cannot be exactly true. When allowance is made for Doppler effect and so on, we recognize that there must be left to every spectra line what is known as the natural width. For example, in a gas at very low pressure (as in the nebulæ), there would be no self-reversal of lines if the lines were infinitely sharp, for then no atom would absorb light unless it were exactly at rest with respect to the emitting atom, which it would never be.

It would be natural to ascribe such phenomena as the natural width of the spectral lines to a lack of complete definition of the states, but this is not the only possible explanation. We must consider the possibility that even when two states are exactly defined, the phenomenon occurring in the transition between these states is not uniquely determined.

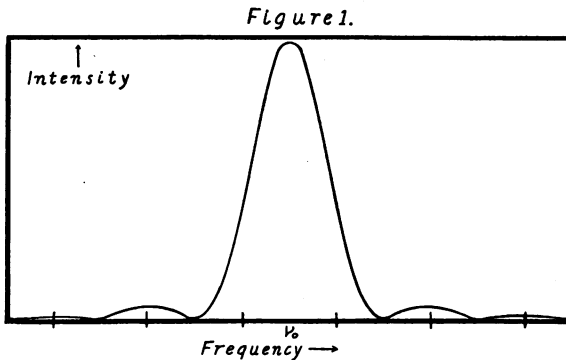
In much of the physics of this century, there is an element of indeterminism. It is possible that this will later disappear when we take into account for an indefinite time the whole sequence of events in a system, and in all systems with which it is connected. At present we may take the view that when, for example, we know exactly the energy difference between two states, and nothing more, we can say only that the most probable value of the energy given out in the transition is that difference.

The first step in resolving the indeterminism with which we are here confronted is to ascertain quantitatively the probability that the energy which is lost differs by a certain amount from that most probable value. We shall see that for this purpose we must know, not only the energies of the states, but also their temporal duration. We are familiar with the effect upon the behavior of a system of the proximity in space of other systems; we are less familiar with the effect upon a certain event of the

proximity in time of other events, yet the ideas which we are about to develop are largely those of classical physics.

First, we must assert with all possible emphasis that what we know about the *states* of a system is a deduction from the whole set of phenomena which the system exhibits. In other words, all that we observe is the behavior of a system, after which we invent a set of states which will account in the simplest possible manner for the phenomena observed. For example, we obtain a large number of spectral lines belonging to some gas and then find that these can be interpreted to be the result of transitions between a relatively small number of individual states.

Ordinarily it is advantageous to assume the smallest number of states which will account for the whole behavior, and the search for this minimum number of states is identical with the search for the number Ω ,



which plays so fundamental a part in statistics and thermodynamics. In some cases the particular states which it is necessary to assume are unique. A system with such properties may be called a "definitely quantized" system. In other systems, such as a monatomic gas which has only translational energy, the particular states may be assumed more or less arbitrarily, and only the minimum number of states necessary to account for the observed phenomena of the system is unique. Such a system may be called "arbitrarily quantized."

Returning to the case of the spectral lines, we may regard Bohr's law as an exact limiting law when the several events occurring within a system are infinitely remote from one another in time. In such case, the lines being infinitely sharp, there is no difficulty in ascertaining the states which account for these lines. When, however, the individual states are of short duration the lines are broadened. Can we still explain the phenomena without assuming a larger number of exactly specified states? Our answer, which we shall attempt to justify both qualitatively and quantitatively in the following pages, is that we can.

We shall assume that the states are exact but that the phenomenon observed in a given transition is never twice the same. Thus the energies ϵ_B and ϵ_A of two states are regarded as absolutely fixed, but the energy emitted in the transition will not be equal to $\epsilon_B - \epsilon_A$. It will fluctuate about this value, the average fluctuation being greater the shorter the lives of the individual states. We thus abandon the law of conservation of energy for an individual transition.

Generalizing now from the particular case of spectral lines to all phenomena of all systems, we may postulate the following as the first basic quantum law: I. *All of the phenomena of any system of finite energy may be interpreted by the assumption of a finite number of exactly determined states.*

As a corollary of this law, since the system must always be in one state or another, we may state as the second quantum law: II. *The transition from one state to another is instantaneous.* This corollary is of much significance since, both in the older theories of light and in the newer theories of mechanics, it has often been assumed that an exact mathematical statement concerning an undulatory field would require a knowledge of how the field is built up and how it dies out. According to our view, each field (at a given point) appears instantly, disappears instantly, and remains constant for the time of duration of the given state.

Finally, we may state a third independent quantum law which at present has only an empirical basis, and which we shall not use further in this paper, but is of vital importance to thermodynamics: III. *When the minimum number of states required to account for the behavior of an isolated system has been ascertained, each state will, in the course of time, be found just as often as any other state.*

The Uncertainty in Phenomena Caused by the Finite Life of States.—In classical electromagnetic theory, a system passing, through loss of radiation, from a state B to a state A was believed to generate in space a field, undulating according to the laws of simple harmonic motion. The Poynting vector derived from this field, integrated over a closed surface, gave the rate of loss of radiant energy.

Similarly, the present known facts of optics may be interpreted by assuming that a system in a given state may pass into any one of a number of other states, and that the probability of any one transition, let us say from B to A , is determined by a field, entirely analogous to the electromagnetic field, undulating with a frequency which is exactly equal to $(\epsilon_B - \epsilon_A)/h$. The integrated Poynting vector now gives what is statistically identical with the rate of flow of energy, namely, the probability that the energy associated with this process will be emitted in unit time.

In the electro-magnetic picture, the undulating field might change in amplitude owing to damping, and might perhaps change also in frequency.

Our present picture is a simple one, for we assume that the undulating field is constant both in amplitude and in frequency, from the instant that the state B appears to the instant that it disappears.

In neither picture does the constancy of frequency over a limited period of time imply the emission of monochromatic light.⁴ Any simple harmonic motion of limited duration is resolved, either by a Fourier analysis, or by physical apparatus such as a prism or a grating, into a whole continuum of frequencies. The amplitude, at a given distance from the central frequency, is determined by the duration of the simple harmonic motion.

In accordance with the newer trend in mechanics, we may assume that not merely a process in which light is evolved but any process whatsoever is associated with a similar undulating field, which will enable us to predict the probability that the process will occur. Since all of such fields have a temporal variation corresponding to simple harmonic motion, it will be understood that the remarks we are about to make concerning the process of light emission may be applied without essential change to all processes.

Let us assume that, just as in classical theory the squares of the amplitudes corresponding to different frequencies represent the amount of light emitted at these frequencies, so in our present case the squares of these amplitudes represent statistically the amount of energy emitted at the various frequencies when the system is made to drop repeatedly from the state B to the state A .

For simplicity, we shall assume that when a system is in the state B , it has not recently been in that state and will not for a long time appear in that state again. Our mathematical problem then consists in fitting a Fourier integral to a curve whose ordinate is zero from minus infinity to a certain point, follows a simple sine curve for a limited number of periods, and is then zero to plus infinity. The calculation is further simplified by assuming that the number of periods is integral and, though finite, is large. The necessary mathematical steps are given in Mathematical Note I. The result is that if P , is the probability that light of frequency ν will be emitted when the system passes from state B to state A ,

$$P_{\nu} = \frac{\sin^2 \pi \tau (\nu - \nu_0)}{\tau \pi^2 (\nu - \nu_0)^2}, \quad (1)$$

where ν_0 is $(\epsilon_B - \epsilon_A)/h$ and τ is the life of the state B .

This equation is plotted in figure 1. It is of the same form as the familiar diffraction curve for a single slit, showing again the essential similarity of time and space. One way to obtain a spectrum with intensities corresponding to figure 1 would be to interrupt a beam of monochromatic light with a shutter operating at a frequency, say, of 10^{-11} or

10^{-12} seconds, when a prism or a grating would give an intensity curve duplicating figure 1.

The General Problem of Uncertainty.—We have seen that by employing methods which are essentially classical we have arrived at the idea that even though the possible states of a system are exactly defined, there is no certainty regarding the phenomena which occur in transitions between these states. Our next step is to show that the uncertainty thus found entirely coincides with the quantitative statement of the uncertainty discovered by Heisenberg.

The *width* of a spectral line may be defined so that the product of this width by the maximum ordinate is equal to the total area under the intensity curve. The area determined by Equation 1 is unity, the maximum ordinate is τ and therefore the width, which we may call $\Delta\nu$ and use as a measure of the uncertainty in frequency, is,

$$\Delta\nu = \frac{1}{\tau}. \quad (2)$$

It is readily seen that in the present case the width coincides with that distance with respect to which the curve is periodic.

Hitherto we have assumed the life of a given state to be always the same. We shall show, however, (Mathematical Note II) that if the life is variable, following the same law as that of radioactive change, we obtain precisely the same equation for $\Delta\nu$ when $\bar{\tau}$ represents the average life. In such case, τ may be considered as the temporal uncertainty of the process, as $\Delta\nu$ represents the uncertainty in frequency. Representing by $\Delta\epsilon$ the uncertainty in the emitted energy, we may write,

$$\Delta\epsilon = h\Delta\nu, \quad (3)$$

an equation which according to the new mechanics has a far wider range of validity than the mere processes of light emission. Combining Equations 2 and 3

$$\bar{\tau}\Delta\epsilon = h. \quad (4)$$

This is one of the important relations of Heisenberg.

We may remark parenthetically, although it is aside from our main topic, that the equation for diffraction from a slit, which corresponds to Equation 1, leads to a similar relation between the width of the diffraction pattern and the width of the slit. Light diffracted by the slit suffers a change in momentum, which is not due to a change in its scalar magnitude, but to a change in direction. The change in momentum corresponding to the width of the diffraction curve we may call Δp and regard as a measure of the uncertainty in momentum due to the slit. The width of the slit l

may be regarded as the uncertainty in position of the light as it passes through. The calculation is entirely similar to the one we have just made, and gives

$$l\Delta p = h. \quad (5)$$

This is the second of the Heisenberg uncertainty relations which, however, we shall presently meet again in a case which has a more direct bearing upon our main problem.

Returning now to figure 1, we shall next examine the case in which a number of such curves overlap. Let us consider, as an example, the familiar case of a molecule which can exist in various vibrational states equally spaced in energy. We may assume that the energies are so nearly alike that the various states occur equally often. For further simplicity, we may ignore rotational levels, and we may also think only of those transitions which occur between the various other states and the zero state. We then have a series of emission lines of equal intensity, equally spaced in frequency, and all having the same width if the various states have the same life. Supposing now, without changing the energy levels, we gradually diminish equally the life of each state (in a similar way, but in a less random way, than would result from an increase in the number of molecular collisions). The emission lines now broaden. A remarkable phenomenon occurs when $1/\tau$ just equals the frequency difference between the spectral lines. Except at the beginning and end of the series, the curves now add exactly to give a horizontal straight line (see Mathematical Note III); in other words, except for the end portions, we obtain a continuous spectrum of constant intensity.

A similar phenomenon is deduced from the diffraction equation. Consider a plane wave of monochromatic light passing in a normal direction through a large number of equal, parallel, long and narrow slits, equally spaced. To one observing in the normal direction from the further side, the slits, if properly spaced, will disappear and seem to be replaced by a field of uniform illumination.

Returning to the vibrational spectrum, it is evident that the whole central portion of this spectrum, which in the particular circumstances that we have just described is entirely uniform, gives no evidence of any particular quantum states. Any section of it, however, can be interpreted by the assumption of a minimum number of quantum states, and this minimum number may be obtained by taking the first state at any arbitrary point and laying off the others at equal intervals, this interval being $1/\tau$. The example that we have given illustrates the sharp transition from the cases in which we determine *what* states must be postulated to account for the behavior of a system, to those cases in which we only determine *how many* states must be postulated. In other words, it is

the transition between systems which are definitely quantized and those which are arbitrarily quantized.

From this example, we obtain a clear picture of the meaning of quantization even in so extreme a case as that of a simple monatomic gas. We may, if we choose, interpret the phenomena exhibited by the gas by assuming an infinite number of perfectly distinct states, but while such a number of states may be sufficient, it is not necessary. Just as in the case of the vibrators, there is a minimum number of discrete states which will explain the observed phenomena, and here again the spacing of these states with respect to energy is determined by the relation

$$\tau\Delta\epsilon = h.$$

In this case, τ , the life of the states, is determined as before by the frequency of events. But these events are now the collisions of the molecules with each other or with the enclosing walls.

As with the vibrators, one of the states may be chosen quite arbitrarily and then the others are laid off at the proper intervals. This produces no ambiguity in determining Ω , except in the immediate neighborhood of the zero state, or the state of lowest energy. Regarding conditions in this small neighborhood, we know nothing at present except that they are of no importance in the great majority of systems with which we have to deal. It remains to show that the minimum number of states which we thus set up is identical with the number of cells into which the phase space is ordinarily divided for statistical purposes, which, in turn, has been shown by Schrödinger to be equal to the number of solutions of his mechanical equations.

Before doing so, lest there be any confusion, we may point out that, in the general case, the states of which we have been speaking are states of the system as a whole. Each event that we have to consider is some transition from one state to another state of the whole system. It sometimes happens, however, and this is true for the ideal vibrator, for the ideal monatomic gas, and for radiation, that we can assign states to the individual particles; and that the properties, such as the energy, of these states are not materially affected by the presence of other particles. In such ideal cases, our procedure is greatly simplified, for if the several atoms are alike and therefore can be assigned the same series of possible states, the minimum number of states Ω for the whole system is merely the number of ways in which all the atoms may be distributed among the several states, in such a way as is consistent with whatsoever conditions are imposed.

In order to show that the minimum number of atomic states which we derive is the same as the number of cells ordinarily used in statistics, let us begin by adopting Ehrenfest's expedient of dealing with a hypothetical

one-dimensional gas. Here we shall say that the life of an atom in a given state is the time that elapses after striking one wall before meeting the opposite wall, assuming that collisions between molecules may be neglected. This is the same for all particles of the same velocity v and if l is the distance between walls,

$$\tau = \frac{l}{v}, \quad (6)$$

but from the fundamental relationship (4),

$$\tau \Delta \epsilon = h,$$

where $\Delta \epsilon$ is the difference of energy between successive states in the particular region of velocity we are considering. This difference in energy requires a difference in momentum Δp . According to the laws of mechanics,

$$\Delta \epsilon = v \Delta p. \quad (7)$$

Eliminating τ and $\Delta \epsilon$ from these three equations, we find finally,

$$l \Delta p = h. \quad (8)$$

If we regard the individual states as defined merely by the momentum, nothing being said about the position of the particle, l may be regarded as the uncertainty in position, while Δp , the difference between the states, is the uncertainty in momentum. Just the same equations are obtained for photons when we consider a one-dimensional Hohlraum.

Proceeding to three dimensions, we may choose arbitrarily three perpendicular directions, x , y and z . We may then define the states of a particle by allowing the momenta p_x , p_y and p_z to assume independently certain discrete values. The problem is to determine the distance apart of these successive values of momentum.

Let us consider two adjacent states in which p_y and p_z are the same, but differing by Δp_x . Designating by $\Delta_x \epsilon$ the difference in energy, we have from mechanics,

$$\Delta_x \epsilon = v_x \Delta p_x, \quad (9)$$

where v_x is, in states of this neighborhood, the velocity in the direction x .

The determination of τ , or what we may here call τ_x , is by no means so obvious as in the one-dimensional case. We may, however, consider the gas as contained in a rectangular box with dimensions l_x , etc., and write in a somewhat formal way,

$$\tau_x = \frac{l_x}{v_x}. \quad (10)$$

Now combining the last two equations with the fundamental Equation 4, we find,

$$l_x \Delta p_x = h. \quad (11)$$

We thus see that our quantization corresponds entirely with the ordinary partition of a phase space into cells.

We shall not at present discuss the more complicated case in which the pressure is so high that the life of a state is determined by collisions between particles rather than by collisions with walls. It should, however, be remarked that the complex phenomenon of the broadening of spectral lines through pressure is probably, in its simplest form, merely the result of shortening the lives of the states through collisions. Hitherto it has been possible to give only a vague definition of a collision, but we may now define it precisely. A collision is any event depending upon the proximity of other molecules which consists in a change of state of one or more molecules. Since the states are usually very close together these collisions occur thousands or millions of times as frequently as the very pronounced collisions of the old kinetic theory. Intermediate between these two are such collisions as have been experimentally studied by Knauer and Stern,⁵ where a molecular ray is slightly deflected by gas at low pressure. They found such collisions to occur much more frequently than the collisions of classical theory.

Mathematical Note I.—Starting with a function which has the value

$$f(t') = \alpha \sin 2\pi\nu_0 t' \quad (12)$$

between $t' = -\tau/2$ and $t' = +\tau/2$ (where τ is an integral multiple of the period $1/\nu_0$), and which has the value zero at all other points, this function may be replaced by the Fourier integral

$$f(t) = 2 \int_0^{+\infty} \int_{-\infty}^{+\infty} f(t') \cos 2\pi\nu(t' - t) dt' d\nu, \quad (13)$$

or in differential form,

$$\frac{\partial f(t)}{\partial \nu} = 2 \int_{-\infty}^{+\infty} f(t') \cos 2\pi\nu(t' - t) dt'. \quad (14)$$

The limits of integration may be replaced by $-\tau/2$ and $+\tau/2$, since $f(t')$ is zero outside these limits, and substituting Equation 12, we may perform the integration and obtain

$$\frac{\partial f(t)}{\partial \nu} = \frac{2\alpha\nu \sin \pi\tau(\nu - \nu_0)}{\pi(\nu^2 - \nu_0^2)} \sin 2\pi\nu t. \quad (15)$$

If only those frequencies are significant which are very close to ν_0 , as is

the case when τ contains a large number of periods, the equation reduces to the simpler form,

$$\frac{\partial f(t)}{\partial \nu} = \frac{\alpha \sin \pi \tau (\nu - \nu_0)}{\pi (\nu - \nu_0)} \sin 2\pi \nu t. \quad (16)$$

The coefficient which does not involve t may be regarded as a sort of amplitude, a_ν , that is,

$$a_\nu = \frac{\alpha \sin \pi \tau (\nu - \nu_0)}{\pi (\nu - \nu_0)} \quad (17)$$

The energy at each frequency is proportional to a_ν^2 , the integral of which over all values of ν proves to be $\alpha^2 \tau$. We may therefore normalize the expression for a_ν^2 if we divide by this integral. Thus, if I_ν is the ratio between the energy at a given frequency to the total energy,

$$I_\nu = \frac{\sin^2 \pi \tau (\nu - \nu_0)}{\tau \pi^2 (\nu - \nu_0)^2}. \quad (18)$$

Since we have assumed that all the significant frequencies are nearly the same, we may replace I_ν by P_ν , where $P_\nu d\nu$ represents the probability that when the transition occurs between state B and state A , the light emitted will have a frequency lying between ν and $\nu + d\nu$.

Mathematical Note II.—It is possible that when we assume the existence of precise quantum states, the life of each state should also be regarded as exactly determined. In that case, it would be necessary to consider such experiments as those of Wien as showing only an *apparent* variability in the life of a state, in the same sense that a study of the energy emitted in a transition shows an apparent variability in the difference of energy between the two states. Nevertheless, we may, for the moment, assume that if we have a large number of identical systems undergoing the same transition, the fraction of these transitions occurring per unit time, $\partial x / \partial t$, follows the simple law of exponential decay,

$$\frac{\partial x}{\partial t} = \frac{1}{\bar{\tau}} e^{-t/\bar{\tau}}, \quad (19)$$

where $\bar{\tau}$ is well known to be the average life. If τ is the actual life in any one system, then all the systems in which the transition occurs at the end of time t have $\tau = t$ and the fraction of these is

$$dx = \frac{1}{\tau} e^{-\tau/\bar{\tau}} d\tau. \quad (20)$$

If we write $P_{\nu, \tau}$ for the characteristic probability of Equation 1 for systems of life τ ,

$$P_{\nu, \tau} = \frac{\sin^2 \pi \tau (\nu - \nu_0)}{\tau \pi^2 (\nu - \nu_0)^2} \quad (21)$$

Then the characteristic probability at the frequency ν , of all the systems, is obtained by integrating $P_{\nu, \tau} dx$. By the last two equations,

$$P_{\nu} = \int_{\tau=0}^{\tau=\infty} \frac{e^{-\tau/\bar{\tau}} \sin^2 \pi \tau (\nu - \nu_0)}{\bar{\tau} \tau \pi^2 (\nu - \nu_0)^2} d\tau. \quad (22)$$

This is a known integral (see Bierens De Haan, Tables D'Intégrales Définies, Leiden, 1867, Table 365, 4) and,

$$P_{\nu} = \frac{-\log[1 + 4\pi^2 \bar{\tau}^2 (\nu - \nu_0)^2]}{4\pi^2 \bar{\tau}^2 (\nu - \nu_0)^2}. \quad (23)$$

The total integral of P_{ν} over all values of ν must, of course, be unity, and is so found when the integration is performed (De Haan, Table 134, 12). Since the area under the curve of P_{ν} is unity the width of the curve $\Delta\nu$, as we have defined it, is the reciprocal of the maximum ordinate. Finding this ordinate to be $\bar{\tau}$ by making $\nu = \nu_0$,

$$\Delta\nu = \frac{1}{\bar{\tau}}, \quad (24)$$

which is the same as Equation 2, except that $\bar{\tau}$ is now the mean life. Equation 23 gives curves which, when spaced at a distance equal to their width, do not, like the curves of Equation 1, add together to give a straight line, but an undulating curve.

Mathematical Note III.—The proof that curves corresponding to Equation 1 add together to a straight line when uniformly spaced at a distance $\Delta\nu = 1/\tau$ is as follows. Consider an abscissa at a distance d from one of the maxima. If n assumes all integral values, the sum of the ordinates at this point will be

$$\sum P = \sum_{n=-\infty}^{n=+\infty} \frac{\tau \sin^2 \pi \tau \left(d + \frac{n}{\tau}\right)}{\left[\pi \tau \left(d + \frac{n}{\tau}\right)\right]^2} = \sum_{n=-\infty}^{n=+\infty} \frac{\tau \sin^2(\pi \tau d + n\pi)}{(\pi \tau d + n\pi)^2} \quad (25)$$

Since the term $n\pi$ contributes nothing to the sine it may be omitted, and removing from the sum the invariant quantities, we find,

$$\sum P = \frac{\tau \sin^2 \pi \tau d}{\pi^2} \sum_{n=-\infty}^{n=+\infty} \frac{1}{(\tau d + n)^2} \quad (26)$$

This sum is known, and is equal to $\pi^2/\sin^2 \pi r d$, so that at every point, the sum of the ordinates amounts to τ .

¹ Lewis and Mayer, these PROCEEDINGS, 14, 569 and 575 (1928).

² W. Heisenberg, *Zeits. Physik*, 43, 172 (1927).

³ E. Schrödinger, *Phys. Zeits.*, 27, 95 (1926).

⁴ For a discussion of this phenomenon, see W. C. Mandersloot, *Jahrb. Radioakt. und Elektronik*, 13, 1 (1916). Since, however, our assumptions are different from his, our equations are of a different character.

⁵ Knauer and Stern, *Zeits. Physik.*, 39, 764 (1926).

THE TRANSFORMATION SPECTRUM OF THE RUBY*

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Read before the Academy November 19, 1928

The luminescence of the ruby under cathodo- and also under photo-excitation has been investigated, among others, by G. C. Schmidt,¹ J. Becquerel,² Miethe,³ DuBois and Elias,⁴ Mendenhall and Wood,⁵ K. S. Gibson.⁶ As the result of their studies the following characteristics have been established:

1. The activator is chromium, presumably in the form of a chromium oxide, in solid solution in aluminium oxide.

2. The sensitiveness of Al_2O_3 to activation by chromium is extreme—so much so that it is difficult to procure or produce Al_2O_3 which does not glow with the characteristic ruddy glow of the ruby when subjected to cathodo-bombardment.

3. Strictly pure Al_2O_3 does not exhibit luminescence of this type.

4. The most striking feature of the luminescence spectrum is the well-known doublet of narrow bands at 0.6918μ and 0.6932μ .

5. Coinciding as to wave-length with these are narrow absorption bands and it is possible by suitable adjustments to get reversals suggestive of the reversals of the bright lines of flame spectra.

6. In addition to these bands there are in the spectrum various other narrow bands and that these correspond to absorption regions is strikingly shown by maps of the red portion of the spectrum by DuBois and Elias.

7. The bands of both luminescence and absorption are narrowed to line-like proportions by cooling the crystal under observation to the temperature of liquid air, a fact that was utilized by DuBois and Elias in their search for the Zeeman effect.

The similarity of the phenomena summarized above to those brought out in our own studies of the luminescence of the uranyl salts and of solid