need for experimental verification has entirely vanished, for in the deduction we assumed as an experimental fact that the Peltier heat is reversible, that is, that on reversing the direction of current through a junction a development of heat becomes a precisely equal absorption. This assumption about the nature of the Peltier heat was also made by Kelvin, and is evidently distinct from the additional assumption contained in his thermodynamic argument. Perhaps the simplest experimental method of showing the reversibility of the Peltier heat would be to show that the e. m. f. of a couple is the same on open and closed circuit. The reversibility of the Thomson heat with direction of current flow does not seem to be necessarily involved in relation (1) and (2), except as E and σ are by implication determined only by temperature and material.

[1] This matter is discussed in greater detail in: P. W. Bridgman, *Phys. Rev.*, **14**, 306–347, 1919.

ON THE NATURE OF THE TRANSVERSE THERMO-MAGNETIC EFFECT AND THE TRANSVERSE THERMO-ELECTRIC EFFECT IN CRYSTALS

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It is known that there is a close formal parallelism between the Ettingshausen temperature gradient set up in an isotropic conductor at right angles to an electric current and a magnetic field, and Kelvin's temperature difference between two opposite sides of a crystal rod, the length of which is oblique to the crystal axis, and in which a longitudinal electric current flows. Not only are these effects geometrically similar, but both reverse sign when the direction of current flow changes, and are, as far as known, proportional to current strength. The parallelism may be carried through for the other effects. The analogue of the Nernst transverse e. m. f. under a longitudinal heat current is the longitudinal e. m. f. in a crystal produced by a transverse temperature difference; the analogue of the Hall effect is the transverse potential gradient in a crystal carrying a longitudinal electric current arising from the fact that the lines of current flow are not perpendicular to the equipotential surfaces, and similarly the analogue of the Righi-LeDuc transverse temperature gradient is the transverse temperature gradient in a crystal carrying a longitudinal heat current arising from the fact that the lines of heat flow are not perpendicular to the surfaces of constant temperature. One might therefore be tempted to look for an underlying similarity of mechanism in a crystal and an isotropic substance made non-isotropic by a magnetic field; it is the purpose of this note to show that in spite of the close parallelism there is a fundamental difference of sign in the relations connecting the Ettingshausen with the Nernst coefficient and their corresponding crystal analogues, which indicates that the effects must be essentially different in character.

There are two diametrically opposite ways of regarding the Ettingshausen effect. The first of these I presented to the Solvay Congress in 1924, and published later in the *Physical Review*.[1] According to this point of view the Ettingshausen temperature difference between two opposite sides of a plate carrying a longitudinal electric current in a magnetic field is not accompanied by a transverse flow of heat from the hot to the cold side of the plate, but the temperature difference is maintained without heat flow, by what I called a "thermomotive" force, precisely as the two terminals of a battery on open circuit are maintained at a difference of electric potential by the "electro-motive" force of the battery. The difference of temperature between the two sides of the plate may be utilized to operate a thermodynamic engine, allowing heat to flow from the hot side through the engine to the cold side, the return flow being from *cold to hot* in the plate. When such a heat current flows between the sides of the plate, a longitudinal e. m. f. is set up by the Nernst effect (the Nernst effect and the Ettingshausen effect thus appear as one the inverse of the other), and the longitudinal electric current flowing against this Nernst e. m. f. puts into the system the energy extracted by the thermodynamic engine. On equating the energy put in to the energy extracted, the relation $Q = kP/\tau$ is found between the Nernst coefficient Q, the Ettingshausen coefficient P, thermal conductivity k, and absolute temperature τ. This relation is checked by experiment within the accuracy of the measurements.

The second point of view with regard to the Ettingshausen effect is that of Lorentz,[2] also presented to the Solvay Congress. According to this, the Ettingshausen temperature difference is accompanied by a transverse current of heat in the plate by conduction from the hot to the cold side. This point of view may also be made to give a quantitative relation. The argument which I now give is not that employed by Lorentz, but is patterned after one given by me in the preceding paper on the application of thermodynamics to the thermo-electric circuit. The continual conduction of heat in the plate means a continual dissipation of available energy, which means an increase in the entropy of the entire universe, which under these conditions demands a continual rise of the average temperature of the plate. This continual rise of the temperature of the plate must be at the expense of energy put into the system by the current, and the source of this is obviously the Nernst e. m. f. associated with the transverse heat flow. Working out the quantitative relations gives exactly the same result as before except for a difference of sign, the result now being $Q = - kP/\tau$. The reason for the difference of sign is obvious, for according to the second point of view the current puts energy into the system when the transverse heat current in the plate is from hot to cold, whereas according to the first point of view, energy is put in when the heat current is from cold to hot. Since the negative sign is directly contrary to experiment, the first point of view must be the correct one, and the necessity for the concept of thermo-motive force is established.

Lorentz, in the discussion which he added in the printed Solvay Report several years after the meeting of the Congress, clearly formulated the essential difference between these two points of view, and recognized the necessity of the concept of thermo-motive force. He also commented on the care which must be used in applying the le Chatelier principle to cases like this. Suppose that we apply a transverse temperature difference to opposite sides of a plate in a magnetic field. This temperature difference is accompanied by a transverse heat flow, which gives rise to a longitudinal e. m. f. by the Nernst effect. Now complete the external circuit, allowing a longitudinal electric current to be driven by the Nernst e. m. f. This electric current will produce a transverse temperature difference by the Ettingshausen effect. In what direction is it? It is natural to expect that it must be in such a direction as to diminish the applied temperature difference. But detailed analysis will show at once that such a diminution of the temperature difference would demand the minus sign in the relation between Q and P which is given by the second point of view, and which is contrary to experiment. The Ettingshausen effect is in such a direction that the transverse temperature difference becomes greater. The le Chatelier principle can be maintained only by noting that, in spite of the greater transverse temperature difference, the transverse heat flow has become less, because of the thermomotive force tending to drive heat up the temperature gradient. In other words, in order to maintain the le Chatelier principle, heat flow must be regarded as the fundamental thing and not temperature difference. It is obvious that the le Chatelier principle might easily prove a false guide in fresh fields.

The transverse temperature difference in crystals predicted by Kelvin may now be subjected to an analysis exactly like that above for the Ettingshausen effect. The transverse temperature difference is an experimental fact; the question is whether this temperature difference is maintained without heat flow by a thermo-motive force in the crystal, or whether there is a continual flow of heat and so continual dissipation of energy when a steady current flows obliquely in a crystal. The answer to this can be given by examining the sign of the inverse effects. If opposite sides of a long crystal bar oblique to the axis are maintained at a difference of temperature, a longitudinal e. m. f. arises, which will drive a current through the bar on completing the external circuit. This current produces in turn a transverse temperature difference—does this oppose or aid the original temperature difference?

The inverse effect, that is, the longitudinal e. m. f. generated by a transverse temperature difference, is very easy to observe, but as far as I am aware, little attention has been paid to the sign of the effect. I have just found with a single crystal of bismuth that the current generated by the inverse effect is in such a direction as to decrease the original temperature difference. That is, the direct and the inverse effect are connected by a relation with the opposite sign from that of the Nernst-Ettingshausen effects. A crystal is not the seat of a thermo-motive force, but if the transverse difference of temperature is allowed to establish itself, there is a continual heat current, and continual dissipation of energy.

The precise connection between the direct and the inverse effect in a crystal may now be found by the counterpart of the argument already indicated. Consider a crystal plate of breadth b and unit depth carrying a longitudinal current of density i (total current ib), and assume for simplicity that the crystal has rotational symmetry about the crystal axis (as do all metals yet investigated) and that this axis lies in the plane of the face b, and makes an angle θ with the length. Furthermore, suppose the bar so long in comparison with the breadth that the Peltier heat at the ends may be neglected in comparison with the transverse heating effects. Consider now a piece of this plate of unit length. Let T denote the transverse generation of heat per unit area of transverse face per unit current density, and let k denote the transverse thermal conductivity. T and k are in general functions of the angle θ, but the precise nature of the functional relation does not concern us here. The difference of temperature between the two sides of the plate is evidently biT/k and the increase of entropy in unit time due to irreversible heat flow transversely is $(biT/k)(iT/\tau^2) = (bi^2T^2)/k\tau^2$. Denote the longitudinal e. m. f, per unit length per unit transverse heat current by e_t. Then the e. m. f. in this case is e_tiT, the energy input of the current against this e. m. f. is e_ti^2Tb, the entropy rise per unit time due to this energy input is e_ti^2Tb/τ, and equating this to the rise of entropy due to irreversible heat conduction gives

$$e_t = \frac{T}{k\tau},$$

the connection between the direct and the inverse effects.

The sign convention with regard to e_t has already been suggested and the positive sign in the relation just deduced has been found to agree with experiment. A direct (quantitative) experimental verification of the relation is not superfluous, however, as it is conceivable that there might be still some transverse thermomotive action, of less than the full amount. It would be interesting also to find whether the transverse heat current or the transverse temperature gradient is active in the production of the longitudinal e. m. f. This might be tested by finding whether a crystal bar, in which a transverse thermo-motive force is developed by a longitudinal current in a magnetic field, but in which there is no transverse heat flow, is the seat of a longitudinal e. m. f.

It is interesting to carry the analysis a little further. If there is no development of heat within the body of the crystal, the net rise of temperature accounted for by the energy input of the current must be described in thermal terms as due to a difference of the transverse heat at the two faces which are at different temperatures. This gives: $\Delta\tau \frac{d}{d\tau}(iT) = e_t . Ti . ib$, and substituting the values already found for $\Delta\tau$ and e_t gives $\frac{dT}{T} = \frac{d\tau}{\tau}$, or $T = c\tau$, exactly as in the analysis for the ordinary Peltier heat, when the Thomson heat is neglected. We may suspect that this relation will be found not to agree with experiment, and that, therefore, there must be a generation of heat in the body of the crystal when a longitudinal electric current flows across a transverse temperature gradient, that is, a transverse Thomson heat. Denote by σ_t the heat so absorbed per unit time per unit depth per unit temperature difference per unit current density. The equation of energy balance is now

$$\Delta\tau \left[i\sigma_t + \frac{d}{d\tau}(iT) \right] = e_t . Ti . ib,$$

whence

$$\sigma_\tau + \frac{dT}{d\tau} = \frac{T}{\tau}.$$

Expressing T in terms of e_t gives at once

$$\sigma_\tau = -\tau \frac{d(e_t k)}{d\tau}$$

for the transverse Thomson heat.

The precise analogy between these formulas and those for the ordinary thermo-electric circuit is at once obvious, T taking the place of the Peltier heat, σ_t the ordinary Thomson heat, and $e_t k$ the thermoelectric power (that is, the e. m. f. per unit temperature difference) of the couple. The method of argument used here is the same as that in my previous paper on the application of thermodynamics to the thermo-electric circuit, so that the uncertainty in the result of Kelvin arising from neglect of the irreversible effects is avoided, and we may be sure that these relations are the rigorous result of thermodynamic principles, with no assumptions involving the neglect or irreversible aspects of the phenomena.

The formal thermo-magnetic analogy of the Thomson transverse effect in crystals is an absorption of heat by a heat current flowing transversely in a bar carrying a longitudinal electric current of density i in a perpendicular magnetic field in amount equal to the fraction PHi/T of itself per unit length measured transversely. This may be proved at once from the equation of energy balance.

[1] P. W. Bridgman, *Phys. Rev.*, Dec., 1924, 644–651. Report of the Fourth Solvay Congress, "Conductibilité Électrique des Métaux," 352–354.

[2] H. A. Lorentz, *Fourth Solvay Congress*, 354–360.

ON THE RED SHIFT OF SPECTRAL LINES THROUGH INTERSTELLAR SPACE

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A. *Introduction.*—It is known that very distant nebulae, probably galactic systems like our own, show remarkably high receding velocities whose magnitude increases with the distance. This curious phenomenon promises to provide some important clues for the future development of our cosmological views. It may be of advantage, therefore, to point out some of the principal facts which any cosmological theory will have to account for. Then a brief discussion will be given of different theoretical suggestions related to the above effect. Finally, a new effect of masses upon light will be suggested which is a sort of gravitational analogue of the Compton effect.

B. *Discussion of the Observational Facts.*—(1) E. Hubble[1] has shown recently that the correlation between the apparent velocity of recession and the distance is roughly linear, corresponding to 500 km./sec. per 10^6 parsecs. Large deviations occur for the nearest nebulae, which may be attributed to their peculiar motions. The most recent observations by M. Humason[2] seem to indicate that for very large distances (50×10^6 light years) the individual deviations become so great (3000 km./sec. out of 8000 km./sec.) that they hardly can be due to peculiar motions and must, therefore, be accounted for in some other way.

(2) The relative shift of frequency $\frac{\Delta\nu}{\nu}$ representing the velocity of recession is apparently independent of the frequency. The available range