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THE IMPULSE MOMENT OF THE LIGHT QUANTUM

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§1. For some years it has been customary to treat the light quantum as a particle and to emphasize those characteristics of the electron which point to its wave-like character. This idea, beautifully substantiated by the success of wave dynamics, has led Beck[1] to assert that for each property of the electron we may hope to find a corresponding property of the light quantum, and vice versa. The spin of the electron, discovered independently by Goudsmit and Uhlenbeck[2] and by Bichowsky and Urey,[3] should then have its analogue in the structure of the quantum. We feel that this idea presents difficulties, but we shall develop some of its consequences and shall discuss available data which have a bearing on it.

After the ideas which we shall present had been developed, a paper by Jordan[4] came to hand in which he accounts for the interaction of polarized light with analyzers by applying the concepts of the statistical interpretation of quantum dynamics. He determined the probability that a quantum which has passed an appropriate polarizing apparatus will be transmitted by an analyzer set at any desired angle with the polarizer. Formally, he has found it possible to associate with polarized light a "quantum-dynamical magnitude" descriptive of its state of polarization. This enables him to derive the probability that it passes the analyzer by Pauli's method for determining the probabilities that the spin vector of the electron entering a magnetic field will take a position parallel or antiparallel to the lines of force. Granting for the moment that the polarization of the quantum and the spin of the electron are related phenomena, the exact correspondence postulated by Jordan seems to us only a special case of more general possibilities. Here we shall not adopt the language of the statistical interpretation of atomic mechanics, but shall speak of individual atoms and quanta, subject to laws which render their behavior definite. A translation of the *physical* ideas involved into the terminology of the theories advocated by Heisenberg, Dirac, and Jordan will probably not be difficult. The concepts with which we deal are the energy, momentum and angular momentum of atoms and quanta. We must examine to what extent this is allowable. Schrödinger[5] has pointed out that the concept of atomic energy may be replaced by that of frequency of the matter-waves. The interpretation of momentum in terms of the wave frequency is simple. For a single particle moving with velocity v the relation between momentum M and energy E ($= h\nu$) is $M = h\nu\, v/c^2$. The extension to other systems is obvious. As for the interpretation of the "angular momentum of an atom," Hund's theory of spectral terms was founded on the vector properties of the quantum numbers, s, l, j, m, etc. Wigner[6] and Hund[7] have pointed out the way in which the enumeration of spectral terms can be carried out in wave mechanics without reference to angular momentum. Their conclusions are in general agreement with the original derivation of the theory and, therefore, it seems relatively unimportant whether we speak in terms of wave mechanics or of additions of angular momentum vectors. The situation is quite similar to the discussion of a branch of analysis in the language of geometry. With this understanding of the meaning of energy, momentum and angular momentum we proceed to discuss some suggested theories of light quanta.

Beck's concept of unpolarized light is an aggregate of quanta with intrinsic impulse moment vectors oriented at random, while a linearly polarized beam is analogous to a beam of electrons which have been oriented by a magnetic field. Beck makes no assumptions as to the magnitude of the impulse moment vector, in any case, except that it is not zero in linear light; nor does he specify its orientation in the various types of polarized light. In what follows we shall designate this vector by $\mathbf{r}h/2\pi$ and shall call r the *radiation* quantum number.

Our problem is to examine the values which it can assume in various types of light, consistent with known data, and to suggest experiments which may yield further information.

§2. *The Selection Principle for* j.—In the older theories the selection rules for the inner quantum number, j, could be derived by considering the conservation of angular momentum in the system, atom + field of radiation. The quantum emitted had angular momentum $h/2\pi$ or 0 according as the change in j was ± 1 or 0. In all cases the direction of j

was preserved during emission. If we assume that $r = 1$ for every quantum the same selection rule is obtained. The transitions in which j changes by unity occur just as in the older theory, but those in which j is unchanged in magnitude are accompanied by a change in its direction so that the closing side of the angular momentum triangle equals $h/2\pi$, representing the impulse moment of the light. The transition $0 \longrightarrow 0$ is impossible on this theory because a system devoid of impulse moment cannot give angular momentum to the radiation field and still have $j = 0$. This condition could not be derived in the older theory by using the correspondence principle or considerations of angular momentum, but can be derived by matrix mechanics. Of course if the q's of the matrix theory are interpreted as characteristic of the individual atom, the angular momentum changes in every emission can be calculated, resulting in a confirmation of older theory. On the other hand, the statistical interpretation of quantum mechanics means that the q's of matrix theory are related to probabilities of radiation, i.e., to average amplitudes from an aggregation of atoms. Until the interpretation of the q's is more definitely known, no decision as to the nature of the individual quanta can be definitely made. For example, linearly polarized light from an aggregate of atoms may be a superposition of linear quanta; or of circular or elliptic quanta with *appropriate phase relations.* The decision as to whether r is always one, or whether it is sometimes one and sometimes zero, is closely bound up with a decision between the macroscopic statistical interpretation of the q's and the view that they are characteristic of the individual atom.

An important fact in favor of the assumption that r is always one is that linearly polarized light can raise Hg atoms from 1^1S_0 to 2^3P_1, a transition involving a gain of $h/2\pi$ in angular momentum.[8] This is hard to explain in terms of quanta for which $r = 0$, unless we abandon the principle that $\mathbf{j} + \mathbf{r}$ is a conservative quantity.

It remains to consider the possibility that r is sometimes 2, 3, ... or perhaps $^1/_2$, $^3/_2$, ... No case is known in which $\Delta j = 2, 3, \ldots.$ except in the presence of external fields. It seems possible that in such cases the integer j assigned to the terms by empirical methods may not have the significance of angular momentum. Half integral values seem to be excluded by the selection principle for j unless radical changes in the interpretation of j are introduced.

Incidentally the state of polarization of the light emitted by individual atoms has not been determined for obvious experimental reasons. An almost impossible experiment, which would contribute valuable information, is to send a beam of atoms through an inhomogeneous magnetic field and excite the resonance radiation of the Stern-Gerlach beams after they emerge from the field. A much more reasonable experiment would be to place a resonance bulb in a magnetic field strong enough to produce complete orientation of the atoms (perhaps a hundred gauss). Let the field be removed very quickly. Presumably the atoms retain their orientation for a time which is very long compared to τ, the life of an excited atom, so that their resonance radiation will inform us as to the polarization of the light emitted at various angles with the j-vectors of the atoms. For example, the mean free path in a mercury resonance bulb at -20 °C. is of the order of 500 cm., which is traversed in about 10^{-3} sec. It might be thought that the limiting factor would be the field of the neighboring Hg atoms. On the average, one atom is present in every cube whose side is 1 μ. It must be remembered that experiments on the polarization of resonance radiation in the absence of a magnetic field, indicate almost complete persistence of the orientation of an excited atom. It is possible that with suitable arrangements of Kerr cells and alternating magnetic fields the experiment could be performed by continuous wave methods. A very desirable modification of this experiment would be one in which the exciting light and the magnetic field are suddenly cut off with a slight lag of the magnetic field behind the light, the resonance radiation being observed only after the removal of the field. Such an experiment would be very difficult if possible at all, but would furnish a definite test of the polarization of the light from an atom oriented by a magnetic field after its excitation.

§3. We see from the preceding section that the selection principle for j does not permit us to decide between the hypotheses, $r = 1$ and $r = 1$ or 0. We now consider emission in a magnetic field of strength suitable to produce the ordinary Zeeman effect. The older theory stated that when Δm, the change of the magnetic quantum number, is $\pm$ 1 the light has angular momentum $h/2\pi$; but when $\Delta m = 0$, the angular momentum is zero. In the first case the emitted light has the properties of a circularly polarized wave with axis parallel to the field and in the latter case it resembles the light from a linear oscillator moving parallel to the lines of force. Let us examine whether these experimental results can be explained by quanta all of which have angular momentum $h/2\pi$. Anticipating we find that this can be done by postulating a certain type of coherence between the individual quanta of a monochromatic beam.

Formally, we may consider the atom and the field-producing magnet as a single system having the quantized angular momentum $\mathbf{J}$ (in quantum units) $= \mathbf{j}_f + \mathbf{j}$, where $\mathbf{j}_f$ is the angular momentum of the field-producing mechanism, i.e., the electrons flowing in the magnet coils. j_f and j precess about J with the frequency of the Larmor precession and m is very nearly the projection of j on J because J and j_f nearly coincide in direction, both being very large compared with j. The various quantized orientations of j with respect to j_f, following the rules for coupling of vectors inside the atom, give the usual spatial orientations of j with respect to J. This model gives us a useful understanding of the mechanism of orientation in a magnetic field. It suggests that this mechanism is the same as that which governs the coupling of the vectors belonging to individual electrons in the atom. Now that the quantization of the helium atom[9] seems to be achieved by means of the Schrödinger theory, there can be little doubt that this orientation can be described by the methods of wave mechanics applied to a model composed of one very large and one very small rotator. The problem is to follow the structure of the ψ function for such a model while the angular momentum of the large rotator grows from zero to a value very large compared to $h/2\pi$ according to any mathematically simple law of increase. It may be that the orientation can be followed by applying the relativistic wave equation to the atom alone, retaining terms of a higher order than those used in previous discussions of the way in which the Larmor precession is set up.

When $\Delta m = \pm 1$, $\Delta J = \pm 1$ and the emitted light is circularly polarized with axis along J. When $\Delta m = 0$, or $\Delta J = 0$, it is linearly polarized with the electric vector parallel to J. This model is very artificial, but if results from it can be trusted, it shows that the polarization of the light emitted by a field-free atom is in agreement with the predictions of the older theory. But, just as in §2, it may be that the linearly polarized light of the p-components is made up of light from which $r = 1$. We see that when $\Delta m = \pm 1$, the r-vector is parallel to J, i.e., very nearly parallel to H. Light of the s-components seen along the field will be left or right circularly polarized according as r is parallel or antiparallel to the direction of flight. If an s-component is observed perpendicular to the field the r-vector stands perpendicular to the direction of flight and the electric vector. In the elliptic light emitted at an angle ϑ with the field the r-vector stands at an angle ϑ to the direction of flight and in a plane perpendicular to the plane defined by the line of flight and the major axis of the ellipse. The case of the p-components is more difficult. Here the r-vectors lie in a plane perpendicular to H. Let φ be an azimuth measured in this plane and let the eye of the observer be located in the line $\varphi = 0$, looking toward the origin. Quanta having their r-vectors along the lines $\varphi = 90°$ and $\varphi = 270°$ will give linearly polarized light along $\varphi = 0$, but quanta with r-vectors at 0° and 180° appear circularly polarized when emitted along $\varphi = 0$. Intermediate orientations of the r-vector give elliptic light along $\varphi = 0$. But now we make one of two assumptions: (1) The phase relations of the circular and elliptic quanta emitted along $\varphi = 0$ are such that the resultant light is linearly polarized; (2) only the quanta with r-vector at $\varphi = 90°$ and $\varphi = 270°$ are emitted along $\varphi = 0$. Hypothesis (2) is incapable of explaining the 80% vertical linear polarization of Hg resonance radiation in the absence of a magnetic field, when the exciting electric vector is vertical.[10] Hypothesis (1) does not seem satisfactory to the authors as compared with the simple assumption that $r = 0$ for linear light, but it is difficult to disprove on purely experimental grounds. Ideas similar to (1) have been suggested by Hanle[11] and by Breit, Ruark and Brickwedde.[12] Indeed (1) is simply a special case of the general idea that phases and amplitudes of a macroscopic radiation field have only a statistical significance. The usual formulation is that at a given point in a beam of light moving parallel to the x axis, E_y^2 and E_z^2, together with the phases of E_y and E_z, determine the number and the *resultant* polarization of the quanta arriving at that point.

With the aid of hypothesis (1) and using quanta for which $r = 1$, it is possible to explain all the cases of polarization of resonance radiation which have yielded to previous theories as well as the polarization of light emitted in an electric field. In discussing resonance radiation in a magnetic field we meet with cases where the exciting light has a polarization different from that which would be emitted by the vapor toward the source of the exciting light if excited by electronic impact while in the field. For example, Hanle has illuminated a mercury resonance bulb by both circular and elliptic light. When no field is present the character of the light emitted at a small angle (20°) with the incident beam is approximately the same as that of the exciting light. On applying a field parallel to the incident beam, circularly polarized light is unaffected; but if the incident light be elliptically polarized, the resonance radiation contains circularly polarized light and also depolarized light. Evidently some of the atoms in the resonance bulb have been raised to the Zeeman level $m = 1$ and re-emit circularly polarized light. The fate of the others which give rise to the depolarized light is uncertain; perhaps they are re-oriented after absorption to the positions $m = 0$ and $m = -1$. The character of the light emitted by these atoms lies beyond our present knowledge, and probably will not be amenable to theory until we understand the way in which atoms arrive at their quantized orientations when a magnetic field is applied.

The theory of the spinning quantum may be applied to aid in the interpretation of experiments on the polarization of radiation excited by unidirectional electron impacts. When λ 2536.7 of Hg is excited in this way, the radiation is partially polarized, with the electric vector perpendicular to the exciting beam of electrons. This means that the r-vectors of the quanta lie parallel and antiparallel to the electron beam. This is in some measure understandable if we suppose that the impulse moment vectors of the electrons lie in the direction of the accelerating field, being space-quantized in analogy to the orientation of hydrogenic atoms in the Stark effect. Of course, many other factors are involved in the complete explanation of this experiment. Skinner[13] has already suggested that the spin of the electron plays an important part in experiments of this kind. Further experiments in which atoms are bombarded by electrons while exposed to a magnetic field parallel to the electron beam are much to be desired.

§4.—It is apparent from what precedes that if r is always 1 there may be several kinds of linearly polarized light. In the normal Zeeman effect, the s-component of higher frequency is composed of quanta with r-vectors parallel to H, while that of lower frequency has r-vectors antiparallel to H. We shall refer to these two types when observed transversely as positive and negative linear quanta, respectively. If the origin of such quanta is unknown they may be called right or left handed, according as the r-vector points to the right or left of the observer. A mixture of these two types may be called racemic light composed of linear quanta. Further we may have linear light composed of circular or elliptic quanta; and, finally, if r is sometimes zero, we may have a "zero" variety of linear light. Similarly, there may be several varieties of circular or elliptic light. Whenever polarized light is produced from unpolarized light by gross polarizing apparatus, we probably deal with racemic varieties. *However, the light composing the various lines of a Zeeman pattern is generally non-racemic.* Thus the Zeeman pattern is to be regarded as a sort of Stern-Gerlach pattern for quanta. We wish to suggest several experiments to test the validity of these views.

A.—Let the light from a mercury resonance bulb placed in a strong magnetic field shine on a second resonance bulb in which the earth's field is very carefully neutralized. Only the p-component will have the frequency necessary to excite the second bulb. If r is zero for this component, there should be no absorption, but if r is 1, absorption is possible. It must be understood in this as in the succeeding experiments that we are speaking of an idealized case in which the line has no fine structure, the Zeeman effect being the $^3/_2$-normal triplet. In practice difficulties will arise because this line is composed of five fine structure components, some of which behave abnormally in the magnetic field.[10]

B.—Consider two resonance bulbs lying on the same east and west line and in the same horizontal plane, in magnetic fields of identical strength. Let the field on the first bulb be directed to the north while that on the second can be placed at any azimuth in the horizontal plane. Initially let it point north. Let the first bulb be illuminated by a mercury arc. Light from the first bulb falling on the second should be absorbed in a quite ordinary way and the emitted light should have the same character as that from the first. Now let the second field point south. A quantum of the s-component of higher frequency will leave the first bulb with r-vector pointing north. There are atoms in the second bulb which have suitable energy levels for absorbing this frequency, but in absorbing it they would acquire angular momentum from the light directed toward the north, whereas the transition to the upper Zeeman level would require an increase of angular momentum directed to the south. Similar relations hold for the *s*-component of lower frequency. Since the angular momentum vector for the field-producing mechanism (j_f of §3), does not change in the emission process, it appears reasonable that it cannot change in absorption, so that the angular momentum cannot be brought to its normal value by a contribution from the field. Therefore, an absorption of the kind described would be impossible if angular momentum is to be conserved in each individual process.

C.—Place the resonance bulbs of the previous experiment on the same north and south line. Let the field on one point north and that on the other point south. The light of the *s*-component of higher frequency emitted by the first bulb will have its *r*-vectors pointing north. But if atoms in the second resonance bulb are carried to the upper Zeeman level of 2^3P_1 they should receive a quantum with angular momentum vector pointing south. Similar relations hold for the *s*-component of lower frequency. Again there is no reason to believe that this angular momentum would be supplied by the field, so that absorption should not occur. Indeed, it is known from experiments on the Zeeman effect in resonance radiation that the wrong sign of circularly polarized light will not be absorbed.[14] We have included this experiment simply because it has not been performed with a resonance lamp as the source of the polarized light and there may be some doubt as to what would happen in this case.

D.—It may be interesting to consider how non-racemic monochromatic linear light (or, for that matter, non-racemic circular or elliptic light) can be obtained. Let a resonance bulb in a strong magnetic field be illuminated by a source which coincides with only one of the *s*-components. If the element in the resonance bulb is properly chosen, e. g., Hg, there will be only one component of the Zeeman pattern in the resonance light. In other cases there will be several lines in the emitted light such as those discussed by Foote, Ruark and Mohler.[15] Experiments in which a single Zeeman component is used as a source have an interest far wider than the proof or disproof of the theories here presented. A typical experiment of this kind bearing on the theory of the spinning quantum is as follows: Take right-handed axes with OY vertical. Let a linearly polarized quantum of one of the *s*-components of $1^1S_0 - 2^3P_1$ fall on a resonance bulb in the direction OX, the electric vector being vertical, and the *r*-vector along OZ. Will the resonance radiation along OZ be partly or wholly circularly polarized?

The authors wish to emphasize that this paper is not a defense of any particular theory. It aims to show that the theory of spinning quanta can explain a great variety of experimental results and to suggest further experiments on which it must stand or fall.

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THE SYMMETRY OF THE STRESS-TENSOR OBTAINED BY SCHROEDINGER'S RULE

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In the recent developments of the calculus of variations used in the new quantum theory the problem arises of finding a general expression for a world-function such that a symmetrical stress-energy tensor may be derived from it by means of Schroedinger's rule.[1]

We shall consider here the analogous problem for the case of a Euclidean space of three dimensions in which the rectangular coördinates of a selected point P are x, y, z. To simplify matters we shall consider a world-function L which depends only on the first derivatives of the components u, v, w of a single vector q associated with the point P. Applying the rule used by Schroedinger in his discussion of Gordon's equations, but with the necessary modifications appropriate for a space of three dimensions, we may associate with L a tensor T with mixed components[2] of type

$$T_{23} = u_2 \frac{\partial L}{\partial u_3} + v_2 \frac{\partial L}{\partial v_3} + w_2 \frac{\partial L}{\partial w_3} + v_1 \frac{\partial L}{\partial w_1} + v_2 \frac{\partial L}{\partial w_2} + v_3 \frac{\partial L}{\partial w_3}$$

the suffixes 1, 2, 3 being used to denote differentiations with respect to x, y, z, respectively.

In order that this tensor may be symmetric the relations

$$T_{23} = T_{32}, \qquad T_{31} = T_{13}, \qquad T_{12} = T_{21}$$