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PRACTICAL PHYSICS

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INDUSTRIAL SERIES

PRACTICAL PHYSICS

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EIGHTH IMPRESSION

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PRACTICAL PHYSICS

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FOREWORD

In the field of adult education the needs of the students are so varied and their educational backgrounds are so diverse that conventional textbooks are often not satisfactory. To be most valuable for these groups the textbooks must be simple and practical, readily understandable by all and yet able to supplement experience at many levels. The Pennsylvania State College Industrial Series was originated in 1941 in an attempt to provide books having these desirable characteristics. The earlier volumes of the series were prepared by members of the staff of the School of Engineering. This volume is the first contribution from the staff of the School of Chemistry and Physics. Other books are expected from the School of the Liberal Arts.

The increased need for technically trained personnel has emphasized the shortage of persons with basic training in physics. To provide trainees with the minimum technical skill and knowledge of physical principles necessary for effective participation in war industries and in the armed services, short courses have been set up, covering only the principles of most immediate use. For example, in extension classes of the Engineering, Science and Management War Training Program, The Pennsylvania State College has given training in a course of basic mathematics and physics to more than 8,000 students in the period from 1941 to 1943. It was realized early that a new concise physics text was needed for this purpose. Consequently the present volume was prepared.

"Practical Physics" is designed to present in streamlined form the major concepts of general physics so that the book can be used in accelerated programs of resident college study, in the ESMWT Program, in classes for special service groups, and in extension and vocational training. It seems probable that such training will continue for a considerable period.

The Editor of this book, Dr. Marsh W. White, Professor of Physics at The Pennsylvania State College, is key supervisor of physics in extension in addition to his campus duties. In the ESMWT Program, he and the other authors, Drs. Robert L. Weber, Kenneth V. Manning, and R. Orin Cornett, have been responsible, with others, for the supervision of the physics instruction given in the 150 extension centers operated by The Pennsylvania State College. The material of the book includes, therefore, the results of wide experience in the field of college physics and its practical applications in industry.

DAVID B. PUGH,
Director of Arts and Science Extension.

THE PENNSYLVANIA STATE COLLEGE,
May, 1943.

PREFACE

This book was prepared in response to a wartime need for a concise textbook in general physics at the introductory level. The emergency conditions brought streamlined courses with shortened hours and special service group and adult training programs, all demanding practical and condensed physics courses. Hence in this book emphasis is placed on those parts of physics that are basic to practical use in engineering, war industry, technical work, and the armed services.

The simplest algebra and the trigonometric functions of a right triangle (the latter contained in this book) constitute the extent of the mathematics used. A distinctive part of the plan of the book is its design to utilize in a logically progressive manner the simple mathematical material that is needed. This enables the students to study and review the mathematics concurrently with the physics. To achieve this integration of mathematics and physics the order of topics in the book is somewhat unusual, with heat and fluid physics preceding the mechanics of solids.

Another feature of this book is the inclusion at the end of each chapter of one or more experiments that have been planned to illustrate the topics considered in the chapter. These experiments are designed to utilize simple and readily available apparatus. The experiments may be performed either as conventional demonstrations or, preferably, as cooperative group exercises. Large-scale apparatus is particularly desirable.

Emphasis has been given throughout the book to the proper use of significant figures. Each topic is illustrated by one or more solved problems. The summary given at the end of each chapter enables the instructor and students to make a quick and systematic review of the material covered. The questions and problems in each chapter are graded in order of increasing difficulty, with answers given to alternate problems. The British system of units is stressed wherever possible, the metric system being employed only as a basis for those units commonly met in science.

The text material has been tried out in lithoprinted form in more than 400 classes, and constructive criticisms have been made by many of the 200 instructors who have used the preliminary editions. This group includes college instructors, high-school teachers, and professional engineers temporarily doing extra-time teaching. Grateful acknowledgment is made to them for these suggestions.

A considerable number of people have been involved in the preparation of the material of this book. Much of the first draft was written by Dr. Robert F. Paton, Associate Professor of Physics at the University of Illinois, and Dr. J. J. Gibbons, Assistant Professor of Physics at The Pennsylvania State College. Dr. Harold K. Schilling, Associate Professor of Physics at The Pennsylvania State College, was largely responsible for many of the group experiments; some of the earlier work on them was done by Dr. C. R. Fountain, formerly of the George Peabody College. Others who have made valuable contributions, especially to the review material and illustrations, are Dr. Ira M. Freeman, of Central College, Dr. Paul E. Martin, formerly Professor of Physics and Mathematics at Muskingum College, and Dr. Harry L. Van Velzer and Dr. Wayne Webb, Assistant Professors of Physics at The Pennsylvania State College.

It is a pleasure to acknowledge the courtesy of the instrument companies and others who have freely granted permission for the use of their illustrations. The following have been particularly helpful: Central Scientific Company, General Electric Company, The National Bureau of Standards, Stromberg-Carlson Telephone Manufacturing Company, C. J. Tagliabue Manufacturing Company, Weston Electrical Instrument Company, Westinghouse Electric and Manufacturing Company, and the U.S. Army Signal Corps.

The Editor expresses his appreciation to his colleagues, Drs. Robert L. Weber, Kenneth V. Manning, and R. Orin Cornett, whose sustained efforts have made possible the completion of this book. The present edition is due almost solely to the work of Dr. Manning and Dr. Weber.

MARSH W. WHITE,
Editor.

THE PENNSYLVANIA STATE COLLEGE,
May, 1943.

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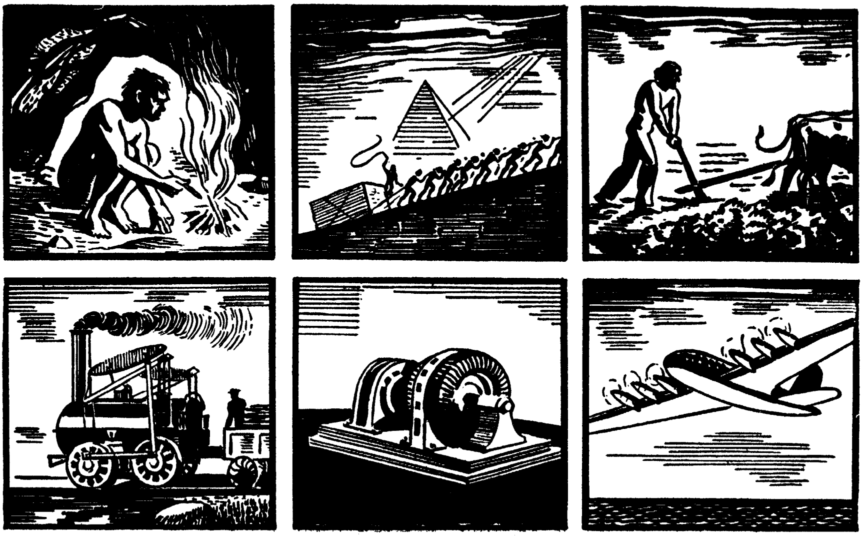


THE FIELDS AND USES OF PHYSICS

The story of man's advancing civilization is the story of his study of nature and his attempt to apply the knowledge so gained to improve his environment. Primitive man was born, lived, and died with little change in his manner of living from generation to generation. Occasional discoveries led to slow advances but no *systematic* attempt was made to study the laws of nature. The existence of such laws was hardly suspected. The use of tools, first of stone and later of metal, the domestication of animals, the development of writing and counting, all progressed slowly since rapid advance was not possible until systematic gathering of data and experimental verification of theories were introduced.

Much of our science has its roots in the speculations of the Greeks but their failure to check conclusions by experiment prevented the rise of a true science. Not until a little over three centuries ago did man adopt the scientific method of studying his environment. Great progress was made, and in the succeeding centuries the development of civilization has become increasingly more rapid.

The rise of all the natural sciences has been almost simultaneous; in fact, many of the prominent scientists have excelled in more than one field. We shall confine our attention to the one field of physics. Probably more than any other science, physics has modified the circumstances under which man lives and exemplifies the scientific method. Physics deals not with man himself, but with the things he sees and feels and hears. Physics is usually defined as *the science of matter and energy*. This science deals with the laws of mechanics, heat, sound, electricity, and light which have been applied in numerous combinations to build our machine age.

Mechanics is the oldest and most basic branch of physics. This division of the subject deals with such ideas as inertia, motion, force, and energy. Their interrelationships of especial interest are the laws dealing with the effects of forces upon the form and motion of objects, since these principles apply to all devices and structures such as machines, buildings, and bridges. Mechanics includes the properties and laws of both solids and liquids.

The subject of *heat* includes the principles of temperature measurement, the effect of temperature on the properties of materials, heat flow, and thermodynamics—the transformation of heat into work. These studies are of importance in foundries, welding plants, pattern and machine shops, where expansion and shrinkage and heat treating are important. In furnaces and steel mills, temperature measurement and control and the flow of heat are essential matters for the engineer to understand.

The study of *sound* is of importance not only in music and speech but also in war and industry. The acoustical and communications engineer is concerned with the generation, transmission, and absorption of sound. An understanding of scientific principles in sound is of importance to the radio engineer. The safety engineer is greatly concerned with the effects of sound in producing fatigue in production personnel.

Electricity and *magnetism* are fields of physics having innumerable everyday applications, many of which are of peculiar importance in war industries and the armed services. An understanding of the principles involving the sources, effects, measurements, and uses of electricity and magnetism is valuable to the worker in that it enables him to use more effectively the manifold electrical devices now so vital to our efficiency and comfort.

Optics is a division of physics that includes the study of the nature and propagation of light, the laws of reflection from plane and curved mirrors, and the bending or refraction that occurs in the transmission of light through prisms and lenses. Of importance also are the separation of white light into its constituent colors, the nature and types of spectra, interference, diffraction, and polarization phenomena. Photometry involves the measurement of luminous intensities of light sources and of the illumination of surfaces, so useful in industry and in everyday life. The war applications of optical devices are numerous and important, as illustrated by such essential achievements of the optical engineer as gun and bomb sights, range finders, binoculars, and searchlights.

A fascinating portion of physics is that known as "modern physics," which includes electronics, atomic and subatomic phenomena, photoelectricity, x-rays, radioactivity, the transmutations of matter and energy, relativity, and the phenomena associated with electron tubes and the electric waves of modern radio. Many of the devices that are almost commonplace today are applications of one or more of these branches of modern physics. Radio, long-distance telephony, sound amplification, and television are a few of the many developments made possible by the use of electron tubes. Photoelectricity makes possible television, transmission of pictures by wire or radio, talking moving pictures, and many devices for the control of machinery. Examination of welds and castings by x-rays to locate hidden flaws is standard procedure in many war and peacetime industries. The practical application of the developments of physics continues at an ever increasing rate.

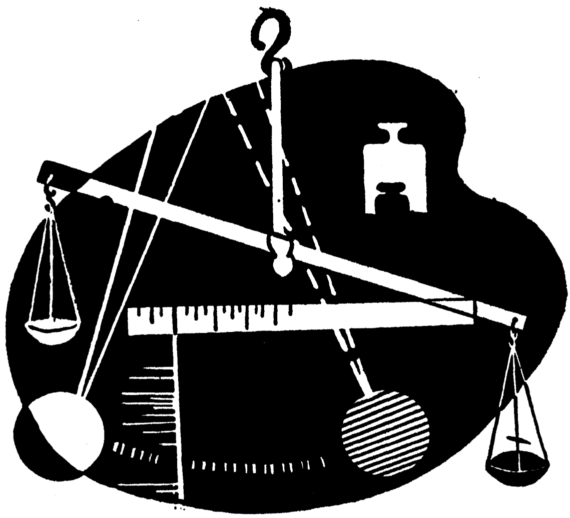
"Practical physics" is, therefore, no idle term, for the laws of physics are applied in every movement we make, in every attempt at communication, in the warmth and light we receive from the sun, in every machine that does our bidding for construction or destruction.

Not only during the war but certainly after actual fighting is over will there be increasing demand for men and women trained in basic physics. It is expected that postwar industrial developments will involve unprecedented applications of physics in industry and will utilize the services of men and women with knowledge of this science to a degree never before visualized. These needs will involve many grades of workers, from high-school graduates to doctors of philosophy, from junior technical aids to the professional engineer. One thing all must have in common—knowledge of the fundamental laws of physics on which so much industrial development and research are based.

The war has placed physics in a peculiarly important position among the sciences. Its present and potential contributions are expected to have a profound effect on the course and outcome of the war. So widely used and so significant are the devices of the physicist that this war is being called a "war of physics." Much of the research that has produced these applications is secret and many of the most important tools of war developed by the physicist cannot even be mentioned. The fact that several hundred physicists are working on war problems in a single research center is startling evidence of the way in which new tools of war are being fashioned by those who are applying the laws of physical science to the war effort.

Practical applications of physics are not all made by those labeled as physicists for the majority of those who apply the principles of physics are called "engineers." In fact most of the branches of engineering are closely allied with one or more sections of physics: civil engineering applies the principles of mechanics; mechanical engineering utilizes the laws of mechanics and heat; electrical engineering is based on the fundamentals of electricity; acoustical engineering and optical engineering are the industrial applications of the physics of sound and light. The alliance between engineering and physics is so close that a thorough knowledge and understanding of physical principles is essential for progress in engineering.

One of the tools common to physics and engineering is mathematics. Principles are expressed quantitatively and most usefully in the language of mathematics. In development and application, careful measurement is essential. If we are to make effective use of the principles and measurements of physical science, we must have a workable knowledge of mathematics. Physics and mathematics are thus the basic "foundations of engineering."



CHAPTER 1

FUNDAMENTAL UNITS; ACCURACY AND SIGNIFICANT FIGURES

Engineering design, manufacture, and commerce today no longer rest on guesswork. Cut-and-try methods have given way to measurement, so that the stone cut in the quarry slips neatly into its prepared place in a building under construction hundreds of miles away. A new spark plug or a piston ring can be purchased in Philadelphia to fit a car made in Detroit. Cooperative planning and the manufacture of interchangeable parts became possible only when people quit guessing and learned to measure.

The Measuring Process. Measuring anything means comparing it with some standard to see how many times as big it is. The process is simplified by using as few standards as possible. These few must be carefully devised and kept. The standard with which other things are compared is called a *unit.* So also are its multiples and submultiples, which may be of more convenient size. The numerical ratio of the thing measured to the unit with which it is compared is called the *numerical measure,* or magnitude, of the thing measured.

Some measurements are *direct,* that is, they are made by comparing the quantity to be measured directly with the unit of that kind, as when we find the length of a table by placing a yard or meter scale beside it. But most measurements are *indirect.* For example, to measure the speed of a plane we measure the distance it travels and the time required, and, by calculation, we find the number of units that represents its speed.

Fundamental Units. Surprising as it may seem, the only kinds of units that are essential in mechanics are those of length, mass, and time. These are arbitrarily chosen, because of their convenience and, hence, are called *fundamental units.* Many other units are based on these three. For example, a unit of length multiplied by itself serves as a unit of area. A unit area multiplied by unit length becomes a unit of volume. A unit of length divided by a unit of time represents a unit of speed. A unit that is formed by multiplying or dividing fundamental units is called a *derived unit.*

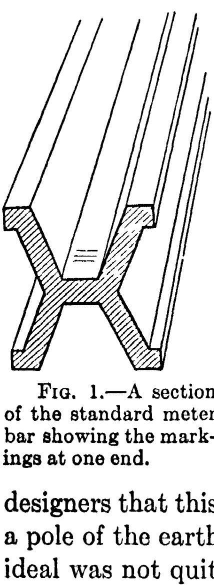


FIG. 1.—A section of the standard meter bar showing the markings at one end.

Length. To specify a distance we must use some unit of length. The unit commonly employed for scientific use and accepted as an international standard is the *meter.* The meter is defined as the distance between two lines on a certain bar of platinum-iridium when the temperature of the bar is that of melting ice (0°C). The prototype meter is kept at the International Bureau of Weights and Measures at Sèvres, France. In order that it could be reproduced if destroyed, it was intended by the designers that this length should be one ten-millionth of the distance from a pole of the earth to the equator, measured along a great circle, but this ideal was not quite realized.

One one-hundredth of the meter is called the *centimeter* (0.01 m), a unit of length that we shall often employ. Other decimal fractions of the meter are the *decimeter* (0.1 m) and the *millimeter* (0.001 m). For large distances the *kilometer* (1,000 m) is employed.

Units of length popularly used in English-speaking countries are the *yard* and its multiples and submultiples. The British or Imperial yard has its legal definition as the distance between two lines on a bronze bar, kept at the office of the Exchequer in London, when its temperature is 62°F. Other common units of length are the mile (1,760 yd), the foot ($\frac{1}{3}$ yd), and the inch ($\frac{1}{36}$ yd).

In the United States the yard is legally defined in terms of the meter: 1 yd = 3,600/3,937m. This leads to the simple approximate relation 1 in. = 2.54 cm. From this simplified relation it is possible to shift from British to metric units on a screw-cutting lathe by the introduction of a gear ratio of 127 to 50 teeth.

Mass. The mass of an object is a measure of the amount of material in it as evidenced by its inertia. (Inertia is the measure of resistance to change of motion.)

The unit of mass chiefly employed in physics is the gram, which is defined as one one-thousandth of the mass of the kilogram prototype—a block made of the same platinum-iridium alloy as the meter prototype and also kept at Sèvres. Fractions and multiples of the gram in common use are named as follows: milligram (0.001 gm), centigram (0.01 gm), decigram (0.1 gm), kilogram (1,000 gm) and the metric ton (1,000 kg or 1,000,000 gm).

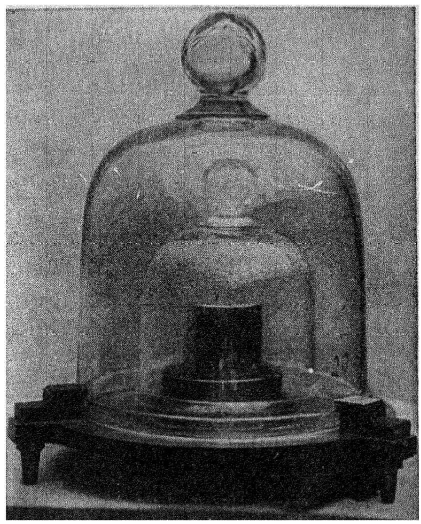


FIG. 2.—The national standard of mass. Kilogram No. 20, a cylinder 39 mm in diameter and 39 mm high, with slightly rounded edges, made of an alloy containing 90 per cent platinum and 10 per cent iridium. It was furnished by the International Bureau of Weights and Measures in pursuance of the metric treaty of 1875.

In the United States the pound, a unit of mass, is legally defined in terms of the kilogram: 1 kg = 2.2046 lb, so that 1 lb equals approximately 454 gm.

Weight. Sir Isaac Newton (1642–1726) pointed out that besides having inertia all material objects have the ability to attract all other objects. As a result of this *universal gravitation* everything on or near the surface of the earth is attracted toward the earth with a force we call *weight.*

The force with which the earth pulls on a mass of 1 lb under standard conditions ($g = 32.16$ ft/sec^2) is called the *weight* of 1 lb or the pound of force. This force is one of the basic units in common usage.

Time. The fundamental unit of time is the mean solar *second.* This is defined as 1/86,400 (NOTE: $86{,}400 = 24 \times 60 \times 60$) of the mean solar day, which is the average, throughout a year, of the time between successive transits of the sun across the meridian at any place. Thus, the time it takes for the earth to turn once on its axis, with respect to the sun, serves as the basis for the unit of time. A properly regulated watch or clock, a pendulum of suitable length, or an oscillating quartz crystal is the working standard for measuring time.

Metric and British Systems. The *metric system* of measure is based on the units: the meter, the kilogram, and the second. It is the one system common to all nations that is used by physicists, chemists, and many engineers. It was legalized for use in the United States by the Metric

Act of 1866, which also included a statement of equivalents of the metric system in British measure. The *British system* of units in popular use is based on the yard, the pound, and the second. Relations between these two systems of units are illustrated schematically in Figs. 3 and 4.

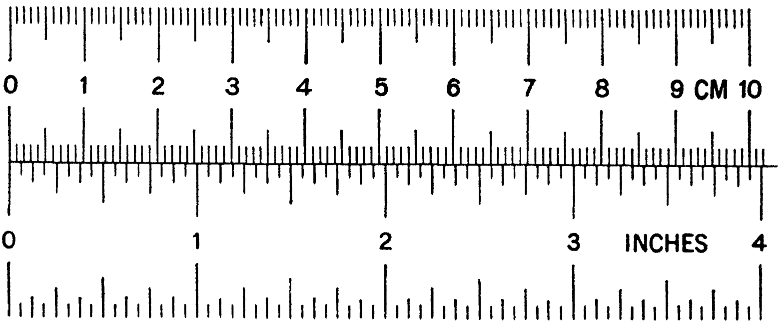


FIG. 3.—Comparison of metric and British units of length.

Since the metric system is a decimal system it is easier to use in computations, conversions within the system being made by shifting the decimal point. No such convenient decimal relationship exists between quantities in our so-called practical system of units, such as yard, foot, inch.

TABLE I. EQUIVALENTS OF CERTAIN UNITS

Centimeter	=	0.3937 inch
Meter	=	39.37 inches (exactly)
Square centimeter	=	0.1550 square inch
Square meter	=	1.196 square yards
Cubic meter	=	1.308 cubic yards
Liter	=	0.2642 gallon
Liter	=	1.057 liquid quarts
Liter	=	0.908 dry quart
Kilogram	=	2.205 pounds, avoirdupois
Inch	=	2.540 centimeters
Yard	=	0.914 meter
Square inch	=	6.451 square centimeters
Square yard	=	0.836 square meter
Cubic yard	=	0.7646 cubic meter
Gallon	=	3.785 liters
Liquid quart	=	0.946 liter
Dry quart	=	1.101 liters
Pound, avoirdupois	=	0.4536 kilogram
Pound, avoirdupois	=	453.6 grams

The centimeter, the gram, and the second are the most commonly used metric units. A system of units based on them is called the *cgs system*. Likewise the British system is often referred to as the *fps system* (foot, pound, second).

Example: Change 115 in. to centimeters.

$$115 \text{ in.} = 115 \text{ in.} (2.54 \text{ cm/in.}) = 292 \text{ cm}$$

If all units are inserted into an equation, they can be handled as algebraic quantities and, when they are handled in this manner, the correct final unit is obtained. This method has an added advantage in that it frequently calls attention to a factor that has been forgotten.

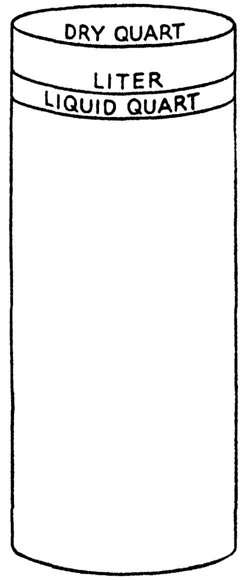


FIG. 4.—Comparison of units of volume.

Example: Convert 165 lb to kilograms.

$$165 \text{ lb} = \frac{165 \text{ lb}}{2.205 \text{ lb/kg}} = 74.8 \text{ kg}$$

Example: Express 50 mi/hr in feet per second.

$$50 \text{ mi/hr} = \frac{(50 \text{ mi/hr})(5{,}280 \text{ ft/mi})}{3{,}600 \text{ sec/hr}} = 73 \text{ ft/sec}$$

Accuracy of Measurements. Measurements of mass, length, and time have been perfected by research to an accuracy that may seem almost fantastic. The length of a meter or the thickness of a transparently thin film can be measured by optical methods with an uncertainty of only 1 part in 1,000,000. Electric oscillators provide standards of time that measure intervals to 1 part in 10,000,000. The range of measurements possible, as well as their accuracy, is important. Masses have been determined, for example, for objects as large as the earth

$$13{,}100{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ lb}$$

and as small as the electron

$$0.000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}001{,}98 \text{ lb.}$$

These long numbers can be expressed much more satisfactorily by using a number multiplied by some power of ten. Thus the mass of the earth is expressed as

$$1.31 \times 10^{25} \text{ lb}$$

and that of the electron as

$$1.98 \times 10^{-30} \text{ lb.}$$

This notation is used frequently in technical work.

The accuracy to which a certain measurement should be taken and the number of figures that should be expressed in its numerical measure are practical questions. Their answers, which depend upon the particular problem, are often more difficult to determine than anything else about the problem. A farmer buying a 160-acre farm need not worry about the boundaries to within a foot or so. But the architect and surveyor who plan a building on Wall Street have to establish boundaries with a precision involving fractions of an inch. The bearings for the crank-shaft of an engine must be more nearly identical in size than the bricks a mason uses in building a house.

Uncertainty in Measurements. The word *accuracy* has various shades of meaning depending on the circumstances under which it is used. It is commonly used to denote the reliability of the indications of a measuring instrument.

As applied to the final result of a measurement, the accuracy is expressed by stating the *uncertainty* of the numerical result, that is, the estimated maximum amount by which the result may differ from the "true" or accepted value.

A few facts should be noted in deciding the accuracy possible and needed in any set of measurements. First, it should be remembered that *no measurement* of a physical magnitude, such as length, mass, or time, *is ever absolutely correct.* It is just as impossible to measure the *exact* volume of the cylinder of an automobile or the space in a building as it is to measure the exact volume of the ocean. It is important also to recognize that all measurements should be taken so that the uncertainty in the final result will not be larger than that which can be tolerated in the completed job.

Significant Figures. The accuracy of a physical measurement is properly indicated by the number of figures used in expressing the numerical measure. Conventionally, only those figures which are reasonably trustworthy are retained. These are called *significant figures.*

Assume that the amount of brass in a sheet of the metal is to be determined. Suppose that the length is measured with a tape measure and found to be (20.2 ± 0.1) ft. The number 20.2 ft has three significant figures and the ±0.1 ft is the way of writing the fact that the length measurement showed that the sheet was not longer than 20.3 nor shorter than 20.1 ft. The width of the sheet measured with the same tape at various places along the sheet gives an average value of (2.90 ± 0.04) ft. Again there are three significant figures and the ±0.04 means that the width is not known closer than to within a range of 0.04 ft, on either side of the value given. The width lies somewhere between 2.86 ft and 2.94 ft.

To measure the thickness of the brass sheet the tape is no longer useful. A vernier caliper or a micrometer gauge is convenient for this purpose. Suppose the average of several readings of the thickness taken at different places on the sheet is (0.0042 ± 0.0001) ft, or (0.050 ± 0.001) in. There are only two significant figures in this result.

The volume is then given by the product of these measurements. If the multiplication is carried out in the customary longhand manner,

$$V = (20.2\ \text{ft})(2.90\ \text{ft})(0.0042\ \text{ft}) \doteq 0.246036\ \text{ft}^3.$$

However the precision implied in writing six digits far exceeds the precision of the original measurements. In order to avoid this situation rules have been set up for deciding the proper number of figures to retain.

Rules for Computation with Experimental Data. There is always a pronounced and persistent tendency on the part of beginners to retain too many figures in a computation. This not only involves too much arithmetic labor but, worse still, leads to a fictitiously precise result as just illustrated.

The following are safe rules to follow and will save much time that would otherwise be spent in calculation; furthermore, their careful use will result in properly indicated accuracies.

RULE I. In recording the result of a measurement or a calculation, *one and one only doubtful digit is retained.*

RULE II. In addition and subtraction, do not carry the operations beyond the first column that contains a doubtful figure.

RULE III. In multiplication and division, carry the result to the same number of significant figures that there are in that quantity entering into the calculation which has the *least* number of significant figures.

RULE IV. In dropping figures that are not significant, the last figure retained should be unchanged if the first figure dropped is less than 5. It should be increased by 1 if the first figure dropped is greater than 5. If the first figure dropped is 5, the preceding digit should be unchanged if it is an even number but increased by 1 if it is an odd number. Examples: $3.4\bar{5}5$ becomes 3.46; $3.4\bar{8}5$ becomes 3.48; $6.79\bar{0}1$ becomes 6.790.

Rules III and IV apply directly to the computation above. The quantity entering into the calculation that has the least number of significant figures is (0.0042), which has only two. Therefore only two significant figures should be retained in the result. The first figure to be dropped is 6 and, since this is greater than 5, we must increase by one the last digit retained. Hence

$$V = 0.25\ \text{ft}^3$$

Example, Illustrating Rule II: Add the following:

Number	Error	Computation
2,807.5	±0.3	2,807.5
0.0648	±0.0006	0.1
83.695	±0.008	83.7
525.0	±0.5	525.0
		3,416.3 Sum

The data indicate that both the first and last quantities have no significant figures after the first decimal place. Hence, the sum can have no significant figure beyond the first decimal place. Note that, even if we had used all the figures in the data, the sum would have been 3,416.2598, a number that does not appreciably differ from the given sum.

In recording certain numbers the location of the decimal point requires zeros to be added to the significant figures. When this requirement leaves doubt as to which figures are significant, we shall *overscore the last significant figure.* This overscored figure is the first digit whose value is doubtful.

Examples:

Length of a page = $22.\bar{7}$ cm (3 significant figures)
Thickness of the page = 0.011 cm (2 significant figures)
Distance to the sun = $9\bar{3},000,000$ mi (2 significant figures)
Speed of light = $299,7\bar{8}0$ km/sec (5 significant figures)

If each of these numbers is expressed in terms of powers of 10, there is no doubt as to the number of significant figures for only the significant figures are then retained. Thus

Length of the page = 22.7×10^1 cm
Thickness of the page = 1.1×10^{-2} cm
Distance to the sun = 9.3×10^7 mi
Speed of light = 2.9978×10^5 km/sec

There are some numbers which, by their definition, may be taken to have an unlimited number of significant figures. For example, the factors 2 and π in the relation,

$$\text{Circumference} = 2\pi\ (\text{radius})$$

In calculations there is frequently need to use data that have been recorded without a clear indication of the number of significant figures. For example, a textbook problem may refer to a "2-lb weight," or in a cooperative experiment a student may announce that he has measured a certain distance as "5 ft." In such cases the values with the appropriate number of significant figures should be written from what is known or assumed about the way in which the measurements were made. If the distance referred to were measured with an ordinary tape measure, it might appropriately be written as 5.0 ft. If it were carefully measured with a steel scale to the nearest tenth of an inch, the distance might be recorded as 5.00 ft. In academic problem work a good rule to follow is to retain three figures unless there is reason to decide otherwise.

A systematic use of the rules given above relating to significant figures results in two advantages: (1) Time is saved by carrying out calculations only to that number of figures which the data justify, and (2) intelligent recording of data is encouraged by noting always the least accurate of a number of measurements needed for a given determination. Attention can then be concentrated on improving the least accurate measurement or, if this is not possible, other measurements need be taken only to an accuracy commensurate with it.

SUMMARY

Engineering is an applied science, which is founded on and utilizes physical measurements.

Measurement means numerical comparison with a standard.

The *mass* of an object is the amount of material in it as evidenced by its inertia. Its *weight* is the force with which it is attracted by the earth.

In scientific work of all countries a *metric* system of units is used. Units in this system are based upon the *centimeter*, *gram*, and *second*.

In the *British* system the *foot*, *pound*, and *second* are the bases of engineering measurements.

Convenient approximate relations for ordinary comparisons are 1 m = 40 in.; 1 in. = 2.54 cm; 1 liter = 1 liquid quart; 1 kg = 2.2 lb; 30 gm = 1 avoirdupois ounce.

The *accuracy* of a numerical measure (65.459 cm) means its reliability. It is expressed by indicating the uncertainty in the measurement (65.459 $\pm$ 0.02 cm), by writing only those figures which are significant (65.46 cm), or by overscoring the first doubtful figure (65.4$\bar{5}$9).

Only *significant figures* are retained in recording data. The last significant figure is the first doubtful digit.

In making calculations there are certain rules to follow, which indicate the number of figures to be retained in the result.

QUESTIONS AND PROBLEMS

1. Define measurement. Give examples of things that can be measured and some that cannot.

2. Convert 1 lb to grams; 2.94 m to feet and inches; 1 day to seconds.

3. Express your height in meters and your weight in kilograms.

4. A shaft is to be turned to a diameter of ⅝ in. Express this in decimal form in inches and in centimeters. *Ans.* 0.625 in.; 1.59 cm.

5. In short-distance running the 440-yd dash is used. How many meters is this?

6. If an industrial process uses 500 tons of iron ore each hour, how many pounds are used per day? per minute? *Ans.* 24,$\bar{0}$00,000 lb/day; 16,$\bar{7}$00 lb/min.

7. A thin circular sheet of iron has a diameter of 14 cm. Find its area. If the material weight is 0.3 kg/m^2, find the weight of the sheet.

8. Why is it necessary to specify the temperature at which comparisons with the standard meter bar are to be made?

9. Suggest several ways in which primary standards of length and mass might be defined in order that, if destroyed, they could be reproduced without loss of accuracy.

10. What is meant by significant figures?

11. Distinguish between mass and weight.

12. Express properly to three significant figures the volume in cubic meters of 1 lb of water. *Ans.* 0.000454 m^3.

13. Express the following quantities with the proper number of significant figures: 3.456 ± 0.2; 746,000 ± 20; 0.002654 ± 0.00008; 6,523.587 ± 0.3; 716.4 ± 0.2; 12.671 good to 5 parts in 1,000. Assume that the errors are correctly stated.

14. Assuming that the following numbers are written with the correct number of significant figures, make the indicated computations, carrying the answers to the correct number of significant figures: (*a*) add 372.6587, 25.61, and 0.43798; (*b*) multiply 24.01 × 11.2 × 3.1416; (*c*) 3,887.6 × 3.1416/25.4.

Ans. 398.71; 846; 482.

EXPERIMENT

Volume Measurements

Apparatus: Meter stick; yardstick; small metric rulers; large table.

To illustrate the principles discussed earlier in this chapter, let us now carry out some actual measurements. It has been emphasized that if measurement is to be "accurate" it must involve rather critical thinking concerning the reliability of the methods employed, the characteristics of the measuring instruments, and the significance of figures appearing as data. To facilitate such thinking it will be advisable to confine our experimentation to very simple cases.

Our general problem is how to measure anything as accurately as possible with the measuring instruments or devices that may be available. What do we mean by "as accurately as possible"? Just how much can we do, or how far can we go, with any given instrument? At what juncture should we be especially careful and when would being very careful be a waste of time?

In particular, how well can we measure the dimensions and volume of a large table top, first with a meter stick and then with a yardstick?

The measurement of a large table top presents distinctly different measurement problems, because the table is longer, the width usually slightly shorter, and the thickness very much shorter than the measuring stick.

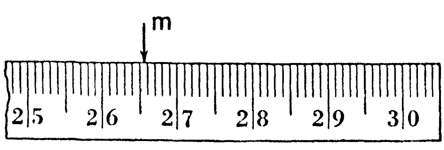


Fig. 5.—In reading a scale, fractions of the smallest division should be estimated.

Suppose we now make a preliminary measurement of each of these dimensions (disregarding bevels at edges, etc.). In doing so it should be remembered that *one can usually (with a bit of practice) estimate the position of a mark with respect to a standard scale down to a fraction of the* *smallest scale division.* Thus arrow m in Fig. 5 is known definitely to be between 26.5 and 26.6, and estimated to be at 26.54. The first three digits are certain, the fourth is uncertain. All four are significant. It would not be correct to report 26.540 or 26.54000 for the position of m. To achieve a result that could legitimately be reported by 26.54000 would require very expensive measuring devices, a great deal of skill and knowledge of techniques, and a vast amount of time.

With all this in mind your results for the preliminary measurements may conceivably look like the figures in the first line of the following table:

Thickness, cm	Width, cm	Length, cm
2.54	83.47	207.16
2.53	83.45	207.03
2.53	83.48	207.30
2.55	83.50	207.27

Suppose we now repeat these measurements several times and record them as in the table. These figures illustrate an important fact, namely, that when one pushes the use of a measuring device to the limit, that is, if one desires results including an estimated significant figure, repeated measurements yield slightly different results. Under such circumstances, therefore, one should always repeat like measurements and then calculate average values, which presumably are much more likely to represent the *true* values, even though those true values are *always* unknown experimentally.

Each student should measure the thickness of the table top independently and record his values. After all measurements have been completed, each should be reported. Note the random distribution of the errors.

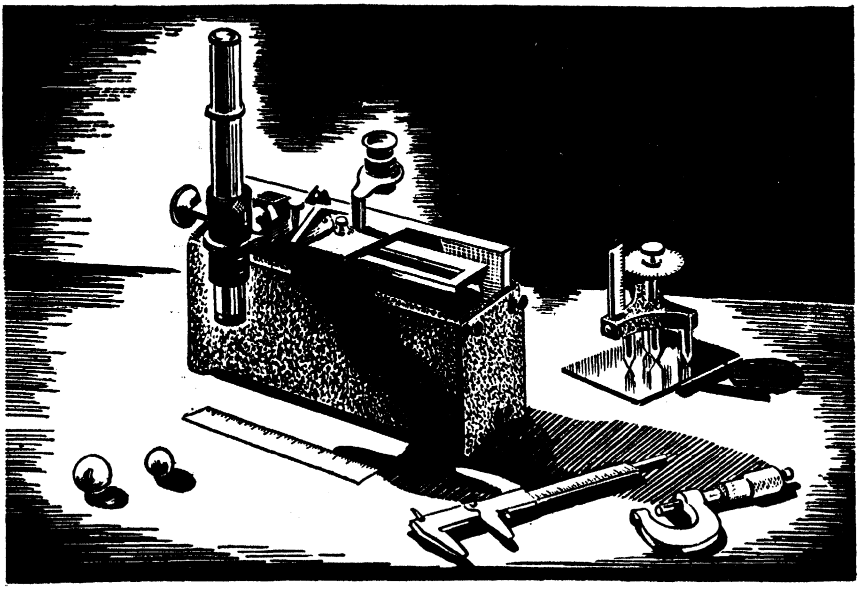
If the results do indeed resemble those given in the accompanying table, we should raise an important question that has been neglected thus far. Is the figure 207.16 for the length really legitimate? The repeated results show that only the first three digits are certain. Furthermore, an analysis of the actual procedure (in laying the meter stick end for end, etc.) makes it very evident that, even though the position of the end of the table can indeed be estimated down to tenths of a millimeter, the *length* of the table, that is, the distance between the ends, cannot so be estimated. Results should therefore have been reported no better than 207.2, 207.0, 207.3, 207.3 cm.

From these results let us calculate the volume $V = TWL$, where T is the thickness, W the width, and L the length of the table top. Multiplying the average values, we obtain a figure with seven digits. How many of these are significant? If we remember that the multiplication of an uncertain digit by either an uncertain or a certain one yields an uncertain product, we shall find that only three digits are significant for the volume—the number of significant digits in the thickness, which is the dimension having the least number of significant digits.

We are now ready to consider another important question concerning the efficiency of our measuring technique. Was there any use in estimating the width to the fourth significant figure, as far as the volume determination is concerned? How about the length? Since the weakest link in our chain is the thickness, with only three significant digits, we could have saved time by measuring the width to millimeters and the length merely to centimeters. Our results, as far as the volume determination is concerned, would have been just as reliable had we recorded:

Thickness	2.54 cm
Width	83.5 cm
Length	207. cm

The use of the British system of units introduces difficulties in measurement and computation, because of the nondecimal character of the fractions involved. If time permits, these difficulties may be observed by repeating the measurements and computations, using the British system of units.



CHAPTER 2

LINEAR MEASUREMENT; ERRORS

Measurement, the comparison of a thing with a standard, usually requires reading a numerical value on an appropriately graduated scale. For the sake of accuracy, that is, to permit the reading of more significant figures, the eye is often aided by some auxiliary device. A simple magnifying glass is frequently useful.

Vernier Principle. Pierre Vernier (1580–1637) introduced a device, now used on many types of instrument. It is an auxiliary scale made to slide along the divisions of a graduated instrument for indicating parts of a division. The vernier is so graduated that a certain convenient number of divisions n on it are equivalent to $n - 1$ divisions on the main scale.

In Fig. 1, 10 divisions on vernier B correspond to 9 divisions on the scale A. This means that the vernier divisions are shorter than the scale divisions by one-tenth of the length of a scale unit. Parts of a division are determined by observing which line on the vernier coincides with a line (any line) on the instrument scale. In Fig. 2, "6" on the vernier coincides with a line on scale A. The reading is therefore

0.3 + 0.06 = 0.36. The essential principle of all verniers is the same and the student who masters the fundamental idea of the vernier can easily understand any special type that he may meet. In brief, the

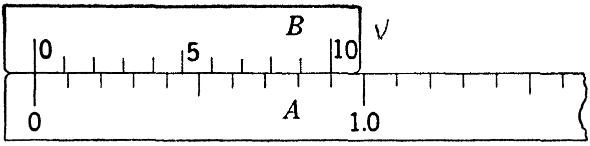


FIG. 1.—A vernier scale.

general principle is that a certain number of vernier divisions will be equal in length to a different number (practically always one less) of scale divisions. Writing this in equation form,

$$nV = (n - 1)S \tag{1}$$

where n is the number of divisions on the vernier scale, V is the length of

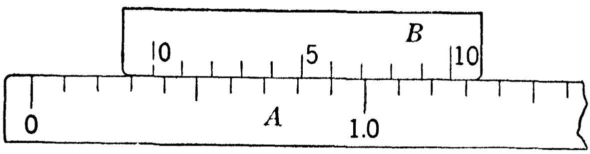


FIG. 2.—Vernier reading 0.36.

one vernier division, and S is the length of the smallest main-scale division.

The term "least count" is applied to the smallest value that can be read directly from a vernier scale. It is equal to the difference in length between a scale division and a vernier division. The definition can be put into the form of a simple equation by rearranging Eq. (1), thus,

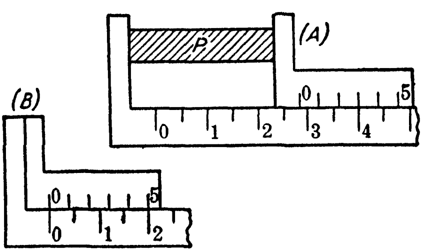


FIG. 3.—A model vernier caliper.

$$\text{Least count} = S - V = \frac{1}{n}S \tag{2}$$

When one has occasion to use a vernier with a new type of scale, the first thing that should be done is to determine the least count of the instrument. To obtain a reading, read first the number that appears on the main scale just in front of the zero of the vernier scale; then note which vernier division coincides with a scale division. Multiplying this latter number by the least count gives the desired fractional part of the least scale division; adding this to the reading first made gives the complete measurement.

Example: What is the least count of the vernier shown in Fig. 3*B*? What is the reading of the caliper measuring the length of *P* in Fig. 3*A*?

Five divisions of the vernier correspond to four divisions on the main scale. Each division of the scale is 0.5 unit. From Eq. (2)

$$\text{Least count} = \frac{S}{n} = \frac{0.5 \text{ unit}}{5} = 0.1 \text{ unit}$$

In Fig. 3*A* the zero of the vernier is beyond the fifth mark. The third mark beyond the zero on the vernier coincides with a line on the main scale. The length of *P* is therefore

$$\begin{aligned} l &= 5(0.5 \text{ unit}) + 3(0.1 \text{ unit}) \\ &= 2.5 \text{ units} + 0.3 \text{ unit} = 2.8 \text{ units} \end{aligned}$$

Vernier Caliper. In design, a vernier caliper is an ordinary rule fitted with two jaws, one rigidly fixed to the rule, the other attached to a vernier scale which slides along the rule. A commercial type of vernier is shown

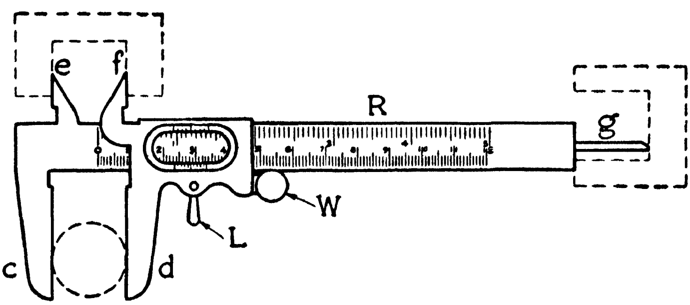


FIG. 4.—A common form of vernier caliper.

in Fig. 4. This instrument has both British and metric scales and is provided with devices to measure internal depths and also diameters of cavities. The jaws *c* and *d* are arranged to measure an outside diameter, jaws *e* and *f* to measure an inside diameter, and the blade *g* to measure an internal depth. The knurled wheel *W* is used for convenient adjustment of the movable jaw and the latch *L* to lock it in position.

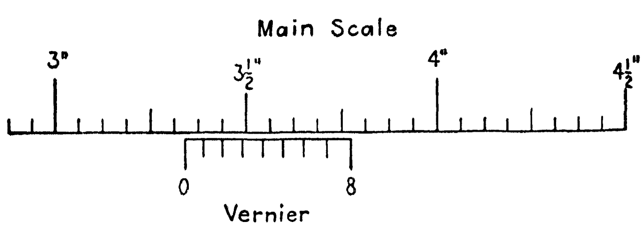


FIG. 5.—A British-scale vernier.

When the jaws of the instrument are together, the zero on the vernier should, of course, coincide with the zero on the scale. On a particular instrument it may not. In that case, whatever reading is indicated when the jaws are in contact, the *zero reading* (which may be either positive or negative) must be subtracted from readings obtained in subsequent use.

Example: Find the reading indicated by the position of the vernier in Fig. 5.

The smallest scale division is $\frac{1}{16}$ in., and eight vernier divisions equal seven main-scale divisions. Hence the least count is one-eighth of $\frac{1}{16}$ in., or $\frac{1}{128}$ in. Since the second vernier division coincides with a main-scale division, the reading of the scale is

$$3 \text{ in.} + \tfrac{5}{16} \text{ in.} + (2)(\tfrac{1}{128}) \text{ in.} = 3\tfrac{42}{128} \text{ in.}$$

Micrometer Caliper. A micrometer caliper employs an accurately threaded screw to determine small distances to a high precision. The instrument, Fig. 6, has a C-shaped frame, one arm of which is drilled and

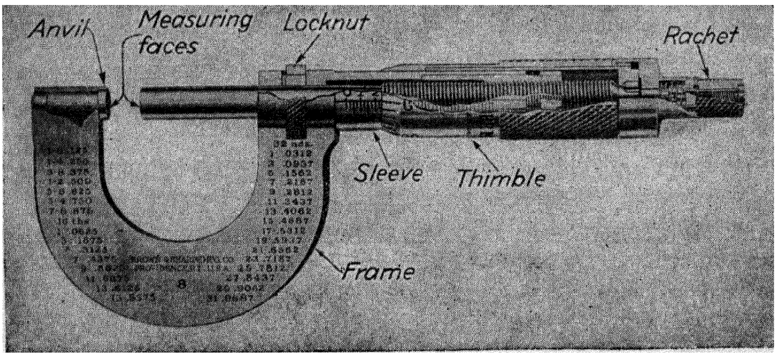


Fig. 6.—A micrometer caliper, cutaway view.

tapped, the other provided with an anvil. A threaded spindle can be advanced through one arm to bear against the anvil attached to the other. The object whose thickness is to be measured is placed between the anvil and the spindle and the screw is rotated until the surfaces are in contact. A reading is taken on the graduated head of the screw. Care should be taken that the contact is not made so tight that the surfaces are dented or the jaws of the instrument sprung. To obtain the zero reading, the object is removed and the screw turned until the micrometer jaws meet. Each revolution of the graduated head advances the spindle by an amount equal to the distance between adjacent threads (the *pitch* of the screw).

Fractions of this distance are read by means of the graduations on the head. For instance, a British micrometer may have a screw whose pitch is $\frac{1}{40}$ in. and a graduated head on the circumference of which 25 equal divisions are marked. This makes it possible to read distances of 0.001 in. directly.

Errors. Physical measurements are always subject to some uncertainty, technically called *error*. There are two classes of error: systematic and erratic. If a distance is repeatedly measured by a scale that is imperfectly calibrated (a yardstick that has shrunk, for example);

the errors in the measured distance will always be similar. This is an example of a *systematic error.*

If one attempted to measure accurately the distance between two fine lines, estimating each time the fraction of the smallest division on the scale, one would probably get slightly different values for each measurement, and these differences would be erratic. These are called *random* or *erratic* errors.

One of the most common sources of error in experimental data is that due to the uncertainty of estimating fractional parts of scale divisions. In spite of this error, such estimations are exceedingly valuable and should always be made, unless there is a good reason to the contrary. For example, if one wishes accurately to note the position of the pointer in Fig. 7, he may record it as 8.4. If he wishes further to record the uncertainty of his estimation of a fraction of the smallest division, he probably may observe that the pointer is nearly halfway between divisions, but not quite so. Furthermore, he can probably note the difference between a reading of 8.4 and 8.6, but not between 8.4 and 8.5. Hence he concludes that the uncertainty of his estimation is about 0.2 of a division, and his reading is recorded 8.4 ± 0.1.

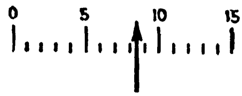


FIG. 7.

Percentage Error. By *percentage error* is meant the number of parts out of each 100 parts that a number is in error. For example, if a 110-yd race track is too long by 0.5 yd, the numerical error is 0.5 yd, the relative error is 0.5 yd in 110 yd and the percentage error is therefore approximately 0.5 per cent. Suppose the same numerical error had existed in a 220-yd track: 0.5 yd in 220 yd is 0.25 yd in 110 yd, or approximately ¼ per cent. This method of determining the approximate percentage error is very desirable, and the habit of making such calculations by a rapid mental process should be cultivated by the student. A more formal statement of the calculation of the percentage error in this case is:

$$\frac{220.5 \text{ yd} - 220.0 \text{ yd}}{220 \text{ yd}} \times 100\% = 0.2\%$$

or in general

$$\text{Percentage error} = \frac{\text{error}}{\text{standard value}} \times 100\% \tag{3}$$

Percentage errors are usually wanted to only one or two significant figures, so that the method of mental approximation or a rough slide-rule computation is quite sufficient for most practical purposes.

It frequently happens that the percentage difference between two quantities is desired when neither of the quantities may be taken as the "standard value." In such cases their average may well be used as the standard value.

Percentage Uncertainty. The *relative uncertainty* or fractional uncertainty of a measurement is the quotient of the uncertainty of measurement divided by the magnitude of the quantity measured. The percentage uncertainty is this quantity expressed as a per cent:

$$\text{Percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\% \qquad (4)$$

The relative uncertainty of a measurement is of greater significance than the uncertainty itself. An uncertainty of an inch in the measurement of a mile race track is of no importance, but an uncertainty of an inch in the diameter of an 8-in. gun barrel is intolerable. The amount of uncertainty is the same, but the relative uncertainty is far greater for the gun barrel than for the race track.

Uncertainty in Computed Results. The uncertainty of a computed result is always greater than that of the roughest measurement used in the calculation. Rules have been set up for determining the uncertainty of the result.

RULE V. The *numerical* uncertainty of the sum or difference of any two measurements is equal to the *sum* of the individual uncertainties.

RULE VI. The *percentage* uncertainty of the product or quotient of several numbers is equal to the *sum* of the *percentage* uncertainties of the several quantities entering into the calculation.

By the use of these rules, we may find the numerical and percentage uncertainties in any computed result if we know the uncertainty of each individual quantity.

Example: From the data recorded in the following table, find the sum and product and determine the numerical and percentage uncertainty of each.

Number, ft	Uncertainty, ft	Percentage uncertainty
20.2	±0.1	0.5
2.9	±0.1	3.4
9.7	±0.2	2.0

From Rule V the sum is (32.8 ± 0.4) ft, and the percentage uncertainty is

$$\frac{0.4 \text{ ft} \times 100\%}{32.8 \text{ ft}} = 1.2\%$$

The product is

$$(20.2 \text{ ft})(2.9 \text{ ft})(9.7 \text{ ft}) = 5\bar{6}0 \text{ ft}^3$$

From Rule VI the percentage uncertainty is 5.9 per cent. To find the numerical uncertainty we use Eq. (4).

$$\text{Percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

$$5.9\% = \frac{\text{uncertainty}}{560 \text{ ft}^3} \times 100\%$$

$$\text{Uncertainty} = \frac{560 \text{ ft}^3 (5.9\%)}{100\%} = 30 \text{ ft}^3$$

The product, therefore, may be expressed as (560 ± 30) ft^3.

SUMMARY

When n divisions on a vernier scale correspond to n-1 divisions on the main scale, the instrument may be read to $(1/n)$th of a division on the main scale.

Physical measurements are always subject to *erratic* errors, detectable by repeating the measurements; and *systematic* errors, detectable only by performing the measurement with different instruments or by a different method.

The rules given in the text are to be followed in making calculations with data from physical measurements.

QUESTIONS AND PROBLEMS

1. Classify the following as to whether they are systematic or erratic errors: (*a*) incorrect calibration of scale; (*b*) personal bias or prejudice; (*c*) expansion of scale due to temperature changes; (*d*) estimating fractional parts of scale divisions; (*e*) displaced zero of scale; (*f*) pointer friction; (*g*) lack of exact uniformity in object repeatedly measured.

2. Determine to one significant figure the approximate percentage error in the following data:

Observed Value	Standard Value
108.	105.
262.	252.
46.2	49.5
339.	336.
460.	450.
0.000011120	0.000011180

Ans. 3 per cent; 4 per cent; −7 per cent; 1 per cent; 2 per cent; −0.5 per cent.

3. The masses of three objects, together with their respective uncertainties, were recorded as $m_1 = 3{,}147.226 \pm 0.3$ gm; $m_2 = 8.23246$ gm $\pm$ 0.10 per cent; $m_3 = 604.279$ gm, error 2 parts in 5,000. Assuming that the errors are correctly given, (*a*) indicate any superfluous figures in the measurements; (*b*) compare the precisions of the three quantities; (*c*) find their sum; (*d*) record their product properly.

4. A certain vernier has 20 vernier divisions corresponding to 19 main-scale divisions. If each main-scale division is 1 mm find the least count of the vernier

Ans. 0.05 mm.

5. *a*. For a vernier and main-scale combination, 10 vernier divisions equal 9 main-scale divisions. What is the least count if the main-scale division equals 1 mm?

b. For a vernier and main-scale combination, 30 vernier divisions equal 28 main-scale divisions. What is the least count in minutes of angle if the main-scale division equals $\frac{1}{2}°$ of angle?

6. *a*. You are given a rule whose smallest divisions are $\frac{1}{8}$ in. and are asked to measure a given length accurately to $\frac{1}{32}$ in. How many divisions will be necessary on the vernier? Make a rough outline schematic sketch to show the vernier set to measure a length of $15\frac{37}{32}$ in.

b. The pitch of a certain micrometer caliper screw is $\frac{1}{32}$ in. If there are 64 divisions on the graduated drum, to what fraction of an inch can readings be determined? *Ans*. 9; 1/2,$\bar{0}$00 in.

7. Two measurements, 10.20 ± 0.04 and 3.61 ± 0.03, are made. What is the error in the result when these data are added? when divided?

8. The masses of two bodies were recorded as $m_1 = (3{,}147.278 \pm 0.3)$ gm and $m_2 = 1.3246$ gm ± 0.1 per cent. Assuming that the errors are properly stated, (*a*) write the numbers properly, omitting any superfluous figures; (*b*) find the sum; (*c*) find the product (each to the proper number of significant figures); (*d*) calculate the uncertainty of the sum and of the product.

Ans. 3,147.3 ± 0.3 gm; 1.325 gm ± 0.1 per cent; 3,148.6 gm; 4,170 gm; ±0.3 gm; ±0.11 per cent.

9. Could a practical vernier be made in which n divisions on the vernier scale corresponded to $n + 1$ divisions on the main scale? Explain.

EXPERIMENT

Length Measurements

Apparatus: Vernier caliper; micrometer caliper; cylindrical cup; thin disk or plate.

In this experiment we shall learn to use vernier and micrometer calipers. With each we should go through the following steps:

1. Examine the instrument carefully with reference to the discussion earlier in this chapter. Just how is it constructed? What can be measured with it?

2. Discover what special care should be taken in using the instrument. In closing the caliper jaws upon an object to be measured, how may good contact be assured without making it too tight or too loose?

3. Evaluate its constants. If it has a vernier scale, what are the values of n, V, and S of Eq. (1)?

4. What is the "zero reading"? How is the final observation to be corrected for this zero reading?

Now let us use the vernier caliper to measure the dimensions of a cylindrical cup and compute its inside volume. Specifically we wish to measure the length, outside diameter, inside diameter, and inside depth.

Each measurement should be repeated for various positions on the object. One reason for doing this has already been discussed, namely, that it is difficult for anyone exactly to duplicate a given measurement. Another reason is that a given object is not exactly uniform with respect to its dimensions. Since it is likely that the cup is longer at some places than at others, an average value should be obtained.

To obtain some estimate of the reproducibility of results, we can compute also the deviation of each individual measurement from the average, that is, the difference between a given measurement and the average. The average of these deviations is a good indication of the reliability of our results, that is, if the average deviation is large, we should not feel so confident of our result as we would were the average deviation small.

The method of obtaining the average deviation of a set of data is illustrated in the following table:

Trial	Length, cm	Deviation, cm
1	8.73	0.00
2	8.71	0 02
3	8.75	0.02
4	8.74	0.01
Sum	34.93	0.05
Average	8.73	0.01

The length may, therefore, be written (8.73 ± 0.01) cm. Since in this case there is no accepted or standard value and hence there is no "error" in the usual sense of that word, we shall call the average deviation the *uncertainty*. What is the percentage uncertainty?

Compute the value of the inside volume of the cup, remembering to observe the rules for significant figures. Also compute the uncertainty and the percentage uncertainty for the volume.

Using the micrometer caliper, make several measurements of the thickness of a thin disk or plate. Determine the average, the uncertainty, and the percentage uncertainty.

The original pyrometer.

CHAPTER 3

TEMPERATURE MEASUREMENT; THERMAL EXPANSION

In many industrial operations it is necessary to heat the material that enters into a process. In such cases a major factor in the success of the procedure is a knowledge of when to stop. In the early stages of the development of heat treatment skilled workers learned to estimate the final stage by visual observation. Such approximate methods yield crude results and hence are not suitable when a uniform product is required. It became necessary to *measure* accurately the factor involved. This concept is known as *temperature*. Many modern industrial processes require precise measurement and control of temperature during the operation. For example, in the metallurgical industries the characteristics of the metal being treated are vitally affected by their temperature history. The most common type of temperature-measuring device, the thermometer, is based upon the expansive properties of certain materials.

Temperature Sensation. To measure temperature it is necessary to set up a new standard procedure, for a unit of temperature cannot be defined in terms of the units of mass, length, and time, or even in a manner strictly similar to that which we have used in defining those units. We can tell something about the temperature of an object by feeling it. If it is very hot, we can sense this even without touching it. But under some conditions our temperature sense is a very unreliable guide. For example, if the hand has been in hot water, tepid water will feel cold; whereas, if the hand has been in cold water, the same tepid water will feel warm. If we go outdoors on a cold day and pick up a piece of wood, it will feel cold. Under the same conditions a piece of steel will feel even colder.

Both of these examples suggest that our sensation of hot or cold depends on the transfer of heat to or away from the hand. The steel feels colder than the wood because it is a better conductor of heat and takes heat from the hand much more rapidly than does the wood.

Temperature Level. The sense we possess for judging whether a thing is hot or cold cannot be used to measure temperature, but it does tell us what temperature is. The temperature of an object is that property which determines the direction of flow of heat between it and its surroundings. If heat flows away from an object, we say that its temperature is above that of the surroundings. If the reverse is true, then its temperature is lower. To answer the question of *how much* higher or lower requires a standard of measure and some kind of instrument calibrated to read temperature difference in terms of that standard. Such an instrument is called a *thermometer.*

Thermometers. There are many possible kinds of thermometers, since almost all the properties of material objects (except mass) change as the temperature changes. The amount of any such change may be used to measure temperature. To be useful, the amount of the change must correspond in some known manner to the temperature change that induces it. The simplest case is the one in which equal changes in the property correspond to equal changes in the temperature. This is practically true for the length of a column of mercury in a glass capillary connected to a small glass bulb.

When a mercury thermometer is heated, the mercury expands more than the glass bulb and rises in the capillary tube. The position of the mercury in the capillary when the bulb is in melting ice is taken as a reference point. Such a reference temperature, chosen because it is easily reproducible, is called a *fixed point.*

If the bulb is placed in contact with something else and the mercury goes above the fixed point set by the melting ice, then that material is at a higher temperature than the melting ice. If the mercury goes below the fixed point, then the temperature is lower. The answer to how much higher or how much lower can be obtained only by selecting another fixed point so that the interval between the two can be divided into a convenient number of units in terms of which temperature changes can be compared or, as we say, measured.

The other fixed point chosen is the boiling point of water. This is the temperature of the water vapor above pure water which is boiling under a pressure of one standard atmosphere. Since the temperature at which water boils depends upon the pressure, it is necessary to define this fixed point in terms of a standard pressure.

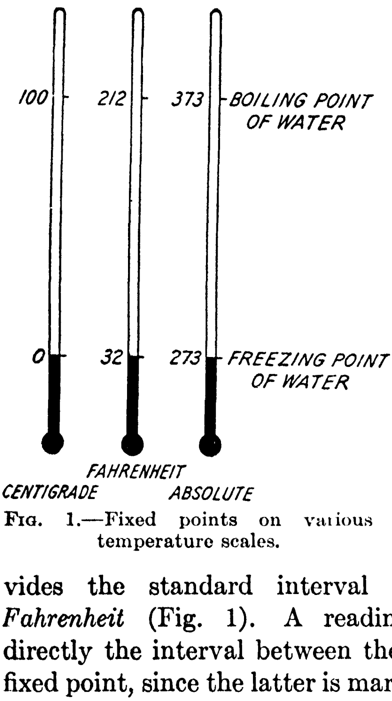


FIG. 1.—Fixed points on various temperature scales.

Many other easily reproducible temperatures may be used as fixed points. For example, the boiling point of oxygen, a very low temperature, and the melting point of platinum, a very high temperature, are sometimes used. The temperatures of such fixed points are based on the primary temperature interval between the freezing point of water and the (standard) boiling point of water.

Common Thermometer Scales. Two thermometer scales are in common use: one, the *centigrade* scale, which divides the standard interval into 100 equal parts called *degrees centigrade;* and the other, the *Fahrenheit* scale, which divides the standard interval into 180 equal parts called *degrees Fahrenheit* (Fig. 1). A reading on the centigrade scale indicates directly the interval between the associated temperature and the lower fixed point, since the latter is marked zero. The Fahrenheit scale is more cumbersome, not only because the standard interval is divided into 180 parts instead of 100, but also because the base temperature, that of melting ice, is marked 32°. The Fahrenheit scale is used in many English-speaking countries, while nearly all others use the centigrade scale. Having the two temperature scales is something of a nuisance, but it is comparatively easy to convert temperatures from one scale to the other.

In science the centigrade scale is used almost exclusively. The centigrade degree is one one-hundredth of the temperature interval between the freezing and boiling points of water at standard pressure. The Fahrenheit degree is one one-hundred-eightieth of the same interval. Therefore, the centigrade degree represents a larger temperature interval than a Fahrenheit degree. One Fahrenheit degree is equal to five-ninths of a centigrade degree.

For any two temperature scales that use the freezing point and boiling point of water as fixed points, the temperature may be converted from one to the other by means of a simple proportion. For centigrade and Fahrenheit scales this relation is

$$\frac{C - 0^\circ}{F - 32^\circ} = \frac{100^\circ - 0^\circ}{212^\circ - 32^\circ}$$

This equation reduces to

$$\frac{C}{F - 32^\circ} = \frac{100}{180} = \frac{5}{9}$$

This may be solved for either C or F to give

$$C = \tfrac{5}{9}(F - 32^\circ) \tag{1}$$

$$F = \tfrac{9}{5}C + 32^\circ \tag{2}$$

Two numerical examples will serve to illustrate the process.

Example: A centigrade thermometer indicates a temperature of 36.6°C. What would a Fahrenheit thermometer read at that temperature? The number of degrees centigrade above 0°C is 36.6. This temperature will be ($\tfrac{9}{5}$) 36.6, or 65.9, Fahrenheit degrees above the freezing po nt of water. The Fahrenheit reading will be 32°F added to this, or 97.9°F.

Example: Suppose a Fahrenheit thermometer indicates a temperature of 14°F, which is 18°F below the freezing point of water. A temperature interval of 18°F is equivalent to an interval of 10°C; hence the corresponding reading of a centigrade thermometer is −10°C.

Absolute Temperature Scale. The absolute temperature scale, whose origin is discussed in Chap. 8, is important in theoretical calculations in physics and engineering. For the present we shall note that temperature when expressed on the absolute scale is designated by degrees Kelvin (°K) and is related to the centigrade temperature by the equation

$$K = 273.16^\circ + C \tag{3}$$

Example: Express 20°C and −5°C on the absolute (or Kelvin) scale

$$K = 273.16^\circ + 20^\circ = 293^\circ\text{K}$$
$$K = 273.16^\circ + (-5^\circ) = 268^\circ\text{K}$$

Properties That Change with Temperature. The fixed temperatures that are used to calibrate thermometers are the melting and boiling points of various substances. The three common states of matter are classified as solid, liquid, and gaseous. Some materials (for example, water) are familiar to us in all three states. The temperature at which a given material melts is always the same at a standard pressure. Boiling also occurs at a definite temperature for a particular pressure.

The property most commonly used in thermometers is expansion. The expansion may be that of a liquid, a solid, or a gas. Mercury-in-glass thermometers may be used over a range from the freezing point of mercury (−38.9°C) to the temperature at which the glass begins to

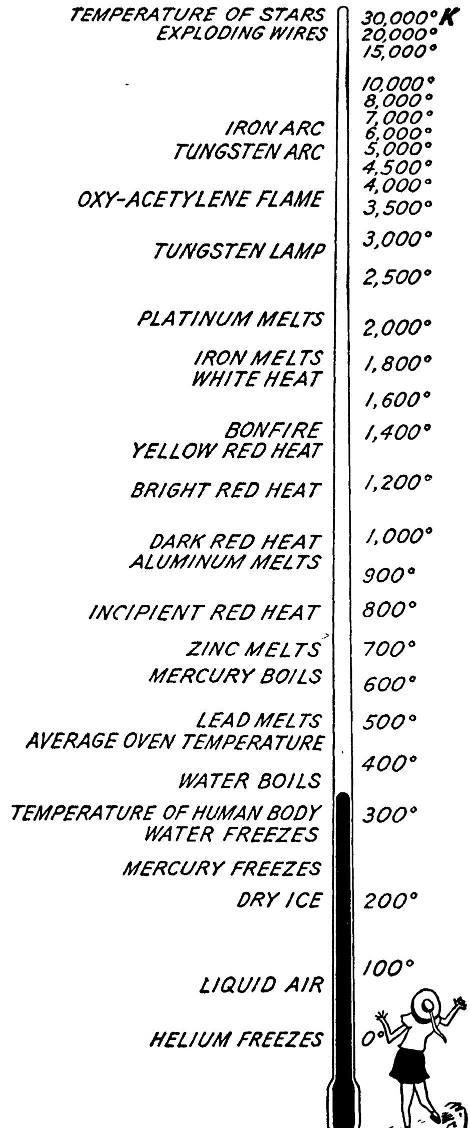


FIG. 2.—The range of temperatures of interest in physics.

soften. For temperatures below the freezing point of mercury other liquids, such as alcohol, may be used. The expansion of a solid or of a gas may be used over a much greater range of temperatures.

The variation of electrical resistance with temperature is often used as a thermometric property. The variation in electric current produced in a circuit having junctions of two different metals (thermocouple) is also used.

At very high temperatures special thermometers, called *pyrometers*, are used. One kind employs the brightness of the hot object (inside of a

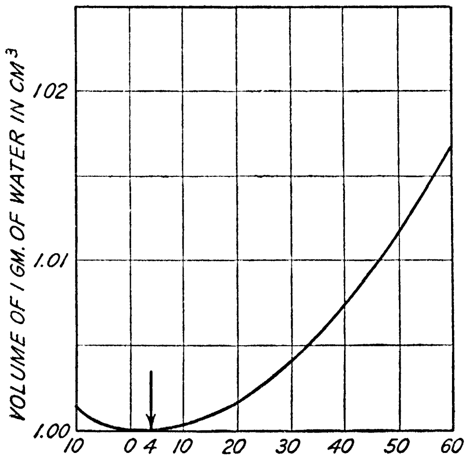


FIG. 3.—The expansion of water.

furnace, for example) to measure the temperature. The color of an object also changes with temperature. As the temperature rises the object first becomes a dull red, at a higher temperature a bright red, and finally, at very high temperatures, white. These changes in color may be used to measure temperature.

Linear Coefficient of Expansion. Nearly all materials expand with an increase in temperature. Water is an exception in that it contracts with rising temperature in the interval between 0 and 4°C. Gases and liquids, having no shape of their own, exhibit only volume expansion. Solids have expansion properties, which, in the case of crystals, may differ along the various axes.

The fractional amount a material will expand for a 1° rise in temperature is called its *coefficient of expansion*. For example, the coefficient of expansion of iron is approximately 0.00001/°C. This means that a bar of iron will increase its length by the 0.00001 fractional part of its original length for each degree centigrade that its temperature increases. An iron rod 50 ft long, when its temperature is changed from 0 to 100°C, increases in length by an amount

$$(0.00001/°\text{C})(100°\text{C})(50\text{ ft}) = 0.05\text{ ft} = 0.6\text{ in.}$$

The coefficient of *linear* expansion of a material is *the change in length per unit length per degree change in temperature.* In symbols

$$\alpha = \frac{L_t - L_0}{L_0 t} \tag{4}$$

where α is the coefficient of linear expansion, L_t is the length at temperature t, and L_0 is the length at 0°C (32°F). Measurements of the change in length and the total length are always expressed in the same unit of length, so that the value of the coefficient will be independent of the length unit used but will depend on the temperature unit used. Hence the value of the coefficient of expansion must be specified as "per degree centigrade" or "per degree Fahrenheit" as the case may be. If we let ΔL represent the change in length of a bar (ΔL is the final length minus the initial length), α the coefficient of expansion, and Δt the corresponding change in temperature, then

$$\Delta L = \alpha L_0 \, \Delta t \tag{5}$$

L_0 being the original length of the rod. The final length of the rod L_t will be

$$L_t = L_0 + \Delta L = L_0 + \alpha L_0 \, \Delta t = L_0(1 + \alpha \, \Delta t) \tag{6}$$

Example: A copper bar is 8.0 ft long at 68°F and has a coefficient of expansion of 0.0000094/°F. What is its increase in length when heated to 110°F?

$$\Delta L = L_0 \alpha \Delta t = (8.0 \text{ ft})(0.0000094/°\text{F})(110°\text{F} - 68°\text{F}) = 0.0032 \text{ ft}$$

Example: A steel plug has a diameter of 10.000 cm at 30.0°C. At what temperature will the diameter be 9.997 cm?

$$\Delta L = L_0 \alpha \, \Delta t$$

$$\Delta t = \frac{10.000 \text{ cm} - 9.997 \text{ cm}}{(10.000 \text{ cm})(0.000013/°\text{C})} = 23.1°\text{C}$$

Hence the required temperature

$$t = 30.0°\text{C} - 23.1°\text{C} = 6.9°\text{C}$$

Volume Coefficient of Expansion. The volume coefficient of expansion for a material is the change in volume per unit volume per degree change in temperature. In symbols

$$\beta = \frac{V_t - V_0}{V_0 t} \tag{7}$$

where β is the volume coefficient of expansion, V_t is the volume at temperature t, and V_0 is the volume at 0°C. The volume coefficient of expansion is very nearly three times the linear expansion coefficient for the same material. By comparing volume coefficients calculated roughly from Table I with those given in Table II, it will be seen that, in general, liquids expand more than solids, but this is not universally true. The coefficients of expansion of all gases are approximately the same. Moreover, this value is much greater than the coefficients of expansion of liquids and solids.

The facts that different solids have different expansion coefficients, and that the coefficient of expansion for a given material may vary somewhat with temperature, lead to many industrial problems. If a structure, for example, a furnace, can be made of materials that expand equally over

TABLE I. COEFFICIENTS OF LINEAR EXPANSION (AVERAGE)

Material	Per °C	Per °F
Aluminum	0.000022	0.000012
Brass	0.000019	0.000010
Copper	0.000017	0.0000094
Glass, ordinary	0.0000095	0.0000053
Glass, pyrex	0.0000036	0.0000020
Invar (nickel-steel alloy)	0.0000009	0.0000005
Iron	0.000012	0.0000067
Oak, with grain	0.000005	0.000002
Platinum	0.0000089	0.0000049
Fused quartz	0.00000059	0.00000033
Steel	0.000013	0.0000072
Tungsten	0.0000043	0.0000024

TABLE II. COEFFICIENTS OF VOLUME EXPANSION OF LIQUIDS

Substance	Per °C	Per °F
Alcohol (ethyl)	0.0011	0.00061
Mercury	0.00018	0.00010
Water (15–100°C)	0.00037	0.00020

wide ranges in temperature, the structure will hold together much better than if such materials cannot be found. When it is impossible to find suitable materials with approximately equal coefficients, allowance must be made for the large forces that arise, owing to the fact that different parts of the structure expand at different rates. Some materials that go together well at one temperature may be quite unsatisfactory at others because their coefficients may change considerably as the temperature changes. The coefficient of expansion for each material is determined for an appropriate range of temperature.

Finding types of glass that have suitable coefficients of expansion and elastic properties has made it possible to enamel metals with glass. This product has wide uses, for enameled ware is much more resistant to corrosion than most of the cheaper metals and alloys. Enameled tanks, retorts, and cooking utensils are familiar examples. If correctly designed, they seldom fail except from mechanical blows or improper use. An enameled dish put over a hot burner will crack to pieces if it boils dry, for the coefficients of expansion of the metal and the glass enamel do not match closely enough, so that at temperatures considerably above the boiling point of water, the stresses that arise are too great to be withstood.

Tungsten is a metal that expands in a manner similar to that of many glasses. Tungsten, platinum, and Dumet (an alloy) are metals often used to seal electrodes through the glass of electric light bulbs, x-ray tubes, and the like.

SUMMARY

The *temperature* of an object is that property which determines the direction of flow of heat between it and its surroundings.

A thermometer scale is established by choosing as *fixed points* two easily reproducible temperatures (ice point and steam point), dividing this interval into a number of equal subintervals, and assigning an arbitrary zero.

Conversions between centigrade and Fahrenheit scale readings are made by the relations

$$F = \tfrac{9}{5}C + 32^\circ$$
$$C = \tfrac{5}{9}(F - 32^\circ)$$

The *fractional change* due to change in temperature is the change in size (length, area, or volume) divided by the original size at some specified temperature.

The *coefficient of expansion* is the fractional change per degree change in temperature. The units, per °C or per °F, must be expressed.

The linear expansion of a material is equal to the product of the coefficient of linear expansion, the original length, and the temperature change. Symbolically

$$\Delta L = \alpha L_0 \,\Delta t$$

QUESTIONS AND PROBLEMS

1. Express a change in temperature of 20°C in terms of the Fahrenheit scale.

2. Convert −14°C, 20°C, 40°C, and 60°C to Fahrenheit readings. Convert 98°F, −13°F, and 536°F to centigrade readings.

Ans. 6.8°F; 68°F; 104°F; 140°F; 37°C; −25°C; 280°C.

3. What is the approximate temperature of a healthy person in °C?

4. Liquid oxygen freezes at −218.4°C and boils at −183.0°C. Express these temperatures on the Fahrenheit scale. *Ans.* −361.1°F; −297.4°F.

5. At what temperature are the readings of a Fahrenheit and a centigrade thermometer the same?

6. From Eq. (5), show that the coefficient of area expansion is approximately twice that of linear expansion and that the coefficient of volume expansion is approximately three times that of linear expansion.

7. Table III gives the coefficient of linear expansion for iron at different temperatures. Explain the meaning and usefulness of such a table.

TABLE III

t, °C	α
0–100	10.5×10^{-6}/°C
100–200	11.5
200–300	13
300–400	15
400–500	14
500–600	16
600–700	16
Above 900	29

8. The coefficient of volume expansion of air at atmospheric pressure is 0.0037/°C. What volume would 10 cm³ of air at 0°C occupy at 100°C? at −100°C? at −200°C? *Ans.* 13.7 cm³; 6.3 cm³; 2.6 cm³.

9. If 40-ft steel rails are laid when the temperature is 35°F, what should be the separation between successive rails to allow for expansion up to 120°F?

10. A steel tape correct at 0°C is used to measure land when the temperature is 25°C. What percentage error will result in length measurements due to the expansion of the tape? *Ans.* 0.03 per cent.

11. A steel wagon tire is 16 ft in circumference at 220°C when it is put onto the wheel. How much will the circumference shrink in cooling to 20°C?

12. A pyrex glass flask of volume 1,000 cm³ is full of mercury at 20°C. How many cubic centimeters will overflow when the temperature is raised to 50°C? *Ans.* 5.1 cm³.

EXPERIMENT

Linear Expansion of Rods

Apparatus: Linear expansion apparatus; boiler; tripod; 0 to 110°C thermometer; rubber tubing; pinchcock; beaker; Bunsen burner; two rods of different materials.

From the illustration already cited it is evident that the proper designing of many types of machines, utensils, instruments, and buildings depends upon accurate knowledge of the coefficients of expansion of the various parts of each and of the materials binding the various parts together.

Since the coefficient of linear expansion of a material is its change in length per unit length per degree change in temperature, its determination requires the measuring of its original length at some definite temperature and its change in length for a given change in temperature.

One type of expansion apparatus commonly available is illustrated by Fig. 4. If the apparatus has been in the laboratory for several hours, it may be assumed that the rod enclosed in the jacket is at room temperature. Record this temperature t_0 and the position P_0 of the pointer on the scale, first making sure that the rod is in contact with the adjustable stop and that the pointer is near the bottom of the scale. Next, allow steam from the boiler to flow slowly through the jacket. When the pointer ceases to rise, it may be assumed that the rod has reached the temperature of the steam. This temperature t is determined by means of

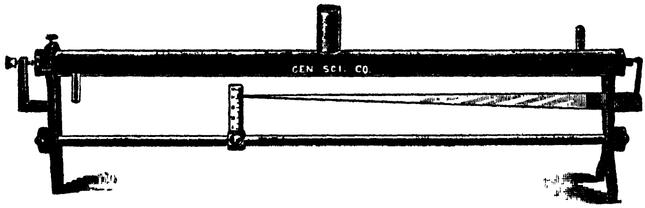


FIG. 4.—Linear-expansion apparatus, mechanical-lever type.

a thermometer inserted in the jacket. The difference between the new position P_t of the pointer on the scale and its original position is the *magnified change* in the length of the rod. The actual change in length ΔL of the rod is equal to the magnified change, $P_t - P_0$, divided by the magnifying power of the lever system, which is the length l_1 of the long lever arm divided by the length l_2 of the short lever arm.

$$\Delta L = \frac{(P_t - P_0)l_2}{l_1}$$

Compute the coefficient of expansion of the rod by using Eq. (5), and compare the value thus obtained with that listed in Table I.

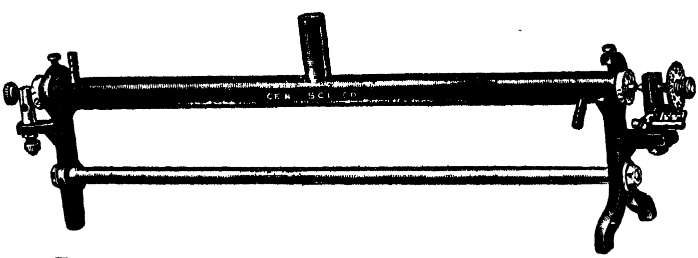
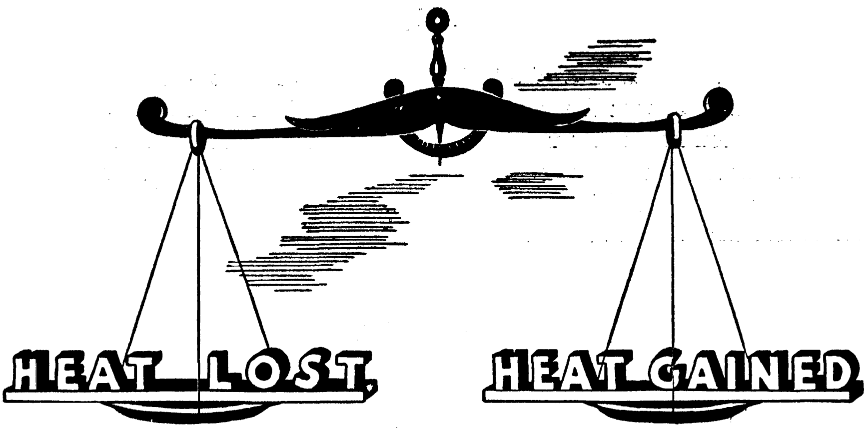


FIG. 5.—Linear-expansion apparatus, micrometer-screw type.

The apparatus shown in Fig. 5 is equipped with a micrometer screw for direct measurement of the expansion of the rod. An electric contact detector is used to adjust the micrometer screw until it is barely in contact with the rod. One connects a dry cell in series with a current-indicating device, such as a telephone receiver or a galvanometer (with a protective resistance), to the two binding posts on the expansion apparatus. The screw is then turned slowly toward the rod until contact is barely made. Several observations of this position are made and recorded. Before allowing the steam to enter the jacket, one *must* turn the screw back until there is no danger that the expanding rod will again make contact. After steam has been issuing from the end of the jacket for at least 1 min, bring the micrometer screw into contact with the rod and make several observations of its position. Compute the change in length ΔL, then determine the coefficient of expansion, using Eq. (5).



CHAPTER 4

HEAT QUANTITIES

After the concept of temperature became understood there were many centuries of scientific development before the real nature of *heat* was established. Even today there are many people who do not carefully observe the distinction between these important technical terms. It was early recognized that a temperature difference between two objects resulted in a flow of heat when they were placed in thermal contact. The real nature of the "thing" that flows under such circumstances has only recently been clearly identified. This development again was due to the *measurement* of heat phenomena.

Meaning of Heat. To raise the temperature of an object, it is necessary either to add heat to it from some source at a higher temperature or to do work on it. It is possible to warm your hands by rubbing them together. The work done against friction is transformed into heat and raises their temperatures. When a wire is broken by bending it back and forth rapidly, some of the work is transformed into heat and the wire gets hot. When a nail is pulled out of a board, work is needed because of the friction between the wood and the nail. The work produces heat, which warms the wood and the nail. Pumping up an automobile tire with a hand pump takes work. Some of this work produces heat which warms the pump, tire, and air. *Heat is a form of energy,* which the molecules of matter possess because of their motion. It must not be confused with temperature, which determines the direction of transfer of heat.

Suppose we dip a pail of water from the ocean. Its temperature is the same as that of the ocean, but the amount of heat (energy) in the pail of water is almost inconceivably smaller than the amount in the ocean. Temperature must be measured in terms of an independently established standard. Heat may be measured in terms of any unit that can be used to measure energy. It is more convenient, however, to measure heat in terms of a unit appropriate to the experiments that involve heat.

Units of Heat. One effect of the addition of heat to water, or any other substance, is a rise of temperature. The amount of heat necessary to raise the temperature of a certain amount of water one degree Fahrenheit is nearly constant throughout the interval between 32 and 212°F. This fact suggests a convenient unit to use in measuring heat. It is called the *British thermal unit* (Btu) and is the amount of heat needed to raise the temperature of one pound of water one degree Fahrenheit. Since the amount is not quite constant throughout the temperature range, it is more precisely defined as the amount of heat needed to change the temperature of a pound of water from 38.7 to 39.7°F.

In the metric system the corresponding unit of heat is called a *calorie*. The calorie is the heat necessary to raise the temperature of one gram of water one degree centigrade (more precisely, from 3.5 to 4.5°C). One Btu is equivalent to approximately 252 calories.

TABLE I. SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Substance	cal/gm °C or Btu/lb °F
Aluminum	0.212
Brass	0.090
Carbon (graphite)	0.160
Copper	0.092
Glass (soda)	0.016
Gold	0.0316
Ice	0.51
Iron	0.117
Lead	0.030
Silver	0.056
Zinc	0.093
Alcohol	0.60
Mercury	0 033
Water (by definition)	1 00

Specific Heat. The heat needed to change the temperature of one pound of a substance one degree Fahrenheit is a characteristic of the substance. The number of Btu's necessary to raise the temperature of one pound of a material one degree Fahrenheit is called the *specific heat*[1]

[1] Some authors call this quantity *thermal capacity* and define specific heat as the ratio of the thermal capacity of the substance to that of water.

of that material. Because of the way the Btu and the calorie are defined, the specific heat of a substance in metric units is the same numerically as when expressed in the British system. This means, for example, that the specific heat of salt, which is 0.204 Btu/lb °F, is also 0.204 cal/gm °C.

Knowing the specific heat S of a material, one can calculate the heat H necessary to change the temperature of a mass M from an initial value t_i to a final value t_f from the relation

$$H = MS(t_f - t_i)$$

or

$$H = MS\,\Delta t \tag{1}$$

Example: How much heat is necessary to raise the temperature of 2.5 lb of alcohol from room temperature (68°F) to its boiling point (78.3°C)? The boiling temperature

$$F = \tfrac{9}{5}C + 32° = \tfrac{9}{5}(78.3°) + 32° = 173°\text{F}$$

Heat required,

$$H = (2.5\ \text{lb})(0.60\ \text{Btu/lb °F})(173°\text{F} - 68°\text{F}) = 160\ \text{Btu}$$

Method of Mixtures. In calorimetry, the determination of heat quantities, one frequently utilizes a simple procedure known as the method of mixtures. In it the heat lost by an object when placed in a liquid is determined by calculating the heat gained by the liquid and its container.

The general equation for use with the method of mixtures expresses the fact that the heat lost by the sample is gained by the water and its container.

$$H_l = H_g \tag{2}$$

The heat lost by the sample H_l is

$$H_l = M_x S_x\,\Delta t_x \tag{3}$$

where M_x is the mass of the sample, S_x the specific heat of the sample, and Δt_x the change in its temperature. The heat gained by the calorimeter and water H_g will be

$$H_g = M_c S_c\,\Delta t_c + M_w S_w\,\Delta t_w \tag{4}$$

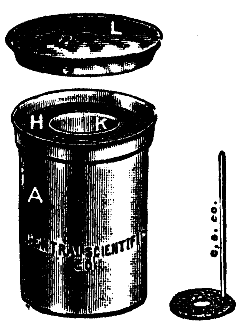


FIG. 1.—Double-walled calorimeter.

where M_c and S_c are the mass and specific heat of the calorimeter, and M_w and S_w are the mass and specific heat of the water in the calorimeter. The temperature change Δt_c refers to the calorimeter, and Δt_w is the change in the temperature of the water. To minimize the exchange of heat with the surroundings, a double-walled vessel (Fig. 1) is usually used in calorimetric experiments.

Example: When 2.00 lb of brass at 212°F are dropped into 5.00 lb of water at 35.0°F, the resulting temperature is 41.2°F. Find the specific heat of brass.

$$
\begin{aligned}
H_l &= H_g \\
M_B S_B \,\Delta t_B &= M_w S_w \,\Delta t_w \\
(2.00 \text{ lb}) S_B (212°\text{F} - 41.2°\text{F}) &= (5.00 \text{ lb})(1 \text{ Btu/lb °F})(41.2°\text{F} - 35.0°\text{F}) \\
S_B &= \frac{(5.00 \text{ lb})(1 \text{ Btu/lb °F})(41.2°\text{F} - 35.0°\text{F})}{(2.00 \text{ lb})(212°\text{F} - 41.2°\text{F})} \\
&= 0.091 \text{ Btu/lb °F}
\end{aligned}
$$

Example: Eighty grams of iron shot at 100.0°C are dropped into 200 gm of water at 20.0°C contained in an iron vessel weighing 50 gm. Find the resulting temperature t.

$$
\begin{aligned}
\text{Heat lost by shot} &= M_x S_x \,\Delta t_x = (80 \text{ gm})(0.12 \text{ cal/gm °C})(100.0°\text{C} - t) \\
\text{Heat gained by water} &= M_w S_w \,\Delta t_w = (200 \text{ gm})(1 \text{ cal/gm °C})(t - 20.0°\text{C}) \\
\text{Heat gained by vessel} &= M_c S_c \,\Delta t_c = (50 \text{ gm})(0.12 \text{ cal/gm °C})(t - 20.0°\text{C})
\end{aligned}
$$

$$
\begin{aligned}
\text{Heat lost} &= \text{heat gained} \\
M_x S_x \,\Delta t_x &= M_c S_c \,\Delta t_c + M_w S_w \,\Delta t_w \\
(80 \text{ gm})(0.12 \text{ cal/gm °C})(100.0°\text{C} - t) &= (200 \text{ gm})(1 \text{ cal/gm °C})(t - 20.0°\text{C}) \\
&\quad + (50 \text{ gm})(0.12 \text{ cal/gm °C})(t - 20.0°\text{C}) \\
t &= 23.6°\text{C}
\end{aligned}
$$

Change of State. Not all the heat that an object receives necessarily raises its temperature. Surprisingly large amounts of energy are needed to do the work of separating the molecules when solids change to liquids and liquids change to gases. Water will serve as a familiar example. In the solid phase water is called ice. Ice has a specific heat of about 0.5 cal/gm °C. Water has a specific heat of 1 cal/gm °C. Water changes from solid to liquid at 0°C (at atmospheric pressure). Large changes in pressure change this melting point appreciably, a fact that is demonstrated when we make a snowball freeze together by squeezing and then releasing the snow.

If a liquid is cooled without being disturbed and without the presence of any of the solid, it is possible to reduce its temperature below the normal freezing point without solidification. The liquid is said to be *supercooled.* If the supercooled liquid is disturbed, it immediately freezes. Water droplets in the air are frequently much below the normal freezing temperature. Clouds of water droplets are more frequent above −10°C; below that temperature ice clouds are more numerous. Water droplets have been observed at temperatures as low as −40°C. Severe icing may result if the supercooled droplets strike an airplane.

Heat of Fusion. To raise the temperature of 1 gm of ice from −1 to 0°C requires ½ cal of heat energy. To raise the temperature of 1 gm of water in the liquid phase from 0 to 1°C requires 1 cal. To melt a gram of ice requires 80 cal, although the temperature does not change while this large amount of heat is being added. The heat needed to change unit mass of a substance from the solid to the liquid state at its melting temperature is called the *heat of fusion.* It is measured in Btu per pound or in calories per gram. The heat of fusion of ice is about 144 Btu/lb, or 80 cal/gm. (NOTE: Whereas specific heats are numerically the same in British and metric units, heats of fusion differ numerically in the two systems of units.)

Heat of Vaporization. Once a gram of ice is melted, 100 cal is required to raise its temperature from the melting point to the boiling point. Though water evaporates at all temperatures, boiling occurs when its vapor pressure becomes as large as atmospheric pressure and bubbles of vapor begin forming under the surface of the liquid. As we continue to add heat at the boiling point, the temperature remains the same until the liquid is changed entirely to vapor. The steps by which a gram of ice is heated through fusion and vaporization are shown to scale in Fig. 2. The amount of heat necessary to change a unit mass of a liquid from the liquid to the vapor phase without changing the tempera-

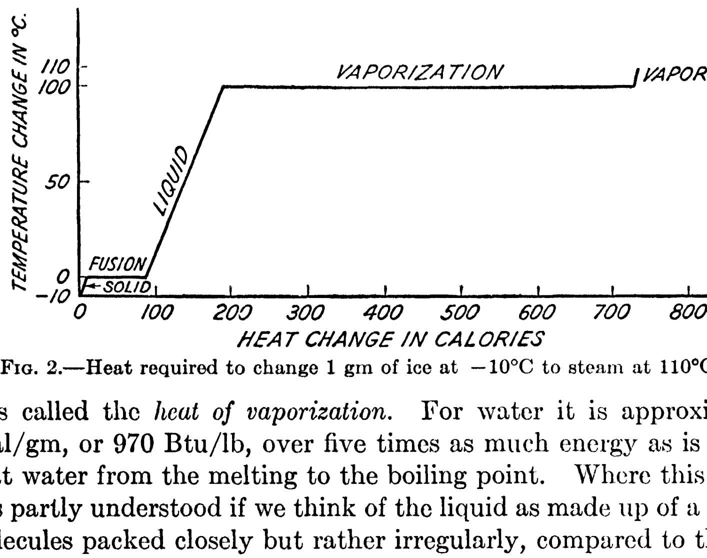


FIG. 2.—Heat required to change 1 gm of ice at $-10°C$ to steam at $110°C$.

ture is called the *heat of vaporization*. For water it is approximately 540 cal/gm, or 970 Btu/lb, over five times as much energy as is needed to heat water from the melting to the boiling point. Where this energy goes is partly understood if we think of the liquid as made up of a myriad of molecules packed closely but rather irregularly, compared to the neat arrangement in the crystals that make up the solid. One gram of water occupies 1 cm^3 of space as a liquid. The same amount of water (and therefore the same number of molecules) in the vapor state at 1 atm of pressure and a temperature of 100°C fills 1,671 cm^3 instead of one. The work to vaporize the water has been done in separating the molecules to much larger distances than in the liquid state.

Example: How much heat is required to change 50 lb of ice at 15°F to steam at 212°F?

Heat to raise temperature of ice to its melting point $= M_i S_i\ (32°F - 15°F)$
$= (50 \text{ lb})(0.51 \text{ Btu/lb °F})(32°F - 15°F) = 430$ Btu

Heat to melt ice $= (50 \text{ lb})(144 \text{ Btu/lb}) = 7200$ Btu

Heat to warm water to its boiling point $= M_w S_w\ (212°F - 32°F)$
$= (50 \text{ lb})(1 \text{ Btu/lb °F})(212°F - 32°F) = 9000$ Btu

Heat to vaporize water $= (50 \text{ lb})(970 \text{ Btu/lb}) = 48{,}000$ Btu

Total heat required:

$$
\begin{array}{r}
4\bar{3}0 \text{ Btu} \\
7{,}\bar{2}00 \\
9{,}\bar{0}00 \\
\underline{4\bar{8}{,}000} \\
6\bar{4}{,}000 \text{ Btu}
\end{array}
$$

Note that in this summation the 4$\bar{3}$0 is negligible and may be disregarded, since there is a doubtful figure in the thousands place in 4$\bar{8}$,000.

Measurement of Heat of Fusion. Heats of fusion and vaporization, like specific heats, are determined by calorimeter experiments. The only change needed in Eqs. (3) and (4) is the addition of a term giving the amount of heat required to change the state. If a mass M of ice is added to a calorimeter containing enough warm water so that the ice all melts, the ice will gain heat and the calorimeter and water will lose an equal amount. The heat gained by the ice will be the heat to melt it, assuming that it is at 0°C when put into the calorimeter, plus the heat to warm it to the final temperature once it is all melted. This is,

$$H_g = M_iL_i + M_iS_w(t_f - 0) \tag{5}$$

where H_g represents heat gained by the mass M_i of melting ice whose heat of fusion L_i is to be measured, S_w is the specific heat of the water which was ice before it melted, and t_f is the final temperature. The heat lost by the calorimeter and the water in it will be

$$H_l = M_cS_c\,\Delta t_c + M_wS_w\,\Delta t_w \tag{6}$$

where the symbols have meanings analogous to those in Eq. (4). The initial temperature should be about as far above room temperature as the latter is above the final temperature. In this case, the heat that is lost to the surroundings while the calorimeter is above room temperature is compensated by that gained while it is below room temperature. Because of the relatively large amount of heat required to melt the ice, the quantity of ice used should be chosen appropriately smaller than the quantity of water. The value of the heat of fusion L_i is determined by equating H_g and H_l from Eqs. (5) and (6), and solving the resulting equation for L_i.

Example: When 150 gm of ice at 0°C is mixed with 300 gm of water at 50°C the resulting temperature is 6.7°C. Calculate the heat of fusion of ice.

Heat lost by water = (300 gm)(1 cal/gm °C)(50°C − 6.7°C) = 1$\bar{3}$,000 cal
Heat to melt ice = (150 gm)L_i
Heat to raise temperature of ice water to final temperature

$$
\begin{aligned}
&= (150\text{ gm})(1\text{ cal/gm °C})(6.7\text{°C} - 0\text{°C}) = 1{,}\bar{0}00\text{ cal} \\
\text{Heat lost} &= \text{heat gained} \\
1\bar{3}{,}000\text{ cal} &= (150\text{ gm})L_i + 1{,}\bar{0}00\text{ cal} \\
L_i &= 80\text{ cal/gm}
\end{aligned}
$$

Phases of Matter. Among the common materials are many that do not have definite melting points; for example, glass and butter. In a furnace, glass will gradually soften until it flows freely even though at ordinary temperatures it is quite solid. When it is solid it may be thought of as a supercooled liquid; it flows, but very slowly. Since it does not have a definite melting point, it does not have a heat of fusion.

The specific heat of glass changes as the temperature rises. Such changes indicate transitions in the arrangement of the molecules. Specific heat measurements may be used by the ceramic engineer in studying the changes in these products as the temperature is varied.

Many materials decompose at high temperatures and therefore do not exist in liquid and gaseous states. Some may exist in the liquid state but decompose before reaching the gaseous state.

Since the chemical elements cannot be decomposed by heating, they are all capable of existing in the solid, liquid, and gaseous states. Many of them have more than one solid state, as in the case of phosphorus, which is known in three solid phases: black, formed at very high pressures, and the more familiar red and yellow forms. Powdered sulphur results from a direct transition of sulphur vapor to the solid state. If this powder is melted and then cooled, it solidifies normally. By lowering the pressure on water with a vacuum pump, one can cause it to boil and freeze at the same time. Ice, solid carbon dioxide, and many other solid substances evaporate. The odor of solid camphor is evidence of its evaporation.

SUMMARY

Heat is a form of energy.

The most commonly used units of heat are the *calorie* and the *British thermal unit.*

The *calorie* is the amount of heat required to change the temperature of 1 gm of water 1°C.

The *British thermal unit* is the amount of heat required to change the temperature of 1 lb of water 1°F.

The *specific heat* of a substance is the amount of heat required to change the temperature of unit mass of the substance one degree (Units: cal/gm °C or Btu/lb °F).

The specific heat of water varies so slightly with temperature that for most purposes it can be assumed constant (1 cal/gm °C) between 0 and 100°C.

The heat lost or gained by a body when the temperature changes is given by the equation

$$H = MS\,\Delta t$$

In a *calorimeter* the heat lost by the hot bodies is equal to the heat gained by the cold bodies ($H_l = H_g$). To reduce the effect of the surroundings, the final temperature of the calorimeter should be as much below (or above) room temperature as it was originally above (or below) room temperature.

The *heat of fusion* of ice is approximately 80 cal/gm, or 144 Btu/lb.

The *heat of vaporization* of water is approximately 540 cal/gm, or 970 Btu/lb. It depends on the temperature at which vaporization takes place.

QUESTIONS AND PROBLEMS

1. How many Btu are required to raise the temperature of 0.50 lb of aluminum from 48 to 212°F?

2. How much heat is required to raise the temperature of 1.5 lb of water in an 8-oz aluminum vessel from 48°F to the boiling point, assuming no loss of heat to the surroundings? *Ans.* $2\bar{6}0$ Btu.

3. What is the specific heat of 500 gm of dry soil if it requires the addition of 2,000 cal to increase its temperature 20°C?

4. From the following data, taken in a calorimeter experiment, what value is obtained for the specific heat of iron?

Mass of iron	320 gm
Mass of calorimeter	55 gm
Specific heat of brass calorimeter	0.090 cal/gm °C
Mass of cold water	350 gm
Temperature of iron, before mixing	95.0°C
Temperature of water, before mixing	15.0°C
Temperature of water and iron after mixing	22.6°C

Ans. 0.116 cal/gm °C.

5. By referring to the definitions, show that 1 Btu is equal to 252 cal.

6. Which produces the more severe burn, boiling water or steam? Why?

7. One frequently places a tub of water in a fruit storage room to keep the temperature above 30°F during a cold night. Explain.

8. How much heat is required to change the temperature of a 10.0-lb block of copper from 50 to 250°F? If the block (at 250°F) is placed in 50 lb of water at 40°F, what will be the resulting temperature? *Ans.* 184 Btu; 43.8°F.

9. A 100-lb casting was cooled from 1300° (red hot) to 200°F by placing it in water whose initial temperature was 50°F. How much water was used? The specific heat of iron is approximately 0.12 Btu/lb °F for this temperature range.

10. Water is heated in a boiler from 100 to 284°F where, under a pressure of 52.4 lb/in.^{2}, it boils. The heat of vaporization for water at 284°F is 511.5 cal/gm, or 920.7 Btu/lb. How much heat is required to raise the temperature and to evaporate 500 gal of water? *Ans.* $4{,}6\bar{1}0{,}000$ Btu.

11. How much energy must be removed by the refrigerator coils from a ½-lb aluminum tray containing 3 lb of water at 70°F to freeze all the water, and then to cool the ice to 10°F? Plot the amount of heat extracted against temperature.

12. Calculate the amount of energy required to heat the air in a house 30 by 50 by 40 ft from 10 to 70°F. The density of air is about 0.08 lb/ft^3 and its specific heat at constant pressure is approximately 0.24 Btu/lb °F. Discuss the assumptions made in your calculations. *Ans.* 69,000 Btu.

EXPERIMENT

Specific Heats of Metals

Apparatus: Double-walled calorimeter; steam generator outfit; Bunsen burner; two thermometers (0 to 110°C and 0 to 50°C); 100 gm each of two different kinds of metal shot; trip scales.

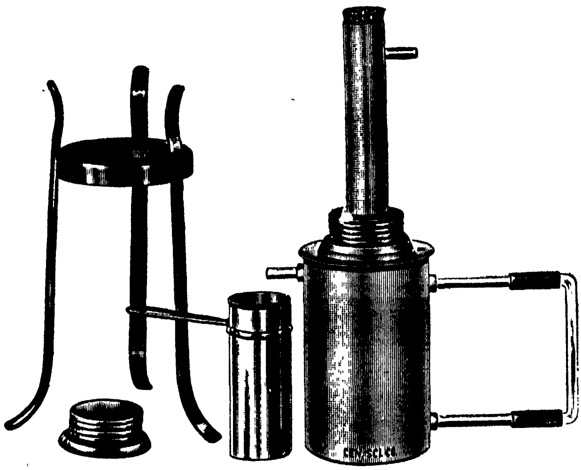
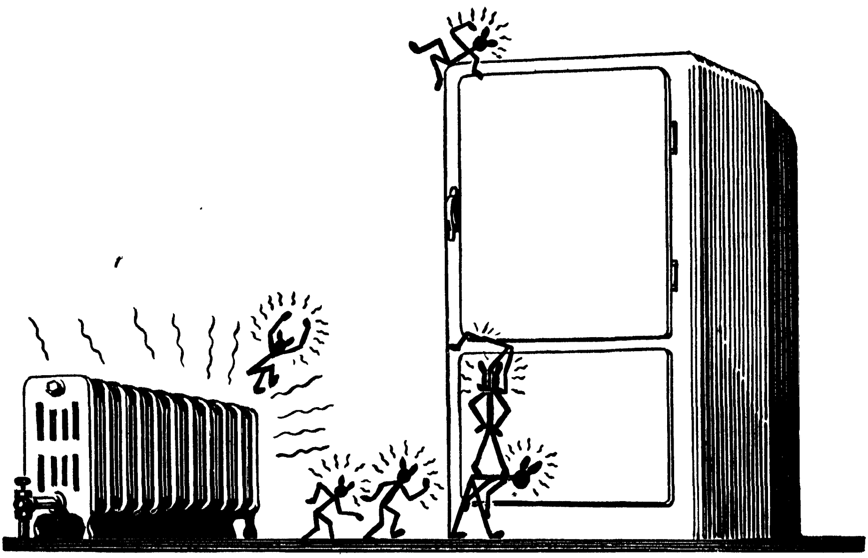


FIG. 3.—Steam generator.

While the water in the boiler is being heated, 100 gm of metal shot M_x is weighed out, poured into the dipper, and placed in the steam generator (Fig. 3) so that the shot can be heated without coming into contact with the hot water or steam. The dipper should be covered during the heating to ensure that the shot will be heated uniformly. While the shot is being heated, weigh the inner calorimeter cup M_c; then pour into it about 100 gm of water M_w whose temperature is 3 or 4° below room temperature, and place it in the outer calorimeter to minimize the gain of heat from the surroundings.

After the shot has reached a temperature above 95°C, record its exact temperature and that of the cold water in the calorimeter. Pour the shot quickly into the water and stir the mixture. Record the highest temperature reached by the water above the shot.

Substitute the data in Eqs. (3) and (4), assume $H_l = H_g$, and solve for the specific heat of the metal shot. (The specific heat of the calorimeter may be obtained from Table I.) Compare the experimental value of the specific heat with the value given in the table. What is the percentage error?



CHAPTER 5

HEAT TRANSFER

Heat is the most common form of energy. The engineer is concerned with it continually. Sometimes he wants to get it from one place to another, sometimes he wants to "bottle it up" for storage. In the first problem, he is confronted with the fact that there are no perfect conductors of heat. The problem of heat storage is complicated by the fact that there are no perfect insulators of heat, so that one cannot confine it.

Heat Flow. Heat is always being transferred in one way or another, wherever there is any difference in temperature. Just as water will run down hill, always flowing to the lowest possible level, so heat, if left to itself, flows down the temperature hill, always warming the cold objects at the expense of the warmer ones. The rate at which heat flows depends on the steepness of the temperature hill as well as on the properties of the materials through which it has to flow. The difference of temperature per unit distance is called the *temperature gradient* in analogy to the idea of steepness of grade, which determines the rate of flow of water.

Types of Heat Transfer. There are three ways in which heat is transferred. Since heat itself is the energy of molecular activity, the simplest mode of transfer of heat, called *conduction*, is the direct communication of molecular disturbance through a substance by means of the collisions of neighboring molecules. *Convection* is the transfer of heat from one place to another by actual motion of the hot material. Heat transfer is accomplished also by a combination of *radiation* and *absorption*. In the former, heat energy is transformed into electromagnetic energy. While in this form, the energy may travel a tremendous distance before being absorbed or changed back into heat. For example, energy radiated from the surface of the sun is converted into heat at the surface of the earth only eight minutes later.

Conduction. Conduction of heat is important in getting the heat from the fire through the firebox and into the air or water beyond. Good heat conductors, such as iron, are used for such jobs. To keep heat in, poor conductors, or insulators, are used, the amount of flow being reduced to the smallest level that is consistent with other necessary properties of the material, such as strength and elasticity. The amount of heat that flows through any body depends upon the time of flow, the area through which it flows, the temperature gradient, and the kind of material. Stated as an equation

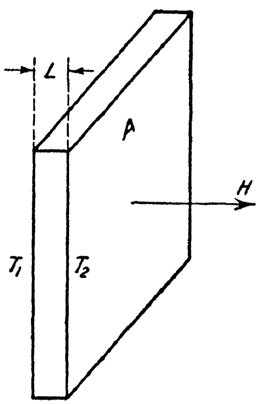


FIG. 1.—Heat conduction through a thin plate.

$$H = KAt\frac{\Delta T}{L} \tag{1}$$

where K is called the *thermal conductivity* of the material, A is the area measured at right angles to the direction of the flow of heat, t is the time the flow continues, and $\Delta T/L$ is the temperature gradient. The symbol ΔT represents the difference in temperature between two parallel surfaces distant L apart (Fig. 1).

In the British system these quantities are usually measured in the following units: H in Btu, A in square feet, t in hours, ΔT in °F, and L in inches. The conductivity K is then expressed in Btu/(ft^2 hr °F/in.). The corresponding unit of K in the metric system is cal/(cm^2 sec °C/cm).

Example: A copper kettle, the circular bottom of which is 6.0 in. in diameter and 0.062 in. thick, is placed over a gas flame. Assuming that the average temperature of the outer surface of the copper is 300°F and that the water in the kettle is at its normal boiling point, how much heat is conducted through the bottom in 5.0 sec? The coefficient of thermal conductivity may be taken as 2,480 Btu/(ft^2 hr °F/in.).

$$H = KAt\frac{\Delta T}{L}$$

The area A of the bottom is

$$A = \pi r^2 = \pi\left(\frac{3.0}{12}\text{ ft}\right)^2 = 0.20\text{ ft}^2$$

$$t = 5.0 \text{ sec} = \frac{5.0}{3{,}600} \text{ hr}$$

$$\frac{\Delta T}{L} = \frac{300°\text{F} - 212°\text{F}}{0.062 \text{ in.}} = 1\bar{4}00 \text{ °F/in.}$$

$$H = \left(2{,}4\bar{8}0 \frac{\text{Btu}}{\text{ft}^2 \text{ hr °F/in.}}\right) (0.20 \text{ ft}^2) \left(\frac{5.0}{3{,}600} \text{ hr}\right) (1\bar{4}00 \text{ °F/in.}) = 9\bar{6}0 \text{ Btu}$$

There are large differences in the conductivities of various materials. Gases have very low conductivities. Liquids also are, in general, quite poor conductors. The conductivities of solids vary over a wide range, from the very low values for asbestos fiber or brick to the relatively high values for most metals. Fibrous materials such as hair felt or asbestos are very poor conductors (or good insulators) when dry; if they become wet, they conduct heat rather well. One of the difficult problems in using such materials for insulation is to keep them dry.

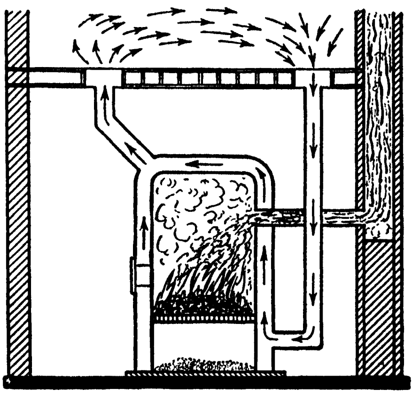


FIG. 2.—Heating by convection.

Under certain conditions good conductors fail to transfer heat readily. This may be caused by an insulating layer of air that sticks to the surface, a layer that can be removed to some extent by vigorous stirring or ventilation. The familiar difficulty we have of keeping warm in a cold wind as compared with cold, still air is an illustration of this. A thin layer of air is one of the most effective of all heat insulators. Surface layers of oxide or other foreign material also impede the flow of heat. Iron, which is a rather good conductor in itself, fails to transfer heat readily when covered by a layer of rust.

Convection. The heating of buildings is accomplished largely through convection. Air heated by contact with a stove (conduction) expands and floats upward through the denser cold air around it. This causes more cold air to come in contact with the stove setting up a circulation, which distributes warm air throughout the room. When these convection currents are enclosed in pipes, one for the ascending hot air and another for the descending cold air, heat from a single furnace can be distributed throughout a large building (Fig. 2). In order to provide a supply of fresh air, the cold-air return pipe is often supplemented or even replaced by a connection to the outside of the building.

In water, as in air, the principal method of heat transfer is convection. If heat is supplied at the bottom of a container filled with water, convection currents will be set up and the whole body of water will be warmed. If, however, the heat is supplied at the top of the container, the water at the bottom will be warmed very slowly.

For example, if the top of a test tube filled with water is placed in a flame, the water in the top of the tube can be made to boil vigorously before the bottom of the tube begins to feel warm to the hand. This is possible only when the test tube is of sufficiently small diameter to prevent the formation of effective convection currents. Convection currents are utilized in hot-water heating systems, in which the hot water rises through the pipes, circulates through the radiators, and sinks again when cooled, forcing up more hot water.

Since convection is a very effective method of heat transfer, it must be considered in designing a system of insulation. If large air spaces are left within the walls of a house, convection currents are set up readily and much heat is lost. If, however, the air spaces are broken up into small, isolated regions, no major convection currents are possible and little heat is lost by this method. For this reason the insulating material used in a refrigerator or in the walls of a house is a porous material—cork, rock wool, or other materials of like nature. They are not only poor conductors in themselves but they leave many small air spaces, which are very poor conductors and at the same time are so small that no effective convection currents can be set up.

Radiation. The transfer of heat by radiation does not require a material medium for the process. Energy traverses the space between the sun and the earth and, when it is absorbed, it becomes heat energy. Energy emitted by the heated filament of an electric lamp traverses the space between the filament and the glass even though there is no gas in the bulb. Energy of this nature is emitted by all bodies. If the temperature of the radiating body is high enough, we can actually see the radiation, for our eyes are sensitive to this type of energy. The fact that objects radiate energy that does not affect the eye is shown by the warmth we get from a stove long before it becomes "red hot."

The rate at which energy is radiated from an object depends upon the temperature of the object, the area of the surface, and upon the condition of the surface. The rate of radiation increases very rapidly as the temperature rises. A piece of ice radiates energy less rapidly than one's hand held near it and thus seems cold, while a heated iron radiates energy faster than the hand and thus seems warm.

Objects whose surfaces are in such condition that they are good absorbers of radiation are also good radiators. A blackened surface will absorb more readily than a polished surface. The blackened surface will also radiate faster than the polished surface if the two are at the same temperature. One can decrease the radiation from a surface by polishing it, or increase the radiation by coating it with suitable absorbing material.

An interesting practical application is illustrated by the method of installing hot-air furnace pipes. For years it was customary to wrap these pipes with asbestos, even after it was known (about 1920) that this practice made the pipes lose heat more rapidly than if they had been left as bright tinned metal. Actual experiments proved definitely that eight or nine layers of asbestos paper have to be applied in order to make the pipe lose less heat than when bare; yet only in the last few years has this fact been utilized commercially. Uncovered galvanized metal pipes are being used in most modern installations. The cold-air returns are likewise being put in with more attention, since it is now recognized that the returning cold air lifts the hot air, causing the circulation.

A thermos bottle (Fig. 3) illustrates how the principles of heat transfer may be used to decrease the amount of heat flowing into (or out of) a container. It consists of two bottles, one inside the other, touching each other only at the neck. The space between the two bottles is evacuated and the surfaces are silvered. Transfers by conduction are minimized by using a very small area of a poorly conducting material, those due to convection are lessened by removing the air. The transfer by radiation is made small because the polished silver acts as a poor emitter for one surface and a poor absorber for the other.

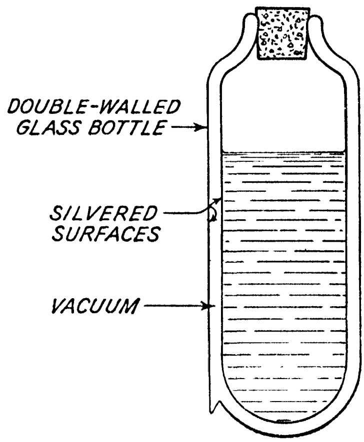


FIG. 3.—Dewar, or thermos, flask.

It is common experience to notice that as a piece of metal is heated sufficiently it begins to glow a dull red (at about 470°C, 880°F). If the heating is continued the color changes from dull red to cherry red, to light red, then to yellow, and finally to a dazzling white (above 1150°C, 2100°F).

As the temperature of a glowing body is increased, the total energy radiated per unit time increases rapidly. With an increase in temperature the color shifts from the red end of the spectrum toward the blue.

These facts suggest two ways of measuring temperatures of hot bodies in terms of their own radiant energy. One method is to collect a certain fraction of this energy, convert it into electrical energy, and then measure the current with an electrical meter. An instrument for measuring temperatures this way is called a *total radiation pyrometer*.

Another method of measuring temperature is merely a refinement of the optical method we use when we observe that iron is "red hot." Even an experienced person probably can judge temperatures by color only to within 50 to 100°C. The human eye cannot judge ratios of intensities accurately. It can, however, match two intensities of the same color very precisely. Advantage is taken of this fact in the design of many instruments. An optical pyrometer provides the eye with a standard (a glowing lamp filament) against which it compares the radiation of an object whose temperature is to be measured. Suitable filters allow only light of one color, usually red, to enter the eye. By varying the current in the filament its temperature can be varied until the radiation received by the eye from the two sources matches. The temperature is then read on a scale that is calibrated in terms of the current in the filament.

SUMMARY

Heat is the most common form of energy.

Energy may appear in any one of several forms (mechanical, electrical, thermal, etc.) and may be changed from one form to another.

The three ways in which heat may be transferred from one place to another are *conduction, convection,* and *radiation.*

Conduction is heat transfer from molecule to molecule through a body, or through bodies in contact.

Convection is heat transfer by means of moving heated matter.

Radiation is heat transfer by means of waves, called electromagnetic waves, which are similar to short radio waves. Radiation passes readily through a vacuum, is partly turned back by polished surfaces, and may be absorbed by vapors or solids or liquids.

Temperature gradient is temperature difference per unit distance along the direction of heat flow. It may have units in degrees centigrade per centimeter, degrees Fahrenheit per inch, etc.

Thermal conductivity K is a quantity that expresses how well a substance conducts heat. It may have units of calories per square centimeter per second for a gradient of 1°C/cm or Btu per square foot per hour for a gradient of 1°F/in.

QUESTIONS AND PROBLEMS

1. Why does a chimney "draw" poorly when a fire is first lighted?

2. Why does iron seem colder to the touch than wood in winter weather?

3. Why is a hollow wall filled with rock wool a better insulator than when filled with air alone?

4. A piece of paper wrapped tightly on a brass rod may be held in a gas flame without being burned. If wrapped on a wooden rod, it burns quickly. Explain.

5. Explain how a thermos flask minimizes energy losses from convection, conduction, and radiation.

6. A certain window glass, 30 in. by 36 in., is ⅛ in. thick. One side has a uniform temperature of 70°F, and the second face a temperature of 10°F. What is the temperature gradient? *Ans.* 480°F/in.

7. The thermal conductivity of window glass is approximately 7.24 Btu/(ft² hr °F/in.) at ordinary temperatures. Find the amount of heat conducted through the window glass of problem 6 in 1 hr.

8. What will be the rise in temperature in 30 min of a block of copper of 500-gm mass if it is joined to a cylindrical copper rod 20 cm long and 3 mm in diameter when there is maintained a temperature difference of 80°C between the ends of the rod? The thermal conductivity of copper is 1.02 cal/(cm² sec °C/cm). Neglect heat losses. *Ans.* 11.3°C.

9. A copper rod whose diameter is 2 cm and whose length is 50 cm has one end in boiling water and the other in a block of ice. The thermal conductivity of the copper is 1.02 cal/(cm² sec °C/cm). How much ice will be melted in 1 hr if 25 per cent of the heat escapes during transmission?

10. How much steam will be condensed per hour on an iron pipe 2 cm in mean radius and 2 mm thick, a 60-cm length of which is in a steam chamber at 100°C, if water at an average temperature of 20°C flows continuously through the pipe? The coefficient of thermal conductivity for iron is 0.18 cal/(cm² sec °C/cm). *Ans.* 364 kg.

DEMONSTRATIONS

Heat Transfer

Apparatus: Convection box; 2 candles; rods or tubes of different materials; paraffin; nails; cans painted differently; insulating material; thermometer; water boiler.

The following simple demonstrations may contribute to a clarification of the fundamental concepts of convection, conduction, and radiation.

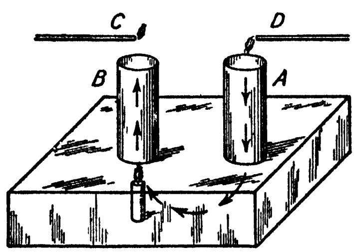


FIG. 4.—Convection currents in air.

They require only simple apparatus, which can easily be assembled and demonstrated to the class by interested students.

Convection. Figure 4 represents a box with two holes in its top. Over each hole is placed a glass or metal tube of large diameter. (A carton or cigar box is suitable. The tubes may be lamp chimneys, or tin cans with tops and bottoms removed.) A burning candle is placed in the box in such a manner that it extends up into tube B. There will be convection currents upward in tube B and downward in A. This may be demonstrated by means of another candle flame as suggested by C and

D in Fig. 4. If chimney *A* is covered (and there are no other holes in the box) the flame in *B* goes out. Why?

Conduction. A metal tube or rod may be shown to be a good conductor by holding it in the hand at one end and putting the other end in a Bunsen flame. Low conductivity can be demonstrated similarly by the use of a glass rod or tube. That different metals have different conductivities may be shown by rods of the same dimensions but of different materials. To one end of each rod a nail is attached with paraffin. When the other ends are heated, the nails will drop after different time intervals. Why?

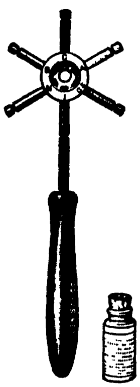
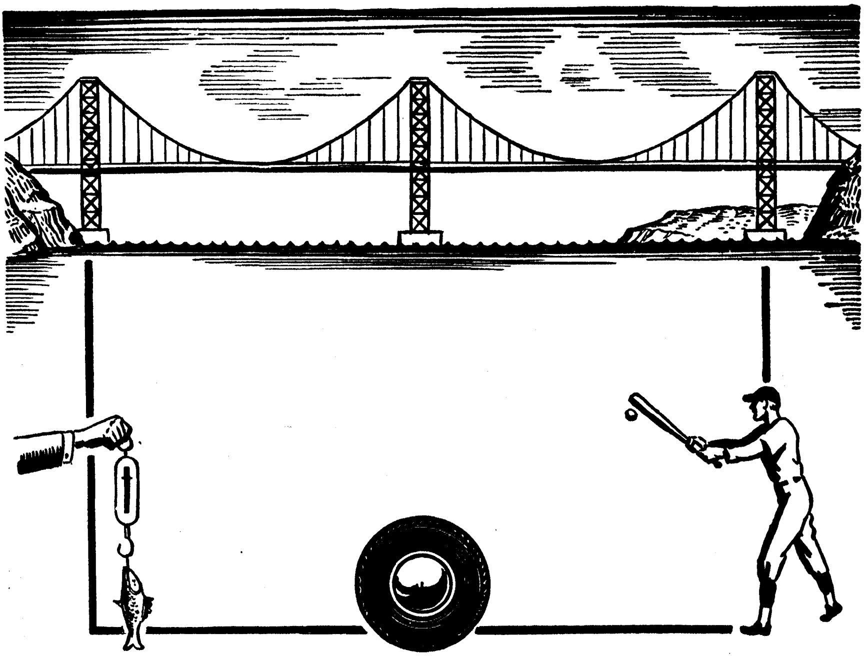


FIG. 5.—Conductometer, an instrument for showing rates of heat transfer in different metals.

Another type of conductometer is shown in Fig. 5. Heat applied to the common junction is conducted to the ends of the rods, which have been dipped into an ignition solution. The sequence of ignitions ranks the different metals according to their thermal conductivities.

Radiation. Tin cans with different coverings or surfaces serve admirably as radiators. They may be painted black, white, aluminized, covered with asbestos paper, polished, etc. After being filled to the same volume with *boiling* water they will cool at different rates. To demonstrate this, record the temperature of the water immediately after it is poured into a can, and again half an hour later. During this time each can should be covered with and rest upon wood, thick cardboard, or other similar insulating material. Which should cool most rapidly? Why? Don't be fooled! The results are not what many persons would anticipate.

Are double-walled calorimeters really more effective as heat retainers than single-walled cups? It is very instructive to observe how much more rapidly water cools in a simple cup than in a calorimeter. Heat transfer of which kind (or kinds) is responsible for the cooling? How may we know experimentally?



CHAPTER 6

PROPERTIES OF SOLIDS

When a structure or a machine is to be built, suitable materials must be chosen for the parts. Each available material is examined to determine whether its properties will meet the demands of a particular application. Some of the properties thus considered are weight, strength, hardness, expansive characteristics, melting point, and elasticity.

Much of the progress in the design of structures has resulted from the discovery, adaptation, or development of new structural materials. As stone, brick, steel, and reinforced concrete replaced the original structural materials, mud and wood, buildings became stronger and taller. Early tools were made of wood, bone, or stone, but the discovery of metals made possible the construction of more intricate and useful devices. The machine age depends largely upon the technology of metals.

Elasticity. Among the most important properties of materials are their elastic characteristics. If, after a body is deformed by some force, it returns to its original shape or size as the distorting force is removed, the material is said to be *elastic.* Every substance is elastic to some degree.

Consider a long steel wire fastened to the ceiling, in such a manner that its upper end is held rigidly in place. To keep the wire taut suppose a stone of sufficient mass is fastened to the lower end of the wire. The force per unit cross section of the wire is defined as the *tensile stress* in the wire. The pound per square inch is the unit in which this stress is commonly measured. To emphasize the fact that stress is stretching force per unit area, it is sometimes called *unit stress.*

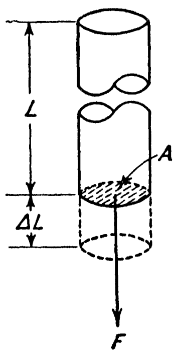


FIG. 1.—Stress and strain in the stretching of a wire.

Let L (Fig. 1) represent the length of the wire when just enough force has been applied to take the kinks out of it. Increasing the stretching force by an amount F will stretch or elongate the wire an amount ΔL. The ratio of the change in length ΔL to the total length L is called the *tensile strain.* Notice that the change in length must be measured in the same unit as the total length if the value of this ratio $\Delta L/L$ is to be independent of the units used.

Hooke's Law. Robert Hooke recognized and stated the law that is used to define a modulus of elasticity. In studying the effects of tensile forces he observed that the increase in length of a body is proportional to the applied force over a rather wide range of forces. This observation may be made more general by stating that *the strain is proportional to the stress.* In this form the statement is known as *Hooke's law.*

If the stress is increased above a certain value, the body will not return to its original size (or shape) after the stress is removed. It is then said to have acquired a *permanent set.* The smallest stress that produces a permanent set is called the *elastic limit.* For stresses that exceed the elastic limit Hooke's law is not applicable.

Young's Modulus. A modulus of elasticity is defined as the ratio of a stress to the corresponding strain. This ratio is a constant, characteristic of the material. The ratio of the tensile stress to the tensile strain is called *Young's modulus.*

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L} \qquad (1)$$

Example: A steel bar, 20 ft long and of rectangular cross section 2.0 by 1.0 in., supports a load of 2.0 tons. How much is the bar stretched?

$$Y = \frac{FL}{A\,\Delta L}$$

Solving for ΔL

$$\Delta L = \frac{FL}{YA}$$

$$F = 2.0 \text{ tons} = (2.0 \text{ tons})(2{,}000 \text{ lb/ton}) = 4{,}000 \text{ lb}$$
$$L = 20 \text{ ft}$$
$$A = (2.0 \text{ in.} \times 1.0 \text{ in.}) = 2.0 \text{ in.}^2$$
$$Y = 29{,}000{,}000 \text{ lb/in.}^2$$
$$\Delta L = \frac{(4{,}\bar{0}00 \text{ lb})(2\bar{0} \text{ ft})}{(2\bar{9}{,}000{,}000 \text{ lb/in.}^2)(2.0 \text{ in.}^2)} = 0.0014 \text{ ft} = 0.017 \text{ in.}$$

Values of Y for several common materials are given in Table I.

TABLE I. VALUES OF YOUNG'S MODULUS

Substance	Young's modulus, lb/in.²	Stress at elastic limit, lb/in.²	Breaking stress, lb/in.²
Aluminum, rolled	10,000,000	25,000	29,000
Aluminum alloy, 20% nickel	9,400,000	23,000	60,000
Iron, wrought	27,500,000	23,500	47,000
Lead, rolled	2,200,000	......	3,000
Phosphor bronze	....	60,000	80,000
Rubber, vulcanized	20	500	2,500
Steel, annealed	29,000,000	40,000	75,000

Note that the physical dimensions of Y are those of force per unit area. Fig. 2 illustrates apparatus for determining Young's modulus by applying successively greater loads to a wire and measuring its elongation.

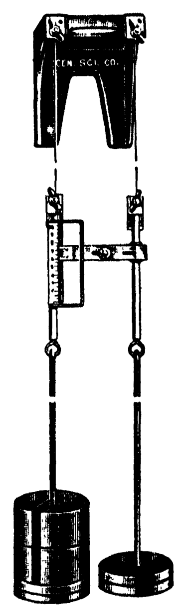


FIG. 2.—Apparatus for determining Young's modulus.

Although stretching a rubber band does increase the restoring force, the stress and strain do not vary in a direct proportion; hence Young's modulus for rubber is not a constant. Moreover, a stretched rubber band does not return immediately to its original length when the deforming force is removed. This failure of an object to regain its original size and shape as soon as the deforming force is removed is called *elastic lag* or *hysteresis* (a lagging behind).

Ordinarily stretching a wire cools it. Rubber gets warmer when stretched and cools when relaxed. This can be verified easily by stretching a rubber band and quickly holding it against the lips or tongue, which are very sensitive to changes in temperature. One would expect then that heating a rubber band would increase the stress. A simple experiment shows this to be true. Suspend a weight by a long rubber band and apply heat to the band with a Bunsen flame played quickly across the band so as not to fire the rubber. The band will contract, lifting the weight. A wire under similar circumstances will expand, lowering the weight. The elastic modulus of a metal decreases as the temperature increases.

Volume Elasticity. Bodies can be compressed as well as stretched. In this type of deformation elastic forces tend to restore the body to its original size.

Suppose that a rubber ball is placed in a liquid confined in a vessel and that a force is applied to the confined liquid, causing the ball to contract. Then the *volume stress* is the increase in force per unit area and the *volume strain* is the fractional change that is produced in the volume of the ball. The ratio (volume stress)/(volume strain) is called the *coefficient of volume elasticity*, or *bulk modulus*.

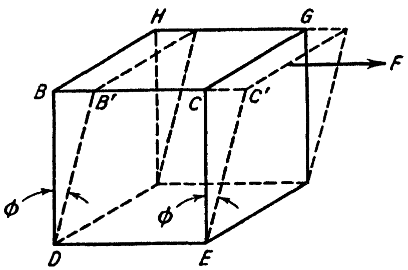


FIG. 3.—Shearing of a cubical block through an angle ϕ by a force F.

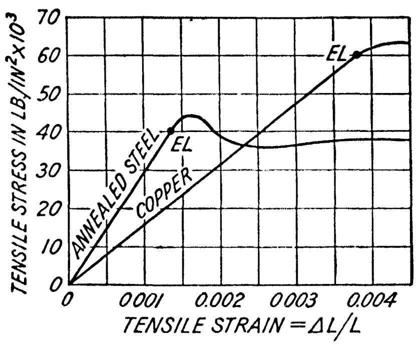


FIG. 4.—Elastic behavior of certain metals.

Elasticity of Shear. A third type of elasticity concerns changes in shape. This is called *elasticity of shear*. As an illustration of shearing strain, consider a cube of material (Fig. 3) fixed at its lower face and acted upon by a tangential force F at its upper face. This force causes the consecutive horizontal layers of the cube to be slightly displaced or sheared relative to one another. Each line, such as BD or CE, in the cube is rotated through an angle φ by this shear. The shearing strain is defined as the angle φ, expressed in radians. (The radian measure of an angle is the ratio of the arc subtended by the angle to its radius.) For small values of the angle, $\varphi = BB'/BD$, approximately. The shearing stress is the ratio of the force F to the area A of the face $BCGH$. The ratio, shearing stress divided by shearing strain, is the *shear modulus* or *coefficient of rigidity*, n.

$$n = \frac{F/A}{\varphi} = \frac{F/A}{BB'/BD} \tag{2}$$

The volume of the body is not altered by shearing strain.

Ultimate Strength. The way in which samples of several different materials are deformed by various loads is illustrated by Fig. 4. For each load the tensile strain is calculated as the ratio of the elongation to the original length. This is plotted against the tensile stress, and a curve is drawn through the points so obtained.

In the region to the left of the elastic limit (EL) the sample obeys Hooke's law and returns to its original length when the stress is removed. The sample will support stresses in excess of the elastic limit, but when unloaded is found to have acquired a permanent set.

If the applied stress is increased slowly, the sample will finally break. The maximum stress applied in rupturing the sample is called the *ultimate strength.* Although the ultimate strength of the sample lies far up on its strain-stress curve, it is seldom safe to expect it to carry such loads in structures. Axles and other parts of machines, which are subject to repeated stress, are never loaded beyond the elastic limit.

Whenever a machine part is subjected to repeated stresses over a long period of time, the internal structure of the material is changed. Each time the stress is applied, the molecules and crystals realign. Each time the stress is removed, this alignment retains some permanent set. As this process continues, certain regions are weakened, particularly around areas where microscopic cracks appear on the surface. This loss of strength in a machine part because of repeated stresses is known as *fatigue.* Since failure due to fatigue occurs much sooner if flaws are present originally than in a perfect part, it is important to detect such flaws, even though they are very slight, before the part is installed. Great care is exercised in testing parts of airplane structures to detect original flaws. In many plants, x-rays are used to detect hidden flaws.

Thermal Expansion. When a structure such as a bridge is put together, the design must take into account changes in shape due to changes in temperature. If such provision is not made, tremendous forces develop that may shatter parts of the structure. Anyone who has ever seen a concrete pavement shattered by these forces on a hot day realizes the violence of such a phenomenon.

It is interesting to consider the forces due to change in temperature of a steel rail. If space is provided for expansion, the change in length is given by

$$\Delta L = \alpha L_0 \, \Delta t$$

If allowance is not made for expansion, forces arise that can be computed from Eq. (1). This gives for a 100-lb/yd rail a force equal to approximately 100 tons. Railroads now frequently weld the rails together into continuous lengths which may be several miles long. Suitable tie fastenings prevent lateral, vertical, or longitudinal motion of the rails, and under elastic restraint the rails experience compression at high temperatures and tension at low temperatures.

Example: A steel rail 40 ft long is fastened rigidly in place when the temperature is 40°F. The area of cross section of the rail is 12 in.² What force must be applied to keep the rail from expanding when the temperature rises to 100°F? The coefficient of linear expansion of steel is 0.0000072/°F.

The increase in length that would occur if no force were applied is

$$\begin{aligned}
\Delta L &= \alpha L_0 \, \Delta t \\
\alpha &= 0.0000072/^\circ\text{F} \\
L_0 &= 40 \text{ ft} \\
\Delta t &= 100^\circ\text{F} - 40^\circ\text{F} = 60^\circ\text{F} \\
\Delta L &= (0.0000072/^\circ\text{F})(40 \text{ ft})(60^\circ\text{F}) \\
&= 0.017 \text{ ft}
\end{aligned}$$

If we use this value in Eq. (1), we may compute the force necessary to compress the rail to its original length. This force is the same as that necessary to extend the rail a similar distance

$$\begin{aligned}
Y &= \frac{FL}{A\,\Delta L} \\
F &= \frac{YA\,\Delta L}{L} \\
Y &= 29{,}000{,}000 \text{ lb/in.}^2 \\
A &= 12 \text{ in.}^2 \\
\Delta L &= 0.017 \text{ ft} \\
L &= 40 \text{ ft} \\
F &= \frac{(29{,}000{,}000 \text{ lb/in.}^2)(12 \text{ in.}^2)(0.017 \text{ ft})}{40 \text{ ft}} \\
&= 1\bar{5}0{,}000 \text{ lb} = 75 \text{ tons}
\end{aligned}$$

Some Further Properties of Matter. Materials possess several characteristics that are closely related to the elastic properties. Among these are ductility, malleability, compressibility, and hardness.

The *ductility* of a material is the property that represents its adaptability for being drawn into wire. *Malleability* is the property of a material by virtue of which it may be hammered or rolled into a desired shape. In the processes of drawing or rolling, stresses are applied that are much above the elastic limit so that a "flow" of the material occurs. For many materials the elastic limit is greatly reduced by raising the temperature; hence processes requiring flow are commonly carried on at high temperature. The *compressibility* of a material is the reciprocal of its bulk modulus.

The property of *hardness* is measured by the *Brinell number.* The Brinell number is the ratio of load, in kilograms weight on a sphere used to indent the material, to the spherical area of the indentation in square millimeters. Among metals, cast lead has one of the smallest Brinell numbers, namely 4.2. Some of the steels have values over 100 times as great.

Importance of Sampling in Testing. One difficulty encountered when attempting to measure the elastic properties of a material is that of providing a uniform sample. If examined under sufficient magnification, no material is found to be uniform (homogeneous). Rock, brick, and concrete show structure that can readily be seen. Elastic constants for such materials should not be taken for samples that are not large compared to the size of the unit structure. Resistance to crushing varies from 800 to 3,800 lb/in.^{2} for concrete, while that of granite varies from 9,700 to 34,000 lb/in.^{2}

SUMMARY

Elasticity is that property of a body which enables it to resist and recover from a deformation.

The smallest stress that produces a permanent deformation is known as the *elastic limit.*

Hooke's law expresses the fact that the strain produced in a body is, within the limits of elasticity, proportional to the applied stress.

A *modulus of elasticity* is found by dividing the stress by the strain.

Young's modulus is the ratio of tensile stress to tensile strain.

$$Y = \frac{F/A}{\Delta L/L}$$

The *coefficient of volume elasticity* or *bulk modulus* is the ratio of volume stress to volume strain.

The *shear modulus* or *coefficient of rigidity* is the ratio of shearing stress to shearing strain.

$$n = \frac{F/A}{\varphi}$$

The *compressibility* of a body is the reciprocal of the bulk modulus.

Brinell hardness number is the ratio of the force applied on a hardened steel ball to the spherical area of indentation produced in a sample.

QUESTIONS AND PROBLEMS

1. Can one use a slender wire in the laboratory to estimate the load capacity of a large cable on a bridge? Explain.

2. A force of 10 lb is required to break a piece of cord. How much is required for a cord made of the same material which is (*a*) twice as long, (*b*) twice as large in diameter and the same length?

3. Young's modulus for steel is about 29×10^6 lb/in.^{2} Express this value in kilograms per square centimeter.

4. In what way do the numerical magnitudes of (*a*) strain, (*b*) stress, and (*c*) modulus of elasticity depend on the units of force and length?

5. How much will an annealed steel rod 100 ft long and 0.040 in.^{2} in cross section be stretched by a force of 1,000 lb?

6. A wire 1,000 in. long and 0.01 in.^{2} in cross section is stretched 4.0 in. by a force of 2,000 lb. What are (*a*) the stretching stress, (*b*) the stretching strain, and (*c*) Young's modulus? *Ans.* 2×10^5 lb/in.^{2}; 0.004; 5×10^7 lb/in.^{2}

7. Fibers of spun glass have been found capable of sustaining unusually large stresses. Calculate the breaking stress of a fiber 0.00035 in. in diameter, which broke under a load of 0.385 oz.

8. A load of 60 tons is carried by a steel column having a length of 24 ft and a cross-sectional area of 10.8 in.^{2} What decrease in length will this load produce? (Consult Table I.) *Ans.* 0.11 in.

9. To maintain 200 in.^{3} of water at a reduction of 1 per cent in volume, requires a force per unit area of 3,400 lb/in.^{2} What is the bulk modulus of the water?

10. A 12-in. cubical block of sponge has two parallel and opposite forces of 2.5 lb each applied to opposite faces. If the angle of shear is 0.020 radian, calculate the relative displacement and the shear modulus.

Ans. 0.87 lb/in.^{2}

11. The bulk (volume) modulus of elasticity of water is 3.0×10^5 lb/in.^{2} What is the change in volume of the water in a cylinder 3 ft long and 2 in. in diameter when there is a compressional force per unit area of 14.3 lb/in.^{2} exerted on a tight piston in the cylinder?

12. Most high-tension cables have a solid steel core to support the aluminum wires that carry most of the current. Assume that the steel is 0.50 in. in diameter, that each of the 120 aluminum wires has a diameter of 0.13 in., and that the strain is the same in the steel and the aluminum. If the total tension is 1 ton, what is the tension sustained by the steel? *Ans.* $5\bar{3}0$ lb.

EXPERIMENT

Elasticity

Apparatus: Meter stick; table clamp; 150-cm rod; weight holder; four 1-kg weights; spring; soft rubber tubing.

Although we have discussed the meaning of elasticity and have contrasted the elastic properties of steel and rubber, it is difficult to realize the significance of this property of matter until we have made quantitative measurements.

Let us take two specimens, one a steel spring, the other a piece of soft rubber tubing about the same length and load capacity as the steel spring, and subject them to the same series of loads.

Mount the steel spring in such a way that the bottom of the weight holder attached to its lower end is just even with the zero on the meter stick, then add weight, 1 kg at a time, and record the corresponding displacements from the "no load" position in the "Down" column of Table II. The spring should not be allowed to bob up and down, but be held in position while the load is being changed and allowed to take its new position gradually. After all the weights have been added and the displacements recorded, the load should be removed 1 kg at a time and the displacements recorded in the "Back" column.

To obtain satisfactory results the spring, and later the rubber tube, should be preloaded. Take the *zero load* reading with the 1-kg weight holder attached to the spring, or rubber tube, that is, consider the weight holder a part of the spring and rubber.

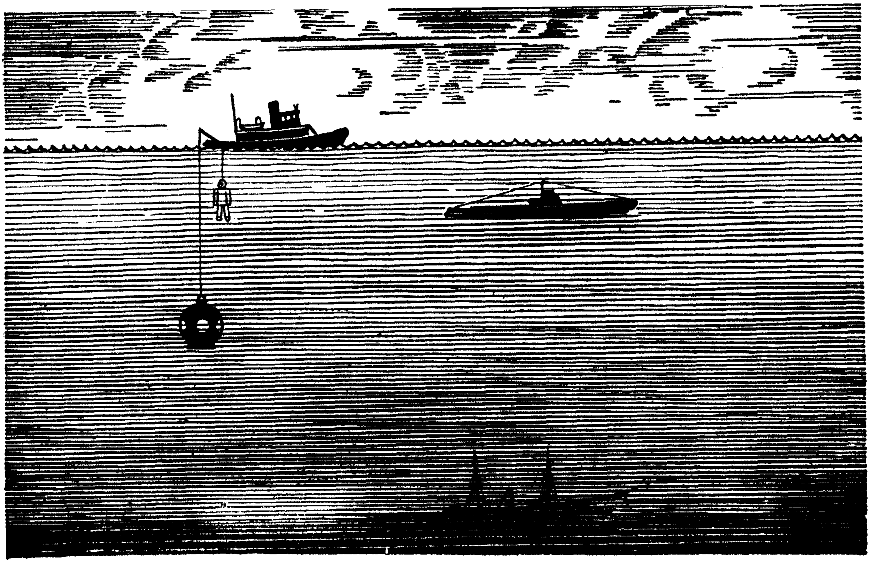
Is the spring perfectly elastic for the loads used, that is, does it return to its original length after the stress is removed?

Plot a curve of displacement against load. If the curve is a straight line passing through the origin, the displacement is proportional to the load. Is this true for your data?

TABLE II

Steel spring				
Load	Displacement		Displacement per gm	
	Down	Back	Down	Back
0				
1 kg				
2 kg				
3 kg				
4 kg				
Rubber tubing				
Load	Displacement		Displacement per gm	
	Down	Back	Down	Back
0				
1 kg				
2 kg				
3 kg				
4 kg				

Substitute the rubber tube for the steel spring and record the readings in the same manner as with the spring. What differences in the characteristic properties of the two materials do the data show? Plot the two sets of data on the same axes and compare the curves obtained. Which material appears to be more perfectly elastic?



CHAPTER 7

PROPERTIES OF LIQUIDS

Materials are commonly classified as solids, liquids, and gases. The class into which a substance falls depends upon the physical conditions surrounding it at the time of observation.

Solids are bodies that maintain definite size and shape. A liquid has a definite size, for it will fill a container to a certain level, forming a free surface, but it does not have a definite shape. Gases have neither definite shape nor definite volume, but completely fill any container no matter how small an amount of gas is put into it. The term "fluid" is applicable to both liquids and gases.

Pressure. *Pressure* is defined as *force per unit area.*

$$P = \frac{F}{A} \tag{1}$$

A unit of pressure may be made from any force unit divided by an area unit. Pressures are commonly expressed in pounds per square inch. Sometimes pressures are expressed in terms of certain commonly observed pressures as, for example, one atmosphere, representing a pressure equal to that exerted by the air under normal conditions, or a centimeter of mercury, representing a pressure equal to that exerted by a column of mercury 1 cm high. The concept of pressure is particularly useful in discussing the properties of liquids and gases.

Fluid Pressure Due to Gravity. The atoms and molecules of which a fluid is composed are attracted to the earth in accordance with Newton's law of universal gravitation. Hence, liquids collect at the bottom of containers, and the upper layers exert forces on the ones underneath. Such attraction for the gas molecules keeps an atmosphere on the surface of the earth.

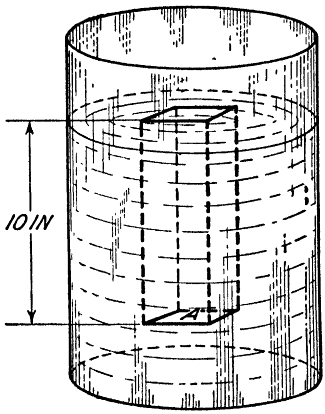


FIG 1.—Pressure in a liquid.

The pressure at a point in a liquid means the force per unit area of a surface placed at the point in question. Imagine a horizontal surface A of unit area (Fig. 1). The weight of the column of liquid directly above this surface is numerically equal to the force per unit area (the pressure) caused by the weight of the liquid. If the liquid is water each cubic foot weighs 62.4 lb, and each cubic inch weighs (62.4/1,728) lb = 0.0361 lb. If we take the area A as 1.0 in.^{2}, the volume above the area is

$$(10 \text{ in.})(1.0 \text{ in.}^2) = 10 \text{ in.}^3,$$

hence the weight of liquid above the area is

$$(0.036 \text{ lb/in.}^3)(10 \text{ in.}^3) = 0.36 \text{ lb}$$

and the pressure on the area is 0.36 lb/in.^{2}.

Weight-density. In computing liquid pressure due to gravity it is helpful to know the weight per unit volume of the liquid. The weight

TABLE I. WEIGHT-DENSITIES OF LIQUIDS AND SOLIDS

	gm/cm^3	lb/ft^3
Alcohol (ethyl) at 20°C	0.79	49.4
Water at 4°C	1.000	62.4
Water at 20°C	0.998	62.3
Gasoline	0.68	42
Mercury	13.6	850
Oak	0.8	50
Aluminum	2.7	169
Copper	8.89	555
Ice	0.92	57
Iron, wrought	7.85	490

per unit volume is called the *weight-density.*

$$d = \frac{W}{V} \tag{2}$$

Values of weight-density for a number of substances are given in Table I.

In order to find the pressure due to a column of liquid, it is sufficient to know the weight-density d and the depth h below the surface.

$$P = hd \tag{3}$$

Example: Find the pressure at the bottom of a tank that is filled with gasoline to a depth of 8.0 ft.

$$\begin{aligned} P &= hd \\ h &= 8.0 \text{ ft} \\ d &= 42 \text{ lb/ft}^3 \\ P &= (8.0 \text{ ft})(42 \text{ lb/ft}^3) = 3\bar{4}0 \text{ lb/ft}^2 \\ &= \frac{3\bar{4}0 \text{ lb/ft}^2}{144 \text{ in.}^2/\text{ft}^2} = 2.\bar{4} \text{ lb/in.}^2 \end{aligned}$$

If the bottom of the tank is 6.0 by 8.0 ft, what force is exerted on it?

$$\begin{aligned} P &= \frac{F}{A} \\ F &= PA \\ P &= 3\bar{4}0 \text{ lb/ft}^2 \\ A &= (6.0 \text{ ft})(8.0 \text{ ft}) = 4\bar{8} \text{ ft}^2 \\ F &= (3\bar{4}0 \text{ lb/ft}^2)(4\bar{8} \text{ ft}^2) = 1\bar{6},000 \text{ lb} \end{aligned}$$

Note that there are only two significant figures in the original data; hence only two significant figures are retained in each result.

Buoyancy; Archimedes' Principle. Everyday observation has shown us that when an object is lowered into water it apparently loses weight and indeed may even float on the water. Evidently a liquid exerts an upward, buoyant force upon a body placed in it. Archimedes, a Greek mathematician and inventor, recognized and stated the fact that a *body wholly or partly submerged in a fluid experiences an upward force equal to the weight of the fluid displaced.*

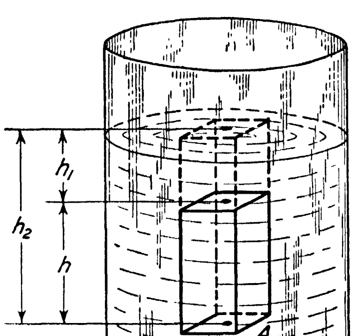


FIG. 2.—The upward force on the bottom of the block is greater than the downward force on the top.

Archimedes' principle can readily be verified experimentally, as indicated at the end of this chapter. One can deduce this principle from a consideration of Fig. 2. Consider a block of rectangular cross section A, immersed in a liquid of weight-density d. On the vertical faces, the liquid exerts horizontal forces, which are balanced on all sides. On the top face it exerts a downward force h_1dA

and on the bottom face an upward force h_2dA. The net upward force on the block is

$$h_2dA - h_1dA = hdA,$$

which is just the weight (volume hA times weight-density d) of the liquid displaced by the block.

The control of submarines depends in part on Archimedes' principle. In submerging the boat, sea water is admitted into ballast tanks and the buoyant force balanced. The boat is brought to the surface by expelling the water from these tanks with compressed air.

Specific Gravity. The *specific gravity* of a body is the ratio of its density to that of some standard substance. The standard usually chosen is water at the temperature of its maximum density, 39.2°F. Thus, if d is the density of the body and d_w the density of the water, the specific gravity (sp. gr.) of the body is

$$\text{Sp. gr.} = \frac{d}{d_w} \tag{4}$$

Since each of the two densities has the same unit, their quotient has no units. Specific gravity is often more convenient to tabulate than density, the values of which in the British and metric systems of units are different. One may easily compute density from specific gravity by the use of Eq. (4).

$$d = (\text{sp. gr.})d_w$$

The units of density thus obtained will be those of the system in which the density of water is expressed.

Since the density of water in metric units is 1 gm/cm³, the density is *numerically* equal to the specific gravity in that system.

Weight-density and Specific Gravity Measurements by Archimedes' Principle. This principle suggests a method for comparing the weight-density of a substance with that of some standard fluid, such as water. The measurement of specific gravity involves the following reasoning which is briefly stated in symbols:

$$\begin{aligned}\text{Sp. gr.} &= \frac{\text{weight-density of substance}}{\text{weight-density of water}} = \frac{W_s/V}{W_w/V} = \frac{W_s}{W_w} \\ &= \frac{\text{weight of body in air}}{\text{loss of weight in water}} \end{aligned} \tag{5}$$

Since the volume of a submerged body is equal to the volume of the displaced water, the ratio of the weight-densities is the same as the ratio of the weight W_s of the sample of the substance to the weight W_w of an equal volume of water. These weights can be determined by weighing the sample in air and in water. The weight in water subtracted from the weight in air gives the loss of weight in water, which is the weight of the water displaced (from Archimedes' principle). Therefore, the specific gravity can be determined by the measurements indicated in Eq. (5).

Example: A metal sphere weighs 35.2 oz in air and 30.8 oz when submerged in water. What is the specific gravity and the weight-density of the metal? From Eq. (5)

$$\text{Sp. gr.} = \frac{\text{weight of sample in air}}{\text{loss of weight in water}}$$

$$\text{Weight in air} = 35.2 \text{ oz}$$

$$\text{Loss of weight in water} = 35.2 \text{ oz} - 30.8 \text{ oz} = 4.4 \text{ oz}$$

$$\text{Sp. gr.} = \frac{35.2 \text{ oz}}{4.4 \text{ oz}} = 8.0$$

From Eq. (4)

$$d = (\text{sp. gr.})d_w$$

$$d_w = 62.4 \text{ lb/ft}^3$$

$$d = (8.0)(62.4 \text{ lb/ft}^3) = 5\bar{0}0 \text{ lb/ft}^3$$

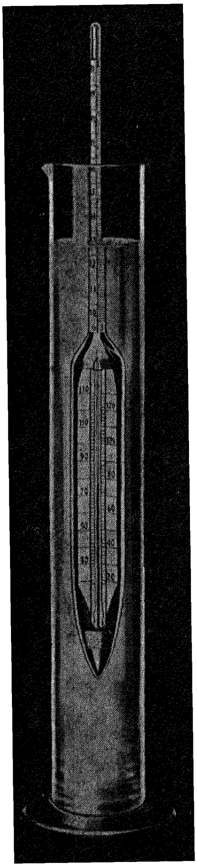


FIG. 3.—Hydrometer.

Quick determinations of the specific gravity of a liquid or solution can be made with a hydrometer. This instrument (Fig. 3) is a glass bulb attached to a narrow stem and weighted so as to remain upright when floating in a liquid. It floats at such a depth as to displace exactly its own weight of the liquid (Archimedes' principle). The stem is calibrated to indicate the specific gravity of the solution, for the smaller this specific gravity the deeper the bulb sinks in the liquid.

External Pressure; Pascal's Law. The pressure previously discussed is that caused by the weight of the liquid. If any external pressure is applied to the liquid, the pressure will be increased beyond that given by Eq. (3). The most common of such external pressures is that due to the atmosphere.

Whenever an external pressure is applied to any fluid at rest, the pressure is increased at every point in the fluid by the amount of the external pressure. This statement is called *Pascal's law,* after the French philosopher who first clearly expressed it. The practical consequences of Pascal's law are apparent in automobile tires, hydraulic jacks, hydraulic brakes, pneumatic drills, and air brakes.

Hydraulic Press. The fact that pressure in a liquid at rest is transmitted by the liquid in all directions unchanged, except by changes in level, has an important application in a machine called the *hydraulic press.* Small forces exerted on this machine cause very large forces exerted by the machine. In Fig. 4, the small force F_1 is exerted on a small area A_1. This increases the pressure in the liquid under the piston by an amount P. The force that this increase of pressure will cause on the large piston will be $F_2 = PA_2$, since the pressure increase under both pistons is the same. Hence,

$$PA_2 = F_2 \quad \text{and} \quad PA_1 = F_1, \quad \text{or} \quad F_2 = \frac{A_2}{A_1} F_1$$

Simply by changing the ratio of A_2 to A_1, the force F_2 may be made as large as is safe for the big piston to carry. Larger pistons require more transfer of liquid and are correspondingly slower in action.

Fluid Flow. The rate of flow of a liquid through a pipe or channel is usually measured as the volume that passes a certain cross section per unit time, as gallons per minute, liters per second, etc. If the average speed of the liquid at section S in Fig. 5 is v, the distance l through which the stream moves in time t is

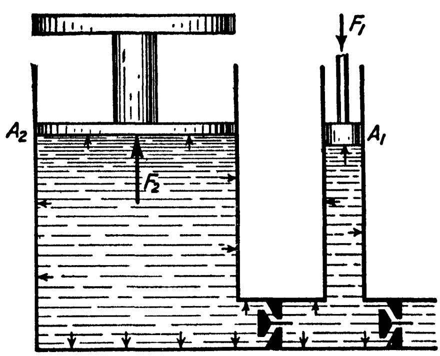


FIG. 4.—Hydraulic press.

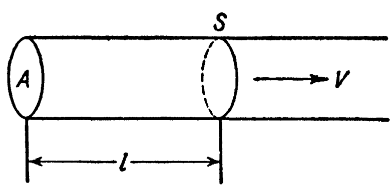


FIG. 5.—Rate of flow of liquid through a pipe.

vt. This may be regarded as the length of an imaginary cylinder which has passed S in time t. If A is the area of the cylindrical section, then its volume is $Al = Avt$, and the volume rate of flow of the liquid is given by

$$R = \frac{Avt}{t} = Av \tag{6}$$

In a fluid at rest the pressures are the same at all points of the same elevation. This is no longer true if the fluid is moving. When water flows in a uniform horizontal pipe, there is a fall in pressure along the pipe in the direction of flow. The reason for this fall in pressure is that force is required to overcome friction. If the liquid is being accelerated, additional force is required.

When the valve of Fig. 6 is closed, water rises to the same level in each vertical tube. When the valve is opened slightly to permit a small rate of flow, the water level falls in each tube, indicating a progressive decrease of pressure along the pipe. If the rate of flow is doubled, the pressure drop is twice as great. The pressure drop and the rate of flow are proportional. Frictional effects are very important when water is distributed in city mains or when petroleum is transported long distances in pipe lines. Pumping stations must be placed at intervals along such lines to maintain the flow.

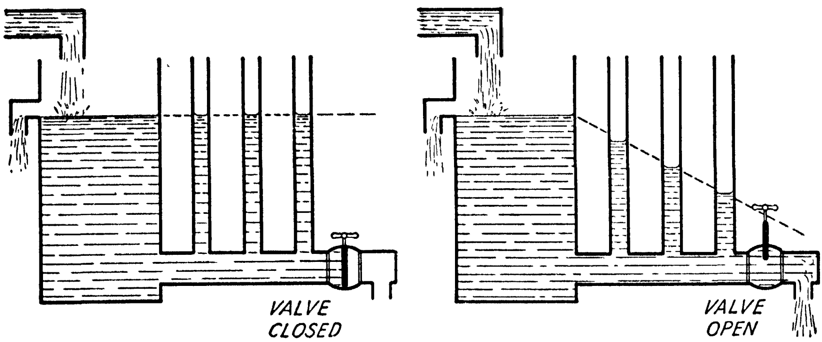


FIG. 6.—Friction causes a fall in pressure along a tube in which a liquid flows.

Pressure and Speed. When water flows through a pipe that has a narrow constriction (Fig. 7), the water necessarily speeds up as it approaches the constriction to keep the mass of liquid passing the cross section there per unit time the same as that passing a cross section anywhere else in the pipe. Hence the speed of the water must increase as it moves from A to B. To cause this acceleration, the pressure at A must be greater than that at B. This is an example of a general rule (*Bernoulli's principle*): whenever the speed of a horizontally moving stream of fluid increases owing to a constriction, the pressure must decrease. High speed is associated with low pressure, and vice versa.

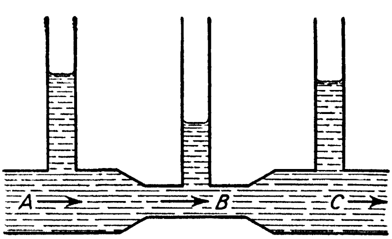


FIG. 7.—Flow through a constriction. Decrease in pressure accompanies increase in speed.

The atomizer on a spray gun and the jets in a carburetor utilize Bernoulli's principle. The curving path of a pitched baseball when spinning is explainable in terms of this principle. A Venturi tube similar to Fig. 7 is used to measure the flow of water or the speed of an airplane in terms of the decrease in liquid or air pressure in the constriction.

The speed of flow of a fluid or that of a body moving relative to a fluid may also be measured by means of a *pitot* tube (Fig. 8). Because of the inertia of the fluid, its impact causes the pressure in tube P to be greater than the static pressure in tube S. The two tubes are connected to a gauge that records the differential pressure. A pitot tube is frequently used to measure the air speed of an airplane. The dial of the pressure gauge can be calibrated to read the speed of the tube relative to the air.

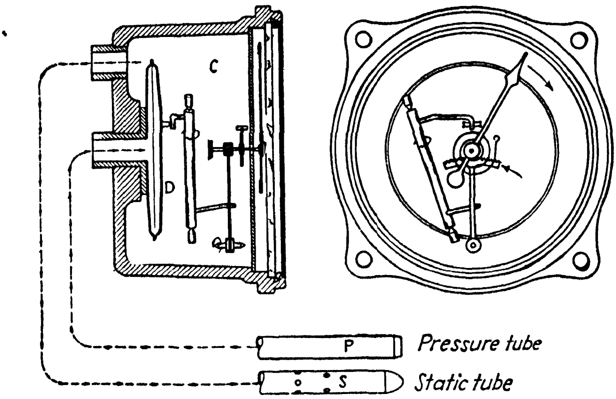


FIG. 8.—Pitot-tube air-speed indicator. The pressure tube P is open at the end while the static tube S is closed at the end but has openings on the side. The pressure gauge has a sealed inner case C and is operated by the pressure-sensitive diaphragm D.

SUMMARY

Pressure is force per unit area.

The *weight-density* of a substance is its weight per unit volume.

At a depth h below the surface, the pressure due to a liquid of weight-density d is

$$P = hd$$

The *specific gravity* of a substance is the ratio of its density to that of water.

Archimedes' principle states that a body wholly or partly submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced.

Pascal's law states that an external pressure applied to a confined fluid increases the pressure at every point in the fluid by an amount equal to the external pressure.

Bernoulli's principle expresses the fact that whenever the speed of a horizontally moving fluid increases due to a constriction, the pressure decreases. A Venturi tube utilizes this principle to measure flow.

QUESTIONS AND PROBLEMS

1. A box whose base is 2.0 ft square weighs 200 lb. What pressure does it exert on the ground beneath it?

2. A vertical force of 4.0 oz pushes a phonograph needle against the record surface. If the point of the needle has an area of 0.0010 in.^{2}, find the pressure on the record in pounds per square inch. *Ans.* 250 lb/in.^{2}

3. What is the pressure at the base of a column of water 40 ft high?

4. A tank 4.0 ft in diameter is filled with water to a depth of 10.0 ft. What is the pressure at the bottom? Find the total thrust on the bottom of the tank. *Ans.* 620 lb/ft^2; 7,800 lb.

5. The piston of a hydraulic lift for cars is 6.0 in. in diameter. The device is operated by water from the city system. What is the water pressure necessary to raise a car if the total load lifted is 3,142 lb?

6. What size piston is to be used in a hydraulic lift, if the maximum load is 5,000 lb and the water pressure is that due to a 100-ft head of water?

Ans. Diameter = 12.1 in.

7. A coal barge with vertical sides has a bottom 40 ft. by 20 ft. When loaded with coal, it sinks 18 in. deeper than when empty. How much coal was taken on?

8. A stone from a quarry weighs 30.0 lb in air, and 21.0 lb in water. What is its (*a*) specific gravity, (*b*) weight-density, and (*c*) volume?

Ans. 3.3; $2\bar{1}0$ lb/ft^3; 0.14 ft^3.

9. A can full of water is suspended from a spring balance. Will the reading of the balance change (*a*) if a block of cork is placed in the water and (*b*) if a piece of lead is placed in the water? Explain.

10. To secure great sensitivity, is a narrow or wide hydrometer stem preferable? Why?

11. Does a ship wrecked in mid-ocean sink to the bottom or does it remain suspended at some great depth? Justify your opinion.

12. Why does the flow of water from a faucet decrease when someone opens another valve in the same building?

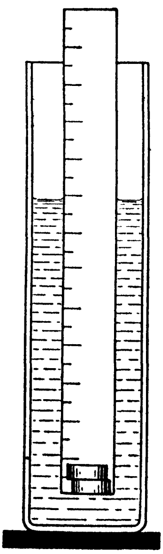


FIG. 9.—Apparatus for measuring liquid pressure.

EXPERIMENTS

Liquid Pressure

Apparatus: A tall glass jar; a 16-in. length of tubing about 2 in. in diameter, sealed at one end and graduated along its length; six 100-gm slotted weights; water; salt solution; hydrometer.

Fill the glass jar about half full of water, place the empty tube in the water as shown in Fig. 9, and record the depth to which it sinks. It will be necessary to hold the tube in a vertical position, but care should be taken to exert no vertical force on it.

If the tube is uniform in diameter, the vertical forces on it include only its weight and the upward force exerted by the water on the bottom of the tube. The latter force, just sufficient to support the weight of the tube, is the product of the pressure and the area of the bottom of the tube, that is, $F = PA$. We can use this relation to find the pressure at the bottom of the tube, since A can be determined and F is the weight of the tube.

If an object is placed in the tube, the latter will sink to a position at which the upward force is equal to the combined weight of tube and object. Again $F = PA$, so that P can be found if F and A are known.

Measure the diameter D of the tube and compute its area of cross section from the relation $A = \frac{1}{4}\pi D^2$. Next, add the 100-gm weights in succession, recording in a table similar to Table II the corresponding depths h to which the tube sinks. For each observation, determine F (remembering that F is equal to the combined weight of the tube and its contents) and calculate the pressure, $P = F/A$.

TABLE II

Weights in tube, W	Fresh water				Salt water	
	Depth, h	Force, F	Pressure, P	Ratio, P/h	Depth, h'	Ratio, h/h'
0						
100						
200						
300						
400						
500						
600						

Do the data that you have recorded indicate that P is proportional to h? In order to answer this question, divide each value of P by the corresponding value of h. If this quotient is essentially constant, P and h are proportional to each other. What does this quotient represent [see Eq. (3)]? Compare its value with that listed in Table I. Plot the graph of P against h. What does the shape of the curve indicate?

Repeat the depth measurements for the same series of loads, using salt water, and record in the sixth column. How do these depths h' compare with the corresponding values of h? Compute the ratio h/h' for each observation. What is the significance of this ratio? Place a hydrometer in the salt water and compare its reading with the value of the ratio h/h'.

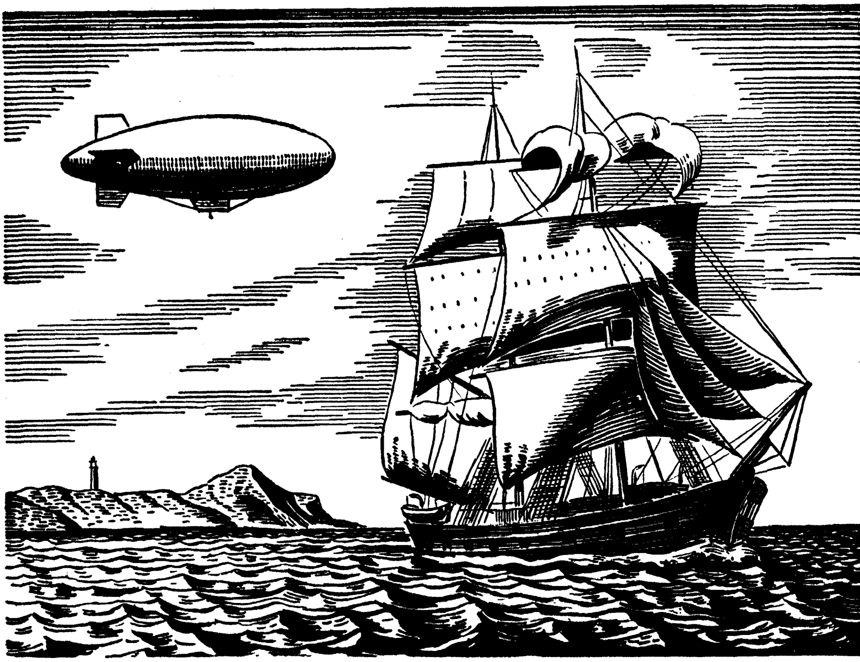
Archimedes' Principle

Apparatus: Platform balance; weights; string; stone or other object to be submerged; graduate; water.

Archimedes' principle indicates that an object partly or completely submerged in a liquid is buoyed up by a force equal to the weight of the displaced liquid. If the object is floating, the buoyant force is exactly equal to its weight, that is, the object sinks just far enough to displace its own weight of liquid. If the object sinks, it displaces its own volume of the liquid, so that the buoyant force is equal to the weight of an equal volume of liquid.

Compute the buoyant force on a submerged object by subtracting its apparent weight (when submerged) from its weight in air.

Determine the volume of liquid displaced by the object by submerging the latter in liquid contained in a graduate and measuring the apparent increase in volume of the liquid. Compare the weight of this amount of liquid with the buoyant force previously evaluated. Is the result in accordance with Archimedes' principle?



CHAPTER 8

GASES AND THE GAS LAWS

Gases consist of molecules whose forces of attraction are comparatively small, so that they do not hold together sufficiently well to form liquids or solids. The molecules are very small, and the distances between them are (on the average) relatively great compared to their size. Air feels soft and smooth, but actually it consists of a large group of discrete particles rather than a continuous, homogeneous substance. The molecules are in constant motion, the speed of the motion increasing as the temperature rises.

A gas exerts pressure on its surroundings because the molecules continually collide with the walls of the container and with each other (Fig. 1). Since the extent of motion of each molecule is limited only by these collisions, a gas will expand until it fills any container in which it is placed.

Gas Laws. Since the pressure that a gas exerts on the walls of the container is caused by collisions of the molecules with the walls, one would expect the pressure to depend upon the number of molecules and upon their speed. The number of molecules depends upon the mass and volume, and the speed depends upon the temperature. The quantitative relation of these factors is given by the equation

$$PV = MRT \tag{1}$$

where P is the pressure, V the volume, M the mass of gas, T the temperature on the absolute scale, and R a constant. This statement may be called the *general gas law*.

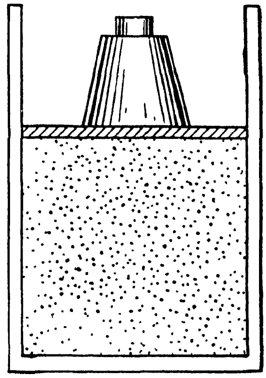


FIG. 1.—Gas pressure is caused by the collision of molecules with the walls of the container.

Absolute Zero of Temperature. If the pressure of a given mass of gas is kept constant, its volume V_t at some temperature t will be

$$V_t = V_0 + \Delta V = V_0 + \beta V_0 \, \Delta t \tag{2}$$

where β is the volume coefficient of expansion of the gas when the pressure is kept constant. Solving Eq. (2) for β gives

$$\beta = \frac{V_t - V_0}{V_0 \, \Delta t} \tag{3}$$

This equation serves to define the volume coefficient of expansion of a gas. It is the change in volume per unit volume at 0°C per degree change in temperature. As long as the pressure on the gas is kept fairly low and the volume at the melting point of ice taken as V_0, the coefficient is approximately constant for all temperatures well above the boiling point of the substance. It is also significant that all gases have practically the same coefficient of expansion.

If the volume of a given mass of gas is held constant, the pressure increases as the temperature increases. The pressure at a temperature t is

$$P_t = P_0 + \Delta P = P_0 + \gamma P_0 \, \Delta t \tag{4}$$

This gives as the definition of γ (gamma), the pressure coefficient,

$$\gamma = \frac{P_t - P_0}{P_0 \, \Delta t} \tag{5}$$

that is, γ is the change in pressure per unit pressure at 0°C per degree change in temperature.

Experiment shows that β and γ are equal to each other and approximately the same for all gases. This somewhat surprising fact shows that, although the masses of the molecules of different gases are quite different, the space between the particles is such a large fraction of the total volume which a gas occupies, that the elastic properties of all gases are alike.

The coefficients, β and γ, have a value of 0.00366/°C. This means that the volume of given mass of gas changes by 0.00366 of its volume at 0°C for each degree change in temperature if its pressure is kept constant. Also, if the volume is kept constant, the pressure will change by the same fraction of the pressure at 0°C.

A gas exerts a pressure because it possesses thermal energy and the molecules are flying about, bumping into the walls of the container. If the gas had no thermal energy at all, the particles would not be moving. The temperature at which no heat would be left in the gas would be an *absolute zero.* No lower temperature could exist because there would be no more heat to take away.

The value of absolute zero can be found by computing the number of times 0.00366 of a pressure at 0°C can be subtracted before the pressure becomes zero. This is about 273 times. As each degree fall in temperature reduces the pressure at 0°C by this fraction, absolute zero must be at −273°C. The volume would also be zero if the pressure had been kept constant while the temperature was reduced, assuming that the fractional change in volume per degree temperature change remained constant. But at very low temperatures, the volume of the molecules themselves becomes appreciable when compared to the volume the gas occupies. The molecules are highly incompressible, so the volume does not approach zero as the temperature is lowered. Absolute zero in degrees Fahrenheit is $\frac{9}{5}$ of 273° below 32°F, the freezing point of water, or −460°F.

TABLE I. DENSITY AND SPECIFIC GRAVITY OF SOME GASES

Gas	Density		Specific gravity (relative to air)
	gm/liter	lb/ft^3	
Air	1.293	0.081	1.000
Carbon dioxide	1.977	0.123	1.529
Hydrogen	0 090	0.0056	0.069
Helium	0.178	0.011	0.138
Nitrogen	1 251	0.078	0.967
Oxygen	1.429	0.089	1.105
Steam 100°C	0 598	0.037	0.462

In Table I are listed the values of density and specific gravity for several gases under standard conditions of pressure and temperature (0°C and one standard atmosphere). Values for other conditions can be derived from those in the table by means of the general gas law.

A convenient form of the general gas law can be obtained by solving Eq. (1) for R, giving

$$R = \frac{PV}{MT} \tag{6}$$

Since R is a constant, PV/MT is constant, so that we may write

$$\frac{PV}{MT} = \frac{P_1V_1}{M_1T_1}$$

Therefore for a given mass of a gas

$$\frac{P_1V_1}{T_1} = \frac{P_0V_0}{T_0} = \frac{P_2V_2}{T_2} = \cdots \tag{7}$$

These relationships are illustrated in Fig. 2. Initially a volume V_1 of gas at a temperature T_1 exerts a pressure P_1. When the pressure is increased to P_2, the temperature remaining the same, the volume is reduced to V. If the temperature is then raised to T_2, the volume increases to V_2, the pressure remaining the same. The temperature must always be measured with respect to absolute zero to use Eq. (6) or Eq. (7), and the pressure must be the absolute pressure, not the differential gauge pressure. The readings of most pressure gauges represent the difference between the absolute pressure and atmospheric pressure. To obtain the absolute pressure, atmospheric pressure must be added to the gauge pressure.

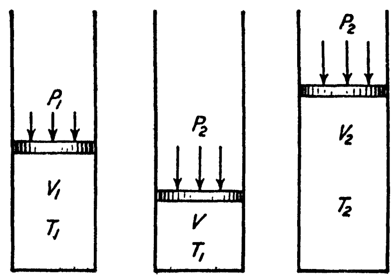


Fig. 2.—The pressure, volume, and temperature relations for a gas.

Boyle's Law and Charles's Laws. Several applications of the general gas law, Eq. (1), under special conditions, are of considerable importance. If the mass of gas and the temperature remain constant, Eq. (1) reduces to

$$PV = K_1 \tag{8}$$

which is known as *Boyle's law.* It may be stated as follows: if the temperature and mass of a gas are unchanged, the product of the pressure and volume is constant. This condition is realized for relatively slow changes in volume and pressure.

If the pressure and the mass of gas remain constant, Eq. (1) becomes

$$V = K_2T \tag{9}$$

Equation (9) is stated in words as follows: the volume of a sample of gas is directly proportional to the absolute temperature if the pressure remains the same.

If the volume and mass of gas remain constant, Eq. (1) reduces to

$$P = K_3T \tag{10}$$

or, in words, the volume remaining the same, the pressure of a sample of gas is directly proportional to the absolute temperature.

These two laws are known as *Charles's laws.*

The Mercury Barometer. A simple device for measuring atmospheric pressure can be made from a glass tube about 3 ft in length, closed at one end. The tube is filled with mercury, stoppered, inverted, and then placed open end down in a vessel of mercury as shown in Fig. 3. When the stopper is removed, some mercury runs out of the tube until its upper surface sinks to a position lower than the top of the tube. The height of the mercury column h is called the *barometric height* and is usually about 30 in. near sea level. Evidently the space above the mercury column contains no air.

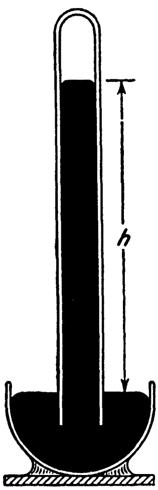


FIG. 3.—Principle of the mercury barometer.

Consider the horizontal layer of mercury particles within the tube and on the same level as the surface outside the tube. The downward pressure on this layer is hd, in which h is the height of the column and d the density of mercury (13.6 gm/cm³). But the upward pressure on this layer must have this same value, since the layer is at rest. Hence the pressure of the atmosphere is equal to the pressure exerted by the mercury column. If, while filling the tube, one allows air to get into the space above the column, the barometer will read too low because of the downward pressure of this entrapped air.

A common type of mercury barometer is shown in Fig. 4. This type is used in technical laboratories, weather observatories, and even on ships for accurate readings of barometric pressures.

Barometer readings are much lower at high altitudes than at sea level; they also vary somewhat with changes in weather. Standard atmospheric pressure supports a column of mercury 76 cm in height, at latitude 45°, and at sea level; hence standard atmospheric pressure = hd = (76.00 cm) (13.596 gm/cm³) = 1033.3 gm/cm². This is approximately 14.7 lb/in.²

Example: The volume of a gas at atmospheric pressure (76 cm of mercury) is 200 in.³ when the temperature is 20°C. What is the volume when the temperature is 50°C and the pressure is 80 cm of mercury?

$$\frac{PV}{T} = \frac{P_1V_1}{T_1}$$

$$V = \frac{P_1T}{PT_1}V_1$$

$$V_1 = 200 \text{ in.}^3$$
$$P_1 = 76 \text{ cm of mercury}$$
$$P = 80 \text{ cm of mercury}$$
$$T_1 = 20°\text{C} + 273 = 293°\text{K}$$
$$T = 50°\text{C} + 273 = 323°\text{K}$$

$$V = \frac{(76 \text{ cm})(323°\text{K})(200 \text{ in.}^3)}{(80 \text{ cm})(293°\text{K})} = 2\bar{1}0 \text{ in.}^3$$

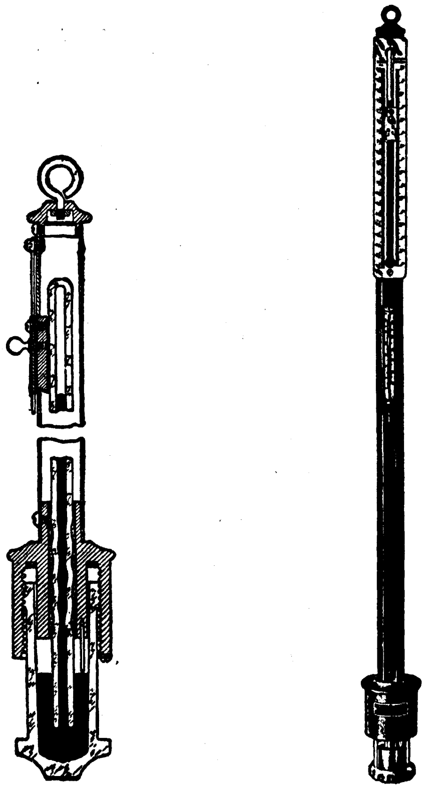


FIG. 4.—A mercury barometer.

SUMMARY

The molecules of a gas occupy only a small fraction of the volume taken up by the gas.

Gases exert pressure on the walls of the container because the molecules collide with the walls.

The *general gas law* states that the product of the pressure and the volume of a sample of gas is proportional to the absolute temperature

$$PV = MRT$$

The *volume coefficient* for a gas is the fractional change in volume per degree change in temperature, when the pressure is constant. (The original volume is that at 0°C.)

The *pressure coefficient* is the fractional change in pressure (based upon the pressure at 0°C) per degree change in temperature.

The volume and pressure coefficients are equal and have nearly the same value for all gases (0.00366/°C).

Absolute zero is that temperature at which (*a*) all molecular activity ceases, (*b*) the volume between gas molecules is reduced to zero, and (*c*) the pressure exerted by molecular activity is zero. Absolute zero is $-273.16°C = -459.7°F$.

Absolute temperature T is measured on a scale beginning at absolute zero. $T = 273.16° + t$, where t is the centigrade temperature.

Boyle's law follows from the general gas law if the temperature is constant. PV is constant if T does not change.

Charles's laws also follow from the general gas law. If the pressure is constant, the volume is directly proportional to the absolute temperature. If the volume is constant, the pressure is directly proportional to the absolute temperature.

QUESTIONS AND PROBLEMS

1. The barometric pressure is 30 in. of mercury. Express this in pounds per square foot and in pounds per square inch. (See Table I of Chap. 7.)

2. Why does air escaping from the valve of a tire feel cool?

3. Change 40°C and −5°C to the absolute scale. Change 45°F and −50°F to the absolute scale.

4. A gas occupies 200 cm^3 at 100°C. Find its volume at 0°C, assuming constant pressure. *Ans.* 146 cm^3.

5. Given 200 cm^3 of oxygen at 5°C and 76 cm of mercury pressure, find its volume at 30°C and 80 cm of mercury pressure.

6. A mass of gas has a volume of 6.0 ft^3 at 40°C and 76 cm of mercury pressure. Find its volume at −15°C and 57 cm of mercury pressure. *Ans.* 6.6 ft^3.

7. A gas occupies 2.0 ft^3 under a pressure of 30 in. of mercury. What volume will it occupy under 25 in. of mercury pressure? Assume that the temperature is unchanged.

8. The volume of a tire is 1,500 $in.^3$ when the pressure is 30 $lb/in.^2$ above atmospheric pressure. What volume will this air occupy at atmospheric pressure? Assume that atmospheric pressure is 15 $lb/in.^2$ How much air will come out of the tire when the valve is removed? *Ans.* $4,\bar{5}00$ $in.^3$; $3,\bar{0}00$ $in.^3$

EXPERIMENT

Expansion of Air

METHOD A

Apparatus: Gas law apparatus illustrated by Fig. 5; large graduate; ice; steam generator; barometer; thermometer.

With the apparatus depicted in Fig. 5 it is possible to study experimentally the changes of volume of air, (1) for constant temperature, and (2) for constant pressure. It is made of glass and consists essentially of three tubes, two (*A* and *C*) open to the atmosphere, the other (*D*) closed. When the apparatus is filled with mercury, air is trapped in tube *D*. With the wooden plunger *E* one can change the relative amounts of mercury in the tubes and thus raise or lower the mercury columns. By means of the scale S one can read the heights of the mercury columns in D and C.

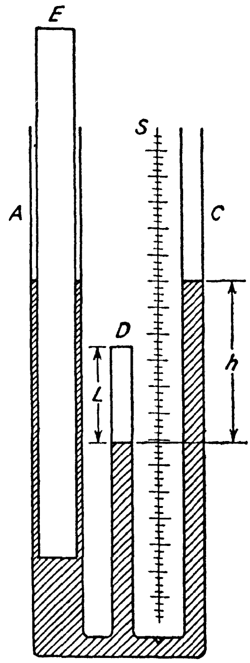


FIG. 5.—Diagram of apparatus for demonstrating Boyle's law.

The apparatus will have been adjusted by the instructor before class time, so that the mercury surface in D is somewhat higher than that in C. If the level in D is higher than that in C by an amount h, the pressure of the gas in D is less than atmospheric pressure by h cm of mercury. As the plunger is pushed down, the pressure of the air in D is increased, becoming equal to atmospheric pressure when the mercury columns are at the same level, and greater than atmospheric pressure by h cm of mercury when the columns are in the positions shown in Fig. 5. Because of the increase in pressure, the entrapped air decreases in volume in accordance with Boyle's law (the temperature remains essentially constant).

The positions of the mercury surfaces in D and C should be recorded in Table II for various positions of the plunger. At the head of the table should be noted the scale reading for the top of the air column in D, and the atmospheric pressure, as read from a barometer, in centimeters of mercury. The total pressure P of the air in D is the sum of P_a, the atmospheric pressure, and P_h, the added pressure produced by the mercury (h cm of mercury). If the air column in D has a uniform cross section, its volume is proportional to its length L. According to Boyle's law, the volume is inversely proportional

TABLE II

Top of air column in D ______
Atmospheric pressure P_a ______

Mercury level in D	Mercury level in C	Difference between levels, $C - D = P_h$	$P_a + P_h = P$	Length of air column in D, L	Product, PL

to the pressure, hence L should be inversely proportional to P. The criterion for the recognition of such a proportion between two variables is that their product shall remain constant. This product should be computed, therefore, and recorded in the last column of the table. What is your conclusion?

Adjust the plunger until the mercury surfaces are at the same level. Place the apparatus in a jar as in Fig. 6 and fill the jar with ice water. Notice that the air in D contracts and that the mercury level in C drops below that in D. Readjust the plunger until the mercury levels in C and D are again the same.

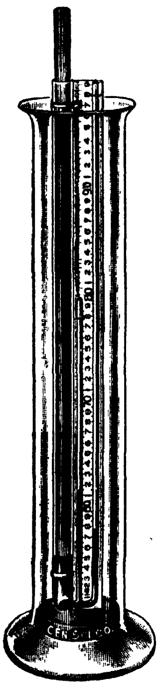


FIG. 6.—Gas law apparatus.

In the procedure just described, the pressure of the air in D was in each case adjusted to equal that of the atmosphere, hence the condition for Charles's law (Eq. 9) was fulfilled. If the procedure is repeated for a series of different temperatures, each value of the volume (or of L) should be proportional to the corresponding absolute temperature T.

Repeat the procedure for a number of different temperatures. The temperature may be raised by bubbling steam through the water. Temperature readings should be made only after the water has been thoroughly stirred to establish a uniform temperature. Record the data as in Table III. Is the ratio T/L essentially constant? Plot a graph, L against T, and interpret it.

METHOD B

Apparatus: Large graduate; water; steam generator; ice; thermometer; capillary tube as shown in Fig. 7; burette clamp; rubber or cork stopper with hole; barometer.

The tube shown in Fig. 7 is of capillary bore having an internal diameter of 1 or 2 mm. In it is trapped some air separated from the external atmosphere by a mercury column. This tube should be slipped into the hole of a stopper which is held by a burette clamp and support rod. In this way the tube can be rotated in a vertical plane with the clamp as a swivel.

TABLE III

Top of air column in D ______

Mercury level in D and C, cm	Length L of air column in D, cm	Temperature, °C	Absolute temperature T, °K	Ratio, T/L

When the tube is oriented as in b, Fig. 7, the gas is under atmospheric pressure (P_a cm of mercury). When in orientation a, the air pressure in the tube is $P_a + P_h$, where P_h is the pressure exerted by a vertical column of mercury h cm high, h being the vertical height of the mercury as illustrated in Fig. 7.

For orientation *c*, the air pressure in the tube is $P_a - P_h$.

For all such orientations the volume of the gas is related to the pressure by the Boyle's law equation, since the temperature is constant.

To study this law, measure L, the length of the air column, and h for different orientations of the tube (include the two vertical positions for which h is easily determined). The values of P_h will be numerically equal to h in centimeters of mercury. (If the mercury column is separated into two or more segments, the length of the mercury column should be determined by adding the lengths of the segments.) Record the data as in Table IV. Refer to the alternative experiment *A* for the interpretation of the data.

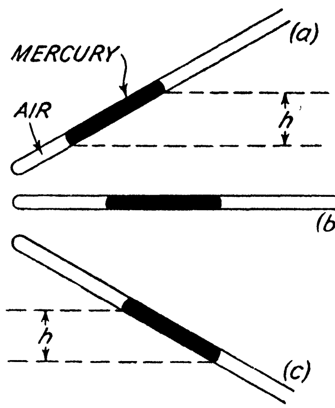


FIG. 7.—A simple capillary-tube type of gas-law apparatus.

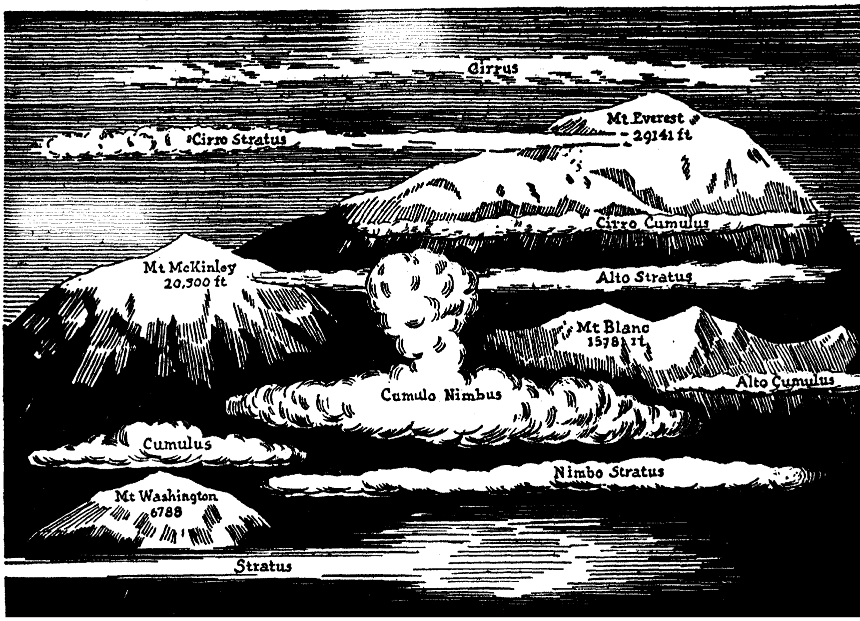
TABLE IV

P_h	$P = P_a + P_h$	L	PL

If the tube is placed vertically in water and then the temperature of the water is changed, the volume and temperature of the air will change, but not the pressure. Take readings of L and t for various temperatures of the water, beginning with ice water. Record the data as in Table V. If the volume of the air column is proportional to the absolute temperature, the ratio in the fourth column should be essentially constant.

TABLE V

t	L	T	T/L



CHAPTER 9

METEOROLOGY

Meteorology is the study of weather and the atmospheric conditions that contribute to it. The phenomena of weather are subjects not only of never-ending interest but of great importance, since weather is one of the chief elements in man's life. Although foreknowledge of weather will not enable us to make any change in the conditions that eventually arrive, yet we can, in many cases, so adjust our activities that adverse weather will produce a minimum of ill effect. The Weather Bureau was established to observe and forecast weather conditions. For many years these reports have been of great value to those engaged in agriculture or marine navigation. At the present time, however, the most important application of meteorology is in connection with airplane flight, both civil and military. The great dependence of the airplane upon the weather makes accurate observation and forecast essential. This need has caused great extension in the number of stations reporting and in the scope of the observations.

The Ocean of Air. The human race lives at the bottom of an ocean of great depth—an ocean of air. Just as the inhabitants of the ocean of water are subject to pressure and water currents, so are we subject to air pressure and air currents. As the pressure in the ocean of water increases as the depth increases ($P = hd$), so also the pressure of the atmosphere increases as the depth below its "surface" increases. As one rises from the bottom of the ocean of air, the pressure decreases.

The pressure of the air is measured by means of a barometer. The most reliable kind of barometer is the mercury type described in Chap. 8. This instrument, however, is not readily portable, and whenever the use requires portability another type called an *aneroid* barometer is used The essential feature of an aneroid barometer (Fig. 1) is a metallic box

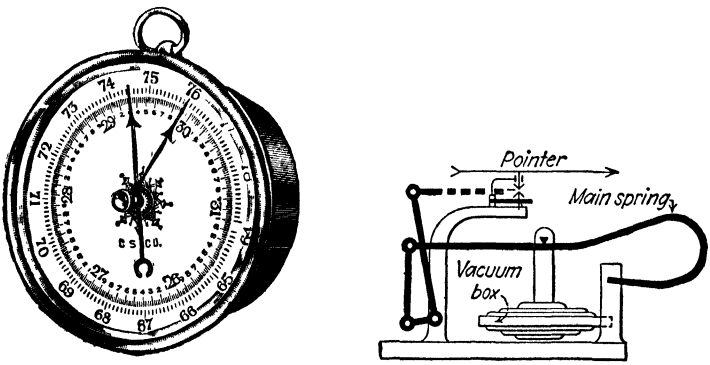


FIG. 1.—The aneroid barometer.

or cell, corrugated in order to make it flexible and partly exhausted of air. This cell tends to collapse under the pressure of air, but a strong spring balances the air pressure and prevents such collapse. As the pressure of the air changes, the free surface of the cell contracts or expands slightly, and this small movement is magnified and transmitted to a needle that moves over a dial.

In the discussion of pressure in Chap. 7, the unit used was a force unit divided by an area unit such as pounds per square inch or pounds per square foot. Pressure may also be expressed in terms of the height of a column of liquid, which is supported by the pressure. Since mercury is commonly used in barometers, air pressure is frequently recorded in inches of mercury. At sea level the average height of the mercury column in the barometer is 29.92 in. Hence we say that normal barometric pressure is 29.92 in. of mercury.

In weather observations another unit of pressure called the *millibar* is now used by international agreement.

$$1 \text{ millibar} = 1{,}000 \text{ dynes/cm}^2$$

To convert from pressures expressed in inches of mercury to pressures expressed in millibars, we may use Eq. (3), Chap. 7.

$$P = hd$$

For normal barometric pressure $h = 29.92$ in.; $d = 13.60$ gm/cm^3.

$$\begin{aligned} P &= (29.92 \text{ in.})(2.54 \text{ cm/in.})(13.60 \text{ gm/cm}^3)(980 \text{ dynes/gm}) \\ &= 1{,}013{,}000 \text{ dynes/cm}^2 = 1{,}013 \text{ millibars} \end{aligned}$$

Normal barometric pressure at sea level is about 1,013 millibars as well as 29.92 in. of mercury. In order to find the pressure in millibars we must multiply the barometric height in inches by the factor 33.86 or $(2.54 \times 13.60 \times 980/1{,}000)$.

The variation in pressure with altitude is a phenomenon with which all are somewhat familiar. If one rides rapidly up a hill, he can feel the change in the pressure at the eardrums—for the pressure inside the ear fails to change as rapidly as that outside. The accompanying table shows the way the atmospheric pressure varies with height above sea level.

TABLE I. RELATIONSHIP BETWEEN PRESSURE AND HEIGHT

Altitude in feet above sea level	Pressure in inches of mercury	Pressure in millibars
Sea level	29.92	1,013.2
1,000	28.86	977.2
2,000	27.82	942.0
3,000	26.81	907.8
4,000	25.84	874.9
5,000	24.89	842.8
6,000	23.98	812.0
7,000	23.09	781.8
8,000	22.22	752.4
9,000	21.38	723.9
10,000	20.53	696.8
15,000	16.88	571.6
20,000	13.75	465.6

Note that, although the decrease in pressure as the altitude increases is not quite uniform, it is approximately 1 in. of mercury per 1,000 ft. This is a convenient figure to remember for rough calculation. For purposes of comparison, observations taken at different levels are always reduced to the equivalent reading at sea level before they are reported.

This variation of pressure with altitude is the basis of the common instrument for measurement of altitude, the altimeter (Fig. 2). It is simply a sensitive aneroid barometer whose dial is marked off in feet above sea level rather than in inches of mercury.

At a single elevation the barometric pressure varies from day to day and from time to time during the day. The lowest sea-level pressure ever recorded is 26.16 in. of mercury (892 millibars), while the highest is 31.7 in. (1,078 millibars). This variation greatly affects the use of an altimeter. If the altitude reading is to be at all reliable, the instrument must be set for the current pressure each time it is to be used. For example, an airplane in taking off from a field at which the pressure is 29.90 in. has an altimeter that is set correctly at the altitude of the field. If it then flies to another field where the pressure is 29.50 in., the altimeter will read 400 ft above the field when the plane lands. Such an error would be disastrous if the pilot were depending upon the instrument for safe landing. In practice the pilot must change the setting en route to correspond to the pressure at the landing field.

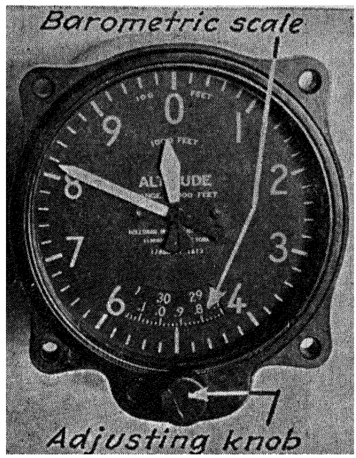


FIG. 2.—A sensitive altimeter The large-hand readings are in hundreds and the small-hand readings are in thousands of feet.

Heating and Temperature. The variations in sea-level pressure in the atmosphere and the resulting air currents are due largely to unequal heating of the surface of the earth. The sun may be considered as the sole source of the energy received, since that received from other sources is so small as to be negligible.

The three methods of heat transfer, conduction, convection, and radiation, were discussed in Chap. 5. Each kind of transfer plays a part in distributing the heat that comes to the earth. Heat comes from the sun to the earth by radiation. A small part of this incoming radiation is absorbed in the air itself. A part is reflected or absorbed by clouds; the remainder reaches the surface of the earth and is there absorbed or reflected.

When heat is absorbed at the surface of the earth, the temperature rises. If no heat were radiated by the earth, the temperature rise would continue indefinitely. However, on the average, over a long period of time and for the earth as a whole, as much energy is radiated as is received. Certain parts of the earth, for example, the equatorial regions, receive more energy than they radiate, while others, such as the polar regions, radiate more than they receive. The balance is maintained by the transfer of heat from one region to the other by convection. The convection currents are set up by unequal heating of the different parts of the surface of the earth.

The unequal heating of adjacent areas may be the result of unequal distribution of the radiation or of unequal absorption of radiation. If the radiation strikes a surface perpendicularly, the amount of energy per unit area is greater than it would be for any other angle. Thus regions (equatorial) where the sun is overhead receive more energy for each square foot of area than do the polar regions where the angle that the rays make with the ground is smaller. In equatorial regions the surface temperature is, on the average, higher than in surrounding regions. The layer of air adjacent to the ground is heated by conduction and expands, becoming less dense than the surrounding air at the same level. The lighter air rises, its place being taken by surrounding colder air; this in turn is heated and rises. The *unequal* heating sets up a circulation that constitutes the major air movement of the world.

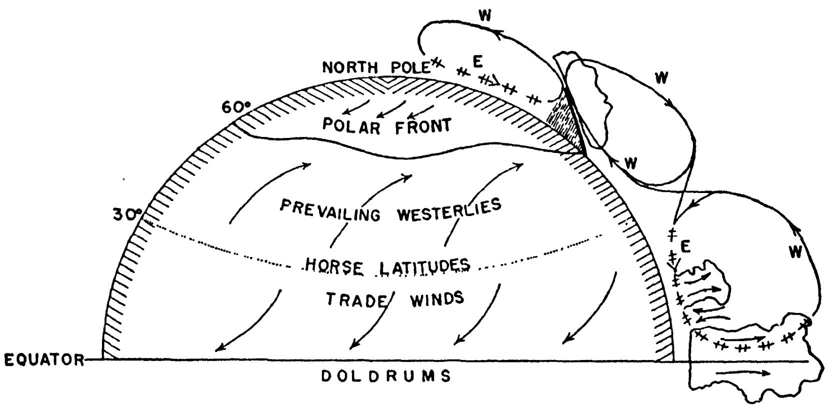


Fig. 3.—General circulation on a uniform earth.

The major circulation of the atmosphere is shown diagrammatically in Fig. 3. Over the equatorial region heated air rises, causing a low-pressure area of calm or light fitful winds, called *doldrums.* Both north and south of the doldrums air rushes in to take the place of the rising air, thus forming the trade winds. If the earth were not rotating, these would be from the north in the Northern Hemisphere and from the south in the Southern Hemisphere. The rotation of the earth, however, causes a deflection of the moving air: to the right in the Northern Hemisphere, to the left in the Southern. Thus the trade winds blow almost constantly from the northeast in the Northern Hemisphere and from the southeast in the Southern.

The air that rises in the doldrums moves out at high altitude and about 25° from the equator begins to descend. This region of descending air is an area of calm or light winds and high pressure, and is called the *horse latitudes.* Part of the descending air moves back toward the equator while the remainder continues to move away from it near the surface.

Again the rotation of the earth causes a deflection to the right (in the Northern Hemisphere), hence the wind comes from the west. The winds of this region are known as *prevailing westerlies.* A part of the air moving out from the equator continues at high level until it reaches the polar area. As it returns toward the equator, it is deflected by the rotation of the earth to form the *polar easterlies.*

Since the atmosphere is principally heated from below, the temperature normally decreases as the altitude increases for several thousand feet; above this region there is little further change. The region of changing temperature is known as the *troposphere;* the upper region of uniform temperature is known as the *stratosphere,* and the surface of separation is the *tropopause.* The altitude of the tropopause varies from about 25,000 ft to 50,000 ft in different parts of the earth, the highest values being above the equatorial regions and the lowest over the poles.

The rate at which the temperature decreases with altitude is called the *lapse rate.* The value of the lapse rate varies over a wide range depending upon local conditions, but the average value is about 3.6°F per 1,000 ft in still air.

If air rises, the pressure to which it is subjected decreases, and it expands. In this process there is little loss of heat to the surroundings or gain from them. In accordance with the general gas law (Chap. 8), the temperature decreases as the air expands. Such a change is called an *adiabatic* change, the word implying "without transfer of heat." In a mass of *rising* air, the temperature decreases faster than the normal lapse rate. If the air is dry, this *adiabatic rate* of decrease is about 5.5°F per 1,000 ft. If the air rises because of local heating, as occurs over a plowed field, it will rise until its temperature is the same as that of the surrounding air at the same level.

Example: The temperature of air at the surface of a plowed field is 80°F, while that over adjacent green fields is 70°F. How high will the air current rise?

The rising air must cool $80° - 70° = 10°\text{F}$ more than the still air. For each 1,000 ft the rising air cools $5.5° - 3.6° = 1.9°\text{F}$ more than the still air. The number of 1,000 ft at which their temperatures will be the same is

$$\frac{10°\text{F}}{1.9°\text{F}} = 5.3$$

$$h = 5.3 \times 1{,}000 \text{ ft} = 5{,}300 \text{ ft}$$

If condensation occurs in the rising air, there is a gain in heat from the heat of vaporization, and therefore the change is no longer adiabatic. During the condensation, therefore, the rising saturated air cools at a smaller *moist-adiabatic rate.*

Cyclones and Anticyclones. As large masses of air move along the surface of the earth, areas of low pressure and other areas of high pressure are formed. The air moves from the high-pressure areas toward the low-pressure areas. As in larger air currents, the rotation of the earth causes the wind to be deflected (to the right in the Northern Hemisphere) so that the air does not move in a straight line from high to low but spirals *out* from the high and spirals *into* the low. The low-pressure area with its accompanying winds is called a *cyclone;* the high-pressure area with its winds is called an *anticyclone.* The deflection to the right causes the winds to move *counterclockwise* in the cyclone and *clockwise* in the

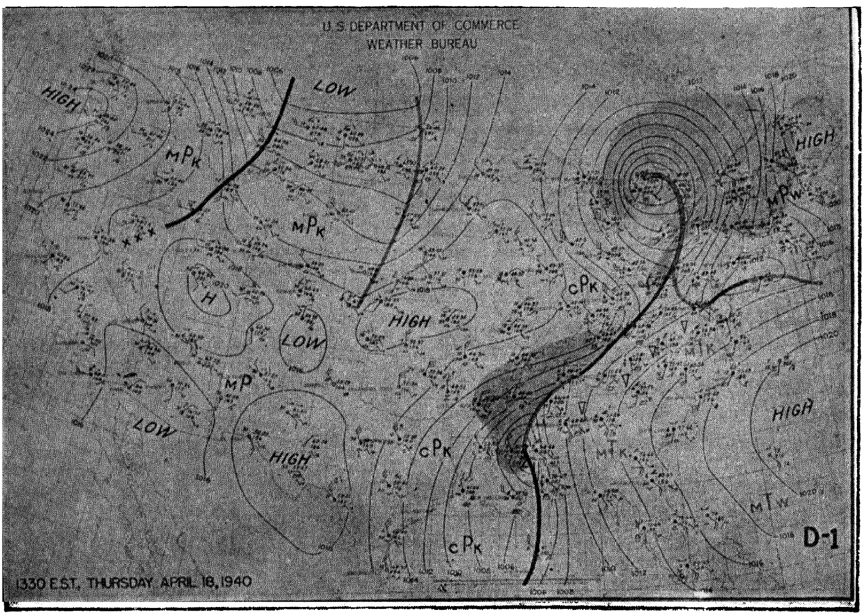


FIG. 4.—A typical weather map.

anticyclone. These high- and low-pressure regions cover very large areas, having diameters of from 200 to 600 miles.

The presence of cyclones and anticyclones is shown on weather maps. Lines called *isobars* are drawn connecting points of equal pressure. Figure 4 is a reproduction of a weather map. Where the isobars are close together, the pressure is changing rapidly and high winds are expected. Where they are far apart, the pressure is more uniform and there is usually less wind.

Humidity. At all times water is present in the atmosphere in one or more of its physical forms—solid, liquid, and vapor. The invisible vapor is always present in amounts that vary over a wide range while water drops (rain or cloud) or ice crystals (snow or cloud) are usually present.

If a shallow pan of water is allowed to stand uncovered in a large room, the water will soon evaporate and apparently disappear although it is still present as invisible vapor. If a similar pan of water is placed in a small enclosure, it will begin to evaporate as before, but after a time the evaporation stops or becomes very slow and droplets begin to condense on the walls of the enclosure. The air is said to be *saturated.* When this condition has been reached the addition of more water vapor merely results in the condensation of an equal amount. The amount of water vapor required for saturation depends upon the temperature; the higher the temperature the greater is the amount of water vapor required to produce saturation. If the air is not saturated, it can be made so either by adding more water vapor or by reducing the temperature until that already present will produce saturation. The temperature to which the air must be cooled, at constant pressure, to produce saturation is called the *dew point.* If a glass of water collects moisture on the outside, its temperature is below the dew point.

When the temperature of the air is reduced to the dew point, condensation takes place if there are present nuclei on which droplets may form. These may be tiny salt crystals, smoke particles, or other particles that readily take up water. In the open air such particles are almost always present. In a closed space where such particles are not present, the temperature may be reduced below the dew point without consequent condensation. The air is then said to be *supersaturated.*

In a mixture of gases, such as air, the pressure exerted by the gas is the sum of the partial pressures exerted by the individual gases. The portion of the atmospheric pressure due to water vapor is called its *vapor pressure.* When the air is saturated, the pressure exerted by the water vapor is the *saturated vapor pressure.* Table 2 in the Appendix lists the pressure of saturated water vapor at various temperatures.

The mass of water vapor per unit volume of air is called the *absolute humidity.* It is commonly expressed in grains per cubic foot or in grams per cubic meter. *Specific humidity* is the mass of water vapor per unit mass of air and is expressed in grams per kilogram, grains per pound, etc. Specific humidity is the more useful since it remains constant when pressure and temperature change, while the absolute humidity varies because of the change in volume of the air involved.

Relative Humidity. *Relative humidity* is defined as the ratio of the actual vapor pressure to the saturated vapor pressure at that temperature. It is commonly expressed as a percentage. At the dew point the relative humidity is 100 per cent. From a knowledge of the temperature and dew point the relative humidity can be readily determined by the use of the table of vapor pressures.

Example: In a weather report the temperature is given as 68°F and the dew point 50°F. What is the relative humidity?

To use Table 2 (Appendix) we must change the temperature to the centigrade scale.

$$C = \tfrac{5}{9}(F - 32°)$$
$$C_1 = \tfrac{5}{9}(68° - 32°) = \tfrac{5}{9}(36°) = 20°$$
$$C_2 = \tfrac{5}{9}(50° - 32°) = \tfrac{5}{9}(18°) = 10°$$

From the table we find the vapor pressures

$$P_1 = 17.6 \text{ mm of mercury} = \text{pressure of saturated vapor}$$
$$P_2 = 9.2 \text{ mm of mercury} = \text{actual vapor pressure}$$

$$\text{Relative humidity} = \frac{P_2}{P_1} = \frac{9.2 \text{ mm of mercury}}{17.6 \text{ mm of mercury}} = 0.52 = 52\%$$

Whenever the temperature of the air is reduced to the dew point, condensation occurs. When the dew point is above the freezing point, water droplets are formed; when it is below, ice crystals are formed. The formation of dew, frost, clouds, and fog are examples of this process. The cooling may be caused by contact with a cold surface, by mixing with cold air, or by expansion in rising air. If the droplets are sufficiently small, the rate of fall is very slow and there is a *cloud*. When the cloud is in contact with the earth's surface, we call it *fog*. One of the most common causes of cloud formation is the expansion and consequent cooling of a rising air column. Each of the small fair-weather clouds of a bright summer day is at the top of a column of rising air. Its base is flat, at the level at which the dew point is reached. The glider pilot may use these clouds as indicators to show the position of the rising currents. Clouds form on the windward side of mountains where the air is forced to rise, while on the leeward side where the air is descending the clouds evaporate.

Whenever the temperature and dew point are close together, the relative humidity is very high and cloud or fog formation is very probable. The pilot, in planning a flight, avoids such areas because of the low visibility and ceiling to be expected there.

SUMMARY

Meteorology is the study of weather and the atmospheric conditions that contribute to it.

Important factors in the weather are *barometric pressure, temperature, wind, humidity.*

The barometric pressure, which is measured in *inches of mercury* or *millibars,* decreases with increase in altitude. The decrease is about 1 in. of mercury per 1,000 ft in the lower levels.

A *millibar* is 1,000 dynes/cm².

The temperature of the air is normally highest at the surface of the earth. The rate at which it decreases with increase in altitude is called the *lapse rate*. Its average value is about 3.6°F per 1,000 ft.

Rising air is cooled by expansion, its temperature decreasing about 5.5°F per 1,000 ft rise for dry air.

A *cyclone* is a low-pressure area with its accompanying winds while an *anticyclone* is a high-pressure area and its winds. In the Northern Hemisphere winds spiral *counterclockwise* into a cyclone and *clockwise* out of an anticyclone.

Isobars are lines on the weather map connecting points of equal barometric pressure.

Absolute humidity is the mass of water vapor per unit volume of air. *Specific humidity* is the mass of water vapor per unit mass of air.

Relative humidity is defined as the ratio of the actual vapor pressure to the saturated vapor pressure at that temperature.

The *dew point* is the temperature to which the air must be cooled, at constant pressure, to produce saturation.

Water vapor condenses to form a *cloud* or *fog* whenever the temperature is reduced to the dew point.

QUESTIONS AND PROBLEMS

1. When side by side, over which will the stronger up current be found during a period of sunshine, a plowed field or a meadow? Why?

2. What is actually meant by the term "falling barometer"?

3. If the temperature is 40°F at the surface, what will it be at 30,000 ft altitude under normal conditions? at 15,000 ft?

4. Why is it impossible to use an altimeter intelligently without knowledge of the terrain and the weather map? Explain fully.

5. What may be the result of flying over mountains, in thick weather, if the altimeter is reading too high?

6. Would an altimeter show increase in altitude if there were no decrease in barometric pressure during a climb? Why?

7. Define relative humidity, specific humidity, dew point.

8. In which case does the air hold more water vapor: (*a*) temperature 32°F, dew point 32°F, (*b*) temperature 80°F, dew point 50°F? What is the relative humidity in each case? *Ans.* 100 per cent; 34 per cent.

9. A decrease of 1 in. of mercury in barometric pressure will cause what change of altitude reading on an altimeter at rest on the ground?

10. How are differences in pressure indicated on the weather map?

11. What kind of weather would you expect to find where the dew point and the air temperature are the same?

EXPERIMENT

Dew Point and Relative Humidity

Apparatus: Sling psychrometer; hair hygrometer; tables.

It is possible to determine the dew point directly by observation of the temperature at which dew first appears on a polished surface as its temperature is reduced. This method is quite inaccurate because of the inability of an observer to determine exactly when the dew first appears.

A more commonly used method is that which makes use of wet-bulb and dry-bulb thermometers. The instrument consists of two thermometers, the bulb of one being covered with cloth that is kept moistened.

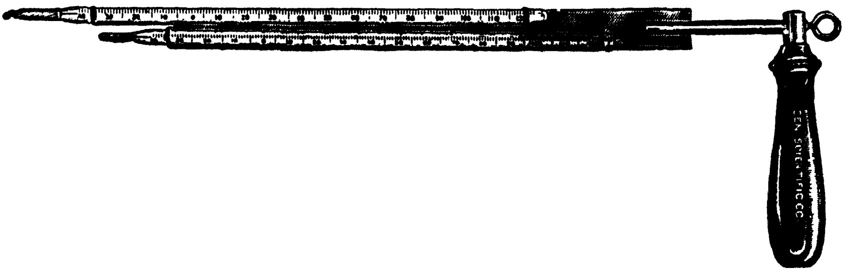


FIG. 5.—Sling psychrometer.

Evaporation causes the temperature of this bulb to be lowered. The rate of evaporation depends upon the relative humidity of the surrounding air and hence the difference in temperature of the two thermometers will give a measure of that quantity. If the wet-bulb thermometer is kept stationary, the air adjacent to it quickly becomes more humid than the surrounding air. In order to get a true reading, the air must move past the bulb. The simplest means of securing this motion is to use the instrument called a *sling psychrometer*, which is depicted in Fig. 5. It consists of two thermometers so mounted that they may be whirled readily. The lowest temperature reached by the wet-bulb thermometer is recorded as the wet-bulb temperature. Tables in a handbook give the relation between the wet- and dry-bulb temperatures and the relative humidity in weather observations.

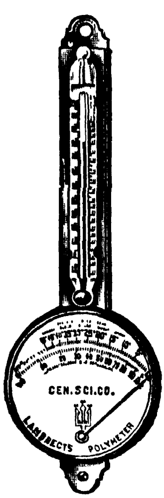


FIG. 6.—Hair hygrometer.

Use the sling psychrometer to determine the dew point and the relative humidity in a room. Wrap gauze around one of the thermometer bulbs, moisten it, and whirl the instrument rapidly for a few minutes. Note the temperature of the two thermometers frequently and continue whirling the instrument until the lowest temperature of the wet bulb is determined. Using the dry-bulb temperature and the difference between the dry- and wet-bulb temperatures, obtain, by the aid of the tables, the dew point and relative humidity of the room.

The hair hygrometer (Fig. 6) is an instrument that reads relative humidity directly. A long hair varies considerably in length under different conditions of humidity. The hair is connected to a suitable system of levers so that its expansion and contraction are communicated to the pointer, which moves over a scale calibrated to read the relative humidity directly. The indications of this type of instrument are usually rather inaccurate. If a hair hygrometer is available, take its reading and compare it with the value obtained by the use of the sling psychrometer.

CHAPTER 10

TYPES OF MOTION

A study of the motions of objects is necessary if we are to understand their behavior and learn to control them. Since most motions are very complex, it is necessary to begin with the simplest of cases. When these simple types of motion are thoroughly understood, it is surprising what complicated motions can be analyzed and represented in terms of a few elementary types.

Speed and Velocity Contrasted. The simplest kind of motion an object can have is motion with constant velocity, a particular case of motion with constant speed. Constant velocity implies not only constant speed but unchanging direction as well. An automobile that travels for 1 hr at a constant velocity of 20 mi/hr *north,* reaches a place 20 mi north of its first position. If, on the other hand, it travels around a race track for 1 hr at a constant speed of 20 mi/hr, it traverses the same distance without getting anywhere. At one instant its velocity may be 20 mi/hr east; at another, 20 mi/hr south.

The statement "An automobile is moving with a *velocity* of 20 mi/hr" is incorrect by virtue of incompleteness, since the direction of motion must be stated in order to specify a velocity. For this reason one should always use the word *speed* when he does not wish to state the direction of motion, or when the direction is changing.

The *average speed* of a body is the distance it moves divided by the time required for the motion. The defining equation is

$$\bar{v} = \frac{s}{t}$$

This may be put in the form

$$s = \bar{v}t$$

where s is the distance traversed, $\bar{v}$ the *average speed,* and t the amount of time. If the speed is constant its value is, of course, identical with the average speed.

If, for example, an automobile travels 200 mi in 4 hr, its average speed is 50 mi/hr. In 6 hr it would travel 300 mi.

Accelerated Motion. Objects seldom move with constant velocity. In almost all cases the velocity of an object is continuously changing in magnitude or in direction, or in both. Motion in which the velocity is changing is called *accelerated motion,* and the rate at which the velocity changes is called the *acceleration.*

The simplest type of accelerated motion, called *uniformly accelerated motion,* is that in which the direction remains constant and the speed changes at a constant rate. The acceleration in this case is equal to the rate of change of speed, since there is no change in direction. Acceleration is called positive if the speed is increasing, negative if the speed is decreasing. Negative acceleration is sometimes called *deceleration.*

Suppose that an automobile accelerates at a constant rate from 15 mi/hr to 45 mi/hr in 10 sec while traveling in a straight line. The acceleration, or the rate of change of speed in this case, is the change in speed divided by the time in which it took place, or

$$a = \frac{(45\text{ mi/hr} - 15\text{ mi/hr})}{10\text{ sec}} = \frac{30\text{ mi/hr}}{10\text{ sec}} = 3.0\text{ mi/hr per sec}$$

indicating that the speed increases 3 mi/hr during each second. Since 30 mi/hr = 44 ft/sec, the acceleration can be written also as

$$\frac{44\text{ ft/sec}}{10\text{ sec}} = 4.4\text{ ft per sec per sec}$$

This means simply that the speed increases 4.4 ft/sec during each second, or 4.4 ft/sec^2.

Using algebraic symbols to represent acceleration a, initial speed v_1, final speed v_2, and time t, the defining equation for acceleration is written

$$a = \frac{v_2 - v_1}{t}$$

Multiplying both sides of the equation by t gives

$$v_2 - v_1 = at$$

which expresses the fact that the *change in speed* is equal to the rate of change of speed multiplied by the time during which it is changing.

The distance traveled during any time is given by the equation

$$s = \bar{v}t$$

but the average speed $\bar{v}$ must be obtained from the initial and final speeds, v_1 and v_2. Since the change of speed occurs at a uniform rate, the average speed $\bar{v}$ is equal to the average of the initial and final speeds, or

$$\bar{v} = \tfrac{1}{2}(v_1 + v_2)$$

In the case under consideration

$$\bar{v} = \tfrac{1}{2}(15 + 45) \text{ mi/hr} = 30 \text{ mi/hr} = 44 \text{ ft/sec}$$

and

$$s = (44 \text{ ft/sec})(10 \text{ sec}) = 440 \text{ ft}$$

Three equations for uniformly accelerated motion have been considered. By combining them, two more useful equations can be obtained. The five equations needed in solving problems in uniformly accelerated motion are

$$s = \bar{v}t \qquad (1)$$
$$\bar{v} = \tfrac{1}{2}(v_1 + v_2) \qquad (2)$$
$$v_2 - v_1 = at \qquad (3)$$
$$s = v_1t + \tfrac{1}{2}at^2 \qquad (4)$$
$$v_2^2 - v_1^2 = 2as \qquad (5)$$

Of these, Eq. (1) is true for all types of motion; the remaining four equations hold only for uniformly accelerated linear motion. That equation should be used in which the quantity to be determined is the only one not known.

Falling Bodies; Acceleration Due to Gravity. The motion of an object under the action of a constant force is uniformly accelerated. A falling stone, since its weight is an essentially constant force, executes a motion in which the acceleration is very nearly constant, if air resistance is neglected.

Observations of the fall of objects reveal that all bodies fall with exactly the same acceleration when the effect of the air is eliminated.

The acceleration of freely falling bodies is so important that it is customary to represent it by the special symbol g. At sea level and 45° latitude, g has a value of 32.17 ft/sec^2, or 980.6 cm/sec^2. For our purposes it is sufficiently accurate to use g = 32 ft/sec^2 or 980 cm/sec^2.

Since a freely falling body is uniformly accelerated, the equations already developed may be applied when air resistance is neglected.

Example: A body starting from rest falls freely. What is its speed at the end of 1.0 sec?

$$a = 32\text{ ft/sec}^2$$
$$v_1 = 0$$
$$t = 1.0\text{ sec}$$

Using Eq. (3)

$$v_2 = v_1 + at = 0 + (32\text{ ft/sec}^2)(1.0\text{ sec}) = 32\text{ ft/sec}$$

Example: How far does a body, starting from rest, fall during the first second?

$$v_1 = 0$$
$$a = 32\text{ ft/sec}^2$$
$$t = 1.0\text{ sec}$$

From Eq. (4)

$$s = v_1t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(32\text{ ft/sec}^2)(1.0\text{ sec})^2 = 16\text{ ft}$$

Table I shows the speed at the end of time t and the distance fallen during time t for a body that starts from rest.

TABLE I

Time, t, sec	Speed (ft/sec) at end of time t	Distance (ft) fallen in time t
1	32	16
2	64	64
3	96	144
4	128	256

When, instead of falling from rest, an object is thrown with initial speed v_1, the first term of Eq. (4) is no longer zero. If it is thrown downward, both v_1 and a have the same direction and hence are given the same algebraic sign. If, however, it is thrown upward, v_1 is directed upward while a is directed downward and thus the latter must be considered as negative.

Example: A body is thrown upward with an initial speed of 40 ft/sec. Find the distance traveled during the first second, the speed at the end of the first second, and the greatest elevation reached by the object.

$$v_1 = 40\text{ ft/sec}$$
$$a = -32\text{ ft/sec}^2$$
$$t = 1\text{ sec}$$

From Eq. (4)

$$s = v_1t + \tfrac{1}{2}at^2 = (40\text{ ft/sec})(1\text{ sec}) + \tfrac{1}{2}(-32\text{ ft/sec}^2)(1\text{ sec})^2 = 24\text{ ft}$$

From Eq. (3)

$$v_2 = v_1 + at = 40\text{ ft/sec} + (-32\text{ ft/sec}^2)(1\text{ sec}) = 8\text{ ft/sec}$$

The time required for the object to reach the highest point in its motion is obtained from Eq. (3). At the highest point the object stops and hence

$$\begin{aligned} v_2 &= 0 \\ v_2 - v_1 &= at \\ 0 - 40\ \text{ft/sec} &= (-32\ \text{ft/sec}^2)t \\ t &= 1.25\ \text{sec} \end{aligned}$$

In 1.25 sec the object will rise a distance

$$\begin{aligned} s &= v_1t + \tfrac{1}{2}at^2 = (40\ \text{ft/sec})(1.25\ \text{sec}) + \tfrac{1}{2}(-32\ \text{ft/sec}^2)(1.25\ \text{sec})^2 \\ &= 50\ \text{ft} - 25\ \text{ft} = 25\ \text{ft} \end{aligned}$$

This is the greatest elevation reached by the object

In the preceding discussion we have assumed that there is no air resistance. In the actual motion of every falling body this is far from true. The frictional resistance of the air depends upon the speed of the moving object. The resistance to a falling stone is quite small for the first one or two seconds but as the speed of fall increases the resistance becomes large enough to reduce appreciably the net downward force on the stone and the acceleration decreases. After some time of uninterrupted fall, the stone is moving so rapidly that the drag of the air is as great as the weight of the stone, so that there is no acceleration. The stone has then reached its *terminal speed*, a speed that it cannot exceed in falling from rest.

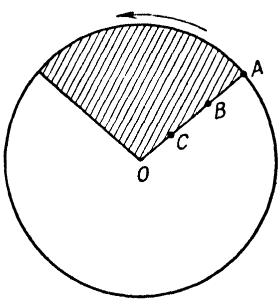


FIG. 1.—Rotation about the axis O. The angular speed is the same for all parts of the disk, but the linear speed increases as the radius increases.

Very small objects, such as dust particles, water droplets, and objects of very low density and large surface, such as feathers, have very low terminal speeds; hence they fall only small distances before losing most of their acceleration. A man jumping from a plane reaches a terminal speed of about 120 mi/hr if he delays opening his parachute. When the parachute is opened, the terminal speed is reduced because of the increased air resistance to about 12 mi/hr which is about equal to the speed gained in jumping from a height of 5 ft. A large parachute produces greater resistance than a smaller one and hence causes slower descent. A plane in a vertical dive without the use of its motor can attain a speed of about 400 mi/hr.

Rotary Motion. Another simple type of motion is that of a disk rotating about its axis. As the disk turns, not all points move with the same speed since, to make one rotation, a point at the edge must move farther than one near the axis and the points move these different distances in the same time. In Fig. 1 the point A has a greater speed than B, and B greater than C.

If we consider the line ABC rather than the points, we notice that the line turns as a whole about the axis. In 1 sec it will turn through a certain *angle* shown by the shaded area. The angle turned through per unit time is called the *angular speed.*

$$\bar{\omega} = \frac{\theta}{t}$$

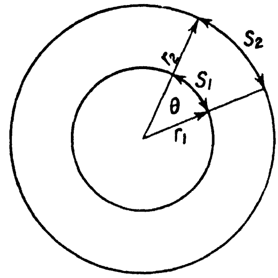


FIG. 2.—The ratio of arc to radius is a measure of the angle. $\theta = \frac{s_1}{r_1} = \frac{s_2}{r_2}$.

where $\bar{\omega}$ (omega) is the average angular speed and θ (theta) is the angle turned through in time t. The angle may be expressed in degrees, in revolutions (1 rev = 360°), or in *radians.* The latter unit is very convenient because of the simple relation between angular motion and the linear motion of the points.

In Fig. 2 is shown an angle with its apex at the common center of two circles. The length of arc cut from the circle depends upon the length of the radius. The ratio of arc to radius is the same for both the circles. This may be used as a measure of the angle

$$\theta = \frac{s}{r}$$

where s is the length of the arc and r is the radius. The unit of angle in this system is the *radian,* which is the angle whose arc is equal to the radius. The length of the circumference is $2\pi r$. Hence

$$360° = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

$$1 \text{ radian} = \frac{360°}{2\pi} = 57.3° \text{ (approximately)}$$

As in the case of linear motion, angular motion may be uniform or accelerated. *Angular acceleration* α (alpha) is the rate of change of angular velocity

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

where ω_1 is the initial and ω_2 the final angular velocity.

In studying uniformly accelerated angular motion, we need five equations similar to those used for uniformly accelerated linear motion:

$$\theta = \bar{\omega}t \tag{6}$$
$$\bar{\omega} = \tfrac{1}{2}(\omega_1 + \omega_2) \tag{7}$$
$$\omega_2 - \omega_1 = \alpha t \tag{8}$$
$$\theta = \omega_1 t + \tfrac{1}{2}\alpha t^2 \tag{9}$$
$$\omega_2^2 - \omega_1^2 = 2\alpha\theta \tag{10}$$

Note that these equations are identical with Eq. (1) to (5) if θ is substituted for s, ω for v, and α for a. These equations hold whatever the angular measure may be, as long as the same measure is used in a single problem. However, only when radian measure is used is there the simple relationship between angular and linear motions given by the equations

$$s = \theta r \tag{11}$$

$$v = \omega r \tag{12}$$

$$a = \alpha r \tag{13}$$

Example: A flywheel revolving at 200 rpm slows down at a constant rate of 2.0 radians/sec². What time is required to stop the flywheel and how many revolutions does it make in the process?

$$\omega_1 = 200 \text{ rpm} = 200(2\pi) \text{ radians/min} = \frac{200(2\pi)}{60} \text{ radians/sec}$$

$$\omega_2 = 0$$

$$\alpha = -2.0 \text{ radians/sec}^2$$

Substituting in Eq. (8)

$$0 - \frac{400\pi}{60} \text{ radians/sec} = (-2.0 \text{ radians/sec}^2)t$$

$$t = 10.5 \text{ sec}$$

Substituting in Eq. (10)

$$0 - \left(\frac{400\pi}{60} \text{ radians/sec}\right)^2 = 2(-2.0 \text{ radians/sec}^2)\theta$$

$$\theta = 110 \text{ radians} = \frac{110}{2\pi} \text{ rev} = 17.5 \text{ rev}$$

SUMMARY

A statement of velocity must specify the direction as well as the speed, for example, 25 mi/hr east, 30 ft/sec southwest.

Acceleration is the rate of change of velocity.

The equations of uniformly accelerated motion have been given for the particular case in which the direction of the motion remains fixed and the speed changes uniformly.

A freely falling body is one that is acted on by no forces of appreciable magnitude other than the force of gravity.

The acceleration of a freely falling body is, at sea level and 45° latitude, 32.17 ft/sec², or 980.6 cm/sec².

The *terminal speed* of a falling object is the vertical speed at which the force of air resistance is just sufficient to neutralize its weight.

For a rotating body the *angular speed* is the angle turned through per unit time by a line that passes through the axis of rotation.

Angular distance, in radians, is the ratio of the arc to its radius.

A *radian* is the angle whose arc is equal to the radius.

Angular acceleration is the rate of change of angular velocity.

Equations of uniformly accelerated angular motion are similar to those for linear motion with angle substituted for distance, angular speed for linear speed, and angular acceleration for linear acceleration.

QUESTIONS AND PROBLEMS

(Use $g = 32$ ft/sec^2 or 980 cm/sec^2; neglect air resistance.)

1. State the relationship between inches and centimeters; centimeters and feet; pounds and kilograms; kilometers and miles.

2. When a batter struck a ball, its velocity changed from 150 ft/sec west to 150 ft/sec east. What was (*a*) the change in speed? (*b*) the change in velocity?

3. A car changes its speed from 20 mi/hr to 30 mi/hr in 5 sec. Express the acceleration in miles per hour per second, feet per minute per second, and feet per second per second.

4. The initial speed of a car having excellent brakes was 30 mi/hr (44 ft/sec). When the brakes were applied it stopped in 2 sec. Find the acceleration and the stopping distance. *Ans.* -22 ft/sec^2; 44 ft.

5. An automobile starts from rest and accelerates 2 m/sec^2. How far will it travel during the third second?

6. A baseball is thrown downward from the top of a cliff 500 ft high with an initial speed of 100 ft/sec. What will be the speed after 3 sec?
Ans. 196 ft/sec.

7. A stone is thrown vertically upward with an initial speed of 96 ft/sec. (*a*) How long does it continue to rise? (*b*) How high does it rise?

8. How much time is required for the baseball of problem 6 to reach the ground? *Ans.* 3.3 sec.

9. What vertical speed will cause a ball to rise just 16 ft? 64 ft? 490 cm?

10. A pulley 18 in. in diameter makes 300 rpm. What is the linear speed of the belt if there is no slippage? *Ans.* $1,\bar{4}00$ ft/min.

11. The belt of problem 10 passes over a second pulley. What must be the diameter of this pulley if its shaft turns at the rate of 400 rpm?

12. A shaft 6 in. in diameter is to be turned in a lathe with a surface linear speed of 180 ft/min. What is its angular speed? *Ans.* $7\bar{2}0$ radians/min.

13. A flywheel is brought from rest to a speed of 60 rpm in ½ min. What is the angular acceleration? What is the angular speed at the end of 15 sec?

14. A wheel has its speed increased from 120 rpm to 240 rpm in 20 sec. What is the angular acceleration? How many revolutions of the wheel are required?
Ans. 0.63 radians/sec^2; 60 rev.

EXPERIMENT

Uniformly Accelerated Motion

Apparatus: Metronome; two grooved inclined planes; marble; supports for the planes; meter stick.

Uniformly accelerated motion has the following characteristics: (1) For motion starting from rest the distance traversed is directly proportional to the time squared. (2) The speed attained is directly proportional to the time. (3) The acceleration is constant.

Presumably, a marble rolling down an inclined plane (Fig. 3) has uniformly accelerated motion. We can be sure of this if its motion has the three characteristics just set forth.

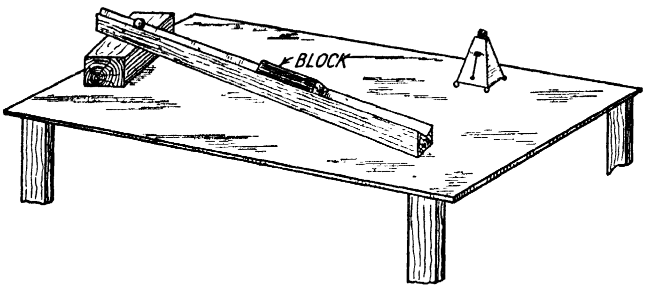


FIG. 3.—Apparatus for experiment on uniformly accelerated motion.

To study the distance-time relation we must measure distance and time intervals. The latter we shall measure in terms of a time unit Δt, which we shall take to be the time between successive ticks of the metronome. The metronome has a scale by which its frequency can be set at a desired value. A frequency of 80 to 90 ticks per minute is satisfactory for this experiment. The distances we shall be interested in are those passed over by a marble rolling down a grooved plane (Fig. 3) in any desired time interval equal to $n\,\Delta t$.

The marble is released near the top of the incline (at a point marked by chalk) at the instant of one tick. Its position after any desired number of time units can be observed by letting the marble strike a heavy block resting in the groove at such a location that the sound of the impact coincides with the tick of the metronome. Usually several trials will be required to determine the proper position of the block. The distance to be measured is that from the starting point to the block.

It is not easy to judge coincidences of clicks, especially after only one or two time intervals, so that it will be found easier to determine the longer distances first.

Table II will be helpful in recording and interpreting data:

TABLE II

Number of time units, n	Mean distances, s_n	n^2	Ratio, s_n/n^2
5	$s_5 =$	25	
4	$s_4 =$	16	
3	$s_3 =$	9	
2	$s_2 =$	4	
1	$s_1 =$	1	

If the relation (1) is true for this motion, there will be a direct proportion between the numbers in the second and third columns of Table II. This means that their ratios, appearing in the fourth column, are constant.

To study the speed-time relationship we shall need two inclines, *A* and *B*, Fig. 4. Incline *B* is nearly level. It should be elevated at one end so that when a marble on it is given a certain speed, it will maintain that speed (without speeding up or slowing down) until it reaches the end of *B*. This means that *B* is tilted just enough to overcome friction, so that it is "level" for practical purposes. If, therefore, a marble starts at point *a* on *A* (Fig. 4), it picks up speed until it arrives at *b*, then it will roll from *b* to *c* at the speed it attained before reaching *b*.

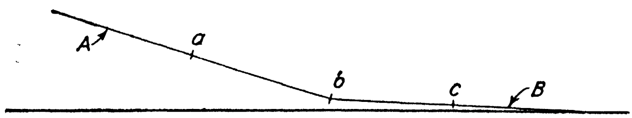


FIG. 4.—Arrangement of grooved inclined planes.

In order to compute the speed with which the marble reaches *b*, it is necessary to determine the distance $\overline{bc}$ which the marble travels along *B* in one time unit Δt. This can be done by releasing the marble at such a position (on *A*) that it reaches *b* in coincidence with one click of the metronome and strikes a block placed on *B* (at *c*) in coincidence with the next click. In this case the distance $\overline{bc}$ is numerically equal to the speed of the marble (while on *B*) expressed in the units $cm/\Delta t$.

The student will find it advantageous to develop a bit of rhythm. Count one, two, three, four, five, etc., audibly with the metronome clicks. After doing this several times one develops a feeling for the timing.

While counting audibly with the metronome, one should release the marble on the count of three. If it is released at the distance s_3 from *b* and the block on *B* is properly placed, the marble should click across *b* on the count of six and should hit the block on the count of seven. In this way the marble is released more accurately at the desired instant because the student can anticipate the time of release. Likewise, he can easily judge coincidences of clicks for the same reason.

Data should be recorded in Table III.

In this table *n* is again the number of time units required for the motion from *a* to *b*, indicating the time required to develop the speed which is determined from the distance $\overline{bc}$ on *B*. The distances in the first column may be taken from Table II. There should be a direct proportion between the numbers in the second and third columns, as indicated by the constant ratios of the fourth column.

The results for column 5 are obtained by subtracting successive values of v_n. These differences $v_5 - v_4$, $v_4 - v_3$, $v_3 - v_2$, $v_2 - v_1$, should be constant, because they equal numerically the acceleration in $cm/\Delta t^2$.

TABLE III

$\overline{ab} = s_n$	n	$\overline{bc} = v_n$ (numerically)	Ratio, v/n	$v_n - v_{n-1}$
	5			
	4			
	3			
	2			
	1			

Great care should be taken to "fit" the adjacent ends of A and B so that the marble does not "jump" at b. Its motion from A to B should be smooth. One end of A (and of B) is beveled slightly to facilitate such fitting. In order to release the marble without imparting any initial speed to it, one should hold it with a light object rather than with the hand.

CHAPTER 11

FORCE AND MOTION

The relation of force to motion was first stated in a comprehensive manner by Sir Isaac Newton, one of the greatest of all scientists, who combined the results of his many diverse observations in mechanics into three fundamental laws, now known as Newton's laws of motion. These laws will be presented in a form consistent with the current terminology of science.

First Law of Motion. *A body at rest remains at rest, and a body in motion continues to move at constant speed in a straight line, unless acted upon by an external, unbalanced force.*

The first part of the law is known from everyday experience; for example, a book placed on a table remains at rest. Though one might be inclined to conclude that the book remains at rest because no force at all acts on it, the realization that the force of gravity is acting on the book leads one to the conclusion that the table exerts a force just sufficient to support the weight of the book. The book remains at rest, therefore, because no *unbalanced* force acts on it.

The second part of the law, which indicates that a body set in motion and then left to itself will keep on moving without further action of a force, is more difficult to visualize. Objects do not ordinarily continue their motion indefinitely when freed from a driving force, because a frictional retarding force always accompanies a motion.

A block of wood thrown along a concrete surface slides only a short distance, because the frictional resistance is great; on a smooth floor it would slide farther; and on ice it would slide a much greater distance. From these examples it appears reasonable that, if friction could be entirely eliminated, a body set in motion on a level surface would continue indefinitely at constant velocity. It is assumed, therefore, that uniform motion is a natural condition requiring no driving force unless resistance to the motion is encountered.

Suppose a dog drags a sled along the ground at constant speed by exerting on it a horizontal force of 50 lb. Since the speed is constant, there must be no unbalanced force on the sled; hence the ground must be exerting a backward force of 50 lb on the sled. The initial force necessary to start the sled is more than 50 lb, for an unbalanced force is required to impart a motion to it. Once the sled is moving, the driving force must be reduced to the value of the retarding force in order to eliminate the acceleration and allow the sled to move at a constant speed.

The acceleration of an object is zero whether it is at rest or moving at constant speed in a straight line; that is, the acceleration of an object is zero unless an unbalanced force is acting on it.

Second Law of Motion. *An unbalanced force acting on a body produces an acceleration in the direction of the force, an acceleration which is directly proportional to the force and inversely proportional to the mass of the body.*

According to the second law, then, the following proportions may be written:

$$a \propto f$$

and

$$a \propto \frac{1}{m}$$

or combined in the forms

$$a \propto \frac{f}{m}$$

and

$$f \propto ma$$

It is common experience that, of two identical objects, the one acted upon by the larger force will experience the greater acceleration. Again, there is no doubt that equal forces applied to objects of unequal mass will produce unequal accelerations, the object of smaller mass having the larger acceleration. It is assumed, of course, either that retarding forces do not exist or that extra force is exerted to eliminate their effect.

These examples illustrate the second law in a qualitative way. More refined, quantitative experiments verify the existence of a direct proportion between force and acceleration, and an inverse proportion between mass and acceleration.

Third Law of Motion. *For every acting force there is an equal and opposite reacting force.* Here the term *acting force* means the force that one body exerts on a second one, while *reacting force* means the force that the second body exerts on the first. It should be remembered that action and reaction, though equal and opposite, can *never* neutralize each other, for they always act on *different* objects. In order for two equal and opposite forces to neutralize each other, they must act on the same object.

A baseball exerts a reaction against a bat which is exactly equal (and opposite) to the force exerted by the bat on the ball. In throwing a light object one has the feeling that he cannot put much effort into the throw, for he cannot exert any more force on the object thrown than it exerts in reaction against his hand. This reaction is proportional to the mass of the object ($f \propto m$) and to the acceleration ($f \propto a$). The thrower's arm must be accelerated along with the object thrown, hence the larger part of the effort exerted in throwing a light object is expended in "throwing" one's arm.

When one steps from a small boat to the shore, he observes that the boat is pushed away as he steps. The force he exerts on the boat is responsible for its motion; while the force of reaction, exerted by the boat on him, is responsible for his motion toward the shore. The two forces are equal and opposite, while the accelerations which they produce (in boat and passenger, respectively) are inversely proportional to the masses of the objects on which they act. Thus a large boat will move only a small amount when one steps from it to shore.

A book lying on a table is attracted by the earth. At the same time it attracts the earth, so that they would be accelerated toward each other if the table were not between them. In attempting to move, each exerts a force on the table, and, in reaction, the table exerts an outward force on each of them, keeping them apart. It is interesting to note that the table exerts outward forces on the book and the earth by virtue of being slightly compressed by the pair of inward forces which they exert on it.

The Force Equation. Suppose that an object is given an acceleration a

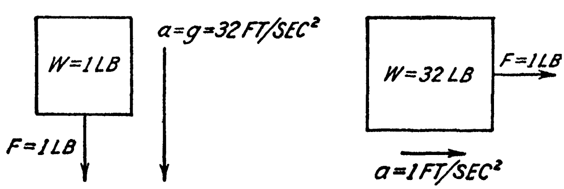


FIG. 1.—$F = \frac{W}{g} a$

by a force F. The weight W of the object is sufficient to give it an acceleration g (32 ft/sec²). Therefore, since the acceleration is proportional to the force causing it,

$$\frac{a}{g} = \frac{F}{W}$$

so that

$$F = \frac{W}{g} a \tag{1}$$

This indicates that the force necessary to produce a given acceleration is just W/g times that acceleration, if W is the weight of the object being accelerated.

Example: Find the force necessary to accelerate a 100-lb object 5.0 ft/sec².

$$F = \frac{W}{g} a = \frac{100 \text{ lb}}{32 \text{ ft/sec}^2} 5.0 \text{ ft/sec}^2 = 16 \text{ lb}$$

Example: A $2\bar{0}0$-gm object is to be given an acceleration of 20 cm/sec². What force is required?

$$F = \frac{W}{g} a = \frac{2\bar{0}0 \text{ gm}}{980 \text{ cm/sec}^2} 20 \text{ cm/sec}^2 = 4.1 \text{ gm}$$

The Dyne. The second law of motion leads to the relation

$$a \propto \frac{f}{m}$$

or

$$f \propto ma$$

Units have already been defined for mass (the gram) and acceleration (centimeter per second per second). If we select a unit of force properly, we can change the above proportion to an equality. As this unit of force we select the force that will cause unit acceleration in a unit mass and to it we give the name *dyne*. A dyne is the force that will give a mass of one gram an acceleration of one centimeter per second per second. *Whenever this set of units is used*

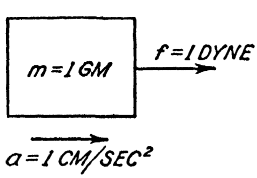


FIG. 2.—$f = ma$.

$$f = ma \tag{2}$$

Since the weight of a 1-gm object is sufficient to cause an acceleration of 980 cm/sec², such an object weighs 980 dynes. A dyne is seen to be a very small force; approximately one one-thousandth the weight of a gram.

Whenever the metric system of units is used in problems involving Newton's second law, Eq. (2) is used. In using Eq. (2) the force must be expressed in dynes (gm × 980 cm/sec²), the mass in grams and the acceleration in centimeters per second per second.

Example: A force of 500 dynes is applied to a mass of 175 gm. In what time will it acquire a speed of 30.0 cm/sec?

$$f = ma$$

hence

$$a = \frac{f}{m} = \frac{500 \text{ dynes}}{175 \text{ gm}} = 2.86 \text{ cm/sec}^2$$

(NOTE: dyne/gm = cm/sec^2)

$$v_2 - v_1 = at$$

where $v_2 = 30.0$ cm/sec, $v_1 = 0$, and $a = 2.86$ cm/sec^2. Then

$$t = \frac{v_2 - v_1}{a} = \frac{30.0 \text{ cm/sec}}{2.86 \text{ cm/sec}^2} = 10.5 \text{ sec}$$

If the force used in Eq. (2) is the weight W of the body, then the acceleration produced is the acceleration due to gravity g.

$$W = mg$$

or

$$m = \frac{W}{g}$$

If W/g is substituted for m in Eq. (2), this equation is found to be the same as Eq. (1). The choice of the equation to use is determined by the units in which the result is to be expressed. In Eq. (1) both F and W are commonly expressed in pounds; in Eq. (2) the force f is expressed in dynes and the mass m in grams.

SUMMARY

Newton's laws of motion:

1. A body at rest remains at rest, and a body in motion continues to move at constant speed in a straight line, unless acted upon by an external, unbalanced force.

2. An unbalanced force acting on a body produces an acceleration in the direction of the force, an acceleration which is directly proportional to the force and inversely proportional to the mass of the body.

3. For every acting force there is an equal and opposite reacting force.

In the equation $F = \frac{W}{g}a$, F and W must be expressed in the same unit of force, a and g in the same units of acceleration. The quantities F and W are most commonly expressed in pounds of force.

A *dyne* is the force that will impart to a 1-gm mass an acceleration of 1 cm/sec^2.

In the equation $f = ma$, f can be expressed in dynes, m in grams, and a in centimeters per second per second.

QUESTIONS AND PROBLEMS

1. Consider an object on a frictionless plane.

a. If the mass is 1 gm and the force 1 dyne, the acceleration is ______.

b. If the mass is 1 gm and the force 5 dynes, the acceleration is ______.

c. If the mass is 5 gm and the force 10 dynes, the acceleration is ______.

d. If the weight is 32 lb and the force 1 lb, the acceleration is ______.
e. If the weight is 320 lb and the force 20 lb, the acceleration is ______.
f. If the weight is 500 lb and the force 10 lb, the acceleration is ______.
g. If the mass is 10.0 gm and the force 9,800 dynes, the acceleration is ______.

2. Does the seat on a roller coaster always support exactly the weight of the passenger? Explain.

3. A 160-lb object is subjected to a constant force of 50 lb. How much time will be required for it to acquire a speed of 80 ft/sec?

4. What force will impart a speed of 40 ft/sec to a 640-lb body in 5.0 sec?
Ans. $1\bar{6}0$ lb.

5. A 500-lb projectile acquires a speed of 2,000 ft/sec while traversing a cannon barrel 16.0 ft long. Find the average acceleration and accelerating force.

6. A rifle bullet (mass 10 gm) acquires a speed of 400 m/sec in traversing a barrel 50 cm long. Find the average acceleration and accelerating force.
Ans. 1.6×10^7 cm/sec²; 1.6×10^8 dynes.

7. A 200-lb man stands in an elevator. What force does the floor exert on him when the elevator is (*a*) stationary; (*b*) accelerating upward 16.0 ft/sec²; (*c*) moving upward at constant speed; (*d*) decelerating at 12.0 ft/sec²?

8. A 1,000-gm block on a smooth table is connected to a 500-gm piece of lead by a light cord which passes over a small pulley at the end of the table. What is the acceleration of the system? What is the tension in the cord?
Ans. 327 cm/sec²; 333 gm.

9. If the gun used to fire the bullet of problem 6 has a mass of 2,000 gm, what will be the acceleration with which it recoils?

EXPERIMENT

Newton's Second Law of Motion

Apparatus: Hall's carriage; 2 pulleys; slotted weights; weight hanger; string; 19-mm rod; oil; clamps; metronome.

Newton's second law of motion asserts (1) that when a constant, unbalanced force acts on a body, the body moves with uniform acceleration and (2) that, for a given mass, the acceleration is directly proportional to the unbalanced force.

In the preceding experiment, we learned how to recognize uniformly accelerated motion. We shall use this method in verifying the fact that a constant (unbalanced) force causes a uniform acceleration.

Figure 3 illustrates the apparatus. The car C, containing objects whose masses total m_1, is propelled by the cord S, to which is attached an object of mass m_2. The small pulleys P_1 and P_2 have little friction and rotational inertia. Pulley P_2 should be mounted far above P_1 in order to provide for a large distance of fall of m_2.

Neglecting frictional forces, we may assume that the car C and the masses m_1 and m_2 are accelerated by the weight of m_2. The total mass accelerated by this force is $M = m_0 + m_1 + m_2$, where m_0 is the mass of the car. Will the system move with uniform acceleration?

To answer this question make use of the technique developed in the preceding experiment. Adjust the metronome so that it ticks 80 times per minute. See that the wheel bearings of the car are well oiled so

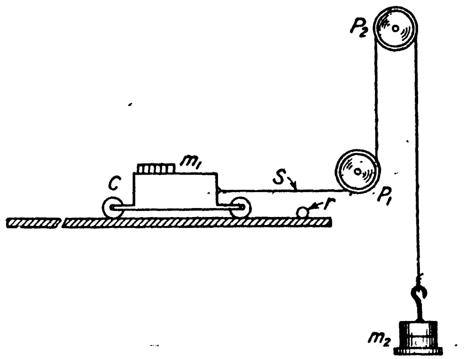


Fig. 3.—Apparatus for demonstration of Newton's second law.

that friction is reduced. In order further to reduce the effect of friction, disconnect the cord and tilt the table so that the car will continue to move uniformly after it is started.

A rod or block r should be placed on the table to stop the car at the proper place. When the rod is correctly located, the sound of the impact of the car with it will be coincident with a click of the metronome. A convenient value of M is roughly 1,500 gm, in which case m_2 can first be used as 80 gm.

By means of repeated trials adjust the position of the rod until the car strikes it one time interval Δt after it is released. Record the distances from the starting point and the number of intervals n in Table I. Repeat

TABLE I

s	n	n^2	s/n^2	$a = 2s/n^2$

the observations for two and three time intervals. Is the distance proportional to the square of the time? Is the acceleration constant? If it is, compute its value. Since the car starts from rest, the distance it travels in a time t is given by

$$s = 0 + \frac{1}{2}at^2, \qquad \text{or} \qquad a = \frac{2s}{t^2}$$

(Note: a will be expressed in cm/$(\Delta t)^2$ if Δt is used as the unit of time.)

Next, change the force, keeping the total mass constant. This may be done by taking some of the mass out of the car and adding it to m_2. In this way the accelerating force is increased, while the total mass in motion remains the same. According to Newton's second law, the acceleration produced should be proportional to the accelerating force.

In Table II record data from the second set of observations taken above. Transfer mass from the car to m_2 to obtain a greater force, and adjust the bar until the car strikes it at the end of the second time interval. Repeat this procedure for a third force. For each force compute the acceleration. Is the acceleration proportional to the applied force?

TABLE II

m_1	$F = m_2$ (numerically)	M	s	a	F/a

CHAPTER 12

FRICTION; WORK AND ENERGY

When there is relative motion between two surfaces that are in contact, frictional forces oppose that motion. These forces are caused by the adhesion of one surface to the other and by the interlocking of the irregularities of the rubbing surfaces. The force of frictional resistance depends upon the properties of the surfaces and the force pushing one against the other.

The effects of friction are both advantageous and disadvantageous. Friction increases the work necessary to operate machinery, it causes wear, and it generates heat, which often does additional damage. To reduce this waste of energy attempts are made to reduce friction by the use of wheels, bearings, rollers, and lubricants. Automobiles and airplanes are streamlined in order to decrease air friction, which is large at high speeds.

On the other hand, friction is desirable in many cases. Nails and screws hold boards together by means of friction. Sand is placed on the rails in front of the drive wheels of locomotives, cinders are scattered on icy streets, chains are attached to the wheels of autos, and special materials are developed for use in brakes—all for the purpose of increasing friction where it is desirable.

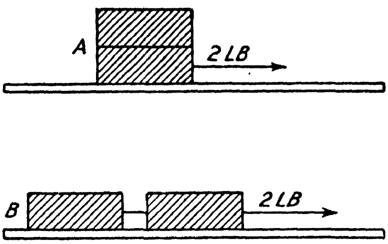


FIG. 1.—Sliding friction is independent of area.

Four kinds of friction are commonly differentiated: starting friction; sliding friction, which occurs when surfaces are rubbed together; rolling friction; and fluid friction, the molecular friction of liquids and gases.

Frictional Forces. When an object is dragged across a table, more force is required to start the motion than to maintain it; hence we say that starting friction is greater than sliding friction. At low speeds the frictional resistance is practically independent of speed. The heat generated at high speeds changes the properties of the surfaces enough to cause an appreciable reduction in the force of friction.

A surprising property of friction is the fact that the force required to overcome sliding friction is (within limits) independent of the area of contact of the rubbing surfaces. This is illustrated in Fig. 1*A* and *B*, where the area of contact is doubled without changing the frictional force.

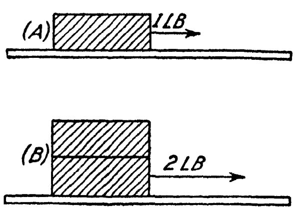


FIG. 2.—The frictional force is directly proportional to the normal force pressing the two surfaces together.

The most important principle of friction is the fact that the force required to overcome sliding friction is directly proportional to the perpendicular force pressing the surfaces together. In Fig. 2 it is seen that the force required to drag a single block is 1 lb, whereas a force of 2 lb is required to drag the block when a similar one is placed on top of it to double the perpendicular force between the rubbing surfaces.

Coefficient of Friction. The ratio of the frictional force to the perpendicular force pressing the two surfaces together is called the *coefficient of friction*. Thus

$$\mu = \frac{F}{N} \tag{1}$$

or

$$F = \mu N$$

where μ is the coefficient of friction, F the force overcoming friction, and N the normal or perpendicular force.

Example: A 65-lb force is sufficient to drag horizontally a 1,200-lb sled on well-packed snow. What is the value of the coefficient of friction?

$$\mu = \frac{65 \text{ lb}}{1{,}200 \text{ lb}} = 0.054.$$

Rolling Friction. Rolling friction is caused by the deformation produced where a wheel or cylinder pushes against the surface on which it rolls. Rolling friction is ordinarily much smaller than sliding friction. Sliding friction at the axle of a wheel is replaced by rolling friction through the use of roller or ball bearings.

Fluid Friction. The friction encountered by solid objects in passing through liquids, and the frictional forces set up within liquids and gases in motion, are examples of fluid friction. The laws of fluid friction differ greatly from those of sliding and rolling friction, for the amount of frictional resistance depends upon the size, shape, and speed of the moving object, as well as on the viscosity of the fluid itself. The frictional resistance encountered by an object moving through a fluid increases greatly with speed; so much so, in fact, that doubling the speed of a boat often increases the fuel consumption per mile by three or four times. The existence of terminal speeds for falling bodies is another result of this increase in fluid friction with speed.

Work. The term *work*, commonly used in connection with innumerable and widely different activities, is restricted in physics to the case in which work is performed by exerting a force that causes a displacement. Quantity of work is defined as the product of the force and the displacement in the direction of the force.

$$\text{Work} = Fs \qquad (2)$$

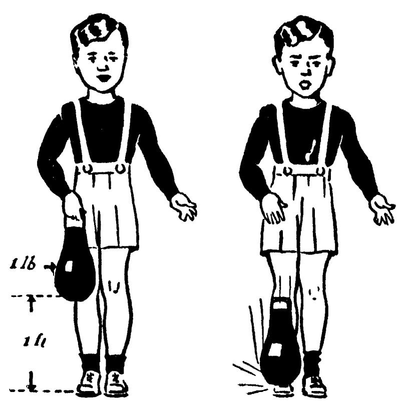


FIG. 3.—One foot-pound.

In the British system, the unit of work is the *foot-pound*, the work done by a force of 1 lb exerted through a distance of 1 ft. In the metric system, work is ordinarily expressed in terms of the *erg* (dyne-centimeter), which is the work done by a force of 1 dyne exerted through a distance of 1 cm. Other units of work are the gram-centimeter and the joule (10^7 ergs).

Example: The sled of the preceding example is dragged a distance of 50 ft. How much work is done?

$$\text{Work} = Fs$$
$$\text{Work} = (65 \text{ lb})(50 \text{ ft}) = 3{,}200 \text{ ft-lb}$$

Energy. The capacity for doing work is called *energy*. Though energy can be neither created nor destroyed, it can exist in many forms and can be transformed from one form to another. The energy possessed by an object by virtue of its motion is called *kinetic energy*, or energy of motion. Energy of position or configuration is called *potential energy*.

Potential Energy. When a man carries a brick to the top of a building, most of the energy that he expends is transformed into heat through friction in the muscles of his body. The work that he accomplishes on the brick (weight of brick times vertical distance) represents energy that can be recovered. By virtue of its position at the top of the building, the brick possesses a capacity for doing work, or potential energy. If allowed to fall, it will gain kinetic energy (energy of motion) as rapidly as it loses potential energy (energy of position) except for the small amount of energy consumed in overcoming the frictional resistance of the air. When the brick strikes the ground, therefore, it will expend in collision an amount of energy nearly equal to the potential energy it had when at the top of the building. This energy is transformed into heat in the collision with the ground.

Example: A 20-lb stone is carried to the top of a building 100 ft high. How much does its potential energy (PE) increase?

It is, neglecting friction, just the amount of work done in lifting the stone, so that

$$PE = Fs = (20 \text{ lb})(100 \text{ ft}) = 2{,}000 \text{ ft-lb}$$

When a spring or a rubber band is stretched, the energy expended in stretching it is converted into potential energy, energy which the spring is capable of giving up because the molecules of which it is composed have been pulled out of their natural pattern and will exert force in order to get back into that pattern. This energy of position should not be thought of as a substance within the spring, but as a condition.

Gasoline possesses potential energy by virtue of the arrangement of the molecules of which it is composed. In an internal-combustion engine, this potential energy of configuration is released through the burning of the gasoline. Most of the energy is transformed into heat, but a portion is converted into useful mechanical work. Even the latter finally takes the form of heat as a result of friction.

Mechanical Equivalent of Heat. Since energy expended in overcoming friction is converted into heat energy, it is not difficult to make measurements of the mechanical equivalent of heat, or the amount of mechanical work necessary to produce unit quantity of heat. It has been found that the expenditure of 778 ft-lb of work against friction is sufficient to produce 1 Btu of heat. Similarly, 4.183 joules (4.183×10^7 ergs) can be converted into 1 cal of heat.

Example: What amount of heat will be produced by the stone of the preceding example if it is allowed to fall to the ground?

$$H = \frac{2,\bar{0}00 \text{ ft-lb}}{778 \text{ ft-lb/Btu}} = 2.6 \text{ Btu}$$

Kinetic Energy. The kinetic energy (KE) of a moving object is the amount of energy it will give up in being stopped. In terms of the weight W of the object and its speed v

$$KE = \frac{1}{2}\frac{W}{g}v^2 \tag{3}$$

or, since $W/g = m$,

$$KE = \tfrac{1}{2}mv^2$$

If a moving body is stopped by a uniform force F, the work done in stopping must be equal to the kinetic energy

$$Fs = \frac{1}{2}\frac{W}{g}v^2 \tag{4}$$

where s is the distance required to stop the object.

Example: What is the kinetic energy of a 3,000-lb automobile which is moving at 30 mi/hr (44 ft/sec)?

$$KE = \frac{1}{2}\frac{W}{g}v^2 = \frac{1}{2}\frac{(3{,}000 \text{ lb})(44 \text{ ft/sec})^2}{32 \text{ ft/sec}^2} = 9\bar{1},000 \text{ ft-lb}$$

Stopping Distance. The fact that the kinetic energy of a moving object is proportional to the square of its speed has an important bearing upon the problem of stopping an automobile. Doubling the speed of the car quadruples the amount of work that must be done by the brakes in making a quick stop.

A consideration of the equation $v_2^2 - v_1^2 = 2as$ shows that, for $v_2 = 0$ (indicating a stop), $s = -v_1^2/2a$, so that the distance in which an automobile can be stopped is likewise proportional to the square of the speed, assuming a constant deceleration. Actually, however, the deceleration accomplished by the brakes is smaller at high speed because of the effect of heat upon the brake linings, so that the increase in stopping distance with speed is even more rapid than is indicated by theoretical considerations.

Example: In what distance can a 3,000-lb automobile be stopped from a speed of 30 mi/hr (44 ft/sec) if the coefficient of friction between tires and roadway is 0.70?

The retarding force furnished by the roadway can be no greater than

$$F = \mu N = (0.70)(3000 \text{ lb}) = 2,\bar{1}00 \text{ lb}$$

Since the work done against this force is equal to the kinetic energy of the car, the stopping distance can be found by substituting in the equation:

$$Fs = \frac{1}{2}\frac{W}{g}v^2$$

$$s = \frac{1}{2}\frac{W}{g}\frac{v^2}{F} = \frac{1}{2}\frac{(3{,}000 \text{ lb})(44 \text{ ft/sec})^2}{(32 \text{ ft/sec}^2)(2{,}100 \text{ lb})} = 43 \text{ ft}$$

Table I shows stopping distances for various speeds, assuming the conditions of the preceding example.

TABLE I

Speed, mi/hr	Stopping Distance, ft
10	4.8
20	19
30	43
40	76
50	$1\bar{2}0$
60	$1\bar{7}0$
70	$2\bar{3}0$
80	$3\bar{1}0$
90	$3\bar{9}0$

The value of the coefficient of friction for rubber on dry concrete is considerably larger than 0.70, the figure assumed; but if the wheels are locked and the tires begin to slip, the rubber melts, and the coefficient of friction becomes much smaller. At the same time it should be remembered that at high speeds the efficiency of the brakes is greatly reduced by the heat developed in the brake linings. In practice, then, an automobile with excellent brakes can often be stopped in shorter distances than those indicated for 10 and 20 mi/hr; whereas at the higher speeds, 60 to 90 mi/hr, the actual stopping distance is several times as large as the theoretical value. At 90 mi/hr, for example, a distance of 1,000 to 1,500 ft (instead of the theoretical value of 390 feet) is required for stopping if the brakes alone are used. The decelerating effect of the motor often exceeds that of the brakes at very high speeds.

The distance in which a freely falling body acquires a speed of 60 mi/hr is 120 ft. In order to stop an automobile which has this speed, therefore, the brakes must dissipate the same energy the automobile would acquire in falling from the top of a 120-ft building.

SUMMARY

$F = \mu N$, where F is the force overcoming friction, μ the coefficient of friction, and N the normal (perpendicular) force.

Work is the product of force and displacement in the direction of the force.

$$\text{Work} = Fs$$

Energy is the capacity for doing work.

According to the *conservation-of-energy principle,* energy can be neither created nor destroyed, only transformed.

Kinetic energy is energy of motion.

$$KE = \frac{1}{2}\frac{W}{g}v^2$$

Potential energy is energy of position or configuration.

QUESTIONS AND PROBLEMS

1. A force of 155 lb is required to start a sled whose weight is 800 lb, while a force of 54 lb is sufficient to keep it moving once it is started. Find the coefficients of starting and sliding friction.

2. A 500-lb piano is moved 20 ft across a floor by a horizontal force of 75 lb. Find the coefficient of friction and the amount of work accomplished. What happens to the energy expended? *Ans.* 0.15; $1,\bar{5}00$ ft-lb.

3. How much work does a 160-lb man do against gravity in climbing a flight of stairs between floors 12 ft apart? Does this account for all of the energy expended?

4. Find the work done in removing 300 gal of water from a coal mine 400 ft deep. *Ans.* $1,0\bar{0}\bar{0},000$ ft-lb, or 1.00×10^6 ft-lb.

5. What is the kinetic energy of a 2,000-lb automobile moving 30 mi/hr? How much heat is produced when it stops?

6. A horizontal force of 6.0 lb is applied to a 10-lb block, which rests on a horizontal surface. If the coefficient of friction is 0.40, find the acceleration. *Ans.* 6.4 ft/sec^2.

7. From how high must a piece of ice be dropped in order to be just melted by friction and the heat of impact?

8. The heat of combustion of canned salmon is 363 Btu/lb. Assuming 30 per cent of this heat is useful in producing bodily energy, how much canned salmon should you eat to lift yourself 100 ft? *Ans.* 0.0012 times your weight.

9. A 100-lb stone is dropped from a height of 200 ft. Find its kinetic and potential energies at 0, 1, and 2 sec after being released, and also upon striking the ground. Notice that the sum of the potential and kinetic energies is constant.

EXPERIMENT

Friction

Apparatus: Friction board; pulley; cord; friction blocks; weights; weight hanger; glass plate; oil.

This experiment is intended to show that (1) the starting frictional force between two solid surfaces is greater than the sliding frictional force; (2) the latter is independent of speed, provided the speed is not excessive; (3) the frictional force depends upon how hard the surfaces are pushed together, that is, upon the perpendicular force between them; (4) it depends upon the nature and condition of the surface; (5) it is nearly independent of the area of the surfaces, unless they are so small as to approximate points or sharp edges. Also we shall observe the effects of wet surfaces, oily surfaces, etc., upon frictional forces.

Use will be made of friction blocks of different materials, on a friction board. Each block has holes to receive weights, and hooks to which cords may be attached. The board upon which the block slides is rather rough on one side, and smoothly sandpapered on the other. Figure 4 illustrates the experimental setup.

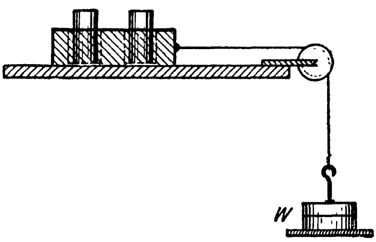


FIG. 4.—Apparatus for measuring coefficient of friction.

To obtain the coefficient of *starting friction*, place slotted weights upon the weight holder until the force F is just sufficient to start the block. Record the values of F and the normal force N, and compute the coefficient of starting friction. Repeat this procedure for (*a*) a different normal force, (*b*) a different friction block, (*c*) the other side of the friction board. Record the data in Table II. How does the frictional force F depend upon N? How does it depend upon the condition of the surfaces? How does the coefficient of friction μ depend upon these factors?

TABLE II

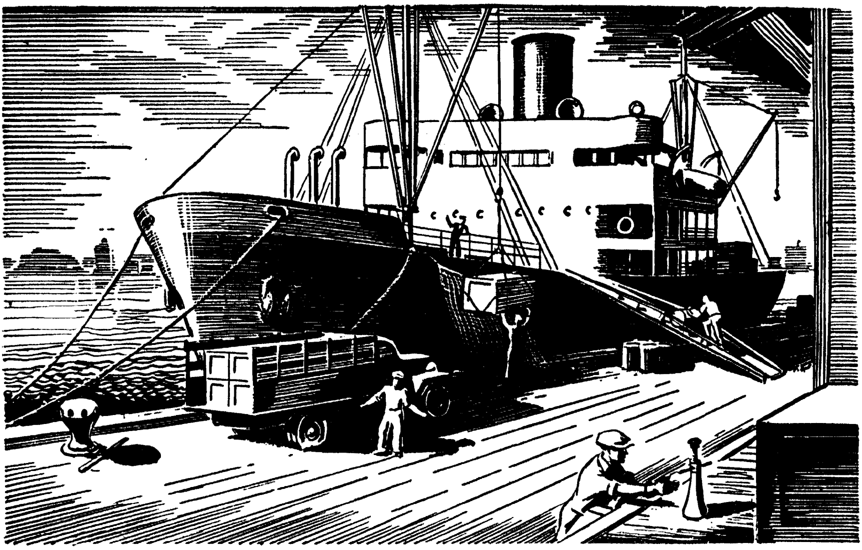
Weight of block, B = ______
Weight of weight holder, F_1 = ______

Load on block, L	$L + B = N$	Load on weight holder, F_2	$F_1 + F_2 = F$	Coefficient $\mu = F/N$

To obtain the coefficient of sliding friction, place slotted weights upon the weight hanger until the block will move uniformly after one starts it. Record as before, using a separate table labeled "sliding friction." Compare F and μ with the values obtained for starting friction.

Repeat this procedure for a different face of the block and compare the results with those previously obtained.

Determine the coefficients of sliding friction for rubber on dry glass, on wet glass, and on glass covered with a thin film of oil. Compare the results.



CHAPTER 13

SIMPLE MACHINES

A machine is a device for applying energy to do work in the way most suitable for a given purpose. No machine can create energy. To do work, it must receive energy from some source, and the maximum work it does cannot exceed the energy it receives.

Machines may receive energy in different forms: mechanical energy, heat, electrical energy, chemical energy, etc. We are here considering only machines that employ mechanical energy and do work against mechanical forces. In the so-called *simple machines,* the energy is supplied by a single force and the machine does useful work against a single resisting force. The former is called the *applied force* and the latter, the *resistance.* The frictional resistance which every machine encounters in action and which causes some waste of energy will be neglected for simplicity in treating some of the simple machines. Most machines, no matter how complex, are made up of one or more of the following simple machines: lever, wheel and axle, inclined plane, pulley, and screw.

Actual Mechanical Advantage. The utility of a machine is chiefly that it enables a person to perform some desirable work by the application of a comparatively small force. The ratio of the force exerted by the machine on a load F_o (output force) to the force exerted by the operator on the machine F_i (input force) is defined as the *actual mechanical advantage* (AMA) of the machine. For example, if a machine is available that enables a person to lift 500 lb by applying a force of 25 lb, its actual mechanical advantage is 500 lb/25 lb = 20. For most machines the AMA is greater than unity.

Ideal Mechanical Advantage. In any machine, because of the effects of friction, the work done by the machine in overcoming the opposing force is always less than the work done *on* the machine. The input work done by the applied force F_i is measured by the product of F_i and the distance s_i through which it acts. The output work is measured by the product of the output force F_o and the distance s_o through which it acts. Hence

$$F_o s_o < F_i s_i$$

Dividing each member of the inequality by $F_i s_o$, we obtain

$$\frac{F_o}{F_i} < \frac{s_i}{s_o}$$

that is, the ratio of the forces F_o/F_i is less than the ratio of the distances s_i/s_o for any machine. If the effects of friction are very small, the value of the output work approaches that of the input work, or the value of F_o/F_i becomes nearly that of s_i/s_o. The *ideal mechanical advantage* (IMA) is defined as the ratio s_i/s_o,

$$IMA = \frac{s_i}{s_o} > \frac{F_o}{F_i} \tag{1}$$

whereas

$$AMA = \frac{F_o}{F_i} < \frac{s_i}{s_o} \tag{2}$$

In a "frictionless" machine the inequalities of Eqs. (1) and (2) would become equalities. Since the forces move these distances in equal times, the ratio s_i/s_o is also frequently called the *velocity ratio.*

Example: A pulley system is used to lift a 1,000-lb block of stone a distance of 10 ft by the application of a force of 150 lb through a distance of 80 ft. Find the actual mechanical advantage and the ideal mechanical advantage.

$$AMA = \frac{F_o}{F_i} = \frac{1{,}000\text{ lb}}{150\text{ lb}} = 6.7$$

$$IMA = \frac{s_i}{s_o} = \frac{80\text{ ft}}{10\text{ ft}} = 8.0$$

Efficiency. Because of the friction in all moving machinery, the work done by a machine is less than the energy supplied to it. From the principle of conservation of energy, energy input = energy output + energy wasted, assuming no energy is stored in the machine. *The efficiency of* *a machine is defined as the ratio of its output work to its input work.* This ratio is always less than 1, and is usually multiplied by 100 and expressed in per cent. A machine has a high efficiency if a large part of the energy supplied to it is expended by the machine on its load and only a small part wasted. The efficiency (eff.) may be as high as 98 per cent for a large electric generator and will be less than 50 per cent for a screw jack.

$$\text{Eff.} = \frac{\text{output work}}{\text{input work}} = \frac{F_o s_o}{F_i s_i}$$

Also, since $\dfrac{F_o s_o}{F_i s_i} = \dfrac{F_o/F_i}{s_i/s_o}$,

$$\text{Eff.} = \frac{AMA}{IMA} \tag{3}$$

Example: What is the efficiency of the pulley system described in the preceding example?

$$\text{Eff.} = \frac{F_o s_o}{F_i s_i} = \frac{(1{,}000\text{ lb})(10\text{ ft})}{(150\text{ lb})(80\text{ ft})} = 0.83 = 83\%$$

Also,

$$\text{Eff.} = \frac{AMA}{IMA} = \frac{6.7}{8.0} = 0.84 = 84\%$$

Note the discrepancy of 1 per cent. Since the distances, s_i and s_o, are quoted to only two significant figures, the second digit in any calculation involving them is doubtful.

Lever. A lever is a bar supported at a point called the *fulcrum* (O, Fig. 1) so that a force F_i applied to the bar at a point A will balance a resistance F_o acting at another point B. To find the relation between F_i and F_o, suppose the bar to turn through a very small angle, so that A moves through a distance s_i and B through a distance s_o. Hence

$$IMA = \frac{s_i}{s_o} = \frac{AO}{BO} \tag{4}$$

The work done by F_i is $F_i s_i$; and the work done against F_o is $F_o s_o$. The conservation-of-energy principle indicates that these are equal, neglecting friction.

In Fig. 1 are shown the three ways in which the applied force, the resistance, and the fulcrum can be arranged to suit the needs of a particular situation.

Example: A force of 5.2 lb is applied at a distance of 7.2 in. from the fulcrum of a nutcracker. What force will be exerted on a nut that is 1.6 in. from the fulcrum?

$$IMA = \frac{7.2\text{ in.}}{1.6\text{ in.}} = 4.5$$

Neglecting friction, we can assume

$$AMA = IMA = 4.5$$

so that

$$\frac{F_o}{F_i} = 4.5$$

and

$$F_o = (4.5)(5.2 \text{ lb}) = 23 \text{ lb}$$

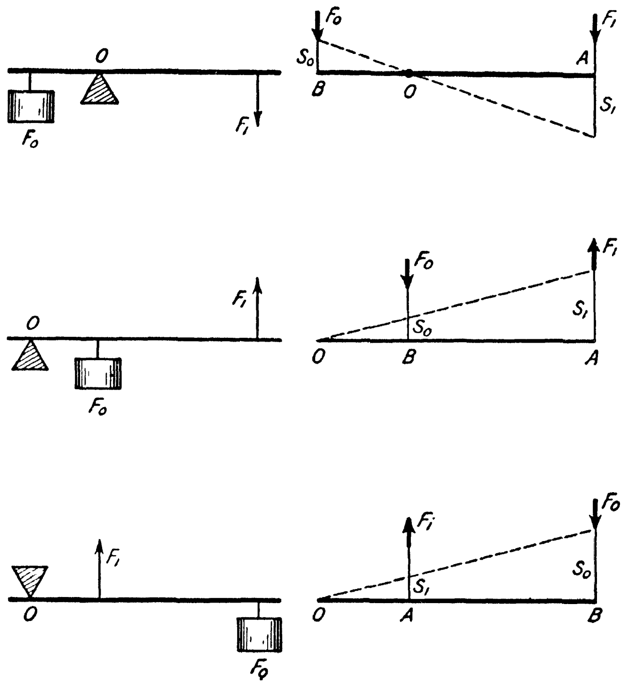


Fig. 1.—Levers.

Wheel and Axle. The wheel and axle (Fig. 2) is an adaptation of the lever. The distances s_i and s_o, for one complete rotation of the wheel, are $2\pi R$ and $2\pi r$, respectively, so that

$$IMA = \frac{s_i}{s_o} = \frac{R}{r} \tag{5}$$

A train of gears is a succession of wheels and axles, teeth on the axle of one meshing with teeth on the wheel of the next (Fig. 3). If the ideal mechanical advantage of the first wheel and axle is R/r and of the second is R'/r', then that of the combination is RR'/rr'.

Example: The wheels of an automobile are 28 in. in diameter and the brake drums 12 in. What will be the braking force necessary (at each drum) to provide a total retarding force of 2,000 lb?

$$IMA = \frac{R}{r} = \frac{12 \text{ in.}}{28 \text{ in.}} = 0.43$$

Assuming no bearing friction, $AMA = 0.43$, so that $F_o/F_i = 0.43$. Here

$$F_o = \frac{2{,}000 \text{ lb}}{4} = 500 \text{ lb}$$

and

$$F_i = \frac{500 \text{ lb}}{0.43} = 1200 \text{ lb}$$

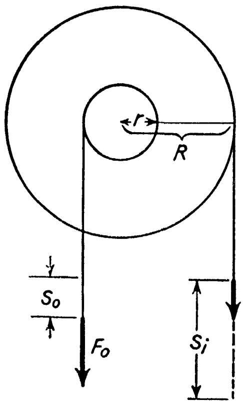


FIG. 2.—Wheel and axle.

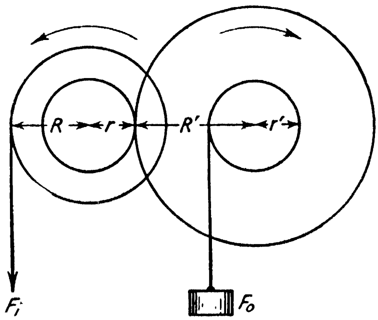


FIG. 3.—Gear train.

Inclined Plane. Let l be the length of an inclined plane (Fig. 4) and h its height. An object of weight W is caused to move up the plane by a force F_i which is parallel to the plane. The distance that the object moves against the force of resistance, its weight, is $s_o = h$, while the distance s_i, through which F_i is exerted, is l. Thus $IMA = l/h$.

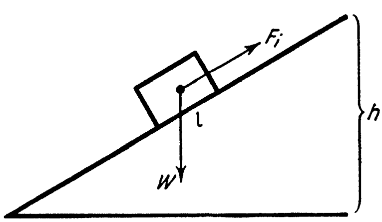


FIG. 4.—Inclined plane.

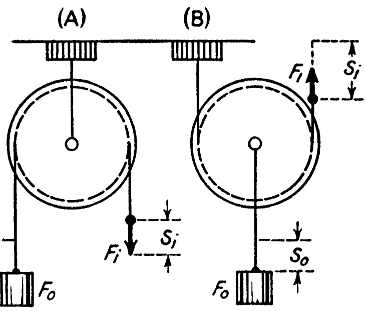


FIG. 5.—Pulleys.

Example: Neglecting friction, what force would be required to move a 300-lb block of ice up an incline 3.00 ft high and 24.0 ft long? Assuming

$$AMA = IMA, \qquad \frac{W}{F_i} = \frac{l}{h}$$

so that

$$F_i = \frac{(300 \text{ lb})(3.00 \text{ ft})}{24.0 \text{ ft}} = 37.5 \text{ lb}$$

Pulleys. In Fig. 5 A and B are shown two ways in which a single pulley can be used. At A is a fixed pulley, which serves to change only the direction of a force. Since $s_o = s_i$, the ideal mechanical advantage is unity. At B is a movable pulley for which, when the two parts of the cord are parallel, $s_i = 2s_o$ and $IMA = 2$.

Several pulleys are frequently used in combination to attain greater mechanical advantage. A common arrangement (Fig. 6) is called the *block and tackle.* It consists of a fixed block with two pulleys or sheaves, a movable block with two sheaves, and a continuous rope. For every foot F_o moves up, each segment of the rope shortens 1 ft, hence F_i must move 4 ft and the ideal mechanical advantage is 4. In general, a combination of pulleys with a continuous rope has an ideal mechanical advantage equal to n, the number of segments connected to the movable pulley.

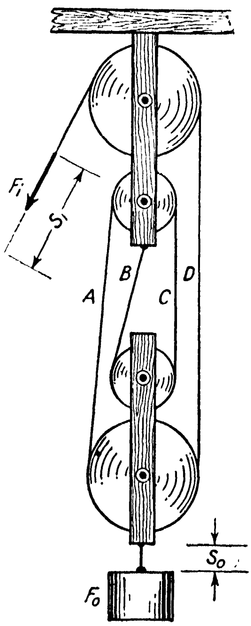


FIG. 6.—Block and tackle.

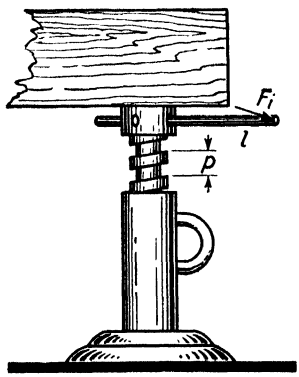


FIG. 7.—Screw jack.

Screw Jack. In a common form of screw jack, an upright screw threads into a stationary base and supports a load at the top, the screw being turned by means of a horizontal bar. The distance between consecutive turns of the thread, measured parallel to the axis of the screw, is called the *pitch* of the screw (Fig. 7). For example, a screw that has four threads per inch has a pitch of ¼ in.

In order to raise a load W a distance P equal to the pitch of the screw, the operator exerts a force F_i (at the end of the bar) through a circular path of length $s = 2\pi l$, where l is the length of the bar. Hence the ideal mechanical advantage of the screw jack is

$$IMA = \frac{2\pi l}{p} \tag{6}$$

The actual mechanical advantage of a screw is usually less than half its ideal mechanical advantage, hence the jack will hold a load at any height without an external locking device. A machine whose efficiency is less than 50 per cent is said to be *self-locking.*

SUMMARY

A *machine* is a device for applying energy at man's convenience.

The *actual mechanical advantage* (AMA) of a machine is the ratio of the force F_o that the machine exerts to the force F_i applied to the machine.

The *ideal mechanical advantage* (IMA) of a machine is defined as the distance ratio: s_i/s_o.

$$\textit{Efficiency} = \frac{\text{work output}}{\text{work input}} = \frac{F_o s_o}{F_i s_i} = \frac{AMA}{IMA}$$

A machine whose efficiency is less than 50 per cent is called *self-locking.*

QUESTIONS AND PROBLEMS

1. What kind of machine would you select if you desired one having a mechanical advantage of 2? of 500 or more? Which machine would likely have the greater efficiency if both machines were as mechanically perfect as it is possible to make them?

2. A man raises a 500-lb stone by means of a lever 5.0 ft long. If the fulcrum is 0.65 ft from the end that is in contact with the stone, what is the ideal mechanical advantage? *Ans.* 6.7.

3. Neglecting friction, what applied force is necessary in problem 2?

4. The radius of a wheel is 2.0 ft and that of the axle is 2.0 in. What force, neglecting friction, must be applied at the rim of the wheel in order to lift a load of 900 lb, which is attached to a cable wound around the axle? *Ans.* 75 lb.

5. A safe weighing 10 tons is to be loaded on a truck, 5.0 ft high, by means of planks 20 ft long. If it requires 350 lb to overcome friction on the skids, find the least force necessary to move the safe.

6. The pitch of a screw jack is 0.20 in., and the input force is applied at a radius of 2.5 ft. Find the ideal mechanical advantage. *Ans.* $9\bar{4}0$.

7. Assuming an efficiency of 30 per cent, find the force needed to lift a load of 3,300 lb with the screw jack of problem 6.

8. A movable pulley is used to lift a 200-lb load. What is the efficiency of the system if a 125-lb force is necessary? *Ans.* 80 per cent.

9. Compare the mechanical advantages of a block and tackle (Fig. 6) when the end of the cord is attached to the upper block and when it is attached to the lower.

10. A block and tackle having three sheaves in each block is used to raise a load of 620 lb. If the efficiency of the system is 69 per cent, what force is necessary? *Ans.* $1\bar{5}0$ lb.

11. A force of 3.0 lb is required to raise a weight of 16 lb by means of a pulley system. If the weight is raised 1 ft while the applied force is exerted through a distance of 8.0 ft, find (*a*) the ideal mechanical advantage, (*b*) the actual mechanical advantage, and (*c*) the efficiency of the pulley system.

12. A man weighing 150 lb sits on a platform suspended from a movable pulley and raises himself by a rope passing over a fixed pulley. Assuming the ropes parallel, what force does he exert? (Neglect the weight of the platform.)
Ans. 50 lb.

EXPERIMENT

Mechanical Advantage, Efficiency

Apparatus: Mechanism hidden in a box; windlass; weights; weight hangers; rope or heavy cord.

This experiment is for the purpose of clarifying by observation the meanings of the terms: ideal mechanical advantage, actual mechanical advantage, and efficiency.

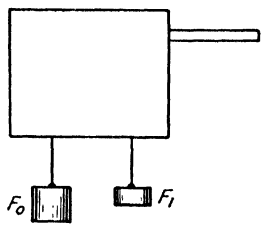


FIG. 8.—An unknown mechanism is hidden within the box. Both the ideal and the actual mechanical advantage can be determined without knowledge of the nature of the machine.

1. In the first part of this experiment use is made of a hidden mechanism (Fig. 8). It is contained by a box with two holes in the bottom. Two cords extend through these holes. When one cord is pulled down the other goes up. The ratio of the distances they move in the same time gives the ideal mechanical advantage—even though it is not known what particular mechanism is in the box. Which is the load cord? Which is the effort cord? What is the ideal mechanical advantage?

The actual mechanical advantage, differing from the ideal because of friction, is given by the ratio of forces, rather than distances. Determine the actual mechanical advantage of the hidden mechanism by applying a load and measuring the force required to keep it moving uniformly after it is started. From the values of the two mechanical advantages, determine the efficiency of the hidden device.

Before examining the mechanism in the box, attempt to establish its identity from the observations you have made. Using the data already obtained, compute the values of input work and output work, and from them determine (again) the efficiency of the mechanism. Is this method of computing the efficiency essentially different from the other? Explain.

Determine the mechanical advantage and efficiency of the windlass illustrated in Fig. 9. Measure the diameter of the axle with a vernier caliper and the diameter of the wheel with a meter stick. Make proper allowances for the thickness of the ropes and the depth of the groove. Calculate the ideal mechanical advantage from the ratio of the diameters.

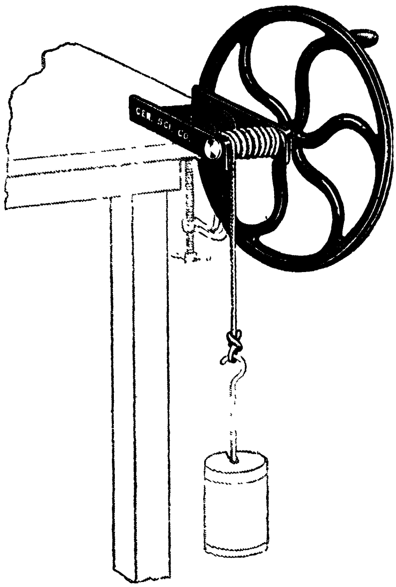
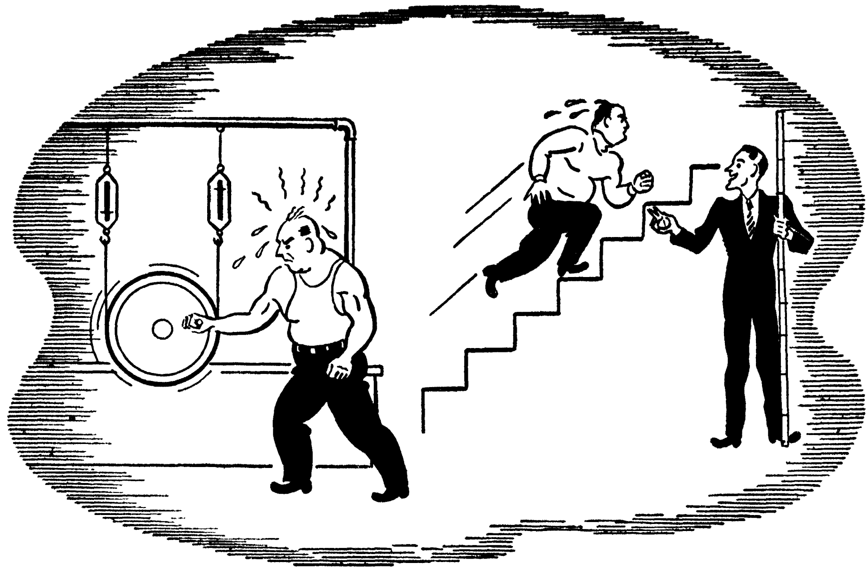


FIG. 9.—Windlass.

Attach a load of 5 to 10 kg to the axle by means of a rope. Attach sufficient weights to a cord passing around the wheel to raise the load at a uniform rate (after it is started by hand). Compute the actual mechanical advantage and the efficiency.



CHAPTER 14

POWER

The rate of production of a man working only with hand tools is quite small, so small that production by these methods does not meet the demand. In order to increase the output, machines were devised. The machine not only enables the operator to make articles that would not otherwise be possible but it also enables him to convert energy into useful work at a much greater rate than he could by his own efforts. Each workman in a factory has at his disposal power much greater than he alone could develop.

Power. In physics the word *power* is restricted to mean the *time rate of doing work.* The average power is the work performed divided by the time required for the performance. In measuring power, both the *work* and the *elapsed time* must be measured.

$$\text{Power} = \frac{\text{work}}{\text{time}} \tag{1}$$

The same work is done when a 500-lb steel girder is lifted to the top of a 100-ft building in ½ min as is done when it is lifted in 10 min. However, the power required is twenty times as great in the first case as in the second, for the power needed to do the work varies inversely as the time. If given sufficient time, a hod carrier can transfer a ton of brick from the ground to the roof of a skyscraper. A hoisting engine can do this work more quickly since it develops more power.

Much of our everyday work is accomplished by using the energy from some source such as gasoline, coal, or impounded water. We often buy the privilege of having energy transformed on our premises. Thus electricity flowing through the grid of a toaster has its electrical energy transformed into heat. Energy is transformed, not destroyed. We pay for the energy that is transformed (not for the electricity, for that flows back to the plant). The amount of energy transformed is the rate of transformation multiplied by the time, Eq. (1).

Units of Power. The British units of power are the *foot-pound per second* and the *horsepower*. A horsepower is defined as 550 ft-lb/sec.

The absolute metric unit of power is the *erg per second*. Since this is an inconveniently small unit, the joule per second, called the *watt*, is commonly used. The watt equals 10^7 ergs/sec. The *kilowatt*, used largely in electrical engineering, is equal to 1,000 watts.

Table I. Units of Power

1 watt = 10^7 ergs per second = 1 joule per second
1 horsepower = 550 foot-pounds per second = 33,000 foot-pounds per minute
1 horsepower = 746 watts
1 kilowatt = 1,000 watts = 1.34 horsepower
1 foot-pound per second = 1.356 watts

Example: By the use of a pulley a man raises a 120-lb weight to a height of 40 ft in 65 sec. Find the average horsepower required.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{(\text{force})(\text{distance})}{\text{time}}$$

$$= \frac{(120\text{ lb})(40\text{ ft})}{65\text{ sec}} = 74\text{ ft-lb/sec}$$

$$1\text{ hp} = 550\text{ ft-lb/sec}$$

Therefore

$$\text{Power} = \frac{74\text{ ft-lb/sec}}{550\,\dfrac{\text{ft-lb/sec}}{\text{hp}}} = 0.13\text{ hp}$$

Measurement of Mechanical Power. The mechanical power output of a rotating machine can be measured by equipping the machine with a special form of friction brake (Prony brake), which absorbs the energy output of the machine and converts it into heat. A simple style of Prony brake suitable for small machines consists of a band that passes around the rotating pulley of the machine and is supported at the ends as shown in Fig. 1. Two screws w serve to tighten or loosen the band, thus regulating the load of the machine, and two spring balances show in terms of their readings, F and F', the forces exerted on the ends of the bands. In operation the band is dragged around by friction at the rim of the rotating pulley and remains slightly displaced. The effective force of friction is equal to the difference of the spring balance readings, $F' - F$. The machine, in opposing friction, does an amount of work $(F' - F)(2\pi r)$ during each rotation, or $(F' - F)(2\pi rn)$ in 1 min, where n is the number of rotations that it makes per minute. If one expresses force in pounds and the radius in feet, the power output of the machine in foot-pounds per minute is $2\pi rn(F' - F)$; or

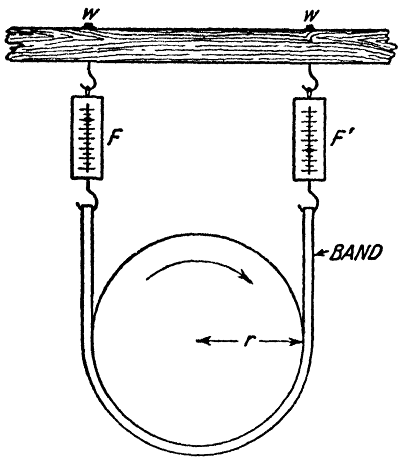


FIG. 1.—Prony brake.

$$\text{Output} = \frac{2\pi rn(F' - F)}{33{,}000}. \qquad (2)$$

This is known as the *brake horsepower*.

Human Power Output. A man who weighs 220 lb may be able to run up a 10-ft flight of stairs in 4 sec. If so, he is able to work at the rate of 1 hp, since

$$(220 \text{ lb})\left(\frac{10 \text{ ft}}{4 \text{ sec}}\right) = 550 \text{ ft-lb/sec} = 1 \text{ hp}$$

A 110-lb boy would have to climb the same height in 2 sec in order to develop the same power. Human endurance will not enable even an athlete to maintain this pace very long. In almost no other way can a man approximate a horsepower in performance. In sustained physical effort a man's power is seldom as great as $\frac{1}{10}$ hp.

Since the muscles of the body are only about 20 per cent efficient, the rate at which they perform useful work is only about one-fifth the rate at which they may be transforming energy. The remainder of the energy is converted into heat, which must be dissipated through ventilation and perspiration. Just as a mechanical engine may have to be "geared down" to match its power output to the requirements of the load, so we as human machines can often best accomplish work by long-continued application at a moderate rate.

Alternative Ways of Writing the Power Equation. The rate at which a machine works depends on several factors, which appear implicitly in Eq. (1). When a machine is working, the average power developed is

$$P = \frac{Fs}{t} \qquad (3)$$

where F is the average force that moves through a distance s in time t.

Equation (3) may be written $P = F(s/t)$, or

$$P = F\bar{v} \tag{4}$$

which shows that the average rate at which a machine works is the product of the force and the average speed. A special use of Eq. (4) is that in which the force is applied by a belt moving with an average speed $\bar{v}$. The belt horsepower is

$$Hp = \frac{F\bar{v}}{33{,}000}$$

where F is the difference between the tensions in the two sides of the belt.

SUMMARY

Power is the time rate of doing work.

A *horsepower* is 550 ft-lb/sec.

A *watt* is 1 joule/sec.

One horsepower is equivalent to 746 watts.

Brake horsepower of an engine is given by

$$Bhp = \frac{2\pi rn(F' - F)}{33{,}000}$$

QUESTIONS AND PROBLEMS

1. A 500-lb safe is suspended from a block and tackle and hoisted 20 ft in 1.5 min. At what rate is the work performed?

2. A horse walks at a steady rate of 3.0 mi/hr along a level road and exerts a pull of 80 lb in dragging a cart. What horsepower is he developing?

Ans. $0.6\bar{4}$ hp.

3. A 10-hp hoisting engine is used to raise coal from a barge to a wharf, an average height of 75 ft. Assuming an efficiency of 75 per cent, how many tons of coal can be lifted in 1 min?

4. A locomotive developing 2,500 hp draws a freight train 1.75 mi long at a rate of 10.0 mi/hr. Find the drawbar pull exerted by the engine.

Ans. 46.8 tons.

5. Find the useful horsepower expended in pumping 5,000 gal of water per minute from a well in which the water level is 40 ft below the discharge pipe.

6. A 2,000-lb car travels up a grade that rises 1 ft in 20 ft along the slope, at the rate of 30 mi/hr. Find the horsepower expended. *Ans.* 8 hp.

7. How heavy a load can a 15-hp hoist lift at a steady speed of 240 ft/min?

8. The friction brake of Fig. 1 is applied to an electric motor. The following data are recorded by an observer: $r = 6$ in; $F' = 55$ lb; $F = 20$ lb; and $n = 1{,}800$ rpm. Compute the horsepower at which the motor is working. *Ans.* 6 hp.

9. A 10-hp motor operates at rated load for 8 hr a day. Its efficiency is 87 per cent. What is the daily cost of operation if electrical energy costs 5 cts per kilowatt-hour?

10. Calculate the horsepower of a double stroke steam engine having the following specifications: cylinder diameter, 12 in; length of stroke, 2 ft; speed, 300 rpm; average steam pressure, 66 lb/in^2. *Ans.* 270 hp.

11. Find the difference in the tensions of the two sides of a belt when it is running 2,800 ft/min and transmitting 150 hp.

EXPERIMENT

Manpower

Apparatus: Dynamometer; slotted kilogram weights; weight hangers; large C clamp.

By means of the apparatus shown in Fig. 2, one can measure his mechanical power, that is, the rate at which he is able to perform mechan-

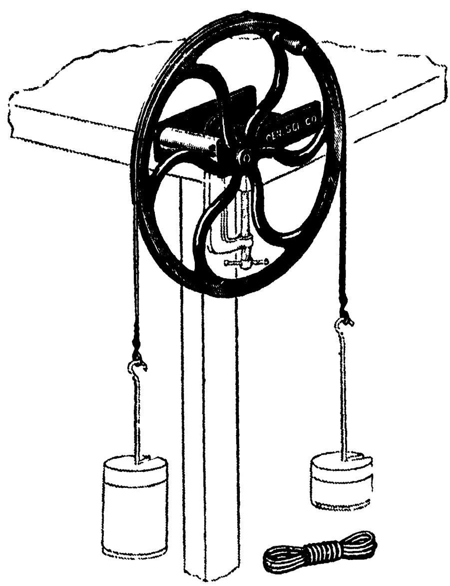


Fig. 2.—Dynamometer.

ical work. The belt that connects the two weight hangers is composite, one-half of its length consisting of metal, the other of leather. If the heavier load is attached to the metal end of the belt, as in Fig. 2, the apparatus is somewhat self-adjusting. When the wheel is turned clockwise, the belt moves with it until the area of contact between the leather and the wheel is insufficient to prevent slipping. When slipping starts, the belt will assume the position in which the frictional force is equal to the difference in the two loads, if the latter are adjusted to suitable values.

Arrange the apparatus as shown in Fig. 2. Measure the circumference of the wheel at the position of the belt. Next, attach the loads to the belt, choosing their values so that they differ by 3 to 6 kg.

The student whose mechanical power is to be measured should turn the wheel at a constant rate. A second student should count the number of revolutions made by the wheel in a given time interval, say 30 sec.

The work done in turning the wheel through one complete revolution is the product of the frictional force F and the circumference C of the wheel. (NOTE: The frictional force is equal to the difference in the two loads.) The total work done during the measured time interval t is, therefore, work $= FCN$, where N is the number of revolutions. The power developed in turning the wheel is

$$P = \frac{\text{Work}}{t} = \frac{FCN}{t}$$

One has the capacity for changing his power at will—within limits. Frequently our rate of working depends upon the total amount of work we have to do. Thus, if we expect to turn the wheel for an hour we will consciously work at a different rate than if we were going to work for only five minutes.

It is interesting to compare one's average "long-time" power with his maximum, or "short-time" power. How much greater than your average power would you estimate your maximum power to be? Measure each and compare them.

One's power depends also upon one's physical condition. How much do you suppose your "maximum power" would be after chinning yourself, say, ten times?

CHAPTER 15

CONCURRENT FORCES; VECTORS

In almost every activity, the engineer must attempt to cause motion or to prevent it or to control it. In order to produce any one of these results a force or perhaps several forces must be applied. Usually several forces act upon every body and the motion is that produced by the combined action of all of them. Many of the problems of the design engineer have to do with the various forces acting upon or within a structure or machine. The problem then frequently resolves itself into the determination of the forces necessary to produce equilibrium in the device.

Equilibrium. The state in which there is no change in the motion of a body is called *equilibrium.* When in equilibrium, therefore, an object has no acceleration. This does not imply that no forces are acting on the object (for nothing is free of applied forces), but that the sum of the forces on it is zero.

A parachute descending with uniform speed is in equilibrium under the action of two forces—the resistance of the air and the combined weight of the parachute and its load—which exactly balance each other. These two forces are in the same (vertical) straight line, but their directions are opposite and their sum is zero.

Forces acting in the same direction can be added arithmetically to find the value of their *resultant,* which is the single force whose effect is equivalent to their combined action. The resultant of two forces in opposite directions is determined by subtracting the numerical value of the smaller force from that of the larger. In order to determine the resultant of two forces that do not act in the same straight line, it is necessary to make use of a new type of addition called *vector* addition. The study of vectors and vector quantities is essential to the solution of problems involving forces in equilibrium.

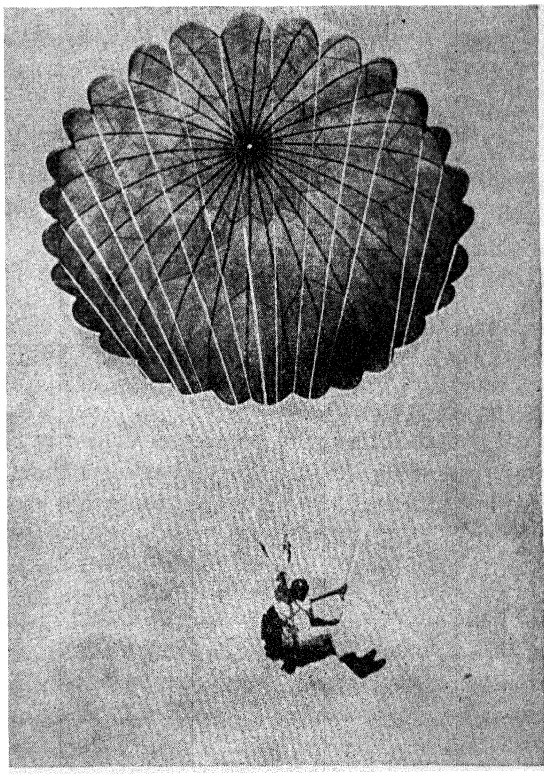


Fig. 1.—An example of equilibrium.

Vector Quantities. *Vector* quantities are those which have both *magnitude* and *direction.* Force, velocity, displacement, and acceleration are vector quantities. In contrast, quantities such as mass, volume, or speed, which have only magnitude, are called *scalar* quantities. We learned to add scalar quantities in grade school by simply adding numbers, but in adding vector quantities we must take their directions into account. Though this process is not so easy as that of adding scalar quantities, its difficulty can be greatly reduced through the use of graphical methods.

Addition of Vectors. It is often convenient to represent vector quantities graphically. A straight line drawn to scale and in a definite direction may be used to represent any vector quantity, and the line is commonly called a *vector* (Fig. 2). When two vectors are not parallel, their resultant (that is, the single vector that is equivalent to them) is found by the parallelogram rule. The resultant of two vectors is represented by the diagonal of a parallelogram of which the two vectors are adjacent sides.

FIG. 2.—A vector: 5 units, east.

Consider, as an example, the addition of two forces, one 3 lb north, the other 4 lb east. By choosing a scale such that arrow F_1 (Fig. 3) of any desired length represents 1 lb, then AB, three times as long, represents in magnitude and direction the force of 3 lb north. Likewise AD represents the force of 4 lb east. Observe that the two arrows are placed tail to tail. Complete the parallelogram $ABCD$. The arrow AC, which is the diagonal of the parallelogram, represents the resultant of AB and AD. It is 5 times as long as F_1, hence the resultant is 5 lb, in the direction AC.

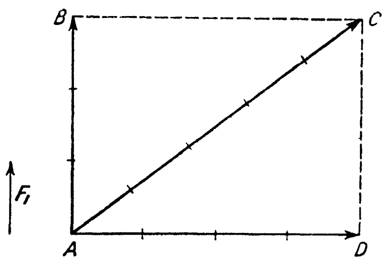


FIG. 3.—The vector AC is the resultant of AB and AD

The resultant of two forces may be greater or less in magnitude than either of them, depending on the angle between them. In Fig. 4A two forces M and N of 2 and 3 units, respectively, are shown separately.

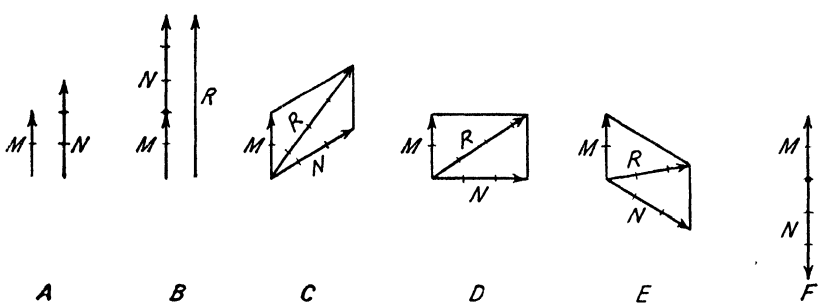


FIG. 4.—The resultant of two vectors depends upon the angle between them.

In B where the forces are in the same direction their resultant is merely their sum. As the angle between the two forces increases, the resultant becomes less, as shown in C, D, and E. At F the resultant is the difference between the forces.

When several forces act at the same point, they are said to be *concurrent* forces. The parallelogram method just described can be used to find the resultant of any set of concurrent forces but it becomes very cumbersome when there are more than two forces. Another graphical method called the *polygon* method is more useful for several forces. In Fig. 3 we found the resultant AC by completing the parallelogram but we can get the same result by moving the vector AB parallel to itself until it coincides with DC. We then have drawn the first vector AD and have placed the tail of the second vector at the head of the first. The resultant AC is the vector that closes the triangle.

This process may be immediately extended to the composition of more than two vectors. The addition of vectors A, B, C, D to give the resultant R should be clear from Fig. 5. Notice that the vectors to be added follow one another head to tail, like arrows indicating a trail. The only place where we may have two arrow points touching is where the head of the resultant arrow R joins the head of the last vector which was added, D. The vectors can be drawn in any order without changing the result.

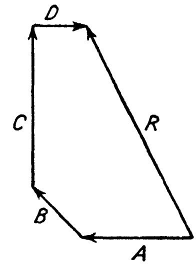


FIG. 5.—Polygon method of vector addition. A = 6 mi, west; B = 4 mi, northwest; C = 8 mi, north; D = 3 mi, east.

We are now in a position to see what we mean when we say that the sum of several forces is zero. This means that the length of the arrow

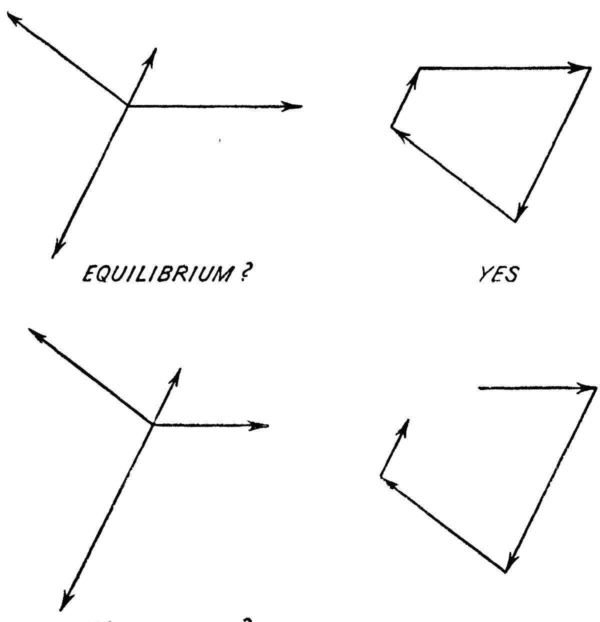


FIG. 6.

representing the resultant is zero. But this can occur only if the head of the last vector to be added comes back to touch the tail of the first vector. (See Fig. 6.) This allows us to state the *first condition of equilibrium* in a rather useful way: *if a body is in equilibrium under the action of several forces, then the vector sum of these forces must be zero,* so that if we add the forces on paper by drawing vectors to scale, these vectors must form a *closed* polygon.

If the resultant of several forces is not zero, the body acted upon is not in equilibrium but it can be set into a condition of equilibrium by adding a single force equal to the resultant but opposite in direction. This force is called the *equilibrant.* In Fig. 5, the equilibrant of the four forces A, B, C, and D is a force equal to R but opposite in direction. If this force were combined with the original four forces the polygon would be closed.

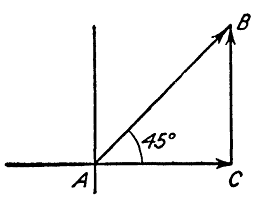


FIG. 7.—Vertical and horizontal components of a vector.

Component Method of Adding Vectors. The ease with which we obtain the resultant of two vectors when they lie at right angles, as in Fig. 3, leads us to attempt to solve the more difficult problem of Fig. 5 by replacing each vector by a pair of vectors at right angles to each other. Each member of such a pair is called a *component* of the original vector. This operation, called *resolution,* is of course just the reverse of *composition* of vectors.

Consider the vector AB (Fig. 7), which makes an angle of 45° with the horizontal. To obtain a set of components of AB, one of which shall be horizontal, draw a horizontal line through the tail of the vector AB. Now from the head of AB drop a perpendicular CB. We see that the vector AB can be considered as the resultant of the vectors AC and CB. The values of the horizontal and vertical components are $AB \cos 45°$ and $AB \sin 45°$. The directions of the arrow heads are important, for we are now considering that AC has been added to CB to give the resultant AB; therefore the arrows must follow head to tail along AC and CB, so that AB can properly be considered as a resultant drawn from the tail of the first arrow AC to the head of the last arrow CB. This resolution into components now allows us to discard the vector AB in our problem and keep only the two components, AC and CB. These two taken together are in every way equivalent to the single vector AB.

What is the advantage of having two vectors to deal with where there was one before? The advantage lies in the fact that a set of vectors making various odd angles with each other can be replaced by two sets of vectors making angles of either 90 or 0° with one another. Each of these two groups of vectors can then be summed up algebraically, thus reducing the problem to one of two vectors at right angles.

It may be of help to amplify one special case of resolution: A vector has no component at right angles to itself. In Fig. 7, imagine that the angle there marked 45° is increased to 90°, keeping the length of the vector AB

constant. Note that the horizontal component AC becomes zero, while the vertical component BC becomes equal to AB itself.

Example: By the method of components find the resultant of a 5.0-lb horizontal force and a 10-lb force making an angle of 45° with the horizontal (Fig. 8).

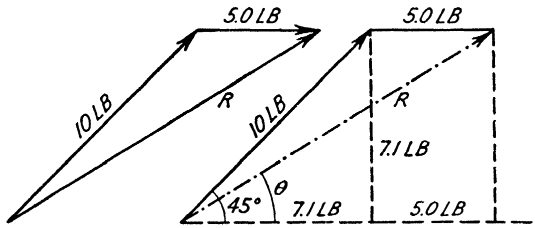


FIG. 8.—Finding a resultant by the method of components.

The horizontal and vertical components of the 10-lb force are (10 lb) cos 45° = 7.1 lb and (10 lb) sin 45° = 7.1 lb. The horizontal component of the 5.0-lb force is 5.0 lb, and its vertical component is zero. There are three forces: one vertical and two horizontal. Since the two horizontal forces are in the same direction, they may be added as ordinary numbers, giving a total horizontal force of 5.0 lb + 7.1 lb = 12.1 lb. The problem is now reduced to the simple one of adding two forces at right angles, giving the resultant

$$R = \sqrt{7.1^2 + 12.1^2}\ \text{lb} = 14.0\ \text{lb}$$

The angle θ which R makes with the horizontal has a tangent $7.1/12.1 = 0.59$, so that $\theta = 30°$.

Example: An object weighing 100 lb and suspended by a rope A (Fig. 9) is pulled aside by the horizontal rope B and held so that rope A makes an angle of 30° with the vertical. Find the tension in ropes A and B.

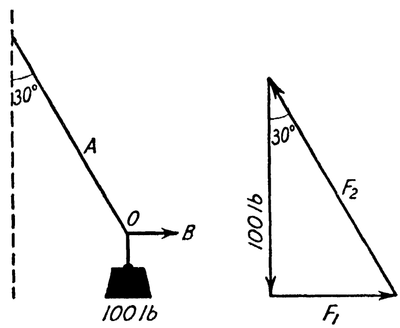


FIG. 9.—Finding a force by the polygon method.

We know that the junction O is in equilibrium under the action of these forces, hence their resultant must be zero. Therefore, the vectors representing the three forces can be combined to form a closed triangle, as shown at the right in Fig. 9. In constructing the vector diagram each vector is drawn parallel to the force that it represents.

In solving the vector triangle it is seen that

$$\frac{F_1}{100\ \text{lb}} = \tan 30° = 0.58$$

so that $F_1 = (100\ \text{lb})(0.58) = 58$ lb. To get F_2, we can put

$$\frac{100\ \text{lb}}{F_2} = \cos 30° = 0.866$$

Therefore,

$$F_2(0.866) = 100\ \text{lb}$$

$$F_2 = \frac{100\ \text{lb}}{0.866} = 116\ \text{lb}$$

That is, in order to hold the system in the position of Fig. 9, one must pull on the horizontal rope with a force of 58 lb. The tension in rope A is then 116 lb. The tension in the segment of rope directly supporting the weight is, of course, just 100 lb.

To solve this problem, we used the straightforward method of adding the vectors to form a closed figure. This method is quite appropriate to such simple cases but, for the sake of illustration, let us now solve the problem again by the more general method of components. In Fig. 10 are shown the same forces, separated for greater convenience of resolution. The horizontal and vertical components of the 100-lb force are, respectively, 0 and 100 lb down. The horizontal and vertical components of F_1 are, respectively, F_1 (to the right) and 0. In finding the components of F_2, we do not yet know the numerical value of F_2, but, whatever it is, the horizontal and vertical components will certainly be $F_2 \sin 30°$ to the left and $F_2 \cos 30°$ up. We now have four forces, two vertical and two horizontal, whose vector sum must be zero to ensure equilibrium. In order that the resultant may be zero the sum of the horizontal components and the sum of the vertical components must (each) be equal to zero.

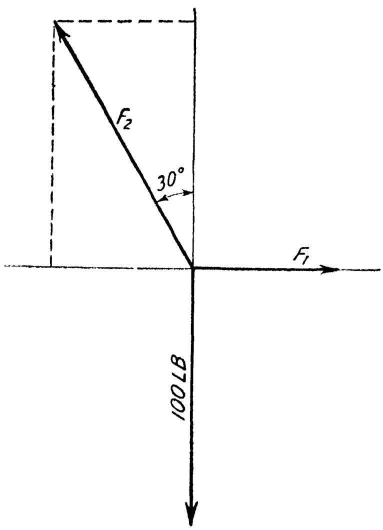


FIG. 10.—Component method of solving the problem of Fig. 9.

Therefore,

$$F_1 - F_2 \sin 30° = 0 \text{ (horizontal)}$$
$$F_2 \cos 30° - 100 \text{ lb} = 0 \text{ (vertical)}$$

If we solve the second equation, we find that $F_2 = 116$ lb, as in the previous solution. By substituting this value in the first equation, we obtain $F_1 = 58$ lb, as before.

Example: A load of 100 lb is hung from the middle of a rope, which is stretched between two walls 30.0 ft apart (Fig. 11). Under the load the rope sags 4.0 ft in the middle. Find the tension in sections A and B.

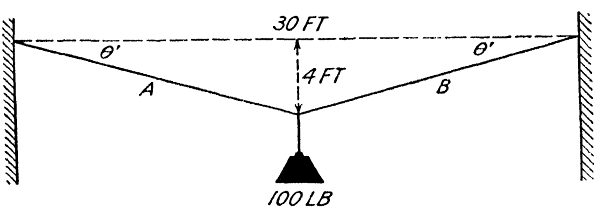


FIG. 11.—Finding the tension of a stretched rope.

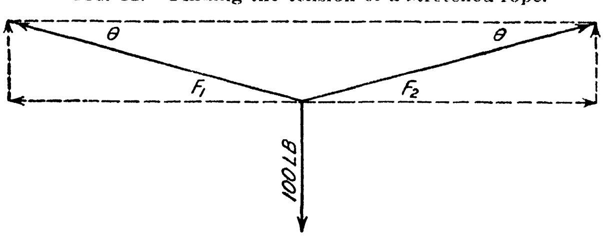


FIG. 12.—Horizontal and vertical components of the forces in a stretched rope.

The mid-point of the rope is in equilibrium under the action of the three forces exerted on it by sections A and B of the rope and the 100-lb weight. A vector diagram of the forces appears in Fig. 12. The horizontal and vertical components of

the 100-lb force are, respectively, 0 and 100 lb downward. The horizontal and vertical components of F_1 are, respectively, $F_1 \cos\theta$ to the left, and $F_1 \sin\theta$ upward. Similarly, the horizontal and vertical components of F_2 are, respectively, $F_2 \cos\theta$ to the right, and $F_2 \sin\theta$ upward. In order that the resultant shall be zero the sum of the horizontal components and the sum of the vertical components must (each) be equal to zero.
Therefore,

$$F_2 \cos\theta - F_1 \cos\theta = 0 \text{ (horizontal)} \tag{1}$$
$$F_1 \sin\theta + F_2 \sin\theta - 100 \text{ lb} = 0 \text{ (vertical)} \tag{2}$$

Since these two equations involve three unknown quantities F_1, F_2, and θ, we cannot solve them completely without more information.

An inspection of Fig. 11 shows that the angle θ' of that figure is identical with the angle θ of Fig. 12. Thus the value of $\sin\theta$ can be determined from the dimensions shown in Fig. 11.

$$\sin\theta = \sin\theta' = \frac{4.0 \text{ ft}}{A}$$

and

$$A = \sqrt{15.0^2 + 4.0^2} \text{ ft} = \sqrt{241} \text{ ft} = 15.5 \text{ ft}$$
$$\sin\theta = \frac{4.0 \text{ ft}}{15.5 \text{ ft}} = 0.26$$

From Eq. (1), $F_1 = F_2$. Substituting in Eq. (2),

$$F_1 \sin\theta + F_1 \sin\theta - 100 \text{ lb} = 0$$
$$2F_1 \sin\theta = 100 \text{ lb}$$
$$2F_1(0.26) = 100 \text{ lb}$$
$$F_1 = \frac{100 \text{ lb}}{2(0.26)} = 1\bar{9}0 \text{ lb}$$

and

$$F_2 = 1\bar{9}0 \text{ lb}$$

It is essential that two things be noticed about the problem just solved: (1) that the value of a function of an angle in the vector diagram was needed in order to carry out the solution; (2) that the value of that function was determined from the geometry of the original problem.

Example: Calculate the force needed to hold a 1,000-lb car on an inclined plane that makes an angle of 30° with the horizontal, if the force is to be parallel to the incline.

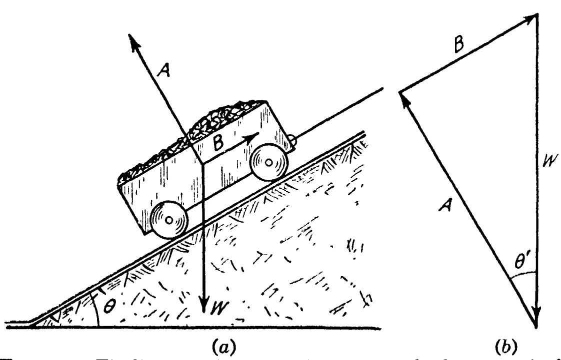


FIG. 13.—Finding the forces acting upon a body on an incline.

The forces on the car include (see Fig. 13) its weight W, the force parallel to the incline B, and the force of reaction A exerted on the car by the inclined plane itself. The last force mentioned is perpendicular to the plane if there is no friction.

Since the car is in equilibrium under the action of the three forces A, B, and W, a closed triangle can be formed with vectors representing them, as in Fig. 13*b*. In the vector diagram, $B/W = \sin \theta'$, so that $B = W \sin \theta'$. The angle θ', however, is equal to angle θ in Fig. 13*a* (Can you prove this?), and we may write $B = W \sin \theta$. Since θ is 30° and $W = 1{,}000$ lb,

$$B = (1{,}000 \text{ lb}) \sin 30° = (1{,}000 \text{ lb})(0.500) = 500 \text{ lb}$$

The value of A, the perpendicular force exerted by the plane, can be found by observing that $A/W = \cos \theta' = \cos \theta$, from which

$$A = W \cos 30° = (1{,}000 \text{ lb})(0.866) = 866 \text{ lb}$$

It should be noticed that W can be resolved into two components that are, respectively, parallel and perpendicular to the incline. These components are, obviously, equal in magnitude and opposite in direction to B and A, respectively.

The relation between the motions of two different objects, called their *relative motion*, can be obtained by taking the vector difference of their velocities measured with respect to some reference body, often the earth. Two cars proceeding in the same direction on a highway each at 30 mi/hr have a relative speed of zero. If, however, car A is traveling east at 30 mi/hr and car B is traveling west at 30 mi/hr, A will have a velocity of 60 mi/hr east relative to B. In this familiar example we have implicitly used the concept that velocity is a vector quantity. In the following problem more explicit use is made of vector methods in describing relative motion.

Example: An airplane is flying 125 mi/hr on a north-to-south course, according to its air-speed indicator and (corrected) compass readings. A cross wind of 30 mi/hr is blowing south 47° west. What are the ground speed and course of the airplane?

Solution is by the method of components. The wind speed can be resolved into 30 cos 47° south and 30 sin 47° west. These components added to the air speed of the airplane, graphically, give a right triangle, one side representing a speed of 145 mi/hr south, the other 22 mi/hr west. The hypotenuse represents the ground speed and course of the airplane: 147 mi/hr in a direction 8°35′ west of south.

SUMMARY

A body is in *equilibrium* when it has no acceleration.

When a body is in equilibrium, the vector sum of all the forces acting on it is zero. This is known as the first condition of equilibrium.

Quantities whose measurement is specified by *magnitude* and *direction* are called *vector* quantities. Those which have only magnitude are called *scalar* quantities.

A vector quantity is represented graphically by a line (vector) drawn to represent its direction and its magnitude on some convenient scale.

The *resultant* of two or more vectors is the single vector that would produce the same result.

The *rectangular components* of a vector are its projections on a set of right angle axes, for example, the horizontal and vertical axes.

Vectors are conveniently added graphically by placing them "head to tail" and drawing the resultant from the origin to the head of the last vector, closing the polygon.

The *component* method of adding vectors is to resolve each into its rectangular components, which are then added algebraically and the resultant found.

QUESTIONS AND PROBLEMS

1. A boat sails 20 mi due east and then sails 12 mi southwest. How far is it from its starting point, and in what direction is it from that point?

2. A ship is sailing 20° north of east at the rate of 14 mi/hr. How fast is it going northward and how fast eastward? *Ans.* 4.8 mi/hr; 13 mi/hr.

3. If a ship is sailing 21° east of north at the rate of 15 mi/hr, what are its component speeds, northward and eastward?

4. If a wind is blowing 17.5 ft/sec and crosses the direction of artillery fire at an angle of 38°, what are its component speeds along, and directly across, the direction of fire? *Ans.* 13.8 ft/sec; 10.8 ft/sec.

5. A boy is pulling his sled along level ground, his pull on the rope being 12 lb. What are the vertical and horizontal components of the force if the rope makes an angle of 21° with the ground?

6. Add the following displacements by the component method: 10 ft directed northeast, 15 ft directed south, and 25 ft directed 30° west of south.
Ans. 30 ft, 10.4°W of S.

7. A boat travels 10 mi/hr in still water. If it is headed 60° south of west in a current that moves it 10 mi/hr due east, what is the resultant velocity of the boat?

8. An airplane is flying at 150 mi/hr on a north to south course according to the compass. A cross wind of 30 mi/hr is blowing south 47° west and carries the airplane west of its course. What are the actual speed and course of the airplane? *Ans.* 172 mi/hr; 7.4°W of S.

9. A weight is suspended by two wires, each inclined 22° with the horizontal. If the greatest straight pull which either wire could sustain is 450 lb, how large a weight could the two support as specified?

10. A boy weighing 80 lb sits in a swing, which is pulled to one side by a horizontal force of 60 lb. What is the tension on the swing rope?
Ans. 100 lb.

11. The angle between the rafters of a roof is 120°. What thrust is produced along the rafters when a 1,200-lb weight is hung from the peak?

12. A rope 100 ft long is stretched between a tree and a car. A man pulls with a force of 100 lb at right angles to and at the middle point of the rope, and moves this point 5.0 ft. Assuming no stretching of the rope, what is the tension on the rope? *Ans.* 500 lb.

13. What is the angle between two equal forces whose resultant is equal to one-half of one of the forces?

14. An airplane leaves the ground at an angle of 15.0°. If it continues in a straight line for half a mile, how high is it then above the level field? Over how much ground has it passed? *Ans.* 684 ft; 2,550 ft.

15. An airplane has an air speed of 150 mi/hr East in a wind which has a speed of 30 mi/hr at 60° South of East. What is the ground speed of the airplane?

16. Two forces of 24 tons and 11 tons, respectively, are applied to an object at a common point and have an included angle of 60°. Calculate the magnitude of their resultant and the angle it makes with the 24-ton force.

Ans. 31 tons; 18°.

EXPERIMENT

Concurrent Forces; Vectors

Apparatus: Pail of sand or other heavy weight; 10-ft length of clothesline; 2-kg spring balance; strong string; hooked weights; one pulley.

a. Suspend a rather heavy load (such as a pail of sand) from a support near the ceiling by means of a light, flexible rope (Fig. 14). We may

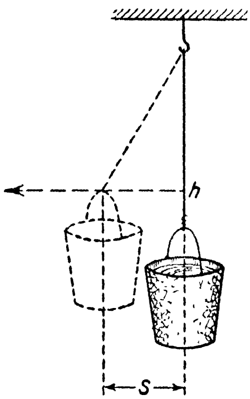


Fig. 14.—Finding the weight of an object by measuring a horizontal force.

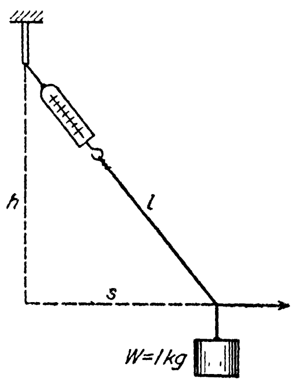


Fig. 15.—Finding the tension in a cord.

measure the weight while it is suspended even though no balance other than a 2,000-gm spring balance is available.

Attach the balance to the rope at h and pull horizontally as indicated by the arrow in Fig. 14 until the load is displaced a convenient measurable distance s. Record the reading of the balance, the distance s, and the length l of the rope. Using the method described in the second example in this chapter, compute the weight of the load.

b. Support a 1-kg weight by means of a cord and the 2-kg spring balance, and then pull the mass aside a distance s by pulling upon a horizontal string, as illustrated in Fig. 15. Measure h and s. Compute the tension in the supporting cord, and compare it with the reading of the spring balance. Note that this can be done without knowing the value of the horizontal force.

c. Attach two ends of a cord of length l to two nails (or other points of support) which are in the same horizontal line, as illustrated by Fig. 16. The length l is considerably greater than the distance ab. Values $ab = 60$ cm and $l = 100$ cm are convenient ones. Hang weight W (say 500 gm) at the middle of the string. What force does each string exert upon point P?

To verify this conclusion remove the string from point b and attach it to a spring balance as indicated by the dotted balance in Fig. 16. If the

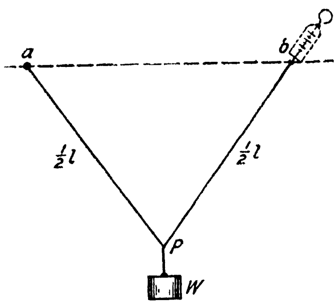


FIG. 16.—Finding the tension in a string supporting a weight at the middle.

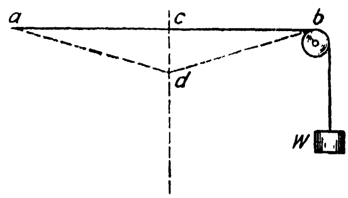
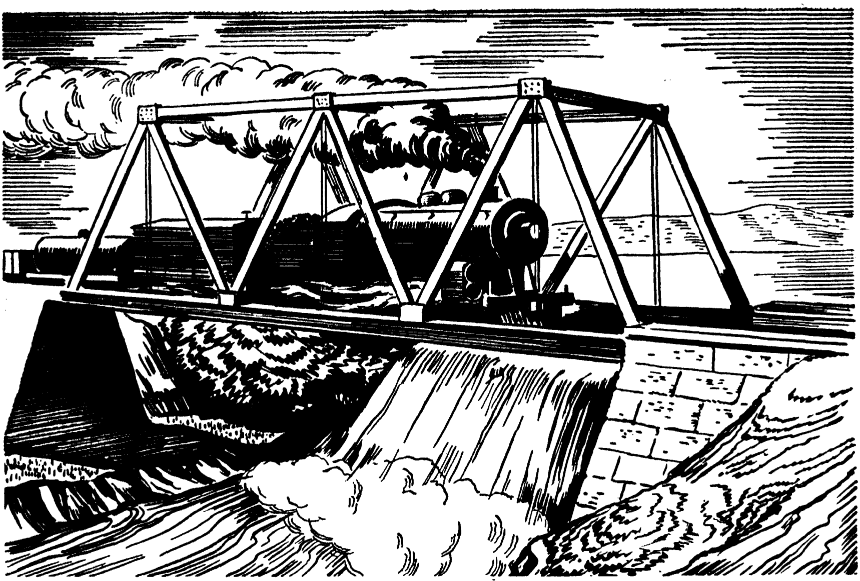


FIG. 17.—Weighing an object by measuring the tension in a cord.

spring balance were attached at a, should it read the same as before? Does it?

Suppose a student were to take the two ends in his hands and pull them apart, that is, virtually increasing the distance ab. How hard would he have to pull in order to lift the point P to within 1 cm of the line ab? Try it. Could the string be "straightened out" that way?

d. Could you weigh an unknown object by the method above? In Fig. 17 a cord passes from hook a over pulley b to an unknown weight W. Line ab is horizontal and is 100 cm long. If a 100-gm weight hung at c pulls the cord down to d (dotted lines), cd being 10 cm, what is the value of the unknown weight W? Obtain a similar set of data and compute W by this method.



CHAPTER 16

NONCONCURRENT FORCES; TORQUE

In the discussion of equilibrium thus far it has been assumed that the lines of action of all the forces intersect in a common point. For most objects this condition will not be realized. For a body to be in equilibrium under the action of a set of nonconcurrent forces more is required than the condition that the vector sum of the forces shall be zero. We must be concerned not only with the tendency of a force to produce linear motion but also with its effectiveness in the production of rotation. The same force applied at different places or in different directions produces greatly different rotational effects. The practical engineer is very much concerned with these effects and must make allowances for them in the design of his structures.

Two Conditions for Equilibrium. Consider an arrangement in which two equal, opposing forces act on a block, as in Fig. 1*a*. It is obvious that, if the block is originally at rest, it will remain so under the action of these two forces. We say, as before, that the vector sum of the forces is zero.

Now suppose that the two forces are applied as in Fig. 1*b*. The vector sum of the forces is again zero; yet it is plain that, under the action of these forces, the block will begin to rotate. In fact, when the vector sum of the applied forces is equal to zero, we can be sure only that the body as a whole will not have a linear acceleration; we cannot be sure that it will not start to rotate, hence complete equilibrium is not assured. In addition to the first condition necessary for equilibrium, then, there is a second one, a condition eliminating the possibility of a rotational

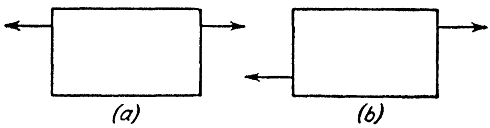


FIG. 1.—Equal and opposite forces produce equilibrium when they have a common line of action (*a*), but do not produce equilibrium when they do not have the same line of action (*b*).

acceleration. The example of Fig. 1*b* indicates that this second condition is concerned with the placement of the forces as well as their magnitudes and directions.

In order to understand the factors that determine the effectiveness of a force in producing rotational acceleration, consider the familiar problem

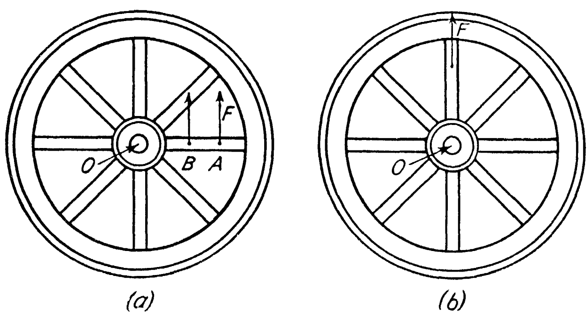


FIG. 2.—A force produces rotational acceleration if its line of action does not pass through the axis of rotation.

of turning a heavy wheel by pulling on a spoke (Fig. 2*a*). It is a matter of common experience that we can set the wheel in motion more quickly by applying a force F at the point A, than by applying the same force at B. The effect of a force in producing rotational acceleration is greater the farther the force is from the axis of rotation, but we should not fall into the elementary error of assuming that this distance is measured from the point of application of the force. In Fig. 2*b* the point of application of the force is just as far from the axle as it was when applied at A in Fig. 2*a*, but now there is no rotational acceleration; F merely pulls the wheel upward. Though the magnitude of the force, its direction, and the distance of its point of application from the axis are the same in the two examples, rotational acceleration is produced in one case and not in the other. The point of application of the force is clearly not the deciding factor.

Moment Arm. The factor that determines the tendency of a force to produce rotational acceleration is *the perpendicular distance from the axis of rotation to the line of action of the force.* We call this distance the *moment arm* of the force. In Fig. 3, the moment arm of the force F is indicated by OP. The line of action of the force is a mere geometrical construction and may be extended indefinitely either way in order to intersect the perpendicular OP. It has nothing to do with the length of the force vector. We now see why the force F in Fig. 2*b* produces no

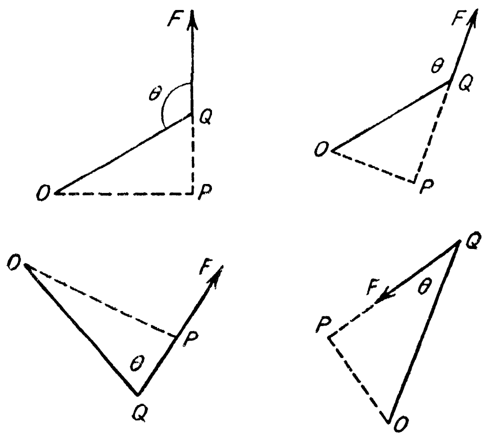


FIG. 3.—Measurement of moment arm.

rotation. Its line of action passes directly through the axis of rotation and the moment arm is therefore zero. The same force F in Fig. 2*a* has the moment arm OA and, therefore, tends to cause rotation.

For a fixed moment arm, the greater the force the greater also is the tendency to produce rotational acceleration. The two quantities, force and moment arm, are of equal importance. Analysis shows that they can be combined into a single quantity, *torque* (also called *moment of force*), which measures the tendency to produce rotational acceleration. Torque will be represented by the symbol L.

Definition of Torque. The torque (moment of force) about any chosen axis is the product of the force and its moment arm. Since torque is the product of a force and a distance, its usual unit in the British system is the *pound-foot.* The inversion of these units from the familiar foot-pound of work serves to call attention to the fact that we are using a unit of torque and not work, although they both have actually the same dimensions.

It is necessary to indicate clearly the direction of the angular acceleration that the torque tends to produce. For example, the torques in

Fig. 3 tend to produce counterclockwise accelerations about O, while the torque in Fig. 4 tends to produce a clockwise acceleration. One may refer to these torques as positive and negative, respectively. Note that a given force may produce a clockwise torque about one axis, but a counterclockwise torque about another axis. The direction of a torque is not known from the direction of the force alone.

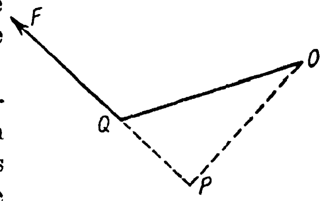


FIG. 4.—A clockwise torque.

Concurrent and Nonconcurrent Forces. Concurrent forces are forces whose lines of action intersect in a common point. If an axis is selected passing through this point, the torque produced by each force of such a set is zero, hence a consideration of torque is not necessary in the study of a set of concurrent forces in equilibrium.

For a set of nonconcurrent forces, there exists no single axis about which no torque is produced by any of the forces. In studying a set of nonconcurrent forces in equilibrium, therefore, it is essential to take into account the relation existing among the torques produced by such a set of forces. This relation is expressed in the second condition for equilibrium.

Second Condition for Equilibrium. For an object to be in equilibrium, it is necessary that the algebraic sum of the torques (about any axis) acting on it be zero. This statement is known as the second condition for equilibrium. It may be represented by the equation

$$\Sigma L = 0 \tag{1}$$

The symbol Σ means "the sum of."

In the first and second conditions for equilibrium we have a complete system for solving problems in statics. If the first condition is satisfied, the vector sum of the forces is zero, and no translational acceleration is produced. If the second condition is satisfied, the algebraic sum of the torques is zero, and there is no rotational acceleration. This does not mean that there is no motion, but only that the forces applied to the body produce no *change* in its motion. While in equilibrium, it may have a uniform motion including both translation and rotation.

Center of Gravity. It can be proved mathematically that for every body, no matter how irregular its shape, there exists a point such that the entire weight of the body may be considered to be concentrated at that point, which is called the *center of gravity*. If a single force could be applied at this point, it would support the object in equilibrium, no matter what its position.

Example: A uniform bar, 9 ft long and weighing 5 lb, is supported by a fulcrum 3 ft from one end as in Fig 5. If a 12-lb load is hung from the left end, what downward pull at the right end is necessary to hold the bar in equilibrium? With what force does the fulcrum push up against the bar?

Consider the bar as an object in equilibrium. The first step is to indicate clearly all the forces that act on it. The weight of the bar, 5.0 lb, can be considered to be concentrated at its middle. A 12-lb force acts downward at the left end of the bar, a force R acts upward at the fulcrum, and there is an unknown downward force F at the right end.

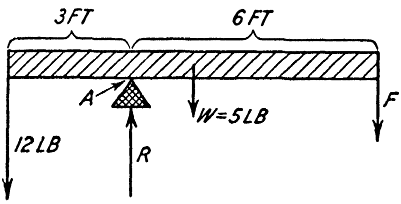


FIG. 5.—Finding the forces acting on a lever.

The first condition for equilibrium indicates that the vector sum of the forces applied to the bar is zero, or that

$$R - 12 \text{ lb} - 5.0 \text{ lb} - F = 0.$$

Without further information we certainly cannot solve this equation, since it has two unknown quantities in it, R and F. Let us set it aside for a moment and employ the second condition for equilibrium, calculating the torques about some axis and equating their algebraic sum to zero. The first thing we must do is to select an axis from which to measure moment arms. This chosen axis need not be any real axle or fulcrum; it may be an axis through any point desired. The wise choice of some particular axis, however, often shortens the arithmetical work.

We shall choose an axis through the point A about which to calculate all the torques. Beginning at the left end of the bar, we have (12 lb)(3.0 ft) = 36 lb-ft of torque, counterclockwise about A. Next, we see that the force R produces no torque, since its line of action passes through the point A. (Is it clear now why we decided to take A as an axis?) Third, the torque produced by the weight of the bar W is (5.0 lb) (1.5 ft) = 7.5 lb-ft, clockwise. Finally, F produces a torque (F)(6.0 ft), clockwise.

Taking the counterclockwise torque as positive and clockwise torque as negative and equating the algebraic sum of all the torques to zero, we write,

$$(12 \text{ lb})(3.0 \text{ ft}) + (R)(0) - (5.0 \text{ lb})(1.5 \text{ ft}) - F(6.0 \text{ ft})$$
$$= 36 \text{ lb-ft} + 0 - 7.5 \text{ lb-ft} - F(6.0 \text{ ft}) = 0$$
$$F(6.0 \text{ ft}) = 28.5 \text{ lb-ft}$$
$$F = 4.8 \text{ lb}$$

Substituting this value in the equation obtained from the first condition for equilibrium, we find $R - 12$ lb $- 5.0$ lb $- 4.8$ lb $= 0$, or $R = 21.8$ lb.

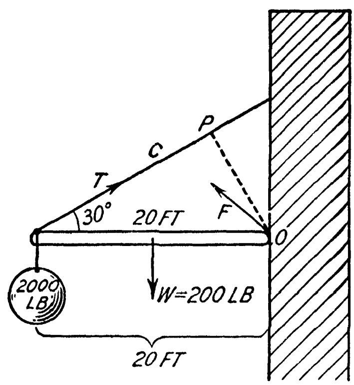


FIG. 6.—The forces acting on a horizontal beam.

Example: A chain C (Fig. 6) helps to support a uniform 200-lb beam, 20 ft long, one end of which is hinged at the wall and the other end of which supports a 1-ton load. The chain makes an angle of 30° with the beam, which is horizontal. Determine the tension in the chain.

Since all the known forces act on the 20-ft beam, let us consider it as the object in equilibrium. In addition to the 200- and 2,000-lb forces straight down, there is the pull of the chain on the beam, and the force F which the hinge exerts on the beam at the wall. Let us not make the mistake of assuming that the force at the hinge is straight up, or straight along the beam. A little thought will convince us that the hinge must be pushing both up and out on the beam. The exact direction of this force, as well as its magnitude, is unknown. The second condition for equilibrium is an excellent tool to employ in such a situation, for if we use an axis through the point O as the axis about which to take moments, the unknown force at the hinge has no moment arm and, therefore, causes no torque. The remarkable result is that we can determine the tension T in the chain without knowing either the magnitude or the direction of the force at O.

The torques about O as an axis are, respectively,

$$(200 \text{ lb})(10 \text{ ft}) = 2{,}000 \text{ lb-ft (counterclockwise)}$$
$$(2{,}000 \text{ lb})(20 \text{ ft}) = 40{,}000 \text{ lb-ft (counterclockwise)}$$
$$(T)(20 \text{ ft}) \sin 30° = (T)(10 \text{ ft}) \text{ (clockwise)}$$

[Note: The moment arm of T is $\overline{OP} = (20 \text{ ft}) \sin 30° = 10$ ft.] Then

$$-(T)(10 \text{ ft}) + 2{,}000 \text{ lb-ft} + 40{,}000 \text{ lb-ft} + (F)(0) = 0$$

so that

$$T = 4{,}200 \text{ lb} = 2.1 \text{ tons}$$

The problem of finding the magnitude and direction of the force at the hinge is left to the student. Suggestion: apply the first condition for equilibrium.

The trick just used in removing the unknown force from the problem by taking torques about the hinge as an axis is a standard device in statics. The student should always be on the lookout for the opportunity to side-step (temporarily) a troublesome unknown force by selecting an axis of torques that lies on the line of action of the unknown force he wishes to avoid.

Couples. In general, the application of one or more forces to an object results in both translational and rotational acceleration. An exception to this is the case in which a single force is applied along a line passing through the center of gravity of the object, in which case there is no rotational acceleration. Another special case is the one in which two equal and opposite forces are applied to the object as in Fig. 1*b*. In this case there is no translational acceleration. Such a pair of forces, resulting in a torque alone, is called a *couple*. The torque produced by a couple is independent of the position of the axis and is equal to the product of one of the forces and the perpendicular distance between them.

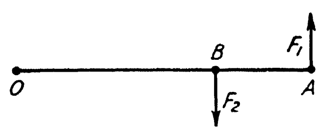


FIG. 7.—Two equal and opposite forces not in the same straight line constitute a couple.

As an example, consider the torque produced by the couple shown in Fig. 7. About the axis O, the torque produced by F_1 is $F_1(\overline{OA})$, and that by $F_2 = -F_2(\overline{OB})$. Since $F_1 = F_2 = F$ the total torque is $F(\overline{OA}) - F(\overline{OB}) = F(\overline{OA} - \overline{OB}) = F(\overline{AB})$. This verifies the statement that the torque produced by a couple is the product of one (either) of the forces and the perpendicular distance between them, a product independent of the location of the axis. A couple cannot be balanced by a single force but only by the application of an equal and opposite couple.

SUMMARY

The *torque* produced by a force is equal to the product of the force and its moment arm.

The *moment arm* of a force is the perpendicular distance from the axis to the line of action of the force.

For an object to be in equilibrium it is necessary (1) that the vector sum of the forces applied to it be zero, and (2) that the algebraic sum of the torques (about any axis) acting on it be zero.

The *center of gravity* of a body is the point at which its weight may be considered to act.

A *couple* consists of two forces of equal magnitude and opposite direction. The torque produced by a couple is equal to the magnitude of one (either) of the forces times the perpendicular distance between them.

PROBLEMS

1. A boy exerts a downward force of 30 lb on a horizontal pump handle at a point 2.0 ft from the pivot. (*a*) What torque is produced? (*b*) What is the torque when the handle makes an angle of 60° with the horizontal?

2. The diameter of a steering wheel is 18.0 in. If the driver exerts a tangential force of 1.0 lb with each hand in turning the wheel, what is the torque? *Ans.* 1.5 lb-ft.

3. In a human jaw, the distance from the pivots to the front teeth is 4.0 in., and the muscles are attached at points 1.5 in. from the pivots. What force must the muscles exert to cause a biting force of 100 lb (*a*) with the front teeth? (*b*) with the back teeth, which are only 2.0 in. from the pivots?

4. A compression of 5.0 lb is applied to the handles of a nutcracker at a distance of 6.0 in. from the pivot. If a nut is 1.0 in. from the pivot, what force does it withstand if it fails to crack? *Ans.* 30 lb.

5. A uniform bridge 100 ft long weighs 10,000 lb. If a 5,000-lb truck is stalled 25 ft from one end, what total force is supported by each of the piers at the ends of the bridge, if they represent the only supports?

6. A man and a boy carry a 90-lb uniform pole 12 ft in length. If the boy supports one end, where must the man hold the pole in order to carry two-thirds of the load? *Ans.* 3.0 ft from the end.

7. With what horizontal force must one push on the upper edge of a 500-lb block of stone whose height is 4.0 ft and whose base is 2.0 ft square, in order to tip it?

8. The weight supported by each of the front wheels of an automobile is 600 lb, while each of the back wheels supports 500 lb. If the distance between front and rear axles is 100 in., what is the horizontal distance of the center of gravity from the front axle? *Ans.* 45.5 in. from front axle.

9. The center of gravity of a log is 6.0 ft from one end. The log is 15 ft long and weighs 150 lb. What vertical force must be applied at each end in order to support it (*a*) horizontally? (*b*) at 30° from the horizontal with the center of gravity nearer the lower end?

10. A 100-lb ladder rests against a smooth wall at a point 15 ft above the ground. If the ladder is 20 ft long and its center of gravity is 8.0 ft from the lower end, what must be the force of friction at the lower end in order to prevent slipping? What is the coefficient of friction if the ladder is at the point of slipping? *Ans.* 35 lb; 0.35.

EXPERIMENT

Nonconcurrent Forces; Torques

Apparatus: Nonuniform board; two spring balances; meter stick; weight.

When forces act upon an extended object their lines of action do not, in general, intersect in a single point. To describe the equilibrium state of such a body we must consider both conditions for equilibrium.

In this experiment we shall consider the forces and torques that act upon a nonuniform board. Since the board is nonuniform, its center of gravity is not necessarily at its mid-point.

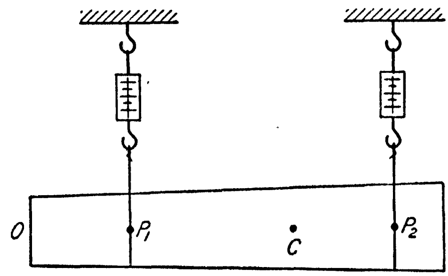


FIG. 8.—Finding the center of gravity and the weight of a nonuniform board.

a. In this part of the experiment we wish to find the weight of the board and the position of the center of gravity. Support the board by means of two spring balances as shown in Fig. 8. Three forces act upon the board: the weight W acting downward at the center of gravity; and the forces F_1 and F_2, which the balances exert upward. Record in a table the readings of the balances and the positions P_1 and P_2 (measured

F_1	F_2	P_1	P_2	$F_1 + F_2 = W$	$F_1(\overline{OP}_1)$	$F_2(OP_2)$	$F_1(\overline{OP}_1) + F_2(\overline{OP}_2)$

from one end O of the board) at which the upward forces act. Take a series of such readings for various positions of P_1 and P_2. From the first condition for equilibrium what do we conclude about the various values of F_1 and F_2? Do our results justify this conclusion? What is the weight of the board? To find the position of the center of gravity of the board we shall consider torques about O as an axis. The forces F_1 and F_2 both

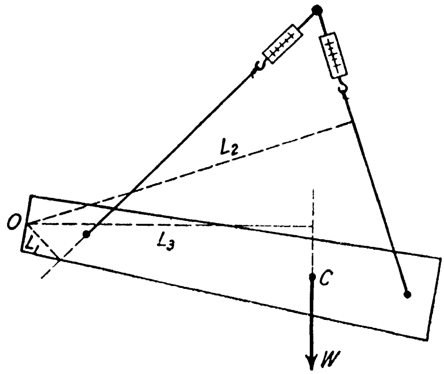


FIG. 9.—Finding the center of gravity of a nonuniform board.

produce counterclockwise torques while W produces a clockwise torque. Therefore

$$F_1(\overline{OP}_1) + F_2(\overline{OP}_2) - W(\overline{OC}) = 0$$

or

$$F_1(\overline{OP}_1) + F_2(\overline{OP}_2) = W(\overline{OC})$$

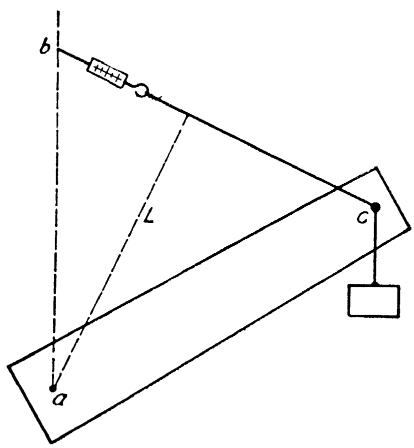


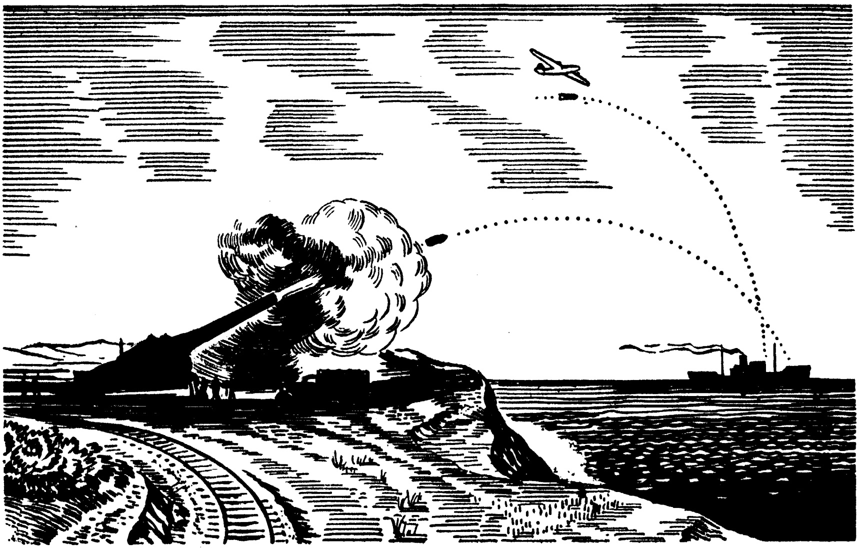
FIG. 10.—A simple crane.

Since the weight W always acts at the same point, the right-hand member of the equation is constant. Since we know W, we can use this equation to find $\overline{OC}$ and thus the position of C.

Hang an unknown weight from the board and adjust the supports so that the board is horizontal. Record the position of each force and the values of the three known forces. Compute the value of the unknown weight (1) by the use of the first condition for equilibrium and (2) by the use of the second condition for equilibrium. Compare the values thus obtained.

b. Support the board as shown in Fig. 9. Record the values of F_1 and F_2 and the value of W obtained in part (*a*). Measure and record the values of the moment arms L_1 and L_2, respectively, about O as an axis. Write the equation of torques about O and compute the value of L_3. How does the position of the center of gravity thus indicated compare with that found in part (*a*)? What is the position of the center of gravity relative to the point of support of the system?

c. If time permits, study the arrangement of the simple derrick shown in Fig. 10. Hang a known weight at *c*. Taking *a* as the axis of torques, measure and record the moment arm of each force. Record also the weight of the board and the reading of the spring balance. Write the equation of torques, considering the tension in the member *bc* as unknown. Solve the equation for the tension and compare it with the balance reading.



CHAPTER 17

PROJECTILE MOTION; MOMENTUM

The science of the motion of missiles that are thrown is called *ballistics*. The study of these motions and of the forces required to produce them, as well as of the forces set up when projectiles strike their targets, is of tremendous importance in the design of all instruments of war, from the soldier's rifle to a naval gun and from the private's helmet to the armor of a tank or battleship. The law of conservation of momentum is used by the ordnance designer both in problems involving the guns and ammunition that shoot the projectiles and in the determination of the forces that are brought into being when the projectiles are stopped.

Projectile Motion. A projectile may be considered as any body which is given an initial velocity in any direction and which is then allowed to move under the influence of gravity. The velocity of the projectile at any instant can be thought of as made up of two parts or components: a horizontal velocity and a vertical velocity. The effect of gravity on the projectile is to change only the vertical velocity while the horizontal velocity remains constant as long as the projectile is moving, if air resistance is neglected.

Suppose we ask ourselves how a stone will move if it is thrown horizontally at a speed of 50 ft/sec. Neglecting air resistance, the stone will travel with a constant horizontal speed of 50 ft/sec until it strikes something. At the same time it will execute the vertical motion of an object falling from rest; that is, beginning with a vertical speed of zero, it will acquire additional downward speed at the rate of 32 ft/sec in each second. It will fall 16 ft during the first second, 48 ft during the next, 80 ft during the third, and so on, just as if it had no horizontal motion. Its progress during the first three seconds is illustrated in Fig. 1. At A the stone has no vertical speed; at B (after 1 sec) its vertical speed is 32 ft/sec; at C, 64 ft/sec; and at D, 96 ft/sec. The curved line $ABCD$ in Fig. 1 is the path that the stone follows and the arrows at B, C, and D represent the velocities at those places. Note that the horizontal arrows are all the same length, indicating the constant horizontal speed while the vertical arrows increase in length to indicate the increasing vertical speed. The vertical arrow at C is twice as long as that at B while that at D is three times as long.

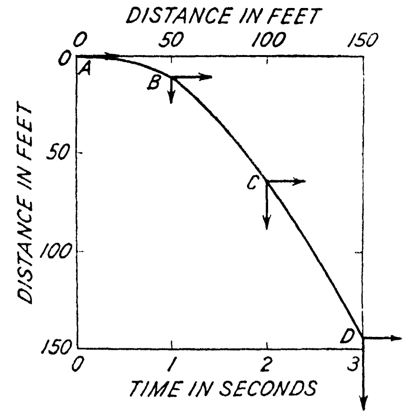


FIG. 1.—Path of a stone thrown horizontally with a speed of 50 ft/sec.

The curve shown in Fig. 1 is called a parabola. As has been indicated, it is traced by the motion of a projectile that executes simultaneously a uniform motion (horizontal) and a uniformly accelerated motion (vertical).

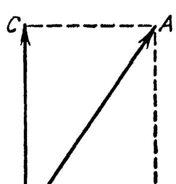


FIG. 2.—Components of velocity.

No matter what may be the initial direction of motion of the projectile, its motion may be broken up into horizontal and vertical parts, which are independent of each other. Suppose a stone is thrown with a speed of 100 ft/sec in a direction 30° above the horizontal. This velocity may be broken up into horizontal and vertical components as shown in Fig. 2. The initial speed in the given direction is represented by the vector OA, but an object that had the *simultaneous* vertical and horizontal speeds represented by OC and OB would follow exactly the same path along the direction OA. In discussing the motion of the stone we may use either the whole speed in the direction OA or the horizontal and vertical parts of the motion. The latter viewpoint simplifies the problem.

Referring to Fig. 3, we find the initial horizontal speed v_h to be (100 ft/sec) cos 30° = (100 ft/sec)(0.866) = 86.6 ft/sec, and the initial vertical speed v_1 to be (100 ft/sec) sin 30° = (100 ft/sec)(0.50) = 50 ft/sec. The problem thus reduces to one of uniform horizontal motion and uniformly accelerated vertical motion. We may ask the distance s the stone rises and its horizontal range.

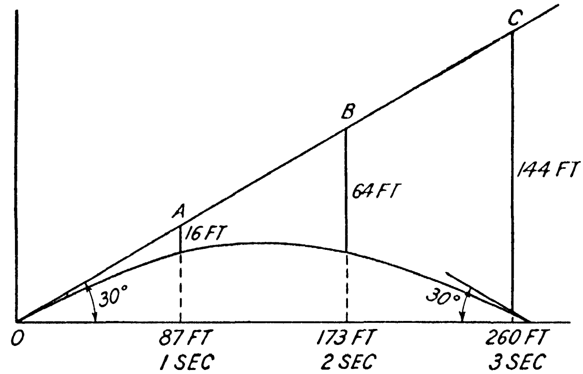


FIG. 3.—Path of a projectile fired at an angle of 30° above the horizontal with an initial speed of 100 ft/sec. Air resistance is neglected. The projectile strikes with a speed equal to the initial speed and at an angle of 30° above the horizontal.

Using Eq. (5), Chap. 10, as applied to the vertical motion,

$$v_2^2 - v_1^2 = 2as; \qquad v_1 = 50 \text{ ft/sec}; \qquad v_2 = 0; \qquad a = -32 \text{ ft/sec}^2$$

$$0 - (50 \text{ ft/sec})^2 = 2(-32 \text{ ft/sec}^2)s$$

$$s = \frac{2{,}500 \text{ ft}^2/\text{sec}^2}{64 \text{ ft/sec}^2} = 39 \text{ ft}$$

The time required to reach this highest point is, from Eq. (3), Chap. 10,

$$v_2 - v_1 = at$$

$$0 - 50 \text{ ft/sec} = (-32 \text{ ft/sec}^2)t$$

$$t = \frac{50 \text{ ft/sec}}{32 \text{ ft/sec}^2} = 1.56 \text{ sec}$$

An equal time will be required for the stone to return to the surface. Hence the time t' elapsed before the stone strikes the surface is

$$t' = 2t = 2 \times 1.56 \text{ sec} = 3.12 \text{ sec}$$

During all this time the stone travels horizontally with a uniform speed of 86.6 ft/sec. The horizontal range R is therefore

$$R = v_ht' = (86.6 \text{ ft/sec})(3.12 \text{ sec}) = 271 \text{ ft}$$

The motion of any projectile, neglecting air resistance, may be treated in this same manner no matter what may be the initial speed and angle of projection. The initial velocity is resolved into vertical and horizontal components and the two are considered separately.

In Fig. 3 we note that the path may be found by considering a uniform motion in the initial direction OC and finding the distance the stone has fallen from this path at each instant. In 1 sec under the action of gravity, the stone falls 16 ft; hence at the end of 1 sec it is 16 ft below A; in 2 sec it falls 64 ft and hence is 64 ft below B, and so on.

In this discussion of the motion of projectiles we have neglected the resistance of the air. For high-speed projectiles the air resistance is no small factor. It reduces the height of flight, the range of the projectile, and the speed of the projectile when it strikes. Figure 4 shows an example of such influence. The dotted curve is the path that the projectile would follow if there were no air resistance, while the solid line shows an actual path. Very long-range guns shoot the projectile at such an angle that a considerable part of the path is in the high

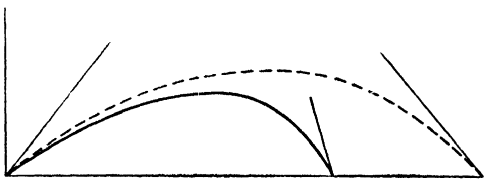


FIG. 4.—Path of a projectile. The dotted curve represents the path that would be followed if there were no air resistance, while the solid line is an actual path. The maximum height, range, and striking speed are decreased, while the striking angle is increased.

atmosphere where air resistance is very small. In the absence of air resistance the maximum horizontal range of a gun is attained when it is fired at an angle of 45° with the horizontal but, because of the change in path due to air resistance, the elevation must be considerably greater than 45° in order to obtain maximum horizontal range. The optimum angle depends upon the size, shape, and speed of the projectile.

During the First World War a gun was developed by the German Army with a range of 75 mi. The initial speed of the projectile was almost 1 mi/sec at an angle of 50° with the horizontal. The shell reached a maximum height of about 27 mi and more than two-thirds of the path was above 13 mi. At such altitude the air resistance is so small that the path is essentially the same as that for no friction. The striking speed of the shell was less than half the initial speed.

In determining the direction and angle of fire of a large gun many factors must be considered if the firing is to be accurate. Among these factors are wind, barometric pressure, temperature, rotation of the earth, shape of the shell, and the number of times the gun has been fired.

Momentum. If a passenger car traveling at a speed of 20 mi/hr strikes a telephone pole, the damage will probably be minor but, if a loaded truck traveling at the same speed strikes it, the damage is much greater. If the speed of the passenger car is 40 mi/hr instead of 20 mi/hr, the damage is also greatly increased. Evidently, the result depends jointly upon the speed and mass of the moving object. The product of the *mass* and *velocity* of a body is called its *momentum*. The defining equation for momentum is

$$M = mv \tag{1}$$

where M is the momentum, m the mass, and v the velocity. Every object in motion has momentum.

In the British system we use W/g in place of m and the equation for momentum becomes

$$M = \frac{W}{g} v \tag{2}$$

where W is the weight and g is the acceleration due to gravity. No special name is assigned to the unit of momentum but it is made up as a composite unit. Since W is in pounds, g in feet per second per second, and v in feet per second, the unit of momentum becomes $\frac{\text{lb}}{\text{ft/sec}^2}$ (ft/sec) = lb-sec. The absolute cgs unit is the gram-centimeter per second.

Example: What is the momentum of a 100-lb shell as it leaves the gun with a speed of 1,200 ft/sec?

$$M = \frac{W}{g} v = \frac{100 \text{ lb}}{32 \text{ ft/sec}^2} (1{,}200 \text{ ft/sec}) = 3{,}800 \text{ lb-sec}$$

Momentum is a vector quantity, its direction being that of the velocity. To find the momentum of a system of two or more bodies we must add their momenta vectorially. Consider two 4-lb balls moving toward each other with equal speeds of 4 ft/sec as shown in Fig. 5. The momentum of A is $M_A = \left(\frac{4 \text{ lb}}{32 \text{ ft/sec}^2}\right)$ (4 ft/sec) = 0.5 lb-sec to the right while that of B is similarly 0.5 lb-sec to the left. The vector sum of the two is zero and hence the momentum of the *system* is zero.

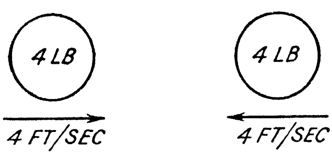


FIG. 5.—Two balls of equal mass having equal but opposite speeds. The momentum of the system is zero.

Conservation of Momentum. According to Newton's first law of motion, the velocity of a body does not change unless it is acted upon by a net force. Since the mass of the body is constant, we find that the momentum does not change unless an external force acts upon the body. The statement that the momentum of a body, or system of bodies, does not change except when an external force is applied, is known as the *law of conservation of momentum*. The use of the law enables us to explain simply the behavior of common objects.

If an external force does act upon a system of bodies, the momentum of the system is changed but, in the process, some other set of bodies must gain (or lose) an amount of momentum equal to that lost (or gained) by the system. In every process where velocity is changed the momentum lost by one body or set of bodies is equal to that gained by another body or set of bodies.

$$\text{Momentum lost} = \text{momentum gained} \tag{3}$$

Let us consider further the balls shown in Fig. 5. If they continue to move toward each other, they will collide and in the collision each will exert a force on the other. The momentum of the system of two balls is zero before the impact. By the law of conservation of momentum it must be zero after the impact. If the balls are elastic, they will rebound and the conservation law requires that the speeds of recoil shall be equal to each other (but not necessarily equal to the original speed) so that the momentum shall remain zero.

The recoil of a gun is an example of conservation of momentum. The momentum of gun and bullet is zero before the explosion. The bullet gains a forward momentum, and hence the gun must acquire an equal backward momentum so that the sum will remain zero.

Example: A 2-oz bullet is fired from a 10-lb gun with a speed of 2,000 ft/sec. What is the speed of recoil of the gun?

The momentum of the gun is equal and opposite to that of the bullet.

$$\frac{10\text{ lb}}{32\text{ ft/sec}^2}\,v = \frac{(2/16\text{ lb})}{32\text{ ft/sec}^2}\,(2{,}000\text{ ft/sec})$$

$$v = \left(\frac{2}{16}\right)\frac{(2{,}000\text{ ft/sec})}{10} = 25\text{ ft/sec}$$

In the firing of the gun, quite obviously, forces are exerted, one on the gun and the other on the projectile. These forces, however, are *internal*, that is, they are within the system of the gun and bullet that we considered. If we consider the bullet alone, the force becomes an external force and causes a change in momentum of the bullet but, in accordance with Newton's third law, an equal and opposite force acts on the gun giving it a momentum equal and opposite to that of the bullet.

Impulse. The change in momentum caused by an *external* force depends upon the amount of the force and also upon the time the force acts. From Newton's second law

$$F = ma$$

$$a = \frac{v_2 - v_1}{t}$$

$$F = m\,\frac{(v_2 - v_1)}{t}$$

$$Ft = mv_2 - mv_1 \tag{4}$$

Thus the product of the force and time is equal to the change in momentum. The product of force and time is called *impulse*. Equation (4) implies that no object can be stopped instantaneously and that the shorter the length of time required for stopping the greater must be the force. A bomb dropped from a height of several thousand feet has very great momentum. As it strikes the steel deck of a ship, it must be stopped in a very short time or it will penetrate the deck. The ordinary steel deck of a ship is unable to supply the force necessary to stop the bomb. Extremely large forces are involved in impacts where a rapidly moving body is stopped quickly.

SUMMARY

A *projectile* is an object which is given an initial velocity and which is then allowed to move under the action of gravity.

In projectile motion the vertical and horizontal motions may be treated separately. The horizontal motion is uniform while the vertical motion is uniformly accelerated if air resistance is neglected.

Momentum is the product of the mass and velocity of a body. It is a vector quantity.

$$M = mv = \frac{W}{g} v$$

Impulse is the product of a force and the time it acts. Impulse is equal to the change in momentum.

$$Ft = mv_2 - mv_1$$

The law of *conservation of momentum* states that the momentum of a body or system of bodies does not change unless an external force acts upon it.

QUESTIONS AND PROBLEMS

1. Why is the rear sight of a long-range rifle adjustable?

2. Why should a shotgun be held tightly against the shoulder when it is fired?

3. What is the momentum of a 160-lb shell if its speed is 2,000 ft/sec?

4. A bomb is dropped from an airplane traveling horizontally with a speed of 210 mi/hr (308 ft/sec). If the airplane is 2,000 ft above the ground, what will be the horizontal distance traversed by the bomb (neglecting air friction)? Where will the airplane be when the bomb reaches the ground, if its course is not changed? *Ans.* 3,4$\bar{5}$0 ft.

5. On an ordinary road surface the frictional force on a 3,000-lb car when the brakes are applied may be as high as 2,500 lb. What time will be required to stop the car with this force from a speed of 30 mi/hr (44 ft/sec)?

6. Find the horizontal range of a shell fired from a cannon with a muzzle velocity of 1,$\bar{2}$00 ft/sec at an angle of 30° above the horizontal.

Ans. 3$\bar{9}$,000 ft.

7. A 40-ton freight car moving with a speed of 15 mi/hr (22 ft/sec) runs into a stationary car of the same weight. If they move off together after the collision, what is their speed?

8. What is the recoil speed of a 9-lb rifle when it projects a 0.6-oz bullet with a speed of 2,400 ft/sec? *Ans.* 10 ft/sec.

9. Why do we seldom observe the recoil of a tightly held gun?

10. A machine gun fires 10 bullets per second into a target. Each bullet weighs 0.5 oz and has a speed of 2,400 ft/sec. Find the force necessary to hold the gun in position and that required to hold the target in position. *Ans.* 23 lb; 23 lb.

EXPERIMENT

Speed of a Rifle Bullet

Apparatus: Block of wood (4 by 4 by 18 in.) suspended by four string supports; 22-caliber rifle; shells; balance; meter stick.

One method that is used to determine the speed of a rifle bullet makes use of the law of conservation of momentum in the collision of the bullet with a block of wood suspended as a ballistic pendulum. If the pendulum is at rest before the impact, the initial momentum is that of the bullet alone, while after the impact the bullet and pendulum move together. Then

$$m_b v = (m_p + m_b)V$$

where m_b is the mass of the bullet, v is its speed before the impact, m_p is the mass of the pendulum, and V is its speed an instant after the impact. The mass of the bullet can be determined by weighing samples of bullets removed from the shell; that of the pendulum can be determined by direct weighing. The speed V cannot be measured directly but can be calculated readily.

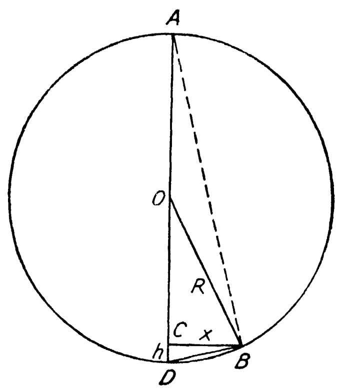


FIG. 6.—Finding the initial speed of a ballistic pendulum from the height to which it rises.

Immediately after the impact the pendulum has a kinetic energy $\frac{1}{2}m_pV^2$. The pendulum swings back until this kinetic energy has all been converted into potential energy as the pendulum rises a distance h. Then

$$\frac{1}{2}m_pV^2 = m_pgh$$
$$V = \sqrt{2gh}$$

Since h is small, a direct measurement is rather inaccurate, but it can be determined by measuring the horizontal distance x through which the block moves. The relation between h and x can be obtained from

Fig. 6, where O is a point of support and R is the length of the pendulum. The triangles ABC and BCD are similar right triangles. Hence

$$\frac{AC}{CB} = \frac{CB}{CD}$$

or

$$\frac{2R - h}{x} = \frac{x}{h}$$

$$2hR - h^2 = x^2$$

Since h is a small distance, its square is very small in comparison to the other terms in the equation and can be neglected. Hence, approximately

$$2hR = x^2$$

$$h = \frac{x^2}{2R}$$

Therefore, in order to determine the speed of the bullet, we must measure:

1. The length of the pendulum from the point of support to its center of gravity.
2. The mass of the bullet.
3. The mass of the pendulum.
4. The horizontal distance through which the pendulum moves.

Any block of wood of dimensions approximating those given may be used as the pendulum. It should be supported by four parallel strings whose length can be adjusted to level the block.

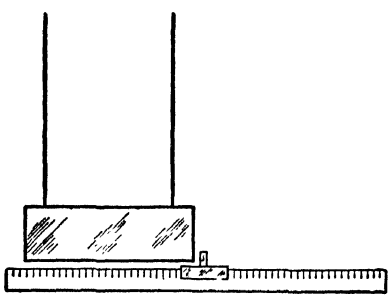
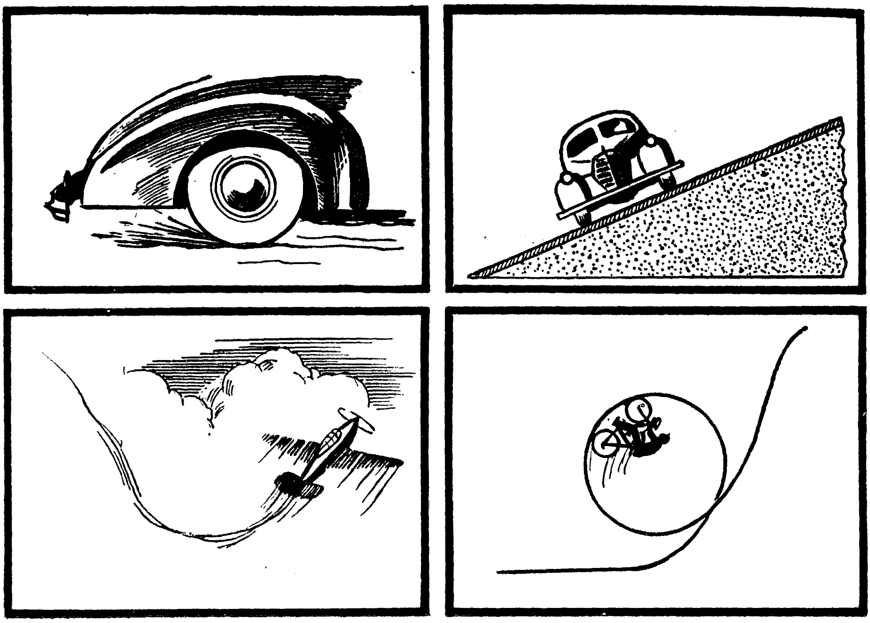


FIG. 7.—Ballistic pendulum, showing scale and rider for measuring the distance the pendulum moves.

The horizontal distance the block moves can be measured by mounting a meter stick horizontally below it and by placing on the stick a light cardboard rider, which rests against the block as shown in Fig. 7. The distance moved by the rider after the impact will be the horizontal distance x.

Fire three shots and use the average distance moved to calculate h. From this value calculate the speed of the pendulum and from it the speed of the bullet.

Compute the kinetic energy of the bullet and also that of the pendulum immediately after the impact. How do the two compare? What has become of the part that does not appear as kinetic energy of the pendulum?



CHAPTER 18

UNIFORM CIRCULAR MOTION

Motion along a straight line seems "natural"; no cause for such action is expected. However, if there is a change in the direction of the motion, some disturbing factor is at once assumed. A force must act to cause a change in the direction of a motion. The simplest type of motion in which the direction changes is uniform circular motion. This sort of motion is frequently found in practice, from the whirling of a stone on a string to the looping of a combat airplane.

Centripetal Force. When an object is moving in a circular path at constant speed, its velocity is changing. According to Newton's laws of motion, therefore, an unbalanced force is acting upon the object. This force, called the *centripetal force,* is directed toward the center of the circular path. Since the speed of motion is constant, the centripetal force serves to change only the *direction* of the motion. It is interesting to note that the only direction in which an unbalanced force can be applied to a moving object without changing its speed is at right angles to its direction of motion.

The magnitude of the centripetal force is given by the relation

$$F_c = \frac{W}{g}\frac{v^2}{r} \tag{1}$$

in which W is the weight of the moving object, v is its linear speed, g is the gravitational acceleration, and r is the radius of the circular path. The force is expressed in pounds if the other quantities are in the customary British units.

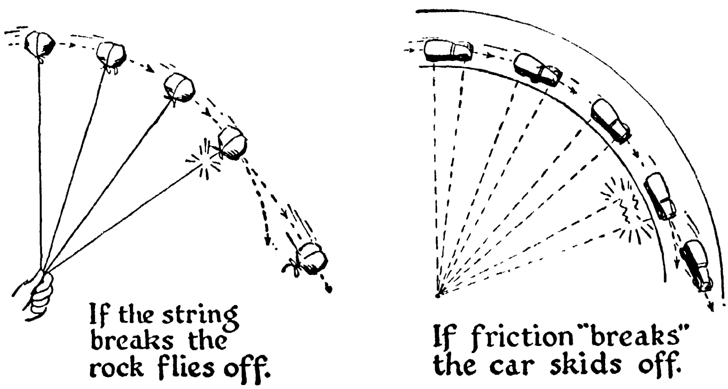


FIG. 1.

In Eq.(1), W/g can be replaced by its equivalent, the mass m of the moving body. Thus

$$f_c = \frac{mv^2}{r} \tag{2}$$

If m is in grams, v in centimeters per second and r in centimeters, the force is expressed in dynes.

An inspection of Eqs. (1) and (2) discloses that the centripetal force necessary to pull a body into a circular path is directly proportional to the square of the speed at which the body moves, while it is inversely proportional to the radius of the circular path. Suppose, for example, that a 10-lb. object is held in a circular path by a string 4 ft long. If the object moves at a constant speed of 8 ft/sec,

$$F_c = \frac{W}{g}\frac{v^2}{r} = \frac{(10 \text{ lb})(8 \text{ ft/sec})^2}{(32 \text{ ft/sec}^2)\ (4 \text{ ft})} = 5 \text{ lb}$$

If the speed is doubled, F_c increases to 20 lb. If, instead, the radius is decreased from 4 to 2 ft, F_c increases to 10 lb. If at any instant the string breaks, eliminating the centripetal force, the object will retain the velocity it has at the instant the string breaks, traveling at constant speed along a direction tangent to the circle. The direction taken by the sparks from an emery wheel is an illustration of this fact.

No Work Done by Centripetal Force. Work has been defined as the product of force and the displacement in the direction of the force. Since centripetal force acts at right angles to the direction of motion, there is no displacement in the direction of the centripetal force, and it accomplishes no work. Aside from the work done against friction, which has been neglected, no energy is expended on or by an object while it is moving at constant speed in a circular path. This fact can be verified much more simply by the observation that, if its speed is constant, its kinetic energy also is constant.

Action and Reaction. Newton's third law expresses the observation that for every force there is an opposite and equal force of reaction. When an object not free to move is acted upon by an external force, it is pushed or pulled out of its natural shape. As a consequence it exerts an *elastic* reaction in an attempt to resume its normal shape. On the other hand, the action of a force upon a free object results in an acceleration, a changing of its motion. By virtue of its tendency to continue a given state of motion, the object exerts an inertial reaction upon the agent of the accelerating force. It reacts, then, against the thing that changes its motion.

Just as the elastic reaction of a stretched body is equal and opposite to the stretching force, so the inertial reaction of an accelerated body is opposite and equal to the accelerating force. It should be remembered, however, that a force of reaction is exerted *by* the reacting object, not on it.

Centrifugal Reaction. A string that constrains an object to a circular path exerts on the object the centripetal force that changes its velocity. In reaction against this change of motion, the object pulls outward on the string with a force called the *centrifugal reaction.* This force, which is exerted by the object in its tendency to continue along a straight path, is just equal in magnitude to the inward (centripetal) force.

As the speed of a flywheel increases, the force needed to hold the parts of the wheel in circular motion increases with the square of the speed, as indicated by Eq. (1). Finally the cohesive forces between the molecules are no longer sufficient to do this, and the wheel disintegrates, the parts flying off along tangent directions like mud from an automobile tire. The stress is greatest near the center of the wheel, where the entire inward force must be sustained.

When a container full of liquid is being whirled at a uniform rate, the pail exerts an inward force on the liquid sufficient to keep it in circular motion (Fig. 2). The bottom of the pail presses on the layer of liquid next to it; that layer in turn exerts a force on the next; and so on. In each layer the pressure (force per unit area) must be the same all over the layer or the liquid will not remain in the layer. If the liquid is of uniform density, each element of volume of weight w in a given layer will experience an inward force $\frac{w}{g}\frac{v^2}{r}$ just great enough to maintain it in that layer and there will be no motion of the liquid from one layer to another. If, however, the layer is made up of a mixture of particles of different densities, the force required to maintain a given element of volume in the layer will depend upon the density of the liquid in that element. Since the inward force is the same on *all* the elements in a single layer, there will be a motion between the layers. For those parts which are less dense than the average the central force is greater than that necessary to hold them in the layer; hence they are forced inward. For the parts more dense than the average the force is insufficient to hold them in the circular path and they will move outward. As rotation continues, the parts of the mixture will be separated, with the least dense nearest the axis and the

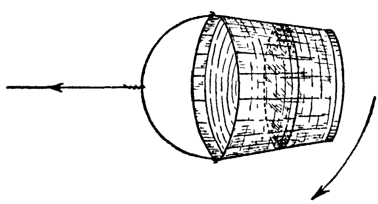


FIG. 2.—Centripetal force on a liquid. The principle of the centrifuge.

most dense farthest from the axis. This behavior is utilized in the centrifuge, a device for separating liquids of different densities. The cream separator is the most common example of the centrifuge but it is very commonly used to separate mixtures of liquids or mixtures of solid in liquid. Very high speed centrifuges may be used to separate gases of different densities.

Airplane test pilots sometimes pull out of a vertical dive at such high speed that the centripetal acceleration becomes several times as large as the gravitational acceleration. Under these circumstances, much of the blood may leave the pilot's brain and flow into the abdomen and legs. This sometimes causes the pilot to lose consciousness during the period of maximum acceleration. In an attempt to avoid fainting from this cause, test pilots often strap tight jackets around their bodies.

Centrifugal Governor. The speed of an engine can be controlled by centripetal force through a *governor* (Fig. 3). This device consists of a pair of masses C, C attached to arms hinged on a vertical spindle which rotates at a speed proportional to that of the engine. As the speed of rotation increases, the centripetal force necessary to maintain the circular motion of the balls is increased and they are lifted farther from the axis. This motion is used to actuate a valve V, decreasing the supply of steam or fuel. As the speed of the engine decreases, the balls descend, opening the throttle. Thus the engine speed may be kept reasonably constant under varying loads.

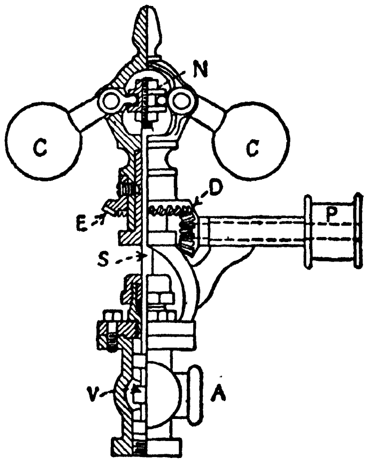


FIG. 3.—A centrifugal governor.

Why Curves Are Banked. A runner, in going around a sharp curve, leans inward to obtain the centripetal force that causes him to turn (Fig. 4*a*). The roadway exerts an upward force sufficient to sustain his

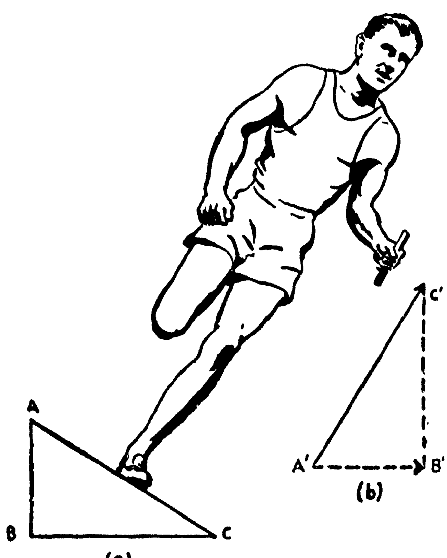


FIG. 4.—The advantage of banking curves.

weight, while at the same time it must supply a horizontal (centripetal) force. If the roadbed is flat, this horizontal force is frictional, so that it cannot be large enough to cause a sharp turn when the surface of the roadway is smooth. If the roadbed is tilted from the horizontal just enough to be perpendicular to the leaning runner, no frictional force is required.

As is shown in Fig. 4b, the force $A'C'$ exerted by the roadway is along the direction of the leaning runner. This force is equivalent to two forces: (1) the upward force $B'C'$ equal to the weight of the runner; (2) the (inward) centripetal force $A'B'$ necessary to cause the runner to turn. If the roadway is tilted as shown, it exerts only a perpendicular force and there is no tendency to slip.

It should be noticed that triangle $A'B'C'$ (in the diagram of forces) is similar to triangle ABC, since the corresponding angles in these two triangles are equal.

By virtue of this fact, we can write $\overline{AB}/\overline{BC} = \overline{A'B'}/\overline{B'C'}$. But $\overline{A'B'} = F_c = \frac{W}{g}\frac{v^2}{r}$, and $\overline{B'C'} = W$, so that $\overline{A'B'}/\overline{B'C'} = v^2/gr$, proving that

$$\frac{\overline{AB}}{\overline{BC}} = \frac{v^2}{gr}$$

where $\overline{AB}/\overline{BC}$ is the ratio of the elevation of the outer edge of the roadway to its horizontal width. Because the ratio $\overline{AB}/\overline{BC}$ depends upon the speed at which the curve is to be traversed, a roadway can be banked ideally for only one speed. At any other speed the force of friction will have to be depended upon to prevent slipping. The banking of highway curves, by eliminating this lateral force of friction on the tires, greatly reduces wear in addition to contributing to safety.

Example: A curve on a highway forms an arc whose radius is 150 ft. If the roadbed is 30 ft wide and its outer edge 4.0 ft higher than the inside edge, for what speed is it ideally banked?

$$\frac{\overline{AB}}{\overline{BC}} = \frac{v^2}{gr}, \qquad \text{hence} \qquad v^2 = \frac{(\overline{AB})(gr)}{\overline{BC}}$$

so that

$$v = \sqrt{\frac{(4.0 \text{ ft})(32 \text{ ft/sec}^2)(150 \text{ ft})}{30 \text{ ft}}} = 25 \text{ ft/sec}$$

SUMMARY

In *uniform circular motion* (*a*) the speed v is constant; (*b*) the direction of the motion is continually and uniformly changing; (*c*) the acceleration a_c is constant in magnitude and is directed toward the center of the circular path ($a_c = v^2/r$, where r is the radius of the circle).

The *centripetal force*, the inward force that causes the central acceleration, is given by

$$F_c = \frac{W}{g}\frac{v^2}{r} \text{ or } f_c = \frac{mv^2}{r}.$$

The *centrifugal reaction* is the outward force exerted *by* the moving object on the agent of its centripetal force. The magnitude of the centrifugal reaction is equal to that of the centripetal force.

The proper banking of a curve to eliminate the necessity for a sidewise frictional force is given by the relation $\overline{AB}/\overline{BC} = v^2/gr$, where $\overline{AB}/\overline{BC}$ is the ratio of the elevation of the outer edge of the roadway to its horizontal width, v is the speed for which the curve is banked, and r is its radius.

QUESTIONS AND PROBLEMS

1. Show that the units of v^2/r are those of acceleration.

2. A ball weighing 2.5 lb is whirled in a circular path at a speed of 12 ft/sec. If the radius of the circle is 5.7 ft, what is the centripetal force?
Ans. 2.0 lb.

3. At what speed must the ball of problem 2 be whirled in order to double the centripetal force?

4. Compute the minimum speed which a pail of water must have in order to swing in a vertical circle of radius 3.8 ft without splashing. *Ans.* 11 ft/sec.

5. Find the ratio $\overline{AB}/\overline{BC}$ for a curve to be traversed at 30 mi/hr, if r is 40 ft.

6. An aviator loops the loop in a circle 400 ft in diameter. If he is traveling 120 mi/hr, how many g's does he experience?
Ans. 4.8 g's in addition to gravity.

7. A 3,200-lb automobile is moving 10 ft/sec on a level circular track having a radius of 100 ft. What coefficient of friction is necessary to prevent the car from skidding?

8. At the equator the centripetal acceleration is about 3 cm/sec^2. How fast would the earth have to turn to make the apparent weight of a body zero?
Ans. 18 rev/day.

EXPERIMENT

Centripetal and Centrifugal Forces

Apparatus: Centripetal force apparatus; meter stick; hooked weights.

The apparatus to be used in this experiment is shown in Fig. 5, G being a glass tube 15 cm long, through which is threaded a string about 125 cm in length.

Bodies m_1 and m_2 of unequal weight are attached to the ends of the string; m_1 being a rubber ball, m_2 being a heavier hooked weight.

Hold the tube horizontally with m_1 and m_2 nearly equidistant from the tube. Since m_2 is greater in weight than m_1, the forces applied to the ends of the string are not balanced; m_2 will go down, drawing m_1 up. How then can m_1 exert such a force upon the string, and therefore upon m_2, that m_2 will go up instead of down? The answer is that, if m_1 is twirled in a horizontal, circular path (Fig. 5), the centripetal force necessary to constrain it to this path will be supplied (through the action of the string) by the weight of m_2. Similarly, the *centrifugal reaction* of m_1, which is the force with which it pulls outward on the string, serves to support m_2 against the action of gravity.

The centripetal force on m_1, which is equal (and opposite) to the centrifugal reaction of m_1 on the string, is given by

$$F_c = \frac{W_1 v^2}{gr}$$

where W_1 is the weight of m_1, v is its speed, and r is the radius of the circular path.

FIG. 5.—Demonstrating centripetal force.

If m_2 is just supported by the outward pull of m_1 on the string, we can write $F_c = W_2$. Our experiment, therefore, will consist of measuring v and r, and using their values and those of W_1 and g to compute F_c, which we will compare with W_2, the weight of m_2. Assuming that W_2 is the correct value for F_c, our final step is to determine the percentage error in our experimentally determined value of F_c.

The experimental procedure is as follows: Hold the tube G in a vertical position and twirl m_1 above your head in a horizontal plane in such a manner that m_2 is supported a few centimeters below the tube. The motion should be begun with the tube at arm's length and *above* the head. After it is under control, more rapid revolution will increase r, the radius of the circle, until it approximates 100 cm and m_1 swings beyond the head —a consideration of some importance, alas! Try to achieve this with as little motion of the tube as is possible. While m_2 remains at a fixed position, have another member of the class count the number of revolutions made by m_1 in, say, 1 min. Next, grasp the string at the lower end of the tube in order to secure the position of m_2, then measure the radius of motion of m_1, calling it r. Record N, the number of revolutions in 1 min, r, the radius, and the values of m_1 and m_2. Next, compute T, the time (in seconds) of one revolution of m_1, and use it to determine v by observing that $v = 2\pi r/T$.

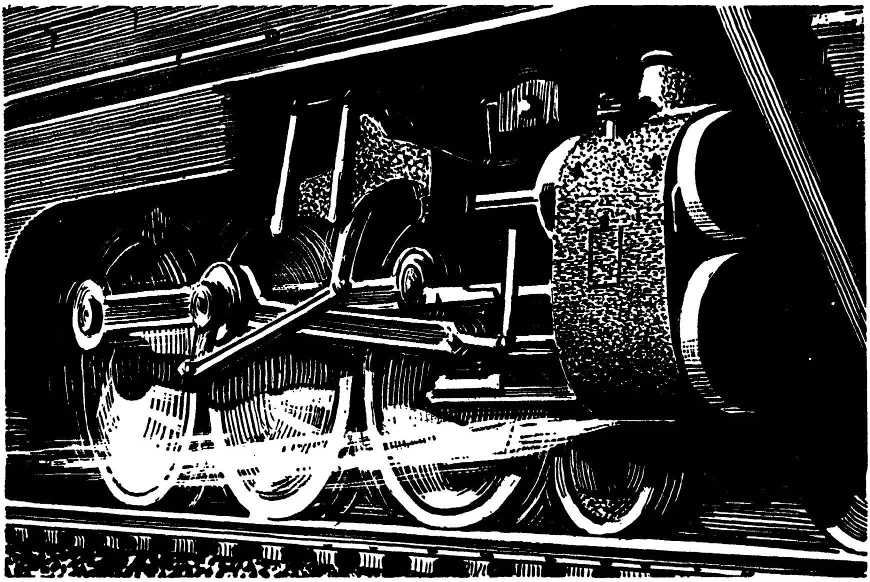
Finally, compute F_c and determine the percentage error, using W_2 as the correct value. Repeat the experiment and computations several times, using circular paths of different radii. List your data as in the table following:

Trial	N	r	T	v	m_1	m_2	F_c	Error, %
1								
2								
3								
4								

Are your values for F_c consistently greater (or smaller) than W_2, or do you obtain values both above and below the correct value? Does this suggest that your error is predominantly *systematic* or is it *erratic?*

Where is the centripetal force in this experiment? Which of the following statements are correct?

1. The force pulling upward upon m_2 is centrifugal force.
2. The force exerted by m_1 on the string is centrifugal force.
3. The centrifugal force pulls outward on m_1.
4. There is no outward force *on* m_1.
5. The centripetal force is the force exerted by the string upon m_1.



CHAPTER 19

ROTARY MOTION; TORQUE; MOMENT OF INERTIA

In almost all engines or motors, energy is transformed from heat or electrical energy to mechanical energy by turning a shaft or wheel. In order to study these machines it is necessary to understand the action of torque in changing *angular motion.*

Moment of Inertia. It has been found that a force is necessary to change the motion of a body, that is, to produce an acceleration. A greater force is required to give an acceleration to a large mass than to cause the same acceleration in a smaller one. If a body is to be caused to rotate about an axis, a *torque* about that axis must be applied. The angular acceleration produced by a given torque depends not only upon the mass of the rotating body but also upon the *distribution of mass* with respect to the axis. In Fig. 1 a bar with adjustable weights W_1 and W_2 is supported on an axle. If a string is wrapped around the axle and a weight W is hung on the string, the axle and rod will rotate. The rate of gain in speed of rotation will be much greater when W_1 and W_2 are near the axle, as shown by the dots, than when they are near the ends of the rod. The mass is not changed by this shift but the distribution of mass is altered and the *rotational inertia* is changed.

If a small body of mass m (W/g) is located at a distance r from the axis its *moment of inertia* I (also called *rotational inertia*) is the product of the mass and the square of the radius. Symbolically,

$$I = mr^2 = \frac{W}{g} r^2$$

In an extended body each particle of matter in the body contributes to the moment of inertia an amount $(W/g)r^2$. The moment of inertia I is the sum of the contributions of the individual elements.

$$I = \frac{W_1}{g} r_1^2 + \frac{W_2}{g} r_2^2 + \frac{W_3}{g} r_3^2 + \cdots$$

$$I = \sum\left(\frac{W}{g} r^2\right) \tag{1}$$

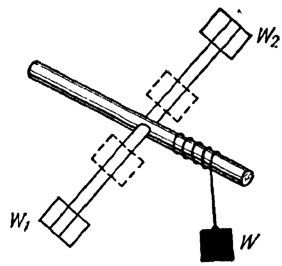


FIG. 1.—Angular acceleration produced by a torque depends upon the distribution of mass.

where Σ means the sum of the products for all the particles of the body. The unit of moment of inertia is made up as a composite unit. Since W is in pounds, g in feet per second per second and r in feet, the unit of moment of inertia becomes $\dfrac{\text{lb}}{\text{ft/sec}^2}(\text{ft})^2 =$ lb-ft-sec^2.

For many regular bodies the moment of inertia can be expressed quite simply in terms of the total mass of the body and its dimensions. A few of these are listed in Table I.

TABLE I. MOMENT OF INERTIA OF REGULAR BODIES

The mass of the body is W/g

Body	Axis	Moment of inertia
Thin ring of radius r	Through center, perpendicular to plane of ring	$\frac{W}{g} r^2$
Thin ring of radius r	Along any diameter	$\frac{1}{2}\frac{W}{g} r^2$
Disk of radius r	Through center, perpendicular to plate	$\frac{1}{2}\frac{W}{g} r^2$
Disk of radius r	Along any diameter	$\frac{1}{4}\frac{W}{g} r^2$
Cylinder of radius r	Axis of the cylinder	$\frac{1}{2}\frac{W}{g} r^2$
Sphere of radius r	Any diameter	$\frac{2}{5}\frac{W}{g} r^2$
Uniform thin rod of length l	Perpendicular to rod at one end	$\frac{1}{3}\frac{W}{g} l^2$
Uniform thin rod of length l	Perpendicular to rod at the center	$\frac{1}{12}\frac{W}{g} l^2$

Each of these formulas is found by adding up the products $(W/g)r^2$ of Eq. (1) for the particles of that particular body. Notice that the value of the moment of inertia depends upon the position of the axis chosen.

Example: What is the moment of inertia of a 50-lb cylindrical flywheel whose diameter is 16 in.?

For a cylinder about its axis

$$I = \frac{1}{2}\frac{W}{g}r^2$$

$$r = 8 \text{ in.} = \frac{2}{3} \text{ ft}$$

$$I = \frac{1}{2}\frac{50 \text{ lb}}{32 \text{ ft/sec}^2}\left(\frac{2}{3}\text{ ft}\right)^2 = 0.35 \text{ lb-ft-sec}^2$$

Newton's Laws for Angular Motion. The laws of rotary motion are very similar to those for linear motion. The first law applies to a condition of equilibrium. *A body does not change its angular velocity unless it is acted upon by an external, unbalanced torque.* A body at rest does not begin to rotate without a torque to cause it to do so. Neither does a body that is rotating stop its rotation or change its axis unless a torque acts. A rotating wheel would continue to rotate forever if it were not stopped by a torque due to friction.

An unbalanced torque about an axis produces an angular acceleration, about that axis, which is directly proportional to the torque and inversely proportional to the moment of inertia of the body about that axis. In the form of an equation this becomes

$$L = I\alpha \tag{2}$$

where L is the unbalanced torque, I is the moment of inertia, and α is the angular acceleration. Torque must always be referred to some axis as are also moment of inertia and angular acceleration. In Eq. (2) we must be careful to use the same axis for all three quantities. As in the case of the force equation for linear motion, we must be careful to use a consistent set of units in Eq. (2). The angular acceleration must be expressed in radians per second per second. The torque should be expressed in pound-feet and the moment of inertia in pound-feet-(second)^2.

Example: A flywheel, in the form of a uniform disk 4 ft in diameter, weighs 600 lb. What will be its angular acceleration if it is acted upon by a net torque of 75 lb-ft?

$$I = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\frac{600 \text{ lb}}{32 \text{ ft/sec}^2}(2 \text{ ft})^2 = 37.5 \text{ lb-ft-sec}^2$$

$$L = I\alpha$$

$$225 \text{ lb-ft} = (37.5 \text{ lb-ft-sec}^2)(\alpha)$$

$$\alpha = 6.0 \text{ radians/sec}^2$$

In radian measure the angle is a ratio of two lengths and hence is a pure number. The unit "radian," therefore, does not always appear in the algebraic handling of units

Example: If the disk of the preceding example is rotating at 1,200 rpm what torque is required to stop it in 3 min?

From Eq. (8) of Chap. 10,

$$\omega_2 - \omega_1 = \alpha t$$
$$\omega_2 = 0$$
$$\omega_1 = 1{,}200 \text{ rpm} = 20 \text{ rps} = 40\pi \text{ radians/sec}$$
$$t = 3 \text{ min} = 180 \text{ sec}$$
$$0 - 40\pi \text{ radians/sec} = \alpha(180 \text{ sec})$$
$$\alpha = -\frac{40\pi}{180} \text{ radians/sec}^2$$
$$L = I\alpha = (37.5 \text{ lb-ft-sec}^2)\left(-\frac{40\pi}{180} \text{ radians/sec}^2\right) = -26.2 \text{ lb-ft}$$

The negative sign is consistent with a retarding torque.

For every torque applied to one body there is an equal and opposite torque applied to another body. If a motor applies a torque to a shaft, the shaft applies an equal and opposite torque to the motor. If the motor is not securely fastened to its base, it may turn in a direction opposite to that of the shaft. If an airplane engine exerts a torque to turn the propeller clockwise, the airplane experiences a torque tending to turn it counterclockwise and this torque must be compensated by the thrust of the air on the wings. For twin-engined planes the two propellers turn in opposite directions and so avoid a net torque.

Work, Power, Energy. If a torque L turns a body through an angle θ, the work done is given by the equation

$$\text{Work} = L\theta \tag{3}$$

Since power is work per unit time,

$$P = \frac{\text{Work}}{t} = \frac{L\theta}{t} = L\omega = L \times 2\pi n \tag{4}$$

The kinetic energy of rotation of a body is given by the equation

$$KE = \tfrac{1}{2}I\omega^2 \tag{5}$$

Frequently a body has simultaneous linear and angular motions. For example, the wheel of an automobile rotates about its axle but the axle advances along the road. It is usually easier to work with the kinetic energy of such a body if we consider the two parts: (1) due to translation of the center of mass ($\frac{1}{2}mv^2$) and (2) due to rotation about an axis through the center of mass ($\frac{1}{2}I\omega^2$).

Example: What is the kinetic energy of a 5-lb ball whose diameter is 6 in., if it rolls across a level surface with a speed of 4 ft/sec?

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$v = \omega r, \qquad \omega = \frac{v}{r} = \frac{4 \text{ ft/sec}}{\frac{1}{4} \text{ ft}} = 16 \text{ radians/sec}$$

From Table I,

$$I = \frac{2}{5}\frac{W}{g}r^2 = \frac{2}{5}\frac{5\text{ lb}}{32\text{ ft/sec}^2}\left(\frac{1}{4}\text{ ft}\right)^2 = \frac{1}{256}\text{ lb-ft-sec}^2$$

$$KE = \frac{1}{2}\frac{5\text{ lb}}{32\text{ ft/sec}^2}(4\text{ ft/sec})^2 + \frac{1}{2}\left(\frac{1}{256}\text{ lb-ft-sec}^2\right)(16\text{ radians/sec})^2$$
$$= 1.3\text{ ft-lb} + 0.5\text{ ft-lb} = 1.8\text{ ft-lb}$$

Where only a limited amount of energy is available, it is divided between energy of translation and energy of rotation. The way in which the energy is divided is determined by the distribution of mass. If two cylinders of equal mass, one being solid but the other hollow, roll down an incline, the solid cylinder will roll faster. Its moment of inertia is less than that of the hollow cylinder and hence the kinetic energy of rotation is smaller than that of the hollow cylinder; but the kinetic energy of translation is greater than that of the hollow cylinder. Hence the solid cylinder has a greater speed.

Angular Momentum. In motions of rotation *angular momentum* appears in much the same way as linear momentum appears in motions of translation. Just as linear momentum is the product of mass and velocity, the angular momentum of a body is defined as the product of its moment of inertia and its angular velocity.

$$\text{Angular momentum} = I\omega \qquad (6)$$

The angular momentum of a body remains unchanged unless it is acted upon by an external torque. This is the law of *conservation of angular momentum.* The action of a flywheel depends upon this principle. It is intended to cause a motor to maintain constant speed of rotation. Since it has a large moment of inertia, it requires a large torque to change its angular momentum. During the time the motor is speeding up, the flywheel supplies a resisting torque; when the motor slows down, the flywheel applies an aiding torque to maintain its speed.

If the distribution of mass of a rotating body is changed, the angular velocity must change to maintain the same angular momentum. Suppose a man stands on a stool that is free to rotate with little friction (Fig. 2). If he is set in rotation with his hands outstretched, he will rotate at a constant rate. If he raises his arms, his moment of inertia is decreased and his rate of rotation increases.

Another consequence of the principle of conservation of angular momentum is that a rotating body maintains the same plane of rotation unless acted upon by a torque. A top does not fall over when it is spinning rapidly for there is not sufficient torque to cause that change in angular velocity. The rotation of the wheels helps maintain the balance of a bicycle or motorcycle. The barrel of a gun is rifled to cause the bullet to spin so that it will not "tumble." A gyroscope maintains an apparently unstable position because of its angular momentum. The gyrocompass has no torque acting upon it only when its axis is parallel to the axis of the earth. It, therefore, turns until its axis is in that position pointing to the true north and remains there as long as it continues to turn.

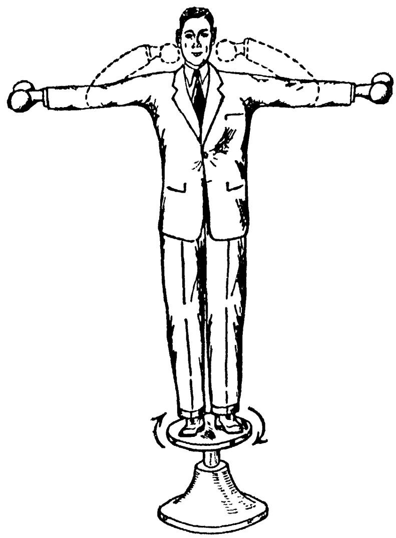


FIG. 2.—Conservation of angular momentum.

Comparison of Linear and Angular Motions. In our discussion of motions and forces we have found the equations of angular motion to be quite similar to those of linear motion. We can obtain them directly from the equations of linear motion if we make the following substitu-

TABLE II. CORRESPONDING EQUATIONS IN LINEAR AND ANGULAR MOTION

	Linear	Angular
Velocity	$\bar{v} = \frac{s}{t}$	$\bar{\omega} = \frac{\theta}{t}$
Acceleration	$\bar{a} = \frac{v}{t}$	$\alpha = \frac{\omega}{t}$
Uniformly accelerated motion	$v_2 - v_1 = at$ $s = v_1t + \frac{1}{2}at^2$ $v_2^2 - v_1^2 = 2as$	$\omega_2 - \omega_1 = \alpha t$ $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ $\omega_2^2 - \omega_1^2 = 2\alpha\theta$
Newton's second law	$F = ma$	$L = I\alpha$
Momentum	$M = mv$	Angular momentum $= I\omega$
Work	Work $= Fs$	Work $= L\theta$
Power	$P = Fv$	$P = L\omega$
Kinetic energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$

tions: θ for s, ω for v, α for a, L for F, I for m. In Table II are listed a set of corresponding equations.

SUMMARY

The *moment of inertia* (rotational inertia) of a body about a given axis is the sum of the products of the mass and square of the radius for each particle of the body

$$I = \sum \left(\frac{W}{g} r^2\right)$$

For angular motion *Newton's laws* may be stated:

1. A body does not change its angular velocity unless it is acted upon by an external, unbalanced torque.

2. An unbalanced torque about an axis produces an angular acceleration about that axis, which is directly proportional to the torque and inversely proportional to the moment of inertia of the body about that axis.

$$L = I\alpha$$

3. For every torque applied to one body there is an equal and opposite torque applied to another body.

In angular motion the *work* done by a torque L in turning through an angle θ is

$$\text{Work} = L\theta$$

The *power* supplied by a torque is

$$P = L\omega$$

Kinetic energy of rotation is given by the equation

$$KE = \tfrac{1}{2}I\omega^2$$

For a rolling body the total kinetic energy, both translational and rotational, is

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Angular momentum is the product of moment of inertia and angular velocity.

$$\text{Angular momentum} = I\omega$$

The *law of conservation of angular momentum* states that the angular momentum of a rotating body remains unchanged unless it is acted upon by an external, unbalanced torque.

In all of the foregoing equations the angles must be expressed in radian measure.

QUESTIONS AND PROBLEMS

1. Why is most of the mass of a flywheel placed in the rim?

2. Considering the earth as a uniform sphere of 6.00×10^{21} tons mass and 4,000 mi radius, calculate its moment of inertia about its axis of rotation. *Ans.* 6.73×10^{37}.

3. What are the advantages of an automobile brake drum with a large diameter over one with a smaller diameter?

4. A uniform circular disk 3 ft in diameter weighs 960 lb. What is its moment of inertia about its usual axis? *Ans.* 34 lb-ft-sec².

5. The disk of problem 4 is caused to rotate by a force of 100 lb acting at the circumference. What is the angular acceleration?

6. If the disk of problems 4 and 5 starts from rest, what is its angular speed at the end of 10 sec? What is the linear speed of a point on the circumference? *Ans.* 44 radians/sec; 66 ft/sec.

7. The rotor of an electric motor has a moment of inertia of 25 lb-ft-sec². If it is rotating at a rate of 1,200 rpm, what frictional torque is required to stop it in 1 min?

8. What is the initial kinetic energy of rotation of the rotor in problem 7? What becomes of this energy when it is stopped as indicated? *Ans.* $2\bar{0}0,000$ ft-lb.

9. A 16-lb bowling ball is rolling without slipping down an alley with a speed of 20 ft/sec. What is its kinetic energy (*a*) of translation, (*b*) of rotation? What is its total kinetic energy?

10. A motor running at a rate of 1,200 rpm can supply a torque of 4.4 lb-ft. What power does it develop? *Ans.* 1.0 hp.

11. What is the angular momentum of the rotor of problem 7?

EXPERIMENT

Torque; Moment of Inertia

Apparatus: Heavy disk with mounting having little friction; weights; string; stop watch or metronome.

Mount a heavy disk on an axle so that it is free to turn with the axle as shown in Fig. 3. The axle should be supported by ball bearings or cone pivots to minimize friction. Wind a string on the axle from which to suspend weights to apply a torque to the disk.

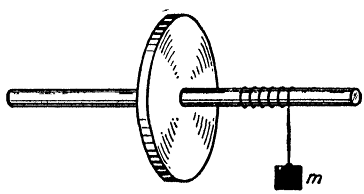


FIG. 3.—Disk and axle accelerated by a torque supplied by m.

As the weight falls, its potential energy is converted into kinetic energy of rotation of the disk and kinetic energy of translation of the weight. In descending a distance h the weight loses potential energy mgh and

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \tag{7}$$

where m is the mass of the descending body, v is its final speed, I is the moment of inertia of the disk, and ω is its final angular speed. We shall use this equation to determine I.

Balance the wheel so that when there is no weight on the string it will stay in any position. To eliminate the effect of friction hang small weights on the string and adjust their value until the wheel turns uniformly *after it is started.* Keep these weights attached to the string during the observations.

Add a known weight (a suitable value depends upon the size of the disk and should be determined by trial) to those on the string and time a suitable number of revolutions of the disk starting from rest, using a stop watch or metronome as a timer. From Eqs. (6) and (7), Chap. 10, we can determine the final angular speed ω_2.

$$\theta = \bar{\omega}t \qquad \bar{\omega} = \frac{\theta}{t}$$

where θ is the whole angle turned through, in radians, and $\bar{\omega}$ is the *average* angular speed.

$$\bar{\omega} = \tfrac{1}{2}(\omega_2 + \omega_1)$$

The initial angular speed ω_1 is zero since the disk started from rest. Then

$$\omega_2 = 2\bar{\omega} = 2\frac{\theta}{t}$$

Since the string is wound around the axle, there is a relation between v and ω given by

$$v = \omega r$$

where r is the radius of the axle. Also

$$h = 2\pi rN$$

where N is the number of revolutions.

From these calculations we know all the quantities in Eq. 7 except I and we can solve the equation to find it.

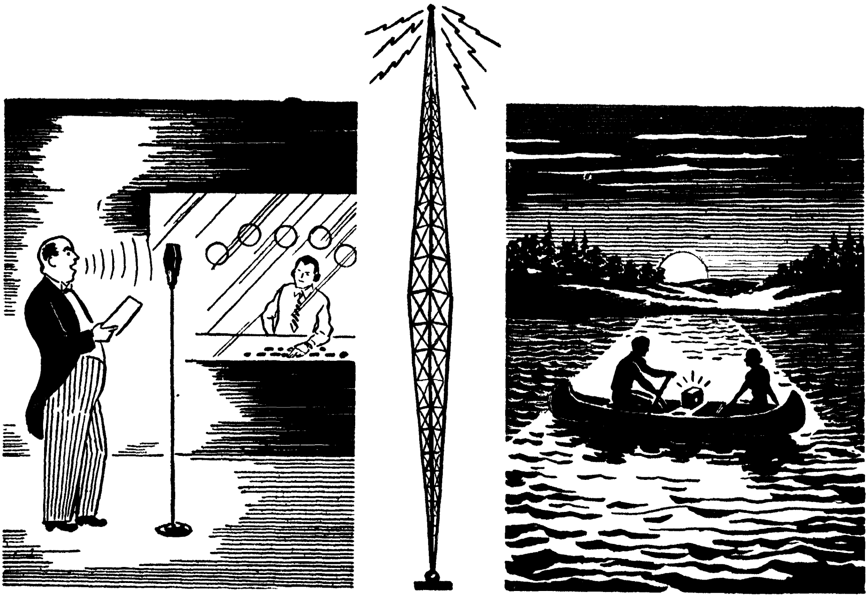
Since the disk is uniform, we can also compute its moment of inertia by the formula given in Table I

$$I = \tfrac{1}{2}MR^2$$

where M is the mass of the disk and R is its external radius. Make this calculation and compare the result with the experimental value.

If a disk such as that just described is not available, a bicycle wheel may be substituted. It will be most satisfactory if the tire is replaced by a lead rim but it will give satisfactory results even if this is not available. Since most of the mass is concentrated in the rim, an approximate value for its moment of inertia can be calculated from the equation

$$I = MR^2$$



CHAPTER 20

VIBRATORY MOTION; RESONANCE

Three types of motion have been treated in the earlier chapters. The simplest is that of an object in equilibrium, a motion consisting of constant speed and unchanging direction. The second type of motion, which is produced by the action of a constant force, is that in which the direction is constant and the speed increases uniformly. Projectile motion was discussed as a combination of these two simple types of motion. The third type of motion discussed is uniform circular motion, that produced by a (centripetal) force of constant magnitude directed inward along the radius of the circular path of the moving object.

It is clear that the forces we commonly observe are not always zero, constant in magnitude and direction, or constant in magnitude and of rotating direction; so that, consequently, the motions commonly observed are not always uniform rectilinear, uniformly accelerated, uniform circular, or even combinations of the three. In general, the forces acting on a body vary in both magnitude and direction, resulting in complicated types of nonuniformly accelerated motion, which cannot be investigated in an elementary physics course.

Simple Harmonic Motion (S.H.M.). A type of motion that is particularly important in practical mechanics is the to-and-fro or vibrating motion of objects stretched or bent from their normal shapes or positions. It is fortunate that this motion, though it is produced by a varying force, can be analyzed rather easily and completely by elementary methods. Suppose that a steel ball is mounted on a flat spring, which is clamped in a vise as in Fig. 1. Pull the ball sideways, bending the spring, and you will observe a restoring force that tends to move the ball back toward its initial position. This force increases as the ball is pulled farther away from its original position; in fact, the restoring force is directly proportional to the displacement from the position of equilibrium. The direct proportionality of restoring force to displacement distinguishes simple harmonic motion from all other types. *Simple harmonic motion* is that type of vibratory motion in which the restoring force is proportional to the displacement and is always directed toward the position of equilibrium.

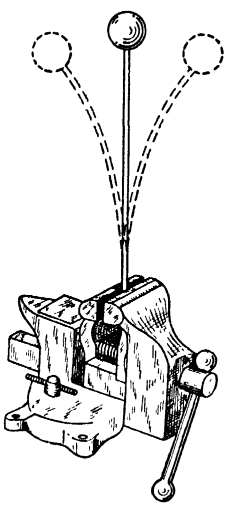


FIG. 1.—A ball and spring in simple harmonic motion.

Period, Frequency, Amplitude. The *period* of a vibratory motion is the time required for a complete to-and-fro motion or oscillation. For a simple harmonic motion, the time required for one complete oscillation depends upon two factors: the stiffness of the spring (or other agency) that supplies the restoring force, and the mass of the vibrating object. The stiffness of the spring is measured by the so-called *force constant* K, which is the force per unit displacement. This is obtained by dividing the applied force by the displacement it produces. For example, in Fig. 1, if a force of 0.2 lb is required to move the mass a distance of 3.0 in. from its equilibrium position, the force constant is $\frac{0.2 \text{ lb}}{0.25 \text{ ft}} = 0.8$ lb/ft. By Hooke's law, the force will be proportional to the displacement, so that a force of 0.4 lb will displace the ball 6 in., if this does not exceed the elastic limit of the spring.

The period of vibration, that is, the time required for a complete oscillation, is given by the equation,

$$T = 2\pi \sqrt{\frac{W}{gK}} \tag{1}$$

in which T is the period, W is the *weight* of the vibrating object, and K is the force constant, measured in gravitational units, pounds per foot or grams per centimeter.

Reference to the equation above will show that, if the object is replaced by another four times as heavy, the period will be doubled. If, instead, the spring is replaced by another four times as stiff, the period is halved.

Example: A 5.0-lb ball is fastened to the end of a flat spring (Fig. 1). A force of 2.0 lb is sufficient to pull the ball 6.0 in. to one side. Find the force-constant and the period of vibration.

$$K = \frac{2.0\ \text{lb}}{0.50\ \text{ft}} = 4.0\ \text{lb/ft}$$

$$T = 2\pi\sqrt{\frac{5.0\ \text{lb}}{(32\ \text{ft/sec}^2)(4.0\ \text{lb/ft})}} = 1.2\ \text{sec}$$

It should be noticed that the weight of the spring has not been considered. More accurate results will be obtained if about one-third the weight of the spring is included in W.

The *frequency* n of the vibratory motion is the number of complete oscillations occurring per second. The frequency is the reciprocal of the period: $n = 1/T$.

The *amplitude* of a vibratory motion is the maximum displacement from the equilibrium position. In simple harmonic motion the period does not depend upon the amplitude.

Another example of simple harmonic motion is the up-and-down vibration of an object suspended vertically by a spiral spring. At the equilibrium position, the spring is stretched just enough to support the weight of the object. If the object is pulled below this position, it is acted upon by a restoring force (since the pull of the spring exceeds that of gravity), which is proportional to the displacement from the equilibrium position. Likewise, if the object is lifted above the equilibrium position, the weight exceeds the pull of the spring by an amount proportional to the displacement from the equilibrium position, so that the conditions for simple harmonic motion are satisfied.

Acceleration and Speed in S.H.M. At the positions of greatest displacement, that is, at the end points of the motion, the vibrating object comes momentarily to a stop. It should be noticed that, at the instant when its speed is zero, the object is acted upon by the maximum restoring force, so that the acceleration is greatest when the speed is zero. The restoring force (and therefore the acceleration) decreases as the object moves toward the equilibrium position, where the acceleration is zero and the speed greatest. The direction of the acceleration reverses as the object passes through the equilibrium position, increasing as the displacement increases and reaching a maximum again at the other extreme of displacement.

Thus far in the discussion of simple harmonic motion the effect of friction has been neglected. Since the frictional force always opposes the motion, its effect is to reduce the amplitude (maximum displacement) of the motion, so that it gradually dies out unless energy is constantly supplied to it from some outside source.

Resonance. Suppose that the natural frequency of vibration of the system represented in Fig. 1 is 10 vib/sec. Now imagine that, beginning with the system at rest, we apply to it a to-and-fro force, say, 25 times per second. In a short time this force will set the system to vibrating regularly 25 times a second, but with very small amplitude, for the ball and spring are trying to vibrate at their natural rate of 10 vib/sec.

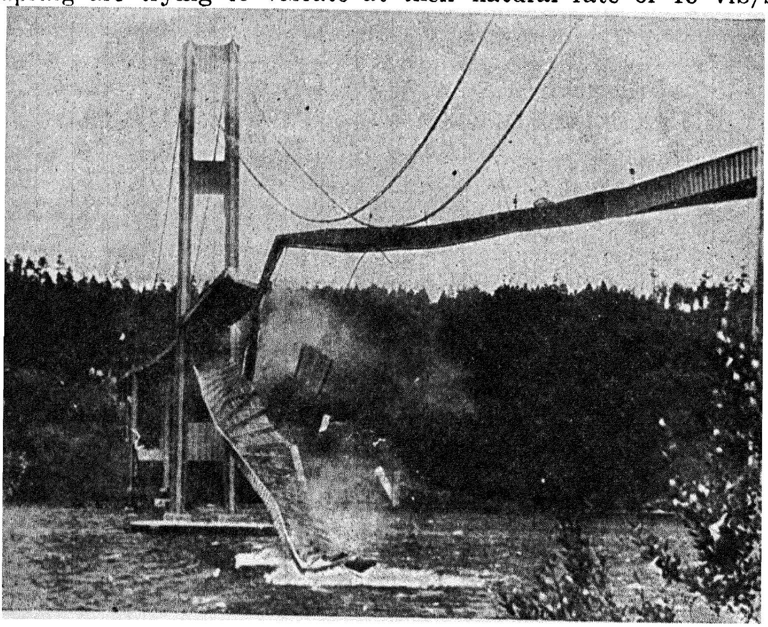


FIG. 2.—Dangerous resonance. Excessive vibration caused the collapse of the bridge.

During part of the time, therefore, the system is so to speak, "fighting back" against the driving force, whose frequency is 25/sec. We call the motion of the system in this case a *forced* vibration.

Now suppose that the alternation of the driving force is gradually slowed down from 25/sec to 10/sec, the natural frequency of the system, so that the alternations of the driving force come just as the system is ready to receive them. When this happens, the amplitude of vibration becomes very large, building up until the energy supplied by the driving force is just enough to overcome friction. Under these conditions the system is said to be in *resonance* with the driving force.

A small driving force of proper frequency can build up a very large amplitude of motion in a system capable of vibration. We have all heard car rattles that appear only at certain speeds, or vibrations set up in dishes, table lamps, cupboards, and the like by musical sounds of particular frequency. A motor running in the basement will often set certain pieces of furniture vibrating.

This problem of resonant vibrations may become particularly important with heavy machinery. The problem is to find the mass that is vibrating in resonance with the machinery and change its natural frequency by changing its mass or its binding force (force constant).

A most common example of resonance is furnished by radio circuits. When one tunes his radio receiver, he is in effect altering what would correspond to the spring constant in a mechanical system. By thus changing the natural frequency, one can bring the circuit into resonance with the desired electrical frequency transmitted by the sending station. The forced vibrations from all other frequencies have such small amplitudes that they do not produce any noticeable effect.

Another Description of S.H.M. It is enlightening to establish a comparison between simple harmonic motion and uniform circular motion. Suppose that in Fig. 3 the object A is executing uniform circular motion in a vertical circle. Let the object be illuminated from vertically overhead so that the shadow of A appears directly below it on the floor at B. The shadow B will execute simple harmonic motion along the line CD. The circle on which A travels uniformly is called the *reference circle*. Simple harmonic motion can thus be described as the motion of the projection on a diameter of a point that moves at constant speed in a circle.

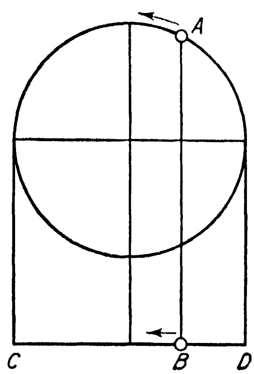


Fig. 3.—Circle of reference.

SUMMARY

Simple harmonic motion is that type of vibratory motion in which the restoring force is proportional to the displacement and is always directed toward the position of equilibrium.

The *period* of a vibratory motion is the time required for one complete oscillation: $T = 2\pi \sqrt{W/gK}$.

The *frequency* is the number of complete oscillations per second.

The *amplitude* of the motion is the maximum displacement from the equilibrium position.

Resonance occurs when a periodic driving force is impressed upon a system whose natural frequency of vibration is the same as that of the driving force. When this happens, the amplitude of vibration builds up until the energy supplied by the driving force is just sufficient to overcome friction in the system.

The motion of the projection of a point that moves at constant speed on the "circle of reference" describes simple harmonic motion.

QUESTIONS AND PROBLEMS

1. What is the force constant of a spring that is stretched 11.0 in. by a force of 5.00 lb? *Ans.* 5.45 lb/ft.

2. What is the period of vibration of a mass of 10.0 lb if it is suspended by the spring of problem 1? *Ans.* 1.50 sec.

3. The spring of problem 2 weighs 1.5 lb. Use this fact to improve your answer for problem 2. What percentage error is introduced in the answer to problem 2 by neglecting the weight of the spring?

Ans. 1.54 sec; 2.6 per cent.

4. A 1,000-gm cage is suspended by a spiral spring. When a 200-gm bird sits in the cage, the cage is pulled 0.50 cm below its position when empty. Find the period of vibration of the cage (*a*) when empty, (*b*) when the bird is inside.

Ans. 0.32 sec; 0.35 sec.

5. Find the maximum speed and acceleration for the vibration of problem 2, assuming that the amplitude of motion is 2.0 in.

Ans. 0.70 ft/sec; 2.9 ft/sec^2.

6. The drive wheels of a locomotive whose piston has a stroke of 2 ft make 185 rpm. Assuming that the piston moves with S.H.M., find the speed of the piston relative to the cylinder head, at the instant when it is at the center of its stroke. *Ans.* 19.4 ft/sec.

7. A 50-gm mass hung on a spring causes it to elongate 2 cm. When a certain mass is hung on this spring and set vibrating its period is 0.568 sec. What is the mass attached to the spring? *Ans.* 200 gm.

8. A 200-gm mass elongates a spring 4.9 cm. What will be the period of vibration of the spring when a 400-gm mass is attached to it? (Neglect the mass of the spring.) What will be the maximum speed of the vibrating mass if the amplitude is 3 cm? *Ans.* 0.63 sec; 30 cm/sec.

9. A body whose mass is 5 kg moves with S.H.M. of an amplitude 24 cm and a period of 1.2 sec. Find the speed of the object when it is at its mid-position, and when 24 cm away. What is the magnitude of the acceleration in each case?

Ans. 126 cm/sec; 0; 0; 6$\bar{6}$0 cm/sec^2.

10. A 100-lb mass vibrates with S.H.M. of amplitude 12 in. and a period of 0.784 sec. What is its maximum speed? its maximum kinetic energy? its minimum kinetic energy? *Ans.* 7.8 ft/sec; 96 ft-lb; 0.

EXPERIMENT

Simple Harmonic Motion; Resonance

Apparatus: Spring; weights.

This experiment is essentially a study of the significance of Eq. (1) of this chapter. From that equation it follows that the frequency of oscillation n of a loaded spring executing simple harmonic motion is given by

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{gK}{W}}$$

The fact that for such motion the frequency is inversely proportional to the square root of the weight can be illustrated with the spring used in the experiment of Chap. 6.

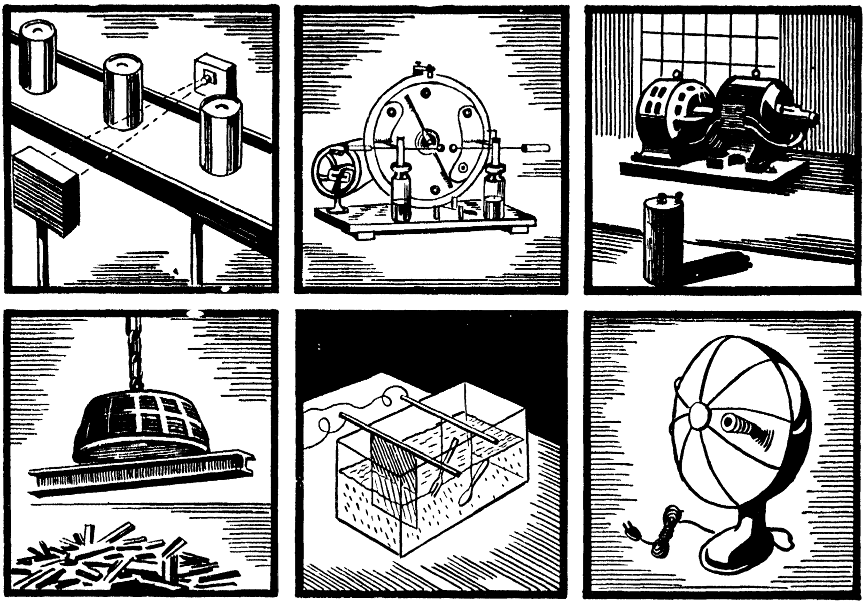
Suspend a load of 1 kg on the spring. Pull the load 10 or 15 cm below the point of equilibrium and then release it. Count the number of *complete* vibrations in, say, a half minute. Then compute the number of vibrations per second. Do this for several different loads and record the results in the accompanying table. If n and $\sqrt{W}$ are indeed inversely proportional, their product (column 4) should be constant. Is it?

W	$\sqrt{W}$	n	$n\sqrt{W}$	$\frac{1}{2\pi}\sqrt{\frac{gK}{W}}$

For a given load, compare the frequency of vibration obtained when the initial displacement is small (5 to 10 cm) with that resulting from an initial displacement several times as large. Does the frequency of the vibration depend on its amplitude?

Compute the force constant K of the spring from data obtained when it was used in the experiment of Chap. 6. Using this value, compute the frequency of vibration to be expected under the conditions of this experiment. Record your calculated values in the last column of the table. How do they compare with the observed frequencies?

To study resonance, suspend the spring and mass from your finger. Now move the finger up and down with an amplitude of several inches and a frequency much greater than the "natural" frequency of the spring and that particular mass. Is the response (the motion) of the mass very large? Now move the finger with a frequency much less than the natural one. The amplitude and frequency of vibration of the mass are approximately equal to those of the finger. Next, move the finger with a frequency approximately equal to the natural frequency of the system. The amplitude of oscillation of the spring and mass will be much larger than that of the finger. This is the condition of resonance. Try it for different loads. Mention some practical advantages and also some dangerous disadvantages of this phenomenon of resonance.



CHAPTER 21

SOURCES AND EFFECTS OF ELECTRIC CURRENT

The present era is one that may properly be characterized as the age of electricity. Homes and factories are lighted electrically; communication by telegraph, telephone, and radio depends upon its use; and the industrial applications of electricity extend from the delicate instruments of measurement and control to giant electric furnaces and powerful motors. People seek recreation at motion-picture houses and theaters whose operations utilize electric current in many ways, and it is probable that television in future years will be as commonplace as is the radio of today. Electricity 's a useful servant of man—a practical means of transforming energy to the form in which it serves his particular need. The effects of electricity both at rest and in motion are well known, and the means to produce these effects are readily available.

Electrification. If a piece of sealing wax, hard rubber, or one of many other substances is rubbed with wool or cat's fur, it acquires the ability to attract light objects such as bits of cork or paper. The process of producing this condition in an object is called *electrification,* and the object itself is said to be electrified or charged with electricity.

There are two kinds of electrification. If two rubber rods, electrified by being rubbed against fur, are brought near each other, they will be found to repel each other. A glass rod rubbed with silk will attract either of the rubber rods, although two such glass rods will repel each other. These facts suggest the first law of electrostatics: *Objects that are similarly charged repel each other, bodies oppositely charged attract each other.*

The electrification produced in a glass rod by rubbing it with silk is arbitrarily called *positive* electrification, while that produced in the rubber rod with wool is called *negative* electrification. It is ordinarily assumed that uncharged objects contain equal amounts of positive and negative electricity. When glass and silk are rubbed together, some negative electricity is transferred from the glass to the silk, leaving the glass rod

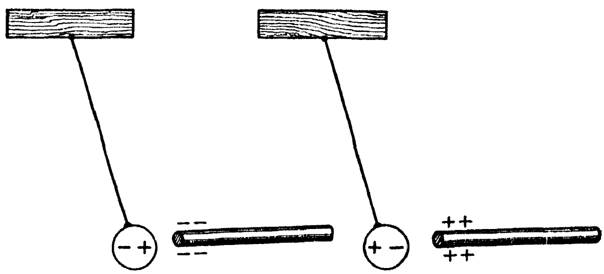


FIG. 1.—A charged body brought near a light insulated conductor causes charges in the conductor to separate. This results in an attraction of the conductor by the charge.

with a net positive charge, and the silk with an equal net negative charge. Similarly, hard rubber receives negative electricity from the wool with which it is rubbed, causing the rod to be negatively charged and leaving the wool positive. Though a similar explanation could be made by assuming a transfer of positive electricity, it can be shown that in solids only negative electricity is transferred.

The attraction of a charged object for one that is uncharged is illustrated in Fig. 1. The separation of positive and negative electricity within the uncharged object is produced by the charged object, which exerts a force of repulsion on the like portion of the charge and an attraction on the unlike. At *a* the negatively charged rod causes the adjacent side of the uncharged object to become positively charged, while the opposite side becomes negatively charged. Because the unlike charge is nearer the rod, the force of attraction will exceed that of repulsion and produce a net attraction of the uncharged object by the rod. At *b* is shown the case in which a positively electrified glass rod is used. It should be remembered that the changes described here do not alter the total amounts of positive and negative electricity in the uncharged object. No charge is gained or lost; all that occurs is a shift of negative electricity toward one side of the object, making that side predominantly negative and leaving the other side predominantly positive.

The Electron Theory. According to modern theory all matter is composed of *atoms*, tiny particles that are the building blocks of the universe. There are many kinds of atoms, one for each chemical element. Each atom consists of a *nucleus*, a small, tightly packed, positively charged mass, and a number of larger, lighter, negatively charged particles called *electrons*, which revolve about the nucleus at tremendous speeds (Fig. 2). The centripetal force necessary to draw these electrons into their nearly circular paths is supplied by the electrical attraction between them and the nucleus. The latter is said to consist of a number of *protons*, each with a single positive charge, and possibly one or more *neutrons*, which have no charge. Thus the positive charge on the nucleus

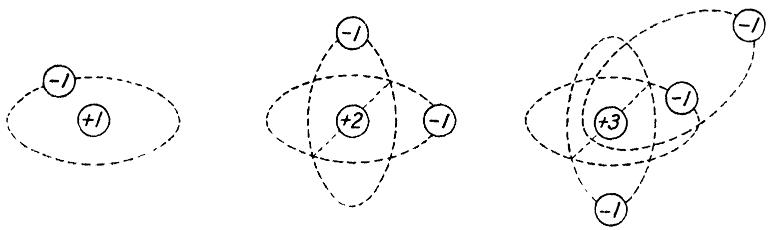


FIG. 2.—Each atom consists of a positively charged nucleus surrounded by electrons. The three simplest atoms, hydrogen, helium, and lithium, are represented diagrammatically.

depends upon the number of protons that it contains, called the *atomic number* of the atom. A neutral atom contains equal numbers of electrons and protons. Each electron carries a single negative charge of the same magnitude as the positive charge of a proton, so that the attraction between the nucleus of an atom and one of the electrons will depend on the number of protons in the nucleus. An electron has a mass of $8.994(10)^{-28}$ gm. Since the mass of a proton is about 1,840 times that of an electron, the nucleus may be thought of as practically unaffected by the attraction of the electrons. The number of electrons in an atom, along with their arrangement, determines the chemical properties of the atom.

From the idea that like charges repel and unlike charges attract, it appears that a nucleus consisting of positive charges could not be expected to cling together as a unit. The explanation is that, at very short distances, two protons will attract each other, clinging together tightly, even though at larger distances they repel each other.

A solid piece of material consists of an inconceivably large number of atoms clinging together. Though these atoms may be vibrating about their normal positions as a result of thermal agitation, their arrangement is not permanently altered by this motion. Also present in solids are numbers of *free electrons,* so-called because they are not permanently attached to any of the atoms. The number and freedom of motion of these electrons determines the properties of the material as a conductor of electricity. A good conductor is a material containing many free electrons whose motion is not greatly impeded by the atoms of which the material is composed. As a result of the repulsive forces between them, free electrons spread throughout the material, and any concentration of them in any one region of the material will tend to be relieved by a motion of the electrons in all directions away from that region until an equilibrium distribution is again reached.

In the best conductors, the outer electrons of the atoms can easily be removed, so that a free electron, colliding with an atom, often causes such an electron to leave the atom. When this happens, the electron ejected becomes a free electron, moving on, while its place in the atom is taken by the next free electron that encounters the atom. An *insulator,* or poor conductor, is a substance which contains very few free electrons and whose atoms have no loosely held orbital electrons.

The reason for describing electrification as occurring through the *transfer of negative electricity* can now be seen. An uncharged object contains a large number of atoms (each of which has equal numbers of electrons and protons) along with some free electrons. If some of these free electrons are removed, the object is considered to be positively charged, though actually this means that its negative charge is below normal, since it still contains more electrons than protons. If extra free electrons are gained by an object, it is said to be negatively charged, since it has more negative charge than is normal. The "normal" or uncharged condition of a body is that obtained by connecting it to the earth.

Electric Current. Consider a circular loop of copper wire. The wire consists of a tremendous number of copper atoms along with a large number of free electrons. If energy is supplied to make these free electrons move around the circuit continuously, an *electric current* is said to be produced in the wire. It is to be emphasized that a *source of electric current is simply a device for causing electrons to move around a circuit.* The electrons themselves are already in the circuit, hence a source of electric current merely causes a motion of electrons but does not produce them. Since electrons repel each other, a motion of those in one part of the circuit will cause those next to them to move, relaying the motion around the circuit. The individual electrons in a current-carrying wire move with a relatively low speed (about 0.01 cm/sec for a current of 1 amp in a copper wire 1 mm in diameter), but the impulse of the electron movement travels around the circuit with a speed approaching that of light (186,000 mi/sec).

Sources of Electric Current. Let us consider some of the methods by which electrons can be caused to move around a circuit. In Fig. 3 is shown an electric circuit consisting of a dry cell, a push button, and a

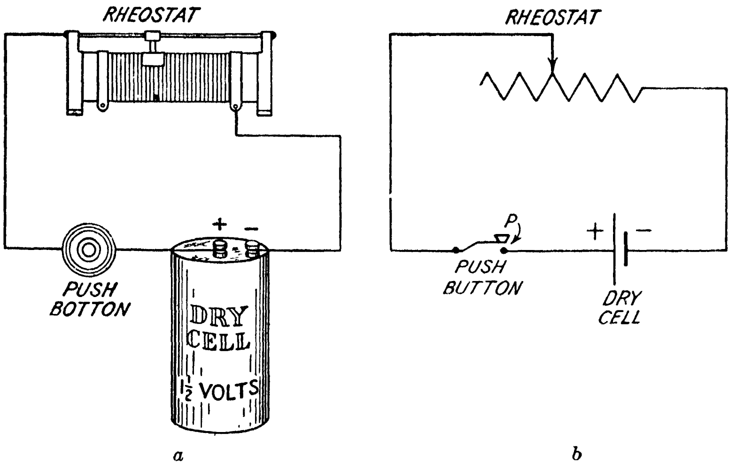


FIG. 3.—(a) A simple electric circuit; (b) a schematic diagram of the simple circuit.

rheostat. The electrons are forced out of the negative terminal of the cell and around the circuit, returning to the positive terminal of the cell to be again "pumped" through. Since the electrons leaving the cell must push those just ahead (and thus those on around the circuit), the cell furnishes the driving force for the electrons throughout the circuit by propelling each as it comes through. The cell thus does work on the electrons, communicating to them the energy released in the interaction of chemicals within it.

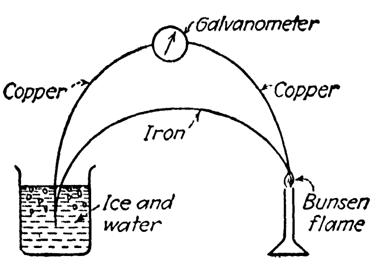
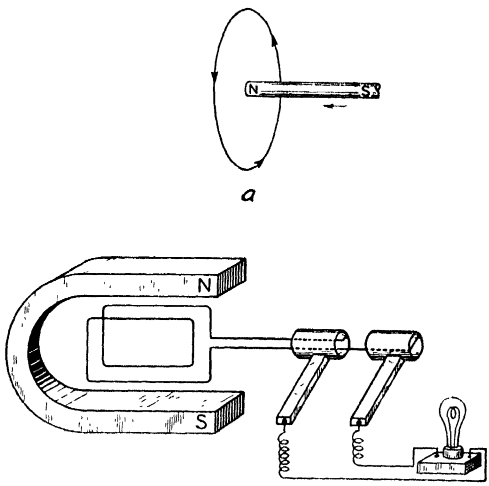


FIG. 4.—A thermocouple.

Direction of Flow. As has been explained, an electric current consists of a stream of electrons. Since they carry negative charges of electricity, their direction of motion is from the negative terminal of the source, through the external circuit and back to the positive terminal. Because for many years the flow was not understood, it was assumed that the flow is from positive to negative. It is still customary to speak of this "conventional" flow from positive to negative as the direction of the current.

A source of electric current in which heat is transformed into electrical energy is shown in Fig. 4. In the diagram there is shown a wire loop consisting of a piece of iron wire joined to a piece of copper wire. One of the junctions is heated by a flame, causing electrons to flow around the circuit. The flow will continue as long as one junction is at a higher temperature than the other junction. Such a device, consisting of a pair of junctions of dissimilar metals, is called a *thermocouple.*

The principle upon which the main source of electric currents depends is illustrated by the following. If one end of a bar magnet is plunged into a loop of wire, the electrons in the latter are caused to move around the wire, though their motion continues only while the magnet is moving (Fig. 5*a*). If the magnet is withdrawn, the electrons move around the loop in the opposite direction. The discovery of this means of producing



a

b

FIG. 5.—(*a*) An electric current is produced by thrusting a magnet into a loop of wire; (*b*) a simple generator.

an electric current with a moving magnet has led to the development of the *electric generator.* A very simple generator is shown in Fig. 5*b*. It consists of a stationary magnet between whose poles a coil of wire is rotated. The two ends of the coil are joined, through rotating contacts, to an incandescent lamp. During one-half of a rotation of the coil the electrons move in one direction through the lamp filament, while during the next half rotation they move in the opposite direction. Such a generator is said to produce an *alternating* current.

If light falls on a clean surface of certain metals, such as potassium or sodium, electrons are emitted by the surface. This phenomenon is called the *photoelectric* effect. If such a metallic surface is made a part of an electric circuit, such as that in Fig. 6, the electric current in the circuit is controlled by the light. If the light is bright, the current will be greater than if the light is dim. This device is known as a *photoelectric* *cell* and serves as a basis for most of the instruments that are operated or controlled by light such as television, talking moving pictures, wire or radio transmission of pictures, and many industrial devices for counting, rejecting imperfect pieces, control, etc.

Electrification through friction, as described earlier in the chapter, can bring about transfers of small quantities of electricity; yet it is not commercially important as a means of sustaining an electric current.

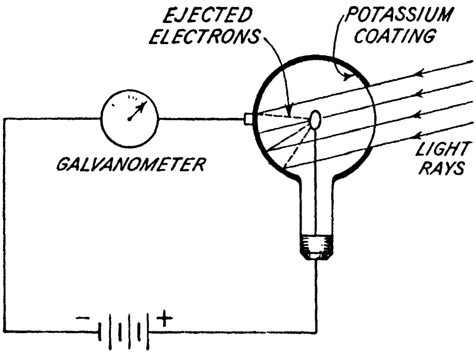


FIG. 6.—A photoelectric cell

In all these sources of electric current some type of energy is used to set the electrons in motion. Chemical, mechanical, thermal, or radiant energy is transformed into electrical energy.

Effects of Electric Current. The circuit in Fig. 7 consists of a battery E in series with a piece of high-resistance wire R; an incandescent lamp L; a cell Z containing metal electrodes (a and b) immersed in water to which a few drops of sulphuric acid have been added; and a key K, which opens and closes the circuit. A magnetic compass C is directly over the wire.

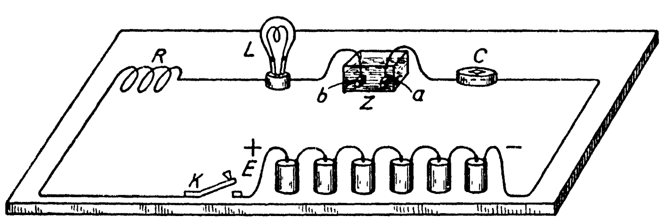


FIG. 7.—A circuit showing three effects of an electric current.

If the key K is closed, the battery produces a flow of electrons from its negative terminal through Z, L, R, K, and back to the positive terminal of the battery. As a result of the flow, several changes occur in the various parts of the circuit. The wire R becomes warm, and the filament of wire in the incandescent lamp becomes so hot that it begins to glow. The water in Z presents a very interesting appearance. Bubbles of gas are coming from the surfaces of the electrodes a and b (twice as much from a as from b). Tests show that hydrogen gas is being given off by a, and oxygen by b. Since oxygen and hydrogen are the gases that combine to form water and since the water in Z is disappearing, it is natural to conclude that the water is being divided into its constituents (hydrogen and oxygen) by the action of the electric current. This device Z is called an *electrolytic cell.*

The compass C, which points north (along the wire in Fig. 7) when the key is open, is deflected slightly to one side when the key is closed. This indicates that a magnetic effect is produced in the vicinity of an electric current.

The heat produced in R and L, the decomposition of water in the cell Z, and the deflection of the compass needle can be accomplished only at the expenditure of energy. By means of the electrons that it drives around the circuit, the battery E communicates energy to the various parts of the circuit. Electrons forced through R and L encounter resistance to their motion because of their collisions with the atoms of the material in R and L. These collisions agitate the atoms, producing the atomic-molecular motion that we call *heat.* In ways that will be discussed later, the electrons cause the decomposition of the water in Z and the deflection of the compass C, and the energy that they expend in these processes is furnished by the battery.

A phenomenon so simple as the deflection of a compass needle hardly indicates the importance of the magnetic effect of an electric current, for it is this magnetic effect that makes possible the operation of electric motors as devices by means of which electric currents perform mechanical work. The magnetic effect makes possible also the radio, telephone, telegraph, and countless other important electrical devices.

Unit of Electric Current. The *ampere,* the practical unit of electric current, is legally defined in terms of the rate at which it will cause metallic silver to be deposited in an electrolytic cell (0.00111800 gm/amp-sec).

Since the ampere is a unit of current, or rate of flow of electricity, a logical unit of *quantity of electricity* is the amount transferred in 1 sec by a current of 1 amp. The *coulomb,* then, is the quantity of electricity which in 1 sec traverses a cross section of a conductor in which there is a constant current of 1 amp. The total quantity of electricity (in coulombs) that passes through a source of electric current in a time t is

$$Q = It \tag{1}$$

where I is the current in amperes, t is the time in seconds, and Q is the quantity of electricity.

Potential, Voltage. The work (in foot-pounds) done by a pump on 1 ft^3 of water which passes through it is numerically equal to the difference in the pressures (in pounds per square foot) at the inlet and outlet of the pump. By analogy, then, we can think of the difference of electric "pressure" (potential) across the terminals of a source of electric current as measured by the work done on each coulomb of electricity transferred. The difference of potential across which 1 coulomb of electricity can be transferred by 1 joule of energy is called the *volt*. Thus, if the difference in potential between two points is 10 volts, exactly 10 joules of energy is necessary to transfer each coulomb of electricity from one of the points to the other. It will be remembered that 1 joule equals 10^7 ergs (dyne-centimeters). When the difference of potential between two points is expressed in volts, it is often referred to as the *voltage* between those points.

Resistance. The electrical resistance of a conductor is the ratio of the potential difference across its terminals to the current produced in it. The practical unit of electrical resistance is the *ohm*, which is the resistance of a conductor in which a current of 1 amp can be maintained by a potential difference of 1 volt. By definition, then,

$$R = \frac{E}{I} \tag{2}$$

where R is the resistance expressed in ohms.

SUMMARY

The electron theory suggests that all matter is composed of atoms, each atom consisting of a nucleus of protons (positive) and neutrons (uncharged), which is surrounded by a group of electrons (negative) whirling about the nucleus in small orbits at tremendous speeds.

A substance is a good conductor of electricity if it contains many free electrons and the outer electrons of its atoms are easily removable.

A source of electric current does not produce electricity but only a motion of electrons, many of which are distributed throughout all conductors. Electrons will not flow of their own accord along a conductor. Energy must be expended to move them.

A sustained electric current can be produced (*a*) chemically, (*b*) magnetically, (*c*) thermoelectrically, (*d*) photoelectrically.

The effects of electric current include the following: (*a*) heating effect, (*b*) magnetic effect, (*c*) chemical effect.

The practical unit of electric current is the *ampere*, which is defined legally as the current that will deposit 0.00111800 gm of silver per second.

The *coulomb* is the quantity of electricity which, in 1 sec, traverses any given cross section of a conductor in which there is a current of exactly 1 amp.

The *volt* is the potential difference across which 1 coulomb of electricity can be transferred by 1 joule of energy.

The *resistance* of a conductor is defined as the ratio $R = E/I$, where E is the difference of potential across its terminals and I is the current through it.

The *ohm* is the electrical resistance of a conductor in which a current of 1 amp can be maintained by a difference of potential of 1 volt.

The *conventional* direction of flow of electricity is from positive to negative (outside the source), while the actual direction of flow is from negative to positive. Whenever reference is to the actual direction of flow, the phrase *electron flow* will be employed, otherwise the reference is intended to be to conventional, or + to − flow.

QUESTIONS AND PROBLEMS

1. What is it that flows in an electric current?

2. Name four important types of sources of electric current.

3. Name three important effects of electric current.

4. Explain why a source of electric current should not be thought of as a source of electricity.

5. When one pays his so-called *electric* bill, is he paying for electricity, electric energy, or electric power? Explain.

6. Explain how a good conductor differs from a poor conductor, or insulator.

7. A current of 0.7 amp is maintained in an electrolytic cell. If each coulomb of electricity that passes through the cell causes 0.00033 gm of copper to be deposited on the negative electrode, how much copper will be deposited in 20 min?

8. If the potential difference across the terminals of the cell in problem 7 is 5.0 volts, how many joules of energy are furnished to it by the electric current during the 20 min? *Ans.* 4,$\bar{2}$00 joules.

9. What is the resistance of the cell of problems 7 and 8?

10. If increasing the difference of potential across the cell (of problems 7 and 8) to 10 volts causes the current to rise to 1.25 amp, what is the resistance of the cell under these conditions? *Ans.* 8 ohms.

EXPERIMENTS

Sources and Effects of Electric Current

Apparatus: Iron wire; flashlight lamp bulb; copper sulphate; carbon rods; battery or dry cells; switch; magnetic compass; two battery jars; water; copper plates; sulphuric acid; zinc plates; galvanometer; magnet; coil of wire.

At this juncture students will probably find a series of descriptive experiments or demonstrations more valuable than formal experiments involving quantitative measurements. The following may be performed with apparatus which, with the exception of a galvanometer, is rather simple and readily available.

a. Chemical Source. Dip a plate of zinc and a plate of copper into a tumbler of dilute sulphuric acid. To these plates connect wires with battery clips (or even paper clips) and connect these in turn through a protective resistance to a galvanometer, a current-indicating instrument.

b. Magnetic Source. Connect the ends of a coil of wire to the galvanometer. Thrust a magnet through the coil while observing the galvanometer.

c. Thermoelectric Source. Connect two wires of different material to the galvanometer in series with a protective resistance. Twist the free ends of the wires together and hold the joint in a flame.

d. Effects of an Electric Current. The circuit is diagramed in Fig. 8. At *B* is shown an ordinary storage battery or a half dozen dry cells, *S* a switch, *W* a length of small-diameter iron wire, *A* an electrolytic acid

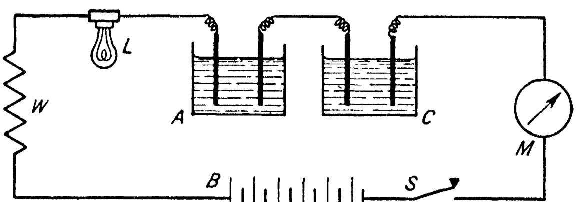


FIG. 8.—Arrangement of apparatus to show chemical, heating, and magnetic effects of electric current.

cell, *C* an electrolytic copper sulphate cell, *L* a flashlight bulb, and *M* a magnetic compass. With the exception of the compass these are all connected by copper wire in series. Hence the current is the same in all these devices, although a different effect is produced in each of them.

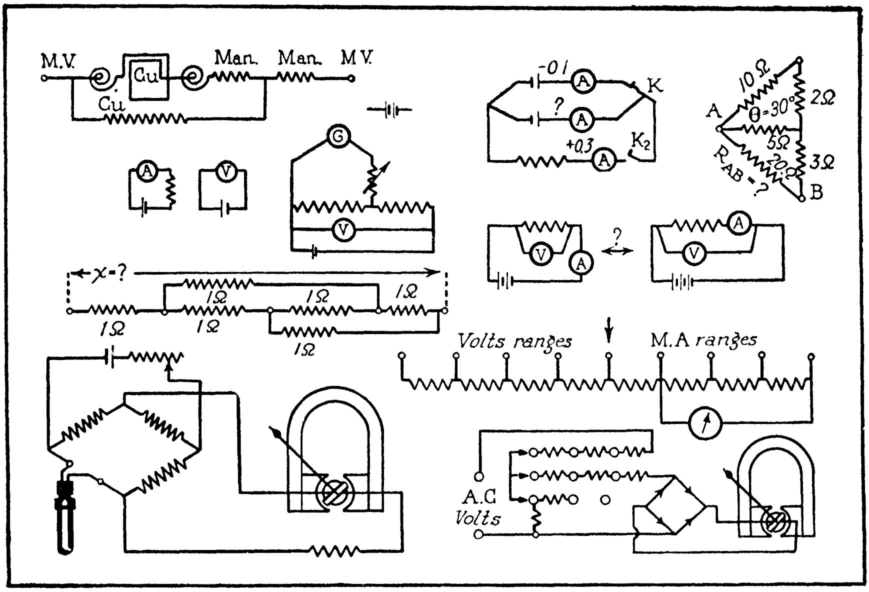
In *W* the main effect observable is the generation of heat. The rise in temperature can be felt directly by touching the wire.

The filament in *L* is heated so much that a very high temperature is produced—high enough for the radiation of white light.

The electrolytic cell *A* consists of a tumbler of dilute sulphuric acid into which dip two identical plates. When electricity passes through the cell, chemical action takes place, as is evidenced by the evolution of bubbles at the electrodes.

In cell *C* the solution is copper sulphate. The electrodes are carbon rods, tied together, but separated by the thickness of a rubber band. The chemical effect of the current is shown by the deposition of copper on that carbon rod which is the negative terminal.

When a magnetic compass is placed above or below the wire (as illustrated at *M*), the needle is deflected, provided the wire itself does not lie in an east-west direction. This shows that when electricity passes through a conductor the latter is surrounded by a magnetic field.



CHAPTER 22

OHM'S LAW; RESISTANCE; SERIES AND PARALLEL CIRCUITS

The practical applications of electricity are almost entirely those which depend upon the effects produced by the flow of electricity, that is, electric current. In order to apply and to control the heating, chemical, or magnetic effects the engineer must control the current. The most important of the laws related to electric current is Ohm's law. From this law and its extensions many vital circuit relationships can be determined.

Ohm's Law. In the preceding chapter the resistance of a conductor is defined as the difference of potential across its terminals divided by the current through it. Ohm's law is the statement that this quotient (resistance) is *constant* for a given conductor so long as its temperature and other physical conditions are not changed.

$$R = \frac{E}{I} = \text{a constant} \tag{1}$$

Ohm's law makes the relation between E, I, and R particularly useful, since it indicates that R is constant under uniform physical conditions.

The relation $R = E/I$ and its derived forms, $E = IR$ and $I = E/R$, are commonly referred to as forms of Ohm's law.

Example: The difference of potential across the terminals of an incandescent lamp is 6 volts. If the current through it is 1.5 amp, what is its resistance?

From the definition $R = \frac{E}{I}$ it is seen that $R = \frac{6 \text{ volts}}{1.5 \text{ amp}} = 4$ ohms.

Now suppose that one wishes to determine what current will be maintained in the lamp if the difference of potential is increased to 8 volts. Ohm's law indicates that the resistance R will remain the same (4 ohms) when the voltage is increased; hence we can write

$$I = \frac{E}{R} = \frac{8 \text{ volts}}{4 \text{ ohms}} = 2 \text{ amp}$$

Note that it is impossible to solve this problem without using Ohm's law, that is, the fact that R is constant.

Ammeters and Voltmeters. An instrument designed to measure electric current in amperes is called an *ammeter*. One designed to measure

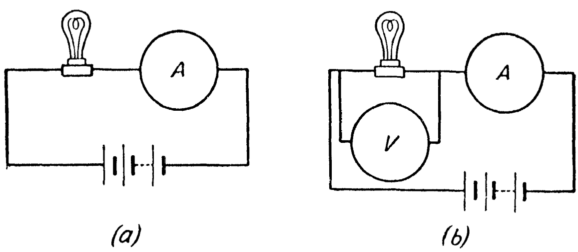


FIG. 1.—The methods of connecting ammeters and voltmeters in a circuit.

difference of potential in volts is called a *voltmeter*. The electrical principles involved in the operation of these instruments will be discussed in a later chapter. For the present it will be sufficient to consider only how they are used.

In Fig. 1*a* is shown the method of connecting an ammeter in such a way as to measure the current through an incandescent lamp. Note that the ammeter carries the current to be measured. In Fig. 1*b* a voltmeter has been added to the circuit to measure the difference of potential across the lamp. A voltmeter is connected across the two points whose potential difference is to be measured.

The circuit of Fig. 1*b* illustrates one of the simplest methods of measuring resistance (ammeter-voltmeter method). If the voltmeter indicates a difference of potential of 10 volts, and the ammeter a current of 2 amp, the resistance of the lamp is $R = \frac{10 \text{ volts}}{2 \text{ amp}} = 5$ ohms.

Because the voltmeter must carry a small current in order to indicate the voltage, the current through the ammeter is slightly larger than that in the lamp. For the present it will be assumed that the error thus involved in measuring the current through the lamp is very small, that is, the current through the voltmeter is negligible in comparison with that through the lamp.

Resistances in Series. Suppose that a box contains three coils of wire whose resistances are r_1, r_2, and r_3, respectively, and which are connected in series as shown in Fig. 2. If one were asked to determine the resistance of whatever is inside the box without opening it, he would probably place it in the circuit shown, measuring the current I through the box and the voltage E across it. He would then write $R = E/I$, where R is the resistance of the part of the circuit inside the box.

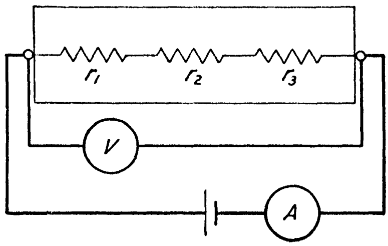


FIG. 2.—Resistances in series.

Let us now determine the relation of R, the combined resistance, to the individual resistances r_1, r_2, and r_3. The current through each of these resistances is I, since the current is not divided in the box. The voltages across the individual resistances are $e_1 = Ir_1$, $e_2 = Ir_2$, and $e_3 = Ir_3$. The sum of these three voltages must be equal to E, the voltage across the box; thus $E = e_1 + e_2 + e_3$ or $E = Ir_1 + Ir_2 + Ir_3$. This can be written $E = I(r_1 + r_2 + r_3)$, or $r_1 + r_2 + r_3 = E/I$; but this is identical with $R = E/I$, so that

$$R = r_1 + r_2 + r_3. \tag{2}$$

It has been shown that the combined resistance of three resistances in series is the sum of their individual resistances. This is true for any number of resistances.

Example: The resistances of four incandescent lamps are measured by the ammeter-voltmeter method and found to be 10.0, 4.0, 6.0, and 5.0 ohms, respectively. These lamps are connected in series to a battery, which produces a potential difference of 75 volts across its terminals. Find the current in the lamps and the voltage across each.

Using Ohm's law, it is assumed that the resistances of the lamps will remain the same when placed in the new circuit (in reality, the resistance of a lamp changes somewhat when the current through it is changed, since this changes its temperature). This gives

$$R = (10 + 4 + 6 + 5) \text{ ohms} = 25 \text{ ohms}$$

so that

$$I = \frac{E}{R} = \frac{75 \text{ volts}}{25 \text{ ohms}} = 3.0 \text{ amp}$$

The voltage across each lamp is the product of its resistance and the current. Thus

$$e_1 = (3.0 \text{ amp})(10 \text{ ohms}) = 30 \text{ volts}$$
$$e_2 = (3.0 \text{ amp})(4.0 \text{ ohms}) = 12 \text{ volts}$$
$$e_3 = (3.0 \text{ amp})(6.0 \text{ ohms}) = 18 \text{ volts}$$

and

$$e_4 = (3.0 \text{ amp})(5.0 \text{ ohms}) = 15 \text{ volts}$$

Resistances in Parallel. Suppose that a box contains a group of three resistances r_1, r_2, and r_3 in parallel, as shown in Fig. 3. The resistance of the combination will be $R = E/I$, where E is the voltage across the terminals of the box and I is the total current through it. Since the voltage across each of the resistances is E, the voltage across the terminals of the box, the currents through the individual resistances are, respectively,

$$i_1 = \frac{E}{r_1}, \qquad i_2 = \frac{E}{r_2}, \qquad i_3 = \frac{E}{r_3}$$

The sum of these three currents must be the total current I, so that

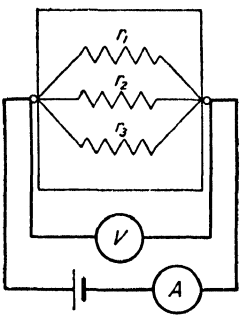


FIG. 3.—Resistances in parallel.

$$i_1 + i_2 + i_3 = I$$

or

$$\frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3} = I$$

This can be written

$$E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = I$$

or

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{I}{E}$$

Since $E/I = R$, we know that $I/E = 1/R$, so that

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \tag{3}$$

Thus the reciprocal of the combined resistance of a group of resistances in parallel is equal to the sum of the reciprocals of their individual resistances.

Example: The values of three resistances are measured by the ammeter-voltmeter method and found to be 10, 4.0, and 6.0 ohms, respectively. What will be their combined resistance in parallel?

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{10 \text{ ohms}} + \frac{1}{4.0 \text{ ohms}} + \frac{1}{6.0 \text{ ohms}}$$
$$= (0.10 + 0.25 + 0.17)/\text{ohm} = 0.52/\text{ohm}$$
$$\frac{1}{R} = 0.52/\text{ohm}, \qquad R = \frac{1}{0.52} \text{ ohms} = 1.9 \text{ ohms}$$

the combined resistance. Note that the resistance of the combination is smaller than any one of the individual resistances.

From the study of resistances in series and parallel it is seen that one can use the relation $R = E/I$ and Ohm's law in connection with groups of resistances as well as single resistances. One can consider any group of resistances not containing a battery as a single resistance $R = E/I$, where E is the voltage across the terminals to which external connection is made and I is the current at those terminals.

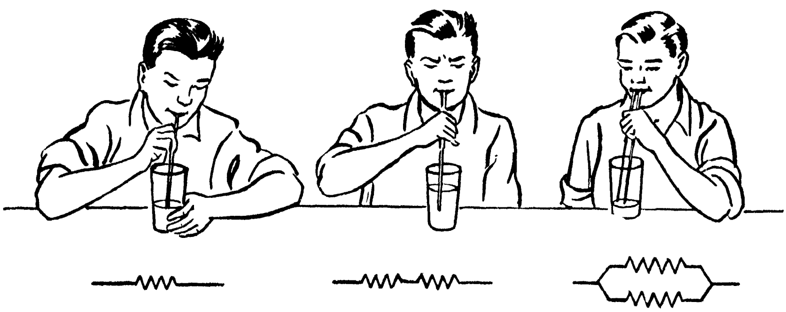


FIG. 4.

Emf and Internal Resistance. Thus far reference has been made to the voltage across the terminals of a battery without indicating the fact that this voltage depends upon the current supplied by the battery. When there is no current in a battery, the voltage E_m across its terminals is a maximum which is called its *emf*, or no-load voltage. The abbreviation emf represents *electromotive force*, but the use of "force" in this term is so misleading that it is desirable to use the abbreviation emf without any thought of the original term.

When a current is being maintained by a battery, the voltage across its terminals is $E_m - Ir$, where E_m is the emf of the battery, I the current, and r its internal resistance. Thus a battery of low internal resistance can supply a large current without much decrease in its terminal voltage. It should be noted that the effect of the internal resistance of a battery is the same as if it were a small resistance r in series with the battery, but inside the terminals. If R is the resistance of the external circuit to which the terminals of the battery are connected,

$$E_m - Ir = IR \tag{4}$$

Example: The internal resistance of a battery is 0.1 ohm and its emf 10.0 volts. What will be the current when a resistance of 4.0 ohms is connected across the terminals of the battery?

$$E_m - Ir = IR, \qquad \text{or} \qquad E_m = IR + Ir = I(R + r)$$

so that

$$I = \frac{E_m}{R + r} = \frac{10.0 \text{ volts}}{(4.0 + 0.1) \text{ ohms}} = \frac{10.0 \text{ volts}}{4.1 \text{ ohms}} = 2.4 \text{ amp}$$

Note that this is slightly less than 2.5 amp, the value obtained if the internal resistance of the battery is neglected. The voltage across the terminals of the battery is

$$E_m - Ir = 10.0 \text{ volts} - (2.4 \text{ amp})(0.1 \text{ ohm}) = 10.0 \text{ volts} - 0.2 \text{ volt} = 9.8 \text{ volts}$$

If a resistance of 1 ohm is connected across the terminals of the same battery, the current is $\frac{10.0 \text{ volts}}{1.1 \text{ ohms}} = 9.1$ amp, or 0.9 amp less than the value obtained when internal resistance is neglected.

When a reverse current is maintained in a battery by a source of higher voltage, the voltage across the terminals of the battery is $E_m + Ir$.

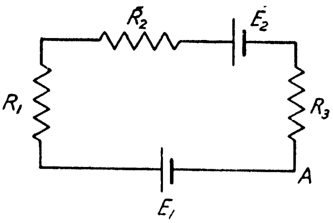


FIG. 5.—A series circuit, $\Sigma E = \Sigma IR$.

Applications to an Entire Circuit. If one goes completely around any closed path in a circuit, returning to the starting point, the sum of the various voltages across the batteries (or generators) in the path (counted negative if they oppose the current) is equal to the sum of the voltages across the (external) resistances in the path; that is,

$$\Sigma E = \Sigma(IR) \tag{5}$$

The symbol Σ means "the summation of," so that

$$\Sigma E = E_1 + E_2 + E_3 + \cdots$$

and

$$\Sigma(IR) = I_1R_1 + I_2R_2 + I_3R_3 + \cdots$$

Consider the circuit of Fig. 5. Let us begin at A and follow the path of the current through E_1, R_1, R_2, E_2, and R_3, returning to A.

$\Sigma E = \Sigma(IR)$, or $E_1 + E_2 = I_1R_1 + I_2R_2 + I_3R_3$ where I_1 is the current through R_1, etc. Since $I_1 = I_2 = I_3$,

$$E_1 + E_2 = I_1(R_1 + R_2 + R_3)$$

Note that E_2 will be a negative number, since it is the voltage across a reversed battery.

Example: Find the current in the circuit of Fig. 5 if $E_1 = 6$ volts, $E_2 = -2$ volts, $R_1 = 2$ ohms, $R_2 = 4$ ohms, $R_3 = 2$ ohms.

From the preceding paragraph

$$E_1 + E_2 = I_1(R_1 + R_2 + R_3)$$

so that

$$I_1 = \frac{E_1 + E_2}{R_1 + R_2 + R_3} = \frac{(6 - 2) \text{ volts}}{(2 + 4 + 2) \text{ ohms}}$$

$$= \frac{4 \text{ volts}}{8 \text{ ohms}} = 0.5 \text{ amp}$$

Emf's in a Closed Circuit. In the preceding section the relation $\Sigma E = \Sigma(IR)$ was stated. In this equation, each voltage E is the potential difference across the terminals of a battery or generator. Remembering that $E = E_m - Ir$, where r is the internal resistance, we can write

$$\Sigma(E_m - Ir) = \Sigma(IR)$$

or

$$\Sigma E_m = \Sigma(IR) + \Sigma(Ir)$$

If all the resistances (internal as well as external) are included in $\Sigma(IR)$, the relation becomes $\Sigma E_m = \Sigma(IR)$. Whenever emf is considered, internal resistance must be taken into account. When there are no branches in the circuit, as in the case of Fig. 5, $I_1 = I_2 = I_3$, so that $\Sigma E_m = I\Sigma R$, or I times the total resistance.

Length, Cross Section, Resistivity. Dr. Georg Simon Ohm, who formulated the law that bears his name, also reported the fact that the resistance of a conductor depends directly upon its length, inversely upon its cross-sectional area, and upon the material of which it is made.

From the study of resistances in series, one would expect that to change the length of a piece of wire would change its resistance, as it can be thought of as a series of small pieces of wire whose total resistance is the sum of the resistances of the individual pieces,

$$R = r_1 + r_2 + r_3 + \cdots$$

The resistance of a piece of uniform wire is directly proportional to its length.

Consider a wire 1 ft in length and having a cross-sectional area of 0.3 in.^{2} By thinking of this as equivalent to three wires (1 ft in length) having cross-sectional areas of 0.1 in.^{2} *connected in parallel*, we may infer that

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

or, since $r_1 = r_2 = r_3$,

$$\frac{1}{R} = \frac{3}{r_1} \qquad \text{and} \qquad r_1 = 3R$$

showing that the resistance of one of the small wires is three times as great as that of the large wire. This suggests (but does not prove) that the resistance of a wire is inversely proportional to the cross section, a fact that was verified experimentally by Ohm himself.

Using $R \propto l$ and $R \propto 1/A$, as indicated at the beginning of this section, we can write $R \propto l/A$, where l is the length and A the cross-sectional area of a uniform conductor. Since conductors of identical size and different materials have different values of resistance, it is useful to define a quantity called the *resistivity* of a substance. It is sometimes so defined as to make $R = \rho(l/A)$, in which ρ is the resistivity, sometimes called specific resistance.

Solving this equation for ρ gives $\rho = R(A/l)$. If A and l are given values of unity, it is seen that ρ is numerically equal to the resistance of a conductor having unit cross section and unit length.

If R is in ohms, A in square centimeters, and l in centimeters, then ρ is expressed in ohm-centimeters.

Example: The resistance of a copper wire 2,500 cm long and 0.09 cm in diameter is 6.7 ohms at 20°C. What is the resistivity of copper at this temperature?

From $R = \rho(l/A)$,

$$\rho = R\frac{A}{l} = \frac{(6.7\text{ ohms})}{2{,}500\text{ cm}}\frac{\pi(0.09\text{ cm})^2}{4} = 1.7 \times 10^{-6}\text{ ohm-cm}$$

Example: What is the resistance of a copper wire (at 20°C) which is 100 ft in length and has a diameter of 0.024 in.? (0.024 in. = 0.002 ft.) From the preceding example

$$\rho = 1.7 \times 10^{-6}\text{ ohm-cm} = \frac{1.7 \times 10^{-6}\text{ ohm-cm}}{30.5\text{ cm/ft}} = \frac{1.7(10^{-6})}{30.5}\text{ ohm-ft.}$$

$$R = \rho\frac{l}{A} = \frac{1.7(10^{-6})}{30.5}\text{ ohm-ft}\frac{(100\text{ft})(4)}{\pi(0.002\text{ ft})^2} = 1.8\text{ ohms}$$

The unit of resistivity in the British engineering system of units differs from that just given, in that different units of length and area are employed. The unit of area is the circular mil, the area of a circle 1 mil (0.001 in.) in diameter, and the unit of length is the foot. Thus the resistivity of a substance is numerically equal to the resistance of a sample of that substance 1 ft long and 1 circular mil in area, and is expressed in ohm-circular mil per foot. This unit is frequently referred to as "ohms per circular mil foot." Since the area of a circle in circular mils is equal to the square of its diameter in mils (thousandths of an inch), $R = \rho(l/d^2)$, where d is the diameter of the wire in mils, l its length in feet, and ρ the resistivity in ohm-circular mil per foot.

Example: Find the resistance of 100 ft of copper wire whose diameter is 0.024 in. and whose resistivity is 10.3 ohm-circular mil/ft. (NOTE: d = 0.024 in. = 24 mils.)

$$R = \rho\frac{l}{d^2} = \frac{(10.3\text{ ohm-circular mil/ft})(100\text{ ft})}{(24^2)\text{ circular mils}} = 1.8\text{ ohms,}$$

agreeing with the result of the preceding example.

SUMMARY

Ohm's law consists of the statement that the resistance of a conductor, defined as the ratio E/I, is constant so long as its temperature and other physical conditions are not changed.

The relation $R = E/I$, and its derived forms $E = IR$ and $I = E/R$, are expressions of the definition of R as the ratio E/I. Ohm's law is utilized when R in these equations is assumed to remain constant while E and I are changed.

An *ammeter* is a device for measuring current in amperes. Since it measures the current that it carries, it must be placed in series with the circuit in which the value of the current is desired.

A *voltmeter* measures difference of potential in volts. It is connected across (in parallel with) the part of the circuit whose voltage is to be measured.

The combined resistance of a *series*-connected group of resistances is the sum of the individual resistances,

$$R = r_1 + r_2 + r_3 + \ . \ . \ .$$

The reciprocal of the combined resistance of a *parallel*-connected group of resistances is equal to the sum of their reciprocals,

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdot \cdot \cdot$$

For any closed circuit,

$$\Sigma E = \Sigma(IR)$$

The *emf* of a source of current is the voltage across its terminals when it is supplying no current.

The *internal resistance* of a source of current (battery or generator) causes a voltage drop within the source, so that the voltage across its terminals is $E_m - Ir$, where E_m is its emf and r its internal resistance.

The resistance of a uniform wire is given by $R = \rho(l/A)$, where ρ is the *resistivity* in ohm-centimeters, l the length in centimeters, and A the cross-sectional area in square centimeters, or by $R = \rho(l/d^2)$, where ρ is the resistivity in ohm-circular mil per foot, l is the length in feet, and d the diameter in mils.

QUESTIONS AND PROBLEMS

1. How does emf differ from potential difference?

2. A resistance forms part of a series circuit. How is the resistance of the circuit affected if a second resistance is connected (*a*) in series with the first? (*b*) in parallel with the first?

3. Why is copper or silver used in electric bus bars rather than a less expensive material such as iron?

4. Why is it more dangerous to touch a 500-volt line than a 110-volt line? Why is it dangerous to have an electric switch within reach of a bathtub?

5. In a circuit like that of Fig. 1*b* the ammeter indicates 0.75 amp and the voltmeter 50 volts. What is the resistance of the lamp, neglecting the fact that the voltmeter carries a small part of the current?

6. If the voltmeter of problem 5 carries a current of 0.001 amp for eac volt indicated by it, what is the actual current through the lamp and the corrected value of its resistance? *Ans.* 0.7 amp; 71 ohms.

7. What is the resistance of the voltmeter of problem 6?

8. In problem 5 the current is increased to 1.00 amp. What will now be the reading of the voltmeter? *Ans.* 67 volts.

9. A dry cell when short-circuited will furnish a current of about 30 amp. If its emf is 1.5 volts, what is the internal resistance? Should the cell be allowed to deliver this current for more than a brief time? Why? An ordinary household electric lamp takes about 1 amp. Would it be safe to connect such a lamp directly to a dry cell? Why?

10. An incandescent lamp is designed for a current of 0.60 amp. If a potential difference of 110 volts is necessary to sustain that amount of current, what is the resistance of the lamp? *Ans.* $1\bar{8}0$ ohms.

11. Find the resistance of a combination formed by 5.0 ohms and 7.0 ohms in parallel.

12. The combination of problem 11 is connected in series with another pair of 4.0 and 3.0 ohms in parallel. What is the total resistance? *Ans.* 4.6 ohms.

13. A battery of emf 5 volts and internal resistance 0.2 ohm is connected to the combination of problem 11. Find the total current and that in each resistance.

14. The terminal voltage of a battery is 9.0 volts when supplying a current of 4.0 amp, and 8.5 volts when supplying 6.0 amp. Find its internal resistance and emf. *Ans.* 0.25 ohm; 10 volts.

15. Find the resistance of 5,000 ft of copper wire of diameter 0.011 in. The resistivity of copper is 10.3 ohm-circular mil/ft.

16. What will be the diameter of a copper wire whose resistance is 20 ohms and whose length is 500 ft? *Ans.* 16 mils = 0.016 in.

EXPERIMENT

Ohm's Law; Resistance Combinations

Apparatus: Panel apparatus for study of Ohm's law; flexible connectors; three dry cells or storage battery.

Ohm's law indicates that for a given conductor the quotient E/I (called the resistance R) is constant so long as its temperature and other physical conditions are not changed.

One purpose of this experiment is to give the student observable proof of Ohm's law. Another is to show that the resistance of a uniform conductor is proportional to its length and inversely proportional to its cross-sectional area. A third purpose is to verify experimentally the equations derived in this chapter for computing the effective resistance of series and parallel combinations of resistances.

Use will be made of the apparatus shown schematically by Fig. 6. The conductors to be investigated are indicated by the lines *a, b, c, d, e.*

Current is obtained by connecting three dry cells or a storage battery at B. The switch should be closed only when readings are being taken.

From the wiring diagram it is evident that if a connector is placed between p_1 and p_3 and switch S is closed, a current I will flow through the fuse F, the ammeter A, the rheostat M, and the conductor a. If a connector is placed between p_3 and p_2, another between p_9 and p_{10} (the dotted lines indicate flexible, removable connectors) the voltmeter V will indicate the potential difference E across a. By means of the rheostat M the current can be changed to different values.

Following the procedure just described, determine several corresponding values of E and I for conductor a, recording them in the table on

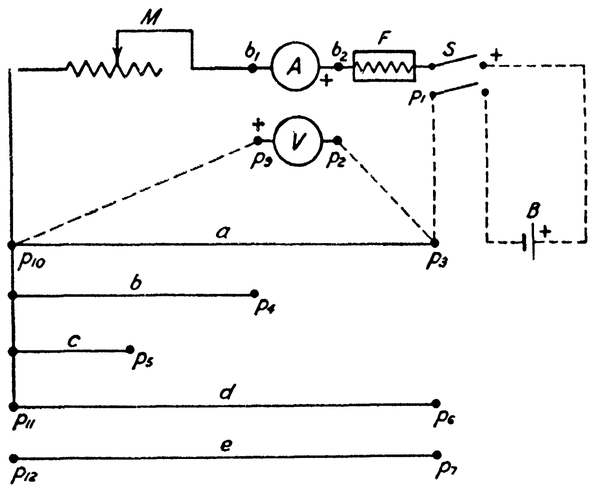


FIG. 6.—Diagram of a panel used in checking the laws of resistance.

page 218 and computing the ratio E/I. Does the ratio remain essentially constant, even though E and I are changed?

Conductor b is half as long as a and twice as long as c, but all three conductors are of the same material and have the same area of cross section. Make sets of measurements for b and c similar to those above and decide whether or not the values of resistance E/I are related to those of a as one would expect.

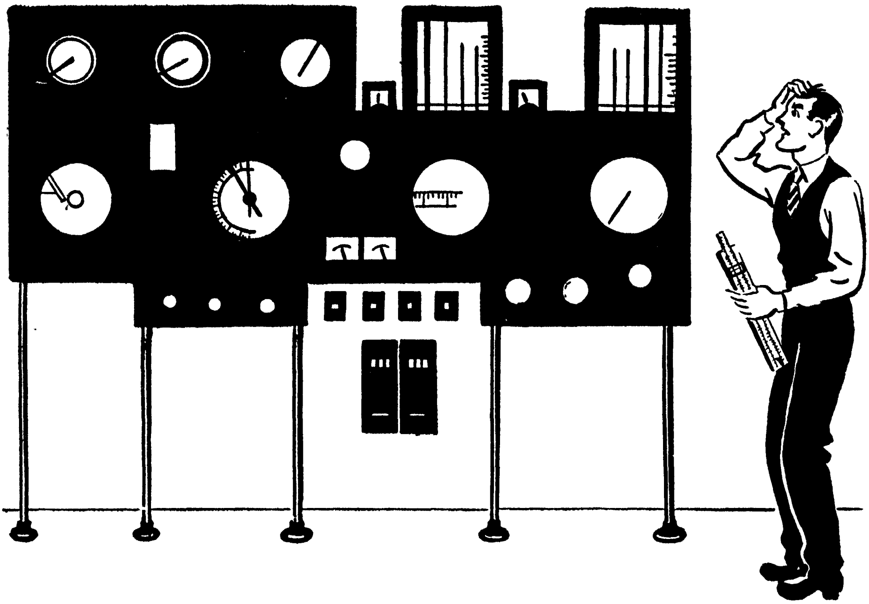
Conductor d is of the same material as a and has the same length, but its area of cross section is four times as great. How should its resistance compare with that of a? Verify this experimentally.

Determine the resistance of conductor e, which is made of a different material. To do this, connect p_{11} to p_{12} and p_1 to p_7.

In order to determine the combined resistance of two conductors in series, connect p_6 to p_7 and p_1 to p_{12}. Which conductors are connected in series? What is their combined resistance? Compare this with the result obtained from their individual resistances.

To arrange the same conductors in parallel, connect p_{11} to p_{12} and p_6 to p_7 and p_1. Determine the resistance of the combination and check it against the result obtained by the use of the equation expressing the resistance of a parallel combination.

Conductor	I	E	E/I
a			
b			
c			
d			
e			
d and e, in series			
d and e, in parallel			



CHAPTER 23

ELECTRICAL MEASURING INSTRUMENTS

Practically all electrical measurements involve either the measurement or detection of electric current. The measurement of electric current can be accomplished by means of any one of the three principal effects of current: heating effect, chemical effect, or magnetic effect; yet for the sake of accuracy and convenience the magnetic effect (Fig. 1) is utilized almost universally in electrical measuring instruments.

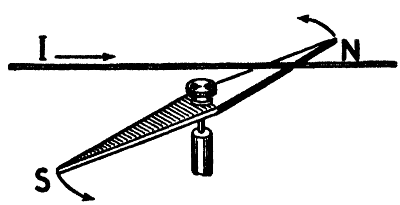
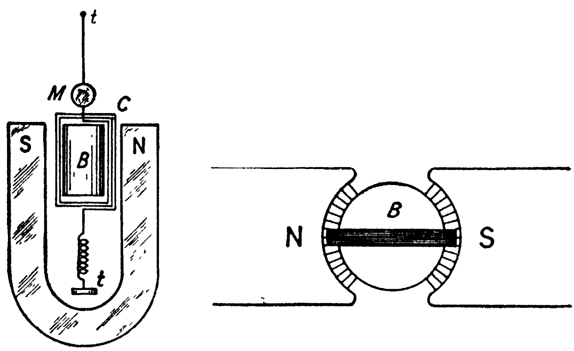


FIG. 1.—The original electric indicator, Oersted, 1819.

Galvanometers. The basic electrical instrument is the galvanometer, a device with which very small electric currents can be detected and measured. The d'Arsonval, or permanent-magnet-moving-coil type of galvanometer, is shown in Figs. 2 and 3. In Fig. 2 a coil C is suspended between the poles N and S of a U-shaped magnet by means of a light metallic ribbon. Connections are made to the coil at the terminals marked t. The cylinder of soft iron B

serves to concentrate and increase the field of the magnet, and the mirror M is used to indicate the position of the coil, either by reflecting a beam of light or by producing an image of a scale to be viewed through a low-power telescope.

When a current is set up in a coil that is between the poles of a magnet, the coil is acted upon by a torque, which tends to turn it into a position perpendicular to the line joining the poles. If a current is set up in the coil (as viewed from above) in Fig. 2*b*, the coil will turn toward a position at right angles to the position shown. In turning, however, it must twist the metallic ribbon that supports it; hence it turns to the position in which the torque exerted on it by the magnet is just neutralized by the reaction of the twisted ribbon.



(a) (b)

FIG. 2.—Permanent-magnet, moving-coil type of galvanometer.

The torque exerted on the coil by the magnet is proportional to the current in the coil, and the torque of reaction of the ribbon is proportional to the angle through which it is twisted. Since these torques are equal and opposite when the coil reaches the equilibrium position, the angle through which the coil turns is proportional to the current through it; that is, $\theta \propto I$, where θ is the angular deflection of the coil. From this we can write $I = k\theta$, so that $k = I/\theta$, where k is called the *current sensitivity* of the galvanometer.

For sensitive galvanometers of the type shown in Fig. 3, which are read with telescope and scale, the current sensitivity k is expressed in microamperes per millimeter deflection on a scale 1 m from the mirror, so that it is numerically equal to the current in *microamperes* (millionths of an ampere and commonly abbreviated μa) required to cause a 1-mm deflection of the image on a scale 1 m distant. For the most sensitive types of commercial d'Arsonval galvanometers, k is about 0.00001 μa/mm, or 10^{-11} amp/mm. The term *current sensitivity* is somewhat misleading, since k is low for sensitive galvanometers.

Portability, ruggedness, and convenience of operation are obtained in the d'Arsonval galvanometer by mounting the moving coil on jeweled pivots, attaching a pointer to the coil, and replacing the metallic ribbon

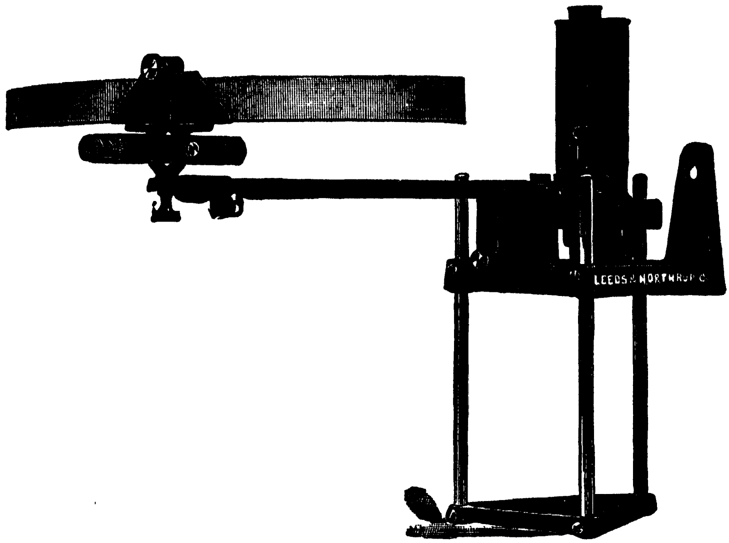


FIG. 3.—Laboratory galvanometer with telescope and scale.

suspension by two spiral springs as shown in Fig. 4. The springs, besides balancing the magnetic torque exerted on the coil, provide its external connections. The current sensitivity of an instrument of this type is

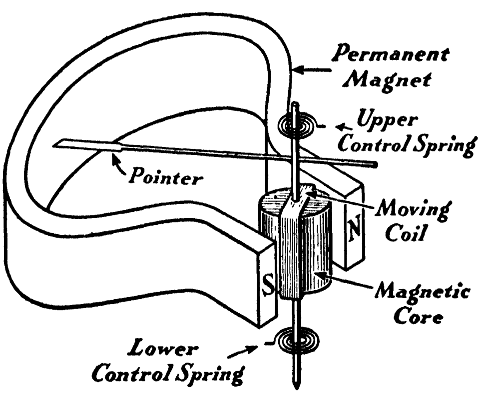


FIG. 4.—Diagrammatic representation of a portable-type galvanometer.

expressed in microamperes per division of the scale over which the pointer moves.

Example: A galvanometer of the type shown in Fig. 3 has a current sensitivity of 0.002 μa/mm. What current is necessary to produce a deflection of 20 cm on a scale

1 m distant? $I = k\theta$, where θ is in millimeters (on a scale 1 m away), so that

$$I = (0.002\ \mu\text{a/mm})(200\ \text{mm}) = 0.4\ \mu\text{a}$$

This is equivalent to 0.0000004 amp. On a scale twice as far away, the deflection would be twice as great.

Example: A current of 2×10^{-4} amp causes a deflection of 10 divisions on the scale of a portable-type galvanometer. What is its current sensitivity?

$$k = \frac{I}{\theta} = \frac{0.0002\ \text{amp}}{10\ \text{divisions}} = \frac{200\ \mu\text{a}}{10\ \text{divisions}}$$
$$= 20\ \mu\text{a/division}$$

Example: If the moving coil of the galvanometer of the first example has a resistance of 25 ohms, what is the potential difference across its terminals when the deflection is 20 cm?

$$E = IR = (0.0000004\ \text{amp})(25\ \text{ohms}) = 0.00001\ \text{volt}$$

Example: What current will cause a full-scale deflection (100 divisions) of a portable galvanometer for which $k = 20\ \mu$a/division?

$$I = k\theta = (20\ \mu\text{a/division})(100\ \text{divisions})$$
$$= 2{,}000\ \mu\text{a} = 0.0020\ \text{amp}$$

Example: Find the potential difference across the galvanometer of the preceding example if its resistance is 5.0 ohms.

$$E = IR = (0.0020\ \text{amp})(5.0\ \text{ohms}) = 0.010\ \text{volt}$$

Voltmeters. In the last example it is seen that a potential difference of 0.01 volt across the terminals of the galvanometer causes a current resulting in a full-scale deflection of 100 divisions. This means that the instrument can be thought of as a voltmeter with which voltages up to 0.010 volt can be measured. Since deflection, current, and potential difference are in direct proportion, each division represents either 0.0001 volt or 2×10^{-5} amp. If this meter were intended to be used primarily in measuring potential difference, it would be called a millivoltmeter, and its scale would be marked 0 to 10 mv (millivolts), each 10 divisions representing 1 mv or 0.001 volt.

On the other hand, if it were intended primarily for use in measuring current, it would be called a milliammeter, and its scale might be marked 0 to 2 ma (milliamperes) in 100 divisions, each 5 divisions representing 0.1 ma or 0.0001 amp.

In order to use this galvanometer as a voltmeter registering to 10 volts, it is necessary only to increase its resistance until a potential difference of 10 volts is just sufficient to produce in it a current of 0.002 amp, or enough for a full-scale deflection. Hence

$$R = \frac{E}{I} = \frac{10\ \text{volts}}{0.002\ \text{amp}} = 5{,}000\ \text{ohms}$$

so that the resistance of the meter (5 ohms) must be increased by the addition of a series resistance r of 4,995 ohms, as in the diagram of Fig. 5. The scale of the instrument should be labeled 0–10 volts, so that each division represents 0.1 volt. If a potential difference of 5 volts is applied to the terminals of this instrument, the current is

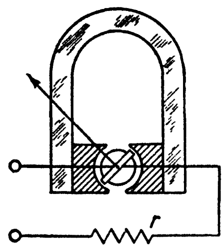


Fig. 5.—Circuit of a voltmeter.

$$I = \frac{E}{R} = \frac{5 \text{ volts}}{5{,}000 \text{ ohms}} = 0.001 \text{ amp}$$

Since 0.002 amp is the full-scale current, the deflection will be just half scale, or 50 divisions, indicating 5 volts on the 0–10 volt scale. It should be noticed that the resistance of the voltmeter is $R = r + R_g$, where r is the series resistance and R_g is that of the galvanometer.

Example: What series resistance should be used with a similar galvanometer in order to employ it as a voltmeter of range 0 to 200 volts?

$$R = \frac{E}{I} = \frac{200 \text{ volts}}{0.002 \text{ amp}} = 100{,}000 \text{ ohms}$$

total resistance, obtained by making $r = 99{,}995$ ohms. Each division on this instrument will represent 2 volts, and its scale will be labeled 0 – 200 volts.

Ammeters. It was pointed out that the portable galvanometer of 5 ohms internal resistance and requiring 0.002 amp for a full-scale deflection can be used as a milliammeter for measurements of 0 to 2 ma, since 2 ma = 0.002 amp. In order to use it as an ammeter for measurements up to 2 amp, it is necessary to connect a low resistance, called a shunt, across its terminals, as in Fig. 7*a*. The resistance r may or may not be included. Let us assume for the present that it is omitted. In order to be deflected full scale, the galvanometer must carry just 0.002 amp; hence the shunt S must carry the remainder of the 2-amp current, or 1.998 amp.

The potential difference across the galvanometer is

$$E = IR = (0.002 \text{ amp})(5 \text{ ohms}) = 0.01 \text{ volt},$$

which must be the same as that across S, thus

$$R_s = \frac{E}{I_s} = \frac{0.01 \text{ volt}}{1.998 \text{ amp}} = 0.005005 \text{ ohm}$$

This resistance is so small that a short piece of heavy copper wire might be used for S in this case. If a larger value of shunt resistance is desired, a resistance r can be placed in the circuit (Fig. 7*a*). This effectively adds to the resistance of the galvanometer, making it necessary to use a higher potential difference to cause a full-scale current. Suppose r is 10 ohms.

For a full-scale current of 0.002 amp through the meter, a potential difference of (0.002 amp)(15 ohms) = 0.03 volt is now required, since the combined resistance of the galvanometer and r is 15 ohms. In order to carry 1.998 amp for a potential difference of 0.03 volt, the resistance

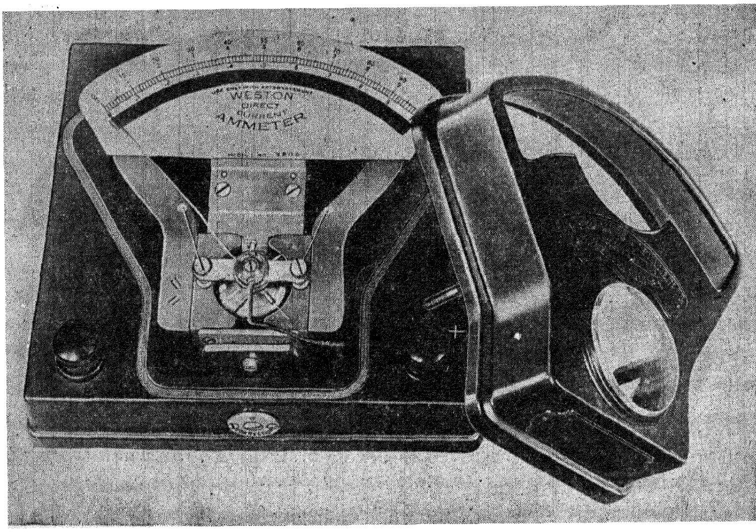


FIG. 6.—A commercial ammeter.

of the shunt must be $R_s = \dfrac{0.03 \text{ volt}}{1.998 \text{ amp}} = 0.015015$ ohm, or three times as much as when r is omitted.

In practice, since it is very difficult to make the resistance R_s exactly a certain value when it is to be very low, one commonly obtains a shunt

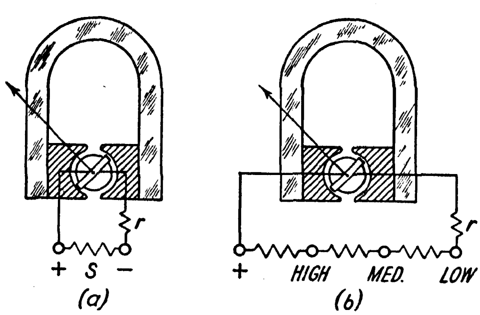


FIG. 7.—Ammeter circuits.

whose resistance is slightly larger than is needed and then adjusts the value of the resistance r to make the meter operate as desired. For example, if a shunt of resistance 0.02 ohm were available, one could utilize it by increasing r. A current of 1.998 amp through a shunt of

0.02-ohm resistance results in a potential difference of (1.998 amp) (0.02 ohm) = 0.03996 volt. This potential difference must cause a current of 0.002 amp through the galvanometer and r combined. Their combined resistance must be, then, $\frac{0.03996 \text{ volt}}{0.002 \text{ amp}} = 19.98$ ohms. Since the resistance of the meter is 5 ohms, that of r must be increased to (19.98 − 5) ohms = 14.98 ohms. The accurate adjustment of a resistance of this size is not difficult.

A galvanometer may be employed as an ammeter of several different ranges through the use of a number of removable shunts, or by the use of a circuit such as that in Fig. 7*b*. Connection is made to the + terminal and to one of the three terminals marked high, medium, and low, respectively. The advantage of this circuit is that the shunt connections are permanently made, eliminating the error due to the variation of contact resistance when a removable shunt is used.

Meter-range Formulas. In order to increase the range of a voltmeter by a factor n, one introduces in series with it a resistance R_m given by

$$R_m = (n - 1)R_v \tag{1}$$

in which R_v is the resistance of the voltmeter.

Example: A voltmeter has a resistance of 250 ohms and a range of 0 to 10 volts. What series resistance will provide it with a range of 0 to 50 volts?

Since n = 50 volts/10 volts = 5.

$$R_m = (n - 1)R_v = (5 - 1)(250 \text{ ohms}) = 1{,}000 \text{ ohms}.$$

In order to increase the range of an ammeter by a factor n, one connects in parallel with it a resistance

$$R_s = \frac{R_a}{n - 1} \tag{2}$$

Example: What shunt resistance should be used with an ammeter whose resistance is 0.048 ohm in order to increase its range from 0 to 1 amp to a range of 0 to 5 amp?

$$R_s = \frac{R_a}{n - 1} = \frac{0.048 \text{ ohm}}{4} = 0.012 \text{ ohm}$$

Effects of Meters in the Circuit. When an ammeter is inserted in a circuit in order to provide a measurement of the current, the current to be measured is changed by the introduction of the resistance of the ammeter into its path. It is essential that the change in current thus caused shall be a very small fraction of the current itself, that is, the resistance of the ammeter must be a small fraction of the total resistance of the circuit.

Similarly, when a voltmeter is connected across a potential difference whose value is desired, the potential difference is changed by the effect of the voltmeter. When the voltmeter is thus placed in parallel with a portion of the circuit, the resistance of the combination so formed is less than without the voltmeter, hence the potential difference across that part of the circuit is decreased and the total current increased. The voltmeter introduces two errors: changing the current in the circuit and reducing the potential difference that is to be measured. In order that these errors shall be small, it is essential that the resistance of the voltmeter shall be very large in comparison with that across which it is connected. This will ensure also that the current through the voltmeter will be small in comparison with that in the main circuit.

The Potentiometer. Suppose that the potential difference between two points in a circuit is desired. If one connects a voltmeter to these two points, the potential difference between them is changed because of the current taken by the voltmeter. It has been shown that the voltmeter reading is an accurate indication of the desired potential difference *only*

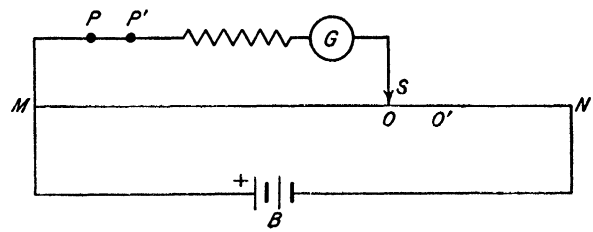


FIG. 8.—Circuit illustrating the principle of the potentiometer.

when the voltmeter current is very small in comparison with that in the main circuit.

If one desires to measure the potential difference between two points in a circuit in which the current is extremely small, as for example, in the grid circuit of a radio tube, he cannot use an ordinary voltmeter because the voltmeter would draw a current comparable with that in the circuit. A device for measuring potential differences which does not draw current from the source being measured is the *potentiometer*, a diagram of which appears in Fig. 8.

A consideration of Fig. 8 will aid in illustrating the principle of the potentiometer. The battery B causes a steady current in a uniform straight wire MN, so that there is a potential difference between the points M and O. If the sliding key S is depressed, therefore, there will be a current in the galvanometer circuit.

Now suppose that the section PP' of the galvanometer circuit is removed and a battery E is inserted with its + terminal at P. If the emf of this battery is equal to the potential difference across MO, there will now be no current in the galvanometer circuit. In practice, of course, the point O is located by sliding S until the galvanometer shows no deflection. The current in MN is not changed by its connection to the galvanometer circuit, since there is no current in the galvanometer circuit.

If E is replaced by a battery whose emf is slightly larger, say, E', one can eliminate the current in the galvanometer circuit by moving S to a position O', where the potential difference across MO' is equal to E'. If, however, the emf to be measured is larger than that of the battery B, no point on the slide-wire can be found such that the current in the galvanometer circuit becomes zero. A battery must be selected for B whose emf is larger than any to be measured. Since the wire MN is uniform, the resistance of a part of it, say MO, is proportional to the length MO, so that the ratio MO'/MO is equal to the ratio of the voltages across MO' and MO, respectively, and $E'/E = MO'/MO$.

Once the ratio E'/E is evaluated, E' can be determined if the emf E of the first battery is known. A standard cell is ordinarily used as the source of an accurately known emf.

The commercial potentiometer is arranged as a direct-reading instrument. The standard cell is first connected at PP' and dials set to read its emf. The current in MN is then adjusted by means of a variable resistance until the galvanometer does not deflect when the switch S is closed. After this adjustment has been made, the standard cell is replaced by the unknown emf and the dials turned until the galvanometer deflection is zero. The reading of the dials is then the value of the unknown voltage.

The Wheatstone Bridge. A very important device for measurement of resistance is the *Wheatstone bridge*, a diagram of which is shown in Fig. 9. It consists essentially of four resistances, one of which is the unknown. The values of the resistances are adjusted until there is no deflection of the galvanometer when the switches are closed. Then, since B and C are at the same potential,

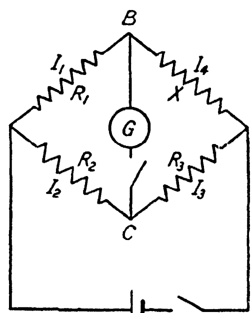


FIG. 9.—Conventional diagram of a Wheatstone bridge.

$$I_1R_1 = I_2R_2 \tag{3}$$

and

$$I_4X = I_3R_3 \tag{4}$$

Since there is no current in the galvanometer $I_1 = I_4$ and $I_2 = I_3$. Dividing Eq. (4) by Eq. (3),

$$\frac{X}{R_1} = \frac{R_3}{R_2} \tag{5}$$

$$X = R_1\frac{R_3}{R_2} \tag{6}$$

To measure X it is not necessary to know R_3 and R_2 individually but merely their ratio. Commercial Wheatstone bridges are built with a known adjustable resistance that corresponds to R_1 and ratio coils that can be adjusted at will to give ratios that are convenient multiples of 10, usually from 10^{-3} up to 10^3. In some instruments both battery and galvanometer are built into the same box as the resistances, with binding posts for connection to the unknown resistance.

SUMMARY

The *current sensitivity* of a galvanometer is $k = I/\theta$, where I is the current and θ is the deflection of the galvanometer. θ is measured either in millimeters deflection on a scale 1 m distance from the axis of rotation, or simply in scale divisions.

A *voltmeter* consists of a portable-type galvanometer, a series resistance of proper value, and a scale calibrated to indicate potential difference in volts.

An *ammeter* is formed by connecting a (shunt) resistance of proper value across the terminals of a portable galvanometer. The scale is calibrated to indicate current in amperes.

The introduction of a meter into a circuit changes the conditions in that circuit. It is essential that the variation thus introduced be small in comparison with the quantity to be measured, unless, of course, the condition of the circuit with the meter in place is desired.

The *potentiometer* is an instrument with which the potential difference between two points can be determined without changing the current between them. The potentiometer simply compares potential differences, since a known potential difference must be available in order to determine an unknown potential difference with this instrument.

The *Wheatstone bridge* is a device for the measurement of an unknown resistance by comparison with a known resistance.

QUESTIONS AND PROBLEMS

1. A portable galvanometer is given a full-scale deflection by a current of 0.00100 amp. If the resistance of the meter is 7.0 ohms, what series resistance must be used with it to measure voltages up to 50 volts?

2. It is desired to employ the galvanometer of problem 1 as a milliammeter of range 0 to 50 ma. What shunt resistance should be placed across it?

Ans. 0.14 ohm.

3. If the lowest shunt resistance available in problem 2 is four times as large as desired, what can be done to achieve the desired result?

4. What is the current sensitivity of a galvanometer that is deflected 20 cm on a scale 250 cm distant by a current of 3.00×10^{-5} amp?

Ans. 0.375 μa/mm.

5. What is the current sensitivity of the galvanometer of problem 1 if its scale has 50 divisions?

6. What essential differences are there between the common types of galvanometers and ammeters? between ammeters and voltmeters? How are ammeters connected in a circuit? How are voltmeters connected? Is it desirable for an ammeter to have a high resistance or a low one? Should a voltmeter have a high resistance or a low one?

7. An ammeter with a range of 5 amp has a voltage drop across it at full-scale deflection of 50 mv. How could it be converted into a 20-amp meter?

8. A certain 3-volt voltmeter requires a current of 10 ma to produce full-scale deflection. How may it be converted into an instrument with a range of 150 volts? *Ans.* 14,700 ohms.

9. A millivoltmeter with a resistance of 0.8 ohm has a range of 24 mv. How could it be converted into (*a*) an ammeter with a range of 30 amp? (*b*) a voltmeter with a range of 12 volts?

10. A certain meter gives a full-scale deflection for a potential difference of 0.05 volt across its terminals. The resistance of the instrument is 0.4 ohm. (*a*) How could it be converted into an ammeter with a range of 25 amp? (*b*) How could it be converted into a voltmeter with a range of 125 volts?

Ans. 0.00201 ohm; 999.6 ohms.

11. Sketch the essential parts of a simple potentiometer. Explain fully how it may be used to measure an unknown emf. Explain why its readings give the true emf of a cell, rather than its terminal potential difference.

12. Sketch the wiring diagram showing the essential parts of the Wheatstone bridge. Describe its operation and derive the *working* equation for its use. (Be careful to *justify* each step of the derivation.)

13. A simple slide-wire potentiometer consisting of a 2-m wire with a resistance of 5 ohms is connected in series with a working battery of emf 6 volts and internal resistance 0.2 ohm and a variable rheostat. What must be the value of the resistance in the rheostat in order that the potentiometer may be "direct reading," that is, for the potential difference per millimeter of slide wire to be 1 mv?

14. In a potentiometer circuit, MO and ON (Fig. 8) are adjusted to 64.0 and 36.0 cm, respectively, in order to produce zero deflection of the galvanometer when a standard cell of emf 1.0183 volts is in the circuit. When the terminals are connected to the grid and cathode, respectively, of a radio vacuum tube (in operation), MO is changed to 95.0 cm in order to reestablish the condition of zero deflection. What is the potential difference between the elements of the radio tube? *Ans.* 1.51 volts.

EXPERIMENT

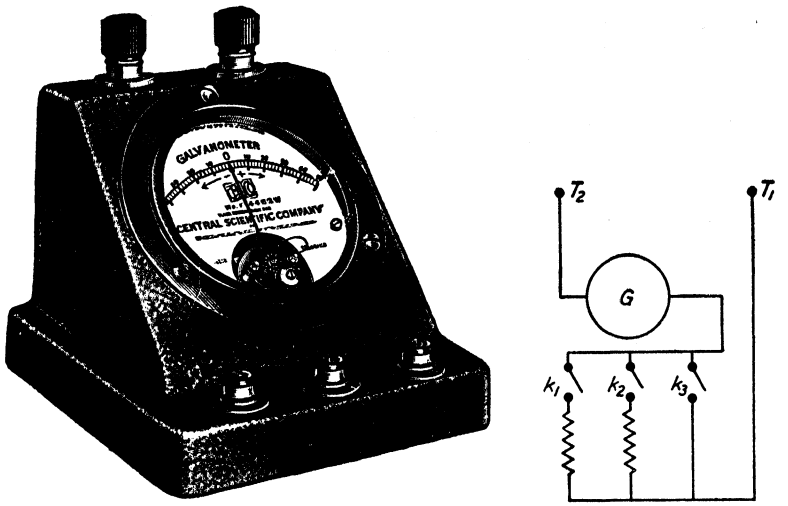
Galvanometers, Multipliers, and Shunts

Apparatus: The panel shown in Fig. 11; multipliers; shunts; dry cell.

The instrument G (Figs. 10 and 11) is a portable-type moving-coil galvanometer, which can be adapted to a variety of uses. The purposes of this experiment are (1) to provide an understanding of the uses of such a multiple-purpose instrument and (2) to apply in an experimental way the method, discussed earlier in this chapter, by which a galvanometer can be made into a voltmeter or an ammeter. The first of these objectives will be treated in this chapter; the second, in Chap. 24.

PART I

The current sensitivity of the galvanometer G is 10 μa/division. Since the full-scale deflection is 50 divisions, the current required for maximum deflection is (50 divisions)(10 μa/division) = 500 μa, or 0.0005 amp. In



(A) (B)

FIG. 10.—A portable galvanometer equipped with keys and resistances to make a multiple-purpose instrument.

order to place the instrument in operation, it is necessary to depress one of the three keys k_1, k_2, k_3. As is evident from an examination of Fig. 10B, when k_3 is depressed, the galvanometer is connected directly to the terminals T_1 and T_2. For this reason one should never depress k_3 without making sure that conditions are such that the current in the galvanometer will not greatly exceed 0.0005 amp, else the instrument might be damaged.

When k_2 is depressed, the galvanometer is connected in series with a resistance of about 197 ohms, or enough to make a total resistance of 200 ohms, since that of the meter itself is about 3 ohms. Finally, k_1 connects the galvanometer in series with a protective resistance of 10,000 ohms, so that voltages as large as 5 volts can safely be applied to T_1 and T_2.

It is imperative that one make a practice of depressing the keys in the order in which they are numbered. Even though it is desired to use the instrument with k_3 depressed, one should first depress k_1 and k_2 in succession, making sure in each case, before depressing the next key, that the deflection does not exceed 1 or 2 divisions.

a. Polarity Indicator. The instrument can be used as a polarity indicator by the use of a simple rule: The terminal (T_1 or T_2) toward which the needle swings is positive with respect to the other.

b. Microämmeter. The instrument (with any key depressed) can be used as a microammeter of range 500/0/500 μa, since the current required for a full-scale deflection is 0.0005 amp, or 500 μa. (The notation used here indicates a range of 500 μa on each side of 0.)

c. Millivoltmeter. Since the total resistance of the instrument is 200 ohms when k_2 is depressed, the potential difference (across T_1 and T_2) necessary to produce a full-scale deflection is

$$(0.0005 \text{ amp})(200 \text{ ohms}) = 0.1 \text{ volt},$$

so that the instrument can be used (with k_2) as a millivoltmeter of range 100/0/100 mv.

d. Voltmeter. With k_1 depressed, the total resistance is 10,000 ohms, so that the potential difference necessary for a full-scale deflection is (0.0005 amp)(10,000 ohms) = 5 volts, hence with k_1 depressed the instrument is a voltmeter of range 5/0/5 volts. Other voltmeter ranges can be provided by the use of external multipliers (series resistances).

e. Ammeter. By connecting a shunt of proper resistance across the terminals of the instrument and depressing k_2 one can convert it into an ammeter. The prepared shunts supplied with the instrument are designed to mount directly on T_1 and T_2. The connections initially attached to T_1 and T_2 are removed and placed on the binding posts of the shunt. The two shunts supplied provide ranges of 0.05/0/0.05 amp (or 50/0/50 ma) and 0.5/0/0.5 amp (500/0/500 ma), respectively.

Let us examine the panel illustrated in Fig. 11 and shown schematically in Fig. 12. Current is supplied to the panel by a dry cell C through a reversing switch S at the lower left of the panel. Let us first make sure that this switch is open.

The decade resistance box R will not be used in this part of the experiment. Hence let us open switches S_1 and S_3. Next, let us close S_2, so that T_1 and T_2, the terminals of the instrument G, are connected to the wires U and V.

Examine the diagram of Fig. 12. When S is closed, the dry cell C is connected in series with the resistances $\overline{MN}$ and $\overline{PQ}$. The latter is variable, since the contact Q can be moved along the rheostat R_1. The maximum resistance afforded by R_1 is obtained when Q is at W, the left end of R_1. If the voltage of the dry cell is about 1.5 volts, that across $\overline{MN}$ can be varied from a maximum of 1.5 volts (when Q is at P) to a minimum

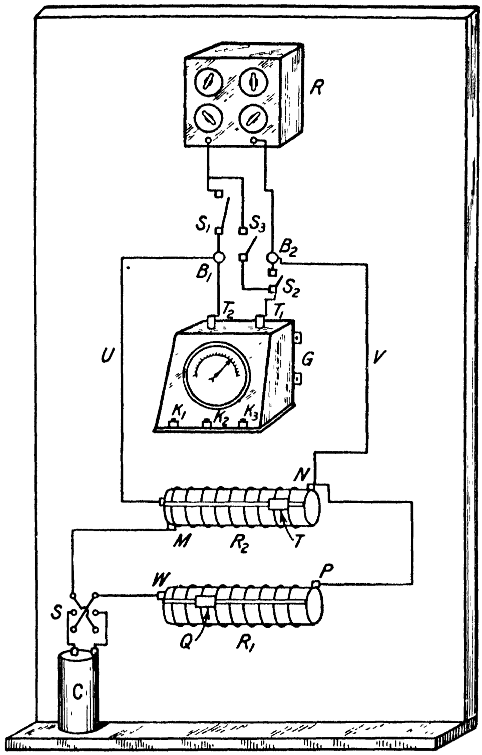


FIG. 11.—A panel arranged for the study of a galvanometer with multipliers and shunts.

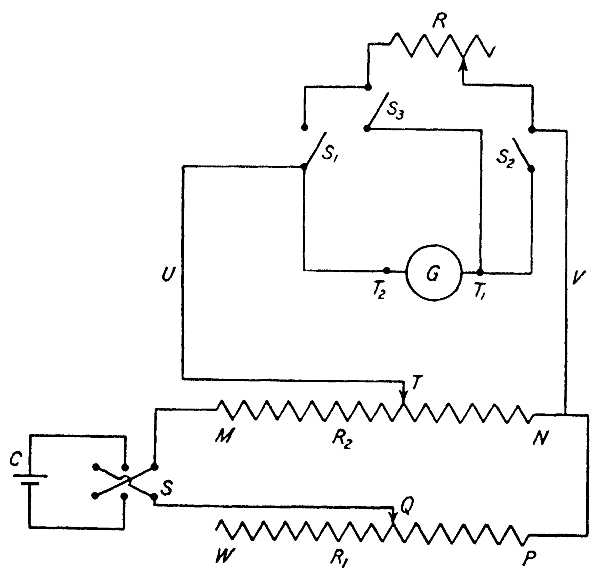


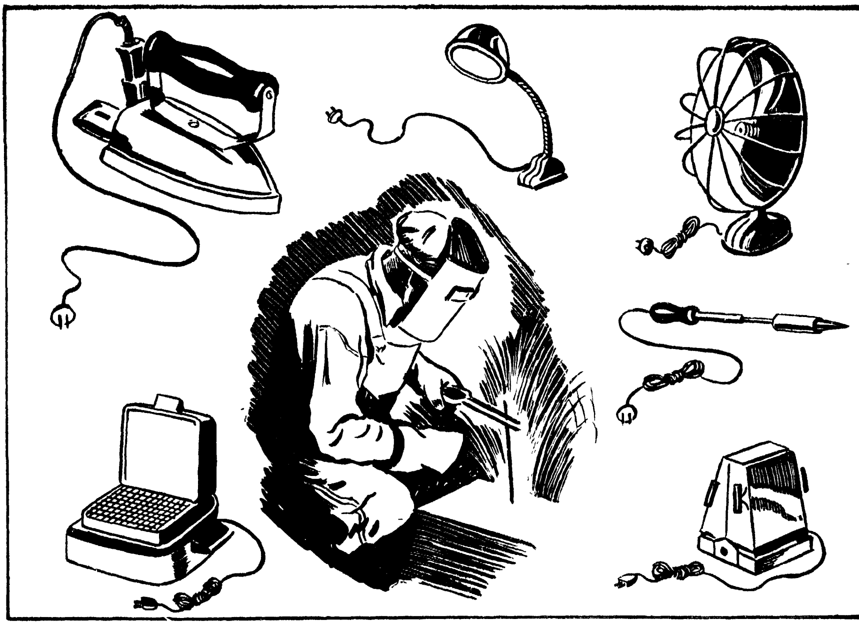
FIG. 12.—Conventional circuit diagram of the panel shown in Fig. 11.

of a few hundredths of a volt (when Q is at W), since the rheostat R_1 has a much larger resistance than that of R_2. The potential difference across $\overline{TN}$ can be made any desired fraction of that across $\overline{MN}$ by moving T along $\overline{MN}$.

1. In order to test the instrument G as a polarity indicator, let us adjust T to a position near the center of $\overline{MN}$, and Q to a position near the center of $\overline{PW}$. Close S, depress k_1, and note the direction of deflection of the needle. If a deflection is not perceptible, depress k_2. As soon as the direction of deflection has been noted, open the switch S in order to avoid unnecessary use of current. Trace the circuit from the positive terminal T_1 or T_2 (that toward which the needle swings) to the dry cell, and verify the indication of polarity.

2. Making sure that S is open, move Q to P and T to M, thereby applying the full voltage of the dry cell to the wires U and V. Now close S and depress k_1. Remembering that, with k_1 depressed, the instrument is a voltmeter of range 5/0/5 volts, determine the reading. Is this a reasonable value for the voltage of a dry cell (under load)? Open the switch S.

Reduce the voltage to be applied to $\overline{MN}$ by moving Q halfway to W, leaving T at M. Depress k_1 to make sure that the deflection is less than 1 division, then depress k_2 in order to use G as a millivoltmeter of range 100/0/100 mv. Record the reading, then open S. Notice the resistances of the rheostats R_1 and R_2, and from the positions of T and Q, estimate the voltage across $\overline{TN}$ and use it as a rough check on the voltage indicated by the reading.



CHAPTER 24

HEATING EFFECT OF AN ELECTRIC CURRENT

The flow of electricity through a wire or other conductor always produces heat. Electric soldering irons, electric welding, electric furnaces, and electric lighting provided by arcs or incandescent lamps are among the important devices and processes that utilize the heating effect of an electric current.

In heating devices the wire in which the useful heat is produced is called the heating element. It is often embedded in a refractory material, which keeps it in place and prevents its oxidation. If the heating element is exposed to air, it should be made of metal that does not oxidize readily. Nickel-chromium alloys have been developed for this purpose.

Joule's Law of Heating. The quantity of heat produced in a given conductor depends, as we might expect, upon the current and the time it is maintained. Still another factor is involved, namely, the resistance of the conductor. If the same current exists for equal intervals of time in pieces of wire having the same dimensions, one of copper and the other of iron, the iron will become hotter than the copper. The iron wire has a resistance greater than that of the copper wire. Experiment shows that the heat produced in a conductor is directly proportional to the resistance of the conductor, to the square of the current, and to the time. This statement is known as *Joule's law* of electric heating.

The energy W converted into heat in a time t by a current I in a conductor of resistance R is given by this law:

$$W = I^2Rt \tag{1}$$

If R is expressed in ohms, I in amperes, and t in seconds, the energy will be given in joules. From the definition of potential difference

$$E = \frac{W}{Q}$$

or

$$W = EQ$$

An amount of energy W (joules) in the form of heat is developed when a quantity of electricity Q (coulombs) passes through a wire whose two ends differ in potential by an amount E (volts). Since

$$Q = It$$
$$W = EQ = EIt = I^2Rt \tag{2}$$

Example: Calculate the energy supplied in 15 min to a percolator using 4.5 amp at 110 volts.

$$\begin{aligned} W &= (110 \text{ volts})(4.5 \text{ amp})(900 \text{ sec}) \\ &= 4.5 \times 10^5 \text{ joules} \end{aligned}$$

Mechanical Equivalent of Heat. Energy is expressed in Eq. (1) in terms of the joule, which is basically a mechanical unit. Energy in the form of heat is measured in terms of the calorie. Experiments are necessary to establish the relation between the joule and the calorie or between any unit of mechanical energy and heat. These experiments have demonstrated the fact that there is a direct proportion between the expenditure of mechanical energy W and the heat H developed. This important law of nature is represented by the equation

$$W = JH \tag{3}$$

where J is the proportionality factor called the *mechanical equivalent of heat*. Relationships for the conversion of heat to mechanical energy are given in the accompanying table.

RELATION OF HEAT TO MECHANICAL WORK

Quantity of Heat	Equivalent Amount of Mechanical Work
1 calorie	= 4.18×10^7 ergs, or 4.18 joules
1 Btu	= 778 foot-pounds
1 Btu	= 1,055 joules
0.239 calorie	= 1 joule

One method of measuring the mechanical equivalent of heat makes use of the electric calorimeter (Fig. 1). This consists of a double-walled calorimeter containing water, into which are inserted a thermometer and a coil of wire. An ammeter is connected in series with the calorimeter, and a voltmeter is connected in parallel with it. By means of a variable resistance the current through the ammeter, and hence that through the calorimeter, is kept at a nearly constant value I for a time t. If, in addition, either the resistance R of the coil in the calorimeter is known or the potential difference E between its ends is read on the voltmeter, the electrical energy supplied can be calculated. From the rise in temperature and the mass of water and calorimeter, the heat developed can be

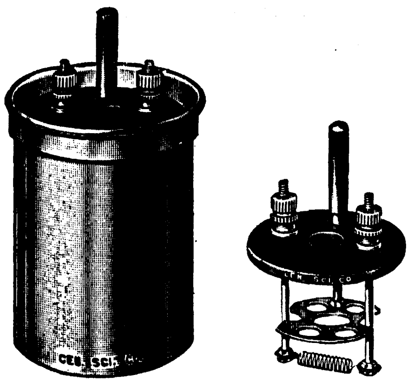


FIG. 1.—Electric calorimeter.

determined. Substituting these values in Eq. (4), the mechanical equivalent of heat can be computed.

$$W = JH = EIt = I^2Rt \tag{4}$$

Experiments that established the fact that heat and mechanical energy are interchangeable are particularly important, since they lead to the acceptance of the law of conservation of energy, the most important principle in the physical sciences.

Example: How many calories are developed in 1 min in an electric heater, which draws 5.0 amp when connected to a 110-volt line?

$$W = (110 \text{ volts})(5.0 \text{ amp})(60 \text{ sec}) = 3.3 \times 10^4 \text{ joules}$$

Since $J = 4.18$ joules/cal

$$H = \frac{3.3 \times 10^4 \text{ joules}}{4.18 \text{ joules/cal}} = 7.9 \times 10^3 \text{ cal}$$

Energy and Power. The relations between power, work, and energy are the same whether we are dealing with electricity, heat, or mechanics.

The production and use of electrical energy involve a series of transformations of energy. Radiation from the sun plays a part in providing potential energy for a hydroelectric plant or the coal for a steam generating plant. In the latter the chemical energy of coal is converted into heat in the furnace, from heat to work by the steam engine, and from work to electrical energy by the generator driven by the steam engine. The energy of the electric current may be converted into work by an electric motor, into heat by an electric range, into light by a lamp. It may be used to effect chemical change in charging a storage battery or in electroplating. The expression $W = EIt$ (as applied to d.c circuits) represents the electrical energy used in any of these cases.

Since power P is the rate of doing work or the rate of use of energy, it may always be obtained by dividing the energy W by the time t which is taken to use or to generate the energy, or (in d.c circuits),

$$P = \frac{W}{t} = \frac{EIt}{t} = EI \tag{5}$$

In practical units, P is the power in joules per second, that is, in watts, if E is given in volts and I in amperes. Thus the power in watts used by a calorimeter (or other electrical device) is found by multiplying the ammeter reading by the voltmeter reading. If the electrical power is entirely used in producing heat in a resistance R, then from Eq. (2),

$$P = \frac{W}{t} = \frac{I^2Rt}{t} = I^2R \tag{6}$$

Units and Cost of Electric Energy. A very practical aspect of the use of any electric device is the cost of operation. It should be noted that the thing for which the consumer pays the utility company is *energy and not power.*

$$\text{Work} = \text{power} \times \text{time}$$

Power of 1 watt used for 1 sec requires 1 joule of energy. This is a rather small unit for practical work. The most frequently used unit is the *kilowatt-hour* (kw-hr), which is the energy used when a kilowatt of power is used for 1 hr. One kilowatt-hour is equal to 3.6×10^6 joules.

A *wattmeter* is used to measure the power in an electric circuit. It has pairs of terminals for both current and voltage connections. Thus its readings are equivalent to the product of current and voltage. The common household electric meter is a *kilowatt-hour meter.* Its readings are a measure of the product of power (in kilowatts) and time (in hours), that is, a measure of the *energy* used.

The cost of electric energy is given by the equation

$$\text{Cost} = \frac{(E_{\text{volts}} I_{\text{amp}} t_{\text{hr}})(\text{cost per kw-hr})}{1{,}000 \text{ watts/kw}}$$

Example: What is the cost of operating a 100-watt lamp for 24 hr if the cost of electrical energy is 5 cents per kilowatt-hour?

$$\begin{aligned} W &= (100 \text{ watts})(24 \text{ hr}) = 2{,}400 \text{ watt-hr} \\ &= 2.4 \text{ kw-hr} \\ \text{Cost} &= (2.4 \text{ hw-hr})(\$0.05/\text{kw-hr}) = \$0.12 \end{aligned}$$

Applications of the Heating Effect. The incandescent lamp is a familiar application of the heating effect of an electric current. A tungsten filament, protected from oxidation by being placed in a vacuum or in an inert gas, is heated by the current to a temperature of about 2700°C, converting a small part of the electrical energy into visible light.

Home lighting circuits and other electrical installations are commonly protected by fuses. These are links of readily fusible metal, usually an alloy of lead and tin. When the current increases above a predetermined safe value, the fuse melts ("burns out") before more valuable equipment is damaged.

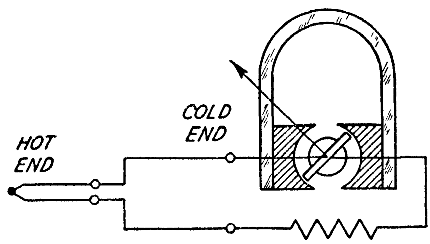


FIG. 2.—Thermocouple pyrometer.

Electric furnaces play an important role in industry. In resistance furnaces, heating is produced by passing the current through metallic conductors or silicon carbide rods which surround the material to be heated, or in some furnaces by using the material itself to conduct the current. Temperatures up to 2500°C are so attained. In arc furnaces the charge is heated, perhaps to 3000°C, by concentrating on it the heat from one or more electric arcs. Both types of furnaces are used to produce steel, silicon carbide (carborundum, a valuable abrasive), and calcium carbide.

Thermoelectricity. Under certain conditions, heat can be transformed directly into electrical energy. If a circuit is formed of two (or more) dissimilar metals the junctions of which are kept at different temperatures, an emf is generated, which produces an electric current in the circuit. The energy associated with the current is derived from the heat required to keep one junction at a higher temperature than the other. The industrial importance of such a circuit is that it provides an accurate and convenient means of measuring temperatures with electric instruments.

In an arrangement called a *thermocouple pyrometer* (Fig. 2) two wires of dissimilar metals are welded together at one end, the other ends being connected to a millivoltmeter. If the cool end (reference junction) of the thermocouple is maintained at a constant and known temperature (often that of an ice bath, 0°C), there will be an increase of emf as the temperature of the warm end of the thermocouple is increased. It is possible to calibrate this system to make it a temperature-measuring device.

Certain alloys are more suitable than the pure metals for thermocouple use, since they produce relatively large emf's and resist contamination. Practical temperature measurements can be made with such thermocouples over the range from −200 to 1600°C.

A number of thermocouples are often connected in series with alternate junctions exposed to the source of heat. Such an arrangement, called a *thermopile*, can be made extremely sensitive—sufficiently so to measure the heat received from a star, or, in a direction finder, to detect the heat from an airplane motor.

SUMMARY

The energy expended in a conductor by an electric current is proportional to the resistance of the conductor, to the square of the current, and to the time.

$$W = I^2Rt$$

The *mechanical equivalent of heat* is the ratio of the energy expended to the heat produced.

$$W = JH \qquad \text{or} \qquad J = \frac{W}{H}$$

Values of J are 4.18 joules/cal or 778 ft-lb/Btu.

Electrical energy is measured by the product $W = EIt$, in which W, E, I, and t are, respectively, in joules, volts, amperes, and seconds.

Electrical power is measured by the product $P = EI$, in which P, E, and I are respectively in watts, volts, and amperes.

A *kilowatt-hour* is the energy expended when 1 kw of power is used for 1 hr.

The cost of electric energy is given by

$$\text{Cost} = \frac{(E_{\text{volts}} I_{\text{amp}} t_{\text{hr}})(\text{cost per kw-hr})}{1{,}000 \text{ watts/kw}}$$

QUESTIONS AND PROBLEMS

1. How much heat would be generated in 10 min by a uniform current of 12 amp through a resistance of 20 ohms?

2. How much energy is used each minute by a d.c. motor carrying 12 amp at 110 volts? *Ans.* 7.9×10^4 joules.

3. A bank of 48 incandescent lamps (in parallel), each having a resistance (hot) of 220 ohms, is connected to a 110-volt circuit. Find (*a*) the power; (*b*) the cost of operating the lamps for 24 hr at 5 cents per kilowatt-hour.

4. If the coils of a resistance box are (each) capable of radiating heat at a rate of 4 watts, what is the highest voltage one could safely apply across a 2-ohm coil? a 200-ohm coil? What is the current in each case?

Ans. 2.8 volts; 28 volts; 1.4 amp; 0.14 amp.

5. A coil of wire having 5.0 ohms resistance is lowered into a liter of water at 10°C, and connected to a 110-volt circuit. How long will it take for the water to come to the boiling point? Neglect the heat required to change the temperature of the wire and the vessel.

6. Find the cost at 1 cent per kilowatt-hour of running an electric furnace for 10 hr if it takes 10,000 amp at 100 volts. *Ans.* $100.

7. In a test on an electric hot plate the temperature of a 1,200-gm copper calorimeter (specific heat, 0.093 cal/gm °C) containing 3 kg of water, rose from 30 to 43.6°C in 4 min. The wattmeter read 875 watts. Find the efficiency of the hot plate.

8. When electrical energy costs 6 cents per kilowatt-hour, how much will it cost to heat 4.5 kg of water from 20°C to the boiling point, if no energy is wasted?

Ans. $0.025.

9. State the fundamental definition of potential difference. Write the defining equation.

10. A current of 4 amp is sent for 3 min through a resistance of 5 ohms, submerged in 600 gm of water in a calorimeter equivalent to 6 gm of water. Compute the rise in temperature of the water. *Ans.* 5.7°C.

11. A motor operates at 100 volts and is supplied with 2 hp from a generator 23.8 ft away. The diameter of the wire connecting the motor and generator is 0.050 in. and its resistivity is 10.5 ohm-circular mil/ft. What is the cost of the energy used in the *line* resistance during an 8-hr day at the rate of 5 cents per kilowatt-hour? What heat would be developed in the wire in this time?

12. What current is taken by an electric hoist operating at 250 volts if it is raising a 2,500-lb load at a uniform speed of 200 ft/min and its over-all efficiency is 25 per cent? What would it cost to operate this device for 3 min at 5 cents per kilowatt-hour? *Ans.* 181 amp; $0.11.

EXPERIMENT

Galvanometers, Multipliers, and Shunts

PART II

Apparatus: Same as that used in Part I, Chap. 23. The instruments and circuits referred to in this experiment are those represented in Figs. 10 to 12 of Chap. 23.

In this experiment the galvanometer G with k_2 depressed will be taken as the basic instrument, and the resistance box R will be used as a multiplier or a shunt in converting the basic instrument into an ammeter or voltmeter of desired characteristics.

The resistance of the basic instrument is 200 ohms, and the current required for a full-scale deflection is 0.0005 amp.

a. Compute the series resistance that must be used with the basic instrument to form a voltmeter of range 5/0/5 volts. Adjust R to this value and connect it in series with G by closing S_3 and opening S_2 (leave S_1 open). Close S, depress k_2, and adjust the control rheostats R_1 and R_2 until the voltage across $\overline{TN}$, as indicated by the reading of G, is 1.0 volt. Check the accuracy of the experimental voltmeter by using G with k_1 depressed to measure the same potential difference. To do this it is necessary to open S_3 and close S_2, removing R from the galvanometer circuit.

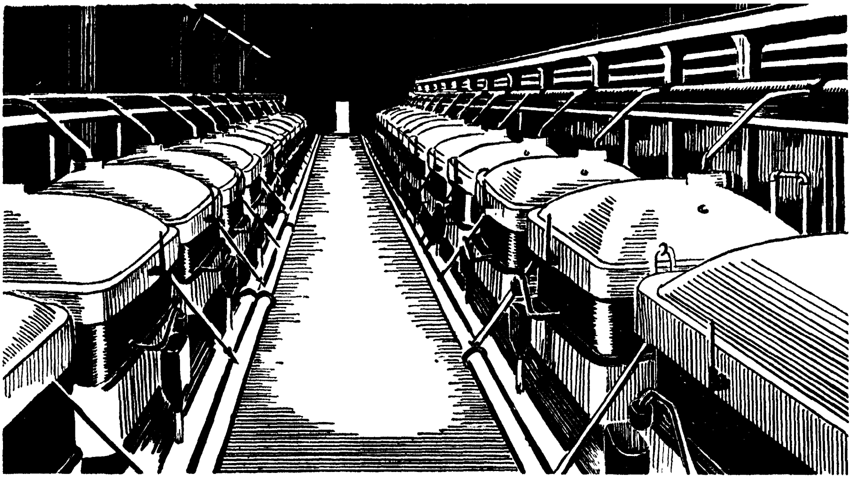
b. Compute the series resistance that must be used with the basic instrument to form a voltmeter of range 0.5/0/0.5 volt. Check by using the external multiplier supplied with the panel. This multiplier is used in series with the basic instrument.

c. Compute the shunt resistance that must be used with the basic instrument to form an ammeter whose range is 0.05/0/0.05 amp. Adjust R to this value and connect it in parallel with the basic instrument by closing S_1, opening S_3, and closing S_2. Remember to test with k_1 before using k_2, *invariably!*

Adjust the reading to full scale by manipulating R_1 and R_2, then check the reading of the experimental ammeter by using the 0.05-amp prepared shunt. To do this, remove the connections to T_1 and T_2, mount the shunt on T_1 and T_2, and place the connections (originally on T_1 and T_2) on the binding posts of the shunt. To check the reading, it will be necessary to disconnect R by opening S_1.

d. Repeat (*c*) for a range of 0.5/0/0.5 amp.

A voltmeter is connected to the two points whose potential difference it is to measure, while an ammeter is connected in series with the circuit in which the current is to be determined. In this experiment, it should be noticed, the experimental voltmeter was used to measure the voltage across $\overline{TN}$; the experimental ammeter was used to measure the current in the circuit which includes the wires U and V.



CHAPTER 25

CHEMICAL EFFECTS OF AN ELECTRIC CURRENT*

The chemical effects of electric currents have widespread and important applications. Chemical action provides a convenient source of electric current in places where power lines are impractical, for there batteries can be substituted. Dry cells in many sizes provide energy for portable electric instruments, and storage batteries are available for purposes that require considerable amounts of energy.

On the other hand, electric energy is used to produce desirable chemical change. Plating of metals to increase attractiveness or to reduce wear or corrosion is common in industry. The purification of copper by electrolytic deposition has long been an established procedure. Aluminum was a laboratory curiosity until an electrical method of extraction was developed to reduce the cost of production. The ever-increasing use of electrical refining methods makes available many new and valuable materials.

Liquid Conductors; Electrolytes. Liquids that are good conductors of electricity are of two classes. Mercury and other metals in the liquid state resemble solid metals in that they conduct electricity without chemical change. Pure water, oils, and organic compounds conduct electricity to only a very small extent. Salts, bases, and acids, fused or in

* Headpiece: Electrolytic cells connected in series to convert sodium chloride brine into chlorine, hydrogen, and caustic soda.

solution, are decomposed by the current and are called *electrolytes* Decomposition by an electric current is called *electrolysis*.

The difference between liquid conductors and liquid insulators may be illustrated by an experiment using the apparatus of Fig. 1. A vessel with electrodes of metal or carbon, a battery B, and an incandescent lamp C are connected in series. If the vessel contains pure water, there will be practically no current, nor will there be a current if sugar solution or glycerin is placed in A. If, however, a solution of salt or of sulphuric acid is placed in the vessel A, a current through the solution will be indicated by the lighting of the lamp C.

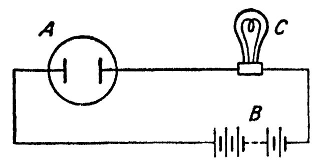


FIG. 1.—Circuit to show the conductivities of liquids.

We have previously discussed the hypothetical picture of an electric current in a metal as a swarm of electrons migrating slowly from the negative pole of a battery to the positive pole, that is, in a direction opposite to that assumed for the conventional current. In many nonmetallic conductors the currents are not swarms of drifting electrons but rather of charged atoms and groups of atoms called *ions*.

Electrolytic Dissociation. In the experiment just proposed, the salt solution differs from the sugar solution in that it has present many ions while the sugar solution does not. When common salt (NaCl) is dissolved in water, its molecules break up or dissociate into sodium ions and chlorine ions. The molecule as a whole has no net charge but in the process of dissolving the chlorine atom takes with it an extra electron giving it a single negative charge, while the sodium atom is thus left with a deficit of one electron, that is, with a single positive charge. If electrodes are inserted into the solution and a battery connected as shown in Fig. 2, the negatively charged chlorine ions will be attracted to the positive terminal while the positively charged sodium ions are attracted to the negative terminal.

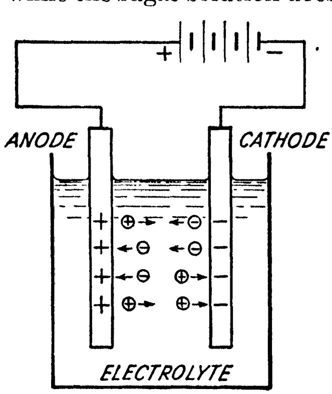


FIG. 2.—Migration of ions in electrolytic conduction.

The current that exists in the cell is the result of the net motion of the ions caused by these attractions. This conduction differs from that in a solid in that both negative and positive ions move through the solution. The electrode at which the current enters the cell is called the *anode*, that by which it leaves is called the *cathode*.

All acids, salts, and alkalies dissociate when dissolved in water and their solutions are thus electrolytes. Other substances, including sugar and glycerin, do not dissociate appreciably and hence their solutions are not conductors.

Electrolytic Decomposition, Electroplating. When an ion in the electrolytic cell reaches the electrode it gives up its charge. If it is a metallic ion such as copper, it is deposited as copper on the negative terminal. Chlorine or hydrogen will form bubbles of gas when liberated. Other materials, such as the sodium already mentioned, react with the water and release a secondary product. Thus the electrolytic cell containing salt solution yields chlorine and hydrogen gases as the product of the decomposition. In Fig. 3 a battery is connected through a slide-wire rheostat to a cell *C* containing water to which a little sulphuric acid has been added and a second cell *D* containing copper sulphate ($CuSO_4$) into

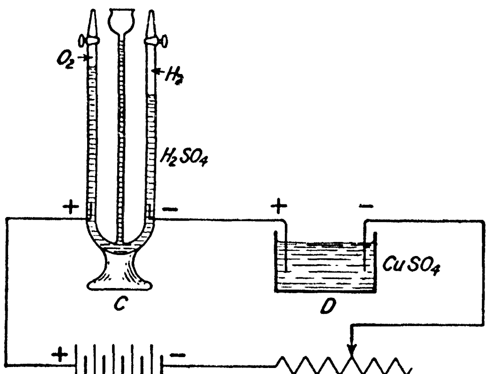


FIG. 3.—Circuit to show decomposition of electrolytes by an electric current.

which copper electrodes have been placed. When the switch is closed, bubbles of gas appear at each of the terminals of cell *C*. If the gases are tested, it is found that hydrogen is set free at the cathode and oxygen at the anode. In the cell *D* a bright deposit of copper soon appears at the cathode while copper is removed from the anode.

When one metal is deposited upon another by electrolysis, the process is known as *electroplating*. This process is very commonly used to produce a coating of silver, nickel, copper, chromium, or other metal. The success of the process in producing a smooth, even layer of metal depends upon such factors as the cleanness of the surface, the rate of deposition, the chemical nature of the solution and the temperature. For each metal there are optimum conditions, which must be set up with the skill born of experience if the best results are to be obtained.

Faraday's Laws of Electrolysis. Quantitative measurements made by Faraday (1833) contributed to the understanding of the processes occurring in electrolytic cells and showed a striking relation between the electrolytic behavior and the chemical behavior of various substances.

Faraday established by experiment the following two laws, which are known, respectively, as Faraday's first and second laws of electrolysis:

First Law: The mass of a substance separated in electrolysis is proportional to the quantity of electricity that passes.

Second Law: The mass of a substance deposited is proportional to the chemical equivalent of the ion, that is, to the atomic weight of the ion divided by its valence. The chemical equivalent of some common ions is illustrated in Fig. 4.

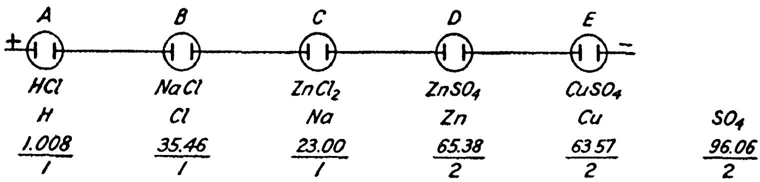


Fig. 4.—Chemical equivalents of ions.

The first law of electrolysis is expressed by the equation

$$m = zQ = zIt \tag{1}$$

in which m is the mass (in grams) of substance deposited by a charge Q (in coulombs). The quantity z is called the *electrochemical equivalent.* By letting Q equal unity, z is seen to be the mass of substance deposited per coulomb. The electrochemical equivalent of silver, which is 0.00111800 gm/coulomb, is taken as the standard. For definiteness in legal matters, the ampere is defined as the unvarying current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0.00111800 gm/sec.

Voltaic Cells. It has been seen that the passage of a current through an electrolytic cell produces chemical changes. The reverse effect is also true. Chemical changes in a cell will produce an electric current in a circuit of which the cell is a part. This fact was verified by an Italian scientist, Volta; hence such cells are called *voltaic* cells.

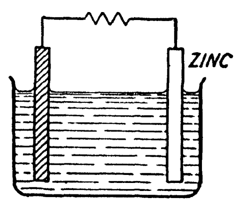


Fig. 5.—The voltaic effect of dissimilar electrodes in an electrolyte.

If a rod of pure zinc is placed in a dilute solution of sulphuric acid (Fig. 5), some of the zinc goes into solution. Each zinc ion so formed leaves behind two electrons on the electrode and thus itself acquires a double positive charge. The attraction of the negatively charged rod for the positively charged ions soon becomes so great that no more zinc can leave the rod and the action stops. A difference of potential is thus set up between the negatively charged rod and the solution, the rod being negative with respect to the solution. If a second zinc rod is placed in the solution, a similar action will take place and it too will acquire a negative potential. When the two rods are connected, no electrons will flow from one to the other for they are at the same potential. If, however, the second zinc rod is replaced by a copper rod, the rate at which the copper dissolves is less than that for the zinc, and, when the action stops, the difference in potential between the solution and the copper is not the same as that between the solution and zinc. Hence, when the copper rod is connected externally to the zinc by a conductor, electrons flow from the zinc to the copper. The cell is a voltaic cell in which copper forms the positive terminal and zinc the negative.

A voltaic cell may be formed by placing any two conductors in an electrolyte, provided that the action of the electrolyte is more rapid on one than on the other. The emf of the cell is determined by the composition of the electrodes and the electrolyte.

Local Action. If a rod of commercial zinc is placed in the acid cell, the action does not stop after a short time as it does with pure zinc. Small pieces of other metals that make up the impurities are embedded in the zinc, and the two metals in contact with each other and the acid form a local cell with a closed circuit. For each such center, chemical action will continue as long as the impurity is in contact with the zinc and hence the rod dissolves rapidly. Such chemical action may cause rapid corrosion of underground pipes, or of imperfectly plated metals when they are in contact with solutions.

Polarization. Whenever a voltaic cell is in action, some kind of material is deposited upon an electrode. In the copper-zinc-sulphuric acid cell hydrogen is liberated at the copper terminal and collects as bubbles of gas. Such deposition of foreign material on an electrode is called *polarization.* It is undesirable in a cell because the internal resistance is increased and also the emf of the cell is decreased. In some cells the materials are so selected that the material deposited is the same as the electrode itself. Such cells are not polarizable. In other cells a depolarizing agent is used to reduce the accumulation of foreign material.

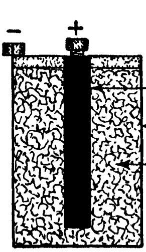


FIG. 6.—Dry cell.

The Dry Cell. The most commonly used voltaic cell is the so-called *dry cell* (Fig. 6). The positive electrode of this cell is a carbon rod and the negative terminal is the zinc container for the cell. A layer of paper moistened with ammonium chloride (NH_4Cl) is placed in contact with the zinc, while the space between this and the central carbon rod is filled with manganese dioxide and granulated carbon moistened with ammonium chloride solution. The ammonium chloride is the electrolyte and the manganese dioxide acts as a depolarizing agent. The cell polarizes when it is used but recovers slowly as the manganese dioxide reacts with the hydrogen. Because of this behavior, the cell should not be used continuously. The emf of the dry cell is slightly more than 1.5 volts.

Cells in Series and in Parallel. A group of cells may be connected either in series or in parallel, or in a series-parallel arrangement. Such a grouping of cells is known as a *battery*, although this word is often loosely used to refer to a single cell.

The laws governing a *series* arrangement of cells are as follows:

1. The emf of the battery is equal to the sum of the emf's of the various cells.
2. The current in each cell is the same.
3. The total internal resistance is equal to the sum of the individual internal resistances.

Cells are said to be connected in parallel when all the positive poles are connected together and all the negative poles are connected together.

The laws governing the *parallel* arrangement of *similar* cells are as follows:

1. The emf of the arrangement is the same as the emf of a single cell.
2. The total internal resistance is equal to $(1/n)$th of the internal resistance of a single cell (n being the number of similar cells).
3. The current delivered to an external resistance is divided among the cells, that through each cell being $(1/n)$th of the total.

In practice, cells are connected in series when their internal resistance is small compared to the external resistance, and in parallel when their internal resistance is appreciable or large compared to the external resistance. That is, cells are connected in series when it is desired to maintain a current in a comparatively high external resistance; in parallel, when a large current is to be produced in a low resistance.

Storage Batteries. Some voltaic cells can be recharged or restored to their original condition by using some other source of emf to force a current in the reverse direction in them. This "charging" current reverses the chemical changes that occur on discharge. Such a cell is called a *storage* cell. The most common type of storage cell is the lead cell (Fig. 7), which is used for automobiles and many other purposes. Both plates are lead grids into which the active material is pressed. The active material is lead oxide (PbO_2) for the positive plate and finely divided metallic lead for the negative electrode. Dilute sulphuric acid is used as the electrolyte. The emf of such a cell is about 2.2 volts.

When the cell maintains a current, the acid reacts with the plates in such a way that a coating of lead sulphate is formed on each plate. As in other types of polarization this process reduces the emf of the cell and, if it is continued long enough, the cell no longer causes a current and is said to be discharged. The reaction also replaces the sulphuric acid with water and hence the specific gravity of the electrolyte decreases during the discharge. Thus the state of charge of the cell can be checked by the use of a hydrometer.

The plates of the lead cell are made with large area and set close together so that the internal resistance is very low. Hence large currents are possible. The current in the starter of an automobile is sometimes as high as 150 amp.

When the storage battery is charged, *chemical energy* is stored up in the cells. The amount of energy that can be stored depends upon the size of the plates. A large cell has exactly the same emf as a small cell but the energy available in it when fully charged is much greater than that in the small cell.

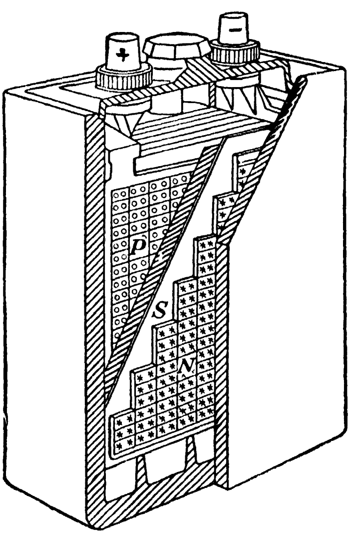


Fig. 7.—A lead storage cell.

Lead storage batteries are very satisfactory when properly cared for but are rather easily damaged by rough handling or neglect. The best service is obtained if they are charged and discharged at a regular rate. The battery is ruined quickly if it is allowed to stand in an uncharged condition.

A lighter and more rugged type of storage battery is the *Edison* cell. Its positive plate is nickel oxide (NiO_2), the negative plate is iron, and the solution is potassium hydroxide. It is more readily portable than the heavy lead cell and can be allowed to stand uncharged for long periods of time without damage. However, it is more expensive than the lead cell and its emf is lower (1.3 volts). It is commonly used in installations where charging is irregular or where weight is an important factor, as in field radio sets and miner's lamps. Its long life and ruggedness have made it a favorite cell for the electrical laboratory.

Nonpolarizing Cells. In certain types of cells the material deposited on each electrode is the same as that of the electrode itself. Such cells have the advantage of not being subject to polarization.

The Daniell cell consists of a zinc plate in zinc sulphate solution and a copper plate in copper sulphate. The two liquids are kept separate either by a porous jar or by gravity, the denser copper sulphate solution being at the bottom of the battery jar. When the cell furnishes a current, zinc goes into solution and copper is deposited. There is a continuous stream of zinc ions in the direction of the current and of SO_4^{--} ions against the current. There is a decrease of Zn and $CuSO_4$ and an increase of Cu and $ZnSO_4$. The Daniell cell is reversible, zinc being deposited and copper going into solution when a current is forced through the cell in the direction to convert electrical energy into chemical energy. When it is prepared in a certain specified way, the cell produces an emf of 1.108 volts.

Standard Cells. The Weston standard cell (Fig. 8) has one electrode of cadmium amalgam in cadmium sulphate, the other of mercury in mercurous sulphate. Weston standard cells are made in two forms. The normal cell contains a saturated cadmium sulphate solution; the unsaturated cell, used as a working standard, has a solution less than saturated. The saturated cell is the basic standard, being reproducible to a very high degree of accuracy, but the variation of its emf with temperature must

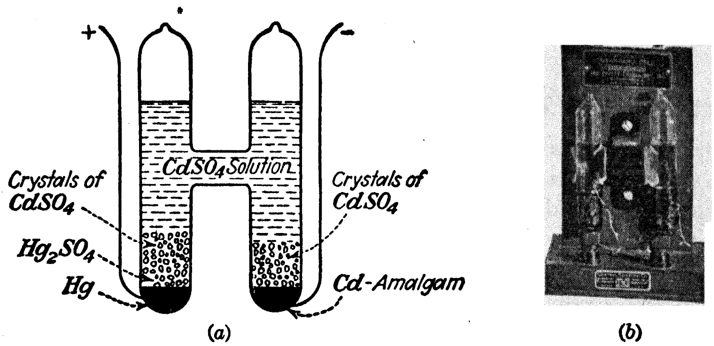


FIG. 8.—Weston standard cell.

taken into account for accurate measurements. The unsaturated cell is not exactly reproducible. Its emf must be checked against a normal cell, but its temperature coefficient is negligible and it is, therefore, a much more practical working standard.

Standard cells are not used for producing appreciable currents but as standards of potential difference. With the aid of special instruments, chiefly potentiometers, an unknown voltage may be accurately measured by comparison with the emf of a standard cell.

SUMMARY

Water solutions of acids, salts, and alkalies are called *electrolytes.* They conduct electricity by the transfer of positive (metallic) ions and negative ions. Univalent atoms gain or lose one electron each in ionization; bivalent atoms gain or lose two electrons.

Faraday's laws of electrolysis are as follows:

1. The mass of a substance deposited by an electric current is proportional to the amount of electrical charge transferred.

2. For the same quantity of electricity transferred, the masses of different elements deposited are proportional to their atomic weights, and inversely proportional to their valences.

A *voltaic cell* consists of two electrodes, of dissimilar substances, in contact with an electrolyte. The substance forming the negative electrode has a greater tendency to dissolve than that forming the positive electrode.

Polarization is the accumulation of layers of foreign substances around the electrodes, which serve to reduce the net emf of the cell.

A *storage cell* is a voltaic cell that can be restored to its initial condition by the use of a reversed or "charging" current.

A *standard cell* is an nonpolarizing voltaic cell made to certain specifications to serve as a standard of potential difference.

QUESTIONS AND PROBLEMS

(Electrochemical Equivalents are Given in Table 3 of the Appendix.)

1. Draw a diagram of a circuit that could be used to silver-plate a key.

2. A steady current of 4.00 amp is maintained for 10.0 min through a solution of silver nitrate. Find how much silver is deposited on the cathode.
Ans. 2.68 gm.

3. How many grams of the following will be deposited or liberated in electrolysis by 96,500 coulombs: (*a*) silver? (*b*) copper? (*c*) oxygen?

4. How much lead changes to lead sulphate per ampere-hour in a lead storage battery? *Ans.* 3.86 gm.

5. A spoon is silver-plated by electrolytic methods. It has a surface area of 20 cm^2 on which a coating of silver 0.0010 cm thick is plated. The density of silver is 10.5 gm/cm^3. (*a*) How many grams of silver are deposited? (*b*) How many coulombs of electricity pass through the solution? (*c*) If a current of 0.1 amp is used, for how long must it be maintained?

6. A battery has an emf of 10 volts and an internal resistance of 3.0 ohms. When connected across a resistance of 12 ohms, what current will it furnish?
Ans. 0.67 amp.

7. Four storage cells, each having an emf of 2.0 volts and an internal resistance of 0.40 ohm, are connected (*a*) in series, (*b*) in parallel, to an external resistance of 10 ohms. What current is furnished by each cell in each of these cases?

8. A battery of four similar cells in series sends a current of 1 amp through a coil having a resistance of 4 ohms. If the emf of the battery is 6 volts, what is the resistance of each cell? *Ans.* 0.5 ohm.

9. An electrolytic cell containing acidulated water, a conductor in a calorimeter, and a galvanometer are connected in series. A current lasting 1 min causes an evolution of 1.0 cm^3 of hydrogen, a rise of 4°C in the calorimeter, and a deflection of 10 divisions on the galvanometer scale. The current is then doubled. Describe the effect in each part of the circuit.

EXPERIMENT

Emf and Internal Resistance

Apparatus: Voltmeter; ammeter; battery; dry cell; rheostat; switch.

In Fig. 9, C is a cell whose emf is E_m and whose internal resistance is r; A is an ammeter and V is a voltmeter; OP is a control rheostat; B is a battery of several cells; and MLN is a single-pole, double-throw switch. This circuit is designed to clarify the concept of terminal potential difference and its relation to emf and internal resistance.

When LM is closed the cell C produces a current the magnitude of which can be varied by changing the resistance OP. When the cell C furnishes no current, the voltage across its terminals is its emf E_m. When the cell furnishes a current, however, there is a drop of potential Ir in the cell. The net potential difference between the terminals is

$$E = E_m - Ir$$

In this equation E_m and r are constants, E and I variables, these being measured by V and A.

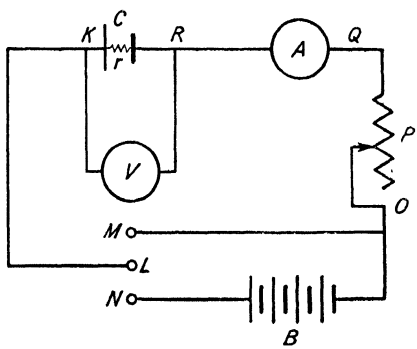


FIG. 9.—Circuit for the study of emf and internal resistance.

Take readings of I and E for approximately 10 different settings of the rheostat OP and record the data in the first two columns of Table I. From such data let us compute E_m and r of the cell. Let us designate the first pair of data by the symbols I_1 and E_1 and the sixth by I_6 and E_6.

TABLE I

Reading	I	E	Computed mean: $E_m =$	$r =$
			$E_m - E$	Ir
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Substituting these particular values in the above equation we obtain a set of linear simultaneous equations

$$E_1 = E_m - I_1 r$$
$$E_6 = E_m - I_6 r$$

Solution of these yields values of E_m and r to be recorded in Table II. The same computational method may then be applied successively to other pairs of data. We thus obtain five values for E_m and r and then compute their average values.

In columns 4 and 5 of Table I are to be recorded the values of $E_m - E$ and Ir. From the theoretical considerations of this chapter, what do you expect as to the relation of the figures in these two columns? Should they be equal? Are they approximately equal? How does $E_m - E$ vary with I? Does E approach E_m as I approaches zero?

TABLE II

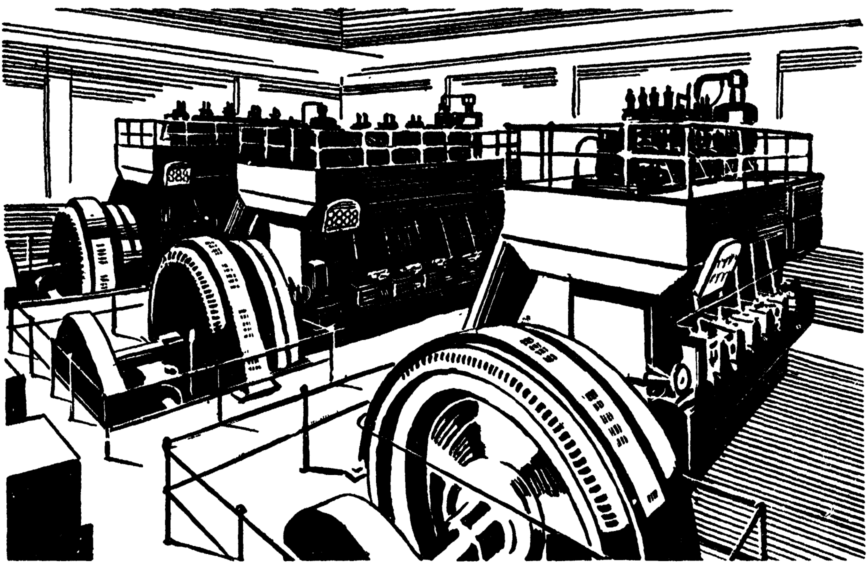
Set of equations	E_m	r
1 and 6		
2 and 7		
3 and 8		
4 and 9		
5 and 10		
Sum		
Mean		

What would happen to E if I passed the value zero and became negative, that is, reversed its direction? To check on your prediction open switch ML and close LN (*after* reversing the connections to A and V, unless they are zero-center instruments). Will this indeed reverse the direction of the current through C? Is the reading of V what you expected? Can E ever be equal to zero? Try to make it zero.

It should be remembered that the potential difference between the terminals of a cell is not identical with its emf. Only for a particular case will they be equal. (What case?)

Plot a curve of E *vs.* I. Is the curve a straight line? At what value of E does it cross the axis? Since this is the value of E for which the current is zero, it should be the emf of the cell. How does this value compare with that of E_m already computed?

It will be instructive for the student to make a similar study (taking fewer data to save time) of combinations of cells in parallel and series connections. Such combinations may be substituted for C in the same circuit.



CHAPTER 26

ELECTROMAGNETIC INDUCTION

Although chemical energy can be used as a direct source of electrical energy, the high cost of the materials required does not permit the use of this effect where large amounts of power must be used. The discovery of the relationships between magnetism and the electric current made possible the development of the electrical industry, for it led to the design of generators for the conversion of mechanical energy into electrical energy and of motors for the transformation of electrical to mechanical energy. In a little over a century since the fundamental discoveries were made the huge electrical industry of today has grown up. This industry is based primarily upon the use of the electric generator to produce electrical energy at low cost and the economical transportation of the energy to the place where it is to be used, there to be converted into other forms of energy.

Magnetism. It is commonly noticed that certain bars of steel attract bits of soft iron. Such a bar is a magnet. If it is placed in a dish of iron filings, bunches of filings cling to the ends of the bar. The magnetism of the bar seems to be concentrated at regions near the ends, called *poles*.

If the bar magnet is suspended so that it is free to turn, it will always take a position with its axis along an approximate north and south line with the same end always to the north. The pole that seeks the north is called a *north-seeking*, or *N*, pole, while the other pole is called a *south-seeking*, or *S*, pole. The steel bar acts as a compass needle; in fact, all magnetic compasses are essentially magnetized steel bars.

If the *N* pole of another magnet is brought near the *N* pole of the suspended magnet, the two poles repel each other; if the *N* pole is brought near the *S* pole of the suspended magnet, there is attraction. This illustrates the general rule that unlike poles attract, but like poles repel each other. The amount of the force of attraction or repulsion is directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them. A pole of unit strength (unit pole) is one that will repel a similar pole at a distance of 1 cm with a force of 1 dyne.

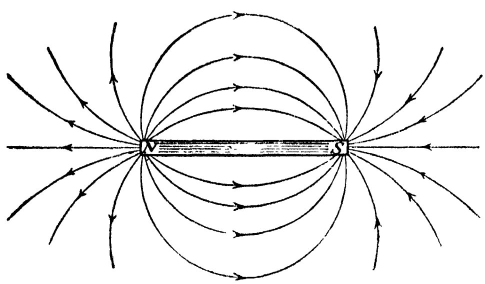


FIG. 1.—Magnetic lines of force.

If a small compass is brought near a magnet, the compass takes a preferred position. When the compass is moved always in the direction its *N* pole points, it traces a path called a *line of force*. If a number of lines of force are thus traced about a magnet, a picture is given of the *magnetic field* (Fig. 1). The strength of the field at any point is the force on a unit *N* pole placed there. The number of lines of force through an area of 1 cm^2 perpendicular to the field is equal to the strength of the field. Where lines of force are close together, the field is strong; where they are farther apart, it is weaker.

Terrestrial Magnetism. The earth acts as a great magnet, the magnetic poles of which are near the geographic poles but do not coincide with them. The magnetic north, as indicated by a compass, therefore, does not correspond exactly to the geographic north at most places on the surface of the earth. The angle by which the magnetic north deviates from the geographic north is called the *variation* (declination). On the map of Fig. 2 are shown lines drawn through points of equal variation. These are called *isogonic* lines. The isogonic line for which the variation is zero is called the *agonic* line. For points east of the agonic line the compass direction is west of north, while for points west of the agonic line the compass direction is east of north. The navigator who uses a magnetic compass must continually make correction for variation.

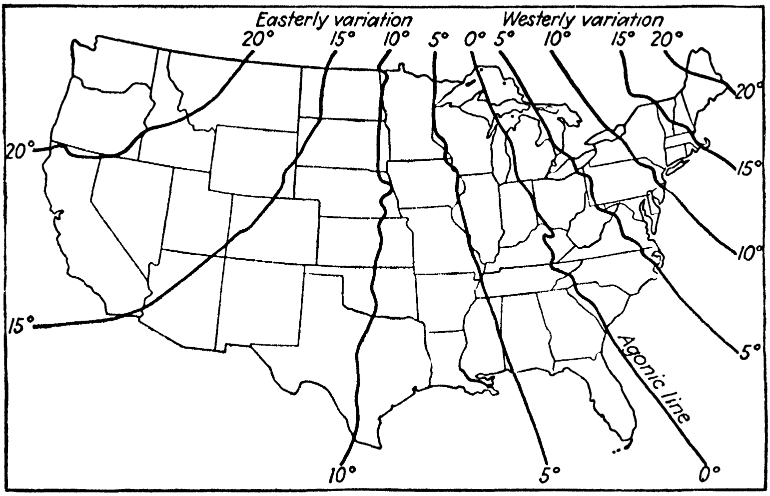


FIG. 2.—Isogonic chart of the United States.

Magnetic Field Associated with an Electric Current. Oersted discovered that when a current is maintained in a conductor the region around it becomes a magnetic field in which a compass needle assumes a preferred orientation. The direction taken by the north-seeking pole (called the

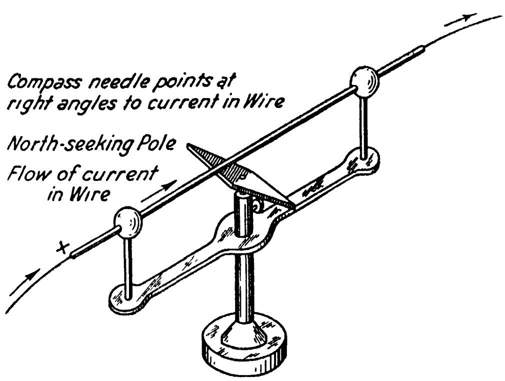


FIG. 3.—This device illustrates Oersted's discovery, 1819. A permanent magnet moves to a position at right angles to a straight wire carrying direct current.

direction of the field) is given by the following right-hand rule: If the right hand grasps the conductor so that the thumb points in the direction of the current, the fingers will point in the direction of the field about the conductor. Figure 3 shows the relation between current and field directions. Were the compass placed above the wire, it would assume a direction opposite to that shown.

A solenoid consists of turns of wire wound in cylindrical form. When a current is maintained in the solenoid, the associated magnetic field is as shown in Fig. 4*c*, being practically uniform within the coil. A solenoid thus acts like a magnet, when a current is maintained in it. To determine

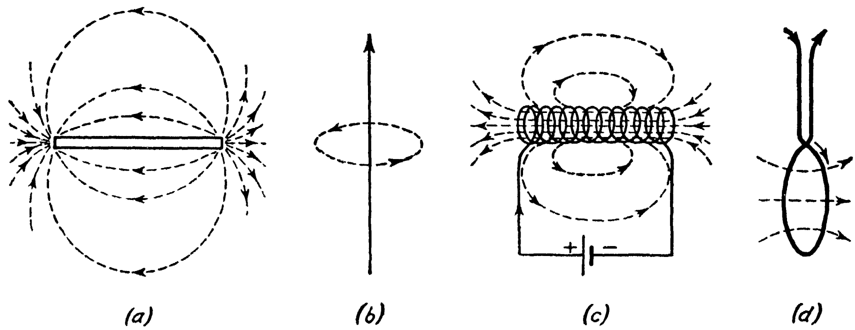


FIG. 4.—Magnetic fields about (*a*) a bar magnet, (*b*) a straight conductor, (*c*) a solenoid, (*d*) a single loop of wire.

the polarity of a solenoid, grasp it in the right hand with the fingers encircling the coil in the direction of the current, then the extended thumb will point to the *N* pole of the solenoid.

If a bar of soft iron is placed in a current-carrying solenoid, it becomes magnetized and remains in that condition as long as the current is maintained. This combination of a solenoid and a soft iron core, called an *electromagnet*, is of tremendous usefulness. It is an essential part of electrical devices such as lifting magnets, generators, motors, transformers, telephone and telegraph instruments and many others.

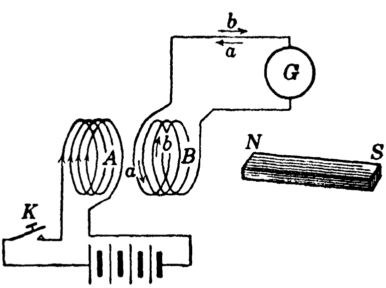


FIG. 5.—Circuit to show induced currents.

Induced Emf's and Currents. In Fig. 5, *B* represents a coil of wire connected to a sensitive galvanometer *G*. If the north pole of a bar magnet is thrust into the coil, the galvanometer will deflect, indicating a momentary current through the coil in the direction specified by arrow *a*. This current is called an *induced current*. As long as the bar magnet remains at rest within the coil, no current will be induced. If, however, the magnet is suddenly removed from the coil, the galvanometer will indicate a current in the opposite direction (arrow *b*).

When the key *K* is closed, producing a current in the coil *A* in the direction shown, a momentary current is induced in coil *B* in a direction (arrow a) opposite to that in A. If K is now opened, a momentary current will appear in B, having the direction of arrow b. In each case there is a current in B only while the current in A is changing. A steady current in A accompanied by a motion of A relative to B is also found to induce a current in B. Observe that in all cases in which a current is induced in B, the magnetic field through B is changing.

Whenever a conductor moves in a magnetic field in such a manner as to cut across the "lines of force" of the field, there is an emf induced in the conductor. The average magnitude of such emf (in volts) is given by the equation

$$E = \frac{N}{10^8}\frac{\Delta\varphi}{t} \tag{1}$$

where N is the number of conductors, $\Delta\varphi$ is the number of lines of force cut, and t is the time required.

Example: A wire 5.0 cm long moves across a uniform magnetic field of 2,000 lines/cm² with a speed of 200 cm/sec. What is the emf induced in the wire?

The number of lines of force cut per second $\Delta\varphi/t$ is the area swept out per second A/t multiplied by the number of lines per unit area

$$\begin{aligned} \frac{\Delta\varphi}{t} &= \frac{A}{t}\,(2{,}000 \text{ lines/cm}^2) = (5.0 \text{ cm})(200 \text{ cm/sec})(2{,}000 \text{ lines/cm}^2) \\ &= 2{,}000{,}000 \text{ lines/sec} \\ N &= 1 \\ E &= 1\,\frac{(2{,}000{,}000)}{10^8} \text{ volt} = 0.02 \text{ volt} \end{aligned}$$

Example: A simple generator has a single coil of 20 turns which makes 30 rotations/sec between two magnetic poles. If the coil links with 25,000 lines of force, what is the average emf induced in the coil as it turns through 180°, starting when all the lines thread the coil?

In this action each conductor cuts each line of force twice, and hence

$$\begin{aligned} \Delta\varphi &= 2 \times 25{,}000 \text{ lines} = 50{,}000 \text{ lines} \\ N &= 20 \\ t &= \text{time of half rotation} = \tfrac{1}{60} \text{ sec} \\ E &= N\frac{\Delta\varphi}{10^8 t} = 20\,\frac{(50{,}000)}{10^8(\frac{1}{60})} \text{ volt} = \frac{(20)(50{,}000)(60)}{10^8} \text{ volt} = 0.6 \text{ volt} \end{aligned}$$

Conservation of Energy. Lenz's Law. An induced current can produce heat or do chemical or mechanical work. The energy must come from the work done in inducing the current. When induction is due to the motion of a magnet or a coil, work is done, therefore the motion must be resisted by a force. This opposing force comes from the action of the magnetic field of the induced current. Hence the induced current is always in such a direction as to oppose by its magnetic action the change inducing the current. This particular example of conservation of energy is called *Lenz's law*.

Generator. Whenever a straight wire, such as AB in Fig. 6, is drawn across a magnetic field, an emf is induced in the conductor. There will be an induced current in the wire if it is made a part of a closed circuit as indicated in the figure. In accordance with Lenz's law the direction of the induced emf is such as to oppose the motion of the conductor. The direction of the current, therefore, depends upon the direction of the field and that of the motion. These three directions are mutually at right angles to each other. A convenient rule for remembering the relations of these directions is Fleming's generator rule: If the thumb, forefinger, and middle finger of the *right* hand are extended so that they are at right angles to each other and the thumb points the direction of the motion while the forefinger points the direction of the magnetic field (flux), then the middle finger points the direction of the induced current.

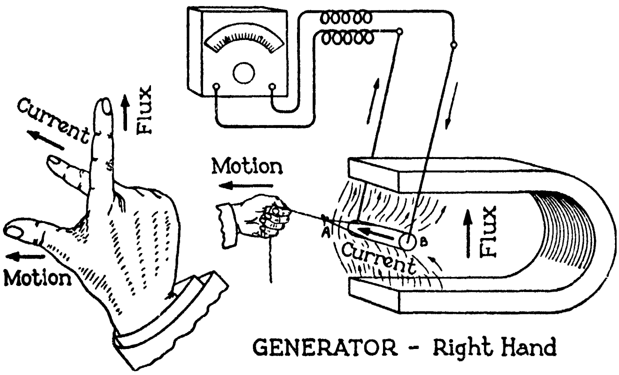


FIG. 6.—Fleming's generator rule.

A *generator* is a machine designed to convert mechanical energy into electrical energy. To accomplish this purpose conductors are made to move across a magnetic field. The simplest generator would be a single coil of wire turning in a uniform magnetic field as in Fig. 7. The loop $ABCD$ turns in a counterclockwise direction starting with the loop vertical. Since the magnetic field is directed from N to S, the generator rule indicates that the current in AB as it moves downward in the first half turn is from B to A and in DC as it moves up at the same time the current is from D to C. The current during this half turn is directed around the loop in the order $BADC$. If the loop continues to turn through a second half turn, AB moves up in front of the S pole and DC moves down before the N pole. During this half turn the current circulates in the opposite direction.

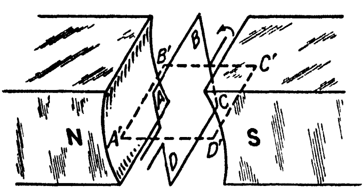


FIG. 7.—A simple generator.

ABCD. Thus the current *alternates* in the coil, reversing direction twice in each complete revolution.

The value of the induced emf, and hence the current, is not constant as the coil turns since it is proportional to the rate at which the lines of force are cut. When the coil is in the vertical position as it turns, both *AB* and *CD* are moving parallel to the field and cutting no lines of force.

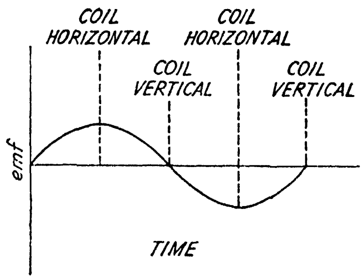


FIG. 8.—Variation of emf in a single coil turning in a uniform magnetic field.

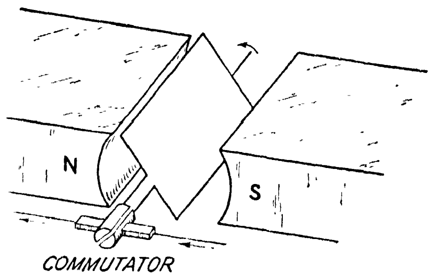


FIG. 9.—A simple generator with a commutator produces a one-direction current in the external line.

Hence at this position the emf is zero. As the coil turns, the rate of cutting increases until its plane is in the horizontal position where the conductors are moving perpendicular to the flux and hence the emf is a maximum. Thereafter it decreases until it becomes zero again when the coil is vertical. The way in which the emf varies during one complete turn of the coil starting from a vertical position is shown in Fig. 8. It starts at zero, rises to a maximum, decreases to zero, rises to a maximum in the opposite direction, and again decreases to zero ready to repeat the cycle. Thus a cycle is completed in each revolution.

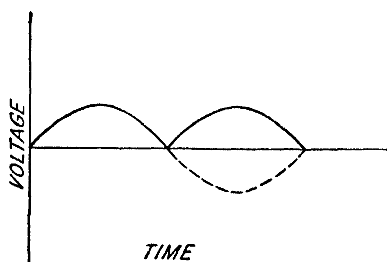


FIG. 10.—Variation of voltage with time in the external line of a simple generator with a commutator.

Such a generator can never have a one-direction current in the coil itself but it is possible to have a one-direction current in the outside circuit by reversing the connections to the outside circuit at the same instant the emf changes direction in the coil. This change in connections is accomplished by means of a *commutator* (Fig. 9). This device is simply a split ring, one side being connected to each end of the coil. Brushes, usually of graphite, bear against the commutator as it turns with the coil. The position of the brushes is so adjusted that they slip from one commutator segment to the other at the instant the emf changes direction in the rotating coil. In the external line there is a one-direction voltage, which varies as shown in Fig. 10. The curve is similar to that of Fig. 8 with the second half inverted. To produce a steady, one-direction current many armature coils are used rather than a single coil. These are usually wound in slots distributed evenly around a soft iron cylinder. These coils are referred to as the *armature*. By this arrangement several coils are always cutting lines of force and the connections are so arranged that those moving in one direction across the field are always joined in series. As the number of coils is increased the number of commutator segments must be increased proportionately.

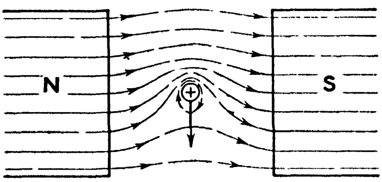


Fig. 11.—Force on a current in a magnetic field.

Motor. When a current-bearing conductor is placed in a magnetic field, the field is distorted as illustrated in Fig. 11. The current in the conductor is directed into the paper. At each point the field is the resultant of that due to the magnet and that due to the current. As a result the field is strengthened above the conductor where the two components are in the same direction and weakened below the conductor where they are in opposite directions. The conductor will experience a force directed from the strong part of the field toward the weaker part. A three-finger rule for remembering the direction of

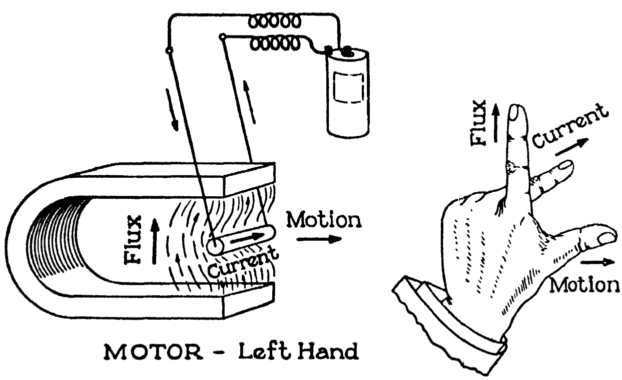


Fig. 12.—The motor rule.

the force is shown in Fig. 12. It is similar to the generator rule except that the *left* hand is used for the motor rule.

The side push that a current-bearing conductor experiences in a magnetic field is the basis of the common electric motor. In construction the motor is similar to the generator having a commutator and an armature wound on a soft iron drum. When a current is maintained in the armature coils, the force on the conductors produces a torque tending to rotate the armature. The amount of this torque depends upon the current, the strength of the magnetic field, the diameter of the drum, and the number and length of the active conductors on the armature. The commutator is used to reverse the current in each coil at the proper instant to produce a continuous torque.

Back Emf in a Motor. Consider an experiment in which an ammeter and an incandescent lamp are connected in series with a small motor (Fig. 13). If the armature is held stationary as the current is turned on, the lamp will glow with full brilliancy but, when the armature is allowed to turn, the lamp grows dim and the ammeter reading decreases.

This shows that the current in a motor is smaller when the motor is running freely than when the rotation of its armature is retarded. The current is diminished by the development of a *back emf*, which acts against the driving emf. That is, every motor is at the same time a

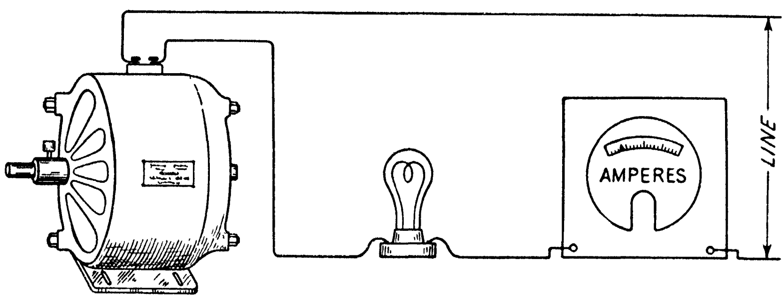


FIG. 13.—Circuit to show the back emf of a motor.

generator. The direction of the induced emf will always be opposite to that impressed on the motor, and will be proportional to the speed of the armature. When the motor armature revolves faster, the back emf is greater and the difference between the impressed emf and the back emf is therefore smaller. This difference determines the current through the armature, so that a motor will draw more current when running slowly than when running fast, and much more when starting than when at normal speed. For this reason adjustable starting resistances in series with the motor are frequently used to minimize the danger of a "burn out" from excessive current while starting.

SUMMARY

A *magnetic field* is any region in which a magnetic pole experiences a force. The field is described by the magnitude and the direction of the force that a unit north-seeking pole would experience in it.

Like magnetic poles repel and unlike poles attract, these forces being proportional to the product of the pole strengths and inversely proportional to the square of the distance between the poles.

The magnetic compass indicates the direction of the magnetic north, which differs from the geographic north by an angle called the *variation*.

When a current is maintained in a conductor, the region around it becomes a magnetic field.

The *right-hand rule:* If the right hand grasps a conductor so that the thumb points in the direction of the current, the fingers will point in the direction of the field about the conductor.

When the magnetic field through a conducting circuit changes, an emf is induced. The average value of this emf is given by

$$E = \frac{N}{10^8}\frac{\Delta\varphi}{t}$$

Lenz's law may be stated: An induced current is always in such a direction as to oppose by its magnetic action the change inducing it.

A *generator* is a machine for converting mechanical energy into electrical energy. Its action depends upon the emf induced when a conductor moves across a magnetic field.

The *motor* operates because of the side push that a current-carrying conductor experiences when placed in a magnetic field.

A *back emf* is produced when the armature of a motor turns in the magnetic field.

QUESTIONS AND PROBLEMS

1. The current in a conductor is directed eastward. What is the direction of the magnetic field (*a*) above the conductor? (*b*) below the conductor? (*c*) to the north? (*d*) to the south?

2. The current in a horizontal helix is counterclockwise as one looks down on it. What is the direction of the field inside the helix? outside the helix?

3. An east-west conductor moves south across a vertical magnetic field directed downward. What is the direction of the induced emf?

4. A conductor carries a current directed eastward in a magnetic field which is directed vertically upward. What is the direction of the force on the conductor?

5. The armature of a motor has a resistance of 0.24 ohm. When running on a 110-volt circuit, it takes 5 amp. What is the back emf?

6. A 1-hp motor having an efficiency of 85 per cent is connected to a 220-volt line. How much current does the motor use? *Ans.* 4 amp.

7. The voltage impressed across the armature of a motor is 115 volts, the back emf is 112.4 volts, and the current is 20 amp. What is the armature resistance?

EXPERIMENT

Electromagnetism

Apparatus: Two coils; galvanometer; switch; dry cell; rheostat; iron cores; St. Louis motor.

The following simple qualitative experiments will be valuable as additions to the student's actual observations and will contribute to the building up of his knowledge of the concepts of electromagnetism.

1. Connect the coil of Fig. 14 through a switch to a dry cell. Insert the half-round core (Fig. 14: 3) in the coil and place a cardboard as shown in Fig. 14: 7. Sprinkle iron filings on the card and tap it gently so that the filings orient themselves in "chains" along the lines of force. Determine the polarity of the "coil magnet" and, with the help of a bar magnet or compass, determine the direction of the magnetic field.

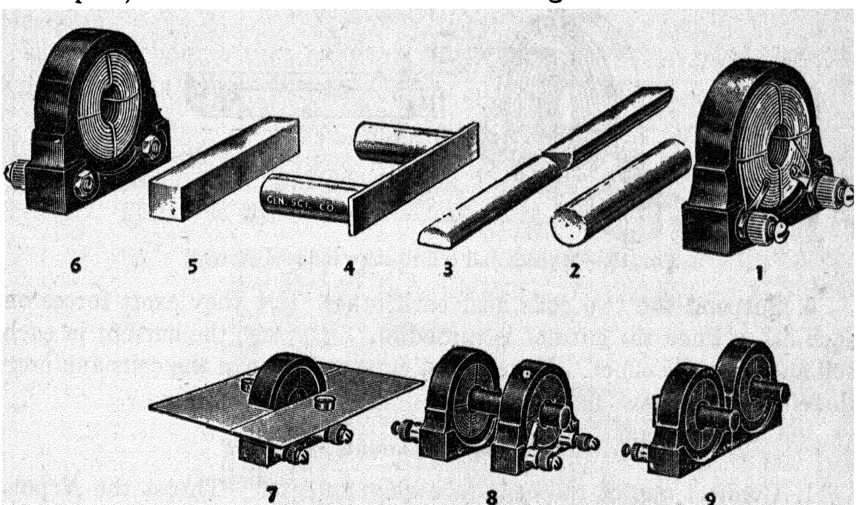


FIG. 14.—Induction-study apparatus.

Reverse the direction of the current and note the direction of the field.

2. Slip the round, soft iron core (Fig. 14: 2) into the coil. When a current is produced in the coil, how does the direction of magnetization of the core depend upon the direction of the field and upon the current? Is the magnet strong enough to support nails, etc.? A bar magnet supported by a string at a distance from the coil serves very well as an indicator of variations of field intensity; that is, stronger fields deflect it farther from normal orientation.

3. Place two coils together and extend the core through both. Connect the coils in series. For the same current, is the electromagnet thus formed stronger than that formed by the use of one coil alone? Does the order of connection of terminals make any difference? Might one coil neutralize the effect of the other?

4. Place the two coils side by side and insert the horseshoe core (Fig. 14: 4). Determine the polarity and compare it with that predicted by the right-hand rule. How does the strength compare with that of the

two-coil magnet of part 3? Does the order of connections affect the strength of the magnet?

5. By means of a string, suspend the coil near a fixed magnet as shown in Fig. 15. What is the effect when the current is turned on? Is the effect in accord with the prediction made by the use of the right-hand rule? Reverse the current and note the effect.

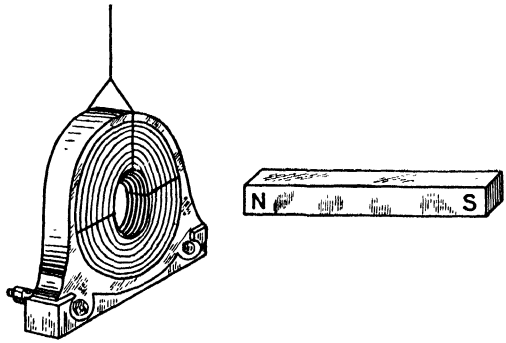


FIG. 15.—Magnet and coil to show induced current.

6. Suspend the two coils near each other. Do they exert forces on each other when the current is turned on? Reverse the current in each coil and note the effect. Reverse the current in one of the coils and note the effect.

Induced Currents

1. Connect one of the coils to a galvanometer. Thrust the N pole of a bar magnet into the coil and note the effect on the galvanometer. Is there an induced current? Is there an induced current while the magnet is stationary within the coil? Withdraw the N pole and note the effect.

Repeat the procedure above using the S pole of the magnet and compare the effects.

Move the magnet across the face of the coil and note the effect.

Use Lenz's law to predict the direction of the current in each case.

2. Thrust the pole of the magnet into the coil quickly and note the deflection of the galvanometer. Again thrust it into the coil slowly and note the result. Compare the two deflections and explain the difference.

3. Connect the second coil in series with a switch and a dry cell. Place the coils back to back and close the switch. What is the effect on the galvanometer? Open the switch and note the effect. What change occurs to cause a current in the galvanometer circuit?

4. With the coils arranged as in part 3 and with the switch closed, quickly pull one coil away from the other and note the effect. Is there a current in the galvanometer when both coils are at rest? Rotate the

plane of one coil through 90°. Does this change cause an induced current? Rotate the coil through 90° in the plane of the coil. Is there an induced current? What change of condition is common to all the tests that produce induced currents?

5. With the coils several inches apart, place the iron core so that it extends through both. Close and open the switch and note the deflections. Remove the iron core and repeat this procedure. Explain the difference in the effects with and without the iron core.

6. Connect a variable rheostat in series with the cell and coil. With the switch closed change the current quickly and note the galvanometer deflection.

List the changes that produced an induced current. What feature is common to all these changes?

St. Louis Motor

1. Connect the St. Louis motor (Fig. 16) in series with a switch and a dry cell. Trace the direction of the current in cell, switch, commutator,

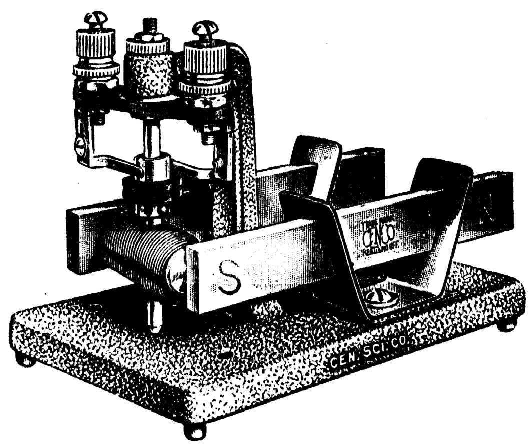
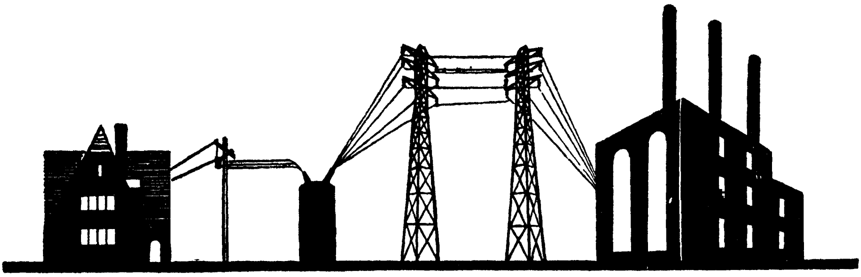


FIG. 16.—St. Louis motor.

and armature. Using the right-hand rule, determine the polarity of the armature. In which direction should the armature rotate? What should be the effect of (*a*) reversing one magnet? (*b*) reversing both magnets? (*c*) reversing the current?

2. Connect the St. Louis motor to the terminals of the galvanometer and rotate the armature. Does the motor function as a generator? Does increasing the speed of rotation of the armature have its predicted effect?



CHAPTER 27

ALTERNATING CURRENT

The use of electrical machinery makes possible the transportation of energy from the place at which it is easily produced to the point at which it is to be used. The electrical energy can there be converted into any other form of energy that best suits the needs of the consumer.

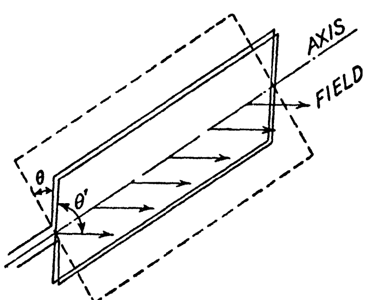


FIG. 1.—A coil in a magnetic field showing the angle θ' it makes with the field and the angle θ it makes with the position in which the emf is zero.

In the early generation of electricity the energy was consumed not far from the generator, and direct-current systems were almost universally used. As it became desirable to transport electrical energy over greater distances, power losses in the lines became excessive and in order to reduce these losses alternating-current systems were set up. At the present time a.c. systems are used almost exclusively in power lines. Where the use requires that direct current be employed, a local rectifying system or motor-generator set is installed.

In Chap. 26 there was described the emf generated in a loop of wire rotating at constant speed in a uniform magnetic field. At any instant, the emf in the loop is $e = E \cos \theta'$, where E is the maximum value of the emf and θ' is the angle between the direction of the magnetic field and the plane of the loop. This is usually written $e = E \sin \theta$, where θ is the angle between the given position of the loop and the position in which its emf is zero. Since the latter is a position at right angles to the magnetic field, $\theta = 90 - \theta'$, showing that $\sin \theta = \cos \theta'$, as was assumed. These angles are shown in Fig. 1, in which the magnetic field is horizontal.

One complete rotation of the loop produces one *cycle* of the emf, causing, therefore, one cycle of current in any circuit connected across its terminals. The number of cycles per second is called the *frequency*.

Effective Values of Current and Voltage. Suppose that a resistance R carries an alternating current whose maximum value is 1.0 amp. Certainly the rate at which heat is developed in the resistance is not so great as if a steady direct current of 1.0 amp were maintained in it.

By remembering that the rate at which heat is developed by a current is proportional to the square of its value ($P = I^2R$), one can see that the average rate of production of heat by a varying current is proportional to the *average value* of the *square* of the current. The square root of this quantity is called the *effective*, or root-mean-square (r.m.s.) current,

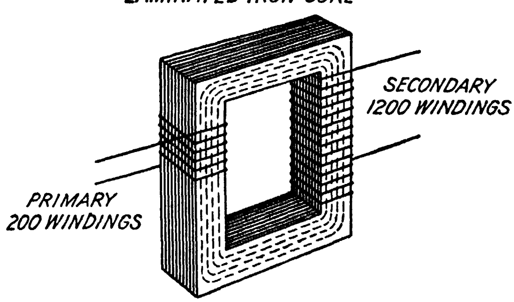


FIG. 2.—Showing the principle of the transformer.

which is equal to the magnitude of a steady direct current that would produce the same heating effect. Thus the value ordinarily given for an alternating current is its effective, or r.m.s. value.

For a current that varies sinusoidally with time, as does that produced by a rotating loop of wire, the effective value is $\frac{1}{2}\sqrt{2}$ times its maximum value; that is, $I_{eff} = 0.707I_{max}$. Similarly, since the effective value of an alternating voltage is defined as its r.m.s. value, $E_{eff} = 0.707E_{max}$ (if the voltage varies sinusoidally).

Example: What is the "peak" value of a 6.0-amp alternating current?

$$I_{eff} = 0.707\,I_{max} = 6.0 \text{ amp}$$

so that

$$I_{max} = \frac{6.0}{0.707} \text{ amp} = 8.5 \text{ amp}$$

Transformers. In Chap. 23 it was explained that a change in the current in one of two neighboring coils causes an emf to appear in the other. It should be emphasized that the emf in the second coil is produced, not by the current in the first coil, but by a change of that current (and the attendant change in the magnetic field in the vicinity).

The induced emf and, therefore, the induced current can be greatly increased by winding the two coils on a closed, laminated iron core, as in Fig. 2. This combination of two coils and an iron core is called a *transformer*. Suppose that an alternating current is maintained in the primary coil P of the transformer. This current is constantly changing; hence the magnetic flux in the iron core also varies periodically, thereby producing an alternating emf in the secondary coil.

In a transformer the voltages across the primary and secondary coils are approximately proportional to their respective numbers of turns; that is, the voltage per turn is nearly the same in the two coils. This makes it possible to obtain very high voltages by the use of a transformer with many times the number of turns in the secondary as are in the primary, for

$$\frac{E_p}{N_p} = \frac{E_s}{N_s} \qquad \text{or} \qquad E_s = \frac{N_s}{N_p} \times E_p \tag{1}$$

where E is used for voltage and N for the number of turns. In practice the secondary voltage is slightly less than the value given above.

Distribution of Electrical Energy. Whenever electrical energy is to be used at any considerable distance from the generator, an a.c. system is used because the energy can then be distributed without excessive loss; whereas, if a d.c. system were used, the losses in transmission would be very great.

In an a.c. system the voltage may be increased or decreased by means of transformers. The terminal voltage at the generator may be, for example, 12,000 volts. By means of a transformer the voltage may be increased to 66,000 volts or more in the transmission line. At the other end of the line "step-down" transformers reduce the voltage to a value that can be safely used. In a d.c. system these changes in voltage cannot readily be made.

One might ask why all this increase and decrease in voltage is needed. Why not use a generator that will produce just the needed voltage, say, 115? The answer lies in the amount of energy lost in transmission. In d.c. circuits, and in the ideal case in a.c. transmission lines, the power delivered is $P = EI$, where E is the (effective) voltage and I the current. (It will be shown later that in a.c. circuits $P = EI$ only in special cases.) If a transformer is used to increase the available voltage, the amount of current available will be decreased. Assuming a transformer to be 100 per cent efficient (a reasonable value is 95 to 99 per cent), the power delivered to the primary is equal to that available at the secondary, or $E_pI_p = E_sI_s$.

Now suppose that a 10-kw generator is to supply energy through a transmission line whose resistance is 10 ohms. If the generator furnishes

20 amp at 500 volts, $P = EI$ = (500 volts)(20 amp) = 10,000 watts, the heating loss in the line is I^2R = (20 amp)^2(10 ohms) = 4,000 watts, or 40 per cent of the original. If a transformer is used to step up the voltage to 5,000 volts, the current will be only 2 amp, and the loss I^2R = (2 amp)^2(10 ohms) = 40 watts, or 0.4 per cent of the original.

A second transformer can be used to reduce the voltage at the other end of the line to whatever value is desired. With 1 or 2 per cent loss in each of the transformers, the over-all efficiency of the system is increased from 60 to 95 per cent by the use of transformers. Thus alternating current, through the use of transformers producing very high voltages, makes it possible to furnish electric power over transmission lines many miles in length.

Self-induction. When a switch is closed connecting a battery to a coil of wire, the current does not instantaneously reach its steady value given by $I = E/R$ but starts at zero and rises gradually to that value. During the time the current is building up, the relation $I = E/R$ does not tell the whole story. In fact it can be shown that $I = E/R$ only when the current I is not changing. In general, therefore, we cannot use the relation $E = IR$ in connection with a.c. circuits.

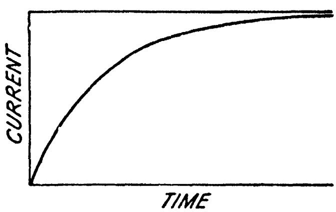


FIG. 3.—Rise of current after the switch is closed in an inductive circuit.

While the current in a coil is increasing, the magnetic field around it is being built up, hence energy is being supplied from the battery to create the magnetic field. The electricity that passes through the coil thus does work in two ways: in passing through the electrical resistance of the coil and in doing its share in building up the magnetic field around the coil. The potential difference across the coil can be divided into two parts, so that $E = IR + e$. Here IR is equal to the work per unit charge done against the electrical resistance of the coil, whereas e is equal to the work per unit charge done in changing the magnetic field. The work per unit charge done by the battery on the electricity passing through it is E.

The equation can be rewritten $e = E - IR$, showing that, as the current I becomes larger, e becomes smaller. It can be proved that e is proportional to the rate at which the current is changing. The constant of proportionality is called the *self-inductance* L of the coil, so that e equals L times the rate of change of the current. When the current is no longer increasing, e is zero, and $E = IR$, since no energy is being used in creating a magnetic field.

The current in a circuit rises as shown in Fig. 3; rapidly at first and then more and more slowly, until any change in it can no longer be detected. It is then said to have reached its maximum, or steady value, for which $E = IR$. The time taken for this to happen is usually a small fraction of a second. When the circuit contains a coil with a closed iron core, the rise may require as much as several tenths of a second. For the current in a coil to decrease, the energy given to the magnetic field must be taken back into the circuit, hence electricity passing through the coil receives e joules per coulomb from the decreasing of the magnetic field. At the same time, it does IR joules per coulomb of work against electrical resistance. The total work done per coulomb is thus

$$E = IR - e$$

while the current is decreasing.

The voltage e is commonly referred to as the emf of self-induction, since its effect is similar to that of an emf opposing or aiding the current.

This emf of self-induction can be accounted for in terms of the ideas presented in connection with induced currents in general, namely, an emf is induced in a coil when there is any change in the magnetic field threading it, whether that change is caused by the motion of a bar magnet, a change in the current in a neighboring coil, or by a change in the current in the coil itself. Since a magnetic field is associated with the current in a coil, any change in that current changes the magnetic field around it; hence an emf opposing the change in the current is induced in the coil. This effect is called *self-induction* or electrical inertia.

The self-inductance of a coil is defined as its emf of self-induction divided by the rate at which the current in it is changing. The unit of self-inductance, called the *henry*, is that of a coil in which an emf of self-induction of 1 volt is produced when the current in it is changing at the rate of 1 amp/sec.

The emf of self-induction is given by the equation

$$e = L \frac{\Delta I}{t} \tag{2}$$

where L is the self-inductance in henrys, e is the induced emf in volts, and $\Delta I/t$ is the rate of change of current. This equation is actually the defining equation for self-inductance.

Capacitance. A simple electrical condenser is formed by placing the surfaces of two metal plates near each other, usually with a sheet of paper, mica, or other insulating material between. If a battery is connected to these plates, though there is essentially no flow of electrons from one of them to the other, electrons do leave one of the plates and enter the + terminal of the battery, while the same number leave the − terminal of the battery and enter the other plate. As this happens, the first plate becomes positive, the second negative; and this continues until the potential difference between the plates is equal to the emf of the battery, after which there is no more current and the condenser is said to be charged.

Note that electricity does not flow through the condenser, but only into and out of the plates that compose it. The *capacitance* of a condenser is the ratio of the amount of electricity transferred, from one of its plates to the other, to the potential difference produced between the plates. The unit of capacitance is the *farad*, which is the capacitance of a condenser that is charged to a potential difference of 1 volt by the transfer of 1 coulomb. A smaller unit is the *microfarad* (μf) which is 10^{-6} farad.

From the definition of capacitance, it is seen that $C = Q/E$ or $Q = CE$, where C is the capacitance of a condenser, Q is the quantity of electricity transferred, and E is the potential difference across its terminals.

In a d.c. circuit, a condenser allows a flow of electricity only until the potential difference across it is equal (and opposite) to the emf in the circuit, after which there is practically no current. In an a.c. circuit, however, electricity can move in one direction, charging the condenser, then in the opposite direction, discharging the condenser and charging it oppositely. This means that an alternating current can be maintained in a circuit containing a condenser.

When an alternating emf is applied to a coil, the tendency of inductance to oppose any change in the current results in a lagging of the changes in current behind the changes in voltage. This is usually expressed by the statement, "The current lags the voltage." In order to calculate the effect of inductance upon the current, it is useful to define a quantity called the *inductive reactance*, $X_L = 2\pi fL$, where L is the value of the inductance in henrys and f is the frequency of the alternating current in cycles per second.

When an alternating emf is applied to a condenser, the tendency of its capacitance is to assist any change in the current, with the result that the changes in current occur ahead (in time) of the changes in emf, so that the current "leads the voltage." In calculating the effect of capacitance upon the current, a quantity called the *capacitive reactance* X_C must be known: $X_C = 1/2\pi fC$, where C is the capacitance in farads.

The effective value of the alternating current in a circuit containing resistance, inductance, and capacitance is

$$I = \frac{E}{Z} \tag{3}$$

where Z is the *impedance* of the circuit, given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{4}$$

Both impedance and reactance are expressed in ohms. In circuits containing no coils or condensers, X_L and X_C are zero, so that $I = E/Z = E/\sqrt{R^2} = E/R$, as in d.c. circuits. (NOTE: When there is no condenser in the circuit, the capacitance is infinite, so that $X_C = 0$.)

In practice it will be found that the inductances of connections and small coils are so small that X_L can be neglected. In cases where coils are wound on iron cores or when the frequency of the current is very high (as in radio circuits), X_L becomes very important and must be taken into account.

Power in A.C. Circuits. In the study of direct current it was learned that $P = EI$ for steady current and voltage. In an a.c. circuit the *average* power is given by

$$P = EI \cos \theta \tag{5}$$

where E and I are the effective values of voltage and current, respectively, and θ is the angle of lag between current and voltage. The factor $\cos \theta$ is called the *power factor*. The angle θ is obtained from the relation

$$\tan \theta = \frac{X_L - X_C}{R}$$

Example: *a.* Find the current through a circuit consisting of a coil and condenser in series, if the following data are given: applied emf, 110 volts, 60.0 cycles/sec; inductance of coil, 1.50 henrys; resistance of coil, 50.0 ohms; capacitance of condenser, 8.0 μf. (NOTE: 8.0 μf $= 8.0 \times 10^{-6}$ farad.) *b.* Find the power developed in the circuit.

$$I = \frac{E}{Z}, \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$R = 50 \text{ ohms}, \qquad X_L = 2\pi f L = 2\pi(60)(1.5) \text{ ohms} = 570 \text{ ohms}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60)(8.0)(10^{-6})} \text{ ohms} = 3\bar{3}0 \text{ ohms}$$

$$Z = \sqrt{(50)^2 + (570 - 330)^2} \text{ ohms} = \sqrt{2{,}500 + (240)^2} \text{ ohms} = 2\bar{4}5 \text{ ohms.}$$

$$I = \frac{E}{Z} = \frac{110 \text{ volts}}{240 \text{ ohms}} = 0.46 \text{ amp}$$

$$P = EI \cos \theta$$

$$\tan \theta = \frac{X_L - X_C}{R} = \frac{2\bar{4}0}{50} = 4.8$$

so that

$$\theta = 78° \text{ and } \cos \theta = 0.20$$

Then

$$\begin{aligned} P &= (110 \text{ volts})(0.46 \text{ amp})(0.20) \\ &= 10.1 \text{ watts} \end{aligned}$$

Resonance. In the equation $Z = \sqrt{R^2 + (X_L - X_C)^2}$ it is seen that, if $X_L = X_C$, $Z = R$ and the current $I = E/R$ as if no inductance or capacitance were present. This is the condition called *resonance*, when the current, for a given voltage and resistance, is a maximum.

A.C. Generators. Almost any d.c. generator will produce an alternating current if the commutator is replaced by a pair of slip rings properly connected to the armature coils. It is simpler and more economical, however, to construct an a.c. generator with the armature

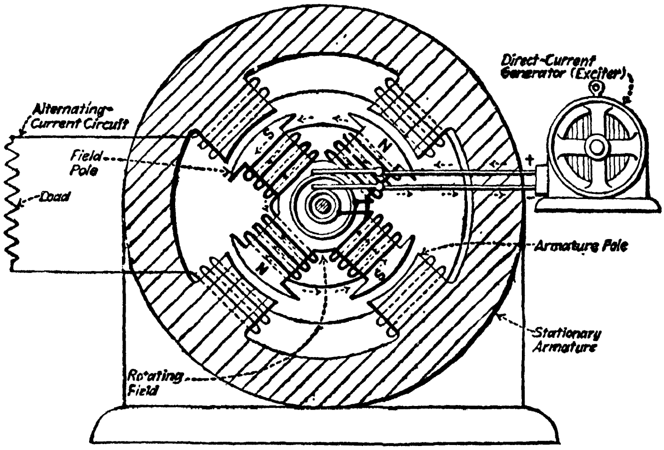


FIG. 4.—Diagram of a four-pole, rotating-field a.c. generator. The magnetic field is excited by a separate d.c. generator.

coils stationary and rotating magnetic poles, and most commercial a.c. generators are so made. A diagram of such a generator is shown in Fig. 4. The emf goes through one complete cycle as a pair of poles passes a coil. The frequency thus depends upon the number of poles and the speed of rotation. The emf produced is proportional to the strength of the field and to the number of turns on the coils as well as to the number of poles and the speed of rotation. In designing the generator the speed and number of poles are fixed to give the desired frequency and the remaining two factors are then adjusted to give the necessary emf. The field magnets are usually excited by current from a small d.c. generator, which is operated as a separate machine.

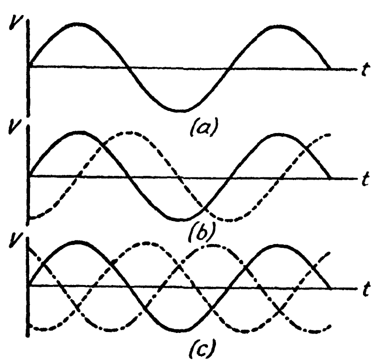


FIG. 5.—Variation of emf with time for (*a*) single-phase generator, (*b*) two-phase generator, (*c*) three-phase generator.

In many a.c. generators the armature is wound with two or three separate coils displaced somewhat in position from each other so that the peak emf is reached at different times. If there are two coils, the emfs vary as shown in the graph of Fig. 5*b*. The emf of one coil is zero when that of the other has its maximum value. Such a machine is called a *two-phase* generator. The more common three-phase generator has three coils so placed that the emf varies as shown in Fig. 5*c*. An advantage of the two- or three-phase machine is the more uniform flow of power.

Induction Motor. The most common type of a.c. motor is the *induction* motor. Its operation depends upon an induced current set up in the closed armature by means of a rotating magnetic field. In Fig. 6 are shown the field connections of a two-phase induction motor. The poles are connected in pairs to the two windings. At the instant that the current is greatest in line 1, pole a is an N pole, c is an S pole, and both b and d are unmagnetized since the current in line 2 is zero (Fig. 5). A quarter cycle later b has become an N pole, d an S pole, while a and c are unmagnetized. After the next quarter cycle, c is an N pole, and later d becomes an N pole. In one complete cycle, therefore, the N pole rotates successively from a to b to c to d, while at the same time the S pole rotates from c to d to a to b. Effectively the magnetic field rotates at the rate of one rotation for each cycle.

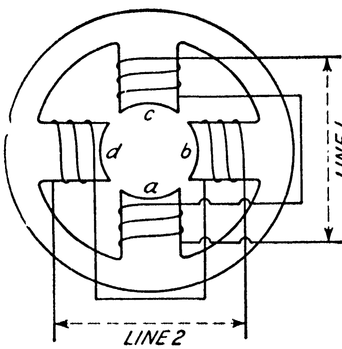


FIG. 6.—Rotating magnetic field of a two-phase induction motor.

If a closed conductor is placed between the poles, a current will be induced in it as the field rotates. The current will be in such direction as to oppose the turning of the field. As a result, there will be a torque tending to turn the conductor, and the machine becomes a motor. Both two-phase and three-phase motors are self-starting but a single-phase motor is not. For such motors a special starting device must be provided.

SUMMARY

For sinusoidal, alternating current,

$$I_{\text{eff}} = 0.707 I_{\text{max}}$$
$$E_{\text{eff}} = 0.707 E_{\text{max}}$$

The voltage per turn in the secondary coil of an efficient transformer is only slightly smaller than in the primary coil. Hence the transformer can be used to step up or step down the voltage at will.

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

The line loss is proportional to the square of the current, so that high voltage and low current are desirable in transmission lines.

The relation $E = IR$ is valid only in the case of a steady current, though it can be used without appreciable error for a.c. circuits in which the frequency is low and there are no condensers or large coils.

Self-inductance L is the ratio of the induced emf to the rate of change of the current.

$$L = \frac{e}{\Delta I/t}$$

The *capacitance* of a condenser is the ratio of the charge to the potential difference.

$$C = \frac{Q}{E}$$

In an a.c. circuit, $I = E/Z$, in which Z, the *impedance* is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = 2\pi fL \qquad \text{and} \qquad X_C = \frac{1}{2\pi fC}$$

where f is frequency, L is inductance, C is capacitance, and X is reactance.

The power developed is

$$P = EI \cos \theta$$

where $\tan \theta = \dfrac{X_L - X_C}{R}$ and $\cos \theta$ is called the *power factor.*

Resonance occurs when $X_L = X_C$, making $Z = R$.

QUESTIONS AND PROBLEMS

1. A condenser has a maximum rating of 550 (peak) volts. What is the highest a.c. voltage (effective) across which it can safely be connected?

2. The primary and secondary coils of a transformer have 500 and 2,500 turns respectively. If the primary is connected to a 110-volt a.c. line, what will be the voltage across the secondary? If the secondary (instead) were connected to the 110-volt line, what voltage would be developed in the smaller coil? *Ans.* 550 volts; 22 volts.

3. Find the power loss in a transmission line whose resistance is 1.5 ohms, if 50 kw are delivered to the line (*a*) at 50,000 volts, (*b*) at 5,000 volts.

4. What is the reactance of a 0.60-henry coil on a 60-cycle line? What is the current if the applied voltage is 110 volts and the coil resistance is 100 ohms? *Ans.* 2$\bar{3}$0 ohms; 0.44 amp.

5. What is the reactance of a 2-μf condenser on a 110-volt, 60-cycle line? What is the current?

6. A 0.10-henry coil (resistance, 100 ohms) and a 10-μf condenser are connected in series across a 110-volt a.c. line. Find the current and the power if the frequency is (*a*) 60 cycles/sec, (*b*) 25 cycles/sec. *Ans.* 0.44 amp; 19 watts; 0.17 amp; 3.1 watts.

7. Compare the growth of currents in inductive and noninductive circuits. Sketch a curve of current *vs.* time for a circuit of high inductance; for one of low inductance.

8. What is the self-inductance of a circuit in which there is induced an emf of 100 volts when the current in the circuit changes uniformly from 1 to 5 amp in 0.3 sec? *Ans*. 7.5 henrys.

9. A steady emf of 110 volts is applied to a coil of wire. When the current has reached three-fourths of its maximum value, it is changing at the rate of 5 amp/sec. At this instant the induced emf is 27.5 volts. Find the self-inductance of the coil.

10. An impressed emf of 50 volts at the instant of closing the circuit causes the current in a coil to increase at the rate of 20 amp/sec. Find the self-inductance of the coil. *Ans*. 2.5 henrys.

11. A certain amount of power is to be sent over each of two transmission lines to a distant point. The first line operates at 220 volts, the second at 11,000 volts. What must be the relative diameters of the line wires if the "line loss" is to be identical in the two cases?

EXPERIMENT

Resistance, Reactance, and Impedance

Apparatus: Choke coil; soft iron core; a.c. ammeter; a.c. voltmeter; electric lamp; condenser.

Measure the d.c. resistance of the choke coil by the voltmeter-ammeter method, using a storage battery as a source.

Connect the same coil in series with a lamp and a.c. ammeter to the a.c. lighting circuit. Place the a.c. voltmeter across the coil. From the ammeter and voltmeter readings compute the impedance Z of the coil by means of Eq. (3). From the value of Z and the resistance R compute the value of the reactance X from the relation $Z^2 = R^2 + X^2$. Since there is no condenser, the reactance is inductive and $X = 2\pi fL$. Compute the value of L.

Repeat this procedure with an iron core in the coil. How does the presence of the iron core affect the impedance? the reactance? the inductance?

Connect a condenser and ammeter in series with the battery. Is there any current? Why?

Connect the condenser, lamp and a.c. ammeter in series to the a.c. line with the a.c. voltmeter across condenser and ammeter. From the ammeter and voltmeter readings compute the impedance of the condenser. Assuming that it consists entirely of capacitive reactance, compute the capacitance from the relation

$$X = \frac{1}{2\pi fC}$$

If a suitable wattmeter is available, the power taken by the lamp, coil, and condenser, separately, may be measured. From the measured power and the voltage and current observations the power factor may be determined.

CHAPTER 28

COMMUNICATION SYSTEMS; ELECTRONICS

Among the most important applications of scientific discoveries to daily life are those that have produced the rapid and revolutionary improvement of communication methods. A century ago messages were sent by foot, horseback, boat, or stagecoach. The development of first the telegraph and later the telephone and radio has brought the most remote parts of the world into close contact. The invention and application of the electromagnet (Chap. 26) made possible the telegraph and telephone. The development of the electron tube made possible long-distance telephony and the rapid extension of radio communication.

Telegraph. In 1837 the American painter and inventor Samuel F. B. Morse devised a system of telegraphy, the basic principle of which was the actuation of an electromagnet by current remotely controlled. Signals are transmitted by manipulating a key in accordance with a code and are received by listening to the clicks made by a sounder (Fig. 1) as its armature A is attracted by the magnet M and then restored to its original position by spring S, when released. Only a single wire is needed between key and sounder, the circuit being completed by connections to ground. In long-distance telegraphy, owing to the resistance of the circuit, the current received may be too small to operate a sounder. In its place is substituted a relay, a similar instrument, the magnet of which is wound with many turns so that a feeble current is sufficient to actuate the armature. When drawn toward the magnet, the armature closes a second circuit through an ordinary sounder operated by a local battery.

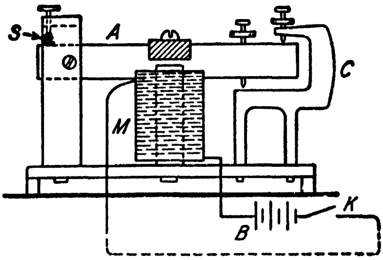


FIG. 1.—A telegraph sounder.

Figure 2 shows the circuit of a telegraph arranged to allow transmission in either direction between two widely separated stations. When the line is idle, switches s and s' are kept closed and the circuit includes the generators B and B', relays R and R', line L, and ground from G to G'. By opening s and depressing key K for short or long intervals, an operator can send a dot-and-dash message from the left-hand station to the other. The current pulses in the relay R' operate the local circuit and

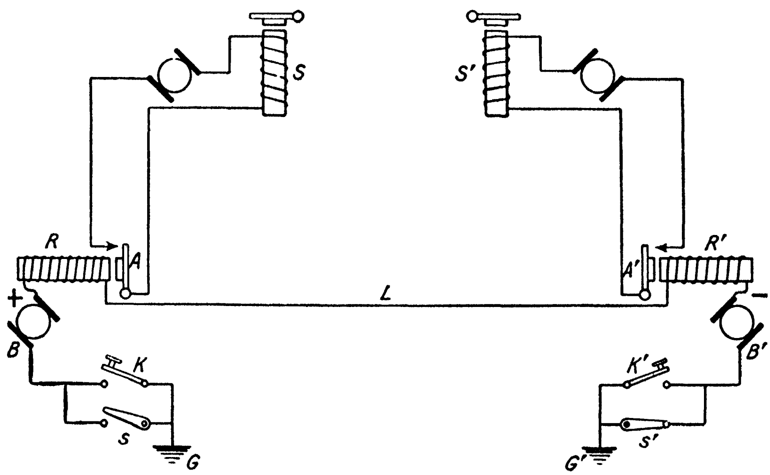


FIG. 2.—A telegraph circuit with relays.

sounder S', or the relay may be used to operate a second line to a more distant station.

Telephone. A telephone circuit for the transmission of speech consists of a transmitter for producing a variable current in response to sound waves, and a receiver for converting this current into sound waves that reproduce the original sounds. The transmitter (Fig. 3) contains carbon granules, through which the current must pass, which are confined in a chamber, one wall of which is a flexible diaphragm. When the voice is directed against this diaphragm, the variations of air pressure alter the area of contact between the carbon granules, thus changing their resistance and producing a corresponding fluctuation in the current.

The receiver is a small electromagnet combined with a permanent magnet and a thin circular diaphragm supported near the poles. Fluctuations in the current in the electromagnet cause vibrations of the diaphragm and thus produce sound. The simplest telephone is a series circuit, including a transmitter, receiver, and battery. A practical circuit is considerably more complex, including an induction coil to reduce line losses, provision for operating a signal bell by superposed

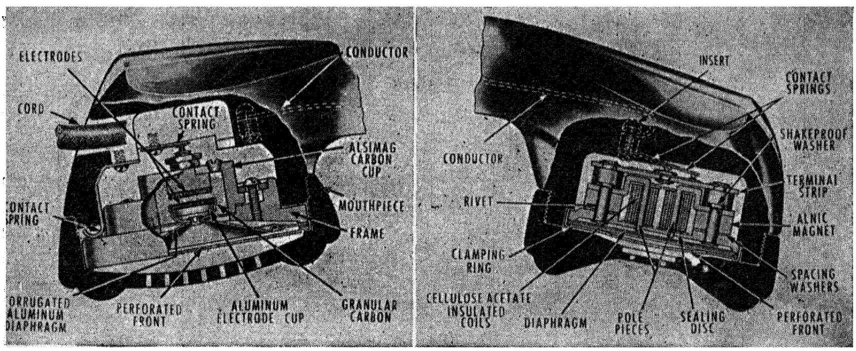


FIG. 3.—Cross section of telephone transmitter and receiver.

alternating current, and electron-tube repeaters to amplify signals that are transmitted over long lines.

Radio. Communication by radio depends upon the production of electric oscillations in a circuit designed to radiate energy in waves. These waves are of the same nature as light or heat waves such as those received from the sun.

Electric Oscillations. In the study of mechanical vibration (Chap. 20) it was found that oscillations can be set up in a body if certain conditions are present. The body must have inertia, a distortion must produce a restoring force, and the friction must not be too great. A mass suspended in air by a spring meets these conditions.

In an electrical circuit analogous conditions are necessary for electrical oscillations. Just as inertia opposes change in mechanical motion, inductance opposes change in the flow of electrons. The building up of charges on plates of condensers causes a restoring force on the electrons in the circuit. Resistance causes electrical energy to be changed into heat, just as friction changes mechanical energy to heat. To produce electrical oscillations it is necessary to have inductance, capacitance, and not too much resistance. As the frequency of mechanical vibrations depends upon the inertia (mass) and the restoring force (force constant), so the frequency of electrical oscillations depends upon inductance and capacitance.

In the circuit of Fig. 4 a capacitance C and an inductance L are connected in series with a sphere gap G. The sphere gap has a high resistance until a spark jumps across but low resistance after it jumps. If the voltage across G is gradually increased, the charge on the condenser will increase. When the voltage across G becomes high enough, a spark will jump and the condenser will then discharge. The current does not stop when the condenser is completely discharged but continues, charging the condenser in the opposite direction. It then discharges again, the current reversing in the circuit. The current oscillations continue until all the energy stored in the condenser has been converted into heat by the resistance of the circuit.

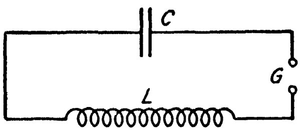


FIG. 4.—Circuit for production of electrical oscillations.

The frequency of the oscillation is determined by the values of L and C and is the frequency for which the impedance of the circuit is the least, that is, the frequency for which the reactance is zero. From Chap. 27,

$$X = 2\pi fL - \frac{1}{2\pi fC} = 0$$

or

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

where L is the inductance in henrys and C is the capacitance in farads.

Resonance. If an alternating voltage is applied to a series circuit in which there is both capacitance and inductance, oscillations are set up the amplitudes of which depend upon the frequency. If the frequency of the impressed voltage is the same as the natural frequency of the circuit, the current will be much larger than for other frequencies. The circuit is then said to be in *resonance*. Figure 5 shows how the current varies with the frequency in such a circuit if the resistance is small (solid curve). If the resistance is increased, the current values are decreased (dotted curve). For a very small range of frequencies the current is rather large, but outside this region the current is small. This response over a very limited range of frequencies makes possible the *tuning* of a radio circuit. The incoming wave produces in the receiver a voltage that varies with a

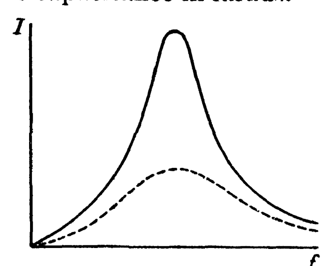


FIG. 5.—Resonance in a series circuit.

fixed frequency, and the circuit is tuned so that its natural frequency is the same as that of the incoming wave. The tuning is usually done by adjusting the value of the capacitance.

Although oscillations can be produced and waves can be detected in several ways, the most satisfactory methods use *electron tubes.*

Thermionic Emission. In metallic conductors there are many free electrons in addition to the atoms and molecules. Both molecules and free electrons take part in the thermal motion. The electrons, being of smaller mass, have much higher average speeds than the molecules. If the temperature is raised sufficiently, many of the electrons have enough speed to leave the metal. The emission of electrons by the heated metal is called *thermionic emission.* The temperature at which appreciable emission takes place depends upon the type of metal and the condition of its surface.

As electrons are emitted by a heated wire, the wire becomes positively charged, while the electrons collect in a "cloud" around it. This charge around the filament is called a *space charge.* Other electrons are attracted by the wire and repelled by the space charge. These effects combine to stop the emission of electrons.

Diode. If a filament and plate are sealed in an evacuated tube, a two-element electron tube, or *diode* (Fig. 6), is formed. When the filament is heated by an electric current, electrons are emitted. If the plate is made positive with respect to the filament, electrons will be attracted to the plate and a current will flow in the tube. If, however, the plate is made negative with respect to the filament, the electrons will be repelled and no current will flow. The diode thus acts as a *valve,* permitting flow in one direction but not in the other. If it is connected in an a.c. line, the diode acts as a rectifier, the current flowing during the half cycle in which the plate is positive.

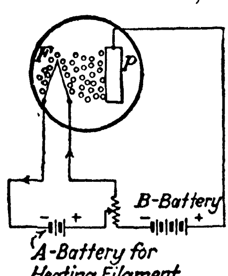


Fig. 6.—A diode.

If the plate is positive with respect to the filament, electrons will flow across, but not all the electrons that come out of the filament reach the plate, because of the space charge. Figure 7 shows a graph of potential against distance across the tube. Because of the space charge, the potential out to A is below the potential of the grid. An electron will reach the plate only if it has sufficient speed as it leaves the filament to reach B, the point of lowest potential, before it is stopped. If the difference of potential E_p between filament and plate is increased, the potential at B rises, and more electrons will be able to reach it. The current depends upon E_p, as is shown in the graph of Fig. 8. At the higher potentials the current no longer increases, because, when A has been pushed back to the filament, all the electrons emitted reach the plate, and further increase in E_p produces no change. *Saturation* has been reached.

If the plate potential is kept constant while the filament temperature is increased, the current increases at first but reaches saturation because of the increase in the electron cloud around the filament.

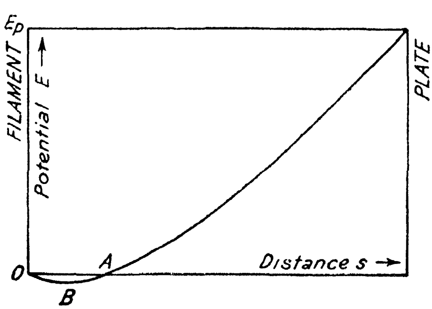


FIG. 7.—Variation of potential between filament and plate.

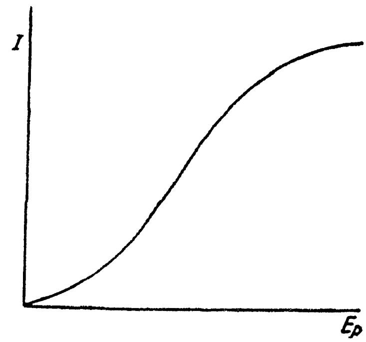


FIG. 8.—Plate current as a function of plate potential.

Triodes. If a third element, the *grid*, is inserted into the tube near the filament, it can be used as a control for the tube current. Such a tube is called a *triode*, or three-element tube. The grid usually consists of a helix, or spiral, of fine wire so that the electrons may freely pass through it. Small variations of the grid potential will cause large changes in the plate current, much larger than those caused by similar changes in the plate potential. If the grid is kept negative with respect to the filament, electrons will not be attracted to the grid itself, and there will be no grid current. A typical variation of plate current with grid potential E_g is shown in Fig. 9. A part of the curve is practically a straight line. If the grid voltage varies about a value in this region, the fluctuations of the plate current will have the same shape as the variation of grid voltage. The tube will *amplify* the disturbance without distorting it.

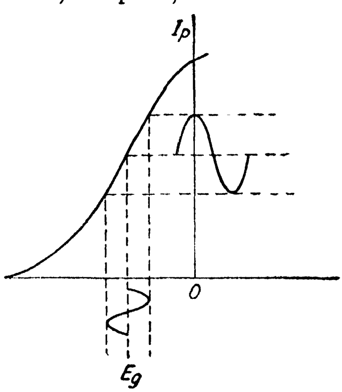


FIG. 9.—Operating characteristics of a vacuum-tube amplifier.

The triode also acts as a *detector* or partial rectifier if the grid voltage is adjusted to the bend of the curve. With this adjustment an increase in grid voltage above the average produces considerable increase in plate current, but a decrease in grid voltage causes little change in plate current. The plate current fluctuates in response to the grid signal, but the fluctuations are largely on one side of the steady current.

In Fig. 10 is shown a simple receiving circuit. When the waves strike the antenna, they set up oscillations in the circuit, which is tuned to the frequency of the waves. This causes the potential of the grid to vary, and the tube acting as a detector permits the flow of a current that pulsates according to the amplitude of the signal. This causes the earphones to emit sound.

In radio work, triode electron tubes are used to produce high-frequency oscillations, to act as detectors or rectifiers, and to act as amplifiers. Tubes of different characteristics are used for each of these purposes.

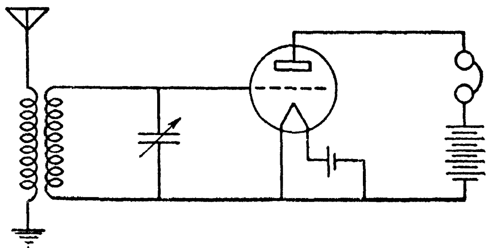


FIG. 10.—Simple receiving circuit.

In many tubes the filament merely acts as a heater of a sleeve that covers it and is insulated from it. The sleeve, or *cathode*, is the element that emits electrons.

For many purposes tubes are constructed with more than three active elements. They are named from the number of active elements, as *tetrode*, *pentode*, etc.

SUMMARY

The *telegraph* transmits signals by use of electromagnets controlled by a key to open and close the circuit. Since the current in a long line is insufficient to operate a sounder, a *relay* is used to operate a local circuit.

The *telephone* transmitter produces variations in an electric current in response to the motion of the diaphragm. The *receiver* is an electromagnet that causes a motion of a magnetic diaphragm in response to the electrical impulse received.

Electric *oscillations* occur in circuits that have inductance, capacitance, and low resistance.

The *frequency* of oscillation is given by the equation

$$f = \frac{1}{2\pi\sqrt{LC}}$$

If a metallic conductor is heated to a sufficiently high temperature, electrons are emitted. This is called *thermionic* emission.

Two-element electron tubes, or *diodes,* act as rectifiers in a.c. circuits.

Three-element electron tubes, or *triodes,* may be used as amplifiers, oscillators, or detectors.

QUESTIONS AND PROBLEMS

1. What conditions are necessary for the production of oscillations?
2. What is electrical resonance?
3. Explain the action of a diode as a rectifier.
4. Explain the action of a triode as an amplifier.
5. Explain the action of a triode as a detector.
6. What inductance must be placed in series with a 2-μf condenser to produce resonance at 60 cycles? at 500 cycles? *Ans.* 3.52 henrys; 0.051 henry.
7. A variable condenser has a range from 0.0000055 to 0.0005 μf. If it is connected in series with a coil whose inductance is 5 millihenrys, what is the frequency range of the circuit?

EXPERIMENT

Characteristics of Electron Tubes

Apparatus: Storage battery; dry cells; rheostat (20 ohm); 2 rheostats (1,000 ohms); 3 S.P.S.T. switches; 1 D.P.D.T. switch; voltmeter (0 to 10 volts); voltmeter (0 to 120 volts); ammeter (0 to 1 amp); milliammeter (0 to 10 ma); electron tube.

The variation of plate current with plate potential for a two-element electron tube can be studied by the use of a circuit similar to that in

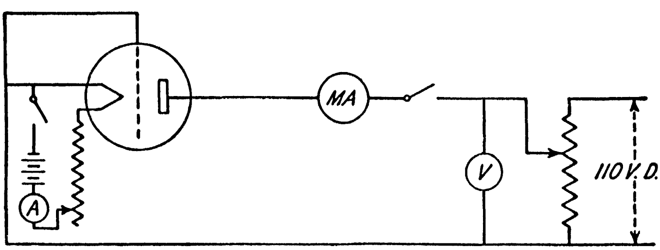


FIG. 11.—Circuit for determining variation of plate current with plate potential.

Fig. 11. The d.c. source may be B batteries, a generator, or other suitable source. Use a simple filament tube (for example, 01A) and connect the grid to the filament. Adjust the filament current to the rated value of the tube Beginning with a plate potential of zero, take a series of readings of plate current and plate potential from zero up to the rated voltage of the tube. Plot a curve of plate current against plate voltage. Does the curve show saturation?

With the same circuit, study the variation of plate current with filament current. With E_p at the rated value for the tube, increase the filament current in uniform steps from zero to the rated value. Plot a curve of plate current against filament current. At what filament current does emission begin?

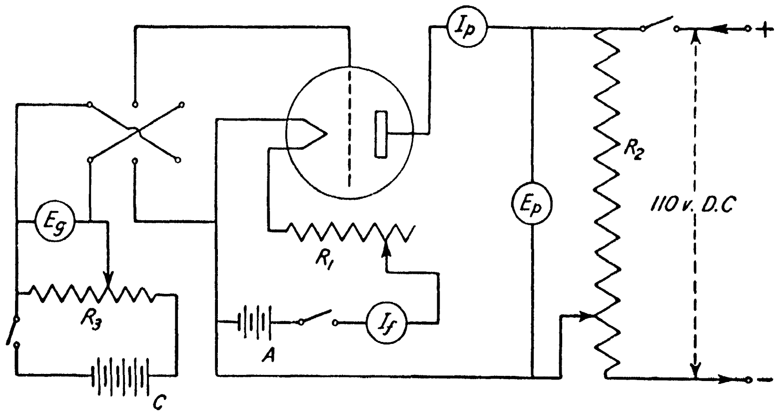
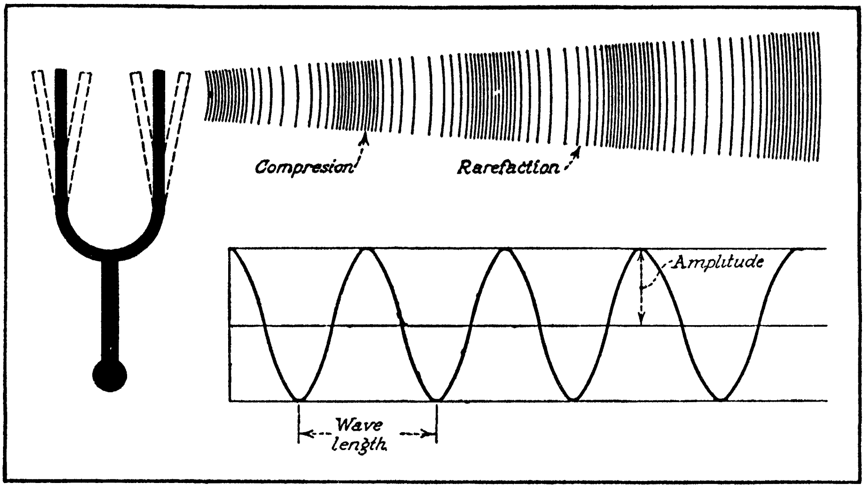


FIG. 12.—Circuit for measurements on a triode.

Use the circuit of Fig. 12 to study the characteristics of the three-element tube. Use the rated values of plate potential and plate current, make the grid negative with respect to the filament, and adjust its value until the plate current is zero. Take a series of readings of grid voltage and plate current, increasing the grid voltage to zero. Then make the grid positive and take several more readings. Plot a curve of plate current against grid voltage. At what grid voltage should this tube be used to be satisfactory as an amplifier?



CHAPTER 29

SOUND WAVES

Many of the phenomena of nature are satisfactorily described in terms of *wave motion.* There are waves on the surface of water; waves are used to show the behavior of light as it is transmitted through space or materials; the radio is dependent upon electromagnetic waves; and all the varied manifestations of sound are explained by the wave theory. The concepts of frequency, amplitude, period, simple harmonic motion, and resonance, which were discussed in Chap. 20, often occur in the consideration of wave motion and should be reviewed in connection with the study of the chapters on sound.

Nature of Sound. Sound may be thought of as an agency capable of affecting the sense of hearing. In order to understand the production and propagation of sound we must examine the physical nature of this agency.

If one plucks a tightly stretched string, he observes that the string vibrates. During the time that the vibrations are seen he also hears a sound, but as soon as the vibration stops the sound is no longer heard; hence he associates the vibration of the string with the sound. All sounds arise from the vibration of material bodies.

Suppose a small rubber balloon is partly inflated and attached to a bicycle pump. If the piston is pushed downward quickly, the balloon expands and the layer of air next to it is compressed. This layer of air will, in turn, compress the layer beyond it and so on. The compression that was started by the expansion of the balloon will thus travel away from the source in the surrounding medium. If the piston is drawn upward, the balloon contracts and the adjacent layer of air is rarefied. As in the case of the compression the rarefaction travels out from the source. If the piston is moved up and down at regular intervals, a succession of compressions and rarefactions will travel out from the source (Fig. 1). Such a regular succession of disturbances traveling out from a source constitutes a *wave motion.* The compression and the following rarefaction make up a *compressional wave.*

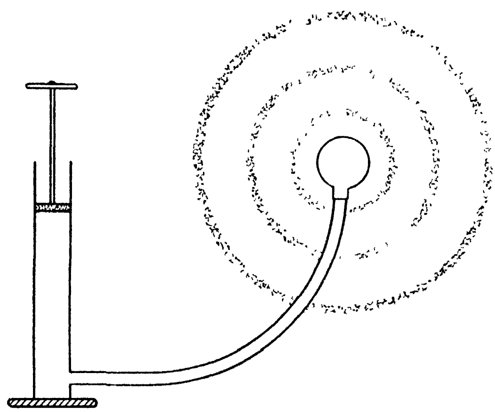


FIG. 1.—Compressional waves produced by an expanding and contracting balloon.

If the up-and-down motion of the piston is made rapid enough, an observer in the neighborhood will be able to hear a sound as the disturbance reaches his ear. These compressional waves are able to cause the sensation of hearing and are referred to as *sound waves.*

In the wave motion no particle travels very far from its normal position. It is displaced a short distance forward, then returned to its initial position, and displaced a short distance backward. In compressional waves the particle thus vibrates back and forth about a normal position, the direction of vibration being parallel to the direction in which the waves travel. Such waves are called *longitudinal* waves. In other types of waves the individual particles may vibrate at right angles to the direction of motion of the wave. Such a wave is called a *transverse* wave. A wave moving along a stretched string is usually of this type. In still other waves the motion of the particles is a combination of the two motions just described. In all these cases the particles of the medium remain close to their normal position while the disturbance moves through the medium.

The Medium. Since a sound wave involves compression and expansion of some material, it cannot proceed without the presence of a material medium. No sound can be transmitted through a vacuum. This fact can be demonstrated experimentally by mounting an electric bell under a bell jar and pumping the air out while the bell is ringing (Fig. 2). As the air is removed, the sound becomes fainter and fainter until it finally ceases, but it again becomes audible if the air is allowed to return.

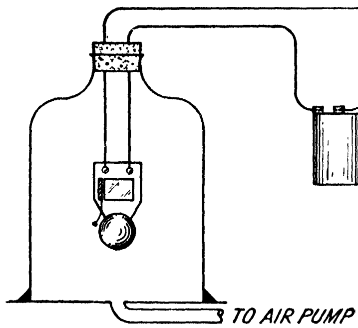


FIG. 2.—Sound is not transmitted through a vacuum.

Sound waves will travel through any elastic material. We are all familiar with sounds transmitted through windows, walls, and floors of a building. Submarines are detected by the underwater sound waves produced by their propellers. The sound of an approaching train may be heard by waves carried through the rails as well as by those transmitted through the air. In all materials the alternate compressions and rarefactions are transmitted in the same manner as they are in air.

Speed of Sound. If one watches the firing of a gun at a considerable distance, he will see the smoke of the discharge before he hears the report. This delay represents the time required for the sound to travel from the gun to the observer (the light reaches him almost instantaneously). The speed of sound may be found directly by measuring the time required for the waves to travel a measured distance. It varies greatly with the

TABLE I. SPEED OF SOUND AT 0°C (32°F) THROUGH VARIOUS MEDIUMS

Medium	ft/sec	m/sec
Air	1,087	331.5
Hydrogen	4,167	1,270
Carbon dioxide	846	258.0
Water	4,757	1,450
Iron	16,730	5,100
Glass	18,050	5,500

material through which it travels. Table I shows values for several common substances.

The speed of sound varies with the temperature of the medium transmitting it. For solids and liquids this change in speed is small and usually can be neglected. For gases, however, the change is rather large. It has been shown that for gases the speeds at any two temperatures are related by the expression:

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \tag{1}$$

where V_1 and V_2 are the speeds and T_1 and T_2 are the respective *absolute* temperatures.

Example: What is the speed of sound in air at 25°C (77°F)?

From Table I the speed at 0°C (32°F) is 1,087 ft/sec.

$$\frac{V_{25^\circ\text{C}}}{1{,}087\ \text{ft/sec}} = \sqrt{\frac{273^\circ + 25^\circ}{273^\circ}} = \sqrt{\frac{298^\circ}{273^\circ}}$$

$$V_{25^\circ\text{C}} = (1{,}087\ \text{ft/sec})\sqrt{\frac{298^\circ}{273^\circ}} = 1{,}137\ \text{ft/sec}$$

For small differences in temperature we can consider the change in speed to be a constant amount for each degree change in temperature, amounting to a difference of about 2 ft/sec per °C (1.1 ft/sec per °F) for temperatures near 0°C. The change is to be added if the temperature increases and subtracted if it decreases.

Refraction of Sound. In a uniform medium at rest sound travels with constant speed in all directions. If, however, the medium is not uniform, the sound will not spread out uniformly but the direction of travel changes because the speed is greater in one part of the medium. The bending of sound due to change of speed is called *refraction.*

The spreading of sound in the open air is an example of this effect. If the air were at rest and at a uniform temperature throughout, the sound would travel uniformly in all directions. Rarely, if ever, does this condition exist, for the air is seldom at rest and almost never is the temperature uniform. On a clear summer day the surface of the earth is heated and the air immediately adjacent to the surface has a much higher temperature than the layers above. Since the speed of sound increases as the temperature rises, the sound travels faster near the surface than it does at higher levels. As a result of this difference in speed the wave is bent away from the surface, as shown in Fig. 3*a*. To an observer on the surface, sound does not appear to travel very far on such a day since it is deflected away from him.

On a clear night the ground cools more rapidly than the air above, hence the layer of air adjacent to the ground becomes cooler than that at a higher level. As a result of this condition sound travels faster at the higher level than at the lower level and consequently is bent downward, as shown in Fig. 3*b*. Since the sound comes down to the surface, it appears to carry greater distances than at other times.

Wind is also a factor in refraction of sound. In discussing the speed of sound in air we assume that the air is stationary. If the air is moving,

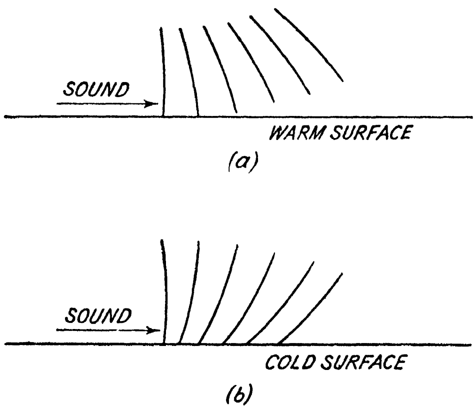


FIG. 3.—Refraction of sound due to temperature difference.

sound travels through the moving medium with its usual speed *relative to the air* but its speed relative to the ground is increased or decreased by the amount of the speed of the air, depending upon whether the air is moving in the same direction as the sound or the opposite. If the air speed is different at various levels, the direction of travel of sound is

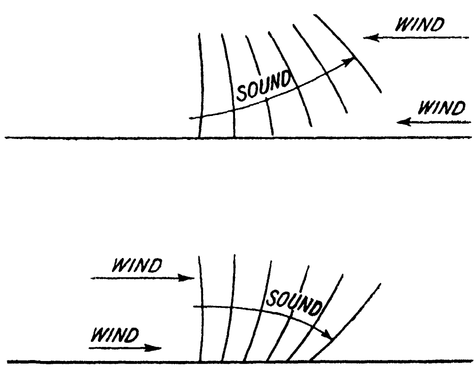


FIG. 4.—Refraction of sound due to wind.

changed, as shown in Fig. 4. Friction causes the wind speed to be lower at the surface than at a higher level, hence sound traveling against the wind is bent upward and leaves the surface while that traveling with the wind is bent downward. As a result, the observer on the surface reports that sound carries farther with the wind than against the wind.

Combinations of the two phenomena just discussed may cause some effects that seem very peculiar. Sound may carry over a mountain and be heard on the other side while similar sounds are not transmitted in the other direction. Frequently sound "skips" a region, that is, it is audible near the source and also at a considerable distance but at intermediate distances it is not audible. Such an effect is quite troublesome in the operation of such devices as foghorns. Refraction effects increase the difficulties in locating airplanes, guns, or submarines by means of sound waves.

Frequency and Wave Length. When waves are sent out by a vibrating body, the number of waves per second is the same as the number of complete vibrations per second of the source. The number of vibrations per second is called the *frequency* of the source and represents as well the frequency of the wave. The *wave length* is defined as the distance between

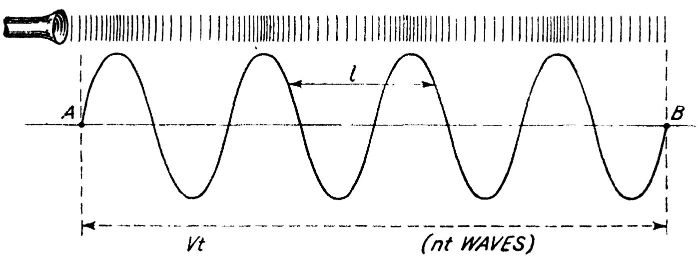


FIG. 5.—Graph showing pressure distribution in a sound wave.

two successive compressions or between two successive rarefactions in the wave motion. The curve in Fig. 5 represents a sound wave. The ordinate of the curve represents, at a single instant, the *pressure* in the medium at each point—higher than normal pressure at the compressions and lower at the rarefactions. The curve is merely a graph of the pressure distribution in the medium and not a picture of the wave. The distance l on this graph is one wave length. It may be measured between one crest and the next, between one trough and the next, or in general, between any point and the next similar one in the wave motion.

There is a simple relation between the frequency n, the wave length l, and the wave speed V. Suppose the source vibrates for a time t. The number of waves sent out will be nt. At the end of this time, the first wave will have reached a point B in Fig. 5. The distance AB is equal to Vt and this distance is equal to the number of waves times the length of each wave. Therefore

$$Vt = ntl$$

or

$$V = nl \qquad (2)$$

This relationship holds for any wave motion whatsoever.

Example: What is the wave length of a sound of frequency 256 vibrations per second?

From Eq. 2,

$$l = \frac{V}{n} = \frac{1,\bar{1}00 \text{ ft/sec}}{256/\text{sec}} = 4.3 \text{ ft}$$

Reflection of Sound Waves. When ripples on water encounter an obstacle, a new set of ripples starts out from the obstruction. The waves are said to be *reflected.* If the surface of the obstacle is at right angles to the direction in which the ripples travel, the reflected ripples will travel back in the direction from which the ripples came (Fig. 6). For other positions of the obstacle the ripples will be reflected in new directions.

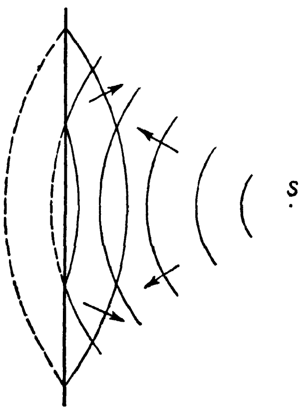


FIG. 6.—Reflection of waves by a plane surface.

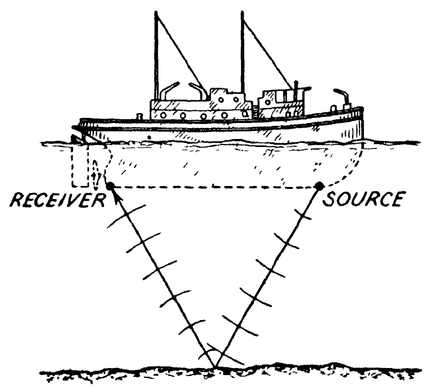


FIG. 7.—Measuring ocean depth by means of a fathometer.

In a similar manner sound waves are reflected from surfaces such as walls, mount ins, clouds, or the ground. A s͡und is seldom heard without accompanying reflections, especially inside a building where the walls and furniture supply the reflecting surfaces. The "rolling" of thunder is largely due to successive reflections from clouds and land surfaces.

The ear is able to distinguish two sounds as separate only if they reach it at least 0.1 sec apart; otherwise, they blend in the hearing mechanism to give the impression of a single sound. If a sound of short duration is reflected back to the observer after 0.1 sec or more, he hears it as a repetition of the original sound; an *echo.* In order that an echo may occur, the reflecting surface must be at least 55 ft away, since sound, traveling at a speed of 1,100 ft/sec will go the 110 ft from the observer to the reflector and return in 0.1 sec.

Use is made of the reflection of sound waves in the *fathometer,* an instrument for determining ocean depths (Fig. 7). A sound pulse is sent out *under water* from a ship. After being reflected from the sea bottom the returned sound is detected by an underwater receiver also mounted on the ship, and the time interval is recorded by a special device. If the elapsed time and the speed of sound in water are known, the depth of the sea at that point can be computed. Measurements may thus be made almost continuously as the ship moves along.

Sound waves may be reflected from curved surfaces for the purpose of making more energy travel in a desired direction, thus making the sound more readily audible at a distance. The curved sounding board placed behind a speaker in an auditorium throws forward some of the sound waves that would otherwise spread in other directions and be lost to the audience. In the same way, a horn may be used to collect sound waves and convey their energy to an ear or other detector.

Interference of Waves: Beats. Whenever two wave motions pass through a single region at the same time, the motion of the particles in the medium will be the result of the combined disturbances of the two sets of waves. The effects due to the combined action of the two sets of waves are known in general as *interference* and are important in all types of wave motion.

If a shrill whistle is blown continuously in a room whose walls are good reflectors of sound, an observer moving about the room will notice that the sound is exceptionally loud at certain points and unusually faint at others. At places where a compression of the reflected wave arrives at the same time as a compression of the direct wave their effects add together and the sound is loud; at places where a rarefaction of one wave arrives with a compression of the other their effects partly or wholly cancel and the sound is faint.

Contrasted with the phenomenon of interference in space, we may have two sets of sound waves of slightly *different frequency* sent through the air at the same time. An observer will note a regular swelling and fading of the sound, which is called *beats*. Since the compressions and rarefactions are spaced farther apart in one set of waves than in the other, at one instant two compressions arrive at the ear of the observer together and the sound is loud. A little later a compression of one set of waves arrives with the rarefaction of the other and the sound is faint. The number of beats occurring each second is equal to the difference of the two frequencies. Thus, in Fig. 8, two sets of waves of frequency 10 vib/sec and 12 vib/sec, respectively, combine and give a resultant sound wave which fluctuates in amplitude 12 minus 10, or 2 times per second. Beats are readily demonstrated by sounding identical tuning forks, one of which has been "loaded" by placing a bit of soft wax on one prong, thus reducing the frequency of this fork slightly.

Absorption of Sound. As a wave motion passes through a medium or from one medium to another, some of the regular motion of particles in the wave motion is converted into irregular motion (heat). This constitutes *absorption* of energy from the wave. In some materials there is very little absorption of sound as it passes through, and in others the absorption is large. Porous materials, such as hair felt, are good absorbers of sound since much of the energy is changed to heat energy in the pores. Whenever it is necessary to reduce the sound transmitted through walls or floors or that reflected from wall, a material should be

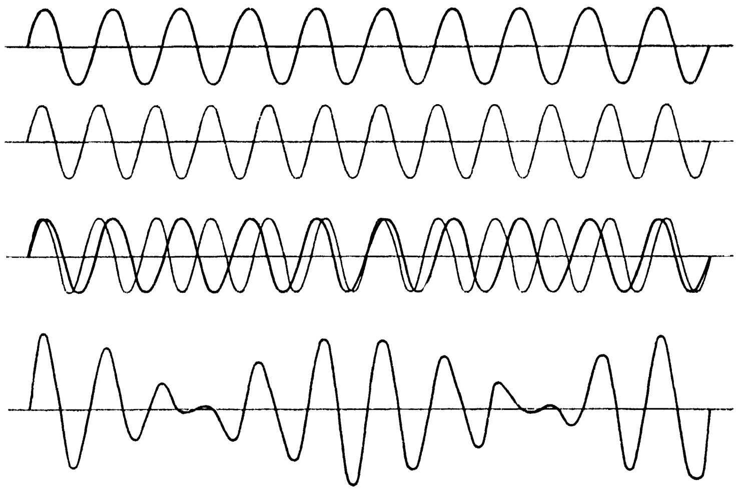


FIG. 8.—Two waves of different frequency combined to cause beats.

used that is a good absorber. Rugs, draperies, porous plasters, felts, and other porous materials are used for this purpose.

SUMMARY

Sound is a disturbance of the type capable of being detected by the ear. It is produced by the *vibration* of some material body.

Sound is transmitted through air or any other material in the form of *longitudinal* (compressional) *waves.*

The *speed* of sound waves in air at ordinary temperatures is about 1,100 ft/sec.

A sound wave may be *refracted* if the speed is not the same in all parts of the medium or if parts of the medium are moving. It may also be refracted as it passes from one medium to another.

The *wave length* is the distance between two successive compressions or between two successive rarefactions.

The *frequency* of a vibrating body is the number of complete vibrations per second. The frequency of the wave motion sent out by a source is the number of waves passing a given point per second. The two frequencies have the same value.

In any wave motion, the velocity, frequency, and wave length are related by the equation, $V = nl$.

The direction of advance of sound waves may be changed by *reflection* from suitable surfaces.

An *echo* occurs when a reflected sound wave returns to the observer 0.1 sec or more after the original wave reaches him, so that a distinct repetition of the original sound is perceived.

Two sets of waves of the same frequency may mutually reinforce or cancel each other at a given place. This is called *interference.*

Beats occur when two sources of *different* frequency are sounded at the same time. The resultant sound periodically rises and falls in intensity as the waves alternately reinforce and cancel each other.

Absorption occurs when the regular motion of the particles in a wave is converted into irregular motion (heat).

QUESTIONS AND PROBLEMS

1. Explain how the distance, in miles, of a thunder storm may be found approximately by counting the number of seconds elapsing between the flash of lightning and the arrival of the sound of the thunder and dividing the result by five.

2. If the earth's atmosphere extended uniformly as far as the moon, how long would it take sound to travel that distance? Take the distance to be 240,000 mi, and use 1,100 ft/sec as the speed of sound. What actually happens to a sound in the earth's atmosphere? *Ans.* 13.3 days.

3. Explain why stroking the tip of a fingernail across a linen book cover produces a musical tone.

4. What will be the wave length in air of the note emitted by a string vibrating at 440 vib/sec when the temperature is 59°F? *Ans.* 2.5 ft.

5. In Statuary Hall of the Capitol at Washington, a person standing a few feet from the wall can hear the whispering of another person who stands facing the wall at the corresponding point on the opposite side, 50 ft away. At points between, the sound is not heard. Explain.

6. By means of Eq. (1) verify the statement that the speed of sound in air increases about 2 ft/sec for each centigrade degree rise in temperature from 0°C.

7. A track worker pounds on a steel rail at the rate of one blow per second. A flagman some distance up the line hears the sound through the rails at the same instant that he hears the previous blow through the air. How far away is he? *Ans.* 1,176 ft.

8. The sound of the torpedoing of a ship is received by the underwater detector of a patrol vessel 18 sec before it is heard through the air. How far away was the ship? Take the speed of sound in sea water to be 4,800 ft/sec. *Ans.* 5 mi.

9. A stone is dropped into a mine shaft 400 ft deep. How much later will the impact be heard? *Ans.* 5.36 sec.

DEMONSTRATION EXPERIMENTS

Apparatus: Tuning fork; rubber mallet; pith ball; spiral spring; tablespoon; metronome; bell jar; air pump; toothed wheel; concave reflector; ripple tank.

Set a tuning fork into vibration by striking it with a rubber mallet. Notice that a sound is produced. Now allow one prong of the vibrating fork to touch a suspended pith ball, which will be thrown aside violently (Fig. 9). This shows that a sounding body is actually in a state of mechanical vibration.

Hang a long spiral spring from the ceiling. Grasp a few coils in one hand in a compressed position and suddenly release them. Observe that the compression passes onward in both directions along the spring

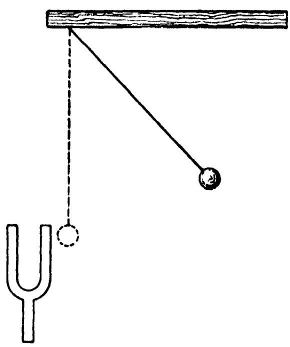


FIG. 9.—Demonstration of the vibration of a sounding fork.

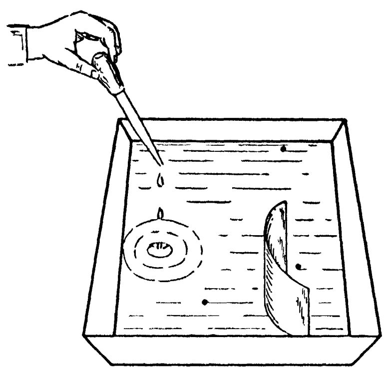


FIG. 10.—Experimenting with the ripple tank.

and that it is reflected repeatedly from the ends. Repeat for a "rarefaction."

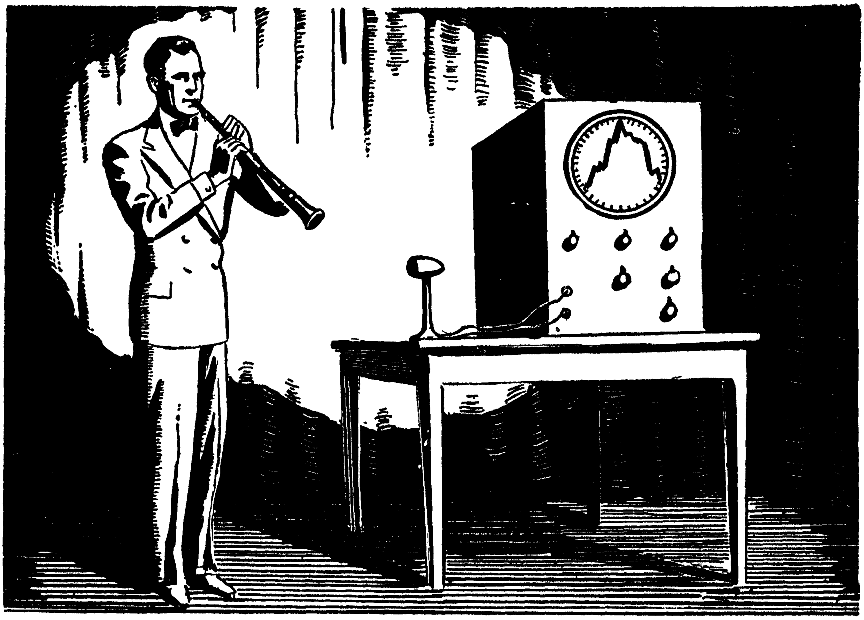
Tie two pieces of string, each about 2 ft long, to the handle of a large silver tablespoon at a point near its center of gravity. Hold the free end of each cord in an ear by means of the finger and strike the suspended spoon against a hard surface. The effective transmission of the vibration through the cords will make the tone of the spoon seem startingly loud and of a quality similar to that of a church bell.

Mount a metronome under a bell jar, supporting it on cork, sponge rubber, or hair felt, so that it does not set the jar itself into vibration. With the metronome sounding, pump out the air and note the fading of the sound, showing that sound cannot be transmitted by a vacuum. Readmitting the air will restore the sound.

Rotate a gear or toothed wheel by means of a variable speed motor or on a hand-driven rotator, while holding a card against the teeth. Does the pitch of the tone change if the speed of the rotation is altered? The same effect may be shown by blowing a jet of air through regularly spaced holes in a disk (siren disk).

Mount a watch at the focus of a concave reflector and turn the reflector in various directions in the classroom. Is the sound much louder in the forward direction? Explain this by discussing with the aid of a diagram the way in which the sound waves are reflected from the curved surface.

Experiment with the reflection of waves from plane and curved surfaces by means of a ripple tank. This may be merely a large photographic tray containing water, and the source of waves may be an eye dropper (Fig. 10). The ripples are easily visible if the tank is illuminated by an unshaded light suspended several feet above it.



CHAPTER 30

ACOUSTICS

The science of *acoustics* includes the production, transmission, and effects of sound. In architectural engineering the term is used in a more restricted sense to refer to the qualities that determine the value of a hall with respect to distinct hearing. We are primarily interested in sound insofar as it affects our sense of hearing.

The hearing mechanism is able to distinguish between sounds that come to the ear if they differ in one or more of the characteristics: *pitch, quality,* and *loudness.* Each of these characteristics is associated with *physical* characteristics of the sound waves that come to the ear.

Pitch and Frequency. *Pitch* is the characteristic of sound by which the ear assigns it a place in a musical scale. The physical characteristic associated with pitch is the *frequency* of the sound wave. A tuning fork that gives a high-pitched sound is found to have a greater frequency of vibration than one giving a lower pitched tone. The range of frequency to which the human ear is sensitive depends somewhat upon the individual but for the average normal ear it is from 20 to 20,000 vib/sec. The upper limit decreases, in general, as the age of the individual increases.

The satisfactory reproduction of speech and music does not require a range of frequencies as great as that to which the ear is sensitive. To have perfect fidelity of reproduction a range of from 100 to 8,000 vib/sec is required for speech and from 40 to 14,000 vib/sec for orchestral music.

The frequency range of most sound-reproducing sytems, such as radio, telephone, and phonograph, is considerably less than that of the hearing range of the ear. A good radio transmitter and receiver in the broadcast band will cover a range of from 40 to 8,000 vib/sec. This range allows it to reproduce speech faithfully but it does detract from the quality of orchestral music. If the frequency range is further restricted, the quality of reproduction is correspondingly reduced.

Although pitch is associated principally with frequency, other factors also influence the sensation. Increase in intensity of the sound causes a decrease in pitch for a fixed frequency, especially at low frequencies. The complexity of the sound wave also influences pitch.

Quality and Complexity. It is a fact of experience that a tone of a given pitch sounded on the piano is easily distinguished from one of exactly the same pitch sounded, for example, on the clarinet. The difference in the two tones is said to be one of *tone quality*. This characteristic of sound is associated with the *complexity* of the sound wave that arrives at the ear.

In Chap. 29 we considered the compressional wave sent out by a balloon that expands and contracts as a piston moves back and forth. The pressure changes in such a wave are represented by a graph (Fig. 5, Chap. 29), which is a simple curve. A few other vibrating bodies send out such simple waves but for most of them the wave is much more complex.

Fundamental and Overtones. Almost all bodies may vibrate in a number of different ways. For example, a stretched string may vibrate in one segment, or in two, or in general in any number of segments, as shown in Fig. 1. Each of these various ways of vibration will have a frequency different from the others. The simplest vibration (one segment) has the lowest frequency and is called the *fundamental*. The more complicated vibrations give higher frequencies and are called *overtones*. In the case of the string, the frequencies of the overtones are two, three, four, etc., times the frequency of the fundamental.

A vibrating body almost always combines several different ways of vibration simultaneously. The sound waves sent out by such a source are quite complex as shown by the graph of such a disturbance in Fig. 2. We may consider such a complex wave as made up of a number of simple waves, one for each manner of vibration of the source. The pressure at each point will be the sum of the pressures of the component waves. Figure 3 shows graphically two simple waves, *a* and *b*, combined to give the complex wave *c*. The ordinate represents the pressure at each point. By adding the ordinates of *a* and *b* for each point we get the ordinate for *c*. Any complex wave can be resolved into a number of simple waves. The more complex the wave the greater is the number of overtones that contribute to it.

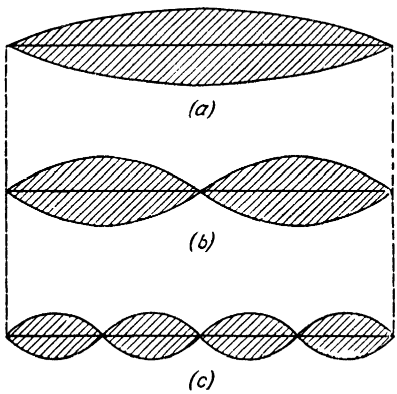


FIG. 1.—A string vibrating in three different forms, (*a*) fundamental, (*b*) first overtone, and (*c*) third overtone.

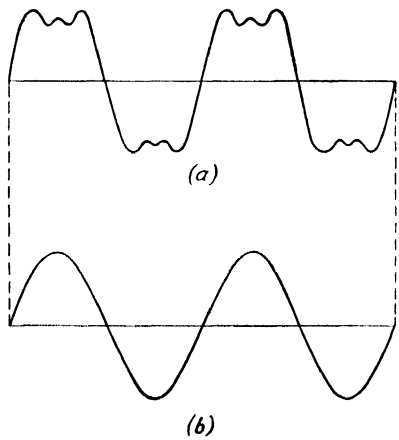


FIG. 2.—Graph of (*a*) a complex wave and (*b*) a simple wave.

The complexity of the wave, which determines the quality of the sound, is controlled by the number and relative intensity of the overtones that are present. A "pure" tone (no overtones) may not be as pleasing as the "rich" tone of a violin, which contains ten or more overtones.

Loudness and Intensity. The *loudness* of a sound is the magnitude of the auditory sensation produced by the sound. The *intensity* of sound

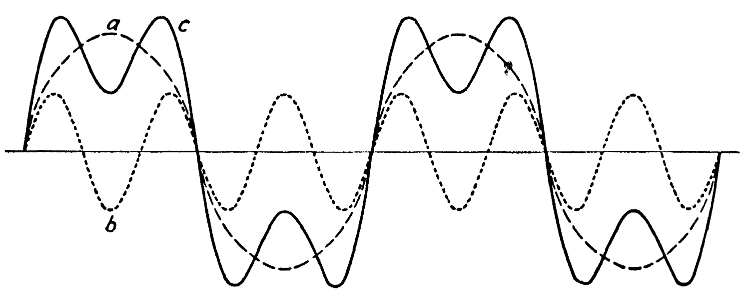


FIG. 3.—Compounding of two simple waves *a* and *b* to form a complex wave *c*.

refers to the rate at which sound energy flows through unit area. It may also be expressed in terms of the changes in pressure since the rate of flow of energy is proportional to the square of the pressure change.

The loudness of sound depends upon both frequency and intensity. For sounds of equal intensity the loudest will be in the frequency region between 3,000 and 4,000 vib/sec for there the sensitivity of the ear is greatest.

The ear is able to hear sounds over an extremely wide range of intensities. For a sound at the threshold of audibility the pressure in the wave varies from normal pressure only by about 0.001 dyne/cm^2, for ordinary speech by about 1 dyne/cm^2, and for the most intense sounds about 1,000 dynes/cm^2. For the most intense sounds the pressure change is about a million times as great as for the least intense. Pressure variations above this maximum do not produce a sensation of hearing but rather one of feeling or pain.

The intensity at the threshold of audibility is almost unbelievably small. At the threshold the rate at which a source of medium pitch supplies energy is so small that a million of them would require about two centuries to produce enough heat to make a cup of coffee.

The measurement of loudness is important for practical purposes but is a difficult one to achieve. The ear is a fair judge of the variation of one intensity to another. This makes it possible to arrange a scale of intensity *ratios*. It happens that the ear judges one sound to be about twice as loud as another of the same frequency when the actual *power* of the second sound is ten times as great as that of the first sound. Hence it is now customary to state the differences in the intensities of two sounds by the *exponent* of 10, which gives the *ratio* of the powers. This exponent is therefore the common logarithm of the ratio of the sound powers. This exponent is called the *bel,* in honor of Alexander Graham Bell, whose researches in sound transmission are famous. If one sound has ten times as much power as a second sound of the same frequency, the difference in their intensities is 1 bel. The bel is an unfortunately large unit and hence the decibel (0.1 bel) is the unit that is generally used in practice. A 26 per cent change in intensity alters the power by 1 decibel. This is practically the smallest change in energy level that the ear can ordinarily detect. Under the best laboratory conditions a 10 per cent (0.4 decibel) change is detectable.

TABLE I. INTENSITIES OF CERTAIN SOUNDS

	Decibels
Barely audible sound	0
Calm evening in country	10
Ordinary conversation	60
Trolley car	80
Boiler factory	100
Threshold of pain	130

A sound that is just audible is usually arbitrarily designated as intensity of zero decibels. The intensities of other familiar sounds are given in Table I.

In a sound wave the particles of air that take part in the vibration move neither far nor fast. For ordinary conversation the maximum velocity of the particle is about 2.4×10^{-2} cm/sec and the maximum displacement is about 3.8×10^{-6} cm. Even for the most intense sounds the maximum displacement is less than 0.004 cm.

We know by experience that the loudness of a sound decreases with distance. For any disturbance carried by waves spreading uniformly in all directions in space, the intensity is inversely proportional to the square of the distance from the source. Thus at a point 3 yd from a given source of sound the intensity will be one-ninth ($1/3^2$) of its value at a distance of 1 yd. This relation holds only if the source of sound is small and if the waves travel uniformly in all directions. In actual practice, reflected sounds usually contribute to the intensity, especially indoors.

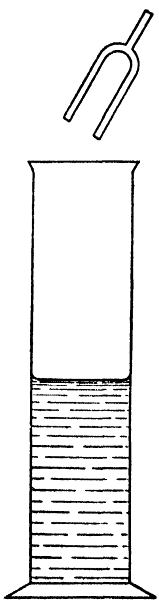


Fig. 4.—Resonance in an air column.

Forced Vibration, Resonance. If a force is applied at regular intervals to an elastic body, the body is caused to vibrate with the frequency of the applied force. Such a vibration is called a *forced vibration.* If the base of a vibrating tuning fork is placed on a table top, the forced vibration of the table increases the intensity of the sound in the region. The sounding board of a piano, the cone of a loud-speaker, and the body of a violin are examples of sound producers whose actions depend upon forced vibration. They are most effective if they respond alike to all frequencies that are applied. They are shaken back and forth by the driving mechanism and their purpose is to set into vibration more air than can the small vibrating object itself. A vibrating string mounted between rigid supports gives a scarcely audible sound but, when it is allowed to agitate a large surface like the back of a violin, the sound is much intensified. The responding surface must be so designed that it has no natural vibration of its own in the range of frequencies for which it is to be used; otherwise an objectionably loud sound will result at that frequency.

If the frequency of the applied force is the same as a natural frequency of vibration of the body, a large amplitude of vibration may be built up This phenomenon is called *resonance* and plays a large part in the production of sound.

If a vibrating tuning fork is held over a tube partly filled with water as shown in Fig. 4, the sound waves will set up vibrations in the air of the tube. These vibrations will have no great amplitude unless the length of the air column is so adjusted that it has a natural frequency of vibration the same as that of the fork. If the length of the air column is adjusted to secure this condition, the sound becomes much louder. The resonance of an air column is used in almost all wind instruments. Vibrations of many frequencies are produced by a reed or by the lips of the player. A few of the many frequencies produce resonance in the air column of the instrument and create the tone heard.

Sound Production. Any vibrating body whose frequency is within the audible range will produce sound provided that it can transfer to the medium enough energy to reach the threshold of audibility. Even though this limit is reached it is frequently necessary to amplify the sound so that it will be readily audible where the listener is stationed. For this purpose sounding boards and loud-speakers may be used, the purpose of each being to increase the intensity of the sound.

When a sounding board is used, the vibrations are transmitted directly to it and force it to vibrate. The combined vibrations are able to impart greater energy to the air than the original vibration alone. If the sounding board is to reproduce the vibrations faithfully, there must be no resonant frequencies, for such resonance will change the quality of sound produced.

The loud-speaker is used to increase the intensity of sound sent out, either by electrical amplification or by resonance. Two general types are used: the direct radiator, such as the cone loud-speaker commonly used in radios, and the horn type. The direct radiator is used more commonly because of its simplicity and the small space required, and is usually combined with electrical amplification. The horn speaker consists of an electrically or mechanically driven diaphragm coupled to a horn. The air column of the horn produces resonance for a very wide range of frequencies and thus increases the intensity of the sound emitted. The horn loud-speaker is particularly suitable for large-scale reproduction.

Sound Detectors. The normal human ear is a remarkably reliable and sensitive detector of sound, but for many purposes mechanical or electrical detectors are of great use. The most common of such detectors is the microphone in which the pressure variations of the sound wave force a diaphragm to vibrate. This vibration, in turn, is converted into a varying electric current by means of a change of resistance or generation of an electromotive force. For true reproduction the response of the microphone should be uniform over the whole frequency range. Such an ideal condition is never realized but a well-designed instrument will approximate this response. Microphones are used when it is necessary to reproduce, record, or amplify sound.

Parabolic reflectors may be used as sound-gathering devices when the intensity of sound is too small to affect the ear or other detectors or where a highly directional effect is desired. The sound is concentrated at the focus of the reflector and a microphone is placed there as a detector. Such reflectors should be large compared to the wave length of the sound received and hence they are not useful for low frequencies.

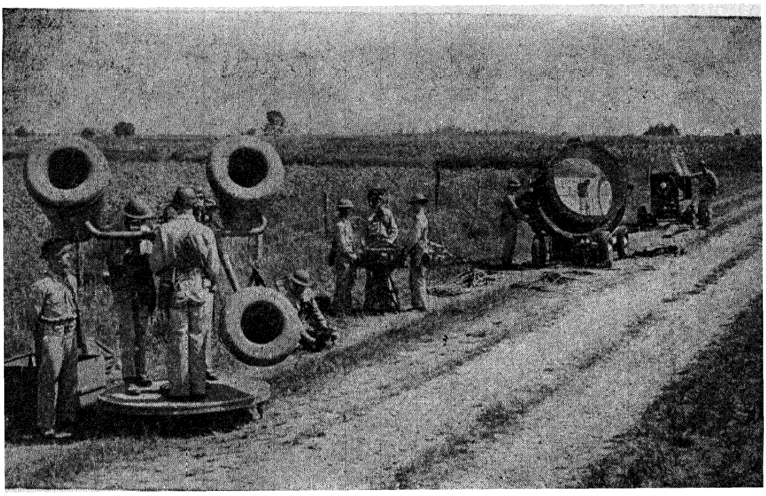


FIG. 5.—Sound locator, control station, searchlight, and power plant set up for operation. The distances between units are smaller than normal. (*Photograph by U.S. Army Signal Corps.*)

Location of Sound. Although a single ear can give some information concerning the direction of a source of sound, the use of two ears is

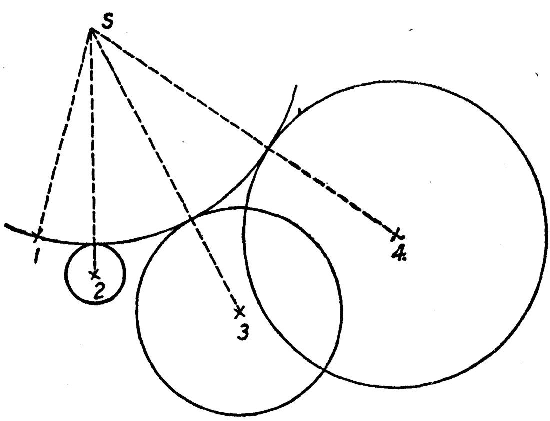


FIG. 6.—Location of sound by multiple observation points. Observers are stationed at posts 1, 2, 3, and 4.

necessary if great accuracy is desired. The judgment of direction is due to a difference between the impression received at the two ears, these differences being due to the differences in loudness or in time of arrival.

This is sometimes called the *binaural* effect. Certain types of sound locators exaggerate this effect by placing two listening trumpets several feet apart and connecting one to each ear. The device is then turned until it is perpendicular to the direction of the sound. In this way the accuracy of location is increased. Such a device may be used to locate airplanes or it may be used under water to locate submarines or other ships. Correction must be made in either case for refraction.

Explosions, such as the firing of a gun or a torpedo blast at sea, may be located quite accurately by the use of a number of observation points. The time of arrival at each station is recorded. Circles are drawn on a chart using each observing point as a center and the distances sound travels in the time after the first impulse is heard as radii. The result is shown in Fig. 6. The arc that is tangent to each of these circles has the source as a center.

Reverberation: Acoustics of Auditoriums. A sound, once started in a room, will persist by repeated reflection from the walls until its intensity is reduced to the point where it is no longer audible. If the walls are good reflectors of sound waves—for example, hard plaster or marble—the waves may continue to be audible for an appreciable time after the original sound stops. The repeated reflection that results in this persistence of sound is called *reverberation.*

In an auditorium or classroom, excessive reverberation may be highly undesirable, for a given speech sound or musical tone will continue to be heard by reverberation while the next sound is being sent forth. The practical remedy is to cover part of the walls with some sound-absorbent material, usually a porous substance like felt, compressed fiberboard, rough plaster, or draperies. The regular motions of the air molecules, which constitute the sound waves, are converted into irregular motions (heat) in the pores of such materials, and consequently less sound energy is reflected.

Suppose a sound whose intensity is one million times that of the faintest audible sound is produced in a given room. The time it takes this sound to die away to inaudibility is called the *reverberation time* of the room. Some reverberation is desirable, especially in concert halls; otherwise the room sounds too "dead." For a moderate-sized auditorium the reverberation time should be of the order of 1 to 2 sec. For a workroom or factory it should, of course, be kept to much smaller values, as sound deadening in such cases results in greater efficiency on the part of the workers, with much less attendant nervous strain.

The approximate reverberation time of a room is found to be given by the expression,

$$T = \frac{0.05V}{A} \tag{1}$$

where T is the time in seconds, V is the volume of the room in cubic feet, and A is the *total absorption* of all the materials in it. The total absorption is computed by multiplying the area, in square feet, of each kind of material in the room by its *absorption coefficient* (see Table II) and adding these products together. The absorption coefficient is merely the fraction of the sound energy that a given material will absorb at each reflection. For example, an open window has a coefficient of 1, since all the sound that strikes it from within the room would be lost to the room. Marble, on the other hand, is found to have a value of 0.01, which means it absorbs only 1 per cent of the sound energy at each reflection. Equation (1) usually gives satisfactory results except for very large or very small halls, for rooms with very large absorption, or for rooms of peculiar shape.

TABLE II. ABSORPTION COEFFICIENTS FOR SOUNDS OF MEDIUM PITCH

Open window	1.00
Plaster, ordinary	0.034
Acoustic plasters	0.20–0.30
Carpets	0.15–0.20
Painted wood	0.30
Hair felt, 1 in. thick	0.58
Draperies	0.40–0.75
Marble	0.01

By means of Eq. (1) we can compute the amounts of absorbing materials needed to reduce the reverberation time of a given room to a desirable value. The absorbing surfaces may be placed almost anywhere in the room, since the waves are bound to strike them many times in any case. In an auditorium, however, they should not be located too close to the performers.

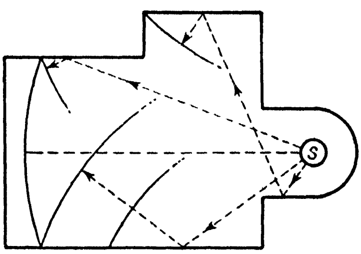


FIG. 7.—Ripple-tank model of an auditorium showing reflections from the walls.

In addition to providing the optimum amount of reverberation, the designer of an auditorium should make certain that there are no undesirable effects due to regular reflection or focusing of the sound waves. Curved surfaces of large extent should in general be avoided, but large flat reflecting surfaces behind and to the sides of the performers may serve to send the sound out to the audience more effectively. Dead spots, due to interference of direct and reflected sounds, should be eliminated by proper design of the room.

The acoustic features of the design of an auditorium may be investigated before the structure is built by experimenting with a sectional model of the enclosure in a ripple tank (Fig. 7). In this way the manner in which waves originating at the stage are reflected can be observed and defects in the design remedied before actual construction is undertaken.

SUMMARY

The *pitch* of a sound is associated with the physical characteristic of *frequency of vibration.* The average human ear is sensitive to frequencies over a range from 20 to 20,000 vib/sec.

A source may vibrate in several different ways. The vibration of lowest frequency is called the *fundamental* while those of higher frequency are called *overtones.*

The *quality* of a sound depends upon the number and relative prominence of the overtones.

The *intensity* of sound is the energy per unit area that arrives each second. For a direct sound from a small source, the intensity varies *inversely as the square of the distance* from the source.

The *loudness* of sound is the magnitude of the auditory sensation.

Forced vibrations occur whenever an applied vibration drives a system back and forth. *Resonance* occurs if the system so acted upon has a natural frequency equal to that of the driving force.

Reverberation is the persistence of sound in an enclosed space, due to repeated reflection of waves. It may be reduced by distributing sound-absorbent materials about in the enclosure.

QUESTIONS AND PROBLEMS

1. What are the wave lengths of the lowest and highest pitched sounds that the average ear can hear?

2. Draw a simple wave and its first harmonic overtone along the same axis, making the amplitude of the latter half as great as that of the fundamental. Combine the two graphically by adding the ordinates of the two curves at a number of different points, remembering that the ordinates must be added algebraically. If the resulting curve is taken to represent a complex sound wave, what feature of the curve reveals the quality of the sound?

3. An experimenter connects two rubber tubes to a box containing an electrically driven tuning fork and holds the other ends of the tubes to his ear. One tube is gradually made longer than the other, and when the difference in length is 7 in. the sound he perceives is a minimum. What is the frequency of the fork? Use $V = 1{,}100$ ft/sec. *Ans.* 943 vib/sec.

4. A concert hall whose volume is 30,000 ft^3 has a reverberation time of 1.50 sec when empty. If each member of an audience has a sound-absorption equivalent to 4 ft^2 of ideal absorbing material (absorption coefficient unity), what will the reverberation time be when 200 people are in the hall assuming that Eq. (1) holds for this hall? *Ans.* 0.832 sec.

5. What is the reverberation time of a hall whose volume is 100,000 ft^3 and whose total absorption is 2,000 ft^2? How many square feet of acoustic wall board of absorption coefficient 0.60 should be used to cover part of the present walls (ordinary plaster) in order to reduce the reverberation time to 2.0 sec, assuming that Eq. (1) holds for this hall? *Ans.* 2.5 sec; 883 ft^2.

DEMONSTRATION EXPERIMENTS

Apparatus: Stretched wire; ripple tank; whistle; tuning fork; glass tube.

Pluck a tightly stretched wire in the middle and note that it vibrates in one segment. Note the pitch of the sound emitted. Again pluck the string but hold a card lightly against the middle while plucking it at a point one-fourth the length from one end. Does the wire vibrate in two segments? How does the pitch compare with the former pitch?

Using the ripple tank described in Chap. 29, produce ripples by dipping into the water at regular intervals a wire bent to form two prongs about 3 in. apart. Note the interference of the two sets of ripples produced. Along certain lines the disturbance is a maximum while along others it is a minimum. From the positions of these lines determine for which ones the waves reinforce each other and for which the waves partly cancel each other.

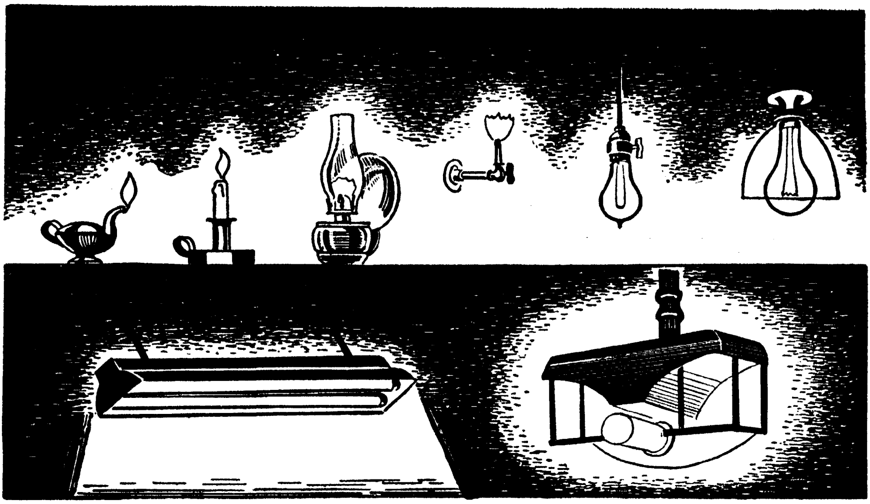
An interference pattern in sound may be formed in a room by blowing a high-pitched whistle continuously. The waves reflected from the wall interfere with the waves coming directly from the whistle. Lines of maximum and minimum sound are set up in the room. If each student stops one ear and moves the head from side to side for a distance of 2 or 3 ft, he will observe the changes in intensity.

Strike an unmounted tuning fork; observe the low intensity of the sound produced while it is held in the hand. Again strike it and hold its base against a board or table top and observe the increased intensity as forced vibrations are set up by the fork.

Tune a wire to the frequency of a vibrating tuning fork by moving an adjustable stop. When they have the same natural frequency, strike the fork and place its base against the stop. The wire will be set into vibration with considerable amplitude because of resonance. A small paper rider placed on the wire will be thrown off by the vibration.

Hold a vibrating tuning fork over an empty glass tube about an inch in diameter closed at one end. Gradually fill the tube with water and note the increase of sound for certain lengths of air column when resonance is produced. Will this condition exist for more than one length?

The advantages of binaural hearing may be demonstrated by having a member of the class plug one ear and attempt to locate a concealed watch by its ticking.



CHAPTER 31

LIGHT; ILLUMINATION AND REFLECTION

Most of our knowledge of our surroundings comes to us by means of sight. Light may be thought of as some agency that is capable of affecting the eye, hence we wish to know more about the physical nature of this agency and to learn something of its behavior and practical uses.

Nature of Light. In order to determine the physical nature of light, we must consider its behavior in as many situations as possible. Certain facts are familiar to all. Light travels in straight lines. Light can pass through transparent substances, such as water, air, and glass, but not through others. Light can pass through empty space, for it reaches us from the sun and stars and, if air is pumped from a transparent bottle, light is still transmitted. Light is reflected at certain surfaces. All these facts and many others are explained satisfactorily upon the assumption that light can be represented as a *wave motion.*

Certain properties are common to all waves. Some of these properties are as follows: The wave travels with a definite speed in a single medium, but at different speeds in various mediums. The wave has a fixed frequency and, in a single medium, a corresponding wave length. The speed V, wave length l, and frequency n are related by the equation $V = nl$. In a uniform medium the waves travel equally in all directions from a source of disturbance. Waves are reflected when they encounter an obstacle. The disturbance travels in straight lines in a uniform medium, but the direction may be changed at the boundary of that medium. Energy is transmitted by the wave. Light waves are found to possess all these properties as well as others.

Speed of Light. Early attempts to measure the speed of light were unsuccessful because its very high value made the measurements impractical. Indirect methods have been devised by which the measurement can be made with great accuracy. In these experiments the speed of light is found to be 186,285 mi/sec or 299,794 km/sec in a vacuum. Thus it takes light $8\frac{1}{3}$ min to travel the 93,000,000 mi from the sun to the earth. The speed is so great that in all except the most accurate experiments the time required for light to travel the short distances involved is smaller than the time intervals that can be measured. In 0.1 sec the distance traveled by light is equal to three-fourths the distance around the earth.

The speed of light in any material medium is found to be less than that in a vacuum. The speed in air is only slightly less than that in a vacuum, in water about three-fourths, and in ordinary glass about two-thirds that in a vacuum.

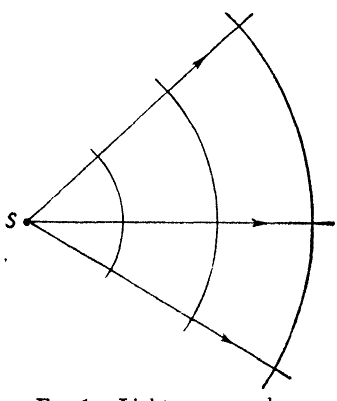


FIG. 1.—Light waves and rays. The concentric arcs represent sections of wave fronts. The straight lines represent rays.

Waves and Rays. Light waves spreading from a small source may be represented by equally spaced spheres with the source as a center. Since every point on each sphere is equidistant from the source, it may be considered to represent, so to speak, the crest of a wave. If we draw a number of straight lines outward from the source, each line will represent the direction along which the wave is advancing at each point. Such lines are called *rays*. Figure 1 shows spherical waves spreading from a small source and also rays drawn to show the direction in which the waves are moving. Notice that the rays always cross the waves perpendicularly. The rays are merely convenient construction lines that often enable us to discuss the travel of light more simply than by drawing the waves.

In Fig. 2 the light from a small source S encounters an obstacle A placed between the source and the screen C. The obstacle casts a shadow; that is, all parts of the screen are illuminated except the area within the curve B. The curve is determined by drawing rays from the source that just touch the edge of the obstacle at each point. If the source is not small or if there is more than one source, the shadow will consist of two parts, a completely dark one where no light arrives at the screen and a gray shadow, which is illuminated from part of the source only. One of the best examples of this is a total eclipse of the sun, which occurs when the moon comes directly between the earth and the sun (Fig. 3). Within the central cone of rays, no light is received from any part of the sun while the surrounding region gets light from part of the sun's disk only. A person located within the central cone experiences a total eclipse and does not see the sun at all; an observer anywhere in the crosslined area sees a crescent-shaped part of the sun—a partial eclipse.

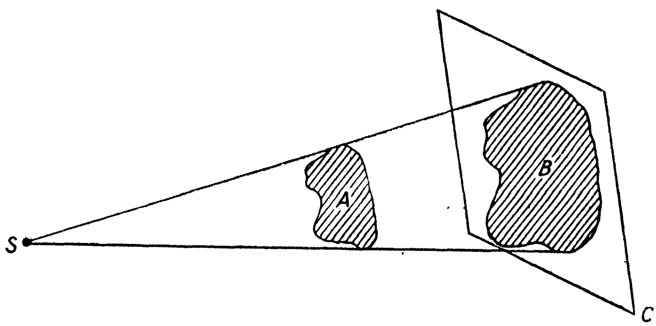


FIG. 2.—Light rays and shadow.

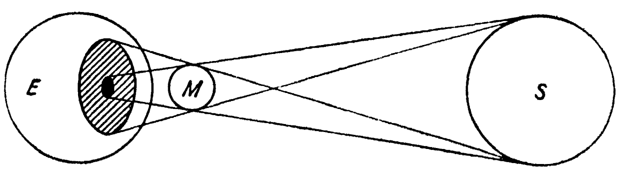


FIG. 3.—An eclipse of the sun.

Illumination. Since modern life is dependent to such a great extent upon artificial lighting, the subject of illumination is a topic of great practical importance in connection with any study of light. We must know how to choose and arrange lamps and other light sources to furnish the proper illumination in our homes, in stores, in factories and offices, and on highways.

The spreading of light waves from a small source is perfectly comparable to the spreading of sound waves under similar circumstances. It was seen (Chap. 29) that, in the case of a small source, the intensity of sound—the amount of sound energy falling on unit surface area in unit time—is inversely proportional to the square of the distance from the source. The same relation holds for *illumination*—the rate at which light energy falls on each unit of area. The geometric reason is the same in both cases, since the area over which the energy is spread increases as the square of the distance from the source. Thus, if E_1, and E_2 are the illuminations at the distances s_1 and s_2, respectively, then

$$\frac{E_1}{E_2} = \frac{s_2^2}{s_1^2} \tag{1}$$

This relation holds provided that the source is small and that the illuminated surface is at right angles to the rays of light.

Example: A small, unshaded electric lamp hangs 6 ft directly above a table. To what distance should it be lowered in order to increase the illumination to 2.25 times its former value?

Substituting in Eq. (1),

$$E_2 = 2.25E_1$$

$$\frac{E_1}{2.25E_1} = \frac{s_2^2}{(6 \text{ ft})^2}$$

$$s_2^2 = \frac{36 \text{ ft}^2}{2.25}$$

$$s_2 = 4 \text{ ft}$$

The illumination produced on a given surface by a light source will obviously depend upon the intensity of the source as well as upon its distance away. The standard of *source intensity* (or *luminous intensity*) is the *standard candle.* This was originally specified in terms of an actual candle of given kind and size and burning in a given manner. The intensity of such a standard source is 1 *candle power* (cp). Nowadays, the actual commercial standards are electric lamps that have been rated by comparison with such a primary standard. A 60-watt electric lamp has a luminous intensity of about 50 cp. This means that it sends forth light energy at the same rate as a concentrated source of 50 standard candles.

Units of illumination can now be defined. The illumination on a surface placed 1 ft from a small source of 1 cp and held perpendicular to the rays is said to be one *foot-candle.* The corresponding metric unit is the illumination 1 m from a source of 1 cp and is called the *meter-candle.* The illumination at a given distance will obviously be doubled if a single candle is replaced by two candles and will be trebled if three candles are used; hence, the complete relation between illumination E, source intensity I, and distance s is

$$E = \frac{I}{s^2} \tag{2}$$

where E is in foot-candles or in meter-candles, I is in candle power, and s is measured in feet or in meters.

Example: It is desired to replace a single 50-cp lamp located 8 ft from a normally illuminated surface by a small fixture containing three 10-cp lamps. How far from the surface should the fixture be placed to give the same illumination as before?

$$E_1 = E_2$$

From Eq. (2),

$$\frac{I_2}{s_1^2} = \frac{I_2}{s_2^2}$$

Substituting,

$$\frac{50 \text{ cp}}{(8 \text{ ft})^2} = \frac{30 \text{ cp}}{s_2^2}$$

$$s_2^2 = \tfrac{30}{50}(8^2) \text{ ft}^2$$

$$s_2 = 6.2 \text{ ft}$$

Lighting. In planning the artificial lighting of a room, the type of work to be done there or the use to which the room is to be put is the determining factor. Experience has shown that certain amounts of illumination are desirable for given purposes. Some figures are given in Table I.

TABLE I. DESIRABLE ILLUMINATION FOR VARIOUS PURPOSES

	Foot-candles
Close work (sewing, drafting, etc.)	20–30
Classrooms, offices, and laboratories	12
Stores	10–15
Ordinary reading	5
Corridors	3–5
Machine shops	4–16

Dull daylight supplies illumination of about 100 ft-candles while direct sunlight when the sun is at the zenith gives about 9,600 ft-candles.

In addition to having the proper amount of illumination it is essential to avoid *glare*, or uncomfortable local brightness such as that caused by a bare electric lamp or by a bright spot of reflected light in the field of vision. Glare may be reduced by equipping lamps with shades or diffusing globes and by avoiding polished surfaces, glossy paper, etc.

Photometers. A *photometer* is an instrument for comparing the luminous intensities of light sources. A familiar laboratory form of

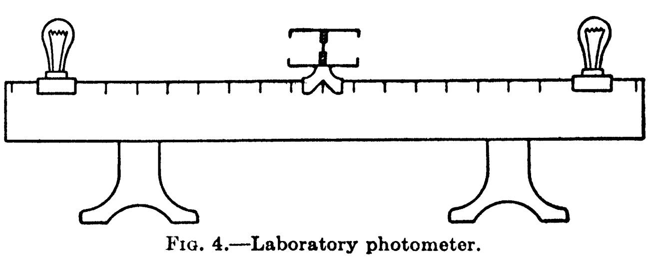


FIG. 4.—Laboratory photometer.

such an instrument usually consists of a long graduated bar with the two lamps to be compared mounted at or near the ends (Fig. 4). A movable dull-surfaced white screen is placed somewhere between the lamps and moved back and forth until both sides of the screen appear to be equally illuminated. When this condition is attained

$$E_1 = E_2$$

From Eq. (2),

$$\frac{I_1}{s_1^2} = \frac{I_2}{s_2^2}$$

or

$$\frac{I_1}{I_2} = \frac{s_1^2}{s_2^2} \tag{3}$$

where I_1 and I_2 are the luminous intensities of the two sources and s_1 and s_2 are their respective distances from the screen. If one source is a standard lamp of known candlepower, that of the other may be found by such comparison.

Example: A standard 48-cp lamp placed 36 in. from the screen of a photometer produces the same illumination there as a lamp of unknown intensity located 45 in. away. What is the luminous intensity of the latter lamp?

Substitution in Eq. (3) gives

$$\frac{I_1}{48 \text{ cp}} = \left(\frac{45 \text{ in.}}{36 \text{ in.}}\right)^2$$

$$I_1 = 75 \text{ cp}$$

Notice that the distances may be expressed in any unit when substituting in the equation, so long as they are both in the same unit.

In order to match the illuminations on the two sides of the photometer screen accurately, some means must be available for making both sides

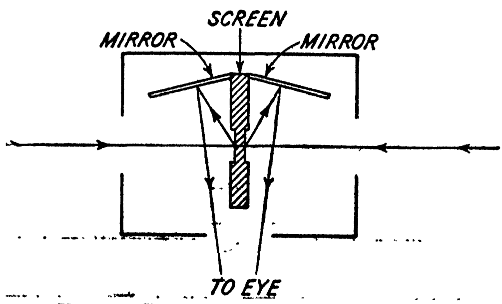


FIG. 5.—Photometer box with mirrors.

visible to the observer at the same time. One method used to accomplish this result is the use of two inclined mirrors as shown in Fig. 5.

A photometer should, of course, be used in a darkened room and there should be no appreciable reflection of light from the surroundings.

Foot-candle Meter. In planning a practical lighting installation for a room, one should take into account not only the direct illumination from all light sources but also the light that is diffused or reflected by the walls and surrounding objects. For this reason it is often very difficult to compute the total illumination at a given point, but this quantity can be measured by the use of instruments known as *foot-candle meters.* The most sensitive and commonly used type of this instrument makes use of the photoelectric effect (Chap. 21). The light falling on the sensitive surface causes an electric current whose value is proportional to the illumination. This current operates an electric meter whose scale is marked directly in foot-candles.

Fig. 6.—Photoelectric foot-candle meter.

Reflection. An object is seen by the light that comes to the eye from the object. If the object is not self-luminous, it is seen only by the light it *reflects.* Only a part of the light falling on a surface is reflected while the remainder passes into the material itself, where it may be either completely absorbed or partly absorbed and partly transmitted. Thus, when light strikes a piece of ordinary glass, about 4 per cent is reflected at the front surface. The remainder passes into the glass where some is absorbed. Again about 4 per cent of the light arriving at the rear surface is turned back, the rest passing through.

It is found by experience that when light, or any wave motion, is reflected from a surface, the reflected ray at any point makes the same angle with the perpendicular, or normal, to the surface as does the incident ray. The angle between the incident ray and the normal to the surface is called the *angle of incidence,* and that between the reflected ray and the normal is called the *angle of reflection* (Fig. 7). The law of reflection may then be stated: *The angle of incidence is equal to the angle of reflection.* This law holds for any incident ray and the corresponding reflected ray. A smooth, or polished, plane surface reflects parallel rays falling on it all in the same direction, while a rough surface reflects them diffusely in many directions (Fig. 8). At each point on the rough surface the angle of incidence is equal to the angle of reflection, but the normals have many directions.

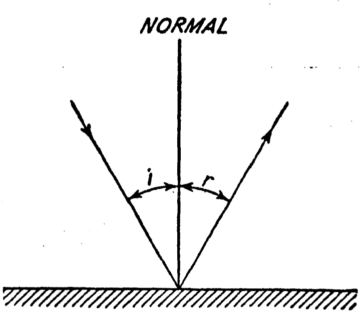


Fig. 7.—Regular reflection. The angle of incidence i is equal to the angle of reflection r.

Figure 9 shows the rays from a small source S and their reflection from a plane mirror. Notice that the ray concept offers a very simple way of describing what happens, while dealing with the waves themselves would be much more cumbersome. There is a point S' behind the mirror from which all the reflected rays *appear* to come. This point is called the *image* of the source. It is as far behind the plane mirror as the source is in front and is located on the normal to the mirror surface through the source S.

The image of an extended source or object in a plane mirror is found by taking one point after another and locating its image. The familiar result is that the complete image is the same size as the object and is placed symmetrically with respect to the mirror (Fig. 10).

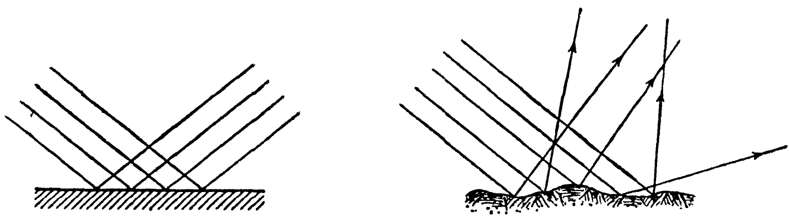


FIG. 8.—Regular and diffuse reflection.

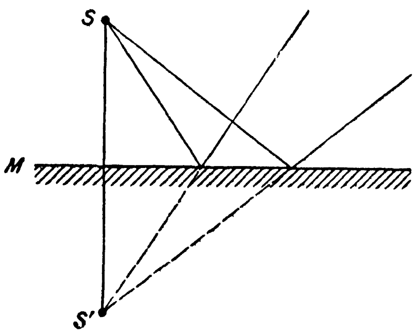


FIG. 9.—Reflection from a plane mirror.

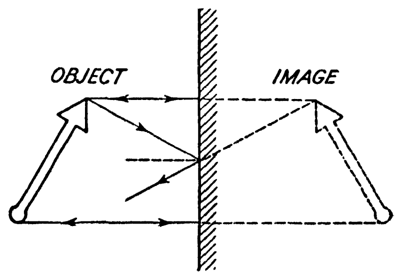


FIG. 10.—Image formed by a plane mirror.

Optical Lever. In many physical and technological instruments, small displacements must be indicated or recorded. One way of magnifying such effects to make them readily measurable is by the use of a ray of light reflected by a small mirror mounted on the moving system, the ray forming a sort of "inertialess" pointer. This arrangement is called an *optical lever*, and is used in such pieces of apparatus as indicating and recording galvanometers, pyrometers, elastometers, and sextants. In Fig. 11, SO represents a ray or narrow beam of light striking the mirror M mounted on a body, for example the coil of a galvanometer, which is to rotate about the axis P. When the mirror is turned through any angle θ, the reflected beam turns through an angle just twice as great. As the mirror turns through the angle θ, the normal also turns through the same angle, decreasing the angle of incidence by θ. The angle between the incident and reflected rays is always twice the angle of incidence. Thus the angle that the reflected ray makes with the incident ray is reduced by 2θ. The position of the reflected beam may be observed on a screen

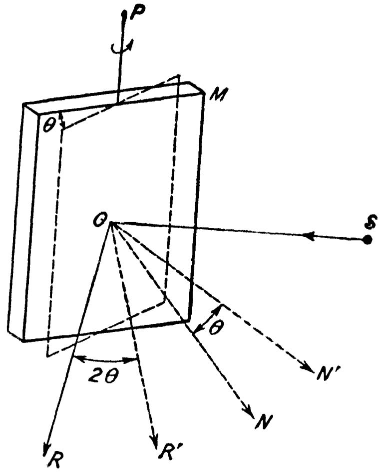


FIG. 11.—The optical lever.

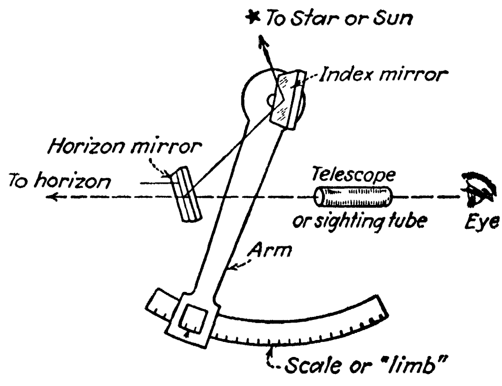


FIG. 12.—The operating principle of the sextant. When the arm is set at zero on the scale, the two mirrors are parallel. Two superposed images of the horizon are seen, one formed by light entering the telescope through the clear part of the horizon mirror, the other by light reflected by the index mirror and the silvered portion of the horizon mirror. In viewing the sun at an angle θ degrees above the horizon, the arm carrying the index mirror is moved through an angle $\theta/2$. The image of the sun then matches that of the horizon, and the angle of elevation, needed to determine latitude, is twice that through which the arm is turned. For aviation use, an artificial (bubble) horizon is employed.

some distance away and from the change in its position the angle of turn may be computed.

Curved Mirrors. If the reflecting surface is curved rather than plane, the same law of reflection holds but the size and position of the image formed are quite different from those of an image formed by a plane mirror.

Curved mirrors are frequently made as portions of spherical surfaces and may be *concave* like a shaving mirror or *convex* like a polished ball. Concave mirrors have wide application because of their ability to make rays of light converge to a *focus*. If rays coming from a point S (Fig. 13) strike the concave spherical mirror, the reflected rays may be constructed by applying the law of reflection at each point, the direction of the normal being that of the radius in each case. All reflected rays will

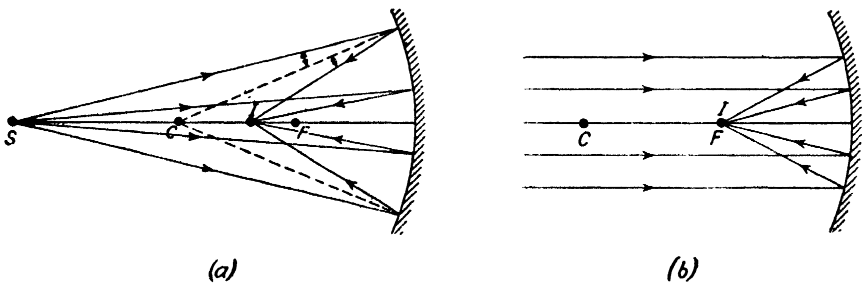


FIG. 13.—Focusing of light by a concave mirror.

be found to pass very nearly through a single point I. If the incoming rays are parallel, that is, if they come from a distant source, the point will be halfway between the mirror and the center of the sphere of which the mirror is a part. This point is then called the *principal focus F* of the concave mirror.

If the spherical mirror is large, the rays are not brought to a focus at a single point. More accurate focusing is obtained if the mirror, in place of being spherical, is part of a surface obtained by rotating a curve called

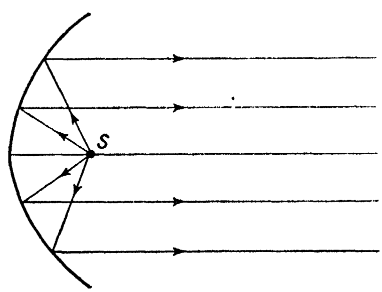


FIG. 14.—Parabolic mirror as used in a searchlight.

a parabola. This type of mirror, called *parabolic,* is widely used where light must be focused by a mirror. The most common use is in the mirror of the automobile headlight. When the filament is placed at the focus of the mirror, the rays sent out form a parallel beam. A very slight shift in the position of the filament causes a marked displacement of the beam. The searchlight mirror and the big reflectors of astronomical telescopes are other applications of the parabolic mirror.

SUMMARY

Light is a disturbance that is capable of affecting the eye.

Light is transmitted by *waves*, which can pass not only through transparent materials such as glass but also through empty space (vacuum).

In a vacuum, the speed of light is about 186,000 mi/sec; in a physical material, the speed is always less than this.

Lines drawn in the direction of travel of light waves are called *rays*. In a uniform material the rays are straight lines.

The *luminous intensity* of a source is measured in *candle power.*

The illumination produced by a point source at a given surface that it illuminates is given by $E = I/s^2$, where I is the luminous intensity of the source and s is the distance from the source to the surface. The illumination E is in *foot-candles* or *meter-candles*, depending upon whether s is given in feet or in meters.

A *photometer* is an instrument for comparing the luminous intensities of two sources. The working equation of the photometer is

$$\frac{I_1}{I_2} = \frac{s_1^2}{s_2^2}$$

A *foot-candle meter* is an instrument that measures illumination directly.

When light is reflected, the reflected ray makes the same angle with the perpendicular to the surface as does the incident ray. This is the *law of reflection.*

The *image* of an object formed by a plane mirror is the same size as the object and is located as far behind the mirror as the object is in front of it.

Parallel rays are focused by a concave spherical mirror to a point known as the *principal focus* of the mirror.

QUESTIONS AND PROBLEMS

1. Radio waves, which are of the same physical nature as light waves and travel with the same speed in empty space, can be made to go completely around the earth. How long does it take for a signal to go around the equator, taking the diameter of the earth to be 8,000 mi?

2. What is the effect on the illumination of a work table if a lamp hanging 4.5 ft directly above it is lowered 1 ft? *Ans.* Increased 65 per cent.

3. An engraver wishes to double the intensity of the light he is now getting from a lamp 55 in. away. Where should the lamp be placed in order to do this? *Ans.* 39 in. away.

4. If a lamp that provides an illumination of 8.0 ft-candles on a book is moved 1.5 times as far away, will the illumination then be sufficient for comfortable reading? *Ans.* No; 3.6 ft-candles.

5. When a diffusing globe is placed over a bare electric lamp of high intensity, the total amount of light in the room is decreased slightly, yet eyestrain may be considerably lessened. Explain.

6. What is the illumination on the pavement at a point directly under a street lamp of 800 cp hanging at a height of 20 ft? *Ans.* 2.0 ft-candles.

7. Find the candle power of a lamp that gives an illumination equal to that of dull daylight on a surface placed 3 ft away. *Ans.* 900 cp.

8. A photometer has a standard 30-cp lamp at one end and a lamp of unknown strength at the other. The two sides of the screen are equally illuminated when the screen is 3 ft from the standard lamp and 5 ft from the unknown. What is the candle power of the latter? *Ans.* 83 cp.

9. At what position on a photometer scale, which is 4 ft long, should a screen be placed for equal illumination by a 20-cp lamp and a 45-cp lamp placed at the two ends of the scale? *Ans.* 1.6 ft from the weaker lamp.

10. What is the total illumination produced by two 60-cp lamps each 4 ft from a surface and one 45-cp lamp 3 ft from this surface if all the light falls on the surface normally? *Ans.* 12.5 ft-candles.

11. Using Fig. 9, prove geometrically that the image point S' is the same distance from the mirror as the object point S.

12. A carpenter who wishes to saw through a straight board at an angle of 45° places his saw at the correct angle by noting when the reflection of the edge of the board seems to be exactly perpendicular to the edge itself. Explain.

13. A narrow beam of light reflected from the mirror of an electrical instrument falls on a scale located 2 m away and placed perpendicular to the reflected rays. If the spot of light moves laterally a distance of 40 cm when a current is sent through the instrument, through what angle does the mirror turn? *Ans.* 5.7°.

14. What illumination will be given on a desk by a 40-cp 1-ft fluorescent lamp placed 18 in. above the surface? (For an extended line source, the illumination decreases as the inverse first power of the distance, $E = I/s$.) *Ans.* 24 ft-candles.

EXPERIMENT

Illumination and Photometry

Apparatus: Bench-type photometer, preferably with Lummer-Brodhun head; standardized lamp; several lamps for unknowns; two 150-volt range voltmeters; two control rheostats. (*Optional*) Foot-candle meter, preferably of photoelectric type.

a. Arrange the photometer and accessories as in Fig. 15. Adjust the rheostat in the standard lamp circuit until the voltage across the lamp is that for which its candle power is known, and keep the voltage across the unknown lamp at some constant value in the range for which it is to be used, generally 110 volts. Move the photometer head back and forth until a match of illumination is attained. Approach the matching point from alternate sides and take as the final setting the average of the positions found by two or more observers.

Repeat for other unknown lamps. Compute the candle power of each from Eq. (3).

b. If a foot-candle meter is available, set up one of the unknown lamps near one end of the room, but not too near any surfaces that reflect appreciable light. With the lamp operating at the same voltage as before, measure the illumination produced at various distances within the range of the foot-candle meter, making certain that the light always

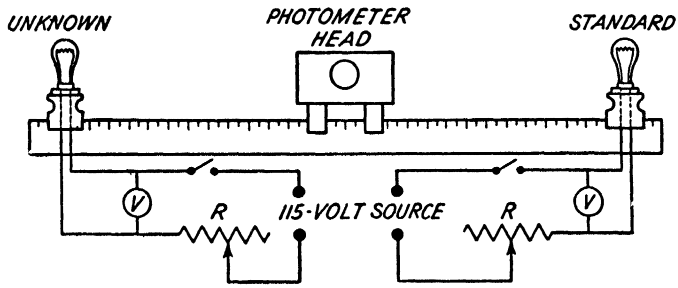


FIG. 15.—Bench photometer, showing electrical connections.

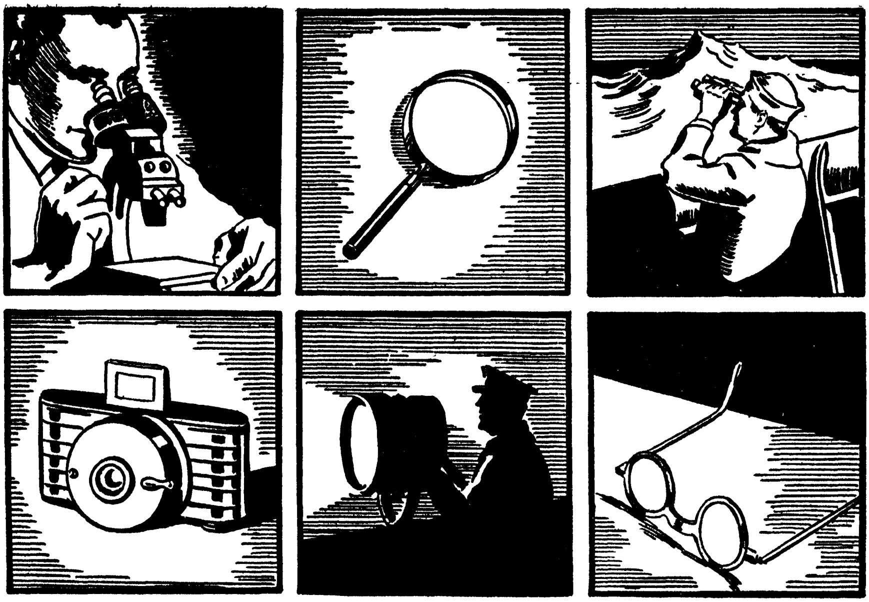
falls perpendicularly on the sensitive surface of the meter. Measure the distance from the lamp to the meter in each case and record the data for each unknown lamp in the first two columns of Table II.

TABLE II. LAMP NO. 1

Distance s, ft	Illumination E, ft-candles	$Es^2 = I$, cp	Candle power from part (*a*)
		Average	

The product Es^2 should, according to Eq. (2), equal the candle power of the lamp, and so the various products for a given lamp should be constant. Compare the average of the values thus obtained with the candle power as determined in part (*a*) of this experiment.

Place the foot-candle meter with the surface perpendicular to the light rays and note the reading. Turn the face of the instrument so that it is no longer perpendicular to the rays. What is the effect on the reading? Explain.



CHAPTER 32

REFRACTION OF LIGHT; LENSES; OPTICAL INSTRUMENTS

The wide variety of optical devices now available, from a simple magnifying lens to a battleship range finder, all owe their design to our knowledge of the bending of light as it passes from one medium to another. The science of optics is an old one, although early progress was made only by trial-and-error methods. Today we have learned how to develop new instruments and to refine old ones by methods based on exact laws and well-known principles.

Refraction. A ray of light passing obliquely from one material into another always experiences an abrupt change of direction at the separating surface. This bending of light rays is called *refraction.* Light advances in a straight line only when it is passing continuously through a uniform substance; the rays are straight lines when, for example, a beam of light is moving through air; but there is a sudden change in direction in each ray when the light enters, say, glass, and another when it leaves (Fig. 1).

This change in direction in the new substance can be explained very simply on the basis of the wave theory of light—in fact, refraction is one of the phenomena that first suggested the wave theory. The explanation is based on the fact that the speed of light in any transparent material is found to be less than its speed in a vacuum. Consider a bundle of parallel rays of light incident obliquely on the plane surface of a piece of glass (Fig. 2). The line MN represents one of the wave surfaces that is about to enter the glass; PQ is a wave surface that has just entered completely. It is found by experiment that the speed of light in glass is only about two-thirds that in air; so, while one side of the wave surface has gone a distance NQ in air, the part traveling entirely in glass has gone a distance MP, which is only two-thirds as great. Since it is found that the wave surfaces remain straight after entry, this must mean that the entire beam swings around somewhat toward the direction

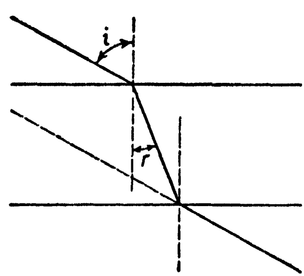


FIG. 1.—Refraction at a plane surface.

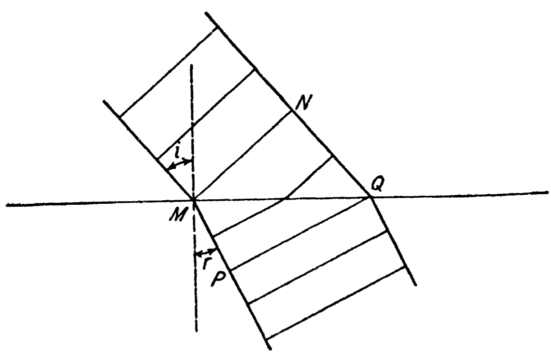


FIG. 2.—Change in the direction of a beam of light on refraction.

of the normal to the surface of the glass. On emerging from the other side of a parallel-surfaced piece of glass, the beam is bent through an equal angle *away* from the normal and so pursues its original direction, although it is now some distance to one side of its initial path in air (Fig. 1).

If the two surfaces of the glass are not parallel to each other, the emergent ray is bent away from the normal as before but, since the direction of the normal has been changed, the emergent ray does not have the same direction as the original ray. The ray has effectively been bent around the thicker part of the glass as shown in Fig. 3.

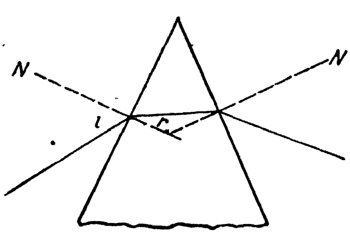


FIG. 3.—Refraction by a prism.

The angle between the incident ray and the normal to the surface is called the *angle of incidence* i and the angle between the refracted ray

and the normal is called the *angle of refraction r*. It has been found experimentally that, for a given pair of substances, the ratio $\sin i/\sin r$ is a constant, independent of the angle at which the original beam is incident. This constant is called the *index of refraction n*. It can be shown that the ratio of the sines of the angles is equal to the ratio of the speed of light in the two mediums.
Thus

$$\frac{\sin i}{\sin r} = n = \frac{V_1}{V_2} \tag{1}$$

where V_1 is the speed of light in the first medium and V_2 is that in the second. This relationship is called the *law of refraction*.

The index of refraction for a given material is usually expressed relative to air or to vacuum. In the latter case it is called the *absolute* index of refraction. Since the speed of light in air is only about 3 parts in 10,000 less than in a vacuum, the two values are very nearly the same.

For materials that are to be used in optical work the index of refraction is a very important property. It must be considered in the design of all lenses or prisms that enter into the various optical instruments. Although we refer to the index of refraction as a constant, its value for a given material depends upon the color of light used. Usually the value given in the tables is for yellow light.

Total Reflection. Imagine a small source of light located under water (Fig. 4) and sending out rays in all directions. Since the speed of light is greater in air than in water, a ray such as SA, coming toward the surface, will be refracted away from the normal on emerging into the air. Another ray SB approaching the surface at a greater angle of incidence will be closer to the surface after emerging. Finally, there will be some ray SD for which the emergent ray will be exactly along the surface, that is, for this particular angle of incidence C the angle of refraction will be 90°. Any ray whose angle of incidence is greater than C will not emerge at all, since the sine of the corresponding angle of refraction would have to be greater than 1 in order to satisfy Eq. (1), and this is impossible. Such a ray does not emerge but is entirely reflected back into the water in accordance with the law of reflection. This is called *total internal reflection*.

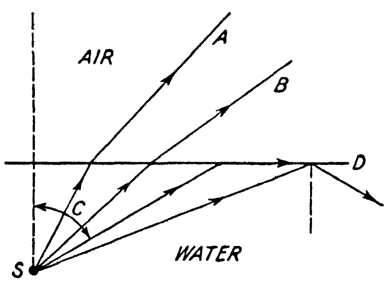


FIG. 4.—Total internal reflection.

For any substance the angle C for which the angle of refraction is 90° is called the *critical angle*. For this angle

$$n = \frac{\sin 90°}{\sin C} = \frac{1}{\sin C}$$

or

$$\sin C = \frac{1}{n}$$

For glass, whose index of refraction is 1.5,

$$\sin C = \frac{1}{1.5} = 0.67$$

or C is about 42°.

Numerous applications of total reflection are made in optical instruments such as periscopes, prism binoculars, etc.

Lenses. A lens is a transparent object with polished surfaces at least one of which is curved. Most lenses used in optics possess two

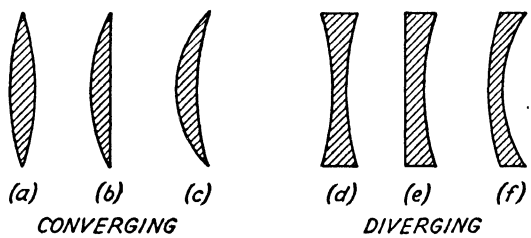


Fig. 5.—Lenses of various forms.

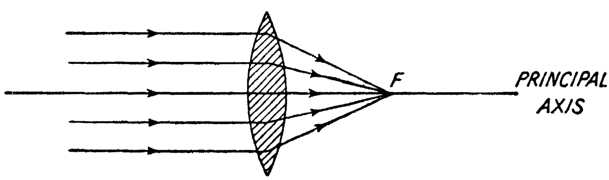


Fig. 6.—Focusing of light by a converging lens.

surfaces which are parts of spheres. The line joining the centers of the two spheres is called the *principal axis* of the lens. Typical lens forms are shown in Fig. 5.

Consider a glass lens such as a of Fig. 5 on which is incident a set of rays from a very distant source on the axis of the lens. These rays will be parallel to the axis. Each ray is bent about the thicker part of the glass. As they leave the lens, they converge toward a point F (Fig. 6). Any lens that is thicker at the middle than at the edge will cause a set of parallel rays to converge and hence is called a *converging* lens. The point F to which the rays parallel to the axis are brought to a focus is called the *principal focus*. The distance from the center of the lens to this point is called the *focal length* of the lens. A lens has two principal foci, one on each side of the lens and equally distant from it.

If a lens such as *d* of Fig. 5 is used in the same manner, the rays will again be bent around the thicker part and in this case will diverge as they leave the lens (Fig. 7). Any lens that is thicker at the edge than at the middle will cause a set of rays parallel to the axis to diverge as they leave the lens and is called a *diverging* lens. The point *F* from which

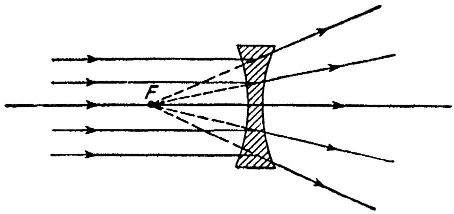


FIG. 7.—The principal focus (virtual) of a diverging lens.

the rays diverge on leaving the lens is the principal focus. Since the light is not actually focused at this point, this focus is known as a *virtual* focus.

If the source is not very distant from the lens, the rays incident upon the lens are not parallel but diverge as shown in Fig. 8. The behavior

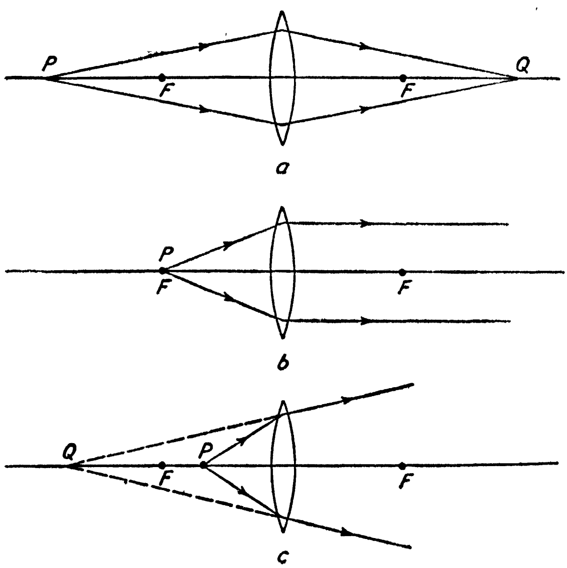


FIG. 8.—Effect of a converging lens on light originating (*a*) beyond the principal focus, (*b*) at the principal focus, and (*c*) within the principal focus.

of the rays leaving a converging lens depends upon the position of the source. If the source is farther from the lens than the principal focus, the rays converge as they leave the lens as shown in Fig. 8*a*; if the source is exactly at the principal focus, the emerging rays will be parallel to the principal axis as shown in *b*. If the source is between the lens and the principal focus, the divergence of the rays is so great that the lens is unable to cause them to converge but merely reduces the divergence. To an observer beyond the lens, the rays appear to come from a point Q rather than from P, as shown in Fig. 8c. The point Q is a virtual focus.

A divergent lens causes the rays emerging from the lens to diverge more than those which enter. No matter what the position of the source, the emergent rays diverge from a virtual focus as shown in Fig. 9.

Image Formation by Lenses. When the rays converge after passing through the lens, an image can be formed on a screen and viewed in that way. Such an image is called a *real* image. If the rays diverge on

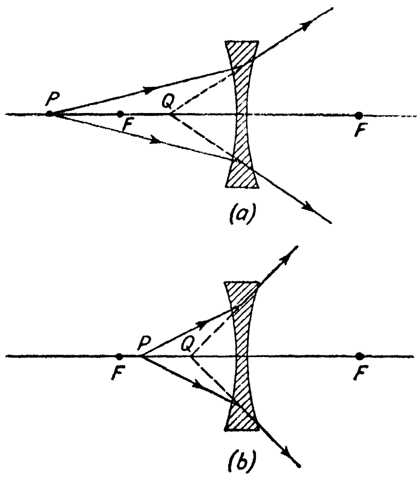


FIG. 9.—Effect of a diverging lens on light originating (a) beyond the principal focus and (b) within the principal focus.

leaving the lens, the image cannot be formed on a screen but can be observed by looking through the lens with the eye. This type of image is called a *virtual* image. Thus Figs. 6 and 8a represent the formation of real images while Figs. 7, 8c, and 9 represent virtual images. Notice that a diverging lens produces only virtual images while a converging lens may produce either real or virtual images, depending upon the location of the object.

Image Determination by Means of Rays. If an object of finite size that either emits or reflects light is placed before a lens, it will be possible under certain conditions to obtain an image of this object. By drawing at least two rays whose complete path we know, the image point corresponding to a given object point may be located graphically. The one fact that must be known is the location of the principal focus of the lens. Suppose we have as in Fig. 10a a converging lens with an object, represented by the arrow, placed some distance in front of it. Let F represent the principal foci on the two sides of the lens. A point on the object, such as the tip of the arrow, may be considered to be the source of any number of rays. Consider the ray from this point which proceeds toward the center of the lens. This ray will continue onward with no change of direction after passing through.

Now consider another ray from the tip of the arrow—one that travels parallel to the axis. What is its path after traversing the lens? We saw from Fig. 6 that all rays parallel to the principal axis which strike a converging lens pass through the principal focus after emerging. Thus the ray we have drawn from the tip of the arrow will, after refraction by the lens, pass through F. If this line is continued, it will cut the ray

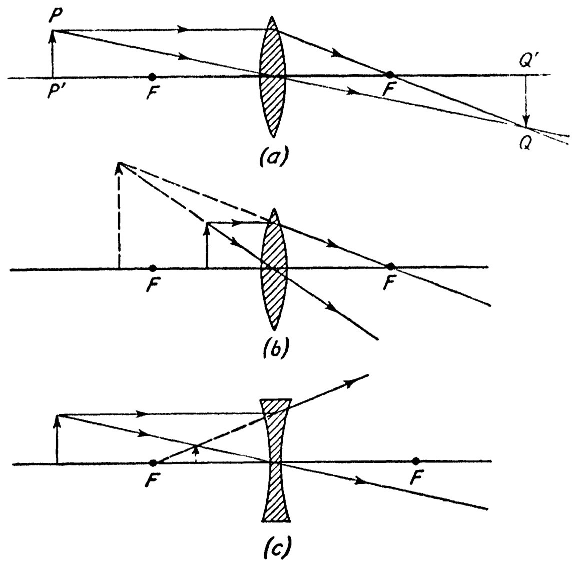


FIG. 10.—Image formation traced by means of ray diagrams.

through the center of the lens at a point Q. This is the image point corresponding to the tip of the arrow. The other image points, corresponding to additional points of the arrow, will fall in the plane through Q perpendicular to the lens axis. In particular, the image of the foot of the arrow will be on the axis if the foot of the arrow itself is so placed. An inverted real image of the arrow will actually be seen if a card is held in the plane QQ'. Inversion takes place also in the sidewise direction so that if the object has any extent in a direction normal to the plane of the figure, right and left will be reversed.

Figure 10*b* shows how to locate the image when an object is placed closer to a converging lens than the focal distance. We have already seen from Fig. 8*c* that this results in a virtual image. The reason, from the point of view of the ray construction, is that the ray through the lens center and the ray passing through F do not intersect on the right of the lens, but diverge instead. However, they *appear* to have come from some point located by projecting them back to the left until they cross. This point is the *virtual image* of the tip of the arrow. The entire virtual image is represented by the dotted arrow. It cannot be formed on a screen but may be viewed by looking into the lens from the right.

In a similar way, the formation of a virtual image by a diverging lens is shown in Fig. 10*c*.

In every example of image formation described, we may see from the graphical construction that

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

The first ratio is called the *lateral magnification*, or simply the magnification. Hence, in symbols,

$$M = \frac{q}{p} \tag{2}$$

where p is the distance of the object from the lens and q is that of the image.

The Thin-lens Equation. It is possible to find the location and size of an image by algebraic means as well as by the graphical method already outlined. Analysis shows that the focal length f of a thin lens, the distance p of the object from the lens, and the distance q of the image are related by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{3}$$

This relation holds for any case of image formation by either a converging or diverging lens provided that the following conventions are observed:

a. Consider f positive for a converging lens and negative for a diverging lens.

b. The normal arrangement is taken to be object, lens, and image, going from left to right in the diagram. If q is negative, this means that the image lies *to the left* of the lens, rather than to the right, and is therefore virtual.

Example: The lens system of a certain portrait camera may be considered equivalent to a thin converging lens of focal length 10 in. How far behind the lens should the plate be located to receive the image of a person seated 50 in. from the lens? How large will the image be in comparison with the object?

Substitution in Eq. (3) gives

$$\frac{1}{50 \text{ in.}} + \frac{1}{q} = \frac{1}{10 \text{ in.}}, \qquad \text{or} \qquad q = 12.5 \text{ in.}$$

From Eq. (2), $M = 12.5 \text{ in.}/50 \text{ in.} = \frac{1}{4}$. The image will be one-fourth as large as the object.

Example: Determine the location and character of the image formed when an object is placed 9 in. from the lens of the previous example.

Substitution in Eq. (3) gives

$$\frac{1}{9\text{ in.}} + \frac{1}{q} = \frac{1}{10\text{ in.}}$$

whence $q = -90$ in.

The negative sign shows that the image lies to the left of the lens and is therefore virtual. It is larger than the object in the ratio

$$M = \frac{90\text{ in.}}{9\text{ in.}} = 10$$

Example: When an object is placed 20 in. from a certain lens, its virtual image is formed 10 in. from the lens. What are the focal length and character of the lens?

Using Eq. (3), we have

$$\frac{1}{20\text{ in.}} + \frac{1}{-10\text{ in.}} = \frac{1}{f}$$

$$f = -20\text{ in.}$$

The negative sign shows that the lens is diverging.

Optical Instruments. We shall describe the principles of a few important optical instruments that consist essentially of lens combinations.

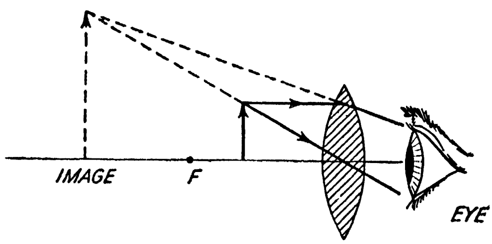


FIG. 11.—A simple magnifier.

The action of a combination of lenses may be found graphically by tracing the rays through the entire system.

Since the eye is the final element in many optical instruments, we consider first the use of a single converging lens in increasing the ability of the eye to examine the details of an object. A lens used in this way is referred to as a *simple magnifier*, or *simple microscope.* The object to be examined is brought just within the focal distance of the lens, and the eye is placed as close beyond the lens as convenient. An enlarged, erect, virtual image of the object is then seen (Fig. 11). Because of the fact that a normal eye is able to see the details of an object most distinctly when its distance is about 10 in., the magnifier should be adjusted so that the image falls at this distance from the eye. The magnification will then be, approximately,

$$M = \frac{10}{f} \tag{4}$$

where f is the focal length of the lens in inches. The magnifier, in effect, enables one to bring the object close to the eye and yet observe it comfortably.

Whenever high magnification is desired, the *compound microscope* is used. It consists of two converging lenses (in practice, lens systems): a so-called *objective* lens of very short focal length and an *eyepiece* of moderate focal length. The objective forms a somewhat enlarged, real image of the object within the tube of the instrument. This image is then examined with the eyepiece, using the latter as a simple magnifier. Thus the final image seen by the eye is virtual and very much enlarged.

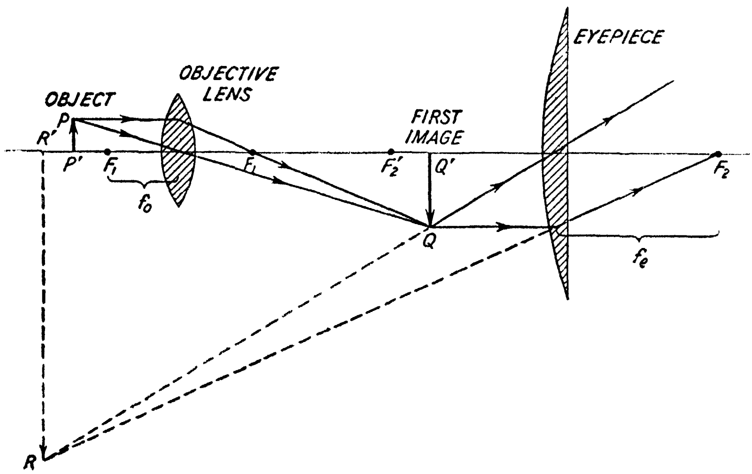


FIG. 12.—Ray diagram for the compound microscope.

Figure 12 shows the ray construction for determining the position and size of the image. The object is placed just beyond the principal focus of the objective lens, and a real image is formed at QQ'. This image is, of course, not caught on a screen but is merely formed in space. It consists, as does any real image, of the points of intersection of rays coming from the object. Next, this image is examined by means of the eyepiece, using the eyepiece as one would a simple magnifier. The position of the eyepiece, then, should be such that the real image QQ' lies just within the principal focus F_2'. Hence the final image RR' is virtual and enlarged and is inverted with respect to the object.

It is possible to prove that with the instrument adjusted to place the final image at a distance of 10 in. the magnifying power is approximately

$$M = \frac{q}{p} \times \frac{10}{f} \tag{5}$$

where p and q are the distances of object and first image, respectively, from the objective, and f is the focal length of the eyepiece—all distances being measured in inches. In practice, the largest magnification employed is usually about 1,500.

The *refracting telescope*, like the compound microscope, consists of an objective lens system and an eyepiece. The instruments differ, however, in that the objective of the telescope has a very large focal length. Light from the distant object enters the objective, and a real image is formed within the tube (Fig. 13). The eyepiece, used again as a simple magnifier, leaves the final image inverted.

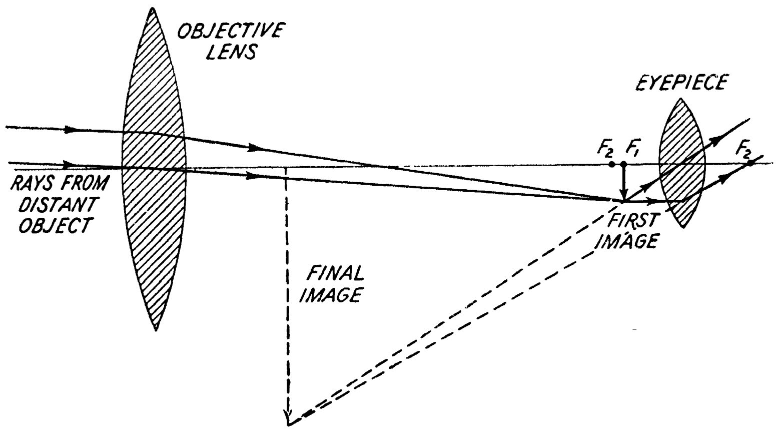


FIG. 13.—Ray diagram for the refracting telescope.

The magnifying power of the instrument may be shown to be

$$M = \frac{f_o}{f_e} \tag{6}$$

where f_o and f_e are the focal lengths of objective and eyepiece, respectively. This formula shows that apparently unlimited values of M may be obtained by making f_o very large and f_e very small. Other factors, however, limit the values employed in practice, so that magnifications greater than about 2,000 are rarely used in astronomy.

Besides its function in magnifying a distant object, thus rendering details more apparent, there is another important feature of the telescope, which is often of greatest importance in astronomy. This is the *light-gathering* power of the instrument, which is one reason for making telescopes with objectives of large diameter, such as the 200-in. telescope now under construction. The amount of light energy collected by an objective is proportional to its area. Since the area of a circle is proportional to the square of its diameter, an objective 200 in. in diameter will gather $(200/0.2)^2 = 1{,}000{,}000$ times as much light energy as the pupil of the eye (0.2 in. in diameter). Thus, stars that are far too faint to be seen with the unaided eye will be visible through a large telescope.

A Galilean telescope (Fig. 14) consists of a converging objective lens L_1, which alone would form a real inverted image QQ' of a distant object practically at its principal focus, and a diverging eyepiece lens L_2. In passing through this concave lens, rays that are converging as they enter are made to diverge as they leave. To an observer the rays appear to

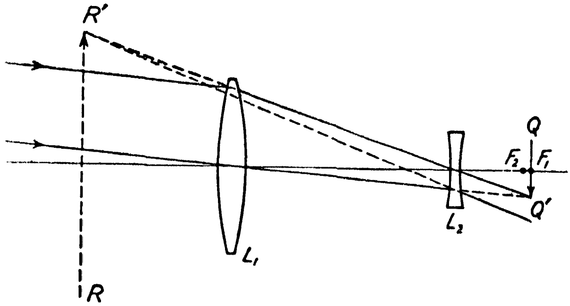


FIG. 14.—A diagram of a Galilean telescope.

come from RR', the enlarged virtual image. With this design of telescope an erect image is secured. The magnification is

$$M = \frac{f_o}{f_e}$$

Two Galilean telescopes are mounted together for opera glasses or for field glasses used in military operations.

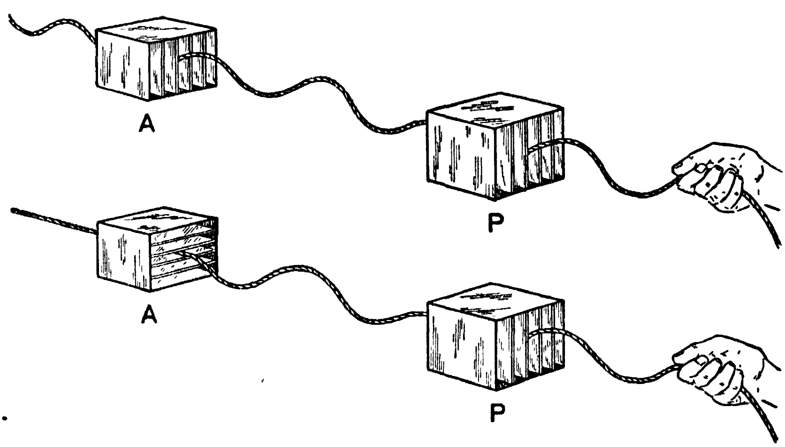


FIG. 15.—Mechanical analogue of polarization.

Polarization. A wave motion can consist of vibrations in the line of propagation (longitudinal vibrations) or vibrations at right angles to that direction (transverse vibrations). Light exhibits the characteristics of a transverse wave motion. Experiments with transverse waves in a rope (Fig. 15) show that a slot P can be used to confine the vibrations to one plane, after which they can be transmitted or obstructed by a second slot A, depending on whether it is placed parallel or perpendicular to the first slot. This suggests that a beam of light might be *plane-polarized;* that is, its vibrations might be restricted to a certain plane.

When it is traveling through a material such as air, glass, or water, the speed of light is the same in all directions, and a light beam may be

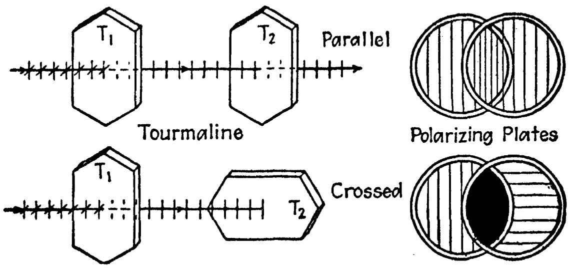


FIG. 16.—Diagram of tourmaline crystals and polarizing plates illustrating polarization by selective absorption.

considered as having vibrations in all directions in a plane perpendicular to its direction of travel. In many other materials such as tourmaline, calcite, quartz, and mica, the speed of transmission or the amount of absorption of a light beam is different for vibrations in different planes.

A tourmaline crystal produces plane-polarized light by selective absorption, transmitting only light whose vibrations are in a particular

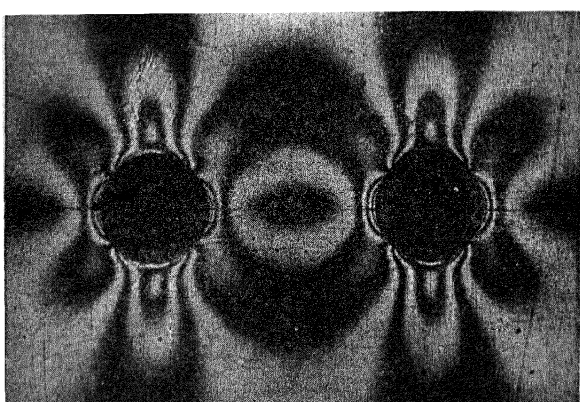


FIG. 17.—The strain pattern about two rivet holes in a member subjected to a vertical tensile stress.

plane (Fig. 16). The polarized light may be examined by a second tourmaline, which will transmit the light when oriented parallel to the first crystal or extinguish it when rotated through 90 degrees. Used thus in pairs, the crystals are referred to as polarizer and analyzer, respectively. Polarizing plates of large area (*polaroids,* Fig. 16) are made by embedding microscopic crystals of herapathite, with their axes properly aligned, in a nitrocellulose sheet. Light is partly plane-polarized when it is reflected from glass or water or when it is scattered by small particles in the air. Polaroid sun glasses or photographic filters are useful in reducing glare by eliminating the plane-polarized component of reflected light.

Glass when under stress transmits light vibrations in certain planes preferentially. Hence laboratory or other glassware can readily be tested for strains by being placed between a polarizer and analyzer. The same principle permits analysis of strains in complicated structures encountered in engineering. A model is built of transparent bakelite, loaded, and examined by suitably polarized light. The regions of greatest strain can be detected as those where there is closest spacing of fringes (Fig. 17).

SUMMARY

Refraction is the abrupt change of direction of a light ray upon passing from one transparent material to another.

The *law of refraction* states that $\sin i/\sin r = n$, a constant called the *index of refraction.* If the light enters from a vacuum, n is called the *absolute* index of refraction of the material. The index of refraction is also equal to the ratio of the velocities of light in the two mediums.

Light incident obliquely on the bounding surface of a transparent material from within will be able to emerge only if the angle is less than the *critical angle* whose value is given by $\sin C = 1/n$.

When the rays of light pass through a lens, they are bent around the thicker part of the lens. Rays parallel to the principal axis of the lens pass through a point called the *principal focus.* The distance of this principal focus from the lens is called the *focal length.*

Under certain conditions, a *converging lens* is able to form a *real image* of an object. Real images may be cast on a screen.

A *diverging lens* always forms virtual images, which cannot be thrown on a screen but may be viewed by the eye.

For any thin lens, $1/p + 1/q = 1/f$. Conventionally, f is to be taken positive for a converging lens and negative for a diverging lens; q is negative for a virtual image.

The *magnifying power* of a lens used as a simple magnifier is $M = 10/f$, where f is the focal length in inches and the lens is adjusted so that the image falls at the distance of most distinct vision, 10 in.

The *compound microscope* consists of a short-focus *objective* and a longer focus *eyepiece.* The magnification is given by

$$M = \frac{q}{p} \times \frac{10}{f}$$

The *astronomical telescope* consists of a long-focus objective and an eyepiece. The magnification is given by $M = f_o/f_e$.

Light is *plane-polarized* when its transverse vibrations are restricted to a certain plane.

QUESTIONS AND PROBLEMS

1. If a pencil standing slantwise in a glass of water is viewed obliquely from above, the part under water appears to be bent upward. Explain.

2. The angle of incidence of a ray of light on the surface of water is 40° and the observed angle of refraction is 29°. Compute the index of refraction. *Ans.* 1.32.

3. A ray of light goes from air into glass ($n = 3/2$), making an angle of 60° with the normal before entering the glass. What is the angle of refraction in the glass? *Ans.* 35.5°.

4. The index of refraction in a certain sample of glass is 1.61. What is the speed of light in this glass? *Ans.* $11\bar{5}$,000 mi/sec.

5. At what angle should a ray of light approach the surface of a diamond ($n = 2.42$) from within, in order that the emerging ray shall just graze the surface? *Ans.* 24.4°.

6. Check the results of the examples on pages, 329–330 by drawing ray diagrams to scale on squared paper. Verify the magnification as well as the position of the image in each case.

7. A converging lens has a focal length of 10 in. Where is the image when the object is (*a*) 20 in. from the lens? (*b*) 5 in. from the lens? How large is the image in each case if the object is 0.50 in. high?

8. A diverging lens has a focal length of -10 in. Where is the image when the object is (*a*) 20 in. from the lens? (*b*) 5 in. from the lens? How large is the image in each case if the object is 0.5 in. high?
Ans. -6.7 in.; -3.3 in.; 0.17 in.; 0.33 in.

9. A screen is located 4.5 ft from a lamp. What should be the focal length of a lens that will produce an image that is eight times as large as the lamp itself? *Ans.* 0.44 ft.

10. A lantern slide 3 in. wide is to be projected onto a screen 30 ft away by means of a lens whose focal length is 8 in. How wide should the screen be to receive the whole picture? *Ans.* 11 ft.

11. A miniature camera whose lens has a focal length of 2 in. can take a picture 1 in. high. How far from a building 120 ft high should the camera be placed to receive the entire image? *Ans.* 240 ft.

12. A photographer wishes to take his own portrait, using a plane mirror and a camera of focal length 10 in. If he stands beside his camera at a distance of 3 ft from the mirror, how far should the lens be set from the plate? *Ans.* 11.6 in.

13. In a copying camera, the image should be of the same size as the object. Prove that this is the case when both object and image are at a distance 2 f from the lens.

14. An object 2 in. high is placed 4 in. from a reading lens of focal length 5 in. Locate the virtual image graphically and determine the magnification.
Ans. -20 in.; 5.

15. A "10X magnifier" is one that produces a magnification of ten times. According to Eq. (4), what is its focal length? How large an image of a flashlight lamp 0.25 in. in diameter will this lens be able to produce on a card held 5 in. away?

16. The focal lengths of the objective and eyepiece of a compound microscope are 0.318 and 1.00 in., respectively, and the instrument is focused on a slide placed 0.35 in. in front of the objective. What magnification is attained?
Ans. 100.

17. The large refracting telescope of the Yerkes Observatory has an objective of focal length 62 ft. If atmospheric conditions do not warrant the use of magnification higher than 1,500, what focal length should the eyepiece have?
Ans. 0.5 in.

EXPERIMENT

Lenses and Optical Instruments

Apparatus: Optical bench, with accessories such as lens holders, illuminated "object," screen, set of thin lenses, etc.; steel rule; squared paper.

The purpose of the following experiments is to observe the formation of images by lenses and lens combinations, to check the lens formula, and to set up and study some lens combinations, such as simple forms of the telescope and compound microscope.

a. Focal Length of a Converging Lens. Determine the focal length of a converging lens by catching on a screen the image of a distant object, such as a chimney or church spire, and measuring the distance from the lens to the screen. The lens selected should have a moderate focal length (6 or 8 in.). Record the focal length thus obtained.

With the object box and screen mounted near the two ends of the bench, move the lens in its holder back and forth between them until a sharp, enlarged image is obtained. Make several settings from opposite sides and take the average. If it is not found possible to obtain an image, the object and the screen should be moved farther apart or a lens of shorter focal length should be used. When the image has been obtained, measure the distances p and q and also the height of any definite part of the object and of the corresponding portion of the image, using the steel rule.

Substitute the values of p and q in the lens equation and solve for f. Compare this value with the one obtained directly by focusing a distant object. Compute the ratio of the measured values of image size and object size and compare it with the value of q/p. Check these findings by making a ray diagram, to scale.

Repeat the entire procedure for one or two other converging lenses.

b. Compound Microscope. Select two converging lenses, one of focal length less than 2 in., another of focal length around 6 or 8 in. Determine the focal lengths roughly by focusing a distant object. Mount on the optical bench, from left to right, a small drawing or picture (postage stamp), the shorter focus lens, and the longer focus lens. Clamp the former (the objective) in position at a distance from the object just greater than the focal length. Move the second lens (the eyepiece) back and forth along the bench until a sharp image is seen when looking through both lenses. Is the image enlarged? Is it erect or inverted? Move the object slightly to one side. Does the image move in the same direction? Notice that the image is distorted and annoyingly fringed with color, particularly near the outer parts. How are these defects minimized in an actual microscope?

c. Refracting Telescope. Here the objective should be a lens of focal length about 20 in. or more and the eyepiece 2 to 4 in. Determine the focal length of each lens approximately by using a distant object. Mount the lenses in holders on the optical bench so that their distance apart is the *sum* of the two focal lengths. Move the eyepiece back and forth until a sharp image of a distant object is obtained (a lamp on the opposite side of the room may be used for this purpose). Is the image larger than the object as seen with the unaided eye? Is the image erect or inverted? How should a refracting telescope be constructed in order to furnish erect images?

APPENDIX

I. FUNDAMENTALS OF TRIGONOMETRY

In the study of vector quantities the use of simple trigonometry is very desirable and hence an elementary knowledge has been assumed in the treatment in this book. The fundamental definitions and principles as applied to right triangles are included in the following discussion.

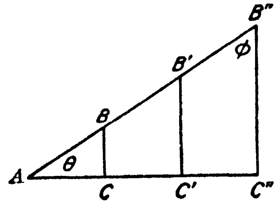


FIG. 1.—Similar right triangles.

In Fig. 1 are shown three right triangles, ABC, $AB'C'$, and $AB''C''$, each of which has the common angle θ. Each side of the triangle $AB'C'$ is longer than the corresponding side of ABC. Since the triangles are of the same shape, the corresponding sides are proportional. That is

$$\frac{AC}{AC'} = \frac{AB}{AB'}$$

or

$$\frac{AC}{AB} = \frac{AC'}{AB'}, \qquad \frac{CB}{AB} = \frac{C'B'}{AB'}, \qquad \text{and} \qquad \frac{CB}{AC} = \frac{C'B'}{AC'}$$

The values of these ratios may be used as measures of the angle θ. For convenience each of the three ratios is given a name and is called a *trigonometric function.* They are named as follows:

$$\frac{\text{Side opposite}}{\text{Hypotenuse}} = \text{sine } \theta \text{ (written sin } \theta)$$

$$\frac{\text{Side adjacent}}{\text{Hypotenuse}} = \text{cosine } \theta \text{ (written cos } \theta)$$

$$\frac{\text{Side opposite}}{\text{Side adjacent}} = \text{tangent } \theta \text{ (written tan } \theta)$$

In the triangles of Fig. 1

$$\sin\theta = \frac{BC}{AB} = \frac{B'C'}{AB'} = \frac{B''C''}{AB''}$$

$$\cos\theta = \frac{AC}{AB} = \frac{AC'}{AB'} = \frac{AC''}{AB''}$$

$$\tan\theta = \frac{BC}{AC} = \frac{B'C'}{AC'} = \frac{B''C''}{AC''}$$

These three functions are very useful in studying vectors for in that study it is necessary to find the lengths of sides of right triangles when one side and an angle is given. For example, if $\overline{AB}$ of Fig. 1 represents a force then

$$\overline{AC} = \overline{AB} \cos \theta$$

and

$$\overline{BC} = \overline{AB} \sin \theta$$

are rectangular components of that force.

The values of the trigonometric functions have been worked out for each angle and are given in Tables 6 and 7 of the Appendix. Values of sines are given in Table 6 for each tenth of a degree from zero to 90°. The first column lists angles in degrees, while the fractions of degrees appear as headings of the columns in the table. To find the sine of 32.4°, find 32° in the first column, and across this row in the column headed by .4 the value of the sine is found to be 0.5358. The same table is used to read the values of cosines, using the angles given in the last column and reading *up* from the bottom. Thus to find the cosine of 55.6°, locate 55° in the last column and look across in this row to the column *above* .6 where the value is observed to be 0.5650. The values of tangents are found in Table 7, which is used in the same manner as the table of sines.

In some tables the angles listed in the first column run only from 0° to 45°. Angles from 45° to 90° are then given in the last column increasing upward. This arrangement is possible because of the relationship between sine and cosine. The sine of any angle is equal to the cosine of 90° minus the angle. Thus

$$\sin \theta = \cos (90° - \theta)$$

and

$$\cos \theta = \sin (90° - \theta)$$

This relation becomes evident from Fig. 1, for the angle ϕ is equal to $90° - \theta$. From the definitions

$$\sin \theta = \frac{BC}{AB}$$

and

$$\cos \phi = \frac{BC}{AB}$$

Hence

$$\sin \theta = \cos \phi = \cos (90° - \theta)$$

Whenever the slide rule is used to obtain cosines, this relationship must be used.

II. GRAPHS

Physical laws and principles express relationships between physical quantities. These relationships may be expressed in words, as is commonly done in the formal statement, or by means of the symbols of an equation or by the pictorial representation called a *graph*. The choice of the means of expression is determined by the use to be made of the information. If calculations are to be made, the equation is usually the most useful. The graph, however, presents to the initiated person a vivid and meaningful picture of the way in which one quantity varies with another.

If a graph is to impart its full meaning, it must be constructed in accordance with standard rules so that it will have the same meaning for every person who inspects it. Some of the rules and suggestions which should prove helpful are given in the following discussion.

In constructing a graph the first step is to select and label carefully a set of axes. It is customary to allow the horizontal axis to represent the quantity to which arbitrary values are assigned while the other variable is plotted along the vertical axis. The name of the quantity and the units in which it is expressed should be clearly lettered along each axis.

The second step is the selection of a scale to be used for each axis. The scale should be so chosen that the range of values being plotted will be of reasonable size, practically filling the page. The scales on the two axes need not be the same; they seldom are. The scales need not always begin with zero; in fact, zero may not appear on the scale unless it is in the range of values being studied. Graphs are usually most easily constructed and read if each space represents a multiple of 2 or of 5. Multiples of 3 or of 7 make the interpretation of the graph difficult and should be avoided wherever possible. When a scale has been selected for each axis, mark values at appropriate equal intervals along each axis, increasing to the right on the horizontal axis and upward on the vertical axis. It is unnecessary to mark a number at each square, but enough marks should be used so that the graph can be read easily.

Each point plotted on the graph represents a pair of corresponding values of the two quantities. It is usually convenient to make out a table of values from which to plot the points. Each plotted point should be marked by a dot surrounded by a small circle. Enough points should be plotted so that a clear picture of the variation is given. A *smooth* curve is then drawn through the average position of the points. It is not necessary that the curve pass through every point, for experimental error causes a scattering of data about the true value. Therefore the points will scatter somewhat on either side of the true curve. Graphs of data representing physical laws should never be drawn as broken lines from point to point.

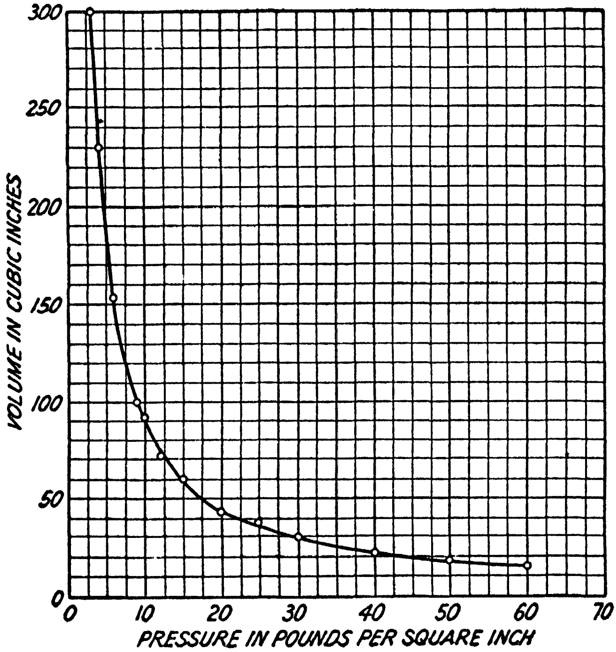


FIG. 2.

Figure 2 is a graph showing the variation of volume V of a confined gas as the pressure P changes. A set of experimental values of pressure and volume are shown in the table.

P, lb/in.^{2}	V, in.^{3}	$1/V$
3	300	0.0033
4	230	0.0045
6	154	0.0065
9	100	0.0100
10	92	0.0109
12	72	0.0139
15	60	0.017
20	44	0.023
25	37	0.027
30	30	0.033
40	22	0.045
50	18	0.056
60	15	0.067

Not only is the graph useful in showing vividly the relationship between the two quantities but it can also be employed to obtain quickly pairs of values other than those which were plotted. For example, from Fig. 2 we may wish to find the volume when the pressure is 35 lb/in.[2] It is necessary merely to observe where the curve cuts the vertical line representing 35 lb/in.[2] and read the value of the horizontal line which intersects the curve there. This is observed to be 26 in.[3] This use of

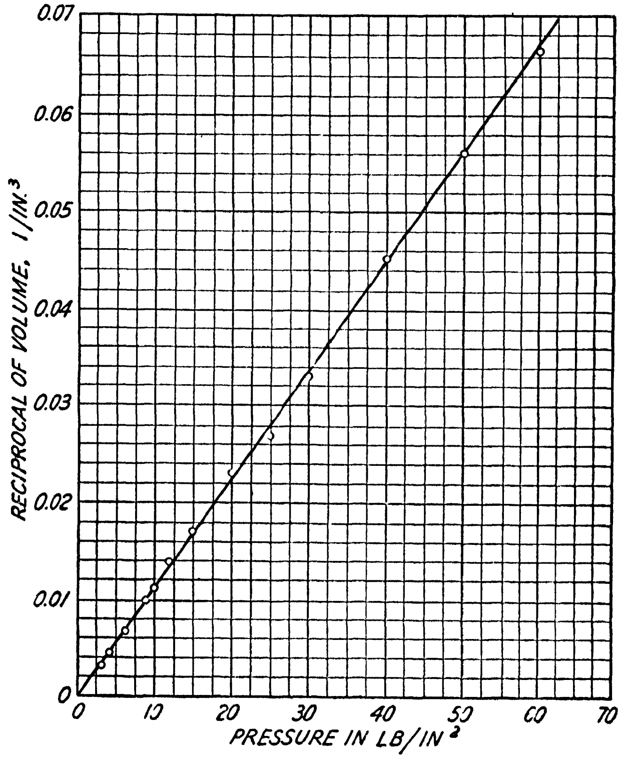


FIG. 3.

graphs to record data in readily available form is common in engineering and technical practice.

When the curve of a graph is a straight line, it is easily identified, more easily drawn, and in many respects more useful than other forms of curve. For these reasons the data are often arranged in such a form that the curve will be a straight line. If, in showing the relationship between P and V, the reciprocal of V is plotted against P, the resulting curve is a straight line as shown in Fig. 3. If it is known from theory that the curve is a straight line, only enough points need be plotted to locate the line. It is then drawn in, using a ruler. Although two points are sufficient to locate a straight line, several others should be plotted when experimental data are used, in order to minimize the effect of experimental error.

III. SYMBOLS USED IN EQUATIONS

Symbol	Use	Chapter
A	Area	5–7, 22
	Total absorption	30
a	Acceleration	10, 11, 17
AMA	Actual mechanical advantage	13
C	Temperature on the centigrade scale	3, 9
	Circumference	14
	Capacitance	27, 28
	Critical angle	32
d	Weight-density	7–9
	Diameter	22
Eff.	Efficiency	13
E	Potential difference, voltage	21–25
	Emf	22, 26, 27
	Illumination	31
e	Instantaneous emf	27
F	Temperature on the Fahrenheit scale	3, 9
	Force	6, 7, 11–15, 16–18
f	Force	11, 18
	Frequency	27
	Focal length	32
g	Acceleration due to gravity	11, 12, 17–20
H	Heat	4, 5, 12, 24
h	Depth	7–9
	Height	13, 17, 19
Hp	Horsepower	14
I	Moment of inertia	19
	Current	21–25, 27
	Source intensity	31
i	Angle of incidence	32
IMA	Ideal mechanical advantage	13
J	Mechanical equivalent of heat	24
K	Temperature on the absolute or Kelvin scale	3
	Thermal conductivity	5
	A constant	8
	Force constant	20
k	Current sensitivity	23
KE	Kinetic energy	12, 19
L	Length	3, 5, 6
	Heat of fusion	4
	Torque	16, 19
	Self-inductance	27, 28
l	Length	7, 13, 22
	Wave length	29–31
M	Mass	4, 8, 11, 19
	Momentum	17

Symbol	Use	Chapter
	Magnification	32
m	Mass	11, 12, 17–19, 25
N	Normal force	12
	Number of revolutions	14, 19
	Number of turns	26, 27
n	Number of vernier divisions	2
	Shear modulus or coefficient of rigidity	6
	Frequency	20, 29–31
	Number of rotations per minute	14
	Factor by which the range of an instrument is increased	23
	Index of refraction	32
P	Pressure	7–9
	Power	14, 19, 24, 27
p	Pitch	13
	Object distance	32
PE	Potential energy	12
Q	Charge or quantity of electricity	21, 24, 25, 27
q	Image distance	32
R	Rate of flow of liquid	7
	Gas constant	8
	Radius	10, 13, 17, 19
	Range of a projectile	17
	Resistance	21–24, 27
r	Radius	10, 13, 18, 19
	Resistance	22, 23, 25
	Angle of refraction	32
S	Length of a main scale division	2
	Specific heat	4
s	Distance	10–14, 31
sp. gr.	Specific gravity	7
T	Absolute temperature	8, 29
	Period	18, 20
	Reverberation period	30
t	Temperature	3, 4
	Time	5, 7, 10, 11, 14, 17, 19, 21, 24–26, 29
V	Length of a vernier division	2
	Volume	3, 7, 8, 30
	Speed, velocity	17, 29–32
v	Speed, velocity	7, 10, 12, 17, 18
$\bar{v}$	Average speed	10, 14
W	Weight	7, 11–13, 17–20
	Work, energy	24, 28
X	Reactance	27
	Unknown resistance	23
x	Horizontal distance	17

Symbol	Use	Chapter
Y	Young's modulus	6
Z	Impedance	27
z	Electrochemical equivalent	25
α (alpha)	Coefficient of linear expansion	3, 6
	Angular acceleration	10, 19
β (beta)	Coefficient of volume expansion	3, 8
γ (gamma)	Pressure coefficient	8
Δ (delta)	Change in	
ΔL	Change in length	3, 6
ΔT	Change in temperature	5
Δt	Change in temperature	3, 4, 6, 8
$\Delta\phi$	Number of lines of force cut	26
ϕ (phi)	Angle of shear	6
μ (mu)	Coefficient of friction	12
ρ (rho)	Resistivity	22
Σ (sigma)	Sum of	16, 19, 22
θ (theta)	Angle	10, 15, 19, 23, 27
	Angle of lag	27
ω (omega)	Angular speed	10, 19
$\bar{\omega}$	Average angular speed	10, 19

TABLE 1. PROPERTIES OF SOLIDS AND LIQUIDS

Substance	Specific gravity	Specific heat, cal/gm °C or Btu/lb °F	Coefficient of linear expansion per °F	Young's modulus, lb/in.^{2}
Alcohol (ethyl)	0.79	0.60		
Aluminum	2.70	0.21	0.000012	10.0×10^6
Brass	8 6	0.09	0.000011	13.1×10^6
Copper	8.9	0.092	0.0000094	18×10^6
Cork	0.22–0 26			
Ether	0.74	0.55		
Glass, crown	2.4–2.8	0 16	0.0000049	
Glass, flint	2.9–5.9	0.12	0.0000044	
Gold	19 3	0.032	0.000028	11.4×10^6
Ice, 0°C	0.92	0.51		
Iron	7.85–7.88	0 117	0.0000067	27.5×10^6
Lead	11.3	0.030	0.000016	2.2×10^6
Mercury	13.6	0.033		
Nickel	8.6–8.9	0.109	0.0000078	30×10^6
Oak	0.8	.....	0.000003	
Pine	0.5	.....	0.000003	
Platinum	21.4	0.032	0.0000049	24.2×10^6
Steel	7.6–7.9	0.118	0.0000072	29.0×10^6
Tin	7.3	0.055	0.000015	6×10^6
Turpentine	0.87	0.46		
Zinc	7.1	0.093	0.000014	13×10^6

TABLE 2. SATURATED WATER VAPOR

Showing pressure P (in millimeters of mercury) and density D of aqueous vapor saturated at temperature t; or showing boiling point t of water and density D of steam corresponding to an outside pressure P

t	P	D	t	P	D
−10	2 0	2 2 $\times 10^{-6}$	80.0	355 1	293.8
− 9	2.1	2 4	85.0	433 5	354.1
− 8	2 3	2.6	90.0	525 8	424.1
− 7	2.6	2.8	91 0	546 1	439.5
− 6	2.8	3.0	92 0	567.1	455.2
− 5	3.0	3.3	93 0	588.7	471.3
− 4	3.3	3.5	94.0	611.0	487.8
− 3	3.6	3.8	95.0	634.0	505
− 2	3.9	4.1	96 0	657.7	523
− 1	4.2	4.5	96.5	669.8	
0	4.6	4.9	97.0	682.1	541
1	4.9	5.2	97.5	694.5	
2	5.3	5.6	98.0	707.3	560
3	5.7	5.9	98.2	712.5	
4	6.1	6.4	98.4	717.6	
5	6.5	6 8	98.6	722 8	
6	7.0	7.3	98.8	728.0	
7	7.5	7.8	99 0	733 3	579
8	8.0	8.3	99.2	738 6	
9	8.6	8.8	99.4	743 9	
10	9.2	9.4	99.6	749.3	
11	9.8	10.0	99.8	754.7	
12	10.5	10.7	100.0	760.0	598
13	11.2	11.4	100.2	765.5	
14	12.0	12.1	100.4	770.9	
15	12.8	12.8	100.6	776.4	
16	13.6	13.6	100.8	781.9	
17	14.5	14.5	101	787.5	618
18	15.5	15.4	102	815.9	639
19	16 5	16.3	103	845.1	661
20	17.6	17.3	104	875.1	683
21	18.7	18.3	105	906.1	705
22	19.8	19.4	106	937.9	728
23	21.1	20.6	107	970.6	751
24	22.4	21.8	108	1,004.3	776
25	23 8	23.0	109	1,038.8	801
26	25.2	24.4	110	1,074.5	827
27	26.8	25.8	112	1,148.7	880
28	28.4	27.2	114	1,227.1	936
29	30.1	28.8	116	1,309.8	995
30	31.8	30.4	118	1,397.0	1,057
35	42.0	39.6	120	1,489	1,122
40	55.1	51.1	125	1,740	1,299
45	71.7	65.6	130	2,026	1,498
50	92.3	83.2	135	2,348	1,721
55	117.8	104.6	140	2,710	1,968
60	149 2	130.5	150	3,569	2,550
65	187.4	161.5	160	4,633	3,265
70	233.5	198.4	175	6,689	4,621
75	289.0	242.1	200	11,650	7,840

TABLE 3. ELECTROCHEMICAL DATA

Element	Atomic mass	Valence	Electrochemical equivalent, gm/coulomb
Aluminum	27.1	3	0.0000936
Copper	63.6	2	0.0003294
Copper	63.6	1	0.0006588
Gold	197.2	3	0.0006812
Hydrogen	1.008	1	0.0000105
Iron	55.8	3	0.0001929
Iron	55.8	2	0.0002894
Lead	207.2	2	0.0010736
Silver	107.9	1	0.00111800

TABLE 4. RESISTIVITIES AND TEMPERATURE COEFFICIENTS

Material	ρ at 20°C, microhm-cm	ρ at 20°C, ohm-circular mil/ft	Temperature coefficient of resistance (based upon resistance at 0°C) per °C
Copper, commercial	1.72	10.5	0.00393
Silver	1.63	9.85	0.00377
Aluminum	2.83	17.1	0.00393
Iron, annealed	9.5	57.4	0.0052
Tungsten	5.5	33.2	0.0045
German silver (Cu, Zn, Ni)	20.–33.	122.–201.	0.0004
Manganin	44.	266.	0.00000
Carbon, arc lamp	6000.	....	−0.0003
Paraffin	3×10^{24}		

TABLE 5. DIMENSIONS AND RESISTANCE OF COPPER WIRE

Gauge No.	Diameter, in.	Diameter, cm	Resistance per 1,000 ft of wire at 20°C, ohms
0	0.3249	0.8251	0.098
1	0.2893	0.7348	0.124
2	0.2576	0.6544	0.156
3	0.2294	0.5827	0.197
6	0.1620	0.4115	0.395
10	0.1019	0.2588	0.999
12	0.0808	0.2053	1.59
16	0.0508	0.1291	4.02
18	0.0403	0.1024	6.39
22	0.0254	0.0644	16.1
28	0.0126	0.0321	64.9

The gauge number referred to is the Brown and Sharpe (B. & S.), also known as the American wire gauge. A study of the above table will reveal the ingenious correlation that exists between gauge numbers, size of wire, and resistance of wire. Each third gauge number halves the area and hence doubles the resistance. Each sixth gauge number halves the *diameter* and hence quadruples the resistance.

TABLE 6. NATURAL SINES AND COSINES

Natural Sines

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement difference		
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89°	
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0872	85	
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81	
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	1736	80	
10	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2588	75	
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	
19	3256	3273	3289	3305	3322	3338	3355	3371	3387	3404	3420	70	
20	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	
22	3746	3762	3778	3795	3311	3827	3843	3859	3875	3891	3907	67	
23	3907	3923	3939	3955	3071	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	4226	65	
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	5000	60	
30	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	5736	55	
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	6428	50	
40	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	
44°	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	7071	45°	
Complement		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cosines

TABLE 6. NATURAL SINES AND COSINES (Continued)

NATURAL SINES

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement difference		
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193	44°	
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314	43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431	42	
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547	41	
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	7660	40	
50	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771	39	
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880	38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986	37	
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090	36	
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	8192	35	
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290	34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387	33	
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480	32	
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572	31	
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	8660	30	9
60	0.8360	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746	29	
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829	28	
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910	27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988	26	
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	9063	25	
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135	24	
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205	23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272	22	
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336	21	
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	9397	20	6
70	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455	19	
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511	18	
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563	17	
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613	16	5
74	9313	9617	9622	9627	9632	9636	9641	9646	9650	9655	9659	15	
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703	14	
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744	13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781	12	
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816	11	
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	9848	10	
80	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877	9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903	8	
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925	7	
83	9925	9928	9930	9932	9934	9936	9938	9940	9940	9943	9945	6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	9962	5	
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976	4	
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986	3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994	2	
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	9998	1	
89°	9998	9999	9999	9999	9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0°	0
Complement		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

NATURAL COSINES

TABLE 7. NATURAL TANGENTS AND COTANGENTS

Natural Tangents

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement difference		
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89°	
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	87	
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	86	
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0875	85	
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	84	
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	83	
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	82	
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	81	
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	1763	80	
10	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	78	
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	77	
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	76	
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	2679	75	
15	0.2679	2698	2717	2736	2754	2774	2792	2811	2830	2849	2867	74	
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	72	
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	71	
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3640	70	
20	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	69	
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	67	
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	66	
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4663	65	21
25	0.4663	4684	4706	4727	4748	4770	4791	4813	4334	4856	4877	64	
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	63	
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	61	
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	5774	60	23
30	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6099	59	
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	57	
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	7002	55	
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	8391	50	29
40	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	49	30
41	8693	8724	8754	8785	8316	8347	8878	8910	8941	8972	9004	48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9825	47	32
43	9325	9358	9391	9424	9557	9490	9523	9556	9590	9623	9567	46	33
44°	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000	45°	34
Complement		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cotangents

TABLE 7. NATURAL TANGENTS (Continued)

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
45°	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	36
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	37
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	38
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	40
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	41
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	43
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	45
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	47
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	49
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	52
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	54
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	57
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	60
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	64
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	68
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	72
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	77
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	82
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	88
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	94
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	14
70	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.250	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.700	25
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	44
79	5.145	4.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	52
80	5.67	5.73	5.79	5.85	5.91	5.98	6.04	6.11	6.17	6.24	7
81	6.31	6.39	6.46	6.54	6.61	6.69	6.77	6.85	6.94	7.03	8
82	7.12	7.21	7.30	7.40	7.49	7.60	7.70	7.81	7.92	8.03	10
83	8.14	8.26	8.39	8.51	8.64	8.78	8.92	9.06	9.21	9.36	14
84	9.51	9.68	9.84	10.0	10.2	10.4	10.6	10.8	11.0	11.2	
85	11.4	11.7	11.9	12.2	12.4	12.7	13.0	13.3	13.6	14.0	3
86	14.3	14.7	15.1	15.5	15.9	16.3	16.8	17.3	17.9	18.5	6
87	19.1	19.7	20.4	21.2	22.0	22.9	23.9	24.9	26.0	27.3	
88	28.6	30.1	31.8	33.7	35.8	38.2	40.9	44.1	47.7	52.1	
89°	57.	64.	72.	82.	95.	115.	143.	191.	286.	573.	
Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	

TABLE 8. LOGARITHMS

Only the mantissa (or fractional part) of the logarithm is given. Each mantissa should be preceded by a decimal point and the proper characteristic.

100–500

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
N	0	1	2	3	4	5	6	7	8	9

TABLE 8. LOGARITHMS (Continued)

500–1000

N	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

INDEX

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D