PROBLEMS

IN

ELECTRICAL ENGINEERING

BY

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PREFACE

THIS collection of problems has been prepared for the use of students at the Massachusetts Institute of Technology, but as the book may be used in other technical schools it seems best to state what ground the problems are intended to cover. At the Institute the book will be used by the third year students in Electrical Engineering, and by the third and fourth year students in the courses of Civil, Mechanical, Mining and Chemical Engineering. The work in these courses includes the theoretical elements of direct and single-phase alternating currents and the theory and operation of direct-current generators and motors. The problems are for the most part what would be called theoretical, although whenever it is possible they apply to some practical engineering question. In many cases certain constants have been exaggerated in order to emphasize some particular point.

At the beginning of each chapter there is a brief statement of the fundamental principles which the problems illustrate. The problems in each chapter and in each section of Chapters XII and XIII have been arranged approximately in their order of difficulty. Many problems which would otherwise be of considerable length have been divided and numbered separately. A large number of simple problems have been included which are suitable for the more elementary courses given to the students in Civil, Mechanical, Mining and Chemical Engineering.

It has been thought best to print the answers to these problems separately. Teachers alone can obtain them from the author.

In conclusion the author wishes to mention the name of Mr. C. H. Porter, Instructor in Electrical Engineering, to whom credit is due for inaugurating and developing the present system of problem work given in the Department of Electrical Engineering at the Institute.

W. V. LYON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
September, 1908.

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INTRODUCTION

In the training of students for the engineering professions, it is certainly fundamental that they should be taught to think straight. Given a certain set of conditions, it becomes necessary to determine from these the essential process to be followed, and then to follow this process logically to the desired result. Consecutive, logical thought is, then, the foundation for successful engineering work. One of the great disadvantages of the purely lecture method of instruction as compared with that by recitations is that the instructor and not the student uses that intellectual power which he is so fortunate as to possess.

But even using the system of instruction by recitations, it is absolutely essential to the securing of the best results in educational work, that students should be required to solve a large number of well selected problems. Where possible these problems should naturally be as closely related to the real problems of engineering as is consistent with the knowledge of the student. It is oftentimes out of the question, however, to give undergraduates the problems arising in certain particular pieces of engineering work, owing to their lack of experience and to the fact that many problems in engineering depend for their successful solution upon a knowledge and use of non-technical features and principles. The fact, then, that a problem is more or less remotely connected with some engineering reality cannot in all cases be urged as a reason for its not being given in the educational training of undergraduate students. If a problem calls for the analysis of certain definite conditions, the bringing into play of the information possessed by the student and bearing on these conditions, and a certain logical thinking essential to the securing of the result, it may be of considerable benefit even though the assumed conditions have no real counterpart in engineering work. In this statement it is of course understood that problems which are puzzles merely are without value for purposes of sound training.

It seems also desirable that the problems should be in a sense graded according to difficulty. Students are frequently seriously and unnecessarily discouraged by being confronted with problems of exceptional difficulty before they have secured the facility and confidence which come from experience in applying the information which they have secured in their regular training.

In much of the engineering work the accuracy of the slide rule is sufficient, and it is desirable that all students should be required to perform the calculations by means of this labor-saving device. Too often is it the case that the work is carried on using the methods of arithmetic or an unnecessary precision is had from the use of logarithm tables of five and six places. It is highly important that students should possess a proper understanding of the principles of precision which apply to the data given in any problem.

Since electrical engineering concerns itself largely with the generation, transmission and distribution of energy, it seems wise in any set of problems to insist strongly upon the energy relations involved. It is frequently true that students go through correctly with the calculations associated with some particular set of conditions without in the least understanding the energy relations involved and the energy results achieved in consequence of the proper carrying out of each individual step.

The use of large numbers of problems in the work in Electrical Engineering has been for many years a feature of the instruction at the Massachusetts Institute of Technology, and the admirable collection of problems which Mr. Lyon has produced illustrates particularly well the general character of the work required in the Junior year of the Electrical Engineering Course. The considerable variety, the sequence so far as difficulty is concerned, and the large number of problems, together with the brief introductory statement of fundamental principles involved in their solution, render this collection eminently satisfactory for purposes of instruction.

H. E. CLIFFORD.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
September, 1908.

CORRECTION SHEET

Chapter I.

Page 2, line 8. Insert after ohm: — that the diameter i approximately 0.1 inch.

Chapter III.

Page 22, problem 19, line 5. Insert after battery: — whose e.m.f. is 15 volts and internal resistance 1 ohm.

Page 23, Fig. 7: — The distances in this Fig. should be read so that the junction point is 0.5 mile from the station, the 40-ampere car is 2 miles from the junction point, and the 70-ampere car is 1.5 miles from the junction point.

Pages 25–28, Figs. 9 to 16 inclusive. The voltages refer to those at the power stations in every case.

Chapter IV.

Page 36, Fig. 26. Change 116 volts to 115 volts.

Chapter V.

Page 62, problem 99, last line. Insert after constant: — and that the voltage across the mains at the balancer is 220 volts under all conditions.

Pages 63 and 64, problems 102 to 108 inclusive. Understand that the power loss in the field windings means the power loss in the entire field circuit.

Chapter XII.

Page 127, Equation (2), third term should be $-\frac{1}{2}\frac{I^2}{C\omega}\cos 2\,\omega t$

Page 131, problem 12, line 2. Should be: $i_3 = 68 \sin (\omega t + 90°)$

Page 140, problem 80, line 3. Change 25-cycle to 60-cycle

Page 144, problem 105, line 3. Change 3.8 amperes to 3.4 amperes.

Page 159, problem 207, last line. Change capacity to condensive reactance.

Chapter XIII.

Page 180, problem 71, last line. Insert after current: — in the second case.

Chapter XIV.

Page 203, problem 52, line 2. Change 157 to 314.

PROBLEMS IN ELECTRICAL ENGINEERING

CHAPTER I

RESISTANCE

THE resistances of ordinary electrical conductors vary directly as their length and inversely as their cross-section. That is, $R \propto \frac{L}{A}$. If V and W represent the volume and weight of the conductor the following proportionalities may also be written:

$$R \propto \frac{V}{A^2}, \qquad \propto \frac{L^2}{V},$$

$$R \propto \frac{W}{A^2}, \qquad \propto \frac{L^2}{W}.$$

If the proper constants are used these proportionalities may be written in the form of equations:

$$R = k\frac{L}{A} = k\frac{V}{A^2} = k\frac{L^2}{V},$$

and

$$R = k'\frac{W}{A^2} = k''\frac{L^2}{W},$$

where

$$\left.\begin{aligned} \frac{W}{V}k' &= k \\ k'' &= \frac{W}{V}k \end{aligned}\right\} \frac{W}{V} = \text{density of conductor.}$$

Since the constants k, k', and k'' are the resistances of conductors of unit dimensions they are known as the specific resistances, or the resistivities, of the material. k is usually expressed as ohms per centimeter cube or as ohms per mil-foot. k per centimeter cube is the resistance between the opposite faces of a centimeter cube; k per mil-foot is the resistance of a wire one foot long and

NOTE.—In all problems marked with an asterisk (*) the approximate method for calculating the resistance given in Chapter I should be used.

of one circular mil cross section, that is, 0.001 inch in diameter. Since the cross sections of circular conductors vary as the squares of their diameters the circular milage of a wire is the square of the diameter expressed in thousandths of an inch.

There is a convenient method for obtaining the approximate resistance of copper wire of the Brown and Sharpe gage. It is based on the fact that the resistance of 1000 feet of No. 10 wire is approximately 1 ohm that diameter is approx 0.1 inch and that the weight is approximately 31.5 pounds. For every third number of smaller wire the resistance doubles and for every third number of larger wire the resistance is halved; thus No. 13 has a resistance of 2 ohms per 1000 feet, No. 16 a resistance of 4 ohms per 1000 feet, and No. 4 a resistance of 0.25 ohm per 1000 feet. The resistance of the second and third numbers is 1.26 and 1.59 respectively times the resistance of the first. Thus the resistance of No. 17 is 5.04 ohms and of No. 18, 6.36 ohms per 1000 feet.

The resistance of the majority of electrical conductors increases with the temperature, and with pure metals this increase is about 0.4 per cent per degree centigrade above zero.

$$R_t = R_0 (1 + at),$$
$$R_{t_1} = R_0 (1 + at_1).$$

If R_0 (resistance at 0 deg. cent.) is eliminated from these equations the difference in temperature $(t_1 - t)$ is found to be

$$(t_1 - t) = \left(\frac{1}{a} + t\right)\left(\frac{R_{t_1} - R_t}{R_t}\right).$$

The formula recommended by the American Institute of Electrical Engineers for calculating the rise in temperature in electrical apparatus by resistance measurements is

$$(t_1 - t) = (238.1 + t)\left(\frac{R_{t_1}}{R_t} - 1\right).$$

PROBLEMS

1. If the resistance of No. 12 copper wire (diameter = 0.0808 inch) is 1.59 ohms per thousand feet, what is the resistance of five miles of No. 00 trolley wire (diameter = 0.365 inch)?

2. If the density of aluminum is 0.097 pound per cubic inch, what will be the cost of 10 miles of 750,000 circular mil aluminum cable at 30 cents a pound?

3. If the resistance of No. 6 copper wire (diameter = 0.162 inch) is 0.394 ohm per thousand feet, what is the resistance per mile of 500,000 circular mil copper cable?

4. A copper rod 1.27 centimeters in diameter and 1.362 meters long has a resistance of 0.000182 ohm. What is the resistance of 10 miles of No. 8 copper wire which has a diameter of 0.1285 inch?

5. An aluminum rod 0.673 inch in diameter and 1.243 meters long has a resistance of 0.000148 ohm. What is the resistance of 3 miles of 750,000 circular mil aluminum cable?

6. Two solenoids are exactly alike in every particular except that one is wound with German silver wire and the other with *IaIa* wire. The resistivity of the German silver wire is 22 times that of copper, while the resistivity of *IaIa* wire is 29 times that of copper. If the resistance of the first coil is 47 ohms, what is the resistance of the second?

7. A No. 0000 copper feeder 0.460 inch in diameter is to be replaced by an aluminum one of the same length and resistance. If the conductivity of aluminum is 62 per cent that of copper, what will be the circular milage of the aluminum feeder?

8. A copper bar 1.87 centimeters in diameter has a resistance of 0.000150 ohm. What will be its resistance when drawn into No. 24 wire (diameter = 0.0201 inch)?

9. The resistance of a copper rod 1 inch in diameter and 10 feet long is 0.000102 ohm. What will be its resistance when drawn into a wire 0.1 inch in diameter?

10. A copper rod 0.75 inch in diameter and 15.1 feet long has a resistance of 0.000274 ohm. What will be its resistance when drawn into a wire 18.0 kilometers long?

11. An aluminum conductor 10 feet long and 1 inch in diameter has a resistance of 0.000165 ohm. How many feet per ohm are there in 250,000 circular mil aluminum cable?

12. A conductor 50 feet long weighing 20 pounds has a resistance of 0.00395 ohm. What is the resistance of a conductor of the same material 200 feet long weighing 10 pounds?

13. For No. 12 copper wire there is 0.0802 ohm per pound and for No. 20 there are 3.28 ohms per pound. If the diameter of No. 12 is 0.0808 inch, what is the diameter of No. 20?

14. A copper bar 5 meters long weighing 20 kilograms has a resistance of 0.000195 ohm. What will be its resistance when drawn into No. 24 wire which is 0.0201 inch in diameter? Copper weighs 0.32 pound per cubic inch.

15. If the specific resistance of copper is 1.62 microhms per centimeter cube at 0 deg. cent., what is its resistivity per mil-foot at this temperature?

16. If the resistance of German silver is 215 ohms per mil-foot, what is its resistance per meter-gram? The specific gravity of German silver is 8.5.

17. If the specific resistance of copper is 1.62 microhms per centimeter cube at 0 deg. cent., what is its resistance per inch cube at this temperature?

18. If the specific resistance of copper is 1.62 microhms per centimeter cube at 0 deg. cent., what is the resistance of a copper wire 5 kilometers long and 4 millimeters in diameter at this temperature?

19. If the resistance of aluminum is 2.74 microhms per centimeter cube, what is the resistance of 5 miles of 460,000 circular mil aluminum cable?

20. If the specific resistance of copper is 1.62 microhms per centimeter cube at 0 deg. cent., what is the resistance of a copper conductor 5 miles long and 0.46 inch in diameter at this temperature?

21. If the resistance of aluminum is 16.5 ohms per mil-foot, what will be the resistance of 5 miles of 250,000 circular mil aluminum cable?

22. If the resistance of copper is 10.2 ohms per mil-foot, what is the resistance of a copper bar 0.512 inch square and 3.674 feet long?

23. If the specific resistance of copper is 0.145 ohm per meter-gram at 0 deg. cent., what will be the resistance, at this temperature, of 10 miles of No. 8 copper wire which weighs 50 pounds per 1000 feet?

24. If the resistance of copper is 1.70 microhms per centimeter cube at 20 deg. cent., what is the resistance of 3 miles of 250,000 circular mil copper cable at this temperature?

25. If the resistance of German silver is 3.5 ohms per meter-gram, what will be the resistance of 1250 feet of No. 20 German silver wire weighing 3.93 pounds?

26. If the specific resistance of copper is 0.145 ohm per meter-gram at 0 deg. cent., what will be the resistance of 10 kilometers of wire 2.5 millimeters in diameter at this temperature? The specific gravity of copper is 8.9.

27. If the specific resistance of mercury is 94 microhms per centimeter cube at 0 deg. cent., what is the resistance, at this temperature, of the mercury contained within a glass tube 1.327 meters long and 0.42 millimeter in diameter?

28. If the resistance of copper is 10.2 ohms per mil-foot, what is the resistance of a copper tube which has inner and outer radii of 0.5 and 0.75 inch respectively and a length of 5.24 feet?

29. A copper tube has an inner diameter of 1 inch and a thickness of 0.25 inch. If the thickness of the tube alone be increased to 0.375 inch by increasing the outer diameter, by what per cent will the resistance of the tube be affected?

30. A copper rod 0.82 inch in diameter has a resistance of 0.000087 ohm. A copper tube of the same length has a thickness of 0.25 inch. What should be the outer diameter of the tube in order that it may have the same resistance as the rod?

31. If the resistance of aluminum is 0.0745 ohm per meter-gram and its density is 0.097 pound per cubic inch, what will be the resistance of 10 miles of 500,000 circular mil aluminum cable?

32. No. 30 *Ia Ia* wire has a resistance of 3005 ohms per 1000 feet and there are 3450 feet per pound. What is its resistance per meter-gram?

33. The resistance of German silver wire is 215 ohms per mil-foot. If No. 16 German silver wire (diameter = 0.051 inch) costs 75 cents a pound, what will be the cost of a coil of No. 16 wire whose resistance is 40 ohms? The density of German silver is 0.31 pound per cubic inch.

34. The resistance of *Ia Ia* wire is 50 microhms per centimeter cube and its specific gravity is 8.4. If double silk covered No. 30 wire (diameter = 0.01 inch) costs $4.50 per pound, what will a coil of this wire having a resistance of 10,000 ohms cost?

35. A copper rod having a length of 3 feet 2.84 inches and an average diameter of 0.405 inch has a resistance of 0.000201 ohm. What is the resistance of copper per mil-foot at this temperature? per centimeter cube?

36. An aluminum bar with an average cross section of 0.185 square inch and a length of 2.956 feet has a resistance of 0.000207 ohm. What is the resistance of aluminum per mil-foot at this temperature? per centimeter cube?

37. A copper wire having a length of 27.42 feet and an average diameter of 0.0508 inch has a resistance of 0.110. What is the resistance of copper per mil-foot at the same temperature? If the specific gravity of this wire is 8.9, what is the resistance of copper per meter-gram at this temperature?

38. A portion of an electrical circuit is composed of 3 coils in series. From the following data find its resistance.

	Length.	Diameter.	Specific Resistance.
	ft.	in.	
1. *Iala* wire	30	0.0142	50 microhms per cm. cube.
2. German silver . . .	100	0.0100	215 ohms per mil-foot.
3. Aluminum	200	0.0201	800 per cent conductivity of German silver.

39. With copper at 16 cents per pound and aluminum at 31 cents per pound is it cheaper to use copper or aluminum for a 5-mile transmission line whose total resistance is 0.74 ohm? The resistance of copper is 10.2 ohms per mil-foot, and aluminum has a conductivity which is 62 per cent that of copper. The densities of copper and aluminum are 0.32 and 0.097 pound per cubic inch respectively. What will be difference in cost.

* **40.** If No. 30 copper wire costs $2.50 per pound, what will a coil of this size cost that has a resistance of 100 ohms (wt. per cubic inch = 0.32 pound)?

* **41.** What is the resistance of 300 feet of No. 18 copper wire?

* **42.** What is the diameter of No. 20 copper wire?

* **43.** What is the smallest size of copper wire that can be used to run 240 feet and return and have the resistance not greater than 0.65 ohm?

* **44.** How many feet of No. 24 German silver wire will be required for a coil of 150 ohms resistance if the conductivity of German silver is 4.5 per cent that of copper?

* **45.** How many feet per ohm are there of No. 28 *IaIa* wire which has a resistivity of about 29 times that of copper?

46. At 0 deg. cent. an aluminum rod 0.914 meter long and 1.57 centimeters in diameter has a resistance of 0.000123 ohm. What is the resistance of aluminum per mil-foot at 20 deg. cent.? The temperature coefficient is 0.4 per cent per degree centigrade.

47. The resistance of copper at 15 deg. cent. is 1.68 microhms per centimeter cube. What is the resistance of 5 miles of No. 10 copper wire (diameter = 0.102 inch) at 85 deg. fahr.? The temperature coefficient is 0.004 per degree centigrade.

48. The resistance of the field of a shunt generator at the room temperature of 62 deg. fahr. is 32.7 ohms. What is its temperature when the resistance is 36.9 ohms?

49. The cold resistance of the armature of a generator (at 20.5 deg. cent.) is 0.046 ohm. After running under load the resistance becomes 0.052 ohm. What is the rise in temperature?

CHAPTER II

OHM'S LAW

If the current has the same value at every point in an electric circuit it is numerically equal to the resultant e.m.f. acting on the circuit divided by the resistance of the circuit. In the preceding sentence the word "circuit" may mean a complete, or closed, electric circuit or it may be applied to a part of the complete circuit. The resultant e.m.f. is the algebraic sum of all the e.m.fs. acting on the circuit. For series circuits the current is the same at all points and the total resistance is the sum of the resistances. For circuits in parallel the impressed e.m.f. is the same for all the branches, and if there are no other e.m.fs. acting in any of them the currents will divide inversely as the resistances. The total current supplied to a number of parallel circuits is evidently equal to the sum of the currents in the several branches.

The e.m.f. of a battery is the potential difference at its terminals when there is no current through it. If the battery is delivering energy a part of its e.m.f. is used to supply the ohmic drop due to the current through the resistance of the battery itself and thus the potential difference at the battery terminals is reduced as the current output increases. If a battery is receiving energy, for example a storage battery which is being charged, the voltage impressed on the battery terminals must be sufficient to balance the e.m.f. of the battery and to supply the ohmic drop necessary to maintain the charging current through the resistance of the battery.

In the problems involving measuring instruments the resistances of the voltmeters and ammeters are assumed to be constant, and thus their deflections are proportional to the currents through them.

PROBLEMS

* **1.** What is the resistance of 5 miles of No. 0000 feeder in parallel with the same length of No. 00 trolley wire?

2. A slide wire of 10 ohms resistance is connected between the points *M* and *N*. Across what portion of this wire must a resistance of 5 ohms be placed so that the total resistance between *M* and *N* shall be 5 ohms?

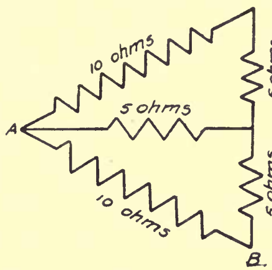

Fig. 1

3. A number of resistances are connected as shown by Fig. 1. Find the resistance between the points *A* and *B*.

4. A storage battery whose e.m.f. is 25 volts and internal resistance 0.5 ohm is connected to a circuit whose resistance is 4 ohms. What current does the battery supply? What is the voltage impressed on the circuit?

5. A battery whose e.m.f. is 6 volts delivers 2 amperes to a resistance of 2.7 ohms connected across its terminals. What is the battery resistance?

6. The open-circuit potential difference of a storage battery is 6.6 volts. This battery will maintain a current of 3 amperes at a potential difference of 5.85 volts. What is the battery resistance?

7. Two batteries whose e.m.fs. are 6.3 volts and 10.5 volts and whose internal resistances are 0.15 ohm and 0.2 ohm respectively are connected in series with a circuit of 2 ohms resistance. What will be the value of the current? If they are connected in series and in opposition what will be the value of the current?

8. A battery whose e.m.f. is 10 volts and internal resistance 1 ohm has connected across its terminals two coils in parallel of

5 ohms and 10 ohms resistance. What is the potential difference at the battery terminals?

9. A storage battery whose e.m.f. is 18 volts will maintain a current of 8.8 amperes through a resistance of 1.95 ohms connected across its terminals. What current will it supply if this circuit be replaced by one having a resistance of 1.14 ohms?

10. A battery whose e.m.f. is 6.3 volts is supplying 1.15 amperes at 5.75 volts to a coil having a resistance of 5 ohms. What current would exist through a coil of 5 ohms resistance placed in parallel with the first?

11. A storage battery which gives 32.5 amperes on short circuit will supply a current of 7.5 amperes to a resistance of 1.2 ohms connected across its terminals. What is the e.m.f. of the battery?

12. A storage battery whose internal resistance is 0.18 ohm has a resistance of 12 ohms permanently connected across its terminals. This battery is supplying a current of 5 amperes at 7.8 volts to another resistance which is connected across its terminals. What is the e.m.f. of the battery?

13. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively are connected in parallel to a circuit to which each is supplying energy. If the current through the first battery is 2 amperes what is the total current supplied?

14. If two batteries which will each supply a current of 25 amperes on short circuit are connected in series in conjunction with a resistance of 2 ohms, a current of 5 amperes will be established through the circuit. If the batteries are connected in series but in opposition with the same resistance only 2 amperes will exist through the circuit. What are the e.m.fs. and internal resistances of the batteries?

15. Each cell of a storage battery has an e.m.f. of 2.1 volts and a resistance of 0.12 ohm. Will the battery supply more current to a circuit whose resistance is 0.09 ohm when the cells are connected in series or in parallel? For what resistance in the external circuit will the battery supply the same current for the series as for the parallel connection?

16. A battery whose e.m.f. is 17 volts and internal resistance 0.21 ohm is supplying 8 amperes to a circuit consisting of a coil of 2.4 ohms in parallel with an unknown resistance. What is the value of this unknown resistance?

17. A battery whose internal resistance is 0.18 ohm is supplying current at a potential difference of 7.2 volts to two coils in parallel. If the resistance of the first is 0.93 ohm and the current through the second is 4.6 amperes, what is the current through the battery? What is the battery e.m.f.?

18. A battery whose internal resistance is 0.5 ohm supplies 3 amperes to a circuit consisting of two coils of 1 ohm and 2 ohms resistance in parallel. What would be the potential difference at the battery terminals if an additional coil of 5 ohms resistance were placed in parallel with the other two?

19. Given a battery whose e.m.f. is E and internal resistance R_B, and two coils of R_x and R_y ohms resistance. For what value of R_x will the current through R_y be the same whether the coils are connected in series or in parallel across the battery terminals?

20. Given a battery of 10 volts e.m.f. and 1 ohm internal resistance and two coils of 5 and 10 ohms resistance. Will the potential across the 10-ohm resistance be greater when the coils are connected in series or in parallel across the battery terminals?

21. In problem 20 what additional resistance must be inserted in series with the battery so that the potential across the 10-ohm resistance will be the same for the series as for parallel connection of the 5 and 10-ohm coils?

22. A storage battery whose internal resistance is 0.3 ohm is connected in parallel with a resistance of 21 ohms across 110-volt mains. If the total current supplied is 32 amperes, what is the e.m.f. of the battery?

23. What resistance must be connected in series with a storage battery whose e.m.f. is 21 volts and internal resistance 0.24 ohm so that a charging current of 5 amperes will be established through the battery when connected across 110-volt mains?

24. Two resistances of 5 and 10 ohms are connected in parallel across an unknown voltage. If the total current is 35 amperes what is the current through each? What is the unknown voltage?

25. When two resistances are connected in series across 110-volt mains they take a current of 3 amperes. If they are connected in parallel across the same voltage the total current is 20 amperes. What are the resistances?

26. A copper tube which has an inner diameter of one inch and a thickness of one-sixteenth inch is filled with mercury. If the resistance of the copper tube alone is 0.000085 ohm what is the resistance of the combination? The specific resistance of mercury is 59 times that of copper.

27. In problem 26 if a potential difference of 1.2 millivolts is applied to the ends of the copper tube filled with mercury, what will be the value of the current? What will be the current density in the copper? in the mercury?

*** 28.** Twenty incandescent lamps each taking approximately 0.5 ampere are installed at a distance of 110 feet from 112-volt mains. What is the smallest size wire that should be run from the mains to the lamps in order that the potential at the lamps may not fall below 109 volts?

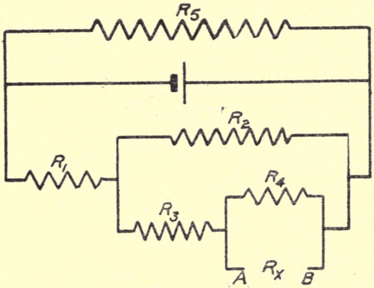

Fig. 2

29. A number of resistances are connected as shown in Fig. 2. The resistance R_x between the terminals A and B is of unknown value. The currents through R_1 and R_2 are 5 and 2 amperes respectively. If the internal resistance of the battery is 0.9 ohm what is its e.m.f.? What is the value of the unknown resistance R_x?

R_1 = 2.0 ohms.
R_2 = 1.4 ohms.
R_3 = 0.7 ohm.
R_4 = 0.3 ohm.
R_5 = 6.5 ohms.

30. Two batteries B_1 and B_2 are supplying energy to a circuit as shown in Fig. 3. Their internal resistances are 0.4 ohm and 1.0 ohm respectively. The currents in three of the branches are

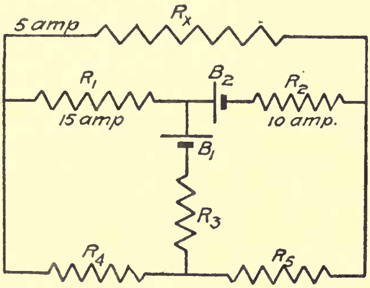

Fig. 3

indicated in the figure. What are the battery e.m.fs. and what is the unknown resistance R_x?

$$R_1 = 0.2 \text{ ohm.}$$
$$R_2 = 0.2 \text{ ohm.}$$
$$R_3 = R_4 = R_5 = 1 \text{ ohm.}$$

31. An unknown resistance is connected between the trolley and the rail of an electric road which is operating at a potential of 550 volts. If the midpoint of this resistance is connected to the rail by a 20-ohm coil there will be a current of 5 amperes through the latter. What is the unknown resistance?

32. Two voltmeters whose resistances are 17,238 ohms and 5,212 ohms are connected in series across 180 volts. What will each instrument read?

33. Two voltmeters having full scale deflections of 150 and 50 volts and resistances of 17,136 and 5,364 ohms respectively are connected in series with an extension coil of 7,500 ohms resistance across 220-volt mains. What will each instrument read?

34. A 150-scale voltmeter whose resistance is 17,527 ohms is connected in series with an unknown high resistance across 220-volt mains. If the voltmeter reads 138.5 volts what is the value of the unknown resistance?

35. In order to measure a difference in potential in the neighborhood of 500 volts a resistance coil of 3000 ohms is connected between the points and across one-third of this coil is connected a voltmeter of 3000 ohms resistance. If the voltmeter reads 138 volts, what is the potential between the given points?

36. It is desired to measure a potential difference in the neighborhood of 600 volts. Only two voltmeters are available, having ranges of 150 volts and 500 volts respectively. They are used in series and the reading of the first is 146 volts. The resistance of this voltmeter is 15,389 ohms and of the other, 48,151 ohms. What is the total potential difference? If the resistance of the first voltmeter had been 18,151 ohms, would the measurement have been practicable?

37. The only available apparatus for measuring a potential difference known to be a little greater than 127 volts is a 15-scale voltmeter of 1,564 ohms resistance, and two coils of 10,000 and 1,000 ohms respectively. The voltmeter can be read to 0.01 volt. What method of measurement will give the best precision? If the potential difference is 132 volts what will the voltmeter read?

38. In problem 37 what is the greatest voltage that can be measured? If the potential difference were approximately 120 volts what method would give the best precision?

39. In order to measure a trolley voltage of about 600 volts with a voltmeter having a 150-volt scale and a resistance of 7,547 ohms, a high resistance of 25,000 ohms is connected from the trolley to the rail and the voltmeter connected across a section of it having a resistance of 10,000 ohms. If the voltmeter reading is 139.5 volts what is the trolley voltage? Could the 25,000-ohm resistance have been used as an extension coil?

40. In problem 39 if the voltmeter had been one having a high resistance, say 15,126 ohms, across what portion of the 25,000-ohm resistance would it have been necessary to connect it in order that it might have read the same, viz., 139.5? Could the 25,000-ohm resistance have been used as an extension coil with this voltmeter?

41. It is desired to use a 25-ampere ammeter which has a resistance of 0.002 ohm with a shunt to give a full scale deflection of 100 amperes. What shunt resistance should be used?

42. The resistance of a millivoltmeter and its connecting leads is 1 ohm and its full scale deflection is 50 millivolts when used with the leads. What is the resistance of an extension coil with which the instrument will give a full scale deflection of 150 volts? of 50 volts?

43. In problem 42 if the connecting leads of the millivoltmeter have a resistance of 0.1 ohm what is the actual voltage across the terminals of the instrument when the pointer registers 30 millivolts? What shunt resistance should be used when the instrument, with its leads, is to be used as an ammeter to give a full scale deflection of 50 amperes? of 100 amperes?

44. It is desired to measure a current of constant value in the neighborhood of 25 amperes, the only available apparatus being a 10-scale ammeter, a 5-scale ammeter, and a shunt of unknown resistance. When the first ammeter and shunt are connected in parallel in the circuit the instrument reads 7.54 amperes and when the other ammeter is used in parallel with the shunt it reads 4.16 amperes. If the resistance of the ammeters be 0.0025 ohm and 0.005 ohm respectively what is the current in the circuit?

45. To increase the range of a 50-scale ammeter whose resistance is 0.0005 it is shunted with a resistance of 0.0003 ohm. If the instrument reads 43.2 amperes what is the total current in the circuit?

46. A direct reading millivoltmeter when connected in series in a circuit indicates the current in milliamperes. What is its resistance?

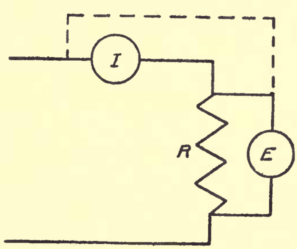

Fig. 4

47. A low resistance, R, is measured with a 25-scale ammeter (I) and a 50-scale millivoltmeter (E) as shown by the full lines in Fig. 4. The resistances of the instruments are 0.001 and 1.0

ohm respectively. If the ammeter reads 18.6 amperes and the voltmeter 47.3 millivolts what is the value of R?

48. In problem 47 if the voltmeter had been connected outside of the ammeter, as shown by the dotted line, and the instruments had read as before, what would have been the value of R?

49. In problems 47 and 48 if it be assumed that $R = \frac{E}{I}$ which method of connecting the voltmeter will give the more accurate result? What is the percentage error for each method?

50. A high resistance, R, is measured with a 50-scale milliammeter (I) and a 150-scale voltmeter (E), as shown by the full lines in figure. The resistances of the instruments are

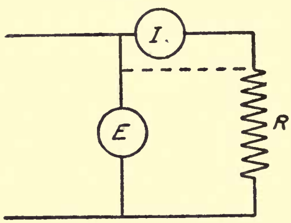

Fig. 5

1 ohm and 17,587 ohms respectively. If the ammeter reads 42.1 milliamperes and the voltmeter 137.4 volts what is the value of R?

51. If it be assumed that $R = \frac{E}{I}$ which method of connecting the voltmeter, inside or outside of the ammeter, will give the better precision when measuring a resistance of about 10,000 ohms? a resistance of 0.001 ohm? For an idea of the comparative resistances of the instruments see problems 47 and 50.

52. A car at the end of a 5-mile trolley line takes 75 amperes. The station voltage is 550 volts and the trolley wire is No. 00, which has a resistance of 0.42 ohm per mile. What is the shortest length of No. 0000 feeder that can be run from the station in parallel with the trolley to give a maximum drop of 80 volts to this car? The resistance of No. 0000 copper wire is 0.259 ohm per mile.

53. Two banks of 110-volt incandescent lamps are connected in series across 220-volt mains. There are 25 lamps in parallel in one bank and 15 lamps in parallel in the other. If the normal current taken by each lamp across 110 volts is 0.9 ampere and its resistance is assumed constant, what will be the voltage across each set of lamps for this unbalanced condition?

54. At the end of a transmission line which has a total resistance of 0.092 ohm there is a constant load of 200 lamps in parallel with a motor load which takes a current that varies from zero to 200 amperes. The normal current taken by one lamp across 110 volts is 0.46 ampere and its resistance may be assumed constant. If the station voltage is 120 what will be the extreme voltages at the load?

55. In problem 54 over what range must the station voltage be adjustable so that the voltage at the load may be kept constant at 110 volts under all conditions of motor load?

CHAPTER III

KIRCHHOFF'S LAWS

Kirchhoff's laws are a development and extension of Ohm's law. They have a wide application in direct and alternating currents, and similar principles are useful in determining the fluxes in some magnetic circuits and the distribution of charges in systems of condensers. They may be stated as follows:

I. At any junction the algebraic sum of the currents is zero; that is, the sum of the currents directed toward a junction equals the sum of the currents directed away from the junction.

II. In any closed electric circuit the algebraic sum of the impressed e.m.fs. and the reactive e.m.fs. equals the algebraic sum of the dissipative voltages. For steady direct currents this may be simplified as follows: In any closed electric circuit the algebraic sum of the impressed e.m.fs. equals the algebraic sum of the ohmic drops.

In applying these laws to the solution of problems involving electric circuits the following simple conventions will be found very useful:

1. Place one arrow on each branch to show the direction of the current assumed in that branch. This arrow may be placed at will. Then at any junction the currents towards the junction will be called positive and those away from the junction will be called negative. The algebraic sum of the currents at any junction is zero. If in the solution of a problem the value of a current is positive the arrow shows the actual direction of the current, on the other hand, if the value of the current is negative the actual direction of flow is opposite to the direction of the arrow.

2. Indicate by plus (+) and minus (−) signs the direction in which each e.m.f. acts or is assumed to act. When passing through an e.m.f. from − to + (a rise in voltage) call it positive, and when passing through an e.m.f. from + to − (a fall in voltage) call it negative. When passing through a resistance in the assumed direction of the current (a fall in voltage) call the

ohmic drop negative, and when passing through a resistance in a direction opposed to the assumed direction of current (a rise in voltage) call the ohmic drop positive. The algebraic sum of all the voltages thus taken around any closed circuit is zero.

A similar convention may be applied to the solution of magnetic circuits. The solution thus obtained will not be exact on account of the magnetic leakage which always occurs to a greater or lesser extent, but it will probably give an indication of the actual condition.

When these laws are applied to the solution of problems involving networks of condensers, each condenser should be treated as a source of e.m.f. and its terminals marked accordingly. A further extension of the first law is also necessary. About any *insulated* junction the charges must all be of the nature of induced charges and their algebraic sum must be zero.

If all these conventions are strictly adhered to the solution of problems will be greatly facilitated.

PROBLEMS

1. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively have their positive terminals connected to one end of a coil of 5 ohms resistance and their negative terminals connected to the opposite end. What current is each battery supplying to this resistance?

2. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively have their positive terminals connected to one end of a variable resistance and their negative terminals connected to the opposite end. For what value of this variable resistance will the currents through the batteries be the same?

3. In problem 2 for what value of the variable resistance will the current through one of the batteries be zero?

4. Two batteries whose e.m.fs. are 5 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively have their positive terminals connected to one end of a slide wire of 10 ohms resistance. The negative terminal of the first battery is attached to the extreme end of the slide wire, while the negative terminal of the second battery may be connected by a slider to any point on the wire. For what position of the slider will the current through the first battery have its greatest value?

5. An electrical circuit is arranged as in problem 4. For what position of the slider will the current through the second battery have its greatest value? its least value?

6. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively are connected in series in conjunction to a resistance of 10 ohms. A resistance of 5 ohms is now connected between the middle point of the 10-ohm resistance and the battery terminals which are in direct contact. What is the current in each part of the circuit?

7. Two batteries whose e.m.fs. are 5 volts and 10 volts and whose internal resistances are 1 ohm and 1.5 ohms respectively have their positive terminals connected to one end of a variable resistance and their negative terminals to the opposite end. For what value of this resistance will the heat generated in it be a maximum?

8. In problem 7 for what value of the variable resistance will the output of the 5-volt battery have its greatest value?

9. In problem 7 for what value of the variable resistance will the output of the 10-volt battery have its greatest value?

10. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 3 ohms and 1 ohm respectively are connected in series in conjunction to a slide wire of 10 ohms resistance. A resistance of 5 ohms is now connected between the battery terminals which are in direct contact and a point on the slide wire. For what point of contact on the slide wire will the current through the 5-ohm resistance be zero? What is the current through each battery for this case?

11. In problem 10 for what point of contact on the slide wire will the current through the 5-ohm resistance have its greatest value? What is this greatest current?

12. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 3 ohms and 1 ohm respectively are connected in series in opposition to a slide wire of 10 ohms. A resistance of 5 ohms is now connected between the battery terminals which are in direct contact and a point on the slide wire. For what point of contact on the slide wire will the current through the 5-ohm resistance have its least value? What is the current through each battery for this case?

13. In problem 12 for what point of contact on the slide wire will the current through the 5-ohm resistance have its greatest value? What is this greatest current?

14. Three dissimilar batteries have e.m.fs. of 10, 15, and 20 volts and internal resistances of 5, 3, and 2 ohms respectively. Their positive terminals are connected to a point A and their negative terminals to a point B. Find the current through each battery and the potential difference between A and B.

15. Two batteries A and B having e.m.fs. of 5 volts and 20 volts and internal resistances of 1 ohm and 2 ohms respectively are connected in parallel between points a and b. The e.m.fs. of both act from a to b. A third battery C is also connected between a and b and its resistance is 5 ohms. Find its e.m.f. in order that the power loss in C may equal the combined power loss in A and B.

16. Three batteries whose e.m.fs. are 10, 15, and 20 volts and whose internal resistances are 1, 2, and 3 ohms respectively have their positive terminals connected together. The negative terminals of the first two batteries are connected to the ends of a slide wire of 10 ohms resistance, and the negative terminal of the third battery is connected, through a resistance of 5 ohms, to a point on the slide wire by means of a slider. For what position of the slider will the current through the third battery have its greatest value?

17. In problem 16 for what position of the slider will the current through the third battery have its least value?

18. Three batteries whose e.m.fs. are 10, 15, and 20 volts and internal resistances 1, 2, and 3 ohms respectively have their positive terminals connected together. The negative terminals of the first and third are connected to the ends of a slide wire of 10 ohms resistance, and the negative terminal of the second is connected, through a resistance of 5 ohms, to a point on the slide wire by means of a slider. For what position of the slider will the potential difference across the terminals of the second battery have its greatest value? its least value?

19. Two batteries whose e.m.fs. are 10 volts and 20 volts and whose internal resistances are 2 ohms and 3 ohms respectively have their positive terminals connected together, and their negative terminals connected to the ends of a slide wire of 10 ohms resistance. A third battery whose e.m.f. is 15 volts and internal res. is 1 ohm has its negative terminal connected to the positive terminals of the other two and its positive terminal connected, through a resistance of 5 ohms, to the slide wire by means of a slider. For what position of the slider will the heating loss in the 5-ohm resistance have its greatest value?

20. In problem 19 for what position of the slider will the heating loss in the 5-ohm resistance have its least value?

21. A trolley road in the form of an oval 5 miles long has two cars on it taking 100 amperes and 50 amperes as shown in Fig. 6. The trolley wire is No. 00, having a resistance of 0.41

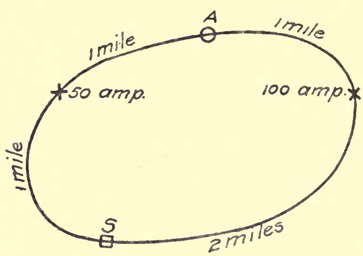

Fig. 6

ohm per mile, and a No. 0000 feeder having a resistance of 0.26 ohm per mile is carried from the station (S) to a point (A), a distance of 1.5 miles. If the resistance of the rail return is neglected and the station voltage is 550, what is the voltage at each car?

22. In problem 21 if the resistance of the rail return is 0.05 ohm per mile, what is the voltage between the trolley wire and the rail at each car?

23. A trolley road extends for a distance of 5 miles out from a 550-volt power station. The trolley wire is No. 00, which has a resistance of 0.41 ohm per mile, and a No. 0000 feeder, which has a resistance of 0.26 ohm per mile, is run in parallel with the trolley wire for a distance of 3 miles out from the station. The latter is tied to the trolley wire every quarter of a mile. The resistance of the rail return is 0.04 ohm per mile. There are two cars out on the line; the first, taking 80 amperes, is 2 miles from the station, and the other, taking 60 amperes, is 4 miles from the station. What is the voltage at each car?

24. The map of a trolley line is given in Fig. 7, and there are cars taking current at the points indicated. The trolley wire is

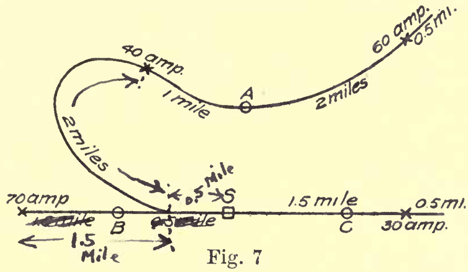

Fig. 7

No. 00, having a resistance of 0.41 ohm per mile, and three No. 0000 feeders, having a resistance of 0.26 ohm per mile, run from the station (S), the first to the point A, a distance of 1 mile, and the others to points B and C, one mile on either side of the station. The latter two are connected to the trolley wire every quarter of a mile. If the station voltage is 550 and the resistance of the rail return is neglected, what is the voltage at each car?

25. In problem 24 if the resistance of the rail return is 0.04 ohm per mile, what is the voltage between the trolley wire and the rail at each car?

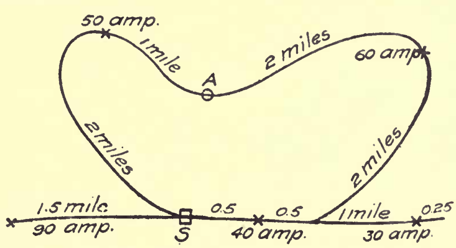

Fig. 8

26. The map of a small trolley road is given in Fig. 8, and there are five cars taking current as indicated. The trolley is all No. 00, having a resistance of 0.41 ohm per mile, and there are two No. 0000 feeders (resistance per mile = 0.26 ohm), one a mile long carried from the station (S) across the loop to a point A, and the other running along the main line 2 miles to the right and 1 mile to the left of the station. The latter is tied to the trolley wire every quarter of a mile. If the resistance of the rail return is 0.04 ohm per mile and the station voltage is 550, what is the voltage between the trolley wire and the rail at each car?

27. A trolley road is in the form of a loop 9 miles long, and the character of the country is such that on leaving the station a car takes 50 amperes for the first 3 miles, 90 amperes for the next 3 miles, and 20 amperes for the remaining 3 miles. Three cars run around the route at regular intervals (3 miles apart). If the resistance of the trolley wire is 0.41 ohm per mile and the resistance of the rail return be neglected, when and at what point will the voltage of the trolley have its least value? If the station voltage is 600 what will be this lowest voltage? If the resistance of the rail return is 0.05 ohm per mile what will be the lowest trolley voltage?

28. A 5-mile trolley road receives energy from a 550-volt power station which is situated at one end of the road. The trolley wire is No. 00 and a No. 0000 feeder is run from the station to a point 3 miles out on the line. The latter is connected to the trolley wire only at each end. The trolley wire has a resistance of 0.41 ohm per mile, the feeder 0.26 ohm per mile, and the rail return 0.04 ohm per mile. There are two cars, each taking 80 amperes, which start at the same time from the ends of the line and run toward each other at the same speed. What is the lowest voltage that exists between the trolley and the rail? At what point and when does the trolley voltage have this minimum value?

29. If in problem 28 the No. 0000 feeder is tied to the trolley every half mile and the cars are running as stated, what is the lowest voltage that exists between the trolley and the rail, at the car nearer the station? At what point and when does the trolley voltage have this minimum value?

30. A single-track trolley road extending 4 miles from the power station has a No. 00 trolley wire (resistance = 0.41 ohm per mile) and a No. 0000 feeder (resistance = 0.26 ohm per mile) which extends 2 miles out from the station. This feeder is tied to the trolley wire every half mile. There are two cars, one a mile and three-quarters from the station taking 90 amperes, and another at the end of the line taking 100 amperes. If the station voltage is 550 and the resistance of the rail return is neglected, what is the voltage at each car?

31. The circuits of a trolley road are arranged as in problem 30. There is a constant load of 30 amperes at the end of the line. At what point between points one and a half and two miles from the station will the voltage at a car taking 90 amperes have its least value?

32. A trolley road is in the form of a loop 9 miles long. The trolley wire is No. 00, and there are 3 miles of No. 0000 feeder extending from the station across the loop to a point on the line 4 miles (by rail) from the station. There are three cars running around the loop 3 miles apart, and each car takes 60 amperes continuously. The resistance of No. 00 trolley wire is 0.41 ohm per mile, of No. 0000 feeder 0.26 ohm per mile, and the resistance of the rail return is 0.035 ohm per mile. If the station voltage is 550 what is the lowest voltage out on the line? At what point and when will the trolley voltage have this least value?

33. The plan of a 230-volt 3-wire distribution system is given in Fig. 9. The normal current taken by each lamp across 110 volts is 1.0 ampere. Assuming that the resistance of the lamps is constant, what is the voltage across each set?

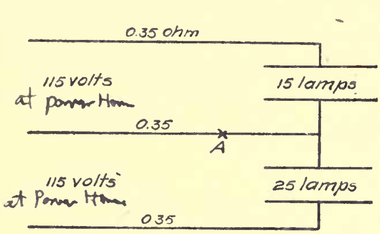

Fig. 9

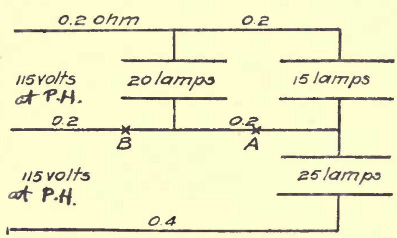

Fig. 10

34. In problem 33 if a fuse in the neutral conductor at A is blown and the resistance of the lamps is assumed constant, what will be the voltage across each set?

35. The plan of a 230-volt 3-wire distribution system is given in Fig. 10. The normal current taken by each lamp across 110 volts is 1.0 ampere. Assuming that the resistance of the lamps is constant, what is the voltage across each set?

36. In problem 35 if a fuse in the neutral conductor at *A* is blown and the resistance of the lamps is assumed constant, what will be the voltage across each set?

37. In problem 35 if a fuse in the neutral conductor at *B* is blown and the resistance of the lamps is assumed constant, what will be the voltage across each set?

38. The plan of a 230-volt 3-wire distribution system is given in Fig. 11. The normal current taken by one lamp across 110 volts is 1 ampere, and its resistance may be assumed constant. *M* is a series motor which is run intermittently. On starting this motor takes 100 amperes, but while running at full rated load it takes 60 amperes. What will be the voltage across the lamps when the motor is starting, running at full load and disconnected from the line? This method of putting the lamps all on one side and the motor on the other is a very bad arrangement. See problem 39.

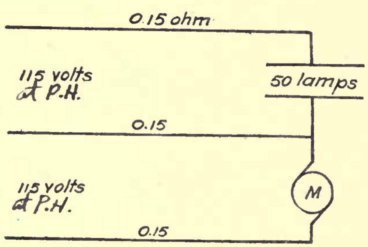

Fig. 11

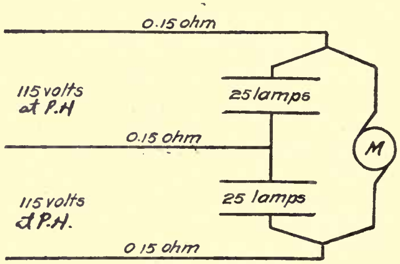

Fig. 12

39. The system given in problem 38 is rearranged as shown in Fig. 12. The lamp load has been divided, half of it on each side, and the 110-volt motor has been replaced by a 220-volt motor which takes 50 amperes on starting and 30 amperes while running at full rated load. Assume that each lamp has the same resistance as given in problem 38. What will be the voltage across each set of lamps when the motor is starting, running at full load and disconnected from the line? This is the best distribution of the given load. See problem 38.

40. The plan of an energy distribution system is given in Fig. 13. There are two lamp loads at the points indicated. The normal current taken by one lamp across 110 volts is 0.48 ampere and its resistance may be assumed constant. If the station voltage is 118 what is the voltage at each load?

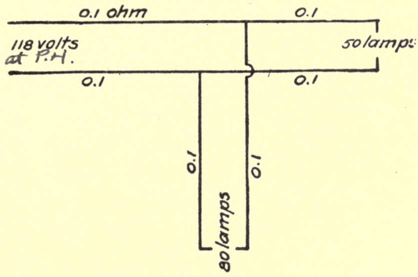

Fig. 13

41. In problem 40 replace the load of 50 lamps by a series motor. On starting this motor takes 50 amperes, but when running under full load the current is 30 amperes. What are the voltages at the 80-lamp load when the motor is starting, running at full load and disconnected from the line?

42. The plan of an energy distribution system is given in Fig. 14. There are lamp loads taking current at the points

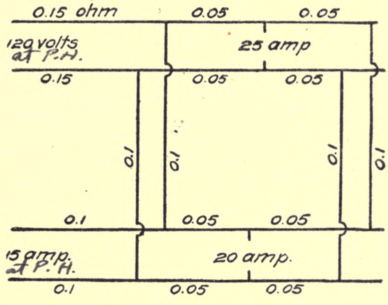

Fig. 14

indicated. If the station voltage is 120 what is the voltage at each load?

43. The plan of an energy distribution system is given in Fig. 15. There is a motor load at M taking 100 amperes. The system is fed from two power stations which are connected in parallel and whose voltages are 117 and 118, as indicated in the figure. What is the voltage at the motor load?

44. If in problem 43 the motor load at M is replaced by a load of 200 lamps what will be the voltage at the lamps? Each lamp takes a normal current of 0.52 ampere at 110 volts and its resistance may be assumed constant. The station voltages are given in problem 43.

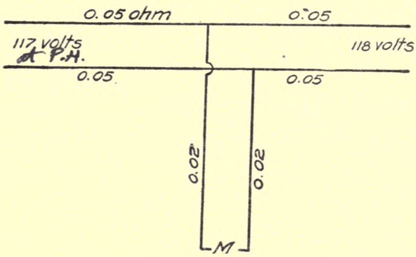

Fig. 15

45. In Fig. 15 if there are also 100 lamps connected in parallel with the motor load at M, what will be the extreme range of voltages at the lamps under the following conditions: The normal current taken by one lamp across 110 volts is 0.52 ampere and its resistance may be assumed constant. The current taken by the motor load varies from 0 to 200 amperes. The station voltages are as given in problem 43.

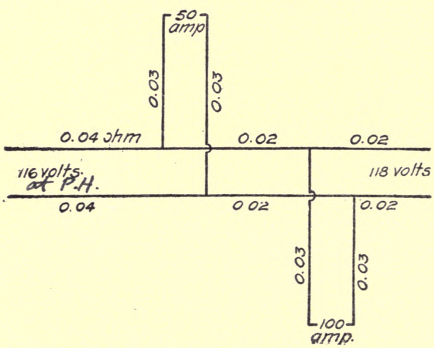

Fig. 16

46. The plan of an energy distribution system is given in Fig. 16, which shows two loads taking current as indicated. The system is fed from two stations which are connected in parallel and whose voltages are 116 and 118, as indicated. What is the voltage at each load?

47. A station voltmeter is connected to a 550-volt d. c. circuit as shown in Fig. 17. The total resistance of the transmission

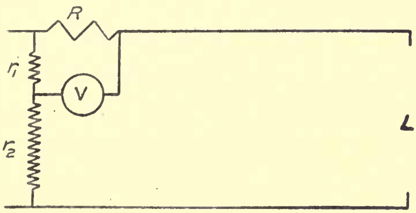

Fig. 17

line is R_0. R is a low resistance connected in series with the line, and a high resistance $(r_1 + r_2)$ is connected across the line. The voltmeter resistance is r. If the current taken by the voltmeter is negligible compared with the load current only, show that the voltmeter will indicate the voltage at the load L if r_1 and r_2 are in the correct ratio. What is this ratio in terms of R and R_o?

If the voltmeter has a 60-volt scale and a resistance of 5,680 ohms, and if the line resistance R_0 is 0.20 ohm and R is 0.024 ohm, what must r_1 and r_2 be so that the voltmeter will be direct reading (*i.e.* voltmeter reading of 50 indicates 500 volts at the load)?

48. Upon the middle member of a magnetic circuit each part of which has a cross section of 5 square inches and a length as shown in Fig. 18, there is a coil of 50 turns which carries

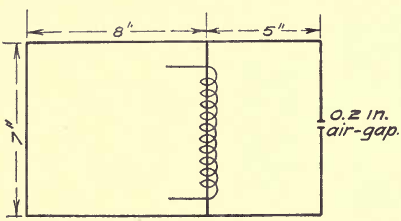

Fig. 18

11 amperes. Assume that μ equals ~~1200~~ for the iron, and neglect all leakage except across the given air gap. What is the flux density in each of the three parts of the magnetic circuit?

49. A magnetic circuit is composed of three parts, on two of which are wound coils *A* and *B*, of 30 and 50 turns respectively. The cross section of the magnetic circuit excepting the middle member is 4.5 square inches. The cross section of the middle member is 6.5 square inches. (See Fig. 19.) The coils *A* and

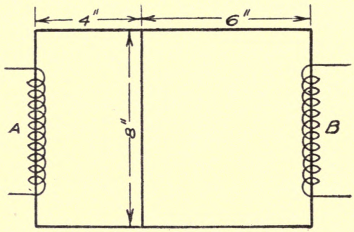

Fig. 19

B are in series carrying 6.5 amperes, and are wound so as to produce magnetic fields that oppose each other on their mutual circuit (in conjunction relative to the middle member). Assume that μ equals 1100 and neglect all leakage. What is the flux density in each part of the magnetic circuit?

50. In problem 49 what would be the new flux distribution if an air gap 0.35 inch long were cut in the middle member?

51. Three condensers are arranged as shown in Fig. 20. What is the ultimate charge on each?

52. Three condensers are arranged as in Fig. 20. What is the total energy of the system before and after the points *A* and *B* are short-circuited?

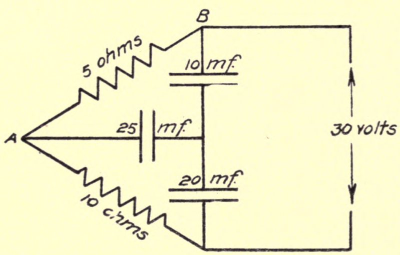

Fig. 20

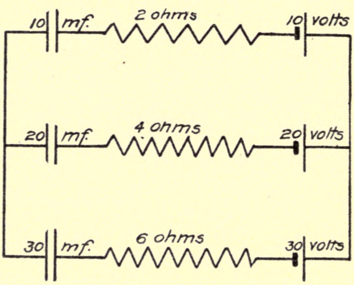

Fig. 21

53. Three condensers are arranged as shown in Fig. 21. What is the ultimate charge on each?

54. Three condensers are arranged as shown in Fig. 22. What is the ultimate charge on each?

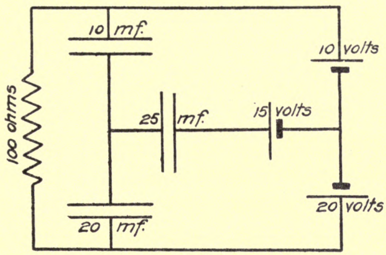

Fig. 22

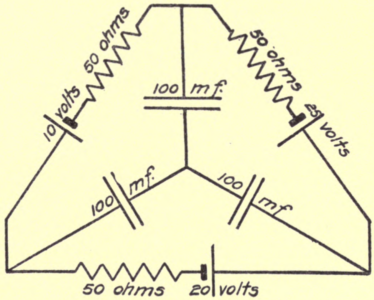

Fig. 23

55. Three condensers are arranged as shown in Fig. 23. What is the ultimate charge on each?

56. Four condensers are arranged as shown in Fig. 24. What are the ultimate charges on each before and after the points *A* and *B* are connected?

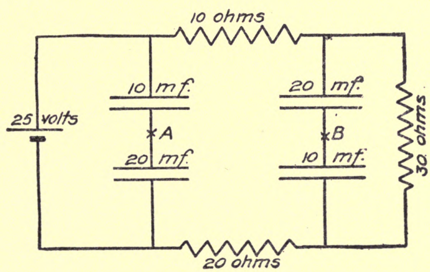

Fig. 24

57. Four condensers are arranged as shown in Fig. 24. By what amount will the combined energy of this system of condensers be changed if the points *A* and *B* are connected?

CHAPTER IV

ENERGY AND POWER

ENERGY is the capacity which a system possesses for doing work. The practical electrical unit of energy is the joule. It is the work that one coulomb of electricity will do when supplied at a difference in potential of one volt. Power is the rate at which energy is supplied to, or by, a system. The practical electrical unit of power is the watt. It is one coulomb per second, that is, one ampere, supplied at a difference in potential of one volt. The kilowatt-hour and the kilowatt are the commercial units of energy and power respectively. The kilowatt-hour is 36×10^5 joules and the kilowatt is 1000 watts.

Power being determined by the product of potential and current is expressed as $P = EI$. This expression for power is often conveniently transformed by means of Ohm's law to $P = I^2 R$ or $P = \frac{E^2}{R}\cdot$ The equivalent resistance, R, is the ratio of the voltage impressed on the circuit to the current in the circuit. It may or may not be the real ohmic resistance of the circuit.

The efficiency of any system is the ratio of the output to the input, or it may be conveniently considered as the ratio of the output to the output plus the losses. The loss in a battery or a transmission line, for example, is given by the square of the current into the resistance of the battery or the line.

The rate at which heat is radiated by a body is proportional to the area of the radiating surface and to the difference in temperature between the surface and the surrounding medium. If the heat is generated by an electric current we may write the following equation which holds after a state of equilibrium is reached, provided the difference in temperature $(T_1 - T_2)$ is small: $I^2 R = KA\,(T_1 - T_2)$. K is the rate at which heat is radiated per unit area per difference of one degree.

PROBLEMS

1. A battery whose e.m.f. is 10 volts and internal resistance 1 ohm is supplying current to a circuit of 4 ohms resistance. What is the power loss within the battery? What is its efficiency for this output?

2. A storage battery which has an open-circuit potential of 6.5 volts delivers 2 amperes at 6.1 volts to a resistance coil connected across its terminals. What per cent of the power delivered to the coil is wasted within the battery?

3. What is the maximum output that can be obtained from a battery whose e.m.f. is 6.3 volts and whose short-circuit current is 14.2 amperes?

4. A battery whose e.m.f. is 10.5 volts and whose internal resistance is 0.5 ohm is supplying current to a variable resistance placed across its terminals. For what value of this resistance will the output of the battery have its greatest value? What is the battery efficiency for this output?

5. If the greatest output that can be obtained from an 8.4-volt storage battery is 50 watts what will be its output when a resistance of 4 ohms is connected across its terminals? What is its efficiency for this output?

6. Two batteries whose e.m.fs. are 10 volts and 15 volts and whose internal resistances are 1 ohm and 2 ohms respectively are connected in series in conjunction with a variable resistance. For what value of this resistance will the combined output of the batteries have its greatest value? What is this output? What is the power loss in each battery? What is their combined efficiency?

7. In problem 6 if the batteries are connected in series, but in opposition, to the variable resistance, what is the greatest output of the combination? For this condition of maximum output what is the output of the 15-volt battery, and what is the power loss in each battery?

8. If the batteries in problem 6 had been connected in parallel to the variable resistance what would have been their maximum output? What would have been the power loss in each battery? What would have been the efficiency of each battery?

9. Across the terminals of a storage battery which has an e.m.f. of 8.7 volts and an internal resistance of 0.3 ohm there is permanently connected a resistance of 7 ohms. If another coil of unknown resistance is connected in parallel with the 7-ohm coil the potential difference at the battery terminals is 7.9 volts. What power is the unknown resistance taking?

10. In problem 9 if the unknown resistance takes a current of 3 amperes what is the potential difference at the battery terminals?

11. In problem 9 what is the greatest power that the coil of unknown resistance can take from the battery?

12. If the selling price of energy is 10 cents per kilowatt-hour what will it cost per day to operate an electric heater that takes 4.3 amperes at 110 volts for 10 hours each day?

13. A small storage battery which has an average e.m.f. of 6.1 volts and a resistance of 0.2 ohm while charging is connected in series with a suitable resistance across 110-volt mains. The battery is charged at a rate of 4 amperes for 15 hours. If the price of energy is 15 cents per kilowatt-hour what does it cost to charge the battery? What would it cost to charge two such batteries in series at the above rates of charge and cost?

14. In problem 13 what part of the energy taken from the mains is wasted in the series resistance when one battery is being charged? when two batteries are being charged?

15. If the safe carrying capacity of No. 10 bare copper wire is 50 amperes what is the safe carrying capacity of No. 00 under the same conditions?

16. If the safe carrying capacity of No. 4 bare aluminum wire is 45 amperes what is the safe carrying capacity of No. 6 copper wire insulated with 0.1-inch weather-proof insulation? Assume that the conductivity of aluminum is 62 per cent of that of copper at the upper temperatures, that the coefficient of radiation of the insulation is 1.3 that of aluminum, and also that the allowable temperature rise of the insulated wire is 70 per cent of that of the bare conductor.

17. A coil of resistance wire whose temperature coefficient is 0.0004 per degree fahrenheit above 75 degrees will reach an ultimate temperature of 150 deg. fahr. if connected across 110 volts. To what temperature will it rise if the impressed voltage is increased 30 per cent? The surrounding air is kept at a constant temperature of 75 deg. fahr.

18. A certain alloy has a specific resistance of 525 ohms per mil-foot at 75 deg. fahr., and a temperature coefficient of 0.0004 per degree fahrenheit above 75 degrees. A resistance coil of this material consisting of 130 feet of No. 10 wire (diameter = 0.102 inch) coiled in an open spiral is connected across 110-volt mains. The temperature of the surrounding air is kept constant at 75 deg. fahr., and the wire radiates heat at the rate of 0.01 watt per square inch per difference in temperature of 1 deg. fahr. What will be the ultimate temperature of the coil? At what rate is energy being supplied to the coil?

19. A resistance coil of 10.8 feet of No. 30 *IaIa* wire (diameter = 0.01 inch) is heated to a temperature of 1100 deg. fahr. when connected across 110 volts. No. 30 *IaIa* wire has a resistance of 3000 ohms per 1000 feet at this temperature. At what rate must energy be supplied from 110-volt mains to a coil of No. 10 *IaIa* wire (diameter = 0.1 inch) to maintain it at the same temperature?

20. A cylindrical tank, 6 feet in diameter, filled to a depth of 4 feet 6 inches with water, is in a room which is kept at a constant temperature of 20 deg. cent.; there is also some means for keeping the water at a uniform temperature. Experiment shows that if the water is heated to 47.2 deg. cent. and then allowed to cool the temperature decreases sensibly uniformly to 45.1 deg. cent. in 2 hours and 30 minutes. Design the most economical (least weight) heating coil of *IaIa* wire which when immersed in the tank and connected across 115 volts will keep the water at a constant temperature of 47 deg. cent. The resistivity of *IaIa* wire is 29 times that of copper, and the allowable current for No. 10 *IaIa* wire under these conditions (temperature, etc.) may be taken as 15 amperes. The chosen wire should be of a stock number (B and S. gauge) and its size may be calculated by the approximate method.

21. The plan of a 3-wire 230-volt distribution system is given in Fig. 25. The lamps each take 0.5 ampere. Find the resistance of one lamp in each set and the total ~~energy~~ power expended in each set of lamps.

22. The plan of a 3-wire 230-volt distribution system is given in Fig. 26. Draw the potential diagram showing the drop in each of the feeders, and find the impressed voltage at each load.

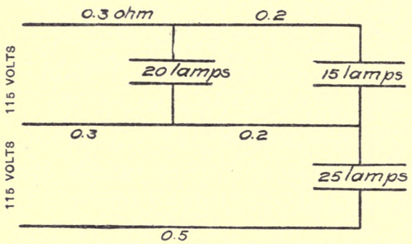

Fig. 25

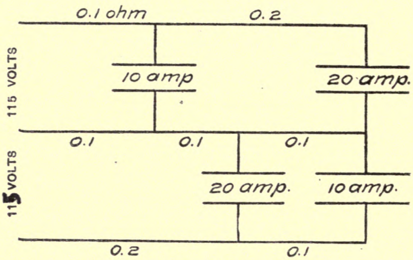

Fig. 26

23. A 3-wire 230-volt system has lamp loads applied at points as shown in Fig. 27. Each lamp takes 0.5 ampere and its candlepower varies approximately as the 6th power of the

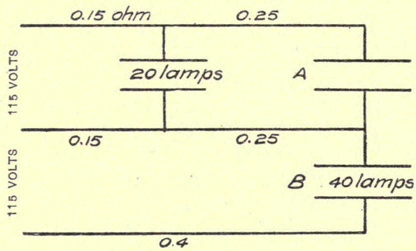

Fig. 27

voltage at its terminals. How many lamps must be turned on at *A* in order that the lamps at *A* and *B* will burn with the same brilliancy?

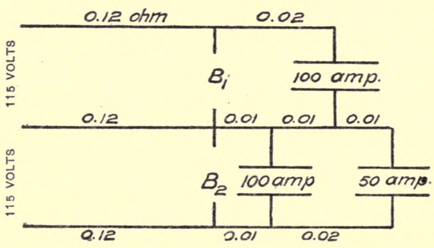

Fig. 28

24. The plan of a 3-wire 230-volt distribution system is given in Fig. 28, and the resistances of the conductors are indicated.

B_1 and B_2 are two storage batteries each of 51 cells in series (2.1 volts electromotive force and 0.001 ohm internal resistance per cell). Find the voltage at each of the lamp loads.

25. The plan of a 3-wire 230-volt system is shown in Fig. 29. Calculate the voltage at each lamp load. What will be the effect of breaking the neutral conductor at *A*.

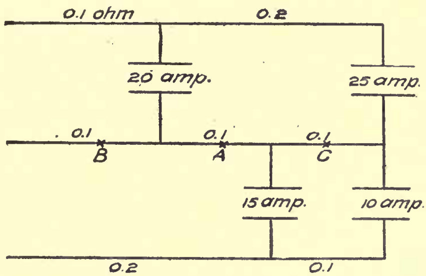

Fig. 29

26. In problem 25 assume that the effective resistances of the lamp loads remain constant. What will be the voltage at each load if the neutral conductor is broken at *B*?

27. In problem 25 assume that the effective resistances of the lamp loads remain constant. What will be the voltage at each load if the neutral conductor is broken at *C*?

28. A 550-volt station delivers 200 amperes to a consumer over a line which has a resistance of 0.30 ohm. What power does the consumer take from the line?

29. A 230-volt generator supplies energy over a short line which has a resistance of 0.54 ohm to a constant load of 15 kilowatts. If the existing line is replaced by one with conductors having double the cross-section, by what per cent will the efficiency of transmission be increased?

***30.** A 230-volt generator supplies energy over a short line which has conductors of No. 10 wire to a constant load of 12 kilowatts. If the potential difference at the load is 214 volts with what size conductors must the existing line be replaced to raise the potential difference at the load to 220 volts?

***31.** A load of 10 kilowatts is to be delivered at a point 190 feet from a 230-volt generator. What is the smallest size of conductors that can be used if the lowest allowable potential difference at the load is 215 volts?

* **32.** A 10-horsepower 220-volt motor which has an efficiency of 88 per cent at full load and fifty 20-candlepower incandescent lamps are installed at a distance of 150 feet from a 230-volt generator. The lamps take 0.3 ampere each at 230 volts and their resistance may be assumed constant. What is the smallest size conductors that should be run from the generator to the load so that the potential at the load will not fall below 220 volts when the motor is delivering 10 horsepower and the lamps are all burning?

33. A 600-volt station supplies energy to a trolley line which has a resistance of 0.41 ohm per mile. What will be the voltage at a car taking 70 amperes 2 miles from the station? What per cent of the power delivered by the station to the line is received at the car?

34. A 550-volt station delivers energy to a trolley line of 0.41 ohm resistance per mile. There is a single car 2 miles out from the station which is taking 90 horsepower from the line. What is the line loss?

35. A 230-volt power station delivers energy to a load taking 200 amperes over a 0.5 mile transmission line having conductors of 500,000 circular mils cross-section. What is the load in kilowatts? What is the efficiency of transmission? Assume that the resistance per mile of conductor is 55,000 ÷ circular mils.

36. In problem 35 if an additional load of 50 kilowatts is to be delivered at the same point, what size of conductors must be run in parallel with the line already constructed in order that the combined efficiency of the transmission shall be 90 per cent?

* **37.** A factory receives 25 kilowatts from a 600-volt power station 0.4 mile away over a line of No. 2 copper wire. Find the potential difference at the factory.

38. What is the maximum rate at which energy can be transmitted to a point 3 miles distant from a power station if the station voltage is 600 and the total line resistance 0.5 ohm per mile? What is the efficiency of transmission?

39. Two power stations 2 miles apart are connected by feeders which have a resistance of 0.05 ohm per mile. The voltage of one station is 550 and of the other 560. Where should a load which is connected across the feeders and whose equivalent resistance is 1 ohm be placed so that it will absorb the maximum possible power?

40. If the maximum rate at which energy can be transmitted from a 600-volt station over a line whose resistance is 0.3 ohm per mile to a certain mill is 100 kilowatts, what will be the transmission efficiency when 25 kilowatts are received at the mill?

41. A 600-volt power station is delivering 150 kilowatts to a transmission line whose resistance is 0.25 ohm. What is the load at the end of the line? What is the efficiency of transmission?

42. A 600-volt power station is to be located on the line between two factories, 2 miles apart, to one of which it delivers 150 amperes and to the other 200 amperes. If the circular milage of the line conductors is 300,000 find the location of the station for maximum efficiency of transmission.

43. A distribution system consists of two copper conductors between which is connected a uniformly distributed load that takes current at the rate of 100 amperes per mile of line. The length of the line is 2.5 miles, the total current is 250 amperes, the voltage at the generator end is 600, and each conductor has a cross-section of 220,000 circular mils. Find the voltage at a point on the line "x" miles from the generator. Find the rate at which a given portion of the line between points x_1 and x_2 is delivering energy.

44. At the end of a transmission line having a resistance of 0.32 ohm there is floated a storage battery of 246 cells. Each cell has a normal e.m.f. of 2.1 volts and an internal resistance of 0.0012 ohm. The station voltage is 550 volts. What current will the battery supply when there is a single load of 300 amperes at the end of the line? What will be the voltage at the load?

45. A 600-volt power station supplies energy to a transmission line of 0.8 ohm resistance at the end of which is floated a storage battery whose e.m.f. is 520 volts and internal resistance 1.2 ohms. When there is a single load of 150 amperes at the end of the line what proportion of it is supplied by the battery?

46. In problem 45 if the power taken at the end of the line is 75 kilowatts what will be the current supplied by the battery?

47. At the end of a transmission line having a resistance of 0.46 ohm there is floated a storage battery each cell of which has a normal e.m.f. of 2.1 volts and an internal resistance of 0.0024 ohm. If a constant load of 250 amperes is taken at the end of the line how many cells are required to keep the voltage at the load 85 per cent of the station voltage, which is 550 volts?

48. A transmission line having a resistance of 0.15 ohm runs from a 550-volt power station (operating continuously) to a load taking 600 amperes for 10 hours each day. There is a storage battery floated continuously across the line at the load. Each cell of this battery has a constant e.m.f. of 2.1 volts and an internal resistance of 0.0004 ohm. How many cells should be used in order that the total charge (current $\times$ time) shall be 5 per cent greater than the total discharge of the battery? What current will the battery supply to the load? What will be the voltage at the load? What is the load on the station when the battery is charging?

49. A 9-mile trolley line which receives its power from a 600-volt station has a storage battery whose e.m.f. is 500 volts and whose internal resistance is 1.5 ohms floated at the end of the line. The trolley wire is No. 00, which has a resistance of 0.41 ohm per mile, and the resistance of the rail return is 0.04 ohm per mile. A single car taking a constant current of 80 amperes is running on the line. What is the voltage at this car when it is 5 miles from the station?

50. In problem 49, with a single car taking 80 amperes at the end of the line, compare its voltage with what it would be if the storage battery were disconnected.

51. Two factories 3 miles apart are supplied with energy from a 600-volt power station situated 1 mile from the one taking the larger current and 2 miles from the other. The first factory takes 300 amperes from the line and the second 200 amperes. The amount of copper for the transmission lines is limited to a total weight of 40,000 pounds. What should be the size of the conductors for each line in order that the resultant efficiency of transmission may be a maximum? What is the maximum efficiency?

$$\text{Resistance per mile of conductor} = \frac{55{,}000}{A}$$

$$\text{Weight per mile of conductor} = \frac{A}{62.5}$$

wherein A is the cross-sectional area of the conductor in circular mils.

52. In problem 51 what should be the size of the conductors for each line in order that the voltages at the loads may be the same? What is the resultant efficiency in this case?

53. A factory running 10 hours per day is supplied with power from a constant potential 600-volt station over a transmission line of 0.24 ohm resistance. The load begins with 50 amperes at 7 a.m. and increases uniformly to 250 amperes at 9 a.m., at which time it becomes constant and remains so until 12 m. From 12 to 1 the factory is shut down. Between 1 and 4 p.m. the load is constant at 250 amperes, and it then increases uniformly to 350 amperes at 6 p.m., when the factory shuts down for the day. What is the all-day efficiency of transmission?

54. An over-compounded generator delivers power over a transmission line of 0.15 ohm resistance to a load which varies between zero and 450 amperes during the day. The no-load generator voltage is 525 volts and the full-load current is 500 amperes. What must be the percentage rise in voltage from no load to full load to exactly compensate for the line drop and thus supply constant voltage at the load?

55. The voltage of an over-compounded generator rises from 500 volts at no load to 550 volts at full load of 250 amperes. This generator supplies energy over a line of 0.25 ohm resistance to a load that increases uniformly from 50 amperes to 250 during the first 2 hours of the day, then remains constant at 250 amperes for 5 hours, and then decreases uniformly to zero during the last 3 hours. What is the total line loss per day? What is the all-day efficiency of transmission?

56. The voltage of an over-compounded generator rises from 500 volts at no load to 550 volts at full load of 500 amperes. This generator supplies power over a line of 0.15 ohm resistance to a load that increases uniformly from zero to 200 kilowatts during the first 4 hours of the day and then remains constant for the remaining 6 hours. What is the all-day efficiency of transmission?

57. A 600-volt power station delivers energy to a transmission line of 0.5 ohm resistance at the end of which is floated a storage battery whose e.m.f. is 530 volts and internal resistance 1.2 ohms. What is the maximum power that can be taken from the line halfway between the station and battery?

58. A 600-volt power station feeds a trolley line 8 miles long at the end of which is floated a storage battery whose e.m.f. is ~~[illegible]~~ 500 volts and internal resistance 1 ohms. A single car is running on the line and is taking a constant current of 60 amperes. If the combined resistance of the trolley and rail return is 0.46 ohm per mile how far will the car be from the station when its voltage has its least value?

59. Deduce the equation connecting the rate at which energy is delivered and the distance to which it will pay to transmit it under the following conditions: The energy is delivered at a constant rate for 10 hours a day and 300 days a year. The selling price of energy at the station is 7 cents per kilowatt-hour. The selling price of energy at the point of delivery is 10 cents per kilowatt-hour. The voltage at the station is 600 and the line efficiency is 90 per cent. Copper costs 20 cents per pound. The cost of the line exclusive of copper is $500 per mile, and 18 per cent of the total cost of the line must be allowed annually for interest and depreciation. Using the equation deduced above find the maximum distance to which it will pay to transmit 100 kilowatts, and the minimum power it will pay to transmit a distance of 2 miles. What will be the cost of the line in each case?

60. Power transmission problem. Data of costs:

Station and equipment $120 per kilowatt.

Transmission line exclusive of copper $500 per mile.

Copper on poles 20 cents per pound.

Generation of energy at switchboard 1 cent per kilowatt-hour.

Depreciation on the entire investment 8 per cent per year.

The station is run at full load capacity 12 hours a day for 300 days in the year, and each consumer is charged with his proportionate share of interest and depreciation on the station and equipment (this is fixed by the power delivered at the switchboard to the consumer's line). The station voltage is 600.

Under the above conditions determine the minimum selling price per kilowatt-hour for 100 kilowatts to be delivered over an independent line to a consumer 2 miles from the station in order that there may be a profit of 10 per cent on the entire investment. At what voltage would this power be delivered to the consumer?

$$\text{Weight per mile of copper wire} = \frac{A}{62.5}.$$

$$\text{Resistance per mile in ohms} = \frac{55{,}000}{A}.$$

Wherein A is the cross-sectional area of the conductor in circular mils.

CHAPTER V

DIRECT-CURRENT GENERATORS AND MOTORS

THE e.m.f. generated in the armature of a direct-current machine, either generator or motor, is proportional to the product of three factors; that is, $E_a \propto SN\phi$; S represents the speed, N the number of series conductors between positive and negative brushes, and ϕ represents the resultant air-gap flux. For a bipolar machine, or for one with a lap winding, the equation for this armature voltage is

$$E_a = \frac{S}{60} N\phi\, 10^{-8},$$

where N is the number of face conductors on the armature, S the speed of the armature in revolutions per minute, and ϕ is the air-gap flux (lines) per pole. The potential difference at the terminals of the armature of a generator is given by

$$E = E_a - IR,$$

where I is the armature current and R is the armature resistance between terminals. The potential difference at the terminals of the armature of a motor is given by

$$E = E_a + IR.$$

When the armature is carrying current there is a magnetomotive force developed (armature reaction) which reduces and distorts the field which would otherwise be produced by the main field windings. The effect of the armature reaction is, in well-designed machines, comparatively small, and since it is at best difficult to predetermine or to measure exactly, it will be neglected in all but a very few of the problems.

The open-circuit characteristic or magnetization curve shows the relation that exists between the armature voltage and the field current when the armature current is zero and the speed is constant. The load characteristic of a generator shows the relation that exists between the terminal voltage and the current supplied to the load when the generator is running at its rated speed.

The field circuit of a shunt generator is in parallel with the external circuit and thus the load current, I_0, equals $I_a - I_f$; I_a is the armature current and I_f is the shunt field current. On the other hand, the field winding of a shunt motor is in parallel with the armature winding, and the line current, I_0, equals $I_a + I_f$.

The electromagnetic torque developed in the armature of a direct-current motor varies as the product of two factors: that is, $D \propto I\phi$; I is the armature current and ϕ is the resultant air-gap flux. The electromagnetic output is proportional to the product of the electromagnetic torque and the speed of the armature. The mechanical output is the electromagnetic output less all of the losses due to the rotation of the armature. These losses are called the stray power. It includes the friction, windage, eddy current, and hysteresis losses. The stray power (P_s) is usually measured by running the machine light as a motor and measuring the electrical input to the armature. The stray power is then given by subtracting the armature copper losses, which are very small at no load, from this input. The stray power varies with the speed and the resultant air-gap flux, but not as the first power of either, although for small variations it is convenient to assume that it does. The stray power is small, varying from about 15 per cent of the output for small motors (1 hp. or thereabouts) to about 4 per cent or 5 per cent of the output for large motors (50 hp. and upwards). Thus a large variation in the stray power will make but a small variation in the efficiency, and in many cases it is sufficiently accurate to assume that the stray power is constant. At no load

$$P_s = E I_a - I_a^2 R = 0.0226 \times 2 \pi DS;$$

D is the electromagnetic torque in pound-feet and S is the speed in revolutions per minute. That is, at no load the stray power is equal to the product of 0.142, the electromagnetic torque and the speed. At any load the output is given by $0.142\ DS - P_s$.

The efficiency of a shunt generator is usually calculated from the formula

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{losses}}.$$

The efficiency of a shunt motor is usually calculated from the formula

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{losses}}{\text{Input}}.$$

PROBLEMS

1. Two shunt motors are *exactly* alike in every particular except that the armature of one is wound with twice the number of turns that are on the other. If the armature resistances are neglected what are the relative speeds and torques of the motors when they are connected in parallel across the same mains and loaded until they each take the same current? Compare the outputs of the motors under this condition.

2. Two shunt motors are *exactly* alike in every particular except that the armature of the first is wound with No. 18 wire and the armature of the second is wound with No. 15 wire. If these motors are connected in parallel across 110-volt mains, compare their outputs, torques, and speeds when the armature heating losses are equal.

3. If the motors in problem 2 are connected in parallel across 110-volt mains and the armatures are rigidly coupled together, compare their armature currents, torques, and armature heating losses when the motors are jointly delivering 10 horsepower.

4. Two *exactly* similar shunt motors are operating in parallel and each is taking the same armature current. If the field strength of the first motor is 75 per cent that of the second motor, compare the speeds, torques, and outputs of the two motors.

5. A separately excited motor, which has an armature resistance of 0.5 ohm, runs at 1200 rev. per min. when the armature takes 10 amperes from 110-volt mains. If the armature is rewound with twice the number of turns but with wire of one-half the cross-sectional area, at what speed will the motor run when the armature takes 5 amperes from 110-volt mains?

6. In problem 5 compare the torques, armature heating losses, and outputs for the two cases.

7. At no load a separately excited motor, which has an armature resistance of 0.2 ohm, runs at 1200 rev. per min. and takes an armature current of 3.5 amperes from 110-volt mains. If the armature is rewound with a 50 per cent greater number of turns the resistance is 0.5 ohm. At what speed must this machine now be driven to supply a charging current of 12 amperes to a storage battery which has an e.m.f. of 66 volts and an internal resistance of 0.8 ohm? The field excitation is maintained constant.

8. If the armature resistance of a shunt generator is neglected what will be the percentage effect on the open-circuit voltage of increasing both the speed and the resistance of the shunt field circuit 50 per cent?

9. A shunt motor and a series motor are connected in parallel. Compare the percentage changes in the speeds, torques, and outputs when the loads are readjusted so that the armature currents are double their former values. Neglect all losses and assume that the magnetization curve for each motor is a straight line.

10. A shunt motor and a series motor are connected in parallel across 220-volt mains. Compare the percentage changes in the speeds, torques, and outputs when the motors are connected in parallel across 110-volt mains and the loads are adjusted so that the currents are unchanged. Neglect all losses and assume that the magnetization curve for each motor is a straight line.

11. A shunt motor takes a total current of 10 amperes from 115-volt mains. If the resistances of the field and armature circuits are 100 ohms and 1 ohm respectively what are the currents in the field and in the armature windings? What are the heating losses in the field and in the armature windings?

12. At no load a 110-volt shunt motor, which has an armature resistance of 0.3 ohm, runs at 1200 rev. per min. and takes an armature current of 2.4 amperes. If the field strength is reduced 25 per cent, a resistance of 1.5 ohms is inserted in series with the armature winding and the motor is loaded until the armature current is 25 amperes, at what speed will the motor run? The motor is connected across 110-volt mains in each case.

13. At no load a small 110-volt shunt motor, which has an armature resistance of 1.8 ohms, runs at a speed of 1500 rev. per min. and takes an armature current of 0.8 ampere from 110-volt mains. What resistance should be inserted in series with the armature across 110-volt mains in order that the speed will be 500 rev. per min. when the armature current is 6 amperes and the field excitation is unchanged?

14. At no load a shunt motor, which has an armature resistance of 0.42 ohm, takes an armature current of 2.4 amperes and runs at 1200 rev. per min. when connected across 110-volt mains. If the field strength is reduced 40 per cent by increasing the resistance of the field circuit, what resistance should be inserted in series with the armature across 110-volts in order that the speed will still be 1200 rev. per min. at no load? In the latter case the armature current is 2.2 amperes.

15. In problem 14 if with the reduced field strength the motor is loaded so that the armature current is 20 amperes, what resistance should be inserted in series with the armature across 110-volts in order that the speed will still be 1200 rev. per min.?

16. A shunt motor which has an armature resistance of 0.85 ohm is connected across 110-volt mains. What is the back e.m.f. generated in the armature when the armature current is 9.6 amperes and the motor is running at 1100 rev. per min.? ~~The field current is 0.4 ampere.~~

17. If the motor in problem 16 is mechanically driven as a generator at a speed of 1250 rev. per min. at what terminal voltage will it deliver a current of 9.6 amperes? The field is maintained unchanged.

18. If the motor in problem 16 is mechanically driven as a generator so that it delivers a current of 9.6 amperes at a terminal potential of 110 volts, what is the e.m.f. generated in the armature winding and at what speed is the generator driven? The field is maintained unchanged.

19. In problem 16 if an e.m.f. of 55 volts is impressed on the armature, at what speed will the motor run when the armature current is 9.6 amperes and the field is maintained unchanged?

20. In problem 16 if the saturation curve of the motor is assumed to be a straight line, at what speed will the motor run when the impressed e.m.f. is 55 volts and the armature current is 9.6 amperes? The resistance of the field circuit is the same as in problem 16.

21. In problem 20 what are the heating losses in the armature and in the field circuit?

22. In problem 16 if the load is readjusted so that the heating loss in the armature winding is equal to that in the field circuit, what current does the motor take from the mains?

23. The armature winding of the motor in problem 16 is connected across a storage battery which has an internal resistance of 0.92 ohm. When the battery is supplying the motor with 9.6 amperes at a terminal potential of 110 volts the motor runs at 1100 rev. per min. At what speed does the motor run when the armature current is 2 amperes? The field of the motor is separately excited and maintained unchanged.

24. In problem 23 at what speed must the motor be mechanically driven in order that it will act as a generator and supply a charging current of 15 amperes to the battery? The field of the motor is maintained unchanged.

25. In problem 23 at what speed will the motor run if the load is decreased until the output of the battery is 200 watts? The field of the motor is maintained unchanged.

26. In problem 23 if the speed of the motor is maintained constant at 1100 rev. per min. and the field is increased 10 per cent, what is the armature current? If the field is increased 30 per cent, what is the armature current?

27. A 5-hp. shunt motor which has an armature resistance of 0.65 ohm is connected across 230-volt mains. At no load this motor takes an armature current of 1.4 amperes and runs at 1200 rev. per min. If the speed is maintained at 1200 rev. per min. and the field is increased 5 per cent, what is the armature current?

28. A 110-volt shunt motor with *constant* field excitation delivers 5 hp. at a speed of 1150 rev. per min. At this time the armature, which has a resistance of 0.15 ohm, takes a current of 42 amperes from the 110-volt mains. What resistance must be inserted in series with the armature across 110-volt mains in order that the motor may give a torque of 20 pound-feet and run at a speed of 500 rev. per min.? Neglect the stray power losses.

29. In problem 28 the greatest allowable armature current is 50 amperes. If a resistance of 0.45 ohm is connected in series with the armature across 110-volt mains what is the greatest output that should be taken from the motor? What are the torque and speed of the motor under these conditions? Neglect the stray power losses.

30. A 10-hp. 110-volt shunt motor, which has an armature resistance of 0.078 ohm, is connected across 110-volt mains and is delivering 9 hp. at a speed of 1175 rev. per min. At this time the armature current is 70 amperes. If the field is increased 50 per cent at what speed will the motor run when it gives a torque of 60 pound-feet? Neglect the stray power.

31. The field of the motor in problem 30 is readjusted so that the motor runs at 1000 rev. per min., when it gives a torque of 50 pound-feet. By what per cent has the field been changed and what current does the armature take under these conditions? Neglect the stray power.

32. A series motor, which has a total resistance of 0.4 ohm, takes a current of 30 amperes from 110-volt mains and runs at a speed of 1200 rev. per min. At what speed will the motor run when the armature current is 20 amperes? Assume that the magnetization curve for this motor is a straight line.

33. The following data are given on an Edison generator whose output is rated as 100 amperes at 115 volts for a speed of 1200 rev. per min. The armature resistance is 0.042 ohm. The no load armature current, when running as a motor at a speed of 1200 rev. per min. is 5.35 amperes if the impressed voltage is 115 volts. What will be the speed when running as a motor and taking an armature current of 80 amperes at an impressed voltage of 110? What resistance should be placed in series with the armature so that the speed will be 1000 rev. per min. for an impressed voltage of 115 and an armature current of 90 amperes? The field is maintained constant.

34. In problem 33 if the field is increased 30 per cent at what speed will this machine run as a motor with an impressed voltage of 115 and an armature current of 100 amperes? At what speed will it be necessary to drive this machine so that it will act as a generator and deliver 100 amperes at 120 volts? The field current is 3.5 amperes, and the air-gap flux is the same as in problem 33.

35. Two such machines (problem 33) are connected in series. If the first is driven as a generator at a speed of 800 rev. per min. and delivers 85 amperes to the second what resistance must be placed in series with the armature winding of the second so that it will run as a motor at a speed of 500 rev. per min. The field currents are each 3.5 amperes, and the air-gap fluxes are the same as in problem 33.

36. The following data are given on a 10 K.W ~~7.5-kw.~~ 135 ~~110~~-volt generator. The armature is 8 inches in diameter and each conductor is 8 inches long (neglecting the end connections). There are 192 face conductors on the armature and 24 commutator segments. The armature resistance is 0.072 ~~0.034~~ ohm. The generator is bipolar and there are 1300 turns in each of the shunt field coils. The no load air-gap flux is 2.44 megalines and the flux density is 30,000 lines per square inch. The full load resultant air-gap flux is 2.55 megalines. The shunt field current is 2.94 amperes at no load and 3.67 amperes at full load. The speed is 1600 rev. per min. What are the no load and full load armature voltages? What is the full load terminal voltage?

37. In problem 36 if the full load brush lead is 2 commutator segments what are the demagnetizing and cross-magnetizing ampere turns on the armature at full load?

38. In problem 36, assuming that the flux density in the air gap is uniform under the pole faces and that the flux lines are radial, what is the force on one conductor when it is under a pole face and is carrying full load current? Full load flux is 2.55 megalines.

39. In problem 36, assuming that the flux density in the air gap is uniform under the pole faces and that the flux lines are radial, what is the electromagnetic torque in pound-feet exerted on the armature when it is carrying full load current? Full load flux is 2.55 megalines.

40. In problem 36 if there are 48 slots in the armature and 10 leakage lines per ampere per inch of embedded conductor (neglect end connections) what is the leakage inductance ~~reactance~~ per slot? What is the inductance ~~reactance~~ per coil on the armature?

41. In problem 36 if the resistance and inductance ~~reactance~~ per coil are 0.0120 ohm and 0.03 mil-henry respectively, how long after the coil is short-circuited, when carrying full load current, will the current have fallen to 5 amperes? The average contact resistance and resistance of the carbon brush during the short circuit of the coil may be taken as 0.004 ohm. Assume that during this time the conductors of this coil are cutting no flux.

42. In problem 36 if the commutator segments are $\frac{7}{16}$ inch wide and the insulation between them $\frac{1}{16}$ inch wide what should be the length of the arc of contact of the brush in order that during short circuit the current in a coil will fall to 5 amperes? Assume that during this time the conductors of this coil are cutting no flux.

43. In problem 36 if an inter-pole, with a cross-section 1 inch by 6 inches, is used to assist commutation what should be the flux density under the inter-pole in order that the current in the armature coil may be reversed to full load value during the period of short circuit? The commutator segments are $\frac{7}{16}$ inch wide, the insulation between them is $\frac{1}{16}$ inch wide, and the arc of brush contact is $\frac{3}{4}$ inch.

44. A 230-volt, 15-hp. shunt motor with a rated speed of 1200 rev. per min. is to be tested under full load. The output is measured by means of a prony brake which has a 2.13-foot arm. What is the greatest pull that the brake balance should be capable of registering? If the normal field current is 2 amperes what is the balance reading when the motor takes 75 amperes from the 230-volt mains? Assume that the full load current is 57 amperes.

45. In problem 44 assume that the motor has an efficiency of 85 per cent and that the copper losses in the armature and field windings are each equal to one-half of the stray power. What are the greatest deflections that ammeters in the field and armature should be capable of registering when the motor is delivering 15 hp.? What are the approximate resistances of the armature and the field windings? What current does the motor take at no load?

46. The output of a motor is measured with a prony brake which has a 2-foot arm. The speed is 1150 rev. per min. and the balance reading (corrected for zero) is 15.8 pounds. The brake arm is inclined 10 degrees from the horizontal, while the balance pulls in a vertical line. What is the brake output of the motor?

47. A 5-hp., 220-volt shunt motor is used to drive a 2-kw., 230-volt shunt generator. Assume that the efficiencies of motor and generator are each 80 per cent and that the copper losses in the armatures and field windings are each equal to the stray power losses in the respective machines. What are the greatest deflections that the instruments used to measure the input to the motor and the output of the generator should be capable of indicating when the generator is delivering rated full load?

48. In problem 47 if the available voltage is 230 volts, calculate the size of the rheostat (resistance and current-carrying capacity) to connect in series with the motor so that it will operate at 220 volts when the generator is delivering rated full load.

49. In problem 47 calculate the size of the rheostat (resistance and current-carrying capacity) to connect in series with the armature of the motor when its resistance is measured with full load current. The available voltage is 230 volts. What are the greatest deflections that the instruments used to determine the armature resistance should be capable of indicating?

50. In problem 47 calculate the size of the rheostat (resistance and current-carrying capacity) to connect in series with the armature of the generator when its resistance is measured with full load current. The available voltage is 230 volts. What are the greatest deflections that the instruments used to determine the armature resistance should be capable of indicating?

51. The magnetization curve of a d. c. machine which has an armature resistance of 1.1 ohms is given in Fig. 30. This curve is for a speed of 1400 rev. per min. What is the resistance of the shunt field circuit when the no load terminal potential is 110 volts? What is the no load terminal potential when the resistance of the shunt field circuit is 130 ohms? In each case the speed of the generator is 1400 rev. per min.

52. If the generator in problem 51 is run at a speed of 1000 rev. per min. what is the no load terminal potential when the resistance of the shunt field circuit is 130 ohms? If this machine is connected across 110-volt mains and run at no load as a motor what is the speed when the resistance of the shunt field circuit is 100 ohms? The no load armature current, which is about 0.7 ampere, may be neglected.

53. The machine whose magnetization curve is given in Fig. 30 is run as a motor with a shunt field resistance of 120 ohms. Neglecting the no load armature current what is the speed of the motor when the impressed voltage is 130 volts? 110 volts? 55 volts?

54. The machine in problem 51 is connected across 110-volt mains and run as a motor under load so that the armature current is 15 amperes and the speed is 1400 rev. per min. If the armature reactions are neglected what is the resistance of the shunt field circuit? What is the total power taken from the mains?

55. The magnetization curve of a shunt generator which has an armature resistance of 1.1 ohms is given in Fig. 30. Assuming that 0.33 ampere in the shunt field is required to neutralize the

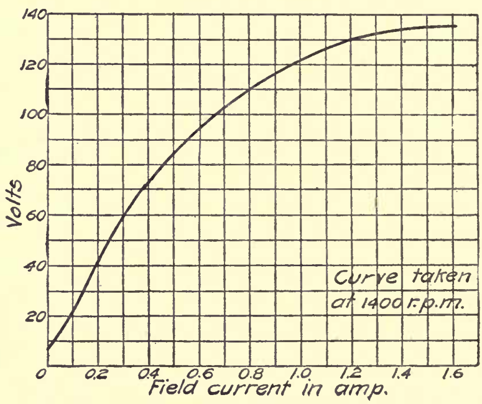

Fig. 30

armature reaction, what field resistance will give a terminal voltage of 115 when the armature current is 10 amperes? With this field resistance what will be the terminal voltage of the machine at no load and at a speed of 900 rev. per min.?

56. The machine in problem 51 is bipolar and has 4500 turns in its shunt field winding and 1200 face conductors on the armature, 25 per cent of which are effective in producing demagnetizing turns. If the shunt field is adjusted to give 110 volts at no load how many series turns are required on each field pole in order that the generator may have a terminal potential of 110 volts when there is an armature current of 10 amperes? Assume the short shunt connection.

57. The machine in problem 51 is bipolar and has 4500 turns in its shunt field and 1200 face conductors, 25 per cent of which are effective in producing demagnetizing turns. The machine is connected across 115-volt mains and run as a motor. When the armature current is 12 amperes, what should be the resistance of the shunt field circuit to give a speed of 1400 rev. per min.?

58. The magnetization curve of a compound motor is given in Fig. 30, in which the scale of abscissas is in shunt field current. The machine is bipolar and has 4500 turns in its shunt field winding and 1200 face conductors on the armature, 25 per cent of which are effective in producing demagnetizing turns. The number of turns in the series coils is 225 and the armature resistance is 1.1 ohms. With an armature current of 10 amperes and an impressed voltage of 115 what is the resistance of the shunt field circuit when the machine runs at a speed of 1400 rev. per min.? With this field resistance what will the speed be when the armature current is 6 amperes?

59. In problem 58 if the series windings are connected differentially what is the resistance of the shunt field circuit when the machine runs at 1400 rev. per min. and takes an armature current of 10 amperes from 115-volt mains?

60. The saturation curve of a 275-kw. 550-volt direct-current generator is given by the following data:

Field current	Armature voltage	Field current	Armature voltage
2.0	250	8.2	585
3.1	350	9.2	610
4.1	425	10.2	630
5.1	480	11.2	650
6.1	520	13.0	680
7.1	555	15.3	705

Plot the portion of the curve used in the calculation to a scale at least as large as 50 volts to the inch.

This generator has 12 poles. The armature is a lap-wound drum winding with 1920 face conductors. The commutator has 960 segments and at full load the brush lead is 5 segments. The shunt field coils have 2700 turns each. The armature has a resistance of 0.025 ohm between terminals. The generator is driven at a speed of 250 rev. per min.

If the no load air-gap flux is 7.50 megalines what is the no load terminal voltage? What is the resistance of the shunt field circuit that will give this open-circuit voltage?

61. In problem 60 assume that the total armature reaction is given by increasing the armature demagnetizing ampere turns by 25 per cent. With the shunt field adjusted to give 600 volts at no load, how many series turns per pole are needed to keep the terminal potential at 600 volts when the generator is delivering full load?

62. A shunt motor has an armature resistance of 0.026 and a field resistance of 16.4 ohms. When running light it takes 11.2 amperes from 115-volt mains. What are the heating losses in the armature and in the field when the motor is running light? What is the stray power?

63. In problem 62 if the stray power is assumed constant what is the horsepower output when the motor takes 150 amperes from 115-volt mains?

64. In problem 62 if the stray power is assumed constant what is the efficiency when running as a generator and delivering 150 amperes at 115 volts? The resistance of the field circuit is reduced to 14 ohms to maintain the terminal voltage.

65. At no load a shunt motor takes an armature current of 4.1 amperes from 115-volt mains. The armature resistance is 0.1 ohm and the field current is 2.5 amperes. What is the stray power? The motor is loaded until the armature current is 50 amperes. If the stray power is assumed constant what is the horsepower output of the motor when loaded? What is the efficiency at this load?

66. A shunt motor is used to drive a lathe and it is observed that the motor takes 22 amperes at 115 volts. When running idle the motor takes 3.2 amperes at 115 volts. In each case the field resistance is 95 ohms and the armature resistance is 0.24 ohm. What power did the motor deliver to the lathe?

67. When a shunt motor is taking 12.6 kw. at 115 volts, with a field current of 4.6 amperes, the measured output is 14.2 hp. What is the efficiency of the motor when delivering 8 hp.? Assume that the field current and the stray power are constant. The armature resistance is 0.05 ohm.

68. A 10-hp., 230-volt shunt motor is rated to take 39 amperes at full load. What is its commercial efficiency? If the full load speed is 1200 rev. per min. what is the full load torque?

69. A shunt motor has an armature resistance of 0.8 ohm and a field resistance of 120 ohms. When the motor takes 12 amperes from 115-volt mains it delivers 1.4 hp. at the pulley. What is the stray power? If the stray power is assumed constant what is the efficiency of the machine when driven as a generator and delivering 12 amperes at 115 volts? For the latter case the resistance of the field circuit is 100 ohms.

70. The stray power of a shunt motor is 346 watts (assumed constant). The armature resistance is 0.44 ohm and the field resistance is 76 ohms. What current will this motor take from 115-volt mains when running light?

71. In problem 70 what horsepower will this motor deliver when taking 18 amperes from 115-volt mains?

72. If the machine in problem 70 is run as a generator and delivers 18 amperes at 115 volts what horsepower is required to drive the generator? The resistance of the field circuit is reduced to 64 ohms in order to maintain the terminal voltage constant.

73. A shunt motor mechanically connected to a centrifugal pump takes 30 amperes from 230-volt mains and runs at 1000 rev. per min. The field current is 1.2 amperes. If this motor is disconnected from the pump and run under no load at a speed of 1000 rev. per min. the armature current is 2.3 amperes. The measured armature resistance is 0.31 ohm. What power did the motor supply to the pump? Assume that the stray power varies directly as the armature voltage.

74. A shunt motor is loaded by means of a prony brake and the following data are taken. The current and impressed voltage are 86.4 amperes and 116 volts respectively. The field current is 4.2 amperes. The speed is 1220 rev. per min. The balance reading corrected for zero is 22 pounds and the brake arm is 2 feet 2 inches. The armature resistance is 0.074 ohm. What are the armature heating loss, the field heating loss, and the stray power? What is the commercial efficiency of the motor at this load?

75. A shunt generator whose output is rated as 25 kw. at 220 volts has an armature resistance of 0.068 ohm. When running light as a motor with an impressed voltage of 210 the armature current is 6.1 amperes. What is its efficiency as a generator when delivering its rated full load? The resistance of the shunt field circuit is 60 ohms. Assume that the stray power is constant.

76. A 10-hp. series motor which has an armature resistance of 0.08 ohm and a field resistance of 0.085 ohm is connected across 110-volt mains. When this motor takes a current of 60 amperes it delivers 7.2 hp. and runs at a speed of 1050 rev. per min. What are the speed, torque, and output when the armature current is 80 amperes? Assume that the magnetization curve for this motor is a straight line and that the stray power is constant.

77. If an e.m.f. of 220 volts is impressed on the armature of a separately excited motor (A) it runs at 1600 rev. per min. and the armature current is 1.5 amperes. If an e.m.f. of 110 volts is impressed on the armature of a separately excited motor (B) it runs at 1200 rev. per min. and the armature current is 2 amperes. If the armatures of these motors are connected in series what is the speed of the first motor (A) when the second motor (B) is mechanically driven as a generator at a speed of 800 rev. per min. and the current through the armatures is 2.5 amperes? The armature resistances of the motors A and B are 3.5 ohms and 2.1 ohms respectively.

78. In problem 77 if the stray power is assumed to vary directly with the speed what is the input to the second motor, B? What is the output of the first motor, A?

79. A generator is rated to deliver 125 amperes at 110 volts for a speed of 1200 rev. per min. When running light as a motor at 1200 rev. per min. it takes an armature current of 9.8 amperes with an impressed voltage of 110. Calculate the efficiency as a generator at one-fourth, one-half, three-fourths full, and 25 per cent overload, assuming that the corresponding field currents are 4.4 amperes, 4.6, 4.9, 5.2, and 5.6 amperes, these being necessary to maintain a constant potential difference of 110 volts. The armature resistance is 0.042 ohm. Assume that the stray power varies directly with the armature voltage. Plot the efficiency curve.

80. The armature current taken by a 25-kw. 110-volt shunt generator when running light as a motor at a speed of 1200 rev. per min. is 13.6 amperes at 109 volts; the armature resistance is 0.016 ohm. This generator is belt driven at 1200 rev. per min. by a 40-hp., 220-volt shunt motor. When the generator is delivering its rated full load the motor operates at an efficiency of 90 per cent. The belt loss is 0.45 hp. and the field current of the generator at full load is 7.8 amperes. What current does the motor take from 220-volt mains when there is full load on the generator? If the motor field loss is 30 per cent of its total loss what is the field current?

81. In problem 80 if the motor armature resistance is 0.051 ohm what is the stray power of the motor for an armature voltage of 220 and a speed of 1200 rev. per min.? Assume that the stray power varies directly as the armature voltage.

82. The following data are given on an Edison generator whose rated output is 100 amperes at 105 volts for a speed of 1200 rev. per min. The armature resistance is 0.052 ohm. At no load when running light as a motor at a speed of 1200 rev. per min. the armature current is 5.3 amperes for an impressed voltage of 110 volts. What is the efficiency of this machine as a generator when delivering one-half rated load at 105 volts? What is the full load efficiency? The field currents for these two loads are 4.5 and 4.9 amperes respectively, these being necessary to maintain a constant terminal potential difference of 105 volts. Assume that the stray power varies directly as the armature voltage.

83. In problem 82 what is the output of the machine when running as a motor at a speed of 1200 rev. per min. if 8.5 kilowatts are supplied to the armature at 115 volts? Assume that the stray power varies directly as the armature voltage.

84. A shunt generator with a rated output of 50 kilowatts at 220 volts has an efficiency of 91 per cent at full load, at which time the field current is 7.1 amperes. The armature resistance is 0.028 ohm. This generator is driven at no load with a terminal potential of 230 volts by a small shunt motor which has an efficiency of 81 per cent. The field current of the generator is 6.6 amperes at no load. What current will the motor take from 115-volt mains? Assume that the stray power of the generator is constant.

85. A 25-hp. shunt motor when running light takes 12.5 amperes from 110-volt mains. The field current is 6.5 amperes. This motor drives a 20-kw. 115-volt shunt generator which has an efficiency of 90 per cent at full load. The belt loss is 0.32 horsepower. What is the motor input at 110 volts when the generator is operating under full load? For this condition the field currents of the motor and generator are each 6.0 amperes. The armature resistances of the motor and generator are each 0.025 ohm. Assume that the stray power of the motor is constant. ~~The first generator is rated as supplying 250 amperes at 115 volts~~.

86. When a compound generator is run at no load as a shunt motor with the series coils short-circuited it takes 14.8 amperes from 110-volt mains and runs at 1000 rev. per min. The resistance of the armature winding is 0.016 ohm, of the shunt field winding 18 ohms, and of the series field winding 0.010 ohm. What is the horsepower input to this machine when driven as a compound generator at a speed of 1000 rev. per min. and delivering 180 amperes at 110 volts? Assume that the stray power varies directly as the armature voltage.

87. When the machine in problem 86 is driven as a shunt generator (series coils cut out) at 1000 rev. per min. and is delivering 180 amperes, the terminal potential falls to 100 volts. What is the horsepower input to the machine under this condition?

88. The armatures of two direct-current shunt generators are coupled together to form a motor-generator set. When running light with an impressed voltage of 550 on the motor the total input is 2.5 kw. The field currents of the motor and generator are 0.93 amperes and 4.2 amperes respectively and the generator potential difference is 110 volts. The armature resistances of the motor and generator are 0.38 ohm and 0.016 ohm respectively. What is the current output of the generator at 110 volts when the motor takes 25.0 kw. from the line at 560 volts, the field currents of the motor and generator being 1.4 and 6.8 amperes respectively? Assume that the stray power of the whole set is constant.

89. The armatures of two separately excited motors which have resistances of 1.2 ohms and 2.0 ohms respectively are connected in series across 110-volt mains. With the armature of the first motor blocked and the second motor running light the current is 0.9 ampere, and the second motor runs at 1585 rev. per min. With the armature of the second motor blocked and the first motor running light the current is 1.4 amperes, and the first motor runs at 1170 rev. per min. Assume that the stray power varies directly ~~with~~ as the square of the speed. At what speed will each motor be running when both armatures are free.

90. In problem 89 if the second motor is mechanically driven at a speed of 1000 rev. per min. in the direction that it freely rotates, at what speed will the first motor run? What is the current taken by the armatures? What is the voltage across each motor? If the second motor is driven at 1000 rev. per min. in the opposite direction, at what speed will the first motor run? What are the armature current and the voltages across each motor?

91. In problem 89 if the first motor is mechanically driven at a speed of 1500 rev. per min. in the direction that it freely rotates, at what speed will the second motor run? What is the current taken by the armatures? What is the voltage across each motor? If the second motor is driven at 1500 rev. per min. in the opposite direction at what speed will the first motor run? What are the armature current and the voltage across each motor?

92. In problem 89 if the first motor is running light what is the greatest torque that the armature of the second will give? What is the speed of each motor?

93. In problem 89 if the second motor is mechanically driven at a speed of 1000 rev. per min. in the direction that it freely rotates what is the armature current when the first motor is delivering 0.[redacted] hp.? At what speed does the first motor run?

94. In problem 89 if the armatures of the motors are mechanically coupled together at what speed will they run? What is the armature current taken by the motors? What is the voltage across each motor?

95. In problem 89 if the armatures of the motors are mechanically coupled together at what speed will they run when their combined output is ~~[illegible]~~ 0.2 hp.?

96. A 110-volt shunt motor, which has an armature resistance of 0.2 ohm, runs at a speed of 1200 rev. per min. and takes an armature current of 3.5 amperes from 110-volt mains at no load. With the same field excitation, what resistance should be inserted in series with the armature across 110-volt mains in order that the speed may be 900 rev. per min. at no load? Assume that the stray power varies directly as the speed.

97. In problem 96, with the same field excitation, what resistance should be inserted in series with the armature across 110-volt mains in order that the motor may deliver 3 hp. at a speed of 900 rev. per min.? Assume that the stray power varies directly as the speed.

98. In problem 96, with the same field excitation, what resistance should be inserted in series with the armature across 110-volt mains in order that the motor may run at 900 rev. per min. and give a torque of 20 pound-feet at the pulley? Assume that the stray power varies directly as the speed.

99. A balancer set consisting of two shunt machines is connected across the mains and neutral of a 220-volt 3-wire system. At no load the armature current is 5 amperes and the potential across each machine is 110 volts. The armature resistances are each 0.04 ohm and the resistance of each conductor between the balancer and the load is 0.03 ohm. There is no neutral conductor running from the power station to the balancer. The load is unbalanced, there being 150 amperes on one side and 50 amperes on the other. What are the voltages between the mains and the neutral at the load? Assume that the ~~stray power losses in the balancer are constant.~~ voltages across mains are 220 Vo[lts] under all conditions.

100. Two similar shunt motors are mechanically coupled together and their armatures are connected in series across 220-volt mains. At no load and with normal field excitations the armature current is 4 amperes. The voltages across them are equal and the motors run at 1200 rev. per min. If the excitation of one motor is increased 30 per cent at what speed do they run? What is the voltage across each armature? Neglect the armature resistance drop. ~~and assume that the stray power is constant.~~

101. Two shunt generators with electromotive forces of 120 volts and 115 volts, armature resistances of 0.05 and 0.04 ohm, and field resistances of 20 and 25 ohms respectively, are connected in parallel and supply a load of 25 kw. What is the electrical efficiency of each machine and what proportion of the total load does each supply?

102. Two similar shunt generators when driven at constant speed operate satisfactorily in parallel; that is, each machine always takes one-half the total load. The fields are adjusted so that the no load terminal voltage of each machine is 120 volts, and when each machine is delivering full load current of 100 amperes the terminal voltage falls to 110 volts. The load characteristic of each machine is a straight line. The armature resistance of each machine is 0.038 ohm and the field resistance is 29 ohms. When the total load is 15 kw. what is the terminal voltage? What are the heating losses in the armature and field ~~windings~~ circuits?

103. In problem 102 if the resistance of the field circuit of one generator is increased to 33 ohms the load characteristic of that generator is reduced 5 per cent at every point. Under this condition what is the load on each machine when the terminal voltage is 110 volts? What are the heating losses in the armature and field ~~windings~~ circuits?

104. In problem 102 if the resistance of the field circuit of one generator is reduced to 25 ohms the load characteristic of that generator is increased 3.5 per cent at every point. Under this condition what is the load on each machine when the total load is 18 kw.? What are the heating losses in the armature and field ~~windings~~ circuits?

105. In problem 102 if a resistance of 0.038 ohm is connected in series with one armature winding, what is the load on each machine when the terminal voltage is 110 volts? What are the heating losses in the armature and field ~~windings~~ circuits? Neglect any difference in the fields that is produced by a difference in the armature reactions.

106. In problem 102 if a resistance of 0.038 ohm is connected in series with one armature winding, what is the load on each machine when the total load is 18 kw.? What are the heating losses in the armature and in the field circuits? ~~windings~~? Neglect any difference in the fields that is produced by a difference in the armature reactions.

107. In problem 102 if the speed of one generator is decreased 3 per cent, what is the load on each machine when the terminal voltage is 114 volts? What are the heating losses in the armature and field circuits? ~~windings~~? Neglect any difference in the fields that is produced by a difference in the armature reactions.

108. In problem 102 if the speed of one generator is increased 3 per cent, what is the load on each machine when the total load is 15 kw.? What is the terminal voltage? What are the heating losses in the armature and field circuits? ~~windings~~? Neglect any difference in the fields that is produced by a difference in the armature reactions.

109. In problem 102 if the load supplied by the generators takes a constant current of 180 amperes, by what per cent must the speed of one generator be increased so that it will take the entire load?

110. In problem 102 if the load supplied by the generators takes a constant power of 18 kw. by what per cent must the speed of one generator be increased so that it will take the entire load?

111. In problem 102 if the equivalent resistance of the load is constant and equal to 0.64 ohm, by what per cent must the speed of one generator be increased so that it will take the entire load?

112. The open-circuit characteristic of a 500-kw. 600-volt compound generator is given by the following data: —

Shunt field current	Armature voltage	Shunt field current	Armature voltage
9.8	520	16.3	630
11.4	555	17.9	650
13.1	585	20.8	680
14.7	610	24.5	705

The armature resistance is 0.017 ohm and the resistance of the series coils is 0.0098 ohm. The shunt field is adjusted so that the open-circuit voltage is 600 volts. As a compound generator the load characteristic is a straight line rising to 630 volts at full load of 800 amperes. With the series coils short-circuited the load characteristic is a straight line, and the terminal voltage falls to 550 volts at full load of 800 amperes. What is the ratio of shunt turns to series turns in the field windings?

113. In problem 112 what resistance should be shunted across the series coils so that the load characteristic will be a straight line with a full load terminal voltage of 600 volts?

114. It is desired to run the generator in problem 112 in parallel with another 500-kw. generator having a load characteristic that rises uniformly from 600 volts at no load to 660 volts at full load. By what per cent should the series coil of the first generator be increased so that the generators will always divide the load equally between them? Assume that each of the added series turns has the same resistance as each of the original series turns.

CHAPTER VI

RESISTANCE

Conductors with Variable Dimensions

THE calculation of the resistance of conductors which have variable dimensions is in general an extremely difficult problem. The method of attack is, first, to determine the lines of current and, second, to divide the conductor into differential volumes whose lengths are measured in the direction of the current and whose areas of cross-section are measured in equipotential planes, that is, perpendicular to the lines of current. The expression for the resistance of this differential volume is given by Ohm's law, and the resistance of the whole conductor is found by integrating over the entire volume. Sometimes it is more direct to divide the conductor into elementary volumes the resistances of which are combined in *series*. In this case the total *resistance* is the sum of the differential *resistances*. Again it is more direct to divide the conductors into elementary volumes the resistances of which are combined in *parallel*. In this case the total *conductance* of the conductor is the sum of the differential *conductances*.

PROBLEMS

1. The insulation resistance of a sample of submarine cable 1127 feet long is 62,900 megohms. What will be the insulation resistance of 50 miles of this cable?

2. A hollow metal cylinder is 10 inches long and has inner and outer radii of 2 and 3 inches respectively. What is its resistance between the cylindrical surfaces? The specific resistance of this metal is 80 microhms per centimeter cube.

3. In problem 2 if a difference in potential of 1 millivolt is applied to the two cylindrical surfaces what will be the current density per square inch at a point 2.5 inches from the axis of the cylinder?

4. In problem 3 what will be the difference in potential between the inner cylindrical surface and a point 2.5 inches from the axis of the cylinder?

5. A cable consists of a copper wire 0.5 inch in diameter separated from a lead sheath by 0.3 inch of rubber insulation. Find its insulation resistance in megohms per mile if the specific resistance of the rubber is 3×10^{16} ohms per centimeter cube.

6. The core of a cable consists of a copper wire having a cross-section of 250,000 circular mils. If the specific resistance of the insulation is 10^{16} ohms per centimeter cube, find the thickness of insulation necessary in order that the cable may have an insulation resistance of 10^4 megohms per mile.

7. A sample of No. 10 wire (diameter = 0.102 inch) 964 feet in length is insulated with 0.075 inch of rubber compound. Its insulation resistance is 51,200 megohms. What will be the insulation resistance per mile of 400,000 circular mil cable covered with 0.25 inch of this compound?

8. A copper cable of 100,000 circular mils cross-section is insulated with 0.1 inch of a rubber compound. What should have been the thickness of the insulation to have doubled its present insulation resistance?

9. A hemispherical shell of copper has radii of 2 and 3 inches. If the resistance of copper is 1.70 microhms per centimeter cube what is the resistance between the spherical surfaces? The current is along radial lines.

10. In problem 9 if a difference in potential of 0.15 millivolts is applied to the two spherical surfaces what will be the current density per square inch at a point 2.5 inches from the center of the shell?

11. In problem 10 what will be the potential difference between the inner spherical surface and a point 2.5 inches from the center of the shell?

12. A semicircular copper annulus has a uniform rectangular cross-section of 5 square centimeters and radii of 10 and 20 centimeters. What is the resistance between the rectangular *ends* of the annulus if the resistance of copper is 1.70 microhms per centimeter cube? The equipotential surfaces are radial planes.

13. If a difference in potential of 5 millivolts is applied to the ends of the annulus in problem 12 what will be the current density per square centimeter at a point 15 centimeters from the axis of the annulus?

14. In problem 13 what will be the potential difference between the rectangular ends of the annulus and the equipotential plane which is perpendicular to the plane of the ends?

CHAPTER VII

FORCE AND POTENTIAL

THE laws of attraction hold universally whether applied to gravitational masses, electric charges, or to magnetic poles. In what follows the word "mass" means gravitational matter, electric charge, or magnetic pole. The force of attraction varies directly as the product of the masses and inversely as the square of the distance between them, and the direction of the force is along the line joining the masses. That is: $F \propto \frac{m_1 m_2}{r^2}$; m_1 and m_2 are gravitational masses, electric charges, or magnetic poles which are *concentrated* at points, and r is the distance between the points. The intensity of the field, or the force, at any point is the force acting on a unit mass placed at the point. If the masses are distributed along lines, over surfaces, or in other manner through space, the line, surface, or solid must be divided into differential elements every part of which may be assumed to be at the same distance from the point at which the force is to be found. Since a force is not completely defined until both its magnitude and direction are determined it is known as a vector quantity. Forces are combined by applying the principles of vector addition or, as it is frequently stated, by applying the principle of the parallelogram of forces. For a graphical solution this latter method is quite satisfactory, but when treating a problem analytically it is usually more direct to resolve each separate or elementary force into components parallel to the coördinate axes. The resultant component along any axis is the algebraic sum, or the integral, as the case may be, of the components of the individual or elementary forces acting along that axis. It should always be borne in mind that forces can only be added algebraically, or integrated, when they act in the *same* direction.

An electric current in a conductor produces a magnetic field.

The strength of the field at any point P due to an elementary length of the conductor dl, which is carrying a current of I amperes, is given by the expression $dF = \frac{I}{10}\frac{dl}{r^2}\cos\theta$; r is the distance from the elementary length of the conductor to the point P; θ is the angle between a normal to the elementary conductor and a line drawn between the elementary conductor and the point P. The direction of the magnetic field due to the current in an elementary length of the conductor at the point P is perpendicular to the plane in which lie the elementary conductor and the point P. The force, or the intensity of the magnetic field at the point P is found by integrating this expression over the entire length of the conductor, provided only that the conductor lies in a single plane.

Potential is defined as being numerically equal to the work done in moving a unit mass against a gravitational, an electric, a magnetic, or an electromagnetic force. The potential at a point due to a system of masses is numerically equal to the work done in moving a unit mass against the force due to the system from a point infinitely remote to the point in question. The difference in potential between two points is numerically equal to the work done in moving a unit mass from one point to the other. The potential at a point due to a concentrated mass m is given by the expression $V = \frac{m}{r}$, where r is the distance from the point to the mass. Potential is a scalar quantity, that is, it has magnitude only. The resultant potential at a point due to a system of concentrated masses, or to a distributed mass, is the algebraic sum, or integral, as the case may be, of the individual or elementary potentials at the point in question.

PROBLEMS

1. Find the force and the potential at the origin due to a concentrated mass of 100 units at the point (10, 0).

2. Find the force and the potential at the origin due to the following system of concentrated masses:

Mass of 30 units at the point (10, 0).
Mass of 40 units at the point (0, 10).

3. Find the force and the potential at the origin due to the following system of concentrated masses:

Mass of 10 units at the point (3, 4).
Mass of 20 units at the point (5, – 5).
Mass of 30 units at the point (– 2, – 4).

4. At the points (8, 6), (–6, 8), and (0, –10) are concentrated the following charges: 10 positive units, 20 positive units, and 30 negative units respectively. Find the potential and the resultant force at the origin. In what direction would a negative charge at the origin tend to move?

5. Three equal masses of 10 units each are concentrated at the apexes of an equilateral triangle of 10-foot sides. Find the resultant force and the potential at the center of the triangle.

6. Three equal masses of 10 units each are concentrated at three of the vertices of a regular tetrahedron. What are the resultant force and the potential at the fourth vertex of the tetrahedron?

7. Two equal concentrated masses of 10 units each are placed 10 units apart. What is the work done in moving a unit mass from a point which is 10 units from each of the concentrated masses to a point which is 5 units from each of them?

8. In problem 7 what is the resultant force acting on the unit mass when it is 10 units from each of the concentrated masses? when it is 5 units from each of the concentrated masses?

9. A mass of 10 units is concentrated at the point (0, 5). What is the expression for the force due to this mass at any point $(x, 0)$ on the axis of abscissas?

10. In problem 9 what are the two components of the force at any point $(x, 0)$ both along the axis of abscissas and in a direction that is perpendicular to this axis.

11. In problem 9 find, by the method of integration, the work done against the force of attraction in moving a *unit* mass from the origin (0, 0) along the axis of abscissas to a point which is infinitely remote.

12. A mass of 10 units is concentrated at the point (10, 0). What is the expression for the force at any point on the circumference of a circle of 5 units radius drawn about the origin as a center?

13. In problem 12 what are the components of this force along a radius of the circle and along a tangent to the circle? Express the equation of the circumference of the circle in polar coördinates as $(5, \theta)$.

14. In problem 12 find the work done in moving a unit mass along the circumference of the circle from the point (5, 0) to the point $(5, \pi,)$ by integrating the tangential component of the force around the semi-circumference.

15. Find the force of attraction due to a rod 10 feet long having a density of 10 units per foot, in a direction parallel to its axis, at a point 6 feet from one end and 8 feet from the other. Consider that the rod is concentrated along its axis.

16. What are the force and the potential due to a rod of uniform linear density m and length L at a point on its axis L units from the nearer end?

17. Find the force and the potential due to a rod of length L and linear density m at a point which is on a perpendicular to the rod at its midpoint and h units from it.

18. Find the force and the potential due to a rod of length L and linear density m at a point which is on a perpendicular to the rod at one end and h units from it.

19. Find the force and the potential at the origin due to a wire of length L and linear density m which is bent into a complete circle about the origin as a center.

20. Find the force and the potential at the origin due to a wire of length L and linear density m which is bent into a semi-circle about the origin as a center.

21. Show that the attraction due to a uniform wire which is bent into the form of a polygon that circumscribes a circle is zero at the center of the circle.

22. A wire of linear density m and length L, which is bent into the form of the quadrant of a circle, has its ends on the coördinate axes and its center at the origin. Where should two concentrated masses, each equal to one-half of the mass of the wire, be placed on the coördinate axes so that they will exert the same force at the origin as does the wire?

23. Two rods of lengths L_1 and L_2 and each of linear density m are placed so as to form the two adjacent sides of a rectangle.

Find the force and the potential at the vertex of the rectangle which is diagonally opposite the junction of the rods.

24. Find the force of attraction between two straight rods each of length L and linear density m lying in the same line with their nearer ends separated by a distance L.

25. In problem 24 find the work done against the force of attraction in moving one of the rods along the line in which they lie to a point that is infinitely remote.

26. A rod of uniform linear density m lies in the axis of abscissas with one end at the point $(0, 0)$ and the other end at the point $(L, 0)$. By integrating the force function find the work done against the force of attraction in moving a unit mass from the point $(2L, 0)$ along the axis of abscissas to the point $(3L, 0)$.

27. A rod of length L and of uniform linear density m lies in the axis of abscissas with its midpoint at the origin. By integrating the force function find the work done against the force of attraction in moving a unit mass from the point $(0, L)$ along the axis of ordinates to the point $(0, 2L)$.

28. A rod of length L and of uniform linear density m lies in the axis of abscissas with its midpoint at the origin. By integrating the force function find the work done against the force of attraction in moving a unit mass from the point $(0, a)$ along a line parallel to the axis of abscissas to a point (b, a).

29. Two rods each of linear density m and length L are placed so as to form the opposite sides of a square. What is the force of attraction that one exerts upon the other?

30. A wire of uniform linear density m is bent into a circle of radius R which lies in the XOY plane and has its center at the origin. What are the force and the potential at a point which is on the axis OZ and h units from the origin?

31. In problem 30, by integrating the force function, find the work done against the force of attraction in moving a unit mass from the point (o, o, a) along the axis OZ to a point (o, o, b).

32. A north pole of 20 units strength is placed 10 inches from a south pole of 10 units strength. What is the force in dynes that one pole exerts upon the other?

33. A north pole of 10 units strength is placed in a uniform field which has an intensity of 500 lines per square inch. What is the force in dynes acting on the pole?

34. At each of the points $(-5, 0)$ and $(5, 0)$ there is a north pole of 20 units strength. What is the force in dynes acting on a north pole of 5 units strength placed at the point $(0, 5)$? In what direction would it tend to move?

35. At the point $(-5, 0)$ there is a north pole of 20 units strength and at the point $(5, 0)$ there is a south pole of 20 units strength. What is the force in dynes acting on a north pole of 5 units strength placed at the point $(0, 5)$? In what direction would it tend to move?

36. A thin bar magnet 20 centimeters long is placed in the axis of abscissas with its center at the origin. The poles of this magnet each have a strength of 30 units and they may be assumed to be concentrated at the ends. What is the force in dynes acting on a unit north pole placed at the point $(10, 20)$?

37. Two thin bar magnets each 20 centimeters long and with poles of 50 units strength are placed so as to form the opposite sides of a square, with their like poles pointing in the same direction. If the poles of the magnets are assumed to be concentrated at the ends what is the force in dynes with which one magnet repels the other?

38. In problem 37 if the like poles of the magnets point in opposite directions what is the force in dynes with which one magnet attracts the other?

39. A thin bar magnet 20 centimeters long and with poles of 50 units strength is pivoted at its center and placed in a uniform field which has an intensity of 200 lines per square centimeter. Assume that the poles of the magnet are concentrated at the ends. When the magnet is perpendicular to the direction of the field what is the torque in centimeter-dynes tending to rotate it about its center?

40. In problem 39 what is the torque tending to rotate the magnet when it makes any angle θ with the direction of the field? What is the torque in centimeter-dynes when $\theta = 45°$?

41. A thin bar magnet 20 centimeters long and with poles of 50 units strength is placed in the axis of abscissas with its center at the origin. Another similar magnet 10 centimeters long and with poles of 20 units strength is placed in the axis of ordinates with its center 15 centimeters from the origin. Assuming that the poles of each magnet are concentrated at the ends what is the torque in centimeter-dynes tending to rotate the shorter magnet about its center?

42. A thin bar magnet 20 centimeters long and with poles of 50 units strength is placed in the axis of abscissas with its center at the origin. Another magnet which is the exact duplicate of the first is placed in the axis of ordinates with its center also at the origin. Assume that the poles of the magnets are concentrated at the ends and that they lie in the same plane. If one of these magnets is fixed, what is the torque in centimeter-dynes tending to turn the other about its center?

43. Find the force due to a rectangular surface distribution of uniform density σ which is bounded by the lines $x = 0$, $x = a$, and $y = -b$, $y = +b$, at the point $(c, 0)$, where c is greater than a.

44. Find the force and the potential due to a circular plate, which has a uniform surface distribution σ, at a point on the axis of the plate h units above the center.

45. Find the force and the potential at the center of a complete annulus of uniform surface density σ and radii a and b.

46. Find the force and the potential at the center of a semicircular annulus of uniform surface density σ and radii a and b.

47. Find the force and the potential at the apex of a right circular cone of uniform volume density ρ. The altitude of the cone is H and the radius of the base is R.

48. A right circular cone, the radius of whose base is 5 units and whose altitude is 12 units, is cut by a plane perpendicular to the axis so that the potentials at the vertex of the cone due to each portion are equal. What is the altitude of the frustrum thus formed and what is the force it exerts at the apex if the uniform density of the mass is 10 units?

49. A right circular cone, the radius of whose base is 3 units and whose altitude is 6 units, is cut by a plane perpendicular to the axis so that each part exerts the same force at the apex of the cone. What is the altitude of the smaller cone thus formed?

What is the force that the frustrum exerts at the apex of the cone if the uniform volume density is 10 units?

50. Find the force and the potential at the center of a spherical shell of radii a and b and of uniform volume density ρ.

51. Find the force and the potential at the center due to a hemispherical shell of radii a and b and of uniform volume density ρ.

52. A straight wire 1 meter long is carrying a current of 100 amperes. Find the force at a point which is equally distant from the ends of the wire and 10 centimeters from the center.

53. A straight wire 10 feet long is carrying a current of 500 amperes. Find the force at a point which is 1 foot from one end of the wire and is on a line that is perpendicular to the wire at this end.

54. A very long, straight conductor is carrying a current of 200 amperes. Find the force at a point which is 5 inches from the center of the conductor and is on a line that is perpendicular to the conductor at the center.

55. A straight conductor 10 feet long is carrying a current of 500 amperes. What is the percentage error introduced by calculating the intensity of the magnetic field at a point that is equally distant from the ends of the conductor and is 1 foot from its center, on the assumption that the conductor is of infinite extent in both directions?

56. In problem 55 what would be the error in calculating the intensity of the field at a similar point 5 feet from the center of the conductor, on the same assumption?

57. In problem 54 what is the work done in carrying a unit pole once around the wire in a circular path which has a radius of 5 inches and the wire as a center?

58. Two long, straight and parallel conductors spaced 3 feet apart are each carrying a current of 500 amperes. If the currents are in the same direction what is the intensity of the magnetic field near the middle of the conductors at a point 1.5 feet from each?

59. In problem 58 if the currents are in opposite directions what will be the force at a point near the middle of the conductors 1.5 feet from each?

60. In problem 58 what will be the force at a point near the middle of the conductors 3 feet from each?

61. In problem 58 if the currents are in opposite directions what will be the force at a point near the middle of the conductors 3 feet from each ?

62. A copper conductor which is carrying a current of 100 amperes is bent into a complete circumference of 5 inches radius. What is the force at a point that is 10 inches from every part of this conductor?

63. In problem 62 what will be the work done in carrying a unit pole from the center of the circumference along a line which is perpendicular to the plane of the coil to a point infinitely remote? This is one-half the magnetomotive force of the coil.

64. A copper conductor is bent into a complete circumference of 10 centimeters radius. What will be the force at the center of the circumference when the conductor carries a current of 100 amperes?

65. A very long, straight conductor is carrying a current of 500 amperes. Find the force at a point which is 10 centimeters from one end of the conductor and is on a line perpendicular to it at this end.

66. A very long, straight, thin copper ribbon 5 centimeters wide is carrying a current of 250 amperes. What will be the intensity of the magnetic field at a point which is in the plane of the ribbon, opposite the center, and 5 centimeters from the nearer edge?

67. A very long, straight, thin copper ribbon 3 inches wide is carrying a current of 200 amperes. What will be the intensity of the magnetic field at a point which is opposite the center of the ribbon and is 3 inches from each edge?

68. A thin copper ribbon 3 inches wide is bent so as to form a complete cylindrical surface of 3 inches radius. Find the intensity of the magnetic field at the center of this cylinder when the ribbon carries a current of 250 amperes. The lines of current are circles.

69. A thin copper ribbon 20 inches wide is bent so as to form a complete cylindrical surface of 1 inch radius. When the ribbon carries a current of 1000 amperes compare the intensities of the magnetic field at the center of the cylinder thus formed and at the center of one end of the cylinder. The lines of current are circles.

70. In problem 68 what is the work done in carrying a unit pole along the axis of the cylinder from the center of one end to the center of the other end?

71. In problem 69 what is the work done in carrying a unit pole along the axis of the cylinder from the center of one end to the center of the other end?

72. In problem 68 what is the work done in carrying a unit pole along the axis of the cylinder from $-\infty$ to $+\infty$? This is the method used to calculate the magnetomotive force of a short coil.

73. In problem 69 what is the work done in carrying a unit pole along the axis of the coil from $-\infty$ to $+\infty$? This is the method used to calculate the magnetomotive force of a long coil.

74. A thin semicircular copper annulus of radii 2 and 4 inches is carrying a current of 200 amperes. What is the intensity of the magnetic field at the center of the annulus? The lines of current are concentric semicircles.

75. A thin copper annulus of radii 5 and 10 centimeters is carrying a current of 250 amperes. What is the intensity of the magnetic field at the center of the annulus? The lines of current are concentric circles.

76. In problem 75 what is the intensity of the magnetic field at a point which is on a line perpendicular to the annulus through its center and x units from the plane of the annulus?

77. In problem 75 what is the work done in carrying a unit pole along the line which is perpendicular to the plane of the annulus through its center from $-\infty$ to $+\infty$? This is the method used to calculate the magnetomotive force of a pancake coil.

78. A thin copper cylinder 5 inches in diameter and 5 feet long is carrying a current of 500 amperes. What is the intensity of the field at any point on the axis of the cylinder? The current is in the direction of the axis of the cylinder.

79. A thin copper cylinder 5 inches in diameter and 5 feet long is divided into two equal parts by a plane in which lies the axis of the cylinder. One of these parts carries a current of 250 amperes. What is the intensity of the field at a point on the axis of the cylinder halfway between the ends? The current is in the direction of the axis of the cylinder.

CHAPTER VIII

THE ELECTROMAGNETIC FIELD

THE magnetomotive force of a coil of wire carrying a steady current is numerically equal to the work done in moving a unit magnetic pole through the coil from plus infinity to minus infinity. The algebraic expression for the magnetomotive force is $\frac{4\pi}{10} NI$, where N is the net number of turns in the coil and I is the current (amperes) in each turn.

For a long solenoid with an air core which has a length of about twenty times its diameter the work done in moving the unit pole from the center of one end to the center of the other is very nearly equal to $\frac{4\pi}{10} NI$ (see Chapter VII, Problems 71 and 73); that is, the difference in the magnetic potentials at the two ends of the coil is approximately equal to the magnetomotive force of the coil. If it is assumed that the flux is uniformly distributed over the cross-section of the coil, the average value of the flux is obtained by dividing the difference in the magnetic potentials at the ends of the coil $\left(\frac{4\pi}{10} NI\right)$ by the reluctance of the magnetic path between the ends of the coil $\left(\frac{l}{a}\right)\cdot$ So that the average value of the flux is approximately $\frac{4\pi}{10} NI \, \frac{a}{l}$, and the average value of the flux density is approximately $\frac{4\pi}{10} \frac{NI}{l}\cdot$ In these formulas and in those that follow, lengths and areas are expressed in centimeters and in square centimeters respectively.

When a coil has a core of magnetic material, the general expression for the flux is $\frac{4\pi}{10} \frac{NI}{Z}$, where $Z = \frac{l}{\mu a}$. Z, the reluctance of the magnetic circuit, is a function of the magnetizing force, but for the sake of simplicity it will be assumed constant in all of the problems in this chapter. If the flux density is relatively low the assumption of constant permeability leads to no great error.

The self-inductance of any electric circuit is measured by the number of linkages of the net series turns, each carrying unit current, with the flux lines produced by these ampere-turns. If all of the flux lines are linked with all of the turns the self-inductance of a long coil is approximately $N \times \frac{4\pi}{10} N \frac{a}{l}$ or $\frac{4\pi}{10} N^2 \frac{a}{l}$ linkages, and of a coil wound on a magnetic core, $\frac{4\pi}{10} N^2 \frac{\mu a}{l}$ linkages. When there are 10^8 linkages per ampere the self-inductance is one henry.

Mutual inductance, or the mutual effect of one electric circuit upon another, is measured by the number of linkages of the net series turns in one circuit with the flux lines produced by unit current in the other circuit. If two coils are wound side by side in the form of a long solenoid, or on the same magnetic core, all of the flux lines produced by one coil may be assumed to be linked with all of the turns in the other coil. Thus for two coils in the form of a long solenoid the mutual inductance is approximately $N_1 \times \frac{4\pi}{10} N_2 \frac{a}{l}$ or $\frac{4\pi}{10} N_1 N_2 \frac{a}{l}$ linkages, and for two coils wound on the same magnetic core the mutual inductance is approximately $\frac{4\pi}{10} N_1 N_2 \frac{\mu a}{l}$ linkages.

The electromagnetic field is a seat of energy, called kinetic energy, since it is produced by the action of an electric current in a circuit. The magnitude of this energy is given by the expression $\frac{1}{2} LI^2$ (joules), where L (henrys) is the self-inductance of the circuit in which a current of I amperes exists. This expression is readily transformed into $\frac{1}{8\pi} \phi^2 Z$ (ergs), where ϕ is the number of flux lines and Z is the reluctance of the magnetic circuit on which the electric circuit is wound. If

$H = \frac{4\pi}{10}\frac{NI}{l}$, that is, the magnetizing force per unit length of the magnetic circuit, the expression for the energy of the field per unit volume may be obtained by transforming this latter expression into $\frac{1}{8\pi}\mu H^2$ (ergs).

PROBLEMS

1. A long solenoid is wound with No. 20 copper wire, of which there are 23 turns per linear inch. What is the flux density per square inch near the center of the coil when the current through it is 1.5 amperes?

2. If the coil in problem 1 has a circular cross-section with a radius of 1 inch, what will be the flux produced within and near the center of the coil per ampere?

3. If the coil in problems 1 and 2 is 1 meter long what will be the number of linkages of flux and turns when the current through the coil is 2 amperes?

***4.** A coil of 1500 turns of No. 24 copper wire is wound in one layer on a wooden cylinder 115 centimeters long and 5 centimeters in diameter. What will be the flux produced within this coil when it is connected across an e.m.f. of 10 volts? How many linkages of flux and turns will there be when the coil carries a current of 0.75 ampere?

5. A solenoid 1 meter long and 4 centimeters in diameter is wound with 1310 turns of copper wire. What is its inductance?

6. A wooden cylinder 30 inches long and 1.5 inches in diameter is completely wound with one layer of No. 28 copper wire. If there are 43 turns per linear inch what is the inductance of the coil?

7. What is the inductance of a long cylindrical coil of 1000 turns of No. 24 copper wire, wound in one layer, if its resistance is 10.2 ohms and there are 33 turns per inch of length? Assume that No. 24 copper wire has a resistance of 0.026 ohm per foot.

8. Find the dimensions of a long cylindrical coil wound with one layer of No. 24 copper wire so that it will have an inductance of 0.002 henry. The length of the coil must be 20 times its diameter. No. 24 copper wire has a resistance of 0.026 ohm per foot, and 33 turns can be wound per inch. What is the resistance of the coil?

9. A long coil which has a length 20 times its diameter is wound with No. 24 copper wire of which there are 33 turns per inch. The coil is so proportioned that the ratio of the resistance in ohms to the inductance in mil-henrys is 5. The resistance of No. 24 copper wire is 0.026 ohm per foot. What are the dimensions of the coil?

10. Find the dimensions of a long cylindrical coil wound with one layer of No. 24 copper wire so that it will have an inductance of 2.5 mil-henrys and a resistance of 10 ohms. No. 24 copper wire has a resistance of 0.026 ohm per foot and 33 turns can be wound per inch. What is the ratio of the length of the coil to its diameter?

11. A closed magnetic circuit of cast iron which has a cross-section of 5 square centimeters and a mean length of 32 centimeters is wound with 2000 turns of No. 30 copper wire. Assuming that the permeability of the iron is constant and equal to 220, what is the inductance of the coil?

12. A soft steel ring with a cross-section of 1 square inch and a mean diameter of 6 inches is wound with 500 turns of No. 20 copper wire. Assuming that the permeability of the steel is constant and equal to 800 what is the inductance of the coil?

13. In problem 12 how long an air-gap must be cut in the magnetic circuit to reduce the inductance of the coil to 0.05 henry?

***14.** A soft steel ring with a mean diameter of 11.1 inches and a cross-section of 0.75 inch radius is to be wound with one layer of copper wire so that the coil thus formed will have an inductance of 0.5 henry and a resistance of 3 ohms. Assuming that the permeability of the steel is 850, with what size wire and with how many turns should this coil be wound?

15. Two coils are wound, one above the other, on a steel ring like that described in problem 14. The first is of No. 20 copper wire and the second is of No. 10, and there are 225 turns in each coil. What is the self-inductance of each coil?

***16.** In problem 15 what will be the flux produced within the coils if the first is alone connected across a constant potential of 2 volts? What will be the number of linkages of this flux and the turns of each coil?

17. In problem 15 what is the mutual inductance of the coils?

18. The inductance of a coil consisting of 350 turns of No. 10 wire wound on a magnetic circuit of assumed constant permeability is 0.12 henry. How many additional series turns are required to increase the inductance to 0.2 henry?

19. Two coils are wound side by side on a steel ring which has a mean diameter of 12 inches and a cross-section with a radius of 0.75 inch. There are 350 turns in the first coil and 1000 in the second. Assume that the permeability of the steel is constant and equal to 800. What is the self-inductance of each coil? What is their mutual inductance?

20. If the coils in problem 19 are connected in series so that they tend to produce magnetic fields in the same direction, what will be their combined inductance?

21. If the coils in problem 19 are connected in series so that they tend to produce opposing magnetic fields, what will be their combined inductance?

22. Two coils are wound side by side on a magnetic circuit whose length is 50 centimeters and area of cross-section 25 square centimeters. The first consists of 800 turns of No. 20 copper wire and the second of 100 turns of No. 10 wire. The coils are connected in series so that they tend to produce opposing magnetic fields. If the permeability of the magnetic circuit is 950 what is the combined inductance of the coils when thus connected?

23. In problem 22 how long an air-gap must be cut in the magnetic circuit so that the self-inductance of the first coil alone will be 2.5 henrys?

24. What is the mutual inductance of any two coils wound side by side on the same magnetic circuit in terms of their self-inductances?

25. Two coils are wound side by side on the same magnetic circuit. The self-inductance of one is L_1 and of the other L_2. What is their combined inductance when connected in series so that they tend to produce magnetic fields in the same direction?

26. What is the combined inductance of the coils in problem 25 if they are connected in series so that they tend to produce opposing magnetic fields?

27. The self-inductance of a coil of copper wire which is wound on a magnetic core built up of steel stampings is 0.15 henry. Another coil of 200 turns is wound on the core and its self-inductance is found to be 0.085 henry. How many turns are there in the first coil? Assume that the permeability of the steel is constant.

28. Two coils, each of an unknown number of turns, are wound on the same magnetic core. If the coils are connected in series in opposition their combined inductance is 1.5 henrys. If the connections of one of the coils are reversed their combined inductance becomes 5 henrys. What is the self-inductance of each coil? Assume that the permeability of the magnetic core is constant.

29. The inductance of a coil consisting of 246 turns of No. 24 copper wire wound in one layer on a magnetic core is 0.025 henry and the resistance of the coil is 4.2 ohms. This wire is removed and the core is rewound with one layer of such a size of copper wire that the resistance is 0.59 ohm and the inductance is 0.005 henry. What size should be used? Use the approximate method for calculating the resistance and choose the gage number that most nearly fulfils the conditions.

30. The inductance of a coil consisting of 232 turns of copper wire wound on a magnetic core is 0.021 henry and its resistance is 0.87 ohm. How many turns of the same wire should be added in series to, or removed from, the coil in order that the ratio of the resistance in ohms to the inductance in henrys may be ~~[illegible]~~ 30? Assume that each turn has the same length.

* **31.** The inductance of a coil consisting of 300 turns of No. 18 copper wire wound on a magnetic core is 0.15 henry and its resistance is 1.2 ohms. What is the least number of additional series turns of No. 14 copper wire that should be wound on this core in order that the flux shall have a value of 450,000 lines when an e.m.f. of 8.5 volts is impressed on the two coils in series? Assume that the length of each of the added turns is 8 inches.

***32.** In problem 31 what is the least number of turns of copper wire that should be wound on the core and connected in parallel with the original turns in order that the flux shall have a value of 450,000 lines and the total current taken by the coils shall not exceed 10 amperes when an e.m.f. of 8.5 volts is impressed on the two coils in parallel? What size wire should be used? Assume that the length of each of the added turns is 8 inches.

33. A coil of 500 turns is wound on a magnetic core which has a cross-section of 5 square inches. The self-inductance of this coil is 2 henrys. How long an air-gap must be cut in the core to reduce the inductance to 0.5 henry? Assume that the permeability of the core is constant and also that the reciprocal of the permeability is negligible compared with unity.

34. A magnetic core of constant length and uniform cross-section has an adjustable air-gap. On this core there is wound a coil of No. 20 copper wire. When the air-gap is reduced to zero the self-inductance of the coil is 3 henrys. When the length of the air-gap is 0.05 inch the self-inductance of the coil is 1 henry. To what must the length of the air-gap be increased to reduce the self-inductance to 0.2 henry? Assume that the permeability of the magnetic core is constant.

35. Two coils are wound side by side on the same magnetic circuit. The first, consisting of 500 turns of No. 20 copper wire, has a resistance of 2.5 ohms and a self-inductance of 0.5 henry, and the second, consisting of 1000 turns of No. 30 wire, has a resistance of 50 ohms. If an e.m.f. of 4.3 volts is impressed on the first coil what e.m.f. must be impressed on the second in order that the resultant flux shall be zero? What will be the flux produced when an e.m.f. of 15 volts is impressed on the second coil alone?

36. A small experimental transformer has a magnetic circuit with a net cross-section of 3.4 square inches and a mean length of 22.5 inches. Assume that the permeability of the iron is 2150. There are four coils, each of which has 232 turns, and a resistance of 0.23 ohm. What is the self-inductance of each coil? If the four coils are connected in series so that they produce magnetic fields in the same direction what is their combined inductance?

37. In problem 36 if the four coils are connected in parallel so that they produce magnetic fields in the same direction, what will be their combined inductance?

38. In a coil which has a self-inductance of 1.2 henrys there is a current of 25 amperes. What is the energy in the electromagnetic field?

39. A coil of 1000 turns wound on a long cylinder has a self-inductance of 0.0025 henry and a resistance of 10 ohms. What is the energy of the electromagnetic field when a flux of 1500 lines has been established through the coil?

40. The field coils of a 4-pole shunt motor are connected in series and each consists of 950 turns. A field current of 5.3 amperes produces an air-gap flux of 2,500,000 lines. Assuming no magnetic leakage, what is the energy of the electromagnetic field when the field current is 5.3 amperes?

41. Two coils are wound side by side on the same magnetic circuit. The first, consisting of 500 turns, has a self-inductance of 0.5 henry, and the second has a self-inductance of 2 henrys. If the coils are connected in series so that they produce magnetic fields in the same direction, what will be the energy of the electromagnetic field when the flux reaches a value of 1,000,000 lines?

42. If the coils in problem 41 had been connected in series so as to produce magnetic fields in opposition, what would have been the energy of the electromagnetic field when the flux reached a value of 1,000,000 lines?

43. In problem 36 what will be the energy of the electromagnetic field if an e.m.f. of 2.5 volts is impressed on one coil alone?

44. In problem 36 the coils are connected for a 2 : 1 ratio of transformation, two coils in series and the other two in parallel. With a current of 10 amperes through each of the coils in series what will be the total energy of the electromagnetic field?

45. In problem 44 what will be the energy of the electromagnetic field if each of the two coils in parallel carries 10 amperes?

46. In problem 36 the four coils are connected in series so that they produce magnetic fields in the same direction. What will be the energy of the electromagnetic field if an e.m.f. of 10 volts is impressed on the outer terminals of the coils?

47. In problem 36 the four coils are connected in parallel so that they produce magnetic fields in the same direction. What will be the energy of the electromagnetic field if an e.m.f. of 2.5 volts is impressed on the coils?

48. A 2-pole shunt-wound generator has 2000 turns per pole in its field winding. A field current of 2.1 amperes produces an air-gap flux of 2,000,000 lines. Assuming no magnetic leakage what is the total inductance of the field winding for this point on the saturation curve? What is the energy of the electromagnetic field when the field current is 2.1 amperes? The field coils are connected in series.

49. A 6-pole shunt generator has 800 turns per pole in its field winding. A field current of 4.3 amperes produces an air-gap flux of 3,000,000 lines. Assuming no magnetic leakage what is the total inductance of the field winding for this point on the saturation curve? What is the energy of the electromagnetic field when the field current is 4.3 amperes? The field coils are connected in series.

CHAPTER IX

THE ELECTROSTATIC FIELD

THE capacity of a condenser is the ratio of the charge (on one plate or surface) to the potential (between the plates or surfaces); that is, $C = \frac{Q}{E}$.

Condensers that are connected in *parallel* have a *common potential difference* across their terminals, and the resultant, or total, charge is the sum of the charges on the individual condensers.

Thus, $$Q_1 = C_1 E,$$
and $$Q_2 = C_2 E,$$
so that $$Q_0 = \Sigma Q = (C_1 + C_2 + \ldots) E = C_0 E;$$

that is, the resultant capacity of a number of condensers in parallel is the sum of the capacities of the individual condensers.

When condensers are about to be connected in parallel the total charge will be the algebraic sum of the charges on the plates that are to be connected to a common terminal.

When each one of a number of condensers that are *not* in parallel has one terminal connected to a common *insulated* point the algebraic sum of the charges on the plates that are thus connected is zero, since whatever charges may exist on these plates are induced charges. This is true only if the condensers were uncharged before being connected.

Condensers that are connected in *series* have the *same charge*, and the resultant, or total, potential across the condensers is the sum of the potentials across the individual condensers.

Thus, $$E_1 = \frac{Q}{C_1},$$
and $$E_2 = \frac{Q}{C_2},$$
so that $$E_0 = \Sigma E = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right) = \frac{Q}{C_0};$$

that is, the reciprocal of the resultant capacity of a number of condensers in series is the sum of the reciprocals of the capacities of the individual condensers.

When a number of condensers are arranged in series in a *closed* circuit the sum of the potentials across the individual condensers, taken in the same direction around the circuit, is zero.

The amount by which the potential energy of a condenser is increased in differential time, dt, is $ei\,dt$; e is the potential of the condenser and i is the charging current. Since $e = \frac{q}{C}$ and $i\,dt = dq$, this expression may be reduced to the more convenient form $\frac{q}{C}\,dq$. The total energy of the condenser is

$$\int_{q=0}^{q=Q} \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}.$$

This expression for the energy is readily transformed into $\frac{1}{2}\,QE$ or $\frac{1}{2}\,CE^2$.

The intensity of the electrostatic field at a distance, l, from a concentrated charge, Q, which is situated in an infinite medium of specific inductive capacity, κ, is given by the expression $F = \frac{Q}{\kappa l^2}$. The potential is given by the expression $E = \frac{Q}{\kappa l}$.

There are two physical theorems which are particularly important in many problems relating to the electrostatic field. They are Gauss' theorem and Laplace's equation. Gauss' theorem states that the total nominal induction over any closed surface is $4\,\pi$ times the total charge within the surface. That is, $\int f_n da$ equals $4\,\pi Q$ over any closed surface; f_n is normal component of the force at a point on the surface, da is a differential area of the surface and Q is the total charge within the surface.

Laplace's equation states that the sum of the second partial derivatives of the potential with respect to x, y and z is zero.

That is
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

The capacity of any system is defined as the ratio of the charge on the system to its potential; that is, $C = \frac{Q}{E}$. The general method of calculating the capacity of a system is to find an expression for the potential in terms of the charge so that in taking this ratio the charge will cancel in numerator and denominator.

Laplace's equation applied to two infinite parallel plates separated by a distance, l, reduces to $\frac{d^2V}{dx^2} = 0$. The solution of this equation is $V = \frac{E}{l}x$, where E is the difference in potential between the plates. The intensity of the field is $F = \frac{dV}{dx} = \frac{E}{l}$. By choosing a closed surface around one of the plates, one part of which is a plane parallel to and between the two plates, it is readily seen by Gauss' theorem that the force at any point between the plates is $4\pi\sigma$, where σ is the surface density of the charge on the plates; thus,

$$\frac{E}{l} = 4\pi\sigma \quad \text{or} \quad E = 4\pi\sigma l,$$

but $$Q = A\sigma, \quad A \text{ being the area of one plate.}$$

$$\therefore C = \frac{A}{4\pi l}.$$

If there is a dielectric with a specific inductive capacity, κ, between the plates the capacity is $\frac{\kappa A}{4\pi l}$.

The force with which one parallel plate attracts the other is the product of the total charge on one plate ($A\sigma$) and the force with which the other plate attracts a unit charge on the first ($\frac{1}{2} \times 4\pi\sigma$). That is, the force is $2\pi A\sigma^2$. This may be transformed into $\frac{2\pi Q^2}{A}$, or, $\frac{AE^2}{8\pi l^2}$. If there is a dielectric between the plates with a specific inductive capacity, κ, these expressions become $\frac{2\pi Q^2}{\kappa A}$ and $\frac{\kappa A E^2}{8\pi l^2}$.

Laplace's equation applied to a cylindrical condenser reduces to $\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0$. The solution of this equation is

$$V = \frac{E}{\log \frac{r_2}{r_1}} \log \frac{r}{r_1},$$

where E is the difference in potential between the cylinders, and r_1 and r_2 are the radii of the inner and outer cylinders respectively. The intensity of the field is

$$F = \frac{dV}{dr} = \frac{E}{\log \frac{r_2}{r_1}} \frac{1}{r}.$$

By Gauss' theorem the force at any point between the two cylinders is $\frac{4\pi Q}{2\pi r}$ or $2\frac{Q}{r}$, where Q is the charge per unit length on the inner cylinder, so that,

$$\frac{E}{\log \frac{r_2}{r_1}} \frac{1}{r} = \frac{2Q}{r},$$

and

$$E = 2Q \log \frac{r_2}{r_1}.$$

The capacity per unit length is

$$\frac{1}{2 \log \frac{r_2}{r_1}}.$$

For a cable of length l which has insulation with a specific inductive capacity, κ, the capacity is

$$\frac{\kappa l}{2 \log \frac{r_2}{r_1}}.$$

In all of these expressions for potential, force, capacity, etc., the quantities involved are in the c.g.s. system. To reduce these expressions for capacity to microfarads divide by 9×10^5.

PROBLEMS

1. Find the force and the potential at a point 10 centimeters from a concentrated charge of 5 electrostatic units.

2. Find the force and the potential at a point 50 centimeters from the center of a sphere of 0.5 inch diameter which is charged to a potential of 50,000 volts above the earth.

3. Two spheres with diameters of 1 inch and 2 inches are charged to potentials of 50,000 volts and 30,000 volts above the earth respectively. With what force will they repel each other when placed 2 feet apart (between centers) in air?

4. If the spheres in problem 3 are first charged as stated and are then immersed in an insulating oil whose dielectric constant is 2.1 what will be the force of repulsion between them when they are 2 feet apart?

5. Two spheres with diameters of 0.5 inch and 1 inch are placed 20 inches apart (between centers) and are charged to potentials of + 30,000 and − 50,000 volts. What is the force and the potential at a point 12 inches from the center of the first sphere and 16 inches from the center of the second?

6. If the spheres in problem 5 are connected by a conducting wire what will be their final potential? What will be the charge on each?

7. If the spheres in problem 5 are first charged as stated and are then immersed in an insulating oil whose dielectric constant is 2.1 what will be the force and the potential at the point located as in problem 5?

8. A sphere 10 inches in diameter is charged to a potential of 100,000 volts above the earth. What is the energy of the sphere in joules due to its electrostatic charge?

9. Two spheres with radii of 10 and 20 centimeters are charged to potentials of 10,000 and 50,000 volts above the earth respectively. They are connected by a wire until they come to the same potential. Find the energy in joules expended in the spark and the connecting wire, and also the final voltage of the system.

10. A condenser of 25 microfarads capacity is charged to a potential difference of 110 volts. What is the heat generated in a wire of 10 ohms resistance which is connected across the terminals of the condenser until it is completely discharged?

11. The capacities of three condensers are 10, 20 and 25 microfarads respectively. What are the greatest and least resultant capacities that can be obtained by combining these condensers?

12. Two condensers of 10 and 15 microfarads capacity connected in parallel are joined in series with one of 25 microfarads capacity. What is the resultant capacity of these condensers?

13. In problem 12 what will be the charge on each condenser when the outer terminals are connected across 110 volts? What is the energy of each condenser?

14. Two condensers of 10 and 20 microfarads capacity are connected in series across 110-volt mains. What is the charge on each? What is the voltage across each condenser? What is the energy of each condenser?

15. In problem 14 if a condenser of 10 microfarads capacity is connected in parallel with the one of 20 microfarads capacity, what will be the voltage across each condenser? What will be the energy of each condenser?

16. The outer terminals of two condensers in series, C_1 and C_2, are connected to the ends of a wire AB of 20 ohms resistance, which is carrying a steady current of 4 amperes. If a wire of 3 ohms resistance, attached to the common terminals of the component condensers, is connected to a point P on the wire AB so that the resistance of AP is 8 ohms, C_1 will lose half its charge and C_2 will gain 60 microcoulombs. Find the capacities of C_1 and C_2.

17. Between the points A and B are connected two sets of condensers in parallel. The first set consists of two condensers in series, each of 20 microfarads capacity. The second set consists of three condensers in series, of 10, 20 and 40 microfarads capacity, connected in the order named. When there is a difference in potential of 110 volts between A and B what is the charge on each condenser? What is the total energy of the system?

18. In problem 17 with a potential difference of 110 volts between A and B what will be the charge on each condenser if the common terimnal of the first two condensers is connected to the common terminal of the 10 and 20-microfarad condensers in the second set? What will be the final energy of the system?

19. Between the points A and B are connected two sets of condensers in parallel. The first set consists of two condensers in series, of 10 and 20 microfarads capacity respectively. The second set also consists of two condensers in series, of 20 and 10 microfarads capacity respectively. The condensers are connected from A to B in the order named, and across these points there is a difference in potential of 110 volts. By what amount is the total energy of the system increased or diminished when the common terminals of the condensers of the first set are connected to the common terminals of the condensers of the second set?

20. A condenser of 20 microfarads capacity which has been charged to a potential difference of 110 volts is connected in parallel with an uncharged condenser of 10 microfarads capacity. What was the energy of the first condenser before being connected to the second? What is the final energy of the two condensers? What is the final voltage across the condensers?

21. A condenser of 20 microfarads capacity which has been charged to a potential difference of 220 volts, and a condenser of 10 microfarads capacity which has been charged to a potential difference of 110 volts, have their positive terminals connected. What is the potential difference across their negative terminals? What are the charges on the condensers before and after the negative terminals are connected? What are the voltages across the condensers after the negative terminals are connected?

22. A condenser of 1 microfarad capacity is charged to a potential difference of 110 volts and its terminals are then connected to those of an uncharged condenser of unknown capacity. If the resulting potential difference across the condensers is 82 volts, measured with an electrostatic instrument, what is the capacity of the second condenser?

23. A condenser of 20 microfarads capacity which has been charged to a potential difference of 110 volts has its positive terminal connected to the negative terminal of a condenser of 15 microfarads capacity which has been charged to a potential difference of 220 volts. What is the potential difference across their outer terminals? If the outer terminals are connected what will be the resulting voltages across the condensers? What will be the energy of each condenser before and after the outer terminals are connected?

24. Two condensers of 10 and 25 microfarads capacity are charged in series across 110-volt mains. They are then disconnected from the mains and from each other without loss of charge and their positive terminals are connected together. What will be the total heat generated in a wire of 100 ohms resistance that is connected across their negative terminals?

25. A condenser with a capacity of 20 microfarads which has been charged to a potential difference of 110 volts has one terminal connected to an uncharged condenser of 15 microfarads capacity. What will be the total heat generated in a wire of 25 ohms resistance that is connected across the outer terminals of the condensers?

26. Two condensers with capacities of 10 and 20 microfarads are connected in series and have their outer terminals attached to the ends of a slide wire of 10 ohms resistance in which there is a current of 5 amperes. The common terminal of the condensers is connected to the slide wire by means of a slider. Find the position of the slider so that (*a*) the total energy of the condensers may be a maximum; (*b*) the total energy may be a minimum; (*c*) the sum of the charges may be a maximum.

27. Two condensers with capacities of 10 and 20 microfarads are connected in series and have their outer terminals attached to the ends of a slide wire of 10 ohms resistance in which there is a current of 5 amperes. The common terminal of the condensers is connected to another condenser of 10 microfarads capacity, the other terminal of which is connected to the slide wire by means of a slider. Find the position of the slider so that the charge on the third condenser will have its greatest value.

28. In problem 27 find the position of the slider so that the total energy of the system will have its greatest value.

29. In problem 27 find the position of the slider so that the total energy of the system will have its least value.

30. Two condensers with capacities of 10 and 20 microfarads are charged in series across 110-volt mains. They are disconnected from the mains, and an outer terminal of one of them is connected to a terminal of an uncharged condenser of 25 microfarads capacity. What will be the total heat generated in a wire of 20 ohms resistance that is connected across the extreme terminals of these three condensers?

31. Two condensers, each of 20 microfarads capacity, which have been charged in parallel to a potential difference of 110 volts, have one of their common terminals connected to a terminal of an uncharged condenser of 15 microfarads capacity. What will be the total heat generated in a wire of 10 ohms resistance that is connected across the outer terminals of this system of condensers?

32. Three condensers with capacities of 10, 20, and 25 microfarads are charged in series across 220-volt mains and are then disconnected from the mains and each other without loss of charge. If the condensers are arranged in parallel with their terminals of like sign connected together, what will be the final potential across them?

33. Three equal condensers, each having a capacity of 20 microfarads, are charged in parallel by an e.m.f. of 110 volts. They are then disconnected without loss of charge, and one of them is connected in series with the other two in parallel, the positive terminal of the single condenser to the common negative terminal of the pair. The outer terminals of the system thus formed are connected by a wire of 10 ohms resistance until the system reaches a state of equilibrium. What will be the total heat generated in this resistance?

34. In problem 33 what will be the initial and final charges on each condenser? What will be the final potential difference across each condenser?

35. Three condensers with capacities of 10, 20, and 25 microfarads are charged in parallel across 220-volt mains. They are disconnected without loss of charge and are reconnected in series with the positive terminal of one condenser in contact with the negative terminal of the next. What will be the total heat generated in a wire of 5 ohms resistance connected across the outer terminals of this system?

36. In problem 35 what will be the final charge and potential of each condenser? Compare the initial and the final polarity of each condenser.

37. Four condensers with capacities of 5, 10, 20, and 25 microfarads respectively are charged in series across 220-volt mains. They are disconnected without loss of charge and joined up in parallel, with the negative terminals of the first two condensers connected to the positive terminals of the others, and the positive terminals of the first two to the negative terminals of the others. What will be the initial and final charges on each condenser? What will be the final voltage across the condensers? ~~Compare the initial and the final polarity of each condenser.~~

38. If the four condensers in problem 37 had been charged in parallel across 110-volt mains and then connected in series in a complete circuit, the positive terminal of one to the negative terminal of the next, what would have been the final charge on each condenser? Compare the initial and the final polarity of each condenser.

39. Between the fixed plates of a parallel plate condenser there is placed, parallel to each, a thin metallic plate of the same area. The fixed plates are 1 inch apart and are kept at a constant potential difference of 10,000 volts. If the middle plate is insulated and 0.2 inch from one of the fixed plates, what is the potential difference between it and each of the fixed plates?

40. Two circular parallel conducting plates 10 inches in diameter, separated by a distance of 0.125 inch, are kept at a constant difference in potential of 1000 volts. How many ergs must be expended in moving them 0.125 inch farther apart?

41. The plates of an absolute electrometer, which are 10 inches in diameter and 0.7 inch apart, are charged to a potential difference of 10,000 volts. Find the force of attraction between them and the charge on each plate in coulombs.

42. The plates of a parallel plate condenser, each of which has an area of 100 square inches, are 0.5 inch apart. They are charged to a potential difference of 5000 volts, and the source of potential is then removed without loss of charge.

What will be the work done in ergs in separating the plates 0.5 inch farther apart?

43. The plates of a guard-ring electrometer, between which there is a dielectric, are charged to a potential difference of 1500 volts and the source of potential is removed without loss of charge. The force of attraction between the plates is 200 dynes, but if the dielectric is removed the force becomes 250 dynes. What is the specific inductive capacity of the dielectric? What is the final difference in potential between the plates?

44. In problem 43 if the potential difference between the plates had been kept constant at 1500 volts what would have been the force between the plates when the dielectric was removed?

45. Two equal and parallel fixed plates, A and B, are kept at potentials E_1 and E_2 and a third similar but movable plate kept at zero potential is placed symmetrically between them. The area of each plate is 100 square inches and the distance between A and B is 1 inch. $E_1 = +5000$ volts and $E_2 = -5000$ volts. Find the work done in joules in moving the middle plate 0.2 inch toward the plate B. What are the forces acting on the middle plate in its initial and in its final position?

46. A parallel plate condenser consists of two plates each with an area of 150 square inches separated by a sheet of dielectric material 0.25 inch thick. The specific inductive capacity of the dielectric is 5.04. What will be the work done against the electric forces in removing the dielectric if the plates are kept at a constant difference in potential of 1000 volts? What is the charge on each plate before and after the dielectric is removed?

47. The plates in problem 46 are first charged to a difference in potential of 500 volts and the source of potential is then removed. What will be the work done against the electric forces in removing the dielectric? What is the potential difference between the plates after the dielectric is removed?

48. Two fixed circular plates, each of area A, placed parallel to one another and separated by a distance l, are kept at a constant potential difference E. A third equal conducting plate of thickness l_1, which is insulated from the others, is inserted between them parallel to each and with its center on the line of their centers. By what amount is the energy of the system thereby increased or diminished?

49. In problem 48 if the plates are charged to a difference in potential of E and the source of potential then removed, by what amount is the energy of the system increased or diminished when the third plate is inserted between them?

50. How many sheets of mica 0.01 of an inch thick separating metallic plates 6 inches in diameter will be necessary to construct a standard 0.5 microfarad condenser? The specific inductive capacity of the mica is 5.04.

51. A condenser is built of 200 circular sheets of tin-foil separated by mica 0.5 millimeter thick. How many tin-foil surfaces will be of one polarity? What must be the diameter of the sheets in order that the condenser may have a capacity of $\frac{1}{3}$ microfarad? The specific inductive capacity of the mica is 5.04.

52. The core of a submarine power cable is 0.5 of an inch in diameter and it is insulated with 0.3 inch of a compound that has a specific inductive capacity of 2.4. What is the capacity of the cable per mile?

53. The copper conductor of a submarine cable has a cross-section of 250,000 circular mils and its insulation has a specific inductive capacity of 2.4. If the capacity of the cable is 0.35 microfarad per mile what is the thickness of its insulation?

54. The core of a high voltage cable has a cross-section of 500,000 circular mils and the insulation is 0.5 inch in thickness and has a specific inductive capacity of 2.2. What is the energy in the electrostatic field for 8 miles of this cable when there is a potential difference of 13,200 volts between the core and the sheath?

55. Between the innermost and outermost of three concentric metallic cylinders there is a difference in potential of 10,000 volts. If the middle one is halfway between the others what is the difference in potential between it and each of the others? The radius of the outermost is three times the radius of the innermost.

CHAPTER X

VARIABLE CURRENTS: RESISTANCE AND INDUCTANCE

When an e.m.f., either constant or variable, is impressed on an electric circuit having resistance and inductance, the general differential equation that must be satisfied is

$$e = Ri + \frac{d}{dt}(N\phi);$$

that is, the impressed e.m.f. must supply not only the ohmic drop but must also supply a component to balance the induced e.m.f. set up by the change in the linkages of the flux and turns. Except in a few unusual cases the number of turns, N, is constant and the equation is then written

$$E = Ri + N\frac{d\phi}{dt}. \tag{1}$$

If the flux, ϕ, is produced by the current, i, alone and is furthermore proportional to it, *i.e.*, the reluctance of the magnetic circuit is assumed constant, the equation (1) may be written

$$e = Ri + L\frac{di}{dt}. \tag{2}$$

If the flux, ϕ, is produced by the combined action of the currents i_1 and i_2, as in the case with two coils wound on the same magnetic circuit, and the reluctance of the magnetic circuit is still assumed constant, the equation (1) may be written

$$e_1 = R_1 i_1 + L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}. \tag{3}$$

There is a similar equation for the second coil.

L_1 is the self-inductance of the coil in which the current i_1

exists, and M is the mutual inductance of the second coil with respect to the first coil. If the coils tend to produce magnetic fields in conjunction, the plus sign before M is taken, but if they tend to produce opposing magnetic fields the minus sign is taken.

The equations (1) and (2) may be transformed into power equations by multiplying each term by i; thus (1) becomes

$$ei = Ri^2 + Ni\frac{d\phi}{dt},$$

and (2) becomes

$$ei = Ri^2 + Li\frac{di}{dt};$$

that is, the rate at which energy is supplied to the circuit is equal to the rate at which energy is *dissipated* in heat plus the rate at which energy is *stored* in the electromagnetic field.

If each term of the equation (2) is multiplied by $i\,dt$ and then integrated it becomes

$$\int_0^t ei\,dt = \int_0^t Ri^2\,dt + \tfrac{1}{2}LI^2,$$

where I is the value of the current at the time t (the upper limit of integration).

If the impressed e.m.f. is constant and equal to E the solution of the equation (2) is

$$i = \frac{E}{R} + K\varepsilon^{-\frac{Rt}{L}}. \qquad (3)$$

The value of K, the constant of integration, will in general be determined if the energy of the electromagnetic field is known at some particular instant.

If the energy of the electromagnetic field is zero, or of some determined value, at the instant before the condition of the electric circuit is altered, it is also zero, or of the same determined value, at the instant after the condition of the electric circuit is altered. That is, the energy of the electromagnetic field cannot be changed by a finite amount in a differential element of time. The current, however, may have different values at these two consecutive instants.

In the equation (3), since a constant e.m.f., E, is *impressed* on the circuit, the energy of the electromagnetic field is zero when $t = 0$, so that $K = -\frac{E}{R}$, and $i = \frac{E}{R}\left(1 - \varepsilon^{-\frac{Rt}{L}}\right)$. $\frac{L}{R}$ is known as the time constant of the circuit.

When more than one constant of integration is involved, as in the case with simultaneous equations of the type of the equation (3), it is not in general sufficient to know the energy of the electromagnetic field at some particular instant alone, but another relation will also be necessary. This is conveniently obtained by writing the fundamental voltage equations in the form of the equation (1) and then eliminating the rate of change of the flux, $\frac{d\phi}{dt}$, which is the same in each equation. This will give a relation between the currents in the circuits that holds for all times.

If a coil which has resistance and self-inductance alone is short-circuited when it is carrying a current I, the differential equation that must be satisfied is a modification of the equation (2), viz., $0 = Ri + L\frac{di}{dt}$, that is, the impressed e.m.f. is zero. The solution of this equation is

$$i = I\varepsilon^{-\frac{Rt}{L}}.$$

PROBLEMS

1. A coil of 1000 turns wound on a magnetic circuit has a resistance of 2 ohms. If an e.m.f. of 25 volts is impressed on the coil, at what rate will the flux be increasing when the current is 5 amperes? when the current is 10 amperes?

2. In problem 1 at what rate is energy being stored in the electromagnetic field when the current is 5 amperes? 10 amperes? 12.5 amperes (*i. e.,* its ultimate value)?

3. In problem 1 at what rate is energy being supplied to the coil when the current is zero? 5 amperes? 12.5 amperes (*i. e.,* its ultimate value)?

4. In problem 1 at what rate is energy being supplied to the coil when the flux is increasing at the rate of 1,000,000 lines per second? at the rate of 100,000 lines per second? At what rate is energy being dissipated in joule heating at each of these instants?

5. An e.m.f. of 30 volts is impressed on an impedance coil which has a resistance of 1 ohm and an inductance of 0.12 henry. What is the time constant of this coil? At what rate does the current begin to increase? What is the ultimate value of the current?

6. In problem 5 at what rate is energy being dissipated in joule heating when the current is increasing at the rate of 100 amperes per second? What is the energy of the electromagnetic field at this instant?

7. In problem 5 at what rate is the current increasing when the energy of the electromagnetic field is one-half of its ultimate value? At what rate is energy being dissipated in joule heating at this instant?

8. In problem 5 at what rate is energy being stored in the electromagnetic field when the current is one-half of its ultimate value?

9. In problem 5 at what rate is energy being supplied to the coil when the energy of the electromagnetic field is one-half of its ultimate value?

10. In problem 5 at what rate is energy being supplied to the coil when the current is increasing at the rate of 50 amperes per second ?

11. In problem 5 what is the energy of the electromagnetic field when the rate at which energy is being dissipated in joule heating is equal to the rate at which it is being stored in the electromagnetic field?

12. An impedance coil has a resistance of 1.2 ohms and an inductance of 0.9 henry. At the instant that this coil is carrying a current of 50 amperes it is short-circuited. At what rate does the current begin to decrease? At what rate is the current decreasing when it reaches a value of 25 amperes?

13. In problem 12 what will be the energy of the electromagnetic field at the instant that the current is decreasing at the rate of 40 amperes per second? At what rate is the energy of the electromagnetic field decreasing at this instant?

14. An impedance coil which has a resistance of 4.4 ohms and an inductance of 1.1 henrys is carrying a steady current of 50 amperes. If the impressed e.m.f. is replaced by a non-inductive resistance of 10 ohms at what rate is the current decreasing at the instant the energy of the electromagnetic field is equal to the total energy that has been dissipated in joule heating since the current began to decrease?

15. In problem 14 what is the value of the current at the instant the energy of the electromagnetic field is equal to the energy that has been dissipated in heating the 10-ohm non-inductive resistance since the current began to decrease?

16. In problem 14 at what rate is the current decreasing at the instant the energy of the electromagnetic field is equal to the energy that has been dissipated in heating the 10-ohm non-inductive resistance since the current began to decrease?

17. An impedance coil has a resistance of 1 ohm and an inductance of 0.1 henry. What is the time constant of the coil? Plot the values of the current during 1 second after an e.m.f. of 10 volts is impressed on the coil. If the impressed e.m.f. were 20 volts in what way would the plot be changed?

18. In problem 17 plot the rate at which energy is supplied to the coil during 1 second after an e.m.f. of 10 volts is impressed on it. If the impressed e.m.f. is doubled in what way is this plot changed?

19. In problem 17 with an e.m.f. of 10 volts impressed on the coil at what rate is the current increasing at the end of 0.1 second? at the end of 1 second?

20. In problem 17 what is the energy of the electromagnetic field 0.1 second after an e.m.f. of 10 volts is impressed on the coil? At what rate is energy being dissipated in joule heating at this instant?

21. An e.m.f. of E volts is impressed on an impedance coil which has a resistance of R ohms and an inductance of L henrys. At what rate is energy being stored in the electromagnetic field at the instant that the e.m.f. is first impressed on the coil? What is the ultimate rate at which energy is being stored in the electromagnetic field?

22. In problem 21 at what time is the rate at which energy is being stored in the electromagnetic field a maximum?

23. In problem 21 at what rate is energy being dissipated in joule heating at the instant that the e.m.f. is impressed on the coil? What is the ultimate rate at which energy is dissipated in heat?

24.[1] An e.m.f. of 15 volts is impressed on an impedance coil which has a resistance of 1 ohm and an inductance of 2 henrys. What is the time constant of the coil? Plot the values of the current during the first 10 seconds.

25.[2] In problem 24 plot the rate at which energy is supplied to the coil during the first 10 seconds.

26.[3] In problem 24 what is the energy of the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

27.[4] In problem 24 at what rate is the current increasing at the end of 5 seconds? at the end of T seconds? (T = time constant.)

28.[5] In problem 24 at what rate is energy being stored in the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

29.[1] An e.m.f. of 15 volts is impressed on an impedance coil which has a resistance of 2 ohms and an inductance of 2 henrys. What is the time constant of the coil? Plot the values of the current during the first 10 seconds.

30.[2] In problem 29 plot the rate at which energy is supplied to the coil during the first 10 seconds.

31.[3] In problem 29 what is the energy of the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

32.[4] In problem 29 at what rate is the current increasing at the end of 5 seconds? at the end of T seconds? (T = time constant.)

33.[5] In problem 29 at what rate is energy being stored in the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

[1–5] Compare the results in these problems, noting the effect of varying R and L.

34.[1] An e.m.f. of 15 volts is impressed on an impedance coil which has a resistance of 1 ohm and an inductance of 4 henrys. What is the time constant of the coil? Plot the values of the current during the first 10 seconds.

35.[2] In problem 34 plot the rate at which energy is supplied to the coil during the first 10 seconds.

36.[3] In problem 34 what is the energy of the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

37.[4] In problem 34 at what rate is the current increasing at the end of 5 seconds? at the end of T seconds? (T = time constant.)

38.[5] In problem 34 at what rate is energy being stored in the electromagnetic field at the end of 5 seconds? at the end of T seconds? (T = time constant.)

39. An impedance coil which has a resistance of 4 ohms and an inductance of 2 henrys is connected in parallel with a non-inductive resistance of 4 ohms across 110 volts until the currents through each are constant. What is the current through the non-inductive resistance at the instant after the impressed e.m.f. is removed and the coil begins to discharge through the non-inductive resistance? What is the energy of the electromagnetic field at this instant and how long will it take for 99 per cent of this energy to be dissipated in joule heating?

40. An impedance coil has a resistance of 5 ohms and an inductance of 0.15 henry. When it is carrying a current of 25 amperes the impressed e.m.f. is replaced by a non-inductive resistance of 10 ohms. What per cent of the initial energy of the electromagnetic field is ultimately dissipated in this non-inductive resistance? If the non-inductive resistance had been one of 2 ohms, what per cent of the initial energy of the electromagnetic field would have been ultimately dissipated in it?

41. In problem 40 what will be the initial voltage across the non-inductive resistance in each case?

42. An impedance coil which has a resistance of 5 ohms and an inductance of 1 henry is connected in parallel with a non-

[1–5] Compare the results in these problems, noting the effect of varying R and L.

inductive resistance of 5 ohms. What is the ratio of the currents in the parallel branches 0.1 second after an e.m.f. of 20 volts is impressed on them? What is the ratio of the currents 1 second after the e.m.f. is impressed on them?

43. An impedance coil which has a resistance of 5 ohms and an inductance of 1 henry is connected in series with a non-inductive resistance of 5 ohms. What is the ratio of the voltages across the coil and the non-inductive resistance 0.1 second after an e.m.f. of 50 volts is impressed on the entire circuit? What is the ratio of the voltages 1 second after the e.m.f. is impressed on the circuit?

44. An e.m.f. of 110 volts is impressed on an inductive circuit. At the end of 3 seconds the current is 37 amperes and at the end of 1 minute it has assumed a sensibly constant value of 44 amperes. What is the resistance of the circuit? What is the inductance of the circuit?

45. The field resistance of a 1-horsepower shunt motor is measured with a 1-scale ammeter and a 150-scale voltmeter which has a resistance of 17,560 ohms. The latter is connected across the field winding terminals inside of the switch which is between the field and the mains. The ammeter is connected so that its reading does not include the current taken by the voltmeter. The current is 0.58 ampere and the potential is 115 volts. The switch is opened so quickly that practically none of the electromagnetic energy of the field is dissipated in the break. What will the ammeter read the instant after the switch is opened? What will be the voltage across the voltmeter at this instant? Will either instrument be deflected in the reverse direction?

46. In problem 45 what is the highest resistance that should be connected across the field so that the potential across it cannot possibly exceed 150 volts when the switch is opened?

47. The resistance of the field winding of a shunt generator is measured with a 10-scale ammeter and a 150-scale voltmeter. As a precaution against puncturing the field winding insulation a resistance of 50 ohms is connected across it. One terminal of this resistance is connected between the ammeter and the switch that is used to connect the field winding to the mains. With the switch closed the instruments indicate 9.2 amperes and

116 volts respectively. The voltmeter is disconnected and the switch opened so quickly that practically none of the energy of the field is dissipated in the break. 1.8 seconds after the switch is opened the potential across the field is measured with the voltmeter and found to be 21 volts. What is the inductance of the field circuit?

48. In problem 47 if the ammeter reads 0.42 amperes 1.8 seconds after the switch is opened what is the inductance of the field circuit?

49. A telegraph circuit is composed of a battery which has an e.m.f. of 10 volts and an internal resistance of 2 ohms, a non-inductive line of 51.6 ohms resistance and a relay of 21.4 ohms resistance and 1.25 henrys inductance. What is the time constant of the circuit? If the armature of the relay is not attracted until the current reaches 0.05 ampere how long after the circuit is closed will it operate?

50. In problem 49 if the armature of the relay is not released until the current falls to 0.03 ampere how long after the relay is short-circuited, when carrying 0.06 ampere, will the armature be released?

51. A relay which has a resistance of 8.4 ohms and an inductance of 0.72 henry is connected by leads having a resistance of 31.6 ohms to a 10-volt constant potential battery. The armature of the relay is attracted when the current reaches 0.15 ampere and is not released until it falls to 0.05 ampere. What is the limiting speed at which the armature can be made to vibrate by alternately short-circuiting the relay and then removing the short circuit?

52. In problem 51 what will be the limiting rate of vibration if the e.m.f. of the battery alone is doubled?

53. In problem 51 if the e.m.f. of the battery is doubled and at the same time the resistance of the leads is increased so that the greatest current through the battery is the same as before, what will be the limiting rate of vibration?

54. Two impedance coils which have no mutual inductance are connected in parallel across an e.m.f. of E volts. What relation must exist between the constants of the coils in order that the currents through them will always be inversely as their resistances?

55. Two impedance coils which have no mutual inductance are connected in series across an e.m.f. of E volts. The resistances and self-inductances of the coils are R_1 and R_2 ohms and L_1 and L_2 henrys respectively. What is the voltage across each coil t seconds after this connection is made?

56. In problem 55 what relation must exist between the constants of the coils in order that the voltages across the coils shall be constant and in the ratio of the resistances of the coils?

57. Two coils are wound side by side on the same magnetic circuit. The first has 500 turns and a resistance of 3.5 ohms, and the second has 1000 turns and a resistance of 15 ohms. What will be the e.m.f. generated in the second coil at the instant that an e.m.f. of 25 volts is impressed on the first coil? What will be the e.m.f. generated in the second coil when the current through the first coil reaches a value of 5 amperes?

58. In problem 57 what is the value of the current at the instant that the e.m.f. generated in the second coil is equal to the e.m.f. impressed on the first?

59. In problem 57 at what rate is energy being supplied to the first coil when the e.m.f. generated in the second coil is 50 volts? 25 volts? zero?

60. In problem 57 at what rate is energy being stored in the electromagnetic field when the e.m.f. generated in the second coil is 50 volts? 25 volts? zero?

61. Two coils are wound side by side on the same magnetic circuit. The first has a resistance of 1.5 ohms and a self-inductance of 0.2 henry, and the second has a resistance of 2 ohms and a self-inductance of 0.3 henry. What is the ratio of the turns in the first coil to those in the second? What is the e.m.f. generated in the second coil at the instant that an e.m.f. of 30 volts is impressed on the first coil? Compare the ratio of the e.m.f. impressed on the first coil to the e.m.f. generated in the second coil at this instant with the ratio of the turns. What is the ultimate value of the e.m.f. generated in the second coil?

62. In problem 61 if an e.m.f. of 30 volts is impressed on the first coil what will be the potential across the second when the current through the first is one-quarter of its ultimate value? three-quarters of its ultimate value? What is the ratio of the e.m.f. impressed on the first coil to the e.m.f. generated in the second for each of these values of the current?

63. In problem 61 if an e.m.f. is impressed on the second coil such as to give the same ultimate value of the current (viz., 20 amperes) what will be the e.m.f. generated in the first coil when the current through the second coil is one-fourth of its ultimate value? three-fourths of its ultimate value? What is the ratio of the e.m.f. impressed on the second coil to the e.m.f. generated in the first for each of these values of the current?

64. The resistances and the self-inductances of two coils which are wound side by side on the same magnetic circuit are 1 and 2 ohms and 0.05 and 0.2 henry respectively. Plot the values of the current through the first coil and the e.m.f. generated in the second coil during 0.5 second after an e.m.f. of 10 volts is impressed on the first coil.

65. In problem 64 what will be the energy of the electromagnetic field when the e.m.f. generated in the second coil has its maximum value? when it is 10 volts? zero?

66. In problem 64 plot the values of the current through the second coil and the e.m.f. generated in the first coil during 0.5 second after an e.m.f. of 10 volts is impressed on the second coil.

67. In problem 64 plot the values of the current through the second coil and the e.m.f. generated in the first coil during 0.5 second after an e.m.f. of 20 volts is impressed on the second coil. In this case the ultimate value of the current is the same as in problem 64.

68. Two coils which are wound side by side on the same magnetic circuit have resistances and self-inductances of 1 and 2 ohms and 0.1 and 0.4 henry respectively. If when the first coil is carrying a current of 25 amperes it is short-circuited, what will be the e.m.f. generated in the second coil at the instant of short circuit? What will be the ultimate value of this generated e.m.f.?

69. In problem 68 plot the values of the current through the first coil and the e.m.f. generated in the second coil during 1 second after short circuit.

70. For a case like that cited in problem 68 show that the e.m.f. generated in the second coil is always equal to the ohmic drop in the first coil multiplied by the ratio of the turns in the second coil to those in the first.

71. Two coils which are wound side by side on a magnetic circuit of constant reluctance are connected in series so that they produce magnetic fields in the same direction. Assuming no magnetic leakage what will be the current through the coils t seconds after an e.m.f. of E volts is impressed on them?

	First Coil.	Second Coil.
Resistance	R_1	R_2
Turns	N_1	N_2
Self-inductance...............	L_1	L_2

72. If the coils in problem 71 are connected so as to produce opposing magnetic fields what will be the current t seconds after the e.m.f. of E volts is impressed on them?

73. On a coil of negligible resistance and of 0.1 henry inductance there is impressed an e.m.f. which is graphically represented as follows: During the first 0.005 of a second it is constant and equal to 100 volts; during the next 0.01 of a second it is reversed in direction but still constant and equal to 100 volts; and thereafter it is of constant magnitude but it is reversed every 0.01 of a second. Plot the values of the current. What is the maximum energy of the electromagnetic field? At what rate is the current changing when it passes through zero?

74. Two coils are wound side by side on a magnetic circuit of constant reluctance so that when connected in parallel they tend to produce opposing magnetic fields. Assuming no magnetic leakage what will be the current in each t seconds after they are connected in parallel across an e.m.f. of E volts? The constants of the coils are given in problem 71.

75. If the coils in problem 74 are connected in parallel so as to produce magnetic fields that are in conjunction, what will be the current in each t seconds after an e.m.f. of E volts is impressed on them?

76. Two coils are wound side by side on the same magnetic circuit and the first is short-circuited. Assuming no magnetic leakage what will be the current in each coil t seconds after an e.m.f. of E volts is impressed on the second coil? The constants of the coils are given in problem 71.

77. In problem 76 what is the total heat dissipated in the first coil when an e.m.f. of E volts is impressed on the second coil?

78. In problem 76 if the second coil is short-circuited when it is carrying a current of I amperes what will be the current in each coil t seconds later?

79. In problem 78 what is the total heat dissipated in each coil?

CHAPTER XI

VARIABLE CURRENTS: RESISTANCE AND CAPACITY

When an e.m.f. which may be either constant or variable is impressed on a circuit consisting of non-inductive resistance and capacity in series, the general voltage equation that must be satisfied is

$$e = Ri + \frac{q}{C}, \tag{1}$$

where q is the charge (coulombs) on the condenser and C is the capacity (farads) of the condenser. This equation (1) may be transformed into a power equation by multiplying each term by i, thus:

$$ei = Ri^2 + \frac{q}{C}\,i,$$

or

$$ei = Ri^2 + \frac{q}{C}\,\frac{dq}{dt};$$

that is, the rate at which energy is supplied to the circuit is equal to the rate at which energy is *dissipated* in heat plus the rate at which energy is *stored* in the electrostatic field.

If each term of this equation (1) is multiplied by $i\,dt$ and then integrated it becomes

$$\int_0^t ei\,dt = \int_0^t Ri^2\,dt + \frac{1}{2}\,\frac{Q^2}{C},$$

where Q is the charge on the condenser at the time t (the upper limit of integration).

Since $i = \dfrac{dq}{dt}$ the equation (1) may be written in the differential form

$$e = R\frac{dq}{dt} + \frac{q}{C}, \tag{2}$$

or, the equation (1) may be differentiated,

$$\frac{de}{dt} = R\frac{di}{dt} + \frac{i}{C}. \tag{3}$$

If the impressed e.m.f. is constant and equal to E, the equations (2) and (3) may be integrated for the values of the charge and the current respectively, thus:

$$q = EC + Q\varepsilon^{-\frac{t}{CR}}, \tag{4}$$

$$i = I\varepsilon^{-\frac{t}{CR}}. \tag{5}$$

The constants of integration, Q and I, will in general be determined if the energy of the electrostatic field is known at some particular instant.

If the energy of the electrostatic field is zero, or of some determined value, at the instant before the condition of the electric circuit is altered, it is also zero, or of the same determined value, at the instant after the condition of the electric circuit is altered. That is, the energy of the electrostatic field cannot be changed by a finite amount in a differential element of time. The current, however, may have, and usually does have, different values at these two consecutive instants of time.

In the equations (4) and (5), since a constant e.m.f., E, is impressed on the circuit, the energy of the condenser is zero when the time is zero and the current is thus equal to $\frac{E}{R}$ at the same instant. Thus the equations for the charge and the current are

$$q = EC\left(1 - \varepsilon^{-\frac{t}{CR}}\right),$$

and

$$i = \frac{E}{R}\varepsilon^{-\frac{t}{CR}}$$

CR is known as the time constant of the circuit.

In some cases, when two or more condensers are in the circuit, it may be necessary to know not only the energy of each condenser when the time is zero but also the values of the current at this instant.

If a charged condenser is allowed to discharge through a non-inductive resistance the differential equations that must be satisfied are modifications of the equations (2) and (3):

$$0 = R\frac{dq}{dt} + \frac{q}{C},$$

and

$$0 = R\frac{di}{dt} + \frac{i}{C};$$

that is, the impressed e.m.f. is zero. The solutions of these equations are

$$q = EC\varepsilon^{-\frac{t}{CR}},$$

and

$$i = \frac{E}{R}\varepsilon^{-\frac{t}{CR}},$$

where E is the potential of the condenser when the time is zero.

PROBLEMS

1. A condenser which has a capacity of 45 microfarads is charged through a resistance of 10,000 ohms by an e.m.f. of 500 volts. What is the charge on the condenser when the current is one-half of its initial value? At what rate is the charge increasing at this instant?

2. In problem 1 at what rate is the charge increasing when the potential energy of the condenser is one-half of its ultimate value?

3. In problem 1 at what rate is the charge on the condenser increasing when the current is decreasing at the rate of 0.01 ampere per second? 0.001 ampere per second?

4. A condenser of 50 microfarads capacity which has been charged to a potential of 500 volts is discharged through a resistance of 10,000 ohms. At what rate does the condenser begin to lose its charge?

5. In problem 4 at what rate is the condenser losing its charge when the potential across its terminals has fallen to 200 volts?

6. In problem 4 at what rate is the condenser losing its charge when the current is diminishing at the rate of 0.008 ampere per second? 0.0008 ampere per second?

7. A condenser which has a capacity of 20 microfarads is charged through a resistance of 100,000 ohms by an e.m.f. of 500 volts. What is the initial charging current? What is the ultimate charge on the condenser?

8. In problem 7 what is the charging current at the instant the condenser has received one-half of its ultimate charge?

9. In problem 7 what is the charging current when the potential energy of the condenser is one-half of its ultimate value?

10. In problem 7 at what rate is energy being delivered to the condenser when the charging current is one-half of its initial value?

11. In problem 7 at what rate is energy being dissipated in heating the resistance at the instant that the condenser has received one-half of its ultimate charge?

12. In problem 7 at what rate is energy being delivered to the system from the source of potential at the instant that the current has its greatest value?

13. In problem 7 at what rate is energy being delivered to the system from the source of potential when the potential energy of the condenser has reached one-half of its ultimate value?

14. A condenser of 100 microfarads capacity which has been charged to a potential of 220 volts is discharged by connecting a resistance of 10,000 ohms across its terminals. What is the initial energy of the condenser? What is the initial value of the current on discharge?

15. In problem 14 what is the current when the charge on the condenser has fallen to one-half of its initial value?

16. In problem 14 at what rate is energy being dissipated in joule heating when the potential energy of the condenser has fallen to one-half of its initial value?

17. In problem 14 what is the total amount of energy that has been dissipated in joule heating up to the instant when the current has fallen to one-half of its initial value?

18. A condenser which has a capacity of 15 microfarads is charged through a resistance of 1,000 ohms by an e.m.f. of 220 volts. At what rate is the current diminishing when it has its greatest value? At what rate is the current diminishing when it has fallen to one-half of its initial value?

19. In problem 18 at what rate is the current diminishing when the charge on the condenser has reached one-half of its ultimate value?

20. In problem 18 at what rate is the current diminishing when the potential energy of the condenser reaches one-half of its ultimate value?

21. A condenser of 45 microfarads capacity which has been charged to a potential of 450 volts is discharged by connecting a resistance of 10,000 ohms across its terminals. At what rate does the current begin to decrease? At what rate is the current diminishing when it has fallen to one-half of its initial value?

22. In problem 21 at what rate is the current diminishing when the energy of the condenser is reduced to one-half of its initial value?

23. In problem 21 at what rate is the current diminishing when the charge on the condenser is reduced to one-half of its initial value?

24.[1] A condenser which has a capacity of 10 microfarads is charged through a resistance of 100,000 ohms by an e.m.f. of 500 volts. What is the time constant of this circuit? Plot the values of the current during the first 5 seconds.

25. In problem 24 what is the energy of the electrostatic field at the end of 1 second?

26.[2] In problem 24 what is the percentage error in assuming that the condenser is fully charged at the end of 5 seconds? at the end of 10 seconds?

27.[3] In problem 24 what is the rate at which the current is decreasing at the end of 1 second? What is the rate at which the charge is increasing at the end of 1 second?

28.[1] A condenser which has a capacity of 10 microfarads is charged through a resistance of 200,000 ohms by an e.m.f. of

[1-3] Compare the results of these problems, noting the effect of varying R and C.

500 volts. What is the time constant of this circuit? Plot the values of the current during the first 5 seconds.

29.[2] In problem 28 what is the percentage error in assuming that the condenser is fully charged at the end of 5 seconds? at the end of 10 seconds?

30.[3] In problem 28 what is the rate at which the current is decreasing at the end of 1 second? What is the rate at which the charge is increasing at the end of 1 second?

31.[1] A condenser which has a capacity of 20 microfarads is charged through a resistance of 100,000 ohms by an e.m.f. of 500 volts. What is the time constant of this circuit? Plot the values of the current during the first 5 seconds.

32.[2] In problem 31 what is the percentage error in assuming that the condenser is fully charged at the end of 5 seconds? at the end of 10 seconds?

33.[3] In problem 31 what is the rate at which the current is decreasing at the end of 1 second? What is the rate at which the charge is increasing at the end of 1 second?

34.[1] A condenser which has a capacity of 20 microfarads is charged through a resistance of 200,000 ohms by an e.m.f. of 500 volts. What is the time constant of this circuit? Plot the values of the current during the first 5 seconds.

35.[2] In problem 34 what is the percentage error in assuming that the condenser is fully charged at the end of 5 seconds? at the end of 10 seconds?

36.[3] In problem 34 what is the rate at which the current is decreasing at the end of 1 second? What is the rate at which the charge is increasing at the end of 1 second?

37. A condenser which has a capacity of 25 microfarads is charged through a resistance of 10,000 ohms. If an e.m.f. of 110 volts is used in what time will the condenser receive 99.9 per cent of its ultimate charge? If an e.m.f. of 500 volts is used in what time will the condenser receive 99.9 per cent of its ultimate charge?

38. In problem 37 compare the rates at which the condenser is charging at the end of 0.2 second for the two impressed e.m.fs.

[1–3] Compare the results of these problems, noting the effect of varying R and C.

39. A condenser of 100 microfarads capacity is charged to a potential of 220 volts and then discharged through a resistance of 10,000 ohms. Plot the current during the first 5 seconds.

40. In problem 39 what is the error in assuming that the condenser is completely discharged at the end of 5 seconds? If this condenser had been discharged through a resistance of 5,000 ohms what would have been the error in assuming that it was completely discharged at the end of 5 seconds?

41. In problem 39 at what rate is the current decreasing at the end of 1 second? At what rate is the charge on the condenser decreasing at the same instant?

42. In problem 39 at what rate is energy being dissipated in heating the resistance at the end of 1 second?

43. A condenser which has a capacity of 30 microfarads is charged through a resistance of 1000 ohms by an e.m.f. of 220 volts. What is the total amount of energy supplied during the first 0.05 second? during the first second?

44. A condenser which has a capacity of 100 microfarads is charged to a potential of 500 volts and then discharged through a resistance of 10,000 ohms. What is the energy dissipated in heat during the first second? during the first 10 seconds?

45. A high resistance is connected across the terminals of a 10-microfarad condenser which has been charged to a potential of 550 volts, and after one minute it is found that the potential of the condenser has fallen to 382 volts. What is the value of the resistance in megohms?

46. A 300-scale voltmeter which has a resistance of 42,840 ohms is permanently connected across the terminals of a condenser of unknown capacity. The condenser is charged to a potential of 232 volts and then disconnected from the source of potential and allowed to discharge through the voltmeter. Five and one-fifth seconds after the condenser begins to discharge the voltmeter reads 67 volts. What is the capacity of the condenser?

47. A 50-scale voltmeter which has a resistance of 5527 ohms is connected in series with a resistance of 50,000 ohms across the terminals of a condenser of unknown capacity. The condenser is charged to a potential of 465 volts and then allowed to discharge through the voltmeter and resistance. 4.6 seconds after the condenser has begun to discharge the voltmeter indicates 12.8 volts. What is the capacity of the condenser?

48. A battery with a constant potential difference of E volts is used to charge a condenser which has a capacity of C farads, through leads of total resistance R. Find the ratio of the total energy delivered by the battery to that received by the condenser.

49. A certain piece of electrical apparatus is operated as follows: A battery with an e.m.f. of 10 volts and an internal resistance of 2 ohms is used to charge a 25-microfarad condenser through a resistance of 8 ohms; the condenser is then discharged through the electrical apparatus to produce the desired effect. What is the efficiency of this combination of battery and condenser? Is there any way in which the efficiency might be improved without altering either the battery or the condenser?

50. A condenser which has a capacity of C farads is charged to a potential of E volts and at time $t = 0$ is connected through a resistance of R ohms to another condenser of capacity C_1 farads which is completely discharged. What is the charge on each condenser at any time t seconds later?

51. Two condensers which have capacities of C_1 and C_2 farads respectively are charged in parallel to a potential of E volts. If the impressed e.m.f. is replaced by a resistance of R ohms at time $t = 0$ what will be the charge on each condenser at any time t seconds later?

52. Two condensers which have capacities of C_1 and C_2 farads respectively are charged in parallel by an e.m.f. of E volts and are then connected in series, with the negative terminal of one in contact with the positive terminal of the other. The outer terminals of the condensers are then connected by a resistance of R ohms. At what rate is energy being dissipated in the resistance t seconds after this connection is made?

53. Two condensers of capacities C_1 and C_2 farads respectively which have been charged in series by an e.m.f. of E volts are reconnected in series with their positive terminals in contact. What is the charge on each condenser t seconds after the negative terminals are connected by a resistance of R ohms?

54. A condenser which has a capacity of C farads is connected in series with a resistance of R ohms to a constant potential battery of E volts. After the condenser is fully charged a resistance of R_1 ohms is connected directly across its terminals, without disconnecting the battery. What is the charge on the condenser t seconds after this final connection is made?

55. A condenser with a capacity of C_1 farads has a resistance of R_1 ohms permanently connected across its terminals. One terminal of this combination is then connected to a resistance of R_2 ohms in series with a condenser of C_2 farads capacity which has been charged to a potential of E volts. What is the quantity on each condenser t seconds after the extreme terminals of the condensers are connected?

56. On a circuit consisting of a condenser of 100 microfarads capacity in series with a resistance of 10,000 ohms there is impressed an e.m.f. which varies uniformly from 0 to 500 volts in the first 10 seconds. At what rate is the current varying when it has a value of 0.0025 ampere?

57. On the terminals of a condenser of 100 microfarads capacity there is impressed an e.m.f. which is graphically represented as follows: During the first fiftieth of a second it increases uniformly from 0 to 200 volts; during the next twenty-fifth of a second it decreases uniformly to -200 volts; during the next twenty-fifth of a second it increases uniformly to $+200$ volts, etc. Plot the values of the current during the first tenth of a second. Plot the rate at which energy is supplied to the condenser (watt curve).

58. On a condenser of 100 microfarads capacity there is impressed an e.m.f. which is graphically represented as follows: During the first fiftieth of a second it increases uniformly from 0 to 200 volts; during the next fiftieth of a second it decreases uniformly to zero; during the next fiftieth of a second it again increases uniformly to 200 volts, etc. Plot the current during the first tenth of a second. Plot the rate at which energy is supplied to the condenser (watt curve).

59. On a series circuit consisting of a non-inductive resistance of 10,000 ohms in series with a condenser of 250 microfarads capacity there is impressed an e.m.f. which increases uniformly from 0 to 500 volts in the first 10 seconds and then remains constant. Plot the values of the current during the first 20 seconds.

60. In problem 59 plot the values of the charge on the condenser during the first 20 seconds.

61. In problem 59 plot the rates at which energy is supplied to the resistance and to the condenser during the first 20 seconds.

62. In problem 59 what is the charge on the condenser at the end of 1 second, and what is the value of the current in the circuit at the end of 11 seconds?

63. In problem 59 what is the energy of the electrostatic field at the end of 10 seconds?

64. On a circuit consisting of a non-inductive resistance of 1 megohm in series with a capacity of 10 microfarads there is impressed an e.m.f. which is constant and equal to 100 volts during the first 10 seconds and then increases uniformly to 200 volts during the next 10 seconds. Plot the values of the current during the first 20 seconds.

65. In problem 64 plot the values of the charge on the condenser during the first 20 seconds.

66. In problem 64 plot the rate at which energy is supplied to the resistance and to the condenser during the first 20 seconds.

67. In problem 64 what is the value of the current in the circuit at the end of 1 second, and what is the charge on the condenser at the end of 11 seconds?

68. In problem 64 what is the energy of the electrostatic field at the end of 15 seconds?

69. An e.m.f. of 500 volts is impressed on a circuit consisting of a non-inductive resistance of 100,000 ohms in series with a condenser of 100 microfarads capacity until the condenser is fully charged. The e.m.f. is then decreased uniformly to zero in 10 seconds. Plot the values of the current during the time that the e.m.f. is variable.

70. In problem 69 plot the values of the charge on the condenser during the time that the e.m.f. is variable.

71. In problem 69 plot the rates at which energy is supplied to the resistance and to the condenser during the time that the e.m.f. is variable.

72. In problem 69 what is the value of the current in the circuit when the impressed e.m.f. is 400 volts, and what is the charge on the condenser at the instant that the impressed e.m.f. becomes zero?

73. In problem 69 what is the energy of the electrostatic field at the instant that the impressed e.m.f. becomes zero?

74. On a circuit consisting of a non-inductive resistance of 100,000 ohms in series with a condenser of 50 microfarads capacity there is impressed an e.m.f. which is constant and equal to 500 volts during the first 10 seconds and then decreases uniformly to zero in the next 10 seconds. Plot the values of the current during the first 20 seconds.

75. In problem 74 plot the values of the charge on the condenser during the first 20 seconds.

76. In problem 74 plot the rates at which energy is supplied to the resistance and to the condenser during the first 20 seconds.

77. In problem 74 what is the current in the circuit at the end of 5 seconds, and what is the charge on the condenser at the end of 20 seconds?

78. In problem 74 what is the energy of the electrostatic field at the end of 20 seconds?

79. On a series circuit consisting of a non-inductive resistance of 100,000 ohms in series with a condenser of 50 microfarads there is impressed an e.m.f. which increases uniformly from zero to 500 volts during the first 10 seconds and then decreases uniformly to zero in the next 10 seconds. Plot the values of the current during the first 20 seconds.

80. In problem 79 plot the values of the charge on the condenser during the first 20 seconds.

81. In problem 79 plot the rates at which energy is supplied to the resistance and to the condenser during the first 20 seconds.

82. In problem 79 what is the value of the current in the circuit at the end of 15 seconds, and what is the charge on the condenser at the same instant?

83. In problem 79 what is the energy of the electrostatic field at the end of 11 seconds?

CHAPTER XII

ALTERNATING CURRENTS: SIMPLE HARMONIC FUNCTIONS

In this chapter all of the electromotive forces and currents are simple harmonic functions. When a simple harmonic e.m.f. is impressed on a circuit consisting of resistance, inductance, and capacity in series, the differential equation that must be satisfied is $E \sin \omega t = Ri + L \frac{di}{dt} + \frac{q}{C}$.

The complete solution of this equation is

$$i = k_1 \varepsilon^{-D_1 t} + k_2 \varepsilon^{-D_2 t}$$

$$+ \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \sin \left(\omega t - \tan^{-1} \left[\frac{L\omega - \frac{1}{C\omega}}{R}\right]\right).$$

After a very brief interval the exponential terms become negligible — in one particular case their sum is zero — so that the current is a simple harmonic function and its equation may be written thus:

$$i = \frac{E}{Z} \sin (\omega t - \theta) = I \sin (\omega t - \theta),$$

where $Z = \sqrt{R^2 + X^2}$ is the impedance of the circuit; $X = L\omega - \frac{1}{C\omega}$ is the reactance of the circuit; $\theta = \tan^{-1} \frac{X}{R}$ is the time lag angle of the current with respect to the e.m.f.; $\omega = 2\pi f$ is the angular velocity of the rotating radius whose projections on a fixed time axis give the instantaneous values of the current or e.m.f.; f is the frequency of the current or e.m.f. E and I are the maximum values of the e.m.f. and current respectively.

For simple harmonic functions the effective values are the maximum values divided by $\sqrt{2}$.

The resultant of a number of alternating voltages acting in series may be found either by adding the instantaneous values as represented by their *equations* or by combining the effective values with due regard to their phase relations, *i.e.* by applying the principles of vector addition. The resultant of the alternating currents in a number of parallel branches may be found by either of these methods.

Ohm's law needs to be modified when applied to alternating-current circuits:

The *impedance* of the circuit must replace the *resistance*, and the *resultant e.m.f.* must be found by one of the methods described above.

Kirchhoff's laws will also have to be modified in a similar manner. The same conventions may be used that are given in Chapter III:

1. At *any instant* the *algebraic* sum of the currents at a junction is zero; or the *vector* sum of the currents at a junction is zero.

2. At *any instant* the *algebraic* sum of the impressed e.m.fs. and the reactive voltages, $-L\frac{di}{dt}$ and $-\frac{q}{C}$, is equal to the algebraic sum of the dissipative voltages, Ri, around any closed circuit; or, as it is more often stated, though not as precisely, the vector sum of the impressed voltages is equal to the vector sum of the reactive and the dissipative voltages around any closed circuit.

If the current is assumed to be a simple harmonic function, $i = I \sin \omega t$, the e.m.f. impressed on a circuit consisting of resistance, inductance, and capacity in series, is given by substituting this value of i in

$$e = Ri + L\frac{di}{dt} + \frac{q}{C},$$

so that

$$e = RI \sin \omega t + L\omega I \cos \omega t - \frac{I}{C\omega} \cos \omega t; \qquad (1)$$

that is, the impressed e.m.f. may be resolved into three components. The ohmic drop, Ri, is in phase with the current;

the component, $L\omega I \cos \omega t$, that overcomes the inductive reactive e.m.f., $- L\omega I \cos \omega t$, is 90 time degrees ahead of the current; the component, $-\frac{I}{C\omega} \cos \omega t$, that overcomes the capacity reactive voltage, $\frac{I}{C\omega} \cos \omega t$, is 90 time degrees behind the current.

If each term in equation (1) is multiplied by i it is transformed into a power equation, thus:

$$ei = RI^2 \sin^2 \omega t + L\omega I^2 \sin \omega t \cos \omega t - \frac{I^2}{C\omega} \sin \omega t \cos \omega t.$$

This may be readily transformed into a better form by applying the trigonometrical formulas for $\sin \frac{x}{2}$ and $\sin 2x$; thus:

$$ei = \frac{RI^2}{2} (1 - \cos 2\,\omega t) + \frac{L\omega I^2}{2} \cancel{\sin} 2\,\omega t - \frac{I^2}{2\,C\omega} \cancel{\sin} 2\omega t. \quad (2)$$

Thus the frequency of the functions which represent the power supplied to the different parts of the circuit is double that of the impressed e.m.f.

The first term of the right-hand side of this equation (2) is the power absorbed in joule heating. The second and third terms respectively represent the power supplied to the electromagnetic and the electrostatic fields. The average power absorbed in joule heating is $\frac{RI^2}{2}$ (I = maximum value of current). The average power supplied to the electromagnetic and to the electrostatic field is zero.

If E, volts, is the resultant *effective* voltage impressed on a circuit which has an *effective resistance* of R ohms and an *effective reactance* of X ohms, the *effective value* of the current is given by the expression

$$I = \frac{E}{\sqrt{R^2 + X^2}} \text{ (amperes)} = \frac{E}{Z} \text{ (amperes)}.$$

When a voltage or a current is designated by the term " volts " or " amperes " the *effective* value is always understood and *not* the maximum nor the average value. The expression for the power becomes I^2R (I = effective value of current). This is readily transformed into $EI \cos \theta$, or into E^2g; wherein

$$\cos \theta = \frac{R}{Z} \text{ and } g = \frac{R}{Z^2}.$$

The power component of the e.m.f. is the component which is in time-phase with the current, that is, it is the component which when multiplied by the current will give the power. Thus it is given by the expression

$$E \cos \theta = E\frac{R}{Z} = IR.$$

Likewise the power component of the current is given by the expression

$$I \cos \theta = \frac{IR}{Z} = Eg,$$

where $g = \dfrac{R}{Z^2}$ is called the conductance. The wattless or reactive component of the e.m.f. is the component which is in quadrature with the current and thus contributes no power. It is given by the expression

$$E \sin \theta = E\frac{X}{Z} = IX.$$

Likewise the wattless component of the current is given by the expression

$$I \sin \theta = I\frac{X}{Z} = Eb,$$

where $b = \dfrac{X}{Z^2}$ is called the susceptance.

In a series circuit the *current* has the *same value* at every point, and thus it is usually best to determine all voltages by their magnitudes and their phase relations with respect to this common current. The two components of the voltage across any portion of a circuit are the ohmic drop (IR) and the reactive drop (IX); R and X are the equivalent resistance and reactance respectively of this portion of the circuit. The first component is in phase with the current and the second component is in quadrature with the current. The voltage across this portion of the circuit leads the current in time by the angle whose tangent is $\frac{X}{R}$, and the value of the effective voltage is $I\sqrt{R^2 + X^2}$.

The resistance of a series circuit is the sum of the resistances of its parts, and the reactance is the algebraic sum of the reactances of its parts; an inductive reactance is positive and a condensive reactance is negative, since they are the cause of voltages that differ in phase by 180 time degrees.

A series circuit containing inductive and condensive reactance is in resonance when the current is in phase with the impressed voltage, that is, when the resultant reactance is zero. This occurs when $L\omega = \frac{1}{C\omega}\cdot$ The current then has its greatest value and is equal to $\frac{E}{R}\cdot$

With parallel circuits the *voltage* across each branch is the *same*, and thus it is usually best to determine all currents by their magnitudes and their phase relations with respect to this common voltage. The two components of the current in any branch that are in phase and in quadrature with the impressed voltage are Eg and Eb respectively; g and b are the conductance $\left(\frac{R}{Z^2}\right)$ and the susceptance $\left(\frac{X}{Z^2}\right)$ of this particular branch. The current in this branch *lags* in time behind the impressed voltage by the angle whose tangent is $\frac{b}{g}$, and the effective value of the current is $E\sqrt{g^2 + b^2}$.

The component of the resultant current that is in phase with the impressed voltage is ΣEg, and the component that is in quadrature with the impressed voltage is ΣEb. Thus the resultant current lags behind the impressed voltage by a time angle whose tangent is $\frac{\Sigma b}{\Sigma g}$, and the effective value of the resultant current is $E\sqrt{g_0^2 + b_0^2}$; $g_0 = \Sigma g$ and $b_0 = \Sigma b$.

Parallel branches are in resonance when the resultant current is in phase with the impressed voltage, that is, when the resultant susceptance is zero. The resultant current then has its least value and is equal to Eg_0.

In any but the simplest problems it is usually an aid in their solution to sketch an approximate vector diagram which will show the phase relations of the currents and voltages.

PROBLEMS INVOLVING INSTANTANEOUS VALUES

1. The equation of an alternating e.m.f. is $e = 143 \sin 314\,t$. What is its maximum value and at what rate is it changing at this value? What is the greatest rate at which it changes and what is its value at this instant?

2. The equation of an alternating e.m.f. is $e = 311 \sin 377\,t$. What is its frequency? What is its effective value? What is its value when it is changing at the rate of 50,000 volts per second?

3. The equation of an alternating e.m.f. is $e = 311 \sin 157\,t$. What is the shortest time interval between successive values of this e.m.f. of +200 volts and +100 volts? At what rate is the e.m.f. changing at each of these values?

4. If the equation of an alternating e.m.f. is $e = 155 \sin (157\,t + 30°)$ what will be the value of t, in seconds, when e first reaches a maximum negative value? What will be the time interval between two such values?

5. The equation of an alternating current is $i = 20 \sin 377\,t$. What is its maximum value and at what rate is it changing at this value? What is the maximum rate of change of the current and what is its value at this instant?

6. The equation of an alternating current is $i = 50 \sin 157\,t$. What is its effective value? What is its value when it is changing at the rate of 5000 amperes per second? At what rate is it changing when it is one-half of its maximum value?

7. The equations of two alternating e.m.fs. are $e = 150 \sin 377\,t$ and $e = 150 \sin (377\,t + 60°)$. If these e.m.fs. are acting in series what is the equation of their resultant? What is the phase angle between the resultant and each of the e.m.fs.? When the resultant is passing through zero what is the value of each of these e.m.fs.?

8. In problem 7 what are the values of the e.m.fs. when their resultant is a maximum? At what rate is each changing at this instant?

9. The equations of three e.m.fs. are $e_1 = 100 \sin 377\,t$, $e_2 = 700 \sin (377\,t + 72°)$, and $e_3 = 666 \sin (377\,t - 90°)$. If these three e.m.fs. are acting in series what is their resultant? What is phase relation of the resultant with respect to e_1?

10. In problem 9 sketch the component e.m.f. waves and their resultant, showing their phase relations with respect to one another. This condition occurs in series resonance.

11. The equations of the alternating currents in two parallel branches are $i_1 = 17 \sin (\omega t - 30°)$ and $i_2 = 26 \sin (\omega t - 60°)$. Sketch these current waves and show their phase relation with respect to each other. What is the equation of their resultant? At the instant when the current in the first branch is passing through zero what is the value of the resultant?

12. The equations of the alternating currents in three parallel branches are $i_1 = 12 \sin \omega t$, $i_2 = 70 \sin (\omega t - 76°)$, and $i_3 = 68 \sin (\omega t + 90°)$. What is the equation of their resultant? What is its phase relation with respect to i_1?

13. In problem 12 sketch the component current waves and their resultant, showing their phase relations with respect to one another. This condition occurs in parallel resonance.

14. A 60-cycle alternating current of 10 amperes effective value exists through a 12-ohm non-inductive resistance. What is the equation of the impressed e.m.f.? Sketch the current and the e.m.f. waves. What direct e.m.f. will cause the same heating loss in the resistance?

15. An impedance coil of negligible resistance is connected across 220-volt 60-cycle mains. If the equation of the current through the coil is $i = 10 \sin \omega t$ what is the equation of the impressed e.m.f.? When the impressed e.m.f. is passing through zero at what rate is the current changing? What is the maximum energy of the electromagnetic field, and what is the impressed e.m.f. at this instant?

16. In problem 15 sketch the current and the e.m.f. waves and show their phase relation. Sketch the curves which represent the energy of the electromagnetic field and the rate at which energy is stored in the field, and show their phase relation.

17. A 60-cycle alternating e.m.f. is impressed across a condenser of 100 microfarads capacity. The equation of the current is $i = 10 \sin \omega t$. What is the equation of the impressed e.m.f.? What is the current when the energy of the condenser is a maximum? What is the energy of the condenser when the current is a maximum?

18. In problem 17 sketch the current and the e.m.f. waves, showing their phase relation. Sketch the curves which represent the energy of the electrostatic field and the rate at which energy is stored in the electrostatic field, and show their phase relation.

19. An impedance coil of negligible resistance and 0.1 henry inductance is connected in series with a non-inductive resistance of 30 ohms across a 60-cycle e.m.f. The equation of the current through this circuit is $i = 6.5 \sin \omega t$. What is the equation of the voltage across the impedance coil?

20. In problem 19 what is the energy of the electromagnetic field when the voltage across the impedance coil is passing through zero? when the voltage across the non-inductive resistance is passing through zero? when the e.m.f. impressed on the entire circuit is passing through zero?

21. In problem 19 sketch the curves which represent the current, the energy of the electromagnetic field and the rate at which energy is stored in the electromagnetic field for one complete cycle, and show their phase relations.

22. A 60-cycle alternating current whose equation is $i = 8 \sin \omega t$ exists in a circuit of 10 ohms resistance and 0.1 henry inductance. What is the equation of the impressed e.m.f.? At what rate is energy being stored in the electromagnetic field at the instant the current is passing through zero and is increasing?

23. A 60-cycle e.m.f. is impressed across a condenser of 50 microfarads capacity, connected in series with a non-inductive resistance of 28 ohms. The equation of the current is $i = 5.2 \sin \omega t$. What is the equation of the voltage across the condenser? across the non-inductive resistance? across the entire circuit?

Sketch the current and the component e.m.f. waves and show their phase relations.

24. In problem 23 what is the energy of the electrostatic field when the voltage across the non-inductive resistance is passing through zero? when the voltage across the condenser is passing through zero? when the impressed voltage is passing through zero?

25. In problem 23 sketch the curves which represent the current, the energy of the electrostatic field and the rate at which energy is stored in the electrostatic field for one complete cycle, and show their phase relations.

26. A series circuit consists of a non-inductive resistance of 10 ohms, a reactance coil with a negligible resistance and an inductance of 0.1 henry, and a condenser of 100 microfarads capacity. The alternating e.m.f. that is impressed on this circuit produces a current whose equation is $i = 20 \sin 377\, t$. What is the equation of the e.m.f. impressed on the circuit? When the current is passing through zero what is the instantaneous value of the impressed e.m.f.? the voltage across the non-inductive resistance? the voltage across the reactance coil? the voltage across the condenser?

27. In problem 26 what is the maximum energy of the electromagnetic field? What is the maximum energy of the electrostatic field? When the energy of the electromagnetic field is zero what is the energy of the electrostatic field?

28. In problem 26 at what rate is the energy of the electromagnetic field changing when the energy of the electrostatic field is a maximum? At what rate is the energy of the electrostatic field changing when the energy of the electromagnetic field is zero?

29. In problem 26 sketch the current and the three component e.m.f. waves for one complete cycle and show their phase relations.

30. In problem 26 sketch the curves which represent the current, the energy of the electromagnetic field and the energy of the electrostatic field for one cycle, and show their phase relations.

31. An e.m.f. whose equation is $e = 155 \sin 377\,t$ is impressed on an impedance coil which has a resistance of 12 ohms and an inductance of 0.1 henry. What is the equation of the current? What is the effective value of the current? When the impressed e.m.f. is passing through zero what is the instantaneous value of the current?

32. In problem 31 what is the maximum energy of the electromagnetic field? What is the impressed e.m.f. at this instant? What is the instantaneous value of the impressed e.m.f. when the energy of the electromagnetic field is zero?

33. In problem 31 sketch the current and the e.m.f. waves for one cycle and show their phase relation. If the frequency of the impressed e.m.f. *alone* is reduced to 25 cycles, sketch the current and the e.m.f. waves and show their phase relation. Compare the magnitudes of the current, and its phase relations with respect to the impressed e.m.f. for the two frequencies.

34. An e.m.f. whose equation is $310 \sin 377\,t$ is impressed on a condenser of 50 microfarads capacity in series with a non-inductive resistance of 40 ohms. What is the equation of the current? What is the effective value of the current? When the current is passing through zero what is the instantaneous value of the impressed e.m.f.? of the voltage across the condenser? When the current is a maximum what is the charge on the condenser?

35. In problem 34 what is the maximum energy of the electrostatic field and what is the impressed e.m.f. at this instant? What is the value of the impressed e.m.f. when the energy of the electrostatic field is zero?

36. In problem 34 sketch the current and the e.m.f. waves for one cycle and show their phase relation. If the frequency of the impressed e.m.f. *alone* is reduced to 25 cycles, sketch the current and the e.m.f. waves and show their phase relation. Compare the magnitudes of the current and its phase relations with respect to the impressed e.m.f. for the two frequencies.

37. A 6-pole alternator which runs at 500 revolutions per minute supplies 20 amperes to an inductive circuit whose constants are $L = 0.1$ henry and $R = 10$ ohms. If the e.m.f. of the alternator is expressed by $e = E \sin \omega t$ what will be the corresponding equation of the current? What is the effective value of the impressed e.m.f.? When the instantaneous value of the current is zero what is the instantaneous value of the impressed e.m.f.? What is the lag angle of the current with respect to the e.m.f.? What is the average rate at which energy is being dissipated in the circuit?

38. An e.m.f. whose equation is $e = 310 \sin 157\, t$ is impressed on a series circuit consisting of a non-inductive resistance of 10 ohms, an impedance coil with a resistance of 2 ohms and an inductance of 0.12 henry, and a condenser of 180 microfarads capacity. What is the equation of the current? What is the effective value of the current? What is the instantaneous value of the impressed e.m.f. when the current is passing through zero? What is the maximum voltage across the impedance coil? What is the maximum charge on the condenser?

39. In problem 38 what is the maximum energy of the electrostatic field and what is the impressed e.m.f. at this instant? What is the maximum energy of the electromagnetic field and what is the impressed e.m.f. at this instant? When the energy of the electromagnetic field is a maximum what is the energy of the electrostatic field?

40. In problem 38 sketch the impressed e.m.f. and the current waves for one cycle and show their phase relation. If the frequency of the impressed e.m.f. *alone* is increased to 60 cycles, sketch the e.m.f. and the current waves and show their phase relation.

41. A 20-pole, 220-volt, alternating-current generator runs at a speed of 150 rev. per min. and supplies current to a series circuit the constants of which are $R = 4$ ohms, $L = 0.1$ henry, and $C = 500$ microfarads. What is the current? What is the phase angle between the e.m.f. impressed on the circuit and the current? What is the effective voltage across the terminals of the condenser and what is the maximum charge on the condenser? Assuming that the terminal voltage of the alternator varies directly with the speed, what will be the effective value of the current if the speed is reduced 50 per cent?

42. The equation of the e.m.f. impressed on an inductive circuit is $e = 310 \sin 377\, t$ and of the current, $i = 35 \sin (377\, t - 60°)$. What are the resistance and the inductance of the circuit?

43. A circuit consists of two parts connected in series across an e.m.f. of 220 volts. The equation of the current is $i = 18 \sin (377\,t + 30°)$ and that of the e.m.f. across the first part of the circuit is $e = 150 \sin (377t + 30°)$. If the total power taken by the circuit is 2 kilowatts what is the equation of the e.m.f. across the second part of the circuit?

44. The equation of the e.m.f. impressed on an inductive circuit whose resistance is 3 ohms is $e = 310 \sin (377\,t + 30°)$. If the power supplied to the circuit is 5 kilowatts what is the equation of the current?

45. If the equation of the current in a circuit is $i = 100 \sin (157\,t + 30°)$ and the equation of the impressed e.m.f. is $e = 310 \sin (157\,t - 30°)$, what is the power supplied to the circuit? What is the reactance of the circuit? What is the resistance of the circuit?

46. The equation of the e.m.f. impressed on a circuit which has a negligible inductance is $e = 310 \sin 377\,t$ and of the current, $i = 15 \sin (377\,t + 60°)$. What are the constants of the circuit?

47. The equation of the e.m.f. impressed on two circuits in parallel is $e = 310 \sin 157\,t$ and of the current in one of them, $i = 25 \sin (157\,t - 30°)$. If the total power supplied to the two circuits is 5200 watts and the current in the other circuit is 50 amperes what is the equation of this current?

48. The equation of the e.m.f. impressed on a circuit is $e = 155 \sin 377\,t$. If the wattless and power components of the current are 10 and 15 amperes respectively what is the equation of the current? What are the resistance and the reactance of the circuit?

49. On a series circuit consisting of a non-inductive resistance, an impedance coil, and a condenser of 50 microfarads capacity there is impressed an e.m.f. whose equation is $e = 310 \sin 377\,t$. The equation of the current is $i = 30 \sin (377\,t + 30°)$. What will be the equation of the current if an e.m.f. whose equation is $e = 310 \sin 157\,t$ is impressed on the circuit?

50. If an e.m.f. whose equation is $e = 310 \sin 157\,t$ is impressed on an impedance coil the current is 20 amperes, but if an e.m.f.

whose equation is $e = 310 \sin 377\, t$ is impressed on this coil the current is reduced to 12 amperes. What are the equations of the current in each case?

SERIES CIRCUITS INVOLVING RESISTANCE AND INDUCTANCE

51. A 220-volt, 25-cycle e.m.f. is impressed on an inductive circuit which has a resistance of 2 ohms and a reactance of 10 ohms at 25 cycles. What is the instantaneous value of the impressed e.m.f. when the energy of the electromagnetic field is zero? What is the maximum energy of the electromagnetic field?

52. In problem 51 if a 220-volt, 60-cycle e.m.f. is impressed on this circuit what is the instantaneous value of the impressed e.m.f. when the energy of the electromagnetic field is zero? What is the maximum energy of the electromagnetic field?

53. In problem 51 if a 110-volt, 25-cycle e.m.f. is impressed on this circuit what is the instantaneous value of the impressed e.m.f. when the energy of the electromagnetic field is zero? What is the maximum energy of the electromagnetic field?

54. A non-inductive resistance takes 10 amperes from 220-volt, 60-cycle mains. What current will it take from 220-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

55. An impedance coil of negligible resistance takes 3 amperes from 220-volt, 60-cycle mains. What current will it take from 220-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

56. What current will an impedance coil which has a resistance of 2.7 ohms and an inductance of 0.05 henry take from 220-volt, 60-cycle mains? from 220-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

57. An impedance coil which has a resistance of 2.5 ohms takes 10 amperes from 220-volt, 60-cycle mains. What current will it take from 110-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

58. Across what direct e.m.f. must an impedance coil which has a resistance of 20 ohms and an inductance of 0.05 henry be connected in order that it may take the same current that it takes from 220-volt, 60-cycle mains?

59. Across what direct e.m.f. must an impedance coil which has a resistance of 20 ohms and an inductance of 0.05 henry be connected in order that it may take the same current that it takes from 220-volt, 25-cycle mains?

60. Across what direct e.m.f. must an impedance coil which has a resistance of 20 ohms and an inductance of 0.05 henry be connected in order that it may take the same power that it takes from 220-volt, 60-cycle mains?

61. Across what direct e.m.f. must an impedance coil which has a resistance of 20 ohms and an inductance of 0.05 henry be connected in order that it may take the same power that it takes from 220-volt, 25-cycle mains?

62. An impedance coil has a resistance of 4 ohms and an inductance of 0.1 henry. What non-inductive resistance must be connected in series with this coil across 110-volt d. c. mains in order that the current will equal that which the coil alone takes when connected across 110-volt, 60-cycle mains? across 110-volt, 25-cycle mains?

63. An impedance coil has a resistance of 4 ohms and an inductance of 0.1 henry. What non-inductive resistance must be connected in series with this coil across 110-volt d. c. mains in order that the coil alone will take the same power that it takes when connected directly across 110-volt, 60-cycle mains? across 110-volt, 25-cycle mains?

64. An impedance coil takes 15 amperes from 220-volt, 60-cycle mains. If the coil is connected across a 32-volt constant potential storage battery the current is 17.5 amperes. What is the energy of the electromagnetic field in the latter case?

65. An impedance coil takes 25 amperes from 110-volt, 60-cycle mains. It also takes 25 amperes if connected across the terminals of a 10.5-volt storage battery. What are the resistance and the inductance of the coil?

66. An impedance coil takes a current of 25 amperes when connected across 220-volt, 60-cycle mains. If this coil is connected in series with a resistance of 5 ohms across 110-volt d. c. mains the current is 17 amperes. What are the resistance and the inductance of the coil?

67. An impedance coil absorbs 300 watts when connected across 220-volt, 60-cycle mains. If this coil is connected in series with a resistance of 8 ohms across 110-volt d. c. mains, it — the coil alone — will also absorb 300 watts. What are the possible values of the resistance and the inductance of the coil?

68. A non-inductive resistance of 10 ohms and an impedance coil of 2 ohms resistance and 0.025 henry inductance are connected in parallel across constant potential a. c. mains. For what frequency will the power taken by the coil equal that taken by the non-inductive resistance?

69. An impedance coil has a resistance of 5 ohms and an inductance of 0.1 henry. What current and what power will this coil take from 220-volt, 60-cycle mains? from 110-volt, 25-cycle mains?

70. An impedance coil which has a resistance of 2 ohms and an inductance of 0.05 henry is connected in series with a non-inductive resistance of 10 ohms across a 110-volt, 60-cycle circuit. What is the current? What is the voltage across the non-inductive resistance? What is the phase angle between the current and the impressed e.m.f.? between the current and the voltage across the coil? the voltage across the non-inductive resistance?

71. What is the inductance of another circuit having a resistance of 5 ohms which will take the same current as in problem 70 from 110-volt, 60-cycle mains? from 220-volt, 25-cycle mains?

72. What is the inductance of another circuit having a resistance of 5 ohms which will take the same power as the entire circuit in problem 70 from 110-volt, 60-cycle mains? from 220-volt, 25-cycle mains?

73. If the coil and non-inductive resistance in problem 70 are connected in series across a 220-volt, 25-cycle circuit what is the current? What is the voltage across the non-inductive resistance? What is the phase angle between the current and the impressed voltage? the voltage across the coil? the voltage across the non-inductive resistance?

74. Two impedance coils have resistances of 5 and 8 ohms and inductances of 0.01 and 0.2 henry respectively. If these coils are connected in series across 220-volt, 60-cycle mains what current will they take? What will be the phase relations of the current and the impressed voltage? the voltage across the first coil? the voltage across the second coil?

75. If the impedance coils in problem 74 are connected in series across 220-volt, 25-cycle mains what current will they take? What will be the phase relations of the current and the impressed voltage? the voltage across the first coil? the voltage across the second coil?

76. An impedance coil which has a resistance of 3.4 ohms takes 7.7 amperes from 220-volt, 60-cycle mains. What should be the resistance and the inductance of another coil which takes the same current and absorbs the same power from 110-volt, 60-cycle mains?

77. An impedance coil takes 10 amperes and absorbs 250 watts when connected across 220-volt, 60-cycle mains. What current will it take when connected across 110-volt, 25-cycle mains? What power will it absorb?

78. An impedance coil takes 8 amperes and absorbs 320 watts when connected across 110-volt, 60-cycle mains. What non-inductive resistance must be connected in series with this coil in order that it may take the same current from 220-volt, 25-cycle mains?

79. An impedance coil takes 250 watts at a power factor of 0.1 from 220-volt, 60-cycle mains. What power will it take from 110-volt, 25-cycle mains? What is the power factor in this case?

80. An impedance coil takes 250 watts at a power factor of 0.1 from 220-volt, 60-cycle mains. What non-inductive resistance must be connected in series with it across 220-volt, 25-cycle mains so that the coil alone will take 250 watts?

81. In problem 80 what non-inductive resistance must be connected in series with the impedance coil so that it will take the same current from 220-volt, 25-cycle mains? from 110-volt, 25-cycle mains?

82. An impedance coil which takes 250 watts at a power factor of 0.1 from 220-volt, 60-cycle mains is connected in series with a non-inductive resistance of 15 ohms across 200-volt, 25-cycle mains. What power does each part of this circuit take? What is the voltage across each part?

83. An impedance coil which takes 20 amperes at a power factor of 0.15 from 220-volt, 60-cycle mains is connected in series with a non-inductive resistance across 220-volt, 25-cycle mains. If the current is 25 amperes what is the voltage across each part of the circuit?

84. In problem 83 what power does each part of the circuit take?

85. An impedance coil takes 25 amperes when connected across 220-volt, 25-cycle mains and 5.4 amperes from 110-volt, 60-cycle mains. What are the resistance and the inductance of the coil?

86. An impedance coil absorbs 250 watts when connected across 220-volt, 60-cycle mains. When it is connected across 110-volt, 25-cycle mains it also absorbs 250 watts. What are the resistance and the inductance of the coil?

87. An impedance coil takes 10 amperes and absorbs 300 watts when connected across 110-volt, 60-cycle mains. What non-inductive resistance must be connected in series with it in order that it may absorb the same power from 110-volt, 25-cycle mains? from 220-volt, 60-cycle mains?

88. Two impedance coils which have resistances of 2.6 and 3.7 ohms take 5 and 10 amperes respectively from 220-volt, 60-cycle mains. What current will they take when connected in series across 220-volt, 60-cycle mains?

89. An impedance coil has a constant resistance of 2 ohms and a variable inductance. What resistance must be connected in series with it so that it will take 5 amperes from 110-volt, 60-cycle mains with a phase angle of 60 degrees between the current and the impressed e.m.f.?

90. An impedance coil which has a resistance of 2.7 ohms and a variable inductance is connected in series with a non-inductive resistance across 220-volt, 60-cycle mains. The circuit is adjusted so that the potential across the coil is 150 volts and the power taken by it is 250 watts. What is the value of the non-inductive resistance?

91. An impedance coil which has a constant resistance of 2 ohms and a variable inductance is connected in series with a 10-ohm non-inductive resistance across 110-volt, 60-cycle mains. For what value of the inductance will the coil absorb 65 watts? What are the power and wattless components of the current with respect to the impressed e.m.f.?

92. An impedance coil which has a constant resistance of 2 ohms and a variable inductance is connected in series with a non-inductive resistance across 110-volt, 60-cycle mains. What is the inductance of the coil when the entire circuit takes 1 kilowatt with a lag angle of 60 degrees between the current and voltage across the coil? What is the non-inductive resistance?

93. A non-inductive resistance of 5 ohms is connected in series with an impedance coil which has a resistance of 2.7 ohms and a variable reactance. What is the inductance of the coil when the total power taken by the circuit from 220-volt, 60-cycle mains is 1240 watts?

94. An impedance coil which has a resistance of 2.6 ohms takes 6.6 amperes from 110-volt, 60-cycle mains. What should be the constants of a similar coil that will absorb the same power and take the same current from 220-volt, 60-cycle mains?

95. An impedance coil which has 180 turns and a negligibly small resistance takes 7.2 amperes from 110-volt, 25-cycle mains. How many turns should be cut out of the circuit in order that the coil may take the same current from 110-volt, 60-cycle mains?

96. An impedance coil which has 108 turns and a resistance of 2.1 ohms takes 50 amperes from 220-volt, 25-cycle mains. Assuming that each turn has the same length, how many turns should be cut out of the circuit in order that the coil may take 50 amperes from 110-volt, 25-cycle mains?

97. An impedance coil which has 108 turns and a resistance of 2.6 ohms takes 30 amperes from 220-volt, 60-cycle mains. Assuming that each turn has the same length, how many turns should be cut out of the circuit in order that the coil may take the same power from 110-volt, 60-cycle mains that it took from the 220-volt, 60-cycle mains?

98. An impedance coil which has 214 turns and a resistance of 4.3 ohms takes 25 amperes from 220-volt, 25-cycle mains

Assuming that each turn has the same length, how many turns should be cut out of the circuit in order that the coil may take 25 amperes from 220-volt, 60-cycle mains?

99. An impedance coil which has 214 turns and a resistance of 4.3 ohms takes 25 amperes from 220-volt, 25-cycle mains. Assuming that each turn has the same length, how many turns should be cut out of the circuit in order that the coil may take the same power from 220-volt, 60-cycle mains that it took from the 25-cycle mains?

100. Two coils of negligibly small resistance are wound on the same magnetic circuit. These coils take 15 and 25 amperes respectively when connected singly across constant potential a. c. mains. What is the ratio of the turns in the coils?

101. Two coils, each of an unknown number of turns, are wound on the same magnetic circuit. The first has a resistance of 3.2 ohms and the second a resistance of 2.5 ohms. These coils take 25 and 35 amperes respectively when connected singly across 110-volt a. c. mains. What is the ratio of the turns in the coils?

102. When a non-inductive resistance and an impedance coil are connected in parallel across 220-volt, 60-cycle mains the currents are 15 amperes and 10 amperes respectively. When connected in series across the same mains the current is 6.9 amperes. What are the resistance and the inductance of the coil?

103. When a non-inductive resistance and an impedance coil which has a resistance of 2 ohms are connected in parallel across a 60-cycle alternating e.m.f. they take 12 amperes and 9.5 amperes respectively. When they are connected in series across the same potential the current is 6.7 amperes. What is the reactance of the coil at 60 cycles?

104. When a non-inductive resistance and an impedance coil which has a negligible resistance are connected in parallel across a 60-cycle alternating e.m.f. they each take 10 amperes. What current will they take when they are connected in series across a 25-cycle e.m.f. of the same effective value?

105. When a non-inductive resistance and an impedance coil are connected in parallel across 220-volt, 60-cycle mains the currents are 13.2 amperes and 9 amperes respectively. When connected in series across 110-volt, 60-cycle mains the current is 3.4 amperes. What are the resistance and the inductance of the coil?

106. When a non-inductive resistance and an impedance coil which has a resistance of 2 ohms are connected in parallel across a 60-cycle alternating e.m.f. they take 30 amperes and 20 amperes respectively. When they are connected in series across a 25-cycle e.m.f. of the same effective value the current is 20 amperes. What is the reactance of the coil at 25 cycles?

107. When a non-inductive resistance of 12 ohms and an impedance coil are connected in parallel across 60-cycle constant potential mains the currents are 18.5 amperes and 15 amperes respectively. When connected in series across the same mains the current is 10.7 amperes. What are the resistance and the reactance of the coil at 60 cycles?

108. When a non-inductive resistance of 10 ohms and an impedance coil are connected in parallel across 60-cycle constant potential mains the currents are 12 amperes and 10 amperes respectively. When connected in series across a 25-cycle e.m.f. of the same effective value the current is 9.1 amperes. What are the resistance and the inductance of the coil?

109. When a non-inductive resistance and an impedance coil are connected in parallel across 220-volt, 60-cycle mains the currents are 10 amperes and 5 amperes respectively. When they are conected in series across 220-volt, 25-cycle mains the current is 7.1 amperes. What are the resistance and the inductance of the coil?

SERIES CIRCUITS INVOLVING RESISTANCE AND CAPACITY

110. A condenser takes 3.2 amperes from 220-volt, 25-cycle mains. What current will it take from 220-volt, 60-cycle mains? from 110-volt, 60-cycle mains?

111. A non-inductive resistance of 20 ohms in series with a condenser of 45 microfarads is connected across 220-volt, 60-cycle mains. What is the impedance of the circuit? What is the power taken by the circuit?

112. If the circuit in problem 111 is connected across 220-volt, 25-cycle mains what are the impedance, the current, and the power?

113. If the circuit in problem 111 is connected across 110-volt, 60-cycle mains what are the impedance, the current, and the power?

114. A 220-volt, 60-cycle e.m.f. is impressed on a circuit consisting of a non-inductive resistance of 20 ohms in series with a condenser which has a reactance of 10 ohms at 60 cycles. What is the instantaneous value of the impressed e.m.f. when the energy of the electrostatic field is zero? What is the maximum energy of the electrostatic field?

115. In problem 114 if a 220-volt, 25-cycle e.m.f. is impressed on the circuit what will be the instantaneous value of the impressed e.m.f. when the energy of the electrostatic field is zero? What is the maximum energy of the electrostatic field?

116. In problem 114 if a 110-volt, 60-cycle e.m.f. is impressed on the circuit what will be the instantaneous value of the impressed e.m.f. when the energy of the electrostatic field is zero? What is the maximum energy of the electrostatic field?

117. A non-inductive resistance of 15 ohms in series with a condenser takes 5 amperes when connected across 220-volt, 60-cycle mains. What will be the ultimate energy of the electrostatic field if a direct e.m.f. of 220 volts is impressed on this circuit? Compare this with the maximum energy of the field when the alternating e.m.f. is impressed on the circuit.

118. A circuit consisting of a non-inductive resistance of 15 ohms in series with a condenser of 60 microfarads is connected in series with another non-inductive resistance of 10 ohms across 220-volt, 60-cycle mains. What is the current? What is the voltage across the 10-ohm non-inductive resistance? What is the phase angle between the current and the impressed voltage? the voltage across the first portion of the circuit? the voltage across the 10-ohm non-inductive resistance?

119. What may be the constants of another circuit having a resistance of 10 ohms which takes the same current as in problem 118 from 110-volt, 60-cycle mains? from 220-volt, 25-cycle mains?

120. What may be the constants of another circuit having a resistance of 10 ohms which takes the same power as in problem 118 from 110-volt, 60-cycle mains? from 220-volt, 25-cycle mains?

121. A non-inductive resistance of 15 ohms in series with a condenser takes 5 amperes from 220-volt, 60-cycle mains? What current will this circuit take from 220-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

122. A circuit consisting of a non-inductive resistance in series with a condenser of 80 microfarads takes 2.5 amperes from 220-volt, 25-cycle mains. What current will this circuit take from 110-volt, 25-cycle mains? from 220-volt, 60-cycle mains?

123. A circuit consisting of a non-inductive resistance in series with a condenser takes 80 watts at a power factor of 0.1 from 220-volt, 60-cycle mains. Across what direct e.m.f. must this circuit be connected in order that the ultimate energy of the electrostatic field may equal the maximum energy of the field when the circuit is connected across the 220-volt, 60-cycle mains?

124. A non-inductive resistance and a condenser are connected in series across 220-volt, 60-cycle mains. They are adjusted so that the current is 8 amperes and the power absorbed is 250 watts. What are the resistance and the capacity?

125. A non-inductive resistance in series with a condenser takes 10 amperes and absorbs 250 watts when connected across 220-volt, 60-cycle mains. What current will this circuit take when connected across 110-volt, 60-cycle mains? What power will it absorb?

126. A non-inductive resistance of 10 ohms and a condenser of 60 microfarads connected in series take 4.9 amperes from 60-cycle mains. What should be the resistance and capacity of another circuit which will take the same current and absorb the same power from 110-volt, 60-cycle mains?

127. A non-inductive resistance in series with a condenser takes 250 watts at a power factor of 0.1 from 220-volt, 60-cycle mains. What power will this circuit take from 110-volt, 25-cycle mains? What is the power factor in the latter case?

128. A non-inductive resistance and a condenser are connected in series across 220-volt, 25-cycle mains. The circuit is adjusted so that the e.m.f. across the capacity is 200 volts and the power absorbed is 450 watts. What are the resistance and the capacity?

129. A non-inductive resistance in series with a condenser takes 5.6 amperes when connected across 220-volt, 25-cycle mains and 5.9 amperes when connected across 110-volt, 60-cycle mains. What are the resistance and the capacity?

130. A circuit consisting of a non-inductive resistance of 2 ohms in series with a condenser takes 5.4 amperes from 220-volt, 25-cycle mains. What are the constants of a similar circuit that takes the same current and absorbs the same power from 220-volt, 60-cycle mains?

131. A non-inductive resistance and a condenser are connected in series across 220-volt, 60-cycle mains. What are the constants of this circuit when it is adjusted to take 250 watts at a power factor of 0.2?

132. A non-inductive resistance and a condenser are connected across 220-volt, 25-cycle mains. What are the constants of the circuit when it is adjusted so that the current is 5.2 amperes and there is a potential difference of 170 volts across the capacity?

133. A non-inductive resistance of 20 ohms in series with a condenser takes 6.5 amperes from 220-volt, 60-cycle mains. What are the constants of a similar circuit that takes the same current and absorbs the same power from 220-volt, 25-cycle mains?

134. A non-inductive resistance in series with a condenser takes 4.2 amperes when connected across 110-volt, 25-cycle mains and 18 amperes when connected across 220-volt, 60-cycle mains. What are the resistance and the capacity?

135. A non-inductive resistance in series with a condenser takes 7 amperes and absorbs 300 watts when connected across 110-volt, 25-cycle mains. What non-inductive resistance must be connected in series with this circuit in order that it may take the same current from 220-volt, 25-cycle mains? from 110-volt, 60-cycle mains?

136. A circuit (c) consisting of a non-inductive resistance in series with a condenser takes 2 amperes and absorbs 120 watts when connected across 110-volt, 25-cycle mains. What non-inductive resistance must be connected in series with this circuit across 220-volt, 60-cycle mains so that the circuit (*c*) will still absorb 120 watts?

137. When a non-inductive resistance of 22 ohms and a circuit consisting of a non-inductive resistance and a condenser in series are connected in parallel across constant potential 25-cycle mains the currents are 10 amperes and 5 amperes respectively. When they are connected in series across a 60-cycle e.m.f. of the same effective value the current is 6.5 amperes. What are the resistance and the capacity?

138. A circuit consisting of a non-inductive resistance of 10 ohms and a variable capacity is connected in series with a non-inductive resistance across 220-volt, 60-cycle mains. The entire circuit is so adjusted that the e.m.f. across the first part, viz., the 10-ohm resistance in series with the capacity, is 150 volts and the power taken by this part is 400 watts. What are the capacity and the second non-inductive resistance?

139. A circuit which consists of a non-inductive resistance of 20 ohms in series with a condenser is connected in series with a non-inductive resistance of 12 ohms across 220-volt, 25-cycle mains. For what capacity will the entire circuit absorb 200 watts? What are the power and wattless components of the current with respect to the impressed voltage?

140. A circuit consisting of a non-inductive resistance of 15 ohms in series with a condenser is connected in series with a non-inductive resistance across 220-volt, 25-cycle mains. What is the capacity when the first portion of the circuit takes 270 watts and there is a lag angle of 60 degrees between the current and the impressed voltage? What is the non-inductive resistance?

141. A circuit consisting of a non-inductive resistance in series with a condenser is connected in series with a non-inductive resistance of 2 ohms across 220-volt, 25-cycle mains. What is the capacity when the entire circuit takes 300 watts at a power factor of 0.2?

142. Two circuits, each consisting of a non-inductive resistance in series with a condenser, take 5 amperes and 10 amperes respectively when connected across 220-volt, 60-cycle mains. The non-inductive resistances are 15 and 10 ohms respectively.

What current will these circuits take when connected in series across 220-volt, 60-cycle mains?

143. A series circuit consists of two parts, the first of which is a resistance of 5 ohms in series with a condenser of 150 microfarads and the second is a resistance of 12.5 ohms in series with a condenser of 500 microfarads. If this circuit is connected across 220-volt, 60-cycle mains what is the phase relation of the current and the impressed voltage? the voltage across the first part of the circuit? the voltage across the second part of the circuit?

144. If the circuit in problem 143 is connected across 220-volt, 25-cycle mains what is the phase relation of the current and the impressed voltage? the voltage across the first part of the circuit? the voltage across the second part of the circuit?

145. Two non-inductive resistances of 10 and 20 ohms respectively are connected in parallel across 110-volt, 60-cycle mains. A condenser is inserted in series with one of these and adjusted so that the resistances absorb the same power. In series with which resistance is the condenser placed and what is the capacity?

SERIES CIRCUITS INVOLVING RESISTANCE, INDUCTANCE, AND CAPACITY

146. A non-inductive resistance of 15 ohms, an impedance coil which has a resistance of 2 ohms and an inductance of 0.1 henry, and a condenser of 120 microfarads capacity are connected in series across 220-volt, 60-cycle mains. What are the current and its phase relation with respect to the impressed voltage? What is the instantaneous value of the impressed voltage when the energy of the electrostatic field is zero? when the energy of the electromagnetic field is zero?

147. A 220-volt, 60-cycle e.m.f. is impressed on a series circuit consisting of an impedance coil which has a resistance of 6 ohms and a reactance of 36 ohms at 60 cycles, and a condenser which has a reactance of 15 ohms at 60 cycles. When the instantaneous value of the impressed e.m.f. is passing through zero what is the energy of the electromagnetic field? When the energy of the electromagnetic field is zero what is the energy of the electrostatic field?

148. In problem 147 if a 220-volt, 25-cycle e.m.f. is impressed on the circuit what will be the energy of the electromagnetic field when the impressed e.m.f. is passing through zero? When the energy of the electromagnetic field is zero what will be the energy of the electrostatic field?

149. In problem 147 if a 110-volt, 60-cycle e.m.f. is impressed on the circuit what will be the energy of the electromagnetic field when the impressed e.m.f. is passing through zero? When the energy of the electromagnetic field is zero what is the energy of the electrostatic field?

150. In problem 147 at what frequency will an e.m.f. of 220 volts produce the greatest current through the circuit?

151. The constants of a series circuit on which is impressed a 110-volt, 60-cycle e.m.f. are $R = 10$ ohms, $L = 0.1$ henry, and $C = 30$ microfarads. At the instant the impressed e.m.f. is passing through zero what is the energy of the electrostatic field? When the energy of the electrostatic field is zero what is the energy of the electromagnetic field?

152. A circuit consists of two coils, A and B, in series with a condenser. The constants of the coil A are $R = 2$ ohms and $L = 0.1$ henry, and of coil B, $R = 20$ ohms and $L = 0.005$ henry. The capacity of the condenser is 80 microfarads. If a 220-volt, 60-cycle e.m.f. is impressed on the circuit what current will be established? What are the magnitude and the phase relation with respect to the current of the voltage across the coil A? across coil B? across the condenser?

153. In problem 152 for what capacity would the current have its greatest value? What would be the voltage across the coil A for this condition?

154. An impedance coil which has a resistance of 20 ohms and an inductance of 0.1 henry is connected in series with a condenser of 100 microfarads capacity across a constant alternating e.m.f. of 220 volts. For what frequency will the voltages across the coil and the condenser be equal?

155. An impedance coil which has a resistance of 1.5 ohms and a reactance of 32 ohms at 60 cycles is connected in series with a condenser of variable capacity across 220-volt, 60-cycle mains. For what capacity will the voltages across the coil and the condenser be equal?

156. In problem 155 if a 220-volt, 25-cycle e.m.f. is impressed on the circuit for what capacity will the voltages across the coil and the condenser be equal?

157. A non-inductive resistance of 10 ohms, an impedance coil which has a resistance of 1.7 ohms and an inductance of 0.1 henry, and a condenser of 60 microfarads capacity are connected in series across 110-volt, 60-cycle mains. What is the current? What is the voltage across each part of the circuit, and what are the phase relations of these voltages with respect to the current?

158. An impedance coil which takes 15 amperes at a power factor of 0.1 when connected across 220-volt, 60-cycle mains is connected in series with a circuit consisting of a non-inductive resistance of 10 ohms in series with a condenser of 120 microfarads capacity. If a 220-volt, 60-cycle e.m.f. is impressed on this entire circuit what will be the voltage across each part?

159. A 110-volt, 60-cycle e.m.f. is impressed on a series circuit which consists of a non-inductive resistance of 10 ohms, an impedance coil of 1 ohm resistance and 0.1 henry inductance, and a condenser of variable capacity. For what capacity will the current be a maximum?

160. An impedance coil has a resistance of 2 ohms and an inductance of 0.1 henry. It is desired to use this coil in series with a condenser to show the phenomena of resonance. If the only electrical circuit available is a 110-volt, 60-cycle one what will be the necessary capacity of the condenser? If the condenser is designed for an allowable effective e.m.f. of 500 volts what resistance will it be necessary to insert in the circuit in order that at resonance the e.m.f. across the condenser may not exceed this amount?

161. An e.m.f. of 110 volts is impressed on a series circuit consisting of a non-inductive resistance of 10 ohms, an impedance coil of 1 ohm resistance and 0.1 henry inductance, and a condenser of 100 microfarads capacity. For what frequency of the impressed e.m.f. does the current have its greatest value? At this frequency what is the voltage across the impedance coil? At this frequency what is the least series resistance required in order that the maximum instantaneous voltage across the condenser shall not exceed 300 volts?

162. An impedance coil and a condenser are connected in series across a constant alternating e.m.f. which has an effective value of 110 volts. By adjusting the frequency of the impressed e.m.f. the current through the circuit can be varied from 10 amperes at a frequency of 25 cycles to a maximum current of 35 amperes at a frequency of 60 cycles. A further increase in the frequency decreases the current. What are the resistance and inductance of the impedance coil? What is the capacity of the condenser?

163. A 220-volt, 60-cycle e.m.f. is impressed on a series circuit which consists of a non-inductive resistance of 5 ohms, an impedance coil of 1.5 ohms resistance and 0.1 henry inductance, and a condenser of variable capacity. For what value of the capacity will the voltage across it have its greatest value? For what value of the capacity will the voltage across the impedance coil have its greatest value? As the capacity is increased which maximum value occurs first, the voltage across the coil or the voltage across the capacity?

164. In problem 163 what additional series resistance is needed in order that the greatest voltage across the condenser shall be 500 volts effective value?

165. An alternating e.m.f. is impressed on a series circuit which consists of a non-inductive resistance of R ohms, an impedance coil of r ohms resistance and L henrys inductance, and a condenser of variable capacity C. For what value of the capacity will the voltage across it have its greatest value? For what value of the capacity will the voltage across the impedance coil have its greatest value? As the capacity is increased which maximum value occurs first, the voltage across the coil or the voltage across the condenser?

166. On a series circuit which consists of a resistance of 10 ohms, an impedance coil of 2 ohms resistance and 0.1 henry inductance, and a condenser of 60 microfarads capacity there is impressed an e.m.f. which varies directly with its frequency. When this e.m.f. is 220 volts its frequency is 60 cycles. For what frequency will the current have its greatest value?

167. In problem 166 for what frequency will the voltage across the condenser have its greatest value?

168. An impedance coil which takes 5 amperes at a power factor of 0.15 from 220-volt, 25-cycle mains is connected in series with a condenser across 220-volt, 60-cycle mains. For what capacity will the current be 5 amperes?

169. An impedance coil takes 250 watts at a power factor of 0.1 when connected across 220-volt, 60-cycle mains. What capacity must be connected in series with this coil so that it will take the same power from 110-volt, 60-cycle mains? from 220-volt, 25-cycle mains?

170. An impedance coil which takes a current of 15 amperes and absorbs 300 watts when connected across 220-volt, 25-cycle mains is connected in series with a condenser. If a 220-volt, 60-cycle e.m.f. is impressed on this circuit for what values of the capacity will the power absorbed by the coil still be 300 watts?

171. An impedance coil which has a resistance of 1.5 ohms is connected in series with a condenser across 220-volt, 60-cycle mains. For what values of the inductance and the capacity will the coil absorb 250 watts and have a potential of 190 volts across its terminals?

172. An impedance coil is connected in series with a condenser across 220-volt, 60-cycle mains. This circuit absorbs 650 watts at a power factor of 0.87 and is so adjusted that the potentials across the coil and the condenser are equal. What are these voltages?

173. An impedance coil is connected in series with a condenser. The potential across the coil is 240 volts and that across the condenser is 190 volts. If the current is 10 amperes and the total power supplied is 2.1 kilowatts what is the impressed voltage?

174. An impedance coil is connected in series with a condenser across 220-volt, 60-cycle mains. This circuit absorbs 800 watts at a power factor of 0.4 and is so adjusted that the potential across the condenser is 350 volts. What is the voltage across the impedance coil?

175. An impedance coil which has a resistance of 10 ohms and a variable inductance is connected in series with a condenser of variable capacity across 220-volt, 60-cycle mains, and the circuit is so adjusted that the potentials across the coil and the condenser are each 220 volts. What is the phase relation of the current and the impressed voltage? the voltage across the impedance coil? the voltage across the condenser? What power does the impedance coil absorb?

176. An impedance coil and a series circuit consisting of a non-inductive resistance and a capacity reactance of 60 ohms are connected in series across 220-volt, 60-cycle mains. The entire circuit is so adjusted that the voltages across each part and the power absorbed by each part are equal. If the entire circuit absorbs 500 watts what is the current? What is the voltage across each part of the circuit? What are the phase relations of the current and the voltage across each part?

PARALLEL CIRCUITS INVOLVING RESISTANCE AND INDUCTANCE

177. An impedance coil which has an inductance of 0.1 henry and a negligible resistance is connected in parallel with a non-inductive resistance across 220-volt, 25-cycle mains. If the total current is 24 amperes what is the current in each branch?

178. An impedance coil which has a resistance of 2.7 ohms takes 15 amperes when connected across 220-volt, 25-cycle mains. If a non-inductive resistance of 18 ohms is connected in parallel with this coil what is the total current taken from 220-volt, 25-cycle mains?

179. Two impedance coils are connected in parallel across 220-volt, 60-cycle mains. The first coil takes 15 amperes and absorbs 300 watts, and the second coil takes 20 amperes and absorbs 850 watts. What is the total current taken from the mains?

180. When a non-inductive resistance of 20 ohms is connected in parallel with an impedance coil across 220-volt, 60-cycle mains the total current taken from the mains is 15.6 amperes. When the resistance and the coil are connected in series across 220-volt, 60-cycle mains the current is 7.2 amperes. What are the resistance and the inductance of the impedance coil?

181. When a non-inductive resistance of 10 ohms is connected in parallel with an impedance coil across 220-volt, 60-cycle mains, the total current taken from the mains is 25 amperes. When the resistance and the coil are connected in series across 220-volt, 25-cycle mains the current is 14 amperes. What are the resistance and the inductance of the coil?

182. Two equal impedance coils with no mutual inductance and negligible resistance take 10 amperes when connected in series across 220-volt, 60-cycle mains. What current will one coil take when connected across 110-volt, 60-cycle mains? What current will the coils take when connected in parallel across 110-volt, 60-cycle mains?

183. Two equal impedance coils with no mutual inductance and negligible resistance take 10 amperes when connected in series across 220-volt, 60-cycle mains. What current will one coil take from 110-volt, 25-cycle mains? What current will they take if connected in parallel across 110-volt, 25-cycle mains?

184. Two equal impedance coils with no mutual inductance each have a resistance of 2.1 ohms. When they are connected in series across 220-volt, 60-cycle mains the current is 25 amperes. What current will one of these coils take when connected across 110-volt, 60-cycle mains? What current will the coils take when connected in parallel across 110-volt, 60-cycle mains?

185. Two equal impedance coils with no mutual inductance each have a resistance of 2.4 ohms. When they are connected in series across 220-volt, 60-cycle mains the current is 15 amperes. What current will one of these coils take from 110-volt, 25-cycle mains? What current will the coils take when connected in parallel across 110-volt, 25-cycle mains?

186. Two equal coils of negligible resistance are wound side by side on the same magnetic circuit. When the coils are connected in series across 220-volt, 60-cycle mains the current is 5 amperes. If one coil is connected across 110-volt, 60-cycle mains what current will it take? What current will the coils take if they are connected in parallel across 110-volt, 60-cycle mains?

187. Two equal coils each with a resistance of 2 ohms are wound side by side on the same magnetic circuit. When they are connected in series across 220-volt, 60-cycle mains the current is 10 amperes. What current will one coil take if connected across 110-volt, 60-cycle mains? What current will these coils take from 110-volt, 60-cycle mains if connected in parallel?

188. Two equal coils each with a resistance of 3.5 ohms are wound side by side on the same magnetic circuit. When the coils are connected in series across 220-volt, 60-cycle mains the current is 25 amperes. What current would one coil take if connected across 110-volt, 60-cycle mains. What current will the coils take when connected in parallel across 110-volt, 60-cycle mains. The coils are connected so as to produce magnetic fields in the same direction.

189. Two equal coils each with a resistance of 3.5 ohms are wound side by side on the same magnetic circuit. When they are connected in series across 220-volt, 60-cycle mains the current is 25 amperes. What current will one coil take when connected across 110-volt, 25-cycle mains? What current will the coils take when connected in parallel across 110-volt, 25-cycle mains? The coils are connected so that they produce magnetic fields in the same direction.

190. Two impedance coils which have resistances of 5 and 20 ohms and inductances of 0.01 and 0.1 henry respectively are connected in parallel across 220-volt, 60-cycle mains. What is the total current taken from the mains? What is the phase relation of the impressed e.m.f. and the total current? the current in the first coil? the current in the second coil?

191. If the coils in problem 190 are connected in parallel across 220-volt, 25-cycle mains what is the total current? What is the phase relation of the impressed e.m.f. and the total current? the current in the first coil? the current in the second coil?

PARALLEL CIRCUITS INVOLVING RESISTANCE AND CAPACITY

192. A non-inductive resistance of 25 ohms is connected in parallel with a condenser of 60 microfarads across 220-volt, 60-cycle mains. What is the total current taken from the mains?

193. A non-inductive resistance and a condenser are connected in parallel across 220-volt, 60-cycle mains. If the current through the resistance is 10 amperes and that taken by the condenser 7.5 amperes what is the total current supplied?

194. A non-inductive resistance and a condenser are connected in parallel across 220-volt, 60-cycle mains. If the entire circuit takes a current of 12 amperes and absorbs 1800 watts what are the currents in the branches?

195. A non-inductive resistance and a condenser connected in parallel across 220-volt, 60-cycle mains take a total current of 10 amperes at a power factor of 0.2. What are the currents in the branches?

196. A circuit consisting of a non-inductive resistance of 20 ohms in series with a condenser of 80 microfarads is connected in parallel with a non-inductive resistance of 75 ohms across constant potential a. c. mains. For what frequency will the power taken by the non-inductive resistance equal that taken by the capacity circuit? What is the resultant impedance of the circuit at this frequency?

197. Two equal condensers take 2.7 amperes when connected in series across 220-volt, 25-cycle mains. What current will they take when connected in parallel across 220-volt, 25-cycle mains? across 220-volt, 60-cycle mains? across 110-volt, 25-cycle mains?

198. When a non-inductive resistance and a circuit consisting of a non-inductive resistance in series with a condenser are connected in parallel across 220-volt, 60-cycle mains the currents are 10 amperes and 5 amperes respectively. When these circuits are connected in series across the same mains the current is 4.2 amperes. What are the resistances and the capacity? What is the total current when these circuits are connected in parallel across 220-volt, 60-cycle mains?

199. When a non-inductive resistance and a circuit consisting of a non-inductive resistance of 20 ohms in series with a condenser are connected in parallel across a 25-cycle constant e.m.f. the currents are 11 and 5 amperes respectively. When these circuits are connected in series across the same mains the current is 4.1 amperes. What are the resistance and the capacity? What is the total impedance of these circuits when they are connected in parallel across a 25-cycle e.m.f.?

200. When a non-inductive resistance and a circuit consisting of a non-inductive resistance in series with a condenser are connected in parallel across 220-volt, 60-cycle mains the currents are 12 amperes and 6 amperes respectively. When these circuits are connected in series across 220-volt, 25-cycle mains the current is 3 amperes. What are the resistances and the capacity?

201. When a non-inductive resistance and a circuit consisting of a non-inductive resistance of 20 ohms in series with a condenser are connected in parallel across 25-cycle constant potential mains the currents are 5.5 amperes and 2.6 amperes respectively. When these circuits are connected in series across a 60-cycle e.m.f. of the same effective value the current is 3.3 amperes. What are the resistance and the capacity?

202. When a non-inductive resistance of 11 ohms and a circuit consisting of a non-inductive resistance and a condenser are connected in parallel across constant potential 60-cycle mains the currents are 20 amperes and 4.1 amperes respectively. When they are connected in series across the same mains the current is 3.7 amperes. What are the resistance and the capacity?

203. A circuit consists of two parallel branches. The first branch has a resistance of 60 ohms and a condensive reactance of 30 ohms at 60 cycles. The second branch has a resistance of 5 ohms and a condensive reactance of 10 ohms at 60 cycles. What is the impedance of this circuit at 60 cycles? What is the phase relation of the impressed voltage and the current in the first branch? the current in the second branch? the total current?

204. If the circuit in problem 203 is connected across 220-volt 25-cycle mains what is the phase relation of the impressed e.m.f. and the current in the first branch? the current in the second branch? the total current? What is the impedance of the circuit at 25 cycles?

205. In problem 203 if the circuit is connected across 220-volt, 60-cycle mains what power does each branch absorb? What is the resultant power factor of the circuit?

206. In problem 203 if the circuit is connected across 220-volt, 25-cycle mains what power does each branch absorb? What is the resultant power factor of the circuit?

207. A circuit which has a resistance of 40 ohms and a condensive reactance of 10 ohms is connected in parallel with a circuit having variable non-inductive resistance and variable capacity. This circuit is so adjusted that the currents in the two branches are equal and displaced in phase by 60 degrees. What are the unknown resistance and the ~~capacity~~? condensive reactance?

208. Two circuits, each consisting of resistance and condensive reactance, are connected in parallel across 220-volt, 60-cycle mains. The circuits are so adjusted that each absorbs 250 watts, and the current in one is double that in the other. If the resultant power factor of these parallel branches is 0.5 what are their resistances and reactances?

209. Two circuits, each containing resistance and condensive reactance, are connected in parallel across 220-volt, 60-cycle mains. The first circuit takes a current of 5 amperes and absorbs 300 watts, and the second circuit takes a current of 3 amperes and absorbs 225 watts. What is the total current taken from the mains?

210. Two circuits, each containing resistance and condensive reactance, are connected in parallel across 220-volt, 60-cycle mains. These circuits are so adjusted that the currents in the branches are equal, and one absorbs three times as much power as the other. The total power taken from the mains is 500 watts, and the power factor of the entire circuit is 0.7. What are the constants of the circuits?

PARALLEL CIRCUITS INVOLVING RESISTANCE, INDUCTANCE, AND CAPACITY

211. An impedance coil which has a negligible resistance and an inductance of 0.1 henry is connected in parallel with a condenser of 60 microfarads capacity across 220-volt, 60-cycle mains. Draw a vector diagram showing the phase relations of the currents and the impressed e.m.f. What are the energies of the electromagnetic and the electrostatic fields at the instant the impressed e.m.f. is passing through zero? At the instant the energy of the electrostatic field is zero what is the energy of the electromagnetic field?

212. An impedance coil which has a resistance of 3 ohms and an inductance of 0.02 henry is connected in parallel with a condenser of 160 microfarads capacity across 220-volt, 60-cycle mains. Draw a vector diagram showing the phase relations of the currents and the impressed e.m.f. What are the energies of the electromagnetic and the electrostatic fields at the instant the impressed e.m.f. is passing through zero?

213. In problem 212 when the energy of the electrostatic field is zero what is the energy of the electromagnetic field?

214. An impedance coil which has a resistance of 12.8 ohms and an inductance of 0.05 henry is connected in parallel with a circuit having a resistance of 15 ohms and a capacity of 100 microfarads. If a 220-volt, 60-cycle e.m.f. is impressed on this circuit what are the currents in the branches? What is the total current? Draw a vector diagram showing the phase relations of the currents and the impressed e.m.f.

215. In problem 214 what are the energies of the electrostatic and the electromagnetic fields at the instant the impressed e.m.f. is passing through zero? At the instant the energy of the electrostatic field is zero what is the energy of the electromagnetic field?

216. An impedance coil which has a resistance of 3 ohms and an inductance of 0.12 henry is connected in parallel with a condenser of 75 microfarads capacity across 220-volt, 60-cycle mains. What current does each branch take and what is the total current supplied?

217. An impedance coil which takes 10 amperes at a power factor of 0.1 from 220-volt, 60-cycle mains is connected in parallel with a circuit which has a resistance of 3.5 ohms and a condensive reactance of 20 ohms at 60 cycles. If a 220-volt, 60-cycle e.m.f. is impressed on this circuit, what is the total current supplied?

218. An impedance coil which has a resistance of 2.6 ohms takes a current of 10 amperes from 220-volt, 60-cycle mains. If a condenser which has a reactance of 30 ohms at 60 cycles is connected in parallel with the coil what is the total current supplied?

219. An impedance coil which has a resistance of 2 ohms and an inductance of 0.1 henry is connected in parallel with a circuit having a resistance of 10 ohms and a capacity of 50 microfarads. For what frequency of the impressed e.m.f. will these circuits absorb the same power?

220. In problem 219 if a 220-volt, 60-cycle e.m.f. is impressed on the circuit what is the current through each branch and what is the total current?

221. An impedance coil has a resistance of 10 ohms and a reactance of 30 ohms at 60 cycles. A second impedance unit has a resistance of 5 ohms and a condensive reactance of 15 ohms at 60 cycles. Will these two impedances take more power from 60-cycle a.c. mains when connected in series or in parallel?

222. An impedance coil has a resistance of 10 ohms and a reactance of 30 ohms at 60 cycles. A second impedance unit has a resistance of 5 ohms and a condensive reactance of 15 ohms at 60 cycles. Will these two impedances take more power from 25-cycle a.c. mains when connected in series or in parallel?

223. For what frequency will the circuits described in problem 222 take the same power when connected ~~in series as when connected~~ in parallel across the same potential?

224. An impedance coil and a condensive impedance unit are connected in parallel across 220-volt, 60-cycle mains. The currents in the branches are 25 amperes and 10 amperes respectively, and the powers absorbed by them are 2000 watts and 200 watts respectively. What is the total current taken from the mains?

225. An impedance coil which has a resistance of 5 ohms and an inductance of 0.08 henry is connected in parallel with a condenser of 60 microfarads capacity. What is the total impedance of this circuit at 60 cycles and what is the power factor?

226. In problem 225 what is the total impedance of the circuit at 25 cycles and what is its power factor?

227. An impedance coil which has a resistance of 4 ohms and an inductance of 0.05 henry is connected in parallel with a circuit which has a resistance of 25 ohms and a condensive reactance of 15 ohms at 60 cycles. What is the total impedance of this circuit at 60 cycles and what is its power factor?

228. In problem 227 what is the total impedance of the circuit at 25 cycles and what is its power factor?

229. A circuit having a non-inductive resistance of 15 ohms and a capacity of 80 microfarads is connected in parallel with an impedance coil which has a resistance of 5 ohms and an inductance of 0.06 henry. If this entire circuit is connected across 220-volt, 60-cycle mains what is the total current? What is the total power absorbed?

230. If the circuit in problem 229 is connected across 220-volt, 25-cycle mains, what is the total current? What is the total power absorbed?

231. In problem 229 what should be the constants of a third branch connected in parallel with the other two across 220-volt, 60-cycle mains in order that the total current taken from the mains may have its least value?

232. In problem 229 what should be the constants of a third branch connected in parallel with the other two across 220-volt, 25-cycle mains in order that the total current taken from the mains may have its least value?

233. In problem 229 what should be the constants of a third circuit connected in series with the parallel branches across 220-volt, 60-cycle mains in order that the current taken from the mains may have its greatest value?

234. In problem 229 what should be the constants of a third circuit connected in series with the parallel branches across 220-volt, 25-cycle mains in order that the current taken from the mains may have its greatest value?

235. An impedance coil has a resistance of 2.4 ohms and takes 10 amperes from 220-volt, 60-cycle mains. What is the capacity of the condenser connected in parallel with this coil across 220-volt, 60-cycle mains that will reduce the total current supplied to its least value?

236. A condenser of 73.3 microfarads capacity and an impedance coil of 0.096 henry inductance and negligible resistance are connected in parallel between the points A and B. These values satisfy the conditions for resonance in the parallel branches. Between B and C is a non-inductive resistance of 5.2 ohms. If a 220-volt, 60-cycle e.m.f. is impressed on AC what are the currents in the three parts of the circuit, and what are their phase relations with respect to the impressed e.m.f.?

237. A condenser of 100 microfarads capacity and an impedance coil which has a resistance of 8 ohms are connected in series across 220-volt, 60-cycle mains and adjusted for the condition of resonance. What is the current? If a non-inductive resistance of 25 ohms is then connected in parallel with the condenser what will be the current in the coil?

238. When two impedances are connected in parallel show that their resultant impedance is equal to the sum of their separate impedances divided by $2\sqrt{2}$, if the impedances are adjusted so that the currents through them are equal and in quadrature?

239. A non-inductive resistance of 5 ohms and an impedance coil which has a resistance of 9 ohms and a reactance of 6 ohms at 60 cycles are connected in parallel across 220-volt, 60-cycle mains. A third circuit connected in parallel with the other two takes a constant current of 20 amperes and is so adjusted that the resultant power-factor of the branches is unity. What are the constants of the third circuit? What is the least current that the third circuit can take and still have the resultant power-factor unity?

240. A non-inductive resistance of 5 ohms and an impedance coil which has a resistance of 6 ohms and a reactance of 4 ohms at 60 cycles, are connected in parallel across 220-volt, 60-cycle mains. A third circuit connected in parallel with the other two takes a constant power of 3 kilowatts and is so adjusted that the resultant power-factor of the three branches is unity. What are the constants of the third branch?

* GENERAL PROBLEMS INVOLVING SERIES AND PARALLEL CIRCUITS

241. An alternator delivers 180 amperes to incandescent lamps, and 92 amperes to start an induction motor. The power-factor of the motor while starting is 0.35. What is the total current delivered by the alternator?

* Many of these problems would in actual practice apply to 3-phase circuits, but in every case it is assumed that the data are given for the equivalent single-phase circuit.

242. An alternating-current series motor takes 5 kilowatts from a 220-volt circuit. If the current is 30.8 amperes at what power-factor is the motor operating?

243. The combined resistance of the armature and field windings of an alternating-current series motor is 0.68 ohm. When the motor is taking 4.52 kilowatts at 200 volts at a power-factor of 0.85 what is its horsepower output? What is its efficiency? The motor losses in addition to the copper losses (I^2R) are 375 watts.

244. The terminal potentials of two alternators are each 120 volts and are displaced 90 degrees in phase. These machines are connected in series and deliver a current of 200 amperes, which lags 15 degrees with respect to their resultant e.m.f. What is the total power delivered and what is the power delivered by each machine? Draw a vector diagram showing the phase relations of the current and the voltages.

245. In problem 244 if the machines deliver 200 amperes at a power-factor of 0.5, what is the total power and what is the power delivered by each machine? Draw a vector diagram showing the phase relations of the current and the voltages.

246. Two alternators, each with terminal potentials of 220 volts, are connected in series and supply 50 kilowatts to a load at a power-factor of 0.8. If their terminal potentials differ in phase by 60 degrees, what part of the load is each alternator supplying? Draw a vector diagram showing the phase relations of the current and the voltages.

247. Two alternators with equal terminal potentials of 220 volts are connected in series and supply a current of 80 amperes to a load of 15.25 kilowatts. The phase difference between their terminal voltages is so adjusted that the load on one of the alternators is zero. What is this phase difference and what is the power-factor of the load?

248. Two alternators with equal terminal voltages are connected in series and supply 60 amperes at 0.5 power-factor to a load of 6.6 kilowatts. If there is a phase difference of 120 degrees between their terminal voltages, what load is each alternator supplying? Draw a vector diagram showing the phase relations of the current and voltages.

249. Two alternators are operating in parallel and are delivering power at 220 volts. The first supplies a current of 50 amperes at a power-factor of 0.8 (lagging with respect to the terminal potential difference) and the second supplies a current of 80 amperes which leads the terminal potential difference by 45 degrees. What per cent of the total load is each machine supplying? Draw a vector diagram showing the phase relations of the currents and the terminal voltage.

250. In problem 249 what is the total current delivered by the machines to the load? What is the power-factor of the load?

251. Two alternators are operating in parallel and supplying an incandescent lamp load with 160 amperes at 220 volts. If the machines divide the load equally and the first operates at a power-factor of 0.8 (lagging current), what is the current supplied by each alternator?

252. Two alternators operating in parallel deliver current at 220 volts to an inductive load of 20 kilowatts, the power-factor of which is 86.6 per cent. If the machine currents are equal and one alternator is operating at unit power-factor, what load does each supply? At what power-factoris the other alternator operating? Draw a vector diagram showing the phase relations of the currents and the terminal voltage.

253. A power station delivers 950 kilowatts at 13,200 volts to a 5-mile transmission line the conductors of which have a cross-section of 50,000 circular mils. If the specific resistance of the conductors is 10.4 ohms per mil-foot what is the power loss in the line at unit power-factor? at 0.85 power-factor? at 0.75 power-factor?

254. If there is an induction motor load taking 750 kilowatts at 0.85 power-factor at the end of the line described in problem 253, what is the efficiency of transmission when the station voltage is 13,200 and the voltage at the load is 11,500?

255. At the end of a 5-mile transmission line which has a resistance of 15 ohms there is an induction motor operating at 0.85 power-factor. This motor is delivering 1000 horse-power at an efficiency of 92 per cent. The voltages at the generating and receiving ends of the line are 13,200 volts and 11,500 volts respectively. What is the efficiency of transmission?

256. A power station delivers 800 kilowatts to a transmission line at 10,000 volts. The resistance of the line is 10 ohms. At the receiving end of the line, where the voltage is 8,800 volts, there is an induction motor load taking 95 amperes. What is the power-factor of the motor load?

257. An over-excited synchronous motor is operating in parallel with an induction motor on a 220-volt, 60-cycle circuit. The synchronous motor takes 3.67 kilowatts from the line at a power-factor of 0.78. The induction motor is operating at a power-factor of 0.85 and delivers 21.7 horsepower at an efficiency of 88 per cent. What is the total line current delivered to these motors? What is the resultant power-factor of the load? The synchronous motor takes a leading current and the induction motor a lagging current.

258. An over-excited synchronous motor is operating at the end of a transmission line which has a resistance of 15 ohms. The motor is taking 560 kilowatts at a power-factor of 0.90. The voltages at the ends of the line are each equal to 11,200. What is the efficiency of transmission? At what power-factor is the station operating?

259. An induction motor is taking 21.8 kilowatts at a power-factor of 0.85 from 220-volt, 60-cycle mains. A synchronous motor, which has a rating of 20 kilovolt-amperes at 220 volts ~~and whose losses, exclusive of the direct-current field loss, are approximately 1800 watts~~, is connected in parallel with the induction motor. If the lowest power-factor at which the synchronous motor can be operated is 0.3, what is the least value that the line current can have? What power is the synchronous motor taking from the line for this condition? The synchronous motor takes a leading current and the induction motor a lagging current.

260. At full load a 20-horsepower, 220-volt, 60-cycle induction motor operates at a power-factor of 0.89 and has an efficiency of 90 per cent. A synchronous motor, which takes a constant load of 25 kilowatts, is operated in parallel with the induction motor on a 220-volt, 60-cycle circuit. At what power-factor should the synchronous motor be operated when the induction motor is delivering full load in order that the resultant power-factor of the entire load may be unity? The synchronous motor takes a leading current and the induction motor takes a lagging current.

261. At full load a 50-horsepower, 220-volt, 60-cycle induction motor operates at a power-factor of 0.90 and has an efficiency of 92 per cent. A synchronous motor which has a rating of 20 kilovolt-amperes at 220 volts and whose losses, exclusive of the direct-current field loss, are approximately 1800 watts, is operated in parallel with the induction motor on a 220-volt, 60-cycle circuit. The least power-factor at which the synchronous motor can be operated while running light is 0.5 When the induction motor is delivering full load, what is the highest resultant power-factor that can be obtained by running the synchronous motor light? The synchronous motor takes a leading current and the induction motor a lagging current.

262. In problem 261, when the induction motor is delivering full load, what is the least output that can be taken from the synchronous motor and have the system adjusted for unit power factor? For this condition assume that the least power-factor at which the synchronous motor can be operated is 0.8.

263. A transmission line has a resistance of 15 ohms and a reactance of 20 ohms at 60 cycles. The station voltage is 13,200. At the end of the line there is an induction motor load of 710 kilowatts taking 69 amperes. What is the efficiency of transmission? At what power-factor is the station operating?

264. In problem 263 draw a vector diagram showing the phase relations of the current and the voltages at the ends of the line. What is the voltage at the load?

265. In problem 263 if the frequency alone is reduced to 25 cycles, what will be the voltage at the load?

266. A transmission line has a resistance of 14 ohms and a reactance of 18 ohms at 60 cycles. The station voltage is 13,200, and a load at the end of the line takes 750 kilowatts at a power-factor of 0.85 (lagging current). What is the current? What is the voltage at the load?

267. In problem 266, if the load takes a leading current, the rest of the data being unchanged, what is the voltage at the load?

268. A transmission line has a resistance of 25 ohms and an inductance of 0.075 henry. The station voltage is 20,000 and at the end of the line there is a constant induction motor load delivering 1740 horsepower at an efficiency of 91 per cent and with a power-factor of 0.87. If the frequency is 60 cycles what is the voltage at the load?

269. In problem 268 what is the efficiency of transmission at a frequency of 60 cycles?

270. In problem 268 if the frequency is reduced to 25 cycles, the power-factor of the induction motor load can be increased to 0.89 and the efficiency to 92 per cent when the motors are delivering 1740 horsepower. What is the voltage at the load when the frequency is 25 cycles and the motors are delivering 1740 horsepower?

271. In problem 270, what is the efficiency of transmission when the frequency is 25 cycles and the motors are delivering 1740 horsepower?

272. A short transmission line has a resistance of 0.25 ohm and a reactance of 0.3 ohm. The station voltage is 240 and at the end of the line there is a constant load of 200 lamps and a variable induction motor load. The lamps each take a normal current of 0.32 ampere at 220 volts and their resistance may be assumed constant. The induction motor takes, on starting, a current of 60 amperes at a power-factor of 0.35. At full load the motor delivers 5 horsepower at an efficiency of 85 per cent and operates at a power-factor of 0.86. Compare the voltages at the load when the lamps alone are taking current and when the induction motor is starting.

273. In problem 272 compare the voltages at the load when the lamps alone are taking current and when the motor is delivering full load.

CHAPTER XIII

SYMBOLIC METHOD: COMPLEX NOTATION

IN many of the more difficult problems in alternating currents, which assume simple harmonic e.m.fs. and currents, it is more convenient in their solution to use the *symbolic method,* or *complex notation.*

The conventions that are here adopted are that positive real quantities are measured to the right along the positive axis of abscissas and that the operator j rotates a vector 90 degrees in the counter clockwise direction. The operator j twice applied, that is the operator j^2, rotates a vector through 180 degrees, or multiplies the vector by -1. Every vector is written as the sum of two vectors which are at right angles to each other. Thus $V = a + jb$ is a vector in the first quadrant if a and b are both positive; in the second quadrant if a is negative and b is positive, and in the fourth quadrant if a is positive and b is negative. The magnitude of the vector, V, is $\sqrt{a^2 + b^2}$, and the angle which it makes with the positive axis of abscissas is $\tan^{-1} \frac{b}{a}$. The resultant, or the sum, of a number of vectors of the form $a + jb$ is $\Sigma a + j\Sigma b$.

A vector will be rotated in the counter clockwise direction through an angle θ if it is multiplied by the operating factor $\cos \theta + j \sin \theta$.

To rationalize the expression $\frac{1}{a + jb}$ multiply numerator and denominator by $a - jb$; this converts it into the expression

$$\frac{a}{a^2 + b^2} - j \frac{b}{a^2 + b^2}.$$

If the vector, or complex expression for the current in a circuit, is taken as $I = i_1 + ji_2$, the power component of the e.m.f. impressed on the circuit is $rI = r\,(i_1 + ji_2)$ and the wattless component of the e.m.f. is $jxI = jx\,(i_1 + ji_2)$ if the current lags and $-jxI = -jx\,(i_1 + ji_2)$ if the current leads. Thus the vector, or complex expression for the impressed e.m.f. is $(r + jx)\,(i_1 + ji_2)$; x is positive when the current lags and negative when the current leads. It should be observed, however, that the impedance, $r + jx$, is *not* a vector.

The general complex equation relating to any portion of a circuit is $\frac{e_1 + je_2}{i_1 + ji_2} = r + jx$; $e_1 + je_2$ is the complex expression for the resultant e.m.f. impressed on the circuit, $i_1 + ji_2$ is the complex expression for the current, and $r + jx$ is the complex expression for the equivalent impedance of the circuit. The effective value of the e.m.f. is $\sqrt{e_1^2 + e_2^2}$; the effective value of the current is $\sqrt{i_1^2 + i_2^2}$ and the current lags behind the e.m.f. by an angle whose tangent is $\frac{x}{r}$. The power supplied to the circuit is given by the expression $e_1 i_1 + e_2 i_2$, where the quantities have the algebraic signs that occur in the complex expressions for the e.m.f. and the current.

For series circuits the complex expression for the resultant impedance is $r_0 + jx_0$; $r_0 = \Sigma r$ and $x_0 = \Sigma x$.

For parallel circuits the complex expression for the resultant admittance is $g_0 - jb_0$, when $g_0 = \Sigma g$ and $b_0 = \Sigma b$.

$g = \frac{r}{r^2 + x^2}$ and $b = \frac{x}{r^2 + x^2}$ are known as the conductance and the susceptance respectively.

COMPLEX NOTATION

1. A vector has a magnitude of 60 and makes an angle of 30 degrees with the axis of reals. If it is in the first quadrant what is its complex expression? If the vector is in the fourth quadrant what is its complex expression?

2. A vector has a magnitude of 50 and makes an angle of 30 degrees with the axis of imaginaries. If it is in the first quadrant what is its complex expression? If the vector is in the second quadrant what is its complex expression?

3. A vector, V, makes an angle of 45 degrees with a vector, V_1, whose complex expression is $-7 + j\,3$. If the magnitude of V is 10 what are its two possible complex expressions?

4. What rotating operator must be applied to the vector whose complex expression is $-5 + j\,10$ in order that it may be made to coincide with the vector whose complex expression is $5 + j\,10$?

5. A vector $V = 10 + j\,15$ is rotated until it makes an angle of 30 degrees with a vector $V_1 = -5 - j\,10$. What are the two possible complex expressions for the first vector after rotation?

6. Two vectors, V_1 and V_2, have complex expressions of $-10 + j\,5$ and $10 + j\,5$ respectively. If a new axis of reals is chosen along V_1 what are the complex expressions for V_1 and V_2, referred to this new axis?

7. A vector has a complex expression of $5 + j\,10$. What is its complex expression after being rotated through a positive angle of 120 degrees? through a negative angle of 120 degrees?

8. What is the angle between two vectors whose complex expressions are $V_1 = -3 + j\,4$ and $V_2 = 4 - j\,3$?

9. The complex expression for a vector V referred to certain perpendicular axes is $-4 + j\,3$. These axes have been rotated through a certain angle from the principal axes so that the positive direction of their axis of reals is coincident with the vector V_1, whose complex expression referred to the principal axes is $-1 + j\,2$. What is the complex expression for V, referred to the principal axes?

10. Through what positive angle must a vector whose complex expression is $-10 - j\,5$ be rotated until it coincides in direction with the vector whose complex expression is $3 + j\,4$?

11. What is the complex expression for the vector which is the sum of two vectors whose complex expressions are $7 + j\,2$ and $-3 - j\,6$. What angle does the resultant make with the positive axis of reals?

12. The complex expressions for two vectors are $8 - j\,6$ and $2 + j\,10$. What is the complex expression of a third vector which, when added to the sum of the first two, will give a resultant whose complex expression is $-3 - j\,6$?

13. The complex expression for a vector is $-8 + j\,6$. What are the complex expressions for two other vectors, each of the same magnitude as the first and so situated that the sum of the three is zero?

14. The complex expressions for two vectors are $V_1 = 10 - j\,20$ and $V_2 = -6 - j\,8$. What is the projection of V_2 upon V_1? What is the projecting line, i.e., the line drawn from the terminus of V_2 perpendicular to and ending in V_1?

15. A vector, V, is the sum of two component vectors. The first has a magnitude of 10 and coincides in direction with $V_1 = 3 - j\,4$. The second has a magnitude of 20 and coincides in direction with $V_2 = -8 + j\,2$. What is the complex expression for V?

SERIES CIRCUITS

16. An e.m.f. whose complex expression is $-132 - j\,176$ produces a current whose complex expression is $5 - j\,10$. Is the reactance in this circuit condensive or inductive? At the instant the impressed e.m.f. is passing through zero what is the value of the current?

17. The complex expression for the e.m.f. impressed on a circuit is $-200 - j\,100$, and for the current, $-50 + j\,30$. What is the power? What are the power and the wattless components of the e.m.f.?

18. The complex expression for the e.m.f. impressed on a circuit is $200 - j\,80$, and for the current, $4 + j\,3$. What are the resistance and the reactance of the circuit?

19. The complex expression for the e.m.f. impressed on a circuit is $-160 + j\,120$, and for the current, $-50 - j\,20$. What is the power supplied? What is the power-factor of the circuit? What is the character of the load?

20. When an e.m.f. whose complex expression is $100 - j\,200$ is impressed on a circuit it produces a current whose complex expression is $-10 - j\,20$. What are the complex expressions for the power and the wattless components of the e.m.f.?

21. The complex expression for the e.m.f. impressed on a circuit is $-200 + j\,100$ and for the current, $-10 - j\,10$. What are the complex expressions for the power and the wattless components of the current?

22. The complex expression for the e.m.f. impressed on a circuit is $-132 + j\,176$. This circuit absorbs 840 watts at a power-factor of 0.8. What are the two possible complex expressions for the current?

23. An impedance coil takes a current of 25 amperes and absorbs 250 watts when connected across 220-volt, 60-cycle mains. What is the complex expression for the impedance of the coil?

24. When an e.m.f. of 220 volts is impressed on an inductive circuit it produces a current whose complex expression may be taken as $12 + j\,20$. The power-factor of the circuit is 0.8. What is the power supplied? What are the resistance and the reactance of the circuit?

25. An impedance coil which absorbs 200 watts when connected across a 20-volt storage battery will absorb 250 watts when connected across an alternating e.m.f. whose complex expression is $200 - j\,100$. What is the complex expression for the current in the latter case?

26. When an e.m.f. whose complex expression is $176 - j\,132$ is impressed on an impedance coil the current is 15 amperes and the coil absorbs 500 watts. What is the complex expression for the current?

27. When an e.m.f. whose complex expression is $-132 + j\,176$ is impressed on an impedance coil, it takes a current of 20 amperes at a power-factor of 0.15. What is the complex expression for the current?

28. In problem 27 what are the complex expressions for the power and wattless components of the current?

29. In problem 27 what are the complex expressions for the power and wattless components of the impressed voltage?

30. If an impedance coil is connected across 220-volt, 60-cycle mains, the complex expressions for the current and the impressed voltage may be taken as $2 - j\,10$ and $220 + j\,0$. If this coil is connected across 220-volt, 25-cycle mains and the complex expression for the impressed voltage is taken as $220 + j\,0$, what is the complex expression for the current?

31. A 220-volt, 60-cycle e.m.f. is impressed on a circuit consisting of a non-inductive resistance in series with a condenser of 80 microfarads capacity. If the complex expression for current is $3 - j\,5$, what is the complex expression for the impressed e.m.f.?

32. A non-inductive resistance and a condenser are connected in series. The complex expression for the impressed e.m.f. is $220 + j\,0$ and for the current, $6 + j\,3$. What are the resistance and the reactance of this circuit?

33. An e.m.f. of 220 volts is impressed on a circuit consisting of a non-inductive resistance in series with a condenser. If the power-factor of this circuit is 0.3 and the complex expression for the current is taken as $10 - j\,5$, what is the power supplied? What is the complex expression for the impedance of the circuit?

34. A non-inductive resistance in series with a condenser takes a current of 5 amperes and absorbs 200 watts when connected across 220-volt, 60-cycle mains. What is the complex expression for the impedance of this circuit?

35. A non-inductive resistance in series with a condenser absorbs 250 watts when connected across 220-volt, 60-cycle mains. If the complex expression for the current is taken as $5 - j\,10$, what is the complex expression for the impressed e.m.f.?

36. An e.m.f. whose complex expression is $-80 + j\,200$ is impressed on a non-inductive resistance and a condenser, which are connected in series. This circuit absorbs 340 watts at a power-factor of 0.2. What is the complex expression for the current?

37. A circuit consists of a non-inductive resistance and a condenser in series. The complex expression for the impressed e.m.f. is $100 + j\,200$, and for the current, $-3 + j\,5$. What are the complex expressions for the voltages across the resistance and the condenser ?

38. The complex expression for the e.m.f. impressed on a circuit consisting of a non-inductive resistance in series with a condenser is $132 - j\,176$. If the current is 6 amperes and the power absorbed by the resistance is 350 watts, what are the complex expressions for the voltages across the resistance and the condenser?

39. If a circuit consisting of a non-inductive resistance in series with a condenser is connected across 220-volt, 60-cycle mains, the complex expressions for the current and the impressed voltage may be taken as $2 + j\,5$ and $220 + j\,0$. If this circuit is connected across 220-volt, 25-cycle mains and the complex expression for the impressed voltage is taken as $220 + j\,0$, what is the complex expression for the current?

40. A non-inductive resistance of 20 ohms in series with a condenser takes a current of 6 amperes from 220-volt, 60-cycle mains. If a 220-volt, 25-cycle e.m.f. is impressed on the circuit, what will be the complex expression for the current referred to the impressed e.m.f. as along the axis of reals?

41. An e.m.f. of 220 volts is impressed on a non-inductive resistance and a condenser, which are connected in series. The complex expression for the current is $10 - j\,4$, and the potential difference across the condenser is 200 volts. What are the complex expressions for the impressed voltage and its components across the resistance and the condenser?

42. The complex expression for the e.m.f. impressed on a circuit consisting of a non-inductive resistance in series with a condenser is $132 + j\,176$, and for the current, $-5 + j\,10$. What are the complex expressions for the voltages across the resistance and the condenser? What is the phase relation of these voltages?

43. An impedance coil, which has a resistance of 2.4 ohms and an inductance of 0.11 henry, is connected in series with a non-inductive resistance of 10 ohms across a 60-cycle circuit. The complex expression for the current is $5 + j\,2$. What are the complex expressions for the impressed voltage and its components across the coil and the resistance?

44. An impedance coil is connected in series with a non-inductive resistance of 1.7 ohms. The complex expression for the impressed e.m.f. is $110 + j\,0$, and for the current, $10 - j\,20$. What are the resistance and the reactance of the impedance coil?

45. An impedance coil and a non-inductive resistance are connected in series. The complex expression for the impressed e.m.f. is $-200 - j\,100$, and for the current, $-20 + j\,8$. If the potential difference across the non-inductive resistance is 94 volts, what are the resistance and the reactance of the coil?

46. An impedance coil and a non-inductive resistance of 0.5 ohms are connected in series. The complex expression for the impressed e.m.f. is $200 + j\,100$, and for the current, $10 - j\,15$. What is the complex expression for the voltage across the impedance coil? What is the phase relation of the voltages across the non-inductive resistance and the impedance coil?

47. When an impedance coil and a non-inductive resistance are connected in series across 110-volt d. c. mains the current is 20 amperes. When they are connected in series across 220-volt, 60-cycle mains, the complex expression for the current may be taken as $12 + j\,20$. If, in the latter case, the potential difference across the non-inductive resistance is 106 volts, what is the complex expression for the voltage across the impedance coil?

48. When an impedance coil and a non-inductive resistance are connected in series across 110-volt d. c. mains the current is 15 amperes and the voltages across them are equal. If they are connected in series across 220-volt, 60-cycle mains, the voltage across the coil is five times that across the non-inductive resistance. If the current is taken as along the axis of reals, what are the complex expressions for the impressed voltage and its components across each part of the circuit?

49. An impedance coil, which takes a current of 25 amperes when connected across 220-volt, 60-cycle mains, is connected in series with a non-inductive resistance across 220-volt, 60-cycle mains. The current is 15 amperes, and if its complex expression is taken as $15 - j\,0$, the impressed voltage is $200 + j\,91.7$. What are the complex expressions for the voltages across the non-inductive resistance and the impedance coil?

50. An impedance coil, which takes a current of 54 amperes when connected across 220-volt, 25-cycle mains, is connected in series with a non-inductive resistance across 220-volt, 60-cycle mains. The current is 20 amperes, and if its complex expression is taken as $12 + j\,16$, the impressed voltage is $-91.7 + j\,200$. What are the complex expressions for the voltages across the non-inductive resistance and the impedance coil?

51. When an impedance coil and a non-inductive resistance are connected across 220-volt, 60-cycle mains they absorb 1000 watts, and the potential across the coil is 200 volts. If the complex expression for the current is taken as $10 + j\,0$, what are the complex expressions for the voltages across the resistance and the coil?

52. An impedance coil, which has a power-factor of 0.2, is connected in series with a non-inductive resistance of 40 ohms across 220-volt, 60-cycle mains. If the potential across the coil is 100 volts, what are the complex expressions for the impressed voltage and its components across the non-inductive resistance and the impedance coil, all referred to the current as along the axis of reals?

53. When two impedance coils are connected in series across 110-volt d. c. mains, the voltage across the first is double that across the second. When these coils are connected in series across 220-volt, 60-cycle mains, the voltage across the first is one-half of that across the second, and the power-factor of the circuit is 0.5. If the current is taken as along the axis of reals what are the complex expressions for the impressed voltage and its components across each coil?

54. When two impedance coils are connected in series across 108-volt d. c. mains the current is 25 amperes and the voltage across the first is double that across the second. When these coils are connected in series across 220-volt, 60-cycle mains the current is 10 amperes and the potential across the first is still 72 volts. If the current is taken as along the axis of reals what are the complex expressions for the impressed voltage and its components across each coil?

55. Two coils, whose impedances at 60 cycles are given by the complex expressions $Z_1 = 3 + j\,10$ and $Z_2 = 10 + j\,3$, are connected in series across 220-volt, 60-cycle mains. What is the voltage across each coil? If these coils are connected in series across 220-volt, 25-cycle mains, what is the voltage across each impedance?

56. When two impedance coils are connected in series across 220-volt, 60-cycle mains, the complex expressions for the current and the impedance voltage are $10 - j\,12$ and $220 + j\,0$, and the voltage across the first coil is 103.4 ~~75 per cent of the voltage across the second coil~~. When these impedance coils are connected in series in the same order across 220-volt, 25-cycle mains the voltage across the first ~~second~~ coil is 123 volts. ~~65 per cent of the voltage across the first coil~~ What are the resistance and the reactance at 60 cycles of each coil?

57. An impedance coil is connected in series with a condenser of 60 microfarads across 60-cycle mains. The complex expression for the impressed voltage is $220 + j\,0$, and for the current, $10 + j\,5$. What are the resistance and the reactance of the impedance coil?

58. An e.m.f., whose complex expression is $-200 + j\,100$, is impressed on an impedance coil connected in series with a condenser of 40 ohms reactance. The complex expression for the current is $-20 - j\,15$. What is the power-factor of this circuit? What is the complex expression for the voltage across the impedance coil?

59. An impedance coil and a condenser are connected in series. The complex expression for the impressed voltage is $100 + j\,200$, and for the current, $-5 + j\,15$. If the potential across the condenser is 350 volts, what are the complex expressions for this voltage and that across the impedance coil?

60. An impedance coil and a condenser, which has a reactance of 53 ohms at 60 cycles, are connected in series across 220-volt, 60-cycle mains. The complex expression for the impressed e.m.f. is $200 - j\,91.7$, and for the current, $15 + j\,5$. What is the complex expression for the voltage across the impedance coil? What is the phase relation of the voltages across the capacity and the impedance coil?

61. A non-inductive resistance of 5 ohms, an impedance coil of 0.1 henry inductance and 2 ohms resistance, and a 100-microfarad condenser are connected in series across a 60-cycle circuit. If the complex expression for the current is $12 + j\,8$, what is the complex expression for the voltage across the non-inductive resistances? across the condenser? across the entire circuit?

What are the phase relations of the current and the impressed voltage and each of its components across the three parts of the circuit?

62. An impedance coil which takes 20 amperes when connected across 220-volt, 60-cycle mains is connected in series with a condenser across 220-volt, 60-cycle mains. The current is 40 amperes and if its complex expression is taken as $40 + j\,0$ the complex expression for the impressed voltage is $132 + j\,176$. What are the complex expressions for the voltages across the impedance coil and the condenser?

63. An impedance coil which takes 25 amperes at 0.15 power-factor when connected across 220-volt, 60-cycle mains is connected in series with a condenser across 110-volt, 60-cycle mains. The current is 30 amperes and if its complex expression is taken as $30 + j\,0$, what are the possible complex expressions for the impressed voltage? What are the possible complex expressions for the voltages across the impedance coil and across the condenser?

64. When an impedance coil and a condenser are connected in series across 220-volt, 60-cycle mains they absorb 450 watts and the potential across the coil is 440 volts. If the complex expression for the current is $6 - j\,8$, what are the possible complex expressions for the voltages across the coil and the condenser?

65. An impedance coil, which has a power-factor of 0.15, is connected in series with a condenser across 220-volt, 60-cycle mains. If the potential across the coil is 450 volts what are the possible complex expressions for the impressed voltage and its components across the impedance coil and the condenser, all referred to the current as along the axis of reals?

66. An impedance coil and a condenser are connected in series across 220-volt, 60-cycle mains and the circuit is so adjusted that the potentials across the coil and the condenser are each 220 volts. If the current is taken as along the axis of reals, what are the complex expressions for the impressed voltage and its components across the coil and the condenser?

67. An impedance coil and a condenser are connected in series. This circuit takes a current of 15 amperes and absorbs

675 watts. The potentials across the coil and condenser are 400 volts and 600 volts respectively. If the current is taken as along the axis of reals, what are the complex expressions for the impressed e.m.f. and its components across the coil and the condenser?

68. An impedance coil and a condenser, which has a reactance of 40 ohms at 60 cycles, are connected in series across 220-volt, 60-cycle mains. If the total power supplied to this circuit is 350 watts and the complex expression for the voltage across the condenser is $500 - j\,200$, what are the possible complex expressions for the current and the voltages across the impedance coil and the entire circuit?

69. An impedance coil and a condenser, which has a reactance of 120 ohms at 25 cycles, are connected in series across 220-volt, 60-cycle mains. If the impressed e.m.f. is taken as along the axis of reals, the complex expression for the current is $10 + j\,5$. What are the possible complex expressions for the impressed voltage and its components across the impedance coil and the condenser?

70. If an impedance coil is connected across 220-volt, 60-cycle mains, the complex expressions for the current and the impressed voltage may be taken as $3 + j\,15$ and $220 + j\,0$. What is the complex expression for another circuit, which, when connected in series with this impedance coil across 220-volt, 60-cycle mains will bring the current into phase with the impressed voltage but will not alter its magnitude?

71. An impedance coil is connected in series with a condenser across 220-volt, 60-cycle mains. The complex expressions for the current and impressed voltage are $5 - j\,15$ and $220 + j\,0$. If the coil and condenser are connected in series across 220-volt 25-cycle mains the effective value of the current remains unchanged. What are the inductance and capacity in the circuit? If the complex expression for the impressed voltage is taken as $220 + j\,0$, what is the complex expression for the current?, in the second case.

72. An impedance coil which has a resistance of 2.5 ohms and a variable reactance is connected in series with a condenser of fixed capacity across a constant potential of 30 volts. The reactance of the coil is adjusted so that the voltage across the condenser has its greatest possible value. If at this time the complex expression for the potential across the impedance coil is taken as $120 - j\,50$, what are the complex expressions for the current and the voltages across the condenser and the entire circuit?

73. An impedance coil and a condenser of variable capacity are connected in series across a constant potential of 30 volts and the capacity is so adjusted that the voltage across the coil has its greatest value. If at this time the complex expression for the voltage across the condenser is taken as $300 + j\,400$, what are the complex expressions for the voltages across the impedance coil and the entire circuit?

74. An impedance coil which has a resistance of 5 ohms and a reactance of 20 ohms at 60 cycles is connected in series with a condenser of variable capacity across 220-volt, 60-cycle mains. The capacity is so adjusted that the voltage across it has its greatest value. If the complex expression for the impressed voltage is $220 + j\,0$, what are the complex expressions for the voltages across the coil and across the condenser?

75. An impedance coil which has a resistance of 3.4 ohms and a variable reactance is connected in series with a condenser having a reactance of 20 ohms at 60 cycles across a 220-volt, 60-cycle circuit. The reactance of the coil is adjusted so that the voltage across the coil has its greatest value. If the complex expression for the impressed voltage is $220 + j\,0$, what are the complex expressions for the voltages across the coil and across the condenser?

76. Two impedances Z_1 and Z_2 are connected in series; the resistance of the first is 3 ohms and the power-factor of the second is 0.4. The total power supplied to the circuit is 8 kilowatts and the complex expression for the current is $30 + j\,20$. What may be the complex expressions for the voltage across the second impedance, Z_2?

77. Two impedance units, which are connected in series across 220-volt, 60-cycle mains, take a current of 10 amperes and absorb 2000 watts. If the potential across each unit is 220 volts, what are the resistance and reactance of each unit? If the complex expression for the current is taken as $10 + j\,0$, what are the complex expressions for the voltages across each unit?

78. Two impedances which are connected in series across 220-volt, 60-cycle mains absorb 1950 watts. The complex expression for the current is $10 + j\,8$ and for the voltage across the second impedance, $200 - j\,100$. What are the resistance and the reactance of each impedance? What is the complex expression for the impressed voltage?

79. A series circuit consists of two parts, the first of which has a resistance of 2 ohms and an inductive reactance of 20 ohms. When connected across certain constant a. c. mains this circuit takes a current of 15.6 amperes. The entire circuit absorbs 870 watts and the potential difference across the second part of the circuit is 180 volts. What are the possible values for the resistance and reactance of the second part of this circuit? What is the smallest possible value of the impressed e.m.f.?

80. The complex expression for the first of two impedances, which are connected in series, is $5 + j\,15$ and for the potential across it, is $100 + j\,50$. The potential across the second is 160 volts and the entire circuit absorbs 750 watts. What are the resistance and the reactance of the second impedance? What may be the complex expressions for the impressed voltage?

81. Two impedances, $Z_1 = 5 + j\,10$ and $Z_2 = 15 - j\,30$, are connected across the mains *AB* of a 3-wire 220-110 volt single-

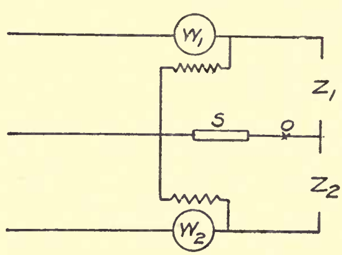

Fig. 31

phase system, as shown in Fig. 31. What will each wattmeter read before and after the switch, *S*, in the neutral is open? What is the voltage across the switch when it is open? Sketch an approximate vector diagram for each case.

82. In problem 81 if the common terminals of the potential coils of the wattmeters are connected to the load side of the switch, S, *i.e.*, to point O, what will the instruments read before and after the switch is open? Compare the power consumed in the impedances with the readings of the corresponding wattmeters for each case.

PARALLEL CIRCUITS

83. An impedance coil which has a negligible resistance is connected in parallel with a non-inductive resistance. The complex expression for the impressed e.m.f. is $100 + j\,200$ and for the total current, $10 + j\,2$. What are the complex expressions for the currents in the coil and the resistance?

84. An impedance coil which has a resistance of 2.4 ohms and an inductance of 0.11 henry is connected in parallel with a non-inductive resistance of 10 ohms across a 60-cycle e.m.f. The complex expression for the total current is $10 + j\,5$. What are the complex expressions for the impressed voltage and the currents in the two branches? What is the complex expression for the single equivalent impedance?

85. An impedance coil is connected in parallel with a non-inductive resistance of 10 ohms. The complex expression for the impressed e.m.f. is $110 + j\,0$, and for the total current, $25 - j\,20$. What are the resistance and the reactance of the impedance coil?

86. An impedance coil and a non-inductive resistance are connected in parallel. The complex expression for the impressed e.m.f. is $-200 - j\,100$ and for the total current, $-20 + j\,10$. If the current in the non-inductive resistance is 10 amperes what are the resistance and the reactance of the coil?

87. An impedance coil and a non-inductive resistance of 5 ohms are connected in parallel. The complex expression for the impressed e.m.f. is $200 - j\,100$ and for the total current, $10 - j\,20$. What is the complex expression for the current in the impedance coil? What is the phase relation of the currents in the non-inductive resistance and the impedance coil?

88. A condenser and a non-inductive resistance are connected in parallel across 220-volt, 60-cycle mains. If the complex expression for the impressed e.m.f. is taken as $132 + j\,176$ the complex expression for the total current is $-5 + j\,10$. What are the complex expressions for the currents in the condenser and the resistance?

89. When a non-inductive resistance of 20 ohms and a condenser are connected in parallel across 220-volt, 60-cycle mains the total current is 12.1 amperes. If the complex expression for the impressed voltage is $132 + j\,176$ what are the complex expressions for the currents in the branches? What is the power taken by this circuit?

90. When a non-inductive resistance and a condenser are connected in parallel across 220-volt, 60-cycle mains, they take a total current of 5.8 amperes and absorb 650 watts. If the current in the capacity branch is taken as along the axis of reals, what are the complex expressions for the impressed voltage, the total current and the current in the non-inductive resistance?

91. When a non-inductive resistance of 20 ohms and a condenser are connected in parallel across 220-volt, 60-cycle mains the total current is 14.3 amperes. What should be the resistance and capacity, which, when connected in series across the same mains, will take the same current and absorb the same power? If the complex expression for the impressed voltage is taken as $220 + j\,0$, what are the complex expressions for the currents in the resistance and the condenser in each case?

92. A non-inductive resistance and a condenser, which are connected in parallel across 220-volt, 60-cycle mains, absorb 800 watts. If the complex expression for the current in the condenser branch is taken as $3 - j\,4$, what are the complex expressions for the impressed voltage, the total current and the current in the non-inductive resistance? What is the phase relation of the currents in the branches?

93. An e.m.f., $E = 100 + j\,50$, is impressed on two impedance circuits in parallel. The complex expressions for the currents in the branches are $10 + j\,5$ and $-5 + j\,10$. What is the total current? What is the power supplied to each of these parallel branches? What is the complex expression for the equivalent admittance of this circuit?

94. An e.m.f., $E = 50 - j\,100$, is impressed on a circuit consisting of two impedances in parallel. The current through one of them is 20 amperes and the complex expression for the current through the other is $15 + j\,5$. If the total power absorbed by these parallel branches is 600 watts, what is the complex expression for each of the impedances?

95. An impedance coil is connected in parallel with a condenser of 60 microfarads capacity, across a 60-cycle e.m.f. The complex expression for the impressed e.m.f. is $220 + j\,0$ and for the total current, $2 - j\,5$. What are the resistance and the reactance of the impedance coil?

96. What are the complex expressions for two impedances which will each absorb 250 watts and take currents that are equal and in quadrature when connected in parallel across 220-volt, 60-cycle mains?

97. Two impedance circuits are connected in parallel across 220-volt, 60-cycle mains. They are so adjusted that the currents are each 10 amperes and in quadrature and the power taken by the first is double that taken by the second. What are the complex expressions for these impedances?

98. Two impedance circuits, whose complex expressions are $5 + j\,25$ and $10 - j\,35$ are connected in parallel. If the current through the first is 8.7 amperes, what is the total current?

99. Two impedance circuits, whose complex expressions are $2 + j\,10$ and $3 - j\,20$ are connected in parallel. If the complex expression for the current through the first is $12 - j\,18$, what is the complex expression for the total current?

100. An e.m.f., $E = 200 + j\,100$, is impressed on two impedance circuits in parallel, the complex expression for one of which is $2 + j\,10$. The other impedance circuit is so adjusted that each branch absorbs the same power and the power-factor of the entire circuit is 0.8. What are the resistance and the reactance of the other branch?

101. An impedance coil and a condenser are connected in parallel across 220-volt, 60-cycle mains. The current in the condenser is 10 amperes, the total current is 5 amperes and the power supplied to the circuit is 300 watts. What are the complex expressions for the total current and currents in the parallel branches referred to the impressed e.m.f. as along the axis of reals?

102. Two circuits of 20 and 30 ohms impedance respectively, take a total current of 5 amperes when connected in parallel across 220-volt, 60-cycle mains. If the power-factor of the first branch is 0.2 what is the power-factor of the second? If the complex expression for the impressed voltage is taken as $220 + j\,0$ what are the possible complex expressions for the currents in the parallel branches?

103. Two impedance circuits are connected in parallel across 220-volt, 60-cycle mains. They are so adjusted that the currents in the branches and in the mains are each 10 amperes and the power taken by one branch is double that taken by the other. If the complex expression for the impressed voltage is taken as $220 + j\,0$, what are the complex expressions for the currents in the parallel branches?

104. An impedance coil and a condenser are connected in parallel across 220-volt, 60-cycle mains and the circuit is so adjusted that the currents in the branches are of the same magnitude and are each equal to the total current supplied to the system. If the circuit absorbs 300 watts, what are the complex expressions for the total current and the currents in the branches referred to the impressed voltage as along the axis of reals?

105. Two impedance circuits, whose complex expressions are $5 + j\,15$ and $10 - j\,30$, are connected in parallel. If the total current supplied to this circuit is 7 amperes, what is the current through each branch?

106. Two impedance circuits are connected in parallel. The first has a complex expression of $4 + j\,18$ and the second takes a leading current of 10 amperes and absorbs 250 watts. If the total current is 4.1 amperes, what are the complex expressions for the total current and the currents in the parallel branches referred to the current in the second branch as along the axis of reals?

107. An impedance coil and a condenser are connected in parallel across 110-volt, 60-cycle mains. The total current is 12 amperes and that in the coil is 15 amperes. The total power supplied to the circuit is 450 watts. What are the possible complex expressions for each of the impedances?

108. The complex expression for the impedance of one of two coils, which are connected in parallel, is $3 + j\,40$. A third circuit, the complex expression for whose impedance is $10 - j\,30$, is connected in series with the first two. The complex expression for the impressed e.m.f. is $200 + j\,100$ and for the current through the third coil, $10 + j\,6$. What is the complex expression for the current through the unknown impedance coil and what are its resistance and reactance?

109. A non-inductive resistance of 5 ohms, an impedance coil of 0.1 henry inductance and 2 ohms resistance, and an 80-microfarad condenser are connected in parallel across a 60-cycle e.m.f. If the complex expression for the total current is $12 + j\,16$, what are the complex expressions for the impressed e.m.f. and the currents in each of the branches? What are the phase relations of the impressed e.m.f. and the currents in the branches?

110. Two impedance circuits, Z_1 and Z_2, are connected in parallel; the resistance of the first is 4 ohms and the power-factor of the second is 0.8. The total power supplied to the circuit is 5 kilowatts, the complex expression for the impressed e.m.f. is $100 + j\,200$ and the current in the first branch is 15 amperes. What are the complex expressions for the currents in the branches if the first branch takes a leading current and the second takes a lagging current?

111. An impedance coil and a condenser, which has a reactance of 40 ohms at 60 cycles, are connected in parallel across 220-volt 60-cycle mains. If the total power is 500 watts and the total current 3 amperes, what are the possible complex expressions for the total current and the currents in the branches, all referred to the impressed voltage as along the axis of reals?

112. A condenser, which has a reactance of 120 ohms at 25 cycles, and an impedance coil are connected in parallel across 220-volt 60-cycle mains. If the complex expression of the impressed e.m.f. is taken as $220 + j\,0$, the complex expression for the total current is $5 + j\,2$. What are the complex expressions for the currents in the two branches?

113. Two impedance circuits whose complex expressions are given by $4 + j\,26$ and $5 - j\,38$ are connected in parallel. If the total current supplied is 2.1 amperes what power does each of the parallel branches absorb?

114. Two impedance circuits are connected in parallel. The complex expression for the first is $17 - j\,20$ and the second has a fixed resistance of 11 ohms and an inductive reactance which may be varied from 3.8 ohms to 38 ohms. For what value of the reactance will the power-factor of the circuit have its least value? Is it possible to adjust the variable reactance so that the circuit will be in resonance?

115. A circuit of impedance, $Z_1 = 4 + j\,20$, is connected in parallel with another of impedance, $Z_2 = r_2 - jx_2$, both r_2 and x_2 being adjustable. Find the maximum value of r_2 which will permit the system to be adjusted for resonance.

116. An e.m.f., whose complex expression is $-200 + j\,100$ is impressed on an impedance coil connected in parallel with a condenser of 40 ohms reactance. The complex expression for the total current is $-2.0 - j\,1.5$. What is the power-factor of this circuit? What is the complex expression for the current in the impedance coil?

117. An impedance coil and a condenser are connected in parallel. The complex expression for the impressed e.m.f. is $100 + j\,200$, and for the total current, $-2 + j\,5$. If the current in the condenser branch is 20 amperes, what are the complex expressions for this current and that in the impedance coil?

118. An impedance coil and a condenser, which has a reactance of 53 ohms at 60 cycles, are connected in parallel across 60-cycle mains. The complex expression for the impressed e.m.f. is $100 - j\,200$ and for the total current $2.5 + j\,0$. What is the complex expression for the current in the impedance coil? What is the phase relation of the currents in the condenser branch and the impedance coil?

119. When an impedance coil and a condenser are connected in parallel across 220-volt, 60-cycle mains, they absorb 350 watts and the current in the coil is 10 amperes. If the complex expression for the total current is $2 + j\,5$ what are the possible complex expressions for the currents in the condenser branch and in the impedance coil?

120. An impedance coil which has a power-factor of 0.15 is connected in parallel with a condenser across constant potential a. c. mains. If the total current is 10 amperes and the current in the coil 15 amperes, what are the complex expressions for the total current and its components in the branches, all referred to the impressed voltage as along the axis of reals?

121. In parallel with an impedance coil, $Z_1 = 2 + j\,10$, is connected a circuit having a resistance of 4 ohms and a variable condensive reactance. Over what range from a leading to a lagging current can the power-factor of this system be varied, and what are the values of the condensive reactance for these limiting conditions?

SERIES-PARALLEL CIRCUITS

122. A non-inductive resistance of 20 ohms and a condenser of 30 ohms reactance are connected in parallel and an impedance coil which has a negligible resistance and a reactance of 15 ohms is connected in series with them. When a certain e.m.f. is impressed on this circuit the current in the non-inductive resistance is 12 amperes. What are the currents in the other branches and what is the impressed e.m.f.? Sketch a vector diagram showing the phase relations of the currents and voltages.

123. A non-inductive resistance and a condenser are connected in parallel and an impedance coil which has a negligible resistance is connected in series with them. This circuit is so adjusted that the voltages across the parallel branches and the impedance coil are each equal to the e.m.f. impressed on the entire circuit. When the impressed e.m.f. is 220 volts the power absorbed by the circuit is 1500 watts. What are the constants of each branch of the circuit?

124. An impedance coil, which has a negligible resistance and a reactance of 30 ohms, is connected in parallel with a condenser of 60 ohms reactance, and another impedance coil whose complex expression is $2 + j\,20$ is connected in series with them. If the complex expression for the impressed e.m.f. is taken as $200 + j\,100$, what are the complex expressions for the currents in each branch? Sketch a vector diagram showing the phase relations of the currents and the voltages?

125. In problem 124, what will be the effect on the currents and the voltages if the condensive reactance alone is reduced to 20 ohms. Sketch a vector diagram for this case.

126. Two impedance circuits whose complex expressions are given by $5 - j\,15$ and $10 + j\,5$, are connected in parallel, and a coil of impedance, $10 + j\,10$, is connected in series with the pair. An e.m.f., whose complex expression is $200 - j\,100$, is impressed on the terminals of this circuit. What are the complex expressions for the voltages across each part of the circuit and for the currents in each of the branches?

127. In problem 126, what is the power absorbed in each of the branches? What is the power-factor of the entire circuit?

128. Two impedance units, Z_1 and Z_2, whose complex expressions are $25 + j\,15$ and $15 - j\,25$, respectively, are connected in parallel, and a third impedance unit, Z_3, whose complex expression is $5 + j\,20$, is connected in series with them. What are the equivalent resistance and reactance of this circuit?

129. Two impedance units, Z_1 and Z_2, whose complex expressions are $25 + j\,5$ and $5 - j\,25$, respectively, are connected in parallel, and an impedance coil Z_3, whose complex expression is $2 + j\,20$, is connected in series with them. If an e.m.f. of 220 volts is impressed on this entire circuit, what are the voltages across each branch and the currents in them?

130. Two impedance units, Z_1 and Z_2, are connected in parallel and an impedance coil, Z_3, is connected in series with the combination. If $Z_1 = 10 + j\,5$, $Z_2 = 5 - j\,20$, $Z_3 = 5 + j\,5$ and the complex expression for the current in Z_1 is $5 - j\,10$, what are the complex expressions for the currents in the second and third impedance units?

131. Two impedance units, Z_1 and Z_2, are connected in parallel and a third impedance unit, Z_3, is connected in series with the combination. $Z_1 = 5 + j\,10$, $Z_2 = 10 - j\,5$, and the complex expression for the current in the first is $10 + j\,10$. If the resistance of the third impedance unit is 2 ohms and the voltage impressed on the system is 220, what is the reactance of the third impedance unit?

132. Two impedance units, Z_1 and Z_2, are connected in parallel and an impedance coil, Z_3, is connected in series with them. $Z_1 = 15 + j\,5$, $Z_2 = 5 - j\,15$ and $Z_3 = 1 + j\,10$. The complex expression for the e.m.f. impressed on this circuit is $-200 + j\,90$. What is the complex expression for the current in each unit? What is the total power supplied to the circuit?

133. Two impedance units, Z_1 and Z_2, are connected in series across a 60-cycle e.m.f. A condenser of what capacity must be placed in parallel with Z_1 in order that the power-factor of the entire circuit may be unity? $Z_1 = 15 + j\,20$, $Z_2 = 20 - j\,15$.

134. A coil, whose impedance is given by the complex expression $2 + j\,8$, is connected in series with a non-inductive resistance of 6 ohms. What non-inductive resistance must be placed in parallel with the impedance coil so that the power-factor of the entire circuit may be 0.8?

135. A coil, whose impedance is given by the complex expression $3 + j\,30$, is connected in parallel with a non-inductive resistance of 20 ohms, and another non-inductive resistance of 5 ohms is connected in series with them. If an e.m.f. whose complex expression is $100 - j\,200$ is impressed on the entire circuit, what are the complex expressions for the voltage across each part of the circuit and for the current in each branch?

136. Two impedance units, Z_1 and Z_2, whose complex expressions are $5 + j\,20$ and $10 - j\,30$, respectively, are connected in series. What condensive or inductive reactance should be placed in parallel with the second impedance so that the power-factor of the entire circuit will be 0.75?

137. Two impedance units, Z_1 and Z_2, are connected in series and another impedance unit, Z_3, is then connected in parallel with Z_2. The complex expression for the e.m.f. impressed on the entire circuit is $200 + j\,95$. The complex expression for Z_1 is $5 + j\,10$ and for the current through it, $15 + j\,5$. If the current in Z_2 is $5 - j\,5$, what are the complex expressions for Z_2 and Z_3? What is the power absorbed in each branch of the circuit?

138. A non-inductive resistance of 10 ohms and a condenser of 80 microfarads capacity are connected in parallel between the points M and N. Between N and P there is an impedance coil which has a resistance of 3 ohms and an inductance of 0.06 henry. A 60-cycle e.m.f. impressed on the terminals MP produces a current of 10 amperes in the non-inductive resistance. Find the complex expressions for all currents and voltages referred to the current in the non-inductive resistance as along the axis of reals.

139. Between the points M and N are connected a non-inductive resistance and a condenser in parallel, and between N and P there is an impedance, whose complex expression is $3 + j\,38$. The complex expression for the impressed e.m.f. is $100 + j\,200$ and for the current between N and P, $6 - j\,1$. What are the complex expressions for the currents in the resistance and condenser?

140. Between the points M and N are connected a non-inductive resistance and a condenser in parallel, and between N and P there is an impedance coil, which has a resistance of 3.5 ohms and a reactance of 17.5 ohms. The currents in the resistance and the capacity branches are 10 and 5 amperes respectively and the power supplied to the entire circuit is 950 watts. If the complex expression for the current in the non-inductive resistance is taken as $10 + j\,0$, what are the complex expressions for the currents in the other branches for the impressed e.m.f. and for its components across $M - N$ and $N - P$?

141. Between the points M and N are connected a non-inductive resistance and a condenser in parallel and between N and P there is an impedance coil. The currents in the non-inductive resistance and the condenser are 5 and 12 amperes respectively. The e.m.f. impressed on the entire circuit is 220 volts and its components across $M - N$ and $N - P$ are each 150 volts. If the complex expression for the current in the non-inductive resistance is taken along the axis of reals, what are the complex expressions for the currents in the other branches, and for the impressed e.m.f., and for its components across $M - N$ and $N - P$?

142. Two impedance units, the complex expression for one of which is $2 + j\,15$, are connected in parallel between the points M and N. Between N and P there is a third unit, which has a resistance of 3 ohms and an inductive reactance of 10 ohms. If the complex expression for the voltage impressed on the terminals $M - P$ is $100 + j\,200$, and for the current in the third unit, $8 + j\,1$, what are the resistance and the reactance of the unknown impedance?

143. Two impedance coils Z_1 and Z_2, whose complex expressions are $3 + j\,15$ and $10 + j\,10$, respectively, are connected in series and a third impedance unit, Z_3, of 10 ohms is connected in parallel with Z_1. What is the complex expression for Z_3 when the power-factor of the entire circuit is unity? ~~What is the voltage across each part of the circuit?~~

144. Two impedance coils, Z_1 and Z_2, whose complex expressions are $5 + j\,10$ and $5 + j\,20$, respectively, are connected in series and a third impedance unit, Z_3, is connected in parallel with Z_2. When an e.m.f. of 220 volts is impressed on the entire circuit the total current is 20 amperes lagging and the total power supplied is 3250 watts. What are the complex expressions for the currents in Z_2 and Z_3 referred to the voltage across them as along the axis of reals?

145. An impedance coil, which has a negligible resistance and a reactance of 40 ohms at 60 cycles, is connected in series with a non-inductive resistance of 30 ohms across 220-volt, 60-cycle mains. A non-inductive resistance is connected in parallel with the impedance coil and adjusted so that the current has its former value, viz., 4.4 amperes. What is the non-inductive resistance?

146. In problem 145, if the complex expression for the impressed voltage is taken as $220 + j\,0$, what are the complex expressions for the currents in each of the branches?

147. Two impedance coils, Z_1 and Z_2, are connected in series and a condenser, having a reactance of 6 ohms, is connected in parallel with Z_1. If an e.m.f. of 110 volts is impressed on this entire circuit the potential across the parallel branches is also 110 volts and the current in Z_2 is 15 amperes. The total power supplied is 600 watts and of this 450 watts are absorbed in Z_2. What is the complex expression for Z_1? What are the complex expressions for the currents in the parallel branches referred to the voltage across the parallel branches as along the axis of reals?

148. A non-inductive resistance and an impedance coil are connected in series and a condenser is connected in parallel with the non-inductive resistance. When an e.m.f. of 220 volts is impressed on the entire circuit and the circuit is so adjusted that the currents in the parallel branches are equal and the voltage across the parallel branches is also equal to and in quadrature with the voltage across the impedance coil, the total power absorbed is 1240 watts. What are the constants of this circuit? Sketch a vector diagram showing the phase relation of the currents and voltages.

149. Between the points A and B there is connected an impedance coil Z_1 whose resistance is 12 ohms, and between B and C are two circuits in parallel, $Z_2 = r_2 - jx_2$ (unknown) and $Z_3 = 0 - j\,20$. The current in AB is 15 amperes and the potential across AB is 220 volts. The voltage across AC is also 220 volts and the total power supplied to the system is 3.3 kilowatts. Find the complex expressions for I_2 and I_3 referred to the current I_1 as along the axis of reals.

* GENERAL PROBLEMS.

150. A 2300-volt generator is delivering power to an induction motor load over a line whose total impedance is $3.2 + j\,4.8$. If the output of the generator is 100 kilowatts at 0.8 power-factor, what is the efficiency of transmission?

151. A 2300-volt generator is delivering power to an induction motor over a transmission line whose total impedance is $4.3 + j\,6.7$. The motor is taking 82.4 kilowatts at a power-factor of 0.90. What is the voltage at the motor?

152. An induction motor takes 150 kilowatts from a transmission line whose total impedance is $3.6 + j\,5.2$. If the current taken by the motor is 75 amperes and the potential impressed on it 2200 volts, what is the voltage impressed on the line at the generating station?

153. The potentials at the generating and receiving ends of a transmission line are each 2200 volts and the line current is 140 amperes. If the complex expression for the total impedance of the line is $1.8 + j\,2.4$, what is the efficiency of transmission?

154. An over-excited synchronous motor is operating at the end of a transmission line whose total impedance is $3.2 + j\,4.8$. If the potential at the generating station is 2200 volts and the

* Many of these problems would in actual practice apply to 3-phase circuits but in every case it is assumed that the data are reduced to the equivalent single-phase circuit.

motor takes 132 kilowatts at a power-factor of 0.86, what is the voltage at the motor? (An over-excited synchronous motor takes a leading current.)

155. An induction motor takes 47.2 amperes from 220-volt, 60-cycle mains when running under full load, and delivers 10 horsepower at an efficiency of 85 per cent. What capacity condenser must be connected in series with this motor across 220-volt 60-cycle mains in order that the power-factor of the system may be unity when there is full load on the motor (assume the same efficiency and power-factor for the motor)? This method of improving the power-factor is never used in practice. Why? (See problem 157.)

156. In problem 155, substitute an over-excited synchronous motor for the condenser, which is connected in series with the induction motor. The synchronous motor is operated under no load and its losses, exclusive of the d. c. field winding loss, are approximately 950 watts. What will be the voltage across each of the motors if the impressed e.m.f. is 220 volts and the system is operating under unit power-factor? This method is even worse than that in problem 155, and would never be used in practice. (See problem 158.)

157. An induction motor takes 24.6 amperes from 220-volt, 60-cycle mains when running under full load, and delivers 4.7 horsepower with an efficiency of 83 per cent. What capacity condenser must be connected in parallel with this motor in order that the power-factor of the system may be unity when the motor is delivering full load?

158. In problem 157, substitute an over-excited synchronous motor for the condenser, which is connected in parallel with the induction motor. The synchronous motor is operated under no load and its losses, exclusive of the d. c. field winding loss, are approximately 1040 watts. What will be the total current and also that taken by the synchronous motor if the system is adjusted for unit power-factor?

159. An induction motor and an over-excited synchronous motor are connected in parallel across 230-volt, 60-cycle mains. The allowable capacity of the synchronous motor at 230 volts is 20 kilovolt-amperes. The induction motor is taking 16.5

kilowatts at a power-factor of 0.86 and the synchronous motor is so adjusted that the power-factor of the entire system is unity. What is the greatest power-factor at which the synchronous motor should be operated? What is the greatest power that the synchronous motor should take from the mains?

160. At the end of a transmission line whose total impedance is $0.34 + j\,0.62$, there are connected a lamp load, which has an equivalent resistance of 4.2 ohms, and a small induction motor. When starting, this motor takes current proportional to the impressed voltage but at a fixed power-factor, and is thus equivalent to an impedance unit whose complex expression is $1.5 + j\,4.8$. If the station voltage is 240 what is the voltage at the load when the motor is starting?

161. At the end of a transmission line, whose total impedance is $0.1 + j\,0.2$, are connected a lamp load taking 100 amperes and an induction motor. When starting, this motor takes 10 kilowatts at 0.4 power-factor. If the potential at the load is to be kept at 230 volts, what will be the necessary impressed voltage at the other end of the line when the motor is starting?

162. In problem 161, when the motor is operating at full load it is equivalent to an impedance unit, $Z = 3.9 + j\,7.1$. If the station voltage is 240 what is the voltage at the load for this condition?

163. At the end of a transmission line, whose total impedance is $0.21 + j\,0.43$, there are a lamp load, which has an equivalent resistance ~~taking 80 amperes~~, and an induction motor, which takes 15 kilowatts at 0.86 power-factor. If the station voltage is 240, what is the voltage at the load?

CHAPTER XIV

NON-SINUSOIDAL E.M.FS. AND CURRENTS

ANY periodic function can be represented by a sum of sine and cosine terms. In the general equation for the e.m.f. the even harmonics will be absent since the two halves of the wave are alike.

Thus: —

$$e = E_1 \sin \omega t + E_3 \sin (3\,\omega t + \alpha_3) + E_5 \sin (5\,\omega t + \alpha_5) + \ldots \tag{1}$$

The angles α_3, α_5, etc., show the phase relations of the harmonics and the fundamental.

An excellent method of plotting the e.m.f. wave is to assign convenient values to ωt, such as $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, etc., and thus fix the ordinates of the fundamental and the harmonics. The ordinates of the resultant e.m.f. will then be given by the algebraic sum of the corresponding ordinates of the fundamental and harmonics. Thus:

$$e_0 = E_1 \sin 0 + E_3 \sin \alpha_3 + E_5 \sin \alpha_5 + \ldots$$

$$e_{\frac{\pi}{6}} = E_1 \sin \frac{\pi}{6} + E_3 \sin \left(\frac{\pi}{2} + \alpha_3\right) + E_5 \sin \left(\frac{5\,\pi}{6} + \alpha_5\right) + \ldots$$

When such an e.m.f. is impressed on a circuit having resistance and reactance the equation for the current is:

$$i = \frac{E_1}{\sqrt{R^2 + X_1^2}} \sin (\omega t - \theta_1) + \frac{E_3}{\sqrt{R^2 + X_3^2}} \sin (3\,\omega t + \alpha_3 - \theta_3)$$

$$+ \frac{E_5}{\sqrt{R^2 + X_5^2}} \sin (5\,\omega t + \alpha_5 - \theta_5) + \ldots$$

$$= I_1 \sin(\omega t - \theta_1) + I_3 \sin(3\omega t + \alpha_3 - \theta_3) + I_5 \sin(5\omega t + \alpha_5 - \theta_5) + \ldots \tag{2}$$

where X_1 is the reactance of the circuit at the fundamental frequency, X_3 is the reactance of the circuit at 3 times the fundamental frequency, X_5 is the reactance of the circuit at 5 times the fundamental frequency, etc.;

$$\theta_1 = \tan^{-1}\frac{X_1}{R},\ \theta_3 = \tan^{-1}\frac{X_3}{R},\ \theta_5 = {}^{-1}\frac{X_5}{R},\quad \text{etc.}$$

The effective value of any periodic e.m.f. is the root-mean-square value and it is given by the expression:

$$\sqrt{\frac{1}{T}\int_o^T e^2\,dt.}$$

Likewise the effective value of any periodic current is given by the expression:

$$\sqrt{\frac{1}{T}\int_o i^2\,dt.}$$

When the equation of the e.m.f. and the current involve higher harmonics (see equations (1) and (2)) the above expression for the effective value of the e.m.f. reduces to:

$$E_0 = \sqrt{\frac{E_1^{\ 2} + E_3^{\ 2} + E_5^{\ 2} + \ldots}{2}} \tag{3}$$

The effective value of the current is:

$$I_0 = \sqrt{\frac{I_1^{\ 2} + I_3^{\ 2} + I_5^{\ 2} + \ldots}{2}} \tag{4}$$

The general expression for the power supplied to any circuit is:

$$\frac{1}{T}\int_o^T ei\,dt,$$

where e and i are the functions which represent the e.m.f. impressed on the circuit and the current in the circuit respectively. When the equations of the e.m.f. and the current involve higher harmonics (see equations (1) and (2)) this expression for the power reduces to:

$$E_0 I_0 \cos\theta_0 = \frac{E_1 I_1}{2}\cos\theta_1 + \frac{E_3 I_3}{2}\cos\theta_3 + \frac{E_5 I_5}{2}\cos\theta_5 + \ldots \tag{5}$$

The values of E_0 and I_0 are given in equations (3) and (4). The equation of the equivalent sine waves of e.m.f. and current are:

$$e = \sqrt{2}\, E_0 \sin \omega t, \quad \text{and} \quad i = \sqrt{2}\, I_0 \sin (\omega t - \theta_0)$$

where E_0, I_0 and θ_0 have the values given in equation (5).

In problems which involve non-sinusoidal e.m.fs. and currents the e.m.fs. and currents *cannot* be correctly represented by vectors as described in Chapter XIII and the symbolic method *cannot* be used in their solution. In such problems Kirchhoff's laws can only be applied to instantaneous values.

PROBLEMS

1. Plot the e.m.f. wave given by the equation $e = 200 \sin 157t + 50 \sin 471\, t$ for one cycle. What is the effective value of this e.m.f.?

2. If the e.m.f. in problem 1 is impressed on a non-inductive resistance of 10 ohms, what is the equation of the current? What is the effective value of the current?

3. What is the maximum value of the e.m.f. in problem 1?

4. In problem 1, at what rate is the e.m.f. changing when its instantaneous value is passing through zero?

5. In problem 2, what is the average rate at which heat is dissipated in the non-inductive resistance?

6. In problem 2, if the equation of the equivalent sine wave is taken as $e = E \sin 157t$, what is the value of E? What is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

7. Plot the e.m.f. wave given by the equation $e = 200 \sin 157t + 50 \sin (471t + 90 \text{ degrees})$ for one cycle. What is the effective value of this e.m.f.?

8. If the e.m.f. in problem 7 is impressed on a non-inductive resistance of 10 ohms, what is the equation of the current? What is the effective value of the current?

9. In problem 7, at what rate is the e.m.f. changing when ~~its instantaneous value is zero?~~ the time is 3

10. In problem 8, what is the average rate at which heat is dissipated in the non-inductive resistance?

11. In problem 8, if the equation of the equivalent sine wave is taken as $e = E \sin 157t$, what is the value of E? What is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

12. Plot the e.m.f. wave given by the equation $e = 200 \sin 157t - 50 \sin 471t$ for one cycle. What is the effective value of this e.m.f.?

13. If the e.m.f. in problem 12 is impressed on a non-inductive resistance of 10 ohms, what is the equation of the current? What is the effective value of the current?

14. What is the maximum value of the e.m.f. in problem 12?

15. In problem 12, at what rate is the e.m.f. changing when its instantaneous value is passing through zero?

16. In problem 13, what is the average rate at which heat is dissipated in the non-inductive resistance?

17. In problem 13, if the equation of the equivalent sine wave is taken as $e = E \sin 157t$, what is the value of E? What is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

18. An e.m.f., whose equation is $e = 200 \sin 157t + 50 \sin 471t$, is impressed on an impedance coil which has a resistance of 5 ohms and an inductance of 0.05 henry. What is the equation of the current? What is the effective value of the current?

19. In problem 18, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

20. In problem 18, plot the e.m.f. and the current waves for one cycle.

21. In problem 18, what is the average rate at which heat is dissipated in the impedance coil?

22. In problem 18, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

23. An e.m.f. whose equation is $e = 200 \sin 157t + 50 \sin (471t + 90 \text{ degrees})$, is impressed on an impedance coil, which has a resistance of 5 ohms and an inductance of 0.05 henry. What is the equation of the current? What is the effective value of the current?

24. In problem 23, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

25. In problem 23, plot the e.m.f. and the current waves for one cycle.

26. In problem 23, what is the average rate at which heat is dissipated in the impedance coil?

27. In problem 23, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

28. An e.m.f., whose equation is $e = 200 \sin 157t - 50 \sin 471t$, is impressed on an impedance coil which has a resistance of 5 ohms and an inductance of 0.05 henry. What is the equation of the current? What is the effective value of the current?

29. In problem 28, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

30. In problem 28, plot the e.m.f. and the current waves for one cycle.

31. In problem 28, what is the average rate at which heat is dissipated in the impedance coil?

32. In problem 28, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

33. An e.m.f., whose equation is $e = 200 \sin 157t + 50 \sin 471t$, is impressed on a circuit which consists of a non-inductive resistance of 20 ohms in series with a condenser of 200 microfarads capacity. What is the equation of the current? What is the effective value of the current?

34. In problem 33, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

35. In problem 33, plot the e.m.f. and the current waves for one cycle.

36. In problem 33, what is the average rate at which heat is dissipated in the resistance?

37. In problem 33, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

38. An e.m.f. whose equation is $e = 200 \sin 157t + 50 \sin (471t + 90 \text{ degrees})$, is impressed on a circuit which consists of a non-inductive resistance of 20 ohms in series with a condenser of 200 microfarads capacity. What is the equation of the current? What is the effective value of the current?

39. In problem 38, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

40. In problem 38, plot the e.m.f. and the current waves for one cycle.

41. In problem 38, what is the average rate at which heat is dissipated in the resistance?

42. In problem 38, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

43. An e.m.f., whose equation is $e = 200 \sin 157t - 50 \sin 471t$, is impressed on a circuit which consists of a non-inductive resistance of 20 ohms in series with a condenser of 200 microfarads capacity. What is the equation of the current? What is the effective value of the current?

44. In problem 43, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

45. In problem 43, plot the e.m.f. and the current waves for one cycle.

46. In problem 43, what is the average rate at which heat is dissipated in the resistance?

47. In problem 43, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 157t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

48. An e.m.f. whose equation is $e = 200 \sin 314t + 50 \sin 942t$, is impressed on a circuit consisting of a condenser of 50 microfarads capacity in series with an impedance coil which has a resistance of 20 ohms and an inductance of 0.05 henry. What is the equation of the current? What is the effective value of the current?

49. In problem 48, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

50. In problem 48, plot curves representing the current, the voltage across the impedance coil and the voltage across the condenser, and show their phase relations.

51. In problem 48, plot the impressed e.m.f. and the current waves for one cycle, showing the phase relations of the fundamentals and the harmonics.

52. In problem 48, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 314t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

53. An e.m.f. has an effective value of 220 volts and a fundamental frequency of 60 cycles. If this e.m.f. consists of a fundamental and a third harmonic which is in phase with the fundamental and has an amplitude one-third as great, what is the equation of the e.m.f.?

54. An e.m.f., whose equation is $e = 215 \sin 314t + 20 \sin (7 \times 314t)$, is impressed on a circuit consisting of a condenser of variable capacity in series with an impedance coil, which has a resistance of 10 ohms and an inductance of 0.1 henry. For what value of the capacity will the current have its greatest value? What is this greatest value of the current?

55. In problem 54, what are the voltages across the impedance coil and the condenser when the current has its greatest value?

56. In problem 54, if a simple harmonic e.m.f. of the same effective value is impressed on the circuit, what is the greatest value that the current can have? Compare this with the result in problem 54. What are the voltages across the impedance coil and the capacity when the current has its greatest value? Compare them with the results in problem 55.

57. A series circuit has a resistance of 10 ohms, an inductance of 0.1 henry and a variable capacity. On the assunption that a simple harmonic 220-volt, 60-cycle e.m.f. is impressed on this circuit, for what value of the capacity will the current have its greatest value? What is the current for this condition? With this value of the capacity what will be the current, if there is impressed on the circuit an e.m.f. of the same effective value and frequency but one having a ~~seventh~~ third harmonic with an amplitude one-tenth that of the fundamental?

58 The constants of a series circuit are R ohms, L henrys and C farads. An e.m.f. whose equation is $e = E_1 \sin \omega t + E_3 \sin (3\omega t + \alpha_3) + E_5 \sin (5\omega t + \alpha_5)$ is impressed on this circuit. If the resistance and inductance are fixed, for what value of the capacity will the current have its greatest value? Compare this with the value of C which gives the condition of resonance when a simple harmonic e.m.f. is impressed on the circuit.

59. An e.m.f. whose equation is $e = 200 \sin 314t + 50 \sin 942t$ is impressed on a circuit consisting of a condenser of 75 microfarads capacity in parallel with an impedance coil, which has a resistance of 3 ohms and an inductance of 0.025 henry.

What is the equation of the current? What is the effective value of the current?

60. In problem 59, what is the phase relation of the fundamental components of the e.m.f. and the current waves? What is the phase relation of the third harmonic components of the e.m.f. and the current waves?

61. In problem 59, plot the curves representing the impressed e.m.f. and the currents in the two branches, and show their phase relations.

62. In problem 59, plot the impressed e.m.f. and current waves for one cycle, showing the phase relations of the fundamentals and the harmonics.

63. In problem 59, if the equation of the equivalent sine wave of the e.m.f. is taken as $e = E \sin 314\, t$, what is the equation of the equivalent sine wave of the current? What is the phase relation of the equivalent sine waves?

64. An e.m.f., whose equation is $e = 215 \sin 314\, t + 20 \sin (7 \times 314\, t)$, is impressed on a condenser of variable capacity connected in parallel with an impedance coil which has a resistance of 10 ohms and an inductance of 0.1 henry. For what value of the capacity will the total current have its least value? What is this least value of the current?

65. In problem 64, what are the currents in the impedance coil and the condenser when the current has its least value?

66. In problem 64, if a simple harmonic e.m.f. of the same effective value is impressed on the circuit, what is the least value that the current can have? Compare this with the result in problem 64. What are the currents in the impedance coil and the condenser when the current has its least value? Compare them with the results in problem 65.

67. A circuit consists of a condenser of variable capacity in parallel with an impedance coil which has a resistance of 3 ohms and an inductance of 0.1 henry. On the assumption that a simple harmonic 220-volt, 60-cycle e.m.f. is impressed on this circuit, for what value of the capacity will the current have its least value? What is the current for this condition? With this value of the capacity, what will be the current if there is impressed on the circuit an e.m.f. of the same effective value and frequency but one having a seventh harmonic with an amplitude one-tenth that of the fundamental?

68. An impedance coil which has a resistance of R ohms and an inductance of L henrys is connected in parallel with a condenser of C farads capacity. An e.m.f. whose equation is $e = E_1 \sin \omega t + E_3 \sin (3\,\omega t + \alpha_3) + E_5 \sin (5\,\omega t + \alpha_5)$ is impressed on this circuit. If the resistance and inductance are fixed, for what value of the capacity will the current have its least value? Compare this with the value of C which gives the condition of resonance when a simple harmonic e.m.f. is impressed on the circuit.

69. Two single-phase generators are rigidly coupled together and are driven at a speed of 1200 rev. per min. One generator has six poles and the other has eighteen poles. The armatures, each having a constant terminal potential of 115 volts, are connected in series and supply current to a circuit connected across their outer terminals. The terminal potentials of the alternators are in phase. If this circuit is a non-inductive resistance of 10 ohms, sketch the resultant waves of current and impressed e.m.f. What are the effective values of the current and the impressed voltage?

70. In problem 69, what is the total load supplied, and what part of it is supplied by each alternator?

71. In problem 69, if the circuit to which the alternators supply current is an impedance coil that has a resistance of 5 ohms and an inductance of 0.025 henry, sketch the waves of the current and the impressed e.m.f. What are the effective values of the current and the impressed e.m.f.?

72. In problem 71, what is the total load supplied and what part of it is supplied by each alternator?

73. In problem 69, if the circuit to which the alternators supply current consists of a non-inductive resistance of 10 ohms in series with a condenser of 100 microfarads capacity, sketch the waves of the current and the impressed e.m.f. What are the effective values of the current and the impressed e.m.f.?

74. In problem 73, what is the total load, and what part of it is supplied by each alternator?

75. An alternating current and a direct current exist simultaneously in the same conductor. If the effective value of the alternating current is 5 amperes and the direct current is 10 amperes, what will an alternating current ammeter indicate when connected in this circuit?

76. In problem 75, what is the average value of the current?

77. A direct-current generator and a 60-cycle alternating-current generator are connected in series and supply current to a circuit connected across their outer terminals. The terminal potentials of the generators are constant and each is 115 volts. If the circuit to which they supply current is a non-inductive resistance of 10 ohms what is the effective value of the current? What is the average value of the current?

78. In problem 77, what is the heating loss in the non-inductive resistance? Sketch the current and the e.m.f. waves. What are the equations of the equivalent sine waves of the current and of the e.m.f.?

79. In problem 77, if the circuit to which the generators supply current is an impedance coil which has a resistance of 5 ohms and an inductance of 0.05 henry, what is the effective value of the current? What is the average value of the current?

80. In problem 79, what is the heating loss in the coil? Sketch the current and the e.m.f. waves. What are the equations of the equivalent sine waves of the current and of the e.m.f.?

81. In problem 77, if the circuit to which the generators supply current consists of a non-inductive resistancee of 20 ohms in series with a condenser of 100 microfarads capacity, what is the effective value of the current? What is the average value of the current?

82. In problem 81, what is the heating loss in the non-inductive resistance? Sketch the current and the e.m.f. waves. What are the equations of the equivalent sine waves of the current and of the e.m.f.?

83. A non-inductive resistance of 10 ohms is connected across 220-volt direct-current mains, and between one end of this resistance and its midpoint there is impressed a 110-volt, 60-cycle e.m.f. What is the effective value of the current in each part of this resistance? What is the maximum value of the current in each part of this resistance? What is the heating loss in the entire resistance?

84. A non-inductive resistance of 10 ohms is connected across 220-volt, 60-cycle mains and between one end of this resistance and its mid-point there is impresssed a 110-volt direct e.m.f. What is the effective value of the current in each part of this resistance? What is the maximum value of the current in each part of this resistance? What is the heating loss in the entire resistance?

85. Two condensers each of 50 mircofarads capacity are connected in series across 220-volt direct-current mains, and across one condenser there is impressed a 110-volt, 60-cycle e.m.f. What is the effective value of the current in each condenser? What is the maximum value of the current in each condenser?

86. In problem 85, what is the maximum charge on each condenser? When the charge on one condenser is a maximum what is the charge on the other?

87. Two condensers each of 50 microfarads capacity are connected in series across 220-volt, 60-cycle mains and across one condenser there is impressed a 110-volt direct e.m.f. What is the effective value of the current in each condenser? What is the maximum value of the current in each condenser?

88. In problem 87, what is the maximum charge on each condenser? When the charge on one condenser is a maximum what is the charge on the other?

89. An e.m.f. increases uniformly from zero to a maximum value of $+E$ in time $\frac{T}{4}$, then decreases uniformly to $-E$ in time $\frac{T}{2}$, and then increases uniformly to $+E$ in time $\frac{T}{2}$, etc. What is the effective value of this e.m.f.? What is the average value? What is the form-factor?

90. An e.m.f. increases uniformly from zero to a maximum value of $+E$ in time $\frac{T}{8}$, remains constant for time $\frac{T}{4}$, then decreases uniformly to $-E$ in time $\frac{T}{4}$, remains constant for time $\frac{T}{4}$, and then increases uniformly to $+E$ in time $\frac{T}{4}$, etc. What is the effective value of this e.m.f.? What is the form-factor?

91. An electrical circuit is composed of such materials that the resistance varies inversely as the square root of the current as represented by the equation $r = \frac{R}{\sqrt{i}}$. R is constant and i is the numerical value of the current. If a simple harmonic e.m.f., $e = E \sin \omega t$, is impressed on this circuit what is the equation of the current? Sketch the current and e.m.f. waves for one cycle. What is the effective value of the current?

92. In problem 91, what is the power supplied to this circuit? What is the equation for the equivalent sine wave of the current? What is the power-factor of the circuit?

93. If the e.m.f. described in problem 89 is impressed on the circuit described in problem 91, what is the equation of the current? Sketch the current and the e.m.f. waves for one cycle. What is the effective value of the current?

94. In problem 93, what is the power supplied to the circuit? What are the equations of the equivalent sine waves of the current and the e.m.f.? What is the power-factor of the circuit?

95. If the e.m.f. described in problem 90 is impressed on the circuit described in problem 91, what is the equation of the current? Sketch the current and the e.m.f. waves for one cycle. What is the effective value of the current?

96. In problem 95, what is the power supplied to the circuit? What are the equations of the equivalent sine waves of the current and the e.m.f.? What is the power-factor of the circuit?